Fiscal Inflation in a Currency Union*

Momo Komatsu
Federal Reserve Board

David Murakami
Milan & Pavia
Bank of England

Ivan Shchapov
IP Paris
CREST

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Abstract

This paper studies how funded and unfunded fiscal expansions propagate in a currency union. A funded fiscal expansion implies future tax adjustments, allowing the central bank to control inflation, whereas an unfunded one lacks such adjustments, requiring the central bank to accommodate fiscally driven inflationary pressures. In euro area local projections, fiscal expansions in Germany and Italy are inflationary domestically and in other countries, while fiscal expansions in France are not and are followed by a more contractionary monetary response. We interpret this heterogeneity through a model with partially unfunded debt in the spirit of Bianchi, Faccini, and Melosi (2023). In the model, funded fiscal expansions imply divergent inflation within the union, whereas unfunded fiscal expansions generate union-wide inflationary spillovers because monetary policy accommodates fiscal inflation. This mechanism maps closely to the empirical patterns. The estimation of the quantitative model confirms that Germany and Italy experienced a higher share of unfunded shock than France since the start of the euro area.

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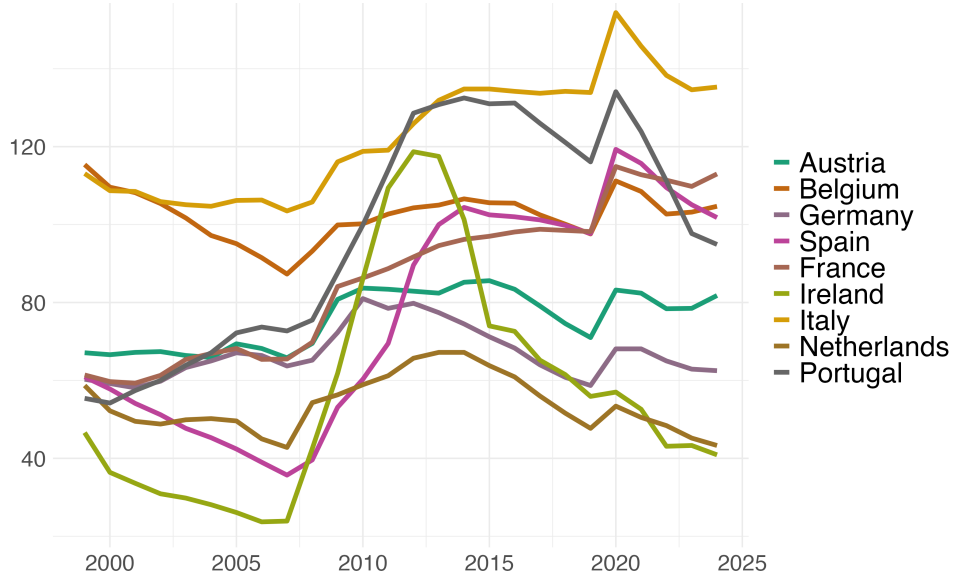
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Komatsu: momo.komatsu@frb.gov | Murakami: david.murakami@bankofengland.co.uk | Shchapov: ivan.shchapov@ensae.fr

1 Introduction

In the last 25 years, the debt-to-GDP ratio in the euro area has been increasing, as shown in Figure 1. The cumulative effects of fiscal expansions during the financial crisis, the debt crisis, the COVID-19 crisis, and the cost-of-living crisis (roughly 2007–2008, 2011–2013, 2020–2021, and 2021–2023, respectively) have contributed to this rise in the debt-to-GDP ratio. A Fisherian model distinguishes between fiscal expansions that are funded by future fiscal adjustments and those that are not. We study the consequences of these two types of fiscal expansions in the context of a currency union.

Figure 1: Debt-to-GDP ratio (in percentages) in the original euro area countries



Notes: The original euro area countries are the member states of the euro area that adopted the euro on January 1, 1999. We exclude Luxembourg and Finland for data availability reasons. Source: Eurostat.

Whether fiscal expansions are funded or unfunded affects a central bank's role in stabilizing the economy. A funded fiscal expansion is backed by future fiscal adjustments, for example, an increase in future taxes. In theory, such an expansion automatically stabilizes demand and therefore has no effect on inflation; in that case, the central bank controls inflation. By contrast, an unfunded fiscal expansion—not backed by future fiscal adjustments—has an inflationary effect on the economy. In this case, the central bank accommodates higher inflation from fiscal pressures, *fiscal inflation*.

In the context of the euro area currency union, with multiple fiscal authorities and one central bank, a natural question arises: how do a country's funded and unfunded fiscal expansions propagate through the rest of the currency union? We first conduct empirical local projections to study the propagation of German, French, and Italian fiscal shocks in the euro area. The results suggest that the central bank accommo-

dates fiscal inflationary pressures originating in Germany and Italy, but not those from France. Second, with a stylized model, we find that the propagation of a French fiscal expansion is qualitatively similar to a funded fiscal shock, whereas the propagation of German and Italian fiscal expansions is similar to unfunded fiscal shocks. Finally, with a quantitative model, we estimate the share of funded and unfunded shocks contributing to the change in debt-to-GDP ratio and find that Germany and Italy have a higher share of unfunded fiscal shocks than France.

The paper makes three contributions. First, we document new evidence on cross-country heterogeneity in fiscal spillovers within the euro area. Using local projections and forecast-error fiscal shocks following Auerbach and Gorodnichenko (2012, 2017), we find that in 2004–2025 fiscal expansions in Germany and Italy increase inflation domestically and in other member countries, whereas fiscal expansions in France lower inflation in other countries. These patterns line up with short-rate local projections: the central bank reacts in a contractionary manner to a fiscal shock in France, while after a fiscal shock in Germany and Italy, the central bank accommodates the resulting inflation, which then spills over to other countries.

Second, we provide a mechanism to explain this heterogeneity in fiscal shock propagations with a two-country New Keynesian currency-union model with partially unfunded debt, building on Bianchi, Faccini, and Melosi (2023). In the model, funded and unfunded fiscal expansions in country A have opposite implications for inflation in country B. A funded expansion shock in country A raises inflation at home but lowers inflation abroad through general-equilibrium effects: the shock boosts consumption and inflation in country A whilst also increasing the government debt, which is backed by future tax adjustments. The increase in government debt in country A increases net savings in country B, depressing demand and inflation in country B. An unfunded expansion shock raises inflation in both countries because fiscal inflation is accommodated by the common central bank. This mechanism rationalizes the empirical contrast between French shocks and German-Italian shocks.

Third, we develop a quantitative extension with habit formation, non-tradables, nominal rigidities, and long-term bonds, and discipline its parameters with Bayesian estimation. The model results preserve the stylized model's qualitative propagation patterns with greater persistence. We estimate the full currency-union model across various country combinations to quantify the historical mix of funded and unfunded shocks in each country, and find that Germany and Italy have a higher share of unfunded fiscal shocks than France.

Related literature. This paper addresses monetary–fiscal interactions in a currency union and therefore bridges two strands of the literature: (i) price-level determinacy and the interaction of monetary and fiscal policy, with Sargent and Wallace (1981), Leeper (1991), Sims (1994), and Woodford (1994) as the pioneers, and (ii) monetary and fiscal policy spillovers in a currency union. Moreover, the empirical exercise contributes to the literature on fiscal policy spillovers.

The literature on price-level determinacy and the interaction of monetary and fiscal policy has been widely studied since, and more recently by Cochrane (2021). Our model framework builds on Bianchi, Faccini, and Melosi (2023) (BFM), which introduced a new class of general equilibrium models that allow for partially unfunded debt. This paper extends this framework to a currency union setting in which countries share a common central bank, but fiscal authorities in each country can issue either funded or unfunded debt.

Among others, Beetsma and Jensen (2005), Galí and Monacelli (2008), Ferrero (2009), and Hjortsoe (2016) study the optimal joint conduct of monetary and fiscal policy in a currency union during asymmetric shocks. More recently, authors like Barbier-Gauchard and Betti (2021) study the spillover effects of fiscal policy in a monetary union. They study different types of fiscal instruments and their spillovers onto the rest of the union. Our paper looks at the difference between funded and unfunded fiscal shocks and how they propagate within the currency union.

Spillovers of fiscal policy are a growing strand of the literature, with contributors like Goujard (2017), Debrun et al. (2021), Born, Müller, and Pfeifer (2020), and Ascari et al. (2024). Many such papers study the channels through which fiscal policy spillovers are transmitted, e.g. trade linkages, financial linkages, or demand spillovers. Our paper links these spillovers and transmission channels to whether fiscal shocks are funded or not.

The rest of the paper is organized as follows. In section 2 we present the empirical analysis and show how shocks from different countries cause different spillovers. Section 3 builds a stylized DSGE model, analyzes the propagation of funded and unfunded shocks, and links those results to the local projections. In section 4 we estimate a quantitative model and investigate the share of funded and unfunded shocks in the euro area. Concluding remarks are in Section 5.

2 Empirical analysis

In this section, we study how fiscal shocks originating in Germany, France, and Italy—the three largest euro area economies—transmit to the domestic economy and to the

rest of the union. Using local projections à la Jordà (2005) and fiscal shocks obtained from forecast errors, we establish three facts. First, fiscal expansions in Germany and Italy are inflationary both domestically and in other euro area countries, whereas fiscal expansions in France are not. Second, monetary policy responses differ by source country: the central bank reacts in a contractionary way to French fiscal expansions, but less so to German and Italian expansions. Third, consistent with this heterogeneous policy response, real rate dynamics also differ by source country. Taken together, these results suggest that the central bank accommodates inflation caused by fiscal expansions in Germany and Italy (*fiscal inflation*), but not fiscal expansions in France.

2.1 Methodology and data

Fiscal shocks. We construct fiscal shocks using the OECD Economic Outlook (OECD, 2025). The OECD publishes the Economic Outlook twice a year, and it includes forecasts of macroeconomic variables as point estimates, including government spending, for the next few quarters. As in Auerbach and Gorodnichenko (2012, 2017) and Boehm (2020), we use those forecasts to construct forecast errors of government spending as a share of gross domestic product (GDP) for each country. We have quarterly data starting from *Economic Outlook No 74* (December 2003), so our sample spans 2004Q1–2025Q4.

We construct forecast errors of government spending using data on government consumption in volume (constant-price) terms. We extract forecasts of government consumption for Q1 and Q2 of each year from the second *Economic Outlook* of the previous year, and forecasts for Q3 and Q4 from the first *Economic Outlook* in the same year. For example, for the forecast of government spending in 2016Q1 and 2016Q2, we consult the second *Economic Outlook* of 2015. We then scale the forecasted growth rate of government spending by GDP to obtain a forecast for the government spending growth rate as a percentage of GDP. The forecast error $FE_{t|t-1}$ is the difference between the realized government-spending growth rate (as a share of GDP) and the forecast made in the previous period. We use realized government-spending growth rates from the most recent *Economic Outlook*.

Even though the forecast error $FE_{t|t-1}$ is already the unpredictable innovation in government-spending growth (as in Auerbach and Gorodnichenko (2017)), we further purge predictable components by regressing $FE_{t|t-1}$ on lags of macroeconomic variables. We use the residuals from this panel regression as the fiscal shock.

Other data. All other data that we use as controls in the local projections are also publicly available at the OECD. We construct quarterly government spending growth

and GDP growth from the government spending and GDP measures above. The harmonized inflation rate and the short-term nominal interest rate are expressed in annualized percentage points. The real rate is the difference between the nominal interest rate and inflation. We focus on the original euro area countries to include countries that have shared a central bank for the longest period.[1]

Local projection specification. We study three types of local projections: (i) country-specific local projections for the effect of a fiscal shock on domestic inflation and real rate, (ii) spillover local projections for the effect of a fiscal shock on inflation and real rate in other countries, and (iii) short rate local projections for the effect of a fiscal shock on the short-term nominal interest rate.

The specification of the country-specific local projections, used for y as inflation and real rate, are as follows:

$$y_{c,t+h} - y_{c,t} = \beta_{c,h}\text{shock}_{c,t} + \sum_{j=0}^{2} \gamma_{c,h,j} X_{c,t-j} + \varepsilon_{c,t+h}, \tag{1}$$

where $c \in \{\text{Germany, France, Italy}\}$. $y_{c,t+h}$ is the inflation or real rate of country c at time $t+h$, and $\text{shock}_{c,t}$ is the fiscal shock, as explained above. $X_{c,t-j}$ includes the control variables: government spending growth, GDP growth, inflation, and their lags.[2] We estimate the response of inflation and the real rate up to $h = 1, 2, \ldots, 8$ quarters ahead with heteroskedastic standard errors. We estimate equation (1) twice for each country, for the inflation and real rate, to obtain the effect of a fiscal shock on domestic inflation and real rate.

The spillover panel local projections estimate the effect of the fiscal shock in the source country on the inflation and real rate in the other countries in the euro area:

$$y_{i\neq c,t+h} - y_{i\neq c,t} = \beta_{c,h}\text{shock}_{c,t} + \sum_{j=0}^{2} \gamma_{i,h,j} X_{i,t-j} + \alpha_{i\neq c} + \varepsilon_{i\neq c,t+h}, \tag{2}$$

where i are all countries in the sample and countries $c \in \{\text{Germany, France, Italy}\}$ are the source country of the fiscal shock. Inflation and the real rate (the dependent variable) and the country fixed effects are of countries other than the source country. The controls include lagged variables from both the receiver and source country and are the same as the country-specific local projections, plus contemporaneous and lagged euro area wide controls for inflation and GDP growth. We cannot add quarterly time

1. Austria, Belgium, Germany, Spain, France, Ireland, Italy, Netherlands, Portugal. We exclude Luxembourg and Finland for data availability reasons.

2. For the real rate local projections, we add the lags of the change of the real rate as controls. Moreover, we include euro area wide GDP and inflation, since those are naturally confounded with the euro area wide short-term interest rate, included in constructing the dependent variable.

fixed effects since the shock is identical across countries and therefore collinear with quarterly time-fixed effects. The spillover panel local projections use Driscoll-Kraay standard errors with GDP weights. With equation (2), we estimate $\beta_{c,h}$ with panel local projections for each source country, which we interpret as the effect of a fiscal shock in the source country on inflation and real rate in other euro area countries.

The short-rate local projections are similar to the country-specific local projections, but with the euro area wide short-term interest rate, i_t, as the dependent variable:

$$i_{t+h} - i_t = \beta_{c,h}\text{shock}_{c,t} + \sum_{j=0}^{2} \gamma_{c,h,j} X_{c,t-j} + \varepsilon_{c,t+h}. \tag{3}$$

For this specification, the controls also include contemporaneous euro area wide inflation and GDP growth rate, and their lags, as well as the lags of the change in the short-term interest rate. We estimate $\beta_{c,h}$ with this equation, which we interpret as the effect of a fiscal shock on the short-term nominal interest rate. As in the country-specific local projections, we use heteroskedastic standard errors. Since the short-term interest rate in the euro area drops below zero in 2014Q2, we consider this as the start of the effective lower bound. We run all short-rate and real-rate local projections with data up to 2014Q1.

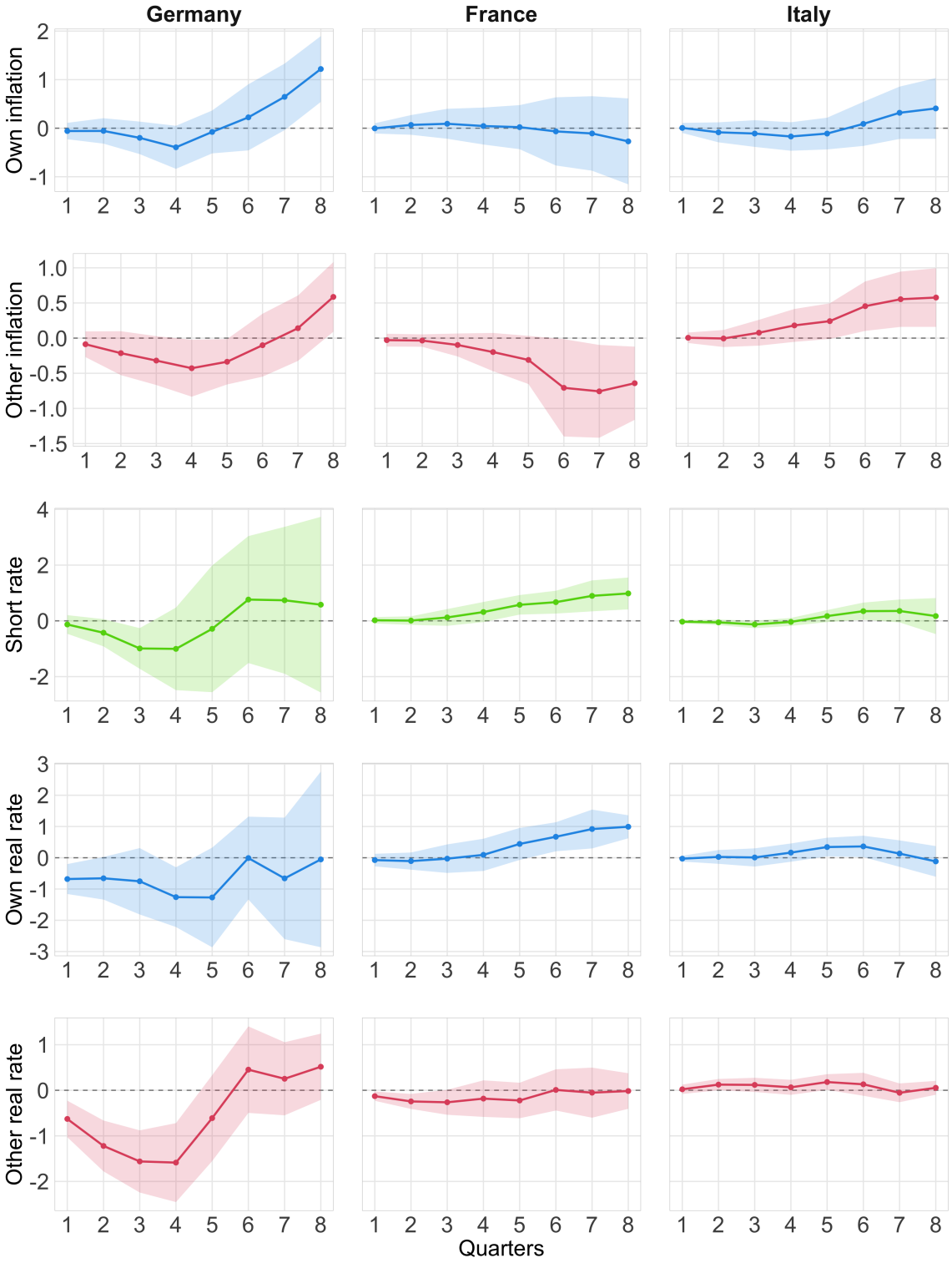
2.2 Results

Figure 2 presents the results from estimating five local projections, on the effect of a fiscal shock on (i) domestic inflation (eq. (1)), (ii) other countries' inflation (eq. (2)), (iii) the short-term interest rate (eq.(3)), (iv) domestic real rate (eq.(1)), and (v) other countries' real rate (eq.(2)). The column labels indicate the source country of the fiscal shock.

The first and the second row in Figure 2 show the effects of a fiscal shock in Germany, France, and Italy on domestic inflation and inflation in other countries in the euro area. Our findings are as follows: fiscal expansions in Germany are inflationary for both the domestic economy and the rest of the euro area. There is a similar pattern for fiscal expansions in Italy, but not significant domestically. In France, fiscal expansions do not have a significant effect on domestic inflation, but they decrease inflation in other countries. These results suggest that the monetary policy stance after fiscal expansions in France is more contractionary than after expansions in Germany or Italy. The responses in the third row of Figure 2 confirm this conjecture: the short-term interest rate increases after a fiscal expansion in France, but not after expansions in Germany or Italy.

The responses of the real rate, as shown in the fourth and fifth row in Figure 2,

Figure 2: Impulse responses to a 1 pp shock in govt spending (% of GDP)



Notes: The own country and short rate local projections use heteroskedastic standard errors. For spillover effects to other countries, the results are pooled estimates from panel local projections with Driscoll-Kraay standard errors and countries weighted by GDP. Shaded area shows 90% confidence interval. Sample period is 2004Q1–2025Q4 for inflation responses, and 2004Q1–2014Q1 for rate responses.

show that fiscal expansions in Germany decrease the real rate domestically and in other countries. Fiscal expansions in France, however, increase the real rate domestically. For Italy, the results are insignificant. The results are in line with the responses for inflation and the short-term interest rate. On one hand, since German fiscal expansions do not move the short-term interest rate, they increase inflation and decrease real rate domestically. Moreover, these effects spill over to other countries in the euro area. On the other hand, French fiscal expansions elicit a contractionary response from the central bank, and hence decrease inflation in other countries sharing the central bank.

In summary, we find that fiscal expansions in Germany, France and Italy have heterogeneous effects on inflation, the short-term interest rate, and the real rate. While fiscal expansions in France elicit an interest hike as a contractionary response from the central bank, expansions in Germany and Italy do not. Hence, expansions in Germany and Italy are inflationary, both domestically and for other countries. In France, domestic inflation remains constant after fiscal expansions, likely because the fiscal boost and the interest hike counteract each other. However, in other countries, inflation decreases due to the interest rate increase. The real rate responses reinforce this narrative: the inflationary effect from the German fiscal expansions and an absent monetary policy response lead to a decline in real rates domestically and in other euro area countries. The real rate in France increases after a fiscal expansion through the contractionary response from the central bank.

The empirical results establish that fiscal shocks in Germany and Italy have an inflationary effect on the rest of the euro area, suggesting that the central bank accommodates inflation. This result alludes to *fiscal inflation* coming from *unfunded* fiscal shocks. By contrast, fiscal shocks in France have a deflationary effect on the rest of the euro area, implying that the central bank reacts to fiscal expansions and that these shocks may be *funded*. In the next section, we use a stylized model to corroborate the link of domestic and spillover inflation movements to funded and unfunded expansions.

As robustness checks, Appendix A includes bilateral spillover responses for Germany, France, and Italy for inflation in Figure 8 and the real rate in Figure 9. Moreover, we do the entire exercise with unpurged shocks, so the pure forecast errors, in Figure 10 and with different standard error specifications in Figure 11. Figure 12 shows the results when all local projections include lags of fiscal shocks as controls and Figure 13 include contemporaneous weighted fiscal shocks of other countries and its lags as controls. Figure 14 shows the results when we include one more period of lags in all the controls. We also run a timing-robust specification in Figure 15 that removes within-semester reallocation variation in forecast errors before estimating the same lo-

cal projections.[3] All results are in line with the baseline results. We also conduct local projections on the sovereign spread in Figure 16.

3 Stylized model

In this section, we present a stylized model of a currency union with partially unfunded debt to analyze the propagation mechanism of funded and unfunded fiscal expansion shocks. In the following section, we build a medium-scale model to perform quantitative exercises.

The stylized model considers a currency union of two countries, A and B, each with a fixed endowment. The countries are inhabited by households who consume goods from both countries with a home bias. Moreover, households in each country hold bonds issued by the fiscal authority of both countries. The fiscal authority in each country collects taxes and issues bonds, and the monetary authority targets an average of the inflation rates in the two countries with an inflation-targeting rule. We allow for partially unfunded debt as in Bianchi, Faccini, and Melosi (2023). Country B variables are denoted with *, except for prices and the nominal interest rate which are the same across the union. Equations are for the economy in country A unless otherwise stated.

3.1 Households

Households derive utility from a consumption bundle and receive an endowment Y. They can smooth consumption by holding bonds issued in countries A and B, subject to portfolio adjustment costs. They face the following maximization problem:

$$\max_{C_t, B_{At}, B_{Bt}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \ln C_t, \tag{4}$$

subject to:

$$\begin{aligned} & P_t C_t + B_{At} + B_{Bt} + P_t T_t \\ & \quad = P_t Y + R_{t-1} \left(B_{A,t-1} + B_{B,t-1} \right) - \frac{\psi_b P_t \left(\overline{b}_A + \overline{b}_B \right)}{2} \left(\delta_b \frac{B_{Bt}}{B_{At}} - 1 \right)^2, \end{aligned} \tag{5}$$

3. For each country and semester (S1: Q1–Q2, S2: Q3–Q4), we compute the quarterly forecast error as before, but replace each quarter's error with the semester average of the two quarterly forecast errors. This keeps the semester-level surprise in government spending but removes within-semester timing reallocation (e.g., if spending was expected in Q3 but realized in Q4). We then apply the same purging step and re-estimate all local projections with the same controls and horizons.

where β is the discount factor, C_t is a bundle of consumption goods and P_t is its price, defined as:

$$C_t = \left[(1-\alpha_I)^{1/\mu}(C_{At})^{(\mu-1)/\mu} + \alpha_I^{1/\mu}(C_{Bt})^{(\mu-1)/\mu}\right], \tag{6}$$

$$P_t = \left[(1-\alpha_I)P_{At}^{1-\mu} + \alpha_I P_{Bt}^{1-\mu}\right]^{1/(1-\mu)}, \tag{7}$$

where C_{At} is a good produced in country A, C_{Bt} a good produced in country B, and P_{At} and P_{Bt} are their prices, respectively. α_I is the share of imported goods and μ is the elasticity of substitution between domestic and imported goods. B_{At} is nominal holdings of bonds issued by the government in country A and B_{Bt} is nominal holdings of bonds issued by the government in country B, and R_t is the gross nominal interest rate in the currency union. We define real bond holdings (as shares of output) as $b_{At} \equiv \frac{B_{At}}{P_t Y}$ and $b_{Bt} \equiv \frac{B_{Bt}}{P_t Y}$. Moreover, $\overline{b}_A = \frac{\overline{B}_A}{\overline{P}\overline{Y}}$ and $\overline{b}_B = \frac{\overline{B}_B}{\overline{P}\overline{Y}}$ are steady-state real bond holdings as shares of output, and $\delta_b = \frac{\overline{b}_A}{\overline{b}_B}$ is the desired portfolio mix. Parameter ψ_b is the cost of deviating from the desired mix. T_t are real lump-sum net taxes.

The cost minimization problem for the consumption bundle results in the per-period relative demand between country A and country B goods:

$$\frac{C_{At}}{C_{Bt}} = \frac{1-\alpha_I}{\alpha_I}\left(\frac{P_{At}}{P_{Bt}}\right)^{-\mu}, \tag{8}$$

using demand equations $C_{At} = (1-\alpha_I)\left(\frac{P_{At}}{P_t}\right)^{-\mu} C_t$ and $C_{Bt} = \alpha_I\left(\frac{P_{Bt}}{P_t}\right)^{-\mu} C_t$. The intertemporal optimization problem results in two Euler equations, one for each bond:

$$1 = \frac{\beta R_t}{1-\psi_b(\overline{b}_A+\overline{b}_B)\left(\delta_b\frac{b_{Bt}}{b_{At}}-1\right)\delta_b\frac{b_{Bt}}{b_{At}^2}}\mathbb{E}_t\left(\Pi_{t+1}^{-1}\frac{C_t}{C_{t+1}}\right), \tag{9}$$

$$1 = \frac{\beta R_t}{1+\psi_b\left(\overline{b}_A+\overline{b}_B\right)\left(\delta_b\frac{b_{Bt}}{b_{At}}-1\right)\delta_b\frac{1}{b_{At}}}\mathbb{E}_t\left(\Pi_{t+1}^{-1}\frac{C_t}{C_{t+1}}\right), \tag{10}$$

where $\Pi_t = P_t/P_{t-1}$ is gross inflation.

We define the terms of trade as the relative price of the good produced in country B to the good produced in country A:

$$S_t = \frac{P_{Bt}}{P_{At}}. \tag{11}$$

3.2 Fiscal authority

The fiscal authority of both countries issues bonds and collects taxes. The nominal budget constraint for the fiscal authority is thus the following:

$$B_t + P_t T_t = R_{t-1}B_{t-1}, \tag{12}$$

where taxes T_t coincide with the real primary surplus and B_t is the supply of bonds. The government follows a fiscal rule:

$$\frac{\tau_t}{\overline{\tau}} = \left(\frac{s_{b,t-1}}{\overline{s}_b}\right)^{\gamma} e^{\zeta_t}, \tag{13}$$

where $\tau_t = \frac{T_t}{Y}$ is the surplus-to-output ratio and $s_{bt} = \frac{B_t}{P_t Y}$ is the real market value of debt as a share of output. Parameter γ determines how strongly the government adjusts primary surpluses with debt levels. ζ_t is a lump-sum tax shock, following an AR(1) process. In Section 3.6 we explain how we introduce partially unfunded government debt.

3.3 Monetary authority

The central bank of the currency union follows a simple inflation-targeting rule:

$$\frac{R_t}{\overline{R}} = \left(\frac{\Pi_t^{MU}}{\overline{\Pi}^{MU}}\right)^{\phi}, \tag{14}$$

where $\Pi_t^{MU} = \Pi_t^{\Theta}\Pi_t^{*1-\Theta} = \left(\frac{P_t}{P_{t-1}}\right)^{\Theta}\left(\frac{P_t^*}{P_{t-1}^*}\right)^{1-\Theta}$ is gross union-wide inflation. Parameter ϕ determines how strongly the central bank adjusts nominal interest rate with inflation movements.

3.4 Equilibrium

The equilibrium of this economy is characterized by a sequence of prices $\{P_{At}, P_{Bt}, R_t\}$ and allocations $\{C_{At}, C_{Bt}, C_{At}^*, C_{Bt}^*, B_{At}, B_{Bt}, B_{At}^*, B_{Bt}^*\}$ such that goods markets clear for the goods produced in countries A and B, and the bond markets clear for bonds supplied by fiscal authorities in both countries.

The goods market clearing conditions for countries A and B are the following:

$$Y = C_{At} + C_{At}^*, \tag{15}$$

$$Y^* = C_{Bt} + C_{Bt}^*. \tag{16}$$

The bond markets clear when the supply of bonds in countries A and B matches demand:

$$B_t = B_{At} + B_{At}^*, \tag{17}$$

$$B_t^* = B_{Bt} + B_{Bt}^*. \tag{18}$$

Finally, we can express the dynamics of net foreign assets in country A with the current account definition:

$$CA_t \equiv P_{At}Y - P_t C_t = B_{Bt} - B_{At}^* - R_{t-1}\left(B_{B,t-1} - B_{A,t-1}^*\right). \tag{19}$$

We linearize the relevant equilibrium conditions, such that the hatted variables are log-deviations from the steady state values. See Appendix B for the full set of equilibrium conditions.

3.5 Existence and uniqueness of a solution

Leeper (1991) shows that for closed economy versions of such models, there are two regions of the parameter space that deliver existence and uniqueness of a stationary solution. In the first region monetary policy is active and stabilizes inflation, and the fiscal authority keeps debt on a stable path: active monetary policy and passive fiscal policy. In the second region fiscal policy does not commit to a stable path of debt, such that monetary policy accommodates inflation driven by fiscal imbalances: passive monetary policy and active fiscal policy. We extend the logic to a currency union version of the model.

To characterize determinacy, we derive the monetary and the fiscal block of the economy. For the monetary block we combine the Fisher equation with the monetary rule followed by the central bank, eq. (14). For the fiscal block, we combine the government budget constraints with the fiscal rules from each country, eq. (13) (in country A) and its counterpart (in country B).

We obtain the Fisher equation by adding the Euler equation of country A with respect to b_{At}, eq. (9), to the Euler equation of country B with respect to b^*_{At}, and applying the goods market clearing conditions (15) and (16). We use the log-linearized versions of the equations from Appendix B:

$$\hat{i}_t = \mathbb{E}_t \hat{\pi}^{MU}_{t+1} + \Theta\psi_b \frac{\overline{b}_A + \overline{b}_B}{\overline{b}_A}\left(\hat{b}_{Bt} - \hat{b}_{At}\right) + (1-\Theta)\psi_b \frac{\overline{b}^*_A + \overline{b}^*_B}{\overline{b}^*_A}\left(\hat{b}^*_{Bt} - \hat{b}^*_{At}\right). \tag{20}$$

The equation links the nominal interest rate to the expected inflation and portfolio adjustment costs. Combining the Fisher equation with the log-linear monetary rule,

$$\hat{i}_t = \phi \hat{\pi}^{MU}_t, \tag{21}$$

leads to the monetary block of the currency union:

$$\mathbb{E}_t \hat{\pi}^{MU}_{t+1} = \phi \hat{\pi}^{MU}_t - \Theta\psi_b \frac{\overline{b}_A + \overline{b}_B}{\overline{b}_A}\left(\hat{b}_{Bt} - \hat{b}_{At}\right) - (1-\Theta)\psi_b \frac{\overline{b}^*_A + \overline{b}^*_B}{\overline{b}^*_A}\left(\hat{b}^*_{Bt} - \hat{b}^*_{At}\right). \tag{22}$$

The coefficient in the monetary rule, ϕ, determines the eigenvalue in the monetary block. For the fiscal block, we combine the log-linear government budget constraints

of country A and B,

$$\hat{s}_{bt} = \frac{1}{\beta}\left[\hat{s}_{b,t-1} + \hat{i}_{t-1} - \hat{\pi}_t - (1-\beta)\hat{\tau}_t\right], \tag{23}$$

$$\hat{s}^*_{bt} = \frac{1}{\beta}\left[\hat{s}^*_{b,t-1} + \hat{i}_{t-1} - \hat{\pi}^*_t - (1-\beta)\hat{\tau}^*_t\right], \tag{24}$$

with fiscal rules of each country,

$$\hat{\tau}_t = \gamma\hat{s}_{b,t-1}, \tag{25}$$

$$\hat{\tau}^*_t = \gamma^*\hat{s}^*_{b,t-1}. \tag{26}$$

The fiscal block of the currency union consists of two equations:

$$\hat{s}_{bt} = \frac{1}{\beta}\left[(1-(1-\beta)\gamma)\hat{s}_{b,t-1} + \hat{i}_{t-1} - \hat{\pi}_t\right], \tag{27}$$

$$\hat{s}^*_{bt} = \frac{1}{\beta}\left[(1-(1-\beta)\gamma^*)\hat{s}^*_{b,t-1} + \hat{i}_{t-1} - \hat{\pi}^*_t\right], \tag{28}$$

for which γ and γ^*, the debt stabilization parameters, determine the eigenvalues.

Equations (22), (27), and (28) describe the currency union model. There is one non-predetermined variable, $\hat{\pi}^{MU}_t$, so there can only be one eigenvalue outside the unit circle for the model to satisfy the Blanchard–Kahn conditions for a determinate equilibrium. The model can either be monetary-led (with $\phi > 1$, placing the eigenvalue from equation (22) outside the unit circle) or fiscal-led by country A (with $\gamma < 1$, placing the eigenvalue from equation (27) outside the unit circle).

In a monetary-led economy, the parameters take the values $\phi > 1$, $\gamma \geq 1$, and $\gamma^* \geq 1$, which means that the central bank controls inflation and both fiscal authorities adjust taxes when debt changes. Tax adjustments to debt changes are sufficiently large that the fiscal authority stabilizes demand by signaling that higher debt levels are matched by future taxation. Hence, an increase in debt does not cause inflation, i.e. the debt is funded, allowing the central bank to control inflation. In a monetary-led economy, monetary policy is active and fiscal policy in both countries is passive.

In a fiscal-led economy, the parameters take the values $\phi \leq 1$, $\gamma < 1$, and $\gamma^* \geq 1$, which means that the economy is fiscal-led by country A. The fiscal authority in country B still adjusts taxes when debt changes, such that a debt increase in country B does not cause inflation. However, with $\gamma < 1$ the fiscal authority in country A does not adjust taxes in response to debt increases, which causes inflation. In this case, the fiscal authority in country A issues unfunded debt. The central bank accommodates this fiscal-led inflation. In this fiscal-led economy, monetary policy is passive, fiscal policy in country A is active, and fiscal policy in country B is passive.[4]

4. When parameters take the values $\phi \leq 1$, $\gamma > 1$, and $\gamma^* \leq 1$, monetary policy is passive, fiscal policy in country A is passive, and fiscal policy in country B is active.

3.6 Introducing partially unfunded debt

This section explains how we model the monetary-led and fiscal-led economy in the above section in one economy, allowing for debt to be *partially* unfunded. We follow the framework developed by BFM but extend it to a currency union setting: we keep track of unfunded government debt in country A, s^F_{bt}, which is a component of the total government debt, s_{bt}. The superscript F denotes that the variable is fiscal-led. To implement an economy in which country A holds unfunded debt, i.e. the fiscal-led economy, we decompose each variable in the monetary rule and fiscal rules as follows:

$$\text{Fiscal rule } A \qquad \frac{\tau_t}{\overline{\tau}} = \left(\frac{s_{b,t-1}}{s^F_{b,t-1}}\right)^{\gamma^M} \left(\frac{s^F_{b,t-1}}{\overline{s}_b}\right)^{\gamma^F} e^{\zeta^M_t + \zeta^F_t}, \tag{29}$$

$$\text{Fiscal rule } B \qquad \frac{\tau^*_t}{\overline{\tau}^*} = \left(\frac{s^*_{b,t-1}}{s^{F*}_{b,t-1}}\right)^{\gamma^{M*}} \left(\frac{s^{F*}_{b,t-1}}{\overline{s}^*_b}\right)^{\gamma^{F*}}, \tag{30}$$

$$\text{Monetary rule} \qquad \frac{R_t}{\overline{R}} = \left[\left(\frac{\Pi_t}{\Pi^F_t}\right)^{\phi^M} \left(\frac{\Pi^F_t}{\overline{\Pi}}\right)^{\phi^F}\right]^{\Theta} \left[\left(\frac{\Pi^*_t}{\Pi^{F*}_t}\right)^{\phi^{M*}} \left(\frac{\Pi^{F*}_t}{\overline{\Pi}^*}\right)^{\phi^{F*}}\right]^{1-\Theta}, \tag{31}$$

for which superscripts M and M^* denote monetary-led variables and parameters, and F and F^* the fiscal-led ones. In the fiscal-led part of the economy, country A's government is not committed to bring debt to a stable path: $\gamma^F \leq 1$. In contrast, country B's government stabilizes debt in the fiscal-led economy, s^{F*}_{bt}, because only country A holds unfunded debt in this economy: $\gamma^{F*} > 1$. The monetary authority accommodates fiscal-led inflation rates, Π^F_t and Π^{F*}_t, with $\phi^F \leq 1$ and $\phi^{F*} \leq 1$. The remaining part of the economy is monetary-led, so that both countries are on a debt stabilizing path, $\gamma^M > 1$ and $\gamma^{M*} > 1$, and the central bank reacts more than one-to-one to inflation $\phi^M > 1$ and $\phi^{M*} > 1$. ζ^M_t is monetary-led, hence the funded fiscal shock and ζ^F_t is the fiscal-led, unfunded fiscal shock in country A. Because of symmetry, we do not present the shocks in country B in the stylized model.

To characterize the determinacy of the model, we construct the monetary block and the fiscal block of the economy, as in the previous section. We linearize the above three equations to obtain:

$$\hat{\tau}_t = \gamma^M \left(\hat{s}_{b,t-1} - \hat{s}^F_{b,t-1}\right) + \gamma^F \hat{s}^F_{b,t-1} + \zeta^M_t + \zeta^F_t, \tag{32}$$

$$\hat{\tau}^*_t = \gamma^{M*} \left(\hat{s}^*_{b,t-1} - \hat{s}^{F*}_{b,t-1}\right) + \gamma^{F*} \hat{s}^{F*}_{b,t-1}, \tag{33}$$

$$\hat{i}_t = \Theta \left[\phi^M \left(\hat{\pi}_t - \hat{\pi}^F_t\right) + \phi^F \hat{\pi}^F_t\right] + (1-\Theta) \left[\phi^{M*} \left(\hat{\pi}^*_t - \hat{\pi}^{F*}_t\right) + \phi^{F*} \hat{\pi}^{F*}_t\right]. \tag{34}$$

Substituting the linear monetary rule that allows for inflation coming from unfunded debt, equation (34), into the Fisher equation (22), we obtain the monetary block of this

economy:

$$\begin{aligned}\mathbb{E}_t\hat{\pi}^{MU}_{t+1} = \phi^M\hat{\pi}^{MU}_t + \Theta\left(\phi^F - \phi^M\right)\hat{\pi}^F_t + (1-\Theta)\left(\phi^{F*} - \phi^M\right)\hat{\pi}^{F*}_t \\ -\Theta\psi_b\frac{\overline{b}_A + \overline{b}_B}{\overline{b}_A}\left(\hat{b}_{Bt} - \hat{b}_{At}\right) - (1-\Theta)\psi_b\frac{\overline{b}^*_A + \overline{b}^*_B}{\overline{b}^*_A}\left(\hat{b}^*_{Bt} - \hat{b}^*_{At}\right),\end{aligned} \tag{35}$$

where we use $\phi^{M*} = \phi^M$. Similarly to the closed-economy case, the parameter on the monetary rule, ϕ^M, determines the eigenvalue in the monetary block. For the fiscal block, we substitute the two fiscal rules allowing for unfunded debt, equations (32) and (33), into the government budget constraints (23) and (24):

$$\hat{s}_{bt} = \frac{1}{\beta}\begin{bmatrix}\left(1-(1-\beta)\gamma^M\right)\hat{s}_{b,t-1} + (1-\beta)\gamma^M\hat{s}^F_{b,t-1} \\ +(1-\beta)\gamma^F\hat{s}^F_{b,t-1} + \hat{i}_{t-1} - \hat{\pi}_t - (1-\beta)\left(\zeta^M_t + \zeta^F_t\right)\end{bmatrix}, \tag{36}$$

$$\hat{s}^*_{bt} = \frac{1}{\beta}\begin{bmatrix}\left(1-(1-\beta)\gamma^{M*}\right)\hat{s}^*_{b,t-1} + (1-\beta)\gamma^{M*}\hat{s}^{F*}_{b,t-1} \\ +(1-\beta)\gamma^{F*}\hat{s}^{F*}_{b,t-1} + \hat{i}_{t-1} - \hat{\pi}^*_t\end{bmatrix}. \tag{37}$$

Again, the debt stabilization parameters γ^M and γ^{M*} characterize the eigenvalues of the fiscal block.

To close the model, we need to pin down the dynamics of fiscal-led inflation $\hat{\pi}^F_t$ and $\hat{\pi}^{F*}_t$ and the fiscal-led government debt $\hat{s}^F_{bt}$ and $\hat{s}^{F*}_{bt}$. To do so, as in BFM we construct a shadow economy which keeps track of the fiscal-led variables. The monetary block of this shadow economy is:

$$\mathbb{E}_t\hat{\pi}^{MU,F}_{t+1} = \phi^F\hat{\pi}^{MU,F}_t - \Theta\psi_b\frac{\overline{b}^F_A + \overline{b}^F_B}{\overline{b}^F_A}\left(\hat{b}^F_{Bt} - \hat{b}^F_{At}\right) - (1-\Theta)\psi_b\frac{\overline{b}^{F*}_A + \overline{b}^{F*}_B}{\overline{b}^{F*}_A}\left(\hat{b}^{F*}_{Bt} - \hat{b}^{F*}_{At}\right). \tag{38}$$

obtained by aggregating the monetary block in each country:

$$\mathbb{E}_t\hat{\pi}^F_{t+1} = \phi^F\hat{\pi}^F_t - \psi_b\frac{\overline{b}^F_A + \overline{b}^F_B}{\overline{b}^F_A}\left(\hat{b}^F_{Bt} - \hat{b}^F_{At}\right), \tag{39}$$

$$\mathbb{E}_t\hat{\pi}^{F*}_{t+1} = \phi^{F*}\hat{\pi}^{F*}_t - \psi_b\frac{\overline{b}^{F*}_A + \overline{b}^{F*}_B}{\overline{b}^{F*}_A}\left(\hat{b}^{F*}_{Bt} - \hat{b}^{F*}_{At}\right), \tag{40}$$

and using $\phi^F = \phi^{F*}$. This parameter governs how the central bank reacts to fiscal-led inflation and determines the eigenvalue of the monetary block in the shadow economy. The fiscal block of the shadow economy is characterized by the government budget constraints combined with the fiscal rules:

$$\hat{s}^F_{bt} = \frac{1}{\beta}\left[\hat{s}^F_{b,t-1} + \hat{i}^F_{t-1} - \hat{\pi}^F_t - (1-\beta)\tau^F_t\right], \tag{41}$$

$$\hat{s}^{F*}_{bt} = \frac{1}{\beta}\left[\hat{s}^{F*}_{b,t-1} + \hat{i}^{F*}_{t-1} - \hat{\pi}^{F*}_t - (1-\beta)\tau^{F*}_t\right], \tag{42}$$

where

$$\hat{\tau}_t^F = \gamma^F \hat{s}_{b,t-1}^F + \zeta_t^F, \tag{43}$$

$$\hat{\tau}_t^{F*} = \gamma^{F*} \hat{s}_{b,t-1}^{F*} \tag{44}$$

and where $\hat{i}_{t-1}^F$ is the part of the nominal interest rate that responds to fiscal-led inflation. Following the monetary rule (34), this component can be written as $\hat{i}_t^F \equiv \Theta\phi^F\hat{\pi}_t^F + (1-\Theta)\phi^{F*}\hat{\pi}_t^{F*}$. The debt stabilization parameters, γ^F and γ^{F*} determine the eigenvalues in the fiscal block of the shadow economy.

The equations that characterize the currency union model with partially unfunded debt are: the monetary and fiscal block of the actual economy, (35)–(37) and the monetary and fiscal block of the fiscal-led, shadow economy, (38), (41), and (42). There are two non-predetermined variables, $\hat{\pi}_t^{MU}$ and $\hat{\pi}_t^{MU,F}$, and two eigenvalues outside the unit circle in equations (35) and (41) such that Blanchard-Kahn conditions are satisfied and there is a unique and stable equilibrium.

As in BFM, we exploit the linearity of the model to prove that the non-fiscal-led components of inflation and debt—so $\hat{\pi}_t^M = \hat{\pi}_t - \hat{\pi}_t^F$, $\hat{\pi}_t^{M*} = \hat{\pi}_t^* - \hat{\pi}_t^{F*}$, $\hat{s}_{bt}^M = \hat{s}_{bt} - \hat{s}_{bt}^F$, and $\hat{s}_{bt}^{M*} = \hat{s}_{bt}^* - \hat{s}_{bt}^{F*}$—are indeed the inflation and debt of a fully monetary-led economy. See Appendix C for details.

3.7 Calibration

The calibration of the stylized model is straightforward and follows the literature. The discount factor β is 0.99, the degree of trade openness α_I is 0.25 to allow for home bias according to Eurostat (2023), and the elasticity of substitution between goods produced in country A and B μ is 1. The countries are of equal size, which means that $\Theta = 0.5$. All shocks have a persistence of 0.5.

Debt adjustment cost ψ_b is 0.001. We calibrate the steady-state real market value of debt as share of output, $\bar{s}_b$, as 0.80, which is the average debt-to-GDP ratio of France and Germany for the period 2004–2024, the main countries and period we focus on in the empirical section (Eurostat Data, 2025). The steady-state ratio of domestic versus foreign bond holdings is $\delta_b = 0.44/0.56$, to match the data for France and Germany for the period 2004-2019 (Bruegel, 2025). The countries are symmetric such that the net foreign asset position is 0 in steady state.

We follow BFM for the parameters specific to the model with partially unfunded debt. The weight on the monetary-led part of inflation, $\phi^M = \phi^{M*} = 2$, whereas the central bank disregards the fiscal-led part of inflation, $\phi^F = \phi^{F*} = 0$. The fiscal governments stabilize the monetary-led part of government debt with $\gamma^M = \gamma^{M*} = 20$ and disregard the fiscal-led part of debt with $\gamma^F = \gamma^{F*} = 0$.

A summary of the calibration of the quantitative model is in Table 4, which includes parameters from the stylized model.

3.8 Results

The results of the stylized model provide intuition on how funded and unfunded fiscal shocks propagate through a currency union. Qualitatively, those results link to the local projections, showing that funded and unfunded fiscal expansions provide similar spillovers to French and German fiscal expansions, respectively. We perform the quantitative exercises, including model estimation and impulse response matching, with the quantitative model in the next section.

Figure 3a shows the impulse responses to a funded fiscal expansion in country A. The fiscal expansion is in the form of a decline in the primary surplus, or taxes. In a closed economy or in a currency union model when the net foreign asset (NFA) position is always 0, a funded fiscal expansion increases debt and has no effect on inflation. The debt increases because it is backed by future tax adjustments.[5] In the currency union model with bonds trade, the increase in domestic debt creates general equilibrium effects through the current account equation (19). The increase in government debt in country A raises country B households' holdings of bonds issued by country A. So, net savings in country A, as expressed by the current account or the NFA position, decreases, and consumption increases. The increased demand pushes inflation upward. On the other hand, the net savings in country B increases. Hence, demand slows down and consumption and inflation decreases. This spillover effect is qualitatively similar to the spillovers that a French fiscal shock induces in the empirical local projections: a fiscal expansion shock in France decreases inflation in other countries.

The central bank targets the average inflation rate across the two countries. Because inflation moves in opposite directions and the countries are of equal size, the nominal interest rate does not move. However, the real interest rate reveals that the monetary policy is expansionary for home and contractionary for country B on impact. Hence, sharing a central bank exacerbates the divergence between the two countries.

Figure 3b shows the impulse responses to an unfunded fiscal expansion in country A. In a closed economy or in a currency union model without bonds trade, an unfunded fiscal expansion adjusts through an inflation increase, decreasing the real value of government debt.[6] In the currency union, we see similarly that inflation in country

5. See BFM for more details on the closed economy. Figure 7a in Appendix A show the responses in a currency union model in which the net foreign asset position is always 0. This setup is isomorphic to imposing bonds adjustment cost only on the foreign bond.

6. See BFM for more details on the closed economy. Figure 7b in Appendix A show the responses in a currency union model in which the NFA position is always 0. In country A, inflation increases and

A increases more than in the funded case and that the real government debt decreases. Since the inflation is fiscal-led, the monetary authority does not react. Households in country B, who also consume goods produced in country A, increase their inflation expectations. Hence, we see an increase in inflation in country B too, as a spillover from the unfunded fiscal expansion in country A, and therefore also a decrease in government debt. These results are qualitatively similar to the spillover results from the local projections with Germany fiscal shocks: a fiscal expansion shock in Germany and Italy increases inflation domestically and in other countries.

Since the central bank does not respond to the increase in fiscal-led inflation, real interest rates in both countries decline. This result is consistent with the findings by BFM in the closed economy, in which the real interest rate responds negatively to unfunded fiscal shocks.

All in all, under a funded fiscal expansion in country A, we see that inflation rates diverge within the currency union, whereas under an unfunded fiscal expansion, the inflation rates move together. Qualitatively, the inflation spillover results from a funded fiscal expansion are similar to that of the French fiscal shock, whereas the results from an unfunded shock are similar to that of the German and Italian fiscal shock. In the next section, we build and estimate a quantitative model to perform impulse response matching. Our goal is to quantify the mix of funded and unfunded shocks in those countries and explaining the different inflation spillovers with those shocks.

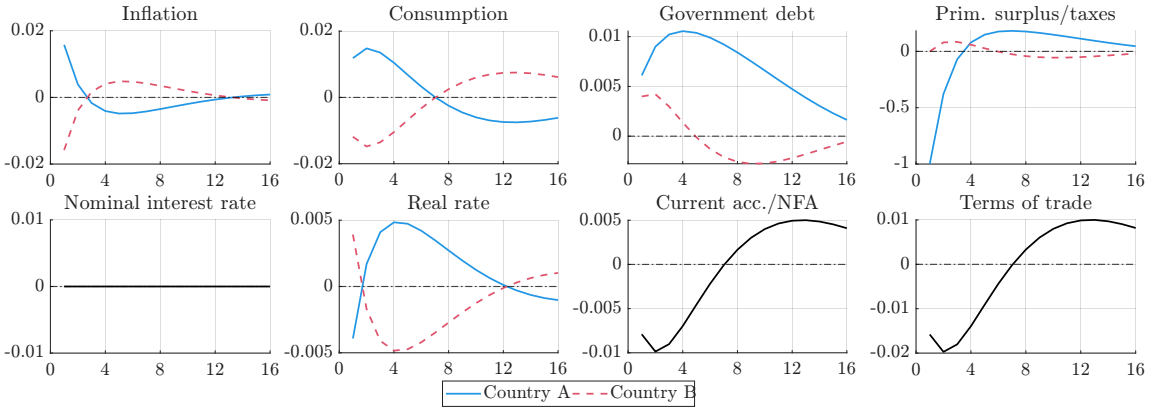
To examine the role of the common central bank, we are currently building a model with flexible nominal exchange rates. See Appendix E. This is still work in progress.

4 Quantitative model

In this section, we build a medium-scale currency union model with partially unfunded debt. The model will feature habit formation by households, production of tradable and non-tradable goods with nominal rigidities, capital formation, and long-term bonds. The aim of the model is to successfully capture the business cycles in the euro area with macroeconomic shocks, such that we can quantify the effect of funded and unfunded fiscal shocks on the economy. The model builds on the stylized model, so we only explain the changed equations.

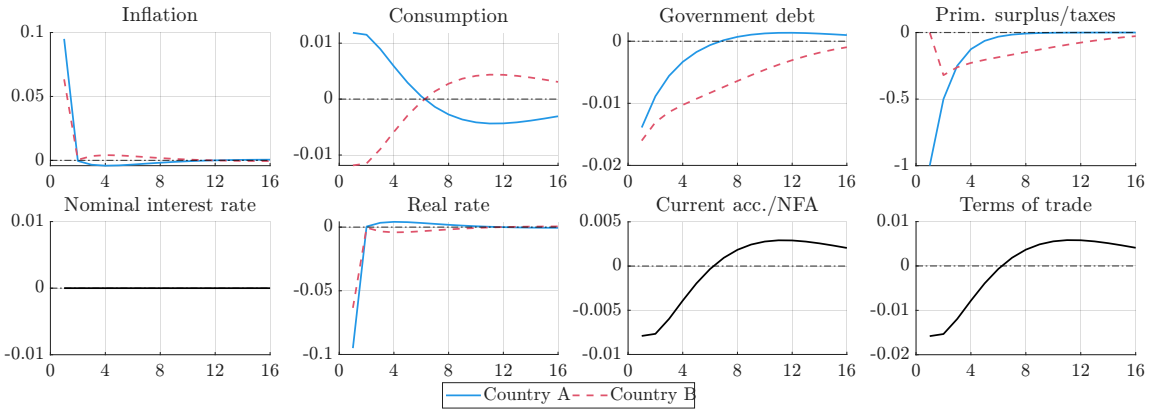
decreases real government debt. In country B, the inflation from country A spills over since the central bank response is absent. Hence, real government debt in country B decreases too.

Figure 3: Impulse responses to funded and unfunded fiscal expansions in country A



(a) Funded fiscal expansion in country A

Notes: Impulse responses to a 1% decline in primary surplus/taxes in country A. The fiscal expansion is funded, i.e. monetary-led, so a shock to ζ_t^M. The y-axis is in terms of percentage deviations from steady state. The x-axis is in quarters. Inflation and interest rates are annualized.



(b) Unfunded fiscal expansion in country A

Notes: Impulse responses to a 1% decline in primary surplus/taxes in country A. The fiscal expansion is unfunded, i.e. fiscal-led, so a shock to ζ_t^F. The y-axis is in terms of percentage deviations from steady state. The x-axis is in quarters. Inflation and interest rates are annualized.

4.1 Households

Households derive utility from consumption C_t and disutility from providing labor L_t. There are external habits in consumption, which means that utility from consumption relates to the previous value of aggregate consumption. The per-period utility function of the household is as follows:

$$\ln\left(C_t - hC_{t-1}\right) - \frac{1}{1+\chi} L_t^{1+\chi} \exp(\varepsilon_t^h), \tag{45}$$

where h is the habit parameter, χ the inverse-Frisch elasticity of labor supply, and ε_t^h is an IID labor supply shock. As in Ferroni, Fisher, and Melosi (2024), we use this shock to accommodate the COVID-19 pandemic period (2020Q1–2021Q2), during which lock-

down policies caused unprecedented and exogenous labor market disruptions. The shock variance is restricted to zero outside the pandemic period using heteroskedastic shock variances, so that ε_t^h does not affect the model's dynamics outside of the COVID quarters.

Households derive income from renting out capital K_t at rate R_{Kt} to tradable goods producers and exerting labor for wages W_t. The price of a unit of capital is Q_t and the depreciation rate is δ. Households can invest in capital through perfectly competitive capital producers as explained below.

Households can save and smooth consumption through one-period government bonds B_t which are in zero net supply, and through long-term government bonds from both country A, B_{At}, and country B, B_{Bt}, in nonzero net supply. The one-period bonds pay interest rate R_t which is set by the central bank. We follow Woodford (2001) and model long-term bonds as a perpetuity with geometrically declining coupons with average maturity of $(1-\beta\rho)^{-1}$ where $\rho \in [0,1]$ is a constant rate of decay. Households can purchase the long-term bonds from country A and B at price P_t^L and P_t^{L*} respectively. Arbitrage conditions determine the price of these bonds, together with country specific risk premium shocks ξ_t^{rp} and ξ_t^{rp*}:

$$R_t = \mathbb{E}_t R_{t+1}^L e^{\xi_t^{rp}} \equiv \mathbb{E}_t \left[\frac{1 + \rho P_{t+1}^L}{P_t^L} \right] e^{\xi_t^{rp}}, \tag{46}$$

$$R_t = \mathbb{E}_t R_{t+1}^{L*} e^{\xi_t^{rp*}} \equiv \mathbb{E}_t \left[\frac{1 + \rho P_{t+1}^{L*}}{P_t^{L*}} \right] e^{\xi_t^{rp*}}. \tag{47}$$

We can then write the per-period budget constraint of the household as:

$$\begin{aligned} P_t C_t + Q_t K_t + P_t^L B_{At} + P_t^{L*} B_{Bt} + B_t + P_t T_t = W_t L_t + \left[R_{Kt} \mu_{K,t} - a(\mu_{K,t}) + Q_t(1-\delta) \right] K_{t-1} \\ + R_{t-1} B_{t-1} + \left(1 + \rho P_t^L\right) B_{A,t-1} + \left(1 + \rho P_t^{L*}\right) B_{B,t-1} - \frac{\psi_b P_t \left(\overline{b}_A + \overline{b}_B\right)}{2} \left(\delta_b \frac{B_{Bt}}{B_{At}} - 1 \right)^2, \end{aligned} \tag{48}$$

where C_t is composed of non-tradable and tradable goods. All other notation is consistent with the stylized model presented in Section 3. The household maximizes its utility choosing consumption, labor, capital, and the three types of bonds, subject to the budget constraint. The optimization problem also gives rise to the labor supply equation:

$$C_t^{\text{eff}} L_t^{\chi} \exp(\varepsilon_t^h) = MRS_t \tag{49}$$

where $C_t^{\text{eff}} = C_t - hC_{t-1}$ is effective consumption and MRS_t is the marginal rate of substitution, which relates to wages as explained in the next subsection. Moreover, households also choose the amount of capital to rent out and the bonds to hold follow-

ing the Euler equations:

$$Q_t = \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \left[R_{K,t+1} \mu_{K,t+1} - a(\mu_{K,t+1}) + (1-\delta) Q_{t+1} \right], \tag{50}$$

$$1 = \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} R_t, \tag{51}$$

$$\lambda_t P_t^L = \beta \mathbb{E}_t \lambda_{t+1} \left(1 + \rho P_{t+1}^L\right) - \psi_b (\overline{b}_A + \overline{b}_B) \left(\delta_b \frac{b_{Bt}}{b_{At}} - 1\right) \delta_b \frac{b_{Bt}}{b_{At}^2}, \tag{52}$$

$$\lambda_t P_t^{L*} = \beta \mathbb{E}_t \lambda_{t+1} \left(1 + \rho P_{t+1}^{L*}\right) + \psi_b \left(\overline{b}_A + \overline{b}_B\right) \left(\delta_b \frac{b_{Bt}}{b_{At}} - 1\right) \delta_b \frac{1}{b_{At}}, \tag{53}$$

where $\lambda_t = \frac{1}{C_t^{\text{eff}}} - \beta h \mathbb{E}_t \frac{1}{C_{t+1}^{\text{eff}}}$ is the marginal utility of consumption. These Euler equations give rise to no-arbitrage conditions across the different asset holdings.

Households also choose the capital utilization rate $\mu_{K,t}$, which determines the intensity at which the capital stock is used in production. Following Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007), varying utilization incurs a resource cost $a(\mu_{K,t})$ per unit of capital, where $a(\cdot)$ satisfies $a(1) = 0$, $a'(1) = \overline{R}_K$, and $a''(1)/a'(1) = \psi_u/(1-\psi_u)$. The parameter $\psi_u \in (0,1)$ governs the curvature of the cost function: as $\psi_u \to 1$, utilization becomes prohibitively costly and effectively fixed at its steady-state value. The first-order condition for utilization equates the rental rate of capital to the marginal cost of utilization:

$$R_{K,t} = a'(\mu_{K,t}) = \alpha \frac{Y_{Tt}}{\mu_{K,t} K_{t-1}}. \tag{54}$$

C_t is a constant elasticity of substitution (CES) aggregate of tradable goods C_{Tt} and non-tradable goods C_{Nt}, and P_t is the price of the aggregate bundle (with Cobb-Douglas aggregation in the limit as $\mu_N \to 1$):

$$C_t = \left[(1-\alpha_N)^{1/\mu_N} (C_{Tt})^{(\mu_N - 1)/\mu_N} + \alpha_N^{1/\mu_N} (C_{Nt})^{(\mu_N-1)/\mu_N} \right]^{\mu_N/(\mu_N-1)}, \tag{55}$$

$$P_t = \left[(1-\alpha_N) P_{Tt}^{1-\mu_N} + \alpha_N P_{Nt}^{1-\mu_N} \right]^{1/(1-\mu_N)}, \tag{56}$$

where C_{Tt} and C_{Nt} are consumption of tradable and non-tradable goods, respectively, and P_{Tt} and P_{Nt} their prices. α_N is the share of non-tradable goods in the consumption basket and μ_N the elasticity of substitution between tradable and non-tradable goods. The bundle of tradable goods C_{Tt} is equivalent to the bundle of consumption goods in the stylized model. Similarly, the price of the tradable goods P_{Tt} is equivalent to the price of the consumption good in the stylized model. The cost-minimization problem

provides the following relative demand between tradable and non-tradable goods:[7]

$$\frac{C_{Tt}}{C_{Nt}} = \frac{1-\alpha_N}{\alpha_N}\left(\frac{P_{Tt}}{P_{Nt}}\right)^{-\mu_N}. \tag{57}$$

4.2 Wage setting

We introduce differentiated labor and wage rigidities as in Christiano, Eichenbaum, and Evans (2005). We assume that each household supplies a unique labor type $L_t(l)$ at nominal wage $W_t(l)$, where $l \in [0,1]$. A labor union then transforms the differentiated labor outputs into aggregate labor L_t using a CES aggregator:

$$L_t = \left(\int_0^1 L_t(l)^{\frac{\epsilon_w - 1}{\epsilon_w}} dl\right)^{\frac{\epsilon_w}{\epsilon_w - 1}}, \tag{58}$$

where ϵ_w is the elasticity of substitution across labor types. The labor union minimizes the cost $\int_0^1 W_t(l)L_t(l)dl$ subject to the aggregation constraint, which yields the labor demand for each type and the aggregate wage index:

$$L_t(l) = \left(\frac{W_t(l)}{W_t}\right)^{-\epsilon_w} L_t, \tag{59}$$

$$W_t = \left(\int_0^1 W_t(l)^{1-\epsilon_w} dl\right)^{\frac{1}{1-\epsilon_w}}. \tag{60}$$

With probability $(1-\theta_w)$, each labor type can reset its wage each period. When the labor type is not re-optimizing its wage, the wage evolves via the following indexation:

$$W_t(l) = W_{t-1}(l)\Pi_{t-1}^{\chi_w}, \tag{61}$$

where Π_t is gross price inflation and $\chi_w \in [0,1]$ the degree of wage indexation. When the labor type is re-optimizing its wage, they solve the following maximization problem:

$$\max_{W_t^{\#}(l)} \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\theta_w)^s \left[\ln C_{t+s} - \frac{1}{1+\chi} L_{t+s}^{1+\chi}\right], \tag{62}$$

subject to the labor type demand (60) and the wage indexation (61), and where $W_t^{\#}(l)$ is the optimal wage for labor type l and $(1-\theta_w)$ is the probability of being able to re-optimize its wage. The first order condition to this problem, after applying symmetry

7. Consequently, the tradable consumption bundle C_{Tt} retains the CES structure from the stylized model, such that $C_{Tt} = \left[(1-\alpha_I)^{1/\mu}(C_{At})^{(\mu-1)/\mu} + \alpha_I^{1/\mu}(C_{Bt})^{(\mu-1)/\mu}\right]^{\mu/(\mu-1)}$, where C_{At} and C_{Bt} denote consumption of goods produced in countries A and B, respectively. The associated demand equations are $C_{At} = (1-\alpha_I)\left(\frac{P_{At}}{P_{Tt}}\right)^{-\mu} C_{Tt}$ and $C_{Bt} = \alpha_I\left(\frac{P_{Bt}}{P_{Tt}}\right)^{-\mu} C_{Tt}$, with analogous expressions for country B households.

condition $W_t(l) = W_t$ and log-linearizing, gives us the standard wage Phillips Curve:

$$\hat{\pi}_t^w = \beta \mathbb{E}_t \hat{\pi}_{t+1}^w + \chi_w \hat{\pi}_{t-1} + \kappa_w \left(\widehat{mrs}_t - \hat{w}_t^{real} \right) + u_t^w, \tag{63}$$

where $\kappa_w \equiv \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w} \frac{1}{1+\chi}$ and u_t^w is a wage markup shock.

4.3 Firms

4.3.1 Capital producers

Capital producers transform final goods into investment subject to adjustment costs as in Christiano, Eichenbaum, and Evans (2005). The optimization problem for the capital producers is as follows:

$$\max_{I_t} \mathbb{E}_t \sum_{t=0}^{\infty} \Lambda_{t,t+1} \left[Q_t - 1 - \frac{\psi_I}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right] I_t, \tag{64}$$

where $\Lambda_{t,t+1} = \beta \frac{C_t}{C_{t+1}}$ is the stochastic discount factor and ψ_I the investment adjustment cost. The first order condition of the capital producers determines the price of capital:

$$Q_t = 1 + \frac{\psi_I}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 + \psi_I \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} - \psi_I \mathbb{E}_t \Lambda_{t+1} \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2. \tag{65}$$

Investment turns into capital K_t, which evolves according to the following law of motion:

$$K_t = (1 - \delta) K_{t-1} + I_t, \tag{66}$$

and we assume that capital is only used in the tradable sector.

4.3.2 Final goods producers

Final goods producers aggregate differentiated intermediate goods from monopolistically competitive firms in both the tradable and non-tradable goods sector, using a CES aggregator:

$$Y_{Tt} = \left(\int_0^1 Y_{Tt}(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{67}$$

$$Y_{Nt} = \left(\int_0^1 Y_{Nt}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{68}$$

where $i \in [0,1]$ represents the continuum of firms in the tradable goods sector, producing goods $Y_{Tt}(i)$, and $j \in [0,1]$ the continuum of firms in the non-tradable sector, producing goods $Y_{Nt}(i)$. ϵ is the elasticity of substitution between the differentiated goods. The cost-minimization problem of the final goods producers yields the follow-

ing demand schedules for each differentiated good:

$$Y_{Tt}(i) = \left(\frac{P_{Tt}(i)}{P_{Tt}}\right)^{-\epsilon} Y_{Tt}, \tag{69}$$

$$Y_{Nt}(j) = \left(\frac{P_{Nt}(j)}{P_{Nt}}\right)^{-\epsilon} Y_{Nt}, \tag{70}$$

where $P_{Tt}(i)$ and $P_{Nt}(j)$ are the price of tradable and non-tradable from firm i and j respectively.

4.3.3 Tradable goods producers

There is a continuum of tradable goods firms $i \in [0,1]$ who produce differentiated intermediate inputs $Y_{Tt}(i)$ for the final goods producer using the following production technology:

$$Y_{Tt}(i) = A_{Tt}(\mu_{K,t} K_{t-1}(i))^{\alpha} L_{Tt}(i)^{1-\alpha}, \tag{71}$$

where α is the output elasticity of capital. A_{Tt} is the labor augmenting technology and follows an AR(1) process. The firms are under monopolistic competition and are subject to Calvo (1983) price rigidities. Hence, the optimization problem of the firm is:

$$\max_{P_{At}(i), L_{Tt}(i), K_t(i)} \mathbb{E}_0 \sum_{s=0}^{\infty} \theta_T^s \Lambda_{t,t+1} \left[\left(\frac{P_{At}(i)}{P_{A,t+s}} - MC_{t+s}(i) \right) Y_{T,t+s}(i) \right], \tag{72}$$

subject to the production function and the demand for differentiated tradable goods (69). $(1-\theta_T)$ is the fraction of firms that can set their price in each period and $MC_t(i)$ is the real marginal cost for firm i. After applying the symmetry conditions $P_{At}(i) = P_{At}$ and $MC_t(i) = MC_t$ and log-linearizing, the optimization problem gives rise to the standard New Keynesian Phillips Curve (NKPC):

$$\hat{\pi}_{Tt} = \kappa_T \widehat{mc}_{Tt} + \beta \mathbb{E}_t \hat{\pi}_{T,t+1} + u_t^T \tag{73}$$

where $\kappa_T \equiv \frac{(1-\theta_T\beta)(1-\theta_T)}{\theta_T}$, $\widehat{mc}_{Tt} = \hat{w}_t^{real} + \hat{l}_{Tt} - \hat{y}_{Tt}$ is the log-linear real marginal cost of the firm, and u_t^T is a tradable goods markup shock.

4.3.4 Non-tradable goods producers

There is a continuum of non-tradable firms $j \in [0,1]$ producing differentiated intermediate inputs $Y_{Nt}(j)$ for the final goods producer. In contrast to the tradable goods producers, they only use labor as a production input:

$$Y_{Nt}(j) = A_{Nt} L_{Nt}(j)^{1-\alpha}. \tag{74}$$

A_{Nt} is the labor augmenting technology and follows an AR(1) process. Similarly to the firms producing tradable goods, the non-tradable goods producers are under monopolistic competition subject to Calvo (1983) price rigidities. The demand equation

for labor and the NKPC are analogous to those of the tradable goods firms.

4.3.5 Fiscal authority

Assuming a zero net supply of one-period government bonds, the nominal budget constraint of the government is as follows:

$$P_t^L B_{At} + P_t T_t = \left(1 + \rho P_t^L\right) B_{A,t-1}, \tag{75}$$

where we redefine $s_{bt} = \frac{P_t^L B_{At}}{P_t Y_t}$ as the debt-to-GDP ratio. The fiscal rule is modified so that the primary surplus explicitly adjusts for government spending, G_t,

$$\frac{\tau_t}{\overline{\tau}} = \left(\frac{s_{b,t-1}}{\overline{s}_b}\right)^{\gamma} \left(\frac{G_t}{\overline{G}}\right)^{-\overline{G}/(\overline{Y}\,\overline{\tau})} e^{\zeta_t/\overline{\tau}}, \tag{76}$$

where G_t is real government spending and follows an AR(1) process in logs, and $\bar{\tau} = \frac{1-\beta}{\beta}\bar{s_b}$ is the steady-state primary surplus-to-output ratio. An increase in government spending reduces the primary surplus, with the elasticity $\bar{G}/(\bar{Y}\bar{\tau})$ reflecting that spending is large relative to the surplus. The fiscal shock ζ_t is scaled by $1/\bar{\tau}$ so that it is measured in units of GDP ratio.

4.4 Equilibrium

The monetary block of the economy remains the same as the stylized model. The equilibrium of this economy is characterized by a sequence of prices

$$\left\{P_{At}, P_{Bt}, P_{Nt}, P_{Nt}^*, W_t, W_t^*, R_t, R_{Kt}, R_{Kt}^*\right\},$$

and allocations

$$\left\{Y_{Tt}, Y_{Tt}^*, Y_{Nt}, Y_{Nt}^*, K_t, K_t^*, \mu_{K,t}, \mu_{K,t}^*, I_t, I_t^*, G_t, G_t^*, C_{Tt}, C_{Tt}^*, C_{Nt}, C_{Nt}^*, L_{Tt}, L_{Tt}^*, L_{Nt}, L_{Nt}^*, B_{At}, B_{Bt}, B_{At}^*, B\right.$$

such that the goods markets are cleared for both country A and B goods, the labor market is cleared, and the bonds market is cleared for bonds supplied by fiscal authorities in both countries. The goods market clearing conditions for country A and B are the following:

$$Y_{Tt} = C_{At} + C_{At}^* + I_t\left[1 + \frac{\psi_I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right] + G_t + a(\mu_{K,t})K_{t-1}, \tag{77}$$

$$Y_{Tt}^* = C_{Bt} + C_{Bt}^* + I_t^*\left[1 + \frac{\psi_I}{2}\left(\frac{I_t^*}{I_{t-1}^*} - 1\right)^2\right] + G_t^* + a(\mu_{K,t}^*)K_{t-1}^*, \tag{78}$$

$$Y_{Nt} = C_{Nt}, \tag{79}$$

$$Y_{Nt}^* = C_{Nt}^*. \tag{80}$$

Labor market clearing conditions for country A and B are the following:

$$L_t = L_{Tt} + L_{Nt}, \tag{81}$$

$$L_t^* = L_{Tt}^* + L_{Nt}^*. \tag{82}$$

We express the dynamics of net foreign assets in country A with the current account definition:

$$\begin{aligned} CA_t \equiv & P_{At} Y_{Tt} - P_{Tt} C_{Tt} - P_{At} I_t \left[1 + \frac{\psi_I}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right] - P_{At} a(\mu_{K,t}) K_{t-1} \\ = & B_{Bt} - B_{At}^* - R_{t-1} \left(B_{B,t-1} - B_{A,t-1}^* \right). \end{aligned} \tag{83}$$

In Appendix D we write down the additional equations associated with the quantitative model and derive the steady state.

4.5 Estimation

We discipline the structural parameters through Bayesian estimation of the two-country model. Following the literature, we obtain the posterior distribution by combining our priors with the model's likelihood function via the Kalman filter. The posterior results of the estimation with Germany as country A and the rest of the euro area as country B are reported in Table 5. We estimate the model separately for each country pair, with country A being either France, Germany, or Italy, and country B being the GDP-weighted aggregate of the remaining euro area countries. This allows us to recover country-specific structural parameters and fiscal shock processes, enabling us to identify the mix of funded versus unfunded fiscal shocks across countries.

Data. We use the publicly available large datasets for the euro area, collected by Barigozzi, Lissona, and Tonni (2025), for 2000Q1–2025Q3. For each country pair, we construct 19 observables: nine per region (real consumption, real government spending, real investment, hours, real GDP, and gross debt-to-GDP in growth rates, wage and CPI inflation, and the long-term interest rate) plus one shared short-term interest rate. The shocks we introduce into our model for the estimation are: technology (tradable and non-tradable), monetary policy, funded and unfunded taxes, risk premium, preference, price markup (tradable and non-tradable), wage markup, investment, and government spending. We add measurement errors for investments, consumption, hours, wages, output, debt, government spending, and inflation in each region.

Priors. Table 4 shows the parameters that are fixed in the estimation. Those parameters either have a direct counterpart in the data or a standard value in the literature.

The discount factor β is 0.99, consistent with a steady-state annualized real rate of 4%. The degree of trade openness, α_I, is 0.25 to match the share of imported goods in the euro area Eurostat (2023). The share of non-tradable goods, α_N, is 0.6 and the elasticity of substitution between tradable and non-tradables, μ_N, is 1 according to literature. The relative country size, Θ, is set to the GDP share of country A in the estimation sample. Capital depreciates at rate $\delta = 0.025$ following Christiano, Eichenbaum, and Evans (2005). As is standard, the output elasticity of capital, α, is 0.33.

The rate of decay of the long-term bond ρ^L is 0.975 to match European Monetary Union data, which implies that the average maturity of the long-term bond portfolio is 10 years. The debt adjustment costs ψ are 0.0074 following Schmitt-Grohé and Uribe (2003). We take the steady-state real market value of debt, $\overline{s}_b = 0.80$, and the steady-state ratio of domestic and foreign bonds, $\delta_b = 0.44/0.56$, from Eurostat Data (2025) and Bruegel (2025) respectively.

For the parameters in the model with partially unfunded debt, we adopt the parameters from BFM: $\phi^M = \phi^{M*} = 2$ and $\phi^F = \phi^{F*} = 0$ to ensure the central bank reacts more than one-to-one to monetary-led inflation and none to fiscal-led inflation. $\gamma^F = 0$ allows for unfunded debt in country A, whereas all other components of debt are funded: $\gamma^M = \gamma^{M*} = \gamma^{F*} = 20$.

Table 5 shows the prior distributions for the parameters we estimate. We estimate region-specific structural parameters for both countries A and B, including the Frisch labor elasticity χ, habit persistence h, Calvo price and wage stickiness (θ_T, θ_N, θ_w), wage indexation χ_w, and the trade elasticity μ. Investment adjustment cost ψ_I and capital utilization cost ψ_u are estimated as common parameters. We also estimate the fiscal rule debt-stabilization coefficients γ^M, γ^{M*}, γ^{F*}, and γ^{FB}, and the Taylor rule coefficient ϕ^M. The prior mean of the Frisch labor elasticity is 2 (Normal), Calvo probabilities are 0.75 (Beta), wage indexation is 0.25 (Beta), and investment adjustment cost is 2 (Gamma). The shock persistence parameters, ρ_X, and standard deviation parameters σ_X assume standard prior distributions.

Posteriors. Table 5 presents the posterior distributions of each parameter for the estimation with Germany as country A and the rest of the euro area as country B. In Table 19 and 20 in Appendix A we show the posterior distributions from an estimation with France and Italy as country A. We observe that the estimated values are all roughly in line with the standard literature.

Figure 4: Estimation results: fixed parameters

Parameter	Description	Value	Source
Households			
β	Discount factor	0.99	literature
α_I	Degree of trade openness	0.25	Eurostat (2023)
μ	Elasticity of subst. goods A vs. B	1	literature
α_N	Share of non-tradable goods	0.6	literature
μ_N	Elasticity of subt. trad. vs. non-trad. goods	1	literature
Θ	Size country A	0.5	–
Firms			
δ	Capital depreciation rate	0.025	Christiano, Eichenbaum, and Evans (2005)
α	Output elasticity of capital	0.33	literature
Monetary			
ϕ^M, ϕ^{M*}	Taylor-rule weight on monetary-led infl.	2	BFM
ϕ^F, ϕ^{F*}	Taylor-rule weight on fiscal-led infl.	0	BFM
Fiscal			
ρ^L	Rate of decay long-term bond	0.975	EMU Criterion
ψ	Debt adjustment cost	0.0074	Schmitt-Grohé and Uribe (2003)
$\bar{s}_b$	Steady-state real market value debt	0.80	Eurostat Data (2025)
δ_b	Steady-state ratio dom. vs. foreign bonds	0.44/0.56	Bruegel (2025)
$\gamma^M, \gamma^{M*}, \gamma^{F*}$	Fiscal rule weight on monetary-led debt	20	BFM
γ^F	Fiscal rule weight on fiscal-led debt	0	BFM

4.6 Results

The simulation results of the quantitative model are in line with the stylized model, but with more persistence as expected. The same set of impulse responses as for the stylized model are in Figure 17a and 17b in Appendix A, where we confirm that under a funded fiscal shock, inflation diverges within the union and under an unfunded fiscal shock inflation rates in both countries increase. Moreover, funded and unfunded government spending shocks (as % of GDP), to match the shock used in local projections, also generate similar propagations as the baseline funded and unfunded fiscal shocks through lump-sum taxes. The responses to funded and unfunded government spending shocks are in Figure 18a and 18b.

We perform Bayesian estimations of the quantitative currency union model with France, Germany, and Italy as country A to investigate the share of funded and unfunded debt in each country historically. Figure 6 presents the historical shock decomposition of debt-to-GDP growth for Germany, France, and Italy. Those are results from three separate estimations of the quantitative currency union model, in which we treat each country as country A and the rest of the euro area as country B. The decomposition results for the rest of the euro area are in Figure 21 in Appendix A. For each shock decomposition, we focus on how domestic monetary-led and fiscal-led fiscal shocks contribute to the fluctuations in the debt-to-GDP ratio, and group all other shocks into one bar.

The historical shock decompositions of the debt-to-GDP ratio growth in Figure 6 show that the majority of fiscal shocks are monetary-led, so funded fiscal shocks for all three countries, though with notable differences in the details. In Germany and Italy,

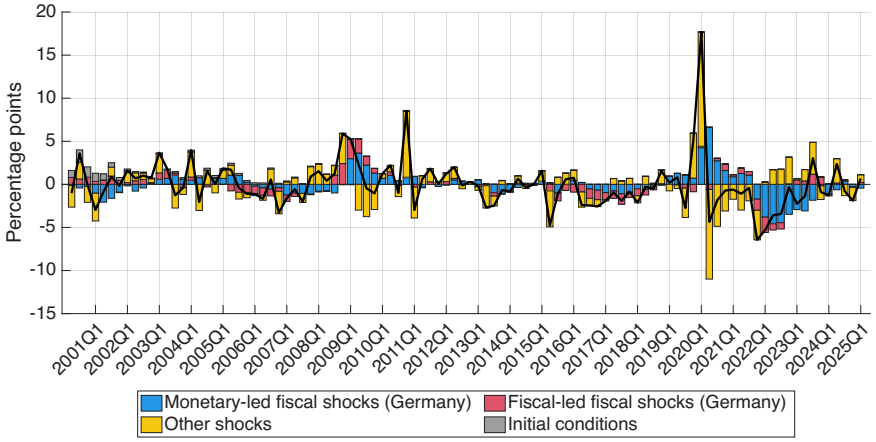
Figure 5: Estimation results: prior and posteriors of the two-country Bayesian estimation (Germany vs. rest of EA)

		Prior			Posterior		
Param.	Description	Type	Mean	Std.	Mode	Median	90% HPD CI
Region A structural							
χ	Inverse Frisch elasticity of labor supply	N	2.00	0.50	3.0670	2.6697	[2.2172, 3.0642]
χ_w	Wage indexation	B	0.25	0.10	0.1035	0.0841	[0.0328, 0.1462]
h	Habit persistence	B	0.70	0.10	0.6298	0.6298	[0.5824, 0.6741]
μ	Elasticity of substitution across tradable goods	N	1.00	0.25	1.2668	1.1760	[1.0001, 1.3622]
θ_T	Calvo probability tradable prices	B	0.75	0.10	0.1218	0.1330	[0.0939, 0.1769]
θ_N	Calvo probability non-tradable prices	B	0.75	0.10	0.4434	0.4265	[0.3775, 0.4901]
θ_w	Calvo probability wages	B	0.75	0.10	0.6183	0.6005	[0.5715, 0.6323]
Region B structural							
χ^*	Inverse Frisch elasticity B	N	2.00	0.50	0.8978	0.8749	[0.5771, 1.1695]
χ^*_w	Wage indexation B	B	0.25	0.10	0.0195	0.0253	[0.0079, 0.0460]
h^*	Habit persistence B	B	0.70	0.10	0.6563	0.6403	[0.5983, 0.6946]
μ^*	Trade elasticity B	N	1.00	0.25	1.5162	1.3896	[1.1709, 1.8266]
θ^*_T	Calvo probability tradable prices B	B	0.75	0.10	0.3368	0.3282	[0.2808, 0.3789]
θ^*_N	Calvo probability non-tradable prices B	B	0.75	0.10	0.4815	0.4637	[0.4132, 0.5290]
θ^*_w	Calvo probability wages B	B	0.75	0.10	0.6964	0.6847	[0.6520, 0.7143]
Common structural							
ψ_I	Investment adjustment cost	G	2.00	0.50	1.8709	2.1100	[1.6886, 2.5075]
ψ_u	Capital utilization cost	B	0.50	0.15	0.0717	0.0697	[0.0479, 0.0919]
Policy							
ϕ^{ML}	Taylor rule inflation coefficient (ML)	N	2.00	0.50	1.0077	1.0093	[1.0054, 1.0136]
γ_A^{ML}	Fiscal rule debt coefficient A (ML, passive=20)	N	20.0	5.00	20.7589	19.9396	[16.2052, 23.7746]
$\gamma_B^{*,ML}$	Fiscal rule debt coefficient B (ML, passive=20)	N	20.0	5.00	17.5548	16.7410	[13.0093, 20.7583]
$\gamma_B^{*,FLA}$	Fiscal rule debt coefficient B (FLA, passive=20)	N	20.0	5.00	15.0952	16.9337	[12.6246, 21.4976]
γ_A^{FLB}	Fiscal rule debt coefficient A (FLB, passive=20)	N	20.0	5.00	31.8133	31.5067	[27.8389, 34.9050]
Shock persistence							
ρ_a	Technology shock persistence	B	0.80	0.10	0.7311	0.7314	[0.7061, 0.7567]
ρ_{rp}	Risk premium shock persistence	B	0.80	0.10	0.9180	0.9171	[0.9001, 0.9326]
ρ_d	Preference shock persistence	B	0.50	0.15	0.0315	0.0388	[0.0140, 0.0683]
ρ_u	Tradable markup shock persistence	B	0.80	0.10	0.9542	0.9573	[0.9260, 0.9852]
ρ_{uN}	NT markup shock persistence	B	0.80	0.10	0.7629	0.7646	[0.7358, 0.7919]
ρ_w	Wage markup shock persistence	B	0.80	0.10	0.5127	0.5120	[0.3872, 0.6173]
ρ_k	MEI shock persistence	B	0.50	0.15	0.1556	0.1238	[0.0641, 0.1893]
ρ_g	Government spending shock persistence	B	0.80	0.10	0.9626	0.9594	[0.9510, 0.9672]
$\rho_{\zeta,ML}$	Funded tax shock persistence A (ML)	B	0.50	0.15	0.7077	0.6887	[0.6091, 0.7707]
$\rho_{\zeta^*,ML}$	Funded tax shock persistence B (ML)	B	0.50	0.15	0.6314	0.6123	[0.5358, 0.6907]
$\rho_{\zeta,FL}$	Unfunded tax shock persistence A (FLA)	B	0.50	0.15	0.2688	0.2210	[0.1342, 0.2974]
$\rho_{\zeta^*,FL}$	Unfunded tax shock persistence B (FLB)	B	0.50	0.15	0.1037	0.1054	[0.0500, 0.1660]
Shock volatilities							
σ_a	St.dev. technology shock	IG	0.50	inf	0.9946	1.0199	[0.8505, 1.2064]
σ_{aN}	St.dev. NT technology shock	IG	0.50	inf	2.0008	2.0311	[1.8477, 2.2181]
σ_{rp}	St.dev. risk premium shock	IG	0.25	inf	0.0302	0.0314	[0.0264, 0.0366]
σ_d	St.dev. preference shock	IG	0.50	inf	6.8362	6.6712	[5.7146, 7.6819]
σ_u	St.dev. tradable markup shock	IG	0.50	inf	3.0445	3.0165	[2.2071, 3.9353]
σ_{uN}	St.dev. NT markup shock	IG	0.50	inf	3.3054	3.4619	[2.7215, 4.0434]
σ_w	St.dev. wage markup shock	IG	0.50	inf	0.3042	0.3352	[0.2595, 0.4127]
σ_k	St.dev. MEI shock	IG	0.50	inf	7.4305	8.4406	[6.5852, 10.6217]
σ_ζ	St.dev. tax shock (all ZETA processes)	IG	0.50	inf	1.4871	1.4822	[1.3728, 1.5971]
σ_g	St.dev. government spending shock (all G processes)	IG	0.50	inf	0.4831	0.4865	[0.4495, 0.5260]
σ_h	St.dev. COVID labor supply shock	G	5.00	2.5	11.7270	11.7296	[9.5560, 13.8181]

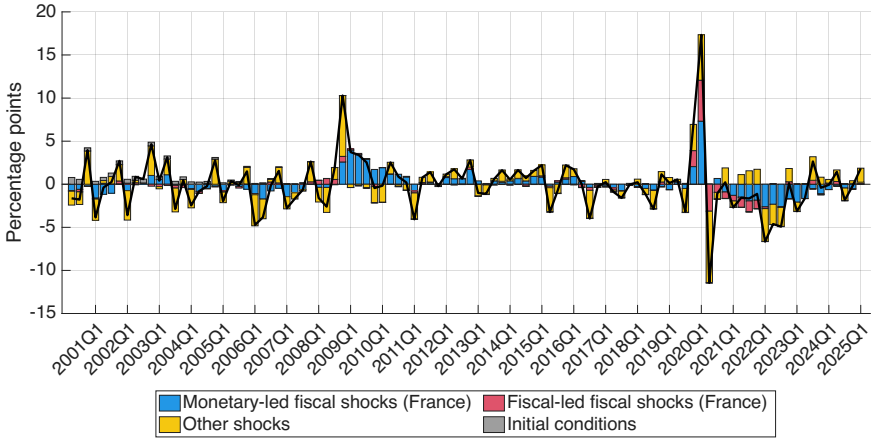
Notes: Results of the estimation of the structural parameters. N = Normal, B = Beta, G = Gamma, IG = Inverse-Gamma, AR coeff. = autoregressive coefficient, St.dev. = standard deviation, HPD = highest posterior density.

the share of fiscal-led, unfunded fiscal shocks is roughly constant over the years. However, in France, unfunded shocks were mostly absent before the COVID-19 pandemic. Averaging over the entire sample period, 33% of the fiscal shocks contributing to the change in debt-to-GDP growth in Germany was unfunded. In Italy, the share was 26% and in France 24%. These results confirm the results from our empirical motivation that Germany and Italy's fiscal shocks seem more unfunded than those of France.

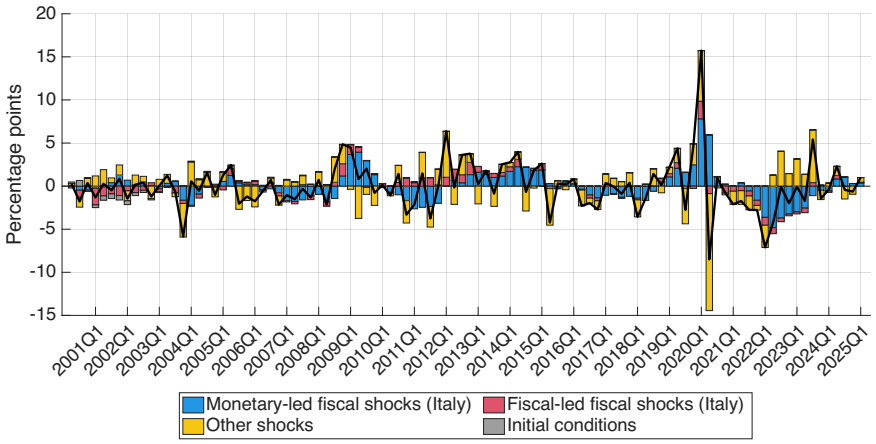
Figure 6: Shock decomposition of debt/GDP growth in Germany, France, and Italy



(a) Debt/GDP growth in Germany. Share monetary-led: 0.67.



(b) Debt/GDP growth in France. Share monetary-led: 0.76.



(c) Debt/GDP growth in Italy. Share monetary-led: 0.74.

Notes: Historical shock decomposition of the debt-to-GDP ratio growth in Germany, France, and Italy in percentage point deviations from the sample mean. We estimate the quantitative two-country model three times, with each of those countries as country A and the rest of the euro area as country B. Monetary-led fiscal shocks are ζ_t^{ML} and g_t^{ML}, and fiscal-led fiscal shocks are ζ_t^{FL} and g_t^{FL}. The x-axis is in quarters.

Appendix A presents some more results of the estimations. Figure 22, 23, and 24 show the historical shock decompositions for the inflation rates. Our work is still work in progress and we are working on constructing a measure for real government transfers to obtain cleaner identification of the funded and unfunded fiscal shocks.

5 Conclusion

This paper studies how funded and unfunded fiscal expansions propagate in a currency union with one central bank and multiple fiscal authorities. We combine evidence from local projections for euro area countries with a two-country New Keynesian model that allows for partially unfunded debt. The central question is whether differences in inflation spillovers across source countries of fiscal shocks can be interpreted through the lens of fiscal backing.

Our empirical analysis documents substantial heterogeneity in spillovers. Fiscal expansions in Germany and Italy are associated with inflationary effects domestically and in other countries, while fiscal expansions in France do not generate the same inflationary spillovers and are accompanied by a more contractionary monetary policy response. These patterns suggest that the central bank accommodates fiscal inflation originating in Germany and Italy, implying fiscal expansions in those countries are less funded, or backed by future tax adjustments, than fiscal expansions in France.

The stylized model provides a mechanism consistent with these facts. Under a funded fiscal expansion, inflation rates diverge across member countries because the common policy rate is too contractionary for some countries and too expansionary for others. Under an unfunded fiscal expansion, inflationary pressures transmit across the union because monetary policy accommodates fiscal inflation. The qualitative mapping between model predictions and local-projection evidence motivates interpreting German and Italian shocks as more unfunded-like and French shocks as more funded-like in the sample period.

The quantitative model extends the mechanism to a richer environment with habit formation, nominal rigidities, non-tradables, capital accumulation, and long-term bonds. Preliminary simulations are consistent with the stylized model and produce more persistent responses. We estimate the full currency-union model across alternative country groupings to quantify the historical mix of funded and unfunded fiscal shocks and find that Germany and Italy have a higher share of unfunded fiscal shocks than France. Our estimation is still a work in progress. We are currently working on constructing a measure for real government transfers to improve the identification of the funded and unfunded shocks.

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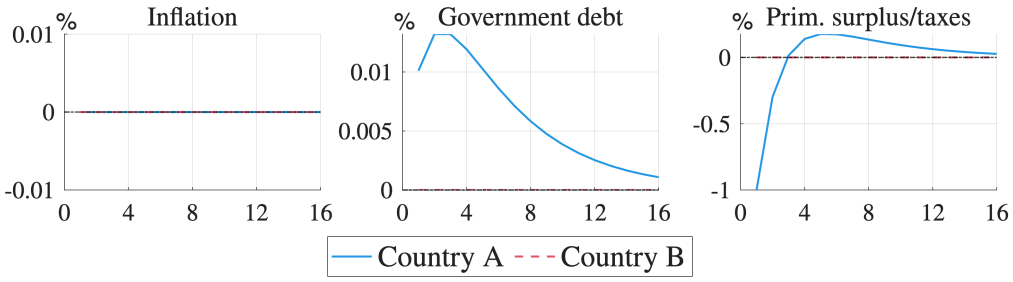
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Appendix

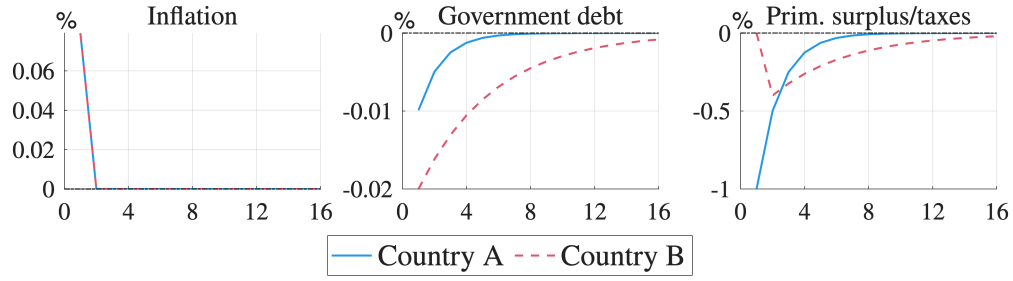
A Additional figures

Figure 7: Impulse responses to funded and unfunded fiscal expansions in country A when $NFA = 0$



(a) Funded fiscal expansion in country A

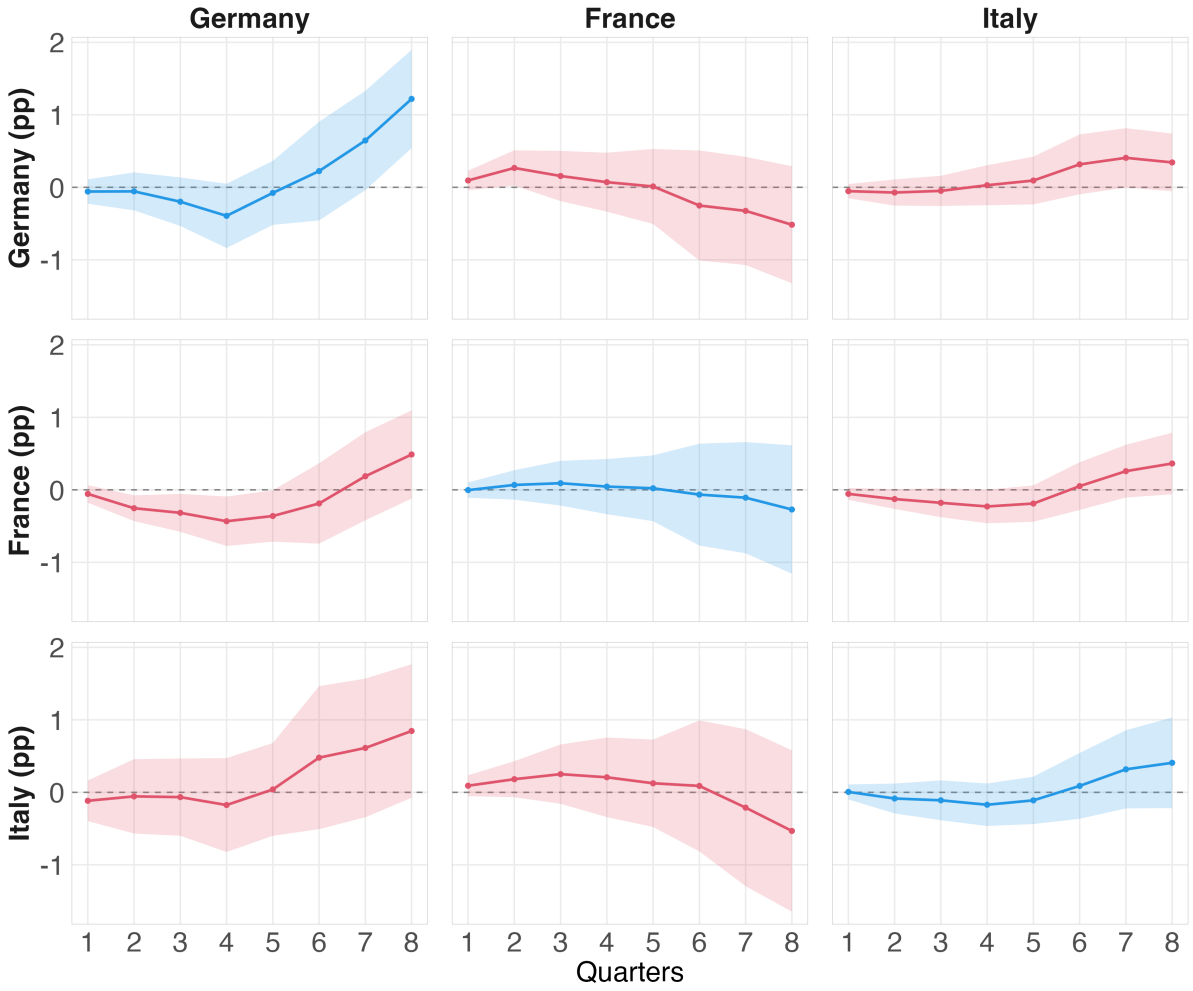
Notes: Impulse responses to a 1% decline in primary surplus/taxes in country A. The fiscal expansion is funded, i.e. monetary-led, so a shock to ζ_t^M. The y-axis is in terms of percentage deviations from steady state. The x-axis is in quarters. Inflation and interest rates are annualized.



(b) Unfunded fiscal expansion in country A

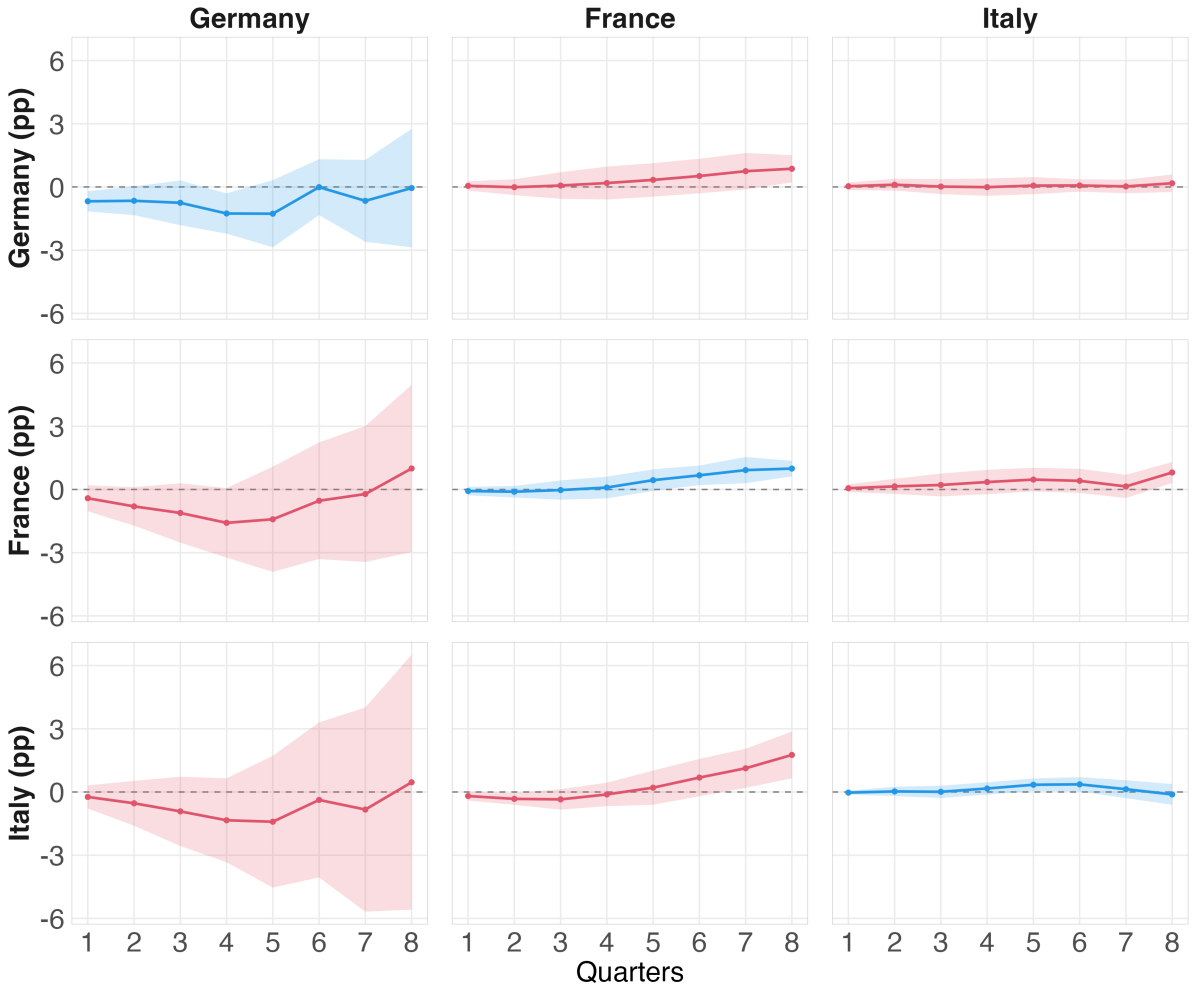
Notes: Impulse responses to a 1% decline in primary surplus/taxes in country A. The fiscal expansion is unfunded, i.e. fiscal-led, so a shock to ζ_t^F. The y-axis is in terms of percentage deviations from steady state. The x-axis is in quarters. Inflation and interest rates are annualized.

Figure 8: Impulse responses of inflation to a 1pp shock in govt spending (% of GDP)



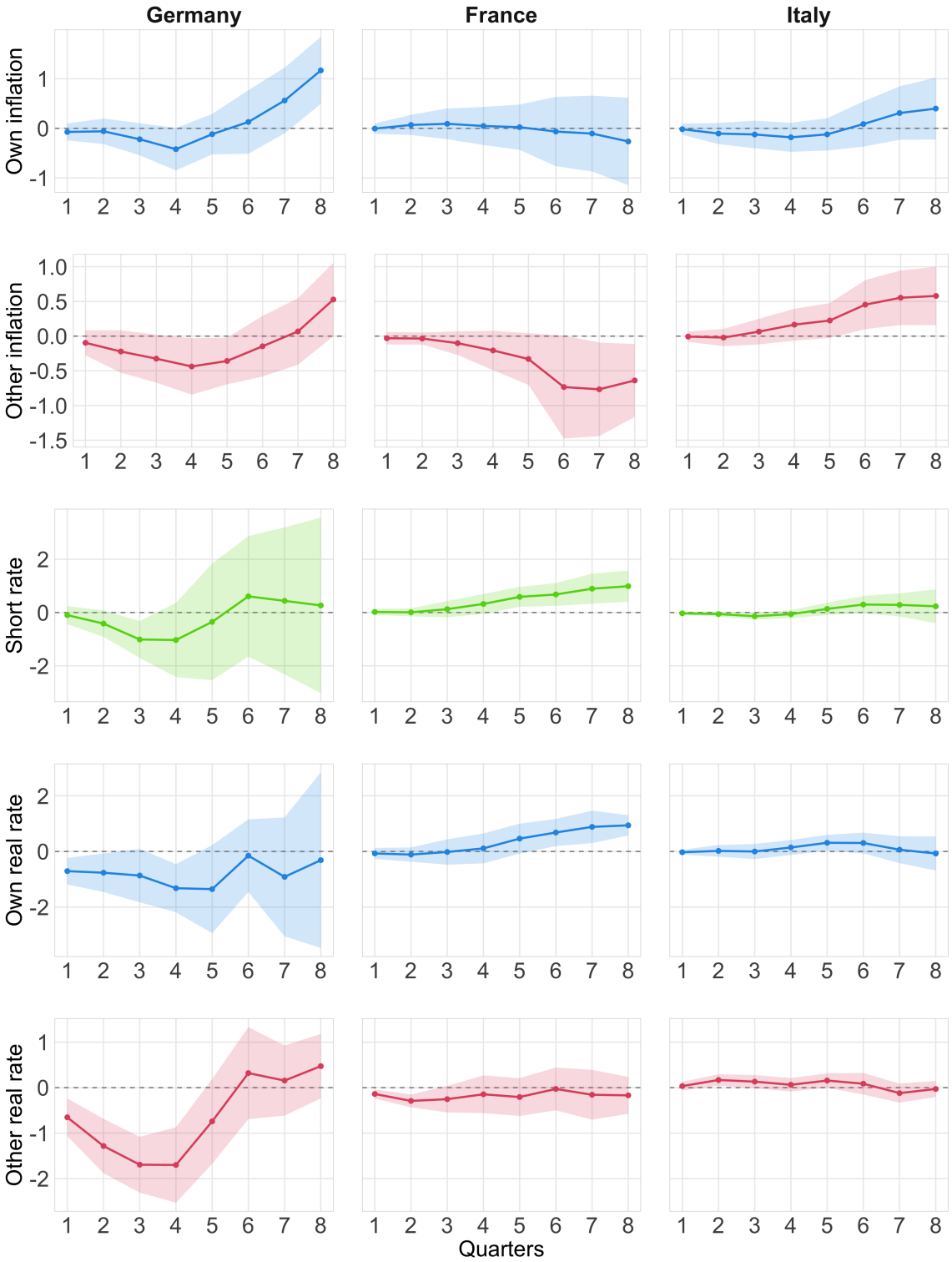
Notes: Pooled estimates from panel local projections on inflation in other countries ($i \neq c$) as response to fiscal shock in country c. Countries are weighted by GDP. Shaded area shows 90% confidence interval. Driscoll-Kraay standard errors.

Figure 9: Impulse responses of real rate to a 1pp shock in govt spending (% of GDP)



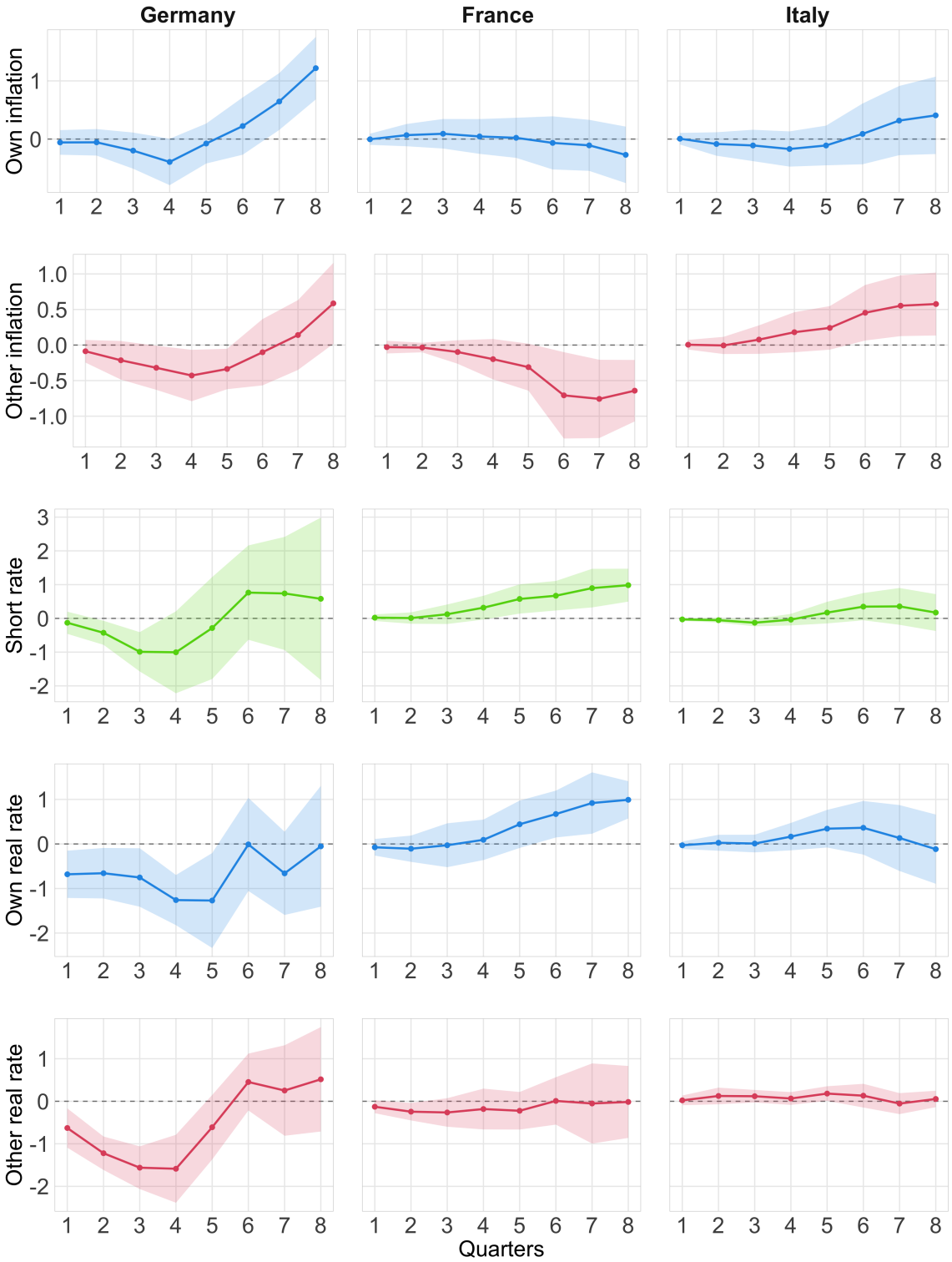
Notes: Pooled estimates from panel local projections on real rate in other countries ($i \neq c$) as response to fiscal shock in country c. Countries are weighted by GDP. Shaded area shows 90% confidence interval. Driscoll-Kraay standard errors.

Figure 10: Impulse responses to a 1 pp shock in govt spending (% of GDP) with unpurged shocks



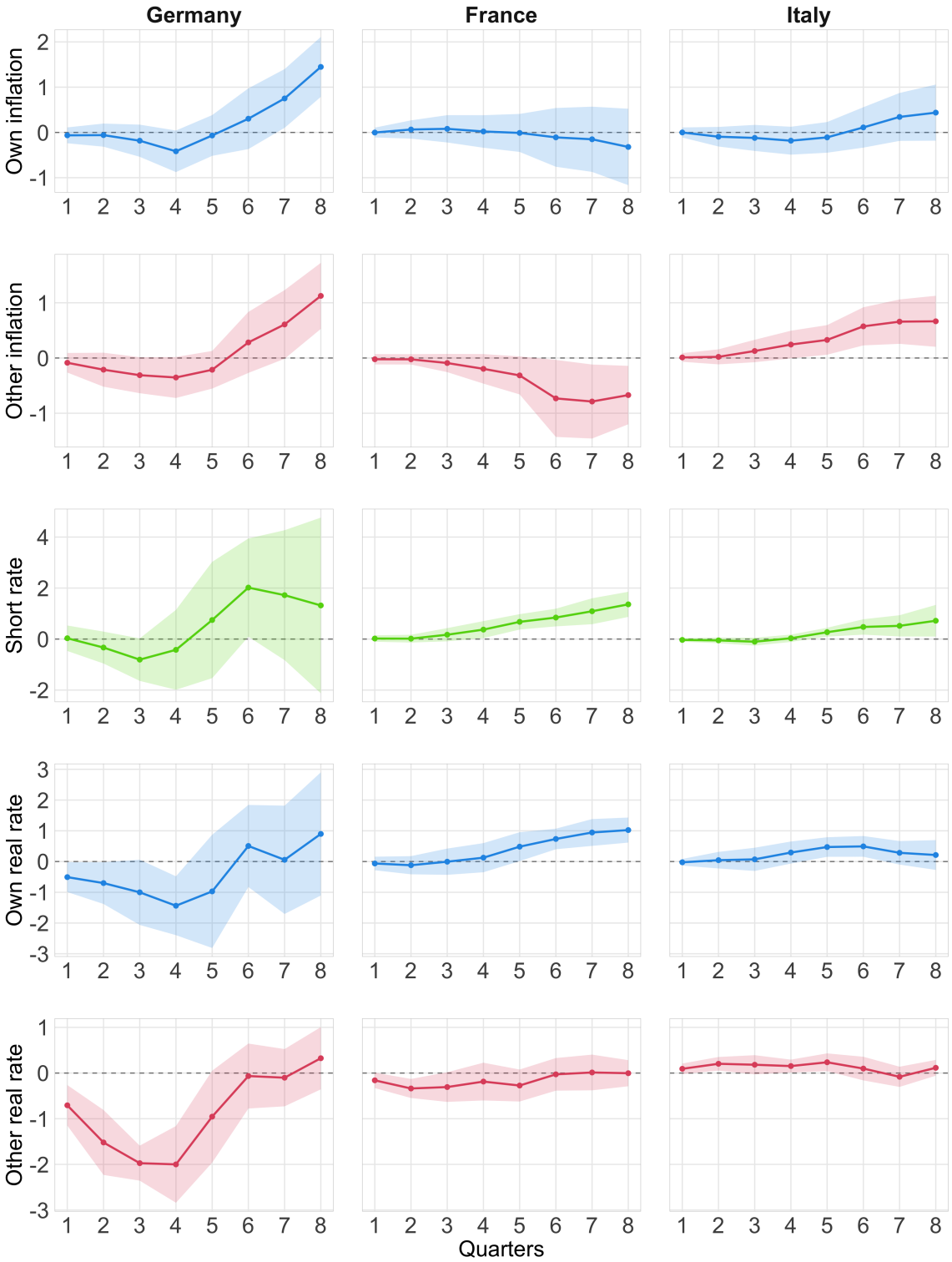
Notes: The own country and short rate local projections use heteroskedastic standard errors. For spillover effects to other countries, the results are pooled estimates from panel local projections with Driscoll-Kraay standard errors and countries weighted by GDP. Shaded area shows 90% confidence interval. Sample period is 2004Q1–2025Q4 for inflation responses, and 2004Q1–2014Q1 for rate responses.

Figure 11: Impulse responses to a 1 pp shock in govt spending (% of GDP) with different standard errors



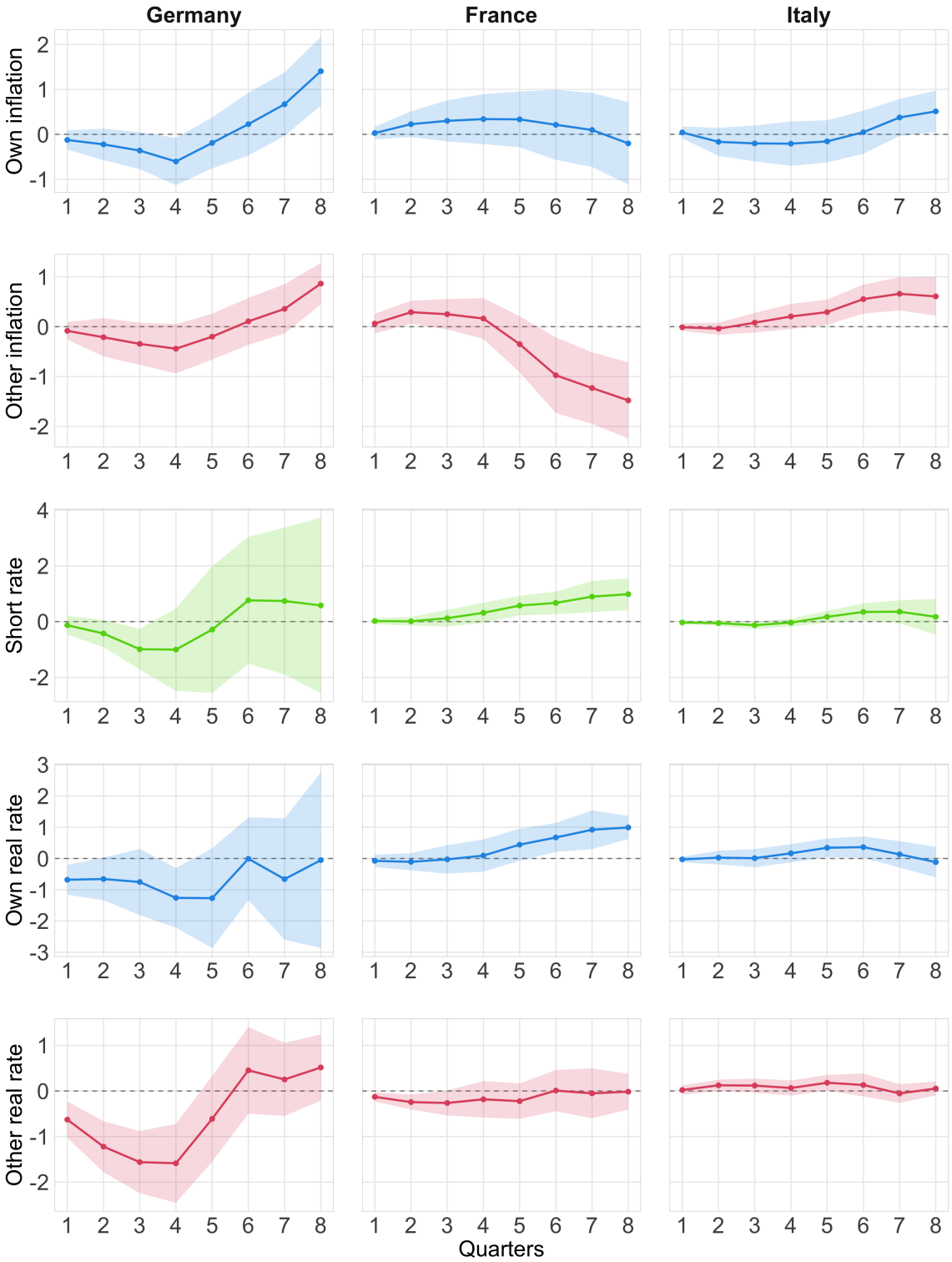
Notes: The own country and short rate local projections use Newey-West HAC standard errors. For spillover effects to other countries, the results are pooled estimates from panel local projections with two-way standard errors and countries weighted by GDP. Shaded area shows 90% confidence interval. Sample period is 2004Q1–2025Q4 for inflation responses, and 2004Q1–2014Q1 for rate responses.

Figure 12: Impulse responses to a 1 pp shock in govt spending (% of GDP) with lags of past shocks as extra controls



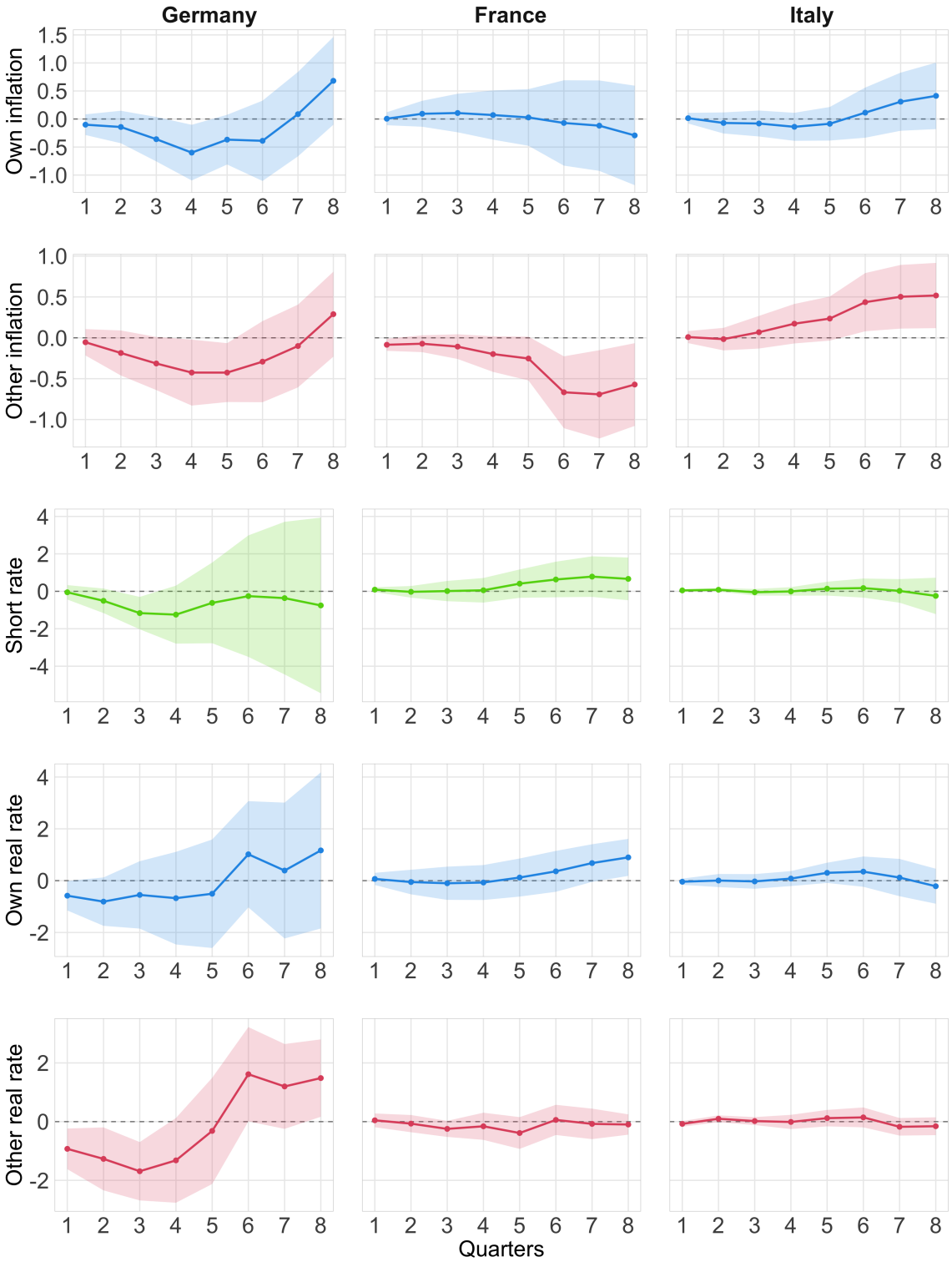
Notes: The own country and short rate local projections use heteroskedastic standard errors. For spillover effects to other countries, the results are pooled estimates from panel local projections with Driscoll-Kraay standard errors and countries weighted by GDP. Shaded area shows 90% confidence interval. Sample period is 2004Q1–2025Q4 for inflation responses, and 2004Q1–2014Q1 for rate responses.

Figure 13: Impulse responses to a 1 pp shock in govt spending (% of GDP) with contemporaneous weighted other-country fiscal shocks and its lags as extra controls



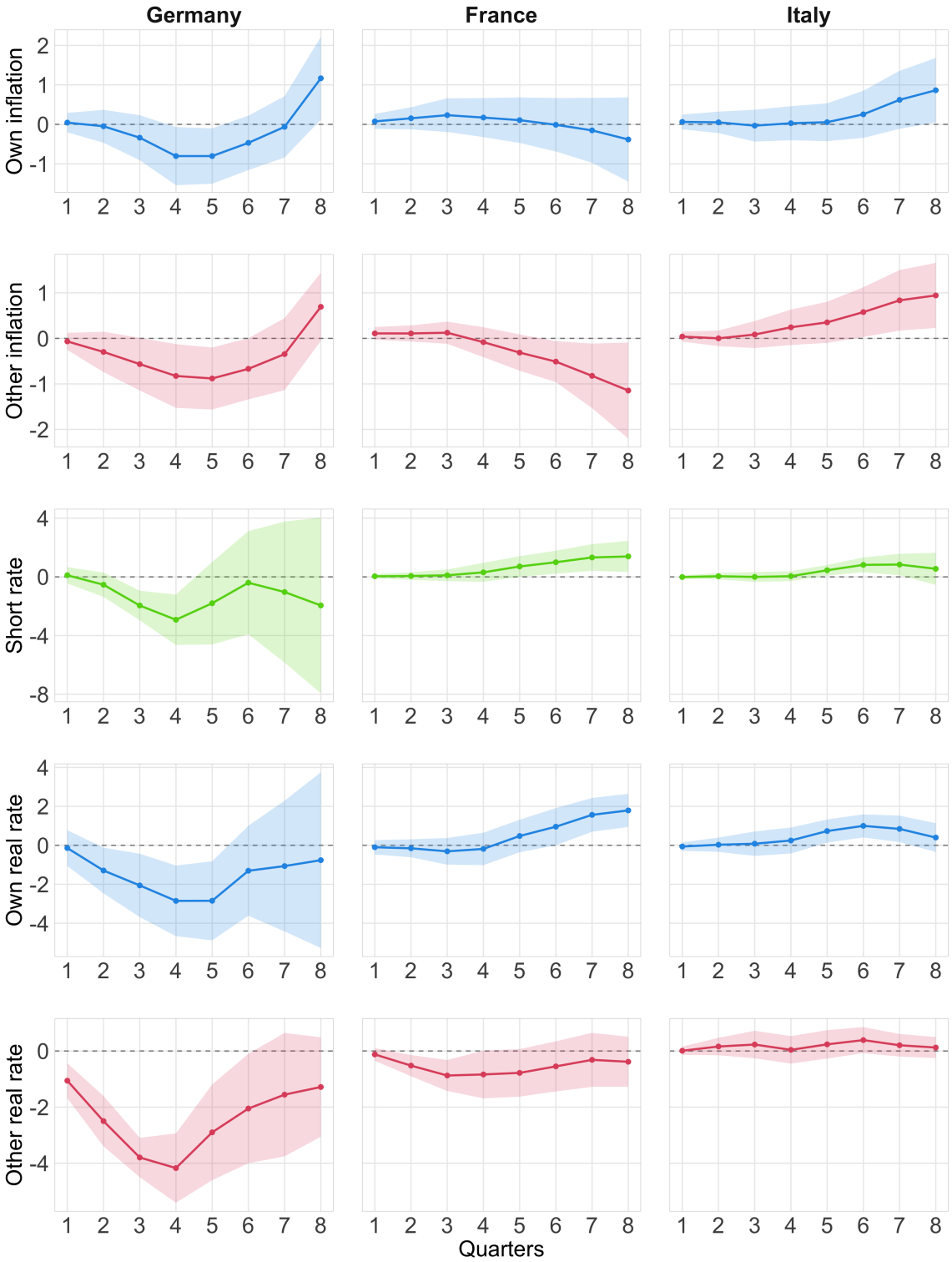
Notes: The own country and short rate local projections use heteroskedastic standard errors. For spillover effects to other countries, the results are pooled estimates from panel local projections with Driscoll-Kraay standard errors and countries weighted by GDP. Shaded area shows 90% confidence interval. Sample period is 2004Q1–2025Q4 for inflation responses, and 2004Q1–2014Q1 for rate responses.

Figure 14: Impulse responses to a 1 pp shock in govt spending (% of GDP) with lags of three periods



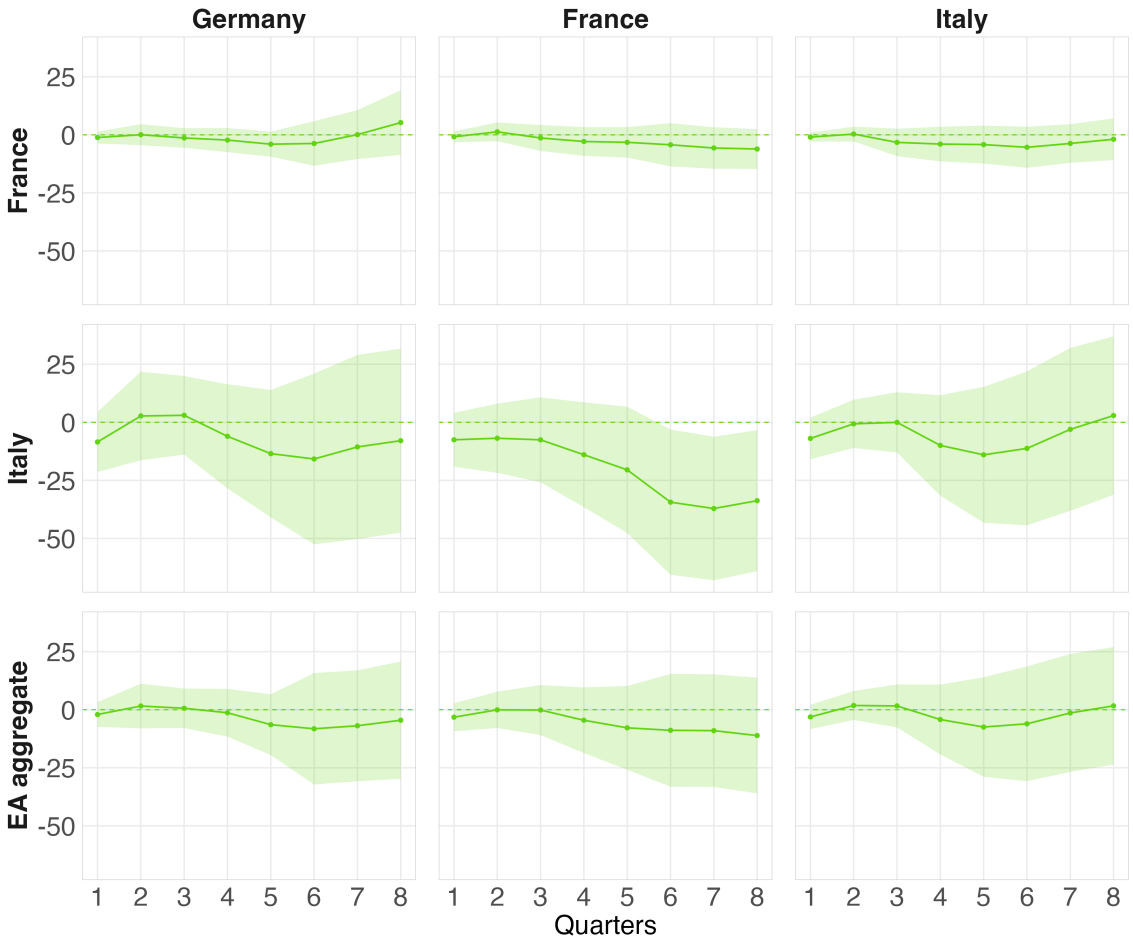
Notes: The own country and short rate local projections use heteroskedastic standard errors. For spillover effects to other countries, the results are pooled estimates from panel local projections with Driscoll-Kraay standard errors and countries weighted by GDP. Shaded area shows 90% confidence interval. Sample period is 2004Q1–2025Q4 for inflation responses, and 2004Q1–2014Q1 for rate responses.

Figure 15: Impulse responses to a 1 pp shock in govt spending (% of GDP) with timing fix



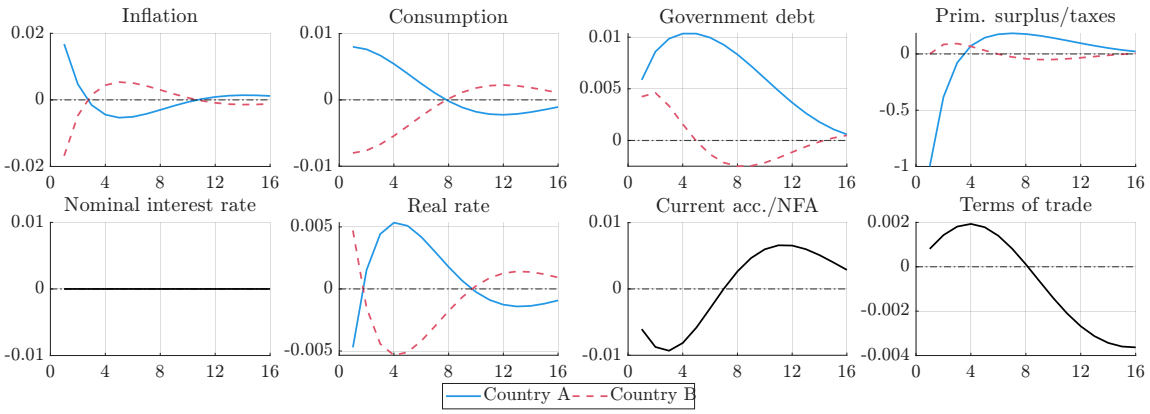
Notes: The own country and short rate local projections use heteroskedastic standard errors. For spillover effects to other countries, the results are pooled estimates from panel local projections with Driscoll-Kraay standard errors and countries weighted by GDP. Shaded area shows 90% confidence interval. Sample period is 2004Q1–2025Q4 for inflation responses, and 2004Q1–2014Q1 for rate responses.

Figure 16: Impulse responses of sovereign spreads to a 1 pp shock in govt spending (% of GDP)



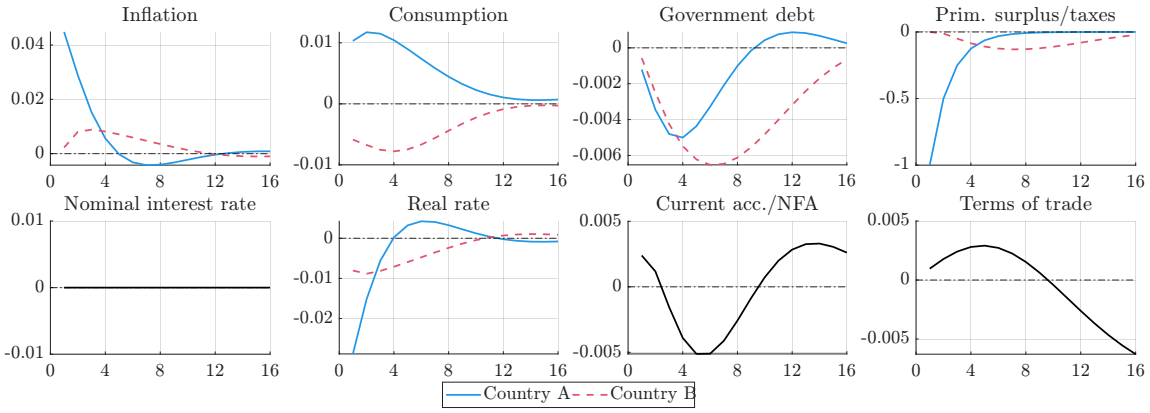
Notes: The dependent variable is the 10-year sovereign bond spread vis-à-vis Germany in basis points. Local projections use heteroskedastic standard errors. Shaded area shows 90% confidence interval. Sample period is 2004Q1–2025Q4. Controls include source country controls, receiving country controls (if not the same as source country), German controls, and euro area aggregate controls.

Figure 17: Impulse responses to funded and unfunded fiscal expansions in country A in the quantitative model



(a) Funded fiscal expansion in country A

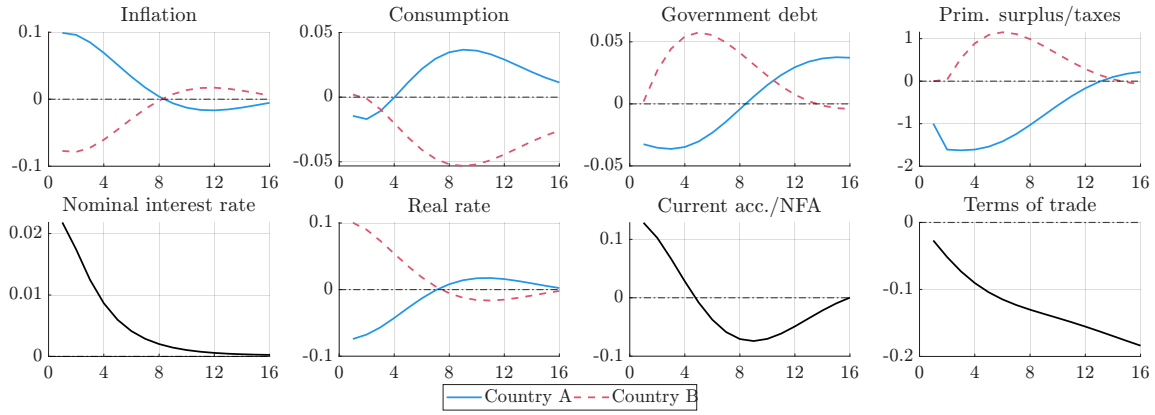
Notes: Impulse responses to a 1% decline in primary surplus/taxes in country A. The fiscal expansion is funded, i.e. monetary-led, so a shock to ζ_t^M. The y-axis is in terms of percentage deviations from steady state. The x-axis is in quarters. Inflation and interest rates are annualized.



(b) Unfunded fiscal expansion in country A

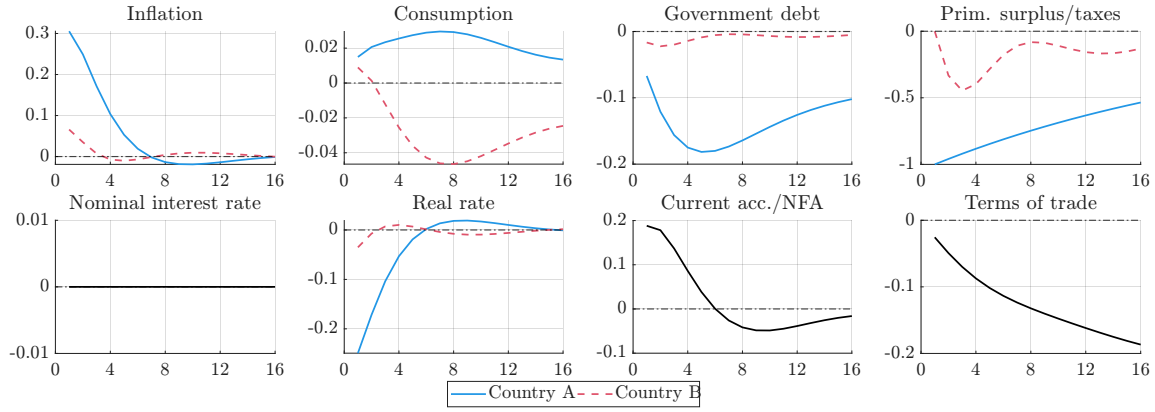
Notes: Impulse responses to a 1% decline in primary surplus/taxes in country A. The fiscal expansion is unfunded, i.e. fiscal-led, so a shock to ζ_t^F. The y-axis is in terms of percentage deviations from steady state. The x-axis is in quarters. Inflation and interest rates are annualized.

Figure 18: Impulse responses to funded and unfunded government spending shocks in country A



(a) Funded government spending shock in country A

Notes: Impulse responses to a 1% increase in government spending (as % of GDP) in country A. The government spending is funded, i.e. monetary-led. The y-axis is in terms of percentage deviations from steady state. The x-axis is in quarters. Inflation and interest rates are annualized.



(b) Unfunded government spending shock in country A

Notes: Impulse responses to a 1% increase in government spending (as % of GDP) in country A. The government spending is unfunded, i.e. fiscal-led. The y-axis is in terms of percentage deviations from steady state. The x-axis is in quarters. Inflation and interest rates are annualized.

Figure 19: Estimation results: prior and posteriors of the two-country Bayesian estimation (France vs. rest of EA)

		Prior			Posterior		
Param.	Description	Type	Mean	Std.	Mode	Median	90% HPD CI
Region A structural							
χ	Inverse Frisch elasticity of labor supply	N	2.00	0.50	1.1164	1.3259	[0.9578, 1.7070]
χ_w	Wage indexation	B	0.25	0.10	0.0953	0.1319	[0.0479, 0.2353]
h	Habit persistence	B	0.70	0.10	0.5970	0.5207	[0.3354, 0.6301]
μ	Elasticity of substitution across tradable goods	N	1.00	0.25	1.9136	1.8224	[1.5669, 2.0835]
θ_T	Calvo probability tradable prices	B	0.75	0.10	0.3433	0.3416	[0.3034, 0.3793]
θ_N	Calvo probability non-tradable prices	B	0.75	0.10	0.4853	0.4688	[0.4190, 0.5182]
θ_w	Calvo probability wages	B	0.75	0.10	0.4496	0.3974	[0.3003, 0.4835]
Region B structural							
χ^*	Inverse Frisch elasticity B	N	2.00	0.50	0.7986	0.8000	[0.5570, 1.0728]
χ^*_w	Wage indexation B	B	0.25	0.10	0.0839	0.0968	[0.0269, 0.1991]
h^*	Habit persistence B	B	0.70	0.10	0.3121	0.2371	[0.1053, 0.3349]
μ^*	Trade elasticity B	N	1.00	0.25	1.2530	1.4072	[1.1952, 1.6205]
θ^*_T	Calvo probability tradable prices B	B	0.75	0.10	0.3829	0.3724	[0.3277, 0.4176]
θ^*_N	Calvo probability non-tradable prices B	B	0.75	0.10	0.5347	0.5216	[0.4791, 0.5676]
θ^*_w	Calvo probability wages B	B	0.75	0.10	0.4495	0.4096	[0.3174, 0.4946]
Common structural							
ψ_I	Investment adjustment cost	G	2.00	0.50	0.7929	0.8171	[0.4924, 1.1287]
ψ_u	Capital utilization cost	B	0.50	0.15	0.0149	0.0210	[0.0062, 0.0385]
Policy							
ϕ^{ML}	Taylor rule inflation coefficient (ML)	N	2.00	0.50	1.6843	1.9651	[1.4643, 2.5868]
γ_A^{ML}	Fiscal rule debt coefficient A (ML, passive=20)	N	20.0	5.00	16.1577	10.1428	[1.0001, 18.4094]
$\gamma_B^{*,ML}$	Fiscal rule debt coefficient B (ML, passive=20)	N	20.0	5.00	25.2408	19.5613	[14.8515, 24.6664]
$\gamma_B^{*,FLA}$	Fiscal rule debt coefficient B (FLA, passive=20)	N	20.0	5.00	2.6321	2.9535	[1.0003, 4.9406]
γ_A^{FLB}	Fiscal rule debt coefficient A (FLB, passive=20)	N	20.0	5.00	19.0856	21.1540	[12.0135, 27.0115]
Shock persistence							
ρ_a	Technology shock persistence	B	0.80	0.10	0.7189	0.7233	[0.6788, 0.7711]
ρ_{rp}	Risk premium shock persistence	B	0.80	0.10	0.9762	0.9765	[0.9678, 0.9850]
ρ_d	Preference shock persistence	B	0.50	0.15	0.1227	0.1660	[0.0876, 0.3301]
ρ_u	Tradable markup shock persistence	B	0.80	0.10	0.8426	0.8168	[0.7179, 0.9265]
ρ_{uN}	NT markup shock persistence	B	0.80	0.10	0.7659	0.7752	[0.7159, 0.8466]
ρ_w	Wage markup shock persistence	B	0.80	0.10	0.7983	0.8050	[0.7175, 0.9042]
ρ_k	MEI shock persistence	B	0.50	0.15	0.1757	0.1691	[0.0853, 0.2611]
ρ_g	Government spending shock persistence	B	0.80	0.10	0.8913	0.8947	[0.8705, 0.9176]
$\rho_{\zeta,ML}$	Funded tax shock persistence A (ML)	B	0.50	0.15	0.5119	0.4866	[0.3724, 0.6106]
$\rho_{\zeta^*,ML}$	Funded tax shock persistence B (ML)	B	0.50	0.15	0.2260	0.2277	[0.1308, 0.3119]
$\rho_{\zeta,FL}$	Unfunded tax shock persistence A (FLA)	B	0.50	0.15	0.1222	0.1487	[0.0660, 0.2290]
$\rho_{\zeta^*,FL}$	Unfunded tax shock persistence B (FLB)	B	0.50	0.15	0.1261	0.1345	[0.0505, 0.2217]
Shock volatilities							
σ_a	St.dev. technology shock	IG	0.50	inf	1.2708	1.2853	[1.0992, 1.4738]
σ_{aN}	St.dev. NT technology shock	IG	0.50	inf	1.8564	1.8369	[1.6714, 2.0183]
σ_{rp}	St.dev. risk premium shock	IG	0.25	inf	0.0340	0.0343	[0.0291, 0.0397]
σ_d	St.dev. preference shock	IG	0.50	inf	4.5668	3.9859	[2.7595, 4.9538]
σ_u	St.dev. tradable markup shock	IG	0.50	inf	0.2234	0.2760	[0.1247, 0.4959]
σ_{uN}	St.dev. NT markup shock	IG	0.50	inf	1.6302	1.7529	[1.4311, 2.0869]
σ_w	St.dev. wage markup shock	IG	0.50	inf	0.6991	0.8442	[0.6023, 1.2074]
σ_k	St.dev. MEI shock	IG	0.50	inf	2.6527	2.7465	[1.7293, 3.7455]
σ_ζ	St.dev. tax shock (all ZETA processes)	IG	0.50	inf	1.5412	1.5419	[1.4223, 1.6635]
σ_g	St.dev. government spending shock (all G processes)	IG	0.50	inf	0.7244	0.7265	[0.6694, 0.7858]
σ_h	St.dev. COVID labor supply shock	G	5.00	2.5	7.0709	8.1520	[6.0968, 10.3628]

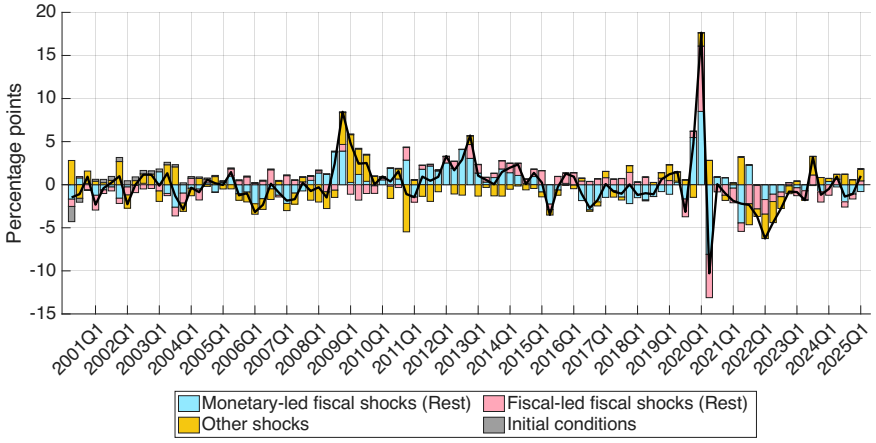
Notes: Results of the estimation of the structural parameters. N = Normal, B = Beta, G = Gamma, IG = Inverse-Gamma, AR coeff. = autoregressive coefficient, St.dev. = standard deviation, HPD = highest posterior density.

Figure 20: Estimation results: prior and posteriors of the two-country Bayesian estimation (Italy vs. rest of EA)

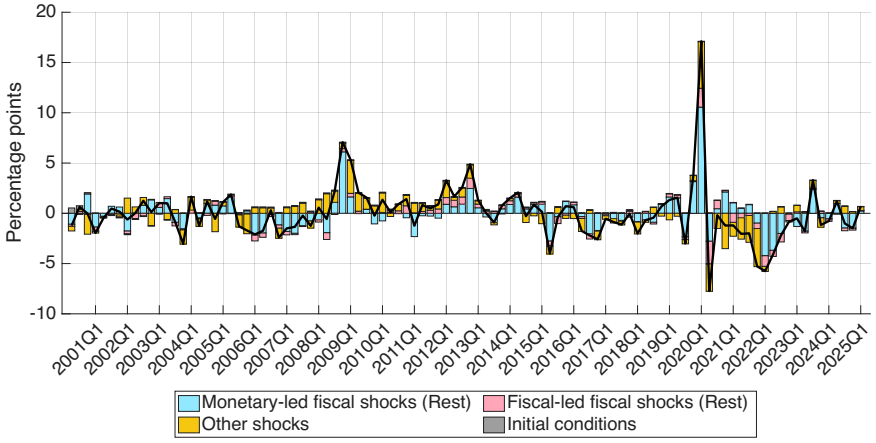
		Prior			Posterior		
Param.	Description	Type	Mean	Std.	Mode	Median	90% HPD CI
Region A structural							
χ	Inverse Frisch elasticity of labor supply	N	2.00	0.50	1.1006	1.2283	[0.9773, 1.5028]
χ_w	Wage indexation	B	0.25	0.10	0.2952	0.2484	[0.1549, 0.3438]
h	Habit persistence	B	0.70	0.10	0.6719	0.6808	[0.6323, 0.7423]
μ	Elasticity of substitution across tradable goods	N	1.00	0.25	2.1013	2.1577	[1.9984, 2.3112]
θ_T	Calvo probability tradable prices	B	0.75	0.10	0.4241	0.4291	[0.3802, 0.4706]
θ_N	Calvo probability non-tradable prices	B	0.75	0.10	0.6227	0.6221	[0.5784, 0.6589]
θ_w	Calvo probability wages	B	0.75	0.10	0.4547	0.4539	[0.4179, 0.4902]
Region B structural							
χ^*	Inverse Frisch elasticity B	N	2.00	0.50	0.0606	0.0860	[-0.0002, 0.2071]
χ_w^*	Wage indexation B	B	0.25	0.10	0.0240	0.0246	[0.0076, 0.0442]
h^*	Habit persistence B	B	0.70	0.10	0.5595	0.5656	[0.5175, 0.6084]
μ^*	Trade elasticity B	N	1.00	0.25	1.3490	1.3079	[1.1620, 1.4757]
θ_T^*	Calvo probability tradable prices B	B	0.75	0.10	0.4637	0.4560	[0.4233, 0.4999]
θ_N^*	Calvo probability non-tradable prices B	B	0.75	0.10	0.7206	0.7107	[0.6658, 0.7479]
θ_w^*	Calvo probability wages B	B	0.75	0.10	0.6251	0.6531	[0.6091, 0.6915]
Common structural							
ψ_I	Investment adjustment cost	G	2.00	0.50	2.0175	1.9848	[1.6333, 2.3479]
ψ_u	Capital utilization cost	B	0.50	0.15	0.0292	0.0316	[0.0171, 0.0471]
Policy							
ϕ^{ML}	Taylor rule inflation coefficient (ML)	N	2.00	0.50	1.0126	1.0137	[1.0035, 1.0239]
γ_A^{ML}	Fiscal rule debt coefficient A (ML, passive=20)	N	20.0	5.00	20.7483	20.7226	[15.8309, 26.0165]
$\gamma_B^{*,ML}$	Fiscal rule debt coefficient B (ML, passive=20)	N	20.0	5.00	15.7268	18.4901	[14.8794, 22.3397]
$\gamma_B^{*,FLA}$	Fiscal rule debt coefficient B (FLA, passive=20)	N	20.0	5.00	27.0773	27.0351	[19.4532, 31.9792]
γ_A^{FLB}	Fiscal rule debt coefficient A (FLB, passive=20)	N	20.0	5.00	19.6802	19.8089	[15.8746, 23.5321]
Shock persistence							
ρ_a	Technology shock persistence	B	0.80	0.10	0.6746	0.6962	[0.6607, 0.7241]
ρ_{rp}	Risk premium shock persistence	B	0.80	0.10	0.9047	0.9027	[0.8848, 0.9210]
ρ_d	Preference shock persistence	B	0.50	0.15	0.1191	0.1189	[0.0618, 0.1767]
ρ_u	Tradable markup shock persistence	B	0.80	0.10	0.9907	0.9836	[0.9616, 0.9950]
ρ_{uN}	NT markup shock persistence	B	0.80	0.10	0.7502	0.7504	[0.7106, 0.7876]
ρ_w	Wage markup shock persistence	B	0.80	0.10	0.5198	0.5059	[0.4378, 0.5812]
ρ_k	MEI shock persistence	B	0.50	0.15	0.2012	0.1966	[0.1044, 0.2821]
ρ_g	Government spending shock persistence	B	0.80	0.10	0.9507	0.9512	[0.9424, 0.9601]
$\rho_{\zeta,ML}$	Funded tax shock persistence A (ML)	B	0.50	0.15	0.6924	0.7223	[0.6569, 0.7838]
$\rho_{\zeta^*,ML}$	Funded tax shock persistence B (ML)	B	0.50	0.15	0.5428	0.5819	[0.5201, 0.6486]
$\rho_{\zeta,FL}$	Unfunded tax shock persistence A (FLA)	B	0.50	0.15	0.1690	0.1785	[0.1063, 0.2504]
$\rho_{\zeta^*,FL}$	Unfunded tax shock persistence B (FLB)	B	0.50	0.15	0.0455	0.0513	[0.0196, 0.0863]
Shock volatilities							
σ_a	St.dev. technology shock	IG	0.50	inf	1.5255	1.5227	[1.3200, 1.7513]
σ_{aN}	St.dev. NT technology shock	IG	0.50	inf	2.0486	2.0247	[1.8443, 2.1957]
σ_{rp}	St.dev. risk premium shock	IG	0.25	inf	0.0279	0.0289	[0.0243, 0.0338]
σ_d	St.dev. preference shock	IG	0.50	inf	5.9321	6.0384	[5.2173, 6.9919]
σ_u	St.dev. tradable markup shock	IG	0.50	inf	0.2205	0.2200	[0.1242, 0.3218]
σ_{uN}	St.dev. NT markup shock	IG	0.50	inf	0.9792	1.0087	[0.8362, 1.1856]
σ_w	St.dev. wage markup shock	IG	0.50	inf	0.4134	0.3960	[0.3144, 0.4839]
σ_k	St.dev. MEI shock	IG	0.50	inf	7.3004	7.3145	[5.8355, 8.8898]
σ_ζ	St.dev. tax shock (all ZETA processes)	IG	0.50	inf	1.5484	1.5676	[1.4353, 1.6976]
σ_g	St.dev. government spending shock (all G processes)	IG	0.50	inf	0.4904	0.4903	[0.4518, 0.5274]
σ_h	St.dev. COVID labor supply shock	G	5.00	2.5	6.5390	9.3333	[5.5991, 12.3669]

Notes: Results of the estimation of the structural parameters. N = Normal, B = Beta, G = Gamma, IG = Inverse-Gamma, AR coeff. = autoregressive coefficient, St.dev. = standard deviation, HPD = highest posterior density.

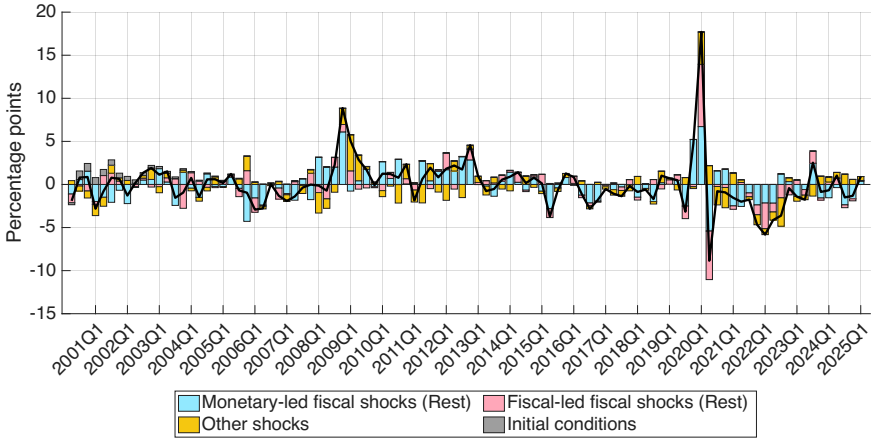
Figure 21: Shock decomposition of debt/GDP growth in rest of euro area



(a) Debt/GDP growth in the rest of the euro area (vs. Germany). Share monetary-led: 0.61.



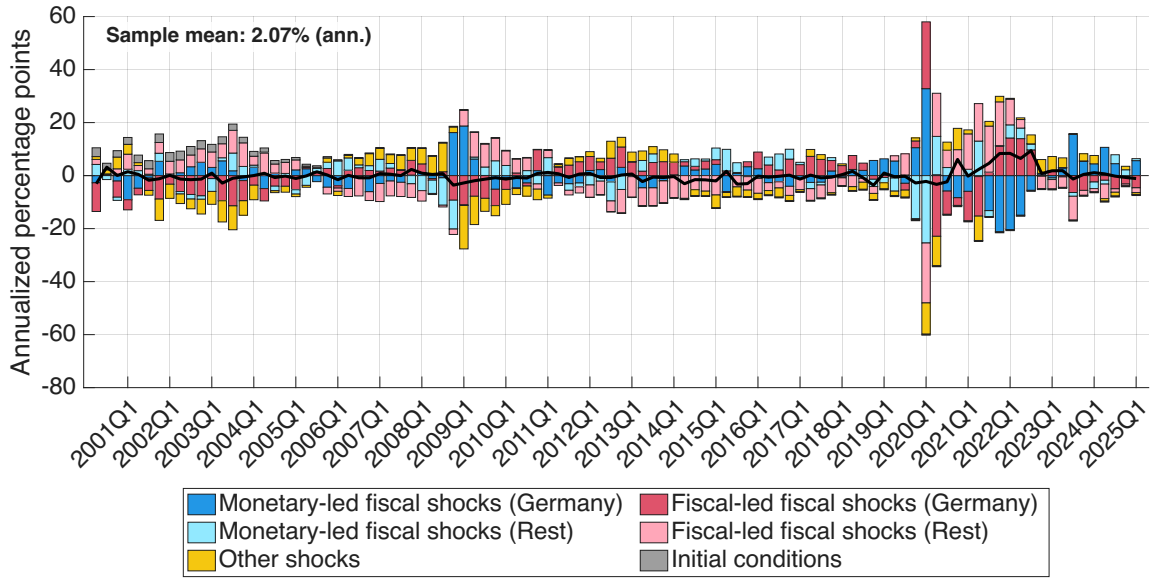
(b) Debt/GDP growth in the rest of the euro area (vs. France). Share monetary-led: 0.79.



(c) Debt/GDP growth in the rest of the euro area (vs. Italy). Share monetary-led: 0.66.

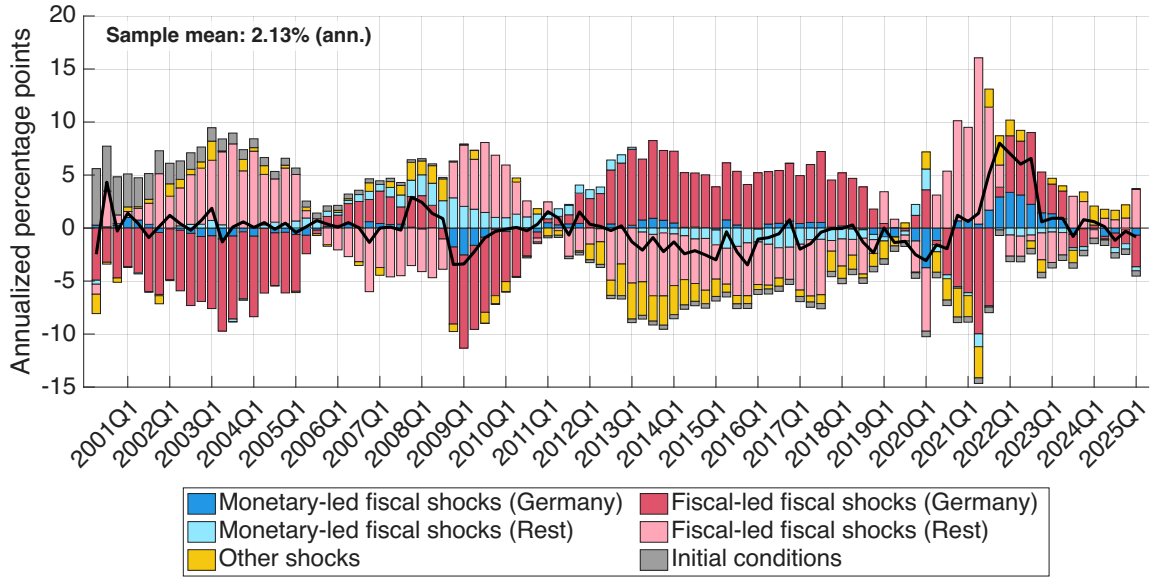
Notes: Historical shock decomposition of the debt-to-GDP ratio growth in Germany, France, and Italy in percentage point deviations from the sample mean. We estimate the quantitative two-country model three times, with each of those countries as country A and the rest of the euro area as country B. Monetary-led fiscal shocks are ζ_t^{ML} and g_t^{ML}, and fiscal-led fiscal shocks are ζ_t^{FL} and g_t^{FL}. The x-axis is in quarters.

Figure 22: Shock decomposition of inflation in Germany and rest of euro area



(a) Annualized inflation rate in Germany

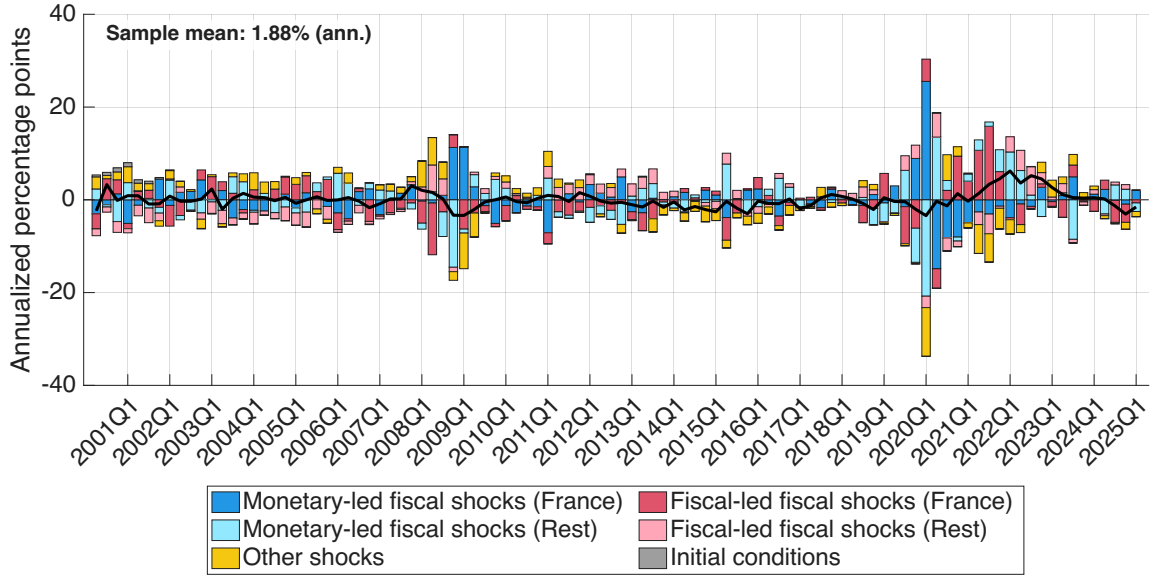
Notes: Historical shock decomposition of the inflation rate in Germany in annualized percentage point deviations from the sample mean. We estimate the quantitative two-country model with Germany as country A and the rest of the euro area as country B. Monetary-led fiscal shocks are shocks to taxes/primary surplus, ζ_t^{ML}, and government spending, g_t^{ML}, and fiscal-led fiscal shocks are shocks to ζ_t^{FL} and g_t^{FL}. The x-axis is in quarters.



(b) Annualized inflation rate in the rest of the euro area

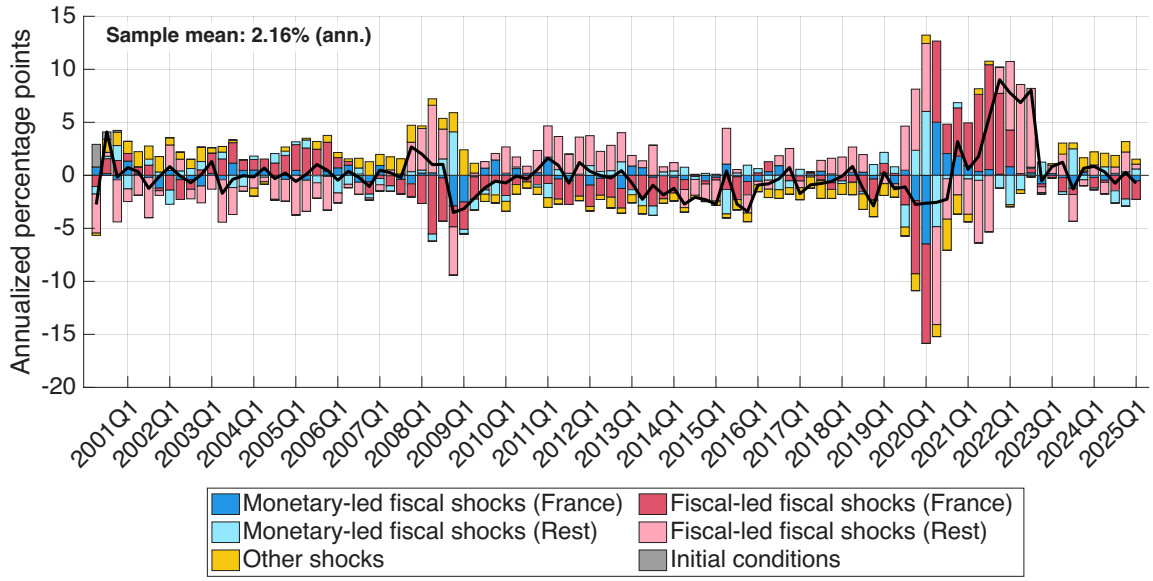
Notes: Historical shock decomposition of the inflation rate in the rest of the euro area in annualized percentage point deviations from the sample mean. We estimate the quantitative two-country model with Germany as country A and the rest of the euro area as country B. Monetary-led fiscal shocks are shocks to taxes/primary surplus, ζ_t^{ML}, and government spending, g_t^{ML}, and fiscal-led fiscal shocks are shocks to ζ_t^{FL*} and g_t^{FL*}. The x-axis is in quarters.

Figure 23: Shock decomposition of inflation in France and rest of euro area



(a) Annualized inflation rate in France

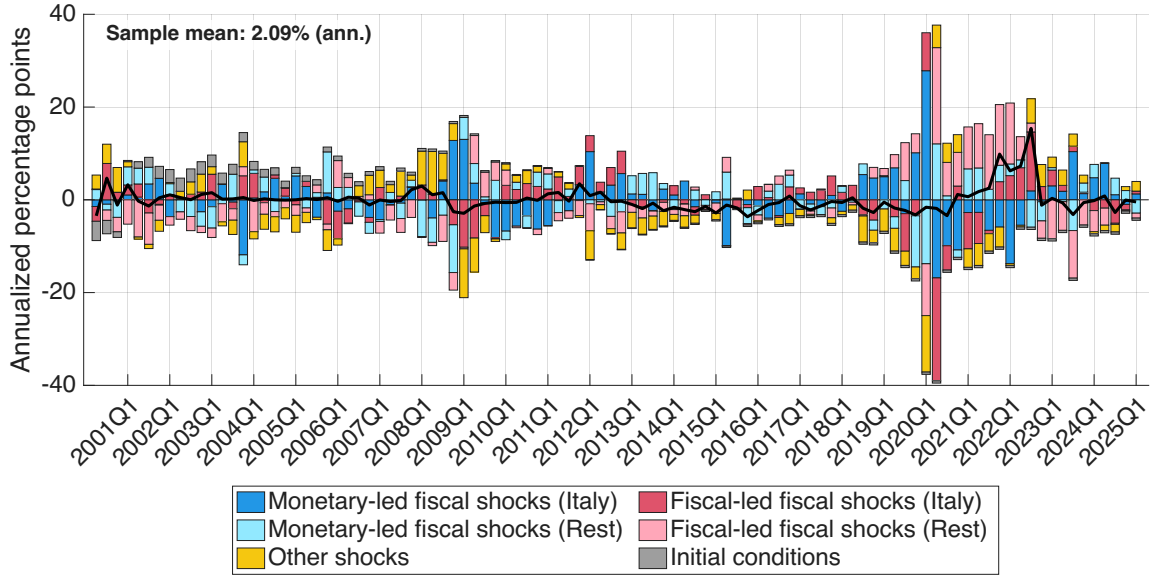
Notes: Historical shock decomposition of the inflation rate in France in annualized percentage point deviations from the sample mean. We estimate the quantitative two-country model with France as country A and the rest of the euro area as country B. Monetary-led fiscal shocks are shocks to taxes/primary surplus, ζ_t^{ML}, and government spending, g_t^{ML}, and fiscal-led fiscal shocks are shocks to ζ_t^{FL} and g_t^{FL}. The x-axis is in quarters.



(b) Annualized inflation rate in the rest of the euro area

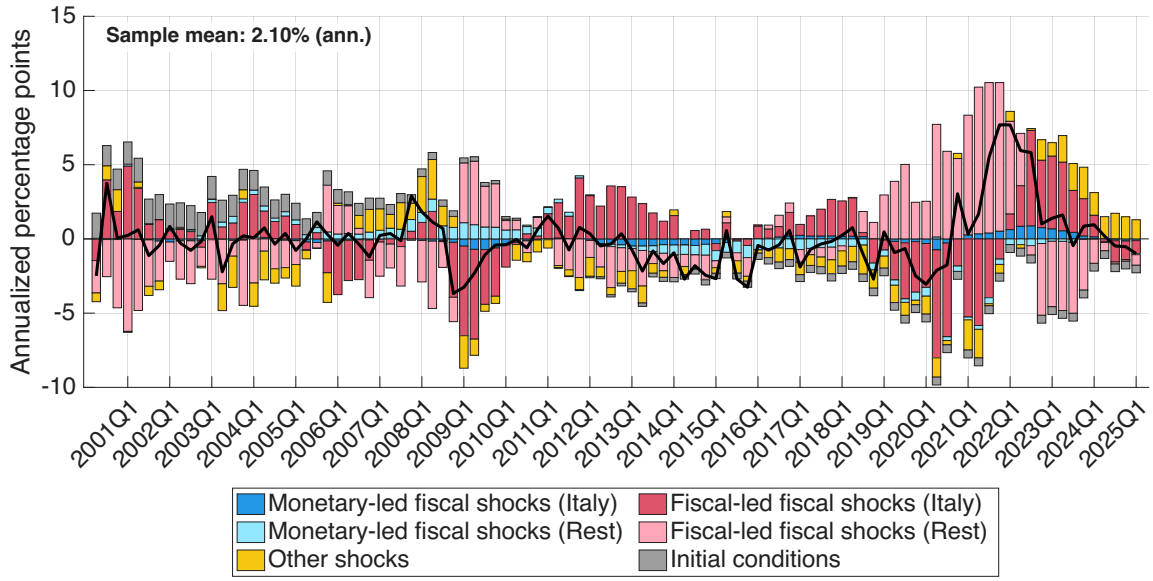
Notes: Historical shock decomposition of the inflation rate in the rest of the euro area in annualized percentage point deviations from the sample mean. We estimate the quantitative two-country model with France as country A and the rest of the euro area as country B. Monetary-led fiscal shocks are shocks to taxes/primary surplus, ζ_t^{ML}, and government spending, g_t^{ML}, and fiscal-led fiscal shocks are shocks to ζ_t^{FL*} and g_t^{FL*}. The x-axis is in quarters.

Figure 24: Shock decomposition of inflation in Italy and rest of euro area



(a) Annualized inflation rate in Italy

Notes: Historical shock decomposition of the inflation rate in Italy in annualized percentage point deviations from the sample mean. We estimate the quantitative two-country model with Italy as country A and the rest of the euro area as country B. Monetary-led fiscal shocks are shocks to taxes/primary surplus, ζ_t^{ML}, and government spending, g_t^{ML}, and fiscal-led fiscal shocks are shocks to ζ_t^{FL} and g_t^{FL}. The x-axis is in quarters.



(b) Annualized inflation rate in the rest of the euro area

Notes: Historical shock decomposition of the inflation rate in the rest of the euro area in annualized percentage point deviations from the sample mean. We estimate the quantitative two-country model with Italy as country A and the rest of the euro area as country B. Monetary-led fiscal shocks are shocks to taxes/primary surplus, ζ_t^{ML}, and government spending, g_t^{ML}, and fiscal-led fiscal shocks are shocks to ζ_t^{FL*} and g_t^{FL*}. The x-axis is in quarters.

B Equations stylized model

We summarize the stylized model with the following equations, linearized around the steady state as $\hat{x}_t = \ln(X_t) - \ln(\overline{X}) \approx \frac{X_t - \overline{X}}{\overline{X}}$.

$$\text{Agg. Euler equation} \quad \hat{c}_t^{MU} = \mathbb{E}_t \hat{c}_{t+1}^{MU} - \left(\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1}^{MU}\right) - \Theta\psi_b \frac{\overline{b}_A + \overline{b}_B}{\overline{b}_A}\left(\hat{b}_{Bt} - \hat{b}_{At}\right) - (1-\Theta)\psi_b \frac{\overline{b}^*_A + \overline{b}^*_B}{\overline{b}^*_A}\left(\hat{b}^*_{Bt} - \hat{b}^*_{At}\right) \tag{B.1}$$

$$\text{Govt. budget constraint A} \quad \hat{s}_{bt} = \frac{1}{\beta}\left[\hat{s}_{b,t-1} + \hat{i}_{t-1} - \hat{\pi}_t - (1-\beta)\hat{\tau}_t\right] \tag{B.2}$$

$$\text{Govt. budget constraint B} \quad \hat{s}^*_{bt} = \frac{1}{\beta}\left[\hat{s}^*_{b,t-1} + \hat{i}_{t-1} - \hat{\pi}^*_t - (1-\beta)\hat{\tau}^*_t\right] \tag{B.3}$$

$$\text{Fiscal rule A} \quad \hat{\tau}_t = \gamma \hat{s}_{b,t-1} + \zeta_t \tag{B.4}$$

$$\text{Fiscal rule B} \quad \hat{\tau}^*_t = \gamma^* \hat{s}^*_{b,t-1} + \zeta^*_t \tag{B.5}$$

$$\text{Monetary rule} \quad \hat{i}_t = \phi \hat{\pi}_t^{MU} \tag{B.6}$$

$$\text{Market clearing A} \quad 0 = (1-\alpha_I)\hat{c}_t + \alpha_I \hat{c}^*_t + 2\mu\alpha_I(1-\alpha_I)\hat{s}_t \tag{B.7}$$

$$\text{Agg. goods market clearing} \quad 0 = \Theta\hat{c}_t + (1-\Theta)\hat{c}^*_t \tag{B.8}$$

$$\text{Bonds market clearing A} \quad \overline{s}_b \hat{s}_{bt} = \overline{b}_A \hat{b}_{At} + \overline{b}^*_A \hat{b}^*_{At} \tag{B.9}$$

$$\text{Bonds market clearing B} \quad \overline{s}^*_b \hat{s}^*_{bt} = \overline{b}_B \hat{b}_{Bt} + \overline{b}^*_B \hat{b}^*_{Bt} \tag{B.10}$$

$$\text{No arbitrage A} \quad \hat{b}_{At} = \hat{b}_{Bt} \tag{B.11}$$

$$\text{No arbitrage B} \quad \hat{b}^*_{At} = \hat{b}^*_{Bt} \tag{B.12}$$

$$\text{Terms of trade} \quad \alpha_I\left(\hat{s}_t - \hat{s}_{t-1}\right) = \hat{\pi}_t - \hat{\pi}_{Tt} \tag{B.13}$$

$$\text{Current account} \quad -\hat{c}_t - \alpha_I \hat{s}_t = \hat{ca}_t \tag{B.14}$$

$$\text{Dynamics of NFA} \quad \hat{ca}_t = \overline{b}_B \hat{b}_{Bt} - \overline{b}^*_A \hat{b}^*_{At} - \frac{1}{\beta}\left(\overline{b}_B \hat{b}_{B,t-1} - \overline{b}^*_A \hat{b}^*_{A,t-1}\right) \tag{B.15}$$

$$\text{Agg. inflation} \quad \hat{\pi}_t^{MU} = \Theta\hat{\pi}_t + (1-\Theta)\hat{\pi}^*_t \tag{B.16}$$

$$\text{Inflation A} \quad \hat{\pi}_t = (1-\alpha_I)\hat{\pi}_{Tt} + \alpha_I \hat{\pi}^*_{Tt} \tag{B.17}$$

$$\text{Inflation B} \quad \hat{\pi}^*_t = \alpha_I \hat{\pi}_{Tt} + (1-\alpha_I)\hat{\pi}^*_{Tt} \tag{B.18}$$

where $\hat{x}_t^{MU} \equiv \Theta\hat{x}_t + (1-\Theta)\hat{x}^*_t$ and bond holdings are in real terms (as shares of output), e.g. $\hat{b}_{At} = \frac{\frac{B_{At}}{P_t Y} - \frac{\overline{B}_A}{\overline{PY}}}{\frac{\overline{B}_A}{\overline{PY}}}$.

Steady state. From the Euler equations, we obtain the steady-state nominal interest rate:

$$\overline{R} = \frac{1}{\beta} \tag{B.19}$$

For simplicity, we impose that in steady state the prices of the goods are unitary:

$$\overline{P}_A = \overline{P}_B = 1 \tag{B.20}$$

From the relative demand equations and the goods market clearing conditions, we get the steady-state consumption values, assuming that the endowments $Y = Y^* = 1$:

$$\overline{C}_A = \overline{C}^*_B = 1 - \alpha_I \tag{B.21}$$

$$\overline{C}_B = \overline{C}^*_A = \alpha_I \tag{B.22}$$

$$\overline{C} = \overline{C}^* = 1 \tag{B.23}$$

With these steady-state values, we can get the government budget constraint in steady state:

$$\overline{B} = \frac{\beta}{1-\beta}\overline{T} \tag{B.24}$$

We calibrate the debt-to-GDP ratio $\frac{\overline{B}}{Y} = \overline{B} = 0.80$. See the Calibration section in the main text for more details.

C Solving the stylized model with partially unfunded debt

In the model with partially unfunded debt in country A, we can keep all equations the same as the model without, except the fiscal rules and the monetary rule which we replace with equations (32), (33), and (34). Moreover, for every equation we add the equivalent equation for the shadow economies.

To prove that the equations for the model with partially unfunded debt are correct, we need to prove that the following is true: first, that $\hat{s}^M_{bt} = \hat{s}_{bt} - \hat{s}^F_{bt}$ and $\hat{s}^{M*}_{bt} = \hat{s}^*_{bt} - \hat{s}^{F*}_{bt}$, i.e. the difference of the overall stock of debt and fiscal-led stock of debt is the monetary-led stock of debt. Second, that $\hat{\pi}^M_t = \hat{\pi}_t - \hat{\pi}^F_t$ and $\hat{\pi}^{M*}_t = \hat{\pi}^*_t - \hat{\pi}^{F*}_t$, i.e. the inflation rate the central bank tries to stabilize in each economy is the overall inflation net of the fiscal-led part of inflation.

To prove those claims, we follow BFM and construct another parallel economy to

characterize the part of inflation the central bank has control over, $\hat{\pi}_t^M$ and $\hat{\pi}_t^{M*}$:

$$\Theta \mathbb{E}_t \hat{\pi}_{t+1}^M + (1-\Theta)\mathbb{E}_t \hat{\pi}_{t+1}^{M*} = \hat{i}_t^M - \Theta\psi_b \frac{\overline{b}_A + \overline{b}_B}{\overline{b}_A}\left(\hat{b}_{Bt} - \hat{b}_{At}\right) - (1-\Theta)\psi_b \frac{\overline{b}_A^* + \overline{b}_B^*}{\overline{b}_A^*}\left(\hat{b}_{Bt}^* - \hat{b}_{At}^*\right) \tag{C.1}$$

$$\hat{s}_{bt}^M = \frac{1}{\beta}\left[\hat{s}_{b,t-1}^M + \hat{i}_{t-1}^M - \hat{\pi}_t^M - (1-\beta)\hat{\tau}_t^M\right] \tag{C.2}$$

$$\hat{s}_{bt}^{M*} = \frac{1}{\beta}\left[\hat{s}_{b,t-1}^{M*} + \hat{i}_{t-1}^M - \hat{\pi}_t^{M*} - (1-\beta)\hat{\tau}_t^{M*}\right] \tag{C.3}$$

$$\hat{\tau}_t^M = \gamma^M \hat{s}_{b,t-1}^M + \zeta_t^M \tag{C.4}$$

$$\hat{\tau}_t^{M*} = \gamma^{M*} \hat{s}_{b,t-1}^{M*} + \zeta_t^{M*} \tag{C.5}$$

$$\hat{i}_t^M = \Theta\phi^M \hat{\pi}_t^M + (1-\Theta)\phi^{M*}\hat{\pi}_t^{M*} \tag{C.6}$$

We obtain the monetary and fiscal blocks of this parallel economy as with other economies in the main text:

$$\begin{aligned}\Theta \mathbb{E}_t \hat{\pi}_{t+1}^M + (1-\Theta)\mathbb{E}_t \hat{\pi}_{t+1}^{M*} &= \Theta\phi^M \hat{\pi}_t^M + (1-\Theta)\phi^{M*}\hat{\pi}_t^{M*} \\ &\quad - \Theta\psi_b \frac{\overline{b}_A + \overline{b}_B}{\overline{b}_A}\left(\hat{b}_{Bt} - \hat{b}_{At}\right) - (1-\Theta)\psi_b \frac{\overline{b}_A^* + \overline{b}_B^*}{\overline{b}_A^*}\left(\hat{b}_{Bt}^* - \hat{b}_{At}^*\right)\end{aligned} \tag{C.7}$$

$$\hat{s}_{bt}^M = \frac{1}{\beta}\left[(1-(1-\beta)\gamma^M)\hat{s}_{b,t-1}^M + \hat{i}_{t-1}^M - \hat{\pi}_t^M - (1-\beta)\zeta_t^M\right] \tag{C.8}$$

$$\hat{s}_{bt}^{M*} = \frac{1}{\beta}\left[(1-(1-\beta)\gamma^{M*})\hat{s}_{b,t-1}^{M*} + \hat{i}_{t-1}^M - \hat{\pi}_t^{M*} - (1-\beta)\zeta_t^{M*}\right] \tag{C.9}$$

We prove $\hat{\pi}_t^M = \hat{\pi}_t - \hat{\pi}_t^F$ by subtracting the equations for the monetary block of the monetary-led economy (C.7) and fiscal-led economy (39) and (40) from the monetary block equation of the overall economy (35):[8]

$$\begin{aligned}&\Theta \mathbb{E}_t\left[\hat{\pi}_{t+1} - \hat{\pi}_{t+1}^M - \hat{\pi}_{t+1}^F\right] + (1-\Theta)\mathbb{E}_t\left[\hat{\pi}_{t+1}^* - \hat{\pi}_{t+1}^{M*} - \hat{\pi}_{t+1}^{F*}\right] \\ &\qquad = \Theta\phi^M\left(\hat{\pi}_t - \hat{\pi}_t^M - \hat{\pi}_t^F\right) + (1-\Theta)\phi^{M*}\left(\hat{\pi}_t^* - \hat{\pi}_t^{M*} - \hat{\pi}_t^{F*}\right)\end{aligned} \tag{C.10}$$

where we multiply the fiscal-led economy equations with the respective country weights before subtracting. Since $\phi^M = \phi^{M*} > 1$, the above equation implies $\hat{\pi}_t - \hat{\pi}_t^M - \hat{\pi}_t^F = 0$ and $\hat{\pi}_t^* - \hat{\pi}_t^{M*} - \hat{\pi}_t^{F*} = 0$, thus $\hat{\pi}_t^M = \hat{\pi}_t - \hat{\pi}_t^F$ and $\hat{\pi}_t^{M*} = \hat{\pi}_t^* - \hat{\pi}_t^{F*}$.

Now, we prove that $\hat{s}_{bt}^M = \hat{s}_{bt} - \hat{s}_{bt}^F$. Substituting the fiscal rule of country A and monetary rule that allow for partially unfunded debt, equations (32) and (34) respectively,

8. Impose the no-arbitrage conditions to simplify the adjustment cost term.

into the country A government budget constraint (23), we obtain:

$$\beta\hat{s}_{bt} = \left(1-(1-\beta)\gamma^{M}\right)\hat{s}_{b,t-1} + (1-\beta)\gamma^{M}\hat{s}^{F}_{b,t-1} - (1-\beta)\gamma^{F}\hat{s}^{F}_{b,t-1} - \hat{\pi}_t - (1-\beta)\left(\zeta^{M}_t + \zeta^{F}_t\right)$$
$$+ \Theta\left[\phi^{M}\left(\hat{\pi}_{t-1} - \hat{\pi}^{F}_{t-1}\right) + \phi^{F}\hat{\pi}^{F}_{t-1}\right] + (1-\Theta)\left[\phi^{M*}\left(\hat{\pi}^{*}_{t-1} - \hat{\pi}^{F*}_{t-1}\right) + \phi^{F*}\hat{\pi}^{F*}_{t-1}\right]. \quad \text{(C.11)}$$

We obtain the equivalent equations for the monetary-led and fiscal-led parallel economies:

$$\beta\hat{s}^{M}_{bt} = (1-(1-\beta)\gamma^{M})\hat{s}^{M}_{b,t-1} + \Theta\phi^{M}\hat{\pi}^{M}_{t-1} + (1-\Theta)\phi^{M*}\hat{\pi}^{M*}_{t-1} - \hat{\pi}^{M}_t - (1-\beta)\zeta^{M}_t \quad \text{(C.12)}$$

$$\beta\hat{s}^{F}_{bt} = (1-(1-\beta)\gamma^{F})\hat{s}^{F}_{b,t-1} + \Theta\phi^{F}\hat{\pi}^{F}_{t-1} + (1-\Theta)\phi^{F*}\hat{\pi}^{F*}_{t-1} - \hat{\pi}^{F}_t - (1-\beta)\zeta^{F}_t \quad \text{(C.13)}$$

Subtract $\beta\hat{s}^{M}_{bt}$ and $\beta\hat{s}^{F}_{bt}$ from $\beta\hat{s}_{bt}$ to obtain:

$$\beta\left(\hat{s}_{bt} - \hat{s}^{M}_{bt} - \hat{s}^{F}_{bt}\right) = \left(1-(1-\beta)\gamma^{M}\right)\left(\hat{s}_{b,t-1} - \hat{s}^{M}_{b,t-1} - \hat{s}^{F}_{b,t-1}\right) - \left(\hat{\pi}_t - \hat{\pi}^{M}_t - \hat{\pi}^{F}_t\right)$$
$$+ \Theta\phi^{M}\left(\hat{\pi}_{t-1} - \hat{\pi}^{M}_{t-1} - \hat{\pi}^{F}_{t-1}\right) + (1-\Theta)\phi^{M*}\left(\hat{\pi}^{*}_{t-1} - \hat{\pi}^{M*}_{t-1} - \hat{\pi}^{F*}_{t-1}\right) \quad \text{(C.14)}$$

Using $\hat{\pi}^{M}_t = \hat{\pi}_t - \hat{\pi}^{F}_t$ and $\hat{\pi}^{M*}_t = \hat{\pi}^{*}_t - \hat{\pi}^{F*}_t$, it must be that $\hat{s}_{bt} - \hat{s}^{M}_{bt} - \hat{s}^{F}_{bt} = 0$ and hence $\hat{s}^{M}_{bt} = \hat{s}_{bt} - \hat{s}^{F}_{bt}$ for all periods. The proof for $\hat{s}^{M*}_{bt} = \hat{s}^{*}_{bt} - \hat{s}^{F*}_{bt}$ is analogous.

D Log-linearized equations (quantitative model)

Since the quantitative model builds onto the stylized model, the equations in Appendix B and C remain the same unless otherwise stated.

Govt. budget constraint A
$$\hat{s}_{bt} = \frac{1}{\beta}\left[\hat{s}_{b,t-1} + \hat{i}^L_t - \hat{\pi}_t - (1-\beta)\hat{\tau}_t\right] \tag{D.1}$$

Govt. budget constraint B
$$\hat{s}^*_{bt} = \frac{1}{\beta}\left[\hat{s}^*_{b,t-1} + \hat{i}^{L*}_t - \hat{\pi}^*_t - (1-\beta)\hat{\tau}^*_t\right] \tag{D.2}$$

Effective consumption A
$$\hat{c}^{\text{eff}}_t = \frac{1}{1-h}\hat{c}_t - \frac{h}{1-h}\hat{c}_{t-1} \tag{D.3}$$

Effective consumption B
$$\hat{c}^{\text{eff}*}_t = \frac{1}{1-h}\hat{c}^*_t - \frac{h}{1-h}\hat{c}^*_{t-1} \tag{D.4}$$

Marginal utility A
$$\hat{\lambda}_t = -\frac{1}{1-\beta h}\left(\hat{c}^{\text{eff}}_t - \beta h \mathbb{E}_t \hat{c}^{\text{eff}}_{t+1}\right) + \frac{1}{1-\beta h}\left(\xi^d_t - \beta h \mathbb{E}_t \xi^d_{t+1}\right) \tag{D.5}$$

Marginal utility B
$$\hat{\lambda}^*_t = -\frac{1}{1-\beta h}\left(\hat{c}^{\text{eff}*}_t - \beta h \mathbb{E}_t \hat{c}^{\text{eff}*}_{t+1}\right) + \frac{1}{1-\beta h}\left(\xi^{d*}_t - \beta h \mathbb{E}_t \xi^{d*}_{t+1}\right) \tag{D.6}$$

Preference shock A
$$\xi^d_t = \rho_d \xi^d_{t-1} + \varepsilon^d_t \tag{D.7}$$

Preference shock B
$$\xi^{d*}_t = \rho_d \xi^{d*}_{t-1} + \varepsilon^{d*}_t \tag{D.8}$$

Agg. Euler equation
$$\mathbb{E}_t \hat{\lambda}^{MU}_{t+1} = \hat{\lambda}^{MU}_t - \left(\hat{i}_t - \mathbb{E}_t \hat{\pi}^{MU}_{t+1}\right) \tag{D.9}$$

Agg. capital Euler equation
$$\hat{q}^{MU}_t = \mathbb{E}_t \hat{\lambda}^{MU}_{t+1} - \hat{\lambda}^{MU}_t + [1-\beta(1-\delta)]\mathbb{E}_t \hat{r}^{MU}_{K,t+1} + \beta(1-\delta)\mathbb{E}_t \hat{q}^{MU}_{t+1} \tag{D.10}$$

No arbitrage capital and ST bonds A
$$\hat{q}_t - [1-\beta(1-\delta)]\mathbb{E}_t \hat{r}_{K,t+1} - \beta(1-\delta)\mathbb{E}_t \hat{q}_{t+1} = -\left(\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1}\right) \tag{D.11}$$

No arbitrage LT and ST bonds A
$$\hat{i}_t = \mathbb{E}_t \hat{r}^L_{t+1} + \xi^{rp}_t - \frac{1-\beta\rho^L}{\beta}\psi_b \frac{\overline{b}_A + \overline{b}_B}{\overline{b}_A}\left(\hat{b}_{Bt} - \hat{b}_{At}\right) \tag{D.12}$$

No arbitrage LT and ST bonds B
$$\hat{i}_t = \mathbb{E}_t \hat{r}^{L*}_{t+1} + \xi^{rp*}_t - \frac{1-\beta\rho^L}{\beta}\psi_b \frac{\overline{b}^*_A + \overline{b}^*_B}{\overline{b}^*_B}\left(\hat{b}^*_{Bt} - \hat{b}^*_{At}\right) \tag{D.13}$$

Risk premium shock A
$$\xi^{rp}_t = \rho_{rp}\xi^{rp}_{t-1} + \varepsilon^{rp}_t \tag{D.14}$$

Risk premium shock B
$$\xi^{rp*}_t = \rho_{rp}\xi^{rp*}_{t-1} + \varepsilon^{rp*}_t \tag{D.15}$$

No arbitrage across LT bonds A $\hat{p}_t^L - \hat{p}_t^{L*} = \beta\rho^L \mathbb{E}_t\left(\hat{p}_{t+1}^L - \hat{p}_{t+1}^{L*}\right) + \left(\xi_t^{rp} - \xi_t^{rp*}\right)$

$$- \frac{1-\beta\rho^L}{\beta}\psi_b \frac{\overline{b}_A + \overline{b}_B}{\overline{b}_A}\left(\hat{b}_{Bt} - \hat{b}_{At}\right) + \frac{1-\beta\rho^L}{\beta}\psi_b \frac{\overline{b}_A + \overline{b}_B}{\overline{b}_B}\left(\hat{b}_{Bt} - \hat{b}_{At}\right) \quad \text{(D.16)}$$

No arbitrage across LT bonds B $\hat{p}_t^L - \hat{p}_t^{L*} = \beta\rho^L \mathbb{E}_t\left(\hat{p}_{t+1}^L - \hat{p}_{t+1}^{L*}\right) + \left(\xi_t^{rp} - \xi_t^{rp*}\right)$

$$- \frac{1-\beta\rho^L}{\beta}\psi_b \frac{\overline{b}_A^* + \overline{b}_B^*}{\overline{b}_A^*}\left(\hat{b}_{Bt}^* - \hat{b}_{At}^*\right) + \frac{1-\beta\rho^L}{\beta}\psi_b \frac{\overline{b}_A^* + \overline{b}_B^*}{\overline{b}_B^*}\left(\hat{b}_{Bt}^* - \hat{b}_{At}^*\right) \quad \text{(D.17)}$$

Return on LT bonds A $$\hat{r}_t^L = \rho^L \beta \hat{p}_t^L - \hat{p}_{t-1}^L \tag{D.18}$$

Return on LT bonds B $$\hat{r}_t^{L*} = \rho^L \beta \hat{p}_t^{L*} - \hat{p}_{t-1}^{L*} \tag{D.19}$$

Labor supply A $$\hat{c}_t^{\text{eff}} + \chi \hat{l}_t + \varepsilon_t^h = \widehat{mrs}_t \tag{D.20}$$

Labor supply B $$\hat{c}_t^{\text{eff}*} + \chi \hat{l}_t^* + \varepsilon_t^{h*} = \widehat{mrs}_t^* \tag{D.21}$$

Demand trad./non-trad. A $$\hat{c}_{Tt} - \hat{c}_{Nt} = -\mu_N \left(\hat{p}_{Tt} - \hat{p}_{Nt}\right) \tag{D.22}$$

Demand trad./non-trad. B $$\hat{c}_{Tt}^* - \hat{c}_{Nt}^* = -\mu_N \left(\hat{p}_{Tt}^* - \hat{p}_{Nt}^*\right) \tag{D.23}$$

Tobin's Q A $$\hat{q}_t = \psi_I \left[\widehat{inv}_t - \widehat{inv}_{t-1} - \beta \left(\mathbb{E}_t \widehat{inv}_{t+1} - \widehat{inv}_t\right)\right] - \xi_t^k \tag{D.24}$$

Tobin's Q B $$\hat{q}_t^* = \psi_I \left[\widehat{inv}_t^* - \widehat{inv}_{t-1}^* - \beta \left(\mathbb{E}_t \widehat{inv}_{t+1}^* - \widehat{inv}_t^*\right)\right] - \xi_t^{k*} \tag{D.25}$$

Capital accumulation A $$\hat{k}_t = (1-\delta)\hat{k}_{t-1} + \delta \left(\widehat{inv}_t + \xi_t^k\right) \tag{D.26}$$

Capital accumulation B $$\hat{k}_t^* = (1-\delta)\hat{k}_{t-1}^* + \delta \left(\widehat{inv}_t^* + \xi_t^{k*}\right) \tag{D.27}$$

MEI shock A $$\xi_t^k = \rho_k \xi_{t-1}^k + \varepsilon_t^k \tag{D.28}$$

MEI shock B $$\xi_t^{k*} = \rho_k \xi_{t-1}^{k*} + \varepsilon_t^{k*} \tag{D.29}$$

Capital services A $$\hat{k}_t^s = \hat{k}_{t-1} + \hat{\mu}_{K,t} \tag{D.30}$$

Capital services B $$\hat{k}_t^{s*} = \hat{k}_{t-1}^* + \hat{\mu}_{K,t}^* \tag{D.31}$$

FOC utilization A $$\hat{\mu}_{K,t} = \frac{1-\psi_u}{\psi_u} \hat{r}_{Kt} \tag{D.32}$$

FOC utilization B $$\hat{\mu}_{K,t}^* = \frac{1-\psi_u}{\psi_u} \hat{r}_{Kt}^* \tag{D.33}$$

Production function trad. A $$\hat{y}_{Tt} = \hat{a}_{Tt} + \alpha \hat{k}_t^s + (1-\alpha)\hat{l}_{Tt} \tag{D.34}$$

Production function trad. B $$\hat{y}_{Tt}^* = \hat{a}_{Tt}^* + \alpha \hat{k}_t^{s*} + (1-\alpha)\hat{l}_{Tt}^* \tag{D.35}$$

Technology trad. A $$\hat{a}_{Tt} = \rho_a \hat{a}_{T,t-1} + \varepsilon_t^T \tag{D.36}$$

Technology trad. B $$\hat{a}_{Tt}^* = \rho_a \hat{a}_{T,t-1}^* + \varepsilon_t^{T*} \tag{D.37}$$

NKPC trad. A $$\hat{\pi}_{Tt} = \beta \mathbb{E}_t \hat{\pi}_{T,t+1} + \kappa_T \widehat{mc}_{Tt} + u_t^T \tag{D.38}$$

NKPC trad. B $$\hat{\pi}^*_{Tt} = \beta \mathbb{E}_t \hat{\pi}^*_{T,t+1} + \kappa_T \widehat{mc}^*_{Tt} + u_t^{T*} \tag{D.39}$$

Tradable goods markup shock A $$u_t^T = \rho_T u_{t-1}^T + \varepsilon_t^{uT} \tag{D.40}$$

Tradable goods markup shock B $$u_t^{T*} = \rho_T u_{t-1}^{T*} + \varepsilon_t^{uT*} \tag{D.41}$$

Marginal cost trad. A $$\widehat{mc}_{Tt} = \hat{w}_t^{real} + \hat{l}_{Tt} - \hat{y}_{Tt} \tag{D.42}$$

Marginal cost trad. B $$\widehat{mc}^*_{Tt} = \hat{w}_t^{*,real} + \hat{l}^*_{Tt} - \hat{y}^*_{Tt} \tag{D.43}$$

Capital demand A $$\hat{y}_{Tt} - \hat{k}_t^s = \hat{r}_{Kt} \tag{D.44}$$

Capital demand B $$\hat{y}^*_{Tt} - \hat{k}_t^{s*} = \hat{r}^*_{Kt} \tag{D.45}$$

Wage Phillips curve A $$\hat{\pi}_t^w = \beta \mathbb{E}_t \hat{\pi}_{t+1}^w + \chi_w \hat{\pi}_{t-1} + \kappa_w \left(\widehat{mrs}_t - \hat{w}_t^{real} \right) + u_t^w \tag{D.46}$$

Wage Phillips curve B $$\hat{\pi}_t^{w*} = \beta \mathbb{E}_t \hat{\pi}_{t+1}^{w*} + \chi_w \hat{\pi}^*_{t-1} + \kappa_w \left(\widehat{mrs}^*_t - \hat{w}_t^{*,real} \right) + u_t^{w*} \tag{D.47}$$

Wage markup shock A $$u_t^w = \rho_w u_{t-1}^w + \varepsilon_t^w \tag{D.48}$$

Wage markup shock B $$u_t^{w*} = \rho_w u_{t-1}^{w*} + \varepsilon_t^{w*} \tag{D.49}$$

Production function non-trad. A $$\hat{y}_{Nt} = \hat{a}_{Nt} + (1-\alpha)\hat{l}_{Nt} \tag{D.50}$$

Production function non-trad. B $$\hat{y}^*_{Nt} = \hat{a}^*_{Nt} + (1-\alpha)\hat{l}^*_{Nt} \tag{D.51}$$

Technology non-trad. A $$\hat{a}_{Nt} = \rho_a \hat{a}_{N,t-1} + \varepsilon_t^N \tag{D.52}$$

Technology non-trad. B $$\hat{a}^*_{Nt} = \rho_a \hat{a}^*_{N,t-1} + \varepsilon_t^{N*} \tag{D.53}$$

NKPC non-trad. A $$\hat{\pi}_{Nt} = \beta \mathbb{E}_t \hat{\pi}_{N,t+1} + \kappa_N \widehat{mc}_{Nt} + u_t^N \tag{D.54}$$

NKPC non-trad. B $$\hat{\pi}^*_{Nt} = \beta \mathbb{E}_t \hat{\pi}^*_{N,t+1} + \kappa_N \widehat{mc}^*_{Nt} + u_t^{N*} \tag{D.55}$$

Non-tradable goods markup shock A $$u_t^N = \rho_N u_{t-1}^N + \varepsilon_t^{uN} \tag{D.56}$$

Non-tradable goods markup shock B $$u_t^{N*} = \rho_N u_{t-1}^{N*} + \varepsilon_t^{uN*} \tag{D.57}$$

Government spending A $$\hat{g}_t = \rho_g \hat{g}_{t-1} + \varepsilon_t^g \tag{D.58}$$

Government spending B $$\hat{g}^*_t = \rho_g \hat{g}^*_{t-1} + \varepsilon_t^{g*} \tag{D.59}$$

COVID labor supply A $$\varepsilon_t^h \sim \mathcal{N}(0, \sigma_h^2 \mathbb{1}_{t \in \text{COVID}}) \tag{D.60}$$

COVID labor supply B $$\varepsilon_t^{h*} \sim \mathcal{N}(0, \sigma_{h*}^2 \mathbb{1}_{t \in \text{COVID}}) \tag{D.61}$$

Fiscal rule A $$\hat{\tau}_t = \gamma \hat{s}_{b,t-1} - \frac{\overline{G}/\overline{Y}}{\overline{\tau}} \hat{g}_t + \frac{1}{\overline{\tau}} \zeta_t \tag{D.62}$$

Fiscal rule B $$\hat{\tau}^*_t = \gamma^* \hat{s}^*_{b,t-1} - \frac{\overline{G}/\overline{Y}}{\overline{\tau}} \hat{g}^*_t + \frac{1}{\overline{\tau}} \zeta^*_t \tag{D.63}$$

CPI inflation A $$\hat{\pi}_t = (1-\alpha_N)\left[(1-\alpha_I)\hat{\pi}_{Tt} + \alpha_I \hat{\pi}^*_{Tt}\right] + \alpha_N \hat{\pi}_{Nt} \tag{D.64}$$

CPI inflation B $$\hat{\pi}^*_t = (1-\alpha_N)\left[(1-\alpha_I)\hat{\pi}^*_{Tt} + \alpha_I \hat{\pi}_{Tt}\right] + \alpha_N \hat{\pi}^*_{Nt} \tag{D.65}$$

Marginal cost non-trad. A $$\widehat{mc}_{Nt} = \hat{w}_t^{real} + \hat{l}_{Nt} - \hat{y}_{Nt} \tag{D.66}$$

Marginal cost non-trad. B $$\widehat{mc}^*_{Nt} = \hat{w}_t^{*,real} + \hat{l}^*_{Nt} - \hat{y}^*_{Nt} \tag{D.67}$$

Market clearing trad. A $$\hat{y}_{Tt} = (1-\alpha_I)\hat{c}_{Tt} + \alpha_I \frac{1-\Theta}{\Theta}\hat{c}^*_{Tt} + 2\mu\alpha_I(1-\alpha_I)\hat{s}_t + \frac{\overline{I}}{\overline{Y}}\widehat{inv}_t + \frac{\overline{G}}{\overline{Y}}\hat{g}_t + \alpha\hat{\mu} \tag{D.68}$$

Market clearing trad. B $$\hat{y}^*_{Tt} = (1-\alpha_I)\hat{c}^*_{Tt} + \alpha_I \frac{\Theta}{1-\Theta}\hat{c}_{Tt} - 2\mu\alpha_I(1-\alpha_I)\hat{s}_t + \frac{\overline{I}}{\overline{Y}}\widehat{inv}^*_t + \frac{\overline{G}}{\overline{Y}}\hat{g}^*_t + \alpha\hat{\mu} \tag{D.69}$$

Market clearing non-trad. A $$\hat{y}_{Nt} = \hat{c}_{Nt} \tag{D.70}$$

Market clearing non-trad. B $$\hat{y}^*_{Nt} = \hat{c}^*_{Nt} \tag{D.71}$$

Current account $$\hat{y}_{Tt} - \hat{c}_{Tt} - \alpha_I\hat{s}_t - \frac{\overline{I}}{\overline{Y}}\widehat{inv}_t - \frac{\overline{G}}{\overline{Y}}\hat{g}_t - \alpha\hat{\mu}_{K,t} = \widehat{ca}_t \tag{D.72}$$

Labor market clearing A $$\hat{l}_t = (1-\alpha_N)\hat{l}_{Tt} + \alpha_N\hat{l}_{Nt} \tag{D.73}$$

Labor market clearing B $$\hat{l}^*_t = (1-\alpha_N)\hat{l}^*_{Tt} + \alpha_N\hat{l}^*_{Nt} \tag{D.74}$$

Aggregate consumption A $$\hat{c}_t = (1-\alpha_N)\hat{c}_{Tt} + \alpha_N\hat{c}_{Nt} \tag{D.75}$$

Aggregate consumption B $$\hat{c}^*_t = (1-\alpha_N)\hat{c}^*_{Tt} + \alpha_N\hat{c}^*_{Nt} \tag{D.76}$$

Steady state. We characterize the steady state around which we log-linearize. We impose zero steady-state inflation ($\overline{\Pi} = 1$) and normalize the steady-state price level as $\overline{P} = 1$. The steady-state gross nominal interest rate is $\overline{R} = \beta^{-1}$.

Long-term bond price. With geometrically decaying coupons with rate ρ^L, the steady-state long-term bond price satisfies

$$\overline{P}^L = \beta\left(1 + \rho^L\overline{P}^L\right) \quad \Longrightarrow \quad \overline{P}^L = \frac{\beta}{1-\beta\rho^L}. \tag{D.77}$$

Defining the real market value of government debt as $\overline{s}_b \equiv \frac{\overline{P}^L\overline{B}}{\overline{P}\,\overline{Y}}$, the steady-state government budget constraint implies

$$\overline{T} = \frac{1-\beta}{\beta}\overline{s}_b \quad \Longleftrightarrow \quad \overline{s}_b = \frac{\beta}{1-\beta}\overline{T}. \tag{D.78}$$

Capital and investment shares. In steady state Tobin's Q is one, and the capital Euler equation implies

$$1 = \beta\left(\overline{R}_K + 1 - \delta\right) \quad \Longrightarrow \quad \overline{R}_K = \frac{1}{\beta} - (1-\delta). \tag{D.79}$$

Using the tradable-sector capital demand condition $\overline{R}_K = \alpha \frac{\overline{Y}_A}{\overline{K}}$, we obtain the steady-state capital-output ratio in the tradable sector,

$$\frac{\overline{K}}{\overline{Y}_A} = \frac{\alpha}{\overline{R}_K}. \tag{D.80}$$

Finally, the capital law of motion implies $\overline{I} = \delta \overline{K}$, so the steady-state investment share (used in the log-linear tradable market clearing and current account equations) is

$$\frac{\overline{I}}{\overline{Y}_A} = \delta \frac{\overline{K}}{\overline{Y}_A} = \delta \frac{\alpha}{\frac{1}{\beta} - (1-\delta)}. \tag{D.81}$$

Government spending share. The steady-state government spending-to-output ratio $\frac{\overline{G}}{\overline{Y}}$ is a calibrated parameter, set to match the average share of government final consumption expenditure in GDP. This ratio appears in the log-linearized tradable market clearing and current account equations.

E Two-country model with flexible nominal exchange rates

To consider the role of the common central bank in a currency union, we also build an equivalent two-country model with flexible nominal exchange rates. We define the nominal exchange rate $\mathcal{E}_t$ as the price of the country B currency in terms of the currency in country A and incur the following changes to the currency union model.

We change the nominal budget constraint of the households in country A to:

$$\begin{aligned} &P_t C_t + B_{At} + \mathcal{E}_t B_{Bt} + P_t T_t \\ &\qquad = P_t Y + R_{t-1} B_{A,t-1} + \mathcal{E}_t R^*_{t-1} B_{B,t-1} - \frac{\psi_b P_t \left(\overline{b}_A + \overline{b}_B\right)}{2} \left(\delta_b \frac{\mathcal{E}_t B_{Bt}}{B_{At}} - 1\right)^2 \end{aligned} \tag{E.1}$$

Then, the two Euler equations become:

$$1 = \frac{\beta R_t}{1 - \psi_b (\overline{b}_A + \overline{b}_B) \left(\delta_b \frac{\mathcal{E}_t b_{Bt}}{b_{At}} - 1\right) \delta_b \frac{\mathcal{E}_t b_{Bt}}{b^2_{At}}} \mathbb{E}_t \left(\Pi^{-1}_{t+1} \frac{C_t}{C_{t+1}}\right) \tag{E.2}$$

$$1 = \frac{\beta R^*_t}{1 + \psi_b \left(\overline{b}_A + \overline{b}_B\right) \left(\delta_b \frac{\mathcal{E}_t b_{Bt}}{b_{At}} - 1\right) \delta_b \frac{1}{b_{At}}} \mathbb{E}_t \left(\Pi^{-1}_{t+1} \frac{C_t}{C_{t+1}} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right) \tag{E.3}$$

where the first Euler equation, with respect to b_{At}, remains unchanged.

The definition of the terms of trade remains $S_t = \frac{P_{Bt}}{P_{At}}$ but the interpretation changes to the relative price of the good produced in country B to the good produced in country A, expressed in the country A currency. Assuming the law of one price holds, the goods

prices in the country B currency are the following:

$$P_{At} = P^*_{At}\mathcal{E}_t \tag{E.4}$$

$$P_{Bt} = P^*_{Bt}\mathcal{E}_t \tag{E.5}$$

We define the real exchange rate as:

$$RER_t \equiv \frac{\mathcal{E}_t P^*_t}{P_t} \tag{E.6}$$

There are now two monetary authorities, one for each country, targeting the domestic gross inflation:

$$\frac{R_t}{\overline{R}} = \left(\frac{\Pi_t}{\overline{\Pi}}\right)^{\phi} \tag{E.7}$$

$$\frac{R^*_t}{\overline{R}^*} = \left(\frac{\Pi^*_t}{\overline{\Pi}^*}\right)^{\phi^*} \tag{E.8}$$

Finally, we modify the current account equation as:

$$CA_t \equiv P_{At}Y - P_tC_t = \mathcal{E}_tB_{Bt} - B^*_{At} - \left(R^*_{t-1}\mathcal{E}_tB_{B,t-1} - R_{t-1}B^*_{A,t-1}\right) \tag{E.9}$$

With the updated Euler equations, the aggregate Euler equations and the no-arbitrage equations in the log-linear system become:

$$\begin{aligned}\hat{c}^{MU}_t = \mathbb{E}_t\hat{c}^{MU}_{t+1} - \left[\Theta\hat{i}_t + (1-\Theta)\hat{i}^*_t - \mathbb{E}_t\hat{\pi}^{MU}_{t+1}\right] \\ - \Theta\psi_b\frac{\bar{b}_A + \bar{b}_B}{\bar{b}_A}\left(\hat{b}_{Bt} + \hat{e}_t - \hat{b}_{At}\right) + (1-\Theta)\psi_b\frac{\bar{b}^*_A + \bar{b}^*_B}{\bar{b}^*_A}\left(\hat{b}^*_{Bt} - \hat{b}^*_{At} - \hat{e}_t\right)\end{aligned} \tag{E.10}$$

$$-\psi_b\frac{\bar{b}_A + \bar{b}_B}{\bar{b}_A}\left(\hat{b}_{Bt} + \hat{e}_t - \hat{b}_{At}\right) - \hat{i}_t = \psi_b\frac{\bar{b}_A + \bar{b}_B}{\bar{b}_B}\left(\hat{b}_{Bt} + \hat{e}_t - \hat{b}_{At}\right) - \hat{i}^*_t - \Delta\hat{e}_{t+1} \tag{E.11}$$

$$-\psi_b\frac{\bar{b}^*_A + \bar{b}^*_B}{\bar{b}^*_A}\left(\hat{b}^*_{Bt} - \hat{b}^*_{At} - \hat{e}_t\right) - \hat{i}_t + \Delta\hat{e}_{t+1} = \psi_b\frac{\bar{b}^*_A + \bar{b}^*_B}{\bar{b}^*_B}\left(\hat{b}^*_{Bt} - \hat{b}^*_{At} - \hat{e}_t\right) - \hat{i}^*_t \tag{E.12}$$

For the aggregate Euler equation, we use the Euler equation in country A with respect to B_{At} and the Euler equation in country B with respect to B^*_{Bt} to avoid adding the nominal depreciation $\Delta\hat{e}_t = \hat{e}_t - \hat{e}_{t-1}$ in the Euler equation. The no-arbitrage equations are the uncovered interest parity (UIP) conditions.

The log-linear monetary rules for each country are:

$$\hat{i}_t = \phi\hat{\pi}_t \tag{E.13}$$

$$\hat{i}^*_t = \phi^*\hat{\pi}^*_t \tag{E.14}$$

Finally, the current account equation becomes:

$$-\hat{c}_t - \alpha_I\hat{s}_t = \overline{b}_B\left(\hat{b}_{Bt} + \hat{e}_t\right) - \overline{b}^*_Ab^*_{At} - \frac{1}{\beta}\left[\overline{b}_B\left(\hat{b}_{B,t-1} + \hat{r}^*_{t-1} + \hat{e}_{t-1}\right) - \overline{b}^*_A\left(b^*_{A,t-1} + \hat{r}_{t-1}\right)\right] \tag{E.15}$$