The Puzzle of Household Economies of Scale and Consumption: A Reassessment of Theory and Evidence*

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Abstract

We revisit the importance of economies of scale in consumption by examining the relationship between household size and per-capita expenditure on food. In influential work, Deaton and Paxson (1998) argue that when household size increases for given expenditure per capita, the per-capita price of public goods such as housing decreases, which induces a positive income effect and a negative substitution effect on the per-capita demand of private goods, such as food. If the income elasticity of the demand of food is large enough and food is a necessity, then the income effect dominates and so the *per-capita demand* of food increases for given per-capita expenditure. When the unit price of food is constant in the quantity of food demanded, the same positive relationship holds between household size and *per-capita expenditure* on food—namely, as household size increases, per-capita expenditure on food increases as well. Given expenditure per capita, Deaton and Paxson (1998), however, estimate a negative relationship between household size and per-capita expenditure on food in a number of countries—the so-called Deaton-Paxson consumption puzzle. In this paper, we address this puzzle by first showing that when food is priced nonlinearly through volume discounts, as is pervasive in developing countries, so that its unit price decreases with the quantity purchased, demand models that allow for economies of scale at the household level in general give rise to an ambiguous, if not altogether negative, relationship between household size and per-capita expenditure on food. Intuitively, when the unit price of food decreases with the quantity demanded of it, a larger household tends to exhibit not only a higher per-capita demand of food when the income elasticity of the demand of food is large enough (the *quantity effect* of economics of scale), but also faces a lower unit price of food (the *price effect* of economics of scale). Due to these two countervailing effects, per-capita expenditure on food may well decrease. Indeed, when quantity discounts are pronounced in that the elasticity of the unit price schedule of food is close to one, per-capita expenditure on food decreases with household size. Yet, as consistent with the conjecture in Deaton and Paxson (1998), the per-capita quantity demanded of food increases with household size. Thus, no puzzle arises. We then provide robust evidence of the importance of price nonlinearities in resolving the Deaton-Paxson consumption puzzle based on data from the well-known Progresa/Oportunidades conditional cash transfer program. Through the lens of the model, we document that nonlinear pricing accounts for approximately half of the measured puzzle in this context.

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1 Introduction

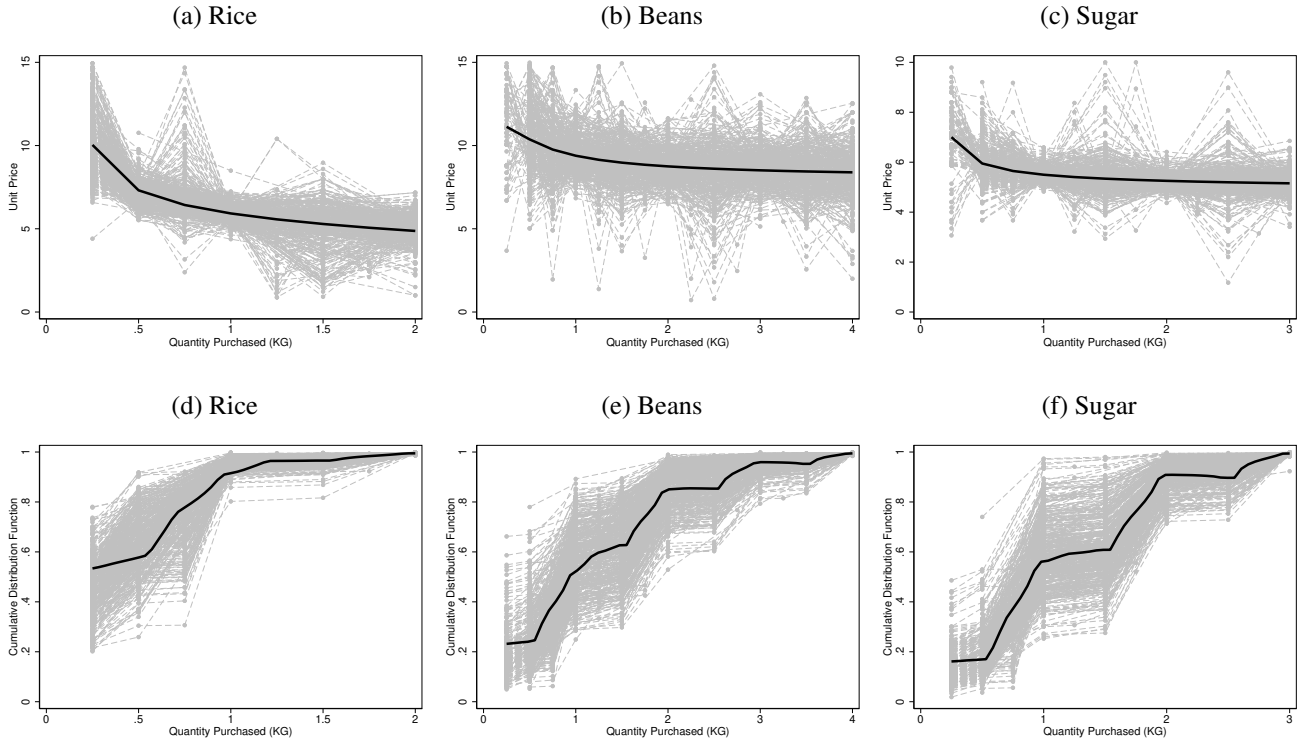
A large literature on household consumption and economies of scale has developed since the influential work of Deaton and Paxson (1998) (hereafter, DP), who highlighted a puzzle concerning the relationship between food expenditure and household size. Standard consumer theory predicts that when households consume both private goods, such as food, and public goods, such as housing, increases in household size, holding per-capita total expenditure constant, reduce the per-capita price of the public good. Such a decline generates a positive income effect and a negative substitution effect on the demand for the private good. When the private good is a necessity, as is typically the case for food, the substitution effect should be small, so the income effect should dominate. As a result, the per-capita consumption of food should increase with household size.

Empirically, however, DP documented the opposite pattern for many countries: per-capita expenditure on food declines with household size, conditional on per-capita total expenditure. This pattern is particularly pronounced in poorer countries, precisely where the theoretical prediction of a positive relationship should be strongest—hence, the puzzle. A large literature has attempted to resolve this puzzle by examining the magnitude of household economies of scale, the difficulty of measuring consumption, and the challenges of estimating demand elasticities—see, for instance, Gan and Vernon (2003). Despite these efforts, the puzzle remains largely unresolved.

In this paper, we revisit the DP puzzle and propose a resolution of it based on a feature of consumer markets that is typically absent from standard demand models and yet very common in actual markets in developing countries: the nonlinear pricing of basic food items. That is, in common settings, the unit price of many food items declines with the quantity purchased of them because of quantity discounts. When such nonlinear pricing is present, the relationship between household size and per-capita expenditure on food becomes ambiguous even when economies of scale are important. The intuition behind this ambiguity is straightforward. If larger households benefit from economies of scale in consumption, then the per-capita *quantity* demanded of food may increase with household size—the *quantity effect*. At the same time, purchasing larger quantities reduces the unit price of food through volume discounts—the *price effect*. As a result, per-capita *expenditure* on food may decline even though per-capita consumption increases. When quantity discounts are pronounced, the price effect dominates, giving rise to the negative relationship between household size and per-capita expenditure on food documented by DP.

We formalize this mechanism within a demand model that allows for the nonlinear pricing of food.

Figure 1: Unit Price and Quantity Purchased for Staples



Note: The top panels report the price schedules across quantities purchased for each village together with an interpolated (solid) line. The bottom panels report the empirical c.d.f of quantities purchased for each village together with an interpolated (solid) line. Quantities are rounded to 0.25 increments. All 656 villages from all survey waves are used.

The model shows that in the presence of quantity discounts, increases in household size can raise the per-capita consumption of food while reducing the per-capita expenditure on it. Thus, the DP puzzle arises because the empirical literature measures economies of scale using information on *expenditure*, whereas the theoretical prediction concerns *consumption*. We then test this mechanism using detailed information on household consumption, purchases, and price data from Mexico, where substantial nonlinearities in the pricing of basic food staples have been documented (see Attanasio and Pastorino, 2020).

Having characterized the theoretical relationship between per-capita demand of food and family size, we begin our empirical exercise by confirming the presence of the DP puzzle in our sample from the evaluation surveys of the Progresa/Oportunidades program. We also document that quantity discounts are a characteristic feature of the pricing of commonly consumed food items. In Figure 1, we report the empirical price schedules for our three staple foods, namely, rice, beans and sugar, for each village in our sample. We calculate these schedules using the 656 villages from all sample waves by first rounding quantities purchased to 0.25 increments and then calculating the mean unit price for each such quantity

for each locality.[1] As the figure shows, the unit price of each of the three staples significantly declines with the quantity purchased in most villages (see top panels) and declines more rapidly over the range of relatively small quantities that most households demand (see the empirical c.d.f.'s in the bottom panels).[2]

Finally, we show that these nonlinear price schedules account for the puzzle. Namely, the negative relationship between household size and per-capita expenditure on food is weaker in markets in which the nonlinear pricing of food is *less* pronounced—so quantity discounts are smaller—and stronger in markets in which the nonlinear pricing of food is *more* pronounced—so quantity discounts are larger—as predicted by our model. Moreover, we demonstrate that when these price nonlinearities are explicitly taken into account in estimating the elasticity of the per-capita quantity demanded of food to household size, we find that the per-capita quantity demanded of food indeed increases with household size, as predicted by the model in the presence of economies of scale.

The rest of the paper proceeds as follows. In Section 2, we discuss the related literature. In Section 3, we review the consumption puzzle identified by DP. In Section 4, we propose our model of demand under nonlinear pricing and prove how it can reconcile the DP puzzle. In Section 5, we replicate the DP puzzle in our data, illustrate how accounting for nonlinearities in the pricing of food can not only attenuate but also resolve the puzzle altogether—in that per-capita expenditure on food decreases with household size but per-capita consumption of food increases with it—and provide evidence in support of the mechanism of our model. Finally, in Section 6, we conclude. The appendix collects all omitted proofs and details.

2 Literature Review

Several papers have sought to reconcile the Deaton–Paxson finding that per-capita food expenditure declines with household size at fixed per-capita total expenditure. This work falls into three broad strands: measurement concerns, considerations about pricing and market structure, and alternative modeling choices. First, several papers examine whether measurement problems could generate the negative size gradient between household size and per-capita food demand. Halliday (2010), for example, argues that household rosters are measured at a point in time, whereas consumption is recalled over longer windows. Therefore, demographic fluidity, or frequent changes in family composition, can lead to mis-measuring the "effective" household size during the recall period. In another paper, Gibson and Kim (2007) focus on recall-based consumption reports, noting that larger households may systematically under-report

[1] This evidence is similar to and extends that reported in Attanasio and Pastorino (2020), who document price schedules using the 506 villages from the original survey waves from October 98 to November 2000.

[2] See also some additional evidence in Table 5 in the appendix.

food purchases due to the complexity of recall. Although both critiques provide evidence that these are potentially legitimate concerns, both studies find that plausible magnitudes for these errors attenuate but do not overturn the basic pattern; see also Brzozowski et al. (2017).

Second, work on prices and market structure examines whether bulk discounts can rationalize the puzzle. Using Swiss data, Abdulai (2003) finds lower food unit values for larger households and interprets this as evidence that bulk discounts—and related scale economies—could reconcile the Deaton–Paxson pattern. However, subsequent transaction-level evidence, holding quality constant and estimating the price–quantity schedule within purchase episodes, finds that bulk-discount elasticities are economically small, on the order of 1–10 percent for common staples. Gibson and Kim (2018) argue this point using transaction-level data for branded packaged foods in urban Papua New Guinea, where holding quality constant by restricting to dominant brands, they find small bulk-discount elasticities. However, in rural markets such as those we study, staple foods like rice, beans, and sugar are commonly sold unpackaged by weight, which allows for more flexible—and potentially larger—quantity discounts, and substantially reduces concerns about quality heterogeneity. Along a similar line, Rao (2000) argues that if poorer households are constrained to purchasing small quantities, they may systematically face higher effective prices, as documented also in Attanasio and Frayne (2006). These considerations underscore the importance of moving beyond average prices to the actual price schedule faced by households.

A third strand revisits the modeling assumptions in DP. Gan and Vernon (2003) question the canonical Barten adult equivalence structure underpinning the DP result, emphasizing that food may embody important household-level economies—such as in food preparation—and that some non-food categories are highly private (see Horowitz, 2002, for a similar argument). In their response, Deaton and Paxson (2003) argue that Gan and Vernon's evidence that housing expenditure may fall faster than food expenditure with household size does not address the core puzzle: at fixed per-capita outlay, larger households should be able to raise per-capita food consumption if food is normal and substitution is limited. Within a Barten framework, the observed decline in per-capita food consumption would require implausibly large scale economies in food or very high price elasticities. In this context, Crossley and Lu (2018) revisit the food preparation argument, showing that larger households shift to more time-intensive food, which can help explain the puzzle when accounting for heterogeneity in time costs across food items.

Finally, Perali (2008) contends the "puzzle" is largely an artifact of specification. Specifically, conditioning on per-capita total expenditure reorders households—for instance, singles move to the top of the ordering—flipping the household-size effect. Using welfare-relevant equivalent income restores the

expected outward shift of food Engel curves with size, with similar patterns across several countries. These analyses underscore how central the mapping from size to needs and the functional forms used to measure Engel relationships are to inference.

Our approach complements this work by incorporating nonlinear pricing directly into the demand problem. When unit prices decline with the quantity purchased, economies of scale at the household level generate two opposing forces: larger households tend to demand more food per capita—through the standard income–substitution channel emphasized by Deaton and Paxson—but they also face lower unit prices. As a result, per-capita expenditure on food can fall even as per-capita consumption rises with household size, reproducing the empirical pattern without any logical contradiction. In what follows, we formalize these comparative statics results, provide sufficient conditions under which the Deaton–Paxson "puzzle" arises, and validate the mechanism we propose using Mexican data where quantity discounts for basic staples are empirically salient, as shown by Attanasio and Pastorino (2020). Indeed, Attanasio and Pastorino document the prevalence of nonlinear pricing for food staples using the same data we employ here, and show that the Progresa/Oportunidades cash transfer intensified these price nonlinearities. The present paper builds on that evidence by developing a theoretical framework that links nonlinear pricing to the DP puzzle and by providing systematic empirical tests of the proposed mechanism.

3 Consumption and Economics of Scale: A Puzzle?

In this section, we first derive the main result in DP on the relationship between the per-capita demand of food and household size, and then characterize the analogous relationship when food is priced non-linearly through quantity discounts. The key takeaway in DP is that larger households should display a *higher per-capita consumption of private goods*, such as food, provided that they do not substitute too much towards effectively cheaper public goods, such as housing. This result follows from an important implication of economies of scale in consumption at the household level for the per-capita demand of food: with total household expenditure per capita (PCE) constant, the demand of food should rise with household size, since food is a private necessity with usually small substitution elasticities.

The evidence that DP provide shows exactly the *opposite* pattern in the data, though. Specifically, at constant per-capita total expenditure, the per-capita demand of food in both rich and poor countries decreases with household size, and it does so the most in the poorest countries, in which substitution away from food should be precisely least likely. Hence, the so-called *DP puzzle*. We note that DP also derive an additional implication of the standard demand model. Namely, at constant PCE and household size,

the demand of food is smaller, the larger the proportion of children in a household. For this implication as well, DP rely on a unitary household model of consumption—an assumption we also maintain in what follows.[3] DP test both of these predictions of the standard model. Whereas the first implication is rejected fairly convincingly in almost all their datasets (see their Table 3), the evidence on the second one is more mixed (see their Table 4). In particular, this second prediction is confirmed in all their data from developed countries and only rejected in data from lower-income countries—namely, Pakistan, rural Thailand, and South Africa (Africans). We thus view the rejection of the first hypothesis as the outstanding DP puzzle and focus our analysis on it. In this precise sense, our approach is not only complementary to existing ones but also more parsimonious in that we show that the DP puzzle can be resolved within the standard unitary model used by DP.

3.1 Demand with Household Economies of Scale in Consumption

DP formalize the demand for private and public goods in the presence of household economies of scale in consumption using a simple version of the model first developed by Barten (1964). To start, suppose that there are two goods, one that is entirely private (food) and another one that is shared and entirely public (housing). A household with n members allocates its total outlay or expenditure x across these two goods. For simplicity, assume that there are only adults in the household. If q_f and q_h are the levels of household consumption of food an housing, respectively, and goods are divided equally, then the household utility function is $n\upsilon(q_f/n, q_h)$ because food, unlike housing, must be shared. The household budget constraint in per-capita terms is given by

$$p_f \left(\frac{q_f}{n}\right) + \left(\frac{p_h}{n}\right) q_h = \frac{x}{n}, \tag{1}$$

where we focus on the quantities that enter the utility function q_f/n and q_h, and their effective prices, p_f and q_f/n. To interpret this budget constraint, note that when two one-person households, identical in their total expenditure x, join to form one two-person household, neither resources per head, x/n, nor the price of the private good, p_f, change. However, the price of the public good is halved, which induces a *positive income effect* and a *negative substitution effect* on the demand of the private good, food. If the private good is a necessity, such as food is, then the substitution effect should be small, and its per-capita consumption will increase. Below, we show that the assumption that p_f is *constant* in the quantity demanded of food is crucial for the argument in DP.

[3]For work that has relaxed this assumption, see the survey in Fafchamps (2005). Gutierrez (2018) rationalizes the DP puzzle by modeling food as a common-pool resource within a collective household framework.

This basic insight extends to richer models of consumption. Suppose that we distinguish food and non-food ("housing"), that both are subject to some scale economies in consumption, and that utility is

$$u = nv\left(\frac{q_f}{\phi_f(n)}, \frac{q_h}{\phi_h(n)}\right). \tag{2}$$

where h is broadly interpreted to be non-food consumption, and the quantities are household *purchases*; $\phi_f(n)$ and $\phi_h(n)$ represent the economies of scale in food and non-food consumption, respectively.

Maximization of (2) subject to the budget constraint in (1) (see, for instance, Deaton and Muellbauer, 1980, p. 200) gives the following food demand function, which corresponds to equation (3) in DP,

$$\frac{q_f}{n} = \frac{\phi_f(n)}{n} g_f\left(\frac{x}{n}, p_f\frac{\phi_f(n)}{n}, p_h\frac{\phi_h(n)}{n}\right). \tag{3}$$

Now, observe that

$$\frac{d\ln\left(\frac{q_f}{n}\right)}{d\ln(n)} = \frac{d\left(\frac{q_f}{n}\right)}{dn}\frac{n^2}{q_f} = \frac{n\phi_f'(n)}{\phi_f(n)} - 1 + \frac{g_{f2}}{g_f}p_f\frac{\phi_f(n)}{n}\left[\frac{n\phi_f'(n)}{\phi_f(n)} - 1\right] + \frac{g_{f3}}{g_f}p_h\frac{\phi_h(n)}{n}\left[\frac{n\phi_h'(n)}{\phi_h(n)} - 1\right].$$

Defining the own-price elasticity of the demand of food as $\varepsilon_{ff} = dq_f/dp_f \cdot p_f/q_f$, the elasticity of the demand of food to the price of housing or, more generally, non-food as $\varepsilon_{fh} = dq_f/dp_h \cdot p_h/q_f$, and the economies of scale in the consumption of good $i = f, h$ as

$$\sigma_i \equiv 1 - \frac{d\ln\left(\phi_i(n)\right)}{d\ln(n)} = 1 - \frac{d\phi_i(n)}{dn}\frac{n}{\phi_i(n)} = 1 - \frac{n\phi_i'(n)}{\phi_i(n)}, \tag{4}$$

we obtain that

$$\frac{d\ln\left(\frac{q_f}{n}\right)}{d\ln(n)} = -\sigma_f - \varepsilon_{ff}\sigma_f - \varepsilon_{fh}\sigma_h = \sigma_h(\varepsilon_{fx} + \varepsilon_{ff}) - \sigma_f(1 + \varepsilon_{ff}), \tag{5}$$

where $\varepsilon_{fx} = dq_f/dx \cdot x/q_f$ is the income elasticity of the demand of food and the second equality in (5) follows from $\varepsilon_{fx} + \varepsilon_{ff} + \varepsilon_{fh} = 0$ by the homogeneity of degree zero of the demand of food, which allows us to substitute ε_{fh} out. As condition (5) makes clear, when food is a highly private good ($\sigma_f \approx 0$) and housing is a highly public good ($\sigma_h \approx 1$), which is the leading case in DP, (5) effectively reduces to $\varepsilon_{fx} + \varepsilon_{ff}$. Hence, whether or not the per-capita demand of food increases with household size, for given per-capita total expenditure, critically depends on the magnitude of the income elasticity of the demand of food (ε_{fx}) and its own price elasticity (ε_{ff}). The larger the income elasticity of the demand of food and the smaller its own price elasticity, the more the per-capita demand of food increases with household size. Since p_f does *not* vary with n or q_f, the *same* relationship holds between per-capita *expenditure* on food and household size. We formalize this result, which summarizes the theoretical structure of DP,

with the following lemma.

Lemma 1. *Suppose that food is a private good ($\sigma_f \approx 0$). When the price of food p_f does not depend on q_f, if ε_{fx} is sufficiently large and ε_{ff} is sufficiently small, $d\ln\left(\frac{q_f}{n}\right)/d\ln(n) > 0$ and $d\ln\left(p_f\frac{q_f}{n}\right)/d\ln(n) > 0$.*

3.2 Evidence on Household Size and Per-Capita Food Expenditure

Whether or not the per-capita demand of food increases with household size, that is, whether the elasticity in (5) is positive for fixed expenditure per capital x/n, is an empirical question, which involves objects whose magnitudes are likely to vary with the level of development of an economy. The general presumption is that in poor countries, the food share of household expenditure is high, the income elasticity of food ε_{fx} is similarly high, but the price elasticity of food ε_{ff} is low in absolute value. As discussed, if food is a pure private good ($\sigma_f = 0$), the expression in (5) is positive whenever the absolute value of the price elasticity of food is smaller than its income elasticity, which should be true for poor consumers, if not rich ones. Intuitively, if food is a necessity, has limited substitutes, and is a private good, then the substitution effect on the demand of food from an increase in household size should be small and so (5) should be positive. Violations of these conditions, if they occur, are more likely for wealthy households, for whom the income elasticity of the demand of food is presumably low.

The evidence reported by DP points to the opposite result, though, in that the coefficient on n is documented by DP to be robustly negative. In particular, DP estimate the relationship between per-capita expenditure on food and household size—for given total per-capita expenditure—in the United States, Great Britain, France, Taiwan, urban and rural Thailand, Pakistan, and South Africa (Africans), controlling for the logarithm of per-capita total expenditure, the ratio to household size of the numbers of household members by age and sex category, and a set of other controls, including household farm status, rural/urban status, region of residence, and the calendar quarter in which the survey was conducted. According to their preferred estimates—from lower income countries and their linear model specification—DP find that the elasticity of per-capita expenditure on food with respect to household size is -0.0548, -0.0565, and -0.0598 for rural Thailand, Pakistan, and South Africa (Africans), respectively.[4]

From this evidence, We draw a few observations that motivate our analysis. The theoretical hypothesis in DP that the per-capita demand of food increases with household size for given expenditure per capita, that is, $d\ln(q_f/n)/d\ln(n) > 0$ for fixed x/n, is predicated on equation (5), which DP never

[4]The Slutsky equation for food can be written as $\varepsilon_{ff} = \varepsilon^h_{ff} - \varepsilon_{fx}w_f$, where ε_{ff} is the (uncompensated) own-price elasticity, ε^h_{ff} is the compensated own-price elasticity, and w_f is the food share. DP report the values of ε^h_{ff} above which the per-capita demand of food increases with household size—see their equation (6)—which range from -.45 for rural Thailand to -.40 for Britain. By the Slutsky formula, they conclude that $\varepsilon_{ff} \geq -0.75$ in rural Thailand and $\varepsilon_{ff} \geq -0.50$ in Britain.

explicitly bring to data . Namely, although the theoretical argument in DP is developed in terms of the per-capita *demand* of food, they estimate Engel curves for food—namely, they focus on the per-capita *expenditure* on food—to test the hypothesis they consider. In general, such an approach is valid when the unit price of food is constant in the quantity demanded of it, as Lemma 1 summarized. However, it is no longer warranted when food items are priced nonlinearly because, for instance, of volume discounts. In this case, the unit price of food items decreases with the quantity demanded of them. Intuitively, in these circumstances, the *price* of food and the *quantity demanded* of it move in opposite directions in response to changes in household size. In particular, households who demand *more* food face *lower* unit prices for it. Therefore, the per-capita demand of food may well *increase* although the per-capita expenditure on it *decreases* with household size, as we show next.

4 A Proposed Resolution: The Nonlinear Pricing of Food

In this section, we show how the nonlinear pricing of food can in principle provide a solution to the DP puzzle. Namely, we allow for the presence of nonlinear pricing of food in the form of unit prices declining with quantity (quantity discounts), and characterize in this case the effect of family size first on the *expenditure* on food and then on the *quantity demanded* of it. As mentioned, when food is priced linearly, this distinction is unnecessary, as the two effects must have the same sign. This equivalence no longer holds under nonlinear pricing.

4.1 The Impact of Economies of Scale on Expenditure

We first demonstrate that when food is priced nonlinearly through volume discounts ($p'_f(q_f) < 0$), the response of *per-capita expenditure* on food to changes in household size is in general quite different from the response of the *per-capita demand* of food, relative to the linear pricing case considered by DP and discussed above. Now, when food is priced at volume discounts, its unit price is a schedule and not a scalar, which declines with the quantity of food purchased. Let then $\varepsilon_{p_f(q_f)q_f} = p'_f(q_f)q_f/p_f(q_f) < 0$ denote the elasticity of the schedule of the unit prices of food, $p_f(q_f)$, with respect to the quantity purchased of it, q_f, which is negative in the presence of quantity discounts, as larger quantities are sold at lower unit prices, hence $= p'_f(q_f) < 0$. Next, observe that

$$\frac{d\left(\frac{q_f}{n}\right)}{dn} = \frac{\left(\frac{dq_f}{dn}n - q_f\right)}{n^2} = \frac{dq_f}{dn}\frac{1}{n} - \frac{q_f}{n^2} \quad \text{and} \quad \frac{d\left(\frac{q_f}{n}\right)}{dn} = \frac{d\ln\left(\frac{q_f}{n}\right)}{d\ln(n)}\frac{q_f}{n^2}.$$

Using the first equality in the equation above to obtain an expression for dq_f/dn, substituting for $d(\frac{q_f}{n})/dn$ using the second equality in the above, and simplifying terms, we obtain that

$$\frac{dq_f}{dn} = \left[\frac{d\left(\frac{q_f}{n}\right)}{dn} + \frac{q_f}{n^2}\right] n = \left[1 + \frac{d\ln\left(\frac{q_f}{n}\right)}{d\ln(n)}\right] \frac{q_f}{n}.$$

Based on these observations, it can be seen that

$$\frac{d\ln\left(p_f(q_f)\frac{q_f}{n}\right)}{d\ln(n)} = \frac{d\ln\left(p_f\left(q_f\right)\right)}{d\ln(n)} + \frac{d\ln\left(\frac{q_f}{n}\right)}{d\ln(n)} = \underbrace{\overbrace{\frac{dp_f\left(q_f\right)}{dq_f}}^{<0}\overbrace{\frac{dq_f}{dn}}^{>0}\frac{n}{p_f(q_f)}}_{\text{price effect } <0} + \overbrace{\frac{d\ln\left(\frac{q_f}{n}\right)}{d\ln(n)}}^{\text{quantity effect}}$$

$$= p_f'(q_f)\left[1 + \frac{d\ln\left(\frac{q_f}{n}\right)}{d\ln(n)}\right]\frac{q_f}{n}\frac{n}{p_f(q_f)} + \frac{d\ln\left(\frac{q_f}{n}\right)}{d\ln(n)} = \frac{p_f'(q_f)q_f}{p_f(q_f)} + \left[1 + \frac{p_f'(q_f)q_f}{p_f(q_f)}\right]\frac{d\ln\left(\frac{q_f}{n}\right)}{d\ln(n)}. \tag{6}$$

Since, as discussed, $\varepsilon_{p_f(q_f)q_f}$ is negative, we can rewrite the last equality in the above as

$$\frac{d\ln\left(p_f(q_f)\frac{q_f}{n}\right)}{d\ln(n)} = -\left|\varepsilon_{p_f(q_f)q_f}\right| + \left[1 - \left|\varepsilon_{p_f(q_f)q_f}\right|\right]\frac{d\ln\left(\frac{q_f}{n}\right)}{d\ln(n)}. \tag{7}$$

When, as in DP, $p_f(q_f)$ does not depend on q_f, $p_f'(q_f)$ is zero. In this case, $d\ln\left(p_f(q_f)\frac{q_f}{n}\right)/d\ln(n)$ is equal to $d\ln\left(\frac{q_f}{n}\right)/d\ln(n)$, whose expression is given in equation (5). Hence, under linear pricing, the response of per-capita expenditure on food to changes in household size has the *same sign* as the response of the per-capita demand of food to changes in household size. This equivalence no longer holds when food is priced nonlinearly, though. Specifically, equation (7) shows that the relationship between household size and per-capita expenditure on food can very well be *negative*, even though $d\ln\left(q_f/n\right)/d\ln(n)$ is *positive*. In particular, it is immediate from (7) that when the nonlinearity of the unit price of food is especially pronounced—in that the elasticity of the unit price of food to the quantity demanded of it is close to one in absolute value—the per-capita expenditure on food *decreases* as household size *increases*.

Proposition 1. *When the price of food is nonlinear in the quantity demanded of it due to volume discounts* ($p_f'(q_f) < 0$)*,* $d\ln\left(p_f(q_f)\frac{q_f}{n}\right)/d\ln(n) < 0$ *if* $\left|\varepsilon_{p_f(q_f)q_f}\right|$ *is close enough to one.*

For later, note that since $T_f(q_f) = p_f(q_f)q_f$, it follows that $T_f'(q_f) = p_f'(q_f)q_f + p_f(q_f)$ and $T_f''(q_f) = p_f''(q_f)q_f + 2p_f'(q_f)$. Thus, the absolute value of $d\ln\left(p_f(q_f)\right)/d\ln(q_f)$, namely, $\left|\varepsilon_{p_f(q_f)q_f}\right|$, *cannot* be greater than 1, otherwise $T_f'(q_f)$ would be negative. But given that total expenditure on food must increase as the quantity purchased of it increases, it must be that $T_f'(q_f) > 0$ and so $\left|\varepsilon_{p_f(q_f)q_f}\right|$ is smaller than 1. We show below that when the pricing of food is highly nonlinear ($\left|\varepsilon_{p_f(q_f)q_f}\right| \approx 1$), as household

size increases, the per-capita quantity demanded of food tends to increase, as DP hypothesized, but the per-capita expenditure on food decreases, as DP measured in the data.[5] Hence, no puzzle arises.

4.2 The Impact of Economies of Scale on Consumption

We now characterize the response of the per-capita demand of food to changes in household size when the unit price of food, $p_f(q_f)$, declines with the quantity demanded of it, q_f. To motivate the form of demand in this case, we start with the case of a constant-elasticity-of-substitution (CES) utility function for households. We consider CES utility for various reasons. It can be thought of as a one-parameter approximation to a general homothetic utility function, which conveniently captures local consumption behavior through a single curvature parameter σ representing the elasticity of substitution between the two commodities we consider (food and non-food). Furthermore, its share weights can be easily modified and interpreted to capture the economies of scale in consumption of interest here. Finally, it is one of the most commonly used utility functions in applied contexts. We start by deriving the per-capita demand of food, which motivates the extension of (3) to the case in which food is priced nonlinearly.

Per-Capita Food Demand. Denote total food expenditure by $T_f(q_f) = p_f(q_f)q_f$. Households maximize

$$u = \; n\left[\alpha_f\left(\frac{q_f}{\phi_f(n)}\right)^{\frac{\rho-1}{\rho}} + \alpha_h\left(\frac{q_h}{\phi_h(n)}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}, \tag{8}$$

$$\text{s.t.} \qquad x = T_f(q_f) + p_h q_h.$$

In this case, the per-capita demand of food is given by

$$\frac{q_f}{n} = \frac{\phi_f(n)}{n}\frac{\frac{x}{n}}{\left\{p_f(q_f)\frac{\phi_f(n)}{n} + \left(p_h\frac{\phi_h(n)}{n}\right)^{1-\rho}\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\frac{\phi_f(n)}{n}\right]^{\rho}\right\}}, \tag{9}$$

which depends on both the average food price $p_f(q_f)$ and the marginal food price $T_f'(q_f)$ evaluated at the total quantity of food demanded by the family. The result in equation (9) is an immediate implication of the household budget constraint being a function of the unit price of food $p_f(q_f)$ and the marginal quantity demanded of food being a function of the marginal price $T_f'(q_f)$. Only when $\rho \approx 0$, which is the pure Leontief case of maximal complementarity between food and non-food consumption, the per-capita demand of food does not depend on the marginal price $T_f'(q_f)$. Intuitively, in this case, the demand for the two goods moves in "lockstep" as the price of either good changes, namely, it is not affected by the

[5] We implicitly maintain that $d\ln(q_f/n)/d\ln(n)$ is bounded for this result to hold. We show below this is indeed the case.

intensity of price discrimination as captured by how quickly marginal expenditure on food changes as the demand for it increases. This setup will provide the leading example in later sections for the situations of interest in which food and non-food are complementary goods.

More generally, equation (9) is an instance of demand when food is priced nonlinearly, motivating the following extension of the canonical Deaton-Paxson demand structure in (3) to

$$\frac{q_f}{n} = \frac{\phi_f(n)}{n} g_f \left(\frac{x}{n}, p_f(q_f)\frac{\phi_f(n)}{n}, T'_f(q_f)\frac{\phi_f(n)}{n}, p_h\frac{\phi_h(n)}{n} \right), \tag{10}$$

when household utility is as in (2) and the household budget constraint is

$$p_f(q_f)\left(\frac{q_f}{n}\right) + \left(\frac{p_h}{n}\right) q_h = \frac{x}{n}, \tag{11}$$

with the unit price of food $p_f(q_f)$ function of q_f. These considerations lead us to the following lemma.

Lemma 2. *When the price of food is nonlinear in the quantity demanded of it due to volume discounts ($p'_f(q_f) < 0$), the per-capita demand of food in general is a function of both its average price $p_f(q_f)$ and its marginal price $T'_f(q_f)$.*

Proof: To convey the logic of the argument most transparently, we purposely keep formalism at a minimum. Consider the profit-maximization problem of a seller facing a population of households (consumers) who differ in their preference for food, say,

$$u = n\upsilon(q_f/\phi_f(n), q_h/\phi_h(n), \theta) = n\tilde{\upsilon}(q_f/\phi_f(n), q_h/\phi_h(n)), \tag{12}$$

where θ is a non-contractible preference parameter that affects household demand of food. Since household demand is derived from the household budget constraint $T_f(q_f) + p_h q_h = x$ in (11), it must depend on the unit price of food $p_f(q_f)$, which determines a consumer total expenditure on food $T_f(q_f) = p_f(q_f)q_f$. Now, the problem of a seller pricing food nonlinearly incorporates as constraint households' incentive-compatibility constraint for the optimal choice of the quantity of food to purchase. Namely, for a schedule $T_f(q_f)$ to be optimal for a seller to offer, it must be (in differential form) that $du/dq_f = T'_f(q_f)$. Then, q_f must depend on $T'_f(q_f)$ as well. □

Differently from the demand structure in DP in (3), the per-capita demand of food in general depends on both the unit price of food $p_f(q_f)$, which changes with the quantity demanded of it q_f, and its marginal price $T'_f(q_f)$, which captures the nonlinearity of the price schedule of food for additional marginal quantities demanded—its negative slope $T''_f(q_f)$ is a common measure of the magnitude of quantity discounts in the nonlinear pricing literature. Under linear pricing, the term $T'_f(q_f)$ is zero and (10) reduces to (3).

Per-Capita Demand for Food and Household Size. We now explore the impact of household size on the per-capita demand of food under nonlinear pricing. Suppose that demand takes the form in (10) and recall that $d\ln\left(\frac{q_f}{n}\right)/d\ln(n) = d\left(\frac{q_f}{n}\right)/dn \cdot n^2/q_f$. Let

$$\varepsilon_{fx} = \frac{dq_f}{dx}\frac{x}{q_f} = \phi_f(n)g_{f_1}\frac{1}{n}\frac{x}{\phi_f(n)g_f} = \frac{g_{f_1}}{g_f}\frac{x}{n} > 0,$$

$$\varepsilon_{ff} = \frac{dq_f}{dp_f(q_f)}\frac{p_f(q_f)}{q_f} = \phi_f(n)g_{f_2}\frac{\phi_f(n)}{n}\frac{p_f(q_f)}{\phi_f(n)g_f} = \frac{g_{f_2}}{g_f}\frac{\phi_f(n)}{n}p_f(q_f) < 0,$$

$$\varepsilon_{fT'_f(q_f)} = \frac{dq_f}{dT'_f(q_f)}\frac{T'_f(q_f)}{q_f} = \phi_f(n)g_{f_3}\frac{\phi_f(n)}{n}\frac{T'_f(q_f)}{\phi_f(n)g_f} = \frac{g_{f_3}}{g_f}\frac{\phi_f(n)}{n}T'_f(q_f) < 0,$$

and

$$\varepsilon_{fh} = \frac{dq_f}{dp_h}\frac{p_h}{q_f} = \phi_f(n)g_{f4}\frac{\phi_h(n)}{n}\frac{p_h}{\phi_f(n)g_f} = \frac{g_{f_4}}{g_f}p_h\frac{\phi_h(n)}{n}.$$

Then, using that $q_f = \phi_f(n)g_f(\cdot) = \phi_f(n)g_f$ by the form of demand and exploiting the homogeneity of demand in that $\varepsilon_{fx} + \varepsilon_{ff} + \varepsilon_{fT'_f(q_f)} + \varepsilon_{fh} = 0$, for the leading case in DP in which $\sigma_f \simeq 0$, we arrive at[6]

$$\frac{d\ln\left(\frac{q_f}{n}\right)}{d\ln(n)} = \frac{\sigma_h\left[\frac{g_{f1}}{g_f}\frac{x}{n} + \frac{g_{f2}}{g_f}p_f(q_f)\frac{\phi_f(n)}{n} + \frac{g_{f3}}{g_f}T'_f(q_f)\frac{\phi_f(n)}{n}\right] + \phi_f(n)\left[\frac{g_{f2}}{g_f}p'_f(q_f) + \frac{g_{f3}}{g_f}T''_f(q_f)\right]\frac{q_f}{n}}{1 - \phi_f(n)\left[\frac{g_{f2}}{g_f}p'_f(q_f) + \frac{g_{f3}}{g_f}T''_f(q_f)\right]\frac{q_f}{n}} \tag{13}$$

maintaining that $d(x/n)/dn = 0$. Further making use of

$$\varepsilon_{p_f(q_f)q_f} = \frac{dp_f(q_f)}{dq_f}\frac{q_f}{p_f(q_f)} = \frac{p'_f(q_f)q_f}{p_f(q_f)} < 0 \quad \text{and} \quad \varepsilon_{T'_f(q_f)q_f} = \frac{dT'_f(q_f)}{dq_f}\frac{q_f}{T'_f(q_f)} = \frac{T''_f(q_f)q_f}{T'_f(q_f)} < 0,$$

we obtain a generalization of the expression in (5) when food is priced nonlinearly.

Proposition 2. *When the price of food is nonlinear in the quantity demanded of it due to volume discounts ($p'_f(q_f) < 0$), the per-capita demand of food increases with household size for given per-capita total expenditure on public ($\sigma_h \approx 1$) and private ($\sigma_f \approx 0$) goods if, and only if,*

$$\frac{d\ln\left(\frac{q_f}{n}\right)}{d\ln(n)} = \frac{\varepsilon_{fx} + \varepsilon_{ff} + \varepsilon_{fT'_f(q_f)} + \left[\varepsilon_{ff}\varepsilon_{p_f(q_f)q_f} + \varepsilon_{fT'_f(q_f)}\varepsilon_{T'_f(q_f)q_f}\right]}{1 - \left[\varepsilon_{ff}\varepsilon_{p_f(q_f)q_f} + \varepsilon_{fT'_f(q_f)}\varepsilon_{T'_f(q_f)q_f}\right]} > 0. \tag{14}$$

Note that the expression in (14) reduces to the corresponding expression (5) in DP when $p_f(q_f)$ is independent of q_f so that $T'_f(q_f) = 0$, namely, when food is priced linearly. Recall that a pure private (excludable) good has $\sigma_i = 0$ and a pure public good has $\sigma_i = 1$. It is immediate that for not too large values of the expression $\varepsilon_{ff}\varepsilon_{p_f(q_f)q_f} + \varepsilon_{fT'_f(q_f)}\varepsilon_{T'_f(q_f)q_f}$, which is positive, in the numerator and

[6]The derivatives in ε_{ff} and $\varepsilon_{fT'_f(q_f)}$ involve specific perturbations of more general Gateaux derivatives, in which the average and marginal price schedules are perturbed by an infinitesimal amount. See the proof of Proposition 2 for details.

denominator of (32), the expression for $d\ln(q_f/n)/d\ln(n)$ is positive and *larger* than when food is linearly priced. In this case, the nonlinear pricing of food reinforces the impact of economies of scale in consumption, thus amplifying the positive effect of household size on the per-capita demand of food. In the next subsection, we show that in a setting with CES utility, this is indeed typically the case.

By combining Propositions 1 and 2, it is immediate to characterize situations in which the response of the per-capita expenditure on food to increases in household size is *negative* in that $d\ln\left(p_f(q_f)\frac{q_f}{n}\right)/d\ln(n) < 0$, as DP have estimated, yet the response of the per-capita demand of food to increases in household size is *positive* in that $d\ln\left(\frac{q_f}{n}\right)/d\ln(n) > 0$, as DP conjectured. We term this configuration of responses—seemingly conflicting from the viewpoint of a linear pricing framework—the *resolution of the DP puzzle*. In any such instance, household economies of scale in consumption imply that the per-capita demand of food indeed increases with household size. The per-capita expenditure on food, however, displays the opposite pattern. It decreases with household size, since households face much lower unit prices for the greater quantity of food they demand as their size expands.

Corollary 1. *When the price of food is nonlinear in the quantity demanded of it due to volume discounts ($p'_f(q_f) < 0$), $d\ln\left(p_f(q_f)\frac{q_f}{n}\right)/d\ln(n) < 0$ and yet $d\ln(\frac{q_f}{n})/d\ln(n) > 0$ if the income elasticity of the demand of food, ε_{fx}, is large enough and the elasticity of the unit price schedule in absolute value, $\left|\varepsilon_{p_f(q_f)q_f}\right|$, is close enough to one.*

4.3 The Canonical Case of Constant-Elasticity Utility

We now show that when food and non-food goods are complements and the nonlinearity of the price of food is pronounced, the per-capita expenditure on food *decreases* with household size, $d\ln\left(p_f(q_f)\frac{q_f}{n}\right)/d\ln(n) < 0$, whereas the per-capita demand of food *increases* with household size, $d\ln\left(\frac{q_f}{n}\right)/d\ln(n) > 0$. To be able to isolate the specific role of the preferences for food and non-food goods for this result, we show that this is indeed the case when utility is of the CES form, so that the complementarity between private and public goods is conveniently summarize by just one parameter, ρ. We proceed by first deriving an exact result on the response of the per-capita demand of food to increases in household size n and then characterizing sharp bounds on this response.

CES Food Demand and Household Size. As shown in the appendix, from equation (9), it follows that

$$\frac{d\ln\left(\frac{q_f}{n}\right)}{d\ln(n)} = \frac{\sigma_h\left\{1-\frac{p_f(q_f)+\rho\left(p_h\frac{\phi_h(n)}{\phi_f(n)}\right)^{1-\rho}\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\right]^{\rho}}{p_f(q_f)+\left(p_h\frac{\phi_h(n)}{\phi_f(n)}\right)^{1-\rho}\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\right]^{\rho}}\right\}-\frac{p_f'(q_f)+\left(p_h\frac{\phi_h(n)}{\phi_f(n)}\right)^{1-\rho}\rho\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\right]^{\rho-1}\frac{\alpha_h}{\alpha_f}T_f''(q_f)}{p_f(q_f)+\left(p_h\frac{\phi_h(n)}{\phi_f(n)}\right)^{1-\rho}\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\right]^{\rho}}q_f}{1+\frac{p_f'(q_f)+\left(p_h\frac{\phi_h(n)}{\phi_f(n)}\right)^{1-\rho}\rho\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\right]^{\rho-1}\frac{\alpha_h}{\alpha_f}T_f''(q_f)}{p_f(q_f)+\left(p_h\frac{\phi_h(n)}{\phi_f(n)}\right)^{1-\rho}\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\right]^{\rho}}q_f}. \tag{15}$$

The last term in the numerator of (15) is negative since $p_f'(q_f) < 0$ and $T_f''(q_f) < 0$. Note that the same term, but with an opposite sign, appears in the denominator of (15). We now argue that $d\ln(q_f/n)/d\ln(n) > 0$, as DP conjectured, under intuitive conditions.

To begin, note that $d\ln(q_f/n)/d\ln(n)$ in (15) is clearly positive when both the numerator and the denominator of it are positive. As for the the numerator of (15), since the last term in it is negative and it is subtracted from the first term $\sigma_h\{\cdot\}$, a sufficient condition for the numerator of (15) to be positive is that this first term is positive or, equivalently, as long as housing is a public good ($\sigma_h > 0$), that

$$(1-\rho)\left(p_h\frac{\phi_h(n)}{\phi_f(n)}\right)^{1-\rho}\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\right]^{\rho} > 0,$$

which reduces to $\rho < 1$ as $T_f'(q_f) > 0$—namely, the requirement that food and non-food be complementary goods. Now, the condition that the denominator of (15) is positive can be expressed as

$$\frac{p_f'(q_f)q_f+\left(p_h\frac{\phi_h(n)}{\phi_f(n)}\right)^{1-\rho}\rho\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\right]^{\rho-1}\frac{\alpha_h}{\alpha_f}T_f''(q_f)q_f}{p_f(q_f)+\left(p_h\frac{\phi_h(n)}{\phi_f(n)}\right)^{1-\rho}\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\right]^{\rho}} > -1. \tag{16}$$

After straightforward manipulations, it is easy to see that this condition reduces to

$$\left[T_f'(q_f)\right]^{1-\rho}+\left(p_h\frac{\phi_h(n)}{\phi_f(n)}\right)^{1-\rho}\left(\frac{\alpha_h}{\alpha_f}\right)^{\rho}\left[1+\rho\frac{T_f''(q_f)q_f}{T_f'(q_f)}\right] > 0. \tag{17}$$

Since $T_f'(q_f)$ is positive and $T_f''(q_f)$ is negative, it is immediate that (17) is satisfied for any given ρ if q_f is small enough, given that the remaining terms on the left side of (17) are all positive.

Proposition 3. *Assume that $\sigma_h > \sigma_f = 0$. For any given degree of complementarity in preferences $\rho \in (0,1)$ for food and non-food goods, there exists an upper bound $\bar{q}(\rho)$ such that for any quantity demanded of food q_f below $\bar{q}(\rho)$, no DP puzzle arises in that $d\ln(p_f(q_f)q_f/n)/d\ln(n) < 0$ and $d\ln(q_f/n)/d\ln(n) > 0$ if the schedule of unit prices of food is close to unit elastic ($|p_f'(q_f)|q_f/p_f(q_f) \to 1$).*

The conditions of Proposition 3 are simple and intuitive. The requirement that the elasticity of sub-

stitution between food and non-food goods ρ be smaller than 1 implies that the two are complementary goods. In this case, the premise of the argument of DP naturally applies, namely, a decrease in the price of non-food goods resulting from an increase in household size has a small substitution effect on the per-capita demand of food but a large income effect on it. Hence, in the DP case, the per-capita demand of food increases. Next, condition $|p_f'(q_f)|q_f/p_f(q_f) \approx 1$ concerns the intensity of price discrimination in the market for food and simply requires the nonlinearity of the prices of food to be strong enough. In this case, food per unit becomes cheaper fast at higher quantities demanded, thus reinforcing the income effect of an increase in household size on the per-capita demand of food just discussed. To summarize, the nonlinear pricing of food amplifies the impact of economies of scale in consumption at the household level because it implies that larger households, who tend to consume more food overall, face lower unit prices for the quantities of food they purchase. This price effect strengthens the positive (income) effect of household size on food consumption.

When non-food is a pure public good ($\sigma_h = 1$), $p_h \approx \phi_f(n)/\phi_h(n)$, and utility is symmetric in food and non-food consumption ($\alpha_h/\alpha_f \approx 1$), the expression in (15) becomes

$$\frac{d\ln\left(\frac{q_f}{n}\right)}{d\ln(n)} = \frac{1 - \frac{p_f(q_f)+\rho\left[T_f'(q_f)\right]^{\rho}}{p_f(q_f)+\left[T_f'(q_f)\right]^{\rho}} - \frac{p_f'(q_f)+\rho\left[T_f'(q_f)\right]^{\rho-1}T_f''(q_f)}{p_f(q_f)+\left[T_f'(q_f)\right]^{\rho}}q_f}{1 + \frac{p_f'(q_f)+\rho\left[T_f'(q_f)\right]^{\rho-1}T_f''(q_f)}{p_f(q_f)+\left[T_f'(q_f)\right]^{\rho}}q_f}.$$

If, in addition $\rho \approx 1$, it follows that the above simplifies to

$$\frac{d\ln\left(\frac{q_f}{n}\right)}{d\ln(n)} = \frac{-\left[p_f'(q_f) + T_f''(q_f)\right]q_f}{p_f(q_f) + T_f'(q_f) + \left[p_f'(q_f) + T_f''(q_f)\right]q_f}. \tag{18}$$

In order to interpret these restrictions, observe that an implication of the form of utility in (12) is that the marginal rate of substitution between food and non-food consumption satisfies $\tilde{v}_f(\cdot)/\tilde{v}_h(\cdot) = T_f'(q_f)\phi_f(n)/[p_h\phi_h(n)]$. If food is purely a private good so that $\phi_f(n) = n$ and $\phi_h(n)$ is approximated as $\phi_h(n) \approx n^{\phi}$ with $0 < \phi < 1$ as in Crossley and Lu (2018), then the normalization $p_h\phi_h(n)/\phi_f(n) = p_h/n^{1-\phi} \approx 1$, with h the numeraire good ($p_h = 1$), is equivalent to evaluating the elasticity of the per-capita demand of food with respect to household size at $n = 1$. Changing the value of n at which the elasticity is evaluated simply amounts to changing the value to which the price of the numeraire is normalized. In practice, in this case, the ratio of food versus non-food consumption is pinned down by the nonlinear marginal price schedule of food $T_f'(q_f)$ alone.[7]

[7]We proceed in this way because there are few credible structural estimates of the functions $\phi_f(n)$ and $\phi_h(n)$ in the literature. As for the values of σ_f and σ_h, using 1960s and 1970s CEX data, Nelson (1988) finds that shelter exhibits by far the largest economies of scale among consumption categories, confirming the qualitative ranking $\sigma_h > \sigma_f$ that underlies DP

Now, the numerator of this expression is positive when food is sold at quantity discounts ($p'_f(q_f) < 0$ and $T''_f(q_f) < 0$). Thus, whether or not $d\ln(q_f/n)/d\ln(n)$ is positive as DP conjectured depends on the sign of the denominator of (18), which is positive if $T''_f(q_f)q_f/T'_f(q_f) < 1$. Note that the latter condition is stronger than the sufficient condition for (17) to hold invoked in the statement of Proposition 3, namely, the requirement that $q_f < \bar{q}(\rho)$. As discussed in the next section, condition (18) is nonetheless satisfied by the prices of the most important food items in our data—rice, beans, and sugar.

A Bounding Approach. Assume now that the unit price schedule for food can be described by $\ln p_f(q_f) = \beta_0 + \beta_1 \ln(q_f)$, with $\beta_1 < 0$, which is a common specification in the nonlinear pricing literature, so that it has a constant elasticity of β_1 with respect to q_f. When the schedule of unit prices for food is of this form as we estimate in our data—see the next section—it can be shown that the response of the per-capita demand of food to increases in household size admits simple lower and upper bounds,

$$\frac{d\ln\left(\frac{q_f}{n}\right)}{d\ln(n)} \in \left(\min\left\{\frac{|\beta_1|}{1-|\beta_1|}, \frac{1-\rho+|\beta_1|\rho}{1-|\beta_1|\rho}\right\}, \max\left\{\frac{|\beta_1|}{1-|\beta_1|}, \frac{1-\rho+|\beta_1|\rho}{1-|\beta_1|\rho}\right\}\right). \tag{19}$$

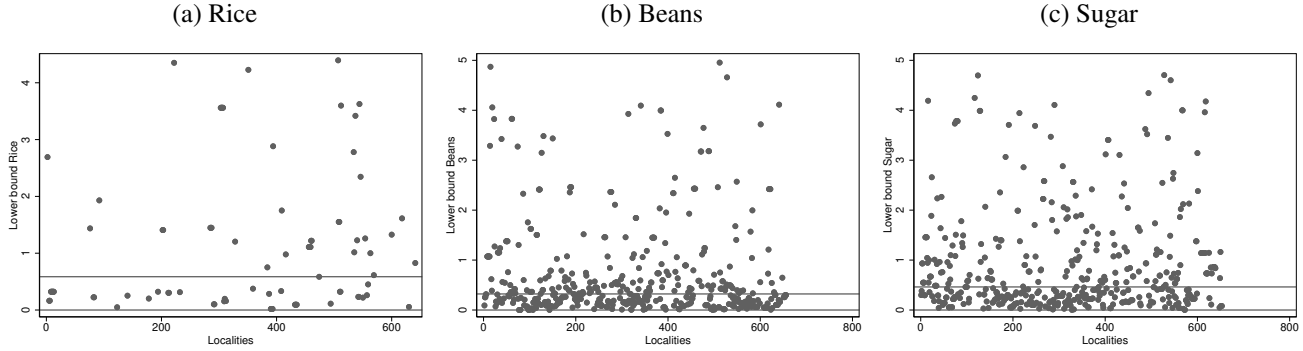
Note that whether food and non-food are weak ($\rho = 1$) or strong ($\rho = 0$) complements, the lower bound to the elasticity of interest in (19) is strictly positive in this case.[8] Hence, as conjectured by DP, the per-capita demand of food *always* increases with household size.

Proposition 4. *For any given degree of complementarity in preferences $\rho \in (0,1)$ for food and non-food goods, the lowest value that the elasticity $d\ln\left(\frac{q_f}{n}\right)/d\ln(n)$ can take is strictly positive when the unit price schedule is log-linear, utility is of the CES form, non-food is a pure public good ($\rho_h = 1$), $p_h \approx \phi_f(n)/\phi_h(n)$, and utility is symmetric in food and non-food consumption ($\alpha_h/\alpha_f \approx 1$).*

An Empirical Validation. Figure 2 considers the three main staples in our data, namely, rice, beans, and sugar, whose prices sharply decline with the quantity demanded of them. The figure shows that the lower bound of the response of the per-capita demand of these staples to increases in household size, as defined in eq. (19), is positive in our data; for details on the estimation of these price schedules, see Section 5.6. We interpret this evidence as the starting point of our empirical analysis of the conjecture that the nonlinearity of the price of food items can help reconcile the DP puzzle, which we now turn to. We will demonstrate that this is precisely the case.

and our analysis. Crossley and Lu (2018) introduce returns-to-scale parameters θ for the consumption of non-food goods and

Figure 2: Lower Bound of Response of Per-Capita Food Consumption to Household Size



Note: Each dot represents the lower bound of the response of per-capita consumption to household size as in (19) in a given locality for the first decile of the quantities purchased in each locality, estimated from a log-linear unit price schedule in each locality (see below for more details on the price schedule estimation). Distributions are top trimmed at the 5%. Horizontal lines are at the median of each distribution across localities.

5 The Consumption Puzzle: Evidence and Proposed Resolution

This section documents the consumption puzzle identified by DP, who estimate Engel curves for food for multiple countries. They find that per-capita food expenditure shares decrease with household size at fixed per-capita total expenditure, despite the plausible existence of economies of scale in consumption. In this section, we revisit this puzzle using data from Progresa/Oportunidades, validate the critical role of nonlinear pricing for the empirical nexus between expenditure on food, food demand, and economies of scale in consumption, and finally demonstrate how the nonlinear pricing of food can resolve the puzzle.

5.1 Traditional Measurement Exercise

The main estimating equation in DP, from which the famous puzzle originates, is the following:

$$w_f = \alpha + \beta \ln\left(\frac{x}{n}\right) + \gamma \ln(n) + \sum_{k=1}^{K-1} \eta_k \frac{n_k}{n} + \zeta \cdot \mathbf{v} + u, \tag{20}$$

where $w_f = p_f q_f / x$ is the budget share of food as well as the ratio of per-capita food expenditure to per-capita total expenditure, and n_k/n is the ratio of the number of household members who belong to one of K age-sex groups to total household size. The vector $\mathbf{v}$ includes a variety of controls and, for all countries considered, measures of the fraction of adults in the household who work.[9] Specifically,

γ for food preparation, with $0 < \theta, \gamma < 1$, but do not provide structural estimates of these parameters.

[8] By the proof of Proposition 3, it is immediate that $d\ln(\frac{q_f}{n})/d\ln(n)$ is positive for low q_f, for which volume discounts tend to be more pervasive in theory (because of the convexity of preferences) and in practice (in our data). The log-linearity of the unit price schedule prevents the ratio $T_f''(q_f)q_f/T_f'(q_f)$ from growing large in absolute value too fast relative to $T_f'(q_f)$, which explains why the sign of the lower bound on $d\ln(\frac{q_f}{n})/d\ln(n)$ is determinate.

[9] Households with more working members could have different food budget shares for several reasons. Workers may eat more of their meals in restaurants, and expenditure on restaurant meals includes a service component that may increase the

DP control for farm status, rural/urban status, region of residence, and, where appropriate, the calendar quarter in which the survey was conducted. The specification in (20) is designed to separate the effects of household size, n, from household composition, as represented by the ratio n_k/n, on (relative) food expenditure. The parameter γ thus captures the conceptual experiment of making a household larger by replication of both people and resources and measuring the effect of this change on the budget share of food. Recall that x/n *is kept fixed* in all exercises.

Differently from DP, in our case, the budget share of food is given by

$$w_f(q_f) = \frac{p_f(q_f)q_f}{x} = \frac{p_f(q_f)q_f}{n} \cdot \frac{n}{x}, \tag{21}$$

from which it is apparent that movements in q_f/n are not equivalent to movements in $w_f(q_f)$, even for given per-capita total expenditure x/n, since $p_f(q_f)q_f/n$ and q_f/n can move in opposite directions.

DP estimate equation (20) by OLS using household budget surveys from seven countries spanning developed and developing economies—namely, United States, Britain, France, Taiwan, Thailand, Pakistan, and South Africa.[10] In our analysis, we follow the parametric approach in DP but additionally exploit village-level variation in price schedules, when food is priced nonlinearly, to identify the role of nonlinear pricing in accounting for the DP puzzle. Specifically, we estimate the main DP regression by OLS in each survey wave, controlling for a similar set of controls as in DP; see more below when we discuss (22), in addition to village effects. We also estimate the DP regression for an additional specification that exploits the panel of households for the Oct-98, Mar-99, and Nov-99 survey waves and includes wave and village effects. Similar to DP, we also present IV estimates obtained by instrumenting (log) PCE with (log) total income and find similar results.

5.2 Data

The data set we use was collected to evaluate the impact of a large and well-known Mexican conditional cash transfer program Progresa/Oportunidades started in 1997. The program provided cash transfers to eligible families with children conditional on behavior such as class attendance by school-aged children, regular visits to health-care centers by young children, and attendance of education sessions on nutrition

food share. On the other hand, the increased commuting expenses associated with work could affect the food share in the opposite direction. Changes in caloric requirements due to the intensity of work must also be accounted for, especially in poor countries in which agricultural employment is common. For the poorer countries, DP include controls for the fractions of adults who work in a variety of activities, including work on a family farm and work as agricultural wage labor.

[10]To address potential measurement error and endogeneity in per-capita expenditure, DP also present IV estimates using log per capita income as an instrument for log PCE. Their IV results are generally similar to OLS, and in some countries—Pakistan and South Africa—yield even more negative estimates of γ. DP also employ non- and semi-parametric approaches to examine the sensitivity of their results to the parametric specification. All approaches yield similar conclusions.

and health by mothers (see Skoufias, 2005).

To evaluate the impact of the program, 506 localities in 7 states were selected, of which 320 were randomly chosen and assigned to early treatment in that the program started in the middle of 1998, and the remaining 186 were assigned to the end of the rollout phase, so the program started in these localities in December 1999. Households in these localities were followed for several periods. In 2003, an additional 150 localities—which brings the total sample of localities to 656—were added to the evaluation sample to create a new control group, since all original evaluation localities were treated by year 2000.

In our analysis, we use the surveys of October 1998, March and November 1999, November 2000, and 2003. We could not use some waves, such as those collected in 1997 and March 2000, because they do not contain information on household expenditure, which we rely on. For most of our empirical analysis, We focus on the sample of control villages for estimating the DP-related regressions in order to abstract from the effects of the program on consumption and expenditure, and on the whole sample of treatment and control villages to estimate price slopes at locality level, so as to have a large enough number of localities.[11] The communities included in the evaluation surveys are small—the average number of households in a locality is just over 50—and remote. Households living in these villages are poor, and food accounts for a substantial share of their consumption. However, not all households within targeted villages were eligible. Eligibility for the program was determined on the basis of a survey that collected information on a set of poverty indicators considered difficult to manipulate, such as the material of the roof or floor of a household's home. On average, about 78% of the households of the villages in the evaluation surveys were considered eligible. The level of poverty of these communities in the evaluation surveys exhibits substantial variation not only within villages but also across them. This heterogeneity is reflected in the variability of the rate of eligible households across villages, for instance.

The evaluation data have been used extensively (see Attanasio et al., 2013 for more details) and are suitable for testing the implications of our model. The data provide a census of the 656 villages in that all households in the relevant localities are surveyed, thereby allowing us to estimate the entire distribution of quantities purchased and prices paid in each village, at least for commodities that are commonly bought. Also, the data are very rich. The consumption and expenditure module of the surveys contains information crucial for the purpose of our paper. Each household is interviewed and asked to report not only the quantity consumed of 36 food commodities during the week preceding an interview but also the quantity purchased of each item and its monetary value. The data also contain information

[11] Results are not affected by these sample restrictions. See robustness checks in the appendix.

about quantities consumed and not purchased—for instance, those acquired through self-production or received as a gift or payment in kind. The food items recorded include fruits and vegetables, grains and pulses, and meat and other animal products, and are supposed to be exhaustive of the foods consumed by households. We focus on commodities that are relatively homogeneous in their quality and are purchased and consumed by most households, as explained below. Given the information available on expenditures and quantities purchased for each recorded item, it is possible to compute their unit values, as defined by the ratios of these variables. From now on, we refer to unit values as unit prices.[12]

Attanasio et al. (2013) discussed some of the measurement issues associated with the construction of these unit values, ranging from measurement error to the heterogeneous quality of goods and the non-linearity of prices in the quantities purchased that we consider here. However, those authors found that average and median unit values well approximate unit prices collected from local stores, which are available for some commodities in the locality surveys. They also found that unit values closely match national data on prices. Naturally, differences in the rate of households eligible for Progresa/Oportunidades across villages are related not only to differences in the distribution of household income across villages but also to differences in the distribution of quantities purchased across them. Such heterogeneity could also account for differences in unit prices and so in the degree of price discrimination across villages, which we exploit to examine whether the existence of price nonlinearities and their intensity affect the relationship between household size and the per-capita demand of food.

5.3 Roadmap of Empirical Strategy

Our empirical strategy to document and resolve the DP puzzle proceeds in five steps. To summarize, we start by replicating the DP puzzle in our data and then proceed to show that accounting for the nonlinearity of the pricing of food through quantity discounts attenuates the puzzle. We conclude by demonstrating that although per-capita expenditure on food *decreases* as household size increases, as DP measured, the per-capita demand of food, through the lens of our model, *increases* as household size increases, as DP conjecture. In this sense, the nonlinearity of the price of food explains the puzzle.

More specifically, we proceed as follows. First, we document that the DP puzzle is present in our data. Namely, consistent with Deaton and Paxson (1998), we find that the food budget share declines with household size, conditional on per-capita expenditure, as documented by DP; see Table 1.

[12] In addition to Attanasio et al. (2013), see Hoddinott and Skoufias (2004) and Cunha et al. (2019) for papers that validate the use of unit prices in the context of Progresa/Oportunidades or data from a similar program. *Programa de Apoyo Alimentario*, which was implemented in localities similar and spatially close to those surveyed in Progresa/Oportunidades'.

Second, we establish that the nonlinear pricing of food is common in the markets we study in the form of volume (quantity) discounts. In particular, we estimate price schedules—regressions of log unit values on log quantities purchased—for staple foods in each locality. A negative slope of a price schedule indicates quantity discounts in that larger purchases command lower unit prices. More negative slopes, that is, steeper price schedules, imply larger discounts and thus more intense nonlinear pricing. See the empirical price schedules reported in Figure 1 above. Estimated locality price slopes are summarized in Figure 5. We validate these estimates using independent price data from the November 2000 evaluation survey, as reported in Table 5 in the appendix.

Third, we show that controlling for the intensity of nonlinear pricing attenuates the DP puzzle—namely, the DP puzzle is *less pronounced* in markets with *less intense* nonlinear pricing. The logic is that by holding fixed the degree of nonlinearity of prices, we can better isolate the true response of per-capita food consumption to household size from the response of per-capita expenditure. Intuitively, according to our conjecture, the DP puzzle is less likely to arise when the nonlinear pricing of food is less important. Specifically, we estimate the DP regression grouping markets in quartiles of the price schedule distribution and also by grouping those in which the estimated slopes of the price schedules of food are above the median. Since price slopes are typically negative reflecting quantity discounts, localities with highest quartile or above-median price slopes experience *less* intense nonlinear pricing, namely, less negative price slopes. Consistent with our model, we find that the DP coefficient on household size is less negative, and so the DP puzzle is less pronounced, in these localities; see Table 2.

Fourth, we test the related key prediction of our model that the DP puzzle should be more severe when nonlinear pricing is more intense. We examine this prediction in two ways. We exploit cross-food variation and using the 36 food items in our data, we estimate both price schedules and DP regressions for each food item separately. We find a positive association between the estimated price slopes and the DP coefficient on household size: food items with steeper quantity discounts exhibit a stronger DP puzzle. Figures 3 and 4 and Figure 8 in the appendix displays this relationship for all items with consumption shares above 1%, separately for staples and non-staples overall and by wave, respectively. We also re-estimate the DP regression for households purchasing small quantities of staple foods—namely, rice, beans, and sugar—for which quantity discounts are most pronounced. Consistent with our model, we find that the DP coefficient is more negative for small-quantity purchasers, as shown in Table 15 in the appendix.

Finally, we directly test the sufficient conditions under which we can reconcile the DP puzzle with

the prediction in DP about the positive impact of household size on the per-capita demand of food in the presence of economies of scale in consumption. Recall that Proposition 3 implies that if the slopes of the price schedules of food items are close to -1, then the per-capita *quantity demanded* of food increases with household size—consistent with economies of scale in food consumption—even as the per-capita *expenditure* on food decreases. We find that price slopes near -1 are indeed common for small purchases; see Table 4 and Figure 5. We also estimate each price component of the right side of (18) nonparametrically. Figure 6 shows that the right side of this equation is almost always positive, which implies that the per-capita demand of food increases with household size, once price nonlinearities are accounted for. The histograms displayed in Figure 6 also convey the precision of these estimates: the distributions are tightly concentrated in the positive region, with relatively small dispersion around the mode. Figure 7 further shows that this relationship holds across the full range of quantities purchased by households, and is especially strong at modal purchased quantities like 1 kilogram.

5.4 Documenting the Consumption Puzzle

We replicate the analysis in DP and document their puzzle about the negative impact of household size on per-capita expenditure on food in our data by estimating

$$w_{f,h} = \alpha + \beta \ln\left(\frac{x_h}{n_h}\right) + \gamma \ln(n_h) + \sum_{k=1}^{K-1} \eta_k \frac{n_{k,h}}{n_h} + \zeta \cdot \mathbf{v_h} + u_h, \tag{22}$$

where h denotes a household and $w_{f,h}$ is the budget share of food as well as the ratio of per-capita food expenditure to per-capita total expenditure (see (21) above), x/n is per-capita expenditure, and n_k/n is the ratio of the number of household members who belong to one of K age-sex groups to total household size. The vector $\mathbf{v}$ includes a variety of controls: we include village effects, or village-time effects depending on the specification—we include them as separate effects not as interactions with explanatory variables—and dummies for whether the household head works and the household is classified as poor by the program. We do not include controls for rural activities given that our sample of villages is predominantly rural.

In Table 1, we use treatment and control villages for the October-1998 wave and only control villages for subsequent waves, since the Progresa/Oportunidades treatment amplifies price nonlinearities (see Attanasio and Pastorino, 2020).[13] We also report results from a village-time fixed effects regression using

[13] Results are not affected by this choice. See the robustness exercises in the appendix

Table 1: Replication of DP Puzzle

	(1) 1998_02	(2) 1999_01	(3) 1999_02	(4) 2000_02	(5) 2003	(6) FE
Log of Household Size	-0.042***	-0.048***	-0.049***	-0.054***	-0.056***	-0.047***
	(0.003)	(0.005)	(0.005)	(0.005)	(0.007)	(0.003)
Log of Per-capita (total) expenditure	-0.088***	-0.100***	-0.111***	-0.091***	-0.115***	-0.099***
	(0.004)	(0.006)	(0.006)	(0.006)	(0.008)	(0.004)
Household head works (dummy)	0.010**	0.010	0.007	0.010*	0.000	0.008**
	(0.004)	(0.006)	(0.006)	(0.005)	(0.005)	(0.004)
Household is classified as poor by the program	0.028***	0.027***	0.023***	0.025***	0.016**	
	(0.002)	(0.004)	(0.004)	(0.004)	(0.007)	
Constant	1.200***	1.235***	1.204***	1.192***	1.271***	1.193***
	(0.022)	(0.036)	(0.027)	(0.030)	(0.043)	(0.018)
Observations	22,780	8,104	8,605	7,994	6,380	21,131
R-squared	0.261	0.273	0.305	0.310	0.374	0.283

Note: Standard errors are clustered at the locality level (*** stands for 1% significance, ** stands for 5% significance, and * for 10% significance). Wave Oct-98 includes both treatment and control villages. Other waves include only controls. FE column reports the results of a fixed effect regression on the balanced panel of households present in the first 3 waves: Oct-98, Mar-99 and Nov-99. See Table 7 in the appendix for the IV results where log of Per-capita (total) expenditure is instrumented with a second-order polynomial of log total income.

a balanced panel for the first three waves (column FE). The village-time fixed effects absorb all locality-specific factors that vary over time—such as local market conditions, seasonal price fluctuations, and village-level shocks—so that identification comes from comparing households of different sizes within the same village at the same point in time. Note that the DP puzzle is replicated in our data: the coefficient on (log) household size is negative for all waves and specifications. Interestingly, the estimates of this coefficient are similar in magnitude to those reported by DP for lower-income countries: -0.054 for Thailand, -0.056 for Pakistan, and -0.059 for South Africa.[14]

5.5 A Potential Reconciliation: Accounting for Price Nonlinearities

Here, we repeat the analysis of the previous section but now by explicitly accounting for the nonlinearity of the price of food. Since "food" is an artificial good that maps into a multiplicity of food items, we proceed by first creating such a map. We start by considering all the food items whose consumption, purchase, and production are collected in the Progresa/Oportunidades data. Overall, food consumption, expenditure, and self-production account for about 80, 60, and 25 percent of total consumption, respectively, for the first October 98 survey wave. See Figure 10 in the appendix for details and Attanasio et al.

[14] A concern with interpreting the coefficient on log household size is that household size is not exogenous: fertility, household splitting, and co-residence decisions may correlate with food consumption patterns. Our village-time fixed effects specification mitigates this concern by comparing households of different sizes within the same village at the same point in time, thereby absorbing village-level confounds such as local market conditions and food availability. However, within-village selection into different household sizes remains a possibility. We note that this concern applies equally to the original DP analysis and to the large subsequent literature. Our main results concern the relative magnitude of the household-size coefficient across villages with different degrees of nonlinear pricing, which is less susceptible to this concern as long as the endogeneity of household size does not differentially correlate with price-schedule intensity.

(2013) for evidence on the other survey waves.[15]

We focus on foods that are frequently consumed in our villages, that is, with a consumption share of at least 1 percent. We then partition all sampled food items into staples and non-staples. Among staple foods—which are a fixture of the local diet and amount to a large fraction of total consumption—we focus on those items that are non-perishable, are little differentiated in the localities we study, and are seldom self-produced. Rice, beans, and sugar fit all those characteristics. These staple goods account for 20 percent of total consumption and are not routinely self-produced. We exclude tortilla and corn grain from the staples we consider, despite them being typical foods for the local diet, because they are substantially self-produced—for instance, about 50 percent of the consumption of tortilla and 35 percent of the consumption corn grain is self-produced. In light of these considerations, rice, beans, and sugar are the staple foods best suited to study the implications of nonlinear pricing via quantity discounts for the role of economies of scale in food consumption, especially since confounding effects from other dimensions of discrimination across consumers and from self-production are arguably minimal.[16]

We proceed by considering the sample of localities for which nonlinear pricing is *less intense*. Specifically, we estimate price schedules for the three staple goods of rice, beans, and sugar in each of our localities, see more details below on the estimation of these price schedules. We then use these estimated locality-specific price slopes as a proxy for the intensity of the nonlinear pricing of food. Specifically, we compute the locality weighted average price slopes for food staples, that is we average the price slopes of rice, beans, and sugar in each locality, using their food share as weights. We then re-estimate the DP regression with the inclusion of interactions between household size and specific percentile dummies from the distribution of the locality weighted average price slope. When we consider quartiles of this distribution (as in column "Effect of Nonlinearities" in Table 2), we are basically estimating the coefficient of log household size in villages with a different degree of nonlinear pricing (from villages with the highest degree of nonlinearity – quartile 1, the excluded category in Table 2 – to quartile 4, the least nonlinear villages). We also consider villages above the median of the locality price slope distribution, which means that they have *less negative* slopes compared to those below the median, which amounts to the nonlinear pricing of food being less intense in those villages (we include the results in the column "Above Median" in Table 2). Finally, we also adopt a stricter criteria to rank nonlinear pricing as we consider villages for which the locality price slopes estimated for rice, beans, and sugar are, at the same

[15] We focus on the 1998 wave because information on consumption out of self-production is available only for this wave.
[16] Importantly, resale is uncommon for these goods, as local data surveyors for the program confirmed to us.

time, above the median of each food distribution (see column "Above Median Strict" in Table 2)[17].

As the table shows, the estimated coefficients of log size for villages with a smaller degree of nonlinearity show a consistent pattern, one in which the coefficients of the interactions are positive and significant, suggesting that when nonlinearities are less pronounced (higher quartiles and above the median), the DP puzzle—a negative coefficient on household size—is indeed less likely to arise.

To gauge the magnitude of these effects, consider a household that doubles in size—for instance, from 3 to 6 members—so that the log change in household size is approximately 0.69. Using the baseline coefficient of -0.058 from column 1 of Table 2 , the food budget share is predicted to decline by about 4 percentage points (-0.058 $\times$ 0.69 $\approx$ -0.040). With average food shares around 60%, this represents roughly a 5% reduction in per-capita food spending. In contrast, using the coefficient from the "Quartile 4" sample in column 1 (-0.033 from -0.058 + 0.025), the decline is only about 2 percentage points (-0.033 $\times$ 0.69 $\approx$ -0.022) so the negative effect is half as large. That is, this difference of approximately 2 percentage points attributable to less intense nonlinear pricing accounts for nearly 50% of the puzzle as measured in the full sample. In this precise sense, quantity discounts help understand the puzzle.

5.6 The Role of Economies of Scale and Price Nonlinearities

This section further explores the main implication of our model, namely, that the nonlinearity of the price of food attenuates, if not altogether resolves, the DP puzzle. Our model predicts that the coefficient on log size in the DP regression should be *more negative* when the price of food is more nonlinear in the quantity demanded of it. We examine this prediction by exploiting the 36 food items available in the Progresa/Oportunidades data.[18] For each food item, we estimate the slope of its price schedule, which measures the degree of nonlinearity of its unit price, and the coefficient on log size, which is the main coefficient of interest in the typical DP regression. We then compare these two sets of estimates to determine whether they are systematically related. Specifically, for each item, we estimate the DP regression in (22) and price schedules, in each village, specified as

$$\ln(p_{fh}) = \beta_{0f} + \beta_{1f} \ln(q_{fh}) + \text{wave}_v + u_{fh}, \tag{23}$$

[17] We do not use higher percentiles of the distribution of village slopes because the sample size becomes very small—for instance, in no locality are price slopes above the top quartile for beans, rice, and sugar at the same time.

[18] 1-tomatoes, 2-onions, 3-potatoes, 4-carrots, 5-leafy vegs, 6-oranges, 7-bananas, 8-apples, 9-lemons, 10-pears, 11-tortilla, 12-corn grain, 13-white bread, 14-sweet bread, 15-loaf of bread, 16-wheat flour, 17-pasta soup, 18-rice, 19-salt cakes, 20-beans, 21-breakfast cereals, 22-chicken, 23-beef and pork, 24-lamb and goat, 25-fish, 26-tinned fish, 27-eggs, 28-milk, 29-cheese, 30-lard, 31-sweets, 32-carbonated drinks, 33-alcohol, 34-coffee, 35-sugar, 36-vegetable oil.

Table 2: Deaton-Paxson Regression and Intensity of Nonlinear Pricing

	(1) Effect on Nonlinearities	(2) Above Median	(3) Above Median Strict
Log of Per-capita (total) expenditure	-0.099***	-0.099***	-0.099***
	(0.004)	(0.004)	(0.004)
Log of Household Size	-0.058***	-0.053***	-0.049***
	(0.005)	(0.004)	(0.003)
Quartile 2 price slope X log HH Size (highly nonlinear)	0.010*		
	(0.005)		
Quartile 3 price slope X log HH Size (medium nonlinear)	0.010*		
	(0.005)		
Quartile 4 price slope X log HH Size (least nonlinear)	0.025***		
	(0.006)		
Above median price slope X log HH size (less nonlinear)		0.010***	
		(0.004)	
Above median price slope (strict) X log HH size (less nonlinear)			0.009*
			(0.005)
Constant	1.193***	1.193***	1.193***
	(0.018)	(0.018)	(0.018)
Observations	21,131	21,131	21,131
R-squared	0.284	0.283	0.283
Panels	Bal 123	Bal 123	Bal 123
Controls	Yes	Yes	Yes

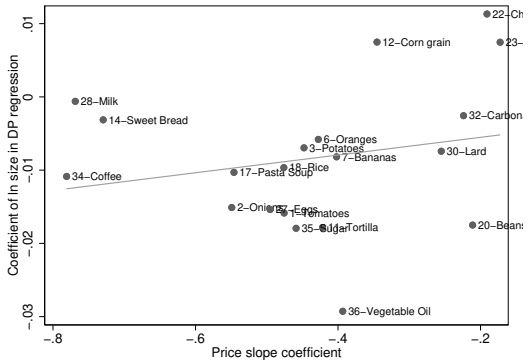
Note: Standard errors are clustered at the locality level (*** stands for 1% significance, ** stands for 5% significance, and * for 10% significance). Wave Oct-98 includes both treatment and control villages. Other waves include only control. Bal 123 means that the sample is a balanced panel of households present in the first three waves: Oct-98, Mar-99 and Nov-99. In column (1) and (2), the quartiles and the "Above Median" dummy are based on the distribution of the locality weighted average price slopes for food staples (that is, we first estimate the price slopes of rice, beans, and sugar for each locality and then average them, in each locality, using their food share as weights). In column (3), "Above Median Strict" uses a stricter criteria under which the locality price slopes estimated for rice, beans, and sugar are, at the same time, above the median of each food distribution (less negative=less nonlinearity). Bootstrap standard errors, which account for the two-stage estimation procedure, yield similar inference (see Table 12 in the appendix). See Table 13 in the appendix for the IV results where log of Per-capita (total) expenditure is instrumented with a second-order polynomial of log total income.

where p_{fh} is the unit value of food item f, q_{fh} is the quantity purchased of it, *wave* are wave fixed effects to control for time effects, and the coefficient β_{1f}, when negative, measures the intensity of price discrimination through volume discounts.

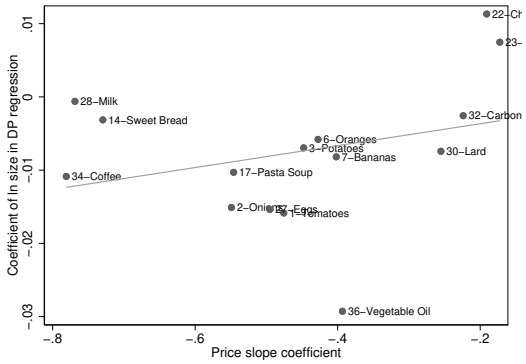
Our model predicts that foods whose prices are more nonlinear—that is, with more negatively sloped price schedules—should have more negative coefficients on log household size in the DP regression. Figure 3 plots estimated price slopes against the estimated DP coefficients on household size for all food items in our data (left panel) and for all non-staple food items (right panel); to ensure that we focus on foods that are commonly consumed in our villages, we consider foods with consumption shares above 1%. As in the analysis above, in Figure 3 we estimate DP coefficients using the specification in (22) on the sample of the first 3 survey waves—the wave October 98 includes both treatment and control villages, other waves include only control—with survey wave and village fixed effects included. Price slopes are estimated using the specification (23) on the sample of all survey waves and households who purchase food quantities below the median with survey wave fixed effects included.

Figure 3: Coefficients on Household Size and Price Slopes for All Goods

(a) All Foods Above 1% Share



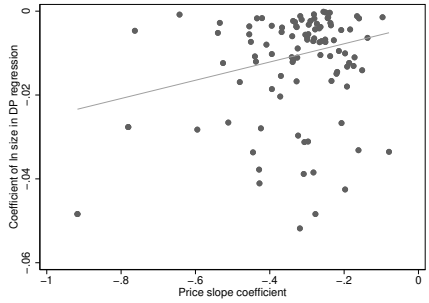
(b) Non-Staples Above 1% Share



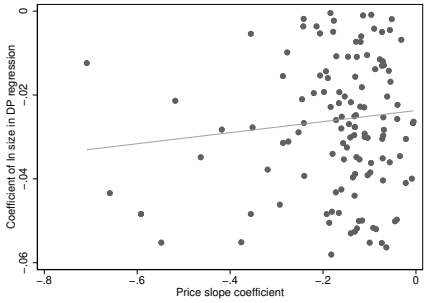
Note: Included food items are 1-tomatoes, 2-onions, 3-potatoes, 6-oranges, 7-bananas, 11-tortilla, 12-corn grain, 14-sweet bread, 17-pasta soup, 18-rice, 20-beans, 22-chicken, 23-beef and pork, 27-eggs, 28-milk, 30-lard, 32-carbonated drinks, 34-coffee, 35-sugar, 36-vegetable oil. Solid lines represent the linear fit between estimated DP coefficients and price slopes. DP coefficients are estimated using the specification in (22) on the sample of the first three survey waves (wave Oct-98 includes both treatment and control villages, other waves include only control) in each locality with survey wave fixed effects included. Price slopes are estimated using the specification (23) on the sample of all survey waves and households who purchases food quantities below the median in each locality, with survey wave fixed effects included. Each dot in the graphs is the average of the locality estimates of DP coefficients and price slopes.

Figure 4: Relationship Between Coefficients on Household Size and Price Slopes for Staples

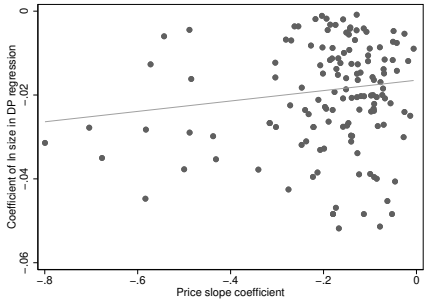
(a) Rice



(b) Beans



(c) Sugar



Note: Each dot represents a locality-level estimated DP coefficient and price slope. DP coefficients and price slopes are estimated in each locality using the specifications in equations (22) and (23), respectively, with survey wave fixed effects included. DP coefficients are estimated in each village using the specification in (22) on the sample of the first 3 survey waves (wave Oct-98 includes both treatment and control villages, other waves include only control) with survey wave fixed effects included. Price slopes are estimated in each village using the specification (23) on the sample of all survey waves with survey wave fixed effects included. Reported estimates are trimmed above 0, that is, we focus on localities for which we found the relevant staple priced nonlinearly (price slope < 0) and the presence of DP puzzle (DP coefficient < 0). The solid line in the graph is a linear fit line weighted by the number of observations in each locality; localities with fewer than 100 observations are excluded. A positive slope for the fitted line confirms the model's prediction that foods with steeper quantity discounts (more negative price slopes) exhibit a stronger DP puzzle (more negative coefficients on household size).

Table 3: DP Puzzle and Treatment Status

	(1) FE-97to2000	(2) Pooled-97to2000	(3) 2003	(4) FE-All	(5) Pooled-All
Interaction Treat with Log of HH Size	-0.006**	-0.008***	-0.014***	-0.009***	-0.011***
	(0.003)	(0.002)	(0.005)	(0.003)	(0.002)
Log of Household Size	-0.044***	-0.046***	-0.055***	-0.052***	-0.055***
	(0.003)	(0.002)	(0.005)	(0.003)	(0.002)
Treatment	0.011**	0.018***	0.025**	0.008	0.016***
	(0.005)	(0.004)	(0.011)	(0.005)	(0.004)
Log of Per-capita (total) expenditure	-0.087***	-0.094***	-0.115***	-0.096***	-0.107***
	(0.002)	(0.002)	(0.003)	(0.002)	(0.002)
Constant	1.135***	1.161***	1.247***	1.178***	1.220***
	(0.011)	(0.010)	(0.017)	(0.010)	(0.009)
Observations	64,200	87,934	32,612	69,258	120,546
R-squared	0.170	0.181	0.273	0.199	0.228
Panels	Bal 1234	Pooled 1234	No	Pooled 12345	Pooled 12345

Note: Standard errors are clustered at the locality level (*** stands for 1% significance, ** stands for 5% significance, and * for 10% significance). FE-97to2000 column reports the results of a fixed effect (with municipal level, the administrative level above village, fixed effects) regression on the balanced panel of households present in the first 4 waves: Oct-98, Mar-99, Nov-99, and Nov-2000 (that is what "Bal 1234" means). Pooled-97to2000 column reports the results over the same 4 waves but using an unbalanced panel (that is what "Pooled 1234" means). Similarly, columns FE-All and Pooled-All reports the results using all the 5 waves available (that is, adding also 2003 to the first 4 waves). Column 2003 reports the results for the 2003 wave only.

For the subset of staples we focus from now on, namely, rice, beans, and sugar, we provide a more granular analysis by estimating price slopes and DP coefficients in each village (with survey wave fixed effects included). We report the corresponding results in Figure 4; Figure 8 represents the same relationship by each wave in our sample, supporting the notion that this relationship is common across waves for each of the three staples—with the exception of rice for the wave of March 1999.

Overall, we find a positive association between the size of the estimated coefficient on household size in the DP regression and the estimated slopes of the schedule of unit prices for each group of foods, as consistent with the main prediction of our model.

So far, we have abstracted from the effects of the Progresa/Oportunidades treatment since evidence shows that the treatment can have an effect on the intensity of nonlinear pricing (see Attanasio and Pastorino, 2020). Also, we did not use survey wave 2003 to ensure we use a consistent set of villages when estimating the DP coefficient on household size.[19] Here, we take advantage of the treatment status and the 2003 survey wave to further test the predictions of our model by evaluating the impact of the treatment on the DP puzzle. Consistent with the evidence in Attanasio and Pastorino (2020), who

[19]To estimate DP coefficients we focus on the panel of villages that were not treated by the program across the first three survey waves. By 2003, the original control villages were all included into the program (since 2000). New control villages were included to the evaluation sample in 2003 and are, therefore, only available in that survey wave.

show that the treatment intensifies the nonlinear pricing of food, our model predicts that the DP puzzle should be more pronounced in treatment villages, where the nonlinearity of the price of food has become stronger—in practice, the estimated DP coefficients in treatment villages should be more negative. We test this prediction by first estimating the DP regression with the inclusion of a dummy for treatment status and an interaction between the treatment dummy and the log of household size in the following augmented DP specification

$$w_{f,h} = \alpha + \beta \ln\left(\frac{x_h}{n_h}\right) + \theta \times 1\{\text{Treat}\} + \gamma \ln(n_h) + \lambda \ln(n_h) \times 1\{\text{Treat}\} + \sum_{k=1}^{K-1} \eta_k \frac{n_{k,h}}{n_h} + \zeta \cdot \mathbf{v_h} + u_h, \quad (24)$$

with results reported in Table 3. In Equation (24), h denotes a household and $w_{f,h}$ is the budget share of foods, x/n is per-capita expenditure, and n_k/n is the ratio of the number of household members who belong to one of K age-sex groups to total household size. The vector $\mathbf{v}$ includes a variety of controls (see above for details). $1\{\text{Treat}\}$ is a dummy that indicates whether a village is treated by Progresa/Oportunidades—namely, receives the corresponding transfer. In this specification, the effect of the treatment on the DP puzzle is retrieved by estimating λ, that is, the parameter of the interaction between $\ln(n_h)$ and $1\{\text{Treat}\}$. Overall, the effect of the treatment on the DP puzzle—namely, the coefficient of the interaction of treatment with log of HH size in Table 3—is consistently negative and statistically significant for all specifications considered, including for the first 4 wave sample and for survey wave 2003 only.[20]

We now turn to explore, through the lens of our model, the sources of the positive association between the DP puzzle and the nonlinearity of the price of food as well as the implications of nonlinear pricing for the response of the per-capita demand of rice, beans, and sugar to increases in household size.

5.7 Reconciling the Puzzle

We now examine the extent to which the sufficient conditions of Proposition 3 are empirically satisfied by performing two distinct exercises for each of our staple goods of interest, namely, rice, beans, and sugar. First, we test whether the condition $|d\ln(p_f(q_f))/d\ln(q_f)| \approx 1$ is verified by estimating equation (23) separately for each good in each of the 656 localities in our sample. We focus on households with purchases in the first decile of the distribution of quantities purchased across villages, for whom price

[20] The treatment could also affect food budget shares through channels other than nonlinear pricing—for instance, through the direct income effect of cash transfers or through changes in household composition induced by the program's conditionalities. However, such alternative channels would need to operate differentially by household size to generate the negative interaction we estimate. Moreover, the fact that the interaction coefficient is negative—indicating that the puzzle is stronger in treated villages—is difficult to reconcile with a pure income effect, which would predict the opposite if food is a necessity.

Table 4: Price Slopes and Sufficient Condition for No Puzzle

	Negative Slope of the Price Schedule	
	(-1.5,-0.5)	(-1.75,-0.25)
Rice		
Locs with *nonlinear* slope	331	
Locs with slope in condition range	192	222
Proportion of locs with condition met	0.580	0.670
Beans		
Locs with *nonlinear* slope	533	
Locs with slope in condition range	156	281
Proportion of locs with condition met	0.293	0.527
Sugar		
Locs with *nonlinear* slope	573	
Locs with slope in condition range	232	384
Proportion of locs with condition met	0.405	0.670

Note: Price slopes are estimated above based on equation (23).

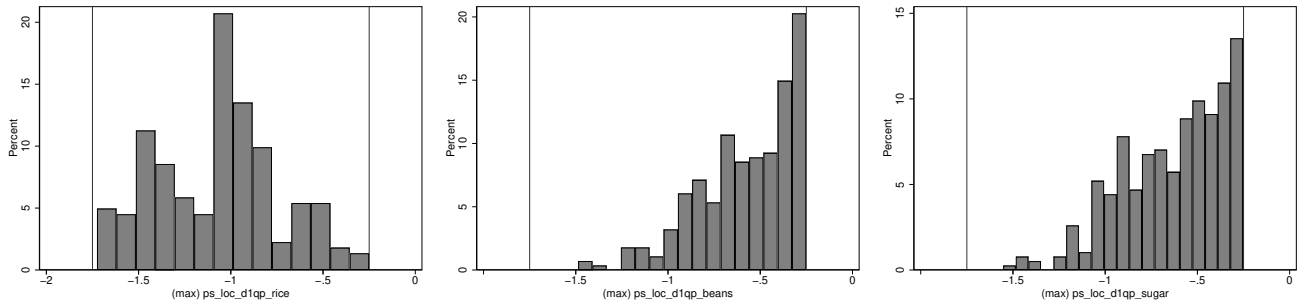
nonlinearities are the largest. Second, we check whether the per-capita demand of food indeed increases with household size through the lens of our model by estimating the right side of equation (18). The latter provides an expression for the elasticity of interest, $d\ln\left(\frac{q_f}{n}\right)/d\ln(n)$, that only depends on the price schedule of a food item, in particular its slope and curvature as measured by $p_f(q_f)$, $p'_f(q_f)$, $T'_f(q_f)$, and $T''_f(q_f)$, and the quantity consumed of it, q_f.

Turning to testing whether the condition that $|d\ln\left(p_f(q_f)\right)/d\ln(q_f)| \approx 1$ or, equivalently, $d\ln\left(p_f(q_f)\right)$-$/d\ln(q_f) \approx -1$ is satisfied, we consider two ranges of values to approximate the distance of the elasticity of the price schedule of a good, $d\ln\left(p_f(q_f)\right)/d\ln(q_f)$, from -1, as before, namely, price slopes between -1.5 and -0.5, and between -1.75 and -0.25. We restrict attention to localities in which the three staples of rice, beans, and sugar are priced nonlinearly, that is, localities with price slopes below zero. Table 4 summarizes the results. Our model's sufficient condition is consistently met for all three staple foods. For the range between -1.75 and -0.25, the condition is met for 67% of localities (222 out of 331) for rice, for 53% of localities (281 out of 533) for beans, and for 67% of localities (384 out of 573) for sugar. Even for the more restrictive notion of distance—the negative range between -1.5 and -0.5—a substantial proportion of localities meet the sufficient condition, ranging from about 30% for beans to around 60% for rice. Hence, the sufficient conditions for reconciling the DP puzzle are robustly satisfied. Figure 5 reports the distribution of the price slopes of the three food items, confirming that slopes are negative in most villages. Hence, the nonlinear pricing of these staples is indeed widespread.

A direct test of whether the implications of our theory are borne out in the data is to estimate equation (18), which relates the response of the per-capita demand of food to increases in household size, $d\ln\left(q_f/n\right)/d\ln(n) > 0$, to the slope and curvature of the price schedule of food, so as to verify whether

Figure 5: Distribution of Price Slopes by Locality for First Decile of Quantities Purchased

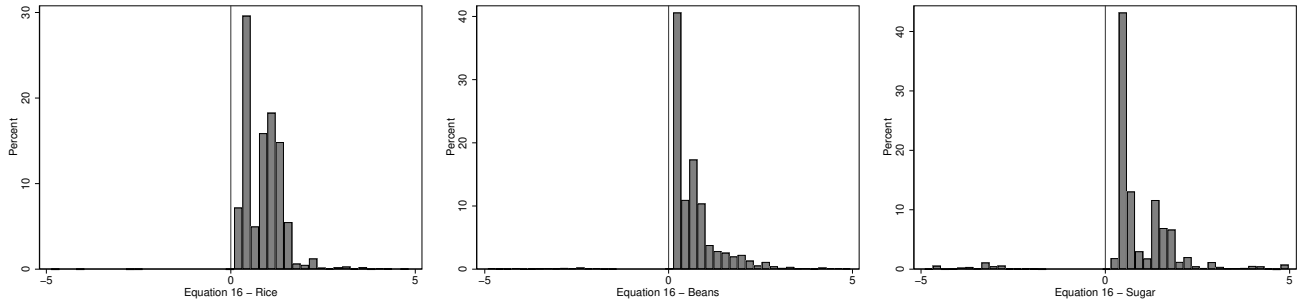
(a) Rice (b) Beans (c) Sugar



Note: The figures report the distribution of price slopes ranging between -1.75 and -0.25 as indicated by the solid vertical lines. As reported in Table 4, the density within this range spans from more than 50 percent for beans to almost 70 percent for rice and sugar. The concentration of slopes near -1 confirms that the sufficient condition for reconciling the puzzle is met in most localities. Conversely, Table 2 shows that in localities with slopes outside this range, the DP puzzle is substantially attenuated.

Figure 6: Nonparametric Estimates of Model-Implied Consumption Per Capita

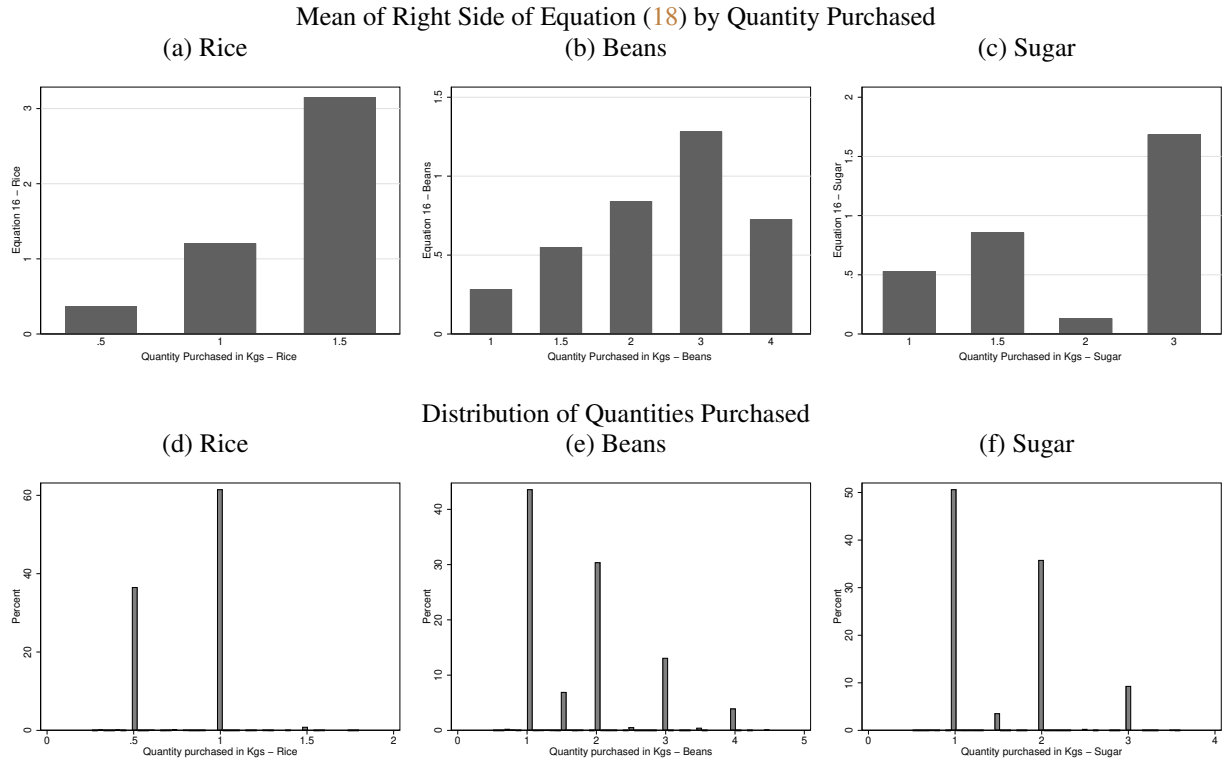
(a) Rice (b) Beans (c) Sugar



Note: These panels display the distribution (empirical p.d.f.) of the values of the right side of equation (18). Distributions are trimmed at the top and bottom 1%.

$d\ln(q_f/n)/d\ln(n) > 0$. To do so, we nonparametrically estimate the components of the right side of (18) that we do not directly observe, that is $p'_f(q_f)$, $T'_f(q_f)$, and $T''_f(q_f)$. We estimate $p'_f(q_f)$ by first nonparametrically regressing the unit price of each food item $p_f(q_f)$ on the quantity purchased of it q_f using the Stata routine `npregress kernel`. We then use the predicted derivatives at each value of q_f as estimates of $p'_f(q_f)$. Similarly, we estimate $T'_f(q_f)$ by first nonparametrically regressing the expenditure on each food item $T_f(q_f)$ on the quantity purchased of it q_f and then using the predicted derivatives at each value of q_f as estimates of $T'_f(q_f)$. We next estimate $T''_f(q_f)$ by taking the first differences of the estimated $T'_f(q_f)$ over ordered values of the quantities purchased—wave fixed effects are included in all these regressions. We finally combine these estimates with the estimated unit values $p_f(q_f)$ and the

Figure 7: Nonparametric Estimates of Model-Implied Per-Capita Consumption per Quantity Purchased



Note: Distributions for the values of the right side of equation (18) and quantities purchased are trimmed at the top and bottom 1%. The bars in the top panels report the mean of the right side of equation (18) at each relevant support point of the distribution of quantity purchased. The bottom panels report the distribution of quantities purchased.

quantities q_f to obtain an estimate of the elasticity of interest in (18) as

$$\frac{d\ln\widehat{(q_f/n)}}{d\ln(n)} = \frac{-\left[\widehat{p'_f(q_f)} + \widehat{T''_f(q_f)}\right]q_f}{\widehat{p_f(q_f)} + \widehat{T'_f(q_f)} + \left[\widehat{p'_f(q_f)} + \widehat{T''_f(q_f)}\right]q_f}.$$

In Figure 6, we report the distribution of these estimated elasticities for our three staple foods, whereas in Figure 7, we report the mean of these estimates across localities for each relevant quantity purchased—the top three panels—as well as the distribution of quantities purchased—the bottom three panels. Note that the estimated elasticity is positive throughout, as it can be seen from the histograms in Figure 6, as consistent with our model. Moreover, the histograms convey the precision of these estimates in that the distributions are tightly concentrated in the positive region with relatively small dispersion, indicating that the positive sign of the estimated $d\ln(q_f/n)/d\ln(n)$ is robust across observations.

To better parse out the relationship between average and marginal prices, on the one hand, and quantities purchased, on the other hand, behind the impact of household size on the quantity demanded of the three staples, we also plot the estimated $d\ln(q_f/n)/d\ln(n)$ against quantity purchased. The values

are above zero for all quantities purchased (see the top panels in Figure 7) including at 1 kilo, where the distribution of quantities purchased is most concentrated (see the bottom panels). Overall, we take these results as evidence supporting the existence of economies of scale in food consumption at the household level as predicted by DP, once nonlinearities in the price of food are taken into account.

6 Conclusion

In this paper we revisit the consumption puzzle documented by DP—that per-capita expenditure on food *decreases* with household size for given expenditure per capita—and propose a resolution of it based on the notion that necessity-type goods like food may be priced nonlinearly due to volume discounts. This form of nonlinear pricing implies that common measurement strategies based on estimating the response of *per-capita consumption* of food to changes in household size by estimating the response of *per-capita expenditure* to such changes are inappropriate, since unit prices and per-capita quantity consumed of food move in opposite directions when economies of scale in consumption at the household level are important but the unit price of food declines with the quantity purchased of it.

Specifically, per-capita expenditure on food may decrease although per-capita consumption of it increases when household size increases, especially when nonlinearities are pronounced. Intuitively, the lower effective price per capita of public goods, triggered by an increase in household size, induces an income effect on the consumption of food that raises its demand. But as higher quantities of food are purchased at a lower unit prices, although per-capita consumption of food increases, its per-capita expenditure may well decrease when quantity discounts are large.

We prove that once the nonlinear pricing of food through quantity discounts is taken into account, indeed no puzzle arises. Namely, using data from Mexico's Progresa/Oportunidades, we first replicate the DP puzzle and then show that the data support our theory, provide evidence of its main mechanism, and demonstrate that, through the lens of our nonlinear pricing model, the economies of scale in consumption at the household level are substantial.

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A Model Appendix

This appendix contains details omitted from the main text.

A.1 Homogeneity of Demand under Linear Pricing

The condition $\varepsilon_{fx} + \varepsilon_{ff} + \varepsilon_{fh} = 0$ can be derived as follows. Recall that

$$q_f = \phi_f(n) g_f \left(\frac{x}{n}, \frac{p_f \phi_f(n)}{n}, \frac{p_h \phi_h(n)}{n} \right) = \phi_f(n) g_f \left(\lambda \frac{x}{n}, \lambda \frac{p_f \phi_f(n)}{n}, \lambda \frac{p_h \phi_h(n)}{n} \right)$$

so that

$$g_f \left(\frac{x}{n}, \frac{p_f \phi_f(n)}{n}, \frac{p_h \phi_h(n)}{n} \right) = g_f \left(\lambda \frac{x}{n}, \lambda \frac{p_f \phi_f(n)}{n}, \lambda \frac{p_h \phi_h(n)}{n} \right).$$

Differentiating both sides by λ and evaluating the resulting expression at $\lambda = 1$ gives

$$g_{f1} \frac{x}{n} + g_{f2} \frac{p_f \phi_f(n)}{n} + g_{f3} \frac{p_h \phi_h(n)}{n} = 0$$

or, equivalently,

$$\frac{g_{f1}}{g_f} \frac{x}{n} + \frac{g_{f2}}{g_f} p_f \frac{\phi_f(n)}{n} + \frac{g_{f3}}{g_f} p_h \frac{\phi_h(n)}{n} = 0.$$

Using

$$\varepsilon_{fx} = \frac{g_{f1}}{g_f} \frac{x}{n}, \varepsilon_{ff} = \frac{g_{f_2}}{g_f} p_f \frac{\phi_f(n)}{n}, \text{ and } \varepsilon_{fh} = \frac{g_{f3}}{g_f} p_h \frac{\phi_h(n)}{n},$$

it follows that $\varepsilon_{fx} + \varepsilon_{ff} + \varepsilon_{fh} = 0$.

A.2 Proof of Lemma 1

Recalling (3) and using that $d \ln \left(\frac{q_f}{n} \right) / d \ln(n) = d \left(\frac{q_f}{n} \right) / dn \cdot n^2 / q_f$, we calculate

$$\begin{aligned} \frac{d \left(\frac{q_f}{n} \right)}{dn} =& \frac{\phi_f'(n) n - \phi_f(n)}{n^2} g_f + \frac{\phi_f(n)}{n} g_{f1} \frac{d}{dn} \left(\frac{x}{n} \right) + \frac{\phi_f(n)}{n} g_{f2} \left[\frac{p_f \phi_f'(n) n - p_f \phi_f(n)}{n^2} \right] \\ &+ \frac{\phi_f(n)}{n} g_{f3} \left[\frac{p_h \phi_h'(n) n - p_h \phi_h(n)}{n^2} \right], \end{aligned}$$

where $d(x/n)/dn = 0$, since in this exercise income is held *fixed* by assumption, that is, x is increased by the same proportion as n. Then, by straightforward algebra, we arrive at

$$\begin{aligned} \frac{d \ln \left(\frac{q_f}{n} \right)}{d \ln(n)} =& \frac{d \left(\frac{q_f}{n} \right)}{dn} \frac{n^2}{q_f} = \frac{n^2}{\phi_f(n) g_f} \frac{\phi_f'(n) n - \phi_f(n)}{n^2} g_f \\ &+ \frac{n^2}{\phi_f(n) g_f} \frac{\phi_f(n)}{n} g_{f2} \left[\frac{p_f \phi_f'(n) n - p_f \phi_f(n)}{n^2} \right] + \frac{n^2}{\phi_f(n) g_f} \frac{\phi_f(n)}{n} g_{f3} \left[\frac{p_h \phi_h'(n) n - p_h \phi_h(n)}{n^2} \right] \end{aligned}$$

or, equivalently,

$$\frac{d\ln\left(\frac{q_f}{n}\right)}{d\ln(n)} = \frac{n\phi_f'(n)}{\phi_f(n)} - 1 + \frac{g_{f2}}{g_f}p_f\frac{\phi_f(n)}{n}\left[\frac{n\phi_f'(n)}{\phi_f(n)} - 1\right] + \frac{g_{f3}}{g_f}p_h\frac{\phi_h(n)}{n}\left[\frac{n\phi_h'(n)}{\phi_h(n)} - 1\right].$$

With

$$\begin{aligned}\varepsilon_{ff} &= \frac{dq_f}{dp_f}\frac{p_f}{q_f} = \phi_f(n)g_{f_2}\frac{\phi_f(n)}{n}\frac{p_f}{\phi_f(n)g_f} = \frac{g_{f_2}}{g_f}p_f\frac{\phi_f(n)}{n} < 0,\\ \varepsilon_{fh} &= \frac{dq_f}{dp_h}\frac{p_h}{q_f} = \phi_f(n)g_{f3}\frac{\phi_h(n)}{n}\frac{p_h}{\phi_f(n)g_f} = \frac{g_{f3}}{g_f}p_h\frac{\phi_h(n)}{n},\end{aligned} \tag{25}$$

and

$$\sigma_i = 1 - \frac{d\ln\left(\phi_i(n)\right)}{d\ln(n)} = 1 - \frac{d\phi_i(n)}{dn}\frac{n}{\phi_i(n)} = 1 - \frac{n\phi_i'(n)}{\phi_i(n)},$$

we obtain the desired relationship, namely,

$$\frac{d\ln\left(\frac{q_f}{n}\right)}{d\ln(n)} = -\sigma_f - \varepsilon_{ff}\sigma_f - \varepsilon_{fh}\sigma_h = \sigma_h(\varepsilon_{fx} + \varepsilon_{ff}) - \sigma_f(1 + \varepsilon_{ff}), \tag{26}$$

where ε_{fx} is defined as

$$\varepsilon_{fx} = \frac{dq_f}{dx}\frac{x}{q_f} = \frac{g_{f1}}{g_f}\frac{x}{n}$$

and the second equality in (26) follows from $\varepsilon_{fx} + \varepsilon_{ff} + \varepsilon_{fh} = 0$ by the homogeneity of degree zero of the demand of food—see Section A.1—that allows to substitute ε_{ff} out from (26).

A.3 The Case of Utility with Constant Elasticity of Substitution

We first derive the per-capita demand for food when food is priced nonlinearly and then characterize the response of this per-capita demand to changes in household size.

Per-Capital Demand for Food Under Nonlinear Pricing. The Lagrangian for this problem is

$$L = n\left[\alpha_f\left(\frac{q_f}{\phi_f(n)}\right)^{\frac{\rho-1}{\rho}} + \alpha_h\left(\frac{q_h}{\phi_h(n)}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} + \lambda[x - T_f(q_f) - p_h q_h]$$

and the first-order conditions with respect to q_f, q_h, and λ are given by

$$\begin{cases} n\left(\frac{\rho}{\rho-1}\right)[\cdot]^{\frac{1}{\rho-1}}\alpha_f\left(\frac{\rho-1}{\rho}\right)\left(\frac{q_f}{\phi_f(n)}\right)^{-\frac{1}{\rho}}\frac{1}{\phi_f(n)} - \lambda T_f'(q_f) = 0\\ n\left(\frac{\rho}{\rho-1}\right)[\cdot]^{\frac{1}{\rho-1}}\alpha_h\left(\frac{\rho-1}{\rho}\right)\left(\frac{q_h}{\phi_h(n)}\right)^{-\frac{1}{\rho}}\frac{1}{\phi_h(n)} - \lambda p_h = 0\\ T_f(q_f) + p_h q_h = x \end{cases}.$$

Simplifying terms, we obtain

$$q_h = \left[\frac{\alpha_h}{\alpha_f}\frac{\phi_f(n)}{\phi_h(n)}\frac{T_f'(q_f)}{p_h}\right]^{\rho}\frac{\phi_h(n)}{\phi_f(n)}q_f = \left[\frac{\alpha_h}{\alpha_f}\frac{T_f'(q_f)}{p_h}\right]^{\rho}\left(\frac{\phi_h(n)}{\phi_f(n)}\right)^{1-\rho}q_f.$$

Substituting this expression into the budget constraint yields

$$T_f(q_f) + p_h\left[\frac{\alpha_h}{\alpha_f}\frac{T_f'(q_f)}{p_h}\right]^{\rho}\left(\frac{\phi_h(n)}{\phi_f(n)}\right)^{1-\rho}q_f = x$$

or, using that $T_f(q_f) = p_f(q_f)q_f$,

$$\frac{q_f}{n} = \frac{\frac{x}{n}}{p_f(q_f) + p_h\left[\frac{\alpha_h}{\alpha_f}\frac{T_f'(q_f)}{p_h}\right]^{\rho}\left(\frac{\phi_h(n)}{\phi_f(n)}\right)^{1-\rho}} = \frac{\phi_f(n)}{n}\frac{\frac{x}{n}}{\left\{p_f(q_f)\frac{\phi_f(n)}{n} + \left(p_h\frac{\phi_h(n)}{n}\right)^{1-\rho}\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\frac{\phi_f(n)}{n}\right]^{\rho}\right\}},$$

which is precisely (9) as desired.

Response of Per-Capital Demand for Food to Changes in Household Size. Suppose now utility is of the CES form. Note that from (9) and

$$\begin{aligned}\frac{q_f}{n} &= \frac{\phi_f(n)}{n}g_f\left(\frac{x}{n}, p_f(q_f)\frac{\phi_f(n)}{n}, T_f'(q_f)\frac{\phi_f(n)}{n}, p_h\frac{\phi_h(n)}{n}\right)\\ &= \frac{\phi_f(n)}{n}\frac{\frac{x}{n}}{\left\{p_f(q_f)\frac{\phi_f(n)}{n} + \left(p_h\frac{\phi_h(n)}{n}\right)^{1-\rho}\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\frac{\phi_f(n)}{n}\right]^{\rho}\right\}},\end{aligned}$$

we have that

$$\frac{g_{f_1}}{g_f} = \frac{\left\{p_f(q_f)\frac{\phi_f(n)}{n} + \left(p_h\frac{\phi_h(n)}{n}\right)^{1-\rho}\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\frac{\phi_f(n)}{n}\right]^{\rho}\right\}}{\left\{p_f(q_f)\frac{\phi_f(n)}{n} + \left(p_h\frac{\phi_h(n)}{n}\right)^{1-\rho}\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\frac{\phi_f(n)}{n}\right]^{\rho}\right\}}\cdot\frac{1}{\frac{x}{n}} = \frac{n}{x},$$

$$\begin{aligned}\frac{g_{f_2}}{g_f} &= -\frac{\frac{x}{n}}{\left\{p_f(q_f)\frac{\phi_f(n)}{n} + \left(p_h\frac{\phi_h(n)}{n}\right)^{1-\rho}\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\frac{\phi_f(n)}{n}\right]^{\rho}\right\}^2}\cdot\frac{p_f(q_f)\frac{\phi_f(n)}{n} + \left(p_h\frac{\phi_h(n)}{n}\right)^{1-\rho}\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\frac{\phi_f(n)}{n}\right]^{\rho}}{\frac{x}{n}}\\ &= -\frac{1}{p_f(q_f)\frac{\phi_f(n)}{n} + \left(p_h\frac{\phi_h(n)}{n}\right)^{1-\rho}\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\frac{\phi_f(n)}{n}\right]^{\rho}},\end{aligned}$$

$$\frac{g_{f_3}}{g_f} = -\frac{\frac{x}{n}\left(p_h\frac{\phi_h(n)}{n}\right)^{1-\rho}\rho\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\frac{\phi_f(n)}{n}\right]^{\rho-1}\frac{\alpha_h}{\alpha_f}}{\left\{p_f(q_f)\frac{\phi_f(n)}{n}+\left(p_h\frac{\phi_h(n)}{n}\right)^{1-\rho}\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\frac{\phi_f(n)}{n}\right]^{\rho}\right\}^2}\cdot\frac{p_f(q_f)\frac{\phi_f(n)}{n}+\left(p_h\frac{\phi_h(n)}{n}\right)^{1-\rho}\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\frac{\phi_f(n)}{n}\right]^{\rho}}{\frac{x}{n}}$$
$$= -\frac{\left(p_h\frac{\phi_h(n)}{n}\right)^{1-\rho}\rho\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\frac{\phi_f(n)}{n}\right]^{\rho-1}\frac{\alpha_h}{\alpha_f}}{p_f(q_f)\frac{\phi_f(n)}{n}+\left(p_h\frac{\phi_h(n)}{n}\right)^{1-\rho}\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\frac{\phi_f(n)}{n}\right]^{\rho}},$$

and

$$\frac{g_{f_4}}{g_f} = -\frac{\frac{x}{n}\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\frac{\phi_f(n)}{n}\right]^{\rho}(1-\rho)\left(p_h\frac{\phi_h(n)}{n}\right)^{-\rho}}{\left\{p_f(q_f)\frac{\phi_f(n)}{n}+\left(p_h\frac{\phi_h(n)}{n}\right)^{1-\rho}\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\frac{\phi_f(n)}{n}\right]^{\rho}\right\}^2}\cdot\frac{p_f(q_f)\frac{\phi_f(n)}{n}+\left(p_h\frac{\phi_h(n)}{n}\right)^{1-\rho}\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\frac{\phi_f(n)}{n}\right]^{\rho}}{\frac{x}{n}}$$
$$= -\frac{\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\frac{\phi_f(n)}{n}\right]^{\rho}(1-\rho)\left(p_h\frac{\phi_h(n)}{n}\right)^{-\rho}}{p_f(q_f)\frac{\phi_f(n)}{n}+\left(p_h\frac{\phi_h(n)}{n}\right)^{1-\rho}\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\frac{\phi_f(n)}{n}\right]^{\rho}}.$$

Therefore, we can rewrite the relevant terms in (31) as

$$\frac{g_{f_1}}{g_f}\frac{x}{n}+\frac{g_{f_2}}{g_f}p_f(q_f)\frac{\phi_f(n)}{n}+\frac{g_{f_3}}{g_f}T_f'(q_f)\frac{\phi_f(n)}{n} = 1-\frac{p_f(q_f)\frac{\phi_f(n)}{n}}{p_f(q_f)\frac{\phi_f(n)}{n}+\left(p_h\frac{\phi_h(n)}{n}\right)^{1-\rho}\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\frac{\phi_f(n)}{n}\right]^{\rho}}$$
$$-\frac{\left(p_h\frac{\phi_h(n)}{n}\right)^{1-\rho}\rho\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\frac{\phi_f(n)}{n}\right]^{\rho-1}\frac{\alpha_h}{\alpha_f}T_f'(q_f)\frac{\phi_f(n)}{n}}{p_f(q_f)\frac{\phi_f(n)}{n}+\left(p_h\frac{\phi_h(n)}{n}\right)^{1-\rho}\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\frac{\phi_f(n)}{n}\right]^{\rho}} = 1-\frac{p_f(q_f)\frac{\phi_f(n)}{n}+\rho\left(p_h\frac{\phi_h(n)}{n}\right)^{1-\rho}\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\frac{\phi_f(n)}{n}\right]^{\rho}}{p_f(q_f)\frac{\phi_f(n)}{n}+\left(p_h\frac{\phi_h(n)}{n}\right)^{1-\rho}\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\frac{\phi_f(n)}{n}\right]^{\rho}}.$$

Also,

$$\left[\frac{g_{f2}}{g_f}p_f'(q_f)+\frac{g_{f3}}{g_f}T_f''(q_f)\right]\frac{q_f}{n} = -\frac{\frac{q_f}{n}\left[p_f'(q_f)+\left(p_h\frac{\phi_h(n)}{n}\right)^{1-\rho}\rho\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\frac{\phi_f(n)}{n}\right]^{\rho-1}\frac{\alpha_h}{\alpha_f}T_f''(q_f)\right]}{p_f(q_f)\frac{\phi_f(n)}{n}+\left(p_h\frac{\phi_h(n)}{n}\right)^{1-\rho}\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\frac{\phi_f(n)}{n}\right]^{\rho}}.$$

We can thus express (31) as

$$\frac{d\ln\left(\frac{q_f}{n}\right)}{d\ln(n)} = \frac{\sigma_h\left\{1-\frac{p_f(q_f)\frac{\phi_f(n)}{n}+\rho\left(p_h\frac{\phi_h(n)}{n}\right)^{1-\rho}\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\frac{\phi_f(n)}{n}\right]^{\rho}}{p_f(q_f)\frac{\phi_f(n)}{n}+\left(p_h\frac{\phi_h(n)}{n}\right)^{1-\rho}\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\frac{\phi_f(n)}{n}\right]^{\rho}}\right\}}{1+\phi_f(n)\frac{q_f}{n}\frac{\left[p_f'(q_f)+\left(p_h\frac{\phi_h(n)}{n}\right)^{1-\rho}\rho\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\frac{\phi_f(n)}{n}\right]^{\rho-1}\frac{\alpha_h}{\alpha_f}T_f''(q_f)\right]}{p_f(q_f)\frac{\phi_f(n)}{n}+\left(p_h\frac{\phi_h(n)}{n}\right)^{1-\rho}\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\frac{\phi_f(n)}{n}\right]^{\rho}}}$$
$$-\phi_f(n)\frac{q_f}{n}\frac{p_f'(q_f)+\left(p_h\frac{\phi_h(n)}{n}\right)^{1-\rho}\rho\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\frac{\phi_f(n)}{n}\right]^{\rho-1}\frac{\alpha_h}{\alpha_f}T_f''(q_f)}{p_f(q_f)\frac{\phi_f(n)}{n}+\left(p_h\frac{\phi_h(n)}{n}\right)^{1-\rho}\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\frac{\phi_f(n)}{n}\right]^{\rho}}$$

or, equivalently, as in (15),

$$\frac{d\ln\left(\frac{q_f}{n}\right)}{d\ln(n)}=\frac{\sigma_h\left\{1-\frac{p_f(q_f)+\rho\left(p_h\frac{\phi_h(n)}{\phi_f(n)}\right)^{1-\rho}\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\right]^{\rho}}{p_f(q_f)+\left(p_h\frac{\phi_h(n)}{\phi_f(n)}\right)^{1-\rho}\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\right]^{\rho}}\right\}-\frac{p_f'(q_f)+\left(p_h\frac{\phi_h(n)}{\phi_f(n)}\right)^{1-\rho}\rho\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\right]^{\rho-1}\frac{\alpha_h}{\alpha_f}T_f''(q_f)}{p_f(q_f)+\left(p_h\frac{\phi_h(n)}{\phi_f(n)}\right)^{1-\rho}\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\right]^{\rho}}q_f}{1+\frac{p_f'(q_f)+\left(p_h\frac{\phi_h(n)}{\phi_f(n)}\right)^{1-\rho}\rho\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\right]^{\rho-1}\frac{\alpha_h}{\alpha_f}T_f''(q_f)}{p_f(q_f)+\left(p_h\frac{\phi_h(n)}{\phi_f(n)}\right)^{1-\rho}\left[\frac{\alpha_h}{\alpha_f}T_f'(q_f)\right]^{\rho}}q_f}. \tag{27}$$

A.4 Proof of Proposition 2

Suppose that demand takes the form in (10)

$$\frac{q_f}{n}=\frac{\phi_f(n)}{n}g_f\left(\frac{x}{n},p_f(q_f)\frac{\phi_f(n)}{n},T_f'(q_f)\frac{\phi_f(n)}{n},p_h\frac{\phi_h(n)}{n}\right)$$

and note that

$$\frac{d\ln\left(\frac{q_f}{n}\right)}{d\ln(n)}=\frac{d\left(\frac{q_f}{n}\right)}{dn}\frac{n}{\frac{q_f}{n}}=\frac{d\left(\frac{q_f}{n}\right)}{dn}\frac{n^2}{q_f}.$$

Then, using that $d\left(\frac{x}{n}\right)/dn=0$ by assumption and $q_f=\phi_f(n)g_f\left(\cdot\right)=\phi_f(n)g_f$ by the form of demand,

$$\begin{aligned}\frac{d\ln\left(\frac{q_f}{n}\right)}{d\ln(n)}&=\frac{n^2}{q_f}\left[\frac{\phi_f'(n)n-\phi_f(n)}{n^2}\right]g_f+\frac{n^2}{q_f}\frac{\phi_f(n)}{n}\left\{g_{f1}\frac{d\left(\frac{x}{n}\right)}{dn}+g_{f2}\frac{d\left[p_f\left(q_f\right)\frac{\phi_f(n)}{n}\right]}{dn}\right.\\&\left.+g_{f3}\frac{d}{dn}\left[T_f'(q_f)\frac{\phi_f(n)}{n}\right]+g_{f4}\frac{d\left[p_h\frac{\phi_h(n)}{n}\right]}{dn}\right\}=\frac{n^2}{\phi_f(n)g_f}\left[\frac{\phi_f'(n)n-\phi_f(n)}{n^2}\right]g_f\\&+\frac{n^2}{\phi_f(n)g_f}\frac{\phi_f(n)}{n}g_{f2}\left[\frac{p_f\left(q_f\right)\phi_f'(n)n-p_f\left(q_f\right)\phi_f(n)}{n^2}+p_f'\left(q_f\right)\frac{dq_f}{dn}\frac{\phi_f(n)}{n}\right]\\&+\frac{n^2}{\phi_f(n)g_f}\frac{\phi_f(n)}{n}\left\{g_{f3}\left[T_f'(q_f)\frac{\phi_f'(n)n-\phi_f(n)}{n^2}+T_f''(q_f)\frac{dq_f}{dn}\frac{\phi_f(n)}{n}\right]+g_{f4}\left[\frac{p_h\phi_h'(n)n-p_h\phi_h(n)}{n^2}\right]\right\}.\end{aligned}$$

This expression simplifies to

$$\begin{aligned}\frac{d\ln\left(\frac{q_f}{n}\right)}{d\ln(n)}&=\frac{n^2}{\phi_f(n)g_f}\left[\frac{\phi_f'(n)n-\phi_f(n)}{n^2}\right]g_f\\&+\frac{n^2}{\phi_f(n)g_f}\frac{\phi_f(n)}{n}g_{f2}\left[p_f\left(q_f\right)\frac{\phi_f'(n)n-\phi_f(n)}{n^2}+p_f'\left(q_f\right)\frac{dq_f}{dn}\frac{\phi_f(n)}{n}\right]\\&+\frac{n^2}{\phi_f(n)g_f}\frac{\phi_f(n)}{n}g_{f3}\left[T_f'(q_f)\frac{\phi_f'(n)n-\phi_f(n)}{n^2}+T_f''(q_f)\frac{dq_f}{dn}\frac{\phi_f(n)}{n}\right]\\&+\frac{n^2}{\phi_f(n)g_f}\frac{\phi_f(n)}{n}g_{f4}p_h\left[\frac{\phi_h'(n)n-\phi_h(n)}{n^2}\right]\end{aligned}$$

or, equivalently,

$$\frac{d\ln\left(\frac{q_f}{n}\right)}{d\ln(n)} = \frac{\phi_f'(n)n}{\phi_f(n)} - 1 + \left[\frac{\phi_f'(n)n}{\phi_f(n)} - 1\right]\frac{\phi_f(n)}{n}\left[\frac{g_{f2}p_f\left(q_f\right) + g_{f3}T_f'(q_f)}{g_f}\right] + \phi_f(n)\left[\frac{g_{f2}}{g_f}p_f'\left(q_f\right) + \frac{g_{f3}}{g_f}T_f''(q_f)\right]\frac{dq_f}{dn} + \frac{g_{f4}}{g_f}p_h\frac{\phi_h(n)}{n}\left[\frac{\phi_h'(n)n}{\phi_h(n)} - 1\right]. \tag{28}$$

Now, observe that

$$\frac{d\left(\frac{q_f}{n}\right)}{dn} = \frac{d\ln\left(\frac{q_f}{n}\right)}{d\ln(n)}\frac{q_f}{n^2} \quad \text{and} \quad \frac{d\left(\frac{q_f}{n}\right)}{dn} = \frac{\left(\frac{dq_f}{dn}n - q_f\right)}{n^2} = \frac{dq_f}{dn}\frac{1}{n} - \frac{q_f}{n^2}$$

so that

$$\frac{dq_f}{dn} = \left[\frac{d\left(\frac{q_f}{n}\right)}{dn} + \frac{q_f}{n^2}\right]n = \left[\frac{d\ln\left(\frac{q_f}{n}\right)}{d\ln(n)}\frac{q_f}{n} + \frac{q_f}{n}\right] = \left[1 + \frac{d\ln\left(\frac{q_f}{n}\right)}{d\ln(n)}\right]\frac{q_f}{n}.$$

Then, it follows from (28) that

$$\frac{d\ln\left(\frac{q_f}{n}\right)}{d\ln(n)} = \frac{\phi_f'(n)n}{\phi_f(n)} - 1 + \left[\frac{\phi_f'(n)n}{\phi_f(n)} - 1\right]\frac{\phi_f(n)}{n}\left[\frac{g_{f2}p_f\left(q_f\right) + g_{f3}T_f'(q_f)}{g_f}\right] + \phi_f(n)\left[\frac{g_{f2}}{g_f}p_f'\left(q_f\right) + \frac{g_{f3}}{g_f}T_f''(q_f)\right]\left[1 + \frac{d\ln\left(\frac{q_f}{n}\right)}{d\ln(n)}\right]\frac{q_f}{n} + \frac{g_{f4}}{g_f}p_h\frac{\phi_h(n)}{n}\left[\frac{\phi_h'(n)n}{\phi_h(n)} - 1\right]$$

or, equivalently,

$$\frac{d\ln\left(\frac{q_f}{n}\right)}{d\ln(n)} = \frac{\begin{array}{c}\frac{\phi_f'(n)n}{\phi_f(n)} - 1 + \left[\frac{\phi_f'(n)n}{\phi_f(n)} - 1\right]\frac{\phi_f(n)}{n}\left[\frac{g_{f2}p_f\left(q_f\right)+g_{f3}T_f'(q_f)}{g_f}\right] + \frac{g_{f4}}{g_f}p_h\frac{\phi_h(n)}{n}\left[\frac{\phi_h'(n)n}{\phi_h(n)} - 1\right] \\ +\phi_f(n)\left[\frac{g_{f2}}{g_f}p_f'\left(q_f\right) + \frac{g_{f3}}{g_f}T_f''(q_f)\right]\frac{q_f}{n}\end{array}}{1 - \phi_f(n)\left[\frac{g_{f2}}{g_f}p_f'\left(q_f\right) + \frac{g_{f3}}{g_f}T_f''(q_f)\right]\frac{q_f}{n}}. \tag{29}$$

Recall that in DP,

$$\sigma_i \equiv 1 - \frac{d\ln\left(\phi_i(n)\right)}{d\ln(n)} = 1 - \frac{d\phi_i(n)}{dn}\frac{n}{\phi_i(n)} = 1 - \frac{n\phi_i'(n)}{\phi_i(n)}.$$

Hence, (29) becomes

$$\frac{d\ln\left(\frac{q_f}{n}\right)}{d\ln(n)} = \frac{-\sigma_f - \sigma_f\frac{\phi_f(n)}{n}\left[\frac{g_{f2}p_f\left(q_f\right)+g_{f3}T_f'(q_f)}{g_f}\right] - \sigma_h\frac{g_{f4}}{g_f}p_h\frac{\phi_h(n)}{n} + \phi_f(n)\left[\frac{g_{f2}}{g_f}p_f'\left(q_f\right) + \frac{g_{f3}}{g_f}T_f''(q_f)\right]\frac{q_f}{n}}{1 - \phi_f(n)\left[\frac{g_{f2}}{g_f}p_f'\left(q_f\right) + \frac{g_{f3}}{g_f}T_f''(q_f)\right]\frac{q_f}{n}}$$

and for $\sigma_f \simeq 0$, this reduces to

$$\frac{d\ln\left(\frac{q_f}{n}\right)}{d\ln(n)} = \frac{-\sigma_h\frac{g_{f4}}{g_f}p_h\frac{\phi_h(n)}{n} + \phi_f(n)\left[\frac{g_{f2}}{g_f}p_f'\left(q_f\right) + \frac{g_{f3}}{g_f}T_f''(q_f)\right]\frac{q_f}{n}}{1 - \phi_f(n)\left[\frac{g_{f2}}{g_f}p_f'\left(q_f\right) + \frac{g_{f3}}{g_f}T_f''(q_f)\right]\frac{q_f}{n}}.$$

Note that the homogeneity property of demand implies that

$$\begin{aligned}\frac{q_f}{n} &= \frac{\phi_f(n)}{n} g_f\left(\frac{x}{n}, p_f(q_f)\frac{\phi_f(n)}{n}, T_f'(q_f)\frac{\phi_f(n)}{n}, p_h\frac{\phi_h(n)}{n}\right) \\ &= \frac{\phi_f(n)}{n} g_f\left(\lambda_o\frac{x}{n}, \lambda_o p_f(q_f)\frac{\phi_f(n)}{n}, \lambda_o T_f'(q_f)\frac{\phi_f(n)}{n}, \lambda_o p_h\frac{\phi_h(n)}{n}\right).\end{aligned}$$

Differentiating both sides by λ_o, evaluating the resulting expression at $\lambda_o = 1$, and dividing by g_f gives

$$\frac{g_{f_1}}{g_f}\frac{x}{n} + \frac{g_{f_2}}{g_f}p_f(q_f)\frac{\phi_f(n)}{n} + \frac{g_{f_3}}{g_f}T_f'(q_f)\frac{\phi_f(n)}{n} + \frac{g_{f_4}}{g_f}p_h\frac{\phi_h(n)}{n} = 0.$$

We can express for $d\ln\left(\frac{q_f}{n}\right)/d\ln(n)$ analogous to that derived by DP as follows. Note that

$$\varepsilon_{fx} = \frac{dq_f}{dx}\frac{x}{q_f} = \phi_f(n)g_{f_1}\frac{1}{n}\frac{x}{\phi_f(n)g_f} = \frac{g_{f_1}}{g_f}\frac{x}{n} > 0,$$

$$\varepsilon_{ff} = \frac{dq_f}{dp_f(q_f)}\frac{p_f(q_f)}{q_f} = \phi_f(n)g_{f_2}\frac{\phi_f(n)}{n}\frac{p_f(q_f)}{\phi_f(n)g_f} = \frac{g_{f_2}}{g_f}\frac{\phi_f(n)}{n}p_f(q_f) < 0,$$

$$\varepsilon_{fT_f'(q_f)} = \frac{dq_f}{dT_f'(q_f)}\frac{T_f'(q_f)}{q_f} = \phi_f(n)g_{f_3}\frac{\phi_f(n)}{n}\frac{T_f'(q_f)}{\phi_f(n)g_f} = \frac{g_{f_3}}{g_f}\frac{\phi_f(n)}{n}T_f'(q_f) < 0,$$

and

$$\varepsilon_{fh} = \frac{dq_f}{dp_h}\frac{p_h}{q_f} = \phi_f(n)g_{f4}\frac{\phi_h(n)}{n}\frac{p_h}{\phi_f(n)g_f} = \frac{g_{f_4}}{g_f}p_h\frac{\phi_h(n)}{n} < 0, \tag{30}$$

it follows that $\varepsilon_{fx} + \varepsilon_{ff} + \varepsilon_{fT_f'(q_f)} + \varepsilon_{fh} = 0$. Therefore,

$$\frac{d\ln\left(\frac{q_f}{n}\right)}{d\ln(n)} = \frac{\sigma_h\left[\frac{g_{f_1}}{g_f}\frac{x}{n} + \frac{g_{f_2}}{g_f}p_f(q_f)\frac{\phi_f(n)}{n} + \frac{g_{f_3}}{g_f}T_f'(q_f)\frac{\phi_f(n)}{n}\right] + \phi_f(n)\left[\frac{g_{f2}}{g_f}p_f'(q_f) + \frac{g_{f3}}{g_f}T_f''(q_f)\right]\frac{q_f}{n}}{1 - \phi_f(n)\left[\frac{g_{f2}}{g_f}p_f'(q_f) + \frac{g_{f3}}{g_f}T_f''(q_f)\right]\frac{q_f}{n}}. \tag{31}$$

Further using that

$$\varepsilon_{p_f(q_f)q_f} = \frac{dp_f(q_f)}{dq_f}\frac{q_f}{p_f(q_f)} = \frac{p_f'(q_f)q_f}{p_f(q_f)} < 0 \quad \text{and} \quad \varepsilon_{T_f'(q_f)q_f} = \frac{dT_f'(q_f)}{dq_f}\frac{q_f}{T_f'(q_f)} = \frac{T_f''(q_f)q_f}{T_f'(q_f)} < 0,$$

we obtain that

$$\begin{aligned}\frac{d\ln\left(\frac{q_f}{n}\right)}{d\ln(n)} &= \frac{\sigma_h\left[\varepsilon_{fx} + \varepsilon_{ff} + \varepsilon_{fT_f'(q_f)}\right] + \left[\frac{\phi_f(n)}{n}\frac{g_{f_2}}{g_f}p_f(q_f)\frac{p_f'(q_f)}{p_f(q_f)}q_f + \frac{\phi_f(n)}{n}\frac{g_{f3}}{g_f}T_f'(q_f)\frac{T_f''(q_f)}{T_f'(q_f)}q_f\right]}{1 - \left[\frac{\phi_f(n)}{n}\frac{g_{f_2}}{g_f}p_f(q_f)\frac{p_f'(q_f)}{p_f(q_f)}q_f + \frac{\phi_f(n)}{n}\frac{g_{f3}}{g_f}T_f'(q_f)\frac{T_f''(q_f)}{T_f'(q_f)}q_f\right]} \\ &= \frac{\sigma_h\left[\varepsilon_{fx} + \varepsilon_{ff} + \varepsilon_{fT_f'(q_f)}\right] + \left[\varepsilon_{ff}\varepsilon_{p_f(q_f)q_f} + \varepsilon_{fT_f'(q_f)}\varepsilon_{T_f'(q_f)q_f}\right]}{1 - \left[\varepsilon_{ff}\varepsilon_{p_f(q_f)q_f} + \varepsilon_{fT_f'(q_f)}\varepsilon_{T_f'(q_f)q_f}\right]},\end{aligned} \tag{32}$$

which is the generalization of the expression in (5) when food is priced nonlinearly.

As for the interpretation of the partial derivative of quantity with respect to the marginal price sched-

ule, denoted throughout as $\partial q_f/\partial T'_f(q_f)$, this derivative can be intuitively understood as a specific perturbation of the more general Gateaux derivative. Specifically, let $T_f(q_f)$ denote the price schedule and let $q^i_f(T_f)$ denote the demand functional dependent on such schedule. We define a specific perturbation $T_{f\epsilon}(q'_f)$ of the price schedule as $T_{f\epsilon}(q'_f) := T_f(q'_f) + \epsilon(q'_f - q_f)$ for all q'_f, where ϵ is a scalar perturbation parameter. The partial derivative of demand with respect to the marginal price is then defined as

$$\frac{\partial q^i_f(T_f)}{\partial T'_f(q_f)} = \lim_{\epsilon \to 0} \frac{q^i_f(T_{f\epsilon}) - q^i_f(T_f)}{\epsilon}. \tag{33}$$

It is important to verify that this limit definition is equivalent to the definition of the "naive" partial derivative $\partial q_f/\partial T'_f(q_f)$. The naive derivative treats $T'_f(q_f)$ as a variable and calculates how demand changes if $T'_f(q_f)$ is increased by a small amount. The Gateaux formulation of the derivative exactly provides this calculation by differentiating the perturbed price schedule with respect to quantity q'_f, namely,

$$\frac{d}{dq'_f} T_{f\epsilon}(q'_f) = \frac{d}{dq'_f} \left[T_f(q'_f) + \epsilon(q'_f - q_f) \right] = T'_f(q'_f) + \epsilon. \tag{34}$$

This result shows that the perturbation $T_{f\epsilon}(q_f)$ effectively shifts the marginal price schedule exactly by the constant ϵ. Thus, the limit term ϵ in the denominator of (33) represents the change in the marginal price ($\Delta T'_f(q_f)$). This makes the limit definition mathematically equivalent to the standard definition of a partial derivative as the ratio of the change in the quantity demanded of food resulting from an infinitesimal (uniform) change in its marginal price $T'_f(q_f)$,

$$\lim_{\epsilon \to 0} \frac{q^i_f(T_{f\epsilon}) - q^i_f(T_f)}{\epsilon} \equiv \frac{\text{Change in Quantity}}{\text{Change in Marginal Price}} = \frac{\partial q_f}{\partial T'_f(q_f)}. \tag{35}$$

A.5 Proof of Proposition 3

By the discussion in the main text, a sufficient condition for the numerator of (15) to be positive is $\rho < 1$—recall that for the CES preferences we posit, food and non-food are complementary goods when $\rho \in (0, 1)$. As for the denominator of (15), using that $T'_f(q_f) = p'_f(q_f)q_f + p_f(q_f)$, it follows that

$$\begin{aligned} &p_f(q_f) + p'_f(q_f)q_f + \left(p_h \frac{\phi_h(n)}{\phi_f(n)} \right)^{1-\rho} \left[\frac{\alpha_h}{\alpha_f} T'_f(q_f) \right]^{\rho} + \left(p_h \frac{\phi_h(n)}{\phi_f(n)} \right)^{1-\rho} \rho \left[\frac{\alpha_h}{\alpha_f} T'_f(q_f) \right]^{\rho-1} \frac{\alpha_h}{\alpha_f} T''_f(q_f) q_f > 0 \\ \Leftrightarrow \; &\left[T'_f(q_f) \right]^{\rho} \left\{ \left[T'_f(q_f) \right]^{1-\rho} + \left(p_h \frac{\phi_h(n)}{\phi_f(n)} \right)^{1-\rho} \left(\frac{\alpha_h}{\alpha_f} \right)^{\rho} \left[1 + \rho \frac{T''_f(q_f) q_f}{T'_f(q_f)} \right] \right\} > 0 \end{aligned}$$

and since $T_f'(q_f) > 0$, the latter displayed inequality is equivalent to

$$\left[T_f'(q_f)\right]^{1-\rho} + \left(p_h \frac{\phi_h(n)}{\phi_f(n)}\right)^{1-\rho} \left(\frac{\alpha_h}{\alpha_f}\right)^{\rho} \left[1 + \rho \frac{T_f''(q_f) q_f}{T_f'(q_f)}\right] > 0,$$

which is exactly (17). Given that $T_f'(q_f) > 0$ and, due to volume discounts, $T_f''(q_f) < 0$, by inspection of (17), it is apparent that this inequality is satisfied for any value of ρ provided that q_f is small enough. The desired result is then an immediate implication of these observations and Proposition 1.

A.6 Proof of Proposition 4

To start, define

$$A(q_f) = \frac{p_f(q_f) + \rho \left[T_f'(q_f)\right]^{\rho}}{p_f(q_f) + \left[T_f'(q_f)\right]^{\rho}} = \frac{\beta_0 q_f^{\beta_1} + \rho \left[\beta_0 q_f^{\beta_1} (1+\beta_1)\right]^{\rho}}{\beta_0 q_f^{\beta_1} + \left[\beta_0 q_f^{\beta_1} (1+\beta_1)\right]^{\rho}} = \frac{1 + \rho \beta_0^{\rho-1} (1+\beta_1)^{\rho} q_f^{\beta_1(\rho-1)}}{1 + \beta_0^{\rho-1} (1+\beta_1)^{\rho} q_f^{\beta_1(\rho-1)}}$$

and

$$\begin{aligned} B(q_f) &= \frac{p_f'(q_f) + \rho \left[T_f'(q_f)\right]^{\rho-1} T_f''(q_f)}{p_f(q_f) + \left[T_f'(q_f)\right]^{\rho}} q_f = \frac{\beta_0 \beta_1 q_f^{\beta_1 - 1} q_f + \rho \left[\beta_0 q_f^{\beta_1} (1+\beta_1)\right]^{\rho-1} \beta_0 \beta_1 q_f^{\beta_1 - 1} (1+\beta_1) q_f}{\beta_0 q_f^{\beta_1} + \left[\beta_0 q_f^{\beta_1} (1+\beta_1)\right]^{\rho}} \\ &= \beta_1 \frac{1 + \rho \beta_0^{\rho-1} (1+\beta_1)^{\rho} q_f^{\beta_1(\rho-1)}}{1 + \beta_0^{\rho-1} (1+\beta_1)^{\rho} q_f^{\beta_1(\rho-1)}}. \end{aligned}$$

We can then express $d \ln\left(\frac{q_f}{n}\right) / d \ln(n)$ as

$$\begin{aligned} \frac{d \ln\left(\frac{q_f}{n}\right)}{d \ln(n)} &= \frac{1 - A(q_f) - B(q_f)}{1 + B(q_f)} = \frac{1 - \frac{1+\rho q_f^{\beta_1(\rho-1)} \beta_0^{\rho-1} (1+\beta_1)^{\rho}}{1 + q_f^{\beta_1(\rho-1)} \beta_0^{\rho-1} (1+\beta_1)^{\rho}} - \beta_1 \frac{1+\rho q_f^{\beta_1(\rho-1)} \beta_0^{\rho-1} (1+\beta_1)^{\rho}}{1 + q_f^{\beta_1(\rho-1)} \beta_0^{\rho-1} (1+\beta_1)^{\rho}}}{1 + \beta_1 \frac{1+\rho q_f^{\beta_1(\rho-1)} \beta_0^{\rho-1} (1+\beta_1)^{\rho}}{1 + q_f^{\beta_1(\rho-1)} \beta_0^{\rho-1} (1+\beta_1)^{\rho}}} \\ &= \frac{1 + q_f^{\beta_1(\rho-1)} \beta_0^{\rho-1} (1+\beta_1)^{\rho} - \left[1 + \rho q_f^{\beta_1(\rho-1)} \beta_0^{\rho-1} (1+\beta_1)^{\rho}\right] - \beta_1 \left[1 + \rho q_f^{\beta_1(\rho-1)} \beta_0^{\rho-1} (1+\beta_1)^{\rho}\right]}{1 + q_f^{\beta_1(\rho-1)} \beta_0^{\rho-1} (1+\beta_1)^{\rho} + \beta_1 \left[1 + \rho q_f^{\beta_1(\rho-1)} \beta_0^{\rho-1} (1+\beta_1)^{\rho}\right]} \\ &= \frac{-\beta_1 + (1 - \rho - \beta_1 \rho) \beta_0^{\rho-1} (1+\beta_1)^{\rho} q_f^{\beta_1(\rho-1)}}{1 + \beta_1 + (1 + \beta_1 \rho) \beta_0^{\rho-1} (1+\beta_1)^{\rho} q_f^{\beta_1(\rho-1)}} = \frac{|\beta_1| + (1 - \rho + |\beta_1| \rho) \beta_0^{\rho-1} (1 - |\beta_1|)^{\rho} q_f^{|\beta_1|(1-\rho)}}{1 - |\beta_1| + (1 - |\beta_1| \rho) \beta_0^{\rho-1} (1 - |\beta_1|)^{\rho} q_f^{|\beta_1|(1-\rho)}}. \end{aligned}$$

Define $R(q_f) = d \ln\left(\frac{q_f}{n}\right) / d \ln(n)$ and observe that $dR(q_f)/dq_f \gtrless 0$ if, and only if,

$$\begin{aligned} &|\beta_1| (1-\rho) (1 - \rho + |\beta_1| \rho) \beta_0^{\rho-1} (1 - |\beta_1|)^{\rho} q_f^{|\beta_1|(1-\rho)-1} \left[1 - |\beta_1| + (1 - |\beta_1| \rho) \beta_0^{\rho-1} (1 - |\beta_1|)^{\rho} q_f^{|\beta_1|(1-\rho)}\right] \\ &- |\beta_1| (1-\rho) \left[|\beta_1| + (1 - \rho + |\beta_1| \rho) \beta_0^{\rho-1} (1 - |\beta_1|)^{\rho} q_f^{|\beta_1|(1-\rho)}\right] (1 - |\beta_1| \rho) \beta_0^{\rho-1} (1 - |\beta_1|)^{\rho} q_f^{|\beta_1|(1-\rho)-1} \gtrless 0 \end{aligned}$$

$$\Leftrightarrow |\beta_1|(1-\rho)(1-\rho+|\beta_1|\rho)\left[1-|\beta_1|+(1-|\beta_1|\rho)\beta_0^{\rho-1}(1-|\beta_1|)^{\rho}q_f^{|\beta_1|(1-\rho)}\right]$$
$$-|\beta_1|(1-\rho)\left[|\beta_1|+(1-\rho+|\beta_1|\rho)\beta_0^{\rho-1}(1-|\beta_1|)^{\rho}q_f^{|\beta_1|(1-\rho)}\right](1-|\beta_1|\rho)\gtrless 0.$$

This latter condition, using that $|\beta_1|(1-\rho)>0$ for $\rho<1$, is equivalent to $(1-\rho)[1-2|\beta_1|]\gtrless 0$.

Thus, $R(q_f)$ strictly *increases* with q_f if $|\beta_1|<1/2$ and strictly *decreases* with q_f if $|\beta_1|>1/2$. Since $R(q_f)|_{q_f=0}=|\beta_1|/1-|\beta_1|$ and

$$R(q_f)|_{q_f\to\infty}=\lim_{q_f\to\infty}\frac{(1-\rho+|\beta_1|\rho)\beta_0^{\rho-1}(1-|\beta_1|)^{\rho}|\beta_1|(1-\rho)q_f^{|\beta_1|(1-\rho)-1}}{|\beta_1|(1-\rho)(1-|\beta_1|\rho)\beta_0^{\rho-1}(1-|\beta_1|)^{\rho}q_f^{|\beta_1|(1-\rho)-1}}=\frac{1-\rho+|\beta_1|\rho}{1-|\beta_1|\rho},$$

it follows that for $|\beta_1|<1/2$,

$$\frac{d\ln\left(\frac{q_f}{n}\right)}{d\ln(n)}\in\left(\frac{|\beta_1|}{1-|\beta_1|},\frac{1-\rho+|\beta_1|\rho}{1-|\beta_1|\rho}\right),$$

whereas for $|\beta_1|>1/2$,

$$\frac{d\ln\left(\frac{q_f}{n}\right)}{d\ln(n)}\in\left(\frac{1-\rho+|\beta_1|\rho}{1-|\beta_1|\rho},\frac{|\beta_1|}{1-|\beta_1|}\right).$$

Condition (19) then immediately follows.

B Results Appendix: Additional Results and Robustness Checks

This section collects additional results, which include robustness checks for the results in the main text as well as additional results that complement some of those discussed there.

B.1 Price Nonlinearities

We validate quantity discounts by using direct information on store prices available in our data (Table 5). The November 2000 survey collects price information as part of the evaluation questionnaire administered at the locality level—namely, information on the prices at one store in each village for rice, beans, corn flour, wheat flour, corn, vegetable oil, and salt. Respondents are asked about both the price per kilo of any such food item purchased and whether the price for a quantity lower than a kilo would be *higher*. If the answer is yes, then respondents are asked for the price of 1/4 of a kilo of the same food item. This information allows us to directly estimate quantity discounts, which we report in Table 5 by food item. As the table shows, quantity discounts at low levels of consumption are indeed substantial.

Table 5: Store-Level Price Information and Price Nonlinearities

Food Item	Observations	Mean Yes Discount
Wheat Flour	216	0.092
Rice	348	0.195
Corn	253	0.083
Vegetal Oil	380	0.055
Beans	330	0.084
Corn Flour	214	0.046
Salt	359	0.050

Note: Data from the November 2000 store survey. "Observations" is the number of localities with price information for each food item. "Mean Yes Discount" is the share of localities where respondents answered "Yes" to the question: "If you buy less than a kilo, would the price per kilo be higher?"

We then present results of regressing log unit values directly on log household size for the 3 staple foods (Table 6).

Table 6: Unit Prices and Household Size

	(1) Log UV Rice	(2) Log UV Beans	(3) Log UV Sugar
Log of Household Size	-0.029***	-0.032***	-0.012***
	(0.004)	(0.004)	(0.002)
Constant	1.864***	2.181***	1.654***
	(0.012)	(0.012)	(0.006)
Observations	64,732	93,738	104,098
R-squared	0.240	0.194	0.112
Controls	Yes	Yes	Yes

Note: Standard errors are clustered at the locality level (*** stands for 1% significance, ** stands for 5% significance, and * for 10% significance). All waves are used and wave effects are included in the estimation.

B.2 Replication of DP Regression

We re-estimate the DP regression using an IV approach to account for the endogeneity of per-capita expenditure (Table 7). We also show results restricting to localities for which the estimated locality price slope is between 0 and -1, which is the range theoretically consistent with our model (Table 8). In Table 9 we provide some evidence on the second theoretical prediction from Deaton and Paxson (1998), that with PCE and household size held constant, the substitution of a child for an adult should decrease the per capita demand for food and thus the food share. In Table 10, we provide DP regression results for the sample of all treatment and control villages.

To control for the age composition of family members,we re-estimate the DP regression using log

Table 7: Replication of DP Puzzle (IV)

	(1) 1998_02	(2) 1999_01	(3) 1999_02	(4) 2000_02	(5) 2003	(6) FE
Log of Household Size	-0.066***	-0.028	-0.037***	-0.055***	-0.060***	-0.042***
	(0.011)	(0.017)	(0.014)	(0.013)	(0.009)	(0.010)
Log of Per-capita (total) expenditure	-0.133***	-0.068**	-0.088***	-0.091***	-0.118***	-0.091***
	(0.019)	(0.030)	(0.024)	(0.020)	(0.017)	(0.017)
Household head works (dummy)	0.015***	0.016	0.009	0.009	-0.001	0.007
	(0.006)	(0.014)	(0.007)	(0.006)	(0.006)	(0.006)
Household is classified as poor by the program	0.023***	0.031***	0.028***	0.024***	0.015*	
	(0.003)	(0.007)	(0.005)	(0.004)	(0.009)	
Observations	19,042	5,958	7,825	7,633	5,466	17,710
R-squared	0.068	0.097	0.129	0.113	0.141	0.097
Weak identification (Kleibergen-Paap) Wald F	98.33	42.72	72.02	109.7	107.3	135.4
Stock-Yogo critical value (10%)	19.93	19.93	19.93	19.93	19.93	19.93
Underidentifaction (Kleibergen-Paap) p-val	0	1.36e-10	0	0	0	0
Overidentification (Hensen J) p-val	0.205	0.811	0.915	0.488	0.446	0.286

Note: Standard errors are clustered at the locality level (*** stands for 1% significance, ** stands for 5% significance, and * for 10% significance). Wave Oct-98 includes both treatment and control villages. Other waves include only controls. FE column reports the results of a fixed effect regression on the balanced panel of households present in the first 3 waves: Oct-98, Mar-99 and Nov-99. log of per-capita (total) expenditure is instrumented with a second-order polynomial of log total income.

Table 8: Replication of DP Puzzle (w/ Locality Price Slope Between 0 and 1)

	(1) 1998_02	(2) 1999_01	(3) 1999_02	(4) 2000_02	(5) 2003	(6) FE
Log of Household Size	-0.036***	-0.056***	-0.064***	-0.059***	-0.037*	-0.062***
	(0.008)	(0.014)	(0.013)	(0.013)	(0.020)	(0.007)
Log of Per-capita (total) expenditure	-0.079***	-0.089***	-0.141***	-0.095***	-0.113***	-0.107***
	(0.010)	(0.016)	(0.013)	(0.017)	(0.016)	(0.009)
Household head works (dummy)	0.013	0.026	0.004	0.010	-0.010	0.014
	(0.008)	(0.021)	(0.015)	(0.018)	(0.013)	(0.009)
Household is classified as poor by the program	0.021***	0.041***	0.020	0.026**	0.000	
	(0.006)	(0.010)	(0.013)	(0.011)	(0.016)	
Constant	1.217***	1.207***	1.347***	1.247***	1.233***	1.263***
	(0.060)	(0.077)	(0.066)	(0.085)	(0.115)	(0.040)
Observations	3,908	1,330	1,332	1,240	1,239	3,348
R-squared	0.185	0.280	0.298	0.287	0.377	0.284

Note: Standard errors are clustered at the locality level (*** stands for 1% significance, ** stands for 5% significance, and * for 10% significance). Wave Oct-98 includes both treatment and control villages. Other waves include only controls. FE column reports the results of a fixed effect regression on the balanced panel of households present in the first 3 waves: Oct-98, Mar-99 and Nov-99. Sample is restricted to those localities for which the estimated slope (for all 3 staple foods: rice, beans, and sugar) of the locality price schedule is between 0 and -1.

Table 9: Replication of DP Puzzle (Young Children)

	(1) 1998_02	(2) 1999_01	(3) 1999_02	(4) 2000_02	(5) 2003	(6) FE
Ratio of Children age 0-4 to HH Size	-0.026***	-0.015	-0.005	-0.024**	-0.006	0.004
	(0.008)	(0.014)	(0.012)	(0.010)	(0.015)	(0.007)
Log of Household Size	-0.043***	-0.048***	-0.055***	-0.055***	-0.057***	-0.049***
	(0.003)	(0.005)	(0.005)	(0.005)	(0.007)	(0.003)
Log of Per-capita (total) expenditure	-0.088***	-0.100***	-0.110***	-0.091***	-0.116***	-0.098***
	(0.004)	(0.006)	(0.006)	(0.006)	(0.008)	(0.004)
Constant	1.179***	1.179***	1.217***	1.174***	1.258***	1.200***
	(0.017)	(0.030)	(0.028)	(0.029)	(0.045)	(0.018)
Observations	22,780	8,104	8,605	7,994	6,380	21,131
R-squared	0.259	0.272	0.303	0.307	0.372	0.283

Note: Standard errors are clustered at the locality level (*** stands for 1% significance, ** stands for 5% significance, and * for 10% significance). Wave Oct-98 includes both treatment and control villages. Other waves include only controls. FE column reports the results of a fixed effect regression on the balanced panel of households present in the first 3 waves: Oct-98, Mar-99 and Nov-99. The variable "Ratio of Children age 0-4 to HH Size" measures the proportion of young children in the household.

adult-equivalent size in place of log household size (Table 11).

Table 10: Replication of DP Puzzle (Treatment and Control Villages)

	(1) 1998_02	(2) 1999_01	(3) 1999_02	(4) 2000_02	(5) 2003	(6) FE
Log of Household Size	-0.042***	-0.048***	-0.044***	-0.054***	-0.061***	-0.045***
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.002)
Log of Per-capita (total) expenditure	-0.088***	-0.096***	-0.095***	-0.092***	-0.103***	-0.091***
	(0.004)	(0.004)	(0.004)	(0.004)	(0.003)	(0.002)
Household head works (dummy)	0.010**	0.005	0.013***	0.013***	0.013***	0.008***
	(0.004)	(0.004)	(0.004)	(0.004)	(0.002)	(0.002)
Household is classified as poor by the program	0.028***	0.027***	0.023***	0.026***	0.033***	
	(0.002)	(0.003)	(0.003)	(0.003)	(0.003)	
Constant	1.200***	1.219***	1.141***	1.176***	1.179***	1.169***
	(0.022)	(0.024)	(0.019)	(0.019)	(0.017)	(0.011)
Observations	22,780	21,088	21,793	20,498	32,576	53,995
R-squared	0.261	0.284	0.288	0.304	0.321	0.283

Note: Standard errors are clustered at the locality level (*** stands for 1% significance, ** stands for 5% significance, and * for 10% significance).FE column reports the results of a fixed effect regression on the balanced panel of households present in the first 3 waves: Oct-98, Mar-99 and Nov-99.

Table 11: DP Regression with Adult Equivalent Size

	(1) Food Share	(2) Rice	(3) Beans	(4) Sugar
Log of Adult Equivalent Size	-0.029***	-0.001**	-0.013***	-0.007***
	(0.002)	(0.001)	(0.001)	(0.001)
Log of Per-capita (total) expenditure	-0.093***	0.001	-0.037***	-0.014***
	(0.004)	(0.001)	(0.002)	(0.002)
Constant	1.118***	0.020***	0.253***	0.115***
	(0.016)	(0.004)	(0.008)	(0.006)
Observations	20,371	20,371	20,371	20,371
R-squared	0.278	0.070	0.203	0.156
Panels	Bal 123	Bal 123	Bal 123	Bal 123
Controls	Yes	Yes	Yes	Yes

Note: Standard errors are clustered at the locality level (*** stands for 1% significance, ** stands for 5% significance, and * for 10% significance). Wave Oct-98 includes both treatment and control villages. Other waves include only control. Bal 123 means that the sample is a balanced panel of households present in the first 3 waves: Oct-98, Mar-99 and Nov-99.

B.3 DP Regression and Nonlinear Pricing Intensity

To take into account that the estimation of the DP regression with nonlinearing pricing intensity involves multiple stages, we present results with bootstrapped standard errors (Table 12). We show results using an IV approach to an IV approach to account for the endogeneity of per-capita expenditure (Table 13). We repeat the same nonlinear pricing intensity analysis on the whole sample (that is, including both treatment and control villages), see results in Table 14.

In Tables 15 to 18, we repeat the same analysis using a different proxy on nonlinear pricing intensity, which is restricting to the sample of households with quantities purchased of staple foods within the first decile (defined in each village).

Finally, in Figure 8 we repeat for each wave the analysis above on the relationship between coefficients on household size and price slopes for staples.

Table 12: Deaton-Paxson Regression and Nonlinear Pricing Intensity (Bootstrap Standard Errors)

	(1) Effect of Nonlinearities	(2) Above Median	(3) Above Median Strict
Log of Per-capita (total) expenditure	-0.099***	-0.099***	-0.099***
	(0.005)	(0.005)	(0.005)
Log of Household Size	-0.058***	-0.053***	-0.049***
	(0.006)	(0.004)	(0.004)
Quartile 2 price slope X log HH Size (highly nonlinear)	0.010*		
	(0.006)		
Quartile 3 price slope X log HH Size (medium nonlinear)	0.010		
	(0.006)		
Quartile 4 price slope X log HH Size (least nonlinear)	0.025***		
	(0.007)		
Above median price slope X log HH size (less nonlinear)		0.010**	
		(0.005)	
Above median price slope (strict) X log HH size (less nonlinear)			0.009
			(0.006)
Observations	21,131	21,131	21,131
Panels	Bal 123	Bal 123	Bal 123
Controls	Yes	Yes	Yes
SE	Bootstrap	Bootstrap	Bootstrap

Note: Bootstrap standard errors in parentheses (199 replications, resampling localities with replacement). All regressions include locality-time fixed effects and control for household demographic composition and head's employment status. *** stands for 1% significance, ** stands for 5% significance, and * for 10% significance. Wave Oct-98 includes both treatment and control villages. Other waves include only control. Bal 123 means that the sample is a balanced panel of households present in the first three waves: Oct-98, Mar-99 and Nov-99. In column (1) and (2), the quartiles and the "Above Median" dummy are based on the distribution of the locality weighted average price slopes for food staples (that is, we first estimate the price slopes of rice, beans, and sugar for each locality and then average them, in each locality, using their food share as weights). In column (3), "Above Median Strict" uses a stricter criteria under which the locality price slopes estimated for rice, beans, and sugar are, at the same time, above the median of each food distribution (less negative=less nonlinearity).

Table 13: Deaton-Paxson Regression and Nonlinear Pricing Intensity (IV)

	(1) Effect on Nonlinearities	(2) Above Median	(3) Above Median Strict
Log of Household Size	-0.055***	-0.046***	-0.044***
	(0.011)	(0.010)	(0.010)
Log of Per-capita (total) expenditure	-0.091***	-0.092***	-0.092***
	(0.017)	(0.018)	(0.017)
Quartile 2 price slope X log HH Size (highly nonlinear)	0.016***		
	(0.006)		
Quartile 3 price slope X log HH Size (medium nonlinear)	0.013**		
	(0.006)		
Quartile 4 price slope X log HH Size (least nonlinear)	0.025***		
	(0.007)		
Above median price slope X log HH size (less nonlinear)		0.009**	
		(0.004)	
Above median price slope (strict) X log HH size (less nonlinear)			0.011*
			(0.006)
Observations	17,710	17,710	17,710
R-squared	0.098	0.097	0.097
Weak identification (Kleibergen-Paap) Wald F	135.7	135.5	135
Stock-Yogo critical value (10%)	19.93	19.93	19.93
Underidentifaction (Kleibergen-Paap) p-val	0	0	0
Overidentification (Hensen J) p-val	0.287	0.282	0.270

Note: Standard errors are clustered at the locality level (*** stands for 1% significance, ** stands for 5% significance, and * for 10% significance). Wave Oct-98 includes both treatment and control villages. Other waves include only control. Bal 123 means that the sample is a balanced panel of households present in the first three waves: Oct-98, Mar-99 and Nov-99. In column (1) and (2), the quartiles and the "Above Median" dummy are based on the distribution of the locality weighted average price slopes for food staples (that is, we first estimate the price slopes of rice, beans, and sugar for each locality and then average them, in each locality, using their food share as weights). In column (3), "Above Median Strict" uses a stricter criteria under which the locality price slopes estimated for rice, beans, and sugar are, at the same time, above the median of each food distribution (less negative=less nonlinearity). Log of per-capita (total) expenditure is instrumented with a second-order polynomial of log total income.

Table 14: Deaton-Paxson Regression and Nonlinear Pricing Intensity (Treatment and Control Villages)

	(1) Effect on Nonlinearities	(2) Above Median	(3) Above Median Strict
Log of Per-capita (total) expenditure	-0.091***	-0.091***	-0.091***
	(0.002)	(0.002)	(0.002)
Log of Household Size	-0.047***	-0.048***	-0.047***
	(0.003)	(0.002)	(0.002)
Quartile 2 price slope X log HH Size (highly nonlinear)	-0.001		
	(0.003)		
Quartile 3 price slope X log HH Size (medium nonlinear)	-0.001		
	(0.003)		
Quartile 4 price slope X log HH Size (least nonlinear)	0.010***		
	(0.003)		
Above median price slope X log HH size (less nonlinear)		0.005**	
		(0.002)	
Above median price slope (strict) X log HH size (less nonlinear)			0.008**
			(0.003)
Constant	1.169***	1.169***	1.169***
	(0.011)	(0.011)	(0.012)
Observations	53,995	53,995	53,995
R-squared	0.283	0.283	0.283
Panels	Bal 123	Bal 123	Bal 123
Controls	Yes	Yes	Yes

Note: Standard errors are clustered at the locality level (*** stands for 1% significance, ** stands for 5% significance, and * for 10% significance). Wave Oct-98 includes both treatment and control villages. Other waves include only control. Bal 123 means that the sample is a balanced panel of households present in the first three waves: Oct-98, Mar-99 and Nov-99. In column (1) and (2), the quartiles and the "Above Median" dummy are based on the distribution of the locality weighted average price slopes for food staples (that is, we first estimate the price slopes of rice, beans, and sugar for each locality and then average them, in each locality, using their food share as weights). In column (3), "Above Median Strict" uses a stricter criteria under which the locality price slopes estimated for rice, beans, and sugar are, at the same time, above the median of each food distribution (less negative=less nonlinearity).

Table 15: DP Regression and Small Quantity Purchases for First Decile of Staple Food Purchases

	(1) FE	(2) D1QP Rice	(3) D1QP Beans	(4) D1QP Sugar	(5) D1QP 3 foods
Log of Household Size	-0.062***	-0.088***	-0.112***	-0.098***	-0.105**
	(0.007)	(0.018)	(0.013)	(0.014)	(0.049)
Log of Per-capita (total) expenditure	-0.107***	-0.094***	-0.145***	-0.129***	-0.098*
	(0.009)	(0.018)	(0.012)	(0.013)	(0.055)
Constant	1.263***	1.170***	1.465***	1.357***	1.138***
	(0.040)	(0.084)	(0.062)	(0.061)	(0.232)
Observations	3,348	370	1,309	1,521	141
R-squared	0.284	0.452	0.378	0.349	0.551
Panels	Bal 123	Bal 123	Bal 123	Bal 123	Bal 123
Controls	Yes	Yes	Yes	Yes	Yes

Note: Standard errors are clustered at the locality level (*** stands for 1% significance, ** stands for 5% significance, and * for 10% significance). Wave Oct-98 includes both treatment and control villages. Other waves include only control. Bal 123 means that the sample is a balanced panel of households present in the first 3 waves: Oct-98, Mar-99 and Nov-99. The prefix D1QP means that the sample includes only households with quantity purchased in the first decile of rice distribution (column 2), beans' (column 3), and sugar's (column 4). "D1QP All foods" means that only households that purchases quantity in the first decile of each of the three distributions, at the same time, are considered.

Table 16: DP Regression and Small Quantity Purchases for First Decile of Staple Food Purchases (IV)

	(1) FE	(2) D1QP Rice	(3) D1QP Beans	(4) D1QP Sugar	(5) D1QP 3 foods
Log of Household Size	-0.042***	-0.165***	-0.079***	-0.105***	-0.115
	(0.010)	(0.047)	(0.021)	(0.019)	(0.131)
Log of Per-capita (total) expenditure	-0.091***	-0.274***	-0.124***	-0.169***	-0.146
	(0.017)	(0.074)	(0.031)	(0.029)	(0.170)
Observations	17,710	1,437	6,640	8,017	457
R-squared	0.097	-0.113	0.124	0.111	0.144
Panels	Bal 123	Bal 123	Bal 123	Bal 123	Bal 123
Controls	Yes	Yes	Yes	Yes	Yes
Weak identification (Kleibergen-Paap) Wald F	135.4	8.780	61.44	54.70	1.568
Stock-Yogo critical value (10%)	19.93	19.93	19.93	19.93	19.93
Underidentifaction (Kleibergen-Paap) p-val	0	0.000783	0	0	0.235
Overidentification (Hensen J) p-val	0.286	0.502	0.0475	0.507	0.0430

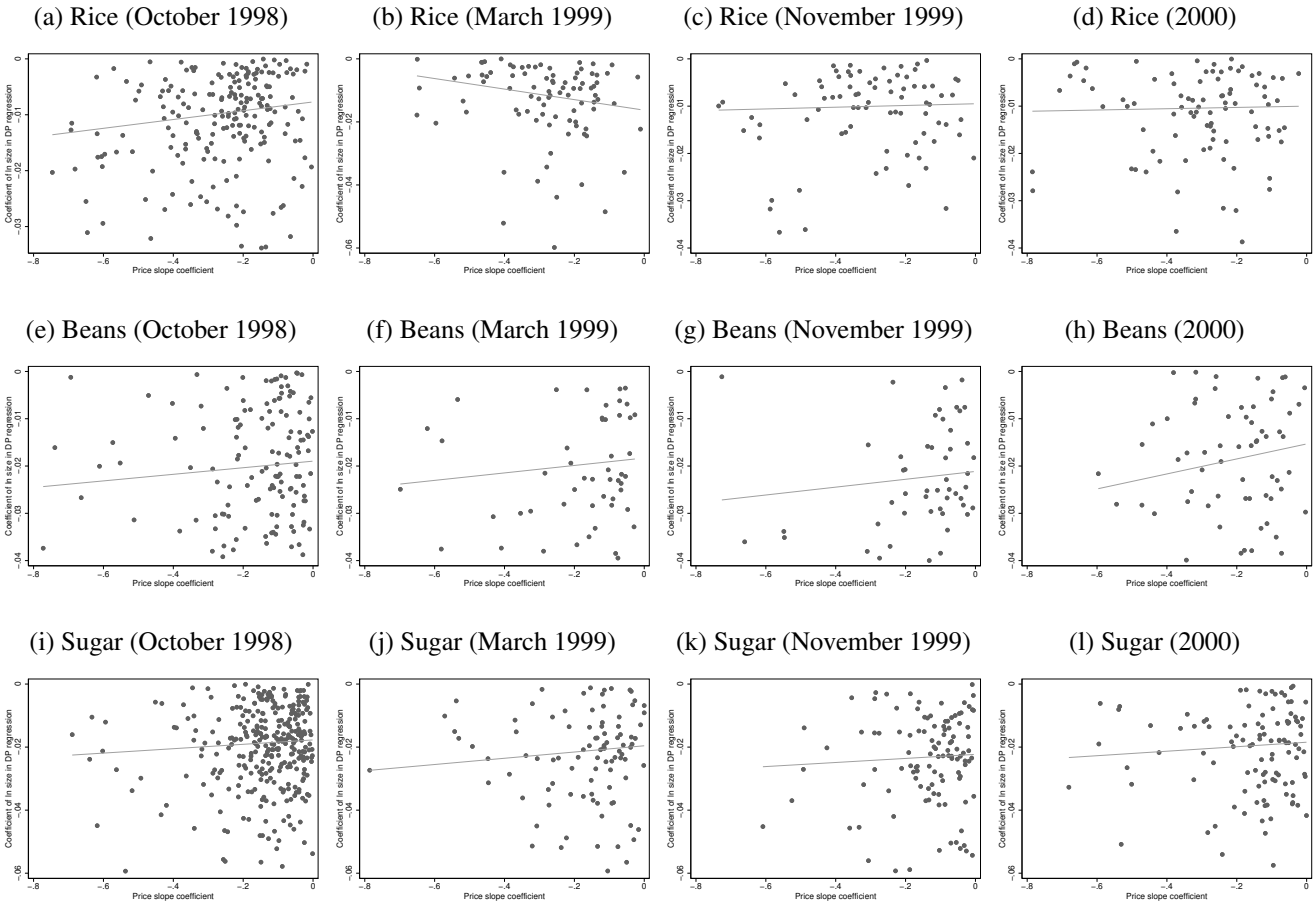
Note: Standard errors are clustered at the locality level (*** stands for 1% significance, ** stands for 5% significance, and * for 10% significance). Wave Oct-98 includes both treatment and control villages. Other waves include only control. Bal 123 means that the sample is a balanced panel of households present in the first 3 waves: Oct-98, Mar-99 and Nov-99. The prefix D1QP means that the sample includes only households with quantity purchased in the first decile of rice distribution (column 2), beans' (column 3), and sugar's (column 4). "D1QP All foods" means that only households that purchases quantity in the first decile of each of the three distributions, at the same time, are considered. Log of per-capita (total) expenditure is instrumented with a second-order polynomial of log total income.

Table 17: DP Regression and Small Quantity Purchases (First Decile of Staple Food Purchases w/ Locality Price Slope Between 0 and 1)

	(1) FE	(2) D1QP Rice	(3) D1QP Beans	(4) D1QP Sugar	(5) D1QP 3 foods
Log of Household Size	-0.062***	-0.088***	-0.112***	-0.098***	-0.105**
	(0.007)	(0.018)	(0.013)	(0.014)	(0.049)
Log of Per-capita (total) expenditure	-0.107***	-0.094***	-0.145***	-0.129***	-0.098*
	(0.009)	(0.018)	(0.012)	(0.013)	(0.055)
Constant	1.263***	1.170***	1.465***	1.357***	1.138***
	(0.040)	(0.084)	(0.062)	(0.061)	(0.232)
Observations	3,348	370	1,309	1,521	141
R-squared	0.284	0.452	0.378	0.349	0.551
Panels	Bal 123	Bal 123	Bal 123	Bal 123	Bal 123
Controls	Yes	Yes	Yes	Yes	Yes

Note: Standard errors are clustered at the locality level (*** stands for 1% significance, ** stands for 5% significance, and * for 10% significance). Wave Oct-98 includes both treatment and control villages. Other waves include only control. Bal 123 means that the sample is a balanced panel of households present in the first 3 waves: Oct-98, Mar-99 and Nov-99. The prefix D1QP means that the sample includes only households with quantity purchased in the first decile of rice distribution (column 2), beans' (column 3), and sugar's (column 4). "D1QP All foods" means that only households who purchases quantity in the first decile of each of the three distributions, at the same time, are considered. Sample is restricted to those localities for which the estimated slope for all 3 staple foods of the locality price schedule is between 0 and -1.

Figure 8: Relationship Between Coefficients on Household Size and Price Slopes for Staples by Wave



Note: Each dot represents a locality-level estimated DP coefficient and price slope. DP coefficients are estimated in each village using the specification in (22) on the sample of the first three survey waves (wave Oct-98 includes both treatment and control villages, other waves include only control). Price slopes are estimated in each village using the specification (23) on the sample of all survey waves. Reported estimates are trimmed above 0, that is, we focus on localities for which we found the relevant staple priced nonlinearly (price slope < 0) and the presence of DP puzzle (DP coefficient < 0). The solid line in the graph is a linear fit line weighted by the number of observations in each locality; localities with fewer than 100 observations are excluded. A positive slope for the fitted line confirms the model's prediction that foods with steeper quantity discounts (more negative price slopes) exhibit a stronger DP puzzle (more negative coefficients on household size).

Table 18: DP Regression and Small Quantity Purchases (Young Children)

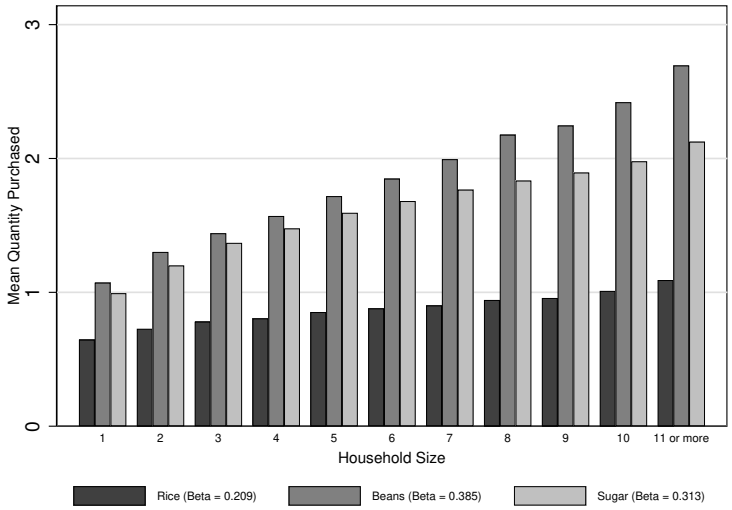
	(1) FE	(2) D1QP Rice	(3) D1QP Beans	(4) D1QP Sugar	(5) D1QP All Foods
Ratio of Children age 0-4 to HH Size	0.004	0.025	0.015	0.007	0.083
	(0.007)	(0.029)	(0.012)	(0.011)	(0.057)
Log of Household Size	-0.049***	-0.073***	-0.076***	-0.079***	-0.101***
	(0.003)	(0.011)	(0.005)	(0.005)	(0.028)
Log of Per-capita (total) expenditure	-0.098***	-0.120***	-0.115***	-0.122***	-0.140***
	(0.004)	(0.012)	(0.006)	(0.005)	(0.029)
Constant	1.200***	1.295***	1.289***	1.317***	1.387***
	(0.018)	(0.055)	(0.029)	(0.025)	(0.132)
Observations	21,131	1,797	8,052	9,711	670
R-squared	0.283	0.464	0.362	0.336	0.605
Panels	Bal 123	Bal 123	Bal 123	Bal 123	Bal 123
Controls	Yes	Yes	Yes	Yes	Yes

Note: Standard errors are clustered at the locality level (*** stands for 1% significance, ** stands for 5% significance, and * for 10% significance). Wave Oct-98 includes both treatment and control villages. Other waves include only control. Bal 123 means that the sample is a balanced panel of households present in the first 3 waves: Oct-98, Mar-99 and Nov-99. The prefix D1QP means that the sample includes only households with quantity purchased in the first decile of rice distribution (column 2), beans' (column 3), and sugar's (column 4). "D1QP All foods" means that only households that purchase quantity in the first decile of each of the three distributions, at the same time, are considered. The variable "Ratio of Children age 0-4 to HH Size" measures the proportion of young children in the household.

B.4 Quantities Purchased and Food Aggregates

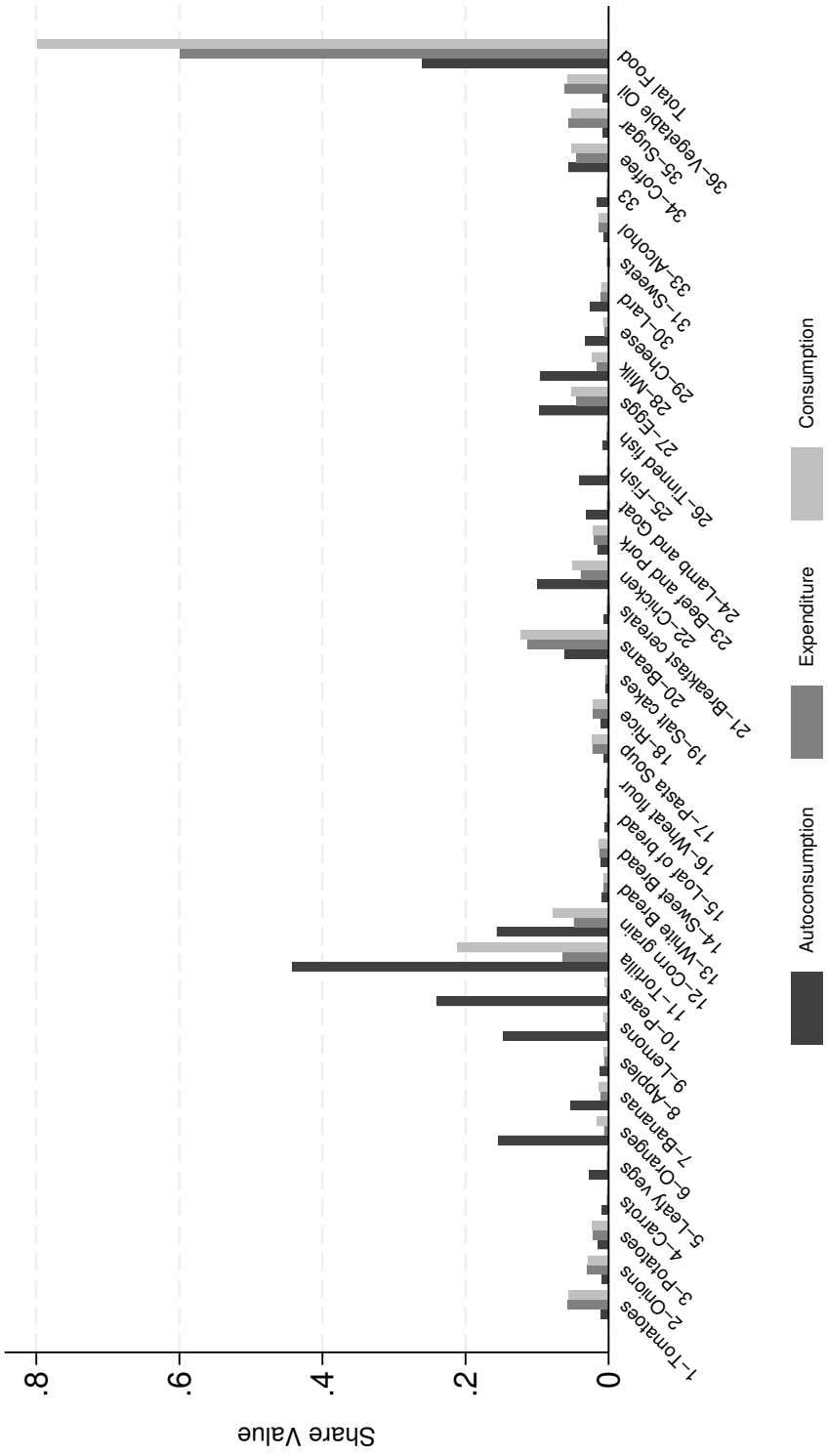
We report direct evidence about the relationship between quantities purchased and household size in Figure 9. Additional details about food consumption, expenditure, and self-production are reported in Figure 10.

Figure 9: Quantities Purchased and Household Size



Note: Mean quantity purchased over household size is calculated using all survey waves. *Betas* report the coefficient of log household size in a regression of log quantity purchased on log household size and controls as described above around equation (22), which include village and survey wave effects.

Figure 10: Food Aggregates Shares



Note: The figure depicts the share of each food item out of the food aggregate considered. To statistics on "autoconsumption" report the proportion of consumption of a good accounted for by self-production. For example, the more than 40 percent of tortilla's autoconsumption share means that out of the total consumption of tortilla, about 40 percent relates to household production of tortilla. Figures refer to the October 1998 survey wave, which is the only wave for which autoconsumption data is available.