Innovation, Financial Frictions, and Hysteresis Effects of Monetary Policy*

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Abstract

Does the financing structure of R&D determine the impact of monetary policy on long-run productivity? We show that it does. Contrary to the literature emphasizing internal finance, we use loan-level and firm-level data to document a significant role for cash flow-based debt in R&D funding. This financing channel introduces a novel margin of adjustment: following a monetary tightening, firms with cash flow-based loans sustain R&D spending significantly better than financially constrained firms. To explore the aggregate implications, we develop an endogenous growth model with nominal rigidities and heterogeneous financial frictions. We find that the availability of external R&D finance determines the "productivity cost" of monetary policy; when firms can access cash flow-based debt, tighter policy induces less persistent productivity losses.

Key Words: Innovation; Monetary policy; Earnings-based borrowing constraints; Financial frictions.

JEL Codes: E22, E32, E44, E52, G32.

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1 Introduction

How does access to external finance shape the long-term effects of monetary policy? Recent research highlights that tight monetary policy can leave lasting scars on productivity by suppressing R&D activity, a phenomenon known as *hysteresis*.[1] In this paper, we investigate the importance of external finance in shaping the transmission of monetary policy, focusing on its short-run impact on R&D and the resulting long-term consequences for productivity.

A key step in understanding the role of external finance is examining how firms fund their innovation activities, as this is crucial for assessing the cyclicality of R&D. It is because, as financial frictions ease, firms become better positioned to smooth their R&D investment over time. Traditional corporate finance literature emphasizes internal funds as the primary source for financing intangible investment due to their non-collateralisable nature.[2] This implies that R&D-intensive firms often face significant financial constraints when funding their innovation activities. However, as shown by Lian and Ma (2021), nearly 80% of debt contracts among US firms are cash flow-based, requiring no physical collateral. This absence of collateral requirements suggests that cash flow-based contracts might be particularly convenient for externally financing R&D activities.

Using detailed firm- and loan-level data, we first show that R&D and debt growth is positively correlated, suggesting that some firms might be utilising external debt to finance their R&D activities.[3] Motivated by this correlation, conditional on monetary policy shocks, we investigate the role of cash flow-based contracts in providing external finance for R&D. Specifically, we assess how these shocks impact firms' R&D expenditures through both demand and financial channels. First, we find that a monetary tightening reduces both cash flow and R&D, reflecting a demand channel where contractionary shocks suppress sales and constrain resources available for R&D investments. However, this average effect masks notable heterogeneity across firms in terms of financial conditions. In particular, we investigate whether access to cash flow-based debt contracts influences the transmission of monetary policy to cash flow and R&D investment. The results show that while cash flow declines similarly across firms, those lacking cash flow-based contracts face more severe reductions in R&D investment —2% more at the peak—highlighting the

[1] See, e.g., Fornaro and Wolf (2023); Garga and Singh (2021); Jordà, Singh, and Taylor (2024); Moran and Queralto (2018).

[2] See, e.g., Haskel and Westlake (2017); Falato, Kadyrzhanova, Sim, and Steri (2022).

[3] We also examine cash and equity as alternative financing strategies. Consistent with prior studies, we find that R&D growth is also positively correlated with cash holdings (Falato et al., 2022) and equity growth (Brown, Martinsson, and Petersen, 2012, 2013). For an overview of alternative financing strategies for R&D, see Hall and Lerner (2009).

importance of the financial channel in shaping the transmission of monetary policy to innovation activities. We interpret these results as follows. Due to the non-collateralisable nature of R&D, firms that rely solely on internal funds for financing (see Falato et al. (2022); Haskel and Westlake (2017)) are highly financially constrained. In contrast, firms with access to cash flow-based contracts benefit from an external funding source, making them better equipped to manage adverse conditions and remain relatively unconstrained. To our knowledge, this is the first study to show how financial frictions create heterogeneous R&D responses to monetary policy shocks. These findings reveal a crucial transmission channel: while monetary tightening suppresses innovation through the demand effect, favorable financial conditions —through access to external finance— counteract this impact and dampen the demand channel.

To rationalise our empirical findings, explore the long-run implications of financial frictions on monetary policy transmission channels; and evaluate the impact of counterfactual monetary policy rules, we develop an endogenous growth model based on Benigno and Fornaro (2018). This model follows Aghion and Howitt (1992), where firms invest in R&D to improve product quality. We choose a vertical innovation model because, as Garcia-Macia, Hsieh, and Klenow (2019) shows, productivity growth is primarily driven by enhancements to existing products rather than the introduction of new ones. We further extend the model by incorporating financial frictions, following Jermann and Quadrini (2012), and integrating cash flow-based borrowing, as outlined in Drechsel (2023).

We simulate the model's response to monetary policy tightening through two scenarios: (i) restricting R&D spending to internal funding only, and (ii) allowing R&D spending to be financed externally through cash flow-based contracts. Our analysis of the short-run effects shows that an unexpected interest rate increase leads to a contraction in the economy, resulting in lower output, inflation, and R&D expenditures. Quantitatively, our model generates a peak response of 0.5% in output following a monetary policy shock, closely aligning with the estimates from Christiano, Eichenbaum, and Evans (2005). Moreover, consistent with our empirical findings, when firms are able to access to external financing, they can sustain part of their investment despite reduced sales. This ability to smooth investment over time contrasts sharply with firms lacking such access, which experience sharper declines in R&D investment.

Our model, with its endogenous growth dynamics, provides a framework to analyse the long-term implications of monetary policy. As emphasised in the literature (Moran and Queralto, 2018; Anzoategui, Comin, Gertler, and Martinez, 2019; León-Ledesma and

Shibayama, 2023), reductions in R&D have significant lasting effects, particularly on productivity and output. The intrinsic link between R&D and productivity suggests that monetary policy can induce hysteresis effects: contractionary shocks suppress R&D activities, which in turn result in persistent declines in productivity and potential output. While these hysteresis effects arise following monetary tightening regardless of firms' financing strategies, the negative impact is less pronounced for firms with access to external financing. Specifically, we find that the decline in productivity is 0.3% smaller, and the damage to output is 0.7% less severe for these firms. This financial flexibility allows firms to sustain their R&D efforts, mitigating the long-term scarring effects of higher interest rates on productivity and output.

Hysteresis effects pose a significant challenge for central banks: while monetary tightening can lower current inflation, it may unintentionally hamper productivity growth, potentially leading to higher inflation in the future. This dynamic raises an essential question: how should monetary policy be conducted when hysteresis effects are at play? Although our empirical analysis centers on the transmission of monetary policy shocks, the mechanisms we uncover hold implications for the broader, systematic aspects of policy conduct. For instance, in response to a trade-off inducing shock (i.e. cost-push shock), a central bank with strict inflation targeting often raises interest rates to control inflation, worsening the output gap as a consequence. Our endogenous growth framework suggests that this trade-off carry over into the long term, as the impact on output and R&D do not revert to their pre-shock trends. Thus, while such policy responses intended to stabilize prices, they can erode the economy's productive capacity over time, resulting in hysteresis effects.

Our analysis suggests that the adverse impacts of these inflation-control measures are moderated under specific conditions: (i) when firms can access to access external finance, allowing them to smooth innovation activities over time, and (ii) when central banks target output alongside inflation. Regarding the first condition, we already discuss that firms with access to external finance are better equipped to maintain R&D during periods of tight monetary policy. As for the second, central banks that incorporate an output target alongside inflation respond less aggressively to inflationary pressures, thereby cushioning the decline in output and thus limiting long-term productivity losses (Moran and Queralto, 2018; Ikeda and Kurozumi, 2019; Garga and Singh, 2021). Consequently, when these two factors –dual mandate policy and external finance access– work in tandem, they help mitigate hysteresis effects without risking prolonged economic overheating. Therefore, our findings suggest that a balanced monetary policy approach can preserve

potential output, reducing reliance on aggressive fiscal subsidies often recommended to support innovation (Benigno and Fornaro, 2018; Fornaro and Wolf, 2023).

Related Literature. This paper contributes to several strands of the literature. First, our work builds on the literature that links short-term economic fluctuations with long-term growth, building on the seminal contributions of Romer (1990) and Aghion and Howitt (1992). Recent studies have explored how business cycle shocks affect potential output and productivity, particularly in the context of post-crisis recoveries (Benigno and Fornaro, 2018; Moran and Queralto, 2018; Anzoategui et al., 2019; Bianchi, Kung, and Morales, 2019; Queralto, 2020; Duval, Hong, and Timmer, 2020; Garga and Singh, 2021). These studies highlight how weak aggregate demand can suppress productivity growth by reducing R&D investment. Our paper extends this literature by demonstrating that not only demand factors but also financial conditions play a critical role in shaping these dynamics.

Second, our paper, in spirit, is closest to the literature strand on the hysteresis effects of monetary policy (Ikeda and Kurozumi, 2019; Jordà et al., 2024; Fornaro and Wolf, 2023; Ma and Zimmermann, 2023; Alves and Violante, 2023). These studies emphasise how temporary shocks can create lasting "scarring" effects on the economy and suggest how central banks can mitigate the long-run consequences of these temporary shocks. We contribute to this literature by demonstrating how the way firms finance their innovation activities plays a pivotal role in mitigating the long-run consequences of adverse shocks.

Third, our focus on access to external finance connects to the heterogeneous monetary policy transmission literature (Anderson and Cesa-Bianchi, 2024; Ottonello and Winberry, 2020; Cloyne, Ferreira, Froemel, and Surico, 2023; Ferreira, Ostry, and Rogers, 2023). In our framework, firms financing R&D expenditures through their internal funds cut their R&D spending sharper than those with cash flow-based financing, showing that how the severity of borrowing constraints amplifies firms' sensitivity to monetary policy shocks, creating heterogeneity in the transmission of these shocks to innovation in the short run and productivity in the long run.

Fourth, given that R&D is a form of intangible investment, our focus on its financing contributes to the literature on the funding of intangible assets (Falato et al., 2022; Brown et al., 2012, 2013) and their macroeconomic implications (Peters and Taylor, 2017; Döttling and Ratnovski, 2023). To the best of our knowledge, this paper is the first to empirically show that firms can use external finance to fund R&D expenditures. Furthermore, we show that the way firms finance their R&D activities significantly shapes their responses to monetary policy shocks.

Finally, our study contributes to the growing research on cash flow-based borrowing, initiated by Lian and Ma (2021). While recent studies have explored the macroeconomic effects of this financing method (Drechsel, 2023; Greenwald, 2019; Ozturk, 2024) and its specific features (Green, 2018; Caglio, Darst, and Kalemli-Özcan, 2021; Gonzalez and Sy, 2024), we show that firms use cash flow-based borrowing to finance R&D and highlight its long-term implications.

Outline. The paper is organized as follows: Section 2 describes the datasets and data treatment. Section 3 presents the empirical framework and micro-level evidence on the role of financial conditions in the transmission of monetary policy to firm-level R&D. Section 4 introduces our endogenous growth model incorporating nominal and financial frictions. Section 5 covers model parameterisation, including firm-level regressions to estimate borrowing constraint tightness. In Section 6, we conduct the baseline analysis, illustrating the main channels explaining our empirical results. Section 7 discusses short- and medium-run policy implications, and Section 8 concludes.

2 Data

In this section, we detail the datasets utilised and describe our focus on the link between R&D and productivity. We then discuss the concept of cash flow-based borrowing, how it operates, and its relevance to R&D financing. Finally, we explain our methodology for identifying cash flow-based borrowers in DealScan.

2.1 Data Sources

Our analyses rely on two key micro-level datasets. The first dataset is Compustat, which provides firm-level data for publicly listed U.S. companies. This dataset offers a comprehensive panel that includes firms' investment expenditures, R&D expenditures, as well as various balance sheet and income statement items, enabling us to measure the critical variables of interest such as size, leverage, liquidity *etc*.

The second dataset is from DealScan, which covers approximately 75% of the total commercial loan market in the United States by volume (Drechsel, 2023). DealScan offers detailed loan-level information, with specific relevance to this paper on aspects such as loan maturity, interest rates, and, crucially, loan covenants.

We merge Compustat data with DealScan data using the linking file of Chava and Roberts (2008). The sample period for our study spans from the first quarter of 1997 to

the third quarter of 2017. Although DealScan provides data from earlier periods, following Greenwald (2019), we begin our sample in 1997Q1 due to the sparse availability of covenant data in DealScan before this date. The sample concludes in 2017Q3, consistent with the latest iteration of Chava and Roberts (2008)'s linking file (April 2018).

The matched Compustat-DealScan sample of R&D-performing firms includes information on 981 unique firms. Within this merged dataset, we impose several sample restrictions: firms with R&D expenditure spells shorter than 16 quarters are excluded, as well as those reporting negative values for assets or sales. Additionally, we remove firms in the finance, insurance, and real estate (FIRE) sectors, along with those in regulated utility industries.

Finally, we use the monetary policy shocks from Jarociński and Karadi (2020), which are cleansed of central bank information shocks. Further details on the data sources and treatment processes can be found in Appendix A.

2.2 R&D - Productivity Link

We focus on R&D expenditures as a key driver of productivity (Romer, 1990; Aghion and Howitt, 1992; Acemoglu, Akcigit, Alp, Bloom, and Kerr, 2018). At the micro level, firms investing in R&D can develop innovative solutions that distinguish them from competitors, leading to market differentiation and, consequently, greater market share. Additionally, R&D promotes the creation of more efficient production processes, which enhances productivity. At the macro level, R&D plays a crucial role in economic growth by driving technological advancements and creating new business models. As these innovations boost productivity across the economy, R&D contributes to higher potential output through time.

Creating an accurate measure for R&D (or intangible capital in general) poses significant challenges, because, R&D expenditures are mostly generated internally and US GAAP regulations do not permit its inclusion on the balance sheet.[4] Therefore, R&D expenditure appears only within the income statement as an item under operating expenses. Taking these factors into account, our main measure is the knowledge capital formed internally as a result of R&D operations.[5]

Following the methodology of Benigno and Fornaro (2018), which builds on the ap-

[4]GAAP, or Generally Accepted Accounting Principles, establishes the standardized accounting rules for preparing, presenting, and reporting financial statements in the U.S. The primary objective of GAAP is to ensure that financial statements are comprehensive, consistent, and comparable across companies.

[5]A discussion on the internally generated intangible capital and externally acquired intangible capital can be found in Appendix A.

proach used by the U.S. Bureau of Labor Statistics, we construct firm j's R&D stock as:

$$k_{j,t}^{RD} = (1-\delta)k_{j,t-1}^{RD} + \texttt{XRDQ}_{j,t} \tag{1}$$

where $k_{j,t}^{RD}$ represents the R&D stock (also referred to as knowledge capital) of firm j at time t, while XRDQ denotes the R&D expenditure sourced directly from Compustat. Consistent with Benigno and Fornaro (2018), we assume an initial R&D stock of zero and we set the depreciation rate, δ, to 15% following the estimations of the U.S. Bureau of Labor Statistics.[6]

2.3 Cash flow-based borrowing

Access to external finance has important implications for firms' investment decisions. Given the information frictions between lenders and borrowers, lenders often impose covenants and collateral requirements as protective measures against the risk of borrowers engaging in financially irresponsible actions, a situation known as moral hazard. These safeguards, in the form of covenants and collateral, act as effective mechanisms to enforce compliance with the terms of the contract, thereby mitigating the potential for moral hazard and ensuring financial stability.

Collateral-based borrowing, also known as asset-based borrowing, is one common method of securing loans. In this approach, discrete assets are pledged as collateral, and in the event of a default, lenders can claim and liquidate these assets to recover their losses. However, this reliance on physical assets makes collateral-based borrowing particularly unsuitable for financing R&D expenditures or other intangible investments, which often lack tangible assets (Falato et al., 2022; Corrado, Haskel, Jona-Lasinio, and Iommi, 2022).

Another key protective measure is covenants, which are contractual stipulations that either mandate or restrict specific actions by the borrower. As highlighted by Chava and Roberts (2008), covenants give lenders the ability to intervene in company management if necessary. Echoing this perspective, Dichev and Skinner (2002) describes covenants as "trip wires," triggering a transfer of control rights when specific conditions, mainly covenant breaches, are met. This arrangement aligns management's actions with the interests of debt holders, reducing the risk of moral hazard.

Lenders often impose covenants tied to the borrower's EBITDA, effectively determining the borrowing capacity based on the firm's current earnings. This type of contract establishes a crucial link between a firm's earnings and borrowing limits and is known

[6] Chiavari and Goraya (2024) shows that excluding all observations from the first five years to avoid sensitivity to the initial condition of knowledge capital does not alter the results.

as *cash flow-based contracts*. Drechsel (2023) demonstrates that covenants are more prevalent in loan agreements that lack specific collateral. Additionally, Ozturk (2024) shows that cash flow-based borrowing is more common among firms with fewer tangible assets. Taken together, these findings suggest that cash flow-based borrowing, characterised by covenants tied to earnings rather than physical collateral, is particularly well-suited for financing R&D activities, which are inherently intangible and not easily collateralisable.[7]

2.4 Identifying Cash flow-based Borrowers

As noted above, we obtain data on financial covenants in commercial loans from DealScan. Instead of bond covenants, we focus on loan covenants, as they tend to be more rigorous and actively enforced compared to bond covenants. Kahan and Tuckman (1993) and Kermani and Ma (2020) state that loan agreements often impose stricter terms than corporate bond issuances, placing significant constraints on a firm's actions, especially regarding borrowing. Furthermore, financial covenants in loans typically require quarterly compliance, whereas those in bonds usually necessitate compliance only when the borrower undertakes specific actions.

We follow the methodology outlined by Drechsel (2023) and Ozturk (2024) to identify cash flow-based borrowers for each firm-quarter observation. Specifically, we classify firms as cash flow-based borrowers if they obtain loans with the financial covenant: **max. Total Debt-to-EBITDA**. This covenant restricts a firm's *total* debt, making it applicable at the firm level rather than the loan level.

3 Empirical Framework

In this section, we present the main set of empirical results leveraging the rich firm- and loan-level information of our matched Compustat-DealScan dataset described in the previous section. To validate that our results are not sample-specific, we first examine simple correlations between R&D expenditures and other financial metrics, specifically cash holdings growth, equity growth, and debt growth. We then proceed to dynamic monetary policy transmission exercises using the local projections approach. In order to show the relevance of demand conditions, we first present the average response of cash flow and R&D expenditures to a tightening monetary policy shock. Then to shed light on the impact of financial frictions, we analyze how monetary policy transmits heterogeneously

[7] Here note that according to Lian and Ma (2021), about 80% of the value of U.S. corporate debt is based on earnings, while only 20% is secured by assets.

across firms based on their R&D financing methods, specifically distinguishing between cash flow-based borrowers and others. This analysis also helps to discipline the model discussed in Section 4.

3.1 How do firms finance R&D?

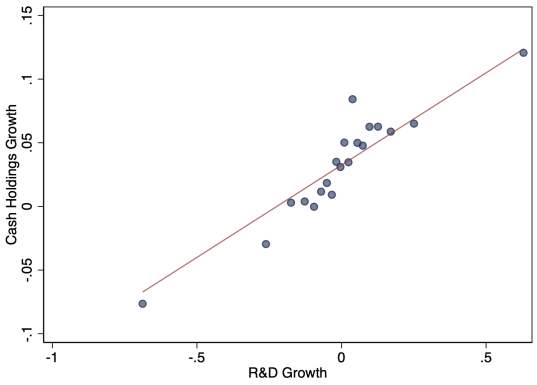
We begin by exploring the correlations between R&D expenditures and firms' financing strategies captured by three key financial metrics: (i) cash holdings, (ii) equity growth, and (iii) debt growth. This analysis aims to ensure our findings are consistent with existing literature and not unique to the matched Compustat-DealScan dataset. To illustrate these relationships, we present scatter plots comparing R&D expenditures with each of these financing strategies. Related regressions with more formal approach can be found in Appendix B where we present the effects of R&D on debt, equity, and cash holdings in Table B.2.

Cash Holdings. Falato et al. (2022) and Corrado et al. (2022) highlight that R&D expenditures are positively correlated with cash holdings. This relationship arises because intangible investments, such as R&D, are difficult to use as collateral due to their limited pledgeability. In frictional capital markets, where external financing incurs high spreads (above the risk-free rate), firms tend to increase their precautionary cash reserves to ensure they have sufficient liquidity for future intangible investments. As illustrated in Figure B.1 - Panel (a), our data aligns with this view, showing a positive correlation between R&D growth and cash holdings growth.

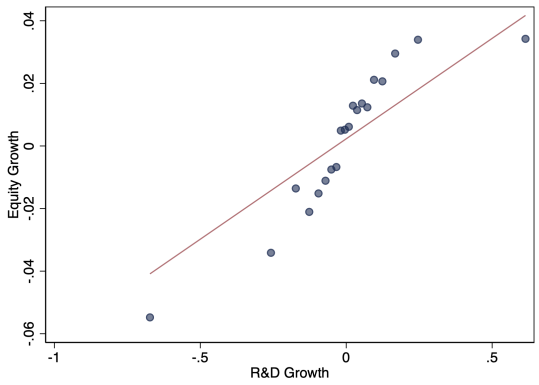
Equity financing. Brown et al. (2013, 2012) demonstrates that easier access to stock market financing is associated with significantly higher long-term R&D investment rates. In line with this, Bianchi et al. (2019), through a theoretical framework, highlights the impact of equity financing shocks on R&D investment. Our data, as depicted in Figure B.1 - Panel (b), corroborates this perspective, revealing a positive correlation between equity growth and R&D expenditure growth.

Debt Financing. Our data reveals a positive, unconditional correlation between debt growth and R&D expenditure growth, as shown in Figure B.1 - Panel (c). This new finding distinguishes our work from existing literature by empirically establishing a relationship that has only been suggested in theoretical frameworks thus far. Specifically, while

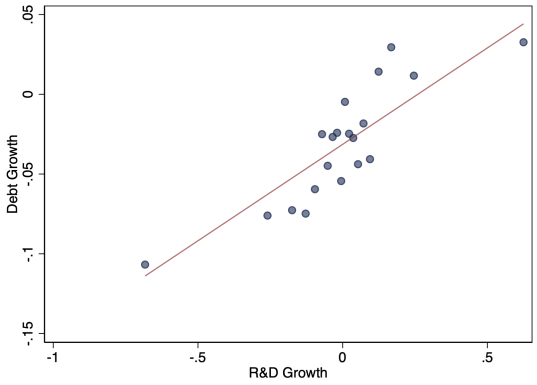
Figure 1
R&D Growth vs. Financing Sources



(a) R&D vs. Cash Holdings



(b) R&D vs. Equity



(c) R&D vs. Debt

Note. This figure reports the binscatter plots between R&D expenditures growth and firms' financing strategies captured by three key financial metrics: (i) cash holdings growth, (ii) equity growth, and (iii) debt growth.

prior research, such as Queralto (2020), demonstrates this connection through credit supply channels within a model, we are the first to empirically validate this positive association between debt growth and R&D expenditures. Building on insights from Falato et al. (2022) and Corrado et al. (2022), who emphasize the limitations of collateral-based financing for intangible investments, our finding suggests that firms may increasingly turn to debt financing, potentially through cash flow-based borrowing, to support R&D investment, circumventing the limitations of traditional collateral-based finance. Our results highlight an important channel by which firms can sustain innovative activities through debt, reinforcing the flexibility offered by cash flow-based credit.

3.2 Impact of Demand Conditions on R&D Expenses

The unconditional positive correlation between R&D growth and debt growth suggests a potential link, but to better establish this connection, we turn to conditional measures based on monetary policy shocks. Our dynamic analysis begins by examining the average effects observed in our firm-level panel data, with a focus on the role of demand conditions. Specifically, we assess how firms' cash flow and R&D activities respond to contractionary monetary policy shocks.In our baseline specification, we use the monetary policy shock series from Jarociński and Karadi (2020), which filters out the non-monetary components from interest rate surprises. Building on recent research on monetary policy transmission (Ottonello and Winberry, 2020; Cloyne et al., 2023), we employ the local projections method (Jordà, 2005) to estimate impulse response functions.

To estimate the average effect, we regress (2) and we interpret β^h as the average effect of interest rates on the variable of interest at horizon h.

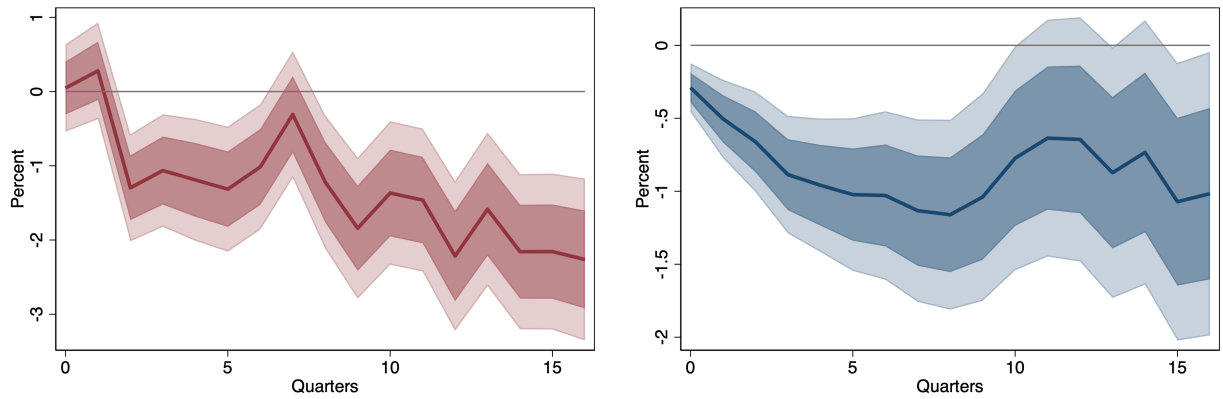
$$y_{j,t+h} - y_{j,t-1} = \alpha_j^h + \beta^h \epsilon_t^m + \sum_{p=1}^{P_Z} \Gamma_p \mathbf{Z}_{j,t-p} + \sum_{p=1}^{P_X} \Gamma_x \mathbf{X}_{t-p} + e_{j,t+h} \tag{2}$$

where $h = 0, 1, ..., H$ denotes the time horizon with $H = 16$ quarters. $y_{j,t+h}$ is the dependent variable of interest: cash flow and R&D expenditures. α_j^h is the firm fixed effect, ϵ_t^m is the quarterly monetary policy surprises. Our baseline specification also controls for a variety of idiosyncratic and aggregate factors. Particularly, $\mathbf{Z}$ is the firm-level covariate set including leverage, size, Tobin's Q, and current assets share, with $P_Z = 1$. $\mathbf{X}$ is the aggregate control variable set, including GDP, inflation, unemployment rate, and the VIX volatility index, with $P_X = 1$.

Figure 2 illustrates the impulse response function over 16 quarters. Following a 25 basis point unexpected increase in the interest rate, cash flow declines significantly, with the effect becoming significant after 2 quarters and persisting throughout much of the forecast horizon. Similarly, R&D activities also decrease significantly in response to the 25 basis point rate hike, with the impact becoming significant immediately. The peak effect occurs between 7 and 9 quarters post-shock, reaching approximately -1. Beyond this peak, the effect diminishes and becomes statistically insignificant after 2 years.

We show that demand conditions are key to the transmission of monetary policy to R&D activities. Both cash flow and R&D expenditures decline in response to contractionary monetary policy shocks, which can be interpreted in two ways. First, tighter policy reduces aggregate demand, lowering the profitability of new product development and weakening innovation incentives (Benigno and Fornaro, 2018; Anzoategui et al., 2019;

Figure 2
AVERAGE RESPONSES



(a) Cash flow **(b) R&D Expenditures**

NOTE. Panel (a) shows the impulse response of cash flow to a 25bp tightening monetary policy shock. Panel (b) shows the impulse response of R&D expenditures. We estimate the impulse responses (β^h) with the local projections specification in (2), namely $y_{j,t+h} - y_{j,t-1} = \alpha_j^h + \beta^h \epsilon_t^m + \sum_{p=1}^{P_Z} \Gamma_p \mathbf{Z}_{j,t-p} + \sum_{p=1}^{P_X} \Gamma_x \mathbf{X}_{t-p} + e_{j,t+h}$, where $h = 0, 1, \ldots, 16$; ϵ_t^m is the quarterly monetary policy shocks from Jarociński and Karadi (2020) ; α_j^h is the firm fixed effect; $\mathbf{Z}$ is the firm level covariate set: leverage, size, Tobin's Q and current assets share; $\mathbf{X}$ is the aggregate control variable set: GDP, inflation, unemployment rate, and the VIX volatility index. The shaded areas display 90 percent confidence intervals and standard errors are Driscoll and Kraay (1998).

Ma and Zimmermann, 2023). Second, reduced aggregate demand translates into lower cash flow for firms through lower sales, leaving them with fewer resources to finance R&D.

Crucially, this average effect masks significant heterogeneity in firms' exposure to financial frictions. Monetary policy impacts innovation through financial conditions (Bianchi et al., 2019; Queralto, 2020), which is the focus of this paper. Our current empirical model, focusing on average effects, does not fully account for the role of financial conditions. In the next section, we introduce heterogeneity to identify the factors and groups driving the observed cash flow and R&D responses.

3.3 Relevance of Financial Frictions on R&D Expenses

The primary aim of this section is to evaluate the role of financial conditions in the transmission of monetary policy to R&D expenditures. While Figure B.1 - Panel (c) shows a positive relationship between R&D expenditures and debt growth, we further investigate whether access to cash flow-based borrowing influences firms' R&D responses to

monetary policy shocks. Specifically, firms using cash flow-based contracts may exhibit different R&D responses to changes in monetary policy compared to those that do not.

To investigate financial heterogeneity, we use our identified measure for cash flow-based borrowers, splitting the sample into two groups: firms classified as cash flow-based borrowers and those that are not. We then examine how the effects of monetary policy differ between these groups by estimating an augmented version of equation (2).

$$y_{j,t+h} - y_{j,t-1} = \alpha_j^h + \beta_1^h \left(\epsilon_t^m \mathcal{I}_{j,t-1}^{cfb}\right) + \beta_2^h \left(\epsilon_t^m \mathcal{I}_{j,t-1}^{non-cfb}\right) + \mathbf{Z}_{j,t-1} + \mathbf{X}_{t-1} + e_{j,t+h} \quad (3)$$

We estimate the impulse responses (β_1^h and β_2^h) with the local projections specification in (3), where $h = 0, 1, \ldots, 16$; ϵ_t^m is the quarterly monetary policy shocks from Jarociński and Karadi (2020). $\mathcal{I}_{j,t-1}^{cfb}$ is a binary variable and takes the value of 1 if a firm is a cash flow-based borrower at time $t-1$.[8] Similarly $\mathcal{I}_{j,t-1}^{non-cfb}$ equals 1 if a firm is not a cash flow-based borrower. α_j^h is the firm fixed effect; $\mathbf{Z}$ is the firm level covariate set, including leverage, size, Tobin's Q and current assets share. $\mathbf{X}$ is the aggregate control variable set: GDP, inflation, unemployment rate, and the VIX volatility index. The shaded areas display 90 percent confidence intervals and standard errors are Driscoll and Kraay (1998).

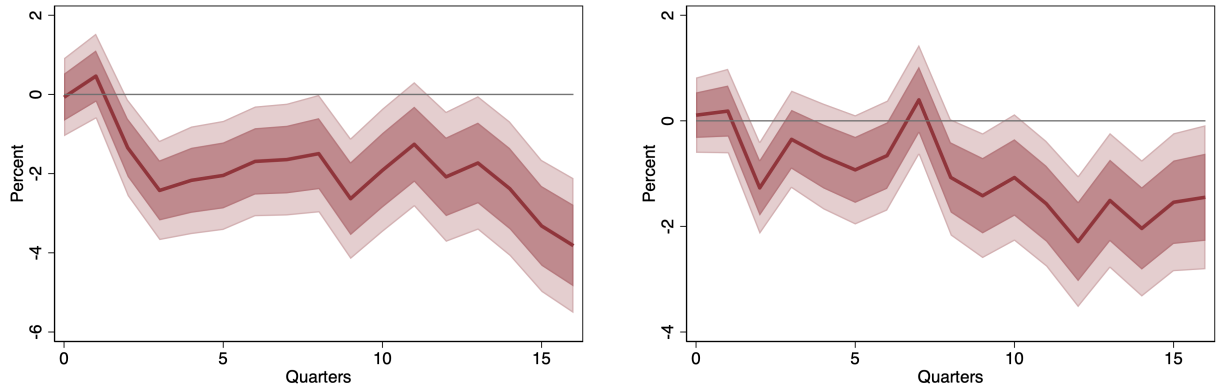
Following the literature on intangible investment (Haskel and Westlake, 2017; Falato et al., 2022), we recognize that the high risk and intangible nature of R&D expenditures make them very difficult to collateralise. As a result, we assume that firms primarily fund their R&D through internal funds, categorising them as financially constrained. In contrast, firms with access to cash flow-based contracts can borrow externally, easing financial frictions and making them relatively unconstrained. Accordingly, we label firms with cash flow-based contracts as the "relatively unconstrained" group ($\mathcal{I}j, t-1^{cfb} = 1$) and those without as the "constrained" group ($\mathcal{I}j, t-1^{non-cfb} = 1$).

Figure B.2 shows that cash flow responses of constrained group and relatively unconstrained group exhibit similar decreases to a tightening shock.[9] Combined with the findings from Figure 2 - Panel (a), it can be concluded that a contractionary monetary policy shock reduces cash flow across both groups, with no significant differential response. This suggests that tightening shocks impact both groups similarly through aggregate de-

[8] Since monetary policy could influence a firm's decision to switch to a cash flow-based contract, to mitigate potential endogeneity we interact the indicator variable at time $t-1$ with the monetary policy shock at time t.

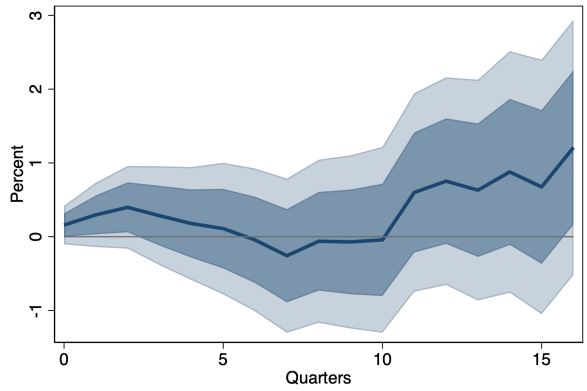
[9] To formally assess statistical differences between the two groups, we conduct additional analysis estimating the relative responsiveness of constrained group, which is found to be significant. Details of this analysis are provided in Appendix B.2.

Figure 3
Impulse Responses: CFB vs. Non-CFB

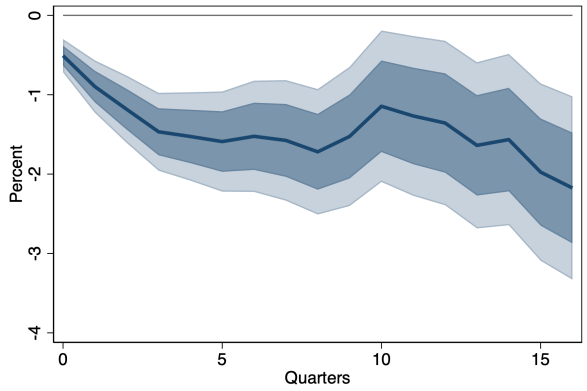


(a) **Cash flow:** CFB

(b) **Cash flow:** Non-CFB



(c) **R&D:** CFB



(d) **R&D:** Non-CFB

Note. Panel (a) shows the impulse response of cash flow to a 25bp tightening monetary policy shock. Panel (b) shows the impulse response of R&D expenditures. We estimate the impulse responses (β_1^h and β_1^h) with the local projections specification in (3), namely $y_{j,t+h} - y_{j,t-1} = \alpha_j^h + \beta_1^h \left(\epsilon_t^m \mathcal{I}_{j,t-1}^{cfb}\right) + \beta_2^h \left(\epsilon_t^m \mathcal{I}_{j,t-1}^{non-cfb}\right) + \mathbf{Z}_{j,t-1} + \mathbf{X}_{t-1} + e_{j,t+h}$, where $h = 0, 1, \ldots, 16$; ϵ_t^m is the quarterly monetary policy shocks from Jarociński and Karadi (2020); $\mathcal{I}_{j,t-1}^{cfb}$, which equals 1 if a firm is a cash flow-based borrower; $\mathcal{I}_{j,t-1}^{non-cfb}$, which equals 1 if a firm is not a cash flow-based borrower; α_j^h is the firm fixed effect; $\mathbf{Z}$ is the firm level covariate set: leverage, size, Tobin's Q and current assets share; $\mathbf{X}$ is the aggregate control variable set: GDP, inflation, unemployment rate, and the VIX volatility index. The shaded areas display 90 percent confidence intervals and standard errors are Driscoll and Kraay (1998).

mand conditions.

For the relevance of financial conditions, next, we examine the responses of R&D expenditures between the two groups. Figure B.2 shows that constrained firms reduce their R&D expenditures more significantly than relatively unconstrained firms in response to a contractionary monetary policy shock. Additionally, Panel (b) of Figure 2 illustrates that, on average, R&D spending declines following a contractionary monetary policy shock. Importantly, while both groups experience reduced cash flow due to the adverse monetary shock, the decline in R&D is primarily driven by non-cash flow-based borrowers. This heterogeneous response can be attributed to differences in financing conditions, as our regression accounts for a comprehensive set of aggregate and firm-level covariates.

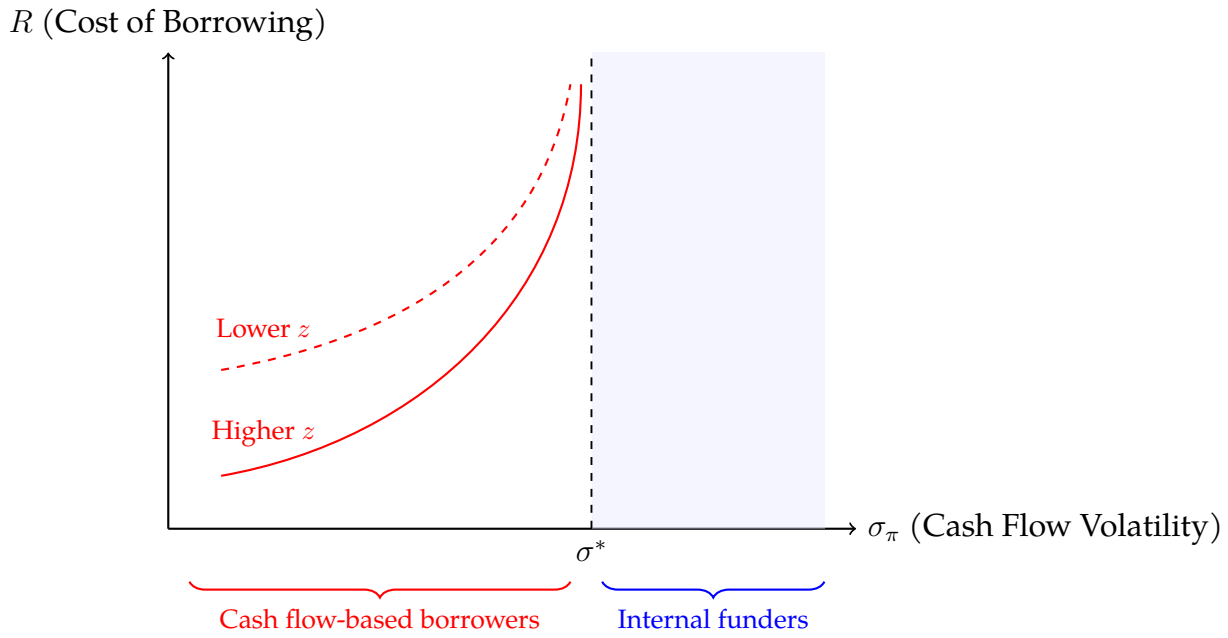
These results suggest insights into the underlying financial friction mechanisms at play. To the extent that firms not utilising cash flow based contracts primarily rely on internal funds to finance R&D expenditures, we interpret the observed heterogeneous sensitivity as evidence that relatively unconstrained group (cash flow-based borrowers) possess an additional tool –borrowing– to buffer against adverse monetary policy shocks. In contrast, constrained group must absorb these shocks through their internal funds. Note that we do not claim that the constrained group entirely refrains from borrowing, they may still finance physical investments through borrowing. However, since they lack access to cash flow-based contracts, they face challenges in collateralising their R&D expenditures. This situation likely leaves them somewhat credit-rationed, particularly when it comes to financing R&D activities.

We also draw an analogy to the recent literature on heterogeneous monetary policy transmission (Anderson and Cesa-Bianchi, 2024; Cloyne et al., 2023). We show that firms without cash flow-based contracts (constrained group) face tighter borrowing constraints compared to those with such contracts (relatively unconstrained). Consequently, when comparing the R&D responses of these two groups, we observe that while R&D investment generally declines following monetary tightening, relatively unconstrained group (firms with access to cash flow-based borrowing) are significantly less sensitive to these policy shifts indicating that more constrained firms exhibit greater responsiveness to monetary policy shocks.

4 Theoretical Setup

Ozgen: belki R degil de level of R&D falan gibi bisey belki
Aydan: Cash volatility wrt different theta mesela? In response to MP shock

Aydan: As it approaches zero, no debt, yani credit rationed



Note. The figure shows how borrowing costs under cash flow-based contracts increase with cash flow volatility. Firms with higher productivity z (solid red curve) face lower borrowing costs compared to lower-productivity firms (dashed red curve). For volatilities above the threshold σ^*, external finance is no longer offered due to excessive risk—resulting in credit rationing. These firms must rely entirely on internal funds (shaded blue region).

Figure 4
Credit Rationing under Cash Flow Volatility and Productivity

In Section 3, we show that for the transmission mechanism from monetary policy to innovation activities, our findings indicate that both demand and financial conditions are significant factors (Blanchard, 2018; Anzoategui et al., 2019; Bianchi et al., 2019; Queralto, 2020). Following a monetary policy tightening, we observe that firms' cash flow and R&D expenditures decline, emphasising the role of demand conditions –a contractionary shock reduces firm profitability, which in turn lowers incentives for productivity-enhancing investments. Additionally, the response of R&D expenditures varies across firms depending on whether they fund through internal funds or external borrowing, suggesting that financial frictions –tightness of borrowing constraints– also impact innovation activities.

Innovation activities are particularly crucial compared to traditional physical asset investments, as they influence the economy's productive capacity, while physical investments have a limited impact on potential output (Blanchard, 2018; Ma and Zimmermann, 2023). Therefore, the response of R&D expenditures has important long-run implications for productivity and potential output, extending beyond its immediate impact on

output. This suggests that monetary policy can exhibit *hysteresis effects*, meaning that temporary monetary shocks may have lasting implications for productivity and potential output through their influence on innovation activities.

Estimating the impact of monetary policy shocks on potential output and productivity poses significant econometric challenges, primarily because it requires longer time horizons than those typically addressed by local projections (Jordà et al., 2024). Additionally, various other monetary policy transmission mechanisms over these extended horizons affect output responses (Christiano et al., 2005; Kroen, Liu, Mian, and Sufi, 2021; Baqaee, Farhi, and Sangani, 2024). Given these complexities, capturing the full range of mechanisms affecting monetary policy's impact on potential output and productivity through R&D expenditures would be empirically challenging. Therefore, to rationalise our empirical results, and to investigate the implications on the supply side of the economy, specifically on the long-run objects such as productivity and potential output, we next build an endogenous growth model with New Keynesian elements.

4.1 Environment

We now develop an endogenous growth New Keynesian model to rationalize our empirical findings and examine how monetary policy affects the supply side of the economy through R&D investment. Our model builds heavily on the endogenous growth model developed in Benigno and Fornaro (2018) which in turn rooted in the seminal work of Aghion and Howitt (1992). Following Aghion and Howitt (1992), firms invest in R&D to enhance product quality. We adopt a vertical innovation approach because, as shown by Garcia-Macia et al. (2019), productivity growth is largely driven by improvements to existing products rather than by the creation of new ones. To introduce a role for monetary policy, we incorporate sticky wages, following the approach of Benigno and Fornaro (2018). Additionally, we extend the model to include cash flow-based borrowing constraints, as outlined by Drechsel (2023), and financial frictions on equity payouts, following Jermann and Quadrini (2012).

The model highlights three key features. First, productivity growth is endogenous and driven by firms' investments in innovation. Second, firms are able to finance their innovation activities by borrowing up to a certain proportion of their cash flows. Third, we introduce sticky wages, which create a role for monetary policy. The presence of nominal rigidities allows output to deviate from its potential level, enabling monetary policy to influence real economic variables.

Time is discrete and infinite. Each period is one year. The economy consists of house-

holds, firms, a government with a balanced budget, and a monetary authority. Within the production sector, a final goods producer sources inputs from intermediate firms. These intermediate firms can invest in innovation to enhance the quality of their products. In our model, intermediate firms can borrow up to a proportion of their cash flows to invest in R&D. We compare this scenario with one in which firms face hard borrowing constraints, preventing access to any external funding sources.

We analyze the perfect foresight transition paths in response to unexpected shifts in the economy. Throughout the simulations, we will compare two scenarios. In the first, firms face a strict borrowing constraint, preventing access to any external funding sources. These firms, labeled "internal funders" in the model, represent the "constrained" group (i.e., firms without cash flow-based contracts) in our empirical analysis. In the second scenario, firms, labeled "borrowers," have access to external funding and represent the "relatively unconstrained" group (i.e., firms with cash flow-based contracts).

4.1.1 Final Good Producer

Perfectly competitive final-goods firms produce Y combining labour, L, and a continuum of intermediate inputs, x_j subject to the below production function:

$$Y_t = (L_t)^{1-\alpha} \int_0^1 A_{j,t}^{(1-\alpha)} x_{j,t}^{\alpha} \, dj. \tag{4}$$

Here, $A_{j,t}$ represents the productivity or quality of input $j \in [0, 1]$. The profit maximization problem for the final goods producer yields the following optimality condition:

$$x_{j,t} = \left(\frac{P_{j,t}}{P_t \alpha (L_t)^{1-\alpha} (A_{j,t})^{1-\alpha}} \right)^{\frac{1}{\alpha-1}} \tag{5}$$

which defines the demand function for intermediate producer j.

4.1.2 Intermediate Producers

Production. Every intermediate input, x_j is produced by a monopolist. Regardless of the product's quality, one unit of final good is required to manufacture one unit of an intermediate good, meaning every producer faces the same marginal cost, P_t. Considering this market structure, it is optimal for the monopolist producing good j to set the price:[10]

$$P_{j,t} = \frac{1}{\alpha} P_t \tag{6}$$

[10]For the detailed discussion of how intermediate producers set their price, please refer to Appendix C.

With some algebraic manipulation, the profits generated by this monopolist for good j can be expressed as:

$$P_{j,t}x_{j,t} - P_t x_{j,t} = P_t \omega A_{j,t} L_t \tag{7}$$

where

$$\omega = \left(\frac{1}{\alpha} - 1\right) \alpha^{\frac{2}{1-\alpha}}.$$

Innovation Process. In equilibrium, each period sees a positive level of research activity directed toward each intermediate good j. In this framework, an intermediate producer investing $I_{j,t}$ units of the final good experiences productivity growth governed by the following law of motion:

$$A_{j,t+1} = A_{j,t} + \chi I_{j,t} \tag{8}$$

where $\chi > 0$ represents the effectiveness of R&D investment. The inclusion of $A_{j,t}$ on the right-hand side reflects the idea that advancing more sophisticated and complex products demands higher investment (*i.e.* higher R&D expenditures), ensuring a stationary growth process over time.

Equity payouts. Following Jermann and Quadrini (2012), adjusting equity payouts incurs costs:

$$\Phi(D_{j,t}) = D_{j,t} + \kappa(D_{j,t} - \overline{D_j})^2, \tag{9}$$

where $\overline{D_j}$ denotes the target payout level, which corresponds to the steady-state value of $D_{j,t}$. This cost can be interpreted in two main ways: first, as direct financial expenses related to share buybacks and equity issuance; and second, as reflecting the tendency of managers to smooth dividends over time. Mechanically, this cost structure allows us to capture the dynamics of how quickly firms can alter their financing methods when financial conditions shift. With a lower κ, firms can swiftly absorb the shocks through changes in equity. As κ increases, the substitution between debt and equity becomes more expensive, leading firms to modify their funding sources more slowly.

Debt contracts. Firms can access debt financing through one-period risk-free bonds, denoted as $B_{j,t+1}$. The effective gross interest rate faced by firms is R_t^b. Following Drechsel

(2023), which builds on the framework of Jermann and Quadrini (2012), firms' interest payments benefit from a tax advantage τ, such that $R_t^b = R_t(1-\tau)$, where R_t is the interest rate received by lenders. The tax advantage τ steers firms to favor debt over equity, encouraging them to borrow up to the limit of their constraints.

Borrowing is subject to the following constraint:

$$\frac{B_{j,t+1}}{R_t} \leq \theta \, CF_{j,t}, \tag{10}$$

where $CF_{j,t} = \omega A_{j,t} L_t - I_{j,t}$.[11]

The earnings-based borrowing constraint (L.14) limits real debt to a multiple θ of current earnings. We focus on the debt-to-earnings formulation, as this covenant type is the most prevalent in loan agreements, surpassing others like the coverage ratio (Drechsel, 2023).

It is important to note that in the internal funding scenario, firms face strict borrowing constraints, specifically $\theta = 0$, indicating that no external borrowing is possible under this condition.

$$\frac{B_{j,t+1}}{R_t} \leq 0. \tag{11}$$

Flow of Funds. In the case where borrowing is available, the corporate finance structure is governed by the following flow of funds constraint:

$$\Phi(D_{j,t}) + B_{j,t+1} = \omega A_{j,t} L_t - \chi I_{j,t} + R_t^b B_{j,t} \tag{12}$$

where equity payouts $\Phi(D_{j,t})$, new debt issuance $B_{j,t+1}$, earnings $\omega A_{j,t} L_t$, investment expenditures $I_{j,t}$, and the cost of servicing debt $R_t^b B_{j,t}$ are all included.

In a scenario with only internal funding, where borrowing is not an option, the flow of funds constraint simplifies to:

$$\Phi(D_{j,t}) = \omega A_{j,t} L_t - \chi I_{j,t} \tag{13}$$

including only equity payouts $\Phi(D_{j,t})$, earnings $\omega A_{j,t} L_t$, and investment expenditures $I_{j,t}$.

[11] It is important to note that investment, $I_{j,t}$, and therefore productivity, $A_{j,t}$, depend on θ_j, as the tightness of the borrowing constraint affects the optimal investment level. For simplicity, we do not express $I_{j,t}$ as $I_{j,t}(\theta_j)$.

Intermediate Producer's Problem for Borrowers. Firm j's objective is to maximize the expected stream of dividends, discounted by the household's discount factor:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \frac{1}{\delta^t} D_{j,t}$$

subject to:

- Law of motion for productivity (L.12): $A_{j,t+1} = A_{j,t} + \chi I_{j,t}$
- Borrowing constraint (L.14): $\frac{B_{j,t+1}}{R_t} \leq \theta \, CF_{j,t}$
- Flow of funds constraint (L.16): $\Phi(D_{j,t}) + B_{j,t+1} = \omega A_{j,t} L_t - \chi I_{j,t} + R_t^b B_{j,t}$

Intermediate Producer's Problem for Internal Funders. For internal funders, firm j's objective remains the same—to maximize the expected stream of dividends—though a strict borrowing constraint applies:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \frac{1}{\delta^t} D_{j,t}$$

subject to:

- Law of motion for productivity (L.12): $A_{j,t+1} = A_{j,t} + \chi I_{j,t}$
- Borrowing constraint (L.15): $\frac{B_{j,t+1}}{R_t} \leq 0$
- Flow of funds constraint (L.17): $\Phi(D_{j,t}) = \omega A_{j,t} L_t - \chi I_{j,t}$

4.1.3 Households

The representative household maximises expected lifetime utility from aggregate consumption, C_t:

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t log C_t$$

$\mathbb{E}$ is the expectations operator conditional on information at time t and $\beta \in (0, 1)$ is the discount factor.

Households finance their expenditure through labour income, holdings of equity shares of firms, dividend payments, and non-state contingent bonds. The representative household's budget constraint is given by:

$$C_t + \frac{B_{t+1}}{1+r_t} + S_{t+1}p_t^s + T_t = w_t L_t + B_t + S_t(D_t + p_t^s)$$

where w_t is the real wage, r_t is the real interest rate. B_t is the holdings of real bonds, S_t is equity shares, D_t is dividend payments received from holdings of shares and p_t^s is the (real) market price of shares. Finally, T_t is the lump-sum taxes.

4.1.4 Wage Setting

Following Fornaro and Wolf (2023) which in turn is based on Galí and Gambetti (2020), we assume that nominal wages evolve according to

$$\frac{W_t}{W_{t-1}} = \bar{g}\left(\frac{L_t}{\bar{L}}\right)^{\xi} \pi_{t-1}^{\varsigma} \tag{14}$$

where π_{t-1}^{ς} denotes lagged price inflation and $\varsigma > 0$ controls the determines the degree to which past inflation affects current wages. In this framework, higher employment exerts upward pressure on wage growth. Nominal wage rigidities serve two functions in the model. First, they create the potential for involuntary unemployment by maintaining positive wages, even when labor demand falls short of household labor supply. Second, these rigidities give monetary policy real traction, as sticky wages slow down price adjustments.

4.1.5 Government

The government's budget constraint, expressed in real terms, is as follows:

$$T_t = \frac{B_{t+1}}{1+r_t(1-\tau)} - \frac{B_{t+1}}{1+r_t} \tag{15}$$

where T_t represents lump-sum taxes imposed on households in consumption units, and the right-hand side of Equation (L.8) corresponds to the tax subsidy provided to firms.

4.1.6 Monetary Policy

The central bank sets the nominal interest rate i_t based on a Taylor rule:

$$\log(1 + i_t) = \log(1 + \bar{i}) + \phi_\pi \pi_t + \epsilon_t^m \tag{16}$$

where ϕ_π represents the weight assigned to inflation in the central bank's response, and ϵ_t^m denotes the monetary policy shock.

4.1.7 Market Clearing

Market clearing for the final good is given by:

$$Y_t = C_t + \int_0^1 I_{j,t}\, dj + \int_0^1 x_{j,t}\, dj \tag{17}$$

This equation states that the total output of final goods in the economy, Y_t, is allocated across three uses: consumption (C_t), investment in R&D ($I_{j,t}$), and intermediate inputs ($x_{j,t}$) used in production. Rearranging the equation, we separate out the intermediate goods as follows:

$$Y_t - \int_0^1 x_{j,t}\, dj = C_t + \int_0^1 I_{j,t}\, dj \tag{18}$$

Here, the left-hand side represents total output less the inputs used in intermediate goods production, effectively giving us the output available for consumption and R&D investment. This quantity can be interpreted as GDP, which reflects the goods and services produced for final use, excluding intermediate inputs.

Next, by substituting the production function for final goods and performing some algebraic manipulation, we can express GDP as:

$$Y_t - \int_0^1 x_{j,t}\, dj = \psi A_t L_t \tag{19}$$

In this equation, ψ is a constant reflecting production efficiency, A_t represents the average productivity level in the economy, and L_t is the total labor used in production. The average productivity, A_t, is defined as the average across all sectors or firms, with $A_t = \int_0^1 A_{j,t}\, dj$, where $A_{j,t}$ is the productivity of firm or sector j at time t. The detailed steps in this derivation are provided in Appendix C.

4.2 Dissecting the Mechanism

4.2.1 Direct vs Indirect Effects

The model developed in the previous section allows us to explore how monetary policy impacts firms differently based on their financing sources. Recall that in Section 3, our empirical findings show that as a response to a common monetary policy shock, cash-flow based borrowers cut their R&D expenditures less than the rest of the sample. This suggests that indirect effects, driven by aggregate demand, can outweigh the direct effects of borrowing costs. Specifically, while one might expect monetary policy to primarily impact cash flow-based borrowers due to increased borrowing costs, internal funders—relying on internal resources—would seem less affected. However, our empirical findings suggest that indirect demand effects dominate these direct effects.

To incorporate this mechanism to the model, we include an equity payout adjustment cost. Note that in the model economy, firms are generally reluctant to cut R&D, knowing today's investment, $I_{j,t}$, directly impacts future productivity ($A_{j,t+1} = A_{j,t} + \chi I_{j,t}$), which boosts future sales ($\omega A_{j,t+1} L_{t+1}$). We begin with the flow of funds for internal funders:

$$\omega A_{j,t} L_t = D_{j,t} + \kappa (D_{j,t} - \overline{D_j})^2 + I_{j,t}.$$

Here, with their sales revenue ($\omega A_{j,t} L_t$) firms can finance R&D investment ($I_{j,t}$) and with remaining funds distributed as dividends ($D_{j,t}$). When a contractionary shock reduces sales revenue, adjustment costs limit dividend cuts, leading firms to reduce R&D investment instead.

For borrowers, the flow of funds is:

$$\omega A_{j,t} L_t = D_{j,t} + \kappa (D_{j,t} - \overline{D_j})^2 - B_{j,t+1} + I_{j,t} + R_t B_{j,t}.$$

In this case, firms use sales revenue for R&D investment, debt servicing ($R_t B_{j,t}$), and dividends. When a contractionary shock lowers sales revenue, borrowers can partially offset the impact by increasing borrowing, though R&D investment may still decline. This mechanism highlights the importance of borrowing in mitigating reductions in innovation, showing how financial conditions critically shape R&D dynamics in response to monetary policy shocks, consistent with empirical observations.

4.2.2 R&D - Cash Flow Relationship

Cash flow-based borrowing constraint. How does the ability to finance R&D through cash flow-based contracts influence the cyclicality of innovation? Before deriving the relationship between cash flow available to the firm and their R&D spending, let us explain the cash flow-based contracts in the model economy. In the model, borrowing constraint is represented as a function of cash flow, which we restate here for clarity:

$$\frac{B_{j,t+1}}{R_t} \leq \theta \, CF_{j,t}$$

where θ measures the tightness of the borrowing constraint. In this economy, internal funding case is a subset of the credit supply state-space. In particular, internal funding case reveals when $\theta = 0$, in which the firm is completely restricted from borrowing, resulting in zero borrowing capacity. However, as $\theta > 0$, external borrowing becomes possible, enabling firms to supplement their internal funds with external finance. Consequently, as θ increases, the borrowing constraint loosens, allowing the firm to borrow more.

Relation between R&D and cash flow. Next, we examine the relationship between the cash flow available to a firm and its R&D spending, focusing on how this relationship differs depending on whether the firm uses retained earnings or external borrowing to finance its R&D expenditures. We begin by considering the case where external borrowing is possible. To determine the partial derivative of cash flow CF_{jt} with respect to I_{jt}, we start with the definition of the firm's available cash flow:

$$CF_{jt} = SR_{j,t} - I_{jt} + \frac{B_{jt+1}}{R_t} - B_{jt}$$

where $B_{j,t+1}$ represents the amount borrowed through a cash flow-based contract, and $SR_{j,t}$ denotes sales revenue, defined as:

$$SR_{j,t} = \omega A_{jt} L_t \tag{20}$$

Then the derivative of CF_{jt} with respect to I_{jt} reads:

$$\frac{\partial CF_{jt}}{\partial I_{jt}} = \frac{-1}{1 - \frac{\theta}{R_t}} < 0 \tag{21}$$

Here notice that the link between CF_{jt} and I_{jt} gets stronger as θ decreases. It is because, as θ decreases, the portion of the increase in I_{jt} absorbed by borrowing gets smaller.

Recall that internal funders case is the subset of this more generalised formulation

$(\theta = 0)$. So for these firms only retained earnings can be the source of financing their R&D expenditures. Again, we first express CF_{jt} as a function of I_{jt} and then compute its partial derivative with respect to I_{jt}. Note that in the internal funding world, CF_{jt} is defined as:

$$CF_{jt} = \omega A_{jt} L_t - I_{jt} \tag{22}$$

It is straightforward to take the partial derivative of CF_{jt} with respect to I_{jt} which reads:

$$\frac{\partial CF_{jt}}{\partial I_{jt}} = -1. \tag{23}$$

This calculation indicates that in the world when there is no external borrowing, (when $\theta = 0$) any increase in I_{jt} is reflected in cash flow one-to-one. The impact of R&D expenditures on the future cash flow is discussed in Appendix D.

Smoothing out R&D expenses. As discussed, when θ increases, the borrowing constraint loosens, allowing firms to smooth out shocks more effectively by accessing external funding to offset temporary cash flow shortfalls. Conversely, as θ decreases, firms become more reliant on internal funds to finance R&D, strengthening the link between cash flow and R&D expenditures. Weakening this link is crucial for firms because R&D investments are typically persistent and long-term. Firms must consistently allocate resources to sustain innovation and maintain a competitive edge. Given the volatility of cash flows, smoothing out shortfalls is essential to avoid disruptions in R&D activities. This underscores the importance of borrowing for firms heavily reliant on R&D. As borrowing constraints ease (*i.e.* θ increases), firms can better manage cash flow fluctuations, ensuring the continuity of their R&D investments. Thus, access to external financing is vital for sustaining long-term innovation.

5 Calibration

This section details the calibration strategy for the model described in Section 4. We divide the parameters into three categories. The first category consists of external parameters, which include standard Taylor rule parameters and parameters related to external finance. These parameters are calibrated using established literature. The second category involves internal parameters, adjusted to match specific empirical moments observed in

the data. The third category comprises estimated parameter, the tightness of the borrowing constraint, which is derived from the relationship between R&D and EBITDA. A summary of the model's parameterisation is provided in Table 1.

External calibration. We define each model period as one year. The Taylor rule coefficient on inflation follows standard values used in the literature. For the dividend adjustment cost (κ) and the tax advantage (τ), we adopt the values from Jermann and Quadrini (2012).

For the Phillips curve slope (ξ) and the inflation inertia parameter (ς), we adopt the calibration from Fornaro and Wolf (2023). Specifically, we set $\xi = 0.19$, reflecting a Calvo wage adjustment model in which wages have a 25% probability of being reset each quarter, consistent with the estimates from Beraja, Hurst, and Ospina (2019). The inflation persistence parameter, ς, is calibrated to 0.5, aligning with the estimates from Barnichon and Mesters (2020).

Table 1

PARAMETERS

Parameter	Description	Value
	External Calibration	
φ_π	Taylor rule coefficient	1.1
κ	Dividend Adjustment Cost Coeff.	0.146
τ	Tax advantage	0.35
ξ	Phillips curve parameter	0.19
ς	Inflation inertia parameter	0.5
	Internal Calibration	
β	Discount factor	0.99
α	Intermediate good share	0.02
χ	Quality of R&D Investment	1.01
	Estimated Parameter	
θ	Borrowing Constraint Tightness	1.24

NOTE. This table outlines the model parameters. The first category lists externally calibrated parameters, based on existing literature. The second category includes internal parameters, adjusted to match empirical moments from the data. The third category presents the estimated tightness of the borrowing constraint, derived from the relationship between R&D and cash flow (21).

Internal calibration. We calibrate the parameters χ, α, and β by targeting three key moments of the full-employment steady state. The parameter χ is set to achieve a steady-state productivity growth rate of 2%. To match the ratio of innovation investment to GDP, which is around 2% in the U.S., we adjust the labor share in gross output, reflecting that R&D investment is a relatively small part of aggregate demand. We calibrate β to 0.995 to ensure a steady-state real interest rate of 2.5%. Furthermore, we set the labor share in gross output to $1 - \alpha$, aligning with the 2.6% R&D-to-GDP ratio, which reflects the long-term average of business R&D spending in the U.S.

Table 2
CALIBRATING BORROWING CONSTRAINT TIGHTNESS

	(1)	(2)	(3)
R&D Expenditures	-2.05***	-3.25***	-2.20***
	(0.30)	(0.25)	(0.30)
Constant	0.52***	0.54***	0.52***
	(0.01)	(0.01)	(0.01)
Observations	9579	9724	9712
R^2	0.769	0.176	0.748
Firm Fixed Effects	yes	no	yes
Firm Controls	yes	yes	yes
Time x Sector FE	yes	no	no

NOTE. This table presents the results from estimating specification (24), expressed as $\mathcal{CF}_{j,t} = \alpha_j + \beta_{s,t} + \gamma\mathcal{RD}_{j,t} + \mathbf{Z}_{j,t-1} + e_{j,t}$, along with its variants discussed in the text. α_j represents firm fixed effects, while $\beta_{s,t}$ captures time-sector fixed effects. $\mathbf{Z}$ includes firm-level covariates: leverage, size, dividend status, Tobin's Q, cash receipts, liquidity, collateral, sales growth, and the proportion of current assets. Standard errors are reported in parentheses. Asterisks indicate statistical significance (*** for $p < 0.01$, ** for $p < 0.05$, * for $p < 0.1$).

Estimating the Limits to Borrowing. This subsection focuses on estimating a crucial parameter of our model: the tightness of the borrowing constraint, θ. This parameter plays a key role as it influences how R&D expenditures impact cash flow and, in turn, how borrowing constraints affect innovation financing. Since θ is not directly observable, we employ a two-stage approach to infer its value.

In the first stage, we estimate the relationship between R&D expenditures and the cash flow available to firms using the following regression:

$$CF_{j,t} = \alpha_j + \beta_{s,t} + \gamma\mathcal{RD}_{j,t} + \mathbf{Z}_{j,t-1} + e_{j,t} \tag{24}$$

We focus on cash flow-based borrowers, as this group is most relevant for our analysis of the partial derivative of cash flow with respect to R&D investment. The coefficient of interest, γ, represents this partial derivative, as derived in (21). The dependent variable $CF_{j,t}$ represents the firm's cash flow, defined as the sum of operating income and changes in total borrowing.[12] The fixed effects α_j control for firm-specific characteristics, while $\beta_{s,t}$ accounts for sector-by-quarter fixed effects to control for time-varying sectoral differences. Additionally, $\mathbf{Z}$ includes firm-level covariates such as leverage, size, dividend status, Tobin's Q, cash receipts, liquidity, collateral, sales growth, and the proportion of current assets.

In the second stage, we use the model-derived expression (21) to back out θ from the estimated γ. Specifically, we employ the following expression:

$$\gamma = \frac{-1}{1 - \frac{\theta}{R}} \tag{25}$$

where R represents the real interest rate in the steady state, set at 2.5%. Using equation (25), we infer θ based on the estimated γ, allowing us to quantify the tightness of the borrowing constraint. This yields a value of $\theta = 1.24$. Full estimation results are provided in Table 2.

6 Hysteresis Effects of Monetary Policy

In this section, we examine how monetary policy can induce hysteresis effects. We begin by outlining how the model developed in Section 4 provides a theoretical foundation for the empirical findings discussed in Section 3. Specifically, we show that our endogenous growth model captures the short-term impact of monetary policy on innovation, which ultimately drives long-term productivity growth.

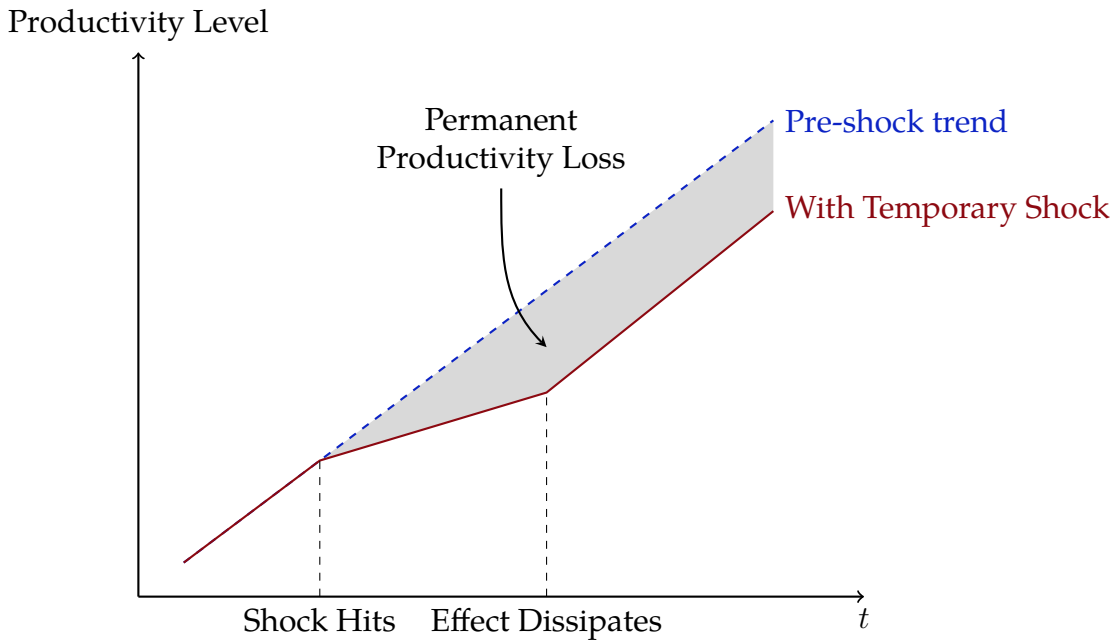
Definition of Hysteresis. To clarify, hysteresis refers to the lasting effects of a temporary shock. In our framework, growth is endogenous, meaning that any reduction in R&D investment directly reduces productivity growth, which in turn affects overall economic growth. As such, a temporary slowdown in R&D investment, regardless of the underlying cause, can lead to a permanent loss in productivity levels, even after the initial shock dissipates.

Hysteresis, therefore, describes the persistence of a temporary shock's effects beyond the resolution of its initial cause. As illustrated in Figure 5, the economy moves along

[12] See Appendix A.3 for precise definitions.

Figure 5

PERMANENT LEVEL EFFECT OF A TEMPORARY SHOCK



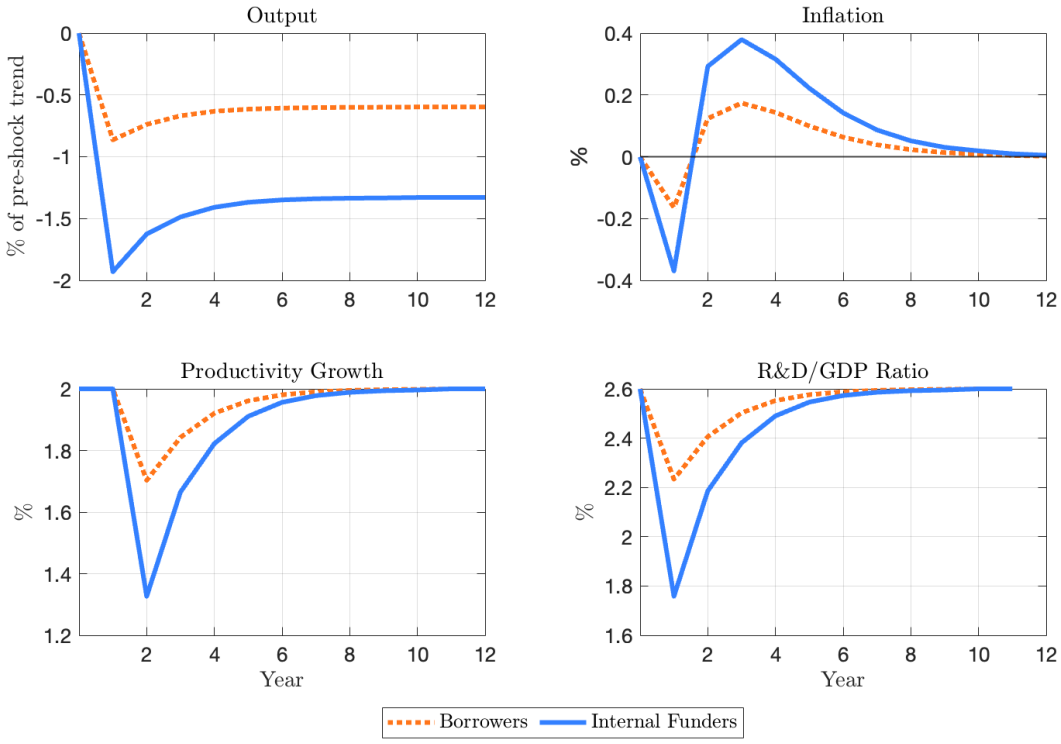
NOTE. The figure illustrates the permanent productivity loss following a temporary decline in the productivity growth rate. After the shock, productivity growth slows, causing the overall level to increase at a reduced pace. Although growth eventually returns to its original rate, the economy continues growing from a lower base, leading to a persistent productivity gap. The shaded region highlights this permanent loss in productivity, as the economy never fully regains the trajectory it would have followed without the shock.

a flatter growth path following a shock.[13] While the shock's direct effects may eventually dissipate and the growth rate returns to its original, steeper trajectory, the economy continues to expand from a permanently lower productivity base. This creates a lasting productivity gap. The shaded region in the figure represents this permanent productivity loss, demonstrating that despite the recovery in growth rates, the economy never fully returns to its pre-shock growth path. This dynamic highlights the significant long-term consequences that short-term disruptions can have on the economy.

Dynamic Transmission of Monetary Policy Shocks. Figure 6 illustrates the dynamic transmission of a contractionary monetary policy shock on output, inflation, total factor productivity (TFP), and R&D investment. The impulse response functions are shown for two cases: R&D financed by retained earnings (blue) and R&D financed by both retained

[13] Note that Figure 5 is for illustrative purposes only. The actual model-generated figure is provided in the Appendix E.

Figure 6
Monetary Policy Shock



Note. The figure illustrates the impulse responses of output, inflation, productivity, and R&D investment to a contractionary monetary policy shock. The responses are compared between borrowers (dotted orange line) and internal funders (solid blue line). Output, R&D expenditures, and productivity are shown as deviations from their pre-shock trends. The monetary policy shock is modeled as an innovation to the Taylor rule, $\epsilon_t^m = 0.0025$, with a decay rate of $\rho_m = 0.5$. The response is computed as the perfect foresight transition following a series of unexpected shocks, starting from the steady state.

earnings and cash flow-based borrowing (orange). Specifically, in the former case, θ in (L.14) is set to 0, effectively prohibiting any borrowing. In the latter case, it is set to 1.24, enabling a limited amount of borrowing.

In our model, an increase in the policy rate reduces demand, which leads to a decline in R&D investment and, hence, a slowdown in productivity growth. This reduction in productivity growth decreases the economy's potential output in the long run, illustrating the non-neutrality of monetary policy. However, the extent of these supply-side effects depends on how R&D is financed.

In the absence of access to cash flow-based borrowing, firms rely solely on retained earnings to finance their R&D. This means they are credit rationed and must fund innovation exclusively through sales revenue. During economic downturns, lower sales lead

to less funds which translates into decreased R&D spending, which amplifies the decline in productivity and output. This magnifies the negative effects of monetary policy shocks on long-term growth, as reduced R&D investment slows future productivity growth.

On the other hand, when firms have access to cash flow-based borrowing, they can better smooth their R&D investment over time, even during downturns. Borrowing allows firms to maintain investment in innovation, which dampens the impact of falling demand on productivity and output. In this case, the exposure to short term fluctuations weakens, as firms are less dependent on immediate cash flows to finance R&D. Therefore, the link between the business cycle and trend growth becomes less pronounced, and productivity growth remains more stable over time.

It is also important to comment on the response of inflation, given that monetary policy's primary objective is price stability. Our findings suggest that when growth is endogenous, monetary tightening can have unintended consequences by adversely affecting productivity, leading to hysteresis effects. While monetary tightening initially reduces inflation, its impact changes over time. In the medium term, contractionary monetary policy can become inflationary as productivity declines. This aligns with the findings of Fornaro and Wolf (2023), which show how tight monetary policy can be "self-defeating" in the medium term.

However, access to cash flow-based borrowing dampens the response of inflation as well. In the short run, it reduces the impact on inflation by dampening the fall in output. In the medium run, it mitigates the decline in trend productivity, reducing inflationary pressures. Our results highlight the role of financial frictions in the transmission.

Quantitatively, our model generates a peak response of 0.5% in output and 0.2% inflation following a monetary policy shock, closely aligning with the estimates from Christiano et al. (2005), which report a comparable decline in both variables after a similar change in the nominal interest rate. However, unlike their model, ours does not produce the typical hump-shaped responses in output and inflation. To capture these dynamics, we could introduce adjustment costs for R&D investment and possibly labor, as R&D adjustment costs are known to be high (Peters and Taylor, 2017). Exploring the interaction between real adjustment costs, financial frictions, and monetary policy shocks presents a promising area for future research, which we intend to pursue. While these extensions are valuable, they fall outside the scope of this paper, and we believe our core findings would remain qualitatively unchanged.

7 Policy Implications

The short-term effects of stabilization policies are well-documented, yet the long-term impacts of monetary policy have received comparatively less attention. Since the onset of the COVID-19 pandemic, economies worldwide have been hit by a series of negative supply shocks, and central banks have raised interest rates to fight the surge in inflation. While academic and policy debate continues on the potential lasting effects of these policy actions, the research in this area remains limited. This section seeks to contribute to this debate by analyzing how changes in monetary policy can affect long-term economic outcomes, particularly in the wake of a positive cost-push shock.

The central question we address here is: If monetary policy affects innovation, what are the implications for its broader implementation? While our empirical analysis in Section 3 relies on monetary policy shocks for the exercises, the underlying mechanisms also apply to the systematic aspects of monetary policy. Specifically, as policymakers respond to cost-push or supply shocks, changes in demand and financial conditions may impact innovation activities differently across firms, especially depending on the extent of cash flow-based borrowing.

To shed light on this, we first explore how central bank mandates affect the magnitude of these hysteresis effects. Then we examine the role of financial development in shaping the hysteresis effects of monetary policy.

7.1 Conduct of Monetary Policy

As previously discussed, temporary shocks in our framework can induce hysteresis through a temporary slowdown in R&D investment. But how does the systematic response of monetary policy to inflation affect long-term productivity? To address this, we analyze the implications of our model under a positive cost-push shock. We compare two types of Taylor rules in this context. The first rule follows a strict inflation-targeting approach, where the monetary authority adjusts the nominal interest rate i_t according to the following rule:

$$\log(1 + i_t) = \log(1 + \bar{i}) + \phi_\pi \pi_t. \tag{26}$$

Here, ϕ_π controls the central bank's responsiveness to inflation and $\bar{i}$ is the steady state interest rate. The second rule adopts a dual mandate, where the central bank targets both inflation and the output gap, adjusting the nominal interest rate based on the following rule:

$$\log(1+i_t) = \log(1+\bar{i}) + \phi_\pi \pi_t + \phi_L (L_t - \bar{L}) \tag{27}$$

In this case, ϕ_L denotes the weight placed on the output gap, measured by the deviation of employment L_t from its steady-state level $\bar{L}$. Figure 7 illustrates the results, with blue and green lines representing strict inflation-targeting rules in borrower and internal-funder economies, respectively. Meanwhile, red and yellow lines correspond to dual-targeting rules (inflation and output) in the same two contexts.

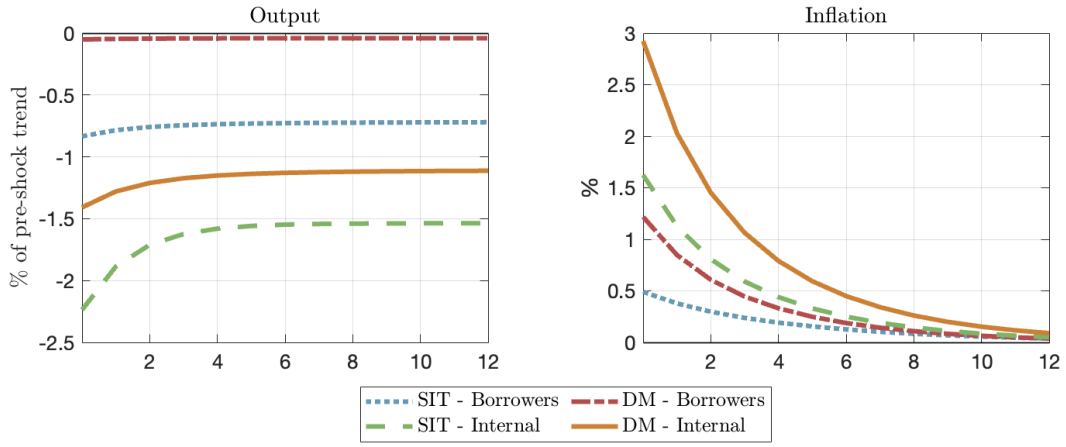
As expected, in all cases, a positive cost-push shock leads to a decline in output and a rise in inflation. The shock is trade-off inducing from the perspective of the monetary policymaker: when the central bank raises interest rates to fight inflation, it amplifies the negative impact on output, worsening the hysteresis effects. This phenomenon is also demonstrated by Fornaro and Wolf (2023), who show that if the central bank responds to a negative supply shock by hiking rates to reduce inflation, it amplifies the shock's negative effects on both output and productivity. Reducing inflation today results in lower future productivity due to reduced innovation, which in turn leads to higher medium-term inflation.

Fornaro and Wolf (2023) suggest that a mix of monetary tightening and fiscal subsidies can help achieve a balance, where the negative impact of interest rate hikes on R&D investment is mitigated by subsidies. This approach could allow for control of both short- and medium-term inflation. However, our results go further, demonstrating that when firms have access to finance, it is possible to mitigate these hysteresis effects without the need for fiscal intervention. Access to external financing allows firms to smooth their innovation activities even when faced with higher interest rates, preserving R&D investment and thus long-term productivity. This reduces the central bank's trade-off between lowering inflation today and risking persistent inflation in the future, as financial access helps maintain potential output without triggering excessive inflationary pressure.

Moreover, a central bank with a dual mandate can fully offset the hysteresis effects of monetary policy, as seen in the red line. This aligns with the findings of Ikeda and Kurozumi (2019), who argue that a welfare-maximizing monetary policymaker should respond more strongly to output than to inflation due to the unintended short-term consequences of interest rate hikes. Our results support this, but we also show that under *conventional* Taylor rule parameters, even when the central bank places greater emphasis on inflation ($\phi_\pi = 1.25$) than the output gap ($\phi_L = 0.35$), access to cash-flow-based borrowing allows firms to mitigate hysteresis effects and preserve potential output with less inflationary pressure.

In this context, access to finance significantly diminishes the negative long-term consequences of monetary policy tightening. Unlike the argument where fiscal subsidies are required to offset the effects of monetary policy on R&D investment, firms with access to finance can smooth out their innovation activity despite higher borrowing costs. This suggests that well-developed financial markets can play a crucial role in cushioning the economy from the adverse productivity effects of inflation-targeting policies. We examine this in the next section.

Figure 7
COST-PUSH SHOCK



NOTE. The figure illustrates the impulse responses of output and inflation to a cost-push shock under two monetary policy frameworks. In the first scenario, the central bank follows a strict inflation-targeting rule, while in the second, it adopts a dual mandate. The blue and green lines represent strict inflation-targeting in borrower and internal-funder economies, respectively. The red and yellow lines depict the responses under a dual mandate with borrower and internal-funder economies, respectively. The responses are computed using a perfect foresight transition following a sequence of shocks, with the economy initially at its steady state.

8 Conclusion

This paper demonstrates that firms' ability to smooth innovation through access to external finance reduces the hysteresis effects of temporary shocks. Empirically, we find that firms with access to cash flow-based borrowing are better able to sustain R&D expenditures in the face of contractionary monetary policy shocks. We then develop a New Keynesian endogenous growth model that aligns with these empirical findings, showing that access to external finance mitigates the long-term scarring effects of monetary tightening on productivity. We argue that when firms can use cash flow-based borrowing to

finance innovation, central banks with a dual mandate –focused on both inflation and output– can offset the hysteresis effects without relying on fiscal measures.

These results highlight the importance of deep financial markets in shielding economies from the adverse long-term effects of interest rate hikes. This is particularly relevant in today's economic environment, where central banks worldwide have raised interest rates in response to inflation driven by severe negative supply shocks. Unlike previous periods of monetary tightening, this cycle is unfolding amid persistently low productivity growth in many advanced economies. Our analysis suggests that, without deep financial markets that allow firms to finance innovation externally, these economies could remain in a low-productivity growth environment for an extended period. While a detailed empirical investigation into these heterogeneities across countries lies beyond the scope of this paper, we believe this is a promising direction for future research.

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Appendix

Online Appendix

"Innovation, Financial Frictions, and Hysteresis Effects of Monetary Policy"

by Aydan Dogan and Ozgen Ozturk

A Data Appendix

This subsection outlines the macro and firm-level variables used in our empirical analysis. We define the variables and conduct the sample selection following established standards in the literature (Benigno and Fornaro, 2018; Ottonello and Winberry, 2020; Cloyne et al., 2023).

A.1 Macro Data

We use three main macroeconomic data sources. For the United States, we obtain macroeconomic data from the Federal Reserve Bank of St. Louis (FRED). For other advanced economies, we gather data from the Organisation for Economic Co-operation and Development (OECD). Additionally, we source the financial development index from the International Monetary Fund (IMF).

From FRED, we directly obtain the real, deseasonalised Research and Development (R&D) series, labeled `Y694RX1Q020SBEA`. We identify U.S. recessionary periods using the "NBER-based Recession Indicators for the United States from the Period following the Peak through the Trough" series, `USREC`. For other advanced economies, we calculate R&D intensity as the ratio of R&D expenditure to GDP, using OECD data.

A.2 Measuring Financial Development

We utilise data on financial development from the IMF's Financial Development Index database, based on the methodology detailed in the corresponding IMF working paper by Svirydzenka (2016). For clarity, we briefly summarise their approach below.

Financial development is measured using a set of indicators that capture the depth, access, and efficiency of both financial institutions and financial markets. The analysis

of financial institutions encompasses three dimensions: *Depth*, which measures size and influence through metrics like private-sector credit to GDP and insurance premiums; *Access*, which evaluates service availability via the number of bank branches; and *Efficiency*, which assesses performance using indicators such as net interest margin and return on assets.

Financial markets are similarly analyzed through *Depth*, capturing market size with metrics like stock market capitalization to GDP; *Access*, which measures inclusiveness by the percentage of market capitalization outside the top 10 firms; and *Efficiency*, reflected in the stock market turnover ratio.

These dimensions combine to form the *Financial Institutions Index* and the *Financial Markets Index*, together creating the comprehensive *Financial Development Index*.

A.3 Firm-level Data

This subsection outlines the firm-level quarterly variables from Compustat that are utilised in the empirical analyses of this paper. The definitions of these variables and their respective roles in the analysis, along with the sample selection methodology, adhere closely to established practices in the literature (Cloyne et al., 2023; Ottonello and Winberry, 2020; **?**). In summary, ratio variables are employed directly as provided in Compustat. In contrast, level variables are adjusted using the aggregate GVA deflator. Some Compustat variables are reported as cumulative figures for the firm's fiscal year; to convert these into quarterly series, we compute the first difference of these variables within each fiscal year. Additionally, if a data series contains a single missing observation, we estimate it through linear interpolation. However, if multiple consecutive observations are missing, no imputation is performed.

Variable Definitions The firm-level variables employed in our empirical analyses are summarised below, with corresponding Compustat variable codes included where available.

1. **Size:** Defined as the logarithm of total real assets (`ATQ`).
2. **Real Sales Growth:** Measured by the log difference in sales (`SALEQ`).
3. **Dividend-Paying Status:** A binary variable that equals one for firm-quarter observations when dividends are paid on preferred stock (`DVPQ`), and zero otherwise.
4. **Cash Flow:** Defined as EBITDA (`OIBDPQ`), deflated by the aggregate GVA deflator.

5. **R&D Expenditures:** Calculated as the ratio of R&D expenditures (XRDQ) to total assets (ATQ).

6. **Leverage:** Measured as the ratio of total debt, including both long-term (DLTTQ) and short-term debt (DLCQ), to total assets (ATQ).

7. **Share of Current Assets:** Defined as the ratio of current assets (ACTQ) to total assets (ATQ).

8. **Cash Flow Available to the Firm:** Calculated as the ratio of the sum of EBITDA (OIBDPQ) and the change in total debt stock (DLCQ + DLTTQ - L.DLCQ - L.DLTTQ) to total assets (ATQ).

9. **Tobin's Q:** Following Cloyne et al. (2023), Tobin's Q is defined as the ratio of total assets at market value to total assets. Market value is computed as the sum of total assets (ATQ), the market value of common shares outstanding (PRCCQ $\times$ CSHOQ), and deferred taxes and investment tax credits (TXDITCQ) minus common equity (CEQQ).

Sample Selection We apply the following sample selection criteria for our empirical analysis.

1. We exclude firms in the finance, insurance, real estate (FIRE) sectors, as well as public administration.

2. We remove firms that are not incorporated in the United States.

3. We further drop firm-quarter observations based on the following conditions to eliminate nonsensical cases:

   (a) Negative capital or assets.

   (b) Acquisitions exceeding 5% of assets, to exclude mergers and acquisitions.

   (c) R&D expenditure spells shorter than 16 quarters.

   (d) Leverage greater than 10 or negative leverage.

   (e) Quarterly real sales growth above 1 or below -1.

   (f) Negative sales or liquidity.

After applying these selection criteria, we winsorise leverage and liquidity observations at the top and bottom 1% of the distribution to reduce the impact of outliers.

B Additional Empirical Exercises

B.1 How do firms finance R&D?

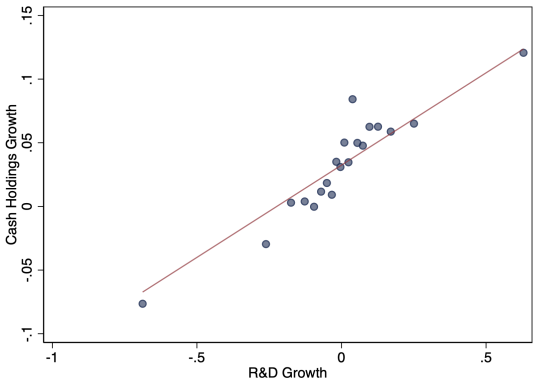
We begin by exploring the correlations between R&D expenditures and firms' financing strategies captured by three key financial metrics: (i) cash holdings, (ii) equity growth, and (iii) debt growth. This analysis aims to ensure our findings are consistent with existing literature and not unique to the matched Compustat-DealScan dataset. To illustrate these relationships, we present scatter plots comparing R&D expenditures with each of these financing strategies. Related regressions with more formal approach can be found in Appendix B where we present the effects of R&D on debt, equity, and cash holdings in Table B.2.

Cash Holdings. Falato et al. (2022) and Corrado et al. (2022) highlight that R&D expenditures are positively correlated with cash holdings. This relationship arises because intangible investments, such as R&D, are difficult to use as collateral due to their limited pledgeability. In frictional capital markets, where external financing incurs high spreads (above the risk-free rate), firms tend to increase their precautionary cash reserves to ensure they have sufficient liquidity for future intangible investments. As illustrated in Figure B.1 - Panel (a), our data aligns with this view, showing a positive correlation between R&D growth and cash holdings growth.

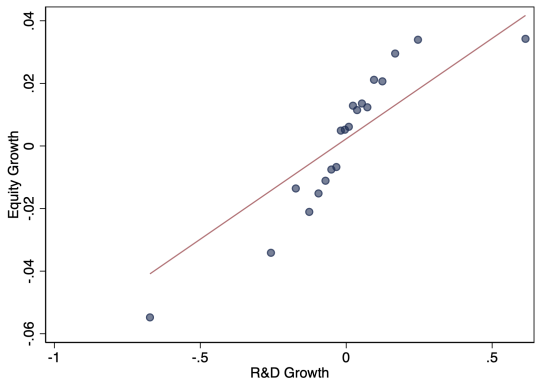
Equity financing. Brown et al. (2013, 2012) demonstrates that easier access to stock market financing is associated with significantly higher long-term R&D investment rates. In line with this, Bianchi et al. (2019), through a theoretical framework, highlights the impact of equity financing shocks on R&D investment. Our data, as depicted in Figure B.1 - Panel (b), corroborates this perspective, revealing a positive correlation between equity growth and R&D expenditure growth.

Debt Financing. Our data reveals a positive, unconditional correlation between debt growth and R&D expenditure growth, as shown in Figure B.1 - Panel (c). This new finding distinguishes our work from existing literature by empirically establishing a relationship that has only been suggested in theoretical frameworks thus far. Specifically, while prior research, such as Queralto (2020), demonstrates this connection through credit supply channels within a model, we are the first to empirically validate this positive association between debt growth and R&D expenditures. Building on insights from Falato et al.

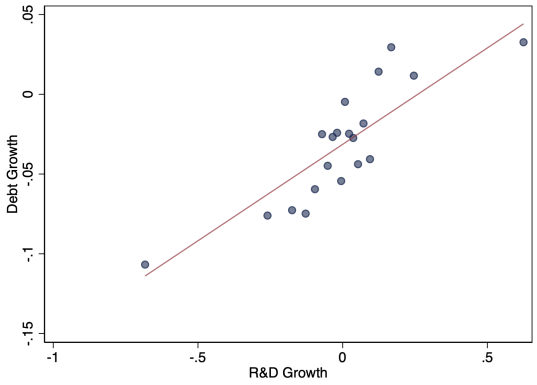
Figure B.1
R&D Growth vs. Financing Sources



(a) R&D vs. Cash Holdings



(b) R&D vs. Equity



(c) R&D vs. Debt

Note. This figure reports the binscatter plots between R&D expenditures growth and firms' financing strategies captured by three key financial metrics: (i) cash holdings growth, (ii) equity growth, and (iii) debt growth.

(2022) and Corrado et al. (2022), who emphasize the limitations of collateral-based financing for intangible investments, our finding suggests that firms may increasingly turn to debt financing, potentially through cash flow-based borrowing, to support R&D investment, circumventing the limitations of traditional collateral-based finance. Our results highlight an important channel by which firms can sustain innovative activities through debt, reinforcing the flexibility offered by cash flow-based credit.

Table B.1
DESCRIPTIVE STATISTICS

	Mean	St. Dev.	P25	Median	P75
	Cash flow-Based Borrowers				
[gray]0.9 Total Assets (logs)	6.856	1.572	5.907	6.880	7.971
Leverage	0.281	0.221	0.131	0.251	0.380
[gray]0.9 Sales Growth	0.010	0.183	-0.046	0.017	0.077
Liquidity	0.028	0.037	0.013	0.028	0.045
[gray]0.9 Sales/Employee	80.816	59.725	47.160	67.006	93.874
$R\&D$ Expenditures	0.015	0.015	0.005	0.010	0.019
[gray]0.9 Cash flow Volatility (10^{-3})	30.632	51.328	8.248	17.409	34.185
R&D Volatility (10^{-3})	1.789	3.617	0.399	0.834	1.773
[gray]0.9 **Observations**	10444				
	Non-Cash flow-Based Borrowers				
Total Assets (logs)	6.909	2.224	5.346	6.876	8.451
[gray]0.9 Leverage	0.277	0.226	0.130	0.243	0.365
Sales Growth	0.013	0.203	-0.053	0.018	0.086
[gray]0.9 Liquidity	0.024	0.044	0.007	0.027	0.044
Sales/Employee	75.519	68.885	36.696	57.418	89.041
[gray]0.9 $R\&D$ Expenditures	0.018	0.019	0.006	0.012	0.025
Cash flow Volatility (10^{-3})	38.072	75.557	7.595	18.037	38.567
[gray]0.9 R&D Volatility (10^{-3})	2.656	4.922	0.443	1.077	2.604
Observations	15893				

NOTE. Summary statistics for asset-based and cash flow-based contracts in the sample. The sample period is from 1997Q1 and 2017Q3. Asset pledgeability refers to the ratio of tangible fixed assets to total assets as in Dinlersoz, Kalemli-Ozcan, Hyatt, and Penciakova (2018) and Cloyne et al. (2023). Profitability is measured as Return-on-Assets as widely used in corporate finance literature. Loan spread is measured in percentage points. The sample consists of 2,236 firms of which 614 firms are asset based borrowers and 1602 are cash flow based borrowers. There are 30,591 loans and 11,457 packages.

B.2 Relative Sensitivity to Monetary Policy Shocks

To formally assess the statistical differences between the two groups, we include Jarociński and Karadi (2020) monetary policy shocks as a separate regressor and interact them with the indicator variable $\mathcal{I}^{rest}_{j,t-1}$, which equals 1 if a firm is not a cash flow-based borrower at time $t-1$.[14] This approach, while limiting our ability to track the average effect of monetary policy, allows us to interpret the significance of the estimated impulse responses for the non-cash flow-based borrowing group as a test of whether their behaviour diverges from that of cash flow-based borrowers.

B.3 Firms' Funding Strategies to Finance R&D

In Section 3.1, we explore the unconditional relationships between R&D expenditures and firms' financing choices, using three main financial indicators: (i) cash holdings, (ii) equity growth, and (iii) debt growth. This appendix takes a more rigorous approach to show that these relationships hold even when accounting for an extensive set of control variables.

To assess how R&D expenditures influence debt, equity, and cash holdings, we estimate the following specification:

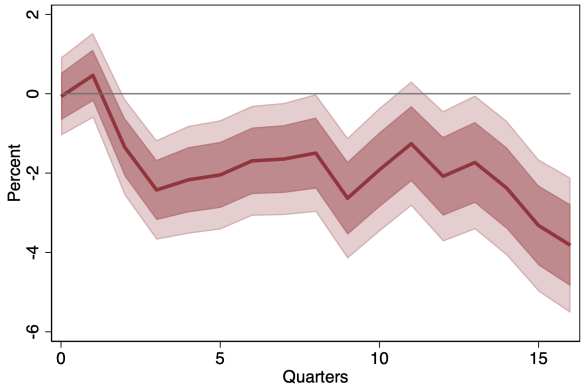
$$y_{j,t} = \alpha_j + \beta_{s,t} + \gamma\left(\mathcal{RD}_{j,t}\right) + \Gamma_p \mathbf{Z}_{j,t-1} + e_{j,t} \tag{B.1}$$

The dependent variable $y_{j,t}$ represents debt, equity, or cash holdings. On the right-hand side, α_j captures firm fixed effects, while $\beta_{s,t}$ accounts for sector-by-quarter fixed effects, controlling for sectoral heterogeneity in the impact of R&D expenditures over time. The coefficient of interest, γ, measures the effect of R&D spending on the outcome variables. Notably, R&D expenditures are normalised by the firm's total assets to allow for cross-firm comparability. Lastly, $\mathbf{Z}$ represents a set of firm-level control variables, including leverage, size, dividend-paying status, Tobin's Q, cash receipts, liquidity, collateral, and the proportion of current assets.

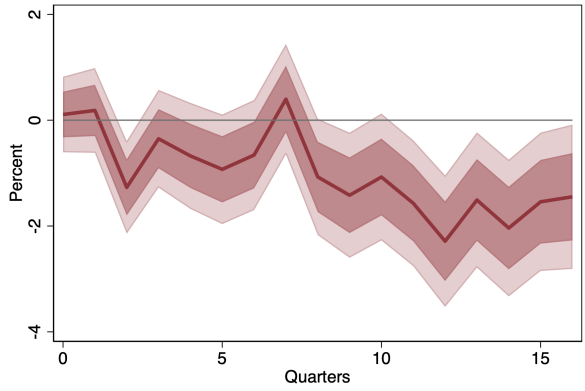
The findings in Table B.2 provide important insights into how firms finance their R&D activities. As shown in columns (1), (2), and (3), R&D spending is positively associated with debt, equity, and cash holdings. These results are consistent with the existing literature, which suggests that firms primarily rely on cash and equity to finance R&D efforts.

[14] Since monetary policy could influence a firm's decision to switch to a cash flow-based contract, to mitigate potential endogeneity we interact the indicator variable at time $t-1$ with the monetary policy shock at time t.

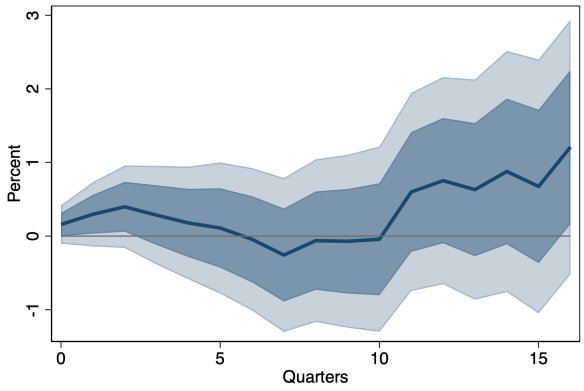
Figure B.2
IMPULSE RESPONSES: CFB VS. NON-CFB FIRMS



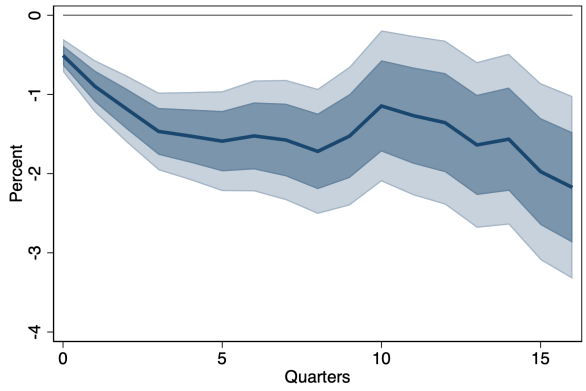
(a) **Cash flow:** CFB



(b) **Cash flow:** Non-CFB



(c) **R&D:** CFB



(d) **R&D:** Non-CFB

NOTE. We estimate the impulse responses (β_1^h and β_2^h) using the local projections specification in (3). $h = 0, 1, \ldots, 16$; ϵ_t^m are quarterly monetary policy shocks from Jarociński and Karadi (2020). $\mathcal{I}_{j,t-1}^{cfb}$ equals 1 if a firm is a CFB borrower at $t-1$; $\mathcal{I}_{j,t-1}^{non-cfb}$ equals 1 otherwise. $\mathbf{Z}$ and $\mathbf{X}$ include the same firm-level and aggregate controls as in Figure 2. Confidence intervals are 90%, with Driscoll-Kraay standard errors.

Table B.2
Impact of R&D on Debt, Equity, and Cash

	(1) Debt	(2) Equity	(3) Cash
$\mathcal{R}\&\mathcal{D}$	0.02***	0.44***	0.12***
	(0.00)	(0.02)	(0.01)
Observations	30368	27090	27104
R^2	0.083	0.109	0.057
Firm Controls	yes	yes	yes
Time x Sector FE	yes	yes	yes
Time x Region FE	no	no	no

NOTE. This table shows the result from estimating (B.1), $y_{j,t} = \alpha_j + \beta_{s,t} + \gamma\left(\mathcal{RD}_{j,t}\right) + \Gamma_p \mathbf{Z}_{j,t-1} + e_{j,t}$, where $y_{j,t}$ represents the dependent variable, focusing on net debt issuance, equity issuance, and cash holdings. α_j is the firm fixed effects, $\beta_{s,t}$ is the sector s by quarter t fixed effects. R&D spending is normalised by the firm's total assets. **Z**, include firm-level covariates which are leverage, size, dividend paying status, Tobin's Q, cash receipts, liquidity, collateral, and the proportion of current assets. The asterisks denote statistical significance ($***$ for $p < 0.01$, $**$ for $p < 0.05$, $*$ for $p < 0.1$).

Moreover, we find a positive relationship between R&D spending and debt growth, indicating that firms may turn to cash flow-based borrowing despite the difficulties in securing external financing for R&D. This challenge arises because R&D investments are inherently risky, with uncertain outcomes, and their intangible nature makes them difficult to collateralise. As a result, firms may prefer cash flow-based financing over traditional collateral-backed debt to support their R&D activities.

Finally, our results remain robust after accounting for region and time fixed effects, as shown in columns (4), (5), and (6).

C Derivations

The profit maximization problem of final good producers implies a demand function for the intermediate good x_{jt}:

$$P_t \alpha \left(L_t\right)^{1-\alpha} \left(A_{jt}\right)^{1-\alpha} x_{jt}^{\alpha-1} = P_{jt}$$

where P_{jt} is the nominal price of intermediate input j. Solving for x_{jt} yields

$$x_{jt} = \left(\frac{\alpha P_t}{P_{jt}}\right)^{\frac{1}{1-\alpha}} L_t A_{jt}.$$

The intermediate goods producer's profit maximization problem involves choosing P_{jt} to maximise the profit $\pi_{jt} = (P_{jt} - P_t)x_{jt}$. Substituting x_{jt} from the demand function, we obtain:

$$\pi_{jt} = (P_{jt} - P_t) \left(\frac{\alpha P_t}{P_{jt}} \right)^{\frac{1}{1-\alpha}} L_t A_{jt}. \tag{C.1}$$

To find the optimal price P_{jt}, we take the first-order derivative of π_{jt} with respect to P_{jt}:

$$\frac{\partial \pi_{jt}}{\partial P_{jt}} = \left(\frac{\alpha P_t}{P_{jt}} \right)^{\frac{1}{1-\alpha}} L_t A_{jt} + (P_{jt} - P_t) \frac{1}{1-\alpha} L_t A_{jt} \left(\frac{\alpha P_t}{P_{jt}} \right)^{\frac{\alpha}{1-\alpha}} \left(\frac{-\alpha P_t}{P_{jt}^2} \right) = 0.$$

Simplifying further, we obtain:

$$\left(\frac{\alpha P_t}{P_{jt}} \right)^{\frac{1}{1-\alpha}} L_t A_{jt} \left(1 + \frac{P_{jt} - P_t}{1-\alpha} \left(\frac{-1}{P_{j,t}} \right) \right) = 0.$$

Rearranging terms, we find the optimal pricing condition:

$$P_{jt} = \frac{1}{\alpha} P_t.$$

D Additional Model Implications

Future effects of R&D on cash flow. For the future effects of investment, as defined in (20), I_{jt} does not appear explicitly in the sales revenue equation for the current period, however, it impacts future sales revenue by improving future productivity A_{jt+1} (recall that productivity law of motion is $A_{jt+1} = A_{jt} + \chi I_{jt}$). Consequently, the partial derivative of current sales revenue with respect to I_{jt} is zero, but the investment I_{jt} has a positive indirect effect on future sales revenue through its influence on A_{jt+1}. To express the relation between CF_{jt+1} and I_{jt}, we start with the definition of cash flow available to the firm for $t+1$:

$$CF_{jt+1} = \omega A_{jt+1} L_{t+1} - I_{jt+1} + \frac{B_{jt+2}}{R_{t+1}} - B_{jt+1}$$

Substituting A_{jt+1} and taking the derivative of $CFjt+1$ with respect to I_{jt} reads:

$$\frac{\partial CF_{jt+1}}{\partial I_{jt}} = \omega \chi Z_{t+1} L_{t+1} > 0$$

For internal funders, iterating one period forward, cash flow at time $t+1$ is $CF_{jt+1} = \omega A_{jt+1} Z_{t+1} L_{t+1} - I_{jt+1}$. Substituting A_{jt+1} from the law of motion into the cash flow formula at time $t+1$:

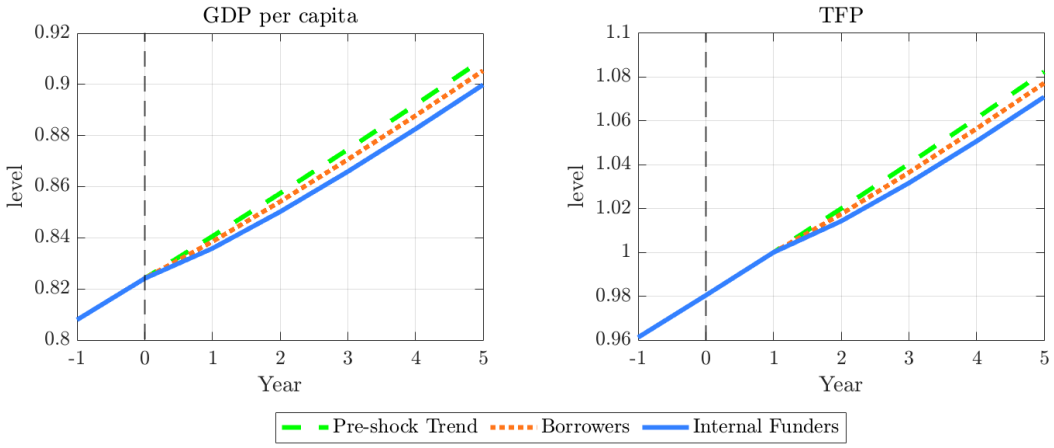
$$CF_{jt+1} = \omega(A_{jt} + \chi I_{jt}) Z_{t+1} L_{t+1} - I_{jt+1}$$

Taking the partial derivative of CF_{jt+1} with respect to I_{jt} shows how current investment affects future cash flow by boosting productivity:

$$\frac{\partial(\text{EBITDA}_{jt+1})}{\partial I_{jt}} = \omega\chi Z_{t+1} L_{t+1} > 0$$

E Further Results

Figure E.1
TFP AND GDP PER CAPITA



NOTE. The figure illustrates the levels of GDP per capita and productivity. The magnitudes are compared between pre-shock trend (dashed green line), borrowers (dotted orange line) and internal funders (solid blue line). The monetary policy shock is modeled as an innovation to the Taylor rule, $\epsilon_t^m = 0.0025$, with a decay rate of $\rho_m = 0.5$. The response is computed as the perfect foresight transition following a series of unexpected shocks, starting from the steady state.

F Intangible capital discussion

Intangible capital can be broadly divided into two categories: internally generated intangible capital and externally acquired intangible capital. Each category has distinct characteristics, with different implications for an organization's strategy, recognition in financial reporting, and accounting treatment.

Internally generated intangible capital. Internally generated intangible capital refers to intangible assets developed within the firm, typically through its own research and development (R&D), employee expertise, and innovation. Examples include proprietary technologies, internally developed patents, trademarks, brand reputation, organizational culture, and employee skills. These assets give firms greater control, as they originate from internal capabilities. However, their development often requires significant investment in innovation and workforce training over extended periods. Current accounting standards (*i.e.* US GAAP regulations) require firms to expense R&D costs as they occur,

meaning these internally generated intangibles may not be fully reflected on the balance sheet.

Externally acquired intangible capital. Externally acquired intangible capital consists of intangible assets purchased or licensed from external sources. These include patents, copyrights, trademarks, and licenses, as well as brand names or customer relationships acquired through mergers and acquisitions. While these assets can be integrated more quickly than internally generated ones, their value may be more dependent on external factors, such as the seller's performance or market conditions. The cost of acquiring these assets is typically capitalised on the balance sheet, and firms can amortise the expense over the asset's useful life, providing clearer financial visibility than with internally generated intangibles.

G 2 Types of Firm World

G.1 Environment

We now develop an endogenous growth New Keynesian model to rationalise our empirical findings and examine how monetary policy affects the supply side of the economy through R&D investment. Our model builds heavily on the endogenous growth model developed in Benigno and Fornaro (2018) which in turn rooted in the seminal work of Aghion and Howitt (1992). Following Aghion and Howitt (1992), firms invest in R&D to enhance product quality. We adopt a vertical innovation approach because, as shown by Garcia-Macia et al. (2019), productivity growth is largely driven by improvements to existing products rather than by the creation of new ones. To introduce a role for monetary policy, we incorporate sticky wages, following the approach of Benigno and Fornaro (2018). Additionally, we extend the model to include cash flow-based borrowing constraints, as outlined by Drechsel (2023), and financial frictions on equity payouts, following Jermann and Quadrini (2012).

The model highlights three key features. First, productivity growth is endogenous and driven by firms' investments in innovation. Second, there are two types of firms that can invest in innovation; constrained firms, denoted by C, and relatively unconstrained firms, denoted by U. C firms face a strict borrowing constraint, preventing access to any external funding sources, they are "internal funders" and represent the firms without cash flow-based contracts in our empirical analysis. U firms on the other hand are able to finance their innovation activities by borrowing up to a certain proportion of their cash

flows. These firms are the "borrowers", and represent the " firms with cash flow-based contracts in our empirical analysis. Third, we introduce sticky wages, which create a role for monetary policy. The presence of nominal rigidities allows output to deviate from its potential level, enabling monetary policy to influence real economic variables.

Time is discrete and infinite. Each period is one year. The economy consists of households, firms, a government with a balanced budget, and a monetary authority. Within the production sector, a final goods producer sources inputs from intermediate firms. These intermediate firms can invest in innovation to enhance the quality of their products. In our model, U type of intermediate firms can borrow up to a proportion of their cash flows to invest in R&D while C firms face hard borrowing constraints, preventing access to any external funding sources.

G.1.1 Final Good Producer

There are two types of perfectly competitive final-goods firms, U and C denoted by i, produce Y^i combining labour, L and continuum of intermediate inputs by i type of firms, x^i_j with the following production function:

$$Y^i_t = (L^i_t)^{1-\alpha} \int_0^1 (A^i_{j,t})^{(1-\alpha)} (x^i_{j,t})^{\alpha} \, dj \tag{G.1}$$

Here, $A^i_{j,t}$ represents the productivity or quality of input $j \in [0,1]$ of an $i = U, C$ type of firm. The profit maximization problem for the final goods producer yields the following optimality condition:

$$x^i_{j,t} = \left(\frac{P^i_{j,t}}{P^i_t \alpha (L^i_t)^{1-\alpha} (A^i_{j,t})^{1-\alpha}} \right)^{1/(\alpha-1)} \tag{G.2}$$

which defines the demand for the intermediate input j produced type $i = U, C$ intermediate producer.

G.1.2 Intermediate Producers

Production. There are two types of intermediate input producers, constrained C and unconstrained, U denoted by i. Every intermediate input, j, is produced by a monopolist, x^i_j. Regardless of the quality of the product, one unit of final good of type i is required to manufacture one unit of intermediate good, which means that every type i intermediate producer faces the same marginal cost P^i_t. Taking into account this market structure, it is

optimal for the i type monopolist producing good j to set the price:[15]

$$P_{j,t}^i = \frac{1}{\alpha} P_t^i \tag{G.3}$$

With some algebraic manipulation, the profits generated by this i type of monopolist for good j can be expressed as:

$$P_{j,t}^i x_{j,t}^i - P_t^i x_{j,t}^i = P_t^i \omega A_{j,t}^i L_t^i \tag{G.4}$$

where

$$\omega = \left(\frac{1}{\alpha} - 1\right) \alpha^{\frac{2}{1-\alpha}}.$$

Innovation Process. In equilibrium, each period sees a positive level of research activity directed toward each intermediate good j by type i producer. In this framework, an i type intermediate producer investing $I_{j,t}^i$ units of the final good experiences productivity growth governed by the following law of motion:

$$A_{j,t+1}^i = A_{j,t}^i + \chi^i I_{j,t}^i \tag{G.5}$$

where $\chi^i > 0$ represents the effectiveness of R&D investment. The inclusion of $A_{j,t}^i$ on the right-hand side reflects the idea that advancing more sophisticated and complex products demands higher investment (*i.e.* higher R&D expenditures), ensuring a stationary growth process over time.

R&D Financing. Constrained and unconstrained firms are different in the way they finance the R&D expenditure. Both constrained and unconstrained firms can issue equity but equity issuance is costly. We follow Jermann and Quadrini (2012) assume that equity adjustment has the following form:

$$\Phi(D_{j,t}^i) = D_{j,t}^i + \kappa(D_{j,t}^i - \overline{D}_j^i)^2, \tag{G.6}$$

where $\overline{D}_j^i$ denotes the target payout level, which corresponds to the steady-state value of $D_{j,t}^i$. This cost can be interpreted in two main ways: first, as direct financial expenses related to share buybacks and equity issuance; and second, as reflecting the tendency of managers to smooth dividends over time. Mechanically, this cost structure allows us to capture the dynamics of how quickly firms can alter their financing methods when

[15] For the detailed discussion of how intermediate producers set their price, please refer to Appendix C.

financial conditions shift. With a lower κ, firms can swiftly absorb the shocks through changes in equity. As κ increases, the substitution between debt and equity becomes more expensive, leading firms to modify their funding sources more slowly.

R&D Financing. *Unconstrained firms* can access debt financing through one-period risk-free bonds, denoted as $B^U_{j,t+1}$. The effective gross real interest rate faced by firms is R^b_t. Following Drechsel (2023), which builds on the framework of Jermann and Quadrini (2012), the interest payments of firms benefit from a tax advantage τ, such that $R^b_t = (1+r_t)(1-\tau)$, where $1+r_t = R_t$ is the gross real interest rate received by the lenders. The tax advantage τ steers firms to favor debt over equity, encouraging them to borrow up to the limit of their constraints.

Borrowing is subject to the following constraint:

$$\frac{B^U_{j,t+1}}{R_t} \leq \theta \, CF^U_{j,t}, \tag{G.7}$$

where $CF^U_{j,t} = \frac{P^U_{j,t}}{P_t}(\omega A^U_{j,t} L^U_t) - I^U_{j,t}$.[16]

The earnings-based borrowing constraint (L.14) limits real debt to a multiple θ of current earnings. We focus on the debt-to-earnings formulation, as this covenant type is the most prevalent in loan agreements, surpassing others such as the coverage ratio (Drechsel, 2023).

Constrained firms, face strict borrowing constraints, specifically $\theta = 0$, indicating that external borrowing is not possible under this condition.

$$\frac{B^C_{j,t+1}}{R_t} \leq 0. \tag{G.8}$$

Flow of Funds. The corporate finance structure of *unconstrained firms* who can borrow externally is governed by the following flow of funds constraint:

$$\Phi(D^U_{j,t}) + R^b_t B^U_{j,t} = \frac{P^U_{j,t}}{P_t}(\omega A^U_{j,t} L^U_t) - I^U_{j,t} + B^U_{j,t+1} \tag{G.9}$$

where equity payouts $\Phi(D^U_{j,t})$, new debt issuance $B^U_{j,t+1}$, earnings $\omega A^U_{j,t} L^U_t$, investment expenditures $I^U_{j,t}$, and the cost of servicing debt $R^b_t B^U_{j,t}$ are all included.

[16] It is important to note that investment by unconstrained firms, $I^U_{j,t}$, and hence their productivity, $A^U_{j,t}$, depend on θ_j, as the tightness of the borrowing constraint affects the optimal investment level. For simplicity, we do not express $I^U_{j,t}$ as $I^U_{j,t}(\theta_j)$.

The flow of funds constraint of *constrained firms*, on the other hand, can be expressed as:

$$\Phi(D^C_{j,t}) = \frac{P^C_{j,t}}{P_t}(\omega A^C_{j,t} L^C_t) - I^C_{j,t} \tag{G.10}$$

including only equity payouts $\Phi(D^C_{j,t})$ in consumption units, earnings $\omega A^C_{j,t} L^C_t$, and investment expenditures $I^C_{j,t}$.

Intermediate Producer's Problem for Unconstrained. The objective of an unconstrained firm j is to maximise the expected stream of dividends, discounted by the discount factor of the household:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \frac{1}{\delta^t} D^U_{j,t}$$

subject to:

- Law of motion for productivity (L.12): $A^U_{j,t+1} = A^U_{j,t} + \chi^U I^U_{j,t}$
- Borrowing constraint (L.14): $\frac{B^U_{j,t+1}}{R_t} \leq \theta \left(\frac{P^U_{j,t}}{P_t}(\omega A^U_{j,t} L^U_t) - I^U_{j,t} \right)$
- Flow of funds constraint (L.16): $\Phi(D^U_{j,t}) + R^b_t B^U_{j,t} = \frac{P^U_{j,t}}{P_t}(\omega A^U_{j,t} L^U_t) - I^U_{j,t} + B^U_{j,t+1}$

Intermediate Producer's Problem for Constrained. For constrained types, the objective of the firm j remains the same: to maximise the expected stream of dividends, although a stricter borrowing constraint applies:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \frac{1}{\delta^t} D^C_{j,t}$$

subject to:

- Law of motion for productivity (L.12): $A^C_{j,t+1} = A^C_{j,t} + \chi^C I^C_{j,t}$
- Borrowing constraint (L.15): $\frac{B^C_{j,t+1}}{R_t} \leq 0$
- Flow of funds constraint (L.17): $\Phi(D^C_{j,t}) = \frac{P^C_{j,t}}{P_t}(\omega A^C_{j,t} L^C_t) - I^C_{j,t}$

G.1.3 Households

The representative household maximises expected lifetime utility from aggregate consumption, C_t:

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t log C_t$$

$\mathbb{E}$ is the expectations operator conditional on information at time t and $\beta \in (0,1)$ is the discount factor.

Households finance their expenditure through labor income, holdings of equity shares of firms, dividend payments, and non-state contingent bonds. The representative household's budget constraint is given by:

$$C_t + \frac{B_{t+1}}{R_t} + S_{t+1} p_t^s + T_t = w_t L_t + B_t + S_t (D_t + p_t^s)$$

where w_t is the real wage, R_t is the real interest rate. B_t is the holdings of real bonds, S_t are equity shares, D_t is dividend payments received from the holdings of shares, and p_t^s is the (real) market price of shares. Finally, T_t is the lump-sum taxes.

The aggregate consumption is a composite of constrained and unconstrained type of final goods and is aggregated with a Cobb-Douglas aggregator:

$$C_t = \left[\frac{(C_t^U)^\rho (C_t^C)^{1-\rho}}{\rho^\rho (1-\rho)^{(1-\rho)}} \right] \tag{G.11}$$

where ρ is the share of goods that unconstrained firms produce in the total aggregate consumption basket of the household.

The corresponding aggregate price index is:

$$P_t = (P_t^U)^\rho (P_t^C)^{1-\rho} \tag{G.12}$$

and the optimal demand for the output of each type of firm can be expressed as:

$$C_t^U = \rho \frac{P_t}{P_t^U} C_t \tag{G.13}$$

$$C_t^C = (1-\rho) \frac{P_t}{P_t^C} C_t \tag{G.14}$$

G.1.4 Wage Setting

Following Fornaro and Wolf (2023) which in turn is based on Galí and Gambetti (2020), we assume that nominal wages evolve according to

$$\frac{W_t}{W_{t-1}} = \bar{g}\left(\frac{L_t}{\bar{L}}\right)^{\xi} \pi_{t-1}^{\varsigma} \tag{G.15}$$

where π_{t-1}^{ς} denotes lagged price inflation and $\varsigma > 0$ controls the degree to which past inflation affects current wages. In this framework, higher employment exerts upward pressure on wage growth. Nominal wage rigidities serve two functions in the model. First, they create the potential for involuntary unemployment by maintaining positive wages, even when labor demand falls short of household labor supply. Second, these rigidities give monetary policy real traction, as sticky wages slow down price adjustments.

G.1.5 Government

The government budget constraint, expressed in real terms, is as follows:

$$T_t = \frac{B_{t+1}}{R_t(1-\tau)} - \frac{B_{t+1}}{R_t} \tag{G.16}$$

where T_t represents lump-sum taxes imposed on households in consumption units, and the right-hand side of Equation (L.8) corresponds to the tax subsidy provided to firms.

G.1.6 Monetary Policy

The central bank sets the nominal interest rate i_t based on a Taylor rule:

$$\log(1+i_t) = \log(1+\bar{i}) + \phi_\pi \pi_t + \epsilon_t^m \tag{G.17}$$

where ϕ_π represents the weight assigned to inflation in the central bank's response, and ϵ_t^m denotes the monetary policy shock.

G.1.7 Market Clearing

Market clearing for the final good is given by:

$$Y_t^i = C_t^i + \int_0^1 I_{j,t}^i \, dj + \int_0^1 x_{j,t}^i \, dj \tag{G.18}$$

This equation states that the total output of each type of final good in the economy, Y_t^i, is allocated across three uses: consumption (C_t^i), investment in R&D ($I_{j,t}^i$), and inter-

mediate inputs ($x^i_{j,t}$) used in production. Rearranging the equation, we separate out the intermediate goods as follows:

$$Y^i_t - \int_0^1 x^i_{j,t}\, dj = C^i_t + \int_0^1 I^i_{j,t}\, dj \tag{G.19}$$

Here, the left-hand side represents the total output less the inputs used in intermediate goods production, effectively giving us the output available for consumption and R&D investment. This quantity can be interpreted as the value-added output of each type, which reflects the goods and services produced for final use, excluding intermediate inputs.

Next, by substituting the production function for final goods and performing some algebraic manipulation, we can express value-added output as:

$$Y^i_t - \int_0^1 x^i_{j,t}\, dj = \psi A^i_t L^i_t \tag{G.20}$$

and the GDP of the economy is the sum of the value added output by each type of firm:

$$Y_t = \frac{P^U_t}{P_t} Y^U_t + \frac{P^C_t}{P_t} Y^C_t \tag{G.21}$$

$$L_t = L^U_t + L^C_t \tag{G.22}$$
$$D_t = D^U_t + D^C_t \tag{G.23}$$
$$B_t = B^U_t + B^C_t \tag{G.24}$$

where $B^C_t = 0$.

OR like Thomas Drechsel:

$$L_t = \rho L^U_t + (1-\rho) L^C_t \tag{G.25}$$
$$D_t = \rho D^U_t + (1-\rho) D^C_t \tag{G.26}$$
$$B_t = B^U_t \tag{G.27}$$

G.2 Generalised Balanced Growth Path (GBGP).

THIS PART IS OLD, UPDATE IT AND MAYBE PUT TO MAIN TEXT

In this model, constrained and unconstrained firms differ in their ability to finance R&D, leading to heterogeneous sectoral productivity growth. As a result, the economy does not converge to a traditional balanced growth path (BGP), in which all real variables

grow at a common constant rate and sectoral composition remains stable.

G.2.1 Definition

Instead, we adopt the weaker notion of a *generalised balanced growth path* (GBGP), consistent with the literature on structural transformation. Under a GBGP, the following conditions hold:

1. Sectoral productivity grows at constant, though potentially unequal, rates:

$$\frac{A^i_{t+1}}{A^i_t} = g^i, \quad \text{for } i \in \{U, C\}, \quad g^U \neq g^C.$$

2. The real interest rate is constant over time:

$$R_t = \bar{R}.$$

   This assumption is consistent with long-run empirical regularities in macroeconomic data (Kaldor facts), and ensures that intertemporal prices are stable in the long run despite structural transformation.

3. Key aggregate variables and firm-level quantities, when expressed in units normalised by an appropriate reference productivity level (e.g., A^i_t or the composite index A_t), converge to stationary stochastic processes. In particular, relative prices, consumption shares, sectoral outputs, and investment rates evolve over time but stabilise in normalised terms.

This GBGP framework allows for analytical tractability and consistent aggregation while capturing the heterogeneous innovation outcomes across constrained and unconstrained firms.

G.2.2 Normalization Strategy

THIS PART IS OLD, UPDATE IT AND MAYBE PUT TO MAIN TEXT To characterise a stationary equilibrium under heterogeneous sectoral productivity growth, we normalise all real variables by appropriate productivity indices. The normalization ensures stationarity of the transformed variables and consistency across equilibrium conditions.

Prices and Numeraire. We normalise all prices by the composite consumption price index $P_t = (P_t^U)^\rho (P_t^C)^{1-\rho}$, which is the dual to the CES aggregator used in final demand. This implies a numeraire choice of $P_t = 1$, and defines relative prices as $p_t^i = P_t^i / P_t$ for $i \in \{U, C\}$. These normalised prices enter demand, first-order conditions, and aggregation equations, ensuring internal consistency and stationarity in the presence of non-unitary sectoral productivity growth.

Consumption. Sectoral consumption C_t^i is normalised by sectoral productivity A_t^i, yielding $c_t^i = C_t^i / A_t^i$. This reflects the fact that consumption is measured in physical units of good i, and utility is defined over these real quantities. Aggregate consumption C_t, which is expressed in final-good units, is normalised by the composite productivity index A_t, so $c_t = C_t / A_t$.

Investment and Dividends. Investment $I_{j,t}^i$ and dividends $D_{j,t}^i$ are normalised by sectoral productivity A_t^i.

Sectoral Output. Sectoral output Y_t^i is normalised by sectoral productivity A_t^i, yielding $y_t^i = Y_t^i / A_t^i$. Output is produced in physical units of the sectoral good and must be quality-adjusted to ensure that production functions and market clearing conditions are stationary in the normalised system.

H Firms Choice: 13 Aug Aydan

Everything is measured in P except for sectoral output i.e. $\omega A_{j,t}^i L_t^i - I_{j,t}^i$

Constrained Firms. These firms cannot borrow. They solve:

$$\max_{\{D_{j,t}^C, I_{j,t}^C\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \frac{1}{\delta_t} D_{j,t}^C$$

subject to:

$$A_{j,t+1}^C = A_{j,t}^C + \chi^C I_{j,t}^C$$
$$D_{j,t}^C + \kappa (D_{j,t}^C - \bar{D}_j^C)^2 = \frac{P_t^C}{P_t} \left(\omega A_{j,t}^C L_t^C - I_{j,t}^C \right) \qquad \{\mu_t^C\}$$

where $R_{t-1} = \frac{\delta_t}{\delta_{t-1}}$

Corresponding Lagrange

$$\mathscr{L}^C = \mathbb{E}_0 \sum_{t=0}^{\infty} \frac{1}{\delta_t} \left\{ D_{j,t}^C + \mu_t^C \left[\frac{P_t^C}{P_t} \left(\omega A_{j,t}^C L_t^C + \frac{A_{j,t}^C}{\chi^C} - \frac{A_{j,t+1}^C}{\chi^C} \right) - D_{j,t}^C - \kappa (D_{j,t}^C - \bar{D}_j^C)^2 \right] \right\}$$

First-order conditions:

- **Productivity:**
$$\frac{\mu_t^C}{\chi^C} = \frac{\pi_{t+1}^C}{\pi_{t+1}} \frac{\mu_{t+1}^C}{R_t} \left[\omega L_{t+1}^C + \frac{1}{\chi^C} \right]$$
Interpretation: The marginal value of productivity accounts only for future returns from higher output (no debt capacity), discounted by its own inflation.

- **Dividends:**
$$1 - \mu_t^C - \mu_t^C \cdot 2\kappa (D_{j,t}^C - \bar{D}_j^C) = 0$$
Interpretation: Same logic as for unconstrained firms — smooth dividend payouts are preferred.

Unconstrained Firms. Each firm $j \in [0,1]$ solves:

$$\max_{\{D_{j,t}^U, I_{j,t}^U, B_{j,t+1}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \frac{1}{\delta_t} D_{j,t}^U$$

subject to:

$$\begin{aligned} A_{j,t+1}^U &= A_{j,t}^U + \chi^U I_{j,t}^U \\ D_{j,t}^U + \kappa (D_{j,t}^U - \bar{D}_j^U)^2 + B_{j,t} &= \frac{P_t^U}{P_t} \left(\omega A_{j,t}^U L_t^U - I_{j,t}^U \right) + \frac{B_{j,t+1}}{R_t^b} \qquad \{\mu_t^U\} \\ \frac{B_{j,t+1}}{R_t} &\leq \frac{P_t^U}{P_t} \theta (\omega A_{j,t}^U L_t^U - I_{j,t}^U) \qquad \{\lambda_t^U\} \end{aligned}$$

where

$$R_t^b = R_t - R_t \tau + \tau$$

Corresponding Lagrange:

$$\begin{aligned} \mathscr{L}^U = \mathbb{E}_0 \sum_{t=0}^{\infty} \frac{1}{\delta_t} \Bigg\{ & D_{j,t}^U + \mu_t^U \left[\frac{P_t^U}{P_t} \left(\omega A_{j,t}^U L_t^U + \frac{A_{j,t}^U}{\chi^U} - \frac{A_{j,t+1}^U}{\chi^U} \right) + \frac{B_{j,t+1}}{R_t^b} - B_{j,t} - D_{j,t}^U - \kappa (D_{j,t}^U - \bar{D}_j)^2 \right] \\ & + \lambda_t \left[\frac{P_t^U}{P_t} \theta \left(\omega A_{j,t}^U L_t^U + \frac{A_{j,t}^U}{\chi^U} - \frac{A_{j,t+1}^U}{\chi^U} \right) - \frac{B_{j,t+1}}{R_t} \right] \Bigg\}. \end{aligned}$$

First-order conditions:

- **Productivity Condition.**

$$\frac{\mu_t^U + \theta\lambda_t}{\chi^U} = \frac{\pi_{t+1}^U}{\pi_{t+1}} \frac{\mu_{t+1}^U + \theta\lambda_{t+1}^U}{R_t} \left[\omega L_{t+1}^U + \frac{1}{\chi^U}\right].$$

- **Dividend Condition.**

$$1 - \mu_t^U [\, 1 + 2\kappa(D_{j,t}^U - \bar{D}_j^U)\,] = 0.$$

- **Borrowing Condition.**

$$\frac{\mu_t^U}{R_t^b} - \frac{\lambda_t}{R_t} = \frac{\mu_{t+1}^U}{R_t}.$$

I De-Trending

I.1 Some preliminaries

The corresponding aggregate price index is:

$$P_t = (P_t^U)^\rho (P_t^C)^{1-\rho} \tag{I.1}$$

and the aggregate consumption is a composite of constrained and unconstrained type of final goods and is aggregated with a Cobb-Douglas aggregator:

$$C_t = \left[\frac{(C_t^U)^\rho (C_t^C)^{1-\rho}}{\rho^\rho (1-\rho)^{(1-\rho)}}\right] \tag{I.2}$$

where ρ is the share of goods that unconstrained firms produce in the total aggregate consumption basket of the household.

I.2 Detrending

Let's derive the trends:

And the relative prices carry, relative productivities $\frac{P^U}{P^C} = \frac{A^C}{A^U}$.

C^C carries A^C and C^U carries A^U. Then we also know the trend for aggregate con-

sumption as it will be a Cobb-Douglas aggregate of the sectoral ones:

$$A_t = \left[(A_t^U)^\rho (A_t^C)^{1-\rho}\right]$$

From the CPI price index, we can derive the trend for relative prices:

$$\frac{P}{P^C} = \left[\frac{A^C}{A^U}\right]^\rho$$
$$\frac{P}{P^U} = \left[\frac{A^U}{A^C}\right]^{1-\rho}$$

From firm's optimal choice of prices, we can derive the trend for real wages:

$$\frac{W^C}{P} = \frac{P^C}{P}(1-\alpha)2^{\alpha/1-\alpha}A^C$$
$$\frac{W^U}{P} = \frac{P^U}{P}(1-\alpha)2^{\alpha/1-\alpha}A^U$$

Notice that, real wage grows by A:

$$\frac{W^C}{P} = \left[\frac{A^C}{A^U}\right]^{-\rho} A^C$$
$$\frac{W^U}{P} = \left[\frac{A^U}{A^C}\right]^{-1+\rho} (1-\alpha)2^{\alpha/1-\alpha}A^U$$

J Normalised equations

$$g_t = \left[(g_t^U)^\rho (g_t^C)^{1-\rho}\right] \tag{J.1}$$

$$c_t = \frac{C_t}{A_t}, \quad c_t^i = \frac{C_t^i}{A_t^i}, \quad d_{j,t}^i = \frac{D_{j,t}^i}{A_t^i}, b_t = \frac{B_t}{A_t}, w_t = \frac{W_t}{P_t A_t} \quad p_t^i = \frac{P_t^i A_t^i}{P_t A_t}, \quad x_{j,t}^i = \frac{X_{j,t}^i}{A_t}$$

where $x \in \{\lambda, \mu\}$

- Optimal demands:

$$\frac{C_t^C}{A_t^C} = (1-\rho)\frac{P_t}{P_t^C}\left[\frac{A^C}{A^U}\right]^{\rho}\left[\frac{A^C}{A^U}\right]^{-\rho}\frac{A_t}{A_t^C}\frac{C_t}{A_t}$$
$$c_t^C = (1-\rho)\frac{1}{p_t^C}c_t \tag{J.2}$$

Similarly:

$$c_t^U = (\rho)\frac{1}{p_t^U}c_t \tag{J.3}$$

- Price index:

$$\frac{P_t}{P_t^C\left[\frac{A^C}{A^U}\right]^{\rho}} = \left(\frac{P_t^U}{P_t^C}\frac{A_t^C}{A_t^U}\right)^{\rho}\left[\frac{A^C}{A^U}\right]^{\rho}\left(\frac{A_t^C}{A_t^U}\right)^{-\rho}$$
$$\frac{1}{p_t^C} = \left(\frac{p_t^U}{p_t^C}\right)^{\rho} \tag{J.4}$$

- Market clearing:

$$c = p^C c^C + p^U c^U \tag{J.5}$$
$$L = L^C + L^U \tag{J.6}$$

- Household Euler equation:

$$\frac{1}{c_t}\frac{\pi_{t+1}}{(1+i_t)} = \beta\,\mathbb{E}_t\left[\frac{1}{c_{t+1}\,g_{t+1}}\right] \tag{J.7}$$

where $g_{t+1} = A_{t+1}/A_t$.

- Inflation:

$$\pi_t = (\pi_t^C)^{1-\rho}(\pi_t^U)^{\rho} \tag{J.8}$$
$$\pi_t^C = \frac{p_t^C}{p_{t-1}^C}\pi_t \tag{J.9}$$
$$\pi_t^U = \frac{p_t^U}{p_{t-1}^U}\pi_t \tag{J.10}$$

- Phillips curve : TBC

$$\pi_t^C = \frac{g_{ss}^C}{g_t^C} \frac{{L_t^C}^\xi}{\bar{L^C}} {\pi_{t-1}^C}^\lambda \tag{J.11}$$

$$\pi_t^U = \frac{g_{ss}^U}{g_t^U} \frac{{L_t^U}^\xi}{\bar{L^U}} {\pi_{t-1}^U}^\lambda \tag{J.12}$$

- Market clearing

$$p_t^C \, c_t^C = p_t^C \left(\omega L_t^C - \frac{g_{t+1}^C - 1}{\chi^C} \right) - \kappa (d_t^C - \bar{d^C})^2 \tag{J.13}$$

$$p_t^U c_t^U = p_t^U \left(\omega L_t^U - \frac{g_{t+1}^U - 1}{\chi^U} \right) - \kappa (d_t^U - \bar{d^U})^2 \tag{J.14}$$

- Unconstrained Firms' FOC: **Productivity:**

$$\frac{\mu_t^U + \theta \lambda_t}{\chi^U} = \frac{(\mu_{t+1}^U + \theta \lambda_{t+1}^U) g_{t+1}}{R_t} \left[\frac{\pi_{t+1}^U}{\pi_{t+1}} \right] \left[\frac{g_{t+1}^U}{g_{t+1}^C} \right]^{\rho - 1} \left[\omega L_{t+1}^U + \frac{1}{\chi^U} \right]$$

note: $\left[\frac{g_{t+1}^U}{g_{t+1}^C} \right]^{\rho-1} = \frac{g_{t+1}}{g_{t+1}^U}$. I think because inflation does not grow in steady state, this should be:

$$\frac{\mu_t^U + \theta \lambda_t}{\chi^U} = \frac{(\mu_{t+1}^U + \theta \lambda_{t+1}^U) g_{t+1}}{R_t} \left[\frac{\pi_{t+1}^U}{\pi_{t+1}} \right] \left[\omega L_{t+1}^U + \frac{1}{\chi^U} \right] \tag{J.15}$$

Borrowing:

$$\frac{\mu_t^U}{R_t^b} - \frac{1}{R_t} \lambda_t = \frac{1}{R_t} \mu_{t+1}^U \tag{J.16}$$

Dividends:

$$1 - \mu_t^U - \mu_t^U \cdot 2\kappa (d_t^U - \bar{d^U}) = 0 \tag{J.17}$$

- Constrained Firms' FOC:

$$\frac{\mu_t^C}{\chi^C} = \frac{\mu_{t+1}^C}{R_t} \left[\frac{\pi_{t+1}^C}{\pi_{t+1}} \right] g_{t+1} \left[\omega L_{t+1}^C + \frac{1}{\chi^C} \right] \tag{J.18}$$

$$1 - \mu_t^C - \mu_t^C \cdot 2\kappa (d_{j,t}^C - \bar{d}_j) = 0 \tag{J.19}$$

- Borrowing rate:

$$R_t^b = \frac{(1+i_t)}{\pi_{t+1}} - \frac{(1+i_t)}{\pi_{t+1}}\tau + \tau \tag{J.20}$$

- Monetary policy follows a taylor rule
- Real interest rate:

$$R_t = \frac{(1+i_t)}{\pi_{t+1}} \tag{J.21}$$

- Equilibrium is finished here. Check number of eqns and vbls. : Now they are the same, but check if this is correct i fleel like we have too many price equations :D

  Definitions to be introduced if interested: GDP, VA, B, T, Inv, For ex:

- Borrowing condition:

$$\frac{b_{t+1}\pi_{t+1}}{p^U(1+i_t)} = \theta(\omega L_t^U - \frac{g_{t+1}^U - 1}{\chi^U}) \tag{J.22}$$

- Firm's input choice:

$$w^C = p^C(1-\alpha)2^{\alpha/1-\alpha} \tag{J.23}$$
$$w^U = p^U(1-\alpha)2^{\alpha/1-\alpha} \tag{J.24}$$

  where w is the real wage.

K Aug 28 - Working Equations in Dynare

1. **Aggregate Growth Rate**

$$g_t = g_{u,t}^{\rho}\, g_{c,t}^{1-\rho}$$

2. **IS Curve**

$$1 = \beta\, R_t\, \frac{c_t}{c_{t+1}\, g_{t+1}}$$

3. **Fisher Equation**

$$R_t = \frac{I_t}{\pi_{t+1}}$$

4. **Price Index**

$$1 = p_{u,t}^{\rho}\, p_{c,t}^{1-\rho}$$

5. **Optimum Demand: Constrained**

$$c_{c,t} = \frac{(1-\rho)\, c_t}{p_{c,t}}$$

6. **Optimum Demand: Unconstrained**

$$c_{u,t} = \frac{\rho\, c_t}{p_{u,t}}$$

7. **Consumption Aggregator**

$$c_t = p_{c,t}\, c_{c,t} + p_{u,t}\, c_{u,t}$$

8. **Labour Market Clearing**

$$L_t = L_{u,t} + L_{c,t}$$

9. **Inflation Index (CPI)**

$$\ln(\pi_t) = \rho\, \ln(\pi_{u,t}) + (1-\rho)\, \ln(\pi_{c,t})$$

10. **Inflation Index: Constrained**

$$\pi_{c,t} = \frac{p_{c,t}}{p_{c,t-1}}$$

11. **Inflation Index: Unconstrained**

$$\pi_{u,t} = \frac{p_{u,t}}{p_{u,t-1}}$$

12. **Phillips Curve: Constrained**

$$\pi_{c,t} = \frac{g_{c,ss}}{g_{c,t-1}}\, \frac{1}{\left(\dfrac{L_{c,t}}{\bar{L}_c}\right)^{\xi}}\, \pi_{c,t-1}^{\iota}$$

13. **Phillips Curve: Unconstrained**

$$\pi_{u,t} = \frac{g_{u,ss}}{g_{u,t-1}} \frac{1}{\left(\frac{L_{u,t}}{\bar{L}_u}\right)^{\xi}} \pi_{u,t-1}^{\iota}$$

14. **Goods Market Clearing: Constrained**

$$c_{c,t} = \Psi L_{c,t} - \frac{g_{c,t} - 1}{\chi_c}$$

where

$$\Psi = \alpha^{\frac{2\alpha}{1-\alpha}}(1 - \alpha^2)$$

15. **Goods Market Clearing: Unconstrained**

$$c_{u,t} = \Psi L_{u,t} - \frac{g_{u,t} - 1}{\chi_u}$$

16. **Productivity Condition: Unconstrained**

$$\frac{\mu_{u,t} + \lambda_t \theta}{\chi_u} = \frac{1}{R_t} \left(\mu_{u,t+1} + \lambda_{t+1}\theta\right) g_{t+1} \frac{\pi_{u,t+1}}{\pi_{t+1}} \left(\omega L_{u,t+1} + \frac{1}{\chi_u}\right)$$

17. **Borrowing Condition: Unconstrained**

$$\frac{\mu_{u,t}}{R_{b,t}} - \frac{\lambda_t}{R_t} = \frac{\mu_{u,t+1}}{R_t}$$

18. **Dividend Condition: Unconstrained**

$$0 = 1 - \mu_{u,t} - \mu_{u,t} \left(2\kappa(d_{u,t} - d_{u,ss})\right)$$

19. **Productivity Condition: Constrained**

$$\frac{\mu_{c,t}}{\chi_c} = \frac{1}{R_t} \mu_{c,t+1} g_{t+1} \frac{\pi_{c,t+1}}{\pi_{t+1}} \left(\omega L_{c,t+1} + \frac{1}{\chi_c}\right)$$

20. **Dividend Condition: Constrained**

$$0 = 1 - \mu_{c,t} - \mu_{c,t} \left(2\kappa(d_{c,t} - d_{c,ss})\right)$$

21. **Borrowing Rate Identity**

$$R_{b,t} = R_t - R_t\tau + \tau$$

22. **Monetary Policy Rule**

$$\ln(R_t) = \ln\left(\frac{g_{ss}}{\beta}\right) + \phi_L \ln\left(\frac{L_t}{\bar{L}}\right) + \epsilon_t^M$$

where ϵ_t^M is persistent

L December 4 –Ozgen

- **"EXTERNALLY FINANCED FIRMS" (WITH A BINDING EARNINGS-BASED CONSTRAINT), AND**
- **"INTERNALLY FINANCED FIRMS" (NO EXTERNAL BORROWING).**

WE WERE NOT LINKING GROWTH TO INVESTMENT IN DYNARE LIST, I ADDED THAT

WE DONT HAVE THE BORROWING CONSTRAINT IN DYNARE LIST

WE ARE NOT DEFINING GOVERNMENT BUDGET CONST IN DYNARE LIST

L.1 Exposition

L.1.1 Final Good Producer

There are two types of perfectly competitive final-goods firms, U and C denoted by i, produce Y^i combining labour, L^i and continuum of intermediate inputs x^i_j of type i, firm j, with the following production function:

$$Y^i_t = (L^i_t)^{1-\alpha} \int_0^1 (A^i_{j,t})^{(1-\alpha)} (x^i_{j,t})^{\alpha} \, dj \tag{L.1}$$

Here, $A^i_{j,t}$ represents the productivity or quality of input $j \in [0,1]$ of an $i = U, C$ type of firm. The profit maximization problem for the final goods producer yields the following optimality condition:

$$x^i_{j,t} = \left(\frac{P^i_{j,t}}{P^i_t \alpha (L^i_t)^{1-\alpha} (A^i_{j,t})^{1-\alpha}} \right)^{1/(\alpha-1)} \tag{L.2}$$

which defines the demand for the intermediate input j produced type $i = U, C$ intermediate producer.

L.1.2 Households

The representative household maximises expected lifetime utility from aggregate consumption, C_t:

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t log C_t$$

$\mathbb{E}$ is the expectations operator conditional on information at time t and $\beta \in (0,1)$ is the discount factor.

Households finance their expenditure through labor income, holdings of equity shares of firms, dividend payments, and non-state contingent bonds. The representative household's budget constraint is given by:

$$C_t + \frac{B_{t+1}}{R_t} + S_{t+1}p_t^s + T_t = w_t L_t + B_t + S_t(D_t + p_t^s)$$

where w_t is the real wage ($w_t = \frac{W_t}{P_t}$), R_t is the real interest rate. B_t is the holdings of real bonds, S_t are equity shares, D_t is dividend payments received from the holdings of shares, and p_t^s is the (real) market price of shares. Finally, T_t is the lump-sum taxes.

The aggregate consumption is a composite of constrained and unconstrained type of final goods and is aggregated with a Cobb-Douglas aggregator:

$$C_t = \left[\frac{(C_t^U)^\rho (C_t^C)^{1-\rho}}{\rho^\rho (1-\rho)^{(1-\rho)}}\right] \tag{L.3}$$

where ρ is the share of goods that unconstrained firms produce in the total aggregate consumption basket of the household.

The corresponding aggregate price index is:

$$P_t = (P_t^U)^\rho (P_t^C)^{1-\rho} \tag{L.4}$$

and the optimal demand for the output of each type of firm can be expressed as:

$$C_t^U = \rho \frac{P_t}{P_t^U} C_t \tag{L.5}$$

$$C_t^C = (1-\rho) \frac{P_t}{P_t^C} C_t \tag{L.6}$$

L.1.3 Wage Setting

Following Fornaro and Wolf (2023) which in turn is based on Galí and Gambetti (2020), we assume that nominal wages evolve according to

$$\frac{W_t^i}{W_{t-1}^i} = g_{ss}^i \left(\frac{L_t^i}{\bar{L}}\right)^\xi (\pi_{t-1}^i)^\varsigma \tag{L.7}$$

where $(\pi_{t-1}^i)^\varsigma$ denotes lagged price inflation of type i and $\varsigma > 0$ controls the degree to which past inflation affects current wages. In this framework, higher employment ex-

erts upward pressure on wage growth. Nominal wage rigidities serve two functions in the model. First, they create the potential for involuntary unemployment by maintaining positive wages, even when labor demand falls short of household labor supply. Second, these rigidities give monetary policy real traction, as sticky wages slow down price adjustments.

L.1.4 Government

The government finances a tax advantage on corporate interest payments through lump-sum taxes levied on households. Let B_{t+1} denote the aggregate face value of one-period real debt issued by firms at time t. Households purchase this debt at the market gross real rate R_t, while firms repay it at the subsidised gross real rate R_t^b, with

$$R_t^b = 1 + (1-\tau)r_t, \qquad 1 + r_t = R_t,$$

so that $R_t^b = R_t(1-\tau) + \tau < R_t$ for $\tau > 0$.

In real terms, firms receive $\frac{B_{t+1}}{R_t^b}$ units of resources when issuing B_{t+1}, whereas households give up $\frac{B_{t+1}}{R_t}$ units of consumption. The government covers the difference between the two, which is interpreted as an interest tax subsidy. Imposing period-by-period budget balance, the government budget constraint in real terms is

$$T_t = \frac{B_{t+1}}{R_t^b} - \frac{B_{t+1}}{R_t}, \tag{L.8}$$

where T_t denotes lump-sum taxes paid by households in units of the consumption good. Thus, tax revenues exactly finance the interest subsidy associated with the wedge between R_t and R_t^b in each period.

L.1.5 Monetary Policy

Monetary policy is conducted by choosing the gross real interest rate R_t according to a simple Taylor-type rule. We assume that the central bank adjusts R_t in response to deviations of inflation and employment from their respective reference levels,

$$\ln(R_t) = \ln\left(\frac{g_{ss}}{\beta}\right) + \phi_\pi \, \ln(\pi_t) + \phi_L \, \ln\left(\frac{L_t}{\bar{L}}\right) + \epsilon_t^m, \tag{L.9}$$

where $\pi_t \equiv P_t/P_{t-1}$ is aggregate inflation, $\bar{L}$ is the reference level of aggregate employment, ϕ_π and ϕ_L measure the strength of the central bank's response to inflation and

the employment gap, respectively, and ϵ_t^M is an exogenous monetary policy shock. ϵ_t^M is persistent and follows $\epsilon_t^M = \rho^M \epsilon_{t-1}^M$ with $\rho^M = 0.5$. The term $\ln(g_{ss}/\beta)$ pins down the steady-state real rate consistent with the long-run growth factor g_{ss} and the household discount factor β.

L.1.6 Intermediate Producers

Production. There are two types of intermediate input producers, constrained C and unconstrained, U denoted by i. Every intermediate input, j, is produced by a monopolist, x_j^i. Regardless of the quality of the product, one unit of final good of type i is required to manufacture one unit of intermediate good, which means that every type i intermediate producer faces the same marginal cost P_t^i. Taking into account this market structure, it is optimal for the i type monopolist producing good j to set the price:[17]

$$P_{j,t}^i = \frac{1}{\alpha} P_t^i \tag{L.10}$$

With some algebraic manipulation, the profits generated by this i type of monopolist for good j can be expressed as:

$$P_{j,t}^i x_{j,t}^i - P_t^i x_{j,t}^i = P_t^i \varphi A_{j,t}^i L_t^i \tag{L.11}$$

where

$$\varphi = \left(\frac{1}{\alpha} - 1\right) \alpha^{\frac{2}{1-\alpha}}.$$

Innovation Process. In equilibrium, each period sees a positive level of research activity directed toward each intermediate good j by type i producer. In this framework, an i type intermediate producer investing $I_{j,t}^i$ units of the final good experiences productivity growth governed by the following law of motion:

$$A_{j,t+1}^i = A_{j,t}^i + \chi^i I_{j,t}^i \tag{L.12}$$

where $\chi^i > 0$ represents the effectiveness of R&D investment. The inclusion of $A_{j,t}^i$ on the right-hand side reflects the idea that advancing more sophisticated and complex products demands higher investment (*i.e.* higher R&D expenditures), ensuring a stationary growth process over time.

[17] For the detailed discussion of how intermediate producers set their price, please refer to Appendix C.

Corporate Finance: R&D Financing. –Constrained Firms Constrained and unconstrained firms are different in the way they finance the R&D expenditure. Both constrained and unconstrained firms can issue equity but equity issuance is costly. We follow Jermann and Quadrini (2012) assume that equity adjustment has the following form:

$$\Phi(D^i_{j,t}) = \kappa(D^i_{j,t} - \overline{D}^i_j)^2, \tag{L.13}$$

where $\overline{D}^i_j$ denotes the target payout level, which corresponds to the steady-state value of $D^i_{j,t}$. This cost can be interpreted in two main ways: first, as direct financial expenses related to share buybacks and equity issuance; and second, as reflecting the tendency of managers to smooth dividends over time. Mechanically, this cost structure allows us to capture the dynamics of how quickly firms can alter their financing methods when financial conditions shift. With a lower κ, firms can swiftly absorb the shocks through changes in equity. As κ increases, the substitution between debt and equity becomes more expensive, leading firms to modify their funding sources more slowly.

Corporate Finance: R&D Financing. –Unconstrained Firms *Unconstrained firms* can access debt financing through one-period risk-free bonds, denoted as $B^U_{j,t+1}$. The effective gross real interest rate faced by firms is R^b_t. Following Drechsel (2023), which builds on the framework of Jermann and Quadrini (2012), the interest payments of firms benefit from a tax advantage τ, such that $R^b_t = 1 + (r_t(1-\tau))$, and $1 + r_t = R_t$ is the gross real interest rate received by the lenders. This yields $R^b_t = R_t - R_t\tau + \tau$. The tax advantage τ steers firms to favor debt over equity, encouraging them to borrow up to the limit of their constraints.

Borrowing is subject to the following constraint:

$$\frac{B^U_{j,t+1}}{R_t} \leq \theta \, \frac{P^U_{j,t}}{P_t}(\varphi A^U_{j,t} L^U_t) - I^U_{j,t}, \tag{L.14}$$

The earnings-based borrowing constraint (L.14) limits real debt to a multiple θ of current earnings. We focus on the debt-to-earnings formulation, as this covenant type is the most prevalent in loan agreements, surpassing others such as the coverage ratio (Drechsel, 2023).[18]

Constrained firms, face hard borrowing constraints (*i.e.* no borrowing), specifically $\theta = 0$, indicating that external borrowing is not possible under this condition.

[18] It is important to note that investment by unconstrained firms, $I^U_{j,t}$, and hence their productivity, $A^U_{j,t}$, depend on θ_j, as the tightness of the borrowing constraint affects the optimal investment level. For simplicity, we do not express $I^U_{j,t}$ as $I^U_{j,t}(\theta_j)$.

$$\frac{B^C_{j,t+1}}{R_t} \leq 0. \tag{L.15}$$

Flow of Funds. The corporate finance structure of *unconstrained firms* who can borrow externally is governed by the following flow of funds constraint:[19]

$$D^U_{j,t} + \kappa(D^U_{j,t} - \bar{D}^U_j)^2 + B_{j,t} = \frac{P^U_t}{P_t}\left(\varphi A^U_{j,t} L^U_t - I^U_{j,t}\right) + \frac{B_{j,t+1}}{R^b_t} \tag{L.16}$$

where equity payouts $D^U_{j,t}$, new debt issuance $B^U_{j,t+1}$, earnings $\varphi A^U_{j,t} L^U_t$, investment expenditures $I^U_{j,t}$, and the cost of servicing debt $B_{j,t}$ are all included.

The flow of funds constraint of *constrained firms*, on the other hand, can be expressed as:

$$D^C_{j,t} + \kappa(D^C_{j,t} - \bar{D}^C_j)^2 = \frac{P^C_t}{P_t}\left(\varphi A^C_{j,t} L^C_t - I^C_{j,t}\right) \tag{L.17}$$

including only equity payouts $D^C_{j,t}$ in consumption units, earnings $\varphi A^C_{j,t} L^C_t$, and investment expenditures $I^C_{j,t}$.

L.2 Intermediate Firms' Problem

Everything is measured in P_t except for sectoral output i.e. $\varphi A^i_{j,t} L^i_t - I^i_{j,t}$. And also let δ_t satisfy $\frac{\delta_{t+1}}{\delta_t} = \frac{C_{t+1}}{\beta C_t}$ so that $\frac{1}{R_t} = \frac{\delta_t}{\delta_{t+1}}$. Also we define $\pi_t = \frac{P_t}{P_{t-1}}$ and $\pi^i_t = \frac{P^i_t}{P^i_{t-1}}$

Constrained Firms. These firms cannot borrow. They solve:

$$\max_{\{D^C_{j,t}, I^C_{j,t}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \frac{1}{\delta_t} D^C_{j,t}$$

subject to:

$$A^C_{j,t+1} = A^C_{j,t} + \chi^C I^C_{j,t}$$
$$D^C_{j,t} + \kappa(D^C_{j,t} - \bar{D}^C_j)^2 = \frac{P^C_t}{P_t}\left(\varphi A^C_{j,t} L^C_t - I^C_{j,t}\right) \qquad \{\mu^C_t\}$$

where $R_{t-1} = \frac{\delta_t}{\delta_{t-1}}$

Corresponding Lagrange

[19] Since, in the equilibrium $B^C_{j,t+1} = 0$, for notational clarity, from now on we express $B^U_{j,t+1}$ as $B_{j,t+1}$.

$$\mathscr{L}^C = \mathbb{E}_0 \sum_{t=0}^{\infty} \frac{1}{\delta_t} \left\{ D_{j,t}^C + \mu_t^C \left[\frac{P_t^C}{P_t} \left(\varphi A_{j,t}^C L_t^C + \frac{A_{j,t}^C}{\chi^C} - \frac{A_{j,t+1}^C}{\chi^C} \right) - D_{j,t}^C - \kappa (D_{j,t}^C - \bar{D}_j^C)^2 \right] \right\}$$

First-order conditions:[20]

- **Productivity:**

$$\frac{\mu_t^C}{\chi^C} = \frac{\pi_{t+1}^C}{\pi_{t+1}} \frac{\mu_{t+1}^C}{R_t} \left[\varphi L_{t+1}^C + \frac{1}{\chi^C} \right]$$

  Interpretation: The marginal value of productivity accounts only for future returns from higher output (no debt capacity), discounted by its own inflation.

- **Dividends:**

$$1 - \mu_t^C - \mu_t^C \cdot 2\kappa (D_{j,t}^C - \bar{D}_j^C) = 0$$

  Interpretation: Same logic as for unconstrained firms — smooth dividend payouts are preferred.

Unconstrained Firms. Each firm $j \in [0,1]$ solves:

$$\max_{\{D_{j,t}^U, I_{j,t}^U, B_{j,t+1}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \frac{1}{\delta_t} D_{j,t}^U$$

subject to:

$$\begin{aligned} A_{j,t+1}^U &= A_{j,t}^U + \chi^U I_{j,t}^U \\ D_{j,t}^U + \kappa (D_{j,t}^U - \bar{D}_j^U)^2 + B_{j,t} &= \frac{P_t^U}{P_t} \left(\varphi A_{j,t}^U L_t^U - I_{j,t}^U \right) + \frac{B_{j,t+1}}{R_t^b} \qquad \{\mu_t^U\} \\ \frac{B_{j,t+1}}{R_t} &\leq \frac{P_t^U}{P_t} \theta (\varphi A_{j,t}^U L_t^U - I_{j,t}^U) \qquad \{\lambda_t\} \end{aligned}$$

where

$$R_t^b = R_t - R_t \tau + \tau$$

[20] In equilibrium all firms of a given type $i \in \{C, U\}$ are identical, so we drop the index j from multipliers and policy functions.

Corresponding Lagrange:

$$\mathscr{L}^U = \mathbb{E}_0 \sum_{t=0}^{\infty} \frac{1}{\delta_t} \Bigg\{ D_{j,t}^U + \mu_t^U \left[\frac{P_t^U}{P_t} \left(\varphi A_{j,t}^U L_t^U + \frac{A_{j,t}^U}{\chi^U} - \frac{A_{j,t+1}^U}{\chi^U} \right) + \frac{B_{j,t+1}}{R_t^b} - B_{j,t} - D_{j,t}^U - \kappa (D_{j,t}^U - \bar{D}_j^U)^2 \right] + \lambda_t \left[\frac{P_t^U}{P_t} \theta \left(\varphi A_{j,t}^U L_t^U + \frac{A_{j,t}^U}{\chi^U} - \frac{A_{j,t+1}^U}{\chi^U} \right) - \frac{B_{j,t+1}}{R_t} \right] \Bigg\}.$$

First-order conditions:

- **Productivity Condition.**

$$\frac{\mu_t^U + \theta \lambda_t}{\chi^U} = \frac{\pi_{t+1}^U}{\pi_{t+1}} \frac{\mu_{t+1}^U + \theta \lambda_{t+1}}{R_t} \left[\varphi L_{t+1}^U + \frac{1}{\chi^U} \right].$$

- **Dividend Condition.**

$$1 - \mu_t^U [\, 1 + 2\kappa (D_{j,t}^U - \bar{D}_j^U) \,] = 0.$$

- **Borrowing Condition.**

$$\frac{\mu_t^U}{R_t^b} - \frac{\lambda_t}{R_t} = \frac{\mu_{t+1}^U}{R_t}.$$

L.3 De-Trending

Some preliminaries

The corresponding aggregate price index is:

$$P_t = (P_t^U)^{\rho} (P_t^C)^{1-\rho} \tag{L.18}$$

and the aggregate consumption is a composite of constrained and unconstrained type of final goods and is aggregated with a Cobb-Douglas aggregator:

$$C_t = \left[\frac{(C_t^U)^{\rho} (C_t^C)^{1-\rho}}{\rho^{\rho} (1-\rho)^{(1-\rho)}} \right] \tag{L.19}$$

where ρ is the share of goods that unconstrained firms produce in the total aggregate consumption basket of the household.

L.3.1 Detrending

Let's derive the trends:

And the relative prices carry, relative productivities $\frac{P^U}{P^C} = \frac{A^C}{A^U}$.

C^C carries A^C and C^U carries A^U. Then we also know the trend for aggregate consumption as it will be a Cobb-Douglas aggregate of the sectoral ones:

$$A_t = \left[(A_t^U)^\rho (A_t^C)^{1-\rho}\right]$$

From the CPI price index, we can derive the trend for relative prices:

$$\frac{P}{P^C} = \left[\frac{A^C}{A^U}\right]^\rho$$
$$\frac{P}{P^U} = \left[\frac{A^U}{A^C}\right]^{1-\rho}$$

From firm's optimal choice of prices, we can derive the trend for real wages:

$$\frac{W^C}{P} = \frac{P^C}{P}(1-\alpha)2^{\alpha/1-\alpha}A^C$$
$$\frac{W^U}{P} = \frac{P^U}{P}(1-\alpha)2^{\alpha/1-\alpha}A^U$$

Notice that, real wage grows by A:

$$\frac{W^C}{P} = \left[\frac{A^C}{A^U}\right]^{-\rho} A^C$$
$$\frac{W^U}{P} = \left[\frac{A^U}{A^C}\right]^{-1+\rho} (1-\alpha)2^{\alpha/1-\alpha}A^U$$

L.3.2 Normalised equations

$$g_t = \left[(g_t^U)^\rho (g_t^C)^{1-\rho}\right] \tag{L.20}$$

$$c_t = \frac{C_t}{A_t}, \quad c_t^i = \frac{C_t^i}{A_t^i}, \quad d_{j,t}^i = \frac{D_{j,t}^i}{A_t^i}, b_t = \frac{B_t}{A_t}, w_t = \frac{W_t}{P_t A_t} \quad p_t^i = \frac{P_t^i A_t^i}{P_t A_t}, \quad x_{j,t}^i = \frac{X_{j,t}^i}{A_t}$$

where $x \in \{\lambda, \mu\}$

- Optimal demands:

$$\frac{C_t^C}{A_t^C} = (1-\rho)\frac{P_t}{P_t^C}\left[\frac{A^C}{A^U}\right]^{\rho}\left[\frac{A^C}{A^U}\right]^{-\rho}\frac{A_t}{A_t^C}\frac{C_t}{A_t}$$
$$c_t^C = (1-\rho)\frac{1}{p_t^C}c_t \tag{L.21}$$

Similarly:

$$c_t^U = (\rho)\frac{1}{p_t^U}c_t \tag{L.22}$$

- Market clearing:

$$c = p^C c^C + p^U c^U \tag{L.23}$$
$$L = L^C + L^U \tag{L.24}$$

- Household Euler equation:

$$\frac{1}{c_t}\frac{\pi_{t+1}}{(1+i_t)} = \beta\,\mathbb{E}_t\left[\frac{1}{c_{t+1}\,g_{t+1}}\right] \tag{L.25}$$

where $g_{t+1} = A_{t+1}/A_t$.

- **Sectoral inflation under trend normalisation**

We define aggregate and sectoral inflation as

$$\pi_t \equiv \frac{P_t}{P_{t-1}}, \tag{L.26}$$
$$\pi_t^i \equiv \frac{P_t^i}{P_{t-1}^i}, \qquad i \in \{U, C\}, \tag{L.27}$$

and the aggregate and sectoral productivity growth factors as

$$g_t \equiv \frac{A_t}{A_{t-1}}, \tag{L.28}$$
$$g_t^i \equiv \frac{A_t^i}{A_{t-1}^i}, \qquad i \in \{U, C\}. \tag{L.29}$$

Normalised sectoral prices are defined as

$$p_t^i \equiv \frac{P_t^i A_t^i}{P_t A_t}, \qquad i \in \{U, C\}. \tag{L.30}$$

Consider the ratio of normalised prices,

$$\frac{p_t^i}{p_{t-1}^i} = \frac{\dfrac{P_t^i A_t^i}{P_t A_t}}{\dfrac{P_{t-1}^i A_{t-1}^i}{P_{t-1} A_{t-1}}} \tag{L.31}$$

$$= \frac{P_t^i}{P_{t-1}^i} \cdot \frac{A_t^i}{A_{t-1}^i} \cdot \frac{P_{t-1}}{P_t} \cdot \frac{A_{t-1}}{A_t} \tag{L.32}$$

$$= \underbrace{\frac{P_t^i}{P_{t-1}^i}}_{\pi_t^i} \underbrace{\frac{A_t^i}{A_{t-1}^i}}_{g_t^i} \underbrace{\frac{P_{t-1}}{P_t}}_{1/\pi_t} \underbrace{\frac{A_{t-1}}{A_t}}_{1/g_t}. \tag{L.33}$$

Hence,

$$\frac{p_t^i}{p_{t-1}^i} = \pi_t^i \, g_t^i \, \frac{1}{\pi_t} \, \frac{1}{g_t}. \tag{L.34}$$

Rearranging for sectoral inflation yields

$$\boxed{\pi_t^i = \frac{p_t^i}{p_{t-1}^i} \, \pi_t \, \frac{g_t}{g_t^i}}, \qquad i \in \{U, C\}. \tag{L.35}$$

The aggregate CPI inflation index is given by the Cobb–Douglas aggregator

$$\pi_t = (\pi_t^U)^{\rho} (\pi_t^C)^{1-\rho} \quad \Longleftrightarrow \quad \ln \pi_t = \rho \ln \pi_t^U + (1-\rho) \ln \pi_t^C. \tag{L.36}$$

Together, these expressions fully characterise the link between normalised prices (p_t^i), sectoral inflations (π_t^i), aggregate inflation (π_t), and the growth factors (g_t, g_t^i).

- Phillips curve

$$\pi_t^C = \frac{g_{ss}^C}{g_t^C} \frac{L_t^C}{\bar{L^C}}^{\xi} \, {\pi_{t-1}^C}^{\varsigma} \tag{L.37}$$

$$\pi_t^U = \frac{g_{ss}^U}{g_t^U} \frac{L_t^U}{\bar{L^U}}^{\xi} \, {\pi_{t-1}^U}^{\varsigma} \tag{L.38}$$

- Market clearing

$$p_t^C\, c_t^C = p_t^C \left(\Psi L_t^C - \frac{g_{t+1}^C - 1}{\chi^C} \right) \tag{L.39}$$

$$p_t^U c_t^U = p_t^U \left(\Psi L_t^U - \frac{g_{t+1}^U - 1}{\chi^U} \right) \tag{L.40}$$

- **Endogenous sectoral growth from R&D**

  The law of motion for sectoral productivity of type $i \in \{U, C\}$ is

$$A_{t+1}^i = A_t^i + \chi^i I_t^i, \tag{L.41}$$

  where $\chi^i > 0$ is the R&D efficiency parameter and I_t^i denotes sectoral R&D investment.

  Define the sectoral growth factor as

$$g_{t+1}^i \equiv \frac{A_{t+1}^i}{A_t^i}, \qquad i \in \{U, C\}. \tag{L.42}$$

  Substituting the law of motion into this definition gives

$$g_{t+1}^i = \frac{A_t^i + \chi^i I_t^i}{A_t^i} \tag{L.43}$$

$$= 1 + \chi^i \frac{I_t^i}{A_t^i}. \tag{L.44}$$

  Let

$$inv_t^i \equiv \frac{I_t^i}{A_t^i} \tag{L.45}$$

  denote R&D intensity in sector i. Then

$$g_{t+1}^i = 1 + \chi^i inv_t^i. \tag{L.46}$$

  From the profit structure of intermediate producers, the flow of resources per unit of sectoral productivity A_t^i is given by

$$\Psi L_t^i - inv_t^i, \tag{L.47}$$

  where φL_t^i is profit per unit A_t^i and inv_t^i denotes R&D expenditure per unit A_t^i.

In normalised units, sectoral goods-market-clearing can be written as

$$c_t^i = \Psi L_t^i - inv_t^i, \qquad i \in \{U, C\}, \tag{L.48}$$

where $c_t^i \equiv C_t^i / A_t^i$ denotes normalised consumption of sector-i goods.

Solving for i_t^i gives

$$inv_t^i = \Psi L_t^i - c_t^i. \tag{L.49}$$

Substituting this into (L.46), we obtain

$$g_{t+1}^i = 1 + \chi^i(\Psi L_t^i - c_t^i). \tag{L.50}$$

Thus endogenous sectoral productivity growth is governed by

$$\boxed{g_{t+1}^i = 1 + \chi^i(\Psi L_t^i - c_t^i), \qquad i \in \{U, C\}.} \tag{L.51}$$

- Unconstrained Firms' FOC: **Productivity:**

$$\frac{\mu_t^U + \theta\lambda_t}{\chi^U} = \frac{(\mu_{t+1}^U + \theta\lambda_{t+1}^U)g_{t+1}}{R_t}\left[\frac{\pi_{t+1}^U}{\pi_{t+1}}\right]\left[\frac{g_{t+1}^U}{g_{t+1}^C}\right]^{\rho-1}\left[\varphi L_{t+1}^U + \frac{1}{\chi^U}\right]$$

note: $\left[\frac{g_{t+1}^U}{g_{t+1}^C}\right]^{\rho-1} = \frac{g_{t+1}}{g_{t+1}^U}$.

Borrowing:

$$\frac{\mu_t^U}{R_t^b} - \frac{1}{R_t}\lambda_t = \frac{1}{R_t}\mu_{t+1}^U \tag{L.52}$$

Dividends:

$$1 - \mu_t^U - \mu_t^U \cdot 2\kappa(d_t^U - \bar{d}^U) = 0 \tag{L.53}$$

- Constrained Firms' FOC:

$$\frac{\mu_t^C}{\chi^C} = \frac{\mu_{t+1}^C}{R_t}\left[\frac{\pi_{t+1}^C}{\pi_{t+1}}\right] g_{t+1}\left[\varphi L_{t+1}^C + \frac{1}{\chi^C}\right] \tag{L.54}$$

$$1 - \mu_t^C - \mu_t^C \cdot 2\kappa(d_{j,t}^C - \bar{d}_j) = 0 \tag{L.55}$$

- Borrowing rate:

$$R_t^b = \frac{(1+i_t)}{\pi_{t+1}} - \frac{(1+i_t)}{\pi_{t+1}}\tau + \tau \tag{L.56}$$

- Monetary policy follows a taylor rule
- Real interest rate:

$$R_t = \frac{(1+i_t)}{\pi_{t+1}} \tag{L.57}$$

- Equilibrium is finished here. Check number of eqns and vbls. : Now they are the same, but check if this is correct i fleel like we have too many price equations :D

  Definitions to be introduced if interested: GDP, VA, B, T, Inv, For ex:

- Borrowing condition:

$$\frac{b_{t+1}\pi_{t+1}}{p^U(1+i_t)} = \theta(\varphi L_t^U - \frac{g_{t+1}^U - 1}{\chi^U}) \tag{L.58}$$

- Firm's input choice:

$$w^C = p^C(1-\alpha)2^{\alpha/1-\alpha} \tag{L.59}$$
$$w^U = p^U(1-\alpha)2^{\alpha/1-\alpha} \tag{L.60}$$

  where w is the real wage.

L.4 Operational Equations in Dynare

1. **Aggregate Growth Rate**

$$g_t = g_{u,t}^{\rho}\, g_{c,t}^{1-\rho}$$

2. **IS Curve**

$$1 = \beta\, R_t\, \frac{c_t}{c_{t+1}\, g_{t+1}}$$

3. **Fisher Equation**

$$R_t = \frac{I_t}{\pi_{t+1}}$$

4. **Price Index**

$$1 = p_{u,t}^{\rho}\, p_{c,t}^{1-\rho}$$

5. **Optimum Demand: Constrained**

$$c_{c,t} = \frac{(1-\rho)\, c_t}{p_{c,t}}$$

6. **Optimum Demand: Unconstrained**

$$c_{u,t} = \frac{\rho\, c_t}{p_{u,t}}$$

7. **Consumption Aggregator**

$$c_t = p_{c,t}\, c_{c,t} + p_{u,t}\, c_{u,t}$$

8. **Labour Market Clearing**

$$L_t = L_{u,t} + L_{c,t}$$

9. **Inflation Index (CPI)**

$$\ln(\pi_t) = \rho\, \ln(\pi_{u,t}) + (1-\rho)\, \ln(\pi_{c,t})$$

10. **Inflation Index: Constrained**

$$\pi_t^C = \frac{p_t^C}{p_{t-1}^C}\, \pi_t\, \frac{g_t}{g_t^C}$$

11. **Inflation Index: Unconstrained**

$$\pi_t^U = \frac{p_t^U}{p_{t-1}^U}\, \pi_t\, \frac{g_t}{g_t^U}$$

12. **Phillips Curve: Constrained**

$$\pi_{c,t} = \frac{g_{c,ss}}{g_{c,t-1}}\, \frac{1}{\left(\dfrac{L_{c,t}}{\bar{L}_c}\right)^{-\xi}}\, \pi_{c,t-1}^{\iota}$$

13. **Phillips Curve: Unconstrained**

$$\pi_{u,t} = \frac{g_{u,ss}}{g_{u,t-1}} \frac{1}{\left(\frac{L_{u,t}}{\bar{L}_u}\right)^{-\xi}} \pi_{u,t-1}^{\iota}$$

14. **Goods Market Clearing: Constrained**

$$c_{c,t} = \Psi L_{c,t} - \frac{g_{c,t} - 1}{\chi_c}$$

15. **Goods Market Clearing: Unconstrained**

$$c_{u,t} = \Psi L_{u,t} - \frac{g_{u,t} - 1}{\chi_u}$$

16. **Productivity Condition: Unconstrained**

$$\frac{\mu_{u,t} + \lambda_t \theta}{\chi_u} = \frac{1}{R_t} \left(\mu_{u,t+1} + \lambda_{t+1}\theta\right) g_{t+1} \frac{\pi_{u,t+1}}{\pi_{t+1}} \left(\varphi L_{u,t+1} + \frac{1}{\chi_u}\right)$$

17. **Borrowing Condition: Unconstrained**

$$\frac{\mu_{u,t}}{R_{b,t}} - \frac{\lambda_t}{R_t} = \frac{\mu_{u,t+1}}{R_t}$$

18. **Dividend Condition: Unconstrained**

$$0 = 1 - \mu_{u,t} - \mu_{u,t} \left(2\kappa(d_{u,t} - d_{u,ss})\right)$$

19. **Productivity Condition: Constrained**

$$\frac{\mu_{c,t}}{\chi_c} = \frac{1}{R_t} \mu_{c,t+1} g_{t+1} \frac{\pi_{c,t+1}}{\pi_{t+1}} \left(\varphi L_{c,t+1} + \frac{1}{\chi_c}\right)$$

20. **Dividend Condition: Constrained**

$$0 = 1 - \mu_{c,t} - \mu_{c,t} \left(2\kappa(d_{c,t} - d_{c,ss})\right)$$

21. **Borrowing Rate Identity**

$$R_{b,t} = R_t - R_t\tau + \tau$$

22. **Endogenous Growth Identity: Unconstrained**

$$g_t^U = 1 + \chi^U\Big(\Psi\, L_{U,t-1} - c_{U,t-1}\Big)$$

23. **Endogenous Growth Identity: Unconstrained**

$$g_t^C = 1 + \chi^C\Big(\Psi\, L_{C,t-1} - c_{C,t-1}\Big).$$

24. **Monetary Policy Rule**

$$\ln(R_t) = \ln\left(\frac{g_{ss}}{\beta}\right) + \phi_\pi \ln(\pi_t) + \phi_L\, \ln\left(\frac{L_t}{\bar{L}}\right) + \epsilon_t^M$$

where ϵ_t^M is persistent

L.5 Derivations for Intermediate Goods Production and Value Added

This appendix derives the equilibrium expressions for intermediate input usage, monopoly profits, aggregate output, and value added for each firm type $i \in \{U, C\}$. Final-good producers are perfectly competitive and combine labour L_t^i and a continuum of intermediate inputs $x_{j,t}^i$ according to:

$$Y_t^i = (L_t^i)^{1-\alpha} \int_0^1 (A_{j,t}^i)^{\,1-\alpha} (x_{j,t}^i)^\alpha \, dj. \tag{L.61}$$

The associated cost minimisation problem yields the inverse demand function for each intermediate variety:

$$x_{j,t}^i = \left(\frac{P_{j,t}^i}{P_t^i \alpha (L_t^i)^{1-\alpha} (A_{j,t}^i)^{1-\alpha}}\right)^{\frac{1}{\alpha-1}}, \tag{L.62}$$

where P_t^i is the aggregate final-good price and $P_{j,t}^i$ is the price of input j.

Optimal Pricing and Input Choice of Intermediate Producers

Intermediate inputs are supplied by monopolists with marginal cost P_t^i. The profit maximisation problem implies the familiar constant-markup monopoly pricing rule:

$$P_{j,t}^i = \frac{P_t^i}{\alpha}. \tag{L.63}$$

Substituting (L.63) into (L.62) gives:

$$\frac{1}{\alpha} = \alpha (L_t^i)^{1-\alpha} (A_{j,t}^i)^{1-\alpha} (x_{j,t}^i)^{\alpha-1} \quad \Rightarrow \quad (x_{j,t}^i)^{1-\alpha} = \alpha^2 (L_t^i)^{1-\alpha} (A_{j,t}^i)^{1-\alpha}.$$

Solving for equilibrium input use yields:

$$x_{j,t}^i = \alpha^{\frac{2}{1-\alpha}} A_{j,t}^i L_t^i. \tag{L.64}$$

Profits of Intermediate Good Producers

Using (L.63) and (L.64), equilibrium profits are:

$$\Pi_{j,t}^i = (P_{j,t}^i - P_t^i) x_{j,t}^i = P_t^i \left(\frac{1}{\alpha} - 1 \right) \alpha^{\frac{2}{1-\alpha}} A_{j,t}^i L_t^i.$$

Define the constant:

$$\varphi \equiv \left(\frac{1}{\alpha} - 1 \right) \alpha^{\frac{2}{1-\alpha}},$$

so that profits simplify to:

$$\Pi_{j,t}^i = P_t^i \, \varphi \, A_{j,t}^i L_t^i. \tag{L.65}$$

Aggregate Output

Substituting equilibrium input use (L.64) into the production technology (L.61):

$$\begin{aligned} Y_t^i &= (L_t^i)^{1-\alpha} \int_0^1 (A_{j,t}^i)^{1-\alpha} \left(\alpha^{\frac{2}{1-\alpha}} A_{j,t}^i L_t^i \right)^{\alpha} dj \\ &= \alpha^{\frac{2\alpha}{1-\alpha}} (L_t^i) \int_0^1 A_{j,t}^i dj. \end{aligned}$$

Define aggregate productivity:

$$A_t^i \equiv \int_0^1 A_{j,t}^i \, dj.$$

Then equilibrium aggregate output is:

$$Y_t^i = \alpha^{\frac{2\alpha}{1-\alpha}} A_t^i L_t^i. \tag{L.66}$$

Value Added

Total intermediate expenditure is:

$$\int_0^1 x_{j,t}^i dj = \alpha^{\frac{2}{1-\alpha}} A_t^i L_t^i.$$

Thus, value added is:

$$Y_t^i - \int_0^1 x_{j,t}^i dj = \left(\alpha^{\frac{2\alpha}{1-\alpha}} - \alpha^{\frac{2}{1-\alpha}}\right) A_t^i L_t^i.$$

Factorising constants gives:

$$Y_t^i - \int_0^1 x_{j,t}^i dj = \alpha^{\frac{2\alpha}{1-\alpha}} (1-\alpha^2) \, A_t^i L_t^i.$$

Define:

$$\Psi \equiv \alpha^{\frac{2\alpha}{1-\alpha}} (1-\alpha^2),$$

so the final expression for value added becomes:

$$Y_t^i - \int_0^1 x_{j,t}^i dj = \Psi \, A_t^i L_t^i. \tag{L.67}$$

Equations (L.64), (L.65), (L.66), and (L.67) correspond to the closed-form equilibrium expressions used in the main text for each firm type $i \in \{U, C\}$.