Pricing to Customers in Firm-to-Firm Relationships

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Abstract

We provide evidence that intermediate-input suppliers price to customers in firm-to-firm relationships. We show that (i) suppliers do not synchronize price adjustments across their different buyers, even for the same product; and (ii) bilateral market shares—particularly the supplier's share within a given buyer relationship—are key determinants of both the frequency and the magnitude of price adjustments (pass-through), in the cross-section and in response to oil price shocks. We rationalize these findings in a model with relationship-specific menu costs and non-constant demand elasticities, which generate heterogeneous pricing behavior across buyers within the same firm. Our results imply that the speed and magnitude of cost pass-through—and therefore inflation dynamics—depend not only on aggregate market concentration, but on the distribution of bilateral dominance within production networks, with important implications for monetary policy in small open economies exposed to commodity price shocks.

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1 Introduction

The nature and extent of firms' pricing rigidity are central to understanding inflation dynamics and the real effects of monetary and fiscal policy (e.g., Pasten, Schoenle, and Weber, 2020; Mongey, 2021). A growing literature shows that the sluggishness with which prices respond to shocks depends on the size of the shock (Cavallo, Lippi, and Miyahara, 2024), the scope of firms' product portfolios (Midrigan, 2011; Bhattarai and Schoenle, 2014), and the degree of market competitiveness (Amiti, Itskhoki, and Konings, 2019; Mongey, 2021). Firms, however, operate within complex production networks in which price adjustment decisions are made at the firm-to-firm level. Thus, the same supplier selling the same product can account for different market shares across its buyers, implying that pricing incentives may vary even *within* a firm-product pair.

In this paper, we study firm-to-firm pricing decisions using unique administrative transaction data that link sellers and buyers at the seller-buyer-variety level and cover the universe of firms in Chile. For each transaction, we observe prices, quantities, and balance sheet information for both the seller and the buyer. This rich structure enables us to construct a simple, intuitive measure of *bilateral market share*. For a given variety, we measure how important a particular seller is for a given buyer by computing the fraction of that buyer's total purchases of that variety that come from that seller. In other words, the seller's bilateral market share captures the weight of seller i in buyer j sourcing of variety v. Similarly, we define the buyer's bilateral market share as the fraction of the seller's total sales of that product that go to buyer j, capturing how important buyer j is to seller i. Using these two key pieces of information, which further characterize one of the main angles of a seller-buyer relationship, we track how suppliers adjust prices across their different customers over time. Specifically, we document five facts on the price setting behavior at the supplier-buyer-variety level.

First, we show that suppliers of a given product do not synchronize their price adjustments to their multiple buyers. That is, when a seller i changes the price of a product, it almost never does so simultaneously for all customers it serves. In fact, only 4% of price adjustments occur simultaneously across the entire portfolio of buyers served by a given seller. This finding extends the evidence on simultaneous price adjustments documented at the multi-product level (Midrigan, 2011; Bhattarai and Schoenle, 2014) to the firm-to-firm dimension. More importantly, it suggests that price rigidities may operate at the seller-buyer-product level, rather than solely at the seller or product level. Accounting for this relationship-specific dimension is therefore crucial for understanding how prices adjust within the complex production networks where firms operate.

Second, we examine how bilateral market shares relate to both the frequency and the magnitude of price adjustments. Specifically, we analyze whether the importance of a seller for a given buyer (or vice versa) shapes pricing dynamics within the relationship. After controlling for a rich set of observed and unobserved determinants of price rigidity at the seller, buyer, industry, and time levels, as well as standard predictors emphasized in the literature, such as supplier industry, price age, price gaps (Campbell and Eden, 2014; Mongey, 2021; Karadi, Schoenle, and Wursten, 2024), and the relationship length (Heise, 2024), we document an important asymmetry. Suppliers that account for a larger share of a buyer's purchases adjust prices *more* frequently and by *larger* amounts. In contrast, when a buyer represents a large share of a supplier's sales, price changes are less frequent and smaller in magnitude.

Third, and building on the two previous facts, we link price-adjustment synchronization with suppliers market shares. Particularly, we show that suppliers with a larger and less concentrated bilateral market share are more prompt to synchronize price changes to their buyers. In other words, sellers whose position is both strong and less heterogeneous across their client base are more prone to adjust prices simultaneously within their portfolio of customers. Hence, and although full synchronization is overall rare in our data, the tightness and internal structure of bilateral relationships emerge as an important determinant of whether price changes are ultimately synchronized across buyers.

To further establish the relevance of bilateral market shares for price adjustment decisions, our fourth fact provides causal evidence from oil price shocks. We restrict the sample to oil-ntensive suppliers that sell oil-related products, e.g., Lubricating petroleum oils, Petroleum gases and other gaseous hydrocarbons, and Gasoline including aviation spiritâto buyers across all other industries in Chile.[1] Although oil suppliers operate in a relatively specific segment of the economy, oil is a pervasive intermediate input across industries. Hence, this exercise allow us to cleanly isolate how shocks to a fundamental cost component are transmitted through bilateral supplier-buyer relationships to the broader Chilean economy. Using a two-stage estimation strategy, we show that oil price increases raise both the probability of positive price adjustments and the magnitude of price revisions, and that these effects are significantly stronger when the supplier holds a larger bilateral market share with the buyer.[2] In particular, the response is economically

[1] Our sample considers more than 3,000 varieties of oil-related products included in 6 product sublcasses: Motor spirit (gasolene) including aviation spirit; Other light petroleum oils and light oils obtained from bituminous minerals (other than crude); Kerosene (including kerosene type jet fuel); Lubricating petroleum oils and oils obtained from bituminous minerals; and Propane and butanes, liquefied, Ethylene, propylene, butylene, butadiene and other petroleum gases or gaseous hydrocarbons, except natural gas.

[2] For illustration, the typical buyers of our six oil-related product subclasses include: (i) premium unleaded gasoline (95 RON), typically sold to national fuel retail chains; (ii) naphtha used as a petrochemical

and statistically different across the distribution of supplier bilateral market shares: suppliers in the upper tail of the market share distribution pass through oil price shocks more aggressively than those in the lower tail. By contrast, buyer bilateral market share plays a much more limited role in this setting, a result that may reflect the specific structure of oil-intensive supply chains.

Our fifth and final fact exploits the exogenous variation in firm-to-firm linkages that occurred in Chile during 2021-2022 to further establish the link between changes in suppliers' bilateral market shares and changes in price-adjustment behavior. Using the full sample, we first document a strong and positive correlation between increases in supplier bilateral market shares and both the frequency and magnitude of price adjustments. This relationship, however, is asymmetric: it vanishes when market shares decline. We then leverage the destruction of supplier-buyer relationships during COVID as an exogenous shock that reshaped supplier-buyer-product market structure. Exploiting this variation, we find that increases in suppliers' bilateral market shares raised the frequency of price adjustment during this period. The estimated effect remains statistically and economically significant even after controlling for other macroeconomic factors associated with the post-pandemic inflation surge occurred in Chile.

Finally, we propose a simple model in the spirit of Gopinath and Itskhoki (2010) and Aruoba et al. (2021) with menu costs and non-constant demand elasticity, to further discipline some of our main findings. A key difference with previous papers is that, to rationalize the evidence, we assume that i) menu costs are specific to the supplier-client relationship and that ii) buyers from different sectors display different elasticity of substitution between the varieties provided by the upstream sector. In our benchmark parametrization of the model, suppliers with higher market share in their buyer purchases of that input, face a smaller super elasticity of demand (threat of losing demand from changing prices), which increases their probability of adjusting prices in the face of oil price shocks. Consistent with our empirical evidence, in the model these suppliers also change prices by a larger magnitude, conditional on adjusting.

Our results have direct macroeconomic implications. Standard models typically discipline cost pass-through using firm-level or sectoral market shares. However, our evidence shows that what matters for price adjustment is not the supplier's aggregate size, but its dominance within specific buyer relationships. This distinction is quantitatively relevant for understanding the transmission of oil price shocks in small open economies. Consider

feedstock, sold to plastics manufacturers; (iii) Jet Fuel A-1, sold to commercial airlines; (iv) synthetic engine oil (e.g. 5W-30), sold to automotive maintenance and servicing firms; (v) liquefied propane for heating, sold to regional gas distribution companies supplying households; and (vi) ethylene, sold to chemical firms producing polyethylene plastics.

two economies with identical distributions of firm-level market shares but different distributions of bilateral market shares. In the first economy, upstream suppliers are highly dominant within their buyer relationships; in the second, sourcing is more competitive across suppliers. Even though aggregate concentration looks identical, the first economy will exhibit faster and stronger pass-through of oil price shocks because dominant suppliers adjust prices more frequently and by larger magnitudes. In our data, Chile displays precisely this pattern in oil-related supply chains, consistent with the relatively rapid transmission of oil price shocks to domestic prices. This suggests that central banks in economies where upstream sectors are characterized by strong bilateral dominance may need to respond more promptly to external cost shocks.

Contribution to the literature. Our paper contributes to the literature studying price setting using microdata (e.g., Nakamura and Steinsson, 2008; Goldberg and Hellerstein, 2011; Bhattarai and Schoenle, 2014; Amiti, Itskhoki, and Konings, 2019; Hong et al., 2023). We provide the first comprehensive evidence that *supplier–buyer–product–specific relationships* are a key determinant of both the frequency and magnitude of price changes. While existing work emphasizes firm- or market-wide measures of market share, we introduce a bilateral measure capturing a supplier's importance within each buyer. This exposes substantial heterogeneity in pricing behavior that is not captured by standard measures of firm size or sectoral market power.

Our findings complement Goldberg and Hellerstein (2011), Amiti, Itskhoki, and Konings (2019), and Mongey (2021), which emphasize the role of aggregate market share, by showing that pricing decisions are fundamentally relationship-specific: the same supplier adjusts prices differently across buyers depending on its bilateral market share. We show that this distinction is quantitatively important for both the extensive margin of price adjustment and pass-through.

We also contribute to the literature on state-dependent pricing. Consistent with Cavallo, Lippi, and Miyahara (2024) and Blanco et al. (2024), we document a sharp increase in the frequency of price adjustment during 2021-2022 in Chile. We extend this evidence by showing that changes in pricing behavior are systematically related to *exogenous*, COVID induced, shifts in the set of suppliers of a given buyer, and that this operates through changes in bilateral market shares.

Our results speak to recent work on market structure, pricing frictions, and monetary non-neutrality. While multisector models with Kimball demand and Calvo pricing (e.g., Pasten, Schoenle, and Weber, 2020; Rubbo, 2023; Afrouzi and Bhattarai, 2023; Baqaee, Farhi, and Sangani, 2024) abstract from within-sector heterogeneity in price rigid-

ity, menu-cost models with non-constant elasticities (e.g., Mongey, 2021; Aruoba et al., 2021) allow adjustment probabilities to vary with market structure. Our contribution is to combine these insights by introducing *relationship-specific* pricing incentives within production networks, consistent with the micro evidence.

Finally, while recent work on granular shocks emphasizes the role of large firms in driving aggregate inflation (e.g., Alvarez-Blaser et al., 2025), our mechanism is distinct. We show that inflation dynamics are shaped by granular pricing decisions at the level of supplier–client relationships, operating primarily through the extensive margin of price adjustment.[3]

2 Data

Our data source consists of a comprehensive firm-to-firm transaction-level dataset, encompassing the universe of Chilean firms. In Chile, all formal businesses have to submit their electronic invoices to the Chilean Internal Revenue Service (SII) in compliance with regulatory mandates since 2018. The main variables we have access to are firms' unique tax identification numbers of both the supplier and the buyer, the date of the transaction, the total value of the transaction, the price per *unit*, the type of product involved in the transaction, and the location of the establishment.[4]

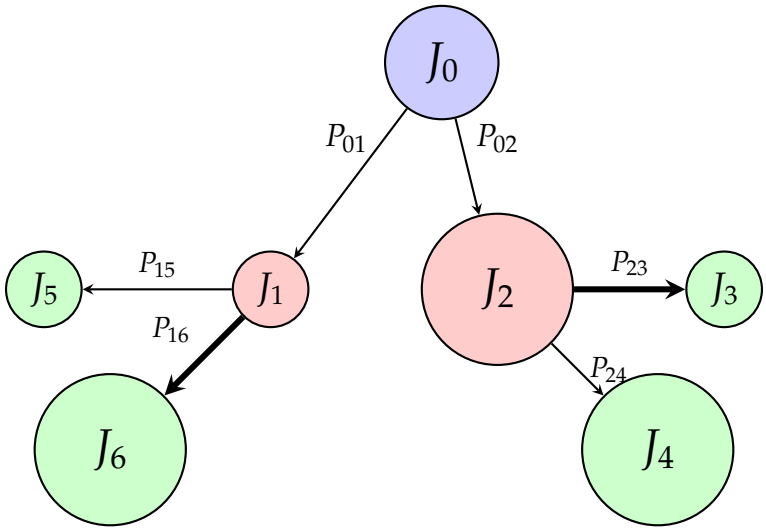
Figure 1 describes the scope and depth of our data, which rely on the information about firm-to-firm transactions. For simplicity, let us assume that each firm supplies one type of product. In the Figure, we observe supplier J_0 selling its product to two different buyers, J_1 and J_2. The price charged to each buyer can differ, and it is represented by P_{01} and P_{02}, respectively. The size of the node indicates a hypothetical size of the firm, and the thickness of the arrow represents the characteristics that are specific to the supplier-buyer pair (e.g., bilateral market share of the supplier in the buyer's purchases, frequency, duration of the relationship). For example, supplier J_1 is a small supplier selling to a small buyer (J_5) and a large buyer J_6. The thicker arrow from J_1 to J_6 indicates a stronger relationship between supplier 1 and buyer 6, compared to supplier 1 and buyer 5. Therefore, beyond information on intermediate input usage, our dataset describes the

[3]Using similar administrative data, Burstein, Cravino, and Rojas (2024) study cross-sectional price dispersion across customers of a given supplier. While their focus is on price levels at a point in time, we study the extensive and intensive margins of price adjustment over time.

[4]To secure the privacy of workers and firms, the CBC mandates that the development, extraction, and publication of the results should not allow the identification, directly or indirectly, of natural or legal persons. All the analysis was implemented by the authors and did not involve or compromise the Chilean IRS.

market structure within supplier-client relationships within a specific input class.

Figure 1: Network graph of an oil supplier (J_0).



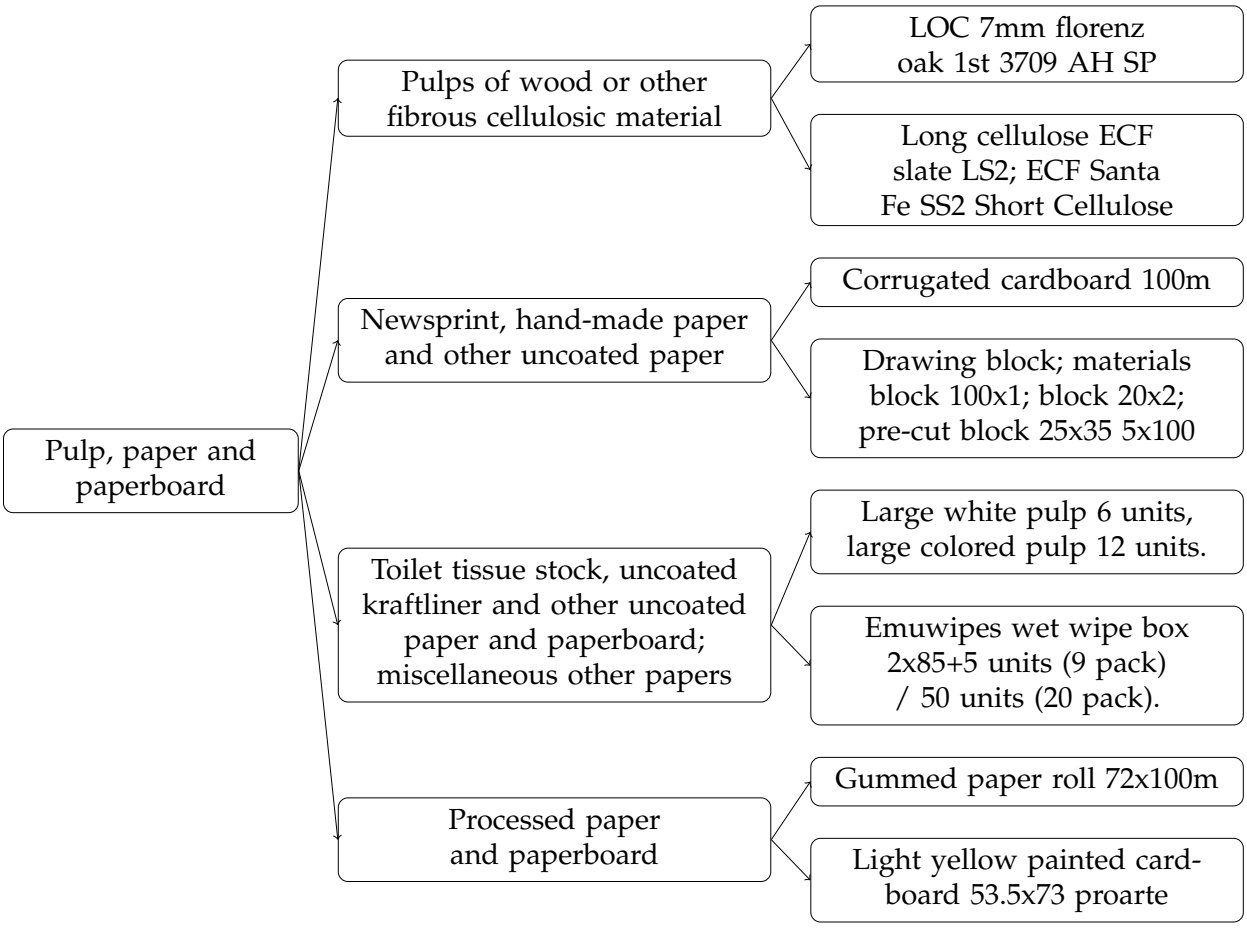
Note: network graph of a supplier J_0. J_1 and J_2 are customers of J_0, while J_3 and J_4 are customers of J_2, and J_5 and J_6 are customers of J_1.

Our dataset is enriched with balance sheet information for both sellers and buyers involved in the transactions. This supplementary data includes total sales, employment figures, industry classifications, and input purchases, among other variables.

Building on our granular data, we further describe the level of disaggregation of the price data for transactions between a supplier and a buyer. The reported information on the type of product or service traded among firms is self-reported and does not always follow standardized product definitions. For example, there might be two firms selling the same type of milk, but one could report it as "two percent milk 1 liter" and the other as "2% milk 1lt". In this example, one would identify these as two different products. To overcome this limitation, we follow the methodology for data cleaning and processing outlined by Acevedo et al. (2023). Specifically, Acevedo et al. (2023) propose an algorithm to classify individual transactions from the firm-to-firm dataset into standardized product categories, such as the Individual Consumption Classification by Purpose (ICCP). The approach relies on a supervised machine learning model on product descriptions and other observables, enabling the assignment of each transaction to a product category contained within the Consumer Price Index (CPI). Figure 2 provides an example of the level of granularity in our data. At the Class level, we have “Pulp, paper, and paperboard”. Classes are further split into Subclasses (e.g., pulps of wood or other fibrous cellulosic material). Standard classification structures then include products and varieties. The classification

algorithm we use groups invoice descriptions into subclasses. Then, within subclasses, we work directly with varieties. Examples of varieties are provided in the last column of Figure 2.

Figure 2: Example of granularity in price data: Classes, Subclasses, and Varieties



This process involves the meticulous identification of prices at the variety level within the transaction data. A "variety" in this context refers to a specific product or service exchanged between firms, with the dataset organized around the triplet ijv, where i represents the seller, j denotes the buyer, and v signifies the variety.

To maintain a focus on significant and recurrent transactions, we have applied the following criteria to the dataset. Varieties included in our analysis are restricted to those associated with products in the official Consumer Price Index (CPI) and Producer Price Index (PPI) baskets. Moreover, each variety must be present in the dataset at least 24 times across any supplier to be considered in our analysis, allowing us to exclude sporadic or non-representative transactions. Price changes ($d \log P$) at the extreme percentiles (1 and 99) have been excluded to eliminate outliers and ensure a robust examination of price

movements. To further validate our data, in Appendix B we replicate the PPI index in Chile using our data, and show that it resembles and traces the official release remarkably well during our data sample.

Invoice data is daily, but we aggregate it to monthly frequency. Our data contains over 142 million monthly transactions for the period January 2018 - May 2023. We observe 15,674 suppliers, 193,479 buyers, 899,595 supplier-buyer pairs, and 230 product subclasses. For more more details see Table A.1 in our Appendix.

Table 1 shows the descriptive statistics of annual sales, numbers of customers, and numbers of product subclasses sold conditioning on sellers. The median firm in the sample displays annual sales of 250 CLP million (approximately 264,000 USD), 3.2 customers, 1.3 product subclasses, and 2.26 product varieties.[5]

Table 1: Descriptive statistics (supplier characteristics)

	Mean	Std. Dev.	p10	p25	p50	p75	p90	Obs.
Average sales (thousands USD)	7,375	140,196	37	92	264	929	3,855	15,674
Number of customers	45.2	963.4	1.0	1.3	3.2	11.3	35.0	15,674
Number of products sold	2.4	3.2	1.0	1.0	1.3	2.5	4.8	15,674
Number of varieties sold	11.57	65.19	1	1.14	2.26	6.42	21	15,674

Note: To convert the values to CLP we can transform each value with an average exchange rate of 950 CLP/USD.

Consistent with the literature (e.g., Gabaix, 2011), the distribution of firm size is highly skewed. There is a long right tail with a handful of very large firms, implying an average sales that is almost thirty times larger than the median sales. A similar pattern is observed for the number of customers each supplier has. While the median firm has 3.2 customers, the average firm sells inputs to 45.2 customers. The average number of product subclasses sold by a supplier is 2.4, while the median is 1.3. The average number of varieties sold by a supplier is 11.57, while the median is 2.26.

3 Facts on price adjustment in supplier-client relationships

Using our firm-to-firm dataset, we document the frequency and magnitude of price adjustments by firms. We study the pricing decisions at the supplier-client-variety p_{ijv}. We also present evidence at the supplier-client-subclass level p_{ijs} to show that our results are not driven by potentially mischaracterized varieties of the same product.

Let us define the frequency of price adjustment f_{ijv} as

[5] As we show in Appendix Figure A.1, suppliers that sell more product subclasses also sell more varieties.

$$f_{ijv} = \frac{\sum_{t=1}^{T_{ijv}} \mathbf{1}(|\Delta p_{ijvt}| > 0.005)}{T_{ijv}}, \tag{1}$$

Where $\mathbf{1}(|\Delta p_{ijvt}| > 0.005)$ is an indicator variable which takes the value of one if the price charged by supplier i to client j for variety v changed by more than 0.5%, and zero otherwise.[6] Where p_{ijvt} is the price seller i charges to buyer j for variety v during month t. To compute the price-adjustment Δp_{ijvt}, we rely on two different criteria. First, we allow for the definition to consider transactions that could occur beyond consecutive months, $\Delta p_{ijvt} = p_{ijv,t} - p_{ijv,t-\gamma_{ijt}}$, where γ_{ijvt} represents the time span between transactions. Each seller-buyer ij pair does not necessarily report a transaction between consecutive months, but rather they could do business only every quarter, once a year, or between irregular (non-stipulated ex-ante) dates. Moreover, in each case, although a transaction is occurring, the price charged does not necessarily change either. For the second definition, we only consider the triplets ijv that account for transactions in consecutive months, i.e., $\Delta p_{ijvt} = p_{ijv,t} - p_{ijv,t-1}$. Throughout the paper, we focus on the first definition. Our results are similar if we use the second definition.

Table 2 presents descriptive statistics on the frequency and magnitude of price adjustments at the supplier level. The average frequency of price adjustments is 0.34. The average magnitude of price changes is 13%. At the supplier-client-variety level, the frequency of price adjustments and the magnitude of price changes are similar. Despite the similarity between f_i and f_{ijv}, there exists great dispersion in the frequency of price adjustment of a given supplier within its buyers. In particular, the average dispersion in suppliers' frequency price adjustment with its buyers, $\sigma_{iv}(f_{ijv})$, is 0.16, about half the value of the average f_i. The dispersion in the magnitude of price adjustment is also large (0.1), which is about 0.91 times the average price change.

[6] We use this threshold to avoid considering monthly price changes that are consequence of price indexation. We used different thresholds and the results stay the same.

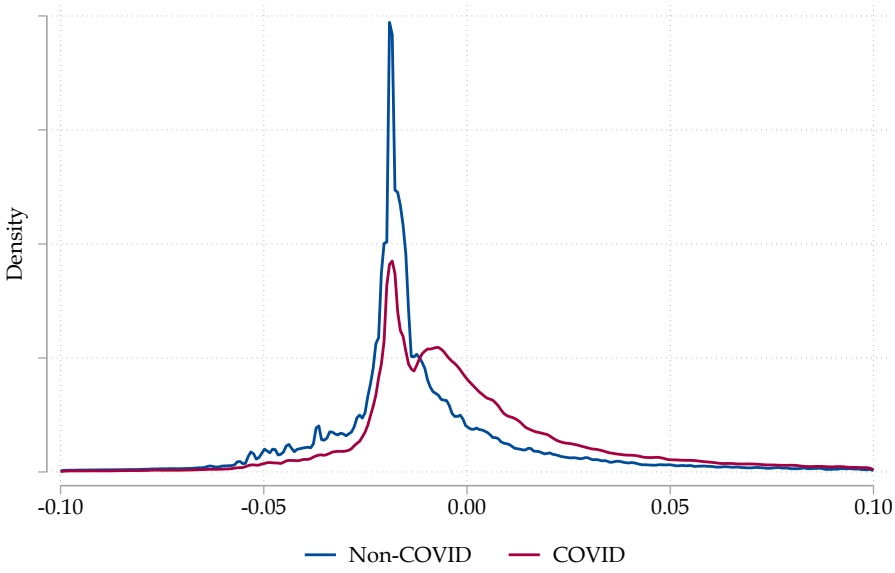
Table 2: Frequency price change at supplier f_i and supplier-client-variety level f_{ijv}

	Mean	SD	P10	P25	P50	P75	P90	N
f_i	0.34	0.28	0.05	0.13	0.25	0.50	0.84	15,684
dp_i	0.13	0.19	0.05	0.07	0.10	0.14	0.21	15,684
f_{ijv}	0.33	0.29	0.00	0.10	0.25	0.50	0.77	10,617,338
dp_{ijv}	0.11	0.20	0.03	0.05	0.07	0.12	0.19	8,514,741
$\sigma_{iv}(f_{ijv})$	0.16	0.10	0	0.08	0.16	0.23	0.28	15,684
$\sigma_{iv}(dp_{ijv})$	0.10	0.19	0	0.02	0.06	0.10	0.21	15,684

Note: We measure the frequency price change f_{ijv} according to Equation (1). There are 15,674 suppliers in our dataset and 10,617,338 supplier-client-variety triplets. $\sigma_{iv}(x_{ijv})$ is the dispersion of measure x_{ijv}, be frequency or magnitude of price change, across all buyers of supplier i. In particular, $\sigma_{iv}(x_{ijv}) = \sqrt{\frac{1}{n_{iv}} \sum_{j \in \mathcal{J}(i,v)} \left(x_{ijv} - \bar{x}_{iv}\right)^2}$, where n_{iv} is the number of buyers of product v for supplier i, $\mathcal{J}(i,v)$ is the set of buyers for supplier i, and $\bar{x}_{iv}$ is the average x across buyers.

We now provide evidence on the change in price-setting behaviour during COVID-19. As documented by Cavallo, Lippi, and Miyahara (2024), the frequency of price adjustment and the magnitude of price adjustment increased during 2020-2022. Figure 3 shows a significant reduction in the mass around zero-price-changes during COVID. This translates into an increase in the average frequency of price changes from 0.3 before COVID-19 to 0.4 during the COVID-19 period (see Table A.2 in our Appendix).

Figure 3: Kernel Distribution of Price Changes



Note: The graph shows the distribution of the log-change of price P_{ijv}. These are all the data where there is an observation before and after COVID for a couple ijv. We extract the fixed effects from the supplier and consumer industries: we obtain the residual (ε_{ijv}) of estimating the following regression $\log P_{ijv} = \alpha + FE_{Ind_i} + FE_{Ind_j} + \varepsilon_{ijv}$.

Not only did the probability of price adjustment increase, but also the magnitude of adjustments rose during the pandemic. This fact is appreciated in the ticker right tail of the distribution in COVID years, compared to non-COVID.

3.1 Synchronization in price adjustment to buyers

Fact 1: *There is little synchronization in price changes from one supplier to different buyers*

Through the granularity of our data, and the firm-to-firm linkages, we could further uncover potential heterogeneous price adjustment patterns at the specific seller-buyer level. In particular, we study the potential simultaneity of price adjustments across the portfolio of buyers j of the same variety v, that each supplier s faces.

Let N_{ivt} be the total number of buyers of a particular seller i for variety v at time t, and K_{ivt} be the number of buyers for variety v that experienced a price adjustment from seller i at time t. Through this definition, we compute the probability of a price change for just one buyer, when the seller has only one customer for variety v, i.e., $Prob(N_{ivt} = K_{ivt} = 1)$. Similarly, we calculate $Prob(N_{ivt} > 1 \cap K_{ivt} = 1)$ and $Prob(N_{ivt} > K_{ivt} > 1)$, which accounts for the probability of a price adjustment for one buyer, when i serves more than

one client, and the probability of a price adjustment for more than one buyer but not for all, respectively. Finally, we can calculate the probability of a simultaneous adjustment across all buyers as $Prob(N_{ivt} = K_{ivt} \cap N_{ivt} > 1)$. The estimated probabilities in these four possible scenarios are presented in Table 3.

Table 3: Simultaneous Price Change Probabilities

Probability	Panel	Cross-Section
$P(K = N = 1)$	17.6%	24.0%
$P(K = 1 \wedge N > 1)$	2.9%	2.1%
$P(N > K > 1)$	6.5%	3.9%
$P(K = N \wedge N > 1)$	5.4%	4.0%
Total	32.4%	34.0%

Note: The *Panel* probabilities are at *ivt* level and the *Cross-Section* are at *iv* level.

Since probabilities are computed at the seller level, aggregating over all possible combinations yields the frequency of price changes reported in Table 2. Strikingly, only 4% of price adjustments occur simultaneously across the entire portfolio of buyers served by seller i. More generally, the evidence suggests highly selective price adjustments at the seller-buyer level. In particular, 2.1% of all adjustments correspond to a single price change, even though the seller serves multiple buyers. An additional 3.9% of the total frequency reflects simultaneous price adjustments by the same seller that are not common across its portfolio of buyers.

Importantly, the data allow us to distinguish between simultaneity that arises in a "mechanical" sense —because a seller has only one buyer for variety v—and a scenario where price adjustment decisions are genuinely coordinated, where all buyers of variety v experience a price change at the same time. Absent this granular $ijv-$level information, one would incorrectly attribute a larger share of price changes to simultaneity, overstating the prevalence of coordinated price adjustments across buyers. To complement this evidence, Table 4 shows the average magnitude of adjustments (in absolute value) for the four categories. While synchronized adjustments (for $N > 1$) are less prevalent in the data, their magnitude is significantly higher than that of all the other categories. This can be interpreted as large cost shocks triggering simultaneous adjustments, which, in turn, cause a large adjustment in the price level.

Table 4: Magnitude of price changes

	Panel		Cross-Section	
Average change	Mean	Std. Dev.	Mean	Std. Dev.
$\Delta Ln(P)\|P(K = N = 1)$	0.054	0.330	0.069	0.255
$\Delta Ln(P)\|P(K = 1 \wedge N > 1)$	0.017	0.131	0.019	0.107
$\Delta Ln(P)\|P(N > K > 1)$	0.027	0.110	0.032	0.103
$\Delta Ln(P)\|P(K = N \wedge N > 1)$	0.068	0.225	0.079	0.210

Note: The *Panel* average changes are at *ivt* level and the *Cross-Section* are at *iv* level.

This empirical evidence has direct implications for the source of price rigidities from a theoretical perspective. If the benefit from resetting a price offsets the cost of changing it, i.e. firms face a menu cost, then a price adjustment would be desirable. In a setup where firms set prices for different products, it has been assumed the presence of economies of scope in price adjustments, where firms face a unique cost for changing prices to all the goods offered (Midrigan, 2011; Alvarez and Lippi, 2014). If this is true, then we should expect a prevalence of simultaneous price adjustments for different buyers of the same firm, which is at odds with our empirical results. Thus, we interpret these results as strong empirical evidence that suggests the presence of idiosyncratic "match-specific" menu costs at the seller-buyer-variety level. The model that we present in Section 4 would build on this evidence.

3.2 Supplier-client market structure

We document key observable match features in the structure of supplier-client relationships. Motivated by Alviarez et al. (2023), we consider two relevant measures of market concentration at the supplier-client level. First, we consider the ratio between a supplier i sales of variety v on a buyer's j total purchases of inputs in the variety v. In particular, we define the supplier's bilateral market share as:

$$s_{ijvy} = \frac{p_{ijvy} q_{ijvy}}{\sum_{k \in Z_j} p_{kjsy} q_{kjsy}},$$

where $p_{ijvy} q_{ijvy}$ represents the sales of product-variety v from seller i to buyer j during year y. The denominator $\sum_{k \in Z_j} p_{kjst} q_{kjst}$ represents the purchases of any input within variety v's subclass s made by buyer j from all possible suppliers of firm j labeled as Z_j.[7]

[7] Following Figure **??**, we interpret s_{ijvy} as the total purchases of buyer j from seller i of a specific product-variety of v, e.g. v ="fish flour" among *the total* purchases of j (from any supplier) of products within the subclass s, e.g. s = "Flours, meals, powders and pellets within year y.

Second, we consider the ratio between a supplier's sales of subclass s on a buyer's total purchases of inputs in that subclass:[8]

$$s_{ijsy} = \frac{p_{ijsy}q_{ijsy}}{\sum_{k \in Z_j} p_{kjsy}q_{kjsy}}.$$

Then, for single-variety suppliers, it must be the case that $s_{ijvy} = s_{ijsy}$. However, for a supplier selling multiple varieties of a given subclass to a buyer, s_{ijvy} is supplier-client-variety specific, while s_{ijsv} is common across varieties sold by supplier i to buyer j. Hence, for different suppliers offering variety v to the same buyer, if $s_{ijvy} > s_{i'jvy}$ indicates that i is a more "dominant" or "relevant" supplier of variety v to buyer j, relative to i'. Likewise, for a given supplier i, if $s_{ijvy} > s_{ij'vy}$ indicates that i is a more relevant supplier to buyer j than to buyer j'.

In addition, and again building on Alviarez et al. (2023), we also measure the bilateral market share of a given buyer j in the sales of a supplier i (x_{ijvy}) as follows

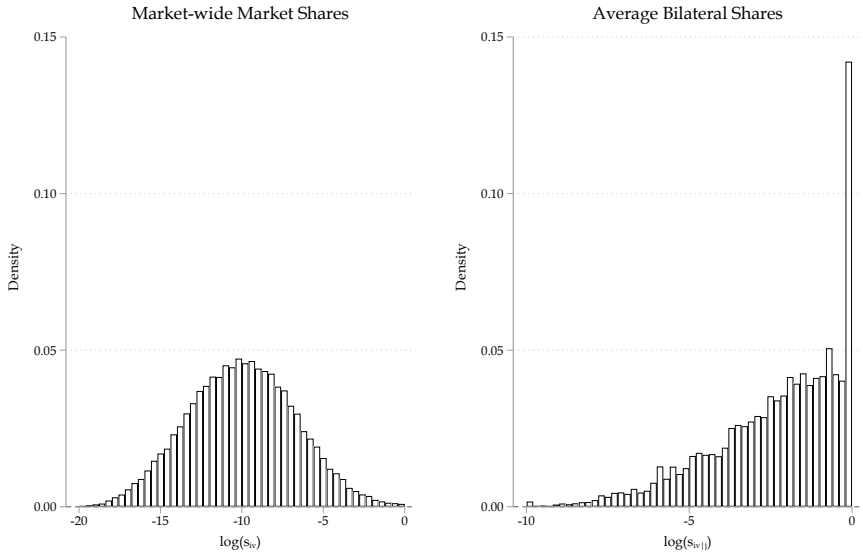
$$x_{ijvy} = \frac{q_{ijvy}}{\sum_{k \in Z_i} q_{iksy}} \quad \text{or} \quad x_{ijsy} = \frac{q_{ijsy}}{\sum_{k \in Z_i} q_{iksy}}$$

In this case, q_{ijvy} corresponds to the total quantities of variety v acquired by buyer j from seller i during the year y. The numerator sums all the acquired quantities of v from all possible sellers of j during year y, which we label as Z_i. We compute this measure at the variety x_{ijvy} and the subclass level x_{ijsy}. Thus, a higher x_{ijsy} is interpreted as a higher or disproportional "sales-dependence" of supplier i on buyer j.

Building on these two measures, we can further stress the relevance of our specific bilateral market share measures to explore their potential relation with price adjustment decisions. First, we can quantify market-wide share of seller-variety iv as a function of the total sales of variety v in the economy, summing across all sellers, for a specific year. In this case, we choose the year 2020. We label this as seller's i sectoral market share $\widehat{s}_{iv} = \frac{\sum_j p_{ijvy}q_{ijsy}}{\sum_j \sum_i p_{ijsy}q_{ijsy}}$. Second, we contrast this with the average bilateral market share across buyers $\widehat{s}_{iv|j} = \frac{\sum_j s_{ijv}}{Z_j}$. The idea behind these two measures, is to stress that while a seller i may be a small player for the overall sales of variety v, i.e., $\widehat{s}_i \approx 0$, the same seller could be the main seller of v for buyer j, $\widehat{s}_{i|j} \approx 1$. This is shown in Figure 4, where the left panel shows that the symmetry of the distribution for $\widehat{s}_i$ shows that both small and big players for the same subclass coexist.

[8] One could aggregate more, as in Alviarez et al. (2023), and consider all the sales made from supplier i to buyer j. However, we consider the product subclass as the relevant market within supplier-client relationships. Table A.9 report the distribution of s_{ij} compared to s_{ijs} and s_{ijv}. As expected, the greater the granularity in the definition of the market, the greater the concentration of suppliers for given buyers.

Figure 4: Market share a the variety level: aggregate vs bilateral



Note: This figure plots the histogram of the logarithm of firms' product market shares. Left panel shows the market shares of the subclasses and right panel shows the average bilateral market share. Both measures correspond to a single supplier per observation, for the year 2020.

On the other hand, the average bilateral market share $\widehat{s}_{i|j}$ (the left panel of the figure) is highly skewed to the right, with a disproportionally mass on 1. This suggests that several buyers have only one supplier for variety v. This circles back to the relevance and granularity of our data in distinguishing the overall or aggregate dominance of a seller within a type of product relative to its importance when examining the specific relationship with their buyers.[9] Moreover, the correlation between the whole market share and the average variety bilateral market share is very small and even negative (see Figures A.6 and A.7 in our Appendix), which indicates that suppliers that sell large amounts of variety v are not necessarily large suppliers in their specific supplier-buyer relationships.

In summary, the relevant measure of market structure within bilateral relationships can be distinctly different from market-wide measures of market concentration, which use aggregated firm-level data. Given the unexplored features of our measures on sellers' and buyers' market shares, we now examine their consequences on price adjustment, first at the cross-sectional level and then by taking into account the full panel dimension.

[9] Figure A.5 in the Appendix shows the same holds at the product subclass level.

3.3 Bilateral market shares and price adjustment

Fact 2: *A larger supplier (buyer) bilateral market share s_{ijv} (x_{ijv}) is associated with a higher (lower) frequency of price adjustment, and with a larger (smaller) magnitude of price adjustment at the supplier-client-variety level.*

We study the relationship between the seller's and buyer's bilateral market shares, i.e., our aforementioned definitions for s_{ijvt} and x_{ijvt}, and the frequency and magnitude of price adjustments within firm-to-firm relationships. We aim to provide specific insights on the average structural determinants of price rigidity while accounting for the relevant unobserved heterogeneity across sellers, buyers, varieties, and time. This latter dimension is relevant, since our sample includes the COVID episode, where the adjustment probabilities changed drastically. We propose the following panel estimation:

$$y_{ijvt} = \alpha + \beta X + FE_i + FE_j + FE_s + +FE_t + \varepsilon_{ijvt} \tag{2}$$

We allow for different definitions of the dependent variable y_{ijvt} to measure different angles of individual prices' flexibility at the ijv level. In particular, the dependent variables are: the (log) level of $ijvt$ prices $Ln(P_{ijvt})$, a dummy equal to one if prices changed upwards $\Delta p_{ijv} > 0$, and a dummy equal to one if prices changed downward instead $\Delta p_{ijv} < 0$. For the intensive margin, we rely on the magnitude of price revisions conditional on adjustment $|\Delta p_{ijv}|\Delta p_{ijv} \neq 0|$ as the dependent variable, where we split this variable depending on the sign of the adjustment. Our controls in $\mathbf{X}$ include the bilateral market share variables (s_{ijvy} and x_{ijt}), the number of transactions between ij, the length of the relationship, and the number of subclasses of products sold by the supplier.[10] Besides these specific ijv controls, we also add the gap to competitors and the age of each price (the number of months since the last p_{ijvt} adjustment). Including the price gap and price age variables allows us to control for additional state-dependent patterns in pricing behavior. The former reflects deviations from a benchmark price level (e.g., the average across competitors), while the latter captures how "stale" the current price is. These indicators are strongly associated with adjustment probabilities, highlighting the role of state-dependent pricing, Campbell and Eden (2014); Karadi, Schoenle, and Wursten (2024).

With respect to the seller and buyers market share, given their high persistence, we measure them at the yearly level y, s_{ijvy}, and x_{ijvy} following the definitions described in

[10] Heise (2024) documents a positive relationship between the price pass-through (intensive margin) in exporter-importer relationships and the age of that relationship. The author explains This pattern in a model illustrates customer relationships within firm-to-firm interactions.

3.2. We saturate regression (2) with a broad set of fixed effects: seller (i), buyer (j), subclass (sub), industry (for both sides) $I_{Ind_{I_i}}$, $I_{Ind_{I_j}}$, and time t (including month and period FEs). The results for the extensive and intensive margins, respectively, are presented in Tables 5 and 6.

Table 5: Panel Estimation - Extensive Margin

	(1) $\Delta p_{ijvt} > 0$	(2) $\Delta p_{ijvt} > 0$	(3) $\Delta p_{ijvt} < 0$	(4) $\Delta p_{ijvt} < 0$
s_{ijvy}		0.017***		0.007***
		(0.000)		(0.000)
x_{ijvy}		-0.007***		0.006***
		(0.001)		(0.001)
Price gap (t-1)	-0.011***	-0.012***	0.000***	0.000**
	(0.000)	(0.000)	(0.000)	(0.000)
Price Age	0.025***	0.013***	-0.015***	-0.018***
	(0.000)	(0.000)	(0.000)	(0.000)
# Transactions		-0.128***		-0.017***
		(0.000)		(0.000)
Relationship Duration		0.063***		0.013***
		(0.000)		(0.000)
Constant	0.202***	0.281***	0.090***	0.089***
	(0.000)	(0.000)	(0.000)	(0.000)
N	130,338,866	130,338,866	130,338,866	130,338,866
Adjusted R2	0.121	0.141	0.125	0.126
Within R2	0.003	0.026	0.003	0.004

Note: The results show the estimation for equation (3). The dependent variable of columns (1) and (2) is constructed as a dummy for positive price changes: $\mathbf{1}(\Delta p_{ijvt} > 0.005)$, ad the dependent variable for columns (3) and (4) as a dummy for negative price changes: $\mathbf{1}(\Delta p_{ijvt} < 0.005)$. All columns included supplier, consumer, subclass, and time fixed effects.

From the results for the extensive margin, in Table 5, we confirm that both the gap to competitors and the age price matter for triggering price adjustments. Relative to the price gap, the results suggest that if a seller's price is higher than the average competitor's, the probability of revising that price upwards decreases. With respect to the age variable, in a context of positive trend inflation, as in our data, prices that have remained constant for several periods are more likely to be revised upwards than downwards, Ball and Mankiw (1994); Karadi and Reiff (2019).

Table 6: Panel Estimation - Intensive Margin

	(1) Ln(P)	(2) Ln(P)	(3) $\lvert\Delta p_{ijv}\rvert\Delta p_{ijv} \neq 0\rvert$	(4) $\lvert\Delta p_{ijv}\rvert\Delta p_{ijv} \neq 0\rvert$	(5) $\lvert\Delta p_{ijv}\rvert\Delta p_{ijv} > 0\rvert$	(6) $\lvert\Delta p_{ijv}\rvert\Delta p_{ijv} > 0\rvert$	(7) $\lvert\Delta p_{ijv}\rvert\Delta p_{ijv} < 0\rvert$	(8) $\lvert\Delta p_{ijv}\rvert\Delta p_{ijv} < 0\rvert$
s_{ijvy}		0.323***		0.001***		0.001***		0.001
		(0.001)		(0.000)		(0.000)		(0.000)
x_{ijvy}		-1.019***		-0.018***		-0.023***		-0.007***
		(0.003)		(0.001)		(0.001)		(0.002)
Price gap (t-1)	0.028***	0.025***	0.004***	0.004***	0.003***	0.003***	0.002***	0.002***
	(0.001)	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Price Age	-0.039***	-0.038***	0.027***	0.016***	0.027***	0.016***	0.029***	0.020***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
# Transactions		-0.041***		-0.036***		-0.036***		-0.034***
		(0.000)		(0.000)		(0.000)		(0.000)
Relationship Duration		0.008***		0.014***		0.013***		0.011***
		(0.000)		(0.000)		(0.000)		(0.000)
Constant	7.671***	7.683***	0.055***	0.092***	0.051***	0.088***	0.066***	0.102***
	(0.000)	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
N	130,338,866	130,338,866	38,779,375	38,779,375	29,636,020	29,636,020	9,115,784	9,115,784
Adjusted R2	0.706	0.708	0.117	0.128	0.135	0.148	0.200	0.204
Within R2	0.002	0.010	0.018	0.031	0.024	0.039	0.012	0.018

Note: The results show the estimation for equation (3). The dependent variable of columns (1) and (2) is constructed as the log level of price p_{ijvt}. Columns (3)-(8) are constructed as the log price change, conditional on absolute, positive, and negative price changes, respectively. All columns included subclass, supplier industry, and consumer industry fixed effects.

The bilateral market power plays a statistically and economically significant role for both margins. As observed in the second column of Table 5, the probability of a price increase rises by 1.7% when a seller i becomes the only supplier of variety v for firm j. Similarly, from column (4) of the same Table, we notice that the probability of price decreases is also positively associated with s_{ijvy}. The interpretation for these results becomes clearer when we look at the intensive margin effects, columns (6) and (8) of Table 6. From these results, we notice that prices of varieties v are adjusted by 0.1% more when $s_{ijvy} = 1$. On the contrary, the effect is not statistically significant for negative price revisions, i.e., these downward adjustments are quantitatively negligible. As noted, in the second column of Table 6, sellers with full bilateral market share charge prices that are 32% more expensive on average to their buyers, relative to sellers with a less relevant position in front of their buyers.

Buyer market power (x_{ijvy}) is negatively associated with both the likelihood and magnitude of positive adjustments. In fact, a higher buyer market power decreases (increases) the probability of positive (negative) price revisions. Moreover, even when prices are revised upwards, the magnitude of such revisions is 2.3% smaller for buyers with full bilateral market power $x_{ijvy} = 1$. This is shown in column (6) of Table 6. Similarly, when prices are revised downwards, buyers with full market power x_{ijvy} allow them to experience an extra drop in prices of approximately 0.1%, as shown in column (8) in Table 6. With respect to the level of prices, and consistent with the previous evidence, a higher

x_{ijvy} reduced the overall price paid by buyers j.[11] These results align with the intuitive notion that bigger and more powerful buyers can bargain for lower prices, which are therefore more likely to decrease. Intuitively, this case is the complete opposite of buyers where the seller exerts a lot of market power on them (a high s_{ijvt}), which is reflected in their different pricing dynamics.

Finally, the number of transactions strongly reduces the probability of upwards revisions, where, even when prices increase, the revision magnitude is less the higher the number of transactions. This is consistent with the presence of explicit or implicit contracts that prevent continuous renegotiation. Likewise, longer relationships are correlated with higher price revisions, plausibly reflecting the fact that in longer-standing links, prices are adjusted more automatically, given the characteristics of the relationship.

We complement our previous analysis by estimating the same specifications at the cross-sectional ijv level. We follow this approach for two main reasons. First, to assess whether key determinants emphasized in the literature also matter for pricing decisions in our data. Since most of these determinants are defined at the seller level, they are absorbed by seller fixed effects in our panel specifications, which motivates using a cross-sectional specification as a robustness check. Second, we examine whether, after controlling for these factors, the structure of relationships within the firmâs network - captured by our s_{ijv} and x_{ijv} measures - remains relevant for price adjustment decisions. The main results are reported in Appendix A.6. Importantly, we show that the probability of price adjustment is strongly related to seller characteristics documented in the literature, in particular the rigidity of input prices Nakamura and Zerom (2010); Gopinath and Itskhoki (2010), and the number of products sold by the firm, Bhattarai and Schoenle (2014). When we add our measures of bilateral market power as additional controls, all previous results continue to hold.

Robustness In Appendix A.7 we discuss two relevant robustness exercises. First, we show that our results also hold when we estimate our main regressions using a subsample of products identified by international product bar codes (blueEXPAND). Second, we ask for the stability of our results when we add a producer-variety fixed effect to the specification, as in Alviarez et al. (2023), to control for average marginal costs at the supplier-variety level.

[11] To make sense of the estimated magnitude, recall that the estimated values of x_{ijvy} are roughly small. In fact, as shown by table A.9, the value for x_{ijvy} at the 95th percentile is approximately 0.07.

3.3.1 Destruction of relationships and price adjustment during 2020-2022

To further support our evidence on the effects of sellers' market share on price adjustment decisions, we rely on the destruction of supplier-client relationships that occurred during 2020-2022 as a source of exogenous variation, which directly affected sellers' market share. Figure A.12 in the Appendix shows the evolution of the number of firm-to-firm linkages, before, during, and after the pandemic years. In the figure, we categorize sellers by size. Notably, across all firm sizes, we observe that a large share of links were broken due to the high economic costs, uncertainty, and mobility restrictions (lockdowns) that occurred during those years in the Chilean economy. Based on this, and relative to the evidence before the pandemic, we assess its potential implications for price adjustment decisions.

First, we show that the bilateral supplier-variety market share iv increased when the number of suppliers to buyer j declined during 2020-2022. We define ΔZ_{js} as the change in the total number of suppliers of buyer j for variety v between 2020-2022 and 2018-2019. We regress this variable on the change of seller's market share Δs_{ijv} for the same years. For completeness, we also control for other relevant variables as the change in the average buyer's market share (Δx_{ijv}) between the same period, the change in the frequency of price adjustments of the inputs of seller i ($\Delta p_{i,inputs}$) and the average initial values of the frequency f^0, the seller's s^0_{ijv} and buyer's market shares x^0_{ijv} between 2018-2019. The results are shown in the first column of Table 7.

According to the results, when the number of suppliers decreased by one, the seller's bilateral market share increased by 5 percentage points on average. Since we interpret the change in the number of j's suppliers as exogenous to supplier i due to the pandemic shock, we use this evidence as a first-stage and recover the predicted Δs_{ijv} to study the implications that changes in bilateral market shares — caused by changes in the number of suppliers of a given producer — have on pricing decisions at the ijv level. Specifically, we explore if changes in bilateral market shares, Δs_{ijv} and Δx_{ijv}, before and after the pandemic, led to significant changes in the frequency of price adjustment from supplier i, variety v, to buyer j. We estimate the following linear probability model regression:

$$\Delta f_{ijv} = \Delta s_{ijv} + \Delta x_{ijv} + \Delta f_{i,inputs} + f^0 + s^0_{ijv} + x^0_{ijv} + FE_j + FE_s + \varepsilon_{ijv},$$

The controls are the same as in the previous regression. We also add fixed effects at the buyer and subclass levels. As discussed, we use the number of links destroyed (ΔZ_{js}) as an instrument to address the potential endogeneity in the change in seller's bilateral market shares and the adjustment frequency. The results are shown in the last

three columns of Table 7.

Δs_{ivt} and x_{ivt} is the change in the bilateral market shares from the period 2018-2019 to the period 2020-2021. Consumer and subclass FEs were used.

Table 7: Changes in frequency price adjustment (Δf_{ijv}) and bilateral market share (Δs_{ijv}, Δx_{ijv}) during the pandemic

	First Stage	Second Stage		
		(1)	(2)	(3)
	Δs_{ijv}	Δf_{ijv}	Δf^{+}_{ijv}	Δf^{-}_{ijv}
ΔZ_{js}	-0.054***			
	(0.001)			
Δs_{ijv}		0.310***	0.489***	-0.181***
		(0.025)	(0.028)	(0.035)
Δx_{ijv}	0.335***	-0.228***	-0.321***	0.073***
	(0.008)	(0.013)	(0.017)	(0.020)
$\Delta f_{i,\text{inputs}}$	0.006***	-0.125***	-0.011***	-0.234***
	(0.001)	(0.003)	(0.003)	(0.005)
f^0	0.008***	-0.744***	-0.456***	-0.485***
	(0.000)	(0.001)	(0.002)	(0.002)
s^0_{ijv}	-0.258***	0.052***	0.083***	-0.028***
	(0.001)	(0.007)	(0.007)	(0.010)
x^0_{ijv}	0.114***	-0.101***	-0.124***	-0.004
	(0.004)	(0.007)	(0.008)	(0.011)
N	1,342,675	1,342,675	863,916	286,024
Adjusted R2	0.216	0.290	-0.041	0.315
F-statistic	9040.5			

Note: The regressions are at a *ivt* level. Δs_{ivt} and x_{ivt} is the change in the bilateral market shares from the period 2018-2019 to the period 2020-2021. Consumer and subclass FEs were used.

From column (1) of the second stage, and consistent with our previous results, we observe that increases in the supplier's bilateral market share lead to larger increases in the frequency of price adjustment.In column (2), we re-estimate the linear probability model, but restricting the dependent variable only to upward price adjustments. Again, increases in bilateral market share also raise the probability of upward revisions. On the other hand, in Column (3), we observe that the probability of a price decrease declined, on average, across relationships in which the supplier became relatively more central to the buyer. We also find that changes in the buyer market share matter, correlated with the frequency with which supplier i changes prices to buyer j.

Taking stock. Besides the unobserved patterns at the firm and time levels, the evidence suggests that specific match characteristics between sellers and buyers matter for price-adjustment decisions. Building on this fact and leveraging the network structure that we can characterize through our data, we now attempt to further characterize the dynamics of pass-through after an input cost shock, while conditioning on ij characteristics.

3.4 Price Synchronization and bilateral market shares

Fact 3. *Suppliers with a larger and less concentrated bilateral market share s_{ijv} are more prompt to synchronize price changes to their buyers.*

Building on the two previous facts, we examine the relationship between bilateral market share and the probability of synchronized price adjustments across buyers. Our hypothesis is that sellers exerting a higher degree of bilateral market power across buyers are more likely to adjust prices simultaneously to multiple clients. Intuitively, when a common cost-push shock hits sellers of the same variety v, those with a higher and more concentrated distribution of s_{ijvy} are precisely the ones that find it optimal to adjust prices to their entire portfolio of buyers at the same time. Similarly, suppliers with higher dispersion in their bilateral market shares across buyers will display less synchronization.

We formalize this intuition with a simple regression linking price synchronization to bilateral market power. Specifically, we construct a dummy variable equal to one whenever synchronization occurs at each specific ivt level. We regress this indicator on the dispersion of seller i's market shares across buyers, $SD(s_{ijvy})$, and on the average market share, $mean(s_{ijvy})$. We estimate these regressions using both the full sample and a subsample conditional on seller i changing its price for at least one buyer j. In all specifications, we include supplier, buyer, and subclass fixed effects. The results are reported in Table 8.

Table 8: Synchronization and bilateral market share

	(1) Synchro	(2) Synchro	(3) Synchro	(4) Synchro	(5) Synchro	(6) Synchro
$sd_{ivt}(s_{ijvy})$	-0.053***	-0.146***			-0.055***	-0.155***
	(0.001)	(0.002)			(0.001)	(0.002)
$mean_{ivt}(s_{ijvy})$			0.009***	0.019***	0.005***	0.016***
			(0.000)	(0.001)	(0.001)	(0.001)
Constant	0.120***	0.213***	0.052***	0.114***	0.119***	0.210***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
N	4,747,579	2,774,833	9,864,721	4,493,646	9,864,721	2,774,833
Adjusted R2	0.148	0.168	0.067	0.075	0.148	0.169
Sample	Whole	Change	Whole	Change	Whole	Change

Note: The regressions are at a *ivt* level. *Synchro* is a dummy equal to 1 if there is price change simultaneity at that pair. $sd_{ivt}(s_{ijvy})$ is the market share dispersion of supplier's i consumers (j), and $mean_{ivt}(s_{ijvy})$ is the mean market share. In the specification, we rely on all sellers independently, whether they have only one buyer for variety v or not. This is why the number of observations drops when we condition on the sd(.) variable. We run this regression for the entire sample and for the sample subject to there being at least one price change ($K > 0$). Supplier, consumer and subclass FEs were used.

From the first two columns, we show that sellers with greater heterogeneity in s_{ijvy} across buyers are less likely to do simultaneous price adjustments. Similarly, and relying on the results from columns (3) and (4), we observe that sellers with a higher average market share are more likely to synchronize price changes across customers. The last two columns indicate that these correlations remain even when we jointly control for both variables. Hence, although price synchronization is rather infrequent, the tightness and complexity of bilateral relationships between a supplier and its (potentially large) pool of clients are important determinants of simultaneous price adjustments.

3.5 Price adjustments to oil price shocks

Fact 4. *A larger supplier (buyer) bilateral market share s_{ijv} (x_{ijv}) increases (decreases) the probability of price adjustment and the magnitude of price adjustment in response to an oil price shock.*

Up to this point, we have provided evidence of unconditional price changes and their correlation with firm-to-firm relationships. In this Section, we study the importance of the nature of firm-to-firm relationships in the transmission of a cost-push shock. We focus on oil price shocks at a specific and relevant point in the production network and examine

their potential effects on both the extensive and intensive margins of price adjustment.[12]

To gain intuition and further explore the richness of our data, we focus on suppliers in nodes J_1 and J_2 of Figure 1. The idea is to provide a quantitative assessment of the propagation dynamics of the same shock at a relevant level of the production network, which we can identify using our granular data. For simplicity, let us assume that nodes J_1 and J_2 in Figure 1 represent *oil-intensive producers* selling oil-related products (e.g., oil engine or gasoline) or services (transportation services) to different customers from different industries, represented by the underneath nodes J_3, J_4, J_5, J_6. We allocate suppliers in the upstream nodes (J_1 and J_2) if their share of total sales of oil-related products, relative to total sales, is bigger than 0.5.[13]

Using this classification, we end up with 27,500 transactions, 121 sellers, 1,555 buyers from different sectors of the economy, 27 product subclasses, and 424 varieties (see the second column of Table A.1 in the appendix, for further details). The resulting subsample has an even more concentrated market structure on the supply side. Table A.9 in the Appendix shows that the seller's bilateral market share is, on average, larger than that of the whole sample. While the average value of s_{ijvt} is 0.25 across the entire sample, it increases to 0.66 in the oil subsample. Similarly, while buyers' market share is often small, our oil subsample also displays a marginally larger value for x_{ijvt}. At the variety level, the average x_{ijvt} is 5% in the oil subsample, whereas it is only 2% in the whole sample.

3.5.1 The extensive margin adjustment

For the extensive margin of price adjustment, we follow Karadi, Schoenle, and Wursten (2024) and estimate the following regression

$$I^{+,-}_{ijv,t+h} = \alpha_i + \alpha_j + \alpha_s + \beta_h \Delta \log \hat{P}^{oil}_{it} + \phi_h (\Delta \log \hat{P}^{oil}_{it} \times W_{ij}) + \gamma_h X_{ijv} + \psi w_t + \epsilon_{ijs}, \qquad (3)$$

In equation (3), $I^{+,-}_{ijv,t+h}$ is a dummy variable that takes the value of one if the log change of prices from period $t+h$ compared to period $t-1$ for supplier i, selling a variety v to customer j, changed, and zero otherwise. The superscript $+,-$ indicates if the price increased ($+$) or decreased ($-$). The variable $\Delta \log \hat{P}^{oil}_{it}$ is the change in the log of oil price that is relevant for supplier i. To account for the relevance of Oil, we quantify supplier i's

[12] Lafrogne-Joussier, Martin, and Méjean (2023) also studies the pass-through of oil price shocks, focusing on the intensive margin of adjustment at the firm level. The authors show that the pass-through is larger for larger firms, indicating oligopoly power in pricing decisions as in Atkeson and Burstein (2008).

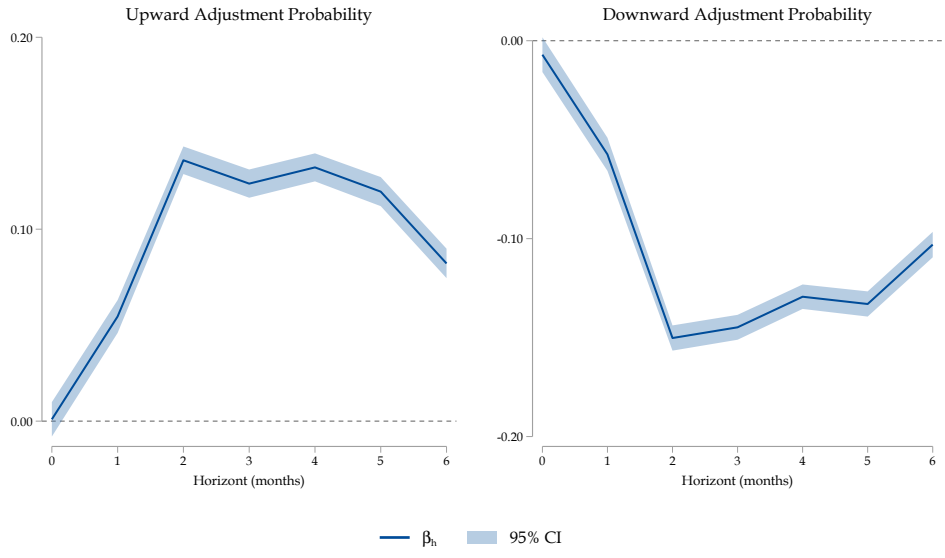
[13] As expected, a typical firm at this node is a gasoline station, transport companies, or a manufacturing firm that redistributes oil to third parties. Recall that the different sizes of these two nodes are only for expository reasons, aiming to resemble different sellers' characteristics, such as sizes, sales, or number of employees.

exposure to Oil as an input by using the share of Oil in total intermediate input purchases.

We instrument the change in oil prices using the oil supply shock series from Baumeister and Hamilton (2019), and we interact the (instrumented) change in oil price $\Delta \log \hat{P}_{it}^{oil}$ with W_{ij}, which can represent i, j, or ij specific (time-invariant) characteristics. We focus particularly on both the seller and buyer market shares (but we could also explore other ij-specific match characteristics, such as the duration of the relationship or the number of transactions). We also add an additional set of controls, X_{ijv}, in each specification, which are the same that we used in equation (2).

Figure 5 shows the estimated average probability of price adjustment β_h up to six months after the oil shock. Initially, we focus on upstream sellers, so we omit the interaction term ϕ_h in (3). The left (right) panel plots the probability of an upward (downward) price revision at different horizons after an oil price increase. As noted, the average probability of a price increase after a one-standard-deviation increase in oil prices is approximately 12% two months later and remains relatively constant for the next six months, when it drops to approximately 8%. Likewise, the probability of reducing prices drops by 15% two months after a 1% increase in oil prices, with a marginal effect of 10% on average six months after.

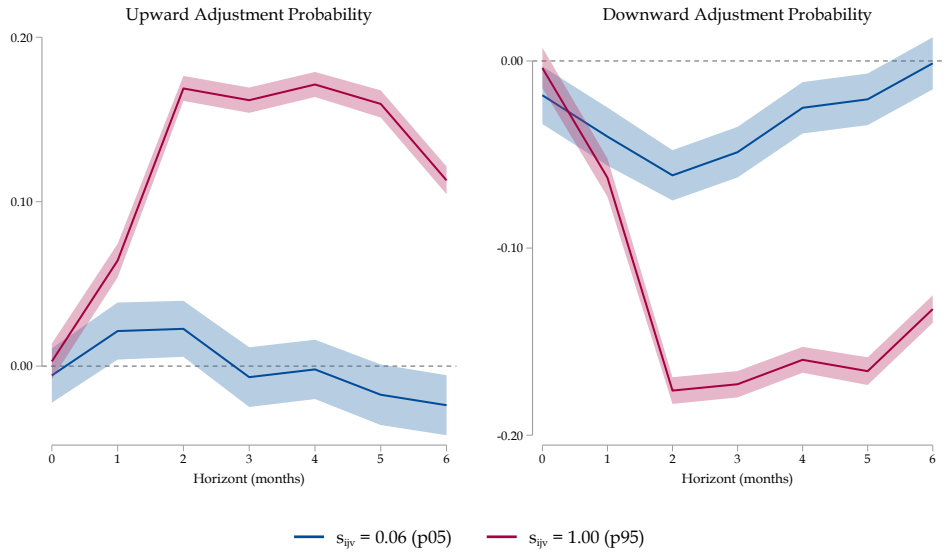
Figure 5: Probability of price adjustment to a 1 SD increase in oil prices



We now analyze the influence of supplier market bilateral shares on price adjustments. We re-estimate (3) but adding the interaction term ϕ_h, where W_{ij} is the ijv bilateral market share s_{ijv}. The left panel of Figure 6 shows the marginal effects for the probability of upward adjustments at the 5th and 95th percentiles of s_{ijv}. The right panel shows the

probability of downward adjustments.

Figure 6: Upward and Downward Adjustment Probability by Supplier Market Share

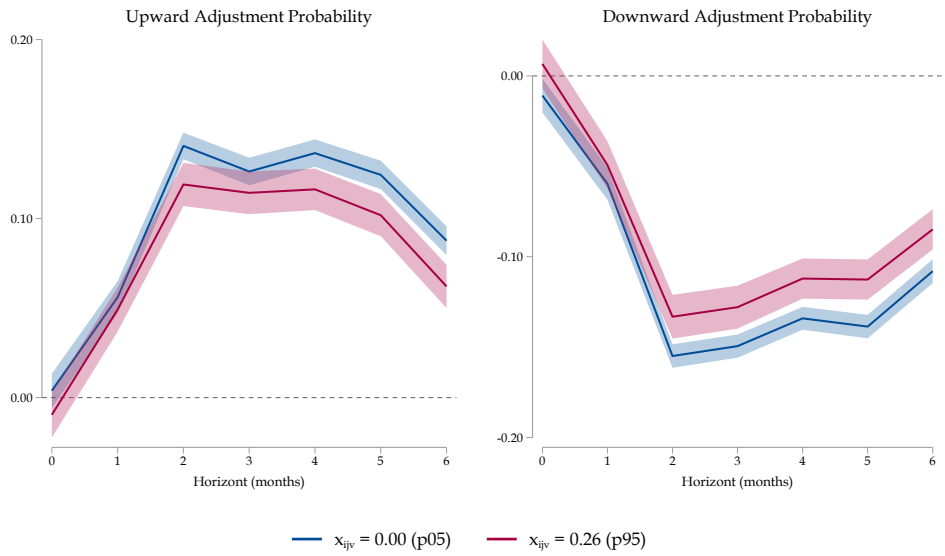


Suppliers with larger bilateral market shares, s_{ijv}, are significantly more likely to raise prices to their customers after a change in oil prices. The probability of positive adjustment increases by almost 20%, on average, for suppliers in the 95th percentile of the ijv bilateral market share distribution. This sharply contrasts the almost quantitatively negligible and barely significant effect of positive adjustment for suppliers at the bottom end of the s_{ijv} distribution. The results for the probability of downward adjustment are roughly similar, where the probability of negative adjustments drops by approximately 18% after a 1% increase in the price of oil for sellers with a high bilateral market share, compared to the (although significant) drop of approximately 4% for the same probability across sellers that only sell a tiny fraction of their varieties to their buyers.[14]

Up to this point, it is clear that the seller's market share has non-negligible effects on the dynamics of price adjustments. We now turn to examining whether the consumer's market share, x_{ijv}, is relevant to the price adjustment probability. We re-estimate equation (3) by defining $W_{ij} = x_{ijv}$; i.e., ϕ_h now accounts for the additional effect that the buyer's market share may have on the adjustment probability. As before, the left panel of Figure 7 shows the marginal effects for the probability of upward adjustments at the 5th and 95th percentiles of x_{ijv}, while the right panel reports the effects for the probability of downward adjustments.

[14] All our results hold if we use s_{ijs} rather than s_{ijv}.

Figure 7: Positive and Negative Price Change by Consumer Market Share



Interestingly, and in sharp contrast with the previous evidence for seller's market share, there are no clear statistical differences in either upward or downward adjustments when we condition on small or large values of x_{ijv}. While the probability of upward (downward) adjustments rises (decreases) after the rise in oil prices, the heterogeneous effect of price adjustments vanishes at this dimension, suggesting that the rate by which prices faced by buyers changed is roughly the same independently of their relative importance to a seller of the same variety v.

3.5.2 The intensive margin adjustment

We now study the intensive margin of price adjustments. We follow Jordà (2005) and Jordà, Schularick, and Taylor (2015), and estimate the following Local Projection-IV regression

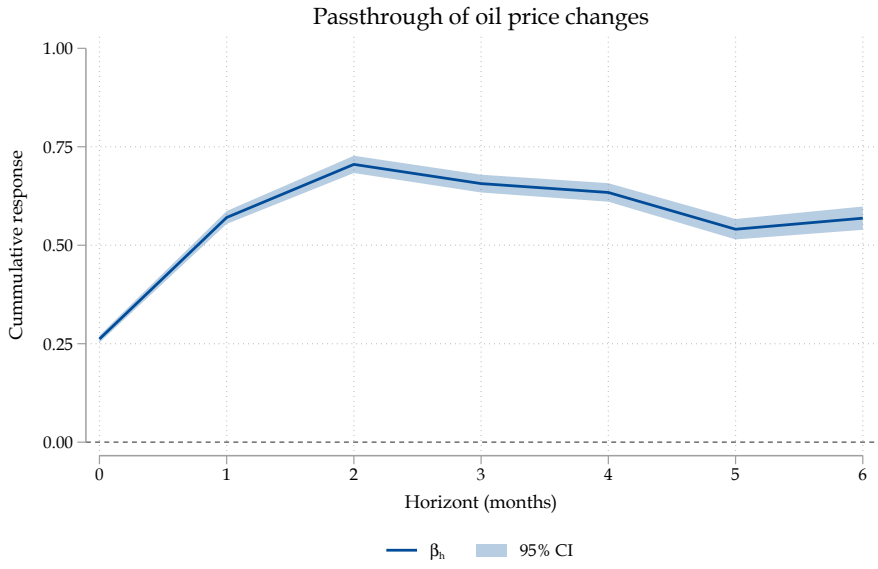
$$\pi^{K,ijv}_{t-1,t+h} = \alpha^h + \beta^K_h(\Delta \log \hat{P}^{oil}_{it}) + \phi^K_h(\Delta \log \hat{P}^{oil}_{it} \cdot W_{ij}) + \sum_{j=1}^{6} \delta^j \pi_{t-j} + \sum_{j=1}^{6} \gamma^j \Delta P_{c,t-j} + \psi X_t + \varepsilon^h_t, \tag{4}$$

The dependent variable in equation (4), $\pi^{ijs}_{t-1,t+h}$, is the log change of prices from period $t+h$ relative to period $t-1$ for supplier i, selling variety v to customer j. As before, $\log \hat{P}^{oil}_{it}$ is the instrumented log price change, and W_{ij} is our market share controls. Following the same strategy as before, initially, we estimate (4) without the interaction term

ϕ_h^K to study how the oil price adjustment passes through to micro prices, β_h^K, at this particular level of the production network.

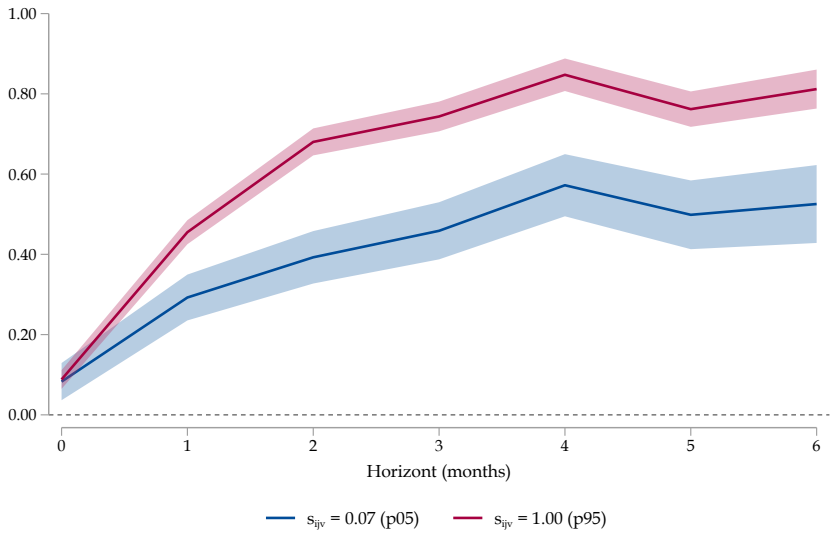
Figure 8, shows the intensive margin response of prices at different time horizons. The figure shows the pass-through of firms that are the largest sellers of oil-intensive goods or services in the economy (nodes J_1 and J_2 in Figure 1). In this case, while the pass-through after one month is more than 50% on average, this value stabilized at 75% two months after the oil price increase. Therefore, the evidence suggests that the pass-through is fairly incomplete even at this point in the network that focuses on large Oil-intensive suppliers.

Figure 8: price response of oil-intensive suppliers (J_1andJ_2)



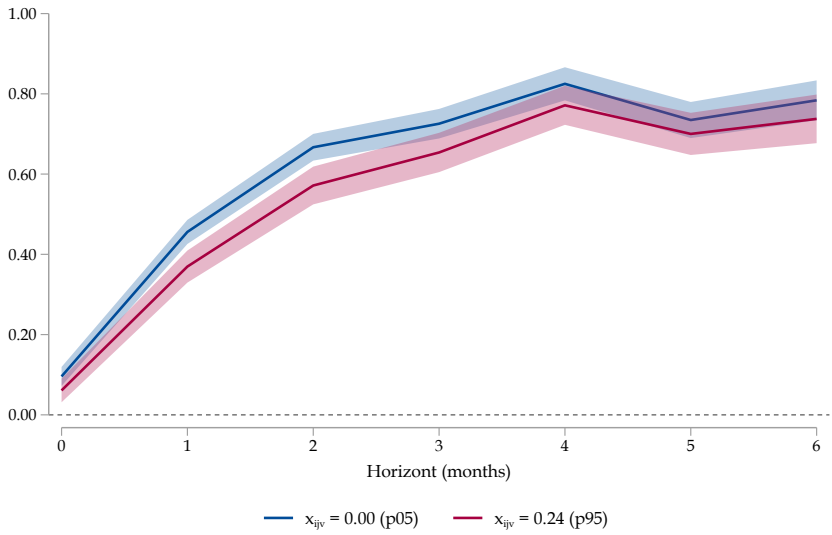
We now include the interaction term in equation (4). As for the extensive margin, we are interested in the pass-through effect of oil prices to micro prices $\frac{\partial \pi^{K,ijv}_{t-1,t+h}}{\partial \Delta \log \hat{P}^{oil}_{it}} = \beta_h^K + \gamma_h^K W_{ij}$, conditioning on different values of W_{ij} across the market share's percentiles. Figure 9 shows the marginal effect where the variable $W_{ij} = s_{ijv}$ is again set at its 5th and 95th percentiles values, for relationships between firms in nodes $J_1 - J_2$ (suppliers) and J_3, J_4, J_5, J_6 buyers.

Figure 9: Intensive Margin by Supplier Market Share



Consistent with the depicted probability of price adjustments, some clear differences arise. Suppliers with a higher bilateral market share pass through their higher costs more aggressively than firms with a small market share. In fact, two months after the oil price change, sellers with higher market power have already passed through approximately 70% of the adjustment to their buyers, in contrast with the roughly 40% of cost pass-through from sellers with a small seller's market share. While the heterogeneous effects are clear at the seller's side, again, such differences are absent when we focus on buyers' market shares x_{ijs}. This is illustrated in Figure 10, which shows that there are no significant differences in the pass-through depending on whether a buyer has a large or small degree of market share with respect to each seller i.

Figure 10: Intensive Margin by Consumer Market Share



In the Appendix, Figures A.10 and A.11, we show that our dynamic results hold if we consider only the period 2018-2020m2, which excludes COVID-19. In light of the large rise in oil prices during the pandemic, it is important to stress that pass-through dynamics remain present, even in less turbulent times.

4 A simple theoretical framework

Taking stock, our facts suggest (i) non-synchronized price adjustment across buyers, (ii) a relevant role for s_{ijvt} in explaining the frequency and the magnitude of price adjustments, (iii) a tight connection between the seller's bilateral market share and the occurrence of simultaneous price adjustments and (iv) an heterogeneous pass-through at the seller level, depending on the degree s_{ijvt}, after a cost-push shock. In this section, we propose a simple theoretical framework attempting to rationalize these facts. For doing so, we rely on two key assumptions: (a) *relationship-specific* menu costs, and (b) *variable-elasticity* demand that makes markups (and therefore, pass-through) and the probability of price adjustment to depend on the supplier's position along the residual-demand curve. We build our model in the spirit of Gopinath and Itskhoki (2010) and the variable-elasticity/menu-cost literature (e.g., Mongey, 2021; Aruoba et al., 2024).

Environment. A continuum of upstream suppliers $i \in [0, 1]$ each produce one intermediate-input variety and sell it to many downstream buyers/sectors $j \in \{1, \ldots, J\}$. The same

supplier can hold different *bilateral market shares* across buyers, which we interpret as different positions on residual demand (and therefore different demand elasticities and curvature). The object of interest is the supplier's share within buyer j's sourcing of that input, s_{ij}.

4.1 Demand with variable elasticities

Downstream aggregation. In each downstream sector j, intermediate varieties are combined through a Kimball aggregator:

$$\int_0^1 G_j\left(\frac{n_{ij}y_{ij}}{Y_j}\right) di \;=\; 1, \tag{5}$$

where y_{ij} is demand by sector j for variety i, Y_j is sector-j total intermediate usage, and n_{ij} is a (relationship- or sector-specific) demand shifter.

Kimball demand implies that the suppliers-buyers *elasticity of demand* ϵ_{ij} and the *super-elasticity* $\sigma_{ij} \equiv \partial \ln \epsilon_{ij} / \partial \ln p_{ij}$ vary with the firm's relative price, i.e., its position on residual demand/market share.[15]

4.2 Pricing, markups, and pass-through

Marginal cost. We assume that each supplier i faces a marginal cost

$$MC_i(z_i, e) \;=\; \frac{W^{1-\phi}e^{\phi}}{z_i}, \tag{6}$$

where W is the wage, e is the (imported) oil/input price, and z_i is the seller's i idiosyncratic productivity.

Flexible-price optimality. Let p_{ij} be the price charged by supplier i to buyer j. Under variable-elasticity demand, each seller i maximizes profits, and derives the flexible (opti-

[15] In Appendix C.1 we show that the elasticity of demand:

$$\epsilon_{ij} = -\frac{G_j'(x_{ij})}{x_{ij}G''(x_{ij})}$$

Hence, the demand elasticity depends on the overall concavitity of the Kimball Aggregator G(.) function and the relative bilateral market share $x_{ij} = \frac{n_{ij}y_{ij}}{Y_j}$. Therefore, for a given seller i, differences in n_{ij} across buyers j would bring differences in elasticities and hence in optimal prices, even abstracting from nominal rigidities.

mal) price set to each customer j, which satisfies the familiar markup rule:

$$p_{ij} \;=\; \mu_{ij}(p_{ij})\, MC_i, \qquad \mu_{ij}(p) \equiv \frac{\epsilon_{ij}(p)}{\epsilon_{ij}(p)-1} \tag{7}$$

In Appendix C.1, we provide all the specific details of the derivation of the optimal price and buyers' demand.

Oil Prices Pass-Through Taking logs and linearizing around a symmetric steady state (where prices are equal to $\overline{p}$) yields an approximate pass-through representation:

$$\ln p_{ij} - \ln \bar{p} \;\approx\; \Psi_{ij} \ln MC_i, \qquad \Psi_{ij} \equiv \left(1 + \frac{\sigma_{ij}}{\epsilon_{ij}-1}\right)^{-1} \in (0,1]. \tag{8}$$

The details of the formal derivation are presented in Appendix C.2. Thus, the specific oil-cost pass-through is:

$$\frac{\partial \ln p_{ij}}{\partial \ln e} \;\approx\; \phi\, \Psi_{ij}.$$

Thus, under Kimball demand the pass-through is incomplete. In fact, the superelasticity $\sigma_{ij} > 0$ which implies $\Psi_{ij} < 1$. Intuitively, given both the curvature and the position of residual demand, the endogenous markup adjustment absorbs part of the cost shock. On the contrary, under CES demand, the constant elasticity implies $\sigma_{ij} = 0$, so $\Psi_{ij} = 1$ and therefore the pass-through is complete.

Link to bilateral market shares. In Kimball systems, both ϵ_{ij} and σ_{ij} vary with the supplier's position on residual demand; we summarize this by writing

$$\Psi_{ij} \;=\; \Psi(s_{ij}) \;=\; \left(1 + \frac{\sigma_{ij}(s_{ij})}{\epsilon_{ij}(s_{ij})-1}\right)^{-1}. \tag{9}$$

This delivers the key testable implication: *pass-through differs across buyer links within the same supplier*, because Ψ_{ij} differs with s_{ij}.

4.3 Relationship-specific menu costs and non-synchronized adjustment

Menu cost. Changing the price to buyer j requires paying a fixed *relationship-specific* menu cost κ_{ij}. This assumption directly matches Fact 1: price changes are rarely synchronized across a supplier's portfolio of buyers, so the adjustment technology must operate at the ij level rather than at the firm level.

State-dependent adjustment. Let $\bar{p}_{ij}$ denote the preset price for link (i, j). Define the profit gain from adjusting (relative to keeping $\bar{p}_{ij}$) as

$$L_{ij} \equiv \pi_{ij}(p^*_{ij}; \text{state}) - \pi_{ij}(\bar{p}_{ij}; \text{state}),$$

where p^*_{ij} is the flexible desired price from (7). A standard second-order approximation (as in Gopinath and Itskhoki (2010)) implies

$$L_{ij} \approx \frac{1}{2} \mathcal{K}_{ij} \left(\ln p^*_{ij} - \ln \bar{p}_{ij} \right)^2, \tag{10}$$

where $\mathcal{K}_{ij} > 0$ is proportional to the curvature of profits with respect to the price for that link. Combining (10) with (8), the gain from adjusting increases with the squared size of the underlying cost shock, scaled by *link-specific* pass-through:

$$L_{ij} \propto \left(\Psi_{ij} \ln MC_i \right)^2.$$

With stochastic menu costs, the probability of adjustment is monotone in L_{ij}:

$$\Pr(\text{adjust } p_{ij}) = \Pr(\kappa_{ij} \leq L_{ij}).$$

Mechanism. A higher supplier bilateral market share s_{ij} changes the local elasticity/curvature of residual demand, shifting Ψ_{ij} and $\mathcal{K}_{ij}$, and therefore affects *both*: (i) the desired pass-through (intensive margin) via (9), and (ii) the incentive to pay the menu cost (extensive margin) via (10). Because menu costs are relationship-specific, a supplier adjusts prices selectively across buyers after the same cost shock, generating the observed lack of synchronization.

4.4 Quantitative illustration

We use the model to map bilateral market shares into (i) pass-through $\Psi(s_{ij})$ and (ii) an adjustment probability through L_{ij} in (10). Fact 4 in our paper illustrates how larger s_{ij} can generate larger pass-through and a higher likelihood of adjustment for sufficiently strong cost shocks, consistent with Facts 2–4.

We calibrate the model in partial equilibrium, given aggregates $(Y_j, P_j, \forall\ j)$. There is a grid of suppliers with different productivity level. The productivity distribution is log normal with the mean and standard deviation of z being 1 and 0.5, respectively. Each supplier i serves each of the J sectors. A downstream representative firm j in sector j is

characterized by a Kimball parameter ψ_j. Therefore, a given supplier i will display, in partial equilibrium, heterogeneous bilateral market shares (s_{ij}) with its buyers.

To construct the loss function we simulate a 40% increase in oil prices e. Figure 11 plots in the y-axis the pass-through and the loss of not adjusting prices for a given supplier i with its different customers. In the x-axis, we show the bilateral market share between supplier i and its buyers $j, \in J$, as a share of each buyer's total purchases of intermediate inputs $s_{ij} = \frac{p_{ij}y_{ij}}{\sum_{k=1}^{K} p_{kj}y_{kj}}$.

Our parametrization follows Aruoba et al. (2024) and sets $\omega = 1.29$ with $\psi_j \in (-1.5, -0.5)$.

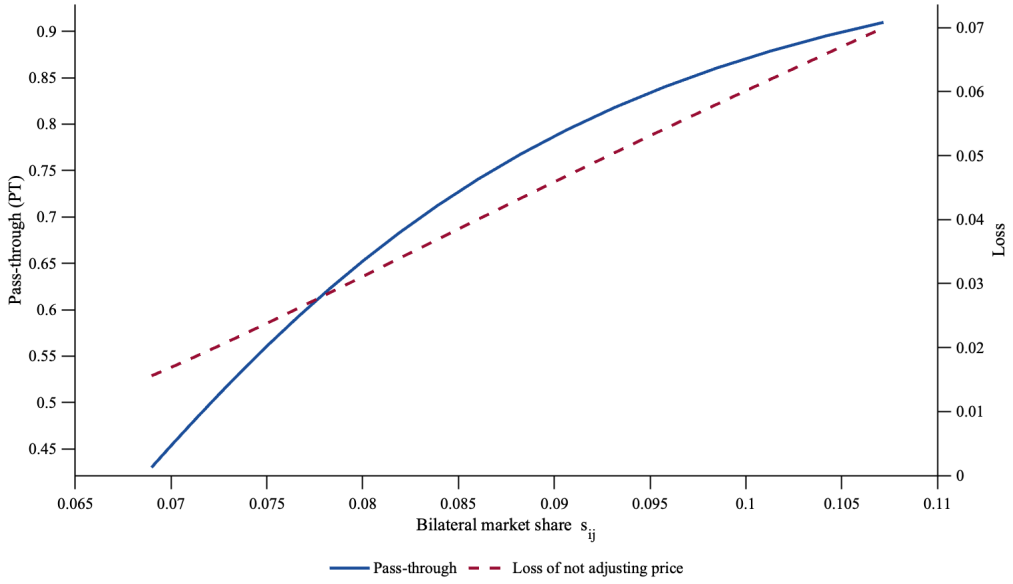


Figure 11: Bilateral market share, pass-through, and loss of not adjusting prices

Under this parametrization, the results show that, consistent with the evidence, higher market share is associated with larger pass-through and higher loss of not adjusting, meaning higher chances of adjusting prices for a given value of the menu cost.

4.4.1 Allowing for Seller specific menu-cost

A natural alternative to the relationship-specific cost κ_{ij} is to assume that adjustment costs operate at the *supplier* level. More specifically, suppose that changing the price to any buyer j requires supplier i to pay a fixed cost κ_i, common across all its buyer relationships. Once this cost is paid, the supplier may reprice any subset of its buyers at no additional cost, i.e., the pricing decision turns into a binary, all-or-nothing choice. In particular, and

given the assumed increase in the oil price, the supplier adjusts if and only if the aggregate gain covers the menu cost,

$$\kappa_i \;\leq\; \sum_{j=1}^{J} L_{ij} \tag{11}$$

Substituting the second-order approximation (10) and the pass-through relationship $\ln p_{ij}^{*} - \ln \bar{p}_{ij} \approx \Psi_{ij} \ln MC_i$ from (8), condition (11) becomes:

$$2\kappa_i \;\leq\; \left(\ln MC_i\right)^2 \bar{\Phi}_i \tag{12}$$

where $\bar{\Phi}_i = \sum_{j=1}^{J} \mathcal{K}_{ij} \Psi(s_{ij})^2$. Hence, the probability of adjustment for any buyer j of supplier i is the same $\Pr\left(\kappa_i \leq \bar{\Phi}_i (\ln MC_i)^2/2\right)$, i.e., depends on the specific supplier-level characteristics and is therefore independent of the ij relationship. Ultimately, under this assumption, the model predicts that any observed price adjustment is synchronized. This is completely at odds with our first fact, where we show that only 4% of price adjustments are synchronized across a supplier's entire portfolio of buyers. Through a relationship-specific menu-cost κ_{ij}, we can allow each ij match to cross its own independent threshold, delivering the non-synchronized pricing patterns that we document.

4.4.2 Synchronization and seller's market share

Our baseline model is also consistent with our third fact, namely that suppliers with a larger average bilateral market share s_{ijv} are more prompt to synchronize price changes with their buyers. As noted from (10) a supplier with uniformly high s_{ijv} faces a larger L_{ij} in every individual relationship, making it more likely that the threshold $\kappa_{ij} \leq L_{ij}$ is met simultaneously across buyers after a given cost shock, further propiciating synchronized pricing adjustment decisions. Although this implication is also true under a common menu-cost κ_i we already shown that this contradicts our first fact.

4.5 Discussion of Model Implications

A key implication of the model concerns the macroeconomic transmission of cost shocks. Because adjustment probabilities and pass-through depend on bilateral market shares rather than aggregate firm size, economies with similar sectoral concentration can display markedly different inflation dynamics. In particular, when upstream suppliers hold high bilateral market shares within their buyer relationships, the effective super-elasticity of demand is lower and price adjustments occur more frequently and with larger magnitude following a cost shock. As a result, oil price shocks propagate more rapidly through the

production network. By contrast, when sourcing is more competitive at the bilateral level, suppliers face stronger demand discipline and adjust prices more gradually, slowing the transmission of cost shocks. Therefore, the distribution of bilateral dominance—rather than aggregate market concentration alone—is more informative for the speed of pass-through in response to upstream shocks.

5 Conclusion

We show that price rigidity is not a firm-level characteristic, but a relationship-level outcome shaped by bilateral market shares. Using administrative data on firm-to-firm transactions, we document five new facts at the supplier-buyer-variety level. First, there is little synchronization in price changes from one supplier to different buyers. Second, a larger supplier bilateral market share within a buyer increases both the frequency and the magnitude of price adjustments, while a larger buyer share reduces them. Third, suppliers with larger and less concentrated bilateral market shares are more likely to synchronize price changes across buyers. Fourth, in response to oil price shocks, relationships with higher supplier market shares display a higher probability and larger magnitude of adjustment, while higher buyer shares mildly dampen responses. Fifth, during the 2021-2022 inflation episode, the frequency of price adjustments increased more in relationships where the supplier's bilateral market share rose relative to 2018-2019.

Our results imply that models of price rigidity—those with menu costs and variable demand elasticities—should move beyond firm-level market power and incorporate relationship-specific exposure within production networks, where bilateral market shares—rather than aggregate firm size—shape adjustment incentives and pass-through. In particular, the transmission of oil price shocks—and thus short-run inflation dynamics—depends critically on the structure of bilateral dominance within production networks, suggesting that monetary authorities in small open economies should account for firm-to-firm market power when assessing the inflationary impact of external cost shocks.

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A Complementary Descriptive Statistics

Figure A.1: Variety and subclasses

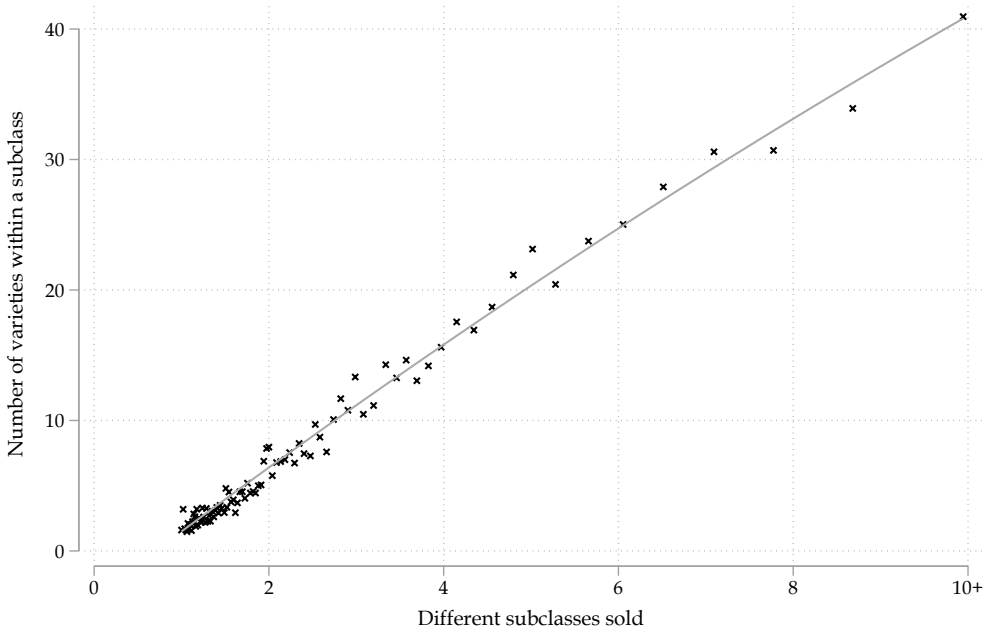


Table A.1: Descriptive Stats - Samples

Variable	Whole Sample	Local Projections
Transactions	142,584,028	27,500
Sellers	15,684	121
Buyers	193,479	1,555
Subclasses	230	27
Varieties	404,322	424
Varieties per Subclass	1774.23	15.70
Buyer-Seller pairs	899,594	1,585
Buyer-variety pairs	479,914	468

Note: We use as a filter that there must be at least 5 transactions per ijv pair. We lose around 9000 issuers, who have very few transactions per pair.

A.1 Complementary evidence to section 3

Table A.2: Price Change: Non-Covid vs Covid periods

Panel A: Probability of Price Change		
Variable	Non-Covid	Covid
F_{ijv}	0.298	0.400
$F_{ijv}\|F_{ijv} \neq 0$	0.466	0.479

Panel B: Magnitude of Price Change		
Variable	Non-Covid	Covid
ΔP_{ijv}	0.012	0.032
$\Delta P_{ijv}\|F_{ijv} \neq 0$	0.019	0.038

Note: The table compares the average values of each variable for the non-COVID and COVID periods. The COVID period covers the months of March 2020 to December 2022. F_{ijv} is the average of F_{ijvt}, which is a dummy equal to 1 if there is a change greater than 0.5%. in time t. ΔP_{ijv} is the average of the monthly change ΔP_{ijvt}. In both cases we show the unconditional changes and the changes, subject to the fact that there was a change in the subsample (e.g., $F_{ijv} \neq 0$)

Figure A.2: Kernel dP (no zeros)

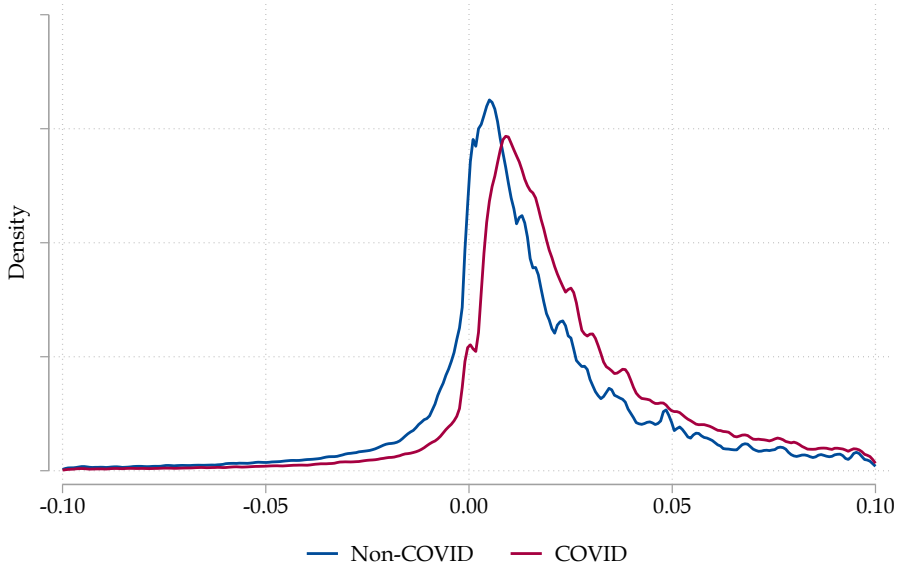


Figure A.3: Kernel dP

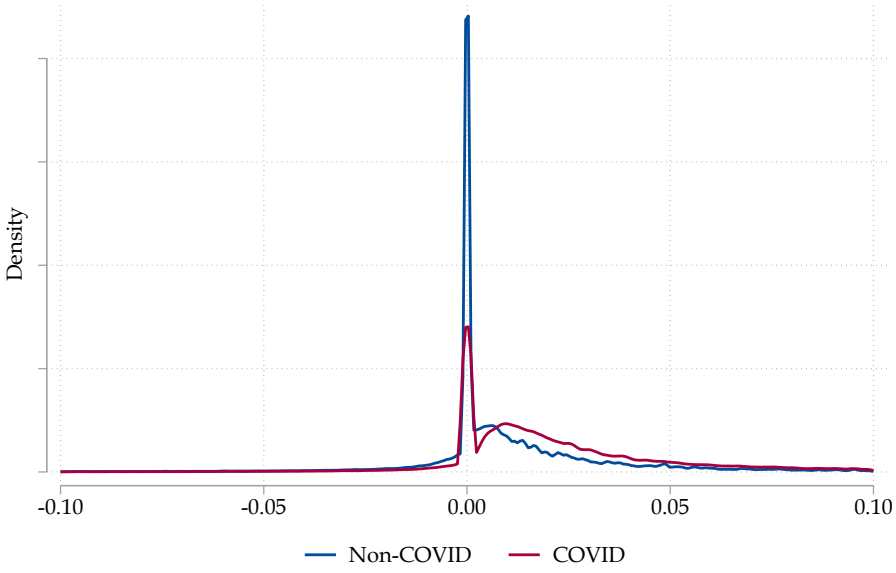
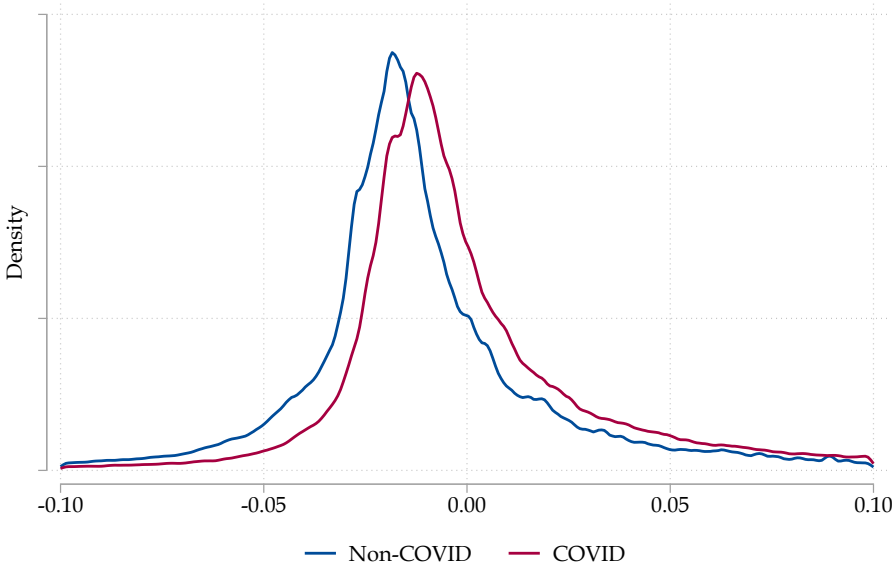


Figure A.4: Kernel dP, excluding zeros and excluding industry FEs



A.2 Complementary evidence to section 3.1.

Here, we present the same analysis as in section 3.1, but at a subclass level. To perform the aggregation, we restricted the sample to subclasses in which the number of varieties was stable in time (i.e., the same number for each month).

Table A.3: Price Change Probability

Probability	Mean	Std. Dev.	p10	p25	p50	p75	p90	Obs.
f_{ivt}	0.308	0.41	0	0	0	0.64	1	1,307,535
f_{iv}	0.318	0.31	0	0.05	0.23	0.5	0.87	65,649

Table A.4: Extended Price Change Probabilities

Probability	Panel	Cross-Section
$P(K = N = 1)$	15.3%	23.7%
$P(K = 1 \wedge N > 1)$	3.4%	2.0%
$P(N > K > 1)$	7.2%	2.9%
$P(K = N \wedge N > 1)$	5.0%	3.1%
Total	30.8%	31.8%

Table A.5: Price Change Probabilities, subject to a Price Change

Probability	Panel	Cross-Section
$P(K = N = 1 \vert K > 0)$	50.0%	74.6%
$P(K = 1 \wedge N > 1 \vert K > 0)$	11.0%	6.4%
$P(N > K > 1 \vert K > 0)$	23.3%	9.3%
$P(K = N \wedge N > 1 \vert K > 0)$	16.7%	9.7%
Total	100%	100%

Table A.6: Price Change Probabilities, subject to a Price Change and more than one consumer per supplier

Probability	Panel	Cross-Section
$P(K = 1 \wedge N > 1 \vert K > 0 \wedge N > 1)$	21.8%	25.3%
$P(N > K > 1 \vert K > 0 \wedge N > 1)$	46.1%	36.5%
$P(K = N \wedge N > 1 \vert K > 0 \wedge N > 1)$	38.2%	40.7%
Total	100%	100%

Here we present a fixed-effect decomposition for the dispersion of the price level and the frequency of price adjustment (similar results hold for the magnitude of price adjustment). Panels A of Tables A.7 and A.8 show that product-time and supplier-client-product-time fixed effects account for 38% and 43%, respectively, of the variation in frequency price adjustment. On the other hand, Panels B show that within supplier-product dispersion, supplier-client-product account for more than 97% of the variation while buyer characteristics only account for the rest.

Table A.7: Extensive Margin Variance Decomposition: Variety Level

	Ln(P)	f_{ijv}	$f_{ijv} > 0$	$f_{ijv} < 0$
Panel A. Overall price dispersion				
Controls				
FE_i	0.4743	0.1741	0.0689	0.1085
FE_j	0.0286	0.0096	0.0040	0.0082
FE_{vt}	0.4679	0.3883	0.4691	0.2248
E_{ijvt}	0.0291	0.4280	0.458	0.6568
Panel B. Within seller-product dispersion				
Controls				
FE_j	0.2546	0.0231	0.0098	0.0129
E_{ijvt}	0.7453	0.9769	0.9902	0.9871
Obs.	88,659,819	88,659,819	88,659,819	88,659,819
Adj. R^2	0.970	0.555	0.524	0.316
Within R^2	0.000	0.000	0.000	0.000

Table A.8: Extensive Margin Variance Decomposition: Subclass level

	Ln(P)	f_{ijv}	$f_{ijv} > 0$	$f_{ijv} < 0$
Panel A. Overall price dispersion				
Controls				
FE_i	0.5007	0.0527	0.0217	0.0299
FE_j	0.0798	0.0408	0.0207	0.0267
FE_{vt}	0.1484	0.066	0.0685	0.0405
E_{ijvt}	0.2711	0.8405	0.889	0.9029
Panel B. Within seller-product dispersion				
Controls				
FE_j	0.2035	0.0486	0.0237	0.0309
E_{ijvt}	0.7965	0.9514	0.9763	0.9691
N	15,164,342	15,164,342	15,164,342	15,164,342
Adjusted R2	0.725	0.149	0.100	0.086
Within R2	0.000	0.000	0.000	0.000

A.3 Complementary evidence to section 3.2.

Figure A.5: Subclass share comparison

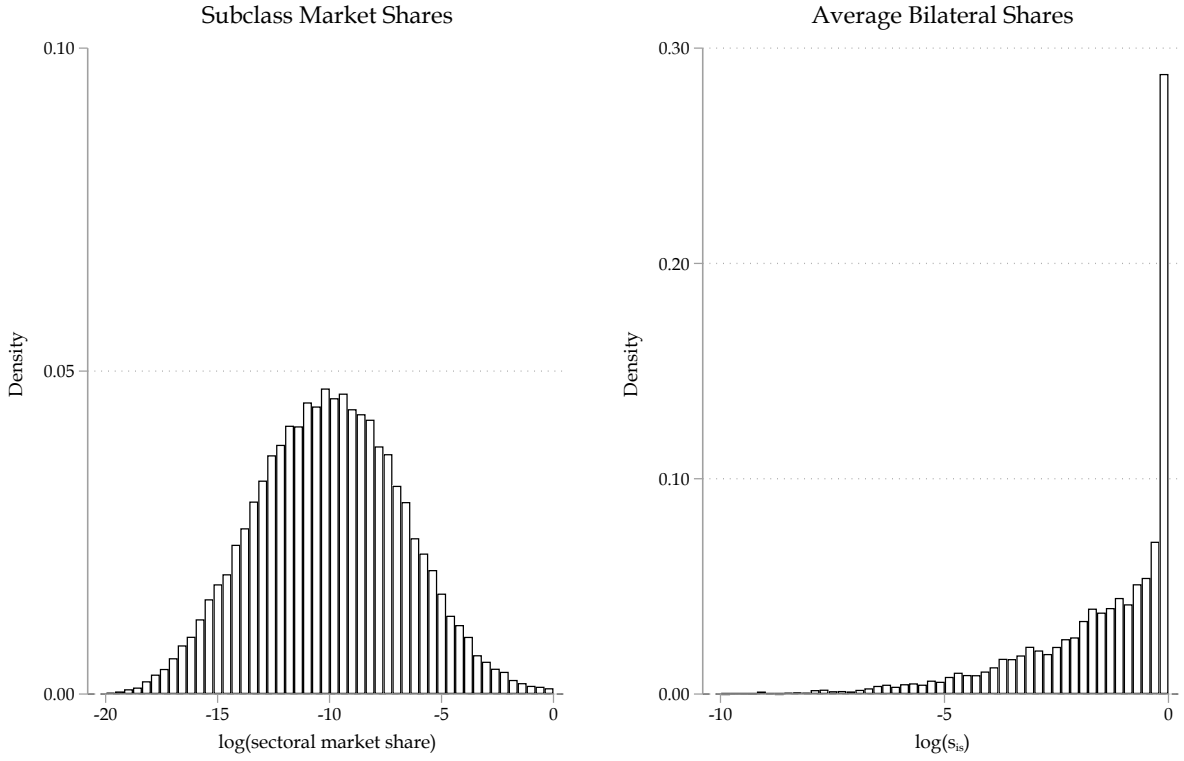


Figure A.6: Scatter Variety Correlation

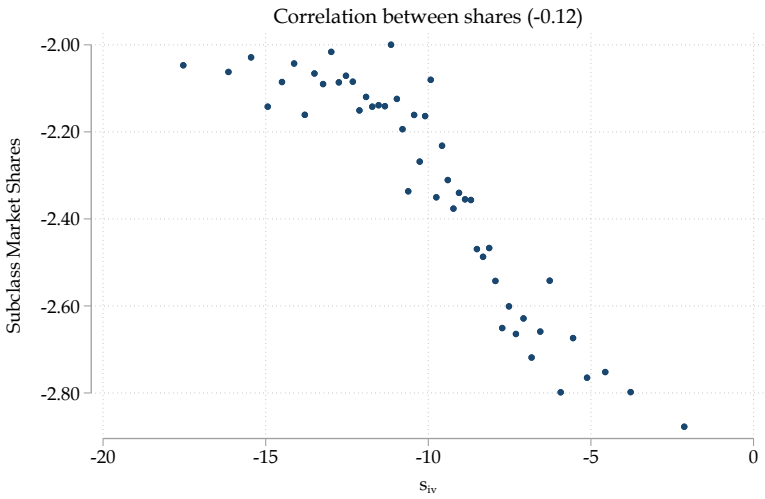
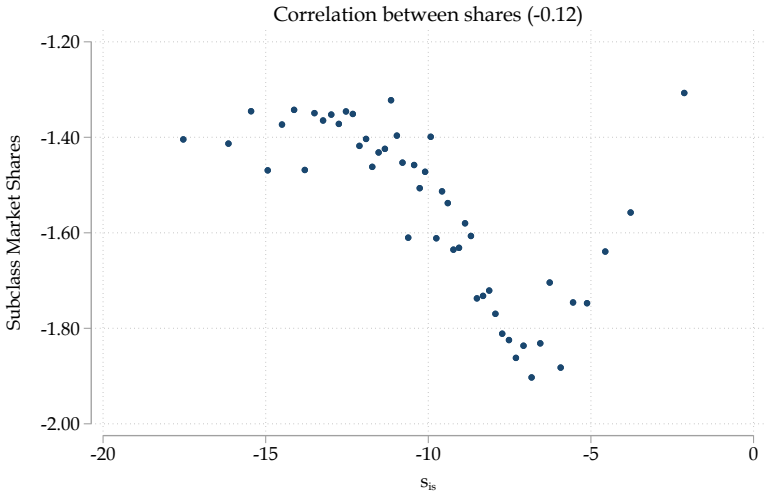


Figure A.7: Scatter Subclass Correlation



A.4 Complementary evidence to section 3.3.

We will update these regressions for $\Delta p_{ijvt} = p_{ijv,t} - p_{ijv,t-1}$ in January.

A.5 Complementary evidence to section 3.4. (Local Projections)

A.6 Cross-sectional evidence

We complement our empirical analysis by estimating a series of cross-sectional regressions that explore the relationship between price rigidity and market structure across

Table A.9: Market Share Distribution

Panel A: Whole sample

Var	Mean	SD	p05	p10	p25	p50	p75	p90	p95	N
s_{ijy}	0.25	0.34	0.00	0.00	0.01	0.08	0.35	1.00	1.00	3,163,905
x_{ijy}	0.02	0.10	0.00	0.00	0.00	0.00	0.00	0.02	0.07	3,163,905
s_{ijsy}	0.73	0.38	0.02	0.06	0.39	1.00	1.00	1.00	1.00	6,538,746
x_{ijsy}	0.03	0.14	0.00	0.00	0.00	0.00	0.00	0.04	0.14	6,538,746
s_{ijvy}	0.15	0.28	0.00	0.00	0.01	0.03	0.14	0.57	1.00	31,006,948
x_{ijvy}	0.01	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.01	31,006,948

Panel B: Local Projections Sub-sample

Var	Mean	SD	p05	p10	p25	p50	p75	p90	p95	N
s_{ijy}	0.66	0.39	0.01	0.03	0.26	0.90	1.00	1.00	1.00	3,509
x_{ijy}	0.05	0.13	0.00	0.00	0.00	0.01	0.03	0.09	0.21	3,509
s_{ijsy}	0.97	0.13	0.87	1.00	1.00	1.00	1.00	1.00	1.00	3,651
x_{ijsy}	0.06	0.15	0.00	0.00	0.00	0.01	0.04	0.14	0.34	3,651
s_{ijvy}	0.78	0.33	0.06	0.15	0.59	1.00	1.00	1.00	1.00	4,246
x_{ijvy}	0.05	0.14	0.00	0.00	0.00	0.01	0.03	0.11	0.26	4,246

supplier-buyer-varieties triplets. This exercise provides an initial, time-invariant description of the main features of firm-to-firm connections and their correlation with pricing outcomes. Specifically, we estimate the following equation:

$$y_{ijv} = \alpha + \beta \mathbf{X} + FE_s + FE_{Ind_{I_i}} + FE_{Ind_{I_j}} + \varepsilon_{ijv}, \tag{13}$$

where we allow the dependent variable to change to measure different angles of individual prices at their flexibility at the ijv level. In particular, and to account for the extensive margin of price adjustments, the dependent variable y_{ijv} includes a dummy equal to one if $\Delta p_{ijv} \neq 0$, and a separate dummy if prices changed upwards or downwards $\Delta p_{ijv} > 0$ and $\Delta p_{ijv} < 0$, respectively. For the intensive margin, we rely on the magnitude of price revisions conditional on adjustment $|\Delta p_{ijv}|\Delta p_{ijv} \neq 0|$ as the dependent variable.

The specification (13) includes fixed effects at the seller's subclass level (s) and both seller (I_i) and buyer industry (I_j) levels. Our controls in $\mathbf{X}$ include the bilateral market share variables (s_{ijv} and x_{ijv}), the number of transactions between ij, the length of the relationship, and the number of subclasses of products sold by the supplier. Importantly, we also include a relevant network-based regressor: the input price adjustment $Inputs_s$ for each seller s. In particular, for the extensive margin $Inputs_s$ accounts for the average frequency by which the prices of inputs of seller i changed, over time in the sample. Similarly, for the intensive margin, $Inputs_s$ is the magnitude by which prices of inputs

of seller i changed, weighted by the relative relevance of inputs in i's production. We quantify such relevance using the average purchases of inputs from i, over time. As we aim to characterize how the average price-stickiness of inputs potentially correlates with the price rigidity of sold products, we abstract from adding fixed effects at the seller level, not to absorb this feature. This variable has proven relevant as a key determinant of price adjustments at the seller level, as discussed by Nakamura and Zerom (2010) and Gopinath and Itskhoki (2010).

Tables A.11 and A.10 show the results for the extensive and intensive margins.

Table A.10: Cross-Section Evidence - Extensive Margin

	(1) $\Delta p_{ijv} \neq 0$	(2) $\Delta p_{ijv} > 0$	(3) $\Delta p_{ijv} < 0$
s_{ijv}	0.018***	0.013***	0.005***
	(0.000)	(0.000)	(0.000)
x_{ijv}	0.056***	0.003*	0.053***
	(0.002)	(0.002)	(0.001)
# Transactions	-0.171***	-0.153***	-0.018***
	(0.000)	(0.000)	(0.000)
Duration	0.159***	0.144***	0.015***
	(0.000)	(0.000)	(0.000)
3-7 subclasses	-0.012***	0.005***	-0.017***
	(0.000)	(0.000)	(0.000)
7+ subclasses	0.030***	0.026***	0.004***
	(0.000)	(0.000)	(0.000)
$Inputs_s$	0.255***	0.124***	0.132***
	(0.001)	(0.001)	(0.000)
Constant	0.109***	0.105***	0.004***
	(0.001)	(0.000)	(0.000)
N	10,301,452	10,301,452	10,301,452
Adjusted R2	0.425	0.350	0.279
Within R2	0.182	0.190	0.029

Note: The results show the estimation for equation (2). The dependent variable of column (1) is constructed as the frequency of price change: $\sum_{t=1}^{T_{ijv}} \mathbf{1}(|\Delta p_{ijvt}| > 0.005)/T_{ijv}$, in which $\mathbf{1}(|\Delta p_{ijvt}| > 0.005)$ is an indicator variable that takes the value of one if the price charged by supplier i to client j for variety v changed by more than 0.5%, and zero otherwise. The dependent variable for columns (2) and (3) follows the same logic, but for $\mathbf{1}(\Delta p_{ijvt} > 0.005)$ and $\mathbf{1}(\Delta p_{ijvt<0.005})$, respectively. All columns included subclass, supplier industry, and consumer industry fixed effects

As noted, the frequency and magnitude of price changes in a supplier's upstream inputs are strongly associated with their own price adjustment behavior. In particular, a (hypothetical) shift from no input changes to a scenario of continuous changes of seller's i input prices over the sample period, i.e., $Inputs_s = 1$, correlates with an increase in the

Table A.11: Cross-Section Evidence - Intensive Margin

	Ln(p_{ijv})	$\lvert\Delta p_{ijv}\rvert \Delta p_{ijv} \neq 0$	$\lvert\Delta p_{ijv}\rvert \Delta p_{ijv} > 0$	$\lvert\Delta p_{ijv}\rvert \Delta p_{ijv} < 0$
s_{ijv}	0.054***	0.010***	0.009***	0.028***
	(0.002)	(0.000)	(0.000)	(0.001)
x_{ijv}	-0.657***	0.062***	0.055***	0.047***
	(0.014)	(0.003)	(0.003)	(0.005)
# Transactions	-0.073***	-0.049***	-0.053***	-0.046***
	(0.001)	(0.000)	(0.000)	(0.000)
Duration	0.177***	0.030***	0.038***	0.019***
	(0.001)	(0.000)	(0.000)	(0.000)
Price gap (t-1)	-0.007***	0.005***	0.006***	-0.003***
	(0.002)	(0.000)	(0.000)	(0.001)
Price Age	-0.223***	0.020***	0.015***	0.038***
	(0.001)	(0.000)	(0.000)	(0.000)
3-7 subclasses	0.109***	-0.016***	-0.014***	-0.011***
	(0.003)	(0.000)	(0.000)	(0.001)
7+ subclasses	0.027***	-0.010***	-0.011***	0.005***
	(0.003)	(0.000)	(0.000)	(0.001)
Input magnitude change	-0.097***	0.028***	0.040***	-0.095***
	(0.018)	(0.002)	(0.003)	(0.007)
Constant	7.591***	0.108***	0.098***	0.120***
	(0.003)	(0.001)	(0.000)	(0.001)
N	10,388,617	8,341,639	8,025,788	3,456,410
Adjusted R2	0.451	0.070	0.079	0.077
Within R2	0.009	0.030	0.036	0.017

Note: The results show the estimation for equation (2). The dependent variable of column (1) is constructed as the log level of price p_{ijv} (time average of p_{ijvt}). Columns (2), (3), and (4) are constructed as the average log price change, conditional on absolute, positive, and negative price changes, respectively. All columns included subclass, supplier industry, and consumer industry fixed effects.

supplier's own probability of revising prices of approximately 26%. Similarly, a 1% increase in upstream prices is associated with a 2.4% increase in the supplier's output prices. Such effects can be observed in columns 1 and 2 of Table A.10 and A.11, respectively. This evidence brings two main implications. First, it provides direct empirical evidence of network propagation in pricing behavior: firms more exposed to upstream input price volatility are themselves more flexible in their pricing. Second, it highlights that such cost pass-through is, on average, incomplete, consistent with the presence of frictions or other factors related to preferences that attenuate the full transmission of costs along the supply chain. In Section 3.5, we revisit this issue using local projections to estimate dynamic pass-through responses in the network.

We next examine the role of bilateral market power. Larger supplier market shares (s_{ijv}) are associated with higher prices and more frequent price adjustments. If a seller accounts for the entire share of a buyer's purchases in a given subclass (i.e., $s_{ijv} = 1$), the probability of an upward price revision increases by 1.3 percentage points, while that of a downward revision rises by 0.5 points. These magnitudes are meaningful: the median frequency of price changes across ijv triplets is 0.25 (see Table 2) in our sample, which even includes the inflationary surge during the pandemic. Interestingly, the increase in downward adjustments may reflect the fact that our sample spans both inflationary and disinflationary periods—an interpretation we revisit in the panel regressions below.

In contrast, larger buyer market shares (x_{ijv}) are associated with lower prices and a higher frequency of downward price revisions. These results align with the intuitive notion that bigger and powerful buyers can bargain for lower prices, which are therefore more likely to decrease. The sign of some effects again suggest that unobserved heterogeneity may be affecting some of the estimates, which motivates the panel approach in the next section.

Finally, other relationship characteristics also matter. More frequent transactions are associated with less frequent price revisions, consistent with the presence of explicit or implicit contracts that prevent continuous renegotiation. Meanwhile, longer relationships are correlated with higher prices, plausibly reflecting the fact that longer-standing links have had more time to incorporate cumulative price revisions. Below, I show the new results of the panel and the cross-section.

A.7 Regressions Robustness

A.7.1 Bar-code Products

We repeated the same specifications as before (equations (2) and (13)), but for the product sample. We denote product as p and are classified according to the EAN13 code, the most common barcode worldwide, used to identify products at points of sale. It consists of 13 digits that encode unique information about the item. This sample is much smaller.

Table A.12: Product Panel Estimation - Extensive Margin

	(1) $\Delta p_{ijvt} > 0$	(2) $\Delta p_{ijvt} > 0$	(3) $\Delta p_{ijvt} < 0$	(4) $\Delta p_{ijvt} < 0$
Price gap (t-1)	-0.103***	-0.102***	-0.002*	-0.002*
	(0.003)	(0.003)	(0.001)	(0.001)
Price Age	0.086***	0.074***	-0.007***	-0.007***
	(0.001)	(0.001)	(0.000)	(0.000)
s_{ijpy}		0.009***		0.011***
		(0.003)		(0.002)
x_{ijpy}		0.035**		-0.011
		(0.016)		(0.009)
# Transactions		-0.193***		-0.009***
		(0.002)		(0.001)
Duration		0.156***		0.011***
		(0.002)		(0.001)
Constant	0.287***	0.287***	0.034***	0.026***
	(0.005)	(0.006)	(0.002)	(0.002)
N	656,334	656,334	656,334	656,334
Adjusted R2	0.131	0.152	0.130	0.131
Within R2	0.026	0.048	0.001	0.002

Table A.13: Product Panel Estimation - Intensive Margin

	(1) Ln(P)	(2) Ln(P)	(3) $\lvert\Delta p_{ijv}\rvert \Delta p_{ijv} \neq 0\rvert$	(4) $\lvert\Delta p_{ijv}\rvert \Delta p_{ijv} \neq 0\rvert$	(5) $\lvert\Delta p_{ijv}\rvert \Delta p_{ijv} > 0\rvert$	(6) $\lvert\Delta p_{ijv}\rvert \Delta p_{ijv} > 0\rvert$	(7) $\lvert\Delta p_{ijv}\rvert \Delta p_{ijv} < 0\rvert$	(8) $\lvert\Delta p_{ijv}\rvert \Delta p_{ijv} < 0\rvert$
Price gap (t-1)	0.156***	0.155***	0.021***	0.019***	0.022***	0.020***	0.002	0.004
	(0.005)	(0.005)	(0.001)	(0.001)	(0.001)	(0.001)	(0.005)	(0.005)
Price Age	-0.018***	-0.026***	0.019***	0.016***	0.018***	0.015***	0.020***	0.016***
	(0.001)	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)	(0.003)	(0.003)
s_{ijpy}		0.336***		0.001		0.009***		-0.032***
		(0.009)		(0.002)		(0.002)		(0.008)
x_{ijpy}		-0.528***		-0.020***		-0.022***		-0.012
		(0.033)		(0.005)		(0.006)		(0.014)
# Transactions		-0.112***		-0.029***		-0.029***		-0.026***
		(0.002)		(0.001)		(0.001)		(0.006)
Duration		0.107***		0.019***		0.019***		0.019***
		(0.003)		(0.001)		(0.001)		(0.006)
Constant	9.125***	9.035***	0.033***	0.044***	0.032***	0.043***	0.059***	0.072***
	(0.008)	(0.009)	(0.002)	(0.002)	(0.002)	(0.002)	(0.005)	(0.010)
N	656,334	656,334	164,333	164,333	149,011	149,011	14,120	14,120
Adjusted R2	0.854	0.856	0.290	0.299	0.341	0.353	0.314	0.317
Within R2	0.004	0.018	0.023	0.037	0.028	0.046	0.007	0.011

Table A.14: Product Cross-Section Evidence - Extensive Margin

	(1) $\Delta p_{ijv} \neq 0$	(2) $\Delta p_{ijv} > 0$	(3) $\Delta p_{ijv} < 0$
s_{ijp}	0.008***	0.003**	0.005***
	(0.002)	(0.002)	(0.002)
x_{ijp}	0.032**	0.032**	-0.001
	(0.015)	(0.013)	(0.007)
# Transactions	-0.319***	-0.295***	-0.024***
	(0.002)	(0.001)	(0.001)
Duration	0.412***	0.380***	0.032***
	(0.002)	(0.002)	(0.001)
3-7 subclasses	-0.008**	-0.018***	0.010***
	(0.004)	(0.004)	(0.002)
7+ subclasses	-0.013***	-0.041***	0.029***
	(0.004)	(0.004)	(0.003)
Price Gap	-0.005*	-0.005*	-0.000
	(0.003)	(0.002)	(0.002)
Price Age	-0.322***	-0.270***	-0.052***
	(0.002)	(0.002)	(0.001)
Input frequency change	-0.059***	-0.171***	0.113***
	(0.013)	(0.012)	(0.009)
Constant	0.377***	0.396***	-0.019***
	(0.009)	(0.008)	(0.006)
N	67,273	67,273	67,273
Adjusted R2	0.698	0.653	0.337
Within R2	0.543	0.523	0.057

Table A.15: Product Cross-Section Evidence - Extensive Margin

	$\text{Ln}(p_{ijv})$	$\lvert\Delta p_{ijv}\rvert\Delta p_{ijv} \neq 0\rvert$	$\lvert\Delta p_{ijv}\rvert\Delta p_{ijv} > 0\rvert$	$\lvert\Delta p_{ijv}\rvert\Delta p_{ijv} < 0\rvert$
s_{ijp}	0.049***	0.033***	0.036***	0.001
	(0.013)	(0.002)	(0.002)	(0.006)
x_{ijp}	-0.175	-0.014	-0.021*	0.015
	(0.175)	(0.011)	(0.011)	(0.021)
# Transactions	0.005	-0.025***	-0.026***	-0.056***
	(0.007)	(0.001)	(0.001)	(0.007)
Duration	0.034***	0.011***	0.012***	0.050***
	(0.010)	(0.002)	(0.002)	(0.008)
Price Gap	0.501***	0.009***	0.009***	0.004
	(0.028)	(0.002)	(0.002)	(0.009)
Price Age	0.024***	0.029***	0.029***	0.004
	(0.009)	(0.001)	(0.001)	(0.005)
3-7 subclasses	0.113***	-0.025***	-0.026***	-0.006
	(0.034)	(0.003)	(0.003)	(0.013)
7+ subclasses	-0.241***	-0.023***	-0.024***	0.028
	(0.039)	(0.004)	(0.004)	(0.021)
Input magnitude change	-3.207**	0.582***	0.449***	0.974***
	(1.295)	(0.097)	(0.104)	(0.250)
Constant	8.529***	0.083***	0.090***	0.034
	(0.053)	(0.006)	(0.006)	(0.024)
N	67,273	58,642	57,714	8,994
Adjusted R2	0.816	0.399	0.432	0.130
Within R2	0.028	0.051	0.055	0.018

A.7.2 Supplier x Subclass Fixed Effects

We repeated the same specification from equations (2), but with FE_{is} (supplier x subclass), instead of FE_i and FE_s separately. We show the results for both our main sample and the product sample (only panel)

Table A.16: FE_{is} Panel Estimation - Extensive Margin

	(1) $\Delta P_{ijv} > 0$	(2) $\Delta P_{ijv} < 0$
Price gap (t-1)	-0.030***	-0.006***
	(0.000)	(0.000)
Price Age	0.015***	-0.017***
	(0.000)	(0.000)
s_{ijvy}	0.024***	0.010***
	(0.000)	(0.000)
x_{ijvy}	0.031***	0.022***
	(0.001)	(0.001)
# Transactions	-0.128***	-0.017***
	(0.000)	(0.000)
Relationship Duration	0.063***	0.013***
	(0.000)	(0.000)
Constant	0.292***	0.091***
	(0.000)	(0.000)
N	131,440,903	131,440,903
Adjusted R2	0.143	0.129
Within R2	0.026	0.004

Table A.17: FE_{is} Panel Estimation - Intensive Margin

	(1) Ln(P)	(2) $\Delta Ln(P)\|I_abs$	(3) $\Delta Ln(P)\|I_pos$	(4) $\Delta Ln(P)\|I_neg$
Price gap (t-1)	0.072***	0.006***	0.005***	0.002**
	(0.001)	(0.001)	(0.001)	(0.001)
Price Age	-0.034***	0.016***	0.016***	0.019***
	(0.000)	(0.000)	(0.000)	(0.000)
s_{ijvy}	0.371***	0.000	0.000*	-0.002***
	(0.000)	(0.000)	(0.000)	(0.000)
x_{ijvy}	-1.619***	-0.026***	-0.032***	-0.011**
	(0.003)	(0.002)	(0.002)	(0.004)
# Transactions	-0.040***	-0.036***	-0.035***	-0.031***
	(0.000)	(0.000)	(0.000)	(0.000)
Relationship Duration	0.010***	0.013***	0.013***	0.010***
	(0.000)	(0.000)	(0.000)	(0.000)
Constant	7.631***	0.090***	0.086***	0.103***
	(0.001)	(0.000)	(0.000)	(0.001)
N	131,440,903	39,102,893	29,891,351	9,176,831
Adjusted R2	0.763	0.139	0.157	0.248
Within R2	0.014	0.030	0.039	0.017

Table A.18: FE_{is} Product Panel Estimation - Extensive Margin

	(1) $\Delta p_{ijvt} > 0$	(2) $\Delta p_{ijvt} < 0$
Price gap (t-1)	-0.130***	-0.002
	(0.003)	(0.001)
Price Age	0.075***	-0.007***
	(0.001)	(0.000)
x_{ijpy}	0.010***	0.012***
	(0.003)	(0.002)
x_{ijpy}	0.091***	0.002
	(0.021)	(0.011)
# Transactions	-0.193***	-0.009***
	(0.002)	(0.001)
Duration	0.156***	0.011***
	(0.002)	(0.001)
Constant	0.331***	0.025***
	(0.006)	(0.002)
N	656,334	656,334
Adjusted R2	0.152	0.132
Within R2	0.049	0.002

Table A.19: FE_{is} Product Panel Estimation - Intensive Margin

	(1) Ln(P)	(2) $\Delta Ln(P)\|I_abs$	(3) $\Delta Ln(P)\|I_pos$	(4) $\Delta Ln(P)\|I_neg$
Price gap (t-1)	0.093***	0.026***	0.028***	0.009
	(0.004)	(0.001)	(0.001)	(0.007)
Price Age	-0.028***	0.016***	0.015***	0.016***
	(0.001)	(0.000)	(0.000)	(0.003)
s_{ijpy}	0.343***	0.002	0.010***	-0.034***
	(0.009)	(0.002)	(0.002)	(0.008)
x_{ijpy}	-1.208***	-0.040***	-0.045***	-0.000
	(0.038)	(0.007)	(0.008)	(0.019)
# Transactions	-0.112***	-0.029***	-0.029***	-0.026***
	(0.002)	(0.001)	(0.001)	(0.006)
Duration	0.110***	0.019***	0.019***	0.020***
	(0.003)	(0.001)	(0.001)	(0.006)
Constant	9.133***	0.034***	0.030***	0.068***
	(0.008)	(0.003)	(0.003)	(0.010)
N	656,334	164,318	148,990	14,094
Adjusted R2	0.863	0.301	0.355	0.316
Within R2	0.019	0.037	0.047	0.011

A.8 Robustness for Changes Including Zeros

Figure A.8: Intensive Margin by Supplier Market Share

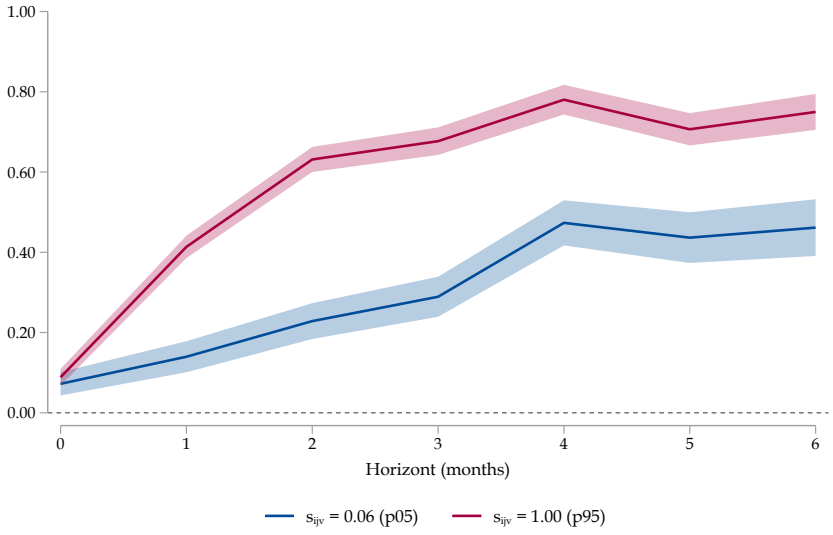
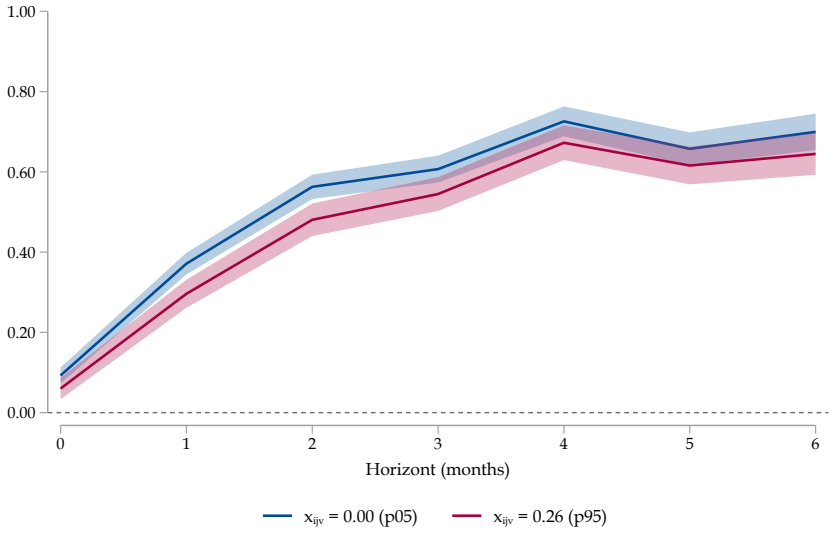


Figure A.9: Intensive Margin by Consumer Market Share



A.8.1 Robustness for Non-Covid Period

Figure A.10: Intensive Margin by Supplier Market Share

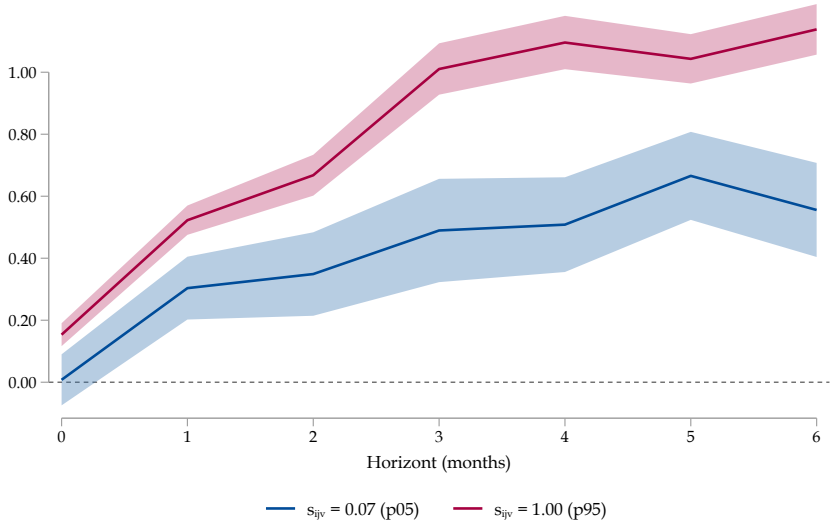


Figure A.11: Intensive Margin by Consumer Market Share

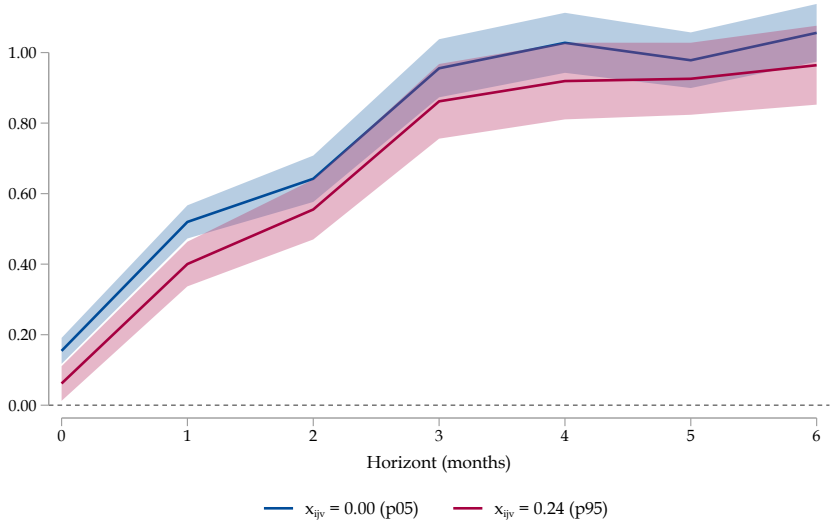
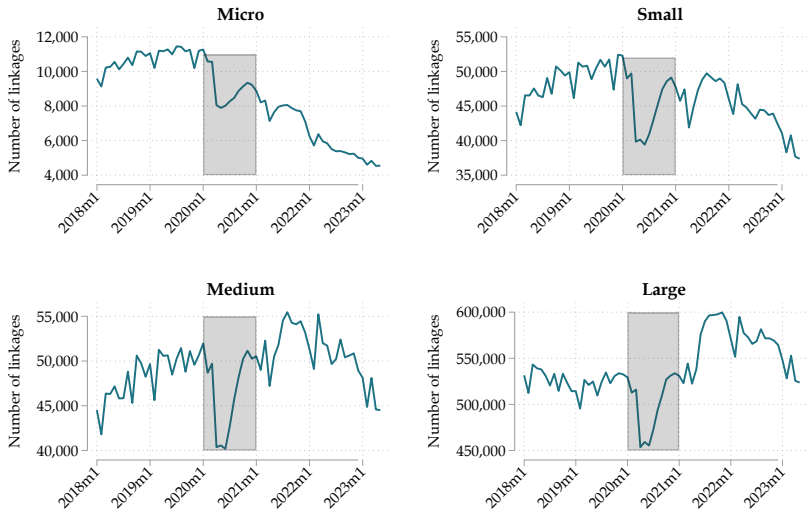
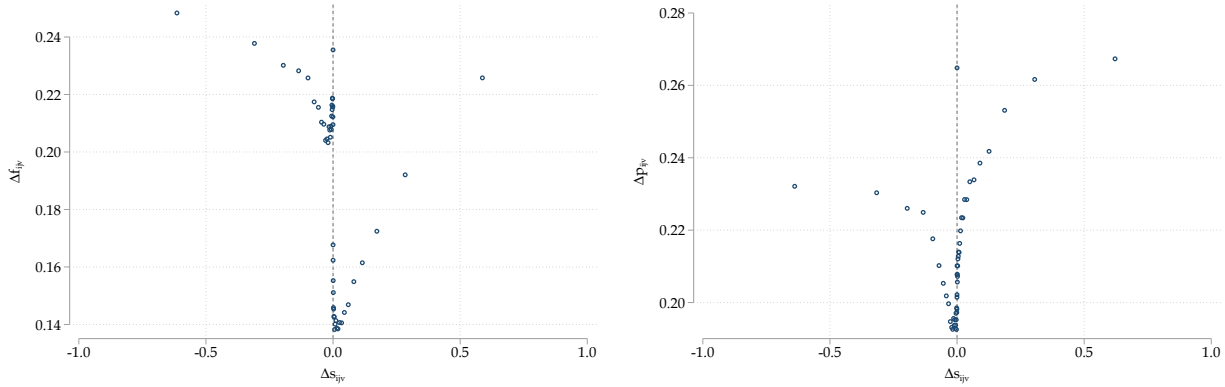


Figure A.12: Evolution of firm-to-firm linkages



A.9 Complementary evidence to section 3.5.

Figure A.13: The rise in price adjustment and bilateral market concentration (variety)



We estimate the following equation to understand the relationship between the change in adjustment frequency and the change in market power.

$$\Delta F_{ijv} = \Delta s_{ijv} + \Delta x_{ijv} + \Delta Inp_{iv} + F^i + s^i + x^i + FE_j + FE_s + \varepsilon_{ijv}$$

where ΔX_{ijv} is the difference between the average values of X for the periods 2020-2021, minus 2018-2019, and X^i is the initial value (average of 2018-2019). F_{ijv} is the price change frequency from supplier i to consumer j in variety v, s_{ijv} and x_{ijv} are our market share

measures, and Inp_{iv} is the change in the input frequency of i. The following table shows the results for the OLS and IV regressions. For the IV, we instrumentalize Δs_{ijv} with $\Delta Links_{jv}$, where $Links_{jv}$ is the total number of links of variety v that the consumer j has in that period.

Table A.20: Change in the frequency price change 2018-2019 vs 2020-2022

	(1) ΔF_{ijv}	(2) ΔF_{ijv}
Δs_{ijv}	-0.086***	0.309***
	(0.002)	(0.025)
Δx_{ijv}	-0.087***	-0.221***
	(0.010)	(0.013)
ΔInp_{iv}	-0.122***	-0.124***
	(0.003)	(0.003)
F^i	-0.741***	-0.744***
	(0.001)	(0.001)
s^i	-0.049***	0.054***
	(0.002)	(0.007)
x^i	-0.040***	-0.086***
	(0.006)	(0.007)
N	1,340,973	1,340,973
Adjusted R^2	0.416	0.291
Sample	All	All
Estimation	OLS	IV

B Validating the dataset

Validating price data with official data

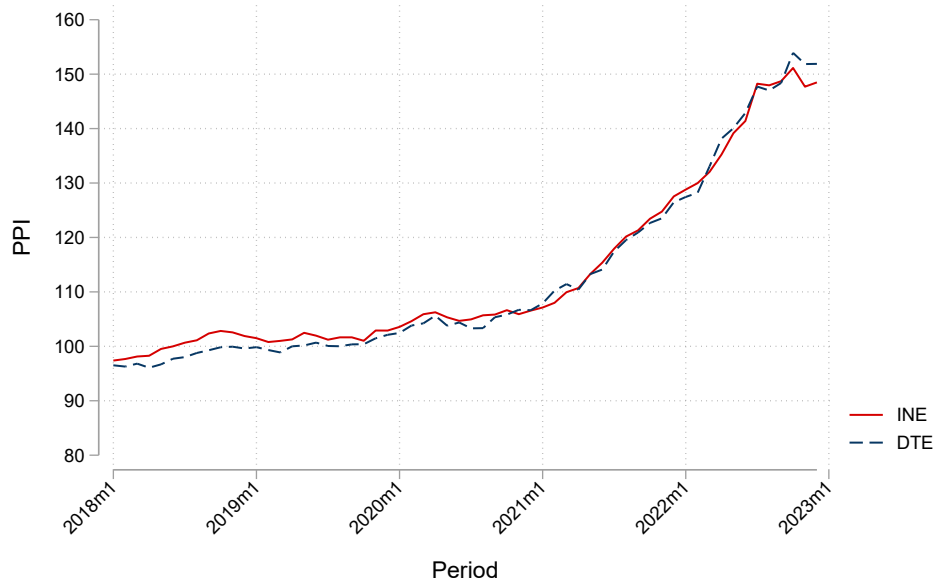
We now proceed to describe the validity and representativeness of our data. We compare the evolution of the official producer price index (PPI^{INE}) and the index we construct using our series of prices (PPI^{c}), which we define as

$$PPI_t^c = \sum_s \omega_s^{INE} P_{st},$$

where ω_s^{INE} is the PPI weight of subclass s in the official PPI by the INE, and P_{st} is the price of the subclass s at time t. We have access to daily transaction-level data which we aggregate to monthly prices following the methodology used by the National Institute of Statistics in Chile (INE). In particular, the subclass price is a geometric average of firm-level prices selling the subclass during a given month.

Figure B.1 depicts the evolution of the official PPI index and the PPI we construct. We are able to successfully replicate the evolution of the official PPI. There are some small differences in levels but the correlation between the series is 0.99.

Figure B.1: Producer Price Index (PPI): official vs constructed



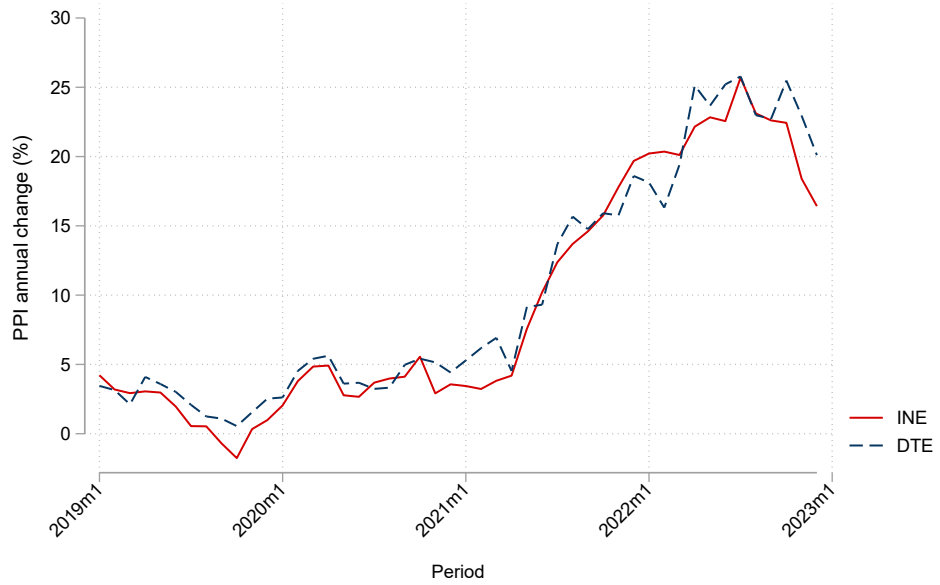
Note: The sum of weights is 0.97 of 100, and there are 165 of 173 products

The next Figure shows the implied annual percent change in prices (inflation). Our annual inflation measure is defined as

$$\pi_t^{PPI} = \sum_s \omega_s^{INE}(p_{st} - p_{st-1})$$

where p_{st} denotes the log of P_{st}. Figure B.2 shows the evolution of PPI's official annual inflation and our proxy of it. We can trace down actual PPI inflation quite well. The correlation between the series is 0.98.

Figure B.2: Annual PPI inflation: official vs constructed

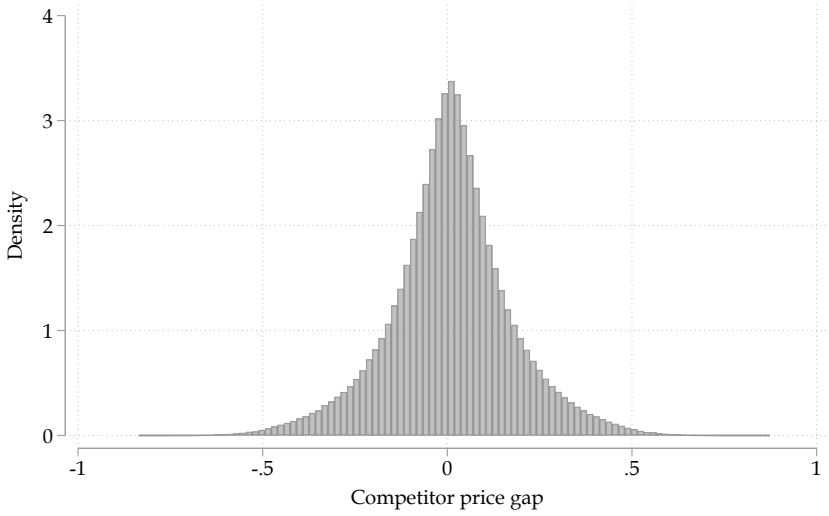


Note: The sum of weights is 0.97 of 100, and there are 165 of 173 products

Competitors price gap

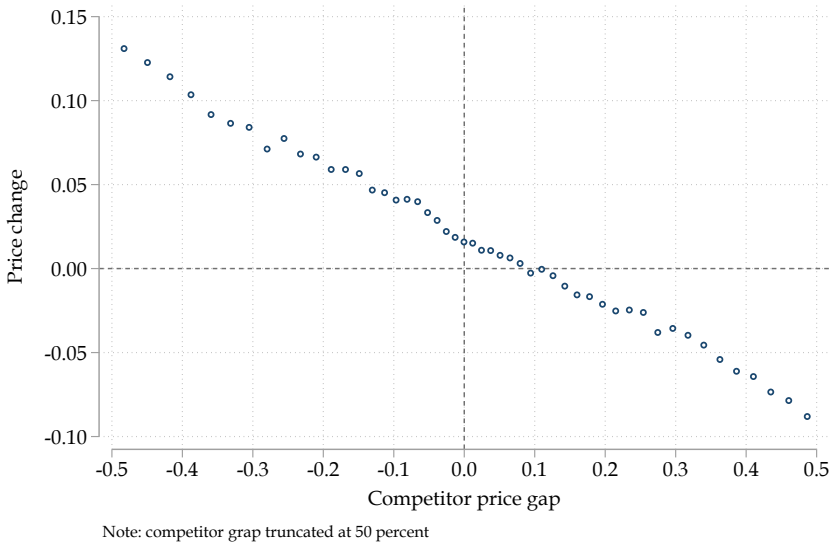
Here, we study the extent of strategic complementarities in our data. Figure B.3 shows the distribution of competitors' price gap, as defined in Karadi, Schoenle, and Wursten (2024). For each firm, we measure the average price charged by competitors in the product subclass market. As seen in Figure B.3, we observe that the price gap is centered around zero, with most of the prices below or above 50% the average price of the market.

Figure B.3: Competitor-price gap (strategic complementarities)



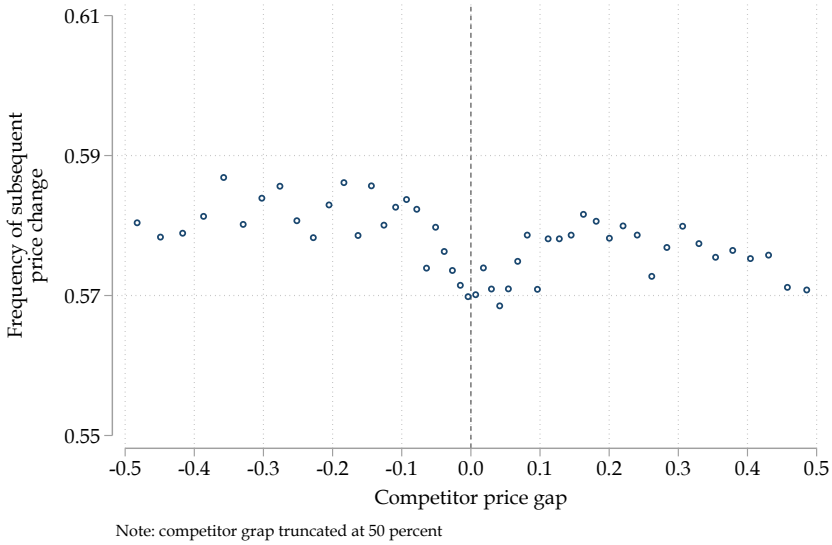
In Figures B.4 and B.5 we confirm the results in Karadi, Schoenle, and Wursten (2024). The magnitude of the price change increases with the distance from the average price of the competition. Additionally, firms are more likely to adjust their price when they deviate from a competitor's price.

Figure B.4: Magnitude of adjustment (t+1) as function of the gap



Version: Filtered of supplier-product FE using OLS

Figure B.5: Magnitude of adjustment as function of the gap



Version: Filtered of supplier-product FE using OLS

As Karadi, Schoenle, and Wursten (2024) emphasizes, these facts are consistent with state-dependent price-setting models in which menu costs are random (e.g., Dotsey, King, and Wolman, 1999; Luo and Villar, 2021; Alvarez, Lippi, and Oskolkov, 2022).

C Model Derivation Details

In this section we will provide the specific derivation for the main ingredients in our model of Section XX.

C.1 Seller's optimal price

Final Good Aggregator Each buyer from sector/industry j, aggregates a continuum of differentiated varieties produced by each seller i according to the Kimball (1995) aggregator:

$$\int_0^1 G_j\left(\frac{n_{ijt}y_{ijt}}{Y_{jt}}\right) di = 1 \tag{14}$$

where y_{ijt} denotes the quantity sold by seller i to buyer j at time t, and n_{ijt} is a buyer-seller specific demand shifter, and Y_{jt} denotes total final output of buyer j at time t. The Kimball aggregator $G(.)$ satisfies $G^{'}(.) > 0$ and $G^{'}(.) < 0$.

Cost Minimization Each buyer/industry j minimizes total expenditure:

$$\min_{\{y_{ij}\}} \int p_{ij}y_{ij}\, di \quad \text{s.t.} \quad \int G\left(\frac{n_{ij}y_{ij}}{Y_j}\right) di = 1. \tag{15}$$

Where the Lagrangian:

$$\mathcal{L} = \int p_{ij}y_{ij}\, di - \lambda_j \left[1 - \int G\left(\frac{n_{ij}y_{ij}}{Y_j}\right) di\right]. \tag{16}$$

The first order condition with respect to y_{ijt} is:

$$p_{ij} = \lambda_j \frac{n_{ij}}{Y_j} G'\left(\frac{n_{ij}y_{ij}}{Y_j}\right). \tag{17}$$

Thus, the price p_{ij} is proportional to how valuable an extra unit of that variety is inside the Kimball aggregator $G(.)$. Rearranging the first-order condition yields the inverse demand curve faced by seller i when selling to buyer j:

$$G'\left(\frac{n_{ij}y_{ij}}{Y_j}\right) = \frac{p_{ij}Y_j}{\lambda_j n_{ij}}. \tag{18}$$

This expression shows that demand depends on the relative price $\frac{p_{ij}}{n_{ij}}$ and on the buyer-specific multiplier λ_j.

Elasticity of demand Let $x_{ij} = \frac{n_{ij}y_{ij}}{Y_j}$, we can differentiate the inverse demand curve (holding λ_j and Y_j fixed from the final-good buyer's problem), we can show:

$$\frac{dp_{ij}}{dy_{ij}} = \frac{\lambda_j}{Y_j^2} n_{ij}^2 G''(x_{ij}) \Rightarrow \frac{dy_{ij}}{dp_{ij}} = \frac{Y_j^2}{\lambda_j n_{ij}^2 G''(x_{ij})} \tag{19}$$

Using this expression and the inverse demand curve previously derived, the elasticity of demand:

$$\epsilon_{ij} \equiv -\frac{dy_{ij}}{dp_{ij}} \frac{p_{ij}}{y_{ij}} = -\left(\frac{Y_j^2}{\lambda_j n_{ij}^2 G''(x_{ij})} \right) G'(x_{ij}) \frac{\lambda_j n_{ij}}{Y_j} \frac{1}{\frac{x_{ij}Y_j}{n_{ij}}} = -\frac{G'(x_{ij})}{x_{ij} G''(x_{ij})}. \tag{20}$$

Demand elasticity therefore depends on the overall concavity of $G(.)$ and the relative bilateral market share x_{ij}. More specifically, for a given seller, differences in n_{ij} across buyers imply differences in elasticities and hence differences in optimal prices, even in the absence of nominal rigidities.

Alternatively, we could derive the inverse demand curve faced by seller i when selling to buyer j as a function of the **aggregate price index** P:

$$G'(x_{ij}) = \frac{p_{ij}}{\lambda_j n_{ij} P} \tag{21}$$

In this case, we just solve the problem for the final buyer to minimize their cost normalized by their total expenditure.[16]

[16]The representative final-good producer chooses $\{y_{ij}\}_{i \in \mathcal{I}_j}$ to maximize real profits:

$$\max_{\{y_{ij}\}} 1 - \int_{i \in \mathcal{I}_j} \frac{p_{ij} y_{ij}}{P Y_j} di \qquad \text{s.t.} \qquad \int_{i \in \mathcal{I}_j} G\left(\frac{n_{ij} y_{ij}}{Y_j} \right) di = 1, \tag{22}$$

where P is the aggregate (final-good) price level and p_{ij} is the ij price of the intermediate input. In other words, we are writing the expression in units of the final good. Let λ_j be, again, the Lagrange multiplier on the aggregator constraint. The Lagrangian is

$$\mathcal{L} = 1 - \int_i \frac{p_{ij} y_{ij}}{P Y_j} di + \lambda_j \left(\int_i G(x_{ij}) di - 1 \right)$$

Multiplying by Y_j the FOC, gives the required demand condition:

$$G'(x_{ij}) = \frac{p_{ij}}{\lambda_j n_{ij} P}. \tag{23}$$

Following Dotsey and King (2005), we specify the Kimball aggregator function as:

$$G\left(\frac{n_{ijt}y_{ijt}}{Y_{jt}}\right) = \frac{\omega_j}{1+\omega_j\psi_j}\left((1+\psi_j)\frac{n_{ijt}y_{ij}}{Y_{jt}} - \psi_j\right)^{\frac{1+\omega_j\psi_j}{\omega_j(1+\psi_j)}} \tag{24}$$

Let $h(x) \equiv (1+\psi_j)x - \psi_j$ and $b \equiv \frac{1+\omega_j\psi_j}{\omega_j(1+\psi_j)}$. Then, the derivative of the Kimball demand (24):

$$G(x) = \frac{\omega_j}{1+\omega_j\psi_j}h(x)^b \quad \Rightarrow \quad G'(x) = \frac{\omega_j}{1+\omega_j\psi_j}\cdot b \cdot h(x)^{b-1}\cdot(1+\psi_j)$$

Using the value of b, the constants terms $\frac{\omega_j}{1+\omega_j\psi_j}\cdot b\cdot(1+\psi_j) = \frac{\omega_j}{1+\omega_j\psi_j}\cdot\frac{1+\omega_j\psi_j}{\omega_j(1+\psi_j)}\cdot(1+\psi_j) = 1$, so we get:

$$G'(x) = h(x)^{b-1} = \left((1+\psi_j)x - \psi_j\right)^{\frac{1-\omega_j}{\omega_j(1+\psi_j)}}. \tag{25}$$

Plugging equation (25) into the inverse demand (21) and raising both sides to the power $\frac{\omega_j(1+\psi_j)}{1-\omega_j}$:

$$(1+\psi_j)x_{ij} - \psi_j = \left(\frac{p_{ij}}{\lambda_j n_{ij} P}\right)^{\frac{\omega_j(1+\psi_j)}{1-\omega_j}}$$

Finally since $x_{ij} = \frac{n_{ij}y_{ij}}{Y_j}$:

$$\frac{n_{ij}y_{ij}}{Y_j} = \frac{1}{1+\psi_j}\left[\left(\frac{p_{ij}}{\lambda_j n_{ij} P}\right)^{\frac{\omega_j(1+\psi_j)}{1-\omega_j}} + \psi_j\right]. \tag{26}$$

This corresponds to equation XX in the main text.

Intermediate producers' price setting As discussed, supplier i faces productivity and cost (oil) shocks. It also produces intermediate goods using labor. The marginal cost is:

$$MC_i(z_i, e) = \frac{W^{1-\phi}e^{\phi}}{z_i} \tag{27}$$

Where we assume the cost of labor is W, and the imported input price is e. Each downstream sectors j demand different varieties from the different suppliers i. As we have already shown, and given the assumed functional form for $G(.)$, the demand from

the downstream buyer/sector j for the particular subclass i, weighted by the demand shocks, is :

$$s_{ij} = \frac{n_{ij}y_{ij}}{Y_j} = \frac{1}{1+\psi_j}\left(\left(\frac{p_{ij}}{\lambda_j n_{ij} P}\right)^{\frac{\omega_j}{1-\omega_j}(1+\psi_j)} + \psi_j\right), \tag{28}$$

Therefore, the optimal price charged by each seller i to customer j, is set by maximizing the profit function of the intermediate input supplier of variety i:

$$\pi(p_i, z_i, n_i, S) = \sum_{j=1}^{J} y_{ij}\frac{1}{P}\left(p_{ij} - \frac{W^{1-\phi}e^{\phi}}{z_i}\right),$$

where S represents the aggregate space (W, P, e).

Giving the optimal demand of the downstream sectors:

$$\pi(p_i, z_i, n_i, S) = \sum_{j=1}^{J}\left(\frac{p_{ij}}{P} - \underbrace{\frac{W^{1-\phi}e^{\phi}}{Pz_i}}_{mc_i}\right)\underbrace{\frac{Y_j}{n_{ij}}\frac{1}{1+\psi_j}\left[\left(\frac{p_{ij}}{\lambda_j n_{ij} P}\right)^{\frac{\omega_j(1+\psi_j)}{1-\omega_j}} + \psi_j\right]}_{y_{ij}} \tag{29}$$

The first-order condition with respect to $\frac{p_{ij}}{P}$ is given by

$$\left(\left(\frac{p_{ij}}{\lambda n_{ij} P}\right)^{\frac{\omega_j(1+\psi_j)}{1-\omega_j}} + \psi_j\right) + \left(p_{ij} - \frac{W^{1-\phi}e^{\phi}}{z_i}\right)\frac{\omega_j(1+\psi_j)}{1-\omega_j}\left(\frac{p_{ij}}{\lambda_j n_{ij} P}\right)^{\frac{\omega_j(1+\psi_j)}{1-\omega_j}-1}\frac{1}{\lambda_j n_{ij} P} = 0 \tag{30}$$

$$\left(\frac{p_{ij}}{\lambda_j n_{ij} P}\right)^{\frac{\omega_j(1+\psi_j)}{1-\omega_j}} + \psi_j = -\left(p_{ij} - \frac{W^{1-\phi}e^{\phi}}{z_i}\right)\left(\frac{\omega_j(1+\psi_j)}{1-\omega_j}\frac{1}{p_{ij}}\right) \tag{31}$$

Using the previous definition for $\epsilon_{ij} = -\frac{G'(x_{ij})}{x_{ij}G''(x_{ij})}$, where $x_{ij} = \frac{n_{ij}y_{ij}}{Y_j}$, we can show that:

$$\epsilon_{ij} = -\frac{\omega_j}{1-\omega_j}(1+\psi_j)\frac{\left(\frac{p_{ij}}{\lambda_j n_{ij} P}\right)^{\frac{\omega_j(1+\psi_j)}{1-\omega_j}}}{\left(\frac{p_{ij}}{\lambda_j n_{ij} P}\right)^{\frac{\omega_j(1+\psi_j)}{1-\omega_j}} + \psi_j} \tag{32}$$

Hence dividing expression (31) by $\left(\frac{p_i}{\lambda n_i P}\right)^{\frac{\omega(1+\psi)}{1-\omega}}$, we can show:

$$-\frac{\omega}{1-\omega}(1+\psi)\frac{1}{\epsilon_{ij}} = -\left(p_i - \frac{W^{1-\phi}e^{\phi}}{z_i}\right)\left(\frac{\omega(1+\psi)}{1-\omega}\frac{1}{p_i}\right) \tag{33}$$

Which we can rearrange to get:

$$p_{ij} = \left(\frac{\epsilon_{ij}}{\epsilon_{ij}-1}\right)\frac{W^{1-\phi}e^{\phi}}{z_i} \tag{34}$$

C.2 Price adjustment decisions

Extensive Margin of Adjustment We derive a closed-form expression for the extensive margin of price adjustments.

We follow Lemma 2 in Gopinath and Itskhoki (2010), where they show that:

$$\begin{aligned} L(z_i, n_i, S) &\equiv \pi(z_i, n_i, S) - \pi(\overline{p}_0|z_i, n_i, S) \\ L(z_i, n_i, S) &= -\frac{1}{2}\frac{\partial^2\pi(z_i, n_i, S)}{\partial p^2}(p(z_i, n_i, S) - \overline{p}_0)^2 + O(||a, e||)^3, \end{aligned}$$

where $\overline{p}_0$ is the preset price under a symmetric equilibrium, where $n_i = W = z_i = e = 1$. Given the profit function $\pi(p|z_i, n_i, S) = p_{ij}y_{ij}(p) - y_{ij}MC$, where $MC = \frac{W^{1-\phi}e^{\phi}}{z_i}$. In this case, the first order condition:

$$\frac{\partial\pi(p|z_i, n_i, S)}{\partial p_{ij}} = y_{ij}(p)\left[1 - \epsilon_{ij}(p) + MC\frac{\epsilon_{ij}(p)}{p}\right]$$

Thus, the second derivative of the profit function can be expressed as:

$$\frac{\partial^2\pi(z_i, n_i, S)}{\partial p^2} = \frac{\partial y_{ij}(p)}{\partial p_{ij}}\left[1 - \epsilon_{ij}(p) + MC\frac{\epsilon_{ij}(p)}{p_{ij}}\right] + y_{ij}(p)\left[-\frac{\partial\epsilon_{ij}}{\partial p_{ij}} + MC\left(\frac{\partial\epsilon_{ij}}{\partial p_{ij}}\frac{1}{p_{ij}} - \frac{\epsilon_{ij}}{p_{ij}^2}\right)\right]$$

$$\frac{\partial^2\pi(z_i, n_i, S)}{\partial p^2} = \frac{\partial y_{ij}(p)}{\partial p_{ij}}\left[1 - \epsilon_{ij}(p) + MC\frac{\epsilon_{ij}(p)}{p_{ij}}\right] + y_{ij}(p_{ij})\left[-\frac{\epsilon_{ij}(p)\sigma_{ij}(p)}{p_{ij}} + \frac{MC}{p^2}\left(\epsilon_{ij}(p)\sigma_{ij}(p) - \epsilon_{ij}(p)\right)\right.$$

$$\frac{\partial^2\pi(a, e)}{\partial p^2} = \frac{\partial y_{ij}(p)}{\partial p_{ij}}\frac{\partial\pi(p_{ij})}{\partial p_{ij}}\frac{1}{y_{ij}(p)} + \frac{y_{ij}\epsilon_{ij}(p)}{p_{ij}}\left[-\sigma_{ij}(p) + \frac{MC}{p_{ij}}\left(\sigma_{ij}(p) - 1\right)\right]$$

Given the FOC at the optimal price p_{ij}, implying $\frac{\partial \pi(p_{ij}|z_i,n_i,S)}{\partial p} = 0$

$$\frac{\partial^2 \pi(z_i,n_i,S)}{\partial p^2} = \frac{y_{ij}\epsilon_{ij}(p)}{p_{ij}}\left[-\sigma_{ij}(p) + \frac{MC}{p_{ij}}\left(\sigma_{ij}(p)-1\right)\right]$$

Since we already prove that:

$$p_{ij} = \frac{\epsilon_{ij}(p_{ij})}{\epsilon_{ij}(p_{ij})-1}MC$$

The second derivate, becomes:

$$\frac{\partial^2 \pi(z_i,n_i,S)}{\partial p^2} = \frac{y_{ij}(p_{ij})(\epsilon_{ij}(p_{ij})-1)}{MC}\left[-\sigma_{ij}(p_{ij}) + \frac{(\epsilon_{ij}(p_{ij})-1)\left(\sigma(p_{ij})-1\right)}{\epsilon_{ij}(p_{ij})}\right]$$

As discussed by Gopinath and Itskhoki (2010), assuming that $\sigma_{ij}(p_{ij})$ is a smooth function, we can rely on the approximation:

$$\frac{\partial^2 \pi(z_i,n_i,S)}{\partial p^2} \approx \frac{\partial^2 \pi(1,1,1)}{\partial p^2} + O(||z_i,n_i,S||)$$

Where the first term to the right of the equality, evaluates the second-order derivate at the symmetric equilibrium, where $y_{ij} = \overline{p}_{ij} = P = W = n = 1$. In this case:

$$\begin{aligned}
\frac{\partial^2 \pi(1,1,1)}{\partial p^2} &= (\epsilon_{ij}(\overline{p}_{ij})-1)\left[-\sigma_{ij}(\overline{p}_{ij}) + \frac{(\epsilon_{ij}(\overline{p}_{ij})-1)\left(\sigma(\overline{p}_{ij})-1\right)}{\epsilon_{ij}(\overline{p}_{ij})}\right] \\
&= -\sigma_{ij}(\overline{p}_{ij})(\epsilon_{ij}(\overline{p}_{ij})-1) + \frac{(\epsilon_{ij}(\overline{p}_{ij})-1)^2\left(\sigma(\overline{p}_{ij})-1\right)}{\epsilon_{ij}(\overline{p}_{ij})} \\
&= \frac{1-\epsilon_{ij}(\overline{p}_{ij})}{\epsilon_{ij}(\overline{p}_{ij})}\left[\sigma_{ij}(\overline{p}_{ij}) + \epsilon_{ij}(\overline{p}_{ij}) - 1\right]
\end{aligned}$$

Finally, defining $\Psi_{ij} = \left[1+\frac{\sigma}{\epsilon-1}\right]^{-1}$, then $\epsilon+\sigma-1 = (\epsilon-1)\Psi^{-1}$ and since at the symmetric equilibrium $\overline{p}_{ij} = 1$, $MC = 1$, then $\frac{\epsilon}{\epsilon-1} = 1$, then:

$$\frac{\partial^2 \pi(1,1,1)}{\partial p^2} = \frac{1-\epsilon_{ij}(\overline{p}_{ij})}{\epsilon_{ij}(\overline{p}_{ij})}\left[\sigma_{ij}(\overline{p}_{ij}) + \epsilon_{ij}(\overline{p}_{ij}) - 1\right] = \frac{1-\epsilon_{ij}(\overline{p}_{ij})}{\epsilon_{ij}(\overline{p}_{ij})}(\epsilon_{ij}-1)\Psi^{-1} = -\frac{\epsilon_{ij}(\overline{p}_{ij})-1}{\Psi}$$

Hence:

$$\frac{\partial^2 \pi(z_i, n_i, S)}{\partial p^2} = \frac{\partial^2 \pi(1,1,1)}{\partial p^2} + O(||z_i, n_i, S||) = -\frac{\epsilon_{ij}(\overline{p}_{ij})-1}{\Psi} + O(||z_i, n_i, S||)$$

So we can rewrite the second order approximation for the profit loss function:

$$\begin{aligned} L(z_i, n_i, S) &\equiv \pi(z_i, n_i, S) - \pi(\overline{p}_0 | z_i, n_i, S) \\ &= -\frac{1}{2}\frac{\partial^2 \pi(z_i, n_i, S)}{\partial p^2}(p(z_i, n_i, S) - \overline{p}_0)^2 + O(||z_i, n_i, S||)^3 \\ &= \frac{1}{2}\frac{\epsilon_{ij}(\overline{p}_{ij})-1}{\Psi}(p(z_i, n_i, S) - \overline{p}_0)^2 + O(||z_i, n_i, S||)^3 \end{aligned}$$

Price pass-through Consider seller i setting a flexible (desired) price to customer j, denoted p_{ij}. As shown, under variable-elasticity demand, the optimal flexible price satisfies:

$$p_{ij} = \mu_{ij}(p_{ij}) \cdot mc_i, \qquad \mu_{ij}(p) \equiv \frac{\epsilon_{ij}(p)}{\epsilon_{ij}(p)-1}, \tag{35}$$

Define log markup

$$m_{ij}(p) \equiv \ln \mu_{ij}(p) = \ln\left(\frac{\epsilon_{ij}(p)}{\epsilon_{ij}(p)-1}\right). \tag{36}$$

Taking logs in (35) yields the flexible pricing condition

$$\ln p_{ij} = m_{ij}(p_{ij}) + \ln mc_i \tag{37}$$

Given the marginal cost function in our model $\ln mc_i = (1-\phi)\ln W + \phi \ln e - \ln z_i$.. Defining the price condition (37) at its steady state value $\overline{p}_0$, i.e., the symmetric equilib-

rium where $p_{ij} = W = e = z_i = 1$ we get $\ln \overline{p}_0 = m_{ij}(\overline{p}_0)$. In this case, the marginal cost is zero, given the assumed input prices and the idiosyncratic shocks in the steady state. Thus, the difference between the desired log price and the steady state price:

$$\ln p_{ij} - \ln \overline{p}_0 = [m_{ij}(p_{ij}) - m_{ij}(\overline{p}_0)] + \ln mc_i$$

If we expand $m_{ij}(p_{ij})$ around the symmetric price:

$$m_{ij}(p_{ij}) \approx m_{ij}(\overline{p}_0) + \left.\frac{\partial m_{ij}(p_{ij})}{\partial \ln p_{ij}}\right|_{\overline{p}_0} [\ln p_{ij} - \ln \overline{p}_0]$$

Then, plugging this into the log differences of prices:

$$\ln p_{ij} - \ln \overline{p}_0 = \left.\frac{\partial m_{ij}(p_{ij})}{\partial \ln p_{ij}}\right|_{\overline{p}_0} [\ln p_{ij} - \ln \overline{p}_0] + \ln mc_i + O(||z_i, n_i, S||)^2$$

$$\left(1 - \left.\frac{\partial m_{ij}(p_{ij})}{\partial \ln p_{ij}}\right|_{\overline{p}_0}\right) [\ln p_{ij} - \ln \overline{p}_0] = \ln mc_i + O(||z_i, n_i, S||)^2$$

Finally, we can express the log-markup $m_{ij}(p)$ as a function of the elasticities. Since $\mu_{ij}(p) = \epsilon_{ij}(p)/(\epsilon_{ij}(p) - 1)$,

$$m_{ij}(p) = \ln \epsilon_{ij}(p) - \ln\big(\epsilon_{ij}(p) - 1\big), \qquad \Rightarrow \qquad \frac{\partial m_{ij}}{\partial \epsilon} = \frac{1}{\epsilon} - \frac{1}{\epsilon - 1} = -\frac{1}{\epsilon(\epsilon - 1)} \tag{38}$$

Given the (bilateral ij) super-elasticity:

$$\sigma_{ij}(p) \equiv \frac{\partial \ln \epsilon_{ij}(p)}{\partial \ln p} \qquad \Longleftrightarrow \qquad \frac{\partial \epsilon_{ij}(p)}{\partial \ln p} = \sigma_{ij}(p)\, \epsilon_{ij}(p) \tag{39}$$

Thus, combining (38)–(39) yields

$$\frac{\partial m_{ij}(p)}{\partial \ln p} = \frac{\partial m_{ij}}{\partial \epsilon} \cdot \frac{\partial \epsilon_{ij}(p)}{\partial \ln p} = \left(-\frac{1}{\epsilon_{ij}(p)\big(\epsilon_{ij}(p) - 1\big)}\right) \cdot \big(\sigma_{ij}(p)\epsilon_{ij}(p)\big) = -\frac{\sigma_{ij}(p)}{\epsilon_{ij}(p) - 1} \tag{40}$$

Intuition Building on Gopinath and Itskhoki (2010), for the second-order approximation, we define $\Psi_{ij} = \left(1 + \frac{\sigma_{ij}}{\epsilon_{ij}-1}\right)^{-1}$. Through the first-order approximation:

$$\ln p_{ij} - \ln \overline{p}_0 \;\approx\; \Psi_{ij} \ln mc_i \tag{41}$$

Hence $\Psi_{ij} < 1$ governs the pass-through given the primitives of demand for each ij seller-buyer match. Moreover, since the cost shocks enters linearly in the (log) price differences:

- Oil Price pass-through: $\frac{\ln p_{ij} - \ln \overline{p}_0}{\partial \ln e} = \phi \Psi_{ij}$
- Wage pass-through: $\frac{\ln p_{ij} - \ln \overline{p}_0}{\partial \ln W} = (1-\phi)\Psi_{ij}$
- Productivity pass-through: $\frac{\ln p_{ij} - \ln \overline{p}_0}{\partial \ln z} = -\Psi_{ij}$

Under CES demand functions ($\sigma_{ij} = 0$), there is complete pass-through ($\Psi_{ij} = 1$), since the elasticity does not change with the relative price. Under Kimball demand functions, the elasticity of demand varies with the firm's relative price, so markups become endogenous, and the pass-through is incomplete ($\Psi_{ij} < 1$). The key object governing pass-through is the *markup elasticity*

$$\frac{\partial \ln \mu_{ij}}{\partial \ln p} = -\frac{\sigma_{ij}}{\epsilon_{ij} - 1}, \tag{42}$$

which captures the extent to which a price increase raises the elasticity of demand and therefore compresses markups. If $\sigma_{ij} > 0$, the elasticity rises with the firm's relative price (demand becomes more price-sensitive as the firm tries to increase its price). This forces the markup down when the firm raises price, i.e., $\frac{\partial m_{ij}(p)}{\partial \ln p} < 0$, so part of a marginal-cost shock is absorbed by markups rather than passed through to prices.

raising the price increases the elasticity faced by the firm, forcing a reduction in the markup and causing part of a marginal-cost shock to be absorbed by markups rather than passed through to prices.

In particular, if $\sigma_{ij} > 0$, the elasticity rises with the firm's relative price (demand becomes more price-sensitive as the firm tries to increase its price). This forces the markup down when the firm raises price, i.e., $\frac{\partial m_{ij}(p)}{\partial \ln p} < 0$. Because part of a marginal-cost increase is absorbed by a lower markup, the optimal price moves less than marginal cost, which is exactly $\Psi_{ij} < 1$.

In our environment, since both ϵ_{ij} and σ_{ij} are functions of the bilateral market share s_{ij}, pass-through is itself a function of market share too:

$$\Psi_{ij}(s_{ij}) = \left(1 + \frac{\sigma_{ij}(s_{ij})}{\epsilon_{ij}(s_{ij}) - 1}\right)^{-1}. \tag{43}$$

Market share affects pass-through through its joint impact on the level and curvature of residual demand. In demand systems with variable elasticity, such as Kimball preferences, both the elasticity of demand ϵ_{ij} and the super-elasticity σ_{ij} are functions of the bilateral market share s_{ij}. In particular, higher market shares are typically associated with lower elasticity (reflecting greater market power) and lower super-elasticity (reflecting lower local curvature of demand). That is why the effect of the market share is actually driven by the price-position along the demand curve for suppliers. In fact, since pass-through is governed by the ratio $\sigma_{ij}/(\epsilon_{ij} - 1)$, the effect of market share on pass-through is, in general, ambiguous and depends on the relative sensitivity of elasticity and curvature to market share. Formally,

$$\frac{\partial}{\partial s_{ij}}\left(\frac{\sigma_{ij}(s_{ij})}{\epsilon_{ij}(s_{ij}) - 1}\right) = \frac{(\epsilon_{ij} - 1)\,\sigma'_{ij}(s_{ij}) - \sigma_{ij}(s_{ij})\,\epsilon'_{ij}(s_{ij})}{(\epsilon_{ij} - 1)^2}, \tag{44}$$

where primes denote derivatives with respect to s_{ij}. Under Kimball demand, both $\epsilon'_{ij}(s_{ij}) < 0$ and $\sigma'_{ij}(s_{ij}) < 0$, implying that market share simultaneously weakens competitive pressure (through a lower elasticity level) and reduces the curvature of demand (through a lower super-elasticity).

The net effect of market share on pass-through therefore depends on which force dominates. If the elasticity effect dominates, in the sense that the decline in ϵ_{ij} is sufficiently strong relative to the decline in σ_{ij}, then

$$\frac{\partial}{\partial s_{ij}}\left(\frac{\sigma_{ij}(s_{ij})}{\epsilon_{ij}(s_{ij}) - 1}\right) > 0 \quad \text{and} \quad \frac{\partial \Psi}{\partial s_{ij}} < 0$$

so that the sensitivity of markups to price changes increases with market share. In this case, firms with higher market shares optimally absorb a larger fraction of marginal-cost shocks through endogenous markup adjustment, resulting in lower pass-through.

Conversely, if the curvature effect dominates, so that $\sigma_{ij}(s_{ij})$ declines sufficiently fast

relative to $\epsilon_{ij}(s_{ij})$, then

$$\frac{\partial}{\partial s_{ij}}\left(\frac{\sigma_{ij}(s_{ij})}{\epsilon_{ij}(s_{ij})-1}\right) < 0 \quad \text{and} \quad \frac{\partial \Psi}{\partial s_{ij}} > 0$$

Higher market shares are associated with more stable markups and higher pass-through. Thus, the model with Kimball demand does not necessarily impose a monotonic relationship between market share and pass-through. It rather delivers a direct decomposition, which highlights how the elasticity and curvature of demand can bring different implications in terms of how the bilateral market share would ultimately affect the pass-through.