Venture Capital Contracts and Heterogeneous Innovation

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Abstract

This paper studies how venture capital (VC) reshapes startups' innovation choices by insuring against default risk, and explores the macroeconomic implications of this mechanism. I develop a dynamic general equilibrium model in which startups choose between conservative (low-risk, low-return) and aggressive (high-risk, high-return) innovation while endogenously selecting their financing mode. Debt financing features state uncontingent repayments and exposes startups to default. By contrast, VC financing is a state-contingent dynamic contract with one-sided limited commitment from startup. Evidence from a new dataset linking VC deals, balance sheets, and patents supports the model predictions: VC-backed startups begin with higher leverage, show greater post-financing profit dispersion, and generate more high-quality patents. Calibrated to financing and innovation data, eliminating VC reduces the aggregate output by 5 percent and the mass of large firms by 11 percent, despite only 0.2 percent of startups ever receiving VC.

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1 Introduction

Innovation and entrepreneurship are key drivers of economic growth, yet financing constraints often limit the ability of young firms to develop and scale new ideas. Venture capital (VC)[1] has emerged to relax these constraints, with VC investment reaching nearly twenty percent of the scale of bank lending[2] as shown in Fig 1. VC-financed startups (VCFS) generate a disproportionate share of high-impact patents and high-growth firms, yet their performance outcomes are dispersed[3]. These patterns suggest that VC financing does not uniformly improve startup performance but instead produces polarized outcomes, with some VCFS achieving extraordinary success while many others remain stagnant. This observation raises fundamental questions about how VC financing reshapes firms' innovation choices and its macroeconomic implications for aggregate growth.

Motivated by these observations, this paper studies how VC reshapes startups' innovation choices through an insurance mechanism. I formalize this mechanism in a general-equilibrium firm-dynamics model where startups search for VC or bank finance and choose between heterogeneous innovation strategies. Debt financing requires *state-uncontingent* repayments and exposes startups to default risk. By contrast, VC financing involves a negotiation that yields a *state-contingent* dynamic contract with one-sided limited commitment on the startup's side. The model predicts that, relative to non-VCFS, VCFS are more leveraged prior to receiving VC financing and, after obtaining VC support, pursue high risk high return innovation that results in more dispersed growth outcomes. I thus take the model to the data, constructing a novel dataset that links VC investment deals to startups' balance sheets and patent records. This dataset reveals empirical patterns consistent with the model's predictions. Finally, I conduct counterfactual experiments using a model calibrated to EU countries. In particular, I quantify the role of VC by eliminating VC financing and varying

[1]VC in this paper refers to venture capitalists who invest in early-stage ideas or firms, collaborate with entrepreneurs to maximize the market value of these ideas, and seek returns through exit strategies.

[2]The ratio of total VC investment to total investment also increased markedly over the same period; see Greenwood et al. (2022).

[3]The fraction of VCFS that receive subsequent rounds of financing has declined significantly in recent years, whereas those VCFS that do advance to later rounds have tended to become exceptionally prominent and successful. (Ewens et al. (2018)) Series A is typically the first round of VC financing for VCFS. A few VCFS have an angel or seed round before Series A, but the majority begin with Series A. Subsequent rounds refer to Series B or M&A/IPO events in which startups successfully attract a new set of investors.

the enforcement power of VC contracts. The model implies that eliminating VC reduces aggregate output by 5 percent and the mass of large firms by 11 percent, despite only 0.2 percent of startups receiving VC.

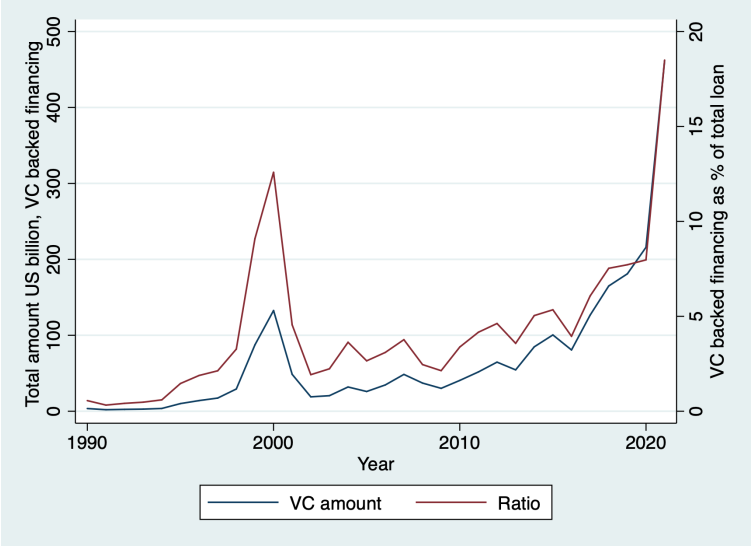


Figure 1: VC investment as % of commercial bank loan

Note: VC backed finance is from author's calculation using CBinsight statistics by aggregation over year. Total outstanding loan data is from FRED with the series *Total Value of Loans for All Commercial and Industry Loans*. Right axis is the simple ratio of new VC investment in each year over the stock level of total outstanding loan in each year.

The paper starts by constructing a quality ladder model (Aghion and Howitt, 1992; Aghion et al., 2001; Akcigit et al., 2022), in which limited liability startups endogenously choose innovation type and search financing sources. The model embeds two key features. First, startups select between heterogeneous innovation strategies: conservative projects yield incremental improvements with high success probability, whereas aggressive projects are riskier but can deliver large productivity gains. Second, startups search for external finance and may, with some probability, match with a venture capitalist. A successful match results in a *state-contingent* VC contract, while unmatched startups rely on debt requiring *state-uncontingent* repayment that exposes them to default.

Startups face an endogenous financing constraint due to limited commitment: they cannot credibly promise to fulfill all future repayment obligations, nor can they freely issue equity to outside investors. As a result, financing is constrained either by default risk under debt or by search and limited-commitment frictions in the VC market. Each startup pays a

search cost to locate a venture capitalist, facing a probability of successful matching. Conditional on the financing outcome, either VC or debt, the startup chooses between aggressive and conservative innovation strategies to improve its productivity.

VC financing is modeled as a dynamic contract with one-sided limited commitment on the startup's side. The contract specifies (i) a contingent innovation strategy, reflecting board control rights that direct strategy, and (ii) contingent transfers, capturing cash-flow rights that incentivize compliance. Startups do not commit to the VC contract and have ability to misappropriate VC investment and deviate from the contract. By contrast, startups without VC funding rely on bank debt. Competitive and risk-neutral banks price loans at break-even rates after evaluating R&D plans, and impose future state uncontingent repayments.

In contrast to VC financing, debt-financed startups (DFS) face a trade-off between growth and default risk arising from the non-negative dividend constraint (Arellano et al., 2019; Ottonello and Winberry, 2020). If they cannot roll over debt while keeping dividends non-negative, they default and exit. This constraint creates a key distinction between conservative and aggressive innovation for DFS. The presence of default risk introduces curvature in the startup's expected payoff with respect to innovation outcomes, effectively making risk-neutral startups behave as if they were risk averse. By contrast, without default risk, VCFS face linear payoffs and, being risk neutral, choose high risk high return innovation to maximize expected returns. As a result, VCFS grow faster on average but also display wider dispersion in outcomes. This mechanism rationalizes the observed pattern in which some VCFS achieve extraordinary success while many others remain stagnant.

To illustrate the model's mechanism, I consider a negative financial shock experiment that highlights how financing structure shapes firms' resilience to adverse shocks. The exercise compares small startups (bottom decile of the size distribution) and median-sized firms when both face a one percent decline in revenue. Relative to small VCFS, small debt-financed startups (DFS) reduce innovation effort by about 0.25 percent and are 15 percent less likely to pursue aggressive innovation. In contrast, mature DFS with deeper financial buffers rely more on debt to smooth the shock and maintain innovation relative to their VC-financed counterparts. For startups without sufficient internal funds, debt finance requires state-uncontingent repayments, so negative shocks can trigger premature exit, pushing them

toward safer, incremental innovation. As firms grow and accumulate wealth, relaxed credit constraints allow them to sustain innovation even during downturns.

Next, I calibrate the model to match observed patterns of startup financing and innovation using a novel firm-level dataset. The dataset links VC investment deals to startups' balance sheets and patent records, allowing me to track firms' performance before and after receiving VC funding. This linkage provides empirical moments to discipline and validate the model and enables the construction of a synthetic control group based on observable firm characteristics to evaluate the impact of VC financing.

Three key predictions emerge from the model. First, the growth trajectories of VCFS become substantially more volatile and more dispersed after VC financing than those of non-VC counterparts. Top-performing VCFS achieve exceptional growth while many others remain mediocre. The empirical pattern is consistent with the model's prediction that VC generates dispersion rather than uniform improvement. Second, the model predicts that VCFS exhibit higher financial leverage prior to matching, reflecting the self-selection of more financially constrained startups into VC search, as these firms stand to benefit most from VC funding. The data confirm this prediction: VCFS consistently display higher leverage prior to receiving VC financing. Lastly, the model implies that the distribution of patent quality for VCFS has heavier right tails, supporting the view that VC induces more aggressive innovation and higher-quality breakthroughs. The empirical evidence mirrors this implication, as the patent-quality distribution reveals that VCFS are disproportionately represented among high-quality patents.

With the calibrated model, I conduct counterfactual experiments to quantify the role of VC. Eliminating VC reduces the mass of large firms (those above the 99th percentile of size in the baseline) by 11 percent and aggregate output by 5 percent, despite only 0.2 percent of startups receiving VC. Second experiment improves the enforcing power in the VC contract and suggests 0.4 percentage aggregate output gain. Lastly, I consider a perfect-sorting scenario in which VC flows exclusively to the most financially constrained startups. This experiment shows that efficient matching reduces the default rate by roughly 0.17 percentage points, relative to a 3 percent points baseline.

The key reason why a small fraction of VC investment generates such a large aggregate

impact is that VC funding tends to occur at critical stages of a startup's lifecycle, particularly in its early growth phase. By absorbing losses in bad states and backloading returns in good states, VCs shift downside risk away from entrepreneurs, eliminating default risk and enabling more aggressive innovation. As these startups expand and eventually separate from VC financing, they become large firms that account for a disproportionate share of aggregate output.

The contribution of this paper is twofold. First, it introduces a general equilibrium model that embeds heterogeneous innovation strategies within a limited-commitment dynamic contract framework. This combination is essential for explaining the empirical patterns of persistence and widening dispersion observed in the data. The dynamic contract not only captures key features of real-world VC agreements (Kaplan and Strömberg, 2003, 2004) but also accounts for the persistent dispersion. Without the dynamic structure, a static contract would generate only a one-time increase in productivity dispersion. Similarly, heterogeneous innovation strategies are essential for producing the differential dispersion between VCFS and DFS. Without heterogeneity in innovation risk, the model would instead resemble the "VC-as-synergy" view,[4] as in Akcigit et al. (2022), yielding higher average growth among VC-financed startups but little dispersion across firms.

Second, this paper provides new evidence on post-financing dynamics. By linking startups' balance sheets to VC investment deals, I show that both cross-sectional dispersion and growth volatility widen after VC financing, shedding light on the behavioral changes induced by VC support. This evidence complements existing studies that emphasize ex-ante heterogeneity by showing that VC also shapes firms' ex-post innovation behavior and growth trajectories.

The next section discusses how this paper relates to the existing literature.

Literature review

A growing literature studies the role of VC in shaping firm behavior, innovation strategies, and long-run outcomes. Early research by Hellmann and Puri (2000) showed how VC financ-

[4]The "VC-as-synergy" view refers to complementarities such as managerial expertise or network support provided by venture capitalists that directly enhance startups' productivity.

ing interacts with product market strategy: venture-backed firms are more likely to introduce innovative products quickly and to professionalize management, underscoring the role of VC as more than passive finance. Subsequent work highlights different mechanism understanding the sources of VC influence. Among those, synergy mechanisms, argues that VCs not only allocate capital but also provide complementary expertise, networks, and governance that enhance entrepreneurial outcomes (Akcigit et al. (2022); Ando (2025)). Another heavily studied channel focuses on VC's superior ability to select the better firm based on firms' characteristics (Catalini et al. (2019); Gompers et al. (2020); Lerner and Nanda (2020)). Collectively, this line of research establishes VC as an institution that reshapes the nature and direction of startups' along lifecycle.

A parallel strand focuses on the contractual arrangements through which VCs and entrepreneurs interact. Classic theories of staged financing and convertible securities emphasized how contracts mitigate informational asymmetries and agency problems (Gompers (1995); Cornelli and Yosha (2003)). More recent work has pushed beyond these static models, analyzing how contracts evolve dynamically with project uncertainty and learning. For example, Cole et al. (2016); Greenwood et al. (2022) highlight how monitor efforts are allocated across states to solve information friction, while Ewens and Farre-Mensa (2022) study how control provisions and cash-flow terms shape startup growth trajectories and exit outcomes. These studies underscore that VC contracts are not simply financial instruments but are designed to manage uncertainty.

This view connects to broader theories of dynamic financing under limited commitment, such as Albuquerque and Hopenhayn (2004). Empirical analyses of VC contracts by Kaplan and Strömberg (2003, 2004) similarly emphasize contingent cash-flow and control rights. Relatedly, studies of firm dynamics under debt (Arellano et al., 2019; Ottonello and Winberry, 2020) and life-cycle finance (Puri and Zarutskie, 2012; Chemmanur et al., 2011) illustrate how non-VC channels lack such insurance, reinforcing the contrast central to this paper.

A closely related line of research studies the insurance role of equity finance more broadly. Étienne de Bettignies and Duchêne (2015) show that venture capital can act as implicit insurance against liquidation risk when compared with debt, highlighting the trade-off between incentives and risk-sharing. My paper extends this literature by formalizing a state-contingent

VC contract that provides explicit insurance against innovation shocks and quantifies its aggregate implications for firm dynamics and productivity.

My paper builds on these strands in two ways. First, whereas much of the prior work emphasizes the presence of VC and its aggregate effects on growth, I focus on how VC investment changes the choice of innovation strategy. In particular, I distinguish between aggressive and conservative innovation and show that the presence of VC financing incentivizes startups to pursue riskier innovation paths that would otherwise be unattractive under debt finance. Second, I incorporate the novelty of VC contracts into a dynamic equilibrium model. Rather than treating VC as an undifferentiated capital source, I explicitly model the contingent nature of VC contracts and their role in insuring against default risk. This allows me to quantify both the direct impact on innovation choices and the general equilibrium implications for firm dynamics and aggregate output.

Relative to the existing literature, my contribution is therefore twofold. Conceptually, I connect the theory of VC financing as an insurance that encourages "risk-taking" with the literature on contractual design under uncertainty, providing a unified framework for understanding VC's role in innovation. Quantitatively, I discipline the model with data on startups' financing and innovation patterns and show that VC financing generates sizable aggregate effects, including more large firms, fewer premature exits, and higher output. By doing so, my paper highlights the insurance and incentive properties of VC contracts as central to their impact on the innovation landscape, thereby extending existing work that has largely studied selection, monitoring, or aggregate growth.

The rest of the paper is organized as follows: Section 2 presents the model and mechanism, including a negative financial shock illustration. Section 3 describes the data, calibration, and empirical validation. Section 4 reports steady-state counterfactual experiments. Section 5 concludes.

2 Model

This section develops a dynamic model of VC finance that formalizes the mechanism introduced in the introduction: VC contracts insure against default risk and encourage startups

to pursue high-risk-high-return innovation. The model has an infinite horizon with discrete time and three sectors: households, startups, and financiers. Households provide labor and consume, but do not save. Startups, protected by limited liability, decide whether to seek VC funding, what type of innovation to undertake to grow, and how much to borrow. Financiers take two forms: competitive banks that lend under default risk, and venture capitalists who sign dynamic contracts subject to one-sided limited commitment from startups. The subsections that follow describe each agent's problem and define equilibrium.

2.1 Household

Households supply labor inelastically and consume a single final good, which is produced as a CES aggregate of differentiated varieties. They own all the firms, but do not have access to saving instruments, so intertemporal trade occurs only through external financiers. Households receive labor income and dividends from firms, which they spend on consumption. The household's problem is formally defined as following:

$$max_{y_{jt}} U = \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\psi}}{1-\psi}$$
$$\text{s.t. } C_t = w_t \bar{L} + \Pi_t \quad \forall t$$
$$C_t = \left(\int_0^1 y_{jt}^{\frac{\eta-1}{\eta}} dj\right)^{\frac{\eta}{\eta-1}}$$

where Π_t is all the profits from firms and financiers, w_t is the wage rate, and I normalize the final good price $P_t = 1 \; \forall \; t$.

2.2 Startups

The startup j is heterogeneous in two dimension, technology level z_{jt} and cash on hand level ω_{jt} at each period. The brief description of the timeline for the startup is given by Figure 2.

The sequence of decisions within a period is as follows. Within each period, a startup first produces with current technology, pays back the debt carried from previous period,

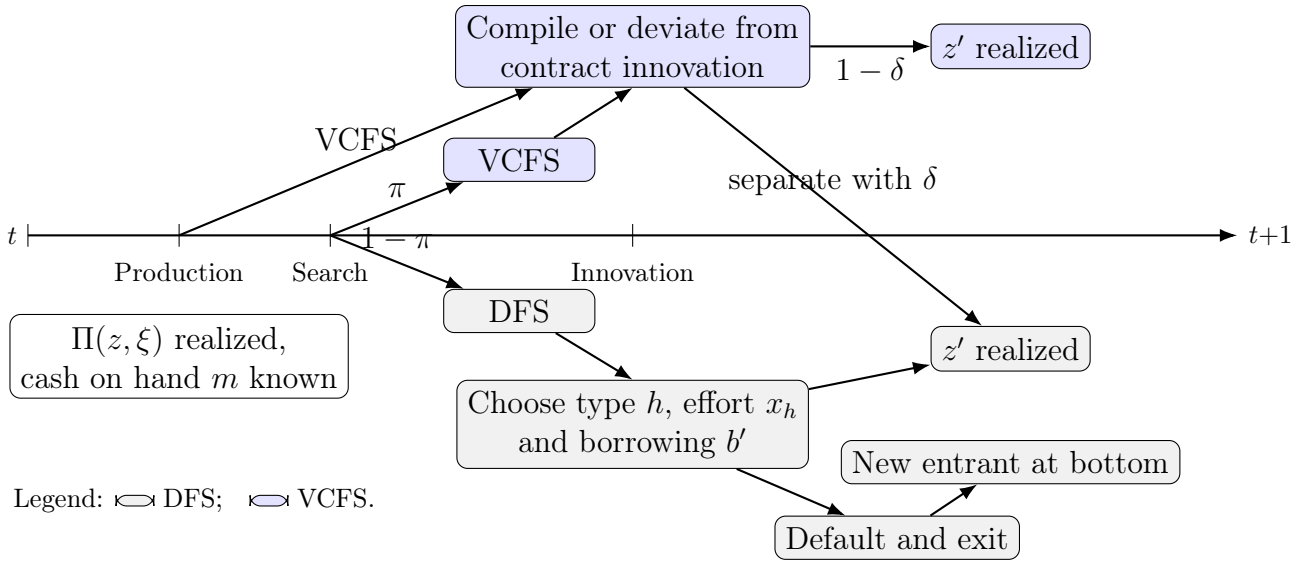


Figure 2: Timeline of startup decisions within period t

and then observes its remaining cash on hand. Given the state (z, ω), it decides whether to pay search cost and search for VC funding, where financially constrained firms searching more intensively. If matched, the startup contracts with a VC. Otherwise, it relies on bank debt. Once the financing source is determined, the startup chooses its innovation strategy and borrows if internal funds are insufficient to cover innovation costs. Finally, innovation outcomes are realized, which determine the startup's next-period technology state. The subsections that follow describe in detail the search process, financing arrangements, and innovation decisions.

2.2.1 Production stage

The production process is static and the production function is Cobb-Douglas using technology and labor as input. Startup's static optimal production decision solves following equation:

$$\pi_{jt} = \max_{p_{jt}, \ell_{jt}} \; p_{jt} y_{jt} - w_t \ell_{jt}$$

$$\text{s.t. } y_{jt} = p_{it}^{-\eta} Y_t$$

$$\text{s.t. } y_{jt} = z_{jt}^{\alpha_z} \ell_{jt}^{1-\alpha_z}$$

After production, a revenue shock ξ hits. The cash on hand level ω_{jt} realizes after production according to $\omega_{j,t} = \pi_{jt} - \xi_{jt} - b_{jt}$, where b_{jt} is the debt carried from last period. For VCFS, I assume that their debt level remains fixed during the duration of the VC contract. In particular, VCFS do not make debt-issuance decisions, and any liquidity shortfalls are covered within the contract. If the startup is not currently VC-financed, it then decides whether to incur the search cost and look for a VC match.

2.2.2 Searching stage

With technology level z and realized cash on hand ω, the startup decides whether to pay a search cost, $c_s(z)$, to search for VC finance. Formally:

$$V(z,\omega) = \max\{V^d(z,\omega)\ ,\ \lambda V^{vc}(z,\omega - c_s(z)) + (1-\lambda)V^d(z,\omega - c_s(z))\} \tag{1}$$

The first term represents the value of not searching and remaining debt-financed. The second term is the expected value of searching, where λ denotes the equilibrium probability of matching with a VC. This probability is determined by the endogenous mass of startups that choose to search. The search cost is given by $c_s(z) = c\, z^{\frac{\alpha}{1-\alpha}}$, where c is cost parameter. Search costs are increasing in technology because more advanced firms face more complex negotiations and higher legal and due-diligence expenses. After the search stage, the startup takes the resulting financing source as given and proceeds to the next stage, where it chooses its innovation strategy and, if necessary, its borrowing level.

2.2.3 Innovation stage

After the searching stage concludes, startups decide on the optimal type of innovation between aggressive innovation and conservative innovation, $h \in \{a, c\}$. Specifically, Aggressive innovation yields a higher expected payoff but also features greater variance in outcomes, capturing the higher technological risk associated with more ambitious projects. Conditional on the chosen type h, the startup selects an innovation effort level $x_h \in [0,1]$. Innovation effort determines the arrival rate of successful innovations, so that x_h coincides with the probability of receiving a successful innovation outcome in the current period.

Conditional on the chosen innovation type h and effort x_h, the startup incurs an innovation cost with two components: a variable cost and a fixed cost. Following Olmstead-Rumsey (2022), the variable cost takes the form. In particular

$$\kappa(z, x_h) = \kappa_{h,v} x_h^2 z^{\kappa_\zeta} + \kappa_{h,f} \ , \text{ for } h \in \{a, c\} \tag{2}$$

where $\kappa_{h,v}$ governs the variable component of the cost and z^{κ_z} captures the fact that innovation becomes more expensive at higher technology levels. The parameter $\kappa_{h,f}$ is a type-specific fixed cost that reflects the setup or overhead costs of undertaking a given innovation process. The presence of a fixed cost plays an important role in the model. First, it breaks the Inada condition in innovation, ensuring that not all firms undertake aggressive innovation, which is consistent with the empirical observation that breakthrough innovations are concentrated among a small set of firms. Second, because fixed costs must be paid up front, financially constrained startups place a higher shadow value on these costs than their unconstrained counterparts, which affects their innovation incentives under financial frictions.

In this stage, the startup also determines how to finance its innovation spending. If it does not secure VC funding, it must cover innovation costs using internal funds or by borrowing from the bank. When internal funds are insufficient, the startup issues debt at the price schedule $q_h(z, x, b')$, which depends on its technology level z, chosen innovation type h, innovation effort x_h, and borrowing amount b'. Banks are assumed to observe the startup's innovation plan when pricing debt. The details of the bond-pricing problem are presented in the next subsection.

By contrast, for VCFS, the financing of innovation is determined within the state-contingent VC contract. The contract specifies the innovation plan and the share of costs borne by the entrepreneur, so VCFS do not make period-by-period borrowing decisions. Instead, they choose whether to comply with the prescribed contract terms.

After the innovation and financing decisions, the startup draws its innovation outcome. I model technological progress using a quality ladder. Technology evolves according to $z' = z g_z^n$ where n is the number of steps after successful innovation and g_z is the technology improve-

ment in each step. The number of steps n is a random variable governed by the probability distribution $f_h(z' \mid z)$. Aggressive innovation features a higher probability of failure but also a higher probability of large technological jumps, resulting in a more dispersed distribution of step sizes than under conservative innovation. Figure 3 illustrates this innovation process.

Aggressive innovation allows startups to make large technological leaps but also carries the possibility of generating no improvement. This reflects the nature of high-risk-high-return activities such as developing next-generation technologies or launching fundamentally new product concepts. By contrast, conservative innovation produces incremental and more predictable progress, consistent with lower-uncertainty activities such as refining production methods or implementing minor design enhancements.

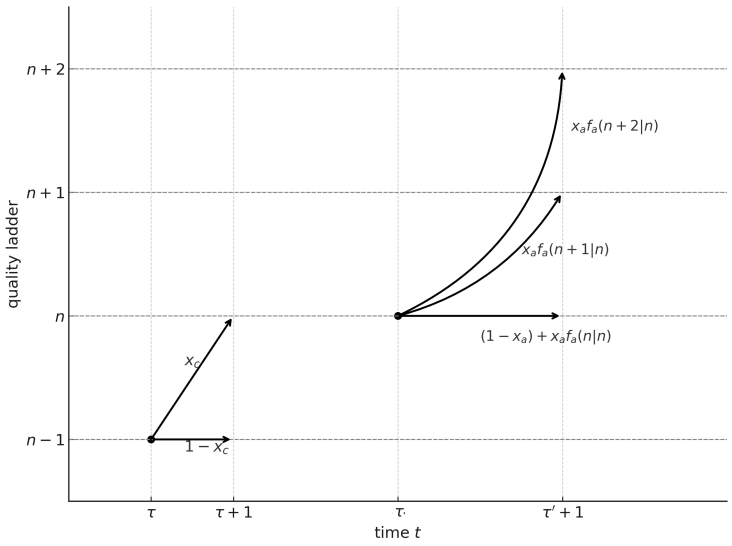


Figure 3: Innovation evolution illustration

Note: This figure illustrates the technology evolution depends on the type of innovation a, c conducted and the effort of innovation x_c, x_a. I assume the effort x coincide with the arrival rate of successful innovation. f_a represents the underlying probability distribution for number of step.

I next formalize the decision problems of DFS and VCFS in the following two subsections.

Debt-financed startups

If the startup does not search at all or does not match with VC, then the startup becomes DFS and the value is given by following equation:

$$
\begin{aligned}
V_h^d(z,\omega) = \max_{x,b'} \quad & div + \beta x \sum_{z'} f_h(z'|z) \int^{\bar{\xi}(z',b')} V(z',\omega') f(\xi') d\xi' \\
& + \beta(1-x) \int^{\bar{\xi}(z,b')} V(z,\omega') f(\xi') d\xi' \qquad (3) \\
\text{subject to:} \quad & div = \omega + q_h(z,x,b')b' - \kappa_h(x) \geq 0 \\
& \omega'(\xi', z', b') = \Pi(z') - b' - \xi'
\end{aligned}
$$

Here the startup chooses its innovation type h, innovation effort x, and borrowing b' in order to satisfy the non-negative dividend constraint. Cash on hand in the next period, ω', evolves with both the realized innovation outcome and the revenue shock ξ'. The cutoff $\bar{\xi}(z,b')$ is the lowerbound of revenue shock that the startup is not able to satisfy limited liability protection. The formal definition of this default cutoff is given by equation 4 following the set up in Ottonello and Winberry (2020). In other word, for any revenue shock $\xi' > \bar{\xi}(z,b')$ given the choice of borrowing b', the startup will not be able to borrow enough money tomorrow to cover the innovation cost and roll-over the debt. In case of negative dividend, the startup exits the market involuntarily and receives zero residual value.

$$
\bar{\xi}(z,b') = \pi(z) - b + max_{h,b',x}\{q_h(z,x,b')b' - \kappa_h(x)\} \qquad (4)
$$

In case of default and exit, the new entrant enter the market from the bottom of the quality ladder with no initial debt. This assumption generates inefficient exit when default.

VC-financed startups

When a startup matches with a VC, they negotiate a state contingent contract specifying how profits are shared and what innovation strategy is pursued. This design mirrors two well-documented features of real-world VC contracts. First, as shown in Kaplan and Strömberg (2003), VC contracts are highly contingent on startup performance, with cash flow rights varying across states rather than being a fixed share. Second, VCs often obtain contingent control rights through board representation, enabling them to influence strategic direction. In the model, this is captured by allowing the VC to specify more aggressive innovation when performance is strong, and conservative innovation otherwise. In addition, I assume that

startups cannot commit to honoring the contract, meaning they may take the innovation funding and walk away. This assumption is motivated by survey evidence from venture capitalists, who cite lack of startup commitment as one of their top concerns in venture financing. Lastly, I assume there's exogenous separation between VC and the startup with probability δ and this captures the average finite investment duration.

Formally, a contract is a state-contingent plan $C = \{d_t(s^t), x_{h,t}(s^t)\}$, consisting of dividend transfers $d_t(s^t)$ to the startup and prescribed innovation effort $x_{h,t}(s^t)$, both of which depend on the history of states. Given any enforceable contract $C^e \in \mathcal{C}$, the startup's value is

$$\begin{aligned} V_t^{vc}(z, m \mid C^e) = d_t(\{s^t\}) + x_{h,t}(\{s^t\}) \sum_{z'} f_h\left(z' \mid z\right) \beta\delta V_{t+1}^{vc}(z' \mid C^e) \\ + (1 - x_{h,t}(\{s^t\}))\beta\delta V_{t+1}^{vc}(z, \mid C^e) + \beta(1-\delta)\mathbb{E}_{x,z'} V^d(z', m) \end{aligned} \tag{5}$$

Given any enforceable contract that specify the contingent payoff from VC to startups, and innovation effort that the startups should conduct, the value is simply the lifetime discounted dividend that the startups receive during the duration of contract. Note that the dividend transfer from VC to startups will implicitly imply the risk-sharing between VC and startups. This is because the VC and startups split the operating profit net the innovation cost, and therefore the decision on dividend transfer incorporates the idea of cashflow right and captures the risk-sharing between two parties.

Solving directly for the optimal contract is infeasible since the state space grows exponentially with history, and enforceability constraints are state-contingent. To address this issue, I follow dual approach proposed in Spear and Srivastava (1987) using the promised value V^{vc} as state variable that summarizes the past history of dividend payment and innovation efforts. Similar to Albuquerque and Hopenhayn (2004), I will first characterize the Pareto frontier of total contract of surplus, and then solve allocation of surplus between VC and startups to get the optimal VC contract. In particular, the total surplus of the contract if conducting the innovation type $h \in \{a, c\}$ is given by Eq. 6.

$$
\begin{aligned}
S_h^C(z, m, V^{VC}) = \max_{x_h, \{V^{VC}(z')\}_{z' \in Z}} & \Pi(z) + x_h \sum_{z' \in Z} f_h(z' \mid z) \beta\delta S^C \left(z', m, V^{VC}(z')\right) \\
& + (1 - x_h)[\beta\delta S^C \left(z, m, V^{VC}(z)\right)] - (1 + r_f)\kappa_h(x_h) + \beta(1 - \delta)\mathbb{E}_{x,z'} V^d(z', m)
\end{aligned} \tag{6}
$$

subject to promise keeping (PK) constraint and enforceability constraint (IC):

$$
\text{[PK]: } V^{vc} \geqslant \beta\delta[x_h \sum_{z' \in Z} f_h(z' \mid z) V^{vc}(z') + (1 - x_h) V^{vc}(z)] + \beta(1 - \delta)\mathbb{E}_{x,z'} V^d(z', m)
$$

$$
\text{[IC]: } V^{vc} \geqslant V^{Aut}(z) + \kappa_h(z, x_h)
$$

When a startup with technology level z is matched with VC, the total profit in current period is $\Pi(z)$. The contract will specify the innovation effort in current period x_h, which is associated with innovation cost $\kappa_h(x_h)$. The second and third term in Eq. 6 represents the continuation surplus when both parties remain inside the contract. Note that startups can collect the innovation cost $\kappa_h(x_h)$ and run away with the penalty of being financially autarky for the rest of the periods. As a result, the capacity of the innovation is limited by the IC constraints and generates the inefficiency in the VC contract.

After solving for the Pareto frontier of total surplus of the contract, I define the optimal VC contract to be the solution of Nash Bargaining problem among all feasible contract that is enforceable. In particular:

$$
\begin{aligned}
\max_{V^{vc}} \ & \left(S^{VC}(z, m, V^{vc}) - S^{VC,o}\right)^{\alpha} \left(V^{vc} - V^d(z, m)\right)^{1-\alpha} \\
\text{s.t.} \ & S^c(z, m, V^{vc}) = S^{VC}(z, m, V^{vc}) + V^{vc}
\end{aligned} \tag{7}
$$

where α and $1 - \alpha$ are bargaining power for VC and startups respectively, and $S^{VC,o}$ is the outside option is VC decides not to invest in the paired startups. The concavity of $S^c(z, m, V^{vc})$ ensures a unique solution for the bargaining problem. Now both the DFS and VCFS problem have been explored. I will describe the financier's problem in the next subsection.

2.3 Bank and VC

I assume a small open economy that takes the international risk-free interest rate r^f as given. Both banks and venture capitalists have unrestricted access to a perfectly elastic supply of funds from an international lender at rate r^f. As a result, financing costs are exogenous and unaffected by domestic conditions.

I assume banks are competitive and set the break-even interest rate. Given technology level and the planned research type and effort, there exists a cutoff $\bar{\xi}(z', b')$ as defined in Eq. 4. In each of possible future states, the bank evaluates the repayment probability according to the probability distribution of revenue shock $\Phi(\xi)$, and set the interest rate schedule according to Eq. 8.

$$q_h(z, x, b') = \frac{RP}{1 + r^f} = \frac{x \sum_{z'} f(z' \mid z)\Phi(\bar{\xi}(z', b')) + (1 - x)\Phi(\bar{\xi}(z, b'))}{1 + r^f} \tag{8}$$

The interest rate schedule depends not only on the current technology level and the amount of debt, but also on the type of innovation and the effort. Under an aggressive innovation strategy, there is a higher probability of producing *useless* innovations even with the same level of effort, but also a greater likelihood of achieving highly successful innovations. Consequently, the interest rate schedule depends in a non-trivial way on both the innovation type and the startup's current position on the quality ladder.

Regarding VC, I assume one representative VC firm with an exogenous fixed mass of managers enters each period. If a manager does not match with a startup or separates from an existing contract, it exits at the end of the period and receives no continuation value. This assumption keeps the VC sector tractable.

Upon matching with a startup and negotiating a VC contract, the manager finances the contract through risk-free borrowing from the same sources as banks. Because the mass of managers is scarce and investors possess strictly positive bargaining power, each manager receives a strictly positive expected share of the contractual surplus, as determined by Eq. 7. This surplus is ultimately rebated to households.

2.4 Equilibrium

Definition 1. *Let $s = \{z, \omega\}$ denotes the individual state of startup. A stationary equilibrium in the baseline model consists a set of policy function $h(s), x_h(s), \ell_h(s), \nu(s)$, an invariant distribution of startups $G(z, \omega)$, prices w, and matching probability λ such that:*

(i) The optimal innovation policy $x_h(s)$ and borrowing policy $b(z, \omega)$ solves DFS problem described by equation 3 and 5. For DFS, the marginal benefit of innovation effort equals to marginal cost of innovation cost plus the shadow value implied by Lagrange multiplier on limited liability constraint.

(ii) The interest schedule $q_h(z, x, b')$ equals to break even interest rate and satisfies zero profit condition. In particular:

$$q_h(z, x, b') = \frac{1}{1 + r_f}[x \sum_{z'} f(z' \mid z)\Phi(\bar{\xi}(z', b')) + (1 - x)\Phi(\bar{\xi}(z, b'))]$$

(iii) The optimal labor demand solves the static profit maximization problem, the marginal product of labor equals to the equilibrium wage rate:

$$\ell(z) = \left[Y \left(\frac{\mu\, w}{(1 - \alpha_z)}\right)^{-\eta}\right]^{\frac{1}{1-\alpha_z+\eta\alpha_z}} z^{\frac{\alpha_z(\eta-1)}{1-\alpha_z+\eta\alpha_z}}, \qquad \mu \equiv \frac{\eta}{\eta - 1}.$$

(iv) The wage clears the labor market:

$$\int \ell(z) dG(z, \omega) = 1$$

(v) The matching probability agrees with the optimal searching decision:

$$\lambda = m_{vc} / \int \nu(s) dG(z, \omega)$$

where ν is indicator function for search decision by type of startup and m_{vc} is fixed exogenous mass of manager in each period.

(vi) The invariant distribution is the solution of discrete time version of Kolmogorov

equation with π^T denote transition probability across states:

$$g(z', m') \;=\; \int_{\mathcal{Z}}\int_{\mathcal{M}} \pi^T(z', m' \mid z, m)\;\; g(z, m)\; dm\, dz.$$

2.5 Model mechanism

In this section, I describe the main model mechanism and present the innovation policy among VCFS and DFS counterpart to understand how does VC insures the default risk and encourage startups to conduct more high-risk-high-return innovation.

First of all, the optimal level of R&D for DFS depends on their technology z_{jt}, the case on hand level ω_{jt}, and the curvature of bond price schedule $q_h(z, x, b')$. Let λ_D be the Lagrangian multiplier of the non-negative dividend constraint in DFS's problem, the optimality condition for DFS can be characterized as:

$$\begin{aligned}
[x]: \quad 0 = {} & \beta\left(\mathbb{E}_{z'}\int^{\bar{\xi}(z',b')} V(z', m')\, dF(\xi') - \int^{\bar{\xi}(z,b')} V(z, m')\, dF(\xi')\right) \\
& + (1+\lambda_D)\left(\frac{\partial q_h(z,x,b')}{\partial x} b' - \kappa'(x)\right), \\
[b']: \quad 0 = {} & (1+\lambda_D)\left(q_h(z,x,b') + \frac{\partial q_h(z,x,b')}{\partial b'}\, b'\right) \\
& + \beta\Bigg\{ x\, \mathbb{E}_{z'}\Bigg[V\big(z', \bar{m}(z',b')\big)\; f\big(\bar{\xi}(z',b')\big)\; \frac{\partial \bar{\xi}(z',b')}{\partial b'} \\
& \qquad - \int^{\bar{\xi}(z',b')} V_m\big(z', m'\big)\; dF(\xi')\Bigg] \\
& + (1-x)\Bigg[V\big(z, \bar{m}(z,b')\big)\; f\big(\bar{\xi}(z,b')\big)\; \frac{\partial \bar{\xi}(z,b')}{\partial b'} \\
& \qquad - \int^{\bar{\xi}(z,b')} V_m\big(z, m'\big)\; dF(\xi')\Bigg]\Bigg\}.
\end{aligned} \tag{9}$$

where $m'(z', \xi'; b') \equiv \Pi(z') - b' - \xi'$ denotes next-period cash on hand, and $\bar{m}(z', b') \equiv \Pi(z') - b' - \bar{\xi}(z', b')$ is the corresponding cash-on-hand cutoff at the default threshold.

Starting with the FOC with respect to innovation effort $[x]$, the first two terms is the

marginal cost and marginal benefit of innovation respectively. With higher amount of the effort, the startup is more likely to receive successful innovation and climb up the quality ladder with higher future value. However, the last term captures the shadow value of extra spending in terms of default risk. When a startup decides to spend more on innovation, the startup will need to issue more debt if the cash on hand is not enough to cover the cost. This in turns increase the probability of default in the next period given higher amount of the cumulative debt. This captures the direct trade-off between innovation cost and benefit, and the indirect impact from the financial friction that leads to insufficient innovation effort from firms.

Next, consider the first-order condition with respect to borrowing b'. The terms inside the expectation capture the *direct effect* of additional borrowing on next-period default risk. An increase in b' lowers next-period cash on hand and therefore reduces the cutoff $\bar{m}(z', b')$, shrinking the continuation region. This effect operates through two channels: a boundary effect, which reflects changes in the default threshold weighted by the density at the cutoff, and an interior effect, which captures the decline in continuation value due to lower cash on hand in non-default states.

The remaining term reflects the *indirect price effect*, as higher borrowing raises default risk and lowers the bond price $q_h(z, x, b')$, thereby increasing the marginal cost of debt issuance. Taken together, these forces imply that when the startup is financially constrained, borrowing decisions internalize default risk and price distortions. Combined with the implication from the $[x]$ condition, limited cash on hand causes the startup's innovation choice to deviate from the efficient level due to the presence of financial frictions.

Figure 4 reports the innovation policy for startups financed by debt. I plot the median startups on the firm productivity distribution to demonstrate the tradeoff faced by DFS. the x-axis represents the cash-on-hand ω_j or startups j, and the left y-axis is the optimal innovation effort level x_h with innovation type labeled by different color. The black dash line represents the optimal innovation type the startup is choosing.

When a DFS gets a consecutive unlucky shock and start cumulating up the debt, the cash-on-hand starts moving from the right to the left end on x-axis. First of all, observe that the DFS will switch from conducting aggressive to conservative innovation. This is

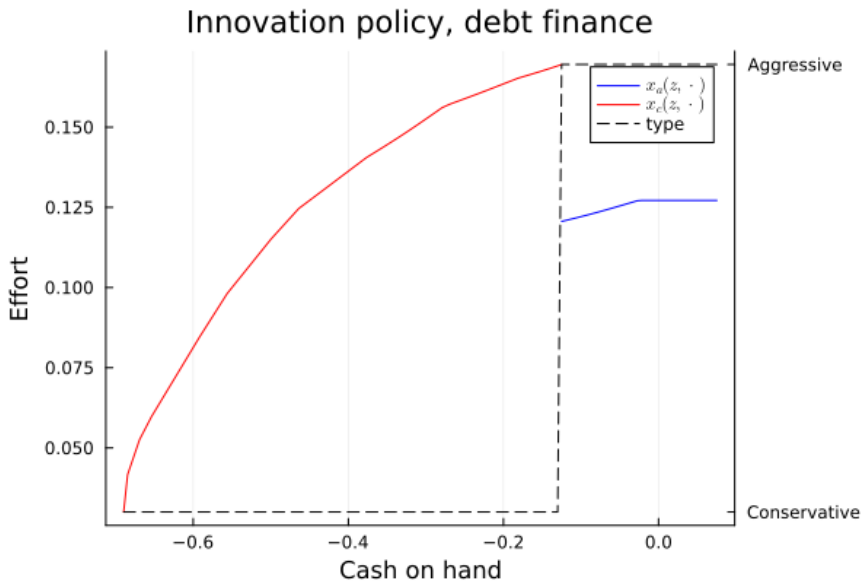


Figure 4: Innovation policy by DFS

Note: Left axis corresponds to innovation effort x_h in the model given type of innovation labeled by color. This figure depicts a typical medium firm in the invariant firm productivity distribution. The discontinuity around the vertical dash line implies the behavior change around the threshold of cash on hand.

the extensive margin switch as aggressive innovation essentially associated with higher risk. When the cash-on-hand is low, the shadow value of each dollar becomes high as startups are closer to default region. If the startups keep doing aggressive innovation, they not only push themselves closer to risky default region because of higher cost of conducting aggressive innovation, but also make themselves more vulnerable because of highly uncertain outcome in aggressive innovation. Second, the startups do not just change their innovation type decision, but also lower the effort when they move closer to risky default region. The reason is similar. When a startup has lower cash-on-hand, the shadow value is higher. However, the marginal benefit is linear since the the innovation effort governs the probability of having successful innovation. The curvature of the shadow value dampens the innovation effort. The lead to lower growth and the effect especially strong when startups are young and small as they tend to be closer to risky default region.

Next, I report the optimal VC contract. For illustration, I focus on the contract negotiated between a median-productivity startup and a VC. Figure 5 plots the Pareto frontier. The x-axis is the value promised to startups V^{vc}, and the y-axis on the left is the total sur-

plus S^c. The blue curve represents the total surplus S^c and the black dash curve represents the contract specified innovation effort x_h. The green vertical line represents the optimal contract that solves the allocation problem described in Eq. 7.

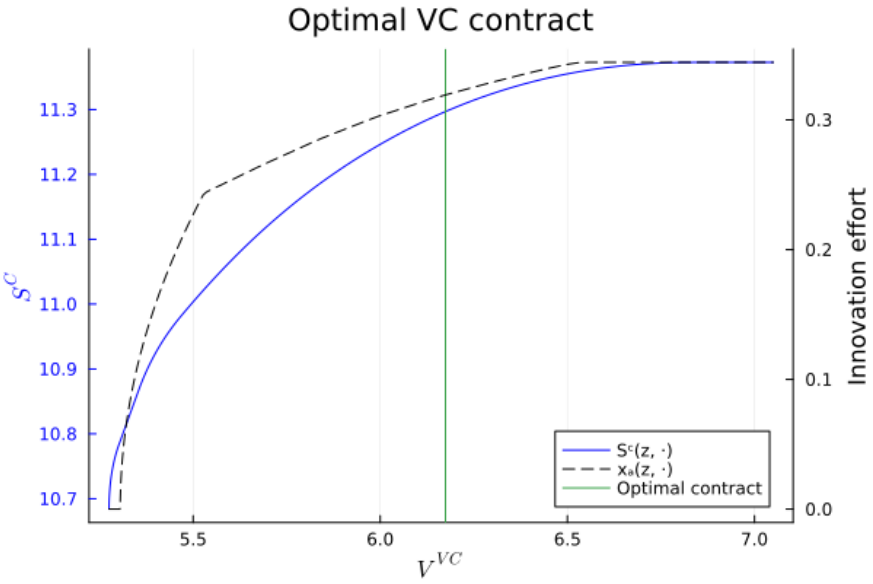


Figure 5: Innovation policy by VCFS

Note: The x-axis is value promised to startups V^{vc}, y-axis on the left is the total surplus of VC contract defined in equation 6, y-axis on the right is the innovation effort specified with each V^{vc}, and green vertical line is solution to the allocation problem in Eq. 7. The figure plot the contract surplus frontier with startup having median technology level in the equilibrium invariant productivity distribution.

The frontier $S^c(z, m, V^{vc})$ is concave in V^{vc}, a standard result driven by the enforceability and promise-keeping constraints. When V^{vc} is low, the enforceability constraint binds. In this region, VC specifies low innovation effort to keep startup in the contract so the slope of the innovation curve is steep. As V^{vc} increases, the promise keeping constraint becomes binding. In particular, while the VC would like to backload dividends where they reward the startup more in later stages when it is larger and more productive, limited liability prevents the use of negative dividends today in exchange for future transfers. This constraint caps the extent of future promises and keeps innovation effort below the efficient level. The efficient benchmark corresponds to the flat segment of the dashed curve at the right end, which the contract never fully achieves.

The bargaining process further shapes outcomes. Because VCs retain part of the surplus, the optimal contract prescribes innovation effort about 5–15% below the efficient level under

the baseline calibration that matches average Series A equity shares. With stronger VC bargaining power, the gap widens as more surplus is shifted away from the startup. From the household's perspective only the total surplus matters since it ultimately receives the rebated profits. Thus, the optimal bargaining power is the one that maximizes innovation efficiency. Importantly, even though innovation effort is below first-best, the VC contract still outperforms debt finance by sponsoring innovation costs and shielding startups from excessive debt accumulation. This enables higher innovation intensity and a greater likelihood of aggressive innovation.

2.6 Differential response to financial shock

To further illustrate how financial frictions affect startup behavior and why VC financing mitigates these constraints, I consider a revenue-shock experiment comparing the resilience of debt- and VC-financed firms. The exercise simulates a one-time, one percent decline in revenue for small startups (bottom decile of the size distribution) and median-sized firms.

Figure 6 plots the responses to a one percent revenue shock. Panels (a)–(e) display, respectively, the responses of innovation effort (intensive margin), the fraction of firms conducting aggressive innovation (extensive margin), borrowing, search, and technology. I simulate a large panel of startups and mature firms and allow the shock to occur randomly in their lifecycle. The solid blue line shows the average percentage change from the pre-shock level for mature firms, while the orange line shows the corresponding change for young startups.

Note that for VCFS, revenue shocks are already internalized in contract negotiations and do not affect innovation strategy. The total surplus from the contract accounts for potential shocks, and profit stream net of these shocks are shared between the VC and the startup. Consequently, the decision rule of VCFS is independent of the realized shock. As a result, the figure reflects the response of DFS relative to VCFS given the size.

Panel (a) shows that innovation effort among young startups falls by about 0.25 percentage points after the shock, with the effect strongest on impact and lasting for two periods. Mature firms, by contrast, display no significant adjustment, consistent with having sufficient buffers to maintain innovation effort. Panel (b) highlights that young DFS adjust not only at the intensive margin but also at the extensive margin: roughly 15 percent fewer

startups pursue aggressive innovation following the shock, and this effect persists for more than five years, while mature firms show no response along the extensive margin. Taken together, these results indicate that young startups absorb adverse shocks by adjusting both intensively and extensively, since the shadow cost of borrowing is high relative to the shadow cost of innovation effort. This mechanism explains why VC financing is particularly valuable for young startups. By alleviating default risk, VC enables young startups to sustain innovation, grow more rapidly, and escape the vulnerable early phase where financial frictions are most severe.

Panels (c) and (d) report borrowing and search responses. Both young and mature DFS increase borrowing to offset lost revenue, particularly when the non-negative dividend constraint binds. However, borrowing plays different roles. Mature firms use debt to smooth temporary shocks without altering innovation, while young startups adjust both financing and innovation. Search behavior is especially responsive among young DFS, with the probability of searching for VC rising by about 10 percentage points, reflecting the large expected gain from matching, albeit at the cost of congestion. Finally, Panel (e) shows the resulting technology gap: following a one percent revenue shock, technology levels among young DFS remain 0.1-0.25 percentage points below their no-shock path after two periods, with the gap persisting for at least five. The implied discounted profit loss is about 1 percent, highlighting the long-term cost of financial frictions.

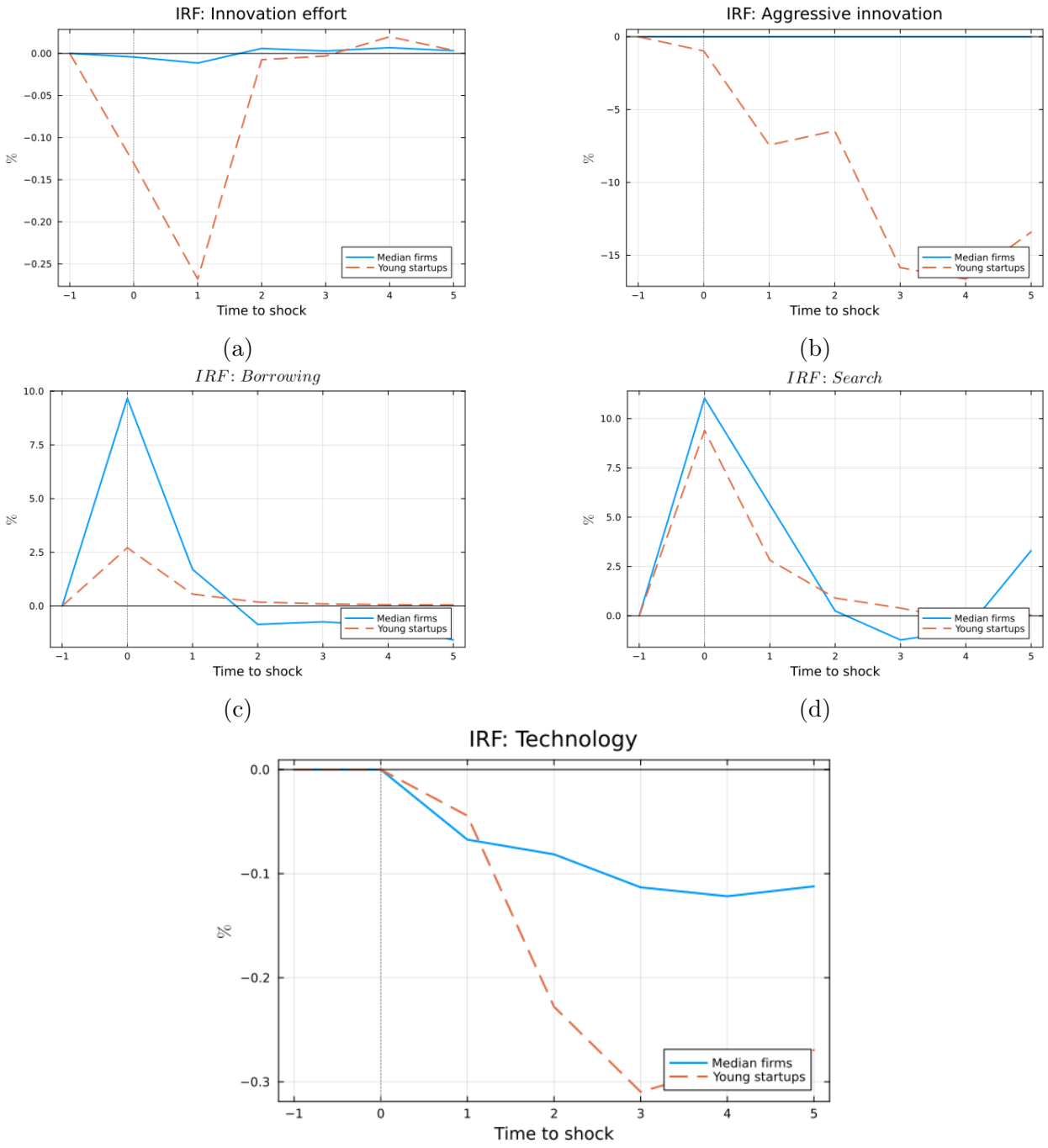


Figure 6: Impulse response functions

Note: The figure plots impulse responses to a one-percent negative revenue shock. The blue solid line shows the percentage change from one year before the shock for mature firms (median of the size distribution), and the orange dashed line for young startups (bottom decile). Panels (a)–(e) report, respectively, responses in innovation effort, the fraction conducting aggressive innovation, borrowing, VC search, and technology level.

3 Calibration and Empirical Validation

After introducing the full model, I discipline the model with startup's financing and innovation patterns and explore the model predictions and corresponding empirical patterns in this section.

3.1 Calibration

To calibrate the model, I divide parameters into two groups. The first group is assigned values commonly used in the growth literature or directly pinned down from data. The second group is chosen to match key moments of startup financing and growth patterns by solving the model. Table 1 reports the parameter values together with their corresponding targets.

The average of main refinancing operations – fixed rate tenders in the EU from 2000 to 2024 serve as the benchmark risk-free rate. The span parameter in the production function, α_z, is set close to values commonly used in the literature, and sensitivity analyses suggest that varying it does not materially affect the model's mechanisms. The risk aversion parameter is fixed at $\psi = 2$, consistent with standard values in the literature, while the exogenous separation rate, $\delta = 0.25$, aligns with the average four-year duration of Series A venture capital contracts.

After pinning down the first set of parameters, I calibrate the model using a simulated method of moments approach to minimize the distance between data moments and model moments. The data moments, described in Section 3.2, rely on startups' R&D and financing information to identify the innovation and financing parameters. On the model side, I first solve for the decision rules described in the previous section and then simulate a large panel of young startups. These simulated firms search, borrow, and innovate according to the policy rules, generating model moments that can be compared with their empirical counterparts.

For innovation related parameters such as the innovation costs $\kappa_a, \kappa_c, \kappa_{a,f}$, the calibration matches the R&D expenditure share and growth rates observed in the data. Because VC funding induces different innovation tendencies, the R&D expenditure gap between VCFS and DFS is informative about the relative cost structure of aggressive versus conservative

Table 1: Calibration

Parameter	Meaning	Value	Source/Moments
Assigned parameters			
r^f	Risk free rate	2.25%	Avg. risk free rate
β	Time discount	0.96	Standard in literature
α_z	Span parameter	0.8	Greenwood et al. (2022)
ψ	Risk aversion	2.0	Standard in literature
δ	VC separation rate	0.25	Avg. investment duration, 4 years
Calibrated parameters			
α	VC bargain power	0.28	VC share in contract
c_s	Search cost	0.0015	Fraction of VCFS, 0.2%
μ_s	Location of revenue shock	-2.45	Debt/sales ratio
σ_s	Std of revenue shock	0.82	Avg. default rate
g_ζ	Step-size of innovation	1.19	Labor productivity growth
κ	Location for aggregate innovation	3.7	Labor productivity growth - VCFS
$\kappa_{a,f}$	Fixed cost for aggregate innovation	0.03	Labor productivity growth - DFS
κ_a	Aggregate innovation cost	1.96	R&D / sales ratio - VCFS
κ_c	Conservative innovation cost	0.49	R&D / sales ratio - DFS

Note: VC share in the contract is computed by VC investment amount/post finance valuation to capture VC's equity share in the contract. The labor productivity is defined by $\frac{sales}{\#\ of\ employee}$
Source: Data compile from Orbis and CB Insights

innovation. In both the data and the model, labor productivity growth is measured as sales per employee. The growth rate differential between VCFS and DFS disciplines the probability distribution of step sizes in aggressive innovation. Since VCFS undertake more aggressive projects, their higher growth rates help pin down the expected number of steps, modeled as a truncated Poisson distribution $f_a(z' \mid z) = \text{Poisson}(\kappa)$ truncated at five steps. This restriction ensures that firms cannot leap to the top of the technology ladder in a single draw.

Turning to the financing side, I use the debt-to-revenue ratio as a proxy for leverage among young startups, which informs the default cutoff $\bar{\xi}$ and helps discipline the shock process. The location parameter of the revenue shock is set to $\mu_s = -2.45$. The average default rate from Ottonello and Winberry (2020) is used to discipline the standard deviation of revenue shocks, which is set to $\sigma_s = 0.82$ to match the equilibrium default probability. The

remaining VC-finance parameters, α and c_s, are chosen to match VC investment moments. In particular, c_s governs equilibrium search effort and determines the fraction of VCFS, while the bargaining power parameter directly pins down the VC's equity share in Series A financing. The resulting model-data moment match is reported in Table 2.

Table 2: Model match

Moments	Data (%)	Baseline (%)
VC share in contract	19.23	22.54
Debt/sales ratio	14.1	11.2
Fraction of VCFS	0.20	0.23
Avg. default rate	3.00	1.25
Labor productivity growth	1.31	1.47
Labor productivity growth - VCFS	7.71	6.21
Labor productivity growth - DFS	3.77	3.68
R&D / sales ratio - VCFS	6.58	8.92
R&D / sales ratio - DFS	3.22	3.35

Note: Debt to sales ratio computed with (long-term debt + short-term debt) / sales. The fraction of VC is # of VCFS / # total firms. Average default rate is computed with $\frac{\#\text{ of defaulting firms}}{\#\text{ total firms}}$. R&D to sales ratio is computed with $\frac{\text{R\&D expenditure}}{\text{sales}}$, following Olmstead-Rumsey (2022) I impute missing value with zero. VC share in contract 19.23% in my dataset is in line with literature ranging from 20% to 40%.
Source: Data compile from Orbis and CB Insights

The model replicates the targeted moments well. In particular, it captures the differences in innovation and financing patterns between VCFS and DFS observed in the data. Both the growth rate and R&D expenditure are higher for VCFS than for DFS in the model.

The next subsection introduces a novel dataset linking balance sheets and VC deals, used to assess the model's untargeted moments.

3.2 Data and Empirical Validation

In this section, I first describe the main dataset used in the paper. I then use the merged dataset to document new empirical patterns in the lifecycle of VC-backed startups that are consistent with the model's implications.

3.2.1 Description

The main data sources are CB Insights, the United States Patent and Trademark Office (USPTO), and Orbis. CB Insights collects information on investment and exit deals from regulatory filings, press releases, and company disclosures, covering transactions across OECD countries. I use these data to identify all VC investment deals, focusing on the first round of financing (Series A). This choice abstracts from the sequential nature of VC financing studied in, for example, Greenwood et al. (2022); Yu (2024), and instead emphasizes how VC financing alters startups' innovation strategies. To evaluate innovation outcomes, I use patent data from the USPTO, which provide comprehensive information on applications, grants, citations, and applicants in the United States. From these data, I construct standard measures of innovation output and quality, including forward citations, originality, and generality. Finally, I use firm-level financial data from Orbis, a comprehensive database of balance sheets and financial statements. I focus on startups in the European Union, where reporting requirements ensure coverage even for relatively young and small firms.

To construct the startup-level panel, I match firms in CB Insights to Orbis using company names, locations, and website URLs, following procedures similar to the NBER Innovation Information Initiative. Company names are standardized by removing spaces and country-specific suffixes (e.g., S.A., GmbH). I then compute Jaro–Winkler distances between names and cross-check locations at the city level. Finally, I verify matches using website information; firms that pass all steps are classified as strong matches. For borderline cases (Jaro–Winkler distance <0.02), I manually verify company details. The resulting dataset with non-missing financial information covers about 1,400 startups.

I then merge these matched firms with USPTO patents and citations, again using name and location matching. The empirical analysis focuses on net income, operating revenue, and employment from Orbis, and on patent counts and quality measures from USPTO. Patent quality is measured using forward citations, originality, and generality indices (Hall et al. (2001)). Originality reflects the diversity of technological fields cited by a patent; generality captures the diversity of fields that cite the patent. Higher scores indicate knowledge that is more interdisciplinary and broadly applicable.

A central challenge is that startups seeking or receiving VC financing may differ systematically from those that do not. To construct a credible counterfactual, I build a synthetic control group following Akcigit et al. (2022). For each VC-financed startup, I select non-VC startups in the same country and four-digit NACE industry with a similar number of employees at the age when the VC-backed firm receives funding.[5] Repeating this procedure for all VC-backed firms yields a matched sample that allows comparison of post-financing outcomes between VCFS and their synthetic controls. To validate the synthetic control construction, I estimate local projection difference-in-differences and report the results in the Appendix Figure A1 and A2. The parallel trends in revenue and profit confirm that VC-financed firms and their synthetic counterparts behave similarly prior to VC funding.

Table 3: Summary Statistics

Variable	**Obs.**	**Mean**	**Std. Dev.**	**Min**	**Max**
Employment (emp)	41,525	15.23	16.99	1	80
R&D spending (rd)	1,067	4009.34	6664.84	0	30,633.38
Firm age (age)	50,487	5.03	2.97	1	12
Debt-to-revenue (debt_rev)	13,619	0.14	0.34	0	2.45
R&D-to-revenue (rd_rev)	973	0.91	2.78	0	24.18
Profit margin (pl_rev)	14,758	-0.47	1.33	-9.49	0.36
Productivity (rev_emp)	20,110	247.45	291.18	10.40	1369.56
Log revenue (lrev)	20,867	7.41	1.33	4.23	10.29
Revenue growth (rev_gr)	16,841	0.12	0.30	-0.64	1.14
Profit growth (pl_gr)	14,522	-0.01	1.12	-2	2
Labor productivity growth (rev_emp_gr)	15,613	0.04	0.25	-0.65	0.85

Note: All variables have been winsorized at 5 percentiles. Labor productivity is a defined with revenue per employee; R&D = research and development. Revenue and profit growth calculated with $\frac{\Delta y}{0.5y+0.5l.y}$ and is bounded between 2 and -2 to avoid extreme value.

Summary statistics for all startups, including VC-financed and their non-VC counterparts, are reported in Table 3, and the geographic and sectoral distribution is shown in Figure 7. The raw dataset spans roughly 180 industries at the NACE four-digit level across 17 countries. Because financial reporting requirements vary across jurisdictions, however, data coverage is uneven. For this reason, the empirical analysis focuses on the 14 countries with mandatory reporting obligations.

[5] I use a 10% bin size as the baseline tolerance; results are robust to stricter thresholds of 5% and 1%.

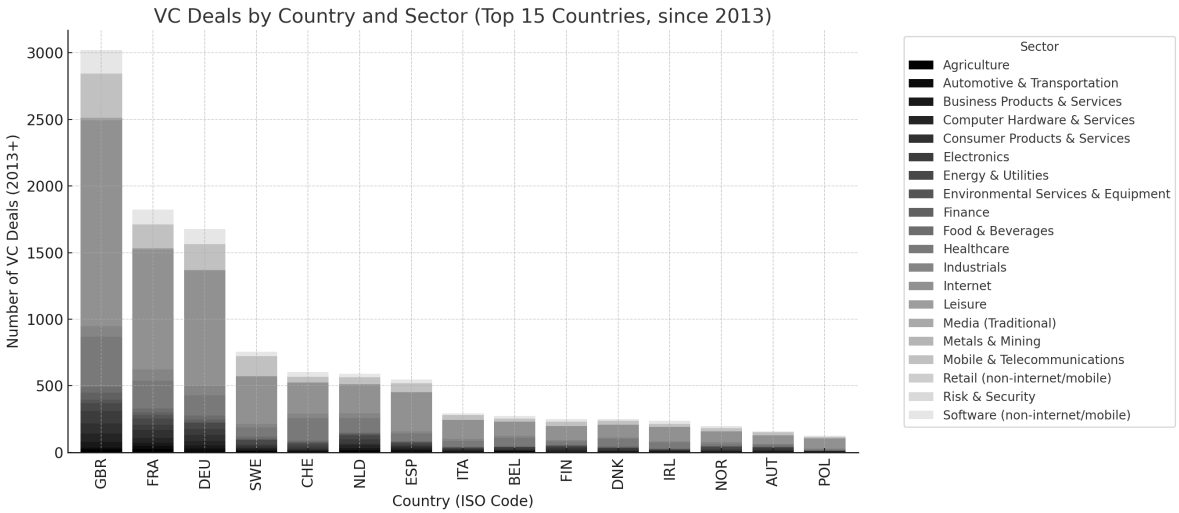


Figure 7: VC deals among EU since 2013

Note: The figure plots Series A deals across EU countries and shows the sectoral composition based on CB Insights classifications.
Source: CB Insights

I will now present the empirical patterns consistent with the model predictions.

3.2.2 Empirical Patterns

I validate the model by comparing its untargeted predictions to the data. The first set of untargeted moments captures the heightened volatility of outcomes after VC financing. Consistent with the model's core mechanism, VC-financed startups exhibit greater volatility in growth and wider dispersion in profits and revenues following investment. The second set of untargeted moments further test the model predictions. First, self-selection into VC generates higher leverage among VCFS prior to financing. Second, mapping patents to successful innovations in the model, VCFS produce patents of higher quality, with higher mass in the right tail of the quality distribution than non-VC counterpart.

Heightened Volatility

The first key untargeted moment is the volatility in startup growth trajectory. The model predicts that VC financing increases the time-series volatility of startup performance. By encouraging aggressive innovation, VCFS are exposed to more uncertain outcomes, which should translate into larger fluctuations in growth and net income relative to non-VCFS.

To test this prediction, I estimate a difference-in-differences specification that compares the standard deviation of revenue growth for VCFS and matched non-VCFS before and after financing. For VCFS, the reference point is the actual year of financing; for non-VCFS, I assign a counterfactual financing year based on their matched VCFS. Because the model abstracts from ex-ante heterogeneity, I remove startup-specific unobserved heterogeneity and time trends by estimating the following fixed-effects regression:

$$\text{growth}_{it} = \alpha + X_{it} + \alpha_i + \alpha_t + \hat{\mu}_{it} \tag{10}$$

where α_i and α_t are startup and time fixed effects. I then use the estimated residual $\hat{\mu}_{it}$ as the stationary growth rate not sourcing from startup's unobservable and compute the standard deviation, $sd(\mu_{i,\tau})$ for $\tau \in \{\text{before, after}\}$, of growth rate before and after for each startups. Finally, I estimate the DiD regression in Equation 11 to compare the volatility in growth rate between VCFS and non-VCFS.

$$sd(\hat{\mu}_{i,\tau}) = \alpha + \alpha_1 VC + \alpha_2 Post + \beta VC \times Post + \epsilon \tag{11}$$

Table 4 reports the results. The estimates indicate that volatility increases significantly for VCFS after financing, while non-VCFS show only modest changes. The DiD coefficient confirms that the rise in volatility is larger among VCFS, consistent with the model's mechanism in which aggressive innovation amplifies both upside potential and downside risk.

Table 4: Comparison of DiD Coefficients: Data vs. Model

Variable	Data	Model	Explained (%)
sd(Profit growth)	0.60	0.47	78.3
sd(Sales growth)	0.23	0.14	60.9

Note: The table presents the DiD coefficient estimates from equation laid out in Section 3.2. The independent variables are the volatility before and after for each startup, and the DiD coefficient captures the mean difference between VC and non-VC before and after VC finance. Both the model and data show increasing volatility in growth rates of revenue and profit after VC finance. The R&D to revenue ratio in data is extremely large due to small sample size and the fact that several VC startups conduct R&D while having almost zero revenue.

Widening Dispersion

In addition to time series volatility, I further examine this pattern using cross-sectional dispersion. After receiving VC financing, startups are more likely to adopt aggressive innovation. Such projects yield highly uneven outcomes. Some firms achieve large jumps in productivity, while others expend resources without moving up the quality ladder.

The model therefore predicts that the dispersion of net income [6] widens after VC financing, as the gap between leaders and laggards increases.

I measure dispersion using the interquartile range (IQR), defined as the difference between the 75th and 25th percentiles. To ensure comparability, I compute the IQR for both VCFS and non-VCFS at the time of financing. For VCFS, this is the actual financing year; for non-VCFS, I assign a counterfactual year based on their matched VCFS counterpart. I then normalize IQRs relative to year 0 (the financing year), so changes are expressed as percentage deviations from the reference year and get rid of scale effect.

Figure 8 shows the normalized IQR before and after financing. Dispersion rises modestly among non-VCFS, reflecting the nature of uncertainty even conservative innovation. In contrast, both the model and the data show a pronounced widening among VCFS, consistent with more aggressive innovation strategies. This is because a subset of startups achieve breakthrough successes, while others fail, amplifying the gap between leaders and laggards.

Differential Financial Leverage

An important prediction of the model is that VCFS tend to exhibit higher financial leverage prior to matching. Startups with high debt-to-revenue ratios are closer to default and thus more financially constrained, making them more likely to search for VC funding than otherwise similar startups with greater borrowing capacity. Because of this self-selection, the average leverage of VCFS is higher than that of non-VCFS. Table 5 reports the data patterns alongside the model's predictions.

I calculate the total debt-to-revenue ratio for both VCFS and their non-VCFS counterparts in the model and in the data. Empirically, VCFS exhibit substantially higher financial

[6] To be consistent with the Orbis balance sheet definition in global format, the net income in the model is defined by revenue subtracts all the operating expenditure including innovation cost and labor cost

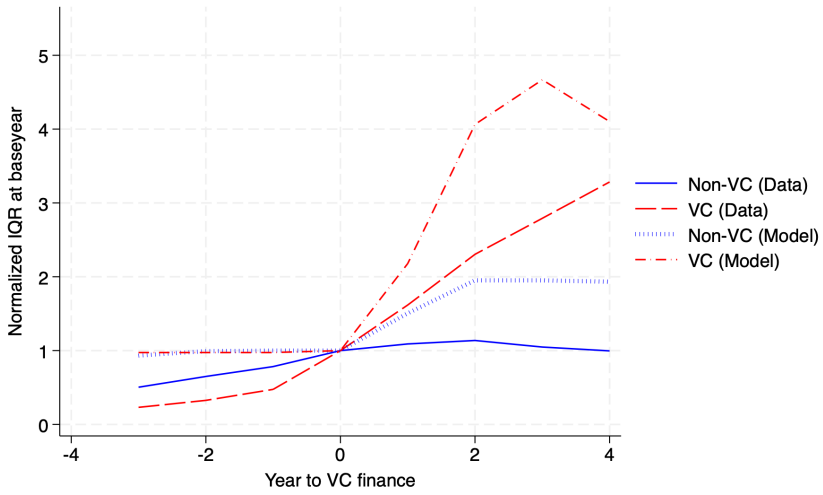


Figure 8: Normalized IQR, model and data

Note: The figure plots the normalized interquartile range (IQR) for VCFS and non-VCFS in both data and model. The x-axis is the relative time to reference year (the year startup receives VC finance), and y-axis is the scale relative to reference year. I first compute the IQR for VCFS and non-VCFS at each year respectively. I then normalized the IQR at each year with the IQR in the reference year. The figure captures how the dispersion evolves among VCFS and non-VCFS.

Table 5: Debt to revenue ratio

	Data	Model	Explained (%)
Non-VC	0.11	0.12	-
VC	0.27	0.16	59

Note: This table reports the level of total debt-to-revenue ratio between VCFS and non-VCFS. The total debt is defined as short-term debt plus long-term debt from startups' balance sheets. The leverage ratio for non-VC firm is targeted.-
Source: Author's calculations based on Orbis data.

leverage, about 2.4 times that of non-VCFS. The model qualitatively reproduces this pattern through the self-selection channel, implying a leverage ratio for VCFS that is roughly 1.3 times larger than for their counterparts. Note that the baseline model considers the random match scenario and abstract from ex-ante heterogeneity in productivity. As a result, 1.3 times larger in financial leverage should be considered lower-bound. The reason is that in reality VC is likely to finance that startups that are more in need of VC finance so to generate more surplus when forming a contract. This direct search type of financing behavior

will amplify the difference in financial leverage between VCFS and non-VCFS.

Better Patent

I next examine the quality of innovation outcomes. In the model, patents correspond to successful innovations, with quality mapped to the step size of the advance. The model predicts that VCFS should generate patents of higher quality, particularly in the right tail of the distribution, since they are more likely to pursue aggressive innovation. To account for differences across years and technology fields, I standardize citations by year and Cooperative Patent Classification, following standard practice in the literature. Figure 9 plots the distribution of forward citations for VCFS and non-VCFS.

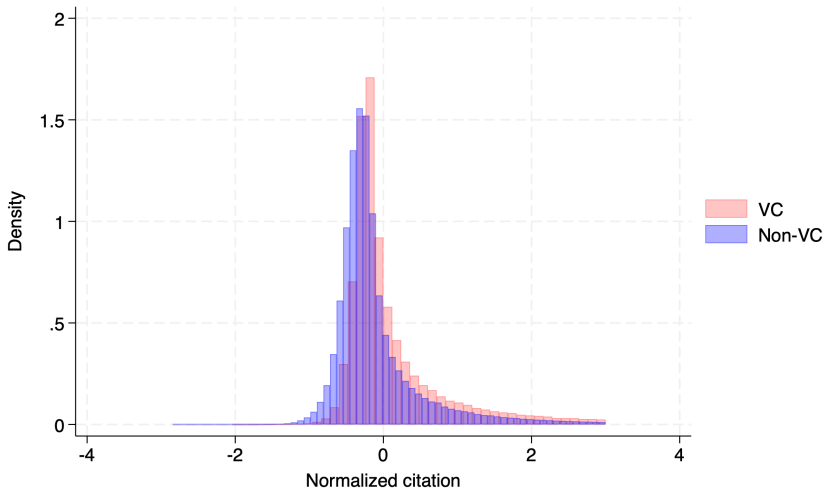


Figure 9: Normalized citation, VCFS vs non-VCFS

Note: The figure plots the normalized forward citation density for VC and non-VCFS. I first compute the number of forward citations for each patent, and then normalize by the number of forward citations within the same CPC in the same year following the suggestion in Hall et al. NBER innovation project. The red bar is density for VCFS and blue for non-VCFS counterpart. Empirical density suggests that VCFS on average has better patent quality and higher mass on the very right tail.
Source: USPTO and CB Insights.

The figure shows that VCFS patents receive on average about 20 more citations or roughly 0.2 standard deviations than patents by non-VCFS. Appendix figures (Figure A6, A7) report similar patterns when patent quality is measured by originality or generality.

I then focus on the right tail of the distribution. Table 6 reports the share of patents

classified as top-quality, defined as being in the top 10% by each quality measure. VCFS are disproportionately represented in this right tail. For example, about 20% of VCFS patents fall into the top 10% by forward citations, compared with only 9.7% for non-VCFS. The superior innovation outcomes of VCFS persist when quality is measured by originality or generality.

Table 6: Top 10% patent

Financing source	Top 10% citation	Top 10% originality	Top 10% generality
VCFS	20.09%	13.72%	12.5%
non-VCFS	9.73%	9.9%	9.94%
t-statistics	120	42.07	28.23

Note: The percentage represents the fraction of patents by either VCFS or non-VCFS falls into each top 10% group. t-statistics suggests the difference between those fractions are significant across groups.
Source: Data compile from Orbis and CB Insights

Taken together, these results support the model's prediction that VC financing not only raises average patent quality but also shifts mass into the upper tail, consistent with a greater tendency on aggressive innovation strategies.

Having established that the model reproduces the key untargeted moments in the data including greater volatility, wider dispersion, higher leverage among VCFS, and superior patent quality, I now use the calibrated model to conduct counterfactual experiments. These experiments allow me to quantify the aggregate role of VC financing and to explore how alternative financing arrangements would affect startup and economic outcomes. Specifically, I examine scenarios without VC financing, with improved matching efficiency, and with revenue shocks to highlight the mechanisms through which VC reshapes innovation and growth.

4 Counterfactual Experimenter

In this section, I conduct a set of counterfactual experiments. In particular, I evaluate the aggregate impact of venture capital by restricting the overall supply of VC, varying the enforcement power of VC contracts, and changing the matching protocol between startups

and VCs. These experiments assess how contract enforcement and the allocation of VC resources reshape startup dynamics and aggregate outcomes.

First, I consider financial frictions. I begin by shutting down VC investment entirely to assess how insurance against early default shapes the firm distribution. I then examine a case in which VC contracts have stronger enforcement, capturing the role of financial development. Second, I consider search frictions. In both the model and the real world, VC financing is shaped by frictions in matching, whether from information problems or congestion. I study the gains from improving targeting, where VC is allocated exclusively to the startups most in need. This experiment has direct implications for the design of government financing programs.

For each scenario, I compute the stationary equilibrium and compare key moments to the baseline steady state. The main results are summarized in Table 7, and I discuss their intuition and implications in the following subsections.

Table 7: Counterfactual analysis

%	No VC	Perfect Enforcement	Perfect Sorting - VC Match
Large firm†	-11.3	0.9	0.4
Default rate	0.17	-0.04	-0.10
Labor productivity			
Average	-1.4	0.1	0.03
VC only	–	2.1	1.3
Aggregate output	-5.1	0.4	0.2

Notes: Values are percentage changes relative to the baseline model with VC investment and heterogeneous innovation. VC only refers to the labor productivity among VCFS only.
Definition: † "Large firm" denotes a firm that eventually exceeds the 99th percentile of the baseline firm-size distribution.

No venture capital investment

In this counterfactual experiment, I shut down VC financing and require all young startups to rely solely on debt. Column 1 of Table 7 shows that the effect on the creation of large startups is dramatic. Although only 0.2% of startups become VCFS in the baseline calibration, eliminating VC reduces the mass of large startups by 11.3%,a decline roughly fifty times

larger than the VC share itself.

The sharp decline is driven by two mechanisms. First, VCFS are hedged against involuntary exit when they are small and most vulnerable to default risk. Second, VC contracts encourage more frequent adoption of aggressive innovation compared to debt finance. Without VC investment, startups climb the quality ladder more slowly and face a higher risk of premature exit, making it much harder for them to reach the upper rungs. This same mechanism generates a large increase in the exit rate. Although the share of VCFS is small, each of these startups is shielded from default risk. Moreover, with VC financing, startups escape the default region more quickly, which keeps their exit rates lower even after separation from VC funding.

Lastly, I examine the aggregate implications by reporting moments such as aggregate output and labor productivity. Although VC investment typically occurs in the early stages and targets small startups, its aggregate impact is substantial. Shutting down VC financing entirely would reduce aggregate output by 5.1%. The steady state comparison underscores the strong role of VC at the macro level. The mechanism is that VCFS grow out of the risky early stage more quickly, shifting the entire firm distribution to the right. Over the lifecycle, this amplification generates a larger pool of influential startups at later stages, magnifying the aggregate impact of VC investment.

Perfect enforced VC contract

Next, I consider a scenario where VC contracts are perfectly enforceable and startups fully commit to honoring the agreement. This counterfactual reflects an environment with stronger financial development and better legal protection. As reported in Column 2 of Table 7, perfect enforcement yields roughly a 2% labor productivity gain among VCFS. This direct effect arises because, without limited commitment, startups cannot deviate or misappropriate innovation funding, allowing contracts to prescribe higher effort levels. With perfect enforcement, the contract specifies not only greater intensity of effort but also more frequent adoption of aggressive innovation. Both the intensive and extensive margins thus accelerate the growth of VCFS.

The consequences for firm dynamics are notable: the model predicts about 1% more

large startups and 0.04% lower default rate, even though VCFS account for only 0.2% of all startups. At the aggregate level, however, the effects are mild. Aggregate output increases only slightly under this counterfactual. The reason is that the punishment for deviation in the baseline model with financial autarky is already severe enough to discourage opportunistic behavior. In the presence of financial frictions, autarky prevents startups from borrowing against shocks, sharply reducing their continuation value. As a result, the gap between limited-commitment contracts and perfectly enforced contracts is narrower than one might expect, leaving only modest room for additional aggregate gains.

Perfect sorting in VC search

The final counterfactual imposes perfect sorting in the VC search process. In the baseline model, all startups that expect some benefit from VC financing search, creating congestion. As a result, some startups close to the default cutoff end up not searching, since they cannot bear the additional search cost as they are unlikely to match successfully. Consequently, VC resources are not always allocated to the startups that need them most. In this counterfactual, I instead assume that VCs are matched with the startups that yield the greatest value gain from financing, formally defined as:

$$i \in \{\max_j V^{vc}(z_j, \omega_j) - V^d(z_j, \omega_j)\}$$

This lead to the implication on who should get the VC finance and provide the implication on government led program like Small Medium Enterprise Instrument (SMEI) in the EU or Small Business Innovation Research program (SBIR) in the US. The counterfactual suggests that the misallocation of VC resources leads to up to 0.2% lower output. When VC resources is targeted and not randomly matched, the startups at the edge of default can survive into later stage in their lifecycle and generate more firms with better technology levels. It turns out that by allocating VC resource based on startups' leverage, the startup default rate lower by 0.11% which is prominent improvement given that VC investment fraction is 0.2%. Essentially, the startups that are hit with consecutive bad shock and fail in innovation can survive in early phase and keep climbing on the quality ladder with perfect sorting in VC

matching.

The counterfactual exercises highlight the aggregate importance of VC financing, despite its small share of startups. To better understand the mechanisms behind these large effects, it is useful to examine how VC financing differentially affects startups at different stages of their lifecycle. In particular, I next compare the responses of young startups and those closer to the median of the productivity distribution. This contrast illustrates why VC financing generates outsized aggregate gains. By insuring the most vulnerable startups and encouraging them to pursue aggressive innovation, VC not only shapes individual trajectories but also amplifies growth through the survival and scaling of successful startups.

5 Conclusion

This paper examines how VC investment reshapes startups' innovation incentives through a novel insurance channel. I develop a general equilibrium firm dynamics model that incorporates heterogeneous innovation strategies and endogenously selected financing sources to quantify the aggregate implications of VC investment. Calibrated to match observed financing and innovation patterns of VCFS and their non-VC counterparts, the model reveals that even though VCFS account for a small fraction of all firms, eliminating VC investment can lower aggregate output by up to 5 percent through its influence on the distribution of firm sizes and innovation types. By insuring young startups against early exit risk and encouraging aggressive, high-risk-high-return innovation strategies, VC shifts the entire firm productivity distribution toward the right. This effect is amplified over the lifecycle, as early-stage innovation decisions compound into persistent differences in firm growth trajectories.

I validate the model's mechanisms using a novel dataset that merges VC investment records with startups' financial statements and patent data. The model predicts three distinctive empirical patterns: (i) higher pre-financing financial leverage among VCFS due to self-selection in search, (ii) greater post-financing dispersion in profitability as a consequence of heterogeneous innovation outcomes, and (iii) higher average patent quality, with a disproportionate share of VCFS patents in the right tail of the quality distribution. The empirical evidence supports these predictions: VCFS exhibit significantly higher leverage prior to VC

financing, a widening dispersion in profits after receiving VC funding, and systematically superior patent quality—consistent with the adoption of high-risk-high-return innovation strategies.

Taken together, the theoretical and empirical analyses suggest that VC financing does far more than alleviate funding constraints but fundamentally alters the strategic innovation calculus of startups. By reducing the downside risk of aggressive innovation, VC accelerates the emergence of "superstar" firms.

References

Aghion, P., Harris, C., Howitt, P., and Vickers, J. (2001). Competition, imitation and growth with step-by-step innovation. *Review of Economic Studies*, 68(3):467–492.

Aghion, P. and Howitt, P. (1992). A model of growth through creative destruction. *Econometrica*, 60(2):323–351.

Akcigit, U., Dinlersoz, E., Greenwood, J., and Penciakova, V. (2022). Synergizing ventures. *Journal of Economic Dynamics and Control*, 143:104427.

Albuquerque, R. and Hopenhayn, H. A. (2004). Optimal lending contracts and firm dynamics. *The Review of Economic Studies*, 71(2):285–315.

Ando, Y. (2025). Dynamics of high-growth young firms and the role of venture capitalists. CES Working Paper No. 25-38.

Arellano, C., Bai, Y., and Kehoe, P. J. (2019). Financial frictions and fluctuations in volatility. *Journal of Political Economy*, 127(5):2049–2103.

Catalini, C., Guzman, J., and Stern, S. (2019). Hidden in plain sight: Venture growth with or without venture capital. Working Paper 26521, National Bureau of Economic Research.

Chemmanur, T. J., Krishnan, K., and Nandy, D. K. (2011). How does venture capital financing improve efficiency in private firms? A look beneath the surface. *Review of Financial Studies*, 24(12):4037–4090.

Cole, H. L., Greenwood, J., and Sanchez, J. M. (2016). Why doesn't technology flow from rich to poor countries? *Econometrica*, 84(4):1477–1521.

Cornelli, F. and Yosha, O. (2003). Stage financing and the role of convertible securities. *Review of Economic Studies*, 70(1):1–32.

Ewens, M. and Farre-Mensa, J. (2022). Private or public equity? The evolving entrepreneurial finance landscape. *Annual Review of Financial Economics*, 14:271–293.

Ewens, M., Nanda, R., and Rhodes-Kropf, M. (2018). Cost of experimentation and the evolution of venture capital. *Journal of Financial Economics*, 128(3):422–442.

Gompers, P. A. (1995). Optimal investment, monitoring, and the staging of venture capital. *Journal of Finance*, 50(5):1461–1489.

Gompers, P. A., Gornall, W., Kaplan, S. N., and Strebulaev, I. A. (2020). How do venture capitalists make decisions? *Journal of Financial Economics*, 135(1):169–190.

Greenwood, J., Han, P., and Sánchez, J. M. (2022). Financing ventures. *International Economic Review*, 63(3):1021–1053.

Hall, B. H., Jaffe, A. B., and Trajtenberg, M. (2001). The NBER patent citations data file: Lessons, insights and methodological tools. NBER Working Paper 8498, National Bureau of Economic Research.

Hellmann, T. and Puri, M. (2000). The interaction between product market and financing strategy: The role of venture capital. *The Review of Financial Studies*, 13(4):959–984.

Kaplan, S. N. and Strömberg, P. (2003). Financial contracting theory meets the real world: An empirical analysis of venture capital contracts. *Review of Economic Studies*, 70(2):281–315.

Kaplan, S. N. and Strömberg, P. (2004). Characteristics, contracts, and actions: Evidence from venture capitalist analyses. *Journal of Finance*, 59(5):2177–2210.

Kerr, W. R., Lerner, J., and Schoar, A. (2014). The consequences of entrepreneurial finance: Evidence from angel financings. *Review of Financial Studies*, 27(1):20–55.

Kochen, F. (2025). Finance over the life cycle of firms. Working Paper.

Lerner, J. and Nanda, R. (2020). Venture capital's role in financing innovation: What we know and how much we still need to learn. *Journal of Economic Perspectives*, 34(3):237–261.

Olmstead-Rumsey, J. (2022). Market concentration and the productivity slowdown. Working Paper.

Ottonello, P. and Winberry, T. (2020). Financial heterogeneity and the investment channel of monetary policy. *Econometrica*, 88(6):2473–2514.

Puri, M. and Zarutskie, R. (2012). On the life cycle dynamics of venture-capital- and non-venture-capital-financed firms. *Journal of Finance*, 67(6):2247–2293.

Spear, S. and Srivastava, S. (1987). On repeated moral hazard with discounting. *The Review of Economic Studies*, 54(4):599–617.

Sui, X. (2025). Uneven firm growth in a globalized world. Working Paper.

Yu, Y. (2024). Venture capital and the dynamism of startups. Unpublished working paper.

Étienne de Bettignies, J. and Duchêne, A. (2015). Product market competition and the financing of new ventures. *Management Science*, 61(8):1849–1867.

A Numerical algorithm

This section describes the algorithm solving the main model.

- Guess a equilibrium wage rate w_0 and matching probability λ_0
  - Taking the initial guess of wage rate w_0 and matching probability λ_0, solve the profit function $\pi(z)$ for each technology level
  - Compute the bond price schedule $q_h(z, x', b')$ which depends on the technology level, type of innovation, and innovation effort.
  - Solve debt-financed startup's optimal R&D type and level of effort using first order condition
  - Update debt-financed startup's value function and policy function using optimal R&D
  - Solve VC-financed startup's optimal R&D type and level of effort using first order condition
  - Update VC-financed startup's value function and policy function using optimal R&D
- Compute the transition matrix using policy function and derive the invariant distribution
- Update the new wage w_1 to clear the labor market.
- Repeat until converge

A.1 Debt finance detail iteration

In this subsection, I describe the detail algorithm to solve the debt financed startup value.

Define the value of debt financed startup with technology level z and cash on hand ω:

$$
\begin{aligned}
V(z,\omega) = \max_{x',b'} \quad & \text{div} + \beta \sum f_x(z'|z) \int^{\xi^(z',b')} V(z',\omega') f(\xi') d\xi' \\
\text{subject to:} \quad & \text{div} = \omega + q(z,x',b')b' - \kappa(x') \geq 0 \\
& \omega'(\xi', z', b') = \Pi(z') - b' - \xi'
\end{aligned}
$$

where q is bond price schedule. Using the equilibrium zero profit condition of bank, $q(\cdot)$ is given by the repayment probability of each state in the future. Here, ξ^* is the upper bound cutoff where the startup is able to pay back the debt and keep non-negative dividend.

$$
q(z,x',b') = \frac{1}{1+r} \sum f_x(z'|z) F(\xi^*(z',b'))
$$

To solve for the ξ^*, consider following equation which pins down the default cutoff at which the startup is unable to roll-over the debt which satisfies the non-negative dividend condition:

$$
\Pi(z) - b - \xi^* + \max_{x',b'} \{q(z,k',b')b' - \kappa_h(x')\} = 0
$$

Let $\bar{\xi}$ denotes the upper-bound of the i.i.d revenue shock, the system implies:

- If $\Pi(z) - b + \max_{x',b'} q(z,x',b')b' > \bar{\xi}$, then the firm will not default for sure.
- If $\Pi(z) - b + \max_{x',b'} q(z,x',b')b' \leq \bar{\xi}$, then :

$$
\xi^*(z,b) = \Pi(z) - b + \max_{x',b'} q(z,x',b')b'
$$

Now, I formally describe the algorithm to solve the value function, policy function, and bond price schedule for startup financed by debt:

1. Solve for the efficient allocation x' and setup grid for b'. The grid of x' depends on z, and the grid of b' depends on (z, x).

2. Iteratively compute borrowing limit and bond price schedule:

   (a) Guess the maximum borrowing:

$$M^n(z) = \max_{x',b'} \{q(z, x', b')b'\}$$
$$\text{subject to: } q(z, x', b')b' \geq \kappa(x')$$

   (b) Update default cutoff ξ^* and the upper bound of debt holding b^L:

$$b^L = \Pi(z) + M^n(z) - \bar{\xi}$$

   If $b < b^L$, then the firm can repay. In this case:

$$F(\xi^{*\{n+1\}}(z, b)) = 1$$

   For $b > b^L$:

$$\xi^{*\{n+1\}}(z, b) = \Pi(z) - b + \max_{k',b'} \{q(z, x', b')b'\}$$

   (c) Update bond price:

$$q^{n+1}(z, x', b') = \frac{1}{1+r} \sum f_h(z'|z) F(\xi^{*\{n+1\}}(z', b'))$$

   (d) Update $M^{n+1}(z)$ accordingly.

   (e) Repeat steps b) to d) until convergence.

3. Guess $V^n(z, w)$.

4. Construct auxiliary function:

$$G^n(z, x', b') = \sum_{z'} f_h(z'|z) \int^{\xi^*(z',b')} V^n(z', w') f(\xi') d\xi'$$

5. Solve unconstrained problem, this would yield the optimal innovation effort level given current return summarized by function G without worrying non-negative dividend constraint. In particular, the borrowing amount is larger than the R&D cost is the only requirement for efficient policy.

$$\hat{V}(z) = \max_{x', b'} \{-\kappa(x') + q(z, x', b')b' + \beta G^n(z, x', b')\}$$

Define efficient cutoff for cash on hand:

$$\hat{w}(z) = -\left[q(z, \hat{x}', \hat{b}')\hat{b}' - \kappa(\hat{x}')\right]$$

for $w > \hat{w}$, the firm is able to achieve optimal level of innovation without NND constraint binding. In this case:

6. For $w \geq \hat{w}(z)$:

$$\begin{aligned} x'(z, w) &= \hat{x}'(z) \\ b'(z, w) &= \hat{b}'(z) \\ V^{n+1}(z, w) &= w - \kappa(\hat{x}') + q(z, x', b')b' + \beta G^n(z, x', b') \end{aligned}$$

7. Set up $w \in [w_{\min}(z), w_{\max}(z)]$ where:

$$w_{\min}(z) = 0.01, \quad w_{\max}(z) = \hat{w}(z) + 0.03$$

8. Update firm value via grid search:

$$V^{n+1}(z,w) = \max_{x',b'} \text{div} + \beta G^n(z,x',b')$$

Subject to $\text{div} = w - \kappa(x') + q(z,x',b')b' \geq 0$

9. Update $G^{n+1}(z,x',b')$:

$$G^{n+1}(z,x',b') = \sum_{z'} f_h(z'|z) \int^{\xi^*(z',b')} V^{n+1}(z',w') f(\xi') d\xi'$$

Split the integration into two parts and use linear interpolation and Gauss-Legendre quadrature as needed. From here, one can observe the trade-off between dividend and future value. As $x \uparrow$, the dividend goes down but the probability of higher z governs by f_h goes up.

10. Apply Howard's improvement 3 times.

11. Iterate steps 4 to 11 until convergence.

A.2 Optimal VC contract

In this section, I show how VC and VCFS jointly negotiate over the optimal contract characterized by $C^* = C(\lambda^h(s^t), x^h(s^t), d(s^t))$ for $h \in \{a, c\}$. Here, (s^t) is the history of states, $d(s^t)$ is the dividend remained in the startups, $\lambda^h(s^t)$ is the probability of conducting the aggressive innovation, and $x^h(s^t)$ is the innovation effort for aggressive and conservative innovation respectively. To determine the optimal VC contract, I will characterize the frontier of total surplus of the contract, and then specify how do VC and startup negotiate over the contract on the Pareto frontier upon matching. Since the choice objects is infinite horizon objects due to all possible future state trajectory, I follow Spear and Srivastava (1987) and formulate the dual approach in following.

Define the total surplus of the contract by

$$S^C(z,\omega,V^{vc}) = V^{vc} + S^{VC}(z,\omega,V^{vc})$$

where z is the technology level of startups,V^{vc} is the promised value to startups, ω is cash on hand level of startup, and S^{VC} is the surplus for VC. The total surplus of the contract is simply the summation of VCFS's value and the surplus going to VC.

I solve frontier of total surplus of VC contract in two steps. First of all, in each period, given the state of technology and cash on hand of VCFS, and promised value, the frontier of the contract is the sum of expected value by providing mixed strategy with conducting both aggressive innovation and conservative innovation. In particular, the frontier is given by following equation:

$$\begin{aligned} S^C(z,\omega,V^{vc}) = \max_{\lambda^A,V_A,V_C} \lambda^A S^{C,A}(z,\omega,V_A) + (1-\lambda^A) S^{C,C}(z,\omega,V_C) \\ \text{subject to } V^{vc} = \lambda^A V_A + (1-\lambda^A) V_C \end{aligned} \tag{12}$$

Here, λ^A is the probability of conducting the aggressive innovation and V_A, V_C are the promised value if aggressive or conservative innovation in conducted eventually. Essentially, the startup is risk neutral and is indifferent between receiving promised value V for sure and receiving expected promised value equals to V^{vc}.

Next, the frontier of total surplus of contract if the aggressive/conservative innovation is eventually conducted is given by following equations:

$$\begin{aligned} S^{C,h}(z,\omega,V^{vc}) = \max_{x_h,\{V^{vc}(z')\}_{z'\in Z}} x_h \sum_{z'\in Z} f_h(z' \mid z)[\Pi(z') + \beta\delta S^C\left(z',\omega,V^{vc}\left(z'\right)\right)] \\ +(1-x_h)[\Pi(z) + \beta\delta S^C\left(z,\omega,V^{vc}\left(z\right)\right)] - (1+r)\kappa_h(x_h) + \beta\delta \mathbb{E}_{x,z'} V^d(z',\omega) \end{aligned} \tag{13}$$

subject to participation constraint (PC), inventive compatibility constraint (IC), and promise keeping constraints (KP):

$$[PC] : V^{vc} \geq V^d(z,\omega) \tag{14}$$

$$[IC] : V^{vc} \geq V^{aut}(z,\omega) + O(x_h) \tag{15}$$

$$[KP] : V^{vc} \geq \beta\delta[x_h \sum_{z'\in Z} f_h(z' \mid z) V^{vc}(z') + (1-x_h) V^{vc}(z)] \tag{16}$$

Observe the PC constraint will pin down the lower bound of support for V^{vc}. For all the promised value lower than this lower bound, the contract has no value since the startup will also seek debt finance instead. After listing out the full contractual problem, I describe the detail solution in each step.

In the first step of proposing mixed strategy of innovation strategy, the FOC for equation 12 is given by:

$$\frac{\partial S^{C,A}(z,V^A)}{\partial V^A} = \frac{\partial S^{C,C}(z,V^C)}{\partial V^A C} \tag{17}$$

$$\frac{\partial S^{C,A}(z,V^A)}{\partial V^A}(V^A - V^C) = S^{C,A}(z,V^A) - S^{C,C}(z,V^C) \tag{18}$$

when $\lambda^A \in (0,1)$, i.e. the interior solution, the solution V^{*A}, V^{*C} does not depends on V, and λ^{*A} is linear in V^{vc} because of the concavity of $S^{C,A}$ and $S^{C,C}$. In other words, the mixed strategy states that in the interior solution the probability of doing the aggressive innovation is bounded by the promised value.

Next, to solve the frontier when conducting aggressive or conservative innovation, the first order condition of problem 13 is given by following:

$$\begin{aligned}
[x_h]: & \sum_{z'} f(z'|z)[\pi(z') + \beta S^C(z', \omega, V^{vc}(z'))] - [\pi(z) + \beta S^C(z, \omega, V^{vc}(z'))] \\
& - (1+r^f)\frac{\partial \kappa_h(x_h)}{\partial x_h} - \lambda_0 \beta (\sum_{z'} f(z'|z) V^{vc}(z') - V^{vc}(z)) - \lambda_1 \frac{\partial \kappa_h(x_h)}{\partial x_h} = 0 \\
[V^{vc}(z')]: & \beta x_h f(z'|z) \partial_{V^{vc}} S^C(z', \omega, V^{vc}(z')) - \beta x_h f(z'|z) \lambda_0 + \lambda_{z'} = 0
\end{aligned} \tag{19}$$

here, λ_0 is Lagrange multiplier on promise keeping constraints. Intuitively, λ_0 measures the shadow cost of raising the startup's promised continuation value. It summarizes the history-dependent state-contingent promises $V^{vc}(z')$ into a single statistic: once λ_0 is determined, the optimal schedule of continuation utilities across future states is pinned down. Similarly, let λ_1 denote the multiplier on the incentive-compatibility constraint. This multiplier captures the shadow cost of enforcing the prescribed innovation effort and preventing

deviation. It reflects the value of relaxing the enforceability condition rather than the direct marginal return to effort. Because continuation values enter linearly in the Lagrangian, the first-order approach with respect to λ_0 allows the problem to be reduced from a high-dimensional choice over $\{V^{vc}(z')\}$ to a tractable dual representation. Moreover, the fact that the contract value enters $[x_h]$ linearly ensures that V^{vc} does not appear directly in the first-order condition for innovation effort, further simplifying computation. I now describe the detail algorithm solving problem 12 and 13 using first order conditions:

1. For each pair of (z, ω, V^{vc}), solve the un-constrained problem (13) without any constraints. Label the solution $x_h^*, V^*(z') \; \forall \; z' \in Z$ and the associated efficient level surplus $S^{C*}(\cdot)$.

2. Using the PC constraint and find the lower bound of promised value V_{low} and set upper bound of promised value V_{high} to be efficient value of startups $S^{C*}(\cdot)$. This is because the VC will never provide startup up more value than efficient level to have negative surplus for themselves.

3. Start by guess a initial function form for $S^{C,A}(z, \omega, V), S^{C,C}(z, \omega, V)$, i.e. the contract surplus if specifying aggressive or conservative innovation to be conducted.

4. Given the initial guess of $S^{C,A}, S^{C,C}$, compute the derivative of $S^{C,A}$ and S^C with respect to V using forward diff. The forward diff is well defined since $S^{C,A}, S^{C,C}$ is concave function.

5. Solve the solution of λ^A, V^A, V^C if interior using 12, otherwise set $\lambda^A = 1$ and $V^A = C$ for $\lambda^A \geq 1$ and set $\lambda^A = 0$ and $V^C = C$ for $\lambda^A \leq 0$

   (a) The solution sensitive to initial guess of non-linear solver. To guarantee the global solution, I start with grid search for some grid points and find a pair of (z, ω, V) that has interior solution.

   (b) Recall that in the interior, the V^A, V^C does not depend on V^{vc}, so once I solved for the first point, I can back out whole solution for $\{\lambda^A(z, \omega, V), V^A(z, \omega, V), V^C(z, \omega, V)\}$ using relation in 17.

6. Update S^C with the solution of $\{\lambda^A(z,\omega,V^{vc}), V^A(z,\omega,V^{vc}), V^C(z,\omega,V^{vc})\}$

7. Solve the first order condition in 19 with following steps to update $S^{C,A}(z,\omega,VV^{vc})$ and $S^{C,C}(z,\omega,V^{vc})$

   (a) Given the promised value V, check if the efficient solution $x_h^*, V^*(z')$ satisfies IC and KP constraints. If it does, than the solution for constrained problem must coincide with the efficient solution. If not, consider the cases when either one of the constraint binds and the case when both constrain bind.

      i. Only KP binds case: In this case, $\lambda_0 > 0, \lambda_1 = 0$. The non-linear solver starts with finding a positive slope $\hat{\lambda}_0$ and solve along the forward diff for $S^{C,h}$ to get the implied $\hat{V}(z')\ \forall z' \in Z$. Given KP binds, the $\hat{x}_h$ can be solved with KP constraint, and then use the first order condition with respect to x_h to check if the solution found.

      ii. Only IC binds case: Observe that the only choice variable enter IC constraints is x_h. In this case we can simply solve x_h with IC constraint, and the solution for $V(z')$ coincides with $V^*(z')$ due to the fact that $S^C(\cdot)$ is increasing and concave in V.

      iii. Both binding case: In this case, I first solve the $\hat{x}_h$ with IC constraint. The non-linear solver than find the $\hat{\lambda}_0$ and corresponding $\hat{V}(z')$ using the FOC. Then evaluate the solution using PK constraint to verify if solution found.

   (b) Evaluate all three cases above, and the solution yielding maximum surplus will be the final solution to constrained problem. Using the result to update both $S^{C,A}(z,V)$ and $S^{C,C}(z,V)$

8. Iterate step 3-5 until the difference between S^C converges below tolerance level.

B Additional Figures and Tables

This section presents the additional figures for robustness check.

Figures on selection validation

This section provides suggestive validation on non-VCFS selection in the empirical validation section. In particular, I consider a local projection difference-in-difference (LPDiD) to validate the selection. Figure A1 and A2 plot the LPDiD coefficient for revenue and profit respectively. The control group constructed in section 3.2 exhibits similar trend in both revenue and profit before VC finance.

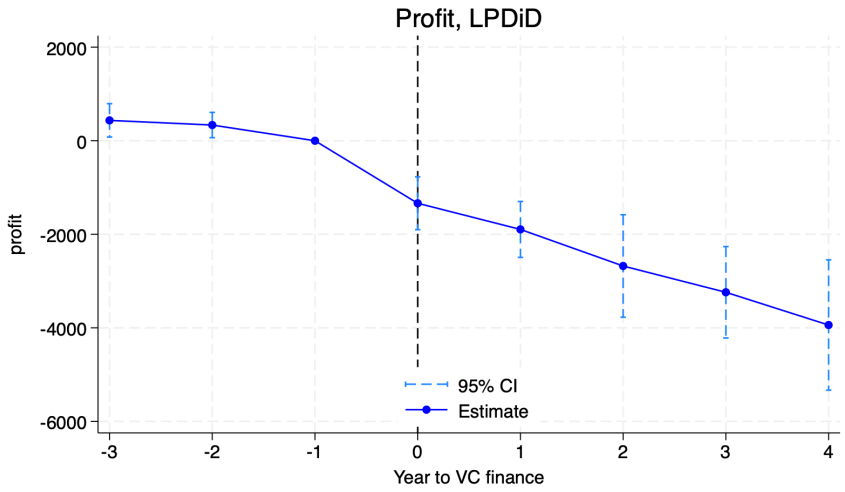


Figure A1: LPDiD, profit

Note: Figure plots the coefficient of local project difference in difference (LPDiD) regression of profit between VCFS and non-VCFS. The result suggests a widening variance in profit after VC finance.
Source: Data compiled from Orbis and CBinsight

Figures on volatility in growth rate

This section provides the suggestive evidence on the widening volatility in growth rate of profit after VC finance. Figure A3 plot the empirical transition matrix for VC and non-VC counterpart among four quartiles measured by profit. The left panel describes the transition among non-VCFS and the right panel describes VCFS. In particular, we see that non-VCFS

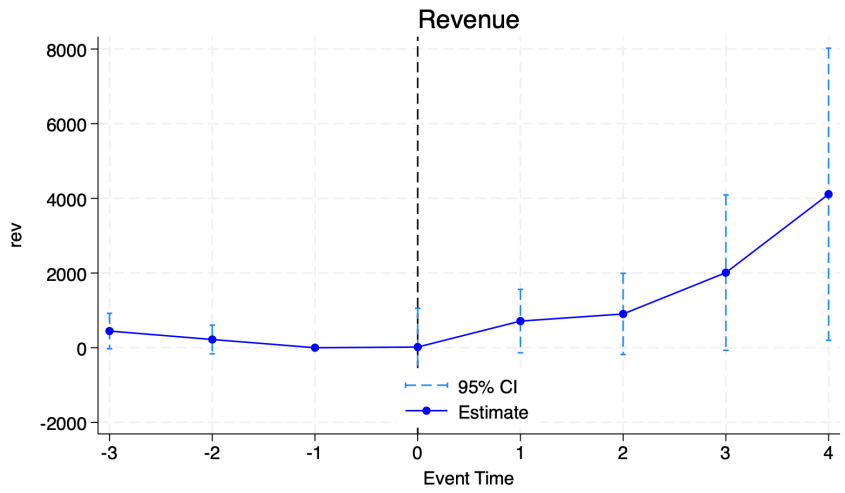


Figure A2: LPDiD, revenue

Note: Figure plots the coefficient of local project difference in difference (LPDiD) regression of revenue between VCFS and non-VCFS. The result suggests a widening variance in revenue after VC finance.
Source: Data compiled from Orbis and CBinsight

are more likely to stay in the same rank in the quartile, i.e. the diagonal element, than transition into different quartile. This provides the suggestion support that non-VCFS, who tends to do more conservative innovation, is more likely to have marginal increment and stay in the same quartile rank. On the other hand, with higher tendency of conducting aggressive innovation, VCFS is more likely to jump around quartile rank.

Figures on cross sectional dispersion

This section provides suggestive evidence on widening dispersion after VC finance. In the main text I report the change of volatility in growth rate, and I report the raw data of profit and revenue with box plot here. The raw data suggests the dispersion measured by IQR widens rapidly after VC finance and is consistent with the model prediction as the result as taking aggressive innovation.

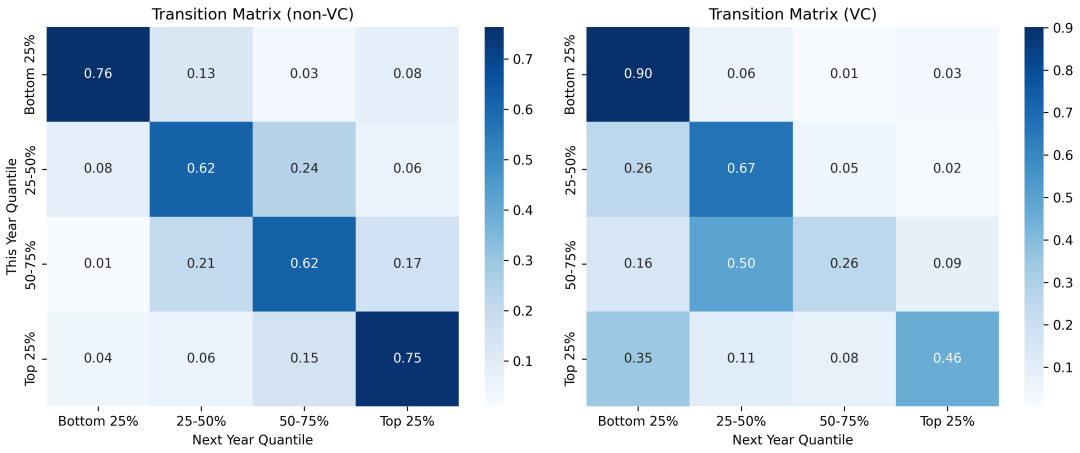


Figure A3: Quartile transition probability, Profit

Note: The figure plots empirical transition matrix along profit quartile rank for VCFS and non-VCFS. I first compute the profit ranking for each firm in each year and compute the transition probability into different quartile in the next period. The VCFS have higher probability of transition into off-diagonal cells which might suggests a highly dispersed performance among VCFS. Source: Data compile from Orbis and CBinsight

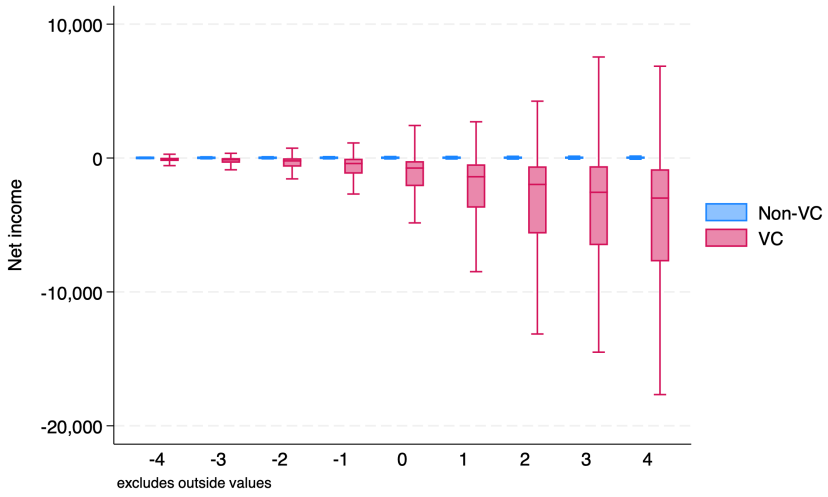


Figure A4: Box plot, profit

Note: Figure plots the 90th, 75th, 50th, 25th, 10th percentile of profit among VCFS and non-VCFS on relative time to reference year. The IQR is the height of box.
Source: Data compiled from Orbis and CBinsight

Figures on additional measure of patent quality

This section provides additional evidence on higher patent quality among VCFS. Following Hall et al. (2001), I construct two standard indices of patent quality. The *originality index*

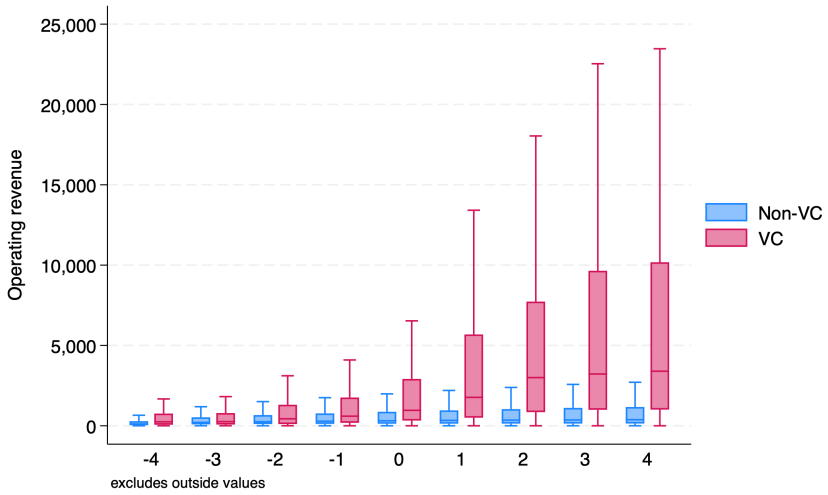


Figure A5: Box plot, revenue

Note: Figure plots the 90th, 75th, 50th, 25th, 10th percentile of revenue among VCFS and non-VCFS on relative time to reference year. The IQR is the height of box.
Source: Data compiled from Orbis and CBinsight

measures how broadly a patent builds on knowledge from different technological fields, while the *generality index* measures how broadly it is used across fields by subsequent patents.

Formally, let s_{ij} denote the share of citations made by patent i to prior patents in technological class j, and let s_{ij}^{f} denote the share of forward citations received by patent i from class j. Both indices are defined as one minus the Herfindahl–Hirschman Index (HHI) of the distribution of citations across classes:

$$\text{Originality}_i = 1 - \sum_j s_{ij}^2, \tag{20}$$

$$\text{Generality}_i = 1 - \sum_j (s_{ij}^{f})^2. \tag{21}$$

A higher originality index indicates that a patent cites prior art from a more diverse set of fields, suggesting the patent combines knowledge across domains. A higher generality index indicates that a patent is cited by subsequent innovations in a wider range of fields, suggesting broader applicability of its knowledge.

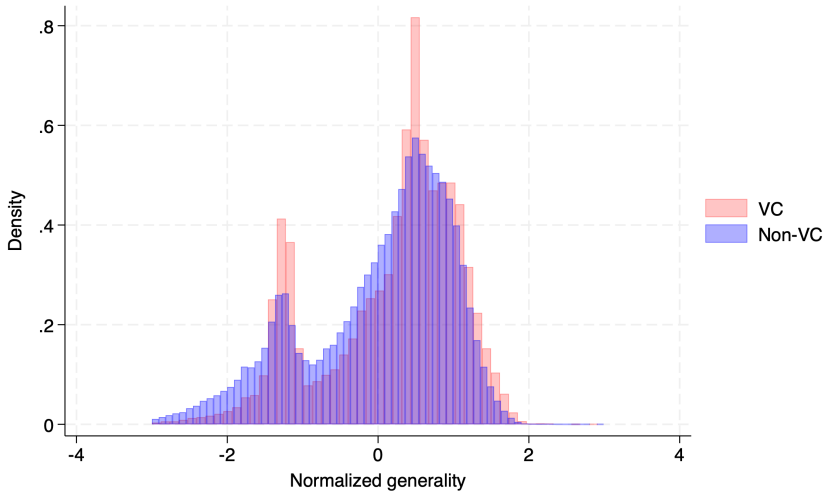


Figure A6: Normalized generality

Note: Figure plots the normalized generality for patent filed by VCFS versus non-VCFS. In particular, each patent's generality is normalized within each year-CPP cohort. Source: Data compiled from USPTO and CBinsight

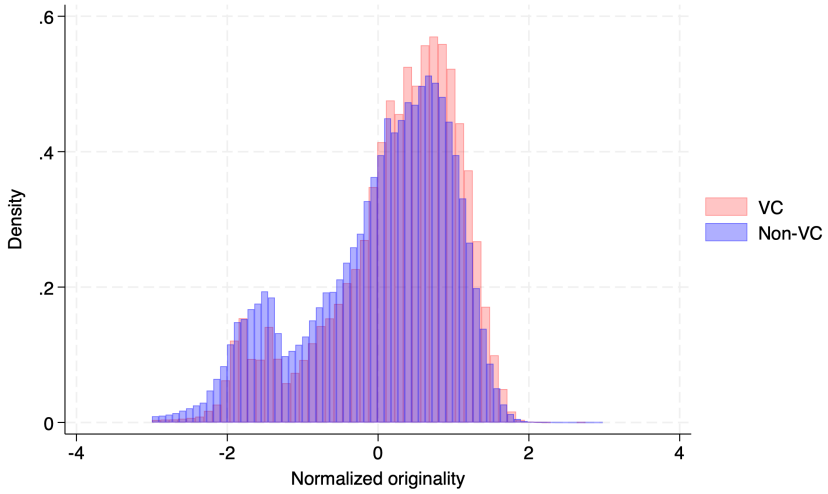


Figure A7: Normalized originality

Note: Figure plots the normalized originality for patent filed by VCFS versus non-VCFS. In particular, each patent's originality is normalized within each year-CPP cohort. Source: Data compiled from USPTO and CBinsight

Tables on comparison between VCFS and non-VCFS

This section provides additional analysis on the comparison between VCFS and non-VCFS. In particular, given the nature of equity-liked financing by VC, the elasticity of innovation

on future productivity and past productivity is different from non-VCFS. For instance, for the non-VCFS relying on debt, the innovation effort depends heavily on the past record of innovation. This is because with better record of successful innovation, the startup is able to cumulate asset and stay away from default region and therefore conduct more innovation. On the other hand, for VCFS the innovation is more correlated with future productivity. This is because the risk neutral VC would lean toward aggressive innovation that yields higher expected future value regardless of past realization of innovation or productivity. Table A1 compare the residual variable correlation after controlling for firm, year fixed effect and some characteristics.

In addition to the correlation, I extend the untargeted moment in the main text beyond the volatility of growth rate between VCFS and non-VCFS. The result is reported in table A2 and A3. In particular, I check the volatility on the level of profit and R&D over sales ratio to support the model mechanism. The model suggests larger volatility in profit because of the tendency of conducting aggressive innovation which is consistent with the data. For the R&D over sales ratio, we also see a huge increase in the volatility, which is driven by the fact that VCFS spends more on R&D while there revenue outcome becomes more volatile after VC finance.

Lastly, regarding to VC finance versus debt finance and equity finance, I follow Kochen (2025) defining equity injection to be negative change in CAPI. I then compare the frequency of using equity finance versus debt finance, and the magnitude of each of those between VCFS and non-VCFS. The result suggests that even those these young startups do issue equity a lot, the magnitude is very small relative to equity finance. In the current model, I implicitly assume the equity issuance cost is prohibitively high for young startups and I argue that the assumption does not depart away from observation where young startups actually issue very limited amount of equity in my sample.

Table A1: Pairwise Correlations of Residuals: Data vs. Model, VC vs. Non-VC

Variable Pair	VC (Data)	VC (Model)	Non-VC (Data)	Non-VC (Model)
Labor Productivity & Profit	0.03	0.25	0.00	0.17
Labor Productivity & Sales	0.32	0.60	0.16	0.43
Labor Productivity & R&D	-0.08	0.29	0.75	0.60
Labor Productivity & Total Debt	0.02	0.02	0.08	0.01
Labor Productivity & R&D/Sales	0.01	0.05	-0.09	0.22
Labor Productivity & Profit/Sales	-0.00	-0.05	0.00	-0.22
Labor Productivity & Profit Growth	0.01	0.01	0.00	0.00
Labor Productivity & Sales Growth	-0.06	0.01	0.00	0.00
Profit & Sales	0.16	0.89	0.13	0.96
Profit & R&D	-0.52	-0.05	0.07	0.06
Profit & Total Debt	-0.06	-0.15	-0.08	-0.58
Profit & R&D/Sales	-0.02	-0.12	-0.04	-0.01
Profit & Profit/Sales	-0.01	0.12	0.00	0.01
Profit & Profit Growth	-0.02	0.08	0.02	0.03
Profit & Sales Growth	-0.06	0.04	0.00	0.01
Sales & R&D	0.11	0.31	0.89	0.25
Sales & Total Debt	0.33	-0.09	0.10	-0.53
Sales & R&D/Sales	0.02	0.16	0.01	0.08
Sales & Profit/Sales	-0.00	-0.16	-0.00	-0.08
Sales & Profit Growth	-0.01	0.07	-0.00	0.02
Sales & Sales Growth	0.04	0.03	0.00	0.01
R&D & Total Debt	-0.12	0.07	0.06	-0.15
R&D & R&D/Sales	0.02	0.96	0.05	0.89
R&D & Profit/Sales	-0.01	-0.96	-0.05	-0.89
R&D & Profit Growth	0.29	0.01	-0.01	-0.00
R&D & Sales Growth	0.23	-0.03	-0.01	-0.02
R&D/Sales & Profit/Sales	-0.86	-1.00	-0.98	-1.00
Profit/Sales & Profit Growth	-0.00	-0.01	0.01	-0.00
Profit/Sales & Sales Growth	0.12	0.03	0.00	0.03
Profit Growth & Sales Growth	0.03	0.00	0.01	0.00

Notes: Correlations rounded to two decimals. Columns 2–3 report correlations for VC firms (data vs. model); columns 4–5 for non-VC firms (data vs. model).

Table A2: Data: Normalized Volatility After VC Finance (Before = 1)

Variable (SD of)	**Non-VC (A/B)**	**VC (A/B)**	**VC / Non-VC Change Ratio**
Profit	1.17	2.10	1.79
R&D / Sales	0.62	1.60	2.58

Note: After/Before refers to the ratio of post-VC to pre-VC dispersion. Column 3 compares relative changes in VC vs. non-VC groups.

Table A3: Model: Normalized Volatility After VC Finance (Before = 1)

Variable (SD of)	**Non-VC (A/B)**	**VC (A/B)**	**VC / Non-VC Change Ratio**
Profit	1.36	2.75	2.02
R&D / Sales	1.80	9.79	5.45

Note: The first two columns report the ratio of volatility (standard deviation) in the post-VC period relative to the pre-VC period (After / Before), for non-VC and VC firms. The third column reports the relative change for VC firms compared to non-VC firms.

Table A4: Equity and Debt Issuance: Frequency and Size

	Before		After	
	Non-VC	VC	Non-VC	VC
Panel A. Equity issuance				
Frequency (mean)	0.45 (0.36)	0.65 (0.36)	0.53 (0.26)	0.71 (0.26)
Size (mean)	24.54 (559)	48.93 (680)	28.12 (1923)	395.47 (8499)
Panel B. Debt issuance				
Frequency (mean)	0.25 (0.43)	0.43 (0.5)	0.24 (0.42)	0.43 (0.5)
Size (mean)	673.94 (8072)	297.90 (1685)	565.55 (26023)	1103.29 (10054)

Notes: Table reports mean values with standard deviations in parentheses. "Before" refers to the pre-financing period; "After" refers to the post-financing period. Equity issuance is based on changes in share capital (CAPI); debt issuance is based on changes in total debt.

C Planner's Allocation and Wedge Decomposition

This section characterizes a frictionless benchmark in which a benevolent planner allocates innovation choices across startups in order to maximize household welfare. The planner internalizes resource costs and exit risk but faces the same technological environment and innovation technology as the decentralized economy. I then compare the planner's first–order conditions to those in the decentralized allocation to obtain wedges for (i) innovation effort, (ii) innovation type, aggressive vs. conservative, and (iii) financing/exit. Finally, we discuss implementability via state-contingent equity-like contracts and the role of commitment frictions.

C.1 Environment and objective

Time is discrete and the measure of potential product lines is normalized to one. For each line, a startup is characterized by a technology level $z \in \mathcal{Z}$. Within the period, given z, the startup can (i) select an innovation type $h \in \{a, c\}$ and an effort $x \in [0, 1]$, and (ii) produce using the realized post-innovation quality. The innovation technology is a Poisson arrival with intensity x; upon success the next quality z' is drawn from the type-specific distribution $f_h(z' \mid z)$; upon failure the line remains at z. The resource cost of innovation is

$$\kappa_h(x; z) \equiv \kappa_{h,v} \frac{x^2 \zeta}{\kappa_\zeta} + \kappa_{h,f},$$

where $\zeta \equiv \zeta(z)$ indexes the technology-dependent marginal difficulty, $\kappa_{h,v} > 0$ and $\kappa_{h,f} \geq 0$.

Product-market competition is monopolistic with CES demand; we take the static profit function $\pi(z)$ as given from the static problem. Households supply labor inelastically and own firms. There is no aggregate saving so maximizing lifetime consumption is equivalent to maximizing the present value of net output.[7]

Let G_t denote the cross-sectional distribution of quality across lines at the beginning of t. The planner chooses measurable allocation rules for each z: innovation type $h_t(z)$ and

[7] All quantities are in units of the final good at date t; the planner discounts with factor $\beta \in (0, 1)$.

effort $x_t(z)$ to maximize

$$\max_{\{h_t(\cdot),x_t(\cdot)\}_{t\geq 0}} \sum_{t=0}^{\infty} \beta^t \int \left[\pi(z) - \kappa_{h_t(z)}(x_t(z);z)\right] \, dG_t(z),$$

subject to the law of motion for G_t implied by the innovation technology and the type-specific transition kernel $f_h(\cdot \mid \cdot)$. Because the static production problem is separable and there are no aggregate shocks, the planner's problem decomposes line by line. We therefore characterize the value function.

C.2 Planner's problem

Define $S^P(z)$ as the planner's value for a product line at quality z:

$$S^P(z) = \max_{h\in\{a,c\},\ x\in[0,1]} \Big\{\pi(z) \ - \ \kappa_h(x;z) + \beta\big[x\,\mathbb{E}_{z'|z,h}S^P(z') + (1-x)\,S^P(z)\big]\Big\}. \tag{22}$$

Equation (22) reflects the fact that the arrival rate of a successful innovation equals x; with success the line jumps to z' according to $f_h(\cdot \mid z)$, otherwise it stays at z.

FOC for innovation effort. Let $\Delta_h^P(z) \equiv \mathbb{E}_{z'|z,h}S^P(z') - S^P(z)$ denote the expected continuation value gain from success relative to failure under type h at z. When the optimal x is interior, the FOC is

$$\kappa_h'(x;z) \ = \ \beta\,\Delta_h^P(z). \tag{23}$$

Because $\kappa_h(\cdot;z)$ is convex in x, (23) pins down a unique $x_h^*(z)$ whenever $\Delta_h^P(z) > 0$; if $\Delta_h^P(z) \leq 0$ the planner sets $x_h^*(z) = 0$.[8]

Type choice. For each type h, let the type-specific objective be

$$\Phi_h(z) \ \equiv \ \max_{x\in[0,1]} \Big\{\pi(z) - \kappa_h(x;z) + \beta\left[x\,\mathbb{E}_{z'|z,h}S^P(z') + (1-x)S^P(z)\right]\Big\}.$$

[8]Convexity follows from the quadratic term in x; the fixed cost $\kappa_{h,f}$ drops out of the marginal condition but affects the type choice and the extensive margin $x > 0$.

The planner chooses $h^*(z) \in \arg\max_{h\in\{a,c\}} \Phi_h(z)$. Using (23), the inner optimum satisfies $x_h^*(z)$ such that $\kappa_h'(x_h^*; z) = \beta\, \Delta_h^P(z)$; substituting back yields

$$\Phi_h(z) \;=\; \pi(z) - \kappa_h(x_h^*; z) + \beta\,[x_h^*\,\Delta_h^P(z) + S^P(z)].$$

The type choice compares the net surplus $\pi(z) - \kappa_h(x_h^*; z) + \beta x_h^* \Delta_h^P(z)$ across h.

Bellman equation in gains form. Rearranging (22) gives a convenient form:

$$(1-\beta)\, S^P(z) \;=\; \pi(z) \;-\; \kappa_{h^*}(x_{h^*}^*; z) \;+\; \beta\, x_{h^*}^*(z)\, \Delta_{h^*}^P(z). \tag{24}$$

Equations (23)–(24) characterize the planner's allocation.

C.3 Decentralized allocation and wedges

I now compare (23) to the decentralized FOCs to obtain wedges. In the decentralized economy, startups (i) face a non-negative dividend (limited liability) constraint when debt-financed, inducing default/exit risk and a state-contingent bond price schedule $q_h(z, x, b')$, and (ii) under VC finance, bargain over a long-term state-contingent contract subject to promised-value and enforceability constraints. These frictions shift the marginal conditions away from (23).

Debt-financed startups (DFS). Let $V^d(\cdot)$ denote the DFS value function and $\lambda_D \geq 0$ the multiplier on the non-negative dividend constraint. The interior FOC for effort (holding the type h fixed) takes the form

$$\kappa_h'(x; z) \;+\; \underbrace{\lambda_D\big[\kappa_h'(x; z) - b'\,\partial_x q_h(z, x, b')\big]}_{\tau_x^D(z)\text{ (innovation wedge under debt)}} \;=\; \beta\, \Delta_h^D(z), \tag{25}$$

where $\Delta_h^D(z) \equiv \mathbb{E}_{z'|z,h} V^D(z') - V^D(z)$. Comparing (25) to the planner's (23) defines the *innovation effort wedge*

$$\tau_x^D(z) \equiv \lambda_D \big[\kappa_h'(x;z) - b'\, \partial_x q_h(z,x,b')\big] \;\geq\; 0 \quad \text{whenever} \quad \lambda_D > 0,$$

which captures (i) the shadow value of on-path resources when close to default and (ii) the adverse feedback from riskier projects on the borrowing rate via $\partial_x q_h < 0$ in states where success variance raises default risk. The type choice h under debt further embeds a *selection wedge*:

$$\tau_h^D(z) \;\equiv\; \Big[\kappa_a(x_a^*;z) - \kappa_c(x_c^*;z)\Big] \;-\; \beta\Big[x_a^*\,\Delta_a^D(z) - x_c^*\,\Delta_c^D(z)\Big] \;-\; \Big(\text{planner's counterpart}\Big),$$

which is typically > 0 in low-cash states because aggressive projects both cost more at the margin and worsen rollover risk.

VC-financed startups (VCFS). Write the total contract surplus (VC + startup) as $S^C(z,V)$ when the promised value to the startup is V. With one-sided limited commitment, promised-value (PK) and enforceability (IC) constraints bind in different regions. The FOC for x on the surplus frontier can be written as

$$\kappa_h'(x;z) \;+\; \underbrace{\Lambda_{\text{IC}}\,\kappa_h'(x;z) \;+\; \Lambda_{\text{PK}}\,\beta\big[\mathbb{E}_{z'|z,h} V(z') - V(z)\big]}_{\tau_x^{VC}(z)\ \text{(innovation wedge under VC)}} \;=\; \beta\,\Delta_h^C(z), \tag{26}$$

where $\Delta_h^C(z) \equiv \mathbb{E}_{z'|z,h} S^C(z', V(z')) - S^C(z, V(z))$, and $\Lambda_{\text{IC}}, \Lambda_{\text{PK}} \geq 0$ are the multipliers on IC and PK. Relative to the planner, VC finance *removes* default/exit risk and the interest-rate externality ($\partial_x q$), but innovation effort remains below the planner whenever PK or IC bind ($\tau_x^{VC} > 0$). Intuitively, the contract cannot (i) extract arbitrary future surplus to back-load transfers (PK), nor (ii) force action when the startup can walk away with the innovation budget (IC).

Financing/exit wedge. Define the *financing wedge* as the expected-value loss from inefficient exit in DFS:

$$\tau_b(z) \;\equiv\; \beta\big[\mathbb{E}_{z'|z,h}S^P(z') - \mathbb{E}_{z'|z,h}S^P(z')\cdot 1\{\text{no exit}\}\big] \;\geq\; 0,$$

which vanishes under VC on-path (no default) but is positive under debt when rollover fails.

C.4 Implementability and efficiency benchmark

I next state a benchmark implementability result.

Proposition 1 (First-best implementability with full commitment)**.** *If (i) the financier can commit to a long-term state-contingent equity-like contract, (ii) the startup cannot divert resources (IC slack), and (iii) intertemporal transfers can be freely back-loaded (PK slack), then there exists a contract that implements the planner's allocation* $\{h^*(z), x^*(z)\}_{z\in\mathcal{Z}}$ *on-path and attains* S^P*.*

Sketch. Construct transfers so that the startup's continuation value equals $V(z) \equiv S^P(z) - S^{VC}(z)$ at every z, with S^{VC} the financier's residual surplus, and set the contract's prescribed type and effort to $(h^*(z), x^*(z))$. PK slack allows any redistribution of surplus across histories; IC slack implies the startup weakly prefers compliance; hence the prescribed (h^*, x^*) and continuation values satisfy both the startup's and financier's incentives. □

Under one-sided commitment and diversion risk, the constraints generate $\tau_x^{VC} > 0$ in regions where PK or IC bind, implying $x^{VC}(z) < x^*(z)$ and—because aggressive a has both higher variance and fixed cost—more frequent selection of c relative to the planner near low z or low promised values.

C.5 Mechanism implications and testable predictions

The wedges summarize the mechanism: (i) under debt, a tight non-negative dividend constraint raises λ_D and amplifies τ_x^D and τ_h^D, lowering both effort and the propensity to choose

a in low-cash states, and inducing inefficient exit (positive τ_b); (ii) under VC, $\tau_b = 0$ on-path, and the removal of $(\partial_x q)$ restores aggressive choices and higher x relative to DFS, but $\tau_x^{VC} > 0$ persists where PK/IC bind, keeping x below the planner.

Empirically, these wedges map to: (1) higher pre-financing leverage among VCFS (self-selection into insurance); (2) a widening dispersion of profits post-financing (greater adoption of high-variance a and higher x); (3) right-tail shifts in patent quality among VCFS; and (4) lower early-phase exit hazards under VC.

D Proof of Contraction of the VC Contract

State, Controls, and Ladder Dynamics

Time is discrete. Inside the VC contract, the startup's state is

$$s \equiv (z, \omega) \in \mathcal{S} := \mathcal{Z} \times \Omega,$$

where z is the position on a discrete quality ladder and ω is cash on hand at the beginning of the period. The ladder $\mathcal{Z}$ is a finite or countable discrete set (the rungs), and $\Omega \subset \mathbb{R}_+$ is compact and Borel.

The VC chooses a control tuple

$$(h, x, d, v'(\cdot)) \in \mathcal{A}(s),$$

where: (i) $h \in \mathcal{H} := \{c, a\}$ denotes innovation type (conservative vs. aggressive), (ii) $x \in \mathcal{X}_h \subset [0, 1]$ is the innovation effort (interpreted as the success probability), (iii) $d \in \mathcal{D} \subset \mathbb{R}_+$ is the transfer from VC to the startup, and (iv) $v'(\cdot) : \mathcal{S} \to \mathcal{V}$ is a Borel-measurable continuation-utility schedule, with $\mathcal{V} \subset \mathbb{R}$ compact.

Ladder dynamics. Given (z, h, x), next period's ladder position z' is generated by a mixture:

$$\Pr\{z' = k \mid z, h, x\} \;=\; (1 - x)\, \mathbf{1}\{k = z\} \;+\; x\, f_h(k \mid z),$$

where $f_h(\cdot \mid z)$ is a probability mass function over $\mathcal{Z}$ describing the distribution of successful step sizes for type h. Thus, with probability $1 - x$, innovation fails and the startup remains at z; with probability x, innovation succeeds and z' is drawn from $f_h(\cdot \mid z)$.

Cash dynamics. Cash on hand evolves according to

$$\omega' \;=\; \Phi\big(\omega, z; h, x, d, \varepsilon\big) \in \Omega, \tag{27}$$

where ε denotes shocks (e.g., revenue or cost). The map Φ is continuous in (ω, h, x, d) for each z and measurable in ε, and ensures $\omega' \in \Omega$ almost surely.

Joint transition. The law of motion for $s' = (z', \omega')$ is defined by the mixture for z' and the mapping Φ for ω', conditional on (s, h, x, d).

Per-period payoffs. The startup's instantaneous utility is $u(d)$, where $u : \mathbb{R}_+ \to \mathbb{R}$ is bounded, continuous, and increasing. The VC's payoff is $\Pi(s; h, x, d)$, bounded and continuous on its domain, capturing residual project returns net of transfer d. Both parties discount at $\beta \in (0, 1)$.

Limited commitment and outside option. At the start of next period in state s', the startup can walk away and obtain outside option $B(s')$, with $B : \mathcal{S} \to \mathbb{R}$ bounded and measurable.

Feasibility and Contract Constraints

A feasible contract in state $s = (z, \omega)$ with promised utility $v \in \mathcal{V}$ is a choice $(h, x, d, v'(\cdot))$ satisfying:

(i) **State transition:** s' is generated by the ladder mixture and cash mapping (27), so $s' \in \mathcal{S}$ almost surely.

(ii) **Promise-keeping (PK):**

$$u(d) \;+\; \beta\,\mathbb{E}\big[\,v'(s')\,\big] \;\geq\; v, \tag{28}$$

where the expectation is taken under the induced law of motion for s' given (s, h, x, d).

(iii) **Limited commitment (LC):**

$$v'(s') \;\geq\; B(s') \qquad \text{for all } s' \text{ in the support of the transition from } (s, h, x, d). \tag{29}$$

Promised-Utility Bellman Operator

Let $J(v, s)$ denote the VC's maximal value when the startup must receive promised utility v today in state s. The Bellman operator is

$$\big(\mathcal{T}J\big)(v, s) \;=\; \sup_{(h,x,d,v'(\cdot))\in\mathcal{A}(s)} \big\{\Pi(s; h, x, d) \;+\; \beta\,\mathbb{E}\big[\,J\big(v'(s'), s'\big)\,\big]\big\} \tag{30}$$

subject to (28)–(29) and the discrete ladder/cash transitions. The continuation schedule $v'(\cdot)$ is a control and must be Borel-measurable with codomain $\mathcal{V}$.

Assumptions

(A1) **Discounting:** $\beta \in (0, 1)$.

(A2) **State and promise sets:** $\mathcal{Z}$ is finite or countable discrete; $\Omega \subset \mathbb{R}_+$ and $\mathcal{V} \subset \mathbb{R}$ are compact and Borel.

(A3) **Action sets:** $\mathcal{H} = \{c, a\}$ is finite; $\mathcal{X}_h \subset [0, 1]$ compact; $\mathcal{D} \subset \mathbb{R}_+$ compact.

(A4) **Ladder transition:** For each (z, h), $f_h(\cdot \mid z)$ is a probability mass function on $\mathcal{Z}$. The induced kernel

$$\Pr\{z' = \cdot \mid z, h, x\} = (1 - x)\,\delta_z(\cdot) + x\,f_h(\cdot \mid z)$$

is measurable in (h, x) for each z.

(A5) **Cash transition:** $\omega' = \Phi(\omega, z; h, x, d, \varepsilon) \in \Omega$, with Φ continuous in (ω, h, x, d) for each z and measurable in ε.

(A6) **Payoffs and outside option:** $u(\cdot)$ and $\Pi(s; h, x, d)$ are bounded and continuous in their arguments; $B(s)$ is bounded and measurable.

(A7) **Feasibility:** For each s, the set $\mathcal{A}(s)$ is nonempty, compact-valued, and upper hemi-continuous in s, with $v'(\cdot)$ restricted to $\mathcal{V}$ and Borel measurable.

Contraction and Uniqueness

Let $\mathcal{B}(\mathcal{V} \times \mathcal{S})$ be the Banach space of bounded real-valued functions on $\mathcal{V} \times \mathcal{S}$ with sup norm $\|J\|_\infty := \sup_{(v,s)} |J(v,s)|$.

Lemma 1 (Monotonicity)**.** *If $J_1 \leq J_2$ pointwise, then $\mathcal{T}J_1 \leq \mathcal{T}J_2$.*

Lemma 2 (β-Discounting)**.** *For any constant $c \in \mathbb{R}$, $\mathcal{T}(J + c) = \mathcal{T}J + \beta c$.*

Proposition 2 (Contraction and Uniqueness)**.** *Under (A1)–(A7), $\mathcal{T}$ is a contraction on $\mathcal{B}(\mathcal{V} \times \mathcal{S})$ with modulus β. Hence there exists a unique fixed point J^*, and value iteration $J_{n+1} = \mathcal{T}J_n$ converges to J^* from any bounded initial guess.*

Proof. Monotonicity and β-discounting (Blackwell's conditions) imply

$$\|\mathcal{T}J_1 - \mathcal{T}J_2\|_\infty \leq \beta \, \|J_1 - J_2\|_\infty.$$

Therefore $\mathcal{T}$ is a contraction with modulus $\beta \in (0, 1)$.

Existence of Optimal Measurable Policies

By Berge's maximum theorem (applied state-by-state) and measurable selection results:

Proposition 3. *For each (v, s), the argmax correspondence of (30) is nonempty, compact-valued, and admits Borel-measurable selections. There exist Markov policies $(h, x, d, v'(\cdot))$ attaining the supremum. The fixed-point value J^* is bounded and continuous in s and weakly concave in v.*