Intangible Capital Around the World

Frederico Belo [*] Yu Li [†] Juliana Salomao [‡] Maria Ana Vitorino [§]

Oct 15, 2025

Abstract

We estimate the value of intangible capital around the world through the valuation approach of a neoclassical model of investment with two heterogeneous types of capital inputs: physical capital and intangible capital. Using data on public listed firms across 77 countries, we infer the importance of intangible capital for the firm's market value in each country. We find that intangible capital accounts for over half of firms' market value globally, with significant cross-country heterogeneity. To understand the drivers of intangible capital accumulation we explore the variation in the economic and legal environment across countries. We find that protection of intellectual property right, cost of contract enforcement significantly influence the cross-country variation in the value of intangible capital.

Key Words: Valuation, Neoclassical Investment, Structural Estimation, Intangible Capital

JEL Classification: D21, D22, E22, E24, G12, G32

[*]Corresponding author: Frederico Belo, INSEAD, Boulevard de Constance 77300, Fontainebleau, France; telephone: +33 1 60 72 45 24. E-mail: frederico.belo@insead.edu.

[†]Shanghai Jiao Tong University, yuli@saif.sjtu.edu.cn.

[‡]University of Minnesota, CEPR and NBER, jsalomao@umn.edu.

[§]INSEAD,maria-ana.vitorino@insead.edu.

1 Introduction

How much does intangible capital contribute to a firm's market value? And what are the economic drivers of intangible capital accumulation? To address these questions, we use cross-country variation in the economic and legal environment and a neoclassical model of investment with two inputs—physical capital (e.g., plants and machinery) and intangible capital (e.g., brand name, stock of knowledge). To quantify the relative importance of intangible capital for firm value across the world, we estimate the model using data from a large set of publicly traded firms in 77 countries spanning the period from 2000 to 2020. We find that, while there is substantial cross-country heterogeneity, intangible capital is consistently a crucial determinant of firm value, accounting for over 50% of a firm's market value in the past decade. In addition, the importance of intangible capital in each country is closely linked to country characteristics. We show that institutions, economic and financial market factors—especially the intellectual property right protection and contracting procedural cost—significantly influence the cross-country variation in the value of intangible capital. Taken together, our findings show that intangible capital is a key driver of firm value, and its accumulation is strongly influenced by a country's institutional environment.

We use a generalized neoclassical model of investment to decompose the market value of a firm between physical and intangible capital. In the model, changing the quantity of the capital inputs is costly, which we capture through standard adjustment cost functions. The market value of the firm is contingent on the shadow price and the quantity of each installed input. These shadow prices capture the replacement cost of the input and can be easily estimated from investment data once we specify an adjustment costs function. If both the operating profit and the adjustment costs functions exhibit homogeneity of degree one, the market value of each input is the product of the input's shadow price and the corresponding stock variable. Consequently, the total market value of the firm becomes the sum of the market values of all inputs. This additive characteristic facilitates a straightforward computation of each input's contribution to the firm's overall value.

We estimate the model using data from a large cross section of publicly traded firms around the world. To measure the firm-level stocks of each capital input, we use accounting information data of listed companies from the Compustat (North America and Global) database. For physical capital, the data is readily available from the firm's balance sheet. For intangible capital, we

follow Eisfeldt and Papanikolaou (2013) and Peters and Taylor (2017), to construct firm-level measures of intangible stock using accounting data related to Selling, General and Administrative (SG&A) expenses. This is a well populated variable in our multi-country database and Lev and Radhakrishnan (2005) document SG&A serves as a comprehensive measure of investment expense of intangible capital, encompassing multiple types of intangible capital. It reflects the value of skilled labor force (capturing training costs), knowledge capital (often including R&D expenditures), brand capital (accounting for advertising expenses), and other operational expenses. We employ the perpetual inventory method to aggregate these expenditures, enabling us to derive the capital stocks for intangible capital.

To estimate the model we minimize the difference between observed and model-generated valuation ratios, specifically the market value of equity plus net debt-to-book value of capital stocks. We estimate adjustment cost parameters for both physical and intangible capital at the individual country and regional levels. For large equity markets, we estimate country specific adjustment cost parameters. These are 17 countries including all major economies in the world. For other countries, we adopt a strategy of pooling these nations into the geographic areas based on United Nations statistics criteria, and estimate region-specific adjustment parameters. In total, our analysis covers 77 countries, whose the listed companies represent 40% of world GDP and 13% of global value added. Due to data availability most of our analysis focus on the last 15 years, the start date varies per country, mostly starting in the mid 2000 and ends in 2020. Leveraging the estimated adjustment cost parameters, we apply our model to decompose the value of firms into physical and intangible capital for each of these countries.

Our main empirical findings can be summarized as follows. First, we show that neoclassical model of investment with two heterogeneous capital inputs aligns well with the data across diverse economies. In major markets where country-specific parameters are estimated, the model effectively accounts for both time-series and cross-sectional variations in valuation ratios across portfolios. In the worldwide, the cross-country time-series R^2 stands at 57%, while the cross-sectional R^2 reaches 87%. The model exhibits robust explanatory power, across sub-samples of continents. In Asia, the time-series R^2 is 60% and a cross-sectional R^2 of 90%. In Europe, the time-series R^2 is 43% and a cross-sectional R^2 of 80%. In Americas, the time-series R^2 is 67% and a cross-sectional R^2 of 88%.

Including intangible capital and allowing for heterogeneity in the capital inputs, the neoclassical model successfully explains the variation of firm valuation across the world.

Second, we find that the estimated adjustment cost of intangible capital varies across countries and that this heterogeneity is important for capturing firm value. Examining larger equity markets, the physical capital adjustment cost parameter varies from 0.78 for Japan to 8.10 for the USA. Across the countries and regions, the average parameter is 3.87 and a standard deviation of 1.73. For Canada, Thailand, and China, the estimated parameters hover around this average, registering values of 3.73, 3.88 and 3.31, respectively. In contrast, the intangible capital adjustment cost parameter displays greater variability, ranging from 2.91 for Japan to 32.54 for China. The cross-country average stands at 11.63, with a standard deviation of 6.10. The Germany, Canada, and Malaysia align closely with this average, featuring estimated values of 10.27, 11.53, and 11.82, respectively.

Third, our findings show that intangible capital significantly contributes to the market value of firms across all countries. Utilizing the estimated parameters of geography-specific and capital-specific adjustment costs, we calculate the market share of intangible capital for each firm and each time point. In the country-specific estimation, the cross country average value of intangible capital represents 53.88% of the firm's market value. Importantly, there exists substantial heterogeneity in the market share of intangible capital, ranging from 68.93% in China to 33.31% in Canada. Apart from China, the top five countries with the highest intangible market shares include the USA (64.39%), the UK (63.09%), Germany (61.87%), and France (61.71%).

Finally, for all countries that the model directly estimates the specific coefficient, the cost of adjusting intangible capital is higher than the cost of adjusting physical capital. In the worldwide, the adjustment cost of intangible capital is twice as large as that of physical capital, where the market share of intangible capital 53.83% surpasses book value 30.90%. Collectively, these findings underscore the pivotal role of intangible capital as a key input in the production and value creation for firms worldwide.

Our results highlight the significance of geographical variation in adjustment costs for understanding differences in market value among firms and their inputs across countries. To investigate the key country characteristics that drive these cross-country differences we investigate

three dimensions of country-level characteristics that may influence adjustment costs: (a) Protection of Intellectual Property Rights, (b) Ease of Contract Enforcement, (c) Economic and Financial Development. We model adjustment costs as a function of these characteristics and re-estimate the neoclassical investment model using this specification. This approach allows us to leverage the heterogeneity of smaller economies within regions, as well as time series variations in firm value and investment rates, to accurately identify the significance of country characteristics. Our findings indicate that the model in which adjustment costs depend on country characteristics fits the data well for all regions except Africa, closely aligning with the parameters in benchmark model.

The extended model translates the cross-country difference in status of institutional infrastructure, economic and financial market development into the amount of capital valuation directly impacted. The marginal compensation value for the under-development of institutional infrastructure and financial market is estimated. Our analysis shows that institutions, particularly the intellectual property rights and the contracting procedures, play significant roles in the cross-country variation of intangible capital value, while similar factors affect physical capital but to a lesser extent. The cross-country variation in intellectual property rights protection contributes to 6.18% standard deviation in variation of firm value, by changing the valuation of intangible capital, while the analogous outcome is 0.74% standard deviation via the channel of impacting physical capital value. The cross-country difference in cost of contract enforcement contributes to 9.65% standard deviation in variation of firm value, by changing the intangible capital value, and 5.69% via changing the physical capital value. Overall, these results emphasize the critical interplay between country characteristics and the valuation of both physical and intangible capital across different economic contexts.

Related Literature. Our research is closely aligned with the extensive literature on valuation and production-based asset pricing, with a specific focus on the role of intangible capital. Hall (2001) discusses the valuation of securities is overly high compared to the price of installed capital in late 1990s. The omitted intangible capital provides a candidate explanation. Belo et al. (2022) provides a decomposition of the value of North American firms, considering physical capital, labor, and two types of intangibles—brand capital and knowledge capital. Crouzet and Eberly (2021) explains the quantitative tension between physical-capital investment rate and the firm valuation,

using the adjustment cost estimates from Belo et al. (2022) to decompose the long-run evolution of firm valuation. Peters and Taylor (2017) incorporate organization capital for the total book capital, adjust the measure for the Tobin's Q, and to explain the total firm investment. Eisfeldt and Papanikolaou (2013) show that firms with more organization capital are riskier than firms with less organization capital. In Baker, Baugh, and Sammon (2023), firm operating activity and equity return are prone to the shift of customer base, suggesting the firm payout is exposed to shocks specific to the customer capital. Eisfeldt, Kim, and Papanikolaou (2020) and Gulen et al. (2022) include intangible capital for Value factor. Adjusted measure of firm valuation ratio improves the asset pricing factor models. Hansen, Heaton, and Li (2012) study the risk characteristics of intangible capital. In international macro-finance, research on the cross-section of equity valuation is emerging. To our knowledge, our paper is the first attempt for quantifying market value of intangible capital in global economy. We find that adjustment cost of intangible capital is larger than that of physical capital in most countries. Furthermore, our cross-country analysis allows us to study the importance of country's fundamental characteristics in driving the importance of each input in the firm's market value.

We investigate whether institutions, specifically, weak enforcement of intellectual property, the complex procedures in contract enforcement, will impede the long-run growth of a country via the friction in accumulating the intangible capital . We examine this question by estimating their roles in determining the adjustment cost of capitals owned by firms. Using a dynamic general equilibrium model, Antill et al. (2023) quantify the extent how plaintiff rights in the litigation influences innovation. Strengthening protection to incumbent patent increases the monopoly rent and the firm value, but discourages the innovation of new entrant. Similarly, Crouzet et al. (2022a) argue that stronger property rights can hinder growth by limiting access to intangible capital. Our empirical examination indicates that countries with stronger patent protections experience lower adjustment costs for intangible capital, which, when keeping investment levels constant, leads to a reduced value of that capital. Djankov et al. (2003) documented the heavier formalism and procedural cost in civil law countries. Acemoglu and Johnson (2005) documented countries end up with low GDP if they are associated with the procedural cost in operating the firm reduces the long-run growth in a country. Our empirical examination supplements the literature, by confirming

the friction in adjusting capital amount can be reduced if the procedural cost is mitigated, both for the physical capital and the intangible capital.

Our work talks to the recent literature of modern corporate sector in international finance. Karabarbounis and Neiman (2014) document the declining labor share of income in national accounting data, both in the United States and globally. Chen, Karabarbounis, and Neiman (2017) investigated the increasing corporate saving in the private corporate sector. Falato et al. (2022) provide an explanation for the simultaneous shift toward the intangible capital and the corporate saving. Altomonte et al. (2021) claim that the frictions in intangible investments can lead to the dispersion of markup at firm-level. Our work investigates the role of capital market environment in the valuation of intangible capital, by quantifying the difficulty of intangible capital investment for countries. High valuation of intangible capital can be the result of its high shadow price.

Our research complements to the extensive literature on cross-country difference of valuation (firm average Q) and investment. What determines the firm value in a country? La Porta et al. (2002) investigate the ownership of the large listed firms in different countries, document that block shareholder impedes the corporate governance and erodes corporate value. The higher firm valuation is the result of the higher cash-flow growth benefiting from the investor protection. McLean, Zhang, and Zhao (2012) extend the investment-Q regression, document the investment is more sensitive to firm value, in countries with better investor protection. Aforementioned research only considers the homogeneous capital, but intangible capital is crucial for modern corporation. Chemmanur and Yan (2009) document the simultaneous advertising expenditure and the firm equity issuance activity. Larkin (2013) discusses how the brand perception mitigates the difficult in firm financing. Acharya, Baghai, and Subramanian (2014) found patent application and economic importance measured by citation increases after a state in the U.S. adopted the Wrongful Discharge Laws. Calcagnini, Ferrando, and Giombini (2014) document firm valuation tends to be lower in economies with stronger protection of employment contract. Bena, Ortiz-Molina, and Simintzi (2022) document increase in firm process innovation when the labor dismissal cost increases. We utilize the model of adjustment cost function that are determined by the institutional development in multiple aspects, to disentangle the joint effects over capital value from the protection of intellectual property right, the efficacy of contract enforcement, the economic development and the fairness of financial market.

The sensitivity of firm value to investment rate elicits the adjustment cost, conveys the shadow price of capital. The estimation doesn't find significant impact from the shareholder rights, dismissal protection and the tenure contract, conditional on the same status of contracting institutions and financial market. Our estimation confirms that the cross-country variation in firm value is largely contributed by status of country in intellectual property protection and contracting enforcement, via changing the value of intangible capital.

The remainder of the paper is structured as follows: Section 2 presents the model, while Section 3 introduces the functional forms and outlines the estimation procedure. Section 4 details the data used. Section 5 discusses the contribution of intangible capital to firm value and the cross-country difference in cost of investing intangible capital. Section 6 examines the heterogeneity of economic and institutional environment in countries, models the cost parameters as a function of country fundamentals. Section 7 addresses various robustness checks, and finally, Section 8 concludes the paper. Additional results and robustness analyses can be found in the Appendix.

2 The Model of the Firm

We consider a neoclassical model of the firm as in Belo et al. (2022)(we use the consistent notation with Belo et al. (2022) whenever possible) with several quasi-fixed inputs. Time is discrete and the horizon is infinite. Firms choose costlessly adjustable inputs (e.g., materials, energy) each period, while taking their prices as given, to maximize operating profits (revenues minus the expenditures on these inputs). Because we treat intangible capital as quasi-fixed inputs, investments in intangible capital is excluded from our definition of operating profits. Then, taking these operating profits as given, firms optimally choose the physical and intangible capital investments, and debt to maximize their market value of equity.

To save on notation, we denote a firm's i set of capital as $\boldsymbol{K_{it}}$ (variables in bold represent a vector). This set includes the physical capital stock (K^P_{it}) and the intangible capital stock (K^I_{it}). Similarly, we denote a firm's i set of investments in the inputs at time t, as $\boldsymbol{I_{it}}$. This set includes the investment in physical capital (I^P_{it}) and the investment in intangible capital (I^I_{it}).

The law of motion of the firm's capital inputs is given by:

$$K^P_{it+1} = I^P_{it} + (1 - \delta^P_{it})K^P_{it} \quad (1)$$

$$K^I_{it+1} = I^I_{it} + (1 - \delta^I_{it})K^I_{it} \quad (2)$$

where δ^P_{it} and δ^I_{it} are the exogenous depreciation rates of physical and intangible capital, respectively.

2.1 Technology

The operating profit function for firm i at time t is $\Pi_{it} \equiv \Pi(\boldsymbol{K_{it}}, \boldsymbol{X_{it}})$, in which $\boldsymbol{X_{it}}$ denotes a vector of exogenous aggregate and firm-specific shocks. Firms incur adjustment costs when investing. The adjustment costs function is denoted $C_{it} \equiv C(\boldsymbol{I_{it}}, \boldsymbol{K_{it}})$. This function is increasing and convex in investment, and decreasing in the capital stocks. For physical and intangible capital inputs these costs include, for example, planning and installation costs, and costs related with production being temporarily interrupted. We assume that the firm's operating profit function and adjustment costs function are both homogeneous of degree one and we specify the functional forms in the empirical section below.

2.2 Taxable Profits and Firm's Payouts

Firms can issue debt to finance their operations.[1] At the beginning of time t, firm i issues an amount of debt, denoted B_{it+1}, which must be repaid at the beginning of time $t+1$. r^B_{it} denotes the gross corporate bond return on B_{it}.

We can write taxable corporate profits, denoted TCP, as operating profits minus intangible capital investments (which are expensed), physical capital depreciation, adjustment costs, and interest expense:

$$TCP_{it} = \Pi_{it} - I^I_{it} - \delta^P_{it}K^P_{it} - C_{it}.$$

Thus, adjustment costs are expensed, consistent with treating them as foregone operating profits.

[1] We include debt in the model to better fit the data, but for parsimonious reasons we keep the financing side of the firm as simple as possible.

Let τ_{it} be the corporate tax rate. The firm' payout, denoted D_{it}, is then given by:[2]

$$D_{it} \equiv (1-\tau_t)[\Pi_{it} - C_{it} - I^I_{it}] - I^P_{it} + B_{it+1} - r^B_{it}B_{it} + \tau_t \delta^P_{it} K^P_{it} + \tau_t (r^B_{it} - 1)B_{it}, \tag{3}$$

in which $\tau_t \delta^P_{it} K^P_{it}$ is the depreciation tax shield, and $\tau_t(r^B_{it} - 1)B_{it}$ is the interest tax shield.

2.3 Equity Value

Firm i takes the stochastic discount factor, denoted $M_{t+\triangle t}$, from period t to $\triangle t$ as given when maximizing its cum-dividend market value of equity:

$$V_{it} \equiv \max_{\{\mathbf{I}_{it+\triangle t}, \mathbf{K}_{it+\triangle t+1}, B_{it+\triangle t+1}\}_{\triangle t=0}^{\infty}} E_t \left[\sum_{\triangle t=0}^{\infty} M_{t+\triangle t} D_{it+\triangle t} \right], \tag{4}$$

subject to a transversality condition given by $\lim_{T\to\infty} E_t[M_{t+T}B_{it+T+1}] = 0$, and the laws of motion for the capital inputs given by equations (1).

Let $P_{it} \equiv V_{it} - D_{it}$ be the ex-dividend equity value. In the online appendix we show that, given the homogeneity of degree one of the operating profit and adjustment costs functions, the firm's value maximization implies that:

$$P_{it} + B_{it+1} = q^P_{it} K^P_{it+1} + q^I_{it} K^I_{it+1}, \tag{5}$$

in which

$$q^P_{it} \equiv 1 + (1-\tau_t)\partial C_{it}/\partial I^P_{it} \tag{6}$$

$$q^I_{it} \equiv (1-\tau_t)\left[1 + \partial C_{it}/\partial I^I_{it}\right] \tag{7}$$

and $\partial C_{it}/\partial x$ denotes the first derivative of the adjustment costs function with respect to variable x, q^P_{it}, and q^I_{it} measure the shadow prices of physical capital and intangible capital, respectively (the Lagrange multipliers of equations (1) to (2)). The valuation equation (5) is simply an extension of

[2]Note that physical capital investment and intangible capital investments are treated differently given the different accounting rules. Investment in physical capital is spread out over time and expensed as depreciation, while the intangible capital costs are mostly treated as expenses at the time that they occur.

Hayashi (1982)'s result to a multi-factor inputs setting. Note that there is subtle difference between the shadow price of the two inputs, which comes from the different tax treatment of the inputs.

According to equation (5) the firm's market value is given by the sum of the value of the firm's installed capital inputs. This additive feature allows us to compute the fraction of firm value that is attributed to each input (henceforth referred simply as "input-shares") in a straightforward manner as follows:

$$\mu_{it}^{P} = \frac{q_{it}^{P} K_{it+1}^{P}}{q_{it}^{P} K_{it+1}^{P} + q_{it}^{I} K_{it+1}^{I}} \tag{8}$$

$$\mu_{it}^{I} = \frac{q_{it}^{I} K_{it+1}^{I}}{q_{it}^{P} K_{it+1}^{P} + q_{it}^{I} K_{it+1}^{I}} \tag{9}$$

The fundamental goal of the empirical analysis is to characterize these input-shares, including their variation across countries and over time.

3 Estimation Methodology

In this section we specify the functional forms and describe the estimation procedure. In subsection 3.1 we discuss the functional forms used to model adjustment costs which define the specification of firm equity value. In subsection 3.2 we describe the estimation procedure used that focuses on using portfolios to minimize the idiosyncratic noise in the data, and the definition of country and regions that we use to estimate the model.

3.1 Functional Forms

The valuation equation (5) only requires the specification of the adjustment costs function, not of the operating profit function. We consider the following quadratic adjustment costs function:

$$C_{it} = \frac{\theta_{P,c(i)}}{2} \left(\frac{I_{it}^{P}}{K_{it}^{P}} \right)^{2} K_{it}^{P} + \frac{\theta_{I,c(i)}}{2} \left(\frac{I_{it}^{I}}{K_{it}^{I}} \right)^{2} K_{it}^{I}, \tag{10}$$

in which $\theta_{P,c(i)}, \theta_{I,c(i)} > 0$ are the parameters that control the magnitude of the adjustment costs of each input. The subscript $c(i)$ is the country (or region as we describe below) of firm's i headquarters. Accordingly, in the baseline of the model, we let the adjustment cost parameters to vary across

countries to capture the cross-country differences in the market environment for physical capital and intangible capital investment. These variation might arise due to differences in the judicial system, culture, regulation of financial sector, among other.[3]

This functional form implies that the shadow prices of the capital inputs can be inferred from firm-level data on investment, capital stocks, and taxes, and are given by:

$$q_{it}^{P} \equiv 1 + (1 - \tau_{c(i),t})\theta_{P,c(i)} \left(\frac{I_{it}^{P}}{K_{it}^{P}} \right) \tag{11}$$

$$q_{it}^{I} \equiv (1 - \tau_{c(i),t}) \left[1 + \theta_{I,c(i)} \left(\frac{I_{it}^{I}}{K_{it}^{I}} \right) \right] \tag{12}$$

We adopt a simple quadratic adjustment cost specification for parsimonious reasons and to avoid parameter proliferation. There are several implicit assumptions in our simple specification, such as using gross flows, smooth, convex and symmetric adjustment costs. See Belo et al. (2022) for a discussion of these assumptions.

Denote the firm's total (effective) dollar amount of capital inputs (physical capital stock and intangible capital stock) as $A_{it} = K_{it}^{I} + K_{it}^{P}$. Accordingly, we write a firm's valuation ratio (VR_{it} $\equiv (P_{it} + B_{it+1}) / A_{it+1}$) as:

$$VR_{it} = q_{it}^{P} \frac{K_{it+1}^{P}}{A_{it+1}} + q_{it}^{I} \frac{K_{it+1}^{I}}{A_{it+1}}. \tag{13}$$

If the adjustment cost coefficients depend on institutions and economic development status, a cost function will summarize this relation. The vector of country-level characteristics $\mathbf{X}_{\mathrm{c}}$ includes various aspects of the market environment and is collected at the national level. The adjustment cost coefficients exogenous for the firm are denoted as $\theta_P(\mathbf{X}_{\mathrm{c}})$ and $\theta_I(\mathbf{X}_{\mathrm{c}})$. Therefore, for a firm i residing in country $c(i)$, the firm valuation ratio is defined as follows:

[3]Capital adjustment cost captures the flexibility in implementing the investment plan. The resource reallocation, internal coordination within the corporation is costly, but not directly reported in the firm cashflow statement.For example, in a country where the business secret is easily leaked to competing firms, firm is unwilling to invest, in the fear or being copied by the cohort firms. When there is new demand for the vehicle-design or cost optimization in assembly, despite the future profit flow is lucrative from the investment action, the concern of being copied by competitors, the expectation of coping competitors in the later round, would push the producers simultaneously postpone or taper the effort in launching the new design/investment. Though one doesn't see the direct payment to installing the new design, the manager and the in-house engineers all spent quite a lot of time in monitoring the industry and contemplating the decision of allocating the investment budget.

$$
\begin{aligned}
V\hat{R}_{it} = \quad & \left[1+\theta_P(\mathbf{X}_{c(i)})\cdot(1-\tau_{c(i),t})\left(\frac{I^P_{it}}{K^P_{it}}\right)\right]\frac{K^P_{it+1}}{A_{it+1}} \\
& +(1-\tau_{c(i),t})\left[1+\theta_I(\mathbf{X}_{c(i)})\cdot\left(\frac{I^I_{it}}{K^I_{it}}\right)\right]\frac{K^I_{it+1}}{A_{it+1}}
\end{aligned}
\quad . \tag{14}
$$

We use this specification to investigate to what extent a universal model can explain the firm valuation ratio in multiple countries jointly. For this state-contingent adjustment cost model, we will estimate the linear approximated function as:

$$
\begin{aligned}
\theta_I(\mathbf{X}) = \quad & \theta_{I,g} + \boldsymbol{\gamma^I}\cdot\mathbf{X}_{\mathrm{c}} \\
\theta_P(\mathbf{X}) = \quad & \theta_{P,g} + \boldsymbol{\gamma^P}\cdot\mathbf{X}_{\mathrm{c}}
\end{aligned}
\quad . \tag{15}
$$

The intercept coefficients $\theta_{I,g}$ and $\theta_{P,g}$ are the global average level of cost for each capital input, as each country characteristic are expressed as the deviation from the global average.[4] The vector of the incremental cost coefficients $\boldsymbol{\gamma^I}$, describes how deviation of a country in institutional environment and economic development, adds to the cost of investing intangible capital. Analogously, the vector γ^P is for investing the physical capital.

We conduct the estimation in steps. First, we directly estimate the country-specific cost parameters, to ensure the minimal assumptions. The main empirical findings will be detailed in Section 5. After verifying the dynamic investment model with intangible capital can reasonably explain the firm investments and valuation in most countries, we next unveil how the institution status determines the cross-country heterogeneity in adjustment costs. Description of countries and the model estimation outcome will be detailed in Section 6. Our strategy enables us to leverage the cross-section and time-series variation in firm value and investment rates, as well as the heterogeneity of economies, to properly identify the importance of country characteristics.

3.2 Estimation Procedure

We estimate the model separately in each country (or region, as explained below). Hence, to save notation, in this section we omit the country/region subscript in the model parameters.

The valuation equation (5) links firm value to the value of its capital inputs. The left-hand side

[4] In other words, $\frac{1}{N_c}\sum_c \mathbf{X}_c = \mathbf{0}$ holds by construction. These coefficients $\theta_{I,g}$ and $\theta_{P,g}$ indicate the level of adjustment costs when all country characteristics are set to their global averages

(LHS) of equation (13) can be directly measured in the data from equity price and debt data (and measures of the capital stocks, which we discuss below). The right hand side (RHS) of equation (13) is the predicted valuation ratio from the model, which we will denote as $\widehat{VR}_{it}$, and depends on firm-level real variables and model parameters.

Equation (13) establishes an exact relationship between a firm's observed valuation ratio and its model-implied valuation ratio at each time-period. However, due to the noise in firm-level data and the sensitivity of their moments to entry and exit and missing observations, using equation (13) and firm-level data to directly estimate the model parameters is challenging. Therefore, we follow the same methodology as Belo et al. (2022) and estimate portfolio-level moments. The portfolio estimation methodology targets the cross-sectional mean at the portfolio level and aligning the realized time series of portfolio-level valuation ratios with the model's predictions, provide robust estimates.

We proceed as follows. In theory, at each point in time, any cross-sectional moment of the observed firm-level valuation ratios in the LHS of equation (13) should be equal to any corresponding cross-sectional moment of the model-implied firm-level valuation ratios in the RHS of equation (13). Accordingly, in each country, for each portfolio j and for each year t, we compute the cross-sectional mean observed and model-implied valuation ratios ($\overline{VR}_{jt}$ and $\widehat{\overline{VR}}_{jt}$, respectively) of the firms in the portfolio as follows:

$$\overline{VR}_{jt} = \sum_{i} \frac{VR_{it}}{N_{jt}}$$

$$\widehat{\overline{VR}}_{jt}\left(\Theta\right) = \sum_{i} \frac{\widehat{VR}_{it}}{N_{jt}} \ , \ i \in \text{portfolio } j,$$

where Θ represents the vector of structural parameters in a given country, $\Theta = [\theta_P, \theta_I]$, and N_{jt} is the number of firms in portfolio j at time t. We target cross-sectional mean valuation ratios because these moments capture the economic behavior of a typical (average) firm in the economy, which is what the theoretical model is designed to study.

We then proceed under the standard assumption that the portfolio-level valuation ratio moments

are observed with error by the econometrician:

$$\overline{VR}_{jt} = \widehat{\overline{VR}}_{jt}\left(\Theta\right) + \varepsilon_{jt}, \tag{16}$$

where ε captures measurement error in the portfolio-level moments.[5] Based on equation (16), in each country, we then estimate the model parameters by minimizing the squared distance between the portfolio-level observed and model-implied valuation ratio moments at each point in time:

$$\widehat{\Theta} = \arg\min_{\Theta} \frac{1}{TN} \sum_{t=1}^{T} \sum_{j=1}^{N} \left(\overline{VR}_{jt} - \widehat{\overline{VR}}_{jt}\left(\Theta\right)\right)^2, \tag{17}$$

where T is the number of years in the sample, and N is the number of portfolios. An attractive feature of our estimation approach is that it corresponds to a simple linear ordinary least squares (OLS) estimation of (modified) portfolio-level average valuation ratios on portfolio-level averages of firm-characteristics. To show this, define the following variable,

$$\overline{VR}^{M}_{jt} = \frac{1}{N_{jt}} \sum_{i \in j} \frac{\left(P_{it} + B_{it+1} - K^{P}_{it+1} - (1-\tau_t)K^{I}_{it+1}\right)}{A_{it+1}}$$

as the modified (M) valuation ratio. In addition, define the variables

$$\overline{IPA}_{jt} = \frac{1}{N_{jt}} \sum_{i \in j} (1-\tau_t) \frac{I^{P}_{it}}{K^{P}_{it}} \frac{K^{P}_{it+1}}{A_{it+1}}$$

and

$$\overline{IKA}_{jt} = \frac{1}{N_{jt}} \sum_{i \in j} (1-\tau_t) \frac{I^{I}_{it}}{K^{K}_{it}} \frac{K^{I}_{it+1}}{A_{it+1}},$$

which are the portfolio-level average physical and intangible capital, respectively, contribution to firm value. We can then write equation (16) simply as:

$$\overline{VR}^{M}_{jt} = \theta_P \overline{IPA}_{jt} + \theta_I \overline{IKA}_{jt} + \varepsilon_{jt} \tag{18}$$

[5] Mismeasured components of the valuation ratio such as the market value of debt and the capital inputs can be better observed by firms than by econometrician. Furthermore, the intrinsic value of equity can temporarily diverge from the market value of equity.

which establishes a linear relation between the portfolio-level modified valuation ratio and portfolio-level characteristics. Thus, our objective function in (17) corresponds to a simple linear OLS regression of equation (18). The functional forms outlined in equation (15) enter linearly in the value ratio of equation (14), allowing us to estimate the set of parameters, global average cost θ_g and incremental cost coefficients γ using the portfolio-level estimation method outlined. Importantly, this method ensures that countries with a large number of firms do not skew the parameter estimates, as each country or each region of the remaining area within continents is represented by an equal number of observations—specifically, the number of portfolios.

3.3 Geographic Areas and Portfolio Sorts

As noted above, the estimation is performed at the portfolio-level and separately for each country or region.

Geographic Areas. To link a firm to a country, we use the firm's headquarter.[6] Ideally, we would like to estimate the model separately for all the countries in the sample. This approach is not possible, however, as the number of firms and observations is too small in some countries. In these cases, we combine multiple countries from a similar geographic area into a region.

Specifically, we define large equity markets as those countries having data for at least 200 firms in the time-series average during the sample period, and medium equity market as the countries with at least 100 firms. We estimate the model for the economies with large-size equity market, and the rest of area in each continent separately.

In the continent of *Asia*, countries (or territories) China, Hong Kong, India, Indonesia, Japan, Malaysia, Singapore, South Korea, Taiwan, Thailand have large-size equity markets, under this criterion. In the remaining area of Asia, Israel, Pakistan, Turkey, Vietnam have the medium-size equity markets. Small-size countries are: Southern Asia (Bangladesh, Sri Lanka), South-Eastern Asia (Philippines), Western Asia (United Arab Emirates, Bahrain, Cyprus, Jordan, Kuwait, Oman, Palestine, Qatar, Saudi Arabia).

In the continent of *Europe*, France, Germany, Poland, United Kingdom, are economies with the large number of listed companies. In the remaining area of Europe, Greece, Italy, Sweden,

[6] For robustness check, we also consider defining the location of firm as its residing country of incorporation. The result are similar.

Switzerland have the medium-size equity market. Small-size countries are: *Southern Europe* (Spain, Croatia, Malta, Serbia, Slovenia), *Eastern Europe* (Bulgaria, Hungary, Romania, Russia, Ukraine), *Northern Europe* (Denmark, Estonia, Finland, Ireland, Iceland, Lithuania, Latvia, Norway), *Western Europe* (Austria, Belgium, Luxembourg, Netherlands, Portugal).

In the continent of *Americas*, Canada, United States of America are large economies. In the remaining area of Americas, Brazil has the medium-size equity market. Small-size countries are: *Latin America and the Caribbean* (Argentina, Chile, Colombia, Cayman Island, Jamaica, Mexico, Peru).

We estimate the adjustment cost parameters for each country and remaining regions separately. This procedure avoids double-accounting of observations. In *Oceania*, we don't have valid observations of listed firms locating in this continent after the large economy Australia is selected out. In *Africa*, South Africa is has the largest equity market and falls under the medium-sized equity market category in our global classification. Other small-size countries in Africa include *Northern Africa* ountries (Morocco and Tunisia) and *Sub-Saharan African countries* (Cote D'ivoire, Kenya, Mauritius, Nigeria).

In summary, we use a simple classification of four regions: *Rest of Asia*, *Rest of Europe*, *Latin America*, and *Africa*. The Oceania sample for estimation includes only one large economy, Australia. Hence, our analysis covers 77 countries across five continents. For simplicity, the model of country-specific adjustment cost assumes countries of a small market locating in the same region have identical adjustment cost. This simple model equates to separated estimations in 21 different geographic areas: 17 countries with large equity markets, and four regions– *Rest of Asia*, *Rest of Europe*, *Latin America*, and *Africa*.[7] This assumption will be relaxed in the formal universal model of state-contingent adjustment cost, where observations in all countries jointly identify the determinant of adjustment cost.

Portfolio sorts. For each one of 21 geographic areas, we form portfolios sorted on the following variables: $\left(\frac{I^P_{it}}{K^P_{it}}\right)\left(\frac{K^P_{it+1}}{A_{it+1}}\right)$, $\left(\frac{I^I_{it}}{K^I_{it}}\right)\left(\frac{K^I_{it+1}}{A_{it+1}}\right)$. Sorting on these variables generates a large dispersion in the explanatory variables which helps for the identification of the model parameters. (In the

[7] Distribution of the 21 geographic areas in the five continents are: *Asia* (China, Hong Kong, India, Indonesia, Israel, Japan, Malaysia, Singapore, South Korea, Taiwan, Thailand, *Rest of Asia*), *Europe* (France, Germany, Poland, United Kingdom, *Rest of Europe*), *Americas* (Canada, United States of America, *Latin America*), *Oceania* and *Africa*.

online appendix, we show that the model results are robust across different sorting variables.) We then follow Fama and French (1993) in constructing the portfolios. Specifically, all firms in each year t are two-way sorted into 9 portfolios based on the quintile of the each sorting variable of each firm for the fiscal year ending in $t-1$. The portfolios are re-balanced at the end of each year. This procedure gives a total of $9 \times 21 = 189$ portfolios from all countries in the sample.

4 Data

In this section, we provide a general description of the data.

We construct firm-level measures of market value, input investment, and stock using the financial reports of publicly traded firms in each country. For firms in the United States and Canada, we collect annual balance sheet information from Compustat North America Annual Fundamentals and stock price data from the Compustat-CRSP linked dataset. For firms in other countries, we gather annual information from Compustat Global Annual Fundamentals and stock prices from Compustat Global Security Daily. [8]

We set the currency as the U.S. dollar for all countries. The frequency is annual and varies by country. We deflate the variables using the country-specific consumer price index. We set the starting-year as the year when the country has sufficient firm-year observations. For major economies, the data spans 2000-2020 (see Table OT.1 for individual country samples, and the starting year for each country or region). The sub-samples of countries in Asia start around 2000, while those for main countries in Europe and Africa start around 2006.[9]. The full records for listed companies in North America date back further; however, for estimation to be comparable to other countries worldwide, we use the sample for Canada and the United States starting from 2000. The end-year is 2020 for all countries and regions.

[8] Additional details about the data sources and the harmonization of measures are available in the Section D of the online appendix.

[9] Gauging the amount of intangible capital requires detailed income statement information that conforms to the international financial statement reporting standard. The sub-sample of listed companies in different countries doesn't start in the same years. This is partly constrained by the adoption of common accounting standard around the year 2005.

4.1 Capital stocks and other variables

4.1.1 Physical Capital

The initial physical capital stock, K_{it}^P, is given by net property, plant, and equipment (data item PPENT). The capital depreciation rate, δ_{it}^P, is calculated as the amount of depreciation (data item DP) divided by the beginning of the period capital stock. We then construct a measure of the firm's capital stock at current prices. Specifically, we construct an investment-price adjusted capital stock that accounts for changes in the real cost of physical capital investment by repricing last period's capital stock using today's price of investment (P_t^P) as $K_{it+1}^P = K_{it}^P(1-\delta_{it})\frac{P_{t+1}^P}{P_t^P} + I_{it+1}$. Following Belo et al. (2022) we infer physical capital investment from the law of motion of capital using this equation (adjusted for inflation). This procedure ensures that the investment and capital stock are consistent with the law of motion for physical capital in the model.

4.1.2 Intangible Capital

Following Eisfeldt and Papanikolaou (2013) we construct a measure of intangible capital based on Selling, General, and Administrative (SG&A) expense data (Compustat data item XSGA).[10] We calculate the installed amount of capital using the perpetual inventory method as follows:

$$K_{it+1}^I = \mathrm{I}_{it+1}^I + (1-\delta^I)\cdot K_{it}^I \cdot \frac{P_{t+1}^I}{P_t^I}. \tag{19}$$

where P_t^I is approximated as the CPI of home country in local currency. The depreciation rate, δ^I, is calibrated as 20%. We set investment expenditure equal to 30% of SG&A expense following Eisfeldt and Papanikolaou (2013) and Peters and Taylor (2017).[11] We set the initial amount of

[10] In the US and Canada, firms report the detailed expenditure of R&D and advertising, estimation could differentiate heterogeneous capital inputs such as knowledge and brand, (see Belo et al. (2022)). Firms in other countries don't universally report these details. The quantity of investment in intangible capital cannot be inferred from the historical records related to goodwill and other intangible assets in balance sheet. These are valuation-based measures. When comparing the valuation of firms across countries, the SG&A expense is the most comparable measure for the investment expenditure in intangible capital.

[11] The fraction of firms disclosing detailed expense of R&D and advertising, is not comparable across countries. Section 7.4 examines the estimation using alternative calibration, by adapting the procedure in Peters and Taylor (2017) in other countries. Section 7.4 examines the model estimation in which the quantity of intangible capital is inferred using the firm operating profit. The conclusion for the importance of intangible capital, and cross-country variation are similar.

capital as

$$K_{i0}^{I} = \frac{I_{i0}^{I}}{g_{\text{Ind}(i)}^{I} + \delta^{I} - \pi_{\text{Ind}(i)}^{I} \cdot (1 - \delta^{I})}. \tag{20}$$

in which I_{i0}^{I} is the firm's investment in intangible capital in the first year in the sample, and $\pi_{\text{Ind}(i)}^{I}$ is the average price growth rate, in the industry, in each country. We let $g_{\text{Ind}(i)}^{I}$ be industry-specific and set it equal to the average growth rate of the SG&A expense in that industry. We consider the first 2-digits of NAICS industry code to classify the industry in each country. Once we have the initial amount of capital, we derive the new amount of capital in equation (19), using the depreciation rate, SG&A expenses, and investment price index. The investment rate on intangible capital is then given by the ratio of the current period investment and the amount of intangible capital at the beginning of the period I_{it}^{I}/K_{it}^{I}.

4.1.3 Additional Variables

We measure the debt value B_{it}, as the book value of net total debt, following Belo et al. (2022). We calculate net debt as long-term debt (Compustat data item DLTT) plus short-term debt (data item DLC), minus cash (data item CHE). When these values are missing, we set the measure to zero. The market value of equity, P_{it}, is the closing price per share (data item PRCCF) multiplied by the number of common shares outstanding (data item CSHO). The market value is based on the year-end price during the firm's fiscal year. All nominal values in local currency are converted into nominal USD using the annual-average exchange rate.

We measure the tax rate, τ_t, as the corporate income tax rate from the Tax Foundation, available for each country. If corporate income tax rate information is unavailable, we use data from Compustat Global-Economic Indicators. Stock variables with subscript t ($t+1$ for debt) are measured and recorded at the end of year t, while flow variables with subscript t are measured over the course of year t and recorded at the end of year $t+1$.

4.2 Summary Statistics

Our sample of 77 countries is representative of the total production across the world. Our sample includes 29,631 firms in total, with sales representing 40.15% of world GDP in the end year of sample period. Table OT.1 reports the descriptive statistics. [12]

Table 1 reports key summary statistics of the observed valuation ratios and their model-implied components according to equation (13), for the major equity markets and regions. The median valuation ratio across all major markets is 1.44, but there is substantial heterogeneity across countries. While China has the highest average valuation ratio, at 2.95, Japan has the lowest, at 0.87.

According to equations (11) and (12), the investment rates determine the shadow prices of the labor and capital inputs. Together with relative magnitude of each capital stock, these variables determine the relative importance of each input for firm value. Columns (3) and (4) in Table 1 show that investment in intangible capital is, on average, higher than investment in physical capital across all countries. The global median investment rate in intangible capital is 25%, with a maximum of 34% in China and a minimum of 20% in Indonesia. The global median investment rate in physical capital is 15%, with a maximum of 24% in the USA and a minimum of 8% in India. In terms of relative magnitudes of the capital stocks, column (5) shows that intangible capital is about 38% of total (physical plus intangible) capital across the globe. Again, this average exhibits substantial variation across countries, with a maximum of 67% in France and a minimum of 21% in Canada.

Column (6) of the table reports the cross-correlations of investment rates. As expected, the investment rates are all positively correlated, ranging from 21% to 37% across major equity markets. These correlations are significantly less than one, suggesting that the investment rates for different types of capital inputs exhibit distinct variations in the data. Distinguishing the shadow prices could help explain the observed variation in market value.

[12] In Asia, the 10 large equity markets and 16 countries in the Rest of Asia, include 16,133 firms, with sales representing 41.73% of GDP in these countries in 2020. For these large countries, per capita GDP in 2020 ranges from $1,849 in India to $56,423 in Singapore. In Europe, the 4 main equity markets and the 27 countries in the Rest of Europe include 5,380 firms, with sales representing 35.28% of GDP in these countries. In Canada, the United States, and 8 countries in Latin America, the sample includes an average of 6,480 firms, with sales representing 44.07% of GDP in these countries. In Australia, listed companies in the sample have sales equal to 23.23% of GDP. For the 9 countries in Africa, the sales of the listed companies in the sample represent around 15.05% of the GDP in these countries.

5 Estimation Results

This section presents the main empirical findings. Subsection 5.1 reports the parameter estimates and the baseline model's overall fit. Subsection 5.2 discusses the model-implied breakdown of firm value, decomposing market value into physical and intangible capital. This analysis underscores the significance of intangible capital in capturing a firm's market. Subsection 5.3 examines the heterogeneity in the market share of intangible capital, distinguishing between quantity and price variations. The analysis highlights the importance of country-specific adjustment costs in explaining cross-country differences in firm value.

5.1 Parameter Estimates and Model Fit

In Table 2, columns (2-3) and (8-9) present the estimates of adjustment cost parameters for the model applied to larger equity markets and regions. All estimates are positive and statistically significant. The intangible capital adjustment cost parameter (θ_I) is consistently higher than the adjustment cost parameter for physical capital (θ_P), but there is considerable heterogeneity across countries in the estimate magnitude of each parameter. The summary statistic for the "world" region shows that the cross-country average adjustment cost coefficient for physical capital is 3.87, while the average adjustment cost coefficient of intangible capital is 11.63.[13] In addition, the dispersion in the estimated adjustment cost coefficient of intangible capital θ_I is more pronounced than that for physical capital. The standard deviation of estimates across countries for physical capital is 1.73, ranging from 0.78 for Japan to 8.10 for USA.[14] In contrast, the cross-country standard deviation for intangible capital estimates is 6.10.

Notably, θ_I estimates are relatively low for European countries such as France (7.75), Germany (10.27), and the U.K. (8.91), but high in North American countries such as United States (16.32)

[13] The cross-country average outcome is calculated including the regions, where there are multiple countries within. As previously mentioned, we assume identical adjustment cost function for countries with the small-size equity market in the same region, considering they have the similar cultural background and legislation origin. This ensures the sufficient sample size to accurately identify the model. Hereafter, the name of terminology of statistics don't differentiate between the countries and the regions of the remaining area in each continent.

[14] As explained in previous section of estimation methodology, within the Latin America, we assume the identical adjustment cost function for the medium-size country Brazil, and other small-size countries. The calculation of standard deviation describes the variation across the 17 large-size countries and the 4 regions where the constituent countries are assumed to be of identical parameters within each region. We explore the more granular estimates in the extended estimation of country-characteristic model.

and Canada (11.53). The pattern is less clear in Asia, where estimates of the intangible capital adjustment cost parameter are low for Japan (2.91), South Korea (5.72), Hong Kong (6.97), and Singapore (6.99), but high for China (32.54), India (19.83), and Taiwan (15.23). In contrast, European countries such as France and Germany show only small differences in the estimated adjustment cost coefficient of intangible capital. The dispersion of the estimated adjustment cost coefficient of intangible capital is much larger in Asia, reflecting the varying levels of economic development and, consequently, the different market environments in which firms operate.

To assess the economic significance of the adjustment cost estimates and evaluate whether the model matches the data with economically reasonable parameter values, we use the functional form specification in equation (10) and the parameter estimates to calculate the realized adjustment costs of each input (denoted as CP and CI). These values are computed as a fraction of a firm's total annual sales as follows:

$$\frac{CP_{it}}{Y_{it}} = \frac{\frac{\theta_{P,c(i)}}{2}\left(\frac{I^P_{it}}{K^P_{it}}\right)^2 K^P_{it}}{Y_{it}} \tag{21}$$

$$\frac{CI_{it}}{Y_{it}} = \frac{\frac{\theta_{I,c(i)}}{2}\left(\frac{I^I_{it}}{K^I_{it}}\right)^2 K^I_{it}}{Y_{it}}. \tag{22}$$

Table 4, columns (2) and (3), report the average realized adjustment costs of each input, computed as the time-series average of cross-sectional medians of the ratios in equations (21) and (22). China stands out as having the highest adjustment cost of intangible capital, at 18.19% of annual sales, followed by 12.13% in the US. For physical capital, the US and European countries top the list.

Table 4 reports the numbers for all firms in our sample (the World). In each year, we calculate the median cost among all firms. In the most recent five years of our sample, the average intangible capital adjustment cost is 5.56% of sales. This cost is higher than the adjustment costs for physical capital, which is about 1.26% of sales. The aggregate cost, expresses as a percentage of the aggregate sales, shows a similar distribution across types of capital and the countries. The fact that the median statistic is higher than the aggregate ratio, reflects that adjustment cost for capital investment matter more for small- to medium-sized firms in each country.

The model with both physical and intangible capital fits the data well when we evaluate model-fitness using the cross-sectional and the time-series fitness measures. In Table 3, the summary panel shows that the cross-sectional R^2 is high, at 87% across all portfolios worldwide, even though the model estimation does not explicitly target this moment. The average time-series R^2 is 57%.

In terms of average valuation ratio errors, the model scaled mean absolute error (m.a.e./$\overline{\text{VR}}$) is 24% on average. Since the subset of portfolios in each country independently identify the adjustment cost coefficients specific to that country, we also report the fitness statistics for individual large countries and regions in Table 2. There is less variation within the country compared to the worldwide. However, the model perform well in most countries, including Indonesia, India, South Korea and United Kingdom. The good model-fit implies that the generalized neoclassical model with intangible and physical capital describes the valuation of firms well across a wide variety of countries.[15]

Overall, the estimation results show that adjustment costs of inputs vary across countries and regions, particularly for intangible capital. The results in Table 2 highlight the importance of quantifying heterogeneous market environments using country-specific adjustment cost parameters.

5.2 The Value of Intangible and Physical Capital

The parameter estimates allow us to compute the model-implied shadow prices of each input and evaluate the contribution of each input to firm value (input shares) based on each input's market value. Specifically, using the estimates reported in Table 2, we compute the model-implied scaled value of each capital input, the values of $q^P_{it}\frac{K^P_{it+1}}{A_{it+1}}$ and $q^I_{it}\frac{K^I_{it+1}}{A_{it+1}}$, for each firm in each year. We then substitute these values into equations (8) and (9) to compute the shares of each capital input.[16] To

[15]There is no simple linear relationship between the firm valuation, the investment rate of physical capital, and that of intangible capital. Especially, the bi-variate distribution of the firm valuation and the intensity of intangible capital, is not the simple monotonic relationship.

Table OT.13 examines the portfolios where firms are grouped using the lagged period investment rate of physical capital, the investment rate of intangible capital. We derive the firms valuation ratio using the parameter estimates in Table 2, then the portfolio-level valuation ratio is calculated for these portfolios of alternative firm characteristic variables. Each firm characteristic variable has 10 decile portfolios. In total, there are 21×20 portfolios cross countries and regions. In these alternative portfolios that are not directly used for model estimation, the measurement error is sufficiently diversified. In each set of portfolios in those 21 countries and regions, the fitness of the model is high, with comparable statistics value to Table 2. The model with the country-specific cost and two capital inputs explain the firm valuation successfully.

[16]Note that, by construction, the input shares sum to 100%. For brevity, we report the share of intangible capital in firm valuation.

characterize the data in a comprehensive yet parsimonious manner, we summarize the properties of the firm-level input shares in the economy. We compute the median of the intangible share in each year and each country, then calculate the mean across years for each country.

In Table 4, Columns (2) and (8) show the estimated importance of intangible capital for each country and region within each continent. Given the firm-level decomposition of capital, we can calculate the worldwide median market share of intangible capital, which is about 53.88. This indicates that the contribution of intangible capital to a firm's market value across the globe is higher than the contribution of physical capital.

Overall, this analysis shows that intangible capital is an important determinant of firms' market values across the world. As summarized in Table 3, we re-estimate the counterfactual model, where the firm uses only physical capital for the capital inputs in production. [17] This counterfactual model assumes zero contribution from intangible capital to firm value. To provide a meaningful comparison of the model fit in terms of R^2, we employ the same set of firms used in the baseline model estimation. Comparing the model fitness of the physical capital-only model, which exhibits mostly negative R^2 values, offers another perspective on the importance of intangible capital in firm value.

When an input is costly to adjust, the installed values of the inputs become valuable to the firm because accumulated capital inputs help avoid adjustment costs in the future. If adjustment costs are zero, the shadow prices of the inputs in equations (11) and (12) are simply one (for physical capital) and $(1-\tau_t)$ (for intangible capital). As a result, the value of each capital input is equal to its book-value (adjusted for the tax rate). Columns (5) and (11) list the book share of intangible capital, $\overline{\mu_I}$, in the counterfactual estimation of the contribution of intangible capital to firm value assuming zero adjustment costs. Compared to the 53.83% average cross-country market share, the cross-country average book share is 30.90% for the major equity markets. China stands out with a 24.22% book share of intangible capital versus a 68.93 market share.

The observed high market value of intangible capital can be attributed to two distinct channels. The first is the quantity channel, where a high book share corresponds to a high market share. This pattern is evident in the United Kingdom, United States and developed European countries. The

[17] Table OT.5 in the online appendix, reports the details of parameter estimates and model fitness.

second channel is the price channel, where a high market share is observed when implied shadow price is high due to the costly investment in intangible capital. This is particularly true for East Asia. Notably, China exhibits the highest intangible investment cost ,θ_I, in the entire sample. These findings highlight the interplay between the quantity and price channels in explaining the market value of intangible capital across different economies.

In summary, the findings presented in this section underscore the importance of the intangible capital input in modern corporations that heavily rely on a high-skill labor force and new technology.

5.3 Heterogeneity of Intangible Capital Value Around the World

The contribution of intangible capital to firm value (μ_I) exhibits substantial dispersion. In Table 4, across the 17 countries and 4 regions where the heterogeneous adjustment cost parameters are directly estimated, the standard deviation is 9.32%. While China sits at the top of the intangible market share, with about 68.93% of the market valuation coming from it, Canada is at the bottom with 33.31. Large economies such as the USA and the UK also have above-average intangible capital market shares, at 64.39 and 63.09, respectively.

Figure 1 provides a visual description of the share of intangible capital across all the countries in our sample. The darkness of the color indicates the share of intangible capital, with darker shades representing higher shares. Countries in the Northern and Western Europe exhibit particularly high intangible market shares, while countries in East Asia show relatively lower shares. Within Asia, the median intangible market share in China is 68.93%, notably higher than Japan's 47.85%. There is significant heterogeneity among individual countries in the contribution of intangible capital to firm value, with a standard deviation of 12.97% across the 77 countries. [18]

The quantity channel and the price channel jointly determine the level of firm value and, correspondingly, the composition of capital in firm value. In countries where the cost coefficient of intangible capital is larger than the worldwide average, intangible capital becomes more important for firm valuation, and vice versa. When adjustment costs vary across countries, the importance of intangible capital in firm value changes accordingly.

If the adjustment cost function is homogeneous across all countries, the shadow price of the

[18] Detailed statistics are provided in Table OT.4.

capital inputs in each country would lead to biased estimation outcomes. We re-estimate the counterfactual model with a homogeneous cost function, summarized in Table 3. [19] The cost parameter for physical capital is 3.63, while that for intangible capital is 11.73. Although these cost coefficients have significantly positive point estimates, this model provides a poor description of the geographical variation in firm value.

In Table 4, columns (6) and (12) present the counterfactual estimation of the contribution of intangible capital to firm value, assuming identical adjustment cost parameters across all countries. Under this assumption, the contribution of intangible capital to firm value ,$\hat{\mu}_I$, for China is 46.51% on average during 2016–2020, compared to the true estimate of 68.93%. A similar underestimation occurs in India. Conversely, assuming global homogeneous capital adjustment costs results in an overestimation for countries like South Korea, where the counterfactual estimate of $\hat{\mu}_I$ is 45.37%, while the true estimate is 40.65%. France exhibits a similar overestimation scenario.

These findings align with the parameters reported in Table 2, which show that adjustment cost parameters for intangible capital in China and India are much higher than the global homogeneous cost, whereas those for South Korea and France are below the global average. In countries such as Thailand and Germany, where the adjustment costs for physical and intangible capital are close to the global average, the counterfactual estimate of $\hat{\mu}_I$ closely matches the true estimate of intangible capital's contribution.

We re-estimate the counter-factual model, where intangible capital has the varying cost, investing in physical capital involves the country-invariant adjustment cost, summarized in Table 3. [20] By comparing this restricted model and the benchmark model, we are able to understand the role of cross-country variation in intangible capital cost, in explaining the firm valuation around the globe. We find the model fitness is much better than the counter-factual model of homogeneous cost, but still distant from the benchmark model with both costs varying in countries.

Overall, the estimation results show that the composition of capital in valuation varies across countries. This geographical variation is jointly determined by the quantity of capital and the heterogeneity of capital adjustment costs.

[19] Table OT.6 in the online appendix, reports the the details of model fitness.
[20] Table OT.7 in the online appendix, reports the details of parameter estimates and model fitness.

6 Economic Drivers of Intangible Capital Investment

Modern corporation is a sophisticate organization of managers, engineers and professionals, not a simple electrical machine. Investment activities involve indirect cost from implementation: internal communication and learning, compliance of legal and industry regulation, lobbying and networking for external resources. The difficulty of verifying the authentic information generates the cost in implementing decisions that involve the high opacity and uncertainty, especially for the intangible capital. Given the same total expenditure, larger fraction evaporates as the adjustment cost, while only a smaller amount is accumulated as the new intangible capital that lasts long for creating profit. The indirect expense for investing intangible capital is more sensitive to the legislation system, efficiency, and financing regulation, in comparison to physical capital.

The findings from the previous section highlight the importance of geographical variation in adjustment costs in explaining cross-country differences in firms' market values and their inputs. In this section, we explore the key factors driving these differences in adjustment costs and input market values. To do so, we re-estimate an universal model using an adjustment cost function that incorporates country-specific characteristics likely to affect firms' ability to invest, and analyze the importance of these characteristics for firm valuation. Subsection 6.1 provides a brief overview of the data used to quantify these country characteristics.

6.1 Country Characteristics Data

Building on previous studies (discussed below), we evaluate three dimensions of country-level characteristics that may impact the adjustment costs of inputs: (a) Protection of Intellectual Property Rights; (b) Ease of Contract Enforcement; (c) Economic and Financial Development.

Protection of Intellectual Property Rights. Antill et al. (2023) quantify the effect to social welfare from the style of legal system in the decision of patent litigation. The patent litigation system in an economy, determines how an incumbent firms can deter the entry of new firm with similar patents. A patent litigation system favoring the existing patent holder encourages more investment in innovation. In Crouzet et al. (2022b), the ease of imitating product design determines the cross-section distribution of firm size, firm value, and the factor share. For intellectual property rights institutions, our primary measure uses expert and professionals' assessment over the respect of

trade secrets and industrial patents, collected by the Institutional Profiles Database. This measure of business environment describes the ease of securing the ownership of the intangible asset that generates exclusive corporate cashflows. Driven by the local culture and historical origin, this measure is of high similarity to copyrights and other intangible goods, the (dis)respect for the industrial counterfeiting, and the summary indicator of IPR protection provided by the WIPO. [21] Hence this is a representative measure describing the ease of creating new intangible capital.

Ease of Contract Enforcement. The complex, costly, and slow legal procedures will undermine trust and increase uncertainty, add unnecessary cost to business activities. For contracting institutions, we use expert and professionals' assessment over the cost of enforcing private contracts, specifically the non-eviction commercial case. Djankov et al. (2003) documented the efficiency of judicial system differs by the legal origin of a country. eg. British colonies with common law systems versus countries with civil law traditions. They found the heavier formalism and procedural cost in civil law countries. Originating from the work by Djankov et al. (2003) , the World Bank maintains the Ease of Business survey to continuously monitor the status of contract enforcement worldwide. Our primary measure is the number of procedures necessary to resolve a court case collecting the commercial debt. Despite the expansion of public service sector and the introduction of automation technologies, the contract complexity is under the slow evolution. Among the sub-sample of countries with early investigation by Djankov et al. (2003) , our measures of procedures cost have strong similarity to the index of legal formalism.

Other than the two primary state variables related to the intellectual property right institution and the contracting institution, we include the out-of-model state variables of economic development and financial market development. These supplementary state variables are income per capita, the population, and the market power of bank. The real GDP per capita is collected using the version of United Nation statistics [22]. The market power of bank is measured as the ratio between the bank

[21] The Variable B6020 is reported as a discrete score in range of {0, 1, 2, 3, 4}, with higher scores indicating stronger legal protections. Institutional Profiles Database reports additional variables related to intellectual property protection, such as industrial counterfeiting (Variable B6021), copyrights and intangible goods ((Variable B6022), and an index (Indicator B602). These variables show a high correlation with the variable we selected, leading us to omit them from our analysis. Similarly, the World Intellectual Property Organization provides an index that is also highly correlated with our chosen variable. The appendix provides the estimation of extended model, in which the supplementary characteristic of product market entry barrier is included, to describe the cost of leaking intangible capital to rivalry companies.

[22] The national account statistics for the territory of Taiwan is from the Penn World Table, imputed to be comparable with other countries in UN statistics.

income and the operating cost, derived using the World Financial Development database. [23]

We collected country characteristics for 73 out of the 77 countries analyzed. [24] Table 5 describes how the adjustment cost coefficients in baseline estimation of Section 5 coincides to the strength in protecting intellectual property right and the procedure cost in contract enforcement. We report the average adjustment cost for the 17 large countries with available direct estimates of individual cost coefficient in previous section. For simple comparison of how country-specific cost parameter determines the adjustment cost, we assume the artificial firm with investment rate $i^I = 20\%$ in intangible capital, use the adjustment cost coefficient $\theta_{I,c}$ to calibrate the cost per unit of installed capital $\frac{C^I}{K^I} = \frac{\theta_{I,c}}{2} \cdot (i^I)^2$. Within a group of countries, the average cost $\frac{1}{N_c} \sum_c \frac{\theta_{I,c}}{2} \cdot (i^I)^2$ is reported. Among the two emerging countries China and India of weak intellectual property protection, the average adjustment cost is 50.94% of installed intangible capital. Among the countries with the highest rating in intellectual property protection, Australia, France , UK , Japan, South Korea, Singapore , USA , the average cost in investing intangible capital is 16.93% of installed intangible capital, smaller in comparison to the countries with the medium rating and weak rating. But we didn't observe significant discrepancy in adjustment cost of physical capital. Analogously, we report the joint distribution of contract procedures and the adjustment cost. We use the 33%, 67% breakpoints of characteristic variables, to classify the countries into group of simple procedures, and of complicate procedures. Countries such as Indonesia and India have lengthy procedures of contract enforcement, coincide to those of high adjustment cost in intangible capital. The observed discrepancy in adjustment cost of physical capital is relatively smaller, in comparison

[23] The income-cost ratio has slow time-series variation in each country. We use the time series averages from 2016 to 2020.

[24] Table OT.8 in the appendix provides an overview of the country characteristics analyzed. The Institutional Profiles Database does not include data for Cayman Islands, Palestine, or Zambia. Romania is not included in the Doing Business survey of United Nations. In Columns (7) to (10) of Table OT.8, we present the correlation of each characteristic with the adjustment cost parameters discussed in subsection 5.1. As expected, strong protection of intellectual property rights is associated with lower adjustment costs, particularly for intangible capital. Conversely, a higher number of procedures correlate with higher adjustment costs. Table OT.9 describes the joint-distribution of country institutions and the firm valuation outcome. These are the descriptive statistics in Table 1, in summarizing the firms for each countries, calculated as the time-series average level within a country. The correlation between the firm valuation and the strength of protecting intellectual property rights is negative -0.44, and weak positive correlation 0.27 to the procedures of contract enforcement. Firm valuation tends to be higher among those countries with weak institutions. However, there is a different landscape between the status of institutions and the firm investment. Strength of protecting intellectual property rights has positive correlation to the firm physical capital investment rate. Both the physical capital investment rate and the intangible capital demonstrate positive correlation to the complexity in contract enforcement. The contrary correlations observed in firm valuation and the firm investment rate is driven by the subtle role of friction in accumulating the capitals, parameterized as adjustment cost.

to the intangible capital.

These descriptive evidence motivates a formal model to separate the effects of institutions, economic and financial development in the adjustment cost of capitals. We specify the dependence of adjustment cost parameters on these variables $\theta(\mathbf{X}) = \theta_g + \boldsymbol{\gamma} \cdot \mathbf{X}_c$, estimate the model with state-contingent cost described in equation (16). Denote the set $\mathbf{X}_c = (\mathbf{X}_1, \mathbf{X}_2)$ as the country characteristics. To begin with the simple state variables, the primary characteristics explaining the adjustment cost are $\mathbf{X}_1 = \{X_z\}_{z \in \mathbf{Z}}$, in which the set $\mathbf{Z}$ include (a) the intellectual property protection, (b) the contract enforcement cost, and (c) the background characteristic of economic development, population, bank power. The $\mathbf{X}_2$ is a fixed effect that reflects the usage of intangible capital is intensive in a given country. [25] This allows for the different steady states among those countries with small amount of intangible capital, and those countries with large amount of intangible capital. The estimation identifies the global average costs θ_g and incremental cost coefficients γ. We detail the estimation outcome in subsection afterward.

6.2 Estimation Results

Here we present the results from the estimation of the model using the adjustment cost parameter specification from equation (15). The results for the baseline specification are reported in columns (1) and (2) of Table 6. In previous description of the one-by-one correlation of each country characteristic with the adjustment costs estimated in subsection 5.1 is presented, we note that the sign of the correlation may also reflect the relationship of that characteristic with other variables. By employing joint estimation to assess the sensitivity of the adjustment cost parameters to all

[25] We categorize countries based on their average book share of intangible capital (K_I/A), classify the group in the highest quartile. Appendix Table OT.10 report the descriptive statistics of country characteristics conditional on their endowed intangible capital. We group the countries into 4 basket, based on the intensity of intangible capital with respect to the physical capital. Among those large countries, the United States, United Kingdom, France and Germany belong to the top quartile of countries, with the high accumulated amount of intangible capital and intensive usage. As shown by the table, there is high protection of intellectual property rights, smaller number of procedures required by completing a standard property law contract, also the bank profitability is much lower. The countries that intensively utilize intangible capital in the production, show adapted institution environment and financial intermediary sector. Alternative robustness estimation show that this conditional effects are approximately close among the lowest three quartiles. In estimation, the lowest intensity group is set as the basis group, latent parameters $(\theta_{I,g} - \gamma^I_{2,n\leq 3}, \gamma^I_{2,n=4} - \gamma^I_{2,n\leq 3})$ are directly estimated. Then we implement the linear transformation using condition $\frac{3}{4}\gamma^I_{2,n\leq 3} + \frac{1}{4}\gamma^I_{2,n=4} = 0$ to infer the parameters $\theta_{I,g}$. For physical capital, we similarly require $\sum_{n=1}^4 \gamma^P_{2,n} = 0$, and the identification of parameters $(\theta_{P,g}, \{\gamma^P_{2,n\leq 3}, \gamma^P_{2,n=4}\})$ is similar. For informative notations, we denote $\gamma^I_{2,n=4}$ as $\gamma^I_{2,\text{Top25\%}}$ and $\gamma^P_{2,n=4}$ as $\gamma^P_{2,\text{Top25\%}}$. The linear model is derived from the local perturbation of adjustment cost function. The detailed derivation of regime switch adjustment cost is provided in the Mathematical appendix.

characteristics simultaneously, we isolate the individual effects of each characteristic.

First, note that the intercepts ($\theta_{I,g}$ and $\theta_{P,g}$), which capture the average global values of each adjustment cost, are significantly higher for intangible capital than for physical capital (11.89 compared to 4.22). This finding aligns with the baseline country-level results. Additionally, countries with stronger protection for trade secrets and industrial patents experience lower costs for investing in intangible capital. The cost of procedures leads to higher adjustment costs for intangible capital. The incremental cost coefficients $\boldsymbol{\gamma}$ assess which characteristics are most critical for capturing the dynamics of a firm's market value. Columns (3) and (4) of Table 6 reports the point estimate $\boldsymbol{\gamma_z} \cdot \sigma(\mathbf{X_{z,c}})$ for the model where the non-binary country characteristic variables are normalized $\frac{X_{z,c}}{\sigma(\mathbf{X_{z,c}})}$with standard deviation as 1. Protection of intellectual property rights have negative point estimate of -2.70. Per 1 standard deviation increase in this characteristic reduces intangible adjustment costs by 2.74 from the global average cost of 11.74. Procedural cost of contract have positive point estimate of 0.73. Per 1 standard deviation increase in this characteristic increases intangible adjustment costs by 4.79. Among other characteristic variables, bank profitability, GDP per capita have large positive point estimates. Higher profits in the banking sector are associated with increased adjustment costs for intangible capital. Finally, wealthier countries, as indicated by higher GDP per capita, exhibit significantly higher adjustment costs. This may reflect the impact of a more expensive labor force. Product innovations in larger countries exhibit scale effects, making competition deterrence more challenging. Consequently, larger countries face higher costs for investing in intangible capital. Qualitatively, Table 6 indicates the cost of intangible capital investment increases with cost of contract enforcement, bank power. but decreases with protection of intellectual property rights. [26]

Overall, we find that the characteristics model fits the data well, with an R^2 of 0.86 for the cross-section and 0.55 for the time series. These values are comparable to the direct estimates in the baseline country-specific models. Figure 4 compares the parameter estimates and fit of sub-sample relative to the benchmark model from subsection 5.1. Notably, the simple universal model of characteristics performs well in most countries and regions except for African countries. We conclude that country fundamentals effectively capture the dynamics of market values for firms

[26] Section 7.5, shows negligible improvement from including other characteristics to describe the country heterogeneity in adjustment cost, reported in Table OT.21 of appendix.

across different countries.

Table 7 summarizes the distribution of firm valuation, composition of capitals across countries that differs in the replacement cost of intangible capital. This adjustment cost parameter of intangible capital is estimated from this universal model that incorporates status of countries in institutions and economic development, and utilizes the firm-level dispersion in investments rates and accumulated quantity of intangible capital within each country. Referring the literature, the firm valuation ratio is defined as the Average-Q, the market valuation in ratio of total amount of capitals.[27]. Among countries, South Korea, Japan, Singapore, the replacement cost of intangible capital is low, the median level of firm valuation ratio is 1.02. Among the costly countries, Indonesia, India, China, the median level of firm valuation ratio is 2.06, much higher than the counterpart statistic in the countries of low cost. Table 7 confirms the the cross-country variation of firm valuation largely aligns with the replacement cost of intangible capital, via lifting up the market price of formed intangible capital.

Next we formally apply the country characteristics model to evaluate the importance of each component of country institutions in determining the value of inputs contributing to a firm's market value. The valuation of intangible capital contributed by country-specific characteristic of institutional environment and economic development is,

$$q_z^I X_{z,c} \cdot K^I = \gamma_z^I \cdot X_{z,c} \cdot (1 - \tau_c) \cdot i^I K^I \tag{23}$$

for characteristic variable $X_{z,c}$, and symmetrically $\{q_z^P X_{z,c} \cdot K^P\}_z$ for the physical capital. Definition equation 23 uses the incremental cost coefficients $\boldsymbol{\gamma^I}$of intangible capital and $\boldsymbol{\gamma^P}$of physical capital. In combination with the composition of capitals and observed investment rates, it translates cross-country differences in institutional infrastructure and financial market development into the resulting impact on capital valuation. This is achieved by applying a correction weight of investment decisions observed in the history.

A significant portion of firm value variation is the value of intangible capital influenced by the protection of intellectual property rights, the cost of contracting. Table 8 reports the value

[27] Different from the physical capital with tax shield, book value of intangible capital has the discount from paying corporate tax continuously. Combining the low replacement cost and discount in paying tax, market price of intangible capital can be smaller than 1.

of intangible capital impacted by country characteristic variables, in share of total firm value. Each component of value, its share in the total firm value, $\frac{q_z^I \cdot K^I}{V} \cdot X_{z,c}$, has the following standard deviations: 6.18% for the protection of intellectual property rights, 9.65% for the cost of contract enforcement and 11.39% for economic development. [28] If a country's value for a specific characteristic is above (or below) the global average, the sign of the impact matches (or is opposite to) the sign of the coefficient γ. For a country characteristic variable that delineates a specific aspect of institutional environment or the development of market infrastructure, a large magnitude for the incremental cost coefficient, combined with substantial cross-country variation, indicates that the characteristic is economically important for explaining the geographical variation in capital value.

This finding aligns with economic intuition, as an efficient legal system and stronger protection of trade secrets and patents can help mitigate the adjustment costs for intangible capital. Additionally, economic development—measured by GDP per capita and population—affects labor costs. Developed countries tend to have a more expensive labor force, resulting in higher adjustment costs and valuations for intangible capital. Since intangible capital is riskier and more challenging to collateralize, the financial sector's ability to finance these investments is crucial. A more robust financial sector can lead to lower adjustment costs for intangible capital, thereby increasing its value.

Turning to the value of physical capital, we observe that the protection of intellectual property rights has a negligible effect on the heterogeneity of physical capital value, while contract enforcement and economic development play a prominent role. However, the cross-country dispersion associated with these characteristics is smaller compared to that of intangible capital. The standard deviation is 5.69% for contract enforcement and 2.63% for economic development. The greater variation in the value of intangible capital associated with intellectual property rights suggests that the difficulties in securing the asset have a significant effect on their valuation.

[28] Figure 5 illustrates the contribution of each characteristic to the value of intangible capital (top panel) and physical capital (bottom panel) across countries and regions.

7 Robustness

7.1 Domestic Firms

Domestic investment and oversea investment, simultaneous determine the total firm value. This paper didn't explicitly estimate heterogeneous capitals operated in different countries. As the simplified examination, Table OT.14 excludes the multinationals from the sample, estimates the model using the domestic firms. We define the domestic firms as firms whose headquarter is identical with the incorporation. Firms residing in Hong Kong are often incorporated in China or Europe. Given the smaller sample of completely local firms, we didn't construct the portfolios for those firms residing and incorporating in Hong Kong. For firms residing in China and incorporating in China, the new parameter estimate of the intangible capital adjustment cost coefficient is 36.20, larger than the outcome of baseline estimation of 30.73. Table OT.18 reports the country-characteristic model using this alternative sample of domestic firms. Estimated distribution of adjustment cost is quantitatively similar, as we observe weak institution is associated with the higher cost.

7.2 Firm-level Method

Section 5 documented substantial variation in the country-specific cost coefficient of intangible capital. In this subsection, we use the alternative method of firm-level equation to estimate the model. This estimation approach is more familiar for the literature, for example Peters and Taylor (2017) discussed when the total firm asset and investment include the knowledge capital and organization capital, firm valuation ratio (total Q, or often named as average Q) can better explains the total investment in the firm-level. We use this estimation to confirm the geographical heterogeneity in capital adjustment cost function.

We estimate the true model of heterogeneous capitals, using the firm-level approach. [29]The firm-level estimation has similar point estimate for country-specific cost coefficient of intangible capital θ_I, but smaller point estimate for the cost coefficients of physical capital θ_P. An intuitive explanation is the attenuation bias. The variation of observed investment of physical capital contains noise irrelevant of the model. The estimator assigns the small θ_P to reduce the equation error. On the

[29]The comparison of the parameters and statistics of fitness are reported in Figure OF.1 and OF.2.

contrary, point estimate of θ_I in the firm-level estimation resembles that of the baseline estimation, especially in countries such as Indonesia, India, Germany, the U.K. and the U.S.. The variation of investment rate and amount of intangible capital is crucial for explaining the variation of firm valuation ratio. As such, the model estimated using the firm-level method has fitness comparable to baseline estimation of portfolio-level approach, with slight deviation.

7.3 Sector-Heterogeneity within Country

We also estimate the model allowing for sector-specific heterogeneity in each country. In the online appendix, we show that within a country, sectors have qualitatively similar point estimates in cost parameters. In Table OT.17, the resulting increase in both the time-series and cross-sectional R^2 values is, in general, very small. This result indicates that cross-country heterogeneity in adjustment costs is important for the model fit, but, perhaps surprisingly, sector heterogeneity plays only a minor role.

7.4 Estimate of Intangible Capital

By the year of 2010, 49 countries out of 77, adopted the IFRS (International Financial Report Standard) or the local standards similar to IFRS. Financial report of listed companies are highly standard, but there exist slight difference in localized practice. Our estimations use the firm information collected by Compustat. Compustat manually harmonizes the financial reports into the format of the Compustat-Global and Compustat-North American. In addition, we take two approaches to address whether heterogeneity in accounting standard is large enough to intervene the model estimation.

(a) In the first approach, based on manual verification and conjecture, we build the alternative measure of quantity of intangible capital, by accruing the XRD expense separately. The estimation outcome using this alternative sample of measured intangible capital is reported in Table OT.16 for the country-specific cost model (benchmark estimation was reported in Table 2), and the country-characteristic model in Table OT.18 (benchmark estimation was reported in Table 6). We observe point estimates and model fitness that are quantitatively similar to the benchmark estimations.

(b) In the second approach, we utilize the firm operating profit, measured as the EBIT, and

the firm valuation, to jointly infer the quantity of intangible capital, by identifying the conversion rate of XSGA and the depreciation rate in intangible capital. Though not targeting the equation of firm valuation, characteristics related to the institutional environment explains the cross-country variation in large extent. Column DeepParam in Table OT.18 report the estimation outcome. The model concludes cost in investing intangible capital is associated to countries with weak protection of intellectual property right, high cost of contract enforcement.

7.5 Model of High-Dimension Characteristics

Aside from the authenticity-model focusing on the institutions, we conduct a systematic review of literature and non-academic discussion, construct the profile of country-level characteristics. This extended model assesses the product market entry barrier, the searching friction in hiring and dismissing employees, the access to physical infrastructure, the friction in monitoring the manager and large shareholder to ensure the dynamically maximized firm valuation. The construction and estimation of high-dimensional model is detailed in the online appendix. [30]

We found positive increase in cost of intangible capital driven by the easy entry of competitors. The weak or zero entry barriers put those industry incumbents, often the large listed companies, in dis-advanced position, in accumulating intangible capital. For the situation in the labor market, the effect of increased usage of tenure contracts on adjustment costs exhibits a large standard error, but the protection of collective dismissal reduces the cost of physical capital in small extent. The threat of suing the large shareholder increases the adjustment cost of physical capital, but have unclear effect to that of intangible capital. We find weak additional improvement from the model of high-dimensional state variables, despite small changes in the incremental cost of institution characteristics. This confirms the sources of cross-country variation in the adjustment cost of capitals are driven by the ease of converting operating expenditure to patents and copyrights that exclusively foster the firm profit, the ease of raising sources and payment for the investment expenditure.

[30] Table OT.20 describes the characteristic variables. Table OT.21 presents the point estimates for the global average cost parameter, the incremental cost coefficients for all characteristics described above.

8 Conclusion

We incorporate intangible capital into the neoclassical model of investment and estimate its contribution of each input for explaining firm market values across 77 countries between 2000 and 2020. For the major markets, where we estimate country specific parameters, the model performs well in explaining both the time-series and the cross-sectional variation of the valuation ratios across portfolios in the worldwide, with time-series R^2 of 57% and a cross-sectional R^2 of 87%.

We find that the importance of the intangible capital for firm value varies across countries and is substantial, ranging from 33.31% to 68.93% . We show that financial markets assign large and positive values to the installed stocks of the capital inputs because they are costly to adjust, thus firm valuation contains the compensation for the cost of adjusting the inputs. The adjustment cost of intangible capital is higher than that of physical capital. When quantifying the market environment for accumulating intangible capital for each country, we observe dispersed point estimates of adjustment cost parameters.

To understand the drivers of intangible and physical capital adjustment costs, we utilize a model in which adjustment cost of capital investment is contingent on country-specific characteristics affecting firms' ability to invest, and analyze the importance of these characteristics for firm valuation. We find that institutions of intellectual property right and contracting significantly influence the cross-country variation in the cost of investing intangible capital and its valuation. While similar factors also impact physical capital, their effect is comparatively less pronounced.

References

Acemoglu, Daron and Simon Johnson (2005), "Unbundling institutions," *Journal of political Economy*, 113 (5), 949–995. 1, OT.19

Acharya, Viral V, Ramin P Baghai, and Krishnamurthy V Subramanian (2014), "Wrongful discharge laws and innovation," *The Review of Financial Studies*, 27 (1), 301–346. 1, OT.19

Altomonte, Carlo, Domenico Favoino, Monica Morlacco, and Tommaso Sonno (2021), "Markups, intangible capital and heterogeneous financial frictions," . 1

Antill, Samuel, Murat Alp Celik, Xu Tian, and Toni M Whited (2023), "The Efficiency of Patent Litigation," *Available at SSRN 4842424*. 1, 6.1, OT.19

Baker, Scott R, Brian Baugh, and Marco Sammon (2023), "Customer churn and intangible capital," *Journal of Political Economy Macroeconomics*, 1 (3), 447–505. 1

Belo, Frederico, Vito D Gala, Juliana Salomao, and Maria Ana Vitorino (2022), "Decomposing firm value," *Journal of Financial Economics*, 143 (2), 619–639. 1, 2, 3.1, 3.2, 4.1.1, 10, 4.1.3

Bena, Jan, Hernan Ortiz-Molina, and Elena Simintzi (2022), "Shielding firm value: Employment protection and process innovation," *Journal of Financial Economics*, 146 (2), 637–664. 1, OT.19

Calcagnini, Giorgio, Annalisa Ferrando, and Germana Giombini (2014), "Does employment protection legislation affect firm investment? The European case," *Economic Modelling*, 36, 658–665. 1, OT.19

Chemmanur, Thomas and An Yan (2009), "Product market advertising and new equity issues," *Journal of Financial Economics*, 92 (1), 40–65. 1, OT.19

Chen, Peter, Loukas Karabarbounis, and Brent Neiman (2017), "The global rise of corporate saving," *Journal of monetary economics*, 89, 1–19. 1

Crouzet, Nicolas, Janice Eberly, Andrea L Eisfeldt, and Dimitris Papanikolaou (2022a), "Intangible capital, non-rivalry, and growth," *Available at SSRN*. 1, OT.19

Crouzet, Nicolas and Janice C Eberly (2021), "Rents and intangible capital: A q+ framework," Technical report, National Bureau of Economic Research. 1

Crouzet, Nicolas, Janice C Eberly, Andrea L Eisfeldt, and Dimitris Papanikolaou (2022b), "A model of intangible capital," Technical report, National Bureau of Economic Research. 6.1

Djankov, Simeon, Rafael La Porta, Florencio Lopez-de Silanes, and Andrei Shleifer (2003), "Courts," *The Quarterly Journal of Economics*, 118 (2), 453–517. 1, 6.1, OT.19

Eisfeldt, Andrea L, Edward Kim, and Dimitris Papanikolaou (2020), "Intangible value," Technical report, National Bureau of Economic Research. 1

Eisfeldt, Andrea L and Dimitris Papanikolaou (2013), "Organization capital and the cross-section of expected returns," *The Journal of Finance*, 68 (4), 1365–1406. 1, 4.1.2, 4.1.2

Falato, Antonio, Dalida Kadyrzhanova, Jae Sim, and Roberto Steri (2022), "Rising intangible capital, shrinking debt capacity, and the US corporate savings glut," *The Journal of Finance*, 77 (5), 2799–2852. 1

Fama, Eugene F. and Kenneth R. French (1993), "Common Risk Factors in the Returns on Stocks and Bonds," 33, 3–56. 3.3

Gulen, Huseyin, Dongmei Li, Ryan H Peters, and Morad Zekhnini (2022), "Intangible capital in factor models," *Available at SSRN 3725002*. 1, OT.12, OT.15, OT.18

Hall, Robert E (2001), "The stock market and capital accumulation," *American Economic Review*, 91 (5), 1185–1202. 1

Hansen, Lars Peter, John C. Heaton, and Nan Li (2012), "Intangible Risk," *NBER Book Chapters*. 1

Hauswald, Robert and Robert Marquez (2006), "Competition and strategic information acquisition in credit markets," *The Review of Financial Studies*, 19 (3), 967–1000. OT.19

Hayashi, Fumio (1982), "Tobin's marginal q and average q: A neoclassical interpretation," *Econometrica: Journal of the Econometric Society*, 213–224. 2.3, C.1

Karabarbounis, Loukas and Brent Neiman (2014), "The global decline of the labor share," *The Quarterly journal of economics*, 129 (1), 61–103. 1

La Porta, Rafael, Florencio Lopez-de Silanes, Andrei Shleifer, and Robert Vishny (2002), "Investor protection and corporate valuation," *The journal of finance*, 57 (3), 1147–1170. 1, OT.19

Larkin, Yelena (2013), "Brand perception, cash flow stability, and financial policy," *Journal of Financial Economics*, 110 (1), 232–253. 1, OT.19

Lev, Baruch and Suresh Radhakrishnan (2005), "The Valuation of Organization Capital," in "Measuring Capital in the New Economy," National Bureau of Economic Research, Inc, 73–110. 1

McLean, R David, Tianyu Zhang, and Mengxin Zhao (2012), "Why does the law matter? Investor protection and its effects on investment, finance, and growth," *The Journal of finance*, 67 (1), 313–350. 1, OT.19

Peters, Ryan H. and Lucian A. Taylor (2017), "Intangible Capital and the Investment-q Relation," *Journal of Financial Economics*, 123 (2), 251–272. 1, 4.1.2, 11, 7.2

Petersen, Mitchell A and Raghuram G Rajan (1994), "The benefits of lending relationships: Evidence from small business data," *The journal of finance*, 49 (1), 3–37. OT.19

Rempel, Mark (2021), "Innovation and firm dynamics with patent wars," . OT.19

A Tables

Table 1: Descriptive Firm Statistics

This table reports the median of firm-level selected characteristics across all firms in the each country (or territory). The same statistics are calculated for the remaining area of each continent. The same statistics for the whole continent is reported below countries within the continent. The same statistics for the full sample of all the countries (World) are reported in the final rows. Statistics of individual countries in the region are reported in the Appendix Table OT.3. Data is winsorized with [2%,98%] by country (or territory). Columns (1) and (7) reports the country and territory, the region, for each continent of Asia, Europe, Americas, Australia and Africa. Firm valuation is Q. Installed physical capital is $K^P$with investment flow equal to I^P. Installed intangible capital is $K^I$with investment flow equal to I^I. The median is calculated in each year, the time-series average statistics are reported in the sub-sample of each geographic area (country, territory and region within the continent, or the continent). In Column (6), the correlation $\rho(\frac{I^P}{K^P}, \frac{I^I}{K^I})$ is calculated using all the observations in the sub-sample.

Country/Region (1)	$\frac{Q}{K^I+K^P}$ (2)	$\frac{I^P}{K^P}$ (3)	$\frac{I^I}{K^I}$ (4)	$\frac{K^I}{K^I+K^P}$ (5)	$\rho(\frac{I^P}{K^P}, \frac{I^I}{K^I})$ (6)
China	2.95	0.17	0.34	0.22	0.32
Hong Kong	1.51	0.18	0.29	0.40	0.22
Indonesia	1.38	0.07	0.20	0.27	0.21
India	1.59	0.08	0.23	0.32	0.26
Japan	0.87	0.12	0.22	0.45	0.37
Malaysia	1.26	0.07	0.25	0.26	0.22
Singapore	1.28	0.16	0.28	0.37	0.23
South Korea	1.00	0.11	0.25	0.25	0.31
Thailand	1.56	0.13	0.24	0.28	0.22
Taiwan	1.77	0.13	0.26	0.26	0.33
Rest of Asia	1.55	0.08	0.22	0.28	0.24
Asia	1.38	0.12	0.24	0.32	0.30
France	1.30	0.22	0.24	0.67	0.26
Germany	1.45	0.21	0.24	0.57	0.26
Poland	1.27	0.15	0.28	0.38	0.32
UK	1.51	0.20	0.25	0.60	0.28
Rest of Europe	1.48	0.18	0.25	0.45	0.26
Europe	1.44	0.19	0.25	0.51	0.27
Canada	1.57	0.18	0.28	0.21	0.31
USA	2.14	0.24	0.25	0.61	0.34
Latin America	1.15	0.09	0.21	0.33	0.33
America	1.86	0.21	0.25	0.53	0.35
Australia	1.65	0.21	0.32	0.34	0.23
Africa	1.53	0.10	0.21	0.42	0.23
World	1.49	0.15	0.25	0.38	0.30

Table 2: Parameter Estimates and Model Fit

This table reports the details of parameter estimates. θ_P and θ_I are, respectively, the physical capital and intangible capital adjustment cost coefficients that are specific to country (or territory, region). s.e. stands for Newey-West standard errors with two lags. Columns (1) and (7) report the country and territory, rest of the region, for each region of Asia, Europe, Americas, Australia and Africa. Columns (2) to (3) report the point estimate and the standard error for parameters specific to the country (or territory,region) in Asia. Columns (8) to (9) report the same statistics for countries in Europe and other regions. We calculate model fit for each country (or territory, other countries in the region). In Columns (3) to (5) and Columns (10) to (12), $XS-R^2$ is the cross-sectional R^2 for the subset of portfolios in the country (or territory, region), $TS-R^2$ is the time-series R^2, and $m.a.e./\overline{VR}$ is the mean absolute valuation error scaled by the absolute value of the ratio.

Country/Region		Point Estimate		Model Fit			Country/Region		Point Estimate		Model Fit		
		θ_P	θ_I	XS-R^2	TS-R^2	$m.a.e./\overline{VR}$			θ_P	θ_I	XS-R^2	TS-R^2	$m.a.e./\overline{VR}$
(1)		(2)	(3)	(4)	(5)	(6)	(7)		(8)	(9)	(10)	(11)	(12)
China		3.31	32.54	0.63	0.23	0.30	France		5.86	7.75	0.67	0.27	0.26
	s.e.	(1.13)	(2.75)					s.e.	(0.92)	(0.89)			
Hong Kong		2.70	6.97	0.68	0.27	0.25	Germany		4.15	10.27	0.74	0.23	0.28
	s.e.	(0.41)	(0.81)					s.e.	(1.13)	(1.44)			
Indonesia		4.48	14.75	0.85	0.49	0.22	Poland		3.09	4.80	0.82	0.35	0.31
	s.e.	(0.61)	(1.61)					s.e.	(0.95)	(0.99)			
India		4.38	19.83	0.81	0.32	0.28	UK		5.48	8.91	0.84	0.54	0.21
	s.e.	(0.78)	(2.02)					s.e.	(0.59)	(0.82)			
Japan		0.78	2.91	0.26	0.12	0.19	Rest of Europe		4.42	11.18	0.81	0.53	0.25
	s.e.	(0.36)	(0.39)					s.e.	(0.58)	(1.01)			
Malaysia		2.77	11.82	0.83	0.39	0.18							
	s.e.	(0.52)	(1.10)				Canada		3.73	11.53	0.84	0.49	0.22
Singapore		1.93	6.99	0.61	0.24	0.23		s.e.	(0.28)	(0.73)			
	s.e.	(0.55)	(0.77)				USA		8.10	16.32	0.84	0.61	0.18
South Korea		0.23	5.72	0.68	0.36	0.14		s.e.	(0.84)	(1.06)			
	s.e.	(0.52)	(0.70)				Latin America		4.50	10.95	0.80	0.61	0.20
Thailand		3.88	10.28	0.80	0.29	0.23		s.e.	(0.71)	(1.32)			
	s.e.	(0.61)	(1.31)										
Taiwan		3.88	15.51	0.81	0.26	0.18	Australia		3.27	10.37	0.75	0.42	0.25
	s.e.	(0.50)	(1.25)					s.e.	(0.47)	(0.99)			
Rest of Asia		6.36	12.03	0.78	0.46	0.17							
	s.e.	(1.26)	(1.72)										
							Africa		3.98	12.75	0.25	0.32	0.21
								s.e.	(0.81)	(1.04)			
Summary of Parameter Estimates and Model Fit													
							World	Mean	3.87	11.63	0.87	0.57	0.23
								S.D.	(1.73)	(6.10)			

Table 3: Summary of Models

This table reports the summary of parameter estimates and measures of fit for the benchmark model and counter-factual models. Columns in **Benchmark Model** report the estimation outcome for the benchmark model of country-specific adjustment cost function. Columns in **Model with Physical Capital Only** report the estimation outcome for the model of country-specific adjustment cost function and single capital input of physical capital. We assume the intangible capital plays no role in the production function nor the adjustment cost function. Columns in **Model with Homogeneous Cost** report the estimation outcome for the model of homogeneous adjustment cost function and two capital inputs. We assume the identical adjustment cost function everywhere. Columns in **Model with Varying Intangible Cost** report the estimation outcome for the model of country-specific adjustment cost in intangible capital and homogeneous adjustment cost in physical capital. Column (1) reports the geographic area, all countries used for estimation (World), the regions of Asia, Europe, Americas, Australia and Africa. In Panel (a) of parameter estimates, Columns (2) to (3), Columns (4) to (5), Columns (6) to (7) and Columns (8) to (9) report the summary statistics of average parameters within the geographic area. θ_P and θ_I are, respectively, the physical capital and intangible capital adjustment cost coefficients. Standard deviation for coefficients cross countries are reported for the models of country-specific adjustment cost function. In Panel (b) of model fit, Columns (2) to (4) report the model fit, $XS - R^2$ is the cross-sectional R^2 for the 301 portfolios in the world (or the subset of portfolios in the region), $TS - R^2$ is the time-series R^2, and $m.a.e./\overline{VR}$ is the mean absolute valuation error scaled by the absolute value of the ratio. Columns (5) to (7), Columns (8) to (10) and Columns (11) to (13) report the same statistics for the counter-factual estimation.

	Benchmark Model			Model with Physical Capital Only			Model with Homogeneous Cost			Model with Varying Intangible Cost		
Panel (a) Summary of Parameter Estimate												
Area		Point Estimate			Point Estimate			Point Estimate			Point Estimate	
		θ_P	θ_I		θ_P	θ_I		θ_P	θ_I		θ_P	θ_I
(1)		(2)	(3)		(4)	(5)		(6)	(7)		(8)	(9)
World	Mean	3.87	11.63	Mean	11.59	-	Mean	3.63	11.73	Mean	3.63	11.73
	S.D.	(1.73)	(6.10)	S.D.	(4.44)	-	S.D.	(0)	(0)	S.D.	(0)	(6.80)
Panel (b) Model Fit												
Area	Model Fit			Counter-factual Model Fit			Counter-factual Model Fit			Counter-factual Model Fit		
	XS-R^2	TS-R^2	$m.a.e./\overline{VR}$	XS-R^2	TS-R^2	$m.a.e./\overline{VR}$	XS-R^2	TS-R^2	$m.a.e./\overline{VR}$	XS-R^2	TS-R^2	$m.a.e./\overline{VR}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
World	0.87	0.57	0.23	0.03	-0.29	0.42	0.35	0.24	0.30	0.47	0.32	0.29
Asia	0.90	0.60	0.23	0.25	-0.13	0.40	0.24	0.15	0.33	0.57	0.40	0.29
Europe	0.80	0.43	0.26	-0.39	-0.56	0.44	0.50	0.28	0.29	0.24	0.12	0.31
Americas	0.88	0.67	0.20	-0.14	-0.32	0.43	0.46	0.38	0.26	0.54	0.42	0.27
Oceania	0.75	0.42	0.25	-0.81	-0.72	0.46	0.83	0.42	0.25	-0.79	-0.15	0.34
Africa	0.25	0.32	0.21	-4.11	-1.45	0.46	0.05	0.26	0.22	-0.85	0.07	0.26

Table 4: Capital Accounting: Share of Intangible

This table reports the contribution of intangible capital in the firm valuation. Columns (1) and (6) report the geographic area (country, territory and region within the continent, or the continent). Columns (2) and (7) report the contribution of intangible capital in the firm valuation for each country, denoted as market share μ_I, reported in unite of percentage (%). The same statistics for the region is reported below countries within the region. The same statistics for the full sample of all the countries (World) are reported in the final rows. Other countries are reported in Table OT.4. Columns (3) to (4), and Columns (8) to (9) report the ratio of adjustment cost of capital investment in firm sale, denoted as C^I/Y for the intangible capital and C^P/Y for the physical capital , reported in unite of percentage (%). Columns (5) and (11) report the percentage of intangible capital in the book capitals, denoted as book share $\overline{\mu}_I$. Columns (6) and (12) report the counter-factual estimation assuming the model of homogeneous adjustment cost function in Table 3, denoted as counterfactual estimated share $\tilde{\mu_I}$. Both the statistics of share and estimated adjustment cost are calculated as the median across firms in the geographic area. The time-series average during the year 2016-2020 are reported.

Country/Region	Share	Adjustment Cost		Counterfactual		Country/Region	Share	Adjustment Cost		Counterfactual	
	μ_I	C^I/Y	C^P/Y	$\overline{\mu}_I$	$\tilde{\mu_I}$		μ_I	C^I/Y	C^P/Y	$\overline{\mu}_I$	$\tilde{\mu_I}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
China	68.93	18.19	1.20	24.22	46.51	France	61.71	7.35	4.46	56.05	70.78
Hong Kong	58.01	4.39	1.55	43.57	62.11	Germany	61.87	7.23	3.05	47.54	63.68
Indonesia	41.56	5.17	1.07	18.96	37.00	Poland	47.29	2.74	0.98	36.03	58.14
India	54.82	3.85	0.83	27.10	45.08	UK	63.09	6.53	3.95	55.67	69.24
Japan	47.85	1.68	0.32	38.32	58.38	Rest of Europe	57.65	6.64	2.55	40.60	58.17
Malaysia	45.86	4.60	0.89	23.94	43.18	Europe	58.33	6.31	2.74	45.21	61.99
Singapore	50.84	3.46	0.96	35.00	53.84						
South Korea	40.90	2.66	0.06	22.81	45.37						
Thailand	40.85	5.41	1.88	23.75	41.77	Canada	33.31	4.76	3.70	16.46	32.23
Taiwan	51.26	5.98	1.63	26.50	44.88	USA	64.39	12.13	6.89	50.85	65.98
Rest of Asia	40.40	3.79	2.45	21.39	40.20	Rest of America	50.40	4.08	1.43	29.37	50.39
Asia	51.91	4.63	0.78	27.26	47.46	Americas	58.81	8.88	4.96	41.75	59.53
						Australia	49.93	9.00	5.63	30.12	49.49
						Africa	60.13	4.64	1.41	36.49	57.47
						World	53.83	5.41	1.30	30.90	51.10
						S.D.	(9.32)	(3.57)	(1.75)	(11.64)	(10.59)

Table 5: Distribution of Country Characteristics and Share of Intangible

This table reports the distribution of intangible capital, adjustment cost across large countries of different status in intellectual property rights protection and contract enforcement cost. Columns (1) , (2) and (3) report the geographic area with intellectual property rights protection of the low, medium and high status, and examples of large countries within the group of type, the country-average cost within the group. Columns (4) to (5) report the example ratio of adjustment cost of capital investment in existing amount of capital assuming investment rate of 20% , denoted as C^I/K^I for the intangible capital and C^P/K^P for the physical capital , reported in unite of percentage (%). Columns (6) to (7) report the equilibrium rate of investment in capitals, denoted as $i^I = I^I/K^I$ for the intangible capital and $i^P = I^P/K^P$ for the physical capital. Columns (8) to (10) report the quantity effect and the shadow-price effect in the market share of intangible capital. Columns (8) reports the quantity effect, described as the book share $\overline{\mu}_I$. Columns (9) reports the contribution of intangible capital in the firm valuation, denoted as market share μ_I, reported in unite of percentage (%). Columns (10) reports the shadow-price effect, the gap between the book share to the market share. Column (11) reports the firm valuation ratio, the average Q in ratio of total amount of capitals. Both the statistics of share and equilibrium adjustment cost are calculated as the median across firms in the geographic area. The time-series average during the year 2016-2020 are reported. Panel (B) reports the statistics for countries of different type of contract enforcement procedure, in the analogous classification and definition.

Panel (A) Statistics in Countries of Different Intellectual Property Right Protection

Country Characteristic			Calibrated Example Cost (%) for $i = 0.20$		Statistics of Firm Decision and Valuation in Equilibrium					
					Investment		Share of Intangible Capital			Firm Value
Type	Large Countries of this Type	IPR X_a	Intan C^I/K^I	Phy C^P/K^P	Intan i^I	Phy i^P	Quantity $\overline{\mu}_I$	Value μ_I	Price $\mu_I/\overline{\mu}_I$	Average Q $\frac{Q}{K^I+K^P}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Weak	China (1) , India (1)	1.00	52.36	7.69	0.20	0.08	25.66	61.88	2.41	2.50
Medium	Indonesia (2) , Canada (3) , Germany (3) , Hong Kong (3) , Malaysia (3) , Poland (3) , Thailand (3) , Taiwan (3)	2.88	21.48	7.17	0.22	0.14	29.59	47.50	1.61	1.42
Strong	Australia (4) , France (4) , Japan (4) , South Korea (4) , Singapore (4) ,UK (4) , USA (4)	4.00	16.85	7.33	0.22	0.19	41.26	54.10	1.31	1.42
[Globe]		[2.88]								

Panel (B) Statistics in Countries of Different Contract Procedure

Country Characteristic			Calibrated Example Cost (%) for $i = 0.20$		Statistics of Firm Decision and Valuation in Equilibrium					
					Investment		Share of Intangible Capital			Firm Value
Type	Large Countries of this Type	Contract X_b	Intan C^I/K^I	Phy C^P/K^P	Intan i^I	Phy i^P	Quantity $\overline{\mu}_I$	Value μ_I	Price $\mu_I/\overline{\mu}_I$	Average Q $\frac{Q}{K^I+K^P}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Simple	Singapore (21), Hong Kong (26) , Australia (28) , France (29) , Malaysia (29), UK (29) , Germany (31) , Japan (32) , South Korea (32) , USA (32)	28.90	17.61	7.05	0.22	0.18	40.39	54.44	1.35	1.41
Medium	Canada (36) , China (37) , Poland (33) , Thailand (36)	35.50	29.57	7.00	0.24	0.14	25.11	47.59	1.90	1.86
Complicate	Indonesia (40) , India (46) , Taiwan (45)	43.67	33.39	8.50	0.19	0.08	24.18	49.21	2.03	1.59
[Globe]		[35.55]								

Table 6: Parameter Estimates of Country-Characteristic Model

This table reports the parameter estimates for the model specification of country-specific state-contingent adjustment cost function (in abbreviation, country-characteristic model). In Columns (1) to (2), Global Cost Coefficients $\theta_{\mathbf{g}}$ and Incremental Cost Coefficient $\boldsymbol{\gamma}$ for each dimension of country characteristic variables are reported. Columns (3) to (4) report the point estimate where the non-binary country characteristic variables are normalized with standard deviation as 1. In Panel (C), $XS-R^2$ is the cross-sectional R^2 for the 21×25 portfolios in the globe, $TS-R^2$ is the time-series R^2, and $m.a.e./\overline{VR}$ is the mean absolute valuation error scaled by the absolute value of the ratio. P-value of t-stat are indicated using * for p<0.10, ** for p<0.05, *** for p<0.010.

	Point Estimate		Point Estimate (Normalized)	
	(1)	(2)	(3)	(4)
Panel (A) Global Cost Coefficient				
	$\theta_{P,g}$	$\theta_{I,g}$	$\theta_{P,g}$	$\theta_{I,g}$
	4.22 ***	11.74 ***	4.22 ***	11.74 ***
s.e.	(0.28)	(0.49)	(0.28)	(0.49)
Panel (B) Incremental Cost Coefficients				
	γ^P	γ^I	γ^P	γ^I
(a) Intellectual Property (X_{1a})				
Secrets and Patents	0.17	-2.70 **	0.17	-2.73 **
s.e.	(0.36)	(0.66)	(0.36)	(0.67)
(b) Contract Enforcement (X_{1b})				
Procedures	0.15 **	0.73 ***	0.95 **	4.79 ***
s.e.	(0.04)	(0.08)	(0.25)	(0.49)
(c) Economic Development (X_{1c})				
GDP per capita	0.20	3.52 ***	0.24	4.15 ***
s.e.	(0.27)	(0.53)	(0.31)	(0.62)
Population	-0.10	0.69 **	-0.17	1.21 **
s.e.	(0.16)	(0.28)	(0.28)	(0.49)
Bank Profit	0.96 *	13.99 ***	0.41 *	6.05 ***
s.e.	(0.64)	(1.22)	(0.28)	(0.53)
Intensity of Intangible Capital (X_2)				
$\mathbf{1}$(Top25%)	3.20 **	8.87 ***	3.20 **	8.87 ***
s.e.	(0.58)	(0.79)	(0.58)	(0.79)
Panel (C) Statistics				
XS-R^2		0.86		0.86
TS-R^2		0.55		0.55
$m.a.e./\overline{VR}$		0.23		0.23

Table 7: Distribution of Adjustment Cost of Intangible Capital and Firm Valuation

This table reports the distribution of adjustment cost of intangible capital across all countries with complete profiles of characteristics related to institutions, economic and financial development. Adjustment cost of intangible capital is estimated using the model of country-specific state-contingent adjustment cost function (in abbreviation, country-characteristic model). Columns (1) , (2) and (3) report the geographic areas of low cost, medium cost and high cost respectively, examples of large countries within the group of type, the country-average cost within the group. Columns (4) to (5) report the example ratio of adjustment cost of capital investment in existing amount of capital assuming investment rate of 20% , denoted as C^I/K^I for the intangible capital and C^P/K^P for the physical capital , reported in unite of percentage (%). Columns (6) to (7) report the equilibrium rate of investment in capitals, denoted as $i^I = I^I/K^I$ for the intangible capital and $i^P = I^P/K^P$ for the physical capital. Columns (8) to (10) report the quantity effect and the shadow-price effect in the market share of intangible capital. Columns (8) reports the quantity effect, described as the book share $\overline{\mu}_I$. Columns (9) reports the contribution of intangible capital in the firm valuation for each area containing the countries of the same type, denoted as market share μ_I, reported in unite of percentage (%). Columns (10) reports the shadow-price effect, the gap between the book share to the market share. Column (11) reports the firm valuation ratio, the average Q in ratio of total amount of capitals. Both the statistics of share and equilibrium adjustment cost are calculated as the median across firms in the geographic area. The time-series average during the year 2016-2020 are reported. Panel (B) reports the statistics for countries of different type of contract enforcement procedure, in the analogous classification and definition.

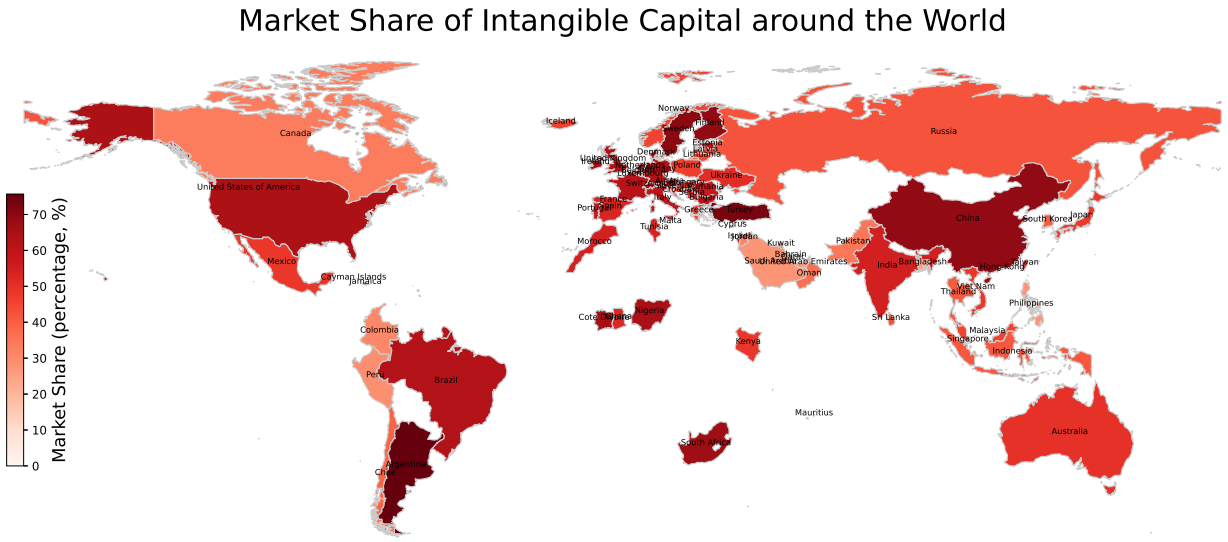
Countries grouped by Adjustment Cost $\hat{\theta}_I$			Calibrated Example		Statistics of Firm Decision and Valuation in Equilibrium					
			Cost (%) for $i = 0.20$		Investment		Share of Intangible Capital			Firm Value
Type	Countries of this Type	IntanCost $\hat{\theta}_I$	Intan C^I/K^I	Phy C^P/K^P	Intan i^I	Phy i^P	Quantity $\overline{\mu}_I$	Value μ_I	Price $\mu_I/\overline{\mu}_I$	Average Q $\frac{Q}{K^I+K^P}$
(1)	(2)	(3)	(4)	(5)	(4)	(5)	(8)	(9)	(10)	(11)
Low	South Korea (3.53), Japan (3.67), Poland (5.83), Hong Kong (7.11), Singapore (7.56); Israel (5.93), South Africa (7.53) and others	4.74	9.49	6.57	0.21	0.15	34.71	44.04	1.27	1.02
Medium	France (8.00), Germany (9.11), Australia (9.24),UK (9.82), Malaysia(10.46),Canada (10.59), Thailand (11.50); Switzerland (7.71) , Greece (8.60) , Viet Nam (11.59) and others	10.18	20.37	8.32	0.21	0.15	30.71	48.66	1.58	1.43
High	ndonesia (14.44) , Taiwan (15.07) , USA (16.09) , India (20.98) , China (32.42) ; Pakistan (14.35) , Italy (15.31) , Sweden (15.92) , Turkey (16.55) , Brazil (19.25) and others	21.38	42.75	10.76	0.21	0.13	28.94	59.63	2.06	2.05
[Globe]		[12.50]								

Table 8: Composition of Country-Specific Value

This table reports the composition of country-characteristic-specific valuation of capitals, in percentage of total firm value. Columns (2a) to (2c), report the share of intangible capital value affected by the characteristic variables in total firm value in each scope of institutional environment and economic development $\gamma_z^I \cdot X_{z,c} \cdot (1-\tau)\left(\frac{I^I}{K^I}\right)\frac{K^I}{V}$ respectively: (a) Protection of Intellectual Property Rights (in abbreviation IPR) , (b) Ease of Contract Enforcement (in abbreviation Contract), and (c) Economic Development (in abbreviation Devp.). Columns (3a) to (3c) report the share of physical capital value affected by the characteristic variables in total firm value in each scope $\gamma_z^P \cdot X_{z,c} \cdot (1-\tau)\left(\frac{I^P}{K^P}\right)\frac{K^P}{V}$ respectively.

Country/Region	Intangible Capital			Physical Capital		
	Characteristic-Specific Value (%)			Characteristic-Specific Value (%)		
	IPR	Contract	Devp.	IPR	Contract	Devp.
(1)	(2a)	(2b)	(2c)	(3a)	(3b)	(3c)
China	8.41	1.77	27.88	-1.22	0.82	2.11
Hong Kong	-0.91	-19.26	13.57	0.16	-10.83	3.34
Indonesia	4.58	6.32	-1.38	-0.78	3.45	-2.01
India	8.20	12.42	-2.04	-1.01	4.90	-2.01
Japan	-14.43	-12.39	-1.01	1.19	-3.26	-2.09
Malaysia	-0.57	-8.28	10.48	0.12	-5.56	2.08
Singapore	-6.40	-22.52	24.79	1.62	-18.17	7.92
South Korea	-12.77	-10.97	-1.48	1.24	-3.41	-1.33
Thailand	-0.69	0.69	4.11	0.17	0.55	-0.17
Taiwan	-0.58	12.05	-1.83	0.14	9.47	-0.88
Rest of Asia	0.25	4.35	8.04	0.23	4.40	2.88
France	-10.80	-17.12	-9.09	1.05	-5.34	-2.24
Germany	-0.91	-9.12	-15.32	0.13	-4.12	-4.02
Poland	-1.08	-6.08	-4.83	0.17	-3.13	-1.72
UK	-7.26	-11.51	-1.76	0.98	-4.97	-1.48
Rest of Europe	-5.87	-7.45	-1.68	0.59	-2.10	-0.37
Canada	-0.61	0.60	1.96	0.13	0.41	-0.43
USA	-7.56	-6.50	8.33	0.80	-2.21	-1.03
Latin America	5.34	5.96	-6.10	-0.42	1.41	-1.06
Australia	-7.48	-13.66	20.46	1.04	-6.04	2.62
Africa	1.17	-9.09	-15.11	-0.28	-2.48	-1.83
Summary						
Large Countries (Territories), Regions of Remaining Area						
Mean	-2.38	-5.23	2.76	0.29	-2.20	-0.08
S.D.	6.18	9.65	11.39	0.74	5.69	2.63

B Figures



Measure: Median of firm variable within Country.
Time-series average statistic during 2016-2020.
Homecountry is Headquarter.
Country (region) has heterogenous adjustment cost parameters.

Figure 1: Contribution of Intangible Capital in Firm Value across Globe

This figure plots the contribution of intangible capital in the firm valuation in individual countries, using the heatmap. The statistics are plotted for countries in Table 4 and Appendix Table OT.4. The statistic for the contribution of intangible capital in the firm valuation are graphed. The statistic is the time-series average of median market share μ_I from the year 2016 to the year 2020, using the available firm-year observations inside the country. The market share μ_I is estimated using the Benchmark model and estimation specification in Table 2 . For countries with insufficient observations of public listed firms, they are omitted in the heatmap.



Figure 2: Distribution of Share of Intangible and Country Characteristics

This figure reports the distribution of intangible capital across countries of different status in intellectual property rights protection (IPR protection), contract enforcement cost (Contract Procedures), and cost coefficient of intangible capital explained by observed country characteristics (Cost of Intangible). Subfigure (1a) reports the example ratio of adjustment cost of capital investment in existing amount of capital assuming investment rate of 20% , denoted as C^I/K^I for the intangible capital. 17 large countries are classified into three types of groups, in weak, medium and strong protection of intellectual property rights. Subfigure (1b) reports the analogous statistic C^P/K^P for the physical capital. Subfigure (1c) reports the contribution of intangible capital in the firm valuation for each area containing the countries of the same type, denoted as market share μ_I, reported in unite of percentage (%), the share of intangible capital in the book capitals, denoted as the book share $\overline{\mu}_I$. Both the statistics of share and equilibrium adjustment cost are calculated as the median across firms in the geographic area. The time-series average during the year 2016-2020 are reported. Subfigures (2a), (2b) and (2c) report the statistics for countries of different type of contract enforcement procedure, in the analogous classification and definition. Subfigures (3a), (3b) and (3c) report the statistics for countries of different adjustment cost of intangible capital, estimated from the country-characteristic model, summarizing the intellectual property rights protection, contract enforcement cost, economic and financial development. Countries are classified into three types of groups, in low, medium and high adjustment cost of intangible capital.

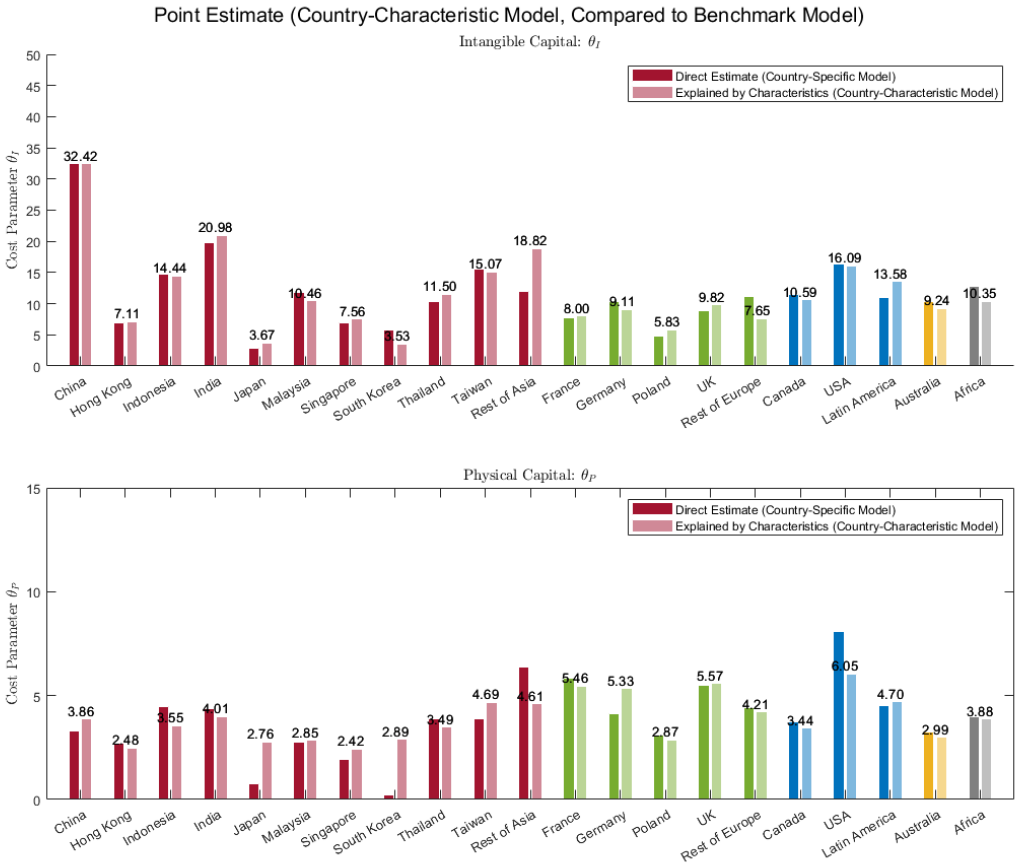


Figure 3: Estimates of Cost Coefficients in Country Characteristic Model

This figure compares the parameter estimates in the baseline model of country-specific adjustment cost and the parameter estimates in the alternative model of country characteristics. The subfigure in first row reports the parameters for intangible capital. The bar of dark color describes the estimation outcome for the baseline model. For each of the 17 countries, 4 regions, the quadratic cost coefficient in the adjustment cost function is estimated. The bar of light color describes the model-implied quadrature cost coefficient using the characteristic model. For each of the 73 countries, the quadratic cost coefficient in the adjustment cost function is a function over the status of intellectual property rights protection, cost of contract enforcement, economic and financial development. For the 56 countries in the 4 regions (Rest of Asia, Rest of Europe, Latin America, Africa), the average outcome of model-implied quadrature cost coefficient are plotted. The subfigure in the second row reports the parameters for the physical capital. The bar of dark red color, the light red color describes the estimation outcomes of the two specifications respectively, for countries and the remaining area in Asia. The dark and light green colors are for countries and the remaining area in Europe. The blue colors are for Americas. The yellow colors are for Australia (Oceania). The gray colors are for Africa.

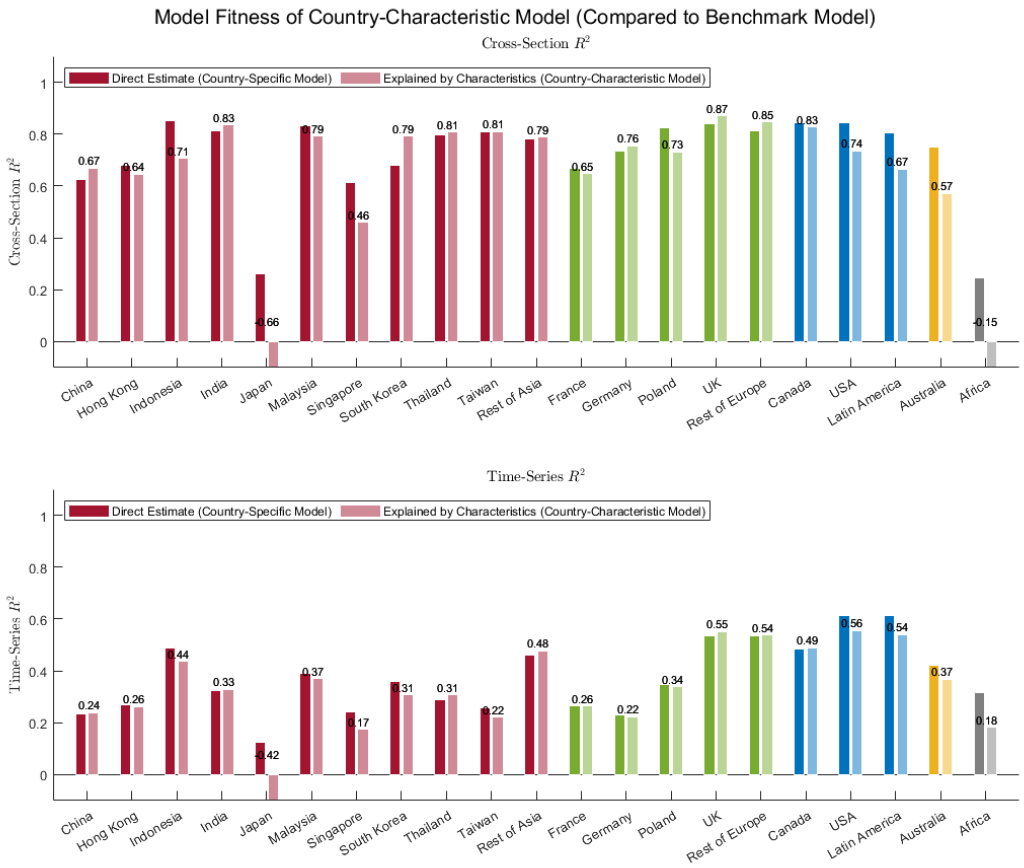


Figure 4: Fitness of Country Characteristic Model

This figure compares the fitness of model of country-specific adjustment cost function, and the alternative model of country characteristics. Estimation outcome of model of country-specific adjustment cost function is from the Baseline estimation. The country-characteristics model is estimated using the portfolio-level moment of the subsample of 17 countries and 56 countries in the 4 regions of remaining area. Country characteristic variables are the respect to the trade secrets and industry patents, cost of contract enforcement, economic and financial development. The subfigure in first row reports the cross-section R^2. The bar of yellow color describes the estimation outcome of the country-specific model. The bar of purple color describes the estimation outcome of the country-characteristics model. The data label of numbers denote the cross-section R^2 of the country-characteristics model. The subfigure in second row reports the time-series R^2. The bar of dark color describes the estimation outcome of the country-specific model. The bar of light color describes the estimation outcome of the country-characteristics model. The data label of numbers denotes the time-series R^2 of the country-characteristics model. Within each geographic area (of 17 countries, Rest of Asia, Rest of Europe, Latin America, Africa), firms are grouped into portfolios. Model-implied valuation is calculated for each model, then the statistics of fitness are calculated using the portfolio-level moments. The construction of portfolios, calculation of statistics are identical with Table 2 in the draft. The bar of dark red color, the light red color describes the estimation outcomes of the two specifications respectively, for countries and the remaining area in Asia. The dark and light green colors are for countries and the remaining area in Europe. The blue colors are for Americas. The yellow colors are for Australia (Oceania). The gray colors are for Africa.

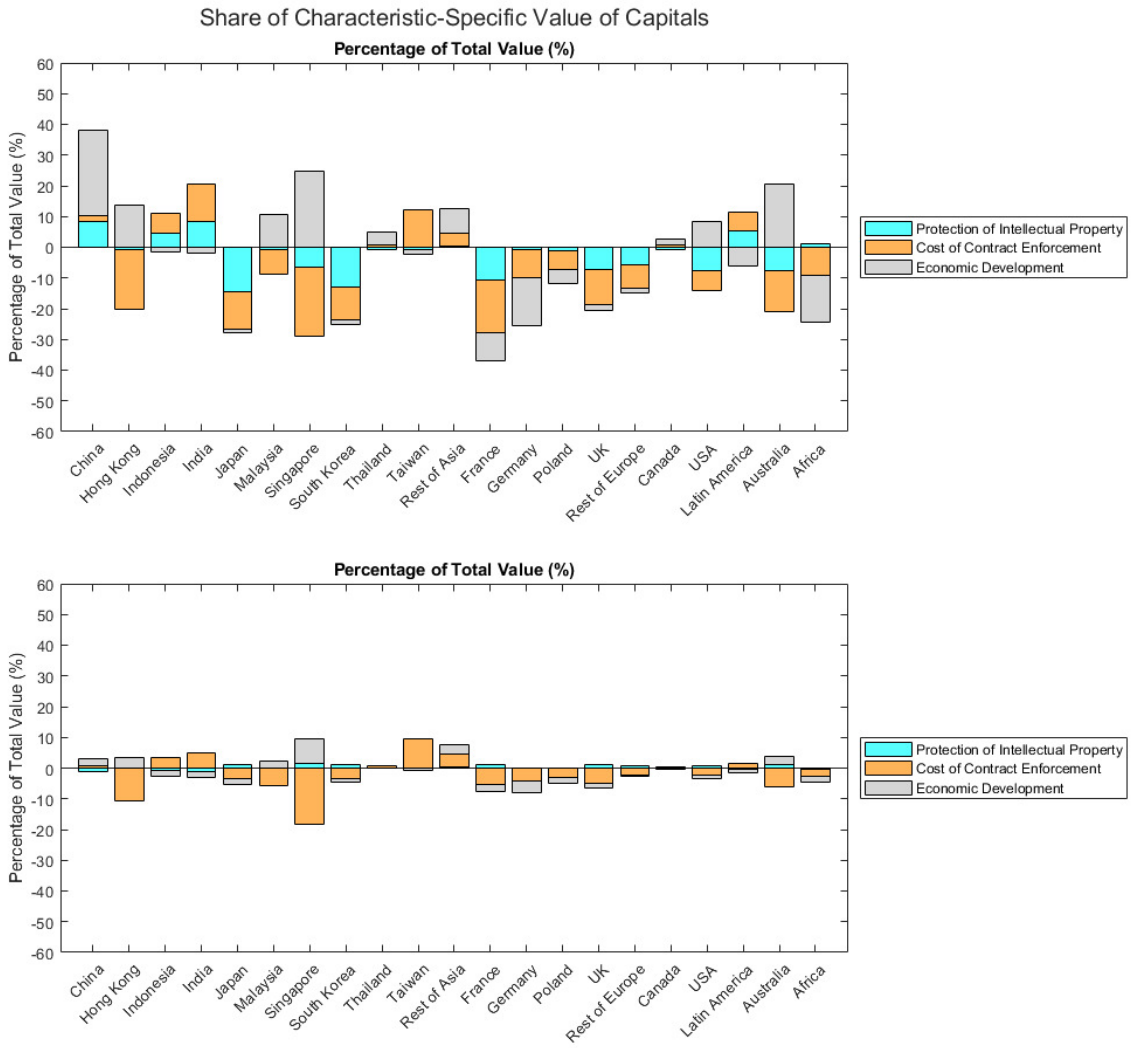


Figure 5: Share of Characteristic-Specific Value

This figure reports the model-implied characteristic-specific valuation for the model of country-specific state-contingent adjustment cost function (in abbreviation, country-characteristic model). The valuation of intangible capital from country-characteristic variables in percentage of the total firm value is reported in the first row. For each aspect z of country-characteristic variables, the cost compensation in value of intangible capital is calculated as $\{\gamma_z^I \cdot X_{z,c} \cdot (1-\tau) \cdot i^I \cdot K^I\}_z$, then its contribution to total firm value $\{\frac{\gamma_z^I \cdot X_{z,c} \cdot (1-\tau) \cdot i^I \cdot K^I}{V}\}_z$ is reported. The component $\frac{(1-\tau) \cdot i^I \cdot K^I}{V} = \frac{1}{T} \sum_t \frac{\sum_i (1-\tau_{c,t}) \cdot i_{it}^I K_{it+1}^I}{\sum_i V_{it}}$ is calculated as the aggregate statistic for each geographic area (of 17 countries, 4 regions of Rest of Asia, Rest of Europe, Latin America, Africa) and each year during 2016-2020. The similar statistic is reported for physical capital in the second row.

C Math Appendix

C.1 Theorem of Firm Valuation Decomposition

This appendix section provides the proof for the valuation equation 5. The proof is direct extension of (Hayashi, 1982) with consideration of multiple types of capital inputs and debt.

Theorem [Hayashi,1982]: Denote $P_{it} \equiv V_{it} - D_{it}$ be the ex-dividend equity value. If profit function $\Pi(.)$ and cost function $C(.)$ are Constant Return to Scale, firm's value maximization implies that

$$P_{it} + B_{it+1} = q^P_{it} K^P_{it+1} + q^I_{it} K^I_{it+1}, \tag{24}$$

in which

$$\begin{aligned} q^P_{it} &= 1 + (1-\tau_t) \cdot \tfrac{\partial C_{it}}{\partial I^P_{it}}, \\ q^I_{it} &= (1-\tau_t) \cdot (1 + \tfrac{\partial C_{it}}{\partial I^I_{it}}). \end{aligned} \tag{25}$$

Proof:

The Bellman equation of firm equity valuation is

$$V(X_{it}, K^P_{it}, K^I_{it}) \equiv \max_{\{L_{it+\triangle t}, \mathbf{I}_{it+\triangle t}, \mathbf{K}_{it+\triangle t+1}, B_{it+\triangle t+1}\}_{\triangle t=0}^{\infty}} D_{it} + E_t \left[M_{t+1} \cdot V(X_{it+1}, K^P_{it+1}, K^I_{it+1}) \right], \tag{26}$$

$$\begin{aligned} s.t. \quad D_{it} \equiv (1-\tau_t)[\Pi_{it} - C_{it} - I^I_{it}] \quad & -I^P_{it} + B_{it+1} - r^B_{it} B_{it} \\ & + \tau_t \delta^P_{it} K^P_{it} + \tau_t (r^B_{it} - 1) B_{it}. \end{aligned} \tag{27}$$

$$K^P_{it+1} = I^P_{it} + (1 - \delta^P_{it}) K^P_{it} \tag{28}$$

$$K^I_{it+1} = I^I_{it} + (1 - \delta^I_{it}) K^I_{it} \tag{29}$$

The X_{it} is vector of state variables of aggregate productivity and firm productivity. Optimal physical capital investment reflects the marginal value of physical capital (K^P_{it+1}):

$$\underbrace{1 + (1-\tau_t)\frac{\partial C_{it}}{\partial I^P_{it}}}_{q^P_{it}\text{: Physical Capital Marginal q}} = E_t \left[M_{t+1} \cdot \frac{\partial V}{\partial K^P}(X_{it+1}, K^P_{it+1}, K^I_{it+1}) \right] \tag{30}$$

Optimal intangible capital investment reflects the marginal value of intangible capital (K^I_{it+1}):

$$\underbrace{(1-\tau_t)\left[1+\frac{\partial C_{it}}{\partial I^I_{it}}\right]}_{q^I_{it}\text{: Intangible Capital Marginal q}} = E_t\left[M_{t+1}\cdot\frac{\partial V}{\partial K^I}(X_{it+1},K^P_{it+1},K^I_{it+1})\right] \tag{31}$$

We assume the Modigliani-Miller environment, total firm valuation is irrelevant of capital structure. Equity value maximization implies the total firm valuation maximization, $r^B_{it}B_{it} + V(X_t,\mathbf{K}_{it}) = \overline{V}(X_t,\mathbf{K}_{it})$. If profit function $\Pi(.)$ and cost function $C(.)$ are Constant Return to Scale w.r.t $\mathbf{K}_{it} = (K^P_{it},K^I_{it})$, firm's value maximization implies that

$$\overline{V}(X_t,a\cdot K^P_{it},a\cdot K^I_{it}) = a\cdot\overline{V}(X_t,K^P_{it},K^I_{it}), \tag{32}$$

Homogeneity of Degree One implies the equation below,

$$\frac{\partial\overline{V}}{\partial K^P}(X_t,a\cdot K^P_{it},a\cdot K^I_{it})\cdot K^P_{it} + \frac{\partial\overline{V}}{\partial K^I}(X_t,a\cdot K^P_{it},a\cdot K^I_{it})\cdot K^I_{it} = a\cdot\overline{V}(X_t,K^P_{it},K^I_{it}), \tag{33}$$

At $a=1$, the total firm valuation is quantity of capital(s) multiplied by the marginal valuation of capital(s). In the Modigliani-Miller environment, total firm valuation is irrelevant of debt. Marginal value of physical capital for the equity valuation is identical with the first-order partial derivative of total firm valuation,

$$\frac{\partial V}{\partial K^P}V(X_t,\mathbf{K}_{it}) = \frac{\partial\overline{V}}{\partial K^P}(X_t,\mathbf{K}_{it}), \tag{34}$$

The similar equation holds for the new amount of intangible capital. Previously, we denote the $q^P_{it}(X_t) = E_t\left[M_{t+1}\cdot\frac{\partial V}{\partial K^P}(X_{it+1},\mathbf{K}_{it+1})\right]$, substituting the equation (34) yields $q^P_{it} = E_t\left[M_{t+1}\cdot\frac{\partial\overline{V}}{\partial K^P}(X_{it+1},\mathbf{K}_{it+1})\right]$, shadow price of physical capital is the expected marginal value for total firm valuation. Replacing the expected marginal value of capital, The expected total firm valuation is written as

$$\begin{aligned} E_t\left[M_{t+1}\cdot\overline{V}(X_{it+1},\mathbf{K}_{it+1})\right] &= E_t\left[M_{t+1}\cdot(\tfrac{\partial\overline{V}}{\partial K^P}(X_{it+1},\mathbf{K}_{it+1})\cdot K^P_{it+1}+\tfrac{\partial\overline{V}}{\partial K^I}(X_{it+1},\mathbf{K}_{it+1})\cdot K^I_{it+1})\right] \\ &= q^P_{it}(X_t)\cdot K^P_{it+1}+q^I_{it}(X_t)\cdot K^I_{it+1} \end{aligned},$$

Recall the total firm valuation is equity plus the debt, we arrive to the expected equity value $E_t\left[M_{t+1}\cdot(V(X_{it+1},\mathbf{K}_{it+1})\right]$ prior to dividend payout,

$$\begin{aligned} E_t\left[M_{t+1}\cdot\overline{V}(X_{it+1},\mathbf{K}_{it+1})\right] &= E_t\left[M_{t+1}\cdot(V(X_{it+1},\mathbf{K}_{it+1})+r^B_{it+1}B_{i,t+1})\right] \\ &= E_t\left[M_{t+1}\cdot(V(X_{it+1},\mathbf{K}_{it+1})\right]+E_t\left[M_{t+1}\cdot r^B_{it+1}B_{i,t+1}\right] \end{aligned},$$

The model considers the one-period debt. The new amount of debt equals the expected repayment in next period, $B_{i,t+1}=E_t\left[M_{t+1}\cdot r^B_{it+1}B_{i,t+1}\right]$. Equity value of current period is the payout and the expected equity value of next period, $V_{it}=D_{it}+E_t\left[M_{t+1}\cdot V(X_{it+1},\mathbf{K}_{it+1})\right]$. Combining these equations, we conclude, if profit function $\Pi(.)$ and cost function $C(.)$ are Constant Return to Scale w.r.t $\mathbf{K}_{it}$, firm's value maximization implies that

$$V_{it}+B_{it+1}=D_{it}+q^P_{it}(X_{it})\cdot K^P_{it+1}+q^I_{it}(X_{it})\cdot K^I_{it+1}, \tag{35}$$

Let $P_{it}\equiv V_{it}-D_{it}$ be the ex-dividend equity value, the equation is further simplified as

$$P_{it}+B_{it+1}=q^P_{it}(X_{it})\cdot K^P_{it+1}+q^I_{it}(X_{it})\cdot K^I_{it+1}, \tag{36}$$

Q.E.D.

C.2 Derivation of Country-Characteristic Model

Linear Approximation: Consider the observed institutional characteristic $\widetilde{X}_{1,c}$, its steady state $\overline{\mathbf{X}}_{\mathbf{1,n}}$ depends on the endowment of intangible capital in that country , which we simplified as discrete value in the range of $n\in\{1,2,3,4\}$, sorted by the relative ranking $\frac{K^I}{K^I+K^P}$. The deviation of institutional characteristic from cohort steady state $\widetilde{\mathbf{X}}_{\mathbf{1,c}}-\overline{\mathbf{X}}_{\mathbf{1,n}}$ have identical incremental effect $\boldsymbol{\gamma}^{\boldsymbol{I}}_{\mathbf{1}}$ to the firm-level adjustment cost. Combining the innate global average cost $\theta_{I,g}$, firm-level

adjustment cost in a country with characteristic profile $\mathbf{X}$ is

$$\begin{aligned}
\theta_I(\mathbf{X}) = &\ \theta_{I,g} + \textstyle\sum_{n=1}^{4} \mathbb{1}(X_{Intan,c} = n) \cdot \boldsymbol{\gamma_1^I} \cdot (\widetilde{\mathbf{X}}_{\mathbf{1,c}} - \overline{\mathbf{X}}_{\mathbf{1,n}}) \\
= &\ \theta_{I,g} + \boldsymbol{\gamma_1^I} \cdot (\widetilde{\mathbf{X}}_{\mathbf{1,c}} - \tfrac{1}{4}\textstyle\sum_{n=1}^{4} \overline{\mathbf{X}}_{\mathbf{1,n}}) + \sum_{n=1}^{4} \mathbb{1}(X_{Intan,c} = n) \cdot \boldsymbol{\gamma_1^I} \cdot (\overline{\mathbf{X}}_{\mathbf{1,n}} - \tfrac{1}{4}\textstyle\sum_{n=1}^{4} \overline{\mathbf{X}}_{\mathbf{1,n}}) \\
= &\ \theta_{I,g} + \boldsymbol{\gamma_1^I} \cdot \mathbf{X}_{\mathbf{1,c}} + \textstyle\sum_{n=1}^{4} \mathbb{1}(X_{Intan,c} = n) \cdot \gamma_{2,n}^I \cdot \\
= &\ \theta_{I,g} + \boldsymbol{\gamma^I} \cdot \mathbf{X_c}
\end{aligned} \tag{37}$$

In this equation 37, the global-demeaned country characteristic is denoted as $\mathbf{X}_{\mathbf{1,c}} = \widetilde{\mathbf{X}}_{\mathbf{1,c}} - \frac{1}{4}\sum_{n=1}^{4} \overline{\mathbf{X}}_{\mathbf{1,n}}$. The unconditional global average steady state is denoted as $\overline{\mathbf{X}}_{\mathbf{1,g}} = \frac{1}{4}\sum_{n=1}^{4} \overline{\mathbf{X}}_{\mathbf{1,n}}$. The fixed effect coefficient $\gamma_{2,n}^I$ summarizes the cohort steady state $\gamma_{2,n}^I = \boldsymbol{\gamma_1^I} \cdot (\overline{\mathbf{X}}_{\mathbf{1,n}} - \frac{1}{4}\sum_{m=1}^{4} \overline{\mathbf{X}}_{\mathbf{1,m}})$. By construction, $\sum_{n=1}^{4} \gamma_{2,n}^I = \sum_{n=1}^{4} \boldsymbol{\gamma_1^I} \cdot (\overline{\mathbf{X}}_{\mathbf{1,n}} - \frac{1}{4}\sum_{m=1}^{4} \overline{\mathbf{X}}_{\mathbf{1,m}}) = \boldsymbol{\gamma_1^I} \cdot (\sum_{n=1}^{4} \overline{\mathbf{X}}_{\mathbf{1,n}} - \frac{1}{4}\sum_{m=1}^{4} \overline{\mathbf{X}}_{\mathbf{1,m}}) = 0$. Under the simple situation where the fixed effects are identical among the lowest 3 quartiles, estimation only requires distinguishing the 4th quartile $\{\gamma_{2,n\leq 3}^I, \gamma_{2,n=4}^I\}$. Under this simple situation, we denote the fixed effect of the top quartile as $\gamma_{2,\text{Top25\%}}^I$ for informative notations. This model can be further simplified as equation 15 where the $\mathbf{X_1}$ is the global-demeaned institutional characteristics, and the indicator variable $\mathbf{X}_2$ for whether endowment of intangible capital in the country belongs to the highest quartile.

Remark: The countries that intensively utilize intangible capital in the production, show adapted institution environment. Approximation with fixed effect incorporates the different steady states. For example, the specialists in evaluating the originality of technology, creation will help law firm to collect evidence for the infringement of intellectual property, or the bank to accurately gauge value for the intangible capital owned by the firm. Such supporting infrastructure has fixed cost in modifying the college education system and the legal system at the country level.

Definition of Valuation contributed by Country Characteristic: The firm valuation ratio (average Q) combines physical and intangible capital were defined in equation 13. The market value of each input can be decomposed into three components,

$$\begin{aligned}
\tfrac{q^I \cdot K^I}{K^P + K^I} = &\ (1-\tau_c)\tfrac{K^I}{K^P+K^I} + (1-\tau_c)\theta_{I,g}\left(\tfrac{I^I}{K^I}\right)\tfrac{K^I}{K^P+K^I} + (1-\tau_c)\cdot\boldsymbol{\gamma^I}\cdot\mathbf{X_c}\cdot\left(\tfrac{I^I}{K^I}\right)\tfrac{K^I}{K^P+K^I} \\
\tfrac{q^P \cdot K^P}{K^P + K^I} = &\ \tfrac{K^P}{K^P+K^I} + (1-\tau_c)\theta_{P,g}\left(\tfrac{I^P}{K^P}\right)\tfrac{K^P}{K^P+K^I} + (1-\tau_c)\cdot\boldsymbol{\gamma^P}\cdot\mathbf{X_c}\cdot\left(\tfrac{I^P}{K^P}\right)\tfrac{K^P}{K^P+K^I}
\end{aligned} . \tag{38}$$

with intangible capital book value $(1-\tau_c)\frac{K^I}{K^P+K^I}$, global value $(1-\tau_c)\theta_{I,g}\left(\frac{I^I}{K^I}\right)\frac{K^I}{K^P+K^I}$ and a characteristic specific value $(1-\tau_c)\cdot\boldsymbol{\gamma^I}\cdot\mathbf{X_c}\cdot\left(\frac{I^I}{K^I}\right)\frac{K^I}{K^P+K^I}$, and analogously for the physical capital. The last component capture the drivers of the heterogeneity in the value of each input that depends on the country characteristics modeled. Taking the observed investment rates and firm valuation, one can use equation 38 to gauge the direct effect over firm valuation from under-developed institutional infrastructure and financial market. Formally, the price of capital in a country c is determined as

$$\begin{aligned} q^I &= (1-\tau_c)+q_g^I+\mathbf{q}_X^I\cdot\mathbf{X_c} \\ q^P &= 1+q_g^P+\mathbf{q}_X^P\cdot\mathbf{X_c} \end{aligned} \tag{39}$$

Given the observed firm investment rates, the common valuation assuming global average cost is determined as

$$q_g^I = \theta_{I,g}\cdot(1-\tau_c)\cdot i^I \quad , \quad q_g^P = \theta_{P,g}\cdot(1-\tau_c)\cdot i^P \tag{40}$$

The marginal compensation value q_z for country characteristic variable $X_{z,c}$ is determined by the incremental cost coefficient γ_z,

$$q_z^I = \gamma_z^I\cdot(1-\tau_c)\cdot i^I \quad , \quad q_z^P = \gamma_z^P\cdot(1-\tau_c)\cdot i^P \tag{41}$$

The valuation of intangible capital contributed by country-specific characteristic of institutional environment and economic development in Equation 23 can be correspondingly derived.

Remark: Ex-post we cannot tell whether the realized low investment rate is discouraged by the anticipated high adjustment cost in the future, or the high discount rate from the risk premium. Such investigation requires fully parameterizing the firm operating profit process (how it is affected by the product market competition and aggregate shocks), and the stochastic discount factor co-moving with the aggregate shocks. It would be meaningful to further quantify the indirect effect through discouraging or distorting the investment rate. This decomposition in equation 39 uses the observed investment rate in realized equilibrium outcome, estimates the component in the price of capital that is directly attributed to country characteristic $q_z^I X_{z,c}$.