The Green Energy Transition in a Putty-Clay Model of Capital

Simon Gilchrist* Joseba Martinez† Natalie Rickard§

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Abstract

The energy transition requires replacing fossil-fuel-dependent capital with green alternatives, but doing so may leave previously productive capital economically unviable. We embed putty-clay technology, where capital-fuel ratios are fixed ex post, within an integrated assessment model to quantify how capital irreversibility reshapes climate policy trade-offs. Four findings emerge. First, achieving given emissions targets requires 55% higher carbon taxes than in standard models, as firms cannot adjust input ratios of existing capital. Second, welfare gains from carbon taxation are substantially smaller in our framework, due to the economic costs of stranding productive capital. Third, when future climate damages are uncertain, the welfare-maximizing fixed tax rises because capital irreversibility makes a higher pre-discovery tax valuable as insurance. Fourth, our framework can tangibly capture the decline in stranded asset valuations, which decline by around 20% under an optimal carbon tax. Grandfathering provisions (giving tax discounts to pre-existing capital) can avert these asset declines, while having minimal impact on aggregate welfare. Our results demonstrate that capital irreversibility fundamentally alters the cost-benefit calculus of climate policy and generates efficiency gains from targeted grandfathering.

JEL Classification: E22, Q54, Q58, H23
Keywords: Climate policy, carbon tax, stranded assets, putty-clay capital, integrated assessment models, grandfathering

*Department of Economics New York University and NBER. Email: sg40@nyu.edu
†London Business School and CEPR. Email: jmartinez@london.edu
‡Bank of England. Email: natalie.rickard@bankofengland.co.uk
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1 Introduction

Fossil fuels met nearly 80% of global energy demand in 2023 despite dramatic cost reductions in renewable technology over the past decade (International Energy Agency, 2024). A fundamental barrier to faster transition is the existing stock of fossil-fuel-dependent capital, such as coal-fired power plants, industrial facilities, and transportation infrastructure with decades-long lifespans. As highlighted in IPCC AR6 (IPCC Working Group III, 2022), a major concern about stranded assets is not only fossil fuel reserves becoming uneconomic, but also the capital infrastructure (power plants, refineries, and industrial facilities) that depends on them. This creates a central tension in climate policy: carbon taxes incentivize green investment and reduce emissions, but may render substantial productive capital economically unviable, with uncertain consequences for economic welfare.

In this paper, we examine how much harder and more costly the climate transition becomes when capital is irreversible. More specifically, we analyze how the fixed nature of fossil fuel reliance of parts of the capital stock alters the tradeoffs between achieving climate goals and mitigating economic costs. We quantify what happens to stranded asset valuations, utilisation patterns, and aggregate economic outcomes. Finally, we examine how carbon tax policy should be designed to account for capital irreversibility.

We answer these questions by embedding putty-clay technology within an integrated assessment model. We build on the IAM frameworks of Nordhaus (1977, 1992) and Golosov, Hassler, Krusell, and Tsyvinski (2014a), incorporating the putty-clay methodology of Gilchrist and Williams (2000) and Wei (2003, 2013). In our framework, firms choose capital-fuel ratios flexibly when investing (*ex ante*), but these ratios are fixed once capital is installed (*ex post*). This contrasts with standard climate models where capital is malleable, so that input ratios can be costlessly adjusted after investment. The putty-clay structure generates three key features absent from standard models: meaningful distinctions between capital vintages, variable capacity utilisation that declines with plant age, and differential short-run versus long-run elasticities of substitution to input prices. These features allow us to coherently define and quantify stranded assets as capital that remains physically productive but becomes economically unviable under climate policy.

Our main findings reveal that capital irreversibility fundamentally reshapes climate policy tradeoffs. First, achieving a given emissions target requires substantially higher carbon taxes, a 55% increase relative to the standard model. With fixed capital-fuel ratios, dirty energy producers cannot adjust existing capital to become more fuel-efficient. Instead, they must invest heavily in new efficient capital while continuing to operate pre-existing fuel-intensive machines, slowing the emissions reduction. This asymmetry between short-run and long-run adjustment possibilities means that the economy bears twin costs under climate policy: resources must be diverted toward accelerated investment in replacement capital, while the existing capital stock continues to generate emissions because it cannot be retrofitted. Standard models with malleable capital miss this distinction entirely, as they assume capital can instantaneously adjust its input ratios, effectively giving the economy access to long-run elasticities even in the short run.

Second, welfare gains are substantially smaller under the optimal carbon tax in the putty-

clay model. This divergence arises from the economic costs of capital stock transformation: consumption declines significantly more under putty-clay as resources are diverted to replace stranded capital, and a substantial portion of pre-existing dirty capital becomes unprofitable to operate. The optimal level of carbon taxation is nonetheless only marginally smaller than under the standard model, and grows over time with economic growth in a similar manner to Golosov et al. (2014a). Optimally, the design of the tax still adds to the cost of fuel usage regardless of the nature of the capital using the fuel, so these differences are due to the different macroeconomic impacts of carbon taxes, rather than a different policy implementation. The putty-clay framework reveals a fundamental tension absent from standard models: carbon policy creates value by reducing climate damages, but also destroys value by rendering productive capital economically obsolete. This capital obsolescence channel operates through newly unproductive capital being endogenously utilised substantially less after policy implementation.

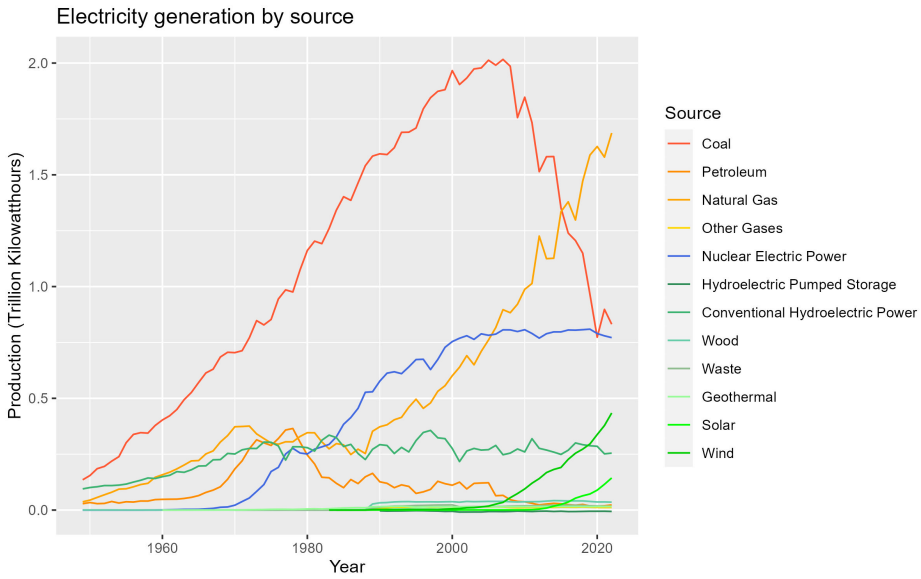
Third, we introduce climate damage uncertainty into the putty-clay framework. At any date, there is a probability that the economy discovers climate damages are more severe than previously understood, capturing the risk of climate tipping points. We find that this risk raises both the level and the welfare gains of the best fixed carbon tax. The mechanism is a form of capital-composition insurance: a higher tax chosen before the bad state is revealed tilts the installed capital stock toward cleaner vintages, and these choices cannot be reversed once damages escalate. This precautionary channel is stronger in the putty-clay model than in standard frameworks precisely because vintage-level input ratios are locked in at installation.

Fourth, our framework provides quantitative measures of stranded asset valuations. Under the optimal fixed carbon tax (a 145% increase), the value of dirty capital installed before policy implementation falls by 40% within seven years, with smaller declines of around 20% under the optimal time-varying tax. Importantly, this decline occurs gradually rather than abruptly. Old machines remain partially utilized in the near term before new capacity comes online, potentially mitigating financial stability concerns. We further study how different policy designs affect the distribution of transition costs across vintages of capital. These valuation losses naturally motivate the question of whether carbon tax policy can be redesigned to protect pre-existing asset owners without sacrificing its environmental effectiveness.

Finally, we analyze grandfathering provisions that give carbon tax discounts to pre-existing capital. Grandfathering is not something that is implemented under optimal taxation, but when considering the fixed tax - which is more consistent with recent carbon policy implementation - grandfathering marginally improves welfare. More importantly, this policy preserves the profitability of inframarginal old machines, reducing the near-term investment requirement and moderating consumption declines. As a result, valuations of stranded assets can rise, rather than decline. This suggests that stranded asset concerns need not preclude ambitious climate policy: pre-existing asset owners may experience attenuated capital losses or even gains under appropriately designed carbon taxes, potentially improving political feasibility while maintaining environmental effectiveness.

The U.S. electricity sector illustrates why capital irreversibility matters for the energy transition. As Figure 1 shows, the sector has been undergoing a substantial transition away from

coal-fired generation since 2000, with renewables growing rapidly. Despite these trends and dramatic cost reductions in renewable technology, fossil fuels still dominate electricity generation. The transition is occurring, but it is slow, precisely because energy capital moves slowly. Long-lived coal plants and industrial facilities cannot simply be repurposed; they must either continue operating with their embedded fuel intensity or be retired entirely.

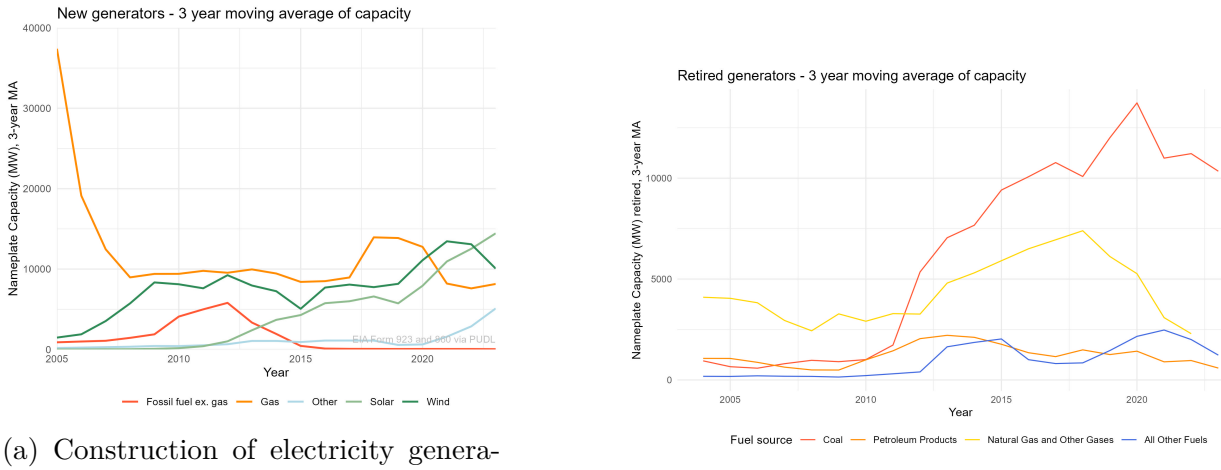


Notes: Total electricity generation measured in trillion kilowatt hours. Data from U.S. Energy Information Administration. The figure shows the absolute decline in coal generation since 2000, rapid expansion of natural gas (especially post-2005), and recent growth in wind and solar from near-zero to substantial levels.

Figure 1: Electricity Generation by Fuel Source, 1950-2022

This slow-moving nature of capital becomes visible when examining the flows of new investment and retirement. Figure 2 reveals a complete reallocation of capital flows: new investment is now concentrated entirely in renewable capacity, while coal-fired capacity is being actively retired. This is not a smooth transformation where existing plants gradually adjust their fuel mix. Rather, the energy transition operates through two distinct margins. New plants embody clean technology from the moment of construction. Old plants, unable to change their embedded fuel requirements, face a choice between continued operation at their original intensity or retirement. The simultaneous observation of substantial new construction and accelerating retirements reflects this fundamental asymmetry between the flexibility available ex ante (when investing) and the fixity that binds ex post (once capital is installed).

A third margin operates alongside construction and retirement: the utilisation of existing capital. Figure 3 compares capacity utilisation for newer versus older coal plants. Older plants exhibit substantially lower and more dispersed utilisation rates. Many operate at very low capacity factors, suggesting they have become economically marginal even though they remain physically productive. This age-utilisation gradient reflects the economic obsolescence that fixed capital-fuel ratios create: as relative prices shift or regulations tighten, older capital



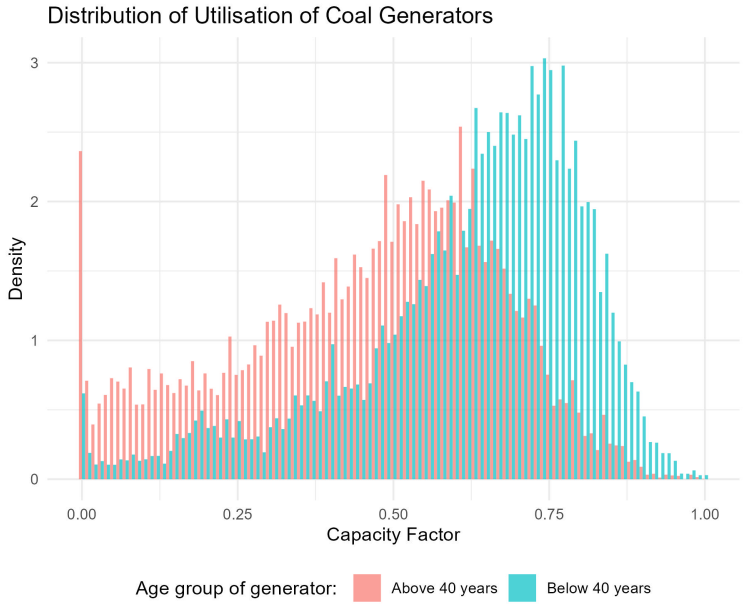
(a) Construction of electricity generators

(b) Retirement of electricity generators

Notes: Three-year moving averages of nameplate capacity in megawatts. Data from U.S. Energy Information Administration. Panel (a) shows new investment is now concentrated in wind and solar. Panel (b) documents substantial retirement of coal-fired capacity.

Figure 2: Construction and Retirement of Electricity Generators, by Fuel Source

with higher embedded fuel intensity becomes increasingly costly to operate and is utilized less intensively before eventual retirement.



Notes: Distribution of capacity factors (annual generation divided by potential generation at full capacity) for coal-fired electricity generators. Data from U.S. Energy Information Administration, 2015-2020 average. Left panel shows plants under 40 years old; right panel shows plants 40+ years old. The age-utilisation gradient motivates the variable capacity utilisation margin in our putty-clay framework.

Figure 3: Capacity utilisation Distribution for Coal Generators, by Plant Age

These patterns underscore that the energy transition operates through margins that are fun-

damental consequences of capital irreversibility: retirement of old assets, reduced utilisation of assets that remain in place, and reallocation of new investment toward clean technology. Standard climate models with malleable capital abstract from these mechanisms by assuming capital can be costlessly adjusted, making vintage distinctions irrelevant and eliminating the stranded asset problem. Our putty-clay framework explicitly models these adjustment margins, calibrating the model to match the observed patterns of vintage-specific utilisation and capital churning. This allows us to analyse questions that cannot be addressed in standard frameworks: how does the fixity of installed capital alter optimal climate policy? How should carbon taxes be designed to account for the destruction of economic value in existing capital? What are the distributional consequences across capital vintages, and can policy design mitigate transition costs while preserving environmental effectiveness?

Related literature Our work connects to several strands of research in climate economics and macroeconomics. The integrated assessment model framework we employ builds on the foundational contributions of Nordhaus (1977, 1991, 1992), who introduced methods for analyzing climate policy within dynamic general equilibrium models. Golosov et al. (2014a) provided important analytical tractability to this approach for studying optimal carbon taxation. Subsequent work has extended these frameworks in multiple directions: incorporating endogenous technical change and directed innovation (Acemoglu, Aghion, Bursztyn, and Hemous (2012); Popp (2002); Hassler, Krusell, and Olovsson (2021)), modeling spatial heterogeneity in climate damages (Cruz and Rossi-Hansberg (2024); Desmet and Rossi-Hansberg (2015); Krusell and Smith (2022)), and characterizing the role of uncertainty and tail risks (Weitzman (2009, 2014); Cai and Lontzek (2019)). Barnett, Brock, and Hansen (2020, 2022); Barnett, Brock, Zhang, and Hansen (2024) have further developed frameworks for incorporating model uncertainty and ambiguity into climate policy analysis. For calibration of climate damage functions and carbon cycle parameters, we draw on Folini, Friedl, Kübler, and Scheidegger (2024) and Dietz, Van Der Ploeg, Rezai, and Venmans (2021). Beyond the more stylized IAMs developed in this literature and in this paper, a large suite of detailed process-based IAMs with rich technological and sectoral structure, drawn upon in the IPCC Sixth Assessment Report IPCC Working Group III (2022), underpin policy evaluation.

A related empirical literature has sought to quantify the economic impacts of climate change and evaluate specific policy interventions. Contributions including Deschênes and Greenstone (2012); Dell, Jones, and Olken (2012); Burke, Hsiang, and Miguel (2015); Nath, Ramey, and Klenow (2024); Bilal and Känzig (2024) use climate variation to identify effects on economic output and other outcomes. A range of empirical evidence more specific to the carbon tax experiments we explore is developed by Metcalf and Stock (2023) and Känzig (2023), while Colmer, Martin, Muuls, and Wagner (2025) provide firm-level evidence on carbon pricing effects. Aghion, Dechezleprêtre, Hémous, Martin, and Van Reenen (2016) and Acemoglu, Hemous, Barrage, and Aghion (2019) provide evidence on directed technical change in response to environmental policy.

Our framework departs from these models by incorporating capital irreversibility through putty-clay technology, following Gilchrist and Williams (2000) and building on the original framework of Johansen (1959). The approach of Gilchrist and Williams (2000) allows for

tractable analysis of both investment irreversibility and variable capacity utilisation, distinguishing it from alternative vintage capital frameworks (Atkeson and Kehoe (1999); Cooley, Hansen, and Prescott (1995)). Applications of this methodology include Gilchrist and Williams (2005) on investment under uncertainty, Gilchrist and Williams (2004) on postwar transitions, and Hurst, Kehoe, Pastorino, and Winberry (2022) on labor market dynamics. We extend the approach of Wei (2003, 2013), who model fuel as a variable input within putty-clay production, to a multi-sector climate model. Recent work has begun incorporating vintage structure into climate models: Meng (2022) and Lanteri and Rampini (2022) emphasize technological lock-in and financing constraints, Hawkins-Pierot and Wagner (2024) study technology choice with vintage capital, Capelle, Kirti, Pierri, and Villegas-Bauer (2023) document within-industry heterogeneity in emissions driven by differences in capital vintages across firms, and Finkelstein Shapiro and Metcalf (2023) show that endogenous green technology adoption by firms can substantially reduce the economic costs of carbon taxation. Our contribution is to incorporate full putty-clay technology across all sectors in an IAM framework, allowing us to analyze how capital irreversibility shapes optimal climate policy, generates endogenous stranded assets, and affects the design of grandfathering provisions.

Our analysis of asset valuations connects to emerging work on climate-related financial risks. Financial regulators and policymakers have raised concerns about potential asset revaluations under climate policy (Carney (2015)), with scenario analyses suggesting substantial fossil fuel reserves may become economically unviable under stringent climate targets. McGlade and Ekins (2015) quantify unburnable fossil fuel reserves under 2°C warming scenarios, while Dietz, Bowen, Dixon, and Gradwell (2016) develop a climate value-at-risk framework for global financial assets. Our focus differs by emphasizing stranded capital, meaning the productive assets that rely on fossil fuels, rather than the reserves themselves. Detailed IAMs including REMIND, assessed in the IPCC Sixth Assessment Report IPCC Working Group III (2022), have also examined stranded asset risks from stringent climate policy. A growing empirical literature documents how investors price climate policy risk, testing whether carbon-intensive firms earn higher returns (Engle, Giglio, Kelly, Lee, and Stroebel (2020); Bolton and Kacperczyk (2021); Berg, Kölbel, Pavlova, and Rigobon (2021); Hsu, Li, and Tsou (2023); Eskildsen, Ibert, Jensen, and Pedersen (2024)). Recent macro-finance models show that while asset prices can, in principle, replicate carbon-tax incentives, equilibrium pricing of climate risk and green finance typically generates cost-of-capital effects that are too small to substitute for explicit carbon pricing (Chittaro, Piazzesi, Sena, and Schneider (2025); Gârleanu and Pedersen (2024); Pedersen (2026)). Our framework provides micro-foundations for valuation dynamics through capital-fuel complementarities, generating stranded assets endogenously in a way that standard models with malleable capital cannot. This allows us to study alternative carbon tax designs—including grandfathering provisions for existing capital—that can reduce transition costs while maintaining environmental effectiveness.

Outline of paper The remainder of the paper proceeds as follows. Section 2 presents the putty-clay integrated assessment model, detailing the three-sector structure, climate damage specification, and household problem. Section 3 describes the calibration strategy and parameter values. Section 4 analyzes the dynamic effects of carbon taxation, comparing outcomes under putty-clay versus standard assumptions. Section 4.2 characterizes the opti-

mal carbon tax and examines grandfathering provisions. Section 5 quantifies stranded asset valuations and their response to different policy designs. Section 6 concludes.

2 Model Description

The model is an integrated assessment model that incorporates the putty-clay technology developed in Gilchrist and Williams (2000). The model has a final goods sector along with dirty and green energy sectors. The final goods sector combines energy, labor and capital to produce final goods. As in Wei (2003), dirty energy uses fossil fuel, labor, and capital as inputs while clean energy uses only capital and labour. These two energy sources are imperfect substitutes in the production of the energy input that is used by the final goods sector.

Within each sector, output is produced using putty-clay technology. Capital goods take the form of machines that are vintage-specific. Without loss of generality, each machine is normalized to employ one unit of labor at maximum capacity. Machines are long-lived and assumed to fully depreciate in any given period according to a Poisson process. The putty-clay technology implies that capital-to-labor and fuel-to-labor ratios are chosen in advance and fixed for the machine's life. Once built, machines are fully utilized as long as the revenue obtained from the machine exceeds the operating cost, which combines fuel and labor costs. All machines within a vintage are identical ex-ante, but are subject to idiosyncratic shocks that determine their productivity once installed. The realized level of productivity is assumed to be permanent and thus embodied in the machine. New machines embody new technologies that improve exogenously over time. Machine productivity may also increase because of growth in disembodied technology. As productivity growth occurs, the cost of operating a machine rises, so older machines are less likely to be utilized.

Following Golosov et al. (2014a), the climate block of the model adds a climate damage function that reduces productivity in all sectors. Climate damage is a function of the carbon stock that accumulates based on fossil fuel usage. The damages are external and hence are not taken into account in the optimality conditions that govern production, investment, and household supply of labor and capital. This section provides a complete specification of the technology and optimality conditions for each of the three sectors: final goods, dirty energy and clean energy, along with the specification of the household problem and the climate damage function.

2.1 Energy sector

The energy sector consists of two types of energy producers. A dirty energy producer produces energy E^D using capital, labour and fossil fuel, and a green energy producer produces energy E^G from renewable sources using only capital and labour.

Dirty and clean energy are assumed to be imperfect substitutes so that the total energy input to the final goods sector is a CES aggregate of the two energy sources:

$$E_t = \left({E_t^D}^{\frac{\epsilon-1}{\epsilon}} + {E_t^G}^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}. \tag{1}$$

Let P_t^G and P_t^D denote the price of clean and dirty energy. Input demands satisfy

$$\frac{E_t^k}{E_t} = \left(\frac{P_t^k}{P_t^E}\right)^{\epsilon} \quad for \quad k = G, D.$$

The cost-minimizing energy bundle then implies the energy price:

$$P_t^E = \left(P_t^{G\,1-\epsilon} + P_t^{D\,1-\epsilon}\right)^{\frac{1}{1-\epsilon}} \tag{2}$$

2.2 Dirty energy sector

The dirty energy sector produces energy from vintage-specific machines that combine labour, capital and fuel. Each machine employs one unit of labor at full capacity. For machines built in period $t-j$, the capital-to-fuel ratio k_{t-j}^D and the fuel-to-labour ratio f_{t-j} are chosen upon installation and hence fixed at time $t-j$.

Technological change in the dirty energy sector reflects three forces: disembodied technology and climate damages that affect machines of all vintages equally and vintage-specific technology embodied in machines. Define aggregate disembodied productivity in the dirty energy sector as $A_{d,t} = Z_{d,t} \times D_t$ where $Z_{d,t}$ denotes disembodied technology and D_t denotes damages from accumulated carbon stocks. Upon installation, machines are subject to an idiosyncratic productivity shock $\theta_{i,t-j}^D$ drawn from a log-normal distribution:

$$log(\theta_{i,t-j}^D) \sim \mathcal{N}(log(\theta_{t-j}^D) - \frac{1}{2}\sigma_D^2, \sigma_D^2)$$

The mean of this distribution $\theta_{t-j}^D = E(\theta_{i,t-j}^D)$ denotes the technology embodied in vintage machines $t-j$.

Given the capacity constraint, $L_{i,t,t-j} \leq 1$, a machine i of vintage j at time t produces dirty energy according to the Leontief production function

$$E_{i,t,t-j}^D = X_{i,t,t-j}^D \min[L_{i,t,t-j}^D(X_{i,t,t-j}^D), 1]$$

where machine labour productivity $X_{i,t,t-j}^D$ is a Cobb-Douglas function of technology, climate damages, and the capital-to-fuel and fuel-to-labor ratios:

$$X_{i,t,t-j}^D = \left(A_{d,t}\theta_{i,t-j}^D\right)^{1-\alpha^D} (k_{t-j}^D)^{\lambda^D} f_{t-j}^{\alpha^D}.$$

2.2.1 Machine utilisation

Let W_t denote the wage rate and P_t^f the price of fuel. The cost of operating a vintage $t-j$ machine at full capacity is $W_t + P_t^f f_{t-j}$. A machine is used ($L(X_{i,t,t-j}^D) = 1$) if machine revenue exceeds cost: $P_t^D X_{i,t,t-j}^D > (W_t + P_t^f f_{t-j})$. Define the cutoff value

$$z_{t,t-j}^D = \frac{1}{\sigma_D}\left[log(W_t + P_t^f f_{t-j}) - log(P_t^D X_{t,t-j}^D) + \frac{1}{2}\sigma_D^2\right], \tag{3}$$

the proportion of machines in $t-j$ that are used in time t is

$$Pr[P_t^D X_{i,t,t-j}^D > (W_t + P_t^f f_{t-j})] = 1 - \Phi(z_{t,t-j}^D).$$

Let $X_{t,t-j}^D = E(X_{i,t,t-j}^D)$ denote the unconditional mean of labour productivity in time t for machines built in period $t-j$. Given the log-normal distribution of machine productivity, the expected output of such a machine conditional on utilisation is

$$E\left[X_{i,t,t-j}^D | P_t^D X_{i,t,t-j}^D > (W_t + P_t^f f_{t-j})\right] = \frac{(1 - \Phi(z_{t,t-j}^D - \sigma_D))}{(1 - \Phi(z_{t,t-j}^D))} X_{t,t-j}^D.$$

2.2.2 Optimal Input Choices and Factor Shares

Dirty machines exogenously fail at rate δ^D per period. Expected net income at $t+s$ from a machine that is installed at time t is

$$\pi_{t+s,t}^D = (1-\delta^D)^{s-1}\left\{\left[1 - \Phi\left(z_{t+s,t}^D - \sigma_D\right)\right] P_{t+s}^D X_{t+s,t}^D - \left[1 - \Phi\left(z_{t+s,t}^D\right)\right]\left(W_{t+s} + P_{t+s}^f f_t\right)\right\}. \tag{4}$$

Define the (ex-dividend) expected market value of a vintage $t-j$ machine at time t as the present value of the expected profit stream $\pi_{t+s,t}^D$, discounted using the household discount factor $m_{t,t+s}$:

$$V_{t,t-j}^D = \sum_{s=1}^{\infty} m_{t,t+s} \pi_{t+s,t-j}^D.$$

Machine producers choose k_t^D and f_t to maximize the expected discounted value of profits of a new machine $V_{t,t}^D$. Owing to sector-specific adjustment costs in the production of new investment goods, the price of a new machine, $P_{I,t}^D$, may deviate from unity. The optimality conditions for k_t^D, f_t may be expressed as

$$P_{I,t}^D k_t^D f_t = \sum_{s=1}^{M} m_{t,t+s}(1-\delta^D)^{s-1}\left\{\lambda^D\left[1 - \Phi\left(z_{t+s,t}^D - \sigma_D\right)\right] P_{t+s}^D X_{t+s,t}^D\right\} \tag{5}$$

$$P_{I,t}^D k_t^D f_t = \sum_{s=1}^{M} m_{t,t+s}(1-\delta^D)^{s-1}\left\{\alpha^D\left[1 - \Phi\left(z_{t+s,t}^D - \sigma_D\right)\right] P_{t+s}^D X_{t+s,t}^D - \left[1 - \Phi\left(z_{t+s,t}^D\right)\right] P_{t+s}^f f_t\right\}. \tag{6}$$

Free entry implies the zero profit condition:

$$P_{I,t}^D k_t^D f_t = \sum_{s=1}^{M} m_{t,t+s}(1-\delta^D)^{s-1}\left\{\left[1 - \Phi\left(z_{t+s,t}^D - \sigma_D\right)\right] P_{t+s}^D X_{t+s,t}^D - \left[1 - \Phi\left(z_{t+s,t}^D\right)\right]\left(W_{t+s} + P_{t+s}^f f_t\right)\right\}. \tag{7}$$

Combining the optimality conditions with the free-entry condition delivers the following labour and fuel share equations:

$$1 - \alpha^D = \frac{\sum_{s=1}^{M} m_{t,t+s}(1-\delta^D)^{s-1}\left\{\left[1 - \Phi\left(z_{t+s,t}^D\right)\right] W_{t+s}\right\}}{\sum_{s=1}^{M} m_{t,t+s}(1-\delta^D)^{s-1}\left\{\left[1 - \Phi\left(z_{t+s,t}^D - \sigma\right)\right] P_{t+s}^D X_{t+s,t}^D\right\}}$$

$$\alpha^D - \lambda^D = \frac{\sum_{s=1}^{M} m_{t,t+s}(1-\delta^D)^{s-1}\left\{\left[1 - \Phi\left(z_{t+s,t}^D\right)\right] P_{t+s}^f f_t\right\}}{\sum_{s=1}^{M} m_{t,t+s}(1-\delta)^{s-1}\left\{\left[1 - \Phi\left(z_{t+s,t}^D - \sigma\right)\right] P_{t+s}^D X_{t+s,t}^D\right\}}$$

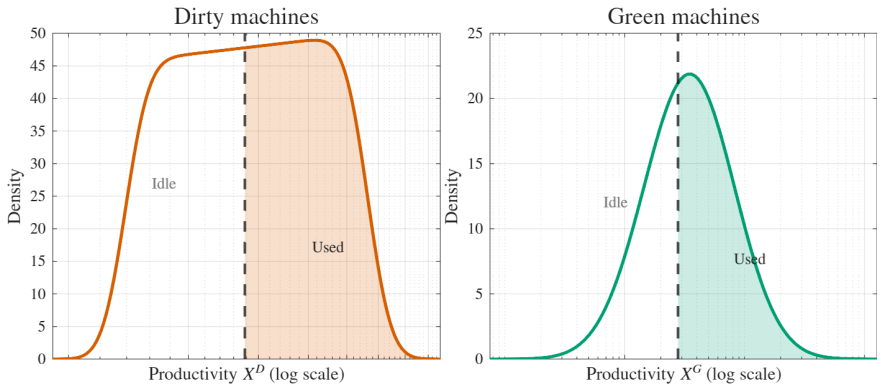


Figure 4: Machine Distribution in Dirty and Green Energy Sectors

Because of the ex-post fixity of capital-to-energy and fuel-to-labour choices, the labour and fuel input cost shares are determined by the ratio of the expected present value of input costs to revenue.

2.2.3 Input Demands and Energy Output

Let Q^D_{t-j} denote the number of dirty energy machines installed at time $t-j$ so that $(1-\delta^D)^{j-1}Q^D_{t-j}$ is the undepreciated stock of vintage $t-j$ machines at time t. The total input demand for labour L^D_t and fuel F_t in the dirty energy sector are obtained by summing the utilisation-adjusted input requirements across vintages:

$$L^D_t = \sum_{j=1}^{M} \left\{ \left[1 - \Phi\left(z^D_{t,t-j}\right)\right] (1-\delta^D)^{j-1} Q^D_{t-j} \right\} \tag{8}$$

$$F_t = \sum_{j=1}^{M} \left\{ \left[1 - \Phi\left(z^D_{t,t-j}\right)\right] \times (1-\delta^D)^{j-1} Q^D_{t-j} f_{t-j} \right\}. \tag{9}$$

Similarly, total energy output from the dirty-energy sector satisfies:

$$E^D_t = \sum_{j=1}^{M} \left\{ \left[1 - \Phi\left(z^D_{t,t-j} - \sigma_D\right)\right] \times (1-\delta^D)^{j-1} Q^D_{t-j} X^D_{t,t-j} \right\}. \tag{10}$$

Figure 4 displays the steady-state distribution of machines for the dirty and green energy sectors along with the steady-state cut-off value determined by $W_t + P^F f_t$ based on the calibration chosen below. Machine revenue $P^D_t X$ is displayed along the horizontal axis. The height of the curve is then determined by the number of undepreciated machines that remain in steady-state for a given productivity X. To obtain this curve we need only to track the steady-state values of $\left[Q^D_{t-j}, X^D_{t-j}\right]_{j=0}^{\infty}$. The shaded area to the right of the cut off represents the measure of machines that are utilized. As the economy grows, the wage rises and the utilisation rate of less productive machines falls to zero. At the same time, new machines with higher productivity are built, expanding the curve to the right. The second line captures the effect of new investment on the machine distribution.

In summary, given a sequence of prices $\left[w_{t+s}, P^D_{t+s}, P^F_{t+s}\right]_{s=0}^{\infty}$ and technological indices $\left[A^d_{t+s}, \theta^d_{t+s}\right]_{s=0}^{\infty}$, along with the state variables $\left[Q^D_{t-j}, X^D_{t-j}\right]_{j=0}^{\infty}$ that summarize the existing distribution of machines, the dirty energy sector can be fully characterized by equation 3, the static optimal utilisation decision, the forward-looking equations 5, 6 and 7 that determine the optimal choices for the capital-to-energy ratio and fuel-to-energy ratio, along with the free entry condition and the backward-looking equations 8, 9 and equation 10 that sum over the existing machine distribution to determine labor and fuel inputs along with dirty energy output.

2.2.4 Vintage Model

An alternative approach to respect the vintage structure of capital is the putty-putty framework formulated by Solow (1959). In this formulation, output per vintage and capital per vintage satisfy

$$\begin{aligned} E^D_{t,t-j} &= A_{d,t}\theta^D_{v,t-j}(K^D_{t,t-j})^{\lambda^D}(F_{t,t-j})^{\alpha^D-\lambda^D}(L^D_{t,t-j})^{1-\alpha^D} \\ K^D_{t,t-j} &= (1-\delta^D)^{t-j-1}I^D_{t-j}. \end{aligned}$$

where I^D_{t-j} are total capital expenditures for vintage $t-j$ and $\theta^{D,v}_{t-j}$ is an index of the technology embodied in the vintage $t-j$ aggregate capital stock. Summing across vintages gives total energy output from the dirty energy sector:

$$E^D_t = \sum_{j=1}^{\infty} E^D_{t,t-j}.$$

Because labor and fuel productivity are equalized across machines both within and across vintages, one can represent the vintage production structure in terms of a capital aggregate that combines with labour, fuel, and technology in a Cobb-Douglas production function. In particular, it is straightforward to show that the vintage model is equivalent to a model with an aggregate capital stock K^D_t subject to investment-specific technological change:

$$K^D_t = (1-\delta^D)K^D_{t-1} + (\theta^{D,v}_{t-1})^{\frac{1}{\alpha}}I^D_t$$

and production function:

$$E^D_t = A^d_t\left(K^D_t\right)^{\lambda^D}(F_t)^{\alpha^D-\lambda^D}(L^D_t)^{1-\alpha^D}.$$

In contrast, in the putty clay model, machine productivities cannot be equalized either within or across vintages. Hence there is no aggregate representation of the capital stock and we therefore need to track the entire distribution of machines through $\left[Q^D_{t-j}, X^D_{t-j}\right]_{j=0}^{\infty}$.

2.3 Green energy sector

The renewable/green energy sector produces green energy E^G_t using only capital and labour, and no fossil fuels, using putty-clay technology. This is the same as the original set-up of Gilchrist and Williams (2000) with the addition of disembodied technological change along

with damages induced by carbon accumulation. The formulation thus mirrors the dirty energy sector absent the fossil-fuel decision and use.

Given a capital intensity k^G_{t-j} for vintage $t-j$, a machine i of vintage $t-j$ produces green energy at time t according to the Leontief production function:

$$E^G_{i,t,t-j} = X^G_{i,t,t-j} \min[L^G_{i,t,t-j}(X^G_{i,t,t-j}), 1]$$

where $X^G_{i,t,t-j}$ is labour productivity:

$$X^G_{i,t,t-j} = \left(A_{g.t}\theta^G_{i,t-j}\right)^{1-\alpha^G} (k^G_{t-j})^{\alpha^G},$$

$A_{g,t} = Z_{g,t}D_t$ denotes disembodied total factor productivity inclusive of carbon damages, and $\theta_{i,t-j}$ is an idiosyncratic draw from the sector-specific log-normal distribution

$$log(\theta^G_{i,t-j}) \sim \mathcal{N}(log(\theta^G_{t-j}) - \frac{1}{2}\sigma^2_G, \sigma^2_G).$$

Here $\theta^G_{t-j} = E(\theta^G_{i,t-j})$ indexes the level of technology embodied in green energy machines.

Projects with productivity $P^G_t X^G_{i,t,t-j} \geq W_t$ are used at full capacity, the rest of the machines are left idle. The proportion of machines of vintage $t-j$ in use is therefore given by:

$$Pr(P^G_t X^G_{i,t,t-j} > W_t | W_t, P^G_t) = 1 - \Phi(z^G_{t,t-j})$$

with

$$z^G_{t,t-j} = \frac{1}{\sigma}\left[log(W_t) - logP^G_t X^G_{t,t-j} + \frac{1}{2}\sigma^2\right] \tag{11}$$

Expected profits at time $t+s$ from a machine built in time t are

$$\pi^G_{t+s,t} = (1-\delta^G)^{s-1}\left\{\left[1 - \Phi\left(z^G_{t+s,t} - \sigma_G\right)\right] P^G_{t+s} X^G_{t+s,t} - \left[1 - \Phi\left(z^G_{t+s,t}\right)\right](W_{t+s})\right\} \tag{12}$$

Define the (ex-dividend) expected market value of a vintage $t-j$ machine at time t as the present value of the expected profit stream $\pi^D_{t+s,t}$, discounted using the household discount factor $m_{t,t+s}$

$$V^G_{t,t-j} = \sum_{s=1}^{\infty} m_{t,t+s}\pi^G_{t+s,t-j}.$$

Machine producers choose k^G_t to maximize $V^G_{t,t}$ resulting in the optimality condition for capital intensity k^G_t

$$P^G_{I,t}k^G_t = \sum_{s=1}^{M} m_{t,t+s}(1-\delta^G)^{s-1}\left\{\lambda^G\left[1 - \Phi\left(z^G_{t+s,t} - \sigma_G\right)\right] P^G_{t+s}X^G_{t+s,t}\right\}. \tag{13}$$

This optimality condition along with the zero profit condition

$$P^G_{I,t}k^G_t = \sum_{s=1}^{M} m_{t,t+s}(1-\delta^G)^{s-1}\left\{\left[1 - \Phi\left(z^G_{t+s,t} - \sigma_G\right)\right] P^G_{t+s}X^G_{t+s,t} - \left[1 - \Phi\left(z^G_{t+s,t}\right)\right](W_{t+s})\right\} \tag{14}$$

deliver the labour share equation:

$$1-\alpha^G = \frac{\sum_{s=1}^{M} m_{t,t+s}(1-\delta^G)^{s-1}\left\{\left[1-\Phi\left(z^G_{t+s,t}\right)\right]W_{t+s}\right\}}{\sum_{s=1}^{M} m_{t,t+s}(1-\delta^G)^{s-1}\left\{\left[1-\Phi\left(z^G_{t+s,t}-\sigma\right)\right]P^G_{t+s}X^G_{t+s,t}\right\}}.$$

Finally the total labour requirement and green energy output are:

$$L^G_t = \sum_{j=1}^{M}\left\{\left[1-\Phi\left(z^G_{t,t-j}\right)\right](1-\delta^G)^{j-1}Q^G_{t-j}\right\} \tag{15}$$

$$E^G_t = \sum_{j=1}^{M}\left\{\left[1-\Phi\left(z^G_{t,t-j}-\sigma^G\right)\right]\times(1-\delta^G)^{j-1}Q^G_{t-j}X^G_{t,t-j}\right\} \tag{16}$$

where Q^G_{t-j} is the quantity of green energy machines of vintage $t-j$.

2.4 Final goods producers

The final goods producers combine capital, labour, and energy to produce output that is either consumed by households or used to produce new machines in the final goods or energy sectors of the economy. The putty-clay version of the final goods problem follows closely the set-up of the dirty energy firm. Let k^{fg}_{t-j} and e^{fg}_{t-j} denote the capital-to-energy and energy-to-labour intensities of machines of vintage $t-j$.

A machine i of vintage $t-j$ produces final goods at time t

$$Y^{fg}_{i,t,t-j} = X^{fg}_{i,t,t-j}\min[L^{fg}_{i,t,t-j}(X^{fg}_{i,t,t-j}),1]$$

where machine labour productivity $X^{fg}_{i,t,t-j}$ is

$$X^{fg}_{i,t,t-j} = \left(A_{fg,t}\theta^{fg}_{i,t-j}\right)^{1-\alpha^{fg}}(k^{fg}_{t-j})^{\lambda^{fg}}e^{\alpha^{fg}}_{t-j}.$$

with

$$log(\theta^G_{i,t-j}) \sim \mathcal{N}(log(\theta^G_{t-j}) - \frac{1}{2}\sigma^2_G, \sigma^2_G)$$

Again, the mean of this distribution $\theta^G_{t-j} = E(\theta^G_{i,t-j})$ denotes the technology embodied in final goods machines of vintage $t-j$.

The cost of operating a vintage $t-j$ machine at full capacity is $W_t + P^e_t e_{t-j}$ Define the cutoff value

$$z^{fg}_{t,t-j} = \frac{1}{\sigma_{fg}}\left[log(W_t + P^e_t e_{t-j}) - log(P^{fg}_t X^{fg}_{t,t-j}) + \frac{1}{2}\sigma^2_{fg}\right], \tag{17}$$

the proportion of machines in $t-j$ that are used in time t is

$$Pr[P^{fg}_t X^{fg}_{i,t,t-j} > (W_t + P^e_t e_{t-j})] = 1-\Phi(z^{fg}_{t,t-j}).$$

Let $X^{fg}_{t,t-j} = E(X^{fg}_{i,t,t-j})$ denote the unconditional mean of labour productivity in time t for machines built in period $t-j$. The expected output of such a machine conditional on utilisation is

$$E\left[X^{fg}_{i,t,t-j}|P^{fg}_t X^{fg}_{i,t,t-j} > (W_t + P^e_t e_{t-j})\right] = \frac{(1-\Phi(z^{fg}_{t,t-j} - \sigma_{fg}))}{(1-\Phi(z^{fg}_{t,t-j}))} X^{fg}_{t,t-j}.$$

Given the failure rate δ^{fg} of final goods machines, expected net income at $t+s$ from a machine that is installed at time t is

$$\pi^{fg}_{t+s,t} = (1-\delta^{fg})^{s-1}\left\{\left[1-\Phi\left(z^{fg}_{t+s,t} - \sigma_{fg}\right)\right] P^{fg}_{t+s} X^{fg}_{t+s,t} - \left[1-\Phi\left(z^{fg}_{t+s,t}\right)\right]\left(W_{t+s} + P^e_{t+s} e_t\right)\right\} \tag{18}$$

Define the (ex-dividend) expected market value of a vintage $t-j$ machine at time t as the present value of the expected profit stream $\pi^D_{t+s,t}$, discounted using the household discount factor $m_{t,t+s}$

$$V^{fg}_{t,t-j} = \sum_{s=1}^{\infty} m_{t,t+s}\pi^{fg}_{t+s,t-j}.$$

Machine producers choose k^{fg}_t and e_t to maximize the expected discounted value of profits of a new machine $V^{fg}_{t,t}$. The optimality conditions for the input ratios k^{fg}_t, e_t may be expressed as

$$P^{fg}_{I,t} k^{fg}_t e_t = \sum_{s=1}^{M} m_{t,t+s}(1-\delta^{fg})^{s-1}\left\{\lambda^{fg}\left[1-\Phi\left(z^{fg}_{t+s,t} - \sigma_{fg}\right)\right] P^{fg}_{t+s} X^{fg}_{t+s,t}\right\} \tag{19}$$

$$P^{fg}_{I,t} k^{fg}_t e_t = \sum_{s=1}^{M} m_{t,t+s}(1-\delta^{fg})^{s-1}\left\{\alpha^{fg}\left[1-\Phi\left(z^{fg}_{t+s,t} - \sigma_{fg}\right)\right] P^{fg}_{t+s} X^{fg}_{t+s,t} - \left[1-\Phi\left(z^{fg}_{t+s,t}\right)\right] P^e_{t+s} e_t\right\}. \tag{20}$$

Free entry implies the zero profit condition:

$$P^{fg}_{I,t} k^{fg}_t e_t = \sum_{s=1}^{M} m_{t,t+s}(1-\delta^{f}g)^{s-1}\left\{\left[1-\Phi\left(z^{fg}_{t+s,t} - \sigma_{fg}\right)\right] P^{fg}_{t+s} X^{fg}_{t+s,t} - \left[1-\Phi\left(z^{fg}_{t+s,t}\right)\right]\left(W_{t+s} + P^e_{t+s} e_t\right)\right\}. \tag{21}$$

We can again combine the optimality conditions for k^{fg}_t and e_t with the free entry condition to obtain expressions for optimal input shares.

Let Q^{fg}_{t-j} denote the number of final goods machines installed at time $t-j$ so that $(1-\delta^{fg})^{j-1}Q^{fg}_{t-j}$ is the undepreciated stock of vintage $t-j$ machines at time t. The total input demand for labour L^{fg}_t and energy E_t in the final goods sector are :

$$L^{fg}_t = \sum_{j=1}^{M}\left\{\left[1-\Phi\left(z^{fg}_{t,t-j}\right)\right](1-\delta^{fg})^{j-1} Q^{fg}_{t-j}\right\} \tag{22}$$

$$E_t = \sum_{j=1}^{M}\left\{\left[1-\Phi\left(z^{fg}_{t,t-j}\right)\right] \times (1-\delta^{fg})^{j-1} Q^{fg}_{t-j} e_{t-j}\right\}. \tag{23}$$

while total final goods output satisfies:

$$Y_t^{fg} = \sum_{j=1}^{M} \left\{ \left[1 - \Phi \left(z_{t,t-j}^{fg} - \sigma_{fg} \right) \right] \times (1 - \delta^{fg})^{j-1} Q_{t-j}^{fg} X_{t,t-j}^{fg} \right\}. \tag{24}$$

2.5 Climate Damage Function

In order to assess the welfare implications of imposing taxes and subsidies to reduce fossil fuel usage, we introduce a climate block into the model which models the endogenous responses of damages from climate change. We model the externality from climate change as reducing the productivity, following the approach of Nordhaus's DICE models. We adopt the tractable approach to modelling the climate block of IAMs proposed by Golosov et al. (2014a). The climate damages are a function of the stock of carbon S_t that has been produced previously by the fossil fuel usage of the dirty energy sector, subject to a reduced form representation of how this stock is partly absorbed and depleted by the environment.

The damage function we use is:

$$D_t = \exp\left(-\zeta \left(\frac{S_t}{\Omega_t} \right) \right) \tag{25}$$

This is similar to Golosov et al. (2014a), as the addition of cumulated emissions to the atmospheric carbon stock (S_t) reduces productivity. Note that this is specified as a function of the industrial emissions, i.e. as deviations from the pre-industrial atmospheric carbon stock and any non-industrial carbon emissions. We allow all sectors, not only the final sector, to be subject to climate damages, with labour productivity reduced by A_t^{fg}, A_t^d and A_t^g being multiplied by D_t. The degree to which the cumulated carbon stock damages productivity is mediated by the parameter ζ, which we set to ensure the initial steady state damages matches the current degree of damage in the economy.

To this, we add the exogenous process Ω_t. This is designed to represent technology improvements which allow the economy to adapt to climate damages in the very long run. This is in the spirit of Barrage (2014)'s suggestion of a technology adjustment which neutralises the emissions from coal use, but we apply this to the cumulated carbon stock rather than the emissions. During the policy experiments, we hold Ω_t constant for an extended period. We allow Ω_t to grow at the rate of S_t under the balanced growth path, so that damages in the very long run do not continue to increase.[1] This allows the model to have a balanced growth rate, but because the growth of Ω_t is very delayed, it does not affect the near-term macroeconomic dynamics. It also does not affect the optimal carbon tax, as by the time it begins to decrease, discounting of future economic outcomes is very substantial.

The carbon stock S_t in excess of pre-industrial levels accumulates based on past fossil fuel use:

[1] Specifically, Ω is initially constant, and then the growth rate gradually increases over time according to a logistic function of the form:

$$g_t^{\Omega} = g_t^{y} * (1 - \exp(-(\Omega_a + \Omega_b * t)))$$

$$S_t = \sum_{s=0}^{t+T} (1 - d_s) F_{t-s} \tag{26}$$

Here $(1 - d_s)$ reflects the depreciation of previous carbon stocks. We follow the depreciation process and calibration of Golosov et al. (2014a), specifically:

$$1 - d_s = (1 - \varphi_L)\varphi_0 + \varphi_L (1 - \varphi_1)^s$$

where the share φ_L depreciates gradually by rate φ_1, $1-\varphi$ exits the atmosphere immediately, and the remainder stays permanently in the atmosphere.

For some of the analysis, we explicitly distinguish between the gradual depreciation ($S_{1,t}$) and permanent ($S_{2,t}$) components of the carbon stock:

$$S_{t,1} = (1 - \varphi_1) S_{t-1,1} + (1 - \varphi_L)\varphi_0 \tilde{F}_t, \tag{27}$$
$$S_{t,2} = \tilde{S}_{t-1,2} + \varphi_L \tilde{F}_t, \tag{28}$$
$$S_t = S_{t,1} + S_{t,2}, \tag{29}$$

2.6 Households

There is a representative household who owns the capital, obtains utility from consumption and disutility from supplying labour. The household can buy and sell sector-specific claims on the portfolio of capital that pay out π_t^x in dividends each period for $x = D, G, fg$ where π_t^x are total profits paid out across vintages in sector x at time t

$$\pi_t^x = \sum_{j=1}^{M} \pi_{t,t-j}^x.$$

Let s_t^x define the number of shares on the portfolio of claims to sector x held by the household, and V_t^x the ex-dividend market value of this portfolio:

$$V_t^x = \sum_{j=1}^{M} V_{t,t-j}^x$$

Households choose a sequence of consumption, labour and portfolio shares to maximize

$$\sum_{t=0}^{\infty} \left\{ \beta^t \frac{1}{1-\eta} \times \left[C_t \left(1 - L_t\right)^{\varphi} \right]^{1-\eta} \right\}$$

subject to

$$C_t + \sum_{x=D,G,fg} s_{t+1}^x V_{t+1}^x = W_t L_t + \sum_{x=D,G,fg} s_t^x \left(\pi_t^x + \left(1 - \delta^x\right) V_t^x \right)$$

The household optimality conditions may be expressed as

$$W_t = \varphi \frac{C_t}{1 - L_t} \tag{30}$$

$$m_{t,t+1} = \frac{1}{1 + r_{t+1}} = \beta \frac{U_1(C_{t+1}, L_{t+1})}{U_1(C_t, L_t)} \tag{31}$$

with rates of return on assets equalized across sectors:

$$1 + r_{t+1} = \frac{(1 - \delta^x) V^x_{t+1} + \pi^x_{t+1}}{V^x_t} \quad for \;\; x = D, G, fg. \tag{32}$$

2.7 Investment, fuel costs and the aggregate resource constraint

Absent adjustment costs, immediate changes in sector-specific technology or taxes imply implausibly large investment and disinvestment patterns across sectors. To address this concern, the model allows for adjustment costs when investment rises above past investment. In particular, increasing investment in sector x relative to last period entails a per-unit of investment cost:

$$C^x_t = \min \left(\frac{\phi_I}{2} \left(\frac{I^x_t}{(1 + g_{x,I} + \delta_x) I^x_{t-1}} \right)^2 - 1, 0 \right)$$

where $g_{x,I}$ is the steady-state growth rate of investment in sector x and sectoral investment reflects both the number of machines and the quality of machines that are constructed:

$$\begin{aligned} I^D_t &= Q^D_t k^D_t f_t \quad \geq 0 \\ I^G_t &= Q^G_t k^G_t \quad \geq 0 \\ I^{fg}_t &= Q^{fg}_t k^{fg}_t e_t \quad \geq 0. \end{aligned}$$

This specification implies that the marginal cost of investment is

$$P^{x,I}_t = 1 + \min \left(\phi_I \left(\frac{I^x_t}{(1 + g_{x,I} + \delta_x) I^x_{t-1}} \right), 0 \right)$$

so that there are zero adjustment costs at the steady-state that rise when investment exceeds its steady-state level. The model also imposes a lower bound of zero on investment so that the investment in past machines is irreversible. The asymmetric nature of the adjustment cost is then consistent with the notion that capacity constraints on the production of new investment goods bind during rapid expansions.

Fossil fuels are assumed to be abundant but costly to extract. In particular, a unit of fuel can be extracted at a constant cost P^F in terms of current units of final goods output. These assumptions imply that output in the final goods sector is used for consumption, investment inclusive of adjustment costs, and fossil fuel extraction:

$$Y_t = C_t + \sum_{x = fg, D, G} \left(1 + C^x_t\right) I^x_t + P^f F_t \tag{33}$$

3 Calibration

Table 1 displays the model parameters chosen for the calibration. The model parameters contain macro parameters that govern household behavior along with parameters that determine the importance of both fuel and energy for the aggregate economy, and the relevance of the putty-clay mechanism. Household parameters include the discount factor β, the intertemporal elasticity of substitution $1/\eta$, and the inverse-Frisch labor supply elasticity. These are chosen to be 0.9925, 1, and 3, respectively, which are standard values in the literature. We calibrate the trend growth rate of final output to two percent, with one percent per year from growth in disembodied technology and one percent from technology embodied in new capital goods. Maximum utilisation rates in the final goods sector are on the order of 0.88 which implies setting $\sigma_{fg} = 0.33$.

Parameter	Value	Description
$1-\alpha^{FG}$	0.55	Labour share in final goods sector
λ^{FG}	0.37	Capital share in final goods sector
$\alpha^{FG}-\lambda^{FG}$	0.08	Energy income share in final goods sector
$1-\alpha^{D}$	0.17	Labour share in dirty energy sector
λ^{D}	0.10	Capital share in dirty energy sector
$\alpha^{D}-\lambda^{D}$	0.72	Fossil fuel share in dirty sector
α^{G}	0.6	Capital share in green sector
ϵ	2	Elasticity of substitution between energy types
$\delta^{FG}=\delta^{G}$	0.10	Depreciation in final goods sector
δ^{G}	0.01	Depreciation in green sector
δ^{D}	0.001	Depreciation in dirty sector
σ_{FG}	0.33	St dev. log productivity in final goods sector
σ_{D}	0.11	St dev. log productivity in dirty sector
σ_{G}	0.76	St dev. log productivity in green sector
ζ	4.35e-05	Climate damage in production function
φ^{L}	0.2	Carbon stock depreciation parameter
φ^{0}	0.40	Carbon stock depreciation parameter
φ^{1}	0.0023	Carbon stock depreciation parameter
Ω_a	0	Adaptation process parameter
Ω_b	0.016	Adaptation process parameter
β	0.9925	Discount rate
$1/\eta$	1	Elasticity of intertemporal substitution (EIS)
φ	3	Leisure preferences
$g_{\theta d}=g_{\theta g}$	1%	Embodied growth in energy sectors
g_{zfg}	1%	Disembodied growth in final goods sector
g_y	2%	GDP growth

Notes: Parameter calibration for the model described in the text.

Table 1: Model Calibration

The depreciation schedule along with the rate of growth of embodied technology and the dispersion in idiosyncratic project outcomes all influence the extent to which the model displays putty-clay features. The depreciation rate in the final goods sector is set to 10 percent per year. The depreciation rate in the green energy sector is set to 1 percent per year, and the depreciation rate in the dirty energy sector is set to 0.1 percent per year to reflect the fact that fossil-fuel plants are longer lived than other forms of capital.

We calibrate σ_d, σ_g and the growth rates of embodied technology in the energy sectors (these growth rates are constrained to be greater than zero and, at most, equal to the growth rate of final output) to match micro evidence on the utilisation-age schedule of plants in the US

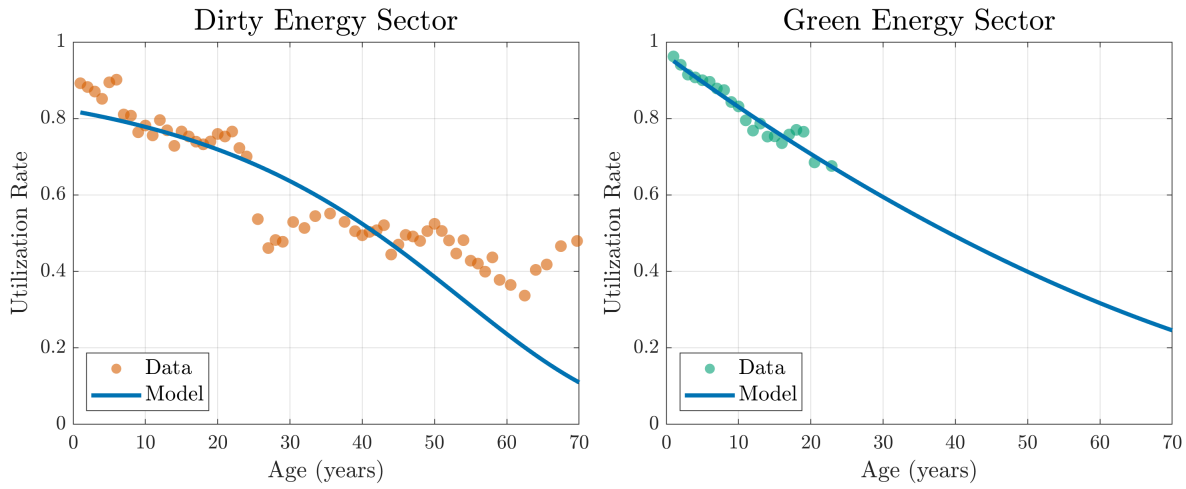


Figure 5: Model fit to age-utilisation microdata

energy sector. Figure 5 plots the fit of the model-implied age-utilisation schedule to the microdata.

The final goods sector has a labor share $1-\alpha^{FG}=0.55$, a capital share $\lambda^{FG}=0.37$ and an energy share $1-\alpha^{FG}-\lambda^{FG}=0.08$. This energy share is consistent with BEA estimates reported for the mid-2000s. Labour share estimates derived from the utility sector suggest setting the labour share for dirty energy $1-\alpha_d=0.17$. The capital share in the dirty energy sector is $\lambda^D=0.10$, which implies a fuel share in the dirty sector of $\alpha^D-\lambda^D=0.72$.[2] We calibrate the capital share in the green energy sector such that the labor share of (non fuel) cost is the same as in the dirty energy sector, which gives $\alpha_g=0.6$. Consistent with recent data, the relative productivity of the dirty versus green energy sector is chosen so that green energy accounts for 17 percent of total energy output.

To analyze green technology transitions, we calibrate a scenario using observed declines in solar photovoltaic costs. Figure 6 shows the evolution of solar panel prices from 1975 to 2023, measured as the log-inverse price to highlight efficiency improvements. Solar efficiency exhibited rapid growth of approximately 20% per year from 1975-1988, slower growth of 5% annually from 1988-2008, and a renewed period of rapid growth at roughly 20% per year since 2008. This dramatic cost reduction—an 88% decline in the levelized cost of electricity for utility-scale solar PV between 2010 and 2021—coincided with substantial increases in solar capacity investment documented in Figure 2. We use these patterns to calibrate a green technology improvement scenario that matches the post-2008 period of rapid cost decline. The details of this green energy transition experiment are provided in Appendix B.

For the climate block, we closely follow prior literature, using the same carbon depreciation

[2]The calibration implies a fuel share of non-capital costs of 76%, consistent with EIA data (see https://www.eia.gov/electricity/annual/html/epa_08_04.html). This calibration is determined jointly with other energy sector parameters to match three moment conditions: green energy quantity share (17%), total energy expenditure ratio (7.6% of output), and fossil fuel expenditure ratio (5.6% of output), as well as age-utilisation profiles for both energy sectors.

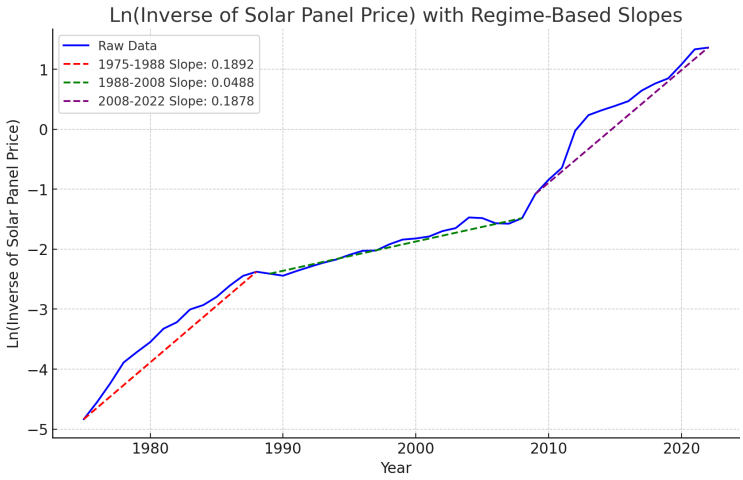


Figure 6: Solar Panel Technology: Log-Inverse Price, 1975-2023

parametrisation as Golosov et al. (2014a). We calibrate the damage function parameter ζ such that climate damages initially reduce output by 3% prior to policy implementation, a magnitude that is somewhat larger than the benchmark DICE calibration of Barrage and Nordhaus (2024), broadly consistent with empirical estimates implying persistent level effects of temperature on output of Nath et al. (2024), but substantially smaller than recent macroeconomic estimates of Bilal and Känzig (2024) that imply much larger long-run output losses from global warming. The long-run adaptation process Ω_t is initially 1, not affecting GDP, and then grows as described in Section 2.5 according to a similar process to Barrage (2014).

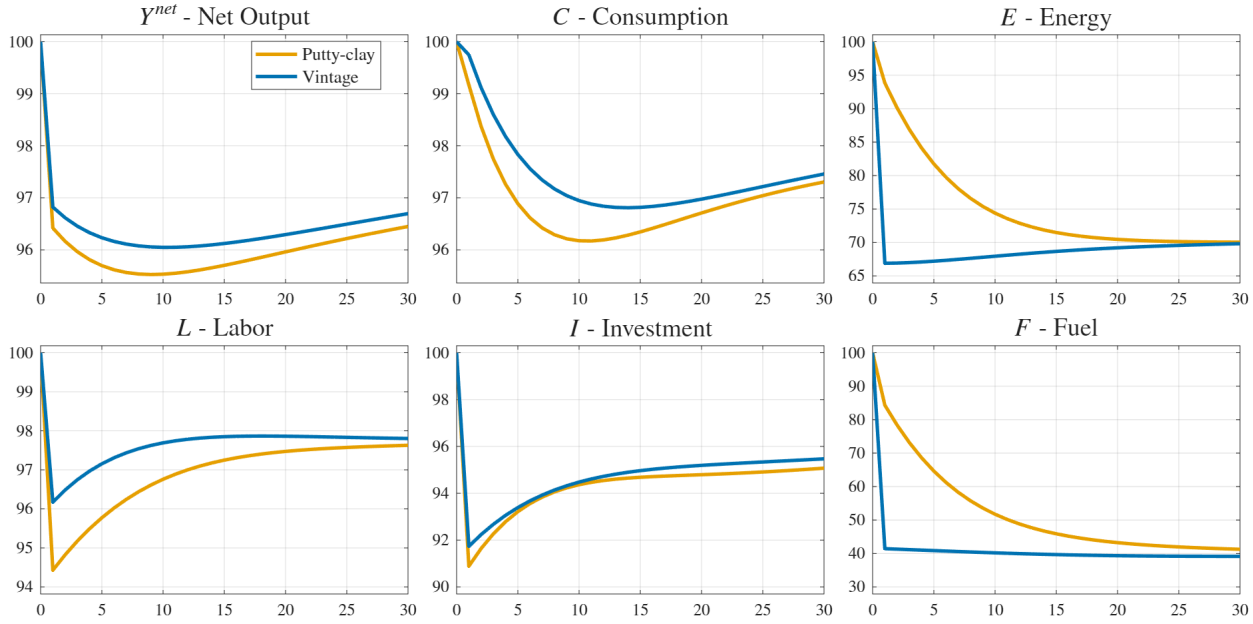
4 Impact of a Carbon Tax in a Putty-Clay framework

In this section we consider the effects of carbon tax policies that seek to reduce fossil-fuel use. In all cases, revenues obtained from such taxes are rebated lump-sum to households.

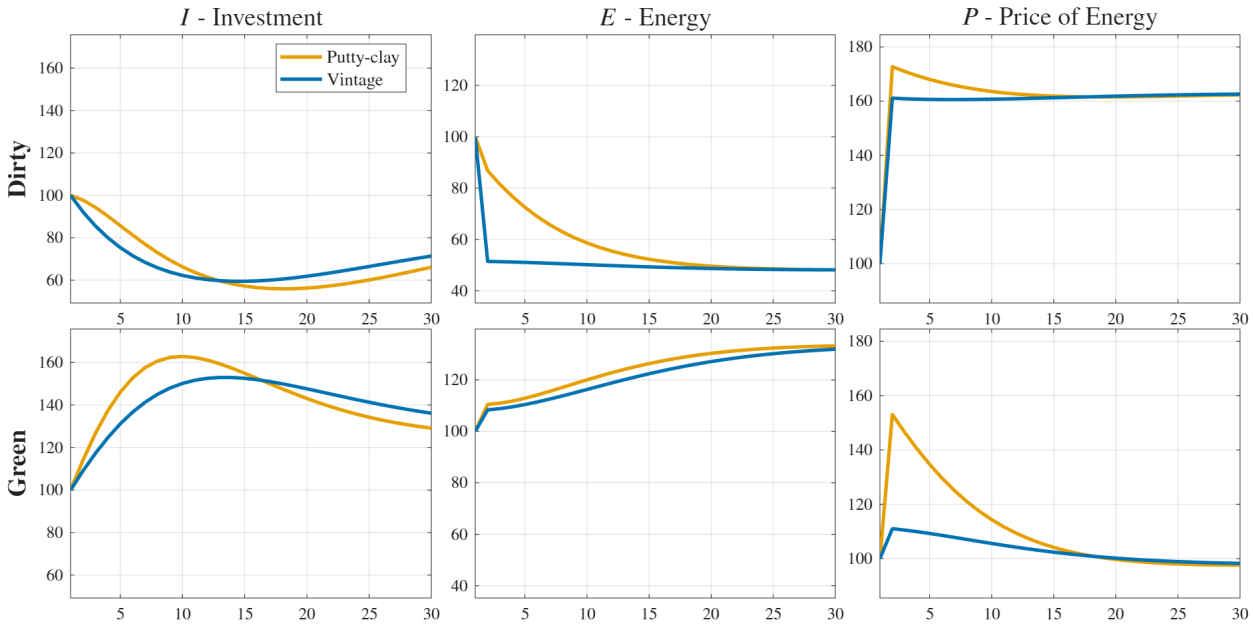
4.1 A Fixed Carbon Tax

The first experiment considers the effect of an unanticipated carbon tax that permanently doubles the price of fossil fuel. This is equivalent to a carbon tax of approximately USD $75 per ton of carbon emissions, well within the range considered in current policy discussions.

Panel A of Figure 7 displays the aggregate responses to this carbon tax. In the short run, the tax causes a substantial contraction in economic activity. In the putty-clay model, output falls by 3.5%, investment by 9%, and labour by 4.5%. In the putty-clay model, there is very little short-run substitutability between energy and other inputs. As a result, total energy only falls gradually in the short-run. As the economy adjusts, energy production continues to fall while output, consumption and investment recover. Labour remains permanently slightly below its initial steady-state however.



(a) Panel A: Aggregate Responses



(b) Panel B: Energy Sector Response

Figure 7: Carbon Tax

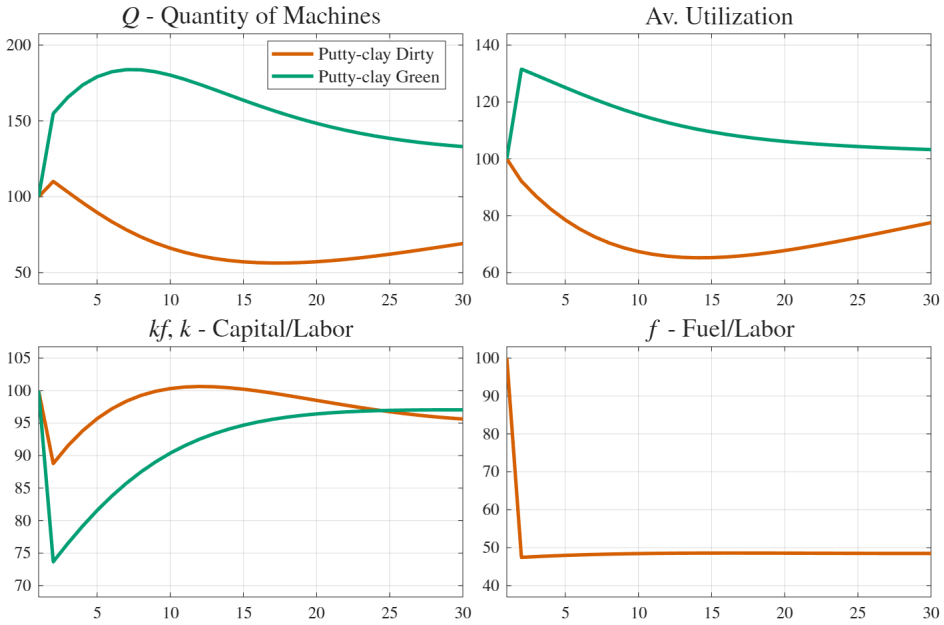


Figure 8: Carbon Tax: Putty-Clay Model

The lack of substitutability between fossil fuels and other inputs also implies sharp rises in energy prices in both the dirty and green energy sectors, along with a gradual decline in dirty energy investment and a shift towards green energy investment. The primary margin of adjustment here is that new machines that are built in the dirty-energy sector reflect the desired long-run fuel-to-labour ratio which falls 50 percent due to the doubling of the fossil fuel price. Thus there are two forces at work to reduce fossil fuel usage: a switch in production towards clean rather than dirty energy and new investments in the dirty-energy sector that embody lower fossil fuel requirements.[3] To enable these transitions, Figure 8 shows how at the machine level, utilisation declines in dirty sectors and new machines installed have sharply lower fuel-labour ratios. Fossil fuel usage gradually declines as these transitions occur.

In contrast to the putty-clay model, the fossil fuel usage and energy production exhibit large immediate declines in the vintage model. The forty percent initial decline in energy production is only a few percent greater than the long-run decline. Similarly, the initial decline in fossil fuel usage is only a few percent above the eventual decline of sixty percent. This adjustment occurs with less disinvestment in the dirty sector and less overall investment in the green sector in comparison to the putty-clay model. Because fossil fuel usage and energy production only adjust gradually in the putty-clay framework, the tax is much more effective in reducing the carbon stock in the vintage model relative to the putty-clay model.

This raises the question of how much larger fossil fuel taxes need to be under the putty-clay framework to achieve the same carbon stock reduction as under the vintage model. Figure

[3]This dynamic of investment shifting overall energy sector capital to be more carbon efficient in both the carbon pricing and green technology improvement cases is consistent with our descriptive evidence and other evidence, such as Bolton and Kacperczyk (2023) that newer capital vintages are more energy efficient.

9 shows the result of a targeting exercise over a 25-year horizon. To achieve the same 2050 carbon stock, fossil fuel taxes need to rise by 55% more under the putty-clay framework. Because the fossil fuel tax takes time to be effective, the near-term carbon stock is relatively high, but converges to a lower level in the long-term as the putty-clay dynamics fade. The economic effects of achieving this target are much more damaging. Output declines more in the short-term and throughout the transition, under the putty-clay model. Consumption declines by nearly 3pp more in the short-term, while investment also declines more sharply and consistently.

4.2 Optimal Carbon Taxation

In the previous section, we outlined results for a fixed carbon tax. As discussed, we view this a reasonable benchmark and approximation of carbon taxes implemented in practice across those economies which have mature systems for pricing carbon emissions. However, this tax is, of course, suboptimal. In this section, we derive the optimal carbon tax under the putty-clay model and show how the macroeconomic impacts of this optimal tax compare.

Solving the planner's problem results in the below optimality conditions for the carbon stock (including its depreciating and permanent components outlined in Section 2.5):

$$
\begin{aligned}
S: &\quad \lambda_{c,t}\frac{\partial Y_t}{\partial S_t} + \lambda_{d,t}\frac{\partial E_t^D}{\partial S_t} + \lambda_{g,t}\frac{\partial E_t^G}{\partial S_t} + \lambda_{s,t} = 0 \\
S^1: &\quad \lambda_{s,t}^1 - \mathbb{E}_t[\beta(1-\varphi_1)\lambda_{s,t+1}^1] - \lambda_{s,t} = 0 \\
S^2: &\quad \lambda_{s,t}^2 - \mathbb{E}_t[\beta(1-\varphi_2)\lambda_{s,t+1}^2] - \lambda_{s,t} = 0
\end{aligned}
$$

Where $\lambda_{c,t}, \lambda_{d,t}, \lambda_{g,t}$ are Lagrange multipliers on consumption, dirty energy and green energy

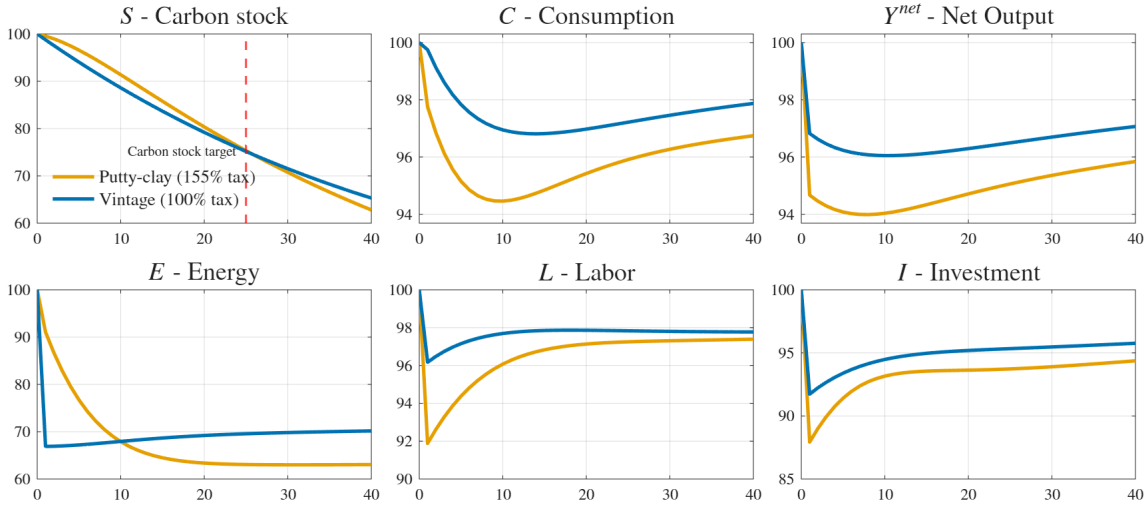


Figure 9: Targeting 2050 carbon stocks

respectively, $\lambda_{s,t}, \lambda^1_{s,t}, \lambda^2_{s,t}$ are Lagrange multipliers for the carbon stock and subcomponents. See Appendix C for details. Combined, these show the dynamic trade-off between consumption and energy outputs today, compared with the long-lasting productivity cost of carbon damaging productivity. The optimality conditions for the dirty energy sector are:

$$
\begin{aligned}
z^D: \quad & \lambda_{d,t}\frac{\partial E^D_t}{\partial z^D_{t,t-j}} - (\lambda_{c,t}P^f_t + ((1-\omega_L)\omega_0\lambda_{s^1,t} + \omega_L\lambda_{s^2,t}))\frac{\partial F_{t,t-j}}{\partial z^D_{t,t-j}} = 0 \\
k^D: \quad & \lambda_{c,t}f_tQ^D_t = \mathbb{E}_t\left[\sum_{k=1}^{M}\beta^k\lambda_{d,t+k}\frac{\partial E^D_{t+k}}{\partial k^D_t}\right] \\
f: \quad & \lambda_{c,t}k^D_tQ^D_t = \mathbb{E}_t\left[\sum_{k=0}^{M}\beta^k\left(\lambda_{d,t+k}\frac{\partial E^D_{t+k}}{\partial f_t} - (\lambda_{c,t+k}P^f_{t+k} + ((1-\omega_L)\omega_0\lambda_{s^1,t+k} + \omega_L\lambda_{s^2,t+k}))\frac{\partial F_{t+k}}{\partial f_t}\right)\right] \\
Q^D: \quad & \lambda_{c,t}k^D_tf_t = \mathbb{E}_t\left[\sum_{k=0}^{M}\beta^k\left(\lambda_{d,t+k}\frac{\partial E^D_{t+k}}{\partial Q^D_t} - \lambda_{c,t+k}P^f_{t+k}\frac{\partial F_{t+k}}{\partial Q^D_t}\right.\right. \\
& \left.\left. - ((1-\omega_L)\omega_0\lambda_{s^1,t+k} + \omega_L\lambda_{s^2,t+k})\frac{\partial F_{t+k}}{\partial Q^D_t} - \lambda_{l,t+k}\frac{\partial L^D_{t+s}}{\partial Q^D_{t+s}}\right.\right.
\end{aligned}
$$

These demonstrate key implications for an optimal carbon tax under a putty-clay framework. Each firm decision that increases fossil fuel usage, namely $\frac{\partial F_{t,t-j}}{\partial z^D_{t,t-j}}$, adjusting utilisation, $\frac{\partial F_{t+k}}{\partial f_t}$, increasing fossil fuel intensity of machines, and $\frac{\partial F_{t+k}}{\partial Q^D_t}$, increasing the number of dirty machines have a shadow price: $((1-\omega_L)\omega_0\lambda_{s^1,t+k} + \omega_L\lambda_{s^2,t+k})$. This shadow price does not depend on the nature of the machine, purely the additional fuel used and resulting emissions. What that means is that the optimal carbon tax does not vary across the distribution of machines captured in the putty-clay framework; the putty-clay dynamics will impact the size of the tax, but this does not need to be different across machines within a period. In addition, this enters in the same manner as the fossil fuel price, so a carbon tax should be levied on fossil fuel consumption in proportion to its emission intensity.

Given this, we can decentralise the planners solution using prices, including a carbon tax τ_t of the form:

$$
\tau_t = \zeta\sum_{s=0}^{\infty}M_{t,t+s}((1-d_s))\left[\frac{Y_{t+s}(1-\alpha^{FG})}{\Omega_{t+s}} + \frac{P^D_{t+s}E^D_{t+s}(1-\alpha^D)}{\Omega_{t+s}} + \frac{P^G_{t+s}E^G_{t+s}(1-\alpha^G)}{\Omega_{t+s}}\right]
$$

Where $M_{t,t+s}$ is the SDF of the representative household, $(1-d_s)$ is the carbon depreciation process described in Section 2.5. As in Golosov, Hassler, Krusell, and Tsyvinski (2014b), the tax grows with the size of the economy (here also taking into account damages in the dirty and green energy subsectors). Our inclusion of the long-term adaptation process Ω_t means that in the (very) long run, the tax stabilises as technology growth to mitigate climate

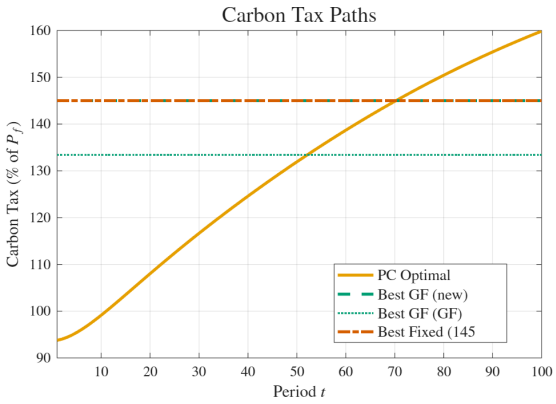


Figure 10: Optimal carbon tax and alternatives

damages offsets future emissions. That said, we calibrate this process such that the medium-term dynamics we explore in this paper are unaffected by the eventual long-run adaptation.

In our quantitative model, the behaviour of this optimal tax is shown in Figure 11. The optimal carbon tax is initially 95% of the cost of fuel, then rising with economic growth to over 150% of the fuel price by 100 years after implementation. To complement this optimal tax, we compare two additional suboptimal rules, the dynamics and nature of which in our view are better approximations of carbon tax policies put in place in the past two decades. The first of these is an 'optimal fixed' carbon tax, so one which remains constant rather than rising with economic growth. We find that an optimised level of a fixed tax is 145%, higher than the benchmark 100% discussed in the previous section. A second is where we 'grandfather' pre-existing assets, which are installed prior to policy implementation. We discuss rationales for the latter further in Section 5. The initial fixed taxes, whether grandfathering in pre-existing assets or not, are much larger initially than the optimal carbon tax.

Figure 11 demonstrates how implementing the optimal tax alters macroeconomic dynamics, the counterpart of Figure 7a's 100% fixed carbon tax. An optimal fixed tax causes a large decrease in output and consumption (larger than that in Section 4, as the optimal fixed tax is larger than our 100% tax benchmark). As the optimal tax grows over time from a lower level, the optimal tax causes a smaller near-term recession. Otherwise, most dynamics are attenuated relative to under the optimised fixed tax. Grandfathering pre-existing assets, which may be useful due to the alleviation of stranded asset issues as discussed in the next section, has the effect of attenuating slightly the worst of the effect of the fixed tax, moving dynamics more towards those of the optimal tax.

4.2.1 Welfare and policy implications

Next, we assess the welfare impact of fossil fuel tax policies. Figure 12 shows the change in welfare under the range of optimal and suboptimal policies, under the vintage and putty clay frameworks, expressed in consumption equivalent terms. The difference in welfare across the two models is substantial. Under the more flexible, vintage model, the carbon tax improves

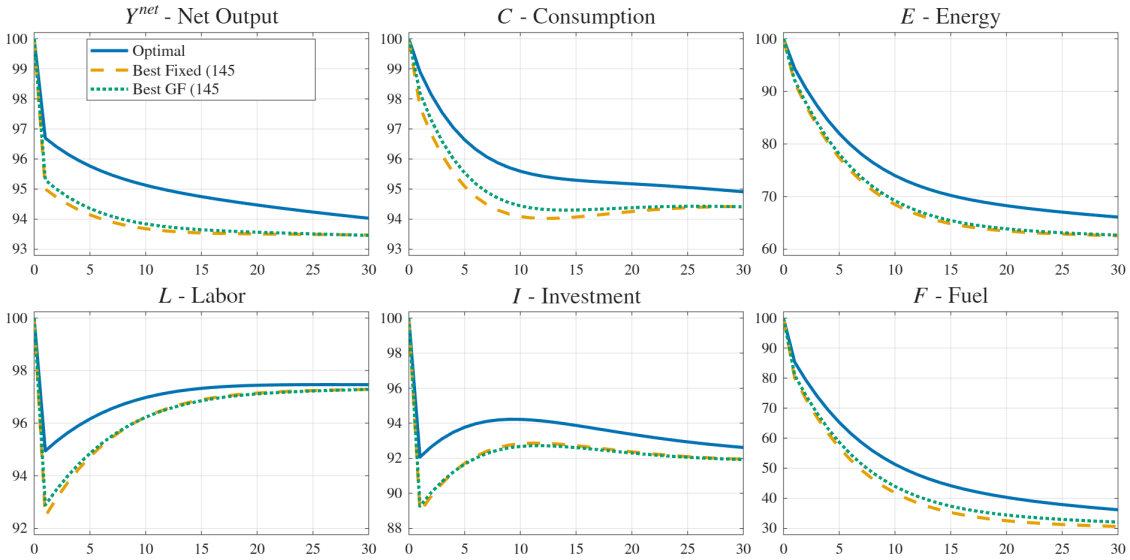


Figure 11: Macro-economic dynamics under optimal policy, and after optimised fixed and grandfathered taxes

welfare as the externality from climate change is reduced. In the putty-clay model, the welfare improvement from carbon taxation is significantly smaller for all values of the tax, and the optimal tax is much smaller. This is because there is a substantial portion of the pre-existing dirty energy capital stock which is now not profitable to use. This represents a large welfare loss from stranded assets which are mothballed in the aftermath of the policy implementation.

Figure 12 further shows the comparison between different levels of fixed taxes, compared to the optimal tax (which varies over time). A fixed tax is worse than the optimal tax, but the differences between the nature of the tax implemented are marginal compared to the welfare gain of imposing any tax. The consumption equivalent welfare increases just under 4.5% under an optimal tax, compared with just under 4.45% under the optimal fixed tax.

4.3 Climate Risk

A growing body of climate science points to the possibility of tipping points, thresholds beyond which climate damages escalate sharply and irreversibly (IPCC Working Group III, 2022). We capture this idea in reduced form by introducing a climate learning hazard. The planner chooses a constant tax $\bar{\tau}$ at date 0. In each subsequent period there is a probability p_ℓ that the economy discovers climate damages are far more severe than previously believed, representing the realization of a climate tail risk such as a tipping point. If this discovery occurs at date D, the damage elasticity is permanently scaled up:

$$\zeta_t = \begin{cases} \zeta & t < D, \\ \kappa_H \, \zeta & t \geq D, \end{cases} \tag{34}$$

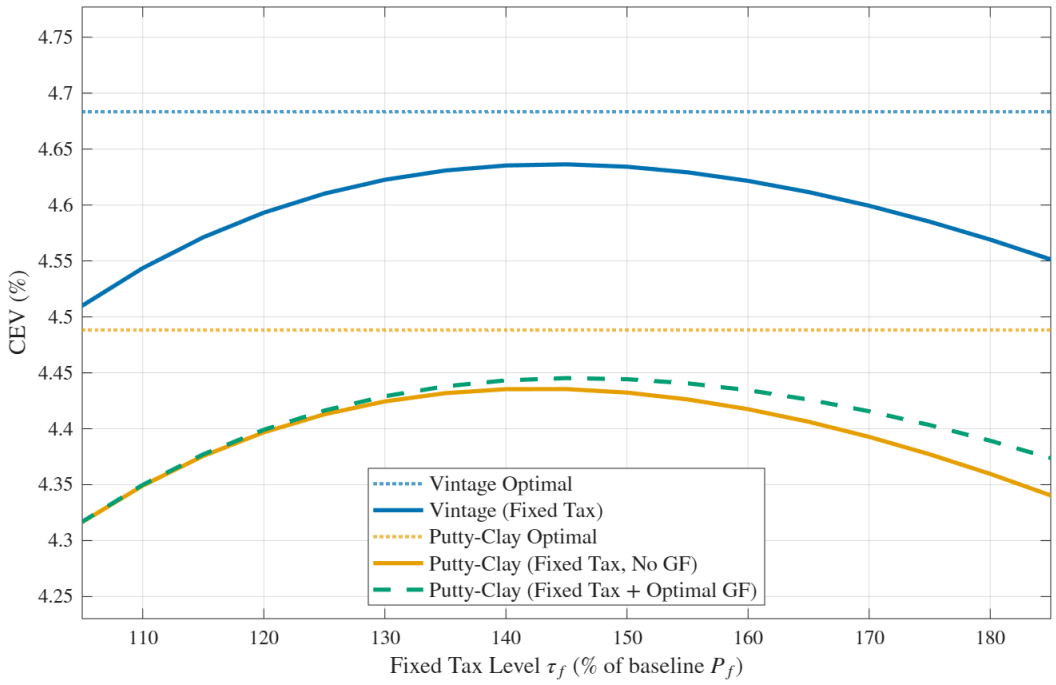


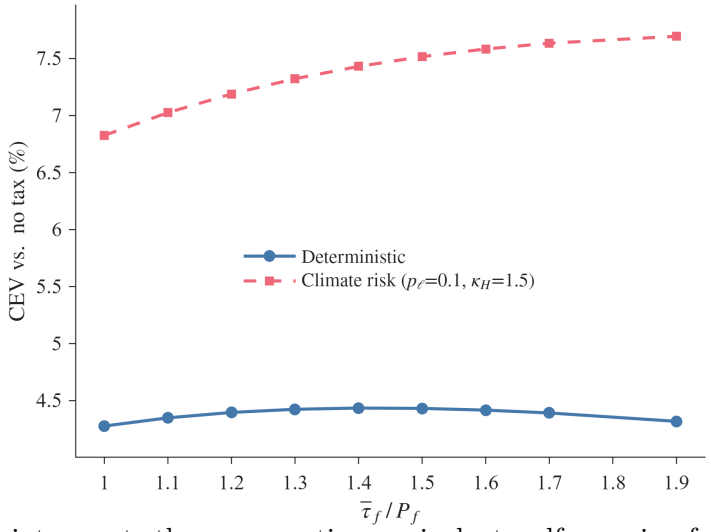
Figure 12: Consumption equivalent welfare for different policies and models

where $\kappa_H > 1$ governs the severity of the bad climate news. Expected welfare averages over the stochastic discovery date D, which follows a geometric distribution with parameter p_ℓ.

Figure 13 plots consumption-equivalent variation (CEV) relative to no tax as a function of $\bar{\tau}/P_f$, for the deterministic case ($p_\ell = 0$) and the climate-risk case ($p_\ell = 0.1$, $\kappa_H = 1.5$).

Two features stand out. First, climate risk shifts the welfare curve upward: for any given tax rate, a non-zero tax provides insurance against the possibility of severe future damages, raising welfare relative to the deterministic case. Second, the welfare-maximizing tax rate increases under climate risk. The best fixed tax rises because the precautionary motive, lowering both the inherited carbon stock and the dirty-capital share before bad news arrives, is stronger when there is a positive probability of transitioning to the high-damage regime. In the putty-clay model this channel is particularly powerful. The planner values a higher pre-discovery tax because it shapes the composition of the installed capital stock in a way that cannot be replicated once the high-damage state is revealed, since vintage-level input ratios are locked in at installation.

In ongoing work we are extending this analysis in two directions. First, we allow the planner to adjust the tax rate conditional on realization of the climate damage revelation, so that the policy response can differ before and after the tipping point is reached. Second, we introduce the risk that a future policy maker removes carbon taxes altogether, capturing political uncertainty about the durability of climate policy. Both extensions interact with capital irreversibility in ways that are absent from the vintage model, and we expect them to reinforce the finding that putty-clay frictions raise the value of early action.



Notes: Each point reports the consumption-equivalent welfare gain of a fixed carbon tax relative to no tax. The solid line is the deterministic benchmark ($p_\ell = 0$). The dashed line adds climate learning risk with discovery hazard $p_\ell = 0.1$ and damage severity multiplier $\kappa_H = 1.5$.

Figure 13: Welfare vs. fixed carbon tax in the putty-clay model: deterministic and climate-risk scenarios.

5 Stranded assets

In this section, we study stock market responses to a carbon tax. A salient concern in the policy debate on carbon tax implementation (Carney (2015), von Dulong, Gard-Murray, Hagen, Jaakkola, and Sen (2023)) is the "stranding" of assets, untapped fossil fuel reserves, and fuel-producing and -burning capital that would be rendered worthless (or at best much less valuable) by taxation. The importance of stranded assets is often motivated by political economy and distributional concerns or by implications for financial stability of large falls in asset values. The concept of a stranded asset is not particularly concrete in a putty-putty world, as we illustrate below. However, with putty-clay technology, we can study the implications of this phenomenon. We focus specifically on capital which is reliant on fossil fuels, the installed machines and infrastructure whose fuel intensity is locked in, rather than unburnable fossil fuel reserves, as this is the dimension of stranding that our framework captures.

We start by defining the value of capital installed prior to the policy change. This is the time t value of the sector I capital stock installed before time t_1, given by the aggregate present discount value of the profits accruing to surviving capital vintages older than t_1:

$$V^I_{t_1,t} = \sum_{j=t-t_1}^{t-t_1+M-1} \sum_{s=1}^{\infty} m_{t,t+s} \pi^I_{t+s,t-j} \left(1-\delta_I\right)^j Q_{I,t-j}, \tag{35}$$

where $I \in \{D, G, fg\}$, and profits in the dirty, green and final goods sectors are given

respectively by Equations 4, 12, and 18.

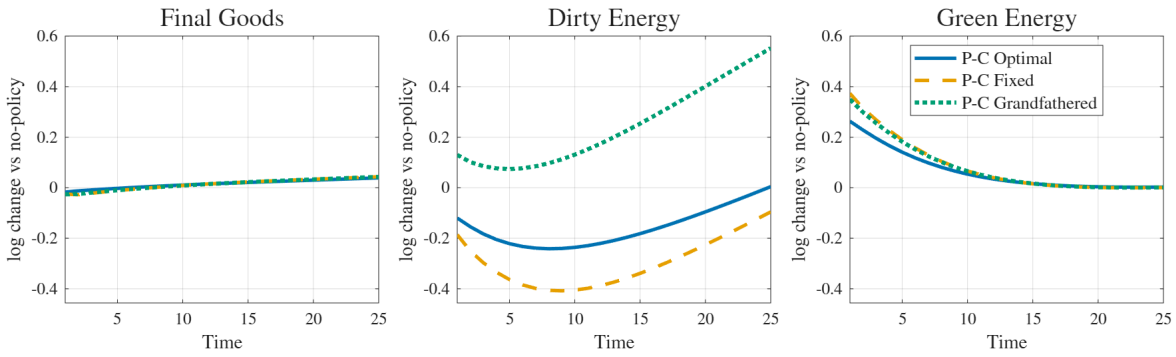
Figure 14 shows the transition path of the value of the capital stocks in each of the three sectors in response to a fuel tax, in log deviation from the balanced growth path trajectory. The upper panel shows the responses under the putty-clay model, and the bottom panel shows the responses under the vintage model, in both cases discussing the impacts of the optimised policies discussed in Section 4.2.

Valuation of the stranded assets in the dirty sector is clear in the putty-clay model. The value of the dirty sector declines on impact, but this is not the peak response. This is because the effect of the doubling of fuel prices on profits is initially offset by a jump in the dirty energy price and a fall in the wage. Over time, dirty assets are stranded in the sense that their value falls persistently, reaching a trough of just over -20% seven years after the initial shock under the optimal tax, -40% under the fixed tax.

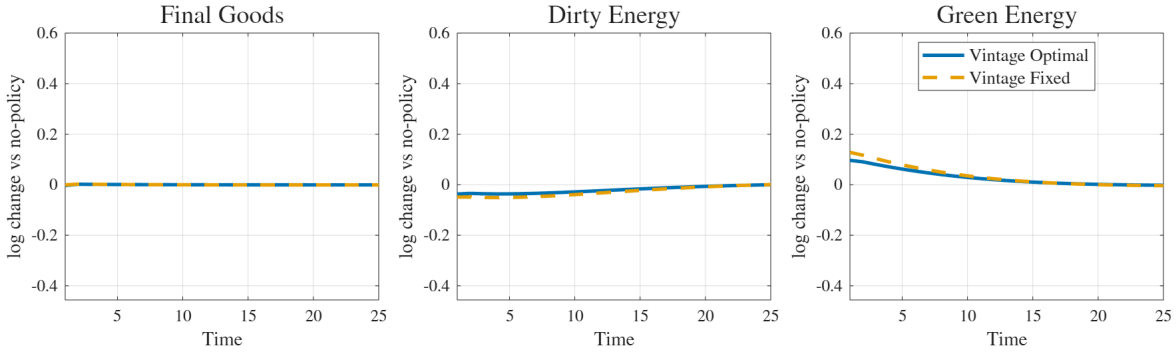
This occurs because old, dirty capital is more heavily dependent on heavily taxed fossil fuels. Though dirty energy prices rise, the resulting rise in revenues is not sufficient to fully offset the rise in costs, and so valuations decline. Because the cost margin on fuel has risen, a portion of intra-marginal machines (which under the balanced growth path would have been used for a more extended period) are switched off, and utilisation falls rapidly. However, this process takes some time to take effect. In the near-term, when new green and more energy efficient dirty machines have not yet been installed, dirty energy prices spike and old dirty machines continue to be used. Once investment in new machines is underway, utilisation drops more sharply, and in tandem, valuations drop further. For more discussion of the underlying drivers and dynamics of the stock market response, see Appendix Section D. These valuation impacts correspond, qualitatively, with findings in the climate finance literature (e.g. Engle et al. (2020), Bolton and Kacperczyk (2021), Berg et al. (2021) and Hsu et al. (2023)) on the response of stock prices to climate policy news and the resulting equilibrium of higher returns in high emitting industries.

The value of the green sector jumps on impact due to the large increases in energy prices, while their only variable input cost - labour - becomes cheaper. Valuation changes in the final goods sector are modest. This is again somewhat consistent with evidence from Bolton and Kacperczyk (2021). However, the perfect information transmission means this is because the pass-through of higher fossil fuel costs is small; not because investors may only be paying attention to scope 1 emissions or particular industries, in a manner perhaps more consistent with imperfect market understanding of carbon risk exposure.

In the vintage model the valuation of old capital is relatively unaffected. This is because assets are not really "stranded"; although their older technology becomes more out of date, the fact at input ratios are not fixed means that the dirty energy sector is able to use old capital in a more fuel efficient way. As a result, energy sector valuations only move slightly and mirror each other. There is a small and transitory fall in value in the dirty sector, and the opposite in the green. The value of the final goods sector also increases modestly, because the negative effect of higher energy prices on profits is more than offset by a fall in wages.



(a) Stock market response in the putty-clay model



(b) Stock market response in the vintage model

Figure 14: Stock market response to fossil fuel tax

Grandfathering stranded assets To make the carbon tax more palatable by avoiding these negative effects on valuations, one option is grandfathering; allowing capital installed prior to policy implementation to have reduced or no carbon tax liability. The dotted green lines in Figure 14 plots the responses of values in the putty-clay model, but using the alternative tax policy in which vintages of capital installed before the tax is introduced pay a 13% lower carbon tax rate. This results in the value of old vintages in the dirty sector increasing on impact, so this alternative policy does not result in stranding of assets. Values in the green and final goods sector respond similarly to the uniform carbon tax case (with more muted responses in both cases). In addition to mitigating the stranded asset valuation issue, there are small positive welfare effects of grandfathering relative to the uniform fixed tax, as shown in Figure 12 and discussed in the previous section. This suggests that a carbon tax policy that partially exempts existing dirty assets from taxation might be preferable from both welfare perspectives and may be more politically feasible.

6 Conclusion

We propose a multi-sector integrated assessment model to understand the economic impacts of a green transition. The key feature of our model is the use of putty-clay technology in each of the sectors. This allows us to model the fixity and irreversibility of capital, producing a

vintage structure and potential underutilisation of old capital. These features of the capital composition are consistent with observations from the data on the energy sector in the US.

Using this model, we study the effectiveness of carbon taxes in reducing fossil fuel usage and provide an explicit comparison to the standard model of capital accumulation that treats fuel to capital ratios as malleable and is widely adopted in this literature. Imposing a uniform carbon tax leads to a much greater contraction in output and consumption along the transition path in the putty-clay model relative to the standard model. Moreover, to achieve the same reduction in carbon stocks at a twenty-five year horizon requires 55% increase in the tax on fossil fuels in the putty-clay model relative to the standard model. We then study optimal policy and show that both the putty-clay model and the standard model imply a gradual introduction of the carbon tax that rises slowly over time. This gradual introduction offers a modest improvement on welfare relative to the gains achieved under the immediate imposition of a uniform tax on fossil fuels.

Although a uniform carbon tax does well in terms of welfare, it also leads to a 40 percent decline in the value of existing assets in the dirty energy sector in the putty-clay version of the model. In contrast, there is almost no decline in the value of assets in the standard model. In the putty-clay model, this decline in asset valuations occurs because capital in the dirty-energy sector is stranded as rising costs of fossil fuel render existing assets unprofitable. This stranding of existing assets does not occur in the standard model where capital is malleable and hence not subject to abandonment. To the extent that policymakers are concerned about such valuation effects, it may be desirable to consider carbon tax policies that grandfather existing assets. Indeed, we show that a modest degree of grandfathering – roughly a 13 percent discount on the carbon tax for existing assets relative to new assets – achieves greater welfare gains than a uniform tax and causes the valuation of existing assets in the dirty energy sector to rise rather than fall over time.

References

Acemoglu, D., P. Aghion, L. Bursztyn, and D. Hemous (2012): "The Environment and Directed Technical Change," *American Economic Review*, 102, 131–166.

Acemoglu, D., D. Hemous, L. Barrage, and P. Aghion (2019): "Climate Change, Directed Innovation, and Energy Transition: The Long-run Consequences of the Shale Gas Revolution," Tech. Rep. 1302, Society for Economic Dynamics, publication Title: 2019 Meeting Papers.

Aghion, P., A. Dechezleprêtre, D. Hémous, R. Martin, and J. Van Reenen (2016): "Carbon Taxes, Path Dependency, and Directed Technical Change: Evidence from the Auto Industry," *Journal of Political Economy*, 124, 1–51, publisher: The University of Chicago Press.

Atkeson, A. and P. J. Kehoe (1999): "Models of energy use: Putty-putty versus putty-clay," *American Economic Review*, 89, 1028–1043.

Barnett, M., W. Brock, and L. P. Hansen (2020): "Pricing uncertainty induced by climate change," *The Review of Financial Studies*, 33, 1024–1066.

——— (2022): "Climate change uncertainty spillover in the macroeconomy," *NBER Macroeconomics Annual*, 36, 253–320.

Barnett, M. L., W. A. Brock, H. Zhang, and L. P. Hansen (2024): "Uncertainty, social valuation, and climate change policy," *University of Chicago, Becker Friedman Institute for Economics Working Paper*.

Barrage, L. (2014): "Sensitivity Analysis for Golosov, Hassler, Krusell, and Tsyvinski (2014): "Optimal Taxes on Fossil Fuel in General Equilibrium"," Supplement to econometrica, vol. 82, no. 1 (2014), Econometric Society, online Supplement.

Barrage, L. and W. D. Nordhaus (2024): "Policies, Projections, and the Social Cost of Carbon: Results from the DICE-2023 Model," *Proceedings of the National Academy of Sciences*.

Berg, F., J. Kölbel, A. Pavlova, and R. Rigobon (2021): "ESG Confusion and Stock Returns: Tackling the Problem of Noise," *SSRN Electronic Journal*.

Bilal, A. and D. R. Känzig (2024): "The Macroeconomic Impact of Climate Change: Global vs. Local Temperature," .

Bolton, P. and M. Kacperczyk (2021): "Do investors care about carbon risk?" *Journal of Financial Economics*, 142, 517–549.

——— (2023): "Global pricing of carbon-transition risk," *The Journal of Finance*, 78, 3677–3754.

Burke, M., S. M. Hsiang, and E. Miguel (2015): "Global non-linear effect of temperature on economic production," *Nature*, 527, 235–239.

Cai, Y. and T. S. Lontzek (2019): "The Social Cost of Carbon with Economic and Climate Risks," *Journal of Political Economy*, 127, 2684–2734, publisher: The University of Chicago Press.

Capelle, D., D. Kirti, N. Pierri, and G. Villegas-Bauer (2023): "Mitigating Climate Change at the Firm Level: Mind the Laggards," IMF Working Paper WP/23/242, International Monetary Fund.

Carney, M. (2015): "Breaking the tragedy of the horizon–climate change and financial stability," *Speech given at Lloyd's of London*, 29, 220–230.

Chittaro, L., M. Piazzesi, M. J. Sena, and M. Schneider (2025): "Asset Returns as Carbon Taxes," Tech. Rep. 34342, National Bureau of Economic Research.

Colmer, J., R. Martin, M. Muuls, and U. J. Wagner (2025): "Does pricing carbon mitigate climate change? Firm-level evidence from the European Union Emissions Trading System," *Review of Economic Studies*, 92, 1625–1660.

Cooley, T. F., G. D. Hansen, and E. C. Prescott (1995): "Equilibrium business cycles with idle resources and variable capacity utilization," *Economic Theory*, 6, 35–49.

Cruz, J.-L. and E. Rossi-Hansberg (2024): "The economic geography of global warming," *Review of Economic Studies*, 91, 899–939.

Dell, M., B. F. Jones, and B. A. Olken (2012): "Temperature shocks and economic growth: Evidence from the last half century," *American Economic Journal: Macroeconomics*, 4, 66–95.

Deschênes, O. and M. Greenstone (2012): "The economic impacts of climate change: evidence from agricultural output and random fluctuations in weather: reply," *American Economic Review*, 102, 3761–73.

Desmet, K. and E. Rossi-Hansberg (2015): "On the Spatial Economic Impact of Global Warming," *Journal of Urban Economics*, 88, 16 – 37.

Dietz, S., A. Bowen, C. Dixon, and P. Gradwell (2016): "'Climate value at risk' of global financial assets," *Nature Climate Change*, 6, 676–679.

Dietz, S., F. Van Der Ploeg, A. Rezai, and F. Venmans (2021): "Are Economists Getting Climate Dynamics Right and Does It Matter?" *Journal of the Association of Environmental and Resource Economists*, 8, 895–921.

Engle, R. F., S. Giglio, B. Kelly, H. Lee, and J. Stroebel (2020): "Hedging climate change news," *The Review of Financial Studies*, 33, 1184–1216.

Eskildsen, M., M. Ibert, T. I. Jensen, and L. H. Pedersen (2024): "In Search of the True Greenium," Working paper.

Finkelstein Shapiro, A. and G. E. Metcalf (2023): "The macroeconomic effects of a carbon tax to meet the US Paris agreement target: The role of firm creation and technology adoption," *Journal of Public Economics*, 218, 104800.

Folini, D., A. Friedl, F. Kübler, and S. Scheidegger (2024): "The climate in climate economics," *Review of Economic Studies*, rdae011.

Gârleanu, N. and L. H. Pedersen (2024): "Climate Risk Pricing," Working paper.

Gilchrist, S. and J. C. Williams (2000): "Putty-Clay and Investment: A Business Cycle Analysis," *Journal of Political Economy*, 108, 928–960.

——— (2004): "Transition Dynamics in Vintage Capital Models: Explaining the Postwar Catch-Up of Germany and Japan," DOI: 10.3386/w10732.

——— (2005): "Investment, capacity, and uncertainty: a putty–clay approach," *Review of Economic Dynamics*, 8, 1–27.

Golosov, M., J. Hassler, P. Krusell, and A. Tsyvinski (2014a): "Optimal Taxes on Fossil Fuel in General Equilibrium," *Econometrica*, 82, 41–88.

——— (2014b): "Optimal taxes on fossil fuel in general equilibrium," *Econometrica*, 82, 41–88, publisher: Wiley Online Library.

Hassler, J., P. Krusell, and C. Olovsson (2021): "Directed Technical Change as a Response to Natural Resource Scarcity," *Journal of Political Economy*, 129, 3039–3072.

Hawkins-Pierot, J. T. and K. R. Wagner (2024): "Technology Lock-In and Optimal Carbon Pricing," *University of California, Berkeley*.

Hsu, P.-H., K. Li, and C.-Y. Tsou (2023): "The pollution premium," *The Journal of Finance*, 78, 1343–1392.

Hurst, E., P. J. Kehoe, E. Pastorino, and T. Winberry (2022): "The distributional impact of the minimum wage in the short and long run," Tech. rep., National Bureau of Economic Research.

International Energy Agency (2024): "World Energy Outlook 2024," .

IPCC Working Group III (2022): "Climate Change 2022: Mitigation of Climate Change," .

Johansen, L. (1959): "Substitution versus fixed production coefficients in the theory of economic growth: a synthesis," *Econometrica: Journal of the Econometric Society*, 157–176.

Krusell, P. and A. A. Smith, Jr. (2022): "Climate Change Around the World," DOI: 10.3386/w30338.

Känzig, D. R. (2023): "The Unequal Economic Consequences of Carbon Pricing," .

Lanteri, A. and A. Rampini (2022): "Financing the Adoption of Clean Technology," .

McGlade, C. and P. Ekins (2015): "The geographical distribution of fossil fuels unused when limiting global warming to 2°C," *Nature*, 517, 187–190.

Meng, K. C. (2022): "Estimating Path Dependence in Energy Transitions," DOI: 10.3386/w22536.

Metcalf, G. E. and J. H. Stock (2023): "The macroeconomic impact of Europe's carbon taxes," *American Economic Journal: Macroeconomics*, 15, 265–286.

Nath, I. B., V. A. Ramey, and P. J. Klenow (2024): "How Much Will Global Warming Cool Global Growth?" .

Nordhaus, W. D. (1977): "Economic Growth and Climate: The Carbon Dioxide Problem," *The American Economic Review*, 67, 341–346, publisher: American Economic Association.

——— (1991): "To slow or not to slow: the economics of the greenhouse effect," *The economic journal*, 101, 920–937, publisher: JSTOR.

——— (1992): "An optimal transition path for controlling greenhouse gases," *Science*, 258, 1315–1319, publisher: American Association for the Advancement of Science.

Pedersen, L. H. (2026): "Carbon Pricing versus Green Finance," *The Journal of Finance*.

Popp, D. (2002): "Induced innovation and energy prices," *American economic review*, 92, 160–180.

Solow, R. (1959): "Investment and technological progress"," *Econometrica: Journal of the Econometric Society*, 157–176.

von Dulong, A., A. Gard-Murray, A. Hagen, N. Jaakkola, and S. Sen (2023): "Stranded Assets: Research Gaps and Implications for Climate Policy," *Review of Environmental Economics and Policy*, 17, 161–169.

Wei, C. (2003): "Energy, the Stock Market, and the Putty-Clay Investment Model," *The American Economic Review*, 93, 311–323.

——— (2013): "A dynamic general equilibrium model of driving, gasoline use and vehicle fuel efficiency," *Review of Economic Dynamics*, 16, 650–667.

Weitzman, M. L. (2009): "On modeling and interpreting the economics of catastrophic climate change," *The Review of Economics and Statistics*, 91, 1–19, publisher: MIT Press.

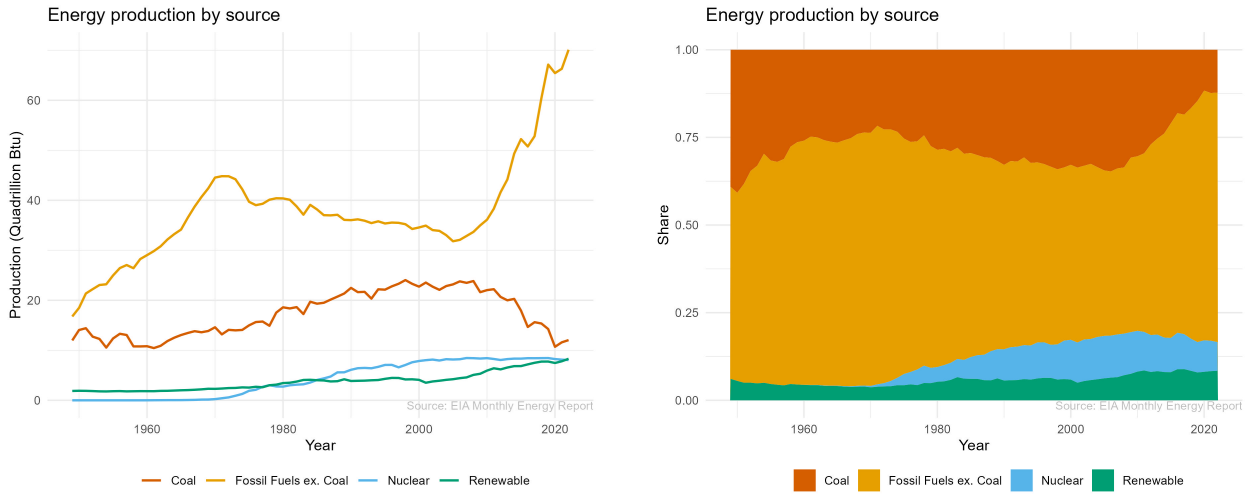
——— (2014): "Fat tails and the social cost of carbon," *American Economic Review*, 104, 544–46.

A Energy Sector Data: Additional Details

This appendix provides detailed documentation of energy production and electricity generation trends in the U.S. economy from 1950-2022. All data are obtained from the U.S. Department of Energy. These patterns complement the stylized facts presented in the introduction and provide comprehensive context for our model calibration.

A.1 Aggregate Energy Production

Figure A.1 shows the evolution of energy production across four sources: coal, fossil fuels excluding coal, nuclear and renewables over the period 1950-2022. The left panel plots production in energy units (Quadrillion BTUs) while the right panel displays the same information as shares of total energy produced.



(a) Levels of energy production

(b) Shares of energy production

Figure A.1: Energy production, by fuel source

Energy production has expanded considerably over the 1950-2022 period with most of this expansion due to an increase in energy production from natural gas which rose from 20 to 70 quadrillion BTUs during this time period. This expansion is especially rapid since the shale boom in the mid-2000s. In contrast, energy production from coal rose gradually from 1950 to 2000 and has since fallen in absolute terms. Nuclear energy expanded in the pre-2000 period but has remained relatively flat since then. Finally renewables have grown gradually in the early part of the sample and exhibit a rapid expansion since 2000.

These trends are seen more clearly in the energy shares plotted in the lower panel of Figure A.1. In the early 2000s, coal accounted for thirty percent of total energy production. Electricity produced from coal has declined significantly as a share and now accounts for only twelve percent. Energy production from fossil-fuels excluding coal has remained relatively stable as a share of total production and currently accounts for seventy percent of total energy. Nuclear energy and renewables combined have risen in importance over time

and now account for 18 percent of total energy production with roughly equal shares at the end of the sample period.

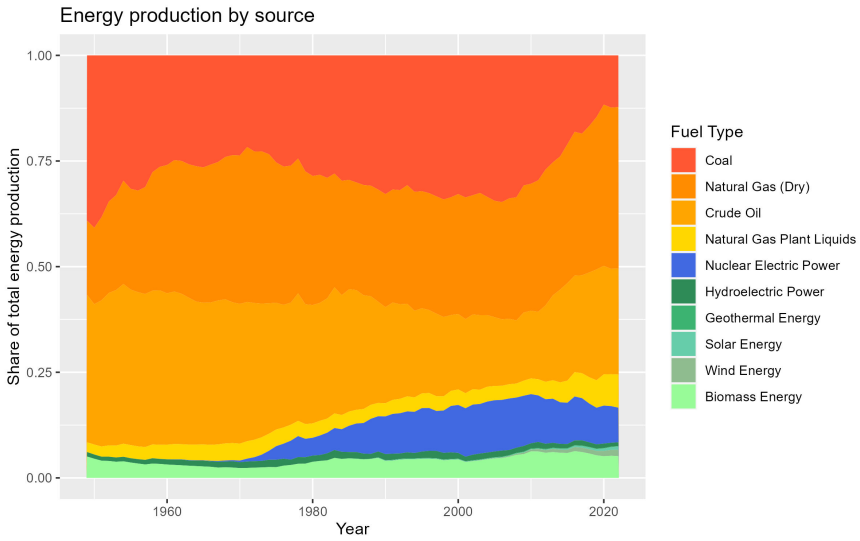


Figure A.2: Shares of energy production in detail

Figure A.2 provides a more detailed breakdown of energy production by source. In particular, renewable energy is broken out by hydro, geothermal, solar, wind and biomass. Among renewables, biomass accounts for the largest share of energy production. Solar and wind are a small but growing fraction of the total.

A.2 Emissions

Figure A.3 shows carbon emissions by fossil fuel source both in levels measured in million metric tons and as a share of total emissions. Total emissions from petroleum has remained relatively flat over time despite the expansion in petroleum use. Emissions from coal rose steadily with production and have now declined while emissions from gas have risen over time.

The emission shares are displayed in the lower panel of Figure A.3. Emission shares differ from production shares due to the fact that fossil fuel emissions differ across fuel types. Emissions from coal are nearly twice as large as emissions from natural gas. Thus, the reduction in coal-powered electricity in favour of gas has had a sizeable effect on total emissions.

A.3 Electricity Sector

To see clearly the transformation from coal to natural gas along with the nascent adoption of green energy it is useful to focus on electricity production. Electricity production accounts for 34 percent of total energy production in 2022. The next largest sector, transportation, accounts for 27 percent of total energy production. Industrial activity which also produces electricity or heat from fossil fuel sources is the third largest sector and accounts for 24 percent. The importance of the electricity sector provides further motivation for describing

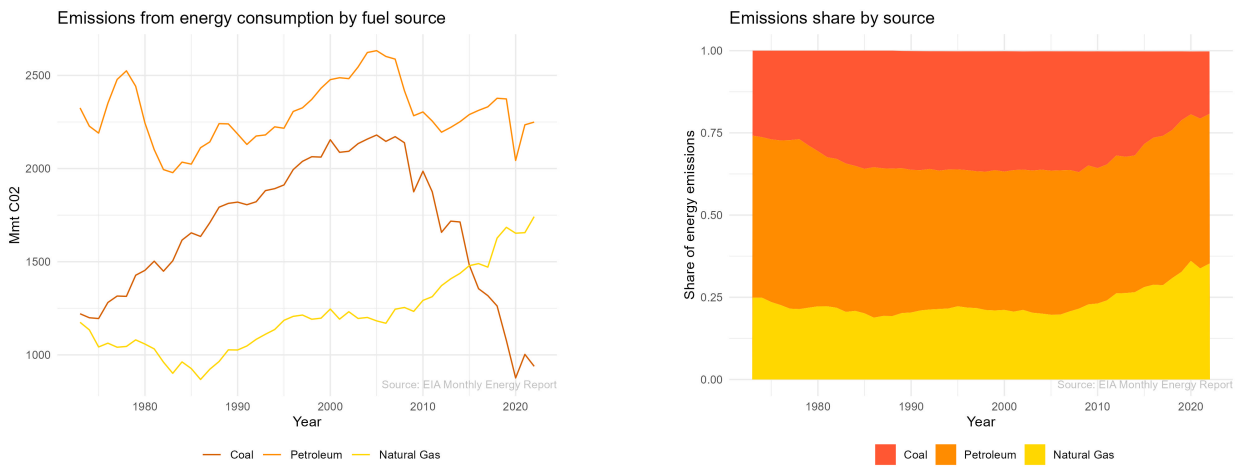


Figure A.3: Emissions from energy consumption, by fuel source

additional details of its transformation over time. Moreover the forces that drive transformation in the electricity sector, namely technological change embodied in capital combined with technological lock-in due to the irreversibility of existing capital choices are also highly relevant for the industrial and transportation sectors.

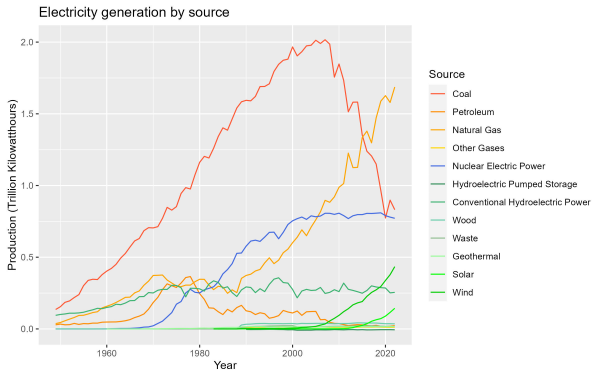
Figure A.4 provides the breakdown of electricity generation by source over the period 1950-2022. The left panel displays the total quantity of electricity measured in trillion kilowatt hours. Again one can clearly see the expansion of electricity generated by coal through 2000 and the contraction in electricity produced from coal that followed. One can also clearly see the rapid expansion in electricity generated from natural gas. The other categories of interest are hydro which has remained stable since 1980 and nuclear which has remained stable since 2000 following a rapid expansion over the 1970-2000 period. Finally electricity produced from wind and more recently solar were negligible before 2005 but have expanded rapidly since then.

In terms of shares, the electricity generated from fossil fuels has fallen from 75 percent to 60 percent of total electricity generation in the period 1980-2022. The rapid expansion in wind and solar implies that these two green energy sources now account for twelve percent of total electricity production, a share comparable to the electricity produced by nuclear energy.

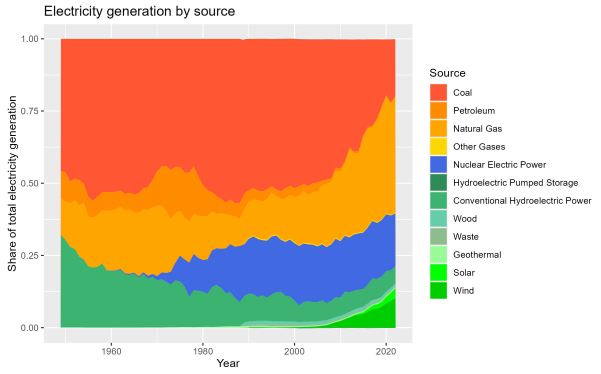
A.4 Gas Technology Evolution

Within the dirty electricity sector, the primary source of technological change is the adoption of more fuel-efficient gas technologies to produce electricity. Early gas-powered electricity plants relied on relatively simple steam turbines that burn gas to heat steam and generate electricity. These technologies have a typical thermal efficiency of 30-40 percent. The combined-cycle gas-powered electricity plants recapture heat exhaust and achieve efficiencies closer to 60 percent.

Figure A.5 illustrates electricity production by gas technology. At the start of the 2000-2022 period, combined cycle generators accounted for fifty percent of gas-generated electricity.



(a) Absolute electricity generation by fuel source



(b) Share of electricity generation by fuel source

Figure A.4: Electricity generation, by fuel source

By the end of the sample period, they account for 85 percent. This expansion occurred as combined-cycle generators replaced the capacity of steam turbines that were either retired or used less intensively.

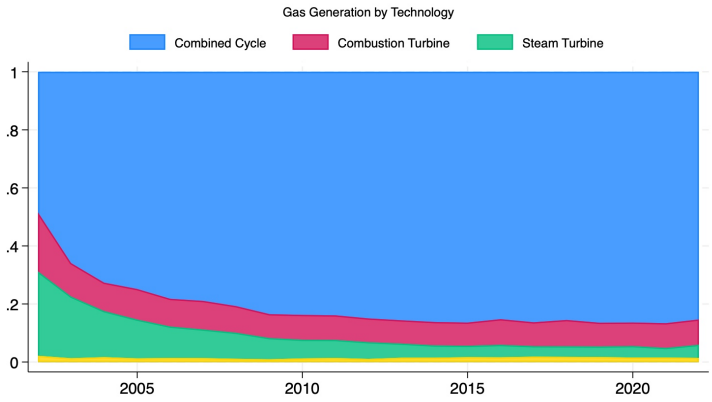


Figure A.5: Electricity Production by Gas Technology

B Green Energy Transition

The first experiment considers the effect of technological change in the green energy sector. In particular, we assume that growth in technology embodied in the green energy sector rises persistently over a thirty-year period. This rise in the growth rate is chosen to match the acceleration in growth exhibited in Figure 6 in the post-2008 period. Although Figure 6 applies directly to solar energy, similar rates of technological change in wind combined with battery storage are leading to rapid gains in technology embodied in clean energy plants. This experiment is therefore informative as to the likely effects of these combined sources of technological change in the green energy sector.

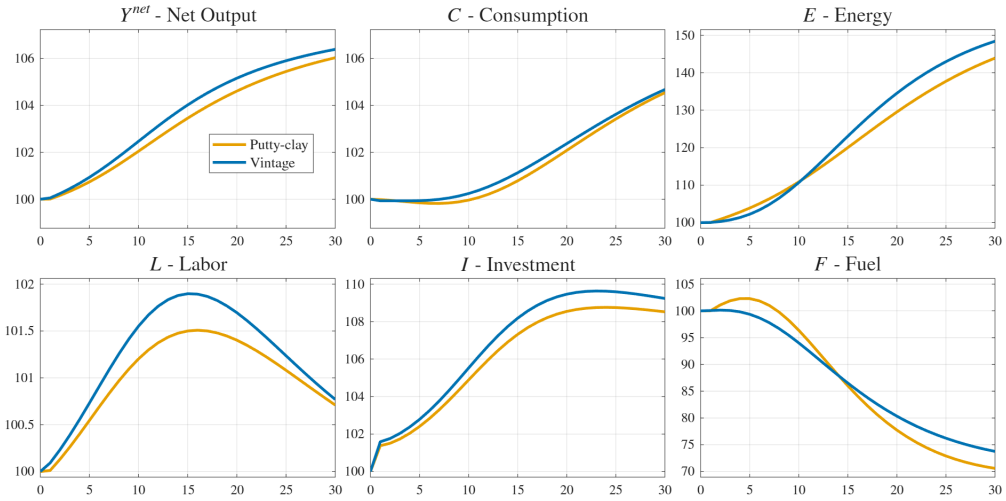
Figure B.1 displays responses for both the putty-clay model (blue) and the vintage model (red). The long-run cumulative effect of this persistent increase in the growth rate of green technology is to cause a substantial increase in green energy machine efficiency. Panel A of Figure B.1 displays the time path of aggregate variables. The increase in machine productivity leads to an expansion in aggregate output, consumption, and investment of approximately eight percent after forty years. The long-run effect on aggregate output reflects two features of the calibration. First, the overall energy share, at 7.5 percent, is relatively small. Second, the initial share of clean energy in energy production is only 20 percent. Because the putty-clay and vintage models share the same steady-state, the long-run effects of an increase in technology are equivalent.

The effects on the energy sector are substantially larger – energy production rises by 180 percent, and since the long-run energy share of output is constant, this implies a decline in energy prices that are equal in magnitude. This long-run transition also implies a shift in production away from dirty energy towards clean energy. Fossil fuel usage declines 30 percent and the carbon stock declines 15 percent over a forty-year horizon. We emphasise that both of these are relative to trend; in absolute terms, the carbon stock continues to rise, but at a slower pace. Although these reductions in fossil-fuel use and carbon stocks are sizeable, this experiment suggests that an increase in the rate of technological change in clean energy as seen in recent experience is unlikely to bring the economy close to net-zero fossil emissions over the next forty years.

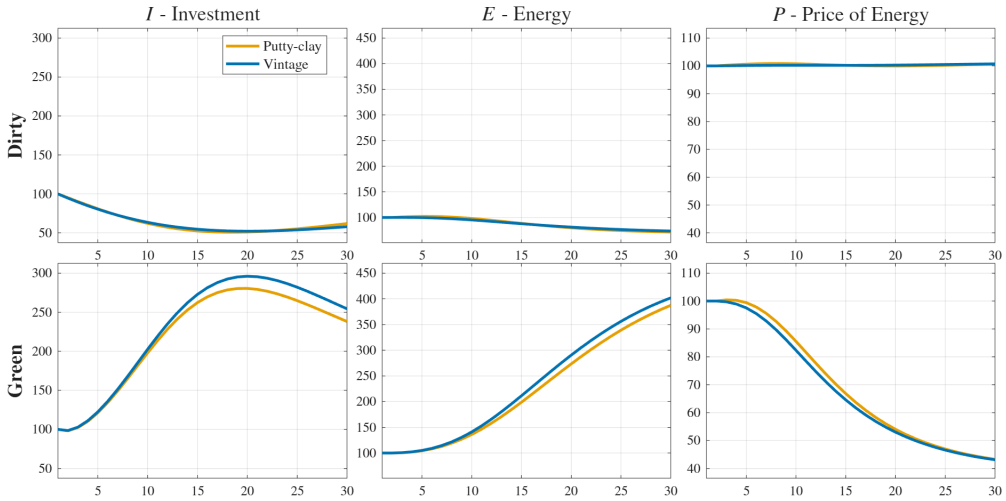
Additional details of the reallocation from dirty to clean energy are shown in panel B of Figure B.1. In the long-run, dirty energy production falls by thirty percent while clean energy production increases by more than 400 percent (starting from an initial low level of 20 percent of total energy). Relatedly, clean energy prices fall by 60 percent while dirty energy prices remain essentially unchanged. [4]

Short-run dynamics differ substantially from the long-run. Because productivity gains occur in the future, there is a reduction in investment and output in the short-run along with a modest rise in consumption. Differences between the putty-clay and vintage model are also relatively minor due to the fact that the transition is gradual. The most distinct difference between the two models is a sharper reduction in dirty energy investment and a smaller

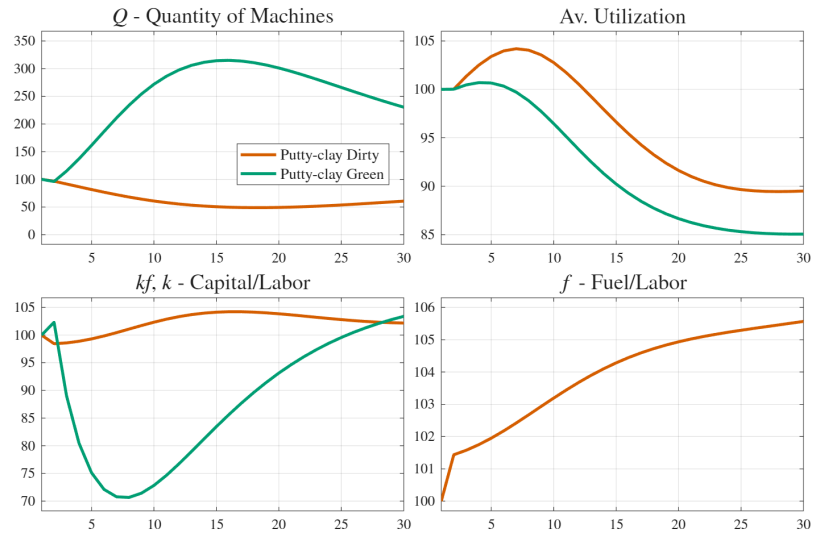
[4] The modest increase of 1 percent in dirty energy prices displayed in panel B of Figure B.1 reflects the imperfect substitutability between the two energy types.



(a) Panel A: Aggregate Responses



(b) Panel B: Energy Sector Response



(c) Panel C: Sectoral Decomposition: Putty-Clay Model

Figure B.1: Technological Change in the Green Energy Sector

expansion in clean energy investment along the adjustment path for the putty-clay version. These differences are most pronounced at the medium run horizon of ten to twenty years. This investment pattern also implies less energy production at these horizons.

Panel C of Figure B.1 shows the time path of variables that are specific to the putty-clay framework. utilisation rates in the green energy sector fall substantially as new machines replace old machines. utilisation rates rise temporarily in the dirty energy sector to compensate for the reduction in capacity due to the delayed investment in clean energy. The putty-clay model also displays the phenomenon of capital widening vs capital deepening. As exogenous machine productivity improves, the economy expands by increasing the quantity of clean machines but reducing their quality as measured by the capital-labor ratio. Over time, this process is reversed and the capital-labour ratio of new machines returns to its long-run value.

In summary, a gradual but persistent improvement in green technology has sizeable effects on energy production and leads to a sizeable reduction in fossil fuel usage. The effects on the aggregate economy are modest due to the size of the energy sector. Most notably, one would require much large gains in clean energy productivity for fossil fuel usage to become inconsequential.

C Optimal carbon tax

This appendix presents the social planner's problem and derives the optimal carbon tax. The Lagrangian below is subject to the model's equilibrium conditions (preferences, technology, climate damage, and carbon stock dynamics) as outlined in the main text.

Social Planning Problem

The social planner maximizes household utility subject to resource constraints, labor allocation, energy aggregation (CES between dirty and green energy), sector-specific production from putty-clay capital, and carbon stock accumulation. The carbon stock is decomposed into fast-decaying (S_t^1) and slow-decaying (S_t^2) components of the carbon cycle.

Planner's problem:

$$
\begin{aligned}
\mathcal{L} \;=\; & \mathbb{E}_t\Bigg[\sum_{s=0}^{\infty}\beta^s\Bigg(U(C_{t+s},L_{t+s}) \\
& + \lambda_{c,t+s}\Big(k^{FG}_{t+s}e_{t+s}Q^{FG}_{t+s}+k^{D}_{t+s}f_{t+s}Q^{D}_{t+s}+k^{G}_{t+s}Q^{G}_{t+s}+C_{t+s}+P^{f}_{t+s}F_{t+s}-Y_{t+s}\Big) \\
& + \lambda_{l,t+s}\Big(L^{FG}_{t+s}+L^{D}_{t+s}+L^{G}_{t+s}-L_{t+s}\Big) \\
& + \lambda_{e,t+s}\Big(\Big(E^{D}_{t+s}{}^{\frac{\epsilon-1}{\epsilon}}+E^{G}_{t+s}{}^{\frac{\epsilon-1}{\epsilon}}\Big)^{\frac{\epsilon}{\epsilon-1}}-E_{t+s}\Big) \\
& + \lambda_{d,t+s}\Big(\sum_{j=1}^{M}\Big(1-\Phi(z^{D}_{t+s,t+s-j}-\sigma^{D})\Big)X^{D}_{t+s,t+s-j}(1-\delta^{D})^{j}Q^{D}_{t+s-j}-E^{D}_{t+s}\Big) \\
& + \lambda_{g,t+s}\Big(\sum_{j=1}^{M}\Big(1-\Phi(z^{G}_{t+s,t+s-j}-\sigma^{G})\Big)X^{G}_{t+s,t+s-j}(1-\delta^{G})^{j}Q^{G}_{t+s-j}-E^{G}_{t+s}\Big) \\
& + \lambda_{s^1,t+s}\Big(S^{1}_{t+s}-(1-\varphi_1)S^{1}_{t+s-1}-(1-\varphi_L)\varphi_0\sum_{j=1}^{M}F_{t+s,t-j}\Big) \\
& + \lambda_{s^2,t+s}\Big(S^{2}_{t+s}-(1-\varphi_3)S^{2}_{t+s-1}-\varphi_L\sum_{j=1}^{M}F_{t+s,t-j}\Big) \\
& + \lambda_{s,t+s}(S_{t+s}-S^{1}_{t+s}-S^{2}_{t+s})\Bigg)\Bigg]
\end{aligned}
$$

C.1 Optimality conditions

Taking first-order conditions with respect to each choice variable and state variable yields the planner's optimality conditions. These include standard conditions for consumption and labor, Euler equations for the two carbon stock components, and sector-specific conditions for utilisation, embodied capital, embodied fuel/energy, and investment in each of the three sectors (final goods, dirty energy, green energy).

Aggregates and carbon stock:

$$
\begin{aligned}
C: &\qquad \lambda_{C,t} = U_c(C_t,L_t) \\
L: &\qquad \lambda_{l,t} = U_l(C_t,L_t) \\
S: &\qquad \lambda_{c,t}\frac{\partial Y_t}{\partial S_t}+\lambda_{d,t}\frac{\partial E^{D}_{t}}{\partial S_t}+\lambda_{g,t}\frac{\partial E^{G}_{t}}{\partial S_t}+\lambda_{s,t}=0 \\
S^1: &\qquad \lambda^{1}_{s,t}-\mathbb{E}_t[\beta(1-\varphi_1)\lambda^{1}_{s,t+1}]-\lambda_{s,t}=0 \\
S^2: &\qquad \lambda^{2}_{s,t}-\mathbb{E}_t[\beta(1-\varphi_3)\lambda^{2}_{s,t+1}]-\lambda_{s,t}=0
\end{aligned}
$$

Dirty energy:

$$
\begin{aligned}
z^D: \quad & \lambda_{d,t}\frac{\partial E_t^D}{\partial z_{t,t-j}^D} - \lambda_{c,t}P_t^f\frac{\partial F_{t,t-j}}{\partial z_{t,t-j}^D} - ((1-\varphi_L)\varphi_0\lambda_{s^1,t} + \varphi_L\lambda_{s^2,t})\frac{\partial F_{t,t-j}}{\partial z_{t,t-j}^D} = 0 \\
k^D: \quad & \lambda_{c,t}f_tQ_t^D = \mathbb{E}_t\left[\sum_{k=1}^{M}\beta^k\lambda_{d,t+k}\frac{\partial E_{t+k}^D}{\partial k_t^D}\right] \\
f: \quad & \lambda_{c,t}k_t^DQ_t^D = \mathbb{E}_t\Bigg[\sum_{k=0}^{M}\beta^k\Big(\lambda_{d,t+k}\frac{\partial E_{t+k}^D}{\partial f_t} - \lambda_{c,t+k}P_{t+k}^f\frac{\partial F_{t+k}}{\partial f_t} \\
& \qquad - ((1-\varphi_L)\varphi_0\lambda_{s^1,t+k} + \varphi_L\lambda_{s^2,t+k})\frac{\partial F_{t+k}}{\partial f_t}\Big)\Bigg] \\
Q^D: \quad & \lambda_{c,t}k_t^Df_t = \mathbb{E}_t\Bigg[\sum_{k=0}^{M}\beta^k\Big(\lambda_{d,t+k}\frac{\partial E_{t+k}^D}{\partial Q_t^D} - \lambda_{c,t+k}P_{t+k}^f\frac{\partial F_{t+k}}{\partial Q_t^D} \\
& \qquad - ((1-\varphi_L)\varphi_0\lambda_{s^1,t+k} + \varphi_L\lambda_{s^2,t+k})\frac{\partial F_{t+k}}{\partial Q_t^D} - \lambda_{l,t+k}\frac{\partial L_{t+s}^D}{\partial Q_{t+s}^D}\Big)\Bigg]
\end{aligned}
$$

Green energy:

$$
\begin{aligned}
z^G: \quad & \lambda_{g,t}\frac{\partial E_t^G}{\partial z_{t,t-j}^G} - \lambda_{l,t}\frac{\partial L_{t,t-j}}{\partial z_{t,t-j}^G} = 0 \\
k^G: \quad & \lambda_{c,t}Q_t^D = \mathbb{E}_t\left[\sum_{k=1}^{M}\beta^k\lambda_{g,t+k}\frac{\partial E_{t+k}^G}{\partial k_t^G}\right] \\
Q^G: \quad & \lambda_{c,t}k_t^G = \mathbb{E}_t\left[\sum_{k=0}^{M}\beta^k\left(\lambda_{g,t+k}\frac{\partial E_{t+k}^D}{\partial Q_t^D} - \lambda_{l,t+k}\frac{\partial L_{t+s}^G}{\partial Q_{t+s}^G}\right)\right]
\end{aligned}
$$

Final goods:

$$
\begin{aligned}
z^{FG}: \quad & \lambda_{c,t}\frac{\partial Y_t}{\partial z^{FG}_{t,t-j}} - \lambda_{e,t}\frac{\partial E_{t,t-j}}{\partial z^{FG}_{t,t-j}} - \lambda_{l,t}\frac{\partial L_{t,t-j}}{\partial z^{FG}_{t,t-j}} = 0 \\
k^{FG}: \quad & \lambda_{c,t} e_t Q^{FG}_t = \mathbb{E}_t\left[\sum_{k=1}^{M} \beta^k \lambda_{c,t+k}\frac{\partial Y_{t+k}}{\partial k^{FG}_t}\right] \\
e: \quad & \lambda_{c,t} k^{FG}_t Q^{FG}_t = \mathbb{E}_t\left[\sum_{k=0}^{M} \beta^k \left(\lambda_{c,t+k}\frac{\partial Y_{t+k}}{\partial e_t} - \lambda_{e,t+k}\frac{\partial E_{t+k}}{\partial e_t}\right)\right] \\
Q^{FG}: \quad & \lambda_{c,t} k^{FG}_t e_t = \mathbb{E}_t\left[\sum_{k=0}^{M} \beta^k \left(\lambda_{c,t+k}\frac{\partial Y_{t+k}}{\partial Q^{FG}_t} - \lambda_{e,t+k}\frac{\partial E_{t+k}}{\partial Q^{FG}_t} - \lambda_{l,t+k}\frac{\partial L^{FG}_{t+s}}{\partial Q^{D}_{t+s}}\right)\right]
\end{aligned}
$$

Energy bundle:

$$
\begin{aligned}
E^G: \quad & \lambda_{g,t} = \left(\frac{E^G_t}{E_t}\right)^{-\frac{1}{\epsilon}} \lambda_{e,t} \\
E^D: \quad & \lambda_{d,t} = \left(\frac{E^D_t}{E_t}\right)^{-\frac{1}{\epsilon}} \lambda_{e,t}
\end{aligned}
$$

Optimality Conditions Given Functional Forms

We now rewrite the optimality conditions in terms of prices and the stochastic discount factor. Define the stochastic discount factor $M_{t,t+s}$, the wage w_t, and energy prices P^D_t, P^G_t, and the aggregate energy price P^E_t. The optimal carbon tax τ_t equals the present discounted value of marginal climate damages from an additional unit of fossil fuel use, accounting for the two-component carbon decay structure.

Defining:

$$
\begin{aligned}
M_{t,t+s} &= \beta^s \frac{U_c(C_{t+s}, L_{t+s})}{U_c(C_t, L_t)} \\
w_t &= U_l / U_c \\
P_t^D &= \frac{\lambda_{d,t}}{\lambda_{c,t}} \\
P_t^G &= \frac{\lambda_{g,t}}{\lambda_{c,t}} \\
P_t^E &= \left({P_t^D}^{1-\epsilon} + {P_t^G}^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}}
\end{aligned}
$$

And the tax rate τ_t as:

$$
\begin{aligned}
\tau_t &= \frac{((1-\varphi_L)\varphi_0 \lambda_{s^1,t} + \varphi_L \lambda_{s^2,t})}{\lambda_{c,t}} \\
&= \zeta \sum_{s=0}^{\infty} M_{t,t+s}((1-\varphi_L)\varphi_0(1-\varphi_1)^s + \varphi_L(1-\varphi_3)^s) \left[\frac{Y_{t+s}(1-\alpha^{FG})}{\Omega_{t+s}} + \frac{P_{t+s}^D E_{t+s}^D (1-\alpha^D)}{\Omega_{t+s}} + \frac{P_{t+s}^G E_{t+s}^G (1-\alpha^G)}{\Omega_{t+s}} \right.
\end{aligned}
$$

The optimality conditions for each sector balance the cost of investment today against the present value of future net revenues from the machine over its lifetime, accounting for depreciation, utilisation decisions, and climate damages.

Dirty energy:

$$
\begin{aligned}
z^D: &\quad \frac{h(z_{t,t-j}^D - \sigma^D)}{h(z_{t,t-j}^D)} E_{t,t-j}^D - (P_t^f + \tau_t) F_{t,t-j} - w_t L_{t,t-j}^D = 0 \\
k^D: &\quad k_t^D f_t Q_t^D = \mathbb{E}_t \left[\sum_{k=1}^{M} M_{t,t+k} \lambda^D P_{t+k}^D E_{t+k,t}^D \right] \\
f: &\quad k_t^D f_t Q_t^D = \mathbb{E}_t \left[\sum_{k=0}^{M} M_{t,t+k} \left(\alpha^D P_{t+k}^D E_{t+k,t}^D - (P_{t+k}^f + \tau_{t+k}) F_{t+k,t} \right) \right] \\
Q^D: &\quad k_t^D f_t Q_t^D = \mathbb{E}_t \left[\sum_{k=0}^{M} M_{t,t+k} \left(P_{t+k}^D E_{t+k,t}^D - (P_{t+k}^f + \tau_{t+k}) F_{t+k,t} - w_{t+k} L_{t+k,t}^D \right) \right]
\end{aligned}
$$

Green energy:

$$
\begin{aligned}
z^G: \quad & \frac{h(z^G_{t,t-j} - \sigma^G)}{h(z^G_{t,t-j})} E^G_{t,t-j} = w_t L_{t,t-j} \\
k^G: \quad & k^G_t Q^D_t = \mathbb{E}_t \left[\sum_{k=1}^{M} M_{t,t+k} \alpha^G P^G_{t+k} E^G_{t+k,t} \right] \\
Q^G: \quad & k^G_t f_t Q^G_t = \mathbb{E}_t \left[\sum_{k=0}^{M} M_{t,t+k} \left(P^G_{t+k} E^G_{t+k,t} - w_{t+k} L^G_{t+k,t} \right) \right]
\end{aligned}
$$

Final goods:

$$
\begin{aligned}
z^{FG}: \quad & \frac{h(z^{FG}_{t,t-j} - \sigma^{FG})}{h(z^{FG}_{t,t-j})} Y_{t,t-j} - P^E_t E_{t,t-j} - w_t L_{t,t-j} = 0 \\
k^{FG}: \quad & k^{FG}_t e_t Q^{FG}_t = \mathbb{E}_t \left[\sum_{k=1}^{M} M_{t,t+k} \lambda^{FG} Y_{t+k,t} \right] \\
e: \quad & k^{FG}_t e_t Q^{FG}_t = \mathbb{E}_t \left[\sum_{k=0}^{M} M_{t,t+k} \left(\alpha^{FG} Y_{t+k,t} - P^E_{t+k} E_{t+k,t} \right) \right] \\
Q^{FG}: \quad & k^{FG}_t e_t Q^{FG}_t = \mathbb{E}_t \left[\sum_{k=0}^{M} M_{t,t+k} \left(Y_{t+k,t} - P^E_{t+k} E_{t+k,t} - w_{t+k} L^{FG}_{t+k,t} \right) \right]
\end{aligned}
$$

Energy bundle:

$$
\begin{aligned}
\frac{P^G_t}{P^E_t} &= \left(\frac{E^G_t}{E_t} \right)^{-\frac{1}{\epsilon}} \\
\frac{P^D_t}{P^E_t} &= \left(\frac{E^D_t}{E_t} \right)^{-\frac{1}{\epsilon}}
\end{aligned}
$$

The above expressions are for a zero-growth economy. Under balanced growth, the optimal Pigouvian tax that decentralizes the social planner's solution is:

$$
\tau_t = \frac{(1 - \varphi_L)\varphi_0 \tilde{\lambda}_{s1,t} + \varphi_L \tilde{\lambda}_{s2,t}}{\tilde{\lambda}_{c,t}}, \tag{36}
$$

where tildes denote detrended variables. With long-run adaptation Ω_t, the tax rate is sta-

tionary.

D Stranded asset valuation discussion

To complement Section 5 and Figure 14, Figure D.1 shows decompositions of sub-components of the valuation of dirty machines which were installed prior to policy announcement. First, we split the overall valuation (equation 35) into two broad components, revenues and costs:

$$V_{t_1,t}^{Rev,D} = \sum_{j=t-t_1}^{t-t_1+M-1} \sum_{s=1}^{\infty} m_{t,t+s}(1-\delta^D)^{s+j-1} \left\{ \left[1 - \Phi\left(z_{t+s,t-j}^D - \sigma_D \right) \right] P_{t+s}^D X_{t+s,t-j}^D \right\} Q_{I,t-j} \quad (37)$$

$$V_{t_1,t}^{Cost,D} = \sum_{j=t-t_1}^{t-t_1+M-1} \sum_{s=1}^{\infty} m_{t,t+s}(1-\delta^D)^{s+j-1} \left\{ \left[1 - \Phi\left(z_{t+s,t-j}^D \right) \right] \left(W_{t+s} + P_{t+s}^f f_{t-j} \right) \right\} Q_{I,t-j} \quad (38)$$

The dynamics of the discounted values of revenues and costs are in the first panel of Figure . Both components rise, but the rise in the discounted value of costs outstrips that of revenues. The second and third panels outline why; though dirty energy prices rise, they do not rise sufficiently to offset the carbon tax imposed, and so the fuel cost margin rises substantially. In response to this rise in cost, panel 4 shows how utilisation falls. Initially, this drop is small relative to the balanced growth path, while new, energy efficient machines are being installed. Once new capacity is added, utilisation falls more sharply. These utilisation dynamics contribute to continued decline in valuations which troughs around 9-10 years after policy implementation.

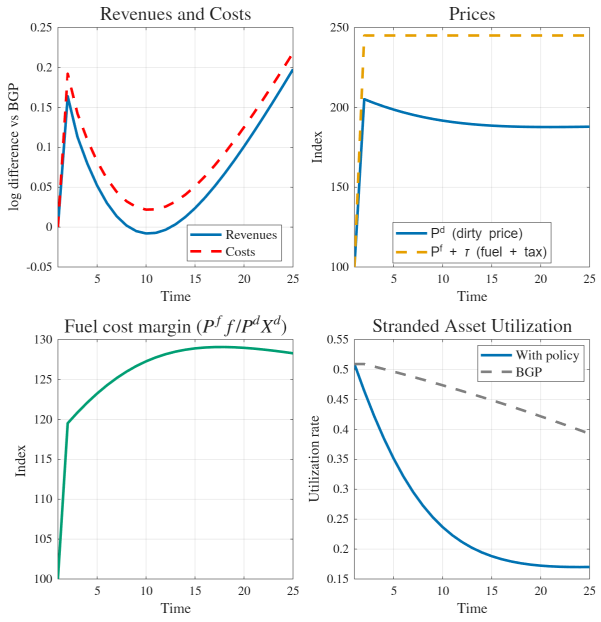


Figure D.1: Decomposition of stranded asset valuation