Macroeconomic Expectations and Limited Awareness

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Abstract

This paper investigates how macroeconomic dynamics and policy transmission are altered when agents are unaware of all structural shocks hitting the economy. We introduce "Limited Awareness" (LA) into a standard New Keynesian model, where otherwise rational agents observe key macroeconomic aggregates but are not aware of the existence of certain disturbances. We show that LA rationalizes existing evidence about overreaction in economic expectations, and naturally give rise to a systematically biased perception of economic relationships—notably, a downward-biased inflation-output tradeoff. We validate this mechanism using actual and forecast data from the Survey of Professional Forecasters and Blue Chip Economic Indicators. Furthermore, LA significantly alters shock propagation: because agents partially conflate supply shocks with demand disturbances, demand shocks have a larger effect on output and a smaller effect on inflation, while supply shocks may lead to a positive comovement between inflation and output, and thus look like demand shock. LA also challenges the desirability of strict inflation-targeting policies, as the misperceived Phillips curve implies a higher sacrifice ratio. Finally, a Structural VAR estimated with macroeconomic and survey data reveals that these misperceived supply shocks are quantitatively highly relevant, accounting for roughly 20 percent of aggregate GDP volatility.

1 Introduction

A cornerstone of standard business cycle models is the assumption that economic agents possess perfect knowledge of their environment. In these frameworks, agents not only understand the true economic structure—the exact mapping from structural shocks to macroeconomic variables—but are also fully aware of which specific shocks are driving economic fluctuations at any

given time. In practice, however, the macroeconomy is continuously subject to a multitude of complex disturbances whose exact number and nature are difficult to discern.[1] This naturally raises a fundamental question: how are macroeconomic dynamics and policy transmission altered when agents are not aware of all the shocks hitting the economy?

This paper addresses this question by introducing the concept of Limited Awareness (LA) into a standard New Keynesian model. Specifically, our primary assumption is that while agents perfectly observe the realizations of key macroeconomic aggregates, such as inflation and output, they remain unaware of the existence of certain underlying structural disturbances. Within this misspecified environment, agents are otherwise fully rational and sophisticated. Consequently, when the economy is subject to an unmodeled shock, agents process this macroeconomic data through a lower-dimensional state space, systematically misattributing the resulting dynamics to the structural shocks included in their simplified view of the world. And, since agents form expectations based on a misperceived view of the world, this may have non trivial implications for economic outcomes and macroeconomic policies.

The paper begins by showing that, within a simple (reduced-form) model, limited awareness naturally leads to specific form of forecast error predictability and biased structural parameters. In particular, we show that limited awareness not only rationalizes existing evidence on *overreaction* of economic expectation (see e.g. Bordalo et. al., 2020), it also generates a systematic *omitted variable bias* in the perceived relationships among economic variables, such as the perceived inflation–output trade-off (i.e., the Phillips multiplier).

To provide empirical support for our analysis, we combine actual data with individual-level forecast data from the Survey of Professional Forecasters (SPF) and the Blue Chip Economic Indicators, and document a systematic bias in forecasters' perceived trade-off between inflation and real economic activity. Specifically, professional forecasters perceive a substantially flatter aggregate supply relationship—or Phillips curve—than the one implied by the objective data-generating process.

We then incorporate limited awareness into a micro-founded New Keynesian model. Within that framework, agents are unaware of aggregate supply shocks and thus interpret all macroeconomic fluctuations strictly through the lens of aggregate demand disturbances. As a result,

[1]The evolution of business cycle theory can be viewed, in part, as the progressive identification and incorporation of novel structural shocks to explain aggregate fluctuations. Early theories explored the role of sunspots and weather-related natural disturbances (e.g., Jevons, 1884). The Real Business Cycle literature subsequently emphasized the role of exogenous aggregate productivity shocks (Kydland and Prescott, 1982). More recently, the macroeconomics literature has vastly expanded the state space of the economy by introducing specialized disturbances such as financial and credit shocks (Kiyotaki and Moore, 1997), news shocks (Beaudry and Portier, 2006), noise shocks (Lorenzoni, 2009), and uncertainty shocks (Bloom, 2009; Bianchi et al., 2018), underscoring the inherent high dimensionality of the macroeconomic environment.

agents erroneously conflate the negative inflation–output co-movement induced by omitted supply shocks with the positive co-movement characteristic of demand shocks. We demonstrate analytically that this process of dynamic misattribution generates a systematically downward-biased estimate of the perceived Phillips multiplier.

This behavioral departure profoundly alters the propagation mechanism of standard macroeconomic shocks. Because agents misinterpret the underlying source of economic fluctuations, their subsequent pricing and consumption decisions diverge significantly from the standard rational expectations equilibrium. Specifically, our model demonstrates that limited awareness *amplifies output volatility in response to demand shocks while simultaneously attenuating inflation volatility*. The intuition is simple: since LA agents form expectations based on a flatter Phillips curve, a demand shock generates changes in expectations that are larger for output and smaller for inflation compared to RE. This translates into a similar effect on aggregate variables.[2]

Compared to RE, in our model, *supply shocks have attenuated effects on both inflation and output*.[3] The intuition is as follows. LA agents interpret supply shocks as demand shocks. Consider a negative supply shock that tends to reduce output and increase inflation. In our model, LA agents interpret such movements as a negative demand shock, which is deflationary. Since inflation expectations move in the opposite direction, the overall inflationary effect is reduced. In fact, we show that when the effect of expectations is sufficiently strong (e.g., when the perceived persistence of demand shocks is high), supply shocks may appear as demand shocks, giving rise to a positive co-movement between inflation and output.

In terms of policy implications, we argue that *limited awareness reduces the scope for adopting inflation-targeting policies*. First, the bias in the perceived slope of the Phillips curve implies that stabilizing inflation is associated with larger output costs than in a rational expectations environment. A direct consequence of the model's response to demand shocks is indeed a higher 'sacrifice ratio'.[4] Second, as explained above, the mis-perception about the nature of the supply shocks generates a 'stabilization effect', so reducing the gains from controlling inflation. Finally, stricter inflation-targeting policies exacerbate expectation biases due to limited awareness, making it more difficult to anchor inflation expectations. This is so since the omitted variable bias in the slope of the Phillips curve gets larger as supply shocks play a greater role in aggregate variable movements compared to demand shocks. And tighter (monetary) stabilization policies effectively

[2]Of course, in determining the GDP response, aggregate demand is key. The result holds as long as monetary policy is sufficiently tight. When the inflation coefficient in the Taylor rule is large, a muted variation in inflation translates into large variations in the real interest rate.

[3]Again, the attenuated effect on output holds as long as monetary policy is sufficiently tight.

[4]The sacrifice ratio measures the percentage of cumulative output lost for each 1 percentage point reduction in inflation (induced by monetary policy).

reduce the role of demand shocks.

In the final part of the paper, we evaluate the quantitative relevance of this mechanism. We estimate a Structural Vector Autoregression (SVAR) model that jointly uses macroeconomic realizations and survey expectations data. By imposing sign restrictions derived directly from our theoretical framework, *we identify these misperceived supply shocks* and show that they account for a substantial fraction of aggregate fluctuations.

Literature Review The literature on macroeconomic expectations is sizeable and fast-growing (e.g., Caroll, 2003; Pesaran et al., 2006; Coibion et al. 2018; and Candia et al., 2023). Our key contribution on the empirical front is to investigate relationships *across aggregate macroeconomic variables.* In particular, we study how the perceived slope of the Phillips curve compares to the true relationship between inflation and unemployment or output. We also observe that the collected evidence indicates the need of a model where agents do not have the 'correct' view of the world; in particular, beyond what macroeconomic models of rational inattention or incomplete information might deliver.[5]

We propose a model with LA agents as a simple departure from Rational Expectations that can explain the macro data. The literature on mis-specified models in macroeconomics is quite large and it has been growing in recent years. Seminal and more recent contributions include: Bray (1982); Branch and Evans (2006); Adam and Marcet (2011); Woodford (2013); Gabaix (2020); Evans and Honkapohja (2001), Molavi (2019) and Hansen and Sargent (2020). To the best of our knowledge, we are the first to study the macro implications of considering limited aware agents.[6] Limited awareness can be seen as a particular case of the vast class of theories based on model misspecification.

Here are the key characteristics that differentiate our model with LA from other approaches. First, differently from models such as Gabaix, Hansen and Sargent and models of rational inattention, LA do not have a correctly specified state space. In fact, the state space considered by LA agents is smaller than the actual one. This also differentiate our approach to that of Adam and Marcet that consider the possibility of 'richer' state spaces than the minimal one under RE.[7]

[5]Calvalho and Necchio (2014), Drager et al. (2016) and Link et al. (2021) study 'qualitative' properties about expectations also considering across-variable relationships. Coibion and Grodnichenko (2012) and Andrade and Le Bihan (2013) study correlations between expectations over output and inflation but do not provide a systematic reading as we do here. Finally, Bandhari et al. (2024) and Melosi (2017) have implicit across variables check but do not read it as a departure from RE.

[6]Seminal papers on unawareness include Modica and Rustichini (1994, 1999), while Schipper (2015) contains a nice and detailed survey of the literature to that date.

[7]On the other hand, as in Adam and Marcet, our agents are 'internally consistent'.

We are hence within the realm of mis-specification. Similarly to other models with miss-specification, a couple of natural and very related questions emerge in our setup. First, how LA form expectations about the future? Second, how do agents 'learn' the laws of motion governing the economy? The macroeconomic literature proposed a series of options, some of them borrowed from game theory, others more related to econometrics. Recently, Molavi (2019) developed a generalization of the *restricted perception equilibria* (RPE) - named Constrained Rational Expectation Equilibrium (CRRE) - where, among other possibilities, agents may project the information obtained from observables into a set of (partially hidden) states that is smaller than the one generating the data, and invoke the limit result of Berk (1966) to determine the parameters determining the evolution of the states.[8] This version of the CRRE is probably the closest case to ours.

Our approach to mis-specification in macro differs and complements CRRE-RPE. First of all, CRRE is a (very general) 'reduced form' approach in that it takes the set of allowed processes governing the observables as given. Our approach specifies in details what LA agents 'think' and then derives the implied perceived process for observables. In this sense, it might constitute a micro-foundation for a subset of models considered by CRRE. Second, and related to the previous point, our approach is mainly *'eductive'* (e.g., Guesnerie, 1992): LA agents believe they know how the economy works and make sophisticated considerations about that, including the behavior of the other agents, the fact that the outcomes satisfy market clearing, and that all agents hold the same (degenerate) mapping between the known shock and the aggregate variables (e.g., see Dutta and Morris, 1997). The CRRE-RPE instead takes an 'evolutive' or 'learning-from-the-data' approach. Third, at least the way it has been implemented so far, CRRE-RPE uses exclusively data on observables and project them into a smaller set of unobservable principal components. We instead *combine both data on observable and expectation data* and impose much more structure on the underlying factors. In particular, the application considered in this paper is motivated by the comparison between expectations and aggregate data, and it posits that the shock ignored by LA agents is a supply shock while the underlying factor considered by the agents must enter into the structural model as a demand shock (and not, say, as an 'optimal' combination of the two shocks).

The model has a few general properties that, in our opinion, might make it a desirable framework for mis-specification in macro. First, LA is a non-parametric concept, thus, it is quite flexible and its general implications do not rely on the specific parametrization chosen. One of them is the 'marginalization result' invoked by Li (2009) that is responsible for the agents' perception of a flatter Phillips Curve in our application. Second, a similar principle allows to predict how expec-

[8] For a similar exercise for static games see Esponda and Pouzo (2016); for dynamic decision problems see Esponda and Pouzo (2021).

tations will change in the case of an increase in the level of awareness. The rule is known - after Karni and Viero (2013) - as *Reverse Bayesianism*. Third, our model satisfies the conditions required in Bohren and Hauser (2024) for a 'well-defined' model of (possibly non-rational) expectations. In particular, our agents have a well defined subjective value of information, they are able to generate 'time-consistent' forecasts and have a well defined ex-ante value of information.

2 A Simple Model of Limited Awareness

To illustrate the fundamental features of Limited Awareness (LA), we consider a stylized macroeconomic environment where agents observe multiple macroeconomic aggregates, but are unaware of the existence of some underlying structural shocks. This implies that agents rely on a misspecified model.

2.1 The True Data-Generating Process (DGP)

The economy is characterized by a vector of observable variables, $\mathbf{x}_t = [y_t, \pi_t]'$, representing output and inflation. The true dynamics of these variables are driven by a vector of unobservable structural shocks, $\mathbf{u}_t = [u_{d,t}, u_{s,t}]'$, representing demand and supply disturbances, respectively. The true state-space representation of the economy is given by:

$$\mathbf{u}_t = \mathbf{R}\mathbf{u}_{t-1} + \boldsymbol{\epsilon}_t, \quad \boldsymbol{\epsilon}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{I}) \tag{1}$$

$$\mathbf{x}_t = \mathbf{A}\mathbf{u}_t + \boldsymbol{\nu}_t, \quad \boldsymbol{\nu}_t \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_\nu) \tag{2}$$

with

$$\mathbf{A} = \begin{bmatrix} a_{y,d} & a_{y,s} \\ a_{\pi,d} & -a_{\pi,s} \end{bmatrix}, \quad \mathbf{R} = \begin{bmatrix} \rho_d & 0 \\ 0 & \rho_s \end{bmatrix}$$

and where all coefficients are assumed to be positive. The structural innovations $\boldsymbol{\epsilon}_t \equiv [\epsilon_{d,t}, \epsilon_{s,t}]'$ are mutually independent and, without loss of generality, normalized to have unit variance. Consequently, the unconditional variance matrix of the structural states $\mathbf{u}_t$ is denoted by $\boldsymbol{\Sigma}_u = \text{diag}(\sigma_d^2, \sigma_s^2)$, where $\sigma_d^2 = \frac{1}{1-\rho_d^2}$ and $\sigma_s^2 = \frac{1}{1-\rho_s^2}$. Finally, $\boldsymbol{\nu}_t$ represents a vector of exogenous, i.i.d. observation errors.

2.2 The Perceived DGP under Limited Awareness

Our main behavioral assumption is that agents have Limited Awareness in the sense that they perceive the economy as being driven by a lower-dimensional state-space, namely a *single aggre-*

gate persistent demand shock $u_{d,t}$. Denoting with a "tilde" the perceived variables and parameters, the agents' perceived state-space model is represented by:

$$\tilde{u}_{d,t} = \tilde{\rho}_d \tilde{u}_{d,t-1} + \tilde{\epsilon}_{d,t}, \quad \tilde{\epsilon}_{d,t} \sim \mathcal{N}(0,1) \tag{3}$$

$$\mathbf{x}_t = \tilde{\mathbf{A}} \tilde{u}_{d,t} + \tilde{\boldsymbol{\nu}}_t, \quad \tilde{\boldsymbol{\nu}}_t \sim \mathcal{N}(\mathbf{0}, \tilde{\boldsymbol{\Sigma}}_\nu) \tag{4}$$

where $\tilde{\rho}_d$ is the perceived persistence of demand shocks and $\tilde{\mathbf{A}} = [\tilde{a}_y, \tilde{a}_\pi]'$ is the vector of perceived structural parameters.[9]

2.3 Over-reaction and Flatter Phillips Curve

Our goal is to present two fairly general implications of the special form of mis-specification we consider in this paper. First, we show that the model generates a precise form of predictable forecast errors: as long as the underlying shocks have different persistence levels, the model tends to generate a negative covariance between Forecast error and forecast revision, commonly named as *over-reaction* (e.g., Bordalo et al. (2020). Second, the specific mis-specification considered in the paper suggests that the parameter estimates will generally be affected by an *omitted variable bias*. In particular, this will generate a bias in the perceived relationship between inflation and output—i.e., the perceived Phillips multiplier $\tilde{\kappa} \equiv \tilde{a}_\pi / \tilde{a}_y$.

Limited-awareness agents may need to estimate (some of) the parameters of their perceived DGP $\tilde{\Theta} = \{\tilde{a}_y, \tilde{a}_\pi, \tilde{\rho}_d, \tilde{\boldsymbol{\Sigma}}_\nu\}$. We make the following two assumptions. First, we assume that agents have access to an infinite amount of data. This assumption is made for convenience, as it allows us to isolate the biases due to limited awareness (misspecification) from those that would arise from small-sample issues or from short-term learning biases starting from an arbitrary initial prior.

Second, we impose that agents must form beliefs that best fit the actual data among the set of beliefs they consider possible within their misspecified framework. Following Esponda and Pouzo (2016) and Molavi (2025), the notion of best fit is formalized in terms of minimizing the Kullback-Leibler Divergence Rate (KLDR) between the actual data and the perceived DGP.[10] In our linear setting with Gaussian shocks, this is equivalent to finding the parameter configuration that minimizes the log-likelihood

$$\max_{\tilde{\Theta}} \mathcal{L}(\tilde{\Theta}) = -\frac{1}{2} \mathbb{E}\left[\log|\boldsymbol{\Omega}| + \mathbf{e}_t' \boldsymbol{\Omega}^{-1} \mathbf{e}_t\right] \tag{5}$$

[9] Of course, the true DGP is affected by how agents form expectations and hence by the perceived parameters. Below we show that our LA model delivers the structure (1) in equilibrium.

[10] This is consistent with the concept of Berk-Nash equilibria. In Appendix A, we show that a similar bias would be obtained if agents were instead estimating parameters with simple moments matching.

where $\mathbf{e}_t = \mathbf{x}_t - \hat{\mathbf{x}}_{t|t-1}$ is the vector of one-step-ahead prediction errors formed under the perceived model, $\mathbf{\Omega}$ is their associated steady-state covariance matrix derived from the Kalman filter, and the mathematical expectation $\mathbb{E}[\cdot]$ is evaluated under the true DGP.

A special case of interest, which admits a simple analytical solution, is one where there is no measurement error in output. In that case, the perceived dynamics of inflation and output simplify to:

$$y_t = \tilde{a}_{y,d}\tilde{u}_{d,t} \tag{6}$$

$$\pi_t = \tilde{a}_{\pi,d}\tilde{u}_{d,t} + \tilde{v}_{\pi,t} \tag{7}$$

Equation (6) implies that agents can use output to fully recover the (hidden) demand shocks, i.e., $\tilde{u}_{d,t} = y_t / \tilde{a}_{y,d}$. If we define $\tilde{\kappa} = \frac{\tilde{a}_{\pi,d}}{\tilde{a}_{y,d}}$, equation (7) can hence be rewritten as:

$$\pi_t = \tilde{\kappa} y_t + \tilde{v}_{\pi,t}. \tag{8}$$

Persistence parameter and Over-reaction. It is easy to show that the estimated persistence of the perceived underlying shock $\tilde{u}_{d,t}$ is given by:

$$\tilde{\rho} = \frac{Cov(y_t, y_{t-1})}{Var(y_{t-1})} = \frac{\rho_d a_{y,d}^2 \sigma_d^2 + \rho_s a_{y,s}^2 \sigma_s^2}{a_{y,d}^2 \sigma_d^2 + a_{y,s}^2 \sigma_s^2} \tag{9}$$

In a number of studies, economists have estimated the following regression, with the target of testing for Rational Expectations:

$$y_{t+1} - \tilde{\mathbf{E}}_t[y_{t+1}] = \alpha + \beta \left(\tilde{\mathbf{E}}_t[y_{t+1}] - \tilde{\mathbf{E}}_{t-1}[y_{t+1}] \right) + e_t \tag{10}$$

The dependent variable represents the forecast error while the regression considered is the forecast revision. Under Rational expectations, forecast errors should not be predictable by any variable belonging to any previous information set of the agent. When regression are performed at individual level however, a common finding is a negative β coefficient (Bordalo et al. (2020)).

We next show that the model naturally delivers such prediction. According to the perceived model, we have: $\mathbf{E}_t[y_{t+1}] = \hat{\rho} y_t$ and $\mathbf{E}_t[y_{t+1}] = \hat{\rho}^2 y_{t-1}$. The estimate of the coefficient in regression (4) solves:

$$\hat{\beta} = \frac{Cov(y_{t+1} - \hat{\rho} y_t)}{Var(\hat{\rho} y_t - \hat{\rho}^2 y_{t-1})} = -\frac{a_{y,d}^2 \sigma_d^2 \; a_{y,s}^2 \sigma_s^2 \; (\rho_d - \rho_s)^2}{\left(a_{y,d}^2 \sigma_d^2 + a_{y,s}^2 \sigma_s^2 \right)^2 (1 - \hat{\rho}^2)} \leq 0 \text{ with equality if and only if } \rho_s = \rho_d.$$

The intuition is as follows. Recall that we normalized the volatility of the innovations to supply and demand to both be the same. This means that *a variation in forecast revision might be generated by either shocks with equal probability*. A fully Bayesian agent would take think into account and adjust his forecast on output by

$$\frac{\rho_s + \rho_d}{2} \left(\tilde{\mathbf{E}}_t[y_{t+1}] - \tilde{\mathbf{E}}_{t-1}[y_{t+1}] \right).$$

On the other hand, from equation (9), the value of $\hat{\rho}$ gives a higher weight to the higher persistence parameter. This is so since the unconditional volatility of the shocks is increasing in the persistence of the shocks (recall indeed, we have $\sigma_d^2 = \frac{1}{1-\rho_d^2}$ and $\sigma_s^2 = \frac{1}{1-\rho_s^2}$. That is the source of 'over-reaction'. Note that - by construction - the LA agent would predict y_{t+1} correctly on average. Regression (4) however captures something different, that our LA agent, by projecting both shocks into one does not capture.

It might be worthwhile stressing that this is a very general result. It does not depend on our normalization, not by the fact that there are only two shocks: any projection to a lower dimensional state space would fail to capture that what moves forecast revision are the innovations while when the persistent parameter is estimated using 'unconditional moments', more persistent parameter receive - certeris paribus - a larger weight, leading to over-reaction.

Flatter Phillips Curve. We now study how LA agents might perceive the relationship between inflation and and output - i.e., the perceived Phillips multiplier $\tilde{\kappa}$. The estimated Phillips multiplier is given by:

$$\tilde{\kappa} = \frac{Cov(y_t, \pi_t)}{Var(y_t)} = \frac{a_{y,d} a_{\pi,d} \sigma_d^2 - a_{y,s} a_{\pi,s} \sigma_s^2}{a_{y,d}^2 \sigma_d^2 + a_{y,s}^2 \sigma_s^2} < a_{\pi,d} / a_{y,d} \equiv \kappa. \tag{11}$$

Notice that in this case, the estimated Phillips multiplier that minimizes the KLDR coincides with what would be obtained by (naive) agents estimating the Phillips curve by simply projecting inflation on output.

Importantly, Equation (11) indicates that in this case the perceived Phillips multiplier is downward biased. This occurs for two reasons. On the one hand, agents (wrongly) attribute to demand shocks the negative inflation-output co-movements generated by supply shocks. Thus, the perceived Phillips multiplier (wrongly) conflates the actual positive inflation-output co-movement due to demand shocks ($a_{y,d} a_{\pi,d} \sigma_d^2$) with the negative co-movement due to supply shocks ($-a_{y,s} a_{\pi,s} \sigma_s^2$). On the other hand, agents (wrongly) attribute to demand shocks the output volatility due to supply shocks, which lowers the perceived relationship between inflation and output for any given inflation process. A downward bias would also arise if agents were observing without error a

different variable, like the nominal interest rate, which negatively co-moves with inflation in response to supply shocks.

In more general cases where all macroeconomic variables could be observed with errors, a closed-form solution for the minimization of the KLDR is typically not available, unless additional simplifying assumptions are imposed.[11] For instance, in Appendix A.2 we show that in the case where measurement errors in inflation and output are homoskedastic, the perceived Phillips multiplier would be downward biased if and only if the actual demand curve is sufficiently steeper than the supply curve, i.e., $-a_{\pi,s}/a_{y,s} > a_{\pi,d}/a_{y,d}$.

In summary, we have shown that two things. First, neglecting some shocks hitting the economy naturally leads to overreaction in economic expectations. Second, limited awareness leads to a bias in the perceived relationship among macroeconomic variables. Under relatively weak assumptions—like perfect observability of output or interest rates—the agents' perceived Phillips multiplier should be downward biased, even though a bias of the opposite sign cannot be excluded. In the next section, we seek to uncover the presence and sign of the bias in the perceived Phillips multiplier by exploiting survey expectations data.

3 Evidence on Macroeconomic Expectations

This section asks whether professional forecasters perceive the same relationship between inflation and real economic activity that is observed in realized data. We use two complementary empirical strategies. The first is structural: we estimate the bias in the perceived slope of the Phillips curve using individual survey forecasts. The second is reduced-form: we compare the inflation–activity tradeoff in realized data and in forecasts after projecting both on monetary policy surprises. Both exercises point in the same direction. Relative to realized data, forecasters behave as if the Phillips curve were flatter.

Data In the main text we present the result based on forecast data from the Survey of Professional Forecasters (SPF), conducted by the Federal Reserve Bank of Philadelphia. Similar results are obtained using data from Blue Chips. The SPF data is a sensible choice for two reasons. First, professional forecasters are informed agents. Second, the SPF data is widely used in the literature and is applied in many contexts. Therefore, our findings are readily comparable to other studies. In the baseline analysis, the sample ranges from 1968:IV to 2019:IV. The end of the sample is chosen to exclude the Covid-19 period. Throughout the paper, we focus on individual forecasts; in the

[11] This is because minimization of the KLDR requires applying the Kalman filter, and in turn solving a non-linear optimization problem.

cases where (mainly for comparison with the relevant literature) we use consensus forecasts, we take the median of individual forecasts.

Realized measures of inflation and unemployment are subject to substantial data revisions. Professional forecasters, who must predict macroeconomic variables in real-time, do not have access to the latest revised data. To avoid this issue, in the baseline analysis we use the second-release real-time data to account for professional forecasters' real-time information. We use second-release data because it is less subject to measurement error than first-release data. Our findings are robust to alternative choices of vintages.

The baseline measure of economic economic activity is the *negative* of the unemployment rate. For robustness, we also consider regressions with the output gap, constructed using the CBO estimate of potential GDP or HP filtered real GDP. Using the output gap as a measure of economic activity raises an identification issue that requires instrumental variables. We postpone this discussion to the robustness section. Finally, in the baseline specification we use GDP deflator to measure inflation.

The mis-perceived Phillips curve. The estimation of the Phillips curve has been proved to be very challenging for a multitude of reasons. One key difficulty is the presence of disturbances, which require the use of convincing instruments. Another is that the choice of controls can be consequential. We therefore begin with a parsimonious baseline specification without controls and then progressively incorporate the terms suggested by the NKPC.

Formally, suppose inflation π_t and economic activity y_t $(= -u_t)$ are linked by

$$\pi_t = \kappa y_t + X_t + \eta_t, \tag{PC}$$

where X_t collects controls, such as expectations or lagged inflation.

Consider now the corresponding *perceived* Phillips curve from the point of view of agent i at time $t-h$:

$$\hat{\mathbb{E}}^i_{t-h}\pi_t = \kappa^e \hat{\mathbb{E}}^i_{t-h} y_t + \hat{\mathbb{E}}^i_{t-h} X_t + \hat{\mathbb{E}}^i_{t-h}\eta_t. \tag{PePC}$$

Crucially, we allow the agent to hold an incorrect perception of the slope governing the relationship between inflation and economic activity. Throughout the quarterly SPF analysis, let h denote the forecast horizon in quarters. Thus, $h = 1, 2, 3$ correspond to forecasts formed at dates $t-1$, $t-2$, and $t-3$ for the realization in period t.

Taking the difference between (PC) and (PePC) yields

$$FE^i_{t-h}(\pi_t) = \kappa FE^i_{t-h}(y_t) + \gamma \hat{\mathbb{E}}^i_{t-h}[y_t] + \widetilde{X}_t + \epsilon^i_t,$$

where $\widetilde{X}_t := X_t - \hat{\mathbb{E}}^i_{t-h} X_t$, $\epsilon^i_t := FE^i_{t-h}(\eta_t)$, and $\gamma := \kappa - \kappa^e$. Intuitively, ϵ^i_t captures innovations to η_t that occur at time t and are not predicted by agent i at $t-h$.

Our baseline specification sets $X_t = 0$, so that there are no controls. The resulting regression is

$$FE^i_{t-h}(\pi_t) = \kappa FE^i_{t-h}(y_t) + \gamma \hat{\mathbb{E}}^i_{t-h}[y_t] + \epsilon^i_t.$$

With the exception of the term involving $\hat{\mathbb{E}}^i_{t-h}[y_t]$, all terms in this baseline equation are *individual* forecast errors. Under RE, individual forecast errors must be uncorrelated with $\hat{\mathbb{E}}^i_{t-h}[y_t]$, therefore simple OLS should yield $\gamma = 0$. It follows that an estimate of γ significantly different from zero indicates a violation of RE. Table 1 reports the baseline OLS estimates, revealing that the coefficient γ is significantly different from zero (in fact, it is always positive) across all specifications.

	(1)	(2)	(3)	(4)	(5)
	h=1	h=2	h=3	Pool	1981-2008
κ	-0.02	0.12	0.11	0.08	0.42***
	(0.26)	(0.20)	(0.17)	(0.19)	(0.13)
γ	0.23***	0.28***	0.30***	0.27***	0.27***
	(0.08)	(0.09)	(0.11)	(0.09)	(0.07)
Observations	5626	5521	5327	16474	7625

Table 1: OLS Regression of deviation between (PC) and (PePC)

Notes: Columns (1)–(3) report the baseline specification $FE^i_{t-h}(\pi_t) = \kappa FE^i_{t-h}(y_t) + \gamma \hat{\mathbb{E}}^i_{t-h}[y_t] + u^i_t$ separately for $h = 1, 2, 3$. Column (4) pools $h = 1, 2, 3$, and column (5) reports the pooled specification on the 1981–2008 subsample. All specifications include forecaster fixed effects. Driscoll and Kraay (1998) standard errors are in parentheses. ***Significant at the 1 percent level. **Significant at the 5 percent level. *Significant at the 10 percent level.

Starting from the no-control baseline, we then consider three richer specifications. In the first one, we let $X_t = \beta \hat{\mathbb{E}}_t \pi_{t+1}$, so that the Phillips curve includes the standard forward-looking NKPC term. Data on the higher-order expectation $\hat{\mathbb{E}}^i_{t-h} \hat{\mathbb{E}}_t \pi_{t+1}$ is not available. We therefore proxy this term with $\hat{\mathbb{E}}^i_{t-h} \pi_{t+1}$. The specification we consider is then

$$FE^i_{t-h}(\pi_t) = \beta \left[\hat{\mathbb{E}}^i_t \pi_{t+1} - \hat{\mathbb{E}}^i_{t-h} \pi_{t+1} \right] + \kappa FE^i_{t-h}(y_t) + \gamma \hat{\mathbb{E}}^i_{t-h}[y_t] + \chi^i_t,$$

where $\chi^i_t := \beta \left[\hat{\mathbb{E}}_t \pi_{t+1} - \hat{\mathbb{E}}^i_t \pi_{t+1} - FE^i_{t-h}(\hat{\mathbb{E}}_t \pi_{t+1}) \right] + \epsilon^i_t$.[12] Similar to the baseline specification,

[12]The term in the square bracket in the definition of χ^i_t can itself be rewritten as the sum of an individual forecast error and an individual forecast revision:

$$\hat{\mathbb{E}}_t \pi_{t+1} - \hat{\mathbb{E}}^i_t \pi_{t+1} - (\hat{\mathbb{E}}^i_{t-h} \hat{\mathbb{E}}_t \pi_{t+1} - \hat{\mathbb{E}}^i_{t-h} \pi_{t+1}) = \underbrace{[\hat{\mathbb{E}}_t \pi_{t+1} - \hat{\mathbb{E}}^i_{t-h} \hat{\mathbb{E}}_t \pi_{t+1}]}_{\textit{individual FE}} - \underbrace{[\hat{\mathbb{E}}^i_t \pi_{t+1} - \hat{\mathbb{E}}^i_{t-h} \pi_{t+1}]}_{\textit{individual forecast revision}}.$$

under the null of RE, $\chi_t^i = \epsilon_t^i$, individual forecast errors must be uncorrelated with $\hat{\mathbb{E}}_{t-h}^i [y_t]$, therefore simple OLS should yield $\gamma = 0$.

Our second richer specification allows for a "hybrid" term in the Phillips curve, which accounts for past inflation:

$$FE_{t-h}^i(\pi_t) = \delta FE_{t-h}^i(\pi_{t-1}) + \beta \left[\hat{\mathbb{E}}_t^i \pi_{t+1} - \hat{\mathbb{E}}_{t-h}^i \pi_{t+1} \right] + \kappa FE_{t-h}^i(y_t) + \gamma \hat{\mathbb{E}}_{t-h}^i [y_t] + \chi_t^i,$$

In addition to documenting a deviation from RE, we are interested in the actual estimate of γ. Unfortunately, OLS is not consistent when $\gamma \neq 0$ and RE is violated. To address this concern, we instrument $\hat{\mathbb{E}}_{t-h}^i [y_t]$, as well as the other regressors, using the contemporaneous value and eight lags of monetary policy shocks constructed from changes in prices of Eurodollar and fed funds futures contracts within a 30-minute window around FOMC announcements.[13] These instruments are valid under the assumption that monetary shocks are uncorrelated with shocks to η_t, and therefore with ϵ_t^i, the individual forecast error about η_t.[14]

The results are reported in Table 2.

Reduced-form approach: the conditional Phillips multiplier. The structural regressions discipline the sign and magnitude of γ, but the same idea can be illustrated in a more model-light way. We project realized inflation and a measure of economic activity on monetary policy surprises and compute the implied Phillips multiplier in the projected data. We then repeat the same exercise using individual forecasts of inflation and economic activity. If forecasters perceive a flatter tradeoff, the Phillips multiplier in the forecast data should be smaller than in the realized data.

Figure 1 provides a reduced-form summary of the same pattern documented in Tables 1 and 2. Conditional on monetary policy surprises, the relationship between inflation and minus unemployment is steeper in the realized data than in the forecast data. This is exactly what one would expect if forecasters perceived a flatter Phillips curve than the one that governs aggregate outcomes.

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[13] The lag length is chosen to ensure instrument relevance.

[14] Note that $\hat{\mathbb{E}}_{t-h}^i \hat{\mathbb{E}}_t \pi_{t+1} = \hat{\mathbb{E}}_{t-h}^i \pi_{t+1}$ if agent i's information set is coarser than the one on which the expectation $\hat{\mathbb{E}}_t \pi_{t+1}$ is based and the law of iterated expectations applies. In that case, χ_t^i collapses to ϵ_t^i. More generally, when $\chi_t^i := \beta \left[\hat{\mathbb{E}}_t \pi_{t+1} - \hat{\mathbb{E}}_t^i \pi_{t+1} - FE_{t-h}^i(\hat{\mathbb{E}}_t \pi_{t+1}) \right] + \epsilon_t^i$, one might worry that the contemporaneous monetary shock is correlated with the term $\beta \left[\hat{\mathbb{E}}_t \pi_{t+1} - \hat{\mathbb{E}}_t^i \pi_{t+1} - FE_{t-h}^i(\hat{\mathbb{E}}_t \pi_{t+1}) \right]$, thereby violating the exclusion restriction. To address this concern, we have also used only lagged monetary policy shocks, excluding the contemporaneous shock, as instruments. The results are qualitatively similar, although the first-stage F statistics are lower, as expected.

	(1) Baseline	(2) NKPC	(3) 1981-2008	(4) Hybrid	(5) IV
One-period ($h = 1$)					
γ	0.23***	0.21***	0.18***	0.19**	0.22***
	(0.08)	(0.08)	(0.06)	(0.08)	(0.07)
obs.	5626	5348	2492	5338	1080
First-Stage F					21.00
All periods ($h = 1, 2, 3$)					
γ	0.27***	0.21***	0.24***	0.15**	0.17**
	(0.09)	(0.08)	(0.07)	(0.07)	(0.08)
obs.	16474	15361	7166	13751	2964
First-Stage F					28.89

Table 2: Phillips curve estimation

Notes: In each panel, column (1) reports the baseline no-control specification. Column (2) adds the forward-looking NKPC term $\beta\left[\hat{\mathbb{E}}^i_t\pi_{t+1} - \hat{\mathbb{E}}^i_{t-h}\pi_{t+1}\right]$. Column (3) re-estimates column (2) on the 1981–2008 subsample. Column (4) adds the hybrid term $\delta FE^i_{t-h}(\pi_{t-1})$. Column (5) estimates the same forward-looking NKPC specification as column (2) by IV, instrumenting $\hat{\mathbb{E}}^i_{t-h}[y_t]$, $FE^i_{t-h}(y_t)$, and $\left[\hat{\mathbb{E}}^i_t\pi_{t+1} - \hat{\mathbb{E}}^i_{t-h}\pi_{t+1}\right]$ with the contemporaneous value and eight lags of high-frequency monetary policy shocks. The upper panel uses $h = 1$, while the lower panel pools $h = 1, 2, 3$. All specifications include forecaster fixed effects. Driscoll and Kraay (1998) standard errors are in parentheses. The first-stage F statistic is reported for the IV column. ***Significant at the 1 percent level. **Significant at the 5 percent level. *Significant at the 10 percent level.

Blue Chip. Table 3 reports the analogous exercise using individual forecasts from *Blue Chip Economic Indicators*, a monthly survey of professional forecasters. We focus on forecasts of CPI inflation and the unemployment rate and use the one-year-ahead panel from 1985 to 2023. In this monthly setting, $h = 1, \ldots, 12$ denotes the number of months between the forecast date and the target date. The results closely mirror the SPF evidence. Across both OLS and IV specifications, the estimated coefficient γ remains positive and quantitatively similar to the quarterly estimates. This monthly evidence therefore points to the same conclusion: professional forecasters behave as if they perceive a flatter Phillips curve than the one observed in realized data.

4 A Limited Awareness New Keynesian Model (LANK)

We now introduce LA agents into an otherwise standard NK model. We consider an economy where economic fluctuations are driven by a demand (monetary policy) and a supply (productivity) shocks. However, our key assumption is that agents are only aware of the presence of demand shocks, while they are unaware of the presence of the supply shock.

Similarly to rational expectations equilibria, agents form expectations on the perceived mapping between the exogenous shocks and the aggregate variables. Our limited awareness assump-

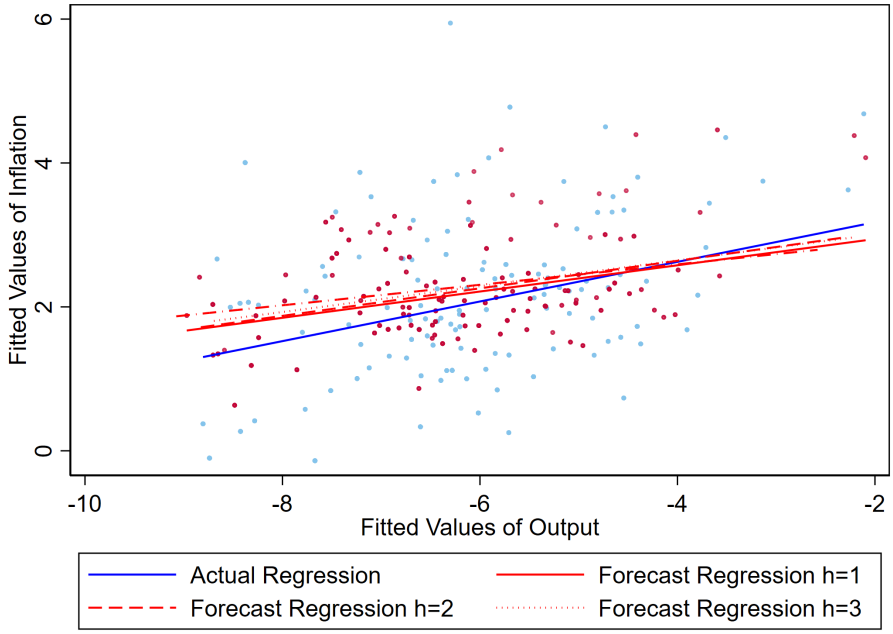


Figure 1: The figure reports a scatter plot and fitted regression lines after projecting the data on monetary policy shocks. Blue denotes realized data, while the red dots and line denote individual forecast data.

Table 3: **OLS and IV Regressions**
(Blue Chip)

	Baseline	NKPC	IV	IV-NKPC
All periods (h = 1-12)				
γ	0.172**	0.174**	0.261**	0.262**
	(0.082)	(0.082)	(0.129)	(0.128)
obs.	12582	12579	8902	8900

Notes: Blue Chip Economic Indicators. Monthly individual forecasts from 1985 to 2023, using the one-year-ahead survey. Here $h = 1, \ldots, 12$ denotes the number of months between the forecast date and the target date. Columns (1) and (2) report the baseline and NKPC specifications estimated by OLS. Columns (3) and (4) report the corresponding IV and IV-NKPC specifications. Standard errors are in parentheses. ***, **, and * denote significance at the 1, 5, and 10 percent levels, respectively.

tions implies that such a mapping does not contemplate supply shocks. In other words, agents' expectations are formed rationally, but based on a misperceived model of the actual economy. The resulting equilibrium outcome, which as usual depends on expectations about future variables, is then different from the outcome that would emerge under fully rational expectations —i.e. when agents are aware of all shocks and use the correct model.

In line with the incomplete information literature, we assume that demand shocks are not observed directly, and hence need to be inferred from observed macro variables. The inferred demand shock is then used to form expectations about future aggregate variables. As shown below, this assumption helps explain in a natural way why expectations of limited aware agents differ from rational expectations, even conditional on demand shocks, consistently with the evidence provided above. To simplify the analysis, we present a model where agents infer the demand shocks looking at the nominal interest rate. This inference strategy would coincide with the optimal one in an economy where the remaining macro variables are observed with noise. In Appendix B we show how the present model can be seen as the limit of a 'noised-up model' setup in the spirit of Lorenzoni (2009) and Angeletos and Lian (2020) where individuals have preference shocks and work in segmented labor markets so that the wage received by a household is only a noisy signal of the aggregate wage, in addition, in each period, households only buy a 'basket' of goods, implying that households only have a noisy signal about the distribution of prices across the economy.

Households. Households are all identical and have preferences given by

$$\hat{\mathbb{E}}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{{N_t}^{1+\varphi}}{1+\varphi} \right)$$

where $C_t = \left(\int_0^1 (C_t^j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}$, $\epsilon > 1$, is a composite good and C_t^j is the good produced by firm j. Households trade nominal bonds at the interest rate $i_t = -\ln Q_t$, where Q_t is the price of the bond, which satisfies

$$Q_t B_{t+1} + \int_0^1 P_t^j C_t^j dj = B_t + W_t N_t + \int_0^1 \Pi_t^j dj,$$

where B_t are nominal bonds' holdings, P_t^j is the price of good j, and W_t and Π_t^j are the nominal wage and nominal profits from firm j, respectively. As in the standard (cashless) NK framework, we let bonds be in zero net supply, that is, in equilibrium $B_t = 0$. Standard arguments imply that,

by letting $\Pi_t := \int_0^1 \Pi_t^j dj$ and defining the price index as

$$P_t := \left(\int_0^1 (P_t^j)^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}},$$

we can rewrite the household's budget constraint as

$$Q_t B_{t+1} + P_t C_t = B_t + W_t N_t + \Pi_t,$$

Expectations will be defined below; for now, we make the simplifying assumption that *households have the same expectations as firms*.[15]

The maximization problem yields:

$$c_t = \hat{\mathbb{E}}_t[c_{t+1}] - \frac{1}{\sigma} \left(i_t - \hat{\mathbb{E}}_t[\pi_{t+1}] \right)$$

and

$$w_t - p_t = \sigma c_t + \phi n_t.$$

Note that in this simple framework, real wage is tightly related to aggregate TFP. This is because, from household's preferences, a low $w_t - p_t$ indicates a low demand for labor. If, in addition, output y_t is observed, a low demand for labor immediately translates into a higher TFP shock a_t. As usual, in macroeconomic models agents have a number of information sources that makes it challenging to keep them with limited information. We assume $w - p$ is perceived as a noisy indicator of the actual real wage, with LA agents showing no interest towards extracting information from it. For simplicity, we do not explicitly incorporate this noise in the real wage. Similarly, by observing the full range of price levels across firms, households might -in principle - recover the distribution of productivity shocks across firms. Again, we assume households 'perceive' prices as noisy signals about the aggregate price level.

From goods market clearing, $y_t = c_t$, thus, the Euler equation becomes

$$y_t = \hat{\mathbb{E}}_t[y_{t+1}] - \frac{1}{\sigma} \left(i_t - \hat{\mathbb{E}}_t[\pi_{t+1}] \right). \tag{IS}$$

[15] We could assume that households hold rational expectations and nothing relevant would change. However, notation would be more cumbersome since households and firms would hold different expectations about inflation and future consumption.

Firms. Firms produce differentiated goods and operate under monopolistic competition. The sole input is labor, the nominal wage is W_t, and (for now) we assume a CRS technology:

$$Y_t^i = A_t^i N_t^i.$$

Each firm faces demand

$$Y_t^i = (P_t / P_t^i)^\epsilon Y_t$$

and sets its price optimally, when allowed to do so, with probability θ. Standard arguments imply that each firm targets its price as a markup over the nominal marginal cost W_t / A_t. Note that, due to Calvo pricing, firms need to forecast the future evolution of the relevant variables, especially the nominal wage. As it is standard in NK models, this inference exercise can be decomposed into predicting future GDP (to recover the real marginal cost) and inflation. We postulate that such expectations are based on the view of the world we describe below.

Information and timing At the beginning of the world, nature draws the mean probability of changing price θ (common to all firms). Then, it draws $\theta^i = \theta \varepsilon_i^\theta$, where ε_i^θ is of mean one. Firms i observes its own θ^i but it cannot observe the average θ. We further assume that firms have access to an infinite amount of data. This assumption effectively bypasses "learning" and disperse beliefs.[16]

More precisely, we can assume that the process for potential output incorporates all productivity shocks of the economy known to the agents. This way, we isolate out the 'neglected' productivity shocks.

$$\pi_t = \beta \hat{\mathbb{E}}_t \pi_{t+1} + \kappa y_t - \psi a_t + u_t. \tag{PC}$$

where $\kappa = \lambda(\sigma + \phi)$ and $\psi = \lambda(1 + \phi)$. For reasons that will be clear below, we added a temporary sock u_t with zero mean and uncorrelated to y_t, unobserved by firms (although firms "know it's there", i.e., they are aware of its existence). This shock can be interpreted as measurement error or a disturbance to the Phillips curve which the central bank does not react to. For notational simplicity, we henceforth set it to zero, but agents believe it to be non-negligible.

4.1 True vs. Perceived Phillips Curve

The "true" Phillips curve. Firms are subject to aggregate TFP shocks. For now we keep expectations unspecified. Log-linearization around the deterministic (efficient) steady state of firm i's

[16] Note that, while agents could theoretically utilize θ^i as a signal for θ, they find little benefit in doing so. This is because, as we will show, estimating θ directly from the Phillips curve will be a dominant approach.

optimal pricing decision gives the price setting rule[17]

$$\pi_t^i = (1-\beta\theta^i)\sum_{n=0}^{\infty}(\theta^i\beta)^n\hat{\mathbb{E}}_t^i[w_{t+n}-p_{t+n}-a_{t+n}] + \sum_{n=0}^{\infty}(\theta^i\beta)^n\hat{\mathbb{E}}_t^i\pi_{t+n}+\mu, \tag{PSR}$$

where $w_{t+n}-p_{t+n}$ is the real wage, a_{t+n} is the (stochastic) TFP (log-)level, and π_{t+n} is aggregate inflation, and $\mu := \ln\frac{\epsilon}{\epsilon-1}$ is the natural logarithm of the steady state markup.

Our assumptions imply that firms will hold the same expectations over aggregate variables. Since the TFP shock is not correlated with θ^i, we can let $\theta^i = \theta\varepsilon_i^\theta$ and log-linearize the latter around its mean (recall $\int_I \varepsilon_i^\theta = 1$):

$$(\theta^i)^n \approx \theta^n(1+n\ln\varepsilon_i^\theta)$$

and, thus,

$$\int_I(\theta^i)^n di \approx \theta^n.$$

This approximation is justified as long as the cross sectional dispersion of θ^i is small. Similarly, we log-linearize the whole expression $(1-\beta\theta^i)\sum_n(\theta^i\beta)^n$. Therefore, if we integrate over the firms changing their price, we get

$$\pi_t^* \approx (1-\beta\theta)\sum_n(\theta\beta)^n\int_I\hat{\mathbb{E}}_t^i[w_{t+n}-p_{t+n}-a_{t+n}]di + \sum_n(\theta\beta)^n\int_I\hat{\mathbb{E}}_t^i\pi_{t+n}di+\mu.$$

Since firms hold the same expectations across future aggregate variables we conclude that

$$\pi_t^* = (1-\beta\theta)\sum_n(\theta\beta)^n\hat{\mathbb{E}}_t[w_{t+n}-p_{t+n}-a_{t+n}] + \sum_n(\theta\beta)^n\hat{\mathbb{E}}_t\pi_{t+n}+\mu. \tag{PSR}$$

Finally, from Calvo pricing, $\pi_t = (1-\theta)\pi_t^*$.

The "perceived" Phillips curve. To derive the process for inflation, we need to describe how firms form expectations. Given that each firm is atomistic, the perceived inflation process is dictated solely by the perception of other firms' price setting behavior (i.e., second order beliefs). We then make the following assumptions:

i) *Each firm knows that all other producers follow the price-setting rule* (PSR);

[17] As it is well known, the expression is a simple rearrangement from the log-linearized version of the firm's first order condition:

$$p_t^i = \mu + (1-\beta\theta^i)\sum_{n=0}^{\infty}(\theta^i\beta)^n\hat{\mathbb{E}}_t^i[w_{t+n}-a_{t+n}].$$

ii) *Each firm observes the productivity shock a_t but (wrongly) assume that it is purely idiosyncratic, and that no other firms received any productivity shock. As a consequence, it believes that its idiosyncratic shock has no aggregate consequences;*

iii) *Each firm thinks (correctly) that all other firms ignore the occurrence of the productivity shock occurred to the firm.*

Note that (i) and (ii) together imply the in the perceived aggregate Philips curve (PSR)a_t disappears. Furthermore, the above assumptions imply that second-order beliefs (i.e., beliefs of beliefs) are homogeneous across firms. This implies that each firm believes that other firms' expectations about aggregate variables are identical to its own. We can then factor out the expectation term and obtain the Perceived Aggregate Price Setting Rule:

$$\hat{\pi}_t^* = (1-\beta\theta)\sum_n (\theta\beta)^n \hat{\mathbb{E}}_t[w_{t+n} - p_{t+n}] + \sum_n (\theta\beta)^n \hat{\mathbb{E}}_t \pi_{t+n} + \mu. \tag{PPSR}$$

Finally, firms also infer that $\hat{\pi}_t = (1-\theta)\hat{\pi}_t^*$.

Standard (textbook) steps imply

$$\begin{aligned} \hat{\mathbb{E}}_t^i \pi_{t+1} = \hat{\mathbb{E}}_t \pi_{t+1} &= (1-\tilde{\theta})\left[(1-\beta\tilde{\theta})\sum_n (\tilde{\theta}\beta)^n \hat{\mathbb{E}}_t[w_{t+1+n} - p_{t+1+n}] + \sum_n (\tilde{\theta}\beta)^n \hat{\mathbb{E}}_t \pi_{t+1+n}\right] \\ &= \tilde{\lambda}\hat{\mathbb{E}}_t\left[w_{t+1} - p_{t+1}\right] + \beta\hat{\mathbb{E}}_t \pi_{t+2}, \end{aligned}$$

where $\tilde{\lambda} = \frac{(1-\tilde{\theta})(1-\beta\tilde{\theta})}{\tilde{\theta}}$.

It is immediate to see that the law of iterated expectations holds at the individual level, namely, our assumptions guarantee that $\hat{\mathbb{E}}_t^i \hat{\mathbb{E}}_{t+1}^i \pi_{t+2} = \hat{\mathbb{E}}_t^i \pi_{t+2}$. The previous expression implies that, if a firm in our model were asked to forecast future inflation π_{t+1}, its answer would be her prediction of the real wage $\hat{\mathbb{E}}_t\left[w_{t+1} - p_{t+1}\right]$, weighted by $\tilde{\lambda}$, and her forecast *at time t* of π_{t+2}.

Of course, we can also write the perceived expected Phillips Curve as follows:

$$\hat{\mathbb{E}}_t \pi_{t+1} = \beta\hat{\mathbb{E}}_t \pi_{t+2} + \tilde{\kappa}\hat{\mathbb{E}}_t\left[y_{t+1}\right], \tag{PPC}$$

where $\tilde{\kappa} = \tilde{\lambda}(\sigma + \phi)$. [18]

[18] We can write all in output-gap terms:

$$\pi_t = \beta\hat{\mathbb{E}}_t \pi_{t+1} + \kappa\tilde{y}_t,$$

where $\tilde{y}_t := y_t - y_t^n$, with y_t^n is the natural level of output, defined as $y_t^n := -\frac{\mu}{\sigma\psi} + \frac{1+\phi}{\sigma\phi}a_t$. Of course, if the agent had access to the correct estimation of the potential output, she could get the correct estimate for κ. In fact, the most useful way to interpret the equations in the main text is as expressed in *perceived output gaps*.

4.2 Parameter Estimation

Agents use their perceived state-state representation of the model to estimate the parameters.

We know focus on how LA firms use the available data to estimate the parameter κ. Given their view of the world, the best way to gain information on κ is to run an OLS regression of π_t on y_t, controlling for expectations about future inflation. All the ingredients are known to the firm and—OLS is optimal given the linearity of the model. When doing so, firms estimate

$$\hat{\kappa} = \frac{Cov\left(\pi_t - \beta \hat{\mathbb{E}}_t \pi_{t+1}, y_t\right)}{Var\left(y_t\right)} = \kappa + \frac{Cov\left(-\psi a_t + u_t, y_t\right)}{Var\left(y_t\right)} = \kappa + \frac{Cov\left(-\psi a_t, y_t\right)}{Var\left(y_t\right)}, \tag{12}$$

where we used (PC) and the fact that u_t is orthogonal to y_t by assumption. This estimation suffers from an omitted-variable bias as long as $Cov(a_t, y_t) \neq 0$. In fact, since $Cov(a_t, y_t) < 0$ in the standard New Keynesian framework, we also know the sign of the bias: $\hat{\kappa} < \kappa$.[19]

4.3 Expectation formation

We are now ready to describe in more detail how firms form expectations in out model. Recall, the true data generation process obeys to the following (classical) equations:

$$\begin{aligned}
EE: \quad & y_t = \tilde{\mathbb{E}}_t y_{t+1} - \frac{1}{\sigma}\left(i_t - \tilde{\mathbb{E}}_t \pi_{t+1}\right); \\
PC: \quad & \pi_t = \beta \tilde{\mathbb{E}}_t \pi_{t+1} + \kappa y_t + s_t, \\
TR: \quad & i_t = \phi_\pi \pi_t + \phi_y y_t + d_t
\end{aligned}$$

RE basically assume that expectations are consistent with all above equations, and the true processes of the two supply and demand shocks.

In our model with **LA**, agents expectations are based on a biased slope in the Phillips curve, namely:

$$\pi_t = \beta \tilde{\mathbb{E}}_t \pi_{t+1} + \hat{\kappa} y_t.$$

Perhaps more importantly however LA agents think that aggregate variables are measurable with respect to demand shocks alone.

Finally, as berifly esxplained at the beginning of this section, agents not only do not observe the underlying fundamental shocks, they also observe aggregate variables with noise. *The only aggregate variable that are able to observe without noise is the nominal interest rate*. They hence use

[19] Whenever GDP is observed without error, running an unconditional regression of inflation to output would deliver the parameter minimizing the KL divergence between the true steady state distribution and that obtainable by our misspecified model in the spirit of Esponda and Pouzo (2021) and Molavi (2019).

the EE to recover (correctly) Y_t and then the PC to estimate aggregate inflation and ultimately - from TR - the demand shock $\hat{d}_t$. Note that since they have a mis-perceived PC and they ignore the presence of supply shocks, agents would in general estimate a wrong level of inflation and hence an incorrect level for the demand shock. LA agents attribute all discrepancies between their estimated values and what they observe to idiosyncratic noise.

When a demand shocks hit the economy, the 'mistake' agents commit is only due to the fact that $\hat{\kappa} \neq \kappa$. When the economy is hit by a supply shock, agents will try to reconcile the evidence they have on the interest rate and hence on output with a level of aggregate inflation and ultimately of demand shock. In the next section we study the positive consequence of LA by comparing the behavior of our economy with that of an otherwise identical economy populated by RE agents.

5 The effect of demand and supply shock with Limited Awareness

In this section we study the effects of demand and supply shocks in our model, comparing them with those from the standard RE model. Recall that we have assumed that neither supply nor demand (monetary policy) shocks are not observed by the agents. Under RE, agents possess sufficient information to filter out the realized shock—whose persistence is assumed to be known—from the observed y_t and π_t.

5.1 Demand Shocks

Let start with a pure demand shock, that is, a situation where the economy is hit only by a monetary policy shock. Recall that agents are aware of such shocks, but do not observe their realizations; they thus "measure" them from aggregate variables. All aggregate variables are observed with error but the nominal interest rate. Agents are able to recover aggregate output by inverting the Euler equation. Then from the Phillips curve they infer inflation and use it to recover demand shocks. Given the mis-specification of the Phillips curve - in general - agents estimate a wrong level of demand shocks, this is exactly however the one that generates the observed interest rate and the correct aggregate level of output in t.

Proposition 1 (Effects of a Monetary Shock)**.** *Suppose $i_t = \phi_\pi \pi_t + \phi_y y_t - d_t$ with $d_t = \rho_d d_{t-1} + \epsilon_t^d$. Further let $\hat{\rho} = \rho_d$ be the perceived persistence of the demand shock and $\kappa > \hat{\kappa} \geq 0$. Then, compared to the standard New Keynesian model with RE agents:*
(i) $\frac{\Delta y_t}{\Delta d_t} (> 0)$ is larger if and only if $\phi_\pi \beta \geq 1$; i.e., the result is reversed iff $\phi_\pi \beta \leq 1$. and
(ii) $\frac{\Delta \pi_t}{\Delta d_t} (> 0)$ is always smaller.

The proof is reported in Appendix C.

The intuition for the role played by $\phi_\pi \beta$ in the output reaction is as follows. An increase in d increases expectations about the future inflation by less than RE. This reduces today inflation by a factor of β and todays' nominal interest rate increases less than RE by a factor of $\beta\phi_\pi$. Output depends on the real interest rate, the reduced response of expectations increase real interest rate by a factor of 1 for each given nominal rate. The condition $\beta\phi_\pi > 1$ implies that the effect on the nominal rate is stronger than that of the expectations and hence the real interest rate increases by less than under RE amplifying the initial demand effect. The opposite occurs when $\beta\phi_\pi < 1$: the reduced change on future expectations about inflation dominates the effect induced on the interest rate, dampening the effect of the demand shock.

Figure 2 reports the results of a persistent and sizable decrease in the d_t leading to an increase in the real interest rate. The blue line reports the response of the benchmark NK model with rational agents. The red solid line reports the effects of the same monetary policy shock when the economy is populated by LA agents. The calibration chosen for this simulation uses a relatively large value for ϕ_π implying $\beta\phi_\pi > 1$. As it can be seen, the presence of limited aware agents amplifies (by about 30% in the example) the reaction of output, while it reduces (by about 10%) the reaction of inflation. The dashed red lines represent limited-aware agents' expectation of the plotted variable one period ahead. The substantial difference between the RE and LA models is due entirely to the fact that LA agents forecast and experience a larger level of inflation (more precisely, a lower decrease of inflation compared to RA agents). As a result, they perceive a higher real interest rate, which in turn depresses output and output expectations.

Finally note that—according to the two most left panels in Figure 2—agents forecast future output correctly. This is due to two factors. First, by assumption, agents observe current output, which they use to infer the realization of the demand shock. In fact, the bottom right panel shows that they slightly underestimate the size of the demand shock. This 'mistake' is, however, fully consistent with the observed output and their mis-specified model. Second, in the example we assume LA agents hold a correct perception of the persistence of demand shocks.[20] In contrast, agents' expectations about inflation are incorrect, which is evident from the discrepancy between the dashed and solid red lines in the top center panel. The difference is mainly due to the fact that LA agents perceive a flatter Phillips curve. The graph also suggests that for demand shocks the gap between expected and actual inflation is small, one order of magnitude smaller than the difference between the RE and LA inflation levels. This is so since expectations constitute an important force that drives actual variables (what we can call a 'mild' self-fulfilling effect).

[20] In case of mis-perceived persistence, the agents will still estimate correctly the current level of GDP while expectations about GDP will be incorrect.

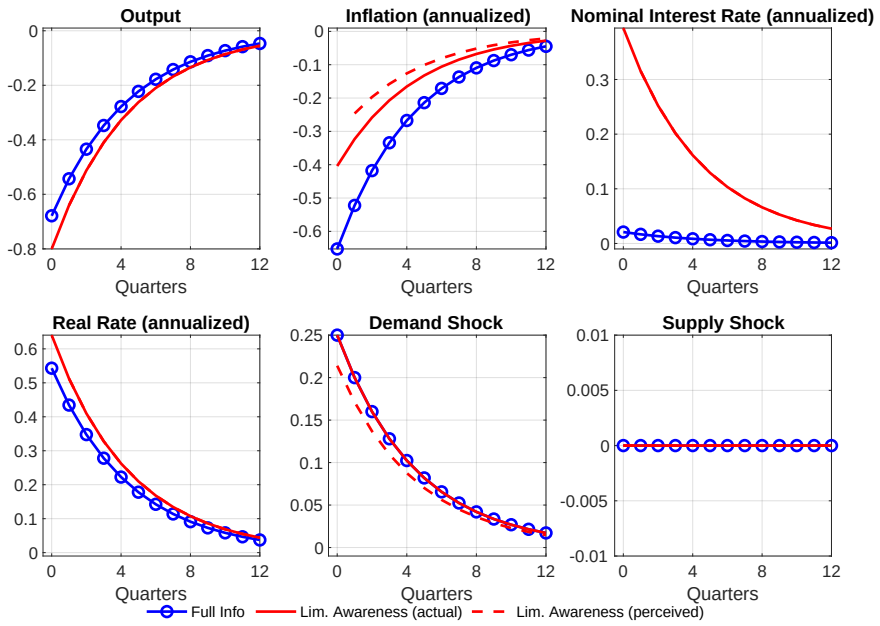


Figure 2: **Contractionary Monetary Shock (Taylor Rule).** Parameters: $\beta = .99$, $\sigma = 2$, $\kappa = .05$, $\hat{\kappa} = .02$, $\phi_\pi = 1.5$, $\phi_y = 0$, $\hat{\rho}_d = \rho_d = .9$. Note that $\phi_\pi \beta = 1.485 > 1$. The figure represents the effects of a persistent and sizable decrease in d_t (the bottom center panel reports $-d_t$), which leads to an initial increase in the real interest rate (bottom left panel). The blue line reports the effect of the shock in the baseline New Keynesian model with RE agents. In that model, the contractionary monetary policy generates a large reaction of inflation (top center panel) and a relatively modest reaction of output (top left panel). The red solid line reports the effect of the same monetary policy in the model with LA agents. The presence of LA agents amplifies (by about 30% in the example) the reaction of output and reduces (by about 10%) the reaction of inflation. The dashed red lines represent LA agents' one-period-ahead expectation of the plotted variable. The graphs illustrate the results of Proposition 1: LA agents experience (and expect) a more moderate reduction of inflation and, hence, a higher real interest rate compared to RA agents.

The mis-specification effect. To gain a deeper insight into the effects of monetary policy, we examine an extreme case where we can perfectly isolate the impact of mis-specification. Suppose the central bank operates under a *perceived* real-interest-rate targeting rule, that is, it adjusts the nominal interest rate so that the *real* interest rate equals d_t:[21]

$$i_t = \hat{\mathbb{E}}_t \pi_{t+1} - d_t, \quad d_{t+1} = \rho_d d_t + \varepsilon_{t+1}.$$

[21] Real-interest-rate targeting is not a special case of the Taylor rule we consider in our propositions. We present purely for didactical reasons.

This rule allows the central bank to control y_t perfectly. In particular, from equation (IS),

$$\begin{aligned} y_t &= \hat{\mathbb{E}}_t y_{t+1} - \frac{1}{\sigma}(i_t - \hat{\mathbb{E}}_t \pi_{t+1}) = \hat{\mathbb{E}}_t y_{t+1} + \frac{1}{\sigma} d_t \\ \Rightarrow \quad y_t &= \frac{1}{\sigma} \sum_{n=0}^{\infty} \hat{\mathbb{E}}_t d_{t+n} = \frac{1}{\sigma} \sum_{n=0}^{\infty} \mathbb{E}_t d_{t+n}, \end{aligned}$$

where in the last equality we have replaced $\hat{\mathbb{E}}_t[\cdot]$ with its RE counterpart $\mathbb{E}_t[\cdot]$ since agents are assumed to know the true process for d_t and, in addition, they can correctly estimate the initial shock.[22]

With this particular choice for monetary policy, output and inflation become

$$y_t = \frac{1}{\sigma(1-\rho_d)} d_t \quad \& \quad \pi_t = \frac{\kappa - \beta\rho_d(\kappa - \hat{\kappa})}{\sigma(1-\rho_d)(1-\beta\rho_d)} d_t,$$

where $\hat{\kappa} < \kappa$.

Figure 3 reports the effects of a contractionary monetary policy. Once again, lines in blue represent the response of the benchmark New Keynesian model with RE agents, while solid red lines represent the response of our model with LA agents. It is apparent from the figure that the only difference between our model and its RE counterpart is in the response of inflation (center top panel). In particular, LA agents underestimate the deflationary effects of future contractionary policies exactly because they perceive the Phillips curve to be flatter. What is more, agents' misperception of the output-inflation trade-off implies that the central bank must implement a more aggressive increase in the nominal interest rate to adhere to its real-interest-rate targeting rule.

Figure 4 provides an intuitive representation of the mechanism. By comparing the red and blue curves, one can see that the only difference between the RE and LA models is that, in the latter, agents perceive a flatter Phillips curve. As a result, an expected future contractionary monetary policy shock has a smaller impact on expected inflation and, in turn, on actual inflation.

Figure 13 in Appendix D reports a graphical representation of the effect studied in Proposition 1. As discussed above, a contractionary monetary policy tends to generate a smaller response of inflation expectations, thus inducing a more modest shift of the Phillips curve. At the same time, when the aggregate demand curve is not vertical, a flatter expected Phillips curve also generates a larger decrease in expected output. This in turn tends to induce a larger shift in aggregate demand. To obtain the total effect on current output, however, one must also consider the more muted increase in the real interest rate caused by the moderate reduction in inflation expectations.

[22] This latter result follows from the fact that, under the specific monetary policy rule we postulated, agents can estimate d_t by simply inverting the IS: $d_t = \sigma(y_t - \hat{\mathbb{E}}_t[y_{t+1}])$.

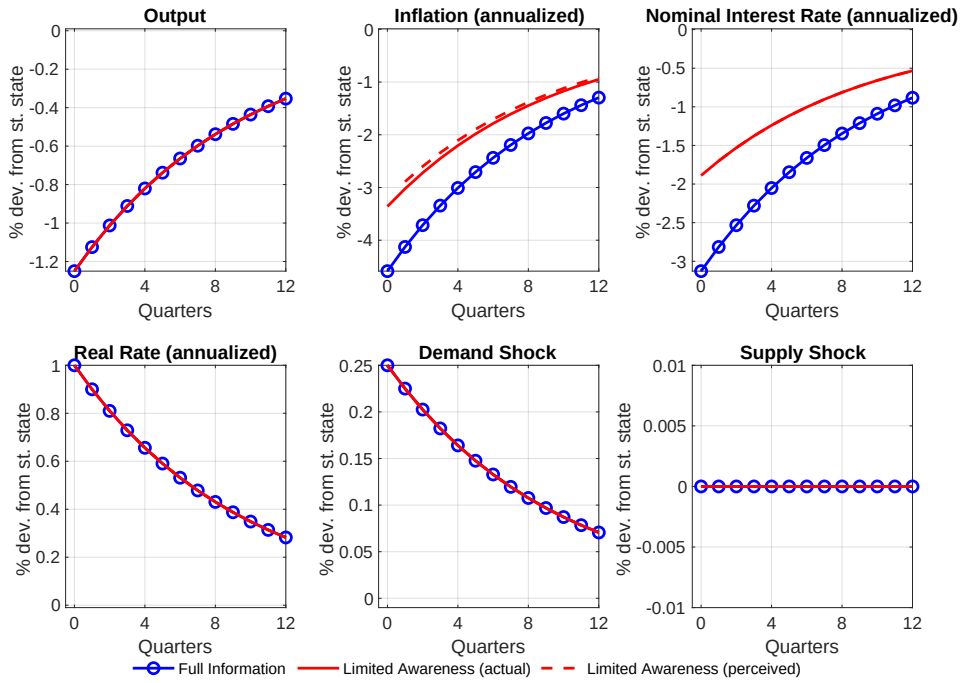


Figure 3: **Contractionary Monetary Shock (Real-Interest-Rate Targeting).** The figure reports the effects of a contractionary monetary policy in the case of a vertical aggregate demand curve, comparing the response between the standard New Keynesian model (blue) and our model with LA agents (solid red). The only difference between the two models is in the inflation response (center top panel), which shows how LA agents underestimate the inflationary effects of future contractionary policies. This is because they perceive a flatter Phillips curve. In turn, since the central bank aims at controlling the real interest rate, the same drop in the real inters rate requires a more aggressive increase in the nominal interest rate.

As stated in the proposition, when $\phi_\pi \beta \geq 1$, the monetary policy shock leads to a larger drop in output compared to the RE benchmark.[23]

The model might suggest a potential ambiguity regarding inflation. On the one hand, the more moderate shift in the Phillips curve tends to generate a smaller decrease in actual inflation and, at the same time, a larger reduction in output. On the other hand, the larger effect on the aggregate demand tends to generate a stronger reaction of both variables. Proposition 1 clarifies that there is no ambiguity: the former effect always dominates, therefore, *the presence of LA agents always implies a more muted inflation response*.

Finally, in order to make directly comparable the LA and RE models, we assumed agents perceive the correct persistence of demand shocks, and focused on the effect induce by the bias in the slope for the Phillips curve. If this is not the case the results are modified in the obvious way. That is, the amplified effect on output compared to RE survives as long as the perceived

[23] In Figure 14 in Appendix E, we report the case where $\phi_\pi \beta < 1$ so that both the effects on output and the effect on inflation are mitigated.

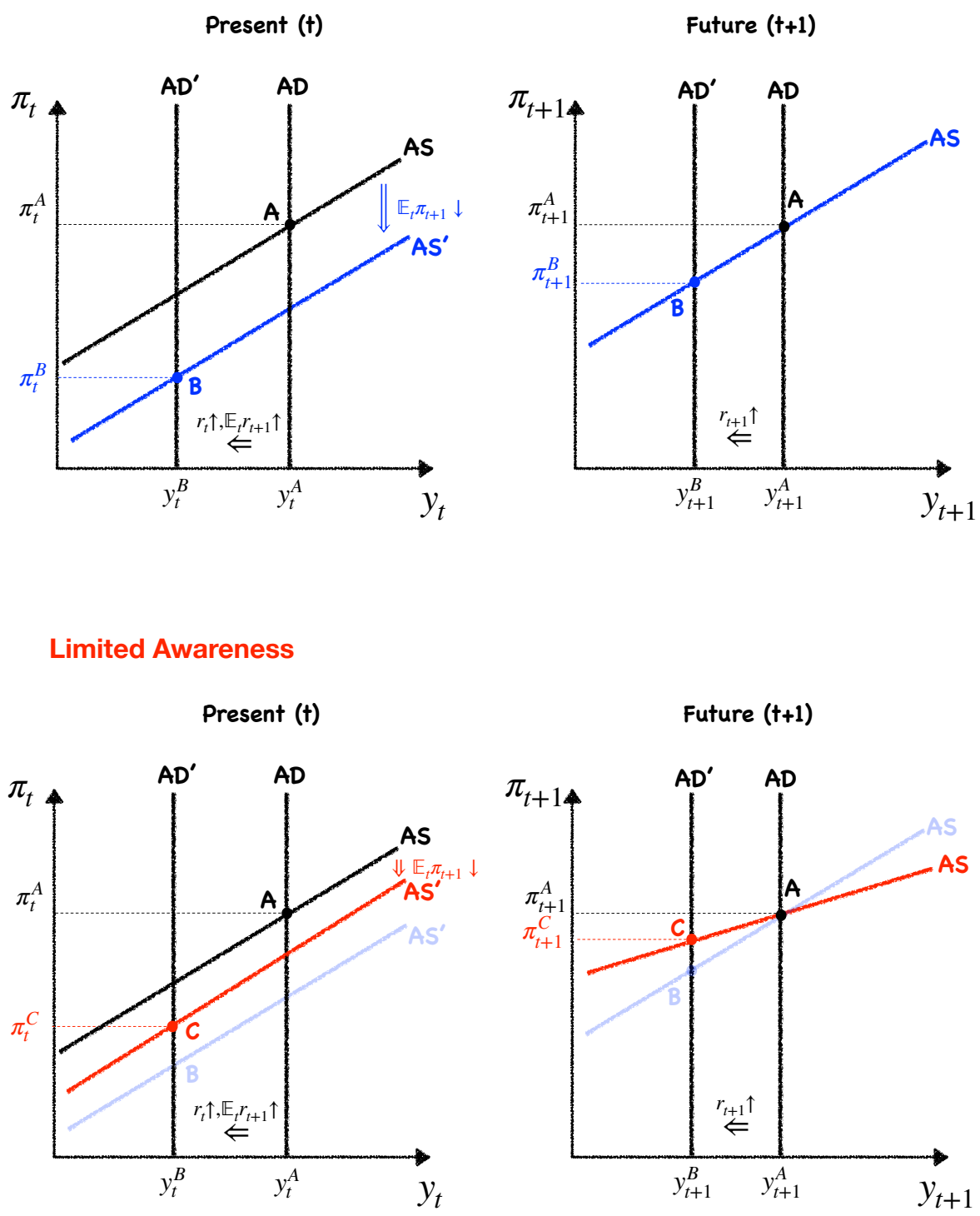


Figure 4: **Demand Shocks with vertical AD.** The figure illustrates the mechanism behind a contractionary monetary policy in the case of a vertical aggregate demand curve, comparing the benchmark New Keynesian model and our model with LA agents. The only difference is due to the fact that LA agents underestimate the inflationary effects of future contractionary policies. This is so since they perceive a flatter Phillips curve. In turn, since the central bank controls output, the same drop in output can be obtained by a milder reduction in the nominal interest rate.

persistence $\hat{\rho}$ is not too low compared to the true ρ_d. Symmetrically, the muted effect on inflation instead survives as long as $\hat{\rho}$ is not too large compared to ρ_d.

5.2 Supply Shocks

We now consider the effect of a supply shock.

Proposition 2 (Effects of a supply shock.)**.** *Assume the central bank follows the interest-rate rule* $i_t = \phi_\pi \pi_t + \phi_y y_t$. *Let the supply shock follow* $a_t = \rho_s a_{t-1} + \epsilon_t^s$. *Let* $\hat{\rho} = \rho_s$ *be the perceived persistence of the demand shock. Finally, assume* $\kappa \geq \hat{\kappa}$. *Then:*
(i) $\frac{\Delta y_t}{\Delta a_t}$ (> 0) *is <u>smaller</u> than under RE if and only if* $\phi_\pi \beta \geq 1$*; i.e, the result is reversed iff* $\phi_\pi \beta \leq 1$*, and*
(ii) $|\frac{\Delta \pi_t}{\Delta a_t}|$ *is <u>smaller</u> than under RE whenever* $\frac{\Delta \pi_t}{\Delta a_t} < 0$ *(regular case);*

The proof is reported in Appendix C.

The intuition for the result is as follows. Consider, for example, a negative supply shock that reduces output and tends to increase inflation. In our model, LA agents interpret such movements as a negative demand shock, which is deflationary. Expectations on inflation decrease, i.e., move in the opposite direction to the underlying shock. The overall inflationary effect is hence mitigated.

Note that the role of $\phi_\pi \beta$ in output reaction is inverted compared to the demands shock. To understand why recall that - as for demand shocks - the overall effect on output purely due to the change in inflation expectations is $\phi_\pi \beta - 1$, that is the implied change in the real interest rate. Under RE, a negative supply shock increases inflation expectations and hence generates a further drop in output whenever $\phi_\pi \beta > 1$. Under LA, inflation expectations decrease. As a consequence, when $\phi_\pi \beta > 1$ this force mitigates the negative effect on output.

Supply Shocks looking like demand shocks. We now show that under certain conditions the misinterpretation of the shock—through changes in expectations—can trigger an aggregate response that is qualitatively similar to that of a demand shock. This effects goes in the direction of one of the key facts listed by Born et al. (2022): in most cases, firm update expectations about prices and quantities in the same direction.

In this case, it would be true that —even for negligible observational errors—agents will never be "surprised" by the observed response of the aggregate variables. Whether expectations make a supply shock look like a demand shock or not will depend on the parameters of the model. Proposition 2 reports the exact conditions for this to happen. Intuitively, the persistence parameter $\hat{\rho}$ perceived by the agents plays an important role. It is also worth mentioning that, conditional on

a given $\hat{\rho}$, *neither the persistence of the demand shock nor the persistence of the supply shock matter for the sign at impact* (of course it will for the shape of the impulse response).

Proposition 3 (Supply shocks like demand shocks.)**.** *Assume that the central bank follows the interest rate rule* $i_t = \phi_\pi \pi_t + \phi_y y_t - d_t$*, with* $d_t = \rho_d d_{t-1} + \epsilon_t^d$*. Let the supply shock function follow* $a_t = \rho_s a_{t-1} + \epsilon_t^s$*. Let* $\hat{\rho} \in (0,1)$ *be the perceived persistence of the demand shock. Finally, assume* $\kappa \geq \hat{\kappa}$*. Then, for a positive supply shock, that is, a realization such that* $a_t > 0$ *and* $d_t = 0$*,*

(i) the response of output $\frac{\Delta y_t}{\Delta a_t}$ *is always positive;*

(ii) the response of inflation $\frac{\Delta \pi_t}{\Delta a_t}$ *is positive if and only if:*

$$\sigma < \frac{\hat{\rho}\hat{\kappa}}{(1-\beta\hat{\rho})(1-\hat{\rho})} - \frac{\phi_y}{1-\hat{\rho}}$$

Figure 5 reports an example where the conditions of Proposition 2 are met.[24] As showed by the red dashed line in the bottom center panel, agents interpret the supply shock as a demand shock. In turn, their mis-perception generates effects on aggregate variables that are akin to a demand shock, i.e., a decrease in both inflation and output. This response is driven by the change in inflation expectations reported in red dashed line in the center top panel. More specifically, as agents misinterpret the negative supply shock as a persistent negative demand shock, they expect a drop in future inflation. If this belief is strong enough, it can offset the direct inflationary effect of the negative supply shock.

It is worth pointing out that, although the conditions in the proposition assume that agents infer the sign and the size of the demand shock by observing output, this assumption is not crucial for the result. In particular, while it is true that the sign of the shock would be reversed if LA agents were to observe inflation instead of output, the model is still capable of producing demand-like movements in the aggregate variables. In this alternative case, wrong output expectations would be key for the co-movement. What is more, under some conditions, this possibility arises even when LA agents infer the realized shock from both variables. Indeed, while observing from output or inflation matters for the sign and the size of the inferred demand shock, the supply shock always generates a same-sign movement in output and inflation expectations.[25]

[24] In Figure 15 in Appendix E, we instead report the case with a low perceived persistence where a supply shock generates the "usual" response: an increase in inflation and a reduction in output.

[25] Of course, the strength of the perceived demand shock is key to overcome the direct effect of the supply shock. For example, if an agent observing both output and inflation were to infer a negligible demand shock, her expectations would change only marginally, allowing the direct effect to prevail. This would cause the movements in aggregate variables to resemble those resulting from a temporary supply shock.

In addition, note that even when such extreme result does not occur the model predicts that the effect of a supply shocks tend to mitigate the inflation movements. Since a supply shock tend to move inflation and output in the opposite direction indeed, the mis-perceived inflation moves expectations in the opposite direction compared to the one generated by the shock, mitigates the effects on the realized inflation. Although also the effect on output is mitigated compared to RE, one can show that for a given volatility of supply shocks a supply shock the model generates a ratio between inflation and income volatility that is smaller than under RE.

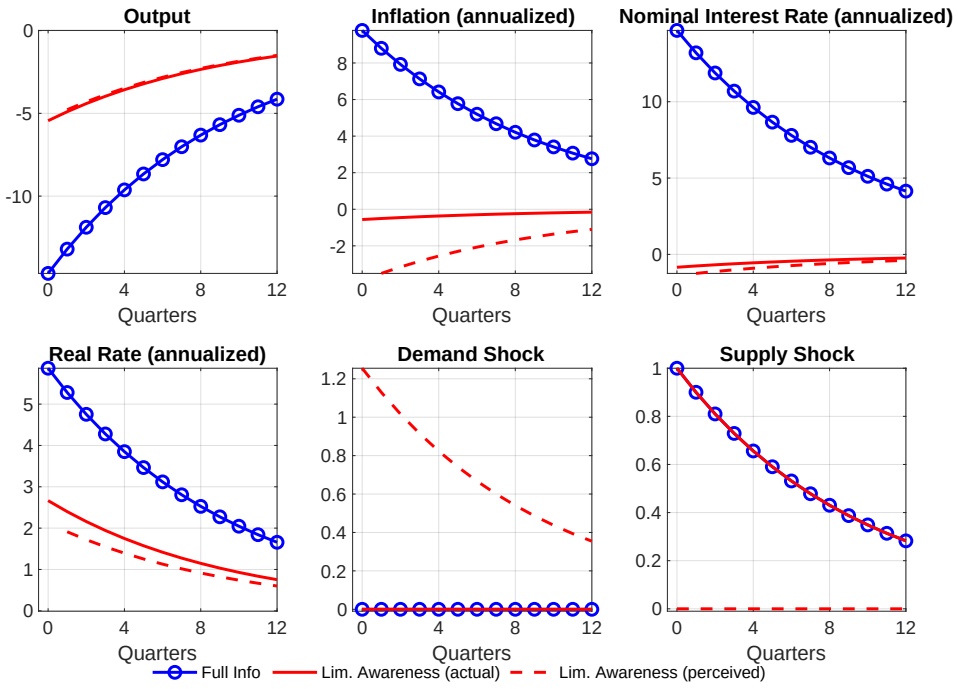


Figure 5: **A supply shock generating effects on inflation and output that look like a demand shock.** The figure reports the effects of a negative supply shock. The blue line reports the effect under RE while the solid red line reports the effects under LA. As it can be seen, a supply shock generates effects on aggregate variables that look qualitatively like a demand shock, namely a decrease in both inflation and input. This effect is driven by the movements of the expectations regarding inflation, reported in red dashed line in the center top panel. The parameters used to draw the figure are: $\kappa = .1$, $\hat{\kappa} = 0.07$; $\beta = .99$; $\sigma = 1$; $\phi_\pi = 1.5$; $\hat{\rho} = \rho_s = .9$

6 Implications for Monetary Policy

In this section we study the implications for monetary policy of our model. In anticipation of what we derive below, we can summarize this section as indicating *three reasons for why in presence of LA about supply shocks the CB should go softer (compared to RE) on inflation control*.

Reason 1. LA implies higher cost of inflation control. Proposition 1 and the simulations showed that, when considering monetary policy shocks (under the plausible assumption $\phi\beta > 1$), the model generates large variations in output and relatively smaller variations in inflation. To generate a similar result in the RE benchmark, one would need to assume either a flat Phillips curve or an aggressive monetary policy stance. Instead, in our LA model, these results follow naturally from the small response in inflation expectations.

The dual side of the coin is that—from a more normative standpoint—the presence of LA agents increases the output cost of a monetary policy aimed at reducing inflation. In Figure 6, we report the average cost of reducing inflation—measured as the (sacrifice) ratio between the cumulative output loss and the cumulative change in inflation—for different levels of persistence of the monetary policy shock. The red solid line represents our model, where we have assumed the Taylor rule $i_t = \phi_\pi \pi_t + \phi_y y_t - d_t$. For comparison, the blue line represents the sacrifice ratio for the standard New Keynesian model, while the green dashed line captures the analogous ratio for a standard New Keynesian model where the true κ equals $\hat{\kappa}$, i.e., the smaller slope perceived by our LA agents. As expected, our model is in between the two benchmarks, with the largest discrepancy occurring for intermediate levels of shock persistence. Since our model differs from its RE counterpart only due to expectations, it is intuitive that for very low levels of persistence the two models give the same results. For very high levels of persistence, the sacrifice ratio of the LA model approximates that of the RE with $\hat{\kappa}$.

Reason 2. Under LA, a tighter monetary policy increases future costs of inflation control. Recall that from condition (12) the bias in the perception of the slope of the Phillips curve is a function of the covariance between output and TFP. Among other things, this covariance is a function of the strength of aggregate demand. Figure 27 reports the values of the estimated $\hat{\kappa}$, for different values of the parameter ϕ_π in the Taylor rule $i_t = \phi_\pi \pi_t + \phi_y y_t - d_t$. Larger values for ϕ_π indicate a more aggressive monetary policy in terms of inflation targeting. Although we did not solve for the dynamic learning problem in the transition between monetary policy regimes, the figure suggests that, to the well-known short-term output costs associated to switching to a stricter monetary policy, one might have to add a novel long-term cost: the increased future costs of reducing inflation caused by a lower estimated/perceived slope of the Phillips curve.

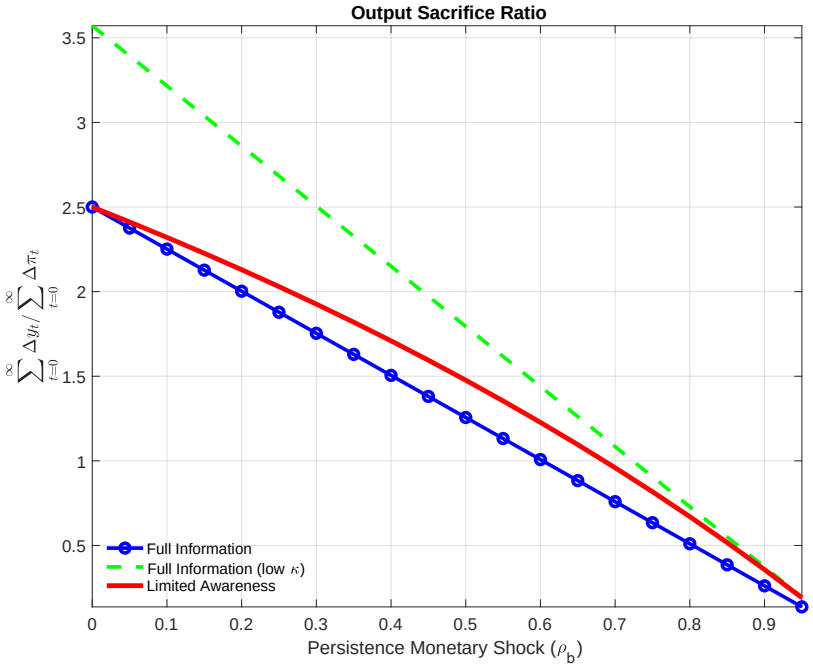


Figure 6: **Output sacrifice ratio.** The figure reports the ratio between the cumulative output loss and the cumulative change in inflation, for different levels of persistence of the monetary shock (ρ_d). The blue line reports the values for the standard New Keynesian model, while the red line reports the measure for the model with limited awareness. For comparison, the figure also reports the results from a standard New Keynesian model where the true κ equals the $\hat{\kappa}$ of our LA model. The simulations assume the following monetary rule: $i_t = \phi_\pi \pi_t + \phi_y y_t - d_t$. The parameters are: $\kappa = .1$, $\hat{\kappa} = .2$, $\beta = .995$, $\sigma = 1$, $\phi_\pi = 1.5$, $\phi_y = .125$, $\rho_s = .5$, $\sigma_d^2 = \frac{.0062}{1-\rho_d^2}$, $\sigma_s^2 = .002$.

Reason 3. Supply Shocks have lower effect on inflation than on output The third reason is more quantitative. A corollary of Proposition 2 is that supply shocks generate lower volatility in output and inflation compared to RE. In Figures 8 and 9 we report two views of the implies relative variance between inflation and output in presence of both supply and demand shocks. As we can see, the quantitative effect of supply shocks dominate that of demand shocks. In addition, not only inflation volatility under LA is lower than that of RE in all cases, the divergence between the two models increases with the volatility of supply shocks.

6.1 Over-reaction in expectations

As discussed in Section 2, our model tends to generates overreaction to economic expectations as long as persistence parameters between demand and supply shocks differ. The following proposition summarized the result.

Proposition 4 (Over-reaction). *(i) If $\rho_s = \rho_d$ we do not have over-reaction in neither inflation nor output. (ii) when $\rho_s \neq \rho_d$ we always have overreaction in output expectations while a sufficient condition to have*

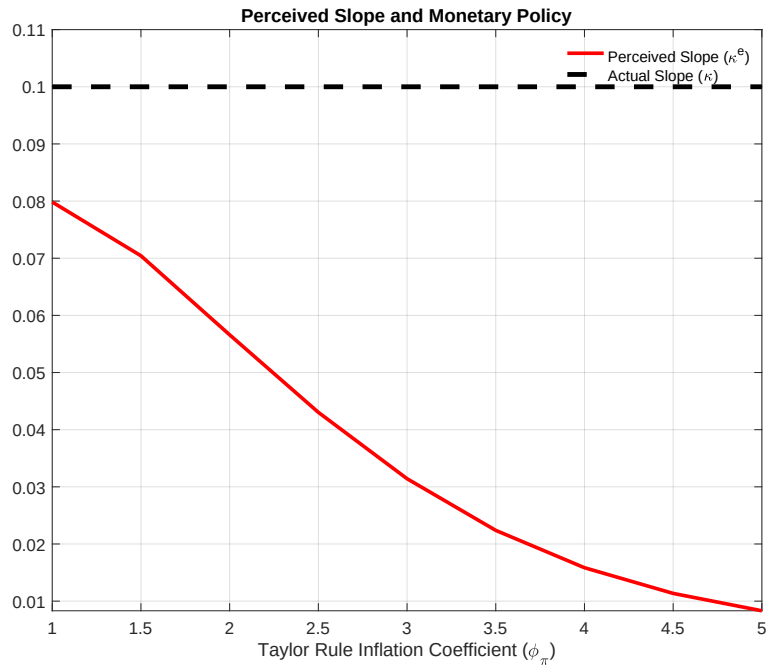


Figure 7: **Perceived slope and monetary policy.** The figure reports (in solid red) the values of the estimated $\hat{\kappa}$ for different values of the parameter ϕ_π in the rule $i_t = \phi_\pi \pi_t + \phi_y y_t - d_t$. Larger values for ϕ_π indicate a more aggressive monetary policy in terms of inflation targeting. In all cases, the model is simulated keeping the true parameter in the Phillips curve set at $\kappa = 0.1$ and considers fixed volatility and persistence of both demand and supply shocks. The parameters used to draw the figure are: $\kappa = .1$, $\beta = .995$; $\sigma = 1$; $\phi_y = 0$; $\rho_d = \rho_s = .5$; $\sigma_d^2 = .0124$; $\sigma_s^2 = .002$

over-reaction in inflation expectations is

$$\frac{\sigma_d^2}{\phi^2 \sigma_s^2} > \frac{\rho_d(1-\rho_s)}{(1-\rho_d)(1-\beta\rho_d)}.$$

The last condition roughly requires that demand shocks have sufficiently low persistence compared to supply shocks. The force generating over-reaction in output expectations has been discussed in Section 2, and directly applies to the fully specified model. Why do we need an extra condition for inflation? Of course, the same force also operates when we consider inflation expectations. At the same time, this effect gets attenuated by the fact that inflation movements are mis-perceived due to bias in the perceived slope of the Phillips curve ($\hat{\kappa} < \kappa$). The condition displayed above implies that demand shocks generate over-reaction in inflation expectations. This suffices since - as shown in the proof of the proposition - supply shocks always generate a negative co-movement between expectation revisions and forecast errors in inflation.

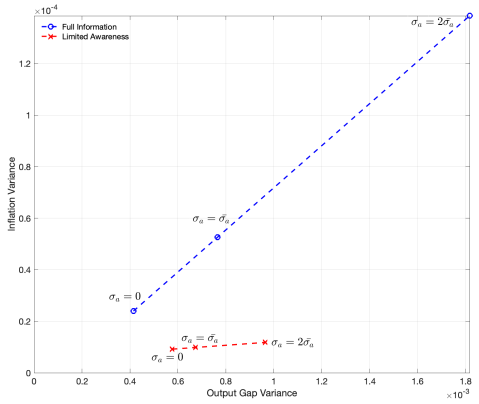


Figure 8: Relative volatility of inflation and output for different volatility levels of the supply shocks given a calibrated volatility of the demand shock. The parameters used in the simulations are: $\phi_\pi = 1.5$, $\phi_y = 0$, $\sigma = 1$, $\beta = .99$ $\rho = .8$, $\kappa = .05$, $\hat{\kappa} = .02$, $\sigma_d = .0045$. The value $\bar{\sigma}_s$ generates the estimated bias in $\hat{\kappa}$

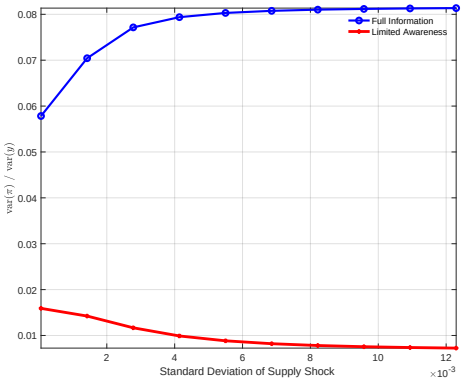


Figure 9: Relative volatility of inflation and output for different volatility levels of the supply shocks given a calibrated volatility of the demand shock. The parameters used in the simulations are: $\phi_\pi = 1.5$, $\phi_y = 0$, $\sigma = 1$, $\beta = .99$ $\rho = .8$, $\kappa = .05$, $\hat{\kappa} = .02$, $\sigma_d = .0045$. The value $\bar{\sigma}_s$ generates the estimated bias in $\hat{\kappa}$

7 A quantitative SVAR Analysis

In this section, our target is to investigate the presence and quantitative significance of mis-perceived supply shocks in the US economy. We consider a reduced-form VAR(p):

$$\boldsymbol{y_t} = \boldsymbol{B x_t} + \boldsymbol{u_t} \quad \text{for} \quad t \in \{1, ..., T\}, \tag{13}$$

where $\boldsymbol{B} = \boldsymbol{A A_+}$, $\boldsymbol{u_t} = \boldsymbol{A \varepsilon_t}$, $\mathbb{E}(\boldsymbol{u_t}) = \boldsymbol{0}$ and $\mathbb{E}(\boldsymbol{u_t u_t'}) = \boldsymbol{\Sigma} = \boldsymbol{A A'}$. We estimated the reduced-form parameter matrices $\boldsymbol{B}$ and $\boldsymbol{\Sigma}$ using a Bayesian approach, based on the Jeffrey prior. Under this uninformative prior, the posterior distribution of the parameters coincides with the (frequentist) likelihood up to a normalization constant, implying that the posterior moments are fully data-driven. We implemented it using the "Empirical Macro Toolbox" by F. Ferroni and F. Canova (Link). The estimation proceeds via a Gibbs sampling algorithm.

We estimate a BVAR(4) model including constants and a linear time-trend using 1.000 draws. Regarding the employed identification strategy, we apply the sign restrictions displayed in Table 4. Restrictions are imposed on the first four quarters.

As common in the literature, we add the excess bond premium (EBP) as a financial indicator to the vector of endogenous variables. As shown by Gilchrist and Zakrajsek (2012), the EBP has strong forecasting ability for economic activity, outperforming other financial indicators. Thus, this particular variable offers a convenient summary of a significant portion of the information excluded from the VAR model, which could potentially have relevance to economic activity. Due

to data availability of the EBP, the VAR in this section is estimated for the period from 1973:Q2 until 2019:Q4. As well, in the baseline specification, we smoothed inflation to get yearly values as arithmetic mean of quarterly values. Details can be found in Appendix F.

We apply the sign restrictions displayed in Table 4 where the focus is on the third column - the misperceived supply shock. The rationale is as follows: The reaction of the *actual* measure of economic activity ($-u_t$) and inflation (π_t) is corresponding to a classic supply shock (as in row one). However, expectations about the two variables are moving in the same direction as if it was a demand shock (as in row two) - highlighting the misperception/limited awareness. The restrictions imposed are for the first 4 periods, i.e. for the first year. In addition to the misperceived supply shock (now in column 3), we additionally identify a typical supply and demand shock (columnn 1 and 2) where the effect on expectations is congruent to the actual response. Again, the restrictions imposed are for the first 4 periods, i.e. for the first year. There are no restrictions imposed on the nominal interest rate or the EBP.

Perhaps it is worthwhile stressing that there is not an obvious way mis-perceived supply shocks can be confused for other shocks under rational expectations. First of all, we should not forget that the main difference with standard models is that - under rational expectations - inflation and expectations of inflation are aligned, while in our shock is identified precisely by a mis-alignment between the two. In addition to this feature, noise shocks about productivity would generate the opposite effect of our mis-perceived supply shocks: agents think that there is a productivity shock that in the end does not realize. This generates demand-like effects both on expectations and on aggregate variables. A news shock on the future productivity, on the other hand, might generate an increase in GDP (in minus unemployment) and at the same time a reduction in inflation in the short run. Of course, it is identified in a very different way and we believe that the introduction of the EBP variable should account for most of such shocks. In addition, the reason why –in the standard NK model under RE – inflation is low today after a news shock is because firms expect low future marginal costs and hence expect low future inflation, while a mis-perceived supply shock is associated with high present and future inflation expectations.

7.1 IRFs, Historical Decomposition, and Forecast Error Variance Decomposition

In Figure 10 we report the impulse-response function of the misperceived (positive) supply shock. As required from the sign restriction inflation movements and expectations are going in opposite directions. In this specification, this feature holds for at least 2 years, well beyond the 4 quarters we imposed. While the EBP does not have any particular pattern, the nominal interest rate increases.

In Figure 11 we report the decomposition of the variance of the forecast error of the different variables over different horizons. On average, mis-specified supply shocks (reported in red) ex-

plain over 20% of the variation in unemployment and expected unemployment while they explain less than 10% of the variation in inflation and associated expectation. If we exclude very few initial periods of inflation, the contribution of mis-perceived supply shocks seems quite stable over horizons.

Figure 12 offers an example of the contribution of the different shocks to movements of unemployment over the time. Positive or negative bars indicate chenges with respect to the previous month. As it can be seem from the figure, mis-perceived supply shocks co-existed with both demand and supply shocks. Perhaps an interesting period where mis-perceived supply shpocks played an important role is between the late 80s and all the 90s. Another intersting period is the pre-covid. In this period, it seems that not all positive supply shocks were fully perceived. Would have been perhaps the coming AI revolution? In Figure 17 in Appendix F we report the historical decomposition of all endogenous variables.

In the appendix we also perform a few robustness checks that include the use of Quarter-to-Quarter inflation as opposed to the smoother yearly version. In addition, we performed the same exercise where the EBP is replaced by the Michigan Survey of Consumers measure of sentiment. For sake of conciseness, we do not report the decomposition for those cases (but of course they are available upon request).

Variable (row) / Shock (column)	Supply shock	Demand shock	Misperceived supply shock	Other shocks
$-u_t$	+	+	+	?
π_t	-	+	-	?
$\mathbb{E}_t[-u_{t+1}]$	+	+	+	?
$\mathbb{E}_t[\pi_{t+1}]$	-	+	+	?
i_t	?	?	?	?
EBP	?	?	?	?

Table 4: BASELINE SIGN RESTRICTIONS *Notes*: A '**-**' means that the response is negative, a '**+**' means that it is restricted to be positive. A '**?**' means that the impact response is unrestricted.

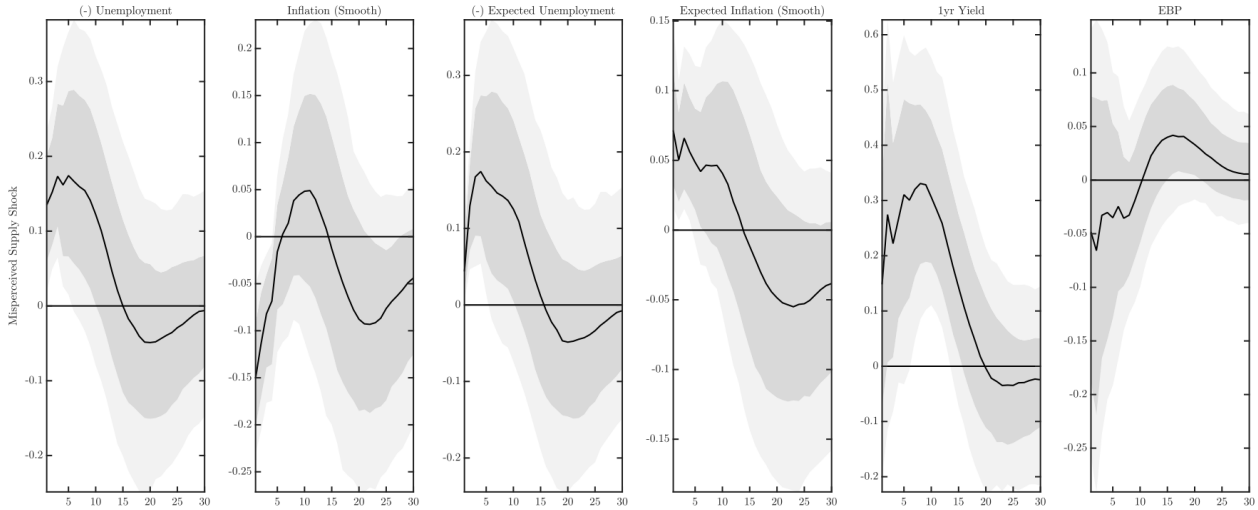


Figure 10: **IRF to Misperceived Supply Shock**: The figure reports the impulse-response function of the misperceived (positive) supply shock. As required from the sign restriction inflation movements and expectations are going in opposite directions. In this specification, this feature holds for at least 2 years, well beyond the 4 quarters we imposed. While the EBP does not have any particular pattern, the nominal interest rate increases. Note: 68% (90%) confidence intervals in dark (light) grey.

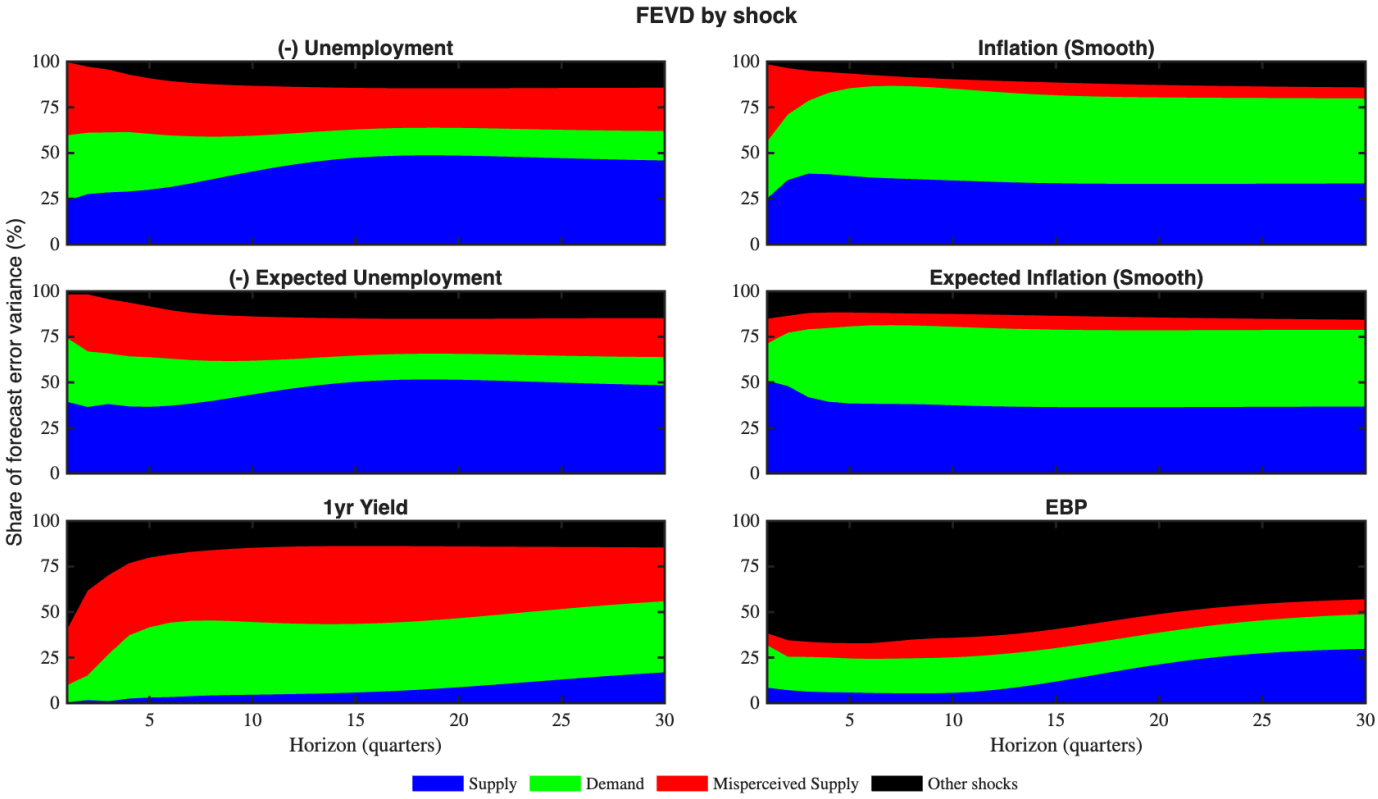


Figure 11: **Forecast Errors Variance Decomposition**: The figure reports the decomposition of the variance of the forecast error of the different variables over different horizons. The decomposition refers to the specification with smooth inflation.

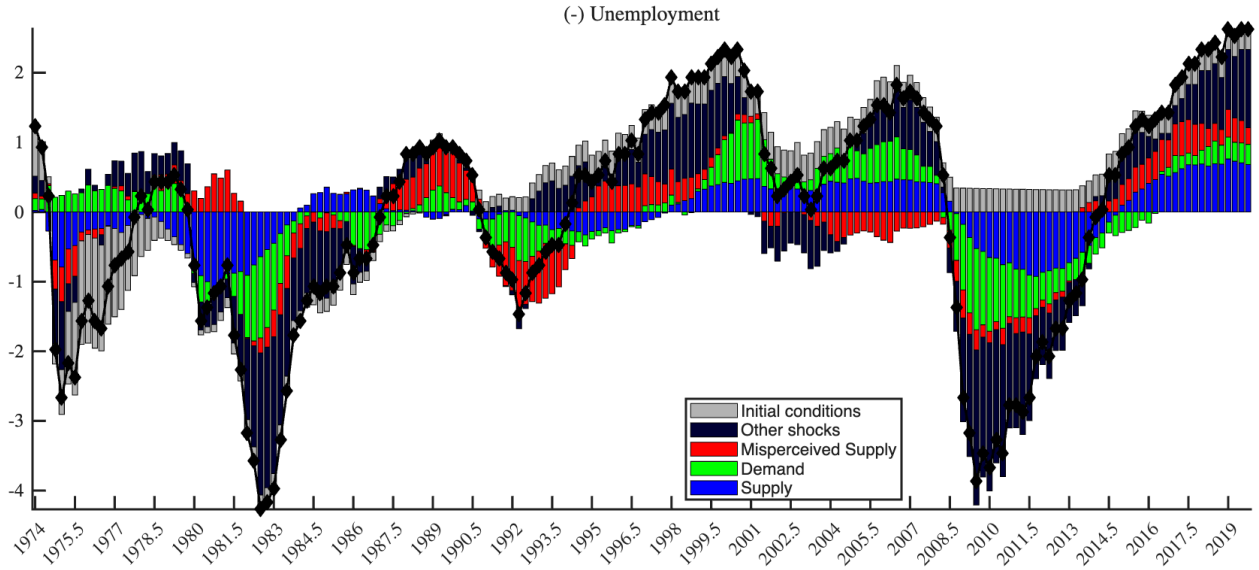


Figure 12: Historical Decomposition: -Unemployment: The time series of the respective endogenous variable is demeaned and represented by the black dots. A 'positive' bar signals an increase compared to the previous month.

8 Concluding Remarks

Do agents understand the determinants of the trade-off between output and inflation? What are the sources of their possible 'mistakes'? What are the implications for monetary policy? We have presented novel empirical evidence suggesting that agents have mis-perceived relationships about the determinants of the aggregate variables. A model with limited awareness can parsimoniously and coherently explain this evidence. In our model, agents are unaware about certain supply shocks. As a result, not only their expectations display over-reaction in the spirit of Bordalo et al. (2020), they also perceive the Phillips curve as being flatter. This occurs because, when estimating the Phillips curve parameters, agents make an omitted variable error by overlooking the existence for supply shocks. Consequently, this leads to a greater cost associated with reducing inflation. Moreover, maintaining strict inflation targeting over an extended period could exacerbate the problem.

While in presence of demand shocks LA commit quantitative mistakes but correctly get the 'nature' of the shock, the reading of supply shocks is more problematic. Within their mis-specified model, LA agents rationalize the observed movements in inflation and output generated by a supply shock by estimating a demand shock. This in particular make expectations about future inflation and output to move in the same direction. An interesting finding is that - under reasonable parameters of the model - if the shocks have enough persistence the effect of expectations is strong enough to make the supply shock displaying qualitative features similar to a demand shock, namely making output and inflation to move in the same direction.

Our setting opens the door to non-trivial implications for forward guidance and for policy communication in general, which we leave to future research. For instance, when agents observe variations in the nominal interest rate that contain information about the future state of the economy, our model generates a rich set of biased expectations and, therefore, non-trivial predictions regarding the signaling role of monetary or fiscal policy. More broadly, the theory we have presented offers detailed predictions on how beliefs, preferences, and economic decisions change in counterfactual scenarios, including when there are shifts in awareness levels. This aspect is crucial for a detailed examination of the impact of macroeconomic policies related to awareness.

By introducing LA into macroeconomic models we gain a final lesson that is not shared by other models proposed in the literature, such as k-level and Gabaix, or models of rational inattention. As remarked by Angeletos and Liam (2022), the main common effect is an attenuation of the GE and itertemporal effects because they reduce the coordination effect generated by common knowledge. The qualitative effects are hence similar to incomplete information under rational expectations. Proposition 1 suggests that LA (and in general mis-specification) can at the same time attenuate the effect of one shock and amplify the effects of another shock on different macro variables. This insight can obviously be applied in other contexts.

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A Appendix A.2: Derivation of the Bias under Moment Matching

In this appendix we derive the bias in the Phillips multiplier when agents estimate parameters by matching theoretical moments of their perceived DGP to empirical counterparts. This approach is consistent with the literature on marginalization over unmodeled dimensions (e.g., Jehiel, 2005, Spiegler, 2016). In this case, since the perceived DGP has a lower dimension than the true DGP, estimation through moment-matching inherently results in an overidentified system. There are more empirical moments to be matched than available parameters, and thus the parameter estimates are highly sensitive to the chosen moments.

As an illustrative example, let's assume that agents seek to match the perceived contemporaneous and first-order covariance matrices to their empirical counterparts.[26] In our simple model, the corresponding moment conditions are given by:

$$\begin{bmatrix} \text{Var}(y_t) & \text{Cov}(y_t, \pi_t) \\ \text{Cov}(\pi_t, y_t) & \text{Var}(\pi_t) \end{bmatrix} = \tilde{\sigma} d^2 \begin{bmatrix} \tilde{a}_y^2 & \tilde{a}_y \tilde{a}_\pi \\ \tilde{a}_y \tilde{a}_\pi & \tilde{a}_\pi^2 \end{bmatrix} + \begin{bmatrix} \tilde{\sigma}_{v,y}^2 & 0 \\ 0 & \tilde{\sigma}_{v,\pi}^2 \end{bmatrix} \tag{14}$$

$$\begin{bmatrix} \text{Cov}(y_t, y_{t-1}) & \text{Cov}(y_t, \pi_{t-1}) \\ \text{Cov}(\pi_t, y_{t-1}) & \text{Cov}(\pi_t, \pi_{t-1}) \end{bmatrix} = \tilde{\rho}_d \tilde{\sigma}_d^2 \begin{bmatrix} \tilde{a}_y^2 & \tilde{a}_y \tilde{a}_\pi \\ \tilde{a}_y \tilde{a}_\pi & \tilde{a}_\pi^2 \end{bmatrix} \tag{15}$$

From eqs.14 and 15, there are 7 empirical moments available but only 6 parameters to estimate Consequently, estimating these parameters requires a choice regarding which specific moments to target.

If only inflation is observed with error moments matching would be equivalent to minimizing the KLDR (and thus OLS estimation) as described in the main text. Instead, if both inflation and output are subject to measurement error, the OLS procedure would instead suffer from standard attenuation bias, since the regressor (output) observed with error. To circumvent this, a plausible estimation strategy—which would deliver a consistent estimate if the perceived DGP were correct—is to rely on the first-order autocovariance matrix (condition 15). Retaining the assumption that agents project inflation onto output, this strategy is equivalent to an Instrumental Variable (IV) approach where lagged output is used as instrument for current output. The resulting estimator is $\tilde{\kappa} = \tilde{\kappa}_{IV} \equiv \text{Cov}(\pi_t, y_{t-1})/\text{Cov}(y_t, y_{t-1})$, yielding:

$$\tilde{\kappa}_{IV} = \frac{a_{y,d} a_{\pi,d} \rho_d \sigma_d^2 - a_{y,s} a_{\pi,s} \rho_s \sigma_s^2}{a_{y,d}^2 \rho_d \sigma_d^2 + a_{y,s}^2 \rho_s \sigma_s^2} < a_{\pi,d}/a_{y,d} \equiv \kappa$$

As in the case illustrated in the main text the perceived Phillips multiplier is downward biased,

[26] According to the perceived DGP, including additional moments would be redundant.

while the exact magnitude of the bias would here depend on the relative persistence (ρ_d vs. ρ_s) of the true underlying shocks.

Yet, since the system is inherently over-identified, there would be other valid procedures from the viewpoint of limited aware agents. For instance, from condition (15), the Phillips multiplier could also be obtained as $\tilde{\kappa} = \tilde{\kappa}_{IV,alt} = \text{Cov}(\pi_t, \pi_{t-1})/\text{Cov}(\pi_t, y_{t-1})$ which yields

$$\tilde{\kappa}_{IV,alt} = \frac{a_{\pi,d}^2 \rho_d \sigma_d^2 + a_{\pi,s}^2 \rho_s \sigma_s^2}{a_{y,d} a_{\pi,d} \rho_d \sigma_d^2 - a_{y,s} a_{\pi,s} \rho_s \sigma_s^2} \gtrless \kappa$$

In this case, the sign of the bias would be ambiguous. Interestingly, if $\text{Cov}(\pi_t, y_{t-1}) > 0$, then this alternative choice of moments leads to an upward bias in the perceived Phillips multiplier.

Appendix A.2: Derivation of the Bias under KLDR Minimization

Analogously to the derivations in Section 2.3, this appendix formally derives the bias in the perceived Phillips multiplier when limited aware agents minimize the Kullback-Leibler Divergence Rate (KLDR) between their misspecified model and the true data. To obtain a closed-form analytical solution, we assume the agents restrict their perceived measurement errors to have the same variance, i.e., $\tilde{\mathbf{\Sigma}}_\nu = \tilde{\sigma}_\nu^2 \mathbf{I}$. Under the perceived model, the unconditional covariance matrix is given by $\tilde{\Gamma}_0 = \tilde{\mathbf{A}}\tilde{\mathbf{A}}' + \tilde{\sigma}_\nu^2 \mathbf{I}$.

In our linear setting with Gaussian shocks, minimizing the static KLDR between the true distribution $\mathcal{N}(\mathbf{0}, \Gamma_0)$ and the perceived distribution $\mathcal{N}(\mathbf{0}, \tilde{\Gamma}_0)$ reduces to minimizing the negative log-likelihood objective function with respect to the perceived parameters:

$$\mathcal{L}(\tilde{\mathbf{A}}, \tilde{\sigma}_\nu^2) = \text{tr}(\tilde{\Gamma}_0^{-1} \Gamma_0) + \ln |\tilde{\Gamma}_0|$$

A well-known result in multivariate statistics establishes that the optimal loading vector $\tilde{\mathbf{A}}$ that minimizes this objective must be proportional to the principal eigenvector of the true covariance matrix Γ_0. Therefore, the perceived Phillips multiplier, $\tilde{\kappa}_{KL} \equiv \tilde{a}_\pi / \tilde{a}_y$, corresponds to the slope of this principal eigenvector.

By definition, the slope of the principal eigenvector $\tilde{\kappa}_{KL}$ must be the solution of the quadratic polynomial

$$P(x) = Cx^2 + (V_y - V_\pi)x - C = 0$$

where V_y, V_π and C denote the variance of output, inflation and their covariance, respectively. Assuming a positive covariance $C > 0$, then $\tilde{\kappa}_{KL}$ corresponds to the positive root of the polyno-

mial. Then, to determine the sign of the bias, notice that the true slope $\kappa = a_{\pi,d}/a_{y,d}$ lies to the right of the root—implying $\tilde{\kappa}_{KL} < \kappa$—if and only if $P(\kappa) > 0$.

Evaluating $P(\kappa)$ using the true analytical moments gives

$$P(\kappa) = \sigma_s^2 a_{y,s}^2 (\kappa - \gamma)(1 + \gamma\kappa)$$

where $\gamma \equiv -a_{\pi,s}/a_{y,s}$ denotes the slope of the "demand" curve in the true DGP.

Assuming standard macroeconomic slopes ($\kappa > 0$ and $\gamma < 0$), the first term $(\kappa - \gamma)$ is strictly positive. Therefore, we have

$$P(\kappa) > 0 \iff 1 + \gamma\kappa > 0 \iff \gamma > -\frac{1}{\kappa}$$

In summary, we can conclude that under a (simplified) KLDR minimization, $\tilde{\kappa}_{KL} < \kappa$ if and only if $\gamma > -1/\kappa$.

B The Island Model

It is well known that - in order to maintain a non-trivial problem of inference in macro models - one need to add some 'noise' (e.g., Lorenzoni, 2009 and Angeletos and Liam, 2018). One possibility is to assume that while the agent's choices are based on the aggregate variable, when thinking about the prices, the agent only remember a *noisy version* of it. The model we presented in the main text can literally be interpreted like this. All variables but the nominal interest rate are remembered with noise when updating, so only the nominal interest rate is used to recover the level of the demand shock.

Alternatively, we could assume that - due to the structure of the economy - agents observes noisy version of the aggregate variables. To show that our main results do not depend on other auxiliary cognitive limitations, we now present a version of our model where households and firms observe noisy signals about aggregate variables.

Setup There is a continuum of households indexed by $h \in [0,1]$ and a continuum of islands indexed by $i \in [0,1]$. In each island, a continuum of firms j produce differentiated goods. The labor market is segmented across islands. We hence denote by w_t^i the equilibrium wage in island i in period t. At the same time, island i receives workers from different households: denote by $L_t^i \subset [0,1]$ the set of households working in island i in period t. In a given period, household h goes to island i to work for a firm in that island. In each period, nature picks a random basket of islands from where household h buys from all firms in the island: we call the basked of islands $B_t^h \subset [0,1]$. Let $D_t^i \subset [0,1]$ the set of households that buy goods from island i.

Household Problem Households have preferences given by

$$\hat{\mathbb{E}}_0^h \sum_{t=0}^{\infty} \beta^t \left(\frac{\chi_t^h (C_t^h)^{1-\sigma} - 1}{1-\sigma} - \xi_t^h \frac{(N_t^h)^{1+\varphi}}{1+\varphi} \right)$$

where $C_t^h = \left(\int_{B_t^h} (C_t^{hi})^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}$, $\epsilon > 1$, is a composite good and C_t^{hi} is itself a composite good produced by firms in island i: $C_t^{hi} = \left(\int_j (C_t^{hij})^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}$, where - in order to simplify notation - we assume the same elasticity across products produced by firms within the same island and across different islands. The preference shocks χ_t^h and ξ_t^h are (not perfectly) correlated across households working in the same island, independent across islands and iid over time. The mentioned correlation is required to generate (purely temporary and non-aggregate) variation in prices and wages across islands. Imperfect correlation across agents guarantees that agents have only imperfect knowledge on the island's situation.

Households trade nominal bonds in a *common market* at the interest rate $i_t = \frac{1}{Q_t} - 1$, where Q_t is the price of the bond. The budget constraint reads:

$$Q_t A_{t+1}^h + \int_{i \in B_t^h} P_t^i C_t^{hi} = A_t^h + W_t^h N_t^h + V_t^h,$$

where A_t^h are nominal bonds' holdings, P_t^i is the price of the composite good from island i, and W_t^h and V_t^h are the nominal wages and profits, respectively, where: $V_t^h := \int_{M_t^h} \int V_t^{ij} dj$, and M_t^h is the portfolio composition of the household across firm-islands.[27] In the above, we defined the island-specific price index as:

$$P_t^i := \left(\int_0^1 (p_t^{ij})^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}.$$

Similarly, we can set

$$P_t^h := \left(\int_{B_t^h} (P_t^i)^{1-\epsilon} dh \right)^{\frac{1}{1-\epsilon}},$$

and write the budget constraint as

$$Q_t A_{t+1}^h + P_t^h C_t^h = A_t^h + W_t^h N_t^h + V_t^h.$$

By definition we can name $P_t^h = P_t u_t^{ph}$, $W_t^h = W_t u_t^{wh}$ and read the prices faced by the household as noisy signals of the aggregate prices. And similarly for profits: $V_t^h = V_t u_t^{vh}$. Even though such shocks have small variance they are still unable to fully reveal the state of the aggregate economy. In addition, since such shocks are temporary they are not useful to make predictions about aggregate variables.

So, on top of their idiosyncratic shocks, and individual past bond holding and consumption bondles $(A_{t-1}^h, \xi_t^h, \chi_t^h, B_t^h)$ households observe[28] $(i_t, P_{t-1}, W_t^h, (P_t^{ij})_{i \in B_t^h}, V_t^h)$. Households do not observe the realization of shocks of the other households in L_t^i where household belongs to and that contribute generating W^h, similarly, they do not observe profits from all islands. We assume that households conjecture the following mapping of the relevant variables to them:[29]

$$\left(P_t, Y_t, W_t^h, P_t^h, V_t^h, i_t \right) = \hat{G}^c \left(P_{t-1}, d_t, \hat{\phi}_t^h \right),$$

[27] We either assume that due to some frictions household cannot fully diversify their portfolio across islands or that households do not pay much attention to the individual profits, they only look at the cumulated ones.

[28] As we will explain below, the household thinks it is able to compute P_{t-1} perfectly.

[29] The household might not need to know the supply shock occurred to the firm she is working for since the firm is small relative to the island and the wage is determined at island level.

where

$$i_t = \Phi(\hat{d}_t, Y_t, P_t, P_{t-1}) - 1.$$

In addition, A_t^h is under control to the household and the evolution of the aggregate A is trivial. In the mind of the consumer, the vector of shocks $\hat{\phi}_t^h$ is meant to capture the shocks $u_t^{kh},\ k = p, w, v$.[30] In each period, agents will rationalize what they observe though their world and will set shocks to preferences and baskets so that to compute $\hat{\phi}_t^h$ so that the observed outcome is consistent with his view of the world. Importantly, *there is no learning (Kalman filter) problem*. LA households believe that all the relevant information to forectast future aggregate variables can be obtained by the observation of the nominal interest rate and their (sophisticated) knowledge on how the economy works. First order conditions read:

$$\chi_t^h (C_t^h)^{-\sigma} = \beta(1+i_t)\hat{\mathbb{E}}_t^h \left[\frac{P_{t+1}^h}{P_t^h} \chi_{t+1}^h (C_{t+1}^h)^{-\sigma} \right]$$

$$W_t^h = P_t^h \frac{\xi_t^h (N_t^h)^{\varphi}}{\chi_t^h (C_t^h)^{-\sigma}}$$

$$C_t^{hij} = C_t^h \left(\frac{P_t^{ij}}{P_t^i} \right)^{-\epsilon} \left(\frac{P_t^i}{P_t^h} \right)^{-\epsilon}$$

Firm Problem Firms are subject to productivity shocks s_t that are thought as only occurring to the firms while they are aggregate, not only to the island, these shocks affect the whole economy. These shocks have positive persistence. On top of their individual shocks and prices, firm ij observes $(i_t, P_t^i, s_t, W_t^i, Y_t^i)$ (but not the realization of the preference shocks of the household or the aggregates in the other islands) and solve the following monopolistic competition problem. When considering other firms within the same island they act as monopolistic competitors, while - since there is a continuum of firms that take P_t^i as given and there is a continuum of islands - they act as price-takers when considering offers coming from other islands.

[30] It is not difficult to add how the agent maps fundamental shocks into the firm prices $(P_t^{ij})_{j\in[0,1], i\in B_t^h}$. The issue only complicates notation.

The demand schedule faced by firm ij is as follows:

$$\begin{aligned} Y_t^{ij}(\hat{p}) &= \int_{D_t^i} C_t^{hij}(\hat{p}) = \int_{D_t^i} C_t^h \left(\frac{\hat{p}}{P_t^i}\right)^{-\epsilon} \left(\frac{P_t^i}{P_t^h}\right)^{-\epsilon} \\ &= C_t \int_{D_t^i} \hat{\chi}_t^h \left(\frac{\hat{p}}{P_t^i}\right)^{-\epsilon} \left(\frac{P_t^i}{P_t^h}\right)^{-\epsilon} \\ &= C_t \int_{D_t^i} \hat{\chi}_t^h \left(\frac{\hat{p}}{P_t^i}\right)^{-\epsilon} \left(\frac{P_t^i}{P_t}\right)^{-\epsilon} (u_t^h)^{\epsilon} \\ &= C_t \left(\frac{\hat{p}}{P_t}\right)^{-\epsilon} \underbrace{\int_{D_t^i} \hat{\chi}_t^h (u_t^h)^{\epsilon}}_{\phi_t^i}. \end{aligned}$$

Optimal price for firm j in island i:

$$p_t^{*ij} = \frac{\epsilon}{\epsilon - 1} \frac{\mathbb{E}_t^{ij} \sum_{k=0}^{\infty} (\beta\theta)^k \frac{W_{t+k}^i}{s_{t+k}} (P_{t+k})^{\epsilon} C_{t+k} \phi_{t+k}^i}{\mathbb{E}_t^{ij} \sum_{k=0}^{\infty} (\beta\theta)^k (P_{t+k})^{\epsilon-1} C_{t+k} \phi_{t+k}^i}. \tag{16}$$

where ϕ_{t+k}^i are island specific shocks that emerge from the cumulation of the preference shocks of the buyers of the island, together with their 'basket shocks'.

Wages. Each household, offers labor according to

$$N_t^h = \left(\frac{\chi_t^h}{\xi_t^h} \frac{W_t^h}{P_t^h} (C_t^h)^{\sigma}\right)^{\frac{1}{\varphi}}$$

So, the wage in island i - and hence the marginal nominal cost of labor - depends both on the preference shocks and on the basket shocks of all workers in L_t^i.

Firms' conjecture is:

$$\left(P_t, Y_t, W_t^i, P_t^i, V_t^i, i_t\right) = \hat{G}^f \left(P_{t-1}, d_t, \hat{\phi}_t^i\right),$$

where

$$i_t = \Phi(\hat{d}_t, Y_t, P_t, P_{t-1}) - 1$$

Moreover P_t^{*ij} is under control of the firm.[31]

Market clearing

[31] Again we could have added with no effort a positive notational cost the firm's mapping for each price adopted by each other firm in the market.

- Bond market clears: $\sum_{h\in H} A_t^h = 0 \; \forall t = 0, 1, \ldots$
- Labor markets clear in each island (plus techology): $\sum_{h\in L_t^i} N_t^h = N_t^i = \sum_j \frac{y_t^{ij}}{s_t} \; \forall i \in I, \forall t$
- Good markets clear for each good: $y_t^{ij} = \int_{D_t^i} C_t^{h,ij} \; \forall j, i, t$

B.1 The limit economy

To see that the model has the potentials to deliver a standard NK model in its log-linearized version, consider the following simpification.

- Suppose there are no preference shocks: $\chi_t^h = \xi_t^h = 0 \; \forall t$.
- Suppose we start from a situation where all households hold the same assets $A_0^h = 0$.
- Suppose that the extraction B_t^h selects the same finite number M of islands for each household. And that each island is selected with equal probability. This implies that *each island have the same demand schedule if either all households are the same or their wealth differ by a zero average shock*.
- Given calvo pricing at firm level, and the fact that household buys all j goods in island $i \in B_t^h$, the distribution of prices faced by each household is the same. In particular, for each $i \in B_t^h$ P_t^i is the same across households.
- Similarly, we assume that portfolio allocations are iid and fully balanced across households: that is, in each period each household holds shares from a finite number M of islands. Or all shares from one island.
- Note that, since households and firms think that s_t is firm-island specific, they do not use s_t to guess aggregate movements if all effects wash out. At the same time, *households and firms all hold identical beliefs about future aggregates and this is common knowledge*

Claim: Under the previous assumptions, all households make the same decisions and hold the same level of assets. Similarly, apart from Calvo pricing, firms in each island behave the same across islands.

As well, each households face the same wage $W_t^i = W_t$, this is so since s_t is the same across islands and - as we will argue next - all islands are identical. All households face the same price P_t^h, this is so since all islands 'charge' the same price index P_t^i (despite products are differentiated, as they enter symmetrically into the utility functions). So households choose the same consumption bundle C_t^h and the same labor N_t^h.

Note that this model implies that households see that the are charged the same price across the M islands they visit. This is a zero probability event in this economy. Positive although small preference shocks allow to manipulate this probability. This is the role of preference shocks in our economy. There is a potential cost of doing it and it is that we might have a non-degenerate distribution of assets across households. Such issue however, disappears in the limit and - especially with log-linearizations. We can hence consider our economy as the approximation of the true one (as in e.g.. Angeletos and Liam, 2018).

Compared to the original model, we created a segmented labor market and prevented households and firms to se the whole distribution of prices. In fact, by symmetry, they do observe the correct price level but they think they are only observing a signal of it. At the same time, they are able to think of Market clearing. They do it assuming that in aggregate $s_t = 1$. They also know that all agents have the same mapping G over aggregate variables.

B.2 Log-linearization and aggregation (sketched)

Let write household foc and demand schedules in log-linearized forms

$$c_t^h = -\frac{1}{\sigma}\left(i_t - \rho - \hat{\mathbb{E}}_t^h[p_{t+1}^h - p_t^h]\right) + \hat{\mathbb{E}}_t^h[c_{t+1}^h]$$

$$w_t^h = p_t^h + \varphi n_t^h + \sigma c_t^h$$

$$c_t^{hij} = c_t^h - \epsilon(p_t^{ij} - p_t^h).$$

The per-period demand schedule faced by firm ij is as follows:

$$y_t^{ij}(\hat{p}) \quad = \quad c_t - \epsilon(\ln \hat{p} - p_t) + \phi_t^i$$

where we accounted for the island specific demand induces by the combination of the basket B_t^h shocks and the preference shocks of the consumers generating the demand, that is $h \in D_t^i$. Importantly, even though TFP shocks occur at firm level they are all the same (although misunderstood)

The optimal pricing is also island specific. This is so since the labor market is segmented at island level, so the wage differ across islands because demand of labor differ and we have labor preference shocks. They affect the wage via the household in the set L_t^i.

$$p_t^{*i} = \mu + (1 - \beta\theta)\sum_n (\theta\beta)^n \hat{\mathbb{E}}_t^i[w_{t+n}^i - s_{t+n}].$$

Then using the household optimality conditions, labor market clearing and the production function we have:

$$w_t^i = w_t^i + \varphi(y_t^i - s_t) - \varphi \int_{L_t^i} n_t^h dh = \varphi(y_t^i - s_t) + \int_{L_t^i} (p_t^h + \sigma c_t^h) dh.$$

And hence:

$$p_t^{*i} = \mu + (1 - \beta\theta) \sum_n (\theta\beta)^n \hat{\mathbb{E}}_t^i [\varphi(y_{t+n}^i - s_{t+n}) + \int_{L_{t+n}^i} (p_{t+n}^h + \sigma c_{t+n}^h) dh - s_{t+n}]$$

It is important to note that all heterogeneities across households are iid and hence not affecting future expectations. In particular, (i) the heterogeneity in y_t^i is perceived as temporary and due to the demand shocks at island level. Similarly, despite today's realizations (ii) each firm in island i not only holds the same expectations as any other firm in the same island, it also holds the same expectations as any firm in other islands. We can hence write

$$p_t^{*i} = \mu + (1 - \beta\theta) \sum_n (\theta\beta)^n \hat{\mathbb{E}}_t [\varphi(y_{t+n}^i - s_{t+n}) + \int_{L^i} (p_{t+n}^h + \sigma c_{t+n}^h) dh - s_{t+n}],$$

where our assumptions imply that $y_{t+n}^i = y_{t+n} + \eta_{t+n}^{i0}$, $p_{t+n}^h = p_{t+n} + \eta_{t+n}^{h1}$ and $c_{t+n}^h = c_t + \eta_{t+n}^{h2}$.

Note that we might have some correlation between y_t^i and the object $\int_{L_{t+n}^i} (p_{t+n}^h + \sigma c_{t+n}^h) dh$. Such correlation however, becomes negligible as the volatility of the shocks goes to zero. Finally, we can integrate over i and get the aggregate Philips curve:

$$\begin{aligned} p_t^* &= \mu + (1 - \beta\theta) \sum_n \hat{\mathbb{E}}_t [\varphi(y_{t+n} - s_{t+n}) + (p_{t+n} + \sigma c_{t+n}) - s_{t+n}] \\ &= \mu + (1 - \beta\theta) \sum_n \hat{\mathbb{E}}_t [\varphi(y_{t+n} - s_{t+n}) + (p_{t+n} + \sigma y_{t+n}) - s_{t+n}] \end{aligned}$$

where in the last row we used the consumption market clearing. Similarly, following Angeletos and Liam (2018), we can log-linearize the permanent income version of the Euler equation, integrate across households using the fact that all households hold the same expectations over the future realization of both aggregate and individual shocks and the zero net supply of bonds to obtain the standard Euler equation in aggregate terms. Again, each contemporaneous correlation between the household idiosincratic variables becomes negligible as the volatility of the shocks goes to zero.

C Proofs

C.1 Proof of Proposition 1

Proof. If we take as given the expectations of the agents, our model of the economy solves:

$$y_t = \tilde{\mathbb{E}}_t \{y_{t+1}\} - \sigma^{-1} \left[\phi_\pi \pi_t + \phi_y y_t - \tilde{\mathbb{E}}_t \{\pi_{t+1}\} - d_t\right] \quad \text{(IS)}$$

$$\pi_t = \beta \tilde{\mathbb{E}}_t \{\pi_{t+1}\} + \kappa y_t + u_t \quad \text{(PC)}$$

If we assume $u_t = 0$, and use the notation $\tilde{\mathbb{E}}_t[x_{t+n}] := x^e_{t+n}$, we have - after rearranging terms so that to isolate expectations and exogenous shocks - we obtain the process of the economy *for given expectations*

$$\begin{aligned} y_t &= \frac{1}{1+\kappa\frac{\phi_\pi}{\sigma}+\frac{\phi_y}{\sigma}} \left[y^e_{t+1} + \frac{1-\beta\phi_\pi}{\sigma}\pi^e_{t+1} + \frac{d_t}{\sigma} \right] \\ \pi_t &= \frac{1}{1+\kappa\frac{\phi_\pi}{\sigma}+\frac{\phi_y}{\sigma}} \left[\left(\beta\left(1+\frac{\phi_y}{\sigma}\right) + \frac{\kappa}{\sigma} \right) \pi^e_{t+1} + \kappa y^e_{t+1} + \frac{\kappa}{\sigma} d_t \right] \end{aligned} \quad (17)$$

Remark. Note that inflation is increased by inflation expectations for two reasons. One is a direct effect (β) while the second is an effect due to income ($\frac{\kappa}{\sigma}$) as higher inflation expectations reduce expected real interest rate. Output is affected by inflation expectations in a sort ambiguous manner. In particular, a very agressive monetary policy ($\phi > \beta^{-1}$) makes inflation expectations to reduce output. This is so precisely since high inflation expectations unambiguously increase inflation. They do so in proportion to β but then if ϕ is large, they have an effect of increasing the nominal interest rate more than one (which is how much the perceived real interest rate decreases for a unit increase in inflation expectations).

Note that in the case of a d_t shock, the agent rightly interprets the shock as coming from the monetary authority. This is so since the shock is exactly a demand shock. If we let the perceived persistence of the monetary shock to be $\hat{\rho}$ and $\hat{d}_t$ is the perceived/estimated value for d_t, we have $\hat{\mathbb{E}}_t[\hat{d}_{t+1}] = \hat{\rho}\hat{d}_t$.

How agents estimate $\hat{d}_t$? Recall, the agent uses observed variables to infer d_t. Since the only variable observed with no error is the nominal interest rate, the agent uses the IS curve. Importantly however, since inflation and output are observed with noise, the agent must also guess what was the choice of the monetary authority, and his guess is:

$$\hat{i}_t = \phi_\pi(\beta\pi^e_{t+1} + \hat{\kappa}\hat{y}_t) + \phi_y\hat{y}_t - \hat{d}_t$$

But this must coincide with what the agent observes, that is: $\hat{i}_t = i_t$. Then:

$$\phi_\pi(\beta\pi^e_{t+1} + \hat{\kappa}\hat{y}_t) + \phi_y\hat{y}_t - \hat{d}_t = \phi_\pi(\beta\pi^e_{t+1} + \kappa y_t) + \phi_y y_t - d_t$$

and so note that from the IS, since both future expectations on inflation and output and the nominal interest rate must coincide with the truth, we must have $\hat{y}_t = y_t$. That is, the estimated value of the monetary shock must be such that *the LA estimation of the present value of aggregate output is correct*. When the persistence parameter is biased, the expectation of future output can be wrong on average, nevertheless, the LA guesses

the correct level of present output. From now onwards we can hence use $\hat{y}_t = y_t$. This also implies that:

$$\hat{d}_t = d_t - (\kappa - \hat{\kappa})\phi_\pi y_t$$

Inverting the IS and considering $y^e_{t+1} = \hat{\rho} y_t$, we get the following estimate for $\hat{d}_t$:

$$\hat{d}_t = (\sigma(1-\hat{\rho}) + \kappa\phi_\pi + \phi_y)y_t + (\phi_\pi\beta - 1)\pi^e_{t+1}.$$

Note that the true persistence of the shock plays no role in determining the impact of the shock in t. What matters is the perceived persistence. So, for notational simplicity, from now on we set $\rho = \hat{\rho}$.

The agents form expectations by solving the following system:

$$\begin{aligned} y^e_{t+1} &= y^e_{t+2} - \frac{1}{\sigma}(\phi_\pi \pi^e_{t+1} + \phi_y y^e_{t+1} - \pi^e_{t+2} - d^e_{t+1}) \\ \pi^e_{t+1} &= \beta\pi^e_{t+2} + \hat{\kappa} y^e_{t+1} \end{aligned}$$

We guess $d^e_{t+1} = \rho\hat{d}_t$, $y^e_{t+n} = \psi_y \rho^n \hat{d}_t$, $\pi^e_{t+n} = \psi_\pi \rho^n \hat{d}_t$, and using the method of indeterminated coefficients we get:

$$\begin{aligned} \psi_y &= \psi_y\rho - \frac{\phi_\pi\psi_\pi}{\sigma} - \frac{\phi_y\psi_y}{\sigma} + \frac{\psi_\pi\rho}{\sigma} + \frac{1}{\sigma} \\ \psi_\pi &= \beta\psi_\pi\rho + \hat{\kappa}\psi_y \end{aligned}$$

Delivering:

$$\psi_y = (1-\beta\rho)\tilde{\Gamma} \tag{18}$$

$$\psi_\pi = \hat{\kappa}\tilde{\Gamma} \tag{19}$$

where:

$$\tilde{\Gamma} = \frac{1}{(1-\beta\rho)(\sigma(1-\rho)+\phi_y) + \hat{\kappa}(\phi_\pi - \rho)}$$

This describes how agents form expectations. (i) Then, using our guesses, $\hat{d}_t = d_t - (k - \hat{k})\phi_\pi y_t$ and $y_t = \psi_y \hat{d}_t$, and using equations in (17), we have:

$$y^{LA}_t = \frac{1-\beta\rho}{(1-\beta\rho)(\sigma(1-\rho)+\phi_y) + \kappa(\phi_\pi - \rho) + (\kappa - \hat{\kappa})\rho(1-\beta\phi_\pi)} d_t$$

which corresponds to the RE value for $\hat{\kappa} = \kappa$. Therefore:

- if $\phi_\pi\beta < 1$, then both numerator and denominator are positive and $y^{LA}_t < y^{RA}_t$ because $\kappa - \hat{\kappa} > 0$.

- if $\phi_\pi \beta > 1$, we need to show that the denominator is positive:

$$\begin{aligned} & (1-\beta\rho)(\sigma(1-\rho)+\phi_y)+\kappa(\phi_\pi-\rho)+(\kappa-\hat{\kappa})\rho(1-\beta\phi_\pi) \\ > \quad & \kappa(\phi_\pi-\rho)-(\kappa-\hat{\kappa})\rho(\beta\phi_\pi-1) \\ > \quad & \kappa(\phi_\pi-\rho)-\kappa\rho(\beta\phi_\pi-1) \\ = \quad & \kappa\phi_\pi-\kappa\rho-\kappa\rho\beta\phi_\pi+\kappa\rho>0 \end{aligned}$$

Hence, again since $\kappa - \hat{\kappa} > 0$ the first point of the proposition is shown.
(ii) Now, consider the second equation of the system and use the definition of $\hat{d}_t$, we have:

$$\pi_t^{LA} = \frac{\kappa(1-\beta\rho)+\hat{\kappa}\beta\rho}{(1-\beta\rho)(\sigma(1-\rho)+\phi_y)+\hat{\kappa}(\phi_\pi-\rho)+(\kappa-\hat{\kappa})\phi_\pi(1-\beta\rho)} d_t := T(\hat{k}) d_t := \frac{N(\hat{\kappa})}{D(\hat{\kappa})} d_t$$

Then:

$$\frac{dT(\hat{\kappa})}{d\hat{\kappa}} = \frac{\beta\rho D(\hat{\kappa}) + N(\hat{\kappa})\rho(1-\beta\phi_\pi)}{D(\hat{\kappa})^2}$$

The denominator is positive whenever $\phi_\pi > 1$. Notice that - after some simplifications - the numerator becomes:

$$\rho(1-\beta\rho)[\beta\sigma(1-\rho)+\beta\phi_y+\kappa] > 0.$$

This is so since $\beta, \rho < 1$. Hence $\frac{T(\hat{\kappa})}{d\hat{\kappa}} > 0$. Therefore, since $\hat{\kappa} < \kappa$, we can conclude that $\pi_t^{LA} < \pi_t^{RE}$, so showing the second point of the proposition.

□

C.2 Proof of Proposition 2

Proof. Our model of the economy solves:

$$\begin{aligned} y_t &= y_{t+1}^e - \frac{1}{\sigma}(\phi_\pi \pi_t + \phi_y y_t - \pi_{t+1}^e) \\ \pi_t &= \beta \pi_{t+1}^e + \kappa y_t - s_t \end{aligned}$$

Rearranging the expressions we have:

$$y_t = \frac{1}{1+\kappa\frac{\phi_\pi}{\sigma}+\frac{\phi_y}{\sigma}} \left(y_{t+1}^e + \frac{1-\beta\phi_\pi}{\sigma}\pi_{t+1}^e + \frac{\phi_\pi}{\sigma} s_t \right) \tag{20}$$

$$\pi_t = \frac{1}{1+\kappa\frac{\phi_\pi}{\sigma}+\frac{\phi_y}{\sigma}} \left[\left(\beta\left(1+\frac{\phi_y}{\sigma}\right)+\frac{\kappa}{\sigma}\right)\pi_{t+1}^e + \kappa y_{t+1}^e - \left(1+\frac{\phi_y}{\sigma}\right) s_t \right] \tag{21}$$

Again, the agent forms expectations based on observed movements of the GDP (or equivalently of the nominal interest rate). The positive supply shock s_t is interpreted as a positive demand shock. This is because the deflationary effect induces a decrease in the nominal interest rate, which translates into an

increase in output. The increase in GDP due to the decrease in nominal interest rate is mis-interpreted as a monetary expansion shock. More precisely, the agents believe that they observe a noisy signal of the true inflation but they believe they are able to extract all relevant information from observing output and knowing the Taylor rule.

Agents hence set $\hat{d}_t$ so that to match the nominal interest rate:

$$\begin{aligned}\phi_\pi(\beta\pi^e_{t+1} + \hat{\kappa}y_t) + \phi_y y_t - \hat{d}_t &= \phi_\pi(\beta\pi^e_{t+1} + \kappa y_t - s_t) + \phi_y y_t \\ \Rightarrow \hat{d}_t &= \frac{\phi_\pi}{1 + \phi_\pi(\kappa - \hat{\kappa})\psi_y} s_t\end{aligned}$$

where in the implication we used $y_t = \psi_y \hat{d}_t$. Recall expressions (18) derived in the proof of Proposition 1:

$$\begin{aligned}\psi_y &= (1 - \beta\rho)\tilde{\Gamma} \\ \psi_\pi &= \hat{\kappa}\tilde{\Gamma}\end{aligned}$$

where:

$$\tilde{\Gamma} = \frac{1}{(1-\beta\rho)(\sigma(1-\rho) + \phi_y) + \hat{\kappa}(\phi_\pi - \rho)}$$

We also know that $y^e_{t+n} = \psi_y \rho^n \hat{d}_t$ and $\pi^e_{t+n} = \psi_\pi \rho^n \hat{d}_t$. Then, we can rearrange equations (20) and (21) to obtain the equilibrium expressions for output and inflation with LA agents and a supply shock:

$$y^{LA}_t = \frac{1}{1 + \kappa\frac{\phi_\pi}{\sigma} + \frac{\phi_y}{\sigma}} \left[\rho \frac{\phi_\pi}{1 + \phi_\pi(\kappa - \hat{\kappa})\psi_y}\psi_y + \rho\frac{1 - \beta\phi_\pi}{\sigma}\frac{\phi_\pi}{1 + \phi_\pi(\kappa - \hat{\kappa})\psi_y}\psi_\pi + \frac{\phi_\pi}{\sigma}\right] s_t \tag{22}$$

$$\pi^{LA}_t = \frac{1}{1 + \kappa\frac{\phi_\pi}{\sigma} + \frac{\phi_y}{\sigma}} \left[\rho\left(\beta\left(1 + \frac{\phi_y}{\sigma}\right) + \frac{\kappa}{\sigma}\right)\frac{\phi_\pi}{1 + \phi_\pi(\kappa - \hat{\kappa})\psi_y}\psi_\pi + \rho\frac{\phi_\pi}{1 + \phi_\pi(\kappa - \hat{\kappa})\psi_y}\kappa\psi_y - \left(1 + \frac{\phi_y}{\sigma}\right)\right] s_t \tag{23}$$

Note that:

$$\frac{\psi_\pi}{\psi_y} = \frac{\hat{\kappa}}{1 - \beta\rho}$$

This is the consequence of agents willinbg to be coherent with their perceived Phllips curve. (i) To compare the effect on inflation we first solve for the Rational expectation coefficients. A RE uses equations (20) and and (21) correctly to form expectations. To make the LA and RE models comparable, we assume that the perceived persistence parameter for demand shock is correct and that the persistence of the supply shock is the same as that of the demand shock. Then we have $\hat{\rho} = \rho$. A RE agent forms expectations by solving the following model for the future periods:

$$\begin{aligned}y^e_{t+1} &= y^e_{t+2} - \frac{1}{\sigma}(\phi_\pi \pi^e_{t+1} + \phi_y y^e_{t+1} - \pi^e_{t+2}) \\ \pi^e_{t+1} &= \beta\pi^e_{t+2} + \kappa y^e_{t+1} - s^e_{t+1}\end{aligned}$$

where all expectations are of period t. We guess $s^e_{t+1} = \rho s_t$, $y^e_{t+n} = \psi^{RE}_y \rho^n s_t$ and $\pi^e_{t+n} = \psi^{RE}_\pi \rho^n s_t$. Then, using the technique of indeterminate coefficients, we get:

$$\psi^{RE}_y = \rho\psi^{RE}_y - \frac{\phi_\pi}{\sigma}\psi^{RE}_\pi - \frac{\phi_y}{\sigma}\psi^{RE}_y + \frac{1}{\sigma}\rho\psi^{RE}_\pi$$
$$\psi^{RE}_\pi = \beta\rho\psi^{RE}_\pi + \kappa\psi^{RE}_y - 1$$

Delivering:

$$\psi^{RE}_y = \frac{\phi_\pi - \rho}{\kappa(\phi_\pi - \rho) + (1-\beta\rho)(\sigma(1-\rho) + \phi_y)}$$
$$\psi^{RE}_\pi = -\frac{\sigma(1-\rho) + \phi_y}{\kappa(\phi_\pi - \rho) + (1-\beta\rho)(\sigma(1-\rho) + \phi_y)}$$

Hence, we have:

$$\begin{aligned}\pi^{RE}_t &= \frac{1}{1 + \kappa\frac{\phi_\pi}{\sigma} + \frac{\phi_y}{\sigma}}\left[\left(\beta\left(1 + \frac{\phi_y}{\sigma}\right) + \frac{\kappa}{\sigma}\right)\rho\psi^{RE}_\pi + \kappa\rho\psi^{RE}_y - \left(1 + \frac{\phi_y}{\sigma}\right)\right]s_t \\ &= -\frac{\sigma(1-\rho) + \phi_y}{\kappa(\phi_\pi - \rho) + (1-\beta\rho)(\sigma(1-\rho) + \phi_y)}s_t \end{aligned} \tag{24}$$

Moreover, recall the equilibrium expression for inflation with LA agents in equation (23):

$$\pi^{LA}_t = \frac{1}{1 + \kappa\frac{\phi_\pi}{\sigma} + \frac{\phi_y}{\sigma}}\left[\rho\left(\beta\left(1 + \frac{\phi_y}{\sigma}\right) + \frac{\kappa}{\sigma}\right)\frac{\phi_\pi}{1 + \phi_\pi(\kappa - \hat{\kappa})\psi^{LA}_y}\psi^{LA}_\pi + \rho\frac{\phi_\pi}{1 + \phi_\pi(\kappa - \hat{\kappa})\psi^{LA}_y}\kappa\psi^{LA}_y - \left(1 + \frac{\phi_y}{\sigma}\right)\right]s_t$$

where:

$$\psi^{LA}_y = \frac{1-\beta\rho}{\hat{\kappa}(\phi_\pi - \rho) + (1-\beta\rho)(\sigma(1-\rho) + \phi_y)}$$
$$\psi^{LA}_\pi = \frac{\hat{\kappa}}{\hat{\kappa}(\phi_\pi - \rho) + (1-\beta\rho)(\sigma(1-\rho) + \phi_y)}$$

Simplifying the expression, we have:

$$\pi^{LA}_t = \frac{\rho\hat{\kappa} - (1-\beta\rho)(\sigma(1-\rho) + \phi_y)}{\hat{\kappa}(\phi_\pi - \rho) + (1-\beta\rho)(\sigma(1-\rho) + \phi_y) + \phi_\pi(\kappa - \hat{\kappa})(1-\beta\rho)}s_t$$

Then:

$$\frac{\Gamma^{RE}_\pi}{\Gamma^{LA}_\pi} = \frac{(\sigma(1-\rho) + \phi_y)}{\rho\hat{\kappa} - (1-\beta\rho)(\sigma(1-\rho) + \phi_y)}\frac{\hat{\kappa}(\phi_\pi - \rho) + (1-\beta\rho)(\sigma(1-\rho) + \phi_y) + \phi_\pi(\kappa - \hat{\kappa})(1-\beta\rho))}{(\kappa(\phi_\pi - \rho) + (1-\beta\rho)(\sigma(1-\rho) + \phi_y)}$$

A necessary and sufficient condition for 'regularity' of supply shock - that is that that they do NOT look

like demand shocks - is the following:

$$\sigma > \frac{\rho\hat{\kappa}}{(1-\beta\rho)(1-\rho)} - \frac{\phi_y}{1-\rho}$$

The proof of this condition is the target of Proposition 3. Then, we have that $\frac{\Gamma_\pi^{RE}}{\Gamma_\pi^{LA}} > 1$ whenever:

$$\begin{aligned}&\rho\hat{\kappa} - (1-\beta\rho)(\sigma(1-\rho)+\phi_y)\\ &\quad > -(\sigma(1-\rho)+\phi_y)\frac{\hat{\kappa}(\phi_\pi-\rho)+(1-\beta\rho)(\sigma(1-\rho)+\phi_y)+\phi_\pi(\kappa-\hat{\kappa})(1-\beta\rho)}{\kappa(\phi_\pi-\rho)+(1-\beta\rho)(\sigma(1-\rho)+\phi_y)}\end{aligned}$$

Rearranging the inequality, we can see that it is always satisfied.
Now, the equilibrium expression for output with RE agents solves:

$$y_t^{RE} = \frac{\phi_\pi - \rho}{\kappa(\phi_\pi-\rho)+(1-\beta\rho)(\sigma(1-\rho)+\phi_y)} s_t$$

Then, recall the equilibrium expression for output with LA agents in equation (25):

$$y_t^{LA} = \frac{1}{1+\kappa\frac{\phi_\pi}{\sigma}+\frac{\phi_y}{\sigma}}\left[\rho\frac{\phi_\pi}{1+\phi_\pi(\kappa-\hat{\kappa})\psi_y^{LA}}\psi_y^{LA} + \rho\frac{1-\beta\phi_\pi}{\sigma}\frac{\phi_\pi}{1+\phi_\pi(\kappa-\hat{\kappa})\psi_y^{LA}}\psi_\pi^{LA} + \frac{\phi_\pi}{\sigma}\right] s_t$$

Simplifying this expression, we get:

$$y_t^{LA} = \frac{\phi_\pi(1-\beta\rho)}{\hat{\kappa}(\phi_\pi-\rho)+(1-\beta\rho)(\sigma(1-\rho)+\phi_y)+\phi_\pi(\kappa-\hat{\kappa})(1-\beta\rho)} s_t$$

We take the ratio of the coefficients in front of s_t:

$$\frac{\Gamma_y^{RE}}{\Gamma_y^{LA}} = \frac{(\phi_\pi-\rho)}{\phi_\pi(1-\beta\rho)}\frac{\hat{\kappa}(\phi_\pi-\rho)+(1-\beta\rho)(\sigma(1-\rho)+\phi_y)+\phi_\pi(\kappa-\hat{\kappa})(1-\beta\rho)}{\kappa(\phi_\pi-\rho)+(1-\beta\rho)(\sigma(1-\rho)+\phi_y)}$$

Since every term is positive, we can write:

$$\begin{aligned}&\frac{\Gamma_y^{RE}}{\Gamma_y^{LA}} > 1\\ &\iff\\ &(\phi_\pi-\rho)(\hat{\kappa}(\phi_\pi-\rho)+(1-\beta\rho)(\sigma(1-\rho)+\phi_y)+\phi_\pi(\kappa-\hat{\kappa})(1-\beta\rho))\\ &\qquad > \phi_\pi(1-\beta\rho)(\kappa(\phi_\pi-\rho)+(1-\beta\rho)(\sigma(1-\rho)+\phi_y))\end{aligned}$$

Simplifying the inequality, we get the necessary and sufficient condition as $\phi_\pi\beta > 1$. □

C.3 Proof of Proposition 3

Proof. Recall the equilibrium expressions for output and inflation with LA agents and a supply shock:

$$y_t^{LA} = \frac{1}{1+\kappa\frac{\phi_\pi}{\sigma}+\frac{\phi_y}{\sigma}}\left[\rho\frac{\phi_\pi}{1+\phi_\pi(\kappa-\hat{\kappa})\psi_y}\psi_y + \rho\frac{1-\beta\phi_\pi}{\sigma}\frac{\phi_\pi}{1+\phi_\pi(\kappa-\hat{\kappa})\psi_y}\psi_\pi + \frac{\phi_\pi}{\sigma}\right]s_t \tag{25}$$

$$\pi_t^{LA} = \frac{1}{1+\kappa\frac{\phi_\pi}{\sigma}+\frac{\phi_y}{\sigma}}\left[\rho\left(\beta\left(1+\frac{\phi_y}{\sigma}\right)+\frac{\kappa}{\sigma}\right)\frac{\phi_\pi}{1+\phi_\pi(\kappa-\hat{\kappa})\psi_y}\psi_\pi + \rho\frac{\phi_\pi}{1+\phi_\pi(\kappa-\hat{\kappa})\psi_y}\kappa\psi_y - \left(1+\frac{\phi_y}{\sigma}\right)\right]s_t \tag{26}$$

Now, take the equation (25), divide both sides by $\phi_\pi\psi_y > 0$ and multiply by $1+\phi_\pi(\kappa-\hat{\kappa})\psi_y > 0$ and $1+\kappa\frac{\phi_\pi}{\sigma}+\frac{\phi_y}{\sigma} > 0$:

$$\frac{y_t^{LA}}{\phi_\pi\psi_y}(1+\phi_\pi(\kappa-\hat{\kappa})\psi_y)(1+\kappa\frac{\phi_\pi}{\sigma}+\frac{\phi_y}{\sigma}) = \left[\rho+\rho\frac{1-\beta\phi_\pi}{\sigma}\frac{\hat{\kappa}}{1-\beta\rho}+\frac{1+\phi_\pi(\kappa-\hat{\kappa})\psi_y}{\sigma\psi_y}\right]s_t$$

Hence, output increases if:

$$\rho+\rho\frac{1-\beta\phi_\pi}{\sigma}\frac{\hat{\kappa}}{1-\beta\rho}+\frac{1}{\sigma\psi_y}+\frac{\phi_\pi(\kappa-\hat{\kappa})}{\sigma} > 0$$

$$\iff$$

$$\frac{1}{\sigma(1-\beta\rho)}(\rho\sigma(1-\beta\rho)+\rho(1-\beta\phi_\pi)\hat{\kappa}+(1-\beta\rho)(\sigma(1-\rho)+\phi_y)+\hat{\kappa}(\phi_\pi-\rho)+\phi_\pi(\kappa-\hat{\kappa})(1-\beta\rho)) > 0$$

$$\iff$$

$$\frac{1}{\sigma}(\sigma+\kappa\phi_\pi+\phi_y) > 0$$

which is always true.

Now, take the equation (23) for inflation, divide both sides by $\phi_\pi\psi_y > 0$ and multiply by $1+\phi_\pi(\kappa-\hat{\kappa})\psi_y > 0$ and $1+\kappa\frac{\phi_\pi}{\sigma}+\frac{\phi_y}{\sigma} > 0$:

$$\frac{\pi_t^{LA}}{\phi_\pi\psi_y}(1+\phi_\pi(\kappa-\hat{\kappa})\psi_y)(1+\kappa\frac{\phi_\pi}{\sigma}+\frac{\phi_y}{\sigma}) = \left[\rho\left(\beta\left(1+\frac{\phi_y}{\sigma}\right)+\frac{\kappa}{\sigma}\right)\frac{\hat{\kappa}}{1-\beta\rho}+\rho\kappa-\left(1+\frac{\phi_y}{\sigma}\right)\frac{1+\phi_\pi(\kappa-\hat{\kappa})\psi_y}{\phi_\pi\psi_y}\right]s_t$$

Hence, inflation increases if:

$$\rho\left[\beta\left(1+\frac{\phi_y}{\sigma}\right)+\frac{\kappa}{\sigma}\right]\frac{\hat{\kappa}}{1-\beta\rho}+\rho\kappa-\left(1+\frac{\phi_y}{\sigma}\right)\frac{1+\phi_\pi(\kappa-\hat{\kappa})\psi_y}{\phi_\pi\psi_y}>0$$

$$\Longleftrightarrow$$

$$\left(\frac{\kappa}{\sigma}+\frac{1+\frac{\phi_y}{\sigma}}{\phi_\pi}\right)\left(\frac{\hat{\kappa}\rho}{1-\beta\rho}-(\sigma(1-\rho)+\phi_y)\right)>0$$

Note that the first term is always positive, while the second is positive if and only if:

$$\sigma<\frac{\hat{\kappa}\rho}{(1-\beta\rho)(1-\rho)}-\frac{\phi_y}{1-\rho}.$$

□

C.4 Proof of Proposition 4

Proof. (ii) We only show the case with different persistence levels. Since agents guess correctly y_t, we have:

$$y^e_{t+1}:=\tilde{E}_t[y_{t+1}|\hat{y}_t]=\hat{\rho}y_t;$$

$$\pi^e_{t+1}:=\tilde{E}_t[\pi_{t+1}|\hat{y}_t]=\frac{\hat{\kappa}}{(1-\hat{\rho}\beta)}\hat{\rho}y_t.$$

In addition, recall that:

$$y_t=\frac{1}{1+\kappa\frac{\phi}{\sigma}}\left[y^e_{t+1}+\frac{1-\phi\beta}{\sigma}\pi^e_{t+1}+\frac{d_t}{\sigma}+\frac{\phi s_t}{\sigma}\right]$$
$$\pi_t=\frac{1}{1+\kappa\frac{\phi}{\sigma}}\left[\left(\beta+\frac{\kappa}{\sigma}\right)\pi^e_{t+1}+\kappa y^e_{t+1}+\frac{\kappa}{\sigma}d_t-s_t\right]$$

This delivers the state space representation:

$$\begin{aligned}\pi^{LA}_t &= A(\hat{\kappa},\hat{\rho})s_t+B(\hat{\kappa},\hat{\rho})d_t\\ y^{LA}_t &= C(\hat{\kappa},\hat{\rho})(\phi s_t+d_t)\end{aligned}$$

$$\begin{aligned}A(\hat{\kappa},\hat{\rho}) &= \frac{\hat{\rho}\hat{\kappa}-(1-\beta\hat{\rho})\sigma(1-\rho)}{E(\hat{\kappa},\hat{\rho})}\\ B(\hat{\kappa},\hat{\rho}) &= \frac{\kappa(1-\beta\hat{\rho})+\hat{\kappa}\beta\hat{\rho}}{E(\hat{\kappa},\hat{\rho})}\\ C(\hat{\kappa},\hat{\rho}) &= \frac{(1-\beta\hat{\rho})}{E(\hat{\kappa},\hat{\rho})}\\ E(\hat{\kappa},\hat{\rho}) &= (1-\beta\hat{\rho})(1-\hat{\rho})\sigma+\kappa(\phi-\hat{\rho})+(\kappa-\hat{\kappa})\hat{\rho}(1-\beta\phi)\end{aligned}$$

Note that, *the state space representation remains the same regardless on wether the perceived persistence is right or wrong*. What really matters is the perceived persistence. This result is consequence of the fact that agents are rational within their model and hence their private expectations obey to the law of iterated expectations.

At the same time, agents believe that they are able to estimate $\frac{\hat{\kappa}}{1-\beta\rho}$ from the unconditional regression of π over y:

$$\frac{cov(\pi^{LA}, y^{LA})}{var(y^{LA})} = \frac{A\phi\sigma_s^2 + B\sigma_d^2}{C\left(\phi^2\sigma_s^2 + \sigma_d^2\right)},$$

where we simplified notation by eliminating the dependence on $\hat{\kappa}, \hat{\rho}$.

Obviously, the proper estimation of $\hat{\kappa}, \hat{\rho}$ are a fixed point problem and solves the equation

$$\frac{\hat{\kappa}}{1-\beta\rho} = \frac{A\phi\sigma_s^2 + B\sigma_d^2}{C\left(\phi^2\sigma_s^2 + \sigma_d^2\right)},$$

together with the definition of the coefficients. That is, the implicit non-linear equation:

$$\frac{\hat{\kappa}}{1-\beta\hat{\rho}} = \frac{\left(\rho\hat{\kappa} - (1-\beta\hat{\rho})\sigma(1-\hat{\rho})\right)\phi\sigma_s^2 + \left(\kappa(1-\beta\hat{\rho}) + \hat{\kappa}\beta\hat{\rho}\right)\sigma_d^2}{(1-\beta\hat{\rho})[\phi^2\sigma_s^2 + \sigma_d^2]}. \tag{27}$$

In addition assuming $s_t = \rho_s s_{t-1} + \epsilon_t^s$; $d_t = \rho_d d_{t-1} + \epsilon_t^d$, we have:

$$\hat{\rho} = \frac{cov(y_t^{LA}, y_{t-1}^{LA})}{var(y^{LA})} = \frac{\rho_s\phi^2\sigma_s^2 + \rho_d\sigma_d^2}{\phi^2\sigma_s^2 + \sigma_d^2}, \tag{28}$$

Over reaction in Inflation Now:

$$\begin{aligned}
\tilde{FE}^{\pi} &= \pi_{t+1} - \pi_{t+1}^e = A[\rho_s s_t + \epsilon_{t+1}^s] + B[\rho_d d_t + \epsilon_{t+1}^s] - \frac{\hat{\kappa}}{1-\hat{\rho}\beta}\hat{\rho}C(\phi s_t + d_t) \\
&= \frac{\hat{\rho}\hat{\kappa} - (1-\beta\hat{\rho})\sigma(1-\rho)}{E}[\rho_s s_t + \epsilon_{t+1}^s] + \frac{\kappa(1-\beta\hat{\rho}) + \hat{\kappa}\beta\hat{\rho}}{E}[\rho_d d_t + \epsilon_{t+1}^s] - \hat{\kappa}\hat{\rho}\frac{1}{E}(\phi s_t + d_t) \\
&= \frac{\hat{\rho}\hat{\kappa} - (1-\beta\hat{\rho})\sigma(1-\rho)}{E}[\rho_s(\rho_s s_{t-1} + \epsilon_t^s) + \epsilon_{t+1}^s] + \frac{\kappa(1-\beta\hat{\rho}) + \hat{\kappa}\beta\hat{\rho}}{E}[\rho_d(\rho_d d_{t-1} + \epsilon_t^d) + \epsilon_{t+1}^s] \\
&\quad -\hat{\kappa}\hat{\rho}\frac{1}{E}(\phi(\rho_s s_{t-1} + \epsilon_t^s) + \rho_d d_{t-1} + \epsilon_t^d)
\end{aligned}$$

and

$$\begin{aligned}
\tilde{FR}^{\pi} &= \pi_{t+1}^e - \tilde{\mathbf{E}}[\pi_{t+1}|y_{t-1}] = \frac{\hat{\kappa}}{1-\hat{\rho}\beta}\hat{\rho}C(\phi s_t + d_t) - \frac{\hat{\kappa}}{1-\hat{\rho}\beta}\hat{\rho}^2 C(\phi s_{t-1} + d_{t-1}) \\
&= \hat{\kappa}\hat{\rho}\frac{1}{E}(\phi s_t + d_t) - \hat{\kappa}\hat{\rho}^2\frac{1}{E}(\phi s_{t-1} + d_{t-1}) \\
&= \hat{\kappa}\hat{\rho}\frac{1}{E}(\phi\rho_s(s_{t-1} + \epsilon_t^s) + \rho_d(d_{t-1} + \epsilon_t^d)) - \hat{\kappa}\hat{\rho}^2\frac{1}{E}(\phi s_{t-1} + d_{t-1})
\end{aligned}$$

In addition:

$$var(\epsilon_t^d) = (1-\rho_d^2)\sigma_d^2; \quad var(\epsilon_t^s) = (1-\rho_s^2)\sigma_s^2$$

From the definitions in the note,

$$\widetilde{FE}^{\pi} = \pi_{t+1} - \pi^{e}_{t+1} = A(\rho_s s_t + \varepsilon^{s}_{t+1}) + B(\rho_d d_t + \varepsilon^{d}_{t+1}) - \frac{\hat{\kappa}}{1-\beta\hat{\rho}}\hat{\rho}\, C(\phi s_t + d_t),$$

and

$$\widetilde{FR}^{\pi} = \pi^{e}_{t+1} - \widetilde{\mathbb{E}}[\pi_{t+1} \mid y_t] = \frac{\hat{\kappa}}{1-\beta\hat{\rho}}\hat{\rho}\, C(\phi s_t + d_t) - \frac{\hat{\kappa}}{1-\beta\hat{\rho}}\hat{\rho}^2 C(\phi s_{t-1} + d_{t-1}).$$

Using

$$C(\hat{\kappa}, \hat{\rho}) = \frac{1-\beta\hat{\rho}}{E(\hat{\kappa}, \hat{\rho})},$$

we get

$$\frac{\hat{\kappa}}{1-\beta\hat{\rho}}\hat{\rho}\, C = \frac{\hat{\kappa}\hat{\rho}}{E}.$$

Define

$$a \equiv A = \frac{\hat{\rho}\hat{\kappa} - (1-\beta\hat{\rho})\sigma(1-\hat{\rho})}{E}, \qquad b \equiv B = \frac{\kappa(1-\beta\hat{\rho}) + \hat{\kappa}\beta\hat{\rho}}{E}, \qquad c \equiv \frac{\hat{\kappa}\hat{\rho}}{E}.$$

Then

$$\widetilde{FE}^{\pi} = a(\rho_s s_t + \varepsilon^{s}_{t+1}) + b(\rho_d d_t + \varepsilon^{d}_{t+1}) - c(\phi s_t + d_t),$$

that is,

$$\widetilde{FE}^{\pi} = (a\rho_s - c\phi)s_t + (b\rho_d - c)d_t + a\varepsilon^{s}_{t+1} + b\varepsilon^{d}_{t+1}.$$

Also,

$$\widetilde{FR}^{\pi} = c(\phi s_t + d_t) - c\hat{\rho}(\phi s_{t-1} + d_{t-1}) = c\phi(s_t - \hat{\rho}s_{t-1}) + c(d_t - \hat{\rho}d_{t-1}).$$

Since ε^{s}_{t+1} and ε^{d}_{t+1} are orthogonal to time-t variables,

$$\operatorname{Cov}\left(a\varepsilon^{s}_{t+1} + b\varepsilon^{d}_{t+1},\, \widetilde{FR}^{\pi}\right) = 0.$$

Hence

$$\operatorname{Cov}\left(\widetilde{FE}^{\pi}, \widetilde{FR}^{\pi}\right) = \operatorname{Cov}\left((a\rho_s - c\phi)s_t + (b\rho_d - c)d_t,\, c\phi(s_t - \hat{\rho}s_{t-1}) + c(d_t - \hat{\rho}d_{t-1})\right).$$

By orthogonality of the s- and d-blocks,

$$\operatorname{Cov}\left(\widetilde{FE}^{\pi}, \widetilde{FR}^{\pi}\right) = c\phi(a\rho_s - c\phi)\operatorname{Cov}(s_t, s_t - \hat{\rho}s_{t-1}) + c(b\rho_d - c)\operatorname{Cov}(d_t, d_t - \hat{\rho}d_{t-1}).$$

Now

$$\operatorname{Cov}(s_t, s_t - \hat{\rho}s_{t-1}) = \operatorname{Var}(s_t) - \hat{\rho}\,\operatorname{Cov}(s_t, s_{t-1}) = \sigma_s^2 - \hat{\rho}\rho_s\sigma_s^2 = (1-\hat{\rho}\rho_s)\sigma_s^2,$$

and similarly

$$\operatorname{Cov}(d_t, d_t - \hat{\rho}d_{t-1}) = (1-\hat{\rho}\rho_d)\sigma_d^2.$$

Therefore

$$\mathrm{Cov}\left(\widetilde{FE}^{\pi}, \widetilde{FR}^{\pi}\right) = c\phi(a\rho_s - c\phi)(1-\hat{\rho}\rho_s)\sigma_s^2 + c(b\rho_d - c)(1-\hat{\rho}\rho_d)\sigma_d^2.$$

Substituting a, b, c,

$$a\rho_s - c\phi = \frac{\rho_s\left[\hat{\rho}\hat{\kappa} - (1-\beta\hat{\rho})\sigma(1-\hat{\rho})\right] - \hat{\kappa}\hat{\rho}\phi}{E},$$

$$b\rho_d - c = \frac{\rho_d\left[\kappa(1-\beta\hat{\rho}) + \hat{\kappa}\beta\hat{\rho}\right] - \hat{\kappa}\hat{\rho}}{E}.$$

Since $c = \hat{\kappa}\hat{\rho}/E$, we obtain

$$\mathrm{Cov}\left(\widetilde{FE}^{\pi}, \widetilde{FR}^{\pi}\right) = \frac{\hat{\kappa}\hat{\rho}}{E^2}\left[\phi(1-\hat{\rho}\rho_s)\sigma_s^2\, S_s + (1-\hat{\rho}\rho_d)\sigma_d^2\, S_d\right]$$

where

$$S_s = \rho_s\big(\hat{\rho}\hat{\kappa} - (1-\beta\hat{\rho})\sigma(1-\hat{\rho})\big) - \hat{\kappa}\hat{\rho}\phi,$$

and

$$S_d = \rho_d\big(\kappa(1-\beta\hat{\rho}) + \hat{\kappa}\beta\hat{\rho}\big) - \hat{\kappa}\hat{\rho}.$$

Equivalently,

$$S_s = \hat{\kappa}\hat{\rho}(\rho_s - \phi) - \rho_s(1-\beta\hat{\rho})\sigma(1-\hat{\rho}),$$

$$S_d = \kappa\rho_d(1-\beta\hat{\rho}) - \hat{\kappa}\hat{\rho}(1-\beta\rho_d).$$

Under

$$0 < \rho_s, \hat{\rho}, \rho_d, \beta < 1, \quad \kappa, \hat{\kappa} > 0 \qquad \phi > 1, \qquad \sigma > 0, \qquad \sigma_s^2 > 0,\ \sigma_d^2 > 0,$$

we have

$$\hat{\kappa}\hat{\rho} > 0, \qquad E^2 > 0, \qquad 1-\hat{\rho}\rho_s > 0, \qquad 1-\hat{\rho}\rho_d > 0, \qquad \phi > 0, \qquad 1-\hat{\rho} > 0, \qquad 1-\beta\hat{\rho} > 0.$$

Hence the sign of the covariance is the sign of

$$\Psi \equiv \phi(1-\hat{\rho}\rho_s)\sigma_s^2\, S_s + (1-\hat{\rho}\rho_d)\sigma_d^2\, S_d.$$

Now,

$$S_s = \hat{\kappa}\hat{\rho}(\rho_s - \phi) - \rho_s(1-\beta\hat{\rho})\sigma(1-\hat{\rho}).$$

Since $\phi > 1 > \rho_s$, we have

$$\rho_s - \phi < 0.$$

Also,

$$\hat{\kappa}\hat{\rho} > 0, \qquad \rho_s > 0, \qquad 1-\beta\hat{\rho} > 0, \qquad \sigma > 0, \qquad 1-\hat{\rho} > 0.$$

Therefore both terms in S_s are strictly negative. For the d-component,

$$S_d = \kappa\rho_d(1-\beta\hat{\rho}) - \hat{\kappa}\hat{\rho}(1-\beta\rho_d),$$

so

$$S_d > 0 \iff \kappa\rho_d(1-\beta\hat{\rho}) > \hat{\kappa}\hat{\rho}(1-\beta\rho_d),$$

that is,

$$\boxed{\kappa > \hat{\kappa}\hat{\rho}\,\frac{1-\beta\rho_d}{\rho_d(1-\beta\hat{\rho})}}.$$

Therefore:

$$\mathrm{Cov}\left(\widetilde{FE}^{\pi}, \widetilde{FR}^{\pi}\right) < 0$$

whenever

$$S_d \leq 0,$$

Using the expression for $\tilde{\kappa}$ and $\hat{\rho}$ we obtain that the condition stated in the main text suffices for $S_d < 0$.

Output Overreaction

$$\tilde{E}_t[y_{t+1}] = \hat{\rho}y_t.$$

Similarly,

$$\tilde{E}_{t-1}[y_{t+1}] = \hat{\rho}^2 y_{t-1}.$$

Using

$$y_t^{LA} = \frac{(1-\beta\hat{\rho})}{E}(\phi s_t + d_t)$$

where

$$E = (1-\beta\hat{\rho})(1-\hat{\rho})\sigma + \kappa(\phi-\hat{\rho}) + (\kappa-\hat{\kappa})\hat{\rho}(1-\beta\phi)$$

we get

$$\begin{aligned}
FR_t^y &= \tilde{E}_t[y_{t+1}] - \tilde{E}_{t-1}[y_{t+1}] = \hat{\rho}y_t - \hat{\rho}^2 y_{t-1} = \frac{1-\beta\hat{\rho}}{E}\hat{\rho}\left[\phi s_t + d_t - \hat{\rho}(\phi s_{t-1} + d_{t-1})\right] \\
&= \frac{1-\beta\hat{\rho}}{E}\hat{\rho}\left[\phi(\rho_s s_{t-1} + \epsilon_t^s) + \rho_d d_{t-1} + \epsilon_t^d - \hat{\rho}(\phi s_{t-1} + d_{t-1})\right]; \\
FE_t^y &= y_{t+1} - \hat{\rho}y_t = \frac{1-\beta\hat{\rho}}{E}\left[\phi s_{t+1} + d_{t+1} - \hat{\rho}[\phi(\rho_s s_{t-1} + \epsilon_t^s) + \rho_d d_{t-1} + \epsilon_t^d]\right] \\
&= \frac{1-\beta\hat{\rho}}{E}\left[\phi(\rho_s s_t + \epsilon_{t+1}^s) + (\rho_d d_t + \epsilon_{t+1}^d) - \hat{\rho}[\phi(\rho_s s_{t-1} + \epsilon_t^s) + \rho_d d_{t-1} + \epsilon_t^d]\right] \\
&= \frac{1-\beta\hat{\rho}}{E}\left[\phi(\rho_s(\rho_s s_{t-1} + \epsilon_t^s) + \epsilon_{t+1}^s) + (\rho_d(\rho_d d_{t-1} + \epsilon_t) + \epsilon_{t+1}^d) - \hat{\rho}[\phi(\rho_s s_{t-1} + \epsilon_t^s) + \rho_d d_{t-1} + \epsilon_t^d]\right].
\end{aligned}$$

Where we used $s_t = \rho_s s_{t-1} + \epsilon_t^s$; $d_t = \rho_d d_{t-1} + \epsilon_t^d$. Finally, recall that

$$\hat{\kappa} = \frac{\left(\rho\hat{\kappa} - (1-\beta\hat{\rho})\sigma(1-\hat{\rho})\right)\phi\sigma_s^2 + \left(\kappa(1-\beta\hat{\rho}) + \hat{\kappa}\beta\hat{\rho}\right)\sigma_d^2}{[\phi^2\sigma_s^2 + \sigma_d^2]}.$$

And

$$\hat{\rho} = \frac{cov(y_t^{LA}, y_{t-1}^{LA})}{var(y^{LA})} = \frac{\rho_s\phi^2\sigma_s^2 + \rho_d\sigma_d^2}{\phi^2\sigma_s^2 + \sigma_d^2},$$

Proposition 5. *Let*

$$FR_t^y = \frac{1-\beta\hat{\rho}}{E}\,\hat{\rho}(x_t - \hat{\rho}x_{t-1}), \qquad FE_t^y = \frac{1-\beta\hat{\rho}}{E}\,(x_{t+1} - \hat{\rho}x_t), \tag{29}$$

where $x_t = \phi s_t + d_t$, and s_t, d_t are AR(1) processes with persistence parameters $\rho_s, \rho_d \in (0,1)$, and unconditional variances σ_s^2, σ_d^2. Then

$$Cov(FE_t^y, FR_t^y) = -\hat{\rho}^2\left(\frac{1-\beta\hat{\rho}}{E}\right)^2 \frac{\phi^2\sigma_s^2\sigma_d^2}{\phi^2\sigma_s^2 + \sigma_d^2}(\rho_s - \rho_d)^2 \leq 0. \tag{30}$$

Moreover, the covariance is strictly negative whenever $\rho_s \neq \rho_d$.

Proof. Define

$$A \equiv \frac{1-\beta\hat{\rho}}{E}, \qquad x_t \equiv \phi s_t + d_t. \tag{31}$$

Then

$$FR_t^y = A\hat{\rho}(x_t - \hat{\rho}x_{t-1}), \qquad FE_t^y = A(x_{t+1} - \hat{\rho}x_t). \tag{32}$$

Hence

$$\begin{aligned} Cov(FE_t^y, FR_t^y) &= A^2\hat{\rho}Cov(x_{t+1} - \hat{\rho}x_t,\, x_t - \hat{\rho}x_{t-1}) && (33)\\ &= A^2\hat{\rho}\Big[Cov(x_{t+1}, x_t) - \hat{\rho}Cov(x_{t+1}, x_{t-1}) - \hat{\rho}Var(x_t) + \hat{\rho}^2Cov(x_t, x_{t-1})\Big]. && (34)\end{aligned}$$

Where

$$\hat{\rho} = \frac{Cov(x_t, x_{t-1})}{Var(x_{t-1})} = \frac{Cov(x_{t+1}, x_t)}{Var(x_t)} = \frac{\phi^2\sigma_s^2\rho_s + \sigma_d^2\rho_d}{\phi^2\sigma_s^2 + \sigma_d^2}.$$

Therefore

$$Cov(FE_t^y, FR_t^y) = -A^2\hat{\rho}^2\frac{\phi^2\sigma_s^2\sigma_d^2}{\phi^2\sigma_s^2 + \sigma_d^2}(\rho_s - \rho_d)^2. \tag{35}$$

□

D Effects of a contractionary monetary policy

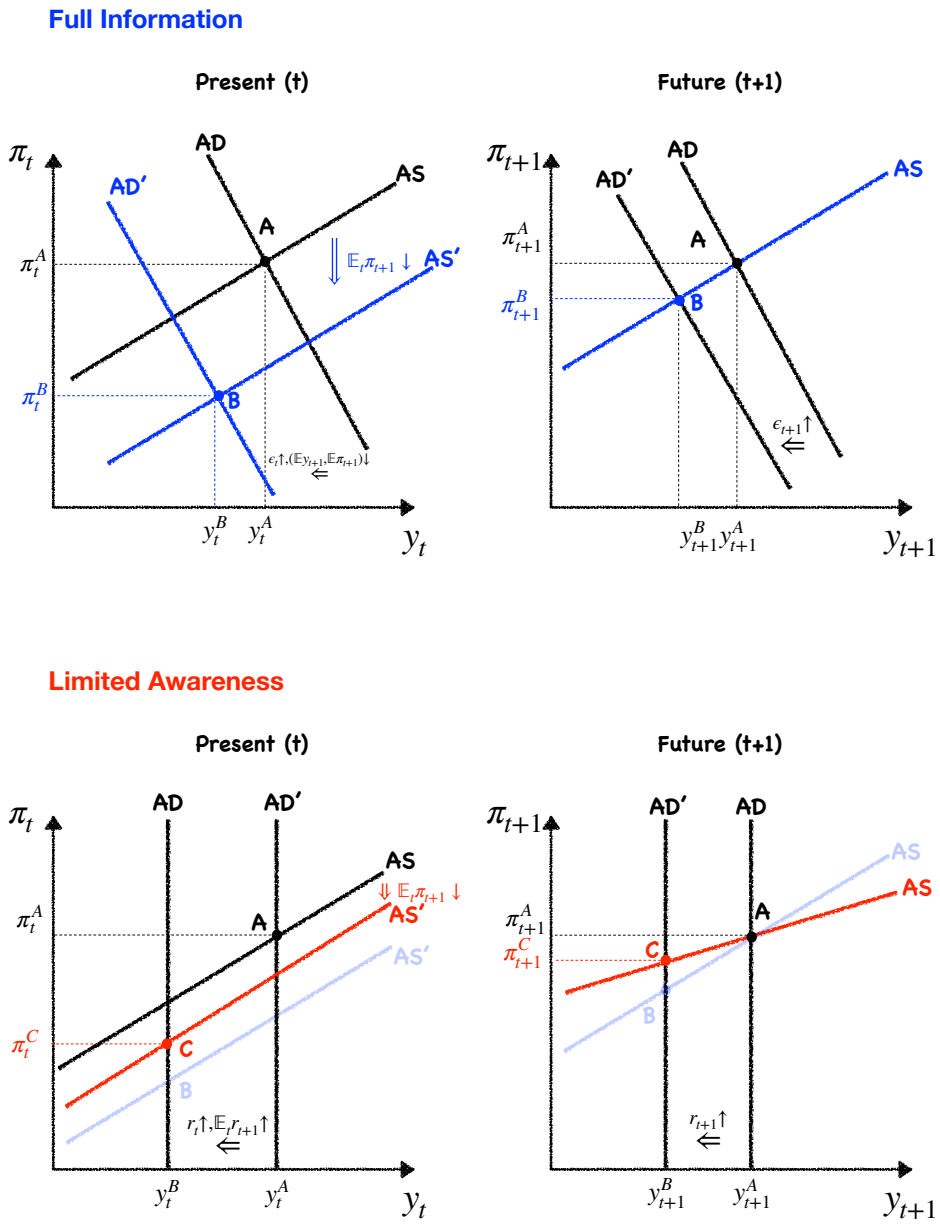


Figure 13: Graphical representation of a contractionary monetary policy (Taylor Rule).

E Additional Figures

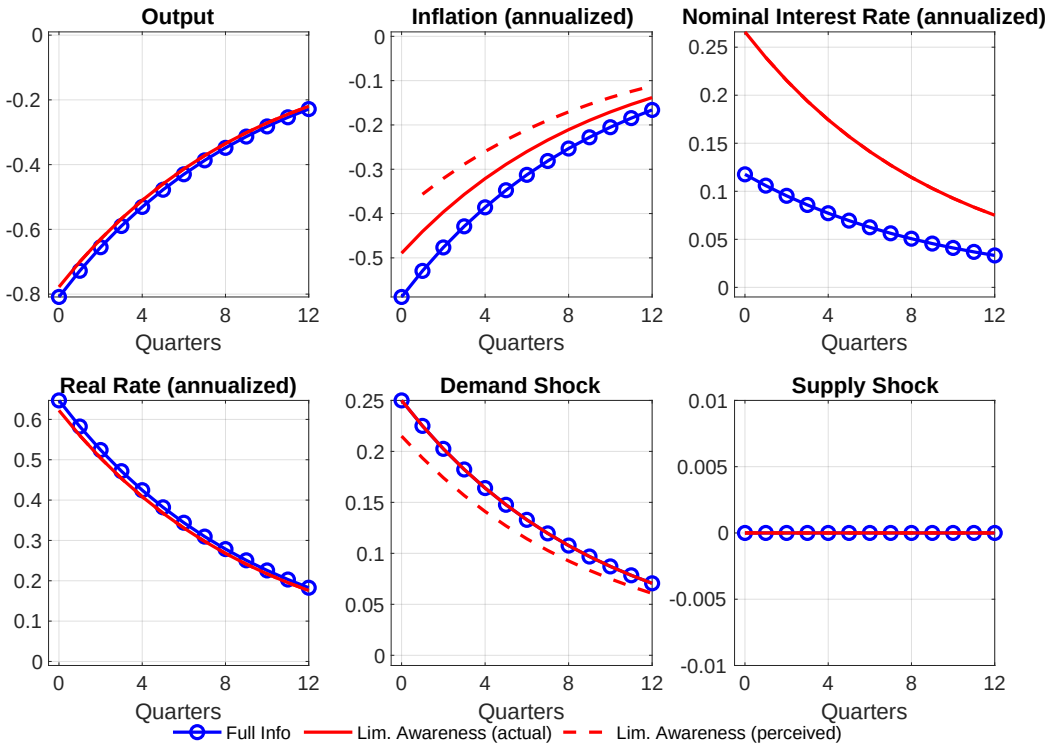


Figure 14: **Contractionary Monetary Shock.** The figure reports the effects of a contractionary monetary policy in the case where $\phi\beta < 1$ and makes a comparison between the standard NK model (blue) and our model with LA agents (solid red). As it can be seen, both the effect on output and inflation are mitigated compared to RE. This effect on output depends on the assumption $\phi\beta < 1$, while the mitigation effect on inflation is always true (see Proposition 1). Parameters: $\beta = .5; \sigma = 2; \kappa = .1; \hat{\kappa} = .07; \phi = 1.5; \rho_d = .9$

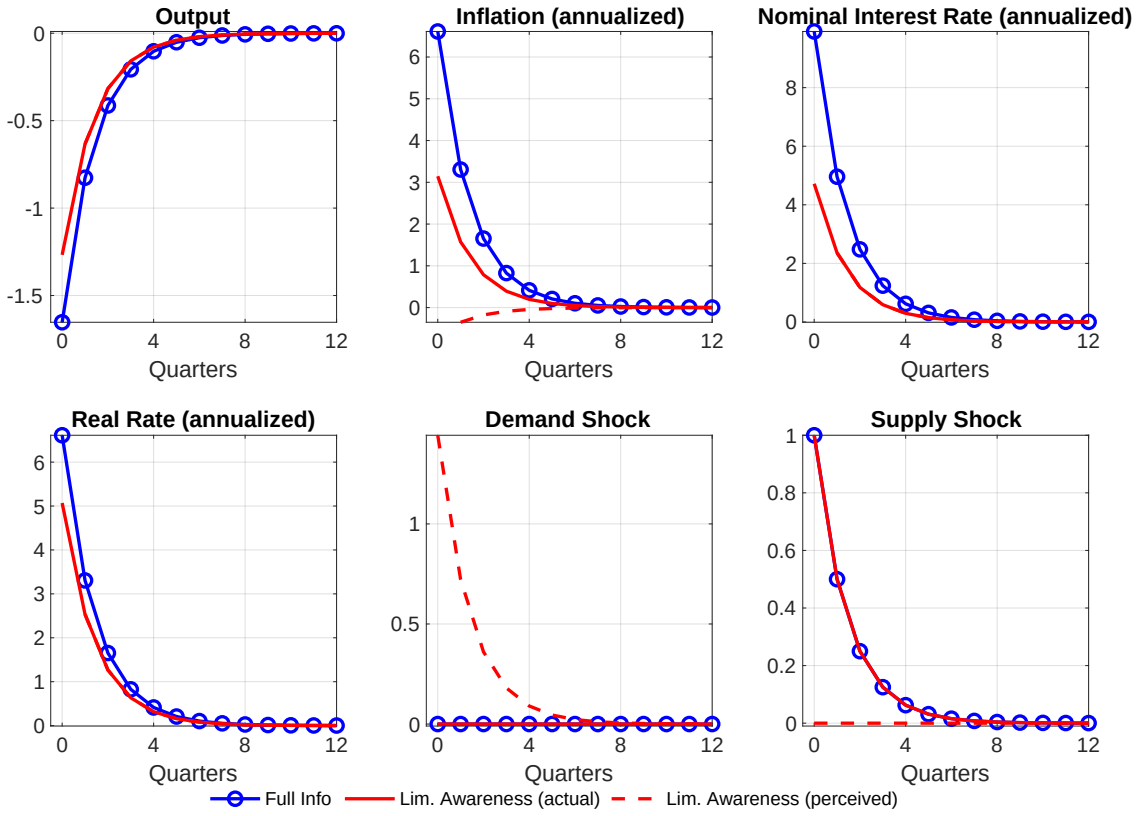


Figure 15: **Effect of a Supply shock with demand and supply shocks with low persistence.** The figure reports the effects of a negative supply shock. The blue line reports the effect under RE while the solid red line reports the effects under LA. As it can be seen, a supply shock generates the 'usual' effects on aggregate variables, namely an increase in inflation and a reduction in input. Compared to Figure 5, we only decreased the persistence parameter of both monetary and supply shocks from 0.5 to 0.9 (per quarter). The parameters used are: $\kappa = .1$, $\hat{\kappa} = 0.07$; $\beta = .99$; $\sigma = 1$; $\phi_\pi = 1.5$; $\rho_d = \rho_s = .5$

F SVAR Analysis

Consider the SVAR(p) with the general form:

$$\boldsymbol{A}^{-1}\boldsymbol{y}_t = \sum_{l=1}^{p} \boldsymbol{A}_l \boldsymbol{y}_{t-l} + \boldsymbol{c} + \boldsymbol{\varepsilon}_t \quad \text{for} \quad t \in \{1, ..., T\}, \tag{36}$$

where $\boldsymbol{y}_t$ is a vector of $(n \times 1)$ endogenous variables, $\boldsymbol{A}_l$ is a $(n \times n)$ matrix of coefficients that captures the lagged effects of the endogenous variables at lag l and $\boldsymbol{c}$ is a vector of constants. Respectively, $\boldsymbol{A}^{-1}$ is a non-singular $(n \times n)$ matrix of contemporaneous interactions between the endogenous variables and $\boldsymbol{\varepsilon}_t$ is a vector of n structural shocks, with $\mathbb{E}(\boldsymbol{\varepsilon}_t) = 0$ and $\mathbb{E}(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t') = \boldsymbol{I}_n$. For the following steps, it is useful to

express Equation (36) as:

$$\boldsymbol{A}^{-1}\boldsymbol{y_t} = \boldsymbol{A_+}\boldsymbol{x_t} + \boldsymbol{\varepsilon_t} \quad \text{for} \quad t \in \{1, ..., T\}, \tag{37}$$

where $\boldsymbol{A_+} = [\boldsymbol{A_1} \quad \cdots \quad \boldsymbol{A_p} \quad \boldsymbol{c}]$ and $\boldsymbol{x_t} = [\boldsymbol{y_{t-1}} \quad \cdots \quad \boldsymbol{y_{t-p}} \quad \boldsymbol{1}]$. We then pre-multiply both sides of Equation (37) by $\boldsymbol{A}$, leading to the reduced-form VAR(p):

$$\boldsymbol{y_t} = \boldsymbol{B}\boldsymbol{x_t} + \boldsymbol{u_t} \quad \text{for} \quad t \in \{1, ..., T\}, \tag{38}$$

where $\boldsymbol{B} = \boldsymbol{A}\boldsymbol{A_+}$, $\boldsymbol{u_t} = \boldsymbol{A}\boldsymbol{\varepsilon_t}$, $\mathbb{E}(\boldsymbol{u_t}) = \boldsymbol{0}$ and $\mathbb{E}(\boldsymbol{u_t}\boldsymbol{u_t'}) = \boldsymbol{\Sigma} = \boldsymbol{A}\boldsymbol{A'}$.

F.1 Estimation: Bayesian

One common estimation technique of the reduced-form parameter matrices $\boldsymbol{B}$ and $\boldsymbol{\Sigma}$ is using a Bayesian approach.

We implement the Bayesian estimation via the "Empirical Macro Toolbox" by F. Ferroni and F. Canova (Link). Specifically, we estimate the reduced-form parameter matrices B and Σ using a Bayesian approach, that is based on the Jeffrey prior. Under this uninformative prior, the posterior distribution of the parameters coincides with the likelihood up to a normalization constant, implying that the posterior moments are fully data-driven. The estimation proceeds via a Gibbs sampling algorithm.

Application. We estimate a BVAR(4) model including constants and a linear time-trend using 1.000 draws. Regarding the employed identification strategy, we apply the sign restrictions displayed in Table 4. Restrictions are imposed on the first four quarters.[32]

F.2 Data and Specification

Given the chosen estimation technique, we estimate the VAR described in Equation (38) using quarterly data for the US from 1968:Q4 to 2019:Q4, i.e. we exclude Covid. As common in the literature, we set $p = 4$ for the baseline estimation.

Baseline variables. The baseline vector of endogenous variables consists of

(i) the negative of the unemployment rate, $-u_t$, as a measure of economic slackness;

(ii) a measure for inflation, π_t, measured as the GDP deflator;

(iii) one-period (i.e., one year) ahead expectations about the (negative) of the unemployment rate, $\mathbb{E}_t[-u_{t+1}]$;

(iv) a measure for inflation expectations (also measured as the GDP deflator), $\mathbb{E}_t[\pi_{t+1}]$, ;

(v) Short-term treasury yields (GS1/GS2), i_t, from FRED.

[32] As we mentioned above, the toolbox allows us to use more sophisticated identification strategies. We used a simple 'sign restriction only' routine, which follows Ramirez(2010).

Measuring inflation and inflation expectations. The SPF dataset provides quarterly forecasts of annualized quarter-over-quarter inflation for different forecast horizons - along with quarter-to-quarter realizations for inflation. Hence, these baseline measures do *not* correspond to year-over-year (YoY) inflation rates. In this report, we will use two different measures:

1. Quarter-to-Quarter measures
2. Smooth measures

For the smooth measures, we proceed as follows: Denoting with π_t the realized (annualised) quarter-to-quarter inflation from period $t-1$ until t, we compute the Year-on-Year realized inflation as the arithmetic average between $t-4$ to t:

$$\pi^{1\text{yr avg}} = \frac{1}{4}[\pi_{t-3} + ... + \pi_t].$$

Regarding inflation expectations, the SPF dataset provides quarterly forecasts of annualized quarter-over-quarter inflation for different forecast horizons. In order to approximate expected cumulative inflation over the next four quarters ($t+1$ to $t+4$), we consider the unweighted average of the next four quarterly inflation forecasts:

$$\mathbb{E}_t[\pi^{1\text{yr avg}}_{t+1:t+4}] = \frac{1}{4}\left(\pi^{\text{exp}}_{t+1} + \pi^{\text{exp}}_{t+2} + \pi^{\text{exp}}_{t+3} + \pi^{\text{exp}}_{t+4}\right).$$

F.3 Smooth inflation & EBP

This is our preferred specification. We use the following set of endogenous variables:

(i) (-) Unemployment rate;

(ii) smoothed inflation (over the past year);

(iii) (-) Expected Unemployment rate;

(iv) smooth inflation expectations (over the next year);

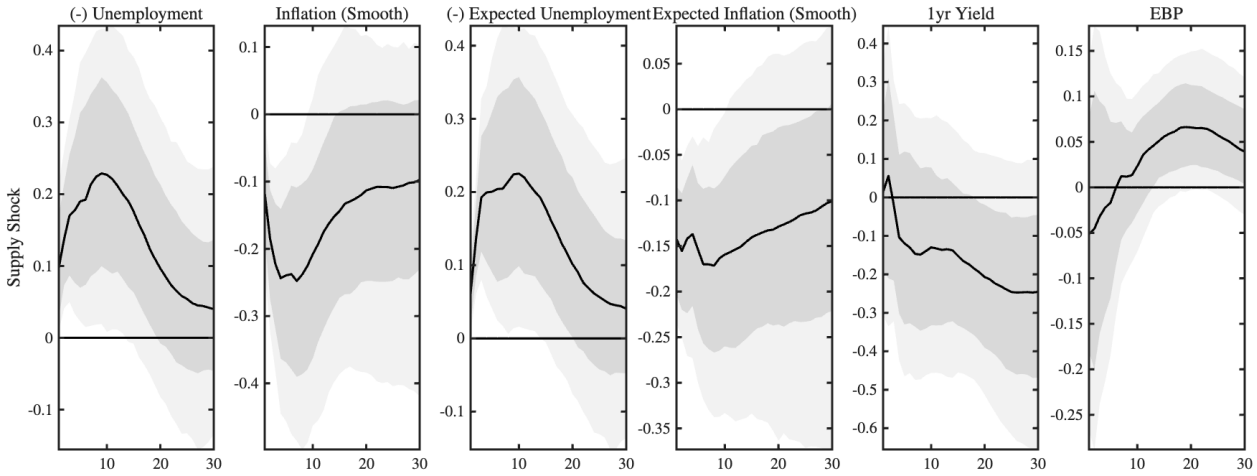
(v) 1yr nominal interest rate (GS1);

(vi) Excess Bond Premium (EBP).

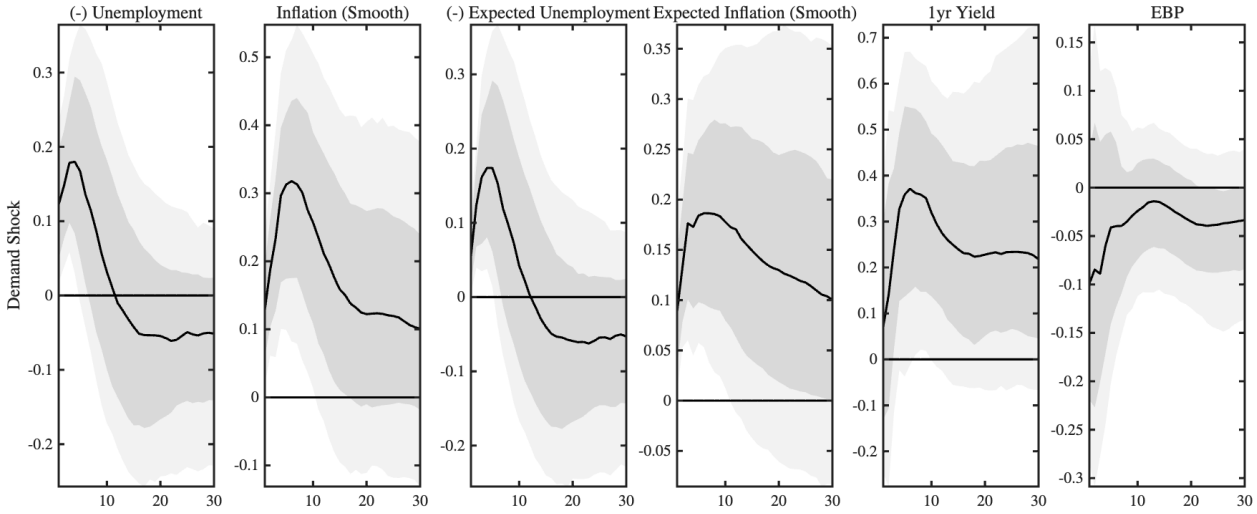
In the main text we reported the IRFs associated to the mis-perceived supply shocks. Figure 18 plots the IRFs to the the classic supply and demand shocks.

F.3.1 Historical decompositions.

After having evaluated the macroeconomic impact of a our new misperceived supply shock, this Subsection decomposes the given endogenous time series into the different structural shocks that drive them according to Equation 36. This so-called historical decomposition (HD) is typically employed to measure the contribution of certain structural shocks to the deviation of the variables from a forecasted path. That is, with HD



(a) IRFs – Classic Supply Shock



(b) IRFs – Classic Demand Shock

Figure 16: IRFs of Supply and Demand Shock

Source: Own illustration. All estimations are performed in Matlab and are based on the VAR Toolbox "Empirical Macro Toolbox" by F. Ferroni and F. Canova. 68% (90%) confidence intervals in dark (light) grey.

we want to answer the question: how much of the deviation of inflation from the predicted path is due, e.g., to typical supply shocks?

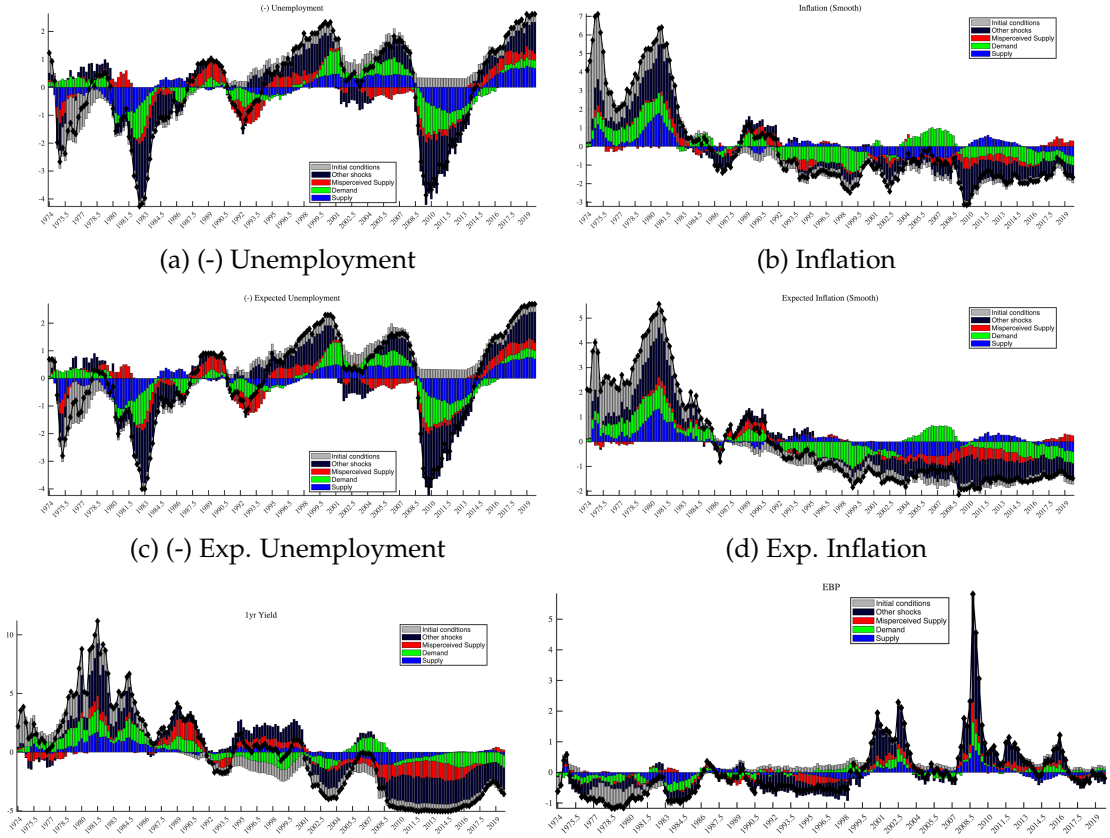
Figure 17 decomposes the respective endogenous time series of the baseline VAR into:

- Initial conditions (in grey), meaning constants and, if added, (deterministic) time-trends;
- Other shocks (in black);[33]

[33]This refers to the residual structural innovations that remain after identifying the three economically interpretable shocks (Supply, Demand, Misperceived Supply). Since the VAR contains five endogenous variables but only three shocks are sign-restricted, the remaining two orthogonal innovations are grouped together under this term and capture the contribution of unidentified shocks to the historical decomposition.

- Classic Demand shock (in green), set-identified in column 2 of Table 4;
- Classic Supply shock (in blue), set-identified in column 1 of Table 4;
- the new Misperceived Supply Shock (in red), set-identified in column 3 of Table 4.

In the main text, we only report the historical decomposition of (minus) unemployment. Figure 17 reports the decomposition of all endogenous variables.



(a) (-) Unemployment (b) Inflation

(c) (-) Exp. Unemployment (d) Exp. Inflation

(e) Interest Rate (GS1) (f) EBP

Figure 17: Historical Decomposition

Source: Own illustration. All estimations are performed in Matlab and are based on the VAR Toolbox "Empirical Macro Toolbox" by F. Ferroni and F. Canova. The time series of the respective endogenous variable is demeaned and represented by the black dots. A 'positive' bar signals an increase compared to the previous month.

F.4 Q-to-Q Inflation & EBP

Variables. For this section, we use the following set of endogenous variables:

(i) (-) Unemployment rate;

(ii) Q-to-Q inflation;

(iii) (-) Expected Unemployment rate;

(iv) One Quarter ahead inflation expectations;

(v) 1yr nominal interest rate (GS1);

(vi) Excess Bond Premium (EBP).

F.4.1 IRFs

Below we plot the impulse response function to the misperceived supply shock. Figure 18 plots the IRFs to the misperceived supply shock as identified above. Similarly, we report the IRFs to a classic supply and demand shock in Figure 19 below.

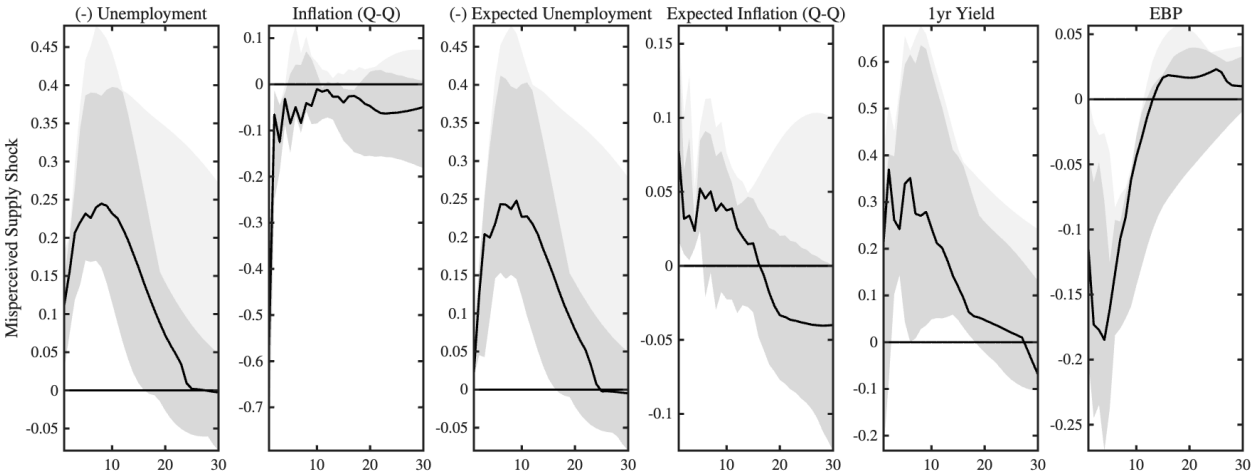


Figure 18: IRFs TO MISPERCEIVED SUPPLY SHOCK

Source: Own Illustration, Data described as above. All estimations are performed in Matlab and are based on the VAR Toolbox "Empirical Macro Toolbox" by F. Ferroni and F. Canova. 68% (90%) confidence intervals in dark (light) grey.

F.4.2 FEVD

Finally, we report the forecast error variance decomposition (FEVD) for each endogenous variables. Using the posterior mean of the estimated BVAR, we report the percentage of forecast error volatility explained by the identified shocks in Table 4 for each endogenous variable. Figure 20 shows the FEVD in a stacked area chart.

F.5 Smooth Inflation & Sentiment

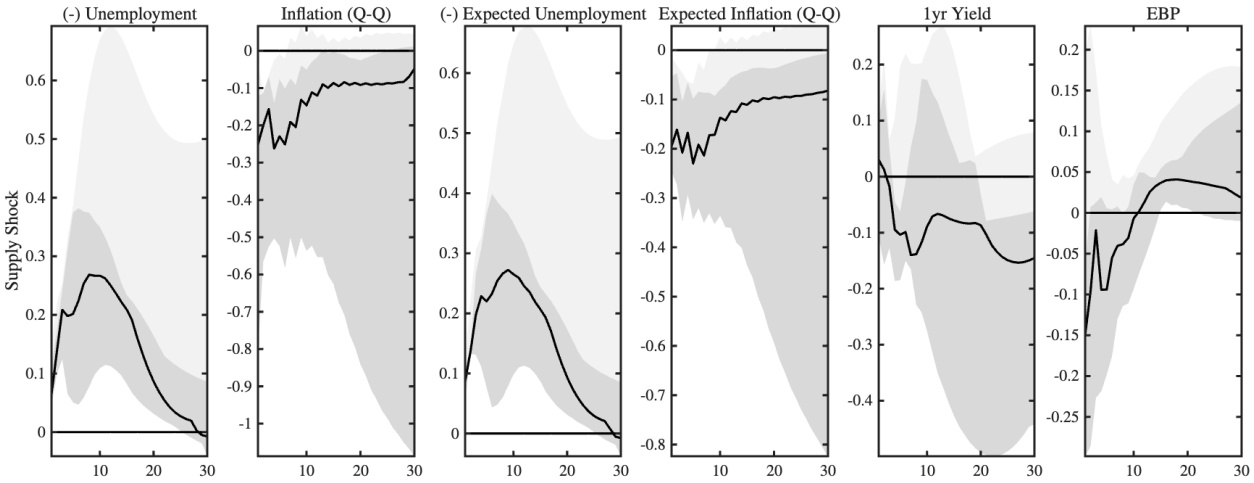
F.5.1 Specification

Variables. For this section, we use the following set of endogenous variables:

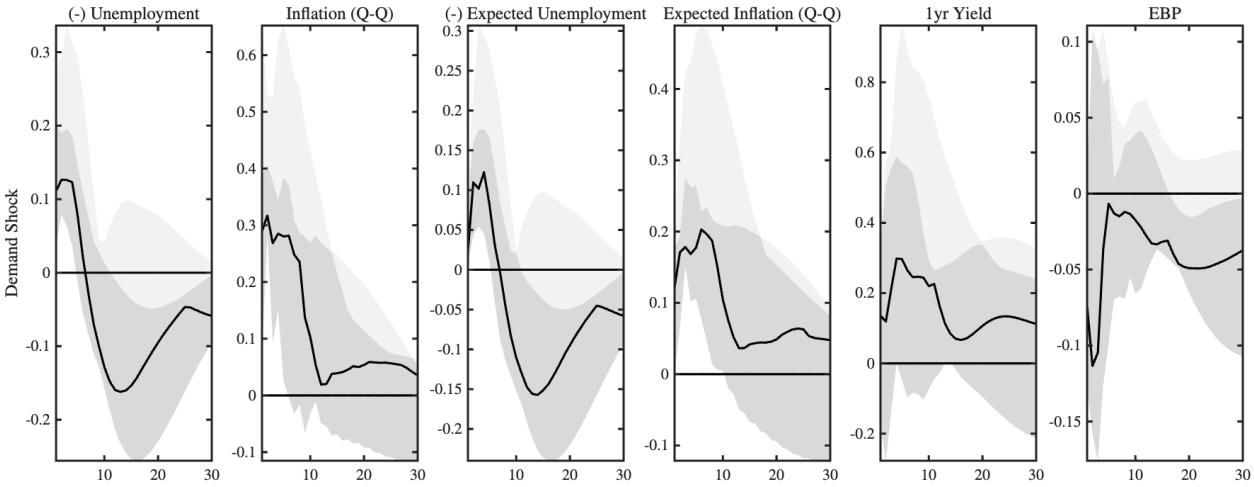
(i) (-) Unemployment rate;

(ii) Smooth inflation (over past year);

(iii) (-) Expected Unemployment rate;



(a) IRFs – Classic Supply Shock



(b) IRFs – Classic Demand Shock

Figure 19: IRFs of Supply and Demand Shock

Source: Own illustration. All estimations are performed in Matlab and are based on the VAR Toolbox "Empirical Macro Toolbox" by F. Ferroni and F. Canova. 68% (90%) confidence intervals in dark (light) grey.

(iv) Smooth inflation expectations (over next year);

(v) 1yr interest rate (GS1);

(vi) a measure of sentiment.

Instead of the EBP, this specification now includes the Michigan Survey of Consumer's Confidence Index as a measure for sentiment.

Due to data availability of the Michican Survey of Consumers, we estimate the BVAR for this specification from 1978:Q1 until 2019:Q4. We apply the same sign restrictions as in the previous Section (i.e., as in Table 4). As for the nominal interest rate, there are no restrictions imposed on the sentiment measure.

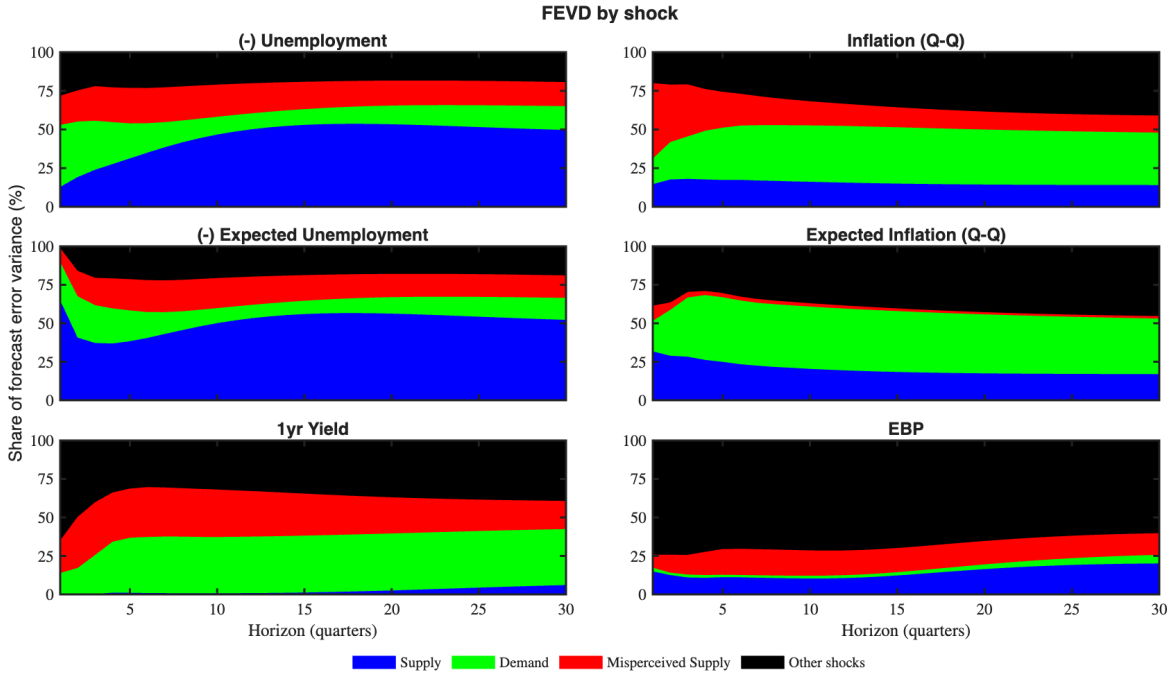


Figure 20: FEVDs

Source: Own Illustration, Data described as above. All estimations are performed in Matlab and are based on the VAR Toolbox "Empirical Macro Toolbox" by F. Ferroni and F. Canova.

F.5.2 IRFs

Below we plot the impulse response function to the misperceived supply shock. Figure 21 plots the IRFs to the misperceived supply shock as identified above. Similarly, we report the IRFs to a classic supply and demand shock in Figure 22 below.

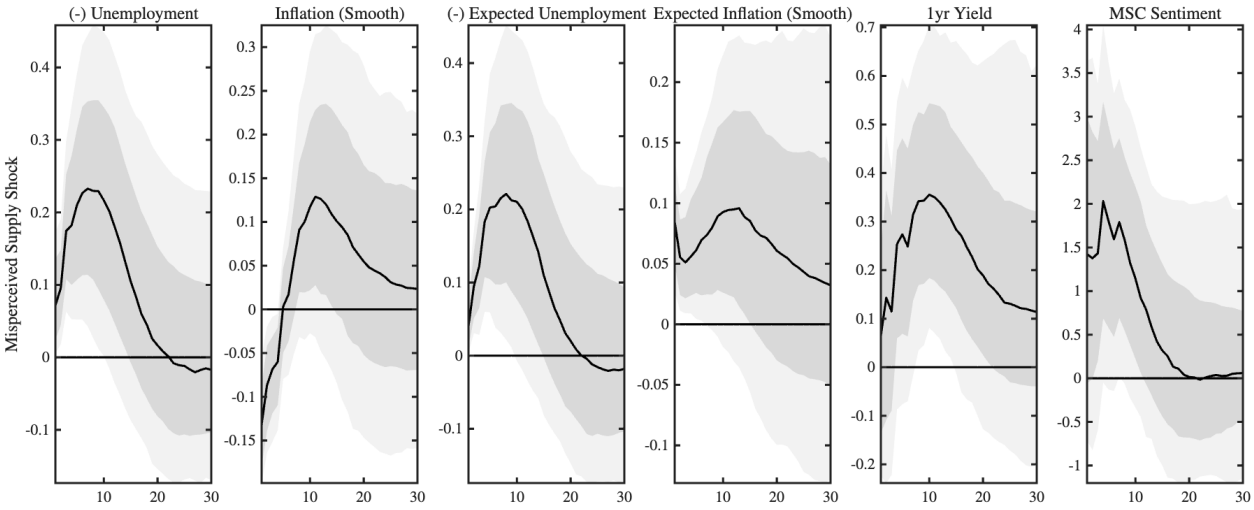
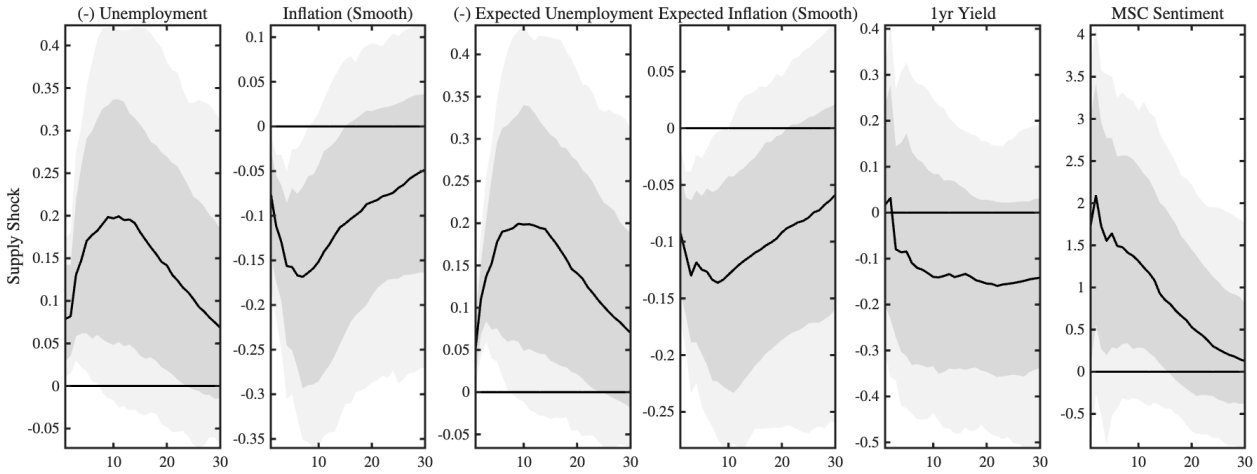


Figure 21: IRFs TO MISPERCEIVED SUPPLY SHOCK

Source: Own Illustration, Data described as above. All estimations are performed in Matlab and are based on the VAR Toolbox "Empirical Macro Toolbox" by F. Ferroni and F. Canova. 68% (90%) confidence intervals in dark (light) grey.



(a) IRFs – Classic Supply Shock



(b) IRFs – Classic Demand Shock

Figure 22: IRFs of Supply and Demand Shock

Source: Own illustration. All estimations are performed in Matlab and are based on the VAR Toolbox "Empirical Macro Toolbox" by F. Ferroni and F. Canova. 68% (90%) confidence intervals in dark (light) grey.

F.5.3 FEVD

Finally, we report the forecast error variance decomposition (FEVD) for each endogenous variables in Figure 23 shows the FEVD in a stacked area chart.

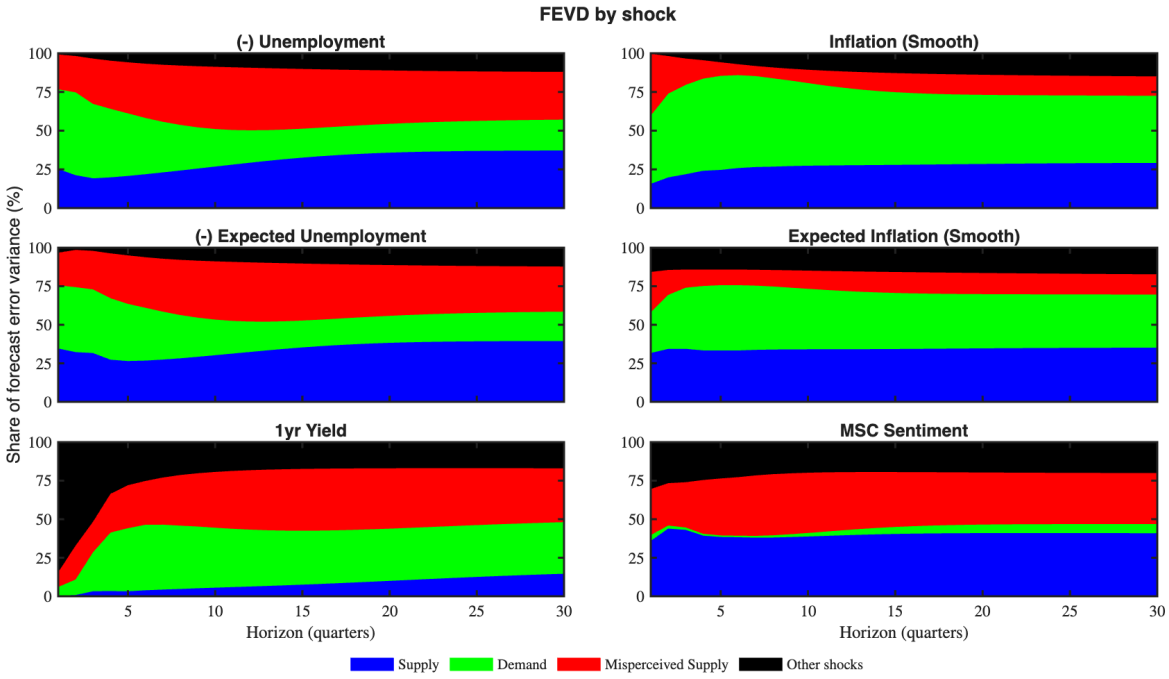


Figure 23: FEVDs

Source: Own Illustration, Data described as above. All estimations are performed in Matlab and are based on the VAR Toolbox "Empirical Macro Toolbox" by F. Ferroni and F. Canova.