Firm Dynamics and Sectoral Reallocation*

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Abstract

The US economy has experienced declining business dynamism alongside substantial sectoral reallocation. To jointly investigate these phenomena, we develop a multisector model with input-output linkages that features endogenous entry and exit, as well as lifecycle profiles of size and exit. We provide conditions under which prices fully offset aggregate shocks. This property implies sectoral exit rates by firm-age are invariant, consistent with the relative stability of exit rates of older US firms. We apply the framework to the decline of US manufacturing. Employment reallocation accounts for one-third of the observed decline in manufacturing entry. We trace this reallocation to three forces: population growth, sectoral technological change, and international trade. Differences in sectoral exit-age profiles imply that, absent changes in population growth, employment reallocation would have been larger. Sectoral technological change accounts for most of the decline in the manufacturing employment share, while international trade plays a substantial role in the decline in the manufacturing firm share.

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1 Introduction

The US economy has experienced declining business dynamism over the past four decades. The firm entry rate has declined by 6 percentage points from 14% in the 1970s to 8% in 2020, the exit rate has declined, and average firm size has increased markedly. Aggregate data, however, mask sharp divergence across US sectors. While both manufacturing and nonmanufacturing experienced comparable declines in firm entry-exit rates, average firm size in nonmanufacturing rose by 50% while it fell by 30% in manufacturing. The employment share of manufacturing declined from 26% in 1978 to 9% in 2019, a reallocation of 8 million jobs from manufacturing to nonmanufacturing. What explains the sectoral patterns?

Sectoral entry rates can be analyzed through the lens of a simple identity:

$$\text{entry rate}_i = \text{pop. growth} + \widehat{\text{emp. share}}_i - \widehat{\text{avg. size}}_i + \text{exit rate}_i, \tag{1}$$

where i indicates sector and hats indicate growth rates.[1] At the aggregate level, the identity shows that population growth drives firm entry rates.[2] For sectoral entry rates, however, the rate of employment reallocation also matters. Any force that drives employment reallocation influences sectoral firm dynamics, which in turn feed back into the pace of reallocation. This paper asks: how does sectoral reallocation interact with firm dynamics?

To address this question, this paper extends the firm dynamics framework in Hopenhayn, Neira and Singhania (2022) to multiple sectors. The framework features firm entry, growth, and endogenous selection. Together these generate realistic firm demographic patterns: average size increases and exit rates decline

[1] This decomposition follows from simple accounting identities. Let $AFS_{it} \equiv N_{it}/M_{it}$ denote average firm size in sector i, where N_{it} is employment and M_{it} is the number of firms. Employment can be written as $N_{it} = L_t s_{it}$, where L_t is population and s_{it} is the sectoral employment share. Taking growth rates gives $\hat{M}_i = \hat{N}_i - \widehat{AFS}_i = g + \hat{s}_i - \widehat{AFS}_i$, where g is population growth. Since firm growth also satisfies $\hat{M}_i = \lambda_i - \xi_i$, where λ_i and ξ_i denote entry and exit rates, respectively, combining these expressions yields $\lambda_i = g + \hat{s}_i - \widehat{AFS}_i + \xi_i$.

[2] A slowdown in US population growth is a key driver of aggregate entry rates (Hopenhayn, Neira and Singhania, 2022; Karahan, Pugsley and Şahin, 2024; Peters and Walsh, 2021). Firm aging combined with firm demographics—the age profiles of exit rates and average firm size—amplifies the effects of the initial population growth shock. The population growth slowdown reduces firm entry relative to the existing stock of incumbent firms, shifting the firm-age distribution towards older firms. Exit rates decline as firms age while average firm size increases with age, so firm aging induced by population growth generates feedback effects that are essential to explaining the decline in US business dynamism.

with firm age. On the sectoral side, the framework incorporates input-output linkages, and rich within-sector firm dynamics. In addition to establishing existence and uniqueness of equilibrium in this richer environment, we provide conditions under which prices fully offset shocks to population growth and sectoral technology. This property has two useful implications. First, sector exit rates by firm-age are invariant to aggregate shocks, consistent with relative stability of exit rates of older firms in US data. Second, prices and firm entry are solved period-by-period without needing to specify a stochastic process for the aggregate state. This makes the framework well-suited for counterfactual analysis.

The framework generates rich transitional dynamics following aggregate and sectoral shocks. Firm entry in a sector depends on both static and dynamic linkages across sectors. Static linkages arise from input-output relationships, as in Baqaee (2018) and Cavalcanti, Mendes and Pannella (2023). Dynamic linkages operate through two channels: own-sector history dependence, as in Hopenhayn et al. (2022), and a cross-sector channel through which entry in one sector depends on the history of entry in all other sectors. The dynamics linkages imply that a shock that affected manufacturing entry in the past will persist in firm entry of nonmanufacturing sectors decades later.

We apply the framework to understand the evolution of US manufacturing. Population growth not only drives sectoral entry rates but also generates sectoral reallocation through differences in firm demographics across sectors, making it an essential force in any analysis of the manufacturing decline. We consider two additional driving forces central to the evolution of manufacturing: changes in sectoral technology, broadly defined to include demand-side factors, and international trade. The structural transformation literature highlights how changes in sectoral technology reallocate resources away from manufacturing.[3] The trade literature documents that rising import competition has contributed to the decline of manufacturing employment (Pierce and Schott, 2016; Fort, Pierce and Schott, 2018).

The quantitative analysis considers all three driving forces: changes in population growth, sectoral technology and international trade. Can the framework with

[3]Supply side drivers of reallocation include improvements in sectoral productivity or factor intensity differences, e.g. Ngai and Pissarides (2007); Acemoglu and Guerrieri (2008). Demand side explanations rely on nonlinearity of Engel curves, e.g. Boppart (2014); Comin, Lashkari and Mestieri (2021); Kongsamut, Rebelo and Xie (2001)

these forces generate both sectoral firm dynamics and the manufacturing decline? What is the role of transitional dynamics? What is the contribution of each force? The model economy is calibrated to US manufacturing and nonmanufacturing sectors at one point in time. The stochastic process for firm employment in each sector is calibrated to match key facts about firm size, firm growth and exit. Given the importance of transitional dynamics, we assume that the economy was at a steady state in 1947. The model is then simulated forward by feeding observed U.S. data from 1947 to 2019 on population growth, sectoral technology and net import shares.

The framework with the three driving forces generates the bulk of the decline in sectoral firm entry rates from 1978 to 2019. The entry rate decline can be decomposed into long-run and transitional effects using the accounting identity (1).[4] A slowdown in population growth generates heterogeneous long-run responses due to sectoral differences in firm demographics. The long-run elasticity of entry to population growth is approximately 1.3 in manufacturing and 2 in nonmanufacturing. These elasticities imply that a 1.75 percentage point change in the rate of population growth will reduce entry by about 2.25pp in manufacturing and 3.5pp in nonmanufacturing. Given the 6.5pp entry rate decline in both sectors, the long-run effect is about one-third for manufacturing and one-half for nonmanufacturing.

Transitional effects account for the remaining 3pp entry decline in nonmanufacturing and 4.3pp in manufacturing. These reflect changes in the firm-age distribution induced by population growth, along with changes in sectoral technology and trade. The 3pp nonmanufacturing decline is almost entirely driven by changes in firm demographics. The 4.3pp manufacturing decline can be decomposed into 1.4pp due to changes in firm demographics, 2.4pp due to employment reallocation, and 0.5pp due to changes in average size growth. The effect of employment reallocation on manufacturing entry (2.4pp) is therefore on par with the long-run effect of population growth (2.25pp).

Beyond entry rates, the framework also replicates the key patterns of sectoral firm dynamics: exit rates, firm aging, and the evolution of average size in both sectors. Counterfactual exercises decompose the contribution of each force. The

[4]The long-run effect is the difference in entry rates across two steady states with different population growth rates.

population growth slowdown drives exit, firm aging and the increase in average size in nonmanufacturing. For manufacturing, all three forces contribute roughly equally to the entry decline, while the decrease in average size is entirely driven by changes in sectoral technology. These technological changes reflect a move away from labor-intensive production in manufacturing, consistent with downsizing within firm-age bins.

A key question is how much of the manufacturing employment decline is due to technology versus trade. To answer this question, we first need to net out the effects of population growth on manufacturing employment. The slowdown in population growth induces reallocation of jobs towards manufacturing, implying that the data understate the extent of the manufacturing employment decline. After incorporating the population growth slowdown, the manufacturing job loss due to technology and trade is closer to 25 million jobs, in contrast to the observed 8 million. Technology is responsible for 80% of the decline in manufacturing employment, with the remaining 20% attributable to trade.

The manufacturing firm share provides another lens on sectoral reallocation. After netting out the effects of population growth, the manufacturing firm share has declined by 6.8pp, with technology responsible for 60% and trade for 40% of this decline. Sectoral spillovers play an important role in the reallocation of firms: 3.5% of nonmanufacturing firms are attributable to the downsizing of manufacturing.

2 Theoretical Framework

There are I production sectors, indexed by $i = 1, \ldots, I$, each producing one of J commodities indexed by $j = 1, \ldots, J$, with $I = J$.

2.1 Consumers

There are L_t identical consumers who supply labor inelastically. Labor is the numeraire. Consumers have preferences over the J-vector of final-good quantities $\mathbf{d}_t$ represented by the utility function $C(\mathbf{d}_t)$. Equivalently, $C(\mathbf{d}_t)$ can be interpreted as an aggregator technology that combines the final-good commodities into a single composite consumption good. Let $\mathbf{p}_t$ denote the corresponding J-vector of

commodity prices. Consumers maximize utility subject to the budget constraint $\mathbf{p}_t \cdot \mathbf{d}_t \leq E_t$, where aggregate expenditure E_t is taken as given by consumers but determined in equilibrium.[5]

2.2 Producers

2.2.1 Static Problem

A firm in sector i with productivity z has a production function

$$q_i(z, n, \mathbf{x}) = z^{1-\eta_i} f_i(n, \mathbf{x})^{\eta_i}, \quad 0 < \eta_i < 1,$$

where n is labor input and $\mathbf{x}$ is a J-vector of intermediate inputs. The aggregator f_i is homogeneous of degree one, differentiable, and increasing in all inputs. Firms also face a fixed cost c_i^f in units of labor, interpreted as an overhead or operating expense that induces endogenous exit.

For any commodity price vector $\mathbf{p}$ we can solve for profit functions $\pi_i(z, \mathbf{p})$, output functions $q_i(z, \mathbf{p})$, and input demand functions $n_i(z, \mathbf{p})$ and $x_{ji}(z, \mathbf{p})$. Because f_i is homothetic, all firms within a sector choose inputs in the same proportions. As a result, firm-level variable profits and input demand functions are separable into a firm-specific component, z, and sector-specific components that depend only on prices. More formally, begin with the unit input requirements for sector i. These are defined as the solution to the cost-minimization problem:

$$c_i(\mathbf{p}) = \min_{n,\mathbf{x}} n + \mathbf{p} \cdot \mathbf{x} \quad \text{subject to } f_i(n, \mathbf{x}) = 1.$$

where $c_i(\mathbf{p})$ is the (constant) marginal cost of the input bundle. Given $c_i(\mathbf{p})$, a firm with productivity z chooses scale y to solve

$$\max_y \; p_i z^{1-\eta_i} y^{\eta_i} - c_i(\mathbf{p}) y - c_i^f,$$

with first-order condition

$$\eta_i p_i z^{1-\eta_i} y^{\eta_i - 1} = c_i(\mathbf{p}).$$

[5]The aggregator $C(\mathbf{d}_t)$ may be non-homothetic so final-good good demand $\mathbf{d}(\mathbf{p}_t, E_t)$ need not scale linearly with E_t.

The optimal scale is

$$y_i(\mathbf{p}, z) = \left(\frac{\eta_i p_i}{c_i(\mathbf{p})}\right)^{\frac{1}{1-\eta_i}} z.$$

Output is therefore

$$q_i(\mathbf{p}, z) = \left(\frac{\eta_i p_i}{c_i(\mathbf{p})}\right)^{\frac{\eta_i}{1-\eta_i}} z. \tag{2}$$

Profits are

$$\pi_i(\mathbf{p}, z) = p_i q_i(\mathbf{p}, z) - c_i(\mathbf{p}) y_i(\mathbf{p}, z) - c_i^f = (1-\eta_i)\left(\frac{\eta_i^{\eta_i} p_i}{c_i(\mathbf{p})^{\eta_i}}\right)^{\frac{1}{1-\eta_i}} z - c_i^f, \tag{3}$$

equal to $1-\eta$ share of revenues, net of the fixed cost.

Let $\zeta_i^w = \frac{\partial c_i}{\partial w}\frac{w}{c_i}$ be the elasticity of marginal cost with respect to the wage. Note that $w = 1$ as labor is the numeraire, so variable firm employment is

$$n_i(\mathbf{p}, z) = \zeta_i^w c_i(\mathbf{p}) y_i(\mathbf{p}, z) = \zeta_i^w \left(\frac{\eta_i p_i}{c_i(\mathbf{p})^{\eta_i}}\right)^{\frac{1}{1-\eta_i}} z, \tag{4}$$

with total firm employment $n_i(\mathbf{p}, z) + c_i^f$. Likewise, demand of intermediate input j for a firm in sector i is

$$x_{ji}(\mathbf{p}, z) = \zeta_i^{p_j} \frac{c_i(\mathbf{p})}{p_j} y_i(\mathbf{p}, z) = \zeta_i^{p_j} \frac{c_i(\mathbf{p})}{p_j}\left(\frac{\eta_i p_i}{c_i(\mathbf{p})}\right)^{\frac{1}{1-\eta_i}} z. \tag{5}$$

2.2.2 Dynamic Problem

A firm's idiosyncratic productivity z evolves according to a sector-specific, persistent, stationary Markov process $F_i(z'|z)$. The exogenous aggregate state $\mathcal{S}_t = (\boldsymbol{\mathcal{T}}_t, L_t)$ follows its own (potentially non-stationary) Markov process, where $\boldsymbol{\mathcal{T}}_t \equiv \{\eta_{it}, c_{it}(\mathbf{p}_t)\}_{i \in I}$ summarizes *sectoral technology*. The value of a firm is

$$v_i(z; \mathbf{p}_t, \mathcal{S}_t) = \max\left\{0, \pi_i(z; \mathbf{p}_t, \mathcal{S}_t) + \beta \mathbb{E}\left[v_i(z'; \mathbf{p}_{t+1}, \mathcal{S}_{t+1})|z, \mathcal{S}_t\right]\right\}, \qquad 0 < \beta < 1$$

for each sector i.[6] Each period, a firm decides to exit or to operate and continue

[6] We assume that this is a small open economy with interest rate R such that $\beta R = 1$. The interest rate is invariant to fluctuations in $C(\mathbf{d}_t)$ in equilibrium if the small open economy borrows or lends in international credit markets in units of the consumption good. This assumption is only

to the following period by comparing the payoff of exiting, normalized to zero, to the continuation payoff.

2.2.3 Firm Entry

Entry costs differ across sectors. Entrants in sector i pay an entry cost c_e^i in units of labor. Upon paying this cost of entry, entrants draw their initial productivity according to distribution function G_i. Expected value for a potential entrant is

$$v_i^e(\mathbf{p}_t, \mathcal{S}_t) = \int v_i(z; \mathbf{p}_t, \mathcal{S}_t) dG_i(z) - c_i^e$$

2.3 Equilibrium Definition

Definition 1. *An equilibrium consists of a price vector* $\mathbf{p}_t$*, firm measures vector* $\boldsymbol{\mu}_t$*, entry mass vector* $\mathbf{m}_t$*, exit thresholds vector* $\mathbf{z}_t^*$*, input demand vectors* $\mathbf{n}_t$ *and* $(\mathbf{x}_{it})_{i=1}^I$*, final demand vector* $\mathbf{d}_t$ *and aggregate expenditure* E_t*, such that firms maximize profits, consumers maximize utility, and:*

1. ***Entry:*** $v_i^e(\mathbf{p}_t, \mathcal{S}_t) \times m_{it} = 0$ *and* $v_i^e(\mathbf{p}_t, \mathcal{S}_t) \leq 0, \quad \forall i.$

2. ***Exit:***
$$v_i(z_{it}^*; \mathbf{p}_t, \mathcal{S}_t) = 0, \quad \forall i. \tag{6}$$

3. ***Law of Motion:*** *For all measurable* $\mathcal{Z}$*,*
$$\mu_{i,t+1}(\mathcal{Z}) = \int \int_{z' \geq z_{i,t+1}^*, z' \in \mathcal{Z}} dF_i\left(z'|z\right) d\mu_{it}(z) + m_{i,t+1} \left(\int_{z' \geq z_{i,t+1}^*, z' \in \mathcal{Z}} dG_i\left(z'\right) \right), \quad \forall i \tag{7}$$

4. ***Good Markets Clearing:***
$$\int q_j(z, \mathbf{p}_t) d\mu_{jt}(z) = d_j(\mathbf{p}_t, E_t) + \sum_i \int x_{ji}(z, \mathbf{p}_t) d\mu_{it}(z), \quad \forall j \tag{8}$$

5. ***Labor Market Clearing:***
$$\sum_i \int n_i(z, \mathbf{p}_t) d\mu_{it}(z) + \sum_i M_{it} c_i^f + \sum_i m_{it} c_i^e = L_t \tag{9}$$

needed when not considering a steady state.

with mass of firms $M_{it} \equiv \int d\mu_{it}(z)$. *Let* $\tilde{\mu}_{it} = \mu_{it}/M_{it}$ *denote probability distributions. If, in addition,* $\tilde{\mu}_{it} = \tilde{\mu}_{i,t+1} = \tilde{\mu}_i^*$ *for all* i*, this is a stationary equilibrium.*

2.4 Solving for Equilibrium

We next develop a simple algorithm to construct the equilibrium. It relies on necessary conditions which have a unique solution. Prices can be obtained considering only the supply side of the model, assuming the free entry conditions of the equilibrium definition hold with equality every period. In other words, we restrict attention to equilibria with strictly positive entry, which is the empirically relevant case for broadly defined sectors.

The construction of the equilibrium relies on a guess and verify argument. Note that prices enter into the profit function just as an argument of the sector-specific variable profits $\tilde{\pi}_i(1; \mathbf{p}_t, \mathcal{S}_t)$,

$$v_i(z; \mathbf{p}_t, \mathcal{S}_t) = \max\left\{0, z\tilde{\pi}_i(1; \mathbf{p}_t, \mathcal{S}_t) - c_i^f + \beta \mathbb{E}\left[v_i(z'; \mathbf{p}_{t+1}, \mathcal{S}_{t+1})|z, \mathcal{S}_t\right]\right\}$$

where

$$\tilde{\pi}_i(1; \mathbf{p}_t, \mathcal{S}_t) = (1 - \eta_{it}) \left(\frac{p_{it}\eta_{it}^{\eta_{it}}}{c_i(\mathbf{p}_t)^{\eta_{it}}}\right)^{\frac{1}{1-\eta_{it}}},$$

is the variable profit of a unit-idiosyncratic productivity firm given the aggregate state at time t. Based on this observation, we conjecture (and verify) that sector-specific variable profits are constant over time (relative to the labor numeraire):

$$\tilde{\pi}_i(1; \mathbf{p}_t, \mathcal{S}_t) = \hat{\pi}_i, \quad \forall t.$$

If so, firm values satisfy

$$v_i(z; \hat{\pi}_i) = \max\left\{0,\ z\hat{\pi}_i - c_i^f + \beta \mathbb{E}_i[v_i(z'; \hat{\pi}_i) \mid z]\right\}, \tag{10}$$

and so does the entrant value $v_i^e(\hat{\pi}_i)$. The latter is strictly increasing in $\hat{\pi}_i$, so there exists a unique vector $(\hat{\pi}_1, \ldots, \hat{\pi}_I)$ which satisfy the free entry conditions,

$$\int v_i(z; \hat{\pi}_i) dG_i(z) = c_i^e, \quad \forall i. \tag{11}$$

Note that each $\hat{\pi}_i$ is found using the value function of the corresponding sector separately. Given these $\hat{\pi}_i$'s for each sector, prices must satisfy

$$\hat{\pi}_i = (1-\eta_{it}) \left(\frac{p_{it}\eta_{it}^{\eta_{it}}}{c_{it}(\mathbf{p}_t)^{\eta_{it}}} \right)^{\frac{1}{1-\eta_{it}}}, \quad \forall i \text{ and } t. \tag{12}$$

Taking logs,

$$\ln \hat{\pi}_i = \theta_{it} + \frac{1}{1-\eta_{it}} \ln p_{it} - \frac{\eta_{it}}{1-\eta_{it}} \ln c_{it}(\mathbf{p}_t), \quad \forall i,$$

with $\theta_{it} = \ln(1-\eta_{it}) + \frac{\eta_{it}}{1-\eta_{it}} \ln \eta_{it}$. For any value of $c_{it}(\mathbf{p}_t)$ and η_{it} we need to find the price p_{it} that satisfies the above equation given $\hat{\pi}_i$. Define the mapping

$$T_i \left(\ln \mathbf{p}_t \right) = (1-\eta_{it}) \left(\ln \hat{\pi}_i - \theta_{it} \right) + \eta_{it} \ln c_{it} \left(\exp \left(\ln \mathbf{p}_t \right) \right), \quad \forall i.$$

This mapping takes the vector $\ln \mathbf{p}_t$ and maps it into a new vector $\ln \mathbf{p}_t'$. It follows immediately that a vector $\mathbf{p}_t^*$ is an equilibrium price vector if and only if it is a fixed point of this mapping. To show that the mapping T_i is a contraction, consider the partial derivative:

$$T_{ik} \left(\ln \mathbf{p}_t \right) = \frac{\partial T_i \left(\ln \mathbf{p}_t \right)}{\partial \ln p_{kt}} = \eta_{it} \left(\frac{p_{kt} x_{ki,t}}{c_{it}(\mathbf{p}_t)} \right),$$

where the last term in brackets is the cost share of input k in sector i, which follows from Shephard's lemma. Because the cost shares sum to one, the row sums of the Jacobian are at most $\eta_{it} < 1$, implying that the mapping T_i is a contraction with modulus $max_i\ \eta_{it}$. Thus, iterations on this mapping converge to a unique fixed point $\mathbf{p}_t^*$, the unique equilibrium price vector.

Given prices $\mathbf{p}_t^*$, we now solve for entry and verify that the remaining equilibrium conditions hold. Start with the good markets clearing conditions (8). These can be written compactly as

$$\mathbf{q}_t = \mathbf{\Omega} \left(\mathbf{p}_t \right) \mathbf{q}_t + \mathbf{d} \left(\mathbf{p}_t, E_t \right)$$

where $\mathbf{q}_t = (Q_{1t}, \ldots, Q_{Jt})$ denotes the vector of sectoral outputs and $\mathbf{\Omega}(\mathbf{p}_t)$ is the input-output matrix of coefficients with (j,i)-th element $\Omega_{ji}(\mathbf{p}_t)$, capturing how

much input j is used per unit of output in sector i. These coefficients reflect firms' optimal input choices at prices $\mathbf{p}_t$, rather than technological constraints. To see how they arise, let $q_i(\mathbf{p}_t, 1)$ denote the output of a firm with unit-idiosyncratic productivity in sector i. By (2) and (5), a firm with productivity z produces $z\, q_i(\mathbf{p}_t, 1)$, and optimally demands $z\, x_{ji}(\mathbf{p}_t, 1)$ units of input j. Thus input use per unit of output is $\Omega_{ji}(\mathbf{p}_t) = \frac{x_{ji}(\mathbf{p}_t,1)}{q_i(\mathbf{p}_t,1)}$.

Manipulating the expression further, the market-clearing condition can be conveniently written as

$$\mathbf{q}_t = \mathbf{\Psi}(\mathbf{p}_t)\, \mathbf{d}(\mathbf{p}_t, E_t), \tag{13}$$

where $\mathbf{\Psi}(\mathbf{p}_t) = [I - \mathbf{\Omega}(\mathbf{p}_t)]^{-1}$ denotes the Leontief inverse matrix, with (j,i)-th element $\Psi_{ji}(\mathbf{p}_t)$, capturing the total (direct and indirect) amount of input j required per unit of final demand in sector i.

Given equilibrium prices $\mathbf{p}_t^*$ determined in (12), equilibrium conditions (7), (9) and (13) yield a system of $J+1$ equations that uniquely determine $J+1$ unknowns, the entry mass vector $(m_{1t}, \ldots, m_{Jt})$ and expenditure E_t. In the expressions that follow, we suppress the dependence on prices for ease of notation.

To solve for entry, decompose total sectoral output into contributions from entrants and incumbents. Combining (13) and the J goods market clearing conditions (7) yields

$$m_{jt} S_j^e q_{jt}^e = \sum_{i=1}^{I} \Psi_{ji,t} d_{it}(E_t) - Q_{jt}^{\text{inc}}, \qquad \forall j \in J. \tag{14}$$

where $S_j^e = 1 - G_j(z_j^*)$ is the entry success rate (the probability an entrant draws $z \geq z_j^*$ and begins production), $q_{jt}^e = \int_{z_j^*} q_j(\mathbf{p}, z) dG_j(z)$ is average output of successful entrants, and Q_{jt}^{inc} is total output by incumbent firms in sector j. This states simply that output by entrants in each sector is equal to total sector output minus the output already produced by incumbents in that sector. This expression pins down entry required by each sector $(m_{1t}, \ldots, m_{Jt})$ up to aggregate expenditure E_t, as all the other terms in this expression are known given prices. The Leontief inverse coefficient $\Psi_{ji,t}$ captures the *static interdependence of entry* across sectors due to input-output linkages, as emphasized by papers such as Baqaee (2018) and Cavalcanti, Mendes and Pannella (2023). The incumbent term Q_{jt}^{inc} captures the dynamic aspect of entry emphasized by Hopenhayn et al. (2022): past entry

matters through the number of incumbent firms and the age distribution of firms when firm characteristics differ by firm age. We refer to this as the *own-sector history dependence of entry*.

The labor market clearing condition, expressed in terms of entrants and incumbents, is

$$L_t = \sum_{j=1}^{J} m_{jt} S_j^e n_{jt}^e + \sum_{j=1}^{J} N_{jt}^{\text{inc}}, \tag{15}$$

where average entrant labor requirement $n_{jt}^e = \int_{z_j^*} n_j(\mathbf{p}, z) dG_j(z) + c_j^f + c_j^e / S_j^e$ and N_{jt}^{inc} is total employment by incumbents in sector j. Substituting $m_{jt} S_j^e$ from (14) into (15) yields an expression that pins down expenditure E_t:

$$\sum_{j=1}^{J} \left(\frac{n_{jt}^e}{q_{jt}^e} \sum_{i=1}^{I} \Psi_{ji,t} d_{it}(E_t) \right) = L_t - \left(\sum_{j=1}^{J} N_{jt}^{\text{inc}} - \sum_{j=1}^{J} \frac{n_{jt}^e}{q_{jt}^e} Q_{jt}^{\text{inc}} \right). \tag{16}$$

This expression has an intuitive interpretation in terms of resources needed to sustain a given level of expenditure E_t. The expression inside the summation on the left-hand side gives, for each sector, the gross output requirement implied by final demand, $\sum_i \Psi_{ji,t} d_{it}(E_t)$, divided by average entrant output q_{jt}^e. This determines the number of entrants that would be needed to produce sectoral output if all production were carried out by entering firms. Multiplying by average entrant requirement n_{jt}^e then yields the labor required in each sector. Summed across all sectors, the left-hand side pins down total resources needed, in units of labor, for a given level of E_t.

The right-hand side shows the amount of resources available in the economy. In addition to the labor not employed by incumbents, $L_t - \sum_{j=1}^{J} N_{jt}^{inc}$, these resources include output produced by incumbent firms. The adjustment term $\sum_{j=1}^{J} n_{jt}^e / q_{jt}^e \times Q_{jt}^{\text{inc}}$ converts incumbent output into units of labor. For sector j, Q_{jt}^{inc} / q_{jt}^e shows the number of entrants that incumbents embody, which multiplied by n_{jt}^e converts it to units of labor. When summed across sectors, this adjustment term shows the amount of resources represented by incumbent firms. Therefore, the term between parentheses on the right-hand-side captures the net resources taken up or freed by incumbents and reflects a dynamic aspect of entry: the history of entry in any given sector will affect current entry in all sectors through

the mass of incumbents firms and their corresponding output and employment, a channel we refer to as the *cross-sector history dependence of entry*.

Equations (14) and (16) form a system of $J+1$ equations and unknowns that are solved sequentially: E_t is determined using (16), and the entry vector $(m_{1t}, \ldots, m_{Jt})$ is solved using (14) as a linear system of J equations and unknowns.[7] These allocations and prices are, therefore, the unique equilibrium of the economy.

Having concluded the construction of the equilibrium, it is useful to compare our characterization of entry to that in Hopenhayn et al. (2022).

Proposition 1 (One-Sector Dynamic Entry Condition)**.** *When the economy consists of a single sector ($J = 1$), the system collapses to the dynamic entry equation in Hopenhayn et al. (2022), where entrants fill the gap between labor supply and incumbent employment:*

$$m_t = \frac{L_t - N_t^{inc}}{S^e n_t^e}.$$

Proof. Evaluating (14) and (16) at $J = 1$ yields

$$m_t S^e q_t^e = \Psi_t d_t(E_t) - Q_t^{\text{inc}}, \tag{17}$$

$$\frac{n_t^e}{q_t^e} \Psi_t d_t(E_t) = L_t - N_t^{\text{inc}} + \frac{n_t^e}{q_t^e} Q_t^{\text{inc}}. \tag{18}$$

Solving (18) for $\Psi_t d_t(E_t)$ and substituting this expression into (17) yields

$$m_t S^e q_t^e = \left(\frac{L_t - N_t^{\text{inc}} + \frac{n_t^e}{q_t^e} Q_t^{\text{inc}}}{n_t^e / q_t^e} \right) - Q_t^{\text{inc}}.$$

Dividing both sides by $S^e q_t^e$ and simplifying, the terms involving Q_t^{inc} cancel, and we obtain the stated expression. ■

2.5 Properties of Equilibrium

We now turn to the properties of the equilibrium, starting with the relationship between firm outcomes and the aggregate state.

[7] In the case of homothetic utility the $J+1$ equations themselves form a linear system.

Proposition 2 (Independence of Firm Values and Exit Thresholds from the Aggregate State)**.** *In the unique equilibrium, firm values are independent of the aggregate state* $(\mathbf{p}_t, \mathcal{S}_t)$ *and depend only on the idiosyncratic productivity shock* z*. It follows that exit thresholds* $\mathbf{z}_t^*$ *are also independent of the aggregate state.*

The proposition follows directly from the value function in (10), where the state space of the firm can be written without reference to $\mathcal{S}_t$ and $\mathbf{p}_t$. Although it may seem surprising that prices fully offset changes in the aggregate state, rendering firm values invariant, this mechanism is a consequence of the free-entry conditions (11). Any residual demand not absorbed by prices is instead accommodated along the entry margin, ensuring market clearing.[8] Note that independence of firm values from the aggregate state does not imply that firm values are constant over time. The invariance property is relative to labor, which is the numeraire, so firm values co-move with wages.

The main implication of Proposition 2 is that exit thresholds are independent of shocks to population growth and sectoral technology. This prediction is consistent with the empirical evidence in Hopenhayn et al. (2022) and Karahan et al. (2024), who find that bulk of variation in aggregate exit rates is due to changes in composition of the firm-age distribution, instead of changes in exit rates conditional on age. At the sector level, Table A-4 in the Appendix shows that exit rates are relatively stable for firms older than five years of age, which represent more than 85 percent of employment. The invariance of exit rates by age is therefore a useful approximation for US sectoral data.

The independence results disciplines the range of possible aggregate mechanisms that are consistent with US sectoral data. As can be seen from equations (10) and (11), Proposition 2 does not hold when there are shocks to entry costs c_i^e, fixed costs c_i^f, the discount rate β, the entrant distribution $G_i(z)$ or the firms' stochastic process $\mathbb{E}_i[\cdot|z]$. In such settings, firm values and exit thresholds would generally depend on these shocks. A shock to these parameters, however, would imply variation in the exit rates of older firms, which is inconsistent with US sectoral data.

[8] This result is closely related to, but distinct from, the property of block recursivity (Menzio and Shi, 2011). Block recursivity allows the endogenous aggregate state to be summarized by prices rather than by the full distribution of firms, as in Hopenhayn (1992). Independence from the aggregate state is a stronger property: it implies that the firm's relevant state space can be characterized without reference to prices altogether.

It is worth highlighting the role played by the homotheticity of the input-aggregator f_i. The resulting separability of firm- and sector-specific components implies that aggregate shocks affect all firms within a sector proportionally, allowing prices to adjust so as to offset the impact of the shock uniformly across firms. This proportionality is less restrictive than it might seem: shocks that impact only a subset of firms in a sector can be accomodated by reclassifying those firms as belonging to a separate sector with its own primitives, thereby preserving the independence result.

A useful implication of the independence result is that the dynamic problem can be solved without imposing assumptions about how firms forecast aggregate dynamics. Counterfactual exercises can therefore be conducted without imposing perfect foresight or modeling beliefs about the future path of $(\mathbf{p}_t, \mathcal{S}_t)$. This allows equilibrium transitions to be computed without specifying a terminal condition, so the solution does not depend on the choice of a terminal steady state.

One final implication of the independence result is that, as in Hopenhayn et al. (2022), all adjustments to changes in population growth occur along the extensive margin through entry. To see this, note that population growth does not appear in the price determination equation (12). Moreover, because population growth does not directly affect variables conditional on firm age, lifecycle profiles remain unchanged.

Proposition 3 (Invariance of Firm-age Profiles to Population Growth)**.** *Firm-age profiles of exit, employment and output within each sector are invariant to changes in population growth.*

Consequently, the effects of population growth operate entirely through compositional changes induced by entry and the resulting shifts in the firm age distribution. In order to connect the model to the empirical measures of firm turnover, we next derive expressions for sectoral entry and exit rates.

2.6 Sectoral Rates of Entry and Exit

The mass of exiting firms in sector i at time t, Ξ_{it}, is determined by the share of incumbents whose productivity falls below the threshold z_i^* (as determined in condition 6):

$$\Xi_{it} \equiv \int F_i\left(z_i^* | z\right) d\mu_{i,t-1}\left(z\right),$$

where $F_i(\cdot|z)$ denotes the conditional productivity distribution. Incumbents receive productivity shocks at the start of the period and exit immediately if their productivity falls below the threshold. Thus, Ξ_{it} accounts for firms that produced in period $t-1$ but exit before producing in period t. As in the data, the mass of exit excludes the mass of potential entrants that exit without producing, $m_{it}(1-S_i^e)$. The sectoral exit rate is therefore:

$$\xi_{it} \equiv \Xi_{it}/M_{i,t-1}.$$

To derive the sectoral entry rate, first decompose the growth rate of the number of firms by separating the contribution of sectoral employment growth from changes in average firm size:

$$\frac{M_{it}}{M_{i,t-1}} = \frac{N_{it}}{N_{i,t-1}} \frac{N_{i,t-1}/M_{i,t-1}}{N_{it}/M_{it}}, \tag{19}$$

The total mass of firms in sector i evolves according to the law of motion (7):

$$M_{it} = (1-\xi_{it})M_{i,t-1} + m_{it}S_i^e. \tag{20}$$

Substituting (20) into (19) and defining the entry rate relative to the previous period's mass, $\lambda_{it} = m_{it}S_i^e/M_{i,t-1}$, yields:

$$\lambda_{it} = \left(\frac{L_t}{L_{t-1}} \frac{s_{it}}{s_{i,t-1}} \frac{N_{i,t-1}/M_{i,t-1}}{N_{it}/M_{it}} - 1 \right) + \xi_{it}, \tag{21}$$

where sectoral employment is $N_{it} = L_t s_{it}$, with s_{it} denoting the share of the population employed in sector i.

Expression (21) is the discrete-time analogue of identity (1). It states that the sectoral entry rate is increasing in population growth L_t/L_{t-1}, increasing in population share growth $s_{it}/s_{i,t-1}$, decreasing in average firm size growth, and increasing in the sectoral exit rate ξ_{it}. The latter two components are themselves functions of the age distribution of firms, as average firm size typically increases with age while the hazard of exit decreases with age. We expand on the lifecycle profile of firms in the next section.

2.7 Population Growth and Sectoral Reallocation

Population growth generates reallocation across sectors due to systematic differences in sectoral firm demographics. To illustrate this mechanism, assume the representative consumer has homothetic preferences and the economy is in a steady state with a constant population growth rate g, such that $L_{t+1} = (1+g)L_t$. We begin by characterizing the lifecycle profile of firms through the following assumptions:

Assumption 1. *The hazard rate of exit is strictly decreasing in firm age.*

Assumption 2. *Output-per-firm is strictly increasing in firm age.*

In the model, these properties emerge as equilibrium outcomes from two distinct channels. First, selection due to the fixed cost ensures that only the most productive (and thus larger) firms survive to older ages. Second, we assume the distribution of entrant idiosyncratic productivity, $G_i(z)$, is stochastically dominated by the ergodic distribution of the incumbent productivity process, $F_i(z'|z)$. Consequently, surviving firms tend to grow toward their long-run mean as they age, a process further reinforced by selection.

Proposition 4 (Population growth and firm share.)**.** *Suppose Assumptions* (1) *and* (2) *hold. Furthermore, suppose that age profiles of output-per-firm are the same across sectors. An increase in population growth raises the relative number of firms in sectors with steeper age-profiles of exit.*

Proof. In a steady state, employment $N_{it} = L_t s_{it}$ and the number of firms M_{it} in each sector grow at the common rate g. The sectoral the entry rate λ_i in (21) is therefore

$$\lambda_i = g + \xi_i(g),$$

where $\xi_i(g)$ is the sector's long-run exit rate, which is a weighted sum of exit rates by age (see Hopenhayn et al., 2022). Because prices are invariant to population growth, firm-age profiles within each sector are unaffected by changes in g. Thus, cross-sector differences in entry rates arise solely from variation in exit-by-age profiles, which determine the long-run exit rate $\xi_i(g)$.

A sector has a steeper exit-by-age profile if, conditional on average age, it exhibits higher exit rates of young firms. An increase in population growth raises the required entry rate λ_i in every sector, thereby shifting each sector's age distribution toward younger

firms. Sectors with steeper exit-by-age profiles exhibit larger increase in their long-run exit rate and therefore experience larger increases in entry rates.

Consider the relative number of firms in sectors i and j:

$$\frac{M_i}{M_j} = \frac{Q_j/M_j}{Q_i/M_i} \times \frac{Q_i}{Q_j}.$$

Homothetic demand implies that relative outputs Q_i/Q_j do not change with changes in population growth. Changes in relative firm numbers therefore operate entirely through output per firm.

Output per firm reflects the firm-age distribution, since older cohorts produce more per firm than younger cohorts. When g rises, sectors with steeper exit profiles experience a larger reweighting toward young, low-output-per-firm cohorts. For a given level of sectoral output, these sectors require more firms; their mass of firms M_i therefore rises relative to sectors with flatter exit profiles.

If, in addition, these sectors have steeper output-per-firm age profiles, then the decline in output per firm induced by the shift toward younger cohorts is larger, further amplifying their increase in firm share. ■

We now turn to the effect of population growth on relative employment across sectors.

Assumption 3. *Output-per-worker is strictly increasing in firm age.*

Output per productive worker in the model is equalized across firms within a sector, so the age profile in output-per-worker is due exclusively to the fixed overhead cost, as smaller firms employ a larger share of their workforce as overhead and younger firms tend to be smaller than older firms, as explained earlier.

Proposition 5 (Population growth and employment share.)**.** *Suppose Assumptions* (1) *and* (3) *hold. Furthermore, suppose that age profiles of output-per-worker are the same across sectors. An increase in population growth raises the relative employment of sectors with steeper age-profiles of exit.*

Proof. Using the decomposition

$$\frac{N_i}{N_j} = \frac{Q_j/N_j}{Q_i/N_i} \times \frac{Q_i}{Q_j}, \tag{22}$$

homothetic demand again implies that relative outputs Q_i/Q_j are invariant to changes in g. Thus, changes in relative employment therefore operate entirely through differences in output per worker.

Output per worker in sector k can be written as

$$\frac{Q_k}{N_k} = \frac{Q_k}{N_k^{\text{prod}}} \times \frac{N_k^{\text{prod}}}{N_k},$$

where the first term is output per productive worker (identical across firms within a sector and independent of firm age), and the second term is the sector's productive-employment share. Because firms employ a fixed amount of non-productive (overhead) labor, the productive-employment share is increasing with firm age: younger firms allocate a larger fraction of their workforce to overhead.

Earlier results established that an increase in g shifts each sector's firm-age distribution toward younger firms, with larger shifts in sectors with steeper exit-by-age profiles. These sectors therefore experience larger declines in their productive-employment share when g rises. Since output per productive worker is age-invariant, output per worker Q_k/N_k falls more sharply in these sectors.

Sectors experiencing larger declines in output per worker require more workers to produce a given amount of sectoral output. Consequently, sectors with steeper exit profiles and steeper output-per-worker age profiles experience larger increases in relative employment when population growth rises. ■

2.8 Special case: A Cobb-Douglas Economy

This section characterizes the effect of sectoral productivity changes on firm dynamics and reallocation in the special case of Cobb-Douglas utility and production functions. As shown earlier in equations (3) and (4), up to some constants, both profits and employment are determined by the quotient

$$\left(\frac{p_i z^{1-\eta_i}}{c_i\left(\mathbf{p}\right)^{\eta_i}}\right)^{\frac{1}{1-\eta_i}}.$$

The following proposition establishes that, in a Cobb-Douglas economy with constant share parameters, firm dynamics are invariant to changes in productivity vectors.

Proposition 6. *Assume the production structure is Cobb-Douglas with constant share parameters and so is consumer utility. For any equilibrium price vector* $\mathbf{p}$*, consider alternative price vector* $\mathbf{p}'$*. There exists a vector with elements* κ_i *such that* $\mathbf{p}'$ *is an equilibrium for the economy with productivities defined by* $z_i' = \kappa_i z_i$*, and the equilibrium has identical firm dynamics properties as the original one.*

Proof. It suffices to assume the vector differs only for one sector. Define κ_i by

$$\frac{p_i'(\kappa_i z)^{1-\eta_i}}{c_i\left(\mathbf{p}'\right)^{\eta_i}} = \frac{p_i z^{1-\eta_i}}{c_i\left(\mathbf{p}\right)^{\eta_i}}.$$

Prices fully offset changes in firm output, leaving revenues, profits, and employment unchanged. In particular,

$$p_i' q_i\left(\mathbf{p}', z'\right) = p_i q_i\left(\mathbf{p}, z\right) \tag{23}$$

$$\pi_i\left(\mathbf{p}', z'\right) = \pi_i\left(\mathbf{p}, z\right), \quad \text{and } n_i\left(\mathbf{p}', z'\right) = n_i\left(\mathbf{p}, z\right).$$

Because revenues are unchanged, the Cobb-Douglas aggregator implies that expenditures on each input are also unchanged, so $p_j' x_{ji}\left(\mathbf{p}', z'\right) = p_j x_{ji}\left(\mathbf{p}, z\right)$ for all j. Total income of the representative agent also remains unchanged, since firm profits are unchanged relative to the wage (normalized to one). Hence consumer expenditure on each good is unchanged, implying $p_i' d_i' = p_i d_i$ for all i. Therefore, for each sector, the total value of sales remains unchanged, and by equation (23) the value of production is also unchanged. It follows that total purchases equal total sales in every sector, and hence all markets clear. ■

This result immediately implies that, in a Cobb-Douglas economy, changes in sectoral aggregate productivity do not affect firm dynamics or reallocation of firms, either during transitions or across steady states.

Proposition 7. *Assume the production function is Cobb-Douglas with constant share parameters and so is the utility function of the representative agent. Suppose the economy is initially at a steady state with price vector* $\mathbf{p}$*. Take any sequence* A_{it} *of sectoral productivity shifters. Then the equilibrium price sequence* $\mathbf{p}_t$ *implies that all firm dynamics are unchanged and there is no aggregate reallocation of labor or firms across sectors.*

Proof. Let $\mathbf{p}_t$ denote the unique price vector satisfying

$$\frac{p_{it} A_{it} z^{1-\eta_i}}{c_i\left(\mathbf{p}_t\right)^{\eta_i}} = \frac{p_i z^{1-\eta_i}}{c_i\left(\mathbf{p}\right)^{\eta_i}},$$

where variables on the right-hand side (without time subscripts) are at their stationary equilibrium values. Following a similar argument as in the previous proof, it then follows that revenues, profits and employment remain unchanged, as do total expenditures on each good. Thus, maintaining the allocation of labor inputs for each firm at its stationary equilibrium level clears all markets. ■

3 The Decline of US Manufacturing

In this section we use the theoretical framework to study the decline in US manufacturing. We divide the US economy into two sectors: Manufacturing and the rest of the economy, which we label Nonmanufacturing. In what follows, we first present facts about the evolution of US manufacturing since the 1970s. We then organize the facts using a simple accounting identity. Finally, we use the model to quantitatively investigate the sources of manufacturing decline.

3.1 Manufacturing Facts

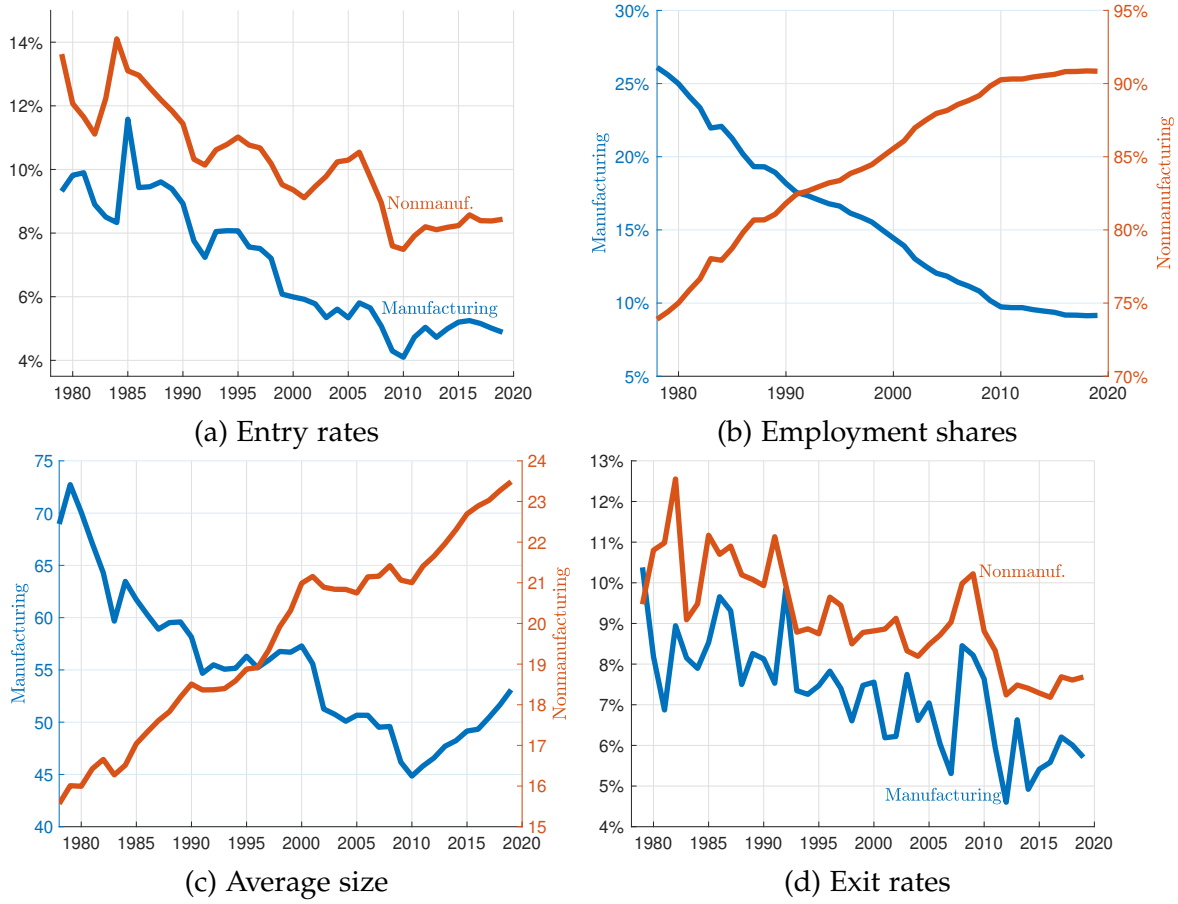
Figure 1 shows the evolution of the sectoral entry rate along with employment shares, average size and exit rates.[9] Recall that these variables are related by the following entry accounting identity:

$$\lambda_i = g + \widehat{\text{Emp. Share}}_i - \widehat{AFS}_i + \xi_i$$

Consistent with the implications of declining population growth, both sectors exhibit declining entry rates over the sample. The consequent changes in firm demographics—aging of the firm distribution combined with age-profiles of exit rates and average size— have feedback effects on entry rates. Therefore, any explanation of the sectoral entry decline must incorporate the slowdown in population growth as a driving force.

While population growth will have reallocation effects through sectoral differences in firm demographics, we need to understand what forces drive employ-

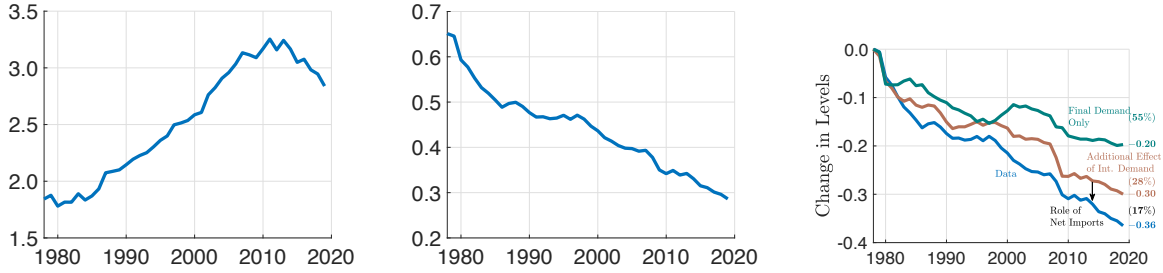
[9]The primary data source is the Business Dynamics Statistics (BDS) produced by the US Census Bureau. This dataset covers the universe of employer firms in the US from 1978 to 2019. We follow the common practice of dropping the first year of BDS data due to suspected measurement error.



(a) Entry rates (b) Employment shares

(c) Average size (d) Exit rates

Figure 1: Sectoral Entry Accounting Variables



(a) Output per Worker (b) Gross Output (c) Output decomposition

Figure 2: Evolution of manufacturing relative to nonmanufacturing

ment reallocation in order to successfully account for the decline in sectoral entry rates. Relative manufacturing employment can be expressed as relative output

divided by relative output per worker,[10]

$$\frac{N_m}{N_{nm}} = \frac{Q_m}{Q_{nm}} \div \frac{Q_m/N_m}{Q_{nm}/N_{nm}} \tag{24}$$

Figures 2a and 2b show the evolution of manufacturing relative output per worker and relative output. Relative output per worker increased by 54%, while relative output decline by 56%. Figure 2c further decomposes the decline in relative output into its sources. A drop in demand for final and intermediate manufacturing goods together are responsible for 83% of the decline, with net imports accounting for the remaining 17%. Taken together, these results suggest a role for two other driving forces in driving employment reallocation. First, an improvement in manufacturing productivity combined with changes in sectoral technology, broadly defined to include final good demand. The second force is an increase in net imports of manufactured commodities.

3.2 Quantitative Analysis

We quantitatively evaluate the ability of the decline in population growth, changes in sectoral technology and increase in net imports to generate the observed decline in manufacturing output. We can then ask a number of quantitative questions. What is the role of steady state vs transitional dynamics? What is the role of firm demographics and sectoral spillovers? How important is each driving force?

Given evolution of relative sectoral output, the quantitative success of the model is determined by how well the model matches the evolution of sectoral firm dynamics, including reallocation of firms. The entry equation (14) relates changes in relative sectoral output to sectoral entry, and therefore to the number of firms in each sector. We calibrate the model to target an initial year and discipline the exercise by keeping constant firm dynamics parameters such as those governing the stochastic process for productivity, entry and fixed costs, and the entry productivity distribution. Therefore there is no apriori reason for the model

[10] Manufacturing employment share and relative employment are closely related:

$$\frac{N_m}{N_m + N_{nm}} = \frac{\frac{N_m}{N_{nm}}}{\frac{N_m}{N_{nm}} + 1}$$

to match subsequent changes in firm dynamics. After the initial year, the evolution of firm dynamics is determined by what the model implies for the effects of the driving forces on firm demographics along with the firm-age distribution, which is determined by entry and exit.

Methodology. The model time-period is set to one year. The input-output structure and sectoral firm dynamics in the model economy are calibrated to US manufacturing and nonmanufacturing sectors in 1978, the first year for which we have firm data. Given the quantitative importance of transitional dynamics, we assume that the economy was at a steady state in 1947 so that 1978 lies on the transition path. This way the firm-age distribution in 1978 reflect the history of entry and exit in both sectors from 1947 onwards. We then simulate the economy forward by feeding the post-1947 evolution of population growth, sectoral technology and net import shares from US data.

Given that population growth in the model refers to working-age population, we map it to data on US civilian labor force growth. We assume the functional forms of utility and sectoral production functions are Cobb-Douglas with time-varying shares. This has the advantage that the corresponding parameters can be interpreted as shares that are observable in the data. The evolution of sectoral technology is inferred from data on final expenditure shares, input factor shares, and sectoral prices.

The final-good demand aggregator over sectoral output is,

$$C(D_M, D_{NM}) = \left(\frac{D_M(\mathbf{p})}{\gamma_t}\right)^{\gamma_t} \left(\frac{D_{NM}(\mathbf{p})}{1-\gamma_t}\right)^{1-\gamma_t},$$

where the subscript i indexes the sector, D_i indicates final demand for the output of sector i, inclusive of imports, and γ_t indicates the expenditure share of manufacturing. The expenditure shares for each sector are set to match the expenditure shares each period in US data. Feeding the expenditure shares over time allows us to capture changes in sectoral output demand coming from changes in final usage.

On the production side, intermediate goods used by sector i are given by the

intermediate good aggregator $X_{it}(\mathbf{x})$,

$$X_{it}(\mathbf{x}) = \left(\frac{X_{M,i}}{\nu_{it}}\right)^{\nu_{it}} \left(\frac{X_{NM,i}}{1-\nu_{it}}\right)^{1-\nu_{it}},$$

where $X_{M,i}$ indicates demand for the manufactured commodity by sector i, inclusive of imports, and $X_{NM,i}$ indicates the same for the nonmanufactured commodity. The time-varying intermediate usage share of the manufacturing good in sector i is denoted ν_{it}. With two sectors, the intermediate usage share of the manufactured commodity by manufacturing is $\nu_{M,t}$ and by nonmanufacturing is $\nu_{NM,t}$. The intermediate good shares are observable in the data, and allow us to capture changes in sectoral output demand due to changes in the input-output structure.

The sectoral input aggregator is Cobb-Douglas in labor and the composite intermediate good

$$g_{it}(n, \mathbf{x}) = A_{it}^{\frac{1}{\eta_{it}}} \left(\frac{n_{it}}{\alpha_{it}}\right)^{\alpha_{it}} \left(\frac{X_{it}(\mathbf{x})}{1-\alpha_{it}}\right)^{1-\alpha_{it}}$$

where η_{it} and α_{it} map to the profit share and the labor share of sector i, which we feed from the data. In order to pin-down sectoral productivity A_{it}, we feed sectoral prices from the data and find the value of A_{it} each period such that the profit of the unit-productivity firm is time-invariant.

As shown in the decomposition in Figure 2c, including net imports is necessary to match the decline in the relative output of manufacturing. We assume that imports and domestic goods are perfect substitutes and feed the import and export shares to the values observed in the data. Under this assumption, the goods market clearing condition for each sector takes the form,

$$(1-\chi_{jt})Q_{jt} = (1-\phi_{jt})\left(D_{jt} + \sum_i \int X_{ji,t}(\mathbf{p}_t, z) d\mu_i(z)\right)$$

where χ_j is the export share of gross output of commodity j, and ϕ_j is the import share of total usage.[11] Qualitatively, the effect of an increase in imports of a

[11]The national income accounts do not split imports based on final consumption or intermediate usage, so we follow NIPA and assign the import share ϕ_j identically to final consumption and intermediate usage. This assignment rule implies domestic and imported goods in final con-

commodity are similar to the effects of a sector-specific decline in labor force growth. While the age-profiles of average size and exit rates are invariant to the increase in imports, firm entry rates decline leading to firm aging in that sector.

Figures A-1, A-2, and A-3 summarize the evolution of all the driving forces that we feed through the model.

Firm dynamics. In order to be consistent with differences in sectoral firm dynamics, we allow the Markov process for the idiosyncratic state z to differ by sector. The Markov process followed by the idiosyncratic state z is AR(1),

$$\log(z_{i,t+1}) = (1-\rho_i)\psi_{ij} + \rho_i \log(z_{it}) + \varepsilon_{i,t+1}; \qquad \varepsilon_{i,t+1} \sim \mathcal{N}(0, \sigma_i^{\varepsilon}), \tag{25}$$

where i indexes the sector and j indexes whether the firm is a high-type or low-type.[12] The parameter ψ_{ij} is the sector-specific long-run mean that varies with type $j \in \{\ell, h\}$, ρ_i is the persistence, and σ_i^{ε} is the standard deviation of shocks. Firms draw their type after paying the sector-specific entry cost c_i^e. Once drawn, the type is fixed throughout the lifetime of the firm. The probability of drawing the high type is parameterized by ω_{ih}. In addition to drawing their type, entrants draw their idiosyncratic state from a lognormal distribution G_i with mean ψ_i^G and standard deviation σ_i^G. The state z then evolves according to the corresponding

sumption and intermediate usage satisfy:

$$d_j(\mathbf{p}) = (1-\phi_j)D_j(\mathbf{p}); \quad d_j^*(\mathbf{p}) = \phi_j D_j(\mathbf{p})$$
$$x_{ji}(\mathbf{p},1) = (1-\phi_j)X_{ji}; \quad x_{ji}^*(\mathbf{p},1) = \phi_j X_{ji}$$

The model equilibrium is consistent with any other split of imports into final and intermediate usage, provided markets clear. Note that in an open economy, the correct measure of income of the representative agent is Gross Domestic Absorption (GDA). GDA measures the total final demand for goods and services within a country's economy regardless of whether those goods and services are produced domestically or imported.

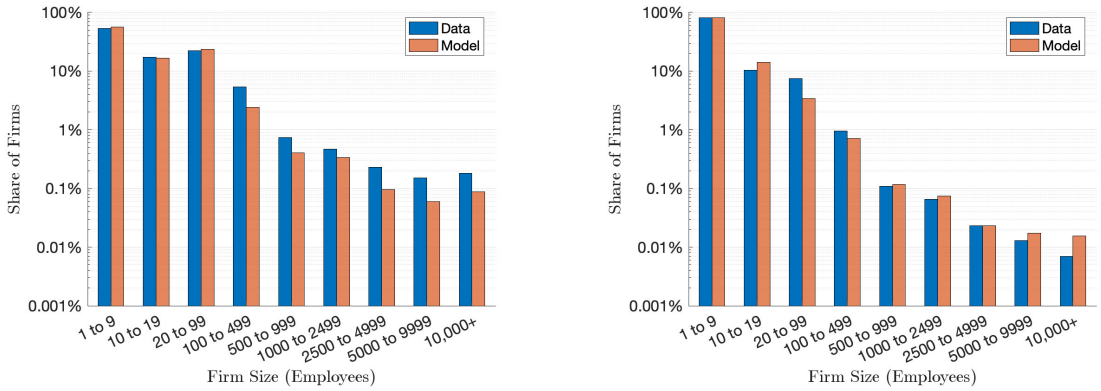
[12] We incorporate permanent differences across firms within each sector by introducing high and low types. This modeling choice addresses three empirical regularities. First, it can help generate the frequency with which young firms achieve large scale, as emphasized by (Luttmer, 2011). Second, it makes the model consistent with the finding in Haltiwanger, Jarmin and Miranda (2013) that firm-age retains explanatory power after controlling for firm size. Third, as documented by Holmes and Stevens (2014); Sterk ⓡ, Sedláček ⓡ and Pugsley (2021), it implies that firms of the same size may respond differently to changes in the economic environment. High-type firms are characterized by both a higher long-run mean in their productivity process and greater labor overhead, corresponding to "superstar" firms in the data. In the estimated model, these high-type firms represent approximately 5 percent of all firms but account for 30 percent of employment and are on average 7 times larger than their low-type counterparts.

AR(1) process.

Together with the overhead parameters for high- and low-type firms in each sector, each sector has 11 parameters that need to be calibrated. The sectoral parameters are calibrated to match firm dynamics in each sector in 1978. The targeted variables for each sector include 1978-83 averages of average firm size, entry rate, average entrant size, 5-year growth and exit rates, share of firms with 1-9 employees, share of entrants with more than 250 employees, and the employment shares of firms with 100+, 1000+ and 10000+ employees.

We jointly calibrate 11 parameters for each sector, but each parameter exerts its primary influence on particular moments. The entry cost predominantly affects the entry rate. Because the entry equation requires matching average entrant size to identify the entry rate, we target both moments. Average entrant size responds mainly to ψ_i^G, the mean of the entrant productivity distribution. The dispersion parameter σ_i^G controls the right tail of this distribution, thereby targeting the concentration of entrants. Turning to the productivity process, the shock variance σ_i^ε determines the probability mass at productivity levels where firms exit, thus targeting the 5-year exit rate. The persistence parameter ρ_i governs convergence rates to long-run means and primarily influences the 5-year growth rate. High-type firms, which are substantially larger than low-type firms, contribute disproportionately to aggregate employment. We therefore calibrate three parameters characterizing these firms—the long-run mean ψ_{ih}, labor overhead c_{fh}, and share at birth ω_h—to match employment-weighted right-tail statistics: the shares employed at firms with 100+, 1000+, and 10,000+ workers. The low-type labor overhead $c_{f\ell}$ primarily pins down the share of firms in the smallest size bin (1 to 4 employees). Table A-1 reports the calibrated parameters, targets, and model fit by sector.

Figure 3 presents the firm size distribution for both sectors in the model and the data. Table A-2 in the Appendix shows firm demographics by sector in both the data and the calibrated model. In both sectors, the age profiles in model and data follow the expected pattern: average size is increasing in age, and exit rates are decreasing. Table A-3 presents the change in average size by age. In the data, average size in manufacturing has declined for all age groups, with older firms experiencing the strongest rate of decline. The model replicates the within-age group decline in manufacturing, because changes in manufacturing factor shares



(a) Manufacturing (b) Nonmanufacturing

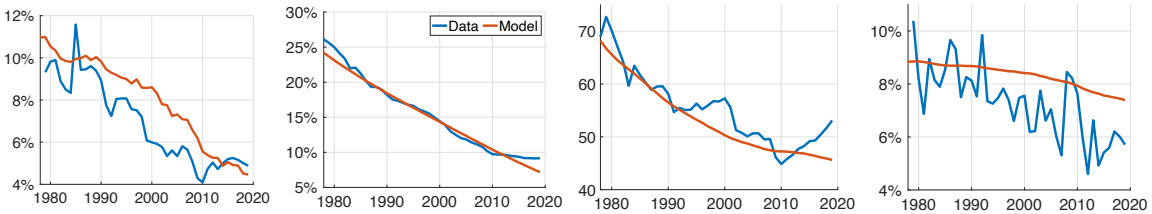
Figure 3: 1978 Firm Size Distribution

fed from the data capture the shift away from labor into intermediates and profits. In nonmanufacturing, the changes in average size within each age group are not statistically different from zero.

Nontargeted Moments. Figure 4 evaluates the model's ability to match the components of the sectoral entry accounting identity (1) for each sector. In manufacturing (Panel A), the model reproduces the key patterns in the data. The entry rate declines steadily from roughly 10 percent in the late 1970s to about 5 percent by the end of the sample. Over the same period, the manufacturing employment share falls from about 25 percent to below 10 percent, while average firm size declines steadily from around 70 to 45 and exit rates fall modestly from roughly 9 to 7 percent. In nonmanufacturing (Panel B), the model captures the opposite reallocation pattern. The employment share rises from roughly 75 percent to about 90 percent, while average firm size increases from about 16 to around 24. Entry rates decline from about 14 percent to roughly 8 percent, accompanied by a modest decline in exit rates from about 10 to 7.5 percent. Overall, the model provides a good quantitative fit to the joint evolution of these moments across sectors.

The Role of Transitional Effects and Firm Demographics. Table 1 decomposes the change in sectoral entry rates between 1978 and 2019 using the entry accounting identity and compares the baseline model, which incorporates firm demographics, to a counterfactual without age heterogeneity. The long-run effect is

Panel A: Manufacturing



(a) Entry rate (b) Emp. share (c) Average firm size (d) Exit rate

Panel B: Nonmanufacturing

(e) Entry rate (f) Emp. share (g) Average firm size (h) Exit rate

Figure 4: Model Fit

computed as the difference between a 1978 steady state and a 2019 steady state, where parameters governing population growth, sectoral technology, and net exports are set to their values in 2019 while all other parameters are held fixed at their baseline 1978 values. The transitional effect is defined as the difference between the total and long-run effects.

The decomposition in Table 1 shows that the model accounts for nearly the entire 6.5 percentage point decline in sectoral entry rates between 1978 and 2019 (column 1). The decomposition is more instructive regarding the underlying mechanisms when we examine how differently this decline is split between long-run and transitional forces across sectors.

A slowdown in population growth exerts a sizable long-run drag on entry, but its impact is not uniform across sectors. Because sectors differ in their firm demographics, particularly in the steepness fo the exit-age profiles as laid out in Proposition 4, the implied long-run elasticity of entry with respect to population growth is about 1.3 in manufacturing and close to 2 in nonmanufacturing. As a result, the same 1.75 percentage point decline in population growth translates into a 2.25 pp drop in manufacturing entry (column 2), but a much larger 3.46 pp decline in nonmanufacturing (column 2). In proportional terms, long-run forces

account for roughly one-third of the total decline in manufacturing (-2.25 out of -6.52) and about one-half in nonmanufacturing (-3.46 out of -6.51).

Table 1: Entry Rate Decomposition: Long-Run vs Transitional Effects

	Model			**No Age Heterogeneity**		
	Total	Long-Run	Transition	Total	Long-Run	Transition
Manufacturing						
Δ LF Growth	-1.75	-1.75	0	-1.75	-1.75	0
$+\ \Delta$ Emp. Share Growth	-2.27	0	-2.27	-2.44	0	-2.44
$-\ \Delta$ AFS Growth	1.57	0	1.57	0.53	0	0.53
$+\ \Delta$ Exit Rate	-1.46	-0.50	-0.96	0	0	0
$+\ \Delta$ Residual	0.53	0	0.53	0.09	0	0.09
Δ Entry Rate	-6.52	-2.25	-4.27	-4.63	-1.75	-2.88
Nonmanufacturing						
Δ LF Growth	-1.75	-1.75	0	-1.75	-1.75	0
$+\ \Delta$ Emp. Share Growth	-0.33	0	-0.33	-0.25	0	-0.25
$-\ \Delta$ AFS Growth	2.30	0	2.30	0.01	0	0.01
$+\ \Delta$ Exit Rate	-2.61	-1.71	-0.90	0	0	0
$+\ \Delta$ Residual	0.48	0	0.47	0.08	0	0.08
Δ Entry Rate	-6.51	-3.46	-3.05	-1.93	-1.75	-0.18

Notes. All values are in percentage points. With entry costs denominated in units of labor, identity (1) holds approximately, $\lambda_i \approx g + \widehat{Emp.Share}_i - \widehat{AFS}_i + \xi_i$. The residual corresponds to changes in the growth rate of labor allocated towards the creation of entrants.

The remaining entry decline is driven by transitional dynamics, which operate through shifts in the firm-age distribution and the reallocation of economic activity across sectors. This effect is stronger in manufacturing, accounting for -4.27 pp of the total decline (column 3), nearly twice the long-run component. The manufacturing transitional component reflects three distinct margins: a -2.27 pp reduction due to employment reallocation growth, a -0.96 pp contribution from the exit rate, and a $+1.57$ pp from changes in the growth rate of average firm size that enters negatively. Notably, the impact of employment reallocation alone is quantitatively as large as the long-run effect of population growth in this sector (-2.27 vs. -2.25, columns 3 and 2).

In nonmanufacturing, transitional effects are smaller in magnitude (-3.05 pp, column 3) but more tightly linked to firm demographics. Nearly the entire tran-

sitional decline in entry is driven by changes in the firm-age distribution, with falling exit rates (-0.90 pp) and rising average firm size ($+2.30$ pp) playing central roles. In contrast to manufacturing, employment reallocation plays only a minor role (-0.33 pp, column 3).

Strikingly, shutting down age heterogeneity sharply attenuates these transitional dynamics. Without firm demographics (columns 4 to 6), the entry decline falls to -4.63 pp in manufacturing and -1.93 pp in nonmanufacturing. The missing decline is almost entirely accounted for by muted responses in exit rates and average firm size (compare columns 3 and 6). This finding underscores that firm demographics are a key mechanism through which aggregate and sectoral shocks propagate into sectoral entry dynamics.

The Role of Driving Forces. Figure 5 decomposes the evolution of manufacturing firm dynamics into the contribution of population growth, sectoral technology and preferences, and net imports. We decompose the three forces cumulatively in the following order: population growth, sectoral technology, then trade. Population growth is a dominant source of variation in aggregate firm dynamics, so its effects must be isolated before attributing changes in sectoral firm dynamics to the remaining forces. We then consider sectoral technology, including sectoral productivity and final good demand, as it has been the primary driver of employment reallocation.

Figure 5a shows that slower population growth alone would reduce manufacturing entry by about 2 percentage points, while changes in sectoral technology and preference further reduce entry to roughly -4.4 pp. We separate the effect of sectoral technology and preferences by decomposing their -2.4 pp contribution into relative price and income effects using a constant elasticity of substitution of 0.01. This low elasticity effectively provides a lower bound on the importance of income effects and is consistent with consumption value-added calculations in Herrendorf, Rogerson and Valentinyi (2013). The decomposition suggests that relative price effects generate about -1 pp of the -2.4 pp decline, with the remaining due to income effects. Finally, we consider the role of rising net imports which reduce entry by a further -2.1 pp. The full model therefore generates an entry rate decline of about -6.5 pp, with each of the three driving forces contributing roughly equally.

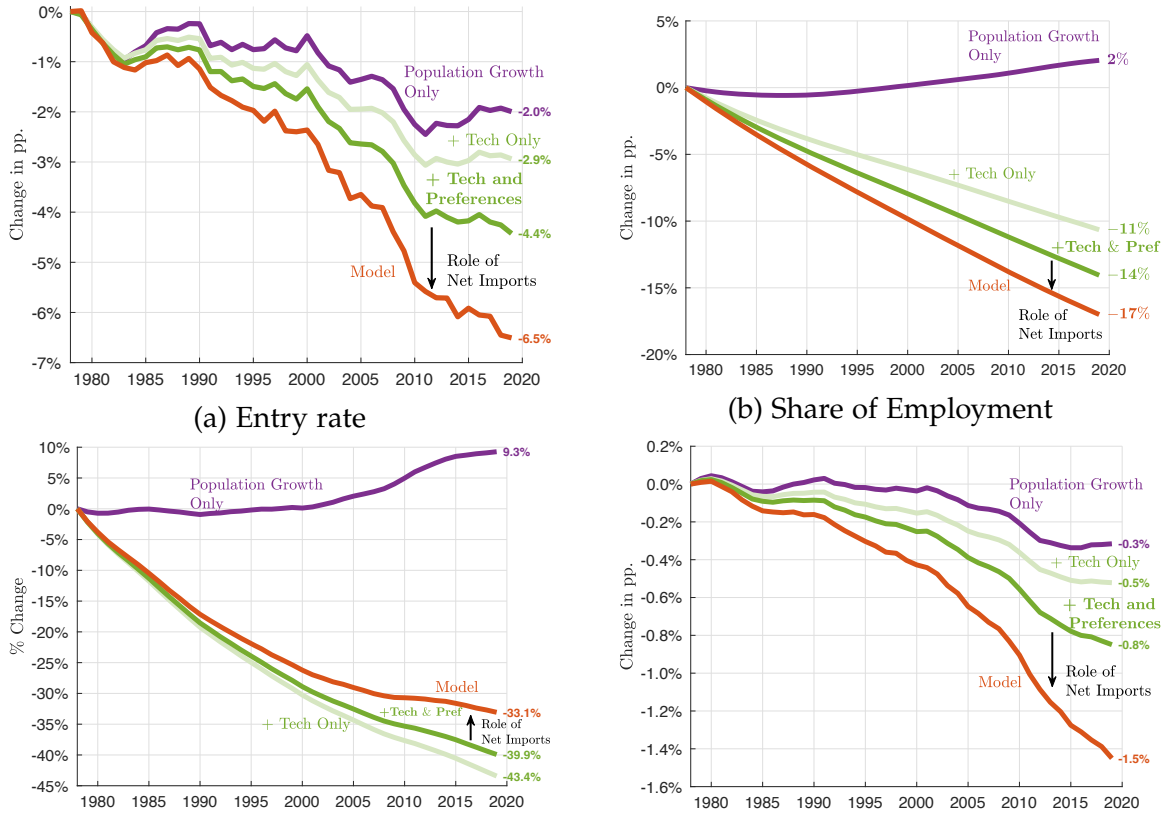
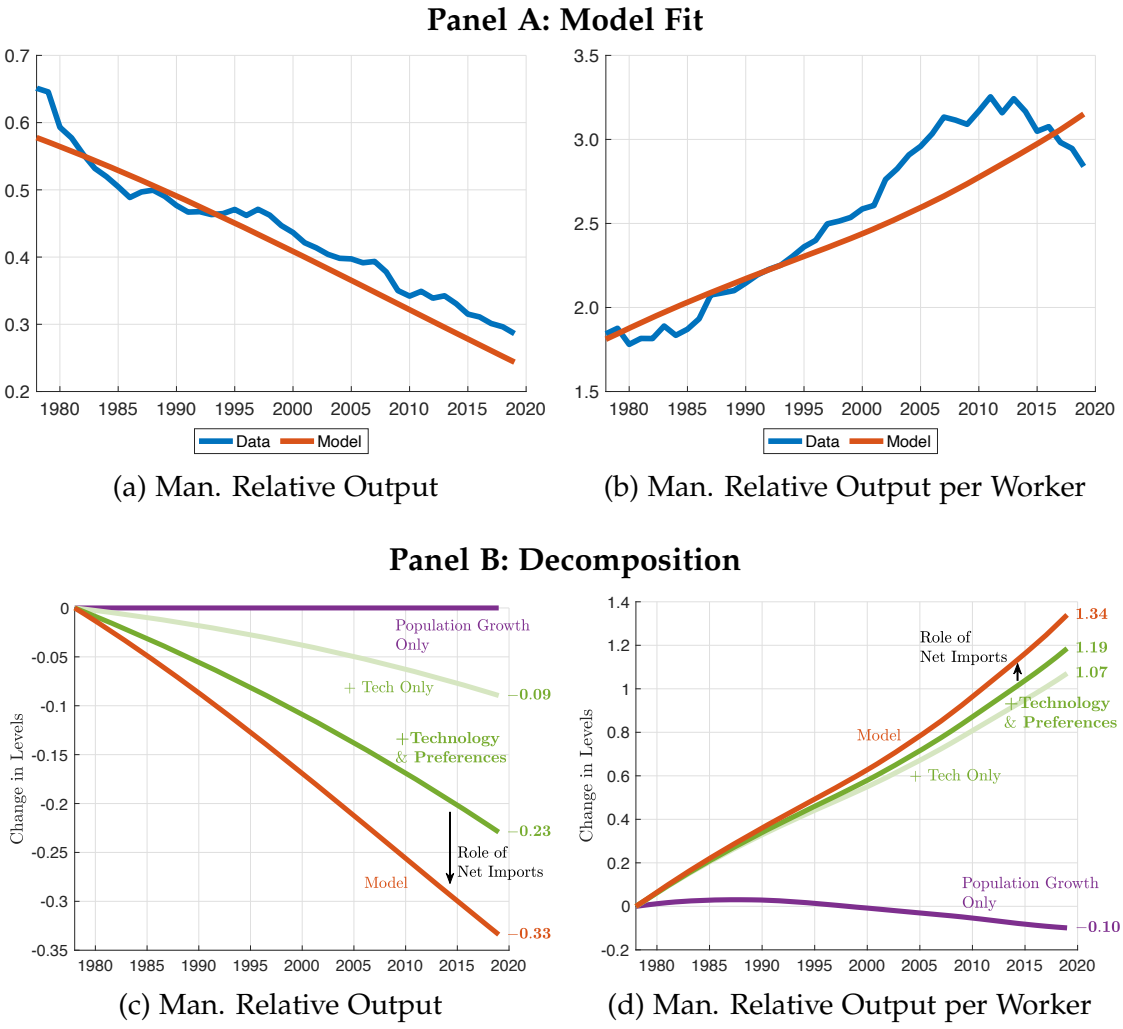


Figure 5: Manufacturing firm dynamics

Figure 5c presents the implications for average firm size. Population growth and net imports—forces that operate primarily through firm aging—both increase in average size. However, changes in sectoral technology reduce firm size within age groups, generating a substantial decline in average size that more than offsets these aging forces. As a result, average firm size in manufacturing falls by about 33 percent in the model. In the absence of firm aging induced by population growth and net imports, the average size decline would be even larger. Figures 5b and 5d show that these forces also account for the decline in manufacturing employment share (about -17 pp) and exit rates (about -1.5 pp) over the period. Overall, Figure 5 highlights how sectoral technology, demographic forces, and trade play distinct roles in shaping the evolution of manufacturing firm dynamics and employment reallocation.

Manufacturing Employment Share. Figure 6 explains the decline in manufacturing employment shares through the lens of the accounting relationship in (24), which relates relative manufacturing employment to relative output and relative output per worker.



(a) Man. Relative Output (b) Man. Relative Output per Worker

(c) Man. Relative Output (d) Man. Relative Output per Worker

Figure 6: Components of Manufacturing Relative Employment Share

Panel A shows that the model reproduces the key patterns in the data. Manufacturing relative output declines from roughly 0.65 in 1978 to about 0.30 by the end of the sample, while relative output per worker rises from about 1.8 to above 3. Panel B decomposes the forces behind these changes. Population growth has no effect on relative output. Changes in sectoral technology and preferences account for part of the decline, reducing relative output by about 0.23 in levels,

but the model matches the full decline of roughly 0.33 only after net imports are included. Imports therefore play a central role in explaining the contraction of manufacturing output relative to the rest of the economy.

Figure 6d shows the corresponding decomposition for relative output per worker. The full model produces an increase in relative output per worker of 1.34, even though population growth slightly reduces relative output per worker (about -0.10). The change in sectoral technology and preferences generates most of the increase, along with a modest positive contribution from net imports. In the model, the shift away from labor-intensive production, captured by rising manufacturing profit shares, raise output per worker in manufacturing relative to nonmanufacturing.

Taken together, Figure 6 shows that the decline in manufacturing employment shares arises from two complementary forces: rising imports reduce manufacturing's share of total output, while technological changes increase manufacturing output per worker relative to the rest of the economy.

Manufacturing jobs. Policy discussions of the decline in U.S. manufacturing often focus on employment levels rather than employment shares. Figure 7 shows the evolution of manufacturing employment levels. Since 1978, U.S. manufacturing employment has fallen from about 18 million to 10 million jobs. The population growth only counterfactual shows that the observed loss of 8 million jobs substantially understates the true magnitude of the decline. The US worker population nearly doubled over the same period, and as Figure 5b shows, the observed slowdown in population growth mechanically raises the manufacturing employment share. Had manufacturing employment kept up with US worker population and manufacturing jobs responded to population-induced reallocation, employment in the sector would have reached roughly 36 million jobs. This implies a loss of 26 million manufacturing jobs, as opposed to the observed 8 million.

To assess the relative contributions of technology, broadly defined to include preferences, and trade to this employment gap, it is therefore essential to incorporate changes in population growth in the counterfactual analysis. The green line cumulates the effect of changes in technology and population growth while holding trade parameters fixed, effectively shutting down the role of trade. The gap between this counterfactual and the population growth counterfactual iso-

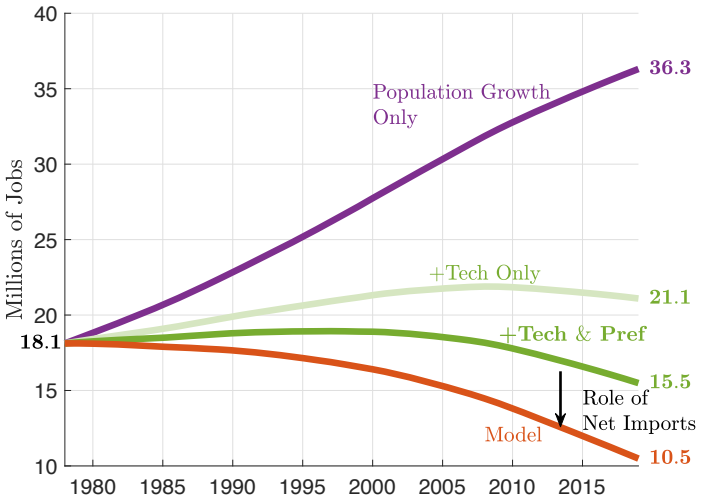


Figure 7: Manufacturing Employment

lates the effects of shifts in sectoral technology. Given that manufacturing would have about 15 million jobs had net imports remained at their initial levels, changes in sectoral technology account for the loss of about 21 million jobs. The loss of the remaining 5 million jobs is attributable to trade. The counterfactual analysis therefore attributes roughly 80% of manufacturing job losses to sectoral technology and 20% to trade.

Manufacturing Firm Share. The evolution of the manufacturing firm share can be understood using an accounting relationship similar to (24). The relative number of manufacturing firms is equal to relative employment divided by relative average size:

$$\frac{M_m}{M_{nm}} = \frac{N_m}{N_{nm}} \div \frac{N_m / M_m}{N_{nm} / M_{nm}} \tag{26}$$

Given that the calibrated model generates the evolution of both sectoral employment shares and average sizes, this accounting relationship implies that it also matches the decline in the manufacturing firm share.

Figure 8 decomposes the decline in the manufacturing share of firms into the contribution of the underlying forces. The slowdown in population growth alone increases the manufacturing firm share by about 3.2 percentage points, opposing the observed decline. This result follows from Proposition 4. In the calibrated model, nonmanufacturing has a steeper exit-by-age profile than manufacturing. Therefore, firm aging induced by the population growth slowdown is more pro-

nounced in nonmanufacturing, which lowers the relative average size and drives up the relative manufacturing firm share.

Changes in sectoral technology, excluding preferences, move firm shares in the opposite direction. Figure 5 shows that changes in technology lower average size in manufacturing without affecting nonmanufacturing much, so the net effect is that relative average size declines. Changes in technology reallocate employment from manufacturing to nonmanufacturing, which reduces relative manufacturing employment. The effect on relative average size is stronger, so the net effect is an increase in the manufacturing firm share. Once the additional effect of employment reallocation due to changing preferences is incorporated, however, the manufacturing firm share declines. As a result, technology and preferences together contribute -4.1 pp, lowering the manufacturing firm share to 0.9 percentage points.

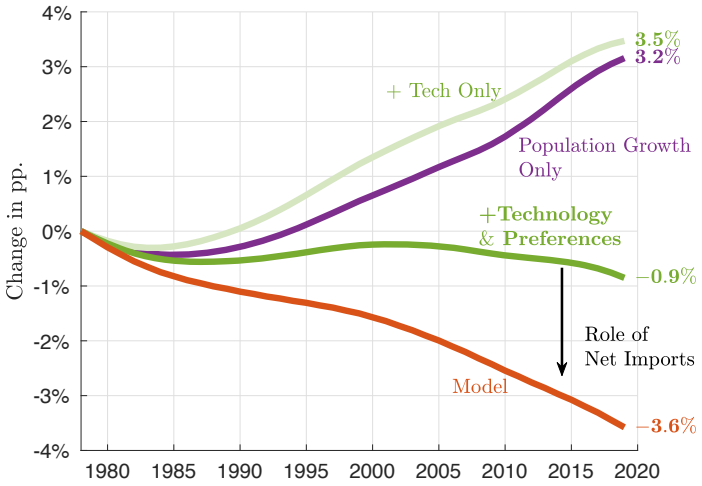
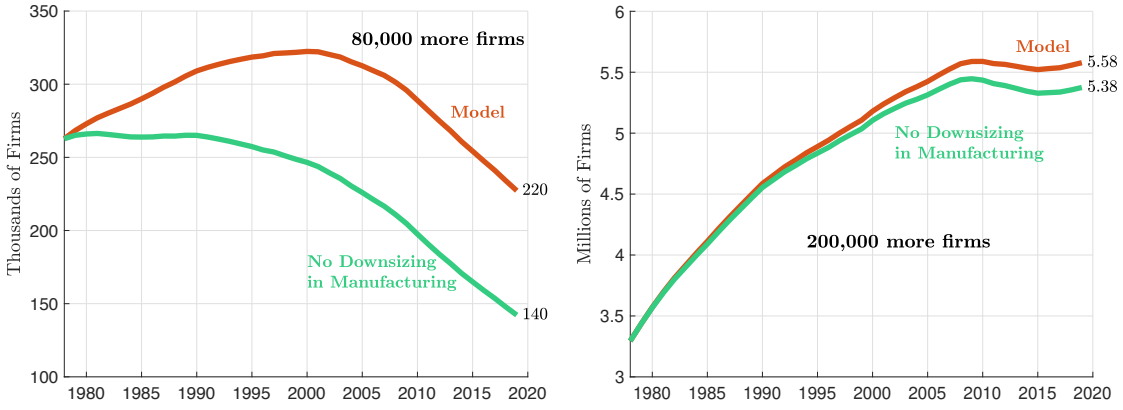


Figure 8: Manufacturing Share of Firms

The full model, however, produces a much larger decline of about 3.6 percentage points. The difference reflects the role of rising net imports, which further reduce domestic manufacturing output and employment relative to nonmanufacturing. Through the accounting relationship above, this additional contraction in employment lowers the number of manufacturing firms required to supply the domestic market. Quantitatively, technology and preferences account for about 60 percent of the total downward forces acting on manufacturing firm shares, while net imports account for the remaining 40 percent. Taken together, the results show that while demographic forces tend to increase manufacturing firm

shares through differential firm aging across sectors, technological change and trade forces drive the long-run contraction in the number of manufacturing firms.



(a) Manufacturing (b) Nonmanufacturing

Figure 9: Sectoral Shock Transmission

Transmission of Sectoral Shocks. The multisector dynamic entry equation implies that a sector's entry rate is determined not only by its own firm demographics, but also by firm demographics in other sectors. Figure 9 illustrates this intersectoral linkage using a counterfactual experiment. The green line depicts a scenario in which average firm size within firm age-groups in manufacturing does not decline over the sample period, implying no downsizing in manufacturing. This counterfactual is implemented by fixing the parameters governing the decline in average firm size—specifically, the labor share of inputs, α_{mt}, and the profit share parameter, η_{mt}—at their baseline levels.

Figure 9a shows that the downsizing shock in manufacturing freed up employment labor to create approximately 80,000 additional manufacturing firms. Importantly, the effect is not confined to manufacturing. As shown in Figure 9b, 200,000 nonmanufacturing firms are due to the decline in manufacturing firm size and the labor reallocation arising from it. In other words, 3.4% of nonmanufacturing firms exist due to the transmission of the downsizing shock that occurred in manufacturing.

4 Final Remarks

This paper develops a multisector framework with input-output linkages, endogenous firm entry and exit, and rich lifecycle dynamics to jointly study declining business dynamism and sectoral reallocation in the US economy. A central insight of the analysis is that sectoral firm dynamics cannot be understood independently of reallocation. Employment shifts across sectors feed back into entry and exit decisions through both static input-output linkages and dynamic channels operating through firm-age distributions.

We applied the framework to study the US manufacturing decline. Extending the framework to explicitly incorporate frictions would be a desirable step to study the joint evolution of firm dynamics and economic development. Such a setting would also allow for policy counterfactuals designed to assess how business dynamism reacts to demographic change, trade policy, or industrial policy along development paths.

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Online appendices

Appendix A

A.1 Data Appendix

- Data Sources:
  1. Firm-level data is from the US Census Business Dynamics Statistics database
  2. All other data series are from the National Income and Product Accounts (NIPA-BEA) and Industry Economic Accounts (IEA-BEA).
     (a) Industry Economic Accounts: Make-Use Framework Tables, "The Use of Commodities by Industries, Before Redefinitions (Producers' Prices) - Sector", 1997-2021
     (b) Industry Economic Accounts: Input-Output Tables, Supplemental Estimate Tables, Historical Make-Use Tables, Use Tables/Before Redefinitions/Producer Value 1947-1996
     (c) GDP by Industry Accounts: Historical Industry Accounts Data, "GDP by Ind VA SIC": Value Added by Industry, Gross Output by Industry, Intermediate Inputs by Industry, the Components of Value Added by Industry, and Employment by Industry, 1947-1997
     (d) Sector Prices are retrieved from the BEA Chain-Type Price Indexes for Gross Output by 2-digit Industry.
        – Non-manufacturing prices are aggregated using Tornqvist index: the growth in the price index is a weighted average of price growth in each underlying sector with weights equal to expenditure shares.
        – Normalized by labor wage growth.
        – Smoothed by fitting an exponential curve with constant growth rate.
- Use the numbers for Manufacturing. Non-Manufacturing is Private Industries minus Manufacturing. Exclude government, scrap, and other used incomparable imports.

- Mapping to sectoral technology parameters:

$$\gamma_i = \frac{\text{Personal Consumption Expenditure in Sector } i \ (PCE_i)}{\sum_j PCE_j}$$

$$\nu_i = \frac{\text{Manufactured Usage in Sector } i}{\text{Total Intermediate Usage in Sector } i}$$

$$\alpha_i \approx \frac{\text{Compensation of Employees in Sector } i}{\text{Compensation of Employees in Sector } i + \text{Total Intermediate Usage in Sector } i}$$

$$1 - \eta_i \approx \frac{\text{Corporate Gross Operating Surplus in Sector } i}{\text{Comp of Emp in } i + \text{Corporate Gross Operating Surplus in } i \ + \text{Total Int Usage in Sector } i}$$

- Compensation of Employees is Wage and Salary accruals plus Supplements to wages and salaries
- Our measure of *Corporate Gross Operating Surplus* differs from the *Gross Operating Surplus* measured in NIPA in that it excludes the government sector, the noncorporate sector, and propietors' income. *Corporate Gross Operating Surplus* = Net Interest (corporate) + Business current transfer payments + Rental Income + Corporate profits before tax with IVA and CCadj + Capital consumption allowance (corporate) + Taxes on production and imports, less subsidies.
- Mapping to international trade parameters:

$$\chi_i = \frac{\text{Sector } i \text{ Exports}}{\text{Sector } i \text{ Gross Output}}$$

$$\phi_j = \frac{\text{Commodity } j \text{ Imports}}{\text{Commodity } j \text{ Total Intermediates} + \text{Commodity } j \text{ Consumption}}$$

A.2 Additional Figures and Tables

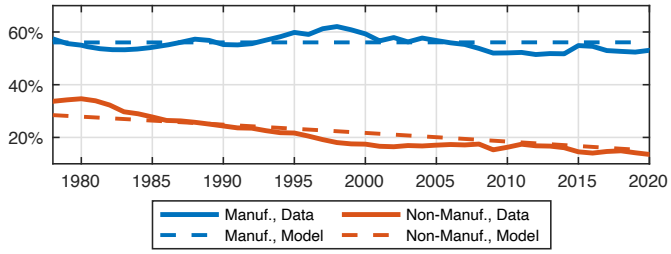
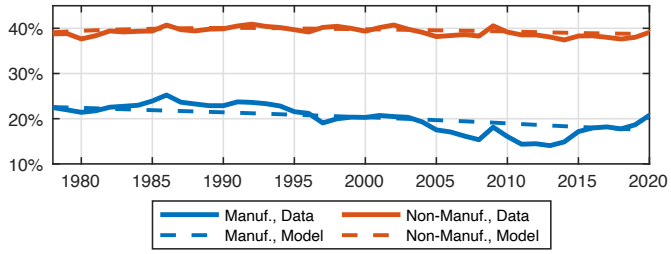
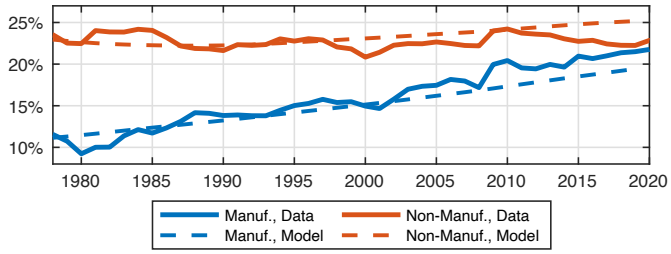
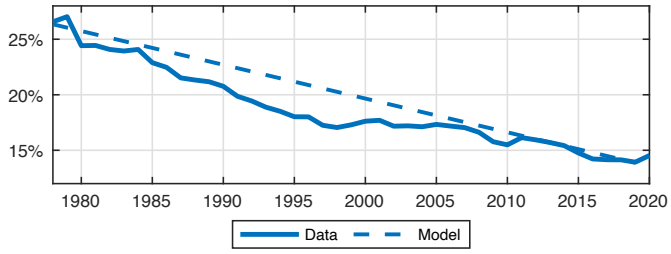
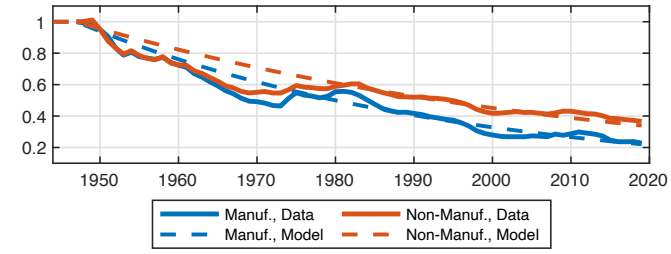
Variable	Data Mapping	Time Series
ν_{it}	Manufactured Usage Share	
α_{it}	Labor Share of Inputs	
$1-\eta_{it}$	Profit Share of Gross Output	
γ_{mt}	Manufacturing Share of Final Good Expenditures	
$p_{it} \to A_{it}$	Sector Prices	

Figure A-1: Drivers: Sectoral Technology, $\mathcal{T}$

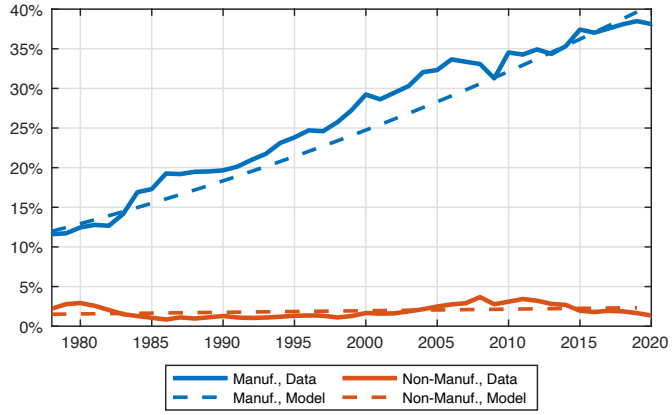
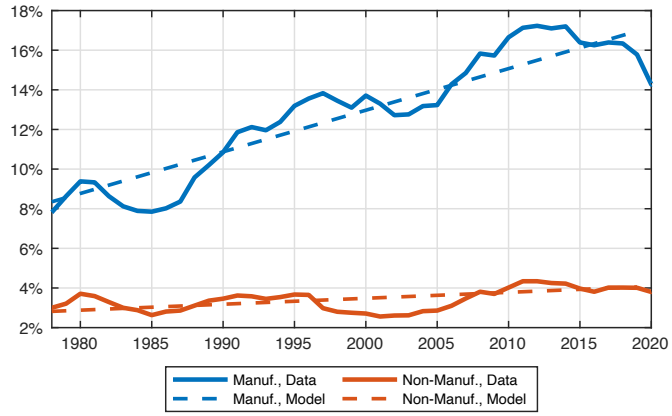
Variable	Data Mapping	Time Series
ϕ_{it}	Import Share of Total Usage	
χ_{it}	Export Share of Gross Output	

Figure A-2: Drivers: International Trade

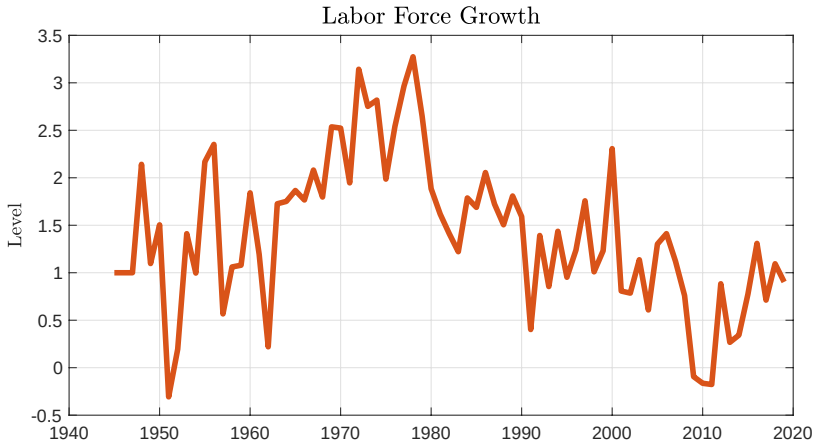
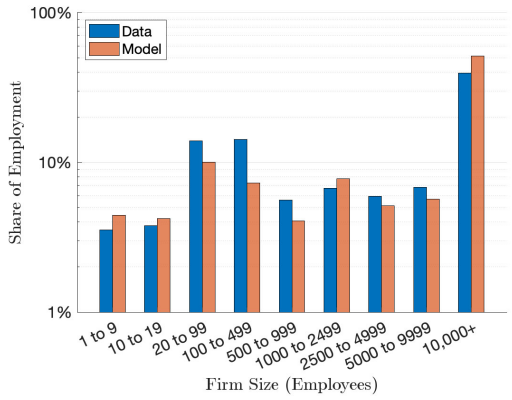
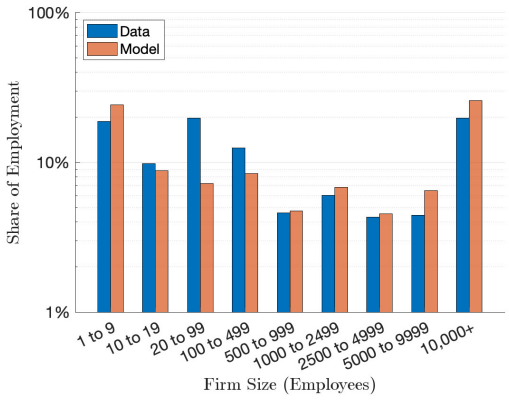


Figure A-3: Drivers: Population Growth

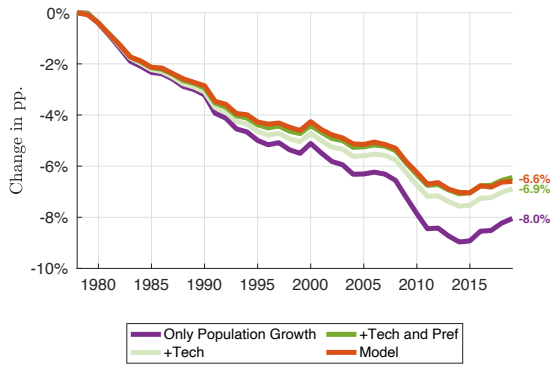


(a) Manufacturing

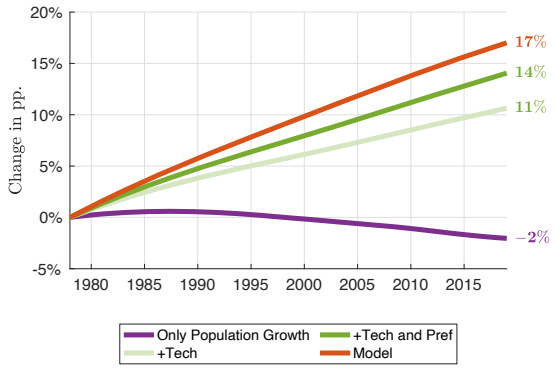


(b) Nonmanufacturing

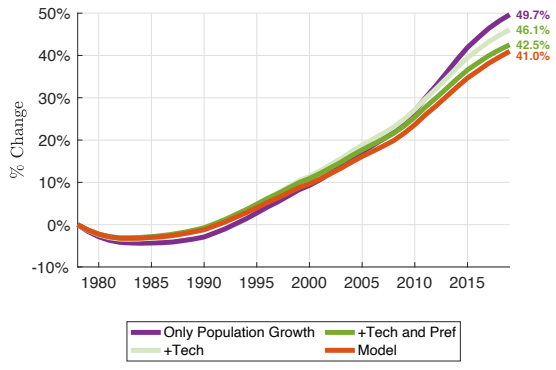
Figure A-4: 1978 Employment Size Distribution



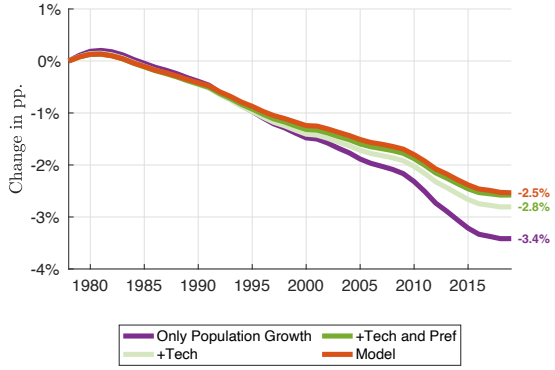
(a) Entry rate



(b) Share of Employment



(c) Average firm size



(d) Exit rate

Figure A-5: Nonmanufacturing firm dynamics

Table A-1: Calibration

Panel A: Parameter Values		
	Manufacturing	Non-Manufacturing
Entry Cost, c_i^e	58.73	9.27
Mean of entrant dist, ψ_i^G	-2.14	-3.71
Std. dev. of entrant dist, σ_i^G	3.50	3.01
AR(1) Persistence, ρ_i	0.96	0.96
AR(1) std. dev., σ_i^ε	1.21	1.04
Long-run mean low-type, $\psi_{i\ell}$	-4.23	-10.64
Long-run mean high-type, ψ_{ih}	-4.23	-1.85
Operating cost low-type, $c_{i\ell}^f$	4.90	0.93
Operating cost high-type, c_{ih}^f	18.12	8.62
Share of high mean startups, ω_{ih}	0.75	0.09

Panel B: Model Fit		
	Manufacturing	
	Data	Model
Average Firm Size	68.96	68.18
Entry Rate	9.29	10.97
Average Entrant Size	11.24	18.57
Concentration of entrants	50.08	49.07
5-year growth rate	84.93	108.25
5-year exit rate	51.83	46.21
Firm Share 1 to 9	52.64	56.60
Employment Share 100+	78.56	80.51
Employment Share 1000+	58.38	68.46
Employment Share 10,000+	38.98	48.44
	Non-Manufacturing	
	Data	Model
Average Firm Size	15.57	17.64
Entry Rate	12.14	14.44
Average Entrant Size	5.28	4.90
Concentration of entrants	25.57	14.17
5-year growth rate	77.55	67.58
5-year exit rate	56.39	55.87
Firm Share 1 to 9	80.12	81.60
Employment Share 100+	53.46	56.08
Employment Share 1000+	36.08	42.94
Employment Share 10,000+	20.62	25.34

Notes: Panel A reports calibrated parameter values. The remaining model parameters are the discount factor $\beta = 0.96$ and labor force growth rate in 1947 $g = 0.01$. Panel B reports data and model moments as time-averages from 1978 to 1983. Concentration of entrants is the share of employment in firms with more than 20 employees within the 0-year old category divided by total employment by 0-year old firms.

Table A-2: Age Profiles by Sector

Panel A: Manufacturing				
	Exit Rate		Avg. Firm Size	
Age	Data	Model	Data	Model
0	0.00	0.00	10.08	16.41
1	21.24	19.24	13.12	18.30
2	15.37	14.25	14.87	20.51
3	12.90	12.07	16.20	23.08
4	11.26	10.78	17.39	25.99
5	10.10	9.91	18.52	29.20
6 to 10	7.82	8.51	22.06	39.39
11 to 15	5.95	7.23	28.02	56.53
16 to 20	5.18	6.59	33.22	68.72
21 to 25	4.94	6.21	36.93	74.65
Above 25	4.18	5.66	111.99	78.72
Panel B: Non-Manufacturing				
	Exit Rate		Avg. Firm Size	
Age	Data	Model	Data	Model
0	0.00	0.00	5.52	4.93
1	25.56	26.70	7.12	5.67
2	16.76	19.64	7.98	6.33
3	13.81	16.32	8.70	6.98
4	11.93	14.17	9.38	7.63
5	10.76	12.57	10.01	8.29
6 to 10	8.63	9.58	11.74	10.58
11 to 15	6.69	6.18	14.80	15.55
16 to 20	5.74	4.29	18.52	22.25
21 to 25	5.29	3.25	22.69	29.94
Above 25	4.58	2.22	67.22	51.02

Notes: Data moments are the average across all years in the sample. Model moments are the time average for the same years as the data moments.

Table A-3: Percentage Change in Average Firm Size by Age Group, 1978-2019

	Manufacturing		Non-Manufacturing	
Age	Data	Model	Data	Model
0	-1.07	-0.64	-0.12	0.04
1	-1.16	-0.83	-0.12	0.04
2	-1.24	-0.99	-0.09	0.04
3	-1.32	-1.13	-0.03	0.04
4	-1.30	-1.25	0.01	0.05
5	-1.34	-1.35	0.02	0.05
6 to 10	-1.82	-1.56	-0.16	0.05
11 to 15	-2.04	-1.73	-0.24	0.07
16 to 20	-2.05	-1.80	-0.09	0.08
21 to 25	-1.48	-1.82	0.19	0.09
Above 25	-1.74	-1.87	-0.60	0.10

Notes: Numbers are annual time trend of log-average firm size for comparable years.

Table A-4: Time trends in exit rates by firm age

Firm Age	Manufacturing	Non-manufacturing
1	-0.169^{***}	-0.179^{***}
	(0.020)	(0.021)
2	-0.101^{***}	-0.092^{***}
	(0.017)	(0.019)
3	-0.060^{***}	-0.066^{***}
	(0.015)	(0.016)
4	-0.037^{**}	-0.045^{***}
	(0.014)	(0.013)
5	-0.024^{*}	-0.031^{**}
	(0.013)	(0.012)
6 to 10	-0.008	-0.011
	(0.012)	(0.010)
11 to 15	0.003	0.016
	(0.014)	(0.010)
16 to 20	0.012	0.023^{*}
	(0.017)	(0.012)
21 to 25	0.002	0.028
	(0.017)	(0.019)
Above 25	-0.016	0.029^{*}
	(0.015)	(0.016)