Inequality, Business Cycles, and Growth: A Unified Theory of Fiscal Stabilization Policies

ALEXANDRE GAILLARD
Brown University

PHILIPP WANGNER
University of Mannheim

March 20, 2026

Abstract: This paper studies the short-run and long-run effects of tax-financed fiscal stimulus programs in a unified heterogeneous-agent New Keynesian growth framework. In this economy, innovative investment generates technology growth, a demand externality drives economic activity, and household heterogeneity shapes aggregate consumption and innovative entrepreneurial investment. We show that the effectiveness of fiscal stimulus programs hinges on the interaction between systematic monetary policy and tax incidence. Only when monetary policy is sufficiently passive in raising interest rates, progressively financed programs raise short- and long-run output by jointly fostering aggregate demand and technology growth. When monetary policy is instead sufficiently aggressive, the same policy creates a trade-off between short-run stabilization gains and long-run output losses. Quantitatively, this trade-off is a robust feature of the U.S. economy: progressively financed expansions in government spending lead to an impact output multiplier above one, a zero multiplier after three years, and sizable long-run output losses relative to the laissez-faire allocation without fiscal intervention.

Keywords: Inequality, Business Cycles, Growth, Fiscal Stabilization Policies.

First Version: February 2022. A previous version was entitled *"The Heterogeneous Agent New Keynesian Growth Synthesis"*. We are extremely grateful to our advisors Christian Hellwig, Patrick Fève, Fabrice Collard, and Nicolas Werquin. We also thank Klaus Adam, Philippe Aghion, Adrien Auclert, Martin Beraja, Corina Boar, Florin Bilbiie, Axelle Ferriere, François Gourio, Rohan Kekre, Dirk Krueger, Moritz Lenel, Chen Lian, Ernest Liu, Iourii Manovskii, Wendy Morrison, Elisa Rubbo, Andreas Schaab, Vincent Sterk, Joe Stiglitz, Paolo Surico, Francesco Zanetti and (seminar) participants at Columbia Business School, Columbia University, CREI, EIEF, FEP Porto, Queen Marry University of London, Tilburg University, University of Essex, University of Exeter, University of Groningen, Universitat Pompeu Fabra, University of Mannheim, Sciences Po, the Federal Reserve Board of Governors, the Federal Reserve Bank of Chicago, the Bank of Italy, the St Gallen Workshop 2023, ESEM 2024, and the Paris Dauphine Workshop 2025 for constructive comments and insightful discussions. Wangner thanks the UniCredit & Universities foundation for financial support. *Author contact information*: Alexandre Gaillard, Brown University; E-Mail: alexandre_gaillard@brown.edu; Philipp Wangner, Department of Economics, University of Mannheim; E-Mail: philipp.wangner@uni-mannheim.de.

1. Introduction

Fiscal expansions are a central policy tool for stabilizing economic activity during recessions. Their effectiveness depends critically on how the associated increase in government spending is financed, in particular the distribution of the tax burden across households. In the United States, spending shocks have historically been financed, on average, through increases in tax progressivity, with relatively larger tax hikes on high-income households as compared to low-income households. Proponents of such policies emphasize their *short-run benefits*, arguing that raising taxes on top-income earners can stimulate aggregate demand effectively. Opponents, however, point to potentially sizable *long-run costs*, arguing that greater tax progressivity may dampen entrepreneurial activity, discourage innovation, and weaken future growth prospects. These contrasting perspectives raise several important questions: Does a trade-off arise between short-run output stabilization and long-run technological progress? If so, which financing schemes can mitigate or avoid it? More broadly, how should the tax burden be distributed across households to maximize welfare?

To make progress on these longstanding questions this paper develops a unified theory of fiscal stimulus programs based on a new Heterogeneous Agent New Keynesian Growth framework (HANK-G), in which household heterogeneity interacts with both business-cycle fluctuations and long-run technological growth. This framework combines three cornerstones of macroeconomic modeling: (i) technological growth is endogenous and arises from innovative investment; (ii) nominal rigidities generate demand-driven fluctuations in economic activity; and (iii) household heterogeneity shapes aggregate consumption and innovation through the joint distribution of marginal propensities to consume and to innovate. Specifically, the paper makes two contributions. First, we characterize the key transmission channels of fiscal stimulus programs in an analytically tractable HANK-G model. Second, we take an extended version of the model to the data, calibrate it to the U.S. economy and quantify the short- and long-run effects of tax-financed expansions in government spending.

The economics of progressively financed fiscal stimulus programs are straightforward. In the short-run, such policies are effective whenever the expansion in private and public consumption outweighs any potential decline in innovative investment, thereby raising output. In the long-run, instead, such policies are effective only if they also stimulate innovative investment and, through it, technological growth. Two opposing forces shape the response of innovative investment. On the one hand, a short-run expansion in private and public consumption raises aggregate demand and therefore the returns to innovation, crowding in innovative investment. On the other hand, higher marginal taxes on top incomes reduce the incentives to innovate, crowding out innovative investment. Whenever output and innovative investment respond in opposite directions, a stabilization trade-off emerges. In that case, progressively financed policies raise output in the short-run, but eventually generate permanent long-run output losses from a lower technology growth relative to the *laissez-faire* allocation without fiscal intervention.

The main policy implication of this paper is that the existence and resolution of such a stabilization trade-off is governed by the interaction of systematic monetary policy and

tax incidence. Theoretically, progressively financed government expansions can deliver both short-run output stabilization and long-run technology growth under sufficiently dovish monetary policy. Under sufficiently hawkish monetary policy, the same policies instead generate a stark short-run versus long-run trade-off. Quantitatively, we show that this trade-off is a robust feature of the U.S. economy. In particular, progressively financed government expansions yield an impact output multiplier of 1.40, a zero multiplier after three years, and a permanent long-run output loss of 0.15 pp.

Analytical Framework. To derive sharp theoretical insights, we start by laying out the main building blocks of the analytical HANK-G framework.

The heterogeneous-agent block features a continuum of households who consume, work, and face idiosyncratic income risk. In the spirit of Bilbiie (2008, 2025), households switch between two states that differ in their access to asset markets, giving rise to a precautionary saving motive. In particular, a fraction of households are unconstrained entrepreneurs (savers) who self-insure through *liquid* bond holdings and, in addition, own innovative firms that accumulate *illiquid* innovative capital. Innovative investment improves product quality and earns a cyclical return premium, in part because of its illiquidity. The remaining fraction of households are hand-to-mouth who consume their disposable income each period. Following Benigno and Fornaro (2018), the firm block consists of a representative final good firm that combines labor with intermediary inputs supplied by entrepreneurs' monopolistic innovative firms. In addition, the New Keynesian block introduces nominal wage rigidities, which turn aggregate fluctuations into cyclical income inequality and idiosyncratic income risk through an unequal earnings incidence. Finally, fiscal policy operates through redistributive transfers and government spending, financed by taxes with heterogeneous incidence across household types, while the Central Bank sets the nominal interest rate according to a feedback rule.

The stationary balanced growth path equilibrium of this economy admits a four-equation representation that comprises: (i) a dynamic IS equation that captures aggregate demand; (ii) an endogenous growth equation that embodies entrepreneurial innovation decisions; (iii) a New Keynesian wage Phillips curve; and (iv) a Taylor-type monetary policy rule. As the four equation representation is purely forward-looking we can determine the aggregate effects of fiscal expansions in a two asset HANK economy in closed form. Beyond its analytical tractability, the key novelty of this representation lies in the unified treatment of the short- and long-run propagation of fiscal stimulus programs within a single framework that preserves direct comparability to existing models studying household heterogeneity, business cycles, and technology growth in isolation.

Theoretical Results. Our results clarify why progressively financed stimulus programs can be powerful in the short-run and, at the same time, harmful in the long-run. To isolate the main propagation mechanisms, we start our theoretical analysis by studying the propagation of progressive – and non-distortionary – fiscal transfers across households, keeping the level of government spending fixed.

To begin, we compare three benchmark economies, each obtained by shutting down one or two key building blocks of the HANK-G framework. In a heterogeneous-agent exogenous growth model without nominal rigidities (HA), progressive redistribution has

no effect on output: it only reshuffles labor supply across household types, leaving aggregate production unchanged. In a HANK model with exogenous growth, progressive redistribution is unambiguously expansionary in the short-run: by shifting income toward high-MPC households, it strengthens aggregate demand and raises output, but leaves long-run technology growth unchanged. In a heterogeneous-agent endogenous growth model without nominal rigidities (HA-G), the opposite holds: progressive redistribution unambiguously contracts both technology growth and output, because it reduces entrepreneurs' innovative investment and, thus, the marginal product of labor.

The full HANK-G framework combines these opposing forces. For a transitory shock to progressive redistribution, we derive closed-form solutions for the impact multipliers for output and technology growth. When fiscal redistribution is persistent and idiosyncratic income risk is present, output unambiguously increases at impact. Central for this result is a counteryclical liquidity premium for innovative investment. By reducing income inequality, progressive redistribution weakens entrepreneurs' precautionary saving motive. As a result, their consumption and innovative investment decline by less, making the reallocation of income toward hand-to-mouth households sufficiently powerful to expand output in the short-run. Whether innovative investment rises or falls is governed by two policy primitives: the persistence of redistribution and the aggressiveness of systematic monetary policy. If redistribution is sufficiently persistent and monetary policy sufficiently dovish, the demand expansion is strong enough to crowd in innovative investment, so both output and technology growth increase. When redistribution is less persistent or monetary policy more hawkish, innovative investment falls, and a trade-off emerges: output rises in the short-run, but as technology growth declines, it eventually falls below the *laissez-faire* allocation without fiscal intervention after some time. The horizon at which this tipping point occurs depends only on the persistence of the redistributive policy and the ratio of the impact output and technology growth multiplier, and is increasing in the latter.

We then extend the analysis to a standard government spending shock that is financed through heterogeneous tax incidence. In this context, government spending itself raises demand, while its financing redistributes resources across household types and thus reshapes private consumption and innovative investment. The sign and size of the output and technology growth multipliers depend jointly on four objects: (i) the persistence of the spending shock, (ii) the presence of idiosyncratic income risk, (iii) the aggressiveness of monetary policy, and (iv) the incidence of the tax burden between hand-to-mouth households and entrepreneurs. These forces generate distinct policy regimes. Under sufficiently dovish monetary policy and progressive tax incidence, government spending can raise both short-run output and long-run growth. Under more hawkish monetary policy and progressive tax incidence, the same shock instead delivers a clear trade-off: short-run output gains at the cost of weaker innovation and lower long-run output. Finally, if the tax incidence is not progressive enough, government spending can lower both short- and long-run output.

Those insights imply that the interaction between systematic monetary policy and tax incidence is the key policy margin in the HANK-G framework. Because hand-to-mouth households shape the short-run demand response while entrepreneurs shape

the long-run output response, the interaction between both policy margins determines whether a stabilization trade-off across horizons emerges.

Quantitative Illustration. While the theoretical results clarify the existence of distinct regimes for the propagation of tax-financed fiscal stimulus programs, a key question is which of these regimes is quantitatively relevant.

To answer this question, we take the HANK-G framework to the data. The quantitative model preserves the same backbone as the theoretical apparatus while enriching the environment only where needed for empirical discipline. It breaks the spender–entrepreneur dichotomy of the analytical model by distinguishing entrepreneurial ability, which is fixed *ex ante*, from hand-to-mouth and saving behavior, which arise from *ex post* heterogeneity within both workers and entrepreneurs. Households face multiple idiosyncratic risks through labor productivity, employment status, and discount-factor shocks, while entrepreneurs additionally face uncertain innovation outcomes and choose investment levels that generate step-wise innovation gains that scale with the aggregate stock of knowledge. These features generate a joint distribution of innovative activity, income, wealth, and MPCs. Finally, we replace the stylized fiscal block of the analytical model with flexible tax and transfer schedules that map more closely to the fiscal instruments used in practice.

We discipline the model using the best available evidence on how taxes shape innovative activity in the U.S.. In particular, we target the average growth rate, key income and wealth inequality moments, and the elasticity of entrepreneurs' patenting and R&D investment with respect to corporate taxation, with the latter disciplined by the estimates of Akcigit et al. (2022). Labor-income risk is calibrated using CPS data, while discount-factor heterogeneity is disciplined to match average MPCs and bond holdings. The model matches the observed distributions of income and wealth, produces a realistic MPC profile for short-run consumption dynamics, and replicates the empirical innovation response to personal income taxation.

We first confirm that the short- versus long-run stabilization trade-off arises under a tax-financed government spending shock. In the HANK-G economy, the long-run costs of the shock financed through greater tax progressivity overtake its short-run gains within just six quarters. This stabilization trade-off is not only a theoretical possibility but a quantitatively robust and economically significant feature of the model, holding across alternative specifications of the Taylor rule, stronger nominal rigidities, a more equal earnings incidence, and a higher persistence of the government spending shock.

Resolving the Stabilization Trade-Off. Finally, we analyze which financing schemes avoid the short- versus long-run stabilization trade-off. First, we vary tax incidence, allowing the government to target the income quantile that bears the bulk of the additional tax burden. Second, we consider debt financing.

In the HANK-G economy, taxing middle-income households is the preferred tax incidence when the objective is to maximize both the short-run output multiplier and the technology-growth multiplier. The reason is simple: this tax scheme preserves the demand response of high-MPC households at the bottom while limiting distortions to innovation at the top. As a result, it not only generates the largest impact effects, but also

delivers the largest output gains even ten years after the spending shock. This result is specific to HANK-G and differs sharply from both HANK and HA-G. In HANK, the MPC channel alone pushes toward taxing the highest bracket. In HA-G, by contrast, that channel is muted, so the cost of taxing top households is no longer offset by stronger demand and higher profits, shifting the preferred incidence toward the very bottom bracket. This conclusion carries over to welfare, which captures both short-run demand gains and the discounted long-run effects of technology on permanent consumption, relative to the *laissez-faire* economy. By this criterion, taxing middle-income households generates substantially larger gains than alternative tax schemes. Relative to an overall increase in tax progressivity, the consumption-equivalent gain amounts to 0.45%, while the output and growth multipliers at the optimal incidence are 1.5 and 2.2, respectively. Strikingly, these gains are broad-based: all households – even the ones directly targeted by the tax increase – benefit from the fiscal expansion.

Finally, we allow the government to finance the spending shock by issuing additional public debt, to be repaid later through lump-sum taxation. In HANK-G, faster growth normally helps reduce the debt burden, whereas slower growth makes it more persistent. Under lump-sum repayment, however, the growth multiplier becomes largely unresponsive, while the short-run demand channel is amplified. As a result, the shock raises both output and technology growth, and no trade-off emerges.

Related Literature. This paper contributes to the extensive literature that studies how household heterogeneity at the micro level affects aggregate outcomes in the short- and long-run. Specifically, we relate to four strands of the literature.

First, our paper relates to the HANK literature. On the analytical side, we relate to Bilbiie (2008, 2020, 2025), Acharya and Dogra (2020), Broer et al. (2020), Ravn and Sterk (2020), Cantore and Freund (2021), and Debortoli and Galí (2024a). As the heterogeneous agent setup is based on limited asset markets participation, our analytical HANK-G model is closest to Bilbiie (2025), however extended by two assets as in Galí et al. (2007) or Bilbiie et al. (2022), a general earnings incidence as in Werning (2015), and sticky wages. On the quantitative side, we relate, among many others, to McKay et al. (2016), Kaplan et al. (2018), Bayer et al. (2019), Auclert (2019), and Auclert et al. (2024). Overall, we synthesize HANK with a model of endogenous growth. We make two contributions: analytically, we derive a closed-form four-equation representation of a two-asset HANK-G model; quantitatively, we calibrate an extended version to the U.S. economy and evaluate the short- and long-run effects of fiscal stimulus programs.[1]

Second, our paper relates to a growing literature that studies the propagation of aggregate shocks and stabilization policies in business cycles models with endogenous growth. Stadler (1990) and Comin and Gertler (2006) were the first to document medium-term business cycles. Building on their work, Benigno and Fornaro (2018), Anzoategui et al. (2019), Garga and Singh (2021), or Fornaro and Wolf (2025) developed a Keynesian growth framework with nominal rigidities.[2] Our contribution is to enrich

[1] As technology growth is driven by innovative investment, our paper shares similar propagation mechanisms as Luetticke (2021), Kekre and Lenel (2022) or Melcangi and Sterk (2025), highlighting the role of investment in heterogeneous agent economies.

[2] Bianchi et al. (2019), Queralto (2020, 2022), and Fornaro and Wolf (2023) are further important papers.

the Keynesian growth framework with household heterogeneity and incomplete markets and, therefore, to provide a comprehensive framework to evaluate the redistributive effects and the efficacy of tax-financed stimulus programs.

Third, by analyzing the efficacy of tax-financed stimulus programs in a heterogeneous agent incomplete markets framework with nominal rigidities, our paper relates to a large literature that identifies key determinants of fiscal multipliers; see, among others, Oh and Reis (2012), Hagedorn et al. (2019), Bilbiie et al. (2024), Auclert et al. (2024), and Ferriere and Navarro (2025). Most closely related to our work is Ferriere and Navarro (2025), who show that fiscal multipliers in the U.S. are larger when government spending is financed by higher tax progressivity. Our paper instead points toward an ambivalent role of stimulus programs, depending on the joint design of systematic monetary policy and tax incidence: while progressively financed stimulus programs improve the short-run recovery from recessions, they may harm the long-run recovery by reducing innovative investment and thus technology growth. Key to this finding is the negative relation between personal and corporate income taxes and innovative activity of entrepreneurs. This mechanism is rooted in the seminal empirical work of Akcigit et al. (2022) and central to many studies analyzing the effects of top tax changes on entrepreneurial activity, innovation, and inequality; see, for instance, Jaimovich and Rebelo (2017), Aghion et al. (2019) or Jones (2022). Relative to these papers, we show that temporarily higher top income taxes have the potential to increase output in the short- and long-run.

Fourth, the HANK-G framework is consistent with a growing empirical literature that documents how discretionary stabilization polices have long-lasting effects on economic activity.[3] Relying on structural vector autoregressions and local projections, Ilzetzki (2024) and Antolin-Diaz and Surico (2025) show that expansions in government spending have positive and long-lasting effects on output, R&D expenditures, and total factor productivity. Cloyne et al. (2025) obtain similar effects for corporate tax cuts.[4] Our results instead point toward a state-dependent role for the short- and long-run effects of government spending shocks with respect to the joint design of systematic monetary policy and tax incidence.

Paper Outline. The paper proceeds as follows. Section 2 develops the analytical HANK-G framework. In a first step, we characterize a balanced growth path, determine long-run and cyclical income inequality, provide a four equation representation of equilibrium dynamics, and derive conditions for local determinacy. In a second step, we use the model to theoretically study the short- and long-run effects of fiscal stimulus programs. Section 3 builds a quantitative extension to the baseline model, calibrates it to U.S. economy, and evaluates the propagation of government spending shocks. Section 4 concludes. Analytical derivations, computational details, and additional results are reported in the Online Appendix.

[3] This strand is motivated and complemented by a vast literature arguing that recessions permanently depress the level of output, i.e., generate *hysteresis* or *scarring*; see, among others, Fatás (2000) and Blanchard et al. (2015). Fatás and Summers (2018) and Cerra et al. (2023) provide a detailed overview.

[4] Related, Moran and Queralto (2018) and Jordá et al. (2024) study the long-run effects of expansionary monetary policy shocks and arrive at a similar conclusion.

2. An Analytical Heterogeneous Agent New Keynesian Growth Economy

We begin with an analytical HANK-G framework to explain the core mechanisms through which fiscal stimulus programs propagate in both the short- and long-run. In this setup, growth emerges endogenously from innovative investment; nominal rigidities generate demand-driven fluctuations in economic activity; and household heterogeneity shapes aggregate consumption and entrepreneurial innovation. Our central policy focus – pure fiscal redistribution and tax incidence of stimulus programs – provides insurance against income risk while impacting simultaneously private consumption and innovation incentives.

2.1 Environment

Time is discrete and indexed by $t \in \{1, 2, \dots\}$. The economy is populated by infinitely lived heterogeneous households, final and intermediary good firms, a government, and a monetary authority.

2.1.1 *Households* At each period t, there is a unit mass of households, indexed by $i \in [0,1]$. Each household derives utility from consumption C_t^i and disutility from labor supply L_t^i, with preferences represented by the following time-separable utility function

$$\mathcal{U}(C_t^i, L_t^i) = \log(C_t^i) - \nu \frac{(L_t^i)^{1+\varphi}}{1+\varphi} ,$$

where $\nu > 0$ governs the disutility of labor and $1/\varphi$ is the Frisch labor supply elasticity. Households discount future periods at a constant rate $\beta \in (0,1)$. They supply differentiated labor services $L_t^i(l)$, where $l \in [0,1]$ indexes their labor type. While households share identical preferences, they differ in their access to financial markets. In particular, households face idiosyncratic shocks that move them between two states: saver (S), and hand-to-mouth (H). Savers hold a portfolio comprising liquid bonds, equity in intermediary-good firms, and illiquid capital from entrepreneurial innovative investment. Because savers own and invest in innovative private businesses, we refer to them as entrepreneurs. By contrast, hand-to-mouth households do not participate in asset markets and rely solely on liquid bonds to buffer income risk. Transitions between states follow a Markov process, with transition probabilities $\mathbb{P}(S_{t+1}|S_t) = s$ and $\mathbb{P}(H_{t+1}|H_t) = h$. In the stationary equilibrium, the mass of hand-to-mouth households is given by $\lambda = \frac{1-s}{2-h-s}$.

At the start of period t, each group – entrepreneurs (S) and hand-to-mouth (H) – pools its members' resources, providing full intra-group insurance against idiosyncratic shocks. Households then observe the aggregate shock and choose to consume and save. At the end of period t, they learn their asset market status for period $t+1$ and pool liquid bond holdings exclusively within their new peer group. Let B_{t+1}^S (respectively, B_{t+1}^H) denote the *aggregate* real bond holdings carried into period $t+1$ by entrepreneurs (respectively, hand-to-mouth households). By construction, B_{t+1}^S collects the liquid assets of two groups: entrepreneurs who stay in S in $t+1$, a mass $(1-\lambda)s$, and households who

move from H in t to S in $t+1$, a mass $\lambda(1-h)$:

$$B^S_{t+1} = (1-\lambda)s\, b^S_{t+1} + \lambda(1-h)\, b^H_{t+1}\,. \tag{1}$$

Similarly, B^H_{t+1} collects the liquid assets of households who remain in the hand-to-mouth state in $t+1$, a mass of λh, and those who move from S in t to H in $t+1$, a mass of $(1-\lambda)(1-s)$:

$$B^H_{t+1} = \lambda h b^H_{t+1} + (1-\lambda)(1-s) b^S_{t+1}\,. \tag{2}$$

Innovative capital is illiquid and cannot be transferred across household types. Entrepreneurs invest an amount $I^S_t(j)$ in each intermediary firm j – firms that are otherwise identical ex ante and ex post – and these investments upgrade the stock of innovative capital, i.e., the quality of intermediary goods, according to the law of motion

$$A^S_{t+1}(j) = A^S_t(j) + \psi I^S_t(j)\,, \quad \text{with} \quad A_0(j) > 0\,, \tag{3}$$

where $\psi \geq 0$ governs how efficiently innovative investment translates into improvements of quality. Since innovation is carried out only by entrepreneurs, aggregate quality and investment in firm j simply scale with their population share, so that $A_t(j) = (1-\lambda)A^S_t(j)$ and $I_t(j) = (1-\lambda)I^S_t(j)$.

Entrepreneurs: Entrepreneurs enter period t with per capita bond holdings of $B^s_t/(1-\lambda)$, carried over from the previous period. These bonds yield a real gross return of R_{t-1}/π_t, where R_{t-1} is the nominal interest rate and $\pi_t = P_t/P_{t-1}$ the gross inflation rate. In addition, entrepreneurs hold innovative capital $A^S_t = \int A^S_t(j)dj$ that earns a real gross return R^A_t, described below. Their capital income also includes equity holdings $\omega^S_t/(1-\lambda)$ in intermediary firms, which pay dividends D_t and can be resold at price q_t. Entrepreneurs pay a lump-sum tax T^S_t. They also supply labor L^S_t at the real wage W_t/P_t. These income sources finance consumption C^S_t, investment in innovative capital I^S_t, and financial asset accumulation in the form of bonds b^S_{t+1} and equity shares ω^S_{t+1}. Entrepreneurs choose an allocation $\mathcal{X}^S_t \equiv \{C^S_t, b^S_{t+1}, A^S_{t+1}, I^S_t, \omega^S_{t+1}\}_{t\geq 0}$ to maximize expected lifetime utility, solving the Bellman equation:

$$V^S(B^S_t, \omega^S_t, A^S_t) = \max_{\{\mathcal{X}^S_t\}} \mathcal{U}(C^S_t, L^S_t) + \beta \mathbb{E}_t \left[V^S(B^S_{t+1}, \omega^S_{t+1}, A^S_{t+1}) + \frac{\lambda}{1-\lambda} V^H(B^H_{t+1}) \right]$$

subject to

$$C^S_t + I^S_t + b^S_{t+1} + q_t \frac{\omega^S_{t+1}}{1-\lambda} = \frac{W_t}{P_t} L^S_t + R^A_t A^S_t + \frac{R_{t-1}}{\pi_t} \frac{B^S_t}{1-\lambda} + (q_t + D_t)\frac{\omega^S_t}{1-\lambda} - \frac{T^S_t}{1-\lambda}\,,$$

$$b^S_{t+1} \geq 0\,, \quad \text{and} \quad (1) - (3)\,,$$

where the value function accounts for the possibility that, in period $t+1$, an entrepreneur may become a hand-to-mouth household.[5]

[5]The value function $V^H(B^H_{t+1})$ is scaled by $\lambda/(1-\lambda)$, reflecting that the relevant state variable – aggregate bond holdings – is expressed in terms of total island-level resources. Accordingly, the relative transition probability $\lambda/(1-\lambda)$ maps aggregate state transitions into individual probabilities $(s, 1-s)$.

Hand-to-mouth: The problem faced by hand-to-mouth households mirrors that of entrepreneurs, except that they lack access to asset markets. Unable to accumulate innovative capital or equity, their choices are limited to consumption C_t^H and liquid bond savings b_{t+1}^H to maximize expected lifetime utility. In addition to labor income and bond returns, they receive government transfers $T_t^H \leq 0$. Their optimal plan, $\mathcal{X}_t^H \equiv \{C_t^H, b_{t+1}^H\}_{t\geq 0}$, satisfies the following Bellman equation:

$$V^H(B_t^H) = \max_{\{\mathcal{X}_t^H\}} \mathcal{U}(C_t^H, L_t^H) + \beta \mathbb{E}_t \left[V^H(B_{t+1}^H) + \frac{1-\lambda}{\lambda} V^S(B_{t+1}^S, \omega_{t+1}^S, A_{t+1}^S) \right]$$

subject to

$$C_t^H + b_{t+1}^H = \frac{W_t}{P_t} L_t^H + \frac{R_{t-1}}{\pi_t} \frac{B_t^H}{\lambda} - \frac{T_t^H}{\lambda} \,,$$

$$b_{t+1}^H \geq 0 \,, \quad \text{and} \quad (1) - (2) \,.$$

Finally, we solve for the financial market equilibrium by imposing Assumption 1, which specifies the nature of risk-sharing and the asset allocation across households.

ASSUMPTION 1. *(A1.a) there is perfect insurance among households in the same state, but not across states; (A1.b) stocks and physical capital are illiquid and cannot be transferred across states; (A1.c) liquid bond holdings are weakly positive, and the positivity constraint of hand-to-mouth agents binds in each period; (A1.d) no bonds are traded in equilibrium.*

$(A1.a)$-$(A1.d)$ jointly ensure a tractable characterization of the bond market equilibrium. Assumptions $(A1.a)$ and $(A1.b)$ imply that all entrepreneurs (respectively, hand-to-mouth households) make identical consumption and saving decisions. Assumption $(A1.c)$ enforces the strict hand-to-mouth constraint by ruling out capital market participation for H households, while Assumption $(A1.d)$ imposes a zero-liquidity limit (see, e.g., Krusell et al. (2011)), eliminating buffer-stock savings for both household types.

2.1.2 *Labor Market* The labor market exhibits two salient features. First, nominal wage rigidity implies that hours worked are demand-determined. This rigidity generates procyclical firm profits, a feature that proves central to generate procyclical investment in an endogenous growth model. Second, fluctuations in hours worked are heterogeneously distributed across households, implying that labor income inequality is cyclical.

Earnings Incidence: Throughout, variables denoted in "hat"-notation represent log-linear deviations from their balanced growth path values. Assumption 2 governs the earnings incidence across household types.

ASSUMPTION 2. *On the balanced growth path, hours worked are uniformly distributed across household types, such that $L^H = L^S = L$. Off the balanced growth path, however, hours may vary disproportionately across types: $\hat{L}_t^H = \mu \hat{L}_t$, where $\mu \in [0, \overline{\mu})$ and $\overline{\mu} > 1/\lambda$.*

The earnings incidence parameter μ controls the allocation of fluctuations in aggregate hours worked across household types.[6] Assumption 2 implies that hours worked by entrepreneurs follow $\hat{L}_t^S = \frac{1-\lambda\mu}{1-\lambda}\hat{L}_t$. Specifically, μ flexibly nests a range of incidence patterns. When $\mu = 1$, changes in aggregate hours worked are distributed uniformly across household types. When $\mu = 0$, hand-to-mouth households remain at steady-state labor supply, and the full adjustment falls on entrepreneurs. By contrast, when $\mu = \lambda^{-1}$, the entire adjustment falls on hand-to-mouth households, echoing a worker-capitalist structure. For intermediate values, the distributional pattern varies: when $\mu \in (0,1)$, hand-to-mouth households adjust hours underproportionally, while entrepreneurs adjust overproportionally; when $\mu \in (1, \lambda^{-1})$ the opposite holds — hand-to-mouth households bear the bulk of adjustment, while entrepreneurs adjust only mildly. Finally, for $\mu \in (\lambda^{-1}, \overline{\mu})$, hours worked by hand-to-mouth households rise overproportionally, while entrepreneurs reduce hours worked in response to higher aggregate labor demand.[7]

Nominal Wage Rigidity: Aggregate labor input L_t is composed of a continuum of differentiated labor varieties indexed by $l \in [0,1]$, which are bundled through a CES aggregator with elasticity of substitution $\epsilon_w > 1$. Each labor variety is supplied by a union l that sets the nominal wage $W_t(l)$ subject to quadratic adjustment costs à la Rotemberg (1982). The union maximizes the utility of a quasi-representative household, which is modeled as a composite of hand-to-mouth and entrepreneur types. The effective labor input for each variety is $L_t(l) = \lambda L_t^H(l) + (1-\lambda)L_t^S(l)$. Demand for labor variety l is given by $L_t(l) = (W_t(l)/W_t)^{-\epsilon_w} L_t^d$, where W_t is the aggregate wage index consistent with cost minimization, and L_t^d is total labor demand by firms. Each union l solves

$$\max_{\{W_t(l)\}} \sum_{t=0}^{\infty} \beta^t \left[\log C_t(l) - \nu \frac{(L_t(l))^{1+\varphi}}{1+\varphi} \right]$$

subject to

$$C_t(l) + I_t(l) = \frac{W_t(l)}{P_t} L_t(l) + R_t^A A_t(l) + D_t(l) - T_t(l) - \frac{\theta}{2}\left(\frac{1}{g^A} \frac{W_t(l)}{W_{t-1}(l)} - 1 \right)^2 A_t \,,$$

where adjustment costs scale with a factor $\theta > 0$ to aggregate technology A_t and wage inflation is normalized to steady state technology growth, $g^A > 1$, ensuring that adjustment costs are absent on the balanced growth path. In equilibrium, unions set identical wages, such that $W_t(l) = W_t$ and $L_t(l) = L_t$ for all l.

[6] Assumption 2 is a two-type counterpart to the general incidence function $\gamma(i, L_t)$ used by Werning (2015), Auclert and Rognlie (2020), Alves et al. (2020), and Patterson (2023). Individual labor income of the form $(W_t/P_t) \cdot L_t \cdot \gamma(i, L_t)$ maps in our setup to $\mu = 1 + \epsilon_{\gamma,L}^H$, where $\epsilon_{\gamma,L}^H$ is the elasticity of $\gamma(i, L_t)$ with respect to aggregate labor L_t in state H. Consistency requires $\epsilon_{\gamma,L}^S = -[\lambda/(1-\lambda)]\epsilon_{\gamma,L}^H$. Moreover, $\overline{\mu}$ ensures that monetary tightening reduces aggregate demand (Bilbiie, 2008, Bilbiie and Straub, 2013).

[7] Empirically, μ reflects heterogeneity in labor market exposure along both the extensive and intensive margins. Exposure at the extensive margin, such as cyclical unemployment risk, has been documented in Storesletten et al. (2004), Guvenen et al. (2014), and Kramer (2022). Exposure at the intensive margin, measured through *worker betas*, are explored in Guvenen et al. (2017). Both strands document that low-income workers disproportionately bear hours adjustments to aggregate shocks, suggesting $\mu > 1$; for monetary policy shocks see, among others, Coglianese et al. (2025), Heathcote et al. (2010), Coibion et al. (2017), Ampudia et al. (2018), or Holm et al. (2021).

2.1.3 *Production* The production sector consists of a representative final good firm and a continuum of intermediary goods producers indexed by $j \in [0,1]$.

Final Good: The final good Y_t^G is produced under perfect competition by aggregating differentiated intermediate inputs $X_t(j)$, each of quality $A_t(j)$, according to

$$Y_t^G = L_t^{1-\alpha} \int_j A_t(j)^{1-\alpha} X_t(j)^{\alpha} dj \,,$$

where $\alpha \in (0,1)$ denotes the share of intermediate inputs in production. The final good is sold at the competitive price P_t, while intermediary goods are purchased at variety-specific prices $P_t(j)$. The final good firm chooses demand for labor L_t^d and intermediate inputs $X_t^d(j)$ to maximize profits, taking factor prices as given.

Intermediary Goods: Intermediary goods are produced by identical firms, operating as a monopolist. In each period t, firm j sets a price $P_t(j)$ to maximize nominal profits $\Theta_t^n(j)$, taking as given the demand for its variety. Specifically, $X_t(j)$ is produced by transforming units of the final good at a constant marginal cost P_t. Firm j's objective is

$$\max_{\{P_t(j)\}} \Theta_t^n(j) \equiv (P_t(j) - P_t) X_t^d(j) \quad \text{subject to} \quad \frac{P_t(j)}{P_t} = \alpha (L_t^d)^{1-\alpha} A_t(j)^{1-\alpha} (X_t^d(j))^{\alpha-1} \,.$$

The firm's optimal pricing decision yields $\alpha P_t(j) = P_t$, where $1/\alpha > 1$ is the gross markup over marginal costs. Substituting this pricing rule into the demand function yields $X_t(j) = \alpha^{\frac{2}{1-\alpha}} L_t A_t(j)$. Aggregating across varieties, gross output is given by $Y_t^G = \alpha^{\frac{2\alpha}{1-\alpha}} L_t A_t$, with $A_t \equiv \int_0^1 A_t(j) dj$ representing the average quality index. Real profits of firm j are $\Theta_t(j) = \Theta_\alpha L_t A_t(j)$, where $\Theta_\alpha = \alpha^{-1}(1-\alpha)\alpha^{\frac{2}{1-\alpha}}$. Finally, returns to innovation are defined as the ratio of profits to quality, $R_t^A(j) \equiv \Theta_t(j)/A_t(j) = \Theta_\alpha L_t = R_t^A$. As a result, returns to innovation are identical across firms and linear in aggregate labor.

2.1.4 *Government* There is no government spending in our benchmark economy, so fiscal policy serves one purpose: redistributing resources across households. The government's budget is balanced when

$$T_t^H = -T_t^S = -T_t \,, \quad \text{with} \quad T_t = T A_t e^{\hat{\tau}_t} \quad \text{and} \quad T > 0 \,,$$

where $\hat{\tau}_t$ is an exogenous, non-distortive, fiscal shock that reallocates net transfers between households. The fiscal shock follows an AR(1) process, $\hat{\tau}_t = \rho \hat{\tau}_{t-1} + \epsilon_t$, with persistence $\rho \in [0,1)$ and i.i.d. innovations ϵ_t. A positive realization ($\hat{\tau}_t > 0$) represents progressive redistribution – from entrepreneurs to hand-to-mouth households – and can be interpreted broadly as fiscal interventions aligned with social safety-net objectives. A negative realization, in contrast, ($\hat{\tau}_t < 0$) represents regressive redistribution–from hand-to-mouth households to entrepreneurs.

2.1.5 *Monetary Authority* We assume that the monetary authority sets the nominal interest rate following a pro-cyclical real interest rate rule of the form

$$R_t = R \mathbb{E}_t \left[\frac{\pi_{t+1}}{\pi} \right] \left(\frac{Y_t / A_t}{Y / A} \right)^{\phi_y} \,, \quad \text{with} \quad \phi_y > 0 \,,$$

and Y_t/A_t denoting stationary net output, as defined below. The parameter $\phi_y > 0$ captures the degree of policy responsiveness: higher values represent a more "hawkish" policy stance, with stronger real interest rate responses to output fluctuations. In the limit, $\phi_y \to \infty$ corresponds to perfect output stabilization.[8]

2.1.6 *Market Clearing* Aggregate labor and consumption are given by $L_t = \lambda L_t^H + (1-\lambda)L_t^S$ and $C_t = \lambda C_t^H + (1-\lambda)C_t^S$, respectively. Aggregate investment is defined as $I_t \equiv (1-\lambda)\int_j I_t^S(j)dj$, and total intermediary output is $X_t \equiv \int_j X_t(j)dj$. Real dividends, which equal net profits in the final goods sector, are zero in all periods, i.e., $D_t = Y_t^G - (W_t/P_t)L_t - (1/\alpha)X_t = 0$. Equity market clearing implies that the entire ownership of firms is held by entrepreneurs, i.e., $\omega_t^S = \omega_{t+1}^S = 1$. Aggregate quality growth is governed by the law of motion $g_{t+1}^A \equiv A_{t+1}/A_t = 1 + \psi \cdot I_t/A_t$. Using the household budget constraints and the equilibrium condition from the labor union's problem, the aggregate resource constraint can be written as

$$Y_t = C_t + I_t + \frac{\theta}{2}\left(\frac{\pi_t^w}{g^A} - 1\right)^2 A_t \,,$$

where net output is defined as $Y_t \equiv Y_t^G - X_t = Y_\alpha L_t A_t$, with $Y_\alpha = (1-\alpha^2)\alpha^{\frac{2\alpha}{1-\alpha}}$.

2.2 Balanced Growth Path

A stationary balanced growth path (BGP) consists of a sequence of trending variables and prices $\{C_t, C_t^S, C_t^H, W_t, D_t, T_t^H, T_t^S, Y_t, Y_t^G, X_t, I_t\}$ that are normalized by the level of endogenous technology A_t. Subsequently, we denote stationary variables in small letters and impose without loss of generality $\pi = 1$. We focus on a BGP with positive household consumption and technology growth.

RESULT 1. *The economy admits a unique BGP with strictly positive individual consumption* $(c^H, c^S) \in \mathbb{R}_+^2$ *and technology growth* $g^A > 1$ *if* $L > L^*$ *and* $T < T^*$*, where*

$$L^* \equiv \frac{\beta^{-1} - 1}{\psi \Theta_\alpha} \,, \qquad T^* \equiv \frac{(1-\lambda)\Theta_\alpha}{\alpha}\left(1 + \alpha\frac{1-\beta}{1-\lambda}\right)L + \frac{1-\beta}{\psi} \,.$$

Result 1 shows that the BGP features strictly positive technology growth whenever hours worked are high enough — formally, when $\nu < \nu^*$. In this case, intermediate firms operate in a sufficiently large market, boosting profits and stimulating innovative investment. The tax bound ensures strictly positive entrepreneur consumption. To solve for the *approximate* equilibrium dynamics, we log-linearize the economy around its BGP.

[8] The real interest rate rule is used, among others, in Fornaro and Wolf (2023), Angeletos et al. (2024, 2026), Auclert et al. (2025) or Collard et al. (2025); additionally, Woodford (2011) and Debortoli and Galí (2024a,b) study the limiting case $\phi_y \to 0$. This particular rule allows us to cleanly isolate how redistributive fiscal policies shape aggregate outcomes in the short- and long-run. We will relax this restriction and discuss the role of more standard feedback rules in Section 3, the economic essence is identical.

2.3 Inequality On and Off the Balanced Growth Path

The economy may feature inequality across households on and off the BGP.

2.3.1 *BGP Inequality* Consumption and income inequalities are measured by the respective ratio of S over H consumption and income, i.e., $\Gamma_c \equiv c^S/c^H$ and $\Gamma_y \equiv y^S/y^H$. When the aggregate consumption share $\gamma_c \equiv c/y$ is below one, income inequality exceeds consumption inequality, since $\Gamma_y = \Gamma_c/\gamma_c + \lambda(1-\gamma_c)/[(1-\lambda)\gamma_c]$.[9] For clarity, the main text focuses on the case $\Gamma_c = 1$, which can be implemented through a transfer T^E from entrepreneurs to hand-to-mouth households, where $0 < T^E < T^*$.

2.3.2 *Cyclical Income Inequality* To derive a measure of cyclical income inequality, we link hand-to-mouth consumption to aggregate income and fiscal redistribution:

$$\hat{c}_t^H = \hat{y}_t^H = \eta\mu\hat{y}_t + (1-\eta)\hat{\tau}_t\,, \quad \text{with} \quad \eta \equiv \frac{\Theta_\alpha L}{\Theta_\alpha L + (\alpha/\lambda)T^E} \in (0,1)\,.$$

Hand-to-mouth consumption fluctuates out of two reasons. First, aggregate income fluctuates over the cycle, affecting individual consumption through heterogeneous earnings exposure. Second, fiscal redistribution shifts resources directly across households. The weight between both channels depends on the steady state earnings-to-income ratio η, which falls with higher steady-state transfers T^E. A larger η amplifies the sensitivity of hand-to-mouth consumption to aggregate income while it simultaneously dampens the sensitivity to fiscal transfers. Income inequality between entrepreneurs and hand-to-mouth households is given by:

$$\hat{y}_t^S - \hat{y}_t^H = \frac{1}{1-\lambda}\frac{y}{y^S}\left((1-\chi)\hat{y}_t - (1-\eta)\hat{\tau}_t\right), \quad \text{with} \quad \chi \equiv \eta\mu\,.$$

Cyclical fluctuations in income inequality stem from two channels: endogenous redistribution via the labor market (χ) and exogenous redistribution via fiscal transfers ($\hat{\tau}_t$). Three cases emerge: (i) if $\chi = 1$, endogenous redistribution is acyclical, (ii) if $\chi > 1$ it is countercyclical, and (iii) if $\chi < 1$ it is procyclical. Progressive fiscal shocks, in contrast, always reduce income inequality. As such, income inequality will be cyclical whenever both channels do not offset each other.

2.4 A Four-Equation HANK-G Representation

We now derive a log-linear four-equation representation of the HANK-G economy. It builds upon (i) an aggregate demand (IS) equation, (ii) an endogenous growth (EG) equation, (iii) a sticky wage Phillips curve, and (iv) a monetary policy rule.

2.4.1 *Aggregate IS Equation*

[9] This observation holds generically in a large class of heterogeneous agent models featuring non-homothetic saving behavior; see, for instance, Straub (2019) or Gaillard et al. (2023).

PROPOSITION 1. *The aggregate IS equation is given by*

$$\hat{y}_t = \underbrace{\zeta_f \mathbb{E}_t\left[\hat{y}_{t+1}\right]}_{\text{① transitory income}} - \underbrace{\zeta_r\left(\hat{i}_t - \mathbb{E}_t\left[\hat{\pi}_{t+1}\right]\right)}_{\text{② real interest rate}} + \underbrace{\zeta_g \hat{g}^A_{t+1} - \zeta_{g'}\mathbb{E}_t\left[\hat{g}^A_{t+2}\right]}_{\text{③ permanent income}} + \underbrace{\zeta_\tau \hat{\tau}_t - \zeta_{\tau'}\mathbb{E}_t\left[\hat{\tau}_{t+1}\right]}_{\text{④ fiscal redistribution}}, \quad \text{(E.1)}$$

where the auxiliary parameters are defined as

$$\begin{aligned}
\zeta_f &= 1 + (1-s)\frac{\gamma_c\chi - 1}{1-\lambda\gamma_c\chi}, \quad \zeta_r = \frac{(1-\lambda)\gamma_c}{1-\lambda\gamma_c\chi}, \\
\zeta_g &= \frac{(1-\lambda)c^S/y + g^A/(\psi y)}{1-\lambda\gamma_c\chi}, \quad \zeta_{g'} = s\frac{g^A}{\psi y}\frac{1}{1-\lambda\gamma_c\chi}, \\
\zeta_\tau &= (1-\eta)\frac{\lambda\gamma_c}{1-\lambda\gamma_c\chi}, \quad \zeta_{\tau'} = \zeta_\tau\left(1 - \frac{1-s}{\lambda}\right).
\end{aligned}$$

The *transitory income channel* ① captures the sensitivity of current demand with respect to future income. Relative to a representative agent economy, the presence of income risk ($s < 1$) implies that the IS equation features compounding $\zeta_f > 1$ (respectively, discounting $\zeta_f < 1$) if endogenous income redistribution is sufficiently countercyclical, i.e., $\chi > \gamma_c^{-1}$ (respectively, $\chi < \gamma_c^{-1}$). Intuitively, when $\chi > \gamma_c^{-1}$ entrepreneurs take into account that their consumption would become relatively more exposed to aggregate income fluctuations if they were to transition to the hand-to-mouth state. This reduces their desire for precautionary savings and expands aggregate demand. The *real interest rate channel* ② reflects the elasticity of today's demand with respect to real interest rate changes. A higher degree of countercyclical endogenous income redistribution (χ) raises the real interest rate elasticity ζ_r. This is the case as redistribution toward hand-to-mouth households mechanically increases the economy-wide average MPC, thereby amplifying the general equilibrium forces of real interest rate movements on aggregate demand.[10] In general, innovative investment ($\gamma_c < 1$) weakens both of these propagation channels as it makes aggregate income fluctuate less with aggregate consumption.

The remaining components focus on the effects of endogenous growth and fiscal redistribution. The *permanent income channel* ③ reflects how current demand responds to changes in both contemporaneous and expected future permanent income. Two opposing forces are at play. On the one hand, a standard permanent income effect raises consumption of savers. On the other hand, innovative investment reduces their current and future consumption, implying a positive feedback from $\hat{g}^A_{t+1}$ but a negative one from $\mathbb{E}_t\left[\hat{g}^A_{t+2}\right]$. The net effect of these opposing forces is positive such that higher permanent income stimulates current aggregate demand, i.e., $\zeta_g - \rho\zeta_{g'} > 0$ due to $\rho < 1$ and $s \leq 1$.[11] Finally, the *fiscal redistribution channel* ④ describes the direct effect of exogenous redistribution on current demand. Progressive fiscal redistribution toward hand-to-mouth

[10] There exist threshold values $(\underline{\mu}, \overline{\mu})$ with $1 < \underline{\mu} < \overline{\mu}$ such that: (a) the elasticity of aggregate demand w.r.t. real interest rates ζ_r is strictly positive iff $\mu < \overline{\mu}$, and (b) the IS equation features compounding iff $\mu > \underline{\mu}$, and discounting iff $\mu < \underline{\mu}$. The threshold values are given by $\underline{\mu} \equiv 1/(\eta\gamma_c)$ and $\overline{\mu} \equiv 1/(\eta\lambda\gamma_c)$.

[11] This channel is more potent for high g^A-economies as they are more sensitive to fluctuations in technology growth when the permanent component of their total income is larger than the transitory part.

households reduces contemporaneous and future consumption of savers, yet naturally increases the consumption of hand-to-mouth households. Overall, the net effect is positive and this channel boosts aggregate demand, i.e., $\zeta_\tau - \rho\zeta_{\tau'} > 0$.

2.4.2 *Endogenous Growth (EG) Equation*

PROPOSITION 2. *The endogenous growth equation is*

$$\hat{g}^A_{t+1} = \underbrace{\xi_y \hat{y}_t + \xi_{y'} \mathbb{E}_t\left[\hat{y}_{t+1}\right]}_{\text{⑤ transitory income}} + \underbrace{\xi_{g'} \mathbb{E}_t\left[\hat{g}^A_{t+2}\right]}_{\text{⑥ permanent income}} - \underbrace{\xi_\tau\left(\hat{\tau}_t - \mathbb{E}_t\left[\hat{\tau}_{t+1}\right]\right)}_{\text{⑦ fiscal redistribution}}, \tag{E.2}$$

where we used $\overline{\xi} = 1/\left[g^A(1-\lambda\gamma_c) + \gamma_c(\lambda - 1)\right] > 0$ *to define the feedback coefficients:*

$$\begin{aligned}
\xi_y &= \overline{\xi} \cdot (g^A - 1)(1 - \lambda\gamma_c\chi)\,, \\
\xi_{y'} &= \overline{\xi} \cdot (g^A - 1)\left[(1-\lambda)\gamma_c(g^A - \beta)/g^A - (1 - \lambda\gamma_c\chi)\right]\,, \\
\xi_{g'} &= \overline{\xi} \cdot (1 - \gamma_c) g^A\,, \\
\xi_\tau &= \overline{\xi} \cdot (1 - \eta)\lambda\gamma_c(g^A - 1)\,.
\end{aligned}$$

Innovative investment – and with it, technology growth – fluctuates because of two forces: a cost-of-funds effect and a market-size effect.

The *cost-of-funds effect* stems from the stochastic discount factor of entrepreneurs and captures the idea that current technology growth moves inversely with their expected consumption growth. When their consumption decreases, the cost of investment—in terms of forgone consumption—rises. In this sense, redistribution away from savers raises their opportunity cost of investing: rather than financing new innovations, they prefer to smooth consumption.

The *market-size effect*, in contrast, affects technology growth through aggregate demand. When future demand is expected to rise, firms anticipate higher sales and greater discounted profits, which in turn stimulate innovative investment. Redistribution toward hand-to-mouth households strengthens this channel because their high marginal propensity to consume translates transfers directly into current demand.

Overall, the cost-of-funds and market-size effects together imply a positive feedback from the *transitory income channel* ⑤ onto technology growth, i.e., $\xi_y + \rho\xi_{y'} > 0$. This arises because the elasticity of aggregate demand with respect to the real interest rates is assumed to be strictly positive, i.e., $\chi < 1/(\lambda\gamma_c)$. The *permanent income channel* ⑥ increases technology growth by lowering expected future consumption of savers and, thus, strengthening consumption smoothing via greater innovative investment. Finally, the *fiscal redistributive channel* ⑦ acts purely on the cost of fund effect as progressive exogenous redistribution reduces savers' consumption and, therefore, technology growth.

2.4.3 *Phillips Curve and Taylor Rule*

In this economy, price and wage inflation are linked through $\hat{\pi}_t = \hat{\pi}^w_t - g^A_t$. Using the unions' optimal wage setting decision, Proposition 3 states the New Keynesian Phillips curve in terms of price inflation.

PROPOSITION 3. *The forward-looking price Phillips curve is given by*

$$\hat{\pi}_t = \beta \mathbb{E}_t \left[\hat{\pi}_{t+1}\right] + \kappa_y \hat{y}_t + \kappa_g \hat{g}^A_{t+1} - \hat{g}^A_t \,, \tag{E.3}$$

where $\kappa_y = \kappa \frac{1+\varphi\gamma_c}{\gamma_c}$, $\kappa_g = \beta - \kappa \frac{1-\gamma_c}{\gamma_c} \frac{g^A}{g^A-1}$, *and* $\kappa = \frac{\epsilon_w - 1}{\theta}(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} L > 0$.

An expansion in aggregate demand increases labor demand, prompting unions to set higher wages, which translates into higher marginal costs and inflation, i.e., $\kappa_y > 0$. As $\gamma_c < 1$, this channel becomes stronger than in a model without endogenous growth, since part of the increase in output is spent on innovative investment. Second, higher technology growth has an ambiguous effect on inflation, i.e., $\kappa_g \gtreqless 0$. On the one hand, technology growth strengthens the forward-looking inflation dynamics through $\beta > 0$. On the other hand, it expands the production frontier and reduces marginal costs and inflation, as captured by $\kappa(1-\gamma_c)g^A/[\gamma_c(g^A-1)] > 0$. If the latter channel dominates, the slope of the Phillips curve flattens or may even reverses, implying that price inflation responds little to aggregate shocks. Finally, the nominal interest rate follows

$$\hat{i}_t = \mathbb{E}_t \left[\hat{\pi}_{t+1}\right] + \phi_y \hat{y}_t \,. \tag{E.4}$$

2.4.4 *Discussion: HANK-G as Unified Framework* The four-equation representation unifies a number of existing frameworks. The model corresponds to (i) a two-agent (TA) Keynesian growth economy with permanent heterogeneity if $s = 1$, and to a representative agent (RA) Keynesian growth economy (Benigno and Fornaro, 2018) if additionally $\lambda = 0, \chi = 1$; (ii) it collapses to an alternative tractable HANK model (Bilbiie, 2025) if $\psi = 0$ and $g^A = 1$ in all periods, to a TANK model (Bilbiie, 2008) if additionally $s = 1$; and (iii) to a baseline RANK model (Galí, 2015) if $\psi = \lambda = 0, s = \chi = 1$. Absent nominal rigidities, one recovers real model variants: (iv) a flexible price HA growth economy (Cozzi, 2018) if $\theta = 0$; (v) a RA growth economy (Aghion and Howitt, 1992) if $\theta = \lambda = 0, s = \chi = 1$; (vi) a tractable Aiyagari (1994) model if $\theta = \psi = 0$; and (vii) a standard RBC if $\theta = \psi = \lambda = 0, s = \chi = 1$. HANK-G is thus well-suited to study short- and long-run effects of fiscal policies while preserving comparability to existing models.[12]

2.5 Determinacy

Before we analyze the transmission of fiscal policies, Proposition 4 states conditions under which the model features a locally determinate equilibrium.

[12] The HANK-G framework can be easily extended by various features that the recent literature has identified to crucially shape the transmission of (demand) shocks and policy effectiveness; among others, cyclical income risk (Bilbiie, 2025), wealth in the utility (Michaillat and Saez, 2021), behavioral frictions and cognitive biases (Angeletos and Lian, 2018, Gabaix, 2020), life-cycle dynamics (Del Negro et al., 2023), default on household debt (Beaudry and Portier, 2018), or a cost channel of monetary policy (Beaudry et al., 2024).

PROPOSITION 4. *Suppose that* $\zeta_f > \underline{\zeta}_f$, *with* $\underline{\zeta}_f < 1$.[13] *The model has a locally determinate solution if and only if*

$$\phi_y > \max\left\{\underline{\phi}_1, \underline{\phi}_2\right\} > 0\,,$$

where the auxiliary parameters are given by:

$$\underline{\phi}_1 = \frac{\zeta_f - 1}{\zeta_r} + \left[1 + (1-s)\frac{\xi_{g'}}{1-\xi_{g'}}\right]\frac{g^A - \beta}{\beta}\,,$$

$$\underline{\phi}_2 = \xi_{g'}\left(\frac{\zeta_f - s}{\zeta_r} + s\frac{g^A - \beta}{\beta}\right)\,.$$

To understand the HANK-G "Taylor-principle", we begin by shutting down the heterogeneous agent and endogenous growth elements in order to recover a standard RANK model. Under our pro-cyclical real interest rate rule, the RANK model has a locally determinate solution if $\phi_y > 0$.[14] In a tractable HANK model the real interest rate rule admits a locally determinate solution if $\phi_y > (\tilde{\zeta}_f - 1)/\tilde{\zeta}_r$, where $\tilde{\zeta}_f = \zeta_f|_{\gamma_c=1}$ and $\tilde{\zeta}_r = \zeta_r|_{\gamma_c=1}$. Hence, $\phi_y > 0$ guarantees determinacy only under a discounted Euler equation ($\tilde{\zeta}_f < 1$), that is, when income inequality is procyclical ($\chi < 1$). Conversely, countercyclical income inequality ($\chi > 1$) leads to a compounded Euler equation and, thus, requires a more aggressive monetary policy reaction.

Given that $\underline{\phi}_1 > \underline{\phi}_2$ under realistic calibrations, we subsequently only discuss the composition of $\underline{\phi}_1$. Relative to HANK, the interaction with endogenous growth acts on four margins. First, compounding requires a higher degree of countercyclical income inequality as $\zeta_f < \tilde{\zeta}_f$ calling for a less aggressive monetary policy; second, $\gamma_c < 1$ decreases the real interest rate elasticity $\zeta_r < \tilde{\zeta}_r$, requiring a more (respectively, less) aggressive monetary policy if $\zeta_f > 1$ (respectively, $\zeta_f < 1$); third, a pure market size effect calls for a more aggressive monetary policy as $(g^A - \beta)/\beta > 0$; fourth, there arises a complementarity between income risk ($s < 1$) and the market size effect $(g^A - \beta)/\beta > 0$ that requires a more aggressive monetary policy. Overall, a discounted Euler equation is not sufficient to induce determinacy under $\phi_y > 0$ if the complementarity between the market size effect and income risk is sufficiently strong.

2.6 On the Ambiguous Effects of Fiscal Redistribution Policies

We now present the main theoretical results of this paper. In particular, we use the HANK-G framework to study how fiscal redistribution propagates in both the short- and the long-run. We consider two main policy experiments. First, we analyze a transitory increase in transfers from entrepreneurs to hand-to-mouth households. To anchor our results, we compare its effects across three benchmark economies: a real heterogeneous agent economy without endogenous growth (HA), a heterogeneous agent New

[13] This assumption is innocuous and made for expositional purposes only. We provide a complete statement of the Proposition in Appendix B.4.

[14] This condition mirrors the Taylor principle, $\phi_\pi > 1$, for a standard inflation feedback rule.

Keynesian economy (HANK), and a real heterogeneous agent economy with endogenous growth (HA-G). We then show how the HANK-G framework integrates these opposing forces, resulting in potentially ambiguous short- and long-run effects on output. Second, we demonstrate how our findings generalize to a government spending shock that is financed with a heterogeneous tax incidence across households.

Throughout this section, the output response is written as $\hat{y}_t = \mathcal{M}_y \hat{\tau}_t$ and the technology response as $\hat{g}^A_{t+1} = \mathcal{M}_g \hat{\tau}_t$, where $\mathcal{M}_y$ and $\mathcal{M}_g$ denote the impact multipliers solving the system of equations (E.1)-(E.4).

2.6.1 *Three Benchmark Heterogeneous Agent Economies* Proposition 5 states the aggregate effects of exogenous progressive redistribution.

PROPOSITION 5. *A transitory exogenous redistribution from entrepreneurs to hand-to-mouth agents, i.e., $\hat{\tau}_t > 0$, affects aggregate outcomes as follows.*

(a) HA: $\theta = \psi = 0$. *In an exogenous growth model with flexible wages, progressive redistribution has no effect on output, i.e., $\mathcal{M}_y = 0$.*

(b) HANK: $\psi = 0$. *In an exogenous growth model with sticky wages, progressive redistribution is expansionary, i.e., $\mathcal{M}_y > 0$.*

(c) HA-GS: $\theta = 0$. *In an endogenous growth model with flexible wages, progressive redistribution is contractionary in short and long-run, i.e., sgn$(\mathcal{M}_y)$ = sgn$(\mathcal{M}_g) < 0$.*

In a heterogeneous agent exogenous growth economy with flexible wages, fiscal redistribution has no impact on output, as the latter is purely supply determined. Specifically, progressive redistribution lowers the labor supply of hand-to-mouth households, whereas it increases the labor supply of entrepreneurs by the same amount. As both effects exactly offset each other and labor is the sole input factor, statement (a) follows.

Nominal rigidities in the form of sticky wages render progressive redistribution expansionary. Intuitively, progressive redistribution from entrepreneurs to hand-to-mouth households raises aggregate demand and, hence, short-run output. This is the case, as progressive redistribution increases the income share of high-MPC households, leading to a powerful Keynesian cross as indirect demand effects gain in importance.

In heterogeneous agent endogenous growth economy with flexible wages, progressive redistribution contracts short-run output and long-run technology growth. In this model class, progressive redistribution reduces technology growth and, thus, the long-run production frontier through a negative cost of funds effect. Intuitively, entrepreneurs smooth their consumption by reducing innovative investment, which triggers a fall in permanent income that additionally reduces current aggregate demand.

2.6.2 *Fiscal Redistribution According to HANK-G*

PROPOSITION 6. *A transitory redistribution from entrepreneurs to hand-to-mouth agents, i.e., $\hat{\tau}_t > 0$, affects aggregate outcomes as follows:*

$$\mathcal{M}_y = \frac{\zeta_\tau - \rho\zeta_{\tau'} - (\zeta_g - \rho\zeta_{g'})\Lambda_\tau}{1 + \phi_y\zeta_r - \rho\zeta_f - (\zeta_g - \rho\zeta_{g'})\Lambda_y}, \quad \mathcal{M}_g = \frac{(\zeta_\tau - \rho\zeta_{\tau'})\Lambda_y - (1 + \phi_y\zeta_r - \rho\zeta_f)\Lambda_\tau}{1 + \phi_y\zeta_r - \rho\zeta_f - (\zeta_g - \rho\zeta_{g'})\Lambda_y},$$

where we have defined $\Lambda_y = (\xi_y + \rho\xi_{y'})/(1-\rho\xi_{g'}) > 0$ *and* $\Lambda_\tau = (1-\rho)\xi_\tau/(1-\rho\xi_{g'}) > 0$.

(a) *If* $\rho = 0$ *or* $s = 1$, *progressive redistribution has no effect on output, i.e.,* $\mathcal{M}_y = 0$, *but reduces technological growth, i.e.,* $\mathcal{M}_g < 0$.

(b) *If* $\rho > 0$ *and* $s < 1$, *progressive redistribution increases output, i.e.,* $\mathcal{M}_y > 0$, *but has an ambiguous effect on technological growth, i.e.,* $\mathcal{M}_g \gtreqless 0$ *if*

$$\rho\frac{1-s}{\lambda}\left[\frac{1-\rho}{\gamma_c} + \rho\mathscr{M}\right] - (1-\rho)(\phi_y - \rho\mathscr{M}) \gtreqless 0\,, \tag{4}$$

where we have defined $\mathscr{M} = (g^A - \beta)/g^A$.

Proposition 6 characterizes the impact multipliers of output and technology growth to a progressive redistribution from entrepreneurs to hand-to-mouth households. There are two channels at play: a consumption channel that stimulates output $\zeta_\tau - \rho\zeta_{\tau'}$, and technology growth $(\zeta_\tau - \rho\zeta_{\tau'})\Lambda_y$; and an investment channel that contracts output $(\zeta_g - \rho\zeta_{g'})\Lambda_\tau$ and technology growth $(1 + \phi_y\zeta_r - \rho\zeta_f)\Lambda_\tau$. Before we explain the intuition behind Proposition 6, let us define the liquidity premium as the difference in returns between illiquid innovative investment and liquid bond holdings, i.e., $\Delta^{LP}_{t+1} \equiv \hat{R}^A_{t+1} - \hat{r}_t = (1-s)\mathbb{E}_t\left[\hat{c}^S_{t+1} - \hat{c}^H_{t+1}\right]$. Based on the cyclicality of Δ^{LP}_{t+1}, two cases emerge.

To begin with, we consider case (a) in which redistribution is either transitory $(\rho = 0)$ *or* there is no income risk as household types are permanent $(s = 1)$, as in a standard TANK model. In this case, progressive redistribution has no effect on output $(\mathcal{M}_y = 0)$, but contracts technology growth $(\mathcal{M}_g < 0)$. The underlying intuition is as follows. Progressive redistribution raises consumption of hand-to-mouth households one to one and, thus, stimulates aggregate consumption. However, progressive redistribution decreases simultaneously consumption and innovative investment of entrepreneurs. As the liquidity premium is acyclical if $\rho = 0$ or $s = 1$, redistribution away from savers reduces their consumption and investment one to one with their income loss. Because aggregate output is the sum of private consumption and investment, it must necessarily remain unchanged.

Consider then case (b) in which redistribution is persistent $(\rho > 0)$ *and* households face income risk $(s < 1)$. In this scenario, progressive redistribution reduces, ceteris paribus, consumption inequality and, hence, weakens precautionary saving motives of entrepreneurs. As a result, illiquid innovative investment becomes relatively more attractive to entrepreneurs who increase their holdings relative to an environment without precautionary savings. The higher investment generates, in turn, a fall in the liquidity premium. As a result, innovative investment of entrepreneurs falls less than one to one with their income loss such that the countercyclical liquidity premium stabilizes investment.[15] This leads to an overall increase in output.

Whether innovative investment increases $(\mathcal{M}_g > 0)$ or falls $(\mathcal{M}_g < 0)$ is captured by the condition in equation (4) and depends on two key policy parameters: (i) the

[15] Luetticke (2021) or Kekre and Lenel (2022) document empirically a counter-cyclical liquidity premium in response to demand shocks.

persistence of fiscal redistribution (ρ) and (ii) the aggressiveness of monetary policy (ϕ_y).[16] The condition in equation (4) can be rewritten in terms of two policy cutoffs, $\rho \gtreqless \rho^*(\phi_y) \in (0,1)$ and $\phi_y \lesseqgtr \phi_y^*(\rho)$, that we spell out in closed form in Appendix B.5.2. Innovative investment raises (respectively, falls) if progressive redistribution is sufficiently persistent (respectively, sufficiently transitory) and monetary policy is sufficiently accommodative (respectively, sufficiently aggressive). Mathematically, both thresholds increase in the respective opposite policy parameter, i.e., $\partial\rho^*/\partial\phi_y > 0$ and $\partial\phi_y^*/\partial\rho > 0$. Intuitively, the more persistent redistribution is, the more persistent is the fall in the liquidity premium such that the demand channel and the market size effect may crowd in innovative investment. On the contrary, the more aggressive monetary policy is in raising real interest rates, the more are demand effects weakened such that innovative investment falls.

2.6.3 *Generalization: Government Spending Shock and Unequal Tax Incidence* We now show how the previous insights extend to a broader set of fiscal stabilization policies that are financed by an unequal tax incidence. Specifically, we derive multipliers to output and technology growth to an expansion in government spending. For this purpose, we modify our setup. The government spends now an amount $s_t^G = S_t^G/A_t$, with a zero spending share on the balanced growth path, i.e., $s^G = 0$. Government spending follows an AR(1) process $\hat{s}_t^G = \rho\hat{s}_{t-1}^G + \epsilon_t^G$, where $\hat{s}_t^G = (s_t^G - s^G)/y$. The government finances its expenditures via lump-sum taxes on both households, such that the budget is balanced in every period. The tax burden is allocated according to the following fiscal rule

$$T_t^H = -T^E A_t + \tau_p S_t^G\,, \quad \text{and} \quad T_t^S = T^E A_t + (1-\tau_p) S_t^G\,,$$

where $\tau_p \in [0,1]$ refers to the inverse tax progressivity, i.e., the lower τ_p is the higher is the tax burden that falls on entrepreneurs.[17]

PROPOSITION 7. *A government spending shock affects aggregate outcomes as follows:*

(a) *If $\rho = 0$ or $s = 1$, government spending has no effect on output, i.e., $\mathcal{M}_y = 0$, but has a weakly negative effect on technological growth, i.e., $\mathcal{M}_g \leq 0$ if $\tau_p \leq 1$.*

(b) *If $\rho > 0$ and $s < 1$, the output multiplier has an ambiguous sign, i.e., $\mathcal{M}_y \gtreqless 0$ if $\tau_p \lesseqgtr \tilde{\tau}_p \in (0,1)$. The response of technology growth is characterized by three regimes:*

(R1) *For $\phi_y > \phi_y^*$, one obtains $\mathcal{M}_g < 0$.*

(R2) *For $\phi_y = \phi_y^*$, one obtains $\mathcal{M}_g = 0$ if $\tau_p = 0$ and $\mathcal{M}_g < 0$ if $\tau_p \in (0,1]$.*

(R3) *For $0 < \phi_y < \phi_y^*$, one obtains $\mathcal{M}_g \gtreqless 0$ if $\tau_p \lesseqgtr \tau_p^*$, where $\tau_p^* < \tilde{\tau}_p < 1$.*

[16] If $s \to 1$, equation (4) reduces to $-(1-\rho)(\phi_y - \rho\mathscr{M}) < 0$, as determinacy requires $\phi_y > \mathscr{M}$ in this case. As a result, we recover statement (a) of Proposition 6, i.e., the case of permanent household heterogeneity.

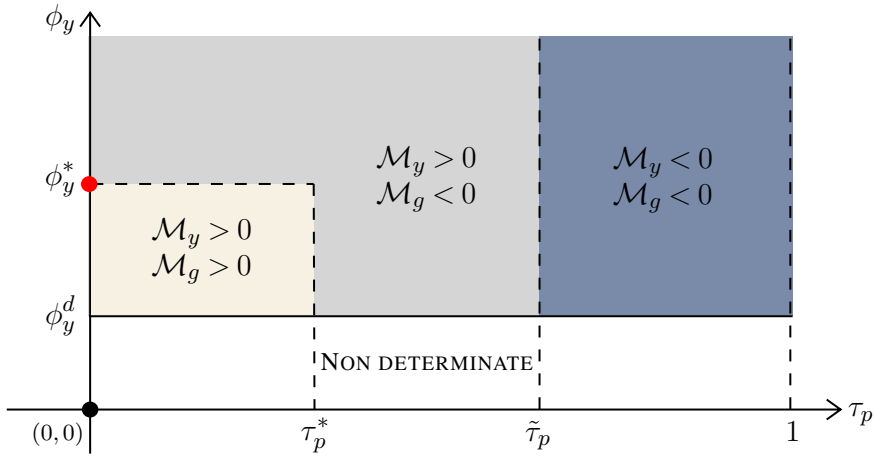
[17] The restriction $\tau_p \in [0,1]$ implies that every household type contributes, at least weakly, to the financing of the government spending shock. We study the more flexible scenario $\tau_p \in \mathbb{R}$ in Online Appendix C. In this case, we combine the financing channel with an additional redistribution channel, i.e., if $\tau_p < 0$ or $\tau_p > 1$ hand-to-mouth or saver households receive a transfer from the counter-party households that "over-finances" the government spending shock.

Closed form expressions for $\mathcal{M}_y$, $\mathcal{M}_g$, ϕ_y^*, $\tilde{\tau}_p$, and τ_p^* are provided in Appendix B.6.[18]

Statement (a) extends the result of Proposition 6: without cyclical liquidity premium, consumption and investment offset each other and output is unchanged, even if transfers do not flow from savers to hand-to-mouth households. In contrast to before, whether technology growth falls or raises depends now on how the tax burden is split across households. If $\tau_p = 1$, only hand-to-mouth households finance government spending and innovative investment of savers is unchanged. In this case, government spending crowds out private consumption of hand-to-mouth households one to one. If $0 < \tau_p < 1$, both household types contribute to the financing of the government spending shock. In this case, private consumption and innovative investment of both household types fall. Finally, if $\tau_p = 0$ only saver households finance the government spending shock. In this case, hand-to-mouth consumption will remain unchanged, and government spending crowds out consumption and innovative investment of savers one-to-one. As a result, innovative investment cannot increase in a TANK economy for $\tau_p \in [0, 1]$.

Statement (b) considers the case of a persistent government spending shock in the presence of income risk. In this case, it is ambiguous whether output and technology increase or decrease. Figure 1 graphically illustrates distinct regimes that depend on the joint design of systematic monetary policy and tax incidence across household types.

FIGURE 1. Stabilization Regimes and Monetary-Fiscal Policy Design for $\rho > 0$ and $s < 1$.



As before, the cyclical liquidity premium allows output to depart from the zero multiplier benchmark result. Whether output increases or falls depends on tax incidence. If hand-to-mouth households bear a sufficiently low burden ($\tau_p < \tilde{\tau}_p < \lambda$), a similar logic as in Proposition 6 applies: consumption inequality between savers and hand-to-mouth households falls such that precautionary saving motives are weakened; as a result, a

[18] The impact multipliers to a government spending shock have an isomorphic analytical form as the ones to a pure fiscal redistribution shock, as in Proposition 6.

countercyclical liquidity premium induces innovative investment to fall by less such that output increases. In contrast, if hand-to-mouth households bear a sufficiently high tax burden ($\tau_p > \tilde{\tau}_p$) precautionary saving motives are strengthened. As a result, savers reduce their innovative investment such that the liquidity premium rises, which leads to a decline in output. Regarding the sign of innovative investment three regimes emerge. In the first regime, monetary policy is sufficiently aggressive ($\phi_y > \phi_y^*$) such that innovative investment will unambiguously fall. This is the case, as the aggregate demand expansion is not sufficient to crowd in investment. In the second regime, monetary policy is neither too aggressive nor too accommodative ($\phi_y = \phi_y^*$). This implies that innovative investment may stay constant whenever the entire tax burden falls on saver households. In this particular case, the aggregate demand expansion from government spending and hand-to-mouth consumption exactly offsets the disincentive effects from higher taxes in shaping innovative investment. Whenever hand-to-mouth households positively contribute in financing the government spending shock, demand effects are too weak such that innovative investment unambiguously falls. Finally, in the third regime, monetary policy is sufficiently accommodative ($0 < \phi_y < \phi_y^*$) such that innovative investment may increase or decrease depending on an interior level of the tax incidence. Specifically, if the financing is sufficiently *progressive* ($\tau_p < \tau_p^* < \tilde{\tau}_p$) innovative investment will increase. In this case, the aggregate demand expansion is strong enough to crowd in investment.

Short- vs. Long-Run Stabilization Trade-Off: Statement (b) of Propositions 6 and 7 tells us that short-run output and technological growth might have opposite signs. As any transitory change in technology growth has a permanent effect on the level of output, a short- versus long-run stabilization trade-off occurs in this scenario. In case of a short-run output expansion ($\mathcal{M}_y > 0$) this implies that there exists a time horizon $z^* > 0$ such that $\ln Y_{t+z} > \ln Y_{t+z}^P$ for $z < z^*$ and $\ln Y_{t+z} < \ln Y_{t+z}^P$ for $z > z^*$, where $\ln Y_{t+z}^P$ denotes the logarithm of output in the counterfactual economy without fiscal shock at period $t+z$. This tipping point is characterized analytically by

$$z^* = \lceil \hat{z} \rceil \,, \quad \text{with} \quad \hat{z} \equiv - \frac{\ln\left(1 - (1-\rho)\dfrac{\mathcal{M}_y}{\mathcal{M}_g}\right)}{\ln \rho} \,,$$

where $\lceil \cdot \rceil$ denotes the ceiling function. It can be seen that the tipping point z^* decreases in $\mathcal{M}_y/\mathcal{M}_g$. The intuition is simple: the more negative impact technology growth is relative to the output expansion the earlier contracts progressive fiscal redistribution the level of output.

Output Maximizing Tax Incidence: To end this section, we analyze which tax incidence, $\tau_p^{\max}(z)$, delivers the greatest stimulus for the level of output Y_{t+z} at horizon $t+z$ in response to a government spending shock in period t.

PROPOSITION 8. *The output maximizing tax incidence $\tau_p^{max}(z)$ is given by:*

(a) *If $\rho = 0$ or $s = 1$, one obtains $\tau_p^{max}(0) \in [0,1]$ and $\tau_p^{max}(z) = 1 \; \forall z \geq 1$.*

(b) *If* $\rho > 0$ *and* $s < 1$*, two regimes emerge based on the threshold* $\overline{\phi}_y > \phi_y^*$*:*

(R1) *If* $0 < \phi_y \leq \overline{\phi}_y$*, one obtains* $\tau_p^{max}(z) = 0 \; \forall z \geq 0$.

(R2) *If* $\phi_y > \overline{\phi}_y$*, one obtains* $\tau_p^{max}(0) = 0$*. Moreover, there exists* $\tilde{z} > 0$ *such that* $\tau_p^{max}(z) = 0 \; \forall z < \tilde{z}$*,* $\tau_p^{max}(z) \in [0,1]$ *for* $z = \tilde{z}$ *and* $\tau_p^{max}(z) = 1 \; \forall z > \tilde{z}$.

Statement (a) of Proposition 8 is a direct implication of statement (a) of Proposition 7. As the impact output multiplier is zero, any tax incidence $\tau_p \in [0,1]$ maximizes the impact level of output. However, as the level of output for any future period is determined by technology growth, it follows that the output maximizing tax incidence must fall completely on hand-to-mouth households. An acyclical liquidity premium therefore calls for regressive redistribution.

Statement (b) discusses the output maximizing tax incidence in the case of a cyclical liquidity premium, where the sign of output and technology growth depends on the monetary-fiscal interaction. Two regimes emerge. The first regime (R1) calls for progressive redistribution. Specifically, when monetary policy is sufficiently accommodative ($\phi_y \leq \overline{\phi}_y$), both the output and the technology growth multiplier are strictly decreasing in τ_p. Consequently, a tax incidence that falls entirely on entrepreneurs maximizes the level of output at any time horizon after the government spending shock. The second regime (R2) underscores the tension between a progressive and a regressive tax incidence. In this regime, the output multiplier continues to decrease in τ_p, whereas the technology growth multiplier increases in τ_p. As a result, a progressive tax incidence maximizes short-run output gains, whereas a regressive tax incidence maximizes long-run output gains. In between both forces, there exists a time horizon $z = \tilde{z}$, where both forces neutralize each other and any tax maximizes the level of output.

Taking stock, our analysis provides new analytical insights on the tension between progressive and regressive fiscal stabilization policies. We highlight that the resolution of this trade-off hinges on four main propagation channels: the persistence of the redistribution shock, idiosyncratic income risk, tax incidence, and systematic monetary policy. Whenever the respective fiscal policy is persistent and households face income risk, output can increase in the short- and long run if monetary policy is sufficiently accommodative and the bulk of the tax incidence falls on entrepreneurs. As such, the combination of these forces generate a powerful progressive regime.

2.7 Discussion and Additional Results

In this section, we place our analysis in the context of the literature, clarify the specific margins on which our approach differs, and outline promising extensions.

2.7.1 *Physical vs. Innovative Capital* The core of our HANK-G framework constitutes a heterogeneous agent economy with two assets, liquid bonds and illiquid innovative capital. In terms of the underlying structure, our model therefore shares similarities to the standard two asset HANK models with liquid bonds and physical capital (Galí et al. (2007), Bilbiie et al. (2022), Kaplan et al. (2018), Auclert (2019)), that are well-suited to

study short-run dynamics in consumption, investment, and output. Compared to these models, our framework with innovative capital offers a distinct yet complementary perspective. Specifically, innovative capital introduces three key differences. First, innovative capital generates a unit root in the technology level such that any transitory shock that alters innovative investment will permanently alter the level of output. In contrast to the existing models, our framework is therefore well-suited to analyze how redistributive fiscal policies can be designed to balance short-run stabilization with long-run technology growth. Second, innovative investment alters technology growth and, therefore, the level of permanent income. As consumption and investment are driven by changes in permanent income, our model further strengthens indirect propagation effects to transitory policy shocks. Finally, a key advantage of our framework lies in its analytical tractability. Due to increasing returns in technology, innovative returns are linear in output. As a consequence, we are able to provide a closed-form solution to a two asset THANK economy.

2.7.2 *Equivalence between Fiscal and Monetary Policy* It is well-known that, in one-asset HANK models, fiscal transfers across households are macroeconomically equivalent to a monetary stimulus in their effects on consumption and output (see, among others, Seidl and Seyrich (2023), Bilbiie et al. (2024), Wolf (2025)). This equivalence breaks down in the HANK-G framework because fiscal transfers also operate through an investment channel: they appear both in the IS equation and the endogenous-growth (EG) equation. Essentially, in this framework a monetary expansion raises both output and technology growth and the redistributive effects of the monetary stimulus weaken or amplify the consumption or investment channel without changing their signs. As a result, a short- vs long-run stabilization trade-off does not occur for a monetary stimulus. We defer a full analysis of monetary policy in HANK-G to a companion paper.

2.7.3 *Departure from Full Information Rational Expectations* There is growing empirical evidence that supports a departure from full information rational expectations (FIRE) benchmark (see, among others, Coibion and Gorodnichenko (2015) or Roth et al. (2023)). To account for this, we follow Gabaix (2020) and Pfäuti and Seyrich (2022) and introduce bounded rationality in the form of cognitive discounting, where $\overline{m} \in [0,1]$ denotes the degree of cognitive discounting. By generating a discounted IS equation, this extension rules out the "Catch-22" (Bilbiie, 2025) in the HANK-G framework: cognitive discounting solves the forward guidance puzzle for output, while counter-cyclical income inequality leads to amplification.[19] Economically, cognitive discounting acts on household expectations and, hence, effectively weakens the market size effect. One can show that Propositions 6 - 8 continue to hold by replacing the persistence of the respective fiscal policy with $\overline{m}\rho < \rho$. As a result, progressive redistribution must be more persistent to raise not only output, but also technology growth.

[19] The forward guidance puzzle means that central bank announcements on the future path of interest rates are theoretically extremely powerful, however, their macroeconomic impact is greatly overestimated as indicated by empirical evidence (Del Negro et al., 2023). More recently, see Eggertsson and Schüle (2024) or Ahn and Holm (2025) for a discussion on the plausibility of the forward guidance puzzle.

3. Quantitative Analysis

We now complement our theoretical results with a quantitative analysis of tax-financed fiscal stimulus programs. Our results so far suggest that progressively financed fiscal stimulus programs might have ambiguous short- and long-run effects. The main takeaway of this section, however, is that, in practice, such programs generate a stark and robust short- versus long-run stabilization trade-off.

To this end, we calibrate an extended version of the HANK-G framework to the U.S. economy. The extended model preserves the same backbone as the theoretical model, enriching the environment only where needed for empirical discipline along three margins: richer household heterogeneity, a more structured entrepreneurial innovation process, and realistic tax and transfer schedules. Subsequently, we first describe model and calibration before we inspect the model's properties. We then show that tax-financed fiscal expansions lead to a stabilization trade-off and close this section with two potential resolutions to the stabilization trade-off.[20]

3.1 Extended HANK-G Model and Calibration

Additional Elements Unlike the theory, we now distinguish between *ex ante* and *ex post* household heterogeneity. Specifically, a time-invariant share Λ_E of households has the ex ante ability to become entrepreneurs, while the remaining share $1-\Lambda_E$ are workers. Both household types supply labor ℓ_{it}, but face ex post shocks to labor productivity h_{it}, employment status $e_{it} \in \{0,1\}$, and discount factor β_{it}. Labor adjusts along both the extensive and intensive margins, with the incidence of employment changes governed by $\mu(h_{it})$ and the adjustment in hours per worker by $\phi(h_{it})$. Workers can save in liquid assets $b_{it} \geq 0$, while entrepreneurs can in addition accumulate a stock of illiquid innovations, $a_{it} \equiv A_{it}/A_t$. Agents die with probability p_d and are replaced by newborns with zero wealth who inherit their parents' ex ante entrepreneurial ability, while e_{it}, β_{it} and h_{it} evolve according to their Markov processes. Bequests are fully taxed, and innovations die with the entrepreneur. Total income of households before taxes y_{it} combines wage payments $w_t h_{it} \ell_{it}$, unemployment benefits $\varsigma w_t h_{it} \ell_{it}$, after-tax entrepreneurial income $(1-\tau_c)\Theta_\alpha L_t a_{it}$ with τ_c denoting a corporate tax, and bond income $r_t b_{it}$:

$$y_{it} = \left[(1-e_{it})\varsigma + e_{it}\right] w_t h_{it} \ell_{it} + \mathbb{I}_{it}^E R_t^A a_{it} + r_t b_{it} \,,$$

where ς denotes the unemployment replacement rate, $\mathbb{I}_{it}^E$ is an indicator for the entrepreneur's type, $R_t^A = (1-\tau_c)\Theta_\alpha L_t$ is the common business income return and w_t and r_t denote the wage and interest rates, respectively.

Only a fraction of entrepreneurs in the economy will hold a positive illiquid stock of innovations, $a_{it} > 0$, depending on the realizations of idiosyncratic shocks. In particular, by investing an amount x_{it}, entrepreneurs generate successful innovations with probability $\zeta(x_{it}) = 1 - \exp(-\gamma x_{it})$. Upon success, the stock of innovations increases by a

[20] For expositional purposes, we keep the description of the extended model in stationary form concise and refer the interested reader to Section D of the Online Appendix for details.

fixed step ηA_t, where $\eta > 0$ is the step size and A_t is the aggregate stock of innovations.[21] Moreover, we incorporate an idiosyncratic investment shock $\iota_{it} \sim F_\iota$, which scales realized previous innovations and generates differences in entrepreneurial income. This modeling device captures the idea that innovative investment does not always translate one-for-one into realized innovation, and large innovations can occur even with modest investment. Normalized innovation capital then evolves as

$$a_{i,t+1} = \iota_{i,t+1} \frac{(1-\delta^A) a_{it}}{g^A_{t+1}} + \eta\, d_{i,t+1} \,,$$

where g^A_{t+1} denotes aggregate *gross* technology growth, δ^A is the depreciation rate of innovation capital and $d_{i,t+1} \in \{0,1\}$ indicates whether the innovation step is realized. Heterogeneous entrepreneurial innovation decisions aggregates into the law of motion for aggregate innovation and thus determine g^A_{t+1}.

The government issues bonds and collects tax revenues. Income taxes and transfers are of the form specified by Ferriere et al. (2023), whereby the average tax rate and transfer amounts are given by

$$\tau(y_{it}) = \exp\left(\log(\vartheta_t)\,(y_{it}/y)^{-2\varrho}\right) \,, \quad \text{and} \quad T(y_{it}) = \underline{T}_t y \frac{2\exp\{-\xi\,(y_{it}/y)\}}{1+\exp\{-\xi\,(y_{it}/y)\}} \,.$$

The parameter $\varrho \in (-\infty, 1)$ reflects the constant rate of tax progressivity, ϑ_t controls the average tax rate in the economy, and y is the steady-state mean income in the economy. If $\varrho = 0$, the income tax schedule is linear with constant marginal tax rate. If $\varrho \in (0,1)$ the tax schedule is progressive, whereas $\varrho < 0$ leads to a regressive tax schedule. For transfers, the level parameter $\underline{T}_t$ measures transfers to a household with zero income as a multiple of steady-state mean income y. Importantly, the parameter ξ determines how quickly transfers phase out with total income. When $\xi = 0$, transfers are a lump sum. As ξ becomes larger, transfers phase out faster. The total tax schedule is defined as: $\mathcal{T}(y_{it}) = \tau(y_{it}) y_{it} - T(y_{it})$ and the after tax income is $y_{it} - \mathcal{T}(y_{it})$. Total revenues of the government are the sum of income taxes net of transfers, corporate taxes, and bequest taxes. They are used to finance an exogenous level of government spending G_t, interest payments on bonds $r_t b^G_t$, and unemployment benefits.

Furthermore, monetary policy sets the nominal interest rate according to a Taylor rule, which depends on gross price inflation π_t and the output gap, i.e.,

$$i_t = r + \phi_\pi\,(\pi_t - 1) + \phi_y\,(y_t/y - 1) \,.$$

Calibration The model is quarterly and calibrated along the BGP of the U.S. economy.

Households. Households have a stationary utility $\mathcal{U}(c_{it}, e_{it}\ell_{it}) = \frac{c_{it}^{1-\sigma}}{1-\sigma} - \nu \frac{(e_{it}\ell_{it})^{1+\varphi}}{1+\varphi}$ with effective discount factor $\widetilde{\beta}_{it} = \beta_{it}(g^A_t)^{1-\sigma}$.[22] We set the inverse EIS to $\sigma = 1.5$ and the

[21] This setup is isomorphic to the analytical model as the law of motion therein can be rewritten as $A_{t+1}(j) = p\big[A_{it}(j) + \eta\big] + (1-p)A_{it}(j)$, where $p = (\psi/\eta)x_{it}$.

[22] Note that the disutility of labor in the non-stationary formulation scales with $A_t^{1-\sigma}$.

inverse Frisch elasticity to $\varphi = 1$. The labor disutility parameter ν is chosen so that employed households work $\ell = 1/3$ on the BGP, and the death probability is $p_d = 0.005$.

To generate heterogeneous MPCs (Carroll et al., 2017), households differ in their discount factor, with $\beta_{it} \in \{\beta_L, \beta_M, \beta_H\}$. Transitions are restricted to adjacent types and follow Krusell and Smith (1998) and Ferriere and Navarro (2025) with persistence $p_\beta = 0.995$, $\beta_L = \beta_M - \Delta^\beta$, and $\beta_H = \beta_M + \Delta^\beta$. We set $\beta_M = 0.97$ and $\Delta^\beta = 0.026$ to match an average MPC out of transitory income of 0.18 and a debt-to-GDP ratio of 40%.

The logarithm of idiosyncratic labor productivity h_{it} follows an AR(1) process, i.e., $log(h_{i,t+1}) = \rho_h log(h_{it}) + \epsilon^h_{it}$, where $\epsilon^h_{it} \sim \mathcal{N}(0, \sigma^2_h)$, with $\sigma^2_h = 0.04$ and $\rho_h = 0.973$, consistent with Storesletten et al. (2004). In addition, we normalize $\int_i h_{it} di = 1$.

To calibrate BGP labor market outcomes, we use CPS panel data from 1989–2022 for individuals aged 20–65, grouped into wage bins consistent with the model's h-heterogeneity. For unemployed individuals, we impute latent wages from observable characteristics. We set job-finding rates to their long-run CPS averages and back out separation rates to match unemployment rates by group. We then use CPS evidence on cyclical movements in employment and hours across wage groups to discipline two key objects: the hours adjustment margin, $\phi(h_{it})$, and the incidence of employment changes, $\mu(h_{it})$. Both imply greater cyclical exposure for low-wage groups. Finally, we set the share of unemployment fluctuations due to separations to $\phi^s \approx 0.3$, following Krusell et al. (2017). The corresponding parameter values are reported in Table D.2 in Online Appendix D.3.

Technology and Innovation. The parameter γ governs the pace of innovation: $1/\gamma$ is the average time required to generate a new idea and is directly related to the elasticity of realized innovations with respect to innovative investment. Although this parameter is inherently difficult to discipline, we follow Jones (2022) and calibrate it using the most recent empirical estimates on the link between entrepreneurial innovation and taxation. In particular, we rely on Akcigit et al. (2022), who use a panel of U.S. states since 1920 to estimate macro-elasticities of invention—measured by patents, citations, and the number of inventors—with respect to taxation. We set $\gamma = 0.44$ to match their estimate of the elasticity of innovation, measured here by patents, with respect to corporate taxation, equal to 0.49. This corresponds to the most conservative lower bound of their estimates for the broader effects of taxation on innovation. For comparison, they estimate a larger elasticity, around 0.8, for patents with respect to the personal net-of-tax rate.

We set the share of potentially innovative entrepreneurs to $\Lambda_E = 0.10$, which roughly corresponds to the share of active entrepreneurs in the U.S. Survey of Consumer Finances (2010). The quarterly depreciation rate of innovation is set to $\delta^A = 0.03$ (about 12.6% annually), a value that lies between the estimate in de Rassenfosse and Jaffe (2018) for patented inventions—based on the revenue streams generated by innovations—and the BEA convention of 15% annual depreciation for R&D expenditures. We assume that the innovation shock ι is Gaussian, i.e., $F_\iota(\iota) \sim \mathcal{N}(1, \sigma^2_\iota)$, where $\sigma^2_\iota = 0.0225$ generates a Pareto tail for income of about 1.7, consistent with Piketty and Saez (2014). Finally, the step size $\eta = 0.5$ pins down an annual growth rate of 2%.

Nominal Rigidity. The labor share is set to $\alpha = 0.4$. We calibrate the NKPC following Ferriere and Navarro (2025) and set $\epsilon_w = 7$, which corresponds to a union's markup of 16.7%. The wage adjustment cost θ is 200 to match a wage Phillips curve slope, ϵ_w/θ, of 0.035.

Fiscal and Monetary Policy. We borrow the progressivity of the tax system and transfer parameters from Ferriere et al. (2023).[23] In particular, the progressivity of the income tax schedule is $\varrho = 0.08$, while $\vartheta = 0.7$ implies an average labor income tax rate of about 23%. Transfers at minimum income are $\underline{T} = 8.3\%$ of average income, with a phase-out parameter $\xi = 4.22$. The corporate tax rate is set to its statutory level, $\tau_c = 0.21$. The UI replacement rate ς is set to 40% of previous labor income, the average across U.S. states between 1990 and 2020. The implied government spending to output ratio is roughly 25%. Finally, monetary policy follows a standard Taylor rule with coefficients $\phi_\pi = 1.5$ on inflation and $\phi_y = 0$ on output. The real interest rate on the BGP is $r = 1\%$.

3.2 Inspecting the Model's Properties

Income and Wealth Inequality. We now examine how well the model matches key long-run features of the data. To this end, Table 1 compares the income and wealth distributions in the data and the model. These distributions matter because the efficacy of fiscal policy hinges on both the underlying forces behind inequality and its overall degree.

TABLE 1. The distribution of income and wealth.

	Income, top share (in %)				Wealth, top share (in %)			
	1%	10%	20%	50%	1%	10%	20%	50%
US data [a]	17	42	58	85	38	73	86	98
Benchmark model	15	38	52	80	35	71	85	99
No entrepreneurs ($\Lambda_E = 0$) [c]	2	19	35	68	11	56	77	98
No innovation shock ($\sigma_\iota^2 = 0$) [b]	12	36	51	79	17	57	75	97

[a] Statistics for income and wealth come from the World Inequality Database (WID).
[b] Recalibrated to match a growth rate of 2% annually.
[c] Recalibrated to match the same average MPC and debt-to-GDP ratio.

The benchmark model (2nd row) generates a pronounced concentration of income and wealth, largely driven by the strong endogenous sorting of entrepreneurs into the top of the distribution. Entrepreneurs account for 45% of the wealthiest 5% of households. The model is therefore consistent with the causal link between innovation and top income shares documented by Aghion et al. (2019). According to their estimates, a 1% increase in patents per capita increases the top 1% income share by ∼0.2%.

We are not the first to emphasize entrepreneurship as an engine of inequality in quantitative models—it is well established that self-employed and business owners are

[23] The latter incorporate refunded tax credits, namely the refunded Additional Child Tax Credit (ACTC), the refunded part of the EITC and state credits, as well as SNAP, housing assistance, and other welfare transfers.

disproportionately represented among the rich. Cagetti and De Nardi (2006), for example, show that among the top 5% of households 68% are business owners (39% when restricted to the self-employed). Consistent with their findings, removing entrepreneurs from the model (3rd row) sharply compresses the income and wealth distributions. A key distinction in our framework, however, is that while 10% of the population are entrepreneurs, only those with a positive number of innovations ($a > 0$) qualify as innovative. By this criterion, just 4.5% of the population are innovators—a small minority. How does this compare to the data? According to the Global Entrepreneurship Monitor, about 11% of U.S. adults (ages 18–64) manage a new business, and roughly one-third of them—around 35%—offer products or services that are new to at least some customers and face little or no direct competition. Extrapolated to the full population, this implies that roughly 3.9% of adults are innovative entrepreneurs. A similar pattern emerges from the 2017 U.S. Annual Business Survey (ABS), which covers 4.6 million active for-profit firms, of which about 43% reported introducing an innovation between 2015 and 2017. Assuming one owner per firm and using the 2016 Survey of Consumer Finances (SCF) estimate that 13.3% of adults are business owners, this corresponds to about 5.7% of the population being owners of innovative firms—a likely upper bound, since some entrepreneurs operate multiple businesses.

Finally, not all innovations—whether in the model or in reality—require large upfront investments. Some entrepreneurs build successful large-scale businesses from modest ideas, while others invest heavily in ventures that fail to take off. In the model, this heterogeneity is captured by the idiosyncratic shocks to innovation, ι. Removing these shocks (4th row) compresses the distributions of income and wealth, underscoring the central role of luck as an engine of concentration in our calibration.

Consumption and Innovative Investment Responses. We now turn to the short-run features of the model and report the model-implied distribution of MPCs across household types and along the income and wealth distributions. We compute the MPC as the marginal change in consumption resulting from a small change in liquid wealth, $\text{MPC}(a_{it}, b_{it}, \mathbf{s}_{it}) = \frac{\partial c(a_{it}, b_{it}, \mathbf{s}_{it})}{\partial b_{it}}$, where $\mathbf{s}_{it}$ is the vector of state variables. We find that unemployed individuals display the highest MPCs (0.38 on average), followed by employed workers (0.16) and, finally, entrepreneurs (0.04). This pattern is consistent with Carroll et al. (2017) and Ganong and Noel (2019), who show that MPCs are highest among the unemployed because borrowing constraints and low liquid wealth make their consumption highly sensitive to income changes. In contrast, entrepreneurs—having accumulated wealth over time—display the lowest marginal propensity to consume.

We then use the model to study how innovative investment responds to tax changes and how these responses compare to the data. Table 3 reports the elasticities of growth, innovative investment, and top income inequality with respect to the corporate tax rate (τ_c) and the progressivity of the personal income tax (ϱ), holding prices, and other transfers or taxes fixed. Progressivity matters here because taxing high-income households effectively targets a segment of the population that is disproportionately entrepreneurial at the intensive margin. Moreover, by reducing the expected returns from future innovations, higher top-income taxes discourage potential entrants ($a_{it} > 0$) at the extensive margin, echoing the mechanism in Jones (2022).

TABLE 2. Quarterly MPC across groups, wealth quantiles, and income quantiles.

GROUP	MPC	MEAN WEALTH	WEALTH Q	MPC	INCOME Q	MPC
Overall population	0.18	1.00	0–20%	0.39	0–20%	0.51
Unemployed	0.38	0.76	20–40%	0.18	20–40%	0.21
Employed	0.16	1.01	40–60%	0.13	40–60%	0.11
Entrepreneurs	0.04	10.15	60–80%	0.08	60–80%	0.08
			80–100%	0.04	80–100%	0.04

TABLE 3. Model response to corporate and personal income taxation.[a]

ELASTICITY	RESPONSE TO AN INCREASE OF TAXATION	
	Corporate taxation τ_c	Income tax progressivity ϱ
Annual net growth rate	-1.19	-0.79
Innovative investment	-0.47[b]	-0.33
Top 1% income share	-0.06	-0.04
Fraction of innovative entrepreneurs	-0.18	-0.02

[a] The elasticity is computed on a one pp increase from 0.20 to 0.21 for τ_c, and from 0.10 to 0.11 for ϱ.
[b] We target the same elasticity as in Akcigit et al. (2022) using patenting as proxy for innovation investment.

Higher taxes substantially weaken incentives to innovate and, in turn, reduce income inequality. We find that a 1% increase in the corporate tax rate lowers the top 1% income share by 0.064% and reduces annual growth by 0.12 percentage points. A similar pattern holds for income tax progressivity: a 1% increase in progressivity reduces growth by 0.8 percentage points and innovative investment by 0.33%.

These elasticities are broadly consistent with recent evidence for the United States. Akcigit et al. (2022) show that higher personal and corporate taxes sharply reduce innovation—measured by both the number of patents and the number of inventors—with effects operating through both weaker incentives and higher inventor mobility. For example, the elasticity of patents with respect to the personal net-of-tax rate ranges from 0.8 to 1.8, while that with respect to the corporate tax rate ranges from 1.3 to 2.8, depending on the specification. While patents do not map directly to growth, the evidence highlights a strong link between taxation and innovation. Our model replicates this hierarchy of effects: corporate taxes exert a larger drag on innovation and growth than personal income taxes.

3.3 Tax-Financed Fiscal Expansions According to HANK-G

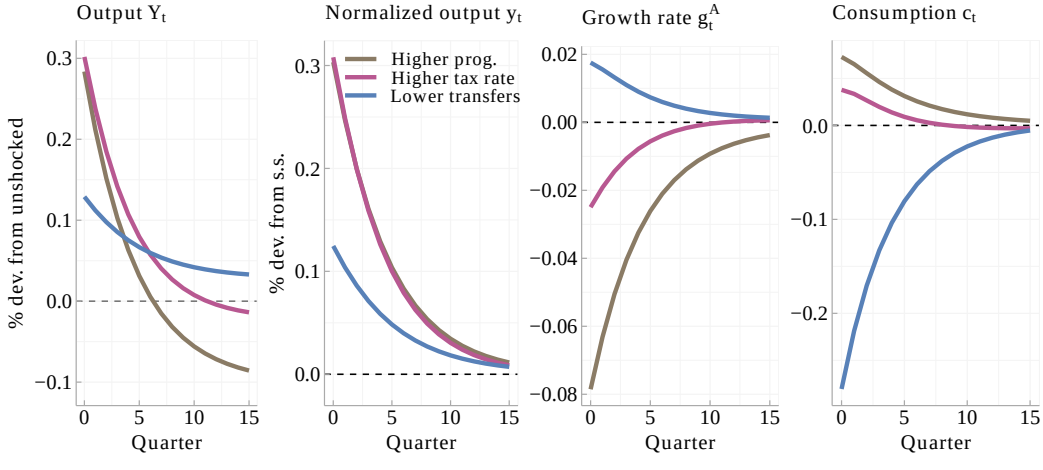
Propositions 6 and 7 show that tax financed fiscal expansions may have opposing short- and long-run effects. In the short-run, progressively financed policies stimulate aggregate demand by reallocating resources toward households with higher marginal propensities to consume and greater cyclical income exposure. In the long-run, however, they may reduce innovation by lowering after-tax returns. As such, both opposing forces give

rise to distinct economic regimes. We now use the extended HANK-G model to assess which of these regimes is, in practice, quantitatively relevant.

A Stark Stabilization Trade-Off We consider a simple fiscal experiment. The government raises public spending by 1% at impact, after which spending gradually returns to its BGP value with persistence $\rho_G = 0.8$. We compare two alternative financing instruments: the increase in spending is financed (i) by an increase in the average tax rate, holding progressivity constant; or (ii) by an increase in tax progressivity according to the law $\varrho_t = \varrho + \phi_G \Delta G_t$, where $\phi_G = 2$ and the residual financing is raised by adjusting the balancing parameter ϑ_t, consistent with a mixed US financing scheme (Ferriere and Navarro, 2025).

Figure 2 displays the resulting dynamics.[24] Financing the government spending increase through higher average taxes or greater tax progressivity produces a stark stabilization trade-off: the short-run expansion is at the cost of weaker technology growth thereafter. Both schemes reduce innovation by lowering the net returns to entrepreneurial activity. The effect is particularly pronounced under greater progressivity: although private consumption rises sharply on impact, weaker investment depresses future output. As a result, the two schemes generate similar short-run gains, but differ markedly in their persistence. Under higher progressivity, the cumulative effects on output turns negative after roughly six quarters, implying a persistent output loss. When spending is financed through higher average taxes instead, the cumulative effects on output remains positive for nearly eleven quarters, although the long-run loss remain limited. Taken together, these results reveal a clear short-run versus long-run trade-off.

FIGURE 2. Aggregate effects of government spending shock across financing schemes.



Note: IRFs to a government spending shock: higher average taxes versus higher tax progressivity.

[24] We provide the complete set of impulse response functions in Appendix D.4.

Sensitivity Analysis We then evaluate how robust the stabilization trade-off is across alternative calibrations regarding the strength of nominal rigidities, the persistence of the fiscal stimulus, the specification of the Taylor rule, and the earnings incidence across households. Part A of Table 4 summarizes our findings. In the baseline calibration, where the fiscal scale parameter ϑ_t adjusts endogenously, the impact output multiplier is $\mathcal{M}_y = 0.31$, while the growth response is negative ($\mathcal{M}_g = -1.16$). The economy eventually experiences a long-run output deviation of about -2%. The switch time z^*—the horizon at which long-run innovation losses dominate the short-run demand expansion—occurs roughly thirteen quarters after the shock.

TABLE 4. Effects of government spending shock: sensitivity analysis.

MODEL VARIANT	PARAMETERS	IMPACT IRF		LONG-RUN DEV.	SWITCH TIME z^*
		$\mathcal{M}_y$	$\mathcal{M}_g$	$\%(Y_\infty/Y_\infty^P - 1)$	
Baseline		0.31	-1.16	-2.08	11.19
A. Adjusting Baseline Parameters					
Higher nominal rigidities	$\theta = 400$	0.35	-0.23	-0.08	21.01
Higher shock persistence	$\rho_g = 0.90$	0.28	-2.07	-8.34	14.23
Taylor rule	$\phi_y = 0.07$	0.28	-1.75	-3.64	10.95
Taylor rule	$\phi_\pi = 2.00$	tba	tba	tba	tba
No earnings incidence	$\mu(h) = 1.00$	0.31	-1.06	-2.04	13.33
B. Model Environment					
HANK model	fixed g^A	0.31	0.00	0.00	–
HA-G model	$\theta = 0$	0.18	-3.92	-9.10	6.31
HA model	fixed g^A, $\theta = 0$	0.18	0.00	0.00	–

Adjusting the core propagation parameters reveals how differently the same spending shock can play out. When nominal rigidities are stronger, wages and prices move even more slowly and the initial demand push bites harder: the impact multiplier climbs to 0.35, long-run losses nearly vanish, and the switch time is pushed far into the future—21 quarters out. Because it calls for higher taxes, making the shock more persistent tells a different story. The short-run boost fades a bit, but the long-run contraction becomes much deeper, with output eventually falling by over eight percent and the switch time arriving somewhat earlier as the scarcity effect surfaces sooner. A more aggressive Taylor rule changes the narrative again: monetary policy leans against the fiscal expansion, the impact multiplier falls to 0.28, and the positive demand window narrows, bringing the switch time forward to about eleven quarters. In contrast, turning off earnings-incidence effects barely moves the results, suggesting that countercyclical inequality is not a major channel.

Finally, in Part B of Table 4 we change the underlying model environment and obtain even sharper contrasts. In a standard HANK model with exogenous innovation growth, impact multipliers remain identical to the baseline, but long-run deviations vanish entirely and no switch time emerges, since the shock has no lasting effect on growth. In the

HA-G model, turning off nominal rigidities weakens the demand response—reducing the impact multiplier to 0.18—while long-run innovation reacts strongly downward, producing a sizable and early long-run contraction of roughly nine percent, with a switch time near six quarters. Finally, in a pure HA model without growth or nominal rigidities, short-run multipliers fall to 0.18, long-run effects disappear, and no switch time exists.

3.4 Resolution of Stabilization Trade-Off

We now analyze two potential resolutions of the short- versus long-run stabilization trade-off emerging under progressively financed fiscal expansions: a heterogeneous tax incidence and additional government debt. The primary and novel insight of this section is that a tax incidence that falls on middle-class households avoids the stabilization trade-off, expands both output and technology growth, and raises welfare of all households relative to the *laissez-faire* allocation without fiscal intervention.

The Role of Tax Incidence In Proposition 7, we show that the efficacy of government spending shocks depends crucially on tax incidence. To account for this observation in our quantitative analysis, we now let the government finance additional spending with a heterogeneous tax incidence across different quantiles of the income distribution. Concretely, we implement a smooth, localized perturbation to the tax and transfer schedule that moves the tax incidence around a chosen point in the income distribution:

$$\tau(y_{it}) = \exp\left(\log(\vartheta_t)\left(\frac{y_{it}}{y}\right)^{-2\varrho}[1-\theta_t\varphi(y_{it})]\right), \quad \varphi(y_{it}) = \exp\left(-\frac{\left(\ln y_{it} - \ln y^*\right)^2}{2\sigma^2}\right).$$

The function $\varphi(\cdot)$ creates a smooth, localized shift in the tax schedule, and allows us to concentrate the additional tax incidence on specific parts of the income distribution. The time-varying scale parameter θ_t is set endogenously to raise a fixed amount of additional revenue. The location parameter y^* pins down where the extra tax burden falls—for example around a chosen income quantile—while σ controls how far the adjustment spreads. A larger σ, for instance, spreads the increase in taxation to a broader range of households with corresponding income levels cenetred around y^*.

Figure 3 displays the effects of two localized tax adjustments used to finance an increase in public spending. The first panel plots the resulting average tax rate (ATR), and the second panel the corresponding marginal tax rate (MTR), which adjusts locally around each threshold to accommodate the perturbation. The two reforms concentrate the additional tax burden at different points of the income distribution: one around $y^* = 0.4$ (roughly the 55th percentile) and the other around $y^* = 1.5$ (about the 95th percentile). Both use the same dispersion, $\sigma = 0.5$, illustrating how shifting the adjustment from middle- to high-income households changes the overall incidence of the reform.

Equipped with our tax-incidence toolkit, we now let the government finance the spending shock with persistence rate of 0.8 by shifting the tax burden (by moving y^*) across different parts of the income distribution—from the 5th to the 95th percentile.

FIGURE 3. Heterogeneous tax incidence

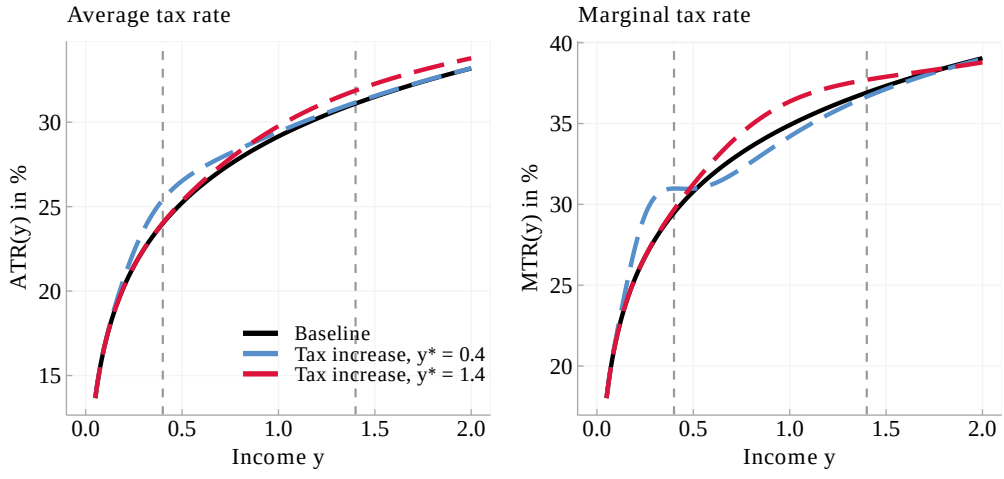


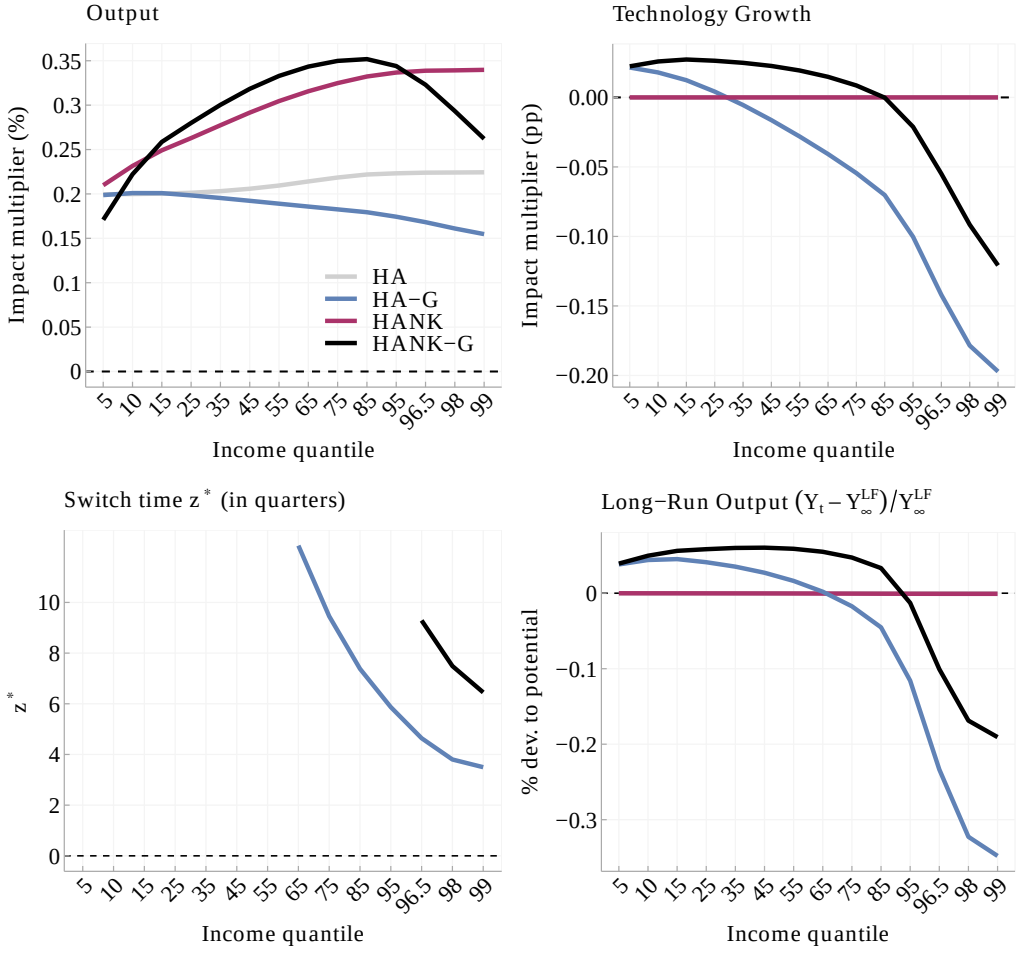
Figure 4 gathers the results. The top panels report the multiplier of normalized output (y_t) and the growth rate (g_t^A). The bottom left panel reports the switch time z^*, whereas the bottom right panel depicts the long-run deviation of output from the laissez faire (unshocked) allocation. As before, we compare in each panel outcomes across four model environments—the full baseline HANK-G economy, a standard HANK model with exogenous growth, the HA-G economy without nominal rigidities, and a HA benchmark with exogenous growth but without nominal rigidities.

We begin with the most intuitive environment: a HA economy. In this economy, the normalized output multiplier hardly moves with the tax incidence—and whatever effects appear in the short-run fade out entirely within a decade. This stands in sharp contrast to the HANK economy, where the strength of the demand response is dictated by the MPC channel. In that world, shifting the tax burden toward the top delivers the largest impact and cumulative multipliers—exactly as predicted by Proposition 5.

The picture changes drastically in a HA-G economy. With endogenous growth but no nominal rigidities, tax incidence matters for the long-run: placing the tax burden at the top produces the largest output losses at five and ten years. The reason is straightforward—taxing higher-income households erodes innovative investment the most, triggering a negative multiplier on growth. A small exception emerges when the incidence falls only on the very bottom, where market-size effects briefly dominate and generate a slight positive response. But overall, in the absence of a private household MPC channel, the short-run multiplier barely varies across incidence patterns; the main differences emerge in the long-run, where the losses are most pronounced.

Once both mechanisms operate jointly—nominal rigidities and endogenous growth—the HANK-G economy delivers a much richer pattern. As in HANK, short-run demand is larger when the tax burden falls on the very top of the income distribution. But with technology growth now being endogenous, this same incidence produces the weakest—and even negative—technology growth multiplier, since innovators bear the brunt

FIGURE 4. Government spending shock and heterogeneous tax incidence.



Note: Panels (a) and (b) display the normalized output and growth multipliers following a 1% government spending shock as a function of the targeted income quantile. Panel (c) shows the switching time z^*, if it exist, while panel (d) shows the long-run deviation from the laissez faire economy, both as a function of the targeted income quantile.

of the tax. These two forces pull in opposite directions, producing a hump-shaped response of output. In this world, the middle class emerges as the sweet spot: financing government spending through taxes on middle-income households maximizes output without choking off innovation later on. Taxing the middle class, in other words, becomes a powerful way to maximize short-run fiscal multipliers without sacrificing long-run technology growth.

Finally, the long-run output deviation from the potential output in the unshocked economy is almost monotonically decreasing in the HA-G economy, whereas in the HANK-G economy the short-run demand expansions generates an initially rising, then flat, and eventually declining long-run output response as tax incidence shifts toward the high-income segment.

Welfare and Decomposition We now compare household welfare across different tax incidence patterns. For this purpose, we compare at date 0 the full transition induced by the government spending shock with the initial stationary balanced-growth path without shock. Let $j = 0$ denote the unshocked and $j = 1$ the shocked economy, respectively. Because aggregate technology growth evolves according to $A^j_{t+1} = g^{A,j}_t A^j_t$, every temporary deviation in the growth rate $g^{A,j}_t$ has a permanent effect on the level of aggregate technology, output, and therefore on lifetime utility. Let $W^j(\mathbf{s})$ denote the date-0 value function in economy j for an individual with state $\mathbf{s}$, where $\mathbf{s}$ is drawn from the initial distribution of the unshocked economy. Aggregate utilitarian welfare is then obtained by integrating over this initial distribution with measure $\mu^0(\mathbf{s})$:

$$W^j = \int W^j(\mathbf{s})\, d\mu^0(\mathbf{s})\,.$$

In practice, these objects are computed directly from the transition path and therefore already embed the full sequence of future allocations and growth rates. We measure the total welfare effect of the government spending shock using the consumption-equivalent variation (CEV):

$$\Delta\text{CEV}^{\text{TOT}} = \left(1 + \frac{W^1 - W^0}{\mathcal{C}^0}\right)^{\frac{1}{1-\sigma}} - 1\,,$$

where $\mathcal{C}^0$ is the discounted utility weight of consumption along the unshocked path.[25]

To distinguish transitional short-run effects from those driven by permanent changes in technology growth, we exploit the fact that welfare can be rescaled by the cumulative growth factor along each path. Let us denote the cumulative growth factor along path j, where T is the horizon by which the economy has effectively converged:

$$Q^j_t \equiv \left(\prod_{s=0}^{T-1} g^{A,j}_s\right)^{1-\sigma}, \quad \text{with} \quad Q^j_0 = 1\,.$$

We can then define $U^j \equiv \frac{W^j}{Q^j}$, which can be interpreted as welfare purged of the level effects induced by technology growth. Using these objects, we construct the counterfactual welfare level $\widetilde{W}^{1|0} \equiv U^1 \cdot Q^0$, which combines the allocation path of the shocked economy with the growth normalization of the unshocked economy. This allows us to define the short-run welfare change as:

$$\Delta\text{CEV}^{\text{SR}} = \left(1 + \frac{W^{1|0} - W^0}{\mathcal{C}^0}\right)^{\frac{1}{1-\sigma}} - 1\,.$$

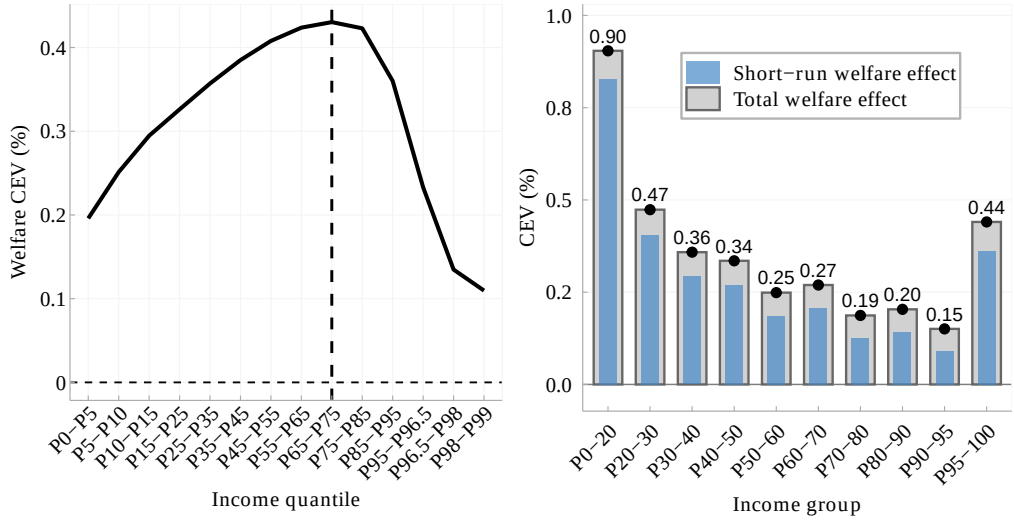
Figure 5 reports the results. The left panel plots total welfare as a function of the income quantile on which the tax incidence falls. It shows that concentrating the tax burden on the upper middle of the income distribution is the welfare-maximizing way

[25] The full derivation is provided in Online Appendix D.5.

to finance the fiscal stimulus. The right panel then shows how the welfare effects of this optimal tax incidence are distributed across the income distribution. Strikingly, even the groups that bear the tax burden experience welfare gains from the fiscal expansion.

The decomposition between short-run and total welfare effects is also informative. For the optimal tax incidence, technology growth is still positive, so the total welfare effect is somewhat higher than the short-run effect. In other words, this tax incidence delivers the largest short-run output gains while inducing positive long-run welfare gains from higher technology growth. This is precisely why taxing the upper middle class emerges as the welfare-maximizing tax incidence.

FIGURE 5. Welfare, tax incidence, and decomposition of gains.



Note: The left panel shows welfare as a function of tax incidence across quantiles; the right panel shows the decomposition of welfare gains and losses across the income distribution, evaluated at the welfare-maximizing tax incidence from the left panel.

The Role of Debt Financing As a final policy experiment, we analyze whether debt financing can avoid the stabilization trade-off. To this end, we assume that the government issues debt on impact and repays it gradually through lump-sum taxes. This setup is attractive because fiscal consolidation is postponed while short-run demand expansions—and the resulting boost to innovation—arrives immediately. If that boost is strong enough, faster productivity growth may offset the future tax burden. In this sense, public debt can become self-financing, echoing recent results Angeletos et al. (2024) but reinforced here as endogenous technology growth helps repay outstanding debt permanently. Table 5 highlights this mechanism. As a reminder, under contemporaneous taxation, long-run output falls and a switch time emerges. With debt financing, the impact multiplier rises, the growth multiplier turns essentially zero on impact (and slightly positive afterwards), and long-run output increases by nearly six percent. No switch time appears: the innovation channel remains positive throughout.

TABLE 5. Debt financing and lump-sum repayment.

FINANCING SCHEME	IMPACT IRF		LONG-RUN DEV.	SWITCH TIME z^*
	$\mathcal{M}_y$	$\mathcal{M}_g$	$\%(Y_\infty/Y_\infty^P - 1)$	
Financed with constant progressivity	0.31	-1.16	-2.08	13.19
Financed with debt and lump-sum repayments	0.36	0.01	5.75	–

Many hybrid financing schemes—combining debt issuance with targeted tax incidence—are also possible. For example, the government could initially finance the shock with debt to harness the short-run demand expansion, and then concentrate the eventual consolidation on the segments of the income distribution that generate the smallest drag on innovation. This exploration lies beyond the scope of this paper.

4. Conclusion

Building on empirical evidence, we develop a unified HANK-G framework in which household heterogeneity, business cycles, and long-run growth interact jointly. We use this framework to reassess the effectiveness of discretionary fiscal stabilization programs, showing that their impact critically depends on the interaction between two key policy margins: systematic monetary policy and the incidence of taxation.

In the first part of the paper, we used an analytical version of the HANK-G framework to identify three distinct stabilization regimes. When monetary policy is sufficiently dovish, a progressive fiscal stabilizer raises output in both the short- and long run. When monetary policy is sufficiently hawkish, the same policy boosts short-run output but generates permanent long-run losses by suppressing innovative investment. Finally, if tax progressivity is weak, output declines in both the short- and long run, regardless of the stance of systematic monetary policy. To assess the plausibility of these regimes quantitatively, we examine, in the second part of the paper, the propagation of government spending shocks in an extended version of the HANK-G model calibrated to the U.S. economy. Notably, the regime featuring a short- versus long-run trade-off emerges as a robust outcome, raising questions about the effectiveness of progressive stabilizers. Shifting the additional tax burden toward middle-class households or financing through public debt avoids this trade-off. In particular, assigning the burden to middle-class households not only mitigates the trade-off by increasing output and technological growth but also improves welfare across all household groups.

This paper provides a first step toward understanding the joint interaction of household heterogeneity, business cycles, and long-run technological growth. Our analysis opens several directions for future research. First, exploring the normative implications of the framework—particularly the joint design of fiscal and monetary policy and the role of taxes and social insurance as automatic stabilizers—appears especially promising. Second, while our middle-class tax proposal is conceptually appealing, assessing its political feasibility requires further investigation. Finally, additional empirical work is needed to structurally identify the potentially opposing short- and long-run effects

of fiscal stabilization policies. Taken together, these avenues underscore the rich set of questions that remain for applied, theoretical, and quantitative evaluation in the context of the HANK-G framework.

References

ACHARYA, SUSHANT AND KESHAV DOGRA (2020): "Understanding HANK: Insights From a PRANK," *Econometrica*, Vol. 88, pp. 1113–1158. [5]

AGHION, PHILIPPE, UFUK AKCIGIT, ANTONIN BERGEAUD, RICHARD BLUNDELL, AND DAVID HEMOUS (2019): "Innovation and Top Income Inequality," *The Review of Economic Studies*, Vol. 86, pp. 1–45. [6, 28]

AGHION, PHILIPPE AND PETER HOWITT (1992): "A Model of Growth Through Creative Destruction," *Econometrica*, Vol. 60, pp. 323–351. [16]

AHN, SEHYOUN AND MARTIN HOLM (2025): "There is no Forward Guidance Puzzle," *Working Paper*. [24]

AIYAGARI, S. RAO (1994): "Uninsured Idiosyncratic Risk and Aggregate Saving," *The Quarterly Journal of Economics*, Vol. 109, pp. 659–684. [16]

AKCIGIT, UFUK, JOHN GRIGSBY, TOM NICHOLAS, AND STEFANIE STANTCHEVA (2022): "Taxation and Innovation in the Twentieth Century," *The Quarterly Journal of Economics*, Vol. 137, pp. 329–385. [4, 6, 27, 30, 89]

ALVES, FELIPE, GREG KAPLAN, BENJAMIN MOLL, AND GIOVANNI L. VIOLANTE (2020): "A Further Look at the Propagation of Monetary Policy Shocks in HANK," *Journal of Money, Credit and Banking*, Vol. 52, pp. 521–559. [10]

AMPUDIA, MIGUEL, DIMITRIS GEORGARAKOS, JIRI SLACALEK, ORESTE TRISTANI, PHILIP VERMEULEN, AND GIOVANNI L. VIOLANTE (2018): "Monetary Policy and Household Inequality," *ECB Working Paper 2170*. [10]

ANGELETOS, GEORGE-MARIOS AND CHEN LIAN (2018): "Forward Guidance without Common Knowledge," *The American Economic Review*, Vol. 108, pp. 2477—-2512. [16]

ANGELETOS, GEORGE-MARIOS, CHEN LIAN, AND CHRISTIAN K. WOLF (2024): "Can Deficits Finance Themselves?" *Econometrica*, Vol. 92, pp. 1351–1390. [12, 37]

——— (2026): "Deficits and Inflation: HANK meets FTPL," *Working Paper*. [12]

ANTOLIN-DIAZ, JUAN AND PAOLO SURICO (2025): "The Long-Run Effects of Government Spending," *The American Economic Review*, Vol. 115, pp. 2376–2413. [6]

ANZOATEGUI, DIEGO, DIEGO COMIN, MARK GERTLER, AND JOSEBA MARTINEZ (2019): "Endogenous Technology Adoption and R&D as Sources of Business Cycle Persistence," *American Economic Journal: Macroeconomics*, Vol. 11, pp. 67–110. [5]

AUCLERT, ADRIEN (2019): "Monetary Policy and the Redistribution Channel," *The American Economic Review*, Vol. 109, pp. 2333–2367. [5, 23]

AUCLERT, ADRIEN AND MATTHEW ROGNLIE (2020): "Inequality and Aggregate Demand," *Working Paper*. [10]

AUCLERT, ADRIEN, MATTHEW ROGNLIE, AND LUDWIG STRAUB (2024): "The Intertemporal Keynesian Cross," *Journal of Political Economy*, Vol. 132, pp. 4068–4121. [5, 6]

——— (2025): "Fiscal and Monetary Policy with Heterogeneous Agents," *Annual Review of Economics*, Vol. 17, pp. 539–562. [12]

BAYER, CHRISTIAN, RALPH LUETTICKE, LIEN PHAM-DAO, AND VOLKER TJADEN (2019): "Precautionary Savings, Illiquid Assets, and the Aggregate Consequences of Shocks to Household Income Risk," *Econometrica*, Vol. 87, pp. 255–290. [5]

BEAUDRY, PAUL, CHENYU HOU, AND FRANCK PORTIER (2024): "Monetary Policy When the Phillips Curve Is Quite Flat," *American Economic Journal: Macroeconomics*, Vol. 16, pp. 1–28. [16]

BEAUDRY, PAUL AND FRANCK PORTIER (2018): "Real Keynesian Models and Sticky Prices," *NBER Working Paper 24223*. [16]

BENIGNO, GIANLUCA AND LUCA FORNARO (2018): "Stagnation Traps," *The Review of Economic Studies*, Vol. 85, pp. 1425–1470. [2, 5, 16]

BIANCHI, FRANCESCO, HOWARD KUNG, AND GONZALO MORALES (2019): "Growth, Slowdowns, and Recoveries," *Journal of Monetary Economics*, Vol. 101, pp. 47–63. [5]

BILBIIE, FLORIN O. (2008): "Limited Asset Markets Participation, Monetary Policy and (Inverted) Aggregate Demand Logic," *Journal of Economic Theory*, Vol. 140, pp. 162–196. [2, 5, 10, 16]

——— (2020): "The New Keynesian Cross," *Journal of Monetary Economics*, Vol. 114, pp. 90–108. [5]

——— (2025): "Monetary Policy and Heterogeneity: An Analytical Framework," *The Review of Economic Studies*, Vol. 92, pp. 2398–2436. [2, 5, 16, 24]

BILBIIE, FLORIN O., DIEGO R. KÄNZIG, AND PAOLO SURICO (2022): "Capital and Income Inequality: an Aggregate-Demand Complementarity," *Journal of Monetary Economics*, Vol. 126, pp. 154–169. [5, 23]

BILBIIE, FLORIN O., TOMMASO MONACELLI, AND ROBERTO PEROTTI (2024): "Stabilization vs. Redistribution: The Optimal Monetary-Fiscal Mix," *Journal of Monetary Economics*, Vol. 147, pp. 1–18. [6, 24]

BILBIIE, FLORIN O. AND ROLAND STRAUB (2013): "Asset Market Participation, Monetary Policy Rules, and the Great Inflation," *The Review of Economics and Statistics*, Vol. 95, pp. 377–392. [10]

BLANCHARD, OLIVIER, EUGENIO CERUTTI, AND LAWRENCE SUMMERS (2015): "Inflation and Activity - Two Explorations and their Monetary Policy Implications," *NBER Working Paper*. [6]

BROER, TOBIAS, NIELS-JAKOB HARBO HANSEN, PER KRUSELL, AND ERIK ÖBERG (2020): "The New Keynesian Transmission Mechanism: A Heterogeneous-Agent Perspective," *The Review of Economic Studies*, Vol. 87, pp. 77–101. [5]

BULLARD, JAMES AND KAUSHIK MITRA (2002): "Learning About Monetary Policy Rules," *Journal of Monetary Economics*, Vol. 49, pp. 1105–1129. [59]

CAGETTI, MARCO AND MARIACRISTINA DE NARDI (2006): "Entrepreneurship, Frictions, and Wealth," *Journal of Political Economy*, Vol. 114, pp. 835–870. [29]

CANTORE, CRISTIANO AND LUKAS B. FREUND (2021): "Workers, Capitalists, and the Government: Fiscal Policy and Income (Re)Distribution," *Journal of Monetary Economics*, Vol. 119, pp. 58–74. [5]

CARROLL, CHRISTOPHER D. (2006): "The Method of Endogenous Gridpoints for Solving Dynamic Stochastic Optimization Problems," *Economics Letters*, Vol. 91, pp. 312–320. [87]

CARROLL, CHRISTOPHER, JIRI SLACALEK, KIICHI TOKUOKA, AND MATTHEW N. WHITE (2017): "The Distribution of Wealth and the Marginal Propensity to Consume," *Quantitative Economics*, Vol. 8, pp. 977–1020. [27, 29, 78]

CERRA, VALERIE, ANTONIO FATÁS, AND SWETA C. SAXENA (2023): "Hysteresis and Business Cycles," *Journal of Economic Literature*, Vol. 61, pp. 181–225. [6]

CLOYNE, JAMES, JOSEBA MARTINEZ, HAROON MUMTAZ, AND PAOLO SURICO (2025): "Short-Term Tax Cuts, Long-Term Stimulus," *Working Paper*. [6]

COGLIANESE, JOHN, MARIA OLSSON, AND CHRISTINA PATTERSON (2025): "Monetary Policy and the Labor Market: A Quasi-Experiment in Sweden," *The American Economic Review*, Vol. 115, pp. 3451–3486. [10]

COIBION, OLIVIER AND YURIY GORODNICHENKO (2015): "Information Rigidity and the Expectations Formation Process: A Simple Framework and New Facts," *The American Economic Review*, Vol. 105, pp. 2644–2678. [24]

COIBION, OLIVIER, YURIY GORODNICHENKO, LORENZ KUENG, AND JOHN SILVIA (2017): "Innocent Bystanders? Monetary Policy and Inequality," *Journal of Monetary Economics*, Vol. 88, pp. 70–89. [10]

COLLARD, FABRICE, PATRICK FÈVE, AND PHILIPP WANGNER (2025): "The Power of Persistence," *Working Paper*. [12]

COMIN, DIEGO AND MARK GERTLER (2006): "Medium-Term Business Cycles," *The American Economic Review*, Vol. 96, pp. 523–551. [5]

COZZI, MARCO (2018): "Optimal Capital Taxation With Incomplete Markets and Schumpeterian Growth," *Working Paper*. [16]

DEBORTOLI, DAVIDE AND JORDI GALÍ (2024a): "Monetary Policy with Heterogeneous Agents: Insights from TANK Models," *NBER Working Paper 32557*. [5, 12]

——— (2024b): "Idiosyncratic Income Risk and Aggregate Fluctuations," *American Economic Journal: Macroeconomics*, Vol. 16, pp. 279–310. [12]

DEL NEGRO, MARCO, MARC P. GIANNONI, AND CHRISTINA PATTERSON (2023): "The Forward Guidance Puzzle," *Journal of Political Economy Macroeconomics*, Vol. 1, pp. 43–79. [16, 24]

EGGERTSSON, GAUTI B. AND FINN D. SCHÜLE (2024): "The Forward Guidance Puzzle is not a Puzzle," *Working Paper*. [24]

FATÁS, ANTONIO (2000): "Do Business Cycles Cast Long Shadows? Short-Run Persistence and Economic Growth," *Journal of Economic Growth*, Vol. 5, pp. 147–162. [6]

FATÁS, ANTONIO AND LAWRENCE H. SUMMERS (2018): "The Permanent Effects of Fiscal Consolidations," *Journal of International Economics*, Vol. 112, pp. 238–250. [6]

FERRIERE, AXELLE, PHILIPP GRÜBENER, GASTON NAVARRO, AND OLIKO VARDISHVILI (2023): "On the optimal design of transfers and income tax progressivity," *Journal of Political Economy Macroeconomics*, Vol. 1, pp. 276–333. [26, 28, 82, 89]

FERRIERE, AXELLE AND GASTON NAVARRO (2025): "The Heterogeneous Effects of Government Spending: It's All About Taxes," *The Review of Economic Studies*, Vol. 92, pp. 1061–1125. [6, 27, 28, 31, 89]

FORNARO, LUCA AND MARTIN WOLF (2023): "The Scars of Supply Shocks: Implications for Monetary Policy," *Journal of Monetary Economics*, Vol. 140, pp. 18–36. [5, 12, 54]

——— (2025): "Fiscal Stagnation," *Working Paper*. [5]

GABAIX, XAVIER (2020): "A Behavioral New Keynesian Model," *The American Economic Review*, Vol. 110, pp. 2271–2327. [16, 24]

GAILLARD, ALEXANDRE, CHRISTIAN HELLWIG, PHILIPP WANGNER, AND NICOLAS WERQUIN (2023): "Consumption, Wealth, and Income Inequality: A Tale of Tails," *SSRN Working Paper 4636704*. [13]

GALÍ, JORDI (2015): "Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework and its Applications," *Princeton University Press*. [16]

GALÍ, JORDI, J. DAVID LÓPEZ-SALIDO, AND JAVIER VALLÉS (2007): "Understanding the Effects of Government Spending on Consumption," *Journal of the European Economic Association*, Vol. 5, pp. 227–270. [5, 23]

GANONG, PETER AND PASCAL NOEL (2019): "Consumer Spending during Unemployment: Positive and Normative Implications," *The American Economic Review*, Vol. 109, pp. 2383–2424. [29]

GARGA, VAISHALI AND SANJAY R. SINGH (2021): "Output Hysteresis and Optimal Monetary Policy," *Journal of Monetary Economics*, Vol. 117, pp. 871–886. [5]

GUVENEN, FATIH, SERDAR OZKAN, AND JAE SONG (2014): "The Nature of Countercyclical Income Risk," *Journal of Political Economy*, Vol. 122, pp. 621–660. [10]

GUVENEN, FATIH, SAM SCHULHOFER-WOHL, JAE SONG, AND MOTOHIRO YOGO (2017): "Worker Betas: Five Facts About Systematic Earnings Risk," *The American Economic Review*, Vol. 107, pp. 398–403. [10]

HAGEDORN, MARCUS, IOURII MANOVSKII, AND KURT MITMAN (2019): "The Fiscal Multiplier," *NBER Working Paper*. [6]

HEATHCOTE, JONATHAN, KJETIL STORESLETTEN, AND GIOVANNI L. VIOLANTE (2010): "The Macroeconomic Implications of Rising Wage Inequality in the United States," *Journal of Political Economy*, Vol. 118, pp. 681–722. [10, 89]

HOLM, MARTIN BLOMHOFF, PASCAL PAUL, AND ANDREAS TISCHBIREK (2021): "The Transmission of Monetary Policy under the Microscope," *Journal of Political Economy*, Vol. 129, pp. 2861–2904. [10]

ILZETZKI, ETHAN (2024): "Learning by Necessity: Government Demand, Capacity Constraints, and Productivity Growth," *The American Economic Review*, Vol. 114, pp. 2436–2471. [6]

JAIMOVICH, NIR AND SERGIO REBELO (2017): "Nonlinear Effects of Taxation on Growth," *Journal of Political Economy*, Vol. 125, pp. 265–291. [6]

JONES, CHARLES I. (2022): "Taxing Top Incomes in a World of Ideas," *Journal of Political Economy*, Vol. 130, pp. 2227–2274. [6, 27, 29]

JORDÁ, ÓSCAR, SANJAY R. SINGH, AND ALAN M. TAYLOR (2024): "The Long-Run Effects of Monetary Policy," *The Review of Economics and Statistics*, pp. 1–49. [6]

KAPLAN, GREG, BENJAMIN MOLL, AND GIOVANNI L. VIOLANTE (2018): "Monetary Policy According to HANK," *The American Economic Review*, Vol. 108, pp. 697–743. [5, 23]

KEKRE, ROHAN AND MORITZ LENEL (2022): "Monetary Policy, Redistribution and Risk Premia," *Econometrica*, Vol. 90, pp. 2249–2282. [5, 19]

KRAMER, JOHN V. (2022): "The Cyclicality of Earnings Growth along the Distribution - Causes and Consequences," *Working Paper*. [10]

KRUSELL, PER, TOSHIHIKO MUKOYAMA, RICHARD ROGERSON, AND AYŞEGÜL ŞAHIN (2017): "Gross Worker Flows over the Business Cycle," *The American Economic Review*, Vol. 107, pp. 3447–76. [27, 84, 89]

KRUSELL, PER, TOSHIHIKO MUKOYAMA, AND ANTHONY A. SMITH (2011): "Asset Prices in a Huggett Economy," *Journal of Economic Theory*, Vol. 146, pp. 812–844. [9]

KRUSELL, PER AND ANTHONY A. SMITH (1998): "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, Vol. 106, pp. 867–896. [27]

LASALLE, JOSEPH P. (1986): "The Stability and Control of Discrete Processes," *Applied Mathematical Science*, Vol. 62, Springer, New York. [59]

LUETTICKE, RALPH (2021): "Transmission of Monetary Policy with Heterogeneity in Household Portfolios," *American Economic Journal: Macroeconomics*, Vol. 13, pp. 1–25. [5, 19]

MCKAY, ALISDAIR, EMI NAKAMURA, AND JÓN STEINSSON (2016): "The Power of Forward Guidance Revisited," *The American Economic Review*, Vol. 106, pp. 3133–3158. [5]

MELCANGI, DAVIDE AND VINCENT STERK (2025): "Stock Market Participation, Inequality, and Monetary Policy," *The Review of Economic Studies*, Vol. 92, pp. 2656–2690. [5]

MICHAILLAT, PASCAL AND EMMANUEL SAEZ (2021): "Resolving New Keynesian Anomalies with Wealth in the Utility Function," *The Review of Economics and Statistics*, Vol. 103, pp. 197—215. [16]

MORAN, PATRICK AND ALBERT QUERALTO (2018): "Innovation, Productivity, and Monetary Policy," *Journal of Monetary Economics*, Vol. 93, pp. 24–41. [6]

OH, HYUNSEUNG AND RICARDO REIS (2012): "Targeted Transfers and the Fiscal Response to the Great Recession," *Journal of Monetary Economics*, Vol. 59, pp. 50–64. [6]

PATTERSON, CHRISTINA (2023): "The Matching Multiplier and the Amplification of Recessions," *The American Economic Review*, Vol. 113, pp. 982—1012. [10]

PFÄUTI, OLIVER AND FABIAN SEYRICH (2022): "A Behavioral Heterogeneous Agent New Keynesian Model," *Working Paper*. [24]

PIKETTY, THOMAS AND EMMANUEL SAEZ (2014): "Inequality in the Long Run," *Science*, Vol. 344, pp. 838–843. [27, 89]

QUERALTO, ALBERT (2020): "A Model of Slow Recoveries from Financial Crises," *Journal of Monetary Economics*, Vol. 114, pp. 1–25. [5]

——— (2022): "Monetary Policy in a Model of Growth," *Working Paper*. [5]

DE RASSENFOSSE, GAÉTAN AND ADAM B JAFFE (2018): "Econometric evidence on the depreciation of innovations," *European Economic Review*, Vol. 101, pp. 625–642. [27, 89]

RAVN, MORTEN O. AND VINCENT STERK (2020): "Macroeconomic Fluctuations with HANK & SAM: An Analytical Approach," *Journal of the European Economic Association*, Vol. 19, pp. 1162–1202. [5]

ROTEMBERG, JULIO J. (1982): "Sticky Prices in the United States," *Journal of Political Economy*, Vol. 90, pp. 1187–1211. [10]

ROTH, CHRISTOPHER, MIRKO WIEDERHOLT, AND JOHANNES WOHLFART (2023): "The Effects of Monetary Policy: Theory with Measured Expectations," *Working Paper*. [24]

SEIDL, HANNAH AND FABIAN SEYRICH (2023): "Unconventional Fiscal Policy in a Heterogeneous-Agent New Keynesian Model," *Journal of Political Economy Macroeconomics*, Vol. 1, pp. 633–664. [24]

STADLER, GEORGE W. (1990): "Business Cycle Models with Endogenous Technology," *The American Economic Review*, Vol. 80, pp. 763–778. [5]

STORESLETTEN, KJETIL, CHRIS I. TELMER, AND AMIR YARON (2004): "Cyclical Dynamics in Idiosyncratic Labor Market Risk," *Journal of Political Economy*, Vol. 112, pp. 695–717. [10, 27, 89]

STRAUB, LUDWIG (2019): "Consumption, Savings, and the Distribution of Permanent Income," *Unpublished Manuscript, Harvard University*. [13]

WERNING, IVÁN (2015): "Incomplete Markets and Aggregate Demand," *Working Paper*. [5, 10]

WOLF, CHRISTIAN K. (2025): "Interest Rate Cuts versus Stimulus Payments: An Equivalence Result," *Journal of Political Economy*, Vol. 133, pp. 1235–1275. [24]

WOODFORD, MICHAEL (2011): "Simple Analytics of the Government Expenditure Multiplier," *American Economic Journal: Macroeconomics*, Vol. 3, pp. 1–35. [12]

ONLINE APPENDIX (NOT FOR PUBLICATION)

Inequality, Business Cycles, and Growth

A Unified Theory of Fiscal Stabilization Policies

The Online Appendix is organized as follows. Appendix A contains detailed derivations of the analytical HANK-G framework, Appendix B provides proofs, and Appendix C incorporates supplementary theoretical results. Finally, Appendix D contains a full description of the quantitative model environment, details the model calibration, shows further impulse response functions, derives the welfare criterion and explains the computational algorithm.

APPENDIX A: Analytical HANK-G Model - Derivations

A.1 Competitive Equilibrium

In Appendix A.1, we characterize the first order conditions to the individual household, firm, and union maximization problems.

A.1.1 *Final and Intermediate Good Firms* The objective of the competitive final good producer is to maximize profits, i.e.,

$$\max_{\{L_t, X_t(j)\}} P_t L_t^{1-\alpha} \int_0^1 A_t(j)^{1-\alpha} X_t(j)^{\alpha} dj - P_t(j) X_t(j) - W_t L_t \,,$$

with corresponding first-order conditions:

$$W_t = (1-\alpha) P_t L_t^{-\alpha} \int_0^1 A_t(j)^{1-\alpha} X_t(j)^{\alpha} dj \,,$$

$$P_t(j) = \alpha P_t L_t^{1-\alpha} A_t(j)^{1-\alpha} X_t(j)^{\alpha-1} \,.$$

Using both conditions, the price-setting problem of an intermediate firm j becomes

$$\max_{\{P_t(j)\}} \Theta_t^n(j) = (P_t(j) - P_t) X_t^d(j) = (P_t(j) - P_t) \left(\frac{P_t(j)}{\alpha P_t} \right)^{\frac{1}{\alpha-1}} L_t A_t(j) \,.$$

The first order condition to this problem is

$$\left(\frac{P_t(j)}{\alpha P_t} \right)^{\frac{1}{\alpha-1}} + \frac{1}{\alpha - 1} (P_t(j) - P_t) \left(\frac{P_t(j)}{\alpha P_t} \right)^{\frac{1}{\alpha-1}-1} \frac{1}{\alpha P_t} = 0 \,,$$

which can be rearranged to $\alpha P_t(j) = P_t$. As a result, one obtains $X_t(j) = \alpha^{\frac{2}{1-\alpha}} L_t A_t(j)$ and consequently $Y_t^G = \alpha^{\frac{2\alpha}{1-\alpha}} L_t A_t$. As such, nominal profits are $\Theta_t^n(j) = \Theta_\alpha P_t L_t A_t(j)$, where $\Theta_\alpha \equiv \alpha^{-1}(1-\alpha)\alpha^{\frac{2}{1-\alpha}}$. Finally, real profits are given by $\Theta_t(j) = \Theta_\alpha L_t A_t(j)$ and returns to innovation $R_t^A(j) = \Theta_t(j)/A_t(j) = \Theta_\alpha L_t = R_t^A$ for each j.

A.1.2 *Households* The first-order conditions for savers are

$$\begin{aligned}
\mathcal{U}_c(C_t^S, L_t^S) &= \Lambda_t^S \,, \\
\Lambda_t^S &= \beta \mathbb{E}_t \left[(1-\lambda) s V_B^S(B_{t+1}^S, \omega_{t+1}^S, A_{t+1}^S) + \lambda(1-s) V_B^H(B_{t+1}^H) \right] + \Xi_t^S \,, \\
\Lambda_t^S \frac{q_t}{1-\lambda} &= \beta \mathbb{E}_t \left[V_\omega^S(B_{t+1}^S, \omega_{t+1}^S, A_{t+1}^S) \right] \,, \\
\Lambda_t^S &= \psi \Omega_t^S \,, \\
\Omega_t^S &= \beta \mathbb{E}_t \left[V_A^S(B_{t+1}^S, \omega_{t+1}^S, A_{t+1}^S) \right] \,,
\end{aligned}$$

together with the complementary slackness condition $\Xi_t^S b_{t+1}^S = 0$, with $\Xi_t^S \geq 0$. Note that Λ_t^S, Ω_t^S and Ξ_t^S are Lagrange multipliers associated with the budget constraint, the LOM of capital, and the individual bond constraint. The Envelope theorem implies

$$\begin{aligned}
V_B^S(B_t^S, \omega_t^S, A_t^S) &= \frac{\Lambda_t^S}{1-\lambda} \frac{R_{t-1}}{\pi_t} \,, \\
V_\omega^S(B_t^S, \omega_t^S, A_t^S) &= \frac{\Lambda_t^S}{1-\lambda} (q_t + D_t) \,, \\
V_A^S(B_t^S, \omega_t^S, A_t^S) &= \Lambda_t^S R_t^A + \Omega_t^S \,.
\end{aligned}$$

Moreover, the first-order conditions for hand-to-mouth households are

$$\begin{aligned}
\mathcal{U}_c(C_t^H, L_t^H) &= \Lambda_t^H \,, \\
\Lambda_t^H &= \beta \mathbb{E}_t \left[\lambda h V_B^H(B_{t+1}^H) + (1-\lambda)(1-h) V_B^S(B_{t+1}^S, \omega_{t+1}^S, A_{t+1}^S) \right] + \Xi_t^H \,,
\end{aligned}$$

together with the complementary slackness condition $\Xi_t^H b_{t+1}^H = 0$, where $\Xi_t^H \geq 0$. The Envelope theorem, in turn, implies

$$V_B^H(B_t^H) = \frac{\Lambda_t^H}{\lambda} \frac{R_{t-1}}{\pi_t} \,.$$

Using the previous expressions, saver and hand-to-mouth FOC's can be rewritten as

$$\begin{aligned}
\frac{1}{C_t^i} &= \Lambda_t^i \,, \quad \text{for} \quad i \in \{S, H\} \,, \\
\Lambda_t^H &= \beta \mathbb{E}_t \left[\frac{R_t}{\pi_{t+1}} \left(h \Lambda_{t+1}^H + (1-h) \Lambda_{t+1}^S \right) \right] + \Xi_t^H \,, \\
\Lambda_t^S &= \beta \mathbb{E}_t \left[\frac{R_t}{\pi_{t+1}} \left(s \Lambda_{t+1}^S + (1-s) \Lambda_{t+1}^H \right) \right] + \Xi_t^S \,, \\
\Lambda_t^S q_t &= \beta \mathbb{E}_t \left[\Lambda_{t+1}^S (q_{t+1} + D_{t+1}) \right] \\
\Lambda_t^S &= \psi \Omega_t^S \,, \\
\Omega_t^S &= \beta \mathbb{E}_t \left[\Lambda_{t+1}^S R_{t+1}^A + \Omega_{t+1}^S \right] \,.
\end{aligned}$$

Assumption 1 implies $\Xi_t^H > 0$ and $\Xi_t^S = 0$. As a result, H households are strictly hand-to-mouth and consume their disposable income in every period, while the Euler equation of savers hods with equality, i.e.,

$$\frac{1}{C_t^S} = \beta \mathbb{E}_t \left[\frac{R_t}{\pi_{t+1}} \left(s \frac{1}{C_{t+1}^S} + (1-s) \frac{1}{C_{t+1}^H} \right) \right] .$$

Finally, the optimal innovative investment decision is given by

$$1 = \beta \mathbb{E}_t \left[\frac{C_t^S}{C_{t+1}^S} \left(1 + \psi R_{t+1}^A \right) \right] .$$

A.1.3 *Union Wage Setting* Differentiated labor inputs are bundled according to a CES aggregator that has the usual form

$$L_t = \left(\int_0^1 L_t(l)^{\frac{\epsilon_w - 1}{\epsilon_w}} dl \right)^{\frac{\epsilon_w}{\epsilon_w - 1}} , \quad \text{with} \quad \epsilon_w > 1 .$$

The final good firm's cost minimization problem is

$$\min_{\{L_t(l)\}_l} \int_0^1 W_t(l) L_t(l) \qquad \text{subject to} \qquad L_t = \left(\int_0^1 L_t(l)^{\frac{\epsilon_w - 1}{\epsilon_w}} dl \right)^{\frac{\epsilon_w}{\epsilon_w - 1}} .$$

Taking the first order conditions and following the standard steps provides us with the following labor demand for variety l

$$L_t(l) = \left(\frac{W_t(l)}{W_t} \right)^{-\epsilon_w} L_t^d ,$$

where the aggregate wage index is defined by

$$W_t \equiv \left(\int_0^1 W_t(l)^{1-\epsilon_w} dl \right)^{\frac{1}{1-\epsilon_w}} .$$

The maximization objective of union l is then given by

$$\max_{\{W_t(l)\}} \sum_{t=0}^{\infty} \beta^t \left[\log C_t(l) - \nu \frac{(L_t(l))^{1+\varphi}}{1+\varphi} \right]$$

subject to

$$L_t(l) = \left(\frac{W_t(l)}{W_t} \right)^{-\epsilon_w} L_t^d ,$$

$$C_t(l) + I_t(l) = \frac{W_t(l)}{P_t} L_t(l) + R_t^A A_t(l) + D_t(l) - T_t(l) - \frac{\theta}{2} \left(\frac{1}{g^A} \frac{W_t(l)}{W_{t-1}(l)} - 1 \right)^2 A_t ,$$

where we have imposed the equilibrium condition for shares, i.e., $\omega_t = 1 \; \forall t$, respectively bonds $b^S_{t+1} = b^H_{t+1} = 0$. The nominal labor income of the average household is

$$W_t(l)L_t(l) = \frac{W_t(l)^{1-\epsilon_w}}{W_t^{-\epsilon_w}} L_t^d \, .$$

The first-order condition is

$$\frac{1}{C_t(l)} \left[\frac{(1-\epsilon_w)L_t(l)}{P_t} - \theta \left(\frac{1}{g^A} \frac{W_t(l)}{W_{t-1}(l)} - 1 \right) \frac{1}{g^A} \frac{A_t}{W_{t-1}(l)} \right] + \nu \epsilon_w \frac{L_t(l)^{1+\varphi}}{W_t(l)}$$
$$+ \beta\theta \mathbb{E}_t \left[\frac{1}{C_{t+1}(l)} \left(\frac{1}{g^A} \frac{W_{t+1}(l)}{W_t(l)} - 1 \right) \frac{1}{g^A} \frac{W_{t+1}(l)}{(W_t(l))^2} A_{t+1} \right] = 0 \, .$$

As unions face identical first-order conditions, we presuppose a symmetric equilibrium with $W_t(l) = W_t$ and, thus, $L_t = L_t(l)$. Multiplying through with W_t and defining gross nominal wage inflation as $\pi^w_t \equiv \frac{W_t}{W_{t-1}}$ provides us with

$$\left(\frac{\pi^w_t}{g^A} - 1 \right) \frac{\pi^w_t}{g^A} - \beta \mathbb{E}_t \left[\frac{C_t}{C_{t+1}} \frac{A_{t+1}}{A_t} \left(\frac{\pi^w_{t+1}}{g^A} - 1 \right) \frac{\pi^w_{t+1}}{g^A} \right] = \frac{\epsilon_w}{\theta} L_t \left(\nu L_t^{\varphi} \frac{C_t}{A_t} - \frac{\epsilon_w - 1}{\epsilon_w} \frac{W_t}{P_t A_t} \right) .$$

A.1.4 *Inflation Dynamics* By substituting the expression for $X_t(j)$ into the final good producer's first order condition for labor, we obtain

$$P_t = \frac{1}{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}} \frac{W_t}{A_t} \, ,$$

such that price inflation, wage inflation, and technology growth are related as follows

$$\pi_t = \frac{\pi^w_t}{g^A_t} \, .$$

A.2 Stationary Equilibrium

After normalizing all trending variables by the level of aggregate technology, the stationary equilibrium is described by the following equations.

Saver Euler Equation	$\frac{g^A_{t+1}}{c^S_t} = \beta \mathbb{E}_t \left[\frac{R_t}{\pi_{t+1}} \left(s \frac{1}{c^S_{t+1}} + (1-s) \frac{1}{c^H_{t+1}} \right) \right]$
Growth Equation	$g^A_{t+1} = \beta \mathbb{E}_t \left[\frac{c^S_t}{c^S_{t+1}} \left(1 + \psi R^A_{t+1} \right) \right]$
Real Wage	$w_t = (1-\alpha) \alpha^{\frac{2\alpha}{1-\alpha}}$
NK Wage Phillips Curve	$\left(\frac{\pi^w_t}{g^A} - 1 \right) \frac{\pi^w_t}{g^A} - \beta \mathbb{E}_t \left[\frac{c_t}{c_{t+1}} \left(\frac{\pi^w_{t+1}}{g^A} - 1 \right) \frac{\pi^w_{t+1}}{g^A} \right] = \frac{\epsilon_w}{\theta} L_t \left(\nu L^\varphi_t c_t - \frac{\epsilon_w - 1}{\epsilon_w} w_t \right)$
Fiscal Transfers	$t^H_t = -t^S_t$
Individual Hours Worked	$L^H_t = (1-\mu) L + \mu L_t \, , \quad L^S_t = \frac{\lambda(\mu-1)}{1-\lambda} L + \frac{1-\lambda\mu}{1-\lambda} L_t$
Aggregate Hours Worked	$L_t = \lambda L^H_t + (1-\lambda) L^S_t$
HtM Consumption	$c^H_t = w_t L^H_t - \frac{t^H_t}{\lambda}$
Saver Consumption	$c^S_t = w_t L^S_t + \frac{R^A_t + d_t - \iota^A_t - t^S_t}{1-\lambda}$
Dividends	$d_t = 0$
Innovative Return	$R^A_t = \Theta_\alpha L_t$
Aggregate Consumption	$\lambda c^H_t + (1-\lambda) c^S_t = c_t$
LOM Technology Growth	$g^A_{t+1} = 1 + \psi \iota^A_t$
Monetary Policy Rule	$R_t = R \mathbb{E}_t \left[\frac{\pi_{t+1}}{\pi} \right] \left(\frac{Y_t / A_t}{Y / A} \right)^{\phi_y}$
Intermediary Inputs	$x_t = \alpha^{\frac{2}{1-\alpha}} L_t$
Gross Output	$y^G_t = \alpha^{\frac{2\alpha}{1-\alpha}} L_t$
Output	$y_t = Y_\alpha L_t$
Resource Constraint	$y_t = c_t + \iota^A_t + \frac{\theta}{2} \left(\frac{\pi^w_t}{g^A} - 1 \right)^2$
Inflation Dynamics	$\pi_t = \frac{\pi^w_t}{g^A_t}$

A.3 Steady State

We consider a full employment steady state ($L > 0$) with positive technology growth (see the proof of Result 1). We assume $\pi = 1$, which implies $\pi^w = g^A$. To begin with, we determine hours worked L using the following equations:

$$
\begin{aligned}
g^A &= \beta\left(1 + \psi\Theta_\alpha L\right) , \\
\nu L^\varphi c &= \frac{\epsilon_w - 1}{\epsilon_w}(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} , \\
Y_\alpha L &= c + \frac{g^A - 1}{\psi} .
\end{aligned}
$$

As a result, hours worked are implicitly characterized by

$$
\nu L^\varphi \left(\left(Y_\alpha - \beta\Theta_\alpha\right) L + \frac{1-\beta}{\psi}\right) = \frac{\epsilon_w - 1}{\epsilon_w}(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} . \tag{5}
$$

To see whether there exists a unique $L > 0$ that satisfies (5), notice that

$$
Y_\alpha - \beta\Theta_\alpha = \alpha^{\frac{2\alpha}{1-\alpha}}(1-\alpha^2)\left(1 - \beta\frac{\alpha}{1+\alpha}\right) , \tag{6}
$$

which is strictly positive because of $\alpha \in (0,1)$ and $\beta < 1$. The left-hand side of (5) is, therefore, strictly increasing in L. Moreover, it is zero for $L = 0$. As a result, there exists a unique positive L that satisfies (5). Based on L, one can determine all remaining steady state values recursively. Steady-state hand-to-mouth and saver consumption, for instance, are given by

$$
\begin{aligned}
c^H &= (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} L - \frac{t^H}{\lambda} \\
c^S &= (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} L \left(1 + \alpha\frac{1-\beta}{1-\lambda}\right) + \frac{1-\beta}{(1-\lambda)\psi} - \frac{t^S}{1-\lambda} .
\end{aligned}
$$

Steady-state hand-to-mouth and saver consumption are positive if

$$
\begin{aligned}
t^H < \bar{t}^H &\equiv \lambda \cdot (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} L > 0 , \\
t^S < \bar{t}^S &\equiv (1-\lambda) \cdot (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} L \left(1 + \alpha\frac{1-\beta}{1-\lambda}\right) + \frac{1-\beta}{\psi} > 0 .
\end{aligned}
$$

There exists a lump-sum redistribution t^E from savers to hand-to-mouth households that equates their steady state consumption levels, i.e., $\Gamma_c = c^S/c^H = 1$. It is given by

$$
t^E = \lambda \cdot (1-\beta)\left[\alpha(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} L + \frac{1}{\psi}\right] < \bar{t}^S .
$$

Finally, steady-state growth is positive, i.e., $g^A > 1$, if

$$
\beta\left(1 + \psi\Theta_\alpha L\right) > 1 \quad \Leftrightarrow \quad L > \frac{\beta^{-1} - 1}{\psi\Theta_\alpha} .
$$

A.4 Log-Linear Stationary Equilibrium

Let us define the following auxiliary parameters:

$$\tilde{s} \equiv \frac{s}{s+(1-s)\Gamma_c}\,, \quad \mathscr{M} \equiv \frac{g^A-\beta}{g^A}\,, \quad \kappa \equiv \frac{\epsilon_w-1}{\theta}(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}L\,, \quad \gamma_c \equiv \frac{c}{y}\,.$$

Log-linearizing the conditions from Appendix A.2 around the steady state, we obtain[26]

Saver Euler Equation	$\hat{c}_t^S = -(\hat{i}_t - \mathbb{E}_t[\hat{\pi}_{t+1}]) + \tilde{s}\mathbb{E}_t\left[\hat{c}_{t+1}^S\right] + (1-\tilde{s})\mathbb{E}_t\left[\hat{c}_{t+1}^H\right] + \hat{g}_{t+1}^A$
Growth Equation	$\hat{g}_{t+1}^A = \hat{c}_t^S - \mathbb{E}_t\left[\hat{c}_{t+1}^S\right] + \mathscr{M}\mathbb{E}_t\left[\hat{R}_{t+1}^A\right]$
Real Wage	$\hat{w}_t = 0$
NK Wage Phillips Curve	$\hat{\pi}_t^w = \beta\mathbb{E}_t\left[\hat{\pi}_{t+1}^w\right] + \kappa\left(\varphi\hat{L}_t + \hat{c}_t\right)$
Fiscal Transfers	$\hat{t}_t^H = -\hat{t}_t^S = -\hat{\tau}_t$
Individual Hours Worked	$\hat{L}_t^H = \mu\hat{L}_t\,, \quad \hat{L}_t^S = \frac{1-\lambda\mu}{1-\lambda}\hat{L}_t$
Aggregate Hours Worked	$\hat{L}_t = \lambda\hat{L}_t^H + (1-\lambda)\hat{L}_t^S$
Hand-to-Mouth Consumption	$\hat{c}_t^H = \frac{wL}{c^H}\left(\hat{w}_t + \hat{L}_t^H\right) - \frac{1}{\lambda}\frac{t^H}{c^H}\hat{t}_t^H$
Saver Consumption	$\hat{c}_t^S = \frac{wL}{c^S}\left(\hat{w}_t + \hat{L}_t^S\right) + \frac{1}{1-\lambda}\left(\frac{R^A}{c^S}\hat{R}_t^A + \frac{y}{c^S}\hat{d}_t - \frac{\iota^A}{c^S}\hat{\iota}_t^A - \frac{t^S}{c^S}\hat{t}_t^S\right)$
Dividends	$\hat{d}_t = 0$
Innovative Return	$\hat{R}_t^A = \hat{L}_t$
Aggregate Consumption	$\lambda c^H\hat{c}_t^H + (1-\lambda)c^S\hat{c}_t^S = c\hat{c}_t$
LOM Technology Growth	$\hat{g}_{t+1}^A = \frac{g^A-1}{g^A}\hat{\iota}_t^A$
Monetary Policy Rule	$\hat{i}_t = \mathbb{E}_t[\hat{\pi}_{t+1}] + \phi_y\hat{y}_t$
Intermediary Inputs	$\hat{x}_t = \hat{L}_t$
Gross Output / Output	$\hat{y}_t^G = \hat{y}_t = \hat{L}_t$
Resource Constraint	$\hat{y}_t = \gamma_c\hat{c}_t + (1-\gamma_c)\hat{\iota}_t^A$
Inflation Dynamics	$\hat{\pi}_t = \hat{\pi}_t^w - \hat{g}_t^A$

[26] We express variables as log deviations from steady state, i.e., $\hat{x}_t = \log x_t - \log x$. We log-linearize the gross rates, which are approximately equal to the net rates. One exception are dividends, which equal zero. We express them as absolute deviations from steady state, relative to steady state output: $\hat{d}_t = (d_t - d)/y$.

APPENDIX B: Analytical Model - Proofs

B.1 Balanced Growth Path - Steady State

B.1.1 *Proof Result 1*

PROOF. The first part is similar to the proof of Proposition 1 in Fornaro and Wolf (2023). A necessary condition for individual saver and hand-to-mouth consumption to be positive is that aggregate consumption is positive. This is the case if

$$c = (Y_\alpha - \beta\Theta_\alpha)\, L + \frac{1-\beta}{\psi} > 0\,,$$

which holds true as $\beta\frac{\alpha}{1+\alpha} < 1$. From Appendix A.3, we know that there exists a positive employment steady state $L > 0$ satisfying

$$\nu L^{\varphi}\left((Y_\alpha - \beta\Theta_\alpha)\, L + \frac{1-\beta}{\psi}\right) = \frac{\epsilon_w - 1}{\epsilon_w}(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}\,, \tag{7}$$

as the left-hand side is strictly increasing in L and takes a zero value for $L = 0$. Provided that $t^H < \bar{t}^H$ and $t^S < \bar{t}^S$ steady state consumption of saver and hand-to-mouth households is positive. Additionally, a positive steady-state growth is guaranteed by

$$L > \frac{\beta^{-1} - 1}{\psi\Theta_\alpha} > 0\,, \tag{8}$$

which is the condition stated in the Proposition. As the left hand side of (7) strictly increases in ν, one can write $L(\nu)$ with $L'(\nu) < 0$. As $\lim_{\nu\to 0} L(\nu) = \infty$ and $\lim_{\nu\to\infty} L(\nu) = 0$, there exists by continuity a unique ν^* such that (8) is satisfied for all $\nu < \nu^*$.[27] □

B.2 Inequality On and Off the Balanced Growth Path

B.2.1 *Inequality on the BGP* To link consumption and income inequality, we use the resource constraint

$$y = \lambda y^H + (1-\lambda)y^S = \lambda c^H + (1-\lambda)c^S + \frac{g^A - 1}{\psi}\,.$$

Rearranging the previous equation by using $c^H = y^H$, we obtain

$$\begin{aligned}
& y^S = c^S + \frac{1}{1-\lambda}\frac{g^A-1}{\psi} \\
\Leftrightarrow\ & \Gamma_y = \frac{c^S}{y^H} + \frac{1}{1-\lambda}\frac{g^A-1}{\psi}\frac{1}{y^H} = \Gamma_c + \frac{1}{1-\lambda}\frac{g^A-1}{\psi y}\frac{y}{y^H} = \Gamma_c + \frac{1-\gamma_c}{1-\lambda}\frac{y}{y^H} = \Gamma_c + \frac{1-\gamma_c}{1-\lambda}\left(\lambda + (1-\lambda)\Gamma_y\right) \\
\Leftrightarrow\ & \Gamma_y = \frac{\Gamma_c}{\gamma_c} + \frac{\lambda}{1-\lambda}\frac{1-\gamma_c}{\gamma_c}\,.
\end{aligned}$$

As a result, this implies $\Gamma_y > \Gamma_c$ if $\gamma_c < 1$.

[27] The wage NKPC also admits a steady state with $L = 0$ that leads, however, to negative technology growth $g^A < 1$ and positive hand-to-mouth consumption only if $t^H < 0$. It is therefore ruled out, i.e., the positive employment steady state is the unique steady state consistent with positive technology growth.

B.2.2 *Cyclical Hand-to-Mouth Consumption* Hand-to-mouth consumption writes

$$c_t^H = w_t L_t^H - \frac{t_t^H}{\lambda} = (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} L_t^H + \frac{T}{\lambda} e^{\hat{\tau}_t} .$$

Log-linearization provides us with

$$\hat{c}_t^H = \frac{1}{c^H}\left(wL^H \hat{L}_t^H + \frac{T}{\lambda}\hat{\tau}_t\right) = \frac{wL^H}{c^H}\mu\hat{L}_t + \frac{1}{c^H}\frac{T}{\lambda}\hat{\tau}_t = \eta\mu\hat{y}_t + (1-\eta)\hat{\tau}_t ,$$

where we have defined the auxiliary parameter η by

$$\eta \equiv \frac{wL^H}{c^H} = \frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} L}{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} L + \frac{T}{\lambda}} ,$$

where $\eta \in (0,1)$ if $T > 0$.

B.2.3 *Cyclical Income Inequality* The aggregate income identity is given by

$$\lambda y_t^H + (1-\lambda) y_t^S = y_t ,$$

which can be written in log-linear terms as

$$\hat{y}_t^S = \frac{y\hat{y}_t - \lambda y^H \hat{y}_t^H}{(1-\lambda)y^S} .$$

Subtracting hand-to-mouth income results in

$$\hat{y}_t^S - \hat{y}_t^H = \frac{y\hat{y}_t - (\lambda y^H + (1-\lambda)y^S)\hat{y}_t^H}{(1-\lambda)y^S} = \frac{1}{1-\lambda}\frac{y}{y^S}\left(\hat{y}_t - \hat{y}_t^H\right) .$$

Finally, substituting for $\hat{y}_t^H = \eta\mu\hat{y}_t + (1-\eta)\hat{\tau}_t$ yields

$$\hat{y}_t^S - \hat{y}_t^H = \frac{1}{1-\lambda}\frac{y}{y^S}\left((1-\eta\mu)\hat{y}_t - (1-\eta)\hat{\tau}_t\right) ,$$

which corresponds to the expression stated in the main text.

B.3 Four-Equation Representation

B.3.1 *Proof Proposition 1*

PROOF. We proceed in two steps: first, we compute consumption of savers $\hat{c}_t^S$ as a function of output $\hat{y}_t$ and technology growth $\hat{g}_{t+1}^A$. Second, we substitute consumption of hand-to-mouth and saver households into the Euler equation. To begin with, we have

$$\hat{c}_t^S = \frac{c\hat{c}_t - \lambda c^H \hat{c}_t^H}{(1-\lambda)c^S} = \left(1 + \frac{\lambda}{1-\lambda}\frac{1}{\Gamma_c}\right)\hat{c}_t - \frac{\lambda}{1-\lambda}\frac{1}{\Gamma_c}\hat{c}_t^H .$$

The aggregate resource constraint implies $\hat{y}_t = \gamma_c \hat{c}_t + (1-\gamma_c)\frac{g^A}{g^A-1}\hat{g}^A_{t+1}$. Substituting in for $\hat{c}_t$ and $\hat{c}^H_t$, we obtain

$$\begin{aligned}\hat{c}^S_t &= \left(1+\frac{\lambda}{1-\lambda}\frac{1}{\Gamma_c}\right)\left(\frac{\hat{y}_t}{\gamma_c} - \frac{1-\gamma_c}{\gamma_c}\frac{g^A}{g^A-1}\hat{g}^A_{t+1}\right) - \frac{\lambda}{1-\lambda}\frac{1}{\Gamma_c}\left(\eta\mu\hat{y}_t + (1-\eta)\hat{\tau}_t\right)\\ &= \frac{(1-\lambda)\Gamma_c + \lambda - \lambda\gamma_c\eta\mu}{(1-\lambda)\Gamma_c\gamma_c}\hat{y}_t - \frac{(1-\lambda)\Gamma_c+\lambda}{(1-\lambda)\Gamma_c}\frac{1-\gamma_c}{\gamma_c}\frac{g^A}{g^A-1}\hat{g}^A_{t+1} - \frac{\lambda(1-\eta)}{(1-\lambda)\Gamma_c}\hat{\tau}_t\\ &= \mathcal{E}_y\hat{y}_t - \mathcal{E}_g\hat{g}^A_{t+1} - \mathcal{E}_\tau\hat{\tau}_t\,,\end{aligned}$$

where $\mathcal{E}_y, \mathcal{E}_g$ and $\mathcal{E}_\tau$ denote partial equilibrium elasticities of $\hat{c}^S_t$ with respect to aggregate income, endogenous technology growth and transfers, i.e.,

$$\mathcal{E}_y \equiv \frac{(1-\lambda)\Gamma_c + \lambda - \lambda\gamma_c\chi}{(1-\lambda)\Gamma_c\gamma_c}\,,\quad \mathcal{E}_g \equiv \frac{(1-\lambda)\Gamma_c+\lambda}{(1-\lambda)\Gamma_c}\frac{1-\gamma_c}{\gamma_c}\frac{g^A}{g^A-1}\,,\quad \mathcal{E}_\tau \equiv \frac{\lambda(1-\eta)}{(1-\lambda)\Gamma_c}\,.$$

The saver Euler equation is given by

$$\hat{c}^S_t = \tilde{s}\mathbb{E}_t\left[\hat{c}^S_{t+1}\right] + (1-\tilde{s})\mathbb{E}_t\left[\hat{c}^H_{t+1}\right] - (\hat{i}_t - \mathbb{E}_t\left[\hat{\pi}_{t+1}\right]) + \hat{g}^A_{t+1}\,.$$

Substituting in for the consumption of saver and hand-to-mouth households results in

$$\begin{aligned}\mathcal{E}_y\hat{y}_t - \mathcal{E}_g\hat{g}^A_{t+1} - \mathcal{E}_\tau\hat{\tau}_t = \tilde{s}\mathbb{E}_t\left[\mathcal{E}_y\hat{y}_{t+1} - \mathcal{E}_g\hat{g}^A_{t+2} - \mathcal{E}_\tau\hat{\tau}_{t+1}\right] + (1-\tilde{s})\mathbb{E}_t\left[\chi\hat{y}_{t+1} + (1-\eta)\hat{\tau}_{t+1}\right]\\ - (\hat{i}_t - \mathbb{E}_t\left[\hat{\pi}_{t+1}\right]) + \hat{g}^A_{t+1}\,,\end{aligned}$$

which can be rearranged to

$$\begin{aligned}\hat{y}_t = &\left(\tilde{s} + (1-\tilde{s})\frac{\chi}{\mathcal{E}_y}\right)\mathbb{E}_t\left[\hat{y}_{t+1}\right] - \frac{1}{\mathcal{E}_y}(\hat{i}_t - \mathbb{E}_t\left[\hat{\pi}_{t+1}\right]) + \frac{1+\mathcal{E}_g}{\mathcal{E}_y}\hat{g}^A_{t+1} - \tilde{s}\frac{\mathcal{E}_g}{\mathcal{E}_y}\mathbb{E}_t\left[\hat{g}^A_{t+2}\right]\\ &+ \frac{\mathcal{E}_\tau}{\mathcal{E}_y}\hat{\tau}_t - \frac{\mathcal{E}_\tau}{\mathcal{E}_y}\left(\tilde{s} - (1-\tilde{s})\frac{1-\eta}{\mathcal{E}_\tau}\right)\mathbb{E}_t\left[\hat{\tau}_{t+1}\right]\,.\end{aligned}$$

The former equation can be restated as

$$\hat{y}_t = \zeta_f\mathbb{E}_t\left[\hat{y}_{t+1}\right] - \zeta_r(\hat{i}_t - \mathbb{E}_t\left[\hat{\pi}_{t+1}\right]) + \zeta_g\hat{g}^A_{t+1} - \zeta_{g'}\mathbb{E}_t\left[\hat{g}^A_{t+2}\right] + \zeta_\tau\hat{\tau}_t - \zeta_{\tau'}\mathbb{E}_t\left[\hat{\tau}_{t+1}\right]\,,$$

where the auxiliary parameters are defined as follows:

$$\begin{aligned}\zeta_f \equiv &1 + (1-\tilde{s})\left(\frac{\chi}{\mathcal{E}_y} - 1\right) = 1 + (1-\tilde{s})\frac{(1-\lambda)\Gamma_c\gamma_c\chi - ((1-\lambda)\Gamma_c + \lambda - \lambda\gamma_c\chi)}{(1-\lambda)\Gamma_c + \lambda - \lambda\gamma_c\chi}\\ = &1 + (1-\tilde{s})\frac{(1-\lambda)\Gamma_c+\lambda}{(1-\lambda)\Gamma_c+\lambda-\lambda\gamma_c\chi}(\gamma_c\chi - 1) = 1 + (1-\tilde{s})\frac{\gamma_c\chi - 1}{1-\lambda\chi y^H/y}\,,\end{aligned}$$

with the last equality is due to $\gamma_c = ((1-\lambda)\Gamma_c + \lambda)\frac{y^H}{y}$.

$$\zeta_r \equiv \frac{1}{\mathcal{E}_y} = \frac{(1-\lambda)\Gamma_c\gamma_c}{(1-\lambda)\Gamma_c + \lambda - \lambda\gamma_c\chi}\,,$$

$$\zeta_g \equiv \frac{1+\mathcal{E}_g}{\mathcal{E}_y} = \frac{\dfrac{(1-\lambda)\Gamma_c\gamma_c(g^A-1)}{(1-\lambda)\Gamma_c\gamma_c(g^A-1)} + \dfrac{(1-\lambda)\Gamma_c+\lambda}{(1-\lambda)\Gamma_c}\dfrac{1-\gamma_c}{\gamma_c}\dfrac{g^A}{g^A-1}}{\dfrac{(1-\lambda)\Gamma_c+\lambda-\lambda\gamma_c\chi}{(1-\lambda)\Gamma_c\gamma_c}}$$

$$= \frac{(1-\lambda)\Gamma_c\gamma_c(g^A-1)+((1-\lambda)\Gamma_c+\lambda)(1-\gamma_c)g^A}{(g^A-1)[(1-\lambda)\Gamma_c+\lambda-\lambda\gamma_c\chi]} = \frac{\dfrac{(1-\lambda)\Gamma_c\gamma_c}{(1-\lambda)\Gamma_c+\lambda} + \dfrac{(1-\gamma_c)g^A}{g^A-1}}{1-\lambda\chi y^H/y}$$

$$= \frac{(1-\lambda)c^S/y + g^A/(\psi y)}{1-\lambda\chi y^H/y},$$

where the last equality follows from $1-\gamma_c = \frac{g^A-1}{\psi y}$.

$$\zeta_{g'} \equiv \tilde{s}\frac{\mathcal{E}_g}{\mathcal{E}_y} = \tilde{s}\frac{g^A}{g^A-1}\frac{((1-\lambda)\Gamma_c+\lambda)(1-\gamma_c)}{(1-\lambda)\Gamma_c+\lambda-\lambda\gamma_c\chi} = \tilde{s}\frac{g^A}{\psi y}\frac{1}{1-\lambda\chi y^H/y},$$

$$\zeta_\tau = \frac{\mathcal{E}_\tau}{\mathcal{E}_y} = \frac{\lambda\gamma_c(1-\eta)}{(1-\lambda)\Gamma_c+\lambda-\lambda\gamma_c\chi},$$

$$\zeta_{\tau'} = \frac{\mathcal{E}_\tau}{\mathcal{E}_y}\left(\tilde{s}-(1-\tilde{s})\frac{1-\eta}{\mathcal{E}_\tau}\right) = \frac{\lambda\gamma_c(1-\eta)}{(1-\lambda)\Gamma_c+\lambda-\lambda\gamma_c\chi}\left(\tilde{s}-(1-\tilde{s})\frac{(1-\lambda)\Gamma_c}{\lambda}\right).$$

The expressions stated in the main text follow directly from setting $\Gamma_c = 1$. This concludes the proof of Proposition 1. □

B.3.2 *Proof Proposition 2*

PROOF. From Appendix A.4 the endogenous technology growth equation is given by

$$\hat{g}^A_{t+1} = \hat{c}^S_t - \mathbb{E}_t\left[\hat{c}^S_{t+1}\right] + \mathscr{M}\mathbb{E}_t\left[\hat{R}^A_{t+1}\right].$$

Substituting in for cyclical saver consumption, $\hat{c}^S_t$, we obtain

$$\hat{g}^A_{t+1} = \mathcal{E}_y\hat{y}_t - \mathcal{E}_g\hat{g}^A_{t+1} - \mathcal{E}_\tau\hat{\tau}_t - \mathbb{E}_t\left[\mathcal{E}_y\hat{y}_{t+1} - \mathcal{E}_g\hat{g}^A_{t+2} - \mathcal{E}_\tau\hat{\tau}_{t+1}\right] + \mathscr{M}\mathbb{E}_t\left[\hat{y}_{t+1}\right],$$

where we have used that $\hat{R}^A_t = \hat{L}_t = \hat{y}_t$. Collecting terms provides us with

$$\hat{g}^A_{t+1} = \xi_y\hat{y}_t + \xi_{y'}\mathbb{E}_t\left[\hat{y}_{t+1}\right] + \xi_{g'}\mathbb{E}_t\left[\hat{g}_{t+2}\right] - \xi_\tau\left(\hat{\tau}_t - \mathbb{E}_t\left[\hat{\tau}_{t+1}\right]\right),$$

where we have the following auxiliary variables

$$\xi_y \equiv \frac{\mathcal{E}_y}{1+\mathcal{E}_g}, \quad \xi_{y'} \equiv \frac{\mathscr{M}-\mathcal{E}_y}{1+\mathcal{E}_g}, \quad \xi_{g'} \equiv \frac{\mathcal{E}_g}{1+\mathcal{E}_g}, \quad \xi_\tau \equiv \frac{\mathcal{E}_\tau}{1+\mathcal{E}_g}.$$

Substituting for the expressions of $\mathcal{E}_y, \mathcal{E}_g, \mathcal{E}_\tau$, and $\mathscr{M}$ and setting $\Gamma_c = 1$ provides us with the expressions in the main text. This concludes the proof of Proposition 2. □

B.3.3 *Proof Proposition 3*

PROOF. From Appendix A.4, the log-linear NKPC is given by

$$\hat{\pi}_t^w = \beta \mathbb{E}_t \left[\hat{\pi}_{t+1}^w\right] + \kappa \left(\varphi \hat{L}_t + \hat{c}_t\right) .$$

Substituting in for $\hat{L}_t$ and $\hat{c}_t$ results in

$$\begin{aligned}\hat{\pi}_t^w &= \beta \mathbb{E}_t \left[\hat{\pi}_{t+1}^w\right] + \kappa \left(\varphi \hat{y}_t + \frac{\hat{y}_t - (1-\gamma_c)\dfrac{g^A}{g^A-1}\hat{g}_{t+1}^A}{\gamma_c}\right) \\ &= \beta \mathbb{E}_t \left[\hat{\pi}_{t+1}^w\right] + \kappa \frac{1+\varphi\gamma_c}{\gamma_c}\hat{y}_t - \kappa \frac{1-\gamma_c}{\gamma_c}\frac{g^A}{g^A-1}\hat{g}_{t+1}^A .\end{aligned}$$

As $\hat{\pi}_t = \hat{\pi}_t^w - \hat{g}_t^A$, one finally obtains

$$\begin{aligned}\hat{\pi}_t &= \beta \mathbb{E}_t \left[\hat{\pi}_{t+1}\right] + \kappa \frac{1+\varphi\gamma_c}{\gamma_c}\hat{y}_t + \left(\beta - \kappa \frac{1-\gamma_c}{\gamma_c}\frac{g^A}{g^A-1}\right)\hat{g}_{t+1}^A - \hat{g}_t^A \\ &= \beta \mathbb{E}_t \left[\hat{\pi}_{t+1}\right] + \kappa_y \hat{y}_t + \kappa_g \hat{g}_{t+1}^A - \hat{g}_t^A ,\end{aligned}$$

where the auxiliary parameters are defined as follows

$$\kappa_y \equiv \kappa \frac{1+\varphi\gamma_c}{\gamma_c} , \qquad \text{and} \qquad \kappa_g \equiv \beta - \kappa \frac{1-\gamma_c}{\gamma_c}\frac{g^A}{g^A-1} .$$

□

B.4 Determinacy

B.4.1 *Proof Proposition 4*

PROOF. To derive conditions under which the model has a determinate, locally unique rational expectations equilibrium, one can neglect all exogenous shocks. Based on Propositions 1-3, the dynamic system can thus be rewritten as

$$\begin{aligned}\hat{y}_t &= \zeta_f \mathbb{E}_t \left[\hat{y}_{t+1}\right] - \zeta_r \hat{r}_t + \zeta_g \hat{g}_{t+1}^A - \zeta_{g'} \mathbb{E}_t \left[\hat{g}_{t+2}^A\right] , \\ \hat{g}_{t+1}^A &= \Phi_y \hat{y}_t + \Phi_{y_+} \mathbb{E}_t \left[\hat{y}_{t+1}\right] + \Phi_{g_+} \mathbb{E}_t \left[\hat{g}_{t+2}^A\right] , \\ \hat{r}_t &= \phi_y \hat{y}_t .\end{aligned}$$

We proceed in three steps. First, we derive a reduced form system for the forward-looking variables $\hat{y}_t$ and $\hat{g}_{t+1}^A$. Second, we determine signs of the eigenvalue conditions. Third, we state conditions on ϕ_y ensuring that both eigenvalues lie inside the unit circle.

Step 1: Derivation Reduced Form System.

Substituting the real interest rate rule into the aggregate IS equation, we obtain

$$\begin{aligned}\hat{y}_t &= \zeta_f \mathbb{E}_t\left[\hat{y}_{t+1}\right] - \phi_y \zeta_r \hat{y}_t + \zeta_g \hat{g}^A_{t+1} - \zeta_{g'} \mathbb{E}_t\left[\hat{g}^A_{t+2}\right] \\ \Leftrightarrow \hat{y}_t &= \eta_0 \mathbb{E}_t\left[\hat{y}_{t+1}\right] + \eta_1 \hat{g}^A_{t+1} - \eta_2 \mathbb{E}_t\left[\hat{g}^A_{t+2}\right] ,\end{aligned}$$

where we have defined the following auxiliary parameters

$$\eta_0 = \frac{\zeta_f}{1+\phi_y\zeta_r} , \quad \eta_1 = \frac{\zeta_g}{1+\phi_y\zeta_r} , \quad \eta_2 = \frac{\zeta_{g'}}{1+\phi_y\zeta_r} .$$

Substituting the former equation into the endogenous growth equation, one obtains

$$\begin{aligned}\hat{g}^A_{t+1} &= \Phi_y\left(\eta_0 \mathbb{E}_t\left[\hat{y}_{t+1}\right] + \eta_1 \hat{g}^A_{t+1} - \eta_2 \mathbb{E}_t\left[\hat{g}^A_{t+2}\right]\right) + \Phi_{y_+} \mathbb{E}_t\left[\hat{y}_{t+1}\right] + \Phi_{g_+} \mathbb{E}_t\left[\hat{g}^A_{t+2}\right] \\ \Leftrightarrow \hat{g}^A_{t+1} &= \frac{\eta_0\Phi_y + \Phi_{y_+}}{1-\eta_1\Phi_y} \mathbb{E}_t\left[\hat{y}_{t+1}\right] + \frac{\Phi_{g_+} - \eta_2\Phi_y}{1-\eta_1\Phi_y} \mathbb{E}_t\left[\hat{g}^A_{t+2}\right] .\end{aligned}$$

Substituting this expression back into the output equation, we obtain

$$\begin{aligned}\hat{y}_t &= \eta_0 \mathbb{E}_t\left[\hat{y}_{t+1}\right] + \eta_1\left(\frac{\eta_0\Phi_y + \Phi_{y_+}}{1-\eta_1\Phi_y} \mathbb{E}_t\left[\hat{y}_{t+1}\right] + \frac{\Phi_{g_+} - \eta_2\Phi_y}{1-\eta_1\Phi_y} \mathbb{E}_t\left[\hat{g}^A_{t+2}\right]\right) - \eta_2 \mathbb{E}_t\left[\hat{g}^A_{t+2}\right] \\ &= \left(\frac{\eta_0 - \eta_0\eta_1\Phi_y + \eta_0\eta_1\Phi_y + \eta_1\Phi_{y_+}}{1-\eta_1\Phi_y}\right) \mathbb{E}_t\left[\hat{y}_{t+1}\right] + \left(\frac{\eta_1\Phi_{g_+} - \eta_1\eta_2\Phi_y - \eta_2 + \eta_1\eta_2\Phi_y}{1-\eta_1\Phi_y}\right) \mathbb{E}_t\left[\hat{g}^A_{t+2}\right] \\ &= \frac{\eta_0 + \eta_1\Phi_{y_+}}{1-\eta_1\Phi_y} \mathbb{E}_t\left[\hat{y}_{t+1}\right] + \frac{\eta_1\Phi_{g_+} - \eta_2}{1-\eta_1\Phi_y} \mathbb{E}_t\left[\hat{g}^A_{t+2}\right] .\end{aligned}$$

As a result, the recursive system of equations is purely forward-looking and can be written in a more compact form as

$$\begin{pmatrix} \hat{y}_t \\ \hat{g}^A_{t+1} \end{pmatrix} = \mathbf{A}_T \begin{pmatrix} \hat{y}_{t+1} \\ \hat{g}^A_{t+2} \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \mathbb{E}_t \begin{pmatrix} \hat{y}_{t+1} \\ \hat{g}^A_{t+2} \end{pmatrix} ,$$

where the auxiliary statistics are given by

$$A_{11} = \frac{\eta_0 + \eta_1\Phi_{y_+}}{1-\eta_1\Phi_y} , \quad A_{12} = \frac{\eta_1\Phi_{g_+} - \eta_2}{1-\eta_1\Phi_y} , \quad A_{21} = \frac{\eta_0\Phi_y + \Phi_{y_+}}{1-\eta_1\Phi_y} , \quad A_{22} = \frac{\Phi_{g_+} - \eta_2\Phi_y}{1-\eta_1\Phi_y} .$$

Step 2: Sign Eigenvalue Conditions.

The dynamic system is determinate if and only if both eigenvalues of $\mathbf{A}_T$ lie inside the unit circle. Following Bullard and Mitra (2002) and LaSalle (1986), this applies if

$$|A_{11}A_{22} - A_{12}A_{21}| < 1 , \quad |A_{11} + A_{22}| < 1 + A_{11}A_{22} - A_{12}A_{21} .$$

To make progress, we first rearrange $A_{11}A_{22} - A_{12}A_{21}$ to

$$\begin{aligned}
A_{11}A_{22} - A_{12}A_{21} &= \frac{(\eta_0 + \eta_1\Phi_{y_+})(\Phi_{g_+} - \eta_2\Phi_y) - (\eta_1\Phi_{g_+} - \eta_2)(\eta_0\Phi_y + \Phi_{y_+})}{(1-\eta_1\Phi_y)^2} \\
&= \frac{\eta_0\Phi_{g_+} - \eta_0\eta_2\Phi_y + \eta_1\Phi_{y_+}\Phi_{g_+} - \eta_1\eta_2\Phi_y\Phi_{y_+} - \eta_0\eta_1\Phi_y\Phi_{g_+} - \eta_1\Phi_{y_+}\Phi_{g_+} + \eta_0\eta_2\Phi_y + \eta_2\Phi_{y_+}}{(1-\eta_1\Phi_y)^2} \\
&= \frac{\eta_0\Phi_{g_+} - \eta_1\eta_2\Phi_y\Phi_{y_+} - \eta_0\eta_1\Phi_y\Phi_{g_+} + \eta_2\Phi_{y_+}}{(1-\eta_1\Phi_y)^2} \\
&= \frac{\eta_0\Phi_{g_+}(1-\eta_1\Phi_y) + \eta_2\Phi_{y_+}(1-\eta_1\Phi_y)}{(1-\eta_1\Phi_y)^2} \\
&= \frac{\eta_0\Phi_{g_+} + \eta_2\Phi_{y_+}}{1-\eta_1\Phi_y}\,.
\end{aligned}$$

Before we verify the previous conditions conditions, we state two intermediary results.

RESULT 2. *It holds that* $sgn(A_{11}A_{22} - A_{12}A_{21}) = sgn(\phi_y)$.

To prove the result, we first simplify the denominator of $A_{11}A_{22} - A_{12}A_{21}$, i.e.,

$$1 - \eta_1\Phi_y = 1 - \frac{\zeta_g}{1+\phi_y\zeta_r}\frac{\mathcal{E}_y}{1+\mathcal{E}_g} = \frac{\phi_y\zeta_r}{1+\phi_y\zeta_r}\,.$$

Regarding the numerator, we obtain

$$\eta_0\Phi_{g_+} + \eta_2\Phi_{y_+} = \frac{\zeta_f\Phi_{g_+} + \zeta_{g'}\Phi_{y_+}}{1+\phi_y\zeta_r} = \frac{1}{1+\phi_y\zeta_r}\left[\zeta_f\frac{\mathcal{E}_g}{1+\mathcal{E}_g} + \tilde{s}\frac{\mathcal{E}_g}{\mathcal{E}_y}\frac{\mathscr{M}-\mathcal{E}_y}{1+\mathcal{E}_g}\right]\,.$$

Note that the term in brackets is positive due to

$$\zeta_f\frac{\mathcal{E}_g}{1+\mathcal{E}_g} + \tilde{s}\frac{\mathcal{E}_g}{\mathcal{E}_y}\frac{\mathscr{M}-\mathcal{E}_y}{1+\mathcal{E}_g} = \tilde{s}\frac{\mathcal{E}_g}{1+\mathcal{E}_g}\frac{\mathscr{M}}{\mathcal{E}_y} + (1-\tilde{s})\frac{\mathcal{E}_g}{1+\mathcal{E}_g}\frac{\chi}{\mathcal{E}_y} > 0\,.$$

Therefore, we have

$$A_{11}A_{22} - A_{12}A_{21} = \frac{1}{\phi_y\zeta_r}\left[\zeta_f\frac{\mathcal{E}_g}{1+\mathcal{E}_g} + \tilde{s}\frac{\mathcal{E}_g}{\mathcal{E}_y}\frac{\mathscr{M}-\mathcal{E}_y}{1+\mathcal{E}_g}\right]$$

such that $\text{sgn}(A_{11}A_{22} - A_{12}A_{21}) = \text{sgn}(\phi_y)$. Additionally, we have the following result.

RESULT 3. $A_{11} + A_{22}$ *is positive if* $\phi_y > \underline{\phi}_y$, *where* $\underline{\phi}_y \equiv -\frac{1-\tilde{s}}{\zeta_r} - \frac{1+\mathcal{E}_g}{\mathcal{E}_g}\left(\frac{\zeta_f - 1}{\zeta_r} + \mathscr{M}\right)$.

First, recognize that we have

$$A_{11} + A_{22} = \frac{\eta_0 + \eta_1\Phi_{y_+} + \Phi_{g_+} - \eta_2\Phi_y}{1-\eta_1\Phi_y}\,.$$

The numerator can be rewritten to

$$\eta_0 + \eta_1\Phi_{y_+} + \Phi_{g_+} - \eta_2\Phi_y = \frac{1}{1+\phi_y\zeta_r}\left[\zeta_f + \zeta_g\Phi_{y_+} + (1+\phi_y\zeta_r)\Phi_{g_+} - \zeta_{g'}\Phi_y\right]$$

$$= \frac{1}{1+\phi_y\zeta_r}\left[\zeta_f + \frac{\mathscr{M}-\mathcal{E}_y}{\mathcal{E}_y} + (1+\phi_y\zeta_r)\frac{\mathcal{E}_g}{1+\mathcal{E}_g} - \tilde{s}\frac{\mathcal{E}_g}{1+\mathcal{E}_g}\right]$$
$$= \frac{1}{1+\phi_y\zeta_r}\left[\zeta_f - 1 + \frac{\mathscr{M}}{\mathcal{E}_y} + (1-\tilde{s})\frac{\mathcal{E}_g}{1+\mathcal{E}_g} + \phi_y\zeta_r\frac{\mathcal{E}_g}{1+\mathcal{E}_g}\right]$$

As a result, we have

$$A_{11}+A_{22} = \frac{1}{\phi_y\zeta_r}\left[\zeta_f - 1 + \frac{\mathscr{M}}{\mathcal{E}_y} + (1-\tilde{s})\frac{\mathcal{E}_g}{1+\mathcal{E}_g} + \phi_y\zeta_r\frac{\mathcal{E}_g}{1+\mathcal{E}_g}\right]$$

Note that the former expression is positive if

$$\zeta_f - 1 + \frac{\mathscr{M}}{\mathcal{E}_y} + (1-\tilde{s})\frac{\mathcal{E}_g}{1+\mathcal{E}_g} + \phi_y\zeta_r\frac{\mathcal{E}_g}{1+\mathcal{E}_g} > 0\,,$$

which is satisfied if $\phi_y > \max\left\{0, \underline{\phi}_y\right\}$.

Step 3: Derivation of HANK-GS Taylor Principle

The first determinacy condition, i.e., $|A_{11}A_{22} - A_{12}A_{21}| < 1$ is satisfied if

$$\phi_y > \frac{\mathcal{E}_g}{1+\mathcal{E}_g}\left[\frac{\zeta_f - \tilde{s}}{\zeta_r} + \tilde{s}\mathscr{M}\right]\,.$$

The second determinacy condition, i.e., $|A_{11}+A_{22}| < 1 + A_{11}A_{22} - A_{12}A_{21}$ is satisfied if

$$\frac{1}{\phi_y\zeta_r}\left[\zeta_f - 1 + \frac{\mathscr{M}}{\mathcal{E}_y} + (1-\tilde{s})\frac{\mathcal{E}_g}{1+\mathcal{E}_g} + \phi_y\zeta_r\frac{\mathcal{E}_g}{1+\mathcal{E}_g}\right] < 1 + \frac{1}{\phi_y\zeta_r}\frac{\mathcal{E}_g}{1+\mathcal{E}_g}\left[\zeta_f - \tilde{s} + \tilde{s}\frac{\mathscr{M}}{\mathcal{E}_y}\right]$$

which can be rearranged to

$$\phi_y > \frac{\zeta_f - 1}{\zeta_r} + [1 + (1-\tilde{s})\mathcal{E}_g]\,\mathscr{M}$$

As a result, model dynamics are locally determinate if

$$\phi_y > \max\left\{\underline{\phi}_y, \frac{\mathcal{E}_g}{1+\mathcal{E}_g}\left[\frac{\zeta_f - \tilde{s}}{\zeta_r} + \tilde{s}\mathscr{M}\right], \frac{\zeta_f - 1}{\zeta_r} + [1 + (1-\tilde{s})\mathcal{E}_g]\,\mathscr{M}\right\}$$

Note that we recover the case stated in the main text when $\underline{\phi}_y \leq 0$, which applies if $\zeta_f > \underline{\zeta}_f = 1 - (1-\tilde{s})\frac{\mathcal{E}_g}{1+\mathcal{E}_g} - \zeta_r\mathscr{M} < 1$. □

B.5 Effects from Progressive Redistribution Policies

B.5.1 *Proof Proposition 5*

Statement (a): In a HA model ($\theta = \psi = 0$) with flexible wages, the labor supply decision of unions is characterized by

$$\nu L_t^{\varphi} c_t = \frac{\epsilon_w - 1}{\epsilon_w}(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}\,,$$

which equals to $\varphi\hat{L}_t + \hat{c}_t = 0$ in log-linear terms. As $\hat{y}_t = \hat{L}_t = \hat{c}_t$ applies, the labor supply equation implies $\mathcal{M}_y = 0$. The aggregate IS equation then provides us with the associated (real) natural interest rate.[28]

Statement (b): In a HANK model ($\psi = 0$), the equilibrium dynamics are summarized by

$$\hat{y}_t = \zeta_f \mathbb{E}_t\left[\hat{y}_{t+1}\right] - \zeta_r\left(\hat{i}_t - \mathbb{E}_t\left[\hat{\pi}_{t+1}\right]\right) + \zeta_\tau \hat{\tau}_t - \zeta_{\tau'}\mathbb{E}_t\left[\hat{\tau}_{t+1}\right] ,$$
$$\hat{i}_t = \mathbb{E}_t\left[\hat{\pi}_{t+1}\right] + \phi_y \hat{y}_t ,$$

Note that $(\zeta_f, \zeta_r, \zeta_\tau, \zeta_{\tau'})$ are now defined under $\gamma_c = 1$ as $\hat{y}_t = \hat{c}_t$ applies. The impact output multiplier is given by

$$\mathcal{M}_y = \frac{\zeta_\tau - \rho\zeta_{\tau'}}{1 + \phi_y\zeta_r - \rho\zeta_f} .$$

The sign of the denominator is positive if $\phi_y > (\rho_\tau\zeta_f - 1)/\zeta_r$, which is for $\rho \in [0,1)$ guaranteed by local determinacy, i.e., $\phi_y > (\zeta_f - 1)/\zeta_r$. As a result, the sign of $\mathcal{M}_y$ is determined by the sign of the numerator. We obtain

$$\zeta_\tau - \rho\zeta_{\tau'} = \frac{\mathcal{E}_\tau}{\mathcal{E}_y}\left(1 - \rho\left[\tilde{s} - (1-\tilde{s})\frac{1-\eta}{\mathcal{E}_\tau}\right]\right) = \frac{\lambda(1-\eta)}{1-\lambda\chi}\left(1 - \rho\left[\tilde{s} - (1-\tilde{s})\frac{(1-\lambda)\Gamma_c}{\lambda}\right]\right) ,$$

where the second equality follows from substituting in for $\mathcal{E}_y$ and $\mathcal{E}_\tau$. As a result, we have $\zeta_\tau - \rho\zeta_{\tau'} > 0$ where the positive sign follows as $\eta \in (0,1)$ and $\chi < 1/\lambda$. Consequently, $\mathcal{M}_y > 0$ holds true.

Statement (c): In a HA-GS model with flexible wages ($\theta = 0$), the equilibrium dynamics are summarized by

$$\hat{g}^A_{t+1} = \frac{\mathcal{E}_y}{1+\mathcal{E}_g}\hat{y}_t + \frac{\mathscr{M} - \mathcal{E}_y}{1+\mathcal{E}_g}\mathbb{E}_t\left[\hat{y}_{t+1}\right] + \frac{\mathcal{E}_g}{1+\mathcal{E}_g}\mathbb{E}_t\left[\hat{g}^A_{t+2}\right] - \frac{\mathcal{E}_\tau}{1+\mathcal{E}_g}\left(\hat{\tau}_t - \mathbb{E}_t\left[\hat{\tau}_{t+1}\right]\right) ,$$
$$\hat{y}_t = \gamma_c\hat{c}_t + (1-\gamma_c)\frac{g^A}{g^A - 1}\hat{g}^A_{t+1} ,$$
$$0 = \varphi\hat{L}_t + \hat{c}_t .$$

Using $\hat{y}_t = \hat{L}_t$, output and technology multipliers are given by

$$\mathcal{M}_y = \frac{1-\gamma_c}{1+\varphi\gamma_c}\frac{g^A}{g^A - 1}\mathcal{M}_g ,$$
$$\mathcal{M}_g = \frac{\mathcal{E}_y(1-\rho) + \rho\mathscr{M}}{1+\mathcal{E}_g(1-\rho)}\mathcal{M}_y - \frac{\mathcal{E}_\tau(1-\rho)}{1+\mathcal{E}_g(1-\rho)} .$$

[28] Note that the result does not rely on the assumptions regarding union wage setting. Specifically, in an alternative setup in which households choose their individual labor supply, we would obtain in log-linear terms $\varphi\hat{L}^i_t + \hat{c}^i_t = \hat{w}_t$, for $i \in \{H, S\}$. Aggregating over household types and abstracting from steady state consumption inequality yields, as above, $\varphi\hat{L}_t + \hat{c}_t = \hat{w}_t = 0$.

The first equation implies $\text{sgn}(\mathcal{M}_y) = \text{sgn}(\mathcal{M}_g)$. Substituting the first into the latter equation provides us with

$$\left(1 - \frac{1-\gamma_c}{1+\varphi\gamma_c}\frac{g^A}{g^A-1}\frac{\mathcal{E}_y(1-\rho)+\rho\mathscr{M}}{1+\mathcal{E}_g(1-\rho)}\right)\mathcal{M}_g = -\frac{\mathcal{E}_\tau(1-\rho)}{1+\mathcal{E}_g(1-\rho_\tau)}\,.$$

The sign of the right-hand side is negative. As a result, $\text{sgn}(\mathcal{M}_g) \geq 0$ if

$$\begin{aligned}(1+\varphi\gamma_c)\left(g^A-1\right)(1+\mathcal{E}_g(1-\rho)) &\geq (1-\gamma_c)g^A\left(\mathcal{E}_y(1-\rho)+\rho\mathscr{M}\right)\,,\\ \Leftrightarrow\quad \frac{(1+\varphi\gamma_c)\left(g^A-1\right)}{(1-\gamma_c)g^A} &\geq \frac{\mathcal{E}_y(1-\rho)+\rho\mathscr{M}}{1+\mathcal{E}_g(1-\rho)}\,. \end{aligned} \tag{9}$$

The derivative of the right hand side with respect to ρ is given by

$$\frac{\partial\left(\dfrac{\mathcal{E}_y(1-\rho)+\rho\mathscr{M}}{1+\mathcal{E}_g(1-\rho)}\right)}{\partial\rho} = \frac{\mathscr{M}(1+\mathcal{E}_g)-\mathcal{E}_y}{\left(1+\mathcal{E}_g(1-\rho_\tau)\right)^2}\,.$$

There are two sub-cases. First, when $\mathscr{M}(1+\mathcal{E}_g)-\mathcal{E}_y \geq 0$, equation (9) holds for $\rho \in (0,1)$ if

$$\frac{(1+\varphi\gamma_c)\left(g^A-1\right)}{(1-\gamma_c)g^A} > \mathscr{M} \quad\Leftrightarrow\quad \frac{1+\varphi\gamma_c}{1-\gamma_c}\left(g^A-1\right) > g^A-\beta\,.$$

The previous equality can be rearranged to

$$g^A > 1 + \frac{1-\gamma_c}{\gamma_c}\frac{1-\beta}{1+\varphi}\,.$$

The resource constraint implies $\gamma_c = 1-(g^A-1)/(\psi Y_\alpha L)$. Substituting into the previous condition yields

$$g^A > 1 + \frac{\dfrac{g^A-1}{\psi Y_\alpha L}}{1-\dfrac{g^A-1}{\psi Y_\alpha L}}\frac{1-\beta}{1+\varphi}\,,$$

which can be rearranged to

$$\begin{aligned}&(g^A-1)\left(1-\frac{g^A-1}{\psi Y_\alpha L}\right) > \frac{g^A-1}{\psi Y_\alpha L}\frac{1-\beta}{1+\varphi}\\ \Leftrightarrow\quad &1 > \frac{1}{\psi Y_\alpha L}\left(g^A-1+\frac{1-\beta}{1+\varphi}\right)\\ \Leftrightarrow\quad &1+\psi Y_\alpha L - \frac{1-\beta}{1+\varphi} > g^A\,.\end{aligned}$$

Using $g^A = \beta(1+\psi\Theta_\alpha L)$, we can rewrite the previous equation as

$$\beta(1+\psi\Theta_\alpha L) < 1 + \psi Y_\alpha L - \frac{1-\beta}{1+\varphi}$$
$$\Leftrightarrow \quad (Y_\alpha - \beta\Theta_\alpha)\psi L > (1-\beta)\left[\frac{1}{1+\varphi} - 1\right],$$

which is always satisfied as $Y_\alpha - \beta\Theta_\alpha > 0$ and the right hand side is strictly negative. Second, when $\mathscr{M}(1+\mathcal{E}_g) - \mathcal{E}_y < 0$, equation (9) holds on $\rho \in (0,1)$ if

$$\frac{(1+\varphi\gamma_c)\left(g^A-1\right)}{(1-\gamma_c)g^A} > \frac{\mathcal{E}_y}{1+\mathcal{E}_g} = \frac{(1-\lambda)\Gamma_c + \lambda - \lambda\gamma_c\chi}{(1-\lambda)\Gamma_c\left(1-\frac{\gamma_c}{g^A}\right) + \lambda(1-\gamma_c)}\frac{g^A-1}{g^A}.$$

After some tedious algebra, the condition can be rewritten as

$$(1-\lambda)\Gamma_c\gamma_c\left(1-\frac{1}{g^A}\right) + \left[(1-\lambda)\Gamma_c\left(1-\frac{\gamma_c}{g^A}\right) + \lambda(1-\gamma_c)\right]\varphi\gamma_c > -\lambda\gamma_c(1-\gamma_c)\chi,$$

which is satisfied as $0 \leq \chi \leq 1/(\lambda\gamma_c)$.

B.5.2 *Proof Proposition 6*

PROOF. Under the full HANK-GS economy with a real interest rate rule, equilibrium dynamics are summarized by

$$\hat{y}_t = \zeta_f \mathbb{E}_t\left[\hat{y}_{t+1}\right] - \phi_y\zeta_r\hat{y}_t + \zeta_g\hat{g}^A_{t+1} - \zeta_{g'}\mathbb{E}_t\left[\hat{g}^A_{t+2}\right] + \zeta_\tau\hat{\tau}_t - \zeta_{\tau'}\mathbb{E}_t\left[\hat{\tau}_{t+1}\right],$$
$$\hat{g}^A_{t+1} = \frac{\mathcal{E}_y}{1+\mathcal{E}_g}\hat{y}_t + \frac{\mathscr{M}-\mathcal{E}_y}{1+\mathcal{E}_g}\mathbb{E}_t\left[\hat{y}_{t+1}\right] + \frac{\mathcal{E}_g}{1+\mathcal{E}_g}\mathbb{E}_t\left[\hat{g}^A_{t+2}\right] - \frac{\mathcal{E}_\tau}{1+\mathcal{E}_g}\left(\mathbb{E}_t\left[\hat{\tau}_t\right] - \mathbb{E}_t\left[\hat{\tau}_{t+1}\right]\right).$$

A. Output and Technology Multipliers.

The impact multipliers are

$$\left(1+\phi_y\zeta_r - \rho\zeta_f\right)\mathcal{M}_y = \left(\zeta_g - \rho\zeta_{g'}\right)\mathcal{M}_g + \zeta_\tau - \rho\zeta_{\tau'},$$
$$\mathcal{M}_g = \Lambda_y\mathcal{M}_y - \Lambda_\tau,$$

where we have defined the auxiliary parameters

$$\Lambda_y \equiv \frac{\mathcal{E}_y(1-\rho) + \rho\mathscr{M}}{1+\mathcal{E}_g(1-\rho)} > 0, \qquad \Lambda_\tau \equiv \frac{\mathcal{E}_\tau(1-\rho)}{1+\mathcal{E}_g(1-\rho)} > 0.$$

As a result, the output and technology multiplier becomes

$$\mathcal{M}_y = \frac{-\left(\zeta_g - \rho\zeta_{g'}\right)\Lambda_\tau + \zeta_\tau - \rho\zeta_{\tau'}}{1+\phi_y\zeta_r - \rho\zeta_f - \left(\zeta_g - \rho\zeta_{g'}\right)\Lambda_y},$$
$$\mathcal{M}_g = \frac{(\zeta_\tau - \rho_\tau\zeta_{\tau'})\Lambda_y - \left(1+\phi_y\zeta_r - \rho\zeta_f\right)\Lambda_\tau}{1+\phi_y\zeta_r - \rho\zeta_f - \left(\zeta_g - \rho\zeta_{g'}\right)\Lambda_y}.$$

B. Sign Denominator Multipliers.

Next, we show that the denominators of both multipliers are strictly positive if the Taylor principle applies. To see this more clearly, we write the dynamic system as

$$\begin{pmatrix} \hat{y}_t \\ \hat{g}^A_{t+1} \end{pmatrix} = \mathbf{A}_T \begin{pmatrix} \mathbb{E}_t\left[\hat{y}_{t+1}\right] \\ \mathbb{E}_t\left[\hat{g}^A_{t+2}\right] \end{pmatrix} + \mathbf{B}_T \hat{\tau}_t \,,$$

where

$$\mathbf{A}_T \equiv \frac{1}{1-\eta_1\Phi_y} \begin{pmatrix} \eta_0 + \eta_1\Phi_{y_+} & \eta_1\Phi_{g_+} - \eta_2 \\ \eta_0\Phi_y + \Phi_{y_+} & \Phi_{g_+} - \eta_2\Phi_y \end{pmatrix} ,$$

$$\mathbf{B}_T \equiv \frac{1}{1-\eta_1\Phi_y} \begin{pmatrix} B_{11} \\ B_{21} \end{pmatrix} = \frac{1}{1-\eta_1\Phi_y} \begin{pmatrix} \eta_3 - \eta_1\Phi_\tau + \rho\left[\eta_1\Phi_{\tau_+} - \eta_4\right] \\ \eta_3\Phi_y - \Phi_\tau + \rho\left[\Phi_{\tau_+} - \eta_4\Phi_y\right] \end{pmatrix} .$$

Note that the additional auxiliary parameters are defined according to

$$\Phi_\tau = \Phi_{\tau_+} = \frac{\mathcal{E}_\tau}{1+\mathcal{E}_g} \,, \quad \eta_3 = \frac{\zeta_\tau}{1+\phi_y\zeta_r} \,, \quad \eta_4 = \frac{\zeta_{\tau'}}{1+\phi_y\zeta_r} \,.$$

As $\mathbb{E}_t\left[\hat{y}_{t+1}\right] = \rho\hat{y}_t$ and $\mathbb{E}_t\left[\hat{g}^A_{t+2}\right] = \rho\hat{g}^A_{t+1}$ hold on the equilibrium path, we have

$$\begin{aligned} \begin{pmatrix} \hat{y}_t \\ \hat{g}^A_{t+1} \end{pmatrix} &= \left(\mathbf{I}_2 - \rho\mathbf{A}_T\right)^{-1} \mathbf{B}_T \hat{\tau}_t \\ &= \frac{1}{(1-\rho A_{11})(1-\rho A_{22}) - \rho^2 A_{12}A_{21}} \begin{pmatrix} 1-\rho A_{22} & \rho A_{12} \\ \rho A_{21} & 1-\rho A_{11} \end{pmatrix} \mathbf{B}_T \hat{\tau}_t \\ &= \frac{1}{(1-\rho A_{11})(1-\rho A_{22}) - \rho^2 A_{12}A_{21}} \frac{1}{1-\eta_1\Phi_y} \begin{pmatrix} (1-\rho A_{22})B_{11} + \rho A_{12}B_{21} \\ \rho A_{21}B_{11} + (1-\rho A_{11})B_{21} \end{pmatrix} \hat{\tau}_t \,. \end{aligned}$$

After some tedious algebra we have

$$\begin{aligned} (1-\rho A_{22})B_{11} + \rho A_{12}B_{21} &= \left(1 - \rho\frac{\Phi_{g_+} - \eta_2\Phi_y}{1-\eta_1\Phi_y}\right)\left(\eta_3 - \eta_1\Phi_\tau + \rho\left[\eta_1\Phi_{\tau_+} - \eta_4\right]\right) \\ &\quad + \rho\frac{\eta_1\Phi_{g_+} - \eta_2}{1-\eta_1\Phi_y}\left(\eta_3\Phi_y - \Phi_\tau + \rho\left[\Phi_{\tau_+} - \eta_4\Phi_y\right]\right) \\ &= (\eta_3 - \rho\eta_4)(1-\rho\Phi_{g_+}) - (\eta_1 - \rho\eta_2)(\Phi_\tau - \rho\Phi_{\tau_+}) \,. \end{aligned}$$

Substituting in for the auxiliary parameters yields

$$\begin{aligned} (1-\rho A_{22})B_{11} + \rho A_{12}B_{21} &= \frac{1}{1+\phi_y\zeta_r}\left\{\left(\zeta_\tau - \rho\zeta_{\tau'}\right)\left(1 - \rho\frac{\mathcal{E}_g}{1+\mathcal{E}_g}\right) - \left(\zeta_g - \rho\zeta_{g'}\right)\frac{(1-\rho)\mathcal{E}_\tau}{1+\mathcal{E}_g}\right\} \\ &= \frac{1}{1+\phi_y\zeta_r}\frac{1+(1-\rho)\mathcal{E}_g}{1+\mathcal{E}_g}\left\{\left(\zeta_\tau - \rho\zeta_{\tau'}\right) - \left(\zeta_g - \rho\zeta_{g'}\right)\Lambda_\tau\right\} \end{aligned}$$

As $1-\eta_1\Phi_y>0$ applies, this implies that $\text{sgn}(\mathcal{M}_y)=\text{sgn}((\zeta_\tau-\rho\zeta_{\tau'})-\left(\zeta_g-\rho\zeta_{g'}\right)\Lambda_\tau)$ if $\det(\mathbf{I}_2-\rho\mathbf{A}_T)>0$. To show the latter statement, note that

$$\begin{aligned}A_{11}A_{22}-A_{12}A_{21}&=\frac{1}{\phi_y\zeta_r}\frac{\mathcal{E}_g}{1+\mathcal{E}_g}\left[\zeta_f-\tilde{s}+\tilde{s}\frac{\mathscr{M}}{\mathcal{E}_y}\right]>0\,,\\ A_{11}+A_{22}&=\frac{1}{\phi_y\zeta_r}\left[\zeta_f-1+\frac{\mathscr{M}}{\mathcal{E}_y}+(1-\tilde{s})\frac{\mathcal{E}_g}{1+\mathcal{E}_g}+\phi_y\zeta_r\frac{\mathcal{E}_g}{1+\mathcal{E}_g}\right]>0\,.\end{aligned}$$

where the strict inequalities are due to the determinacy conditions. Moreover, we rewrite

$$\begin{aligned}\det(\mathbf{I}_2-\rho\mathbf{A}_T)&=(1-\rho A_{11})(1-\rho A_{22})-\rho^2A_{12}A_{21}\\&=1-\rho\left(A_{11}+A_{22}\right)+\rho^2\left(A_{11}A_{22}-A_{12}A_{21}\right)\,.\end{aligned}$$

As $A_{11}+A_{22}>0$ applies, we obtain

$$\begin{aligned}\det(\mathbf{I}_2-\rho\mathbf{A}_T)&=1-\rho\left(A_{11}+A_{22}\right)+\rho^2\left(A_{11}A_{22}-A_{12}A_{21}\right)\\&>1-\rho\left(1+A_{11}A_{22}-A_{12}A_{21}\right)+\rho^2\left(A_{11}A_{22}-A_{12}A_{21}\right)\\&=(1-\rho)\left[1-\rho\left(A_{11}A_{22}-A_{12}A_{21}\right)\right]\\&>0\,,\end{aligned}$$

where the first strict inequality is due to the determinacy condition $A_{11}+A_{22}<1+A_{11}A_{22}-A_{12}A_{21}$ and the second strictly inequality is due to the determinacy condition $A_{11}A_{22}-A_{12}A_{21}<1$. As a result, this implies that

$$\begin{aligned}\text{sgn}(\mathcal{M}_y)&=\text{sgn}((\zeta_\tau-\rho\zeta_{\tau'})-\left(\zeta_g-\rho\zeta_{g'}\right)\Lambda_\tau)\,,\\ \text{sgn}(\mathcal{M}_g)&=\text{sgn}((\zeta_\tau-\rho_\tau\zeta_{\tau'})\Lambda_y-\left(1+\phi_y\zeta_r-\rho\zeta_f\right)\Lambda_\tau)\,,\end{aligned}$$

which is equivalent to stating

$$1+\phi_y\zeta_r-\rho\zeta_f-\left(\zeta_g-\rho\zeta_{g'}\right)\Lambda_y>0\,.$$

C. Sign Numerator Multipliers.

The sign of (the numerator of) $\mathcal{M}_y$ is

$$\begin{aligned}\text{sgn}\left(\mathcal{M}_y\right)&=\zeta_\tau-\rho\zeta_{\tau'}-\left(\zeta_g-\rho\zeta_{g'}\right)\Lambda_\tau\\&=\frac{\mathcal{E}_\tau}{\mathcal{E}_y}-\rho\frac{\mathcal{E}_\tau}{\mathcal{E}_y}\left(\tilde{s}-(1-\tilde{s})\frac{1-\eta}{\mathcal{E}_\tau}\right)-\left[\frac{1+\mathcal{E}_g}{\mathcal{E}_y}-\rho\tilde{s}\frac{\mathcal{E}_g}{\mathcal{E}_y}\right]\frac{\mathcal{E}_\tau(1-\rho)}{1+\mathcal{E}_g(1-\rho)}\\&=\frac{\mathcal{E}_\tau}{\mathcal{E}_y}\frac{1}{1+\mathcal{E}_g(1-\rho)}\left\{(1+\mathcal{E}_g(1-\rho))\left[1-\rho\left(\tilde{s}-(1-\tilde{s})\frac{1-\eta}{\mathcal{E}_\tau}\right)\right]-(1-\rho)\left(1+\mathcal{E}_g-\rho\tilde{s}\mathcal{E}_g\right)\right\}\\&=\frac{\mathcal{E}_\tau}{\mathcal{E}_y}\frac{\rho}{1+\mathcal{E}_g(1-\rho)}\left\{1-(1+\mathcal{E}_g(1-\rho))\left[\tilde{s}-(1-\tilde{s})\frac{1-\eta}{\mathcal{E}_\tau}\right]+(1-\rho)\tilde{s}\mathcal{E}_g\right\}\end{aligned}$$

$$= \frac{\mathcal{E}_\tau}{\mathcal{E}_y} \frac{\rho}{1+\mathcal{E}_g(1-\rho)} \left\{ 1 - \tilde{s} - (1+\mathcal{E}_g(1-\rho)) \left[-(1-\tilde{s}) \frac{1-\eta}{\mathcal{E}_\tau} \right] \right\}$$
$$= \frac{\mathcal{E}_\tau}{\mathcal{E}_y} \frac{\rho(1-\tilde{s})}{1+\mathcal{E}_g(1-\rho)} \left\{ 1 + [1+\mathcal{E}_g(1-\rho)] \frac{1-\lambda}{\lambda} \Gamma_c \right\}.$$

It follows that $\text{sgn}\,(\mathcal{M}_y) = 0$ if $\rho = 0$ or $\tilde{s} = 1$ and $\text{sgn}\,(\mathcal{M}_y) > 0$ otherwise. Regarding the sign of the technology growth multiplier, we obtain

$$\begin{aligned}
\text{sgn}\,(\mathcal{M}_g) &= (\zeta_\tau - \rho_\tau \zeta_{\tau'}) \Lambda_y - \left(1 + \phi_y \zeta_r - \rho \zeta_f\right) \Lambda_\tau \\
&= \left\{ \frac{\mathcal{E}_\tau}{\mathcal{E}_y} - \rho \frac{\mathcal{E}_\tau}{\mathcal{E}_y} \left(\tilde{s} - (1-\tilde{s}) \frac{1-\eta}{\mathcal{E}_\tau} \right) \right\} \frac{\mathcal{E}_y(1-\rho) + \rho \mathscr{M}}{1+\mathcal{E}_g(1-\rho)} - \left(1 + \phi_y \zeta_r - \rho \zeta_f\right) \frac{\mathcal{E}_\tau (1-\rho)}{1+\mathcal{E}_g(1-\rho)} \\
&= \frac{\mathcal{E}_\tau}{\mathcal{E}_y} \frac{1}{1+\mathcal{E}_g(1-\rho)} \left\{ \left[1 - \rho \left(\tilde{s} - (1-\tilde{s}) \frac{1-\eta}{\mathcal{E}_\tau} \right) \right] [\mathcal{E}_y(1-\rho) + \rho \mathscr{M}] - (1-\rho) \left(1 + \phi_y \zeta_r - \rho \zeta_f\right) \mathcal{E}_y \right\}.
\end{aligned}$$

The sign of the previous expression is determined by the term in brackets, i.e.,

$$\begin{aligned}
\text{sgn}\,(\mathcal{M}_g) &= \left[1 - \rho \left(\tilde{s} - (1-\tilde{s}) \frac{1-\eta}{\mathcal{E}_\tau} \right) \right] [\mathcal{E}_y(1-\rho) + \rho \mathscr{M}] - (1-\rho) \left(1 + \phi_y \zeta_r - \rho \zeta_f\right) \mathcal{E}_y \\
&= \left[1 - \rho + \rho(1-\tilde{s}) \frac{\lambda + (1-\lambda)\Gamma_c}{\lambda} \right] [\mathcal{E}_y(1-\rho) + \rho \mathscr{M}] - (1-\rho) \left(1 + \phi_y \zeta_r - \rho \zeta_f\right) \mathcal{E}_y \\
&= (1-\rho) \mathcal{E}_y \left\{ 1 - \rho + \rho(1-\tilde{s}) \frac{\lambda + (1-\lambda)\Gamma_c}{\lambda} - \left(1 + \phi_y \zeta_r - \rho \zeta_f\right) \right\} \\
&+ \left[1 - \rho + \rho(1-\tilde{s}) \frac{\lambda + (1-\lambda)\Gamma_c}{\lambda} \right] \rho \mathscr{M} \\
&= \rho(1-\rho) \mathcal{E}_y \left\{ \zeta_f - 1 + (1-\tilde{s}) \frac{\lambda + (1-\lambda)\Gamma_c}{\lambda} \right\} - (1-\rho)\phi_y \\
&+ \left[1 - \rho + \rho(1-\tilde{s}) \frac{\lambda + (1-\lambda)\Gamma_c}{\lambda} \right] \rho \mathscr{M} \\
&= \rho(1-\rho) \mathcal{E}_y \left\{ (1-\tilde{s}) \left(\frac{\chi}{\mathcal{E}_y} - 1 \right) + (1-\tilde{s}) \frac{\lambda + (1-\lambda)\Gamma_c}{\lambda} \right\} - (1-\rho)\phi_y \\
&+ \left[1 - \rho + \rho(1-\tilde{s}) \frac{\lambda + (1-\lambda)\Gamma_c}{\lambda} \right] \rho \mathscr{M} \\
&= \rho(1-\rho)(1-\tilde{s}) \frac{\lambda + (1-\lambda)\Gamma_c}{\lambda \gamma_c} - (1-\rho)\phi_y + \left[1 - \rho + \rho(1-\tilde{s}) \frac{\lambda + (1-\lambda)\Gamma_c}{\lambda} \right] \rho \mathscr{M}.
\end{aligned}$$

This equation defines a cutoff value ϕ_y^* such that $\text{sgn}\,(\mathcal{M}_g) \gtreqless 0$ if $\phi_y \lesseqgtr \phi_y^*$. It is given by

$$\phi_y^* = \rho(1-\tilde{s}) \frac{\lambda + (1-\lambda)\Gamma_c}{\lambda \gamma_c} + \left[1 + (1-\tilde{s}) \frac{\lambda + (1-\lambda)\Gamma_c}{\lambda} \frac{\rho}{1-\rho} \right] \rho \mathscr{M}.$$

It is straightforward to verify that ϕ_y^* is strictly increasing in ρ. Alternatively, the former equation defines a second order polynomial in ρ, i.e., $a\rho^2 + b\rho + c$, where we have the

following auxiliary parameters

$$
\begin{aligned}
a &\equiv -(1-\tilde{s})\frac{\lambda+(1-\lambda)\Gamma_c}{\lambda}\left(\frac{1}{\gamma_c}-\mathscr{M}\right)-\mathscr{M}<0\\
b &\equiv (1-\tilde{s})\frac{\lambda+(1-\lambda)\Gamma_c}{\lambda\gamma_c}+\phi_y+\mathscr{M}>0\\
c &\equiv -\phi_y<0
\end{aligned}
$$

where the first inequality follows from $\gamma_c \in (0,1)$ and $\mathscr{M} \in (0,1)$ and the second and third from $\phi_y \geq 0$. The determinant of the second order polynomial is

$$
\Delta = b^2-4ac = \left(\mathscr{M}-\phi_y+(1-\tilde{s})\frac{\lambda+(1-\lambda)\Gamma_c}{\lambda\gamma_c}\right)^2+4\phi_y(1-\tilde{s})\frac{\lambda+(1-\lambda)\Gamma_c}{\lambda}\mathscr{M}>0
$$

As such, the second order polynomial has two real valued roots that are given by

$$
\tilde{\rho}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}\,,\quad \text{with}\quad \rho^{min}=\frac{-b+\sqrt{b^2-4ac}}{2a}<\rho^{max}=\frac{-b-\sqrt{b^2-4ac}}{2a}\,,
$$

where the inequality is due to $a<0$. Note that $0<\rho^{min}<1$ as

$$
\begin{aligned}
&-b+\sqrt{b^2-4ac}<0 \;\Leftrightarrow\; 4ac>0\,,\\
&-b+\sqrt{b^2-4ac}>2a \;\Leftrightarrow\; \sqrt{b^2-4ac}>2a+b \;\Leftrightarrow\; 0>a(a+b+c)\,,
\end{aligned}
$$

where the last inequality is satisfied as $a<0$ and

$$
a+b+c=(1-\tilde{s})\frac{\lambda+(1-\lambda)\Gamma_c}{\lambda}\mathscr{M}>0\,.
$$

Moreover, we have $\rho^{max}>1$ due to

$$
-b-\sqrt{b^2-4ac}<2a \;\Leftrightarrow\; -\sqrt{b^2-4ac}<2a+b \;\Leftrightarrow\; 0>a(a+b+c)
$$

As a result, the persistence cutoff is

$$
\rho^*=\frac{-\left[(1-\tilde{s})\frac{\lambda+(1-\lambda)\Gamma_c}{\lambda\gamma_c}+\phi_y+\mathscr{M}\right]+\sqrt{\left(\mathscr{M}-\phi_y+(1-\tilde{s})\frac{\lambda+(1-\lambda)\Gamma_c}{\lambda\gamma_c}\right)^2+4\phi_y(1-\tilde{s})\frac{\lambda+(1-\lambda)\Gamma_c}{\lambda}\mathscr{M}}}{2\left[-(1-\tilde{s})\frac{\lambda+(1-\lambda)\Gamma_c}{\lambda}\left(\frac{1}{\gamma_c}-\mathscr{M}\right)-\mathscr{M}\right]}\,.
$$

The comparative statics of ρ^* with respect to ϕ_y are given by

$$
\frac{\partial\rho^*}{\partial\phi_y}=\frac{-1+\frac{\Delta^{-\frac{1}{2}}}{2}\left[-2\left(\mathscr{M}-\phi_y+(1-\tilde{s})\frac{\lambda+(1-\lambda)\Gamma_c}{\lambda\gamma_c}\right)+4(1-\tilde{s})\frac{\lambda+(1-\lambda)\Gamma_c}{\lambda}\mathscr{M}\right]}{2\left[-(1-\tilde{s})\frac{\lambda+(1-\lambda)\Gamma_c}{\lambda}\left(\frac{1}{\gamma_c}-\mathscr{M}\right)-\mathscr{M}\right]}\,.
$$

As a result, we have $\frac{\partial \rho^*}{\partial \phi_y} > 0$ if

$$\Delta^{\frac{1}{2}} > 2(1-\tilde{s})\frac{\lambda+(1-\lambda)\Gamma_c}{\lambda}\mathscr{M} - \left(\mathscr{M} - \phi_y + (1-\tilde{s})\frac{\lambda+(1-\lambda)\Gamma_c}{\lambda\gamma_c}\right) .$$

This condition is obviously satisfied when the right hand side is weakly negative. In the reverse case, we can rewrite

$$\Delta > \left[2(1-\tilde{s})\frac{\lambda+(1-\lambda)\Gamma_c}{\lambda}\mathscr{M} - \left(\mathscr{M} - \phi_y + (1-\tilde{s})\frac{\lambda+(1-\lambda)\Gamma_c}{\lambda\gamma_c}\right)\right]^2 ,$$

which can be simplified to

$$0 > (1-\tilde{s})\frac{\lambda+(1-\lambda)\Gamma_c}{\lambda}\left(\mathscr{M} - \frac{1}{\gamma_c}\right) - \mathscr{M} ,$$

which holds true as $\gamma_c \in (0,1)$ and $\mathscr{M} < 1$.

D. Short- vs. Long-Run Stabilization Trade-Off:

The output response at period $t+z$ is $\ln\left(\frac{y_{t+z}}{y}\right) = \rho^z \mathcal{M}_y$, respectively the endogenous technology growth response $\ln\left(\frac{g^A_{t+z+1}}{g^A}\right) = \rho^z \mathcal{M}_g$. As a result, we have

$$\begin{aligned}
\ln y_{t+z} &= \ln y + \rho^z \mathcal{M}_y \\
\ln Y_{t+z} &= \ln A_{t+z} + \ln y + \rho^z \mathcal{M}_y \\
\ln g^A_{t+z+1} &= \ln g^A + \rho^z \mathcal{M}_g \\
\ln A_{t+z+1} &= \ln A_{t+z} + \ln g^A + \rho^z \mathcal{M}_g .
\end{aligned}$$

Consequently, we can rewrite

$$\begin{aligned}
\ln Y_{t+z} &= \ln A_{t+z} + \ln y + \rho^z \mathcal{M}_y \\
&= \ln A_{t+z-1} + \ln g^A + \rho^{z-1} \mathcal{M}_g + \ln y + \rho^z \mathcal{M}_y \\
&= \ln A_t + \sum_{i=0}^{z-1} \rho^i \mathcal{M}_g + z \ln g^A + \ln y + \rho^z \mathcal{M}_y .
\end{aligned}$$

In the absence of shocks, we have $\ln\left(\frac{y^{NS}_{t+z}}{y}\right) = 0$ such that we can state

$$\ln y^{NS}_{t+z} = \ln y \;\Leftrightarrow\; \ln Y^{NS}_{t+z} = \ln A^{NS}_{t+z} + \ln y = \ln A_t + z \ln g^A + \ln y .$$

Using the former expression, we can rewrite

$$\ln Y_{t+z} = \begin{cases} \ln Y^{NS}_{t+z} + \sum\limits_{i=0}^{z-1} \rho^i \mathcal{M}_g + \rho^z \mathcal{M}_y & \text{for} \quad \rho > 0 \\ \ln Y^{NS}_{t+z} + \mathcal{M}_g & \text{for} \quad \rho = 0 \end{cases}$$

such that the permanent output loss under a persistent shock with $|\rho| < 1$ is

$$\lim_{z\to\infty}\left(\ln Y_{t+z} - \ln Y_{t+z}^{NS}\right) = \frac{\mathcal{M}_g}{1-\rho}\,.$$

In the case $\mathcal{M}_y > 0$ and $\mathcal{M}_g < 0$, a short- vs. long-run stabilization trade-off occurs. Using the relation that $\sum_{i=0}^{z-1}\rho^i = \frac{1-\rho^z}{1-\rho}$, the time span z^* is thus implicitly defined by

$$\frac{1-\rho^{z^*}}{1-\rho}\mathcal{M}_g + \rho^{z^*}\mathcal{M}_y = 0 \quad \Leftrightarrow \quad \rho^{z^*}\left((1-\rho)\mathcal{M}_y - \mathcal{M}_g\right) = -\mathcal{M}_g\,,$$

which can finally be rearranged to

$$\rho^{z^*} = \frac{1}{1-(1-\rho)\dfrac{\mathcal{M}_y}{\mathcal{M}_g}} \quad \Leftrightarrow \quad z^* = -\frac{\ln\left(1-(1-\rho)\dfrac{\mathcal{M}_y}{\mathcal{M}_g}\right)}{\ln(\rho)} > 0\,.$$

□

B.6 State-Dependent Government Spending Shocks

B.6.1 *Proof Proposition 7* The proof proceeds in several steps. First, we derive expressions for hand-to-mouth and saver consumption. Second, we provide the new DIS and GE equations. Third, we compute output and technology multipliers before we finally derive conditions that determine their respective sign.

A. Policy Functions. Under this formulation, we obtain for HtM consumption

$$c_t^H = w_t L_t^H - \frac{t_t^H}{\lambda} = (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} L_t^H + \frac{T}{\lambda} - \frac{\tau_p}{\lambda} s_t^G\,.$$

Log-linearization provides us with

$$\hat{c}_t^H = \frac{wL^H}{c^H}\hat{L}_t^H - \frac{\tau_p}{\lambda}\frac{y}{c^H}\hat{s}_t^G = \frac{wL^H}{c^H}\mu\hat{L}_t - \frac{\tau_p}{\lambda}\frac{y}{c^H}\hat{s}_t^G = \eta\mu\hat{y}_t - \frac{\tau_p}{\lambda}\Theta\hat{s}_t^G\,,$$

where η is defined as before, $\Theta \equiv \lambda + (1-\lambda)\Gamma_y$, and $\hat{s}_t^G = (s_t^G - s^G)/y$. The aggregate resource constraint now reads

$$\hat{y}_t = \gamma_c\hat{c}_t + (1-\gamma_c)\frac{g^A}{g^A-1}\hat{g}_{t+1}^A + \hat{s}_t^G$$

and saver consumption becomes

$$\begin{aligned}\hat{c}_t^S &= \left(1 + \frac{\lambda}{1-\lambda}\frac{1}{\Gamma_c}\right)\left(\frac{\hat{y}_t - \hat{s}_t^G}{\gamma_c} - \frac{1-\gamma_c}{\gamma_c}\frac{g^A}{g^A-1}\hat{g}_{t+1}^A\right) - \frac{\lambda}{1-\lambda}\frac{1}{\Gamma_c}\left(\eta\mu\hat{y}_t - \frac{\tau_p}{\lambda}\Theta\hat{s}_t^G\right)\\ &= \mathcal{E}_y\hat{y}_t - \mathcal{E}_g\hat{g}_{t+1}^A - \mathcal{E}_s\hat{s}_t^G\,,\end{aligned}$$

where $\mathcal{E}_y, \mathcal{E}_g$ and $\mathcal{E}_s$ are

$$\mathcal{E}_y \equiv \frac{(1-\lambda)\Gamma_c + \lambda - \lambda\gamma_c\chi}{(1-\lambda)\Gamma_c\gamma_c},$$
$$\mathcal{E}_g \equiv \frac{(1-\lambda)\Gamma_c + \lambda}{(1-\lambda)\Gamma_c}\frac{1-\gamma_c}{\gamma_c}\frac{g^A}{g^A-1},$$
$$\mathcal{E}_s \equiv \frac{1}{(1-\lambda)\Gamma_c}\left(\frac{\lambda+(1-\lambda)\Gamma_c}{\gamma_c} - \Theta\tau_p\right).$$

B. DIS and Growth Equation. The saver Euler equation is given by

$$\hat{c}_t^S = \tilde{s}\mathbb{E}_t\left[\hat{c}_{t+1}^S\right] + (1-\tilde{s})\mathbb{E}_t\left[\hat{c}_{t+1}^H\right] - \hat{r}_t + \hat{g}_{t+1}^A.$$

Substituting in for the consumption of saver households, $\hat{c}_t^S$, results in

$$\mathcal{E}_y\hat{y}_t - \mathcal{E}_g\hat{g}_{t+1}^A - \mathcal{E}_s\hat{s}_t^G = \tilde{s}\mathbb{E}_t\left[\mathcal{E}_y\hat{y}_{t+1} - \mathcal{E}_g\hat{g}_{t+2}^A - \mathcal{E}_s\hat{s}_{t+1}^G\right] + (1-\tilde{s})\mathbb{E}_t\left[\chi\hat{y}_{t+1} - \frac{\tau_p}{\lambda}\Theta\hat{s}_{t+1}^G\right] - \hat{r}_t + \hat{g}_{t+1}^A,$$

which can be rearranged to

$$\hat{y}_t = \left(\tilde{s} + (1-\tilde{s})\frac{\chi}{\mathcal{E}_y}\right)\mathbb{E}_t\left[\hat{y}_{t+1}\right] - \frac{1}{\mathcal{E}_y}\hat{r}_t + \frac{1+\mathcal{E}_g}{\mathcal{E}_y}\hat{g}_{t+1}^A - \tilde{s}\frac{\mathcal{E}_g}{\mathcal{E}_y}\mathbb{E}_t\left[\hat{g}_{t+2}^A\right] + \frac{\mathcal{E}_s}{\mathcal{E}_y}\hat{s}_t^G - \frac{\mathcal{E}_s}{\mathcal{E}_y}\left(\tilde{s} + (1-\tilde{s})\frac{\tau_p}{\lambda}\frac{\Theta}{\mathcal{E}_s}\right)\mathbb{E}_t\left[\hat{s}_{t+1}^G\right].$$

The former equation can be restated as

$$\hat{y}_t = \zeta_f\mathbb{E}_t\left[\hat{y}_{t+1}\right] - \zeta_r\hat{r}_t + \zeta_g\hat{g}_{t+1}^A - \zeta_{g'}\mathbb{E}_t\left[\hat{g}_{t+2}^A\right] + \zeta_s\hat{s}_t - \zeta_{s'}\mathbb{E}_t\left[\hat{s}_{t+1}^G\right],$$

where $\zeta_f, \zeta_r, \zeta_g, \zeta_{g'}$ are as before and the new auxiliary parameters are given by:[29]

$$\zeta_s = \frac{\mathcal{E}_s}{\mathcal{E}_y},$$
$$\zeta_{s'} = \frac{\mathcal{E}_s}{\mathcal{E}_y}\left(\tilde{s} + (1-\tilde{s})\frac{\tau_p}{\lambda}\frac{\Theta}{\mathcal{E}_s}\right).$$

Similarly, substituting the consumption policy function into the growth equation yields

$$\hat{g}_{t+1}^A = \frac{\mathcal{E}_y}{1+\mathcal{E}_g}\hat{y}_t + \frac{\mathscr{M}-\mathcal{E}_y}{1+\mathcal{E}_g}\mathbb{E}_t\left[\hat{y}_{t+1}\right] + \frac{\mathcal{E}_g}{1+\mathcal{E}_g}\mathbb{E}_t\left[\hat{g}_{t+2}^A\right] - \frac{\mathcal{E}_s}{1+\mathcal{E}_g}\left(\mathbb{E}_t\left[\hat{s}_t^G\right] - \mathbb{E}_t\left[\hat{s}_{t+1}^G\right]\right).$$

[29] The expressions in the main body of the text follow from the relation $\gamma_c\Theta = 1$, which holds true as $\Gamma_c = 1$ implies $\Gamma_y = (1-\lambda\gamma_c)/[(1-\lambda_c)\gamma_c]$.

C. Output and Technology Multipliers

As a result, the impact multipliers are

$$\left(1+\phi_y\zeta_r-\rho\zeta_f\right)\mathcal{M}_y=\left(\zeta_g-\rho\zeta_{g'}\right)\mathcal{M}_g+\zeta_s-\rho\zeta_{s'}\,,$$
$$\mathcal{M}_g=\Lambda_y\mathcal{M}_y-\Lambda_s\,,$$

where we have defined the auxiliary parameters

$$\Lambda_y\equiv\frac{\mathcal{E}_y(1-\rho)+\rho\mathscr{M}}{1+\mathcal{E}_g(1-\rho)}>0\,,\qquad \Lambda_s\equiv\frac{\mathcal{E}_s(1-\rho)}{1+\mathcal{E}_g(1-\rho)}\geq 0\,.$$

As a result, the output and technology multiplier becomes

$$\mathcal{M}_y=\frac{\zeta_s-\rho\zeta_{s'}-\left(\zeta_g-\rho\zeta_{g'}\right)\Lambda_s}{1+\phi_y\zeta_r-\rho\zeta_f-\left(\zeta_g-\rho\zeta_{g'}\right)\Lambda_y}\,,$$
$$\mathcal{M}_g=\frac{(\zeta_s-\rho\zeta_{s'})\Lambda_y-\left(1+\phi_y\zeta_r-\rho\zeta_f\right)\Lambda_s}{1+\phi_y\zeta_r-\rho\zeta_f-\left(\zeta_g-\rho\zeta_{g'}\right)\Lambda_y}\,,$$

where, in case of $\Gamma_c=1$, the auxiliary variables ate given by:

$$\zeta_s=\frac{1-\tau_p}{1-\lambda\gamma_c\chi}\,,$$
$$\zeta_{s'}=\left(s+(1-s)\frac{1-\lambda}{\lambda}\frac{\tau_p}{1-\tau_p}\right)\zeta_s\,,$$
$$\xi_s=\frac{(g^A-1)(1-\tau_p)}{g^A(1-\lambda\gamma_c)+\gamma_c(\lambda-1)}\,,$$
$$\Lambda_s=(1-\rho)\xi_s/(1-\rho\xi_{g'})\,.$$

D. Sign Output and Technology Multipliers.

The sign of $\mathcal{M}_y$ is

$$\begin{aligned}
\text{sgn}\left(\mathcal{M}_y\right)&=\zeta_s-\rho\zeta_{s'}-\left(\zeta_g-\rho\zeta_{g'}\right)\Lambda_s\\
&=\frac{\mathcal{E}_s}{\mathcal{E}_y}-\rho\frac{\mathcal{E}_s}{\mathcal{E}_y}\left(\tilde{s}+(1-\tilde{s})\frac{\tau_p}{\lambda}\frac{\Theta}{\mathcal{E}_s}\right)-\left[\frac{1+\mathcal{E}_g}{\mathcal{E}_y}-\rho\tilde{s}\frac{\mathcal{E}_g}{\mathcal{E}_y}\right]\frac{\mathcal{E}_s(1-\rho)}{1+\mathcal{E}_g(1-\rho)}\\
&=\frac{\mathcal{E}_s}{\mathcal{E}_y}\frac{1}{1+\mathcal{E}_g(1-\rho)}\left\{\left(1+\mathcal{E}_g(1-\rho)\right)\left[1-\rho\left(\tilde{s}+(1-\tilde{s})\frac{\tau_p}{\lambda}\frac{\Theta}{\mathcal{E}_s}\right)\right]-(1-\rho)\left(1+\mathcal{E}_g-\rho\tilde{s}\mathcal{E}_g\right)\right\}\\
&=\frac{\mathcal{E}_s}{\mathcal{E}_y}\frac{\rho}{1+\mathcal{E}_g(1-\rho)}\left\{1-\left(1+\mathcal{E}_g(1-\rho)\right)\left[\tilde{s}+(1-\tilde{s})\frac{\tau_p}{\lambda}\frac{\Theta}{\mathcal{E}_s}\right]+(1-\rho)\tilde{s}\mathcal{E}_g\right\}\\
&=\frac{\mathcal{E}_s}{\mathcal{E}_y}\frac{\rho}{1+\mathcal{E}_g(1-\rho)}\left\{1-\tilde{s}-\left(1+\mathcal{E}_g(1-\rho)\right)(1-\tilde{s})\frac{\tau_p}{\lambda}\frac{\Theta}{\mathcal{E}_s}\right\}\\
&=\frac{\mathcal{E}_s}{\mathcal{E}_y}\frac{\rho(1-\tilde{s})}{1+\mathcal{E}_g(1-\rho)}\left\{1-\left[1+\mathcal{E}_g(1-\rho)\right]\frac{\tau_p}{\lambda}\frac{\Theta}{\mathcal{E}_s}\right\}
\end{aligned}$$

$$= \frac{1}{\mathcal{E}_y} \frac{\rho(1-\tilde{s})}{1+\mathcal{E}_g(1-\rho)} \left\{ \mathcal{E}_s - [1+\mathcal{E}_g(1-\rho)] \frac{\tau_p}{\lambda} \Theta \right\}.$$

It follows that $\text{sgn}(\mathcal{M}_y) = 0$ if $\rho = 0$ or $\tilde{s} = 1$. In all other cases, the sign of the output multiplier is determined by the sign of the term in brackets, i.e.,

$$\begin{aligned}
\mathcal{E}_s - [1+\mathcal{E}_g(1-\rho)] \frac{\tau_p}{\lambda} \Theta &= \frac{1}{(1-\lambda)\Gamma_c} \left(\frac{\lambda + (1-\lambda)\Gamma_c}{\gamma_c} - \Theta \tau_p \right) - [1+\mathcal{E}_g(1-\rho)] \frac{\Theta \tau_p}{\lambda} \\
&= \frac{\lambda + (1-\lambda)\Gamma_c}{(1-\lambda)\Gamma_c \gamma_c} - \left(\frac{1}{(1-\lambda)\Gamma_c} + \frac{1+\mathcal{E}_g(1-\rho)}{\lambda} \right) \Theta \tau_p \,,
\end{aligned}$$

which is strictly positive if

$$\tau_p < \tilde{\tau}_p \equiv \frac{1}{\Theta} \frac{\dfrac{\lambda + (1-\lambda)\Gamma_c}{(1-\lambda)\Gamma_c \gamma_c}}{\dfrac{1}{(1-\lambda)\Gamma_c} + \dfrac{1+\mathcal{E}_g(1-\rho)}{\lambda}} \,.$$

In the case of $\Gamma_c = 1$, we obtain

$$\tau_p < \frac{1}{\Theta} \frac{\dfrac{1}{\gamma_c}}{\dfrac{1}{\lambda} + (1-\rho)\dfrac{1-\lambda}{\lambda}\mathcal{E}_g} = \frac{\lambda}{1 + (1-\rho)\dfrac{1-\gamma_c}{\gamma_c}\dfrac{g^A}{g^A - 1}} < 1 \,.$$

To determine the sign of the technology growth multiplier, we compute

$$\begin{aligned}
\text{sgn}\left(\mathcal{M}_g\right) &= (\zeta_s - \rho\zeta_{s'})\Lambda_y - \left(1 + \phi_y\zeta_r - \rho\zeta_f\right)\Lambda_s \\
&= \left[\frac{\mathcal{E}_s}{\mathcal{E}_y} - \rho \frac{\mathcal{E}_s}{\mathcal{E}_y} \left(\tilde{s} + (1-\tilde{s}) \frac{\tau_p}{\lambda} \frac{\Theta}{\mathcal{E}_s} \right) \right] \frac{\mathcal{E}_y(1-\rho) + \rho\mathscr{M}}{1+\mathcal{E}_g(1-\rho)} - \left(1 + \phi_y\zeta_r - \rho\zeta_f\right) \frac{\mathcal{E}_s(1-\rho)}{1+\mathcal{E}_g(1-\rho)} \\
&= \frac{\mathcal{E}_s}{\mathcal{E}_y} \frac{1}{1+\mathcal{E}_g(1-\rho)} \left\{ \left[1 - \rho \left(\tilde{s} + (1-\tilde{s}) \frac{\tau_p}{\lambda} \frac{\Theta}{\mathcal{E}_s} \right) \right] \left[\mathcal{E}_y(1-\rho) + \rho\mathscr{M} \right] - (1-\rho)\left(1 + \phi_y\zeta_r - \rho\zeta_f\right)\mathcal{E}_y \right\}.
\end{aligned}$$

We can simplify the term in brackets further to

$$\begin{aligned}
&\left[1 - \rho \left(\tilde{s} + (1-\tilde{s}) \frac{\tau_p}{\lambda} \frac{\Theta}{\mathcal{E}_s} \right) \right] \left[\mathcal{E}_y(1-\rho) + \rho\mathscr{M} \right] - (1-\rho)\left(1 + \phi_y\zeta_r - \rho\zeta_f\right)\mathcal{E}_y \\
=&\left[1 - \rho + \rho(1-\tilde{s}) \left(1 - \frac{\tau_p}{\lambda} \frac{\Theta}{\mathcal{E}_s} \right) \right] \left[\mathcal{E}_y(1-\rho) + \rho\mathscr{M} \right] - (1-\rho)\left(1 + \phi_y\zeta_r - \rho\zeta_f\right)\mathcal{E}_y \\
=&(1-\rho)\mathcal{E}_y \left\{ 1 - \rho + \rho(1-\tilde{s}) \left(1 - \frac{\tau_p}{\lambda} \frac{\Theta}{\mathcal{E}_s} \right) - \left(1 + \phi_y\zeta_r - \rho\zeta_f\right) \right\} \\
&+ \left[1 - \rho + \rho(1-\tilde{s}) \left(1 - \frac{\tau_p}{\lambda} \frac{\Theta}{\mathcal{E}_s} \right) \right] \rho\mathscr{M} \\
=&\rho(1-\rho)\mathcal{E}_y \left\{ \zeta_f - 1 + (1-\tilde{s}) \left(1 - \frac{\tau_p}{\lambda} \frac{\Theta}{\mathcal{E}_s} \right) \right\} - (1-\rho)\phi_y \\
&+ \left[1 - \rho + \rho(1-\tilde{s}) \left(1 - \frac{\tau_p}{\lambda} \frac{\Theta}{\mathcal{E}_s} \right) \right] \rho\mathscr{M}
\end{aligned}$$

$$
\begin{aligned}
&=\rho(1-\rho)\mathcal{E}_y\left\{(1-\tilde{s})\left(\frac{\chi}{\mathcal{E}_y}-1\right)+(1-\tilde{s})\left(1-\frac{\tau_p}{\lambda}\frac{\Theta}{\mathcal{E}_s}\right)\right\}-(1-\rho)\phi_y\\
&+\left[1-\rho+\rho(1-\tilde{s})\left(1-\frac{\tau_p}{\lambda}\frac{\Theta}{\mathcal{E}_s}\right)\right]\rho\mathscr{M}\\
&=\rho(1-\rho)(1-\tilde{s})\left\{\chi-\frac{\tau_p\Theta}{\lambda}\frac{\mathcal{E}_y}{\mathcal{E}_s}\right\}-(1-\rho)\phi_y+\left[1-\rho+\rho(1-\tilde{s})\left(1-\frac{\tau_p}{\lambda}\frac{\Theta}{\mathcal{E}_s}\right)\right]\rho\mathscr{M}\,.
\end{aligned}
$$

As a result, the sign is given by (multiplying by $\mathcal{E}_s$ from above)

$$
\rho(1-\rho)(1-\tilde{s})\left\{\chi\mathcal{E}_s-\frac{\tau_p\Theta}{\lambda}\mathcal{E}_y\right\}-(1-\rho)\phi_y\mathcal{E}_s+\left[(1-\rho)\mathcal{E}_s+\rho(1-\tilde{s})\left(\mathcal{E}_s-\frac{\tau_p\Theta}{\lambda}\right)\right]\rho\mathscr{M}\,.
$$

Collecting terms yields

$$
\left[\rho(1-\rho)(1-\tilde{s})\chi+(1-\rho\tilde{s})\rho\mathcal{M}-(1-\rho)\phi_y\right]\mathcal{E}_s-\rho(1-\tilde{s})\frac{(1-\rho)\mathcal{E}_y+\rho\mathcal{M}}{\lambda}\Theta\tau_p\,.
$$

Substituting in for $\mathcal{E}_s$ delivers finally an equation of the form $\Xi_1-\Xi_2\Theta\tau_p$, where

$$
\begin{aligned}
\Xi_1&\equiv\frac{\lambda+(1-\lambda)\Gamma_c}{(1-\lambda)\Gamma_c\gamma_c}\left(\rho(1-\rho)(1-\tilde{s})\chi+(1-\rho\tilde{s})\rho\mathcal{M}-(1-\rho)\phi_y\right),\\
\Xi_2&\equiv\rho(1-\tilde{s})\frac{(1-\rho)\mathcal{E}_y+\rho\mathcal{M}}{\lambda}+\frac{1}{(1-\lambda)\Gamma_c}\left(\rho(1-\rho)(1-\tilde{s})\chi+(1-\rho\tilde{s})\rho\mathcal{M}-(1-\rho)\phi_y\right).
\end{aligned}
$$

From here the following case distinction applies:

1. $\phi_y>\overline{\phi}_y$ implies $\Xi_2<0$ and $\Xi_1<0$. Thus, we have $\mathcal{M}_g\gtreqless 0$ if $\tau_p\gtreqless\frac{1}{\Theta}\frac{\Xi_1}{\Xi_2}>0$.
2. $\phi_y=\overline{\phi}_y$ implies $\Xi_2=0$ and $\Xi_1<0$. Thus, we have $\mathcal{M}_g<0$.
3. $\overline{\phi}_y>\phi_y>\underline{\phi}_y$ implies $\Xi_2>0$ and $\Xi_1<0$. Thus, we have $\mathcal{M}_g\gtreqless 0$ if $\tau_p\lesseqgtr\frac{1}{\Theta}\frac{\Xi_1}{\Xi_2}<0$.
4. $\phi_y=\underline{\phi}_y$ implies $\Xi_2>0$ and $\Xi_1=0$. Thus, we have $\mathcal{M}_g\gtreqless 0$ if $\tau_p\lesseqgtr 0$.
5. $\overline{\phi}_y>\underline{\phi}_y>\phi_y$ implies $\Xi_2>0$ and $\Xi_1>0$. Thus, we have $\mathcal{M}_g\gtreqless 0$ if $\tau_p\lesseqgtr\frac{1}{\Theta}\frac{\Xi_1}{\Xi_2}>0$.

Statement (a). In the case of $\tilde{s}=1$, i.e. for a TANK model, we obtain

$$
\Xi_1=\frac{\lambda+(1-\lambda)\Gamma_c}{(1-\lambda)\Gamma_c\gamma_c}(1-\rho)\left(\rho\mathcal{M}-\phi_y\right)<0\,,\quad \Xi_2=\frac{1}{(1-\lambda)\Gamma_c}(1-\rho)\left(\rho\mathcal{M}-\phi_y\right)<0\,,
$$

where the signs follows from the determinacy condition. Moreover, for $\rho=0$, we obtain

$$
\Xi_1=-\frac{\lambda+(1-\lambda)\Gamma_c}{(1-\lambda)\Gamma_c\gamma_c}\phi_y<0\,,\quad \Xi_2=-\frac{1}{(1-\lambda)\Gamma_c}\phi_y<0\,.
$$

As a result, we obtain for both cases that $\mathcal{M}_g\gtreqless 0$ if

$$
\tau_p\gtreqless\frac{1}{\Theta}\frac{\Xi_1}{\Xi_2}=\frac{\lambda+(1-\lambda)\Gamma_c}{\Theta\gamma_c}=\frac{1}{\gamma_c}\frac{\lambda+(1-\lambda)\Gamma_c}{\lambda+(1-\lambda)\Gamma_y}=1\,.
$$

As a result, demand effects are never strong enough in a TANK economy, such that technology can only increase if one redistributes sufficiently towards savers, i.e., we are in a regressive regime.

Statement (b). In the first case $\phi_y > \overline{\phi}_y$, the tax threshold is given by

$$\tau_p = \frac{\lambda+(1-\lambda)\Gamma_c}{\Theta\gamma_c} \frac{\frac{1}{(1-\lambda)\Gamma_c}\left(\rho(1-\rho)(1-\tilde{s})\chi+(1-\rho\tilde{s})\rho\mathcal{M}-(1-\rho)\phi_y\right)}{\frac{1}{(1-\lambda)\Gamma_c}\left(\rho(1-\rho)(1-\tilde{s})\chi+(1-\rho\tilde{s})\rho\mathcal{M}-(1-\rho)\phi_y\right)+\rho(1-\tilde{s})\frac{(1-\rho)\mathcal{E}_y+\rho\mathcal{M}}{\lambda}},$$

which can be rearranged to

$$\tau_p = 1 - \frac{\rho(1-\tilde{s})\frac{(1-\rho)\mathcal{E}_y+\rho\mathcal{M}}{\lambda}}{\frac{1}{(1-\lambda)\Gamma_c}\left(\rho(1-\rho)(1-\tilde{s})\chi+(1-\rho\tilde{s})\rho\mathcal{M}-(1-\rho)\phi_y\right)+\rho(1-\tilde{s})\frac{(1-\rho)\mathcal{E}_y+\rho\mathcal{M}}{\lambda}} > 1\,,$$

which implies $\mathcal{M}_g < 0$ on $\tau_p \in [0,1]$. Additionally, $\phi_y = \overline{\phi}_y$ implies $\mathcal{M}_g < 0$. Moreover, for $\overline{\phi}_y > \phi_y > \underline{\phi}_y$ we have $\mathcal{M}_g < 0$ if $\tau_p > \frac{1}{\Theta}\frac{\Xi_1}{\Xi_2}$, which is satisfied throughout on $\tau_p \in [0,1]$ as $\frac{1}{\Theta}\frac{\Xi_1}{\Xi_2} < 0$ in this case. For $\phi_y = \underline{\phi}_y = \phi_y^*$, we obtain $\mathcal{M}_g = 0$ if $\tau_p = 0$ and $\mathcal{M}_g < 0$ if $\tau_p \in (0,1]$. Finally, $\overline{\phi}_y > \underline{\phi}_y > \phi_y$ implies $\mathcal{M}_g \gtrless 0$ if $\tau_p \lessgtr \tau_p^* = \frac{1}{\Theta}\frac{\Xi_1}{\Xi_2} \in (0,1)$. To show that $\tau_p^* < \tilde{\tau}_p$ note that the technology growth multiplier writes $\mathcal{M}_g = \Lambda_y\mathcal{M}_y - \Lambda_s$, where $\Lambda_y > 0$ and

$$\begin{aligned}\Lambda_s = \frac{(1-\rho)\mathcal{E}_s}{1+\mathcal{E}_g(1-\rho)} &= \frac{(1-\rho)}{1+\mathcal{E}_g(1-\rho)}\frac{1}{(1-\lambda)\Gamma_c}\left(\frac{\lambda+(1-\lambda)\Gamma_c}{\gamma_c} - \Theta\tau_p\right)\\ &= \frac{(1-\rho)}{1+\mathcal{E}_g(1-\rho)}\frac{\Theta}{(1-\lambda)\Gamma_c}(1-\tau_p) \geq 0\,.\end{aligned}$$

As a result, $\mathcal{M}_g$ can only be strictly positive if $\mathcal{M}_y$ is strictly positive. This implies $\tau_p^* = \frac{1}{\Theta}\frac{\Xi_1}{\Xi_2} < \tilde{\tau}_p < 1$, which concludes the proof.

B.6.2 *Proof Proposition 8*

PROOF. We begin with statement (a). For $\rho = 0$, Proposition 7 implies $\mathcal{M}_y = 0$. As a result, any tax incidence maximizes the impact output level $\ln Y_t$, i.e., $\tau_p^{\max}(0) \in [0,1]$. For any $z > 0$, the output level can be written as $\ln Y_{t+z} = \ln A_t + z\ln g^A + \ln y + \mathcal{M}_g$. Maximizing the level of output is therefore equivalent to maximizing the technology growth multiplier $\mathcal{M}_g$. Importantly, only its numerator depends on τ_p in a linear way, i.e., $\Xi_1 - \Xi_1\Theta\tau_p$, where $\Xi_1 < 0$ and $\Xi_2 < 0$. As a result, $\mathcal{M}_g$ and $\ln Y_{t+z}$ are maximized under $\tau_p^{\max}(z) = 1\ \forall z > 0$. Similarly, when $\tilde{s} = 1$, we obtain for any ρ that $\mathcal{M}_y = 0$ such that $\tau_p^{\max}(0) \in [0,1]$ follows. For any $z > 0$, however, the output level is now given by

$$\ln Y_{t+z} = \ln A_t + z\ln g^A + \ln y + \sum_{i=0}^{z-1}\rho^i\mathcal{M}_g + \rho^z\mathcal{M}_y\,,$$

which is again equivalent to maximizing $\mathcal{M}_g$ with respect to τ_p. As before, only the numerator of $\mathcal{M}_g$ depends in a linear way on τ_p, i.e., $\Xi_1 - \Xi_1\Theta\tau_p$, where $\Xi_1 < 0$ and $\Xi_2 < 0$ and the result follows.

To prove statement (b), recall that the denominators of $\mathcal{M}_y$ and $\mathcal{M}_g$ are equal and independent of τ_p. Moreover, their numerators are given by

$$\begin{aligned}
\text{NU}(\mathcal{M}_y) &= \frac{1}{\mathcal{E}_y}\frac{1}{1+\mathcal{E}_g(1-\rho)}\rho(1-\tilde{s})\left[\Lambda_1 - \Lambda_2\Theta\tau_p\right] , \\
\text{NU}(\mathcal{M}_g) &= \frac{1}{\mathcal{E}_y}\frac{1}{1+\mathcal{E}_g(1-\rho)}\left[\Xi_1 - \Xi_2\Theta\tau_p\right] ,
\end{aligned}$$

where Ξ_1 and Ξ_2 are defined in the proof of Proposition 7. The other both auxiliary parameters are

$$\Lambda_1 = \frac{\lambda + (1-\lambda)\Gamma_c}{(1-\lambda)\Gamma_c\gamma_c} > 0\,, \quad \Lambda_2 = \frac{1}{(1-\lambda)\Gamma_c} + \frac{1+\mathcal{E}_g(1-\rho)}{\lambda} > 0\,.$$

Additionally, using the rules for geometric series, we can rewrite

$$\ln Y_{t+z} = \ln A_t + z\ln g^A + \ln y + \rho^z\mathcal{M}_y + \frac{1-\rho^z}{1-\rho}\mathcal{M}_g\,.$$

Consequently, we obtain

$$\text{sgn}\left(\frac{\partial \ln Y_{t+z}}{\partial \tau_p}\right) = -\rho^z\rho(1-\tilde{s})\Lambda_2 - \frac{1-\rho^z}{1-\rho}\Xi_2\,.$$

From here, we distinguish two cases. In the first case, $0 < \phi_y \leq \overline{\phi}_y$ implies $\Xi_2 \geq 0$. As a result, $\text{sgn}\left(\frac{\partial \ln Y_{t+z}}{\partial \tau_p}\right) < 0$ such that $\tau_p^{\max}(z) = 0\ \forall z \geq 1$. Moreover, maximizing $\ln Y_t$ is achieved by maximizing $\mathcal{M}_y$, resulting in $\tau_p^{\max}(0) = 0$. Taken together, we have $\tau_p^{\max}(z) = 0\ \forall z \geq 0$. In the second case, $\phi_y > \overline{\phi}_y$ implies $\Xi_2 < 0$. As a result, the overall sign is ambiguous, i.e.,

$$\text{sgn}\left(\frac{\partial \ln Y_{t+z}}{\partial \tau_p}\right) = -\frac{\rho(1-\rho)(1-\tilde{s})\Lambda_2 - \Xi_2}{1-\rho}\left[\rho^z + \frac{\Xi_2}{\rho(1-\rho)(1-\tilde{s})\Lambda_2 - \Xi_2}\right] \gtreqless 0\,.$$

The term in brackets is zero for $z = \tilde{z}$, where

$$\tilde{z} = \frac{\ln\left(-\dfrac{\Xi_2}{\rho(1-\rho)(1-\tilde{s})\Lambda_2 - \Xi_2}\right)}{\ln\rho} > 0\,.$$

As a consequence, we obtain $\text{sgn}\left(\frac{\partial \ln Y_{t+z}}{\partial \tau_p}\right) < 0$ and thus $\tau_p^{\max}(z) = 0\ \forall\ 0 < z < \tilde{z}$, $\text{sgn}\left(\frac{\partial \ln Y_{t+z}}{\partial \tau_p}\right) = 0$ and thus $\tau_p^{\max}(\tilde{z}) \in [0,1]$, and $\text{sgn}\left(\frac{\partial \ln Y_{t+z}}{\partial \tau_p}\right) > 0$ and thus $\tau_p^{\max}(z) = 1\ \forall z > \tilde{z}$. As before, for in the case $z = 0$ maximizing $\ln Y_t$ is achieved by maximizing $\mathcal{M}_y$, resulting in $\tau_p^{\max}(0) = 0$. This completes the proof. □

APPENDIX C: Additional Theoretical Results

[TO BE COMPLETED] The following proposition presents results on the aggregate effects of a government spending shock in the case of $\tau_p \in \mathbb{R}$ and, thus, generalizes the proposition presented in the main text.

PROPOSITION 9 (FISCAL MULTIPLIER). *The impact multipliers to output and technology growth following a government spending shock are given by:*

$$\mathcal{M}_y = \frac{\zeta_{sG} - \rho\zeta_{sG'} - (\zeta_g - \rho\zeta_{g'})\Lambda_{sG}}{1 + \phi_y\zeta_r - \rho\zeta_f - (\zeta_g - \rho\zeta_{g'})\Lambda_y}, \quad \mathcal{M}_g = \frac{(\zeta_{sG} - \rho\zeta_{sG'})\Lambda_y - (1 + \phi_y\zeta_r - \rho\zeta_f)\Lambda_{sG}}{1 + \phi_y\zeta_r - \rho\zeta_f - (\zeta_g - \rho\zeta_{g'})\Lambda_y},$$

where we have used $\Theta = \lambda + (1-\lambda)\Gamma_y$ *to define the new auxiliary variables*

$$\begin{aligned}
\zeta_{sG} &= \frac{1 - \gamma_c\tau_p\Theta}{1 - \lambda\gamma_c\chi}, \\
\zeta_{sG'} &= \left(s + (1-s)\frac{1-\lambda}{\lambda}\frac{\tau_p\Theta}{1/\gamma_c - \Theta\tau_p}\right)\zeta_{sG}, \\
\xi_{sG} &= (g^A - 1)\frac{1 - \gamma_c\tau_p\Theta}{g^A(1-\lambda\gamma_c) + \gamma_c(\lambda - 1)}, \\
\Lambda_{sG} &= (1-\rho)\xi_{sG}/(1 - \rho\xi_{g'}).
\end{aligned}$$

(a) *If* $\rho = 0$ *or* $s = 1$, *government spending has no effect on output, i.e.,* $\mathcal{M}_y = 0$, *but has an ambiguous effect on technological growth, i.e.,* $\mathcal{M}_g \gtreqless 0$ *if* $\tau_p \gtreqless 1/(\Theta\gamma_c)$.

(b) *If* $\rho > 0$ *and* $s < 1$, *government spending has ambiguous effects on output, i.e.,* $\mathcal{M}_y \gtreqless 0$ *if*

$$\tau_p \lesseqgtr \tilde{\tau}_p \equiv \frac{1}{\Theta}\frac{\lambda/\gamma_c}{1 + (1-\rho)\dfrac{1-\gamma_c}{\gamma_c}\dfrac{g^A}{g^A - 1}} \in (0,1).$$

Regarding the response of technology growth, there are three regimes:

(R1) *For* $\phi_y > \overline{\phi}_y$, *one obtains* $\mathcal{M}_g \gtreqless 0$ *if* $\tau_p \gtreqless \tau_p^*$, *where* $\tau_p^* \in \mathbb{R}_+$. *Additionally, for* $\phi_y = \overline{\phi}_y$, *one obtains* $\mathcal{M}_g < 0$.

(R2) *For* $\overline{\phi}_y > \phi_y > \underline{\phi}_y$, *one obtains* $\mathcal{M}_g \gtreqless 0$ *if* $\tau_p \lesseqgtr \tau_p^{**}$, *where* $\tau_p^{**} \in \mathbb{R}_-$.

(R3) *For* $\overline{\phi}_y > \underline{\phi}_y > \phi_y$, *one obtains* $\mathcal{M}_g \gtreqless 0$ *if* $\tau_p \lesseqgtr \tau_p^{***}$, *where* $\tau_p^{***} \in \mathbb{R}_+$. *Additionally, for* $\phi_y = \underline{\phi}_y$, *one obtains* $\mathcal{M}_g \gtreqless 0$ *if* $\tau_p \lesseqgtr 0$.

APPENDIX D: Quantitative Model

This appendix provides the complete specification of the quantitative HANK-G model whose key features are summarized in Section 3 of the main text. Appendix D.1 describes the full model environment, Appendix D.2 states the definition of the BGP equilibrium, Appendix D.3 details the labor-market splitting rule and the calibration of the incidence functions, Appendix D.4 provides the impulse response functions, and Appendix D.5 derives the welfare criterion used for welfare analysis.

D.1 Full Model Environment

As the main text presents a condensed version of the model environment (see Section 3) we spell out the full specification subsequently.

D.1.1 *Households* Household i's utility depends on consumption c_{it}, hours worked ℓ_{it}, and employment status $e_{it} \in \{0,1\}$ according to:[30]

$$\mathcal{U}(c_{it}, e_{it}\ell_{it}) = \frac{c_{it}^{1-\sigma}}{1-\sigma} - \nu \frac{(e_{it}\ell_{it})^{1+\varphi}}{1+\varphi}.$$

A time-invariant share Λ_E of households are *potential entrepreneurs*; the remainder are workers. Both types supply labor hours ℓ_{it}, face labor income risk through shocks to idiosyncratic productivity h_{it} and employment status e_{it}, and differ in their discount factor β_{it}. Workers save only in liquid assets $b_{it} \geq 0$. Entrepreneurs additionally accumulate an illiquid stock of normalized innovation capital $a_{it} \equiv A_{it}/A_t > 0$, representing knowledge, patents, and entrepreneurial skills, which depreciates at rate δ^A.

Agents die with probability p_d each period and are replaced by newborns with zero wealth who inherit their parent's entrepreneurial type, while β and h continue along their Markov processes. Bequests are fully taxed; innovations vanish upon death.

The vector of exogenous states is $\mathbf{s}_{it} \equiv (\beta_{it}, h_{it}, e_{it}, \lambda_i^E)$, where $\lambda_i^E = 1$ flags a potential entrepreneur. The joint distribution over assets and exogenous states is $m_t(a_{it}, b_{it}, \mathbf{s}_{it})$.

Idiosyncratic heterogeneity and Markov processes. We now details the process behind the three dimensions of persistent heterogeneity: the discount factor β_{it}, idiosyncratic labor productivity h_{it}, and employment status e_{it}.

1. *Discount factor.* To generate a realistic cross-sectional distribution of marginal propensities to consume (Carroll et al., 2017), the discount factor takes three values, $\beta_{it} \in \{\beta_L, \beta_M, \beta_H\}$, with $\beta_L = \beta_M - \Delta^\beta$ and $\beta_H = \beta_M + \Delta^\beta$. Transitions are

[30] The corresponding non-stationary utility is $A_t^{1-\sigma}\left[\frac{c_{it}^{1-\sigma}}{1-\sigma} - \nu\frac{(e_{it}\ell_{it})^{1+\varphi}}{1+\varphi}\right]$, so the disutility of labor $\nu A_t^{1-\sigma}$ scales with technology.

restricted to adjacent types, so the Markov transition matrix $\mathbb{P}^{\beta}$ satisfies

$$\mathbb{P}^{\beta} = \begin{array}{c} \\ \beta_L \\ \beta_M \\ \beta_H \end{array} \begin{array}{c} \begin{array}{ccc} \beta_L & \beta_M & \beta_H \end{array} \\ \begin{pmatrix} p_\beta & 1-p_\beta & 0 \\ \frac{1-p_\beta}{2} & p_\beta & \frac{1-p_\beta}{2} \\ 0 & 1-p_\beta & p_\beta \end{pmatrix} \end{array}$$

2. *Labor productivity.* The logarithm of idiosyncratic productivity follows an AR(1) process with autocorrelation ρ_h and innovation ε_{it}^h:

$$\log h_{i,t+1} = \rho_h \log h_{it} + \varepsilon_{it}^h \,, \qquad \varepsilon_{it}^h \sim \mathcal{N}(0, \sigma_h^2) \,.$$

   We normalize $\int_i h_{it}\, di = 1$. In the numerical implementation this process is discretized using the standard Rouwenhorst method into the five productivity groups $\{h_1, \ldots, h_5\}$ described in Table D.1, with transition matrix $\mathbb{P}^h(h_{t+1} \mid h_t)$.

3. *Employment status.* Employment $e_{it} \in \{0, 1\}$ evolves according to a type-h transition matrix that is allowed to vary over time:

$$\mathbb{P}_t^e(\,\cdot \mid h_{it}) = \begin{array}{c} \\ e_t = 0 \\ e_t = 1 \end{array} \begin{array}{c} \begin{array}{cc} e_{t+1} = 0 & e_{t+1} = 1 \end{array} \\ \begin{pmatrix} 1 - \mathbb{P}_t^e(1 \mid 0; h_{it}) & \mathbb{P}_t^e(1 \mid 0; h_{it}) \\ \mathbb{P}_t^e(0 \mid 1; h_{it}) & 1 - \mathbb{P}_t^e(0 \mid 1; h_{it}) \end{pmatrix} \end{array},$$

   where $\mathbb{P}_t^e(1 \mid 0; h_{it})$ is the job-finding probability and $\mathbb{P}_t^e(0 \mid 1; h_{it})$ is the separation probability, both depending on productivity type h_{it}. Their steady-state values and cyclical calibration are described in Appendix D.3.

D.1.2 *Entrepreneurial Innovation.* By investing an amount x_{it}, entrepreneurs generate innovations with success probability

$$\zeta(x_{it}) = 1 - \exp(-\gamma x_{it}) \,.$$

A successful innovation raises the entrepreneur's knowledge stock by a fixed step ηA_t, where $\eta > 0$ is the step size and A_t the aggregate stock of innovation.[31] Independently of the investment decision, realized innovations are scaled by an idiosyncratic success shock $\iota \sim F_\iota$, which generates heterogeneity in innovation outcomes and hence in entrepreneurial income: investment does not always translate one-for-one into a realized innovation, and large realizations can occur even with modest investment.

Let $d_{i,t+1} \in \{0, 1\}$ indicate whether the innovation step is realized. Then normalized innovation capital evolves as

$$a_{i,t+1} = \iota_{i,t+1} \frac{(1 - \delta^A) a_{i,t}}{g_{t+1}^A} + \eta\, d_{i,t+1} \,,$$

[31] This setup is isomorphic to the analytical model in Section 2, but with a different probability specification. The law of motion can be rewritten as $A_{t+1}(j) = p\big(A_{it}(j) + \eta\big) + (1-p)A_{it}(j)$, with $p = (\psi/\eta) I_{it}^S$.

where g^A_{t+1} captures aggregate gross technology growth (obsolescence) and δ^A is the depreciation rate of innovation capital. Equivalently, in case-form:

$$a_{i,t+1} = \begin{cases} \iota_{i,t+1} \dfrac{a_{i,t}(1-\delta^A)}{g^A_{t+1}} + \eta, & \text{with prob. } \zeta(x_{it}), \\ \iota_{i,t+1} \dfrac{a_{i,t}(1-\delta^A)}{g^A_{t+1}}, & \text{with prob. } 1-\zeta(x_{it}). \end{cases}$$

D.1.3 *Income.* An entrepreneur receives after-tax business income $R^A(a_{it}) = (1-\tau_c)\Theta_\alpha L_t a_{it}$ by selling intermediate goods, where τ_c is the corporate tax rate. Total before-tax income combines labor earnings, including unemployment benefits, entrepreneurial profits, and interest income:

$$y_{it} = \big(\varsigma(1-e_{it}) + e_{it}\big) w_t h_{it} \ell_{it} + \lambda^E_i R^A(a_{it}) + r_t b_{it}\,, \tag{10}$$

where ς is the unemployment replacement rate, w_t the real wage rate in the economy, and r_t the real interest rate. Households face an income tax $\tau(y_{it})$ and receive transfers $T(y_{it})$, both functions of total income.

D.1.4 *Household Problem.* The household's recursive problem is

$$V(a_{it}, b_{it}, \mathbf{s}_{it}) = \max_{\substack{c_{it}, x_{it}, \\ b_{it+1} \geq 0}} \mathcal{U}(c_{it}, e_{it}\ell_{it}) + \widetilde{\beta}_{it} \mathop{\mathbb{E}}_{\substack{h_{it+1}, \beta_{it+1}, \\ e_{it+1}}} \Big[\lambda^E_i V^x\big(a_{it}, b_{it+1}, \mathbf{s}_{it+1} | x_{it}\big) + (1-\lambda^E_i) V\big(0, b_{it+1}, \mathbf{s}_{it+1}\big) \Big],$$

$$\text{subject to} \qquad c_{it} + x_{it} + g^A_{t+1} b_{it+1} = y_{it} - \tau(y_{it}) + T(y_{it}) + b_{it}\,, \quad \text{and} \quad (10)\,,$$

where $\widetilde{\beta}_{it} = \beta_{it}(1-p_d)(g^A_{t+1})^{1-\sigma}$ is the growth-normalized discount factor. The entrepreneurial continuation value from innovating is

$$V^x\big(a_{it}, b_{it+1}, \mathbf{s}_{it+1} \mid x_{it}\big) = \int_\iota \Big[(1-\zeta(x_{it})) V\Big(\iota \tfrac{a_{it}(1-\delta^A)}{g^A_{t+1}}, b_{it+1}, \mathbf{s}_{it+1} \Big) + \zeta(x_{it}) V\Big(\iota \tfrac{a_{it}(1-\delta^A)}{g^A_{t+1}} + \eta, b_{it+1}, \mathbf{s}_{it+1} \Big) \Big] dF_\iota(\iota).$$

D.1.5 *Unions and Phillips Curve* To facilitate comparison with the tractable model, nominal rigidity is introduced through union wage setting. Each union k chooses its wage W_{kt} to maximize the utility of a representative member, subject to quadratic wage adjustment costs that enter utility directly:

$$\sum_{t=0}^{\infty} \left(\prod_{s=0}^{t} \widetilde{\beta}_s \right) \left[\big(u(C_t) - \nu(L_{kt})\big) - \frac{\theta}{2} \left(\frac{W_{kt}}{W_{k,t-1}} - g^A \right)^2 \right],$$

where the demand for employment tasks is $L_{kt} = \left(\frac{W_{kt}}{W_t}\right)^{-\epsilon_w} L_t$ and $W_t = \left(\int W_{kt}^{1-\epsilon_w} dk\right)^{\frac{1}{1-\epsilon_w}}$ is the price index for aggregate employment services. Under the above specification, the

Phillips curve is given by

$$\frac{\epsilon_w}{\theta} L_t \left(\nu'(L_t) - u'(C_t) w_t \frac{(\epsilon_w - 1)}{\epsilon_w} \left(1 - \frac{\partial \mathcal{T}(y^d)}{\partial y^d} \right) \right) = \pi_t^w (\pi_t^w - g^A) - \widetilde{\beta} \left[\pi_{t+1}^w (\pi_{t+1}^w - g^A) \right] .$$

The steady-state is characterized by

$$\overline{L} = \nu^{-1} \left(u'(\overline{C}) w_t \frac{\epsilon_w - 1}{\epsilon_w} \left(1 - \frac{\partial \mathcal{T}(y^d)}{\partial y^d} \right) \right) .$$

D.1.6 *Labor-Market Splitting Rule.* Off the balanced growth path, unions distribute labor adjustments across productivity types and between the intensive and extensive margins according to a simple splitting rule. For each productivity type h, total efficiency-weighted labor is given by

$$L_t^h = h(1 - u_t^h)\ell_t^h .$$

We assume that deviations of labor at type h from its steady-state level satisfy

$$\Delta L_t^h \equiv L_t^h - \overline{L}^h = \mu(h)\big(L_t - \overline{L}\big) , \qquad \text{with} \qquad \int_h \mu(h)\, dF_h(h) = 1 ,$$

where $\overline{L}$ and $\overline{L}^h$ denote steady-state aggregate labor and steady-state labor for productivity type h, respectively. The function $\mu(h)$ therefore governs how aggregate labor fluctuations are distributed across productivity groups.

Within each productivity type, all employed workers supply the same number of hours, and steady-state hours are common across types, so that ℓ_t^h is type-specific off steady state while $\overline{\ell}^h \equiv \overline{\ell}$ in the steady state. A lottery then determines which households are employed and which are unemployed.

We further assume that unions split changes in labor demand for type h between hours per worker and the employment rate. In particular,

$$\Delta \ell_t^h = \frac{\phi(h)}{1 - \overline{u}^h} \cdot \frac{\Delta L_t^h}{h} , \qquad \Delta u_t^h = -\frac{1 - \phi(h)}{\ell_t^h} \cdot \frac{\Delta L_t^h}{h} , \tag{11}$$

where $\phi(h) \in [0, 1]$ measures the share of labor adjustment absorbed by hours per worker, while $1 - \phi(h)$ measures the share absorbed by changes in employment.[32]

Unemployment for type h must also be consistent with flows into and out of employment. Letting $\mathbb{P}_t^e(\cdot \mid \cdot; h)$ denote transition probabilities conditional on productivity

[32] These expressions follow from the accounting identity

$$\frac{\Delta L_t^h}{h} = (1 - \overline{u}^h)\Delta \ell_t^h - \ell_t^h \Delta u_t^h .$$

h, the law of motion for unemployment can be written as

$$\Delta u_{t+1}^h = \int_i \Big(\underbrace{\mathbb{P}_t^e(e_{i,t+1}=0 \mid e_{it}=1; h_{it}) \mathbb{1}_{e_{it}=1}}_{\text{job separations among workers of type } h} - \underbrace{\mathbb{P}_t^e(e_{i,t+1}=1 \mid e_{it}=0; h_{it}) \mathbb{1}_{e_{it}=0}}_{\text{jobs found among unemployed of type } h} \Big) \mathbb{P}^h(h \mid h_{it})\, di \,. \tag{12}$$

This expression makes clear that unemployment risk for a given productivity group reflects both separation and job-finding flows, including transitions from other productivity states.[33]

To discipline the decomposition of unemployment fluctuations into separations and job-finding, let $p_t^u(h)$ denote the separation rate that rationalizes the observed change in unemployment for type h, holding job-finding rates fixed at their steady-state values. We then parameterize actual separations as

$$\mathbb{P}_t^e(e'=0 \mid e=1; h) = \phi^s p_t^u(h) + (1-\phi^s)\overline{\mathbb{P}}^e(e'=0 \mid e=1; h) \,, \quad \forall h \,,$$

where $\phi^s \in [0,1]$ controls the share of unemployment variation accounted for by separations. Job-finding rates are then determined residually so as to match the implied unemployment dynamics for each productivity group. This parsimonious structure allows the model to reproduce both the level and the persistence of unemployment risk across workers with different labor income exposure.

D.1.7 *Government and Monetary Policy* The government issues bonds and collects revenues from income taxes, corporate taxes, and bequest taxes. Income taxes and transfers follow Ferriere et al. (2023):

$$\tau(y_{it}) = \exp\Big(\log(\vartheta_t)\big(y_{it}/y\big)^{-2\varrho}\Big) \,, \qquad T(y_{it}) = \underline{T}_t y \frac{2\exp\{-\xi(y_{it}/y)\}}{1+\exp\{-\xi(y_{it}/y)\}} \,, \tag{13}$$

where $\varrho \in (-\infty, 1)$ is the constant rate of tax progressivity ($\varrho = 0$ linear; $\varrho \in (0,1)$ progressive; $\varrho < 0$ regressive), ϑ_t controls the average tax rate, y is BGP mean income, $\underline{T}_t$ is the transfer to a household with zero income as a multiple of y, and ξ governs how quickly transfers phase out. The net tax schedule is $\mathcal{T}(y_{it}) = \tau(y_{it}) y_{it} - T(y_{it})$.

The government budget constraint is

$$g_{t+1}^A \frac{B_{t+1}^G}{A_{t+1}} + \int \Big(\mathcal{T}(y_{it}) + p_d b_{it} - \varsigma w_t h_{it} \ell_{it} \Big) dm_t(a_{it}, b_{it}, \mathbf{s}_{it}) + \tau_c \Theta_\alpha L_t = (1+r_t)\frac{B_t^G}{A_t} + \frac{G_t}{A_t} \,. \tag{14}$$

Monetary policy follows a Taylor rule:

$$i_t = r + \phi_\pi(\pi_t - 1) + \phi_y\left(y_t/y - 1\right) \,. \tag{15}$$

[33] No terms involving population shares appear here because the law of motion is written in integral form over individuals rather than in group-mass form. The integration measure already accounts for the mass of each origin productivity group, while $\mathbb{P}^h(h \mid h_{it})$ maps those individuals into destination type h.

D.2 Balanced Growth Path Equilibrium

DEFINITION 1 (Balanced Growth Path General Equilibrium). Given an initial stock of innovations A_{-1}, a nominal wage W_{-1}, government debt B^G_{-1}, an initial distribution $m_{-1}(a,b,\mathbf{s})$, and exogenous sequences of taxes and transfers, a *balanced growth path general equilibrium* is a sequence of prices $\{P_t, W_t, \pi_t, \pi^w_t, r_t, i_t\}$, a gross technology growth rate g^A_{t+1}, individual policy functions $\{c_{it}(a_{it}, b_{it}, \mathbf{s}_{it})$, $b_{it+1}(a_{it}, b_{it}, \mathbf{s}_{it})$, $x_{it}(a_{it}, b_{it}, \mathbf{s}_{it})\}$, a joint distribution $m_t(a_{it}, b_{it}, \mathbf{s}_{it})$, and aggregates $\{Y_t, L_t, C_t, G_t, B^G_t, T_t, A_t\}$ such that:

1. Households, unions, and firms optimally solve their respective programs.
2. Monetary and fiscal policy follow their rules and constraints (14) and (15).
3. Goods markets clear:
$$I_t + \frac{G_t}{A_t} + \int c(a_{it}, b_{it}, \mathbf{s}_{it})\, dm_t(a_{it}, b_{it}, \mathbf{s}_{it}) = \frac{Y_t}{A_t}\,, \quad \text{with} \quad I_t \equiv \int_i x_{it}\, di\,.$$
4. Bond markets clear: $B_t \equiv \int_i b_{it}\, di = B^G_t$.
5. On the balanced growth path, aggregate technology satisfies
$$g^A = (1-\delta^A) + \eta \int \zeta\big(x_{it}(a_{it}, b_{it}, \mathbf{s}_{it})\big) dm^E_t(a_{it}, b_{it}, \mathbf{s}_{it})\,,$$
where m^E_t is the measure of entrepreneurs.

D.3 Labor-Market Calibration and Incidence Functions

D.3.1 *Steady-State Employment* We use panel data from the *Current Population Survey* (CPS) over 1989–2022, restricting the sample to individuals aged 20–65. Workers are sorted into wage bins that map to the model's productivity groups h. Because wages are not observed for unemployed individuals, we impute latent wages using observable characteristics.

For each h-group, the steady-state job-finding rate, $\bar{\mathbb{P}}^e(e_{it}=1 \mid e_{it}=0; h_{it})$, is set equal to its long-run average in the CPS. Given these values, the steady-state separation rate, $\bar{\mathbb{P}}^e(e_{it}=0 \mid e_{it}=1; h_{it})$, is chosen so that the model matches the long-run unemployment rate of each group observed in the CPS.

D.3.2 *Cyclical Incidence* The CPS also provides empirical counterparts for fluctuations in total employment, ΔL_t, average hours by productivity group, $\Delta \ell^h_t$, and total hours by productivity group, $\Delta L^h_t/h$. We use these series to discipline the two objects governing cyclical labor-market incidence. From equations (11), we compute
$$\phi(h) = \frac{\Delta \ell^h_t (1-\bar{u}^h)}{\Delta L^h_t/h}\,, \qquad \mu(h) = \frac{\Delta L^h_t}{\Delta L_t}\,.$$

The parameter $\phi(h)$ measures the share of labor adjustment absorbed by hours per worker rather than by employment. Its value ranges from 0.17 to 0.32, with lower values

for low-income groups, indicating that labor adjustment at the bottom of the wage distribution occurs primarily through the extensive margin. The incidence function $\mu(h)$ ranges from 0.28 to 1.61 and is highest for the lowest wage bins, implying that low-wage workers bear a disproportionate share of aggregate employment fluctuations.

To discipline cyclical unemployment risk across groups, we use equation (12) to infer the separation rates that are consistent with the observed fluctuations in unemployment, while keeping job-finding rates fixed at their steady-state values. We denote these implied separation rates by $p_t^u(h)$. By construction, they represent the changes in separation rates that would exactly reproduce the observed variation in unemployment for group h if job-finding rates did not move.

Because part of unemployment fluctuations in the data also reflects variation in job-finding rates, we do not impose $p_t^u(h)$ directly as the actual separation rate. Instead, following Krusell et al. (2017), we assume that actual separation rates are given by

$$\mathbb{P}_t^e(e=0 \mid e=1;h) = \phi^s p_t^u(h) + (1-\phi^s)\bar{\mathbb{P}}^e(e=0 \mid e=1;h)\,, \qquad \forall h\,, \tag{16}$$

where ϕ^s measures the share of unemployment fluctuations accounted for by separations. We set $\phi^s \approx 0.3$, so that job separations explain roughly one third of total unemployment variation. Given these separation rates, job-finding rates are then solved residually from equation (12) so that the model matches the observed unemployment dynamics across h-groups.

Table D.1 reports the resulting incidence measures together with the steady-state labor-market statistics by h-group.

TABLE D.1. Labor market statistics across h-groups.

		Incidence		Steady-state		
h-group	Wage bin	$\mu(h)$	$\phi(h)$	$\bar{u}^h$	$\bar{\mathbb{P}}^e(e=1\|e=0;h)$	$\bar{\mathbb{P}}^e(e=0\|e=1;h)$
h_1	$[0.00, 0.16]$	1.61	0.22	0.107	0.497	0.078
h_2	$[0.16, 0.38]$	1.11	0.18	0.069	0.487	0.026
h_3	$[0.38, 0.62]$	1.30	0.17	0.053	0.481	0.026
h_4	$[0.62, 0.84]$	0.64	0.22	0.041	0.475	0.022
h_5	$[0.84, 1.00]$	0.28	0.32	0.028	0.461	0.009

D.4 Impulse Response Functions

[TO BE ADDED]

D.5 Welfare Criterion

We evaluate welfare at the onset of a deterministic MIT shock by comparing the entire transition path of the economy to the initial stationary balanced-growth path. Let aggregate technology in economy $j \in \{0,1\}$ evolve according to

$$A_{t+1}^j = g_t^{A,j} A_t^j\,, \quad A_0^1 = A_0^0 \equiv A_0\,,$$

where $j = 0$ denotes the no-shock economy and $j = 1$ the shocked economy. Period utility for household i is

$$u(C_{it}, L_{it}; A_t) = \frac{C_{it}^{1-\sigma}}{1-\sigma} - \nu A_t^{1-\sigma} \frac{L_{it}^{1+\varphi}}{1+\varphi} .$$

This specification ensures that the household problem can be written in stationary form after normalizing by aggregate technology. Defining normalized consumption by

$$c_{it}^j \equiv \frac{C_{it}^j}{A_t^j} ,$$

where i denotes the household index and μ_t^j the associated cross-sectional distribution over household states, period utility becomes

$$u(C_{it}^j, L_{it}^j; A_t^j) = (A_t^j)^{1-\sigma} \left[\frac{\left(c_{it}^j\right)^{1-\sigma}}{1-\sigma} - \nu \frac{\left(L_{it}^j\right)^{1+\varphi}}{1+\varphi} \right] .$$

It follows that date-0 ex-ante welfare in economy j can be written as

$$\widetilde{W}_0^j = \int \mathbb{E}_0 \left[\sum_{t=0}^{\infty} Q_t^j \beta_t^j(x) \left(\frac{\left(c_t^j(x)\right)^{1-\sigma}}{1-\sigma} - \nu \frac{\left(L_t^j(x)\right)^{1+\varphi}}{1+\varphi} \right) \right] d\mu_t^j(x) ,$$

where

$$Q_t^j \equiv \left(\prod_{s=0}^{t-1} g_s^{A,j} \right)^{1-\sigma} , \qquad Q_0^j = 1 .$$

This representation makes clear that temporary deviations in the gross growth rate $g_t^{A,j}$ have permanent welfare effects through their cumulative impact on the level of aggregate technology A_t^j.

We measure the welfare effect of the MIT shock using a consumption-equivalent variation, defined as the uniform proportional change in consumption along the no-shock path that makes households indifferent between the shocked and no-shock economies. Letting

$$\mathcal{C}^0 \equiv \int \mathbb{E}_0 \left[\sum_{t=0}^{\infty} Q_t^0 \beta_t^j(x) \frac{\left(c_t^0(x)\right)^{1-\sigma}}{1-\sigma} \right] d\mu_t^0(x) ,$$

the corresponding consumption-equivalent variation is

$$\lambda = \left(1 + \frac{\widetilde{W}_0^1 - \widetilde{W}_0^0}{\mathcal{C}^0} \right)^{\frac{1}{1-\sigma}} - 1 .$$

A positive value of λ indicates that the MIT shock raises welfare relative to the initial balanced-growth path, while a negative value indicates a welfare loss.

D.5.1 *Decomposition* To isolate the welfare effect arising from transitional allocations from that due to the permanent change in growth, it is useful to rewrite welfare differences using the no-shock weights Q_t^0 as a reference. Define

$$\mathcal{U}_t^j \equiv \int \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta_t^j(x) \left(\frac{\left(c_t^j(x)\right)^{1-\sigma}}{1-\sigma} - \nu \frac{\left(L_t^j(x)\right)^{1+\varphi}}{1+\varphi} \right) \right] d\mu_t^j(x) \,.$$

Adding and subtracting $\sum_{t=0}^{\infty} Q_t^0 \mathcal{U}_t^1$, welfare differences can be decomposed as

$$\widetilde{W}_0^1 - \widetilde{W}_0^0 = \underbrace{Q_0^0(\mathcal{U}_0^1 - \mathcal{U}_0^0)}_{\text{short-run effects}} + \underbrace{(Q_0^1 - Q_0^0)\mathcal{U}_0^1}_{\text{permanent long-run effects}} \,.$$

The first term measures the welfare effect of the MIT shock when period utilities are valued using the no-shock discount-growth weights. The second term captures the additional welfare effect generated by the fact that temporary deviations in growth rates alter the future path of aggregate technology and therefore the welfare themselves.

D.6 Government Budget Constraint under Debt Financing

When government spending is initially financed through debt issuance, we assume that fiscal adjustment eventually occurs through lump-sum transfers. In particular, transfers T_t jump on impact at the beginning of the transition and then decay geometrically at rate $\rho_T \in (0,1)$. Hence, once the adjustment is triggered, transfers follow

$$T_{t+j} = \rho_T^j T_t, \qquad j \geq 0 \,.$$

Given this structure, the initial transfer adjustment T_t can be pinned down from the intertemporal government budget constraint. Let

$$\mathcal{P}_t^{def,T} = \mathbb{T}_t^y + \mathbb{T}_t^D + \mathbb{T}_t^b - \frac{G_t}{A_t} - \mathbb{S}_t - r_t^b \frac{B_t^G}{A_t}$$

denote the primary deficit net of transfers, where $\mathbb{T}_t^y$, $\mathbb{T}_t^D$, and $\mathbb{T}_t^b$ are tax revenues, G_t/A_t is government spending in efficiency units, $\mathbb{S}_t$ denotes transfers related to social insurance, and $r_t^b B_t^G/A_t$ is interest payments on public debt.

Using the present-value government budget constraint over a horizon of length z, the impact adjustment in transfers satisfies

$$T_t = \frac{\dfrac{\overline{B}}{\overline{A}} \left(g_{t+z}^A - (1+r_t) \displaystyle\prod_{s=1}^{z-1} \frac{1+r_{t+s}}{g_{t+s}^A} \right) - \left(\displaystyle\sum_{j=0}^{z-2} \mathcal{P}_{t+j}^{def,T} \prod_{s=j+1}^{z-1} \frac{1+r_{t+s}}{g_{t+s}^A} + \mathcal{P}_{t+z-1}^{def,T} \right)}{\displaystyle\sum_{j=0}^{z-2} \rho_T^j \prod_{s=j+1}^{z-1} \frac{1+r_{t+s}}{g_{t+s}^A} + \rho_T^{z-1}} \,.$$

This expression gives the size of the initial transfer adjustment required to ensure that the government budget constraint holds in present value, taking as given the path of

interest rates, growth, and primary deficits excluding transfers. Intuitively, T_t is chosen so that the discounted value of future transfer reductions exactly offsets the additional debt issued during the transition.

D.7 Computational Algorithm

We solve for the stationary equilibrium using the following steps.

1. Construct the full state-space grid over liquid assets b, productivity h, the relative stock of innovations a, discount factors β, and employment status e.

2. Guess an initial path for aggregate prices and quantities, namely $\{r_t, L_t, g^A_{t+1}\}$, and compute the implied wage rate w_t.

3. Conditional on these prices, solve the household consumption-saving problem using a modified version of the endogenous grid method (EGM) of Carroll (2006).

   In addition, the optimal innovative investment x satisfies the first-order condition

$$\begin{aligned}
&\frac{\partial \mathcal{I}(a', b', \mathbf{s}', x)}{\partial x} = 0 \\
\Leftrightarrow \quad &\frac{\partial \mathcal{I}(a', b', \mathbf{s}', x)}{\partial x} = p'_x(x) \left[\mathbb{E}\mathcal{V}\left(a' + \iota, b' - \frac{\psi(x)}{g^A_{t+1}}, \mathbf{s}'\right) - \mathbb{E}\mathcal{V}\left(a', b' - \frac{\psi(x)}{g^A_{t+1}}, \mathbf{s}'\right)\right] \\
&+ \left(-\frac{\partial \psi(x)}{\partial x}\right) \frac{1}{g^A_{t+1}} \left((1 - p_x(x)) \mathbb{E}\mathcal{V}_b \left(a', b' - \frac{\psi(x)}{g^A_{t+1}}, \mathbf{s}'\right) + p_x(x) \mathbb{E}\mathcal{V}_b \left(a' + \iota, b' - \frac{\psi(x)}{g^A_{t+1}}, \mathbf{s}'\right)\right) = 0\,,
\end{aligned}$$

   We solve this equation with a standard one-dimensional Newton method based on numerical derivatives.

4. Given the policy rules, construct the transition matrix $\mathbf{M}$ implied by $\mathbb{P}_h$, $\mathbb{P}_\beta$, $\mathbb{P}_e$, and the policy functions $a'(\mathbf{s})$, $b'(\mathbf{s})$, and $x(\mathbf{s})$. We then compute the stationary distribution $\mathcal{G}(\mathbf{s})$ by iterating on

$$\mathcal{G}'(\mathbf{s}) = \mathbf{M}\mathcal{G}(\mathbf{s})$$

   from an initial guess until convergence.

5. Aggregate individual decisions, update prices and quantities, and return to step 2 until the equilibrium conditions are satisfied.

D.7.1 *Transitional Dynamics* We solve the transition path using a standard MIT-shock algorithm over a finite horizon T, chosen sufficiently long so that the economy is arbitrarily close to its terminal steady state by date T.

We begin by guessing a sequence for aggregate prices and quantities,

$$\left\{r^b_t, w_t, L_t, g^A_{t+1}\right\}_{t=0}^{T},$$

together with the path of exogenous shocks and fiscal instruments implied by the experiment under consideration. Taking these sequences as given, we solve the household

problem backward from date T to date 0, using the terminal steady-state value function as the final condition. This backward step delivers time-dependent policy functions for consumption, savings, and innovative investment at each date along the transition.

Given these policy functions, we then simulate the cross-sectional distribution of households forward. Starting from the initial stationary distribution at date 0, $\mathcal{G}_0$, we use the law of motion implied by individual decisions and exogenous transitions to construct the sequence

$$\{\mathcal{G}_t\}_{t=1}^{T} .$$

This forward step yields the implied paths for aggregate savings, labor supply, innovative investment, and all other endogenous aggregates.

We then compare these model-implied aggregates with the guessed equilibrium paths. Prices and quantities are updated using a relaxation scheme based on the equilibrium conditions of the model. In particular, the path of the bond return r_t^b is updated so as to clear the asset market at each date, while the path of innovation-driven growth g_{t+1}^A is updated using the law of motion for aggregate innovative investment. Wage inflation is updated so as to satisfy the New Keynesian Phillips curve along the transition, which in turn pins down the path of wages.

Fiscal variables are updated consistently with the closure rule under consideration. Depending on the experiment, this means adjusting either taxes, transfers, or government debt so that the government budget constraint is satisfied period by period, or in present value when debt is allowed to absorb temporary imbalances.

We iterate on these steps until convergence, that is, until the guessed and model-implied paths for prices and quantities coincide up to a pre-specified tolerance.

D.8 Full Calibration Table

TABLE D.2. Calibrated parameters.

PARAMETER	SYMBOL	VALUE	SOURCE / TARGET
Preferences			
Discount factor	$\{\Delta^\beta, \beta_M, p^\beta\}$	$\{0.026, 0.97, 0.995\}$	Quarterly MPC of 0.18; $B^G/Y = 40\%$
Death probability	p_d	0.005	40-year average working life
Inverse EIS	σ	1.5	Heathcote et al. (2010)
Inverse Frisch elasticity	φ	1.0	Standard
Labor disutility	ν	calibrated	$\ell = 1/3$ on BGP
Labor Market			
Productivity	$\{\rho_h, \sigma_h^2\}$	$\{0.973, 0.04\}$	Storesletten et al. (2004)
Union elasticity	ϵ_w	7	Markup of 16.7%; Ferriere and Navarro (2025)
Wage adjustment cost	θ	200	NKPC slope $\epsilon_w/\theta = 0.035$
Steady-state unemployment	$\bar{u}^h$	Table D.1	CPS data
Job-finding/separation	$\{\mathbb{P}^e, \phi^s\}$	Table D.1	CPS data; Krusell et al. (2017)
Incidence functions	$\{\mu(h), \phi(h)\}$	Table D.1	CPS data
Technology and Innovation			
Share of pot. entrepreneurs	Λ_E	0.10	Fraction of US entrepreneurs (SCF 2010)
Quarterly depreciation	δ^A	0.03	de Rassenfosse and Jaffe (2018)
Innovation step size	η	0.50	Annual BGP growth rate of 2%
Innovation shock	σ_ι^2	0.0225	Income Pareto tail of 1.7 (Piketty and Saez, 2014)
Innovation success rate	γ	0.44	Patent tax elasticity (Akcigit et al., 2022)
Labor share	α	0.40	Share of profits in total income
Fiscal and Monetary Policy			
Tax progressivity	ϱ	0.08	Ferriere et al. (2023)
Average tax scale	ϑ	0.70	Average labor income tax rate ≈23%
Corporate tax	τ_c	0.21	Statutory rate
Transfer level	$\underline{T}$	0.083	8.3% of average income
Transfer phase-out	ξ	4.22	Ferriere et al. (2023)
Taylor rule (inflation, output)	$\{\phi_\pi, \phi_y\}$	$\{1.5, 0\}$	Standard
Steady-state real interest rate	$\bar{r}$	1% (quarterly)	
Unemployment replacement rate	ς	0.40	US state average, 1990–2020
Persistence government spending	ρ_G	0.80	Standard