# The Origins of Top Firms*

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March 2026

**Abstract**

We study the origins and life cycles of firms that reach the top 1 percent of the size distribution and show that they are central for quantifying the aggregate effects of financial frictions. Using longitudinal data, we document that future top firms make sizable capital investments early in life, exhibit strongly backloaded profits, and heavily rely on external financing. By contrast, firms in the bottom 99 percent operate with little capital early on and exhibit fairly flat profit shares over their life cycle. To interpret these patterns, we develop a firm dynamics model with ex-ante heterogeneity, fixed costs in inputs that generate non-homotheticities in production, and forward-looking financing that makes it possible for high-growth potential firms to sustain backloaded profits. Quantitatively, the aggregate losses from financial frictions are driven primarily by distortions that affect the entry and survival of future top firms.

*Keywords*: top 1 percent, firm size distribution, firm dynamics, financial frictions.
*JEL classifications*: E44, O47, G30.

*We are grateful to George Alessandria, Yan Bai, Paco Buera, Lucía Casal, Nicolas Caram, Nicolas Coeurdacier, Jan Eeckhout, Sebastian Fanelli, Manuel García-Santana, Nezih Guner, Hugo Hopenhayn, Jeremy Lise, Yueran Ma, Claudia Macaluso, Virgiliu Midrigan, Kurt Mitman, Matias Moretti, Julian Neira, Ezra Oberfield, Sergio Ocampo, Serdar Ozkan, Alessandra Peter, Tommaso Porzio, Diego Restuccia, Sergio Salgado, Yongs Shin, Riccardo Silvestrini (discussant), Ina Simonovska, Nicholas Trachter, Thomas Winberry, and seminar participants at Bocconi, BSE SF 2024, CEMFI, CREi-UPF, Columbia IPD Workshop, CREi-UPF, FRB of Richmond, JRC European Commission, Lisbon Macro 2025, RIDGE Forum 2025, SED Barcelona 2024, SED Copenhagen 2025, SEU 2025, Spanish Macro Network, UC Davis, University of Rochester, Western University, and the World Bank, for valuable comments and suggestions. We thank Sebastian Medina and Manuel Ruiz for excellent research assistance. Kochen gratefully acknowledges financial support from the Spanish Government under Grant PID2022-142790NB-I00 and the Ramón Areces Foundation. The views expressed are those of the authors and not necessarily those of Banco de España or the Eurosystem. First draft: February 2024

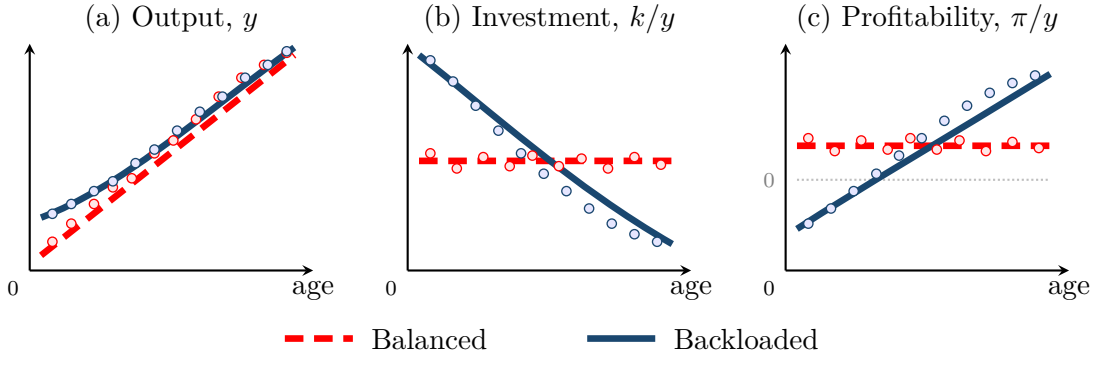
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# 1 Introduction

Top firms, namely the largest firms in the economy, account for a disproportionate share of economic activity (Axtell, 2001; Luttmer, 2007) and have become increasingly important in recent decades (Kwon, Ma, and Zimmermann, 2024). Recent work links their rise to the decline in the aggregate labor share (Autor et al., 2020), the increase in market power (De Loecker, Eeckhout, and Unger, 2020), among other aggregate trends.

In this paper, we argue that understanding the origins and life cycles of top firms is crucial for assessing the aggregate implications of financial frictions. To see why, consider the two views in Figure 1 illustrating firms' rise to the top. Under the *balanced profits view*, firms that reach the top start small, require only moderate investments, and are profitable from their early years. Entry and early expansion can be easily financed, and their scarcity mainly reflects that highly productive firms are rare. Under the *backloaded profits view*, future top firms are initially large, require sizable upfront investments, and become profitable only later in their life cycle. Entry and early expansion are therefore much more difficult to finance, and their scarcity reflects not only productivity differences but also financing constraints at early stages. These views have very different policy implications, yet there is little systematic evidence on the early years and trajectories of top firms.

**Figure 1:** The Rise to the Top – Balanced vs. Backloaded Profits




Using comprehensive longitudinal firm-level data from several countries, we find that firms that reach the top of the size distribution are initially large and make sizable capital investments early in life. They also exhibit heavily backloaded profits: around zero in the first few years, but strongly increasing over time. By contrast, firms in the bottom 99 percent start small and operate with little capital early on, which increases moderately over time, and exhibit positive but fairly flat profit shares over their life cycle. We also find that firms that reach the top rely much more on debt and equity financing, especially in their first years. Taken together, these facts are consistent with the *backloaded profits view* in which future top

firms require large initial investments and substantial external financing in their early years.

We interpret these markedly different patterns using a firm dynamics model with ex ante heterogeneity in productivity and technology, fixed costs in inputs that generate non-homotheticities in production, and forward-looking debt and equity financing. In the model, firms operating a high-growth potential technology require large initial investments and have backloaded profits due to high fixed costs in capital and labor, which are most significant early in their life cycle. As they grow, their average cost declines as fixed costs are amortized over more output, explaining the backloaded profits. By contrast, firms with low-growth potential technologies have low fixed costs. Hence, they require small initial investments, feature fairly flat profit shares, and gradually grow out of their borrowing constraints.

Quantitatively, we find that access to finance plays a central role in shaping the economy's technological composition and aggregate output because it enables high-growth potential firms to cover fixed costs and sustain heavily backloaded profits. As a result, although fixed costs in capital account for less than 0.5% of top firms' total output, their interaction with financial frictions generates very sizable indirect effects, of about 70% of aggregate output, through the entry and survival of firms operating top technologies. Moreover, we find that the aggregate costs of financial frictions are driven almost entirely by distortions affecting top technologies. Indeed, we show that neglecting the early years and trajectories of future top firms severely understates the importance of finance for the macroeconomy. Finally, we study additional implications of top-firm dynamics, including the long-run effects of interest rate changes, technological frontier expansion, and size-based corporate taxation.

**Empirical Evidence.** We use Orbis Historical, an extensive firm-level database with balance sheets and income statements, covering millions of companies worldwide, to document the origins of top firms. Our baseline analysis focuses on Spain, for which the data provides a long panel with excellent coverage and representativeness. We present additional results for countries with sufficiently long panel data, including Belgium, Finland, France, and Sweden.

We define *top* firms as those in the top 1 percent of the output distribution and *bottom* firms as those in the remaining 99 percent. For our longitudinal analysis, we classify firms as top or bottom based on their size at age 20.[1] Hence, we concentrate on firms that survive to age 20 and track their trajectories back in time. Our key empirical findings are as follows.

First, we document stark differences in the output trajectories of top and bottom firms. Importantly, these differences are present in their initial size and subsequent growth. Firms

[1]As the size distribution spans out as firms age, it is worth noting that a firm in the top 1 percent at age 20 also belongs to the top 1 percent of the entire firm size distribution. Furthermore, we observe that the firms' top status is quite persistent after age 20.

that reach the top by age 20 are about 9 times larger than bottom firms at entry, and grow roughly 11 times over their first two decades. In contrast, bottom firms grow only 2 times in their first twenty years. These output dynamics are consistent with ex-ante heterogeneity in both initial scale and growth potential (Sterk, Sedláček, and Pugsley, 2021).

Next, we study the capital and labor usage of top and bottom firms. We document a key new fact: future top firms exhibit high initial input usage and sharply declining input-to-output ratios over their life cycles. By contrast, bottom firms show stable or increasing input-to-output ratios. In terms of capital, firms that reach the top 1 percent by age 20 enter with a high capital-output ratio, about 4.9, which declines by roughly one-half over their first two decades. Firms in the bottom 99 percent, by contrast, start with a much lower capital-output ratio, around 2.3, and it increases by about 20 percent over the same period.

The pattern for labor usage also differs. Early in the life cycle, the labor share, defined as labor costs over output, is similar for top and bottom firms, at about 0.7. Their subsequent paths, however, diverge. For bottom firms, the labor share remains roughly constant, whereas for firms that reach the top 1 percent by age 20 it declines by about 11 percentage points over the first two decades. This pattern is consistent with cross-sectional evidence showing that top firms have lower labor shares (Autor et al., 2020; Kehrig and Vincent, 2021).

These large differences in the use of capital and labor have substantial implications for the dynamics of profits. We find that profit shares, defined as profits over output, are much more backloaded for firms that reach the top 1 percent by age 20 than for firms in the bottom 99 percent. The differences are sizable. For future top firms, the profit share rises sharply from about 0 to 0.26 over their first two decades. For bottom firms, by contrast, it increases only slightly, from roughly 0.09 to 0.13 over the same period.

Finally, we document the use of debt and equity financing by top and bottom firms. Relative to firms in the bottom 99 percent, future top firms borrow more, deleverage less as they age, and rely more on equity financing in their early years. Overall, our findings are consistent with the view that top firms operate technologies that require large initial investments and depend heavily on external financing early in their life cycles. These patterns contrast with the initially smaller investments and gradual growth of bottom firms.

**Theory.** We show that a general-equilibrium firm dynamics model with ex-ante heterogeneity, fixed costs in inputs that generate non-homotheticities in production, and forward-looking financing can account for our novel empirical findings. Firms are *ex-ante heterogeneous* in initial productivity, productivity growth profiles, and technology types, where technology types differ in span of control, input shares, and fixed costs. Technologies are scarce—reflecting

barriers to entry or limited diffusion of frontier knowledge—and hence every period there is a fixed number of potential entrants of each technology type.

We introduce *non-homotheticities* in production by assuming that firms incur input-specific fixed costs in both capital and labor to operate. This formulation is analogous to Stone–Geary preferences (Geary, 1950; Stone, 1954), in which fixed components generate non-homothetic behavior. These fixed costs can be microfounded through input indivisibilities (Banerjee and Duflo, 2005), or complementarities between inputs and productivity.

Firms have access to debt and equity *financing*. Debt is subject to a borrowing constraint with two components: a standard asset-based backward-looking term and a value-based forward-looking term. The forward-looking component allows firms with higher growth potential to borrow more than firms with more modest potential, even when assets and current earnings are similar. Additionally, we incorporate costly equity injections as a second source of forward-looking financing.

Our theory provides a parsimonious explanation for the patterns observed in the data. A central feature is that firms operating technologies with high growth potential have large fixed costs, especially in capital. Importantly, these firms can finance those costs through forward-looking financing because of their high growth potential. As firms grow, fixed costs become smaller relative to output, lowering their capital-output ratio and labor share over the life cycle. These input dynamics, in turn, generate a heavily backloaded profit share that is initially close to zero and rises substantially as firms age. By contrast, bottom firms have lower growth potential and small fixed costs. Therefore, they benefit less from forward-looking financing and face tighter financial constraints early on. As they accumulate assets and relax their borrowing constraints, their capital-output ratio rises. Because financial constraints do not distort labor in the model, their labor share remains roughly flat.

We also examine alternative explanations for the life-cycle trajectories of top firms, including product and labor market power and input-biased productivity growth. We argue that these alternative theories have difficulty in jointly explaining both the input-to-output dynamics and the capital-labor composition.

**Aggregate Implications.** As is standard in the firm dynamics, we calibrate our model to match firm growth, exit rates, and output concentration in the data. However, unlike previous work, we also directly target moments related to our novel findings on the life-cycle dynamics of growth, input use, profit shares, and debt and equity financing for top and bottom firms. For parsimony, in our baseline quantification, we use two types of technologies:

top and bottom.[2] Our model does a good job of quantitatively capturing the life cycle facts documented in the first part of the paper.

We start by conducting three exercises to quantify the aggregate importance of the origins and life cycles of top firms. First, we remove input-specific fixed costs in both our baseline model and a version without financial frictions. We find that these costs have negligible aggregate effects under perfect credit. Yet, in our baseline calibration, their removal increases aggregate output by almost 70%, with nearly all of the increase driven by capital fixed costs. This result points to a strong interaction between access to finance and the large initial capital investments required by top technologies. Consistent with this, when measured directly, these fixed costs account for no more than 0.5% of top firms' total output, but can reach as much as 30% of young top firms' output. This result highlights the importance of accounting for the life cycle trajectories of top firms when assessing their aggregate implications.

In the second exercise, we quantify the losses from financial frictions. We find that removing financing frictions almost doubles aggregate output. This effect operates mainly through changes in the technological composition of the economy. Without financing frictions, there is more entry and higher survival rates for firms operating top technologies. Fewer firms operate bottom technologies because of higher wages in general equilibrium. Both effects lead to a higher share of firms operating top technologies. Indeed, we show that 90% of the total increase in output is explained by the elimination of financial frictions for top technologies.

Lastly, we conduct the opposite exercise and ask the model about the benefits of finance: what happens if we remove external financing, and investment must be entirely self-financed? In this scenario, output contracts by more than 30%. The gains from finance are substantial and are driven primarily by the financing of top firms, especially through the forward-looking component of financing (debt backed by firm value and equity). This exercise reinforces the previous two by showing that the large early investments of top firms, and their reliance on forward-looking financing, are central to the aggregate importance of finance in our model.

Beyond the role of finance, we explore other potentially relevant aggregate implications of top firms' dynamics. We find that the presence of top and bottom firms amplifies the long-run effects of interest rate changes, as top firms are more directly affected by changes in the cost of capital and access to finance that crucially depends on their net present value.

We also show that if growth is driven by technological composition improvements, such as greater diffusion of ideas and lower adoption costs, we should expect a concurrent decline in the labor share, together with an increase in the profit share and firm concentration. These

[2]Firms with top technologies do not necessarily map one-to-one to the top 1 percent of the size distribution, as some firms with bottom technologies may reach the top by size because of favorable ex post shocks.

predictions are consistent with the aggregate trends observed in the last several decades in advanced economies (Ma, Zhang, and Zimmermann, 2025).

Finally, we study size-based corporate taxation in the presence of spillover and concentration externalities. Taxing firms once they become large creates a static distortion (bunching) and a dynamic distortion, as it lowers the value of firms expected to reach the top and thereby reduces their access to financing in the early years, even before they are taxed. If spillovers are moderately large, taxing top firms can significantly reduce output.

**Related Literature.** Our paper contributes to the following strands of literature.

*Top Firms in Macroeconomics.* First, our paper contributes to the literature on top firms in macroeconomics. One strand of this literature studies the relationship between business cycles and top firms. A substantial body of work (e.g., Gertler and Gilchrist, 1994; Crouzet and Mehrotra, 2020) documents how the cyclical properties of large firms differ from those of small firms, while other research (e.g., Gabaix, 2011; Carvalho and Grassi, 2019) investigates the extent to which shocks to top firms influence aggregate fluctuations. Another strand of literature focuses on the long-run implications of the rise of top firms.[3] Existing papers in this literature have studied the relationship between the rise of top firms and market power (e.g., Hall, 2018; Van Reenen, 2018; De Loecker, Eeckhout, and Unger, 2020; Covarrubias, Gutiérrez, and Philippon, 2020), the decline in the labor share (e.g., Autor et al., 2020; Kehrig and Vincent, 2021), and their role in innovation (e.g., Garcia-Macia, Hsieh, and Klenow, 2019; Braguinsky et al., 2023; Ayyagari, Demirgüç-Kunt, and Maksimovic, 2023; Casal, 2025).[4]

Our paper contributes to this literature by documenting novel facts about the first twenty years of the largest 1 percent firms. Hence, unlike most previous work in this literature, we are not only interested in top firms once they are large and mature but also in their early years and their way to the top. In this line, our paper relates to Luttmer (2011) that documents the employment histories of U.S. firms with more than 10,000 employees in 2008 and interprets the relatively young age of large firms through a firm dynamics model with ex-ante heterogeneity and ex-post growth. Perhaps closest to our work is Ma et al. (2025),

[3]Corporate concentration has increased in the U.S. (Kwon, Ma, and Zimmermann, 2024) and the rest of the world (Ma, Zhang, and Zimmermann, 2025) over the last 100 years. Similar trends have been documented for Europe (Bajgar et al., 2023; Koltay, Lorincz, and Valletti, 2023) and Korea (Choi et al., 2025). Other work highlighted divergences between local and national concentration (Rossi-Hansberg, Sarte, and Trachter, 2021) and between product and market concentration (Amiti and Heise, 2025).

[4]The mechanisms driving the rise of top firms are contested and intertwined, with explanations ranging from changes in firms' technology and economies of scale (Aghion et al., 2023; Hsieh and Rossi-Hansberg, 2023; Lashkari, Bauer, and Boussard, 2024; Hubmer and Restrepo, 2022; Firooz, Liu, and Wang, 2025, among others) demographic changes (Hopenhayn, Neira, and Singhania, 2022; Karahan, Pugsley, and Şahin, 2024; Peters and Walsh, 2024), antitrust and regulation (e.g., Philippon, 2019; Akcigit and Ates, 2023), or declining interest rates (Kroen et al., 2022; Chatterjee and Eyigungor, 2023).

which documents the birth rates of the largest companies in the U.S. over time. It finds evidence of special cohorts of entrants in specific sectors, leading to significant shifts in the age distribution of top firms. As in those papers, we also track top firms backward and study their origins and life cycle growth. However, unlike previous work, we document input usage, profits, and financing of the largest 1 percent firms and contrast their trajectories to those in the bottom 99 percent. In addition to our empirical contribution, we argue that these patterns provide key insights into top firms' dynamics and that accounting for them has important macroeconomic implications.[5]

*Financial Frictions and Development.* Our paper also contributes to the literature on financial frictions and development Banerjee and Duflo (2005), Buera, Kaboski, and Shin (2011), Moll (2014), and Midrigan and Xu (2014). Consistent with this literature, we show that financial frictions matter mainly through distortions at the extensive margin (entry and exit of top technologies). Gains from finance and losses from financial frictions are an order of magnitude larger when one accounts for the origins of top firms.

Our paper is particularly related to two contributions. First, Cole, Greenwood, and Sanchez (2016) develops a model in which financial development shapes technology adoption when the most promising technologies have backloaded returns. As in their framework, forward-looking financing is central to sustaining backloaded returns for high-growth firms. Our empirical findings are consistent with this mechanism, and our model quantifies the role of forward-looking financing, showing that it is about twice as important as standard asset-based collateral. Importantly, different from previous work, we discipline our quantitative exercise using data on output, input, and financing over the life cycle of top and bottom firms.

Second, Banerjee and Duflo (2005) argues, in a simple static model with capital-specific fixed costs, that financial frictions can hinder development because more productive firms face higher capital fixed costs that are harder to finance in less developed financial systems. This mechanism is closely related to ours. In our setting, large early capital fixed costs are central to the emergence of top technologies, yet, as our calibration shows, they need not be large in the aggregate to generate sizable output effects through their interaction with financial frictions because they are concentrated in the early years of top firms.

*Life Cycle of Firms.* Our paper also relates and contributes to the literature that empirically and theoretically studies the life cycle of firms (e.g., Davis, Haltiwanger, and Schuh, 1996; Haltiwanger, Jarmin, and Miranda, 2013; Hsieh and Klenow, 2014; Sedláček

[5]Our empirical analysis focuses on Europe, for which we have longitudinal data on balance sheets, income statements, and ownership data. Comparable data is not available for the U.S. beyond specific sectors (e.g., manufacturing, which represents only 10% of GDP) or publicly listed firms.

and Sterk, 2017; Moreira, 2018; Kochen, 2025; Dinlersoz et al., 2024; Eslava, Haltiwanger, and Urdaneta, 2024). Most closely related to our empirical work within this literature is De Haas, Sterk, and Van Horen (2022), which clusters firms based on their initial observable characteristics and studies them *forward*.[6] Our two papers complement each other, as our paper identifies the largest firms at age 20 and tracks them *backward*.[7]

From a theoretical perspective, there has been significant interest in the literature on how backloaded profits are over the life cycle of firms. For example, Atkeson and Kehoe (2005) develops a theory of the life cycle of organization rents where interest rates and backloaded rents play a crucial role. More recently, Van Vlokhoven (2021) studies the aggregate implications of profits becoming more backloaded over the life cycle of firms. Our paper contributes to this literature by providing novel evidence on the path of profits for firms at different points of the size distribution, delivering new insights for modeling and theories on the life cycle of firms. To the best of our knowledge, our paper is the first to document that the profit share is strongly backloaded for the largest 1 percent of firms and relatively flat over the life cycle for the bottom 99 percent.

*Firm Dynamics.* Lastly, our paper contributes to the vast literature on firm dynamics (e.g., Jovanovic, 1982; Hopenhayn, 1992; Hopenhayn and Rogerson, 1993; Melitz, 2003; Luttmer, 2007). Closest to our work is Sterk, Sedláček, and Pugsley (2021), which using insights from the earnings dynamics literature, estimates the importance of ex-ante heterogeneity for firm growth and the firm size distribution. Our contribution to this literature is twofold. On the empirical side, we document novel facts about growth, input usage, and profits over firms' life cycle and uncover strikingly different trajectories for firms at the top and bottom of the size distribution. On the quantitative side, we show that, beyond ex ante heterogeneity in productivity profiles, which is crucial for matching the growth patterns of top and bottom firms, non-homotheticities in production combined with forward-looking financing are essential for explaining the input use and profitability of top firms in the data.[8]

[6]Within the entrepreneurial literature, Guzman and Stern (2015) follows a similar approach using information near the firm foundation to identify the best predictors of firm growth. In Appendix A.5, we perform a similar analysis in our data and find that firms with a large initial size, higher capital-output ratios, and more frequent equity injections early in their life cycle are more likely to reach the top.

[7]From a methodological standpoint, our backward-looking approach resembles that of Halvorsen et al. (2024), which studies the life cycle dynamics of the wealthiest households using Norwegian panel data.

[8]Some early work studying non-homotheticities in production inputs include Lau and Tamura (1972), Hanoch (1975), and Sato (1975). More recently, Lashkari, Bauer, and Boussard (2024) and Eckert, Ganapati, and Walsh (2024) study production non-homotheticities focusing on ICT capital. On the other hand, forward-looking financing in the form of debt and equity has been studied in the context of quantitative firm dynamics models in Cooley and Quadrini (2001), Albuquerque and Hopenhayn (2004), Hennessy and Whited (2007), and Brooks and Dovis (2020).

**Outline** The rest of the paper is organized as follows: Section 2 describe our data and presents descriptive statistics; Section 3 presents our key empirical findings on the origins and life cycle of top firms; Section 4 presents the model; Section 5 describes the predictions of input usage in data and theory; Section 6 presents our calibration strategy; Section 7 presents our aggregate results; and finally, Section 8 concludes.

# 2 Data and Descriptive Statistics

This section describes the data used in the paper. It also describes the sample selection and provides definitions of the main variables used in the analysis. Furthermore, it presents some motivating cross-sectional descriptive statistics.

## 2.1 Orbis Historical

To document the origins of the top firms, we use Orbis Historical, an extensive firm-level database covering millions of companies worldwide. Moody's Bureau van Dijk (BvD) compiles this database by aggregating data from various sources, such as national business registries, and harmonizes it into a globally comparable format. We focus on a sample of European countries with the best coverage in Orbis (Kalemli-Özcan et al., 2024). For our baseline results, we focus on Spain, where Orbis Historical has a long panel of 32 years, between 1992 and 2023, and has excellent coverage, especially from 2000 onward. Figure A.1 in the Appendix shows that Orbis Historical covers, on average across years, 78% of aggregate revenue in Spain, with higher coverage for the most recent years. Furthermore, Figure A.2 shows that the data captures well the firm size distribution reported in official statistics. In addition to our baseline analysis for Spain, we present results for other European countries with sufficiently long panel data in Orbis, such as Belgium, Finland, France, and Sweden.

**Sample Selection** Our analysis focuses on the non-financial private sector.[9] Throughout the paper, we define firms using companies' unconsolidated accounts. We restrict to firm-year observations with available data on wage bill, output, capital, debt, net worth, and profits. We exclude years and 4-digit NACE sectors with fewer than 100 observations to adequately control for time- and sector-fixed effects. After applying this sample selection criterion, we have a sample for Spain comprising 10,867,785 firm-year observations from 1,344,949 firms. For our longitudinal results, we further restrict the analysis to a sample of firms we observe at age 20 and can track backward for at least 10 years, resulting in 3,434,472 firm-year

[9] In detail, we exclude firms in the following sectors: Financial and Insurance (K), Public Administration and Defense (O), Activities of Households as Employers (T), and Extraterritorial Organizations and Undifferentiated Goods and Services (U).

observations from 165,017 firms, of which 1,665 are in the top 1 percent at age 20.

**Variable Definitions** We study the evolution of firm-level output, wage bill, capital, profits, and financing. We convert all nominal variables to real terms using country-specific CPI deflators. We measure output using value-added, defined as revenue minus a comprehensive measure of costs, which excludes labor expenses and capital depreciation. For the wage bill, we use all labor costs. Our baseline measure of capital is equity plus net financial debt. We also provide results for capital measured as tangible assets, as well as tangible and intangible assets. For firms' profits, we use a model-consistent definition of economic profits, obtained by setting values for capital depreciation and the interest rate. We show that our results are similar if we use the variable net income reported in the data. For firms' debt financing, we measure leverage as net financial debt over capital. For equity financing, we measure annual equity injections (negative dividends) over capital. See Appendix A.1 for the exact definition of these variables and further measurement details.

**Firms' Foundation Year** Central to our analysis is the accurate measurement of firm age, defined as the number of years since foundation. Our measure of firm age is based on the incorporation year recorded in Orbis. However, because incorporation years may differ from true foundation years—due to restructurings, mergers and acquisitions (M&A), or regulatory changes—we verified and, when necessary, corrected the foundation year of every firm in the top 1% at age 20 through a large language model (LLM)-assisted search of company websites, Wikipedia, LinkedIn, business registries, and press archives. This procedure revealed discrepancies in 14.2% of cases, which we corrected when constructing our baseline sample of top firms at age 20 (see Figure A.3 for the distribution of foundation and incorporation years). Appendix A.2 describes our verification procedure in detail, discusses some examples, and explains how we assign the foundation year in different cases.

## 2.2 Top Firms

We define top firms as those in the top 1 percent of the output distribution. To account for variation in firm size by year and sector, we use a residualized measure of output when making this classification, which we obtained after controlling for year and NACE 4-digit industry fixed effects. We rank firms across the entire distribution and conditional on age. Thus, we refer to a firm as being in the *top 1 percent at age a* if its output at age $a$ is in the top 1 percent of the output distribution among all firms of age $a$ observed in the data. We use this age-based definition of top firms in our life cycle analysis to track backwards the origins of the top and bottom firms at a given age $a$.

Our longitudinal analysis studies the origins of the top 1 and bottom 99 percent of firms at age 20. We focus on the first twenty years of a firm's life cycle, which is the largest horizon we can track firms backward for at least 10 years without running into small sample concerns. Thus, given our sample period, our longitudinal analysis conditional on size at age 20 focuses on the firms born between 1980 and 1998. As we document below, firms in the top 1 percent at age 20 are also in the top 1 percent of the full-size distribution, reflecting both firm growth over the life cycle and survival bias. Yet, many top firms are older than 20 years, as Figure A.4 in the Appendix shows. While we do not observe the origins of older cohorts in our data, it is worth noting that the top firm status at age 20 is highly persistent. Indeed, Appendix A.3 documents that top firms at age 20 are likely to remain at the top in subsequent years. This evidence suggests that the life cycle trajectories of the top firms at age 20 that we document for the 1980 to 2002 cohorts can provide valuable insights into the trajectories of both older and younger cohorts.

## 2.3 Cross-Sectional Descriptive Statistics

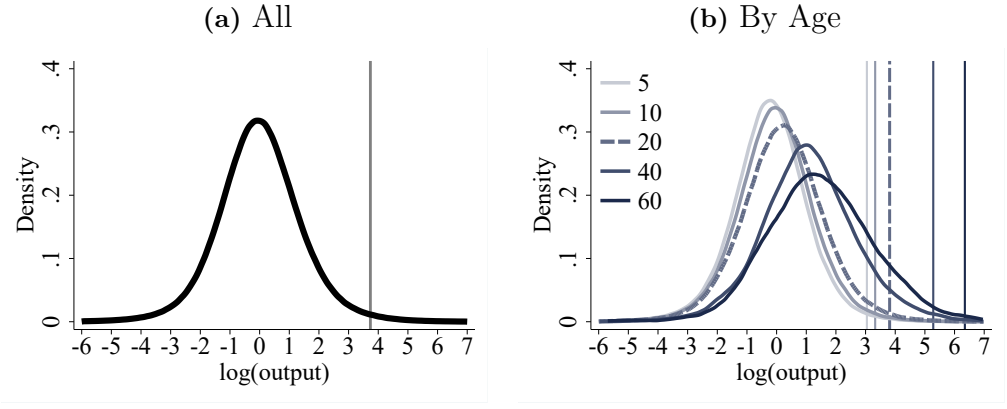
Before presenting our longitudinal analysis, we characterize top firms in the cross-section. We report standard income and balance sheet variables and key ratios we study in the longitudinal analysis between top and non-top firms and document top firms' production sectors and business structures. Overall, the cross-sectional patterns align with previous studies. Yet, it is worth noting that many top firms are privately held and are not particularly concentrated in a specific production sector.

**Output Distribution** Figure 2a shows the distribution of log output for the whole sample of firm-year observations in Spain. The vertical line indicates the 99th percentile. This figure shows the extreme dispersion in the empirical distribution of firm size. To put this in perspective, the 99th percentile of log output is 3.86, implying that a firm in the 99th percentile has 47.7 times the output of the average firm. Alternatively, we can measure the share of production accounted for by the largest firms. Figure A.5 shows that the largest 1 percent of firms account for more than 50% of total output. The high concentration of production by the top firms is well known and has been documented for both the U.S. (Axtell, 2001; Luttmer, 2007) and Europe (Bajgar et al., 2023; Koltay, Lorincz, and Valletti, 2023).

Figure 2b presents the distribution of log output conditional on firm age. The vertical lines represent the 99th percentile of each distribution. The output distribution by age indicates that firms grow over their life cycle and that the distribution spans out as firms get older, consistent with Cabral and Mata (2003) and Sterk, Sedláček, and Pugsley (2021). It is worth noting that the 99th percentile of the log output for age 20 firms is 3.92 (dashed

vertical line), which is higher than the 99th percentile of the full distribution. Hence, the top 1 percent of firms at age 20, studied in our longitudinal analysis below, also belong to the top 1 percent of the entire firm size distribution.

**Figure 2:** Output Distribution




*Notes*: Residualized log output distributions after controlling for sector and year fixed effects. Log output is normalized to zero using the mean of the unconditional distribution. Output is measured as value added using a comprehensive measure of non-labor and non-capital costs. Distributions are approximated by a kernel density. The vertical line corresponds to the 99th percentile of the distribution. Panel (a) shows the distribution for all the firms and panel (b) shows the distribution conditional on firm age.

**Top 1 vs Bottom 99: Size and Main Variables** The top panel of Table 1 presents summary statistics for top and bottom firms, defined using the whole sample. All the ratios reported in the table are weighted by their respective variables in the denominator. Hence, we can interpret them as the ratio for the representative firm in each group.[10] On average, considering output or sales, firms in the top 1 percent of the size distribution are 92 times larger than the average firm in the bottom 99 percent. Top firms' have a higher average capital-output ratio $k/y$, equal to 3.3, while this number is 2.7 for the bottom firms. Notably, firms in the top 1 percent have an average labor share, defined as labor costs over output $wl/y$, of 0.52, 16 p.p. lower than those in the bottom 99 percent, equal to 0.68. This fact is consistent with the evidence in Autor et al. (2017), which documents that superstar firms, the largest and most productive in a given industry, have a low labor share of value added. A perhaps more novel fact is that the profit share, defined as profits over output $\pi/y$, of the top 1 percent firms is two times higher than that of the bottom 99 firms: 0.26 and 0.13, respectively. Top firms also have higher leverage, defined as net financial debt over capital $b/k$, equal to 0.37 relative to 0.28 for the bottom firms.

[10]The average weighted by the denominator is equal to the ratio of the aggregate amounts, which we can interpret as the ratio for the representative firm. For example, the weighted average capital-output ratio is equal to: $\sum_i \frac{y_{it}}{Y_t}\frac{k_{it}}{y_{it}} = \frac{K_t}{Y_t}$, where $Y_t = \sum_i y_{it}$ and $K_t = \sum_i k_{it}$. Note that this measure is less sensitive to outliers than the simple average of the ratios.

**Table 1:** Summary Statistics

| | Bottom 99 | | | Top 1 | | |
|---|---|---|---|---|---|---|
| | Mean | Median | SD | Mean | Median | SD |
| Age | 12.7 | 11.0 | 10.2 | 25.1 | 22.0 | 16.5 |
| Output (USD, millions) | 0.5 | 0.2 | 1.6 | 49.9 | 17.1 | 203.4 |
| Sales (USD, millions) | 2.1 | 0.5 | 13.8 | 196.3 | 50.0 | 865.4 |
| Employment | 11.8 | 5.0 | 71.8 | 629.3 | 220.0 | 2373.1 |
| Capital-output ratio | 2.7 | 1.3 | 5.9 | 3.3 | 1.6 | 5.7 |
| Labor share | 0.68 | 0.71 | 0.28 | 0.52 | 0.56 | 0.33 |
| Profit share | 0.13 | 0.13 | 0.34 | 0.26 | 0.25 | 0.38 |
| Leverage | 0.28 | 0.26 | 0.44 | 0.37 | 0.40 | 0.36 |
| | Share of Bottom 99 | | | Share of Top 1 | | |
| | Obs. | Output | Capital | Obs. | Output | Capital |
| Manufacturing | 0.32 | 0.43 | 0.37 | 0.27 | 0.29 | 0.28 |
| Retail and Transport | 0.31 | 0.26 | 0.20 | 0.28 | 0.25 | 0.17 |
| Services | 0.33 | 0.27 | 0.37 | 0.40 | 0.33 | 0.32 |
| Other | 0.04 | 0.04 | 0.06 | 0.05 | 0.13 | 0.22 |
| Listed Firms | 0.00 | 0.00 | 0.02 | 0.03 | 0.15 | 0.32 |
| Public Firms | 0.13 | 0.37 | 0.36 | 0.54 | 0.61 | 0.46 |
| Private Firms | 0.87 | 0.62 | 0.62 | 0.43 | 0.24 | 0.22 |

*Notes*: Top 1 and Bottom 99 groups are defined by output, as described in Section 2.1. Variables in dollars are in 2015 USD using constant prices at constant exchange rates. The capital-output ratio, the labor share (labor costs over output), and the profit share (profits over output) are weighted by output. The shares of firms by sector and legal status are reported both unweighted (Obs.) and weighted by output and capital. Sectors are based on NACE 2-digit classifications: Manufacturing is `nace` $\in [10, 33]$ and includes Construction `nace` $\in [41, 43]$, Retail and Transport is `nace` $\in [45, 53]$, and Services excluding Retail and Transport is `nace` $\in [55, 82]$, `nace` $\in [85, 88]$, and `nace` $\in [90, 96]$. Other corresponds to those sectors not listed before. Listed Firms are those publicly listed throughout our sample. Public Firms are public limited companies (*Sociedades Anónimas*) and Private Firms are partnerships and private limited companies (*Sociedad de Responsabilidad Limitada*). See Appendix A.1 for definitions and measurement of these variables.

**Top 1 vs Bottom 99: Production Sectors and Legal Status** The bottom panel of Table 1 documents the composition of top and bottom firms across industries and by their legal status. We report the share of observations, output, and capital across these categories, conditional on top and bottom status. Importantly, our data allows us to study all economic sectors, not only manufacturing, and includes public and private firms. By construction, our definition of top firms accounts for average differences across sectors, partly explaining the similarities in the sectoral composition of top and bottom firms. Yet, top firms have a higher share of observations and output in services. There are also significant differences in firms' legal status. Among the top 1 percent firms, 3% are publicly listed in the stock market, 54%

are public companies (analogous to U.S. C-corporations), and 43% are private companies. In contrast, the bulk of the firms in the bottom 99 percent, 87% of them, are private.

To summarize, the largest 1 percent of firms in the cross-section exhibit a higher capital-output ratio, lower labor share, higher profit share, and are more leveraged. They are also more likely to operate in the service sector, with over half being public (C-corporations). In the next section, we exploit the longitudinal dimension of our data to study the origins and life cycle trajectories of top firms at age 20.

# 3 The Origins of Top Firms

This section documents how the top 1 percent of firms start and how they grow. Our analysis focuses on the evolution of firm size and key ratios characterizing firms' use of input and profits. We first describe our empirical strategy and present the empirical specification we use to estimate the life cycle of firms throughout their first two decades. We then present our main results for the top 1 and bottom 99 percent of firms. Lastly, we describe some robustness checks and additional results on the origins of top firms.

## 3.1 Illustrative Examples

To motivate our empirical analysis, Figure 3 shows the life-cycle trajectories of three firms that reached the top of the size distribution: a vertically integrated fashion company (Firm F), a venture capital-backed digital services startup (Firm T), and a foreign multinational in the food industry (Firm M). Despite operating in very different industries, all three companies share the same combination of rapid output growth, sizable upfront capital investments, declining labor shares, and strongly backloaded profits. The econometric analysis that follows confirms that these patterns are a systematic feature of the firms that reach the top.

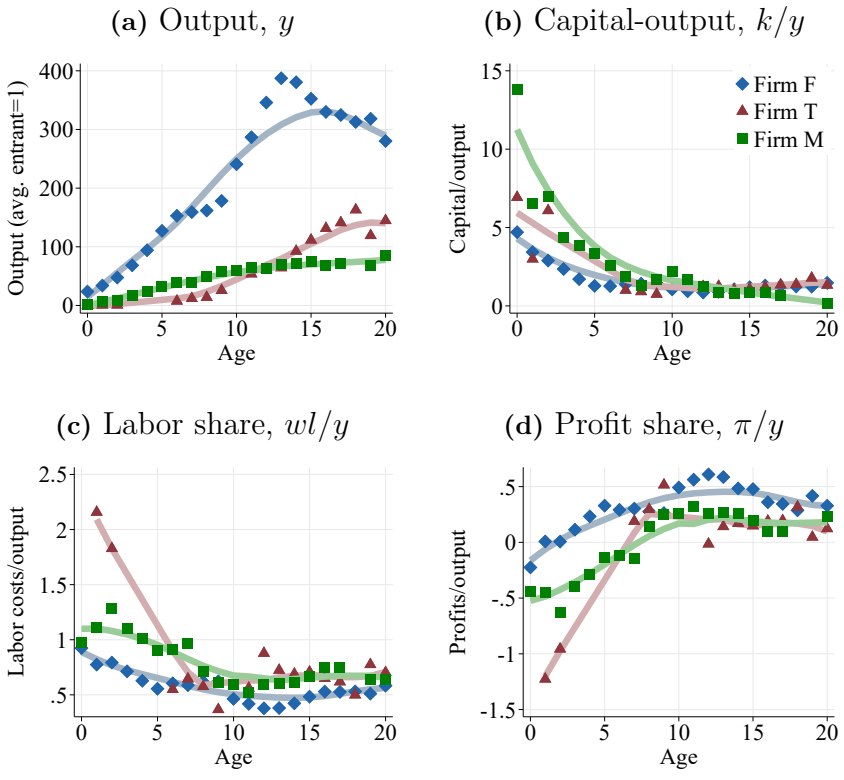
## 3.2 Empirical Specification

We study the origins of firms conditional on their size at age 20. Thus, our analysis focuses on the firms that survived at least to that age. A way to think about our empirical strategy is that we study firms' life cycle *backward*. Given their position in the age 20 size distribution, we trace their trajectories back in time to when they started operating.

We begin by using our residualized measure of output, which controls for average sector and year differences, to classify firms with available data at age 20 into two groups: the top 1 percent and the bottom 99 percent. We then focus on firms observed for at least 10 years between ages 0 and 20 to estimate firm fixed effects more precisely.[11]

[11] In addition to incomplete firm spells, left data censorship is the main reason why we would exclude a

**Figure 3:** The Rise to the Top – Three Examples




*Notes*: Life cycle trajectories for output, inputs, and profit shares of three top firms in our sample. To control for time and sector heterogeneity, the reported numbers are the predicted values after controlling for sector and time fixed effects. Firm F is a vertically integrated fashion company. Firm T is a venture capital-backed digital services startup. Firm M is a foreign multinational in the food industry.

Having classified firms, we study the first twenty years of firms' life cycle by estimating the following non-parametric specification for different dependent variables $x_{it}$

$$x_{it} = \sum_{a \in \mathcal{A}} \left( \beta_a^T T_i + \beta_a^B B_i + \beta_a^R R_i \right) \times D_{it}^a + \alpha_i + \alpha_t + \varepsilon_{it} \tag{1}$$

where $D_{it}^a = \mathbf{1}_{[\text{age}_{it} \in a]}$ is a dummy variable equal to 1 if firm $i$ belongs to age group $a$ at period $t$. The set $\mathcal{A}$ includes nine age groups: 0-4, 5-6, 7-8, 9-10, 11-12, 13-14, 15-16, 17-18, and $\geq 21$. The omitted group, which serves as the reference, is 19-20. The variable $T_i$ equals 1 if firm $i$ is in the top 1 percent of the output distribution at age 20, and 0 otherwise. Analogously, $B_i$ equals 1 if firm $i$ is in the bottom 99 percent, and 0 otherwise. The variable $R_i = 1 - T_i - B_i$ captures the remaining firms, which are included in the regression to estimate the age and time fixed effects using all available data but are not the focus of our longitudinal analysis. Finally, coefficients $\alpha_i$ and $\alpha_t$ represent firm and year fixed effects, respectively.

Some comments about specification (1) are warranted. First, it is worth emphasizing that the top and bottom classifications are fixed and given by firm size at age 20. Second, the firm

firm that we observe at age 20 but do not observe for at least 10 years between age 0 to 20 (e.g., a firm that was age 20 in 1996, we only observe for 5 years in our data from age 16 to 20).

fixed effects control for average differences across firms, ensuring that our main coefficients of interest, $\{\beta_a^T\}_{a\in\mathcal{A}}$ and $\{\beta_a^B\}_{a\in\mathcal{A}}$, capture the average life cycle dynamics of top and bottom firms, respectively. Note that these coefficients capture the difference relative to the omitted age group at the firm level. Hence, to interpret the results graphically in the figures below, we scale the results by adding the average value of the variable of interest at age 19-20 for the top and bottom separately. Lastly, to mitigate the impact of outliers in level variables (e.g., output, $y$) and ratios (e.g., the capital-output ratio, $k/y$), we estimate (1) using the log of the variable of interest (e.g., $x = \log(y)$). For shares, such as the labor share, $wl/y$, and the profit share, $\pi/y$, we run the regression in levels (e.g., $x = wl/y$), excluding values exceeding 2.5 in absolute value.

## 3.3 Life Cycle Growth

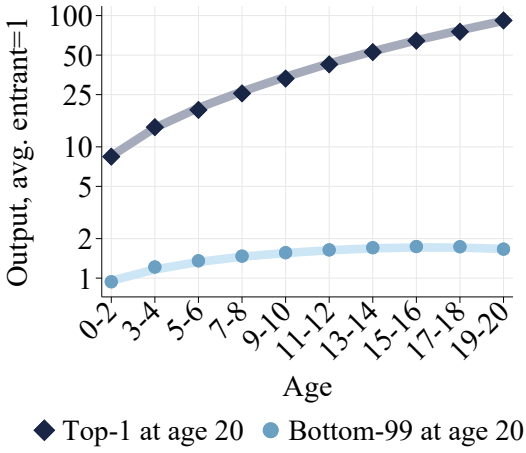
The availability of longitudinal data allows us to move beyond cross-sectional analysis and document novel facts about the origins of top firms. As a first step, we document firms' life cycle growth. Figure 4 presents the output dynamics during the first twenty years of top and bottom firms at age 20, which we estimate using (1).[12] For output, we divide the first age bin into two groups: 0-2 and 3-4. To facilitate interpretation, we normalize the results to the output of the average entrant, defined as firms aged 0-2.

Figure 4 shows remarkably stark differences in the life cycle trajectories of top and bottom firms. Importantly, these differences are present in terms of their *initial size* and life cycle *growth*. On average, firms in the bottom 99 percent at age 20 have a level of output equal to 1.03 times the average entrant at age 0-2, and they grow 1.8 times during the first 20 years. In contrast, firms in the top 1 percent at age 20 are 9.6 times larger than the average entrant at age 0-2, and they grow by a factor of 10.9 times during the first two decades of their life cycle. Hence, top firms at age 20 are not only 9.3 times larger at entry but also grow 6.2 times more than bottom firms. By age 19-20, the average top firm is 104.2 times the size of the average entrant, while the average bottom firm is only 1.8 times the average entrant. The life cycle growth of top and bottom firms is consistent with the view that ex-ante heterogeneity shapes the firm size distribution (Sterk, Sedláček, and Pugsley, 2021).[13]

[12] For these results, given the year firm-level fixed effects in (1), we just use log output and not the residualized variable used to classify top and bottom firms.

[13] For example, with mean-reverting shocks and no ex-ante heterogeneity in growth profiles, top firms at age 20 would, on average, have an entry size comparable to that of the typical entrant. This observation stems from the difficulty that ex-post shocks face in generating a high correlation between output at entry and output at age 20 across firms.

**Figure 4:** Output Over the Life Cycle of Top and Bottom Firms




*Notes*: Output life cycle trajectories of top 1 and bottom 99 percent firms at age 20 estimated using (1). Output is normalized by the size of the average entrant (age=0-2), and the y-axis is on a log scale. The solid lines represent smoothed scatterplots generated through locally weighted regressions. The dashed vertical lines indicate 95% confidence intervals considering firm-level clustered standard errors.

## 3.4 Inputs, Profits, and Finance Over the Life Cycle

In the previous section, we showed that the firms that reach the top of the size distribution start at a much larger scale and grow significantly faster than the rest during their first 20 years. We now study top firms' input usage (capital and labor) and profit trajectories over the same period, comparing them to bottom firms.

**Capital-Output Ratio** Figure 5a presents the trajectory of the capital-output ratio for firms in the top 1 and bottom 99 percent of the size distribution at age 20, obtained from estimating (1). The dynamics of capital relative to output are significantly different between these two groups of firms. Our estimates show that top firms start operating with a considerably high capital stock. The average age 0-4 capital-output ratio of the top 1 percent firms equals 4.9 and is 0.65 log points higher than the ratio at age 19-20. Thus, the capital-out ratio *decreases* over the life cycle of top firms. In contrast, for the bottom 99 percent of firms, the 0-4 capital-output ratio equals 2.3, which is 0.17 log points lower than its age 19-20 value. Hence, the capital-output ratio *increases* as firms age for the bottom 99 of the size distribution. Figure A.7 in the appendix shows that this novel fact on the dynamics of the capital-output ratio for top and bottom firms is robust to alternative measures of capital, such as solely using tangible capital or the sum of tangible and intangible capital.

**Labor Share** Our second key finding concerns top firms' labor share, defined as labor costs over output. Figure 5b reports the life cycle dynamics of firms' labor share for the top 1 and bottom 99 percent firms at age 20, estimated using (1). In this case, the initial labor

shares of top and bottom firms are similar at ages 0-4, around 0.67. Yet, the evolution of the labor share over firms' life cycles is starkly different for top and bottom firms. For the top 1 percent firms, the labor share *decreases* as firms age. Specifically, the labor share of top firms falls by 11 p.p. over the first two decades, from 0.67 to 0.56. The labor share of the bottom 99 percent of firms is nearly constant over the first 20 years of the firm's life cycle. This result for the bottom 99 is consistent with Kehrig and Vincent (2021), which documents roughly constant labor shares at the establishment-level over time. The main novelty of our findings is that a low labor share is not a fixed characteristic of large firms. Instead, the labor share of firms that end up at the top of the size distribution falls as firms grow.[14]

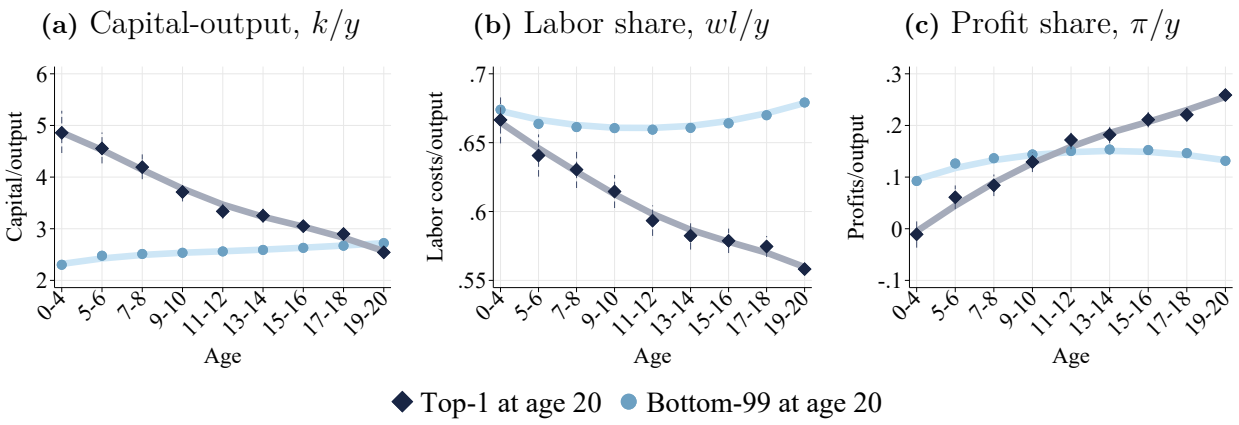
Lastly, it is worth noticing that the decline in the labor share of top firms is considerably smaller than the decline in the capital-to-output ratio. Indeed, Figure A.6 in the Appendix shows that the capital-to-labor costs ratio significantly declines over the life cycle of top firms. In contrast, for firms in the bottom 99, the increase in the capital-to-output ratio and the almost flat labor share imply that the capital-to-labor cost ratio increases over time. As discussed in Section 5, changes in firms' input composition over their life cycle are informative on the factors driving the decline in input usage among top firms.

**Profit Share** After documenting the dynamics of input usage, we now turn our analysis to firms' profits. Figure 5c presents the results from estimating (1) for the profit share, defined as economic profits over output. This figure shows that the profits of the top 1 percent firms at age 20 are much more *backloaded* towards the second decade of the firms' life cycle. Our estimates indicate that the profit share is not statistically different from zero for the first five years of top firms. Yet, the profit share of top firms significantly increases with firm age and reaches 0.26 by age 19-20. In contrast, the profit share of the bottom 99 percent is almost flat, and only modestly increases by 4 p.p., from 0.09 at age 0-4 to 0.13 at age 19-20. Thus, there are remarkable differences in the life cycle dynamics of the profit share by firm size, with top 1 firms' profits having a much steeper life cycle growth than for the bottom 99 percent. Figure A.8 in the appendix shows that this key fact on the life cycle path of the profit share for top and bottom firms is robust to measuring profits by net income.

**Finance** Finally, we document the financing of top and bottom firms over their life cycle. We focus on net financial leverage, defined as financial debt minus cash over capital, and the rate of equity injections over capital that firms receive from their owners (negative dividends over capital). Figure 6 presents the results of estimating (1) for leverage and equity

[14]Consistent with Autor et al. (2017), Table 1 shows that top firms have low labor shares in the cross-section. Our findings on the decline in the labor share of top firms are consistent with Mertens and Schoefer (2024), which shows that firms' labor share falls as they grow over short- and medium-term horizons.

**Figure 5:** Inputs and Profit Share Over the Life Cycle of Top and Bottom Firms



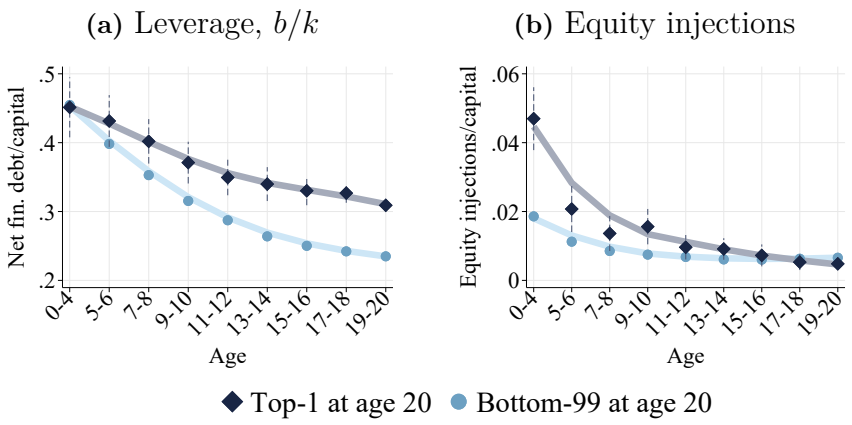

*Notes*: Life cycle trajectories for the inputs and profit shares of the top 1 and bottom 99 percent firms at age 20 estimated using (1). The regression for the capital-output ratio is in logs, while the labor and profit shares are in levels. Results are scaled by adding the average of the omitted group (age 19-20) for the top and bottom firms. The solid lines represent smoothed scatterplots generated through locally weighted regressions. The dashed vertical lines indicate 95% confidence intervals considering firm-level clustered standard errors.

injections. Consistent with the findings in Kochen (2025), we document that leverage and equity injections decrease as firms age. Unlike previous work, we find significant differences in the use of debt and equity financing for top and bottom firms. Regarding debt, Figure 6a shows that while leverage is initially similar, around 0.45, the top 1 percent firms at age 20 deleverage *less* over their life cycle, with average leverage of 0.31 by age 19-20 (vs. 0.23 at age 19-20 for the bottom 99 percent). Figure 6b shows that top firms' annual equity injections over capital equals 0.05 at age 0-4, twice as large as the number for bottom firms, 0.02. Yet, these differences in equity injections fade out as firms age. Hence, relative to bottom firms, the top 1 percent of firms at age 20 deleverage less as they age and rely more on equity financing in their early years. The model described below will be consistent with these facts.

In sum, there are substantial differences in the origins of the top 1 and bottom 99 percent of firms at age 20. The findings for the capital-output ratio are consistent with top firms requiring large early investments, with output subsequently growing faster than capital as firms age. Furthermore, the decline in the labor share over the life cycle of top firms shows that the low labor share in large firms results from significant output growth rather than being a fixed characteristic. As consequence, the profit share of top firms has a considerably steeper life cycle growth than bottom firms and is much more backloaded towards the second decade of the firms' life cycle. Finally, these patterns are accompanied by differences in the use of debt and equity financing, with top firms deleveraging less and relying more on equity injections in their early years.

In the next section, we extend our analysis to other segments of the firm size distribution,

**Figure 6:** Finance Over the Life Cycle of Top and Bottom Firms




*Notes*: Life cycle trajectories for leverage and equity injections over capital for top 1 and bottom 99 percent firms at age 20 estimated using (1). Results are scaled by adding the average of the omitted group (age 19-20) for the top and bottom firms. The solid lines represent smoothed scatterplots generated through locally weighted regressions. The dashed vertical lines indicate 95% confidence intervals considering firm-level clustered standard errors.

countries, and sectors, and perform various robustness checks. Overall, the main results presented above hold across all these additional exercises.

## 3.5 Additional Results and Robustness

We now expand on our baseline results across several dimensions. First, we extend our analysis to the average firm and to other segments of the firm size distribution beyond the top 1 percent and bottom 99 percent, as well as to different economies and production sectors. Additionally, we perform various robustness checks, including restricting the sample to privately held firms, employing a balanced panel sample, and reclassifying top firms using alternative firm size variables. Lastly, we present results for materials usage, which exhibit much more modest variation over firms' life cycles. We also perform a forward-looking analysis that is consistent with our baseline and document potential sources of ex-ante heterogeneity.

**Average Firm** First, to motivate the importance of separately studying the life cycle of top and bottom firms, Figure A.9 in the Appendix presents the average trajectory of input usage and profits for firms that survive to age 20. As expected, the life cycle trajectory for the average firm is close to that of the bottom 99 percent, with an increasing capital-output ratio, an even flatter path for the labor share, and a slightly increasing but much less backloaded profit share. As we now show, the average life cycle trajectory of input usage and profits masks substantial heterogeneity among firms at different points of the size distribution, even beyond the top 1 and bottom 99 percent.

**Beyond the Top 1 and Bottom 99** Our baseline analysis focuses on the origins of the top 1 and bottom 99 percent of firms at age 20. A natural question is whether our findings

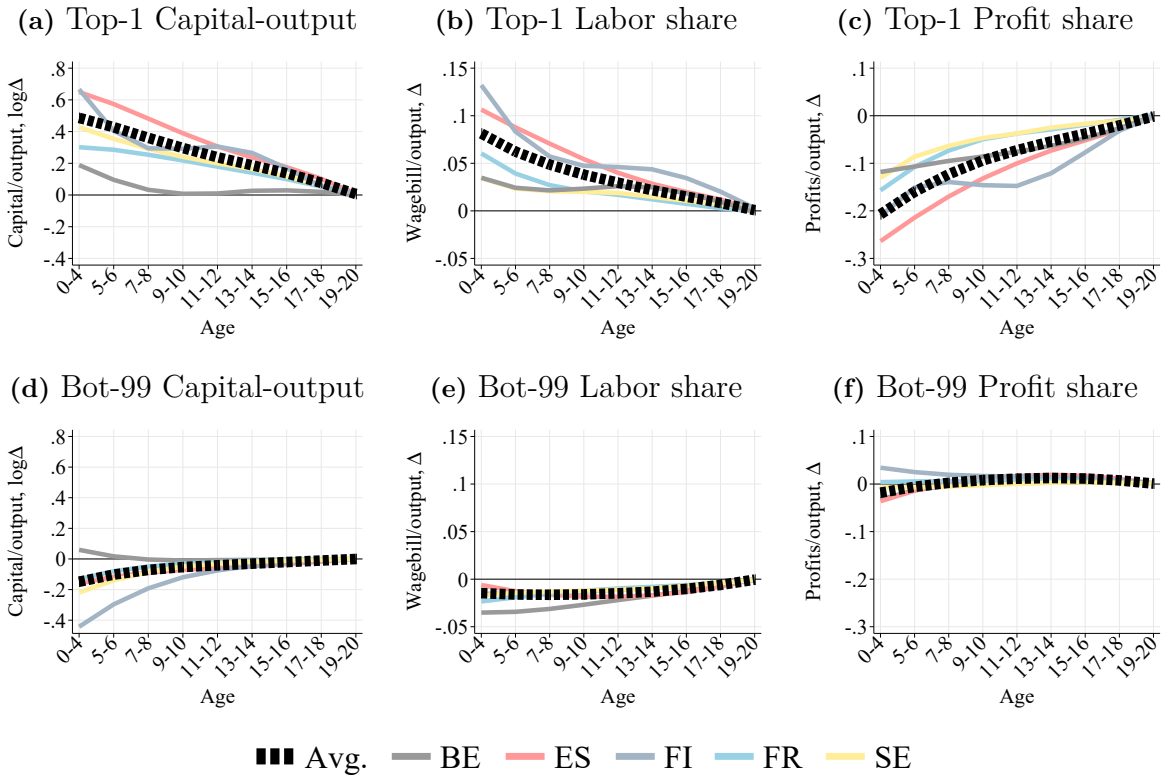
extend to other segments of the firm size distribution. To study this, we estimate a version of regression (1) considering six groups of firms defined by their output at age 20: the top 0.01, percentile 99 to 99.9, percentile 95 to 99, percentile 75 to 95, percentile 50 to 75, and the bottom 50 percent. Figure A.10 presents the results for the capital-output ratio, the labor share, and profit share over firms' life cycle, conditional on their size at age 20 according to these six groups. In this case, we present the results for the firm-level changes relative to the age 19-20 value to simplify the exposition. We report log changes for the capital-output ratio and level changes for the labor and profit share.

The main takeaways from Figure A.10 can be summarized as follows. First, this figure shows that our baseline findings on the input and profits life cycle dynamics for top and bottom firms extend to the entire size distribution. Furthermore, the life cycle profiles vary monotonically with firm size, with the top 0.01 and the bottom 50 percent of firms at age 20 exhibiting the starkest differences. Second, it is worth noting that there is heterogeneity even within the top 1 percent, with firms in the top 0.01 exhibiting steeper decreases in the capital-output ratio and labor share, together with much more backloaded profit shares, than firms in the percentile 99 to 99.9. Third, and perhaps more expected, there is significant heterogeneity among bottom firms. Interestingly, the capital-output ratio and labor share increase as firms age for the bottom 50 percent of the size distribution.

**Other Countries** So far, we have focused on Spain (ES), a country with an ample panel and excellent coverage in Orbis Historical. We now extend our analysis to four additional countries with a sufficiently long panel in Orbis: Belgium (BE), Finland (FI), France (FR), and Sweden (SE). Figure 7 presents the results from estimating (1) for our three main variables of interest separately for each of these five countries, with country-specific classifications for the top 1 and bottom 99 percent firms at age 20. We focus on life cycle changes relative to ages 19–20 to control for average cross-country differences in these variables. The dashed dark line is the average value across countries for the life cycle estimates. Figure 7 shows that our main findings on the origins and life cycle dynamics of top and bottom firms hold in these additional countries. For top 1 percent firms at age 20, we observe a decreasing capital-output ratio, a declining labor share, and a backloaded profit share. In contrast, the bottom 99 percent experience increasing capital-output ratios and labor shares as they age.

**Sectors** Due to data limitations, previous studies have primarily focused on the manufacturing sector (Hsieh and Klenow, 2014; Midrigan and Xu, 2014; Eslava, Haltiwanger, and Urdaneta, 2024). Yet, in developed countries like Spain, the manufacturing sector typically accounts for only 20% to 30% of total output, and its importance has declined over time. An advantage

**Figure 7:** Other Countries
Life Cycle Changes in Inputs and Profit Share of Top and Bottom Firms




*Notes*: Life cycle changes for the inputs and profit shares of the top 1 and bottom 99 percent firms at age 20 estimated using (1), separately for each country. Changes are relative to the omitted group (age 19-20). The selected countries are Belgium (BE), Spain (ES), Finland (FI), France (FR), and Sweden (SE). The dashed dark line is the average value across countries for the life cycle estimates. The lines are smoothed scatterplots generated through locally weighted regressions on the estimated parameters.

of our data is that it covers all sectors. While we use all firms for our baseline estimates, Figure A.11 in the Appendix presents results for the broad four sectors reported in Table 1: manufacturing, retail and transport, services, and other. This figure shows that our key findings about the first two decades of the top 1 percent firms are present in all these sectors.

**Private Firms** Our analysis includes publicly listed, public companies (C-corporations), and privately held firms. Table 1 reports that listed and public firms constitute 56% of the firms and 77% of the output of the top 1 percent. Hence, a natural question is whether listed and public firms primarily drive our findings. Figure A.12 in the Appendix reports the estimates for the life cycle of top and bottom firms at age 20, differentiating by their legal status: private vs. listed and public firms. This figure shows that our key findings on the life cycle of top firms also hold for the sample of privately held firms.

**Balanced Panel** For our baseline results, we use a sample of firms observed for at least ten years from ages 0 to 20 to increase the number of observations and have more precise estimates, especially for top firms. As a robustness check, in Figure A.13 in the Appendix, we

present the results of estimating (1) restricting to a balanced panel and compare the results with our baseline. The results for the balanced sample, while estimated less precisely, are very similar to our baseline numbers.

**Top Firms Under Alternative Measures of Firm Size** Throughout our analysis, we defined top and bottom firms using value-added, considering a comprehensive measure of costs. We now consider alternative definitions of top firms using other outcome variables as a robustness check. Figure A.14 in the Appendix presents results from estimating (1) for definitions of top and bottom firms using revenue and total assets at age 20. This figure shows that our main findings on the life cycle dynamics of top and bottom firms are robust to alternative definitions of firm size.

**Materials** Given the focus of our paper on firms' output, not *revenue*, we measure firm size as revenue minus a comprehensive measure of non-capital and non-labor costs, which includes materials. Contemporaneously developed work has found that materials play a role behind short and medium-run revenue growth (Mertens and Schoefer, 2024) and for the revenue size distribution (Hubmer et al., 2024). To analyze the role of materials usage over the life cycle of top and bottom firms, Figure A.15 in the Appendix presents the results of estimating (1) using materials over revenue as the dependent variable. This figure shows that the materials-to-revenue ratio modestly increases over the life cycle of the top 1 percent of firms at age 20. Yet, the estimates are not statistically different from the age 19-20 value, except for the first age bin. This ratio is roughly constant for the first two decades of firms in the bottom 99 percent at age 20. Thus, we find limited life cycle variation of materials usage at the firm level.

**Forward-looking Analysis** Alternatively, we perform two forward-looking exercises. First, we pair top firms at age 20 with bottom firms at age 20 that have similar initial characteristics, such as initial size, input usage, profits, and financing. By matching each top firm to a bottom pair and estimating (1) on the paired firms, we study the life cycle patterns of firms that are initially similar to top firms but do not reach the top. Figure A.18 shows the results. We find that their growth, input usage, and profits follow a flat profile. These patterns are consistent with the predictions of our theory for top technologies that are unlucky and do not grow, or for bottom ones that are initially unconstrained.

Second, we study how initial size, input usage, profits, and financing predict whether a firm becomes a top firm twenty years later. Table A.3 presents the results. We find that higher initial output, a higher initial capital-output ratio, and greater initial equity injections make it more likely for a firm to reach the top. To a lesser extent, lower initial profitability

and leverage also predict this. These findings hold whether we condition on bottom firms surviving until age 20 or include those that exit before age 20, and they are overall consistent with our baseline exercise, despite it being backward in nature.[15] Appendix A.5 provides further details on both forward-looking empirical exercises.

**Sources of Ex-Ante Heterogeneity** Having documented sharply different trajectories over the life cycle of top and bottom firms, we conclude this section with suggestive evidence on the sources of firm ex-ante heterogeneity. Using firm-owner linkages in Orbis, we classify firms along two dimensions: whether the owner is domestic or foreign, and whether the owner simultaneously owns multiple firms—capturing both business groups and serial entrepreneurs (Brandt et al., 2022; De Vera et al., 2024). Ownership data are available from 2007 onward. For firms older than 20 years, we use the first available ownership record. Results are at the firm level, computed by averaging over time and owners using equity shares.[16]

Table A.2 reveals pronounced differences across groups. We document that firms in the top 1 percent at age 20 are almost 11 times more likely to be foreign-owned than firms in the bottom 99. Further, they are 2.8 times more likely to have owners with multiple firms, highlighting the role of business groups and serial entrepreneurs. The model in the next section captures these differences through ex-ante heterogeneous growth profiles.

# 4 A Model of Top and Bottom Firms

In this section, we develop a firm dynamics model with ex-ante heterogeneity in productivity and production technology, production non-homotheticities in the form of required minimum capital and labor levels, and forward-looking financing, in which financing capacity depends on expected future profitability. We use the model to explain our empirical findings and study the macroeconomic implications of the origins of top firms.

## 4.1 Setup

We study a discrete-time, infinite-horizon economy inhabited by heterogeneous firms. Technologies are scarce, so each period, a mass of potential entrants, $\mathcal{E}$, decides whether to enter the market and begin operations. Firms are born with a technology type $h \in \mathcal{H}$, where the measure of types $\mathcal{E}_h$ satisfies $\sum_h \mathcal{E}_h = \mathcal{E}$.[17] The technology types can vary in their

[15] In other words, if a top firm at age 20 is characterized by certain initial characteristics on average, it does not imply that a firm with these initial characteristics is more likely to be top at age 20, especially considering that some firms may exit before age 20.

[16] For Spain, 94% of the age 20 top 1 percent firms have ownership files, compared to 57% of bottom 99 firms and 43% of the rest. Results are similar focusing solely on majority owners.

[17] The technological scarcity can also be understood as the presence of implicit adoption costs required for technologies to begin operating. In our quantitative exercises, we explore counterfactuals in which there is

productivity process, returns to scale, input shifters, and input non-homotheticities. The mass of potential entrants and their distribution across types are exogenous, characterizing the technological frontier of the economy.

Potential entrants are drawn from a time-invariant distribution $G$ over firms' idiosyncratic states—technology type, assets, and productivity. If potential entrants decide to enter, they produce a unique final good using capital and labor. Aside from production, they can also save in a risk-free asset. The risk-free interest rate $r$ is exogenous, while wages $w$ are determined in equilibrium. To close the labor market, there is an aggregate labor supply function $L^s(w)$, which in equilibrium equals firms' aggregate labor demand $L^d$, determining the wage $w$. We focus on the steady-state equilibrium, where $t$ indexes firm age.

We now provide a detailed description of firms' objective, production technology, financial frictions, and their entry and exit.

**Firms' Objective** Firm $i$ at age $t$ maximizes the net present value of its dividends, $d_{it}$:

$$V_{it} = \mathbb{E}_t \left[ \sum_{s \geq t} \beta^{s-t} d_{i,s} \right]$$

for $t \geq 0$, where $\beta \in (0,1)$ is the firm's subjective discount factor and satisfies $\beta(1+r) \leq 1$.

**Productivity Process** The evolution of firms' productivity is technology-specific, with the productivity process of firm $i$ with technology $h$ at age $t \geq 1$ given by

$$p\left(\mathbf{s}_{it}^h\right) = \exp\left(u_{it}^h + z_{it}\right) \tag{2}$$

where $\mathbf{s}_{it}^h = (u_{it}^h, z_{it})$, $u_{it}^h$ is the *ex-ante* component, and $z_{it}$ is the *ex-post* component.

Similar to Sterk, Sedláček, and Pugsley (2021) and Hopenhayn, Neira, and Singhania (2022), we introduce ex-ante heterogeneity in TFP by assuming that, when born, firms draw an initial ex-ante component $u_{i0}^h \sim \mathcal{N}\left(\underline{\mu}_u^h, \underline{\sigma}_u^2\right)$, which evolves *deterministically* as

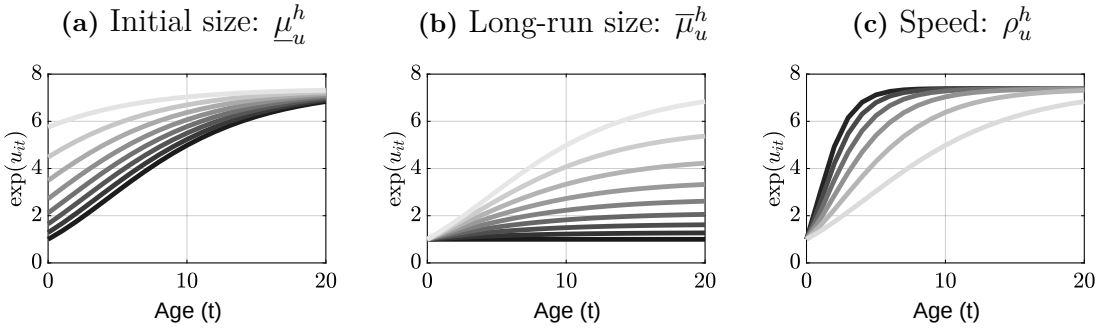
$$u_{it+1}^h = \rho_u^h u_{it}^h + \left(1 - \rho_u^h\right) \overline{\mu}_u^h$$

where the parameters $\left(\rho_u^h, \underline{\mu}_u^h, \overline{\mu}_u^h\right)$ jointly determine the level and growth profile of $h$, as Figure 8 illustrates. $\underline{\mu}_u^h$ defines the initial size across types, $\overline{\mu}_u^h$ specifies the size in the long run, and $\rho_u^h$ determines the speed at which the ex-ante component converges to its long-run value. Lastly, $\underline{\sigma}_u^2$ parameterizes the initial heterogeneity within technologies.

Given an initial value $z_{i0}$, the ex-post component of TFP evolves *stochastically* according

endogenous initial technology selection.

**Figure 8:** Ex-ante TFP Component




*Notes*: The figure shows different paths of the deterministic ex-ante TFP component $\exp(u_{it})$ for different values of $\underline{\mu}_u^h$ (panel a), different values of $\overline{\mu}_u^h$ (panel b), and different values of $\overline{\mu}_u^h$ (panel c). Lighter lines indicate higher values. Numerical example.

to the law of motion

$$z_{it+1} = \rho_z z_{it} + \varepsilon_{it+1},$$

where ex-post shocks are $\varepsilon_{it+1} \sim \mathcal{N}(0, \sigma_\varepsilon^2)$. Hence, notice that a firm can be in the top 1 percent of the size distribution firm if it is lucky (i.e., experiences a sequence of large ex-post shocks) and/or has a high-growth and level profile.

**Production Function** Firms combine capital $k_{it}$ and labor $l_{it}$ to produce output $y_{it}$ using technology

$$y_{it} = p\left(\mathbf{s}_{it}^h\right) f^h\left(k_{it}, l_{it}\right).$$

The function $f^h(k_{it}, l_{it})$ exhibits decreasing returns to scale (DRS) in capital and labor, aggregating the inputs using a constant elasticity of substitution (CES) aggregator. Additionally, there are non-homotheticities in capital and labor (Geary, 1950; Stone, 1954).[18] The function $f^h(k_{it}, l_{it})$ is parameterized as

$$f^h\left(k_{it}, l_{it}\right) = \left[\alpha_h^{\frac{1}{\sigma}}\left(k_{it} - \kappa_k^h\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha_h)^{\frac{1}{\sigma}}\left(l_{it} - \kappa_l^h\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}\theta^h} \tag{3}$$

where $\sigma$ is the elasticity of substitution between capital and labor (with $\sigma < 1$ indicating complements and $\sigma > 1$ indicating substitutes), $\alpha_h$ is the capital share, and $\theta^h \in (0, 1)$ represents the degree of decreasing returns to scale (DRS), which may depend on the technology type. Differences in the DRS across types may reflect both technological heterogeneity and variation in the slope of demand (see Appendix B.1). When $\sigma \to 1$ the function is Cobb-

[18] Other types of parametric non-homothetic production functions have been used by Lau and Tamura, 1972 (Leontief) and Hanoch (1975), Sato (1977), Lashkari, Bauer, and Boussard (2024), and Eckert, Ganapati, and Walsh (2024) (non-homothetic CES).

Douglas, given by $f^h(k_{it}, l_{it}) = \left[\left(k_{it} - \kappa_k^h\right)^{\alpha_h} \left(l_{it} - \kappa_l^h\right)^{1-\alpha_h}\right]^{\theta^h}$. Central to our analysis are the technology-specific non-homotheticities in capital and labor, parameterized by $\kappa_k^h$ and $\kappa_l^h$, respectively. In Appendix B, we discuss how input non-homotheticities can account, for example, for production input indivisibilities (Banerjee and Duflo, 2005) and varying complementarities between inputs and TFP.

**Production Costs** Firms hire labor at wage $w$ and rent capital at rate $R$, such that the firms variables costs are $wl_{it} + Rk_{it}$. Capital depreciates at an exogenous rate $\delta \in [0, 1]$, so in equilibrium, the rental rate of capital is $R = \delta + r$, while wages are determined by firms' labor demand and the aggregate labor supply function. As a result, at period $t$, the firm $i$ profits are $\pi_{it} = y_{it} - Rk_{it} - wl_{it}$. Although it is not explicit, it is important to note that non-homotheticities are equivalent to input-specific fixed costs. Using $\tilde{k}_{it} = k_{it} - \kappa_k^h$ and $\tilde{l}_{it} = l_{it} - \kappa_l^h$, such that $\tilde{f}^h(\tilde{k}_{it}, \tilde{l}_{it})$ is a standard homothetic CES aggregator, we can rewrite profits as $\pi_{it} = y_{it} - (R\tilde{k}_{it} + w\tilde{l}_{it}) - (R\kappa_k^h + w\kappa_l^h)$, where $R\kappa_k^h + w\kappa_l^h$ represents the input-specific fixed costs.

**Financial Assets and Frictions** Over time, firms can accumulate a risk-free asset $a_{it} \geq 0$, which earns a return $r$. This asset can be used as collateral and liquidated in any given period. There are two sources of financial frictions. First, it is costly to issue equity (Hennessy and Whited, 2007). Dividends are given by $d_{it} = \tilde{d}_{it} - \Lambda\left(\tilde{d}_{it}\right)$, where $\tilde{d}_{it} = \pi_{it} + a_{it}(1 + r) - a_{it+1}$ are the dividends before accounting for equity issuance costs and $\Lambda\left(\tilde{d}_{it}\right)$ the equity issuance costs. The issuance cost function is given by

$$\Lambda\left(\tilde{d}_{it}\right) = \begin{cases} \chi_0 + \chi_1 \left|\tilde{d}_{it}\right| & \text{if } \tilde{d}_{it} < 0 \\ 0, & \text{otherwise,} \end{cases}$$

where $\chi_0, \chi_1 \geq 0$ are positive parameters. Second, firms can rent a limited amount of capital $k_{it} \leq \bar{k}^h\left(a_{it}, \mathbf{s}_{it}^h\right)$. We parameterize the maximal amount of capital that a firm can rent, $\bar{k}^h\left(a_{it}, \mathbf{s}_{it}^h\right)$, as

$$\bar{k}^h\left(a_{it}, \mathbf{s}_{it}^h\right) = \underbrace{\lambda_a a_{it}}_{\text{backward-looking}} + \underbrace{\lambda_v \max\left\{0, V^{h*}\left(\mathbf{s}_{it}^h\right)\right\}}_{\text{forward-looking}}, \tag{4}$$

where $\lambda_a \geq 1$ captures how much capital can be rented using assets as collateral (Kiyotaki and Moore, 1997), which is backward looking, and $\lambda_v \geq 0$ is how much additional capital can be rented against the value of the firm, which is forward looking (Albuquerque and Hopenhayn, 2004; Brooks and Dovis, 2020). For simplicity, we assume that the forward-looking component of the collateral constraint depends on the firm's unconstrained value, $V^{h*}(\mathbf{s}_{it}^h)$, which we

define later.[19] Crucially, this value reflects both current and future profits. The equity issuance and the forward-looking borrowing allows firms to fund themselves against current assets and future expected profits, which can vary significantly depending on different growth profiles. Importantly, the set of parameters $(\chi_0,\chi_1,\lambda_a,\lambda_v)$ governing the degree of financial development are common across technology types.

**Entry** Firms are born with a technology type $h$, which has a measure $\mathcal{E}_h$, and draw initial productivity $u_{i0}^h$ and assets $a_{i0}$ from a distribution $G^h\left(a_{i0}, u_{i0}^h\right)$.[20] Following Kochen (2025), $u_{i0}$ is drawn from $u_{i0}^h \sim \mathcal{N}\left(\underline{\mu}_u^h, \underline{\sigma}_u^2\right)$. Once $u_{i0}$ is determined, the firm's initial asset position is given by $a_{i0} = \iota k_0^{h*}\left(u_{i0}^h, 0\right)$, where $k_0^{h*}\left(u_{i0}^h, 0\right)$ represents the initial unconstrained level of capital, and $\iota \in [0,1]$ indicates how close the firm is from initially having enough assets to be unconstrained, which is drawn from $\iota \sim \text{Beta}\left(\bar{\iota}, 1\right)$. Thus, conditional on $h$, the parameters $\{\underline{\mu}_u^h, \sigma_u^2, \bar{\iota}\}$ characterize the distribution $G^h\left(a_{i0}, u_{i0}^h\right)$. After being born, the firm decides whether to enter. If it enters, it starts operating in the next period; otherwise, it exits with value $V_e\left(a_{i0}\right) = (1+r)a_{i0}$.

**Exit** Firms can exit for several reasons. First, as in standard models of firm dynamics with financing frictions, firms of type $h$ exit with an exogenous probability $\eta^h \in [0,1]$. This assumption is needed because, when $\beta(1+r) = 1$, becoming unconstrained is an absorbing state, which would make the stationary distribution degenerate (see, e.g., Khan and Thomas (2013)). Second, a firm exits endogenously if the value of continuing $V^h$ (defined below) falls below the exit value $V_e\left(a_{it}\right)$. Finally, a firm must exit if it cannot satisfy the capital non-homotheticity condition, $\bar{k}^h\left(a_{it}, \mathbf{s}_{it}^h\right) < \kappa_k^h$, meaning firms cannot remain dormant while waiting to meet this condition.[21] In all cases, the exit value equals the liquidation value of assets, $V_e\left(a_{it}\right) = (1+r)a_{it}$.[22]

## 4.2 Firm Problem

The firm solves a static profit maximization problem and a dynamic savings decision, which can be written recursively.

[19] Using the unconstrained value precludes potential multiplicity concerns and yields a cleaner separation between the asset-based and earnings-based components of the collateral constraint. The cost of this approach is that we abstract from the feedback loop between the collateral constraint and firm value, which eliminates additional curvature in the profit function.

[20] Note that $G$ represents the initial distribution across and within types, while $G^h$ is the distribution conditional on type $h$.

[21] This assumption is consistent with standard firm dynamics models, where firms cannot suspend operations to avoid the fixed cost while waiting to become profitable.

[22] Since $\beta(1+r) \leq 1$, the (weakly) best alternative upon exit is to consume the assets immediately.

**Profit Maximization** Given the idiosyncratic states $(a_{it}, \mathbf{s}_{it}^h)$, a firm $i$ with type $h$ technology has per-period optimal profits given by

$$\begin{aligned} \pi^h\left(a_{it}, \mathbf{s}_{it}^h\right) = \max_{k_{it}, l_{it}} \quad & p\left(\mathbf{s}_{it}^h\right) f^h\left(k_{it}, l_{it}\right) - Rk_{it} - wl_{it} \\ \text{s.t.} \quad & k_{it} \leq \bar{k}^h\left(a_{it}, \mathbf{s}_{it}^h\right), \end{aligned} \tag{5}$$

such that capital $k^h\left(a_{it}, \mathbf{s}_{it}^h\right)$ and labor $l^h\left(a_{it}, \mathbf{s}_{it}^h\right)$ policy functions solve this problem. To compute firm's collateral constraint, it is helpful to define the unconstrained solution. The unconstrained profits $\pi^{h*}\left(\mathbf{s}_{it}^h\right)$ are the solution to problem (5) when capital is not subject to the collateral constraint. The policy functions $k^{h*}\left(\mathbf{s}_{it}^h\right)$ and $l^{h*}\left(\mathbf{s}_{it}^h\right)$ denote the optimal unconstrained choices of capital and labor, respectively. To find collateral constraint $\bar{k}^h\left(a_{it}, \mathbf{s}_{it}^h\right)$, we compute iteratively the unconstrained value $V^{h*}\left(\mathbf{s}_{it}^h\right) = \max \pi^{h*}\left(\mathbf{s}_{it}^h\right) + \left(1-\eta^h\right)\beta\mathbb{E}\left[\max\left\{V^{h*}\left(\mathbf{s}_{it+1}^h\right), 0\right\}\right]$ using the unconstrained profits. Appendix B provides more details of the constrained and unconstrained solutions.

**Incumbent Recursive Problem** Let $\tilde{V}^h$ be the value of the incumbent firm $i$ of technology type $h$ with states $\left(a_{it}, u_{it}^h, z_{it}\right)$ at the beginning of period $t$ such that

$$\tilde{V}^h\left(a_{it}, u_{it}^h, z_{it}\right) = \max\{ \underbrace{V^h\left(a_{it}, u_{it}^h, z_{it}\right)}_{\text{continue}}, \underbrace{V_e\left(a_{it}\right)}_{\text{close}} \}, \tag{6}$$

where $V_e(a_{it}) = a_{it}(1+r)$ and $V^h$ denotes the value of continuing operations instead of closing the firm. It follows that the value of continuing $V^h$ is given by

$$V^h\left(a_{it}, u_{it}^h, z_{it}\right) = \max_{a_{it+1},\ \tilde{d}_{it}} \ \tilde{d}_{it} \ - \Lambda(\tilde{d}_{it}) + \beta\mathbb{E}\left[\left(1-\eta^h\right)\tilde{V}^h\left(a_{it+1}, u_{it+1}^h, z_{it+1}\right) + \eta^h V_e\left(a_{it+1}\right)\right]$$

where

$$\begin{aligned} \tilde{d}_{it} &= \pi^h\left(a_{it}, u_{it}^h, z_{it}\right) + a_{it}\left(1+r\right) - a_{it+1} \\ u_{it+1}^h &= \rho_u^h u_{it}^h + \left(1-\rho_u^h\right)\overline{\mu}_h \\ z_{it+1} &= \rho_z z_{it} + \varepsilon_{it+1}, \quad \varepsilon_{it+1} \sim N\left(0, \sigma_\varepsilon^2\right). \end{aligned} \tag{7}$$

If there is a feasible solution (i.e., $\bar{k}^h(a_{it}, \mathbf{s}_{it}^h) \geq \kappa_k$ is satisfied), the policy function $a^h\left(a_{it}, u_{it}^h, z_{it}\right)$ solves the incumbent's problem. Otherwise, the firm exits with $V_e(a_{it})$.

**Entry Problem** A firm enters if the expected value of entering and operating exceeds the outside option of retaining the initially endowed assets. Thus, the value of the potential

entrant $i$ with technology $h$ and states $(a_{i0}, u^h_{i0})$ is

$$V^h_0\left(a_{i0}, u^h_{i0}\right) = \max\{ \underbrace{\mathbb{E}\left[\tilde{V}^h\left(a_{i0},\ \rho^h_u u^h_{i0} + \left(1-\rho^h_u\right)\overline{\mu}_h, \varepsilon_z\right)\right]}_{\text{enter}} ,\ \underbrace{a_{i0}(1+r)}_{\text{not enter}} \}. \tag{8}$$

where the value of not entering is keeping the firm wealth plus the risk free return. Importantly, unlike the free-entry condition, technologies are scarce. The availability of different technologies, represented formally by the measures $\{\mathcal{E}_h\}_{h\in\mathcal{H}}$, can be interpreted as the technological frontier of the economy. Changes in the technological composition—formally, the distribution of these measures—can have significant aggregate implications, which we study in our quantitative applications.

### 4.3 Competitive Equilibrium

A *stationary competitive equilibrium* in this economy consists of: (i) wage $w$ and interest rates $\{r, R\}$; (ii) incumbent firms' policy functions $\left\{a^h\left(a_{it}, \mathbf{s}^h_{it}\right), k^{h*}\left(\mathbf{s}^h_{it}\right), l^{h*}\left(\mathbf{s}^h_{it}\right)\right\}_{h\in\mathcal{H}}$; (iii) potential entrants' entry and incumbents' exit decisions; (iv) initial distribution of idiosyncratic states $G$, types measures $\{\mathcal{E}_h\}_{h\in\mathcal{H}}$ and distribution of incumbent firms that produce $\Omega^h\left(a_{it}, \mathbf{s}^h_{it}\right)$; and (v) aggregate labor supply function $L^s(w)$ such that:

- Given prices, the policy functions solve the incumbent firms' profit maximization (5) and dynamic problem (7).
- Entry choices are consistent with problem (8), and exit decisions are consistent with the exit decision (6) and the capital non-homotheticity feasibility.
- The labor market clears, i.e., $L_s(w) = \sum_{h\in\mathcal{H}} \int l^h\left(a_{it}, \mathbf{s}^h_{it}\right) \mathrm{d}\Omega^h$.
- The rental rate of capital is determined by the risk-free rate and the capital depreciation rate such that $R = r + \delta$.
- For each $h \in \mathcal{H}$, the distribution $\Omega^h(a_{it}, \mathbf{s}^h_{it})$ is implicitly defined by the equilibrium mapping $\mathbf{\Omega}^h = \mathcal{T}\left[\mathbf{\Omega}^h\right]$, which is determined by the exit and entry decisions, policy functions, and the initial distribution of firms.

## 5 Firms Input Usage in Data and Theory

In this section, we contrast the model's implications for input usage over the life cycle with our empirical findings for top and bottom firms. Additionally, we analyze alternative hypotheses related to changes in product and labor market power over the life cycle, as well as input-biased growth, which are abstracted from in our baseline model.

## 5.1 Input Usage and Profits in the Baseline Model

Our stylized theory provides two potential sources through which the capital-output, labor share, and input mix can vary across the life-cycle. Let $\mu_i$ be the lagrange multiplier on the capital collateral constraint in problem (5), then the capital-output ratio in the model is

$$\frac{k_{it}}{y_{it}} = \theta^h \alpha_h \underbrace{\left(\frac{1}{R+\mu_{it}}\right)}_{\text{capital costs}} \underbrace{\left[\alpha_h + (1-\alpha_h)\left(\frac{w}{R+\mu_{it}}\right)^{1-\sigma}\right]^{-1}}_{\text{input complementarities}} \underbrace{\frac{1}{\left(1-\kappa_k^h/k_{it}\right)}}_{\text{K non-homotheticity}} . \tag{9}$$

The first term on the right-hand side of (9) depends on the returns to scale $\theta^h$ and capital elasticity $\alpha_h$, which are fixed over time.

The second term relates to the cost of capital. The higher the cost, the lower the capital-output ratio. The cost of capital consists of the rental rate, which is fixed over time, and the shadow cost of capital, which reflects how binding the capital collateral constraint is. The shadow cost of capital can vary over a firm's life cycle. For example, as firms age and become less constrained, $\mu_{it}$ decreases, leading to an increase in the capital-output ratio. Given a decline in the shadow cost of capital, the magnitude of this increase depends on the elasticity of substitution $\sigma$, with a higher $\sigma$ implying a faster increase in $k/y$. An increase in the capital-output ratio as firms age aligns with the empirical observations for the bottom firms but not for the top firms, as we document in the empirical section.

An additional source of variation in the capital-output ratio comes from input non-homotheticities. For firms that produce, we know that $\kappa_k^h > \overline{k}^h \geq k_{it}$, which implies that $\kappa_k^h/k_{it} \in [0,1)$ and that $\frac{1}{\left(1-\kappa_k^h/k_{it}\right)} \geq 1$. Importantly, as $k_{it}$ increases this term will decrease. For example, if firms initially operate with levels of capital close to the necessity level $\kappa_k^h$, then the $k/y$ ratio will be large at the start, but as they grow, it will gradually decline. This pattern aligns with the life-cycle dynamics of $k/y$ for top firms, suggesting, through the lens of the theory, that non-homotheticities may play a significant role for firms that reach the top of the size distribution, while potentially being much less important for bottom firms.

Now we turn to the labor share. In the model, the firm-level labor share is

$$\frac{wl_{it}}{y_{it}} = \theta^h (1-\alpha_h) \underbrace{\left[(1-\alpha_h) + \alpha_h \left(\frac{R+\mu_{it}}{w}\right)^{1-\sigma}\right]^{-1}}_{\text{input complementarities}} \underbrace{\frac{1}{\left(1-\kappa_l^h/l_{it}\right)}}_{\text{L non-homotheticities}} . \tag{10}$$

Analogous to the $k/y$ ratio, there is a fixed component that depends on the shape of the production function, $(1-\alpha_h)$ and $\theta^h$, which remains constant over time. Although labor is not directly distorted by financial frictions, it is indirectly affected by them if the input

aggregator is not Cobb-Douglas ($\sigma \neq 1$). Moreover, the labor share can also vary if there are labor input non-homotheticities ($\kappa_l^h > 0$).

Next, we discuss how financial frictions can distort the labor share. Assuming that financial frictions become less relevant as firms grow and $\mu_{it}$ decreases, the change in the labor share will depend crucially on the parameter $\sigma$, which determines the degree of complementarity and substitution between capital and labor. A sensible assumption is that as firms age and grow, they become less financially constrained, reducing $\mu_{it}$. In this case, if $\sigma > 1$, the labor share would decrease as labor is substituted away in favor of capital, which becomes relatively cheaper. In contrast, if $\sigma < 1$, the labor share would increase, as capital and labor are complements. A third possibility is that $\sigma \approx 1$, in which case the labor share remains roughly unchanged. This last possibility is consistent with the dynamics of the labor share of bottom firms, which experience an increase in $k/y$ over time while maintaining a stable labor share. On the other hand, for top firms, labor and capital must be substitutes to match the decline in the labor share; however, this would counterfactually imply an increase in $k/y$.

There is an additional term affecting the labor share, which depends on the necessary level of labor required for production, $\kappa_l^h$. This term implies that if firms start with employment levels close to $\kappa_l^h$ they will have initially a large labor share, which over time will decline as firms grow. This pattern is consistent with the labor share trajectories we observe for top firms, but not for bottom firms. Similar to the capital-output ratio, the model suggests that a stylized explanation for the labor share decline among top firms is the presence of input non-homotheticities in high-growth technology types. In addition, we observe that $k/wl$ declines sharply for top firms, suggesting that capital non-homotheticities are larger than labor non-homotheticities.[23]

Finally, the profit share is

$$\frac{\pi_{it}}{y_{it}} = 1 - \underbrace{\left(\frac{wl_{it}}{y_{it}} + R\frac{k_{it}}{y_{it}}\right)}_{\text{inputs usage}} = 1 - \underbrace{\left(\frac{w\tilde{l}_{it}}{y_{it}} + R\frac{\tilde{l}_{it}}{y_{it}}\right)}_{\text{inputs usage net of non-h}} - \underbrace{\left(\frac{w\kappa_l^h}{y_{it}} + R\frac{\kappa_k^h}{y_{it}}\right)}_{\text{input-specific fixed costs}} \tag{11}$$

which shows that this share is determined by the input usage, i.e., the sum of the labor share and capital share. For example, if input usage is decreasing over the life-cycle, the profits path over the life cycle is going to be backloaded, as we observe for top firms. Importantly, non-homotheticities appear as input-specific fixed costs become less important as firms grow. The input usage costs not related to non-homotheticities may vary over time if distortions, such as financial frictions, change over the life cycle.

[23]The capital-to-labor costs ratio is $\frac{k_{it}}{wl_{it}} = \frac{\alpha_h}{1-\alpha_h}\left(R+\mu_{it}\right)^{-\sigma} w^{\sigma-1} \frac{\left(1-\kappa_l^h/l_{it}\right)}{\left(1-\kappa_k^h/k_{it}\right)}$.

Overall, increasing capital usage combined with a flat labor share and a mildly decreasing profit share—reflecting financial frictions that limit capital accumulation in the initial years and a capital-labor elasticity of substitution of approximately $\sigma \approx 1$—is consistent with the life cycle of bottom firms. For top firms, on the other hand, decreasing input usage and backloaded profits over the life cycle are driven by high growth and non-homotheticities, especially in capital.[24] A key implication is that when non-homotheticities are large, input usage is high and profits are low in the early years; as firms grow, these non-homotheticities become less important and profitability rises. Financing in the early years can therefore play a critical role, particularly financing based on the future profitability of the firm. Quantifying this mechanism is one of our main exercises below.

## 5.2 Alternative Theories of Top Firms' Declining Input Usage

We now discuss alternative explanations for the life-cycle pattern of input usage among top firms. In Appendix B.3, we extend the static firm problem by allowing firms' output prices and wages to be elastic to their output and labor choices. Changes in these elasticities can capture, for example, changes in output and labor market power as firms age.[25] Furthermore, we allow for TFP growth to be labor and capital augmented. As we focus on explanations beyond non-homotheticities for the top firms' decline in input usage, we assume that technology is homothetic and there are no financial frictions for these cases. Appendix B.3 analytically derives the input-to-output ratios for these extensions of the model. Table 2 summarizes the main results regarding input usage over firms' life cycle in the data, the baseline model, and under these alternative theories.

First, increasing markups as firms grow can explain the decline in $k/y$ and $wl/y$. Nevertheless, variable markups imply a proportional change in both input-to-output ratios while maintaining the input mix unchanged. This prediction is at odds with the data where $k/wl$ significantly declines. Second, an increase in the shadow cost of labor as firms grow, as in Mertens and Schoefer (2024), can explain the decline in the labor share. The capital-output ratio will also fall if capital and labor are complements ($\sigma < 1$). However, such complementarities would imply a smaller decrease in $k/y$ relative to $wl/y$ (see (24) in the Appendix), which is once again inconsistent with top firms' life-cycle patterns. More generally, any combination of changes in labor and product market power over the life cycle of firms would be unable to account for the observed decline in $k/wl$.

[24] The non-homothetic technology explanation for the life-cycle input usage of top firms does not depend on the elasticity of substitution between capital and labor, a value that remains highly contested (see, for example, Karabarbounis and Neiman (2013) and Oberfield and Raval (2021)).

[25] Changing elasticities can also reflect variations in input adjustment costs over time.

Lastly, regarding input-augmenting TFP growth, equations (22) and (23) in the Appendix show that the capital-output and labor share depend on the capital-to-labor augmented TFP ratio only if $\sigma \neq 1$. Regardless of the value of $\sigma$, capital-biased growth will cause the labor share and the capital-output ratio to move in opposite directions. Therefore, changes in capital- and labor-augmented TFP cannot account for the simultaneous decline in the capital-output ratio and the labor share of top firms.

In sum, as Table 2 shows, it is challenging for these alternative theories to explain the decline in top firms' input usage over their life cycle. We conclude that non-homotheticities in inputs combined with high growth offer a parsimonious explanation for the joint declines in $k/y$ and $wl/y$ among top firms, without relying on a convoluted combination of factors or taking a stance on whether capital and labor are complements or substitutes. Having established that our model can account for these patterns, we now turn to its quantitative implications.

**Table 2:** Input Usage Over Firms' Life Cycle in Data, Model, and Alternative Theories

| | | Life Cycle $\Delta$ | | |
|---|---|---|---|---|
| | $\sigma$ | $k/y$ | $wl/y$ | $k/wl$ |
| *Data* | | | | |
| Top 1 percent firms | | $(-)$ | $(-)$ | $(-)$ |
| Bottom 99 percent firms | | $(+)$ | $(\approx)$ | $(+)$ |
| *Baseline Model* | | | | |
| High growth + non-homotheticity* | any value | $(-)$ | $(-)$ | $(-)$ |
| $K$ shadow costs decrease | $\sigma \approx 1$ | $(+)$ | $(\approx)$ | $(+)$ |
| *Alternative Theories for Top Firms* | | | | |
| Markups increase | any value | $(-)$ | $(-)$ | $(=)$ |
| $L$ shadow costs increase | $\sigma < 1$ | $(-)$ | $(-)$ | $(+)$ |
| $K$-biased growth | $[\sigma < 1, \sigma > 1]$ | $[(-),(+)]$ | $[(+),(-)]$ | $[(-),(+)]$ |

*Notes*: $(+)$ implies that the variable increases over firms' life cycle, $(-)$ decreases, and $(=)$ denotes no change. *Data* refers to the evidence documented in Section 3, *Model* to the baseline model implications, and *Alternative Theories for Top Firms* to the alternative hypotheses discussed in the text for the top firms input composition. *For $k/wl$ to decline, $\kappa_k$ must be large relative to $\kappa_l$.

# 6 Parameterization

This section describes our calibration strategy. First, we assign some parameters to standard values in the literature. Second, we calibrate 27 parameters, some common and

some technology $h$-specific, so the model is consistent with 92 moments from the data. We calibrate the model to Spain, targeting standard firm dynamics moments and the key novel facts of top and bottom firm dynamics documented in Section 3.

## 6.1 Assigned Parameters

We set the interest rate $r = 0.02$, and firms' subjective discount factor to $\beta = 1/(1.04)$. For the capital depreciation rate, we use a value of $\delta = 0.05$. For simplicity, we assume $H = 2$. Hence, there are two types of technologies: $h$=1 denotes the high-growth technologies, and $h$=2 the bottom, modest-growth technologies. We set the exogenous probability of firm exit by technology type to $(\eta^1, \eta^2) = (0.04, 0.06)$ such that, beyond endogenous exit, the technological composition improves as firms age. Following Clementi and Palazzo (2016), we assume that the aggregate labor supply function is given by $L^s(w) = \bar{L}w^\gamma$, with wage elasticity $\gamma = 2$. We calibrate $\bar{L}$ so that $w = 1$ in the initial steady state.

## 6.2 Calibrated Parameters and Model Fit

We calibrate the remaining parameters so that the model replicates key cross-sectional and longitudinal moments and those capturing top and bottom firm dynamics. Table 3 presents the 92 moments used in the calibration. Table 4 reports the 26 calibrated parameter values. We first describe the targeted moments, second explain our calibration strategy, and third describe the model fit.

**Targeted Moments** Panel (a) of Table 3 includes the more standard, cross-sectional, and longitudinal firm dynamics moments we target in our calibration exercise. These moments include cross-sectional exit rates by firm age and size, the total output share of the top 1 and top 10 percent of firms, the average and standard deviation of output growth, and the autocorrelation of log output at different horizons. Panel (b) reports the key novel moments on top and bottom firm dynamics which we directly target. We compute these moments in the model-simulated data using the same strategy as in the empirical section.[26] The first set of moments includes the initial level and growth of output, the capital-output ratio, labor share, and profit share over the life cycle of bottom and top firms at age 20, which we documented in Section 3. The second set of moments relates to firms' debt and equity financing. We target the average leverage of top and bottom firms and their equity injections relative to capital at ages 0-4 and 19-20.

[26] For example, we use the same definition of top 1 and bottom 99 firms percent firms at age 20 as in the data. Thus, while the top and bottom classifications are highly correlated with the $h = 1$ and $h = 2$ technologies, the mapping is not direct due to compositional changes in the size distribution.

**Table 3:** Targeted Moments

| | Data | Model |
|---|---|---|
| *(a) Cross-sectional and Longitudinal Moments* | | |
| Exit rate, all | 0.08 | 0.08 |
| Exit rate, Top-1 at age 0-2 | 0.08 | 0.05 |
| Exit rate, Top-1 at age 19-20 | 0.03 | 0.04 |
| Exit rate, Bot-99 at age 0-2 | 0.12 | 0.13 |
| Exit rate, Bot-99 at age 19-20 | 0.06 | 0.06 |
| Output share, Top-1 | 0.49 | 0.64 |
| Output share, Top-10 | 0.75 | 0.71 |
| Log output growth, Mean | 0.06 | 0.05 |
| Log output growth, Std. Dev. | 0.18 | 0.16 |
| Log output, Autocorr. 1-year | 0.91 | 0.88 |
| Log output, Autocorr. 3-year | 0.84 | 0.81 |
| Log output, Autocorr. 5-year | 0.79 | 0.76 |
| *(b) Life Cycle of Top and Bottom Firms* | | |
| $y/\bar{y}_{0\text{-}4}$, Top-1 and Bot-99 at age 20 | Figure 9a | |
| $k/y$, Top-1 and Bot-99 at age 20 | Figure 9b | |
| $wl/y$, Top-1 and Bot-99 at age 20 | Figure 9c | |
| $\pi/y$, Top-1 and Bot-99 at age 20 | Figure 9d | |
| $b/k$ avg., Top-1 at age 20 | 0.42 | 0.48 |
| $b/k$ avg., Bot-99 at age 20 | 0.33 | 0.32 |
| $\mathbb{1}(\tilde{d}<0)\lvert\tilde{d}\rvert/k$ age 0-4, Top-1 at age 20 | 0.05 | 0.05 |
| $\mathbb{1}(\tilde{d}<0)\lvert\tilde{d}\rvert/k$ age 19-20, Top-1 at age 20 | 0.00 | 0.00 |
| $\mathbb{1}(\tilde{d}<0)\lvert\tilde{d}\rvert/k$ age 0-4, Bot-99 at age 20 | 0.02 | 0.03 |
| $\mathbb{1}(\tilde{d}<0)\lvert\tilde{d}\rvert/k$ age 19-20, Bot-99 at age 20 | 0.01 | 0.00 |

*Notes:* Data moments from Spain. Model moments are computed from model simulated data following the same empirical strategy as in Section 3. Top-x and Bot-99 denote the top x and bottom 99 percent of the firm size distribution, respectively. Unless an age group is specified, firms are ranked across the entire distribution. $y/\bar{y}_{0\text{-}2}$ is output relative to average entrant. $k/y$ is the capital-output ratio. $wl/y$ is the labor share. $\pi/y$ is the profit share. $b/k$ is leverage, where debt in the model is $b = k - a$. $\mathbb{1}(\tilde{d}<0)|\tilde{d}|/k$ denotes equity injections relative to capital and is equal to zero if the firm pays dividends ($\tilde{d} > 0$).

**Calibrated Parameters** Table 4 reports the calibrated parameters. We classify the parameters into four groups: productivity, technology, entrants, and finance. We make some initial assumptions to reduce the dimension of the parameter space. First, without loss of generality, we assume that the bottom technologies' initial ex-ante TFP equals $\underline{\mu}_u^2 = 0$. Second, as we analytically show in Section 5, our model requires an elasticity of substitution between capital and labor close to unity, so we set $\sigma = 1$ for both types of technologies. Third, given the life cycle trajectory of the capital-output ratio and the labor share for the bottom 99, we set the non-homothetic input costs of $h$=2 technologies to zero: $(\kappa_k^2, \kappa_l^2) = \mathbf{0}$.

Lastly, without loss of generality, we fix the mass of potential entrants every period to $\mathcal{E} = 1$, and hence we have that $\mathcal{E}_2 = 1 - \mathcal{E}_1$.

After the previous assumptions, we are left with 21 parameters. We find the remaining parameters in a three-step routine to minimize the distance between the models in the data and the model. In the first step, we use a minimization algorithm to jointly find 14 parameters governing the productivity process $(\overline{\mu}_u^1, \overline{\mu}_u^2, \underline{\mu}_u^1, \rho_u^1, \rho_u^2, \underline{\sigma}_u, \rho_z, \sigma_z)$, the non-homotheticities of top firms $(\kappa_k^1, \kappa_l^1)$, and firms access to finance, $(\lambda_a, \lambda_v, \chi_0, \chi_1)$. For each combination of these 14 parameters, in the second step, we set four technology parameters $\{(\theta^h, \alpha^h)\}_{h=1}^2$ to directly match four moments: the capital-output ratio and labor share of top and bottom firms at age 19-20. We target these moments using the expressions in (9) and (10), where we allow for some dampening to account for the potentially higher capital costs and the presence of the capital and labor non-homothetic terms. Lastly, the third step chooses the three parameters governing the distribution of potential entrants, $(\mathcal{E}_1, \iota_0^1, \iota_0^2)$, to minimize the distance between the targeted moments in data and model.

**Table 4:** Calibrated Parameters

| | | $h=1$ | $h=2$ | Description |
|---|---|---|---|---|
| *Productivity* | $\overline{\mu}_u^h - \underline{\mu}_u^h$ | 1.622 | 0.085 | Ex-ante TFP growth $u$ |
| | $\underline{\mu}_u^h$ | 0.399 | 0 | Initial ex-ante TFP $u$ |
| | $\rho_u^h$ | 0.970 | 0.976 | Persistence, ex-ante TFP $u$ |
| | $\underline{\sigma}_u$ | 0.031 | | Std. Deviation, initial TFP $u$ |
| | $\rho_z$ | 0.784 | | Persistence, ex-post shock $z$ |
| | $\sigma_z$ | 0.029 | | Std. Deviation, ex-post shock $z$ |
| *Technology* | $\sigma$ | 1 | | CES capital-labor |
| | $\theta^h$ | 0.758 | 0.920 | DRS |
| | $\alpha^h$ | 0.245 | 0.250 | Capital share |
| | $\kappa_k^h$ | 15.51 | 0 | Capital non-homotheticity |
| | $\kappa_l^h$ | 0.654 | 0 | Labor non-homotheticity |
| *Entrants* | $\mathcal{E}_h$ | 0.016 | 1-$\mathcal{E}_1$ | Share of $h$-type entrants |
| | $\iota_0^h$ | 0.200 | 0.216 | Asset distribution at entry |
| *Finance* | $\lambda_a$ | 1.435 | | Collateral constraint |
| | $\lambda_v$ | 0.040 | | Forward-looking constraint |
| | $\chi_0$ | 0.721 | | Equity cost, fixed |
| | $\chi_1$ | 0.070 | | Equity cost, linear |

Given our model's characteristics, it is not possible to directly match all parameters to specific moments. Yet, in what follows, we briefly describe which moments are most informative for each group of parameters. First, within the productivity parameters, those governing

the ex-ante TFP process $(\{\overline{\mu}_u^h, \underline{\mu}_u^h, \rho_u^h\}_{h=1}^2, \underline{\sigma}_u)$ are primarily informed by the dynamics of output over the life cycle of top and bottom firms (Figure 9a). Notably, the initial differences in firm size between top and bottom firms are crucial to identifying the importance of ex-ante heterogeneity across firms.[27] The parameters for the ex-post shocks, $(\rho_z, \sigma_z)$, are mainly determined by output cross-sectional and longitudinal moments, such as the average and dispersion of output growth and the autocorrelations of log output (Panel (a), Table 3).

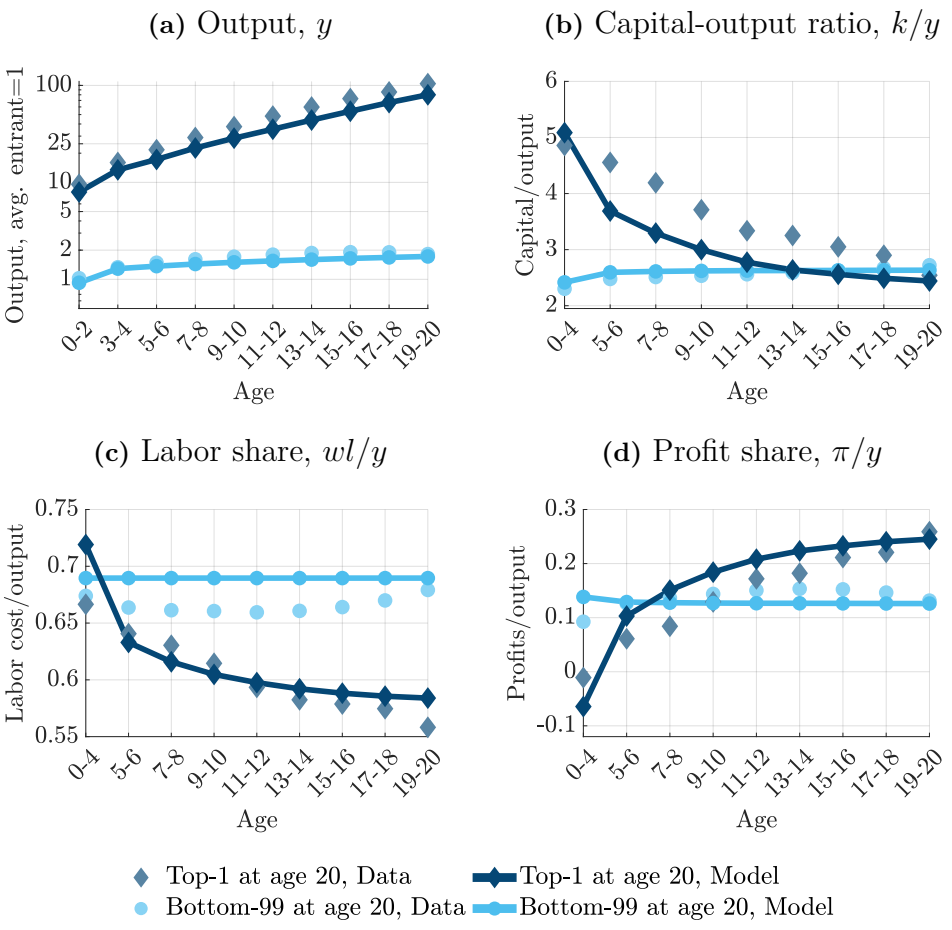
The technology parameters are mostly informed by our novel life cycle facts on top and bottom firms' input usage. As described above, the parameters governing the degree of DRS and the capital share, $\{(\theta^h, \alpha^h)\}_{h=1}^2$, are directly chosen to match four moments of top and bottom firms input usage at age 19-20. The non-homothetic terms for the top technologies, $(\kappa_k^1, \kappa_l^1)$, are mostly informed by the life cycle path of the capital-output ratio (Figure 9b), labor share (Figure 9c), and profit share (Figure 9d) of the top 1 percent of firms at age 20. The parameters governing the distribution of entrants, $(\mathcal{E}_1, \iota_0^1, \iota_0^2)$, are informed by both cross-sectional moments on the size distribution, such as the top 1 percent share of output, as well moments directly related to entrants exit rates and initial growth (Panel (a), Table 3). Lastly, the parameters governing the degree of financial frictions in the model, $(\lambda_a, \lambda_v, \chi_0, \chi_1)$, are pinned down by the moments on the use of debt and equity financing of top and bottom firms (Panel (b), Table 3).

**Model Fit** Table 3 shows that our model closely aligns with the observed cross-sectional and longitudinal moments and the life cycle trajectories of top and bottom firms as Figure 9 illustrates. While the dynamics of top firms in our model are somewhat faster, the panels in this figure show that our model correctly matches the trajectories of top and bottom firms' output, input usage, and profits.

What explains these patterns? Figure B.2 in the appendix illustrates the key factors driving the life-cycle profiles of top and bottom firms in the model. In the absence of input non-homotheticities, top firms' capital-output ratio (Figure B.2a) and labor share (Figure B.2b) would be mostly flat over their life cycle, which would be at odds with the data. On the other hand, in the absence of financing frictions, the capital-output ratio of bottom firms would be flat over their life cycle (Figure B.2c). In contrast, it is increasing in the data and the baseline model. Finally, financing frictions do not affect the labor share for bottom firms (Figure B.2d) because the capital-labor substitution, $\sigma$, is equal to 1. Thus, consistent with our qualitative analysis in Section 5, the quantitative model requires that top firms' technologies be non-homothetic and that capital usage be initially low due to financing

[27] In Appendix B.5, we provide further intuition on how the initial level and growth profile of top and bottom firms inform the ex-ante heterogeneity.

**Figure 9:** Life Cycle of Top and Bottom Firms in Data and Model




*Notes*: Life cycle trajectories for the inputs and profit shares of the top 1 and bottom 99 percent firms at age 20 estimated using (1). Model moments are computed from model simulated data. The regression for the capital-output ratio is in logs, while the labor and profit shares are in levels. Results are scaled by adding the average of the omitted group (age 19-20) for the top and bottom firms.

frictions, to match the life-cycle profiles of input usage across top and bottom firms.

# 7 Macroeconomic Implications

This section examines the macroeconomic implications of top and bottom firm dynamics. First, we study the role of financial frictions for aggregate outcomes and the importance of accounting for the empirical patterns documented above. Next, we explore aggregate implications beyond finance, including the long-run effects of interest rate changes, growth through expanding the technological frontier, and size-dependent corporate taxation.

## 7.1 The Role of Top Firms Financing

We perform several experiments to assess the role of finance for top firm growth, focusing on two dimensions: its interaction with non-homotheticities, and whether forward-looking financing is especially important for these firms.

### 7.1.1 Non-Homotheticities and Finance

In our quantification, we show that non-homotheticities in capital and labor are crucial for capturing the input usage of top firms at their origins and over their subsequent life-cycle trajectories. Importantly, due to imperfect credit access, some of these firms may be unable to finance these costs early in their life cycle if they are large enough. In this section, we evaluate the magnitude of these costs and how they interact with financing frictions.

We begin by measuring *directly* how large the fixed costs of top technologies, $\{\kappa_k^h, \kappa_l^h\}$, are. We express them as a percentage of total output for all top-technology firms: $\left\{ \frac{R\kappa_k^{h_{\text{top}}}}{\int_{i\in h_{\text{top}}} y_i \, \mathrm{d}\Omega^{h_{\text{top}}}}, \frac{w\kappa_l^{h_{\text{top}}}}{\int_{i\in h_{\text{top}}} y_i \, \mathrm{d}\Omega^{h_{\text{top}}}} \right\}$, where $\Omega^{h_{\text{top}}}$ is the distribution of firms conditional on being of the top $h$-type ($h = 1$ in our calibration). We also compute the same ratios conditional on being young (ages 0 to 4): $\left\{ \frac{R\kappa_k^{h_{\text{top}}}}{\int_{i\in h_{\text{top}}} y_i \, \mathrm{d}\Omega^{h_{\text{top}}|\text{age 0–4}}}, \frac{w\kappa_l^{h_{\text{top}}}}{\int_{i\in h_{\text{top}}} y_i \, \mathrm{d}\Omega^{h_{\text{top}}|\text{age 0–4}}} \right\}$, where $\Omega^{h_{\text{top}}|\text{age 0–4}}$ is the distribution of firms conditional on being of the top $h$-type and of age 0 to 4. We find that in the aggregate these costs are negligible for top technologies (0.5% and 0.3% of output for capital and labor, respectively); however, they are very large early in the life cycle (20.7% and 12.5% of output for capital and labor, respectively). Thus, non-homotheticities are relevant for top firms in their early stages but become less important as they grow, accounting for only a modest share of their aggregate output.

Alternatively, we can assess the relevance of non-homotheticities *indirectly* by setting $\{\kappa_k^h\}_{h\in\mathcal{H}} = \mathbf{0}$ and $\{\kappa_k^h, \kappa_l^h\}_{h\in\mathcal{H}} = \mathbf{0}$. Table 5, Panel (b), presents the results. When shutting down only capital fixed costs, we observe a remarkable increase in aggregate output of 68%. Although capital fixed costs are negligible and initially affect only a small subset of firms (i.e., measure $\mathcal{E}_1 = 0.016$), they prevent many top technologies from being financed, leading to their early exit and deterring their entry, which compounds into substantial aggregate effects. Indeed, the table shows that when non-homotheticities are absent, the measure of incumbent top technologies ($h = 1$) increases threefold. In general equilibrium, wages rise, inducing a significant share of bottom technologies ($h = 2$) to exit and deter their entry. Lastly, we find that removing labor non-homotheticities, in addition to those in capital, does not lead to significant aggregate consequences.

Importantly, in Table 5, Panel (b), third column, we do the same experiment in an economy with perfect credit (i.e., no collateral constraint). In this case, removing non-homotheticities increases aggregate output by only 1%, mostly due to deadweight losses. It is therefore the interaction between non-homotheticities and financial frictions that generates these very large effects on aggregate output: by precluding the financing of fixed costs in the early stages of the life cycle of top firms, financial frictions compound into substantial aggregate losses.

**Table 5:** Non-Homotheticities and Finance

*Panel (a): Non-homothetic costs as a share of output*

| | Capital ($\kappa_k$) | Labor ($\kappa_l$) |
|---|---|---|
| Top-$h$ ($h = 1$), all ages | 0.005 | 0.003 |
| Top-$h$ ($h = 1$), age 0–4 | 0.207 | 0.125 |

*Panel (b): Aggregate implications of removing non-homothetic costs*

| | Relative to model with non-h costs (= 1) | | |
|---|---|---|---|
| | Baseline | | Perfect credit |
| | $\{\kappa_k^h\}_h = \mathbf{0}$ | $\{\kappa_k^h, \kappa_l^h\}_h = \mathbf{0}$ | $\{\kappa_k^h, \kappa_l^h\}_h = \mathbf{0}$ |
| Output, $Y$ | 1.68 | 1.69 | 1.01 |
| Capital, $K$ | 1.55 | 1.56 | 0.97 |
| Labor, $L$ | 1.36 | 1.35 | 0.97 |
| Wage, $w$ | 1.16 | 1.16 | 1.00 |
| Incumbents, Top-$h$ | 3.61 | 3.61 | 1.00 |
| Incumbents, Bot-$h$ | 0.46 | 0.46 | 1.00 |

*Notes*: Panel (a) quantifies the direct aggregate importance of non-homotheticities based on the size of input-specific fixed costs relative to aggregate output for top firms and top firms at ages 0 to 4. Panel (b) assesses the indirect importance of non-homotheticities, along with several other aggregate outcomes, by removing non-homotheticities from the baseline model (columns 1 and 2) and from the model with perfect credit (column 3). In Panel (b), all aggregate values are normalized to the model with non-homotheticities.

### 7.1.2 Losses from Financial Frictions

The previous exercises suggest that the losses from financing frictions are sizable, through their interactions with non-homothetic input costs—especially in capital—that are large and difficult to finance early in the life cycle of top firms. To measure these losses more directly, we conduct complementary experiments that remove financing frictions and examine their implications for aggregate outcomes. We do so for all firms, but also separately for top and bottom technologies, to analyze which are most affected by financing frictions.

Table 6 shows that removing financing frictions for all firms increases aggregate output by 90 percent and more than doubles the capital stock, indicating significant losses from financing frictions. These losses are mostly driven by changes in the measure of firms, especially top technologies. The losses are sizable, especially given that Spain (the calibrated economy) is a relatively high-income country.

Interestingly, most of the losses stem from financing frictions distorting top technologies. Removing financial frictions only for top technologies increases output by 83 percent, whereas

**Table 6:** Removing Financial Frictions

| | Relative to Baseline (= 1) | | |
|---|---|---|---|
| | All | Top | Bottom |
| Output, $Y$ | 1.90 | 1.83 | 1.12 |
| Capital, $K$ | 2.24 | 2.11 | 1.35 |
| Labor, $L$ | 1.49 | 1.43 | 1.11 |
| Wage, $w$ | 1.22 | 1.20 | 1.05 |
| Incumbents, Top-$h$ | 3.61 | 3.61 | 0.82 |
| Incumbents, Bot-$h$ | 1.35 | 0.45 | 1.35 |

*Notes*: All values are normalized to the baseline model. In the Top exercise, we remove financing frictions only for top technologies, while in the Bottom exercise, we remove them for bottom technologies only.

removing them only for bottom technologies increases output by just 12 percent. These losses are largely driven by top firms being unable to meet their financing needs early in their life cycle—namely, the non-homothetic input costs—which precludes their entry and precipitates their early exit. Even though these firms are few (1.6% of potential entrants each period), due to their very high growth potential, they have a substantial aggregate impact.

Overall, consistent with the previous exercise, financing frictions play a very important role for top technologies and can create large costs even in relatively advanced economies by distorting their entry and exit. The losses for bottom firms, by contrast, are more aligned with the standard modest losses from financing frictions when accounting for average firm growth dynamics (see, e.g., Midrigan and Xu (2014)).

### 7.1.3 Gains from (Forward-Looking) Finance

While the previous two experiments focus on the losses from financing frictions, in this section we study the gains from financing, in particular forward-looking financing. To this end, we compare the aggregate losses from removing different sources of financing in our model with top and bottom firm dynamics to those in an alternative economy calibrated to match the life cycle of the average firm.

In the model, firms can finance production through retained earnings (savings) and external funds, either through debt or equity. Debt financing consists of a standard backward-looking component based on the firm's assets and a forward-looking component, which depends on current and expected profits. Equity financing is inherently forward-looking, as it depends on the firm's value. We quantify the importance of forward-looking debt and equity and backward-looking debt by shutting down each of these sources of financing at a time. We also quantify the overall importance of finance by comparing the baseline economy to a

counterfactual of financial autarky.

**Table 7:** Aggregate Implications of Forward-Looking Financing

| | Top-Bot Firms Model | | | Average Firm Model | | |
|---|---|---|---|---|---|---|
| | No FL | No BL | Autarky | No FL | No BL | Autarky |
| *Relative to Baseline (=1)* | | | | | | |
| Output, $Y$ | 0.81 | 0.90 | 0.69 | 1.02 | 0.93 | 0.92 |
| Capital, $K$ | 0.82 | 0.84 | 0.66 | 0.99 | 0.86 | 0.81 |
| Labor, $L$ | 0.89 | 0.94 | 0.82 | 1.01 | 0.95 | 0.95 |
| Profits, $\Pi$ | 0.71 | 0.90 | 0.57 | 1.06 | 1.01 | 1.05 |
| Wage, $w$ | 0.94 | 0.97 | 0.91 | 1.01 | 0.97 | 0.97 |
| Leverage, $B/K$ | 0.84 | 0.24 | 0.00 | 0.93 | 0.12 | 0.00 |
| Equity, $X/K$ | 0.00 | 1.56 | 0.00 | 0.00 | 1.33 | 0.00 |
| Incumbents, $h = 1$ | 0.50 | 0.79 | 0.27 | - | - | - |
| Incumbents, $h = 2$ | 1.01 | 1.08 | 1.01 | 1.26 | 0.95 | 1.18 |

*Notes*: "Top-Bot Firms Model" refers our baseline model that targeting the life cycle of top and bottom firms. "Average Firm Model" corresponds to the model where we target the average firm life cycle. "No FL" refers to the counterfactual where neither type of forward looking financing, equity and debt, is available ($\chi_0 \to \infty$ and $\lambda_v = 0$). "No BL" denotes the case with no backward-looking debt ($\lambda_a = 1$), and hence firms cannot borrow against their assets. "Autarky" is the counterfactual with no financing ($\chi_0 \to \infty$, $\lambda_v = 0$, and $\lambda_a = 1$). All values are normalized relative to the baseline model.

Table 7 summarizes the results from this exercise in our top and bottom firms model and the alternative average firm model. The results are steady-state comparisons relative to each model baseline calibration. The first and fourth columns show the results from shutting down debt and equity forward-looking financing (No FL), which we obtain by setting $\chi_0 \to \infty$ and $\lambda_v = 0$. We find large reductions in aggregate output, of 19%, and significant changes in the composition of firms absent forward-looking financing in our top and bottom firms model. Since top technologies ($h = 1$) have backloaded profits, they rely much more on this type of financing and hence removing this source of funds reduces the measure of top incumbents by 50%. There is little change in bottom technology incumbents, which rely much less on forward-looking financing and benefit from lower wages in general equilibrium. In contrast, in the average firm model, removing forward-looking financing has almost zero effects, and if anything, aggregate output increases by 2%. These starkly different results highlight the importance of forward-looking access to finance for top firm dynamics, characterized by high-growth and backloaded returns.

The second and fifth columns of Table 7 present the results for the counterfactual in which firms cannot borrow against their assets (No BL), corresponding to $\lambda_a = 1$. Without backward-looking financing, aggregate output is 10% and 7% lower in our top and bottom firms model and the average firm economy, respectively. Hence, the output losses are much

more comparable across models and are significant despite firms using more equity financing to compensate for the decline in debt. There are also substantial compositional changes in our top and bottom firms model in the absence of backward-looking debt, with the measure of top technologies falling by 21%. In contrast, bottom technologies rise by 8% due to lower wages. Yet, it is worth noting that the losses in the counterfactual without forward-looking financing are almost twice as large as those in the counterfactual without backward-looking debt in our top and bottom firms model.

Finally, we assess the overall gains of finance for the macroeconomy by analyzing the output losses in a financial autarky counterfactual where we shut down all sources of external funds (case with $\chi_0 \to \infty$, $\lambda_v = 0$, and $\lambda_a = 1$). Columns three and six of Table 7 (Autarky) present these results. Our model with top and bottom firms predicts sizable losses of 31% in the counterfactual without financing. These considerable losses are driven both by a large decline in aggregate capital and by significant compositional changes among incumbent firms, with 73% fewer top technologies in equilibrium. In contrast, the average firm model predicts much smaller losses, of only 8%, under financial autarky. Hence, the losses from a complete lack of financing are almost four times larger in our top and bottom firms model than in the alternative average firm economy. These results highlight the importance of taking into account top-firm dynamics when quantifying the role of finance for the macroeconomy.

In sum, our results show that forward-looking debt and equity play an essential role in the financing of top firms. Intuitively, with forward-looking financing, top firms' input decisions are close to being undistorted despite having large initial investments due to non-homotheticities, as they can borrow against their high future profits. However, when this source of financing is unavailable, their low initial profits and high input requirements hinder their growth, deter entry, and increase their exit, significantly reducing their prevalence in the economy, which results in considerable aggregate output losses. Forward-looking financing plays a much more muted role in the counterfactual average firm model that ignores the origins and life cycle trajectories of top firms.

## 7.2 Additional Macroeconomic Implications

In this section, we explore other relevant macroeconomic implications of our model.

### 7.2.1 Long-Run Consequences of Interest Rate Changes

Motivated by secular changes in the risk-free rate over the past decades (Bernanke, 2005; Summers, 2014), we study the long-run aggregate implications of real interest rate increases in an economy with top firms, comparing our baseline model to the alternative calibrated to

average firm life-cycle dynamics.[28]

The first column of Table B.1 presents the results when $r$ increases by 1.5 percentage points, holding the discount factor $\beta$ fixed. Aggregate output drops by 3.2% in our model, relative to only 1.0% in the average firm model. With $\beta$ fixed, a higher interest rate operates through two channels: it raises the cost of capital ($R = r + \delta$), reducing output, but it also increases the return on savings, strengthening self-financing incentives and reducing misallocation. In the average firm model these forces nearly offset, yielding a contraction of around 1%. In the top-bottom model, however, the contraction is two to three times larger, because the higher cost of capital effectively raises the capital-specific fixed cost for top technologies ($R\kappa_k^h \uparrow$), disproportionately discouraging their entry.

An increase in $r$ may also pass through to the discount factor $\beta$—for example, through foreign ownership of domestic firms. The third column of Table B.1 reports results when we additionally increase the implicit interest rate $r_\beta = 1/\beta - 1$ by 1.5 percentage points ($\beta \downarrow$). The output contraction is now sizable in both models, though larger in the top-bottom model (17.1% versus 14.4%). A lower discount factor reduces the incentive to self-finance and lowers firm values, limiting how much top firms can borrow against future profits. Consequently, the measures of both top and bottom firms decline.

Overall, accounting for top firms is important when assessing the long-run aggregate effects of interest rate changes, as these firms transmit shocks through distinct channels: forward-looking financing rather than self-financing, and input-specific fixed costs that amplify changes in the cost of capital.

### 7.2.2 Technological Frontier Changes

Next, we analyze how exogenous changes to the technological frontier—specifically, the distribution of potential entrant measures $\{\mathcal{E}_h\}_{h\in\mathcal{H}}$—affect aggregate outcomes. Holding the total measure of potential entrants $\mathcal{E}$ fixed, we vary the share that are top technologies ($\mathcal{E}_1$). A greater availability of top technologies can be linked, for example, to better diffusion of ideas, both of which have been argued to be key drivers of economic growth (Lucas and Moll, 2014; Perla and Tonetti, 2014).

Figure B.4a shows how aggregate output changes with the availability of top technologies in the economy, i.e., as $\mathcal{E}_1$ varies. As expected, greater availability of high-growth technologies translates into higher aggregate output, capital, labor, and profits. Due to the change in the technology mix, specifically a higher prevalence of top technologies, labor doesn't increase as

[28] While in a closed economy changes in $r$ can be associated, e.g., with changes in growth expectations, we interpret this as a shock exogenous to a small open economy.

much as output, which, in Figure B.4b, translates into a lower labor share as the availability of top technologies increases.[29] In line with this pattern, Figure B.4c, we observe that the profit share increases.

Finally, Figure B.4d shows that output concentration at the top increases with the availability of top technologies; however, this relationship is non-monotone. If top technologies become more common, up to a certain point, several of them will populate the bottom 99 percent of the distribution, limiting how much concentration can increase as they become more prevalent. Thus, if an economy grows because of the higher availability of top technologies, our model predicts increased concentration in economic activity, lower labor shares, and higher profit shares, even without firms gaining greater market power.

### 7.2.3 Top Firms' Taxation with Externalities

Finally, we explore how taxing or subsidizing the largest firms affects aggregate outcomes. Following the literature on size-dependent distortions (e.g., Guner, Ventura, and Xu (2008)), we consider a size-dependent corporate tax: firms with output $y_{it}$ above a threshold $\bar{y}$ pay a flat tax $\tau \in \mathbb{R}$ on their profits (if they are positive), $\tau\pi_{it}$, while firms below the threshold are not taxed. Firms above the threshold receive a subsidy if $\tau < 0$. We assume tax revenues, or costs, are transferred to the representative household.

To motivate the use of taxes or subsidies on top firms, we introduce two competing externalities in the economy. On the one hand, top firms can generate technological spillovers to other firms (parametrized by $\Gamma_1$), providing an incentive for subsidizing them. On the other hand, we allow for potential concentration externalities (parametrized by $\Gamma_2$), where increased concentration may reduce aggregate productivity (Aghion et al., 2005), providing a rationale for taxing them. Appendix B.7 provides further details on the setup with externalities.

The tax creates two types of distortions. First, a static size-dependent distortion arises as a mass of firms bunch their production levels around the threshold $\bar{y}$ (see Figure B.3). The intuition is that firms subject to the tax may find it optimal to reduce their output to stay below the threshold if doing so increases their profits. Second, dynamic distortions arise because the tax affects the value of firms that are already at the top or are likely to reach the top in the future (e.g., firms with top technologies), distorting entry, financing, and savings. We set $\bar{y}$ as the output level in the 99th percentile in the no-tax steady state.[30] We vary the tax rate $\tau \in [-0.4, 0.4]$ for different levels of spillovers and concentration externalities.

[29] As argued for the U.S. in recent decades by Autor et al. (2020), Kehrig and Vincent (2021), and Hubmer and Restrepo (2022), the decline in the aggregate labor share in our experiment is also driven by the reallocation of economic activity to top firms, rather than by an increase in market power.

[30] For simplicity, we keep it fixed across different values of $\tau$. Alternatively, allowing the threshold to vary with $\tau$ would make the tax scheme an additional equilibrium object.

Figure B.5 shows results for aggregate output, the labor share, and concentration. The gray line depicts the case with no externalities ($\Gamma_1 = \Gamma_2 = 0$). Here, the tax parameter $\tau$ is negatively related to output but has non-monotonic effects on the labor share and concentration. A subsidy ($\tau < 0$) encourages the entry of top technologies without generating the rise in concentration and labor share decline observed after a frontier shift (see Section 7.2.2), though output gains remain modest (less than 10%). On the other hand, a higher tax ($\tau > 0$) decreases output, raises the labor share, and significantly reduces concentration.

We then introduce the two competing externalities: top-technology spillovers ($\Gamma_1 = 0.4$, green line) and concentration externalities ($\Gamma_2 = 0.4$, red line).[31] Under strong spillovers, a subsidy can increase output by about 25%, while a high tax can collapse the economy by as much as 75%—accompanied, notably, by a *decline* in the labor share and a *rise* in concentration. Under strong concentration externalities, the opposite holds: taxing top firms can increase output, raise the labor share, and reduce concentration. These results highlight that the consequences of corporate taxes on top firms depends critically on the nature of the underlying externalities.

# 8 Conclusions

This paper asks how the origins and life cycles of top firms shape the aggregate implications of financial frictions. We answer this question through three contributions: documenting new empirical facts, developing a firm dynamics model that accounts for them, and conducting a quantitative assessment of their macroeconomic consequences.

Empirically, using longitudinal firm-level data, we uncover striking differences in life-cycle growth, input usage, and profitability across the firm size distribution. We find that future top firms are initially large and make sizable capital investments early in life. They also exhibit heavily backloaded profits, close to zero in the first few years, but strongly increasing over time. By contrast, firms in the bottom 99 percent start small and operate with little capital early on, which increases moderately over time, and exhibit positive but fairly flat profit shares over their life cycle. We also find that firms that reach the top rely much more on debt and equity financing, especially in their first years.

We show that a firm dynamics model with ex-ante heterogeneity in productivity and technology, fixed costs in inputs that generate non-homotheticities in production, and forward-looking financing can account for these empirical patterns. The central feature of our model is that firms with high growth potential face high input-specific fixed costs that are most

[31] An elasticity of $\Gamma_1 = 0.4$ implies that a 1% increase in top firms' aggregate productivity raises the common TFP component $P$ by 0.4%, in line with Ottonello and Winberry (2024).

significant early in their life cycle, making them strongly reliant on external financing. As these firms grow, such costs become small relative to their size, leading to declining input-to-output ratios and an increasing profit share. In contrast, low-growth-potential firms have lower fixed costs. Hence, they require small initial investments, feature fairly flat profit shares, and slowly increase their capital-output ratio over time.

Our quantitative results show that access to finance plays a central role in shaping the economy's technological composition and aggregate output because it enables high-growth potential firms to cover fixed costs and sustain heavily backloaded profits. Indeed, our model predicts that the aggregate costs of financial frictions are driven almost entirely by distortions affecting the entry and survival of high-growth potential firms. We further explore the macroeconomic implications of our top-firm dynamics model through changes in interest rates, shifts in the technological frontier, and size-dependent corporate taxation.

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# Online Appendix for

# The Origins of Top Firms

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## Table of Contents

**A Data Appendix** **1**

A.1 The Orbis Database . . . . . 1

A.2 Firms' Foundation Year . . . . . 4

A.3 Top Status Persistence . . . . . 7

A.4 Additional Figures and Tables . . . . . 8

A.5 Top Firms Pairs and Predictors . . . . . 17

**B Model Appendix** **20**

B.1 Nested CES Model . . . . . 20

B.2 Profit Maximization Problem . . . . . 21

B.3 Alternative Theory for Top Firms' Declining Input Shares . . . . . 23

B.4 Input Non-Homotheticity Microfoundations . . . . . 24

B.5 Ex-Ante Heterogeneity and Top and Bottom Firms Growth . . . . . 25

B.6 Input Usage Over The Life Cycle: Quantitative Model . . . . . 26

B.7 Top Firms' Taxation with Externalities . . . . . 26

B.8 Additional Figures and Tables . . . . . 29

# A Data Appendix

## A.1 The Orbis Database

This appendix provides definitions and additional results regarding the Orbis database. We primarily focus on Spain, the country we use for our baseline results.

### A.1.1 Variable Definitions and Measurement

**Firm Age** Following Gopinath et al. (2017), firm $i$'s age at time $t$ is defined as $age_{it} = t - \tau_{i0} + 1$, where $\tau_{i0}$ is the year of incorporation reported in Orbis. The plus one term accounts for incomplete reporting spells at entry. For some companies, the incorporation year in the data and the actual foundation year may differ because of changes in the company's legal name resulting from restructurings, mergers, and acquisitions (M&A). In addition, as we focus on unconsolidated accounts, there could be cases of subsidiaries of older parent companies that are not truly new firms. As explained in Appendix A.2 below, we address this issue by searching for the true foundation year of all the top 1 percent firms at age 20 in our sample using a large language model (LLM)-assisted search of publicly available information.

**Wage bill** Regarding firms' income statement variables, we measure firms' wage bill as

$$wl_{it} = \texttt{staf}_{it}$$

where $\texttt{staf}_{it}$ are total labor costs of firm $i$ at year $t$, using Orbis acronyms. For what follows, we follow BvD acronyms to refer to the variables in Orbis Historical.

**Output** We measure firms' output as value-added using a comprehensive measure of costs. In detail, we measure the output of firm $i$ at time $i$ as the sum of EBITDA (earnings before interest, taxes, depreciation, and amortization), acronym `ebta`, and labor costs

$$\begin{aligned} y_{it} &= \texttt{ebta}_{it} + \texttt{staf}_{it} \\ &= (\texttt{oppl}_{it} + \texttt{depr}_{it}) + \texttt{staf}_{it} \\ &= (\texttt{opre}_{it} - \texttt{cost}_{it} - \texttt{oope}_{it}) + \texttt{depr}_{it} + \texttt{staf}_{it} \\ &= \underbrace{\texttt{opre}_{it}}_{\text{Operating revenue}} - \underbrace{(\texttt{cost}_{it} - \texttt{staf}_{it} - \texttt{depr}_{it}) - \texttt{oope}_{it}}_{\text{Costs net of labor and capital}} \end{aligned} \tag{12}$$

where, as the last expression in (12) shows, it can be easily shown using the definitions of the variables in Orbis that our measure of output is equal to the sum of operating revenue (`opre`) minus a comprehensive measure of costs excluding those related to labor and capital: costs of goods sold (`cost`) net of labor costs (`staf`) and capital depreciation (`depr`) plus other operating expenses (`oope`).[32]

[32] This definition of output coincides to that used in Boar, Gorea, and Midrigan (2023), with the exception that we do not subtract for taxes.

**Capital, Equity, and Debt** It is important to note that capital and the rest of the balance sheet variables are reported at the end of each year. Hence, as in Hsieh and Klenow (2009), we measure the stock of the variable at $t$ as the average between the end-of-year values of $t-1$ and $t$. For our baseline results, we follow Kochen (2025) and measure capital as equity plus net financial debt

$$k_{it} = a_{it} + b_{it}.$$

Using Orbis acronyms, equity is equal to

$$a_{it} = (0.5 + 0.5L) \times (\texttt{toas}_{it} - \texttt{culi}_{it} - \texttt{ncli}_{it})$$

where `toas` denotes total assets, `culi` is current liabilities, `ncli` is non-current liabilities. The term $(0.5 + 0.5L)$ indicates the average between current and previous year balance sheet variables, where $L$ is the lag operator.

We measure firms' net financial debt as

$$b_{it} = (0.5 + 0.5L) \times (\texttt{loan}_{it} + \texttt{ltdb}_{it} - \texttt{cash}_{it})$$

wher `loan` is short-term financial debt (payable within a year), `ltdb` is long-term financial debt, and `cash` denotes the firm's cash and cash equivalents.

In addition to our baseline definition of capital, we also present results for capital measured by tangible fixed assets, $(0.5 + 0.5L) \times \texttt{tfas}_{it}$, and alternatively by the sum of tangible and intangible fixed assets, $(0.5 + 0.5L) \times (\texttt{tfas}_{it} + \texttt{ifas}_{it})$. As Appendix A.2 of Kochen (2025) shows, our baseline measure of capital is broadly equal to the sum of tangible and intangible assets plus inventories (`stok`).

**Profits** Our baseline analysis focuses on a model-consistent definition of economic profits. Noting that the firm's capital is the sum of equity plus debt, $k_{it} = a_{it} + b_{it}$, we can rewrite economic profits, defined in (5), as

$$\begin{aligned} \pi_{it} &= y_{it} - wl_{it} - Rk_{it} \\ &= y_{it} - wl_{it} - \delta k_{it} - rb_{it} - ra_{it} \end{aligned} \tag{13}$$

where the rental cost of capital is $R = r + \delta$. We take the definition of economic profits in (13) to the data by setting $\delta = 0.05$ and $r = 0.02$, as in the model, and then directly measuring output $y_{it}$, labor costs $wl_{it}$, capital $k_{it}$, and equity $a_{it}$ as defined above. Lastly, we measure the cost of debt $rb_{it}$ using net financial expenses: $-\texttt{fipl}_{it} = \texttt{fiex}_{it} - \texttt{fire}_{it}$, where `fiex` are financial expenses and `fire` financial revenue.

In addition to the analysis using economic profits, we present results for profits measured by net income of the year, available in the data and equal to

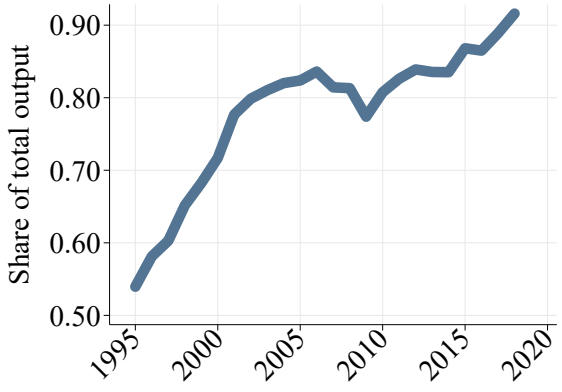
$$\begin{aligned} \pi_{it} &= \texttt{pl}_{it} \\ &= \texttt{plat}_{it} + \texttt{extr}_{it} \end{aligned}$$

where `plat` is profits after taxation and `extr` is extraordinary and other profit.

### A.1.2 Coverage and Representativeness for Spain

**Coverage** To analyze the coverage of Orbis Historical, we follow Kalemli-Özcan et al. (2024) and compute the share of total revenue in our data relative to the *OECD STAN Database*. As we study the non-financial private sector, we focus on matched two-digit ISIC and year pairs in both Orbis and OECD STAN. Figure A.1 reports the share of total revenue (turnover) in our data relative to the official statistics in OECD STAN. The figure shows that coverage increases by year. The firms in our sample represent 54% of aggregate revenue in 1995 and 92% by 2018. On average, across all years, Orbis Historical covers 78% of total revenue in Spain, considering the matched two-digit sectors.

**Figure A.1:** Orbis Historical Share of Spain Total Revenue



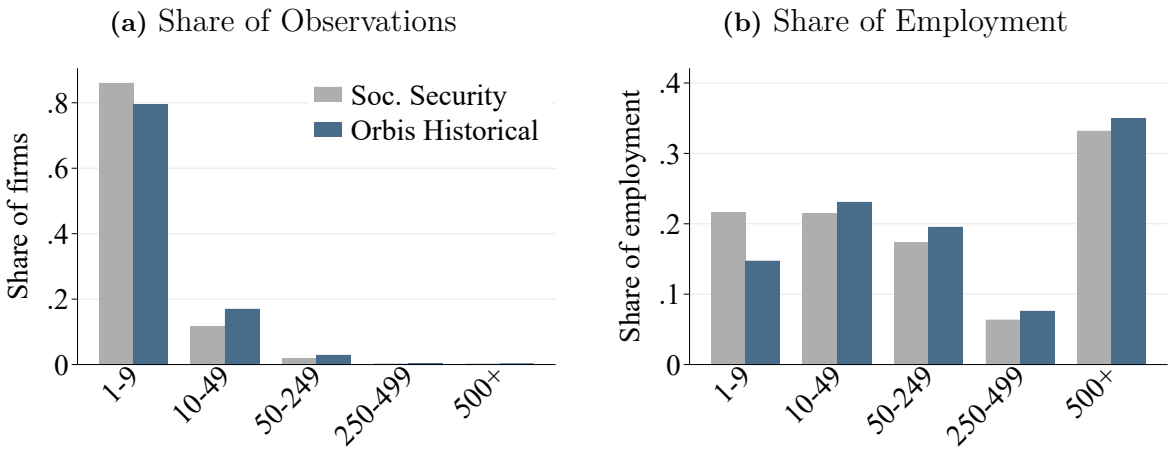

*Notes*: Share of total revenue relative to OECD STAN data for matched 2-digit sector-year pairs.

**Size Distribution** To test the representativeness of the data in terms of the size distribution, we compare the share of firms and share of total employment by firm size in Orbis Historical to official statistics from the Spanish Social Security (*Estadística de Empresas Inscritas en la Seguridad Social*). As before, we focus on employer firms. Figure A.2a reports the share of observations by five different employment bins across these two datasets for 2019. Figure A.2b presents the share of total employment. These two panels show that while Orbis Historical slightly underrepresents the smallest firms (1-9 employees), it captures the full employment distribution quite well.

### A.1.3 Firm-Owner Linkages

In addition to our longitudinal analysis, in Section 3.5, we present a taxonomy for top and bottom firms given the characteristics of their owners. For that, we use firm-owner linkages in Orbis Historical, available from 2007 onward, which include annual records with owners' names and equity shares. The data also specifies whether the owner is an individual, another company, a financial institution, or a government. Given firms' sometimes complex ownership

**Figure A.2:** Employment Distribution in Orbis Historical and Social Security Data




*Notes*: Shares for Spain in the year 2019 considering employer firms. Soc. Security data comes from Estadística de Empresas Inscritas en la Seguridad Social published by the Spanish Labor Ministry.

structures, we identify firms' ultimate owners by sequentially matching the ownership files as in Peter (2021) and Kochen (2024). The main objective of this iterative procedure is to account for the cases where other companies own firms by assigning the respective ownership shares of the parent company.[33] Indeed, for the sample of Spanish firms we study, 20% of the direct owners are other companies. As we elaborate below, solely studying direct owner linkages would underestimate the importance of foreign and multiple firm owners.

## A.2 Firms' Foundation Year

The year of incorporation of the current legal entity reported in Orbis may differ from the firm's true foundation year due to corporate restructurings, mergers and acquisitions (M&A), or changes in legal form due to regulation. In addition, because we focus on unconsolidated accounts, subsidiaries of older parent companies may appear as young firms even though they are part of the parent firm's core operations. These discrepancies can be more common among the largest firms in the economy. Since our life-cycle analysis requires an accurate measure of firm ages, we undertook a systematic verification of the foundation year for all top 1 percent firms in our sample at age 20. For Spain, we searched for a total of 2,334 companies, including an initial sample of 1,740 from the top 1 percent and additional companies from the top 2 percent. Whenever the verified foundation year and the incorporation year do not coincide, we use the true foundation year to compute firm age.

**Examples** To illustrate the importance of properly measuring companies' foundation years, we present examples we identified in our verification analysis. For this Appendix, we focus on Spain, but similar considerations apply to the other countries in our sample.

[33]Hence, for example, if 100% of company A is owned by company B, the procedure assigns the ownership information of B to A. If x% of company A is owned by company B, only x% of the ownership of B will be transferred to company A.

- *Corporate Restructuring and Subsidiaries.* One reason for corporate restructuring is that, as companies grow, they divide their operations into separate legal entities. For example, "Telefónica de España S.A.U.", the largest telecommunications company in Spain, separated its domestic and international operations thirty years ago. Hence, in the data, we observe an incorporation year in the late 1990s. However, according to the company website, this company was founded in 1924. Another example in the data is a large energy producer founded in the 1900s that has two relatively recent legal entities, one specializing in energy generation and the other in distribution. In both of these examples, we assigned the true foundation year to be that of the older parent company.
- *M&A.* Another common case is Spanish-founded companies that change legal entities, and sometimes brands, when foreign companies acquire them. In this case, we kept the foundation year of the original Spanish company that was later acquired. For mergers that result in a new legal entity, we set the foundation year to the oldest foundation year of the two pre-merge companies.
- *Regulation.* Finally, regulatory changes can also drive companies to change their legal entities. In our sample, we identified eight well-known Spanish football clubs, most founded at the beginning of the 20th century, that had to convert to Sociedad Anónima Deportiva (SAD) in the early 1990s because of Spain's 1990 Sports Law (Ley del Deporte 10/1990).

**LLM-Assisted Search** Given the scale of this task, which involves searching for information on more than 2,000 firms, we employed the large language model *Claude Opus 4.6* (Anthropic) as the primary research assistant. The LLM was given access to web search capabilities and instructed to query company websites, Wikipedia, LinkedIn company pages, Spanish business registries (Informa D&B, Axesor, eInforma), the official gazette of the commercial registry (BORME), and press archives. Each company was researched individually, and the LLM was required to document the source for every verified founding year. We conducted this process under continuous supervision, working on batches of 100 companies at a time. In addition to the foundation year, we asked the LLM to report whether the firm originated in Spain, if it is foreign-owned, and whether a parent company owns it. In those cases, it also had to report the parent's name and foundation year, and whether the company is independent of the parent's core operations, following the rules described below. Once the LLM retrieved all the information, we "manually" verified all companies with discrepancies between their incorporation and foundation years. We identified and corrected very few mistakes.

**Classification Rules** When reporting the previous variables, we instructed the LLM to apply the following classification rules that distinguish among the different types of firms in the data. For standalone Spanish domestic companies, the verified foundation year records the true operational founding year of the business, even when this predates the current legal

entity by decades. For foreign multinational companies that established a Spanish subsidiary from scratch, we measure firm age since entering into the Spanish market; the foundation year therefore records the year the Spanish entity was incorporated, while the parent company's founding year is recorded separately. For Spanish-founded businesses later acquired by foreign companies, the original Spanish founding year is preserved as the foundation year to reflect the true age of the business activity. When a parent company was itself formed by merging older entities, the founding year of the oldest constituent is used.

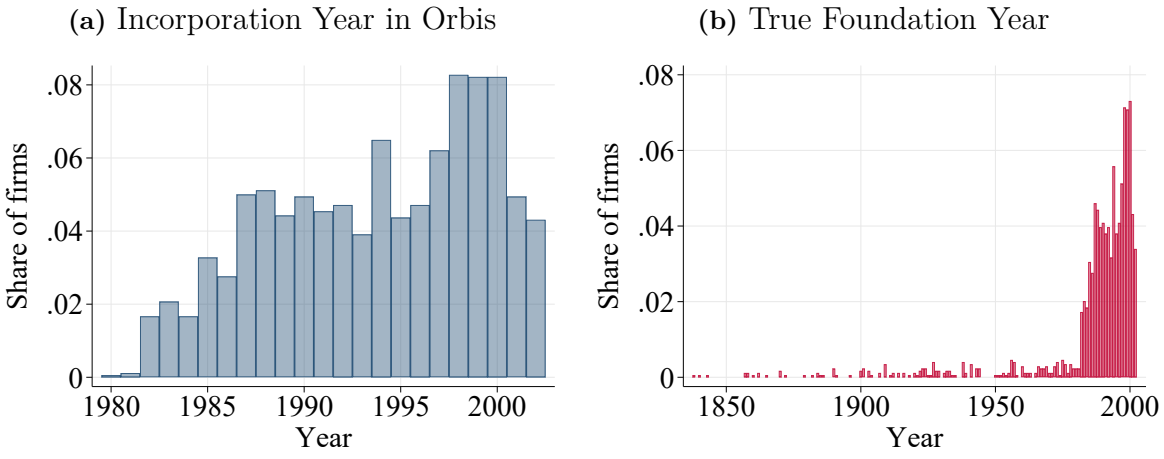
**Business Groups and Subsidiaries** Equipped with the verified foundation years of the top companies at age 20, along with additional variables obtained from the LLM-assisted search, we need to determine the foundation years of companies within business groups (i.e., that are owned by a parent company). For Spanish business groups, we treat the companies that are independent of the parent company's core activities as different firms, with their own foundation years. When they are not independent and are merely an organizational unit within the group, we set the subsidiary's foundation year to match that of the parent company. We asked the LLM to determine whether a company is independent using the following four criteria. 1) Distinct market-facing activity: if the entity's name combines the parent name with functional suffixes (e.g., Finance, Leasing, Capital, Services, etc.), it is not considered an independent company. 2) Brand identity: if the company name corresponds to a recognized separate brand, it is considered an independent firm. 3) Economic purpose: if the entity serves primarily as a legal or financial vehicle for the group, it is not considered independent. 4) Standalone viability: if the parent could sell the company as a standalone business unit, it is considered an independent firm. A firm is then classified as independent when the preponderance of these criteria points toward a distinct, self-standing firm.

**Foreign Multinationals** For companies owned by foreign multinationals, we use the Spanish subsidiary's date of incorporation as the foundation year, as our interest is to measure firm age since entering the domestic market. We show that our results are robust to excluding all foreign-owned companies from our analysis.

**Results** Following these steps, from the initial sample of 1,740 firms in the top 1 percent at age 20, we corrected for the foundation year and excluded 247 (14.2%) from this group. For those 247 cases, the difference in foundation years was considerable, averaging 52 years. To analyze these differences, Figure A.3a presents the cohort distribution of this sample using the incorporation year in the data. Given our sample selection, focusing on firms we observe at age 20 and can track back for at least 10 years, these are firms with cohorts between 1980 and 2002. Figure A.3b shows the distribution of corrected foundation years. It shows that several firms date back to the 1800s and the first half of the 1900s, underscoring the importance of accurately measuring the foundation years of top firms. Once we excluded companies with an incorrect measure of firm age, we recomputed the firm size distribution

conditional on age and defined the final sample as the top 1 percent of firms at age 20.

**Figure A.3:** Cohort Distribution of Initial Sample of Top 1 at Age 20




*Notes*: Distribution of incorporation and foundation year for the initial sample of 1,740 top 1 firms at age 20 in Spain. Incorporation year is directly taken from the Orbis data. Foundation year is the one we uncover from our LLM-assisted search.

## A.3 Top Status Persistence

We now analyze the transitions of firms between the top and bottom of the size distribution depending on their age. We focus on firms in their early stages—ages 1 to 5—and at age 20, which is the maximum age considered in our baseline life-cycle analysis. Table A.1 presents the main results. We find that young top firms are much less likely to remain at the top many years later, even when conditional on survival, compared to top firms at age 20. The top firms between ages 1 and 5, conditional on surviving, have a 0.6 probability of remaining in the top 20 years later, while the probability of age-20 top firms is 0.8. Furthermore, we find that young top firms are much less representative, in terms of the number of firms and their output share, of top firms many years later than top firms at age 20. Top firms between ages 1 and 5 represent 25% of the firms and 65% of the output of top firms 20 years later, while top firms at age 20 represent 65% of the firms and 92% of the output, respectively.

**Table A.1:** Top Transition and Representation

| | Top at Age 1-5 | Top at Age 20 |
|---|---|---|
| $P(\text{Top at age } a+20 \mid \text{Top at age } a)$ | 0.62 | 0.81 |
| Share of top firms at age $a+20$ | 0.25 | 0.65 |
| Share of top firms' output at age $a+20$ | 0.51 | 0.92 |

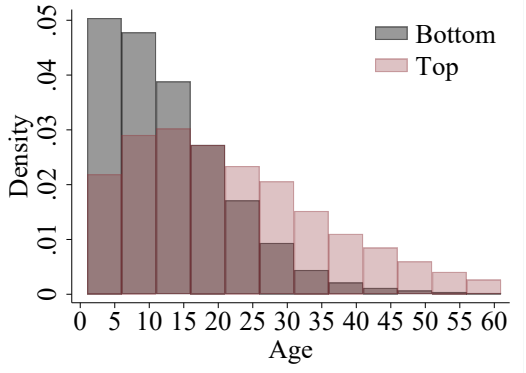
*Notes*: The first line shows the probability that a top firm at age $a$ is at the top of the entire distribution 20 years later. The results are shown for top firms at ages 1 to 5 ($a \in \{1,2,3,4,5\}$) and at age 20 ($a = 20$) in columns one and two, respectively. The second and third lines show the share of firms and the output share within top firms at age $a+20$, respectively, that correspond to firms that were top at age $a$, where $a \in \{1,2,3,4,5\}$ and $a = 20$. We aggregate for ages 1 to 5 by taking the average of the estimates.

These patterns suggest that there is substantial mobility between the top and bottom when firms are very young. In Section 3, we study whether the life cycle differs between firms that transition from the bottom to the top compared to those that remain at the top. Secondly, since top firms at age 20 are very likely to remain at the top and are quite representative of much older top firms, the early-life patterns of top firms at age 20, which are the ones we track back in our analysis of the origins of top firms in Section 3, are likely to be similar to those of much older top firms.[34]

## A.4 Additional Figures and Tables

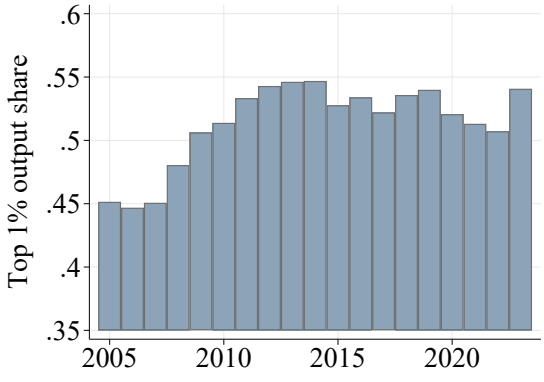
This subsection presents additional figures and tables referenced throughout the paper.

**Figure A.4:** Age distribution: all and top firms




*Notes*: The figure shows the age distribution for all the firms and conditional on top firms.

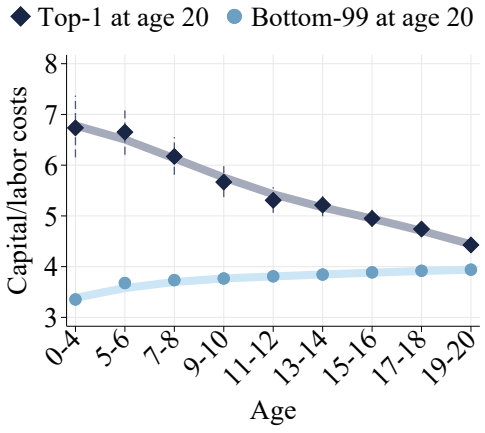
**Figure A.5:** Output Share of Top 1 Percent Firms Over Time




*Notes*: Top 1 percent separately defined for each year controlling for sector fixed effects. Results for Spain using Orbis Historical. Firm-level output is measured as value added, defined as revenue minus a comprehensive measure of non-capital and non-labor costs.

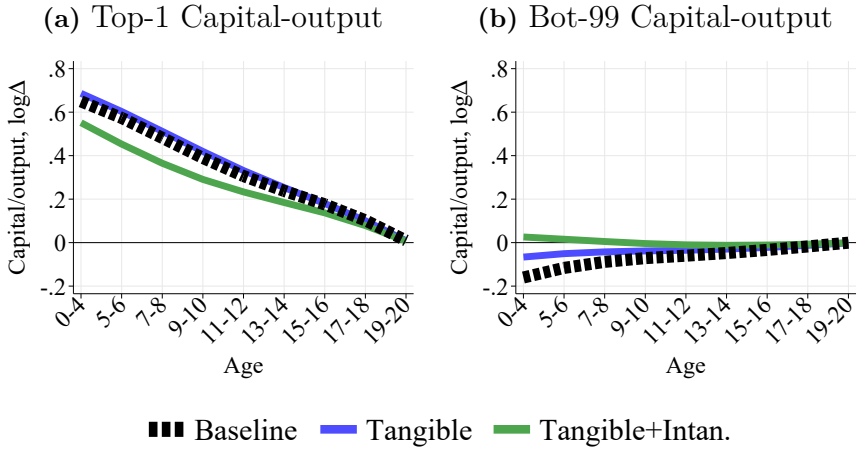
[34]Notice that if there are strong cohort effects, older firms may have behaved differently. Then, our results are likely to be very informative of older top firms' early life cycle if we condition on cohort effects.

**Figure A.6:** Capital-to-Labor Costs Ratio of Top and Bottom Firms



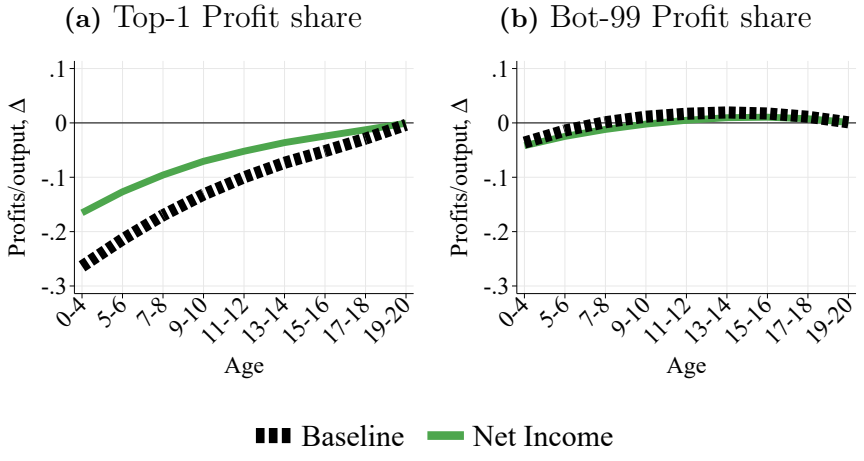

*Notes*: Life cycle trajectories of the capital-labor costs ratio of top 1 and bottom 99 percent firms at age 20 are estimated using (1). Results are scaled by adding the average of the omitted group (age 19-20) for the top and bottom firms. The solid lines are smoothed scatterplots using locally weighted regressions.

**Figure A.7:** Alternative Measures for Firms' Capital
Life Cycle Changes in Capital-Output Ratio of Top and Bottom Firms




*Notes*: Life cycle changes for the capital-output ratio of top 1% and bottom 99% firms at age 20 considering two alternative measures of firms' capital. Changes are relative to the omitted group (age 19-20). The first alternative measure is tangible capital (`tfas`). The second measure is the sum of tangible and intangible capital (`tfas` + `ifas`). The dashed dark line denotes the baseline estimation. All the lines are smoothed scatterplots generated through locally weighted regressions on the estimated parameters.

**Figure A.8:** Alternative Measure for Firms' Profits
Life Cycle Changes in Profit Share of Top and Bottom Firms



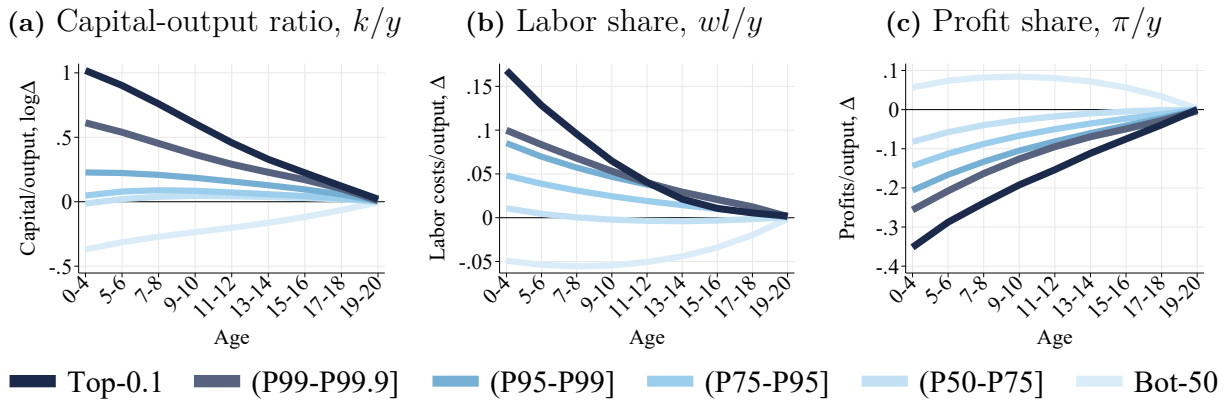

*Notes*: Life cycle changes for the profit share of top 1% and bottom 99% firms at age 20 considering firms' net income (variable `pl` in Orbis) as alternative measure for profits. Changes are relative to the omitted group (age 19-20). The dashed dark line denotes the baseline estimation. All the lines are smoothed scatterplots generated through locally weighted regressions on the estimated parameters.

**Figure A.9:** Inputs and Profit Share Over the Life Cycle of the Average Firm



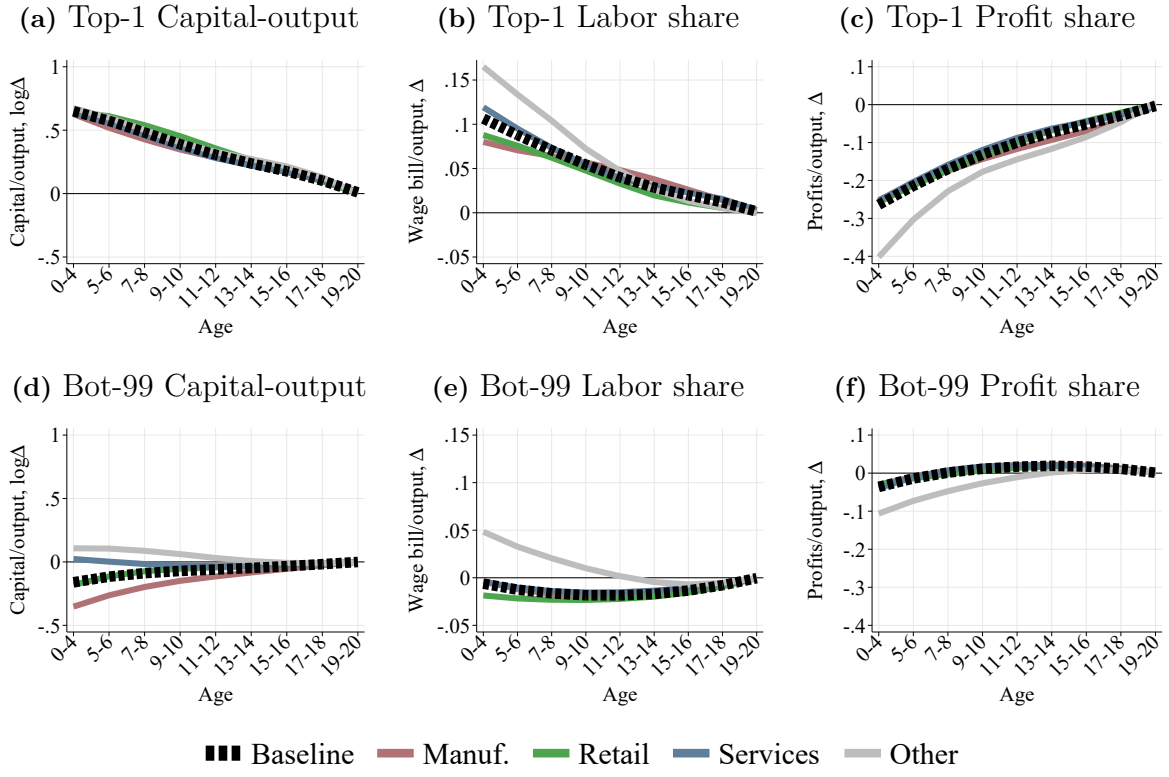

*Notes*: Life cycle trajectories for the inputs and profit shares of the average firm estimated using a version of (1) putting together all the firms that survive to age 20. The regression for the capital-output ratio is in logs, while the labor and profit shares are in levels. Results are scaled by adding the average of the omitted group (age 19-20). The solid lines represent smoothed scatterplots generated through locally weighted regressions. The dashed vertical lines indicate 95% confidence intervals considering firm-level clustered standard errors.

**Figure A.10:** Life Cycle Changes in Inputs and Profit Share by Size at Age 20



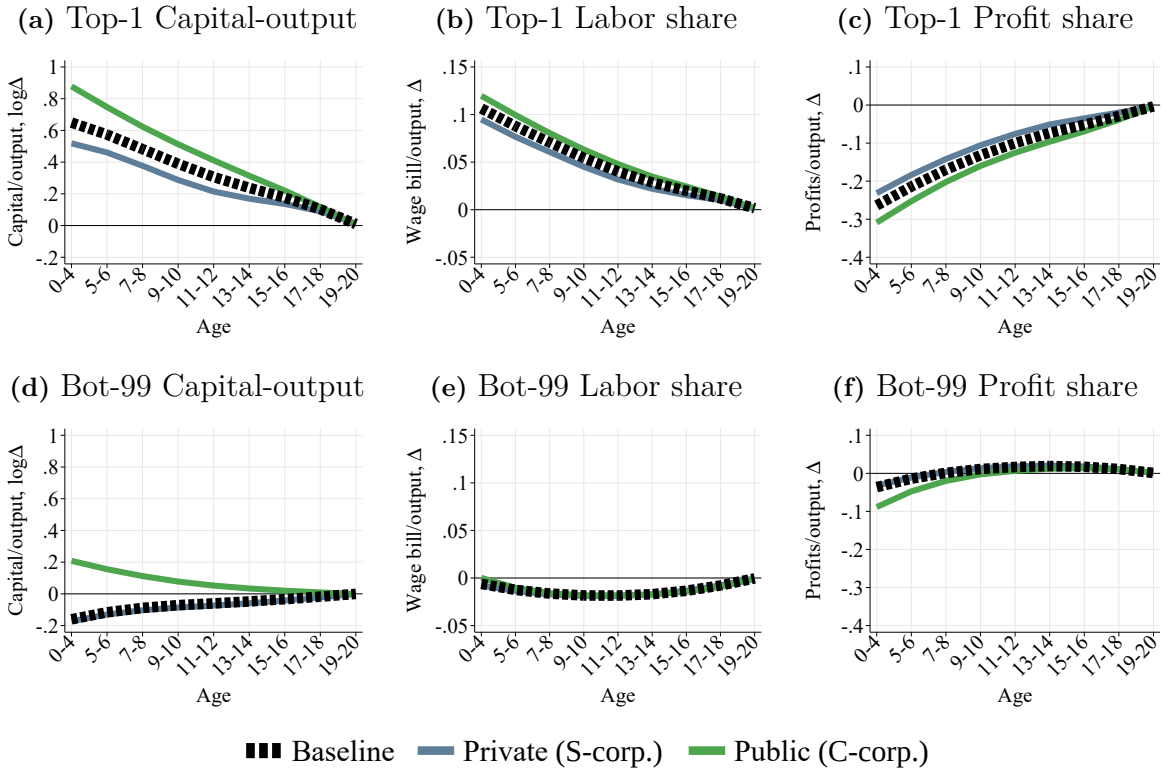

*Notes*: Life cycle changes for the inputs and profit shares of the top 0.01, percentile 99 to 99.9, percentile 95 to 99, percentile 75 to 95, percentile 50 to 75, and the bottom 50 percent firms at age 20. Life cycle changes are estimated using a version of regression (1) with these six groups of firms. Changes are relative to the omitted group (age 19-20). The lines are smoothed scatterplots generated through locally weighted regressions on the estimated parameters.

**Figure A.11:** Heterogeneity by Sector
Life Cycle Changes in Inputs and Profit Share of Top and Bottom Firms



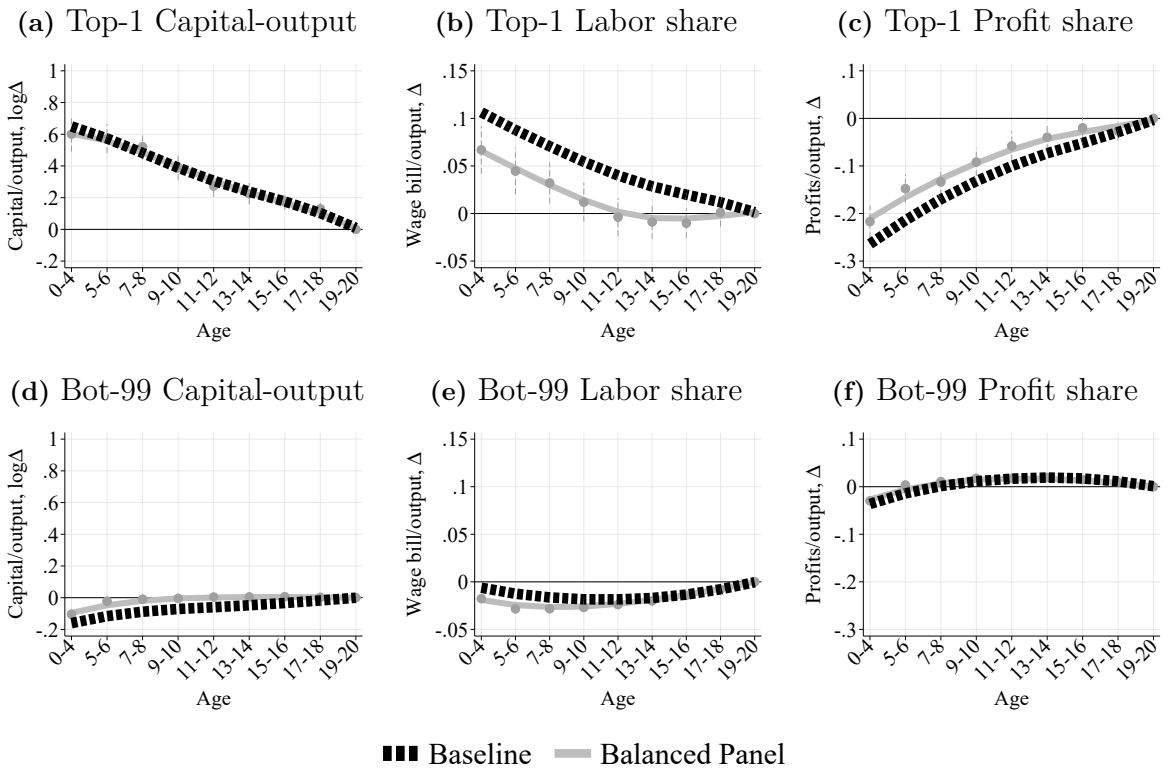

*Notes*: Life cycle changes for the inputs and profit shares of the top 1 and bottom 99 percent firms at age 20 estimated using a modified version of (1) with interactions by sector. Changes are relative to the omitted group (age 19-20). See Table 1 for the definition of the sectors. The dashed dark line denotes the baseline estimation. All the lines are smoothed scatterplots generated through locally weighted regressions on the estimated parameters.

**Figure A.12:** Heterogeneity by Legal Status
Life Cycle Changes in Inputs and Profit Share of Top and Bottom Firms



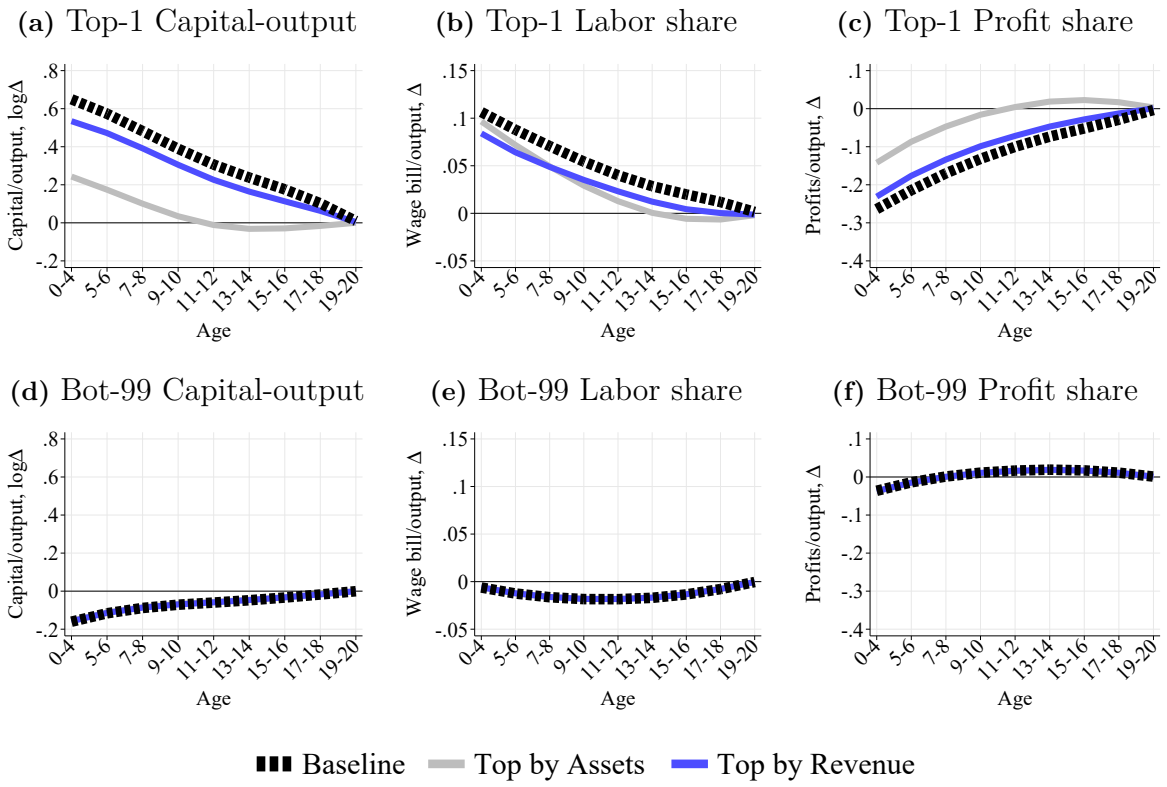

*Notes*: Life cycle changes for the inputs and profit shares of the top 1 and bottom 99 percent firms at age 20 estimated using a modified version of (1) with a public firm dummy. Changes are relative to the omitted group (age 19-20). Public Firms are public limited companies (*Sociedades Anónimas*), analogous to U.S. C-corporations, and Private Firms are partnerships and private limited companies (*Sociedad de Responsabilidad Limitada*), analogous to U.S. S-corporations. The dashed dark line denotes the baseline estimation. All the lines are smoothed scatterplots generated through locally weighted regressions on the estimated parameters.

**Figure A.13:** Balanced Panel
Life Cycle Changes in Inputs and Profit Share of Top and Bottom Firms



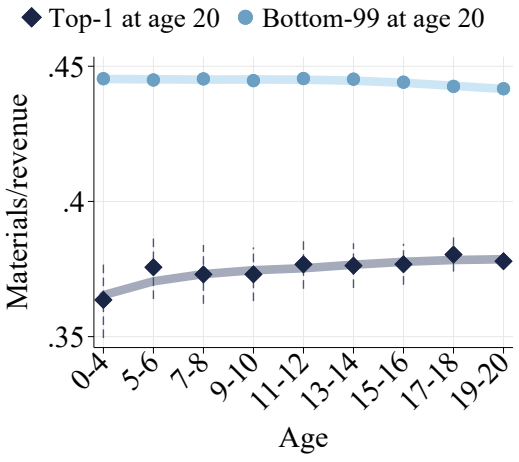

*Notes*: Life cycle trajectories for the inputs and profit shares of the top 1 and bottom 99 percent firms at age 20 estimated using (1) for a balanced panel of firms. The solid lines represent smoothed scatterplots generated through locally weighted regressions. The dashed vertical lines indicate 95% confidence intervals considering firm-level clustered standard errors. The dashed dark line denotes the baseline estimation.

**Figure A.14:** Top and Bottom by Assets and Revenue
Life Cycle Changes in Inputs and Profit Share of Top and Bottom Firms




*Notes*: Life cycle trajectories for the inputs and profit shares of the top 1 and bottom 99 percent firms at age 20 estimated using (1) for a balanced panel of firms. The solid lines represent smoothed scatterplots generated through locally weighted regressions. The dashed vertical lines indicate 95% confidence intervals considering firm-level clustered standard errors. The dashed dark line denotes the baseline estimation.

**Figure A.15:** Materials-to-Revenue Ratio of Top and Bottom Firms




*Notes*: Life cycle trajectories of the materials-to-revenue ratio of top 1 and bottom 99 percent firms at age 20 estimated using (1). Results are scaled by adding the average of the omitted group (age 19-20) for the top and bottom firms. The solid lines represent smoothed scatterplots generated through locally weighted regressions. The dashed vertical lines indicate 95% confidence intervals considering firm-level clustered standard errors.

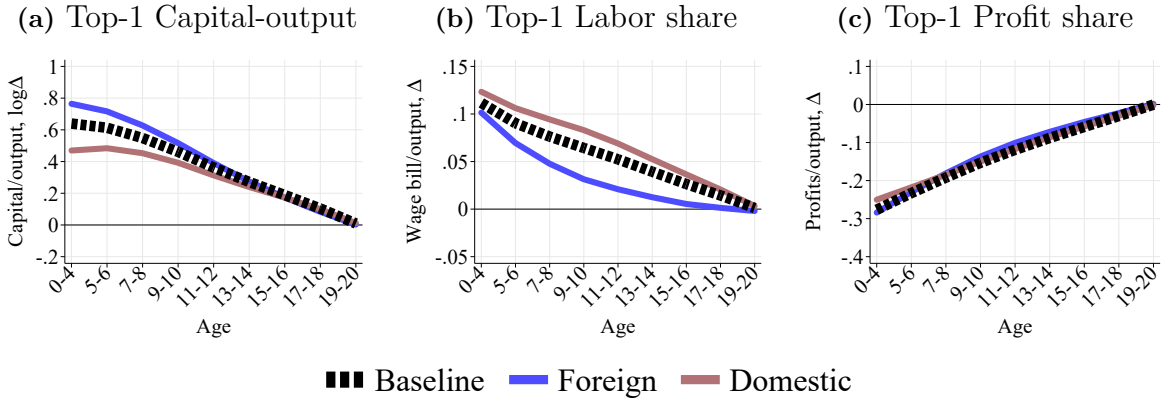
**Table A.2:** Firms' Taxonomy by Owners' Characteristics

| | By Size at Age 20 | | |
|---|---|---|---|
| | Top-1 | Bot-99 | Rest |
| Domestic owner | 0.62 | 0.80 | 0.59 |
| Foreign owner | 0.33 | 0.03 | 0.05 |
| Multiple firms | 0.74 | 0.26 | 0.23 |
| Multiple firms + Domestic | 0.49 | 0.23 | 0.19 |
| Multiple firms + Foreign | 0.22 | 0.01 | 0.03 |

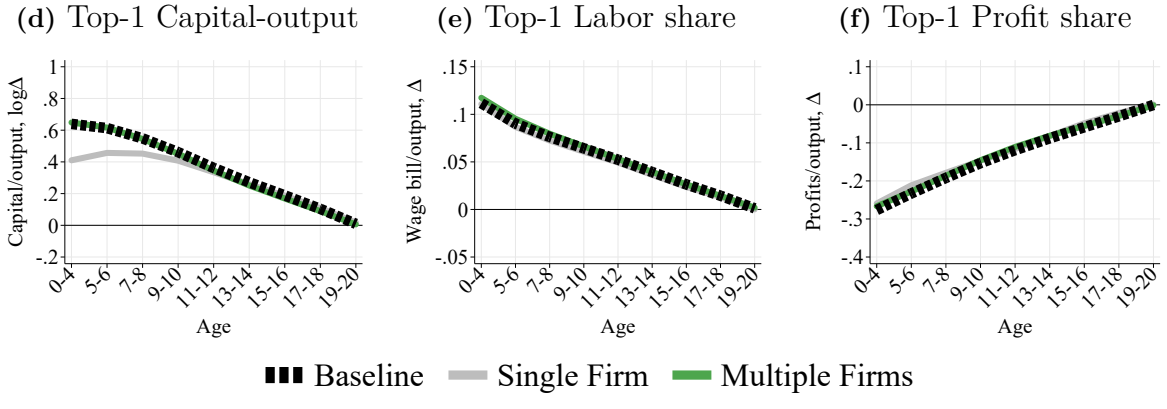
*Notes*: Share of firms by owners' characteristics. Domestic (Foreign) owner equals one if the owner is registered in (outside) Spain. Both variables are zero if the owner's country is unknown. Multiple firms is equal to one if the owner owns at least one other firm inside Spain in the same year. Shares are at the firm level, computed after taking each firm's equity-weighted average over different owners and years and then averaging across firms. Results using ultimate owners after three rounds of matching.

**Figure A.16:** Top Firms by Owners' Characteristics

Life Cycle Changes in Inputs and Profit Share of Top Firms by Foreign Status




Life Cycle Changes in Inputs and Profit Share of Top Firms by Multiple Firm Status



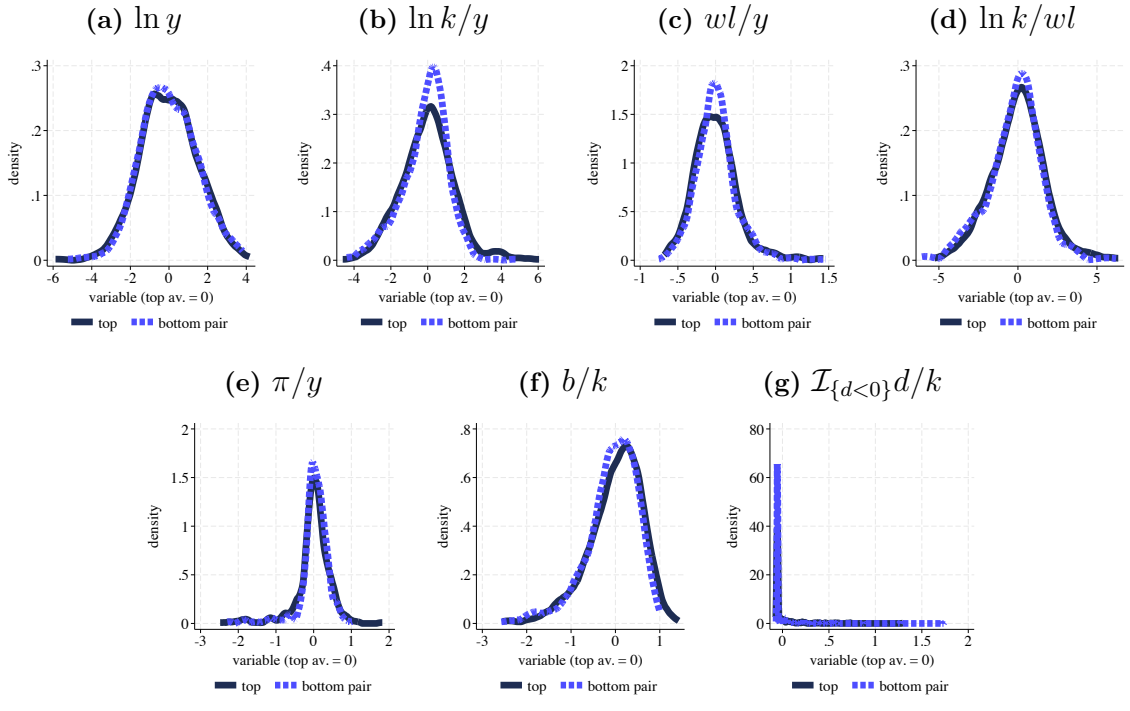

*Notes*: Life cycle trajectories for the inputs and profit shares of the top 1 firms at age 20 estimated using a modified version of (1). The solid lines represent smoothed scatterplots generated through locally weighted regressions. The dashed dark line denotes the baseline estimation.

## A.5 Top Firms Pairs and Predictors

In this section, we study firms that are similar to top firms in their early years (i.e., bottom pairs) but end up in the bottom 20 years later. Furthermore, we examine which observable characteristics predict whether firms are top at age 20.

**Pairs Analysis** Using a propensity score matching approach, we match each top firm with the closest bottom firm in its initial years. The variables used for matching are output level ($y$), input usage ($k/y$, $wl/y$, and $k/wl$), profitability ($\pi/y$), and financing ($b/k$, $\mathcal{I}_{\{d<0\}}d/k$), after removing sector and year fixed effects. We condition on bottom firms that survive to age 20 to perform the life cycle analysis. To obtain the propensity scores, we run a logit regression using these variables and then match each observation to its nearest neighbor, ensuring that propensity scores differ by no more than 0.05. This threshold guarantees that all top firms in this sample are matched, allowing us to retain a relatively large sample of more than 550 top firms. However, we find that 14.5% of the paired bottom firms are matched with more than one top firm, with a maximum of six matches for three of the paired bottom firms.

**Figure A.17:** Top and Bottom Pairs Matching Fit



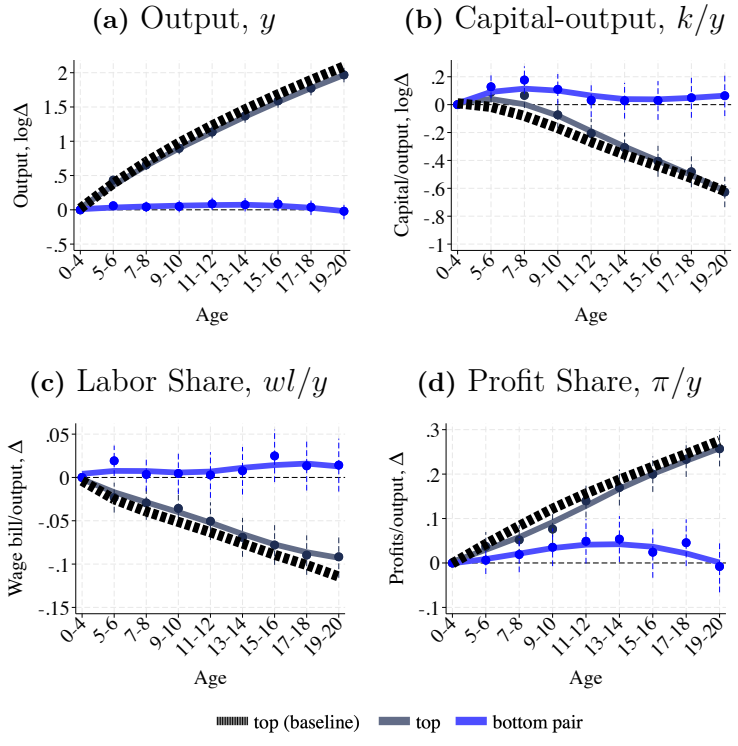

*Notes*: All variables exclude sector and year fixed effects, and are normalized such that the top firms' average is 0. The sample of top firms, is the one used for the matching. Details regarding the pairing are in the text.

Figure A.17 shows the distribution of top firms and their bottom pairs are very similar across all the variables used for the matching procedure.[35] We estimate an empirical model

[35] In the first step, we estimate a logit model, $\text{top}_i = \mathcal{L}(\beta \mathbf{x}_{i1} + \varepsilon_{it})$, where $\text{top}_i$ indicates whether firm $i$ is a top firm at age 20, and $\mathbf{x}_{i1}$ is a vector of characteristics of firm $i$ at the first age group. Using the estimated

based on (1) for the top and bottom pairs. Figure A.18 presents the results for output, input usage, and profits. Panel (a) shows that, as expected, the output growth of bottom firms that were similar to top firms at the start remains flat. Panels (b) and (c) show that their capital-output ratio and labor share are roughly flat as well. Consequently, in panel (c), the profit share is also flat. Furthermore, the matched top firms behave very similarly to our baseline estimates. This is consistent with large initial shares being driven by input-specific fixed costs, which become relatively smaller as firms grow.

**Figure A.18:** Life Cycle of Top Firms and Their Bottom Pairs




*Notes*:Life cycle trajectories of inputs and profit shares for the top 1% and their pairs in the bottom 99% of firms at age 20, estimated using (1), with the omitted age group being [0,4]. The regression for the output and capital-output ratio is in logs, while the labor and profit shares are in levels. The solid lines represent smoothed scatterplots generated through locally weighted regressions. The thick black dashed line is the baseline profile for top firms. The dashed vertical lines indicate 95% confidence intervals considering firm-level clustered standard errors.

**Predictors** Next, we examine what initial observable characteristics predict whether a firm becomes a top firm at age 20. We use the same set of variables from the pair analysis as predictors, residualized by projecting them onto year and sector fixed effects. We estimate a probit model conditional on firms surviving until age 20 and another including exit. Table A.3 shows the results. Firms with a larger initial size, a higher capital-output ratio, and greater equity injections are more likely to reach the top. On the other hand, the initial labor share, profitability, and leverage are not significant predictors. When including all variables

probabilities, $\hat{p}_i = \mathcal{L}(\beta \mathbf{x}_{i0})$, we match top firms with bottom firms such that, for any top firm $i$, the bottom pair $j$ is determined as $j = \arg\min_{\{j:\text{top}_j=0\}} |\hat{p}_i - \hat{p}_j| < 0.05$.

simultaneously, we find that lower profitability and slightly lower leverage increase the likelihood of reaching the top at age 20. These findings are overall consistent with our baseline exercise, even when including firms that exit before reaching age 20.

**Table A.3:** Predictors of Top Firms

| | $\beta: \ \mathcal{P}(\beta \mathbf{x}_1) = \text{pr}[\text{top} = 1 \mid \mathbf{x}_1]$ | | | | | | |
|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| *(a) Conditional on Survival* | | | | | | | |
| $\ln y$ | 0.599*** | | | | | | 0.653*** |
| | (0.00) | | | | | | (0.000) |
| $\ln k/y$ | | 0.057*** | | | | | 0.113*** |
| | | (0.00) | | | | | (0.000) |
| $wl/y$ | | | 0.035 | | | | 0.167 |
| | | | (0.53) | | | | (0.191) |
| $\pi/y$ | | | | -0.043 | | | -0.213** |
| | | | | (0.34) | | | (0.01) |
| $b/k$ | | | | | 0.015 | | -0.081** |
| | | | | | (0.47) | | (0.04) |
| $\mathbb{I}_{\{d<0\}}\lvert d\rvert/k$ | | | | | | 0.449*** | 0.705*** |
| | | | | | | (0.00) | (0.00) |
| Observations | 98,712 | 75,306 | 98,144 | 73,640 | 72,920 | 66,470 | 64,331 |
| Pseudo R-2 | 0.32 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.36 |
| *(b) Including Exit* | | | | | | | |
| $\ln y$ | 0.507*** | | | | | | 0.550*** |
| | (0.00) | | | | | | (0.00) |
| $\ln k/y$ | | 0.065*** | | | | | 0.150*** |
| | | (0.00) | | | | | (0.00) |
| $wl/y$ | | | -0.065 | | | | 0.173 |
| | | | (0.19) | | | | (0.11) |
| $\pi/y$ | | | | 0.011 | | | -0.097 |
| | | | | (0.80) | | | (0.17) |
| $b/k$ | | | | | 0.005 | | -0.137*** |
| | | | | | (0.78) | | (0.00) |
| $\mathbb{I}_{\{d<0\}}\lvert d\rvert/k$ | | | | | | 0.411** | 0.550** |
| | | | | | | (0.00) | (0.00) |
| Observations | 234,305 | 178,402 | 232,419 | 174,292 | 171,520 | 147,311 | 142,526 |
| Pseudo R-2 | 0.28 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.31 |

*Notes*: The table shows the coefficients $\beta$ and their p-values in parentheses for the probit model, where the outcome variable indicates whether the firm is top at age 20. The value of the predictors correspond to observable characteristics between age 0 and 4. Panel (a) conditions on firms that survive to age 20 and panel (b) includes firms that exit before age 20.

# B Model Appendix

## B.1 Nested CES Model

Alternatively, we can assume that each firm type $h \in \mathcal{H}$ produces a differentiated variety of the $h$-type good, where the elasticity of substitution across varieties is $\nu_h$ and across firm types is $\nu$. We show that the steady-state equilibrium of this model is isomorphic to ours, with $\nu_h$ mapping into the returns to scale $\theta^h$; however, these elasticities can be relevant for counterfactuals.

**Demand.** For the type-$h$ good, the aggregator is

$$y_h = \left[ \int \tilde{z}_i \, y_{hi}^{\frac{\nu_h - 1}{\nu_h}} \, di \right]^{\frac{\nu_h}{\nu_h - 1}},$$

where $y_h$ is $h$-type aggregate output and $y_{hi}$ is the output of firm $i$ of type $h$. Across types, the aggregator is

$$Y = \left[ \int \gamma_h^{\frac{1}{\nu}} \, y_h^{\frac{\nu - 1}{\nu}} \, dh \right]^{\frac{\nu}{\nu - 1}},$$

where $Y$ is aggregate output. The price index for the type-$h$ good is

$$P_h = \left[ \int \tilde{z}_i \, p_{hi}^{1 - \nu_h} \, di \right]^{\frac{1}{1 - \nu_h}},$$

and the aggregate price index is

$$P = \left[ \int \gamma_h \, P_h^{1 - \nu} \, dh \right]^{\frac{1}{1 - \nu}}.$$

The demand for each variety and its inverse are

$$y_{hi} = \gamma_h \, \tilde{z}_i^{\nu_h} \left( \frac{p_{hi}}{P_h} \right)^{-\nu_h} \left( \frac{P_h}{P} \right)^{-\nu} Y,$$

$$p_{hi} = P_h^{\left(1 - \frac{\nu}{\nu_h}\right)} \left( \frac{y_{hi}}{Y} \, \gamma_h^{-1} \right)^{-\frac{1}{\nu_h}}.$$

We assume that aggregate output is the numéraire, so that $P = 1$.

**Supply.** The firm chooses $k$ and $l$ to solve

$$\max_{k \in [\underline{k}^h, \bar{k}^h], \, l} \; p_{hi} \, y_{hi} - w l - R k,$$

where $p_{hi}$ is given by the inverse demand above and the production function is $y_{hi} = \hat{z}_i \, f^h(k_i, l_i)$, with $\hat{\theta}^h$ denoting its returns to scale. Using the inverse demand, we can write revenue as

$$p_{hi} \, y_{hi} = P_h^{\left(1 - \frac{\nu}{\nu_h}\right)} \, (Y \, \gamma_h)^{\frac{1}{\nu_h}} \; \tilde{z}_i \, y_{hi}^{\frac{\nu_h - 1}{\nu_h}}$$

$$= \underbrace{P_h^{\left(1-\frac{\nu}{\nu_h}\right)} \left(Y\,\gamma_h\right)^{\frac{1}{\nu_h}}}_{\text{aggregate}} \quad \underbrace{\tilde{z}_i\,\hat{z}_i^{\,\frac{\nu_h-1}{\nu_h}}}_{\text{TFP}} \quad \underbrace{f^h(k_i,l_i)^{\frac{\nu_h-1}{\nu_h}}}_{\text{input aggregator}}.$$

The TFP term can therefore be determined by demand or supply, and the effective returns to scale, $\frac{\nu_h-1}{\nu_h}\,\hat{\theta}^h$, reflect both the curvature of the demand function and the technology.

**Equilibrium and counterfactuals.** Our baseline steady-state equilibrium maps to this model when we set $\gamma_h = y_h/Y$, where $Y$ and $y_h$ are the steady-state aggregate and $h$-type output quantities, so that $P_h = 1$ and the aggregate term in the revenue function equals one. The steady-state model is then isomorphic to ours when

$$\theta^h = \frac{\nu_h - 1}{\nu_h}\,\hat{\theta}^h \qquad \text{and} \qquad p(\mathbf{s}_i^h) = \tilde{z}_i\,\hat{z}_i^{\,\frac{\nu_h-1}{\nu_h}}.$$

Importantly, the two models are not equivalent across counterfactuals, since the elasticities $\{\nu_h, \nu\}$ can play a role by changing relative prices across $h$-type goods (in our baseline model the relative price remains fixed).

## B.2 Profit Maximization Problem

For ease of notation, we remove the time $t$ and individual firm $i$ subscripts.

**Capital and Labor** The FOC of problem (5) are

$$\theta^h\,(1-\alpha)^{\frac{1}{\sigma}}\,\frac{y}{(x^h)^{\frac{\sigma-1}{\sigma}}}\left(l-\kappa_l^h\right)^{-\frac{1}{\sigma}} = w$$
$$\theta^h\alpha^{\frac{1}{\sigma}}\,\frac{y}{(x^h)^{\frac{\sigma-1}{\sigma}}}\left(k-\kappa_k^h\right)^{-\frac{1}{\sigma}} = R+\mu,$$

where $\mu$ is the lagrange multiplier of the capital collateral constraint. Combining the FOC, we get that the optimal labor choice as a function of capita is

$$l = \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{R+\mu}{w}\right)^{\sigma}\left(k-\kappa_k^h\right)+\kappa_l.$$

Using this, the input aggregator $x^h$ can be written as

$$x^h = \underbrace{\left[\alpha^{\frac{1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}}\left[\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{R+\mu}{w}\right)^{\sigma}\right]^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}}_{A_k}\left(k-\kappa_k^h\right)$$
$$= A_k\left(k-\kappa_k^h\right).$$

Finally, using the FOC of capital we get the optimal capital choice as

$$k = \left[\frac{\theta^h\alpha^{\frac{1}{\sigma}}p\left(\mathbf{s}^h\right)}{A_k^{\frac{\sigma-1}{\sigma}-\theta^h}\left(R+\mu\right)}\right]^{\frac{1}{1-\theta^h}} + \kappa_k^h.$$

To find the *unconstrained* solution we set $\mu = 0$, then the optimal labor and capital choices are

$$l^* = \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{R}{w}\right)^{\sigma}(k^* - \kappa_k) + \kappa_l^h \tag{14}$$

$$k^* = \left[\frac{\theta^h \alpha^{\frac{1}{\sigma}} p\left(\mathbf{s}^h\right)}{\left(A_k^*\right)^{\frac{\sigma-1}{\sigma}-\theta^h} R}\right]^{\frac{1}{1-\theta^h}} + \kappa_k^h, \tag{15}$$

where $A_k^* = \left[\alpha^{\frac{1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}}\left[\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{R}{w}\right)^{\sigma}\right]^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$. To solve for the *constrained* problem, first, we need to check if the solution is constrained and feasible. Thus, if $k^* < \bar{k}^h\left(a, s^h\right)$, then $k = k^*$; otherwise, $k = \bar{k}^h\left(a, s^h\right)$. Moreover, if $\bar{k}^h\left(a, s^h\right) > \kappa_k^h$, the solution is feasible; otherwise, we set the value of the firm to $-\infty$, forcing it to exit.[36] Under the constrained solution, we know that $\mu > 0$, which we find implicitly from

$$\bar{k}^h\left(a, \mathbf{s}^h\right) = \left[\frac{\theta^h \alpha^{\frac{1}{\sigma}} p\left(\mathbf{s}^h\right)}{A_k^{\frac{\sigma-1}{\sigma}-\theta^h}(R+\mu)}\right]^{\frac{1}{1-\theta^h}} + \kappa_k^h.$$

Once we find $\mu$, we can compute the optimal solution as

$$l = \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{R+\mu}{w}\right)^{\sigma}\left(\bar{k}^h\left(a, \mathbf{s}^h\right) - \kappa_k\right) + \kappa_l^h \tag{16}$$

$$k = \bar{k}^h\left(a, \mathbf{s}^h\right). \tag{17}$$

**Input Usage** We characterize now $k/wl$, $wl/y$, and $k/y$ ratios. Combining the FOC we can find that

$$\begin{aligned}\frac{l-\kappa_l^h}{k-\kappa_k^h} &= \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{R+\mu}{w}\right)^{\sigma} \\ \frac{k}{wl} &= \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{w^{1-\frac{1}{\sigma}}}{R+\mu}\right)^{\sigma}\left(\frac{k/\left(k-\kappa_k^h\right)}{l/\left(l-\kappa_l^h\right)}\right).\end{aligned} \tag{18}$$

Using the labor FOC and the relationship between capital labor we can rewrite the labor share as

$$\frac{wl}{y} = \theta^h(1-\alpha)\left[(1-\alpha) + \alpha\left(\frac{R+\mu}{w}\right)^{1-\sigma}\right]^{-1}\frac{l}{\left(l-\kappa_l^h\right)}. \tag{19}$$

Analogously, we can write the capital-output ratio as

$$\frac{k}{y} = \theta^h \alpha\left(\frac{1}{R+\mu}\right)\left[\alpha + (1-\alpha)\left(\frac{w}{R+\mu}\right)^{1-\sigma}\right]^{-1}\frac{k}{\left(k-\kappa_k^h\right)}. \tag{20}$$

[36] An alternative assumption is that if the firm is unable to satisfy the capital non-homotheticity requirement, it remains dormant for that period and may produce later if it manages to obtain sufficient funds.

## B.3 Alternative Theory for Top Firms' Declining Input Shares

To simplify the notation we drop $i$ firm and $t$ time subscripts, and $h$ technology-type superscripts. We assume that the firm doesn't face financial frictions and the technology is homothetic, but their price is elastic $p(y)$ to output and wages are elastic $w(l)$ to labor, and input efficiency can change over time $z^j$ for $j = \{k, l\}$. Then, the firms solve the profit maximization problem

$$\max_{y,l,k} p(y)\, y - w(l)\, l - Rk - c_F \tag{21}$$

subject to

$$\begin{aligned} y &= p(\mathbf{s})\, x^{\theta} \\ x &= \left[\alpha^{\frac{1}{\sigma}} (z_k k)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}} (z_l l)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}. \end{aligned}$$

The FOC of problem (21) are

$$\begin{aligned} [l]: \quad & \left[\frac{\partial p(y)/p(y)}{\partial y/y} + 1\right] \frac{\partial y}{\partial l} p(y) = \left[\frac{\partial w(l)/w(l)}{\partial l/l} + 1\right] w(l) \\ [k]: \quad & \left[\frac{\partial p(y)/p(y)}{\partial y/y} + 1\right] \frac{\partial y}{\partial k} p(y) = R. \end{aligned}$$

We denote $\mu_w = \frac{\partial w(l)/w(l)}{\partial l/l} + 1$ as the proportional shadow cost of labor (i.e., if $\mu_w = 1$, there is no shadow cost), and $\frac{\partial p(y)/p(y)}{\partial y/y} + 1 = \frac{1}{\mu_y}$, where $\mu_y$ represents the markup (i.e., if $\mu_y = 1$ there is no markup). Thus, the FOC can be rewritten as

$$\begin{aligned} \frac{1}{\mu_y} \theta (1-\alpha)^{\frac{1}{\sigma}} z_l \frac{(z_l l)^{-\frac{1}{\sigma}}}{x^{\frac{\sigma-1}{\sigma}}} y p(y) &= \mu_w w(l) \\ \frac{1}{\mu_y} \theta \alpha^{\frac{1}{\sigma}} z_k \frac{(z_k k)^{-\frac{1}{\sigma}}}{x^{\frac{\sigma-1}{\sigma}}} y p(y) &= R. \end{aligned}$$

Combining the FOC we get the relationship

$$\frac{z_k k}{z_l l} = \frac{\alpha}{1-\alpha} \left(\frac{\mu_w w(l)}{R} \frac{z_k}{z_l}\right)^{\sigma}.$$

Again doing some algebra and combining with the FOC we find

$$\begin{aligned} \frac{w(l)\, l}{p(y)\, y} &= \frac{1}{\mu_y} \theta (1-\alpha)^{\frac{1}{\sigma}} \left[\alpha^{\frac{1}{\sigma}} \left(\frac{z_k k}{z_l l}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}}\right]^{-1} - \frac{(\mu_w - 1)\, w(l)\, l}{p(y)\, y} \\ \frac{k}{p(y)\, y} &= \frac{1}{\mu_y} \theta \alpha^{\frac{1}{\sigma}} \frac{1}{R} \left[\alpha^{\frac{1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}} \left(\frac{z_l l}{z_k k}\right)^{\frac{\sigma-1}{\sigma}}\right]^{-1}. \end{aligned}$$

Combining the previous three equations, we get the $wl/py$, $k/py$, and $k/wl$ ratios:

$$\frac{w(l)\,l}{p(y)\,y} = \theta(1-\alpha)\frac{1}{\mu_y}\left[\alpha\left(\frac{R}{\mu_w w(l)}\right)^{1-\sigma}\left(\frac{z_l}{z_k}\right)^{1-\sigma}+(1-\alpha)\right]^{-1} - \frac{(\mu_w-1)\,w(l)\,l}{p(y)\,y} \tag{22}$$

$$\frac{k}{p(y)\,y} = \frac{\theta\alpha}{R}\frac{1}{\mu^y}\left[\alpha+(1-\alpha)\left(\frac{\mu_w w(l)}{R}\right)^{1-\sigma}\left(\frac{z_k}{z_l}\right)^{1-\sigma}\right]^{-1} \tag{23}$$

$$\frac{k}{w(l)\,l} = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{z_l}{z_k}\right)^{1-\sigma}\left(\frac{\mu_w}{R}\right)^{\sigma}(w(l))^{\sigma-1} \tag{24}$$

## B.4 Input Non-Homotheticity Microfoundations

In this section, we provide a microfoundation for the production technology with input non-homotheticities using homothetic technologies. Without loss of generality, we simplify the non-homothetic technology to be $y = p(\mathbf{s})(x-\kappa_x)^{\theta}$, where $x$ input rental price is $p_x$, and there are no financing frictions. For this technology, we can solve analytically for the $x/y$ ratio as:

$$\frac{x}{y} = \frac{\theta}{p_x}\left[1+\frac{\kappa_x}{\left[\frac{\theta p(\mathbf{s})}{p_x}\right]^{\frac{1}{1-\theta}}}\right]. \tag{25}$$

**Input Indivisibilities** First, we can microfound this technology by assuming that firms produce using of two types of inputs $x$: a standard divisible input, $\tilde{x}$, and an indivisible input with a renting cost of $p_x\kappa_x$. The production technology is $y = p(\mathbf{s})\,\tilde{x}^{\theta}\mathbf{1}_x$ such that $\mathbf{1}_x = 1$ if the indivisible input was purchased. Thus, the firm profits are now

$$\pi = \max\left\{\underbrace{\max_{\tilde{x}} p(\mathbf{s})\,\tilde{x}^{\theta} - p_x(\tilde{x}+\kappa_x)}_{\text{rent indivisible input}}, \underbrace{0}_{\text{not produce}}\right\}.$$

Thus, if the firm decides to produce, the optimal choice of $\tilde{x} = \left[\frac{\theta p(\mathbf{s})}{R}\right]^{\frac{1}{1-\theta}}$ and input-to-output ratio is simply $\frac{\tilde{x}}{y} = \frac{\theta}{p_x}$. However, if we redefine the input as $x = \tilde{x}+\kappa_x$, then doing some algebra we get

$$\frac{x}{y} = \frac{\theta}{p_x} + \frac{\kappa_x}{y} = \frac{\theta}{p_x}\left[1+\frac{\kappa_x}{\left[\frac{\theta p(\mathbf{s})}{p_x}\right]^{\frac{1}{1-\theta}}}\right],$$

which is equivalent to (25).

**Input-TFP Complementarities** Next, we show how a technology featuring *learn-by-doing*, which decreases as firms grow, can serve as a microfoundation for the non-homothetic technology. Now we assume that the firm's TFP is $p(\mathbf{s}, x)$ and the production technology is

$y = p(\mathbf{s}, x) x^{\theta}$, then the profits are

$$\pi = \max_{x} p(\mathbf{s}, x) x^{\theta} - p_x x$$

the FOC is

$$\frac{\partial p(\mathbf{s}, x)}{\partial x} x^{\theta} + \theta p(\mathbf{s}, x) x^{\theta-1} = p_x.$$

Note that the term $\frac{\partial p(\mathbf{s},x)}{\partial x} x^{\theta}$ captures additional marginal benefit if there are complementarities. Also, the input-to-output ratio is now

$$\frac{x}{y} = \frac{\theta}{p_x} + \frac{\epsilon_{p,x}}{p_x},$$

where $\epsilon_{p,x} = \frac{\partial p(\mathbf{s},x)/p(\mathbf{s},x)}{\partial x/x}$ is the elasiticity of the productivity to the input choice. We can interpret this elasticity as capturing potential *learn-by-doing*. Finally, if we parametrize this elasticity as $\epsilon_{p,x} = \theta \frac{\kappa_x}{\left[\frac{\theta p(\mathbf{s})}{p_x}\right]^{\frac{1}{1-\theta}}}$, such that the complementarities are lower as productivity grows, the input usage in equilibrium will be equivalent to (25).

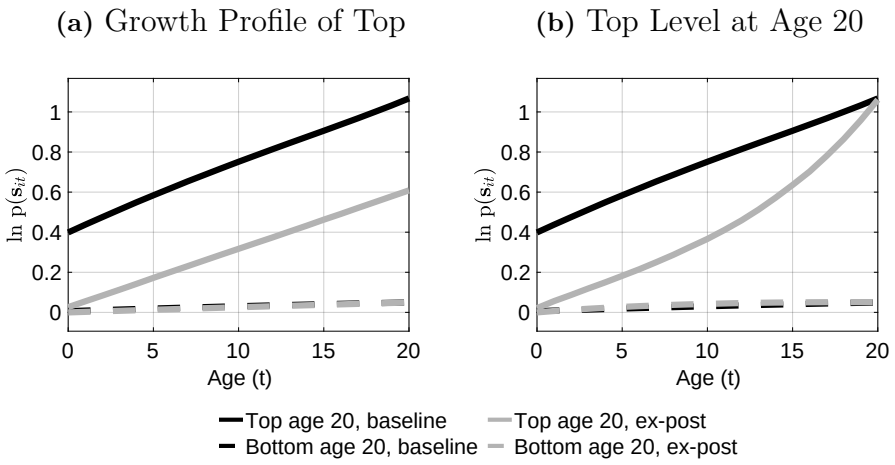
## B.5 Ex-Ante Heterogeneity and Top and Bottom Firms Growth

To provide intuition on how the initial level and growth profile of top and bottom firms inform the ex-ante heterogeneity component of our baseline TFP process, we present a simple example comparing a simplified version of our TFP process with a standard TFP process that includes only ex-post shocks. Figure B.1 shows the TFP life cycle path for our baseline TFP process without entry and exit selection and within type heterogeneity in the ex-post component ($\underline{\sigma}_u = 0$) and a TFP process without the ex-ante component, i.e., $p(\mathbf{s}_{it}) = \exp(z_{it})$ with $z_{it+1} = (1 - \rho_z)\mu_z + \rho_z z_{it} + \varepsilon_{it+1}$. For the baseline TFP process we use the parameters of the TFP process and the measure of potential entrants $\{\mathcal{E}_1, \mathcal{E}_1\}$ calibrated in Section 6, to match the observed initial output level and growth profile of top and bottom firms.

Panel (a) compares the simplified baseline TFP process with a process that excludes the ex-ante component, calibrated to roughly match the life-cycle profile of bottom firms and the growth profile of top firms (i.e., slope) implied by our baseline process. To match the top firms' growth profile, we need an extremely persistent shock ($\rho_z = 0.99$) and very low volatility ($\sigma_z = 0.05$). Importantly, even if this unusual calibration allows us to match the growth profile of top firms, it remains far from fitting its level.

On the other hand, in Panel (b), we calibrate the ex-post process to fit the life-cycle profile of bottom firms and the TFP level of top firms at age 20 implied by our baseline process. This can be achieved with a relatively standard persistence ($\rho_z = 0.9$) and a relatively high volatility ($\sigma_z = 0.17$). In this case, the TFP process with only ex-post shocks implies an extremely high growth profile for top firms at age 20, as the initial shocks dissipate over time,

**Figure B.1:** TFP Process: Baseline vs. Ex-Post Heterogeneity




*Notes*: The figure shows the average life-cycle profile of total factor productivity (TFP) for top 1% and bottom 99% firms at age 20 in the baseline model–assuming entry and exit selection margins and no ex-ante within-type heterogeneity (i.e., $\underline{\sigma}_u = 0$)–with top and bottom firms represented by black solid and dashed lines, respectively. These profiles are compared to those obtained when the TFP process includes an ex-ante component $p(\mathbf{s}_{it}) = \exp(z_{it})$ with $z_{it+1} = (1-\rho_z)\mu_z + \rho_z z_{it} + \varepsilon_{it+1}$, shown by gray solid and dashed lines, respectively. Panel (a) shows the life-cycle profiles for the baseline TFP process and an alternative TFP process where the ex-post shocks have high persistence and low volatility ($\rho_z = 0.99, \sigma_z = 0.05, \mu_z = 0.3$) such that it matches the bottom firms life cycle profile and growth profile of top firms (i.e., slope). Panel (b) shows the life-cycle profiles for the baseline TFP process and an alternative TFP process where the ex-post shocks have lowe persistence and high volatility ($\rho_z = 0.90, \sigma_z = 0.17, \mu_z = 0.07$), such that it matches the bottom firms life cycle profile and the top firms' TFP level at age 20.

causing the initial size of top firms at age 20 to be close to the average.

These examples illustrate that ex-ante heterogeneity in our model is informed by both the initial levels and growth profiles of top and bottom firms.

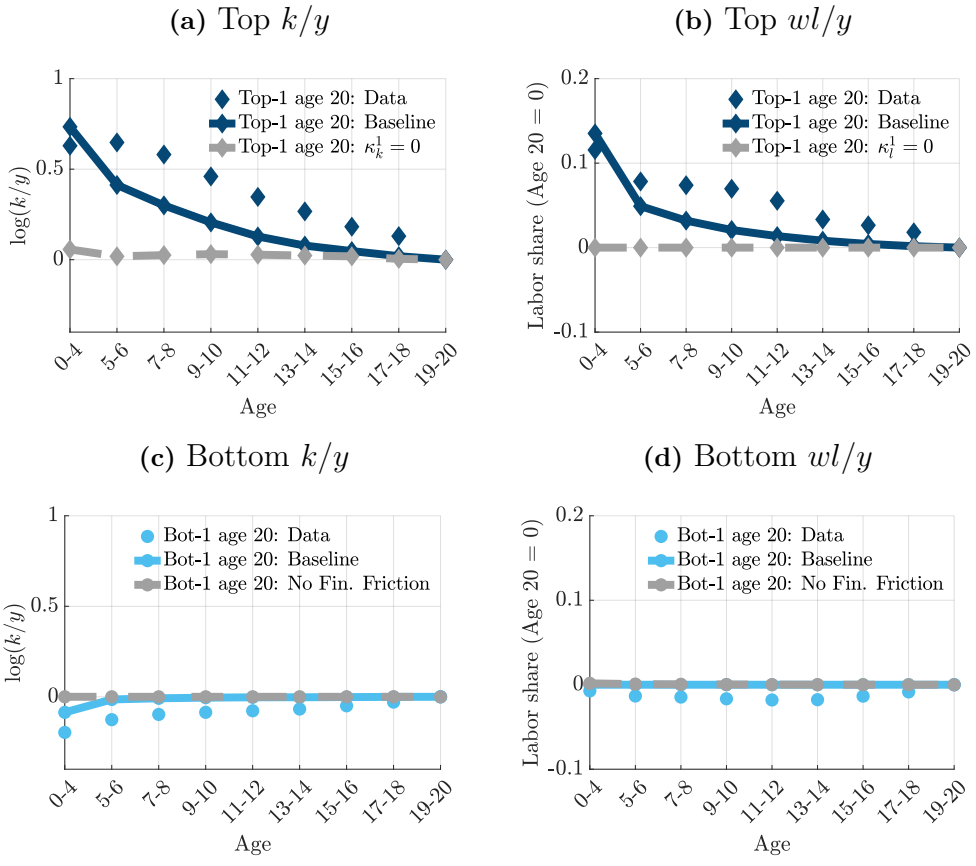
## B.6 Input Usage Over The Life Cycle: Quantitative Model

In this section, we study the determinants of input usage profiles among top and bottom firms over their life cycle in the quantitative model. First we analyze how the life cycle input usage profile of top firms changes when input non-homotheticities are removed. Figure B.2, Panel (a) and (b), present the results, for reference we include the data and the baseline model patterns. We find that, in the absence of non-homotheticities, the input usage of top firms remains flat or slightly increases, contradicting the data, which show a sharp decline in input usage. Next, Panel (c) and (d) display the life cycle input usage patterns of bottom firms when financing frictions are removed. In this scenario, bottom firms' input usage remains roughly flat, as it is no longer distorted. This profile is consistent with the labor share pattern but contradicts the capital-output ratio pattern.

## B.7 Top Firms' Taxation with Externalities

We provide additional details on the firm problem under size-dependent taxation.

**Figure B.2:** Role of Non-Homotheticities and Financing Frictions




*Notes*: The figure shows the life cycle pattern of input usage, capital and labor, for top and bottom firms, respectively, in the data (dots) and using the model-simulated data. Panel (a) shows the top firms input usage when setting $\kappa_k^1 = 0$ and $\kappa_l^1 = 0$ (gray lines). The trajectory is in logs and is normalized to equal 0 at age 20. Panel (b) shows the bottom firms input usage when there are no financial frictions (gray lines). The trajectory is in percentage points and it is normalized to 0 at age 20.

**Firm problem** Denote the profit function $\pi_{it}$, given state variables $(\mathbf{s}_{it}^h, a_{it})$, such that

$$\pi_{it}(y) \equiv \max_{l,k \leq \bar{k}^h\left(a_{it},\mathbf{s}_{it}^h\right)} p\left(\mathbf{s}_{it}^h\right) f^h(k,l) - wl - Rk$$

subject to $p\left(\mathbf{s}_{it}^h\right) f^h(k,l) = y$, where $\{k_{it}(y), l_{it}(y)\}$ solve the standard cost minimization problem. Denote $y^* \equiv \arg\max_y \pi_{it}(y)$. For $\tau > 0$, the firms decide between maximizing output and potentially paying taxes (if $y^* > \bar{y}$) or producing at the threshold and not paying corporate taxes (i.e., bunching):

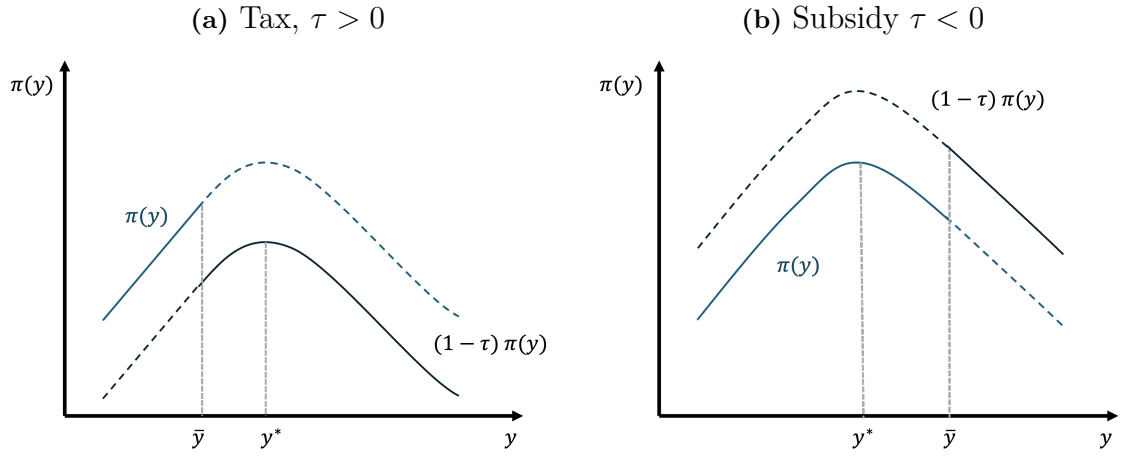
$$\max\left\{\pi_{it}(\bar{y}), \mathbb{I}_{\{y^*>\bar{y}\}}(1-\tau)\pi_{it}(y^*) + \mathbb{I}_{\{y^*\leq\bar{y}\}}\pi_{it}(y^*)\right\}.$$

Analogously, if $\tau < 0$ (subsidy) the choice is

$$\max\left\{\pi_{it}(\bar{y})(1-\tau), \mathbb{I}_{\{y^*>\bar{y}\}}(1-\tau)\pi_{it}(y^*) + \mathbb{I}_{\{y^*\leq\bar{y}\}}\pi_{it}(y^*)\right\}.$$

As a result, the tax may induce some firms to scale down to avoid paying it, while a subsidy may encourage others to scale up to receive it. Figure B.3 illustrates this distortion for the case of tax, in panel (a), and a subsidy, in panel (b).

**Figure B.3:** Top Firms' Static Distortions from Taxation




Beyond the static output distortion, taxes also affect firm value, influencing $\bar{k}^h\left(a_{it}, \mathbf{s}_{it}^h\right)$ disproportionately more for firms that are currently at the top or more likely to reach it. Additionally, by distorting firm returns, taxes impact equity financing and savings. These financing distortions can, in turn, lead to greater capital distortions for such firms. Furthermore, it is straightforward to see that distortions in firm value would also impact entry and exit decisions. The rest of the firm problem remains unchanged, as tax revenues are not rebated to firms.

**Tax revenues** Corporate tax revenues, $\text{TR} = \int_{y_i > \bar{y}} \tau \pi_i \, \mathrm{d}i$, are not rebated to firms; instead, it is as if they were rebated to households. Since we assume that the labor supply decision is not affected by income effects (e.g., GHH preferences), it remains undistorted by changes in output and tax revenues.[37]

**Externalities** There are two sources of externalities. Assume firm level productivity now is $Pp_i$ where $P$ is an aggregate common component, that is

$$P = \bar{P} \underbrace{\left( \int_{i \in h_{\text{top}}} p_i \mathrm{d}i \right)^{\Gamma_1}}_{\text{top tech spillovers}} \underbrace{\left( \frac{\int_{i \in \text{top } 1\%} y_i \mathrm{d}i}{Y} \right)^{-\Gamma_2}}_{\text{concentration}},$$

where $\bar{P}$ is a scaling parameter calibrated such, given externality parameters $\{\Gamma_1, \Gamma_2\}$, $P = 1$ when $\tau = 0$.

First, the top-technology spillover component of $P$ depends on the aggregate level of

[37] More generally, we can assume that $L^s \equiv \bar{L} \frac{w^{\gamma_L}}{(Y + \text{TR})^{\gamma_Y}}$, where $\gamma_L$ and $\gamma_Y$ capture substitution and wealth effects, respectively. In the baseline, we assume $\gamma_L = 2$ and $\gamma_Y = 0$.

top technologies, $\int_{i\in h_{\text{top}}} p_i \, \mathrm{d}i = \Omega_{\text{top}} \mathbb{E}\left[p_i \mid i \in \text{top}\right]$, which reflects both the measure of top-technology firms, $\Omega_{\text{top}}$, and their average productivity. This component can be interpreted as the potential benefit of having more advanced technologies in the economy, which may spill over to the rest in various ways. It is an externality because top firms do not internalize that their entry could affect the productivity of other firms. The importance of this externality depends on $\Gamma_1$, which is set to $\Gamma_1 = 0$ in the baseline.

The concentration externality component of $P$ depends on the share of top 1% firms output, $\int_{i\in\text{top }1\%} \frac{y_i}{Y} \mathrm{d}i$. The idea is that higher concentration could hinder aggregate productivity, as argued by the work that studies the relationship between market power and innovation (Aghion et al., 2005). The largest firms do not internalize that, by capturing a greater share of output, they reduce the productivity of other firms. The importance of this externality depends on $\Gamma_2$, which is set to $\Gamma_2 = 0$ in the baseline.

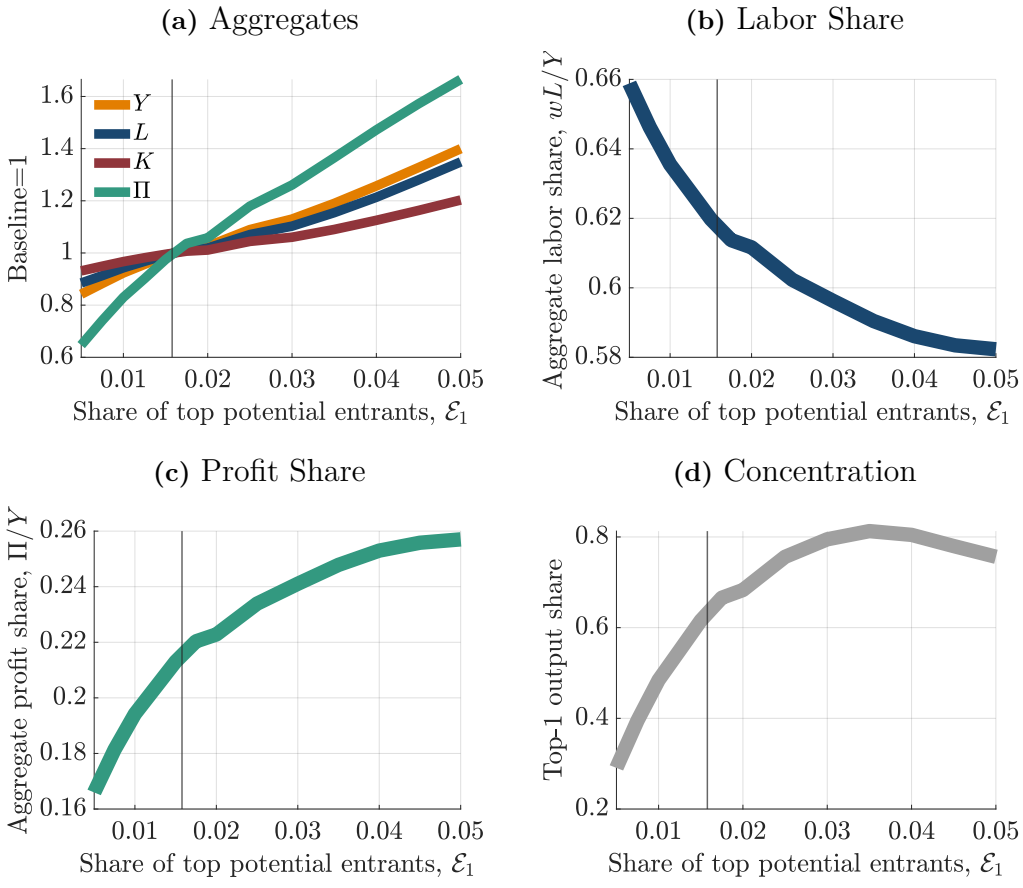
## B.8 Additional Figures and Tables

**Table B.1:** Long-Run Consequences of Interest Rate Increases

| | Top-Bot Firms Model | | Average Firm Model | |
|---|---|---|---|---|
| | $r \uparrow$ | $r \uparrow, \beta \downarrow$ | $r \uparrow$ | $r \uparrow, \beta \downarrow$ |
| *Relative to Baseline (=1)* | | | | |
| Output, $Y$ | 0.968 | 0.829 | 0.990 | 0.856 |
| Capital, $K$ | 0.985 | 0.755 | 0.993 | 0.766 |
| Labor, $L$ | 0.987 | 0.893 | 0.994 | 0.902 |
| Profits, $\Pi$ | 0.814 | 0.754 | 0.817 | 0.832 |
| Wage, $w$ | 0.993 | 0.948 | 0.996 | 0.949 |
| Incumbents, $h = 1$ | 0.905 | 0.690 | - | - |
| Incumbents, $h = 2$ | 1.031 | 0.922 | 0.957 | 0.824 |

*Notes*: 'Top-Bot Firms Model" refers our baseline model that targeting the life cycle of top and bottom firms. "Average Firm Model" corresponds to the model where we target the average firm life cycle. For each calibration, all variables are normalized relative to the baseline quantification. The interest rate increases ($r \uparrow$) from 2% to 3.5%, while the implicit discount rate decreases from $r_\beta$ to $r_\beta + 1.5\%$, where $r_\beta = 1/\beta - 1$, in both models, such that $\beta \downarrow$.

**Figure B.4:** Technological Frontier Changes




*Notes*: Steady state comparisons in GE solving for new equilibrium wage $w$. Aggregate variables in Panel (a) are normalized relative to the baseline economy.

**Figure B.5:** Top Firms' Subsidies and Taxes with Externalities




*Notes*: Steady state comparisons in GE solving for new equilibrium wage $w$. Aggregate output in Panel (a) are normalized relative to the no-tax steady state, which is the same across counterfactuals. $\Gamma_1 = 0.4$ is the case with strong positive technological spillovers. $\Gamma_2 = 0.4$ is the economy with strong negative concentration externalities.