Lost in Transition: Financial Barriers to New Technologies*

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Abstract

Advanced economies are reshaped by transitions from old to new technological paradigms. These transitions rely on complementary innovations that spread new technologies across industries. What happens when a financial crisis disrupts investment in these innovations? Using a Schumpeterian endogenous growth model with financial frictions, we show that investment in new technologies is more sensitive to tighter financial conditions than investment in mature technologies. The key mechanism is path dependence: firms build expertise in the technologies they previously developed, leading young firms to self-select into emerging technologies. As young firms are more financially constrained, shocks to financing disproportionately depress innovation in new technologies. We validate this mechanism using causal evidence on green patenting by German firms around the Global Financial Crisis (GFC). The aggregate fraction of green innovation (patenting in carbon reducing technologies) stagnated after the GFC. Quantifying our model using the causal estimates from the micro data implies that the slowdown of the sorting of young firms into new technologies accounts for about half of the macro trends in green patenting.

Keywords: Endogenous Growth, Technological Progress, Climate Change, Financial Frictions, Innovation, Creative Destruction.

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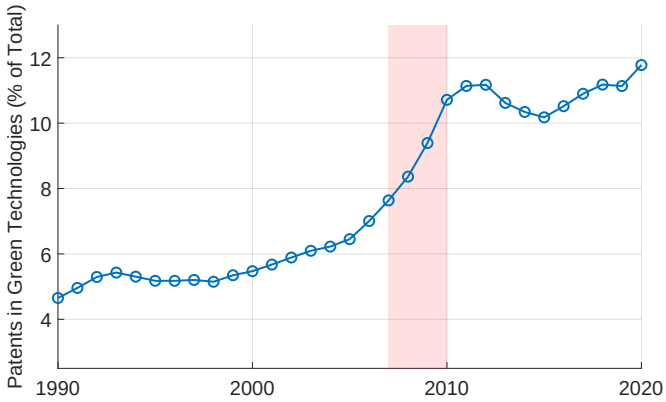
1. Introduction

Modern economies are frequently reshaped by technological transitions in which new paradigms of production gradually replace old ones. Examples include the transition from the typewriter to the computer, from steam power to electricity, and from the internal combustion engine to the electric vehicle. These episodes of structural change can be set in motion by general purpose technologies, shifts in consumer demand, or policy interventions, but their success ultimately depends on whether the new technology can be adapted to a wide range of uses and production processes. That, in turn, requires a sustained flow of complementary innovations that improve the technology, lower its costs, and enable its diffusion across firms and industries (Griliches 1957, Helpman and Trajtenberg 1994, Bresnahan and Trajtenberg 1995, Acemoglu et al. 2016, Ribeiro 2025). A natural concern is that disruptions to investment in these complementary innovations may be especially costly, because they slow the reallocation of economic activity toward new technologies.

This paper argues that tighter financial constraints—such as those induced by a financial crisis—disproportionately depress innovation in new technologies during technological transitions. Our mechanism is *not* that new technologies are inherently harder to finance than mature technologies. Rather, it is because the financial crisis disproportionately hurts the kind of firms most active in the transition, namely younger, start-up firms. This rests on two forces. First, innovation is path dependent across technology vintages at the firm level: firms find it cheaper to continue innovating within the technology vintage in which they have previously succeeded. As a result, early in a technological transition, younger firms self-select into innovation in emerging technologies, while older firms are more likely to innovate in the mature technologies with which they are familiar. Second, younger firms are more financially constrained than older firms, because they have not yet developed the lending relationships or securities-market access of mature firms (Gertler and Gilchrist 1994, Cloyne et al. 2023). Taken together, a tightening of financial conditions disproportionately depresses innovation in new technologies by mostly affecting the firms that select into them. The costs of financial crises are therefore *state dependent*: they are especially large early in a technological transition, when young firms play a central role in the diffusion of the new technology.

We analyze the effect of the self-selection of young firms into new technologies on the susceptibility to financial shocks in three steps. To motivate our main argument, we use global patent data to provide empirical evidence on three stylized innovation facts from before, during, and after the recent Global Financial Crisis. First, we find that there is a clear positive relationship between the age of technologies and the age of innovators: the younger a technology, the greater the fraction of patents originating from young firms. Second, innovation by young firms was disproportionately affected by the Global Financial Crisis. Third, patenting in mature technologies were robust to the Global Financial Crisis, while technologies that were in the midst of a transition fell sharply below trend. Together, these

Figure 1. Global Trend: Green Patents as a Percentage of Total Patents



Notes: The figure shows the share of patent families classified as green under the Y02 climate-change mitigation classification, using PATSTAT data (granted applications by date of filing). We focus on patents that were taken out in two or more jurisdictions ("multi-patent families") to screen out low value patents. Appendix B, Figure B1, shows that the break in green patenting trends around 2011 is similar when restricting attention to triadic patents (those granted at the EPO, USPTO, and Japan Patent Office); weighting patents by forward citations; restricting the sample to OECD countries; when examining trends within individual countries and using all patents.

three facts suggest that financial disruptions can derail technological transitions because the firms that drive early-stage innovation in new technologies are the one exposed to tight credit.

In the second part of the analysis, we formalize the argument in a model of firm dynamics and creative destruction in the spirit of Klette and Kortum (2004), where long-run growth is driven by firms developing higher-quality versions of goods that other firms initially produce. We depart from the standard models in two main respects. First, goods can be produced with either a modern or an old technology vintage, which differ in their marginal costs. When firms innovate they decide how much of their research and development (R&D) to direct towards the development of higher-quality goods that rely on the old and the new vintage, respectively. Innovation exhibits path-dependence: the higher the number of products that a firm already produces using the older vintage , the lower the cost of further innovation in the old vintage (and vice versa for the newer vintage). As a result, total new vintage innovation is a function of both the profitability of producing with the new vintage and the fraction of the economy's products that is already produced in the new vintage. Second, firms in our model are subject to a simple credit constraint: they are only able to finance R&D up to a firm-specific multiple of their earnings (Drechsel 2023, Lian and Ma 2021). The multiple increases stochastically as firms mature and build a reputation (e.g., a track record of repayment). This makes younger firms' innovation more elastic to overall credit conditions than mature firms.

In the third part of the analysis, we apply this framework to understand the contribution of tight credit and the self-selection of young firms into new technologies to the recent slowdown of *green innovation*. Green innovation is a prime example of an emerging technology in transition. Prior to the 2008-2009 Global Financial Crisis, there was a steady switch of

patenting towards technologies related to climate change mitigation (Figure 1). The share of multi-patent families (i.e. technologies that were patented in more than one country, to assure quality) steadily rose from just over 4% to around 11%. That rise juddered to a halt after the financial crisis, and green innovation hovered at around 10-11% for the decade that followed.[1] It was not until 2020 that the green patenting share exceeded its crisis peak.

We perform two complementary analyses to show that our proposed mechanism explains a significant share of the slowdown of green innovation. First, we provide a causal validation of the key mechanisms in our model using micro data on patenting (both green and non-green) and firm-bank links for a sample of German firms before and after the Global Financial Crisis. Second, we use the micro data to discipline the parameters of the Schumpeterian model, and plot the effect of a financial tightening calibrated to the one observed in Germany after the Global Financial Crisis, in order to quantify the extent to which the self-selection of young firms into new technologies can explain the aggregate slowdown of green patenting.

The micro analysis uses data on German firms for which we observe exposure to the credit-supply contraction of a large bank, Commerzbank. Although innovation is not typically financed directly through bank borrowing, variation in firms' pre-crisis bank relationships provides a useful difference-in-differences design for identifying the effect of tighter credit on subsequent patenting, following Chodorow-Reich (2014) and Huber (2018). This design differences out common post-crisis changes in the profitability of green innovation—such as changes in policy support, energy prices, or investor demand—and isolates the differential effect of tighter credit on more exposed firms. We show that firms more exposed to the contraction in credit reduced patenting only in green technologies. The reason is not that exposed firms selectively cut green innovation. Rather, the effect is entirely driven by younger firms, which reduce both green and non-green innovation by similar magnitudes, while older firms' innovation is unaffected. Since younger firms accounted for a disproportionate share of green innovation before the crisis, the overall decline in green patenting among exposed firms reflects the self-selection of young firms into innovation in the emerging green technology, as predicted by the model.

Quantitatively, the model implies that the self-selection of young firms into new technologies can account for a large share of the slowdown in green patenting after the Global Financial Crisis, similar to the global slowdown shown in Figure 1. Calibrating the model to match the decline in credit supply implied by the European Central Bank's Lending Supply Indicator (Altavilla et al. 2019), we find that it can replicate roughly half of the slowdown in green patenting as a share of total patenting. Finally, a counterfactual exercise shows that if the crisis had occurred just five years later in the transition, the greater maturity of green innovators would have reduced its effect on green innovation by about 20%.

[1] The stagnation occurred just after the financial crisis in 2010, which is plausible because of the lag between a fall in R&D and subsequent patenting.

Related Literature This paper relates to several strands of literature. Our theoretical exercise combines models where innovative technologies arrive in vintages (e.g. Bresnahan and Trajtenberg 1995, Acemoglu et al. 2016, Ribeiro 2025) with models where financial frictions impede optimal innovation decisions (e.g. King and Levine 1993, Rajan and Zinglas 1998, Aghion et al. 2018, and more recently Akcigit et al. 2023).

A related literature particularly focuses on financial crises. Garcia-Macia (2015) explains that firms are unable to fund investment in intangible assets during financial crises, as their collateral value is hard to collect. Similarly, the models in Ates and Saffie (2013, 2014) claim that financial turmoil affects technological progress through lowering the ability of banks to observe project quality under imperfect information. In Queraltó (2013), financial crises increase the costs of financial intermediation through balance sheet deterioration á la Gertler and Kiyotaki (2010), which reduces the entrepreneurial entry. Schmitz (2021) adds that a crisis' effect on innovation is amplified by the fact that small and young firms are most affected by credit tightness, as these produce more radical innovation. We contribute to this literature by looking at the effect of credit constraints on the direction – modern versus old –of innovation, in younger versus mature firms, and from there at the overall effects of credit tightening on the speed of a transition toward modern production technologies.

The key assumption in our model that young firms face greater financial constraints is rooted in a long literature. Gertler and Gilchrist (1994) note that informational frictions in finance are particularly strong for young firms. This is because young firms have yet to establish a sufficiently long credit history, as pointed out by (e.g.) Evans and Jovanovic (1989), Haltiwanger et al. (2013), Davis and Haltiwanger (2019) and Chaney et al. (2012). Recent evidence supports this idea. Cloyne et al. (2023) find that young firms' investments are more responsive to monetary policy shocks, a result confirmed for employment by Bahaj et al. (2019). Clymo and Rozsypal (2022) similarly find that young firms exhibit more business cycle sensitivity, consistent with tighter financial constraints.

Our application to green patenting during the Global Financial Crisis is closest to the literature that jointly studies endogenous technological change and the transition to clean production. The seminal reference is Acemoglu et al. (2016). Our key novelty is that we introduce a path dependence that ties the economy's capacity to engage in green innovation to the prevalence of green production. This causes green innovation to concentrate among young firms in the transition, which in turn drives its sensitivity to tight credit.

More broadly, the application contributes to the literature that analyzes the interaction between climate and directed technological change after Acemoglu et al. (2012).[2] Existing work in this literature primarily focuses on how policy can induce an optimal transition to clean production (see, e.g., Hémous 2016, Aghion et al. 2016, Fried 2018, Acemoglu et al. 2023). Hémous and Olsen (2021) provide a recent review. We contribute to this literature by introducing credit constraints and firm dynamics, looking at how credit tightening can slow

[2]This work, in turn, builds on the long literature on growth and climate change, starting with Nordhaus (1994). See Golosov et al. (2014) and the recent reviews by Caggese et al. (2023) and Hassler et al. (2018).

down the transition of the overall economy towards a clean economy by directly affecting green innovation by younger and smaller firms.

We consider our theoretical exercise complementary to papers that explain why clean technology adoption might be difficult for financially constraint firms. Lanteri and Rampini (2023), for example, show that older shipping firms were more likely to adopt clean technologies in 2022 than younger firms, and rationalize this with a model in which green adoption comes at high up-front costs. Capelle et al. (2025) show formally that investments by that financially constraint firms have a pecking order, where they first choose to invest in cheaper (polluting) capital. Our application is different in the sense that we focus on green innovation rather than green capital adoption. In addition, our contribution is that even in absence of *any* innate difference in the cost of financing green or dirty innovation, green innovation is endogenously more sensitive to financial constraints when the economy is in transition.

Our micro data analysis relates to the recent literature on the financial costs of climate change and on green finance. Bolton et al. (2022) show that financiers require compensation for climate risk. Our focus is instead on how finance – or the lack of it – affects firms' choice to innovate green versus dirty, and from there the steady state firm-size distribution and aggregate growth and welfare. We also relate to De Haas et al. (2024), who show that firms that were rejected from a loan application were less likely to adopt green management technologies and to invest in green practices. De Haas and Popov (2020) find that industries in countries with better-developed financial systems transition faster to the use of low-carbon production technologies. Our contribution over and above these papers is to demonstrate that green innovations are particularly severely hampered by financial frictions.

A related literature notes that there has recently been a dramatic rise in "green investors": investors that allocate a large fraction of their capital to climate-change related projects. Bolton and Kacperczyk (2021) show that low-emission firms have lower expected returns on U.S. capital markets, implying that their valuations are higher and cost of capital are lower. Berk and van Binsbergen (2021) finds that the impact of green investors on cost of capital remains small. Gormsen et al. (2021) argue that the green wedge in capital costs has increased rapidly between 2016 and 2022. Accetturo et al. (2022) and Zazzaro et al. (2024) show that green investments are responsive to changes in credit supply, consistent with our findings. Patozi (2023) notes that green investors might affect monetary policy transmission. A review on the interaction between climate change and finance is provided in Giglio et al. (2021).

Our empirical strategy builds on work that identifies the real effects of financial tightness by exploiting variation in the lenders that firms rely on. Examples include Chodorow-Reich (2014), Acharya et al. (2015), Bentolila et al. (2015), Franklin et al. (2015) and Giroud and Mueller (2015), who analyze the employment effects of tightening of credit supply using variation in firm-level exposure. It is similarly related to Almeida et al. (2012), Greenstone et al. (2014), Adelino et al. (2015), Aghion et al. (2015), Paravisini et al. (2015), and De Ridder (2019). These papers use exposure to credit shocks to analyze the effect on employment,

investments, exports, output and patenting. To our knowledge, ours is the first paper to usethat methodology to study the differential effect of tight credit on (non) green patenting.

Outline The remainder of this paper proceeds as follows. We provide the motivating evidence on the stylized facts of patenting around the Global Financial Crisis in Section 2. The Schumpterian growth model with path dependence and financial constraints is presented in Section 3. The causal evidence in support of the model's mechanisms and the quantitative application to green innovation are presented in Section 4. Section 5 concludes.

2. Motivating Evidence

This section presents three stylized facts that motivate the theoretical framework in Section 3: young firms are more likely to innovate in young technologies; innovation by young firms has declined since the Global Financial Crisis; and innovation in young technologies has also declined since the Crisis. To document these facts, we use patents as our measure of innovation. Patents have well known drawbacks as many innovations are not patented and many patents are not innovations. But they are a well validated measure that are commonly used in the literature. Our data include all patents granted by the USPTO between 1976 and 2020 that are assigned to private businesses. We measure firm age as the number of years since a firm's first patent and begin the analysis after 1990. We classify patents as coming from 'young' firms if the firm first appears in the patent data fewer than fifteen years earlier.[3]

2.1. Young Firms Invest in Young Technologies

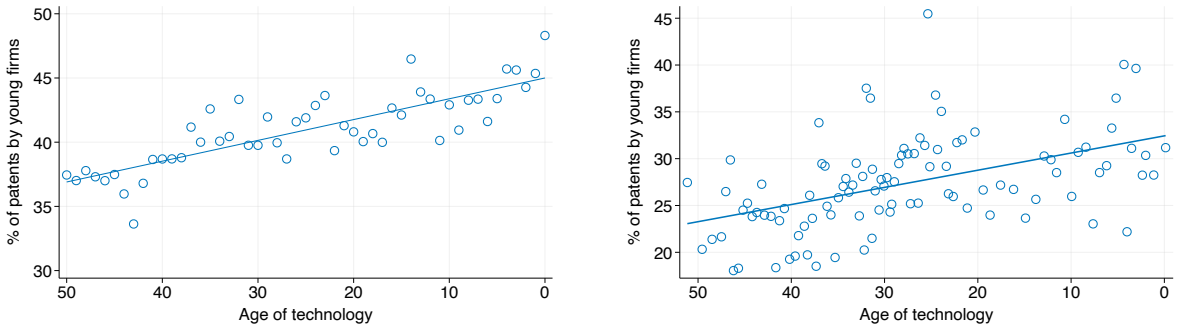
We begin by showing that younger firms contribute disproportionately to innovation in emerging technologies. We measure the age of a technology by the number of years since it first recorded at least 20 patents in a single year.[4] To identify technologies, we use the detailed Cooperative Patent Classification (CPC) codes assigned to each patent. We retain inventional CPC codes, which capture the patent's core inventive content, and define technologies at the four-digit CPC level. This procedure yields 130 technologies with initial transitions after 1990.

Figure 2 presents the relationship between firm age and the age of technologies. It presents a binned scatter plot on the fraction of patents in a technology that originates from

[3]Patents are taken from Patentsview. Each patent is associated with bibliographical information and, in particular, its list of disambiguated assignees. We remove patents with no declared assignee (independent inventors) or where the assignee is not a private business. Because foreign firms may have patented in other jurisdictions, we restrict attention to assignees located in the United States. We define a *young firm* as one that filed its first patent less than 10 years ago at every point in time. Because our dataset starts in 1976, we can identify young innovative firms only over the period 1990–2020. The resulting dataset contains approximately 3.2 million individual patents filed by 0.2 million assignees, of which 43% are classified as young at the time of filing.

[4]The stylized fact that young firms innovate disproportionately in young technologies is robust to alternative thresholds for defining technology age. We require a minimum number of patents in a single year because some technologies exhibit negligible patent counts in scattered early years, whereas once a technology experiences a burst of patenting, its presence in the data usually becomes sustained.

Figure 2. Technology Age and Share of Young firms



(a) Without year fixed effects (b) With year fixed effects

Notes: The figure plots the relationship between technology age and the share of young firms in a technology-year panel, where technologies are defined at the 4-digit CPC level and the sample is 1990–2006. Observations are grouped by technology age into 50 bins, and each marker reports the mean share of young firms within a bin.

young firms against the number of years that have passed since the technology received its first burst of patents (on the inverse scale). It presents a binned scatter plot of the fraction of patents in a technology that originates from young firms (y-axis) against the (inverse) age of the technology class on the x-axis (as measured by the number of years that have passed since the technology received its first burst of patents). We perform this analysis on the 1990 to 2006 crisis period, to abstract from the impact of the Global Financial Crisis.

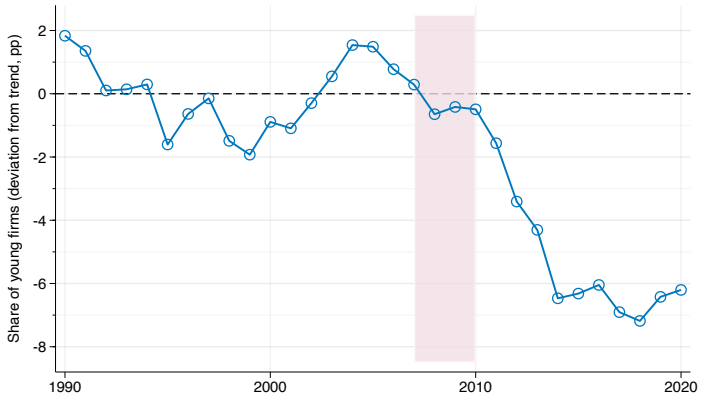
Figure 2 shows clearly that there is a larger fraction of innovation from young firms in less mature technology classes. The left-hand panel presents a simple scatter plot with a least squares fit, while the right-hand panel removes year fixed effects to control for aggregate changes in aggregate patenting over time. The relationship is economically and statistically significant. The gradient in the left panel implies that for each additional year of technology age, there is a 0.14 percentage point reduction (with a standard error of 0.07) in the fraction of patents from young firms. The year dummies reduce the tightness of the fit, but leave the slope of the relationship quantitatively unchanged.

2.2. Young-Firm Innovation Fell after the Global Financial Crisis

The second stylized fact is that innovation by young firms fell disproportionately after the Global Financial Crisis. To show this, we next analyze the contribution of young firms to overall innovation for every year between 1990 and 2020. As before, a young firm here is defined as a firm that filled its first patent less than 10 years prior. Figure 3 presents the result. It plots plots the share of young firms in overall patenting from 1990 to 2022, after removing a simple linear trend estimated on the 1990–2006 pre-crisis period.[5] The figure reveals a striking break: the share of young firms' patenting falls by around seven percentage points after the crisis, from around 40% to 33%.

[5]The results are qualitatively robust to alternative adjustments for time trends (i.e. higher order polynomials).

Figure 3. Share of Patents from Young Firms



Notes: For each year t, a young firm is defined as an assignee for which the first patent has a filing year t_0 such that $t - t_0 \leq 10$. The sample includes all granted USPTO patents with a US-based assignee. The raw time series has been detrended using 1990–2006 data.

As an aside, we should note that the decline in young firms' patenting is not merely a by-product of the secular decline in the number of young firms in the U.S. documented by (Decker et al., 2016). Appendix Figure B3 plots deviations from trend in young firms' patenting alongside deviations from trend in the overall number of young firms, measured both by firm counts and employment. Although both economy-wide series trend downward over the full sample, neither displays a discrete break comparable to that in the patent data.

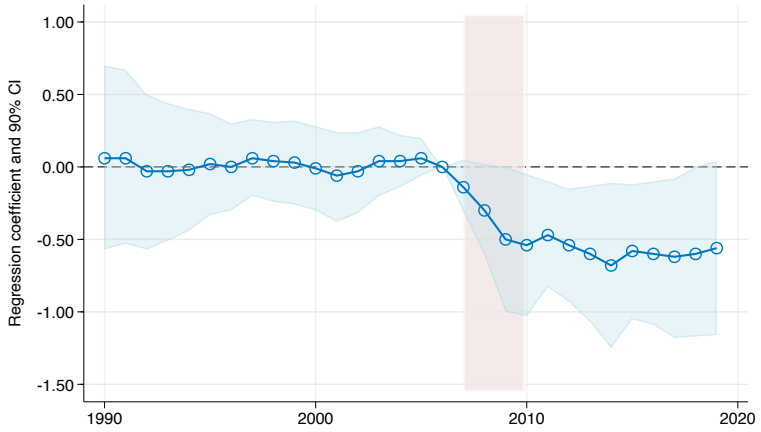
Case Study Examples To illustrate the patterns in Figure 2 and Figure 3, we consider two examples of young technology classes in the pre-crisis period: solar panels and micro sensors. These are in the bottom quintile of the technology class age distribution. [6] Young firms accounted for 55% of all patents of solar patents on the eve of the financial crisis in 2007. After the crisis, that share fell by about 10 percentage points to 45% by 2014. A similar pattern is visible in micro-sensors with the young-firm patent share peaking at 43% in 2008, before declining sharply to about 25% by 2014. In other words, both technologies were in the early diffusion phase, and saw their young-firm innovation base erode markedly after the crisis.

2.3. Innovation in Young Technologies Fell after the Global Financial Crisis

The third stylized fact is that after the crisis, innovation in young technology classes slowed by much more than expected compared to mature technology classes. While this follows naturally from the first two facts, the remainder of this section establishes it more rigorously using an event-study design.

[6] In 2007, we calculate the age of solar photovoltaic module systems (CPC class H02S), as seven years old and micro-sensors (microelectromechanical manufacturing, CPC class B81C) as nine years old as they reached their "transition" threshold of 20 patents in 2000 and 1998 respectively.

Figure 4. Impact of the Global Financial Crisis on Young versus Mature Technologies



Notes: Unit of observation is a CPC4 technology-year. The dependent variable is $\ln_{c,t} - \ln \widehat{n}_{c,t}$, where $\widehat{n}_{c,t}$ is obtained by fitting a Gompertz flow curve on ages 0–10 and scaling by the observed total number of patents in each year. The regression includes CPC4, year, and age fixed effects; 2006 is the omitted year. Standard errors are clustered at the CPC4 level.

We proceed in two steps. First, for each four-digit technology class, we construct a predicted path for annual patenting. This is necessary because patenting in emerging technologies is expected to slow mechanically as those technologies mature. Simply showing that patenting in young technologies flattened after the crisis, as in Figure 1 for green patenting, would therefore risk overstating the effect of the crisis itself. Second, we examine how the gap between observed and expected patenting evolves around the Global Financial Crisis, and whether that gap widens more for young than for mature technologies.

To do so, we construct a technology-specific counterfactual path for patenting that nets out aggregate changes in overall USPTO activity. Define $\widehat{n}_{c,t}$ as the predicted path of the number of patents in technology c in year t if there had not been a financial crisis. For each technology, we fit a Gompertz flow curve as a function of technology age using only the first ten years of its life cycle. This specification captures the standard hump-shaped pattern of technological diffusion, with patenting rising, peaking, and then declining as the technology matures.[7] The estimated curve therefore provides a benchmark for how patenting in that technology would have evolved absent the crisis, based on its own early trajectory. Details on the counterfactual estimation procedure are provided in Appendix C.

We then assess how the gap between the actual number of patents $n_{c,t}$ and the predicted patent number evolves for both mature and emerging technologies around the Global Financial Crisis. Specifically, we estimate the following specification:

$$\ln n_{c,t} - \ln \widehat{n}_{c,t} = \sum_{\tau \neq 2006} \beta_\tau \, \mathbb{I}_{t=\tau} \, Young_c + \psi_c + \chi_t + \eta_{a_{c,t}} + \varepsilon_{c,t}, \tag{1}$$

[7] Gompertz processes are a standard reduced-form representation of technological diffusion and life-cycle dynamics (Geroski, 2000): they imply an S-shaped cumulative pattern and a single-peaked (hump-shaped) flow, closely matching the empirical evolution of activity in many technologies (Griliches, 1957; Ribeiro, 2025).

where $Young_c$ indicates a young technology class at the time of the financial crisis (i.e. with $t_0 > 1990$). The specification includes fixed effects ψ_c for each of the technology classes, year fixed effects χ_t, and technology's age fixed effects $\eta_{a_{c,t}}$.

Figure 4 reports the estimated coefficients β_τ with 90% confidence intervals. The pre-crisis coefficients are close to zero, supporting the parallel-trends assumption. After 2008, a clear and persistent negative gap opens up: young technologies fall progressively further below their counterfactual path relative to mature technologies.

Discussion Taken together, the three facts paint a coherent picture: young firms are the primary innovators in emerging technologies (Fact 1); the Global Financial Crisis disproportionately curtailed innovation by young firms (Fact 2), and the technologies in the midst of a transition—those most reliant on young-firm innovation—experienced the sharpest decline relative to their counterfactual trajectories (Fact 3). The next section develops a theory of path dependent innovation and financial constraints that dis consistent with these stylized facts.

3. Framework

We analyze an economy in which there are two vintages of technologies: an old and a new technology vintage. Our goal is to quantitatively study the economy's transitional dynamics after a shock to the relative profitability of producing with the new vintage, and the effect of a temporary tightening of financial conditions in the middle of that transition. In this section, we start by setting up the economy and describing the properties of the economy along an initial balanced growth path, where production with the new technology vintage is still expensive. We then explain that early in a transition, induced by a decline in the cost of producing with the new technology vintage, young firms are responsible for a large share of innovation in the new technology. In Section 4 we test that production, and show that a simulated financial tightening disproportionately affects the modern technology as a result.

3.1. Preferences

The economy comprises of a continuum of identical households with unit mass that choose the path of consumption to maximize the utility along:

$$U = \int_0^\infty \exp(-\rho t) \ln C_t \, dt$$

where C_t denotes consumption and ρ denotes the discount rate. The household supplies a single unit of labor inelastically. Time is continuous and indexed by t, which we suppress when convenient. Consumption is a Cobb-Douglas aggregate of a continuum of intermediate goods of measure 1. Each good j can be produced by any firm i in the set of firms I_j that

owns a patent to a production method to produce that good. The intermediate goods are competitively aggregated along:

$$Y = \exp \int_0^1 \left(\ln \sum_{i \in I_j} q_{ij} y_{ij} \right) dj.$$

where $Y = C$ is aggregate output, which is fully consumed. y_{ij} is the amount of good j that is produced by firm i, and q_{ij} denotes the quality of the patent that firm i owns. Higher quality patents enable firms to produce goods from which the consumer derives more utility per unit. Beyond differences in quality, the output of good j of the various producers is perfectly substitutable for the consumer, and firms compete on prices. Consumers will thus only demand the good produced by the firm that offers the lowest quality-adjusted price p_{ij}/q_{ij}.

3.2. Production

Goods can be produced with either a modern or an old technology vintage. These vintages determine the marginal cost of producing a good. Whether firm i produces good j with the modern or the old technology is determined by its patent: the production method q_{ij} either enables it to do so with one of the two. The production function associated with producing a good using technology vintage h, where $h \in \{m, o\}$ for modern and old, $y_{ij} = l_{ij} a^h$. Labor l_{ij} is remunerated at the wage rate w, which the firm takes as given. Normalizing the productivity of the modern technology vintage to unity, $a^o > 0$ determines the relative marginal cost of producing with either technology:

$$mc^h = \begin{cases} w & \text{if } h = m \\ w/a^o & \text{if } h = o \end{cases}$$

We think of the transition as being driven by an exogenous change in the relative productivity of the modern vintage: while a^o may initially be (substantially) larger than unity, at some point it becomes cheaper to produce goods with the modern vintage, meaning a^o falls below one.

In the context of a green transition, the drivers of a^o could include taxes and subsidies. In Acemoglu et al. (2016), for example, firms produce with patents that enable them to produce with either a clean or a dirty production function (akin to our vintages), where clean production is linear in labor and dirty production (d) requires an additional fossil input o_{ij}. The production function reads:

$$y_{ij} = \left(l_{ij}\right)^{\eta} \cdot \left(o_{ij}\right)^{1-\eta} \quad \Rightarrow \quad o^*_{ij} = \left(\frac{1-\eta}{\eta} \frac{w}{p^0} \right) l_{ij},$$

where $\eta \in (0,1)$ governs the factor shares in production with the dirty technology and o^*_{ij} denotes the firms' profit-maximizing use of the fossil input. The variable p^0 is the relative price of the dirty input in units of numeraire. An increase in p^0 reflects an increase in the cost of using the dirty input, potentially as the result of an increased emission tax. That raises the relative costs of producing with dirty rather than clean production vintages. Inserting optimal fossil inputs into the production function gives the production function $y_{ij} = a^d l_{ij}$, where the relative productivity a^d equals

$$a^d = \left(\frac{1-\eta}{\eta} \frac{w}{p^0} \right)^{1-\eta}.$$

3.3. Static Optimization

The firm that offers the lowest quality-adjusted price to consumers faces the demand function

$$y_{ij} = Y p_{ij}^{-1}. \tag{2}$$

The unit-elastic demand function applies as long as, adjusted for quality differences, the firm sets its price at a lower level than its closest competitor. The profit-maximizing action is thus to limit-price the firm that is able to set the closest quality-adjusted price. Because the marginal cost of producing a good depend on whether the producer's production method relies on the new or the old technology, the limit price depends on the technology of both the leading and that of the closest competitor firm. If the production method of both firms relies on the same technology vintage their marginal costs are equal, and the leading firm charges a markup λ_{ij} over marginal costs, where λ_{ij} is the quality gap between the leading and the follower firm. If the leading firm uses a different technology vintage than its closest competitor, marginal costs across the two firms differ, and the limit-price markup reflects that. Respectively denoting the technology vintage of the producer by h and the technology used by its closest competitor by h', the markup charged by firm μ_{ij} equals

$$\mu_{ij} = \begin{cases} \lambda_{ij} & \text{if } h = h' \\ \lambda_{ij}/a^o & \text{if } h = m,\ h' = o \\ \lambda_{ij} \cdot a^o & \text{if } h = o,\ h' = m. \end{cases} \tag{3}$$

It follows that the average markup on goods produced with the modern technology rises with the relative cost of producing goods using the old technology. When the modern technology differs from the old one in relying less on fossil inputs, this effect could arise from policies such as a carbon tax, which raise the markups of green producers while lowering those of dirty producers.

3.4. Innovation

As in Klette and Kortum (2004), firms may hold the production technologies for multiple intermediate goods. To expand their product portfolios, firms invest in research and development (R&D). A successful R&D effort grants the firm a frontier patent that allows it to produce a randomly drawn product j using a production method with quality

$$q_{ij} = q_{i'j}\lambda_{ij}, \tag{4}$$

where i' denotes the firm that previously owned the highest-quality patent for product j. In what follows, we assume that all innovations entail the same quality step, i.e. $\lambda_{ij} = \lambda$.[8]

Firms' R&D investments are directed to the two technological vintages, in the sense that firms decide what fraction of their research to allocate towards the development of production methods based on the modern technology vintage, and what fraction to devote to the old technology vintage. This decision is shaped by three factors. First, the expected cost of obtaining a patent that relies on a given technology vintage decreases with the firm's prior experience using that vintage. In other words, firms that are already the leading producers of several products based on vintage $h \in \{m, o\}$ find it easier to expand the range of products they offer with that same technology. Second, the intrinsic cost of innovating in each direction may differ, even abstracting from firms' prior experience. Third, the expected markups associated with producing under the old and the modern vintage differ (depending on a^o), implying that the value of owning a patent also differs across technologies.

Formally, firms choose the Poisson arrival rates of patents with production methods that rely on the modern technology vintage (x_i^m) and the Poisson arrival rate for old vintage patents (x_i^o). To raise either, firms must employ researchers rd_i^h along the following R&D cost function:

$$rd_i^h = \phi^h \left(x_i^h\right)^{\psi} (n_i^h)^{1-\psi}, \tag{5}$$

where $\phi^h > 0$ is a cost shifter, $\psi > 1$ is a scalar, and where n_i^h denotes the number of goods that are produced with technology vintage h for which firm i is currently the producer.

The final term in equation (5) introduces path dependence in the direction of a firm's R&D investments. Firms that have previously developed production methods based on technology vintage h achieve a higher arrival rate of new patents in that vintage for the same research expenditure. This, in turn, leads them to allocate a greater fraction of their overall R&D to that technology. This specification builds on the standard assumption in, e.g., Klette and Kortum (2004) and Luttmer (2011), that larger firms are more effective innovators because their existing production activities generate organizational capital that facilitates

[8] We focus on calibrations where $(a^o)^{-1} > \lambda > a^0$ so that a firm that develops a patent to produce a good j is able to become the producer of that product irrespective of the technology vintage of its patent, and of the technology vintage of the patent of the previous producer. See De Ridder (2024) for an example of a framework in which differences in marginal costs across firms can prevent firms with the highest-quality patent of a good from becoming the producer.

further innovation. In addition, the inclusion of a firms' current product count equation (5) ensures that firms of different sizes *grow* at the same rate—a well-established fact in the firm-dynamics literature known as Gibrat's Law. In our setting, these spillovers are technology-vintage specific.

Assumption 1. *The cost of achieving Poisson arrival rate of patents that rely on technology vintage h, x^h, declines in the number of products that a firm already produces using vintage h.*

3.5. Financial Constraints

Innovation by incumbent firms is subject to a cash-in-advance constraint and is financed by funds provided by competitive creditors. These creditors are willing to lend at the rate r_t, which is the return on household deposits. We assume that if a firm attempts to default on its credit obligations, creditors can recover only a fraction θ_i of firm i's revenue py_i for debt repayment. Hence, the creditors' participation constraint is satisfied only when the firm's debt d_i does not exceed the recoverable portion of revenue:

$$d_i \leq \theta_i p y_i. \tag{6}$$

This formulation is consistent with a growing empirical literature documenting that financial constraints typically depend on earnings flows rather than on firms' capital assets (see, e.g., Lian and Ma 2021, Drechsel 2023).[9] The model abstracts from a decline in financial constraints through retained earnings (see, e.g., Moll 2014): profits are paid out in dividends to the household every period.

The fraction of revenue that borrowers can appropriate, θ_i, has both an idiosyncratic and an aggregate component: $\theta_i = \vartheta_i \cdot \Theta_t$, where Θ_t summarizes the aggregate state of financial conditions. The idiosyncratic component, ϑ_i, reflects the number and quality of a firm's relationships with creditors. Through variation in the aggregate financial condition, Θ_t, we introduce a financial shock outside of the balanced growth path. At Poisson rate $\nu^{\uparrow}$, firms form new lending relationships—or, equivalently, existing creditors improve their screening technologies—thereby increasing the share of revenue they can pledge. Conversely, at rate $\nu^{\downarrow}$, firms lose lending relationships, which reduces the amount of revenue they can borrow against. Entrants begin with the weakest credit access, $\vartheta_i = \underline{\vartheta}$. The key assumption is that firms are more likely to face an easing of their financial constraints rather than a worsening of their financial constraints over time:

Assumption 2. *The Poisson rate at which firms face an increase in θ exceeds the rate at which firms face a decrease in θ, that is, $\nu^{\uparrow} > \nu^{\downarrow}$.*

[9] Lian and Ma (2021) show that collateral requirements in debt contracts commonly take the form of covenants restricting ratios of debt or interest payments to accounting profits. Equation (6) models borrowing constraints as a function of revenue rather than profits because accounting profits (e.g., EBITDA) include factor payments for capital and therefore do not correspond to the economic rents in our framework. This is also how covenants are modeled in Ottonello and Winberry (2024).

Assumption 2 yields that younger firms, on average, are more financially constraint than older firms. The fraction of revenue that creditors can jointly capture is bounded above by $\overline{\vartheta}$, beyond which ϑ_i cannot further improve.

3.6. Entry, Exit, and Creative Destruction

Firms endogenously enter and exit the economy. There is a continuum of potential entrants that employ rd^{en} researchers to develop patents that rely on the modern technology vintage and rd^{ed} on patents that rely on the older technology vintage:

$$rd_e^m = \phi_e^m \left(e^m\right)^\psi \quad , \quad rd_e^o = \phi_e^o \left(e^o\right)^\psi$$

where e^m denotes the entry rate of firms that initially operate using the clean technology and e^o is the rate at which firms operating an old-technology product line enter. As with incumbent innovation, entrants become the owner of the highest-quality patent of a random product which is a γ multiple of the level of quality of the previous producer. Without loss of generality, we assume that entrants' innovations are of the type – modern or old – in which the innovation was directed.

Firms exit the economy when they cease to be the producer of any of the economy's goods. A firm will stop producing one of its products if an innovator, be it an entrant or an incumbent, develops a higher-quality version of one of the goods that it produces. A multi-product firm that is hit by creative destruction stops producing one of its products. Single-product firms that see their sole product creatively destroyed exit. This happens at the rate of creative destruction, τ:

$$\tau = \int_0^1 \left(x_i^c + x_i^d\right) di + e^c + e^d. \tag{7}$$

As each creatively destroyed product's quality improves by a multiple λ, aggregate productivity growth in the economy is given by

$$g = \tau \ln \lambda. \tag{8}$$

3.7. Dynamic Optimization

Let $V_t(J, \theta)$ denote the value function of a firm with financial state θ and patent portfolio J, which is a multiset detailing both the technology vintage with which the firm produces, and the technology with which its second closest competitor produces. The Bellman equation reads as follows:

$$rV(J,\theta) - \dot{V}(J,\theta) = \max_{\substack{x^m, x^o \\ d \leq \theta p y}} \left\{ \begin{array}{c} \sum_{j \in J} \pi^{h_j h'_j} Y \\ +\nu^{\uparrow}\left[V_t(J,\theta^{\uparrow}) - V_t(J,\theta)\right] + \nu^{\downarrow}\left[V_t(J,\theta^{\downarrow}) - V_t(J,\theta)\right] \\ +\tau n^m \, \mathbb{E}\left[V_t(J \setminus \{m,h'\},\theta) - V_t(J,\theta)\right] \\ +\tau n^o \, \mathbb{E}\left[V_t(J \setminus \{o,h'\},\theta) - V_t(J,\theta)\right] \\ +x^m \, \mathbb{E}\left[V_t(J \cup \{m,h'\}) - V_t(J,\theta)\right] \\ +x^o \, \mathbb{E}\left[V_t(J \cup \{o,h'\}) - V_t(J,\theta)\right] \\ -w\left(n^m \phi^m (x^m)^{\psi} + n^o \phi^o (x^o)^{\psi}\right) - F(J,\theta) \end{array} \right\} \tag{9}$$

The first row captures the firm's instantaneous flow profit, given by the sum of product-level operating profits $\pi^{h_j h'_j} Y$ across all patents $j \in J$, where $\pi^{h_j h'_j} = (\mu^{h_j h'_j} - 1)/\mu^{h_j h'_j}$ and where, in turn, markups are given by (3) for $\lambda_{ij} = \lambda$. The second row describes the expected change in firm value due to an improvement in its financial maturity: with Poisson rate $\nu^{\uparrow}$, the firm transitions from θ to a more favorable financial state $\theta^{\uparrow}$, generating a corresponding jump in value $V_t(J,\theta^{\uparrow}) - V_t(J,\theta)$. The third row mirrors this for deterioration in financial maturity: with probability flow $\nu^{\downarrow}$, the firm moves to a tighter constraint $\theta^{\downarrow}$, reducing its continuation value by $V_t(J,\theta^{\downarrow}) - V_t(J,\theta)$. The next two rows represent creative destruction acting on the firm's patent portfolio: existing modern-vintage products are displaced by outside innovators at rate τn^m, and old-vintage products at rate τn^o, with the loss of a patent reducing firm value from $V_t(J,\theta)$ to the value of the portfolio with that patent removed. The following two rows capture the firm's own R&D investments: by choosing innovation intensities x^m and x^o, the firm can acquire new modern- and old-vintage patents, each increasing expected firm value by the jump from $V_t(J,\theta)$ to the value with the new patent added. The first term in the last row subtracts R&D costs, which depend on the number of product lines n^m and n^o and the convex cost function $w_t\left(n^m \phi^m (x^m)^{\psi} + n^o \phi^o (x^o)^{\psi}\right)$. Finally, we follow Akcigit and Kerr (2018) and assume that firms face a fixed operating cost $F(J,\theta)$ that–for now–subsumes changes in the option value of R&D that arise as a result of changes in firm size. This keeps the solution to the model analytically tractable.[10]

Proposition 1. *The value V_t of a firm grows at a constant rate g over the balanced growth path*

$$\dot{V}(J,\theta) = gV(J,\theta)$$

and can be expressed as the product of aggregate income Y and a time-invariant function of the number of modern and old-vintage products that the firm produces:

$$V_t(J,\theta) = (n^m V^m + n^o V^o) Y, \tag{10}$$

[10] In absence of this simplification, the optimal R&D in either technology is quantitatively similar, but there exists no closed form solution for the value of owning a product, complicating the analytical analysis.

where V^m and V^o respectively denote the present value of expect profits from producing a modern or old vintage product. The first-order condition for the arrival rate of ideas in technology vintage $h \in \{m, o\}$ is given by

$$x^h(J,\theta) = \left(\frac{Y}{w}\frac{V^h(\theta)}{\phi^h \psi\,(1+\mathcal{M}(J,\theta))}\right)^{\frac{1}{\psi-1}} n^h \tag{11}$$

where $\mathcal{M}(J,\theta) \geq 0$ is the shadow price of a financing R&D expenditures, which declines in a firm's financial maturity θ. The optimal entry rate is given by

$$e^h = \left(\frac{Y}{w}\frac{V^h(\theta)}{\phi_e^h \psi\,(1+\mathcal{M}^e)}\right)^{\frac{1}{\psi-1}} \tag{12}$$

where $\mathcal{M}^e$ is the financing shadow price for entrants.

The proposition shows that a firms' innovation rates increase in the value of becoming a good's producer, which is the sum of the discounted value of the good's expected flow of profits. The optimal innovation rates decline in the cost of R&D, which is a combination of the cost of hiring researchers and the shadow cost of obtaining the necessary financing.

3.8. Stationary Distributions of Maturity, Size, and Technology-Type

The joint distribution of three firm-level state variables are relevant for the equilibrium: the distribution of financial maturity θ, its joint distribution with the number of modern and old vintage products that a firm produces, and the fraction of products that is produced with the old and the new technology vintage. a firm is characterized by three state variables: the number of modern product lines (n^m), the number of old product lines (n^o), and financial maturity (θ). Firms can move across these states because their financial conditions improve or deteriorate, they gain products through innovation, and they lose products through creative destruction. Firms that lose their last product exit; entry replenishes mass at the lowest financial-maturity level, with entrants starting either as modern or old producers.

Given innovation and entry rates, these forces define a Markov process over the full state space. The stationary distribution is the invariant distribution of that process: it is the unique cross-section that is reproduced by the model's own transition dynamics. A convenient way to write this is to separate incumbent transitions from entrant inflows. Let M be the vector of masses over active firm states, let T be the matrix with transition rates implied by innovation, destruction, financial-state movements, and let E be the vector of entry inflows. The law of motion is $\dot{M} = TM + E$. In a stationary equilibrium, $\dot{M} = 0$, so that

$$M = -T^{-1}E.$$

That is, the stationary cross section is just the one that exactly balances within-state reallocation and entry inflows. Details on computation are provided in Appendix D.

Defining the stationary mass of firms of a combination of firm states by $M_{n^m,n^o}(\theta)$, we can then calculate key moments such as the stationary equilibrium fraction of products that is produced with the modern technology vintage:

$$F = \frac{\sum_\theta \sum_{n^m,n^o} n^m \, M_{n^m,n^o}(\theta)}{\sum_\theta \sum_{n^m,n^o} (n^m + n^o) \, M_{n^m,n^o}(\theta)}. \tag{13}$$

3.9. Aggregate Variables

We next characterize the economy's aggregate variables. The equilibrium wage is given by

$$w = \exp\left(\int_0^1 \ln q_j dj\right) (a^o)^{1-F} \left(\exp\left(\overline{\ln \mu^m}\right)^F \exp\left(\overline{\ln \mu^o}\right)^{(1-F)}\right)^{-1}, \tag{14}$$

where the first two terms together form the standard CES productivity index based on the quality at which goods are produced, while the variables in the third term are

$$\overline{\ln \mu^m} = F \ln \lambda + (1-F) \ln(\lambda \cdot a^o) \quad \text{and} \quad \overline{\ln \mu^d} = F \ln \ln(\lambda / a^o) + (1-F) \ln \lambda.$$

The third term in (14) captures the degree to which wages are suppressed by the fact that firms have market power and thus charge markups, and the fact labor is combined with other inputs in the production of dirty products.

Turning to aggregate output, the economy's GDP is determined by productivity and the fraction of labor employed to produce goods (L^p):

$$Y = L^p \exp\left(\int_0^1 \ln q_j dj\right) (a^o)^{1-F} \mathcal{W}. \tag{15}$$

Derivations are provided in Appendix A. The term $\mathcal{W}$ captures the productivity loss that arises when firms have heterogeneous markups, because products that sell at a high markup are demanded in inefficiently low quantities. A term similar to the latter is derived in Peters (2020). Production labor is determined by clearance of the labor market. Households supply their single unit of labor inelastically, so that

$$1 = L^m + L^o + L^{rd} + L^e, \tag{16}$$

where L^c and L^d is the labor used to produce clean and dirty intermediate goods while L^{rd} and L^e denote the labor required for innovation by incumbents and entrants. Incumbent R&D labor is given by

$$L^{rd} = \phi \sum_\theta \sum_{n_m,n_o} \left[n_m \, M_{n_m,n_o}(\theta) \, x_m(n_m,n_o,\theta)^\psi + n_o \, M_{n_m,n_o}(\theta) \, x_o(n_m,n_o,\theta)^\psi \right], \tag{17}$$

and entrant R&D labor is

$$L^e = \phi_e \left((e^m)^\psi + (e^o)^\psi \right). \tag{18}$$

3.10. Equilibrium

The following definition characterizes the equilibrium:

Definition 1. *The economy is in a balanced growth path equilibrium if for every t and for every firm maturity level k, the variables e^m, e^o, τ, r, L^p, $\mathcal{W}$ and the functions $x^m(j,\theta)$, $x^o(j,\theta)$, V^m, and V^o are constant while Y and w grow at the constant rate g that satisfies (8) with creative destruction rate (7). The value of owning a firm is given by (10), while innovation rates for incumbents and entrants are respectively given by (11) and (12). The equilibrium fraction of modern products F is given by (13). The interest rate obeys the Euler equation, output and wage are respectively given by (14) and (15). Appendix A provides expressions for L^p, using labor market clearance (16) and markup dispersion $\mathcal{W}$.*

3.11. Financial Tightening During a Technological Transition

In the following sections, we show that a tightening of financial conditions that raises the cost of innovation for young firms disproportionately affects innovation in the modern technology vintage when the economy is in a technological transition. We define such a transition as a phase during which the fraction of goods produced with the modern vintage, F, is rising. This occurs whenever the combined flow of incumbent and entrant innovation directed toward the modern vintage exceeds the number of modern products that are displaced through creative destruction, τF.

A technological transition begins after the relative profitability of producing with the old vintage falls. In the model, this happens when the relative cost of producing with the old technology, $\tilde{p}^o$, rises. This reduces the markup from producing old-vintage goods and increases the markup from producing modern-vintage goods. As a result, incumbent and entrant innovation directed toward the modern vintage rises, while innovation directed toward the old vintage falls. The fraction F therefore starts to increase and continues to do so until the economy converges to its final level of modern production.

Early in the transition, young firms account for a disproportionately large share of modern innovation relative to what they do along the balanced growth path. The reason is path dependence. Total modern innovation by firms of a given maturity depends on both their innovation intensity and the share of their product portfolio that already uses the modern vintage, $M^m(k)$. At the start of the transition, incumbent firms still produce mainly with the old technology, so $M^m(k)$ is low for the firms that were already active before the transition. The initial rise in modern innovation is therefore driven primarily by entrants. Over time, these entrants expand and become incumbents themselves, but their average maturity remains below that of the firms that were active before the transition.

The below-average age of firms that innovate in the modern technology makes the transition especially vulnerable to financial shocks. We model such shocks as an increase in κ, which raises financing costs $\tilde{r}$, with a larger effect on young firms than on old firms. Importantly, the elasticity of any innovation rate with respect to $\tilde{r}(k)$ is the same across technology types, and equal to $\frac{1}{1-\psi}$. The reason a rise in κ depresses total modern innovation more than total old-vintage innovation is therefore not that modern innovation is intrinsically more finance-sensitive. It is that, during the transition, modern innovation is disproportionately carried out by the young firms whose financing costs rise the most.

4. Application: Green Innovation After the Global Financial Crisis

In this section, we use the framework in Section 3 to quantify how much the self-selection of young firms into green innovation can explain the slowdown in green innovation after the Global Financial Crisis. Our goal is twofold. First, we use micro data to scrutinize the empirical validity of the model's mechanisms (Section 4.1). Second, we use the micro data to discipline the model's parameters (Section 4.2) and then study the path of green innovation after a financial shock of the size experienced after the Global Financial Crisis (Section 4.3).

4.1. Micro Evidence

We use micro data on the heterogeneous exposure of German firms to the Global Financial Crisis to study its effect on green patenting in subsequent years and thereby validate the model's predictions. Green technologies are a natural example of the model's "modern" technology, as their share in total patenting had risen steadily over the two decades before the crisis. German firms are particularly well suited for this analysis because, relative to their American counterparts, they rely more heavily on bank credit than on market finance (Huber 2018). As we explain below, this creates plausibly exogenous variation in firms' exposure to the crisis through the identity of their bank relationships, allowing us to isolate the effect of tighter credit while controlling for confounding macroeconomic factors.

The model delivers three predictions for green innovation during the crisis. First, it predicts that young firms are responsible for a disproportionate share of total green innovation. That is, the fraction of green patents that can be attributed to young firms exceeds that fraction for non-green patents prior to the crisis. Second, the average negative average effect of exposure to the crisis on green panting should exceed the effect on non-green patenting. Third, it predicts that young firms cut their overall patenting while mature firms (whose financial constrains may not bind) do not, so that the larger effect of exposure to tight credit on green patenting is driven by the age composition of green innovators.

Strategy To test these predictions, we combine firm-level measures of green versus non-green innovation based on IPC patent classification codes with data on whether firms relied

on loans from to Commerzbank prior to the Global Financial Crisis. We use the fact that firms that relied on Commerzbank were disproportionately subject to a tightening of their credit constraint, as shown in Huber (2018).[11] This enables us to assess the effect of tight credit on (green) innovation while holding constant other macroeconomic variables that might affect firms' investments in (green) innovation, such as energy prices, fiscal policy including subsidies for emissions-reducing technology, or changes in investor preferences.

Commerzbank was a large German bank prior to the crisis, with a market share of around 9% in 2006. It incurred significant trading losses on the U.S. subprime market in 2008, causing the bank's equity to fall by 68%. These losses forced Commerzbank to cut its assets significantly. As Basel II regulation assigns a significant risk weight to corporate loans, reduced credit supply to corporations are an effective way of doing so. This led Commerzbank to reduce its lending stock by 35% between 2006 and 2013, which is around three times the average cut in lending stock by German banks after the Global Financial Crisis. Given the long-term nature of relationships between firms and banks, induced by the fact that repeated interaction improves the ability of banks to screen and monitor lenders (e.g., Boot 2000), this means that firms that relied on credit from Commerzbank prior to the Global Financial Crisis faced a greater tightening of their credit constraint.

The empirical strategy proceeds in three steps. First, we assess the relationship between age and firms' propensity to patent in green technologies for German firms in the years prior to the Global Financial Crisis. Second, we perform a difference-in difference estimation to assess the extent to which the flow of green and non-green patenting react differently to the Global Financial crisis in firms with high versus low pre-crisis dependence upon Commerzbank lending. Third, we perform the triple-difference-in-difference regression, where we assess whether the effect of exposure to Commerzbank on patenting is conditional on firm age. To implement the latter two, we estimate the parameters of the following equation using Silva and Tenreyro (2006)'s Pseudo-Poisson Maximum Likelihood estimator:

$$E_t[y_{it} \mid X_{it}, i, t] = \exp\left(\sum_{\tau \neq \tau_0} \gamma_t \mathbb{I}_{t=\tau} \text{Commerzbank Exposure}_i + \beta_t' \tilde{X}_{it} + \phi_i + \psi_t\right), \tag{19}$$

for the difference-in-difference, where y_{it} denotes firm i's flows of green and dirty patents in year t, and $\mathbb{I}_{t=\tau}$ is the indicator function so that γ captures the time-varying effect of pre-crisis dependence on loans from Commerzbank on the flow of patents. That is, we expect the coefficients γ_t to be negative in the aftermath of Commerzbank's 2008-2012 reduction in credit supply. Time effects ψ_t control for macroeconomic events common to all firms during the global financial crisis, such as changes in energy prices, fiscal incentives for green

[11] Huber (2018) looks at the effects of Commerzbank exposure on employment, investments and overall patenting. Its Online Appendix B provides a detailed analysis of the trading losses incurred by Commerzbank and the effect this had on lending. Huber also provides extensive balance checks to show that firms that rely on Commerzbank loans have similar pre-crisis characteristics to other firms.

Table 1: Descriptive Statistics

	Mean	St. Dev.	Median	10th Pct.	90th Pct.
Value added	73,384,421	6.14E+08	11,249,385	3,342,691	97,015,680
Employment	969.3	10831.9	144	35	1145
Operating Profits	73384413	6.14E+08	11249381	3342692	97015688
Fixed Assets	61749246	6.2E+08	6091660	622625.9	96746492
Wage Bill	28111845	2.72E+08	3912469	1055326	36397191

Notes: Summary statistics for the merged Dafne-Creditreform-Patstat dataset.

innovation, or overall changes in investor preferences. For the triple-difference-in-difference, the estimation equation is

$$E_t[y_{it} \mid X_i, i, t] = \exp\left(\sum_{\tau \neq \tau_0} \gamma_t \mathbb{I}_{t=\tau} \text{Commerzbank Exposure}_i \times D_i^{Age} + \beta_t' \tilde{X}_{it} + \phi_i + \psi_t\right), \quad (20)$$

where D_i^{Age} is a dummy that equals 1 for firms that are below the median age of firms in the sample. We satiate the estimation equation, i.e., we include the baseline interaction terms between time and either age or Commerzbank exposure as part of the set of controls $\tilde{X}_{it}$.

Data To implement the empirical strategy, we combine firm-level data on exposure to Commerzbank with firm-level accounting data and data on patenting.

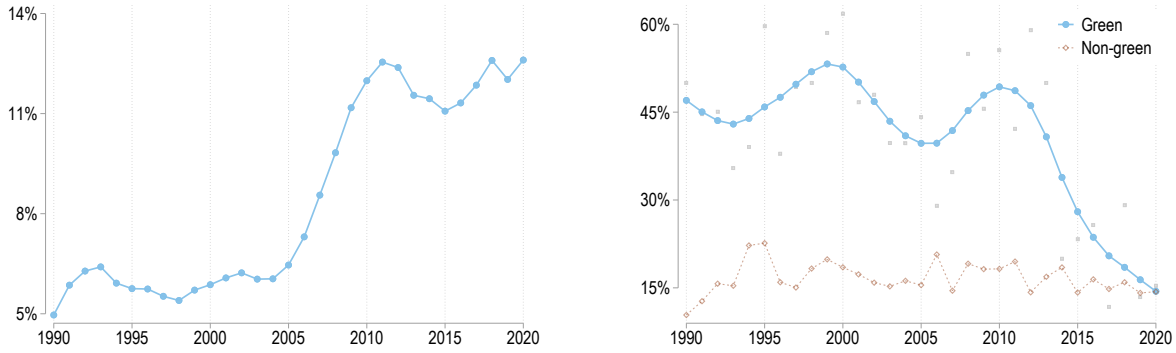
We measure exposure to Commerzbank from the fraction of a firm's pre-crisis relationship banks that is a branch of Commerzbank. Relationship banks (*Hausbanken*) are the primary banks that firms obtain banking services from, and are based on firms' response to surveys from credit rating agency Creditreform in 2006. Following Huber (2018), we construct a measure of reliance on Commerzbank that equals the portion of a firms' set 2006 relationship banks H_i that is a Commerzbank branch:

$$\text{Commerzbank Exposure}_i = \sum_{h \in H_{i,2006}} \frac{\mathbb{I}_{h=\text{Commerzbank Branch}}}{\mid H_{i,2006} \mid},$$

where the indicator function equals one if bank h is a Commerzbank branch. The index is naturally bounded between 0 and 1 and equals 0.17 on average, with a 90th percentile of 0.5. This means that few firms rely solely on Commerzbank as their relationship bank.

The accounting data come from Dafne, the German subset of Bureau van Dijk's Orbis database, and include firms' balance sheet and income statement information. Patent data come from the 2018 edition of PATSTAT, which provides worldwide coverage of patents from more than 90 patent offices. We merge the two datasets using the Bureau van Dijk firm identifier, which is available in both sources. We restrict the sample to firms that file at least one patent by 2018 and for which we observe continuous, non-missing data on employment, wages, loans, value added, and fixed assets between 2007 and 2012. The resulting sample contains 601 firms observed over 18 years. Table 1 reports summary statistics.

Figure 5. Green Patenting Trends During Global Financial Crisis in Germany



(a) Green Patents (% of Total Patents)

(b) Younger Firms (% of Total Patents)

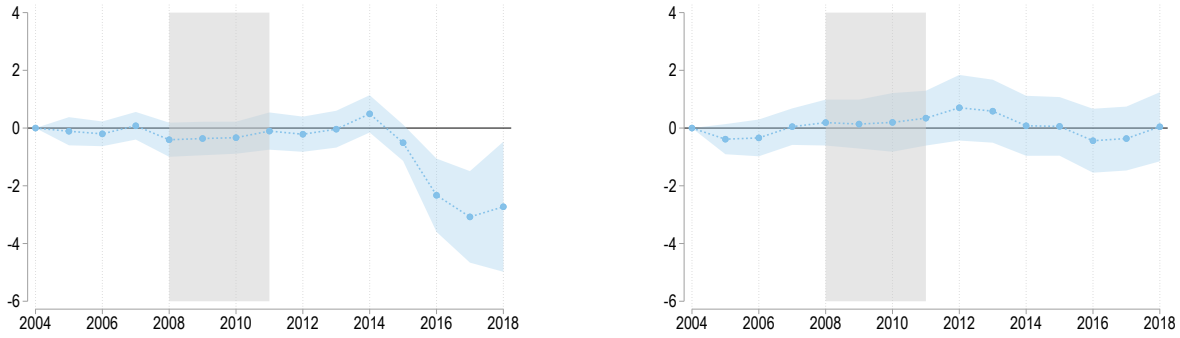
Notes: The left-hand figure plots the fraction of German patent applications (out of total applications) that relates to green technologies. This figure is based on the universe of firms in PATSTAT. The right-hand figure plots the percentage of patents that comes from below-median aged firms separately for green patents (blue-circled lines) and for non-green patents (brown-squared lines). This figure is based on the matched PATSTAT-Dafne-Creditreform dataset. Lines plot 5-period moving averages—the underlying raw data is plotted with grey scatters for green patents.

We measure green innovation by assigning patents to green and non-green technologies using the OECD classification proposed by Haščič and Migotto (2015). They identify green technologies based on International Patent Classification (IPC) codes. The classification includes four broad groups: environmental management, water-related adaptation, biodiversity protection, and climate change mitigation, which accounts for the largest share of green IPC codes. We classify a patent as green whenever at least one of its IPC codes falls in the OECD definition. A firm's annual patent count is then defined as the number of green and non-green patent applications it files in a given calendar year.

Results Before turning to the model's predictions, we first verify that Germany experienced a slowdown in the rise of green patents as a share of total patenting. Figure 5a shows that green patents accounted for 4.9% of all German patents in 1990. This share rose only modestly over the following decade, before accelerating rapidly from 2005 onward, increasing from 6.5% to a peak of 12.5%. Consistent with the global patterns documented in Figure 1 of the Introduction, that rise stalled in Germany after 2011. Between 2011 and 2020—the last year we plot because of patent grant delays—green patent applications accounted for between 11% and 12.6% of total patents.

Turning to the first prediction of the model, Figure 5b confirms that younger firms played an outsized role in green patenting in the years prior to the crisis. Because we need to observe firm age to analyze the contribution of young firms, we perform this analysis on the subset of patents that is part of the merged PATSTAT-Dafne-Creditreform dataset. The blue-circled line plots a moving average of the fraction of all green patents that are linked to firms with below-median age, while the grey scatters plot the underlying data. Between 1990 and 2010,

Figure 6. Effect of Commerzbank Exposure on Patent Applications



(a) Green Patents (b) Non-Green Patents

Notes: The figure plots the effect of exposure to Commerzbank on green patents (left) and non-green patents (right). Patents are green if at least one IPC code relates to environmental management, water-related adaptation technologies, biodiversity and ecosystem health, or climate-change mitigation (OECD classification; Haščič and Migotto 2015). Confidence bounds are at the 95% level based on firm-clustered standard errors.

young firms are responsible for around 45% of green patents, with peak contributions of over 61%. In contrast, the contribution of young firms to patenting in non-green technologies is more modest, averaging only 16.7% per year. Thus, younger firms appear to be sorting in green innovation, as predicted by our framework.

Turning to the drivers of the slowdown of green patenting, Figure 6 provides the results of the difference-in-difference estimation of equation (19). It lends evidence in support of the model's prediction that green innovation declines because it is more sensitive to a tightening of financial conditions. The left-hand figure plots the effect of greater exposure to the Global Financial Crisis through Commerzbank on green patents and the right-hand figure does the same for non-green patents. The grey shaded area covers the period 2008 to 2011, which is when firms that relied on loans from Commerzbank were actively subject to tighter credit than other firms.[12] Confidence bounds are at the 95% level using firm-clustered standard errors. The figure shows that exposure to the crisis has no meaningful effect on innovating in non-green technologies, whereas it had a significantly negative effect on green innovation. The point estimates are large: the elasticity of green patenting with respect to Commerzbank dependence is about 3 by 2018. Note, however, that few firms rely solely on loans for Commerzbank – the average value of the exposure variable is 0.17 with a 90th percentile of 0.5. A firm with an average (90th percentile) dependence on Commerzbank will therefore have 28% (45%) fewer green patents than a firm that has no direct exposure.[13]

[12] Biermann and Huber (2024) note that debt of firms with exposure to Commerzbank remained suppressed after that, and had not rebounded by the end of their sample in 2015.

[13] Despite the modes sample size, the results in Figure 6 are robust to the inclusion of detailed fixed effects. In Figure B4 in the Supplementary Appendix we show that the effect of the crisis on both green and non-green patents is qualitatively and quantitatively robust to the addition of additional controls such as size-year fixed effects and detailed industry-year fixed effect.

Table 2: Effect of Commerzbank Exposure on Green and Non-Green Pattenting by Age

	Average Effect		Age Below Median		Age Above Median	
	Green	Non-Green	Green	Non-Green	Green	Non-Green
Commerzbank Exposure	-0.506***	-0.107	-0.886***	-0.664*	0.950*	0.303
(Effect by Age)	(0.166)	(0.334)	(0.321)	(0.350)	(0.555)	(0.474)

Notes: PPML regressions of patent counts. The coefficient measures the effect of Commerzbank Exposure interacted with a dummy that equals 1 after 2012. Middle and right columns additionally interact with age dummy D_i^{age}. Firm-clustered standard errors are in parentheses. *, **, *** measure significance at the 10, 5, 1% level, respectively. All columns include firm and year fixed effects. Green innovation regressions rely on 2700 observations, non-green innovation regressions rely on 7,080 observations.

Turning to the third test of the mechanism, we assess whether firm age drives the negative effect of Commerzbank exposure on green patenting in Figure 6. To do so, we estimate the triple-difference specification in equation (20). The results are reported in Table 2. From left to right, the columns show the effect of Commerzbank exposure for the full sample, for firms below the median age, and for firms above the median age. The table shows that young firms reduce both green and non-green innovation, with coefficients of -0.886 and -0.664, respectively. By contrast, mature firms do not reduce innovation in either category. This is precisely what the model predicts: the overall negative effect of Commerzbank exposure on green innovation is driven by the combination of: (1) younger firms cutting *all* innovation in response to tight financial conditions, and (2) younger firms specializing in green innovation.

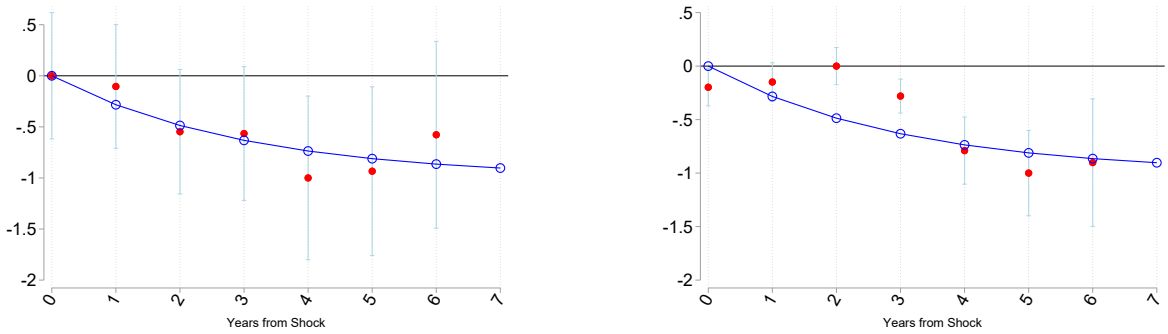
4.2. Model Quantification

We next use this micro evidence to quantify the model. Our goal is to measure how much the endogenous self-selection of younger firms into new technologies contributes to the differential effect of tighter financial conditions on new versus old technologies. Specifically, we ask whether a uniform tightening of aggregate financial conditions in the model can account for a substantial share of the slowdown of green patenting relative to overall patenting around the Global Financial Crisis in Germany.

Approach Our quantification strategy proceeds in two steps. First, we structurally estimate the model to match a combination of pre-1998 moments from the German micro data and the German economy more broadly, as well as the pre-Global Financial Crisis rise in green patenting by young firms documented in Figure 5. The pre-1998 moments pin down the initial steady state, while the remaining moments discipline the transition from non-green to green innovation in the counterfactual scenario without a financial crisis. Second, in the main experiment, we subject the model to a tightening of credit supply equal in magnitude to that experienced in Germany after 2007.

Innovation and General Parameters We discipline the model's innovation parameters, as well as the discount rate ρ, using a either external calibration and structural estimation.

Figure 7. Lagged Effect of Financial Shock on Patenting: Model versus Data



(a) Green Patents (b) Non-Green Patents

Notes: The figure compares the post-shock responses of exposure to Commerzbank (Figure 6, red scatters) to the effect of a financial shock on patenting in the model (blue-solid lines). To solely reflect the model's ability to match the lagged effect of the shock, the axes are normalized such that the lowest coefficient is zero, and the peak effect is -1. As ζ gives implementation lags for both innovation types, the model lags are equal in both plots.

We set the discount rate ρ to 0.02, a standard value in the literature (e.g. Akcigit and Kerr (2018), Acemoglu et al. 2018, Aghion et al. 2020). Given the target for the growth in total factor productivity along the balanced growth path of 1.6% (as we motivate below), this implies a real interest rate of 3.6%.

We set the curvature of R&D costs, ψ, to 2. This parameter can be linked to the elasticity of R&D with respect to user costs, ϵ, using that $\psi = (1-\epsilon)/\epsilon$. A large literature estimates this elasticity using firm-level responses to R&D tax credits. For example, Appelt et al. (2022) study micro evidence across OECD countries and find elasticities around -0.6, which would imply $\psi = 2.67$. More broadly, Bloom et al. (2002) discuss values of the elasticity between -0.5 and -1.0, implying ψ between 2 and 3. We set $\psi = 2$, consistent with Acemoglu et al. (2018), Akcigit and Kerr (2018), and De Ridder (2024).

The innovation step λ determines average markups, and thus the profit share in the economy's GDP. We set λ to 1.1 to achieve a profit share of around 9%, consistent with estimates in Barkai (2020).

To calibrate the probability that a firm develops a patent in the non-directed technology, α, we target the probability that a single-patent firm innovates in the dirty (green) technology conditional on its initial patent being green (dirty). In the model, the ratio of this probability to the probability of innovating in the original technology corresponds to α. Using German single-patent firms between 1985 and 1995, we estimate $\alpha = 0.049$.

We set the rate at which innovations are implemented, ζ, to 1/3. This implies an average three-year lag between a finance-induced reduction in R&D investment and the subsequent decline in patenting. This lag is consistent with a large empirical literature on the delay between R&D and innovation, going back to Pakes and Griliches 1980, which finds lags of up to five years. The resulting impulse response of innovation in the model is also consistent with our own micro estimates in Section 4.1. Figure 7 plots the share of the full effect of a

financial shock on patenting that is realized in each year after the shock, and shows that the model matches the timing of the effect on patenting well.

The remaining innovation parameters are not directly identified from the data and are therefore estimated as part of a larger indirect inference exercise that includes the calibration of the green/dirty profitability. The first is ϕ, the cost shifter governing research and development (R&D). We pin down this parameter by targeting average German TFP growth of 1.6% between 1985 and 1995. We identify the corresponding parameter for entrant R&D, ϕ^e, by targeting the average rate of entry in German over 1985-1995, which was 11.4%.[14]

Green vs. Non-Green Production Cost Parameters We next turn to the relative cost of producing with the non-green technology, the counterpart in the application to the relative productivity of producing with an old vintage in the model, a^o. Two values must be calibrated. First, we set the initial value of a^o so that the model matches the share of patents that is green in the German micro data between 1985 and 1995, which is 10.6%. Second, because a decline in a^o triggers the transition toward green innovation, we also need to discipline its value in the final steady state. Motivated by canonical endogenous growth models with clean and dirty production—especially Acemoglu et al. (2016)—in which the share of products produced with the non-green technology is proportional to emissions, we set the terminal value of a^o so that the share of products produced with the polluting technology falls by 65% relative to the initial steady state. This corresponds to the decline in emissions required by 2035, relative to 2010, to meet the climate target of limiting global warming to 1.5 degrees Celsius (IPCC 2023).

In addition, we must discipline the path of a^o between the initial and final steady states. We allow a^o to follow a sigmoidal transition path and pin down its half-life by targeting the increase in green patenting between the start of the transition and the Global Financial Crisis. Taking the late-1997 Kyoto Protocol agreement as the start of the transition, we target a 2.1-fold increase in the share of green patenting in total patenting, in line with Figure 5. Although the financial crisis may itself have affected the path of non-green production costs—for example through lower oil prices or reduced European green subsidies following austerity, see Hofman and Huisman (2012)—we do not allow the crisis to alter the path of a^o. This is because our objective is to assess whether the self-selection of young firms into green innovation can account for the slowdown in green patenting relative to overall patenting, holding other forces constant.

Steady State Financial Parameters There are four financial parameters to be calibrated. We choose parameter values to assure that the model replicates our findings for both the importance of age in green innovation prior to the crisis, and to match the effect of age on exposure to financial shocks in our difference-in-difference regressions. The first two parameters govern the rate at which firms upgrade or downgrade in terms of their financial

[14] Because direct measures of business dynamism are unavailable for these years, we construct the entry rate as the share of firms filing a patent for the first time relative to all firms with a patent history in a given year.

Table 3: Quantification Strategy Initial and Final Balanced Growth Path

Par.	Description	Target	Approach	Value
ψ	R&D cost curvature	R&D cost elasticity	External	2.00
ρ	Discount rate	Risk-free interest rate	External	.020
λ	Innovation step size	Profit share in GDP	External	1.10
α	Innovation spillover	Patenting specialization	External	.951
ζ	Implementation lag	Patenting impulse response	External	.333
ϕ	R&D productivity scalar	Productivity growth	Indirect inf.	133
ϕ_e	Entry cost scalar	Entrants as % of firms	Indirect inf.	48.1
a_0^o	Relative o cost (initial S.S.)	Relative value of green patents	Indirect inf.	1.09
a_1^o	Relative o cost (final S.S.)	Decline in emissions	Indirect inf.	.913
T	Duration $a_0^o \to a_1^o$ (years)	Pre-2008 rise green patents	Indirect inf.	31.2
$\nu^{\uparrow}$	Fin. maturity increase prob.	Pre-2008 young firm green pat.	Indirect inf.	.129
$\nu^{\downarrow}$	Fin. maturity decrease prob.	Moments of age distribution	Indirect inf.	.050
$\overline{\theta}$	Upper bound financial mat.	Commerzbank Diff-in-Diff	Indirect inf.	.005
$\underline{\theta}$	Lower bound financial mat.	Commerzbank Diff-in-Diff	Indirect inf.	.100

Notes: See text for details on the numerical values of each of the parameters, and for a summary of the computational algorithm that we use to implement the structural estimation.

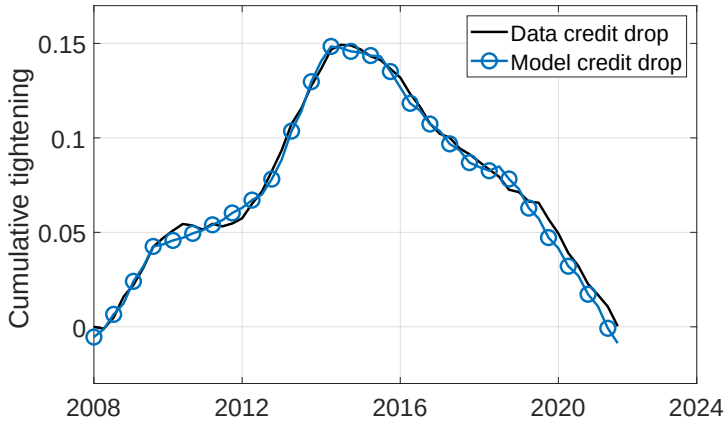
maturity, $\nu^{\uparrow}$ and $\nu^{\downarrow}$. We choose these parameters so that the average age of a below-median age firm is 16.3 years, in line with the data, and to match the contribution of below-median aged firms to green innovation prior to the financial crisis (47%, see Figure 5b). The second two parameters are the value for θ_i for the least-mature firms ($\underline{\theta}$) as well as the maximum value of $\overline{\theta}$.[15] We choose these values in order to match the linear regression of an age-CommerzBank interaction on the effect of overall patenting, and to match the fact that firms with above-median age are unaffected by Commerzbank-like shock (21% additional drop in credit) in the difference in difference regression.

Computational Implementation To find the parameters of the steady states and the baseline transition between them we deploy the following algorithm. We first guess a vector with values for each of the parameters above. We then solve the model for the initial steady state, collect the relevant moments, and find the change in a^o that delivers the targeted increase in green production. We then solve for the transition using a shooting algorithm, and collect moments for the financial parameter calibration. We then compare these moments to the targets, and iterate until complete. Further details on computation of the equilibrium and the transition are provided in Appendix D. Table 3 lists the resulting parameter values.

Shock to Credit Having estimated the model's key parameters, we now summarize our approach to disciplining the shock to credit following the Global Financial Crisis. Our objective is to find the path of aggregate credit conditions, Θ_t in equation (6), so that the model replicates the decline in credit supply during the crisis. A key identification challenge of the tightening of credit conditions is that the Global Financial Crisis and the subsequent Euro

[15] We specify the grid of θ_i as having N^{θ} equidistance values between these two. We set N^{θ} to 8, which is the highest value for which computation time remained reasonable at the time of writing.

Figure 8. Tightening of Lending Supply in Model and Data



Notes: Implied tightening of credit supply from the ECB Lending Standards Index (Altavilla et al. 2019).

area created a deep recession, that affected both supply and demand for credit. This means that data on new loans issued or loans outstanding are are not suitable for our calibration.

To isolate the change in credit that is due to a tightening of lending conditions, we use the Lending Supply Indicator developed by Altavilla et al. (2019) at the European Central Bank. They aggregate data from bank-level surveys on changing in lending standards into a single index, and we use fitted values of changes in credit outstanding on a regression of this indicator as our target for lending. The black-solid line in Figure 8 presents the implied reduction in credit outstanding from 2008 to 2022. It shows that credit standards caused a gradual reduction of loan supply in the Euro area, which peaked around 2013. In subsequent years there was a gradual easing of credit, reaching baseline values around 2022. The blue-circled line shows that our calibration for the path of Θ_t induces a closely matched path of credit tightening. In the quantitative experiment of the subsequent section, we allow Θ_t to remain at its steady state value of unity from 2022 onwards.

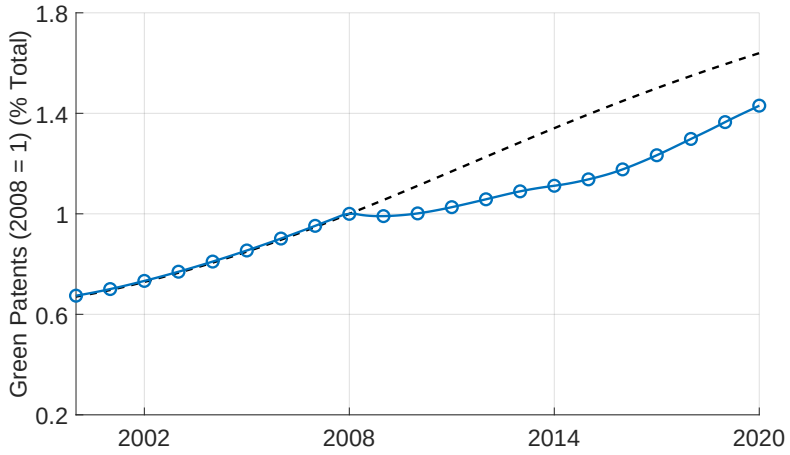
4.3. Preliminary Results

With the quantified model in hand, we turn to a final counterfactual exercise: if we simulate a tightening of financial constraints large enough to match Figure 8, can the self-selection of young firms into green technologies during the transition explain a meaningful part of the slowdown in green patenting that we observe in the data?

Main exercise Figure 9 gives the main answer. The blue-circled line plots the share of new patents directed toward the green technology, normalized to 1 at the onset of the financial shock. The black dashed line shows the counterfactual path of that share in the absence of a financial crisis.

The figure shows that the model can explain a large share of the slowdown in green patenting observed in the data. On impact, the model predicts a flattening of the green patent

Figure 9. Simulation of the Effect of a Financial Crisis in 2008 on Green Patents as % of Total



Notes: The figure plots the effect of a financial tightening on the path of the share of green patents in total patents in response to a 10-year financial tightening of financial constraints Θ matching the path plotted in Figure 8.

share, closely resembling the slowdown in green innovation visible in both the German and global patent series (Figures 5 and 1). One difference is that the model generates an immediate effect of the shock on patenting. This is because the shock immediately lowers the Poisson arrival rate of ideas. If we instead impose a minimum lag of two or three years between R&D investment and innovation, the timing lines up well with the data.

Over longer horizons, however, the self-selection of young firms into new technologies explains a smaller share of the path of green patenting. By 2015, for example, the model predicts that the green patent share resumes growing at roughly the same rate as on the baseline path, although from a persistently lower level because of the path dependence of R&D. In the data, by contrast, green patenting remains subdued until the late 2010s.

Over the full sample, the model explains well over half of the slowdown in green patenting. That it does not account for the entire decline is not surprising. The mechanism we study is not the only force shaping green innovation after 2008, and the post-crisis macroeconomic environment changed along several other dimensions that likely mattered as well. Examples include the sharp decline in energy prices after 2007 and cuts to European green subsidies during the austerity period (Hofman and Huisman 2012). Other financial forces may also have played a role, such as shifts in investor demand for green innovation or changes in risk appetite and regulation. The point of Figure 9 is therefore not that financial constraints explain the whole slowdown. It is that, holding these other forces constant, the self-selection of financially constrained young firms into green innovation can account for a substantial share of the decline.

Counterfactual: Crisis Timing A central implication of the theory is that the cost of financial crises is state dependent: crises are more damaging when the economy is still in the early stages of a technological transition. If a financial crisis arrives later in the transition, firms

Figure 10. Counterfactual: Financial Crisis in 2013 instead of 2008 and Green Patenting



Notes: The plot presents the deviation from trend of the path of green patenting (as a percentage of total patenting) in response to a 10-year financial tightening of financial constraints Θ matching the path plotted in Figure 8. Solid-circled line: baseline, with the financial crisis starting 10 years after the start of the transition (2008). Orange-dashed line: counterfactual, with the financial crisis starting 15 years after the start of the transition (2013).

with a comparative advantage in the new technology have had more time to mature and are therefore less financially constrained. The same logic applies to the transition from non-green to green technology. Had the Global Financial Crisis occurred just five years later, it would have produced a noticeably smaller slowdown in green innovation.

Figure 10 illustrates this state dependence. It plots the path of the green share of patenting, relative to a counterfactual with no financial crisis. The two lines differ only in the timing of the crisis. The blue-circled line is the baseline case, in which the crisis hits in 2008, ten years after the start of the transition. The orange line shows the alternative case in which the crisis hits in 2013, fifteen years after the start of the transition. In both cases, the economy is subjected to the same path for the decline in Θ_t from the onset of the crisis onward.

The figure shows that a financial shock arriving five years later causes a substantially smaller disruption to green innovation. The decline in the green patenting share is somewhat smaller at the peak of the crisis, although the difference is modest. This reflects a general equilibrium effect: because green innovation is less affected by tighter financial constraints, financially mature firms have less incentive to step *up* their green innovation efforts. The larger difference emerges after year 8. When the crisis occurs later in the transition, the recovery is faster, and from year 10 onward the green patenting share remains 15 to 25% closer to the no-crisis path for two decades.

5. Conclusion

This paper proposes that new technology vintages are disproportionately affected by financial disruptions when the economy is in transition towards a new technological steady state. We hypothesize that younger firms have a comparative advantage in innovation in emerging

technology vintages at the early stages of a transition, as they have not accumulated expertise in mature technology vintages that older firms have. As young firms are financially constrained, a tightening of overall credit will thus endogenously affect innovation in new technology vintages more than innovation in mature ones.

We develop this argument using a Schumpeterian growth model with firm dynamics where: (i) firms expand their portfolio of products by engaging in innovation directed at either modern or old technology vintages in order to improve existing product lines; (ii) the choice between modern or old vintage innovation is path dependent, in the sense that it becomes increasingly less costly for a firm to innovate in a given vintage if they already produce a large number of products using that vintage; (iii) firms are subject to credit constraints that depend on their age, with less mature firms facing the tighter constraints.

Our main finding is that a financial crisis has a state-dependent effect on innovation in new technologies. As there is no intrinsic difference between financing costs of innovation directed at modern versus old vintages, both would be equally affected in the steady state. This changes during a transition towards modern vintages, when young firms—who lack incumbents' expertise in mature technologies—disproportionately carry innovation in the new direction.

We validate these mechanisms using micro evidence on green patenting by German firms around the Global Financial Crisis. We exploit firms' pre-crisis dependence on Commerzbank—a large German bank that cut its lending stock by 35% after incurring severe subprime losses—as plausibly exogenous variation in credit tightening. We document three facts consistent with the model's predictions. First, young firms accounted for a disproportionate share of green patenting prior to the crisis—around 45% of green patents versus only 17% for non-green patents. Second, Commerzbank exposure significantly reduced green patenting in subsequent years, with an elasticity of around 3 by 2018, while leaving non-green patenting unaffected. Third, this asymmetric effect is driven entirely by young firms, who cut both green and non-green patenting sharply in response to tighter credit, while older firms are unaffected.

Quantifying the Schumpeterian framework to match the rise of green patenting in the years the Global Financial Crisis, and simulate the effect of a tightening of credit constraints early in the transition. We find that the decline in credit induced by the financial crisis is able to explain a significant share of the slowdown in green patenting after 2008. Moreover, the financial crisis induces a persistent delay in the diffusion of the modern technology vintage, with long-lasting consequences for productivity and, in the green application, for cumulative emissions.

References

Accetturo, A., Barboni, G., Cascarano, M., Garcia-Appendini, E., and Tomasi, M. (2022). Credit supply and green investments. *Available at SSRN 4217890*.

Acemoglu, D., Aghion, P., Barrage, L., and Hémous, D. (2023). Climate change, directed innovation, and energy transition: The long-run consequences of the shale gas revolution. Technical report, National Bureau of Economic Research.

Acemoglu, D., Aghion, P., Bursztyn, L., and Hemous, D. (2012). The environment and directed technical change. *American economic review*, 102(1):131–166.

Acemoglu, D., Akcigit, U., Alp, H., Bloom, N., and Kerr, W. (2018). Innovation, reallocation, and growth. *American Economic Review*, 108(11):3450–91.

Acemoglu, D., Akcigit, U., Hanley, D., and Kerr, W. (2016). Transition to clean technology. *Journal of political economy*, 124(1):52–104.

Acharya, V. V., Eisert, T., Eufinger, C., and Hirsch, C. W. (2015). Real effects of the sovereign debt crisis in europe: Evidence from syndicated loans. *Available at SSRN 2612855*.

Adelino, M., Schoar, A., and Severino, F. (2015). House prices, collateral, and self-employment. *Journal of Financial Economics*, 117(2):288–306.

Aghion, P., Bergeaud, A., Boppart, T., Klenow, P. J., and Li, H. (2020). A theory of falling growth and rising rents. *Working Paper, November 2020*.

Aghion, P., Bloom, N., Lucking, B., Sadun, R., and Van Reenen, J. (2015). Growth and decentralization in bad times preliminary.

Aghion, P., Dechezleprêtre, A., Hemous, D., Martin, R., and Van Reenen, J. (2016). Carbon taxes, path dependency, and directed technical change: Evidence from the auto industry. *Journal of Political Economy*, 124(1):1–51.

Aghion, P., Howitt, P., and Levine, R. (2018). Financial development and innovation-led growth. *Handbook of finance and development*, 1:1–28.

Akcigit, U., Alp, H., and Ercan, E. (2023). Navigating stormy waters: Crises, selection and productivity dynamics under financial frictions.

Akcigit, U. and Kerr, W. R. (2018). Growth through heterogeneous innovations. *Journal of Political Economy*, 126(4):1374–1443.

Almeida, H., Campello, M., Laranjeira, B., and Weisbenner, S. (2012). Corporate debt maturity and the choice between cash and lines of credit. *Critical Finance Review*.

Altavilla, C., Paries, M. D., and Nicoletti, G. (2019). Loan supply, credit markets and the euro area financial crisis. *Journal of Banking & Finance*, 109:105658.

Appelt, S., Bajgar, M., Criscuolo, C., and Galindo-Rueda, F. (2022). Micro-data based insights on trends in business r&d performance and funding: Findings from the oecd microberd+ project.

Ates, S. T. and Saffie, F. E. (2013). Project heterogeneity and growth: The impact of selection.

Ates, S. T. and Saffie, F. E. (2014). Fewer but better: Sudden stops, firm entry, and financial selection.

Bahaj, S., Pinter, G., Foulis, A., and Surico, P. (2019). Employment and the collateral channel of monetary policy.

Barkai, S. (2020). Declining labor and capital shares. *Journal of Finance, Forthcoming*.

Bentolila, S., Jansen, M., Jiménez, G., and Ruano, S. (2015). When credit dries up: Job losses in the great recession.

Berk, J. and van Binsbergen, J. H. (2021). The impact of impact investing.

Biermann, M. and Huber, K. (2024). Tracing the international transmission of a crisis through multinational firms. *The Journal of Finance*, 79(3):1789–1829.

Bloom, N., Griffith, R., and Van Reenen, J. (2002). Do r&d tax credits work? evidence from a panel of countries 1979–1997. *Journal of Public Economics*, 85(1):1–31.

Bolton, P., Halem, Z., and Kacperczyk, M. (2022). The financial cost of carbon. *Journal of Applied Corporate Finance*, 34(2):17–29.

Bolton, P. and Kacperczyk, M. (2021). Do investors care about carbon risk? *Journal of financial economics*, 142(2):517–549.

Boot, A. W. (2000). Relationship banking: What do we know? *Journal of financial intermediation*, 9(1):7–25.

Bresnahan, T. F. and Trajtenberg, M. (1995). General purpose technologies 'engines of growth'? *Journal of econometrics*, 65(1):83–108.

Caggese, A., Sampreet, G., Villegas-Sanchez, C., and Chiavari, A. (2023). Climate change, firms, and aggregate productivity. *Working Paper*.

Capelle, D., Diaz, E. E., Kirti, D., Villegas-Bauer, G., and Banerjee, S. (2025). Financial constraints and the effectiveness of green financial policies. Technical report, International Monetary Fund.

Chaney, T., Sraer, D., and Thesmar, D. (2012). The collateral channel: How real estate shocks affect corporate investment. *The American Economic Review*, 102(6):2381–2409.

Chodorow-Reich, G. (2014). The Employment Effects of Credit Market Disruptions: Firm-level Evidence from the 2008-9 Financial Crisis. *The Quarterly Journal of Economics*, 129(1):1–59.

Cloyne, J., Ferreira, C., Froemel, M., and Surico, P. (2023). Monetary policy, corporate finance, and investment. *Journal of the European Economic Association*, page jvad009.

Clymo, A. and Rozsypal, F. (2022). *Firm cyclicality and financial frictions*. CFM, Centre for Macroeconomics.

Davis, S. J. and Haltiwanger, J. C. (2019). Dynamism diminished: The role of housing markets and credit conditions. Technical report, National Bureau of Economic Research.

De Haas, R., Martin, R., Muuls, M., and Schweiger, H. (2024). Managerial and financial barriers to the green transition. *Management Science*.

De Haas, R. and Popov, A. (2020). Financial development and green growth. *ECB Working*.

De Ridder, M. (2019). Intangibles investment and the persistent effect of financial crises on output. In *Technical Report*.

De Ridder, M. (2024). Market power and innovation in the intangible economy. *American Economic Review*, 114(1):199–251.

Decker, R. A., Haltiwanger, J., Jarmin, R. S., and Miranda, J. (2016). Declining business dynamism: What we know and the way forward. *American Economic Review*, 106(5):203–207.

Drechsel, T. (2023). Earnings-based borrowing constraints and macroeconomic fluctuations. *American Economic Journal: Macroeconomics*, 15(2):1–34.

Evans, D. S. and Jovanovic, B. (1989). An estimated model of entrepreneurial choice under liquidity constraints. *Journal of political economy*, 97(4):808–827.

Franklin, J., Rostom, M., Thwaites, G., et al. (2015). The banks that said no: banking relationships, credit supply and productivity in the uk. Technical report.

Fried, S. (2018). Climate policy and innovation: A quantitative macroeconomic analysis. *American Economic Journal: Macroeconomics*, 10(1):90–118.

Garcia-Macia, D. (2015). The Financing of Ideas and the Great Deviation. Job market paper, Stanford University.

Geroski, P. A. (2000). Models of technology diffusion. *Research policy*, 29(4-5):603–625.

Gertler, M. and Gilchrist, S. (1994). Monetary policy, business cycles, and the behavior of small manufacturing firms. *The Quarterly Journal of Economics*, 109(2):309–340.

Gertler, M. and Kiyotaki, N. (2010). Financial intermediation and credit policy in business cycle analysis. *Handbook of Monetary Economics*, 3(3):547–599.

Giglio, S., Kelly, B., and Stroebel, J. (2021). Climate finance. *Annual Review of Financial Economics*, 13:15–36.

Giroud, X. and Mueller, H. (2015). Firm Leverage and Unemployment during the Great Recession. *MIT Working Paper*.

Golosov, M., Hassler, J., Krusell, P., and Tsyvinski, A. (2014). Optimal taxes on fossil fuel in general equilibrium. *Econometrica*, 82(1):41–88.

Gormsen, N., Huber, K., and Oh, S. (2021). Climate capitalists.

Greenstone, M., Mas, A., and Nguyen, H.-L. (2014). Do credit market shocks affect the real economy? Technical report, NBER.

Griliches, z. (1957). Hybrid corn: an exploration in the economics of technological change. *Econometrica*, 25(4):501.

Haltiwanger, J., Jarmin, R. S., and Miranda, J. (2013). Who creates jobs? small versus large versus young. *Review of Economics and Statistics*, 95(2):347–361.

Haščič, I. and Migotto, M. (2015). Measuring environmental innovation using patent data. *OECD Working Paper*.

Hassler, J., Krusell, P., and Olovsson, C. (2018). The consequences of uncertainty: climate sensitivity and economic sensitivity to the climate. *Annual Review of Economics*, 10:189–205.

Helpman, E. and Trajtenberg, M. (1994). A time to sow and a time to reap: Growth based on general purpose technologies.

Hémous, D. (2016). The dynamic impact of unilateral environmental policies. *Journal of International Economics*, 103:80–95.

Hémous, D. and Olsen, M. (2021). Directed technical change in labor and environmental economics. *Annual Review of Economics*, 13:571–597.

Hofman, D. M. and Huisman, R. (2012). Did the financial crisis lead to changes in private equity investor preferences regarding renewable energy and climate policies? *Energy Policy*, 47:111–116.

Huber, K. (2018). Disentangling the effects of a banking crisis: Evidence from german firms and counties. *American Economic Review*, 108(3):868–898.

IPCC (2023). Climate change 2023 synthesis report summary for policymakers. *CLIMATE CHANGE 2023 Synthesis Report*.

King, R. G. and Levine, R. (1993). Finance, entrepreneurship and growth. *Journal of Monetary economics*, 32(3):513–542.

Klette, T. J. and Kortum, S. (2004). Innovating firms and aggregate innovation. *Journal of political economy*, 112(5):986–1018.

Lanteri, A. and Rampini, A. A. (2023). Financing the adoption of clean technology.

Lian, C. and Ma, Y. (2021). Anatomy of corporate borrowing constraints. *The Quarterly Journal of Economics*, 136(1):229–291.

Luttmer, E. G. (2011). On the mechanics of firm growth. *The Review of Economic Studies*, 78(3):1042–1068.

Moll, B. (2014). Productivity losses from financial frictions: can self-financing undo capital misallocation? *The American Economic Review*, 104(10):3186–3221.

Nordhaus, W. D. (1994). *Managing the global commons: the economics of climate change*, volume 31. MIT press Cambridge, MA.

Ottonello, P. and Winberry, T. (2024). Capital, ideas, and the costs of financial frictions. Technical report, National Bureau of Economic Research.

Pakes, A. and Griliches, Z. (1980). Patents and r&d at the firm level: A first report. *Economics letters*, 5(4):377–381.

Paravisini, D., Rappoport, V., Schnabl, P., and Wolfenzon, D. (2015). Dissecting the effect of credit supply on trade. *The Review of Economic Studies*, 82(1):333–359.

Patozi, A. (2023). Green transmission: Monetary policy in the age of esg. *Available at SSRN 4327122*.

Peters, M. (2020). Heterogeneous markups, growth, and endogenous misallocation. *Econometrica*, 88(5):2037–2073.

Queraltó, A. (2013). A model of slow recoveries from financial crises. *FRB International Finance Discussion Paper*, (1097).

Rajan, R. G. and Zinglas, L. (1998). Financial dependence and growth. *The American Economic Review*, 88(3):559–586.

Ribeiro, B. (2025). Growth with new and old technologies. EIEF Working Paper.

Schmitz, T. (2021). Endogenous growth, firm heterogeneity and the long-run impact of financial crises. *European Economic Review*, 132:103637.

Silva, J. S. and Tenreyro, S. (2006). The log of gravity. *The Review of Economics and statistics*, pages 641–658.

Zazzaro, A., Oliviero, T., and Rondinella, S. (2024). Are green firms more financially constrained? the sensitivity of investment to cash flow.

Online Appendix for 'Lost in Transition'

Appendix A. Proofs and Derivations

A.1. Derivations of the Aggregate Variables

To find the equilibrium wage, start from the expression for aggregate output:

$$\ln Y = \int_0^1 \ln q_j + \ln y_j dj,$$

where firm subscripts are omitted for readability. Inserting unit-elastic demand gives:

$$\ln Y = \int_0^1 \ln q_j dj + \ln Y - \int_0^1 \ln p_j dj,$$

Note that the unit measure of products can be separated into the fraction F that is produced with the new technology vintage, and the remainder $1-F$ which is produced with the old technology vintage:

$$\ln Y = \int_0^1 \ln q_j dj + \ln Y - \int_0^F \ln p_j dj - \int_F^1 \ln p_j dj.$$

Further note that, by the law of large numbers, a fraction F of all products is produced with a new technology second-best competitor, regardless of whether the first-best (producing) firm uses the new or old technology vintage itself. Inserting optimal prices using the values for marginal costs and markups (3) therefore yields

$$0 = \int_0^1 \ln q_j dj - F \ln w - (1-F) \ln w/a^o - F\left(F \ln \lambda + (1-F) \ln(\lambda a^o)\right) - (1-F)\left(F \ln(\lambda a^o) + (1-F) \ln \lambda\right)$$

Isolating w on the left-hand side and grouping terms yields (14).

Next we derive an expression for aggregate output from the source constraint. Define L^p as the total amount of labor devoted to goods production. Labor market equilibrium requires

$$L^p = \int_0^1 l_j dj$$

where, again, firm subscripts are omitted. Inserting production and demand functions yields

$$L^p = \int_0^F y_j dj + \int_F^1 y_j/a^o dj \quad \Rightarrow \quad L^p = Y\left(\int_0^F (p_j)^{-1} dj + \int_F^1 p_j^{-1}/a^o dj\right)$$

Inserting optimal prices gives

$$L^p = \frac{Y}{w}\left(F\left[F\lambda + (1-F)\lambda/a^o\right] + (1-F)\left[F\lambda \cdot a^o + (1-F)\lambda\right]\right)$$

Inverting the function and inserting the equilibrium wage (14) yields

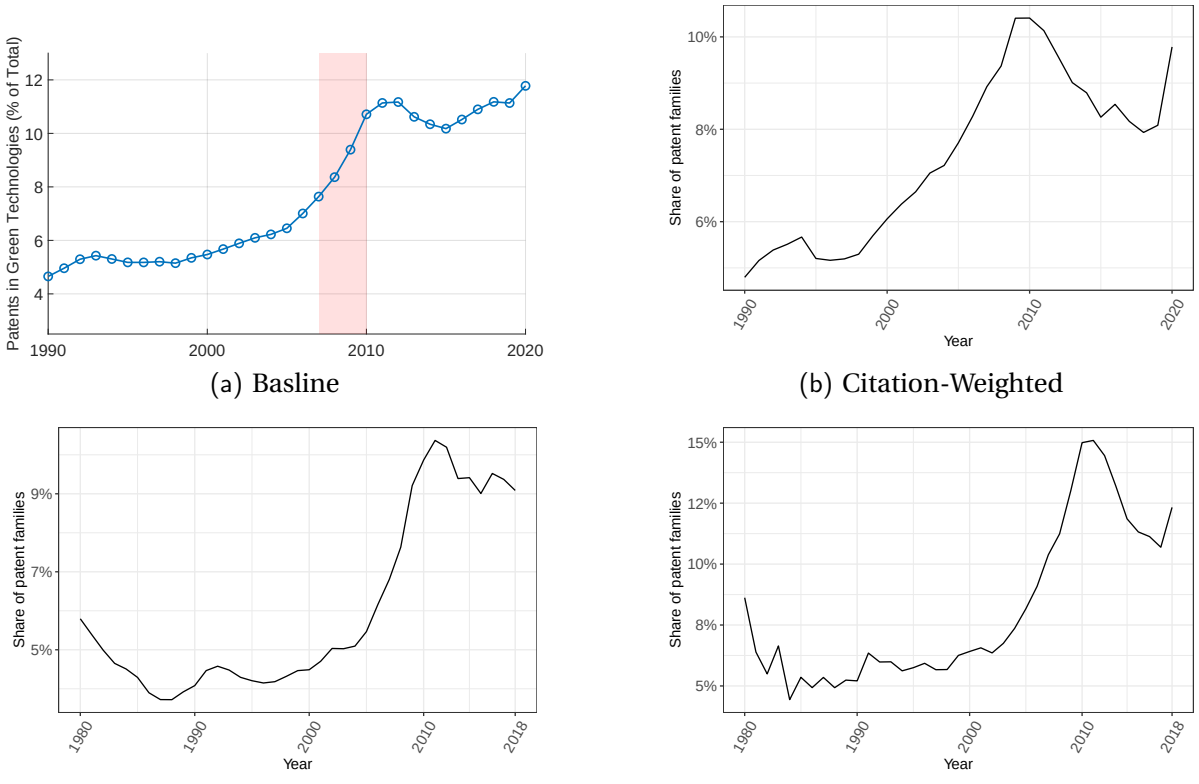
$$Y = L^p \exp\left(\int_0^1 \ln q_j dj\right)\left(\frac{\left(\exp\left(\overline{\ln \mu^m}\right)^F \exp\left(\overline{\ln \mu^o}\right)^{(1-F)}\right)^{-1}}{F\left[F\lambda + (1-F)\lambda/a^o\right] + (1-F)\left[F\lambda \cdot a^o + (1-F)\lambda\right]}\right)(a^o)^{1-F}$$

A.2. Proof of Proposition 1

Available on request.

Appendix B. Additional Empirical Results

Figure B1. Alternative Trends in Green Patents as a Percentage of Total Patents

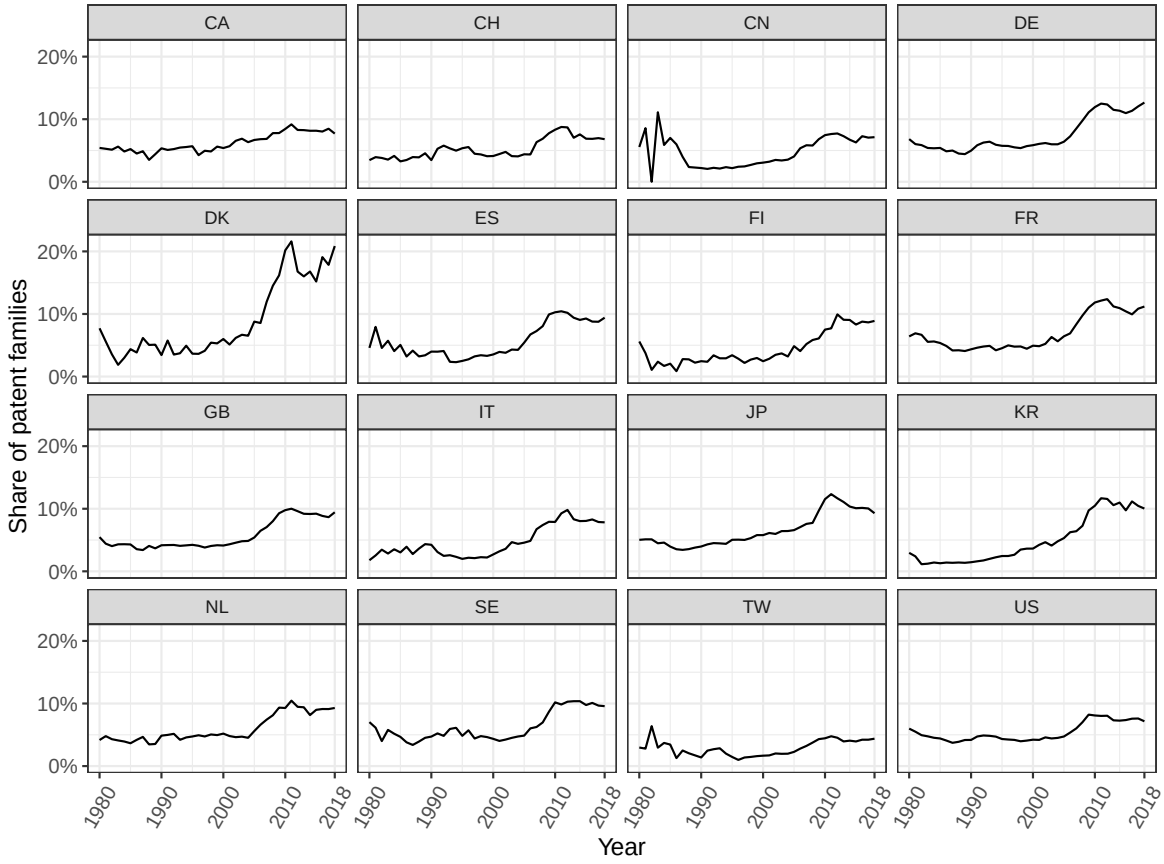


(a) Basline (b) Citation-Weighted

(c) OECD (d) Triadic

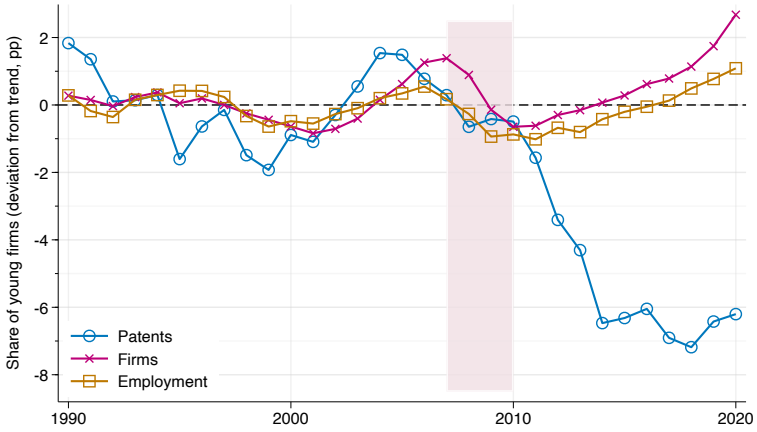
Notes: Source: % patents in environmental management, water-related adaption technologies, biodiversity and ecosystem health, climate-change mitigation (OECD classification - Hascic and Migotto 2015) - PATSTAT data. Triadic patents are patents filed at each of the European Patent Office, the U.S. Patent Office and the Japan Patent Office.

Figure B2. Global Trend: Green Patents as a Percentage of Total Patents



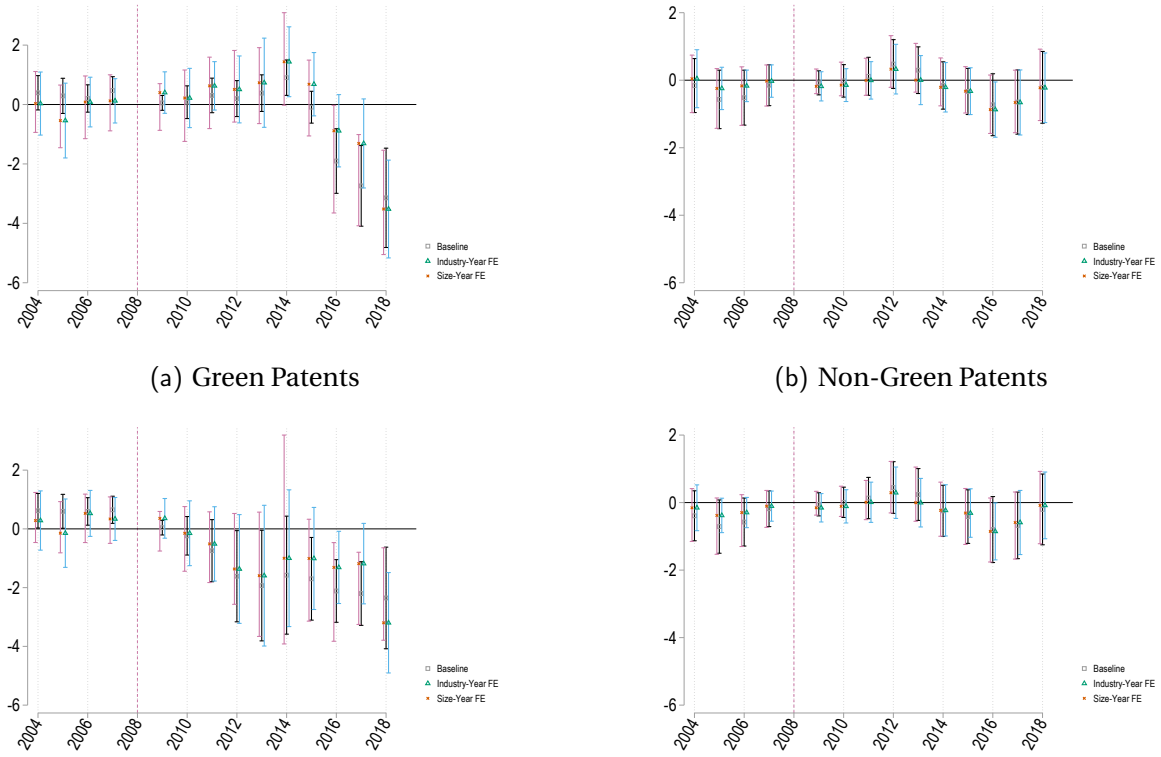
Notes: Source: % patents in environmental management, water-related adaption technologies, biodiversity and ecosystem health, climate-change mitigation (OECD classification - Hascic and Migotto 2015) - PATSTAT data. CA: Canada, CH: Switzerland, CN: China, DE: Germany, DK: Denmark, ES: Spain, FI: Finland, FR: France, GB: United Kingdom, IT: Italy, JP: Japan, KR: Republic of Korea, NL: Netherlands, SE: Sweden, TW: Taiwan, US: United States.

Figure B3. Share of patents, firms and employment from young firms



Notes: . The orange dashed line reports the share of firms younger than 15 years, while the blue dashed line reports the share of total employment in firms younger than 15 years. Each raw series is detrended using data over the 1990–2006 period.

Figure B4. Robustness: Effect of Global Financial Crisis on Patent Applications



(a) Green Patents

(b) Non-Green Patents

(c) Green Patents

(d) Non-Green Patents

Notes: The figure plots the effect of exposure to Commerzbank on non-green patents (left) and green patents (right). Patents are classified as green if at least one IPC code relates to environmental management, water-related adaption technologies, biodiversity and ecosystem health, climate-change mitigation (OECD classification - Haščič and Migotto 2015). Confidence bounds are at the 95% level based on firm-clustered standard errors.

Appendix C. Counterfactual Construction for Fact 3

To isolate deviations specific to a given technology's development path, we proceed in terms of patent *shares* to net out aggregate changes in the level of patenting. Let $n_{c,t}$ denote the number of patents in technology c in year t, and let $N_t = \sum_c n_{c,t}$ be the total number of USPTO patents in year t. We define the technology-year patent share as $S_{c,t} = n_{c,t}/N_t$.

Our counterfactual is obtained by fitting, separately for each technology, a parametric *Gompertz flow* curve as a function of technology age. Gompertz processes are a standard reduced-form representation of technological diffusion and life-cycle dynamics (Geroski, 2000): they imply an S-shaped cumulative pattern and a single-peaked (hump-shaped) flow, closely matching the empirical evolution of activity in many technologies (Griliches, 1957; Ribeiro, 2025). Letting a denote age, we model the patent share flow as

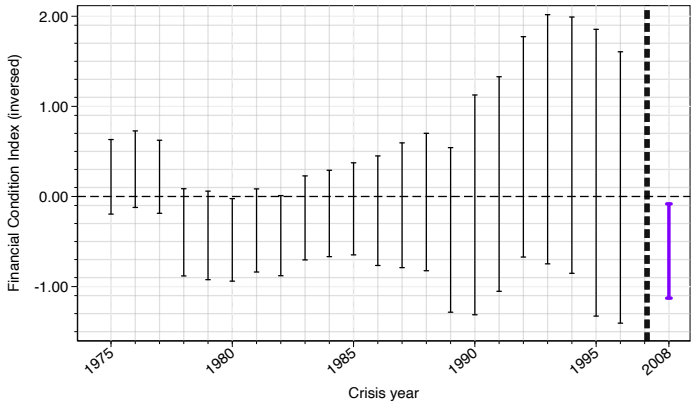
$$f(a;\, K, b, m) \;=\; K\, b \exp\Big(-b(a-m) - \exp\big(-b(a-m)\big)\Big), \qquad K \in (0,1],\; b > 0,$$

where K scales the overall mass, b governs the steepness of the rise and decline, and m indexes the age at which patenting peaks. We estimate (K_c, b_c, m_c) by nonlinear least squares using only observations from ages 0–10. The technology-specific predicted patent *counts* are then obtained by rescaling to the observed aggregate patent total: $\widehat{n}_{c,t} = \widehat{S}_{c,t}\, N_t$. Intuitively, $\widehat{n}_{c,t}$ captures how much a technology would be expected to patent in year t given (i) its own early-life trajectory and (ii) the contemporaneous aggregate level of patenting.

To assess whether the estimated break is specific to the 2008 crisis, we conduct two placebo exercises. First, Figure B5 in the Appendix repeats the event-study analysis using *fake* crisis years between 1975 and 1996. For each placebo date, we re-estimate the specification and summarize its medium-run impact by averaging the coefficients from crisis+6 to crisis+11 (corresponding to the 2014–2019 window in Figure 4 for the true GFC). The 2008 estimate is the largest in absolute value among all placebo dates.

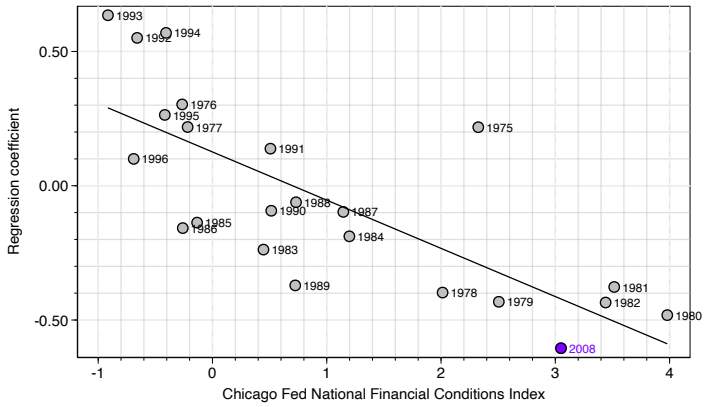
Some placebo dates nonetheless coincide with episodes of financial stress. Figure B6 therefore relates the placebo estimates to an external index of financial conditions. The correlation is negative: episodes of tighter credit are associated with a more negative post-crisis gap for young technologies six to eleven years later. This cross-placebo correlation reinforces the interpretation that it is the financial dimension of the crisis, rather than some other contemporaneous shock, that is responsible for the differential impact on emerging technologies.

Figure B5. Placebo crisis dates and medium-run effects on young versus mature technologies



Notes: This figure reports a placebo exercise based on eq (1). For each placebo crisis year between 1975 and 1996, we re-estimate the event-study specification treating that year as the crisis date, and compute the average of the estimated coefficients β_τ from placebo+6 to placebo+11. This corresponds to the medium-run effect of the placebo crisis on the patenting gap of young relative to mature technologies. The figure also reports the corresponding estimate for the actual 2008 Global Financial Crisis. A more negative value indicates a larger post-crisis decline in patenting for young technologies relative to mature ones.

Figure B6. Correlation between placebo coefficient and financial stress



Notes: This figure plots the placebo estimates from Figure B5 for each crisis year against the Chicago Fed's National Financial Conditions Index. The index is transformed so that higher values correspond to tighter financial conditions.

Appendix D. Computational Appendix

This appendix summarizes the computational algorithm used in the quantitative exercise. The firm-level state is (N_m, N_o, θ), where N_m and N_o denote the number of modern and old product lines and θ indexes financial maturity. The same state space is used in the stationary equilibrium, in the transition problem, and in the auxiliary firm-level simulations.

1. *Steady states and structural estimation.*

   (a) Computation starts from a set of structural parameters, numerical settings, and empirical targets. These objects determine the moments targeted in the initial steady state, the emissions-reduction target used to discipline the final steady state, the length of the transition horizon, and the financial-shock environment used in the crisis exercises.

   (b) The initial steady state is estimated by nonlinear least squares over $(\phi, \phi_{em}, \tilde{p}_o, \chi_{up})$. For each candidate parameter vector, the model is solved and the resulting moments are compared with their targets. The moments used in this stage are the quarterly entry rate, the growth rate, the incumbent modern-innovation share, and the mean age of firms at or below median financial maturity.

   (c) Each evaluation of the objective requires solving the stationary equilibrium as a nonlinear system in the 15 unknowns

   $$(V_o, V_m, \omega, \tau, C, e_o, e_m, x_o, x_m, F, L_o, L_m, w, g, \text{rdint}).$$

   The inner fixed point works as follows:

      i. Given a guess for the aggregate equilibrium objects, compute state-by-state innovation rates for all firm states. The R&D first-order conditions are solved analytically. Unconstrained states use the closed-form interior solution, while constrained states use the closed-form solution associated with a binding financial budget, so no nested state-by-state nonlinear solver is required.

      ii. Construct the sparse generator for firm dynamics over all active states and the exit state. This generator incorporates product gains, product losses, movements across financial-maturity states, exit after the last product is lost, and entry at the lowest financial-maturity state.

      iii. Recover the stationary distribution by solving

      $$\begin{bmatrix} T \\ \mathbf{1}' \end{bmatrix} K = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

      iv. Aggregate the resulting distribution to obtain entry, aggregate innovation, labor allocations, the modern-product share, growth, and R&D intensity. The

same stationary generator is also used to recover the firm-age moment by financial-maturity bin, which feeds back into the initial steady-state objective.

v. Return the associated equilibrium residuals and iterate on the 15 unknowns until the system converges.

(d) After the initial steady state has been estimated and solved, the implied modern share F is used to define the target for the final steady state. The latter is then estimated by a one-dimensional nonlinear least-squares problem over $\tilde{p}_o$, so that the model matches the final modern-share target implied by the emissions-reduction objective.

(e) To reduce computation time, the parameter search can be accelerated through warm starts and a faster screening pass. However, the final reported estimates are always re-evaluated with the baseline equilibrium routine before they are treated as official results.

2. *Transition, policy-speed calibration, and crisis computation.*

(a) Once the initial and final steady states have been solved, the transition algorithm builds a finite time grid with step size Δt. The paths for relative prices and innovation step sizes are then parameterized so that $\tilde{p}_o$ and λ move gradually from their initial to their final values over the policy transition.

(b) Before solving the main transition, the speed of the policy transition is itself calibrated with a one-dimensional search routine. Each candidate adjustment speed implies a different path for $\tilde{p}_o$ and λ, and hence a different no-shock transition. The chosen value minimizes the squared gap between the model-implied pre-crisis green-patent index and its empirical target.

(c) The baseline transition without a financial shock is solved as a fixed point over entire time paths. The initial guess is either a linear interpolation between the two steady states or, when available, a warm start from a previously computed transition. Each iteration has four steps:

i. *Forward pass.* For every date on the grid, compute state-level innovation rates, entry rates, and the period-specific sparse generator, then update the cross-sectional distribution of firms over (N_m, N_o, θ) using the law of motion implied by that generator.

ii. *Aggregation.* Given the updated distribution, compute product shares, markups, wages, output, labor allocations, growth, and R&D intensity date by date.

iii. *Backward pass.* Update the value of old and modern products using backward recursion.

iv. *Update step.* Use the new value-function paths to update the guess for the next iteration. The fixed-point iteration is accelerated with an Anderson step and is repeated until the value functions, output, and wages converge.

(d) The crisis block adds a time-varying financial shock. A quarterly path for the multiplier Θ_t on θ is estimated from lending data by optimizing over a low-dimensional knot parameterization. The optimized knot values are then interpolated to a full quarterly series, mapped into model periods, and, after the calibrated path ends, the multiplier returns to one.

(e) The crisis transition is then solved using the same fixed-point algorithm, but under imperfect foresight. Before the shock arrives, firms evaluate innovation and entry decisions using the no-shock transition as the expected path; once the shock hits, they update decisions using the realized crisis path. The same calibrated shock profile is also reused for the late-crisis counterfactual, which shifts the timing of the financial shock while holding its shape fixed.