An odd approximation of inflation for state-dependent pricing models

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Abstract

We show that, in a broad class of state-dependent pricing models, aggregate inflation can be approximated by an odd (third-order) polynomial in the average price gap—the cross-sectional mean of the difference between firms' optimal reset prices and their listed prices. The approximation, derived from first principles under mild assumptions on firms' value functions and adjustment hazards, preserves the key economic forces of state-dependent pricing—movements in the frequency of price adjustment and the selection effect—while collapsing the aggregate state from the full cross-sectional distribution of price gaps to a single scalar. This dimensionality reduction allows models that would otherwise require global solution methods to be solved using standard perturbation techniques. We embed the approximation in a standard general equilibrium setting with state-dependent pricing, yielding a tractable framework with a nonlinear Phillips curve whose slope steepens endogenously with the size of the shock. We show that the approximate model closely replicates the impulse responses of the globally solved model across shock sizes and is capable of accurately tracking the time series of inflation, including the post-2020 surge. Quantitative exercises show that aggressive monetary tightening is considerably more effective—and less costly in output—when the economy is hit by large supply-side disturbances, such as the energy price shocks that drove the post-2020 inflation surge, a result that linear models systematically miss.

Keywords: Inflation; Non-linear dynamics; State-dependence; Menu-cost models

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Non-technical summary

A central challenge in macroeconomics is aggregating the pricing decisions of heterogeneous firms. In state-dependent pricing models—where firms adjust prices only when the benefit of doing so exceeds a random fixed cost—aggregate inflation depends on the entire cross-sectional distribution of firms' price gaps, an infinite-dimensional object. Tracking this distribution requires computationally intensive global solution methods, which has limited the use of these models in applied work, despite their ability to generate the nonlinear inflation dynamics observed during episodes such as the post-pandemic surge.

This paper shows that, under mild assumptions, aggregate inflation in a broad class of state-dependent pricing models can be well approximated by a simple formula: an odd polynomial of the average price gap, the single number summarizing how far firms' listed prices are, on average, from their desired prices. The linear term captures the standard, Calvo-like pass-through of cost shocks into inflation. The cubic term captures the nonlinearities that distinguish state-dependent pricing (Caballero and Engel (1993); Alvarez et al. (2016)) from time-dependent pricing (Calvo 1983): when large shocks push many firms far from their optimal prices, both the frequency of price adjustment and the selection of which firms adjust rise sharply, amplifying the inflation response well beyond what a linear model would predict. Crucially, this approximation reduces the state of the economy from the full distribution of price gaps to a single variable, allowing the model to be solved with standard perturbation methods (e.g., Dynare) rather than computationally demanding global techniques.

We validate the approximation quantitatively in two settings. First, in a Golosov and Lucas (2007) model calibrated to Belgian manufacturing data, the approximate model closely replicates both the impulse responses and the time series of aggregate inflation generated by the fully solved model—including the sharp rise and fall of inflation during the post-2020 period—while the standard Calvo model captures only about two-thirds of the observed surge. Second, we embed the approximation in a standard New Keynesian model (Galí 2015), obtaining a tractable framework with a nonlinear Phillips curve. Policy experiments show that the sacrifice ratio—the output cost of reducing inflation—is much smaller when shocks are large than what the Calvo model implies, because the endogenous rise in the adjustment frequency makes monetary tightening considerably more effective. These findings suggest that central banks should not simply "look through" large supply shocks: ignoring the nonlinear amplification inherent in state-dependent pricing can lead to significant underestimation of inflation risks and overestimation of the costs of policy intervention.

1 Introduction

Canonical time-dependent models of price setting (e.g., Taylor 1980; Calvo 1983), widely used for the analysis of inflation dynamics and monetary policy, imply a linear relationship between inflation and movements in fundamentals. As such, they cannot reproduce the nonlinear patterns that emerge following large aggregate cost or demand shocks—patterns that the recent inflation surge made particularly evident. By contrast, models with state-dependent pricing—such as menu-cost models or models with state-dependent information frictions—naturally generate these nonlinearities, especially the sharp increases in the frequency of price adjustment that lead to nonlinear amplification of inflation in response to large shocks (Gagliardone et al., 2025a). Despite their empirical appeal, state-dependent models remain rarely used in applied macroeconomic work. A key deterrent is computational: their equilibrium requires tracking the full cross-sectional distribution of price gaps—an infinite-dimensional state variable—using global solution methods that are computationally intensive, difficult to estimate, and challenging to embed within standard DSGE frameworks. In environments with strategic complementarities in price setting and aggregate shocks, the computational burden is even greater, as the equilibrium involves a nested fixed-point problem between firms' pricing decisions and the aggregate price level—a structure closely related to mean-field games (Alvarez et al. 2023).

This paper develops a tractable approximation of aggregate price dynamics for a broad class of state-dependent pricing models. Starting from first principles of firms' pricing behavior, we show that, under mild assumptions on the firm's value function and the structure of adjustment hazards, aggregate inflation can be approximated by an odd (third-order) polynomial in the average price gap, x_t—defined as the cross-sectional mean of the difference between firms' optimal reset prices and their listed prices:

$$\pi_t \simeq \phi_0 \cdot x_t + \phi_1 \cdot x_t^3.$$

This mapping—the *odd approximation* of inflation—preserves the core economic mechanisms of state-dependent pricing while sharply reducing computational complexity. The approximation accounts for both movements in the frequency of price adjustment and the selection effect—the hallmarks of state-dependence—and therefore nests both time- and state-dependent pricing frameworks as special cases. When shocks are small, the cubic term is negligible and inflation is approximately linear in the price gap, as in Calvo models. When large aggregate shocks move the average price gap substantially, the cubic term becomes salient and captures the nonlinearities induced by the endogenous response of the adjustment frequency and the selection effect. Crucially, the approximation collapses the aggregate state from the entire cross-sectional

distribution of price gaps to a single state variable: the average price gap. This dimensionality reduction, akin to a Krusell-Smith-type quasi-aggregation result (Krusell and Smith 1998), makes it possible to solve environments that would otherwise require global methods using standard perturbation techniques, such as those implemented in Dynare.

Our theoretical results rest on three primitive conditions. First, the firm's indirect value function depends on the price gap—the difference between the optimal reset price and the listed price—as the sole idiosyncratic state variable (Lemma 1). Second, the reset price is a local optimum, so that the first derivative of the value function vanishes at zero gap. Third, the hazard rate of price adjustment depends on the value gain from resetting, which, combined with the first two conditions, yields a quadratic approximation of the firm-level hazard in the price gap (Lemma 2). These conditions are satisfied by a wide class of models, including the menu-cost frameworks studied by Caballero and Engel (2007), Golosov and Lucas (2007), and the Calvo-plus model of Nakamura and Steinsson (2010). Using the quadratic hazard and the statistical properties of cross-sectional cumulants, we show that aggregate inflation can be projected onto the odd moments of the price-gap distribution (Theorem 1). Under the additional assumption that the cross-sectional distribution of price gaps has approximately zero skewness, the third moment reduces to a polynomial in the first moment alone, yielding the odd approximation (Corollary 1). Importantly, the coefficients of the approximation are structural parameters determined by the steady-state pricing problem—specifically, by the intercept and curvature of the generalized hazard function and the cross-sectional variance of idiosyncratic shocks—so no additional free parameters are required to capture nonlinearities (Corollary 2).

We embed the odd approximation in two progressively richer general-equilibrium environments to validate its quantitative performance and illustrate its practical value. We first consider a variant of the Golosov and Lucas (2007) (henceforth GL) model, augmented with random menu costs and exogenous free-adjustment opportunities as in the Calvo-plus framework of Nakamura and Steinsson (2010). In this environment, the approximate model delivers a compact two-equation representation of the supply side—the odd-polynomial approximation of inflation and a closed-form law of motion for the average price gap—that replaces the distributional law of motion required by the global solution. Calibrating the model to moments from Belgian manufacturing microdata, we show that the approximate model closely replicates the impulse responses of inflation, output, and the frequency of price adjustment generated by the globally solved model across small, medium, and large monetary shocks. The accuracy across untargeted shock sizes is not a coincidence: it follows from Proposition 1, which establishes that the polynomial coefficients are structural objects determined at the steady state, so a single calibration simultaneously governs dynamics at all shock magnitudes.

We then perform a feed-in exercise using a time series of aggregate marginal-cost shocks recovered from Belgian manufacturing data. The approximate model tracks the observed dynamics of PPI inflation remarkably well—both during the low-inflation pre-COVID period and during the post-2020 surge and decline—with performance nearly identical to the globally solved menu-cost model and far superior to the Calvo model, which captures only about two-thirds of the observed inflation peak. A decomposition based on Corollary 1 reveals that, in normal times, inflation is driven primarily by the linear (Calvo) term. During inflation surges, however, the cubic term and the selection effect become quantitatively important and at times dominant, accounting for the amplification that the Calvo model cannot replicate.

We next embed the odd approximation in a standard New Keynesian model (Galí 2015) where the central bank sets the short-term interest rate according to a policy rule. Unlike the GL model, there is mutual feedback between pricing behavior and the real economy: marginal costs, and hence price gaps, are jointly determined in general equilibrium. Despite this added complexity, the approximate model remains easy to solve.

The odd approximation makes explicit the notion of a *nonlinear Phillips curve* in models with state-dependent pricing. The linear term ($\phi_0 x_t$) plays the role of standard linear term in the Calvo (1983) model, while the cubic term ($\phi_1 x_t^3$) generates an endogenous, state-dependent steepening: the *effective* slope of the Phillips curve rises with the square of the aggregate shock. This nonlinearity has a direct bearing on the interpretation of cost-push shocks. In the standard log-linearized NK model, a residual cost-push shock absorbs all discrepancies between observed inflation and the prediction of the linear Phillips curve. Our framework reveals that part of what the linear model attributes to exogenous cost-push shocks is in fact the cubic term of the odd approximation—an endogenous, higher-order function of the very same fundamentals that enter the linear term. Ignoring this endogeneity overstates the role of unexplained shocks in driving inflation and understates the scope for monetary policy to influence inflation through its effect on the price gap.

Quantitative exercises show that the nonlinear NK model delivers strongly nonlinear inflation responses to cost-push shocks relative to the canonical model with Calvo pricing. Because the endogenous rise in the adjustment frequency and the selection effect increase monetary neutrality when shocks are large, the sacrifice ratio—the output cost of reducing inflation by one percentage point—becomes state-dependent. We conduct two policy experiments that illustrate the practical implications, using cost-push shocks as a reduced-form stand-in for supply-side disturbances such as energy price increases.

First, we study whether central banks should "look through" large but transitory supply shocks. Under an accommodative monetary policy, a sequence of cost-push shocks generates an

inflation surge roughly twice as large in the menu-cost model as in the Calvo model, because the latter ignores the endogenous amplification through the extensive margin and the selection effect. A policymaker relying on a linear model would systematically underestimate inflation risks and might delay the policy response.

Second, we evaluate the inflation-output tradeoff under aggressive monetary tightening. In the menu-cost model, tighter policy reduces inflation significantly more, and with smaller output losses, than in the Calvo model. The sacrifice ratio is substantially lower when shocks are large, because the steepening of the Phillips curve means that a given tightening buys more disinflation per unit of foregone output. Akin to the results in Karadi et al. (2025) and Ascari et al. (2025), these exercises suggest that optimal monetary policy should itself be state-dependent, with the aggressiveness of the policy response calibrated to the magnitude of prevailing shocks.

The remainder of the paper is organized as follows. Section 2 presents the general theoretical framework, derives the odd approximation, and establishes the structural characterization of the impulse response functions. Section 3 embeds the approximation in the GL model, compares transitional dynamics with the globally solved model, and performs the inflation accounting and decomposition exercise using Belgian manufacturing data. Section 4 incorporates the approximation into the New Keynesian model, discusses the nonlinear Phillips curve and the interpretation of cost-push shocks, and presents the policy experiments on "looking through" large shocks and the state-dependent sacrifice ratio. Proofs and derivations are collected in the Appendix.

2 The general framework

We develop an analytical approximation of inflation tailored to capture aggregate price dynamics in models with state-dependent nominal rigidities, with time-dependent models arising as a limiting case. Under a mild set of assumptions on the primitives of the firms' pricing problem, we show that inflation can be characterized as a function of moments of the distribution of price gaps—the difference between firms' target (aka, ideal) prices and their listed prices. Our key aggregation result echoes the insight of Krusell and Smith (1998) that, in models with heterogeneous agents, aggregate dynamics can be accurately approximated by functions of low-order cross-sectional moments. In our setting, the first three moments of the price-gap distribution—mean, variance, and skewness—are sufficient in the most general case. Under additional distributional assumptions, movements in the average price gap alone suffice as a single state variable to accurately capture aggregate inflation. Our approach therefore provides

a tractable solution that sidesteps well-known computational challenges associated with solving models with heterogeneity (Moll 2025).
The proofs of all lemmas, theorems, propositions, and corollaries are reported in Appendix A.

2.1 The pricing block

In each period t, the economy is populated by a continuum of firms indexed by $f \in [0,1]$ selling a single differentiated product under imperfect competition facing a demand function with arbitrary (albeit finite) elasticity of substitution across products. Firms maximize intertemporal profits by choosing factor demands and pricing policies subject to nominal rigidities.

Throughout the paper, we use lowercase letters to denote the logarithms of the corresponding uppercase variables. We denote by $p_t(f) \equiv \ln(P_t(f))$ the firm's price and by $p_t \simeq \ln(P_t)$ the log-linearized aggregate price index (defined in Section 2.3).

Profit maximization. We consider general cost functions that allow us to express firm's nominal marginal cost as $mc^n(f) = \xi_t(f) + \nu y_t(f)$. Here $\xi_t(f)$ denotes a cost shifter, which depends on both an aggregate component (e.g, wages and intermediate inputs prices) and an idiosyncratic component (e.g., firm-level productivity, firm's appeal, or both). We allow $\xi_t(f)$ to follow a general Markov process with drift. $y_t(f)$ denotes firm-level output with the elasticity $\nu := \partial mc^n_t(f)/\partial y_t(f) \geq 0$ indexing short-run returns to scale (with $\nu = 0$ indicating constant returns).[1]

As is standard (e.g., Alvarez et al. 2023), we use a quadratic approximation to characterize the (real) flow profit of firm f at date t around the flexible (static) optimal price, denoted by $p^o_t(f)$:

$$\Pi_t(f) \equiv \Pi_t(p^o_t(f) - p_t(f)) := -\frac{B}{2}\left(p^o_t(f) - p_t(f)\right)^2, \tag{1}$$

where $B > 0$ is a constant normalizing the profit function by the firm's steady state profits. Specifically, up to a second-order approximation of profits, cost minimization implies that the flexible-price optimal price is given by a markup over the nominal marginal cost of production. Specifically, we have that:

$$p^o_t(f) = (1-\Xi)\left(\mu + \xi_t(f)\right) + \Xi\, p_t, \tag{2}$$

where $\mu > 1$ denotes the steady state markup and the parameter $\Xi \in [0,1]$ captures the curvature of the profit function, which can be driven by strategic complementarities in price settings as well

[1] To fix ideas, with Cobb-Douglas production technology and Hicks neutral productivity we have that $mc^n_t(f) = \left(w_t - \log(1+\nu)\right) - (1+\nu)a_t(f) + \nu y_t(f)$, where the first and second term denotes the aggregate and idiosyncratic component of the cost shifter, respectively.

as decreasing returns to scale production. In both cases, a firm's pricing decisions are influenced not only by its own unit cost of production but also by its competitors' pricing behavior.[2]

Denote by $\beta \in (0,1)$ the firm's discount factor. Each firm pays a fixed cost $\chi_t(f)$ upon changing its price. As in Caballero and Engel (2007), the fixed cost is a random variable, i.i.d. between firms and time, drawn from an exogenous and continuous distribution with finite density $G(\chi)$.

Given the realization of the menu-cost $\chi_t(f)$ and the firms' problem is to jointly choose whether to adjust prices, where $\mathbb{I}_t(f) = \{0,1\}$ and, conditional on adjustment, to the reset price:

$$V_t(f) = \max_{\{p_{t+i}(f),\mathbb{I}_{t+i}(f)\}_{i\geq 0}} \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \Big\{ \Pi_t(f) - \chi_{t+i}(f)\, \mathbb{I}_{t+i}(f) \Big\}. \tag{3}$$

We denote by $\{p^\star_{t+i}(f),\, \mathbb{I}^\star_{t+i}(f)\}_{i\geq 0}$ the policies that maximize the value of the firm.

The reset price. The first order condition of the problem (3) with respect to the price at the optimal reset price $p^\star_t(f)$ is given by:

$$0 = \left\{ \frac{\partial \Pi_t(f)}{\partial p_t(f)} + \beta \mathbb{E}_t \left(\frac{\partial V_{t+1}(f)}{\partial p_t(f)} \right) \right\} \Bigg|_{p_t(f)=p^\star_t(f)} \tag{4}$$

Iterating the first-order condition forward, we obtain an expression for $p^\star_t(f)$ as a function of current and expected static optimal prices:

$$p^\star_t(f) = \frac{\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left(\prod_{j=1}^{i} (1 - \mathbb{I}^\star_{t+j}(f)) \right) p^o_{t+i}(f)}{\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left(\prod_{j=1}^{i} (1 - \mathbb{I}^\star_{t+j}(f)) \right)} \tag{5}$$

where $\mathbb{E}_t \prod_{j=1}^{i} (1 - \mathbb{I}^\star_{t+j}(f))$ denotes the probability that a firm adjusts its price between t and $t+j$. Intuitively, as in Calvo (1983), because of nominal rigidities, firms recognize that the price set today may remain in effect for several periods, so expectations about future economic conditions influence current pricing decisions. Unlike the Calvo model, the probability of non-adjustment

[2] As shown in Gagliardone et al. (2025b), the parameter Ξ is given by the product of the strength of strategic price complementarities and a term that declines as firms' short-run returns to scale decrease. Under a broad class of models with imperfect competition—including monopolistic competition with variable elasticity of demand (Kimball 1995), static oligopoly (Atkeson and Burstein 2008), and dynamic oligopoly (Wang and Werning 2022)—variable markups imply that a firm's elasticity of demand declines as its market share increases. This feature generates strategic complementarities in price setting, which dampen the response of prices to marginal cost (Amiti et al. (2019)). Below, we introduce the parameter $\Omega \in [0,1)$ to capture the strength of these complementarities. Decreasing returns to scale in production ($\nu > 0$) produce a similar dampening effect through macroeconomic complementarities. Intuitively, aggregate shocks induce broad increases in marginal costs, compressing relative cost differences across firms. Consequently, the gap between a firm's optimal price and the aggregate price level rises by less, reducing the desired relative price adjustment.

is firm- and time-specific and varies as a function of the current and expected price gaps. These probabilities modify the effective discount factor in a time-varying way (Dotsey and King 2005): when the probability of a price adjustment in the near future is high, firms place less weight on expected cost and demand conditions beyond that horizon. and profit flows beyond that horizon have a low weight on today's pricing decisions. We write equation (5) in recursive form as:

$$\Lambda_t(f)\, p_t^\star(f) = p_t^o(f) + \beta\, \mathbb{E}_t \left(\left(1 - h_{t+1}(f)\right) \Lambda_{t+1}(f)\, p_{t+1}^\star(f) \right) \tag{6}$$

where the discounting $\Lambda_t(f)$—the denominator of the present value—can be recursively derived as:

$$\Lambda_t(f) = 1 + \beta\, \mathbb{E}_t \left\{ \left(1 - h_{t+1}(f)\right) \Lambda_{t+1}(f) \right\}. \tag{7}$$

Price adjustment decision. To characterize the discrete choice problem of adjusting the price we define the following firm-level objects:

(*i*) The firms' price gap—the log-deviation of today's optimal reset price (conditional on adjusting) relative to the price the firm is entering the period:

$$x_t(f) := p_t^\star(f) - p_{t-1}(f),$$

which corresponds to the firm's price change conditional on adjustment.

(*ii*) The reset gap—the difference between the static and dynamic optima:

$$x_t^\star(f) := p_t^o(f) - p_t^\star(f).$$

Throughout the paper, we assume homogeneity: $x_t^\star(f) = x_t^\star$ (Alvarez et al. 2016).[3]

(*iii*) The probability of price adjustment between two consecutive periods (aka, the hazard rate) before observing the realization of the random menu cost but conditional on a the realization of the price gap :

$$h_t(x) := \mathbb{E}_t\{\mathbb{I}_t^\star(f) \mid x_t(f) = x\}.$$

The following lemma characterizes the value function and the decision rules of the firm as a function of the firm's price gap.

Lemma 1 (Price gaps as forcing variable)**.** *The indirect value function* $x_t(f) \mapsto \mathcal{V}_t(x_t(f)) \in \mathbb{R}$ *satisfies the following conditions:*

[3]This is a mild assumption that holds under commonly used production technologies (e.g., Cobb Douglas or CES) featuring Hicks neutral productivity which is and i.i.d. process or evolves as a random walk. When these hold, we can write $p_t^\star(f) = p_t^o(f) + \Psi_t$. In these cases, the continuation value Ψ_t is independent of f.

(i) Recursive representation:

$$\mathcal{V}_t(x) = \Pi(x_t^\star + x) + \beta\, \mathbb{E}_t\Big[h_{t+1}(x')\Big(\mathcal{V}_{t+1}(0) - \chi_{t+1}(f)\Big) + \big(1 - h_{t+1}(x')\big)\,\mathcal{V}_{t+1}(x')\Big],$$

(ii) Interior optimum: $\mathcal{V}_t'(0) = 0, \quad \mathcal{V}_t''(0) < 0.$

(iii) Optimality principle: $V_t(f) = \max\Big\{\mathcal{V}_t(x_t(f)),\ \mathcal{V}_t(0) - \chi_t(f)\Big\}$

Moreover, if $x_t^\star \equiv x^\star$ and the conditional law of motion of $x_{t+1}(f)$ is time-homogeneous (stationary environment), then $\mathcal{V}_t(\cdot) \equiv \mathcal{V}(\cdot)$ and $h_t(\cdot) \equiv h(\cdot)$.

Lemma 1 states that, the price gap $x_t(f)$—or, equivalently, $p_t^\star$ given p_{t-1}—is the only idiosyncratic variable relevant for firm's f price adjustment decision in period t. Moreover, in a stationary environment, the indirect value function and hazard rate are time-independent functions of the price gap.

When the conditions in Lemma 1 hold, we can derive a second-order approximation the hazard rate as a convex (quadratic) polynomial in the firm's price gap.

Lemma 2 (Quadratic approximation of the hazard rate)**.** *The probability price adjustment between any two consecutive periods can be approximated up to second order by:*

$$h_t(x_t(f)) = h_t(0) + \phi_t x_t^2(f) + o(x_t^2(f)) \tag{8}$$

where $h_t(0) \geq 0$ and $\phi_t := -\frac{1}{2}(1 - h_t(0))G'(0)\mathcal{V}_t''(0) \geq 0$ and $o(x)/x \to 0$ as $x \to 0$.
In a stationary environment, $h_t(0) = h(0)$ and $\phi_t = \phi$.

The quadratic approximation of the hazard mirrors the results in Alvarez et al. (2022).[4] Note that $h(0)$ could be zero or greater than zero when firms are given the possibility of a free price adjustment, as, e.g., in the Calvo-plus model of Nakamura and Steinsson (2010)). Empirical evidence lends strong support in favor of this result. For example, using various micro-level datasets Gagnon et al. 2013; Gautier et al. 2023; Karadi et al. 2024, document a convex relationship between probability of price adjustments and some empirical approximations of firm-level price gaps. Gagliardone et al. 2025a provide direct evidence in favor of the quadratic approximation.

[4]Note that, the linear term in the hazard, drops because of the optimality condition ($\mathcal{V}_t'(0) = 0$). Note also that the quadratic approximation holds in a broad class of state dependent pricing models, e.g., the menu-cost frameworks studied by Caballero and Engel (1993; 2007), Golosov and Lucas (2007), and the Calvo-plus model of Nakamura and Steinsson (2008). It also encompasses time-dependent pricing models as special cases. For example, the Calvo (1983) model arises when the upper bound of the menu-cost distribution is arbitrarily large, firms' hazard rates are uniform across firms and independent of their state ($\phi = 0$), and $h(0)$ is recalibrated to match the steady state frequency.

2.2 The odd approximation of aggregate inflation

Under the second-order approximation of the profits, the aggregate price index, p_t, can be written as the cross-sectional simple average of firm-level prices, $p_t(f)$. It follows that aggregate inflation, $\pi_t := p_t - p_{t-1}$ is given by:

$$\begin{aligned}\pi_t &= \int_0^1 \left(p_t(f) - p_{t-1}(f)\right) df \\ &= \int_0^1 \mathbb{I}_t^\star(f)\, df \cdot \int_0^1 x_t(f)\, df + \mathbb{C}\text{ov}\left(\mathbb{I}_t^\star(f), x_t(f)\right) \\ &= h_t \cdot x_t + \mathbb{C}\text{ov}\left(h_t(f), x_t(f)\right) \end{aligned} \tag{9}$$

The first line shows that aggregate inflation—the average of firm-level price adjustments—is equal to the product of a firm's adjustment probability and its price change conditional on adjustment. The second line decomposes inflation into (i) the product of the average frequency of price adjustment $h_t := \int_0^1 \mathbb{I}_t^\star(f)\, df$, (the *extensive margin*) and the average price gap $x_t := \int_0^1 x_t(f)\, df$ (the *intensive margin*) and (ii) the cross-sectional covariance between the two variables (the *selection effect*): firms that are more likely to adjust are also those that change their prices the most conditional on adjustment (Caballero and Engel 2007; Golosov and Lucas 2007). The third line uses the law of iterated expectations to rewrite the covariance. Note that Equation (9) holds generally for both time- and state-dependent pricing models. Movements in the average frequency and in the covariance term arise from the state-dependent nature of firms' pricing decisions. In time-dependent models (e.g., Taylor (1980) and Calvo (1983)), these margins are absent, as the frequency is exogenous and the covariance is zero.

The theorem and corollary that follow lay the foundation for the central result of the paper: aggregate inflation can be approximated by projecting onto the moments of the price gap distribution. This approach substantially reduces the dimensionality of the state space—from an infinite-dimensional distribution to a finite set of low-order moments—while preserving the key economic forces underlying Equation (9). Define the cross-sectional distribution of gaps at time t and its moments by:

$$F_t(x) := \Pr\left(x_t(f) \leq x\right), \qquad m_{k,t} := \int x^k\, dF_t(x) = \mathbb{E}_t\left[x_t(f)^k\right].$$

Theorem 1 (Projection onto moments)**.** *Inflation satisfies the following representation in terms of the odd moments of the distribution of price gaps:*

$$\pi_t = h_t(0)\, m_{1,t} + \phi_t\, m_{3,t} + u_t. \tag{10}$$

where $u_t := \int x\, r_t(x)\, dF_t(x)$ *is an approximation error and* $r_t(x) = o(x^2)$ *pointwise.*

If additionally there exists a $C < \infty$ such that $supp(F_t) \subseteq [-\bar{x}_t, \bar{x}_t]$ with $\bar{x}_t^2 \leq C\,\sigma_t^2$, where the variance is defined as $\sigma_t^2 \equiv m_{2,t} - m_{1,t}^2$, then the error term is of the order $o(\sigma_t^3)$.

To obtain Equation (10), we use the quadratic approximation of $h_t(f)$ to express the adjustment probabilities as a function of the price gap and then make use of the statistical properties of cumulants to characterize cross-sectional integrals as a function of lower-order moments of the distribution of price gaps. This step allows us to aggregate across firms by linking inflation to the cross-sectional moments of the gap distribution. The approximation error of the projection is due to the average approximation error of the individual hazards (which is smaller than quadratic), and therefore depends on the cross-sectional variance of the price gap distribution. Provided that the inaction region can be bounded—as is typically the case,— the error converges to zero in the limit of no heterogeneity at an order of magnitude faster than the variance.

The following corollary of Theorem (1) discusses the economic intuition focusing on a stationary environment.

Corollary 1 (Odd approximation of inflation)**.** *Let $x_t := m_{1,t}$ denote the mean of the distribution of price gaps at time t, $\sigma_t^2 \equiv m_{2,t} - x_t^2$ its variance, and $\psi_t \equiv m_{3,t} - x_t^3 - 3x_t\sigma_t^2$ its skewness. In a stationary environment, inflation satisfies the following decomposition:*

$$\pi_t = \underbrace{\left(h(0) + \phi\sigma_0^2\right)x_t}_{\text{Calvo term}} + \underbrace{\phi\left((\sigma_t^2 - \sigma_0^2)x_t + x_t^3\right)}_{\text{Cubic term}} + \underbrace{\mathbb{C}\text{ov}(h_t(f), x_t(f))}_{\text{Selection term}} + o(\sigma_t^3) \tag{11}$$

where the selection term is given by:

$$\mathbb{C}\text{ov}(h_t(f), x_t(f)) = \underbrace{2\phi\sigma_0^2 x_t}_{\text{Linear effect}} + \underbrace{\left(2\phi(\sigma_t^2 - \sigma_0^2)x_t + \phi\psi_t\right)}_{\text{Cubic effect}}$$

The economic intuition behind formula (10) is as follows. The first term captures the first-order inflationary impact of aggregate shocks around the steady state. As in a Calvo (time-dependent) model, this is precisely precisely the product of the steady-state frequency ($h(0) + \phi\sigma_0^2$) and the aggregate shock (x_t). The other two terms arise from the state-dependent nature of the pricing behavior of firms. The "cubic term" captures the nonlinearities due to the increase in the average frequency of price changes in response to the aggregate shock. The aggregate shock x_t affects the frequency *directly* by shifting the the price gap distribution along the (quadratic) hazard function: this gives raise to the cube x_t^3. As we discuss below, the shock also affects the frequency indirectly by increasing the variance of price gaps away from its steady state value ($\sigma_t^2 - \sigma_0^2$). Finally, our approximation accounts for the selection effect. As we show in Appendix A, the covariance between firm-level gaps and hazards is given by a first-order term that captures selection in steady

state ($2\phi\sigma_0^2 x_t$), and a higher-order term ($2\phi(\sigma_t^2-\sigma_0^2)x_t+\phi\psi_t$) which scales non-linearly with the size of the shock and depends on depends on the third central moment (unscaled skewness). Finally, the approximation error keeps track of the change in the inaction region due to the aggregate shock, which has an impact on inflation that is smaller than first order.

With small aggregate shocks, the inflation is driven by the Calvo term and the steady-state component of the covariance, which both have first-order effects. The cubic term is negligible and the mapping between average price gaps and inflation (i.e., expected price changes) is approximately linear. Thus, the approximation is consistent with the near-equivalence between state-dependent and time-dependent models documented in Gertler and Leahy (2008), Alvarez et al. (2017), and Auclert et al. (2024). With large shocks, the cubic term becomes significant and the variance scaled up non-linearly, capturing the nonlinearities arising from the state-dependent nature of firms' price adjustments.

Importantly, the approximation in Equation (10) preserves the key economic forces underlying the inflation general decomposition presented in Equation (9). To see this, note that the cross-sectional average of the firm-level (quadratic) hazards gives the following expression for the average frequency of price adjustment in period t:[5]

$$h_t \simeq h(0) + \phi\left(\sigma_t^2 + x_t^2\right) \tag{12}$$

Using this expression inside Equation (10) and re-arranging we obtain that:

$$\pi_t \simeq h_t \cdot x_t + \mathbb{C}\text{ov}\left(h_t(f), x_t(f)\right).$$

As one wants, aggregate inflation is approximated by interaction between the pure extensive margin and intensive margins and a selection effect. Due to the state-dependent nature of firms' pricing decisions, the extensive margin and the selection effect are endogenous objects that scale (non-linearly) with the size aggregate shock (i.e., with the average price gap).

Nonlinear impulse response functions. The inflation approximation is particularly well suited for computing the impulse response of inflation to aggregate shocks that directly or indirectly affect firms' pricing decisions (e.g., cost-push, monetary, or TFP shocks), accounting for nonlinearities. We show that, the impulse response function is approximated we can recover the IRF of the price level (the path of inflation), up to third-order, from knowledge of the path of

[5]More precisely, because the hazard is quadratic, simple averaging yields the average frequency of price changes gives $h_t = h(0) + \phi\left(\sigma_t^2 + x_t^2\right) + o(\sigma_t^2)$, where the approximation error $o(x_t^2 + \sigma_t^2) = o(\sigma_t^2)$ vanishes faster than the dispersion in price gaps. As discussed below, an aggregate shock increases the frequency of adjustment directly through the squared displacement of the mean gap and indirectly through an increase in dispersion. The term $o(\sigma_t^2)$ reflects the fact that aggregate shocks may shift the inaction region, but these effects are typically small—indeed smaller than the variance of price gaps.

the average price gap and the steady-state derivatives of the higher-order moments to the shock.

Specifically, we study the dynamic response of aggregate inflation to a small aggregate disturbance of size δ, starting from a symmetric stochastic steady state with $\delta = 0$. To make explicit the dependence on the shock, let $x(f;\delta)$ denote the price gap after the aggregate disturbance, $x(\delta) := \mathbb{E}[x(f;\delta)]$ the corresponding average gap, and F^δ the cross-sectional distribution of $x(f;\delta)$. We assume the distribution admits a first-order expansion in the sense of signed measures:

$$F^\delta = F_0 + \delta \widehat{F} + o(\delta),$$

where F_0 denotes the symmetric steady-state distribution and $\widehat{F}$ is the signed "impulse" measure capturing the linearized effect of the shock. Moreover, as in Alvarez et al. (2016), we require that the signed measure is antisymmetric, i.e. $\widehat{F}(A) = -\widehat{F}(-A)$ for all Borel sets A. The key implication of the antisymmetry is that the first-order displacement of the cross-sectional distribution does not change even moments (variance, kurtosis, etc.) at first order.

Define $\pi(\delta) = p(\delta) - p_0$ the growth rate of the aggregate price in response to the same shock. The next proposition characterizes the trajectory of the average frequency and of aggregate inflation as (even and odd) polynomials of the mean of the price gap distribution.

Proposition 1 (Impulse response functions)**.** *Up to a third-order approximation, the impulse response function of the average frequency of price adjustment and aggregate inflation to a small aggregate shock of size δ are given, respectively, by an even polynomial and an odd polynomial of the mean of the price gap distribution:*

$$h(\delta) = \alpha_0 + \alpha_2 x^2(\delta) + o(\delta^3) \tag{13}$$

$$\pi(\delta) = \pi_0 + \beta_1 x(\delta) + \beta_3 x^3(\delta) + o(\delta^3) \tag{14}$$

where $\pi_0 \equiv o(\sigma_0^3)$, the coefficients of the polynomials are:

$$\alpha_0 \equiv h(0) + \phi\sigma_0^2, \qquad \frac{\alpha_2}{\phi} \equiv 1 + \frac{\sigma_{xx}^2}{2}$$

$$\beta_1 \equiv h(0) + 3\phi\sigma_0^2 + \phi\psi_x, \qquad \frac{\beta_3}{\phi} \equiv 1 + \frac{3}{2}\sigma_{xx}^2 + \frac{1}{6}\psi_{xxx}$$

and the derivatives of the moments are defined as:

$$\sigma_{xx}^2 \equiv \frac{\partial^2\sigma^2}{\partial\delta^2}\left(\frac{\partial\delta}{\partial x}\right)^2\bigg|_{\delta=0}, \qquad \psi_x \equiv \frac{\partial\psi}{\partial\delta}\frac{\partial\delta}{\partial x}\bigg|_{\delta=0}, \qquad \psi_{xxx} \equiv \left(\frac{\partial^3\psi}{\partial\delta^3}\left(\frac{\partial\delta}{\partial x}\right)^3 + \frac{\partial\psi}{\partial\delta}\frac{\partial^3\delta}{\partial x^3}\right)\bigg|_{\delta=0}.$$

The following Corollary provides closed-form expressions for the coefficient of the impulse response functions, $\alpha_0, \alpha_2, \beta_1, \beta_3$, as a function of parameters that govern the steady state of the model: the intercept and curvature of the generalized hazard function around optimum reset gap

($h(0)$ and ϕ) and the dispersion of the idiosyncratic shocks affecting firm's marginal costs—i.e., cross-sectional variance of $\xi_t(f)$.

Corollary 2 (Closed-form coefficients)**.** *Let $\epsilon(f) \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_\epsilon^2)$ be the idiosyncratic disturbance, normally distributed with mean zero in the cross-section. Denote by $h_0 \equiv h(0)$. Assume that $\epsilon(f) \perp x(f;0)$, $\sigma_\epsilon^4 \simeq 0$, and $\psi(\delta) \simeq \psi(0)$ approximately constant. The impulse response function of inflation to an aggregate shock that moves the price gaps by δ is characterized, to a third-order approximation, by the following coefficients:*

$$\begin{aligned}
\alpha_0 &\simeq h_0 + \phi \frac{h_0(1-h_0)\sigma_\epsilon^2}{h_0^2 + 6\phi\sigma_\epsilon^2} \\
\frac{\alpha_2}{\phi} &\simeq 1 + \frac{h_0\left(h_0^2 + 6\phi\sigma_\epsilon^2\right)\left(h_0^2(1-h_0) + 6\phi\sigma_\epsilon^2(1-2h_0)\right)}{\left(h_0^2(1-h_0) + 3\phi\sigma_\epsilon^2(2-3h_0)\right)^2} \\
\beta_1 &\simeq h_0 + 3\phi \frac{h_0(1-h_0)\sigma_\epsilon^2}{h_0^2 + 6\phi\sigma_\epsilon^2} \\
\frac{\beta_3}{\phi} &\simeq 1 + 3\frac{h_0\left(h_0^2 + 6\phi\sigma_\epsilon^2\right)\left(h_0^2(1-h_0) + 6\phi\sigma_\epsilon^2(1-2h_0)\right)}{\left(h_0^2(1-h_0) + 3\phi\sigma_\epsilon^2(2-3h_0)\right)^2}.
\end{aligned}$$

Corollary 2 has direct quantitative relevance. The parameters $h_0, \phi, \sigma_\epsilon^2$ can be disciplined using steady-state moments of the price gap distribution (or price changes, under standard invertibility results) and the frequency of price adjustment, which are commonly targeted moments used in the literature.[6]

2.3 Closing the general equilibrium

Household. There is a representative household that chooses consumption C_t and labor supply L_t to maximize life-time utility,

$$\max_{C_t, L_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big(\frac{C_t^{1-\gamma}}{1-\gamma} - \frac{L_t^{1+\varphi}}{1+\varphi} \Big),$$

where $\gamma > 0$ denotes the inverse of the intertemporal elasticity of substitution in consumption, and $\varphi > 0$ denotes the inverse Frisch elasticity of labor supply. The aggregate consumption C_t is

[6] As shown in the Appendix, to derive these closed-form solutions above we considered perturbations of the firm-level gaps (before adjustment) of the form $x_t'(f) = x_t(f) + \delta + \epsilon(f)$. Then trace down the effect on the distribution of price gaps (after adjustment): $x_{t+1}(f) = (1 - h(x_t'(f)))x_t'(f)$. In this setting, , using the fact that the price distribution does not move in the absence of the aggregate shock ($\mathbb{E}((x_t'(f))^k) = \mathbb{E}((x_{t+1}(f))^k)$ when $\delta = 0$), we can characterize the steady state moments of the price gap distribution in terms of moments of the ergodic distribution of idiosyncratic shocks. To simply the closed-form expressions, Corollary 2 assumes that the the aggregate shock does not affect the skewness of the price gap distribution ($\psi(\delta) \simeq 0$). In the Appendix, we present a more general proposition that relaxes this assumption.

a Kimball aggregator of the differentiated goods produced by firms:

$$\int_0^1 \Upsilon\left(\frac{C_t(f)}{C_t}\right) df = 1, \tag{15}$$

where $\Upsilon(\cdot)$ is a continuous and twice-differentiable function, strictly increasing and strictly concave, and homogeneous of degree one with $\Upsilon(1) = 1$. The associated aggregate price index, P_t, is implicitly defined as:

$$\int_0^1 \Upsilon \circ (\Upsilon')^{-1}\left(\Upsilon'(1)\frac{P_t(f)}{P_t}\right) di = 1$$

The household can transfer resources intertemporally by purchasing one-period nominally risk-less discount bonds, B_t, which yield a gross nominal yield $i_t > 1$. The intertemporal budget constraint is given by

$$P_t C_t = W_t L_t + \Pi_t - B_t + i_t B_{t-1} \tag{16}$$

where W_t denotes the nominal wage and Π_t the aggregate dividend (net of menu costs) accruing from the ownership of the production sector. Finally, households' expenditure need to satisfy a cash-in-hand constraint

$$P_t C_t \leq M_t, \tag{17}$$

where M_t denotes its nominal money balance. This constraint might or might not be binding in equilibrium. The household optimality conditions for labor and the Euler equation for consumption are given by:

$$\frac{W_t}{P_t} = C_t^{\gamma} L_t^{\varphi} \tag{18}$$

$$\frac{1}{\beta} \geq \mathbb{E}_t\left\{\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \frac{P_t}{P_{t+1}} i_{t+1}, \right\} \tag{19}$$

where the latter holds with equality when the cash-in-hand constraint is slack. Moreover, the market clearing condition holds for all firms:

$$C_t(f) = Y_t(f). \tag{20}$$

Monetary policy. To close the model we need to take a stance on how the interest rate i_t and money supply M_t are determined. The choice of the policy block is model specific, as we discuss in the applications presented below.

3 Accounting for inflation

We conduct quantitative exercises to illustrate the computational advantages and practical applications of our inflation approximation in models with state-dependent pricing. We begin by comparing the transitional dynamics generated by a globally solved menu-cost model with those implied by our approximate model. We then perform a feed-in exercise designed to account for the time series of aggregate inflation and to decompose it into its linear and nonlinear components. To these ends, we employ a variant of Golosov and Lucas (2007) (henceforth GL), a simple general equilibrium framework in which random menu costs generate state-dependent pricing, and the aggregate component of firms' marginal cost (wages) is entirely driven by the exogenous evolution of the money supply.

3.1 The Golosov-Lucas model

We consider a simplified version of the household problem in Section (2.3), assuming a unit intertemporal elasticity of substitution and infinitely elastic labor supply ($\gamma = 1$ and $\varphi = 0$). Preferences are therefore log-additive, $U(C_t, L_t) = \log(C_t) - L_t$, subject to the budget constraint, the cash-on-hand constraint, and the Kimball aggregator (Equations 16—15). The central bank controls aggregate money supply, which follows a random walk:

$$m_t = m_{t-1} + g_t, \qquad g_t \sim N(0, \sigma_g) \tag{21}$$

We refer to g_t as a monetary (or money) shock. Assuming the cash-on-hand constraint binds in equilibrium, nominal wages move one-for-one with money, $m_t = w_t$. Combining this with labor supply optimality and the aggregate resource constraint (Equations 18 and 20), aggregate demand satisfies:

$$y_t = m_t - p_t. \tag{22}$$

Equations (21) and (22) fully characterize the demand-side of the model.

On the supply side, firms hire labor to produce output using a constant-returns-to-scale technology with Hicks neutral log productivity composed by an idiosyncratic component, $a_t(f)$, and an aggregate component a_t. Both components evolve as random walks with innovations given by $\varepsilon_t(f) \overset{\text{iid}}{\sim} N(0, \sigma_\varepsilon)$ and $\nu_t \overset{\text{iid}}{\sim} N(0, \sigma_\nu)$. It follows that firm's nominal log marginal cost is given by:

$$mc_t^n(f) = m_t - a_t(f) - a_t, \tag{23}$$

and therefore also evolves as a random walk.[7]

[7] As discussed in Section 2.1 and shown in Section 4, the results underlying our approximation of hazards

The pricing problem is nested within the general framework presented in Section 2.1. Firms face nominal rigidities in the form of random menu costs of price adjustment. Furthermore, as in the Calvo-plus model of Nakamura and Steinsson (2010), they have random opportunities to adjust their price without paying any fixed cost with probability $h(0) \in (0, 1)$. Moreover, Kimball's demand generates a scope for strategic complementarities in price setting.

The optimal reset price is given by the forward-looking, recursive expression in Equation (6), with the static (flexible-price) optimum given by Equation (2), a convex combination of the firm's own marginal cost realization and competitors' prices. Moreover, the assumptions behind Lemma 1 apply and we can characterize firm's adjustment choices as a function of the firms' price gap $x_t(f)$. Specifically, assuming stationarity, the optimal hazard rate is given by:

$$h(x_t(f)) = h(0) + \big(1 - h(0)\big)\mathbb{P}_\chi\Big\{\mathcal{V}(0) - \chi_t(f) \geq \mathcal{V}(x_t(f))\Big\}. \tag{24}$$

In the first line, the second term is the probability of adjusting conditional on receiving no free adjustment opportunity. This depends on whether the difference between the values of adjusting, net of the menu cost, exceeds the value of not adjusting.

The global solution. The general equilibrium of the model and its dynamics are characterized by the law of motion for money (Equations 21), the aggregate demand equation (Equation 22), aggregate inflation (Equation 9), and the cross-sectional distribution of firms' price gaps F_t.

As is standard in state-dependent pricing models, no general closed-form solution exists, and the equilibrium must be computed numerically using global methods. Doing so requires overcoming two distinct but closely related computational challenges.

(i) An infinite-dimensional state space—Because aggregate inflation depends on the *entire* cross-sectional distribution of price gaps, F_t, the state of the firm's problem is an infinite dimensional object. Following an aggregate shock—such as a monetary innovation g_t—the entire distribution shifts. Because both the probability of adjustment and the size of price changes, $h_t(f)$ and $x_t(f)$, depend on a firm's position within this distribution, computing aggregate inflation requires tracking the full measure F_t, an infinite-dimensional object. Its equilibrium evolution is governed by:

$$F_{t+1}(x') = \int \Big[h(x)\,\Phi(x' - x^\star - g_t) + \big(1 - h(x)\big)\,\Phi(x' - x - g_t)\Big]dF_t(x),$$

where Φ denotes the cumulative distribution function of idiosyncratic productivity shocks. This operator combines endogenous *selection*—firms that adjust jump to the reset price—with stochastic

and inflation extend to general environments in which marginal cost follows a general Markov process (e.g, an ARMA process) and production exhibits non-constant returns to scale.

drift for non-adjusting firms. Because adjustment probabilities depend nonlinearly on gaps, the transition law is itself nonlinear in F_t. In particular, aggregation cannot be reduced to a finite set of sufficient statistics: the evolution of moments depends on higher-order features of the distribution. This curse of dimensionality implies that equilibrium computation requires either discretizing the distribution or simulating a large panel of firms, both of which are computationally intensive.

(ii) A fixed-point over distributions—The second challenge is even more demanding. Firms are rational and forward-looking. In the presence of strategic complementarities, the optimal reset price $p_t^\star(f)$ depends on expected future aggregate prices. Yet future aggregate prices depend on future price-gap distributions, which in turn depend on firms' collective pricing policies. Firms must therefore form expectations over the evolution of an infinite-dimensional distribution whose dynamics depend on their own behavior. Computing the equilibrium and transitional dynamics of the model thus requires solving a nonlinear fixed-point problem over functions: firms' optimal policies depend on forecasts of F_t, while the evolution of F_t depends on those same policies. In this sense, the model has the structure of a mean-field game (Alvarez et al. 2023).[8]

Even in the stylized equilibrium environment of the GL model, these two challenges render global solutions computationally burdensome and limit the feasibility of standard quantitative exercises—such as estimation, filtering, or counterfactual policy analysis—that are routinely conducted in time-dependent pricing models.

The approximate solution. We build on the results in Section 2 to approximate pricing block of the model. The following system of equations in endogenous variables $\left\{y_t, m_t, a_t, \pi_t, p_t, \Lambda_t, p_t^\star, p_t^o\right\}$ and exogenous shocks $\{g_t, \nu_t\}$ characterizes the aggregate equilibrium dynamics of the model:

[8] See Moll (2025) for a discussion of the computational challenges posed by heterogeneous-agent models under rational expectations, and Alvarez et al. (2023) for a mean-field treatment in state-dependent pricing models.

Golosov-Lucas model with odd approximation

$$y_t = m_t - p_t \tag{GL–1}$$

$$x_t = p_t^\star - p_{t-1} \tag{GL–2}$$

$$p_t = \pi_t + p_{t-1} \tag{GL–3}$$

$$p_t^o = (1-\Omega)(\mu + m_t - a_t) + \Omega p_t \tag{GL–4}$$

$$\pi_t = \beta_1 x_t + \beta_3 x_t^3 \tag{GL–5}$$

$$h_t = \alpha_0 + \alpha_2 x_t^2 \tag{GL–6}$$

$$\Lambda_t p_t^\star = p_t^o + \beta\, \mathbb{E}_t \left\{ (1-h_{t+1}) \Lambda_{t+1} p_{t+1}^\star \right\} \tag{GL–7}$$

$$\Lambda_t = 1 + \beta\, \mathbb{E}_t \left\{ (1-h_{t+1}) \Lambda_{t+1} \right\} \tag{GL–8}$$

$$a_t = a_{t-1} + v_t \tag{GL–9}$$

$$m_t = m_{t-1} + g_t \tag{GL–10}$$

Equations (GL–1) is the aggregate demand equation. Equations (GL–2) through (GL–3) are the definition of the aggregate price gap $x_t := \int x dF_t(x)$, an identity which defines the aggregate price index, and the aggregate static optimum—a convex combination of aggregate marginal cost (Equation 2). (GL–5) and (GL–6) follow from Proposition 1 and Corollary 2, which characterize the dynamics of the average adjustment frequency and aggregate inflation as functions of the aggregate price gap. To obtain (GL–7) and (GL–8), we take cross-sectional averages of Equations (6) and (7) and used the assumption that idiosyncratic shocks are random walk, yielding recursive representations for the aggregate reset price and the effective discount factor.[9] Equations (GL–9) and (GL–10) are the dynamics of the exogenous processes for money and productivity.

The recursive representation in (GL–1)—(GL–10) makes explicit how the approximation developed in Section 2 reduces the infinite-dimensional state space of the global model to a one-dimensional aggregate state, thereby substantially lowering the computational burden. While the global solution requires tracking the full distribution of price gaps, the approximate model summarizes cross-sectional heterogeneity through its first moment. In this sense, the economy evolves as if governed by a representative firm whose state variable is the average price gap. The resulting system can be solved using standard perturbation methods, such as those implemented in Dynare, preserves the nonlinear dynamics of these aggregate variables induced by state-dependent pricing up to third-order accuracy.

[9] See Appendix A.9 for the derivations.

3.2 Comparison of transitional dynamics

We assess the ability of the approximate model to replicate the transitional dynamics of inflation, output, and the frequency of price adjustment generated by the globally solved menu-cost model in response to aggregate money shocks of varying sizes.

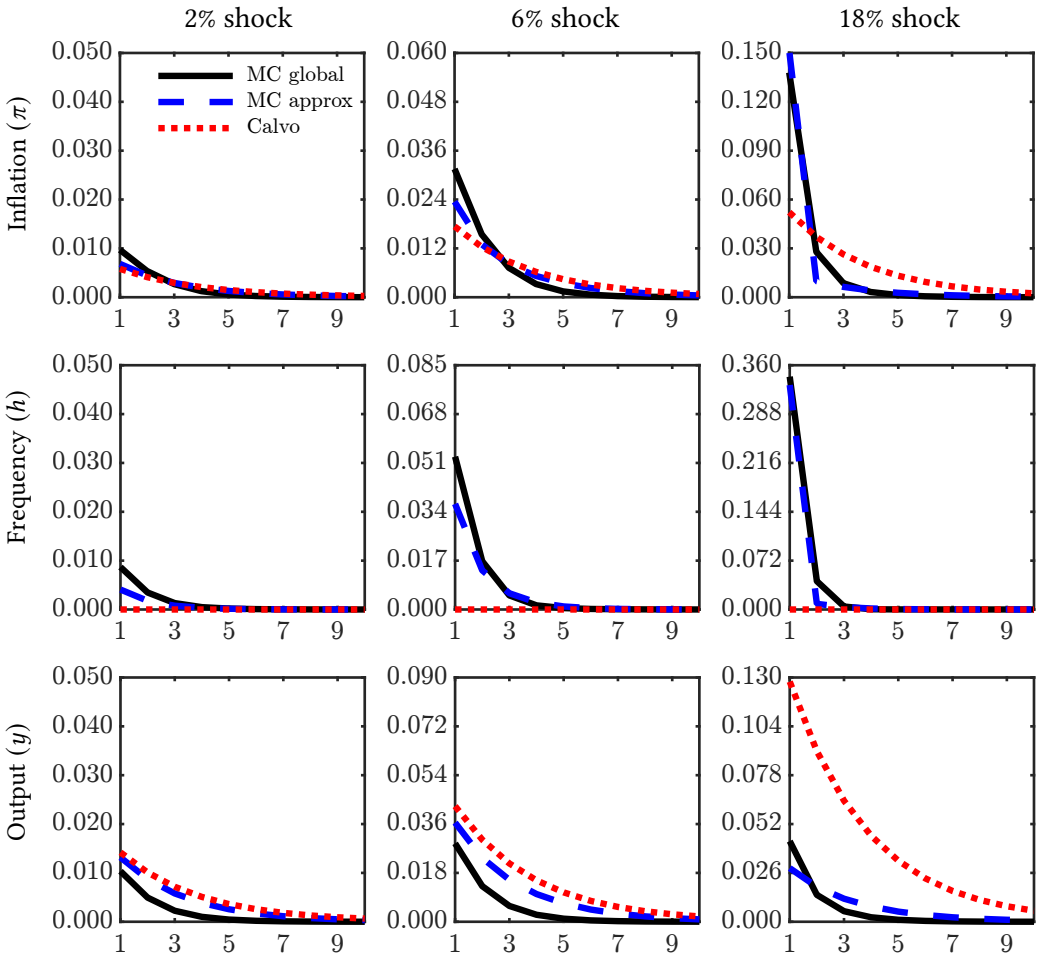
We calibrate the global menu-cost model following Gagliardone et al. (2025a), targeting empirical moments from the joint distribution of quarterly price changes and price gaps in the Belgian manufacturing sector. The elasticity of substitution and discount factor are set to conventional values, $\{\vartheta, \beta\} = \{6, 0.99\}$. The IRFs are computed using global solution methods.

For the approximate model, we retain ϑ and β and calibrate the coefficients $\{\alpha_0, \alpha_2, \beta_1, \beta_3\}$ to match the IRFs of inflation, output, and adjustment frequency generated by the calibrated global model in response to a moderate (6 percent) money shock. A medium-sized shock provides a natural calibration benchmark: it is large enough to reveal nonlinearities while remaining free of distortions driven by extreme responses. We additionally impose the restriction that the average quarterly frequency of price adjustment coincides across the global and approximate models. The IRFs of the approximate model are obtained via a third-order perturbation expansion of the policy functions around the deterministic steady state, as implemented in Dynare.

As a benchmark, we also compare both models to the Calvo model, which corresponds to the first-order approximation of the menu-cost model. The Calvo model is calibrated by setting $\bar{\chi}$ arbitrarily large and choosing the free-adjustment probability to match the average quarterly frequency of price adjustment in the data. As is standard, the Calvo model is solved in Dynare using a first-order perturbation expansion.

Figure 1 reports the impulse responses of inflation, output, and the average frequency of price adjustment across the three models following one-time, unanticipated money shocks of different magnitudes, $g_0 = 2, 6, 18$ percent. The global menu-cost and Calvo models are shown as blue and red dashed lines, respectively. As is well known, the two models exhibit observationally equivalent linear pass-through in response to small shocks (Gertler and Leahy 2008; Auclert et al. 2024; Gagliardone et al. 2025a). As the shock size increases, however, the response of inflation becomes markedly nonlinear in the menu-cost model, while remaining proportional in the Calvo model. The amplification of nonlinearities is evident: on impact, an 18 percent shock generates an inflation response roughly six times larger than that of a 6 percent shock, despite being only three times as large—a convexity that the log-linear Calvo model, by construction, cannot replicate. These nonlinearities arise because large shocks displace firms into regions of the price-gap distribution where the generalized hazard function is steep, increasing the average frequency of price adjustment. Similar mechanisms govern the dynamic response of output.

Figure 1: Impulse responses of the GL model to money shocks of varying sizes



Notes. Impulse-Response Functions of aggregate inflation (π), average frequency of price adjustment (h), and real aggregate output (y) in the Golosov-Lucas model to money shocks of different magnitudes ($g_0 = \{2, 6, 18\}$ percent).

The impulse responses of the approximate menu-cost model are displayed as solid black lines. First note that, as one would like, the approximate model closely matches the global menu-cost model's IRFs for the medium-size shock (6 percent) used in calibration. More importantly, it also closely tracks the (untargeted) IRFs of the global menu-cost model for both small and large shocks (2 and 18 percent shocks, respectively), despite the fact that these moments are untargeted in the calibration. This is important property of the model, thar derives from Proposition 1: given a calibration of the model's parameters, the analytical mapping between shocks and endogenous variables does not depend on the size of the shock. For small shocks, the average price gap is small, the cubic term in the odd approximation is negligible relative to the linear term, so inflation is effectively a linear function of the average gap, as in Calvo. The cubic term becomes quantitatively important in response to large shocks and the approximate model continues to track the global menu-cost solutions closely. The approximate model also captures the faster price-level passthrough associated with larger shocks, reflecting the endogenous rise in the frequency of adjustment and the correspondingly greater degree of monetary neutrality.

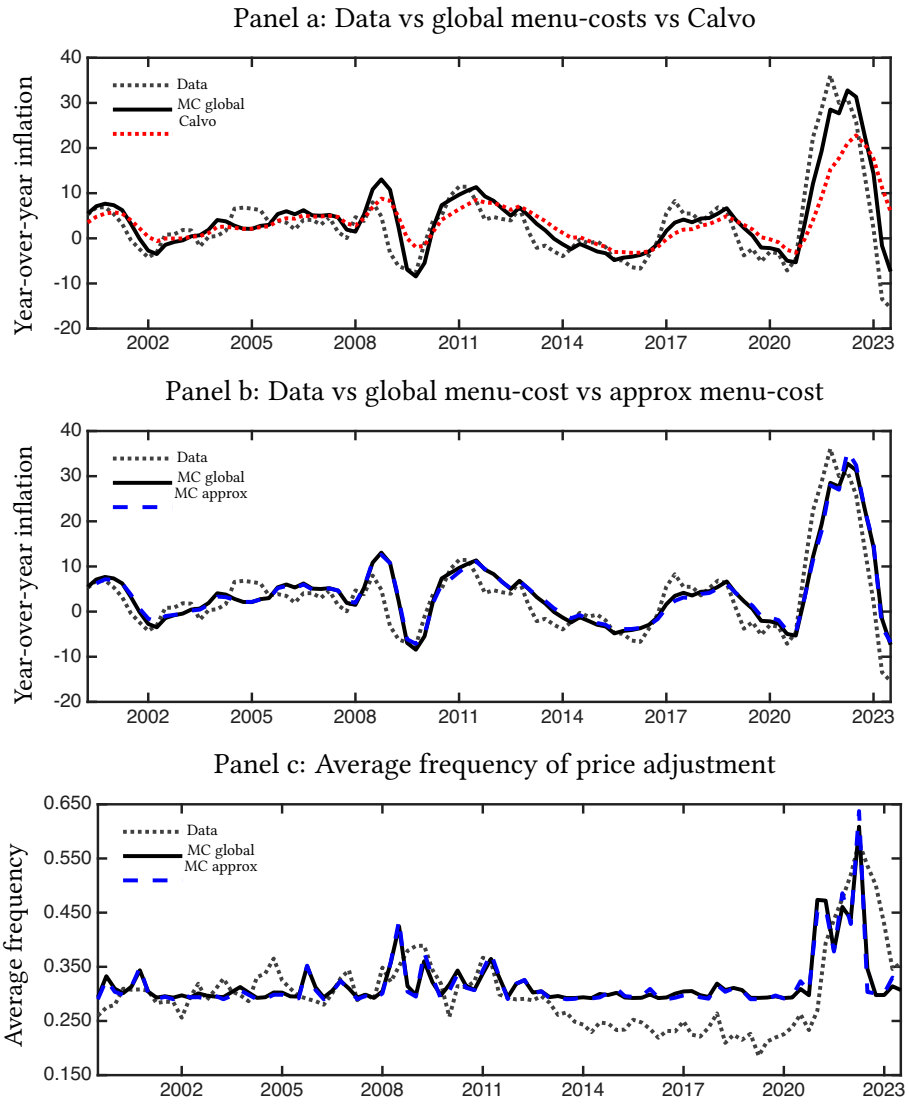
3.3 Inflation accounting and decomposition

We now assess the ability of our approximate menu-cost model to replicate the observed time series of aggregate inflation. In the GL model, nominal marginal cost evolves as an exogenous variable. We leverage this structure to perform a feed-in exercise to account for inflation and decompose inflation into the contribution of the extensive margin, extensive margin, and the selection effect.

Accounting for inflation. Following the approach in Gagliardone et al. (2025a), we use microdata on prices and production costs to construct a measure of producer price inflation (PPI), π_t, and an index of nominal marginal costs for the Belgian manufacturing sector, mc_t^n.[10] Using the marginal cost series, the quantitative exercise proceeds as follows. We begin in 1999:Q1, assuming the economy is in steady state. We then feed the model a shock to the aggregate component of marginal cost, given by the log change in the nominal marginal cost index, Δmc_t^n, between 1999:Q1 and 1999:Q2. Consistent with the model's structure, we assume that the logarithm of the aggregate component of firms' marginal costs follows a random walk with drift. Given this shock, we solve the model to obtain the average price gap, X_t, and the corresponding inflation response, assuming

[10] We compute domestic PPI inflation as a Törnqvist price index, averaging firms' quarterly price changes weighted by smoothed sales shares. Similarly, we construct an aggregate nominal cost index, mc_t^n, by aggregating and concatenating average changes in a proxy for firm-level nominal marginal costs, which exploits variation in average variable costs. See Gagliardone et al. (2025a) for a detailed description of the data and procedure used to construct these variables.

Figure 2: Accounting for inflation

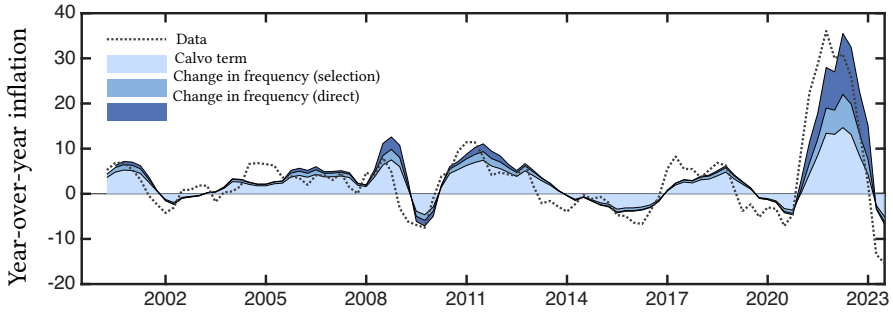


Notes. This figure compares the time series of manufacturing PPI inflation in the data (gray dotted line) with simulated inflation series generated by different models, all fed with the same sequence of aggregate cost shocks recovered from the data. Panel a reports inflation from the global menu-cost model (black solid line) and from the global Calvo model (red dotted line). Panel b adds inflation from the approximate menu-cost model (blue dashed line). Panel c decomposes plots the time-series of the frequency of price adjustment in the data against the model-based frequency generated by the global and approximate model. Panel (d) provides a decomposition of the inflation dynamics reproduced by the global model into a Calvo, a cubic term, and a covariance term.

all future aggregate shocks are unanticipated, as in a standard impulse-response analysis. Updating the average price gap to the new equilibrium level, we repeat the procedure sequentially for each quarter through 2023:Q4, the final quarter in our sample.

Figure 2 (cont'ed): Accounting for inflation

Panel d: Inflation decomposition



Notes. See previous page

Figure 2, panel a, plots year-over-year inflation in the data (black dotted line) together with the model-based inflation series from the menu-cost model solved with global methods (red dashed line) and from the Calvo model (blue solid line). As shown in Gagliardone et al. (2025a), the menu-cost model reproduces inflation dynamics well, both during the low-inflation regimes and during inflation surges. By contrast, the Calvo model matches inflation reasonably well when the economy experiences moderate shocks but captures only about two-thirds of the observed increase in inflation when shocks are large—such as during the global financial crisis and the pandemic—despite being fed the same sequence of cost shocks.

Figure 2, panels b and c, show that our approximate menu-cost model (solid black line) closely replicates the dynamics of inflation and frequency of the global model and provides a substantially better fit compared to the Calvo model, particularly during the inflation surge. Importantly, the approximate model captures the nonlinearities implied by firms' state-dependent pricing behavior, reproducing the rapid acceleration and subsequent deceleration of inflation in response to large shocks.

Inflation decomposition. Figure 2, panel d, decomposes the time-series of inflation into the three components identified in Corollary 1: the Calvo term, which captures the linear mapping from average price gaps to inflation at the steady-state adjustment frequency; the cubic term, which captures nonlinearities arising from the endogenous response of the average adjustment frequency to aggregate shocks; and the selection term, which reflects the covariance between firm-level price gaps and hazard rates.

In normal times, inflation is driven primarily by movements along the intensive margin, and—much as in a time-dependent model—the Calvo term is sufficient to accounts for most of the

variation in inflation. Nonlinear terms become quantitatively important, and at times dominant, during inflation surges such as the global financial crisis and the COVID-19 pandemic. In these episodes, both the endogenous increase in the frequency of price adjustment and a sizable selection effect contribute meaningfully to inflation dynamics. The decomposition shows that both forces are quantitatively important to generate the observed amplification and therefore cannot be neglected.

4 A textbook NK model with state-dependent pricing

As we have noted, in the equilibrium of the GL model, wages and nominal marginal cost vary one for one with the money supply exogenously set by a monetary authority. In this section, we relax these convenient but unrealistic assumptions. We embed state-dependent pricing in a conventional three equation New Keynesian model (e.g., Galí 2015) where the central bank uses the short term interest rate as the policy tool. The model features exogenous cost-push shocks—a standard reduced-form representation of supply-side disturbances such as oil or energy price increases (Clarida et al. 1999; Galí 2015)—whose nonlinear transmission through the pricing block is the focus of the quantitative exercises that follow. Unlike the GL model, there is mutual feedback between pricing behavior and the real economy. This mutual feedback enriches aggregate dynamics but also further complicates the solution of the model using global methods. As we show, however, the approximate model remains easy to solve.

We first show that the approximate model closely captures the dynamic propagation of shocks generated by the same model, where the pricing block is solved with global methods. We then use the calibrated model to perform policy exercises that illustrate how, when shocks are large, the sacrifice ratio under different monetary policy stances (accommodative or more aggressive) differs radically in the state-dependent model relative to those produced by the linear Calvo model.

4.1 The three equation model

As in Section 2.3, a representative household has preferences over consumption and labor with intertemporal elasticity of substitution in consumption and inverse Frisch elasticity of labor supply given by $\gamma, \varphi > 0$ (Equation 2.3). We drop the cash-in-advance constraint and instead allow the household to transfer resources intertemporally by purchasing one-period nominally risk-less discount bonds paying a gross nominal yield $i > 1$. The central bank intervenes to stabilize

inflation using to the real rate rule (Holden 2024; Galí 2025):

$$r_t = (\phi_\pi - 1)\mathbb{E}_t\{\pi_{t+1}\} + \nu_t, \tag{26}$$

where ν_t denotes a money shock that evolves according to:

$$\nu_t = \rho_\nu \, \nu_{t-1} + \varsigma_t. \tag{27}$$

and the nominal rate is given by the Fisher equation $i_t = r_t + \mathbb{E}_t\{\pi_{t+1}\}$. The policy coefficient $\phi_\pi > 1$ ensures determinacy and it is consistent with the optimal policy under discretion (Clarida et al. 1999).

On the supply side, firms' production and pricing decisions are the same as those in GL, with a few differences. First, we allow the aggregate component of productivity, a_t, to evolve as AR(1) process with persistence ρ_a. Second, we introduce an exogenous cost-push shock, u_t, which evolves according to:

$$u_t = \rho_u \, u_{t-1} + \varrho_t. \tag{28}$$

The global and approximate solutions. The general equilibrium of the model and its dynamics are characterized by the law of motion for the cost-push, aggregate productivity, and money shocks, the Euler equation (19), the policy rule in (26), aggregate inflation (Equation 9), and the cross-sectional distribution of firms' price gaps F_t.

To keep things simple, we work with a standard log-linearized aggregate demand equation:

$$y_t = \mathbb{E}_t\{y_{t+1}\} - \gamma^{-1}\big(r_t - \ln(\beta)\big). \tag{29}$$

obtained log-linearizing the household consumption Euler Equation (19) and using the market clearing condition and the Fisher equation. However, as in GL, the state-dependent pricing block of the model admits no general closed-form solution for the supply side, and the equilibrium must be computed numerically using global methods if nonlinearities want to be accounted for. The computational burden is now even greater than GL because the wage rate is endogenously determined in general equilibrium, rather than pinned down by the money stock via the cash-on-hand constraint. This implies that firm's (and aggregate) marginal costs are jointly determined together with the demand side of the model.

The following system of equations in endogenous variables $\{y_t, r_t, u_t, a_t, \pi_t, p_t, \Lambda_t, p_t^\star, p_t^o\}$ and exogenous shocks $\{\varrho_t, \nu_t\}$ characterizes the aggregate equilibrium dynamics of the model:

New Keynesian model with odd approximation

$$y_t = \mathbb{E}_t\{y_{t+1}\} - \gamma^{-1}\big(r_t - \ln(\beta)\big) \tag{NK–1}$$

$$r_t = (\phi_\pi - 1)\mathbb{E}_t\{\pi_{t+1}\} + \nu_t \tag{NK–2}$$

$$x_t = p_t^\star - p_{t-1} \tag{NK–3}$$

$$p_t = \pi_t + p_{t-1} \tag{NK–4}$$

$$p_t^o = (1-\Omega)\big(\mu + (\gamma+\varphi)y_t - a_t + u_t\big) + p_t \tag{NK–5}$$

$$\pi_t = \beta_1 x_t + \beta_3 x_t^3 \tag{NK–6}$$

$$h_t = \alpha_0 + \alpha_2 x_t^2 \tag{NK–7}$$

$$\Lambda_t p_t^\star = p_t^o + \beta\,\mathbb{E}_t\left\{(1-h_{t+1})\Lambda_{t+1}p_{t+1}^\star\right\} \tag{NK–8}$$

$$\Lambda_t = 1 + \beta\,\mathbb{E}_t\left\{(1-h_{t+1})\Lambda_{t+1}\right\} \tag{NK–9}$$

$$u_t = \rho_u\, u_{t-1} + \varrho_t \tag{NK–10}$$

$$a_t = \rho_a\, a_{t-1} + \eta_t \tag{NK–11}$$

$$\nu_t = \rho_\nu \nu_{t-1} + \varsigma_t \tag{NK–12}$$

The system of equations resembles the one in GL. (NK–1) and (NK–2) are the log-linear aggregate demand equation and the real-rate rule. Compared to the GL, this model features a richer cost dynamic. To obtain (NK–5) we started from the general expression in Equation (2), and use the household's labor supply condition and market clearing to substitute for the nominal wage. The remaining equations are similar to those introduced in the GL model: the aggregate reset price and associated discounting, the quadratic approximation of the GHF and the cubic approximation of inflation as a function of the average price gap, and the law of motions of shocks. The rest of the equations are in common with the approximate GL model, net of the dynamics of the cost-push, aggregate productivity, and money shocks.

A few observations follow. First, the odd approximation of inflation makes explicit the notion of a *nonlinear Phillips curve* in models with state-dependent pricing. In the textbook three-equation NK model with time-dependent pricing (Calvo 1983), the Phillips curve summarizes the supply side by relating inflation linearly to real marginal cost. With state-dependent pricing, however, the elasticity of inflation to real marginal cost—what would be the slope in the textbook model—is itself an endogenous object: it varies nonlinearly with the size of the aggregate shock through the endogenous response of the frequency of price adjustment and the selection effect (Blanco et al. 2024; Karadi et al. 2025). The odd approximation captures these nonlinearities in

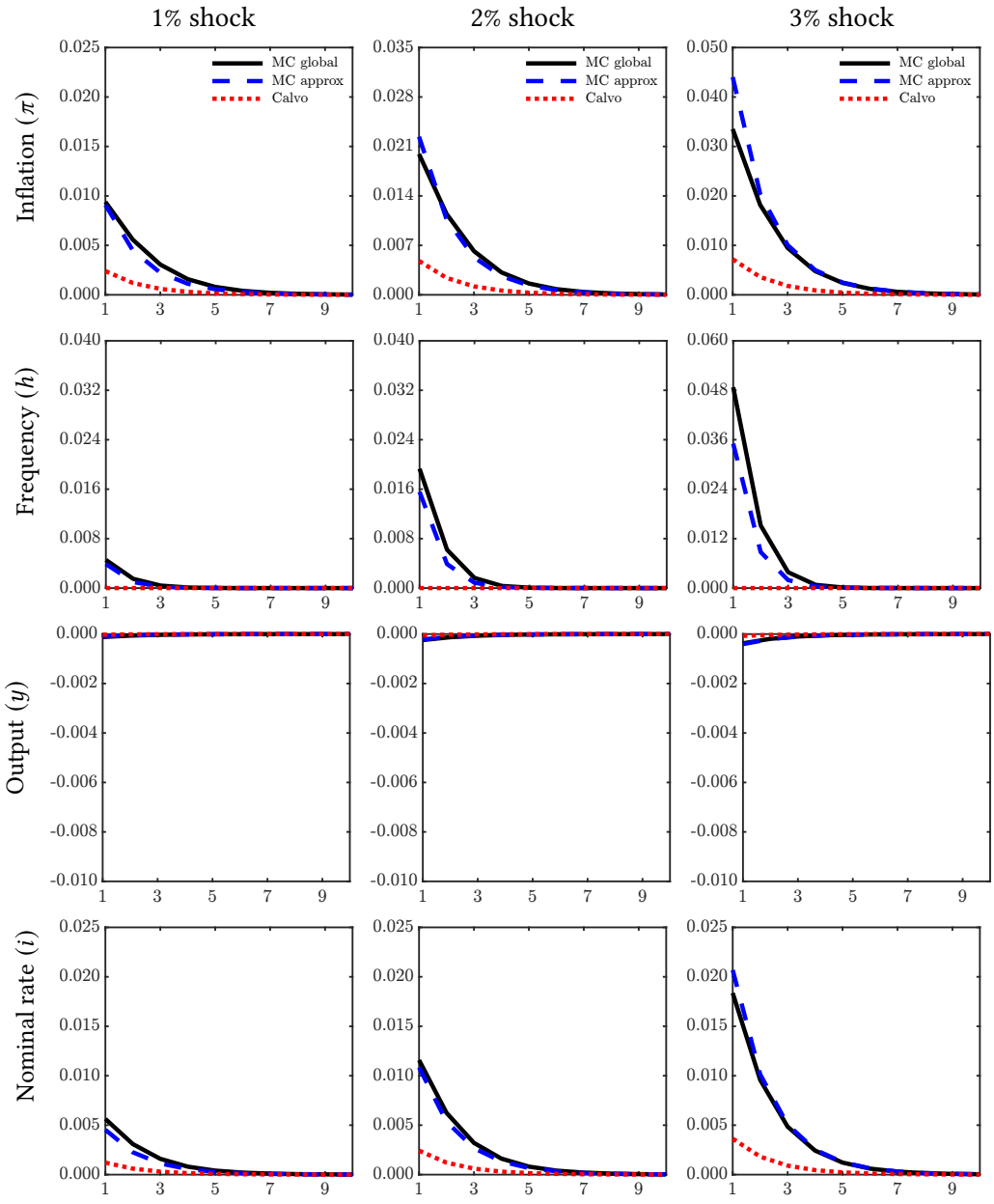
closed form, accounting for both the direct movement of the frequency and the associated selection effects. The average price gap is driven by real marginal cost, thus the linear coefficient β_1 plays the role of the textbook slope while the cubic term $\beta_3 x_t^3$ generates the state-dependent steepening: the *effective* slope of the Phillips curve, $\beta_1 + 3\beta_3 x_t^2$, rises with the square of the aggregate shock.[11] Quantitatively, as shown in Corollary 2, the cubic coefficient β_3 is disciplined by the same structural parameters that govern steady-state pricing behavior, so no additional free parameters are required to capture the nonlinearity.

Second, the nonlinearity of the Phillips curve has a direct bearing on the interpretation of cost-push shocks. In the standard log-linearized NK model, a residual "cost-push shock" augmenting the NKPC absorbs discrepancies between observed inflation and the prediction of the linear relationship $\pi_t = \kappa \widehat{mc}_t + \beta \mathbb{E}_t\{\pi_{t+1}\}$. The odd approximation makes explicit how part of what the linear model attributes to the NKPC cost-push shock is not a genuine exogenous disturbance but rather the nonlinear response of inflation to the same fundamentals that drive the linear term. Specifically, writing $\pi_t = \beta_1 x_t + \beta_3 x_t^3$ and projecting onto the linear model $\pi_t = \beta_1 x_t + \tilde{u}_t$ yields $\tilde{u}_t = \beta_3 x_t^3$: the "cost-push shock" recovered by linear filtering is in fact the cubic term of the odd approximation—an endogenous, higher-order function of the very same average price gap that enters the linear term. In this sense, the cost-push shock is not exogenous at all; it is a systematic, predictable component of inflation that scales with the cube of the aggregate disturbance. Ignoring this endogeneity leads to two practical errors: it overstates the role of unexplained shocks in accounting for inflation dynamics, and it understates the scope for monetary policy to influence inflation through its effect on the price gap. The decomposition in Corollary 1 provides a structural accounting that separates the genuinely exogenous cost-push term from the endogenous nonlinear amplification, thereby disciplining the interpretation of reduced-form estimates of the Phillips curve.

Comparison of transitional dynamics. To begin, we can show that the approximate model can replicate the transitional dynamics of the NK model with state-dependent pricing solved using global methods. Figure 3 plots the impulse responses in global model (black solid lines) and the approximate menu-cost model (blue dashed lines) in response to cost push shocks of different sizes. The latter is solved via stochastic simulations using Dynare using a third-order approximation. The calibration procedure for the global model is the same as in Section 3. We

[11] This result is in line with the inflation-accelerator mechanism emphasized by Blanco et al. (2024): a feedback loop between inflation and the frequency of price adjustment that amplifies the pass-through of cost shocks when the economy is far from the stochastic steady state. The authors focus on movements of the frequency and assume away selection effects. Our odd approximation accounts for both effects, which the quantitative exercises shown in Section 3 are both important to rationalize the data.

Figure 3: Impulse responses of the NK model to cost push shocks of varying sizes



Notes. Impulse-Response Functions of aggregate inflation (π), real aggregate output (y), average frequency of price adjustment (h), and nominal rate (i) in the three-equation NK model to cost push shocks of different magnitudes ($u_0 = \{1, 2, 3\}$ percent).

compute the IRFs under accommodation setting $\phi_\pi = 1.01$, which allows us to hold the real rate constant at its steady state level and focus on the real side of the model abstracting from policy feedback. The odd approximation parameters are calibrated to match the IRFs of inflation, frequency, and output produced by the global model in response to a moderately small cost push shock (2 percent).[12] As benchmark, Figure 3 also reports the IRFs obtained by a Calvo model—the first-order approximation of both models—to illustrate the nonlinearities resulting from state-dependence (dotted red line). The menu-cost model exhibits nonlinear inflation dynamics and increasing monetary neutrality. The changes in the frequency of price adjustment account for this pattern, through their effect on aggregate inflation. As it was the case for the GL, our third-order approximate model accurately matches these transitional dynamics.

4.2 Monetary policy tradeoffs under state-dependent pricing

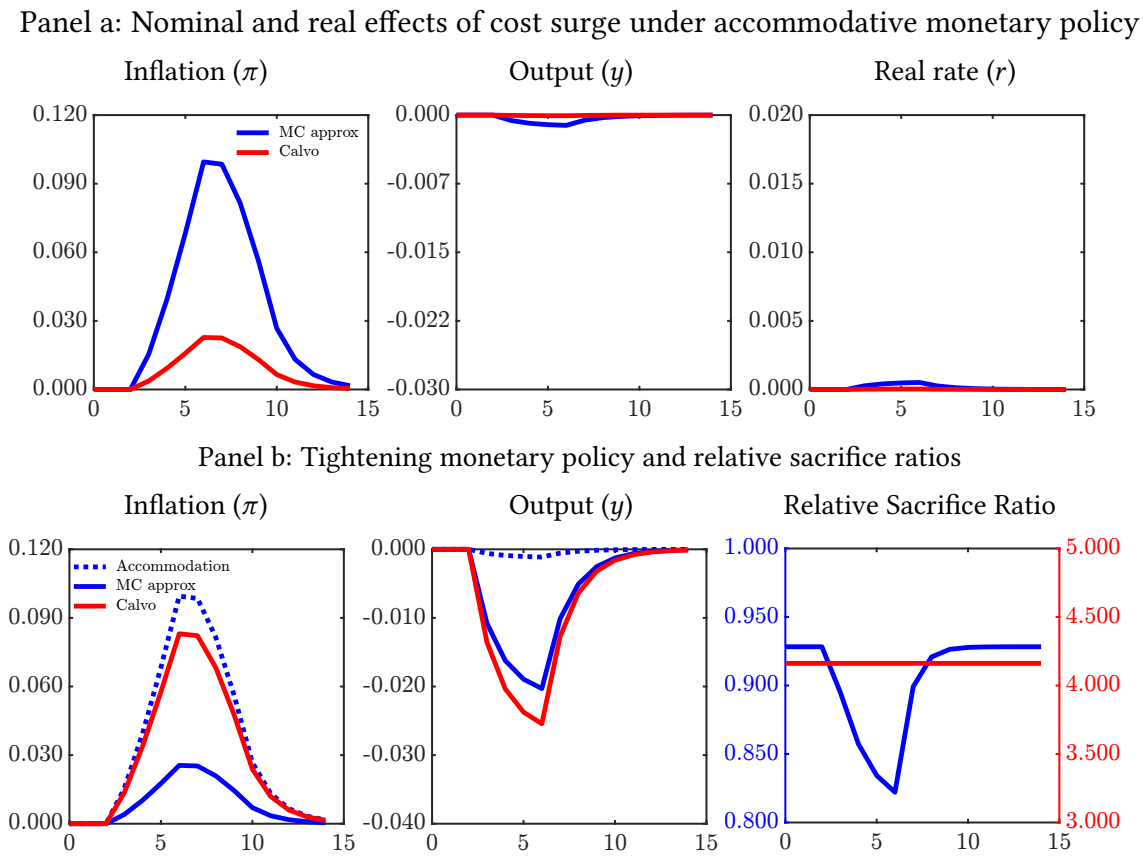
We conduct two policy experiments to evaluate how the transmission of large cost shocks differs across models under alternative monetary policy rules. We treat the menu-cost model as the data-generating process and compare it to the Calvo model—what an economist would infer if they ignored nonlinearities and relied on a first-order approximation of the system.

Looking through large shocks. The first exercise studies the response of the economy to supply shocks under a highly accommodative monetary policy ($\phi_\pi = 1.01$). We start with the menu-cost model and feed in a sequence of transitory, unexpected cost-push shocks of 5 percent each. We then repeat the same exercise using the Calvo model. The blue lines in Figure 4, panel a, show the responses of year-over-year inflation, output (in logs), and the average frequency of price adjustment in the menu-cost model. The red dashed lines represent the responses of the same variables in the Calvo model.

We start the shocks in the fourth quarter, given that we are analyzing year-over-year inflation. First, note that real output responds little in both models because, under the accommodative rule, the real interest rate remains nearly constant. Inflation, however, rises substantially more in the menu-cost model than in the Calvo model. In the menu-cost economy, the sequence of shocks generates a rise and fall in inflation, peaking in quarter seven at 10 percent, comparable to the surge observed in the aftermath of the pandemic (between 2020 and 2022). This is about twice as much as the increase in inflation produced by the Calvo model. The key difference,

[12] Relative to the GL model—where wage dynamics are determined directly by the exogenous evolution of the money supply—the NK model exhibits stronger general equilibrium amplification due to the endogenous response of wages. As a result, smaller shocks are sufficient in the NK framework to generate quantitatively meaningful fluctuations in inflation.

Figure 4: Looking through large inflationary shocks and sacrifice ratios under different monetary policy regimes



Notes. Panel a presents the impulse-response functions of aggregate inflation (π), real aggregate output (y), average frequency of price adjustment (h) to a sequence of cost-push shocks under monetary policy accommodation ($\phi_\pi = 1.01$). The red line represent the IRFs obtained from the log-linearized model (Calvo model). The blue line represent the IRFs obtained from the approximate menu-cost model. In panel b, we switch to a tight monetary policy ($\phi_\pi = 2$). The blue dotted lines in the left and central panels represent the impulse responses under accommodation; the solid lines the responses under tight money for Calvo (red) and for the approximate menu-cost model (blue). The right panel depicts the relative sacrifice ratios of Calvo and the menu-cost model.

of course, is that in the menu-cost economy, the frequency of price adjustments rises due to the cost push shocks, amplifying their pass-through both through the extensive margin (direct effect) and the selection effect (indirect effect).

This exercise speaks directly to the debate on whether central banks should respond to large but transitory supply shocks. A common view early in the pandemic was that inflationary pressures would be modest and would fade once the transitory shocks subsided, so monetary policy should simply "look through" these shocks. However, ignoring the nonlinear effects triggered by large shocks—as the Calvo framework does—can result in high inflationary costs if monetary policy is overly complacent, even if these shocks are transitory, as this exercise illustrates.

The sacrifice ratio. The question is then: should the central bank respond more aggressively to large shocks, and what are the costs of doing so? To address this question, in Panel b, we study the inflation-output trade-off under different monetary policy rules. In particular, we consider the same kind of experiment as above, but we switch to a tight monetary policy by increasing ϕ_π from 1.01 to 2. We then compare inflation and output responses to those observed under the accommodative policy.

The dashed blue line represents the response of inflation under the accommodative policy for the menu-cost economy (the same as Panel a). We adjust the sequence of shocks fed to the Calvo model in order to generate similar inflationary pressure in the log-linearized environment. Tighter monetary policy leads to a significantly larger drop in inflation in the menu-cost model than in the Calvo model (left panel). At the same time, output falls less in the menu-cost model (middle panel). This implies a much smaller sacrifice ratio. To illustrate this, for each model, we compute the relative sacrifice ratio (RSR_t)

$$\text{RSR}_t = \left(y_t^{accom} - y_t^{tight}\right) / \left(\pi_t^{accom} - \pi_t^{tight}\right),$$

where "relative" indicates that we measure the output cost of tightening against the accommodative baseline. The right panel displays the RSR. As before, the blue line refers to the menu-cost model (left axis) and the red line to the Calvo model (right axis). Two findings stand out. First, the menu-cost model features a lower sacrifice ratio than the Calvo model, even though both models are calibrated to the same average frequency of price adjustment. Second, the sacrifice ratio is approximately constant in the Calvo model—consistent with the log-linear Phillips curve, which ties the inflation-output trade-off to a fixed coefficient—whereas it falls in the menu-cost model as the shock unfolds. The decline reflects the endogenous rise in the frequency of price adjustment triggered by large shocks: as nominal rigidity falls, the Phillips curve steepens and the output sacrifice needed to reduce inflation diminishes. This suggests that when shocks are large,

the real output cost of a more aggressive monetary policy stance is substantially smaller than the Calvo model implies.

5 Conclusion

This paper develops a tractable approximation of aggregate inflation in state-dependent (menu-cost) models. The approach collapses the infinite-dimensional cross-sectional distribution of price gaps into a single scalar sufficient statistic, thereby allowing the use of standard perturbation methods in place of computationally intensive global solution techniques. The coefficients governing the linear and nonlinear dynamics of inflation map directly to the structural parameters of the underlying pricing problem. The resulting approximate model delivers a nonlinear Phillips curve whose effective slope increases with the magnitude of aggregate shocks. Its nonlinear impulse responses can be computed straightforwardly using standard perturbation methods, such as those implemented in Dynare. Using this framework, we show that state-dependent models can generate quantitatively distinct policy implications relative to the linearized Calvo benchmark when the economy is subject to large aggregate disturbances.

Several extensions are left for future work. First, the scalar-sufficient-statistic representation makes the odd approximation well suited for structural estimation using standard Bayesian methods applied to observables on inflation and average price gaps. Second, embedding the approximation in richer environments—multi-sector economies, heterogeneous firms with persistent idiosyncratic productivity shocks, or models with nominal wage rigidities—would allow the framework to speak more directly to the cross-sectional heterogeneity documented in micro price-setting data. Third, and perhaps most importantly, the nonlinear Phillips curve implied by the odd approximation opens the door to a formal analysis of optimal monetary policy under state-dependent pricing, where the trade-off between stabilizing inflation and the output gap is itself a function of the state of the economy. This finding is consistent with Karadi et al. (2025), who characterize Ramsey-optimal monetary policy in a menu-cost model and show that the central bank should lean disproportionately against inflation after large cost-push shocks—precisely because the endogenous rise in repricing frequency lowers the sacrifice ratio. Our exercise is the positive-analysis counterpart: for a given comparison of policy rules, the output cost of the tighter stance is substantially lower in the menu-cost model than the Calvo model implies. Ascari et al. (2025) also reach a similar conclusion in an estimated NK model for the euro area that incorporates state-dependent price and wage setting, concluding that a more aggressive monetary policy during the 2021–2022 inflation surge would have entailed only limited additional output costs.

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An Odd Approximation of Inflation

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Appendix

A Derivations

A.1 Proof of Formula (STATIC TARGET)

The bundle of inputs is assembled into final goods using the Kimball aggregator:

$$\int_0^1 \Upsilon\left(\frac{Y_{i,t}}{Y_t}\right) di = 1,$$

where $\Upsilon(\cdot)$ is strictly increasing, strictly concave, and satisfies $\Upsilon(1) = 1$. Taking as given the demand Y_t, each firm minimizes costs subject to the aggregate constraint:

$$\min_{Y_{i,t}} \int_0^1 P_{i,t} Y_{i,t} di \qquad \text{s.t.} \int_0^1 \Upsilon\left(\frac{Y_{i,t}}{Y_t}\right) di = 1.$$

Denoting by ψ the Lagrange multiplier of the constraint, the first-order condition of the problem is:

$$P_{i,t} = \psi \Upsilon'\left(\frac{Y_{i,t}}{Y_t}\right) \frac{1}{Y_t} \tag{A.1}$$

Define implicitly the aggregate price index P_t as:

$$\int_0^1 \Upsilon \circ (\Upsilon')^{-1}\left(\Upsilon'(1)\frac{P_{i,t}}{P_t}\right) di = 1.$$

Evaluating the first-order condition (A.1) at symmetric prices, $P_{i,t} = P_t$, we get $\psi = \frac{P_t Y_t}{\Upsilon'(1)}$. Replacing for ψ, we recover the demand function:

$$\frac{P_{i,t}}{P_t} = \frac{1}{\Upsilon'(1)} \Upsilon'\left(\frac{Y_{i,t}}{Y_t}\right). \tag{A.2}$$

Therefore, the demand function faced by firms when resetting prices is:

$$Y_{i,t} = \left[(\Upsilon')^{-1}\left(\Upsilon'(1)\frac{P^o_{i,t}}{P_t}\right)\right] Y_t$$

Taking logs of equation (A.1) and differentiating, we obtain the following expression for the residual elasticity of demand:

$$\epsilon_{fit} \equiv -\frac{\partial \ln Y_{i,t}}{\partial \ln P^o_{i,t}} = -\frac{\Upsilon'\left(\frac{Y_{i,t}}{Y_t}\right)}{\Upsilon''\left(\frac{Y_{i,t}}{Y_t}\right)\cdot\left(\frac{Y_{i,t}}{Y_t}\right)} \tag{A.3}$$

Denote the steady-state residual demand elasticity by $\epsilon = -\frac{\Upsilon'(1)}{\Upsilon''(1)}$. Then the derivative of the residual demand elasticity $\epsilon_{i,t}$ in (A.3) with respect to $\frac{Y_{i,t}}{Y_t}$, evaluated at the steady state, is given by:

$$\epsilon' = \frac{\Upsilon'(1)\left(\Upsilon'''(1)+\Upsilon''(1)\right)-\left(\Upsilon''(1)\right)^2}{\left(\Upsilon''(1)\right)^2} \leq 0. \tag{A.4}$$

The equation above holds with equality if the elasticity is constant (e.g., under CES preferences). Log-linearizing the Lerner index around the steady state and using equation (A.4), we have that, up to a first-order approximation, the log-markup (in deviation from the symmetric steady state) is equal to:

$$\mu_{i,t} - \mu = \frac{\epsilon'}{\epsilon(\epsilon-1)}\left(y_{i,t} - y_t\right)$$

Finally, log-linearizing the demand function (A.1), we obtain:

$$p^o_{i,t} - p_t = \frac{\Upsilon''(1)}{\Upsilon'(1)}(y_{i,t} - y_t)$$

and use it to replace the log difference in output:

$$\mu_{i,t} - \mu = -\Gamma\left(p^o_{i,t} - p_t\right)$$

where the sensitivity of the markup to the relative price is given by $\Gamma \equiv -\frac{\epsilon'}{\epsilon(\epsilon-1)}\frac{\Upsilon'(1)}{\Upsilon''(1)} \geq 0$. In the text, we set $\mu = 0$ from the normalization of the steady-state real marginal cost.

A.2 Proof of Reset price formula (Equation 5)

Proof. Define, for any gap $x \in \mathbb{R}$, the value of not changing the price at date t as

$$\widetilde{\mathcal{V}}_t(x) \;:=\; \Pi(x) \;+\; \beta\,\mathbb{E}_t\big[\,V_{t+1}(f)\mid x\,\big], \qquad \beta\in(0,1). \tag{A.5}$$

Hence, the optimal reset gap

$$x^\star_t(f) \;\in\; \arg\max_x \widetilde{\mathcal{V}}_t(x)$$

is independent of $\chi_t(f)$, and the corresponding optimal reset price is

$$p^\star_t(f) \;=\; p^o_t(f) - x^\star_t(f).$$

Under our assumptions, $\widetilde{\mathcal{V}}_t(\cdot)$ is differentiable and the maximizer is interior, so that the first-order condition holds:

$$\widetilde{\mathcal{V}}_t'\big(x_t^\star(f)\big) = 0. \tag{A.6}$$

In terms of the value of adjusting and not adjusting, the value function is

$$V_t(f) = \max\Big\{\widetilde{\mathcal{V}}_t\big(x_t^o(f)\big),\ \widetilde{\mathcal{V}}_t\big(x_t^\star(f)\big) - \chi_t(f)\Big\}. \tag{A.7}$$

Because $\widetilde{\mathcal{V}}_t\big(x_t^\star(f)\big) - \chi_t(f)$ is independent of x, and $Pr\{\widetilde{\mathcal{V}}_t\big(x_t^o(f)\big) = \widetilde{\mathcal{V}}_t\big(x_t^\star(f)\big) - \chi_t(f)\} = 0$ (the distribution of χ is continuous), the derivative of the expectation of the maximum is:

$$\frac{\partial\, \mathbb{E}_t(V_{t+1}(f)|x)}{\partial x} = \mathbb{I}\Big\{\widetilde{\mathcal{V}}_{t+1}(x) > \widetilde{\mathcal{V}}_{t+1}\big(x_t^\star(f)\big) - \chi_{t+1}(f)\Big\} \cdot \widetilde{\mathcal{V}}_{t+1}'(x). \tag{A.8}$$

Hence, differentiating (A.5) with respect to x and using that $\partial(p_{t+1}^o(f) - p)/\partial x = 1$ yields

$$\widetilde{\mathcal{V}}_t'(x) = \Pi'(x) + \beta\ \mathbb{E}_t\Big[\big(1 - \mathbb{I}_{t+1}^\star(f)\big)\ \widetilde{\mathcal{V}}_{t+1}'\big(p_{t+1}^o(f) - p\big)\Big] \quad \text{with } p = p_t^o(f) - x. \tag{A.9}$$

Evaluate (A.9) at $x = x_t^\star(f)$, so that $p = p_t^\star(f)$, and use (A.6) to obtain

$$0 = \Pi'\big(p_t^o(f) - p_t^\star(f)\big) + \beta\ \mathbb{E}_t\Big[\big(1 - \mathbb{I}_{t+1}^\star(f)\big)\ \widetilde{\mathcal{V}}_{t+1}'\big(p_{t+1}^o(f) - p_t^\star(f)\big)\Big]. \tag{A.10}$$

Next, apply (A.9) one period ahead to the derivative $\widetilde{\mathcal{V}}_{t+1}'\big(p_{t+1}^o(f) - p_t^\star(f)\big)$, substitute into (A.10), and use the law of iterated expectations. Repeating this substitution N times yields, for any $N \geq 0$,

$$\begin{aligned} 0 = \mathbb{E}_t\left[\sum_{i=0}^{N} \beta^i \left(\prod_{j=1}^{i}\Big(1 - \mathbb{I}_{t+j}^\star(f)\Big)\right) \Pi'\big(p_{t+i}^o(f) - p_t^\star(f)\big)\right] \\ + \beta^{N+1}\ \mathbb{E}_t\left[\left(\prod_{j=1}^{N+1}\Big(1 - \mathbb{I}_{t+j}^\star(f)\Big)\right) \widetilde{\mathcal{V}}_{t+N+1}'\big(p_{t+N+1}^o(f) - p_t^\star(f)\big)\right], \end{aligned} \tag{A.11}$$

where by convention $\prod_{j=1}^{0}(\cdot) = 1$.

Because $\beta \in (0,1)$ and $\widetilde{\mathcal{V}}_t'(\cdot)$ is integrable under the maintained assumptions that guarantee $V_t(f)$ is finite,[13] the second line of (A.11) vanishes as $N \to \infty$. Taking limits in (A.11) gives the infinite-horizon first-order condition

$$0 = \mathbb{E}_t\left[\sum_{i=0}^{\infty} \beta^i \left(\prod_{j=1}^{i}\Big(1 - \mathbb{I}_{t+j}^\star(f)\Big)\right) \Pi'\big(p_{t+i}^o(f) - p_t^\star(f)\big)\right]. \tag{A.12}$$

Under $\Pi(x) = -Bx^2$ we have $\Pi'(x) = -Bx$, and (A.12) becomes

$$0 = -B\ \mathbb{E}_t\left[\sum_{i=0}^{\infty} \beta^i \left(\prod_{j=1}^{i}\Big(1 - \mathbb{I}_{t+j}^\star(f)\Big)\right) \big(p_{t+i}^o(f) - p_t^\star(f)\big)\right].$$

[13] For instance, it suffices that $\mathbb{E}_t\,|p_{t+i}^o(f)| < \infty$ for all i and that the value functions are well-defined and differentiable.

Divide by $-B \neq 0$ and rearrange to isolate $p_t^\star(f)$:

$$p_t^\star(f)\ \mathbb{E}_t\left[\sum_{i=0}^{\infty}\beta^i\left(\prod_{j=1}^{i}\left(1-\mathbb{I}_{t+j}^\star(f)\right)\right)\right]=\mathbb{E}_t\left[\sum_{i=0}^{\infty}\beta^i\left(\prod_{j=1}^{i}\left(1-\mathbb{I}_{t+j}^\star(f)\right)\right)p_{t+i}^o(f)\right].$$

Hence,

$$p_t^\star(f)=\frac{\mathbb{E}_t\left[\sum_{i=0}^{\infty}\beta^i\left(\prod_{j=1}^{i}\left(1-\mathbb{I}_{t+j}^\star(f)\right)\right)p_{t+i}^o(f)\right]}{\mathbb{E}_t\left[\sum_{i=0}^{\infty}\beta^i\left(\prod_{j=1}^{i}\left(1-\mathbb{I}_{t+j}^\star(f)\right)\right)\right]}. \tag{A.13}$$

□

A.3 Proof of Lemma 1

Proof of Lemma 1 (State Variable). Fix a firm f and a date t. We work conditional on the aggregate state (suppressed in the notation) and on the firm's current draw of the adjustment cost $\chi_t(f)$, which is observed by the firm before choosing $\mathbb{I}_t(f)$ and $p_t(f)$. Define the static gap between the frictionless optimal price and the price at the beginning of the period:

$$x_t^o(f) := p_t^o(f) - p_{t-1}(f). \tag{A.14}$$

Then,

$$\begin{aligned} x_t^o(f) &= p_t^o(f) - p_{t-1}(f) \\ &= \left(p_t^o(f) - p_t^\star(f)\right) + \left(p_t^\star(f) - p_{t-1}(f)\right) \\ &= x_t^\star(f) + x_t(f). \end{aligned} \tag{A.15}$$

Under the Lemma's assumption that $x_t^\star(f) = x_t^\star$ for all f, (A.15) becomes

$$x_t^o(f) = x_t^\star + x_t(f). \tag{A.16}$$

Define the value of not changing price as:

$$\widetilde{\mathcal{V}}_t(x) := \Pi(x) + \beta\ \mathbb{E}_t\left[V_{t+1}(f)\ \middle|x\right]. \tag{A.17}$$

Because profits depend on $p_t(f)$ only through $p_t^o(f) - p_t(f)$ (by (1)), and because the law of motion for future primitives is Markov, $\widetilde{\mathcal{V}}_t(\cdot)$ is a well-defined function of the scalar gap x (given the aggregate state at t). The value of adjusting is

$$\max_{x\in\mathbb{R}}\ \widetilde{\mathcal{V}}_t(x)\ -\ \chi_t(f). \tag{A.18}$$

Under differentiability and an interior optimum, it satisfies

$$\widetilde{\mathcal{V}}_t'\left(x_t^\star(f)\right)=0, \qquad \widetilde{\mathcal{V}}_t''\left(x_t^\star(f)\right)<0. \tag{A.19}$$

In terms of the value of adjusting and not adjusting, the value function is

$$V_t(f) = \max\left\{\widetilde{\mathcal{V}}_t\big(x_t^o(f)\big),\ \widetilde{\mathcal{V}}_t\big(x_t^\star(f)\big) - \chi_t(f)\right\}. \tag{A.20}$$

Hence, the optimal decisions satisfy

$$p_t^\star(f) = p_t^o(f) - x_t^\star(f). \tag{A.21}$$

$$\mathbb{I}_t^\star(f) = 1 \quad \Longleftrightarrow \quad \widetilde{\mathcal{V}}_t\big(x_t^\star(f)\big) - \chi_t(f) \ \geq\ \widetilde{\mathcal{V}}_t\big(x_t^o(f)\big). \tag{A.22}$$

Under the Lemma's assumption $x_t^\star(f) = x_t^\star$ for all f, define the value function in terms of the dynamic gap as the re-centered version of $\widetilde{\mathcal{V}}_t$:

$$\mathcal{V}_t(x) := \widetilde{\mathcal{V}}_t\big(x_t^\star + x\big), \qquad x \in \mathbb{R}. \tag{A.23}$$

Using (A.16) and (A.23),

$$\widetilde{\mathcal{V}}_t\big(x_t^o(f)\big) = \widetilde{\mathcal{V}}_t\big(x_t^\star + x_t(f)\big) = \mathcal{V}_t\big(x_t(f)\big), \tag{A.24}$$

and also

$$\mathcal{V}_t(0) = \widetilde{\mathcal{V}}_t(x_t^\star). \tag{A.25}$$

Substituting (A.24)–(A.25) into (A.20) yields:

$$V_t(f) = \max\left\{\mathcal{V}_t\big(x_t(f)\big),\ \mathcal{V}_t(0) - \chi_t(f)\right\}. \tag{A.26}$$

Differentiate (A.23):

$$\mathcal{V}_t'(x) = \widetilde{\mathcal{V}}_t'\big(x_t^\star + x\big), \qquad \mathcal{V}_t''(x) = \widetilde{\mathcal{V}}_t''\big(x_t^\star + x\big).$$

Evaluating at $x = 0$ and using (A.19) with $x_t^\star(f) = x_t^\star$ gives

$$\mathcal{V}_t'(0) = 0, \qquad \mathcal{V}_t''(0) < 0,$$

which is condition (1). Fix $x \in \mathbb{R}$ and interpret it as the current dynamic gap $x_t(f) = x$. If the firm does not adjust at t, the flow payoff is $\Pi(x_t^\star + x)$, and by definition of $\widetilde{\mathcal{V}}_t$,

$$\widetilde{\mathcal{V}}_t(x_t^\star + x) = \Pi(x_t^\star + x) + \beta\ \mathbb{E}_t\left[V_{t+1}(f)\ \middle|\ x_t(f) = x\right]. \tag{A.27}$$

The left-hand side equals $\mathcal{V}_t(x)$ by (A.23). Let

$$x' \equiv x_{t+1}(f) \tag{A.28}$$

denote next period's dynamic gap implied by inaction at t (i.e. given that the price carried into $t+1$ is $p_t(f) = p_{t-1}(f)$). Its conditional law is pinned down by the Markov evolution of $p_{t+1}^o(f)$ (and thus of $p_{t+1}^\star(f)$ under the common-reset-gap assumption) together with the fixed inherited

price. At date $t+1$, the firm again chooses whether to adjust. Using (A.26),

$$V_{t+1}(f) = \max\left\{\mathcal{V}_{t+1}(x'),\ \mathcal{V}_{t+1}(0) - \chi_{t+1}(f)\right\}.$$

Write this maximum using the optimal decision $\mathbb{I}^{\star}_{t+1}(f)$:

$$V_{t+1}(f) = \mathbb{I}^{\star}_{t+1}(f)\Big(\mathcal{V}_{t+1}(0) - \chi_{t+1}(f)\Big) + \big(1 - \mathbb{I}^{\star}_{t+1}(f)\big)\mathcal{V}_{t+1}(x').$$

Take conditional expectations given x', and use the definition of the hazard:

$$\mathbb{E}_{t+1}\big[V_{t+1}(f) \mid x'\big] = \mathbb{E}_{t+1}\Big[h_{t+1}(x')\Big(\mathcal{V}_{t+1}(0) - \chi_{t+1}(f)\Big) + \big(1 - h_{t+1}(x')\big)\mathcal{V}_{t+1}(x') \mid x'\Big].$$

Substituting this into (A.27) (and using the law of iterated expectations) yields (1). □

A.4 Proof of Lemma 2

Proof of Lemma 2 (Quadratic hazard). Fix t and condition on the event $\{x_t(f) = x\}$ and consider the case when firms adjust only depending on the realization of the menu cost (no free-price adjustment). From Lemma 1

$$V_t(f) = \max\{\mathcal{V}_t(x_t(f)),\ \mathcal{V}_t(0) - \chi_t(f)\},$$

the firm adjusts at date t if and only if the adjustment branch weakly dominates the inaction branch:

$$\begin{aligned}\mathbb{I}^{\star}_t(f) = 1 &\iff \mathcal{V}_t(0) - \chi_t(f) \geq \mathcal{V}_t(x_t(f)) \\ &\iff \chi_t(f) \leq \Delta_t(x_t(f)),\end{aligned} \tag{A.29}$$

where we define the gain from adjusting at gap x by

$$\Delta_t(x) := \mathcal{V}_t(0) - \mathcal{V}_t(x). \tag{A.30}$$

By definition of the hazard,

$$h_t(x) := \mathbb{E}_t\big[\mathbb{I}^{\star}_t(f) \mid x_t(f) = x\big] = \Pr\big(\mathbb{I}^{\star}_t(f) = 1 \mid x_t(f) = x\big)\,.$$

Using (A.29) and the fact that $\chi_t(f) \sim G$ is i.i.d. and independent of $x_t(f)$,

$$h_t(x) = \Pr(\chi_t(f) \leq \Delta_t(x)) = G(\Delta_t(x))\,. \tag{A.31}$$

Moreover,

$$\Delta_t(0) = \mathcal{V}_t(0) - \mathcal{V}_t(0) = 0 \quad \Longrightarrow \quad h_t(0) = G(0).$$

Since $\mathcal{V}_t$ is twice differentiable at 0 and $\mathcal{V}'_t(0) = 0$, Taylor expansion at 0 yields

$$\mathcal{V}_t(x) = \mathcal{V}_t(0) + \mathcal{V}'_t(0)x + \frac{1}{2}\mathcal{V}''_t(0)x^2 + o(x^2) = \mathcal{V}_t(0) + \frac{1}{2}\mathcal{V}''_t(0)x^2 + o(x^2). \tag{A.32}$$

Subtracting from $\mathcal{V}_t(0)$ gives

$$\Delta_t(x) = \mathcal{V}_t(0) - \mathcal{V}_t(x) = -\frac{1}{2}\mathcal{V}_t''(0)x^2 + o(x^2). \tag{A.33}$$

Since G is differentiable at 0,

$$G(y) = G(0) + G'(0)\, y + o(y) \qquad (y \to 0). \tag{A.34}$$

Let $y = \Delta_t(x)$. By (A.33), $\Delta_t(x) = O(x^2)$, hence $\Delta_t(x) \to 0$ as $x \to 0$ and $o(\Delta_t(x)) = o(x^2)$. Substituting (A.33) into (A.34) yields

$$\begin{aligned} h_t(x) &= G(\Delta_t(x)) \\ &= G(0) + G'(0)\left(-\frac{1}{2}\mathcal{V}_t''(0)x^2 + o(x^2)\right) + o(x^2) \\ &= h_t(0) - \frac{1}{2}G'(0)\mathcal{V}_t''(0)\, x^2 + o(x^2). \end{aligned}$$

Therefore,

$$h_t(x) = h_t(0) + \phi_t x^2 + o(x^2), \qquad \phi_t := -\frac{1}{2}G'(0)\mathcal{V}_t''(0).$$

In a stationary environment, $\mathcal{V}_t''(0)$ is time-invariant (and G is time-invariant by assumption), hence $h_t(0) = h(0)$ and $\phi_t = \phi$.

In the no free price adjustment case, $h_t(0) = 0$. To allow for a probability of free price adjustment, define an independent Bernoulli process N_t with intensity $(1 - \theta_t^0)$:

$$\mathbb{I}_t^\star(f) = 1 \iff \chi_t(f) \le \Delta_t(x_t(f)) \vee \Delta N_t = 1, \tag{A.35}$$

With free price adjustments, $h_t(0) = (1 - \theta_t^0) > 0$ and $\phi_t := -\frac{1}{2}\theta_t^0 G'(0)\mathcal{V}_t''(0)$.

□

A.5 Proof of Theorem 1

Theorem 1. Taking conditional expectations and using law of iterated expectations:

$$\pi_t = \mathbb{E}_t\left[\mathbb{I}_t^\star(f)x_t(f)\right] = \mathbb{E}_t\left[\mathbb{E}_t\left[\mathbb{I}_t^\star(f) \mid x_t(f)\right] x_t(f)\right] = \mathbb{E}_t[h_t(x_t(f))\, x_t(f)] = \int x\, h_t(x)\, dF_t(x).$$

Therefore

$$\pi_t = \int \Big(h_t(0) + \phi_t x^2 + r_t(x)\Big) x\, dF_t(x) = h_t(0) \underbrace{\int x\, dF_t(x)}_{m_{1,t}} + \phi_t \underbrace{\int x^3\, dF_t(x)}_{m_{3,t}} + \underbrace{\int x\, r_t(x)\, dF_t(x)}_{=:R_t}.$$

Hence it remains to show $R_t = o(\sigma_t^3)$. Write the remainder as

$$r_t(x) = x^2 \varepsilon_t(x), \qquad \text{where} \qquad \varepsilon_t(x) := \begin{cases} \dfrac{r_t(x)}{x^2}, & x \neq 0, \\ 0, & x = 0. \end{cases}$$

Then the Taylor approximation of the hazard is equivalent to $\varepsilon_t(x) \to 0$ as $x \to 0$, and

$$R_t = \int x^3 \, \varepsilon_t(x) \, dF_t(x).$$

Fix $\eta > 0$. Since $\varepsilon_t(x) \to 0$ as $x \to 0$, there exists $\delta = \delta(\eta) > 0$ such that $|\varepsilon_t(x)| \leq \eta$ for all $|x| \leq \delta$. Split the integral into a "local" part and a "tail" part:

$$|R_t| \leq \int_{|x| \leq \delta} |x|^3 \, |\varepsilon_t(x)| \, dF_t(x) + \int_{|x| > \delta} |x|^3 \, |\varepsilon_t(x)| \, dF_t(x) =: R_{t,1} + R_{t,2}.$$

For the local part, by construction and monotonicity of the integral:

$$R_{t,1} \leq \eta \int_{|x| \leq \delta} |x|^3 \, dF_t(x) \leq \eta \int |x|^3 \, dF_t(x).$$

Using the additional assumption, there exists $\bar{x}_t < \infty$ such that $\text{supp}(F_t) \subseteq [-\bar{x}_t, \bar{x}_t]$. Hence:

$$\int |x|^3 \, dF_t(x) \leq \bar{x}_t \int x^2 \, dF_t(x) = \bar{x}_t \, m_{2,t}.$$

Moreover, since $|m_{1,t}| \leq \bar{x}_t$, we have

$$m_{2,t} = \sigma_t^2 + (m_{1,t})^2 \leq \sigma_t^2 + \bar{x}_t^2.$$

Letting $\sigma_t^2 \equiv m_{2,t} - m_{1,t}^2$, the condition $\bar{x}_t^2 \leq C m_{2,t}$ can be equivalently stated as $\bar{x}_t \leq C \, \sigma_t$ because C is arbitrary and $m_{1,t} \leq 2\bar{x}_t$. It follows that $\bar{x}_t = O(\sigma_t)$ and $m_{2,t} = O(\sigma_t^2)$, so that

$$\int |x|^3 \, dF_t(x) \leq \bar{x}_t \, m_{2,t} = O(\sigma_t) \cdot O(\sigma_t^2) = O(\sigma_t^3).$$

The above implies $R_{t,1} \leq \eta \, O(\sigma_t^3)$.

For the tail part, as $h_t(\cdot) \in [0, 1]$, the remainder $r_t(x) = h_t(x) - h_t(0) - \phi_t x^2$ is bounded on $[-\bar{x}_t, \bar{x}_t]$, hence $\varepsilon_t(x) = r_t(x)/x^2$ is bounded on $\{x : \delta \leq |x| \leq \bar{x}_t\}$. Define

$$M_{t,\delta} := \sup\{|\varepsilon_t(x)| : \ \delta \leq |x| \leq \bar{x}_t\} < \infty.$$

Then

$$R_{t,2} \leq M_{t,\delta} \int_{|x| > \delta} |x|^3 \, dF_t(x) \leq M_{t,\delta} \, \bar{x}_t^3 \, \Pr(|x| > \delta).$$

Using Chebyshev's inequality and $m_{2,t} = O(\sigma_t^2)$,

$$\Pr(|x| > \delta) \leq \frac{\mathbb{E}[x^2]}{\delta^2} = \frac{m_{2,t}}{\delta^2} = O(\sigma_t^2)$$

Also $\bar{x}_t^3 = O(\sigma_t^3)$ because $\bar{x}_t \leq C\sigma_t$. Therefore

$$R_{t,2} \leq M_{t,\delta} \, O(\sigma_t^3) \cdot O(\sigma_t^2) = O(\sigma_t^5) = o(\sigma_t^3).$$

Finally, putting together the bounds,

$$|R_t| \le \eta\, O(\sigma_t^3) + o(\sigma_t^3).$$

Since $\eta > 0$ is arbitrary, this implies $R_t = o(\sigma_t^3)$. □

A.6 Proof of Corollary 1

Proof of the odd approximation in Corollary 1. Let $X_t := x_t(f)$ denote the cross-sectional random variable with distribution F_t, and let

$$x_t := \mathbb{E}[X_t] = m_{1,t}, \qquad \sigma_t^2 := \mathbb{V}\mathrm{ar}(X_t), \qquad \psi_t := \mathbb{E}[(X_t - x_t)^3]$$

be, respectively, the mean, variance, and skewness of F_t. In a stationary environment the odd-approximation theorem gives

$$\pi_t = h(0)\, m_{1,t} + \phi\, m_{3,t} + o(\sigma_t^3). \tag{A.36}$$

Add and subtract $\phi\, m_{2,t} m_{1,t}$:

$$\begin{aligned} h(0)m_{1,t} + \phi m_{3,t} &= \big(h(0) + \phi m_{2,t}\big)m_{1,t} + \phi\big(m_{3,t} - m_{2,t}m_{1,t}\big) \\ &= \big(h(0) + \phi m_{2,t}\big)x_t + \mathbb{C}\mathrm{ov}\big(h(0) + \phi X_t^2,\ X_t\big), \end{aligned} \tag{A.37}$$

where in the last step we used

$$\mathbb{C}\mathrm{ov}\big(h(0) + \phi X_t^2,\ X_t\big) = \phi\Big(\mathbb{E}[X_t^3] - \mathbb{E}[X_t^2]\,\mathbb{E}[X_t]\Big) = \phi\big(m_{3,t} - m_{2,t}m_{1,t}\big).$$

Define the (quadratic-hazard) selection term as

$$\mathbb{C}\mathrm{ov}(h_t(f), x_t(f)) := \mathbb{C}\mathrm{ov}\big(h(0) + \phi X_t^2,\ X_t\big).$$

Next, use $m_{2,t} = \sigma_t^2 + x_t^2$ to rewrite the first component:

$$\big(h(0) + \phi m_{2,t}\big)x_t = \big(h(0) + \phi\sigma_t^2\big)x_t + \phi x_t^3.$$

Finally, add and subtract $\phi\sigma_0^2 x_t$:

$$\big(h(0) + \phi\sigma_t^2\big)x_t = \big(h(0) + \phi\sigma_0^2\big)x_t + \phi(\sigma_t^2 - \sigma_0^2)x_t.$$

Putting these identities into (A.36) yields

$$\pi_t = \big(h(0) + \phi\sigma_0^2\big)x_t + \phi\Big((\sigma_t^2 - \sigma_0^2)x_t + x_t^3\Big) + \mathbb{C}\mathrm{ov}(h_t(f), x_t(f)) + o(\sigma_t^3),$$

which is (10). It remains to express the selection term in moments. Using the definition of skewness,

$$\begin{aligned} m_{3,t} = \mathbb{E}[X_t^3] &= \mathbb{E}\left[(x_t + (X_t - x_t))^3\right] \\ &= x_t^3 + 3x_t\,\mathbb{E}[(X_t - x_t)^2] + \mathbb{E}[(X_t - x_t)^3] \\ &= x_t^3 + 3x_t\sigma_t^2 + \psi_t, \end{aligned}$$

and $m_{2,t}m_{1,t} = (\sigma_t^2 + x_t^2)x_t = x_t^3 + \sigma_t^2 x_t$. Therefore

$$m_{3,t} - m_{2,t}m_{1,t} = 2\sigma_t^2 x_t + \psi_t,$$

so that

$$\mathbb{C}\mathrm{ov}(h_t(f), x_t(f)) = \phi\big(m_{3,t} - m_{2,t}m_{1,t}\big) = 2\phi\sigma_t^2 x_t + \phi\psi_t,$$

□

A.7 Proof of Proposition 1

Proof of Proposition 1. For any bounded measurable test function φ,

$$\int \varphi(x)\, dF^{\delta}(x) = \int \varphi(x)\, dF_0(x) + \delta \int \varphi(x)\, d\widehat{F}(x) + o(\delta).$$

If φ is even (i.e. $\varphi(x) = \varphi(-x)$), then antisymmetry implies

$$\int \varphi(x)\, d\widehat{F}(x) = 0.$$

To see this, write $\int \varphi(x)\, d\widehat{F}(x) = \int \varphi(-x)\, d\widehat{F}(x) = -\int \varphi(x)\, d\widehat{F}(x)$, hence the integral is zero. Applying this with $\varphi(x) = x^{2j}$ yields

$$m_{2j}(\delta) = m_{2j}(0) + o(\delta), \qquad j = 1, 2, \ldots$$

In particular, $m_2'(0) = 0$. Since $x(0) = m_1(0) = 0$ at the symmetric steady state, it follows that

$$\left.\frac{\partial \sigma^2}{\partial \delta}\right|_{\delta=0} = \left.\frac{\partial m_2}{\partial \delta}\right|_{\delta=0} - 2x(0)\left.\frac{\partial x}{\partial \delta}\right|_{\delta=0} = 0.$$

Thus, the variance has no first-order response to δ (and likewise any even centered moment). Because $\sigma^2(\delta)$ is locally an even function of δ, it admits

$$\sigma^2(\delta) = \sigma_0^2 + \tfrac{1}{2}\sigma_{\delta\delta}^2(0)\,\delta^2 + o(\delta^2).$$

Moreover, $x(\delta) = x_\delta(0)\,\delta + o(\delta)$ is locally invertible in δ (as $x_\delta(0) \neq 0$). Substituting $\delta = (\partial\delta/\partial x)|_{\delta=0}\; x(\delta) + o(x(\delta))$ yields

$$\sigma^2(\delta) = \sigma_0^2 + \frac{1}{2}\left.\frac{\partial^2\sigma^2}{\partial\delta^2}\left(\frac{\partial\delta}{\partial x}\right)^2\right|_{\delta=0} x^2(\delta) + o\big(x^2(\delta)\big) = \sigma_0^2 + \frac{1}{2}\sigma_{xx}^2\, x^2(\delta) + o\big(x^2(\delta)\big).$$

At $\delta = 0$, symmetry implies $\psi(0) = 0$. Moreover $\psi(\delta)$ is an odd functional of δ under an antisymmetric first-order perturbation, so $\psi(\delta)$ is locally odd as well. Therefore, using local invertibility of $x(\delta)$ and the (odd) Taylor expansion of $\psi(\delta)$, we obtain the odd expansion in $x(\delta)$:

$$\psi(\delta) = \psi_x\, x(\delta) + \frac{1}{6}\psi_{xxx}\, x^3(\delta) + o\big(x^3(\delta)\big),$$

where the coefficients are defined as:

$$\psi_x \equiv \left.\frac{\partial\psi}{\partial\delta}\frac{\partial\delta}{\partial x}\right|_{\delta=0}, \qquad \psi_{xxx} \equiv \left.\left(\frac{\partial^3\psi}{\partial\delta^3}\left(\frac{\partial\delta}{\partial x}\right)^3 + \frac{\partial\psi}{\partial\delta}\frac{\partial^3\delta}{\partial x^3}\right)\right|_{\delta=0}.$$

From Theorem 1, aggregate inflation satisfies

$$\pi(\delta) = \int x\, h(x)\, dF^{\delta}(x) = h(0)\, x(\delta) + \phi\, m_3(\delta) + o\big(\sigma^3(\delta)\big),$$

At the steady state, because $x(0) = m_3(0) = 0$ by symmetry, it holds $\pi_0 \equiv \pi(0) = o(\sigma_0^3)$. Therefore,

$$\pi(\delta) = \pi_0 + h(0)\, x(\delta) + \phi\, m_3(\delta) + o\big(x^3(\delta)\big),$$

Next, use the relation between third moments and cumulants

$$m_3(\delta) = x^3(\delta) + 3x(\delta)\sigma^2(\delta) + \psi(\delta),$$

Substituting into $\pi(\delta)$ gives

$$\pi(\delta) = \pi_0 + h(0)\, x(\delta) + \phi\Big[x^3(\delta) + 3x(\delta)\sigma^2(\delta) + \psi(\delta)\Big] + o\big(x^3(\delta)\big).$$

Now plug in the expansions of variance and skewness and obtain:

$$3x(\delta)\sigma^2(\delta) = 3\sigma_0^2 x(\delta) + \frac{3}{2}\sigma_{xx}^2 x^3(\delta) + o\big(x^3(\delta)\big).$$

Collecting terms yields

$$\pi(\delta) = \pi_0 + \underbrace{\Big(h(0) + 3\phi\sigma_0^2 + \phi\psi_x\Big)}_{\beta_1}\, x(\delta) + \underbrace{\phi\left[1 + \frac{3}{2}\sigma_{xx}^2 + \frac{1}{6}\psi_{xxx}\right]}_{\beta_3}\, x^3(\delta) + o\big(x^3(\delta)\big).$$

Finally, substitute $x(\delta) = x_\delta(0)\,\delta + o(\delta)$ into the approximation error, obtaining $o(\delta^3)$. □

A.8 Variance and skewness

We now prove the following stronger result.

Proposition 2. *Let $\epsilon(f) \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_\epsilon^2)$ be the idiosyncratic disturbance, normally distributed with mean zero in the cross-section. Assume that $\epsilon(f) \perp x(f;0)$ and $\sigma_\epsilon^4 \simeq 0$. Then:*

$$\sigma_0^2 \simeq \frac{h_0(1-h_0)\sigma_\epsilon^2}{h_0^2 + 6\phi\sigma_\epsilon^2}, \quad \sigma_{xx}^2 \simeq \frac{2v_2}{a^2}, \quad \psi_x \simeq \frac{d_1}{a}, \quad \psi_{xxx} \simeq \frac{6d_3}{a^3} + \frac{6\phi d_1}{a^4}$$

where:

$$a = 1 - h_0 - 3\phi \frac{h_0 \sigma_\epsilon^2}{h_0^2 + 6\phi\sigma_\epsilon^2}$$
$$v_2 = h_0(1-h_0) - 6h_0\phi \frac{h_0 \sigma_\epsilon^2}{h_0^2 + 6\phi\sigma_\epsilon^2}$$
$$d_1 = 3h_0(1-h_0) \frac{h_0 \sigma_\epsilon^2}{h_0^2 + 6\phi\sigma_\epsilon^2}$$
$$d_3 = h_0(1-h_0)(2h_0 - 1) + 2\phi \frac{h_0 \sigma_\epsilon^2}{h_0^2 + 6\phi\sigma_\epsilon^2}(1 + 3h_0 - 9h_0^2)$$

Proof. Let X denote the gap at the *beginning* of the period (post-adjustment from the previous period), ϵ an idiosyncratic shock, and δ a small aggregate shock. Define the pre-adjustment gap:

$$Y := X + \delta + \epsilon.$$

Conditional on $Y = y$, the firm adjusts with probability:

$$\Pr(\mathbb{I}_\delta = 1 \mid Y = y) = h(y), \qquad h(y) = h_0 + \phi y^2 + o(y^2) \quad (y \to 0), \qquad h_0 := h(0),$$

so the post-adjustment (post-decision) gap is:

$$X_\delta := (1 - \mathbb{I}_\delta)\, Y. \tag{A.38}$$

Pre-adjustment moments. Let

$$Z := X + \epsilon, \qquad \text{so that} \qquad Y = Z + \delta.$$

Denote by M_k the raw moments of the *pre-adjustment* distribution of Z (at $\delta = 0$):

$$M_k := \mathbb{E}[Z^k], \qquad k \geq 1.$$

As X has symmetric distribution and ϵ is Gaussian, $M_1 = M_3 = 0$. By symmetry of Z:

$$\mathbb{E}[Y] = \delta, \tag{A.39}$$
$$\mathbb{E}[Y^2] = M_2 + \delta^2, \tag{A.40}$$
$$\mathbb{E}[Y^3] = 3M_2\delta + \delta^3, \tag{A.41}$$
$$\mathbb{E}[Y^4] = M_4 + 6M_2\delta^2 + O(\delta^4), \tag{A.42}$$
$$\mathbb{E}[Y^5] = 5M_4\delta + 10M_2\delta^3 + O(\delta^5). \tag{A.43}$$

Post-adjustment moments. Denote by $m_k(\delta)$ the raw moments of the *post-adjustment* distribution of X_δ:

$$m_k(\delta) := \mathbb{E}[(X_\delta)^k], \qquad k \geq 1.$$

Since $X_\delta = Y$ with probability $1 - h(Y)$ and $X_\delta = 0$ with probability $h(Y)$, for any integer $k \geq 1$,

$$\mathbb{E}\big[(X_\delta)^k \mid Y\big] = (1 - h(Y))\, Y^k.$$

Hence,

$$m_k(\delta) = \mathbb{E}\big[(1 - h(Y))Y^k\big] = (1 - h_0)\,\mathbb{E}[Y^k] - \phi\ \mathbb{E}[Y^{k+2}] + o\big(\mathbb{E}[|Y|^{k+2}]\big), \tag{A.44}$$

where the remainder comes from averaging the $o(y^2)$ term in $h(y)$.

Post-adjustment mean. Using (A.44) with $k = 1$ and (A.39)–(A.41),

$$\begin{aligned} x(\delta) \equiv m_1(\delta) &= (1 - h_0)\,\mathbb{E}[Y] - \phi\ \mathbb{E}[Y^3] + o(\delta^3) \\ &= (1 - h_0)\delta - \phi(3M_2\delta + \delta^3) + o(\delta^3) \\ &= \underbrace{\big(1 - h_0 - 3\phi M_2\big)}_{=:a}\ \delta - \phi\,\delta^3 + o(\delta^3). \end{aligned} \tag{A.45}$$

Since $a \neq 0$, $x(\delta)$ is locally invertible. Solving (A.45) for $\delta = \delta(x)$ yields

$$\delta(x) = \frac{1}{a}\,x + \frac{\phi}{a^4}\,x^3 + o(x^3), \tag{A.46}$$

hence

$$\delta_x(0) = \frac{1}{a}, \qquad \delta_{xxx}(0) = \frac{6\phi}{a^4}. \tag{A.47}$$

Post-adjustment variance. Using (A.44) with $k = 2$ and (A.40)–(A.42),

$$\begin{aligned} m_2(\delta) &= (1 - h_0)\,\mathbb{E}[Y^2] - \phi\ \mathbb{E}[Y^4] + o(\delta^2) \\ &= (1 - h_0)(M_2 + \delta^2) - \phi(M_4 + 6M_2\delta^2) + o(\delta^2) \\ &= \underbrace{\big((1 - h_0)M_2 - \phi M_4\big)}_{=:m_2(0)=\sigma_0^2} + \big(1 - h_0 - 6\phi M_2\big)\delta^2 + o(\delta^2). \end{aligned} \tag{A.48}$$

Moreover

$$\sigma^2(\delta) = m_2(\delta) - x(\delta)^2.$$

Using (A.45), $x(\delta)^2 = a^2\delta^2 + o(\delta^2)$, hence

$$\sigma^2(\delta) = \sigma_0^2 + v_2\,\delta^2 + o(\delta^2), \qquad v_2 := \big(1 - h_0 - 6\phi M_2\big) - a^2. \tag{A.49}$$

With $a = 1 - h_0 - 3\phi M_2$,

$$v_2 = h_0(1 - h_0) - 6h_0\phi M_2 - 9\phi^2 M_2^2. \tag{A.50}$$

Therefore $\sigma^2_{\delta\delta}(0) = 2v_2$ and

$$\sigma^2_{xx} := \frac{\partial^2\sigma^2}{\partial\delta^2}\left(\frac{\partial\delta}{\partial x}\right)^2\bigg|_{\delta=0} = 2v_2 \cdot \delta_x(0)^2 = \frac{2v_2}{a^2} = \frac{2\big(h_0(1 - h_0) - 6h_0\phi M_2 - 9\phi^2 M_2^2\big)}{\big(1 - h_0 - 3\phi M_2\big)^2}.$$

Steady-state relation between gaps and shocks. Using $\epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2)$ and $\epsilon \perp X$,

$$M_2 = \mathbb{E}[(X+\epsilon)^2] = \mathbb{V}\mathrm{ar}(X) + \mathbb{V}\mathrm{ar}(\epsilon) = \sigma_0^2 + \sigma_\epsilon^2. \tag{A.51}$$

At $\delta = 0$, the second-moment from (A.48) satisfies

$$\sigma_0^2 = (1-h_0)M_2 - \phi M_4. \tag{A.52}$$

Moreover, by independence and Gaussianity,

$$M_4 = \mathbb{E}[(X+\epsilon)^4] = \mathbb{E}[X^4] + 6\sigma_0^2\sigma_\epsilon^2 + 3\sigma_\epsilon^4. \tag{A.53}$$

Without aggregate shocks, because the distribution does not change, the moments remain constant. Hence, the steady-state transition of gaps pre-shock and pre-adjustment to post-adjustment is given by:

$$\mathbb{E}(X^k) = \mathbb{E}[(1-h(Z))Z^k]$$

To eliminate $\mathbb{E}[X^4]$ from M_4, evaluate at $k = 4$:

$$\mathbb{E}[X^4] = \mathbb{E}[(1-h(Z))Z^4] = (1-h_0)M_4 + o(\sigma_\epsilon^4),$$

which drops terms proportional to sixth moments (order σ_ϵ^6 under Gaussianity).

Combining with (A.53) yields

$$M_4 = \frac{6\sigma_0^2\sigma_\epsilon^2 + 3\sigma_\epsilon^4}{h_0} + o(\sigma_\epsilon^4). \tag{A.54}$$

Substituting (A.51) and (A.54) into (A.52) gives

$$\sigma_0^2 = \frac{h_0(1-h_0)\sigma_\epsilon^2 - 3\phi\sigma_\epsilon^4}{h_0^2 + 6\phi\sigma_\epsilon^2} \;+\; o(\sigma_\epsilon^4). \tag{A.55}$$

Consequently,

$$M_2 = \sigma_0^2 + \sigma_\epsilon^2 = \frac{h_0\sigma_\epsilon^2 + 3\phi\sigma_\epsilon^4}{h_0^2 + 6\phi\sigma_\epsilon^2} \;+\; o(\sigma_\epsilon^4), \qquad M_4 = \frac{3(2-h_0)\sigma_\epsilon^4}{h_0^2 + 6\phi\sigma_\epsilon^2} \;+\; o(\sigma_\epsilon^4). \tag{A.56}$$

Plugging (A.56) into (A.49) gives the formula for σ_{xx}^2 in terms of $h_0, \sigma_\epsilon^2, \phi$.

Post-adjustment skewness. Using (A.44) with $k = 3$ and (A.41)–(A.43),

$$\begin{aligned} m_3(\delta) &= (1-h_0)\,\mathbb{E}[Y^3] - \phi\;\mathbb{E}[Y^5] + o(\delta^3) \\ &= (1-h_0)(3M_2\delta + \delta^3) - \phi(5M_4\delta + 10M_2\delta^3) + o(\delta^3). \end{aligned} \tag{A.57}$$

Let the centered third moment be

$$\psi(\delta) := \mathbb{E}[(X_\delta - x(\delta))^3] = m_3(\delta) - 3x(\delta)m_2(\delta) + 2x(\delta)^3.$$

Collecting terms up to δ^3 yields the odd expansion

$$\psi(\delta) = d_1\,\delta + d_3\,\delta^3 + o(\delta^3), \tag{A.58}$$

with

$$d_1 = 3h_0(1-h_0)M_2 + 9(1-h_0)\phi M_2^2 - (2+3h_0)\phi M_4 - 9\phi^2 M_2 M_4, \tag{A.59}$$

$$d_3 = h_0(1-h_0)(2h_0-1) + 2\phi M_2(1+3h_0-9h_0^2) - 3\phi^2(18h_0M_2^2 + M_4) - 54\phi^3 M_2^3. \tag{A.60}$$

Therefore,

$$\psi_\delta(0) = d_1, \qquad \psi_{\delta\delta\delta}(0) = 6d_3. \tag{A.61}$$

Recall the definitions

$$\psi_x := \psi_\delta\,\delta_x|_{\delta=0}, \qquad \psi_{xxx} := \left(\psi_{\delta\delta\delta}(\delta_x)^3 + \psi_\delta\,\delta_{xxx}\right)\Big|_{\delta=0}.$$

Using (A.47) and (A.61),

$$\psi_x = \frac{d_1}{a} = \frac{d_1}{(1-h_0-3\phi M_2)}, \tag{A.62}$$

and

$$\psi_{xxx} = \frac{6d_3}{a^3} + \frac{6\phi d_1}{a^4}. \tag{A.63}$$

Simplifications. Using $\sigma_\epsilon^4 \simeq 0$, we can further simplify the above formulas as:

$$\sigma_0^2 \simeq \frac{h_0(1-h_0)\sigma_\epsilon^2}{h_0^2 + 6\phi\sigma_\epsilon^2}, \qquad M_2 \simeq \frac{h_0\sigma_\epsilon^2}{h_0^2 + 6\phi\sigma_\epsilon^2},$$

and $M_2^2 = M_4 = O(\sigma_\epsilon^4) \simeq 0$ given our approximation order.

Replace into (A.49) to obtain:

$$\sigma_{xx}^2 \;\simeq\; \frac{2h_0\,(h_0^2 + 6\phi\sigma_\epsilon^2)\,\left(h_0^2(1-h_0) + 6\phi\sigma_\epsilon^2(1-2h_0)\right)}{\left(h_0^2(1-h_0) + 3\phi\sigma_\epsilon^2(2-3h_0)\right)^2}. \tag{A.64}$$

Plugging into (A.59)–(A.60):

$$d_1 = 3h_0(1-h_0)\frac{h_0\sigma_\epsilon^2}{h_0^2 + 6\phi\sigma_\epsilon^2} \tag{A.65}$$

$$d_3 = h_0(1-h_0)(2h_0-1) + 2\phi\frac{h_0\sigma_\epsilon^2}{h_0^2 + 6\phi\sigma_\epsilon^2}(1+3h_0-9h_0^2) \tag{A.66}$$

□

A.9 Derivation of aggregate reset price and discounting

Start from Equation (5) for the reset price:

$$p_t^\star(f) = \frac{\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left(\prod_{j=1}^{i} \left(1 - \mathbb{I}_{t+j}^\star(f)\right)\right) p_{t+i}^o(f)}{\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left(\prod_{j=1}^{i} \left(1 - \mathbb{I}_{t+j}^\star(f)\right)\right)}.$$

Define the (firm-level) present-value of the hazards as

$$\Lambda_t(f) := \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left(\prod_{j=1}^{i} \left(1 - \mathbb{I}_{t+j}^\star(f)\right)\right). \tag{A.67}$$

Multiplying the two yields:

$$\begin{aligned}
\Lambda_t(f)\, p_t^\star(f) &= \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left(\prod_{j=1}^{i} \left(1 - \mathbb{I}_{t+j}^\star(f)\right)\right) p_{t+i}^o(f) \\
&= p_t^o(f) + \beta\, \mathbb{E}_t \left[\left(1 - \mathbb{I}_{t+1}^\star(f)\right) \sum_{i=0}^{\infty} \beta^i \Big(\prod_{j=1}^{i} \left(1 - \mathbb{I}_{t+1+j}^\star(f)\right)\Big) p_{t+1+i}^o(f) \right] \\
&= p_t^o(f) + \beta\, \mathbb{E}_t \left[\left(1 - \mathbb{I}_{t+1}^\star(f)\right) \Lambda_{t+1}(f)\, p_{t+1}^\star(f) \right] \\
&= p_t^o(f) + \beta\, \mathbb{E}_t \left[\left(1 - h_{t+1}(f)\right) \Lambda_{t+1}(f)\, p_{t+1}^\star(f) \right],
\end{aligned}$$

where the last line applies the law of iterated expectations. The reset price can be decomposed into:

$$p_t^o(f) = \bar{p}_t^o - (1 - \Omega)\, a_t(f) \tag{A.68}$$

where $\bar{p}_t^o$ is common across firms and $a_t(f)$ is the idiosyncratic component, which follows a random walk. Letting $\varepsilon_t(f)$ denote the i.i.d. shock to $a_t(f)$, it holds by symmetry of the distribution of $\varepsilon_t(f)$, oddness of $\varepsilon_t(f)$, and evenness of the hazard (up to third order) that:

$$\mathbb{C}\mathrm{ov}(h_{t+i}(f), \varepsilon_{t+i}(f)) = 0, \qquad i \geq 0$$

Then, up to third order, the dynamic reset price can be written as:

$$p_t^\star(f) = \bar{p}_t^\star - (1 - \Omega)\, a_t(f), \tag{A.69}$$

where $\bar{p}_t^\star$ is common across firms. Integrating over $f \in [0, 1]$:

$$\int_0^1 \Lambda_t(f)\, p_t^\star(f)\, df = \int_0^1 p_t^o(f)\, df + \beta\, \mathbb{E}_t \left[\int_0^1 \left(1 - h_{t+1}(f)\right) \Lambda_{t+1}(f)\, p_{t+1}^\star(f)\, df \right]. \tag{A.70}$$

Using (A.69), we have

$$\int_0^1 \Lambda_t(f)\, p_t^\star(f)\, df = \bar{p}_t^\star \int_0^1 \Lambda_t(f)\, df - (1 - \Omega) \int_0^1 \Lambda_t(f)\, a_t(f)\, df.$$

Under symmetry and evenness, $\Lambda_t(f)$ is (to third order) an even function of the symmetric idiosyncratic innovations, while $a_t(f)$ is odd. Hence $\int_0^1 \Lambda_t(f)\, a_t(f)\, df = 0$ up to third order, and therefore

$$\int_0^1 \Lambda_t(f)\, p_t^\star(f)\, df = \Lambda_t\, \bar{p}_t^\star.$$

Moreover, since $\int_0^1 a_t(f)\, df = 0$, (A.69) implies $p_t^\star = \bar{p}_t^\star$, so the left-hand side equals $\Lambda_t p_t^\star$ up to third order. From (A.68) and $\int a_t(f)\, df = 0$, we get $\int p_t^o(f)\, df = p_t^o = \bar{p}_t^o$. Using (A.69) at $t+1$ and the same symmetry/evenness argument, the idiosyncratic term drops out:

$$\int_0^1 \big(1 - h_{t+1}(f)\big)\, \Lambda_{t+1}(f)\, p_{t+1}^\star(f)\, df = p_{t+1}^\star \int_0^1 \big(1 - h_{t+1}(f)\big)\, \Lambda_{t+1}(f)\, df.$$

Finally, since both $\Lambda_{t+1}(f)$ and $h_{t+1}(f)$ are even, the covariance is fourth order implying:

$$\int_0^1 \big(1 - h_{t+1}(f)\big)\, \Lambda_{t+1}(f)\, df = (1 - h_{t+1})\, \Lambda_{t+1} - \mathbb{C}\mathrm{ov}_f\big(h_{t+1}(f), \Lambda_{t+1}(f)\big) = (1 - h_{t+1})\, \Lambda_{t+1}$$

up to third order. Substituting these results into (A.70) gives, up to terms of order strictly higher than three, gives the cross-sectional aggregates of Equations (6) and (7):

$$\Lambda_t p_t^\star = p_t^o + \beta\, \mathbb{E}_t \left\{ (1 - h_{t+1}) \Lambda_{t+1} p_{t+1}^\star \right\}$$
$$\Lambda_t = 1 + \beta\, \mathbb{E}_t \left\{ (1 - h_{t+1}) \Lambda_{t+1} \right\},$$

used in the system of recursive equations that characterize the approximate solutions of the menu-cost models in Sections 3 and 4.