Bills of Exchange in a Supply Chain

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Abstract

We study a trade credit chain in which firms finance input purchases by issuing IOUs that circulate as settlement instruments and as collateral. Even when IOU holdings are perfectly diversified against idiosyncratic default risks, limited commitment induces endogenous default by downstream firms and makes the collateral value of receivables depend on downstream firms' incentives to default on their own payables. This valuation channel propagates shocks upstream by tightening collateral constraints and downstream by depressing trade-credit supply when default diverts pledged receivables to creditors, generating distortions in input use and strategic complementarity in default incentives. We characterize how heterogeneity across firms or sectors shapes equilibrium distortions and over- and under-investment patterns. We show that self-consistent beliefs about default can generate multiple equilibria, implying fragility to coordination failures. In a line network, a small local productivity shock can trigger default contagion and discontinuous drops in investment. In a general production network, we illustrate how financial frictions interact with input–output linkages to produce non-monotone output patterns and a structured matrix of inefficiency wedges. We also study policy interventions that shift default costs and thereby coordinate failure-state repayment incentives; among efficiency-implementing default profiles, the least-cost intervention maximizes default probabilities.

Keywords: trade credit; bills of exchange; collateral; endogenous default; contagion; production networks; financial networks.

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1 Introduction

Trade credit plays a crucial role in facilitating economic transactions across supply chains. In this paper, we study a chain of trade credit where firms acquire inputs for investment from upstream suppliers and subsequently supply inputs to downstream firms. This chain of trade credit is linked to a chain of collateral: firms issue trade credit in the form of IOUs backed by their own output and by receivables from downstream firms' IOUs. In our model, "circulation" means that these IOUs serve as the medium of payment in input transactions and that receivables form a collateral chain, rather than literal endorsement or resale of the same IOU across multiple trades.

A key mechanism we investigate is how idiosyncratic risks propagate through the trade credit chain. Under limited commitment, firms may default when facing unfavorable output realizations. The propagation operates through two distinct channels. *Upstream propagation* works through financing and collateral. When a firm becomes more likely to repay poorly in adverse states, its promises become less valuable as settlement instruments and as collateral. This makes it more expensive for the firm to finance input purchases with trade credit. As a result, the firm cuts back on inputs purchased from its upstream supplier, and the supplier's balance sheet weakens because the receivables it holds have lower pledgeable value and are less valuable as balance-sheet assets. In this way, a deterioration in one firm's repayment incentives and collateral value tightens financing conditions for firms upstream.

Downstream propagation works through equilibrium prices and quantities, driven by endogenous trade-credit supply. When the receivables a firm would obtain from supplying inputs on credit are less valuable—as settlement instruments, balance-sheet assets, and collateral, in particular because upon failure-state default the pledged fraction is seized by its creditors—the firm has weaker incentives to extend trade credit and to produce. These adjustments change the prices and quantities faced by its downstream customers, altering their investment incentives and the collateral positions they carry forward.

These two propagation channels interact to create a feedback loop and a strategic complementarity in default incentives. When agents expect more widespread default, the collateral value of receivables falls and financing tightens upstream, making default more attractive and reinforcing the initial expectation. Simultaneously, as the valuation of new receivables falls, firms reduce the supply of trade credit and production, which shifts equilibrium prices and quantities and weakens downstream collateral positions. The resulting coordination problem is fundamentally over failure-state repayment behavior (default intensities $\{D_i\}$) and the implied collateral coverage of receivables. While IOU prices $\{v_i\}$ adjust endogenously with default risk, movements in v_i offset rather than reinforce the effect of default on buyers' funding terms (the expected-repayment per unit of funds raised, τ_i), so the main source of complementarity runs through collateral coverage and the value of receivables rather than through IOU-price dynamics.

Section 4 characterizes this multiplicity in a cycle network where both channels operate. Section 5

illustrates the two channels separately: productivity shocks propagate upstream through collateral, while riskiness shocks also generate downstream effects through the trade-credit supply incentive. Section 6 shows how the upstream channel alone can trigger default cascades on a line network.

The mechanism we study extends beyond the historical institution of bills of exchange. In modern supply chains, trade credit still takes the form of bilateral payables and receivables rather than literal endorsement of the same bill across multiple trades. These claims are often financed by banks through factoring or reverse factoring. In a standard supply-chain-finance program, the buyer approves an invoice, a bank pays the supplier early against that approved receivable, and the buyer pays the bank later. Such arrangements do not eliminate interfirm credit; they refinance it. The underlying bilateral payable-receivable link remains, so bank intermediation complements rather than replaces interfirm credit chains.

Recent evidence shows that these claims remain central to firms' liquidity management and to shock transmission along production networks. Using Swedish administrative data, Jacobson and von Schedvin (2015) show that losses on trade receivables following customer bankruptcy raise suppliers' own bankruptcy risk, while Amberg et al. (2021) show that firms buffer liquidity shocks by drawing more credit from suppliers and cutting credit to customers. More recently, Amberg et al. (2024) show that supply-chain-finance programs—in which banks advance funds against approved receivables—relax suppliers' liquidity constraints and raise sales, employment, and investment. Modern institutional arrangements thus differ from historical bills of exchange, but they do not eliminate the underlying chain of interfirm claims.

Our environment is therefore best viewed as a model of chains of receivables and payables whose value depends on downstream repayment and that can be pledged, discounted, or financed by banks. Historically, bills of exchange used in the late 18th century Amsterdam banking system provide a particularly transparent antecedent for this mechanism, and the real-bills doctrine offers a natural benchmark for thinking about its welfare properties. Bills could be resold with each seller acting as a guarantor, creating a "credit wrapper" where all signatories were jointly liable. Acceptance fees were low, implying perceived low risk. However, the 1763 crisis revealed the fragility of this system: market participants were bound through balance sheets even when no goods were traded, and a single bank could simultaneously hold receivables and payables for the same bill, amplifying default impacts. The real-bills doctrine held that unrestricted discounting of "real bills" should be stabilizing because such claims were backed by real transactions. Our analysis challenges this view: even when claims are safe in expectation, their use in a collateral chain can generate financial contagion. The lender's valuation over collateral depends on default risk assessment, and in a collateral chain this creates feedback loops that can impair collateral values and generate inefficiencies.

Our analysis yields several key results. First, heterogeneity across sectors generates systematic patterns of inefficiency: in cycle networks with alternating productivity or riskiness, we show that high-productivity or high-success-probability firms may underinvest while low-productivity or low-success-probability firms

overinvest, with the pattern depending on relative default intensities. Second, equilibrium multiplicity arises: multiple self-consistent equilibria can coexist for the same fundamentals, creating fragility to coordination failures. Third, small local shocks can trigger default contagion on line networks: a productivity dip at one firm reduces the collateral value held by upstream neighbors, triggering their default and cascading upstream through the collateral channel. Fourth, in general production networks with triangular input-output structures, final output exhibits non-monotone patterns across sectors, reflecting the interaction of rising prices, changing default intensities, and input-use wedges.

The analysis also has sharp policy implications. Section 8 studies interventions that shift reputational (or legal) default costs and thereby act directly on failure-state default probabilities. We characterize the set of default policies that implement the efficient allocation—typically low-dimensional in cycle networks, while in connected networks with multi-sourcing efficiency generically requires eliminating default—and show that the least-cost efficient intervention is the one that maximizes default probabilities within that efficient set.

Our work contributes to several literatures. First, on financial contagion along credit chains (Allen and Gale, 2000; Acemoglu et al., 2013; Elliott et al., 2014): existing work focuses on intermediation chains where fixed default costs or run risk cause contagion ex post. We study a trade credit chain where default arises endogenously and affects ex ante investment decisions. (Kiyotaki and Moore, 2004; Bigio, 2023; Altinoglu, 2021) also study supply chain credit; Bigio (2023) focuses on settlement delay, while we emphasize the role of default. Relative to Altinoglu (2021), we emphasize a distinct mechanism: trade credit is tied to production linkages and to a collateral chain in which receivables are pledgeable assets. Endogenous default affects IOU prices and valuations over receivables, which in turn distort real input demands. This joint treatment of financial and production networks delivers sharp cross-sectional predictions for input-use wedges and allows small shocks to trigger discontinuous default regimes and contagion along supply chains.

Second, on production networks (Baqaee and Farhi, 2019, 2020): this literature studies how microeconomic heterogeneity aggregates to macroeconomic outcomes through input-output linkages. Related empirical work shows that financial constraints amplify the propagation of small shocks through supplier-customer networks (Demir et al., 2024). Our contribution is to study financial networks jointly with production networks, where financial frictions are micro-founded through limited commitment and endogenous default decisions. This joint analysis reveals how collateral chains interact with production linkages to generate inefficiency patterns and contagion.

Third, on collateral constraints (Geanakoplos, 2010; Fostel and Geanakoplos, 2012, 2014; Ordoñez, 2018): this literature studies how collateral requirements affect asset prices, leverage cycles, and financial stability. We contribute by studying how collateral frictions affect financial contagion specifically within supply chain networks, where the collateral value of receivables depends on downstream firms' default decisions, creating feedback loops that propagate shocks.

2 Model

This section formalizes a production economy with trade credit in which (i) intermediate goods are exchanged at date $t = 1$ against privately issued IOUs that are settled at $t = 2$, and (ii) firms can pledge only a fraction of their future resources. The objective is to isolate a mechanism through which *endogenous default* and the *collateral value of receivables* jointly determine the equilibrium price of IOUs and, through financing conditions, real input demands.

The analysis relies on three assumptions. First, entrepreneurs are risk neutral and IOU holdings are perfectly diversified, so IOU prices equal expected repayments. Second, markets for goods are segmented so that firms must use IOUs to transact at $t = 1$ (no frictionless spot transfer of claims on the date-2 numéraire). Third, limited commitment implies that only a fraction of own output and a fraction of receivables can be seized by creditors.

These primitives deliver a tractable mapping from production decisions to leverage and default incentives, which pins down the state-contingent repayment schedule and the IOU price v_i. Section 3 then uses these objects to characterize competitive equilibrium and to study how financial linkages (through receivables as collateral) distort real allocations.

2.1 Environment and timing

The economy is populated by a continuum of risk-neutral entrepreneurs organized in N sectors, indexed by $i \in \{1, \ldots, N\}$. Time is discrete with two dates, $t = 1, 2$.

At $t = 1$, sector-i entrepreneurs ("firm i") (a) choose the quantity y_i of intermediate good i supplied to the market, (b) choose a vector of intermediate inputs $\{x_{ij}\}_{j=1}^{N}$ used in their own long-term project, and (c) choose the face value a_i of IOUs they issue to finance input purchases.

At $t = 2$, idiosyncratic production risk is realized, final output is produced (if successful), IOUs are settled, and consumption takes place. The final good is the numéraire.

2.2 Technology

Intermediate-good supply. In period $t = 1$, firm i can produce $y_i \geq 0$ units of intermediate good i at a labor/effort cost $c_i k(y_i)$, where $c_i > 0$ and $k(\cdot)$ is strictly increasing, strictly convex, twice continuously differentiable, and satisfies $k(0) = 0$.

Final-good production. Firm i also has a long-term risky technology. If it acquires inputs $\{x_{ij}\}_{j=1}^{N}$ at $t = 1$, then at $t = 2$ it produces

$$z_i f_i(\{x_{ij}\}_{j=1}^{N}) = z_i \prod_{j=1}^{N} \left(\frac{x_{ij}}{\alpha_{ij}} \right)^{\alpha_{ij}}. \tag{1}$$

where $\alpha_{ij} \geq 0$ and $\sum_{j=1}^{N} \alpha_{ij} = \alpha < 1$. Production succeeds with probability $q_i \in (0,1)$, yielding output as above; with probability $1 - q_i$ output is zero. Shocks are independent across firms.

2.3 Trade at $t = 1$ and IOUs

Intermediate goods are traded competitively at $t = 1$. Let p_j denote the competitive price of intermediate good j in units of the date-2 numéraire.

Market segmentation and inside money. Because the numéraire good is available only at $t = 2$ and there is no frictionless spot transfer of an *outside*, risk-free claim on the date-2 numéraire at $t = 1$ (trading of endogenously issued IOUs is allowed), firms must finance purchases of inputs by issuing IOUs (inside money). An IOU issued by firm i is a promise to deliver one unit of the date-2 numéraire.

Let v_i denote the $t = 1$ competitive price of one unit of face value of firm-i IOUs. Risk neutrality and diversification (made explicit below) imply that v_i equals the expected repayment per unit of face value.

Two clarifications are useful. First, "circulation" refers to the fact that these privately issued IOUs serve as the settlement instrument for intermediate-good transactions at $t = 1$, and that the receivables generated by those transactions form a *collateral chain*. The model does not feature literal endorsement or resale of the same IOU across multiple trades. Second, entrepreneurs within each sector form a continuum, so an individual firm is atomistic with respect to the market price of its sector's IOUs: it takes (p_i, v_i) as given when choosing $(y_i, \{x_{ij}\}, a_i)$. In equilibrium, v_i is pinned down by the sector-i repayment schedule implied by the equilibrium collateral coverage ratio $r_i = \gamma p_i y_i / a_i$.

This environment features two interacting networks: (i) a *production network* given by the physical input matrix $\{x_{ij}\}$, and (ii) a *credit network* given by the issuance and holding of IOUs along trade relationships.

2.4 Limited commitment and pledgeable resources

Firms cannot commit to repay more than what creditors can seize at $t = 2$. Pledgeable resources have two components.

- **Own output.** In the success state, creditors can seize a fraction $\theta \in (0,1)$ of firm i's final output, i.e., $\theta z_i f_i(\{x_{ij}\})$. In the failure state, own output is zero.
- **Receivables (collateral chain).** When firm i supplies y_i at price p_i, it receives IOUs from its customers. We assume these receivables are perfectly diversified across a continuum of downstream buyers, so the aggregate repayment from receivables is deterministic and equals its expected value. Denote by $p_i y_i$ the *market value* of this diversified receivables portfolio (in units of the date-2 numéraire), equal under risk neutrality to expected repayment; this value is distinct from the face values of individual

downstream IOUs and is determined in general equilibrium. Creditors can seize a fraction $\gamma \in (0,1)$ of this amount, i.e., $\gamma p_i y_i$.

Consequently, the face value of IOUs issued by firm i, denoted a_i, is bounded above by pledgeable resources in the success state:

$$a_i \leq \gamma p_i y_i + \theta z_i f_i(\{x_{ij}\}). \tag{2}$$

This constraint ensures that firm i is solvent in the success state.

2.5 Repayment and default

Repayment on firm-i IOUs depends on the realization of its idiosyncratic shock.

- **Success (probability q_i).** Firm i repays the full face value a_i.
- **Failure (probability $1 - q_i$).** Firm i has no own output; the maximum repayment creditors can obtain is the pledgeable portion of receivables, $\gamma p_i y_i$.

To keep notation consistent across the analytical and numerical sections, we represent the default decision using a function $\delta(\cdot)$. Define the *collateral coverage ratio*

$$r_i \equiv \frac{\gamma p_i y_i}{a_i}.$$

Then we let $D_i \in [0,1]$ denote the default decision/intensity in the failure state:

$$D_i \equiv \delta(r_i), \qquad \delta(r) = \begin{cases} 1 & \text{if } r < 1, \\ d \in [0,1] & \text{if } r = 1, \\ 0 & \text{if } r > 1. \end{cases} \tag{3}$$

Thus $D_i = 1$ corresponds to default upon failure ($a_i > \gamma p_i y_i$), $D_i = 0$ to full repayment upon failure ($a_i < \gamma p_i y_i$), and the knife-edge case $r_i = 1$ allows a mixed/indifferent default decision $d \in [0,1]$.

In Section 7 we consider a general supply-chain network and, for the purpose of numerical illustration, allow an *interior* default intensity $D_i = \delta(r_i) \in (0,1)$ with $r_i \equiv \gamma p_i y_i / a_i$. This smooth reduced-form specification captures the idea that repayment upon failure may occasionally be supported by non-pledgeable resources (e.g., reputational capital, liquidity buffers, or renegotiation/rollover), so that default upon failure need not be deterministic even when $r_i < 1$. In that case, the expected failure-state repayment fraction $D_i r_i + (1 - D_i) = 1 - D_i(1 - r_i)$ exceeds the collateral recovery r_i only when the firm is under-collateralized and does not default with certainty (i.e., when both $r_i < 1$ and $D_i < 1$). The knife-edge rule in (3) is the limiting benchmark.

Let v_i be the $t = 1$ price of one unit of face value of firm-i IOUs. Under risk neutrality and diversification,

$$v_i = q_i \cdot 1 + (1 - q_i) \left[(1 - D_i) + D_i \frac{\gamma p_i y_i}{a_i} \right]. \tag{4}$$

2.6 Firm problem

Given input prices $\{p_j\}_{j=1}^N$ and IOU prices $\{v_i\}_{i=1}^N$, firm i chooses $(y_i, \{x_{ij}\}_{j=1}^N, a_i)$ to maximize expected consumption net of effort costs:

$$\begin{aligned}\max_{y_i,\{x_{ij}\},a_i} \quad & q_i\Big[p_i y_i + z_i f_i(\{x_{ij}\}) - a_i\Big] \\ & + (1-q_i)\Big[(1-\gamma)p_i y_i + (1-D_i)(\gamma p_i y_i - a_i)\Big] - c_i k(y_i)\end{aligned} \tag{5}$$

subject to the date-1 budget constraint

$$\sum_{j=1}^{N} p_j x_{ij} \leq v_i a_i, \tag{6}$$

and the pledgeability constraint (2). The budget constraint states that firm i finances input purchases by issuing IOUs with face value a_i; since each unit of face value sells at price v_i, total financing proceeds equal $v_i a_i$.

2.7 Discussion and interpretation

The model links real production decisions to financial conditions through a collateral channel. Each firm is simultaneously a borrower (issuing IOUs to pay suppliers) and a lender (holding receivables from customers). Limited commitment implies that default in the failure state destroys creditors' expected recovery and therefore lowers the IOU price v_i. The private marginal cost of financing inputs depends on the wedge $[q_i + (1-q_i)(1-D_i)]/v_i$, so default risk can simultaneously dilute repayment incentives and lower v_i; conditional on default, weaker collateral reduces v_i and tightens financing. Because receivables are pledgeable, financing capacity depends on downstream repayment values, generating endogenous intersectoral linkages in the credit network even when idiosyncratic shocks are diversified.

This structure captures the logic of bills-of-exchange systems in which short-term private claims serve as settlement instruments and, through the collateral chain, can be repeatedly pledged. The key economic feature is not runs or fixed default costs, but rather an endogenous feedback between leverage, default incentives, and the collateral value of trade receivables.

3 Equilibrium

This section defines a competitive equilibrium and characterizes how endogenous default and the collateral value of trade receivables shape input supply and input demand across sectors. The goal is to make explicit the mapping from default incentives to equilibrium prices and real allocations, which will later discipline the efficiency analysis and the comparative statics.

Throughout, we maintain the assumptions introduced in Section 2, in particular risk neutrality and diversified IOU holdings (so IOU prices equal expected repayments), competitive trade in intermediate goods at $t = 1$, and limited commitment as summarized by (2)–(4). In addition, to isolate distortions from default incentives, we restrict attention to equilibria in which the success-state pledgeability constraint (2) is slack (equivalently, its Lagrange multiplier is zero), i.e.,

$$a_i < \gamma p_i y_i + \theta z_i f_i(\{x_{ij}\}).$$

When (2) binds, limited pledgeability generates the standard borrowing-capacity (and hence output) distortion; we abstract from it to focus on how failure-state default affects IOU prices and real input choices.

The key economic distinction is that IOU prices $\{v_i\}$ are pinned down by each issuer's repayment schedule implied by the equilibrium collateral coverage ratio—and hence by its default incentives—whereas the expected value of receivables, $p_i y_i$, is determined in general equilibrium and reflects downstream financing conditions through equilibrium goods prices and quantities.

3.1 Supply and Demand

Input supply. Supplying inputs y_i allows firm i to lend to other firms and to receive IOUs in return. The benefit of lending consists of expected repayment at date $t = 2$ and of the contribution of pledgeable revenues to firm i's borrowing capacity. However, when firm i defaults, the pledgeable fraction of these revenues is seized by its creditors.

The first-order condition with respect to y_i is

$$c_i k'(y_i) = p_i \left[(1-\gamma) + \gamma \left(q_i + (1-q_i)(1-D_i)\right)\right] = p_i \left[1 - \gamma(1-q_i)D_i\right]. \tag{7}$$

The term $q_i + (1-q_i)(1-D_i)$ is the probability that the pledged portion of receivables accrues to the entrepreneur (seizure occurs only in the failure state and only when the firm defaults). The right-hand side captures the *downstream propagation* channel: when firm i expects to default ($D_i = 1$), the pledged fraction of loan revenues is seized by its creditors, so the entrepreneur internalizes a lower effective return from receivables. This lower effective return depresses firm i's incentive to supply inputs on credit, which shifts equilibrium prices and quantities and affects downstream buyers' investment decisions and collateral positions. When firm i defaults, the expected return from lending is reduced, reflecting the loss of pledgeable income.

Default ($D_i = 1$) reduces the effective return from supplying inputs because, upon failure, the entrepreneur loses the pledged portion of receivables, $\gamma p_i y_i$, to creditors. Under the maintained restriction that the success-state pledgeability constraint (2) is slack, increases in y_i do not relax the borrowing constraint at the margin, so the right-hand side of (7) captures only the expected-return effect.

Input demand. Intermediate inputs raise expected final output when the project succeeds. Their cost reflects the expected repayment on issued IOUs, discounted by default risk and scaled by their value v_i. Issuing IOUs tightens the borrowing constraint, while higher output relaxes it through pledgeability.

Using the budget constraint (6) as an equality, the first-order condition with respect to x_{ij} is

$$q_i z_i \frac{\partial f_i}{\partial x_{ij}}(\{x_{ij}\}) = [q_i + (1-q_i)(1-D_i)]\,\frac{p_j}{v_i}. \tag{8}$$

Figure 1 provides a schematic graphical summary of the equilibrium input-supply schedule implied by (7) and the aggregate input-demand schedule implied by (8); the exact expression for aggregate demand is given in the discussion below.

The right-hand side of (8) has a transparent interpretation. To finance one unit of input expenditure, $p_j x_{ij}$, firm i issues IOUs with face value $a_i = p_j x_{ij}/v_i$. In the success state (probability q_i), the firm repays the full face value; in the failure state (probability $1-q_i$), it repays in full only if it does not default (probability $1-D_i$). Thus the entrepreneur's expected repayment per unit of funds raised is

$$\tau_i \equiv \frac{q_i + (1-q_i)(1-D_i)}{v_i}.$$

In the numerical illustration in Section 7 (which allows an interior default rule $D_i = \delta(r_i)$), τ_i need not be monotone across sectors in either D_i or r_i, because changes in collateral coverage affect both the default term in the numerator and the IOU price in the denominator.[1] This object combines a *default-dilution* term in the numerator and an *IOU discount* term in the denominator. Holding v_i fixed, a higher default intensity D_i lowers expected repayment and therefore lowers the entrepreneur's private marginal cost of financing inputs (risk-shifting). At the same time, the market price v_i declines when collateral coverage deteriorates, raising τ_i. In the knife-edge default regimes below, moving from full repayment upon failure ($D_i = 0$, hence $v_i = 1$) to default upon failure ($D_i = 1$, hence $v_i < 1$) reduces τ_i from 1 to $q_i/v_i < 1$. However, *conditional on default* ($D_i = 1$), a decline in pledgeable receivables ($p_i y_i$) lowers v_i and therefore increases q_i/v_i, tightening financing and reducing input demand. Through market clearing and the collateral chain, these changes in buyers' demand feed back to suppliers upstream by altering the value of the receivables they hold and can pledge.

Default regimes and IOU values. For given (p_i, y_i, a_i), the failure-state default decision is summarized by (3). If $a_i \le \gamma p_i y_i$, firm i repays in full even upon failure, so $D_i = 0$ and (4) implies $v_i = 1$. If instead $a_i > \gamma p_i y_i$, firm i defaults upon failure, so $D_i = 1$ and (4) implies

$$v_i a_i = q_i a_i + (1-q_i)\gamma p_i y_i \quad \text{and} \quad v_i < 1.$$

[1] To see the basic forces, note that the IOU price equals expected repayment per unit of face value: $v_i = q_i + (1-q_i)\big[(1-D_i) + D_i r_i\big]$, where r_i is the collateral coverage ratio. Hence v_i exceeds $q_i + (1-q_i)(1-D_i)$ whenever $D_i > 0$ and $r_i > 0$, implying $\tau_i < 1$ under interior default.

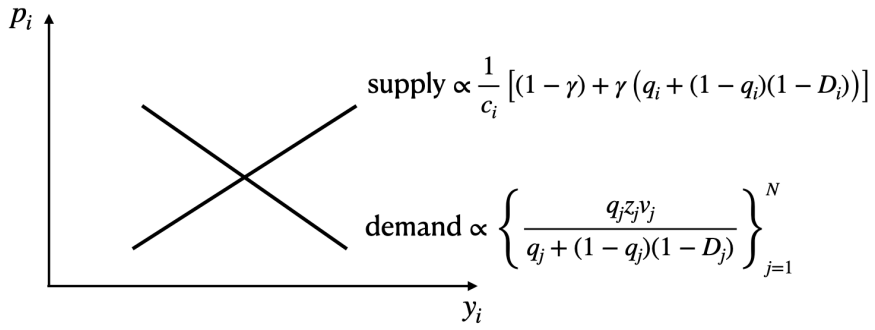


Figure 1: Demand and supply of intermediate good i.

Thus, conditional on default, higher pledgeable receivables ($p_i y_i$) increase the repayment in the failure state and raise the IOU price v_i.

3.2 Competitive Equilibrium

Definition 1. A competitive equilibrium consists of production choices $\{y_i\}$, input demands $\{x_{ij}\}$, financial choices $\{a_i\}$, default decisions $\{D_i\}$, prices $\{p_i\}$, and IOU values $\{v_i\}$ such that, for all i:

1. **Optimization:** Given (p_i, v_i), the allocation $(y_i, \{x_{ij}\}, a_i)$ solves firm i's problem as described in Section 2. In particular, it satisfies the budget constraint (6), the pledgeability constraint (2) (slack by maintained assumption), and the first-order conditions (7) and (8). This takes (p_i, v_i) as parametric at the level of an individual firm; v_i is pinned down in equilibrium by (4) evaluated at the sector-i equilibrium coverage ratio.

2. **Input Market Clearing:** The supply of intermediate good j equals the aggregate demand from all other sectors:
$$y_j = \sum_{i=1}^{N} x_{ij} \quad \forall j. \tag{9}$$

3. **Default and IOU pricing:** Default decisions satisfy (3) and IOU prices satisfy (4).

Input supply in sector i, derived from (7), depends on the default decision of firms in that sector. Aggregate demand for input i, obtained by summing (8) across buyers, depends on default decisions and IOU values in all sectors.

Figure 1 illustrates the determination of equilibrium in the market for the intermediate input produced in sector i. The vertical axis reports the price p_i of the input, while the horizontal axis reports the quantity supplied, y_i.

The upward-sloping curve represents the supply of intermediate goods in sector i, as characterized by the first-order condition (7). Abstracting from binding pledgeability constraints in the success state, the supply

schedule satisfies

$$p_i(y_i) = \frac{c_i k'(y_i)}{(1-\gamma) + \gamma\left(q_i + (1-q_i)(1-D_i)\right)} = \frac{c_i k'(y_i)}{1-\gamma(1-q_i)D_i}.$$

The upward slope comes from the convexity of k (i.e. $k'' > 0$), while the level of the schedule scales with c_i and the effective expected return from lending to buyers, $1-\gamma(1-q_i)D_i$. When firm i defaults ($D_i = 1$), the pledgeable fraction of loan revenues is seized by creditors, reducing the expected payoff from supplying inputs. As a result, default shifts the supply curve inward by lowering the effective return to production and lending.

The downward-sloping curve represents aggregate demand for input i, obtained by summing the optimal input demands (8) across all purchasing sectors $j = 1, \ldots, N$. In equilibrium, aggregate demand is proportional to

$$\sum_{j=1}^{N} \alpha_{ji} \frac{q_j z_j f_j v_j}{q_j + (1-q_j)(1-D_j)},$$

which follows from (8) and Cobb–Douglas technology: $q_j z_j \partial f_j / \partial x_{ji} = \left[q_j + (1-q_j)(1-D_j)\right] p_i / v_j$ implies $p_i x_{ji} = \alpha_{ji} \frac{q_j z_j f_j v_j}{q_j + (1-q_j)(1-D_j)}$. where α_{ji} captures the technological importance of input i in sector j's production. Demand depends not only on technological parameters and productivity, but also on the default decisions of downstream sectors. When firms in sector j default, the expected cost of repayment on IOUs they issue falls, which raises their demand for intermediate inputs and shifts the demand curve outward.

The intersection of the two curves determines the equilibrium price p_i and quantity y_i. The figure highlights that both supply and demand are shaped by default risk, but through distinct channels. Default in sector i primarily affects supply by reducing the expected return from lending, while default in downstream sectors affects demand by lowering the effective cost of financing input purchases. Through these channels, financial linkages propagate shocks across sectors even when borrowing constraints do not bind.

3.3 Efficiency

This subsection defines a frictionless (First-Best) benchmark allocation used to measure inefficiency wedges. Because our environment features market segmentation at $t = 1$ and limited commitment, this benchmark should be read as a reference point rather than as the outcome of a planner subject to the same trading and pledgeability frictions.

Efficiency requires that the *expected* marginal productivity of each input equal its marginal cost. For all i, j, define

$$\omega_{ij} \equiv \frac{MP_i(\text{input } j)}{MC_j(\text{input } j)} = \frac{q_i z_i \frac{\partial f_i}{\partial x_{ij}}(\{x_{ij}\})}{c_j k'(y_j)}.$$

We say the allocation is (First-Best) efficient if $\omega_{ij} = 1$ for all i, j; that is, for all sectors i and j, the expected marginal product of input j in producing good i equals the marginal cost of producing input j.

The marginal cost in sector j is the marginal cost of producing one additional unit of y_j in units of final consumption. From the input-supply first-order condition (7),

$$MC_j = c_j k'(y_j) = [(1-\gamma) + \gamma\,(q_j + (1-q_j)(1-D_j))]\,p_j.$$

It reflects the technological cost of production, adjusted for the fact that a fraction γ of revenues from input sales is seized when default occurs.

The marginal rate of substitution for firm i with respect to input j is the marginal contribution of that input to expected consumption. From the input-demand first-order condition (8),

$$MP_{ij} = q_i z_i \frac{\partial f_i}{\partial x_{ij}}(\{x_{ij}\}) = [q_i + (1-q_i)(1-D_i)]\,\frac{p_j}{v_i}.$$

This term captures the effective price of inputs once default risk and the value of IOUs are taken into account. Therefore,

$$\omega_{ij} = \frac{[q_i + (1-q_i)(1-D_i)]\,\frac{1}{v_i}}{(1-\gamma) + \gamma\,[q_j + (1-q_j)(1-D_j)]}. \tag{10}$$

Financial linkages distort production whenever default risk or IOU values differ across sectors. Input use in sector i depends not only on technology but also on default decisions in sectors i and j and on the endogenous value of IOUs.

Proposition 1. *No distortion is induced by financial linkages and production is efficient in equilibrium in the following cases:*

1. $q_i = 1$ *for all* i*;*
2. $\gamma p_i y_i \geq a_i$ *so that* $D_i = 0$ *for all* i*;*
3. $z_i = z$, $c_i = c$, $0 < q_i = q < 1$, *and* $\sum_j \alpha_{ij} = \sum_j \alpha_{ji} = \alpha$ *for all* i.

The knife-edge restriction $\sum_j \alpha_{ij} = \sum_j \alpha_{ji} = \alpha$ in case (3) is best interpreted as a *balanced gross exposure* condition: each sector has the same overall intensity of input purchases (row sum) and the same overall intensity of sales on credit to downstream buyers (column sum), even though the bilateral pattern of shares α_{ij} can be arbitrarily asymmetric. From Cobb–Douglas, sector i's total expenditure on intermediate inputs satisfies $\sum_j p_j x_{ij} = (a_i/\alpha) v_i \sum_j \alpha_{ij} = a_i v_i$, i.e., equals the market value of the IOUs it issues. From market clearing, sector i's total sales revenue satisfies $p_i y_i = \sum_j \alpha_{ji}(a_j/\alpha) v_j$; when every column sum equals α, a symmetric profile $a_j v_j \equiv av$ delivers $p_i y_i = av$ for all i. Thus each sector's *gross* borrowing and *gross* lending positions are the same in equilibrium (no node is systematically a net debtor or net creditor), so the discount on trade credit applies symmetrically to what the firm pays and to what it receives. This shuts down cross-sectional dispersion in IOU values and therefore eliminates the relative-price wedge in (10); the

network can be highly asymmetric, but only the overall exposure summarized by the row and column sums matters for efficiency in this symmetric environment.

In case (3), all sectors are symmetric: $q_i = q$, $D_i = 1$, $p_i = p$, and $v_i = v$ for all i. Hence,

$$\mathrm{MC}_j = [(1-\gamma) + \gamma q]\, p, \qquad \mathrm{MP}_{ij} = \frac{q}{v} p.$$

Using the equilibrium IOU pricing condition $v = q/[1-(1-q)\gamma]$, we obtain

$$\frac{q}{v} = (1-\gamma) + \gamma q,$$

so that $\mathrm{MP}_{ij} = \mathrm{MC}_j$ for all i, j. See Appendix B for detailed proof of case (3) of the proposition.

Although default occurs in equilibrium and IOUs trade at a discount ($v < 1$), each firm both pays and receives IOUs at the same discounted rate. As a result, the financial wedge created by default risk cancels out in relative prices, leaving production decisions undistorted and the equilibrium allocation efficient.

4 Pattern of Inefficiency induced by Financial Linkages

The objective of this section is to characterize *how* inefficiency arises once sectors are asymmetric, and to clarify the *pattern* of wedges induced by financial linkages. The central question is not whether default occurs, but rather how heterogeneity across sectors translates into cross-sectional dispersion in IOU values and hence into over- and under-investment along the supply chain.

In the cycle network analyzed below, both propagation channels are active. The upstream channel operates through default-contingent funding terms: a sector's default decision D_j and the value of its pledgeable receivables pin down its IOU price v_j, which scales the effective cost of financing inputs. The downstream channel operates through the incentive to extend trade credit: when a firm anticipates default, the pledged fraction of the receivables it acquires is seized by its creditors, lowering the effective return from supplying on credit. Together, these channels create a strategic complementarity in default decisions that can support multiple self-fulfilling equilibria, as characterized in Proposition 4.

We impose three assumptions to obtain sharp comparative statements. First, we focus on the same frictionless competitive environment as in Sections 2–3 and maintain the restriction that pledgeability constraints are slack in the success state. Second, we restrict attention to a simple *cycle* production network in which each sector uses only one intermediate input. Third, we consider structured heterogeneity across sectors (either alternating productivity or alternating project success probabilities).

These restrictions deliver an analytically tractable laboratory in which IOU values are "slow-moving" state variables along the credit loop: a change in failure-state default incentives in one location shifts trade-credit supply and the collateral value of receivables elsewhere, which feeds back into real input choices and default

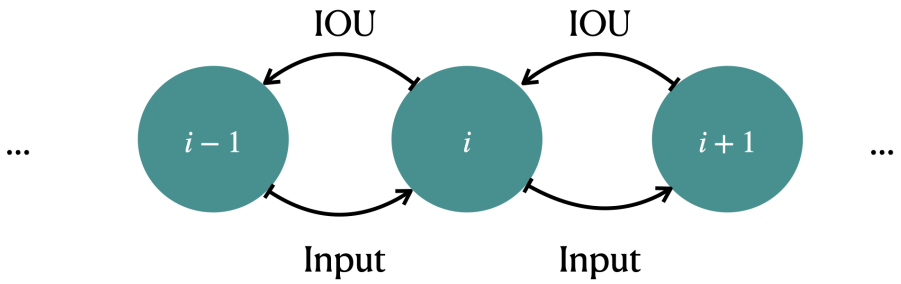


Figure 2: Cycle production network and the induced credit linkages.

incentives. IOU prices co-move endogenously with default risk, but because they enter buyers' funding terms in the denominator, their adjustment partly absorbs changes in expected repayment. The key driver of multiplicity is therefore the feedback in default incentives and collateral coverage, not the sensitivity of IOU prices per se.

4.1 A Cycle Network with Alternating Productivity

To illustrate the mechanism, consider a production network where the intermediate good produced in sector $i-1$ is used only by sector i, for $i = 1, \ldots, N$ (with $0 \equiv N$). Thus each sector uses a single intermediate input from its upstream neighbor, and the production function for final-good production in sector i can be written as $f_i(x_{i-1})$.

The production network arranges sectors on a directed cycle. The induced credit network runs in the opposite direction, because firms borrow from upstream suppliers while holding receivables from downstream buyers. Figure 2 summarizes these linkages.

We assume heterogeneity lies only in productivity: $z_i \in \{z_H, z_L\}$ alternates across sectors, with $z_H > z_L > 0$, while $f_i(\cdot) = f(\cdot)$, $q_i = q \in (0,1)$, and $c_i = 1$ for all i. In a stationary equilibrium there are therefore two types of sectors, H and L, and equilibrium objects take two values: (a_H, v_H, p_H, y_H) and (a_L, v_L, p_L, y_L). Here a_T is the face value of IOUs issued by type $T \in \{H, L\}$, v_T is the IOU price, p_T is the intermediate-good price, and y_T is input supply.

Recall that in the general network the input-use wedge ω_{ij} in (10) is indexed by the *final-good producer* i (the buyer / input demander, whose productivity is z_i) and the *input supplier* j (the producer of the intermediate good). In the N-cycle each firm uses a single upstream input, so we can write $\omega_i \equiv \omega_{i,i-1}$ (with $0 \equiv N$). Under alternating productivities, we denote by ω_H (respectively ω_L) the wedge faced by a high- (respectively low-) productivity *buyer* i, i.e., $\omega_H \equiv \omega_{H,L}$ and $\omega_L \equiv \omega_{L,H}$; the supplier type is therefore pinned down by the cycle and suppressed in the shorthand. Likewise, (a_T, v_T) refer to IOUs *issued by* type-T firms (buyers), whereas (p_T, y_T) refer to the price and quantity of the intermediate good *produced by* type-T

firms (suppliers). In a cycle the same sector is both a buyer (from its upstream neighbor) and a supplier (to its downstream neighbor), so the relevant type depends on the role being described.

From Proposition 1 it follows that when $z_H = z_L$ the equilibrium is efficient even though all firms default[2]. We show in what follows that when instead $z_H > z_L$ the equilibrium is inefficient and the nature of the inefficiency depends on whether the heterogeneity among sectors, captured by the difference $z_H - z_L$ is small or large.

Small productivity heterogeneity: both types default

When productivity differences are small, the equilibrium can feature default upon failure for both types. In this regime, the wedge created by discounted IOUs does not cancel across types: the two IOU prices differ, so one type faces a lower effective cost of inputs and the other a higher one. The result is a systematic *reallocation* of investment away from the high-productivity sectors and toward the low-productivity sectors.

Proposition 2. *Consider the cycle network with alternating productivity. Fix $z_H > z_L$ and consider a stationary equilibrium in which both types default upon failure ($D_H = D_L = 1$). Then:*

1. *Both types default upon failure: $D_H = D_L = 1$;*
2. *High-productivity firms underinvest and low-productivity firms overinvest: $\omega_H > 1 > \omega_L$;*
3. *The following inequalities hold: $a_H > a_L$ and $v_H < v_L$;*
4. *If such an equilibrium exists for z_H, it also exists for all $z'_H \in (z_L, z_H)$.*

Proof. See Appendix C.

When $z_H > z_L$, high-productivity firms have greater demand for intermediate inputs produced by low-productivity L firms. To finance this higher investment, H firms issue more IOUs: $a_H > a_L$. Consequently, H firms have lower collateral coverage in the failure state, depressing their IOU value: $v_H < v_L$.

This valuation wedge creates asymmetric financing costs. L firms, with higher-valued IOUs, face a lower effective cost of financing inputs (through the factor $1/v_L$ in their demand condition, equivalently q/v_L in the wedge), leading them to overinvest relative to efficiency. Conversely, H firms face a higher effective cost (through q/v_H in the wedge), causing them to underinvest despite their higher productivity.

The mechanism operates entirely through the collateral channel: even though both firm types default with the same probability $(1-q)$, the endogenous difference in pledgeable receivables generates heterogeneous IOU values, distorting the real allocation.

[2]When $z_H = z_L$ we have $a_i = a$ and $v_i = v$ for all i,so that $\gamma v a < a$,and all firms default.

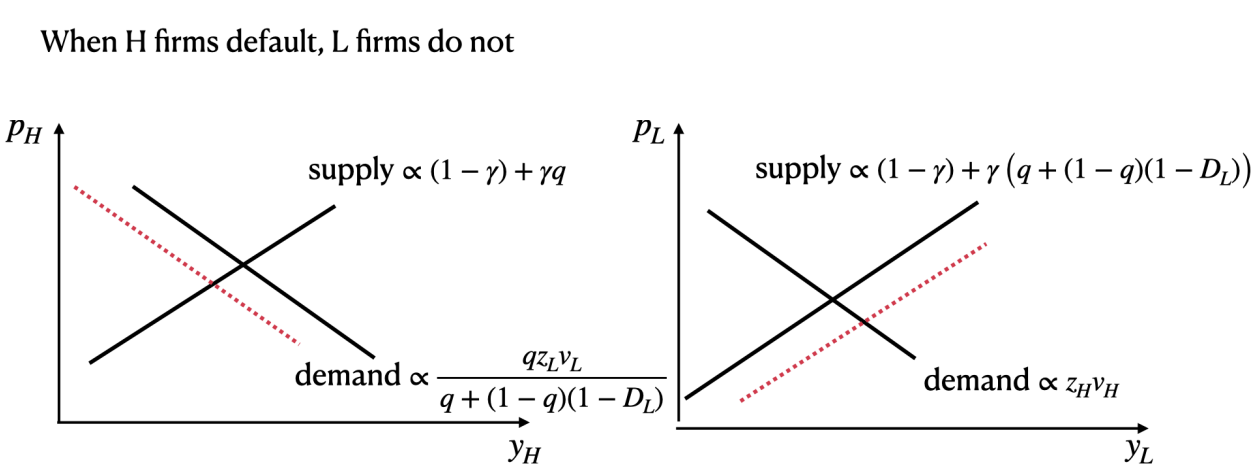


Figure 3: Demand and supply when L firms do not default.

Large productivity heterogeneity: only H types default

When the productivity gap is large, the L types receive sufficiently valuable receivables in equilibrium that they can repay even in the failure state. This change in default behavior alters both (i) their return from supplying inputs and (ii) the buyer-side expected-repayment term $\tau_L = [q+(1-q)(1-D_L)]/v_L$ that governs their input demand. In this regime the pattern of distortions reverses relative to the all-default case: H types overinvest while L types underinvest.

Figure 3 illustrates how the supply and demand schedules shift when L firms switch from defaulting upon failure to being always solvent.

Proposition 3. *Consider the cycle network with alternating productivity. Suppose $z_H \geq \underline{z}_H$, where $\underline{z}_H$ is the (implicit) threshold value of z_H above which the L type is solvent upon failure (equivalently, $a_L \leq \gamma p_L y_L = \gamma v_H a_H$) in the only-$H$-default regime; see Appendix D for its characterization. Then there exists a stationary equilibrium in which:*

1. *Only high-productivity firms default upon failure: $D_H = 1$ and $D_L = 0$;*
2. *The distortion pattern reverses: high-productivity firms overinvest and low-productivity firms underinvest, i.e., $\omega_H < 1 < \omega_L$;*
3. *The following inequalities hold: $a_H > a_L$ and $v_H < v_L = 1$;*
4. *If such an equilibrium exists for z_H, it also exists for all $z'_H > z_H$.*

Proof. See Appendix D.

When the productivity gap is sufficiently large ($z_H \geq \underline{z}_H$), high-productivity H firms demand so many inputs from L suppliers that the pledgeable value of L firms' receivables, $\gamma p_L y_L$, becomes large enough to cover their IOU obligations even in the failure state. Consequently, L firms become solvent ($D_L = 0$) and their IOUs trade at par ($v_L = 1$).

This regime shift changes wedges in (10) by shifting both the *buyer-side* expected-repayment term, $\tau_i \equiv [q_i + (1 - q_i)(1 - D_i)]/v_i$, and the *supplier-side* return term, $1 - \gamma(1 - q_j)D_j$. When L firms become solvent ($D_L = 0$ and hence $v_L = 1$), their input-supply condition becomes $k'(y_L) = p_L$ (rather than $k'(y_L) = p_L[(1 - \gamma) + \gamma q]$). Since $[(1 - \gamma) + \gamma q] < 1$, eliminating default raises the effective return from supplying on credit and shifts the L-supply schedule *outward* (more supply for a given price), consistent with Figure 3.

The reversal in investment distortions follows from how solvency changes the two components of the cross-type wedges. For ω_L (an L buyer using the H input), the supplier-side term is unchanged because H firms still default, so the denominator remains $(1 - \gamma) + \gamma q$. What changes is the buyer-side term: when L firms switch from default upon failure to full repayment, τ_L jumps from q/v_L to $1/v_L = 1$ (with $v_L = 1$), raising the numerator in (10) and implying $\omega_L > 1$ (underinvestment). For ω_H (an H buyer using the L input), the buyer-side term remains $\tau_H = q/v_H$, but the supplier-side term switches from $(1 - \gamma) + \gamma q$ to 1 when L firms become solvent; this is the same force behind the outward shift in L supply. As a result, $\omega_H = \tau_H = q/v_H < 1$ (overinvestment).

Fragility (for intermediate levels of firms' heterogeneity)

The transition between the regimes above can be discontinuous in the L-type failure-state default intensity D_L. When equilibrium beliefs switch from a regime in which L types default upon failure to one in which they are solvent, D_L jumps discretely (from $D_L = 1$ to $D_L = 0$). This discrete shift implies that the L-type IOU value jumps from a discount to par (v_L jumps from $v_L < 1$ to $v_L = 1$). Because D_L enters both the L-type input-supply condition and the buyer-side expected-repayment term that governs input demand, equilibrium input supplies and input demands shift discretely as well. As a result, the distortion pattern reverses: the economy switches from the all-default pattern (H types underinvest and L types overinvest) to the only-H-default pattern (H types overinvest and L types underinvest). For intermediate productivity gaps, both regimes can be self-consistent, generating equilibrium multiplicity.

Hence, for intermediate values of z_H, both regimes can be self-consistent: an equilibrium in which both types default and an equilibrium in which only H types default. This multiplicity reflects a coordination problem over the collateral value of receivables and, ultimately, over whether L firms default upon failure (i.e., over D_L): a shift in beliefs can trigger a jump in D_L and therefore in v_L, with discrete changes in input supplies and wedges. IOU prices move with default risk but do not constitute the amplifying feedback.

Proposition 4. *Consider the cycle network with alternating productivity. Assume the functional forms* $f(x) = x^{\alpha}/\alpha$ *and* $k(x) = x^{1+\kappa}/(1+\kappa)$ *with* $\alpha \in (0,1)$ *and* $\kappa > 0$*. Under suitable parameter restrictions, there exists a non-empty interval* $[\underline{z}_H, \bar{z}_H]$ *with* $z_L < \underline{z}_H < \bar{z}_H$ *such that for* $z_H \in [\underline{z}_H, \bar{z}_H]$*, multiple equilibria coexist:*

1. *An all-default equilibrium as characterized in Proposition 2, in which both types default upon failure* ($D_H = D_L = 1$)*;*
2. *An only-H-default equilibrium as characterized in Proposition 3, in which only high-productivity firms default* ($D_H = 1$, $D_L = 0$)*;*
3. *These equilibria feature opposite distortion patterns: in the all-default equilibrium, H firms underinvest and L firms overinvest; in the only-H-default equilibrium, H firms overinvest and L firms underinvest.*
4. *Additionally, for each* z_H *in this multiplicity region there exists a mixed-strategy equilibrium in which a fraction* $D_L(z_H) \in (0,1)$ *of L-type firms default upon failure. Along this mixed-equilibrium branch,* $D_L(z_H)$ *varies continuously with* z_H*. In particular, there exists a (knife-edge) mixed equilibrium with* $D_L = \gamma$ *that delivers efficiency,* $\omega_H = \omega_L = 1$*.*

Proof. See Appendix E.

The multiplicity arises from the interaction between upstream and downstream propagation. The key state variable is the L-type failure-state default intensity D_L, because it enters directly in the return from supplying on credit: when D_L is expected to be high, L firms are more likely to lose the pledged fraction of the receivables they acquire, so the effective return to lending falls (downstream channel) and L-type input supply contracts. The key upstream effect of a higher D_L on H's receivables operates via L's IOU value v_L and L's demand for the H good: a change in D_L affects L's funding terms and thus its demand schedule; that transmits to $p_H y_H$ (the value of receivables held by H firms, equal to $v_L a_L$ by the budget identity) and hence to v_H, and back to the L-type solvency condition $a_L \leq \gamma v_H a_H$, closing the strategic complementarity.

IOU prices for L firms, v_L, move endogenously with D_L but are not the source of the strategic complementarity. In the buyer-side funding term $\tau_L = [q + (1-q)(1-D_L)]/v_L$, a higher expected D_L lowers the numerator (expected repayment) while the induced discount in v_L works in the opposite direction, partially offsetting the change in τ_L. Conversely, when L firms expect to remain solvent, they supply more, H firms' receivables and v_H rise, and L firms' collateral coverage is high enough that default upon failure is not optimal. Both self-fulfilling beliefs are consistent for intermediate productivity gaps.

For intermediate productivity gaps $z_H \in [\underline{z}_H, \bar{z}_H]$, both self-fulfilling beliefs are consistent with equilibrium, generating multiplicity. The transition between regimes is discontinuous because the L-type default regime changes discretely (from $D_L = 1$ to $D_L = 0$ across the two pure equilibria), shifting input supplies

and collateral values and causing the distortion pattern to reverse. The associated movement in v_L (from a discount to par) is an equilibrium implication of this regime change rather than its cause.

Mixed-strategy equilibria interpolate between these pure strategies in the sense that, within the mixed-equilibrium branch, $(D_L, v_H, a_H, a_L, \omega_H, \omega_L)$ move smoothly with z_H. This does *not* rule out discontinuities across equilibrium types: for a given z_H in the multiplicity region, the all-default equilibrium has $v_L < 1$ while the mixed equilibrium has $v_L = 1$. The special case $D_L = \gamma$ achieves efficiency by exactly offsetting the distortion from default risk: the effective financing costs align so that marginal products equal marginal costs for both firm types, restoring the first-best allocation.

Figure 4 illustrates the three types of equilibria and how their properties vary with z_H. The figure displays eight plots in a 2×4 grid: the top row shows variables for high-productivity (H) firms (D_H, v_H, a_H, ω_H), while the bottom row shows variables for low-productivity (L) firms (D_L, v_L, a_L, ω_L). Each plot compares three equilibrium regimes: the all-default equilibrium (solid line, labeled "Low type defaults"), the only-H-default equilibrium (dashed line, labeled "Low type no default"), and the mixed equilibrium (dotted line). The figure reveals several key features. First, there exists a multiplicity region (approximately $z_H \in [1.3, 1.7]$) where all three equilibria coexist, demonstrating the coordination problem over L firms' default behavior. Second, the mixed equilibrium interpolates smoothly between the two pure-strategy equilibria, with D_L varying continuously from 0 to 1. Third, the efficiency wedges ω_H and ω_L in the mixed equilibrium cross the efficient level ($\omega = 1$) at intermediate values of z_H, confirming that the mixed equilibrium can achieve efficiency when $D_L = \gamma$. Fourth, the distortion patterns reverse between the two pure-strategy equilibria: in the all-default equilibrium, H firms underinvest ($\omega_H > 1$) and L firms overinvest ($\omega_L < 1$), while in the only-H-default equilibrium, H firms overinvest ($\omega_H < 1$) and L firms underinvest ($\omega_L > 1$). Finally, the IOU values reflect the default regimes: $v_L = 1$ when L firms do not default or in the mixed equilibrium, while $v_H < 1$ throughout, reflecting that H firms always default in the failure state.

5 Propagation

Throughout this section we focus on an all-default equilibrium (all firms default when projects fail), allowing us to isolate the propagation mechanisms. The productivity shock illustrates pure upstream propagation through the collateral channel: a local productivity decline reduces the collateral value of the IOUs it issues—and therefore reduces the collateral available to upstream firms—lowering upstream IOU values and tightening upstream funding terms. The riskiness shock illustrates both channels: it propagates upstream through collateral and downstream through the trade-credit supply incentive, as a firm that anticipates failure-state default internalizes that pledged receivables will be seized by creditors, lowering the private return from supplying on credit. Appendix G reports the full riskiness-shock exercise; here we summarize

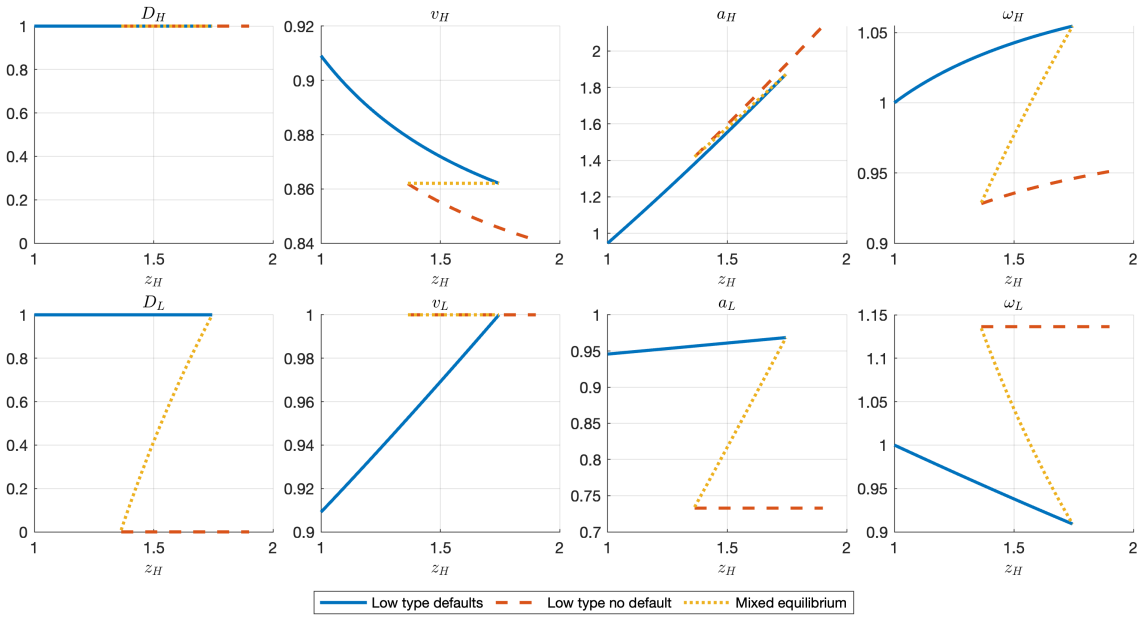


Figure 4: Equilibrium multiplicity and comparative statics with respect to z_H. The figure displays eight plots showing how equilibrium variables vary with z_H for three equilibrium types: all-default (solid line, "Low type defaults"), only-H-default (dashed line, "Low type no default"), and mixed equilibrium (dotted line). Top row: H-type firm variables (D_H, v_H, a_H, ω_H). Bottom row: L-type firm variables (D_L, v_L, a_L, ω_L). Parameters: $\gamma = 0.6$, $q = 0.8$, $\kappa = 1$, $\alpha = 0.4$, $z_L = 1$, and $z_H \in [1, 1.9]$.

the key downstream mechanism. When a single firm becomes riskier (a lower success probability q_i), its effective return from supplying inputs on credit falls because, in the failure state, the pledged fraction of the receivables it acquires is seized by creditors. The input-supply condition (7) therefore shifts inward: at a given price, the firm supplies less, and in equilibrium the input it provides becomes more expensive and scarcer for its downstream customer. These price–quantity changes tighten the downstream customer's financing and investment conditions, generating downstream propagation through the trade-credit supply incentive even though IOU values do not recurse mechanically from v_i to v_{i+1}.

Propagation triggered by a productivity change

Consider a cycle network with $N = 50$ firms where productivity alternates between $z = 0.5$ and $z = 1$ along the chain. A negative productivity shock hits a single firm, reducing its productivity. Figure 5 illustrates how this local shock propagates through the network, comparing equilibrium outcomes (solid blue) with efficient allocations (dotted orange).

The figure displays seven plots showing key variables along the chain. The top row shows exogenous parameters (z and q), while subsequent rows show endogenous variables: IOU values (v), face values (a), goods prices (p), investment levels (x), and expected output. The shock creates a sharp discontinuity around the affected firm, with effects propagating primarily *upstream* through financing and collateral. In this exercise there is no visible downstream propagation in goods prices or quantities (consistent with Figure 5).

Upstream propagation. The shock propagates upstream through the collateral channel. When firm i experiences a negative productivity shock (a lower z_i), it demands less of the input produced by firm $i-1$. Market clearing therefore lowers the traded value $p_{i-1}y_{i-1}$ on link $(i-1, i)$. This traded value is the relevant collateral object for the upstream supplier: it pins down the pledgeable value of firm $i-1$'s receivables from firm i, $\gamma p_{i-1}y_{i-1}$, and hence its IOU price v_{i-1} through the IOU pricing equation (4):

$$v_{i-1} = q_{i-1} + (1 - q_{i-1}) \left[(1 - D_{i-1}) + D_{i-1} \frac{\gamma p_{i-1} y_{i-1}}{a_{i-1}} \right].$$

Throughout this section we hold fixed the all-default regime ($D_j = 1$ for all j), so a decline in collateral coverage on link $(i-1, i)$ reduces v_{i-1} and tightens firm $i-1$'s financing conditions.

The relevant scaling object in the input-demand condition (8) is the expected repayment per unit of funds raised, $\tau_{i-1} \equiv [q_{i-1} + (1 - q_{i-1})(1 - D_{i-1})] / v_{i-1}$. Under all-default, $\tau_{i-1} = q_{i-1}/v_{i-1}$, so when v_{i-1} falls, τ_{i-1} rises and firm $i-1$ faces a higher effective marginal cost of inputs. Equivalently, under Cobb–Douglas, input expenditure is proportional to the reciprocal object $v_{i-1} / [q_{i-1} + (1 - q_{i-1})(1 - D_{i-1})]$ (as in Figure 1), so a fall in v_{i-1} reduces firm $i-1$'s demand for its own upstream input. This depresses $p_{i-2}y_{i-2}$, lowers the pledgeable value of receivables further upstream, and causes the shock to cascade.

(No) downstream propagation in this experiment. The productivity term z_i enters firms' opti-

mality conditions through *input demand* (8) (and hence through the proportionality factor highlighted in Figure 5), not through the *input-supply* condition (7). Since throughout this section we hold fixed the all-default regime ($D_i = 1$), a local decline in z_i primarily operates by lowering firm i's demand for its upstream input and (mechanically) its own *final-good* output $z_i f_i(\cdot)$; the channel that moves upstream collateral is reduced input demand and the traded value $p_{i-1}y_{i-1}$, not firm i's intermediate-good supply y_i. This depresses p_{i-1} and upstream collateral values as described above. By contrast, the downstream buyer's input price p_i is essentially unchanged in Figure 5, so there is no meaningful downstream adjustment in firm $i+1$'s input demand or IOU value in this calibration.

Equilibrium vs. efficiency. The figure reveals that equilibrium investment and output (solid blue lines) deviate significantly from efficient levels (dotted orange lines), particularly in the region affected by the shock. In the efficient allocation, the real adjustment is concentrated at the directly affected firm i: its production (and hence investment) responds *more* than in equilibrium, while activity elsewhere remains close to its pre-shock level. By contrast, the equilibrium response is attenuated at the shock location but spreads along the network, with sharp discontinuities driven by coordination failures and endogenous valuation effects (Section 3). Accordingly, the gap between equilibrium and efficiency is largest around the shock location, where financial linkages transform a local productivity change into non-local distortions in prices, IOU values, and real allocations.

6 Contagion

Shocks may also propagate through *default contagion*. In contrast to the cycle network where both channels generate strategic complementarity, the line network analyzed here exhibits contagion primarily through the upstream channel: a deterioration in downstream collateral values triggers an upstream cascade of defaults via the coupled recursion for IOU values $\{v_i\}$ and default decisions $\{D_i\}$ (firms' solvency thresholds). The key difference relative to the stationary cycle examples is that here a small local shock can trigger a *regime switch* in default decisions $\{D_i\}$, producing discontinuous changes in IOU values and real activity.

We consider a non-stationary environment where firms are lined on a chain. Index firms by $i = 1, \ldots, n$ and interpret $i+1$ as the (unique) downstream buyer of firm i's output. Firm i finances its input purchases by issuing IOUs with face value a_i, sells them at price v_i, and uses the proceeds to buy intermediate inputs from firm $i-1$ at goods price p_{i-1}. Let y_i denote the quantity of good i traded between i and its downstream buyer $i+1$ (market clearing implies the buyer's input demand equals the supplier's output). With a single supplier, the buyer i's input purchases are pinned down by the standard budget identity:

$$p_{i-1}y_{i-1} = v_i a_i, \qquad i \geq 2. \tag{11}$$

Thus the real-side object $p_{i-1}y_{i-1}$ is exactly the funds raised $v_i a_i$ and pins down firm i's funding on link

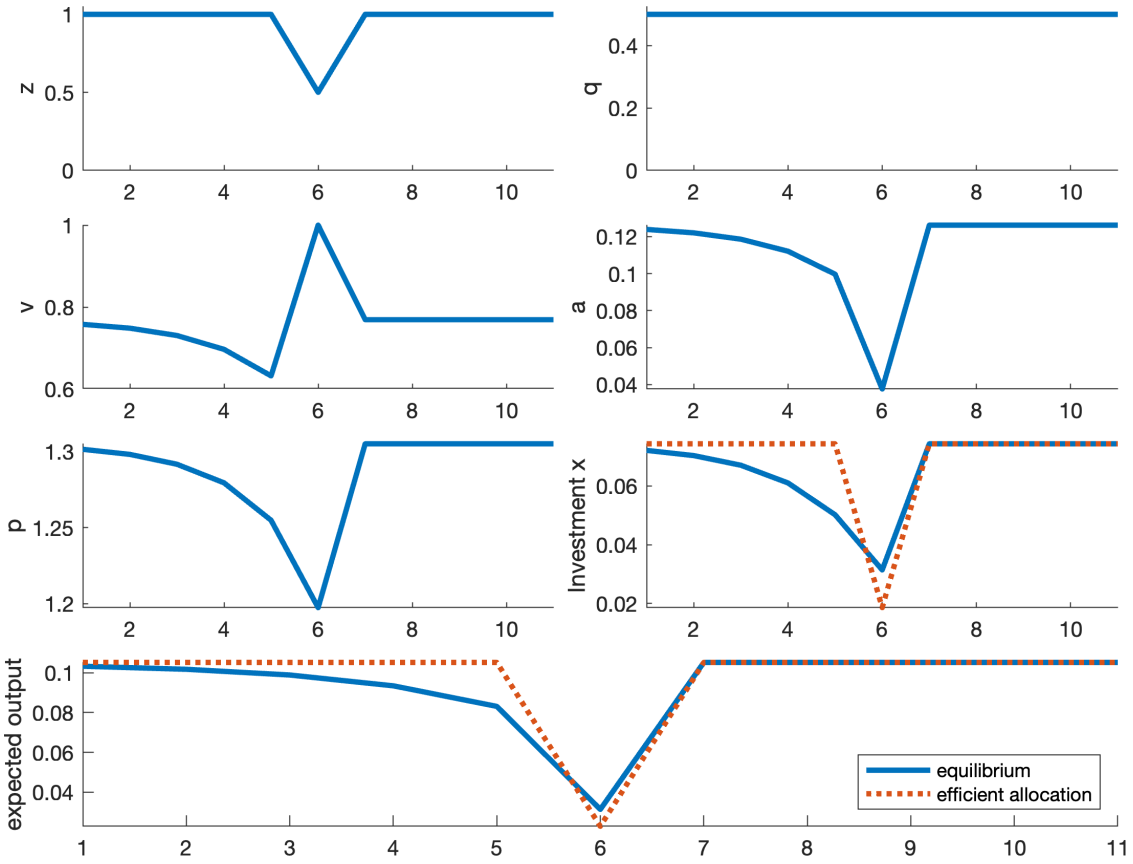


Figure 5: Propagation triggered by a productivity change. The figure displays seven plots showing how equilibrium variables (solid blue lines) and efficient allocations (dotted orange lines) respond to a negative productivity shock affecting a single firm in a cycle network. Top row: exogenous parameters z and q. Subsequent rows: IOU values v, face values a, goods prices p_{i-1} (the input price faced by firm i), investment levels x, and expected output. The shock creates discontinuities that propagate upstream through the collateral channel. Parameters: $\alpha = 0.6$, $\gamma = 0.7$, $\kappa = 0.1$, $q = 0.5$ (all firms), z alternates between 0.5 and 1, $N = 50$ firms.

$(i-1, i)$. Firm i's pledgeable receivables are a *separate* object, $\gamma p_i y_i$, on the downstream link $(i, i+1)$ (with $p_i y_i = v_{i+1} a_{i+1}$ by the downstream budget identity); $p_{i-1} y_{i-1}$ does not by itself determine $\gamma p_i y_i$, which is determined in equilibrium. Using the downstream budget identity, $p_i y_i = v_{i+1} a_{i+1}$, this collateral object can also be written as $\gamma v_{i+1} a_{i+1}$.

Equilibrium recursion on a line

This environment is a specialization of the general equilibrium conditions from Section 7. In particular, each firm uses a *single* input, so the intermediate-good market on link $(i, i+1)$ is summarized by the price–quantity pair (p_i, y_i).

Input supply. Specializing the supply first-order condition (7) to the line gives

$$k'(y_i) = p_i \left[(1-\gamma) + \gamma \left(q + (1-q)(1-D_i) \right) \right], \qquad i = 1, \ldots, n-1. \tag{12}$$

When default becomes more likely (higher D_i), supplying a given y_i requires a higher price p_i.

Input demand. Specializing the demand first-order condition (8) to a single input buyer (firm $i+1$) yields

$$q z_{i+1} f'(y_i) = \left[q + (1-q)(1-D_{i+1}) \right] \frac{p_i}{v_{i+1}}, \qquad i = 1, \ldots, n-1. \tag{13}$$

The term p_i / v_{i+1} is the effective (financing-adjusted) marginal cost of purchasing input i for firm $i+1$.

Default and IOU values. Default and IOU pricing are governed by the same collateral object as elsewhere in the paper. In particular, firm i's pledgeable receivables are $\gamma p_i y_i$, and the default rule (3) implies that firm i defaults in the failure state if and only if its face value exceeds collateral:

$$D_i = \mathbf{1}\{a_i > \gamma p_i y_i\}.$$

Equivalently, using the budget identity (11) for its downstream buyer, $\gamma p_i y_i = \gamma v_{i+1} a_{i+1}$, which matches the code normalization used in the numerical example. Finally, IOU values satisfy (4):

$$v_i = q + (1-q) \left[(1-D_i) + D_i \frac{\gamma p_i y_i}{a_i} \right]. \tag{14}$$

Taken together, (12)–(13), (11), and (14) form a coupled system in (p, y, v, a, D): a local shock changes downstream demand, which moves (p_i, y_i), thereby changing collateral values $\gamma p_i y_i$ and potentially triggering discrete switches in default decisions $\{D_i\}$; those switches feed back into both supply (via (12)) and the effective cost of finance (via v_{i+1} in (13)).

We keep the baseline assumption (as in earlier sections) that the success-state pledgeability constraint is slack, so default incentives are governed by the failure-state collateral comparison in (3) and the valuation equation (4).

Reference (no-default) profile. To construct a transparent benchmark, we choose a productivity sequence $\{z_i^*\}$ that supports a no-default equilibrium with $D_i = 0$ and hence $v_i = 1$ for all $i \geq 2$. In this benchmark, the line conditions simplify. From (12), we have $k'(y_i) = p_i$. From (13) with $v_{i+1} = 1$ and $D_{i+1} = 0$, we have $qz_{i+1}f'(y_i) = p_i$. Combining yields

$$qz_{i+1}f'(y_i) = k'(y_i),$$

so the real allocation coincides with the efficient one on each link.

Solvency requires $a_i \leq \gamma p_i y_i$ for all $i \geq 2$. Under $v_{i+1} = 1$, the budget identity (11) implies $p_i y_i = a_{i+1}$, so the no-default feasibility condition becomes $a_i \leq \gamma a_{i+1}$. Using again (11) and $p_i = k'(y_i)$, this can be written in real-side terms as

$$\frac{1}{\gamma} y_{i-1} k'(y_{i-1}) \leq y_i k'(y_i), \qquad i = 2, \ldots, n-1. \tag{15}$$

In the numerical example below we impose (15) with equality to pin down a reference profile, normalize $z_2^* = 1$, and then introduce a small negative shock at an interior firm: with $n = 15$ and $j = n - 5 = 10$, we set $z_{10} = 0.99\, z_{10}^*$.

Why a small shock causes large real effects

Figure 6 shows that the small change in z_{10} produces a large, discontinuous change in default and real allocations. The mechanism is a feedback loop between (i) goods-market outcomes (p, y), which determine collateral values $\gamma p_i y_i$, (ii) the default decision D_i, and (iii) financing conditions through IOU values v_i.

Drop in z_{10}.

- *Real-side contraction on link* $(9, 10)$. A lower z_{10} reduces firm 10's marginal product of the input purchased from firm 9. The demand condition (13) therefore shifts inward, lowering equilibrium activity on the $9 \to 10$ link: both the traded quantity y_9 and the associated revenue $p_9 y_9$ fall.

- *Collateral/default channel at firm 9.* The decline in $p_9 y_9$ reduces the pledgeable value of firm 9's receivables, $\gamma p_9 y_9$. If the shock is large enough to violate the solvency condition $a_9 \leq \gamma p_9 y_9$, firm 9 switches from solvent to default in the failure state ($D_9 : 0 \to 1$). This regime switch produces a discrete fall in the IOU value v_9 via (14).

- *Amplification back toward the shocked firm.* Once D_9 rises, the supply condition (12) requires a higher price p_9 to induce any given y_9 (the effective expected return from supplying falls). This shifts the supply schedule on the $9 \to 10$ link, reinforcing the initial contraction in y_9.

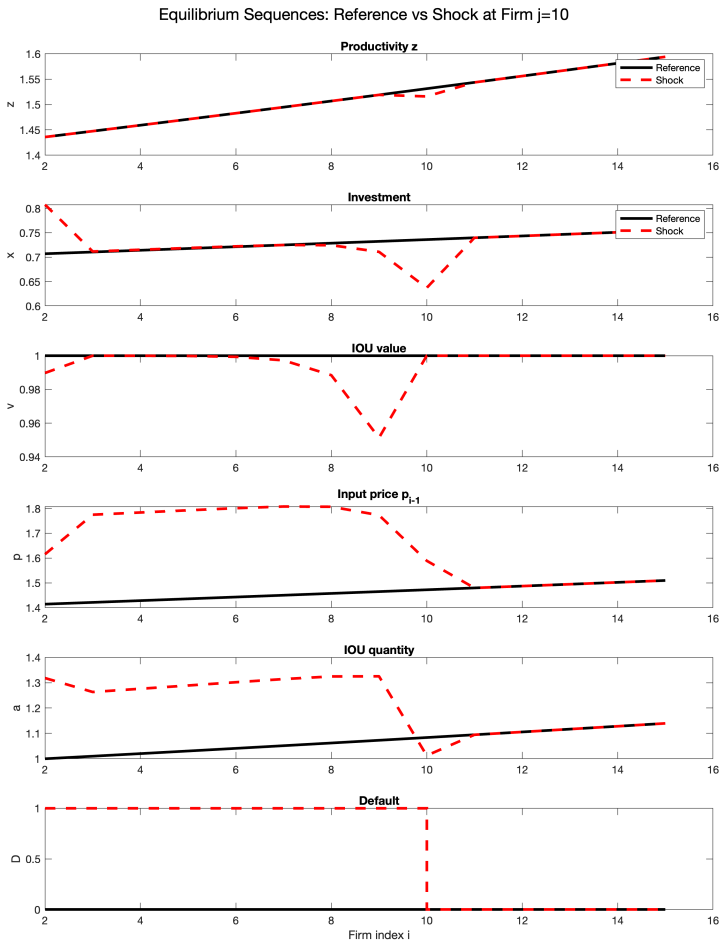


Figure 6: Default contagion from a small productivity dip at firm $j = 10$ on a line network. Black solid lines show the reference equilibrium (constructed to have no default, $D_i = 0$), while red dashed lines show the post-shock equilibrium after setting $z_{10} = 0.99\, z_{10}^*$. Panels report, as functions of the firm index i, productivity z_i, investment x_i (equal to the upstream output y_{i-1}), IOU value v_i, the input price faced by firm i (i.e. p_{i-1}, the price on link $(i-1, i)$), IOU face value a_i, and default D_i. Parameters: $\gamma = 0.99$, $q = 0.8$, $\alpha = 0.4$, $\kappa = 2$, $n = 15$.

Cascade to firms $i < 10$.

- *Tighter financing for upstream purchases.* The drop in v_9 raises the effective marginal cost p_8/v_9 faced by firm 9 when buying input 8 (see (13) with $i = 8$). This reduces firm 9's demand for good 8, lowering (p_8, y_8) and hence lowering collateral values $\gamma p_8 y_8$ for firm 8.
- *Upstream default cascade.* As (p_i, y_i) fall for $i = 8, 7, \ldots$, collateral coverage deteriorates and additional firms can cross the default threshold $a_i \leq \gamma p_i y_i$, turning on $D_i = 1$. Each new default further depresses IOU values v_i, amplifying the increase in effective input costs p_{i-1}/v_i faced by upstream buyers.
- *Real activity and prices propagate upstream.* The joint movement in v (financing) and in supply incentives (12) implies that the shock is transmitted upstream through both prices and quantities, generating large real effects relative to the initial 1% productivity dip.

Overall, the key object for contagion is the traded value $p_i y_i$. A small local demand shock lowers $p_9 y_9$, which reduces collateral $\gamma p_9 y_9$ and can trigger a discrete default switch at firm 9; the resulting fall in v_9 raises effective upstream input costs and pushes the contraction further upstream, generating a disproportionately large change in real activity relative to the original shock.

Construction of the example. Appendix H describes how we construct the reference no-default profile $\{z_i^*\}$ and compute the post-shock equilibrium shown in Figure 6.

7 General Supply Network

We now return to a general production network with N sectors. Sector i produces an intermediate good i and uses a vector of intermediate inputs $\{x_{ij}\}_{j=1}^N$ in final-good production. We maintain the environment and primitives from Sections 2–3: final-good production is Cobb–Douglas with input shares $\alpha_{ij} \geq 0$ and row sums $\sum_j \alpha_{ij} = \alpha$, and each sector has productivity z_i, success probability q_i, and input-supply cost $c_i k(y_i)$.

Equilibrium conditions. In competitive equilibrium, sector-i firms choose input supply y_i, input demands x_{ij}, IOU issuance a_i, and default intensity $D_i \in [0, 1]$. The key equilibrium objects are intermediate-good prices p_i and IOU values v_i.

Optimal input supply and input demand are characterized by the same first-order conditions as in Section 3, namely (7) and (8), now indexed by (i, j) in the general network. Default in the failure state is summarized by (3), and IOU values satisfy the expected-repayment pricing condition (4).

For later use it is convenient to record two Cobb–Douglas identities (see Appendix A). First, optimal input demands satisfy the spending-share identity

$$p_j x_{ij} = \alpha_{ij} \frac{a_i}{\alpha} v_i. \tag{16}$$

Second, substituting (16) into market clearing yields the value form

$$p_j y_j = \sum_{i=1}^{N} p_j x_{ij} = \sum_{i=1}^{N} \alpha_{ij} \frac{a_i}{\alpha} v_i. \tag{17}$$

Define the collateral coverage ratio

$$r_i \equiv \frac{\gamma p_i y_i}{a_i}.$$

In the numerical exercise below we impose an *interior* (failure-state) default intensity via a smooth reduced-form rule $D_i = \delta(r_i) \in (0,1)$, decreasing in collateral coverage, as discussed in Section 2 around (3). As throughout, D_i should be read as the probability of default conditional on the failure state, and IOU values are pinned down by (4). We use the parametric specification

$$\delta(r) = \frac{e^{-\beta \min(1,r)} - e^{-\beta}}{1 - e^{-\beta}}, \qquad \beta > 0, \tag{18}$$

so that $\delta(0) = 1$ and $\delta(r) \downarrow 0$ as $r \uparrow 1$.

Inefficiency wedge. We use the same input-use wedge ω_{ij} defined in Section 3.3. In the general network, combining (7) and (8) yields

$$\omega_{ij} = \frac{q_i + (1 - q_i)(1 - D_i)}{[1 - (1 - q_j) D_j \gamma]\, v_i}. \tag{19}$$

Equation (19) is equivalent to (10) since $(1-\gamma) + \gamma\,[q_j + (1-q_j)(1-D_j)] = 1 - (1-q_j) D_j \gamma$. Heterogeneity in default intensities $\{D_i\}$ and IOU values $\{v_i\}$ therefore generates a pattern of over- and under-use of inputs across the network.

Numerical illustration: a triangular production network

We illustrate these forces using a numerical example. The production network is *triangular* (lower-triangular) in the sense that the input-share matrix α_{ij} is lower-triangular. This imposes a directed acyclic production network (no input loops), with sectors naturally ordered from upstream to downstream. In this structure, more upstream sectors sell intermediate goods to a larger number of downstream sectors than the number of sectors from which they purchase intermediates; conversely, downstream sectors buy from a larger set of suppliers but sell to fewer buyers. Hence upstream sectors tend to have more receivables relative to payables (higher collateral coverage), and are less likely to default (low D_i). Default rates rise toward downstream sectors. For this illustration, we parameterize intermediate-good supply costs as $c_i k(y_i)$ with

$$k(y) = \frac{y^{1+\kappa}}{1+\kappa} \qquad \Rightarrow \qquad k'(y) = y^{\kappa},$$

so κ governs the curvature of the marginal cost schedule.

Figure 7 highlights three patterns. First, equilibrium intermediate-goods output declines sharply toward downstream sectors and lies below the efficient benchmark, with the gap widening downstream. This pattern reflects weaker collateral coverage and greater default risk in downstream sectors, which tighten financing conditions and depress production. Second, final-goods output is comparatively flat in equilibrium but falls short of the efficient benchmark, especially in downstream sectors. Because the calibration holds $q_i z_i$ constant across sectors, this pattern does not reflect exogenous differences in final-good productivity; instead, it is generated endogenously by the interaction of financing frictions, default, and network position. Third, the lower panels show how these output distortions map into default and input wedges. Default probabilities are negligible in upstream sectors and rise only for sufficiently downstream firms. The wedge matrix ω_{ij} in the lower-right panel reflects relative default intensities across the buyer i and supplier j. Using (19), we can decompose the wedge as

$$\omega_{ij} = \frac{\tau_i}{1-(1-q_j)D_j\gamma}, \qquad \tau_i \equiv \frac{q_i + (1-q_i)(1-D_i)}{v_i}.$$

The wedge formula and the interpretation in terms of over- and under-use apply only on *active* input links (those with $\alpha_{ij} > 0$). In the triangular network, $\alpha_{ij} = 0$ for $j > i$, so the corresponding cells in the plotted matrix should be read as inactive/irrelevant rather than as distortions. It is useful to note that default enters the supplier-side term only through

$$\widetilde{D}_j \equiv (1-q_j)\gamma D_j \in [0, (1-q_j)\gamma],$$

so that $1-(1-q_j)\gamma D_j = 1-\widetilde{D}_j$. Throughout the policy discussion below, "controlling D_j" is equivalent to controlling $\widetilde{D}_j$ within this feasible range. The supplier-side term depends only on D_j: holding i fixed, a lower default intensity in sector j (equivalently higher collateral coverage r_j) raises $1-(1-q_j)D_j\gamma$ and therefore lowers ω_{ij}. In the triangular network, upstream sectors have higher collateral coverage and lower default, so wedges are typically lower for upstream suppliers (a clear gradient across columns of the matrix). The buyer-side term τ_i captures expected repayment per unit of funds raised. Under interior default, $\tau_i < 1$ because (4) implies

$$v_i = \big[q_i + (1-q_i)(1-D_i)\big] + (1-q_i)D_i r_i > q_i + (1-q_i)(1-D_i),$$

so $\tau_i = \big[q_i + (1-q_i)(1-D_i)\big]/v_i \in (0,1)$. Hence when suppliers are very safe ($D_j \simeq 0$), we have $\omega_{ij} \simeq \tau_i < 1$, implying over-use of those inputs. More generally,

$$\omega_{ij} > 1 \quad \Longleftrightarrow \quad 1-(1-q_j)D_j\gamma < \tau_i, \tag{20}$$

so $\omega_{ij} > 1$ is not automatic when j is risky: it requires sufficiently high supplier default relative to the

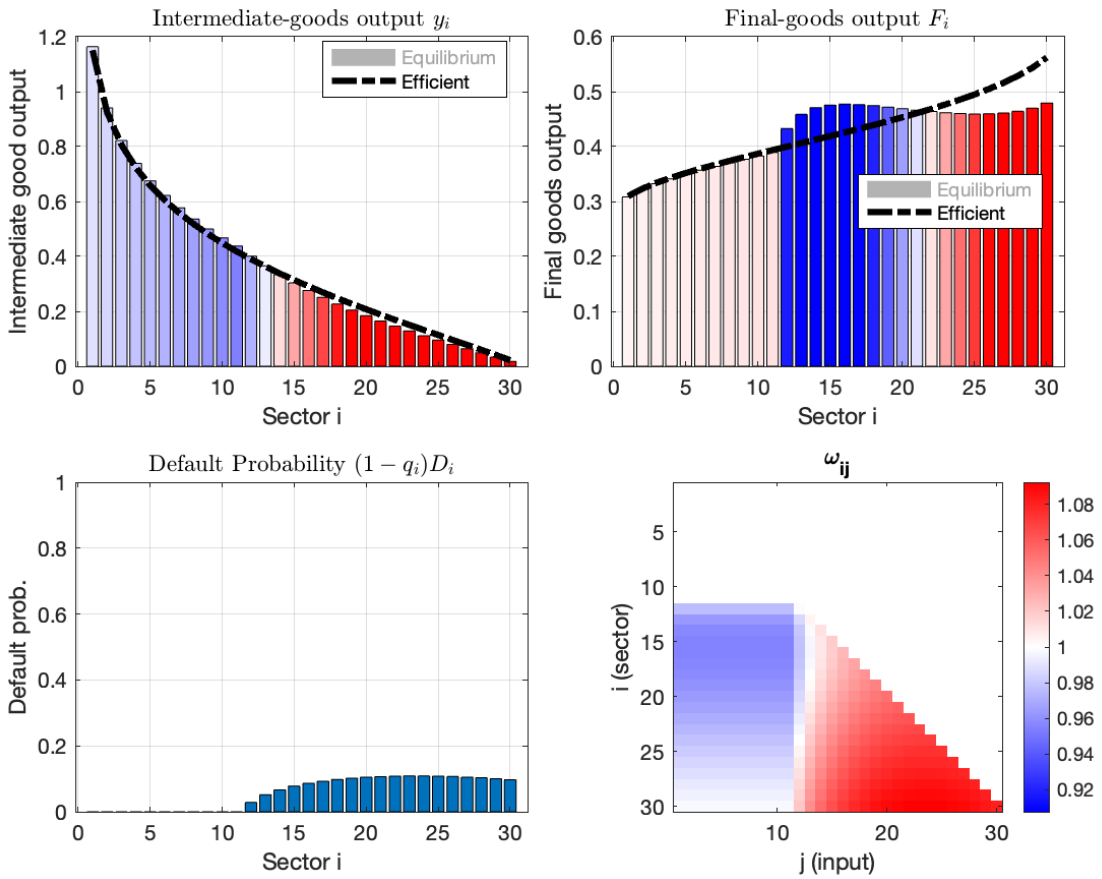


Figure 7: Multisector equilibrium in a triangular production network. The four panels report: intermediate-goods output y_i (top-left), final-goods output $F_i \equiv q_i z_i f_i$ (top-right), default probability $(1-q_i)D_i$ (bottom-left), and the wedge matrix ω_{ij} (bottom-right). In the top panels, the bars denote the equilibrium allocation and the dashed line denotes the efficient benchmark obtained by imposing $D_i = 0$ for all i. The network is lower-triangular: $\alpha_{ij} = 0$ for $j > i$, so the network is directed and acyclic (no loops), with rows normalized to sum to $\alpha = 0.8$. Parameters: $N = 30$, $\kappa = 0.1$ (so $k'(y) = y^{0.1}$), $\gamma = 0.8$, $\beta = 0.5$, $c_i = 1$ for all i, and heterogeneous pledgeability with q_i increasing linearly from 0.7 to 0.9 while $z_i = 0.8/q_i$, so $q_i z_i = 0.8$ for all i. The interior default rule is $D_i = \delta(r_i)$ with $r_i = \gamma p_i y_i / a_i$ and $\delta(\cdot)$ given by (18).

buyer-side term. In the calibration in Figure 7, condition (20) is equivalent to

$$D_j > \frac{1-\tau_i}{(1-q_j)\gamma}.$$

Because v_i appears in the denominator of τ_i, a decline in v_i raises τ_i ceteris paribus (holding D_i fixed). Under the interior default rule, $\tau_i(r_i) \in (0,1)$ for $r_i \in (0,1)$ and satisfies $\tau_i(r_i) \to 1$ as $r_i \downarrow 0$ or $r_i \uparrow 1$; the limit as $r_i \to 0$ is what underpins τ_i being close to one for low collateral coverage (e.g., downstream buyers in a triangular network).[3] Thus wedges exceed one precisely for downstream buyers i paired with sufficiently default-prone downstream suppliers j, producing the downstream–downstream region of under-use. Across buyers i, however, τ_i need not vary monotonically with D_i because default both dilutes repayment (lowering τ_i holding v_i fixed) and affects the IOU price v_i (raising τ_i as v_i falls). This interaction helps explain why the pattern down a given column (varying buyers i holding j fixed) can be less sharp.

8 Policy implications: regulating default incentives

This section characterizes interventions that operate directly on default incentives. The idea is that policymakers can affect the *reputational* (or legal) costs of default and therefore can influence firms' default intensities. We model such interventions as allowing the policymaker to choose sector-specific default intensities $\{D_i\}_{i=1}^N$, where $D_i \in [0,1]$ is the probability of default *conditional on the idiosyncratic production failure state* in sector i (i.e. conditional on the event that the firm produces zero final output, as in the failure state described in Section 2). Intuitively, the policy instrument is a sector-specific reputational penalty for default—which may be negative (a subsidy), thus encouraging default—so that policy can shift the default probability in the failure state without directly changing the technological primitives.

We hold fixed the competitive environment, technologies, and pledgeability γ as in the baseline model. The only change is that the equilibrium default rule (e.g. $D_i = \delta(r_i)$ in Section 7, or the step rule in (3)) is replaced by an exogenous policy choice of $\{D_i\}$.[4]

Because wedges arise from default risk and IOU valuations, a natural policy objective is to pick $\{D_i\}$ so that the resulting competitive equilibrium eliminates the financial-linkage wedges relative to the First-Best benchmark. Using the wedge ω_{ij} in (10), this amounts to imposing $\omega_{ij} = 1$ on all active input links. The propositions below characterize, in tractable network structures, the set of exogenous default probabilities $\{D_i\}$ that can implement efficiency once the rest of the equilibrium equalities hold.

Two-sector alternating (two-cycle) economy.

[3] Using $v_i = (1-(1-q_i)D_i) + (1-q_i)D_i r_i$ gives $\tau_i = \frac{1-(1-q_i)D_i}{1-(1-q_i)D_i(1-r_i)}$.

[4] Formally, given $\{D_i\}$, equilibrium is obtained by solving the same system as in laissez-faire for $(p_i, y_i, \{x_{ij}\}, a_i, v_i)$, but omitting the equations that endogenize D_i.

Consider the alternating-productivity cycle in Section 4 specialized to $N = 2$ sectors, H and L, with common success probability $q \in (0,1)$, common pledgeability $\gamma \in (0,1)$, and $z_H > z_L > 0$. Let (y_H^*, y_L^*) denote the intermediate-good quantities in the efficient allocation. Define the productivity ratio evaluated at the efficient allocation:

$$r \equiv \frac{z_L f(y_H^*)}{z_H f(y_L^*)}.$$

Proposition 5. ***(Implementing the First-Best benchmark in a two-sector cycle).*** *In the $N = 2$ alternating economy, the set of exogenous default policies $(D_H, D_L) \in [0,1]^2$ that can implement the efficient allocation is one-dimensional: it can be indexed by a single scalar (e.g. D_L), with D_H pinned down by*

$$\frac{D_L}{1-(1-q)\gamma D_L} = r\,\frac{D_H}{1-(1-q)\gamma D_H}. \tag{21}$$

Equivalently, solving (21) *for D_H as a function of D_L yields*

$$D_H(D_L) = \frac{\frac{1}{r}D_L}{1+(1-q)\gamma D_L\left(\frac{1}{r}-1\right)}, \tag{22}$$

with the symmetric expression obtained by swapping H and L.

Equation (21) constrains the transformed failure-state default intensity $D/[1-(1-q)\gamma D]$ across the two sectors. Equivalently, since $\widetilde{D} \equiv (1-q)\gamma D$, we have

$$\frac{D}{1-(1-q)\gamma D} = \frac{1}{(1-q)\gamma}\cdot\frac{\widetilde{D}}{1-\widetilde{D}},$$

so (21) is equivalent to an odds-ratio restriction in $\widetilde{D}$ (the common factor $1/[(1-q)\gamma]$ cancels). The closed-form mapping (22) makes explicit how choosing one sector's default probability determines the other's along the efficient-implementing set.

In a two-cycle, efficiency amounts to neutralizing a *relative* wedge between the two links. Once the efficient real allocation is fixed, the policy problem is essentially to align the effective repayment factors on the two sides of the cycle, leaving one degree of freedom.

There is a continuum of efficient default policies. Hence the relevant design question is not whether one can restore efficiency, but which efficient policy is cheapest to implement given a cost of enforcement (e.g. increasing in the reputational penalties required to reduce default).

An N-cycle network.

Consider an N-cycle production network where each sector uses a single supplier (a directed cycle). Let $(\{x_{ij}^*\}, \{y_i^*\})$ be the efficient allocation. Define

$$K_i \equiv \frac{c_i k'(y_i^*) y_i^*}{\alpha q_i z_i f_i(\{x_{ij}^*\})},$$

The constants $\{K_i\}$ are pinned down by primitives and the efficient allocation, and summarize how the sector-i supply margin scales relative to its output at the target allocation.

Proposition 6. ***(Implementing the First-Best benchmark in an N-cycle network).*** *Impose efficiency on each active link of the cycle, $\omega_{i,i-1} = 1$ for all i (with $0 \equiv N$). Then:*

1. *For any admissible D_1 (i.e. $D_1 \in [0,1]$ such that the implied $\{D_i\}_{i=2}^N$ lie in $[0,1]$), the remaining $\{D_i\}_{i=2}^N$ are pinned down by the closed-form expression*

$$D_i(D_1) = \frac{1}{(1-q_i)\gamma} \cdot \frac{\left(\prod_{m=2}^{i} K_m\right)^{-1} \frac{(1-q_1)\gamma D_1}{1-(1-q_1)\gamma D_1}}{1 + \left(\prod_{m=2}^{i} K_m\right)^{-1} \frac{(1-q_1)\gamma D_1}{1-(1-q_1)\gamma D_1}}, \qquad i = 2, \ldots, N \tag{23}$$

   (so the efficient-default set is at most *one-dimensional).*

2. *In an N-cycle with a single supplier per buyer, the efficient allocation implies*

$$\prod_{i=1}^{N} K_i = 1, \tag{24}$$

   so the recursion closes automatically around the cycle: any admissible D_1 generates a consistent default profile implementing efficiency. Here, "cycle closure" means that if one starts from a candidate value of the transformed variable $\widetilde{D}_1/(1-\widetilde{D}_1)$ and applies the link-by-link update implied by efficiency around the directed cycle, the value obtained upon returning to sector 1 coincides with the starting value. When (24) *holds, one full traversal leaves $\widetilde{D}_1/(1-\widetilde{D}_1)$ unchanged, so iterating through the cycle any number of times yields the same implied default profile $\{D_i\}$.*

The closed form (23) makes explicit that a single degree of freedom (the choice of D_1) pins down the entire default profile, subject to feasibility. The appendix derives (23) by introducing $\widetilde{D}_i \equiv (1-q_i)\gamma D_i$ (i.e. γ times the ex ante default probability). In our wedge expressions, $\widetilde{D}$ is the convenient policy-relevant object: on the supplier side it enters through $1-\widetilde{D}_j$, and on the buyer side it also enters through the additional factor $1+\widetilde{D}_i(K_i-1)$ (see Appendix I). Thus controlling D_i amounts to controlling $\widetilde{D}_i \in [0,(1-q_i)\gamma]$. The proof iterates a multiplicative recursion in the transformed variable $\widetilde{D}_i/(1-\widetilde{D}_i)$ implied by link-by-link efficiency.

In a cycle, the efficiency conditions link default incentives across sectors through a multiplicative recursion. In an N-cycle with one supplier per buyer, the product restriction (24) is implied by the efficient allocation and the cycle structure (it is not an additional restriction), so the recursion closes without imposing a knife-edge constraint.

Even when efficiency is attainable with positive default, the efficient set has low dimension. This sharp characterization is useful for policy because it reduces the design problem to choosing a point on a one-parameter frontier subject to feasibility ($D_i \in [0,1]$).

General production networks with multi-sourcing.

Return to the general network in Section 7. Suppose at least one buyer uses multiple suppliers: there exists i with $|S_i| \geq 2$, where $S_i \equiv \{j : \alpha_{ij} > 0\}$. Impose the efficiency requirement $\omega_{ij} = 1$ on all *active* links (i, j) with $\alpha_{ij} > 0$. Using the decomposition in (19):

$$\omega_{ij} = \frac{\tau_i}{1 - (1 - q_j)D_j\gamma}, \qquad \tau_i \equiv \frac{q_i + (1 - q_i)(1 - D_i)}{v_i},$$

efficiency for a fixed buyer i requires that the supplier-side terms $1 - (1 - q_j)D_j\gamma$ be equalized across all $j \in S_i$. If the active-link structure is sufficiently connected so that these equalizations propagate across the economy (so that $1 - (1 - q_j)D_j\gamma$ is common across j), then the efficiency conditions become globally restrictive.

Proposition 7. ***(Generic implementation of the First-Best benchmark in connected networks with multi-sourcing).*** *Assume: (i) there exists at least one buyer with $|S_i| \geq 2$; (ii) the active-link structure is sufficiently connected that the link-by-link efficiency conditions imply a common supplier-side term $1 - (1 - q_j)D_j\gamma = M$ for all j; and (iii) the sectoral constants $\{K_i\}$ implied by the efficient allocation are not all equal to one. Then the only policy vector $\{D_i\}$ that can implement efficiency is*

$$D_i = 0 \quad \textit{for all } i.$$

(In the knife-edge case where $K_i = 1$ for all i, there is a one-parameter family indexed by M, subject to feasibility $D_i = (1 - M)/[(1 - q_i)\gamma] \in [0, 1]$.)

Multi-sourcing turns the condition $\omega_{ij} = 1$ on each active link into equalization constraints across suppliers. Once these constraints percolate through a connected network, the policy effectively must pick a *single* repayment factor for all sectors. With generic heterogeneity in the real margins summarized by $\{K_i\}$, the only way to reconcile all efficiency conditions is to set that repayment factor to one, which corresponds to eliminating default.

In dense production networks where firms commonly multi-source, restoring efficiency may require interventions that raise the reputational (or legal) costs of default enough to eliminate it, rather than fine-tuning heterogeneous default incentives across sectors. This underscores that the scope for "targeted" default management is structurally limited in realistic networks.

Cost-effective implementation. Suppose the policymaker can achieve lower default intensities only by raising the enforcement or reputational penalty associated with default, so the policy cost is weakly *decreasing* in each D_i.[5] Then, among the (potentially low-dimensional) set of default policies that satisfy the efficiency

[5]Equivalently, write the instrument as a penalty Π_i with $D_i = D_i(\Pi_i)$ decreasing; then minimizing total policy cost in Π is equivalent to selecting, among feasible efficient policies, the one that allows the largest default probabilities.

conditions in Propositions 5–7, the most cost-effective efficient intervention is the one that maximizes default probabilities within that set (subject to feasibility). Any further reduction in default is costly (requires higher penalties) but yields no additional efficiency gain once wedges are already neutralized.

9 Conclusion

We study a trade credit chain under limited commitment in which trade credit circulates as settlement instruments and as collateral. Even when idiosyncratic risk is diversified and IOUs are "safe" in expectation, endogenous default in the failure state lowers a firm's valuation over the receivables it holds. This valuation effect tightens financing conditions for its trading partners and can propagate shocks along the supply chain, generating inefficient input use and fragility through equilibrium multiplicity and discontinuous switches in default regimes (and the associated repricing of trade credit).

Section 8 studies policies that operate directly on default incentives by changing the reputational (or legal) costs of default, and therefore by shifting sectoral default intensities $\{D_i\}$. Modeling such interventions as choosing $\{D_i\}$ exogenously and letting the rest of the competitive equilibrium adjust as in laissez-faire, we show that the set of efficient default policies is typically low-dimensional in cycle networks, while in connected networks with multi-sourcing efficiency generically requires eliminating default ($D_i = 0$ for all i). When enforcement costs are increasing in the reduction of default, the most cost-effective efficient intervention is the one that maximizes default probabilities within the set of $\{D_i\}$ that satisfy the implementation conditions.

References

Daron Acemoglu, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. Systemic risk and stability in financial networks. Technical report, National Bureau of Economic Research, 2013.

Franklin Allen and Douglas Gale. Financial contagion. *Journal of Political Economy*, 108(1):1–33, 2000.

Levent Altinoglu. The origins of aggregate fluctuations in a credit network economy. *Journal of Monetary Economics*, 117:316–334, 2021.

Niklas Amberg, Tor Jacobson, Erik von Schedvin, and Robert Townsend. Curbing shocks to corporate liquidity: The role of trade credit. *Journal of Political Economy*, 129(1):182–242, 2021.

Niklas Amberg, Tor Jacobson, and Yingjie Qi. Supply-chain finance: An empirical evaluation of supplier outcomes. Technical Report 435, Sveriges Riksbank Working Paper Series, 2024.

David Rezza Baqaee and Emmanuel Farhi. The macroeconomic impact of microeconomic shocks: Beyond hulten's theorem. *Econometrica*, 87(4):1155–1203, 2019.

David Rezza Baqaee and Emmanuel Farhi. Productivity and misallocation in general equilibrium. *The Quarterly Journal of Economics*, 135(1):105–163, 2020. doi: 10.1093/qje/qjz030.

Saki Bigio. A theory of payments-chain crises. Technical report, National Bureau of Economic Research, 2023.

Banu Demir, Beata Javorcik, Tomasz K. Michalski, and Evren Ors. Financial constraints and propagation of shocks in production networks. *The Review of Economics and Statistics*, 106(2):437–454, 2024.

Matthew Elliott, Benjamin Golub, and Matthew O Jackson. Financial networks and contagion. *American Economic Review*, 104(10):3115–53, 2014.

Ana Fostel and John Geanakoplos. Tranching, cds, and asset prices: How financial innovation can cause bubbles and crashes. *American Economic Journal: Macroeconomics*, 4(1):190–225, 2012. doi: 10.1257/mac.4.1.190.

Ana Fostel and John Geanakoplos. Endogenous collateral constraints and the leverage cycle. *Annual Review of Economics*, 6(1):771–799, 2014. doi: 10.1146/annurev-economics-080213-041426.

John Geanakoplos. The leverage cycle. *NBER macroeconomics annual*, 24(1):1–66, 2010.

Tor Jacobson and Erik von Schedvin. Trade credit and the propagation of corporate failure: An empirical analysis. *Econometrica*, 83(4):1315–1371, 2015.

Nobuhiro Kiyotaki and John Moore. Credit chains. Technical report, 2004.

Guillermo Ordoñez. Sustainable shadow banking. *American Economic Journal: Macroeconomics*, 10(1):33–56, 2018. doi: 10.1257/mac.20150346.

A Cobb–Douglas identities (no symmetry)

This appendix records two algebraic identities that hold in the general network under Cobb–Douglas production. They do *not* rely on symmetry.

Lemma 1 (Spending shares and face values under Cobb–Douglas)**.** *Fix any sector i and any competitive equilibrium in the general network with Cobb–Douglas technology $f_i(\{x_{ij}\}) = \prod_j (x_{ij}/\alpha_{ij})^{\alpha_{ij}}$ and row sum $\sum_j \alpha_{ij} = \alpha$. Then the spending-share identity holds:*

$$p_j x_{ij} = \alpha_{ij} \frac{a_i}{\alpha} v_i,$$

and face values satisfy

$$\frac{a_i}{\alpha} = \frac{q_i z_i f_i(\{x_{ij}\})}{q_i + (1 - q_i)(1 - D_i)}. \tag{25}$$

Proof. The spending-share identity is (16). For (25), start from the demand condition (8), multiply by x_{ij}, and use the Cobb–Douglas property $x_{ij}\,\partial f_i/\partial x_{ij} = \alpha_{ij} f_i$ to obtain

$$q_i z_i \alpha_{ij} f_i(\{x_{ij}\}) = [q_i + (1-q_i)(1-D_i)]\,\frac{p_j x_{ij}}{v_i}.$$

Substitute the spending-share identity $p_j x_{ij} = \alpha_{ij}(a_i/\alpha)v_i$ and cancel α_{ij} to get (25). □

B Proof of Proposition 1: Case (3)

This appendix proves efficiency in case (iii) of Proposition 1, where $z_i = z$, $c_i = c$, $0 < q_i = q < 1$, and $\sum_j \alpha_{ij} = \sum_j \alpha_{ji} = \alpha$ for all i.

Equilibrium conditions

We first record a convenient equilibrium system. The Cobb–Douglas step that delivers the face-value identity for a_i does not use symmetry (Lemma 1); symmetry is used only in the next subsection.

An equilibrium is characterized by the system

$$c_i k'(y_i) = [1 - (1-q_i)D_i\gamma]\,p_i, \tag{26}$$

$$\frac{a_i}{\alpha} = \frac{q_i z_i f_i(\{x_{ij}\})}{q_i + (1-q_i)(1-D_i)}, \tag{27}$$

$$p_j x_{ij} = \alpha_{ij}\frac{a_i}{\alpha}v_i, \tag{28}$$

$$p_i y_i = \sum_{j=1}^{N} \alpha_{ji}\frac{a_j}{\alpha}v_j, \tag{29}$$

$$D_i = \delta\!\left(\frac{\gamma p_i y_i}{a_i}\right), \tag{30}$$

$$v_i = q_i + (1-q_i)\left[(1-D_i) + D_i\frac{\gamma p_i y_i}{a_i}\right], \tag{31}$$

for all $i = 1, \ldots, N$, where $f_i(\{x_{ij}\}) = \prod_j (x_{ij}/\alpha_{ij})^{\alpha_{ij}}$.

Symmetric equilibrium

Under the assumptions of case (iii), we guess and verify a symmetric equilibrium in which

$$p_i = p, \quad y_i = y, \quad a_i = a, \quad v_i = v, \quad D_i = D \qquad \text{for all } i.$$

Using $\sum_j \alpha_{ij} = \sum_j \alpha_{ji} = \alpha$, equations (28) and (29) imply

$$py = av. \tag{32}$$

Since $v \leq 1$ and $\gamma < 1$, (32) implies $\gamma \frac{py}{a} = \gamma v < 1$. Hence, by (30), $D_i = 1$ for all i: all firms default. That is, for each firm, the value of IOUs received from buyers equals the value of IOUs issued to finance input purchases.

Moreover, from (28),

$$\frac{x_{ij}}{\alpha_{ij}} = \frac{1}{p}\frac{a}{\alpha}v \quad \text{for all } i, j,$$

so that

$$f_i(\{x_{ij}\}) = \left(\frac{1}{p}\frac{a}{\alpha}v\right)^{\alpha}.$$

Substituting into (27) and using symmetry yields

$$\frac{a}{\alpha} = z\left(\frac{1}{p}\frac{a}{\alpha}v\right)^{\alpha}. \tag{33}$$

Default and IOU values

Using (32), the default condition (30) becomes

$$D = \delta(\gamma v).$$

Here $\delta(\cdot)$ is the step function defined in (3), distinct from the smooth approximation used in Section 7. Since $v \leq 1$ and $\gamma < 1$, we have $\gamma v < 1$, so the step function yields $D = 1$ in equilibrium.

Given $D = 1$, equation (31) implies

$$v = q + (1 - q)\gamma v, \tag{34}$$

which has the unique solution

$$v = \frac{q}{1 - (1 - q)\gamma} < 1.$$

Characterization and efficiency

The first-order condition for input supply (26) reduces to

$$ck'(y) = [1 - (1 - q)\gamma]\, p. \tag{35}$$

Equations (33), (32), (34), and (35) jointly determine (y, p, a, v).

Finally, efficiency follows from the fact that, in the symmetric equilibrium,

$$\omega_{ij} = \frac{[q + (1 - q)(1 - D)]\,\frac{1}{v}}{(1 - \gamma) + \gamma\,[q + (1 - q)(1 - D)]} = 1 \quad \text{for all } i, j,$$

so marginal products equal marginal costs for every input in every sector. □

C Proof of Proposition 2

Proof. The proof proceeds in four steps.

Step 1: Equilibrium system under all-default regime. When both types default upon failure ($D_H = D_L = 1$), the equilibrium is characterized by the following system. From the first-order conditions (7) and (8), we have:

$$k'(y_H) = p_H[(1-\gamma) + \gamma q], \tag{36}$$

$$k'(y_L) = p_L[(1-\gamma) + \gamma q], \tag{37}$$

$$z_H f'(y_L) = \frac{p_L}{v_H}, \tag{38}$$

$$z_L f'(y_H) = \frac{p_H}{v_L}. \tag{39}$$

Market clearing requires $y_H = x_L$ and $y_L = x_H$. The budget constraint and market clearing imply:

$$p_H y_H = v_L a_L, \tag{40}$$

$$p_L y_L = v_H a_H. \tag{41}$$

IOU prices satisfy:

$$v_H = q + (1-q)\frac{\gamma v_L a_L}{a_H}, \tag{42}$$

$$v_L = q + (1-q)\frac{\gamma v_H a_H}{a_L}. \tag{43}$$

Solving (42)–(43) yields:

$$v_H = \frac{q[1 + (1-q)\gamma\frac{a_L}{a_H}]}{1-(1-q)^2\gamma^2}, \tag{44}$$

$$v_L = \frac{q[1 + (1-q)\gamma\frac{a_H}{a_L}]}{1-(1-q)^2\gamma^2}. \tag{45}$$

Step 2: Establishing $a_H > a_L$ in equilibrium. We prove $a_H > a_L$ by contradiction. Suppose $a_H \leq a_L$. Then (44)–(45) imply $v_H \geq v_L$.

Consider two cases:

Case 1: $y_H > y_L$. Then $k'(y_H) > k'(y_L)$, so $p_H > p_L$ by (36)–(37). From (38)–(39), we have:

$$\frac{z_H f'(y_L)}{z_L f'(y_H)} = \frac{(p_L/v_H)}{(p_H/v_L)} = \frac{p_L v_L}{p_H v_H}.$$

Since $v_H \geq v_L$ and $p_H > p_L$, the right side is less than 1. But f' is decreasing and $z_H > z_L$, so the left side exceeds 1 if $y_H > y_L$, a contradiction.

Case 2: $y_H \leq y_L$. Then $p_H \leq p_L$. From (40)–(41):

$$p_H y_H = v_L a_L \quad \text{and} \quad p_L y_L = v_H a_H.$$

Since $p_H \leq p_L$ and $y_H \leq y_L$, we have $p_H y_H \leq p_L y_L$, which implies $v_L a_L \leq v_H a_H$. Substituting (44)–(45):

$$q[a_L + (1-q)\gamma a_H] \leq q[a_H + (1-q)\gamma a_L].$$

Simplifying:

$$a_L[1-(1-q)\gamma] \leq a_H[1-(1-q)\gamma].$$

Since $1-(1-q)\gamma > 0$, this implies $a_L \leq a_H$. Combined with our assumption $a_H \leq a_L$, we have $a_H = a_L$, which yields $v_H = v_L$ from (44)–(45). But then (38)–(39) imply $z_H f'(y_L) = z_L f'(y_H)$, contradicting $z_H > z_L$ and $y_H \leq y_L$ (since f' is decreasing).

Therefore, $a_H > a_L$ in equilibrium, which immediately implies $v_H < v_L$ from (44)–(45).

Step 3: Efficiency wedges. The efficiency wedges are:

$$\omega_H = \frac{q z_H f'(x_H)}{k'(y_L)} = \frac{q p_L / v_H}{p_L[(1-\gamma)+\gamma q]} = \frac{q}{v_H[(1-\gamma)+\gamma q]},$$
$$\omega_L = \frac{q z_L f'(x_L)}{k'(y_H)} = \frac{q p_H / v_L}{p_H[(1-\gamma)+\gamma q]} = \frac{q}{v_L[(1-\gamma)+\gamma q]}.$$

Since $v_H < v_L$, we have $\omega_H > \omega_L$. Moreover, when all firms are symmetric ($a_H = a_L$), we showed in Proposition 1 that $v = q/[1-(1-q)\gamma]$ and $\omega = 1$. Since $1-(1-q)\gamma = (1-\gamma)+\gamma q$, the symmetric benchmark gives:

$$\omega = \frac{q}{v[(1-\gamma)+\gamma q]} = 1 \quad \text{when} \quad v = \frac{q}{(1-\gamma)+\gamma q}.$$

Since $v_H < \frac{q}{(1-\gamma)+\gamma q} < v_L$ (by (44)–(45) with $a_H > a_L$), we conclude $\omega_H > 1 > \omega_L$.

Step 4: Monotonicity and a (possibly non-existent) upper boundary. We prove the monotonicity claim in item (iv) in a way that is consistent with market clearing. Let $m(\cdot) \equiv k'(\cdot)/f'(\cdot)$, which is strictly increasing because k' is increasing and f' is decreasing. Combining the demand conditions (38)–(39) with the supply conditions (36)–(37) gives

$$m(y_L) = z_H\,[(1-\gamma)+\gamma q]\,v_H, \tag{46}$$
$$m(y_H) = z_L\,[(1-\gamma)+\gamma q]\,v_L. \tag{47}$$

Thus, for given IOU values (v_H, v_L), y_H is pinned down by (47), while y_L is increasing in z_H through (46).

Next, using the budget equalities (40)–(41) and $p_T = k'(y_T)/[(1-\gamma)+\gamma q]$ from (36)–(37), we obtain

$$a_H = \frac{y_L k'(y_L)}{[(1-\gamma)+\gamma q]\,v_H}, \qquad a_L = \frac{y_H k'(y_H)}{[(1-\gamma)+\gamma q]\,v_L}.$$

Since (v_H, v_L) are determined by (44)–(45) as functions of the ratio a_L/a_H, the equilibrium can be summarized by a fixed point in that ratio. The key comparative-static is that, holding the ratio (hence (v_H, v_L) and y_H) fixed, an increase in z_H raises y_L via (46), increases a_H, and therefore lowers the implied ratio a_L/a_H. Hence the mapping that takes a trial ratio a_L/a_H to the implied ratio (from the equilibrium conditions) shifts downward pointwise in z_H. By continuity and monotonicity of this mapping, if a fixed point exists at some z_H, then at least one fixed point exists for all $z'_H \in (z_L, z_H)$; thus the set of z_H for which an all-default equilibrium exists is downward closed. This proves item (iv).

If there exists a value $\bar{z}_H > z_L$ at which the L-type failure-state solvency condition becomes binding, then it is characterized by:

$$a_L = \gamma v_H a_H \quad \Leftrightarrow \quad \frac{a_L}{a_H} = \frac{q}{\frac{1}{\gamma} - \gamma(1-q)}.$$

Whether such a $\bar{z}_H$ exists (and exceeds z_L) depends on additional parameter restrictions, as clarified in Proposition 4. For Proposition 2, we only need the monotonicity conclusion: the set of z_H for which an all-default equilibrium exists is downward closed, so existence at z_H implies existence for all $z'_H \in (z_L, z_H)$. □

D Proof of Proposition 3

Proof. The proof proceeds in five steps.

Step 1: Equilibrium system under only-H-default regime. When only high-productivity firms default upon failure ($D_H = 1$ and $D_L = 0$), the IOU price of L types is $v_L = 1$ (since they repay in full even upon failure), while $v_H \in (q, 1)$ satisfies:

$$v_H = q + (1-q)\frac{\gamma a_L}{a_H}. \tag{48}$$

The first-order conditions for supply are:

$$k'(y_L) = p_L, \tag{49}$$

$$k'(y_H) = p_H[(1-\gamma) + \gamma q], \tag{50}$$

where the difference reflects that L firms never default while H firms do.

Notation reminder. Here y_T denotes the equilibrium quantity of the intermediate good *produced* by type $T \in \{H, L\}$ and p_T is its equilibrium price. Market clearing implies that type-L firms use the type-H intermediate input, so $x_L = y_H$, while type-H firms use the type-L input, so $x_H = y_L$. The IOU value v_T is the price of IOUs *issued* by type T (the buyer), so it scales the buyer's effective input cost in the demand FOC.

The demand conditions are:

$$qz_L f'(y_H) = \frac{p_H}{v_L} = p_H, \tag{51}$$

$$z_H f'(y_L) = \frac{p_L}{v_H}. \tag{52}$$

Market clearing requires $y_H = x_L$ and $y_L = x_H$. The budget constraints give:

$$p_H y_H = a_L, \tag{53}$$

$$p_L y_L = v_H a_H. \tag{54}$$

Step 2: Establishing the lower bound on a_H/a_L and $a_H > a_L$. For this to be an equilibrium we must have:

(i) L types are solvent upon failure: $a_L \leq \gamma p_L y_L$;

(ii) H types default upon failure: $a_H > \gamma p_H y_H$.

Using the budget identities (53)–(54), these are equivalently

$$a_L \leq \gamma p_L y_L = \gamma v_H a_H \qquad \text{and} \qquad a_H > \gamma p_H y_H = \gamma a_L.$$

Define the issuance ratio $R \equiv a_H/a_L$. Condition (ii) is $R > \gamma$, while condition (i) is $a_L \leq \gamma v_H a_H$, equivalently $1 \leq \gamma v_H R$. We will show below that (i) implies $R \geq \frac{1-(1-q)\gamma^2}{\gamma q} > 1$, and hence $R > \gamma$ automatically. Thus it suffices to impose the L-type solvency condition. Substituting (48) into $1 \leq \gamma v_H R$ (and using $a_L/a_H = 1/R$), we obtain

$$a_L \leq \gamma \left[q + (1-q)\frac{\gamma a_L}{a_H} \right] a_H = \gamma q a_H + (1-q)\gamma^2 a_L.$$

Rearranging yields:

$$a_L \left[1 - (1-q)\gamma^2 \right] \leq \gamma q a_H.$$

Hence

$$\frac{a_H}{a_L} \geq \frac{1-(1-q)\gamma^2}{\gamma q}. \tag{55}$$

Since $1 - \gamma^2(1-q) > \gamma q$ (because $(1-\gamma)(1+\gamma) > q\gamma(1-\gamma)$ reduces to $1+\gamma > q\gamma$, which always holds), we have:

$$\frac{1-\gamma^2(1-q)}{\gamma q} > 1,$$

confirming that $a_H > a_L$ is necessary for this equilibrium. Moreover, (48) then implies $v_H < v_L = 1$.

Step 3: Efficiency wedges. The efficiency wedges are:

$$\omega_H = \frac{qz_H f'(x_H)}{k'(y_L)} = \frac{qp_L/v_H}{p_L} = \frac{q}{v_H},$$
$$\omega_L = \frac{qz_L f'(x_L)}{k'(y_H)} = \frac{p_H}{p_H[(1-\gamma)+\gamma q]} = \frac{1}{(1-\gamma)+\gamma q} = \frac{1}{1-\gamma(1-q)}.$$

Since $v_H > q$, we have $\omega_H = q/v_H < 1$, so H types overinvest. Since $(1-\gamma)+\gamma q < 1$ (equivalent to $1-\gamma(1-q) < 1$), we have $\omega_L > 1$, so L types underinvest.

This reverses the distortion pattern from Proposition 2: in the all-default regime, H firms underinvested and L firms overinvested; here, H firms overinvest and L firms underinvest.

Step 4: Monotonicity of a_H/a_L in z_H. We show that a_H/a_L is increasing in z_H by exploiting the equilibrium structure.

First, the L-type demand and H-type supply conditions, (51) and (50), imply

$$\frac{k'(y_H)}{f'(y_H)} = qz_L\left[(1-\gamma)+\gamma q\right],$$

so y_H (and hence p_H) is pinned down independently of z_H. Therefore $a_L = p_H y_H$ is constant as z_H varies.

Second, the H-type demand condition (52) and the L-type supply condition (49) imply

$$\frac{k'(y_L)}{f'(y_L)} = z_H\, v_H.$$

Thus y_L is increasing in the product $z_H v_H$.

Finally, the budget identity (54) gives $v_H a_H = p_L y_L = k'(y_L)y_L$, so

$$a_H = \frac{k'(y_L)y_L}{v_H}.$$

Since a_L is constant and $v_H = q + (1-q)\gamma a_L/a_H$ from (48), an increase in z_H (which raises the right-hand side of $k'(y_L)/f'(y_L) = z_H v_H$ for any given v_H) must raise the equilibrium y_L and therefore raise $k'(y_L)y_L$. This increases a_H and hence increases a_H/a_L.

Therefore, once an only-H-default equilibrium exists at some z_H, it continues to exist for all higher $z'_H > z_H$.

Step 5: Threshold $\underline{z}_H$. The threshold $\underline{z}_H$ is defined as the value at which condition (55) holds with equality:

$$\frac{a_H}{a_L} = \frac{1-\gamma^2(1-q)}{\gamma q}. \tag{56}$$

Since a_H/a_L is increasing in z_H (from Step 4), for $z_H > \underline{z}_H$, the ratio a_H/a_L exceeds this threshold, ensuring L types remain solvent while H types default. □

E Proof of Proposition 4

Proof. The proof proceeds in five steps. We assume the functional forms $f(x) = x^\alpha/\alpha$ and $k(x) = x^{1+\kappa}/(1+\kappa)$ with $\alpha \in (0,1)$ and $\kappa > 0$ to obtain explicit characterizations.

Step 1: Mixed equilibrium conditions. For a mixed equilibrium with $D_L \in (0,1)$, the equilibrium conditions are:

$$k'(y_L) = p_L\left[(1-\gamma) + \gamma(q + (1-q)(1-D_L))\right], \tag{57}$$

$$k'(y_H) = p_H\left[(1-\gamma) + \gamma q\right], \tag{58}$$

$$v_H = q + (1-q)\frac{\gamma a_L}{a_H}, \tag{59}$$

$$v_L = 1, \tag{60}$$

$$z_H f'(x_H) = \frac{p_L}{v_H}, \tag{61}$$

$$q z_L f'(x_L) = \left[q + (1-q)(1-D_L)\right] p_H, \tag{62}$$

$$a_L = \gamma v_H a_H. \tag{63}$$

From (59) and (63), we obtain:

$$v_H = \frac{q}{1-(1-q)\gamma^2}, \tag{64}$$

$$\frac{a_L}{a_H} = \frac{\gamma q}{1-(1-q)\gamma^2}. \tag{65}$$

Note that (65) matches the upper bound from Proposition 2.

Step 2: Threshold characterization using functional forms. With $f(x) = x^\alpha/\alpha$ and $k(x) = x^{1+\kappa}/(1+\kappa)$, we have $f'(x) = x^{\alpha-1}$ and $k'(x) = x^\kappa$. It is convenient to define

$$\theta \equiv (1-\gamma) + \gamma q = 1 - \gamma(1-q).$$

The thresholds $\underline{z}_H$ and $\bar{z}_H$ are defined by the L-type failure-state solvency boundary $a_L = \gamma v_H a_H$ (see Propositions 3 and 2). At this boundary, $v_L = 1$ and (64) implies $v_H = q/[1-(1-q)\gamma^2]$.

(a) Upper threshold $\bar{z}_H$ (end of all-default regime). In the all-default regime ($D_H = D_L = 1$), both types satisfy $k'(y_T) = p_T\theta$, so $p_T = k'(y_T)/\theta$. Using market clearing and $v_L = 1$, the demand conditions imply

$$z_L f'(y_H) = p_H, \qquad z_H f'(y_L) = \frac{p_L}{v_H},$$

which under the functional forms become

$$y_H^{1+\kappa-\alpha} = z_L\theta, \qquad y_L^{1+\kappa-\alpha} = z_H\theta v_H.$$

Therefore,

$$\frac{y_H}{y_L} = \left(\frac{z_L}{z_H v_H}\right)^{\frac{1}{1+\kappa-\alpha}}.$$

Next, the boundary condition $a_L = \gamma v_H a_H$ together with the budget identities implies $(p_H y_H)/(p_L y_L) = \gamma$. Since $p_T = k'(y_T)/\theta$, this is

$$\left(\frac{y_H}{y_L}\right)^{1+\kappa} = \gamma \qquad \Rightarrow \qquad \frac{y_H}{y_L} = \gamma^{1/(1+\kappa)}.$$

Combining the two expressions for y_H/y_L yields

$$\bar{z}_H = \frac{z_L}{v_H}\,\gamma^{\frac{\alpha}{1+\kappa}-1} := z_L \frac{1-(1-q)\gamma^2}{q}\,\gamma^{\frac{\alpha}{1+\kappa}-1}. \tag{66}$$

(b) Lower threshold $\underline{z}_H$ (start of only-H-default regime). In the only-H-default regime ($D_H = 1$, $D_L = 0$), the L type has $k'(y_L) = p_L$ while the H type still satisfies $k'(y_H) = p_H\theta$. Using market clearing and $v_L = 1$, the demand conditions become

$$q z_L f'(y_H) = p_H, \qquad z_H f'(y_L) = \frac{p_L}{v_H},$$

which under the functional forms imply

$$y_H^{1+\kappa-\alpha} = q z_L \theta, \qquad y_L^{1+\kappa-\alpha} = z_H v_H.$$

Hence

$$\frac{y_H}{y_L} = \left(\frac{q z_L \theta}{z_H v_H}\right)^{\frac{1}{1+\kappa-\alpha}}.$$

Moreover, $a_L = \gamma v_H a_H$ and the budget identities imply again $(p_H y_H)/(p_L y_L) = \gamma$. Now $p_H y_H = y_H^{1+\kappa}/\theta$ while $p_L y_L = y_L^{1+\kappa}$, so

$$\left(\frac{y_H}{y_L}\right)^{1+\kappa} = \gamma\theta \qquad \Rightarrow \qquad \frac{y_H}{y_L} = (\gamma\theta)^{1/(1+\kappa)}.$$

Equating the two expressions for y_H/y_L yields

$$\underline{z}_H = \frac{q z_L}{v_H}\,\theta^{\frac{\alpha}{1+\kappa}}\,\gamma^{\frac{\alpha}{1+\kappa}-1} := z_L\left[1-(1-q)\gamma^2\right]\,\theta^{\frac{\alpha}{1+\kappa}}\,\gamma^{\frac{\alpha}{1+\kappa}-1}. \tag{67}$$

Step 3: General mixed equilibria and threshold bounds. For any $D_L \in (0,1)$, the mixed equilibrium satisfies (57)–(63). The efficiency wedges are:

$$\omega_H = \frac{q z_H f'(x_H)}{k'(y_L)} = \frac{q p_L/v_H}{p_L\left[(1-\gamma)+\gamma(q+(1-q)(1-D_L))\right]} = \frac{q}{v_H\left[(1-\gamma)+\gamma(q+(1-q)(1-D_L))\right]}, \tag{68}$$

$$\omega_L = \frac{q z_L f'(x_L)}{k'(y_H)} = \frac{\left[q+(1-q)(1-D_L)\right]p_H}{p_H\left[(1-\gamma)+\gamma q\right]} = \frac{q+(1-q)(1-D_L)}{(1-\gamma)+\gamma q} = \frac{1-(1-q)D_L}{1-\gamma(1-q)}. \tag{69}$$

Using (64), ω_H simplifies to:

$$\omega_H = \frac{q\left[1-(1-q)\gamma^2\right]}{q\left[(1-\gamma)+\gamma(q+(1-q)(1-D_L))\right]} = \frac{1-(1-q)\gamma^2}{(1-\gamma)+\gamma(q+(1-q)(1-D_L))}.$$

For $\omega_H = \omega_L = 1$ (efficiency), it is enough to impose $\omega_L = 1$ using (69):

$$\frac{1-(1-q)D_L}{1-\gamma(1-q)} = 1 \quad \Longleftrightarrow \quad (1-q)D_L = \gamma(1-q) \quad \Longleftrightarrow \quad D_L = \gamma.$$

Substituting $D_L = \gamma$ into (68) (and using (64)) verifies that $\omega_H = 1$ as well. Therefore, when $D_L = \gamma$, the allocation is efficient.

Step 4: Parameter restrictions for non-empty multiplicity region. For the multiplicity region $[\underline{z}_H, \bar{z}_H]$ to be non-empty and both thresholds to exceed z_L, we require $\underline{z}_H < \bar{z}_H$ and $\underline{z}_H > z_L$.

From (67) and (66), $\underline{z}_H < \bar{z}_H$ holds when:

$$\underline{z}_H < \bar{z}_H \quad \Longleftrightarrow \quad q\,\theta^{\frac{\alpha}{1+\kappa}} < 1,$$

where $\theta = 1-\gamma(1-q) \in (0,1)$. Since $q \in (0,1)$, this inequality holds automatically.

For $\underline{z}_H > z_L$, we need:

$$\left[1-(1-q)\gamma^2\right]\,\theta^{\frac{\alpha}{1+\kappa}}\,\gamma^{\frac{\alpha}{1+\kappa}-1} > 1.$$

This condition is *not always true* and depends on $(\alpha, \kappa, \gamma, q)$. For example, when $\gamma = 0.5$, $q = 0.5$, and $\alpha/(1+\kappa) = 1$, the left side equals $\left(1 - 0.5 \cdot 0.5^2\right) \times 0.75 \times 1 = 0.875 \times 0.75 \approx 0.656$, so the condition fails.

Similarly, $\bar{z}_H > z_L$ requires:

$$\frac{1-(1-q)\gamma^2}{q}\,\gamma^{\frac{\alpha}{1+\kappa}-1} > 1,$$

which also depends on parameters.

Therefore, the multiplicity region exists under suitable parameter restrictions. Sufficient conditions include: γ not too close to 0 or 1, q bounded away from 0, and $\alpha/(1+\kappa)$ in an appropriate range. More precisely, we require:

(i) $\gamma \in (\gamma_{\min}, 1)$ where $\gamma_{\min}$ ensures $\underline{z}_H > z_L$;

(ii) $q \in (q_{\min}, 1)$ where $q_{\min}$ ensures both thresholds exceed z_L;

(iii) $\alpha/(1+\kappa)$ sufficiently small to ensure the threshold ordering.

Step 5: Efficiency properties of mixed equilibria. From Step 3, we established that when $D_L = \gamma$, the wedges satisfy $\omega_H = \omega_L = 1$, achieving the first-best allocation. This efficient mixed equilibrium exists when z_H takes a specific value $z_H^* \in [\underline{z}_H, \bar{z}_H]$ that satisfies the mixed-equilibrium conditions with $D_L = \gamma$.

For other values of $D_L \in (0,1)$, the mixed equilibria feature distortions. From (69), $\omega_L(D_L) = \frac{1-(1-q)D_L}{1-\gamma(1-q)}$ decreases monotonically in D_L, while $\omega_H(D_L)$ increases monotonically in D_L (see (68)). Both wedges equal one exactly at $D_L = \gamma$.

The existence of this continuum of mixed equilibria, including the efficient one, highlights that the coordination problem over default behavior can be resolved in ways that partially or fully offset the distortions from limited commitment and default risk. □

F Alternating Riskiness in a Cycle Network

In this example, we keep productivity constant at z and instead assume that investment success probabilities alternate between q_L and q_H.

Equilibrium conditions (all-default regime). Let p_H and p_L denote the intermediate-good prices charged by H and L suppliers, respectively, and let x_H (resp. x_L) denote the quantity of intermediate input purchased by an H (resp. L) firm from its upstream supplier. In a stationary equilibrium with default upon failure ($D_H = D_L = 1$), the two active links satisfy the supply and demand conditions specialized from (7) and (8):

$$k'(x_H) = p_L \left[(1-\gamma) + \gamma q_L\right], \tag{70}$$

$$zf'(x_H) = \frac{p_L}{v_H}, \tag{71}$$

$$k'(x_L) = p_H \left[(1-\gamma) + \gamma q_H\right], \tag{72}$$

$$zf'(x_L) = \frac{p_H}{v_L}. \tag{73}$$

IOU values satisfy

$$v_H = q_H + (1-q_H)\frac{\gamma v_L a_L}{a_H}, \tag{74}$$

$$v_L = q_L + (1-q_L)\frac{\gamma v_H a_H}{a_L}. \tag{75}$$

Budget identities imply $p_L x_H = v_H a_H$ and $p_H x_L = v_L a_L$.

Benchmark (symmetric riskiness). When all firms are ex ante identical ($q_H = q_L = q$), the IOU valuation equations reduce to $v = q + (1-q)\gamma v$, hence $v = q/(1-\gamma+\gamma q)$. The wedge is therefore $\omega = q/(v\left[(1-\gamma)+\gamma q\right]) = 1$, so the equilibrium allocation is efficient.

We now deviate from the benchmark to $q_H > q > q_L$ but the deviation from q is small. In this case, both firms default.

Proposition 8. *In a stationary equilibrium with alternating riskiness, slack pledgeability constraints, and default upon failure, high-success-probability firms underinvest and low-success-probability firms overinvest:* $\omega_H > 1 > \omega_L$.

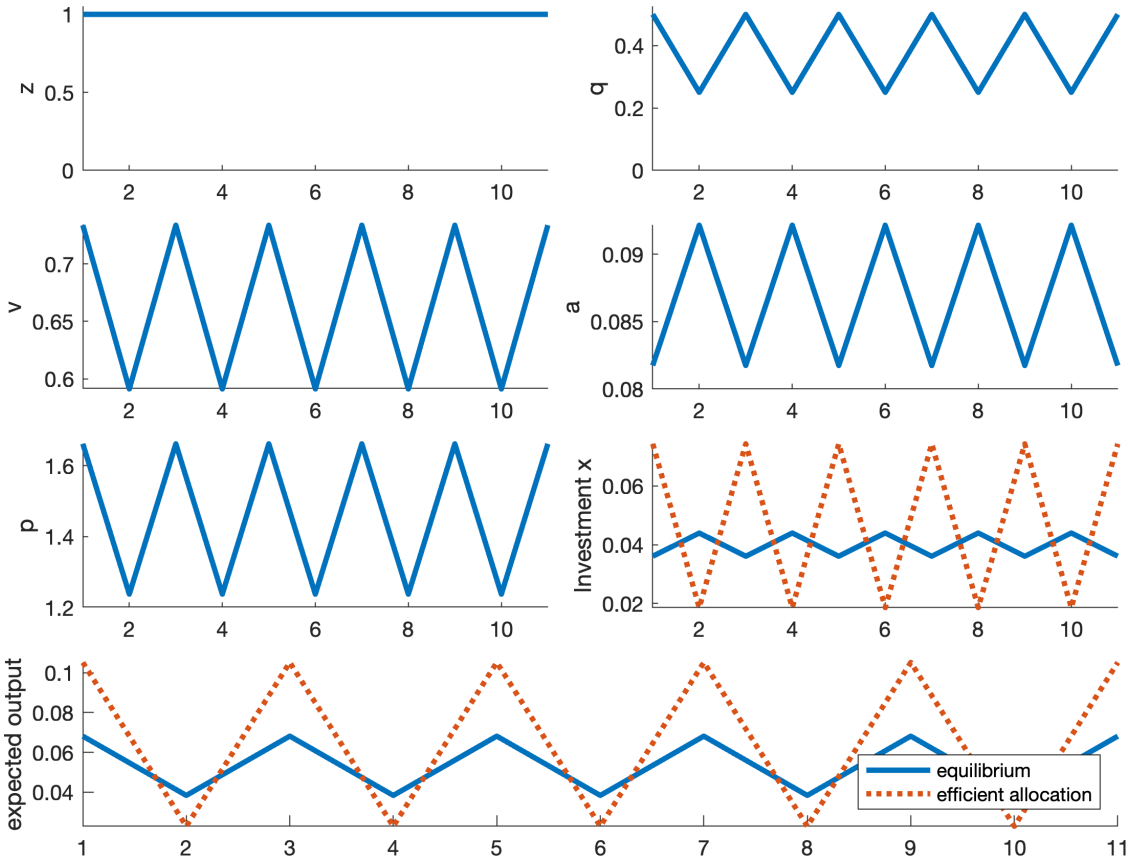


Figure 8: Alternating investment riskiness

The proof is in Appendix F.1.

In the limiting case $q_H = 1$ and q_L close to one, safe firms can invest less than risky firms when collateral is sufficiently pledgeable (Proposition 9).

Figure 8 illustrates the equilibrium allocation. The main take away is that firms with safe investment may invest less than firm with risky investment.

$$\frac{\frac{zf'(I_H)}{k'(I_H)}}{\frac{zf'(I_L)}{k'(I_L)}} = \frac{\left[q_L + (1-q_L)q_H \frac{\gamma a_H}{a_L}\right][(1-\gamma)+\gamma q_H]}{\left[q_H + (1-q_H)q_L \frac{\gamma a_L}{a_H}\right][(1-\gamma)+\gamma q_L]}$$

$I_H < I_L$ if and only if

$$\frac{\left[q_L + (1-q_L)q_H \frac{\gamma a_H}{a_L}\right][(1-\gamma)+\gamma q_H]}{\left[q_H + (1-q_H)q_L \frac{\gamma a_L}{a_H}\right][(1-\gamma)+\gamma q_L]} > 1$$

$$\left[q_L + (1-q_L)q_H \frac{\gamma a_H}{a_L}\right][(1-\gamma)+\gamma q_H] > \left[q_H + (1-q_H)q_L \frac{\gamma a_L}{a_H}\right][(1-\gamma)+\gamma q_L]$$

If $q_H = 1$

$$q_L + (1 - q_L)\frac{\gamma a_H}{a_L} > (1 - \gamma) + \gamma q_L$$
$$(1 - q_L)\frac{\gamma a_H}{a_L} > (1 - \gamma)(1 - q_L)$$
$$\frac{a_H}{a_L} > \frac{1 - \gamma}{\gamma}$$

Proposition 9. *Suppose $q_H = 1$ and q_L is close to one. Then safe firms underinvest and risky firms overinvest. Moreover, risky firms invest more than safe firms whenever $\gamma > 1/2$.*

Proof. See Appendix F.2. So far we assume that q_H and q_L are close to q so that both types of firms default.

F.1 Proof of Proposition 8

Proof. The proof proceeds in four steps. We consider a cycle network with alternating riskiness where productivity is constant at z but success probabilities alternate between q_H and q_L with $q_H > q_L$.

Step 1: Equilibrium system under all-default regime. When both types default upon failure ($D_H = D_L = 1$), the equilibrium is characterized by the following system. To streamline notation, let ϕ_T denote the purchasing power of one unit of face value of type-T IOUs in units of the intermediate good they buy; investment is $x_T \equiv \phi_T a_T$. From the first-order conditions for supply and demand, we have:

$$k'(\phi_H a_H)\,\phi_H = v_H\left[(1 - \gamma) + \gamma q_L\right], \tag{76}$$
$$k'(\phi_L a_L)\,\phi_L = v_L\left[(1 - \gamma) + \gamma q_H\right], \tag{77}$$
$$zf'(\phi_H a_H) = \frac{1}{\phi_H}, \tag{78}$$
$$zf'(\phi_L a_L) = \frac{1}{\phi_L}. \tag{79}$$

IOU prices satisfy:

$$v_H = q_H + (1 - q_H)\frac{\gamma v_L a_L}{a_H}, \tag{80}$$
$$v_L = q_L + (1 - q_L)\frac{\gamma v_H a_H}{a_L}. \tag{81}$$

Step 2: Solving for IOU prices. From (80) and (81), we solve the coupled system. Substituting (81) into (80):

$$v_H = q_H + (1 - q_H)\frac{\gamma}{a_H}\left[q_L a_L + (1 - q_L)\gamma v_H a_H\right] = q_H + (1 - q_H)\gamma q_L\frac{a_L}{a_H} + (1 - q_H)(1 - q_L)\gamma^2 v_H.$$

Rearranging:

$$v_H\left[1 - (1 - q_H)(1 - q_L)\gamma^2\right] = q_H + (1 - q_H)\gamma q_L\frac{a_L}{a_H}.$$

Similarly, substituting (80) into (81):

$$v_L = q_L + (1-q_L)\frac{\gamma}{a_L}\left[q_H a_H + (1-q_H)\gamma v_L a_L\right] = q_L + (1-q_L)\gamma q_H \frac{a_H}{a_L} + (1-q_H)(1-q_L)\gamma^2 v_L.$$

Rearranging:

$$v_L\left[1-(1-q_H)(1-q_L)\gamma^2\right] = q_L + (1-q_L)\gamma q_H \frac{a_H}{a_L}.$$

Therefore:

$$v_H = \frac{q_H + (1-q_H)\gamma q_L \frac{a_L}{a_H}}{1-(1-q_H)(1-q_L)\gamma^2}, \tag{82}$$

$$v_L = \frac{q_L + (1-q_L)\gamma q_H \frac{a_H}{a_L}}{1-(1-q_H)(1-q_L)\gamma^2}. \tag{83}$$

Step 3: Establishing the inequality for a_H/a_L. From (76)–(77) and (78)–(79), we combine the conditions. From (78) and (79), since f' is decreasing, if $\phi_H a_H < \phi_L a_L$, then $f'(\phi_H a_H) > f'(\phi_L a_L)$, which implies $\phi_H < \phi_L$.

From (76) and (77):

$$\frac{k'(\phi_H a_H)\,\phi_H}{k'(\phi_L a_L)\,\phi_L} = \frac{v_H\left[(1-\gamma)+\gamma q_L\right]}{v_L\left[(1-\gamma)+\gamma q_H\right]}.$$

Since k' is increasing and Step 1 implies $\phi_H < \phi_L$ when $\phi_H a_H < \phi_L a_L$, if $\phi_H a_H < \phi_L a_L$ then the left side is less than 1, so:

$$\frac{v_H\left[(1-\gamma)+\gamma q_L\right]}{v_L\left[(1-\gamma)+\gamma q_H\right]} < 1.$$

Substituting (82) and (83):

$$\frac{\left[q_H + (1-q_H)\gamma q_L \frac{a_L}{a_H}\right]\left[(1-\gamma)+\gamma q_L\right]}{\left[q_L + (1-q_L)\gamma q_H \frac{a_H}{a_L}\right]\left[(1-\gamma)+\gamma q_H\right]} < 1.$$

Cross-multiplying and rearranging yields a quadratic inequality in the ratio $X \equiv a_H/a_L$. Let $\bar{X}(q_H, q_L, \gamma)$ denote the unique positive root of the associated quadratic; the inequality above is equivalent to $X > \bar{X}$. We do not need a closed-form expression for $\bar{X}$ for the qualitative wedge result.

Step 4: Efficiency wedges. The efficiency wedges are:

$$\omega_H = \frac{q_H/v_H}{(1-\gamma)+\gamma q_L}, \tag{84}$$

$$\omega_L = \frac{q_L/v_L}{(1-\gamma)+\gamma q_H}. \tag{85}$$

In the symmetric benchmark $q_H = q_L = q$, we have $v_H = v_L = v$ and the IOU pricing condition becomes $v = q + (1-q)\gamma v$, so $v = q/[1-(1-q)\gamma] = q/[(1-\gamma)+\gamma q]$. Substituting into the expressions above yields $\omega_H = \omega_L = 1$. For a small deviation with $q_H > q_L$, the equilibrium varies continuously in (q_H, q_L) and the two wedges move in opposite directions, yielding $\omega_H > 1 > \omega_L$. This establishes the qualitative statement in Proposition 8. □

F.2 Proof of Proposition 9

Proof. With $q_H = 1$, safe firms never default and therefore have $v_H = 1$. Risky firms have $v_L = q_L + (1 - q_L)\gamma a_H/a_L < 1$. The Euler equations imply

$$\frac{zf'(I_H)}{k'(I_H)} = \frac{1}{(1-\gamma) + \gamma q_L} > 1$$

for safe firms and

$$\frac{q_L z f'(I_L)}{k'(I_L)} = \frac{q_L}{v_L} < 1$$

for risky firms, hence safe firms underinvest and risky firms overinvest. Moreover, when q_L is close to one, equilibrium allocations satisfy a_H/a_L close to one; then the condition for risky firms to invest more than safe firms reduces to $a_H/a_L > (1-\gamma)/\gamma$, which holds whenever $\gamma > 1/2$. □

G Propagation Triggered by Project Riskiness

This example illustrates the equilibrium response when the investment of a single firm along the chain becomes riskier. Figure 9 illustrates the equilibrium allocation and prices. The increase in investment riskiness spreads not only upstream but also downstream. The investment of the downstream neighbor of the firm which becomes riskier can be hit the hardest even though the firm itself does not become riskier. This is again because of the valuation channel. The firm whose investment becomes riskier is more likely to default in the failure state; when it does, the pledged fraction of the receivables it acquires is seized by creditors, lowering the private return from supplying on credit and dampening downstream investment.

Upstream propagation. When firm i's success probability q_i decreases, the upstream effect operates through the financing wedge $\tau_i \equiv [q_i + (1-q_i)(1-D_i)]/v_i$ (defined above). In particular, q_i tightens financing both directly (through the numerator in (8)) and indirectly through the IOU value v_i (via (4) and, in general, the induced default decision $D_i = \delta(r_i)$). Even holding the default regime fixed in this example, a decline in q_i reduces v_i and raises τ_i, lowering firm i's input demand. In addition, the reduction in v_i reduces the value of receivables held by upstream firm $i-1$, depressing v_{i-1} and propagating the shock upstream through the collateral channel.

Downstream propagation. The impact on downstream firms operates through general-equilibrium prices and quantities rather than through a direct recursion from v_i to v_{i+1}. In the model, firm $i+1$'s IOU value is pinned down by the pledgeable value of *its own* receivables from its downstream buyer (firm $i+2$), not by the IOUs it owes to firm i. A decline in q_i reduces firm i's demand for inputs and changes equilibrium prices and investment along the chain; these changes affect firm $i+1$ through its goods-market conditions and through the value of the receivables it ultimately holds, generating downstream propagation.

The figure shows that equilibrium investment and output (solid blue lines) deviate from efficient levels (dotted orange lines), with the downstream neighbor of the affected firm experiencing the largest impact due to the valuation channel.

H Construction of the Contagion Example

This appendix describes how Figure 6 is constructed. The objective is to build a reference line economy in which *no firm defaults* and IOUs trade at par, and then show that a small local productivity dip can trigger a cascade of defaults and a discontinuous drop in investment.

Step 0: Parameters and functional forms. We adopt the following functional forms: $f'(x) = x^{\alpha-1}$ with $\alpha = 0.4$ and $k'(x) = \kappa x$ with $\kappa = 2$. We set $\gamma = 0.99$, $q = 0.8$, and $n = 15$ firms indexed by $i = 1, \ldots, n$.

Step 1: Reference (no-default) equilibrium profile. We construct a productivity sequence $\{z_i^*\}$ increasing with i so that, in the reference equilibrium, all firms are solvent in the failure state: $D_i^* = 0$ and $v_i^* = 1$ for all $i \geq 2$. In this regime, the equilibrium conditions reduce to the efficient ones. In particular, with $v_i^* = 1$ and $D_i^* = 0$, the line-network supply and demand conditions (12)–(13) imply

$$p_{i-1}^* = k'(x_i^*) \qquad \text{and} \qquad p_{i-1}^* = q z_i^* f'(x_i^*),$$

so $q z_i^* f'(x_i^*) = k'(x_i^*)$ for all $i \geq 2$. The budget identity (11) further implies $a_i^* = p_{i-1}^* x_i^*$. To ensure solvency along the chain, we impose the collateral threshold with equality, equivalently

$$a_i^* = \gamma a_{i+1}^* \quad \Longleftrightarrow \quad x_{i+1}^* k'(x_{i+1}^*) = \frac{1}{\gamma}\, x_i^* k'(x_i^*) \qquad (i \geq 2),$$

which pins down a rapidly increasing sequence of investments $\{x_i^*\}$ and hence of productivities $\{z_i^*\}$.

Step 2: Choosing the boundary condition at firm 2. Firm 1's choice pins down only a normalization for the reference (no-default) profile: any positive choice of x_2^* can be used to initialize the recursion, with the implied input price $p_1^* = k'(x_2^*)$ and productivity z_2^* adjusted accordingly. For concreteness, we normalize $x_2^* \equiv 1/\sqrt{\kappa}$ (so $k'(x_2^*) = \sqrt{\kappa}$), set $p_1^* \equiv k'(x_2^*)$, and use the demand condition $p_1^* = q z_2^* f'(x_2^*)$ to back out z_2^*. Finally, we set the face value a_2^* using the budget identity $a_2^* = p_1^* x_2^*$.

Step 3: Introducing the productivity dip. We set $j = n - 5 = 10$ and reduce productivity by one percent at that location: $z_{10} = 0.99\, z_{10}^*$, keeping $z_i = z_i^*$ for $i \neq 10$.

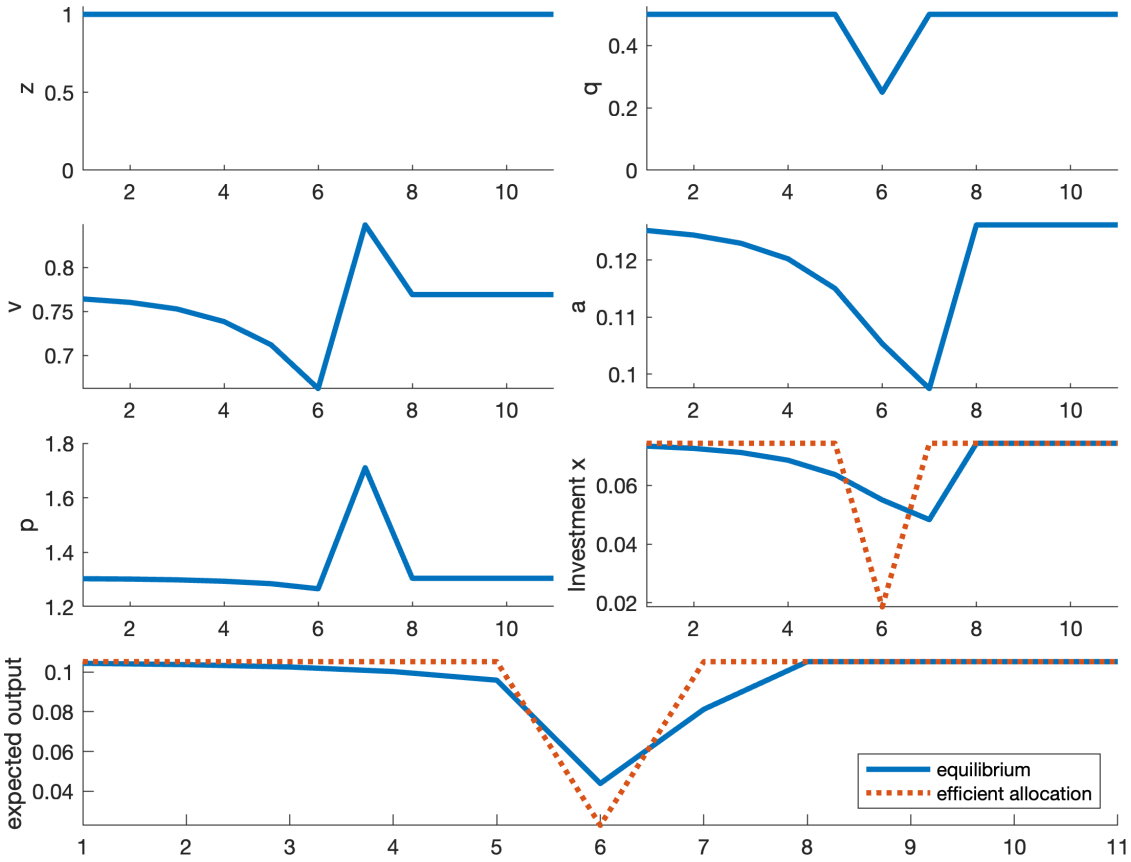


Figure 9: Propagation triggered by a change in project riskiness. The figure displays seven plots showing how equilibrium variables (solid blue lines) and efficient allocations (dotted orange lines) respond when a single firm's success probability q decreases. The shock propagates upstream through the collateral channel and downstream through the valuation channel. Top row: exogenous parameters z and q. Subsequent rows: IOU values v, face values a, goods prices p_{i-1} (the input price faced by firm i), investment levels x, and expected output. The downstream neighbor of the affected firm experiences the largest impact. Parameters: $\alpha = 0.6$, $\gamma = 0.7$, $\kappa = 0.1$, $q = 0.5$ (baseline, all firms), $z_i = 1$ for all i, $N = 50$ firms.

Step 4: Solving for the post-shock equilibrium. Given the shocked $\{z_i\}$, the equilibrium is computed by iterating the coupled recursion: (i) a backward pass updating (D_i, v_i) using the default rule $D_i = \mathbf{1}\{a_i > \gamma p_i y_i\}$ (equivalently $\mathbf{1}\{a_i > \gamma v_{i+1} a_{i+1}\}$ using (11)) and the valuation recursion (14); (ii) an update of prices/quantities using the line-network supply and demand conditions (12)–(13) together with the budget identity (11). Iteration continues until (a, v, p, y) converge. The resulting profiles are plotted in Figure 6 (black solid for the reference, red dashed for the shocked equilibrium).

I Proofs for Section 8

A useful representation of wedges under exogenous default. Fix a target allocation $(\{x^*_{ij}\}, \{y^*_i\})$ and consider a competitive equilibrium in which this allocation is implemented under an exogenous default policy $\{D_i\}$. Define $s_i \equiv 1 - q_i$. Since D_i is the probability of default conditional on failure, $s_i D_i = (1-q_i)D_i$ is the ex ante probability of default. Define $\widetilde{D}_i \equiv s_i \gamma D_i$, i.e. $\widetilde{D}_i$ is γ times the ex ante default probability, and

$$K_i \equiv \frac{c_i k'(y^*_i) y^*_i}{\alpha q_i z_i f_i(\{x^*_{ij}\})}.$$

Proof of the representation. We combine the general-network equilibrium conditions in Section 7, evaluated at the implemented allocation.

Step 1: A convenient expression for face values. Under Cobb–Douglas, the demand condition (8) and the spending-share identity (16) imply (Lemma 1) that

$$\frac{a_i}{\alpha} = \frac{q_i z_i f_i(\{x^*_{ij}\})}{q_i + (1-q_i)(1-D_i)} = \frac{q_i z_i f_i(\{x^*_{ij}\})}{1 - s_i D_i}. \tag{86}$$

Step 2: Collateral coverage. Using the supply condition (7), we can write

$$p_i = \frac{c_i k'(y^*_i)}{1 - \widetilde{D}_i}.$$

Therefore,

$$r_i \equiv \frac{\gamma p_i y^*_i}{a_i} = \gamma \, \frac{(1 - s_i D_i)}{1 - \widetilde{D}_i} \, K_i.$$

Step 3: IOU values. Using (4) and the expression for r_i above,

$$\begin{aligned} v_i &= q_i + (1 - q_i)\left[(1 - D_i) + D_i r_i\right] = (1 - s_i D_i) + s_i D_i r_i \\ &= (1 - s_i D_i)\left[1 + \frac{s_i D_i \gamma K_i}{1 - \widetilde{D}_i}\right] = (1 - s_i D_i)\, \frac{1 + \widetilde{D}_i (K_i - 1)}{1 - \widetilde{D}_i}. \end{aligned}$$

Step 4: Wedges. Finally, substituting this expression for v_i into (10) and using the simplification $(1-\gamma)+\gamma\left[q_j+(1-q_j)(1-D_j)\right]=1-\widetilde{D}_j$, we obtain the representation

$$\omega_{ij}=\frac{1-\widetilde{D}_i}{(1-\widetilde{D}_j)\left[1+\widetilde{D}_i(K_i-1)\right]}. \tag{87}$$

□

Proof of Proposition 6

Proof. In an N-cycle, impose efficiency on each active link: $\omega_{i,i-1}=1$ for all i (with $0\equiv N$).

A convenient transformation. The link-by-link efficiency restriction implies a nonlinear recursion in $\widetilde{D}_i$. A simple algebraic transformation converts it into a linear multiplicative recursion and yields the closed form.

Using (87), $\omega_{i,i-1}=1$ is equivalent to

$$1-\widetilde{D}_{i-1}=\frac{1-\widetilde{D}_i}{1+\widetilde{D}_i(K_i-1)}.$$

Rearranging gives the recursion

$$\widetilde{D}_{i-1}=\frac{\widetilde{D}_iK_i}{1+\widetilde{D}_i(K_i-1)}. \tag{88}$$

Dividing both sides of (88) by $1-\widetilde{D}_{i-1}$ yields the linear update

$$\frac{\widetilde{D}_{i-1}}{1-\widetilde{D}_{i-1}}=K_i\,\frac{\widetilde{D}_i}{1-\widetilde{D}_i}. \tag{89}$$

Iterating (89) forward yields, for $i=2,\dots,N$,

$$\frac{\widetilde{D}_i}{1-\widetilde{D}_i}=\left(\prod_{m=2}^{i}K_m\right)^{-1}\frac{\widetilde{D}_1}{1-\widetilde{D}_1},$$

which implies the closed form (23) after converting back from $\widetilde{D}_i=(1-q_i)\gamma D_i$.

Finally, cycle closure requires that the recursion is consistent when indices are taken modulo N. Equivalently, applying (89) around the cycle implies that $\widetilde{D}_1/(1-\widetilde{D}_1)$ must equal $\left(\prod_{m=1}^{N}K_m\right)\widetilde{D}_1/(1-\widetilde{D}_1)$. Thus the recursion closes for any admissible $\widetilde{D}_1$ whenever $\prod_{m=1}^{N}K_m=1$. Moreover, once the one-cycle consistency holds, iterating the update around the cycle r times would multiply $\widetilde{D}_1/(1-\widetilde{D}_1)$ by $\left(\prod_{m=1}^{N}K_m\right)^r=1$, so the implied profile is invariant to how many times one traverses the cycle.

In the N-cycle with a single supplier per buyer, this product condition is implied by the efficient allocation and the ring structure (it is not an additional restriction). To see this, first note that each sector i has only one active input link, $(i,i-1)$, so $\alpha_{i,i-1}=\alpha$. Cobb–Douglas therefore implies the standard identity

$$x^*_{i,i-1}\,\frac{\partial f_i}{\partial x_{i,i-1}}(\{x^*_{i,j}\})=\alpha_{i,i-1}f_i(\{x^*_{i,j}\})=\alpha f_i(\{x^*_{i,j}\}).$$

Second, because good $i-1$ is used only by buyer i in the directed cycle, market clearing implies $y^*_{i-1} = x^*_{i,i-1}$. Efficiency ($\omega_{i,i-1} = 1$) therefore gives

$$c_{i-1}k'(y^*_{i-1})y^*_{i-1} = q_i z_i\, x^*_{i,i-1}\frac{\partial f_i}{\partial x_{i,i-1}}(\{x^*_{i,j}\}) = \alpha q_i z_i f_i(\{x^*_{i,j}\}).$$

Using the definition of K_{i-1}, this implies

$$K_{i-1} = \frac{c_{i-1}k'(y^*_{i-1})y^*_{i-1}}{\alpha q_{i-1}z_{i-1}f_{i-1}(\{x^*_{i-1,j}\})} = \frac{q_i z_i f_i(\{x^*_{i,j}\})}{q_{i-1}z_{i-1}f_{i-1}(\{x^*_{i-1,j}\})}.$$

Taking products over $i = 1, \ldots, N$ (with indices modulo N), each term $q_i z_i f_i(\{x^*_{i,j}\})$ appears once in a numerator and once in a denominator, so the product cancels to $\prod_{i=1}^N K_i = 1$. This establishes (24) and therefore closure for any admissible D_1. □

Proof of Proposition 5

Proof. This is the special case of Proposition 6 with $N = 2$ and common q. In this case, (89) reduces to

$$\frac{\widetilde{D}_L}{1-\widetilde{D}_L} = K_H\,\frac{\widetilde{D}_H}{1-\widetilde{D}_H}.$$

Since here $\widetilde{D}_T = (1-q)\gamma D_T$, dividing by the common factor $(1-q)\gamma$ gives

$$\frac{D_L}{1-(1-q)\gamma D_L} = K_H\,\frac{D_H}{1-(1-q)\gamma D_H}.$$

In the two-sector alternating environment, evaluating the coefficient K_H at the efficient allocation yields $K_H = r$ (as defined in the main text), so we obtain (21). Solving that equation for D_H as a function of D_L gives (22). □

Proof of Proposition 7

Proof. Under link-by-link efficiency on the active input set, (19) implies that for any buyer i and any two active suppliers $j, k \in S_i$, we must have

$$1-(1-q_j)\gamma D_j = 1-(1-q_k)\gamma D_k,$$

because $\omega_{ij} = 1 = \omega_{ik}$ holds with the same buyer-side term τ_i. Under the connectivity assumption, these pairwise equalizations propagate across the active-link structure as follows. Consider the undirected "supplier-overlap" graph on sectors in which we draw an edge between j and k if there exists some buyer i with $j, k \in S_i$ (so buyer i multi-sources from both j and k on the active set). For any edge (j, k), the displayed equality implies that the supplier-side term $1-(1-q_\ell)\gamma D_\ell$ takes the same value at $\ell = j$ and $\ell = k$. By connectivity of this overlap graph, any two sectors can be joined by a path $j = j_0, j_1, \ldots, j_m = k$ along which these

equalities hold, so iterating yields $1-(1-q_j)\gamma D_j = 1-(1-q_k)\gamma D_k$ for all j,k. Hence there exists a common constant M such that $1-(1-q_j)\gamma D_j = M$ for all j, equivalently $1-\widetilde{D}_j = M$ for all j.

Now impose $\omega_{ij}=1$ on all active links and use (87). Since $1-\widetilde{D}_i = M$ for all i as well, for any active link (i,j) we have

$$1=\omega_{ij}=\frac{M}{M\left[1+\widetilde{D}_i(K_i-1)\right]}=\frac{1}{1+\widetilde{D}_i(K_i-1)}.$$

Hence $\widetilde{D}_i(K_i-1)=0$ for every i. If the $\{K_i\}$ are not all equal to one, this forces $\widetilde{D}_i=0$ for all i, i.e. $D_i=0$ for all i because $\widetilde{D}_i=(1-q_i)\gamma D_i$ with $(1-q_i)\gamma>0$. In the knife-edge case $K_i=1$ for all i, the condition reduces to $1-\widetilde{D}_i=M$, i.e. $D_i=(1-M)/[(1-q_i)\gamma]$, with feasibility requiring $D_i\in[0,1]$. □