Carrots or Sticks?
Short-Time Work vs. Lay-Off Taxes*

Preliminary and incomplete - do not circulate

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Abstract

While unemployment insurance systems are widely used to insure workers against income losses after separations, it is well known that they can inefficiently increase separations in the labor market. There are two common policy instruments that can counter this known problem: lay-off taxes and short-time work schemes. This study provides a search-and-matching framework to evaluate which of the two is the better policy tool. We show that if only a few firms are financially constrained, lay-off taxes are better because they do not distort working hours in the economy. With a large share of financially constrained firms, short-time work emerges as the superior tool, as lay-off taxes have trouble deterring separations in financially constrained firms. Additionally, short-time work can help provide insurance against income losses to risk-averse workers that constrained firms cannot afford to provide in their wage contracts. Calibrating the model to the U.S economy, we find that short-time work is the superior policy instrument if 24% of firms in the economy or more are financially constrained.

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1 Introduction

It has long been understood that unemployment insurance systems can inefficiently increase separations in the labor market. To counter this inefficiency, governments commonly have two main policy instruments at their disposal. On the one hand, there are lay-off taxes that, in effect, punish firms for firing workers. On the other, there is the short-time work system that rewards the retainment of endangered workers through subsidizing and enabling hours reductions[1].

This raises the natural question, which of the two governments should employ. Existing literature highlights that lay-off taxes have many desirable properties. (Cahuc and Zylberberg, 2008) and Blanchard and Tirole (2008) show in implicit contract frameworks that lay-off taxes can implement the planner solution. Short-time work on the other hand, has the problem of introducing new inefficiencies into the economy through the distortion of working hours (e.g. Burdett and Wright (1989), Stiepelmann (2024)). However, these results crucially hinge on the absence of financial constraints for firms. This assumption is widespread in the search and matching literature, but clearly is at odds with reality.

In this study, we relax this assumption in a rich yet analytically tractable DMP framework and allow for a share of firms to be financially constrained. Using the Ramsey policy approach, we then determine the welfare consequences of optimally set short-time work benefits and optimally set lay-off taxes.

Our main result is that lay-off taxes are indeed superior when the share of financially constrained firms is small. Short-time work emerges as the superior policy instrument if the share of constrained firms becomes sufficiently large. This is a consequence of two main channels. Firstly, lay-off taxes have trouble deterring separations in financially constrained firms, while short-time work can still operate as effectively as before. Secondly, with financial constraints, firms lose their ability to insure risk-averse workers against negative income shocks. Short-time work can then partially mitigate this and provide insurance against income shocks in the firms' stead.

Quantitatively, after calibrating the model to the U.S. economy, we find that short-time work is the superior policy instrument if 40% of firms in the economy or more are financially constrained.

The backbone of our model is a canonical Diamond-Mortensen-Pissarides type search and matching model (Mortensen and Pissarides (1993), Mortensen and Pissarides (1994)) with idiosyncratic productivity shocks, endogenous separations, and generalized Nash-

[1]Short-time work systems are pervasive, e.g., in European countries where they were utilized during the Great Recession and the Covid period. Lay-off taxes are implemented, e.g., in the U.S. through an experience-rated unemployment insurance system.

Bargaining between workers and firms. We augment the standard model with three realistic key ingredients: Risk aversion on the worker side, financial constraints on the firm side, and flexibly adjustable working hours. Workers and firms are randomly matched, form expectations over their match productivity, and bargain over wages, working hours, and separation productivity thresholds. Productivity shocks are i.i.d. and realize after firms and workers complete bargaining.

Government implements an unemployment insurance system under which unemployed workers are paid lump sum benefits and either a lay-off tax system or a short-time work scheme. Further, it collects taxes on firms to finance the systems.

In the presence of lay-off taxes, firms have to pay lump-sum lay-off taxes once worker and firm separate after productivity falls below the separation threshold. To replicate the U.S. experience-rated unemployment insurance system, we assume that lay-off taxes are backloaded to the next period, thereby avoiding firm bankruptcies.

The short-time work scheme consists of two main components: eligibility conditions and benefit payments. Workers become eligible for short-time work once they agree to reduce their working hours below a threshold specified by the government. Short-time work benefits compensate for lost income by providing fixed payments for each hour not worked relative to normal working hours. Short-time work effectively acts as a subsidy paid directly to workers. Working hours under short-time work are also determined through bargaining between firms and workers. In this setup, firms and workers have an incentive to reduce working hours inefficiently to attract more government support.

We assume that a share of firms is financially constrained. Specifically, we assume that these firms can never borrow more than the expected discounted value of the firm. These constraints have direct welfare implications. With risk-averse workers and risk-neutral firms, firms would like to offer workers insurance against low productivity shocks and commit to paying the worker a fixed (consumption equivalent) wage, no matter how low or high productivity turns out to be. The worker is then willing to accept a slightly lower wage in return for the insurance. This implies that for sufficiently low productivity realizations, firms make ex-post losses. However, if the firm is financially constrained, firms cannot make ex-post losses and lose their ability to fully insure workers. Low enough productivity shocks make the borrowing constraint binding and the shocks pass through to the worker's wage fully. Workers are not committed to staying in the match and quit once they hit their participation constraint (i.e. once the value from quitting, becoming unemployed, and looking for a new job is greater than staying). Therefore, the welfare effect of financial constraints is twofold: Firstly, firms cannot provide as much insurance to workers as they would like, and secondly, workers quit sooner.

We derive closed-form expressions for optimal lay-off taxes and optimal short-time work

schemes, depending on how many firms are financially constrained. The theoretical results show that the sole purpose of lay-off taxes is to offset the fiscal externality of the unemployment insurance system on financially unconstrained firms. However, lay-off taxes cannot correct inefficient separations or address the lack of insurance for workers in financially constrained firms. The intuition is straightforward: once financial constraints become binding, firms can no longer absorb shocks, which are instead passed on to workers in the form of reduced income. Even though firms would like to keep the worker employed to avoid future lay-off taxes, workers quit unitarily when income falls enough, rendering the lay-off tax ineffective[2].

By contrast, short-time work is particularly effective in supporting financially constrained firms. By supplementing the income of workers who experience reduced hours, it incentivizes workers to remain attached to the firm. In addition, it provides workers with income insurance, where constrained firms can no longer provide it. We proceed to show that short-time work cannot simply be replaced by providing loans to financially constrained firms. This is because where short-time work increases the joint surplus of firms and workers by providing insurance to workers, loans merely shift funds across periods and therefore have no bearing on a worker's wage and her decision to quit. Unlike short-time work, loans therefore do not prevent inefficient separations.

When the share of financially constrained firms is low, lay-off taxes are clearly preferable to short-time work, as the distortionary cost of reduced working hours under short-time work outweighs the cost of insufficient support for financially constrained firms under a lay-off tax regime. However, as the prevalence of financially constrained firms increases, the trade-off shifts. Both theoretically and quantitatively, we demonstrate that short-time work becomes the superior policy instrument once a sufficiently large share (more than 24%) of firms face financial constraints.

Related literature We contribute to four branches of the literature. Firstly, we add to the literature about optimal unemployment insurance. The trade-off between the benefits of unemployment insurance and adverse effects on job-search, and separations has been a recurring theme in the economic literature (e.g. Shavell and Weiss (1979), Baily (1978)). How UI should be used optimally has therefore been examined in seminal papers such as Hopenhayn and Nicolini (1997) and Chetty (2006). More recently Landais et al. (2018) how UI should vary with labor market tightness and Kroft and Notowidigdo (2016) show evidence that moral hazard costs of UI are procyclical. We contribute to this literature by evaluating policy tools that can mitigate the adverse externalities of unemployment insurance.

[2]In the U.S., workers who quit following a substantial deterioration in job conditions remain eligible for unemployment insurance benefits.

Secondly, there is a branch of the literature that concerns itself with lay-off taxes, often emphasizing their effectiveness in overcoming UI fiscal externalities. Blanchard and Tirole (2008) and Cahuc and Zylberberg (2008) propose frameworks in which lay-off taxes can decentralize the planner solution. Closely related to our work, Jung and Kuester (2015) and Michau (2015) take a Ramsey planner approach and examine optimal unemployment insurance with lay-off taxes and vacancy subsidies in a DMP. Duggan et al. (2023) show that lay-off taxes can act as a stabilizer over the business cycle. Postel-Vinay and Turon (2011), on the other hand, show that employers can use severance packages to coax workers into quitting and avoid lay-off taxes. Ratner (2013) makes the point that the experience-rated UI system in the U.S. reduces layoffs but also hampers hires. Similarly, Johnston (2021) finds that increases in lay-off taxes lead to less hiring. We contribute to this literature by introducing firm borrowing constraints and showing that they reduce the effectiveness of lay-off taxes.

Thirdly, there is a growing body of research on short-time work, though the literature remains divided on its overall usefulness. Some studies emphasize potential inefficiencies. For example, Burdett and Wright (1989) argue that short-time work encourages inefficient reductions in working hours. Cooper et al. (2017) highlight the risk of subsidizing employment at unproductive firms, while Giupponi and Landais (2022) raise concerns about impeding beneficial worker reallocations, though they also note that short-time work may support firms in maintaining efficient levels of labor hoarding. Cahuc et al. (2021) raise concern about the potential windfall effects of short-time work.

Other studies focus on the potential advantages of short-time work. Balleer et al. (2016) show that it can introduce valuable flexibility at the intensive margin of employment. Giupponi et al. (2021) argue that short-time work complements unemployment insurance by insuring against different types of labor market shocks. Similarly, Braun and Brügemann (2017) analyze optimal unemployment insurance and short-time work jointly within an implicit contract model.

Despite these recent advancements, the relationship between short-time work and unemployment insurance remains insufficiently understood, as emphasized in a comprehensive review by Cahuc (2024). Addressing this gap, Stiepelmann (2024) introduces the analysis of optimal short-time work policy and optimal unemployment insurance into a search-and-matching framework. Building on this foundation, our paper contributes to this branch of the literature by enhancing the understanding of the interplay between short-time work and unemployment insurance by highlighting the pivotal role of financial constraints.

Finally, there is a literature on firms' financial constraints. Drechsel (2023) argues that earnings-based borrowing constraints react more strongly to shocks. A part of the literature shows that financial constraints matter for monetary shock transmissions (e.g.

Ottonello and Winberry (2020)) or innovation (e.g. Cascaldi-Garcia et al. (2023)). In the labor market, fewer financial constraints are empirically shown to lead to higher employment (e.g. Duygan-Bump et al. (2015), Fonseca and Van Doornik (2022)). We contribute to this literature by incorporating financial constraints into a search and matching framework with endogenous employment response.

In incorporating financial constraints, we depart from the classical approach of Wasmer and Weil (2004) or Iliopulos et al. (2019). This strand of the literature treats hiring costs as working capital that must be financed. As a result, financial constraints make it more difficult to fund hiring activities and thereby tend to depress vacancy posting. In contrast, in our model, financial constraints do not directly affect the hiring process. Instead, financially constrained firms face difficulties fulfilling wage contracts that yield a negative ex-post value of a worker to the firm, thereby leading to excess separations.

2 Model

2.1 Basic Assumptions

Set-Up Time is discrete. There is a unit mass of workers. Workers can be either employed or unemployed. While unemployed, workers search for vacant jobs and receive unemployment benefits b. Unemployed workers and firms with vacancies are matched randomly. The number of contacts is governed by the Cobb-Douglas matching function $m(v,n) = \bar{m} \cdot v^{(1-\gamma)} \cdot (1-n)^{\gamma}$, where v is the mass of vacancies and n is the mass of employed workers. We denote the job-finding rate by f, the vacancy-filling rate by q, and the separation rate by ρ. Let θ denote labor market tightness, i.e. $\theta = \frac{v}{1-n}$.

A worker-firm match produces output y according to the production function

$$y(\epsilon, h) = \epsilon \cdot h^{\alpha} - (\mu_{\epsilon} - \epsilon) \cdot k_f$$

where ϵ is realization of a productivity shocj $\tilde{\epsilon}$ satisfying shock $\log \tilde{\epsilon} \sim \mathcal{N}(\mu, \sigma^2)$ with cdf $G(\epsilon)$ and pdf $g(\epsilon)$. New productivity shocks arrive with probability λ. h is the number of working hours worked by the worker, and A is the total factor productivity. The term $(\mu_{\epsilon} - \epsilon) \cdot k_f$ is a cost shock. While workers are employed they receive a salary $w(\epsilon)$ and suffer dissutility $\phi(h) = \frac{h^{(1+\psi)}}{1+\psi}$ from working h hours.

Firm profits go to firm owners of whom there is a mass of ν_f.

Preferences Workers are risk-averse with a concave flow utility function over consumption, net of work disutility $u(c - \phi(h))$. Because utility is defined over consumption-equivalent units, we introduce notation for production in consumption-equivalent units

for use in later expressions and let $z(\epsilon, h) = y(\epsilon, h) - \phi(h)$. Firm owners have the same flow utility function u but draw utility only from their consumption c_f (i.e. $u(c_f)$).

Government Policy The government can choose between two policy regimes: the Short-Time Work (STW) regime and the Lay-Off Tax (LT) regime. Since the chosen policy regime has implications for firm and worker value, bargaining, and equilibrium, the remaining model assumptions are described in two separate sections for each respective policy regime in the following.

2.2 The Model with Lay-off Taxes

Government Under the LT regime, firms have to pay lay-off taxes F once the worker and firm jointly agree to separate. Importantly, firms pay no lay-off taxes if the worker unilaterally leaves the match. In the following, let ρ denote the probability that firms and workers separate. The lay-off tax F and the unemployment insurance benefits b are set by the government. To finance the policy regime, firms pay a lump-sum tax s whenever their productivity changes. The government budget constraint is

$$n^s \cdot s = (1 - n) \cdot b - n^s \cdot \rho \cdot F$$

where n^s is the mass of matches that receive a new productivity shock.

Firms There are two types of firms - financially constrained and financially unconstrained firms. Firms are financially constrained with probability p. When constrained, once the productivity shock has realized to ϵ, a firm can borrow no more than its expected value, conditional on its realized productivity. Specifically,

$$y(\epsilon, h(\epsilon)) - w^c(\epsilon) \geq -\beta \cdot \left[\lambda \cdot \bar{J} + (1 - \lambda) \cdot J^c(\epsilon)\right]$$

must hold.$\bar{J}$ is the expected firm value once a new shock arrives, and $J^c(\epsilon)$ is the value of a constrained firm with productivity draw ϵ. The constrained (monthly) wage function $w^c(\epsilon)$ will be made explicit in the bargaining section below. The bargained-over hours functions will depend on productivity ϵ but not on the financial constraint. Anticipating this, we save on notation and denote $h^u(\epsilon)$ and $h^c(\epsilon)$ as $h(\epsilon)$. Note that with persistent shocks ($\lambda < 1$), a low realization of productivity will lead to a tighter borrowing constraint.

The value of an unconstrained firm *after* productivity realizes to ϵ is

$$J^u(\epsilon) = y(\epsilon, h(\epsilon)) - w^u(\epsilon) + \beta \cdot \left[\lambda \cdot \bar{J} + (1 - \lambda) J^u(\epsilon)\right]$$

and the value of a constrained firm *after* productivity realizes to ϵ that is

$$J^c(\epsilon) = y(\epsilon, h(\epsilon)) - w^c(\epsilon) + \beta \cdot \left[\lambda \cdot \bar{J} + (1-\lambda) \cdot J^c(\epsilon)\right]$$

where $\bar{J}$ is the expected firm value *before* the shock has realized and *before* it becomes known if the firm is constrained. It is given by

$$\bar{J} = -s + (1-p)\left(\int_{\epsilon_s^u}^{\infty} J^u(\epsilon)\, dG(\epsilon)\right) + p\left(\int_{\epsilon_s^c}^{\infty} J^c(\epsilon)\, dG(\epsilon)\right) \quad - \rho \cdot \beta \cdot F$$

Firms separate from a worker once the productivity drops below the separation threshold ϵ_s^u or ϵ_s^c for the unconstrained and constrained case, respectively. With probability ρ, firms and workers separate, and the firm has to pay a lay-off tax F. In accordance with the experience rating system in the US, we assume that lay-off taxes are back loaded so that financial constraints do not bind when paying the lay-off tax. The lay-off tax is therefore discounter by β.[3]

Firms can freely enter the labor market and post vacancies at cost k_v. The mass of vacancies v must therefore solve

$$\frac{k_v}{q} = \beta \cdot \bar{J}$$

Workers Employed workers can work at financially constrained and unconstrained firms. The value of a worker at an unconstrained firm *after* productivity realizes to ϵ is

$$V^u(\epsilon) = u(w^u(\epsilon) - \phi(h(\epsilon))) + \beta \cdot \left[\lambda \bar{V} + (1-\lambda) V^u(\epsilon)\right]$$

The value of a worker at a constrained firm *after* productivity realizes to ϵ is

$$V^c(\epsilon) = u(w^c(\epsilon) - \phi(h(\epsilon))) + \beta \cdot \left[\lambda \bar{V} + (1-\lambda) V^c(\epsilon)\right]$$

where $\bar{V}$ is the expected worker value *before* a new productivity shock has realized It is given by

$$\bar{V} = (1-p)\left(\int_{\epsilon_s^u}^{\infty} V^u(\epsilon)\, dG(\epsilon)\right) + p\left(\int_{\epsilon_s^c}^{\infty} V^c(\epsilon)\, dG(\epsilon)\right) + \rho \cdot U.$$

[3]Note that theoretically, the firm could stay financially constraint and thus would get bankrupt by the lay-off tax. We assume that this cannot happen.

With probability ρ a worker becomes unemployed. The Value of an unemployed worker U is given by

$$U = u(b) + \beta \cdot \left[f \cdot \bar{V} + (1 - f) \cdot U \right]$$

Bargaining With financial constraints, bargaining takes place before both - the realization of productivity ϵ and whether the firm is constrained or not - become known. For each possible state $\epsilon \in \mathbb{R}_+$ worker and firm agree on the total monthly wage functions $w^u(\epsilon)$ and $w^c(\epsilon)$, the hours functions $h^u(\epsilon)$ and $h^c(\epsilon)$, on the separation threshold when unconstrained ϵ_s^u, as well as the separation threshold when constrained ϵ_s^c.

Further, there is no commitment to the contract on the workers' side. This means that if the outside option of the worker becomes weakly better than staying with the match, the worker quits unitarily:

$$V^u(\epsilon) \leq U, \qquad V^c(\epsilon) \leq U$$

Formally, the bargaining outcome solves

$$\max_{\mathcal{X}} \quad \bar{J}^{1-\eta} \left(\bar{V} - U \right)^{\eta}$$

subject to

$$\begin{aligned} V^u(\epsilon) \geq U, \quad V^c(\epsilon) \geq U \qquad & \text{(commitment problem)} \\ y(\epsilon, h^c(\epsilon)) - w^c(\epsilon) \geq -\beta \left[\lambda \bar{J} + (1 - \lambda) J^c(\epsilon) \right] \qquad & \text{(financial constraint).} \end{aligned}$$

where the choice set is given by

$$\mathcal{X} = \left\{ w^u(\epsilon), w^c(\epsilon), h^u(\epsilon), h^c(\epsilon), \epsilon_s^u, \epsilon_s^c \right\}.$$

where η denotes the bargaining power of workers. Note that firms and workers have to take the commitment problem of the worker and the financial constraints of the firm into account when writing their contracts. Appendix C.1 derives the bargaining outcomes in detail.

Since workers are risk-averse and firms are risk-neutral, the bargaining outcome is that firms insure workers against low-productivity states. Workers will accept a lower average wage in exchange for insurance. As long as the firm is not constrained (either because it is an unconstrained firm or because constraints do not bind) the firm will *fully* insure the worker against productivity risk and the total monthly wage $w(\epsilon)$ will be set to a constant consumption equivalent c^w, equating worker utility across all realizations of ϵ

that do not lead to binding constraints. c^w is pinned down by the condition

$$u'(c(\epsilon)) = u'(c^w)$$

Once a firm becomes constrained and the constraint binds, it instead pays the maximum monthly wage $c^w(\epsilon)$ it can still afford:

$$c^w(\epsilon) = z(\epsilon) + \lambda \cdot \beta \cdot \bar{J}$$

The hours function that worker and firm agree on is, as already mentioned, the same for constrained and unconstrained firms. It is pinned down by

$$\alpha \cdot \epsilon \cdot h(\epsilon)^{\alpha-1} = h(\epsilon)^{\psi}$$

both, in the constrained and the unconstrained case. The equilibrium, therefore, only contains one general hours Function $h(\epsilon)$. Since $h(\epsilon)$ equates the marginal disutility of work and the marginal product of labor, working hours are always set efficiently.

Note that because under bargaining, each productivity level ϵ will imply a working hours level $h(\epsilon)$, we write $z(\epsilon, h(\epsilon))$ simply as $z(\epsilon)$. The job-destruction equations pin down the bargained-over separation thresholds of constrained and unconstrained matches

$$z(\epsilon_s^u) + [1 - \beta \cdot (1 - \lambda)] \cdot \beta \cdot F + \frac{u(c^u) - u(b)}{u'(c^w)} - c^w + \frac{\lambda - \eta \cdot f}{1 - \eta} \cdot \frac{k_v}{q} = 0$$

$$\frac{u(c^w(\epsilon_s^c)) - u(b)}{u'(c^w)} + (\lambda - f) \cdot \frac{\eta}{1 - \eta} \cdot \frac{k_v}{q} = 0$$

Note that F enters the separation condition for unconstrained firms only. In the unconstrained case, firms insure workers' income against idiosyncratic productivity shocks such that a low idiosyncratic productivity level is not passed on to the worker. As a result, the worker has no incentive to quit unilaterally $V^u(\epsilon_s^u) > U$, and firms have to pay the lay-off tax. A sufficiently low productivity realization in a financially constrained firm is passed directly to the worker's income. For low enough productivity levels, the continuation value of employment falls below the worker's outside option, such that $V^c(\epsilon_s^c) \leq U$, and the worker quits unilaterally. Although the firm would prefer to retain the worker in order to avoid the backloaded layoff tax, it lacks the financial resources to offer a wage that satisfies the worker's participation constraint. Consequently, the layoff tax does not affect the separation decision

Labor Markets The separation rate ρ is given by $\rho = (1-p) \cdot \rho^u + p \cdot \rho^c$ where $\rho^u = G(\epsilon_s^u)$ and $\rho^c = G(\epsilon_s^c)$. The steady-state law of motion for employment is

$$n = (1-\lambda) \cdot n + (1-\rho) \cdot n^s,$$

where n^s denotes the number of matches that received a new shock

$$n^s = f \cdot (1-n) + \lambda \cdot n$$

The mass of unemployment workers can be expressed as $u = 1 - n$. The mass of workers who work for unconstrained firms n^u, and the mass of workers who work at constrained firms n^c are given by:

$$n^u = \frac{1-p}{\lambda} \cdot (1-\rho^u) \cdot n^s$$
$$n^c = \frac{p}{\lambda} \cdot (1-\rho^c) \cdot n^s$$

Equilibrium A steady state Equilibrium consists of the working hours function $h(\epsilon)$, the consumption equivalent c^w paid as the monthly wage to workers without binding constraints, the consumption-equivalent monthly wage $c^w(\epsilon)$ paid when financial constraints bind, the separation thresholds ϵ_s^u and ϵ_s^c, the productivity level at which the borrowing constraint becomes binding ϵ^p and labor market flows, i.e the job-finding rate f, the vacancy-filling rate q and the separation rate ρ^u and ρ^c as well as n. The exact equations pinning down equilibrium and their derivation are delegated to Section C in the appendix.

2.3 The Model with Short-Time Work

Government Under the STW regime, firms become eligible for short-time work benefits if they set their working hours on short-time work $h_{\text{stw}}(\epsilon)$ below a threshold D. In this case, the worker receives $s_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon))$ worth of benefits. $\bar{h}$ is a parameter that reflects the normal working hour level. In essence, for every hour the worker works less than she would under normal circumstances, she receives compensation s_{stw}. Choosing an hours threshold D is a close implementation of what short-time work rules mandate in e.g. Germany.[4] Enforcing working hours reduction can help screen out firms that are actually productive from receiving subsidies because only unproductive firms will find hours reductions optimal (Teichgräber et al. (2022)).

In principle, government chooses the policy parameters D, s_{stw} and b. However, since

[4] In practice, this is implemented as a minimum reduction of hours threshold - which in our model is given by $\frac{D-h}{h}$ and is implicitly chosen by the government through D.

the hours functions $h^u(\epsilon)$, $h^c(\epsilon)$, $h^u_{\text{stw}}(\epsilon)$ and $h^c_{\text{stw}}(\epsilon)$ will always be strictly decreasing in ϵ, choosing D is equivalent to setting the eligibility productivity threshold ϵ_{stw} that will induce the same hours threshold D. The resulting government budget constraint is

$$\begin{aligned} n^s \cdot s = \frac{1-p}{\lambda} \cdot n^s \cdot \int_{\epsilon^u_s}^{\max\{\epsilon_{\text{stw}},\epsilon^u_s\}} s_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon))\, dG(\epsilon) \\ + \frac{p}{\lambda} \cdot n^s \cdot \int_{\epsilon^c_s}^{\max\{\epsilon_{\text{stw}},\epsilon^c_s\}} s_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon))\, dG(\epsilon) \\ + (1-n) \cdot b \end{aligned}$$

where n^s is the mass of matches that receive a new productivity shock. ϵ^u_s and ϵ^c_s are the separation thresholds of matches with access to short-time work of unconstrained and constrained matches, respectively. The max operators in the integral limits are necessary because the government can choose an eligibility condition that is so tight that all matches separate without gaining access to short-time work first.

Firms Again, firms can be either financially constrained or unconstrained. With short-time work, they can also currently be on short-time work or not. Firms are financially constrained with probability p. When constrained, once the productivity shock has realized to ϵ, a firm can borrow no more than its expected value, conditional on its realized productivity. Specifically, without access to short-time work

$$y(\epsilon, h(\epsilon)) - w^c(\epsilon) \geq -\beta \cdot \left[\lambda \cdot \bar{J} + (1-\lambda) \cdot J^c(\epsilon)\right]$$

and with short-time work

$$y(\epsilon, h_{\text{stw}}(\epsilon)) - w^c_{\text{stw}}(\epsilon) \geq -\beta \cdot \left[\lambda \cdot \bar{J} + (1-\lambda) \cdot J^c_{\text{stw}}(\epsilon)\right]$$

must hold. $\bar{J}$ is the expected firm value once a new shock arrives and $J^c(\epsilon)$ is the value of a constrained firm without short-time work and $J^c_{\text{stw}}(\epsilon)$ is the value of a constrained firm on short-time work. The constrained monthly wage function $w^c(\epsilon)$ will be made explicit in the bargaining section below. Like in the model with lay-off taxes, the bargained-over hours functions will depend on productivity ϵ but not on the financial constraint. Anticipating this, we save on notation and denote $h^u(\epsilon)$ and $h^u(\epsilon)$ as $h(\epsilon)$ and $h^u_{\text{stw}}(\epsilon)$ and $h^c_{\text{stw}}(\epsilon)$ as $h_{\text{stw}}(\epsilon)$. Further we write $z(\epsilon)$ instead of $z(\epsilon, h(\epsilon))$ and $z_{\text{stw}}(\epsilon)$ instead of $z(\epsilon, h_{\text{stw}(\epsilon)})$. The value of firm *without* short-time work *after* productivity realizes to ϵ that is unconstrained is

$$J^u(\epsilon) = y(\epsilon, h(\epsilon)) - w^u(\epsilon) + \beta \cdot \left[\lambda \bar{J} + (1-\lambda) J^u(\epsilon)\right]$$

and the value of an unconstrained firm *with* short-time work *after* productivity realizes is

$$J^u_{\text{stw}}(\epsilon) = y(\epsilon, h_{stw}(\epsilon)) - w^u_{\text{stw}}(\epsilon) + \beta \cdot \left[\lambda \bar{J} + (1-\lambda) J^u_{\text{stw}}(\epsilon)\right].$$

The value of firm *without* short-time work *after* productivity realizes to ϵ that is constrained is

$$J^c(\epsilon) = y(\epsilon, h(\epsilon)) - w^c(\epsilon) + \beta \cdot \left[\lambda \bar{J} + (1-\lambda) J^c(\epsilon)\right]$$

and the value of a constrained firm *with* short-time work *after* productivity realizes is

$$J^c_{\text{stw}}(\epsilon) = y(\epsilon, h_{stw}(\epsilon)) - w^c_{\text{stw}}(\epsilon) + \beta \cdot \left[\lambda \bar{J} + (1-\lambda) J^c_{\text{stw}}(\epsilon)\right]$$

where $\bar{J}$ is the expected firm value *before* the shock has realized. It is given by

$$\begin{aligned}\bar{J} = -s + (1-p)\left(\int_{\max\{\xi^u_s, \epsilon_{\text{stw}}\}}^{\infty} J^u(\epsilon)\, dG(\epsilon) + \int_{\epsilon^u_s}^{\epsilon_{\max\{\epsilon^u_s, \text{stw}\}}} J^u_{\text{stw}}(\epsilon)\, dG(\epsilon)\right) \\ + p\left(\int_{\max\{\xi^c_s, \epsilon_{\text{stw}}\}}^{\infty} J^c(\epsilon)\, dG(\epsilon) + \int_{\epsilon^c_s}^{\max\{\epsilon^c_s, \epsilon_{\text{stw}}\}} J^c_{\text{stw}}(\epsilon)\, dG(\epsilon)\right)\end{aligned}$$

The max operators in the integral bounds $\max\{\xi^i_s, \epsilon_{\text{stw}}\}$ reflect that a sufficiently strict eligibility threshold ϵ_{stw} can exclude certain matches from accessing short-time work, even if their productivity would allow them to survive with such support but not without it. In this case, otherwise, viable matches are denied access to the short-time work system, leading to unnecessary separations. Firms can freely enter the labor market and post vacancies at cost k_v. The mass of vacancies v must therefore solve

$$\frac{k_v}{q} = \beta \cdot \bar{J}$$

Workers The value of a worker at an unconstrained firm *without* short-time work *after* productivity realizes to ϵ is

$$V^u(\epsilon) = u(w^u(\epsilon) - \phi(h(\epsilon))) + \beta \cdot \left[\lambda \bar{V} + (1-\lambda) V^u(\epsilon)\right]$$

and the value of a worker at an unconstrained firm *with* short-time work *after* productivity realizes is

$$V^u_{\text{stw}}(\epsilon) = u(w^u_{\text{stw}}(\epsilon) + s_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon)) - \phi(h_{\text{stw}}(\epsilon))) + \beta \cdot \left[\lambda \bar{V} + (1-\lambda) V^u_{\text{stw}}(\epsilon)\right]$$

The value of worker *without* short-time work *after* productivity realizes to ϵ who works at a constrained firm is

$$V^c(\epsilon) = u(w^c(\epsilon) - \phi(h(\epsilon))) + \beta \cdot \left[\lambda \bar{V} + (1-\lambda) V^c(\epsilon)\right]$$

and the value of a worker *with* short-time work *after* productivity realizes who works at a constrained firm is

$$V^c_{\text{stw}}(\epsilon) = u(w^c_{\text{stw}}(\epsilon) + s_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon)) - \phi(h^c_{\text{stw}}(\epsilon))) + \beta \cdot \left[\lambda \bar{V} + (1-\lambda) V^c_{\text{stw}}(\epsilon)\right]$$

where $\bar{V}$ is the expected worker value *before* a new productivity shock has realized. It is given by

$$\begin{aligned}\bar{V} = (1-p) &\left(\int_{\max_{\{\epsilon_{\text{stw}}, \xi^u_s\}}}^{\infty} V^u(\epsilon)\, dG(\epsilon) + \int_{\epsilon^u_s}^{\max\{\epsilon_{\text{stw}}, \epsilon^u_s\}} V^u_{\text{stw}}(\epsilon)\, dG(\epsilon) \right) \\ &+ p \left(\int_{\max\{\epsilon_{\text{stw}}, \xi^c_s\}}^{\infty} V^c(\epsilon)\, dG(\epsilon) + \int_{\epsilon^c_s}^{\max\{\epsilon_{\text{stw}}, \epsilon^c_s\}} V^c_{\text{stw}}(\epsilon)\, dG(\epsilon) \right) \\ &+ \rho \cdot U\end{aligned}$$

where ρ is the separation rate. The Value of an unemployed worker U is given by

$$U = u(b) + f \cdot \bar{V} + (1-f) \cdot U$$

Bargaining Bargaining takes place before both - the realization of productivity ϵ and whether the firm is constrained or not - become known. With short-time work, there are a few more items that workers and firms bargain about than in the lay-off tax regime.

Further, there is no commitment to the contract on the workers' side. This implies that workers will unilaterally choose to leave the match once the outside option of becoming unemployed offers a weakly higher expected utility than remaining employed at the firm.

$$\begin{aligned} V^u(\epsilon) \leq U, \qquad V^u_{stw}(\epsilon) \leq U \\ V^c(\epsilon) \leq U, \qquad V^c_{stw}(\epsilon) \leq U \end{aligned}$$

In the case with and without short-time work, respectively. For each possible state $\epsilon \in \mathbb{R}_+$ worker and firm agree on the total monthly wage functions $w^u(\epsilon)$ and $w^c(\epsilon)$, as well as the total monthly wages paid in both cases while on short-time work $w^u_{\text{stw}}(\epsilon)$ and $w^c_{\text{stw}}(\epsilon)$. The hours functions, likewise, are bargained over separately - for times in which the firm has no access to short-time work ($h^u(\epsilon)$ and $h^c(\epsilon)$) and for times on short-time work ($h^u_{\text{stw}}(\epsilon)$ and $h^c_{\text{stw}}(\epsilon)$). Lastly, the separation thresholds are also agreed upon for both cases: access

to short-time work and no access to short-time work. We denote the separation thresholds without short-time work by ξ_s^u and ξ_s^c. ϵ_s^u, and ϵ_s^c denote the separation thresholds while on short-time work.

Formally, the bargaining outcome solves

$$\max_{\mathcal{X}} \quad \bar{J}^{1-\eta}\left(\bar{V}-U\right)^{\eta}$$

Subject to

$$\begin{aligned}
V^u(\epsilon) \geq U \quad , V^c(\epsilon) \geq U \quad , V^u_{\text{stw}}(\epsilon) \geq U \quad , V^c_{\text{stw}}(\epsilon) \geq U \quad &\text{(commitment problem)} \\
y(\epsilon, h^c(\epsilon)) - w^c(\epsilon) \geq -\beta\left[\lambda\bar{J} + (1-\lambda)J^c(\epsilon)\right] \quad &\text{(financial constraint)} \\
y(\epsilon, h^c_{\text{stw}}(\epsilon)) - w^c_{\text{stw}}(\epsilon) \geq -\beta\left[\lambda\bar{J} + (1-\lambda)J^c_{\text{stw}}(\epsilon)\right] \quad &\text{(financial constraint, STW)}
\end{aligned}$$

where the choice set is given by

$$\mathcal{X} = \{w^u(\epsilon),\, w^c(\epsilon),\, w^u_{\text{stw}}(\epsilon),\, w^c_{\text{stw}}(\epsilon),\, h^u(\epsilon),\, h^c(\epsilon),\, h^u_{\text{stw}}(\epsilon),\, h^c_{\text{stw}}(\epsilon),\, \epsilon^u_s,\, \epsilon^c_s,\, \xi^u_s,\, \xi^c_s\}.$$

Here, η denotes the bargaining power of workers. Firms and workers take both the worker's participation constraints and the firm's financial constraints into account when writing contracts. Appendix D.1 derives the bargaining outcomes.

Since workers are risk-averse and firms are risk-neutral, the bargaining outcome is still that firms insure workers against low-productivity states, while workers will accept a lower average wage in exchange for insurance. Much like in the lay-off tax model, as long as the firm is not constrained (either because it is an unconstrained firm or because constraints do not bind) the firm will *fully* insure the worker against productivity risk and the total monthly wage $w(\epsilon)$ will be set to a constant consumption equivalent c^w, equating worker utility across all realizations of ϵ that do not lead to binding constraints. This also includes the state where workers are on STW:

$$u'(c^w(\epsilon)) = u'(c^w_{stw}(\epsilon)) = u'(c^w)$$

Once a firm becomes constrained and the constraint binds, it instead pays the maximum monthly wage $c^w(\epsilon)$ it can still afford. This will depend on whether the firm has access to short-time work or not:

$$\begin{aligned}
c^w(\epsilon) &= z(\epsilon) + \lambda \cdot \frac{k_v}{q} \\
c^w_{\text{stw}}(\epsilon) &= z_{\text{stw}}(\epsilon) + s_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon)) + \lambda \cdot \frac{k_v}{q}.
\end{aligned}$$

When the firm goes on short-time work, the worker is compensated for the reduced working hours by the government. This means that when the firm has a binding borrowing constraint and can only afford to pay the worker $c_{\text{stw}}(\epsilon)$ in monthly wages, going on short-time work can still increase the worker's income. The hours function that worker and firm agree on also differs depending on whether the firm is on short-time work or not. The two cases are pinned down by the conditions

$$\begin{aligned}\alpha \cdot \epsilon \cdot h(\epsilon)^{\alpha-1} &= h(\epsilon)^{\psi} \\ \alpha \cdot \epsilon \cdot h_{\text{stw}}(\epsilon)^{\alpha-1} &= h_{\text{stw}}(\epsilon)^{\psi} + s_{\text{stw}}.\end{aligned}$$

Again, the hours functions turn out to be independent of financial constraints and equilibrium; therefore, they only contain the two general hours functions $h(\epsilon)$ and $h_{\text{stw}(\epsilon)}$. Note that without short-time work, the bargaining outcome for working hours equates to the marginal product of labor and the marginal disutility from labor and thus sets working hours efficiently. With short-time work, working hours take the subsidy s_{stw} into account, and hours are set lower than the efficient number of working hours. This means that through inefficiently low working hours, the short-time work scheme introduces a distortion into the economy. The bargained-over separation thresholds are pinned down by the job-destruction equations

$$\begin{aligned} z(\xi_s^u) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + \frac{\lambda - \eta f}{1-\eta} \cdot \frac{k_v}{q} &= 0 \\ z_{\text{stw}}(\epsilon_s^u) + s_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon_s^u)) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + \frac{\lambda - \eta f}{1-\eta} \cdot \frac{k_v}{q} &= 0 \\ \frac{u(c(\xi_s^u)) - u(b)}{u'(c^w)} + (\lambda - f)\frac{\eta}{1-\eta} \cdot \frac{k_v}{q} &= 0 \\ \frac{u(c_{\text{stw}}(\epsilon_s^c)) - u(b)}{u'(c^w)} + (\lambda - f)\frac{\eta}{1-\eta} \cdot \frac{k_v}{q} &= 0 \end{aligned}$$

In unconstrained firms, the separation decision is determined as a bargaining outcome: firms and workers separate once their joint surplus becomes negative. Importantly, short-time work benefits are not included in the worker's utility function. The rationale is straightforward—under STW, the firm continues to insure the worker's income against idiosyncratic productivity shocks. The worker receives a constant consumption equivalent c^w, regardless of the realization of ϵ. As a result, the worker has no incentive to quit unilaterally. STW operates by reducing the wage burden on the firm required to sustain this consumption level. In effect, STW lowers the cost of providing income insurance, thereby reducing the firm's incentive to initiate separations.

In unconstrained firms, the worker's commitment problem becomes binding: low productivity realizations are passed through to the worker's income. If income falls sufficiently, the worker chooses to quit the match. Short-time work increases the worker's available

income in such cases, thereby increasing the worker's incentive to remain with the firm.

Labor Markets The separation rate ρ is given by $\rho = (1-p) \cdot \rho^u + p \cdot \rho^c$ where $\rho^u = G(\epsilon_s^u) + G(\max\{\xi_s^u, \epsilon_{\text{stw}}\}) - G(\max\{\epsilon_s^u, \epsilon_{\text{stw}}\})$ and $\rho^c = G(\epsilon_c^c) + G(\max\{\xi_s^c, \epsilon_{\text{stw}}\}) - G(\max\{\epsilon_s^c, \epsilon_{\text{stw}}\})$. The remaining worker flows are given by $f = \frac{m}{1-n}$ and $q = \frac{m}{v}$. The steady-state law of motion for employment is

$$n = (1-\lambda) \cdot n + (1-\rho) \cdot n^s,$$

where n^s denotes the number of matches that received a new shock

$$n^s = f \cdot (1-n) + \lambda \cdot n$$

Unemployment can be denoted as $u = 1 - n$. The number of workers who work for unconstrained firms n^u, and the number of workers who work at constrained firms n^c are given by:

$$n^u(p) = \frac{1-p}{\lambda} \cdot (1-\rho^u) \cdot n^s$$
$$n^c(p) = \frac{p}{\lambda} \cdot (1-\rho^c) \cdot n^s$$

Equilibrium A steady state Equilibrium consists of the working hours functions $h(\epsilon)$, $h_{\text{stw}}(\epsilon)$, the consumption equivalent c^w paid as the monthly wage to workers without binding constraints, the consumption-equivalent wage $c^w(\epsilon)$ and c^w_{stw} paid when financial constraints bind, the separation thresholds ξ_s^u, ξ_s^c, ϵ_s^u and ϵ_s^c and labor market flows, i.e the job-finding rate f, the vacancy-filling rate q and the separation rates ρ^u and ρ^c as well as n. The exact equations pinning down equilibrium and their derivation are delegated to Section D in the appendix.

3 Optimal Policy

3.1 The Ramsey Problem

To show the differences in how short-time work and lay-off taxes act against the fiscal externality of UI on separations, we take a Ramsey planner approach. We set up separate Ramsey problems for the lay-off tax regime and the short-time work regime and proceed to compare optimal policies and their implications. In both cases, the Ramsey planner maximizes welfare, subject to the equilibrium constraints stated in Sections C and D in

the appendix. To make the theoretical results more consice, we henceforth assume that $\beta \to 1$ for the remainder of the paper.

To state the welfare function formerly, recall that there is a mass of firm owners v^f. Firm owners have the same preferences over consumption as workers and receive equal shares of firm profits in the economy. Further, let Ω denote the *average* loss of production in consumption equivalent units due to hours distortions:

$$\Omega = \frac{1}{1-\rho}\left((1-p)\cdot \underbrace{\int_{\epsilon_s^u}^{\max\{\epsilon_{\text{stw}},\epsilon_s^u\}} \Omega(\epsilon)\, dG(\epsilon)}_{:=(1-\rho^u)\Omega^u} + p\cdot \underbrace{\int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}},\epsilon_s^c\}} \Omega(\epsilon)\, dG(\epsilon)}_{:=(1-\rho^c)\Omega^c}\right)$$

where

$$\Omega(\epsilon) = z(\epsilon) - z_{\text{stw}}(\epsilon).$$

Stiepelmann (2024) shows that $\Omega \geq 0$, $\frac{\partial \Omega}{\partial s_{\text{stw}}} \geq 0$ and $\frac{\partial \Omega}{\partial \epsilon_{\text{stw}}} \geq 0$.

The welfare function is given by

$$\begin{aligned} W(\pi) = {} & n^u \cdot u(c^w) + n^c \cdot u^c + (1-n)\cdot u(b) \\ & + v^f \cdot u((n\cdot(z-\Omega) - n^u \cdot c^w - n^c \cdot e^c - s(b) - \theta\cdot(1-n)\cdot k_v)/v^f) \end{aligned} \tag{1}$$

where n^u is the mass of workers employed at firms that do not hit their constraint and n^c is the mass of workers whose employers have a binding borrowing constraint. u^c is the *average* utility of a worker at a firm that has hit its borrowing constraint. z is the *average* production, net of work-disutility, and Ω is the *average* loss of production in consumption equivalent units due to hours distortions: Under the lay-off tax regime, $\Omega = 0$ will always hold. The *average* total monthly wage of a worker at a firm that has hit its borrowing constraint is e^c. The UI-tax $s(b) = (1-n)\cdot b$ is used to finance the UI system.

π is the vector of policy parameters the planner can choose, i.e. $\pi = (b, F)$ in the lay-off tax regime and $\pi = (b, \epsilon_{\text{stw}}, s_{\text{stw}})$ under the short-time work regime. The problem of the Ramsey planner is given by

$$\max_{\pi} W(\pi) \quad \text{s.t. equilibrium conditions are fulfilled}$$

The full problem is stated in Section E in the appendix. Since the analysis does not focus on distributional effects between firm owners and workers, we set the mass of firm owners ν^f such that the consumption equivalent of workers always equals the consumption of firm owners.

3.2 The Optimal Lay-Off Tax Regime

Before we turn our attention to how F is set optimally, we first look at how the planner sets b in the presence of a lay-off tax. This allows us to illustrate how a lay-off tax interacts with the fiscal externalities caused by unemployment insurance.

Setting the unemployment insurance level b must balance a trade-off. On the one hand, the Ramsey planner would like to insure the risk-averse worker against income losses after a separation. If there were no welfare costs to increasing b, the Ramsey planner would like to set b to a level that fully equates the utility of employed and unemployed workers. However, as is well known, unemployment insurance leads to fiscal externalities.

Increasing b will lead to more separations as the workers' outside option (i.e., U) becomes more attractive. This will be true for workers at unconstrained and constrained firms. This decreases the expected firm value, and vacancy posting falls as well. On top of that, with risk-averse workers, constrained firms will lose some of their ability to insure workers against bad productivity shocks because with higher b, the continuation value of a firm will be smaller, further tightening the borrowing constraint. The resulting trade-off between more worker insurance and greater fiscal externalities is balanced by the Ramsey planner's first order condition.

Proposition 1. *The optimal unemployment insurance benefit b given lay-off tax F must fulfill the following first-order condition*

$$\underbrace{(1-n)\cdot\left(\frac{u'(b)-u'(c^w)}{u'(c^w)}\right)}_{\text{Income insurance to unemployed, MIUB}} = \underbrace{\left(-\frac{\mathrm{d}f}{\mathrm{d}b}\right)^{ge}\cdot u\cdot L_v(b)}_{\text{Less vacancy posting, MLV}} + \underbrace{\left(\frac{\mathrm{d}\epsilon_s^u}{\mathrm{d}b}\right)^{ge}\cdot\frac{\partial n^u(p)}{\partial\epsilon_s^u}\cdot L_s^u(b,F)}_{\text{More separation in unconstrained firms, MLS}^u}$$
$$+ \underbrace{\left(\frac{\mathrm{d}\xi_s^c}{\mathrm{d}b}\right)^{ge}\cdot\frac{\partial n^c(p)}{\partial\epsilon_s^c}\cdot L_{s,\xi}^c(b)}_{\text{More separations in constrained firms, MLS}^c} + \underbrace{MIE_b(b)}_{\text{Less income insurance to employed}}$$

where L_v denotes the social value of hiring an additional worker, L_s^u and L_s^c denote the social value of the marginal match in an unconstrained and constrained firm,

$$L_v(b)=\left(\frac{\eta-\gamma}{(1-\gamma)(1-\eta)}\cdot\bar{J}+\frac{1}{f}\cdot b\right),\quad L_s^u(b,F)=\lambda\cdot\left(\frac{1}{f}\cdot b-\beta\cdot F\right),\quad L_s^c(b)=\frac{\lambda}{f}\cdot b$$

and MIE_b denotes the social costs form the insurance loss in unconstrained firms when UI benefits are set more generous.

$$MIE_b(b)=n^c(p)\cdot\frac{1}{1-\rho^c}\cdot\left(-\left(\frac{\mathrm{d}\theta}{\mathrm{d}b}\right)^{ge}\right)\cdot\left(\int_{\epsilon_s^c}^{\epsilon^p}\lambda\cdot\frac{\gamma}{f}\cdot\frac{u'(c(\epsilon))-u'(c^w)}{u'(c^w)}\,dG(\epsilon)\right)\cdot k_v$$

PROOF: See Section F.6 in the appendix.

The term labeled $MUIB$ represents the marginal social benefit of a higher unemployment benefits level b, stemming from improved income insurance for unemployed workers.

The term labeled MLV captures the marginal social loss associated with a decline in hiring induced by an increase in unemployment insurance benefits. The term comprises two components. First, L_v denotes the social value of hiring one additional worker. Second, $\left(-\frac{\mathrm{d}f}{\mathrm{d}b}\right)^{ge} \cdot u$ represents the reduction in the number of new hires. Intuitively, higher unemployment benefits raise the worker's outside option, which puts upward pressure on wages and reduces the value of a worker for the firm. As a result, firms find it less profitable to post vacancies, leading to a decline in job creation.

The MLS terms denote the marginal social loss resulting from increased separations caused by UI benefits. Specifically, MLS^u captures the social cost arising from increased separations in unconstrained firms, while MLS^c captures the corresponding cost in constrained firms. Intuitively, higher unemployment insurance benefits raise workers' outside options, exerting upward pressure on wages. As a result, unconstrained firms want to initiate separations early. In constrained firms, workers are less willing to accept income reductions and thus quit into unemployment sooner.

The term MLS^u consists of two components. First, L_s^u denotes the social value of the marginal match at an unconstrained firm. Second, $\left(\frac{\mathrm{d}\epsilon_s^u}{\mathrm{d}b}\right)^{ge} \cdot \frac{\partial n^u(p)}{\partial \epsilon_s^u}$ represents the number of additional separations in unconstrained firms.

Similarly, MLS^c consists of L_s^c, the social value of the marginal match at a constrained firm, and $\left(\frac{\mathrm{d}\epsilon_s^c}{\mathrm{d}b}\right)^{ge} \cdot \frac{\partial n^c(p)}{\partial \epsilon_s^c}$ which denotes the number of additional constrained matches lost due to higher UI benefits.

Finally, MIE^c labels the social loss of worker income insurance in financially constrained firms consisting of two parts: $\left(-\left(\frac{\mathrm{d}\theta}{\mathrm{d}b}\right)^{ge}\right) \cdot \hat{IE}_\theta$ denotes the average social loss arising from financially constrained firms' reduced capacity to provide income insurance[5] while n^c denotes the number of constrained firms. Intuitively, higher UI benefits reduce the continuation value of a worker for a firm. Firms can thus borrow less in bad periods, reducing their ability to insure the worker.

Note that, in general, all these loss terms consist of the marginal social effect of b on the respective threshold (or labor market tightness in the case of MIE^c) multiplied by the mass of affected workers.

[5]Note that ϵ^p in IE_θ denotes the productivity level at which financial constraints become binding for a constrained firm.

The lay-off tax F enters the policy trade-off exclusively through the MLS^u term. Importantly, it does not affect any terms associated with financially *constrained* firms. The intuition is straightforward: once firms become financially constrained, they lose their ability to insure workers against adverse productivity shocks. As described in Section 2, low productivity is then fully passed through to the worker's income. If the income falls sufficiently, the value of remaining employed falls below the value of unemployment ($V^c(\epsilon) < U$) and the worker will choose to quit into unemployment. Since the firm cannot sustain a wage that the worker is willing to work for, it cannot prevent the worker from quitting. Thus the firm has no means to avoid paying lay-off taxes rendering lay-off tax ineffective.

As a result, lay-off taxes can only mitigate the negative fiscal externalities of UI by combating separations in *unconstrained* firms. This means that when $p = 0$, lay-off taxes can induce the efficient number of separations in the economy. As p increases, the lay-off taxes gradually lose their effectiveness until they lose their bite completely once $p = 1$.

On the upside, the planner can choose to completely eliminate this part of fiscal UI externalities. This can be done with the optimal Lay-off tax stated in Proposition 2.

Proposition 2. *The optimal backloaded lay-off tax F is determined by its FOC:*

$$F = \frac{1}{\beta} \cdot \frac{1}{f} \cdot b + BE_{lt}(b) \tag{2}$$

PROOF: See Section F.5 in the appendix.

The first component captures the fiscal externality of the unemployment insurance system. It reflects the expected UI benefits a worker receives upon entering unemployment, and thus represents the fiscal cost that each separation imposes on the UI system. BE_{lt} is a bargaining effect - an expression acknowledging that setting F will have general equilibrium effects on other equilibrium variables such as worker flows via the bargaining process. Note that the optimal lay-off tax implements the efficient number of separations in unconstrained firms as shown in Corollary 1.

Corollary 1. *(i) Optimal Lay-off taxes fully eliminate socially inefficient separations in* unconstrained *firms.*
(ii) If $p = 0$, then all inefficient separations are eliminated in the economy

PROOF: See Section F in the appendix.

The proof shows that the optimal lay-off tax sets the Lagrange multiplier for the separation condition of unconstrained firms equal to zero.

Note that our model extends the insights of Cahuc and Zylberberg (2008) and Blanchard and Tirole (2008) by embedding optimal lay-off taxes within a modern search-and-matching framework of the labor market. Both of these earlier studies show that lay-off taxes can decentralize the social planner's allocation in settings that focus solely on the separation margin. By contrast, search-and-matching models feature both a hiring margin and a separation margin. In the absence of financial constraints (i.e., when $p = 0$), our model confirms that the logic underlying lay-off taxes - namely, their ability to correct the fiscal externality associated with unemployment insurance - carries over to this richer framework by eliminating inefficient separations. Jung and Kuester (2015) show that a combination of a vacancy subsidy and a layoff tax can decentralize the planner's solution in a search-and-matching model, as the vacancy subsidy additionally operates on the vacancy-posting margin.

3.3 The Optimal Short-Time Work Regime

Under the short-time work regime the Ramsey planner chooses s_{stw} and ϵ_{stw} (through D). Again, we first turn to how the planner sets the level of UI benefits b in the presence of the short-time work scheme. Setting the unemployment insurance level b must balance the same trade-off as before.

Proposition 3. *The optimal unemployment insurance benefit b given a short-time work scheme $(s_{stw}, \epsilon_{stw})$ must fulfill the following first-order condition:*

$$\underbrace{(1-n)\cdot\left(\frac{u'(b)-u'(c^w)}{u'(c^w)}\right)}_{\textit{Income insurance to unemployed, MUIB}} = \underbrace{\left(-\frac{\mathrm{d}f}{\mathrm{d}b}\right)^{ge}\cdot u\cdot L_v(b)}_{\textit{Less vacancy posting, MLV}} + \underbrace{\left(-\frac{\mathrm{d}\epsilon_s^u}{\mathrm{d}b}\right)^{ge}\cdot\frac{\partial n^u(p)}{\partial\epsilon_s^u}\cdot L_s^u(b,s_{stw})}_{\textit{More separations in unconstr. firms, MLS}^u}$$
$$+\underbrace{\mathbb{1}(\epsilon_{stw}\geq\epsilon_s^c)\cdot\left(-\frac{\mathrm{d}\epsilon_s^c}{\mathrm{d}b}\right)^{ge}\cdot\frac{\partial n^c(p)}{\partial\epsilon_s^c}\cdot L_s^c(b,s_{stw})}_{\textit{More separations in constr. firms with STW, MLS}_\epsilon^c}$$
$$+\underbrace{\mathbb{1}(\epsilon_{stw}\leq\xi_s^c)\cdot\left(-\frac{\mathrm{d}\xi_s^c}{\mathrm{d}b}\right)^{ge}\cdot\frac{\partial n^c(p)}{\partial\xi_s^c}\cdot L_{s,\xi}^c(b)}_{\textit{More separations in constr. firms without STW, MLS}_\xi^c}$$
$$+\underbrace{MIE_b^c}_{\textit{Less income insurance to employed workers}}$$

where L_v denotes the social value of hiring an additional worker

$$L_v = \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \bar{J} + \frac{1}{f} \cdot b \right)$$

L_s^u, L_s^c and $L_{s,\xi}^c$ denote the social value of a marginal match in a constrained firm, in an unconstrained firm with access to STW and an unconstrained firm without access to STW,

$$L_s^u = \left(\frac{\lambda}{f} \cdot b - s_{stw} \cdot \left(\bar{h} - h_{stw}(\xi_s^u) \right) \right) \quad with \quad i \in \{u, c\}, \quad L_{s,\xi}^c = \frac{\lambda}{f} \cdot b$$

and MIE_b^c denotes the total insurance loss from marginally increasing UI benefits in constrained firms

$$MIE_b^c = n^c(p) \cdot \left(-\frac{\mathrm{d}\theta}{\mathrm{d}b} \right)^{ge} \cdot \frac{1}{1-\rho^c} \cdot \left(\int_{\max\{\epsilon_{stw}, \xi_s^c\}}^{\epsilon^p} \lambda \cdot \frac{\gamma}{f} \cdot \frac{u'(c(\epsilon)) - u'(c^w)}{u'(c^w)} \, dG(\epsilon) \right.$$
$$\left. + \int_{\epsilon_s^c}^{\max\{\epsilon_{stw}, \epsilon_s^c\}} \lambda \cdot \frac{\gamma}{f} \cdot \frac{u'(c_{stw}(\epsilon)) - u'(c^w)}{u'(c^w)} \, dG(\epsilon) \right) \cdot k_v$$

PROOF: See Section G.6 in the appendix.

Like with the lay-off tax regime, the marginal social benefit of more generous unemployment benefits ($MUIB$) has to equal the marginal welfare losses, i.e. the marginal loss through too few vacancies being posted (MLV), too many separations in both unconstrained and constrained firms (MLS^u and MLS^c) and loss of insurance in constrained firms (MIE^c) caused by an increase in UI benefits. Importantly, constrained firms may feature two distinct separation margins: one at the threshold where matches separate with short-time work support, denoted by $i = \epsilon$, and another where they separate without STW support, denoted by $i = \xi$. Therefore the marginal welfare loss from increased separations in financially constrained firms is divided into firms with and without STW support: $MWL^c = MWL_\epsilon^c + MWL_\xi^c$.

However, there is a key difference between the FOCs under the lay-off tax regime (Proposition 1) and the FOCs under the short-time work regime. The lay-off tax parameter F entered only the MLS^u term, and the lay-off tax could only counteract the adverse effect of unemployment benefits on separations in unconstrained firms. By contrast, the short-time work parameters $(s_{\text{stw}}, \epsilon_{\text{stw}})$ appear in all the terms except the MLV term. Unlike lay-off taxes, short-time work has an effect not only on separations in unconstrained firms, but can also act on constrained firms, counteracting inefficient high separations and inefficiently low insurance provision.

The core problem for financially constrained firms is their inability to absorb negative productivity shocks, which forces them to pass the resulting income loss directly onto workers. If the decline in income is large enough, workers may choose to quit. Short-time work functions as a subsidy that partially replaces this lost income, thereby incentivizing workers to remain with the firm. Through this channel, STW not only reduces inefficient separations, captured by the marginal social loss from separations (MLS) terms but also enhances income insurance by acting on the MIE_b^c term.

A second difference is that the eligibility threshold ϵ_{stw} determines which firms have access to STW, leading to the MLS^c-term being split into two parts. If the eligibility condition is strict, i.e., $\epsilon_{\text{stw}} < \xi_s^c$, then some firms that could otherwise survive with STW support are excluded from the system. Naturally, UI benefits also influence the separation threshold without STW support. Moreover, if ϵ_{stw} is set even more strictly to $\epsilon_{\text{stw}} < \epsilon_s^c$, then constrained firms are entirely excluded from the STW system. As a result, unemployment insurance benefits can no longer influence separations under the STW regime, eliminating the corresponding term from the equation.

It is clear from Proposition 3 that short-time work has the advantage of acting on the loss terms of constrained firms. On the downside, as argued in the bargaining paragraph of Section 2.3, firms on short-time work will distort their working hours, in turn leading to welfare losses. The Ramsey planner has to trade off the benefits of short-time work as a tool that can counteract the fiscal externalities of UI in both unconstrained and constrained firms and the adverse effects of hours distortions. This is reflected in how optimal short-time work is set.

We begin by examining how the short-time work eligibility threshold ϵ_{stw} should be determined. The following proposition establishes that there are three candidates for the optimal eligibility threshold. The optimal threshold can be found by comparing the welfares induced by these three candidates. As discussed in section 2.3, the optimal eligibility condition in terms of working hours D is uniquely implied by the optimal ϵ_{stw}.

Proposition 4. *Assume that the Ramsey planner never wants to use short-time work to impede vacancy posting by destroying the efficiency of the match.*[6] *The optimal eligibility threshold $\epsilon_{stw}^{\star}$ is one of three candidates pinned down by the following conditions:*

Candidate 1

[6]In principle, this could happen in extreme cases of deviations from the Hosios condition. In this case, too many vacancies could introduce inefficiencies to such an extent that the Ramsey planner would use short-time work to destroy vacancies. Since this is neither realistic nor the focus of our analysis, we exclude this case. The formal corresponding assumption is stated as Inequality in Section G.5 in the appendix.

Suppose it is optimal to set $\epsilon_{stw} \geq \xi_s^u$ (ignoring constrained firms with a productivity level between ξ_s^u and ξ_s^c). If

$$MWL_{\epsilon_{stw}} + BE_{stw,2} \quad > \quad MIE^c_{\epsilon_{stw}}, \tag{3}$$

then the optimal threshold is given by $\epsilon_{stw} = \xi_s^c$.
Otherwise, the optimal threshold solves

$$MWL_{\epsilon_{stw}} + BE_{stw,2} \quad = \quad MIE^c_{\epsilon_{stw}}. \tag{4}$$

Here, $MWL_{\epsilon_{stw}} = n \cdot \frac{\partial \Omega}{\partial \epsilon_{stw}}$ denotes the total welfare loss from marginally increasing STW benefits, and $MIE^c_{\epsilon_{stw}}$ denotes the marginal welfare gain from providing income insurance:

$$MIE^c_{stw} = n^s \cdot \frac{p}{\lambda} \cdot g(\epsilon_{stw}) \cdot \frac{\frac{u(c_{stw}(\epsilon_{stw})) - u(c(\epsilon_{stw}))}{c_{stw}(\epsilon_{stw}) - c(\epsilon_{stw})} - u'(c^w)}{u'(c^w)} \cdot [c_{stw}(\epsilon_{stw}) - c(\epsilon_{stw})]\,.$$

<u>*Candidate 2*</u>
Suppose it is optimal to set $\epsilon_{stw} \leq \xi_s^u$ (letting all firms in need access short-time work). Then the optimal threshold is given by $\epsilon_{stw} = \xi_s^u$.

<u>*Candidate 3*</u>
Let $I = [\max\{\epsilon_s^c, \xi_s^u\}, \xi_s^c]$. Suppose it is optimal to set $\epsilon_{stw} \in I$ (partial exclusion of constrained firms in need of STW). If

$$\underbrace{L^c_{s,\epsilon}(\epsilon_{stw})}_{\text{Social value of preserved match}} \quad > \quad \underbrace{\frac{MWL^u_{\epsilon_{stw}}}{MMS^c_{\epsilon_{stw}}}}_{\substack{\text{Social cost of hours distortion} \\ \text{in unconstrained firms} \\ \text{per match retained}}} + BE_{stw,2} \quad \forall \epsilon \in I, \tag{5}$$

then the optimal threshold is given by $\epsilon_{stw} = \xi_s^c$.
If

$$L^c_{s,\epsilon}(\epsilon_{stw}) \quad < \quad \frac{MWL^u_{\epsilon_{stw}}}{MMS^c_{\epsilon_{stw}}} + BE_{stw,2} \quad \forall \epsilon \in I, \tag{6}$$

then the optimal threshold is given by $\epsilon_{stw} = \max\{\epsilon_s^c, \xi_s^u\}$.
Otherwise, the optimal threshold is given by ϵ_{stw} that solves

$$L^c_{s,\epsilon}(\epsilon_{stw}) \quad = \quad \frac{MWL^u_{\epsilon_{stw}}}{MMS^c_{\epsilon_{stw}}} + BE_{stw,2} \tag{7}$$

Here, $L_s^c(\epsilon_{stw})$ denotes the social value of an unconstrained match retained by marginally loosening the participation constraint. $MMS^c_{\epsilon_{stw}} = n^s \cdot \frac{p}{\lambda} \cdot g(\epsilon_{stw})$ denotes the number of constrained matches retained with a looser eligibility condition. $MWL^u_{s_{stw}} = n^s \cdot \frac{1-p}{\lambda} \cdot \frac{\partial \Omega}{\partial \epsilon_{stw}}$ denotes the additional welfare cost of working hours distortions in unconstrained firms when loosening the eligibility condition. Finally, $BE_{stw,2}$ captures general equilibrium effects through the generalized Nash bargaining process.

PROOF: See Section G.5 in the appendix.

To explain why Proposition 4 can only characterize candidates we first discuss the basic trade-off the planner faces. For this, one intuitive additional result is needed, namely that constrained firms will always separate at even higher productivity levels than their unconstrained counterparts as stated by the following lemma:

Lemma 1. *Constrained firms separate for at higher productivity levels than unconstrained firms:*

$$\epsilon_s^c > \epsilon_s^u \quad \textit{and} \quad \xi_s^c > \xi_s^u$$

PROOF: See Section H in the appendix.

Remember that any mass of firms on short-time work introduces socially sub-optimal distortion of working hours into the economy. This makes it socially costly to allow firms that would not have separated even without short-time work benefits access to short-time work.

When choosing ϵ_{stw}, the planner therefore faces a trade-off. The threshold can be set leniently enough to allow constrained matches that would otherwise separate access to short-time work—that is, $\epsilon_{\text{stw}} \geq \xi_s^c$. However, in this case a mass of unconstrained matches with productivity $\epsilon \in [\xi_s^u, \xi_s^c]$ will also gain access to short-time work, even though they would not separate in the absence of the program, thereby introducing socially costly distortions in working hours in unconstrained firms.

Alternatively, to avoid these distortions, the planner could choose to ignore constrained firms and set the eligibility threshold to $\epsilon_{\text{stw}} = \xi_s^u$. At this threshold unconstrained firms separate without access to short-time work. In this case, constrained firms with $\epsilon \in [\max\{\epsilon_s^c, \xi_s^u\}, \xi_s^c]$ are excluded from the STW system even though they are in greater need of support than unconstrained firms.

Since it depends on the exact parameters which of these is the lesser evil, Proposition 4

can only characterize candidates for the optimal eligibility threshold.

With this in mind, we now turn to explaining the rationale behind each of the three candidates for the optimal eligibility threshold. One possibility is that the eligibility condition should be set so loose that all firms in need of support can access short-time work. In this case candidate 1 is optimal. To provide access to short-time work to all firms in need, $\epsilon_{\text{stw}} = \xi_s^c$ is sufficient. The question remains whether an even looser eligibility condition can be optimal. Because short-time work can provide income insurance to workers in financially constrained firms even before the match reaches the separation margin, it may be socially efficient to adopt a more generous eligibility condition. However, this effect could be outweighed by the additional working hours distortion introduced to both constrained and unconstrained firms, in which case the threshold remains at ξ_s^c. Proposition 4 characterizes the conditions under which either of these cases is optimal. Our numerical results presented in Section 5, indeed, show that quantitatively, the distortion effect dominates and that $\epsilon_{\text{stw}} = \xi_s^c$ will be optimal.

A second possibility is to set the eligibility threshold to include only unconstrained firms that are actually in need of short-time work, thereby excluding all constrained firms with a productivity level between ξ_s^u and ξ_s^c from the system. In this case, candidate 2 is optimal. The optimal threshold is given by $\epsilon_{\text{stw}} = \xi_s^u$. As argued by Stiepelmann (2024) $\epsilon_{\text{stw}} < \xi_s^u$ cannot be optimal as this would exclude the most productive firms in need of support from short-time work. Increasing the threshold cannot be optimal either, as this would not impede additional separations but introduce additional distortions in working hours.

A third possibility is that the eligibility criterion for short-time work (STW) should be set sufficiently strict that some financially constrained firms are excluded, even though STW is necessary to retain their workers but not strict enough to exclude all financially constrained firms. In this case, candidate 3 is optimal. In this scenario, all unconstrained firms that require STW for worker retention can participate in short-time work, along with some that could survive without it. The Ramsey planner must balance the social gains from protecting constrained firms through a looser eligibility threshold against the additional hours distortions imposed on unconstrained firms where STW is not needed to retain workers. This trade-off is captured by Inequalities 5 and 6 and depends on the value of p.

A higher p increases the share of workers in financially constrained firms. When p is large, the social value of retaining these workers dominates the costs of distorting hours in unconstrained firms, as the marginal cost of distortion MWL_{stw}^u declines with p while the number of constrained firms retained MMS_{stw}^c rises. In this case, the eligibility threshold is set sufficiently loose to allow all unconstrained firms for which STW is needed to retain

workers to participate, i.e., $\epsilon_{stw} = \xi_s^c$. Conversely, when p is small, hours distortion costs dominate, and the planner effectively ignores STW, setting $\epsilon_{stw} = \xi_s^u$. For intermediate values of p, the optimal threshold might balance the social benefits of worker retention against the social costs of hours distortions in unconstrained firms. Our numerical results in section 5 show that either the social value of worker retention or the social cost of working hours distortion dominate for almost all values of p.

Next, we turn to the optimal level of STW benefits s_{stw}, characterized by following proposition:

Proposition 5. *The optimal short-time work subsidy s_{stw} is pinned down by the first order condition*

$$\bar{s}_{stw} = \underbrace{\frac{T_{UI}}{T_{STW}} \cdot b}_{\textit{Fiscal Ext.}} + \underbrace{\frac{MIE^c_{s_{stw}}}{MMS_{s_{stw}}}}_{\textit{Insurance}} - \underbrace{\frac{MWL_{s_{stw}}}{MMS_{s_{stw}}}}_{\textit{Distortion}} + \underbrace{BE_{stw,3}}_{\textit{Bargaining Effect}}$$

Here, $T_{UI} = \frac{1}{f}$ denotes the expected duration that a worker spends on the UI system, $T_{STW} = \frac{1}{\lambda}$ denotes the expected duration that a workers spends on the STW system, $MIE^c_{s_{stw}}$ denotes the marginal welfare gain from increasing insurance provided to workers in unconstrained firms:

$$MIE^c_{s_{stw}} = \frac{n^c(p)}{1-\rho^c} \cdot \int_{\epsilon_s^c}^{\max\{\epsilon_s^c, \epsilon_{stw}\}} \left(\frac{u'(c_{stw}(\epsilon)) - u'(c^w)}{u'(c^w)} \right) (\bar{h} - h_{stw}(\epsilon))\, dG(\epsilon)$$

Further, $MWL_{\epsilon_{stw}} = n \cdot \frac{\partial \Omega}{\partial \epsilon_{stw}}$ denotes the marginal welfare loss from hours distortion from increasing STW benefits. $MMS_{s_{stw}} = MMS^c_{s_{stw}} + MMS^u_{s_{stw}}$ denotes the marginal number of matches saved with an increase of STW benefits, where

$$MMS^u_{s_{stw}} = n^s \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot \frac{\partial \epsilon_s^u}{\partial s_{stw}}, \qquad MMS^c_{s_{stw}} = \mathbb{1}\{\epsilon_{stw} \geq \epsilon_s^c\} \cdot n^s \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot \frac{\partial \epsilon_s^c}{\partial s_{stw}}$$

denote the marginal number of matches saved in constrained and unconstrained firms. The net-transfer of firms and workers is weighted by how many matches of constrained and unconstrained firms can be retained with an increase in STW benefits:

$$\bar{s}_{stw} = \frac{MMS^u_{s_{stw}}}{MMS_{s_{stw}}} \cdot s_{stw} \cdot \left(\bar{h} - h_{stw}(\epsilon_s^u)\right) + \frac{MMS^c_{s_{stw}}}{MMS_{s_{stw}}} \cdot s_{stw} \cdot \left(\bar{h} - h_{stw}(\epsilon_s^c)\right)$$

PROOF: See Section G.4 in the appendix.

Proposition 5 reveals two motives for the use of short-time work (STW) as well as two

associated costs. The first motive is the fiscal externality correction arising from the unemployment insurance (UI) system. STW helps internalize the fiscal externality that UI imposes on separation decisions in both constrained and unconstrained firms. In unconstrained firms, STW reduces inefficient separations by lowering the cost of insuring workers when the match is hit by a low-productivity shock. In financially constrained firms, STW partly compensates workers for the associated income loss and thereby reduces their incentive to quit.

The optimal level of STW benefits depends on the expected duration of time T_{stw} spent in the STW and UI systems. If the time a match spends in STW T_{stw} is relatively long, the Ramsey planner sets lower benefits in order to encourage reallocation toward more productive matches. Conversely, if workers tend to spend a long time in the UI system, higher STW benefits become desirable, as preserving the match and waiting for the firm's recovery may restore productivity.

The second term reflects the insurance motive for STW. It shows that STW can provide income insurance to workers in financially constrained firms. Here, $MIE^c_{s_{stw}}$ denotes the marginal insurance effect—the total welfare gain from increasing income insurance through higher STW benefits. As in the standard optimal UI framework, the planner would ideally like to fully insure workers against income shocks. However, the Ramsey planner faces a trade-off: increasing STW benefits improves insurance but simultaneously weakens separation incentives. With full insurance, workers would have no incentive to quit following a negative shock.

The optimal level of STW benefits therefore balances the insurance motive against the behavioral distortion arising from inefficient worker retention. This trade-off is reflected in the formula by the fact that $MIE^c_{s_{stw}}$ is scaled by the marginal number of matches preserved when STW benefits increase $MMS_{s_{stw}}$. If STW is highly effective at preventing separations, providing insurance through STW becomes more distortionary, since higher benefits lead to a larger increase in inefficient retention. In this case, the optimal level of STW benefits is lower.

Turning to the costs of STW, the first downside arises from hours distortions. STW distorts firms' working hours choices, generating a marginal welfare loss captured by MWL. The Ramsey planner must therefore trade off the employment stabilization benefits of worker retention against the efficiency costs of distorted hours. In the formula, this trade-off is reflected by the fact that the marginal welfare loss MWL is scaled by the marginal number of matches preserved, $MMS_{s_{stw}}$.

A second limitation, also highlighted in Stiepelmann (2024), is that STW does not address distortions in hiring incentives. Unlike unemployment insurance or hiring subsidies, STW primarily affects separation decisions but does not directly stimulate job creation.

This is reflected in the formula by the absence of a term capturing improvements in hiring incentives.

It is noteworthy that the Ramsey planner sets the *average net* subsidy $\bar{s}_{\text{stw}}$ optimally and then adjusts the actual benefits s_{stw} to achieve this optimal level. Since constrained firms have higher separation thresholds ($\epsilon_s^c > \epsilon_s^u$), it will hold that financially constrained firms reduce working hours less and get a smaller net subsidy: $s_{stw} \cdot \left(\bar{h} - h_{\text{stw}}(\epsilon_s^c)\right) < s_{stw} \cdot \left(\bar{h} - h_{\text{stw}}(\epsilon_s^u)\right)$. The net subsidies are weighted according to how many firms the Ramsey planner can save with an increase of short-time work benefits: $MMS^i_{s_{stw}}$.

Note that the optimal short-time work benefits depend on the fraction of workers in financially constrained firms. First, we have seen that financially constrained firms get a smaller net-subsidy. If the fraction of financially constrained firms rises then the benefits must rise to accommodate for the smaller reduction in working hours. Second, with more firms in financially constrained firms the social welfare gain from providing insurance $MIE^c_{s_{stw}}$ rises and thus the optimal benefits.

Having disentangled how the Ramsey planner uses lay-off taxes and short-time work to counteract the fiscal externalities created by unemployment insurance, it is now time to determine which is the superior policy tool. From a theoretical point of view, this cannot be definitively determined. While lay-off taxes do not introduce any inefficiencies into the economy and can in fact induce the efficient number of separations in unconstrained firms, they cannot counteract fiscal externalities on constrained firms. Short-time work, on the other hand, can act on constrained firms as well. This, though, comes at the price of distorted working hours. To determine which is more effective in practice, we focus on the quantitative analysis of the model in the remainder of the paper.

Corollary 2. *If there are no constrained firms in the economy (i.e., $p = 0$), the optimal lay-off tax increases welfare more than the optimal short-time work scheme.*

PROOF: This follows from the fact that lay-off taxes introduce no distortions but can fully eliminate inefficient separations in unconstrained firms. All inefficiencies that short-time work can act on but lay-off taxes cannot, drop out when $p = 0$.

It is also clear that short-time work is the superior policy instrument if all firms are constrained.

Corollary 3. *If there are no unconstrained firms in the economy (i.e., $p = 1$), the optimal short-time work scheme increases welfare more than the optimal lay-off tax.*

PROOF: This follows from the fact that the only inefficiency that short-time work can act on, i.e., inefficient separations in unconstrained firms, cannot occur anymore. Short-time

work can still act against inefficient separations in constrained firms and provide worker insurance.

In all cases with $p \in (0,1)$, it is ambiguous which of the two policy regimes is superior without quantitative analysis, to which we turn next. It is clear, however, that as p increases lay-off taxes gradually lose their advantage over short-time work.

3.4 Why Loans Cannot Replace Short-Time Work

The theoretical analysis shows that short-time work does particularly well when a lot of firms are financially constrained. A common critique of using short-time work to address inefficiencies arising from financial constraints is, that such constraints could instead be alleviated by providing loans to firms or workers. However, the financial friction we consider cannot be resolved through the provision of loans.

In our model, the financial friction arises from the implicit assumption that financially constrained firms cannot write fully state-contingent contracts that insure them against low-productivity realizations. In low-productivity states, firms do not have the resources to pay workers enough to sustain the match. However, a Ramsey planner would prefer to retain some of these matches, as workers may inefficiently quit in order to receive UI benefits. Providing loans does not resolve this inefficiency because loans merely shift funds across periods and therefore do not alter the fundamental surplus generated by the match and thus the financial constraint.

To see this, assume the government offers a loan to the firm. For simplicity assume that the government loans sum L to the worker that has to be repaid if the firm gets productive in the next period and that does not have to be repaid, if it is not productive. We assume that the loan is fairly priced so that

$$L = \beta \cdot \lambda \cdot R \cdot L + \beta \cdot 0 \quad \Leftrightarrow \quad R = \frac{1}{\beta \cdot \lambda}.$$

Consider the impact of the loan on the financial constraint:

$$y(\epsilon, h(\epsilon)) + L - w^c(\epsilon) \geq -\beta \cdot \left[\lambda \cdot \left(\bar{J} - R \cdot L\right) + (1-\lambda) \cdot J^c(\epsilon)\right]$$

Inserting the fairly priced loan into the formula shows that loans do not alter the financial constraint:

$$y(\epsilon, h(\epsilon)) + L - w^c(\epsilon) \geq L - \beta \cdot \left[\lambda \cdot \bar{J} + (1-\lambda) \cdot J^c(\epsilon)\right]$$

The reason for this is that loans do not alter the fundamental surplus of firms and workers. The loan proves ineffective. This is because the constraint faced by the firm is not a simple liquidity constraint. To sustain employment, the joint surplus of firms and workers must increase. Short-time work achieves this by compensating workers for lost income when hours are reduced, thereby raising the joint surplus of the match.

4 Calibration

We calibrate the model to the U.S. economy at monthly frequency. We calibrate for *CRRA* utility, i.e. $u(c) = \frac{c^{1-\phi}}{1-\phi}$. For ϕ, we set the standard value of 2. We set the labor elasticity of the production function α to 0.65 following Christoffel and Linzert (2010). The elasticity of matching with respect to unemployment γ is set to the standard value of 0.65. As customary, we set the worker bargaining power η to 0.65 as well to fulfill the Hosios condition (Hosios (1990)). Note that 0.65 is well within the range of estimates reported by Petrongolo and Pissarides (2001). The disutility of labor parameter ψ is set to 1.5 to target a Frisch elasticity of labor supply of 0.65 as in Domeij and Floden (2006).

We set the shock arrival probability λ to 0.07, so that a firm will remain at any drawn productivity level for $1/0.07 = 14.29$ months on average. This choice is informed by the estimate of yearly plant-level autocorrelation of productivity of around 0.4 by Abraham and White (2006).[7]

The location of the productivity shock distribution μ is set to -0.80 to normalize the average wage w to 1. Since the effective dispersion of productivity shocks is governed by the cost shock parameter c_f, which is calibrated internally, we set σ to 0.12 following Krause and Lubik (2007).

The remaining parameters are calibrated internally. The calibration targets are shown in Table 1. All targets are met exactly.

[7] In the model, the autocorrelation of productivity is given by $1 - \lambda$. Therefore, $\lambda = 0.07$ implies a monthly autocorrelation of 0.93, which implies a yearly autocorrelation of 0.42.

Table 1: Model calibration targets

Parameter	**Description**	**Value**
Worker flows		
q	Vacancy-filling rate	0.3381
f	Job-finding rate	0.41
ρ^{eff}	Effective monthly separation rate	0.03
Policy parameters		
b^{rep}	UI replacement rate	0.45
F^{rep}	Lay-off tax coverage of UI payments	0.5

Notes: $b^{\text{rep}} \equiv \frac{b}{w}$ denotes the unemployment insurance (UI) replacement rate, i.e. benefits b relative to the average wage w (normalized to 1 in the calibration). The lay-off tax coverage target is reported as $\frac{f}{b}F^{\text{cal}}$, i.e. what fraction of expected UI payments to a laid off worker is covered by the lay-off tax F.

For the purpose of calibration, we set p to 0.5. We target a UI replacement rate $b^{\text{rep}} = \frac{b}{w}$ of 0.45 as reported by Engen and Gruber (2001). Following Jung and Kuester (2015), we target a lay-off tax coverage F^{rep} of 0.5, which means that the lay-off tax covers 50% of the expected UI payments transferred to a worker after a lay-off.[8] Given these policy parameter targets to inform the calibration of b and F, we calibrate the remaining parameters (matching efficiency $\bar{m}$, cost shock scale c_f, and vacancy posting cost k_v) to match standard worker flow targets. Note that since ρ is the probability of separation conditional on a new shock arriving, the effective separation rate ρ^{eff} is different from ρ. It is given by $\rho^{\text{eff}} = \frac{n \cdot \lambda \cdot \rho + (1-n) \cdot f \cdot \rho}{n_0}$, where n_0 is the mass of matches at the beginning of a period. It includes the mass of already existing matches n and new matches $(1-n) \cdot f$. Existing matches draw a new shock with probability λ, while all new matches draw a productivity shock and, like every other match, will separate with probability ρ once productivity realizes.

Table 2 summarizes the calibrated and set parameters of the model.

[8]Since the expected duration of unemployment is $1/f$, the expected UI payments per unemployed worker are given by $\frac{b}{f}$, implying that $F^{\text{rep}} = \frac{F \cdot f}{b}$.

Table 2: Model Parameters: Calibrated and Set Values

Parameter	Description	Value
Calibrated Parameters		
$\bar{m}$	Matching efficiency parameter	0.383
c_f	Cost shock parameter	0.407
k_v	Vacancy posting cost	0.086
b	Unemployment benefit level	0.450
$\bar{h}$	Regular working hours level	0.953
F^{cal}	Lay-off tax	0.549
Set Parameters		
ϕ	Coefficient of relative risk aversion	2
α	Labor elasticity in production function	0.65
γ	Elasticity of matching w.r.t. unemployment	0.65
η	Worker bargaining power	0.65
ψ	Disutility of labor parameter	1.5
λ	Shock arrival probability	0.07
μ	Location of productivity shocks	-0.80
σ	Scale of productivity shocks	0.12
p	Fraction of financially constrained firms	0.5

5 Quantitative Results

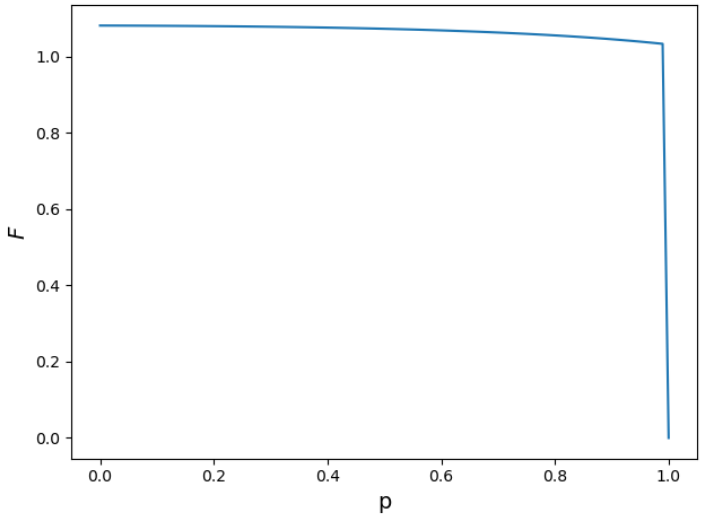
5.1 Taking UI as given

As a first benchmark, we compute the optimal lay-off tax and short-time work parameters chosen by a Ramsey planner who takes the calibrated UI system with its replacement rate of 45% as given. To calculate the Ramsey optimal policy parameters, we maximize the welfare function (Equation 1) numerically. The details are described in Section J in the appendix.

Figure 1 shows the results for the optimal lay-off tax F across $p \in [0, 1]$. The result is what FOC pinning down the optimal lay-off tax (Equation 2) lets us expect. The planner exactly offsets the fiscal externality on unconstrained firms $\frac{\lambda}{f} \cdot b$. Since this does not depend on p in any way, the level of the optimal lay-off tax F hardly changes across p. The slight downward slope is explained by general-equilibrium feedback effects in $BE_{\text{lt}}(b)$. At $p = 1$, lay-off taxes lose their bite completely as they can no longer target any inefficiencies. To illustrate this, we show the optimal lay-off tax to be $F = 0$. In fact, any lay-off tax $F \in \mathbb{R}_+$ is optimal.

Figure 2 shows the Ramsey optimal short-time work parameters across p. The way that the Ramsey planner sets the eligibility threshold reflects the trade-off described in Section

Figure 1: Optimal Lay-Off Tax



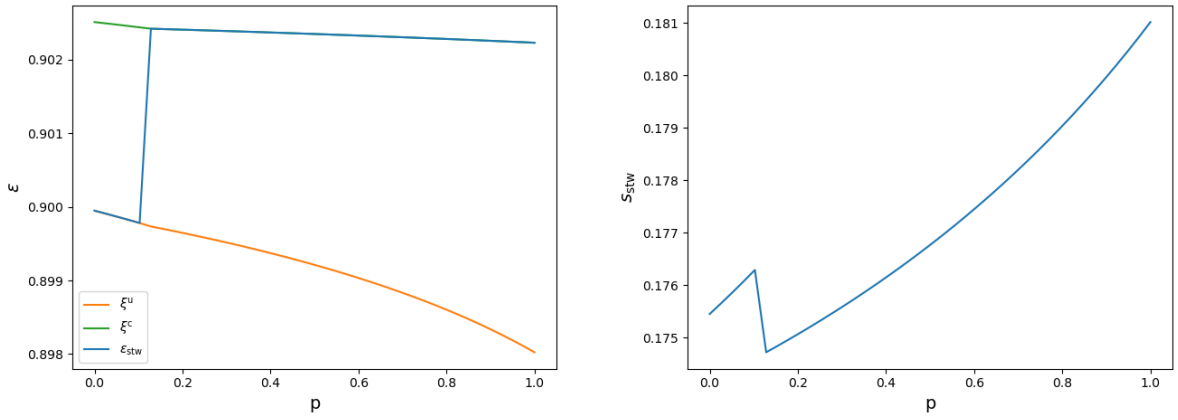
Notes: Optimal lay-off tax F set by the Ramsey planner across $p \in [0, 1]$. The calibrated UI parameter of $b = 0.45$ is taken as given.

3.3 under Proposition 4: While there are only a few constrained firms in the economy, it is very costly to protect all firms, including constrained ones, from premature separations. Setting the eligibility constraint high enough to allow all constraint firms onto short-time work before they hit their separation threshold ξ_s^c will allow many unconstrained firms onto short-time work and collect windfall profits, even though they would not separate without it. As discussed, this introduces inefficient working hours distortions, and as long as only a few firms are constrained, these distortions outweigh the benefit of protecting these few constrained firms. The optimal eligibility threshold is therefore ξ_s^u. However, as the mass of constrained firms grows along the x-axis of Figure 2, the benefits from protecting constrained firms eventually outweigh the added distortion from protecting excess unconstrained firms. At this point ($p = 0.128$), the eligibility condition jumps to the threshold ξ_s^c at which constrained firms without short-time work separate.

The optimal generosity parameter s_{stw} is increasing in p except at the kink at $p = 0.128$. This is because, as discussed in the context of Proposition 5 in Section 3.3, the planner targets the *average* net subsidy. As constrained firms have less margin to adjust their hours, this means that s_{stw} has to rise in p as long as ϵ_{stw} does not change much. Additionally, the planner can provide more insurance with higher s_{stw} which becomes more valuable as p increases, also contributing to the upward slope. However, when ϵ_{stw} jumps to ξ_s^c, many more firms are suddenly on short-time work, introducing greater hours distortion to the economy. To counteract that distortion, the planner reduces the s_{stw} as the more lenient eligibility threshold is introduced, explaining the kink.

Aggregate outcomes of the economies associated with both policy regimes are shown in

Figure 2: Optimal Short-time Work



(a) Optimal ϵ_{stw} **(b)** Optimal s_{stw}

Notes: Optimal eligibility threshold ϵ_{stw} and optimal short-time work replacement rate s_{stw} set by the Ramsey planner across $p \in [0, 1]$. ξ_s^u is the separation threshold of an unconstrained firm without access to short-time work. ξ_s^c is the separation threshold of a constrained firm without access to short-time work. The calibrated UI parameter of $b = 0.45$ is taken as given.

Figure 8 in appendix B.

Having established how the Ramsey planner sets both lay-off taxes and short-time work, it is now time to discuss which is the superior policy tool. Figure 2 shows the respective welfare induced by the optimal lay-off tax and the optimal short-time work schemes across p on the left. Unsurprisingly, welfare is decreasing in p in both cases as higher shares of firms become constrained. As expected, when there are no or only a few constrained firms, the lay-off tax is the superior policy tool. Crucially, however, the slopes are different for the two policy regimes: as p increases, lay-off taxes gradually lose effectiveness, leading to a faster decline of welfare. The two curves have different slopes and intersect at $p = 0.275$. The interpretation is simple: If more than 27% of firms are financially constrained, short-time work becomes the superior policy instrument; for smaller shares of constrained firms, lay-off taxes are the better tool.

For better interpretation, Figure 3 shows consumption equivalent variation, i.e., by what share consumption would need to be increased in the economy without either short-time work or lay-off taxes to induce the same welfare level as the economy with the respective optimal policy, on the right. To calculate consumption equivalent variation Δ^{policy} we solve

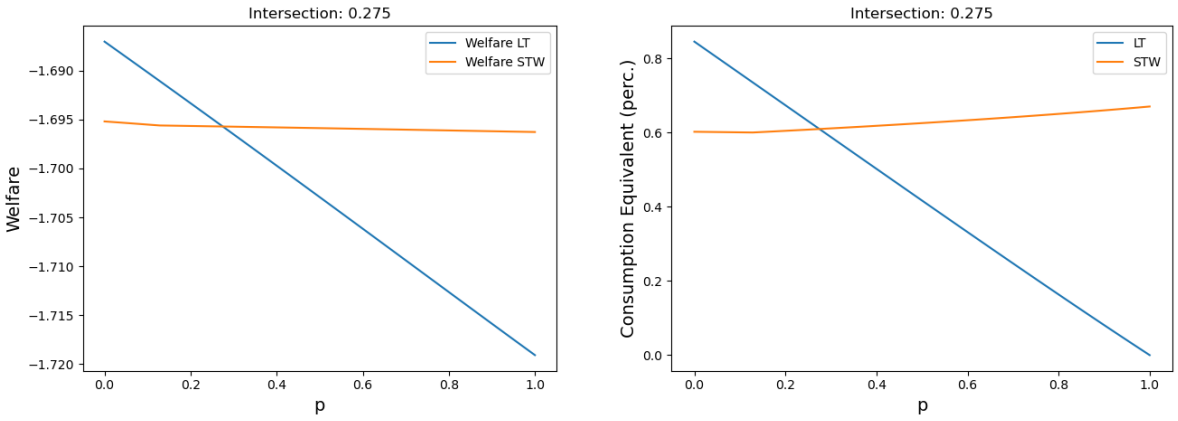
$$u(c_0 \cdot (1 + \Delta^{\text{policy}})) = u(c_{\text{policy}})$$

where $c_0 = u^{-1}(W(0))$ and $c_{\text{policy}} = u^{-1}(W(\pi^\star))$. $W(0)$ is welfare without either short-

time work or lay-off taxes (with the calibrated UI parameter $b = 0.45$) and $W(\pi^\star)$ is welfare under the respective optimal policy.

While welfare improvements in terms of consumption equivalent variation intersect at the exact same point as raw welfare functions, they carry two additional messages. Firstly, the resulting improvements of the respective superior policy scheme are substantial at every p, ranging around 0.6-0.8%. Secondly, as p increases, short-time work becomes not only preferable to lay-off taxes, but improvements in terms of consumption equivalent variation even increase. This stems from the effect that short-time work helps firms provide insurance, which becomes increasingly important for higher p.

Figure 3: Welfare Comparison



(a) Welfare (b) Consumption Equivalent

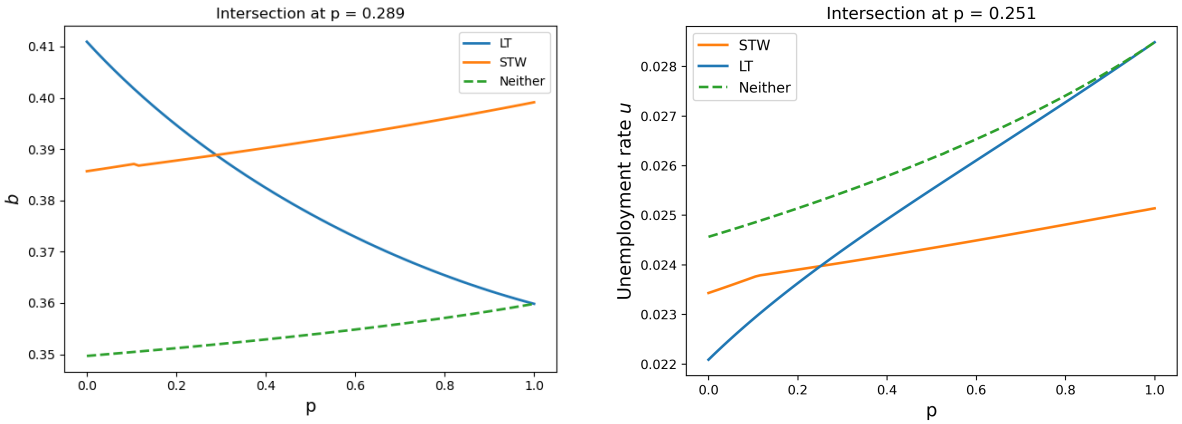
Notes: Plot (a) on the left shows welfare across $p \in [0, 1]$ of the economy with the optimal lay-off tax and the optimal short-time work regimes, respectively. The calibrated UI parameter of $b = 0.45$ is taken as given. Plot (b) on the right shows improvements of optimal short-time work and optimal lay-off taxes over the economy without either of these two policies but with the calibrated UI parameter in terms of consumption equivalent variation.

5.2 Full Optimization

We now turn to the full optimization problem, where the Ramsey planner optimally chooses all the optimal UI parameter and the optimal lay-off tax and short-time work parameters simultaneously. As a first benchmark, we compute how the Ramsey planner sets the optimal unemployment benefit b absent both lay-off taxes and short-time work. Figure 4 shows the result as a green dashed line. Without lay-off taxes that could counteract the fiscal externality of the unemployment benefit, the optimal unemployment benefit b is set to a lower level compared to the calibrated value of 0.45 ranging between 0.35 and 0.36. It slightly increases in p indicating that the planner values higher insur-

ance more than lowering unemployment when faced with more separations and therefore higher unemployment because of more constrained firms.

Figure 4: Optimal Unemployment Benefits



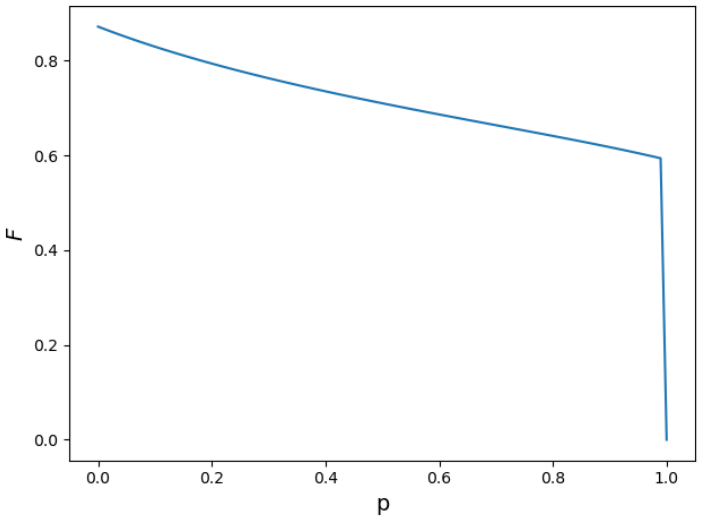
(a) Optimal Unemployment Benefits b (b) Unemployment Rate u

Notes: The optimal unemployment benefit b set by the Ramsey planner across $p \in [0, 1]$ without either lay-off taxes or short-time work (dashed green line), while simultaneously setting the lay-off tax (blue line) and while simultaneously setting short-time work (orange line) is shown in plot (a) on the left. The resulting unemployment rate u across p in the three cases is shown in plot (b) on the right.

Having established how unemployment benefits are set in the absence of supplementary policies, we now turn to the problem of jointly optimizing the unemployment benefit b and the lay-off tax F. The first thing to note, is that b is set considerably higher than without supplementary policies for $p < 1$ as shown by the blue line in Figure 4. This is because the planner now has the lay-off tax at her disposal to counteract the fiscal externality of the unemployment benefit, which allows her to set a higher unemployment benefit and thus provide more insurance to workers. The second thing to note is that b sharply decreases with p until it reaches the same level as without supplementary policies at $p = 1$. The reason behind this downward slope is that with more constrained firms, the lay-off tax that can only act on unconstrained matches becomes less and less effective at counteracting the fiscal externality of the unemployment benefit. This forces the planner to set a lower unemployment benefit to reduce the externality. Indeed, as the right panel of Figure 4 shows, the unemployment rates associated with the regime with no supplementary policies and the regime with the optimal lay-off tax converge as p increases until the two curves meet at $p = 1$ as well.

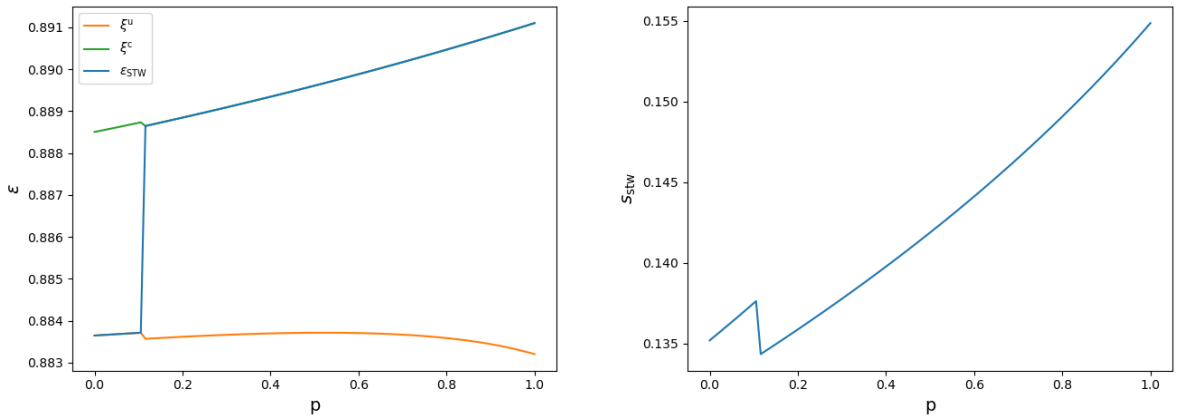
A secondary effect of this is that the optimal lay-off tax F shown in Figure 5 also decreases in p as the planner sets lower and lower unemployment benefits and thus has to counteract a smaller and smaller fiscal externality in unconstrained matches. At $p = 1$, the lay-off tax loses its bite completely and the planner sets the same unemployment benefit as without the option to set a lay-off tax.

Figure 5: Optimal Lay-Off Tax



Notes: The optimal lay-off tax F across $p \in [0, 1]$ when the Ramsey planner optimally chooses the optimal lay-off tax F and the optimal unemployment benefit b simultaneously.

Figure 6: Optimal Short-time Work



(a) Optimal ϵ_{stw} **(b)** Optimal s_{stw}

Notes: Optimal eligibility threshold ϵ_{stw} and optimal short-time work replacement rate s_{stw} set by the Ramsey planner across $p \in [0, 1]$ while setting the optimal unemployment benefit b simultaneously. ξ_s^u is the separation threshold of an unconstrained firm without access to short-time work. ξ_s^c is the separation threshold of a constrained firm without access to short-time work.

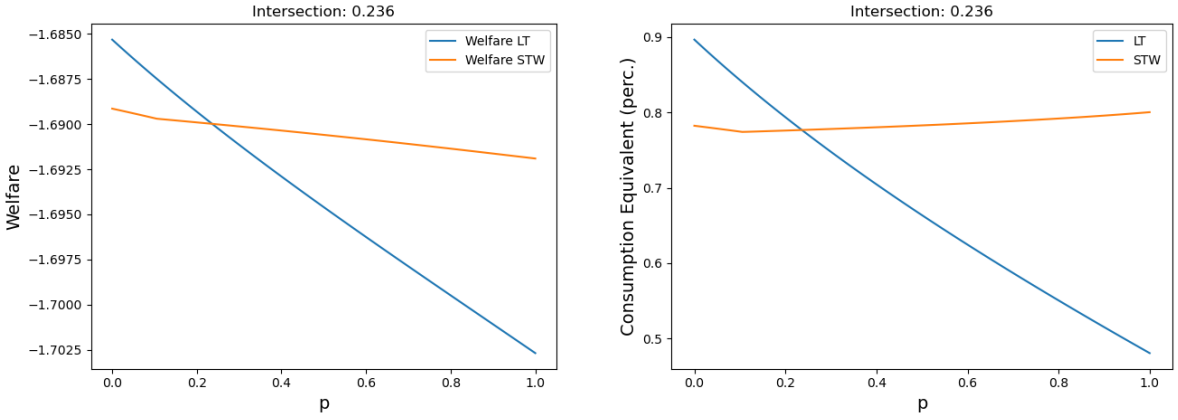
Next, we turn to the joint optimization of the unemployment benefit b and the short-time work parameters ϵ_{stw} and s_{stw}. Like with the lay-off tax, the optimal unemployment benefit b with short-time work shown in Figure 4 as an orange line is set at a higher level than without supplementary policies. Unlike with the lay-off tax, however, the optimal unemployment benefit b increases with p. The reason for this pattern is that with more constrained firms, more matches are at risk of separation leading higher welfare gains from providing insurance through the unemployment benefit. Since short-time work remains effective at counteracting the fiscal externality of the unemployment benefit even with more constrained firms, the planner is not kept from increasing b by increasing numbers of separations as with the lay-off tax. As the right panel of Figure 4 shows, the unemployment rate under the short-time work regime remains much lower for higher p than in the other regimes as a result.

This dynamic also explains how the optimal short-time work parameters ϵ_{stw} and s_{stw} shown in Figure 6 are set differently than in the case without optimizing b. The qualitative patterns of the optimal eligibility thresholds persist: the planner still initially sets $\epsilon_{\text{stw}} = \xi_s^u$ before transitioning to $\epsilon_{\text{stw}} = \xi_s^c$ as p increases for the same reasons described in Section 5.1. However, there are quantitative differences. Both ξ_s^u, and with it the eligibility threshold, now increase with p, reflecting that higher unemployment benefits lead to higher separation thresholds and thus firms are in need of support sooner. Meanwhile, the optimal short-time work generosity parameter s_{stw} increases far more steeply in p than in the case where b is held fixed. This sharp increase arises because the higher unemployment benefits amplify the fiscal externality that the planner must counteract, necessitating a much higher short-time work generosity to keep the average net subsidy at the optimal level.

Aggregate outcomes of the economies associated with the different policy regimes are shown in Figure 9 in appendix B.

Finally, we compare the welfare of the optimal lay-off tax and the optimal short-time work when the Ramsey planner optimally sets the unemployment benefit b simultaneously. Figure 7 shows the results. As expected, the welfare of both optimal policies is higher than without optimizing b. The improvement over an economy with calibrated UI parameters but without either lay-off taxes or short-time work in terms of consumption-equivalent variation is around 0.8% with short-time work instead of around 0.6% and is close to 0.9% with lay-off taxes instead of around 0.8% for $p = 0$. In particular, in the case of full optimization, short-time work becomes the superior policy instrument at an even lower share of financially constrained firms of $p = 0.236$ compared to $p = 0.275$ when b is held fixed.

Figure 7: Welfare Comparison



(a) Welfare

(b) Consumption Equivalent

Notes: Plot (a) on the left shows welfare across $p \in [0, 1]$ of the economy with the optimal lay-off tax and the optimal short-time work regimes, respectively, while the Ramsey planner optimally chooses the optimal unemployment benefit b simultaneously. Plot (b) on the right shows improvements of optimal short-time work and optimal lay-off taxes over the economy without either of short-time work or lay-off taxes as well as without setting b optimally; but with the calibrated UI parameter in terms of consumption equivalent variation.

6 Conclusion

We set out by asking the simple question, which of the two policy tools that policy makers in many countries already employ - lay-off taxes or short-time work - is better at counteracting the fiscal externalities of unemployment insurance. While existing literature emphasizes desirable properties of lay-off taxes, we show that their effectiveness is highly sensitive to the assumption that firms are unconstrained in their ability to smooth shocks.

By introducing firm-level financial constraints into a rich yet tractable Diamond-Mortensen-Pissaridis search-and-matching framework (DMP) with endogenous separations, risk-averse workers, and flexible working hours, we show that lay-off taxes struggle with counteracting these externalities when firms are constrained. Short-time work schemes, on the other hand, can act on constrained firms but have the disadvantage of introducing new inefficiencies into the economy through distorting working hours.

Our theoretical analysis, grounded in a Ramsey policy approach, delivers closed-form expressions for optimal lay-off taxes and STW subsidies as a function of the share of financially constrained firms. We show that lay-off taxes are the superior policy instrument with no or only a few financially constrained firms, but that short-time work benefits are better if sufficient firms are financially constrained. Quantitatively, we find that when 24% or more of firms are financially constrained, short-time work dominates lay-off taxes in welfare terms.

Policy recommendations must therefore depend on the prevalence of financial constraints in an economy. Anecdotally, it seems highly plausible that in the U.S., where a lay-off tax is implemented through experience-rated unemployment insurance, financial constraints play a smaller role than in countries like Germany or France with less developed financial sectors and smaller firms where short-time work is widely used. Empirically determining the exact extent of financial constraints in different countries and the implications for optimal policy is therefore key.

Another interesting way forward is to explore how our results change over the business cycle. If the extent to which firms are borrowing constrained or the share of constrained firms changes in downturns, there could be a case for relying more on short-time work during downturns and more on lay-off taxes during good times.

We view this as a fruitful ground for future research.

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A Table of Model Parameters, Variables and Functions

Symbol	Description
n	Mass of employed workers
$u = 1 - n$	Mass of unemployed workers
v	Mass of posted vacancies
$\theta = \frac{v}{1-n}$	Labor market tightness
f	Job-finding rate
q	Vacancy-filling rate
ρ	Separation rate
ρ^u, ρ^c	Separation rates (unconstrained/ constrained)
ρ^F	Separation with lay-off tax
ρ^L	Separation via bankruptcy
p	Probability firm is constrained
λ	Probability of productivity shock
$\tilde{\epsilon}$	Productivity shock (lognormal)
$G(\epsilon), g(\epsilon)$	CDF and PDF of productivity shocks
$\bar{m}$, γ	Matching function parameters
A	Total factor productivity
α	Output elasticity (hours)
ψ	Inverse Frisch elasticity
ν_f	Mass of firm owners
$\bar{h}$	Reference hours under STW
D	Eligibility threshold for STW
ϵ_{stw}	Productivity threshold for STW
b	Unemployment benefits
F	Lay-off tax
s	Lump-sum tax per shock
s_{stw}	STW subsidy per hour gap
k_v	Cost of posting a vacancy
η	Bargaining power of workers
ϕ	Risk aversion

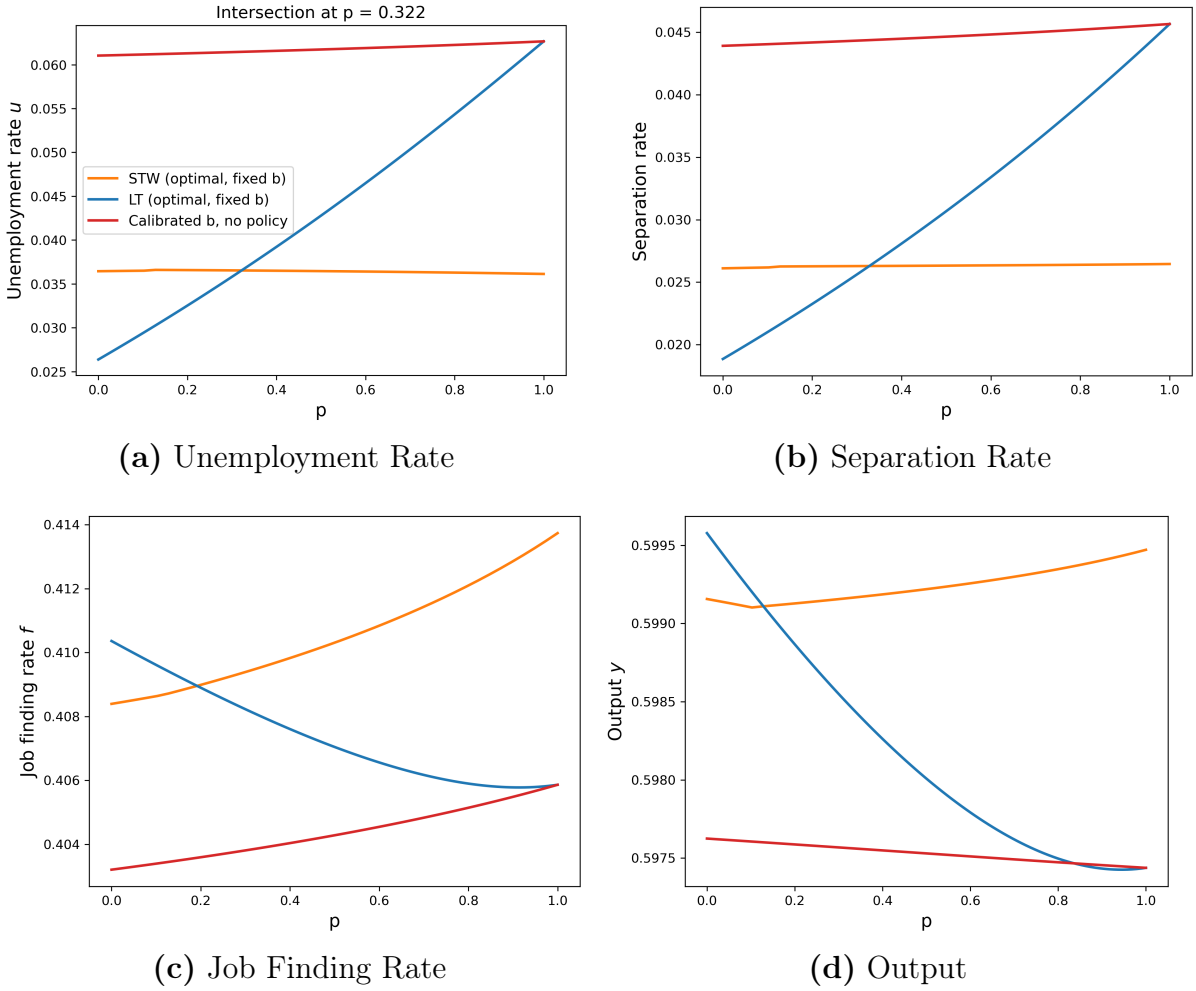
Table 3: Model Parameters, Variables and Functions Part 1

Symbol	Description
u/ c	unconstrained/ constrained
$m(v,n)$	Matching function
$\phi(h)$	Disutility of labor: $\frac{h^{1+\psi}}{1+\psi}$
$u(c)$	Worker utility from consumption net of disutility
$y(\epsilon,h)$	Output
$z(\epsilon,h)$	Output net of disutility (cons.-eq. units)
$z(\epsilon)$	Shorthand for $z(\epsilon,h(\epsilon))$
$z_{\text{stw}}(\epsilon)$	Output net of disutility under STW hours
u^c	Average utility of a worker at a constrained firm
e^c	Average total wage net of disutility paid at a constrained firm
$\Omega(\epsilon)$	Welfare Costs STW in match at productivity ϵ
Ω^u	Average welfare loss STW in unconstrained firms
Ω^c	Average welfare loss STW in constrained firms
Ω	Average welfare loss net of disutility of all firms
$h(\epsilon)$	Hours worked (non-STW)
$h_{\text{stw}}(\epsilon)$	Hours worked under STW
c^w	Constant consumption-equivalent wage
$c^w(\epsilon)$	Constrained consumption wage
$c^w_{\text{stw}}(\epsilon)$	STW-constrained wage
$w^u(\epsilon), w^c(\epsilon)$	(Monthly) Wage functions (u/ c, no STW)
$w^u_{\text{stw}}(\epsilon), w^c_{\text{stw}}(\epsilon)$	(Monthly) Wage (u/ c) under STW
$V^u(\epsilon), V^c(\epsilon)$	Worker value (u/ c, no STW)
$V^u_{\text{stw}}(\epsilon), V^c_{\text{stw}}(\epsilon)$	Worker value (STW)
$\bar{V}$	Expected worker value
U	Value of unemployment
$J^u(\epsilon), J^c(\epsilon)$	Firm value (u/ c, no STW)
$J^u_{\text{stw}}(\epsilon), J^c_{\text{stw}}(\epsilon)$	Firm value (u/ c) under STW
$\bar{J}$	Expected firm value
$\epsilon^u_s, \epsilon^c_s$	Separation thresholds (u/ c, under STW)
ξ^u_s, ξ^c_s	Separation thresholds (u/ c, no STW)
ϵ^p	Productivity where constraint binds
n^s	Mass of matches receiving new shock
n^u, n^c	Mass of workers at u/c firms

Table 4: Model Parameters, Variables and Functions Part 2

B Aggregate Outcomes

Figure 8: Aggregate Outcomes



(a) Unemployment Rate

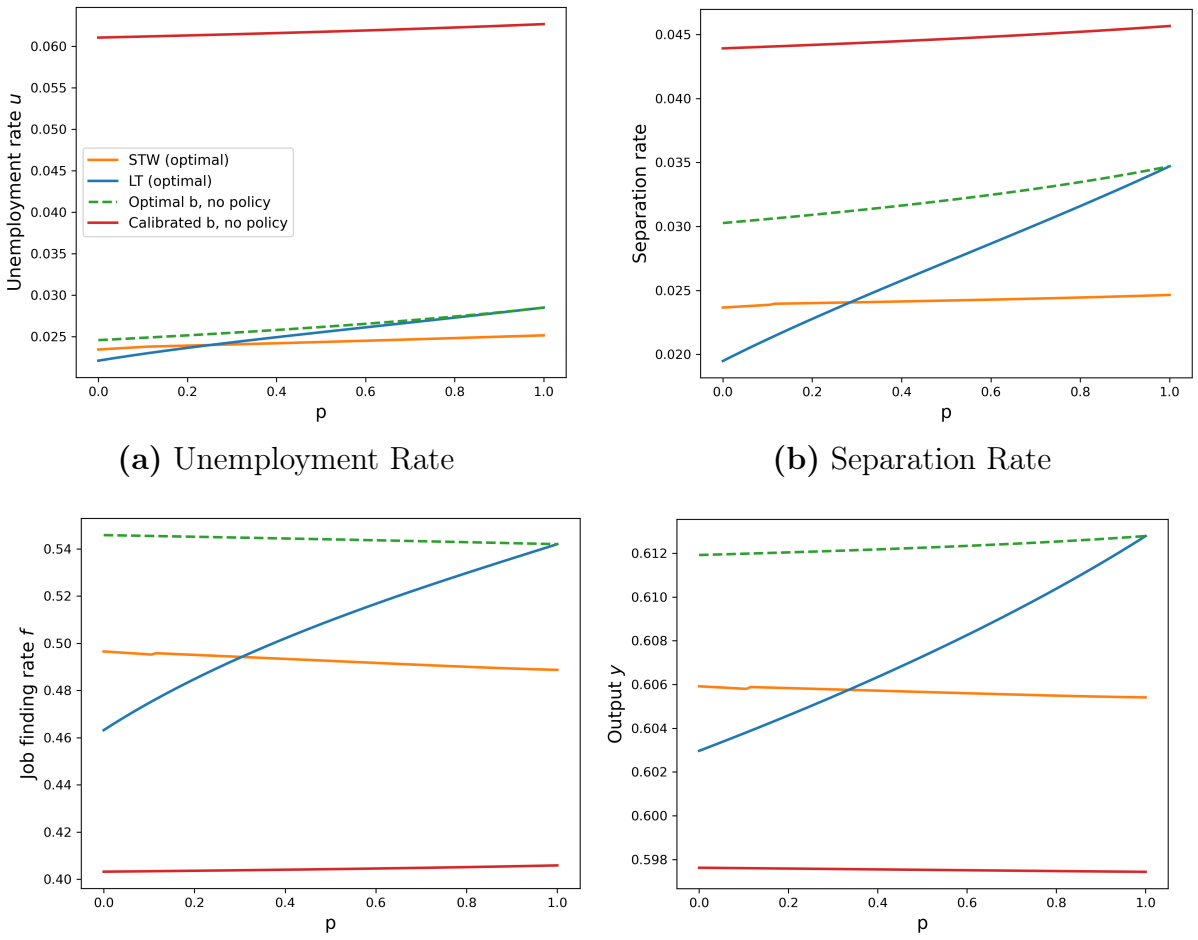
(b) Separation Rate

(c) Job Finding Rate

(d) Output

Notes: Aggregate outcomes for the different policy regimes taking the calibrated unemployment benefit level of $b = 0.45$ as given against the share of constrained firms, p. The red lines are associated with a regime with no lay-off tax and no short-time work, the blue lines with a regime with optimal UI benefits, and the orange lines with optimal short-time work. **(a)** Unemployment rate, **(b)** separation rate, **(c)** job finding rate and **(d)** output.

Figure 9: Aggregate Outcomes - Full Optimization



(a) Unemployment Rate

(b) Separation Rate

(c) Job Finding Rate

(d) Output

Notes: Aggregate outcomes for the different policy regimes (calibrated UI, optimal UI, optimal UI with optimal LT and optimal UI with optimal STW) as a function of the share of constrained firms, p. **(a)** Unemployment rate, **(b)** separation rate, **(c)** job finding rate and **(d)** output.

C Equilibrium Lay-off Tax

C.1 Bargaining

Nash-Bargaining Problem:

$$\max_{w^u(\epsilon),w^c(\epsilon),h^u(\epsilon),h^c(\epsilon),\epsilon_s^u,\epsilon_s^c} \bar{J}^{(1-\eta)} \left(\bar{V}-U\right)^{\eta}$$

subject to

1. Financial constraint of financially constrained firms:

$$y(\epsilon,h(\epsilon)) - w^c(\epsilon) + \beta \cdot \left[\lambda \cdot \bar{J} + (1-\lambda) \cdot J^c(\epsilon)\right] \geq 0$$

2. Commitment problem (Worker):

$$V^c(\epsilon) > U, \quad V^u(\epsilon) > 0$$

First of all, note that we can rewrite the financial constraint as

$$z(\epsilon) - c^c(\epsilon) + \beta \cdot \left[\lambda \cdot \bar{J} + (1-\lambda) \cdot J^c(\epsilon)\right] > 0$$

Second, we can integrate the commitment problem of the worker into the value functions for the firm and worker surplus. For $i \in \{c,u\}$, we can rearrange the condition as:

$$u(c^i(\epsilon)) - u(b) + (\lambda - f) \cdot \beta \cdot (\bar{V}-U) \geq 0$$

Let $\epsilon_s^{i,\text{cp}}$ denote the threshold at which the worker wants to separate unitarily:

$$u(c^i(\epsilon_s^{i,\text{cp}})) - u(b) + (\lambda - f) \cdot \beta \cdot (\bar{V}-U) = 0$$

We can rewrite the value functions of the firm and the surplus of the worker as:

$$
\begin{aligned}
\bar{J} = &-s + (1-p)\cdot\int_{\max\{\epsilon_s^{u,B},\epsilon_s^{u,\text{cp}}\}}^{\infty}\frac{1}{1-\beta\cdot(1-\lambda)}\cdot\left(z(\epsilon)-c^u(\epsilon)+\lambda\cdot\beta\cdot\bar{J}\right)dG(\epsilon)\\
&+(1-p)\cdot G(\max\{\epsilon_s^{u,B},\epsilon_s^{u,\text{cp}}\})\cdot\beta\cdot F\\
&+p\cdot\int_{\max\{\epsilon_s^{c,B},\epsilon_s^{c,\text{cp}}\}}^{\infty}\frac{1}{1-\beta\cdot(1-\lambda)}\cdot\left(z(\epsilon)-c^c(\epsilon)+\lambda\cdot\beta\cdot\bar{J}\right)dG(\epsilon)\\
&+p\cdot G(\max\{\epsilon_s^{c,B},\epsilon_s^{c,\text{cp}}\})\cdot\beta\cdot F\\
(\bar{V}-U) = &(1-p)\cdot\int_{\max\{\epsilon_s^{u,B},\epsilon_s^{u,\text{cp}}\}}^{\infty}\frac{1}{1-\beta\cdot(1-\lambda)}\cdot\left(u(c^u(\epsilon))-u(b)+\lambda\cdot\beta\cdot(\bar{V}-U)\right)dG(\epsilon)\\
&+p\cdot\int_{\max\{\epsilon_s^{c,B},\epsilon_s^{c,\text{cp}}\}}^{\infty}\frac{1}{1-\beta\cdot(1-\lambda)}\cdot\left(u(c^c(\epsilon))-u(b)+\lambda\cdot\beta\cdot(\bar{V}-U)\right)dG(\epsilon)
\end{aligned}
$$

Note that $\epsilon_s^{i,B}$ denotes the separation threshold in the bargaining outcome. Separations occurs either when the productivity falls below the bargained separation threshold or when the commitment constrained of the worker is violated $\epsilon < \max\{\epsilon_s^{c,B},\epsilon_s^{c,\text{cp}}\}$. Finally, note that it is equivalent to maximize over $w^i(\epsilon)$ and $c^i(\epsilon)$ as

$$c^i(z) = w^i(\epsilon) - \phi(h^i(\epsilon)).$$

Set up Kuhn-Tucker Problem:

$$\max_{c^c(\epsilon),c^u(\epsilon),h^c(\epsilon),h^u(\epsilon),\epsilon_s^{c,B},\epsilon_s^{u,B}}\mathcal{B}$$

where

$$\mathcal{B} = \left(\bar{J}\right)^{\eta}\cdot\left(\bar{V}-U\right)^{1-\eta} - \int_0^{\infty}\mathcal{M}(\epsilon)\left[z(\epsilon)-c^c(\epsilon)+\beta\cdot\left[\lambda\cdot\bar{J}+(\lambda-f)\cdot J^c(\epsilon)\right]\right]d\epsilon$$

With Kuhn-Tucker conditions for a maximum:

$$
\begin{aligned}
&(I) \quad \mathcal{M}(\epsilon)\leq 0\\
&(II) \quad \mathcal{M}(\epsilon)\cdot\left[z(\epsilon)-c^c(\epsilon)+\beta\cdot\left[\lambda\cdot\bar{J}+(1-\lambda)\cdot J^c(\epsilon)\right]\right]=0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial\mathcal{B}}{\partial c^u(\epsilon)} &= -(1-\eta)\cdot(1-p)\cdot g(\epsilon)\cdot\left(\frac{\bar{V}-U}{\bar{J}}\right)^{\eta}\\
&\quad+\eta\cdot(1-p)\cdot g(\epsilon)\cdot u'\left(c^u(\epsilon)\right)\cdot\left(\frac{\bar{J}}{\bar{V}-U}\right)^{1-\eta}\\
&= 0
\end{aligned}
$$

$$\iff \frac{1-\eta}{\eta} \cdot \frac{\bar{V}-U}{\bar{J}} = u'\left(c^u(\epsilon)\right) \quad \Rightarrow \quad u'\left(c^u(\epsilon)\right) = u'(c^w)$$

Note that the firm wants to perfectly insure workers against idiosyncratic productivity shocks as long as they are unconstrained.

Define the joint surplus as

$$\bar{S} = \bar{J} + \frac{\bar{V}-U}{u'(c^w)}$$

The resulting surplus splitting rule gives:

$$J = (1-\eta)\cdot\bar{S}, \quad \frac{\bar{V}-U}{u'(c^u)} = \eta\cdot\bar{S}$$

$$\begin{aligned}\frac{\partial\mathcal{B}}{\partial c^c(\epsilon)} = &-(1-p)\cdot g(\epsilon)\cdot(1-\eta)\cdot\left(\frac{\bar{V}-U}{\bar{S}}\right)^{\eta} \\ &+(1-p)\cdot g(\epsilon)\cdot u'\left(c^u(\epsilon)\right)\cdot U\cdot\left(\frac{J}{\bar{V}-U}\right)^{1-\eta} \\ &+\mathcal{M}(\epsilon) \overset{!}{=} 0\end{aligned}$$

Insert the surplus splitting rule:

$$\begin{aligned}\Rightarrow \quad &-(1-p)\cdot g(\epsilon)\cdot\left(\frac{\eta}{1-\eta}\cdot u'(c^w)\right)^{\eta} \\ &+(1-p)\cdot g(\epsilon)\cdot u'\left(c^c(\epsilon)\right)\cdot\eta \\ &+\mathcal{M}(\epsilon)\cdot\left(u'(c^w)\cdot\frac{1-\eta}{\eta}\right)^{1-\eta} = 0\end{aligned}$$

$$\Leftrightarrow (1-p)\cdot g(\epsilon)\cdot\eta\cdot\left(u'(c^c(\epsilon)) - u'(c^w)\right) = -\mathcal{M}(\epsilon)\cdot\left(u'(c^u)\cdot\frac{\eta}{1-\eta}\right)^{1-\eta}$$

Note that if $c^c(\epsilon) < c^w$, then the RHS of the equation is positive due to risk aversion. Thus, firms and workers gain joint surplus until $c^c(\epsilon) = c^w$.

If

$$z(\epsilon) - c^w + \beta\cdot\left[\lambda\cdot\bar{J} + (1-\lambda)\cdot J^c(\epsilon)\right] \geq 0$$

then the constraint is non-binding and $\mathcal{M}(\epsilon) = 0$. ✓

If

$$z(\epsilon) - c^w + \beta \cdot \left[\lambda \cdot \bar{J} + (1-\lambda) \cdot J^c(\epsilon)\right] < 0$$

then the constraint is binding and $\mathcal{M}(\epsilon) < 0$. ✓

Thus, it is optimal for the firm to pay as much as it can:

$$J^c(\epsilon) = z(\epsilon) - c^c(\epsilon) + \beta \cdot \left[\lambda \cdot \bar{J} + (1-\lambda) \cdot J^c(\epsilon)\right] = 0$$

$$\Leftrightarrow \quad c^c(\epsilon) = z(\epsilon) + \lambda \cdot \beta \cdot \bar{J}$$

Using the surplus splitting rule, the Nash-Bargaining problem can be simplified to

$$\max_{w^u(\epsilon), w^c(\epsilon), h^u(\epsilon), h^c(\epsilon), \epsilon_s^u, \epsilon_s^c} (1-\eta)^{(1-\eta)} \cdot (u'(c^w)\eta)^\eta \cdot \bar{S}$$

That is, we just have to maximize the joint surplus with the remaining contracts:

$$\begin{aligned}
\bar{S} = &-s \\
&+ (1-p) \cdot \int_{\max\{\epsilon_s^{u,B}, \epsilon_s^{u,\text{cp}}\}}^{\infty} \frac{1}{1-\beta\cdot(1-\lambda)} \cdot \left(z(\epsilon) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + (1-\eta\cdot f)\cdot\beta\cdot\bar{S} \right) dG(\epsilon) \\
&- (1-p) \cdot G(\max\{\epsilon_s^{u,B}, \epsilon_s^{u,\text{cp}}\}) \cdot \beta \cdot F \\
&+ p \cdot \int_{\epsilon^p}^{\infty} \frac{1}{1-\beta\cdot(1-\lambda)} \cdot \left(z(\epsilon) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + (1-\eta\cdot f)\cdot\beta\cdot\bar{S} \right) dG(\epsilon) \\
&+ p \cdot \int_{\max\{\epsilon_s^{c,B}, \epsilon_s^{c,\text{cp}}\}}^{\epsilon_p} \frac{1}{1-\beta\cdot(1-\lambda)} \cdot \left(z(\epsilon) + \frac{u(c^c(\epsilon)) - u(b)}{u'(c^w)} - c^c(\epsilon) + (1-\eta\cdot f)\cdot\beta\cdot\bar{S} \right) dG(\epsilon) \\
&- (1-p) \cdot G(\max\{\epsilon_s^{c,B}, \epsilon_s^{c,\text{cp}}\}) \cdot \beta \cdot F
\end{aligned}$$

The FOC for working hours in unconstrained firms is:

$$\frac{\partial \mathcal{B}}{\partial h^u(\epsilon)} = (1-p) \cdot g(\epsilon) \cdot \left(\frac{\partial y(\epsilon, h^u(\epsilon))}{\partial h^u(\epsilon)} - \phi'(h^u(\epsilon)) \right) = 0$$

$$\Longleftrightarrow \quad \frac{\partial y(\epsilon, h^u(\epsilon))}{\partial h^u(\epsilon)} = \phi'(h^u(\epsilon))$$

The FOC for working hours in constrained firms is:

$$\frac{\partial \mathcal{B}}{\partial h^c(\epsilon)} = p \cdot g(\epsilon) \cdot \left(\frac{\partial y(\epsilon, h^c(\epsilon))}{\partial h^c(\epsilon)} - \phi'(h^c(\epsilon)) \right) \cdot \frac{u'(c^c(\epsilon))}{u'(c^w)} = 0$$

$$\Longleftrightarrow \quad \frac{\partial y(\epsilon, h^c(\epsilon))}{\partial h^c(\epsilon)} = \phi'(h^c(\epsilon))$$

Suppose that $\epsilon_s^{u,B} > \epsilon_s^{u,cp}$

$$\frac{\partial \mathcal{B}}{\partial \epsilon_s^{u,b}} = -(1-p)\cdot g(\epsilon_s^{u,b})\cdot \Bigg[\frac{1}{1-\beta\cdot(1-\lambda)}\cdot\Big(z(\epsilon_s^{u,b}) + \frac{u(c^w)-u(b)}{u'(c^w)} - c^w + (\lambda-\eta f)\cdot\beta\cdot\bar{S}\Big) + \beta\cdot F\Bigg] = 0$$

$$\iff z(\epsilon_s^{u,b}) + (1-\beta\cdot(1-\lambda))\cdot\beta\cdot F + \frac{u(c^w)-u(b)}{u'(c^w)} - c^w + (\lambda-\eta f)\cdot\beta\cdot\bar{S} = 0$$

Insert free-entry Condition:

$$z(\epsilon_s^{u,b}) + (1-\beta\cdot(1-\lambda))\cdot\beta\cdot F + \frac{u(c^w)-u(b)}{u'(c^w)} - c^w + \frac{\lambda-\eta f}{1-\eta}\cdot\left(\frac{k_v}{q}\right) = 0$$

Note that workers do not have a commitment problem $\epsilon_s^{u,B} > \epsilon_s^{u,cp}$, as workers would never quit under the insurance constraint of the firm:

$$V(\epsilon) - U = u(c^w) - u(b) + (\lambda - f)\cdot\beta\cdot(\bar{V}-U) > 0$$

Workers in unconstrained firms always have a positive surplus!

Suppose that $\epsilon_s^{c,B} \geq \epsilon_s^{c,cp}$

$$\frac{\partial \mathcal{B}}{\partial \epsilon_s^{c,B}} = -(1-p)\cdot g(\epsilon_s^{c,B})\cdot \Bigg[\frac{1}{1-\beta\cdot(1-\lambda)}\cdot\Big(z(\epsilon_s^{c,B}) + \frac{u\left(c^c(\epsilon_s^{c,B})\right)-u(b)}{u'(c^w)} - c^c(\epsilon_s^{c,B}) + (\lambda-\eta f)\cdot\beta\cdot\bar{S}\Big) + F\Bigg] = 0$$

$$\iff z(\epsilon_s^{c,B}) + (1-\beta\cdot(1-\lambda))\cdot\beta\cdot F + \frac{u(c^c(\epsilon_s^{c,B}))-u(b)}{u'(c^w)} - c^c(\epsilon_s^{c,B}) + (\lambda-\eta f)\cdot\beta\cdot\bar{S} = 0$$

Insert free-entry condition:

$$z(\epsilon_s^{c,B}) + (1-\beta\cdot(1-\lambda))\cdot\beta\cdot F + \frac{u(c^c(\epsilon_s^{c,B}))-u(b)}{u'(c^w)} - c^c(\epsilon_s^{cp}) + \frac{\lambda-\eta f}{1-\eta}\cdot\frac{k_v}{q} = 0$$

The participation constraint of workers can be written with free-entry condition as:

$$z(\epsilon_s^{c,cp}) + \frac{u(c^c(\epsilon_s^{c,cp}))-u(b)}{u'(c^w)} - c^c(\epsilon_s^{c,cp}) + \frac{(\lambda-\eta f)}{1-\eta}\cdot\frac{k_v}{q} = 0$$

Note that to avoid paying future lay-off taxes, firms would like workers to stay within the firm for negative surplus values. Thus, the worker decides to leave the firm unilaterally

$\epsilon_s^{c,B} < \epsilon_s^{c,cp}$.

C.2 Job Creation and Wage Equation

The notation of this section follows Appendix E for lay-off taxes.

(I) Value of an unconstrained firm after the idiosyncratic productivity shock has realized:

$$\begin{aligned} & J^u(\epsilon) = z(\epsilon) - c^w + \beta \cdot \left[\lambda \bar{J} + (1-\lambda) \cdot J^u(\epsilon)\right] \\ \Leftrightarrow \quad & J^u(\epsilon) = \frac{1}{1-\beta\cdot(1-\lambda)} \left[z(\epsilon) - c^w + \lambda \cdot \beta \cdot \bar{J}\right] \end{aligned}$$

(II) Value of a constrained firm without binding constraints after the idiosyncratic productivity shock has realized:

$$\begin{aligned} & J^c(\epsilon) = z(\epsilon) - c^w + \beta \cdot \left[\lambda \bar{J} + (1-\lambda) \cdot J^c(\epsilon)\right] \\ \Leftrightarrow \quad & J^c(\epsilon) = \frac{1}{1-\beta\cdot(1-\lambda)} \left(z(\epsilon) - c^w + \lambda \cdot \beta \cdot \bar{J}\right) \end{aligned}$$

(III) Value of a constrained firm with binding constraints after the idiosyncratic productivity shock has realized:

$$\begin{aligned} & J^c(\epsilon) = z(\epsilon) - c^w(\epsilon) + \beta \cdot \left[\lambda \bar{J} + (1-\lambda) \cdot J^c(\epsilon)\right] \\ \Leftrightarrow \quad & J^c(\epsilon) = \frac{1}{1-\beta\cdot(1-\lambda)} \left(z(\epsilon) - c^w(\epsilon) + \lambda \cdot \beta \cdot \bar{J}\right) \end{aligned}$$

(IV) Value of a firm before the idiosyncratic productivity shock has realized:

$$\bar{J} = -s + (1-p) \cdot \int_{\epsilon_s^u}^{\infty} J^u(\epsilon)\, dG(\epsilon) + p \cdot \int_{\epsilon_s^c}^{\infty} J^c(\epsilon)\, dG(\epsilon) - \rho \cdot F$$

Inserting (I)-(III) into (IV) gives:

$$\begin{aligned} \Leftrightarrow \quad \bar{J} = -s + \frac{1}{1-\beta\cdot(1-\lambda)} \Big[& (1-\rho)\cdot z - (1-p)\cdot(1-\rho^u)\cdot c^w - p\cdot(1-\rho^c)\cdot e^c \\ & + (1-p)\cdot\lambda\cdot\beta\cdot\bar{J}\Big] - (1-p)\cdot\rho^u\cdot F \end{aligned}$$

Here z denotes the average production (without distortions) and e^c denotes the average cost of a constrained worker for a firm. Both are defined in section E of the appendix.

(V) Value of an unconstrained worker after the idiosyncratic productivity shock has

realized:

$$\begin{aligned} & V^u(\epsilon) = u(c^w) + \beta \cdot \left[\lambda \bar{V} + (1-\lambda) \cdot V^u(\epsilon)\right] \\ \Leftrightarrow \quad & V^u(\epsilon) = \frac{1}{1-\beta \cdot (1-\lambda)} \left(u(c^w) + \lambda \cdot \beta \cdot \bar{V}\right) \end{aligned}$$

(VI) Value of a constrained worker without binding constraints after the idiosyncratic productivity shock has realized:

$$\begin{aligned} & V^c(\epsilon) = u(c^w) + \beta \cdot \left[\lambda \bar{V} + (1-\lambda) \cdot V^c(\epsilon)\right] \\ \Leftrightarrow \quad & V^c(\epsilon) = \frac{1}{1-\beta \cdot (1-\lambda)} \left(u(c^w) + \lambda \cdot \beta \cdot \bar{V}\right) \end{aligned}$$

(VII) Value of a constrained worker with binding constraints after the idiosyncratic productivity shock has realized:

$$\begin{aligned} & V^c(\epsilon) = u(c^w(\epsilon)) + \beta \cdot \left[\lambda \bar{V} + (1-\lambda) \cdot V^c(\epsilon)\right] \\ \Leftrightarrow \quad & V^c(\epsilon) = \frac{1}{1-\beta \cdot (1-\lambda)} \left(u(c^w(\epsilon)) + \lambda \cdot \beta \cdot \bar{V}\right) \end{aligned}$$

(VIII) Value of a worker before the idiosyncratic productivity shock has realized:

$$\bar{V} = (1-p) \cdot \int_{\epsilon_s^u}^{\infty} V^u(\epsilon)\, dG(\epsilon) + p \cdot \int_{\xi_s^c}^{\infty} V^c(\epsilon)\, dG(\epsilon) + \rho \cdot U$$

Inserting (V)-(VII) into (VIII) gives:

$$\begin{aligned} \Leftrightarrow \quad \bar{V} = \frac{1}{1-\beta \cdot (1-\lambda)} \Big[& (1-p)(1-\rho^u) \cdot u(c^w) + p \cdot (1-\rho^c) \cdot u^c \\ & + \beta \cdot \left[(1-\rho) \cdot \lambda \cdot \beta \cdot \bar{V} + \rho \cdot U\right] \Big] \end{aligned}$$

u^c is defined as the average utility of a worker in a constrained firm. Its definition can be found in appendix E.

(IX) Unemployment:

$$U = u(b) + \beta \cdot \left[f \cdot \bar{V} + (1-f) \cdot U\right]$$

Next, we turn to calculating the joint surplus.

A) First, let us calculate the surplus of the worker after the idiosyncratic productivity

shock has been realized:

$$\begin{aligned}
& V^i(\epsilon) - U = u(c^i(\epsilon)) - u(b) + (\lambda - f) \cdot V^i(\epsilon) + (\lambda - f) \cdot \beta \cdot \bar{V} - (\lambda - f) \cdot \beta \cdot U \\
\Leftrightarrow \quad & V^i(\epsilon) - U = u(c^i(\epsilon)) - u(b) + (\lambda - f) \cdot \beta \cdot (V^i(\epsilon) - U) + (\lambda - f) \cdot \beta \cdot (\bar{V} - U) \\
\Leftrightarrow \quad & V^i(\epsilon) - U = \frac{1}{1 - \beta \cdot (1 - \lambda)} \left(u(c^i(\epsilon)) - u(b) + (\lambda - f) \cdot \beta \cdot (\bar{V} - U) \right)
\end{aligned}$$

B) Next, we calculate the expected surplus of the worker, before the shocks have been realized:

$$\begin{aligned}
\bar{V} - U &= (1 - \rho) \cdot \int_{\epsilon_s^u}^{\infty} (V^u(\epsilon) - U) \; dG(\epsilon) + p \cdot \int_{\epsilon_s^c}^{\infty} (V^c(\epsilon) - U) \; dG(\epsilon) \\
&= \frac{1}{1 - \beta \cdot (1 - \lambda)} \Bigg[(1 - \rho) \cdot (1 - \rho^u) \cdot (u(c^u) - u(b)) \\
&\quad + p \cdot (1 - \rho^c) \cdot (u(c^c) - u(b)) \\
&\quad + (1 - \rho) \cdot (\lambda - f) \cdot \beta \cdot (\bar{V} - U) \Bigg]
\end{aligned}$$

C) Next, we can calculate the joint surplus from the expected value of a worker for a firm and the expected surplus of the worker:

$$\begin{aligned}
S &= \bar{J} + \frac{\bar{V} - U}{u'(c^u)} \\
&= -s + \frac{1}{1 - \beta \cdot (1 - \lambda)} \Bigg[(1 - \rho) \cdot z \\
&\quad + (1 - \rho) \cdot (1 - \rho^u) \cdot \left(\frac{u(c^u) - u(b)}{u'(c^u)} - c^u \right) \\
&\quad + p \cdot (1 - \rho^c) \cdot \left(\frac{u(c^c) - u(b)}{u'(c^u)} - e^c \right) \\
&\quad + (1 - \rho) \cdot \lambda \cdot \beta \cdot \bar{J} + (1 - \rho) \cdot (\lambda - f) \cdot \beta \cdot \frac{\bar{V} - U}{u'(c^u)} \Bigg] - \rho \cdot \beta \cdot F
\end{aligned}$$

Using the surplus splitting rule, we can rewrite the equation above as:

$$\begin{aligned} S = -s + \frac{1}{1-\beta\cdot(1-\lambda)}\Bigg[&(1-\rho)\cdot z \\ &+ (1-\rho)\cdot(1-\rho^u)\cdot\left(\frac{u(c^u)-u(b)}{u'(c^u)} - c^u\right) \\ &+ p\cdot(1-\rho^c)\cdot\left(\frac{u(c^c)-u(b)}{u'(c^u)} - e^c\right) \\ &+ (1-\rho)\cdot(\lambda-\eta f)\cdot\beta\cdot S\Bigg] - \rho\cdot\beta\cdot F \end{aligned}$$

Next, we want to replace the tax in the surplus equation. We can do this by using the budget constraint of the government:

$$\begin{aligned} n^s\cdot s &= -n^s\cdot\rho\cdot F + (1-n)\cdot b \\ \Leftrightarrow \quad s &= -\rho\cdot F + \frac{1-n}{n^s}\cdot b \\ \text{Insert } n^s &= \frac{\lambda}{1-\rho}\cdot n \\ \Rightarrow \quad s &= \rho\cdot F + \frac{1}{\lambda}\cdot(1-\rho)\cdot\frac{1-n}{n}\cdot b \end{aligned}$$

Inserting it into the equation for the joint surplus gives:

$$\begin{aligned} S = \frac{1}{1-\beta\cdot(1-\lambda)}\Bigg[&(1-\rho)\cdot\left(z - \frac{1-n}{n}\cdot b\right) + (1-p)\cdot(1-\rho^u)\cdot\left(\frac{u(c^w)-u(b)}{u'(c^w)} - c^w\right) \\ &+ p\cdot(1-\rho^c)\cdot\left(\frac{u(c^c)-u(b)}{u'(c^w)} - e^c\right) + (1-\rho)\cdot(\lambda-\eta f)\cdot\beta\cdot S\Bigg] \\ &-\frac{1}{\lambda}\cdot(1-\rho)\cdot\frac{1-n}{n}\cdot b + \rho\cdot(1-\beta)\cdot F \end{aligned}$$

Next, insert the free-entry condition: $\frac{k_v}{q} = \beta\cdot\bar{J}$

$$\frac{1}{1-\eta}\cdot\frac{k_v}{q}=\frac{\beta}{1-\beta\cdot(1-\lambda)}\cdot\Bigg[(1-\rho)\cdot z$$
$$+(1-p)\cdot(1-\rho^u)\cdot\left(\frac{u(c^w)-u(b)}{u'(c^w)}-c^w\right)$$
$$+p\cdot(1-\rho^c)\cdot\left(\frac{u(c^c)-u(b)}{u'(c^w)}-e^c\right)$$
$$+(1-\rho)\cdot\left(\frac{\lambda-\eta f}{1-\eta}\cdot\frac{k_v}{q}\right)\Bigg]$$
$$-\frac{\beta}{\lambda}\cdot(1-\rho)\cdot\frac{1-n}{n}\cdot b+\rho\cdot\beta\cdot(1-\beta)\cdot F$$

The subsequent derivation of the wage equation is completely analogous to the derivation under STW regime described in Section D.2. The wage equation thus reads:

$$(1-p)\cdot(1-\rho^u)\cdot(1-\eta)\cdot\left(\frac{u(c^w)-u(b)}{u'(c^w)}+\eta\cdot c^w\right)=$$
$$\eta\cdot[(1-\rho)\cdot z+(1-\rho)\cdot\theta\cdot k_v]$$
$$-\frac{1-\beta\cdot(1-\lambda)}{\lambda}\cdot(1-\rho)\cdot\frac{1-n}{n}\cdot b+\frac{1-\beta\cdot(1-\lambda)}{\lambda}\cdot\rho\cdot\beta\cdot(1-\beta)\cdot F$$
$$-p\cdot(1-\rho^c)\cdot(1-\eta)\cdot\left(\frac{u(c^c)-u(b)}{u'(c^w)}+\eta\cdot e^c\right)$$

D Equilibrium STW

D.1 Bargaining

Nash-Bargaining Problem:

$$\max_{\substack{w^u(\epsilon),w^c(\epsilon),w^u_{\text{stw}}(\epsilon),w^c_{\text{stw}}(\epsilon),\\ h^u(\epsilon),h^c(\epsilon),h^u_{\text{stw}}(\epsilon),h^c_{\text{stw}}(\epsilon),\\ \epsilon^u_s,\epsilon^c_s,\xi^u_s,\xi^c_s}} \bar{J}^{1-\eta}\cdot(\bar{V}-U)^{\eta}$$

Subject to:

1. Financial constraints of financially constrained firms:

$$y(\epsilon,h(\epsilon))-w^c(\epsilon)\geq\beta\cdot\left[\lambda\cdot\bar{J}+(1-\lambda)\cdot J^c(\epsilon)\right]$$
$$y(\epsilon,h_{\text{stw}}(\epsilon))-w^c_{\text{stw}}(\epsilon)\geq\beta\cdot\left[\lambda\cdot\bar{J}+(1-\lambda)\cdot J^c_{\text{stw}}(\epsilon)\right]$$

2. Workers have a commitment problem:

$$U > V^u(\epsilon), \quad U > V^u_{\text{stw}}(\epsilon)$$
$$U > V^c(\epsilon), \quad U > V^c_{\text{stw}}(\epsilon)$$

Second, we can integrate the commitment problem of the worker into the value functions for the firm and worker surplus. For $i \in \{\text{stw}, \text{no stw}\}$ and $j \in \{u, c\}$, we can reformulate the condition as:

$$u(c_i^j(\epsilon)) - u(b) + (\lambda - f) \cdot \beta \cdot (\bar{V} - U) \geq 0$$

Let $\epsilon_s^{j,\text{cp}}$ denote the threshold at which the worker wants to separate from a firm with STW support:

$$u(c_i^j(\epsilon_s^{j,\text{stw,cp}})) - u(b) + (\lambda - f) \cdot \beta \cdot (\bar{V} - U) = 0$$

Likewise, we can determine the threshold at which workers leave firms without STW support, $\xi_s^{j,\text{cp}}$, as:

$$u(c_i^j(\xi_s^{j,\text{cp}})) - u(b) + (\lambda - f) \cdot \beta \cdot (\bar{V} - U) = 0$$

We can rewrite the value functions of the firm and the surplus of the worker as:

$$\begin{aligned}
\bar{J} = -s + (1-p) \cdot \Bigg[& \int_{\max\{\xi_s^{u,B}, \xi_s^{u,\text{cp}}, \epsilon_{\text{stw}}\}}^{\infty} \frac{1}{1 - \beta \cdot (1-\lambda)} \left(z(\epsilon) - c^u(\epsilon) + \lambda\beta\bar{J} \right) dG(\epsilon) \\
& + \int_{\max\{\epsilon_s^{u,B}, \epsilon_s^{u,cp}, \epsilon_{\text{stw}}\}}^{\max\{\epsilon_s^{u,B}, \epsilon_s^{u,cp}, \epsilon_{\text{stw}}\}} \frac{1}{1 - \beta \cdot (1-\lambda)} \left(z_{\text{stw}}(\epsilon) - c^u_{\text{stw}}(\epsilon) + \lambda\beta\bar{J} \right) dG(\epsilon) \Bigg] \\
+ p \cdot \Bigg[& \int_{\max\{\xi_s^{c,B}, \xi_s^{c,\text{cp}}, \epsilon_{\text{stw}}\}}^{\infty} \frac{1}{1 - \beta \cdot (1-\lambda)} \left(z(\epsilon) - c^c(\epsilon) + \lambda\beta\bar{J} \right) dG(\epsilon) \\
& + \int_{\max\{\epsilon_s^{c,B}, \epsilon_s^{c,cp}, \epsilon_{\text{stw}}\}}^{\max\{\epsilon_s^{c,B}, \epsilon_s^{c,cp}, \epsilon_{\text{stw}}\}} \frac{1}{1 - \beta \cdot (1-\lambda)} \left(z_{\text{stw}}(\epsilon) - c^c_{\text{stw}}(\epsilon) + \lambda\beta\bar{J} \right) dG(\epsilon) \Bigg]
\end{aligned}$$

$$
\begin{aligned}
&(\bar{V}-U)\\
&=(1-p)\cdot\Bigg[\int_{\max\{\xi_s^{u,B},\xi_s^{u,\mathrm{cp}},\epsilon_{\mathrm{stw}}\}}^{\infty}\frac{1}{1-\beta\cdot(1-\lambda)}\left(u(c^u(\epsilon))-u(b)+\lambda\beta(\bar{V}-U)\right)dG(\epsilon)\\
&\quad+\int_{\max\{\epsilon_s^{u,B},\epsilon_s^{u,\mathrm{cp}}\}}^{\max\{\epsilon_s^{u,B},\epsilon_s^{u,cp},\epsilon_{\mathrm{stw}}\}}\frac{1}{1-\beta\cdot(1-\lambda)}\left(u(c^u_{\mathrm{stw}}(\epsilon))+s_{\mathrm{stw}}(\bar{h}-h_{\mathrm{stw}}(\epsilon))-u(b)+\lambda\beta(\bar{V}-U)\right)dG(\epsilon)\Bigg]\\
&\quad+p\cdot\Bigg[\int_{\max\{\xi_s^{c,B},\xi_s^{c,\mathrm{cp}},\epsilon_{\mathrm{stw}}\}}^{\infty}\frac{1}{1-\beta\cdot(1-\lambda)}\left(u(c^c(\epsilon))-u(b)+\lambda\beta(\bar{V}-U)\right)dG(\epsilon)\\
&\quad+\int_{\max\{\epsilon_s^{c,B},\epsilon_s^{c,\mathrm{cp}}\}}^{\max\{\epsilon_s^{c,B},\epsilon_s^{c,cp},\epsilon_{\mathrm{stw}}\}}\frac{1}{1-\beta\cdot(1-\lambda)}\left(u(c^c_{\mathrm{stw}}(\epsilon))+s_{\mathrm{stw}}(\bar{h}-h_{\mathrm{stw}}(\epsilon))-u(b)+\lambda\beta(\bar{V}-U)\right)dG(\epsilon)\Bigg]
\end{aligned}
$$

Note that $\xi_s^{u,B}, \xi_s^{c,\mathrm{B}}, \epsilon_s^{u,B}$ and $\epsilon_s^{c,\mathrm{B}}$ denote the separation thresholds at bargaining outcome. To be consistent with the bargaining problem under lay-off taxes, we assume that $\epsilon_s^{i,\mathrm{cp}}$ and $\xi_s^{i,\mathrm{cp}}$ are exogenous to the bargaining problem.

Note that we can rewrite the problem so that we optimize over consumption equivalents instead of salaries. $c^i_{\mathrm{stw}}(\epsilon)$ denotes the consumption equivalent paid from the firm to the worker on STW. $c^i(\epsilon)$ is the consumption equivalent consumed by the worker.

$$
\max_{\substack{c^u(\epsilon),c^c(\epsilon),c^u_{\mathrm{stw}}(\epsilon),c^c_{\mathrm{stw}}(\epsilon),\\ h^u(\epsilon),h^c(\epsilon),h^u_{\mathrm{stw}}(\epsilon),h^c_{\mathrm{stw}}(\epsilon),\\ \xi_s^{u,B},\xi_s^{c,B},\epsilon_s^{u,B},\epsilon_s^{c,B}}}\left(\bar{J}\right)^{1-\eta}\cdot\left(\bar{V}-U\right)^{\eta}
$$

Financial constraints of financially constrained firms

$$
\begin{aligned}
y(\epsilon,h(\epsilon))-w^c(\epsilon)&\geq\beta\cdot\left[\lambda\cdot\bar{J}+(1-\lambda)\cdot J^c(\epsilon)\right]\\
y(\epsilon,h_{\mathrm{stw}}(\epsilon))-w^c_{\mathrm{stw}}(\epsilon)&\geq\beta\cdot\left[\lambda\cdot\bar{J}+(1-\lambda)\cdot J^c_{\mathrm{stw}}(\epsilon)\right]
\end{aligned}
$$

Set up Kuhn-Tucker Conditions:

$$
\begin{aligned}
&\left(\bar{J}\right)^{\eta}\cdot\left(\bar{V}-U\right)^{1-\eta}-\int_0^{\infty}\mathcal{M}(\epsilon)\left[z(\epsilon)-c^c(\epsilon)+\beta\cdot\left[\lambda\cdot\bar{J}+(1-\lambda)\cdot J^c(\epsilon)\right]\right]dG(\epsilon)\\
&\quad-\int_0^{\infty}\mathcal{M}_{\mathrm{stw}}(\epsilon)\left[z_{stw}(\epsilon)-c^c_{\mathrm{stw}}(\epsilon)+\beta\cdot\left[\lambda\cdot\bar{J}+(1-\lambda)\cdot J^c_{\mathrm{stw}}(\epsilon)\right]\right]dG(\epsilon)
\end{aligned}
$$

With Kuhn-Tucker conditions for a maximum:

(A) (i) $\mathcal{M}(\epsilon)\leq 0$

(ii) $\mathcal{M}(\epsilon)\left[z(\epsilon)-c^c(\epsilon)+\beta\cdot\left[\lambda\cdot\bar{J}+(1-\lambda)\cdot J^c(\epsilon)\right]\right]=0$

(B) (i) $\mathcal{M}_{\mathrm{stw}}(\epsilon)\leq 0$

(ii) $\mathcal{M}_{\text{stw}}(\epsilon)\left[z_{stw}(\epsilon) - c^c_{\text{stw}}(\epsilon) + \beta \cdot \left[\lambda \cdot \bar{J} + (1-\lambda)\cdot J^c_{\text{stw}}(\epsilon)\right]\right] = 0$

FOC for $c^u(\epsilon)$:

$$\begin{aligned}\frac{\partial \mathcal{B}}{\partial c^u(\epsilon)} = &-(1-\eta)\cdot(1-p)\cdot g(\epsilon)\left(\frac{\bar{V}-U}{\bar{J}}\right)^{\eta} \\ &+\eta\cdot(1-p)\cdot g(\epsilon)\cdot u'(c^u(\epsilon))\left(\frac{\bar{J}}{\bar{V}-U}\right)^{1-\eta} = 0\end{aligned}$$

Unconstrained firms want to insure workers against idiosyncratic productivity shocks, outside STW:

$$\iff \quad \frac{1-\eta}{\eta}\cdot\frac{\bar{V}-U}{\bar{J}} = u'\left(c^u(\epsilon)\right) \quad \Rightarrow \quad u'(c^u(\epsilon)) = u'(c^u(\epsilon')) = u'(c^u)$$

FOC for $c^u_{\text{stw}}(\epsilon)$:

$$\begin{aligned}\frac{\partial \mathcal{B}}{\partial c^u_{\text{stw}}(\epsilon)} = &-(1-\eta)(1-p)\cdot g(\epsilon)\left(\frac{\bar{V}-U}{\bar{J}}\right)^{\eta} \\ &+\eta\cdot(1-p)\cdot g(\epsilon)\cdot u'\left(c^u_{\text{stw}}(\epsilon)\right)\cdot\eta\cdot\left(\frac{\bar{J}}{\bar{V}-U}\right)^{1-\eta} = 0\end{aligned}$$

Unconstrained firms want to insure workers against idiosyncratic productivity shocks, also on STW:

$$\begin{aligned}\iff \quad & \frac{1-\eta}{\eta}\cdot\frac{\bar{V}-U}{\bar{J}} = u'\left(c^u(\epsilon)\right) \\ \Rightarrow \quad & u'\left(c^u_{\text{stw}}(\epsilon)\right) = u'\left(c^u_{\text{stw}}(\epsilon')\right) = u'(c^w)\end{aligned}$$

FOC for $c^c(\epsilon)$:

$$\begin{aligned}\frac{\partial \mathcal{B}}{\partial c^c(\epsilon)} = &-(1-p)\cdot g(\epsilon)\cdot(1-\eta)\cdot\left(\frac{\bar{V}-U}{\bar{J}}\right)^{\eta} \\ &+(1-p)\cdot g(\epsilon)\cdot u'(c^c(\epsilon))\cdot\eta\cdot\left(\frac{\bar{J}}{\bar{V}-U}\right)^{1-\eta} + \mathcal{M}(\epsilon) \\ &\stackrel{!}{=} 0\end{aligned}$$

Insert the surplus splitting rule:

$$
\begin{aligned}
&- (1-p) \cdot g(\epsilon) \cdot \left(\frac{\eta}{1-\eta} \cdot u'(c^w) \right)^{\eta} \\
&+ (1-p) \cdot g(\epsilon) \cdot u'(c^c(\epsilon)) \cdot \eta \cdot \left(\frac{1-\eta}{\eta} \cdot \frac{1}{u'(c^w)} \right) + \mathcal{M}(\epsilon) \stackrel{!}{=} 0
\end{aligned}
$$

$$
\iff \quad (1-p) \cdot g(\epsilon) \cdot \eta \cdot (u'(c^c(\epsilon)) - u'(c^w)) = -\mathcal{M}(\epsilon) \cdot \left(u'(c^w) \cdot \frac{\eta}{1-\eta} \right)^{1-\eta}
$$

Note: If $c^c(\epsilon) < u'(c^w)$, then the RHS of the equation is positive.
Thus, firms and workers gain joint surplus until $c^c(\epsilon) = c^w$.

If

$$
z(\epsilon) - c^w + \beta \cdot \left[\lambda \cdot \bar{J} + (1-\lambda) \cdot J^c(\epsilon) \right] \geq 0,
$$

then the constraint is non-binding and $\mathcal{M}(\epsilon) = 0$. ✓

If

$$
z(\epsilon) - c^w + \beta \cdot \left[\lambda \cdot \bar{J} + (1-\lambda) \cdot J^c(\epsilon) \right] < 0,
$$

then the constraint is binding and $\mathcal{M}(\epsilon) < 0$. ✓

Thus, it is optimal for the firm to pay as much as it can:

$$
J^c(\epsilon) = z(\epsilon) - c^c(\epsilon) + \beta \cdot \left[\lambda \cdot \bar{J} + (1-\lambda) \cdot J^c(\epsilon) \right] = 0
$$

$$
\iff \quad c^c(\epsilon) = z(\epsilon) + \lambda \cdot \beta \cdot \bar{J}
$$

Using the same arguments, we get that under STW

$$
c_{\text{stw}}(\epsilon) = c^w \quad \text{if the financial constraint is non-binding,}
$$

and

$$
c_{\text{stw}}(\epsilon) = \epsilon_{\text{stw}}(\epsilon) + s_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon)) + \lambda \cdot \beta \cdot \bar{J} \quad \text{when it is binding.}
$$

Using the surplus splitting rule, we can simplify the Nash-Bargaining problem to:

$$
\max_{h^u(\epsilon), h^c(\epsilon), h^u_{\text{stw}}(\epsilon), h^c_{\text{stw}}(\epsilon), \xi^u_s, \xi^c_s, \xi^{u^c}_s, \xi^{c,c}_s} (1-\eta)^{1-\eta} \cdot (u'(c^w) \cdot \eta)^{\eta} \cdot \bar{S}
$$

That is, we just have to maximize the joint surplus:

$$
\begin{aligned}
\bar{S} = &-s \\
&+ (1-p) \cdot \int_{\max\{\xi_s^{u,B}, \xi_s^{u,cp}, \epsilon_{stw}\}}^{\infty} \frac{1}{1-\beta\cdot(1-\lambda)} \Big(z(\epsilon) + \frac{u(c^w)-u(b)}{u'(c^w)} - c^w + (1-\eta\cdot f)\cdot\beta\cdot\bar{S} \Big) dG(\epsilon) \\
&+ (1-p) \cdot \int_{\max\{\epsilon_s^{u,B}, \epsilon_s^{u,cp}\}}^{\max\{\epsilon_s^{u,B}, \epsilon_s^{u,cp}, \epsilon_{stw}\}} \frac{1}{1-\beta\cdot(1-\lambda)} \Big(z(\epsilon) + \frac{u(c^w)-u(b)}{u'(c^w)} - c^w \\
&\qquad + s_{stw}(\bar{h} - h_{stw}(\epsilon)) + (1-\eta\cdot f)\cdot\beta\cdot\bar{S} \Big) dG(\epsilon) \\
&+ p \cdot \int_{\epsilon^p}^{\infty} \frac{1}{1-\beta\cdot(1-\lambda)} \Big(z(\epsilon) + \frac{u(c^w)-u(b)}{u'(c^w)} - c^w + (1-\eta\cdot f)\cdot\beta\cdot\bar{S} \Big) dG(\epsilon) \\
&+ p \cdot \int_{\max\{\xi_s^c, \xi_s^{c,cp}, \epsilon_{stw}\}}^{\epsilon^p} \frac{1}{1-\beta\cdot(1-\lambda)} \Big(z(\epsilon) + \frac{u(c^c(\epsilon))-u(b)}{u'(c^w)} - c^c(\epsilon) + (1-\eta\cdot f)\cdot\beta\cdot\bar{S} \Big) dG(\epsilon) \\
&+ p \cdot \int_{\max\{\epsilon_s^c, \epsilon_s^{c,cp}\}}^{\max\{\epsilon_s^c, \epsilon_s^{c,cp}, \epsilon_{stw}\}} \frac{1}{1-\beta\cdot(1-\lambda)} \Big(z_{stw}(\epsilon) + \frac{u(c^c_{stw}(\epsilon))-u(b)}{u'(c^w)} \\
&\qquad - c^c(\epsilon) + (1-\eta\cdot f)\cdot\beta\cdot\bar{S} \Big) dG(\epsilon)
\end{aligned}
$$

Note that without financial constraints, the firm smooths the consumption equivalent consumed by the worker:

$$
\begin{aligned}
c^w = c^u_{\text{stw}}(\epsilon) &= c^u_{\text{stw}}(\epsilon) + s_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon)) \\
\Leftrightarrow \quad c^u_{\text{stw}} &= c^w - s_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon))
\end{aligned}
$$

STW makes it less expensive to smooth consumption equivalents consumed by the worker. The FOC for working hours in unconstrained firms outside STW is:

$$
\begin{aligned}
\frac{\partial \mathcal{B}}{\partial h^u(\epsilon)} &= (1-p)\cdot g(\epsilon) \cdot \left(\frac{\partial y(\epsilon, h^u(\epsilon))}{\partial h^u(\epsilon)} - \phi'(h^u(\epsilon)) \right) = 0 \\
\Leftrightarrow \quad \frac{\partial y(\epsilon, h^u(\epsilon))}{\partial h^u(\epsilon)} &= \phi'(h^u(\epsilon))
\end{aligned}
$$

The FOC for working hours in constrained firms *outside* STW is:

$$
\begin{aligned}
\frac{\partial \mathcal{B}}{\partial h^c(\epsilon)} &= p\cdot g(\epsilon) \cdot \left(\frac{\partial y(\epsilon, h^c(\epsilon))}{\partial h^c(\epsilon)} - \phi'(h^c(\epsilon)) \right) \cdot \frac{u'(c^c(\epsilon))}{u'(c^w)} = 0 \\
\Leftrightarrow \quad \frac{\partial y(\epsilon, h^c(\epsilon))}{\partial h^c(\epsilon)} &= \phi'(h^c(\epsilon))
\end{aligned}
$$

The FOC for working hours in unconstrained firms *on* STW is:

$$\frac{\partial \mathcal{B}}{\partial h^u_{\text{stw}}(\epsilon)} = (1-p)\cdot g(\epsilon)\cdot\left(\frac{\partial y(\epsilon, h^u_{\text{stw}}(\epsilon))}{\partial h^u_{\text{stw}}(\epsilon)} - \phi'(h^u_{\text{stw}}(\epsilon)) - s_{\text{stw}}\right) = 0$$
$$\Leftrightarrow \quad \frac{\partial y(\epsilon, h^u_{\text{stw}}(\epsilon))}{\partial h^u_{\text{stw}}(\epsilon)} = \phi'(h^u_{\text{stw}}(\epsilon)) + s_{\text{stw}}$$

The FOC for working hours in constrained firms *on* STW is:

$$\frac{\partial \mathcal{B}}{\partial h^c(\epsilon)} = p\cdot g(\epsilon)\cdot\left(\frac{\partial y(\epsilon, h^c(\epsilon))}{\partial h^c(\epsilon)} - \phi'(h^c(\epsilon)) - s_{\text{stw}}\right)\cdot\frac{u'(c^c(\epsilon))}{u'(c^w)} = 0$$
$$\Leftrightarrow \quad \frac{\partial y(\epsilon, h^c(\epsilon))}{\partial h^c(\epsilon)} = \phi'(h^c(\epsilon)) + s_{\text{stw}}$$

Suppose that

$$\max\{\xi^{u,B}_s,\, \xi^{u,\text{cp}}_s,\, \epsilon_{\text{stw}}\} = \xi^{u,B}_s$$

Then the FOC for the separation threshold of the unconstrained firm without STW support is:

$$\frac{\partial \mathcal{B}}{\partial \xi^{u,B}_s} = -(1-p)\cdot g(\xi^{u,B}_s)\cdot\frac{1}{1-\beta\cdot(1-\lambda)}\left(z(\xi^{u,B}_s) + \frac{u(c^w)-u(b)}{u'(c^w)} - c^w + (\lambda-\eta f)\cdot\beta\cdot\bar{S}\right) = 0$$

$$\Longleftrightarrow\; z(\xi^{u,B}_s) + \frac{u(c^w)-u(b)}{u'(c^w)} - c^w + (\lambda-\eta f)\cdot\beta\cdot\bar{S} = 0$$

Insert free-entry Condition:

$$z(\xi^{u,B}_s) + z(\xi^{u,B}_s) + \frac{u(c^w)-u(b)}{u'(c^w)} - c^w + \frac{(\lambda-\eta f)}{1-\eta}\cdot\frac{k_v}{q} = 0$$

Note that $\xi^{u,B}_s > \xi^{u,\text{cp}}_s$ must hold, as workers would never quit under the insurance constraint of the firm:

$$V(\epsilon) - U = u(c^w) - u(b) + (\lambda - f)\cdot\beta\cdot(\bar{V} - U) > 0$$

It always gets positive surplus!

Suppose that

$$\max\{\xi^{c,B}_s,\, \xi^{c,\text{cp}}_s,\, \epsilon_{\text{stw}}\} = \xi^{c,B}_s$$

Then the FOC for the separation threshold of the unconstrained firm without STW

support is:

$$\frac{\partial \mathcal{B}}{\partial \xi_s^{c,B}} = -(1-p)\cdot g(\xi_s^{c,B})\cdot\Bigg[\frac{1}{1-\beta\cdot(1-\lambda)}\cdot\Big(z(\xi_s^{c,B}) + \frac{u\left(c^c(\xi_s^{c,B})\right)-u(b)}{u'(c^w)} - c^c(\xi_s^{c,B}) + (\lambda-\eta f)\cdot\beta\cdot\bar{S}\Big)\Bigg] = 0$$

$$\iff z(\xi_s^{c,B}) + \frac{u(c^c(\xi_s^{c,B}))-u(b)}{u'(c^w)} - c^c(\xi_s^{c,B}) + (\lambda-\eta f)\cdot\beta\cdot\bar{S} = 0$$

Insert free-entry condition:

$$z(\xi_s^{c,B}) + \frac{u(c^c(\xi_s^{c,B}))-u(b)}{u'(c^w)} - c^c(\xi_s^{c,B}) + \frac{(\lambda-\eta f)}{1-\eta}\cdot\frac{k_v}{q} = 0$$

The participation constraint of workers can be written with free-entry condition as:

$$z(\xi_s^{c,\mathrm{cp}}) + \frac{u(c^c(\xi_s^{c,\mathrm{cp}}))-u(b)}{u'(c^w)} - c^c(\xi_s^{c,\mathrm{cp}}) + \frac{(\lambda-\eta f)}{1-\eta}\cdot\frac{k_v}{q} = 0$$

Note that these are the same conditions. This implies that given the inability to insure workers against idiosyncratic productivity shocks, separations for a constrained firm without access to STW are efficient.

Suppose that

$$\max\{\epsilon_s^{u,B}, \epsilon_s^{u,\mathrm{cp}}, \epsilon_{\mathrm{stw}}\} = \epsilon_s^{u,B}$$

Then the FOC for the separation threshold of the unconstrained firm with STW support is:

$$\frac{\partial \mathcal{B}}{\partial \epsilon_s^{u,B}} = -(1-p)\cdot g(\epsilon_s^{u,B})\cdot\Bigg[\frac{1}{1-\beta\cdot(1-\lambda)}\cdot\Big(z_{stw}(\epsilon_s^{u,B}) + \frac{u(c^w)-u(b)}{u'(c^w)} - c^w + s_{\mathrm{stw}}\cdot(\bar{h}-h_{\mathrm{stw}}(\epsilon_s^{u,B})) + (\lambda-\eta f)\cdot\beta\cdot\bar{S}\Big)\Bigg] = 0$$

$$\iff z_{stw}(\epsilon_s^{u,B}) + \frac{u(c^w)-u(b)}{u'(c^w)} - c^w + s_{\mathrm{stw}}(\bar{h}-h_{\mathrm{stw}}(\epsilon_s^{u,B})) + (\lambda-\eta f)\cdot\beta\cdot\bar{S} = 0$$

Insert free-entry Condition:

$$z_{stw}(\epsilon_s^{u,B}) + \frac{u(c^w)-u(b)}{u'(c^w)} - c^w + \frac{\lambda-\eta f}{1-\eta}\cdot\frac{k_v}{q} = 0$$

Note that $\epsilon_s^{u,B} > \epsilon_s^{u,\mathrm{cp}}$ must hold, as workers would never quit under the insurance con-

straint of the firm:

$$V(\epsilon) - U = u(c^w) - u(b) + (\lambda - f) \cdot \beta \cdot (\bar{V} - U) > 0$$

It always gets positive surplus!

Suppose that

$$\max\{\epsilon_s^{c,B},\, \epsilon_s^{c,\text{cp}},\, \epsilon_{\text{stw}}\} = \epsilon_s^{c,B}$$

Then the FOC for the separation threshold of the constrained firm with STW support is:

$$\frac{\partial \mathcal{B}}{\partial \epsilon_s^{c,B}} = -(1-p) \cdot g(\epsilon_s^{c,B}) \cdot \Bigg[\frac{1}{1 - \beta \cdot (1-\lambda)} \cdot \Big(z_{stw}(\epsilon_s^{c,B}) + \frac{u\left(c_{\text{stw}}^c(\epsilon_s^{c,B})\right) - u(b)}{u'(c^w)} - c_{\text{stw}}^c(\epsilon_s^{c,B}) + (\lambda - \eta f) \cdot \beta \cdot \bar{S} \Big) \Bigg] = 0$$

$$\iff z(\epsilon_s^{c,B}) + \frac{u(c_{\text{stw}}^c(\epsilon_s^{c,B})) - u(b)}{u'(c^w)} - c_{\text{stw}}^c(\epsilon_s^{c,B}) + (\lambda - \eta f) \cdot \beta \cdot \bar{S} = 0$$

Insert free-entry condition:

$$z_{stw}(\epsilon_s^{c,B}) + \frac{u(c_{\text{stw}}^c(\epsilon_s^{c,B})) - u(b)}{u'(c^w)} - c_{\text{stw}}^c(\epsilon_s^{c,B}) + \frac{\lambda - \eta f}{1 - \eta} \cdot \frac{k_v}{q} = 0$$

The participation constraint of workers can be written with free-entry condition as:

$$z_{stw}(\epsilon_s^{c,\text{cp}}) + \frac{u(c_{\text{stw}}^c(\epsilon_s^{c,\text{cp}})) - u(b)}{u'(c^w)} - c_{\text{stw}}^c(\epsilon_s^{c,\text{cp}}) + \frac{\lambda - \eta f}{1 - \eta} \cdot \frac{k_v}{q} = 0$$

Note that these are the same conditions. This implies that, given the inability to insure workers against idiosyncratic productivity shocks, separations for a constrained firm with access to STW are efficient.

D.2 Job Creation and Wage Equation

This section derives the job creation and the equation for STW. It follows its notation part E of the appendix.

(I) Value of an unconstrained firm outside STW after the idiosyncratic productivity shock has realized:

$$\begin{aligned} & J^u(\epsilon) = z(\epsilon) - c^w + \beta \cdot \left[\lambda\, \bar{J} + (1-\lambda)\, J^u(\epsilon)\right] \\ \Leftrightarrow \quad & J^u(\epsilon) = \frac{1}{1 - \beta \cdot (1-\lambda)} \left(z(\epsilon) - c^w + \lambda \beta \bar{J}\right) \end{aligned}$$

(II) Value of an unconstrained firm on STW after the idiosyncratic productivity shock has realized:

$$\begin{aligned} & J^u_{\text{stw}}(\epsilon) = z_{\text{stw}}(\epsilon) + s_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon)) - c^w + \beta \left[\lambda \bar{J} + (1-\lambda) J^u_{\text{stw}}(\epsilon)\right] \\ \Leftrightarrow \quad & J^u_{\text{stw}}(\epsilon) = \frac{1}{1-\beta\cdot(1-\lambda)} \left(z_{\text{stw}}(\epsilon) - c^w + \lambda\beta\bar{J}\right) \end{aligned}$$

(III) Value of a constrained firm outside STW with non-binding constraints after the idiosyncratic productivity shock has realized:

$$\begin{aligned} & J^c(\epsilon) = z(\epsilon) - c^w + \beta \cdot \left[\lambda \bar{J} + (1-\lambda) J^c(\epsilon)\right] \\ \Leftrightarrow \quad & J^c(\epsilon) = \frac{1}{1-\beta\cdot(1-\lambda)} \left(z(\epsilon) - c^w + \lambda\beta\bar{J}\right) \end{aligned}$$

(IV) Value of a constrained firm outside STW with binding constraints after the idiosyncratic productivity shock has realized:

$$\begin{aligned} & J^c(\epsilon) = z(\epsilon) - c^w(\epsilon) + \beta \left[\lambda \bar{J} + (1-\lambda) J^c(\epsilon)\right] \\ \Leftrightarrow \quad & J^c(\epsilon) = \frac{1}{1-\beta\cdot(1-\lambda)} \left(z(\epsilon) - c^w(\epsilon) + \lambda\beta\bar{J}\right) \end{aligned}$$

(V) Value of a constrained firm on STW after the idiosyncratic productivity shock has realized:

$$\begin{aligned} & J^c_{\text{stw}}(\epsilon) = z_{\text{stw}}(\epsilon) - c^w_{stw}(\epsilon) + \beta \cdot [\lambda J + (1-\lambda) J^c_{\text{stw}}(\epsilon)] \\ \Leftrightarrow \quad & J^c_{\text{stw}}(\epsilon) = \frac{1}{1-\beta\cdot(1-\lambda)} \left(z_{\text{stw}}(\epsilon) - c^w_{stw}(\epsilon) + \lambda\beta J\right) \end{aligned}$$

(VI) Value of a firm before the idiosyncratic productivity shock has realized:

$$\begin{aligned} \bar{J} = & -s + (1-p) \cdot \left(\int_{\epsilon_{\text{stw}}}^{\infty} J^u(\epsilon)\, dG(\epsilon) + \int_{\epsilon^u_s}^{\epsilon_{\text{stw}}} J^u_{\text{stw}}(\epsilon)\, dG(\epsilon) \right) \\ & + p \cdot \left(\int_{\max\{\epsilon_{\text{stw}}, \xi^c_s\}}^{\infty} J^c(\epsilon)\, dG(\epsilon) + \int_{\epsilon^c_s}^{\max\{\epsilon_{\text{stw}}, \epsilon^c_s\}} J^c_{\text{stw}}(\epsilon)\, dG(\epsilon) \right) \end{aligned}$$

Inserting (I) - (IV) into (V) gives:

$$
\begin{aligned}
\bar{J} = & -s \\
& + \frac{1}{1-\beta\cdot(1-\lambda)}\Bigg[(1-\rho)\cdot(z-\Omega) \\
& - (1-p)\cdot(1-\rho^u)\cdot\left(c^w - \int_{\epsilon_s^u}^{\epsilon_{\text{stw}}} s_{\text{stw}}(\bar{h}-h_{\text{stw}}(\epsilon))\,dG(\epsilon)\right) \\
& - p\cdot(1-\rho^c)\cdot\left(e^c - \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}},\epsilon_s^c\}} s_{\text{stw}}(\bar{h}-h_{\text{stw}}(\epsilon))\,dG(\epsilon)\right) \\
& + (1-\rho)\cdot\lambda\cdot\beta\cdot\bar{J}\Bigg]
\end{aligned}
$$

Here z denotes the average production (without distortions) and ec denotes the average cost of a constrained worker for a firm. Both are defined in section D of the appendix.

(VII) Value of an unconstrained worker outside STW after the idiosyncratic productivity shock has realized:

$$
\begin{aligned}
& V^u(\epsilon) = u(c^w) + \beta\cdot\left[\lambda\,\bar{V} + (1-\lambda)\,V^u(\epsilon)\right] \\
\Leftrightarrow \quad & V^u(\epsilon) = \frac{1}{1-\beta\cdot(1-\lambda)}\left(u(c^w) + \lambda\beta\bar{V}\right)
\end{aligned}
$$

(VIII) Value of an unconstrained worker on STW after the idiosyncratic productivity shock has realized:

$$
\begin{aligned}
& V^u_{\text{stw}}(\epsilon) = u(c^w) + \beta\cdot\left[\lambda\,\bar{V} + (1-\lambda)\,V^u_{\text{stw}}(\epsilon)\right] \\
\Leftrightarrow \quad & V^u_{\text{stw}}(\epsilon) = \frac{1}{1-\beta\cdot(1-\lambda)}\left(u(c^w) + \lambda\beta\bar{V}\right)
\end{aligned}
$$

(IX) Value of a constrained worker outside STW with non-binding constraints after the idiosyncratic productivity shock has realized:

$$
\begin{aligned}
& V^c(\epsilon) = u(c^w) + \beta\cdot\left[\lambda\,\bar{V} + (1-\lambda)\,V^c(\epsilon)\right] \\
\Leftrightarrow \quad & V^c(\epsilon) = \frac{1}{1-\beta\cdot(1-\lambda)}\left(u(c^w + \lambda\beta\bar{V}\right)
\end{aligned}
$$

(X) Value of a constrained worker outside STW with binding constraints after the id-

iosyncratic productivity shock has realized:

$$\begin{aligned} & V^c(\epsilon) = u(c^w(\epsilon)) + \beta \cdot \left[\lambda \bar{V} + (1-\lambda) V^c(\epsilon)\right] \\ \Leftrightarrow \quad & V^c(\epsilon) = \frac{1}{1-\beta\cdot(1-\lambda)} \left(u(c^w(\epsilon)) + \lambda\beta\bar{V}\right) \end{aligned}$$

(XI) Value of a constrained worker on STW after the idiosyncratic productivity shock has realized:

$$\begin{aligned} & V^c_{\text{stw}}(\epsilon) = u(c_{\text{stw}}(\epsilon)) + \beta \cdot \left[\lambda \bar{V} + (1-\lambda) V^c_{\text{stw}}(\epsilon)\right] \\ \Leftrightarrow \quad & V^c_{\text{stw}}(\epsilon) = \frac{1}{1-\beta\cdot(1-\lambda)} \left(u(c_{\text{stw}}(\epsilon)) + \lambda\beta\bar{V}\right) \end{aligned}$$

(XII) Value of a worker before the idiosyncratic productivity shock has realized:

$$\begin{aligned} \bar{V} = (1-p) \cdot \Bigg(& \int_{\epsilon_{stw}}^{\infty} V^u(\epsilon)\, dG(\epsilon) \\ & + \int_{\epsilon_s^u}^{\epsilon_{stw}} V^u_{\text{stw}}(\epsilon)\, dG(\epsilon) \Bigg) \\ & + p \cdot \Bigg(\int_{\epsilon_p}^{\infty} V^c(\epsilon)\, dG(\epsilon) \\ & + \int_{\max\{\xi_s^c, \epsilon_{stw}\}}^{\epsilon_p} V^c(\epsilon)\, dG(\epsilon) \\ & + \int_{\epsilon_s^c}^{\max\{\epsilon_s^c, \epsilon_{stw}\}} V^c_{\text{stw}}(\epsilon)\, dG(\epsilon) \Bigg) \\ & + \rho \cdot U \end{aligned}$$

Inserting (VII) - (XI) into (XII) gives:

$$\begin{aligned} \bar{V} = \frac{1}{1-\beta\cdot(1-\lambda)} \Big[& (1-p)(1-\rho^u) \cdot u(c^w) + p \cdot (1-\rho^c) \cdot u^c \\ & + (1-\rho) \cdot \lambda \cdot \beta \cdot \bar{V} + \rho \cdot U \Big] \end{aligned}$$

Here u^c denotes the average utility of a constrained worker for a firm. Both are defined in section E of the appendix.

(XIII) Unemployment:

$$U = u(b) + \beta \cdot \left[f \cdot \bar{V} + (1-f) \cdot U\right]$$

Next, we want to calculate the joint surplus of firms and workers.

(A) The surplus of the worker after the idiosyncratic productivity shock has realized is:

$$\begin{aligned}
& V_i^j(\epsilon) - U = u(c_i^j(\epsilon)) - u(b) + (\lambda - f) \cdot V_i^j(\epsilon) + (\lambda - f) \cdot \beta \cdot \bar{V} - (\lambda - f) \cdot \beta \cdot U \\
\Leftrightarrow \quad & V_i^j(\epsilon) - U = u(c_i^j(\epsilon)) - u(b) + (\lambda - f) \cdot \beta \cdot (V_i^j(\epsilon) - U) + (\lambda - f) \cdot \beta \cdot (\bar{V} - U) \\
\Leftrightarrow \quad & V_i^j(\epsilon) - U = u(c_i^j(\epsilon)) - u(b) + (\lambda - f) \cdot (V_i^j(\epsilon) - U) + (\lambda - f) \cdot \beta \cdot (\bar{V} - U) \\
\Leftrightarrow \quad & V_i^j(\epsilon) - U = \frac{1}{1 - \beta \cdot (1 - \lambda)} \Big(u(c_i^j(\epsilon)) - u(b) + (\lambda - f) \cdot \beta \cdot (\bar{V} - U) \Big)
\end{aligned}$$

for $i \in \{\text{stw}, \text{no stw}\}$ and $j \in \{u, c\}$.

(B) The surplus of the worker before shock realizations are known:

$$\begin{aligned}
\bar{V} - U = (1 - p) \cdot & \left(\int_{\epsilon_{\text{stw}}}^{\infty} V^u(\epsilon) - U \, dG(\epsilon) + \int_{\epsilon_s^u}^{\epsilon_{\text{stw}}} V_{\text{stw}}^u(\epsilon) - U \, dG(\epsilon) \right) \\
& + p \cdot \left(\int_{\epsilon_p}^{\infty} V(\epsilon) - U \, dG(\epsilon) + \int_{\max\{\xi_s^c, \epsilon_{\text{stw}}\}}^{\epsilon_p} V^c(\epsilon) - U \, dG(\epsilon) \right. \\
& \left. + \int_{\epsilon_s^c}^{\max\{\epsilon_s^c, \epsilon_{\text{stw}}\}} V_{\text{stw}}^c(\epsilon) - U \, dG(\epsilon) \right)
\end{aligned}$$

Inserting A into B gives:

$$\begin{aligned}
\bar{V} - U = \frac{1}{1 - \beta \cdot (1 - \lambda)} \Big[& (1 - p)(1 - \rho^u) \cdot (u(c^w) - u(b)) \\
& + p \cdot (1 - \rho^c) \cdot (u^c - u(b)) \\
& + (\lambda - f) \cdot \beta \cdot (\bar{V} - U) \Big]
\end{aligned}$$

(C) Finally, we can calculate the joint surplus before shock realizations are known:

$$\begin{aligned}
S &= J + \frac{\bar{V} - U}{u'(c^w)} \\
&= -s + \frac{1}{1 - \beta \cdot (1 - \lambda)} \left[(1 - \rho)(1 - \rho^u) \cdot \left(\frac{u(c^w) - u(b)}{u'(c^w)} - c^w \right. \right. \\
&\quad \left. + \frac{1}{1 - \rho^u} \int_{\epsilon_s^u}^{\epsilon_{stw}} s_{stw} \cdot (\bar{h} - h_{\text{stw}}(\epsilon)) \, dG(\epsilon) \right) \\
&\quad + p(1 - \rho^c) \cdot \left(\frac{u^c - u(b)}{u'(c^w)} - e^c \right. \\
&\quad \left. + \frac{1}{1 - \rho^c} \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}}, \epsilon_s^c\}} s_{\text{stw}} (\bar{h} - h_{\text{stw}}(\epsilon)) \, dG(\epsilon) \right) \\
&\quad \left. + (1 - \rho)\lambda\beta J + (1 - \rho)(\lambda - f) \cdot \beta \cdot \frac{\bar{V} - U}{u'(c^w)} \right]
\end{aligned}$$

Next, we start deriving the job-creation condition. Inserting the surplus splitting rule

$$J = \eta \cdot S, \quad \frac{\bar{V} - U}{u'(c^w)} = (1-\eta) \cdot S$$

into the equation for the joint surplus gives:

$$\begin{aligned}
S &= J + \frac{\bar{V} - U}{u'(c^w)} = \eta S + (1-\eta) S \\
&= -s + \frac{1}{1 - \beta \cdot (1-\lambda)} \Bigg[(1-\rho) \left(z - \Omega - \frac{1-n}{n} b \right) \\
&\quad + (1-p)(1-\rho^u) \left(\frac{u(c^w) - u(b)}{u'(c^w)} - c^w \right) \\
&\quad + (1-p) \int_{\epsilon_s^u}^{\epsilon_{stw}} s_{stw} \cdot (\bar{h} - h_{\text{stw}}(\epsilon))\, dG(\epsilon) \\
&\quad + p(1-\rho^c) \left(\frac{u^c - u(b)}{u'(c^w)} - e^c \right) \\
&\quad + p \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}}, \epsilon_s^c\}} s_{\text{stw}} (\bar{h} - h_{\text{stw}}(\epsilon))\, dG(\epsilon) \Bigg] \\
\Leftrightarrow \quad S &= -s + \frac{1}{1 - \beta \cdot (1-\lambda)} \Bigg[(1-\rho) \left(z - \Omega - \frac{1-n}{n} b \right) \\
&\quad + (1-p)(1-\rho^u) \left(\frac{u(c^w) - u(b)}{u'(c^w)} - c^w \right) \\
&\quad + (1-p) \int_{\epsilon_s^u}^{\epsilon_{stw}} (\bar{h} - h_{\text{stw}}(\epsilon))\, dG(\epsilon) \\
&\quad + p(1-\rho^c) \left(\frac{u^c - u(b)}{u'(c^w)} - e^c \right) \\
&\quad + p \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}}, \epsilon_s^c\}} s_{\text{stw}} (\bar{h} - h_{\text{stw}}(\epsilon))\, dG(\epsilon) \\
&\quad + (1-\rho)(\lambda - \eta \cdot f) \cdot \beta \cdot S \Bigg]
\end{aligned}$$

Next, we want to replace the tax. To do this, we need to know the budget constraint of the government:

$$\begin{aligned}
n^s \cdot s = \frac{n}{1-\rho} \cdot \Bigg(& (1-p) \int_{\epsilon_s^u}^{\epsilon_{stw}} s_{\text{stw}} (\bar{h} - h_{\text{stw}}(\epsilon))\, dG(\epsilon) \\
& + p \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}}, \epsilon_s^c\}} s_{\text{stw}} (\bar{h} - h_{\text{stw}}(\epsilon))\, dG(\epsilon) \Bigg) \\
& + (1-n) \cdot b
\end{aligned}$$

Inserting the employment equation

$$n = (1-\rho) \cdot \frac{n^s}{\lambda}$$

gives:

$$\begin{aligned} s = \frac{1}{\lambda} \cdot \Bigg(& (1-p) \int_{\epsilon_s^u}^{\infty} s_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon))\, dG(\epsilon) \\ & + p \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}}, \epsilon_s^c\}} s_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon))\, dG(\epsilon) \Bigg) \\ & + \frac{(1-\rho)}{n^s} \cdot b \end{aligned}$$

Inserting the expression for the number of firms that receive a shock

$$n^s = \frac{n}{1-\rho} \cdot \lambda$$

gives:

$$\begin{aligned} s = \frac{1}{\lambda} \cdot \Bigg(& (1-p) \int_{\epsilon_s^u}^{\infty} s_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon))\, dG(\epsilon) \\ & + p \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}}, \epsilon_s^c\}} s_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon))\, dG(\epsilon) \Bigg) \\ & + \frac{1}{\lambda} \cdot (1-\rho) \cdot \left(\frac{1-n}{n} \cdot b \right) \end{aligned}$$

Inserting the tax into the surplus equation gives:

$$
\begin{aligned}
S = \frac{1}{1-\beta\cdot(1-\lambda)}\Bigg[&(1-\rho)\,(z-\Omega)\\
&+(1-p)(1-\rho^u)\left(\frac{u(c^w)-u(b)}{u'(c^w)}-c^w\right)\\
&+p(1-\rho^c)\left(\frac{u^c-u(b)}{u'(c^w)}-e^c\right)\\
&+(1-\rho)(\lambda-f)\cdot\beta\cdot S\Bigg]\\
&-\left(\frac{1}{\lambda}-\frac{1}{1-\beta\cdot(1-\lambda)}\right)\cdot(1-p)\int_{\epsilon_s^u}^{\epsilon_{stw}}(\bar{h}-h_{\text{stw}}(\epsilon))\,dG(\epsilon)\\
&-\left(\frac{1}{\lambda}-\frac{1}{1-\beta\cdot(1-\lambda)}\right)\cdot p\int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}},\epsilon_s^c\}}s_{\text{stw}}(\bar{h}-h_{\text{stw}}(\epsilon))\,dG(\epsilon)\\
&-\frac{1}{\lambda}\cdot(1-\rho)\cdot\frac{1-n}{n}\cdot b
\end{aligned}
$$

Inserting the free-entry condition

$$\frac{k_v}{q}=\beta\cdot J$$

gives:

$$
\begin{aligned}
\frac{1}{1-\eta}\cdot\frac{k_v}{q}=\frac{\beta}{1-\beta\cdot(1-\lambda)}\Bigg[&(1-\rho)\left(z-\Omega-\frac{1-n}{n}\cdot b\right)\\
&+(1-p)(1-\rho^u)\left(\frac{u(c^w)-u(b)}{u'(c^w)}-c^w\right)\\
&+p(1-\rho^c)\left(\frac{u^c-u(b)}{u'(c^w)}-e^c\right)\\
&+(1-\rho)\cdot\frac{\lambda-\eta\cdot f}{1-\eta}\cdot\frac{k_v}{q}\Bigg]
\end{aligned}
$$

This is the job-creation condition used in the Ramsey problem for STW in Appendix E.

Next, we want to calculate the wage-equation. Remember the surplus splitting rule from Nash-Bargaining:

$$\eta\cdot\bar{J}=(1-\eta)\cdot\frac{\bar{V}-U}{u'(c^w)}$$

Inserting the tax into the value of a worker for a firm gives:

$$\begin{aligned}\bar{J} = \frac{1}{1-\beta\cdot(1-\lambda)}\Big[&(1-\rho)(z-\Omega) - p(1-\rho^u)e^w - (1-p)(1-\rho^c)e + (1-\rho)\lambda\beta\bar{J}\Big] \\ &- \left(\frac{1}{\lambda} - \frac{1}{1-\beta\cdot(1-\lambda)}\right)\cdot(1-p)\int_{\epsilon_s^u}^{\epsilon_{stw}} s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon))\, dG(\epsilon) \\ &- \left(\frac{1}{\lambda} - \frac{1}{1-\beta\cdot(1-\lambda)}\right)\cdot p\int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}},\epsilon_s^c\}} s_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon))\, dG(\epsilon) \\ &- \frac{1}{\lambda}\cdot(1-\rho)\cdot\frac{1-n}{n}\cdot b\end{aligned}$$

Likewise, we can calculate the surplus of a worker:

$$\begin{aligned}\bar{V} - U = \frac{1}{1-\beta\cdot(1-\lambda)}\Bigg[&(1-p)(1-\rho^u)\left(u(c^w) - u(b)\right) + p(1-\rho^c)\left(u^c - u(b)\right) \\ &+ (1-\rho)(\lambda - f)\beta(\bar{V} - U)\Bigg]\end{aligned}$$

Next, we can insert the value of a worker for a firm and the surplus of a worker into the surplus splitting rule:

$$\begin{aligned}\frac{\eta}{1-\beta\cdot(1-\lambda)}\cdot\Big[&(1-\rho)(z-\Omega) - p(1-\rho^u)\cdot c^w - (1-p)(1-\rho^c)\cdot e^c \\ &+ (1-\rho)\cdot\lambda\cdot\beta\cdot\bar{J}\Big] \\ &- \left(\frac{1}{\lambda} - \frac{1}{1-\beta\cdot(1-\lambda)}\right)\cdot(1-p)\int_{\epsilon_s^u}^{\epsilon_{stw}} s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon))\, dG(\epsilon) \\ &- \left(\frac{1}{\lambda} - \frac{1}{1-\beta\cdot(1-\lambda)}\right)\cdot p\int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}},\epsilon_s^c\}} s_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon))\, dG(\epsilon) \\ &- \frac{1}{\lambda}\cdot(1-\rho)\cdot\frac{1-n}{n}\cdot b \\ = \frac{1-\eta}{1-\beta\cdot(1-\lambda)}\cdot\frac{1}{u'(c^w)}\cdot\Big[&(1-p)(1-\rho^u)\cdot\left(u(c^w) - u(b)\right) \\ &+ p(1-\rho^c)\cdot\left(u^c - u(b)\right) \\ &+ (1-\rho)(\lambda - f)\beta(\bar{V} - U)\Big]\end{aligned}$$

Insert the surplus splitting rule again: $\eta\cdot\bar{J} = (1-\eta)\cdot\frac{\bar{V}-U}{u'(c^w)}$

$$
\begin{aligned}
\frac{\eta}{1-\beta\cdot(1-\lambda)}\cdot\Big[\quad & (1-\rho)(z-\Omega)-p(1-\rho^u)\cdot c^w-(1-p)(1-\rho^c)\cdot e^c \\
& +(1-\rho)\cdot\lambda\cdot\beta\cdot\bar{J}\Big] \\
& -\left(\frac{1}{\lambda}-\frac{1}{1-\beta\cdot(1-\lambda)}\right)\cdot(1-p)\int_{\epsilon_s^u}^{\epsilon_{stw}} s_{stw}(\bar{h}-h_{\text{stw}}(\epsilon))\,dG(\epsilon) \\
& -\left(\frac{1}{\lambda}-\frac{1}{1-\beta\cdot(1-\lambda)}\right)\cdot p\int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}},\epsilon_s^c\}} s_{\text{stw}}(\bar{h}-h_{\text{stw}}(\epsilon))\,dG(\epsilon) \\
& -\frac{1}{\lambda}\cdot(1-\rho)\cdot\frac{1-n}{n}\cdot b \\
= & \frac{1-\eta}{1-\beta\cdot(1-\lambda)}\cdot\frac{1}{u'(c^w)}\cdot\Big[(1-p)(1-\rho^u)\cdot(u(c^w)-u(b)) \\
& +p(1-\rho^c)\cdot(u^c-u(b)) \\
& +(1-\rho)(\lambda-f)\cdot u'(c^w)\cdot\frac{\eta}{1-\eta}\cdot\beta\cdot\bar{J}\Big]
\end{aligned}
$$

$$
\begin{aligned}
\Leftrightarrow\quad & \frac{\eta}{1-\beta\cdot(1-\lambda)}\Big[(1-\rho)(z-\Omega)-p(1-\rho^u)\cdot c^w-(1-p)(1-\rho^c)\cdot e^c \\
& +(1-\rho)\cdot\lambda\cdot f\cdot\beta\cdot\bar{J}\Big] \\
& -\left(\frac{1}{\lambda}-\frac{1}{1-\beta\cdot(1-\lambda)}\right)\cdot(1-p)\int_{\epsilon_s^u}^{\epsilon_{stw}} s_{stw}(\bar{h}-h_{\text{stw}}(\epsilon))\,dG(\epsilon) \\
& -\left(\frac{1}{\lambda}-\frac{1}{1-\beta\cdot(1-\lambda)}\right)\cdot p\int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}},\epsilon_s^c\}} s_{\text{stw}}(\bar{h}-h_{\text{stw}}(\epsilon))\,dG(\epsilon) \\
& -\frac{1}{\lambda}\cdot(1-\rho)\cdot\frac{1-n}{n}\cdot b \\
= & \frac{1-\eta}{1-\beta\cdot(1-\lambda)}\cdot\frac{1}{u'(c^w)}\Big[(1-p)(1-\rho^u)(u(c^w)-u(b)) \\
& +p(1-\rho^c)(u^c-u(b))\Big]
\end{aligned}
$$

$$
\begin{aligned}
\Leftrightarrow \quad & \eta \cdot \Big[(1-\rho)(z-\Omega) + (1-\rho)\cdot f \cdot \beta \cdot \bar{J}\Big] \\
& - \left(\frac{1-\beta\cdot(1-\lambda)}{\lambda} - 1\right) \cdot (1-p) \int_{\epsilon_s^u}^{\epsilon_{stw}} s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon))\, dG(\epsilon) \\
& - \left(\frac{1-\beta\cdot(1-\lambda)}{\lambda} - 1\right) \cdot p \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}},\epsilon_s^c\}} s_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon))\, dG(\epsilon) \\
& - \frac{1-\beta\cdot(1-\lambda)}{\lambda} \cdot (1-\rho) \cdot \frac{1-n}{n} \cdot b \\
= & (1-p)(1-\rho^u)(1-\eta) \cdot \left(\frac{u(c^w) - u(b)}{u'(c^w)} + \eta \cdot c^w\right) \\
& + p(1-\rho^c)(1-\eta) \cdot \left(\frac{u^c - u(b)}{u'(c^w)} + \eta \cdot e^c\right)
\end{aligned}
$$

Inserting the free-entry condition again, we get the wage equation from Appendix E for STW.

$$
\begin{aligned}
& (1-p)(1-\rho^u)(1-\eta) \cdot \frac{u(c^w) - u(b)}{u'(c^w)} + \eta \cdot c^w \\
& - \left(\frac{1-\beta\cdot(1-\lambda)}{\lambda} - 1\right) \cdot (1-p) \int_{\epsilon_s^u}^{\epsilon_{stw}} s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon))\, dG(\epsilon) \\
& - \left(\frac{1-\beta\cdot(1-\lambda)}{\lambda} - 1\right) \cdot p \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}},\epsilon_s^c\}} s_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon))\, dG(\epsilon) \\
& - \frac{1-\beta\cdot(1-\lambda)}{\lambda} \cdot (1-\rho) \cdot \frac{1-n}{n} \cdot b \\
= & \eta \cdot \Big[(1-\rho)(z-\Omega) - (1-\rho)\cdot \frac{1-n}{n} \cdot b + (1-\rho)\cdot\theta\cdot k_v\Big] \\
& - p(1-\rho^c)(1-\eta) \cdot \left(\frac{u^c - u(b)}{u'(c^w)} + \eta \cdot e^c\right)
\end{aligned}
$$

E The full Ramsey Problems

E.1 Lay-off Tax

For $\beta \to 1$ the Ramsey planner's problem reads:

$$
\begin{aligned}
\max_{b, s_{\text{stw}}, \epsilon_{\text{stw}}} \; & n^u \cdot u(c^w) + n^c \cdot u^c + (1-n)\cdot u(b) \\
& + v^f \cdot u((n\cdot z - n^u \cdot c^w - n^c \cdot e^c - s(b) - \theta\cdot(1-n)\cdot k_v)/v^f)
\end{aligned}
$$

subject to the following constraints:

(I) Number of unconstrained workers:

$$
n^u = \frac{1-p}{\lambda} \cdot (1-\rho^u) \cdot n^s
$$

(II) Separation rate, unconstrained workers:

$$\rho^u = G(\epsilon_s^u)$$

(III) Number of constrained workers:

$$n^c = \frac{p}{\lambda} \cdot (1 - \rho^c) \cdot n^s$$

(IV) Separation rate, constrained workers:

$$\rho^c = G(\epsilon_s^c)$$

(V) Aggregate separation rate:

$$\rho = (1 - p) \cdot \rho^u + p \cdot \rho^c$$

(VI) Number of firms that are hit by a shock:

$$n^s = \theta \cdot q(\theta) \cdot (1 - n) + \lambda \cdot n$$

(VII) Total employment:

$$n = n^u + n^c$$

(VIII) Average utility of constrained worker:

$$u^c = \frac{1}{1 - \rho^c} \left((1 - G(\epsilon^p)) \cdot u(c^w) + \int_{\epsilon_s^c}^{\epsilon^p} u(c(\epsilon))\, dG(\epsilon) \right)$$

(IX) Average cost of a constrained worker for a firm:

$$e^c = \frac{1}{1 - \rho^c} \left[(1 - G(\epsilon^p)) \cdot c^w + \int_{\epsilon_s^c}^{\epsilon^p} c^w(\epsilon)\, dG(\epsilon) \right]$$

(X) Average production (without distortions):

$$z = \frac{1}{1 - \rho} \left((1 - p) \int_{\epsilon_s^u}^{\infty} z(\epsilon)\, dG(\epsilon) + p \int_{\epsilon_s^c}^{\infty} z(\epsilon)\, dG(\epsilon) \right)$$

(XI) Job-creation condition:

$$\begin{aligned}\frac{1}{1-\eta}\cdot\frac{k_v}{q}=\frac{1}{\lambda}\Bigg[&(1-\rho)\cdot\left(z-\frac{1-n}{n}\cdot b\right)\\&+(1-p)(1-\rho^u)\cdot\left(\frac{u(c^w)-u(b)}{u'(c^w)}-c^w\right)\\&+(1-p)(1-\rho^c)\cdot\left(\frac{u^c-u(b)}{u'(c^w)}-e^c\right)\\&+(1-\rho)\cdot\frac{\lambda-f\cdot\eta}{1-\eta}\cdot\frac{k_v}{q}\Bigg]\end{aligned}$$

(XII) Wage:

$$\begin{aligned}WE=&(1-p)\cdot(1-\rho^u)\cdot\left(\eta\cdot c^w+(1-\eta)\cdot\frac{u(c^w)-u(b)}{u'(c^w)}\right)\\&-\eta\cdot(1-\rho)\cdot\left(z-\frac{1-n}{n}\cdot b+\theta\cdot\frac{k_v}{q}\right)\\&+p\cdot(1-\rho^c)\cdot\left[(1-\eta)\cdot\frac{u^c-u(b)}{u'(c^w)}+\eta\cdot e^c\right]=0\end{aligned}$$

(XIII) Separation condition unconstrained firm:

$$z(\epsilon_s^u)+F+\frac{u(c^w)-u(b)}{u'(c^w)}-c^w+\frac{\lambda-\eta\cdot f}{1-\eta}\cdot\frac{k_v}{q}=0$$

(XIV) Threshold at which the constraint is binding:

$$z(\epsilon^p)+\lambda\cdot\frac{k_v}{q}=c^w$$

(XV) Separation condition constrained firm:

$$\frac{u(c^w(\epsilon_s^c))-u(b)}{u'(c^w)}+(\lambda-f)\cdot\frac{\eta}{1-\eta}\cdot\frac{k_v}{q}=0$$

(XVI) Consumption of constrained worker:

$$c^w(\epsilon)=z(\epsilon)+\lambda\cdot\frac{k_v}{q}$$

E.2 Short-time Work

For $\beta \to 1$ the Ramsey planner's problem reads:

$$\max_{b, s_{\text{stw}}, \epsilon_{\text{stw}}} n^u \cdot u(c^w) + n^c \cdot u^c + (1-n) \cdot u(b) \\ + v^f \cdot u((n \cdot z - n^u \cdot c^w - n^c \cdot e^c - s(b) - \theta \cdot (1-n) \cdot k_v)/v^f)$$

subject to the following constraints:

(I) Number of unconstrained workers:

$$n^u = \frac{1-p}{\lambda} \cdot (1-\rho^u) \cdot n^s$$

(II) Separation rate (unconstrained workers):

$$\rho^u = G(\epsilon_s^u)$$

(III) Number of constrained workers:

$$n^c = \frac{p}{\lambda} \cdot (1-\rho^c) \cdot n^s$$

(IV) Separation rate (constrained workers):

$$\rho^c = G\left(\max\{\xi_s^c, \epsilon_{\text{stw}}\}\right) - G\left(\max\{\epsilon_{\text{stw}}, \epsilon_s^c\}\right) + G(\xi_s^c)$$

(V) Aggregate separation rate:

$$\rho = (1-p) \cdot \rho^u + p \cdot \rho^c$$

(VI) Number of firms that are hit by a shock:

$$n^s = \theta \cdot q(\theta) \cdot (1-n) + \lambda \cdot n$$

(VII) Total employment:

$$n = n^u + n^c$$

(VIII) Average utility of a constrained worker:

$$u^c = \frac{1}{1-\rho^c}\left((1-G(\epsilon^p))\cdot u(c^w) + \int_{\max\{\epsilon_{\text{stw}},\xi_s^c\}}^{\epsilon_p} u(c^w(\epsilon))\,dG(\epsilon) + \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}},\epsilon_s^c\}} u(c^w_{\text{stw}}(\epsilon))dG(\epsilon)\right)$$

(IX) Average cost of a constrained worker for a firm:

$$e^c = \frac{1}{1-\rho^c}\left[(1-G(\epsilon^p))\cdot c^w + \int_{\max\{\epsilon_{\text{stw}},\xi_s^c\}}^{\epsilon_p} c^w(\epsilon)\,dG(\epsilon) + \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}},\epsilon_s^c\}} \left(c^w_{\text{stw}}(\epsilon) - s_{\text{stw}}(\bar{h}-h_{\text{stw}}(\epsilon))\right)dG(\epsilon)\right]$$

(X) Average production (without distortions):

$$z = \frac{1}{1-\rho}\left((1-p)\int_{\xi_s^u}^{\infty} z(\epsilon)\,dG(\epsilon) + p\int_{\max\{\xi_s^c,\epsilon_{\text{stw}}\}}^{\infty} z(\epsilon)\,dG(\epsilon) + p\cdot\int_{\epsilon_s^c}^{\max\{\epsilon_s^c,\epsilon_{\text{stw}}\}} z(\epsilon)\,dG(\epsilon)\right)$$

(XI) Average distortion of working hours:

$$\Omega = \frac{1}{1-\rho}\left((1-p)\int_{\epsilon_s^u}^{\epsilon_{\text{stw}}} \Omega(\epsilon)\,dG(\epsilon) + p\int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}},\epsilon_s^c\}} \Omega(\epsilon)\,dG(\epsilon)\right)$$

$$\text{with}\quad \Omega(\epsilon) = \bar{z}(\epsilon) - z_{\text{stw}}(\epsilon)$$

(XII) Job-creation condition:

$$\frac{1}{1-\eta}\cdot\frac{k_v}{q} = \frac{1}{\lambda}\Bigg[(1-\rho)(z-\Omega-\frac{1-n}{n}b) + p(1-\rho^u)\left(\frac{u(c^w)-u(b)}{u'(c^w)} - c^w\right) + (1-p)(1-\rho^c)\left(\frac{u^c-u(b)}{u'(c^w)} - e^c\right) + (1-\rho)\cdot\frac{\lambda-\eta\cdot f}{1-\eta}\cdot\frac{k_v}{q}\Bigg]$$

(XIII) Wage:

$$\begin{aligned} WE = (1-p)\cdot(1-\rho^u)\left(\eta\cdot c^w + (1-\eta)\cdot\frac{u(c^w)-u(b)}{u'(c^w)}\right) \\ -\eta(1-\rho)\left(z-\frac{1-n}{n}b+\theta\cdot k_v\right) \\ +p\cdot(1-\rho^c)\left[(1-\eta)\cdot\frac{u^c-u(b)}{u'(c^w)}+\eta\cdot e^c\right]=0 \end{aligned}$$

(XIV) Separation condition for unconstrained firm without STW:

$$z(\xi_s^u)+\frac{u(c^w)-u(b)}{u'(c^w)}-c^w+\frac{\lambda-\eta\cdot f}{1-\eta}\cdot\frac{k_v}{q}=0$$

(XV) Separation condition for unconstrained firm with STW:

$$z_{stw}(\epsilon_s^u)+s_{\text{stw}}(\bar{h}-h_{\text{stw}}(\epsilon_s^u))+\frac{u(c^w)-u(b)}{u'(c^w)}-c^w+\frac{\lambda-\eta\cdot f}{1-\eta}\cdot\frac{k_v}{q}=0$$

(XVI) Threshold at which the constraint is binding:

$$z(\epsilon_p)+\lambda\cdot\frac{k_v}{q}=c^w$$

(XVII) Separation condition, constrained firm without STW:

$$\frac{u(c^w(\xi_s^c))-u(b)}{u'(c^w)}+(\lambda-f)\cdot\frac{\eta}{1-\eta}\cdot\frac{k_v}{q}=0$$

(XVIII) Separation condition, constrained firm with STW:

$$\frac{u(c_{stw}^w(\epsilon_s^c)))}{u'(c^w)}+(\lambda-f)\cdot\frac{\eta}{1-\eta}\cdot\frac{k_v}{q}=0$$

(XIX) Consumption, constrained worker outside STW

$$c^w(\epsilon)=z(\epsilon)+\lambda\cdot\frac{k_v}{q}$$

(XX) Consumption, constrained worker on STW

$$c^w_{stw}(\epsilon) = z_{stw}(\epsilon) + s_{stw}(\bar{h} - h_{stw}(\epsilon)) + \lambda \cdot \frac{k_v}{q}$$

F Ramsey FOCs with Lay-off Tax

In the following, multipliers from the Lagrangian, implied by the maximization problem from the previous section, are denoted by λ_{idx}, the index depending on the constraint. Here, λ_n denotes the Lagrange multiplier for the total employment equation, λ_{n^u} for the number of unconstrained firms, λ_{n^c} for the number of constrained firms, λ_{n^s} for the number of firms that received a shock, λ_θ for the job-creation condition, λ_c for the wage equation, $\lambda_{\epsilon^u_s}$ for the separation condition of unconstrained firms and $\lambda_{\epsilon^c_s}$ for the separation condition of constrained firms. Every other equation listed in the Ramsey problem for the lay-off tax is assumed to be plugged in.

F.1 Employment (lay-off tax)

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial n^s} &= -\lambda_{n^s} + \frac{1-p}{\lambda} \cdot (1-\rho^u) \cdot (u(c^w) - u'(c^w) \cdot c^u) \\
&\quad + \frac{p}{\lambda} \cdot (1-\rho^c) \cdot (u(c^w) - u'(c^w) \cdot e^c) \\
&\quad + \frac{1-p}{\lambda} \cdot (1-\rho^u) \cdot \lambda_{n^u} + \frac{p}{\lambda} \cdot (1-\rho^c) \cdot \lambda_{n^c} = 0 \\
\frac{\partial \mathcal{L}}{\partial n^u} &= -\lambda_{n^u} + \lambda_n = 0 \\
\frac{\partial \mathcal{L}}{\partial n^c} &= -\lambda_{n^c} + \lambda_n = 0 \\
\frac{\partial \mathcal{L}}{\partial n} &= -\lambda_n + \lambda_{n^s} \cdot (\lambda - q(\theta) \cdot \theta) + u'(c^w) \cdot [z + b + \theta \cdot c] - u(b) \\
&\quad + \frac{1-\rho}{\lambda} \cdot \frac{b}{n^2} \cdot \lambda_\theta + \eta(1-\rho) \cdot \frac{b}{n^2} \cdot \lambda_c = 0 \\
\Leftrightarrow \quad \frac{\lambda_n}{u'(c^w)} &= (\lambda - q(\theta) \cdot \theta) \cdot \frac{\lambda_{n^s}}{u'(c^w)} + [z + b + \theta \cdot c] - \frac{u(b)}{u'(c^u)} \\
&\quad + \frac{1-\rho}{\lambda} \cdot \frac{b}{n^2} \cdot \frac{\lambda_\theta}{u'(c^w)} + \frac{\lambda \cdot b}{n^2} \cdot \frac{\lambda_c}{u'(c^w)} + \eta \cdot \frac{b}{n^2} \cdot \frac{\lambda_c}{u'(c^w)}
\end{aligned}$$

F.2 Optimal Job Creation Condition (lay-off tax)

Before we begin, let us define the insurance effect as:

$$\tilde{IE}_\theta = \frac{p}{\lambda}\left(\int_{\epsilon_s^c}^{\epsilon^p} \lambda \cdot \frac{\gamma}{f} \cdot \frac{u'(c(\epsilon)) - u'(c^w)}{u'(c^w)}\, dG(\epsilon)\right) \cdot k_v$$
$$IE_\theta = n^s \cdot u'(c^w) \cdot \left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \tilde{IE}_\theta$$

The FOC for labor market tightness denotes:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \theta} = &-k_v(1-n)u'(c^w) + IE_\theta \cdot k_v + (1-\gamma)q(\theta)(1-n)\lambda_{n^s} \\ &- \frac{\gamma}{1-\eta}k_v\lambda_\theta + \frac{\lambda\gamma - f\eta}{(1-\eta)f}\left(\frac{1-\rho}{\lambda}\lambda_\theta - \lambda_{\epsilon_s^u}\right) \\ &- \left(\lambda \cdot \frac{\gamma}{f}u'(c(\epsilon_s^u)) + \frac{\lambda\gamma - f}{(1-\eta)f}\eta\right)\lambda_{\epsilon_s^c} \\ &- \lambda_c \cdot \frac{\partial WE}{\partial \theta}\end{aligned}$$

$$\Leftrightarrow \quad \frac{\lambda_{n^s}}{u'(c^w)} = \frac{1+\chi}{1-\eta} \cdot \frac{k_v}{q}$$

with

$$\begin{aligned}\chi = \frac{1}{u \cdot u'(c^w)} \cdot \Bigg[&\frac{1}{f} \cdot \frac{1}{1-\eta} \cdot \left(\gamma\lambda_\theta - (\lambda\gamma - f\eta)\left(\frac{1-\rho}{\lambda}\lambda_\theta - \lambda_{\epsilon_s^u}\right)\right) \\ &+ \left(\lambda \cdot \frac{\gamma}{f} \cdot u'(c^w(\epsilon_s^u)) + \frac{\lambda\gamma - f}{(1-\eta)f} \cdot \eta\right)\lambda_{\epsilon_s^c} \\ &+ \frac{1}{k_v} \cdot \left(\lambda_c \cdot \frac{\partial WE}{\partial \theta} - IE_\theta\right)\Bigg]\end{aligned}$$

Using the FOCs for employment and labor market tightness gives:

$$\begin{aligned}u'(c^w) \cdot \frac{1+\chi}{1-\gamma} \cdot \frac{k_v}{q} = &\frac{1-p}{\lambda} \cdot (1-\rho^u) \cdot (u(c^w) - u(b) - u'(c^w) \cdot c^w) \\ &+ \frac{p}{\lambda} \cdot (1-\rho^c) \cdot (u^c - u(b) - u'(c^w) \cdot e^c) \\ &+ u'(c^w) \cdot \frac{1-\rho}{\lambda} \cdot \left(z + b + \frac{\lambda - \gamma f + \chi(\lambda - f)}{1-\gamma} \cdot \frac{k_v}{q}\right) \\ &+ \frac{1-\rho}{\lambda} \cdot \frac{\lambda_\theta}{n^s} \cdot \frac{b}{n} + \frac{1-\rho}{n^s} \cdot \frac{b}{n} \cdot \lambda_c\end{aligned}$$

Rearranging gives the *Optimal Job Creation Condition:*

$$\begin{aligned}\frac{1+\chi}{1-\gamma}\cdot\frac{k_v}{q} = \frac{1-\rho}{\lambda}\cdot(z+b) + \frac{1-\rho}{\lambda}\cdot\frac{\lambda_\theta}{n^s\cdot u'(c^w)}\cdot\frac{b}{n} + \frac{1-\rho}{\lambda}\cdot\frac{\lambda\cdot\lambda_c}{n^s\cdot u'(c^w)}\cdot\frac{b}{n} \\ + \frac{1-p}{\lambda}\cdot(1-\rho^u)\cdot\left(\frac{u(c^w)-u(b)}{u'(c^w)} - c^w\right) \\ + \frac{p}{\lambda}\cdot(1-\rho^c)\cdot\left(\frac{u^c-u(b)}{u'(c^w)} - e^c\right) \\ + \frac{1-\rho}{\lambda}\cdot\frac{\lambda-\gamma f+\chi\cdot(\lambda-f)}{1-\gamma}\cdot\frac{k_v}{q}\end{aligned}$$

Subtracting the decentralized job-creation condition from the optimal gives:

$$\begin{aligned}\left(\chi-\frac{\eta-\gamma}{1-\eta}\right)\cdot\frac{1}{1-\gamma}\cdot\frac{k_v}{q} = \left(1+\frac{\lambda_\theta+\eta+\lambda\cdot\lambda_c}{n^s\cdot u'(c^w)}\right)\cdot\frac{1-\rho}{\lambda}\cdot\frac{b}{n} \\ + \frac{(1-\rho)\cdot(\lambda-f)}{\lambda}\cdot\left(\chi-\frac{\eta-\gamma}{1-\gamma}\right)\cdot\frac{k_v}{q}\end{aligned}$$

Rearranging gives:

$$\begin{aligned}\left(\chi-\frac{\eta-\gamma}{1-\eta}\right)\cdot\frac{1}{1-\gamma}\cdot\frac{k_v}{q} &= \left(1+\frac{\lambda_\theta+\eta\cdot\lambda\cdot\lambda_c}{n^s\cdot u'(c^w)}\right)\cdot\frac{\frac{1-\rho}{\lambda}}{\rho+(1-\rho)\cdot\frac{f}{\lambda}}\cdot\frac{b}{n} \\ &= \left(1+\frac{\lambda_\theta+\eta\cdot\lambda\cdot\lambda_c}{n^s\cdot u'(c^w)}\right)\cdot\frac{b}{f}\end{aligned}$$

F.3 Optimal separation condition (unconstrained firms)

FOC for the separation threshold of unconstrained firms:

$$\begin{aligned}-\frac{\partial\mathcal{L}}{\partial\epsilon_s^u} = \frac{n}{1-\rho}\cdot(1-p)\cdot g(\epsilon_s^u)\cdot u'(c^w)\cdot\left(z(\epsilon_s^u)+\frac{u(c^w)}{u'(c^w)}-c^w-z\right) \\ + \lambda_\theta\cdot\frac{1-p}{\lambda}\cdot g(\epsilon_s^u)\cdot\left[z(\epsilon_s^u)+\frac{u(c^w)-u(b)}{u'(c^w)}-c^w-\frac{1-n}{n}\cdot b+\frac{\lambda-\eta f}{1-\eta}\cdot\frac{k_v}{q}\right] \\ + \lambda_c\cdot\frac{\partial WE}{\partial\epsilon_s^u}+\frac{1-p}{\lambda}\cdot g(\epsilon_s^u)\cdot n^s\cdot\lambda_{n^u}+\lambda_{\epsilon_s^u}\cdot\frac{\partial S_{lt}^u(\epsilon_s^u)}{\partial\epsilon_s^u}=0\end{aligned}$$

Insert for λ_{n^u} and subtract decentralized separation condition of unconstrained firms:

$$\begin{aligned}g(\epsilon_s^u)\cdot\frac{1-p}{\lambda}\cdot n^s\cdot\left[z(\epsilon_s^u)+\frac{u(c^w)}{u'(c^w)}-c^w+\frac{\lambda_n}{u'(c^w)}\right] \\ - \frac{\lambda_\theta}{u'(c^w)}\cdot\frac{1-p}{\lambda}\cdot g(\epsilon_s^u)\cdot\left[\lambda F+\frac{1-n}{n}\cdot b\right] \\ + \frac{\lambda_c}{u'(c^w)}\cdot\frac{\partial WE}{\partial\epsilon_s^u}+\frac{\lambda_{\epsilon_s^u}}{u'(c^w)}\cdot\frac{\partial S_{\mathrm{lt}}^u(\epsilon_s^u)}{\partial\epsilon_s^u}=0\end{aligned}$$

Insert for λ_n:

$$
\begin{aligned}
& g(\epsilon_s^u) \cdot \frac{1-p}{\lambda} \cdot n^s \left[z(\epsilon_s^u) + \frac{u(c^w)}{u'(c^w)} - c^w - z \right] \\
& \quad + g(\epsilon_s^u) \cdot \frac{1-p}{\lambda} \cdot n^s \left[(\lambda - q(\theta) \cdot \theta) \cdot \frac{\lambda_{n^s}}{u'(c^w)} + (z + b + \theta \cdot c) - \frac{u(b)}{u'(c^w)} \right. \\
& \qquad \left. + \frac{1-\rho}{\lambda} \cdot \frac{b}{n^2} \cdot \frac{\lambda_\theta}{u'(c^w)} + \eta \cdot \frac{b}{n^2} \cdot \frac{\lambda_c}{u'(c^w)} \right] \\
& \quad + \frac{\lambda_\theta}{u'(c^w)} \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \left[\lambda F + \frac{1-n}{n} \cdot b \right] \\
& \quad + \frac{\lambda_c}{u'(c^w)} \cdot \frac{\partial WE}{\partial \epsilon_s^u} + \frac{\lambda_{\epsilon_s^u}}{u'(c^w)} \cdot \frac{\partial S^u_{\text{stw}}(\epsilon_s^u)}{\partial \epsilon_s^u} = 0
\end{aligned}
$$

$$
\begin{aligned}
\iff \quad & z(\epsilon_s^u) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + b + (\lambda - q(\theta) \cdot \theta) \cdot \frac{\lambda_{n^s}}{u'(c^w)} + \theta \cdot c \\
& - \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \left(\lambda F + \frac{1-n}{n} \cdot b - \frac{b}{n} \right) \\
& + \frac{\lambda_c}{n^s \cdot u'(c^w)} \left(\eta \cdot \frac{b}{n} + \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial WE}{\partial \epsilon_s^u} \right) \\
& + \frac{\lambda_{\epsilon_s^u}}{n^s \cdot u'(c^w)} \cdot \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial S^u_{\text{lt}}(\epsilon_s^u)}{\partial \epsilon_s^u} = 0
\end{aligned}
$$

Inserting λ_n^s gives the *Optimal Separation Condition*:

$$
\begin{aligned}
& z(\epsilon_s^u) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + b + \frac{\lambda - \gamma \cdot f + \chi \cdot (\lambda - f)}{1-\gamma} \cdot \frac{k_v}{q} \\
& - \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \cdot (\lambda \cdot F - b) + \frac{\lambda_c}{n^s \cdot u'(c^w)} \cdot \left(\eta \cdot \frac{b}{n} + \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial WE}{\partial \epsilon_s^u} \right) = 0
\end{aligned}
$$

Subtracting the decentralized separation condition and rearranging gives:

$$
\begin{aligned}
\lambda \cdot F = b & + (\lambda - \theta q(\theta)) \cdot \left(\chi - \frac{\eta - \gamma}{1-\gamma} \right) \cdot \frac{k_v}{q} \\
& + \frac{\lambda_{\epsilon_s^u}}{n^s \cdot u'(c^w)} \cdot \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial S^u_{\text{lt}}(\epsilon_s^u)}{\partial \epsilon_s^u} \\
& - \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \cdot (\lambda F - b) \\
& + \frac{\lambda_c}{n^s \cdot u'(c^w)} \left(\eta \cdot \frac{b}{n} + \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial WE}{\partial \epsilon_s^u} \right)
\end{aligned}
$$

$$\iff 0 = \left(1 + \frac{\lambda_\theta}{u'(c^w) \cdot n^s}\right) \cdot \left(b + (\lambda - f) \cdot \frac{\frac{1-\rho}{\lambda}}{\rho + (1-\rho) \cdot \frac{f}{\lambda}} \cdot \frac{b}{n} - \lambda \cdot F\right)$$
$$+ \frac{\lambda_c}{n^s \cdot u'(c^w)} \left(\eta \cdot \frac{b}{n} + \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial WE}{\partial \epsilon_s^u}\right)$$
$$+ \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\lambda_{\epsilon_s^u}}{n^s \cdot u'(c^w)} \cdot \frac{\partial S_{\text{lt}}^u(\epsilon_s^u)}{\partial \epsilon_s^u}$$

$$\iff \quad 0 = \left(1 + \frac{\lambda_\theta}{u'(c^w) \cdot n^s}\right) \cdot \left(b + (\lambda - f) \cdot \frac{b}{f} - \lambda F\right)$$
$$+ \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\lambda_{\epsilon_s^u}}{n^s \cdot u'(c^w)} \frac{\partial S_{\text{stw}}^u(\epsilon_s^u)}{\partial \epsilon_s^u}$$
$$+ \frac{\lambda_c}{n^s \cdot u'(c^w)} \cdot \left(\eta \cdot \frac{b}{n} + \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial WE}{\partial \epsilon_s^u}\right)$$

$$\iff \quad 0 = \left(1 + \frac{\lambda_\theta}{u'(c^w) \cdot n^s}\right) \cdot \left(\frac{\lambda}{f} \cdot b - \lambda F\right)$$
$$+ \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\lambda_{\epsilon_s^u}}{n^s \cdot u'(c^w)} \cdot \frac{\partial S_{\text{lt}}^u(\epsilon_s^u)}{\partial \epsilon_s^u}$$
$$+ \frac{\lambda_c}{n^s \cdot u'(c^w)} \cdot \left(\eta \cdot \frac{b}{n} + \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial WE}{\partial \epsilon_s^u}\right)$$

Rearranging gives the Lagrange multiplier for unconstrained firms:

$$-\frac{\lambda_{\epsilon_s^u}}{n^s \cdot u'(c^w)} = \frac{1-p}{\lambda} \cdot \frac{g(\epsilon_s^u)}{\frac{\partial S_{\text{stw}}^u(\epsilon_s^u)}{\partial \epsilon_s^u}} \cdot \left(\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \left(\frac{\lambda}{f} \cdot b - \lambda F\right)\right.$$
$$\left. + \frac{\lambda_c}{n^s \cdot u'(c^w)} \cdot \left(\eta \cdot \frac{b}{n} + \frac{1}{g(\epsilon_s^u)} \cdot \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial WE}{\partial \epsilon_s^u}\right)\right)$$

F.4 Optimal separation condition (constrained firms)

The FOC for the separation threshold in the constrained firm is:

$$\begin{aligned}
-\frac{\partial \mathcal{L}}{\partial \epsilon_s^c} = &\frac{n}{1-\rho} \cdot p \cdot g(\epsilon_s^c) \cdot u'(c^w) \left(z(\epsilon_s^c) + \frac{u(c^w(\epsilon_s^c)) - u(c^w)}{u'(c^w)} - c^w(\epsilon_s^c) - z \right) \\
&+ \lambda_\theta \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \left[z(\epsilon_s^c) + \frac{c^w(\epsilon_s^c) - u(b)}{u'(c^w)} - c^w(\epsilon_s^c) - \frac{1-n}{n} \cdot b + \frac{\lambda - \eta f}{1-\eta} \cdot \frac{k_v}{q} \right] \\
&+ \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s \cdot \lambda_{n^c} \\
&+ \lambda_c \cdot \frac{\partial WE}{\partial \epsilon_s^c} \\
&+ \lambda_{\epsilon_s^c} \cdot \frac{\partial S_{\text{lt}}^c(\epsilon_s^c)}{\partial \epsilon_s^c} = 0
\end{aligned}$$

Note that we can reformulate the decentralized separation threshold of a worker in a constrained firm as:

$$\begin{aligned}
& \frac{u(c^w(\epsilon_s^c)) - u(b)}{u'(c^w)} + (\lambda - f) \cdot \frac{\eta}{1-\eta} \cdot \frac{k_v}{q} = 0 \\
\iff \quad & \frac{u(c^w(\epsilon_s^c)) - u(b)}{u'(c^w)} + (\lambda - f) \cdot \frac{\eta}{1-\eta} \cdot \frac{k_v}{q} = c^w(\epsilon_s^c) - z(\epsilon_s^c) - \lambda \cdot \frac{k_v}{q} \\
\iff \quad & z(\epsilon_s^c) + \frac{u(c^w(\epsilon_s^c)) - u(b)}{u'(c^w)} - c^w(\epsilon_s^c) + \frac{\lambda - \eta f}{1-\eta} \cdot \frac{k_v}{q} = 0
\end{aligned}$$

This simplifies the equation to:

$$\begin{aligned}
-\frac{\partial \mathcal{L}}{\partial \epsilon_s^c} = &\frac{n}{1-\rho} \cdot p \cdot g(\epsilon_s^c) \cdot \left(z(\epsilon_s^c) + \frac{u(c^w(\epsilon_s^c))}{u'(c^w)} - c^w(\epsilon_s^c) - z \right) \\
&+ \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s \cdot \frac{\lambda_{n^c}}{u'(c^w)} \\
&- \frac{\lambda_\theta}{u'(c^w)} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot \frac{1-n}{n} \cdot b \\
&+ \frac{\lambda_c}{u'(c^w)} \cdot \frac{\partial WE}{\partial \epsilon_s^c} + \frac{\lambda_{\epsilon_s^c}}{u'(c^w)} \cdot \frac{\partial S^c(\epsilon_s^c)}{\partial \epsilon_s^c} \stackrel{!}{=} 0
\end{aligned}$$

Inserting λ_n^s gives the *Optimal Separation Condition*

$$\begin{aligned}
-\frac{\partial \mathcal{L}}{\partial \epsilon_s^c} = &z(\epsilon_s^c) + \frac{u(c^w(\epsilon_s^c))}{u'(c^w)} - c^w(\epsilon_s^c) + b + \frac{\lambda - \gamma \cdot f + \chi \cdot (\lambda - f)}{1-\gamma} \cdot \frac{k_v}{q} \\
&+ \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \cdot b \\
&+ \frac{\lambda_c}{n^s \cdot u'(c^w)} \left(\eta \cdot \frac{b}{n} + \frac{1}{g(\epsilon_s^c)} \cdot \frac{\lambda}{p} \cdot \frac{\partial WE}{\partial \epsilon_s^c} \right) \\
&+ \frac{\lambda}{p} \cdot \frac{1}{n^s \cdot g(\epsilon_s^c)} \cdot \frac{\lambda_{\epsilon_s^c}}{u'(c^w)} \cdot \frac{\partial S_{\text{lt}}^c(\epsilon_s^c)}{\partial \epsilon_s^c} = 0
\end{aligned}$$

Subtracting the decentralized separation condition gives:

$$\begin{aligned}0 = b &+ (\lambda - \theta q(\theta)) \cdot \left(\chi - \frac{\eta-\gamma}{1-\eta}\right) \cdot \frac{1}{1-\gamma} \cdot \frac{k_v}{q} \\ &+ \frac{\lambda \cdot \epsilon_s^c}{n^s \cdot u'(c^w)} \cdot \frac{1}{\rho} \cdot \frac{1}{\lambda} \cdot \frac{\partial S_{\text{lt}}^c(\epsilon_s^c)}{\partial \epsilon_s^c} \cdot \frac{1}{g(\epsilon_s^c)} \\ &+ \frac{\lambda \cdot \theta}{n^s \cdot u'(c^w)} \cdot b + \frac{\lambda_c}{n^s \cdot u'(c^w)} \left(\eta \cdot \frac{b}{n} + \frac{1}{g(\epsilon_s^c)} \cdot \frac{\lambda}{p} \cdot \frac{\partial WE}{\partial \epsilon_s^c}\right)\end{aligned}$$

Rearranging gives back the Lagrange multiplier:

$$\begin{aligned}-\frac{\lambda_{\epsilon_s^c}}{n^s \cdot u'(c^w)} = \frac{p}{\lambda} \cdot \frac{g(\epsilon_s^c)}{\frac{\partial S_{\text{stw}}^c(\epsilon_s^c)}{\partial \epsilon_s^c}} \cdot \Bigg(&\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \frac{\lambda}{f} \cdot b \\ &+ \frac{\lambda_c}{n^s \cdot u'(c^w)} \cdot \left(\eta \cdot \frac{b}{n} + \frac{1}{g(\epsilon_s^c)} \cdot \frac{\lambda}{p} \cdot \frac{\partial WE}{\partial \epsilon_s^c}\right)\Bigg)\end{aligned}$$

F.5 The Optimal Lay-Off Tax

We start from

$$\frac{\partial \mathcal{L}}{\partial F} = -\lambda_{\epsilon_s^u} = 0$$

Note that the optimal lay-off tax sets the Lagrange multiplier for the separation condition equal to zero. This implies that the lay-off tax implements the optimal separation threshold for unconstrained firms. Then

$$\begin{aligned}-\frac{\lambda_{\epsilon_s^u}}{n^s \cdot u'(c^w)} = \frac{p}{\lambda} \cdot \frac{g(\epsilon_s^u)}{\frac{\partial S_{\text{lt}}^u(\epsilon_s^u)}{\partial \epsilon_s^u}} \cdot \Bigg(&\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \left(\frac{\lambda}{f} \cdot b - \lambda \cdot F\right) \\ &+ \frac{\lambda_c}{n^s \cdot u'(c^w)} \left(\eta \cdot \frac{b}{n} + \frac{\lambda}{g(\epsilon_s^u)} \cdot \frac{\lambda}{1-p} \cdot \frac{\partial WE}{\partial \epsilon_s^u}\right)\Bigg)\end{aligned}$$

This leads to:

$$\begin{aligned}F &= \frac{1}{f} \cdot b + BE_{lt} \\ BE_{lt} &= \frac{1}{\lambda} \frac{1}{\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right)} \cdot \frac{\lambda_c}{n^s \cdot u'(c^w)} \cdot \left(\eta \cdot \frac{b}{n} + \frac{1}{g(\epsilon_s^u)} \cdot \frac{\lambda}{1-p} \cdot \frac{\partial WE}{\partial \epsilon_s^u}\right)\end{aligned}$$

F.6 Optimal UI given Lay-off Tax

From the FOC of UI, we can derive the optimality condition for unemployment insurance. With an optimal lay-off tax, the Lagrange multiplier for the separation condition of the

unconstrained firm is equal to zero.

$$
\begin{aligned}
(1-\eta)\cdot(u'(b)-u'(c^w)) = \lambda_\theta\cdot(1-\rho)\cdot\left(\frac{1-n}{n}+\frac{u'(b)}{u'(c^w)}\right)\\
+\left(\lambda_{\epsilon_s^c}+\lambda_{\xi_s^c}\right)\cdot\left(-\frac{u'(b)}{u'(c^w)}\right)\\
+\lambda_c\cdot(1-\rho)\cdot\left(\eta\cdot\frac{1-n}{n}+(1-\eta)\cdot\frac{u'(b)}{u'(c^w)}\right)
\end{aligned}
$$

To get better insight, we need to determine the Lagrange multipliers for λ_θ, λ_c. We already know the Lagrange multipliers for the separation conditions of constrained firms:

$$
\begin{aligned}
-\frac{\lambda_{\epsilon_s^u}}{n^s\cdot u'(c^w)} = \frac{1-p}{\lambda}\cdot g(\epsilon_s^u)\cdot\frac{1}{\frac{\partial S^u_{\text{stw}}(\epsilon_s^u)}{\partial\epsilon_s^u}}\cdot\left[\left(1+\frac{\lambda_\theta}{n^s\cdot u'(c^w)}\right)\cdot\left(\frac{\lambda}{f}\cdot b-\lambda\cdot F\right)\right.\\
\left.+\frac{\lambda_c}{n^s\cdot u'(c^w)}\cdot\left(\eta\cdot\frac{b}{n}+\frac{1}{g(\epsilon_s^u)}\cdot\frac{\lambda}{1-p}\cdot\frac{\partial WE}{\partial\epsilon_s^u}\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
-\frac{\lambda_{\epsilon_s^c}}{n^s\cdot u'(c^w)} = \frac{p}{\lambda}\cdot g(\epsilon_s^c)\cdot\frac{1}{\frac{\partial S^c_{\text{stw}}(\epsilon_s^c)}{\partial\epsilon_s^c}}\cdot\left[\left(1+\frac{\lambda_\theta}{n^s\cdot u'(c^w)}\right)\cdot\frac{\lambda}{f}\cdot b\right.\\
\left.+\frac{\lambda_c}{n^s\cdot u'(c^w)}\cdot\left(\eta\cdot\frac{b}{n}+\frac{1}{g(\epsilon_s^c)}\cdot\frac{\lambda}{p}\cdot\frac{\partial WE}{\partial\epsilon_s^c}\right)\right]
\end{aligned}
$$

First, let us find an expression for λ_c:

$$
\begin{aligned}
\frac{\partial\mathcal{L}}{\partial c^w} = -\Bigg(&(1-p)\cdot(1-\rho^u)\cdot\frac{u(c^w)-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\\
&+p\cdot(1-\rho^c)\cdot\frac{u^c-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\Bigg)\cdot\lambda_\theta\\
&-(1-\eta)\cdot\Bigg((1-p)\cdot(1-\rho^u)\cdot\left(1-(1-\eta)\cdot\frac{u(c^w)-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\right)\\
&\quad-p\cdot(1-\rho^c)\cdot(1-\eta)\cdot\frac{u^c-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\Bigg)\cdot\lambda_c\\
&+\frac{u(c^w)-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\cdot\lambda_{\epsilon_s^u}\\
&+\frac{u(c_{\text{stw}}(\epsilon_s^c))-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\cdot\lambda_{\epsilon_s^c}=0
\end{aligned}
$$

Next we can solve for the Lagrange multiplier of the wage equation:

$$\begin{aligned}\lambda_c = -\frac{1}{\left(\frac{\partial WE}{\partial c^w}\right)^{\text{total}}}\Bigg[&\frac{\partial \epsilon_s^u}{\partial c^w}\cdot\frac{1-p}{\lambda}\cdot g(\epsilon_s^u)\cdot n^s\cdot u'(c^w)\cdot\left(\frac{\lambda}{f}\cdot b-\lambda\cdot F\right)\\&+\frac{\partial \epsilon_s^c}{\partial c^w}\cdot\frac{p}{\lambda}\cdot g(\epsilon_s^c)\cdot n^s\cdot u'(c^w)\cdot\frac{\lambda}{f}\cdot b\\&+\left(-\frac{\partial S^{\text{total}}}{\partial c^w}\right)\cdot\lambda_\theta\Bigg]\end{aligned}$$

Insert λ_c into the Lagrange multipliers for the separation conditions. To do that, let us rewrite the Lagrange multipliers of the separation conditions:

$$\begin{aligned}\lambda_{\epsilon_s^u} = -\frac{1}{\frac{\partial S^u_{\text{stw}}(\epsilon_s^u)}{\partial \epsilon_s^u}}\cdot\Bigg(&n^s\cdot u'(c^w)\cdot\frac{1-p}{\lambda}\cdot g(\epsilon_s^u)\cdot\left(1+\frac{\lambda_\theta}{n^s\cdot u'(c^w)}\right)\cdot\left(\frac{\lambda}{f}\cdot b-\lambda\cdot F\right)\\&+\lambda_c\cdot\left(\frac{\partial n}{\partial \epsilon_s^u}\cdot\left(-\frac{\partial WE}{\partial n}\right)+\frac{\partial WE}{\partial \epsilon_s^u}\right)\Bigg)\\\lambda_{\epsilon_s^c} = -\frac{1}{\frac{\partial S^c_{\text{stw}}(\epsilon_s^c)}{\partial \epsilon_s^c}}\cdot\Bigg(&n^s\cdot u'(c^w)\cdot\frac{p}{\lambda}\cdot g(\epsilon_s^c)\cdot\left(1+\frac{\lambda_\theta}{n^s\cdot u'(c^w)}\right)\cdot\frac{\lambda}{f}\cdot b\\&+\lambda_c\cdot\left(\frac{\partial n}{\partial \epsilon_s^c}\cdot\left(-\frac{\partial WE}{\partial n}\right)+\frac{\partial WE}{\partial \epsilon_s^c}\right)\Bigg)\end{aligned}$$

Inserting λ_c gives:

$$\begin{aligned}\lambda_{\epsilon_s^u} = -\frac{1}{\frac{\partial S^u_{\text{stw}}(\epsilon_s^u)}{\partial \epsilon_s^u}}\cdot\Bigg(&n^s\cdot u'(c^w)\cdot\frac{1-p}{\lambda}\cdot g(\epsilon_s^u)\cdot\left(1+\frac{\lambda_\theta}{n^s\cdot u'(c^w)}\right)\cdot\left(\frac{\lambda}{f}\cdot b-\lambda\cdot F\right)\\&+\left(\frac{\partial n}{\partial \epsilon_s^u}\cdot\left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}}+\left(\frac{\partial c^w}{\partial \epsilon_s^u}\right)^{\text{total}}\right)\\&\cdot\Bigg[\frac{\partial \epsilon_s^u}{\partial c^w}\cdot\frac{1-p}{\lambda}\cdot g(\epsilon_s^u)\cdot n^s\cdot u'(c^w)\cdot\left(\frac{\lambda}{f}\cdot b-\lambda\cdot F\right)\\&+\frac{\partial \epsilon_s^c}{\partial c^w}\cdot\frac{p}{\lambda}\cdot g(\epsilon_s^c)\cdot n^s\cdot u'(c^w)\cdot\frac{\lambda}{f}\cdot b+\left(-\frac{\partial S^{\text{total}}}{\partial c^w}\right)\cdot\lambda_\theta\Bigg]\Bigg)\end{aligned}$$

$$\lambda_{\epsilon_s^c} = \frac{1}{\frac{\partial S^c_{\text{stw}}(\epsilon_s^c)}{\partial \epsilon_s^c}} \cdot \Bigg(n^s \cdot u'(c^w) \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot \left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \frac{\lambda}{f} \cdot b + \left(\left(\frac{\partial n}{\partial \epsilon_s^c}\right) \cdot \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \epsilon_s^c}\right)^{\text{total}} \right) \cdot \Bigg[\frac{\partial \epsilon_s^u}{\partial c^w} \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot n^s \cdot u'(c^w) \cdot \left(\frac{\lambda}{f} \cdot b - \lambda \cdot F\right) + \frac{\partial \epsilon_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s \cdot u'(c^w) \cdot \frac{\lambda}{f} \cdot b + \left(-\frac{\partial S^{\text{total}}}{\partial c^w}\right) \cdot \lambda_\theta \Bigg] \Bigg)$$

Insert λ_c into the Lagrange multipliers for the separation conditions. To do that, let us rewrite the Lagrange multipliers of the separation conditions:

$$\lambda_{\epsilon_s^u} = -\frac{1}{\frac{\partial S^u_{\text{stw}}(\epsilon_s^u)}{\partial \epsilon_s^u}} \cdot \Bigg(- n^s \cdot u'(c^w) \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot \left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \left(\frac{\lambda}{f} \cdot b - \lambda \cdot F\right) + \lambda_c \cdot \left(\frac{\partial n}{\partial \epsilon_s^u} \cdot \left(-\frac{\partial WE}{\partial n}\right) + \frac{\partial WE}{\partial \epsilon_s^u} \right) \Bigg)$$

$$\lambda_{\epsilon_s^c} = -\frac{1}{\frac{\partial S^c_{\text{stw}}(\epsilon_s^c)}{\partial \epsilon_s^c}} \cdot \Bigg(n^s \cdot u'(c^w) \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot \left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \frac{\lambda}{f} \cdot b + \lambda_c \cdot \left(\frac{\partial n}{\partial \epsilon_s^c} \cdot \left(-\frac{\partial WE}{\partial n}\right) + \frac{\partial WE}{\partial \epsilon_s^c} \right) \Bigg)$$

Inserting λ_c gives:

$$\lambda_{\epsilon_s^u} = -\frac{1}{\frac{\partial S^u_{\text{stw}}(\epsilon_s^u)}{\partial \epsilon_s^u}} \cdot \Bigg(n^s \cdot u'(c^w) \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^u) \cdot \left(\lambda + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \left(\frac{\lambda}{f} \cdot b - \lambda \cdot F\right) + \left(\frac{\partial n}{\partial \epsilon_s^u} \cdot \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \epsilon_s^u}\right)^{\text{total}} \right) \cdot \Bigg[\frac{\partial \epsilon_s^u}{\partial c^w} \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot n^s \cdot u'(c^w) \cdot \left(\frac{\lambda}{f} \cdot b - \lambda \cdot F\right) + \frac{\partial \epsilon_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s \cdot u'(c^w) \cdot \frac{\lambda}{f} \cdot b + \left(-\frac{\partial S^{\text{total}}}{\partial c^w}\right) \cdot \lambda_\theta \Bigg] \Bigg)$$

$$\lambda_{\epsilon_s^c} = -\frac{1}{\frac{\partial S^c_{\text{stw}}(\epsilon_s^c)}{\partial \epsilon_s^c}} \cdot \left(n^s \cdot u'(c^w) \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot \left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \frac{\lambda}{f} \cdot b \right.$$
$$+ \left(\left(\frac{\partial n}{\partial \epsilon_s^c}\right) \cdot \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \epsilon_s^c}\right)^{\text{total}} \right)$$
$$\cdot \left[\frac{\partial \epsilon_s^u}{\partial c^w} \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot n^s \cdot u'(c^w) \cdot \left(\frac{\lambda}{f} \cdot b - \lambda \cdot F\right) \right.$$
$$\left. \left. + \frac{\partial \epsilon_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s \cdot u'(c^w) \cdot \frac{\lambda}{f} \cdot b + \left(-\frac{\partial S^{\text{total}}}{\partial c^w}\right) \cdot \lambda_\theta \right] \right)$$

Now we have everything to calculate λ_θ from the two equations:

$$(1) \quad \left(\chi - \frac{\eta - \gamma}{1 - \eta}\right) \cdot \frac{1}{1-\gamma} \cdot \frac{k_v}{q} = \left(1 + \frac{\lambda_\theta + \eta \cdot \lambda \cdot \lambda_c}{n^s \cdot u'(c^w)}\right) \cdot \frac{b}{f}$$

$$(2) \quad \chi = \frac{1}{1-\eta} \cdot \frac{1}{u \cdot f} \cdot \frac{1}{u'(c^w)} \cdot \left[\gamma - (\lambda\gamma - f\eta) \cdot \left(\frac{1}{\lambda}(1-\rho)\lambda_\theta - \lambda_{\epsilon_s^u}\right)\right]$$
$$+ \frac{1}{u} \cdot \frac{1}{u'(c^w)} \cdot \left(\lambda \cdot \frac{\gamma}{f} \cdot u'(c^w) + \left(\frac{1}{1-\eta} \cdot \frac{\lambda\gamma - f}{f}\right) \cdot \eta\right) \cdot \lambda_{\epsilon_s^c}$$
$$+ \frac{1}{u} \cdot \frac{1}{u'(c^w)} \cdot \lambda_c \cdot \frac{\partial WE}{\partial \theta} \cdot \frac{1}{k_v}$$
$$- \frac{1}{u} \cdot \frac{1}{u'(c^w)} \cdot IE_\theta$$

We can rearrange (1) to:

$$\chi \cdot k_v = (1-\gamma) \cdot q \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \left(1 + \frac{\lambda_\theta + \eta \cdot \lambda \cdot \lambda_c}{n^s \cdot u'(c^w)}\right) \cdot \frac{b}{f} \right)$$

Note that

$$(1-\gamma) \cdot q = \frac{\partial f}{\partial \theta}$$

This follows from the fact that:

$$f'(\theta) = q(\theta) + \theta \cdot q'(\theta) = q(\theta) \cdot \left(1 + \frac{q'(\theta) \cdot \theta}{q(\theta)}\right) = q(\theta) \cdot (1-\gamma)$$

So we can write:

$$\chi \cdot k_v = \left(\frac{\partial f}{\partial \theta}\right) \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \left(1 + \frac{\lambda_\theta + \eta \cdot \lambda \cdot \lambda_c}{n^s \cdot u'(c^w)}\right) \cdot \frac{b}{f} \right)$$

Inserting χ gives:

$$\begin{aligned}
&\left[\gamma - (\lambda\gamma - f\eta)\cdot\frac{1}{\lambda}\cdot(1-\rho)\right]\cdot\frac{1}{1-\eta}\cdot\frac{k_v}{f}\cdot\lambda_\theta \\
&= u'(c^w)\cdot\left(\frac{\partial f}{\partial\theta}\right)\cdot u\cdot\left(\frac{\eta-\gamma}{(1-\gamma)(1-\eta)}\cdot\frac{k_v}{q} + \left(1+\frac{\lambda_\theta+\eta\cdot\lambda\cdot\lambda_c}{n^s\cdot u'(c^w)}\right)\cdot\frac{b}{f}\right) \\
&\quad +(\lambda\gamma - f\eta)\cdot\frac{1}{1-\eta}\cdot\frac{k_v}{f}\cdot(-\lambda_{\epsilon_s^u}) \\
&\quad +\left(\lambda\cdot\frac{\gamma}{f}\cdot u'(c^w) + \left(\frac{1}{1-\eta}\cdot\frac{\lambda\gamma-f}{f}\right)\cdot\eta\right)\cdot k_v\cdot(-\lambda_{\epsilon_s^c}) \\
&\quad +\frac{\partial WE}{\partial\theta}\cdot(-\lambda_c) \quad + IE_\theta
\end{aligned}$$

Inserting $\lambda_{\epsilon_s^u}, \lambda_{\epsilon_s^c}, IE_\theta$ gives

$$\begin{aligned}
&\left[\gamma - (\lambda\gamma - f\eta)\cdot\frac{1}{\lambda}\cdot(1-\rho)\right]\cdot\frac{1}{1-\eta}\cdot\frac{k_v}{f}\cdot\lambda_\theta \\
&= u'(c^w)\cdot\frac{\partial f}{\partial\theta}\cdot u\cdot\left(\frac{\eta-\gamma}{(1-\gamma)(1-\eta)}\cdot\frac{k_v}{q} + \left(1+\frac{\lambda_\theta}{n^s u'(c^w)}\right)\cdot\frac{b}{f}\right) \\
&\quad + u'(c^w)\cdot n^s\cdot\left(-\frac{\partial\epsilon_s^u}{\partial\theta}\right)^{\text{total}}\cdot g(\epsilon_s^u)\cdot\frac{1-p}{\lambda}\cdot\left(1+\frac{\lambda_\theta}{n^s u'(c^w)}\right)\cdot\left(\frac{\lambda}{f}\cdot b - \lambda F\right) \\
&\quad + u'(c^w)\cdot n^s\cdot\left(-\frac{\partial\epsilon_s^c}{\partial\theta}\right)^{\text{total}}\cdot g(\epsilon_s^c)\cdot\frac{p}{\lambda}\cdot\left(1+\frac{\lambda_\theta}{n^s u'(c^w)}\right)\cdot\frac{\lambda}{f}\cdot b \\
&\quad + \lambda_\theta\cdot\left(-\frac{\partial c^w}{\partial\theta}\right)^{\text{total,2}}\cdot\left(-\frac{\partial S}{\partial c^w}\right)^{\text{total}} \\
&\quad + u'(c^w)\cdot n^s\cdot\left(1+\frac{\lambda_\theta}{n^s u'(c^w)}\right)\cdot\tilde{IE}_\theta
\end{aligned}$$

with

$$\begin{aligned}
\left(-\frac{\partial\epsilon_s^u}{\partial\theta}\right)^{\text{total}} &= \left(-\frac{\partial\epsilon_s^u}{\partial\theta}\right) + \left(\frac{\partial c^w}{\partial\theta}\right)^{\text{total}}\cdot\left(-\frac{\partial\epsilon_s^u}{\partial c^w}\right) \\
&\quad + \left(-\frac{\partial\epsilon_s^u}{\partial\theta}\right)\cdot\left[\left(\frac{\partial n}{\partial\epsilon_s^u}\right)\cdot\left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial\epsilon_s^u}\right)^{\text{total}}\right]\cdot\left(\frac{\partial\epsilon_s^u}{\partial c^w}\right) \\
&\quad + f'(\theta)\cdot\frac{\partial n}{\partial f}\cdot\frac{\partial c^w}{\partial n}\cdot\frac{\partial\epsilon_s^u}{\partial c^w} \\
\left(-\frac{\partial\epsilon_s^c}{\partial\theta}\right)^{\text{total}} &= \left[\left(-\frac{\partial\epsilon_s^c}{\partial\theta}\right) + \left(\frac{\partial c^w}{\partial\theta}\right)^{\text{total}}\cdot\left(-\frac{\partial\epsilon_s^c}{\partial c^w}\right)\right. \\
&\quad + \left(-\frac{\partial\epsilon_s^c}{\partial\theta}\right)\cdot\left[\left(\frac{\partial n}{\partial\epsilon_s^c}\right)\cdot\left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial\epsilon_s^c}\right)^{\text{total}}\right]\cdot\frac{\partial\epsilon_s^c}{\partial c^w} \\
&\quad \left. + f'(\theta)\cdot\frac{\partial n}{\partial f}\cdot\frac{\partial c^w}{\partial n}\cdot\frac{\partial\epsilon_s^c}{\partial c^w}\right]
\end{aligned}$$

$$\left(-\frac{\partial c^w}{\partial \theta}\right)^{\text{total},2} = \left(-\frac{\partial c^w}{\partial \theta}\right)^{\text{total}} + \left(-\frac{\partial \epsilon_s^u}{\partial \theta}\right) \cdot \left(\frac{\partial n}{\partial \epsilon_s^u} \cdot \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \epsilon_s^u}\right)^{\text{total}}\right)$$
$$+ \left(-\frac{\partial \epsilon_s^c}{\partial \theta}\right) \cdot \left(\frac{\partial n}{\partial \epsilon_s^c} \cdot \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \epsilon_s^c}\right)^{\text{total}}\right)$$
$$+ f'(\theta) \cdot \frac{\partial n}{\partial f} \cdot \frac{\partial c^w}{\partial n}$$

This allows us to calculate the Lagrange multiplier for the job-creation condition

$$M \cdot \lambda_\theta = u'(c^w) \cdot \frac{\partial f}{\partial \theta} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \left(1 + \frac{\lambda_\theta}{n^s u'(c^w)}\right) \cdot \frac{b}{f}\right)$$
$$+ u'(c^w) \cdot n^s \cdot \left(-\frac{\partial \epsilon_s^u}{\partial \theta}\right)^{\text{total}} \cdot g(\epsilon_s^u) \cdot \frac{1-p}{\lambda} \left(\frac{\lambda}{f} b - \lambda F\right)$$
$$+ u'(c^w) \cdot n^s \cdot \left(-\frac{\partial \epsilon_s^c}{\partial \theta}\right)^{\text{total}} \cdot g(\epsilon_s^c) \cdot \frac{p}{\lambda} \cdot \frac{\lambda}{f} b$$
$$+ u'(c^w) \cdot n^s \cdot \tilde{IE}_\theta$$

with

$$M = \left[\gamma - (\lambda\gamma - f \cdot \eta) \cdot \frac{1}{\lambda} \cdot (1-\rho)\right] \cdot \frac{1}{1-\eta} \cdot \frac{k_v}{f}$$
$$+ \left(-\frac{\partial f}{\partial \theta} \cdot u \cdot \frac{b}{f}\right)$$
$$+ \left(\frac{\partial \epsilon_s^u}{\partial \theta}\right)^{\text{total}} \cdot g(\epsilon_s^u) \cdot \frac{1-p}{\lambda} \cdot \left(\frac{\lambda}{f} b - \lambda \cdot F\right)$$
$$+ \left(\frac{\partial \epsilon_s^c}{\partial \theta}\right)^{\text{total}} \cdot g(\epsilon_s^c) \cdot \frac{p}{\lambda} \cdot \frac{\lambda}{f} b$$
$$+ \left(\frac{\partial c^w}{\partial \theta}\right)^{\text{total}} \cdot \left(-\frac{\partial S}{\partial c^w}\right)^{\text{total}}$$
$$+ \tilde{IE}_\theta$$

Note: $\frac{1}{M}$ denotes the general equilibrium effect of an increase of the joint surplus on θ:

$$\left(\frac{\partial \theta}{\partial S}\right)^{\text{ge}} = \frac{1}{M}$$

Rearranging for λ_θ gives:

$$\begin{aligned}\lambda_\theta = u'(c^w) \cdot \left(\frac{\partial f}{\partial S}\right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta-\gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \frac{\lambda}{f} \cdot b\right) \\ + u'(c^w) \cdot \left(-\frac{\partial \epsilon_s^u}{\partial S}\right)^{\text{ge}} \cdot \frac{\partial n^u}{\partial \epsilon_s^u} \cdot \left(\frac{\lambda}{f} b - \lambda F\right) \\ + u'(c^w) \cdot \left(-\frac{\partial \epsilon_s^c}{\partial S}\right)^{\text{ge}} \cdot \frac{\partial n^c}{\partial \epsilon_s^c} \cdot \frac{\lambda}{f} b \\ + u'(c^w) \cdot n^c \cdot \left(\frac{\partial \theta}{\partial S}\right)^{\text{ge}} \cdot \hat{IE}_\theta\end{aligned}$$

Note that

$$\left(\frac{\partial f}{\partial S}\right)^{\text{ge}} = \frac{1}{M} \cdot \frac{\partial f}{\partial S}$$

$$\left(\frac{\partial \epsilon_s^u}{\partial S}\right)^{\text{ge}} = \frac{1}{M} \cdot \left(\frac{\partial \epsilon_s^u}{\partial S}\right)^{\text{total}}$$

$$\left(\frac{\partial \epsilon_s^c}{\partial S}\right)^{\text{ge}} = \frac{1}{M} \cdot \left(\frac{\partial \epsilon_s^c}{\partial S}\right)^{\text{total}}$$

Further define:

$$\hat{IE}_\theta = \frac{1}{1-\rho^c} \cdot \int_{\epsilon_s^c}^{\epsilon^p} \lambda \cdot \frac{\gamma}{f} \cdot \frac{u'(c_{\text{stw}}(\epsilon)) - u'(c^w)}{u'(c^w)} \, dG(\epsilon)$$

Finally, we can calculate the Lagrange multiplier for λ_c by inserting into λ_θ:

$$\begin{aligned}\lambda_c = \frac{-1}{\left(\frac{\partial S}{\partial c^w}\right)^{\text{total}}} \cdot \Bigg\{ \\ \left(\frac{\partial \epsilon_s^u}{\partial c^w} + \left(\frac{\partial S}{\partial c^w}\right)^{\text{total}} \cdot \left(\frac{\partial \epsilon_s^u}{\partial S}\right)^{\text{ge}}\right) \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot n^s \cdot u'(c^w) \cdot \left(\frac{\lambda}{f} b - \lambda F\right) \\ + \left(\frac{\partial \epsilon_s^c}{\partial c^w} + \left(\frac{\partial S}{\partial c^w}\right)^{\text{total}} \cdot \left(\frac{\partial \epsilon_s^c}{\partial S}\right)^{\text{ge}}\right) \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s \cdot u'(c^w) \cdot \frac{\lambda}{f} b \\ + \left(\frac{\partial S}{\partial c^w}\right)^{\text{total}} \cdot \left(-\frac{\partial f}{\partial S}\right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta-\gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \left(1 + \frac{\lambda_\theta}{n^s u'(c^w)}\right) \cdot \frac{b}{f}\right) \\ - n^c \cdot \left(\frac{\partial S}{\partial c^w}\right)^{\text{ge}} \cdot \left(\frac{\partial \theta}{\partial S}\right) \cdot \hat{IE}_\theta \Bigg\}.\end{aligned}$$

Simplifying:

$$\lambda_c = -\frac{u'(c^w)}{\left(\frac{\partial WE}{\partial c^w}\right)^{\text{total}}}\Bigg[\left(\frac{\partial \epsilon_s^u}{\partial c^w}\right)^{\text{ge}} \cdot \frac{\partial n^u}{\partial \epsilon_s^u} \cdot \left(\frac{\lambda}{f}b - \lambda F\right) + \left(\frac{\partial \epsilon_s^c}{\partial c^w}\right)^{\text{ge}} \cdot \frac{\partial n^c}{\partial \epsilon_s^c} \cdot \frac{\lambda}{f}b$$
$$+ \left(-\frac{\partial f}{\partial c^w}\right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \frac{b}{f}\right) + n^c \cdot \left(-\frac{\partial \theta}{\partial c^w}\right)^{\text{ge}} \cdot \hat{IE}_\theta\Bigg]$$

Following the same arguments, we can express the Lagrange multiplier for the separation conditions as:

$$\lambda_{\epsilon_s^u} = -\frac{u'(c^w)}{\left(\frac{\partial S^u_{\text{stw}}(\epsilon_s^u)}{\partial \epsilon_s^u}\right)^{\text{total}}}\Bigg[\left(\frac{\partial n^u}{\partial \epsilon_s^u}\right)^{\text{ge}} \cdot \left(\frac{\lambda}{f}b - \lambda F\right) + \left(\frac{\partial \epsilon_s^c}{\partial \epsilon_s^u}\right)^{\text{ge}} \cdot \frac{\partial n^c}{\partial \epsilon_s^c} \cdot \frac{\lambda}{f}b$$
$$+ \frac{\partial c^w}{\partial \epsilon_s^u} \cdot \left(-\frac{\partial f}{\partial c^w}\right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \frac{b}{f}\right)$$
$$+ n^c \cdot \frac{\partial c^w}{\partial \epsilon_s^u} \cdot \left(-\frac{\partial \theta}{\partial c^w}\right)^{\text{ge}} \cdot \hat{IE}_\theta\Bigg]$$

$$\lambda_{\epsilon_s^c} = -\frac{u'(c^w)}{\left(\frac{\partial S^c_{\text{stw}}(\epsilon_s^c)}{\partial \epsilon_s^c}\right)^{\text{total}}}\Bigg[\frac{\partial \epsilon_s^u}{\partial \epsilon_s^c} \cdot \left(\frac{\partial n^u}{\partial \epsilon_s^u}\right)^{\text{ge}} \cdot \left(\frac{\lambda}{f}b - \lambda F\right)$$
$$+ \left(\frac{\partial n^c}{\partial \epsilon_s^c}\right)^{\text{ge}} \cdot \frac{\lambda}{f}b + \frac{\partial c^w}{\partial \epsilon_s^c} \cdot \left(-\frac{\partial f}{\partial c^w}\right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \frac{b}{f}\right)$$
$$+ n^c \cdot \frac{\partial c^w}{\partial \epsilon_s^c} \cdot \left(-\frac{\partial \theta}{\partial c^w}\right)^{\text{ge}} \cdot \hat{IE}_\theta\Bigg]$$

For convenience, the optimal UI benefits are replicated here:

$$(1-n)\cdot(u'(b) - u'(c^w)) = \lambda_\theta \cdot (1-\rho) \cdot \left(\frac{1-n}{n} + \frac{u'(b)}{u'(c^w)}\right) \cdot \left(-\frac{u'(b)}{u'(c^w)}\right)$$
$$+ (\lambda_{\epsilon_s^u} + \lambda_{\epsilon_s^c}) \cdot \left(-\frac{u'(b)}{u'(c^w)}\right)$$
$$+ \lambda_c \cdot (1-\rho) \cdot \left(\eta \cdot \frac{1-n}{n} + (1-\eta) \cdot \frac{u'(b)}{u'(c^w)}\right)$$

Inserting the Lagrange multiplier gives:

$$
\begin{aligned}
&(1-n)\cdot(u'(b)-u'(c^w)) \\
&\qquad = u'(c^w)\cdot\left[\frac{\partial c^w}{\partial \epsilon_s^u}\cdot\left(-\frac{\partial f}{\partial c^w}\right)^{\text{ge}}\cdot u\cdot\left(\frac{\eta-\gamma}{(1-\gamma)(1-\eta)}\cdot\frac{k_v}{q}+\frac{b}{f}\right)\right] \\
&\qquad + u'(c^w)\cdot\left[\left(\frac{\partial \epsilon_s^u}{\partial b}\right)^{\text{ge}}\cdot\frac{\partial n^u}{\partial \epsilon_s^u}\cdot\left(\frac{\lambda}{f}b-\lambda F\right)\right] \\
&\qquad + u'(c^w)\cdot\left[\left(\frac{\partial \epsilon_s^c}{\partial b}\right)^{\text{ge}}\cdot\frac{\partial n^c}{\partial \epsilon_s^c}\cdot\frac{\lambda}{f}b\right] \\
&\qquad + u'(c^w)\cdot\left[n^c\cdot\left(-\frac{\partial \theta}{\partial b}\right)^{\text{ge}}\cdot I\hat{E}_\theta\right].
\end{aligned}
$$

With:

$$
\left(\frac{\partial \epsilon_s^u}{\partial b}\right)^{\text{ge}}\cdot\frac{\partial n^u}{\partial \epsilon_s^u} = \underbrace{\left(\frac{\partial \epsilon_s^u}{\partial b}\right)^{\text{ge}}\cdot\frac{\partial n^u}{\partial \epsilon_s^u}}_{\text{direct effects}} + \underbrace{\left[\frac{\partial S}{\partial b}\cdot\left(\frac{\partial \epsilon_s^u}{\partial S}\right)^{\text{ge}}+\frac{\partial \epsilon_s^c}{\partial b}\cdot\left(\frac{\partial \epsilon_s^u}{\partial \epsilon_s^c}\right)^{\text{ge}}+\frac{\partial c^w}{\partial b}\cdot\left(\frac{\partial \epsilon_s^u}{\partial c^w}\right)^{\text{ge}}\right]\cdot\frac{\partial n^u}{\partial \epsilon_s^u}}_{\text{indirect effect}}
$$

$$
\left(\frac{\partial \epsilon_s^c}{\partial b}\right)^{\text{ge}}\cdot\frac{\partial n^c}{\partial \epsilon_s^c} = \underbrace{\left(\frac{\partial \epsilon_s^c}{\partial b}\right)^{\text{ge}}\cdot\frac{\partial n^c}{\partial \epsilon_s^c}}_{\text{direct effects}} + \underbrace{\left[\frac{\partial S}{\partial b}\cdot\left(\frac{\partial \epsilon_s^c}{\partial S}\right)^{\text{ge}}+\frac{\partial \epsilon_s^u}{\partial b}\cdot\left(\frac{\partial \epsilon_s^c}{\partial \epsilon_s^u}\right)^{\text{ge}}+\frac{\partial c^w}{\partial b}\cdot\left(\frac{\partial \epsilon_s^c}{\partial c^w}\right)^{\text{ge}}\right]\cdot\frac{\partial n^c}{\partial \epsilon_s^c}}_{\text{indirect effect}}
$$

$$
\left(\frac{\partial f}{\partial b}\right)^{\text{ge}} = \underbrace{\left(\frac{\partial S}{\partial b}\cdot\left(-\frac{\partial f}{\partial S}\right)^{\text{ge}}\right)}_{\text{direct effect}} + \underbrace{\left[\frac{\partial \epsilon_s^u}{\partial b}\cdot\frac{\partial c^w}{\partial \epsilon_s^u}+\frac{\partial \epsilon_s^c}{\partial b}\cdot\frac{\partial c^w}{\partial \epsilon_s^c}+\frac{\partial c^w}{\partial b}\right]\cdot\left(\frac{\partial f}{\partial c^w}\right)^{\text{ge}}}_{\text{indirect effect}}
$$

$$\left(\frac{\partial\theta}{\partial b}\right)^{\text{ge}} = \underbrace{\frac{\partial S}{\partial b}\cdot\left(\frac{\partial\theta}{\partial S}\right)^{\text{ge}}}_{\text{direct effect}} + \underbrace{\left[\frac{\partial\epsilon_s^u}{\partial b}\cdot\frac{\partial c^w}{\partial\epsilon_s^u}+\frac{\partial\epsilon_s^c}{\partial b}\cdot\frac{\partial c^w}{\partial\epsilon_s^c}+\frac{\partial c^w}{\partial b}\right]\cdot\left(\frac{\partial\theta}{\partial c^w}\right)^{\text{ge}}}_{\text{indirect effect}}$$

F.7 Optimal UI under Optimal Lay-off Tax

From the FOC of UI, we can derive the optimality condition for unemployment insurance. In contrast to the section where we take lay-off taxes as given, the government implements the optimal lay-off tax. This implies that the Lagrange multiplier of the separation condition in unconstrained firms is equal to zero. This is also fundamental for the optimality condition for the UI benefits:

$$\begin{aligned}(1-\eta)\cdot(u'(b)-u'(c^w)) = \lambda_\theta\cdot(1-\rho)\cdot\left(\frac{1-n}{n}+\frac{u'(b)}{u'(c^w)}\right)+\lambda_{\epsilon_s^u}\cdot\left(-\frac{u'(b)}{u'(c^w)}\right)\\ +\lambda_c\cdot(1-\rho)\cdot\left(\eta\cdot\frac{1-n}{n}+(1-\eta)\cdot\frac{u'(b)}{u'(c^w)}\right)\end{aligned}$$

The Lagrange multiplier for the separation condition of unconstrained firms is equal to

$$\begin{aligned}-\frac{\lambda_{\epsilon_s^c}}{n^s\cdot u'(c^w)} = \frac{p}{\lambda}\cdot g(\epsilon_s^c)\cdot\frac{1}{\frac{\partial S^c_{\text{stw}}(\epsilon_s^c)}{\partial\epsilon_s^c}}\cdot\Bigg[\left(1+\frac{\lambda_\theta}{n^s\cdot u'(c^w)}\right)\cdot\frac{\lambda}{f}\cdot b\\ +\frac{\lambda_c}{n^s\cdot u'(c^w)}\cdot\left(\eta\cdot\frac{b}{n}+\frac{1}{g(\epsilon_s^c)}\cdot\frac{\lambda}{p}\cdot\frac{\partial WE}{\partial\epsilon_s^c}\right)\Bigg]\end{aligned}$$

First, let us find an expression for λ_c:

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial c^w} = -\Bigg((1-p)\cdot(1-\rho^u)\cdot\frac{u(c^w)-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\\ +p\cdot(1-\rho^c)\cdot\frac{u^c-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\Bigg)\cdot\lambda_\theta\\ -(1-\eta)\cdot\Bigg((1-p)\cdot(1-\rho^u)\cdot\left(1-(1-\eta)\cdot\frac{u(c^w)-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\right)\\ -p\cdot(1-\rho^c)\cdot(1-\eta)\cdot\frac{u^c-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\Bigg)\cdot\lambda_c\\ +\frac{u(c_{\text{stw}}(\epsilon_s^c))-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\cdot\lambda_{\epsilon_s^c}=0\end{aligned}$$

Insert Lagrange multipliers for separation condition:

$$
\begin{aligned}
&- \Bigg((1-p) \cdot (1-\rho^u) \cdot \frac{u(c^w) - u(b)}{u'(c^w)} \cdot \frac{u''(c^w)}{u'(c^w)} \\
&\quad + p \cdot (1-\rho^c) \cdot \frac{u^c - u(b)}{u'(c^w)} \cdot \frac{u''(c^w)}{u'(c^w)} \Bigg) \cdot \lambda_\theta \\
&- (1-\eta) \cdot (1-p) \cdot (1-\rho^u) \cdot \left(1 - (1-\eta) \cdot \frac{u(c^w) - u(b)}{u'(c^w)} \cdot \frac{u''(c^w)}{u'(c^w)} \right) \\
&\quad \cdot p \cdot (1-\rho^c) \cdot (1-\eta) \cdot \frac{u^c - u(b)}{u'(c^w)} \cdot \frac{u''(c^w)}{u'(c^w)} \cdot \lambda_c \\
&+ \frac{\partial \epsilon_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot \Bigg((n^s \cdot u'(c^w) + \lambda_\theta) \cdot \left(\frac{\lambda}{f} \cdot b \right) \\
&\quad + \lambda_c \cdot \left(\eta \cdot \frac{b}{n} + \frac{1}{g(\epsilon_s^c)} \cdot \frac{\lambda}{p} \cdot \frac{\partial WE}{\partial \epsilon_s^c} \right) \Bigg) = 0
\end{aligned}
$$

Rearranging gives:

$$
\begin{aligned}
0 = \lambda_\theta \cdot & \frac{\partial S^{\text{total}}}{\partial c^w} \\
& + \lambda_c \cdot \frac{\partial WE^{\text{total}}}{\partial c^w} \\
& + \frac{\partial \epsilon_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s \cdot u'(c^w) \cdot \frac{\lambda}{f} b
\end{aligned}
$$

with

$$
\begin{aligned}
\frac{\partial S^{\text{total}}}{\partial c^w} = \Bigg[& p(1-\rho^u) \frac{u(c^w) - u(b)}{u'(c^w)} \frac{u''(c^w)}{u'(c^w)} \\
& - (1-p)(1-\rho^c) \frac{u^c - u(b)}{u'(c^w)} \frac{u''(c^w)}{u'(c^w)} \\
& + \frac{\partial \epsilon_s^c}{\partial \theta} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot \frac{\lambda}{f} b \Bigg]
\end{aligned}
$$

and

$$\frac{\partial WE^{\text{total}}}{\partial c^w} = \Bigg[-(1-\eta)(1-p\rho^u)\left(1-(1-\eta)\frac{u(c^w)-u(b)}{u'(c^w)}\frac{u''(c^w)}{u'(c^w)}\right) + (1-(1-p)\rho^c)(1-\eta)\frac{u^c-u(b)}{u'(c^w)}\frac{u''(c^w)}{u'(c^w)} + \frac{\partial \epsilon_s^c}{\partial c^w}\cdot\frac{p}{\lambda}\cdot g(\epsilon_s^c)\cdot\lambda_c\left(\eta\frac{b}{n}+\frac{1}{g(\epsilon_s^c)}\cdot\frac{\lambda}{p}\cdot\frac{\partial WE}{\partial \epsilon_s^c}\right)\Bigg]$$

Next we can solve for the Lagrange multiplier of the wage equation:

$$\lambda_c = -\frac{1}{\left(\frac{\partial WE}{\partial c^w}\right)^{\text{total}}}\Bigg[\frac{\partial \epsilon_s^c}{\partial c^w}\cdot\frac{p}{\lambda}\cdot g(\epsilon_s^c)\cdot n^s\cdot u'(c^w)\cdot\frac{\lambda}{f}\cdot b + \left(-\frac{\partial S^{\text{total}}}{\partial c^w}\right)\cdot\lambda_\theta\Bigg]$$

Now we have everything to calculate λ_θ from the two equations:

$$(1)\quad \left(\chi-\frac{\eta-\gamma}{1-\eta}\right)\cdot\frac{1}{1-\gamma}\cdot\frac{k_v}{q} = \left(1+\frac{\lambda_\theta+\eta\cdot\lambda\cdot\lambda_c}{n^s\cdot u'(c^w)}\right)\cdot\frac{b}{f}$$

$$\begin{aligned}(2)\quad \chi = {} & \frac{1}{1-\eta}\cdot\frac{1}{u\cdot f}\cdot\frac{1}{u'(c^w)}\cdot\left[\gamma-(\lambda\gamma-f\eta)\cdot\left(\frac{1}{\lambda}(1-\rho)\lambda_\theta\right)\right] \\ & + \frac{1}{u}\cdot\frac{1}{u'(c^w)}\cdot\left(\lambda\cdot\frac{\gamma}{f}\cdot u'(c^w)+\left(\frac{1}{1-\eta}\cdot\frac{\lambda\gamma-f}{f}\right)\cdot\eta\right)\cdot\lambda_{\epsilon_s^c} \\ & + \frac{1}{u}\cdot\frac{1}{u'(c^w)}\cdot\lambda_c\cdot\frac{\partial WE}{\partial\theta}\cdot\frac{1}{k_v} \\ & - \frac{1}{u}\cdot\frac{1}{u'(c^w)}\cdot IE_\theta\end{aligned}$$

We can rearrange (1) to:

$$\chi\cdot k_v = (1-\gamma)\cdot q\cdot\left(\frac{\eta-\gamma}{(1-\gamma)(1-\eta)}\cdot\frac{k_v}{q}+\left(1+\frac{\lambda_\theta+\eta\cdot\lambda\cdot\lambda_c}{n^s\cdot u'(c^w)}\right)\cdot\frac{b}{f}\right)$$

Note that

$$(1-\gamma)\cdot q = \frac{\partial f}{\partial\theta}$$

This follows from the fact that:

$$f'(\theta) = q(\theta)+\theta\cdot q'(\theta) = q(\theta)\cdot\left(1+\frac{q'(\theta)\cdot\theta}{q(\theta)}\right) = q(\theta)\cdot(1-\gamma)$$

So we can write:

$$\chi \cdot k_v = \left(\frac{\partial f}{\partial \theta}\right) \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \left(1 + \frac{\lambda_\theta + \eta \cdot \lambda \cdot \lambda_c}{n^s \cdot u'(c^w)}\right) \cdot \frac{b}{f}\right)$$

Inserting χ gives:

$$\begin{aligned}
&\left[\gamma - (\lambda\gamma - f\eta) \cdot \frac{1}{\lambda} \cdot (1-\rho)\right] \cdot \frac{1}{1-\eta} \cdot \frac{k_v}{f} \cdot \lambda_\theta \\
&= u'(c^w) \cdot \left(\frac{\partial f}{\partial \theta}\right) \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \left(1 + \frac{\lambda_\theta + \eta \cdot \lambda \cdot \lambda_c}{n^s \cdot u'(c^w)}\right) \cdot \frac{b}{f}\right) \\
&\quad + \left(\lambda \cdot \frac{\gamma}{f} \cdot u'(c^w) + \left(\frac{1}{1-\eta} \cdot \frac{\lambda\gamma - f}{f}\right) \cdot \eta\right) \cdot k_v \cdot (-\lambda_{\epsilon_s^c}) \\
&\quad + \frac{\partial WE}{\partial \theta} \cdot (-\lambda_c) \quad + IE_\theta
\end{aligned}$$

Inserting $\lambda_{\epsilon_s^c}, IE_\theta$ gives

$$\begin{aligned}
&\left[\gamma - (\lambda\gamma - f\eta) \cdot \frac{1}{\lambda} \cdot (1-\rho)\right] \cdot \frac{1}{1-\eta} \cdot \frac{k_v}{f} \cdot \lambda_\theta \\
&= u'(c^w) \cdot \frac{\partial f}{\partial \theta} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \left(1 + \frac{\lambda_\theta}{n^s u'(c^w)}\right) \cdot \frac{b}{f}\right) \\
&\quad + u'(c^w) \cdot n^s \cdot \left(-\frac{\partial \epsilon_s^c}{\partial \theta}\right)^{\text{total}} \cdot g(\epsilon_s^c) \cdot \frac{p}{\lambda} \cdot \left(1 + \frac{\lambda_\theta}{n^s u'(c^w)}\right) \cdot \frac{\lambda}{f} \cdot b \\
&\quad + \lambda_\theta \cdot \left(-\frac{\partial c^w}{\partial \theta}\right)^{\text{total,2}} \cdot \left(-\frac{\partial S}{\partial c^w}\right)^{\text{total}} \\
&\quad + u'(c^w) \cdot n^s \cdot \left(1 + \frac{\lambda_\theta}{n^s u'(c^w)}\right) \cdot \tilde{IE}_\theta
\end{aligned}$$

with

$$\begin{aligned}
\left(-\frac{\partial \epsilon_s^c}{\partial \theta}\right)^{\text{total}} &= \left[\left(-\frac{\partial \epsilon_s^c}{\partial \theta}\right) + \left(\frac{\partial c^w}{\partial \theta}\right)^{\text{total}} \cdot \left(-\frac{\partial \epsilon_s^c}{\partial c^w}\right)\right. \\
&\quad \left. + \left(-\frac{\partial \epsilon_s^c}{\partial \theta}\right) \cdot \left(\frac{\partial n}{\partial \epsilon_s^c} \cdot \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \epsilon_s^c}\right)^{\text{total}}\right)\right] \cdot \frac{\partial \epsilon_s^c}{\partial c^w} \\
&\quad + f'(\theta) \cdot \frac{\partial n}{\partial f} \cdot \frac{\partial c^w}{\partial n} \cdot \frac{\partial \epsilon_s^c}{\partial c^w}
\end{aligned}$$

$$\begin{aligned}
\left(-\frac{\partial c^w}{\partial \theta}\right)^{\text{total,2}} &= \left(-\frac{\partial c^w}{\partial \theta}\right)^{\text{total}} \\
&\quad + \left(-\frac{\partial \epsilon_s^u}{\partial \theta}\right) \cdot \left(\frac{\partial n}{\partial \epsilon_s^c} \cdot \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \epsilon_s^c}\right)^{\text{total}}\right) \\
&\quad + f'(\theta) \cdot \frac{\partial n}{\partial f} \cdot \frac{\partial c^w}{\partial n}
\end{aligned}$$

This allows us to calculate the Lagrange multiplier for the job-creation condition

$$\begin{aligned} M \cdot \lambda_\theta = u'(c^w) \cdot \frac{\partial f}{\partial \theta} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \left(1 + \frac{\lambda_\theta}{n^s u'(c^w)}\right) \cdot \frac{b}{f} \right) \\ + u'(c^w) \cdot n^s \cdot \left(-\frac{\partial \epsilon_s^c}{\partial \theta} \right)^{\text{total}} \cdot g(\epsilon_s^c) \cdot \frac{p}{\lambda} \cdot \frac{\lambda}{f} b \\ + u'(c^w) \cdot n^s \cdot \tilde{IE}_\theta \end{aligned}$$

with

$$\begin{aligned} M = \left[\gamma - (\lambda\gamma - f \cdot \eta) \cdot \frac{1}{\lambda} \cdot (1-\rho) \right] \cdot \frac{1}{1-\eta} \cdot \frac{k_v}{f} \\ + \left(-\frac{\partial f}{\partial \theta} \cdot u \cdot \frac{b}{f} \right) \\ + \left(\frac{\partial \epsilon_s^c}{\partial \theta} \right)^{\text{total}} \cdot g(\epsilon_s^c) \cdot \frac{p}{\lambda} \cdot \frac{\lambda}{f} b \\ + \left(\frac{\partial c^w}{\partial \theta} \right)^{\text{total}} \cdot \left(-\frac{\partial S}{\partial c^w} \right)^{\text{total}} \\ + \tilde{IE}_\theta \end{aligned}$$

Note: $\frac{1}{M}$ denotes the general equilibrium effect of an increase of the joint surplus on θ:

$$\left(\frac{\partial \theta}{\partial S} \right)^{\text{ge}} = \frac{1}{M}$$

Rearranging for λ_θ gives:

$$\begin{aligned} \lambda_\theta = u'(c^w) \cdot \left(\frac{\partial f}{\partial S} \right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \frac{\lambda}{f} \cdot b \right) \\ + u'(c^w) \cdot \left(-\frac{\partial \epsilon_s^c}{\partial S} \right)^{\text{ge}} \cdot \frac{\partial n^c}{\partial \epsilon_s^c} \cdot \frac{\lambda}{f} b \\ + u'(c^w) \cdot n^c \cdot \left(\frac{\partial \theta}{\partial S} \right)^{\text{ge}} \cdot \hat{IE}_\theta \end{aligned}$$

Note that

$$\left(\frac{\partial f}{\partial S} \right)^{\text{ge}} = \frac{1}{M} \cdot \frac{\partial f}{\partial S}$$

$$\left(\frac{\partial \epsilon_s^c}{\partial S} \right)^{\text{ge}} = \frac{1}{M} \cdot \left(\frac{\partial \epsilon_s^c}{\partial S} \right)^{\text{total}}$$

Further define:

$$\hat{IE}_\theta = \frac{1}{1-\rho^c} \cdot \int_{\epsilon_s^c}^{\epsilon^p} \lambda \cdot \frac{\gamma}{f} \cdot \frac{u'(c_{\text{stw}}(\epsilon)) - u'(c^w)}{u'(c^w)} \, dG(\epsilon)$$

Finally, we can calculate the Lagrange multiplier for λ_c by inserting into λ_θ:

$$\lambda_c = \frac{-1}{\left(\frac{\partial S}{\partial c^w}\right)^{\text{total}}} \Bigg[\left(\frac{\partial \epsilon_s^c}{\partial c^w} + \left(\frac{\partial S}{\partial c^w}\right)^{\text{total}} \cdot \left(\frac{\partial \epsilon_s^c}{\partial S}\right)^{\text{ge}} \right) \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s \cdot u'(c^w) \cdot \frac{\lambda}{f} b$$

$$+ \left(\frac{\partial S}{\partial c^w}\right)^{\text{total}} \cdot \left(-\frac{\partial f}{\partial S}\right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \left(1 + \frac{\lambda_\theta}{n^s u'(c^w)}\right) \cdot \frac{b}{f} \right)$$

$$+ n^c \cdot \left(\frac{\partial S}{\partial c^w}\right)^{\text{ge}} \cdot \left(-\frac{\partial \theta}{\partial c^w}\right) \cdot \hat{IE}_\theta \Bigg]$$

Simplifying:

$$\lambda_c = -\frac{u'(c^w)}{\left(\frac{\partial WE}{\partial c^w}\right)^{\text{total}}} \Bigg[\left(\frac{\partial \epsilon_s^u}{\partial c^w}\right)^{\text{ge}} \cdot \frac{\partial n^u}{\partial \epsilon_s^u} \cdot \left(\frac{\lambda}{f} b - \lambda F\right) + \left(\frac{\partial \epsilon_s^c}{\partial c^w}\right)^{\text{ge}} \cdot \frac{\partial n^c}{\partial \epsilon_s^c} \cdot \frac{\lambda}{f} b$$

$$+ \left(-\frac{\partial f}{\partial c^w}\right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \frac{b}{f} \right) + n^c \cdot \left(-\frac{\partial \theta}{\partial c^w}\right)^{\text{ge}} \cdot \hat{IE}_\theta \Bigg]$$

Following the same arguments, we can express the Lagrange multiplier for the separation condition as:

$$\lambda_{\epsilon_s^c} = -\frac{u'(c^w)}{\left(\frac{\partial S_{\text{stw}}^c(\epsilon_s^c)}{\partial \epsilon_s^c}\right)^{\text{total}}} \Bigg[\frac{\partial \epsilon_s^u}{\partial \epsilon_s^c} \cdot \left(\frac{\partial n^u}{\partial \epsilon_s^u}\right)^{\text{ge}} \cdot \left(\frac{\lambda}{f} b - \lambda F\right) + \left(\frac{\partial n^c}{\partial \epsilon_s^c}\right)^{\text{ge}} \cdot \frac{\lambda}{f} b$$

$$+ \frac{\partial c^w}{\partial \epsilon_s^c} \cdot \left(-\frac{\partial f}{\partial c^w}\right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \frac{b}{f} \right) + \frac{\partial c^w}{\partial \epsilon_s^c} \cdot \left(-\frac{\partial \theta}{\partial c^w}\right)^{\text{ge}} \cdot \hat{IE}_\theta \Bigg]$$

For convenience, the optimal UI benefits are replicated here:

$$(1-n) \cdot (u'(b) - u'(c^w)) = \lambda_\theta \cdot (1-\rho) \cdot \left(\frac{1-n}{n} + \frac{u'(b)}{u'(c^w)} \right) \cdot \left(-\frac{u'(b)}{u'(c^w)} \right)$$

$$+ \lambda_{\epsilon_s^c} \cdot \left(-\frac{u'(b)}{u'(c^w)} \right)$$

$$+ \lambda_c \cdot (1-\rho) \cdot \left(\eta \cdot \frac{1-n}{n} + (1-\eta) \cdot \frac{u'(b)}{u'(c^w)} \right)$$

Inserting the Lagrange multiplier gives

$$(1-n)\cdot(u'(b)-u'(c^w)) = u'(c^w)\cdot\Bigg[\left(-\frac{\partial f}{\partial b}\right)^{\text{ge}}\cdot u\cdot\left(\frac{\eta-\gamma}{(1-\gamma)(1-\eta)}\cdot\frac{k_v}{q}+\frac{b}{f}\right) + \left(\frac{\partial \epsilon_s^c}{\partial b}\right)^{\text{ge}}\cdot\frac{\partial n^c}{\partial \epsilon_s^c}\cdot\frac{\lambda}{f}b + n^c\cdot\left(-\frac{\partial\theta}{\partial b}\right)^{\text{ge}}\cdot\hat{IE}_\theta\Bigg]$$

with

$$\left(\frac{\partial \epsilon_s^c}{\partial b}\right)^{\text{ge}}\cdot\frac{\partial n^c}{\partial \epsilon_s^c} = \underbrace{\left(\frac{\partial \epsilon_s^c}{\partial b}\right)^{\text{ge}}\cdot\frac{\partial n^c}{\partial \epsilon_s^c}}_{\text{direct effects}} + \underbrace{\left[\frac{\partial S}{\partial b}\cdot\left(\frac{\partial \epsilon_s^c}{\partial S}\right)^{\text{ge}}+\frac{\partial c^w}{\partial b}\cdot\left(\frac{\partial \epsilon_s^c}{\partial c^w}\right)^{\text{ge}}\right]\cdot\frac{\partial n^c}{\partial \epsilon_s^c}}_{\text{indirect effect}}$$

$$\left(\frac{\partial f}{\partial b}\right)^{\text{ge}} = \underbrace{\left(\frac{\partial S}{\partial b}\cdot\left(-\frac{\partial f}{\partial S}\right)^{\text{ge}}\right)}_{\text{direct effect}} + \underbrace{\left[\frac{\partial \epsilon_s^c}{\partial b}\cdot\frac{\partial c^w}{\partial \epsilon_s^c}+\frac{\partial c^w}{\partial b}\right]\cdot\left(\frac{\partial f}{\partial c^w}\right)^{\text{ge}}}_{\text{indirect effect}}$$

$$\left(\frac{\partial \theta}{\partial b}\right)^{\text{ge}} = \underbrace{\frac{\partial S}{\partial b}\cdot\left(\frac{\partial \theta}{\partial S}\right)^{\text{ge}}}_{\text{direct effect}} + \underbrace{\left[\frac{\partial \epsilon_s^c}{\partial b}\cdot\frac{\partial c^w}{\partial \epsilon_s^c}+\frac{\partial c^w}{\partial b}\right]\cdot\left(\frac{\partial \theta}{\partial c^w}\right)^{\text{ge}}}_{\text{indirect effect}}$$

Inserting the Lagrange multiplier gives

$$(1-n)\cdot\frac{(u'(b)-u'(c^w))}{u'(c^w)} = \left(-\frac{\partial f}{\partial b}\right)^{\text{ge}}\cdot u\cdot L_v + \left(\frac{\partial \epsilon_s^c}{\partial b}\right)^{\text{ge}}\cdot\frac{\partial n^c}{\partial \epsilon_s^c}\cdot L_s^c + n^c\cdot\left(-\frac{\partial\theta}{\partial b}\right)^{\text{ge}}\cdot\hat{IE}_\theta$$

Note that when we set the lay-off tax optimally, then UI will not impose any distortions on the separation condition of unconstrained firms. In the context of Proposition 1, this means that $MLS^u = 0$, bolstering the fact that optimal lay-off taxes can implement the optimal number of separations.

G Ramsey FOCs with STW

In the following, multipliers from the Lagrangian, implied by the maximization problem from the previous section, are denoted by λ_{idx}, the index depending on the constraint. Here, λ_n denotes the Lagrange multiplier for the total employment equation, λ_{n^u} for the number of unconstrained firms, λ_{n^c} for the number of constrained firms, λ_{n^s} for the number of firms that received a shock, λ_θ for the job-creation condition, λ_c for the wage

equation, $\lambda_{\epsilon_s^u}$ for the separation condition of unconstrained firms with STW support, $\lambda_{\xi_s^c}$ for the separation condition of constrained firms without STW support, and $\lambda_{\epsilon_s^c}$ for the separation condition of constrained firms with STW support. Every other equation listed in the Ramsey problem for STW is assumed to be plugged in.

G.1 Employment

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial n^s} &= -\lambda_{n^s} + \frac{1-p}{\lambda}(1-\rho^u)\cdot(u(c^w) - u'(c^w)\cdot c^w) \\
&\quad + \frac{p}{\lambda}(1-\rho^c)\cdot(u^c - u'(c^w)\cdot e^c) \\
&\quad + \frac{1-p}{\lambda}\cdot(1-\rho^c)\cdot\lambda_{n^u} + \frac{p}{\lambda}(1-\rho^c)\cdot\lambda_{n^c} = 0
\end{aligned}$$

$$\frac{\partial \mathcal{L}}{\partial n^u} = -\lambda_{n^u} + \lambda_n = 0$$

$$\frac{\partial \mathcal{L}}{\partial n^c} = -\lambda_{n^c} + \lambda_n = 0$$

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial n} &= -\lambda_n + \lambda_{n^s}\cdot(\lambda - q(\theta)\cdot\theta) \\
&\quad + u'(c^w)\cdot[(z-\Omega) + b + \theta\cdot k_v] - u(b) \\
&\quad + \frac{(1-\rho)}{\lambda}\cdot\frac{b}{n^2}\cdot\lambda_\theta + \eta(1-\rho)\cdot\frac{b}{n^2}\cdot\lambda_c
\end{aligned}$$

$$\begin{aligned}
\Leftrightarrow \quad \frac{\lambda_n}{u'(c^w)} &= (\lambda - q(\theta)\cdot\theta) + [z - \Omega + b + \theta\cdot k_v] - \frac{u(b)}{u'(c^w)} \\
&\quad + \frac{1-\rho}{\lambda}\cdot\frac{b}{n^2}\cdot\frac{\lambda_\theta}{u'(c^w)} + \eta\cdot(1-\rho)\cdot\frac{b}{n^2}\cdot\frac{\lambda_c}{u'(c^w)}
\end{aligned}$$

G.2 Job Creation

Before we start, let us define the Insurance effect:

$$IE_\theta = n^s\cdot u'(c^w)\cdot\left(1 + \frac{\lambda_\theta}{n^s\cdot u'(c^w)}\right)\cdot\tilde{IE}_\theta$$

$$\begin{aligned}
\tilde{IE}_\theta = \frac{p}{\lambda}\Bigg(&\int_{\max\{\xi_s^c,\epsilon_{stw}\}}^{\epsilon_p}\frac{\lambda}{f}\cdot\gamma\cdot\frac{u'(c(\epsilon)) - u'(c^w)}{u'(c^w)}\,dG(\epsilon) \\
&+ \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}},\epsilon_s^c\}}\frac{\lambda}{f}\cdot\gamma\cdot\frac{u'(c_{\text{stw}}(\epsilon)) - u'(c^w)}{u'(c^w)}\,dG(\epsilon)\Bigg)\cdot k_v
\end{aligned}$$

The FOC for the labor market tightness can be written as:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \theta} = &-k_v(1-n)\cdot u'(c^w) + IE_\theta + (1-\gamma)\cdot q(\theta)\cdot(1-n)\cdot\lambda_{n^s} \\ &- \frac{1}{1-\eta}\cdot\gamma\cdot k_v + \frac{\lambda_\theta}{f} + \frac{1}{1-\eta}\cdot\frac{\lambda\gamma - f\eta}{f}\cdot\left(\frac{1}{\lambda}\cdot(1-\rho)\cdot\lambda_\theta - \lambda_{\epsilon_s^u}\right) \\ &- \left(\lambda\cdot\frac{\gamma}{f}\cdot u'(c^w_{stw}(\epsilon^c_s)) + \left(\frac{1}{1-\eta}\cdot\frac{\lambda\gamma - f}{f}\right)\eta\right)\cdot\lambda_{\epsilon^c_s} \\ &- \left(\lambda\cdot\frac{\gamma}{f}\cdot u'(c^w(\xi^c_s)) + \frac{1}{\lambda}\cdot\left(\frac{1}{1-\eta}\cdot\frac{\lambda\gamma - f}{f}\right)\eta\right)\cdot\lambda_{\xi^c_s} \\ &- \lambda_c\cdot\frac{\partial WE}{\partial\theta}\end{aligned}$$

$$\iff \quad \frac{\lambda_{n^s}}{u'(c^w)} = \frac{1+\chi}{1-\gamma}\cdot\frac{k_v}{q}$$

$$\begin{aligned}\text{with}\quad \chi = &\frac{1}{(1-n)\cdot u'(c^w)}\cdot\frac{1}{f} \\ \cdot\quad &\frac{1}{1-\eta}\cdot\left(\gamma\cdot\lambda_\theta - (\lambda\cdot\gamma - f\cdot\eta)\cdot\left(\frac{1}{\lambda}\cdot(1-\rho)\cdot\lambda_\theta - \lambda_{\epsilon_s^u}\right)\right) \\ &+ \frac{1}{(1-n)\cdot u'(c^w)}\left(\lambda\cdot\frac{\gamma}{f}u'(c^w_{stw}(\epsilon^c_s)) + \left(\frac{1}{1-\eta}\cdot\frac{\lambda\gamma - f}{f}\right)\eta\right)\cdot\lambda_{\epsilon^c_s} \\ &+ \frac{1}{(1-n)\cdot u'(c^w)}\left(\lambda\cdot\frac{\gamma}{f}u'(c^w(\xi^c_s)) + \left(\frac{1}{1-\eta}\cdot\frac{\lambda\gamma - f}{f}\right)\eta\right)\cdot\lambda_{\xi^c_s} \\ &+ \frac{\lambda_c}{(1-n)\cdot u'(c^w)}\cdot\frac{\partial WE}{\partial\theta}/k_v \\ &- \frac{1}{(1-n)\cdot u'(c^w)}\cdot IE_\theta/k_v\end{aligned}$$

Collecting terms from FOCs for employment and job-creation conditions gives:

$$\begin{aligned}u'(c^w)\cdot\frac{1+\chi}{1-\gamma}\cdot\frac{k_v}{q} = &\frac{1-p}{\lambda}\cdot(1-\rho^u)\cdot(u(c^w) - u(b) - u'(c^w)\cdot c^w) \\ &+ \frac{p}{\lambda}\cdot(1-\rho^c)\cdot(u^c - u(b) - u'(c^w)\cdot e^c) \\ &+ \left(\frac{1-\rho}{\lambda}\cdot\frac{b}{n}\cdot\frac{\lambda_\theta}{n^s}\right) + \left(\frac{1-\rho}{\lambda}\cdot\frac{\lambda_c}{n^s}\cdot\eta\cdot\lambda_c\cdot\frac{b}{n}\right) \\ &+ u'(c^w)\cdot\frac{1-\rho}{\lambda}\cdot\left(z - \Omega + b + \frac{\lambda - \gamma f + \chi(1-f)}{1-\gamma}\cdot\frac{k_v}{q}\right)\end{aligned}$$

Optimal Job Creation Condition:

$$\begin{aligned}\frac{1+\chi}{1-\gamma}\cdot\frac{k_v}{q} = \frac{1-\rho}{\lambda}\cdot(z-\Omega+b)&\\ +\frac{1-\rho}{\lambda}\cdot\frac{b}{n}\cdot\frac{\lambda_\theta}{n^s\cdot u'(c^w)}&\\ +\frac{1-\rho}{\lambda}\cdot\frac{b}{n}\cdot\frac{\lambda_c\cdot\lambda}{n^s\cdot u'(c^w)}&\\ +\frac{1-p}{\lambda}\cdot(1-\rho^u)\cdot\left(\frac{u(c^w)-u(b)}{u'(c^w)}-c^w\right)&\\ +\frac{p}{\lambda}\cdot(1-\rho^c)\cdot\left(\frac{u^c-u(b)}{u'(c^w)}-e^c\right)&\\ +\frac{1-\rho}{\lambda}\cdot\frac{\lambda-\gamma f+\chi(\lambda-f)}{1-\gamma}\cdot\frac{k_v}{q}&\end{aligned}$$

Subtracting the decentralized job-creation condition from the optimal gives:

$$\begin{aligned}\left(\chi-\frac{\eta-\gamma}{1-\eta}\right)\cdot\frac{1}{1-\gamma}\cdot\frac{k_v}{q} &= \left(1+\frac{\lambda_\theta+\eta\cdot\lambda\cdot\lambda_c}{n^s\cdot u'(c^w)}\right)\cdot\frac{1-\rho}{\lambda}\cdot\frac{b}{n}\\ &\quad+\frac{(1-\rho)(\lambda-f)}{\lambda}\cdot\left(\chi-\frac{\eta-\gamma}{1-\eta}\right)\cdot\frac{1}{1-\gamma}\cdot\frac{k_v}{q}\end{aligned}$$

Rearranging gives:

$$\begin{aligned}\left(\chi-\frac{\eta-\gamma}{1-\eta}\right)\cdot\frac{1}{1-\gamma}\cdot\frac{k_v}{q} &= \left(1+\frac{\lambda_\theta+\eta\cdot\lambda\cdot\lambda_c}{n^s\cdot u'(c^w)}\right)\cdot\frac{\frac{1-\rho}{\lambda}}{\rho+(1-\rho)\cdot\frac{f}{\lambda}}\cdot\frac{b}{n}\\ &= \left(1+\frac{\lambda_\theta+\eta\cdot\lambda\cdot\lambda_c}{n^s\cdot u'(c^w)}\right)\cdot\frac{b}{f}\end{aligned}$$

G.3 Lagrange Multipliers for Separation Condition

Optimal separation condition unconstrained firms

$$\begin{aligned}&-\frac{\partial\mathcal{L}}{\partial\epsilon_s^u}\\ &=\frac{n}{1-\rho}\cdot(1-p)\cdot g(\epsilon_s^u)\cdot u'(c^w)\cdot\left(z(\epsilon_s^u)-\Omega(\epsilon_s^u)+\frac{u(c^w)}{u'(c^w)}-c^w-(z+\Omega)\right)\\ &\quad+\lambda_\theta\cdot\frac{1-p}{\lambda}\cdot g(\epsilon_s^u)\cdot\left[z(\epsilon_s^u)-\Omega(\epsilon_s^u)+\frac{u(c^w)-u(b)}{u'(c^w)}-c^w\right.\\ &\quad\left.-\frac{1-n}{n}\cdot b+\frac{\lambda-\eta f}{1-\eta}\cdot\frac{k_v}{q}\right]\\ &\quad+\lambda_c\cdot\frac{\partial WE}{\partial\epsilon_s^u}+\frac{1-p}{\lambda}\cdot g(\epsilon_s^u)\cdot n^s\cdot\lambda_{n^u}+\lambda_{\epsilon_s^u}\cdot\frac{\partial S^u_{\text{stw}}(\epsilon_s^u)}{\partial\epsilon_s^u}\overset{!}{=}0\end{aligned}$$

Insert for λ_{n^u}:

$$\begin{aligned}
\iff \quad & g(\epsilon_s^u) \cdot \frac{1-p}{\lambda} \cdot n^s \\
& \cdot \left[z(\epsilon_s^u) - \Omega(\epsilon_s^u) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w - (z + \Omega) + \frac{\lambda_{n^u}}{u'(c^w)} \right] \\
& - \frac{\lambda_\theta}{u'(c^w)} \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot \left[s_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon_s^u)) + \frac{1-n}{n} \cdot b \right] \\
& + \frac{\lambda_c}{u'(c^w)} \cdot \frac{\partial WE}{\partial \epsilon_s^u} + \frac{\lambda_{\epsilon_s^u}}{u'(c^w)} \cdot \frac{\partial S_{\text{stw}}^u(\epsilon_s^u)}{\partial \epsilon_s^u} \stackrel{!}{=} 0
\end{aligned}$$

$$\begin{aligned}
\iff \quad & z(\epsilon_s^u) - \Omega(\epsilon_s^u) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + b + (\lambda - q(\theta) \cdot \theta) \cdot \frac{\lambda_{n^s}}{u'(c^w)} + \theta \cdot c \\
& - \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \cdot \left(s_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon_s^u)) + \frac{1-n}{n} \cdot b - \frac{b}{n} \right) \\
& + \lambda_c \cdot \frac{1}{n^s \cdot u'(c^w)} \cdot \left(\eta \cdot \frac{b}{n} + \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial WE}{\partial \epsilon_s} \right) \\
& + \lambda_{\epsilon_s^u} \cdot \frac{1}{n^s \cdot u'(c^w)} \cdot \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial S_{\text{stw}}^u(\epsilon_s^u)}{\partial \epsilon_s^u} = 0
\end{aligned}$$

Insert for λ_{n^s} :

$$\begin{aligned}
\iff \quad & z(\epsilon_s^u) - \Omega(\epsilon_s^u) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + b \\
& + (\lambda - q(\theta) \cdot \theta) \cdot \frac{\lambda_{n^s}}{u'(c^w)} + \theta \cdot c \\
& - \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \left(s_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon_s^u)) + \frac{1-n}{n} \cdot b - \frac{b}{n} \right) \\
& + \frac{\lambda_c}{n^s \cdot u'(c^w)} \left(\eta \cdot \frac{b}{n} + \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial WE}{\partial \epsilon_s^u} \right) \\
& + \frac{\lambda_{\epsilon_s^u}}{n^s \cdot u'(c^w)} \cdot \frac{1}{n^s} \cdot \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial S_{\text{stw}}^u(\epsilon_s^u)}{\partial \epsilon_s^u} = 0
\end{aligned}$$

Insert λ_{n^s}

$$\begin{aligned}
\iff & z(\epsilon_s^u) - \Omega(\epsilon_s^u) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + b \\
& + \frac{\lambda - \gamma f + \chi(\lambda - f)}{1-\gamma} \cdot \frac{k_v}{q} \\
& - \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \left(s_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon_s^u)) - b \right) \\
& + \lambda \cdot \frac{\lambda_c}{n^s \cdot u'(c^w)} \left(\eta \cdot \frac{b}{n} + \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial WE}{\partial \epsilon_s^u} \right) \\
& + \frac{\lambda_{\epsilon_s^u}}{n^s \cdot u'(c^w)} \cdot \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial S_{\text{stw}}^u(\epsilon_s^u)}{\partial \epsilon_s^u} = 0
\end{aligned}$$

Subtract decentralized separation condition:

$$\begin{aligned}
s_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon_s^u)) = b & + (\lambda - \theta q(\theta)) \cdot \left(\chi - \frac{\eta - \gamma}{1-\eta} \right) \cdot \frac{1}{1-\gamma} \cdot \frac{k_v}{q} \\
& + \frac{\lambda_{\epsilon_s^u}}{n^s \cdot u'(c^w)} \cdot \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial S_{\text{stw}}^u(\epsilon_s^u)}{\partial \epsilon_s^u} \\
& - \frac{\lambda_\theta \cdot \theta}{n^s \cdot u'(c^w)} \cdot \left(s_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon_s^u)) - b \right) \\
& + \frac{\lambda_c}{n^s \cdot u'(c^w)} \left(\eta \cdot \frac{b}{n} + \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial WE}{\partial \epsilon_s^u} \right)
\end{aligned}$$

Hence

$$\begin{aligned}
0 = & \left(1 + \frac{\lambda_\theta}{u'(c^w) \cdot n^s} \right) \\
& \cdot \left(b + (\lambda - f) \cdot \frac{(1 - \rho/\lambda)}{(\rho + (1-\rho) \cdot f/\lambda)} \cdot \frac{b}{n} - s_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon_s^u)) \right) \\
& + \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\lambda_{\epsilon_s^u}}{n^s \cdot u'(c^w)} \cdot \frac{\partial S_{\text{stw}}^u(\epsilon_s^u)}{\partial \epsilon_s^u} \\
& + \frac{\lambda_c}{n^s \cdot u'(c^w)} \left(\eta \cdot \frac{b}{n} + \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial WE}{\partial \epsilon_s^u} \right) \\
\iff \quad 0 = & \left(1 + \frac{\lambda_\theta}{u'(c^w) \cdot n^s} \right) \cdot \left(\frac{\lambda}{f} \cdot b - s_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon_s^u)) \right) \\
& + \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\lambda_{\epsilon_s^u}}{n^s \cdot u'(c^w)} \cdot \frac{\partial S_{\text{stw}}^u(\epsilon_s^u)}{\partial \epsilon_s^u} \\
& + \frac{\lambda_c}{n^s \cdot u'(c^w)} \left(\eta \cdot \frac{b}{n} + \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial WE}{\partial \epsilon_s^u} \right)
\end{aligned}$$

And finally:

$$-\frac{\lambda_{\epsilon_s^u}}{n^s \cdot u'(c^w)} = \frac{(1-p)\cdot g(\epsilon_s^u)}{\lambda} \cdot \frac{1}{\frac{\partial S^u_{\text{stw}}(\epsilon_s^u)}{\partial \epsilon_s^u}} \cdot \Bigg(\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \left(\frac{\lambda}{f} \cdot b - s_{\text{stw}} \cdot \left(\bar{h} - h_{\text{stw}}(\epsilon_s^u)\right)\right) + \frac{\lambda_c}{n^s \cdot u'(c^w)} \cdot \left(\eta \cdot \frac{b}{n} + \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial WE}{\partial \epsilon_s^u}\right)\Bigg)$$

Optimal separation condition constrained firms

$$\begin{aligned} -\frac{\partial \mathcal{L}}{\partial \epsilon_s^c} = \Bigg[& \frac{n}{1-\rho} \cdot p \cdot g(\epsilon_s^c) \cdot u'(c^w) \left(z(\epsilon_s^c) - \Omega(\epsilon_s^c)\right. \\ & \left. + \frac{u(c_{\text{stw}}(\epsilon_s^c))}{u'(c^w)} - c_{\text{stw}}(\epsilon_s^c) - (z+\Omega)\right) \\ & + \lambda_\theta \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \left[z(\epsilon_s^c) - \Omega(\epsilon_s^c) + \frac{u(c_{\text{stw}}(\epsilon_s^c)) - u(b)}{u'(c^w)} - c^w(\epsilon_s^c) \right. \\ & \left. - \frac{1-n}{n} \cdot b + \frac{\lambda - \eta f}{1-\eta} \cdot \frac{k_v}{q} \right] \\ & - \lambda_c \cdot \frac{\partial WE}{\partial \epsilon_s^c} + \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s \cdot \lambda_{n^c} \Bigg] \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c) + \lambda^c_{\epsilon_s^c} \cdot \frac{\partial S^u_{\text{stw}}(\epsilon_s^c)}{\partial \epsilon_s^c} = 0 \end{aligned}$$

Subtract decentralized separation condition (constrained firms):

$$\begin{aligned} -\frac{\partial \mathcal{L}}{\partial \epsilon_s^c} = \Bigg[& \frac{n}{1-\rho} \cdot p \cdot g(\epsilon_s^c) \left(z_{\text{stw}}(\epsilon_s^c) - \Omega(\epsilon_s^c) + \frac{u(c_{\text{stw}}(\epsilon_s^c)) - u(c^w)}{u'(c^w)} - (z+\Omega)\right) \\ & + \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s \cdot \frac{\lambda_{n^c}}{u'(c^w)} \\ & - \frac{\lambda_\theta}{u'(c^w)} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \left[s_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon_s^c)) + \frac{1-n}{n} \cdot b \right] \\ & + \frac{\lambda_c}{u'(c^w)} \cdot \frac{\partial WE}{\partial \epsilon_s^c} \Bigg] \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c) + \frac{\lambda^c_{\epsilon_s^c}}{u'(c^w)} \cdot \frac{\partial S^c_{\text{stw}}(\epsilon_s^c)}{\partial \epsilon_s^c} = 0 \end{aligned}$$

Insert λ_{n^c} and λ_n

$$\begin{aligned}
\iff -\frac{\partial \mathcal{L}}{\partial \epsilon_s^c} = \Bigg[& \frac{p}{\lambda} \cdot n^s g(\epsilon_s^c) \left(z_{\text{stw}}(\epsilon_s^c) - \Omega(\epsilon_s^c) + \frac{u(c_{\text{stw}}(\epsilon_s^c))}{u'(c^w)} - u(c^w) - (z+\Omega) \right) \\
& + \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s \cdot \Bigg((\lambda - q(\theta) \cdot \theta) \cdot \frac{\lambda_{n^s}}{u'(c^w)} \\
& + [z - \Omega + b + \theta \cdot c] - \frac{u(b)}{u'(c^w)} \\
& + \frac{1-\rho}{\lambda} \cdot \frac{b}{n^2} \cdot \lambda_\theta \cdot \frac{1}{u'(c^w)} + \eta \cdot \frac{b}{n^2} \cdot \frac{\lambda_c}{u'(c^w)} \Bigg) \Bigg] \\
& - \frac{\lambda_\theta}{u'(c^w)} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot \left[s_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon_s^c)) + \frac{1-n}{n} \cdot b \right] \\
& + \frac{\lambda_c}{u'(c^w)} \cdot \frac{\partial WE}{\partial \epsilon_s^c} \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c) + \frac{\lambda^c_{\epsilon_s^c}}{u'(c^w)} \cdot \frac{\partial S^c_{\text{stw}}(\epsilon_s^c)}{\partial \epsilon_s^c} = 0
\end{aligned}$$

Insert λ_{n^s}

$$\begin{aligned}
-\frac{\partial \mathcal{L}}{\partial \epsilon_s^c} = \Bigg[& z_{\text{stw}}(\epsilon_s^c) - \Omega(\epsilon_s^c) + \frac{u(c_{\text{stw}}(\epsilon_s^c))}{u'(c^w)} - c_{\text{stw}}(\epsilon_s^c) + b \\
& + \frac{\lambda - \gamma \cdot f + \chi \cdot (\lambda - f)}{1-\gamma} \cdot \frac{k_v}{q} \\
& - \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \left[s_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon_s^c)) - b \right] \\
& + \frac{\lambda_c}{n^s \cdot u'(c^w)} \left(\eta \cdot \frac{b}{n} + \frac{1}{g(\epsilon_s^c)} \cdot \frac{\lambda}{p} \cdot \frac{\partial WE}{\partial \epsilon_s^c} \right) \Bigg] \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \xi_s^c) \\
& + \frac{\lambda}{p} \cdot \frac{1}{n^s \cdot g(\epsilon_s^c)} \cdot \frac{\lambda^c_{\epsilon_s^c}}{u'(c^w)} \cdot \frac{\partial S^c_{\text{stw}}(\epsilon_s^c)}{\partial \epsilon_s^c} = 0
\end{aligned}$$

And finally:

$$\begin{aligned}
-\frac{\lambda_{\epsilon_s^c}}{n^s \cdot u'(c^w)} = \frac{p}{\lambda} \cdot \frac{g(\epsilon_s^c)}{\frac{\partial s^c_{\text{stw}}(\epsilon_s^c)}{\partial \epsilon_s^c}} \cdot \Bigg(& \left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \right) \cdot \left(\frac{\lambda}{f} \cdot b - s_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon_s^c)) \right) \\
& + \frac{\lambda_c}{n^s \cdot u'(c^w)} \left(\eta \cdot \frac{b}{n} + \frac{1}{g(\epsilon_s^c)} \cdot \frac{\lambda}{p} \cdot \frac{\partial WE}{\partial \epsilon_s^c} \right) \Bigg) \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c)
\end{aligned}$$

Analogously, it can be derived that

$$\begin{aligned}
-\frac{\lambda_{\xi_s^c}}{n^s \cdot u'(c^w)} = \frac{p}{\lambda} \cdot \frac{g(\xi_s^c)}{\frac{\partial s^c_{\text{stw}}(\xi_s^c)}{\partial \xi^c}} \cdot \Bigg(& \left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \right) \cdot \left(\frac{\lambda}{f} \cdot b \right) \\
& + \frac{\lambda \cdot \lambda_c}{n^s \cdot u'(c^w)} \left(\eta \cdot \frac{b}{n} + \frac{1}{g(\xi_s^c)} \cdot \frac{\lambda}{p} \cdot \frac{\partial WE}{\partial \xi^c} \right) \Bigg) \cdot \mathbb{1}(\epsilon_{\text{stw}} \leq \xi_s^c)
\end{aligned}$$

G.4 Optimal STW Benefits

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial s_{\text{stw}}} = &\frac{p}{\lambda}\cdot n^s \cdot \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}},\epsilon_s^c\}} \left(u'(c_{\text{stw}}(\epsilon)) - u'(c^w)\right)\left(\bar{h} - h_{\text{stw}}(\epsilon)\right)\, dG(\epsilon) \\ &+ \lambda_\theta \cdot \frac{p}{\lambda}\cdot \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}},\epsilon_s^c\}} \left(\frac{u'(c_{\text{stw}}(\epsilon)) - u'(c^w)}{u'(c^w)}\right)\left(\bar{h} - h_{\text{stw}}(\epsilon)\right)\, dG(\epsilon) \\ &- \frac{n^s\cdot(1-\rho)}{\lambda}\cdot u'(c^w)\cdot\frac{\partial\Omega}{\partial s_{\text{stw}}} - \lambda_\theta\cdot\frac{(1-\rho)}{\lambda}\cdot\frac{\partial\Omega}{\partial s_{\text{stw}}} - \lambda_c\cdot\frac{\partial WE}{\partial s_{\text{stw}}} \\ &- \lambda_{\epsilon_s^u}\cdot\frac{\partial S^u_{\text{stw}}(\epsilon_s^u)}{\partial s_{\text{stw}}} - \lambda_{\epsilon_s^c}\cdot\frac{\partial S^c_{\text{stw}}(\epsilon_s^c)}{\partial s_{\text{stw}}} = 0\end{aligned}$$

Insert Lagrange multiplier separation condition:

$$\begin{aligned}&\left(1+\frac{\lambda_\theta}{n^s\cdot u'(c^w)}\right)\cdot\left(\frac{p}{\lambda}\int_{\epsilon_s^c}^{\max\{\epsilon_s^c,\epsilon_{\text{stw}}\}}\frac{u'(c_{\text{stw}}(\epsilon)) - u'(c^w)}{u'(c^w)}\cdot(\bar{h}-h_{\text{stw}}(\epsilon))\,dG(\epsilon)\right. \\ &\qquad\left.-(1-\rho)\cdot\frac{\partial\Omega}{\partial s_{\text{stw}}}\right) - \frac{\lambda_c}{n^s\cdot u'(c^w)}\cdot\frac{\partial WE}{\partial s_{\text{stw}}} \\ &+\frac{(1-p)}{\lambda}\cdot g(\epsilon_s^u)\cdot\frac{\partial\epsilon_s^u}{\partial s_{\text{stw}}}\cdot\left(\left(1+\frac{\lambda_\theta}{n^s\cdot u'(c^w)}\right)\cdot\left(\frac{\lambda}{f}\cdot b - s_{\text{stw}}\cdot(\bar{h}-h_{\text{stw}}(\epsilon_s^u))\right)\right. \\ &\qquad\left.+\frac{\lambda_c}{n^s\cdot u'(c^w)}\cdot\left(\eta\cdot\frac{b}{n}+\frac{1}{g(\epsilon_s^u)}\cdot\frac{\lambda}{1-p}\cdot\frac{\partial WE}{\partial\epsilon_s^u}\right)\right) \\ &+\frac{p}{\lambda}\cdot g(\epsilon_s^c)\cdot\frac{\partial\epsilon_s^c}{\partial s_{\text{stw}}}\cdot\left(\left(1+\frac{\lambda_\theta}{n^s\cdot u'(c^w)}\right)\cdot\left(\frac{\lambda}{f}\cdot b - s_{\text{stw}}\cdot(\bar{h}-h_{\text{stw}}(\epsilon_s^c))\right)\right. \\ &\qquad\left.+\frac{\lambda_c}{n^s\cdot u'(c^w)}\cdot\left(\eta\cdot\frac{b}{n}+\frac{1}{g(\epsilon_s^c)}\cdot\frac{\lambda}{p}\cdot\frac{\partial WE}{\partial\epsilon_s^c}\right)\right) = 0\end{aligned}$$

Finally:

$$\begin{aligned}\bar{s}_{\text{stw}} = &\underbrace{\frac{\lambda}{f}b}_{\text{Fiscal Ext.}} \\ &\underbrace{+\frac{\mathbb{1}\{\epsilon_{\text{stw}}\geq\epsilon_s^c\}}{MMS}\cdot n^s\cdot\frac{p}{\lambda}\int_{\epsilon_s^c}^{\epsilon_{\text{stw}}}\left(\frac{u'(c_{\text{stw}}(\epsilon)) - u'(c^w))}{u'(c^w)}\right)(\bar{h}-h_{\text{stw}}(\epsilon))\,dG(\epsilon)}_{\text{Insurance}} \\ &\underbrace{-\frac{n^s}{MMS}\cdot\frac{1}{\lambda}\cdot(1-\rho)\frac{\partial\,\Omega}{\partial\,s_{\text{stw}}}}_{\text{Distortion}} + \underbrace{BE_{\text{stw},3}}_{\text{Bargaining Effect}}\end{aligned}$$

$$\bar{s}_{\text{stw}} = \frac{MMS^u}{MMS}s_{\text{stw}}(\bar{h}-h_{\text{stw}}(\epsilon_s^u)) + \frac{MMS^c}{MMS}s_{\text{stw}}(\bar{h}-h_{\text{stw}}(\epsilon_s^c))$$

where

$$MMS^u = \frac{(1-p)}{\lambda} \cdot g(\epsilon_s^u) \cdot \frac{\partial \epsilon_s^u}{\partial s_{\text{stw}}} \cdot n^s, \quad MMS^c = \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c) \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot \frac{\partial \epsilon_s^c}{\partial s_{\text{stw}}} \cdot n^s,$$

$$MMS = MMS^c + MMS^u$$

and

$$\begin{aligned} BE_{\text{stw},3} = & \frac{1}{\left(1 + \frac{\lambda_\theta}{n^s\, u'(c^w)}\right)} \\ & \times \left[- \frac{\lambda_c}{\varphi\, n^s\, u'(c^w)} \frac{\partial WE}{\partial s_{\text{stw}}} + \frac{\varphi^u}{\varphi} \frac{\lambda_c}{n^s\, u'(c^w)} \left(\eta \frac{b}{n} + \frac{1}{g(\epsilon_s^u)} \frac{\lambda}{1-p} \frac{\partial WE}{\partial \epsilon_s^u} \right) \right. \\ & \left. + \frac{\varphi^c}{\varphi} \frac{\lambda_c}{n^s\, u'(c^w)} \left(\eta \frac{b}{n} + \frac{1}{g(\epsilon_s^c)} \frac{\lambda}{p} \frac{\partial WE}{\partial \epsilon_s^c} \right) \right] \end{aligned}$$

G.5 Optimal Eligibility Condition

Assumption throughout: $\epsilon^p \geq \epsilon_{\text{stw}} \geq \epsilon_s^u$

Further assume that:

$$n^s \cdot u'(c^w) + \lambda_\theta - \eta \cdot \lambda_c > 0, \qquad n^s \cdot u'(c^w) + \lambda_\theta > 0$$

These conditions exclude the case that the Planner would want to use STW's hours distortions to destroy production and thus reduce vacancy posting. The Planner might want to do that when the Hosios condition is not fulfilled. For reasonable parameter values, however, the condition should hold anyway. Recall that by Lemma 1 $\epsilon_s^c > \epsilon_s^u$ and $\xi_s^c > \xi_s^u$.

Case I:

FOC for the eligibility threshold:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \epsilon_{stw}} &= -n^s \cdot u'(c^w) \cdot \frac{1-p}{\lambda} \cdot (1-\rho^u) \cdot \frac{\partial \Omega_u}{\partial \epsilon_{stw}} u'(c^w) \\ &\quad - (\lambda_\theta - \eta \cdot \lambda_c) \cdot \frac{1-p}{\lambda} \cdot (1-\rho^u) \cdot \frac{\partial \Omega_u}{\partial \epsilon_{stw}} \\ &= - (n^s \cdot u'(c^w) + \lambda_\theta - \eta \cdot \lambda_c) \cdot \frac{1-p}{\lambda} \cdot (1-\rho^u) \cdot \frac{\partial \Omega_u}{\partial \epsilon_{stw}} \end{aligned}$$

Under reasonable parameter values, we get:

$$\frac{\partial \mathcal{L}}{\partial \epsilon_{stw}} < 0 \quad \text{thus it is optimal to set} \quad \epsilon_{stw} = \epsilon_s^u.$$

Case II:

FOC for the eligibility threshold:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \epsilon_{stw}} = & -n^u \cdot \frac{\partial \Omega_u}{\partial \epsilon_{stw}} \cdot u'(c^w) \\
& + \frac{n}{1-\rho} \cdot p \cdot g(\epsilon_{stw}) \cdot u'(c^w) \\
& \quad \cdot \left(z_{\text{stw}}(\epsilon_{stw}) + \frac{u(c_{\text{stw}}(\epsilon_{stw}))}{u'(c^w)} - c_{\text{stw}}(\epsilon_{stw}) - (z+\Omega) \right) \\
& + \lambda_\theta \cdot \frac{(1-p)}{\lambda} \cdot (1-\rho^u) \cdot \frac{\partial \Omega_u}{\partial \epsilon_{stw}} \\
& + \frac{p}{\lambda} \cdot g(\epsilon_{stw}) \\
& \quad \cdot \Bigg[z(\epsilon_{\text{stw}}) + \frac{u(c_{stw}(\epsilon_{\text{stw}})) - u(b)}{u'(c^w)} \\
& \qquad - c_{stw}(\epsilon_{stw}) - \frac{1-n}{n} \cdot b + \frac{\lambda - \eta \cdot f}{1-\eta} \cdot \frac{k_v}{q} \Bigg] \\
& - \lambda_c \cdot \frac{\partial WE}{\partial \epsilon_{\text{stw}}} + \frac{p}{\lambda} \cdot g(\epsilon_{\text{stw}}) \cdot n^s \cdot \lambda_{n^c} \overset{!}{>} 0.
\end{aligned}$$

Insert n^u

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \epsilon_{\text{stw}}} = & -n^s \cdot (1-\rho^u) \cdot \left(\frac{1-p}{\lambda} \right) \cdot \frac{\partial \Omega_u}{\partial \epsilon_{\text{stw}}} \cdot u'(c^w) \\
& + n^s \cdot \frac{p}{\lambda} \cdot g(\epsilon_{\text{stw}}) \cdot u'(c^w) \cdot \left(z_{\text{stw}}(\epsilon_{\text{stw}}) + \frac{u(c_{\text{stw}}(\epsilon_{\text{stw}}))}{u'(c^w)} \right) \\
& - n^s \cdot \frac{p}{\lambda} \cdot g(\epsilon_{\text{stw}}) \cdot u'(c^w) \cdot (c_{\text{stw}}(\epsilon_{\text{stw}}) + z + \Omega) \\
& + \lambda_\theta \cdot \frac{g(\epsilon_{\text{stw}})}{\lambda} \cdot (1-p)(1-\rho^u) \cdot \frac{\partial \Omega_u}{\partial \epsilon_{\text{stw}}} \\
& + \frac{p}{\lambda} \cdot g(\epsilon_{\text{stw}}) \cdot \Bigg(z_{\text{stw}}(\epsilon_{\text{stw}}) + \frac{u(c_{\text{stw}}(\epsilon_{\text{stw}})) - u(b)}{u'(c^w)} - c_{\text{stw}}(\epsilon_{\text{stw}}) - \frac{1-n}{n} \cdot b \\
& \qquad + \frac{\lambda - \eta \cdot f}{1-\eta} \cdot \left(\frac{k_v}{q} \right) \Bigg) \\
& - \lambda_c \cdot \frac{\partial WE}{\partial \epsilon_{\text{stw}}} + \frac{p}{\lambda} \cdot g(\epsilon_{\text{stw}}) \cdot n^s \cdot \lambda_{n^c} \overset{!}{>} 0
\end{aligned}$$

$$
\begin{aligned}
\Leftrightarrow\ & -\left(n^s \cdot u'(c^w) + \lambda_\theta\right) \cdot (1-\rho^u) \cdot \left(\frac{1-p}{\lambda}\right) \cdot \frac{\partial \Omega_u}{\partial \epsilon_{\text{stw}}} \\
& + n^s \cdot \frac{p}{\lambda} \cdot g(\epsilon_{\text{stw}}) \cdot u'(c^w) \\
& \cdot \left(z_{\text{stw}}(\epsilon_{\text{stw}}) + \frac{u(c_{\text{stw}}(\epsilon_{\text{stw}}))}{u'(c^w)} - c_{\text{stw}}(\epsilon_{\text{stw}}) - (z+\Omega) + \frac{\lambda_{n^c}}{u'(c^w)} \right) \\
& + \frac{p}{\lambda} \cdot g(\epsilon_{\text{stw}}) \cdot \left[z_{\text{stw}}(\epsilon_{\text{stw}}) + \frac{u(c_{\text{stw}}(\epsilon_{\text{stw}})) - u(b)}{u'(c^w)} - c_{\text{stw}}(\epsilon_{\text{stw}}) \right. \\
& \qquad \left. - \frac{1-n}{n} \cdot b + \frac{\lambda - \eta \cdot f}{1-\eta} \cdot \left(\frac{k_v}{q}\right) \right] \\
& - \lambda_c \cdot \frac{\partial WE}{\partial \epsilon_{\text{stw}}} \overset{!}{>} 0
\end{aligned}
$$

Insert λ_{n^c} and subtract decentralized separation condition for constrained firms:

$$
\begin{aligned}
\Leftrightarrow\quad & -\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \frac{(1-p)}{\lambda} \cdot (1-\rho^u) \cdot \frac{\partial \Omega_u}{\partial \epsilon_{\text{stw}}} \cdot \frac{1}{g(\epsilon_{\text{stw}})} \\
& + \frac{p}{\lambda} \cdot \left[b + z_{\text{stw}}(\epsilon_{stw}) + \frac{u(c_{\text{stw}}(\epsilon_{stw})) - u(b)}{u'(c^w)} - c_{\text{stw}}(\epsilon_{stw}) \right. \\
& \left. + \frac{\lambda - \eta \cdot f + \chi \cdot (\lambda - f)}{1-\eta} \cdot \left(\frac{k_v}{q}\right) \right] \\
& - \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \cdot \frac{p}{\lambda} \cdot \left[s_{\text{stw}} \cdot (h - h_{\text{stw}}(\epsilon_{stw})) - b \right] \\
& - \frac{\lambda_c}{n^s \cdot u'(c^w)} \cdot \left(\frac{\partial WE}{\partial \epsilon_{\text{stw}}} \cdot \frac{1}{g(\epsilon_{\text{stw}})} \cdot \frac{\partial S_{\text{stw}}(\epsilon_{\text{stw}})}{\partial \epsilon_{\text{stw}}} + \frac{p}{\lambda} \cdot \eta \cdot \frac{b}{n} \right) \overset{!}{>} 0
\end{aligned}
$$

Subtract decentralized separation condition:

$$
\begin{aligned}
\Leftrightarrow\quad & -\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \frac{(1-p)}{\lambda} \cdot (1-\rho^u) \cdot \frac{\partial \Omega_u}{\partial \epsilon_{\text{stw}}} \cdot \frac{1}{g(\epsilon_{\text{stw}})} \\
& + \frac{p}{\lambda} \cdot \left[z_{\text{stw}}(\epsilon_{\text{stw}}) - z_{\text{stw}}(\epsilon_s) + \frac{u(c_{\text{stw}}(\epsilon_{\text{stw}})) - u(c_{\text{stw}}(\epsilon_s^c))}{u'(c^w)} \right. \\
& \quad \left. - s_{\text{stw}} \cdot (h - h_{\text{stw}}(\epsilon_s^c)) + b + (1-\rho) \cdot (\lambda - f) \cdot \left(\chi - \frac{\eta - \gamma}{1-\eta}\right) \cdot \frac{1}{1-\gamma} \cdot \frac{k_v}{q} \right] \\
& + \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \cdot \frac{p}{\lambda} \cdot \left[z_{\text{stw}}(\epsilon_{\text{stw}}) - z_{\text{stw}}(\epsilon_s^c) + \frac{u(c_{\text{stw}}(\epsilon_{\text{stw}})) - u(c_{stw}(\epsilon_s^c))}{u'(c^w)} \right. \\
& \left. - s_{\text{stw}} \cdot (h - h_{\text{stw}}(\epsilon_s^c)) + b \right] \\
& - \frac{\lambda_c}{n^s \cdot u'(c^w)} \cdot \left(\frac{\partial WE}{\partial \epsilon_{\text{stw}}} \cdot \frac{1}{g(\epsilon_{\text{stw}})} \cdot \frac{\partial S_{\text{stw}}(\epsilon_{\text{stw}})}{\partial \epsilon_{\text{stw}}} + \frac{p}{\lambda} \cdot \eta \cdot \frac{b}{n} \right) \overset{!}{>} 0
\end{aligned}
$$

Inserting for $(\chi - \frac{\eta-\gamma}{1-\eta}) \cdot \frac{1}{1-\gamma} \cdot \frac{k_v}{q}$ gives us the final condition. We can distinguish between three cases:

(A)

$$
\begin{aligned}
& g(\epsilon_{\text{stw}}) \cdot \frac{p}{\lambda} \cdot n^s \cdot \Bigg[z_{\text{stw}}(\epsilon_{\text{stw}}) - z_{\text{stw}}(\epsilon_s^c) \\
& \quad + \frac{u(c_{\text{stw}}(\epsilon_{\text{stw}})) - u(c_{\text{stw}}(\epsilon_s^c))}{u'(c^w)} + \frac{\lambda}{f} \cdot b - s_{\text{stw}} \cdot \big(h - h_{\text{stw}}(\epsilon_s^c)\big) \Bigg] \\
& > n^s \cdot \frac{1-p}{\lambda} \cdot (1-\rho) \cdot \frac{\partial \Omega_u}{\partial \epsilon_{\text{stw}}} + BE \\
& \forall\, \epsilon_{\text{stw}} \text{ in case 2} \quad \Rightarrow \quad \epsilon_{\text{stw}} = \xi_s^c
\end{aligned}
$$

(B)

$$
\begin{aligned}
& g(\epsilon_{\text{stw}}) \cdot \frac{p}{\lambda} \cdot n^s \cdot \Bigg[z_{\text{stw}}(\epsilon_{\text{stw}}) - z_{\text{stw}}(\epsilon_s^c) \\
& \quad + \frac{u(c_{\text{stw}}(\epsilon_{\text{stw}})) - u(c_{\text{stw}}(\epsilon_s^c))}{u'(c^w)} + \frac{\lambda}{f} \cdot b - s_{\text{stw}} \cdot \big(h - h_{\text{stw}}(\epsilon_s^c)\big) \Bigg] \\
& < n^s \cdot \frac{1-p}{\lambda} \cdot (1-\rho) \cdot \frac{\partial \Omega_u}{\partial \epsilon_{\text{stw}}} + BE \\
& \forall\, \epsilon_{\text{stw}} \text{ in case 2} \quad \Rightarrow \quad \epsilon_{\text{stw}} = \max\{\xi_s^c, \xi_s^u\}
\end{aligned}
$$

(C)

$$
\begin{aligned}
& g(\epsilon_{\text{stw}}) \cdot \frac{p}{\lambda} \cdot n^s \cdot \Bigg[z_{\text{stw}}(\epsilon_{\text{stw}}) - z_{\text{stw}}(\epsilon_s^c) \\
& \quad + \frac{u(c_{\text{stw}}(\epsilon_{\text{stw}})) - u(c_{\text{stw}}(\epsilon_s^c))}{u'(c^w)} + \frac{\lambda}{f} \cdot b - s_{\text{stw}} \cdot \big(h - h_{\text{stw}}(\epsilon_s^c)\big) \Bigg] \\
& = n^s \cdot \frac{1-p}{\lambda} \cdot (1-\rho) \cdot \frac{\partial \Omega_u}{\partial \epsilon_{\text{stw}}} + BE
\end{aligned}
$$

BE is defined as:

$$
\begin{aligned}
BE = g(\epsilon_{\text{stw}}) \cdot \frac{\lambda_c}{u'(c^w)} \cdot \left(\frac{\partial WE}{\partial \epsilon_{\text{stw}}} \cdot \frac{1}{g(\epsilon_{\text{stw}})} \cdot \frac{\partial S_{\text{stw}}(\epsilon_{\text{stw}})}{\partial \epsilon_{\text{stw}}} + \eta \cdot \frac{b}{n} \right) \\
\Bigg/ \left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \right)
\end{aligned}
$$

Case III:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \epsilon_{\text{stw}}} = & \, g(\epsilon_{\text{stw}}) \cdot \frac{p}{\lambda} \cdot n^s \cdot \Big(u(c_{\text{stw}}(\epsilon_{\text{stw}})) - u(c(\epsilon_{\text{stw}})) \\ & - u'(c^w) \cdot [c_{\text{stw}}(\epsilon_{\text{stw}}) - c(\epsilon_{\text{stw}})] \Big) \\ & - n \cdot \frac{\partial \Omega}{\partial \epsilon_{\text{stw}}} \\ & + \lambda_\theta \cdot \frac{p}{\lambda} \cdot \Big(u(c_{\text{stw}}(\epsilon_{\text{stw}})) - u(c(\epsilon_{\text{stw}})) \\ & - u'(c^w) \cdot [c_{\text{stw}}(\epsilon_{\text{stw}}) - c(\epsilon_{\text{stw}})] \Big) \\ & - \lambda_\theta \cdot \frac{1-\rho}{\lambda} \cdot \frac{\partial \Omega}{\partial \epsilon_{\text{stw}}} \\ & + \lambda_c \cdot \frac{\partial WE}{\partial \epsilon_{\text{stw}}} \overset{!}{\geq} 0\end{aligned}$$

$$\begin{aligned}\Leftrightarrow \quad & \left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \left(\frac{1-\rho}{\lambda} \cdot n^s \cdot \frac{\partial \Omega}{\partial \epsilon_{\text{stw}}} + BE\right) \\ & \overset{!}{\geq} \left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \frac{p}{\lambda} \cdot g(\epsilon_{\text{stw}}) \cdot n^s \\ & \cdot \left(\frac{\dfrac{u(c_{\text{stw}}(\epsilon_{\text{stw}})) - u(c(\epsilon_{\text{stw}}))}{c_{\text{stw}}(\epsilon_{\text{stw}}) - c(\epsilon_{\text{stw}})} - u'(c^w)}{u'(c^w)} \right) \\ & \cdot [c_{\text{stw}}(\epsilon_{\text{stw}}) - c(\epsilon_{\text{stw}})]\end{aligned}$$

$$\begin{aligned}\Leftrightarrow \quad & \left(\frac{1-\rho}{\lambda} \cdot n^s \cdot \frac{\partial \Omega}{\partial \epsilon_{\text{stw}}} + BE\right) \overset{!}{\geq} \cdot \frac{p}{\lambda} \cdot g(\epsilon_{\text{stw}}) \cdot n^s \\ & \cdot \left(\frac{\dfrac{u(c_{\text{stw}}(\epsilon_{\text{stw}})) - u(c(\epsilon_{\text{stw}}))}{c_{\text{stw}}(\epsilon_{\text{stw}}) - c(\epsilon_{\text{stw}})} - u'(c^w)}{u'(c^w)} \right) \\ & \cdot [c_{\text{stw}}(\epsilon_{\text{stw}}) - c(\epsilon_{\text{stw}})]\end{aligned}$$

If the equation holds with strict inequality, the cost of hours distortion exceeds the benefit of providing additional insurance to workers in constrained firms. Consequently, the Ramsey planner chooses not to allow firms and workers to enter short-time work (STW) when they could survive without reaching the threshold $\epsilon_{\text{stw}} = \xi_s^c$. Conversely, if the equation holds with exact equality, the participation threshold is determined by balancing the additional cost of hours distortions against the benefit of providing extra insurance. This concludes the proof of Proposition 4.

G.6 Optimal Unemployment Insurance with STW

From the FOC of UI, we can derive the optimality condition of unemployment insurance:

$$\begin{aligned}(1-n)\cdot(u'(b)-u'(c^w)) = \lambda_\theta(1-\rho)\cdot&\left(\frac{1-n}{n}+\frac{u'(b)}{u'(c^w)}\right)\\ &+\left(\lambda_{\epsilon_s^u}+\lambda_{\epsilon_s^c}+\lambda_{\epsilon^g}\right)\cdot\left(-\frac{u'(b)}{u'(c^w)}\right)\\ &+\lambda_c(1-\rho)\cdot\left(\eta\cdot\frac{1-n}{n}+(1-\eta)\cdot\frac{u'(b)}{u'(c^w)}\right)\end{aligned}$$

To get better insight, we need to determine the Lagrange multipliers for λ_θ, λ_c. We already know the Lagrange multipliers for the separation conditions:

$$\begin{aligned}-\frac{\lambda_{\epsilon_s^u}}{n^s\cdot u'(c^w)} = \frac{1-p}{\lambda}\cdot&\frac{g(\epsilon_s^u)}{\frac{\partial S^u_{\text{stw}}(\epsilon_s^u)}{\partial \epsilon_s^u}}\\ &\cdot\left(\left(1+\frac{\lambda_\theta}{n^s\cdot u'(c^w)}\right)\cdot\left(\frac{\lambda}{f}\cdot b - s_{stw}\cdot\left(\bar{h}-h_{\text{stw}}(\epsilon_s^u)\right)\right)\right.\\ &\left.+\frac{\lambda_c}{n^s\cdot u'(c^w)}\cdot\left(\eta\cdot\frac{b}{n}+\frac{1}{g(\epsilon_s^u)}\cdot\frac{\lambda}{1-p}\cdot\frac{\partial WE}{\partial\epsilon_s^u}\right)\right)\end{aligned}$$

$$\begin{aligned}-\frac{\lambda_{\xi_s^c}}{n^s\cdot u'(c^w)} = \frac{p}{\lambda}\cdot&\frac{g(\xi_s^c)}{\frac{\partial S^c_{\text{stw}}(\xi_s^c)}{\partial \xi_s^c}}\cdot\left(\left(1+\frac{\lambda_\theta}{n^s\cdot u'(c^w)}\right)\cdot\frac{\lambda}{f}\cdot b\right.\\ &\left.+\frac{\lambda_c}{n^s\cdot u'(c^w)}\left(\eta\cdot\frac{b}{n}+\frac{1}{g(\xi_s^c)}\cdot\frac{\lambda}{p}\cdot\frac{\partial WE}{\partial\xi_s^c}\right)\right)\cdot\mathbb{1}(\epsilon_{\text{stw}}\le\xi_s^c)\end{aligned}$$

$$\begin{aligned}-\frac{\lambda_{\epsilon_s^c}}{n^s\cdot u'(c^w)} = \frac{p}{\lambda}\cdot&\frac{g(\epsilon_s^c)}{\frac{\partial S^c_{\text{stw}}(\epsilon_s^c)}{\partial \epsilon_s^c}}\cdot\left(\left(1+\frac{\lambda_\theta}{n^s\cdot u'(c^w)}\right)\cdot\left(\frac{\lambda}{f}\cdot b - s_{stw}\cdot\left(\bar{h}-h_{\text{stw}}(\epsilon_s^c)\right)\right)\right.\\ &\left.+\frac{\lambda_c}{n^s\cdot u'(c^w)}\cdot\left(\eta\cdot\frac{b}{n}+\frac{1}{g(\epsilon_s^c)}\cdot\frac{\lambda}{p}\cdot\frac{\partial WE}{\partial\epsilon_s^c}\right)\right)\cdot\mathbb{1}(\epsilon_{\text{stw}}\ge\epsilon_s^c)\end{aligned}$$

First, let us find an expression for λ_c:

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c^w} = -\Bigg(& (1-p)\cdot(1-\rho^u)\cdot\frac{u(c^w)-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)} \\
& + p\cdot(1-\rho^c)\cdot\frac{u^c-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\Bigg)\cdot\lambda_\theta \\
& - (1-\eta)\cdot(1-p)\cdot(1-\rho^u)\cdot\left(1-(1-\eta)\cdot\frac{u(c^w)-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\right) \\
& - (1-p)\cdot(1-\cdot\rho^c)\cdot(1-\eta)\cdot\frac{u^c-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\cdot\lambda_c \\
& + \frac{u(c^w)-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\cdot\lambda_{\epsilon_s^u} \\
& + \frac{u(c(\xi_s^c))-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\cdot\lambda_{\xi_s^c} \\
& + \frac{u(c_{\text{stw}}(\epsilon_s^c))-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\cdot\lambda_{\epsilon_s^c} = 0
\end{aligned}
$$

Insert Lagrange multipliers for separation conditions

$$
\begin{aligned}
& -\left(p(1-\rho^u)\cdot\frac{u(c^w)-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}+(1-p)(1-\rho^c)\cdot\frac{u^c-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\right)\lambda_\theta \\
& -(1-\eta)\cdot\Bigg((1-p)\cdot(1-\rho^u)\cdot\left(1-(1-\eta)\cdot\frac{u(c^w)-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\right) \\
& -p\cdot(1-\rho^c)\cdot(1-\eta)\cdot\frac{u^c-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\Bigg)\lambda_c \\
& +\frac{\partial\epsilon_s^u}{\partial c^w}\cdot\frac{1-p}{\lambda}\cdot g(\epsilon_s^u)\left(n^s\cdot u'(c^w)+\lambda_\theta\right)\cdot\left(\frac{\lambda}{f}\cdot b - s_{\text{stw}}\cdot(\bar{h}-h_{stw}(\epsilon_s^u))\right) \\
& +\lambda_c\cdot\left(\eta\cdot\frac{b}{n}+\frac{1}{g(\epsilon_s^u)}\cdot\frac{\lambda}{1-p}\cdot\frac{\partial WE}{\partial\epsilon_s^u}\right) \\
& +\frac{\partial\xi_s^c}{\partial c^w}\cdot\frac{p}{\lambda}\cdot g(\xi_s^c)\left(n^s\cdot u'(c^w)+\lambda_\theta\right)\cdot\left(\frac{\lambda}{f}\cdot b+\lambda_c\cdot\left(\eta\cdot\frac{b}{n}-\frac{1}{g(\xi_s^c)}\cdot\frac{\lambda}{p}\cdot\frac{\partial WE}{\partial\xi_s^c}\right)\right) \\
& \cdot\mathbb{1}(\epsilon_{stw}\leq\xi_s^c) \\
& +\frac{\partial\xi_s^c}{\partial c^w}\cdot\frac{p}{\lambda}\cdot g(\xi_s^c)\left(n^s\cdot u'(c^w)+\lambda_\theta\right)\cdot\left(\frac{\lambda}{f}\cdot b - s_{\text{stw}}\cdot(\bar{h}-h_{stw}(\xi_s^c))\right) \\
& +\lambda_c\cdot\left(\eta\cdot\frac{b}{n}+\frac{1}{g(\xi_s^c)}\cdot\frac{\lambda}{p}\cdot\frac{\partial WE}{\partial\xi_s^c}\right)\cdot\mathbb{1}(\epsilon_{stw}\geq\epsilon_s^c)=0
\end{aligned}
$$

Rearranging yields

$$
\begin{aligned}
0 = \lambda_\theta \cdot \frac{\partial S^{\text{total}}}{\partial c^w} + \lambda_c \cdot \frac{\partial WE^{\text{total}}}{\partial c^w} \\
+ \frac{\partial \epsilon_s^u}{\partial c^w} \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot n^s \cdot u'(c^w) \cdot \left(\frac{\lambda}{f} \cdot b - s_{stw} \cdot (\bar{h} - h_{\text{stw}}(\epsilon_s^u)) \right) \\
+ \frac{\partial \xi_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\xi_s^c) \cdot n^s \cdot u'(c^w) \cdot \frac{\lambda}{f} \cdot b \cdot \mathbb{1}(\epsilon_{\text{stw}} \leq \xi_s^c) \\
+ \frac{\partial \epsilon_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s \cdot u'(c^w) \cdot \left(\frac{\lambda}{f} \cdot b - s_{stw} \cdot (\bar{h} - h_{\text{stw}}(\epsilon_s^c)) \right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c)
\end{aligned}
$$

with

$$
\begin{aligned}
\frac{\partial S^{\text{total}}}{\partial c^w} &= (1-p) \cdot (1-\rho^u) \cdot \frac{u(c^w) - u(b)}{u'(c^w)} \cdot \frac{u''(c^w)}{u'(c^w)} \\
&\quad - p \cdot (1-\rho^c) \cdot \frac{u^c - u(b)}{u'(c^w)} \cdot \frac{u''(c^w)}{u'(c^w)} \\
&\quad + \frac{\partial \epsilon_s^u}{\partial c^w} \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot \left(\frac{\lambda}{f} b - s_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon_s^u)) \right) \\
&\quad + \frac{\partial \xi_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\xi_s^c) \cdot \frac{\lambda}{f} \cdot b \cdot \mathbb{1}(\epsilon_{\text{stw}} \leq \xi_s^c) \\
&\quad + \frac{\partial \epsilon_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot \left(\frac{\lambda}{f} b - s_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon_s^c)) \right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c)
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial WE^{\text{total}}}{\partial c^w} &= -(1-\eta) \cdot (1-p) \cdot (1-\rho^u) \cdot \left(1 - (1-\eta) \cdot \frac{u(c^w) - u(b)}{u'(c^w)} \cdot \frac{u''(c^w)}{u'(c^w)} \right) \\
&\quad + p \cdot (1-\rho^c) \cdot (1-\eta) \cdot \frac{u^c - u(b)}{u'(c^w)} \cdot \frac{u''(c^w)}{u'(c^w)} \\
&\quad + \frac{\partial \epsilon_s^u}{\partial c^w} \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot \lambda_c \cdot \left(\eta \cdot \frac{b}{n} + \frac{1}{g(\epsilon_s^u)} \cdot \frac{\lambda}{1-p} \cdot \frac{\partial WE}{\partial \epsilon_s^u} \right) \\
&\quad + \frac{\partial \xi_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\xi_s^c) \cdot \lambda_c \cdot \left(\eta \cdot \frac{b}{n} - \frac{1}{g(\xi_s^c)} \cdot \frac{\lambda}{p} \cdot \frac{\partial WE}{\partial \epsilon_s^c} \right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \leq \xi_s^c) \\
&\quad + \frac{\partial \epsilon_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot \lambda_c \cdot \left(\eta \cdot \frac{b}{n} - \frac{1}{g(\epsilon_s^c)} \cdot \frac{1}{p} \cdot \frac{\partial WE}{\partial \epsilon_s^c} \right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c)
\end{aligned}
$$

Collecting terms:

$$\begin{aligned}\lambda_c = \frac{-1}{\left(\frac{\partial WE}{\partial c^w}\right)^{\text{total}}} \Bigg[& \frac{\partial \epsilon_s^u}{\partial c^w} \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot n^s \cdot u'(c^w) \left(\frac{\lambda}{f} \cdot b - s_{\text{stw}} \cdot \left(\bar{h} - h_{\text{stw}}(\epsilon_s^u) \right) \right) \\ & + \frac{\partial \xi_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\xi_s^c) \cdot n^s \cdot u'(c^w) \left(\frac{\lambda}{f} \cdot b \right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \leq \xi_s^c) \\ & + \frac{\partial \epsilon_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s \cdot u'(c^w) \left(\frac{\lambda}{f} \cdot b - s_{\text{stw}} \cdot \left(\bar{h} - h_{\text{stw}}(\epsilon_s^c) \right) \right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c) \\ & + \left(-\frac{\partial S_{\text{total}}}{\partial c^w} \right) \cdot \lambda_\theta \Bigg]\end{aligned}$$

Insert λ_c into the Lagrange multipliers for the separation conditions. To do that, let us rewrite the Lagrange multipliers of the separation conditions:

$$\begin{aligned}\lambda_{\epsilon_s^u} = & -\frac{1}{\frac{\partial S_{\text{stw}}^u(\epsilon_s^u)}{\partial \epsilon_s^u}} \Bigg(n^s \cdot u'(c^w) \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^u) \left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \right) \\ & \cdot \left(\frac{\lambda}{f} \cdot b - s_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon_s^u)) \right) \\ & + \lambda_c \left(\frac{\partial n}{\partial \epsilon_s^u} \cdot \left(-\frac{\partial WE}{\partial n} \right) + \frac{\partial WE}{\partial \epsilon_s^u} \right) \Bigg)\end{aligned}$$

$$\begin{aligned}\lambda_{\xi_s^c} = & \frac{\mathbb{1}(\epsilon_{\text{stw}} \leq \xi_s^c)}{\frac{\partial S_{\text{stw}}^c(\epsilon_s^c)}{\partial \epsilon_s^c}} \Bigg(n^s \cdot u'(c^w) \cdot \frac{p}{\lambda} \cdot g(\xi_s^c) \left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \right) \cdot \frac{\lambda}{f} \cdot b \\ & + \lambda_c \left(\frac{\partial n}{\partial \xi_s^c} \cdot \left(-\frac{\partial WE}{\partial n} \right) + \frac{\partial WE}{\partial \xi_s^c} \right)\end{aligned}$$

$$\begin{aligned}\lambda_{\epsilon_s^c} = & -\frac{\mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c)}{\frac{\partial S_{\text{stw}}^c(\epsilon_s^c)}{\partial \epsilon_s^c}} \Bigg(n^s \cdot u'(c^w) \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \right) \\ & \cdot \left(\frac{\lambda}{f} \cdot b - s_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon_s^c)) \right) \\ & + \lambda_c \left(\frac{\partial n}{\partial \epsilon_s^c} \cdot \left(-\frac{\partial WE}{\partial n} \right) + \frac{\partial WE}{\partial \epsilon_s^c} \right) \Bigg)\end{aligned}$$

Inserting λ_c gives:

$$
\begin{aligned}
\lambda_{\epsilon_s^u} = \\
&- \frac{1}{\frac{\partial S^u_{\text{stw}}(\epsilon^u_s)}{\partial \epsilon^u_s}} \Bigg[n^s u'(c^w) \cdot \frac{1-p}{\lambda} \cdot g(\epsilon^u_s) \left(1 + \frac{\lambda_\theta}{n^s u'(c^w)}\right) \left(\frac{\lambda}{f} b - s_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon^u_s))\right) \\
&+ \left(\frac{\partial n}{\partial \epsilon^u_s} \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \epsilon^u_s}\right)^{\text{total}}\right) \\
&\cdot \left(\frac{\partial \epsilon^u_s}{\partial c^w} \cdot \frac{1-p}{\lambda} \cdot g(\epsilon^u_s) \cdot n^s u'(c^w) \left(\frac{\lambda}{f} b - s_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon^u_s))\right)\right) \\
&+ \frac{\partial \xi^c_s}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\xi^c_s) \cdot n^s u'(c^w) \cdot \frac{\lambda}{f} b \cdot \mathbb{1}(\epsilon_{\text{stw}} \leq \xi^c_s) \\
&+ \frac{\partial \epsilon^c_s}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\epsilon^c_s) \cdot n^s u'(c^w) \left(\frac{\lambda}{f} b - s_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon^c_s))\right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon^c_s) \\
&+ \left(-\frac{\partial S^{\text{total}}}{\partial c^w}\right) \cdot \lambda_\theta \Bigg) \Bigg]
\end{aligned}
$$

$$
\begin{aligned}
\lambda_{\xi_s^c} = \\
&\frac{\mathbb{1}(\epsilon_{\text{stw}} \leq \xi^c_s)}{\frac{\partial S^c_{\text{stw}}(\xi^c_s)}{\partial \xi^c_s}} \Bigg[- n^s u'(c^w) \cdot \frac{p}{\lambda} \cdot g(\xi^c_s) \left(1 + \frac{\lambda_\theta}{n^s u'(c^w)}\right) \left(\frac{\lambda}{f} b - s_{\text{stw}}(\bar{h} - h_{\text{stw}}(\xi^c_s))\right) \\
&+ \left(\frac{\partial n}{\partial \xi^c_s} \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \xi^c_s}\right)^{\text{total}}\right) \\
&\cdot \left(\frac{\partial \epsilon^u_s}{\partial c^w} \cdot \frac{1-p}{\lambda} \cdot g(\epsilon^u_s) \cdot n^s u'(c^w) \left(\frac{\lambda}{f} b - s_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon^u_s))\right)\right) \\
&+ \frac{\partial \xi^c_s}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\xi^c_s) \cdot n^s u'(c^w) \cdot \frac{\lambda}{f} b \cdot \mathbb{1}(\epsilon_{\text{stw}} \leq \xi^c_s) \\
&+ \frac{\partial \epsilon^c_s}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\epsilon^c_s) \cdot n^s u'(c^w) \left(\frac{\lambda}{f} b - s_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon^c_s))\right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon^c_s) \\
&+ \left(-\frac{\partial S^{\text{total}}}{\partial c^w}\right) \cdot \lambda_\theta \Bigg) \Bigg]
\end{aligned}
$$

$$
\begin{aligned}
&\lambda_{\epsilon_s^c} = \\
&\quad - \frac{\mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c)}{\frac{\partial S_{\text{stw}}^c(\epsilon_s^c)}{\partial \epsilon_s^c}} \Bigg[n^s u'(c^w) \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \left(1 + \frac{\lambda_\theta}{n^s u'(c^w)}\right) \left(\frac{\lambda}{f} b - s_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon_s^c))\right) \\
&\quad + \left(\frac{\partial n}{\partial \epsilon_s^c} \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \epsilon_s^c}\right)^{\text{total}}\right) \\
&\quad \cdot \left(\frac{\partial \epsilon_s^u}{\partial c^w} \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot n^s u'(c^w) \left(\frac{\lambda}{f} b - s_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon_s^u))\right)\right) \\
&\quad + \frac{\partial \xi_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\xi_s^c) \cdot n^s u'(c^w) \cdot \frac{\lambda}{f} b \cdot \mathbb{1}(\epsilon_{\text{stw}} \leq \xi_s^c) \\
&\quad + \frac{\partial \epsilon_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s u'(c^w) \left(\frac{\lambda}{f} b - s_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon_s^c))\right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c) \\
&\quad + \left(-\frac{\partial S^{\text{total}}}{\partial c^w}\right) \cdot \lambda_\theta \Bigg) \Bigg]
\end{aligned}
$$

Now we have everything to calculate λ_θ from two equations:

$$
\begin{aligned}
&(1) \quad \left(\chi - \frac{\eta - \gamma}{1 - \eta}\right) \cdot \frac{1}{1-\gamma} \cdot \frac{k_v}{q} = \left(1 + \frac{\lambda_\theta + \eta \cdot \lambda \cdot \lambda_c}{n^s \cdot u'(c^w)}\right) \cdot \frac{b}{f} \\
&(2) \quad \chi = \frac{1}{1-\eta} \cdot \frac{1}{u \cdot f} \cdot \frac{1}{u'(c^w)} \cdot \left[\gamma - (\lambda\gamma - f\eta) \cdot \left(\frac{1}{\lambda}(1-\rho)\lambda_\theta - \lambda_{\epsilon_s^u}\right)\right] \\
&\qquad + \frac{1}{u} \cdot \frac{1}{u'(c^w)} \cdot \left(\lambda \cdot \frac{\gamma}{f} \cdot u'(c^w) + \left(\frac{1}{1-\eta} \cdot \frac{\lambda\gamma - f}{f}\right) \cdot \eta\right) \cdot \lambda_{\xi_s^c} \\
&\qquad + \frac{1}{u} \cdot \frac{1}{u'(c^w)} \cdot \left(\lambda \cdot \frac{\gamma}{f} \cdot u'(c^w) + \left(\frac{1}{1-\eta} \cdot \frac{\lambda\gamma - f}{f}\right) \cdot \eta\right) \cdot \lambda_{\epsilon_s^c} \\
&\qquad + \frac{1}{u} \cdot \frac{1}{u'(c^w)} \cdot \lambda_c \cdot \frac{\partial WE}{\partial \theta} \cdot \frac{1}{k_v} \\
&\qquad - \frac{1}{u} \cdot \frac{1}{u'(c^w)} \cdot IE_\theta
\end{aligned}
$$

We can rearrange (1) to:

$$
\chi \cdot k_v = (1-\gamma) \cdot q \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \left(1 + \frac{\lambda_\theta + \eta \cdot \lambda \cdot \lambda_c}{n^s \cdot u'(c^w)}\right) \cdot \frac{b}{f}\right)
$$

Note that

$$
(1-\gamma) \cdot q = -\frac{\partial f}{\partial \theta}
$$

with

$$\begin{aligned} f'(\theta) &= q(\theta) + \theta \cdot q'(\theta) \\ &= q(\theta) \cdot \left(1 + \frac{q'(\theta) \cdot \theta}{q(\theta)}\right) \\ &= q(\theta) \cdot (1 - \gamma) \end{aligned}$$

Inserting χ gives:

$$\begin{aligned} & [\gamma - (\lambda\gamma - f \cdot \eta)] \cdot \frac{1}{\lambda} \cdot (1 - \rho) \cdot \frac{1}{1 - \eta} \cdot \frac{k_v}{f} \cdot \lambda_\theta \\ &= u'(c^w) \cdot \left(\frac{\partial f}{\partial \theta}\right) \cdot u \cdot \left(\frac{\eta - \gamma}{(1 - \gamma)(1 - \eta)} \cdot \frac{k_v}{q} + \left(1 + \frac{\lambda_\theta + \eta \cdot \lambda \cdot \lambda_c}{n^s \cdot u'(c^w)}\right) \cdot \frac{b}{f}\right) \\ &\quad + \frac{1}{f} \cdot (\lambda\gamma - f \cdot \eta) \cdot k_v \cdot (-\lambda_{\epsilon_s^u}) \\ &\quad + \left(\lambda \cdot \frac{\gamma}{f} \cdot u'(c^w(\epsilon_s^c)) + \left(\frac{1}{1 - n} \cdot \frac{\lambda\gamma - f}{f}\right) \cdot \eta\right) \cdot k_v \cdot (-\lambda_{\xi_s^c}) \\ &\quad + \left(\lambda \cdot \frac{\gamma}{f} \cdot u'(c(\xi_s^c)) + \left(\frac{1}{1 - n} \cdot \frac{\lambda\gamma - f}{f}\right) \cdot \eta\right) \cdot k_v \cdot (-\lambda_{\epsilon_s^c}) \\ &\quad + \frac{\partial WE}{\partial \theta} \cdot (-\lambda_c) \\ &\quad + IE_\theta \end{aligned}$$

Insert $\lambda_{\epsilon_s^u}, \lambda_{\epsilon_s^c}, \lambda_{\xi_s^c}, IE_\theta$:

$$
\begin{aligned}
&\left[\gamma - (\lambda\gamma - f\cdot\eta)\cdot\frac{1}{\lambda}\cdot(1-\rho)\right]\cdot\frac{1}{1-\eta}\cdot\frac{k_v}{f}\cdot\lambda_\theta \\
&= u'(c^w)\cdot\left(\frac{\partial f}{\partial\theta}\right)\cdot u\cdot\left(\frac{\eta-\gamma}{(1-\gamma)(1-\eta)}\cdot\frac{k_v}{q} + \left(1+\frac{\lambda_\theta}{n^s u'(c^w)}\right)\cdot\frac{b}{f}\right) \\
&\quad + u'(c^w)\cdot n^s\cdot\left(-\frac{\partial\epsilon_s^u}{\partial\theta}\right)^{\text{total}}\cdot g(\epsilon_s^u) \\
&\quad \cdot\frac{1-p}{\lambda}\left(1+\frac{\lambda_\theta}{n^s u'(c^w)}\right)\left(\frac{\lambda}{f}b - s_{\text{stw}}(\bar{h}-h_{\text{stw}}(\epsilon_s^u))\right) \\
&\quad + u'(c^w)\cdot n^s\cdot\left(-\frac{\partial\xi_s^c}{\partial\theta}\right)^{\text{total}}\cdot g(\xi_s^c)\cdot\frac{p}{\lambda}\left(1+\frac{\lambda_\theta}{n^s u'(c^w)}\right)\cdot\frac{\lambda}{f}b\cdot\mathbb{1}(\epsilon_{\text{stw}}\le\xi_s^c) \\
&\quad + u'(c^w)\cdot n^s\cdot\left(-\frac{\partial\epsilon_s^c}{\partial\theta}\right)^{\text{total}}\cdot g(\epsilon_s^c) \\
&\quad \cdot\frac{p}{\lambda}\left(1+\frac{\lambda_\theta}{n^s u'(c^w)}\right)\left(\frac{\lambda}{f}b - s_{\text{stw}}(\bar{h}-h_{\text{stw}}(\epsilon_s^c))\right)\cdot\mathbb{1}(\epsilon_{\text{stw}}\ge\epsilon_s^c) \\
&\quad + \lambda_\theta\cdot\left(-\frac{\partial c^w}{\partial\theta}\right)^{\text{total}}\cdot\left(-\frac{\partial S}{\partial c^w}\right)^{\text{total}} \\
&\quad + n^s\cdot u'(c^w)\cdot\left(1+\frac{\lambda_\theta}{n^s u'(c^w)}\right)\cdot\tilde{IE}_\theta
\end{aligned}
$$

Denote:

$$
\begin{aligned}
\left(-\frac{\partial\epsilon_s^u}{\partial\theta}\right)^{\text{total}} &= \left(-\frac{\partial\epsilon_s^u}{\partial\theta}\right) + \left(\frac{\partial c^w}{\partial\theta}\right)^{\text{total}}\cdot\left(-\frac{\partial\epsilon_s^u}{\partial c^w}\right) \\
&\quad + \left(-\frac{\partial\epsilon_s^u}{\partial\theta}\right)\cdot\left[\left(\frac{\partial n}{\partial\epsilon_s^u}\right)\cdot\left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial\epsilon_s^u}\right)^{\text{total}}\right]\cdot\left(\frac{\partial\epsilon_s^u}{\partial c^w}\right) \\
&\quad + f'(\theta)\cdot\frac{\partial n}{\partial f}\cdot\frac{\partial c^w}{\partial n}\cdot\frac{\partial\epsilon_s^u}{\partial c^w}
\end{aligned}
$$

$$
\begin{aligned}
\left(-\frac{\partial\xi_s^c}{\partial\theta}\right)^{\text{total}} &= \left[\left(-\frac{\partial\xi_s^c}{\partial\theta}\right) + \left(\frac{\partial c^w}{\partial\theta}\right)^{\text{total}}\cdot\left(-\frac{\partial\xi_s^c}{\partial c^w}\right)\right. \\
&\quad + \left(-\frac{\partial\xi_s^c}{\partial\theta}\right)\cdot\left[\left(\frac{\partial n}{\partial\xi_s^c}\right)\cdot\left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial\xi_s^c}\right)^{\text{total}}\right]\cdot\left(\frac{\partial\xi_s^c}{\partial c^w}\right) \\
&\quad \left. + f'(\theta)\cdot\frac{\partial n}{\partial f}\cdot\frac{\partial c^w}{\partial n}\cdot\frac{\partial\xi_s^c}{\partial c^w}\right]\cdot\mathbb{1}(\epsilon_{\text{stw}}\le\xi_s^c)
\end{aligned}
$$

$$
\begin{aligned}
\left(-\frac{\partial \epsilon_s^c}{\partial \theta}\right)^{\text{total}} = \Bigg[& \left(-\frac{\partial \epsilon_s^c}{\partial \theta}\right) + \left(\frac{\partial c^w}{\partial \theta}\right)^{\text{total}} \cdot \left(-\frac{\partial \epsilon_s^c}{\partial c^w}\right) \\
& + \left(-\frac{\partial \epsilon_s^c}{\partial \theta}\right) \cdot \left[\left(\frac{\partial n}{\partial \epsilon_s^c}\right) \cdot \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \epsilon_s^c}\right)^{\text{total}}\right] \cdot \frac{\partial \epsilon_s^c}{\partial c^w} \\
& + f'(\theta) \cdot \frac{\partial n}{\partial f} \cdot \frac{\partial c^w}{\partial n} \cdot \frac{\partial \epsilon_s^c}{\partial c^w}\Bigg] \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c)
\end{aligned}
$$

$$
\begin{aligned}
\left(-\frac{\partial c^w}{\partial \theta}\right)^{\text{total,2}} = & \left(-\frac{\partial c^w}{\partial \theta}\right)^{\text{total}} \\
& + \left(-\frac{\partial \epsilon_s^u}{\partial \theta}\right) \cdot \left[\left(\frac{\partial n}{\partial \epsilon_s^u}\right) \cdot \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \epsilon_s^u}\right)^{\text{total}}\right] \\
& + \left(-\frac{\partial \xi_s^c}{\partial \theta}\right) \cdot \left[\left(\frac{\partial n}{\partial \xi_s^c}\right) \cdot \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \xi_s^c}\right)^{\text{total}}\right] \\
& + \left(-\frac{\partial \epsilon_s^c}{\partial \theta}\right) \cdot \left[\left(\frac{\partial n}{\partial \epsilon_s^c}\right) \cdot \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \epsilon_s^c}\right)^{\text{total}}\right] \\
& + f'(\theta) \cdot \frac{\partial n}{\partial f} \cdot \frac{\partial c^w}{\partial n}
\end{aligned}
$$

This allows us to calculate the Lagrange multiplier for the job-creation condition:

$$
\begin{aligned}
& M \cdot \lambda_\theta = \\
& u'(c^w) \cdot \frac{\partial f}{\partial \theta} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \left(1 + \frac{\lambda_\theta}{n^s u'(c^w)}\right) \cdot \frac{b}{f}\right) \\
& \quad + u'(c^w) \cdot n^s \cdot \left(\frac{\partial \epsilon_s^u}{\partial \theta}\right)^{\text{total}} \cdot g(\epsilon_s^u) \cdot \frac{1-p}{\lambda}\left(\frac{\lambda}{f} b - s_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon_s^u))\right) \\
& \quad + u'(c^w) \cdot n^s \cdot \left(-\frac{\partial \xi_s^c}{\partial \theta}\right)^{\text{total}} \cdot g(\xi_s^c) \cdot \frac{p}{\lambda} \cdot \frac{\lambda}{f} b \cdot \mathbb{1}(\epsilon_{\text{stw}} \leq \xi_s^c) \\
& \quad + u'(c^w) \cdot n^s \cdot \left(-\frac{\partial \epsilon_s^c}{\partial \theta}\right)^{\text{total}} \cdot g(\epsilon_s^c) \\
& \quad \cdot \frac{p}{\lambda}\left(\frac{\lambda}{f} b - s_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon_s^c))\right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c) \\
& \quad + u'(c^w) \cdot n^s \cdot \tilde{IE}_\theta
\end{aligned}
$$

with

$$
\begin{aligned}
M = &\left[\gamma - (\lambda\gamma - f\cdot\eta)\cdot\frac{1}{\lambda}\cdot(1-\rho)\right]\cdot\frac{1}{1-\eta}\cdot\frac{k_v}{f}\\
&+\left(-\frac{\partial f}{\partial\theta}\cdot u\cdot\frac{b}{f}\right)\\
&+\left(\frac{\partial\epsilon_s^u}{\partial\theta}\right)^{\text{total}}\cdot g(\epsilon_s^u)\cdot\frac{1-p}{\lambda}\left(\frac{\lambda}{f}b - s_{\text{stw}}(\bar{h}-h_{\text{stw}}(\epsilon_s^u))\right)\\
&+\left(\frac{\partial\xi_s^c}{\partial\theta}\right)^{\text{total}}\cdot g(\xi_s^c)\cdot\frac{p}{\lambda}\cdot\frac{\lambda}{f}b\cdot\mathbb{1}(\epsilon_{\text{stw}}\le\xi_s^c)\\
&+\left(\frac{\partial\epsilon_s^c}{\partial\theta}\right)^{\text{total}}\cdot g(\epsilon_s^c)\cdot\frac{p}{\lambda}\left(\frac{\lambda}{f}b - s_{\text{stw}}(\bar{h}-h_{\text{stw}}(\epsilon_s^c))\right)\cdot\mathbb{1}(\epsilon_{\text{stw}}\ge\epsilon_s^c)\\
&+\left(\frac{\partial c^w}{\partial\theta}\right)^{\text{total}}\cdot\left(-\frac{\partial S}{\partial c^w}\right)^{\text{total}}\\
&+\tilde{IE}_\theta
\end{aligned}
$$

Note. $\frac{1}{M}$ denotes the general equilibrium effect of an increase of the joint surplus on θ:

$$\left(\frac{\partial\theta}{\partial S}\right)^{\text{ge}} = \frac{1}{M}$$

Rearranging for λ_θ gives:

$$
\begin{aligned}
&M\cdot\lambda_\theta =\\
&u'(c^w)\cdot\left(\frac{\partial f}{\partial S}\right)^{\text{ge}}\cdot u\cdot\left(\frac{\eta-\gamma}{(1-\gamma)(1-\eta)}\cdot\frac{k_v}{q}+\frac{\lambda}{f}\cdot b\right)\\
&\quad+u'(c^w)\cdot n^s\cdot\left(\left(-\frac{\partial\epsilon_s^u}{\partial S}\right)^{\text{ge}}\cdot g(\epsilon_s^u)\cdot\frac{1-p}{\lambda}\left(\frac{\lambda}{f}\cdot b - s_{\text{stw}}\cdot(\bar{h}-h_{\text{stw}}(\epsilon_s^u))\right)\right)\\
&\quad+u'(c^w)\cdot n^s\cdot\left(\left(-\frac{\partial\xi_s^c}{\partial S}\right)^{\text{ge}}\cdot g(\xi_s^c)\cdot\frac{p}{\lambda}\cdot\frac{\lambda}{f}\cdot b\cdot\mathbb{1}(\epsilon_{\text{stw}}\le\xi_s^c)\right)\\
&\quad+u'(c^w)\cdot n^s\\
&\quad\cdot\left(\left(-\frac{\partial\epsilon_s^c}{\partial S}\right)^{\text{ge}}\cdot g(\epsilon_s^c)\cdot\frac{p}{\lambda}\cdot\left(\frac{\lambda}{f}\cdot b - s_{\text{stw}}\cdot(\bar{h}-h_{\text{stw}}(\epsilon_s^c))\right)\cdot\mathbb{1}(\epsilon_{\text{stw}}\ge\epsilon_s^c)\right)\\
&\quad+u'(c^w)\cdot n^s\cdot\left(\frac{\partial\theta}{\partial S}\right)^{\text{ge}}\cdot\tilde{IE}_\theta
\end{aligned}
$$

Define:

$$
\begin{aligned}
\hat{IE}_\theta = \frac{1}{1-\rho^c}\cdot\Bigg[&\int_{\max\{\epsilon_{\text{stw}},\xi_s^c\}}^{\epsilon^p}\lambda\cdot\frac{\gamma}{f}\cdot\frac{u'(c^w(\epsilon))-u'(c^w)}{u'(c^w)}\,dG(\epsilon)\\
&-\int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}},\epsilon_s^c\}}\lambda\cdot\frac{\gamma}{f}\cdot\frac{u'(c_{\text{stw}}(\epsilon))-u'(c^w)}{u'(c^w)}\,dG(\epsilon)\Bigg]\cdot k_v
\end{aligned}
$$

Note:

$$\left(\frac{\partial f}{\partial S}\right)^{\text{ge}} = \frac{1}{M} \cdot \frac{\partial f}{\partial S}$$

$$\left(\frac{\partial \epsilon_s^u}{\partial S}\right)^{\text{ge}} = \frac{1}{M} \cdot \left(\frac{\partial \epsilon_s^u}{\partial S}\right)^{\text{total}}$$

$$\left(\frac{\partial \xi_s^c}{\partial S}\right)^{\text{ge}} = \frac{1}{M} \cdot \left(\frac{\partial \xi_s^c}{\partial S}\right)^{\text{total}}$$

$$\left(\frac{\partial \epsilon_s^c}{\partial S}\right)^{\text{ge}} = \frac{1}{M} \cdot \left(\frac{\partial \epsilon_s^c}{\partial S}\right)^{\text{total}}$$

$$\begin{aligned}
\lambda_c = -\frac{1}{\left(\frac{\partial WE}{\partial c^w}\right)^{\text{total}}} \Bigg[& \left(\frac{\partial \epsilon_s^u}{\partial c^w} + \left(\frac{\partial S}{\partial c^w}\right)^{\text{total}} \left(\frac{\partial \epsilon_s^u}{\partial S}\right)^{\text{ge}}\right) \\
& \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot n^s u'(c^w) \left(\frac{\lambda}{f} b - s_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon_s^u))\right) \\
& + \left(\frac{\partial \xi_s^c}{\partial c^w} + \left(\frac{\partial S}{\partial c^w}\right)^{\text{total}} \left(\frac{\partial \xi_s^c}{\partial S}\right)^{\text{ge}}\right) \\
& \cdot \frac{p}{\lambda} \cdot g(\xi_s^c) \cdot n^s u'(c^w) \cdot \frac{\lambda}{f} b \cdot \mathbb{1}(\epsilon_{\text{stw}} \leq \xi_s^c) \\
& + \left(\frac{\partial \epsilon_s^c}{\partial c^w} + \left(\frac{\partial S}{\partial c^w}\right)^{\text{total}} \left(\frac{\partial \epsilon_s^c}{\partial S}\right)^{\text{ge}}\right) \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \\
& \cdot n^s \cdot u'(c^w) \left(\frac{\lambda}{f} b - s_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon_s^c))\right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c) \\
& + \left(\frac{\partial S}{\partial c^w}\right)^{\text{total}} \left(\frac{\partial f}{\partial S}\right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \left(1 + \frac{\lambda_\theta}{n^s u'(c^w)}\right) \cdot \frac{b}{f}\right) \\
& - n^c \cdot \left(\frac{\partial S}{\partial c^w}\right)^{\text{total}} \left(\frac{\partial \theta}{\partial S}\right)^{\text{ge}} \cdot \hat{IE}_\theta \Bigg]
\end{aligned}$$

Following the same arguments, we can express the Lagrange multiplier for the separation conditions as:

$$\begin{aligned}
\lambda_{\epsilon_s^u} = -\frac{u'(c^w)}{\left(\frac{\partial S^u_{\text{stw}}(\epsilon^u_s)}{\partial \epsilon^u_s}\right)^{\text{total}}} \Bigg[& \left(\frac{\partial n}{\partial \epsilon^u_s}\right)^{\text{ge}} \cdot \left(\frac{\lambda}{f} b - s_{\text{stw}}\left(\bar{h} - h_{\text{stw}}(\epsilon^u_s)\right)\right) \\
& + \left(\frac{\partial \xi^c_s}{\partial \epsilon^u_s}\right)^{\text{ge}} \cdot \frac{\partial n}{\partial \xi^c_s} \cdot \frac{\lambda}{f} b \cdot \mathbb{1}(\epsilon_{\text{stw}} \leq \xi^c_s) \\
& + \left(\frac{\partial \epsilon^c_s}{\partial \epsilon^u_s}\right)^{\text{ge}} \cdot \frac{\partial n}{\partial \epsilon^c_s} \left(\frac{\lambda}{f} b - s_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon^c_s))\right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon^c_s) \\
& + \frac{\partial c^w}{\partial \epsilon^u_s} \left(\frac{\partial f}{\partial c^w}\right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \frac{b}{f}\right) \\
& + n^c \cdot \frac{\partial c^w}{\partial \epsilon^u_s} \left(\frac{\partial \theta}{\partial c^w}\right)^{\text{ge}} \cdot \hat{IE}_\theta \Bigg]
\end{aligned}$$

$$\begin{aligned}
\lambda_{\xi_s^c} = -\frac{u'(c^w)}{\left(\frac{\partial S^c_{\text{stw}}(\xi^c_s)}{\partial \xi^c_s}\right)^{\text{total}}} \Bigg[& \left(\frac{\partial \xi^u_s}{\partial \xi^c_s}\right)^{\text{ge}} \cdot \frac{\partial n}{\partial \xi^u_s} \left(\frac{\lambda}{f} b - s_{\text{stw}}(\bar{h} - h_{\text{stw}}(\xi^u_s))\right) \\
& + \left(\frac{\partial n}{\partial \xi^c_s}\right)^{\text{ge}} \cdot \frac{\lambda}{f} b \cdot \mathbb{1}(\epsilon_{\text{stw}} \leq \xi^c_s) \\
& + \left(\frac{\partial \epsilon^c_s}{\partial \xi^c_s}\right)^{\text{ge}} \cdot \frac{\partial n}{\partial \epsilon^c_s} \left(\frac{\lambda}{f} b - s_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon^c_s))\right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon^c_s) \\
& + \frac{\partial c^w}{\partial \xi^c_s} \left(\frac{\partial f}{\partial c^w}\right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \frac{b}{f}\right) \\
& + n^c \cdot \frac{\partial c^w}{\partial \xi^c_s} \left(\frac{\partial \theta}{\partial c^w}\right)^{\text{ge}} \cdot \hat{IE}_\theta \Bigg]
\end{aligned}$$

$$\begin{aligned}
\lambda_{\epsilon_s^c} = -\frac{u'(c^w)}{\left(\frac{\partial S^c_{\text{stw}}(\epsilon^c_s)}{\partial \epsilon^c_s}\right)^{\text{total}}} \Bigg[& \left(\frac{\partial \epsilon^u_s}{\partial \epsilon^c_s}\right)^{\text{ge}} \cdot \frac{\partial n}{\partial \epsilon^u_s} \left(\frac{\lambda}{f} b - s_{\text{stw}}(\bar{h} - h_{\text{stw}}(\xi^u_s))\right) \\
& + \left(\frac{\partial \xi^c_s}{\partial \epsilon^c_s}\right)^{\text{ge}} \cdot \frac{\partial n}{\partial \epsilon^c_s} \cdot \frac{\lambda}{f} b \cdot \mathbb{1}(\epsilon_{\text{stw}} \leq \epsilon^c_s) \\
& + \frac{\partial n}{\partial \epsilon^c_s} \left(\frac{\lambda}{f} b - s_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon^c_s))\right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon^c_s) \\
& + \frac{\partial c^w}{\partial \epsilon^c_s} \left(\frac{\partial f}{\partial c^w}\right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \frac{b}{f}\right) \\
& + n^c \cdot \frac{\partial c^w}{\partial \epsilon^c_s} \left(\frac{\partial \theta}{\partial c^w}\right)^{\text{ge}} \cdot \hat{IE}_\theta \Bigg]
\end{aligned}$$

For convenience, the optimal UI benefits are reproduced here:

$$\begin{aligned}(1-\eta)\cdot(u'(b)-u'(c^w)) = \lambda_\theta(1-\rho)\cdot\left(\frac{1-n}{n}+\frac{u'(b)}{u'(c^w)}\right)\\ +(\lambda_{\epsilon_s^u}+\lambda_{\xi_s^c}+\lambda_{\epsilon_s^c})\cdot\left(-\frac{u'(b)}{u'(c^w)}\right)\\ +\lambda_c\cdot(1-\rho)\cdot\left(\eta\cdot\frac{1-n}{n}+(1-\eta)\cdot\frac{u'(b)}{u'(c^w)}\right)\end{aligned}$$

Inserting the Lagrange multiplier gives:

$$\begin{aligned}&(1-n)(u'(b)-u'(c^w))\\ &=u'(c^w)\Bigg[\left(\frac{\partial c^w}{\partial b}\right)^{\text{ge}}\left(-\frac{\partial f}{\partial c^w}\right)\cdot u\cdot\left(\frac{\eta-\gamma}{(1-\gamma)(1-\eta)}\cdot\frac{k_v}{q}+\frac{b}{f}\right)\\ &\quad+\left(\frac{\partial \epsilon_s^u}{\partial b}\right)^{\text{ge}}\cdot\frac{\partial n}{\partial \epsilon_s^u}\cdot\left(\frac{\lambda}{f}\cdot b-s_{\text{stw}}\cdot(\bar{h}-h_{\text{stw}}(\epsilon_s^u))\right)\\ &\quad+\left(\frac{\partial \xi_s^c}{\partial b}\right)^{\text{ge}}\cdot\frac{\partial n}{\partial \xi_s^c}\cdot\frac{\lambda}{f}\cdot b\cdot\mathbb{1}(\epsilon_{\text{stw}}\leq\xi_s^c)\\ &\quad+\left(\frac{\partial \epsilon_s^c}{\partial b}\right)^{\text{ge}}\cdot\frac{\partial n}{\partial \epsilon_s^c}\cdot\left(\frac{\lambda}{f}\cdot b-s_{\text{stw}}\cdot(\bar{h}-h_{\text{stw}}(\epsilon_s^c))\right)\cdot\mathbb{1}(\epsilon_{\text{stw}}\geq\epsilon_s^c)\\ &\quad+n^c\left(-\frac{\partial\theta}{\partial b}\right)^{\text{ge}}\cdot\hat{IE}_\theta\Bigg]\end{aligned}$$

with

$$\begin{aligned}\left(\frac{\partial \epsilon_s^u}{\partial b}\right)^{\text{ge}}\cdot\frac{\partial n}{\partial \epsilon_s^u} &= \underbrace{\left(\frac{\partial \epsilon_s^u}{\partial b}\right)^{\text{ge}}\left(\frac{\partial n}{\partial \epsilon_s^u}\right)}_{\text{direct effects}}\\ &+\underbrace{\left[\frac{\partial S}{\partial b}\left(\frac{\partial \epsilon_s^u}{\partial S}\right)^{\text{ge}}+\frac{\partial \xi_s^c}{\partial b}\left(\frac{\partial c^w}{\partial \xi_s^c}\right)^{\text{ge}}+\frac{\partial \epsilon_s^c}{\partial b}\left(\frac{\partial c^w}{\partial \epsilon_s^c}\right)^{\text{ge}}+\frac{\partial c^w}{\partial b}\left(\frac{\partial \epsilon_s^u}{\partial c^w}\right)^{\text{ge}}\right]\cdot\frac{\partial n}{\partial \epsilon_s^u}}_{\text{indirect effect}}\end{aligned}$$

$$\left(\frac{\partial \xi_s^c}{\partial b}\right)^{\text{ge}} \cdot \frac{\partial n}{\partial \xi_s^c} = \underbrace{\left(\frac{\partial \xi_s^c}{\partial b}\right)^{\text{ge}} \left(\frac{\partial n}{\partial \xi_s^c}\right)}_{\text{direct effects}} + \underbrace{\left[\frac{\partial S}{\partial b}\left(\frac{\partial \xi_s^c}{\partial S}\right)^{\text{ge}} + \frac{\partial \epsilon_s^u}{\partial b}\left(\frac{\partial c^w}{\partial \epsilon_s^u}\right)^{\text{ge}} + \frac{\partial \epsilon_s^c}{\partial b}\left(\frac{\partial c^w}{\partial \epsilon_s^c}\right)^{\text{ge}} + \frac{\partial c^w}{\partial b}\left(\frac{\partial \xi_s^c}{\partial c^w}\right)\right] \cdot \frac{\partial n}{\partial \xi_s^c}}_{\text{indirect effect}}$$

$$\left(\frac{\partial \epsilon_s^c}{\partial b}\right)^{\text{ge}} \cdot \frac{\partial n}{\partial \epsilon_s^c} = \underbrace{\left(\frac{\partial \epsilon_s^c}{\partial b}\right)^{\text{ge}} \left(\frac{\partial n}{\partial \epsilon_s^c}\right)}_{\text{direct effects}} + \underbrace{\left[\frac{\partial S}{\partial b}\left(\frac{\partial \epsilon_s^c}{\partial S}\right)^{\text{ge}} + \frac{\partial \epsilon_s^u}{\partial b}\left(\frac{\partial c^w}{\partial \epsilon_s^u}\right)^{\text{ge}} + \frac{\partial \xi_s^c}{\partial b}\left(\frac{\partial c^w}{\partial \xi_s^c}\right)^{\text{ge}} + \frac{\partial c^w}{\partial b}\left(\frac{\partial \epsilon_s^c}{\partial c^w}\right)\right] \cdot \frac{\partial n}{\partial \epsilon_s^c}}_{\text{indirect effect}}$$

$$\left(-\frac{\partial f}{\partial b}\right)^{\text{ge}} = \underbrace{\frac{\partial S}{\partial b}\left(-\frac{\partial f}{\partial S}\right)^{\text{ge}}}_{\text{direct effect}} + \underbrace{\left[\frac{\partial \epsilon_s^u}{\partial b} \cdot \frac{\partial c^w}{\partial \epsilon_s^u} + \frac{\partial \xi_s^c}{\partial b} \cdot \frac{\partial c^w}{\partial \xi_s^c} + \frac{\partial \epsilon_s^c}{\partial b} \cdot \frac{\partial c^w}{\partial \epsilon_s^c} + \frac{\partial c^w}{\partial b}\right] \cdot \left(-\frac{\partial f}{\partial c^w}\right)^{\text{ge}}}_{\text{indirect effect}}$$

$$\left(-\frac{\partial \theta}{\partial b}\right)^{\text{ge}} = \underbrace{\frac{\partial S}{\partial b}\left(-\frac{\partial \theta}{\partial S}\right)^{\text{ge}}}_{\text{direct effect}} + \underbrace{\left[\frac{\partial \epsilon_s^u}{\partial b} \cdot \frac{\partial c^w}{\partial \epsilon_s^u} + \frac{\partial \xi_s^c}{\partial b} \cdot \frac{\partial c^w}{\partial \xi_s^c} + \frac{\partial \epsilon_s^c}{\partial b} \cdot \frac{\partial c^w}{\partial \epsilon_s^c} + \frac{\partial c^w}{\partial b}\right] \cdot \left(-\frac{\partial \theta}{\partial c^w}\right)^{\text{ge}}}_{\text{indirect effect}}$$

H Proof of Lemma 1

Separation condition: unconstrained matches

$$z_{\text{stw}}(\epsilon_s^u) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + s_{\text{stw}}(\bar{h} - h(\epsilon_s^u)) + \frac{\lambda - \eta \cdot f}{1 - \eta} \cdot \frac{k_v}{q} = 0$$

Separation condition: constrained matches

$$\frac{u(c_{\text{stw}}(\epsilon_s^c)) - u(b)}{u'(c^w)} + \frac{\eta}{1 - \eta} \cdot (\lambda - f) \cdot \frac{k_v}{q}$$

Remember:

$$c_{\text{stw}}(\epsilon) = z_{\text{stw}}(\epsilon_s^c) + s_{\text{stw}}(\bar{h} - h(\epsilon_s^c)) + \lambda \cdot \frac{k_v}{q}$$

This allows us to rewrite the separation condition of constrained firms on STW as:

$$z_{\text{stw}}(\epsilon_s^c) + \frac{u(c_{\text{stw}}(\epsilon_s^c)) - u(b)}{u'(c^w)} - c_{\text{stw}}(\epsilon_s^c) + s_{\text{stw}}(\bar{h} - h(\epsilon_s^c)) + \frac{\lambda - \eta \cdot f}{1 - \eta} \cdot \frac{k_v}{q} = 0$$

T.b.s.: $\epsilon_s^c > \epsilon_s^u$ **(the proof for** $\xi_s^c > \xi_s^u$ **is completely analogous)**

Note: $c^w > c_{\text{stw}}(\epsilon)$

$$\begin{aligned}
\epsilon_s^c > \epsilon_s^u \Leftrightarrow \quad & z_{\text{stw}}(\epsilon) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + s_{\text{stw}}(\bar{h} - h(\epsilon)) + \frac{\lambda - \eta f}{1 - \eta} \cdot \frac{k_v}{q} \\
& \geq z_{\text{stw}}(\epsilon) + \frac{u(c_{\text{stw}}(\epsilon)) - u(b)}{u'(c^w)} - c_{\text{stw}}(\epsilon) \\
& + s_{\text{stw}}(\bar{h} - h(\epsilon)) + \frac{\lambda - \eta f}{1 - \eta} \cdot \frac{k_v}{q}
\end{aligned}$$

$$\Leftrightarrow \quad \frac{u(c^w)}{u'(c^w)} - c^w > \frac{u(c_{\text{stw}}(\epsilon))}{u'(c^w)} - c_{\text{stw}}(\epsilon)$$

$$\Leftrightarrow \quad \frac{u(c^w) - u(c_{\text{stw}}(\epsilon))}{u'(c^w)} > c^w - c_{\text{stw}}(\epsilon)$$

$$\Leftrightarrow \quad u(c^w) - u(c_{\text{stw}}(\epsilon)) > (c^w - c_{\text{stw}}(\epsilon)) \cdot u'(c^w)$$

$$\Leftrightarrow \quad \int_{c_{\text{stw}}(\epsilon)}^{c^w} u'(c)\, dc > \int_{c_{\text{stw}}(\epsilon)}^{c^w} u'(c^w)\, dc$$

$$\Leftrightarrow \quad \int_{c_{\text{stw}}(\epsilon)}^{c^w} [u'(c) - u'(c^w)]\, dc > 0 \qquad \checkmark \quad \text{(due to risk aversion)}$$

I Calibration

To calibrate the model, we solve the following system of equations for parameters, taking targets as exogenous. Note that we solve for some endogenous model variables, too. This works because the system of equations pins down the model solution for these variables,

along with the parameters to be calibrated.

Exogenous:

$$f,\ q, \rho, b_{\text{rep}}, n$$

Endogenous:

$$k_f, \bar{m}, b, \epsilon_s^c, \epsilon_s^u, c^w, \omega$$

(I)

$$\rho = (1-p) \cdot G(\epsilon_s^u) + p \cdot G(\epsilon_s^c)$$

(II)

$$0 = z(\epsilon_s^u) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + \frac{\lambda - \eta f}{1-\eta} \cdot \frac{k_v}{q}$$

(III)

$$0 = z(\epsilon_p) + \lambda \cdot \frac{k_v}{q} - c^w$$

(IV)

$$\frac{u(c(\epsilon_s^u)) - u(b)}{u'(c^w)} + (\lambda - f) \cdot \frac{\eta}{1-\eta} \cdot \frac{k_v}{q} = 0$$

(V)

$$\begin{aligned} \frac{1}{1-\eta} \cdot \frac{k_v}{q} = \frac{1}{\lambda} \Bigg[& (1-\rho) \cdot \left(z - \frac{1-\eta}{\eta} b \right) \\ & + (1-p)(1-\rho^u) \cdot \left(\frac{u(c^w) - u(b)}{u'(c^w)} - c^w \right) \\ & + p(1-\rho^c) \cdot \left(\frac{u(c^c) - u(b)}{u'(c^w)} - e^c \right) \\ & + (1-\rho) \cdot \frac{\lambda - f\eta}{1-\eta} \cdot \frac{k_v}{q} \Bigg] \end{aligned}$$

(VI)

$$
\begin{aligned}
&(1-p)\cdot(1-\rho^u)\cdot\left(\eta\cdot c^w+(1-\eta)\cdot\frac{u(c^w)-u(b)}{u'(c^w)}\right)\\
&=\eta\cdot(1-\rho)\cdot\left(z+\frac{1-n}{n}\cdot b+\theta\cdot k_v\right)\\
&\quad-p\cdot(1-\rho^c)\cdot\left[(1-\eta)\cdot\frac{u^c-u(b)}{u'(c^w)}+\eta\cdot e^c\right]
\end{aligned}
$$

(VII)

$$
\begin{aligned}
\omega=\frac{1}{1-\rho}\cdot\Bigg((1-p)\cdot\int_{\epsilon_s^u}^{\infty}\left[c^w+\phi(h(\epsilon))\right]dG(\epsilon)\\
+p\cdot\int_{\epsilon_p}^{\infty}\left[c^w+\phi(h(\epsilon))\right]dG(\epsilon)\\
+p\cdot\int_{\epsilon_s^c}^{\epsilon_p}\left[c(\epsilon)+\phi(h(\epsilon))\right]dG(\epsilon)\Bigg)
\end{aligned}
$$

(VIII)

$$
b=b_{\text{rep}}\cdot\omega
$$

J Optimization

To optimize for policy parameters, we use the following algorithm: First we use the global Covariance Matrix Adaptation Evolution Strategy (CMA-ES) with 8 random starting points. Then we select the best 4 solutions and use a local trust region optimization algorithm to further optimize these candidates. As the local optimization algorithm we rely on BOBYQA (Bound Optimization BY Quadratic Approximation). Finally, we select the best solution among these 4 candidates as the optimal policy parameters. This procedure is conducted independently for each value of p. We rely on an evenly spaced grid for p of 96 points between 0 and 1.