# Private Credit and the Tailoring of Loan Contracts

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**Abstract**

We document three facts about corporate lending from banks and private credit funds. First, compared to banks, funds rely more on equity for their financing and retain a significantly larger share of originated loans on their balance sheet. Second, private credit loans are more tailored. Third, lower-quality firms—those that are more volatile and less profitable—are more likely to borrow from private credit and receive more tailored contracts. We rationalize these facts in a model where banks and private credit compete to attract heterogeneous firms by offering loan contracts with an endogenous degree of tailoring. Leverage constraints lead banks to specialize in standardized loans, which are easier to securitize. Lower-quality firms value tailoring more and therefore sort disproportionately into private credit. The model predicts that most private credit borrowers could obtain a bank loan, but choose fund financing because tailored contracts mitigate financial frictions. We then use policy changes to validate model predictions about how bank leverage requirements and securitization costs affect competition between banks and funds, firm sorting, and equilibrium contract tailoring.

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# 1 Introduction

Direct lending to firms by investment funds, often referred to as *private credit*, has grown substantially over the past decade. A growing literature studies both the forces behind this expansion and the fundamental differences between banks and private credit lenders. Yet a central question remains: what drives the allocation of firms between banks and private credit lenders, and why do the contracts they receive differ so markedly in their degree of tailoring?

This question is particularly salient given the rapid expansion of private credit and its distinctive emphasis on bespoke (tailored) financing. Private credit funds assets under management have grown from just a few hundred billion dollars in the early 2000s to more than $2.5 trillion today, with direct lending now accounting for a large share of new corporate borrowing (Avalos, Doerr, and Pinter, 2025). Borrowers and private equity sponsors increasingly report choosing private credit not only for speed and certainty of execution but also for highly tailored structures—such as PIK features and bundled debt-equity claims—that are difficult to arrange through syndicated bank markets.[1] At the same time, banks remain the dominant providers of corporate credit and continue to serve many of the same broad classes of borrowers. Understanding the economic boundary between these two forms of intermediation is therefore important both for corporate finance and for the changing structure of the financial system.

In this paper, we develop and test a unified framework in which differences in funding structure, origination costs, and regulation across intermediaries endogenously generate both the allocation of firms across lender types and the complexity of the contracts they receive. Our central idea is that banks and private credit funds differ not only in the cost and regulation of their balance sheets, but also in the types of contracts they can efficiently originate, hold, and distribute. Banks, because they are more highly levered and more reliant on funding models that favor standardized and distributable claims, have a comparative advantage in relatively simple lending. Private credit funds, by contrast, rely more heavily on equity financing and retain a larger share of loans on balance sheet, making them better suited to originate and hold more tailored contracts.

We begin by documenting three facts about bank and private credit lending. First, private credit funds retain a substantially larger share of originated loans on their bal-

[1] Such reporting is common in industry-facing articles, for example https://www.institutionalinvestor.com/article/2bswsphkclc75samrsikg/portfolio/private-equity-funds-fuel-growth-in-private-credit and https://www.chronograph.pe/unpacking-the-structural-drivers-of-private-credit-growth/.

ance sheet than banks, which more often securitize and distribute exposures. Second, private credit contracts exhibit more non-standard, tailored features than bank loans. Third, riskier and less profitable firms are more likely to borrow from private credit and to receive more tailored contracts. Taken together, these facts suggest that the boundary between bank and private credit lending is shaped not only by who can fund a borrower, but also by who can efficiently provide borrower-specific contract design.

We rationalize these facts in a model in which banks and private credit funds compete for heterogeneous firms by offering loan contracts with an endogenous degree of tailoring. Tailoring improves firm outcomes by mitigating financing frictions, but it also makes loans harder to standardize and thus securitize. Because leverage constraints make it more costly for banks to retain such loans on balance sheet, banks optimally specialize in simpler, more standardized lending. Private credit funds, which are less levered and more oriented toward hold-to-maturity lending, specialize instead in contracts with greater borrower-specific tailoring. Firms facing greater financial frictions value such tailoring more and therefore sort disproportionately into private credit.

The model delivers a sharp implication for how observed differences between bank and private credit borrowers should be interpreted. Although private credit borrowers are observably riskier and receive more customized contracts, most of them could still obtain a bank loan in the absence of private credit funds. The value created by private credit arises primarily not from lending to firms that are entirely excluded from bank credit, but from offering more suitable, tailored contracts to firms that would otherwise borrow from banks on less appropriate terms. In this sense, our framework cautions against the interpretation that the risk differences between bank and private credit borrowers imply that the two forms of lending are only weak substitutes. Instead, banks and private credit funds may compete for many of the same firms while differentiating themselves through contract design.

We then take the model to the data. The models mechanisms map directly into empirical predictions for how intermediary constraints should affect lender competition, firm sorting, and equilibrium contract design. In particular, tighter bank capital or leverage constraints should push banks toward more standardized lending and create room for private credit to expand in more tailored segments of the market. Likewise, a decline in securitization costs should strengthen banks comparative advantage in standardized loans, reshaping which borrowers are served by banks versus funds and how much tailoring is observed in equilibrium. We test these predictions using policy variation that shifts bank balance-sheet capacity and securitization incentives, and ex-

amine how these shocks affect lending by banks and private credit funds, the types of firms each serves, and the complexity of the contracts they offer.

**Related Literature.** Our paper relates first to the growing literature on private credit and direct lending. Existing work documents the rapid expansion of private debt markets, describes their institutional structure, and studies why firms borrow from nonbanks rather than traditional banks. For example, Chernenko, Erel, and Prilmeier (2022) show that nonbank lenders serve observably riskier middle-market firms, while Ivashina (2025) emphasize the broader role of private debt in the financial ecosystem and its emergence as an important substitute for bank-intermediated credit. Consistent with recent empirical findings such as Jang, Kim, and Sufi (2025), our framework predicts that private credit funds lend to firms that differ systematically from bank borrowers in their risk profile and, additionally, that they receive substantially more customized contracts. Our contribution is to show that these observed differences need not imply that private credit primarily serves firms excluded from bank lending. In our counterfactual without private credit institutions, the vast majority of fund borrowers could still obtain a bank loan. The value created by private credit instead arises mainly from offering tailored loan solutions to firms that would otherwise borrow from banks, rather than from lending only to firms that cannot access bank credit.

Our analysis also connects to recent empirical work on how private credit lending technology differs from bank lending. Recent evidence suggests that direct lenders rely on intensive monitoring, covenant-based control, and relationship-style lending, and that borrowers value the flexibility and customization offered by these arrangements. Jang (2025) and Block et al. (2024) provide evidence on these distinctive lending technologies and on how private debt funds position themselves relative to banks and syndicated lending markets. We build on this literature by treating tailoring as an equilibrium object: rather than taking contractual differences as given, we study how intermediary balance-sheet structure, securitization capacity, and regulation jointly shape the degree of contractual tailoring that emerges in equilibrium.

More broadly, the paper speaks to work on the interaction between bank regulation and nonbank credit supply. A growing literature shows that tighter constraints on banks can shift credit intermediation toward private credit vehicles, including BDCs, and can reshape the transmission of shocks through bank–nonbank funding chains. Chernenko, Ialenti, and Scharfstein (2025) show that private credit funds are substantially less leveraged than banks, arguing against the view that private credit growth is driven simply by higher leverage outside the banking system. Our mechanism is

consistent with this evidence, but shows that bank leverage regulation can still be an important driver of private credit growth. In our framework, funds optimally rely more on equity than banks, while tighter bank leverage constraints make it more costly for banks to retain loans on balance sheet and push them toward securitizing more standardized loans. This creates room for private credit lenders to expand precisely in the provision of more tailored loan contracts. In this sense, our paper complements the existing empirical literature by clarifying how differences in intermediary balance-sheet structure and regulation shape not just credit quantities, but also firm sorting and contract design.

Finally, our paper connects to broader theories of financial intermediation that emphasize how lenders' funding structures and lending technologies shape their comparative advantage across borrowers and contract types (Kashyap, Rajan, and Stein, 2002; Boot and Thakor, 2000). In particular, Kashyap, Rajan, and Stein (2002) highlight how banks' balance-sheet structure gives them a distinctive funding advantage, while Boot and Thakor (2000) emphasize how intermediaries can differentiate themselves through lending technologies and the provision of more relationship-oriented financing. We bring these perspectives to a setting with competing banks and private credit funds, and show how differences in funding structure and regulation jointly determine firm sorting and equilibrium contract tailoring.

# 2 Institutional Setting and Motivating Facts

Private credit refers to direct corporate lending by nonbank investment vehicles, typically outside public debt and broadly syndicated loan markets. Relative to banks, private credit lenders rely less on deposit funding, operate under a different regulatory regime, and often specialize in more bespoke, hold-to-maturity lending. These institutional differences make private credit a natural setting for studying how intermediary funding structure and regulation shape both lender-borrower matching and contract design.

Because much of private credit operates through opaque private vehicles, we focus empirically on publicly traded Business Development Companies (BDCs), a prominent and observable segment of the market. Unlike most private credit funds, these publicly traded BDCs disclose relatively rich information on balance sheets, portfolio holdings, and loan characteristics in their 10-Ks; we use this data, as cleaned by Robinson and Wallskog (2025). The observability makes BDCs a tractable empirical laboratory for comparing private credit with bank lending. At the same time, BDCs

do not represent the entire private credit universe, so our empirical results should be interpreted as speaking most directly to this visible segment of the market.

We compare the investment behavior of BDCs to that of banks whose lending appears in DealScan, which covers large bank lending to companies. We restrict to DealScan deals in which lenders are U.S. banks, borrowers are U.S. firms, and the deal is marked as an origination rather than an amendment.

Against this institutional backdrop, we document three facts about bank and private credit (i.e., BDC) lending that motivate the model and guide the empirical analysis.

**Fact 1: Banks and private credit funds differ in leverage and loan retention.** Banks are substantially more levered than private credit funds. As documented in Matvos, Piskorski, and Seru (2026), private credit funds typically operate with assets-to-equity ratios of between 1.25 and 1.5, whereas for banks the corresponding ratio is around 9, suggesting banks operate with around six times more leverage.

This difference remains stark even when focusing on public BDCs, which we use as a lens to study private credit. BDCs are subject to a regulatory limit that caps debt-to-equity at 2:1 (and 1:1 prior to a 2018 reform), the U.S. banking sector operates with non-equity liabilities of around 9 times equity. Thus, even relative to the maximum permitted leverage of a BDC, banks operate with more than four times as much leverage.[2]

Public BDCs also raise equity much more frequently. For instance, on average, banks issue equity only 0.03 times per year, while the average BDC issues equity 0.25 times per year. Conditional on issuing equity in a seasoned equity offering (SEO), BDCs issue stock equivalent to 11% of their assets, while banks only issue 2% of their asset value.[3] Moreover, Avalos, Doerr, and Pinter (2025) document that BDCs also have a lower cost of equity than banks. Indeed, greater equity issuance by BDCs and their lower cost of equity is consistent with the fact that BDCs operate with significantly less leverage than banks.

Finally, in relation to loan retention, Davydiuk, Marchuk, and Rosen (2024) highlight that unlike banks, which are more likely to operate a strategy of originate-to-distribute, BDCs typically hold loans until repayment or maturity.

**Fact 2: Tailored loans are less likely to be securitized and are more prevalent in private credit.** We leverage deal-level data for 2010 through 2023 both bank

[2]To compute the banking sectors leverage ratio, we use aggregate equity and non-equity liabilities from the FDICs Quarterly Banking Profile.

[3]We calculate these figures based on data from SDC Platinum.

and BDC investments at origination.[4] We consider a simple measure of "tailoring": the presence of at least one non-standard feature, where we consider pricing grids, payment in kind (PIK) interest, and equity stakes.

Table 1 presents the prevalence of these contract features by lender type. Within bank lending, deals with more tailoring are less likely to feature Term Loan Bs, and hence less likely to be securitized;[5] 10.8% of bank deals that feature Term Loan Bs also feature tailoring, in the form of a pricing grid; that rate is 11.7% for bank deals that do not feature Term Loan Bs, and are thus less likely to be securitized. Meanwhile, BDCs deals feature more tailoring than either type of bank loan, with 18.2% of BDC deals featuring tailoring, as a mix of PIK interest and equity stakes. Across the board, tailoring is more common in larger deals, as show in Panel B of Table 1, where we value-weight deals.[6] Panels C and D present similar (albeit larger) shares, restricting to borrowers found in Compustat, whom we study below.

[4]For banks, we take all tranches of a deal at origination with the same total deal value. For BDCs, we take all securities (debt and equity) listed for a borrower company in the first year a BDC lists the borrower company in their Schedule of Investments. In the BDC data, we restrict to deals featuring at least one debt security and require a positive, non-missing deal value. We focus on the 2010-2023 window, as that is where we have the best coverage of BDC investments.

[5]Nadauld and Weisbach (2012) document that Term Loan B facilities are the loan tranches most likely to be securitized through CLOs.

[6]The distribution of deal size values is extremely skewed. We winsorized deal size at the 99th percentile, i.e., at $4.7 billion. Note that we use nominal deal values.

**Table 1:** Tailoring rates by lender and securitization

| | Percent of deals with any: | | | |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| | Tailoring | Pricing grid | PIK interest | Equity stakes |
| **Panel A: All deals, deal-weighted** | | | | |
| Bank deals *[N = 31,039]* | 11.6 | 11.6 | - | - |
| . . . securitized *[N = 4,815]* | 10.8 | 10.8 | - | - |
| . . . not securitized *[N = 26,224]* | 11.7 | 11.7 | - | - |
| BDC deals *[N = 19,580]* | 18.2 | - | 5.4 | 15.4 |
| **Panel B: All deals, value-weighted** | | | | |
| Bank deals *[N = 31,039]* | 25.8 | 25.8 | - | - |
| . . . securitized *[N = 4,815]* | 17.2 | 17.2 | - | - |
| . . . not securitized *[N = 26,224]* | 28.8 | 28.8 | - | - |
| BDC deals *[N = 19,580]* | 30.8 | - | 10.6 | 25.5 |
| **Panel C: Compustat deals, deal-weighted** | | | | |
| Bank deals *[N = 9,876]* | 27.3 | 27.3 | - | - |
| . . . securitized *[N = 1,931]* | 17.5 | 17.5 | - | - |
| . . . not securitized *[N = 7,945]* | 29.7 | 29.7 | - | - |
| BDC deals *[N = 1,122]* | 19.6 | - | 5.2 | 16.8 |
| **Panel D: Compustat deals, value-weighted** | | | | |
| Bank deals *[N = 9,876]* | 36.9 | 36.9 | - | - |
| . . . securitized *[N = 1,931]* | 24.5 | 24.5 | - | - |
| . . . not securitized *[N = 7,945]* | 41.1 | 41.1 | - | - |
| BDC deals *[N = 1,122]* | 24.5 | - | 11.4 | 17.2 |

Note: This table presents the share of originating bank and private credit deals that feature tailoring (i.e., having a pricing grid *or* having PIK interest *or* having equity stakes).
Panels A and B present deal- and deal value-weighted rates for four groups: all bank deals in DealScan (for U.S. borrowers and banks), the bank deals we label as securitized (i.e., have a Term Loan B present), the bank deals we label as not securitized (i.e. do not have a term loan B present), and all debt deals in our BDC sample. Panels C and D restrict to borrowers found in Compustat (at any time). Note that some BDC deals feature both PIK interest *and* equity stakes, and so the sum of columns (3) and (4) is larger than column (1)).

We formalize the comparisons between likely-securitized and non-securitized bank deals, and between bank and BDC deals, in regressions in Table 2, where we control for deal year and deal size. Among bank deals in the same year and with similar deal

sizes, those that feature tailoring are less likely to also feature Term Loan Bs (and thus less likely to be securitized). Across all bank and BDC deals in the same year and with similar deal sizes, those that feature more tailoring are more likely to be BDC deals.

**Table 2:** Tailoring rates by lender and securitization

| Dependent Variable: | Securitized | | | BDC deal | | |
|---|---|---|---|---|---|---|
| Sample: | Bank deals | | | All deals | | |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| Tailored | -0.008 | -0.118*** | -0.110*** | 0.162*** | 0.136*** | 0.005*** |
| | (0.006) | (0.007) | (0.011) | (0.006) | (0.003) | (0.001) |
| Year FEs | x | x | x | x | x | x |
| Value Pctl FEs | | x | | | x | |
| Weight | Deal | Deal | Value | Deal | Deal | Value |
| N | 31,039 | 31,037 | 31,039 | 50,619 | 50,619 | 50,619 |

Note: This table demonstrates that tailoring tends to be more common amongst non-securitized bank deals (columns (1)-(3)) and among BDC deals (columns (4)-(5)). In columns (1)-(3), we restrict the sample to bank deals and regress an indicator for the deal likely being securitized (as measured by the presence of any Term Loan Bs) on our tailoring measure (i.e., an indicator for a deal featuring a pricing grid, PIK interest, or equity stake). In columns (4)-(5), we study our full sample of bank and BDC deals and regress an indicator for the deal being a BDC deal on the tailoring measure.

Robust standard errors in parentheses. $^{*}$ $p < 0.05$, $^{**}$ $p < 0.01$, $^{***}$

All columns include deal year fixed effects. Columns (2) and (4) include deal size (value) percentile fixed effects (based on the distribution of deal size in our full sample of bank and BDC deals). All columns except (3) and (6) are deal-weighted; columns (3) and (6) are value-weighted.

**Fact 3: Riskier firms sort toward private credit and receive more tailored contracts.** Among publicly traded firms borrowing from banks or public BDCs, firms with higher volatility and lower profitability are more likely to borrow from BDCs and to receive more tailored contracts from those BDCs. The riskiest firms receive the most tailored BDC loans, while safer firms are more likely to borrow from banks.

Table 3 presents this sorting, for the sample of borrowers for whom we observe both return on assets (ROA) and stock return volatility in Compustat and CRSP, respectively, in the year before the deal. The table presents analogous regressions to those in Table 2, with the distinction that we now control for deal size less granularly, since the sample is markedly smaller. Columns (1)-(3) demonstate that lower-ROA and higher volatility borrowers tend to borrow from BDCs rather than banks. Conditional on borrowing from a BDC, those with low ROAs and higher stock return

volatility are more likely to have deals featuring tailoring.

**Table 3:** Sorting of borrowers to BDC and tailored deals

| Dependent Variable: | BDC deal | | | Tailoring | | |
|---|---|---|---|---|---|---|
| Sample: | All deals | | | BDC deals | | |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| ROA | -0.455*** | -0.177*** | -0.120*** | -0.795*** | -0.811*** | -0.743*** |
| | (0.045) | (0.031) | (0.015) | (0.110) | (0.117) | (0.122) |
| EBITDA/Assets | -0.400*** | -0.165*** | -0.100*** | -0.871*** | -0.900*** | -0.817*** |
| | (0.045) | (0.031) | (0.013) | (0.102) | (0.105) | (0.113) |
| Stock return volatility | 0.151*** | 0.031*** | 0.027*** | 0.160** | 0.159** | 0.167* |
| | (0.016) | (0.010) | (0.003) | (0.078) | (0.077) | (0.095) |
| Year FEs | x | x | x | x | x | x |
| Value Ventile FEs | | x | | | x | |
| Weight | Deal | Deal | Value | Deal | Deal | Value |
| N | 6,031 | 6,031 | 6,031 | 375 | 375 | 375 |

Note: This table demonstrates that lower return on asset (ROA, net income over assets), lower EBITDA-to-assets, and higher stock return volatility Compustat borrowers more often receive BDC deals than bank deals (columns (1)-(3)) and, among BDC deals, receive more tailored deals (columns (4)-(5)). In columns (1)-(3), we restrict the sample to bank and BDC deals for Compustat borrowers and regress an indicator for the deal being a BDC deal on the borrower's ROA, EBITDA/assets, and annualized monthly stock return volatility from the year before the deal (separately); we winsorize ROA, EBITDA/assets, and annualized monthly stock return volatility at the bottom and top percentile of their distributions for our entire sample of Compustat borrowers. In columns (4)-(5), we study only the BDC deals and regress an indicator for the deal featuring tailoring on the ROA, EBITDA/assets, and stock return volatility.
Mean ROA for columns (1)-(3): 2.3%; for columns (4)-(6): -11.1%.
Mean EBITDA/assets for columns (1)-(3): 11.1%; for columns (4)-(6): -2.0%.
Mean stock return volatility for columns (1)-(3): 37.1%; for columns (4)-(6): 56.7%.
Robust standard errors in parentheses. $^{*}$ $p < 0.05$, $^{**}$ $p < 0.01$, $^{***}$
All columns include deal year fixed effects. Columns (2) and (4) include deal size (value) ventile fixed effects (based on the distribution of deal size in our full sample of bank and BDC deals). All columns except (3) and (6) are deal-weighted; columns (3) and (6) are value-weighted.

# 3 Theory

We study a two-period economy with three agents: financial intermediaries (banks and investment funds), firms, and households. Households allow us to close the economy in general equilibrium, but because we will initially focus on a steady state without aggregate risk, this part of the economy will play a minor role.

There is a unit mass of heterogeneous firms, initially fully owned by diversified entrepreneurs. Firms are endowed with a project that has an expected output $\bar{y}$ and volatility $\sigma$, heterogeneous across firms. To operate, firms must make an initial investment of one dollar, which entrepreneurs must fully finance from the financial sector.

A continuum of banks and investment funds perfectly compete to finance firms. These two types of financial intermediaries have access to the same investment opportunities. They can buy safe assets from the government, which yield a return $r^f = 1/q_b - 1$, originate loans, and buy or sell loans on the secondary market.

Loan contracts between lenders and firms are defined by a pair $(R,x)$. $R$ is a non-contingent repayment component, akin to a traditional bank loan, which we refer to as the vanilla component of the loan. We capture loan tailoring using the variable $x$, which denotes the fraction of equity that accrues to the lender, and we think of capturing any contractual feature of a lending deal that introduces state-contingency in the repayment plan of the firm (such as PIK, pricing grids, or literal equity stakes). An entrepreneur who chooses to borrow with a contract $(R,x)$ pledges to repay $R$ and gives up a fraction of firm equity $x$ to the lender, retaining only the residual fraction $(1-x)$.

When a lender originates a loan, it incurs a cost $C_o^i(x)$, potentially heterogeneous across the two lender types $i \in \{b,f\}$. Securitizing a loan and selling it on the secondary market also involves a cost $C_s(x)$. Such costs depend on the extent to which loans are tailored, capturing the idea that tailored loans are harder to originate and to securitize.

Banks and funds differ in their liability structure. Investment funds can raise equity from households frictionlessly, and therefore act as a veil for households (i.e., they operate in a Modigliani-Miller environment). Banks can raise costly equity from households, and incur a cost $\lambda_e$ per dollar raised. They can also finance themselves by borrowing at the risk-free rate,[7] but face a regulatory leverage constraint that the loans on their balance sheet cannot exceed a multiple $\phi$ of their equity.

## 3.1 Investment Funds

Investment fund's objective:

$$\max -e_F + \beta E[M_1 \Pi_1]$$

[7]Here, we do not explicitly model bank deposits, but we can think of banks' ability to borrow large sums at the risk-free rate as stemming from their deposit market power. Giving banks the ability to borrow at below the risk-free rate would break the zero-profit environment that we are focusing on.

Time-0 budget:

$$\underbrace{e_F}_{\text{equity}} = \underbrace{q_b B}_{\text{bonds}} + \underbrace{L_o}_{\text{held loans}} + \underbrace{C_o^f(x) L_o}_{\text{origination costs}} + \underbrace{q_l L_s}_{\text{purchased loans}} \tag{1}$$

Time-1 equity payoff[8]:

$$\underbrace{\Pi_1}_{\text{equity}} = \underbrace{B}_{\text{bonds}} + \underbrace{L_o f(y,R,x)}_{\text{held loans}} + \underbrace{L_s f(y,R,x)}_{\text{purchased loans}} \tag{2}$$

We now derive the optimality conditions for the investment fund. The fund simply acts as a veil for the household, and thus prices most assets.

**FOC bonds B**:

$$q_b = \beta E[M_1] \tag{3}$$

**FOC loan origination $L_0$** (this will be the **PC fund zero-profit loan pricing schedule**):

$$\underbrace{1 + C_o^f(x)}_{\text{effective origination cost}} = \underbrace{\beta E[M_1 f(y,R,x)]}_{\text{risk adj. expected return}} \tag{4}$$

**FOC loan purchase $L_s$**:

$$\underbrace{q_l}_{\text{secondary loan price}} = \underbrace{\beta E[M_1 f(y,R,x)]}_{\text{risk adj. expected return}} \tag{5}$$

Notice that funds will optimally choose to keep all the loans they originate on balance sheet. Selling loans to banks, who have costly external equity, would be suboptimal.

*Note:* the loan pricing equation (5) pins down $q_l$, the market price of a bank loan $(R,x)$ purchased on the secondary market by a fund. Instead, the zero-profit condition at origination, (4), pins down the lending rate charged by banks to firms, $R$, given a complexity choice $x$. That is, it pins down a zero-profit rate-complexity surface on $R(x)$).

*Note:* $R$ cannot be directly interpreted as a lending rate here when $x > 0$. Indeed, $R$ is the repayment amount on the loan component. If $x = 0$, then the bank puts \$1 in loan and gets back $R$. But if $x > 0$, part of the initial dollar investment is in equity.

[8]Remark: here, technically, there is a continuum of originated and purchased loans by type, but it is irrelevant because of zero profit. We could also equivalently assume that each fund specializes in a specific x and firm type.

Precisely, if $x$ is the share of equity that is owned by the bank, then the value of the equity shares owned by the lender is $e^{lender} = x\beta\bar{y}\bar{Y}(\omega)$. So this means the loan principal is $(1 - x\beta\bar{y}\bar{Y}(\omega))$, and therefore the actual bank must have put in equity $e^{lender} = \frac{x}{1-x}e$. Therefore, the =effective lending rate $\tilde{R}$ is $\tilde{R} = \frac{R}{1-e^{lender}} = \frac{R}{1-x\beta\bar{y}\bar{Y}(\omega)}$.

## 3.2 Banks

Banks can raise costly equity (they pay $1+\lambda_e$ for each dollar of equity they raise). This captures the fact that banks are large and complex organizations, that rarely engage in IPO or SEO. They can borrow at the risk-free rate $q_b$ frictionlessly, which captures bank's ability to borrow cheaply using deposits. However, they face a leverage constraint set by the regulator, which limits how much lending can be financed with debt. This combination of equity issuance cost and leverage constraints makes banks financially constrained, creating an incentive to free up their balance sheets by distributing loans to less constrained investment funds.

Bank's objective:

$$\max \quad -(1+\lambda_e)e + \beta E[M_1\Pi_1]$$

Time-0 budget:

$$\underbrace{L_o(1-zq_l)}_{\text{held loans}} + \underbrace{C_o^b(x)L_o}_{\text{origination costs}} + \underbrace{C_s^b(x)zL_o}_{\text{resale costs}} = \underbrace{e}_{\text{equity}} + \underbrace{q_b b}_{\text{borrowing}} \tag{6}$$

Time-1 budget:

$$\underbrace{\Pi_1}_{\text{equity}} = \underbrace{(1-z)L_o f(y,R,x)}_{\text{held loans}} - \underbrace{b}_{\text{borrowing}} \tag{7}$$

And a leverage constraint:

$$(1-z)L_o \leq \phi e \tag{8}$$

Notice that $L_s$ denotes the *sale* of loans on the secondary market as opposed to the *purchase* as in the case for funds.[9]

Where $f(y,R,x)$ is a loan-return function to be defined in the next section, which depends on the income realization $y$ (sum of aggregate and small firms component,

[9]In practice, we want $L_s > 0$ as long as financing is cheaper for funds than for banks. However, with different assumptions, it could be that funds sell loans to banks (e.g., if funds face a higher equity issuance cost than banks). Also, in such an environment, there could be a two-way purchase-selling, since these are effectively different assets due to the different degree of chosen tailoring in banks and PC funds.

idiosyncratic washes out on average but the idiosyncratic shocks are still relevant of course), on the lending rate $R$, and on the loan complexity $x$.

**Proposition 1:** *The problem of the bank is scale-invariant and proportional to $L_0$. For any choice of bank size $L_0$, the bank makes zero profit when offering a contract $(R,x)$ to a firm $(\bar{y},\sigma)$ under the following condition:*

$$\underbrace{1+\lambda_e\frac{(1-z)}{\phi}}_{\text{cost of funds}}+\underbrace{C_o^b(x)}_{\text{origination cost}}+\underbrace{C_s^b(x)z}_{\text{resale cost}}=\beta E\left[M_1 f(y,R,x)\right] \tag{9}$$

**Proof:** *See Appendix A.1.*

### 3.2.1 Optimal bank securitization choice

Notice that the bank zero profit condition is a function $g(z,R,x)=0$. It implicitly defines a function $R(z,x)$. Now, for each $x$, the relevant portion of the $g$ function is the upper envelope w.r.t. $z$. That is, if a firm wants a loan with complexity $x$, they do not care whether the loan is securitized or not, i.e., $z$ does not affect the firm's payoff. Therefore, the firm will simply pick the $z$ that minimizes $R$ given $x$. $g(z,R,x)=0$ implies that there are other zero-profit combinations that banks could offer with suboptimal securitization choice, but they are irrelevant in equilibrium.

Lender's payoff $f$ must be increasing in $R$ at the optimum.[10] Therefore, the lower $R$ is achieved by choosing $z$ to minimize the LHS (given $x$).

$$\min -z\frac{\lambda_e}{\phi}+C_s^b(x)z$$

We get the following bang-bang solution:

- keep on balance sheet ($z=0$) if the loan is sufficiently complex $C_s^b(x)>\frac{\lambda_e}{\phi}$
- securitize the loan ($z=1$) if the loan is sufficiently vanilla $C_s^b(x)\leq\frac{\lambda_e}{\phi}$

Where notice that the securitization threshold captures how leverage-constrained the bank is, highlighting the trade-off between securitization costs and equity capital cost. A higher equity issuance cost $\lambda_e$ or a tighter leverage constraint $\phi$ pushes the bank towards more securitization.

[10] Notice that $f$ may not be globally increasing in $R$, as higher $R$ increases repayment in survival regions but also enlarges the default region. However, for any optimal contract, $f$ must be increasing in $R$. If not, we could achieve a Pareto improvement by lowering $R$ and raising the values for both the lender and the entrepreneur.

## 3.3 Relevant zero-profit conditions for the entrepreneur

The entrepreneur who wants to raise capital faces the following zero-profit conditions.

**PC pricing schedule.** Private credit funds will offer any combination $(R,x)$ such that:

$$\underbrace{1}_{\text{cost of funds}} + \underbrace{C_o^f(x)}_{\text{origination cost}} = \beta E\left[M_1 f(y,R,x)\right] \tag{4}$$

Banks offer a segmented zero-profit schedule.

**Bank vanilla loan pricing schedule.** For low complexity loans that are securitized ($C_s^b(x) \leq \frac{\lambda_e}{\phi}$), banks offer any combination $(R,x)$ such that:

$$\underbrace{1}_{\text{cost of funds}} + \underbrace{C_o^b(x)}_{\text{origination cost}} + \underbrace{C_s^b(x)}_{\text{resale cost}} = \beta E\left[M_1 f(y,R,x)\right] \tag{10}$$

**Bank complex loan pricing schedule.** Instead, for high complexity loans ($C_s^b(x) > \frac{\lambda_e}{\phi}$), banks offer any combination $(R,x)$ such that:

$$\underbrace{1+\frac{\lambda_e}{\phi}}_{\text{cost of funds}} + \underbrace{C_o^b(x)}_{\text{origination cost}} = \beta E\left[M_1 f(y,R,x)\right] \tag{11}$$

Notice that all of these three schedule imply a function $R(x)$.

## 3.4 Entrepreneur Problem

The entrepreneur needs to make both "continuous" and "discrete" choices. It has to choose whether to borrow from a bank using a vanilla loan, from a bank using a complex loan, from a fund, or not to operate at all. Then, conditional on that choice, it needs to pick the desired contract $(x,R)$ among those offered by lenders (i.e., among those on the zero-profit condition).

The payoff of an entrepreneur with output $y$ who took out an $(R,x)$ loan is the following:

$$\pi(x,R,y) = \begin{cases} (1-x)(y-R) & if \quad y \geq R \\ 0 \quad if \quad y < R \end{cases}$$

While the payout for the lender is:

$$f(x,R,y) = \begin{cases} R + x(y-R), & \text{if } y \geq R \\ \delta y, & \text{if } y < R \end{cases}$$

In principle, the problem of the entrepreneur is the following:

$$V = \max\{V_b^{securitized}, V_b^{held}, V_f, 0\}$$

which capture, respectively, the firm's value conditional on borrowing from a bank (with a vanilla or a complex loan), from a fund, and 0 captures the outside value of not operating. In practice, exit is never voluntary. If the contract set is not empty, that is, if there is any combination of $(R,x)$ that makes at least one type of lender break-even, then the firm obtains positive value (strictly positive as long as $x \in [0,1)$). Therefore, exit only occurs for firms that are unable to obtain any loan.

Recall that, because of perfect competition across lenders, banks will offer any combination of $(R,x)$ to break even, including any combinations that rely on securitization or on holding loans on their balance sheet. Therefore, there are two equivalent ways to think about the choice between securitization and holding on balance sheet. The first is that banks offer a segmented zero-profit condition, where securitization choice is made optimally as per Section 3.2.1. The second is to treat these as separate banks, with banks of one type securitizing loans and banks of the other type holding them on balance sheet, and letting firms optimally choose which lender to borrow from.

For each option $i$ of the three above, the entrepreneur chooses the optimal $(x_i, R_i)$ subject to the lender's specific zero-profit condition:

$$V_i = \max_{x_i, R_i} \quad \beta E[M_1 \pi(x_i, R_i, y)] \tag{12}$$
$$s.t. \, Zero \, Profit \, Condition \, of \, lender \, i$$

Consider a generic zero-profit condition $1 + C(x) = \beta E[M_1 f(x,R,y)]$

Denote derivatives by $\pi_x, \pi_R$, etc.

FOC-x reads:

$$\beta E[M_1 \pi_x] = \lambda \left( \beta E[M_1 f_x] - C'(x) \right) \tag{13}$$

FOC-R reads:

$$E[M_1 \pi_R] = \lambda E[M_1 f_R] \tag{14}$$

Intuitively, $\lambda$ captures the marginal value of a dollar for a firm (that is, how con-

strained it is). The two FOCs are telling us that on both dimensions of the loan we need the choice to reflect how much value it can create for the firm and the bank, and equalizing that to $\lambda$. For instance, $\lambda = \frac{E_y[M_1\pi_R]}{E[M_1 f_R]}$ is saying that $R$ has to be chosen so that, at the margin, lowering $R$ creates an amount of value for the entrepreneur that is equivalent to $\lambda$ times the value it destroys for the bank.

The two FOCs together imply the optimal choice of complexity in PC loans:

$$\underbrace{\frac{E[M_1\pi_x]}{(E[M_1 f_x] - \beta^{-1}C'(x))}}_{\text{firm-to-fund marg. value of } x} = \underbrace{\frac{E_y[M_1\pi_R]}{E[M_1 f_R]}}_{\text{firm-to-fund marg. value of } \pi} = \underbrace{\lambda}_{\text{shadow value of equity}} \tag{15}$$

We can also rewrite this optimality condition in terms of the **MRS between complexity and loan rates**:

$$\underbrace{\frac{E[M_1\pi_x]}{E[M_1\pi_R]}}_{\text{MRS firm}} = \underbrace{\frac{\left(E[M_1 f_x] - \beta^{-1}C'(x)\right)}{E[M_1 f_R]}}_{\text{MRS fund}} \tag{16}$$

Of course, Equation (16) only holds when $x > 0$, and may break in corner solutions where the optimal contract is a vanilla loan with $x = 0$. We examine these subcases in greater detail in the following section, after specifying the functional forms.

# 4 Functional Forms and Solution

In this section, we make some simplifying assumptions to generate additional insights. Throughout, we will assume that there is no aggregate risk ($M_1 = 1$).

We further assume that firms' output follows a log-normal distribution:

$$y = \tilde{\omega}\bar{y}, \quad ln\tilde{\omega} \sim N(\mu, \sigma)$$

where $\bar{y}$ and $\sigma$ can potentially be heterogeneous across firms. We set $\mu = -\frac{1}{2}\sigma^2$, so that $\mathbb{E}[\tilde{\omega}] = 1$ for all firms, and expected output is governed by $\bar{y}$.

Before proceeding further, it is useful to define a few objects that are normalized by $\bar{y}$. First, we denote by $\omega = R\bar{y}$ the normalized vanilla loan payoff. Notice that $\omega$ is the threshold such that for any realization $\tilde{\omega} < \omega$ the firm is in default.

The total payoff for equity-holders, which include the entrepreneur and possibly a lender-shareholder, is $y - R = (\tilde{\omega} - \omega)\bar{y}$ if $\tilde{\omega} > \omega$, and zero otherwise. We can then

compactly denote the expected payoff of equity, normalized by $\bar{y}$, as follows:

$$Y(\omega) \equiv \mathbb{E}[(\tilde{\omega} - \omega)\mathbf{1}\{\tilde{\omega} \geq \omega\}]$$

We also denote the value of expected loss from bankruptcy frictions, again normalized by $\bar{y}$, as:

$$B(\omega) \equiv (1-\delta)\,\mathbb{E}[\omega\,\mathbf{1}\{\omega < \omega\}]$$

We now also specify the functional forms for origination and securitization costs. Origination costs are as follows:

$$C_o^i(x) = c_o^i + \tau^i \bar{y} Y(\omega) x, \quad i \in \{f, b\}$$

And securitization costs as follows:

$$C_s(x) = c_s + \eta \bar{y} Y(\omega) x$$

These functional forms assume that originating or securitizing a loan entails a fixed cost even for vanilla loans, and an additional cost which increases with loan tailoring $x$, and which is proportional to the firm's equity value $\bar{y}Y(\omega)$[11]. This proportionality allows us to derive neater analytical solutions. Economically, it can be interpreted as capturing the fact that the cost of originating or securitizing an equity tranche depends on its value, although such proportionality may not be exact in practice.

We solve the entrepreneur problem for a cost function with generic parameters $\{c_o, c_x\}$:

$$C(x) = c_o + c_x \bar{y} Y(\omega) x$$

As outlined in Table 4, this generic specification nests the three possible financing regimes for a firm.

**Table 4:** Cost structure across financing regimes

| Financing regime | $c_o$ | $c_x$ |
|---|---|---|
| Private credit | $c_o^f$ | $\tau^f$ |
| Bank held | $c_o^b + \lambda_e/\phi$ | $\tau^b$ |
| Bank securitized | $c_o^b + c_s$ | $\tau^b + \eta$ |

[11] Technically, the equity value is $\beta \bar{y} Y(\omega)$, but we can achieve this by renormalizing $\tau^i$, or we can add $\beta$, whatever is cleaner.

When borrowing from a private credit fund, as in Equation (4), the parameters $\{c_o, c_x\}$ simply map to the fund origination cost parameters $c_o^f$ and $\tau^f$. When borrowing from a bank with a loan held on balance sheet, origination costs are augmented to also capture the shadow value of financing for the bank, $\lambda_e/\phi$, as in Equation (11). Finally, when borrowing from a bank with a securitized loan, the cost for bank financing does not show up, but tailoring now entails an additional cost when securitizing the loan, as shown in Equation (10).

Under the functional forms outlined above, we can also more neatly characterize the threshold of tailoring derived in Section 3.2.1 beyond which banks choose to hold on balance sheet: $c_s + \eta\bar{y}Y(\omega)x > \lambda_e/\phi$. That is, banks hold on their balance sheet any loan with $x > \frac{\lambda_e/\phi - c_s}{\eta\bar{y}Y(\omega)}$.

### 4.0.1 Excluded lenders

Looking at Table 4, we can already draw some conclusions about which lenders will originate which type of loans, and which lenders will be excluded.

Recall zero-profit (here with $\leq$ to better convey intuition):

$$\beta^{-1}(1+c_o) \leq [1 - Y(\omega) - B(\omega) + (1-c_x)xY(\omega)]\,\bar{y}$$

We get the following results (assuming $\lambda_e > 0, \phi > 0$):

1. The lender of type $i$ with the lower origination cost $c_o^i$ originates all vanilla loans. Notice that vanilla loans are always securitized by banks, so $\lambda/\phi$ is irrelevant here.

2. If we further assume $\tau^f \leq \tau^b$, we also get that banks originate some tailored loans if and only if $c_o^b < c_o^f$.

3. If $c_s > \lambda/\phi$, no loan is ever securitized.

## 4.1 Optimality conditions

Using the more compact notation and the functional forms introduced in the previous section, we can write the entrepreneur problem as follows:

$$\max_{\omega,x} \quad (1-x)\,\bar{y}Y(\omega)$$

$$\text{s.t.} \quad \beta^{-1}(1+c_o) = [\underbrace{1 - Y(\omega) - B(\omega)}_{L(\omega)\text{: vanilla loan payoff}} + \underbrace{(1-c_x)xY(\omega)}_{\text{equity stake net payoff}}]\,\bar{y}$$

$$\omega \geq 0, \quad 0 \leq x \leq 1$$

We start by characterizing the optimality conditions for an interior solution, in which the firm borrows with a tailored loan ($0 < x < 1$). The first-order condition for $x$ simplifies to $\lambda = 1/(1 - c_x)$, where $\lambda$ is the Lagrange multiplier associated to the zero-profit condition, which captures the marginal value of a dollar for the firm—or shadow value of firm equity. Intuitively, when $x > 0$, the firm is effectively issuing costly equity to the lender at cost $c_x$.

Plugging the expression for $\lambda$ into the first-order condition for $\omega$ yields an optimality condition that pins down $\omega$, the default threshold, by trading off tailoring and bankruptcy costs, as characterized in Proposition 2.

**Proposition 2:** *The interior optimum default threshold $\omega$ when borrowing from a lender with tailoring cost $c_x$ solves:*

$$\underbrace{c_x\left(-Y'(\omega)\right)}_{\text{tailoriong costs}} = \underbrace{B'(\omega)}_{\text{bankruptcy costs}} \tag{17}$$

**Proof:** *See Appendix A.2.*

Increasing $\omega$, and thus $R$, shifts the financing mix toward the vanilla loan and away from the equity stake. The right-hand side term is mechanical: increasing the default threshold $\omega$ raises bankruptcy costs. The left-hand side term embeds the fact that a higher $R$ relaxes the zero-profit constraint, thus allowing for a reduction in $x$, which reduces the tailoring costs associated with it. At the margin, reducing tailoring costs increases the firm's value by $c_x\left(-Y'(\omega)\right)$, where recall that $Y'(\omega) < 0$.

All forces that are not frictions cancel out since we are solving for the optimal contract (i.e., we are moving along the Pareto frontier, and the zero-profit condition in levels effectively defines on which point of the Pareto frontier we'll land: the one where all the surplus goes to the borrower).

Higher $c_x$ leads to higher $\omega$ and higher $R$, because it changes the optimal trade-off between complexity cost and bankruptcy costs, by making bankruptcy look more attractive in relative terms, and therefore shifting the optimal contract towards less $x$ and higher $R$.

**Corollary to Proposition 2:** *The optimality condition in Equation (17) characterizing the interior optimum default threshold can be rewritten as:*

$$\underbrace{\frac{\varphi(q)}{1-\Phi(q)}}_{\text{survival hazard}} = \frac{c_x}{1-\delta}\sigma \tag{18}$$

*where $\varphi$ and $\Phi$ are the standard normal PDF and CDF, and $q$ is the standardized default threshold: $q \equiv \frac{\ln\omega-\mu}{\sigma} = \frac{\ln\omega+\frac{1}{2}\sigma^2}{\sigma}$.*

**Proof:** *See Appendix A.2.1.*

The survival hazard $\frac{\varphi(q)}{1-\Phi(q)}$ captures how quickly the survival probability $1-\Phi(q)$ shrinks at the default threshold $q$, and is high when a small increase in $q$ reduces the survival probability a lot. The right-hand side captures how high tailoring costs are relative to bankruptcy costs. Intuitively, a high survival hazard means that shifting the financing mix away from the vanilla component, by lowering $R$—and thus lowering $\omega$ and $q$—would reduce the probability of bankruptcy significantly. Choosing such $q$ as the optimal threshold can only be optimal if tailoring costs are high relative to bankruptcy costs, as shifting the financing mix towards the equity stake would be too costly. The scaling factor $\sigma$ emerges because here we are solving for $q$ in the standardized normal space. A higher $\sigma$ effectively reduces the survival hazard compared to the standardized normal, as the density of $\omega$ is more spread out, which raises the optimal survival hazard in the normalized $q$ space.

The optimal degree of loan tailoring, $x$, is obtained residually from the lender's zero-profit condition. Once we solve for the optimal default threshold $q$ implied by Equation (18), we can immediately find $\omega$, and we can then plug this into the zero-profit condition to obtain:

$$x(\omega) = \frac{S-L(\omega)}{(1-c_x)Y(\omega)} \tag{19}$$

where $S \equiv \frac{1+c_o}{\beta\bar{y}}$ is the normalized vanilla origination cost, and $L(\omega) \equiv 1-Y(\omega)-B(\omega)$ is the normalized vanilla loan payoff. The numerator captures the extent to which the initial loan alone, with a rate $R = \omega\bar{y}$, is sufficient to cover the amount advanced by the lender in the first period, inclusive of the baseline origination cost. More specifically, $(S-L(\omega)$ captures the financing gap left by the vanilla loan. If $S > L(\omega)$, the expected payoff of the vanilla loan is insufficient to cover the amount advanced, and for the lender to break even, a portion of equity stake has to be issued. Each dollar of financing gap is offset by the issuance of a portion $\frac{1}{(1-c_x)Y(\omega)}$ of firm equity, where $Y(\omega)$ captures the value of the equity stake, and $c_x$ corrects for the costs associated with originating and possibly securitizing such stake.

The value of $x(\omega)$ found in Equation (19) is useful to characterize the regions in which the solution is indeed interior. There are three possibilities. If $x(\omega) \in (0,1)$, then the solution is interior: the optimal contract involves a tailored loan with $(\omega, x)$ pin down by Equations (18) and (19). If $x(\omega) < 0$, then we are at a corner solution where the optimal contract is a vanilla loan with $x = 0$. Intuitively, by solving for the FOC without imposing the constraint $x > 0$, we implicitly allow the firm to issue negative equity i.e., pay out to shareholders finance such payout with vanilla debt, and gain $c_x$ in the process. As this is of course not possible, we get a corner solution in which $\omega$ is pinned down by the zero-profit condition, which for a vanilla loan reads $L(\omega) = S$.[12] Finally, if $x(\omega) > 1$, then the firm is unable to obtain any contract from the lender. That is, the contract set is empty: there is no combination of $(\omega, x) \in R^+ \times [0,1]$ that makes the lender break even.[13]

# 5 Quantitative Results

In this section, we first estimate the model using our empirical results from Section 2, and we then study competition between funds and banks in the calibrated model and under some policy shocks.

## 5.1 Estimation

There are two groups of parameters to calibrate. The first comprises the risk-free discount rate $\beta$ and the recovery rate $\gamma$, which are standard and calibrated externally to 0.96 and 0.7 respectively. The second set is comprised of the parameters needed to populate the cost functions of Table 4, which govern competition among banks and credit funds. We estimate these parameters to match some of the empirical moments documented in Section 2.

First of all, we let $\psi = \lambda_e/\phi$ denote bank liability-related parameters, which capture the extent of financial constraints on banks, since the model does not allow us to identify $\lambda_e$ and $\phi$ separately and only depend on their ratio. Of the remaining 7 parameters of Table 4, we can only identify 6: $c_s$ cannot be identified. Therefore, we just assume it is zero for the estimation exercise (if we externally calibrate it to be positive, this will simply shift down the calibrated values for $\psi$ and $c_o^b$).

[12] Notice that such $\omega$ must exist. One may be worried that vanilla is not feasible, but if at $\omega^*$ we get $x < 0$ is needed to satisfy zero-profit, then some vanilla loans are for sure feasible.

[13] To see this, notice that as $x \to 1$, $\omega$ implied by (18) remains optimal, but such $\omega$ is not feasible no matter how close to unity $x$ gets.

Then we have six parameters of the cost functions to calibrate, which determine the extent of competition in the lending market. Three relate to levels of costs: $c_o^b$, $c_o^f$, and $\psi$. Three relate to slope costs of tailoring: $\tau^b$, $\tau^f$, and $\eta$.

We identify these parameters using six moments of the data, as per the following table.

**Table 5:** Model calibrated to match empirical moments from Section 2

| Parameter | Value | Target | Data | Model |
|---|---|---|---|---|
| $c_o^f$ | 0.040 | Frac. of PC borrowers | 0.158 | 0.170 |
| $\psi = \lambda_e/\phi$ | 0.001 | Frac. of bank-held borrowers | 0.032 | 0.028 |
| $c_o^b$ | 0.003 | Frac. of bank-sec borrowers | 0.610 | 0.595 |
| $\tau^f$ | 0.274 | Tailoring in PC | 0.700 | 0.739 |
| $\tau^b$ | 0.505 | Tailoring in bank-held | 0.300 | 0.246 |
| $\eta$ | 0.270 | Tailoring in bank-sec | 0.050 | 0.000 |

The estimated parameter values clarify how the empirical moments are interpreted through the model. In the data, we observe that firms obtain loans of all the three types possible, and that loans exhibit a tailoring order as follow: syndicated bank loans are the least tailored, bank held loans have an intermediate level of tailoring, and private credit loans are the most tailored. To match the pecking order of tailoring, the parameters are estimated so that $\tau^f < \tau^b < \tau^b + \eta$, reflecting a higher cost of tailoring in banks due to both the origination technology ($\tau^b$) and—for securitized loans—the securitization cost $+\eta$. Secondly, for all lenders to be active in the model, the order of the cost parameters $c_o$ must be the flip side of that of $c_x$. Otherwise, one type of loan would dominate another, and no firm would ever seek credit from the dominated lender. Therefore, origination costs for plain vanilla loans are highest in private credit, and they are lower in banks, particularly so for securitized loans who are not affected from the equity shadow cost: $c_o^f > c_o^b + \psi > c_o^b$.

## 5.2 Model solution

The model reveals clear sorting patterns of firms, with a pecking order that depends on their profitability and volatility. Figure 1 shows the optimal choice of lender for each firm $(\bar{y}, \sigma)$. We report the inverted volatility, $\sigma^{-1}$, on the y-axis, to make the figure more interpretable. Firms close to the origin, with high volatility and especially

low profitability, are unable to obtain any loans and exit the market. As firms become marginally more profitable or less risky, they borrow from private credit funds. Funds have a comparative advantage in lending with highly tailored loans, a type of contract that is particularly valuable for high-risk, low-profitability firms. Indeed, as shown in Panel A of Figure 3, firms right across the exit boundary obtain highly tailored loans. As firms improve further, they obtain tailored loans from banks and as they improve yet more, they obtain vanilla loans from banks which are subsequently securitized.

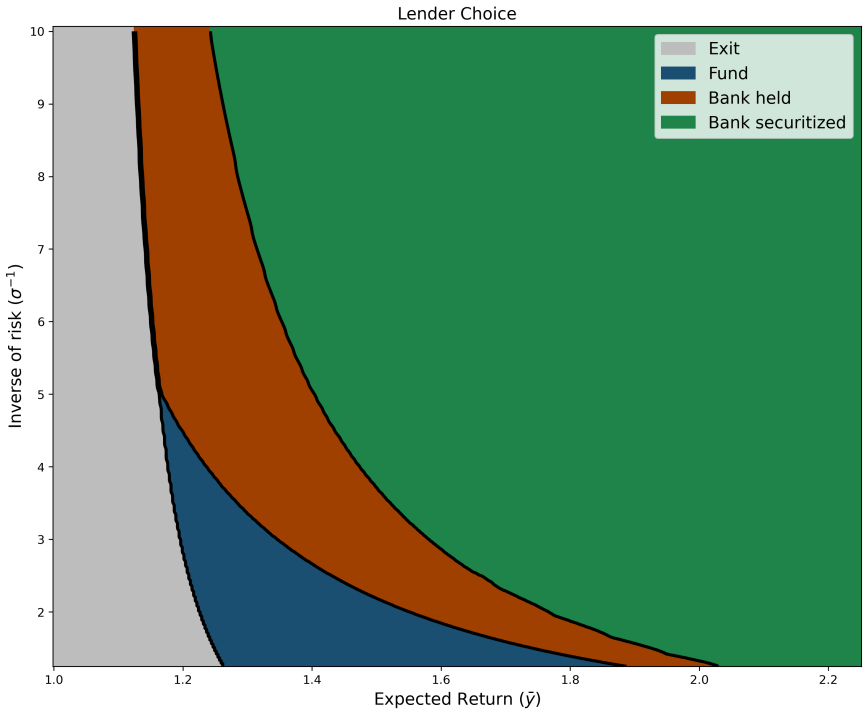



**Figure 1:** Optimal choice of the entrepreneur for each $(\bar{y}, \sigma)$. The x-axis reports firm expected output, $\bar{y}$, while the y-axis reports the inverse of its volatility, $\sigma^{-1}$.

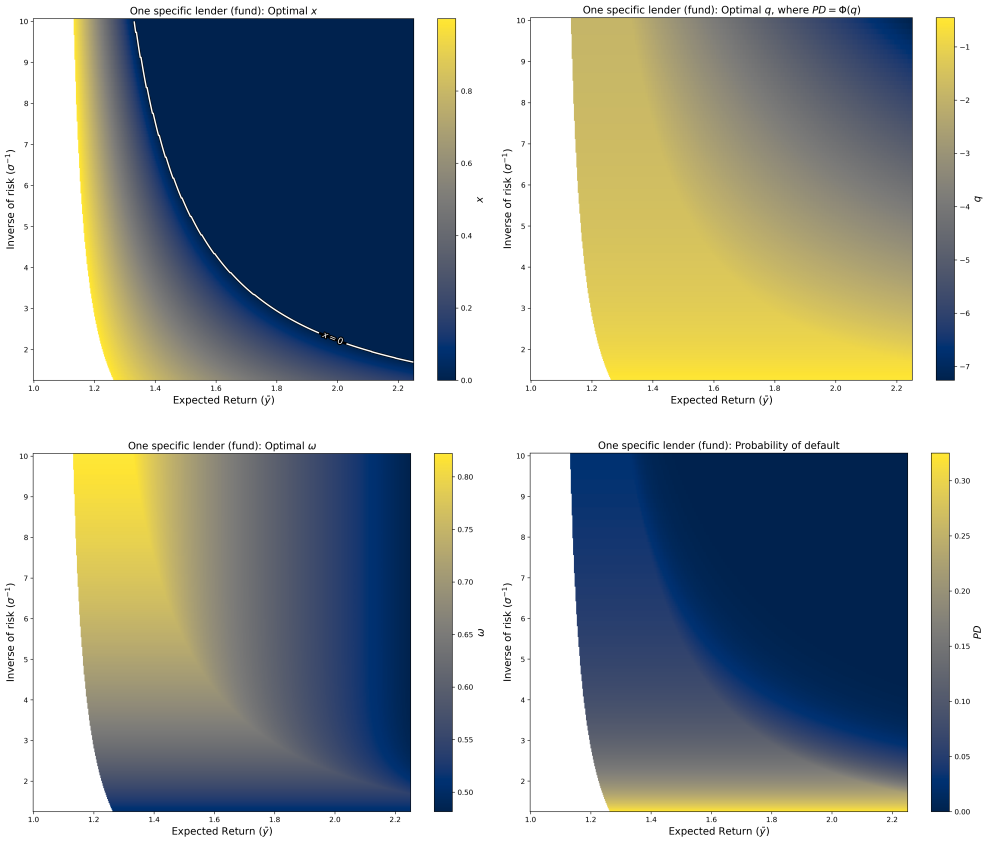



**Figure 2:** Optimal policies for all firms conditional on borrowing from the private credit fund. Tailoring choice $x$ (top-left panel), standardized default threshold $q$ (top-right panel), default threshold $\omega$ (bottom-left panel), and probability of default $PD = \Phi(q)$ (bottom-right panel).

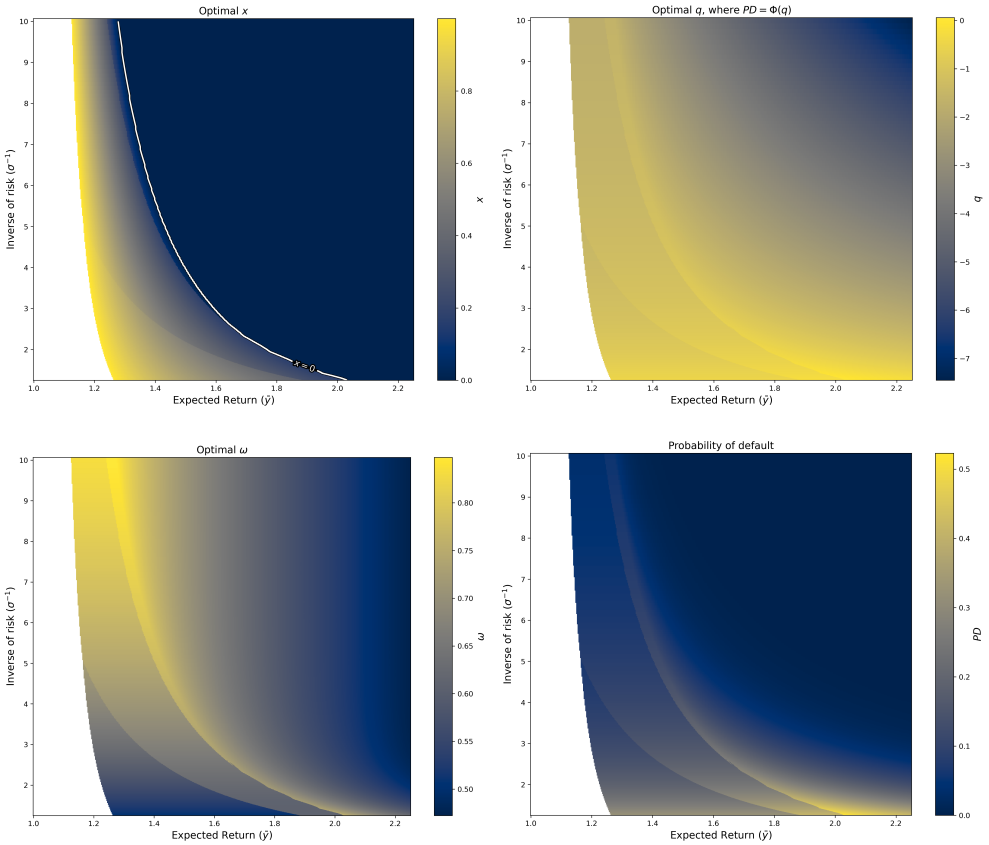



**Figure 3:** Optimal policies for all firms conditional on their optimal lender choice. Tailoring choice $x$ (top-left panel), standardized default threshold $q$ (top-right panel), default threshold $\omega$ (bottom-left panel), and probability of default $PD = \Phi(q)$ (bottom-right panel).

Policy functions conditional on borrowing from one specific lender (in this case, the private credit fund) are reported in Figure 2. We notice that as firms become more profitable and less risky they quickly reduce the optimal degree of tailoring $x$, and then gradually reduce the default threshold $q$. Figure 3 reports the policies conditional on the optimal lender choice. We notice similar patterns as in Figure 2, but also that some discontinuities emerge at the boundary where firms switch lenders. Moving from the top-right towards the origin, we see that firms initially borrow with vanilla loans with low $q$ and thus low probability of default. As they move towards the origin, they gradually start to demand tailored loans ($x > 0$) as their default threshold increases. At some point, they find it optimal to get a loan that is held on the bank's balance sheet, and at such boundary we observe a sharp increase in tailoring and a decline in the probability of default. A similar pattern occurs at the boundary where firms switch from bank-held loans to private credit funds.

## 5.3 Removing private credit counterfactual

The counterfactual of removing private credit is straightforward to implement in the model. It is sufficient to solve an alternative problem of the firm with:

$$V = \max\{V_b^{securitized}, V_b^{held}, 0\}$$

This allows us to derive what would happen to firms in the model absent private credit financing, thus answering the question of whether private credit simply substitutes bank lending or instead allows more firms to obtain financing.

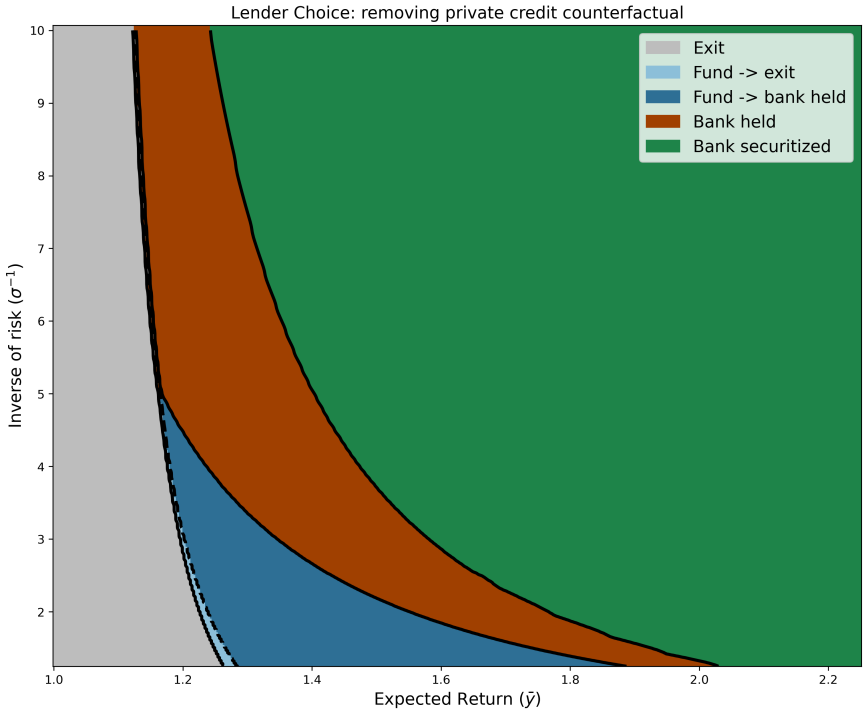



**Figure 4:** Optimal choice of the entrepreneur for each $(\bar{y}, \sigma)$. The x-axis reports firm expected output, $\bar{y}$, while the y-axis reports the inverse of its volatility, $\sigma^{-1}$.

We find that private credit funds primarily lend to firms that could obtain credit from banks, rather than providing credit to firms that would otherwise be excluded from credit markets. More precisely, 96.3% of firms that borrow from a private credit fund would switch to banks if funds were not available. All of these firms would be financed under a bank loan held on balance sheet, which is the closest substitute for highly tailored private credit loans. Instead, only 3.7% of private credit borrowers would be forced to exit. This means that most firms borrow from private credit thanks to the value they obtain from receiving a tailored contract.

Among firms that exit when only banks lend, only 4.6% are able to obtain a private

credit loan. Even among firms that are unable to obtain a bank loan but should operate under first best, this figure stands at 5.7%.[14]

**Table 6:** Counterfactual financing regime for private credit borrowers

| Exit | Bank held | Bank securitized |
|---|---|---|
| 3.7% | 96.3% | 0.0% |

**Table 7:** Counterfactual aggregate change in consumption and decomposition of the two drivers

| Total | = | Entrants | + | Switchers |
|---|---|---|---|---|
| 0.375% | | 0.014% | | 0.361% |

Table 7 shows that, within the model, the vast majority of welfare gains generated by private credit lenders occur through switchers—firms that funds attract from banks thanks to their ability to offer tailored solutions more affordably. Only a fraction of the welfare gains occur through firms that are only able to get funding thanks to private credit and would otherwise exit.

## 5.4 Policy counterfactuals

We now run two policy counterfactuals to study how these affect lending by funds and banks, optimal contracts, and borrowers' self-selection between the two. The first is a tightening of banks' leverage constraint, $\phi$, which makes it more costly for banks to hold loans on their balance sheet. The second is a reduction in the cost of securitizing loans.

### 5.4.1 Tightening in banks' leverage constraint

In the first counterfactual exercise, we study the effects of a tightening in the leverage limit for banks, $\phi$, from 10 to 5. This effectively increases the shadow value of banks' balance-sheet funding, $\lambda_e/\phi$, from 0.5% to 1%. Figure 5 visually shows the optimal lender choice, highlighting a transition of borrowers out of bank balance sheet loans after the shock. Table 8 reports more detailed statistics about changes in borrower selection and optimal contracts after the shock.

A tightening of banks' leverage limit substantially reduces bank balance sheet lending, which collapses from 16.3% of total borrowers to only 2.3%. About two-thirds

[14] This assumes in first best $C(x) = 0$.

of these borrowers flock to private credit, and about a third continue borrowing from banks, but now with loans that get securitized. The fraction of exiters slightly increases, but the increase is so small that it is not visible in Figure 5. Among firms that borrow with bank-held loans in the baseline, firms that were previously close to the boundary with private credit flow to private credit: this raises the average quality of the pool of borrowers in private credit. Conversely, bank borrowers that were previously close to the boundary between getting a balance sheet and a securitized loan now get securitized. Because these borrowers are of lower quality than previously securitized borrowers, the average securitized loan is of lower quality.

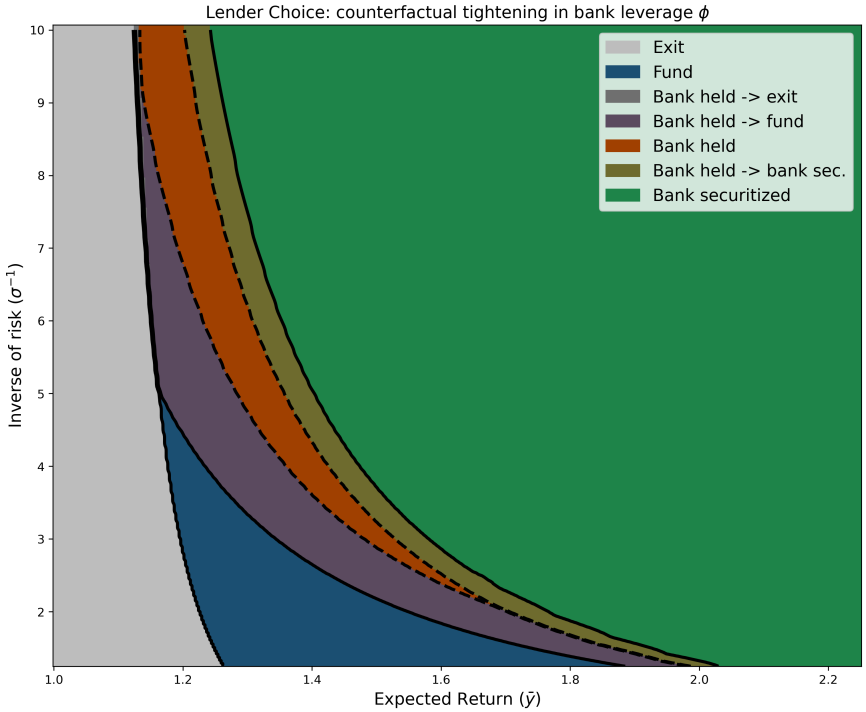



**Figure 5:** Optimal choice of the entrepreneur for each $(\bar{y}, \sigma)$ before and after the tightening of bank leverage constraint $\phi$. The x-axis reports firm expected output, $\bar{y}$, while the y-axis reports the inverse of its volatility, $\sigma^{-1}$.

We now turn to the optimal contracts $(q,x)$ chosen by borrowers from the three types of lenders. To understand, it is useful to note that changes in leverage requirements affect the optimal $(q,x)$ of private credit and securitized loan borrowers only through the selection of firms that borrow from these lenders. Instead, tighter leverage requirements directly affect bank-held loans, as they change the $\{c_o, c_x\}$ cost structure by increasing $c_o$. In the new equilibrium, private credit loans appear less tailored. Intuitively, as the average quality of fund borrowers has increased, the new borrowers demand less costly tailoring, lowering average tailoring levels and increasing default

thresholds. The opposite occurs in the pool of securitized loans. As lower-quality borrowers now get securitized, these borrowers now opt for slightly more tailored loans than in the baseline. While these loans are still close to vanilla, the percentage increase of tailoring compared to the baseline is massive, as most securitized borrowers obtained a plain vanilla $x = 0$ loan in the baseline. In summary, the leverage tightening makes bank lending polarize between a growing mass of securitized and relatively standard loans to high-quality borrowers, and a shrinking mass of balance sheet lending that now looks more similar to private credit lending, with a lower quality pool and highly tailored lending solutions.

**Table 8:** Counterfactual tightening in bank leverage constraint $\phi$ from 10 to 5.

| | Baseline | Counterfactual | % change |
|---|---|---|---|
| *Fraction of firms* | | | |
| Fund | 0.220 | 0.316 | 0.096 |
| Bank held | 0.163 | 0.023 | -0.140 |
| Bank sec. | 0.448 | 0.491 | 0.043 |
| *Average* $\bar{y}$ | | | |
| Fund | 1.448 | 1.487 | 2.671 |
| Bank held | 1.575 | 1.377 | -12.577 |
| Bank sec. | 1.926 | 1.904 | -1.113 |
| *Average* $\sigma$ | | | |
| Fund | 0.597 | 0.558 | -6.381 |
| Bank held | 0.437 | 0.243 | -44.313 |
| Bank sec. | 0.369 | 0.378 | 2.293 |
| *Average* $x$ | | | |
| Fund | 0.701 | 0.633 | -9.712 |
| Bank held | 0.365 | 0.452 | 23.606 |
| Bank sec. | 0.001 | 0.007 | 703.619 |
| *Average* $q$ | | | |
| Fund | -0.748 | -0.816 | 9.036 |
| Bank held | -0.864 | -1.291 | 49.477 |
| Bank sec. | -1.717 | -1.612 | -6.142 |

### 5.4.2 Reduction in securitization costs

In the second counterfactual exercise, reported in Table 9, we study the effects of a reduction in the securitization cost $\eta$ from 0.1 to 0.05. The first thing we notice from

the table is that all values pertaining to funds are unchanged. Changes in securitization costs directly affect only firms borrowing from a bank with a securitized loan, by effectively lowering $c_x$ in the securitized loan cost structure $\{c_o, c_x\}$. Indirectly, through selection, lower securitization costs also affect the pool of firms borrowing from a bank with a balance-sheet loan. Instead, they do not alter the pool of fund borrowers, because funds compete primarily with bank balance-sheet loans, as shown in Figure 6, and the cost $\{c_o, c_x\}$ associated with holding a loan on the balance sheet for banks does not change.

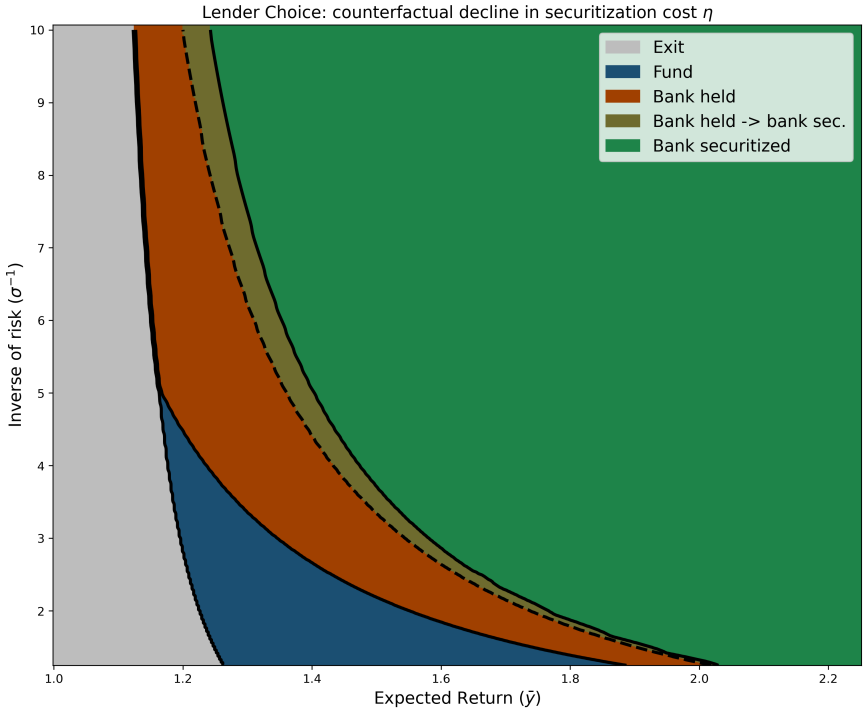



**Figure 6:** Optimal choice of the entrepreneur for each $(\bar{y}, \sigma)$ before and after the reduction in securitization costs $\eta$. The x-axis reports firm expected output, $\bar{y}$, while the y-axis reports the inverse of its volatility, $\sigma^{-1}$.

As securitization becomes more attractive, the highest-quality bank borrowers who previously received balance-sheet loans now find it more attractive to be securitized. Because all firms initially securitized are of higher quality than balance sheet firms, this transition lowers the average quality of firms in both groups.[15]

[15]This pattern mimics the well-known joke about two schools in Boston.

**Table 9:** Counterfactual decline in securitization cost $\eta$ from 0.1 to 0.05.

| | Baseline | Counterfactual | % change |
|---|---|---|---|
| *Fraction of firms* | | | |
| Fund | 0.220 | 0.220 | 0.000 |
| Bank held | 0.163 | 0.138 | -0.025 |
| Bank sec. | 0.448 | 0.473 | 0.025 |
| *Average* $\bar{y}$ | | | |
| Fund | 1.448 | 1.448 | 0.000 |
| Bank held | 1.575 | 1.569 | -0.384 |
| Bank sec. | 1.926 | 1.909 | -0.860 |
| *Average* $\sigma$ | | | |
| Fund | 0.597 | 0.597 | 0.000 |
| Bank held | 0.437 | 0.445 | 1.899 |
| Bank sec. | 0.369 | 0.370 | 0.302 |
| *Average x* | | | |
| Fund | 0.701 | 0.701 | 0.000 |
| Bank held | 0.365 | 0.391 | 7.074 |
| Bank sec. | 0.001 | 0.011 | 1282.718 |
| *Average q* | | | |
| Fund | -0.748 | -0.748 | -0.000 |
| Bank held | -0.864 | -0.845 | -2.152 |
| Bank sec. | -1.717 | -1.675 | -2.444 |

# 6 Conclusion

We document three facts about corporate lending by banks and private credit funds. First, funds retain a significantly larger share of originated loans on their balance sheets than banks. Second, private credit loans are more tailored. Third, lower-quality firms—those that are more volatile and less profitable—are more likely to borrow from private credit and to receive more tailored contracts. We rationalize these facts in a model where banks and private credit funds compete to attract heterogeneous firms by offering loan contracts with an endogenous degree of tailoring. Leverage constraints lead banks to specialize in standardized loans, which are easier to securitize. Lower-quality firms value tailoring more and therefore sort disproportionately into private credit. The model predicts that many private credit borrowers could obtain a bank loan, but choose fund financing because tailored contracts mitigate financial frictions. We then use policy changes to validate the models predictions about how bank leverage requirements and securitization costs affect competition between banks and funds, firm sorting, and equilibrium contract tailoring.

More broadly, our results highlight how changes in financial structure shape the allocation of credit across intermediaries. As private credit expands, a larger share of lending to riskier firms is intermediated by institutions that differ from banks in leverage, funding structure, and loan retention. These differences matter for how regulation and other policy interventions affect the distribution of credit across lenders, and therefore for the transmission of monetary policy and the design of financial stability policy. Our framework provides a way to study these forces jointly, linking intermediary balance-sheet structure to contract design, borrower sorting, and the changing boundaries of credit intermediation.

# References

Avalos, Fernando, Sebastian Doerr, and Gabor Pinter. 2025. "The global drivers of private credit." *BIS Quarterly Review* :13.

Block, Joern, Young Soo Jang, Steven N Kaplan, and Anna Schulze. 2024. "A survey of private debt funds." *The Review of Corporate Finance Studies* 13 (2):335–383.

Boot, Arnoud WA and Anjan V Thakor. 2000. "Can relationship banking survive competition?" *The Journal of Finance* 55 (2):679–713.

Chernenko, Sergey, Isil Erel, and Robert Prilmeier. 2022. "Why do firms borrow directly from nonbanks?" *The Review of Financial Studies* 35 (11):4902–4947.

Chernenko, Sergey, Robert Ialenti, and David Scharfstein. 2025. "Bank capital and the growth of private credit." *Available at SSRN* 5097437.

Davydiuk, Tetiana, Tatyana Marchuk, and Samuel Rosen. 2024. "Market discipline in the direct lending space." *The Review of Financial Studies* 37 (4):1190–1264.

Ivashina, Victoria. 2025. "Private Credit: What Do We Know?" *Unpublished Working Paper* .

Jang, Young Soo. 2025. "Are Direct Lenders More Like Banks or Arms-Length Investors?" *Unpublished Working Paper* .

Jang, Young Soo, Dasol Kim, and Amir Sufi. 2025. "The Lending Technology of Direct Lenders in Private Credit." Tech. rep., National Bureau of Economic Research.

Kashyap, Anil K, Raghuram Rajan, and Jeremy C Stein. 2002. "Banks as liquidity providers: An explanation for the coexistence of lending and deposit-taking." *The Journal of Finance* 57 (1):33–73.

Matvos, Gregor, Tomasz Piskorski, and Amit Seru. 2026. "Private Credit, Balance Sheets and Financial Stability." Tech. rep., National Bureau of Economic Research.

Nadauld, Taylor D and Michael S Weisbach. 2012. "Did securitization affect the cost of corporate debt?" *Journal of Financial Economics* 105 (2):332–352.

Robinson, David T and Melanie Wallskog. 2025. "Why is Private Lending So Popular?" *Working Paper* .

# A Theory

## A.1 Proof of Proposition 1

**Proposition 1:** *The problem of the bank is scale-invariant and proportional to $L_0$. For any choice of bank size $L_0$, the bank makes zero profit when offering a contract $(R,x)$ to a firm $(\bar{y},\sigma)$ under the following condition:*

$$\underbrace{1+\lambda_e\frac{(1-z)}{\phi}}_{\text{cost of funds}}+\underbrace{C_o^b(x)}_{\text{origination cost}}+\underbrace{C_s^b(x)z}_{\text{resale cost}}=\beta E\left[M_1 f(y,R,x)\right]$$

**Proof:**

We show that the bank problem is scale-invariant, which combined with free-entry implies a zero-profit condition (or otherwise banks would choose infinite size and make infinite profits).

First, we need to show that the leverage constraint always binds. Suppose by contraddiction that it does not bind. Then, we can show that the value of the bank can be improved by reducing $e$ by one unit and increasing $b$ by $q_b^{-1}$ units. This leaves time-0 budget unchanged. Reduces cash-flows tomorrow by $q_b^{-1}$, which reduces bank value by $\beta E[M_1]q_b^{-1} = 1$. But reducing equity by 1 increases bank value by $1+\lambda_e$. C.V.D.

Now, we can write $e = \frac{(1-z)}{\phi}L_0$.

From time-0 budget, we can write $b$ as:

$$\begin{aligned} q_b b &= \underbrace{L_o(1-zq_l)}_{\text{held loans}} + \underbrace{C_o^b(x)L_o}_{\text{origination costs}} + \underbrace{C_s^b(x)zL_o}_{\text{resale costs}} - \underbrace{e}_{\text{equity}} \\ &= \underbrace{L_o(1-zq_l)}_{\text{held loans}} + \underbrace{C_o^b(x)L_o}_{\text{origination costs}} + \underbrace{C_s^b(x)zL_o}_{\text{resale costs}} - \frac{(1-z)L_0}{\phi} \\ &= L_0\Big[\underbrace{(1-zq_l)}_{\text{held loans}} + \underbrace{C_o^b(x)}_{\text{origination costs}} + \underbrace{C_s^b(x)z}_{\text{resale costs}} - \frac{(1-z)}{\phi}\Big] \end{aligned} \tag{20}$$

That is:

$$b = L_0 \frac{\left[(1-zq_l) + C_o^b(x) + C_s^b(x)z - \frac{(1-z)}{\phi}\right]}{q_b} \tag{21}$$

But then we can rewrite bank problem as follows.

$$\max L_0 \left[ -\underbrace{(1+\lambda_e)\frac{(1-z)}{\phi}}_{\text{equity issuance}} + \beta E\left[M_1\left[\underbrace{(1-z)f(y,R,x)}_{\text{held loans}} - \underbrace{\frac{(1-zq_l)+C_o^b(x)+C_s^b(x)z-\frac{(1-z)}{\phi}}{q_b}}_{\text{borrowing}}\right]\right]\right]$$

Which is scale invariant!

**Bank zero-profit condition**

We can simplify bank profits by noticing that the borrowing term is non-stoochastic and that $q_b = \beta E[M_1]$.

Therefore:

$$\max L_0 \left[ -\underbrace{(1+\lambda_e)\frac{(1-z)}{\phi}}_{\text{equity issuance}} + \beta E\left[M_1\left[\underbrace{(1-z)f(y,R,x)}_{\text{held loans}}\right]\right] - \underbrace{\left[(1-zq_l) + C_o^b(x) + C_s^b(x)z - \frac{(1-z)}{\phi}\right]}_{\text{borrowing}} \right]$$

Either profits are infinite and $L_0$ is infinite, or we get a well-defined zero-profit condition:

$$\underbrace{-(1+\lambda_e)\frac{(1-z)}{\phi}}_{\text{equity issuance}} + \beta E\left[M_1\left[\underbrace{(1-z)f(y,R,x)}_{\text{held loans}}\right]\right] - \underbrace{\left[(1-zq_l) + C_o^b(x) + C_s^b(x)z - \frac{(1-z)}{\phi}\right]}_{\text{borrowing}} = 0$$

Which we can rearrange as:

$$\underbrace{\lambda_e\frac{(1-z)}{\phi}}_{\text{equity issuance}} + \underbrace{\left[(1-zq_l) + C_o^b(x) + C_s^b(x)z\right]}_{\text{borrowing}} = \beta E\left[M_1\left[\underbrace{(1-z)f(y,R,x)}_{\text{held loans}}\right]\right]$$

That is:

$$\underbrace{1+\lambda_e\frac{(1-z)}{\phi}}_{\text{cost of funds}} + \underbrace{C_o^b(x)}_{\text{origination cost}} + \underbrace{C_s^b(x)z}_{\text{resale cost}} = \underbrace{zq_l}_{\text{sold loans}} + \beta E\left[M_1\left[\underbrace{(1-z)f(y,R,x)}_{\text{held loans}}\right]\right]$$

Now recall from bond loan pricing that:

$$q_l = \beta E\left[M_1\left[(1-z)f(y,R,x)\right]\right]$$

Therefore, the **bank zero profit loan pricing condition is**:

$$\underbrace{1+\lambda_e\frac{(1-z)}{\phi}}_{\text{cost of funds}} + \underbrace{C_o^b(x)}_{\text{origination cost}} + \underbrace{C_s^b(x)z}_{\text{resale cost}} = \beta E\left[M_1 f(y,R,x)\right]$$

## A.2 Proof of Proposition 2

**Proposition 2:** *The interior optimum default threshold $\omega$ when borrowing from a lender with tailoring cost $c_x$ solves:*

$$\underbrace{c_x\left(-Y'(\omega)\right)}_{\text{tailoriong costs}} = \underbrace{B'(\omega)}_{\text{bankruptcy costs}} \tag{17}$$

**Proof:**

Recall the problem of a firm borrowing from a lender $\{c_o, c_x\}$.

$$\max_{\omega,x} \quad (1-x)\,\bar{y}\,Y(\omega)$$

$$\text{s.t.} \quad \beta^{-1}(1+c_o) = [\underbrace{1-Y(\omega)-B(\omega)}_{\text{vanilla loan net payoff}} + \underbrace{(1-c_x)xY(\omega)}_{\text{equity stake net payoff}}]\,\bar{y}$$

$$\omega \geq 0, \quad 0 \leq x \leq 1$$

The first order conditions for an interior solution read as follows:

$$x: \; -Y(\omega) + \lambda\,(1-c_x)Y(\omega) = 0$$

$$\omega: (1-x)Y'(\omega) + \lambda[-Y'(\omega) - B'(\omega) + (1-c_x)xY'(\omega)] = 0$$

$$\lambda: \; 1 - Y(\omega) - B(\omega) + (1-c_x)xY(\omega) = (1+c_o)/(\beta\bar{y})$$

The FOC for x simplifies to: $\lambda = 1/(1-c_x)$. This simply captures the fact that, when $x > 0$, the firm is effectively issuing equity through the lender at cost $c_x$, which pins down the shadow value of equity $\lambda$.

We can plug the FOC-x into the FOC-$\omega$, and get:

$$(1-c_x)(1-x)Y'(\omega) + [-Y'(\omega) - B'(\omega) + (1-c_x)xY'(\omega)] = 0$$

This can be directly rearranged as in Equation (17), which is effectively as the earlier Equation (16) with MRS equalization, just now with the more compact notation and the functional forms.

### A.2.1 Proof of Corollary to Proposition 2

**Corollary to Proposition 2:** *The optimality condition in Equation* (17) *characterizing the interior optimum default threshold can be rewritten as:*

$$\underbrace{\frac{\varphi(q)}{1-\Phi(q)}}_{\text{survival hazard}} = \frac{c_x}{1-\delta}\sigma \tag{18}$$

*where* $\varphi$ *and* $\Phi$ *are the standard normal PDF and CDF, and* $q$ *is the standardized default threshold:* $q \equiv \frac{\ln\omega - \mu}{\sigma} = \frac{\ln\omega + \frac{1}{2}\sigma^2}{\sigma}$.

**Proof:**

Let us start from Equation (17):

$$c_x\left(-Y'(\omega)\right) = B'(\omega)$$

Where recall the definitions of $Y(\omega)$ and $B(\omega)$.

$$Y(\omega) \equiv \mathbb{E}[(\tilde{\omega}-\omega)\mathbf{1}\{\tilde{\omega}\geq\omega\}] = \int_{\omega}(\tilde{\omega}-\omega)\,f(\tilde{\omega})d\tilde{\omega}$$

$$B(\omega) \equiv (1-\delta)\,\mathbb{E}[\omega\,\mathbf{1}\{\omega<\omega\}] = \int^{\omega}\tilde{\omega}f(\tilde{\omega})d\tilde{\omega}$$

Where $f$ and $F$ denote the PDF and CDF of $\omega$. Using Leibniz' rule,

$$Y'(\omega) = -\underbrace{\mathbb{P}(\omega\geq\omega)}_{\text{survival probability}} = -(1-F(\omega)),$$

$$B'(\omega) = (1-\delta)\underbrace{\omega f(\omega)}_{\text{threshold}\,\times\,\text{density}}.$$

Plugging these into (17) we obtain:

$$c_x\underbrace{(1-F(\omega))}_{\text{survival prob.}} = (1-\delta)\underbrace{\omega f(\omega)}_{\text{threshold}\,\times\,\text{density}}. \tag{22}$$

Notice that what matters for tailoring costs is the CDF, while what matters for the bankruptcy costs is the PDF. Intuitively, the benefit from reducing tailoring costs scales with $c_x$ and with the survival probability, as a higher survival probability increases the value of the equity stake. Instead, the cost of increasing bankruptcy costs scales with $1-\delta$ and is increasing in $\omega f(\omega)$, as at the margin we increase the probability of default by $f(\omega)$ and it induces a cost $(1-\delta)\omega$.

We can also rearrange Equation (22) as follows:

$$\underbrace{\frac{\omega f(\omega)}{1-F(\omega)}}_{\substack{\text{survival hazard at } \omega \\ \text{scaled by } \omega}} = \frac{c_x}{(1-\delta)} \tag{23}$$

Note that all the above derivations to obtain Equations (17) and (22) do not depend on the distribution of $\tilde{\omega}$. We now make use of the log-normality of $\tilde{\omega}$ to further

specialize Equation (23). To do so, define $q$ as the standardized version of $\omega$:

$$q \equiv \frac{\ln \omega - \mu}{\sigma} = \frac{\ln \omega + \frac{1}{2}\sigma^2}{\sigma}.$$

Then

$$1 - F(\omega) = \underbrace{1 - \Phi(q)}_{\bar{\Phi}(q)}, \qquad f(\omega) = \frac{1}{\omega\sigma}\varphi(q),$$

so (22) becomes

$$c_x \bar{\Phi}(q) = (1-\delta)\,\omega \cdot \frac{1}{\omega\sigma}\varphi(q)$$

Therefore, the cutoff for $q$ (and thus $\omega$) is pin down by:

$$\underbrace{\frac{\varphi(q)}{1-\Phi(q)}}_{\text{survival hazard}} = \frac{c_x}{1-\delta}\sigma \tag{18}$$

# B Solution Algorithm

1. **Step 1.** For each $\sigma$, use Equation (18) to get the interior solution for $q^*$, which does not depend on $\bar{y}$. Then, use such $q^*$ to get $\omega^*$

2. **Step 2.** For each $(\bar{y}, \sigma)$, use the value of $\omega^*$ found in Step 1 and the zero-profit condition to back out the implied $x^*$

   - **Complex loan.** If $x^* \in (0,1)$, we are done. We get an interior solution for a complex loan
   - **Exit.** If $x^* > 1$, the firm has to exit, as the lender cannot propose any financing solution and break-even
   - **Vanilla loan.** If $x^* < 0$, then we are at the corner solution where the firm borrowers though a vanilla loan $x = 0$, and the value for $\omega$ is determined by the zero-profit condition.

Recall the zero-profit condition:

$$[1 - Y(\omega) - B(\omega) + (1-c_x)xY(\omega)]\,\bar{y} = \beta^{-1}(1+c_o)$$

Therefore, we get:

$$x(\omega) = \frac{S - L(\omega)}{(1-c_x)Y(\omega)}$$

where recall $S = \frac{1+c_o}{\beta \bar{y}}$ is the normalized vanilla origination cost, and $L(\omega) = 1 - Y(\omega) - B(\omega)$ is the normalized vanilla loan payoff.