Inelastic Capital*

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Abstract

How does an economy absorb a rise in the demand for savings? In the standard neoclassical growth model, where consumption and investment goods are produced in the same sector, long-run saving shocks are absorbed entirely through higher capital quantities rather than higher asset prices. In a general multi-sector model, with heterogeneous factor intensities and/or limited factor reallocation across sectors, this one-sector result breaks down and the long-run supply of capital is no longer perfectly elastic. We derive closed-form formulas for the long-run supply elasticity of capital quantity and asset prices in such models. After calibrating the model to U.S. sectoral data, we find that roughly half of a long-run increase in the demand for savings is absorbed through higher capital prices rather than higher capital quantity, owing to the importance of specialized high-skill labor in intangible investment. The model helps explain the "missing investment" of the last decades—when low interest rates translated into high valuations rather than higher investment. Our results also have implications for the incidence of capital taxation.

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Introduction

The neoclassical growth model treats capital formation as a frictionless process: a single final good can be used either for consumption or to accumulate capital. A well-understood implication is that the supply curve for capital goods is vertical (i.e., the relative price of capital goods must be one). As a result, shocks in the capital market (e.g., changes in preferences, taxes, etc) are entirely absorbed by real investment, without any valuation effects.

This prediction, however, is at odds with the data, where capital asset prices are much more volatile than quantities, even at long horizons. For instance, the global savings glut of the 2000s (see, e.g., the role of international capital flows Bernanke, 2007; demographics Auclert et al., 2021; and rising inequalities Mian et al., 2021) had little effect on capital formation in the US, but instead led to large revaluations. The q-theory of investment slows down the response of investment, but only in the short-run. A literature has emerged on the topic, emphasizing explanations related to rising markups and mismeasured intangible investments (see, e.g., Gutiérrez and Philippon, 2017; Farhi and Gourio, 2018; Crouzet and Eberly, 2023).

This paper takes a different route: we emphasize the fact that the production of capital requires specialized labor (scientists, managers, financiers) whose aggregate supply is inherently limited. For instance, roughly half of aggregate investment in the US is now accounted for by the production of "intellectual property" and "organizational services", both of which are high-wage, labor-intensive sectors. Even traditional investment sectors such as "construction" are much more labor-intensive than the economy as a whole. In both cases, scaling up aggregate investment implies a rise in the marginal cost of capital.

Using a sectoral production model, we show that this labor bottleneck in capital formation generates an upward-sloping supply curve for capital, even in the long-run. What this means is that, generically, shocks in the capital market are absorbed in part by prices, and in part by quantities. Formally, we calculate comparative statics formulas for the response of investment and relative prices to a demand shock (i.e., a shift in capital valuations that leaves technology unchanged). We then map those formulas to US input-output data using a sufficient statistic approach. We conclude that the world is roughly halfway between the neoclassical growth model (perfectly elastic capital) and an endowment economy (inelastic capital), depending on the time horizon and capital type considered.

Model. In Section 1, we start by calculating the long-run effect of a capital income tax shock on the capital stock as well as the price of capital relative to consumption. We start with a two sector model (consumption and investment) with two factors of production (labor and capital) in the spirit of Uzawa (1961). Production is Cobb-Douglas and labor supply is CES. We show that whenever investment require more labor than consumption or (ii) labor is imperfectly mobile across sectors, the cost of producing capital goods *rises* with aggregate investment (i.e., the capital

supply curve is upward sloping). Intuitively, as the production of investment goods increases, it puts upward pressure on the price of the factor it uses most intensively. As a result, factor prices endogenously move against investment good producers, which dampens capital accumulation.

Two sets of sufficient statistics govern the response of quantities and prices to demand shocks: (i) sectoral cost shares, (ii) the labor supply elasticity across sectors. We extend the theory to account for arbitrary production functions, multiple sectors, labor types, capital types, etc. In each case we describe which additional information is needed to quantify the model (for instance how cost shares respond to shocks when moving away from Cobb-Douglas).

Measurement. In Section 2, we use data from the national accounts to study the factor intensity of the consumption and investment goods sectors. In a companion paper (Gomez and Gouin-Bonenfant, 2025), we combine sectoral production data with the BEA's make-use matrices, combined with employment and compensation data from the BLS's Current Population Survey to construct a consolidated sectoral dataset. We allocate 100% of aggreate value-added to three broad sectors: consumption, tangible investment, intangible investment and calculate the labor embedded (both directly and indirectly) in sectoral output. We further decompose the within-sector labor content by education levels (i.e., no-college, college, phd).

For the purpose of this paper, two important facts emerge. First, the labor share has always been higher in the investment sector, the divergence has increased over time. Even over the 1970–1990 period, where intangible capital investment was more limited, the fact that the construction sectors is very labor-intensive made aggregate investment more labor-intensive that aggregate consumption. As the importance of r&d and the management sector increased, this total labor content gap has increased. Second, we highlight the fact that intangible-capital producing sectors disproportionately employ high-education and high-wage workers.

Quantitative predictions. In Section 3, we bring the model to the data in order to quantify the propagation demand shocks in a quantitative extension of the stylized model. We add three ingredients: (i) multiple types of labor and capital, (ii) type-specific labor supply elasticity, (iii) type-specific internal adjustment costs. For the calibration of sectoral technology in the model, we use our sectoral dataset combined with assumptions on the link between sectoral ownership of capital and sectoral use of capital services. For the calibration of the labor supply system, we draw on external evidence on sectoral labor supply elasticities.

The calibrated model predicts a low aggregate capital supply elasticity of roughly 1 in the short-run and 3 in the long-run. This low level of elasticity has first-order implications on the incidence of macro shocks, cutting the quantity effect of a given demand shock by half relative to the neoclassical growth model. We decompose the resulting price effect into the contribution of factor costs by type.

Finally, we use the calibrated model to quantify the incidence of corporate taxes in our econ-

omy. In the neoclassical growth model, capital tax cuts have a large effect on investment, and end up being born entirely by workers via higher wages. Instead, the model predicts that most of the incidence of a permanent tax cut is born by capitalists (via revaluation effects) and investment-sector workers (via higher wages). The real effect of the tax cut is modest, in line with the empirical evidence from Goolsbee (1998b) on the aggregate effects of investment tax credits.

Literature review. This paper revisits Uzawa (1961)'s insight that macroeconomic dynamics depend on technology asymmetry in the production of consumption versus investment. We consider the minimal departure from his model that can account for US data (mainly multiple types of capital and labor and imperfect labor mobility) and calculate the effect of demand shocks in the capital market on quantities and prices. This represents a fundamental question in medium-run macro and finance: whether we live in an inelastic economy (Lucas' orchard) or an infinitely-elastic one (neoclassical growth model) has radically different implications for welfare incidence and optimal policy (more on this shortly).

The core insight is that sectoral technology differences dampen the response of real investment in response to a demand shock, even in the long-run. Our contribution is to map the theory to the data using a sufficient statistic approach and generate quantitative predictions. Our measurement exercise builds on an accounting framework described in details in a companion paper Gomez and Gouin-Bonenfant (2025) (US data over 1960-2020). We document the fact that investment is more labor-intensive than consumption, and increasingly so with the rising important of r&d and organizational in investment. Earlier empirical work that measure the difference in the labor share in the consumption sector versus the investment sector include Chari et al. (1996), Cummins and Violante (2002) and Valentinyi and Herrendorf (2008) — our key contribution is to extend the definition of investment goods to intangibles, including organization capital (e.g., Eisfeldt and Papanikolaou, 2013; Peters and Taylor, 2017; Eisfeldt, Falato and Xiaolan, 2023; Corrado, Haskel, Jona-Lasinio and Iommi, 2022).[1] Note that we aggregate the economy activity in broad expenditure sectors (consumption versus investment) rather than measuring the full production and investment network at the industry-level (as in VanLoem-Foster, Canaudo, Malmberg). The reason is that we are interested in demand shocks (which lift the demand for capital goods), rather than industry-level shocks to productivity and markups (which reallocate inputs between industries).

Finally, our calibrated model provides an alternative interpretation of weak investment in the US post-2000, a period marked by low interest rates and high equity valuations. The existing literature focuses mostly on the role of evolving markups and market power (e.g., Gutiérrez and

[1]Intangible capital is thought to have distinct properties, relative to structures and equipment. In Eisfeldt and Papanikolaou (2013), intangible capital is unique because it is embedded in key talents. In Crouzet, Eberly, Eisfeldt and Papanikolaou (2022), intangible capital tends to be non-rival and is characterized by limited excludability, which prevents firms from capturing all rents. We instead focus on a distinct inputs in the production of intangible capital: it requires specialized labor in finite supply.

Philippon, 2017; Crouzet and Eberly, 2019; Barkai, 2020; Ball and Mankiw, 2023; De Ridder, 2024). Our analysis instead focuses only on Walrasian forces (i.e., price adjustments and sectoral reallocation of finite resources). Our theory is *not* about a disconnect between investment and q, but rather an inelastic supply of capital goods, which dampens the real effect of demand shocks. As discussed in Mussa (1977), our core channel, which works via equilibrium price adjustments, differs from models with capital adjustment cost functions (see, e.g., Lucas Jr, 1967; Uzawa, 1969; Hayashi, 1982).

1 Theory

We study a two-sector economy, that produces consumption and investment goods. We derive sufficient-statistic formula for the long-run elasticity of capital prices and quantity to shocks in the demand for savings. We show that saving shocks are partly absorbed through capital prices, instead of capital quantity, if there is heterogeneity in factor intensity across the two sectors or if there is limited reallocation of factors across the two sectors.

1.1 Stylized model

We start by considering a simple model that already delivers realistic long-run elasticities of capital prices and quantity to saving shocks. Production in each sector requires a combination of labor and capital. The model differs from the benchmark neoclassical growth model in two ways: the consumption and investment sectors have heterogeneous factor intensities and there is limited reallocation of labor across the two sectors. We introduce these two ingredients because they capture the main forces omitted by the standard one-sector model that make the long-run supply of capital less than perfectly elastic.

Supply side. We consider an infinite-horizon economy populated by representative firms and households. There are two factors, capital K and labor L, and two final goods, C and I. Production in each sector is Cobb-Douglas with constant return to scale:

$$C = A_C K_C^{\alpha_C} L_C^{1-\alpha_C}, \qquad \alpha_C \in (0,1), \qquad \text{(production– } C \text{ sector)}$$
$$I = A_I K_I^{\alpha_I} L_I^{1-\alpha_I}, \qquad \alpha_I \in (0,1), \qquad \text{(production– } I \text{ sector)}$$

A key difference with the usual one-sector neoclassical growth model is that we allow the exponent on capital to differ across the two sectors. Empirically, we will show in the next section that the consumption sector has a higher capital intensity than the investment sector; that is, $\alpha_C > \alpha_I$. Consumption is the numeraire and we denote by p_K the price of the investment good relative to the consumption good. The output of the investment sector adds to the total stock of capital,

$K = K_C + K_I$, which depreciates at rate δ:

$$K' = (1-\delta)K + I, \qquad \delta \in (0,1). \tag{capital accumulation}$$

Demand side. Household have CRRA utility with elasticity of intertemporal substitution ψ and impatience parameter β. Households can hold capital or one-period bonds (in zero net supply) with interest rate r. Household consumption choice implies the Euler equation:

$$\beta(1+r')\left(\frac{C'}{C}\right)^{-\frac{1}{\psi}} = 1. \tag{Euler}$$

We denote r_K the rental rate of capital and r the interest rate on one-period bonds. We assume that there is a wedge τ (interpretable as a capital income tax or some intermediation cost) that drives a gap between what firms pay and what households receive; that is, firms pay r_K while households receive $(1-\tau)r_K$. The no-arbitrage condition between capital and bonds gives

$$(1+r')p_K = (1-\tau)r'_K + (1-\delta)p'_K. \tag{capital valuation}$$

Households also supply a fixed quantity of labor to firms. Labor is imperfect reallocated across sectors. We denote w_C and w_I the wage in the consumption and investment sectors. Formally, the representative household choose a quantity of labor in each sector to maximize labor income $w_C L_C + w_I L_I$ subject to the constraint

$$1 = \left(\pi\, L_C^{\frac{1+\phi}{\phi}} + (1-\pi)\, L_I^{\frac{1+\phi}{\phi}}\right)^{\frac{\phi}{1+\phi}}, \qquad \pi \in (0,1),\ \phi \geq 0. \tag{labor supply}$$

The parameter ϕ corresponds to the elasticity of sectoral labor supply: $\phi = 0$ means fixed shares: labor cannot get reallocated across sectors. In contrast, the case $\phi \to \infty$ implies perfect mobility, i.e. $1 = L_C + L_I$.

This specification for the sectoral labor supply could be micro-founded via a Roy-Fréchet selection model. Formally, consider a continuum of households that draw a productivity pair (a_C, a_I) in each sector according to a bivariate Frechet distribution with shape ϕ. Each household chooses the sector in which they maximize their labor income — $a_C w_C$ if they work in the consumption sector and $a_I w_I$ if they work in the investment sector. Then it is well known that the labor supply side of the economy is equivalent to the one specified in (labor supply) for a representative household (see, for instance, McFadden, 1974, Eaton and Kortum, 2002).

Equilibrium. An equilibrium is a sequence $\{K_C, K_I, K, L_C, L_I\}$ and prices $\{w_C, w_I, r_K, p_K, r\}$ such that: (i) firms choose capital and labor to minimize cost and set output price equal to unit cost; (ii) households choose consumption and labor supply in each sector to maximize labor income; (iii) the market for capital and labor clear (iii) the equations for (capital accumulation) and (capital valuation)

are satisfied. There are three intertemporal equations (i.e. equations that relate variables across different periods): (capital valuation), (capital accumulation), and (Euler). The rest of the conditions constitutes the intratemporal block of our model, which pins down allocation and prices in each time given the state of the economy, as summarized by p_K and K.

In the rest of the section, we particularly focus on the steady-state of the economy ($C = C'$, $p_K = p'_K$ and $K = K'$). In this case, the intertemporal equations simplify to:

$$r = \frac{1}{\beta} - 1, \quad I = \delta K, \quad p_K = (1-\tau)\frac{r_K}{r+\delta}.$$

The first equation says that the long-run interest rate is pinned down by the households preference parameters. The second equation says that the long-run investment-to-capital ratio is pinned down by the depreciation rate of capital. The third equation says that the long-run price of capital is ratio between its cash-flow and the sum of the interest rate and the depreciation rate. One way to understand this equation is to realize that capital is an intermediary input; like all intermediary inputs, its price is equal to its marginal return in production, with the key subtlety that capital can be used in all future periods, and so its price corresponds to the present value of its future returns.

Comparative statics. We are interested in how capital prices and quantities respond to a long-run shift in the demand for capital, which we capture through a change in τ. Note that the parameter τ only affects the equilibrium through the capital valuation equation — in particular, it corresponds to a shift in the demand for capital leaving the supply side completely unchanged. Changes in τ are actually isomorphic to changes in the impatience parameter of households, and, more generally, can be thought of anything that changes the demand for savings holding fixed the supply of the economy.[2]

In the neoclassical growth model, consumption and investment goods are produced with the same technology, so the supply side pins down the relative price of investment goods. In steady state, (capital valuation) then implies that, for any decrease in capital tax τ, the gross rental rate r_K decreases one-for-one so that the after-tax rental rate $r_K(1-\tau)$ is unchanged. The capital stock then adjusts so that its marginal product equals to this new rental rate. Hence, the long-run elasticity of the capital-good price is zero, and the long-run elasticity of the capital stock is $1/(1-\alpha)$, the inverse of one minus the Cobb-Douglas exponent on capital (i.e., the inverse of the labor share).

In a multi-sector model, however, the technology side no longer pins down the relative price of capital. When demand for savings increases, the economy reorients factors of productions form the consumption to the investment sector, but that increases the relative cost of producing investment good relative to consumption good. As a result, the supply of capital is not perfectly

[2]More precisely, for any endogenous object X such as capital quantity or its price, we have

$$\frac{\mathrm{d}\log X}{\mathrm{d}\log\left(\beta^{-1}\right)} = \frac{1+r}{r+\delta}\frac{\mathrm{d}\log X}{\mathrm{d}\log(1-\tau)}.$$

elastic and some of the shock in the demand for capital is absorbed through a higher capital prices rather than higher quantity.

The following proposition determines the long-run elasticity of capital prices and quantity in the stylized model. We denote λ_C and λ_I the steady-state nominal output shares of the two sectors and λ_K and λ_L the steady-state income shares of capital and labor.

Proposition 1.1 (Long-run elasticities)**.** *The long-run elasticities of the price and quantity of capital with respect to capital-demand shock are:*

$$\frac{\mathrm{d}\log p_K}{\mathrm{d}\log(1-\tau)} = \underbrace{\frac{\alpha_C-\alpha_I}{1-\alpha_I}}_{\text{technology asymmetry}} + \underbrace{\frac{1}{1+\phi}\frac{1-\alpha_C}{\alpha_C}\frac{\lambda_K}{\lambda_C}}_{\text{reallocation friction}},$$

$$\frac{\mathrm{d}\log K}{\mathrm{d}\log(1-\tau)} = \frac{1}{\lambda_L}\Big(1 - \underbrace{(\alpha_C-\alpha_I)\left(\frac{\lambda_C}{1-\alpha_I}+\frac{\lambda_I}{\alpha_C}\right)}_{\text{technology asymmetry}} - \underbrace{\frac{1}{1+\phi}\frac{1-\alpha_C}{\alpha_C}\lambda_K}_{\text{reallocation friction}}\Big).$$

The proposition characterizes the long-run elasticities of the capital price and quantity to a shift in capital demand. In the special case of the one-sector neoclassical growth model ($\alpha_C = \alpha_I$ and $\phi = \infty$), the proposition gives the classical result: the elasticity of the capital price is zero, while the elasticity of the capital stock is the inverse of the labor share.

Outside of this special case, the elasticity of the capital price is the sum of two components. The first reflects *technology asymmetry*, and is positive when $\alpha_C > \alpha_I$, i.e., when the investment sector is more labor-intensive than the consumption sector. Intuitively, if this is the case, anything that makes labor more expensive relative to capital (such as a shift in the demand for capital) raises the cost of producing investment goods relative to consumption goods. This is exactly the same logic as in the Stolper-Samuelson theorem in international trade: the price of a good increases with the price of the factor used more intensively in its production.

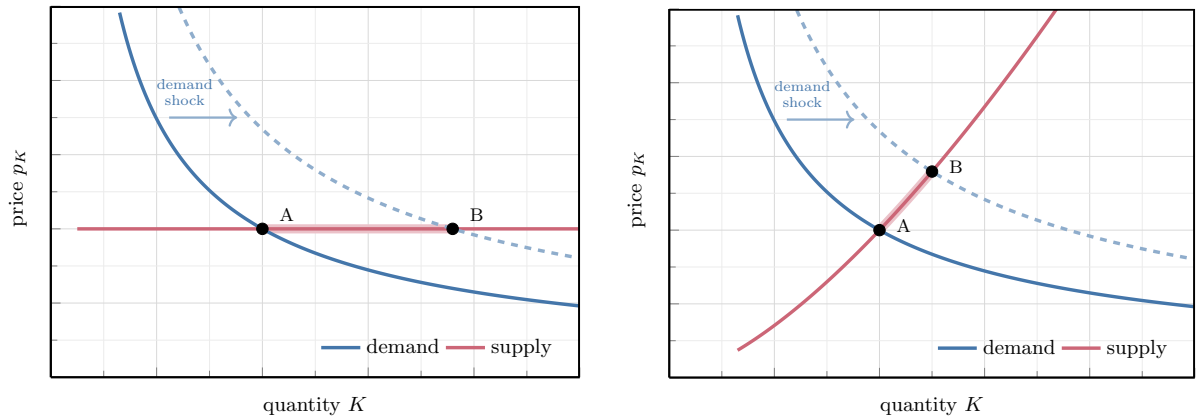
The second term reflects *limited labor mobility*, and is positive as long as ϕ is finite. Intuitively, as soon as $\phi < \infty$, it is costly to reallocate labor from C to I, which creates a bottleneck when the economy tries to respond to a shock to the demand for savings by producing more capital. This bids up the price of labor in the investment sector relative to the consumption sector, which in turn pushes up the relative price of capital.

The long-run elasticity of capital quantity is the sum of three terms. The first corresponds to the one-sector neoclassical benchmark, while the remaining two are dampening terms with exactly the same interpretations as the two components of the price elasticity.[3]

Figure 1 illustrates the situation graphically, by comparing a shift in the demand for capital

[3] In fact, in this model there is a direct mapping between the elasticity of capital prices and the extent to which the quantity response is dampened:

$$\frac{\mathrm{d}\log K}{\mathrm{d}\log(1-\tau)} = \frac{1}{\lambda_L}\left(\frac{\lambda_K}{\alpha_C}\,\mathrm{d}\log(1-\tau) - \lambda_C\,\mathrm{d}\log p_K\right).$$



(a) Neoclassical-growth model: $\alpha_C = \alpha_I$ and $\phi = \infty$ (b) Stylized model: $\alpha_C > \alpha_I$ or $\phi < \infty$

Figure 1: Equilibrium in the capital market after a shift in capital demand

Figure 1 plots the long-run supply and demand curves in the capital market. The long-run *demand curve* for capital is the set of (p_K, K) pairs that satisfy the intratemporal block together with $p_K = (1-\tau)r_K/(r+\delta)$: it tells us the maximum price households are willing to pay for each K. The long-run *supply curve* collects (p_K, K) pairs that satisfy the intratemporal block together with $I = \delta K$: it is the K the economy can deliver at a given p_K. In the neoclassical benchmark (symmetric technologies, perfect mobility), the supply curve is flat, since the relative price is pinned down by technology. With $\alpha_C \neq \alpha_I$ or $\phi < \infty$, supply slopes up: raising the output of the investment sector relative to consumption sector raises its relative cost.

in the one-sector neoclassical growth model versus our stylized two-sectors model. The more inelastic the capital supply (a steeper supply curve), the more the shock is absorbed through prices rather than quantities. In the neoclassical limit, with perfectly elastic capital supply, demand shocks translate entirely into quantities. In a more realistic model, where the supply of capital is not perfectly elasticity, demand shocks are absorbed by a mix of higher quantity and higher capital prices.

Response of factor prices. The positive elasticity of the capital price to demand shocks has direct implications for the incidence of saving (capital-tax) shocks. In the one-sector neoclassical growth benchmark, a decrease in the capital tax is fully absorbed by higher capital quantity (instead of higher capital prices): in the long-run, the post-tax rental rate of capital remains the same while the wages of workers increases, reflecting the higher marginal revenue of labor. In our stylized model, however, capital is less inelastic, and so the incidence of lower capital taxes is partly tilted towards capitalists.

To see why, use the fact that, due to unit cost pricing in each sector, the weighted change in output prices in the economy must equal the weighted change in factor prices:

$$\lambda_I \, \mathrm{d} \log p_K = \lambda_K \, \mathrm{d} \log r_K + \lambda_L \, \mathrm{d} \log w,$$

where $\mathrm{d} \log w$ denotes the change in the aggregate wage—the income-share-weighted average of w_C and w_I. Because the consumption good is the numeraire, the left-hand-side only shows the change in the price of the investment good.

Now, in the long-run, the steady state valuation equation implies $\mathrm{d}\log p_K = \mathrm{d}\log r_K + \mathrm{d}\log(1-\tau)$. and so we can directly write the change in factor prices to changes in the price of capital:

$$\mathrm{d}\log w = \frac{\lambda_K}{\lambda_L}\,\mathrm{d}\log(1-\tau) - \frac{\lambda_K - \lambda_I}{\lambda_L}\,\mathrm{d}\log p_K.$$

In the neoclassical benchmark, $\mathrm{d}\log p_K = 0$, so a decrease in taxes does not affect the post-tax compensation of capital holders while wages increase by λ_K/λ_L. Put differently, workers entirely capture the effect of lower taxes through higher wages. However, in our stylized model with $\alpha_C \neq \alpha_I$ and/or $\phi < \infty$, part of the shock is absorbed by p_K, so the pre-tax rental rate falls by *less than* one-for-one and wages rise by *less than* λ_K/λ_L. In incidence terms, some of the shock is borne by capital holders (though higher post-tax capital payments) rather than workers.[4] We will quantify the welfare incidence of capital taxes more precisely in the calibrated model in Section 3.

1.2 Extensions

Our stylized model departs from the neoclassical growth benchmark along two axes: (i) heterogeneous factor intensities across the consumption and investment sectors; and (ii) limited sectoral labor mobility. The extensions below show that adding realistic ingredients preserves this mechanism and, if anything, further dampens the long-run response of investment to demand shocks.

Multiple labor types In the stylized model there is one fixed factor. In reality there are several: multiple skill-types of labor and, potentially, land. Land is supplied inelastically within each sector ($\phi_{\text{land}} = 0$): housing sites, industrial plots, and farmland are geographically fixed and cannot relocate across sectors, consistent with the sector-specific capital assumption ($\xi = 0$). In Appendix C we extend the model to allow for these additional factors. The key takeaway is that introducing multiple labor types adds *composition* differences between the consumption and investment sectors. Even if total labor shares coincide across sectors, the mix of types need not, which makes reallocation from C to I more inelastic and magnifies the price and quantity effects described above.

Arbitrary production functions The stylized model assumes Cobb-Douglas production. Appendix C derives the formulas for arbitrary production functions. In general, the long-run elasticities of p_K and K depend on the relevant substitution elasticities. A higher elasticity of substitution between capital and labor increases the quantity response of K, as in the neoclassical benchmark.

[4] More precisely, the change in the net income earned by capital holder who maintains their stock of capital constant is

$$\mathrm{d}\log\left((1-\tau)r_K\right) - \frac{\lambda_I}{\lambda_K}\,\mathrm{d}\log p_K = \frac{\lambda_K - \lambda_I}{\lambda_K}\,\mathrm{d}\log p_K.$$

Nevertheless, heterogeneity in factor intensities ($\alpha_C \neq \alpha_I$) or reallocation frictions ($\phi < \infty$) continues to dampen the adjustment of K to demand shocks. Moreover, the price elasticity increases with the elasticity of substitution of the consumption sector relative to the investment sector. Intuitively, the less flexible the production technology in the investment sector relative to the consumption sector, the more costly it is to build more capital goods relative to consumption goods.

Internal investment In the stylized model, firms purchase capital directly from the investment sector. In the data, installed capital also uses internal inputs (engineering time, downtime, materials). We therefore interpret the investment sector broadly as the technology that transforms purchased goods together with any internal inputs into installed capital. This relabeling is without loss under separability: the technologies for consumption and for installation do not share inputs.

We could also allow the installation technology to depend on the existing capital stock. Although this makes the production function of consumption and capital goods non-separable (since the same capital is used twice in the production process), it does not alter the steady-state comparative statics as long as the adjustment function is homothetic. This is because, in the long-run, the creation of capital must still equal δK, which pins down the investment as a fraction of the existing capital, and so the price of installed capital relative to the price of investment goods (one in standard specifications). While capital adjustment costs do not matter for long-run elasticities, they matter for transitory dynamics, as we will show in the calibrated model in Section 3.

Markups So far we assumed prices equal marginal cost. Appendix A.3.5 allows for exogenous markups that may differ across C and I. The core logic is unchanged: what matters for the long run are *cost shares* rather than income shares. Empirically, identifying markups is therefore essential to separate profit shares from capital shares when mapping the theory to data.

Knowledge spillovers We have treated TFP as fixed in the baseline model. However, growth theories often incorporate the idea that producing more capital may raise TFP (e.g., learning by doing or spillovers of intangible capital). This type of effect would increase the long-run elasticity of K to demand shocks. The price response depends on the *difference* in spillovers across sectors because p_K is the price of capital relative to the consumption good (the numeraire): stronger spillovers in I dampen the price elasticity, whereas stronger spillovers in C amplify it. Appendix A.3.6 develops these results.

2 Measuring factor shares in the consumption and investment sectors

In the previous section, we showed that the long-run supply elasticity of capital depends on heterogeneity between the investment and consumption sectors. We now provide empirical evidence

on the factors required to produce consumption and investment goods in the U.S. economy. Our main finding is that producing investment goods is more labor-intensive. Moreover, intangible forms of investment rely disproportionately on specialized labor, which is in finite supply in the economy.

2.1 Datasets

Input-output tables. We begin with the BEA Use Table, available from 1963 to 2023 (U.S. Bureau of Economic Analysis, 2024a). Each row reports the uses of a commodity. The first set of columns reports the use of the commodity as an intermediate input by different industries. The remaining columns report final uses: private consumption; private investment-split into equipment, structures, and intellectual property products (IPP); government consumption; government investment; and exports/imports. The BEA also reports a Make Table, which indicates, for each commodity (column), how much of it is produced by each industry (row).

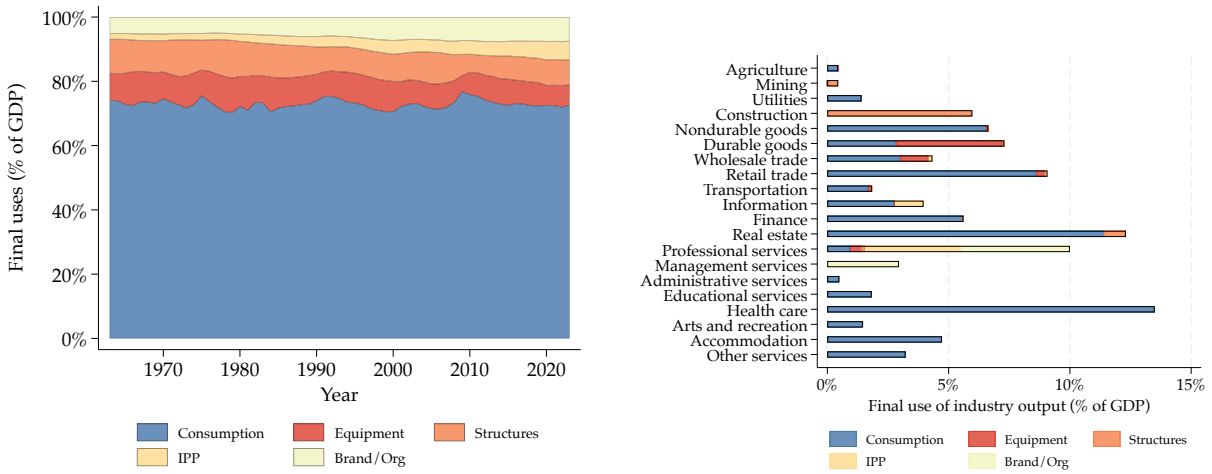
For all results below, we work at the level of the 71 BEA "summary commodities," since this level is available annually back to 1963. In Appendix B.2, we show that the results are similar when applying the same methodology to the more detailed 303-commodity classification (at least in 2017, when both levels of aggregation are available).

We modify the input-output tables as follows. Following Corrado et al. (2009), we adopt a broader definition of intangible investment than the BEA. In particular, we reclassify firm expenditures on organizational capital-expenditures related to workforce human capital, distribution systems, logistics, product design, and customer/brand capital-from intermediate expenditures to investment (Atkeson and Kehoe, 2005; Eisfeldt and Papanikolaou, 2013; Crouzet and Eberly, 2021; Crouzet et al., 2022). Concretely, we focus on two commodities purchased by industries: "Miscellaneous professional, scientific, and technical services" and "Management of companies and enterprises." The first category includes externally purchased services such as design services, management consulting, advertising, and computer system design. The second captures the service output of establishments that manage other units within a company.

Figure 2a plots the composition of aggregate investment as a fraction of GDP. Total investment averages about 20% of GDP and is fairly stable over time. As is well known, investment in equipment and structures has declined secularly, while investment in intellectual property products and organizational services has risen.

Figure 2b plots, for each BEA commodity, the breakdown of final uses between consumption and our four investment categories. Unsurprisingly, structures are concentrated in construction, equipment in durable goods, and IPP/organizational services in information and professional services.

KLEMS. We merge the input-output data with the Integrated Industry-Level Production Account (U.S. Bureau of Economic Analysis, 2024b), which allocates each industry's revenues into



(a) Final uses over time (% of GDP)

(b) Final uses by commodity (2014–2023)

payments to high-school-educated workers, college-educated workers, capital income, and intermediate expenditures, annually from 1963 to 2023.

We modify KLEMS in two ways. First, while KLEMS splits labor expenditures into non-college versus college, we further split the college category into (i) college degree holders and (ii) post-college degrees, using the industry-level split of labor income reported in the Current Population Survey.

Second, we split what KLEMS reports as "capital income" into compensation to land, equipment, structures, IPP, and organizational capital. We assume land payments account for 18% of total revenues in agriculture, 24% in residential housing, and zero elsewhere. We then allocate the remaining capital income across equipment, structures, IPP, and organizational capital in proportion to industry investment rates: using BEA Fixed Asset Tables for equipment/structures/IPP and the input-output tables for organizational services.[5]

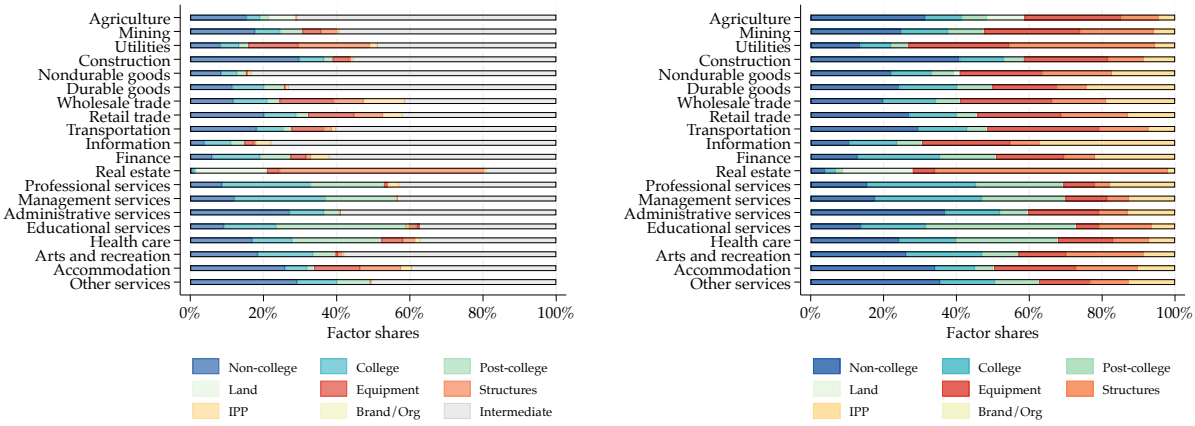
2.2 Methodology

We compute factor shares of the consumption versus investment sectors in three steps, which we sketch below. In the first step, we convert factor shares at the industry level (as reported in KLEMS) into factor shares at the commodity level. Indeed, it is important to realize that, there no one-to-one mapping between the concept of industries and commodities used by BEA: industries typically produce multiple commodities (as shown in the Make Table). For our baseline results, we adopt the common commodity-technology assumption: we assume that each commodity has a unique input structure, regardless of which industry produces it. This allows us to infer commodity level factor shares from industry-level factor shares after inverting the make table (see

[5]In steady state, capital income equals the investment rate multiplied by $(r+\delta)/(r+g)$, where r is the interest rate, δ the depreciation rate, and g the growth rate. We therefore split capital income shares using these scaled investment rates for each capital type.

Appendix B.1 for more details).

The second step is to convert factor shares at the commodity level into *consolidated* factor shares, which account for both direct and indirect factor usage. More precisely, the consolidated share of a factor for some commodity includes not only the factor used in the commodity's own value added, but also the factor embodied in all intermediate inputs along the production chain. To do so, we multiply direct-requirements factor shares by the Leontieff inverse of the direct commodity by commodity requirement matrix (see Appendix B.1 for more details). Figure 3, which contrasts direct-requirements factor shares at the industry level (Figure 3a) with total-requirements shares at the commodity level (Figure 3b).



(a) Direct-requirements factor shares

(b) Total-requirements factor shares

Figure 3: Factor shares and final uses by commodity

In the third step, we obtain consolidated factor shares for each final-use sector (consumption or investment sector) by aggregating the commodity-level consolidated factor shares, with weights corresponding to each commodity's share in the sector's final expenditures — its share in total consumption spending for the consumption sector, and its share in total investment spending for each investment sector.

2.3 Results

Table 2 reports the resulting factor shares. The first row reports factor shares for the aggregate private economy (a final-use-weighted average across commodities used in private consumption or private investment). We find an average labor share of 49%, which is somewhat lower than common benchmark numbers (see, e.g., Koh et al. (2020))-mainly because, relative to much of the literature, we include housing and utilities, which have relatively low labor shares. The next two rows report factor shares for the consumption and investment sectors. The labor share is 45% in consumption versus 59% in investment, a difference of 14 pp. Most of this gap reflects differences in skilled labor: the college labor share is 11 pp higher in investment than in consumption.

The remainder of the table reports factor shares for the four investment subcategories (equipment, structures, IPP, and organization). The higher labor share of investment is driven almost entirely by intangible investment: the average labor share for tangible investment (equipment + structures) is 52%, while it rises to 67% for intangible investment (IPP + organization). The other columns report capital shares by asset type. One key finding is that consumption and investment have similar intensities for equipment and organizational capital. However, consumption relies more on structures (reflecting housing consumption), while investment-through IPP-producing sectors-relies more on IPP capital, consistent with these sectors reinvesting part of their own output.

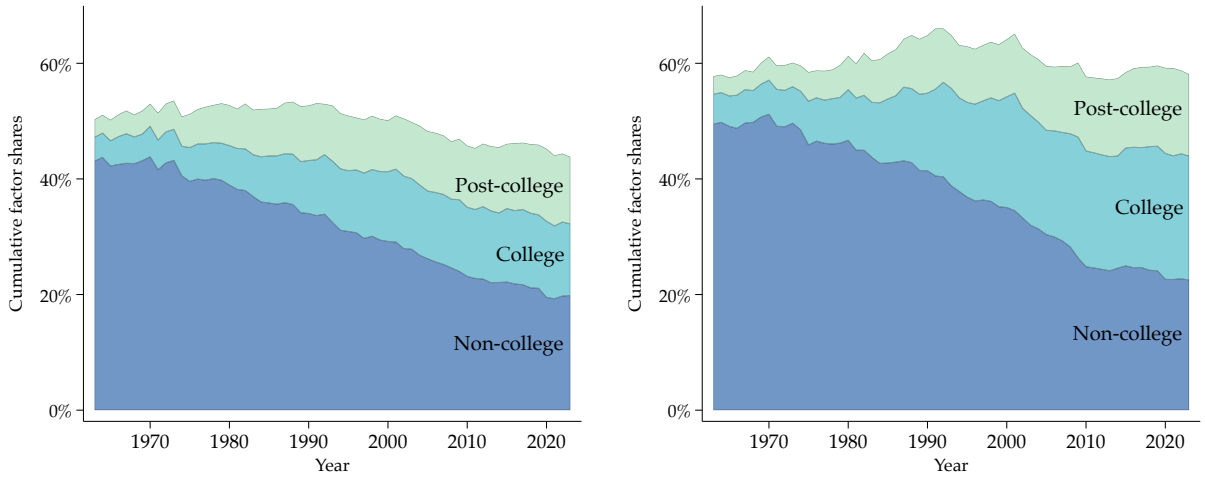
Table 1: Factor shares by sector (2014–2023)

	GDP share	Factor shares							
		Labor			Land	Capital			
		Non-college	College	Post-college		Equipment	Structures	IPP	Brand/Org
$C + I$	1.00	0.22	0.15	0.12	0.03	0.12	0.15	0.09	0.12
C	0.73	0.21	0.13	0.12	0.04	0.12	0.19	0.07	0.12
I	0.27	0.24	0.21	0.14	0.00	0.12	0.06	0.12	0.12
$I_{\text{Equipment}}$	0.07	0.23	0.16	0.10	0.00	0.14	0.07	0.16	0.14
$I_{\text{Structures}}$	0.08	0.37	0.12	0.06	0.00	0.16	0.09	0.05	0.14
I_{IPP}	0.05	0.15	0.28	0.19	0.00	0.08	0.04	0.17	0.09
$I_{\text{Brand/Org}}$	0.08	0.17	0.30	0.23	0.00	0.07	0.04	0.11	0.09

Figure 4 plots labor shares (and their subcomponents) over the full sample, 1963–2023. The striking pattern is that while the labor share declines in the consumption sector, it remains roughly constant in the investment sector. As a result, the gap in labor shares between investment and consumption rises from 7 pp at the start of the sample (1962–1968) to 14 pp at the end (2014–2023). In both sectors, an increasing share of labor compensation goes to college-educated workers relative to non-college workers. This shift is more pronounced in investment, where payments to college workers rise more rapidly than in consumption. In Appendix B.2, we show that the growing labor-share gap between investment and consumption is entirely driven by organizational services, which have a high labor share. If these services are instead treated as intermediate inputs, as in the BEA's baseline classification, labor shares decline similarly in both sectors.

2.4 Robustness

In constructing factor shares for the consumption and investment sectors, we necessarily make a number of assumptions. We now examine robustness to alternative choices along several dimensions. Table 2 reports results for both the total labor share and the college-educated (or above) labor share. Across specifications, labor shares remain higher in investment than in consumption, though the magnitude of the gap ranges from roughly 10 pp to 20 pp. We discuss each exercise in



(a) Consumption sector

(b) Investment sector

Figure 4: Evolution of labor share in consumption and investment sectors

turn.

Table 2: Labor share by sector (2014–2023)

	C	I	$I - C$
Panel A: Total labor share			
Baseline	0.50	0.61	+0.11
Without capitalizing organizational services	0.54	0.64	+0.10
Capitalizing financial services	0.48	0.58	+0.10
Including government sector	0.53	0.61	+0.08
Cost-shares instead of income-shares	0.67	0.81	+0.14
Using common industry-technology assumption	0.51	0.59	+0.08
Panel B: College (or above) labor share			
Baseline	0.18	0.23	+0.05
Without capitalizing organizational services	0.20	0.22	+0.01
Capitalizing financial services	0.17	0.23	+0.06
Including government sector	0.21	0.23	+0.02
Cost-shares instead of income-shares	0.24	0.31	+0.07
Using common industry-technology assumption	0.19	0.22	+0.03

Without capitalizing organizational services. The first exercise follows the BEA's current treatment and does not capitalize organizational services. Under this definition, the difference in labor shares across sectors is substantially smaller-by about 7 percentage points. This is unsurprising, since the treatment of intangible investment is central to the baseline difference in labor shares between investment and consumption. One concern might therefore be that our results are sensitive to this recategorization. However, in the calibrated model we allow different types of capital to have different depreciation rates, with intangible capital depreciating faster than structures. From

this perspective, organizational services need not be viewed as categorically different from intermediate inputs, but rather as an intermediate form of investment characterized by a higher depreciation rate. Allowing for multiple investment goods with heterogeneous depreciation schedules is therefore a natural modeling choice.

Capitalizing organizational and financial services. Conversely, we consider a broader definition of intangible capital than Corrado et al. (2022), by also capitalizing part of the services in commodity 521CI ("Federal Reserve banks, credit intermediation, and related activities") and commodity 523 ("Securities and commodity contracts intermediation and brokerage"). The motivation is that these services may reflect costs firms incur-beyond the purchase of physical investment goods-to install productive capital. This alternative classification has little effect on our results, largely because these intermediate expenditures account for less than 3 percent of GDP.

Collapsing private and public sectors. In a third check, we incorporate government activity. As in the private sector, government output can be decomposed into government consumption and government investment. Under this classification, we find that labor shares are higher in the consumption sector. This result is driven in part by the large role of defense spending in government consumption, which tends to be relatively labor-intensive.

Markups. We next examine robustness to markups. In our baseline accounting, we treat the portion of revenues not paid to labor as compensation for capital, interpreted as the return to past investment expenditures. However, some of this residual may instead reflect markups-pure profits unrelated to factor payments (Barkai, 2020). If so, our procedure may overstate capital shares. For our purposes, the relevant object is the difference in cost shares between consumption and investment, which governs technological asymmetry in the model (see Appendix A.3.5). To assess robustness, we adjust factor shares using markup estimates from De Loecker et al. (2020) for public firms, and then average them across industries-implicitly assuming that markups in Compustat are representative for the broader economy. Accounting for markups increases the difference in labor cost shares between investment and consumption, reflecting higher estimated markups in investment-oriented industries.

Common industry-technology assumption. To infer commodity-level factor shares from industry-level factor shares (the ones reported in KLEMS), we used the common commodity-technology assumption, which posits that technology is commodity specific. An alternative is use the common-technology assumption, which posits that technology is industry specific. The factor shares of a commodity can then be constructed as the weighted average of industry-level factor shares, where the weights come from the BEA make matrix. Note that, under this alternative, the labor-share gap between the investment and consumption sectors is smaller. The reason is that, by construc-

tion, commodity-level factor shares are obtained by smoothing out industry-level factor shares, reducing the heterogeneity in factor shares across commodities.

3 Quantitative model

We now calibrate an extended version of the stylized model from Section 1, quantify the implied dynamic response to demand shocks, and finally simulate the welfare incidence of capital taxation.

We introduce $N = 4$ capital types (equipment, structures, IPP, organizational) and $F = 4$ fixed factors (non-college labor, college labor, post-college labor, and land), yielding $1 + N = 5$ sectors (consumption and four investment goods). The heterogeneity in factors shapes how demand shocks affect the income of different factors. For instance, a demand shock for intangible capital would disproportionately benefit scientists, and a housing demand shock would favor construction workers, etc.

Table 3: Linearized model equations

Intratemporal block	Equation
C pricing	$0 = \sum_m \alpha_{CK_m} \, \mathrm{d}\log r_{K_m} + \sum_f \alpha_{CL_f} \, \mathrm{d}\log w_{Cf}$
$I_{n'}$ pricing	$\mathrm{d}\log p_{I_{n'}} = \sum_m \alpha_{I_{n'}K_m} \, \mathrm{d}\log r_{K_m} + \sum_f \alpha_{I_{n'}L_f} \, \mathrm{d}\log w_{I_{n'}f}$
K_{Cn} demand	$\mathrm{d}\log K_{Cn} = \mathrm{d}\log C + \sum_{m\neq n} \alpha_{CK_m} \, \mathrm{d}\log \frac{r_{K_m}}{r_{K_n}} + \sum_f \alpha_{CL_f} \, \mathrm{d}\log \frac{w_{Cf}}{r_{K_n}}$
$K_{I_{n'}n}$ demand	$\mathrm{d}\log K_{I_{n'}n} = \mathrm{d}\log I_{n'} + \sum_{m\neq n} \alpha_{I_{n'}K_m} \, \mathrm{d}\log \frac{r_{K_m}}{r_{K_n}} + \sum_f \alpha_{I_{n'}L_f} \, \mathrm{d}\log \frac{w_{I_{n'}f}}{r_{K_n}}$
L_{Cf} demand	$\mathrm{d}\log L_{Cf} = \mathrm{d}\log C + \sum_m \alpha_{CK_m} \, \mathrm{d}\log \frac{r_{K_m}}{w_{Cf}} + \sum_{f'\neq f} \alpha_{CL_{f'}} \, \mathrm{d}\log \frac{w_{Cf'}}{w_{Cf}}$
$L_{I_{n'}f}$ demand	$\mathrm{d}\log L_{I_{n'}f} = \mathrm{d}\log I_{n'} + \sum_m \alpha_{I_{n'}K_m} \, \mathrm{d}\log \frac{r_{K_m}}{w_{I_{n'}f}} + \sum_{f'\neq f} \alpha_{I_{n'}L_{f'}} \, \mathrm{d}\log \frac{w_{I_{n'}f'}}{w_{I_{n'}f}}$
Intertemporal block	**Equation**
K_{sn} valuation	$\mathrm{d}\log p_{K_{sn},t} = \frac{\lambda_{K_{sn}}}{\lambda_{W_{sn}}} \mathrm{d}\log r_{K_{sn},t+1} - \frac{\lambda_{I_{sn}}}{\lambda_{W_{sn}}} \mathrm{d}\log p_{I_n,t+1} + \frac{1}{1+r} \mathrm{d}\log p_{K_{sn},t+1} - \mathrm{d}\log(1+r_{t+1}) + \frac{\lambda_{K_{sn}}}{\lambda_{W_{sn}}} \mathrm{d}\log(1-\tau_n)$
K_{sn} accumulation	$\mathrm{d}\log K_{sn,t+1} = \big(1-(1-\theta_n)\delta_n\big) \, \mathrm{d}\log K_{sn,t} + (1-\theta_n)\delta_n \, \mathrm{d}\log I_{sn,t}$
I_{sn} investment-q	$\mathrm{d}\log I_{sn,t} = \mathrm{d}\log K_{sn,t} + \frac{1}{\theta_n} \mathrm{d}\log \frac{p_{K_{sn},t}}{p_{I_n,t}}$
C euler equation	$\mathrm{d}\log C_t = \mathrm{d}\log C_{t+1} - \psi \, \mathrm{d}\log(1+r_{t+1})$
L_f accumulation	$\mathrm{d}\log L_{f,t+1} = (1-\nu) \, \mathrm{d}\log L_{f,t} + \nu\eta \, \mathrm{d}\log \frac{p_{L_f,t}}{p_{L,t}}$
L_{sf} accumulation	$\mathrm{d}\log L_{sf,t+1} = \mathrm{d}\log L_{f,t+1} + (1-\mu) \, \mathrm{d}\log \frac{L_{sf,t}}{L_{f,t}} + \mu\phi_f \, \mathrm{d}\log \frac{p_{L_{sf},t}}{p_{L_f,t}}$
L_{sf} valuation	$\mathrm{d}\log p_{L_{sf},t} = \frac{r}{1+r} \mathrm{d}\log w_{sf,t+1} - \mathrm{d}\log(1+r_{t+1}) + \frac{1}{1+r} \mathrm{d}\log p_{L_{sf},t+1}$
L_f valuation	$\mathrm{d}\log p_{L_f,t} = \frac{\alpha_{CL_f}\lambda_C}{\lambda_{L_f}} \mathrm{d}\log p_{L_{Cf},t} + \sum_n \frac{\alpha_{I_nL_f}\lambda_{I_n}}{\lambda_{L_f}} \mathrm{d}\log p_{L_{I_nf},t}$

Table 3 lists the linearized model equations, where $\mathrm{d}\log x$ denotes log-deviations from steady state. Indices: $n, m \in \{1, \ldots, N\}$ are capital types; $f \in \{1, \ldots, F\}$ are fixed factor types; $s \in \{C, I_1, \ldots, I_N\}$ are sectors.

To simulate transition dynamics, we augment the model with a dynamic labor supply block where cohorts sort into education groups and then across sectors in response to relative wages.

We also include sector-specific capital stocks subject to internal adjustment costs, aimed to match firm-level evidence on sluggish investment responses to Tobin's q.

Linearized equations. We focus directly on the linearized model around the steady-state in response to an arbitrary sequence of type-specific demand shocks $\{\{d\tau_{n,t}\}_{n=1}^{N}\}_{t=0}^{\infty}$. As in the stylized model, this shock maps directly to a capital income tax shock, but is also isomorphic to Euler equation shocks. Since the model is a direct extension of the stylized model from Section 1, we list the linearized model equations in Table 3, and refer the reader to Appendix A for formal derivations of the extensions. The new parameters of the labor block will be introduced formally in the calibration section below.

The equations are broken down into the intratemporal block (which determines the allocation given the state of the economy) and the intertemporal block (which pins down the evolution of the state). The states are the stocks of capital and labor, and the co-states are the associated present values of future income flows from these factors. With N capital types, F labor types, and $1 + N$ sectors, the system comprises $7 + 7N + 2NF + N^2F = 131$ equations in as many unknowns.

3.1 Calibration

First, we calibrate the demand side of the model externally: a real annual return to $r = 0.05$ and a unit intertemporal elasticity of $\psi = 1$. The goal is to first isolate the unique predictions of the supply side of the model (i.e., how does production of consumption and investment goods respond to relative prices). Then, we consider alternatives to the euler equation such as: small-open economy (fixed r), as well as a non-homothetic euler equation meant to capture the dynamics of MPCs in the data.

Second, we calibrate the sectoral share and output shares using the BEA data described in Section 2, averaged over the 2017-2023 period. We do the same for the depreciation rates from the BEA over the 2017-2023 period, which averages 14% for equipment, 3% for structures, and 24% for IPP. For organization capital, we impute the same depreciation rate as IPP.

Table 4 reports the calibration of the remaining parameters (i.e., the labor supply elasticities, and the capital adjustment parameter). Below, we go over the existing evidence in detail and describe the moments that we seek to match.

Calibration of the dynamic labor block. Our representation of aggregate labor supply is meant to capture the rich dynamics of education choices, comparative advantage in the labor market, as well as reallocation frictions in a parsimonious way. The parameters to calibrate are: (ν, μ), which governs the speed at which education choices and sectoral allocations reach their long-run values (for fixed prices); and $(\eta, \{\phi_f\}_{f=1}^{F})$ which governs the long-run. Notice that ϕ_f is exactly the sectoral labor supply elasticity in the stylized model, but now we allow it to differ across education groups.

Table 4: Baseline calibration and empirical targets.

Parameter	Value	Targeted moment	Reference
α, λ, δ	Table 1	sectoral factor/output shares	BEA-KLEMS
ν	0.03	worklife of 36 years	Adão et al. (2024)
η	0.4	education choice elasticity of 0.4	Adão et al. (2024)
μ	0.3	95% reallocation in 8 years	Artuç et al. (2010)
ϕ_f	(3, 2, 1, 0)	long-run labor-supply ranges	see text
θ_n	(0.12, 0.04, 0.18, 0.18)	quadratic adjustment cost of 1.5	Chodorow-Reich (2026)

To fix ideas, it is useful to consider a partial equilibrium experiment around the steady-state where education-sector specific wages permanently changes: an arbitrary perturbation $\mathrm{d}\log w_{sf,t}$ at all $t \geq 0$. The horizon-dependent aggregate labor supply is:

$$\mathrm{d}\log L_{sf,t} = \underbrace{\left(1-(1-\nu)^t\right)\eta\ \mathrm{d}\log\frac{w_{L_f}}{w_L}}_{\text{education choice}} + \underbrace{\left(1-(1-\mu)^t\right)\phi_f\ \mathrm{d}\log\frac{w_{L_{sf}}}{w_{L_f}}}_{\text{sectoral labor supply}}.$$

The first term captures reallocation across education groups, while the second captures reallocation across sectors within an education group. In this parameterization, (ν, η) govern the education margin, μ governs the speed of within-cohort reallocation, and ϕ_f governs the long-run within-education sectoral elasticity. The effective elasticity at horizon t is therefore $\left(1-(1-\mu)^t\right)\phi_f$. Because the second term is written in relative wages, we do not rely on a small-sector assumption in the exposition.

We set $\nu = 0.03$ and $\eta = 0.4$ using Adão et al. (2024, Table 2), who calibrate a death rate of $1/36$ and a long-run labor-supply elasticity of 0.4, following Wiswall and Zafar (2015). For the speed of within-cohort reallocation, we follow Artuç et al. (2010), who estimate inter-sectoral switching costs of roughly two times annual income using CPS data. In their calibrated model, 95 percent of labor reallocation following a tariff elimination is completed within eight years (Artuç et al., 2010, p. 1042); under our transition law, this implies $\mu \approx 0.31$. We round to $\mu = 0.3$.

We anchor the elasticity vector at college-educated workers and set $\phi_{\text{coll.}} = 2$, the central estimate from the Roy-Fréchet occupational sorting literature: Hsieh et al. (2019, Table II) calibrate $\theta = 2$, Burstein et al. (2019, Table 2) estimate $\theta = 1.78$, and Galle et al. (2023, Table 2) estimate $\kappa \approx 2$, with a range of 1.95–2.15 across specifications. We set $\phi_{\text{non-coll.}} = 3$, one unit above, reflecting the greater cross-sector mobility of workers with less specialized human capital.

For post-college workers, we set $\phi_{\text{post-coll.}} = 1$. Goolsbee (1998a, p. 6) and Fernández-Villaverde et al. (2025, Table 1) document short-run supply elasticities of 0.1–0.2 for scientists and engineers, but the post-college group also includes managers, lawyers, and physicians, who face lower barriers to sectoral reallocation. The effective short-run elasticity $\mu\phi_{\text{post-coll.}} = 0.3$ lies above the scientist-specific range while remaining well below the general-worker benchmark. Finally, $\phi_{\text{land}} = 0$. The resulting vector $\phi = (3, 2, 1, 0)$ implies an income-weighted average $\bar{\phi} \approx 2$,

consistent with the occupational sorting consensus.

Calibration of adjustment cost parameter. We calibrate θ to match external evidence on capital adjustment costs. The investment-q equation shows that $1/\theta$ is the elasticity of the investment rate to Tobin's q. Chodorow-Reich (2026) synthesizes the empirical literature; the cleanest IV estimate, from Cummins et al. (1996) who use tax reforms as instruments, finds a coefficient of 0.65, implying an adjustment cost scalar of 1.5. Our installation function is equivalent to a quadratic adjustment cost model (Appendix A), so we solve for type-specific θ_n from the nonlinear mapping. Given our BEA depreciation rates, this gives $\theta = 0.12$ for equipment, 0.04 for structures, and 0.18 for IPP and organizational capital.[6]

3.2 How elastic is the supply of capital?

We start by quantifying the response of capital and valuations to a permanent demand shock $\mathrm{d}\log(1-\tau) = 1_{t\geq 0}$ and focus on aggregate quantities and prices.[7] Throughout this section, we will refer to Table 5, which summarizes the term structure of these responses in the calibrated model (e.g., on impact, at a planning horizon of 10 years, and in the long-run). To tease out which of the two forces emphasized in Section 1 matters more quantitatively, we calibrate two alternative models where we add one ingredient at a time to the neoclassical growth model. Similarly, we consider a model extension with only capital adjustment costs (e.g., Hayashi's q-theory of investment).

The bottom panel of Table 5 reports three alternative closures that probe the role of trade openness and demand-side assumptions. The first extension opens goods markets by introducing imports as a CES composite with domestic factors, with an Armington trade elasticity $\sigma_M = 3$ and sector-specific import shares from BEA domestic input–output tables (2017–2023 average; see Table A3). The remaining two are demand-side extensions that vary how much desired saving responds to the shock. The first fixes the interest rate (isomorphic to $\psi \to \infty$ at the first order). The second replaces the representative-agent Euler equation with a perpetual-youth overlapping-generations structure augmented with hand-to-mouth spenders, following Angeletos et al. (2024), which raises the marginal propensity to consume and dampens the savings response to asset price movements.[8]

Overall, the calibrated model predicts a weak response of capital accumulation, even in the long-run (i.e., 1.37 versus 1.94 in the neoclassical growth model). Over a more realistic plan-

[6] Adjustment costs are larger for faster-depreciating capital because a given friction $\chi = 1.5$ requires higher θ when δ is larger. The resulting heterogeneity is consistent with evidence that intangible capital has higher adjustment costs (Belo et al., 2022; Crouzet and Eberly, 2023).

[7] We define $\mathrm{d}\log K_t \equiv \sum_n \frac{\lambda_{I_n}}{\lambda_I} \mathrm{d}\log K_{n,t}$ and $\mathrm{d}\log p_{K,t} \equiv \sum_n \frac{\lambda_{I_n}}{\lambda_I} \mathrm{d}\log p_{K_n,t}$ the aggregate quantity and price responses.

[8] The OLG-spender demand block is parametrized by (ψ, ρ, ω), where ω is the share of hand-to-mouth spenders and ρ governs the implicit discounting of perpetual-youth savers (Appendix A). We set $\psi = 1$ (same as the baseline representative-agent model) and fix $(\rho, \omega) = (2/3, 1/3)$. This specification nests the representative agent when $\omega = 0$ and $\rho = 1/(1+r)$.

Table 5: Summary of price and quantity responses (permanent demand shock)

	On impact	Medium-run		Long-run	
	$\mathrm{d}\log p_{K,0}$	$\mathrm{d}\log p_{K,10}$	$\mathrm{d}\log K_{10}$	$\mathrm{d}\log p_{K,\infty}$	$\mathrm{d}\log K_{\infty}$
Calibrated model	1.18	0.51	0.71	0.45	1.37
Neoclassical growth model	0.00	0.00	1.53	0.00	1.94
+capital adjustment costs	0.15	0.04	1.47	0.00	1.94
+technology asymmetry	0.02	0.16	1.37	0.19	1.66
+reallocation frictions	1.21	0.37	1.02	0.24	1.73
Demand-side extensions					
perfectly elastic demand $\psi = \infty$	1.14	0.62	0.79	0.45	1.37
olg-spender demand	0.83	0.39	0.59	0.26	0.91
capital income tax $\tau_0 = 0.2$	1.18	0.51	0.71	0.45	1.37
capital income tax $\tau_0 = 0.3$	1.18	0.51	0.71	0.45	1.37
Supply-side extensions					
imported inputs $\sigma_M = 3$	1.18	0.46	0.86	0.41	1.67
imported inputs $\sigma_M = 5$	1.24	0.47	0.96	0.42	1.94
capital-labor elasticity $\sigma_{KL} = 0.5$	1.03	0.43	0.45	0.34	0.74
capital-labor elasticity $\sigma_{KL} = 1.5$	1.27	0.55	0.90	0.53	1.93
productivity spillovers $\epsilon_C = 0.1$	1.24	0.61	0.74	0.59	1.41
productivity spillovers $\epsilon_I = 0.1$	1.16	0.46	0.75	0.35	1.54
labor supply elasticities $\phi + 1$	1.14	0.43	0.76	0.38	1.44
labor supply elasticities $\phi - 1$	1.27	0.70	0.60	0.62	1.19

Responses to a permanent demand shock $\mathrm{d}\log(1-\tau) = 1$. The calibrated model has four capital types, four fixed factors, sector-specific capital, adjustment costs, asymmetric technologies, and reallocation frictions. Benchmark rows add ingredients sequentially to the neoclassical growth model. Demand-side extensions vary savings behavior; supply-side extensions vary production technology and factor mobility. σ_{KL} denotes the capital-labor elasticity of substitution, σ_M the Armington trade elasticity, ϵ_C, ϵ_I sectoral productivity spillover elasticities, and $\phi \pm 1$ shifts each sectoral labor supply elasticity by ± 1 relative to the baseline vector.

ning horizon of ten years (i.e., the medium-run), the gap between the calibrated model and the neoclassical growth model is even more severe (0.71 versus 1.53). What this means is that back-of-the-envelope calculations regarding the predicted effect of a policy on capital accumulation is materially lower in the calibrated model relative to the benchmark.

This weak response of real quantities imply a large response of relative prices: a significant part of the demand shock is capitalized in higher capital goods prices. Recall that, in the neoclassical growth model, there is no change in the relative price of capital goods versus consumption goods. In the calibrated model, a fraction 0.45 of the demand shock is absorbed by higher prices in the long-run. What this means is that, even after the economy has fully adjusted to the demand shock, capital goods prices remain sensitive to demand conditions. In the short-run capital goods owners experience a large revaluation gain of 1.18 , which decays slowly (i.e., 0.51 after ten years).

Decomposition of the long-run using sufficient statistic. We now decompose the long-run price effect into its fundamental drivers. We focus on the long-run since it is invariant to the level of capital adjustment costs θ and intertemporal substitution ψ. This allows us to decompose the long-run price effect into the role of technology asymmetry ($\alpha_C > \alpha_I$), reallocation frictions ($\phi < \infty$),

and the heterogeneity of capital and labor types.

$$\frac{\mathrm{d}\log p_K}{\mathrm{d}\log(1-\tau)} = \underbrace{\frac{\alpha_{CK}-\alpha_{IK}}{1-\alpha_{IK}}}_{=0.19} + \underbrace{\frac{1}{1+\phi}\frac{1-\alpha_{CK}}{\alpha_{CK}}\frac{\lambda_K}{\lambda_C}}_{=0.24} + \underbrace{\mathrm{Cov}_K}_{=-0.01} + \underbrace{\mathrm{Cov}_L}_{=0.03},$$

The first two terms are the sufficient statistic formulas from Proposition 1.1, corresponding to "technology asymmetry" and "reallocation frictions" in Table 5. Unlike in the stylized model, the shares $\alpha_{CK}, \alpha_{IK}, \lambda_K$ are capital share weighted averages over capital types, and the labor supply elasticity $\phi \equiv \sum_f (\lambda_{L_f}/\lambda_L)\,\phi_f$ is the labor share weighted average over labor types.

The two remaining terms account for the effect of type heterogeneity.[9] The term Cov_K captures the covariance between the demand composition $\alpha_{CK_n} - \lambda_{K_n}/\lambda_K$ and the Leontief multipliers $\left((\mathbf{I} - \boldsymbol{\alpha}_{IK})^{-1}\mathbf{1}\right)_n$ across capital types; Cov_L captures the heterogeneity of labor supply elasticities across education groups. These formulas are derived in Appendix C.3. Each term is constructed as the difference between two nested model predictions: Cov_K is the gap between the full model (N capital types) and the $N = 1$ aggregated model, while Cov_L is the gap between the $N = 1$ model with F labor types and the $N = 1$, $F = 1$ model with a single aggregate labor supply elasticity ϕ. Table 5 reports the corresponding dynamic contribution terms. The key takeaway is that the long-run valuation response of 0.45 is roughly equally split between technological asymmetry and labor reallocation frictions, with a smaller contribution from supply elasticity heterogeneity across education groups.

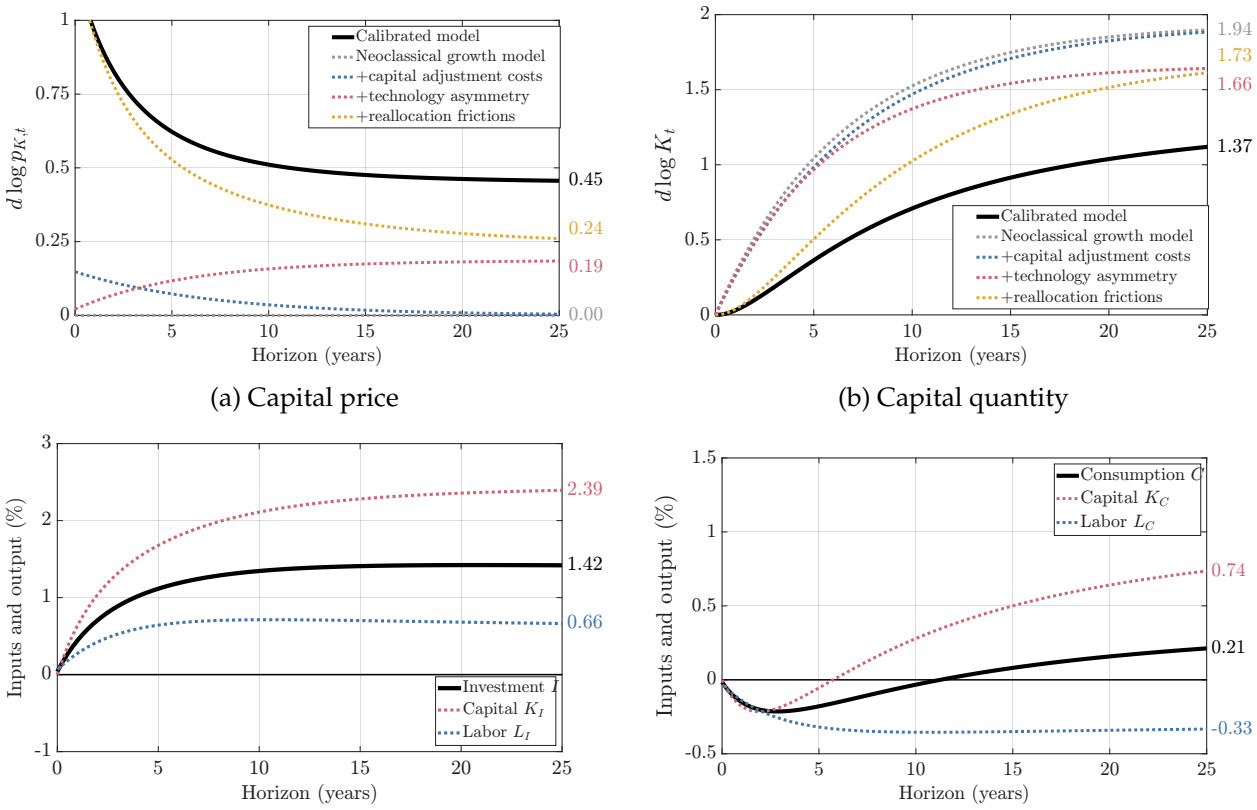
Transition dynamics. What does the model predict in terms of transition dynamics? Figure 5 plots the transition path for prices and quantities. Focusing on the valuation response first, we find that the price response on impact is much larger than in the long-run: 1.18 versus 0.45. To tease out which economic force leads to this overshoot, we quantify the benchmark models, and compare their predictions with the calibrated model, as we did for the long-run.

We find that both capital adjustment costs and labor reallocation frictions matter disproportionately in the short-run. As discussed earlier, adjustment costs have no long-run implication. And for the importance of labor reallocation frictions in the short-run, recall that labor supply rise over the horizon, as governed by the parameters $\mu, \nu \in (0,1)$. In contrast, the technology asymmetry force builds slowly over time, alongside capital accumulation. See Table 5 for the short-run responses in these special cases.

[9] The formulas for the two terms that capture the marginal contribution of labor and capital heterogeneity are

$$\mathrm{Cov}_K \equiv \sum_{n=1}^{N}\left(\alpha_{CK_n} - \frac{\lambda_{K_n}}{\lambda_K}\right)\left(\left((\mathbf{I} - \boldsymbol{\alpha}_{IK})^{-1}\mathbf{1}\right)_n - \frac{1}{1-\alpha_{IK}}\right)$$

$$\mathrm{Cov}_L \equiv \frac{(1-\alpha_{CK})\lambda_K}{\alpha_{CK}\lambda_C}\left(\frac{\phi}{1+\phi} - \frac{1-\lambda_K}{(1-\alpha_{CK})(1-\alpha_{IK})}\sum_{f=1}^{F}\frac{\phi_f}{1+\phi_f}\frac{\alpha_{CL_f}\alpha_{IL_f}}{\lambda_{L_f}}\right).$$



(a) Capital price

(b) Capital quantity

(c) Investment sector

(d) Consumption sector

Figure 5: Transition dynamics of a permanent demand shock

Panels (a)–(b) plot the impulse response of the aggregate capital price p_K and quantity K; the calibrated model and special cases are described in Table 5. Panels (c)–(d) plot the sectoral reallocation of inputs (types aggregated using cost shares) and output in the calibrated model.

A few remarks are worth noting regarding the path of capital accumulation. First, the gap between capital stock in the calibrated model versus in the neoclassical growth model is large throughout the model horizon: it is not just a long-run result. Second, the forces that drive this gap are mostly capital adjustment costs and labor frictions in the short-run, and technology asymmetry in the long-run. Panels (c)–(d) of Figure 5 illustrate the sectoral reallocation that makes the rise in capital accumulation possible. On impact, both capital and labor shift from the consumption sector to the investment sector, leading to a temporary decline in consumption. Over time, however, the rising capital stock reverses this pattern: as capital becomes more abundant, inputs gradually flow back to the consumption sector. After roughly 15 years, consumption begins to rise above its initial level, and in the long-run both sectors expand.

3.3 Welfare incidence of capital income taxes

We now consider the incidence of a permanent capital income tax cut, which represents a transfer of resources away from the government and towards the household sector. We compute the household welfare effect (i.e., willingness to pay for the tax cut, in units of consumption) using

the envelope condition: the change in welfare equals the mechanical change in income holding quantities fixed. For a permanent tax cut $\mathrm{d}\log(1-\tau_n) = 1$ for all $t \geq 0$, the household sector welfare gain is

$$\mathcal{W} = \underbrace{\sum_{n=1}^{N} \lambda_{K_n} \frac{1+r}{r}}_{\text{direct effect of tax cut}} + \underbrace{\sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \left(\sum_{n=1}^{N} \lambda_{K_n} \mathrm{d}\log r_{K_n,t+1} - \sum_{n=1}^{N} \lambda_{I_n} \mathrm{d}\log p_{I_n,t+1} + \sum_{f=1}^{F} \lambda_{L_f} \mathrm{d}\log w_{f,t+1} \right)}_{\text{welfare effect due to relative price changes}}.$$

The first term in is the mechanical tax benefit to capital owners, equal to the present value of the permanent tax cut. The second term captures general equilibrium price effects: changes in rental rates r_{K_n} and investment good prices p_{I_n} (which affect net capital income) and wages w_f (which affect labor and land income).

We define the government welfare effect as $\mathcal{G} = -\sum_n \lambda_{K_n} \frac{1+r}{r}$, which represents the cost of the mechanical cost of the tax cut (i.e., lost resources holding constant the amount of capital income being taxed). An important remark is that $\mathcal{W} + \mathcal{G} = 0$: the tax cut is purely redistributive. This holds for small shocks around the undistorted equilibrium because the second term (the welfare effect due to relative price changes) sums up to zero. Indeed, the linearized unit cost pricing conditions state that factor price changes must offset when aggregated using income shares.[10]

An important remark is that the more inelastic aggregate capital is (i.e., the larger the revaluation effect is), the more capitalists benefit from the tax cut. Holding discount rates fixed (for instance in a small open economy), the capitalists welfare gain equal would be precisely the impact rise in the value of their existing capital stock. When discount rates respond to tax cuts, this adds a wedge between revaluation and welfare effect. Nonetheless, the insight remains: in economies with inelastic capital supply (i.e., where p_K rises substantially while K adjusts slowly) the incidence of the tax cut falls disproportionately on capitalists rather than workers. The welfare analysis is first-order: it evaluates the incidence of a small shock around the undistorted steady state. Second-order (Harberger triangle) corrections require specifying a pre-existing tax distortion and are zero when the initial equilibrium is efficient.

Results. Table 6 reports welfare incidence as a share of government revenue loss. The calibrated model implies that workers bear 0.65 of the tax incidence and capitalists 0.35, compared to 0.79 and 0.21 in the neoclassical benchmark. The gap in capitalist incidence between the two models is +13 pp. This difference arises because inelastic capital supply generates substantial revaluation gains: the tax cut raises asset prices rather than just quantities. Among capitalists, tangible and intangible capital owners benefit roughly equally (0.20 and 0.08 , respectively), while landowners capture a small but positive share (0.06) reflecting both the mechanical tax benefit and rising land rents.

[10] Importantly, this budget balance holds even with imported intermediates ($\alpha_{\cdot,M} > 0$), since import prices are fixed at world levels ($\mathrm{d}\log p_M = 0$) and therefore do not affect domestic factor incomes.

Table 6: Welfare incidence of capital income tax.

	Workers				Capitalists			
	Total	Non-col.	Col.	Post-col.	Total	Tang.	Intan.	Land
Calibrated model	0.65	0.27	0.23	0.16	0.35	0.20	0.08	0.06
Neoclassical growth model	0.79	0.34	0.24	0.20	0.21	0.09	0.07	0.04
+capital adjustment costs	0.78	0.34	0.24	0.20	0.22	0.10	0.08	0.04
+technology asymmetry	0.80	0.34	0.25	0.21	0.20	0.09	0.07	0.04
+reallocation frictions	0.72	0.31	0.22	0.19	0.28	0.13	0.10	0.04
Demand-side extensions								
perfectly elastic demand $\psi = \infty$	0.70	0.28	0.24	0.17	0.30	0.17	0.07	0.06
olg-spender demand	0.48	0.18	0.17	0.12	0.52	0.34	0.12	0.06
capital income tax $\tau_0 = 0.2$	0.73	0.30	0.25	0.18	0.38	0.22	0.09	0.07
capital income tax $\tau_0 = 0.3$	0.77	0.31	0.27	0.19	0.41	0.24	0.10	0.07
Supply-side extensions								
imported inputs $\sigma_M = 3$	0.64	0.25	0.22	0.16	0.36	0.21	0.09	0.07
imported inputs $\sigma_M = 5$	0.62	0.24	0.22	0.16	0.38	0.22	0.09	0.07
capital-labor elasticity $\sigma_{KL} = 0.5$	0.69	0.29	0.23	0.17	0.31	0.16	0.07	0.07
capital-labor elasticity $\sigma_{KL} = 1.5$	0.63	0.25	0.22	0.16	0.37	0.23	0.09	0.06
productivity spillovers $\epsilon_C = 0.1$	0.77	0.31	0.26	0.19	0.37	0.21	0.09	0.07
productivity spillovers $\epsilon_I = 0.1$	0.71	0.29	0.25	0.18	0.34	0.20	0.08	0.06
labor supply elasticities $\phi + 1$	0.66	0.27	0.23	0.16	0.34	0.20	0.08	0.06
labor supply elasticities $\phi - 1$	0.64	0.26	0.22	0.16	0.36	0.21	0.09	0.06

Table 6 reports welfare incidence of a capital income tax cut as a share of government revenue loss. Workers are split by education and capitalists by asset type. See Table 5 note for model descriptions.

The extensions rows show that the calibrated model's incidence is robust to alternative closures. Under a fixed interest rate ($\psi = \infty$), the worker share rises only modestly from 0.65 to 0.70. With open goods markets, workers bear 0.64 in the calibrated model, confirming that import competition does not fundamentally alter the incidence. The OLG-spender demand extension, which raises the marginal propensity to consume, produces worker incidence of 0.48 .

Validation. A central prediction of the model is that aggregate investment demand shocks are partially absorbed by capital prices, implying a finite supply elasticity. We validate this prediction against nine empirical studies that measure the investment supply response across capital types (equipment, structures). See Appendix C.1 for full paper descriptions, exact regression coefficients, and model experiment details.

One key challenge in practice is that our ideal measure of the relative price of investment goods p_I would be consistent with the pricing equations under no productivity or markup shocks.

A pattern in Table 7 is that producer-price-based measures can be unreliable. The House et al. (2022) equipment–price row yields a negative price share (-0.07), meaning the estimated price response has the wrong sign—likely reflecting quality adjustment, hedonic mismeasurement, or import competition in producer price indices. By contrast, the wage-based rows for the same papers yield price shares of 0.40–0.45, consistent with model predictions. This motivates our focus on producer-wage-based measures, which are less susceptible to measurement issues in capital

Table 7: Model replication of evidence on pass-through of demand shocks to capital prices

Paper	Capital	Sample	Variable definitions		Horizon	Price share	
			Quantity	Price		Data	Model
Goolsbee (1998b)	equip.	1959–1988	expenditures	prod. prices	1 yr	0.48	0.51
					2 yr	0.34	0.39
			expenditures	prod. wage	1 yr	0.45	0.33
House and Shapiro (2008)	equip.	2000-2006	expenditures	prod. prices	0 qtr	0.11	0.69
House et al. (2022)	equip.	1959–2009	expenditures	prod. prices	0 qtr	−0.07	0.56
			expenditures	prod. wages	0 qtr	0.40	0.37
House et al. (2022)	struct.	1959–2009	expenditures	prod. prices	0 qtr	0.38	0.55
Topel and Rosen (1988)	struct.	1947–1984	housing starts	valuation	1 yr	0.37	0.44
					2 yr	0.27	0.35
					3 yr	0.26	0.30
Antonova et al. (2025)	equip.	1947–1990	military output	prod. prices	1 yr	0.53	0.51
		1991–2018			1 yr	0.71	0.44

Price share $\equiv \mathrm{d}\log p/(\mathrm{d}\log I + \mathrm{d}\log p)$: the fraction of the expenditure response absorbed by prices rather than real quantities. Values near zero indicate elastic supply; values near one indicate inelastic supply. Negative values signal that prices moved in the wrong direction (measurement error in deflators). See Appendix C.1 for paper descriptions, regression coefficients, and model experiment details.

goods deflators.

4 Conclusion

This paper examines how the labor intensity in the investment sector affects the long-run elasticity of capital supply. In our model, producing investment goods requires specialized labor, making capital supply less responsive to changes in asset prices. Unlike the standard neoclassical model, where capital can be frictionlessly accumulated, our framework predicts that the supply of capital becomes inelastic as the share of intangible investment rises.

Using data from the BEA and Compustat, we document that the investment sector has become more labor-intensive, particularly in terms of college-educated workers. This pattern contrasts with the consumption sector, where the labor share has declined. Our findings suggest that the rising labor intensity in investment goods dampens the elasticity of capital supply, leading to higher asset price sensitivity in response to saving shocks.

We then build and calibrate a realistic two-sector economy to quantify the supply elasticity of capital. We find that the long-run supply elasticity of capital is finite, around 1.5, as opposed to the infinite elasticity in the neoclassical model. This lower elasticity has important implications for tax incidence: capital owners and investment workers bear a larger share of the tax burden than production workers. As a result, corporate tax cuts disproportionately benefit those involved in capital formation, exacerbating inequality in intangible-intensive economies.

References

Adão, Rodrigo, Martin Beraja, and Nitya Pandalai-Nayar, "Fast and Slow Technological Transitions," *Journal of Political Economy Macroeconomics*, 2024, *2* (2), 183–227.

Angeletos, George-Marios, Chen Lian, and Christian K. Wolf, "Can Deficits Finance Themselves?," *Econometrica*, 2024, *92* (5), 1351–1390.

Antonova, Nadja, Ralph Luetticke, and Karsten Muller, "The Military Multiplier," 2025. CESifo Working Paper No. 11882.

Artuç, Erhan, Shubham Chaudhuri, and John McLaren, "Trade Shocks and Labor Adjustment: A Structural Empirical Approach," *American Economic Review*, 2010, *100* (3), 1008–1045.

Atkeson, Andrew and Patrick J Kehoe, "Modeling and measuring organization capital," *Journal of Political Economy*, 2005, *113* (5), 1026–1053.

Auclert, Adrien, Hannes Malmberg, Frédéric Martenet, and Matthew Rognlie, "Demographics, Wealth, and Global Imbalances in the Twenty-First Century," Technical Report, National Bureau of Economic Research 2021.

Ball, Laurence and N Gregory Mankiw, "Market Power in Neoclassical Growth Models," *The Review of Economic Studies*, 2023, *90* (2), 572–596.

Barkai, Simcha, "Declining Labor and Capital Shares," *The Journal of Finance*, 2020, *75* (5), 2421–2463.

Belo, Frederico, Vito D. Gala, Juliana Salomao, and Maria Ana Vitorino, "Decomposing Firm Value," *Journal of Financial Economics*, 2022, *143* (2), 619–639.

Benhabib, Jess and Kazuo Nishimura, "The Hopf bifurcation and the existence and stability of closed orbits in multisector models of optimal economic growth," *Journal of Economic Theory*, 1979, *21* (3), 421–444.

Bernanke, Ben S, "Global imbalances: recent developments and prospects," *Bundesbank Lecture*, 2007.

Burstein, Ariel, Eduardo Morales, and Jonathan Vogel, "Changes in Between-Group Inequality: Computers, Occupations, and International Trade," *American Economic Journal: Macroeconomics*, 2019, *11* (2), 348–400.

Chari, Varadarajan V, Patrick J Kehoe, and Ellen McGrattan, "The Poverty of Nations: A Quantitative Exploration," 1996.

Chodorow-Reich, Gabriel, "The Neoclassical Theory of Firm Investment and Taxes: A Reassessment," 2026. Working paper, Harvard University.

Corrado, Carol, Charles Hulten, and Daniel Sichel, "Intangible capital and US economic growth," *Review of Income and Wealth*, 2009, *55* (3), 661–685.

_ , Jonathan Haskel, Cecilia Jona-Lasinio, and Massimiliano Iommi, "Intangible capital and modern economies," *Journal of Economic Perspectives*, 2022, *36* (3), 3–28.

Crouzet, Nicolas and Janice C Eberly, "Understanding Weak Capital Investment: The Role of Market Concentration and Intangibles," Technical Report, National Bureau of Economic Research 2019.

_ and Janice Eberly, "Intangibles, markups, and the measurement of productivity growth," *Journal of Monetary Economics*, 2021, *124*, S92–S109.

_ and _ , "Rents and intangible capital: A q+ framework," *The Journal of Finance*, 2023, *78* (4), 1873–1916.

__ , **Janice C Eberly, Andrea L Eisfeldt, and Dimitris Papanikolaou**, "The economics of intangible capital," *Journal of Economic Perspectives*, 2022, *36* (3), 29–52.

Cummins, Jason G and Giovanni L Violante, "Investment-specific technical change in the United States (1947–2000): Measurement and macroeconomic consequences," *Review of Economic Dynamics*, 2002, *5* (2), 243–284.

Cummins, Jason G., Kevin A. Hassett, and R. Glenn Hubbard, "Tax Reforms and Investment: A Cross-Country Comparison," *Journal of Public Economics*, 1996, *62* (1-2), 237–273.

Eaton, Jonathan and Samuel Kortum, "Technology, Geography, and Trade," *Econometrica*, 2002, *70* (5), 1741–1779.

Eisfeldt, Andrea L and Dimitris Papanikolaou, "Organization capital and the cross-section of expected returns," *The Journal of Finance*, 2013, *68* (4), 1365–1406.

__ , **Antonio Falato, and Mindy Z Xiaolan**, "Human capitalists," *NBER Macroeconomics Annual*, 2023, *37* (1), 1–61.

Farhi, Emmanuel and François Gourio, "Accounting for Macro-Finance Trends: Market Power, Intangibles, and Risk Premia," *Brookings Papers on Economic Activity*, 2018, pp. 147–250.

Fernández-Villaverde, Jesús, Yang Yu, and Francesco Zanetti, "Defensive Hiring and Creative Destruction," NBER Working Paper 33588, National Bureau of Economic Research 2025.

Galle, Simon, Andrés Rodríguez-Clare, and Moises Yi, "Slicing the Pie: Quantifying the Aggregate and Distributional Effects of Trade," *Review of Economic Studies*, 2023, *90* (1), 331–375.

Gomez, Matthieu and Emilien Gouin-Bonenfant, "How Much Labor Is There in Capital?," 2025. Working paper.

Goolsbee, Austan, "Does Government R&D Policy Mainly Benefit Scientists and Engineers?," *American Economic Review*, 1998, *88* (2), 298–302.

__ , "Investment Tax Incentives, Prices, and the Supply of Capital Goods," *Quarterly Journal of Economics*, 1998, *113* (1), 121–148.

Gutiérrez, Germán and Thomas Philippon, "Investmentless Growth: An Empirical Investigation," *Brookings Papers on Economic Activity*, 2017, pp. 89–169.

Hayashi, Fumio, "Tobin's marginal q and average q: A neoclassical interpretation," *Econometrica*, 1982, *50* (1), 213–224.

House, Christopher L, Ana-Maria Mocanu, and Matthew D Shapiro, "Stimulus Effects of Investment Tax Incentives: Production versus Purchases," 2022. NBER Working Paper 23391.

__ **and Matthew D Shapiro**, "Temporary Investment Tax Incentives: Theory with Evidence from Bonus Depreciation," *American Economic Review*, 2008, *98* (3), 737–768.

Hsieh, Chang-Tai, Erik Hurst, Charles I Jones, and Peter J Klenow, "The Allocation of Talent and US Economic Growth," *Econometrica*, 2019, *87* (5), 1439–1474.

Jr, Robert E Lucas, "Adjustment costs and the theory of supply," *Journal of Political Economy*, 1967, *75* (4, Part 1), 321–334.

Koh, Dongya, Raül Santaeulàlia-Llopis, and Yu Zheng, "Labor Share Decline and Intellectual Property Products Capital," *Econometrica*, 2020, *88* (6), 2609–2628.

Loecker, Jan De, Jan Eeckhout, and Gabriel Unger, "The Rise of Market Power and the Macroeconomic Implications," *The Quarterly Journal of Economics*, 2020, *135* (2), 561–644.

McFadden, Daniel, "The measurement of urban travel demand," *Journal of Public Economics*, 1974, *3* (4), 303–328.

Mian, Atif R, Ludwig Straub, and Amir Sufi, "The Saving Glut of the Rich," *Quarterly Journal of Economics*, 2021, *136* (3), 1851–1890.

Mussa, Michael, "External and internal adjustment costs and the theory of aggregate and firm investment," *Economica*, 1977, *44* (174), 163–178.

Peters, Ryan H and Lucian A Taylor, "Intangible capital and the investment-q relation," *Journal of Financial Economics*, 2017, *123* (2), 251–272.

Ridder, Maarten De, "Market Power and Innovation in the Intangible Economy," *American Economic Review*, 2024, *114* (1), 199–251.

Topel, Robert and Sherwin Rosen, "Housing Investment in the United States," *Journal of Political Economy*, 1988, *96* (4), 718–740.

U.S. Bureau of Economic Analysis, "Input-Output Accounts Data," 2024. Accessed: 2024-03-20.

__ , "KLEMS Industry-Level Productivity Data," 2024. Accessed: 2024-03-20.

Uzawa, Hirofumi, "On a two-sector model of economic growth," *The Review of Economic Studies*, 1961, *29* (1), 40–47.

__ , "Time preference and the Penrose effect in a two-class model of economic growth," *Journal of Political Economy*, 1969, *77* (4, Part 2), 628–652.

Valentinyi, Akos and Berthold Herrendorf, "Measuring factor income shares at the sectoral level," *Review of Economic Dynamics*, 2008, *11* (4), 820–835.

Wiswall, Matthew and Basit Zafar, "Determinants of College Major Choice: Identification Using an Information Experiment," *Review of Economic Studies*, 2015, *82* (2), 791–824.

Appendix

A Appendix for Section 1

A.1 General model

We discuss the long-run elasticity of capital and prices in a general model.

A.1.1 Equilibrium

Model setup. There are $N+1$ sectors in the economy. The first sector produces consumption while the other sectors produces N types of investment goods. Firms are homogeneous in each sector and they produce output using N capital types and F fixed factors. The production functions are:

$$\begin{aligned} C &= F_C\left(\{K_{Cm}\}_{m=1}^N, \{L_{Cf}\}_{f=1}^F\right), \\ I_n &= F_{I_n}\left(\{K_{Im}\}_{m=1}^N, \{L_{If}\}_{f=1}^F\right) \qquad \forall n \end{aligned}$$

and they are homogeneous of degree one (constant returns to scale). Firms choose inputs to maximize profits. The economy is perfectly competitive. Because of constant return to scales, this implies that firm profits are driven to zero in equilibrium.

Each fixed factor f is supplied by the representative household in the consumption and investment sectors according to the function

$$1 = \left(\left(1 - \sum_{n=1}^N \pi_{nf}\right) L_{Cf}^{\frac{1+\phi_f}{\phi_f}} + \sum_{n=1}^N \pi_{nf} L_{I_n f}^{\frac{1+\phi_f}{\phi_f}}\right)^{\frac{\phi_f}{1+\phi_f}}$$

The representative household supplies each fixed factor f across sectors to maximize total labor income.

To determine equilibrium quantities, it will be useful to work with cost functions rather than production functions. Denote r_{K_n} the rental rate of the capital of type n, w_{Cf} the wage of the fixed factor f in the consumption sector, and $w_{I_n f}$ the wage of the fixed factor f in the investment sector. The cost function of the representative firms in each sector is

$$\begin{aligned} \mathcal{C}_C(\{r_{K_m}\}_{m=1}^N, \{w_{Cf}\}_{f=1}^F) &= \min_{\{K_{Cm}\}_{n=1}^N, \{L_{Cf}\}_{f=1}^F} \left\{\sum_{m=1}^N r_{K_m} K_{Cm} + \sum_{f=1}^F w_{Cf} L_{Cf}\right\} \quad \text{s.t. } 1 = F_C\left(\{K_{Cm}\}_{m=1}^N, \{L_{Cf}\}_{f=1}^F\right) \\ \mathcal{C}_{I_n}(\{r_{K_m}\}_{m=1}^N, \{w_{If}\}_{f=1}^F) &= \min_{\{K_{I_n m}\}_{m=1}^N, \{L_{I_n f}\}_{f=1}^F} \left\{\sum_{m=1}^N r_{K_m} K_{I_n m} + \sum_{f=1}^F w_{I_n f} L_{I_n f}\right\} \quad \text{s.t. } 1 = F_{I_n}\left(\{K_{I_n m}\}_{m=1}^N, \{L_{I_n f}\}_{f=1}^F\right) \end{aligned}$$

Denote a_{CK_m} and a_{CL_f} the quantity of capital m and fixed factor f necessary to produce one unit of consumption good (i.e. the minimizer of the cost minimization problem). Similarly, denote $a_{I_n K_m}$ and $a_{I_n L_f}$ the quantity of capital m and fixed factor f necessary to produce one unit of investment of type m. These unit requirements can be obtained as the derivative of the cost function with respect to factor prices.

Consumption is the numeraire. Denote p_{K_n} the relative price of capital of type n. Firms are perfectly

competitive with linear return to scale so they set output price equal to unit cost. This implies:

$$1 = \sum_{m=1}^{N} a_{CK_m} r_{K_m} + \sum_{f=1}^{F} a_{CL_f} w_{Cf} \qquad \text{(unit cost pricing – } C \text{ sector)}$$

$$p_{K_n} = \sum_{m=1}^{N} a_{I_n K_m} r_{K_m} + \sum_{f=1}^{F} a_{I_n L_f} w_{I_n f} \qquad \text{(unit cost pricing – } I_n \text{ sector)}$$

The optimality condition for the firm problem gives

$$L_{Cf} = a_{CL_f} C \qquad \forall f \qquad \text{(labor demand – } C \text{ sector)}$$

$$L_{I_n f} = a_{I_n L_f} I_n \qquad \forall f, n \qquad \text{(labor demand – } I \text{ sector)}$$

$$K_n = a_{CK_n} C + \sum_{m=1}^{N} a_{I_m K_n} I_m \qquad \forall n \qquad \text{(capital demand)}$$

The optimality condition for the household problem gives

$$L_{Cf} = \left(1 - \sum_{n=1}^{N} \pi_{nf}\right) \left(\frac{w_{Cf}}{w_f}\right)^{\phi_f} \qquad \forall f \qquad \text{(labor supply – } C \text{ sector)}$$

$$L_{I_n f} = \pi_{nf} \left(\frac{w_{I_n f}}{w_f}\right)^{\phi_f} \qquad \forall f, n \qquad \text{(labor supply – } I \text{ sector)}$$

$$w_f = \left(\left(1 - \sum_{n=1}^{N} \pi_{nf}\right) w_{Cf}^{\frac{\phi_f}{1+\phi_f}} + \sum_{n=1}^{N} \pi_{nf} w_{I_n f}^{\frac{\phi_f}{1+\phi_f}}\right)^{\frac{1+\phi_f}{\phi_f}} \quad \forall f \qquad \text{(wage index)}$$

The set of equations (unit cost pricing – C sector), (unit cost pricing – I_n sector), (labor demand – C sector), (labor demand – I sector), (capital demand), (labor supply – C sector), (labor supply – I sector), (wage index) corresponds to the intratemporal block of the model. Given a state of the economy determined by the quantity and price of capital ($\{K_n, p_{K_n}\}_{n=1}^{N}$), this system of $2N + 1 + 2FN + 3F$ equations determines $2N + 1 + 2FN + 3F$ equilibrium quantities: $F(N+1)$ factor allocations ($\{L_{Cf}, \{L_{I_n f}\}_{n=1}^{N}\}_{1}^{F}$), $N + F(N+1) + F$ factor prices ($\{r_{K_n}\}_{n=1}^{N}, \{w_{Cf}, \{w_{I_n f}\}_{n=1}^{N}, w_f\}_{1}^{F}$) and $N+1$ sectoral outputs ($C, \{I_n\}_{n=1}^{N}$).

The model is completed by an intertemporal block, which determines the growth rate of consumption, capital accumulation, and capital valuation

$$\beta(1+r')\left(\frac{C'}{C}\right)^{-\frac{1}{\psi}} = 1 \qquad \text{(Euler)}$$

$$K'_n = (1-\delta_n) K_n + I_n \qquad \forall n \qquad \text{(capital accumulation)}$$

$$(1+r) p_{K_n} = (1-\tau') r'_{K_n} + (1-\delta_n) p'_{K_n} \qquad \forall n \qquad \text{(capital valuation)}$$

Substituting the solution of the intratemporal block into these two equations delivers a coupled system of forward- and backward-looking equations in p_{K_n} and K_n. This coupled system of difference equations characterizes the evolution of the economy.

Steady-state. In steady state, the equations in the intertemporal block simplify to

$$r = \frac{1}{\beta} - 1; \quad K_n = \frac{I_n}{\delta_n} \quad \forall n; \qquad p_{K_n} = \frac{(1-\tau_n) r_{K_n}}{r + \delta_n} \quad \forall n.$$

This determines K_n and p_{K_n} given I_n and r_{K_n}. Given that intratemporal block determines I_n and r_{K_n} given K_n and p_{K_n}, this system of equations fully summarizes the model.

It will be useful to define a set of notations. Denote λ_C and λ_{I_n} sectoral shares, i.e. the fraction of total nominal output accounted by each sector in steady-state. Denote α_{CL_f} and $\alpha_{I_n L_f}$ the intensity of factor f in the consumption and investment sector, respectively (i.e., the factor's expenditure share in firm costs). Finally, denote $\{\lambda_{K_n}\}_{n=1}^N$ and $\{\lambda_{L_f}\}_{f=1}^F$ factor shares as the fraction of aggregate income earned by each factor in steady state. Given the market clearing for capital and labor above, these factor shares are related to sectoral shares and factor intensities via

$$\lambda_{K_n} = \alpha_{CK_n} \lambda_C + \sum_{m=1}^N \alpha_{I_m K_n} \lambda_{I_m} \quad \forall n; \qquad \lambda_{L_f} = \alpha_{CL_f} \lambda_C + \sum_{m=1}^N \alpha_{I_m L_f} \lambda_{I_m} \quad \forall f. \tag{A.1}$$

Note that the steady-state version of equations in the intertemporal block implies

$$\lambda_{K_n} = \frac{r + \delta_n}{\delta_n} \lambda_{I_n} \quad \forall n$$

and so together with the previous equation this pins down sectoral shares as the solution of a linear system.

Linearized model. We now consider a perturbation around the steady state due to a perturbation in $\mathrm{d}\log(1-\tau_n)$. We now solve for the perturbed version of the model. Differentiating (unit cost pricing – C sector) and (unit cost pricing – I_n sector) gives:

$$\begin{aligned} 0 &= \sum_{m=1}^N \alpha_{CK_m} \mathrm{d}\log r_{K_m} + \sum_{f=1}^F \alpha_{CL_f} \mathrm{d}\log w_{Cf}, \\ \mathrm{d}\log p_{K_n} &= \sum_{m=1}^N \alpha_{I_n K_m} \mathrm{d}\log r_{K_m} + \sum_{f=1}^F \alpha_{I_n L_f} \mathrm{d}\log w_{I_n f} \qquad \forall n \end{aligned}$$

where we used the fact that, as a result of the envelope theorem, the change in factor intensities does not have a first-order effects on unit costs. Next, we define the Allen-Uzawa elasticity of substitution between factor f and f' for the representative firm in the consumption sector as

$$\sigma_{CL_f L'_f} \equiv \frac{1}{\alpha_{CL'_f}} \frac{\partial \log a_{CL_f}}{\partial \log w_{Cf'}}, \qquad f \neq f'.$$

We set the same definition for the investment sector, and when one of the factors is a capital type instead of a fixed factor. Note that these cross-factor elasticities of substitution are symmetric ($\sigma_{CL_f L'_f} = \sigma_{CL'_f L_f}$) and that the own-price elasticity of factor f can be obtained as a weighted sum of the Allen-Uzawa elasticities of substitution with respect to other factors; that is,

$$\frac{\partial \log a_{CL_f}}{\partial \log w_{Cf}} = -\sum_{n=1}^N \alpha_{CK_n} \sigma_{CL_f K_n} - \sum_{f' \neq f} \alpha_{CL'_f} \sigma_{CL_f L'_f}. \tag{A.2}$$

Armed with these elasticities of substitutions, we can differentiate the equations for factor demand in each sector. Linearizing (capital demand) gives

$$\begin{aligned}\mathrm{d}\log K_n = \frac{\alpha_{CK_n}\lambda_C}{\lambda_{K_n}}\left(\mathrm{d}\log C + \sum_{m\neq n}\alpha_{CK_m}\sigma_{CK_nK_m}\,\mathrm{d}\log\left(\frac{r_{K_m}}{r_{K_n}}\right) + \sum_{f=1}^{F}\alpha_{CL_f}\sigma_{CK_nL_f}\,\mathrm{d}\log\left(\frac{w_{Cf}}{r_{K_n}}\right)\right)\\ + \sum_{n'=1}^{N}\frac{\alpha_{I_{n'}K_n}\lambda_{I_{n'}}}{\lambda_{K_n}}\left(\mathrm{d}\log I_{n'} + \sum_{m\neq n}\alpha_{I_{n'}K_m}\sigma_{I_{n'}K_nK_m}\,\mathrm{d}\log\left(\frac{r_{K_m}}{r_{K_n}}\right) + \sum_{f=1}^{F}\alpha_{I_{n'}L_f}\sigma_{I_{n'}K_nL_f}\,\mathrm{d}\log\left(\frac{w_{I_{n'}f}}{r_{K_n}}\right)\right) \quad \forall n\end{aligned}$$

Linearizing (labor demand – C sector) and (labor demand – I sector) gives

$$\begin{aligned}\mathrm{d}\log L_{Cf} &= \mathrm{d}\log C + \sum_{m=1}^{N}\alpha_{CK_m}\sigma_{CK_mL_f}\,\mathrm{d}\log\left(\frac{r_{K_m}}{w_{Cf}}\right) + \sum_{f'\neq f}\alpha_{CL_{f'}}\sigma_{CL_{f'}L_f}\,\mathrm{d}\log\left(\frac{w_{Cf'}}{w_{Cf}}\right) \quad \forall f\\ \mathrm{d}\log L_{I_nf} &= \mathrm{d}\log I_n + \sum_{m=1}^{N}\alpha_{I_nK_m}\sigma_{I_nK_mL_f}\,\mathrm{d}\log\left(\frac{r_{K_m}}{w_{I_nf}}\right) + \sum_{f'\neq f}\alpha_{I_nL_{f'}}\sigma_{I_nL_{f'}L_f}\,\mathrm{d}\log\left(\frac{w_{I_nf'}}{w_{I_nf}}\right) \quad \forall f,n\end{aligned}$$

Finally, linearizing (labor supply – C sector), (labor supply – I sector), and (wage index) gives

$$\begin{aligned}\mathrm{d}\log L_{Cf} &= \phi_f\,\mathrm{d}\log\left(\frac{w_{Cf}}{w_f}\right) \quad \forall f\\ \mathrm{d}\log L_{I_nf} &= \phi_f\,\mathrm{d}\log\left(\frac{w_{I_nf}}{w_f}\right) \quad \forall f,n\\ \mathrm{d}\log w_f &= \frac{\alpha_{CL_f}\lambda_C}{\lambda_{L_f}}\,\mathrm{d}\log w_{Cf} + \sum_{n=1}^{N}\frac{\alpha_{I_nL_f}\lambda_{I_n}}{\lambda_{L_f}}\,\mathrm{d}\log w_{I_nf} \quad \forall f\end{aligned}$$

This set of equations constitutes the linearized version of the intratemporal block of the model. It constitutes a *linear* system of $1+2N+(3+2N)F$ that determines, given the state of the economy $(\{\mathrm{d}\log K_n, \mathrm{d}\log p_{K_n}\}_{n=1}^N)$ at some point in time, a set of $1+2N+(3+2N)F$ equilibrium quantities: N rental rates of capital $\{\mathrm{d}\log r_{K_n}\}_{n=1}^N$, $(2+N)F$ fixed factor prices $\{\mathrm{d}\log w_f, \mathrm{d}\log w_{Cf}, \{\mathrm{d}\log w_{I_nf}\}_{n=1}^N\}_1^F$, $(1+N)F$ fixed factor allocations $(\{\mathrm{d}\log L_{Cf}, \{\mathrm{d}\log L_{I_nf}\}$ and $1+N$ sectoral outputs $(\mathrm{d}\log C, \{\mathrm{d}\log I_n\}_{n=1}^N)$.

Note that adding up the equations for capital and labor demands using factor shares as weights gives a version of Hulten's theorem in our setup:

$$\lambda_C\,\mathrm{d}\log C + \sum_{n=1}^{N}\lambda_{I_n}\left(\mathrm{d}\log p_{K_n} + \mathrm{d}\log I_n\right) = \sum_{n=1}^{N}\lambda_{K_n}\left(\mathrm{d}\log r_{K_n} + \mathrm{d}\log K_n\right) + \sum_{f=1}^{F}\lambda_{L_f}\,\mathrm{d}\log w_f$$

Finally, linearizing the equations in the intertemporal block gives

$$\begin{aligned}\mathrm{d}\log C' - \mathrm{d}\log C &= \psi\,\mathrm{d}\log(1+r') && \text{(Euler equation)}\\ \mathrm{d}\log K_n' &= (1-\delta_n)\,\mathrm{d}\log K_n + \delta_n\,\mathrm{d}\log I_n \quad \forall n\\ \mathrm{d}\log p_{K_n} + \mathrm{d}\log(1+r') &= \frac{r+\delta_n}{1+r}\left(\mathrm{d}\log(1-\tau_n') + \mathrm{d}\log r_{K_n}'\right) + \frac{1-\delta_n}{1+r}\,\mathrm{d}\log p_{K_n}' \quad \forall n.\end{aligned}$$

Combining the intertemporal block and intratemporal blocks delivers a coupled system of forward- and backward-looking *linear* equations in $\mathrm{d}\log p_{K_n}$ and $\mathrm{d}\log K_n$.

Solving for the equilibrium path. Consider a permanent change in $\{\mathrm{d}\log(1-\tau_n)\}_{n=1}$. To solve for the transitory dynamics of the model, we set up the linearized model as the following system

$$Ax' = Bx + Cz' \tag{A.3}$$

where z is a vector of the $\{\mathrm{d}\log(1-\tau_n)\}_{n=1}^N$, x is a vector of $1+4N+(3+2N)F$ endogenous variables: $3N$ capital prices, quantities, and rental rates $\{\mathrm{d}\log p_{K_n}, \mathrm{d}\log K_n, \mathrm{d}\log r_{K_n}\}_{n=1}^N$, $(2+N)F$ fixed factor prices $\{\mathrm{d}\log w_f, \mathrm{d}\log w_{Cf}, \{\mathrm{d}\log w_{I_nf}\}_{n=1}^N\}_{f=1}^F$, $(1+N)F$ fixed factor sectoral allocations $\{\mathrm{d}\log L_{Cf}, \{\mathrm{d}\log L_{I_nf}\}_{n=1}^N\}_{f=1}^F$ and $1+N$ sectoral outputs $(\mathrm{d}\log C, \{\mathrm{d}\log I_n\}_{n=1}^N)$. The first $2N$ rows of the system encodes the intertemporal equations while rest encodes the intratemporal block. The system can then be solved given a sequence of demand shocks $(\mathrm{d}\log(1-\tau_t))_{t\geq 0}$ using the usual Gensys algorithm.

We can directly solve for the long-run comparative statics in response to a permanent change in $\{\mathrm{d}\log(1-\tau_n)\}_{n=1}$ by solving the linear system

$$x = (A-B)^{-1}Cz$$

We denote by η a solution to $\det(A-\eta B) = 0$ (i.e., an eigenvalue of $A^{-1}B$). A solution is $\eta = 0$, and is associated with the intratemporal equations. We ignore these trivial eigenvalues as they do not matter for dynamics. If there are (i) as many nontrivial solutions $\eta > 1$ as there are forward-looking variables and (ii) as many nontrivial solutions $\eta < 1$ as there are backward-looking equations, then the equilibrium path is saddle-path stable. For any initial values, the linearized equilibrium path converges to the unique steady-state.

A.2 Capital elasticity in stylized model

We now derive the exact elasticity of capital supply and demand in the stylized model. In the stylized model, the intratemporal block is

$$\begin{aligned}
0 &= \alpha_C \,\mathrm{d}\log r_K + (1-\alpha_C)\,\mathrm{d}\log w_C \\
\mathrm{d}\log p_K &= \alpha_I \,\mathrm{d}\log r_K + (1-\alpha_I)\,\mathrm{d}\log w_I \\
\mathrm{d}\log(r_K K) &= \frac{\alpha_C\lambda_C}{\lambda_K}\,\mathrm{d}\log C + \frac{\alpha_I\lambda_I}{\lambda_K}\,\mathrm{d}\log(p_K I) \\
\phi\,\mathrm{d}\log\left(\frac{w_C}{w}\right) + \mathrm{d}\log w_C &= \mathrm{d}\log(C) \\
\phi\,\mathrm{d}\log\left(\frac{w_I}{w}\right) + \mathrm{d}\log w_I &= \mathrm{d}\log(p_K I) \\
\mathrm{d}\log w &= \frac{(1-\alpha_C)\lambda_C}{\lambda_L}\,\mathrm{d}\log w_C + \frac{(1-\alpha_I)\lambda_I}{\lambda_L}\,\mathrm{d}\log w_I
\end{aligned}$$

Given $\mathrm{d}\log p_K$ and $\mathrm{d}\log K$, this system of six equations determines six unknowns: rental rate of capital $\mathrm{d}\log r_K$, wages $\mathrm{d}\log w_C, \mathrm{d}\log w_I, \mathrm{d}\log w$, and sectoral output $\mathrm{d}\log C$ and $\mathrm{d}\log I$. Solving this system gives

$$\begin{aligned}
\mathrm{d}\log r_K = {} & \frac{(1-\alpha_C)\lambda_I\left(\alpha_I\lambda_L + \phi(\alpha_I-\alpha_C)\lambda_C\right)}{\alpha_C(1-\alpha_I)\lambda_C\lambda_L + \alpha_I(1-\alpha_C)\lambda_I\lambda_L + \phi\lambda_I\lambda_C(\alpha_I-\alpha_C)^2}\,\mathrm{d}\log p_K \\
& - \frac{(1-\alpha_C)(1-\alpha_I)\lambda_K\lambda_L}{\alpha_C(1-\alpha_I)\lambda_C\lambda_L + \alpha_I(1-\alpha_C)\lambda_I\lambda_L + \phi\lambda_I\lambda_C(\alpha_I-\alpha_C)^2}\,\mathrm{d}\log K
\end{aligned} \tag{A.4}$$

$$\mathrm{d}\log I = -\frac{\alpha_I}{\alpha_I - \alpha_C}\,\mathrm{d}\log p_K \;+\; \frac{(1-\alpha_C)\lambda_K}{(\alpha_I - \alpha_C)\lambda_I}\,\mathrm{d}\log K \;+\; \frac{\lambda_K}{(\alpha_I - \alpha_C)\lambda_I}\,\mathrm{d}\log r_K \tag{A.5}$$

Substituting (A.4) into (A.5), we write the reduced-form coefficients as $\mathrm{d}\log r_K = \varepsilon_{r,p}\,\mathrm{d}\log p_K + \varepsilon_{r,K}\,\mathrm{d}\log K$ and $\mathrm{d}\log I = \varepsilon_{I,p}\,\mathrm{d}\log p_K + \varepsilon_{I,K}\,\mathrm{d}\log K$.

Proof of Proposition 1.1. In steady state, capital accumulation requires $\mathrm{d}\log I = \mathrm{d}\log K$ and capital valuation requires $\mathrm{d}\log p_K = \mathrm{d}\log r_K + \mathrm{d}\log(1-\tau)$. Substituting the intratemporal solutions gives the long-run supply and demand curves for capital (cf. Figure 1):

$$\mathrm{d}\log K = \frac{\varepsilon_{I,p}}{1-\varepsilon_{I,K}}\,\mathrm{d}\log p_K \qquad \text{(supply)}$$

$$\mathrm{d}\log p_K = \frac{\varepsilon_{r,K}}{1-\varepsilon_{r,p}}\,\mathrm{d}\log K + \frac{1}{1-\varepsilon_{r,p}}\,\mathrm{d}\log(1-\tau) \qquad \text{(demand)}$$

The *supply curve* collects the (p_K, K) pairs satisfying the intratemporal block together with $I = \delta K$: it is the capital stock the economy can sustain at each p_K. When $\varepsilon_{I,K} < 1$ (heterogeneous factor intensities or imperfect labor mobility), the supply curve slopes upward: raising the output of the investment sector relative to the consumption sector bids up its relative cost. The *demand curve* collects the (p_K, K) pairs satisfying the intratemporal block together with $p_K = (1-\tau)r_K/(r+\delta)$: it is the maximum price households are willing to pay for each K. Because more capital depresses the rental rate ($\varepsilon_{r,K} < 0$), the demand curve slopes downward.

Eliminating $\mathrm{d}\log K$ using the supply curve and solving:

$$\frac{\mathrm{d}\log p_K}{\mathrm{d}\log(1-\tau)} = \frac{1-\varepsilon_{I,K}}{(1-\varepsilon_{I,K})(1-\varepsilon_{r,p}) - \varepsilon_{I,p}\,\varepsilon_{r,K}}, \qquad \frac{\mathrm{d}\log K}{\mathrm{d}\log(1-\tau)} = \frac{\varepsilon_{I,p}}{(1-\varepsilon_{I,K})(1-\varepsilon_{r,p}) - \varepsilon_{I,p}\,\varepsilon_{r,K}}.$$

To obtain the proposition, we substitute the explicit formulas for the four intratemporal coefficients from (A.4)–(A.5). The key simplification uses the identity $\lambda_I(\alpha_C - \alpha_I) + (1-\alpha_C)\lambda_K = \alpha_C\lambda_L$, which follows from the steady-state share equation $\lambda_K = \alpha_C\lambda_C + \alpha_I\lambda_I$. This identity lets the common denominator of $\varepsilon_{r,p}$, $\varepsilon_{r,K}$, $\varepsilon_{I,p}$, and $\varepsilon_{I,K}$ cancel, yielding the closed-form expressions in the proposition. □

All four intratemporal coefficients can be expressed in terms of a single sufficient statistic, the elasticity of investment to capital at fixed prices:

$$\varepsilon_{I,K} = \frac{(1-\alpha_C)\lambda_K\,(\alpha_I\lambda_L - (\alpha_C - \alpha_I)\lambda_C\,\phi)}{\alpha_I(1-\alpha_C)\lambda_L + (\alpha_C - \alpha_I)\lambda_C\lambda_L + \phi\lambda_I\lambda_C(\alpha_C - \alpha_I)^2}. \tag{A.6}$$

In the numerator, $\alpha_I\lambda_L$ captures *capital expansion*: additional capital raises output in both sectors, expanding investment through the capital share α_I. The term $(\alpha_C - \alpha_I)\lambda_C\phi$ captures *reallocation friction*: when $\alpha_C > \alpha_I$, absorbing more capital requires reallocating labor across sectors, which is costly when $\phi > 0$, dampening the investment response. When $\alpha_C = \alpha_I \equiv \alpha$, the reallocation term vanishes and $\varepsilon_{I,K} = \alpha$: additional capital is absorbed at fixed relative prices and the supply curve is perfectly elastic. Heterogeneous factor intensities lower $\varepsilon_{I,K}$ below α, and imperfect labor mobility amplifies this effect when $\alpha_C > \alpha_I$, making the supply curve less elastic.

The remaining three coefficients follow by substituting (A.4) into (A.5) and matching coefficients on

$\mathrm{d}\log p_K$ and $\mathrm{d}\log K$, and can all be expressed in terms of $\varepsilon_{I,K}$:

$$\varepsilon_{I,p} = \frac{\varepsilon_{I,K} - \alpha_I}{\alpha_I - \alpha_C}, \qquad \varepsilon_{r,p} = \frac{\lambda_I}{\lambda_K}\varepsilon_{I,K}, \qquad \varepsilon_{r,K} = \frac{(\alpha_I - \alpha_C)\lambda_I}{\lambda_K}\varepsilon_{I,K} - (1 - \alpha_C). \tag{A.7}$$

The intratemporal system also pins down consumption. By Hulten's theorem, $\lambda_C \,\mathrm{d}\log C + \lambda_I(\mathrm{d}\log p_K + \mathrm{d}\log I) = \lambda_K(\mathrm{d}\log r_K + \mathrm{d}\log K) + \lambda_L \,\mathrm{d}\log w$. Using the zero-profit conditions to express $\mathrm{d}\log w$ in terms of $\mathrm{d}\log r_K$ and $\mathrm{d}\log p_K$, the rental rate terms cancel and $\lambda_C \,\mathrm{d}\log C = \lambda_K \,\mathrm{d}\log K - \lambda_I \,\mathrm{d}\log I$. Substituting:

$$\mathrm{d}\log C = \varepsilon_{C,p} \,\mathrm{d}\log p_K + \varepsilon_{C,K} \,\mathrm{d}\log K, \quad \varepsilon_{C,p} = -\frac{\lambda_I}{\lambda_C}\varepsilon_{I,p}, \quad \varepsilon_{C,K} = \frac{\lambda_K - \lambda_I\varepsilon_{I,K}}{\lambda_C}. \tag{A.8}$$

Consumption falls when p_K rises (resources shift toward investment) and rises with K by more than the direct capital-income effect (λ_K/λ_C), net of the induced investment response.

Stability. Substituting the intratemporal solutions into the capital valuation and accumulation equations yields a 2×2 system $Ax_{t+1} = Bx_t + Cz_{t+1}$ in $x = (\mathrm{d}\log p_K, \mathrm{d}\log K)'$, as in the general representation (A.3). Saddle-path stability requires that $A^{-1}B$ has one eigenvalue inside and one outside the unit circle (Benhabib and Nishimura, 1979). As shown in the proof below, the $1/\psi$ terms from the Euler equation enter A and B symmetrically, so $\det(A - B)$ — and hence the stability condition — is independent of ψ. The following result shows that stability is governed by the same expression that determines long-run price elasticity.

Proposition A.1 (Stability Condition)**.** *The balanced growth path is saddle-path stable if and only if*

$$\frac{\alpha_C - \alpha_I}{1 - \alpha_I} + \frac{1}{1 + \phi}\frac{1 - \alpha_C}{\alpha_C}\frac{\lambda_K}{\lambda_C} > 0. \tag{A.9}$$

The condition (A.9) shares the sign of the long-run price elasticity in Proposition 1.1. This connection is not coincidental: at the bifurcation boundary where (A.9) equals zero, capital supply becomes perfectly inelastic and the price response diverges. When the condition fails (LHS < 0), both eigenvalues exceed one in absolute value (since their product is $1/\beta > 1$ by (A.11), they cannot both lie inside the unit circle). With two unstable roots and only one jump variable (p_K), no bounded equilibrium exists.

Three remarks on the stability condition. First, $\alpha_C > \alpha_I$ is sufficient but not necessary: even if the investment sector is more capital-intensive, the reallocation friction ($\phi < \infty$) provides a stabilizing force. Second, when labor is perfectly mobile ($\phi \to \infty$), the second term vanishes and (A.9) reduces to $\alpha_C > \alpha_I$, the classical Uzawa condition. Third, when $\alpha_C = \alpha_I$, stability holds for any finite ϕ since the reallocation term is strictly positive.

Proof. Substituting the intratemporal solutions (A.4)–(A.5) into the capital valuation and accumulation equations gives a 2×2 system $Ax_{t+1} = Bx_t + Cz_{t+1}$ with $x = (\mathrm{d}\log p_K, \mathrm{d}\log K)'$, $z = \mathrm{d}\log(1 - \tau)$, and $\beta = 1/(1 + r)$:

$$A = \begin{pmatrix} \beta(1-\delta) + \beta(r+\delta)\varepsilon_{r,p} - \frac{\varepsilon_{C,p}}{\psi} & \beta(r+\delta)\varepsilon_{r,K} - \frac{\varepsilon_{C,K}}{\psi} \\ 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 - \frac{\varepsilon_{C,p}}{\psi} & -\frac{\varepsilon_{C,K}}{\psi} \\ \delta\varepsilon_{I,p} & 1 - \delta(1 - \varepsilon_{I,K}) \end{pmatrix}, \quad C = \begin{pmatrix} \beta(r+\delta) \\ 0 \end{pmatrix}. \tag{A.10}$$

The first row encodes the capital valuation equation; the Euler equation $\mathrm{d}\log(1 + r_t) = \psi^{-1}(\mathrm{d}\log C_{t+1} - \mathrm{d}\log C_t)$ contributes the $1/\psi$ corrections, with consumption expressed in terms of the state via (A.8). The

second row is the capital accumulation equation, which is ψ-independent. Since the $1/\psi$ terms enter A and B symmetrically, $A - B$ is independent of ψ: long-run comparative statics and stability do not depend on the EIS.

Saddle-path stability requires that $A^{-1}B$ has one eigenvalue inside and one outside the unit circle.

Since A is upper-triangular with $A_{22} = 1$, its determinant is $\det(A) = A_{11}$. Using $\varepsilon_{r,p} = (\lambda_I/\lambda_K)\varepsilon_{I,K}$ from (A.7) together with $\lambda_I/\lambda_K = \delta/(r+\delta)$, the $(1,1)$ entry simplifies to $A_{11} = \beta(1-\delta(1-\varepsilon_{I,K})) - \varepsilon_{C,p}/\psi$, which is strictly positive since $\varepsilon_{C,p} < 0$ (from (A.8)). For B, expanding the determinant gives $\det(B) = (1-\delta(1-\varepsilon_{I,K}))(1-\varepsilon_{C,p}/\psi) + \delta\varepsilon_{I,p}\varepsilon_{C,K}/\psi$. Substituting the expressions for $\varepsilon_{I,p}$, $\varepsilon_{C,p}$, and $\varepsilon_{C,K}$ from (A.7) and (A.8), and using $\lambda_I/\lambda_K = \delta/(r+\delta)$, one can verify that $\det(B) = \det(A)/\beta$. The eigenvalue product is therefore

$$\eta_1\eta_2 = \frac{\det(B)}{\det(A)} = \frac{1}{\beta} \quad \text{for all } \psi \in (0,\infty]. \tag{A.11}$$

Since $1/\beta = 1 + r > 1$, at least one eigenvalue must exceed one in absolute value.

It remains to show that exactly one eigenvalue lies inside the unit circle. The characteristic polynomial evaluated at $\eta = 1$ gives $\det(B-A)/\det(A)$. Since $A - B$ is ψ-independent (the $1/\psi$ terms cancel), this reduces to the condition (A.9). When (A.9) holds, $\det(B-A) < 0$, so $\eta = 1$ lies between the two eigenvalues—one inside, one outside the unit circle—establishing saddle-path stability. □

Impact response. When stability holds, the economy jumps to the saddle path at $t = 0$. Since capital is predetermined ($\mathrm{d}\log K_0 = 0$) while prices adjust freely, the impact response of p_K is

$$\frac{\mathrm{d}\log p_{K,0}}{\mathrm{d}\log p_{K,\infty}} = \frac{1-\eta_s}{\delta\,(1-\varepsilon_{I,K})}, \tag{A.12}$$

where $\eta_s \in (0,1)$ is the stable eigenvalue. To derive (A.12), consider the accumulation equation (second row of (A.10)):

$$\mathrm{d}\log K_{t+1} = (1-\delta(1-\varepsilon_{I,K}))\ \mathrm{d}\log K_t + \delta\varepsilon_{I,p}\ \mathrm{d}\log p_{K,t}. \tag{A.13}$$

On the saddle path, deviations from the new steady state decay at rate η_s:

$$\begin{aligned}\mathrm{d}\log K_t &= \mathrm{d}\log K_\infty\left(1-\eta_s^t\right),\\ \mathrm{d}\log p_{K,t} &= \mathrm{d}\log p_{K,\infty} + \left(\mathrm{d}\log p_{K,0} - \mathrm{d}\log p_{K,\infty}\right)\eta_s^t.\end{aligned}$$

Substituting into (A.13) and matching coefficients on η_s^t gives the eigenvector condition

$$\mathrm{d}\log p_{K,0} - \mathrm{d}\log p_{K,\infty} = \frac{1-\eta_s-\delta(1-\varepsilon_{I,K})}{\delta\varepsilon_{I,p}}\ \mathrm{d}\log K_\infty.$$

The long-run supply curve $(1-\varepsilon_{I,K})\,\mathrm{d}\log K_\infty = \varepsilon_{I,p}\,\mathrm{d}\log p_{K,\infty}$ allows us to substitute $\mathrm{d}\log K_\infty/\varepsilon_{I,p} = \mathrm{d}\log p_{K,\infty}/(1-\varepsilon_{I,K})$, giving

$$\frac{\mathrm{d}\log p_{K,0}}{\mathrm{d}\log p_{K,\infty}} = 1 + \frac{1-\eta_s-\delta(1-\varepsilon_{I,K})}{\delta(1-\varepsilon_{I,K})} = \frac{1-\eta_s}{\delta(1-\varepsilon_{I,K})}.$$

The ratio (A.12) compares two speeds: $1-\eta_s$, the convergence rate along the saddle path, and $\delta(1-\varepsilon_{I,K})$, the rate of capital mean-reversion at fixed prices. Asset prices overshoot their long-run level when the former exceeds the latter. The derivation uses only (A.13), which is ψ-independent, so (A.12) holds for

all ψ. However, η_s itself depends on ψ as described next.

Stable eigenvalue. By (A.11), the eigenvalues of $A^{-1}B$ have product $\eta_s\eta_u = 1+r$ and sum $1+\Sigma$, where

$$\Sigma = 1 + r + \frac{\delta(r+\delta)(1-\varepsilon_{I,K})\,\psi}{(1-\delta(1-\varepsilon_{I,K}))\,\psi + (1+r)\lambda_I\varepsilon_{I,p}/\lambda_C} \cdot \frac{\mathrm{d}\log(1-\tau)}{\mathrm{d}\log p_K}, \tag{A.14}$$

with $\mathrm{d}\log p_K / \mathrm{d}\log(1-\tau)$ the long-run price elasticity from Proposition 1.1. The stable eigenvalue is

$$\eta_s = \frac{1+\Sigma - \sqrt{(1+\Sigma)^2 - 4(1+r)}}{2}. \tag{A.15}$$

Define the *convergence rate*

$$\zeta \equiv \frac{\Sigma - r}{1+\Sigma} = 1 - \frac{1+r}{1+\Sigma}. \tag{A.16}$$

Since $\eta_s\eta_u = 1+r$ and $\eta_s + \eta_u = 1+\Sigma$, $\zeta = 1 - \eta_s + \eta_s^2/(1+\Sigma) \approx 1-\eta_s$. On the saddle path, the remaining gap to steady state shrinks by a factor η_s each period, so $1-\eta_s$ is the per-period convergence rate; ζ approximates it to second order in η_s.

Derivation of (A.14). Since A in (A.10) is upper-triangular with $A_{22} = 1$,

$$\mathrm{tr}(A^{-1}B) = B_{22} + \frac{B_{11} - A_{12}B_{21}}{A_{11}}.$$

Using $\varepsilon_{r,p} = (\lambda_I/\lambda_K)\varepsilon_{I,K}$ from (A.7) and the steady-state relation $\lambda_I/\lambda_K = \delta/(r+\delta)$, the $(1,1)$ entry simplifies to

$$A_{11} = \frac{1-\delta(1-\varepsilon_{I,K})}{1+r} + \frac{\lambda_I\varepsilon_{I,p}}{\lambda_C\psi},$$

where we substitute $\varepsilon_{C,p} = -\lambda_I\varepsilon_{I,p}/\lambda_C$ from (A.8). Both A_{11} and $B_{11} - A_{12}B_{21}$ are affine in $1/\psi$, so $\Sigma = \mathrm{tr}(A^{-1}B) - 1$ is a ratio of affine functions of ψ. The denominator coefficients are read off from A_{11}. As $\psi \to 0$, the Euler equation freezes consumption, which pins $\mathrm{d}\log p_{K,t} = -(\varepsilon_{C,K}/\varepsilon_{C,p})\,\mathrm{d}\log K_t$; substituting into the accumulation equation yields $\Sigma \to 1+r$, fixing the constant term. As $\psi \to \infty$ the $1/\psi$ corrections vanish, and direct computation gives the numerator coefficient $\delta(r+\delta)(1-\varepsilon_{I,K}) \cdot \mathrm{d}\log(1-\tau)/\mathrm{d}\log p_K$, completing the derivation. □

The convergence correction $\Sigma - 1 - r$ is the product of two factors. The second, $\mathrm{d}\log(1-\tau)/\mathrm{d}\log p_K$, is the inverse long-run price elasticity from Proposition 1.1: a steeper supply-demand intersection means a stronger restoring force. The first factor increases in ψ from 0 (when $\psi \to 0$, consumption smoothing freezes adjustment: $\Sigma \to 1+r$, $\eta_s \to 1$) to $\delta(r+\delta)(1-\varepsilon_{I,K})/(1-\delta(1-\varepsilon_{I,K}))$ (when $\psi \to \infty$, the interest rate is pinned and capital mean-reverts at the speed set by δ and the supply-curve slope $1-\varepsilon_{I,K}$). At intermediate ψ, the denominator term $(1+r)\lambda_I\varepsilon_{I,p}/\lambda_C$ captures the drag from consumption smoothing.

Figure A1 illustrates the transition dynamics in the stylized model. After a permanent demand shock shifts the demand curve from D to D', the economy jumps from the old steady state A to the impact point B on the saddle path, then converges along the saddle path to the new steady state C. The slope of the saddle path at C is determined by the stable eigenvalue η_s.

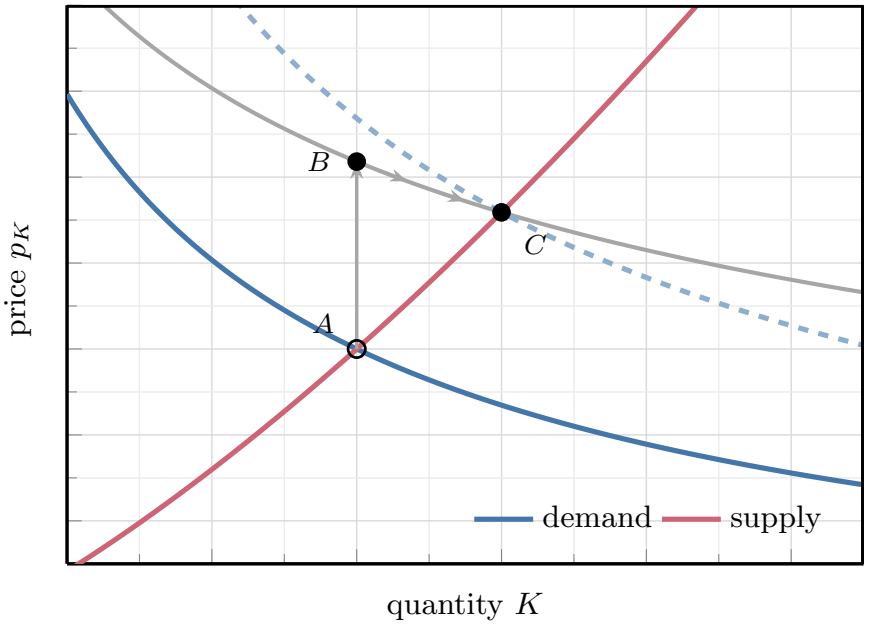


Figure A1: Phase diagram: transition dynamics after a permanent demand shock

A.3 Extensions

A.3.1 OLG-spender demand-side

We now depart from representative-agent CRRA Euler equation and replace it with an "OLG-spender demand-side", which is parametrized by three-parameter (ψ, ρ, ω):

$$\lambda_C \, \mathrm{d}\log C_0 = -(1-\omega)\frac{\lambda_L - (1-\psi)\lambda_C}{1+r} \sum_{t=0}^{\infty} \rho^t \, \mathrm{d}\log(1+r_{t+1}) + (1-\omega)(1-\rho)\,\lambda_W \,\mathrm{d}\log V_{-1}$$
$$+ (1-(1-\omega)\rho)\,\lambda_L \,\mathrm{d}\log w_0 + (1-\omega)\rho(1-\rho)\lambda_L \sum_{t=0}^{\infty} \rho^t \,\mathrm{d}\log w_{t+1}.$$

The microfoundation is that a fraction of households and hand-to-mouth spenders, and the rest are perpetual-youth savers. The parameter $\psi \geq 0$ represents the savers' EIS; $\rho \in (0,1)$ governs the implicit discounting; $\omega \in (0,1)$ represents the share of spenders. Regardless of the microfoundation, the equation provides a theory of consumption that can flexibly match evidence on intertemporal MPCs. This specification nests the representative-agent case when $\omega = 0$ and $\rho = \frac{1}{1+r}$. In the calibration below, we set $\omega = 1/3$ and $\rho = 2/3$ and choose ψ so that the model matches the baseline representative-agent present value of $\mathrm{d}\log(1+r_t)$.

A.3.2 Capital adjustment costs

We can easily augment the model above to have internal adjustment costs. Formally, assume that the capital accumulation equation is

$$K_n' = (1-\delta_n)K_n + I_n^{1-\theta_n} K_n^{\theta_n} \quad \forall n,$$

where $\theta_n \in (0,1)$ parametrizes the strength of adjustment costs, with $\theta_n = 0$ being the frictionless case. Denoting p_{K_n} the installed capital price and p_{I_n} the price of the investment good, the optimality condition for capital installment is

$$(1-\theta_n)\left(\frac{I_n}{K_n}\right)^{-\theta_n} p_{K_n} = p_{I_n} \quad \forall n \qquad \text{(investment-q equation)}$$

To derive the loglinearized valuation equation for type-n capital, we start from:

$$\begin{aligned}
(1+r_{t+1})p_{K_n,t} &= (1-\tau_n)r_{K_n,t+1} + \left(1-\delta_n+\theta_n\left(\tfrac{I_{n,t+1}}{K_{n,t+1}}\right)^{1-\theta_n}\right)p_{K_n,t+1} \\
\lambda_{W_n}\,\mathrm{d}\log\left((1+r_{t+1})p_{K_n,t}\right) &= \lambda_{K_n}\,\mathrm{d}\log((1-\tau_n)r_{K_n,t+1}) + (\lambda_{W_n}-\lambda_{K_n})\left(\mathrm{d}\log p_{K_n,t+1} + \tfrac{(1-\theta_n)\delta_n}{1-(1-\theta_n)\delta_n}\theta_n\,\mathrm{d}\log\tfrac{I_t}{K_t}\right) \\
&= \lambda_{K_n}\,\mathrm{d}\log((1-\tau_n)r_{K_n,t+1}) + (\lambda_{W_n}-\lambda_{K_n})\left(\mathrm{d}\log p_{K_n,t+1} + \tfrac{(1-\theta_n)\delta_n}{1-(1-\theta_n)\delta_n}\,\mathrm{d}\log\tfrac{p_{K_n,t+1}}{p_{I_n,t+1}}\right) \\
&= \lambda_{K_n}\,\mathrm{d}\log((1-\tau_n)r_{K_n,t+1}) + (\lambda_{W_n}-\lambda_{K_n})\left(-\tfrac{(1-\theta_n)\delta_n}{1-(1-\theta_n)\delta_n}\,\mathrm{d}\log p_{I_n,t+1} + \tfrac{1}{1-(1-\theta_n)\delta_n}\,\mathrm{d}\log p_{K_n,t+1}\right)
\end{aligned}$$

The second equation uses the definition $\lambda_{W_n} = (1+r)\frac{p_{K_n}K_n}{Y}$, which in steady-state is $\lambda_{W_n} = \frac{1+r}{r+(1-\theta_n)\delta_n}\lambda_{K_n}$.
Next, we use the steady-state relationship $\lambda_{I_n} = \frac{(1-\theta_n)\delta_n}{r+(1-\theta_n)\delta_n}\lambda_{K_n}$ to obtain:

$$\begin{aligned}
\lambda_{W_n}\,\mathrm{d}\log\left((1+r_{t+1})p_{K_n,t}\right) &= \lambda_{K_n}\,\mathrm{d}\log((1-\tau_n)r_{K_n,t+1}) + \lambda_{K_n}\tfrac{1-(1-\theta_n)\delta_n}{r+(1-\theta_n)\delta_n}\left(-\tfrac{(1-\theta_n)\delta_n}{1-(1-\theta_n)\delta_n}\,\mathrm{d}\log p_{I_n,t+1} + \tfrac{1}{1-(1-\theta_n)\delta_n}\,\mathrm{d}\log p_{K_n,t+1}\right. \\
&= \lambda_{K_n}\,\mathrm{d}\log((1-\tau_n)r_{K_n,t+1}) - \lambda_{I_n}\,\mathrm{d}\log p_{I_n,t+1} + \lambda_{W_n}\frac{1}{1+r}\,\mathrm{d}\log p_{K_n,t+1}, \\
\mathrm{d}\log p_{K_n,t} + \mathrm{d}\log(1+r_{t+1}) &= \tfrac{\lambda_{K_n}}{\lambda_{W_n}}\left(\mathrm{d}\log(1-\tau_n) + \mathrm{d}\log r_{K_n,t+1}\right) - \tfrac{\lambda_{I_n}}{\lambda_{W_n}}\,\mathrm{d}\log p_{I_n,t+1} + \frac{1}{1+r}\,\mathrm{d}\log p_{K_n,t+1}.
\end{aligned}$$

The solution method for the equilibrium along a perturbation path is exactly the same as in the baseline model above. The only difference is that we now have N supplementary variables corresponding to the producer price of capital goods, $\{p_{I_n}\}_{n=1}^N$, and N new equations corresponding the the N linearized investment-q equations:

$$\mathrm{d}\log I_n - \mathrm{d}\log K_n = \frac{1}{\theta_n}(\mathrm{d}\log p_{K_n} - \mathrm{d}\log p_{I_n}) \quad \forall n$$

Moreover, the valuation and accumulation equations become:

$$\begin{aligned}
\mathrm{d}\log p_{K_n} &= \frac{\lambda_{K_n}}{\lambda_{W_n}}\left(\mathrm{d}\log(1-\tau_n') + \mathrm{d}\log r_{K_n}'\right) - \frac{\lambda_{I_n}}{\lambda_{W_n}}\,\mathrm{d}\log p_{I_n}' - \mathrm{d}\log(1+r') + \frac{1}{1+r}\,\mathrm{d}\log p_{K_n}' \quad \forall n \\
\mathrm{d}\log K_n' &= (1-(1-\theta_n)\delta_n)\,\mathrm{d}\log K_n + (1-\theta_n)\delta_n\,\mathrm{d}\log I_n \quad \forall n
\end{aligned}$$

where we use the notation $\lambda_{W_n} \equiv (1+r)\frac{p_{K_n}}{r_{K_n}}\lambda_{K_n}$ to denote the market value of installed capital of type n relative to total output.

It is important to note that internal adjustment costs do not change long-run elasticities because, in steady state, the investment-q optimality condition simplifies to

$$\mathrm{d}\log p_{K_n} = \mathrm{d}\log p_{I_n} \quad \forall n.$$

and so the long-run elasticity of the price of installed capital is the same as the one of the price of the

investment good.

A.3.3 Sector-specific capital

In the baseline calibration, we allow capital to be sector-specific: each sector accumulates its own capital stock with adjustment costs. Capital of type n in sector $s \in \{C, I_1, \ldots, I_N\}$ accumulates according to:

$$K_{sn,t+1} = (1 - \delta_n) K_{sn,t} + I_{sn,t}^{1-\theta_n} K_{sn,t}^{\theta_n} \quad \forall s, n.$$

The q-theory investment equation applies at the sectoral level:

$$\mathrm{d} \log I_{sn} - \mathrm{d} \log K_{sn} = \frac{1}{\theta_n} (\mathrm{d} \log p_{K_{sn}} - \mathrm{d} \log p_{I_n}) \quad \forall s, n.$$

Rental rates equal sectoral marginal products, which can differ across sectors:

$$r_{K_{sn}} = \alpha_{sK_n} \frac{p_s Y_s}{K_{sn}} \quad \forall s, n.$$

The aggregate capital stock of type n is defined as a weighted sum of sectoral capital:

$$K_n = \frac{\lambda_{K_{Cn}}}{\lambda_{K_n}} K_{Cn} + \sum_{m=1}^{N} \frac{\lambda_{K_{I_m n}}}{\lambda_{K_n}} K_{I_m n} \quad \forall n,$$

where $\lambda_{K_{Cn}} = \alpha_{CK_n} \lambda_C$ is capital income from type n in the consumption sector, $\lambda_{K_{I_m n}} = \alpha_{I_m K_n} \lambda_{I_m}$ is capital income from type n in investment sector m, and $\lambda_{K_n} = \lambda_{K_{Cn}} + \sum_m \lambda_{K_{I_m n}}$ is total capital income from type n.

Note that while capital is sector-specific during the transition, rental rates equalize in the long run: the steady-state valuation equation implies $r_{K_{sn}} = (r + (1 - \theta_n)\delta_n)\, p_{K_n}$ for all sectors s, where $p_{K_n} = p_{I_n}$ in steady state. The linearized system includes $2N(N+1)$ additional sectoral capital variables (rental rates, prices, and investment for each sector-type pair), which allow for transitional dynamics where sectoral rental rates and prices diverge before converging to their common long-run values.

A.3.4 Dynamic labor supply

We extend the static labor supply model to capture two margins of adjustment: education choice and sectoral allocation. The model now has $F + F(N+1) = F(N+2)$ labor state variables and an equal number of human wealth co-state variables. We introduce four new parameters: $\nu \in [0, 1]$ governs the speed of education choice, $\mu \in [0, 1]$ governs the speed of sectoral allocation, $\eta > 0$ is the education choice elasticity, and $\phi_f > 0$ is the sectoral supply elasticity for education type f.

The F education-level state and co-state variables are the total labor supply by education type, $\{L_{f,t}\}_{f=1}^{F}$, and the corresponding human wealth values, $\{p_{L_f,t}\}_{f=1}^{F}$. The $F(N+1)$ sector-education state and co-state variables are the labor allocations by sector and education, $\{L_{Cf,t}, \{L_{I_n f,t}\}_{n=1}^{N}\}_{f=1}^{F}$, and the corresponding human wealth values, $\{p_{L_{Cf},t}, \{p_{L_{I_n f},t}\}_{n=1}^{N}\}_{f=1}^{F}$.

The education choice dynamics are governed by

$$L_{f,t+1} = (1-\nu)L_{f,t} + \nu\pi_f \left(\frac{p_{L_f,t}}{p_{L,t}}\right)^{\eta} \qquad \forall f,$$
$$p_{L,t} = \left(\sum_{f=1}^{F} \pi_f p_{L_f,t}^{\frac{\eta}{1+\eta}}\right)^{\frac{1+\eta}{\eta}},$$

where π_f are the steady-state education shares and $p_{L,t}$ is the aggregate human wealth index. The sectoral allocation dynamics within each education group are

$$L_{Cf,t+1} = (1-\mu)L_{Cf,t} + \mu\left(1-\sum_{n=1}^{N}\pi_{nf}\right)\left(\frac{p_{L_{Cf},t}}{p_{L_f,t}}\right)^{\phi_f} L_{f,t+1} \qquad \forall f,$$
$$L_{I_nf,t+1} = (1-\mu)L_{I_nf,t} + \mu\pi_{nf}\left(\frac{p_{L_{I_nf},t}}{p_{L_f,t}}\right)^{\phi_f} L_{f,t+1} \qquad \forall f,n,$$
$$p_{L_f,t} = \left(\left(1-\sum_{n=1}^{N}\pi_{nf}\right)p_{L_{Cf},t}^{\frac{\phi_f}{1+\phi_f}} + \sum_{n=1}^{N}\pi_{nf}p_{L_{I_nf},t}^{\frac{\phi_f}{1+\phi_f}}\right)^{\frac{1+\phi_f}{\phi_f}} \qquad \forall f,$$

where π_{nf} are the steady-state sector-education shares. The human wealth valuations satisfy

$$(1+r_{t+1})p_{L_{Cf},t} = w_{Cf,t+1} + p_{L_{Cf},t+1} \qquad \forall f,$$
$$(1+r_{t+1})p_{L_{I_nf},t} = w_{I_nf,t+1} + p_{L_{I_nf},t+1} \qquad \forall n,f.$$

Linearizing these equations around steady state gives the education choice dynamics:

$$\mathrm{d}\log L_{f,t+1} = (1-\nu)\,\mathrm{d}\log L_{f,t} + \nu\eta\,\mathrm{d}\log\left(\frac{p_{L_f,t}}{p_{L,t}}\right) \qquad \forall f,$$

the sectoral allocation dynamics:

$$\mathrm{d}\log\frac{L_{Cf,t+1}}{L_{f,t+1}} = (1-\mu)\,\mathrm{d}\log\frac{L_{Cf,t}}{L_{f,t}} - (1-\mu)\left(\mathrm{d}\log L_{f,t+1} - \mathrm{d}\log L_{f,t}\right) + \mu\phi_f\,\mathrm{d}\log\left(\frac{p_{L_{Cf},t}}{p_{L_f,t}}\right) \qquad \forall f,$$
$$\mathrm{d}\log\frac{L_{I_nf,t+1}}{L_{f,t+1}} = (1-\mu)\,\mathrm{d}\log\frac{L_{I_nf,t}}{L_{f,t}} - (1-\mu)\left(\mathrm{d}\log L_{f,t+1} - \mathrm{d}\log L_{f,t}\right) + \mu\phi_f\,\mathrm{d}\log\left(\frac{p_{L_{I_nf},t}}{p_{L_f,t}}\right) \qquad \forall f,n,$$

and the human wealth valuations:

$$\mathrm{d}\log p_{L_{Cf},t} = \frac{r}{1+r}\,\mathrm{d}\log w_{Cf,t+1} - \mathrm{d}\log(1+r_{t+1}) + \frac{1}{1+r}\,\mathrm{d}\log p_{L_{Cf},t+1} \qquad \forall f,$$
$$\mathrm{d}\log p_{L_{I_nf},t} = \frac{r}{1+r}\,\mathrm{d}\log w_{I_nf,t+1} - \mathrm{d}\log(1+r_{t+1}) + \frac{1}{1+r}\,\mathrm{d}\log p_{L_{I_nf},t+1} \qquad \forall f,n,$$
$$\mathrm{d}\log p_{L_f,t} = \frac{\alpha_{CL_f}\lambda_C}{\lambda_{L_f}}\,\mathrm{d}\log p_{L_{Cf},t} + \sum_{n=1}^{N}\frac{\alpha_{I_nL_f}\lambda_{I_n}}{\lambda_{L_f}}\,\mathrm{d}\log p_{L_{I_nf},t} \qquad \forall f.$$

For the sake of simplicity, in the extensions below, we only consider the special case in which (i) there is only one type of capital (ii) there is only one type of labor (iii) the production functions in each sector

is Cobb-Douglas in capital and labor. We denote α_C and α_I the Cobb-Douglas exponents on capital in the consumption and investment sectors, respectively.

A.3.5 Markups

In the baseline model, we assumed that the representative firm made zero profit. We can easily consider the case in which representative firm sets output price as some (exogenous) markup above the unit cost instead. More precisely, we assume that the representative firm in each sector prices its output as a markup times the unit cost of manufacturing it. Denote μ_C and μ_I the markup in the consumption and investment sectors, respectively. The market clearing equations become

$$r_K K = \alpha_C \frac{C}{\mu_C} + \alpha_I \frac{p_K I}{\mu_I}, \qquad \text{(market clearing – capital)}$$

$$w_C L_C = (1-\alpha_C)\frac{C}{\mu_C} \qquad \text{(labor demand – consumption labor)}$$

$$w_I L_I = (1-\alpha_I)\frac{p_K I}{\mu_I} \qquad \text{(labor demand – investment labor)}$$

In particular, the last three equations yield the following relationship between factor shares and sectoral shares:

$$\lambda_K = \frac{\alpha_C}{\mu_C}\lambda_C + \frac{\alpha_I}{\mu_I}\lambda_I$$
$$\lambda_L = \frac{1-\alpha_C}{\mu_C}\lambda_C + \frac{1-\alpha_I}{\mu_I}\lambda_I$$

Note that factor shares no longer sum to one, reflecting that part of aggregate income is pure profits.

Proposition A.2 (Case with Markup)**.** *We have*

$$\frac{\mathrm{d}\log p_K}{\mathrm{d}\log(1-\tau)} = \frac{\alpha_C-\alpha_I}{1-\alpha_I} + \frac{\mu_C}{1+\phi}\frac{1-\alpha_C}{\alpha_C}\frac{\lambda_K}{\lambda_C}$$
$$\frac{\mathrm{d}\log K}{\mathrm{d}\log(1-\tau)} = \frac{\alpha_I}{1-\alpha_I} + \frac{\phi}{1+\phi}\frac{1-\alpha_C}{\alpha_C}\frac{\lambda_K}{\lambda_L}$$

Proof of Proposition A.2. Log-linearized version of the intratemporal system is:

$$\begin{aligned}
0 &= \alpha_C\,\mathrm{d}\log r_K + (1-\alpha_C)\,\mathrm{d}\log w_C \\
\mathrm{d}\log p_K &= \alpha_I\,\mathrm{d}\log r_K + (1-\alpha_I)\,\mathrm{d}\log w_I \\
\mathrm{d}\log(r_K K) &= \frac{\alpha_C\frac{\lambda_C}{\mu_C}}{\lambda_K}\,\mathrm{d}\log(C) + \frac{\alpha_I\frac{\lambda_I}{\mu_I}}{\lambda_K}\,\mathrm{d}\log(p_K I) \\
\phi\,\mathrm{d}\log\left(\frac{w_C}{w}\right) + \mathrm{d}\log w_C &= \mathrm{d}\log(C) \\
\phi\,\mathrm{d}\log\left(\frac{w_I}{w}\right) + \mathrm{d}\log w_I &= \mathrm{d}\log(p_K I) \\
\mathrm{d}\log w &= \frac{\frac{(1-\alpha_C)}{\mu_C}\lambda_C}{\lambda_L}\,\mathrm{d}\log w_C + \frac{\frac{(1-\alpha_I)}{\mu_I}\lambda_I}{\lambda_L}\,\mathrm{d}\log w_I
\end{aligned}$$

Hulten's theorem (obtained by taking weighted average of the market clearing equations) now gives

$$\lambda_K \,\mathrm{d}\log K = \frac{\lambda_C}{\mu_C}\,\mathrm{d}\log C + \frac{\lambda_I}{\mu_I}\,\mathrm{d}\log I$$

$$\begin{aligned}\mathrm{d}\log(r_K K) &= \mathrm{d}\log(p_K I) + \frac{\frac{\alpha_C\lambda_C}{\mu_C}}{\lambda_K}\,\mathrm{d}\log\frac{C}{p_K I}\\ \mathrm{d}\log w &= \mathrm{d}\log(p_K I) + \frac{\frac{(1-\alpha_C)\lambda_C}{\mu_C}}{\lambda_L}\,\mathrm{d}\log\frac{C}{p_K I}\end{aligned}$$

Now we also have $\mathrm{d}\log C/I = \frac{\lambda_L}{\lambda_C}\,\mathrm{d}\log K$.

Now, in steady state,

$$\begin{aligned}\mathrm{d}\log I &= \mathrm{d}\log K\\ \mathrm{d}\log p_K &= \mathrm{d}\log r_K + \mathrm{d}\log(1-\tau)\end{aligned}$$

Plugging the steady state equation for capital valuation into pricing gives

$$\begin{aligned}0 &= -\frac{\alpha_C}{1-\alpha_I}\,\mathrm{d}\log(1-\tau) + \alpha_C\,\mathrm{d}\log w_I + (1-\alpha_C)\,\mathrm{d}\log w_C\\ \mathrm{d}\log p_K &= -\frac{\alpha_I}{1-\alpha_I}\,\mathrm{d}\log(1-\tau) + \mathrm{d}\log w_I\\ \mathrm{d}\log C &= \frac{\lambda_K - \frac{\lambda_I}{\mu_I}}{\frac{\lambda_C}{\mu_C}}\,\mathrm{d}\log K\\ \mathrm{d}\log I &= \mathrm{d}\log K\end{aligned}$$

Plugging these into market clearing for labor gives

$$\begin{aligned}\left(-\phi\frac{(1-\alpha_I)\frac{\lambda_I}{\mu_I}}{\lambda_L} - \alpha_C\right)\mathrm{d}\log\left(\frac{w_I}{w_C}\right) &= -\frac{\alpha_C}{1-\alpha_I}\,\mathrm{d}\log(1-\tau) + \frac{\lambda_K - \frac{\lambda_I}{\mu_I}}{\frac{\lambda_C}{\mu_C}}\,\mathrm{d}\log K\\ \phi\frac{(1-\alpha_C)\frac{\lambda_C}{\mu_C}}{\lambda_L}\,\mathrm{d}\log\left(\frac{w_I}{w_C}\right) &= -\frac{\alpha_I}{1-\alpha_I}\,\mathrm{d}\log(1-\tau) + \mathrm{d}\log K\end{aligned}$$

Substituting $\mathrm{d}\log K$ from the first equation using the second one gives

$$\alpha_C(1+\phi)\,\mathrm{d}\log\left(\frac{w_I}{w_C}\right) = \frac{\lambda_K}{\frac{\lambda_C}{\mu_C}}\,\mathrm{d}\log(1-\tau) \implies \mathrm{d}\log\left(\frac{w_I}{w_C}\right) = \frac{1}{1+\phi}\frac{\lambda_K}{\alpha_C\frac{\lambda_C}{\mu_C}}\,\mathrm{d}\log(1-\tau)$$

and so

$$\begin{aligned}\frac{\mathrm{d}\log K}{\mathrm{d}\log(1-\tau)} &= \frac{\alpha_I}{1-\alpha_I} + \frac{\phi}{1+\phi}\frac{1-\alpha_C}{\alpha_C}\frac{\lambda_K}{\lambda_L}\\ \frac{\mathrm{d}\log p_K}{\mathrm{d}\log(1-\tau)} &= \frac{\alpha_C-\alpha_I}{1-\alpha_I} + \frac{1}{1+\phi}\frac{1-\alpha_C}{\alpha_C/\mu_C}\frac{\lambda_K}{\lambda_C}\end{aligned}$$

□

A.3.6 Knowledge spillovers

We now consider technological spillovers arising from endogenous growth. Specifically, total factor productivity (TFP) in both the consumption and investment sectors increases with the aggregate capital stock. Let $A_C(K)$ and $A_I(K)$ denote TFP in the consumption and investment sectors, respectively:

$$C = A_C(K)\, K_C^{\alpha_C} L_C^{1-\alpha_C},$$
$$I = A_I(K)\, K_I^{\alpha_I} L_I^{1-\alpha_I}.$$

In the main text, we assume that A_C and A_I were fixed. Here, we assume that they are functions of the quantity of capital. To first order, what matters is the local elasticity of productivity with respect to aggregate capital:

$$\epsilon_C \equiv \frac{\partial \log A_C}{\partial \log K}, \qquad \epsilon_I \equiv \frac{\partial \log A_I}{\partial \log K}.$$

Proposition A.3 (Long-run elasticities with knowledge spillovers)**.** *We have*

$$\frac{\mathrm{d}\log K}{\mathrm{d}\log(1-\tau)} = \frac{1}{1-\dfrac{\epsilon_I}{1-\alpha_I}}\left(\frac{\alpha_I}{1-\alpha_I} + \frac{\phi}{1+\phi}\,\frac{1-\alpha_C}{\alpha_C}\,\frac{\lambda_K}{\lambda_L}\right),$$

$$\frac{\mathrm{d}\log p_K}{\mathrm{d}\log(1-\tau)} = \frac{\alpha_C-\alpha_I}{1-\alpha_I} + \frac{1}{1+\phi}\,\frac{1-\alpha_C}{\alpha_C}\,\frac{\lambda_K}{\lambda_C} + \left(\epsilon_C - \frac{1-\alpha_C}{1-\alpha_I}\,\epsilon_I\right)\,\frac{\mathrm{d}\log K}{\mathrm{d}\log(1-\tau)}.$$

The elasticity of capital is the same as in the baseline model, except for an accelerator: higher K raises A_I (through ϵ_I), which makes it easier to create additional capital, which then raises A_I even more etc.

For relative prices, what matters is the differential effect of K on sectoral productivity. If $\epsilon_C > \epsilon_I = 0$ (only the consumption sector benefits), the capital elasticity is unchanged, and the relative price of investment goods rises more than in the baseline. Conversely, if $\epsilon_I > \epsilon_C = 0$ (only the investment sector benefits), the capital elasticity is larger, and the relative price response is dampened, as investment becomes more productive as its output expands. When $\epsilon_C = \frac{1-\alpha_C}{1-\alpha_I}\,\epsilon_I$—i.e., when the elasticity of unit cost to capital is the same in both sectors—the relative price response coincides with the baseline model. This holds, for instance, when $\alpha_C = \alpha_I$ and $\epsilon_C = \epsilon_I$.

Proof of Proposition A.3. The log-linearized version of this intratemporal system is:

$$\begin{aligned}
0 &= -\epsilon_C\,\mathrm{d}\log K + \alpha_C\,\mathrm{d}\log r_K + (1-\alpha_C)\,\mathrm{d}\log w_C \\
\mathrm{d}\log p_K &= -\epsilon_I\,\mathrm{d}\log K + \alpha_I\,\mathrm{d}\log r_K + (1-\alpha_I)\,\mathrm{d}\log w_I \\
\mathrm{d}\log(r_K K) &= \frac{\alpha_C\lambda_C}{\lambda_K}\,\mathrm{d}\log(C) + \frac{\alpha_I\lambda_I}{\lambda_K}\,\mathrm{d}\log(p_K I) \\
\phi\,\mathrm{d}\log\left(\frac{w_C}{w}\right) + \mathrm{d}\log w_C &= \mathrm{d}\log(C) \\
\phi\,\mathrm{d}\log\left(\frac{w_I}{w}\right) + \mathrm{d}\log w_I &= \mathrm{d}\log(p_K I) \\
\mathrm{d}\log w &= \frac{(1-\alpha_C)\lambda_C}{\lambda_L}\,\mathrm{d}\log w_C + \frac{(1-\alpha_I)\lambda_I}{\lambda_L}\,\mathrm{d}\log w_I
\end{aligned}$$

Hulten's theorem (obtained by taking weighted average of the market clearing equations) now gives

$$(\lambda_K + \epsilon_C \lambda_C + \epsilon_I \lambda_I)\, \mathrm{d}\log K = \lambda_C\, \mathrm{d}\log C + \lambda_I\, \mathrm{d}\log I$$

Now, imposing the steady state conditions $\mathrm{d}\log I = \mathrm{d}\log K$ and $\mathrm{d}\log p_K = \mathrm{d}\log r_K + \mathrm{d}\log(1-\tau)$ on this set of equations gives

$$\begin{aligned}
0 &= -(\alpha_C \frac{\epsilon_I}{1-\alpha_I} + \epsilon_C)\, \mathrm{d}\log K - \frac{\alpha_C}{1-\alpha_I}\, \mathrm{d}\log(1-\tau) + \alpha_C\, \mathrm{d}\log w_I + (1-\alpha_C)\, \mathrm{d}\log w_C \\
\mathrm{d}\log p_K &= -\frac{\epsilon_I}{1-\alpha_I}\, \mathrm{d}\log K - \frac{\alpha_I}{1-\alpha_I}\, \mathrm{d}\log(1-\tau) + \mathrm{d}\log w_I \\
\mathrm{d}\log C &= \frac{\lambda_K + \epsilon_C \lambda_C + \epsilon_I \lambda_I - \lambda_I}{\lambda_C}\, \mathrm{d}\log K \\
\mathrm{d}\log I &= \mathrm{d}\log K
\end{aligned}$$

Plugging these into market clearing for labor gives

$$\begin{aligned}
\left(-\phi \frac{(1-\alpha_I)\lambda_I}{\lambda_L} - \alpha_C\right) \mathrm{d}\log\left(\frac{w_I}{w_C}\right) &= -\frac{\alpha_C}{1-\alpha_I}\, \mathrm{d}\log(1-\tau) + \left(1 - \frac{\lambda_L}{\lambda_C}\right)\left(1 - \frac{\epsilon_I}{1-\alpha_I}\right) \mathrm{d}\log K \\
\phi \frac{(1-\alpha_C)\lambda_C}{\lambda_L}\, \mathrm{d}\log\left(\frac{w_I}{w_C}\right) &= -\frac{\alpha_I}{1-\alpha_I}\, \mathrm{d}\log(1-\tau) + \left(1 - \frac{\epsilon_I}{1-\alpha_I}\right) \mathrm{d}\log K
\end{aligned}$$

Substituting $\mathrm{d}\log K$ from the first equation using the second one gives

$$\alpha_C(1+\phi)\, \mathrm{d}\log\left(\frac{w_I}{w_C}\right) = \frac{\lambda_K}{\lambda_C}\, \mathrm{d}\log(1-\tau) \implies \mathrm{d}\log\left(\frac{w_I}{w_C}\right) = \frac{1}{1+\phi} \frac{\lambda_K}{\alpha_C \lambda_C}\, \mathrm{d}\log(1-\tau)$$

Plugging this equation into the second one gives

$$\frac{\mathrm{d}\log K}{\mathrm{d}\log(1-\tau)} = \frac{1}{1 - \frac{\epsilon_I}{1-\alpha_I}} \left(\frac{\alpha_I}{1-\alpha_I} + \frac{\phi}{1+\phi} \frac{1-\alpha_C}{\alpha_C} \frac{\lambda_K}{\lambda_L}\right)$$

And the relative price is now

$$\frac{\mathrm{d}\log p_K}{\mathrm{d}\log(1-\tau)} = \frac{\alpha_C - \alpha_I}{1-\alpha_I} + (1-\alpha_C) \frac{\mathrm{d}\log w_I/w_C}{\mathrm{d}\log(1-\tau)} + \left(\epsilon_C - \frac{1-\alpha_C}{1-\alpha_I}\epsilon_I\right) \frac{\mathrm{d}\log K}{\mathrm{d}\log(1-\tau)}$$

substituting out $\mathrm{d}\log(w_I/w_C)$ and $\mathrm{d}\log K$ obtained above gives the result.

□

B Appendix for Section 2

B.1 Methodology details

We first give more details about the methodology to compute factor shares at the sectoral level, as briefly described in the main text. We denote the commodity x industry use table by U and the industry x commodity make table by M, which we obtain from the BEA input-output tables. We denote Ω the matrix of direct factor shares for each industry, which we obtain from KLEMS.

The first step is to convert the direct factor shares at the industry level to factor shares at the commodity

level. We do so using

$$\tilde{\Omega} = D'\Omega$$

where D is the matrix allowing to go from industry factor shares to commodity factor shares. With the common industry-technology assumption, this matrix is constructed as

$$D \equiv M \cdot \text{col}(M)^{-1}$$

where $\text{col}(M)$ denotes the diagonal matrix where the nth diagonal element corresponds to the sum of the nth column of the matrix M (i.e. total output by commodity). In contrast, with the common commodity-technology assumption, this matrix is constructed as

$$D \equiv \left(M' \cdot \text{col}(M')^{-1}\right)^{-1}.$$

The second step is to convert direct factor shares into consolidated factor shares. To do so, we first construct the direct requirement commodity by commodity matrix as $U \cdot \text{col}(U)^{-1} \cdot D$. Each column of the matrix represents, for a commodity, the share of expenditures spent on intermediary commodities. The matrix of *consolidated* factor shares by commodity is then obtained by the Leontieff inverse of this direct requirement matrix:

$$\Psi = (I - U \cdot \text{col}(U)^{-1} \cdot D)^{-1}\tilde{\Omega}$$

The third step is to aggregate these consolidated factor shares at the commodity level to the sector level. Denote by β_C the vector of the shares of total consumption spending on each commodity and, similarly, denote by β_I the vector of the shares of total investment spending on each commodity (both are reported as additional columns in the BEA use table). We obtain the vector of factor shares of the consumption and investment sector as the weighted average of the factor shares of the commodities used in each sector:

$$\alpha_C = \beta_C'\Psi, \qquad \alpha_I = \beta_I'\Psi.$$

B.2 Additional robustness checks

We now discuss two additional robustness checks not mentioned in the main text.

First, in the main text, we did all of our analysis at the BEA "summary" level which includes 71 commodities. This allows us to conduct the analysis in a unified methodology from 1963 to 2023. For 2017, the BEA also reports make-use tables using the more detailed 402 commodity classifications.

To examine whether this more granular data changes the result, we report the results using the summary versus detailed classification in 2017. Table A1 shows that the results are very similar, indicating that using the summary industry classification — necessary to extend the analysis back to 1963 — does not materially affect our findings.

Table 2 in the main text reports the robustness checks for end-of-sample average labor shares (2017–2023). Table A2 reports analogous robustness checks for the change in labor shares between the first six years of the sample (1962–1968) and the last six years (2017–2023), using the same set of exercises discussed in the main text (see the discussion of Table 2).

Table A1: Comparing summary and detailed tables

	C	I	Difference
Panel A: Total labor share			
Baseline	0.47	0.59	+0.12
Using detailed input-output tables	0.47	0.58	+0.11
Panel B: College (or above) labor share			
Baseline	0.23	0.33	+0.09
Using detailed input-output tables	0.24	0.32	+0.08

There are two main takeaways. First, essentially all of the increase in the labor share of the investment sector relative to the consumption sector comes from the treatment of organizational services. If organizational services are not capitalized, the investment–consumption labor-share gap is basically stable over time. Second, the other robustness checks do not materially alter the magnitude of the increase in this gap: across specifications, the rise in the investment–consumption labor-share difference ranges from about 5 pp to 10 pp.

Table A2: Change in labor income shares between 1963 and 2023 by sector

	ΔC	ΔI	Difference
Panel A: Total labor share			
Baseline	−0.06	+0.00	+0.07
Without capitalizing organization services	−0.03	−0.04	−0.01
Also capitalizing financial services	−0.07	0.00	+0.07
Including government sector	−0.06	−0.01	+0.05
Cost-shares instead of income-shares	−0.06	+0.03	+0.10
Common industry-technology assumption	−0.06	−0.01	+0.06
Panel B: College (or above) labor share			
Baseline	+0.17	+0.27	+0.11
Without capitalizing organization services	+0.20	+0.25	+0.05
Also capitalizing financial services	+0.16	+0.26	+0.11
Including government sector	+0.16	+0.26	+0.10
Cost-shares instead of income-shares	+0.24	+0.37	+0.13
Common industry-technology assumption	+0.17	+0.26	+0.08

B.3 Domestic production

Right now, we implicitly treat imported commodities as if they were produced with the same factor shares as domestically produced commodities.

Instead, we can follow the BEA methodology and compute domestic input–output tables, which considers imports as a separate factor. This perspective lets us separate inputs purchased from abroad from payments to domestic labor and capital,

We compute these domestic factor shares using the BEA methodology. We first subtract from the use matrix the import matrix, which a matrix that reports the use of the imports by industries and final uses. We construct this import matrix by assuming that, for each commodity, each use of the imported commodity is

proportional to the total use reported in the use table.

Table A3 reports the factor shares of each sector, splitting between domestic factor shares and imports. One important takeaway is that imports account for a very large part of equipment investment, but very little of structures, IPP, and organization investment.

Table A3: Domestic factor shares by sector (2017-2023)

Sector	Factor shares								
	Labor			Land	Capital				Import
	High school	College	Master/PhD		Equipment	Structures	IPP	Brand/Org	
$C + I$	0.19	0.14	0.12	0.03	0.11	0.14	0.07	0.11	0.10
C	0.19	0.12	0.11	0.04	0.11	0.18	0.06	0.11	0.08
I	0.21	0.19	0.13	0.00	0.10	0.05	0.10	0.10	0.14
$I_{\text{Equipment}}$	0.15	0.11	0.06	0.00	0.09	0.04	0.11	0.09	0.34
$I_{\text{Structures}}$	0.35	0.11	0.06	0.00	0.15	0.08	0.04	0.13	0.08
I_{IPP}	0.14	0.26	0.18	0.00	0.07	0.04	0.16	0.08	0.07
$I_{\text{Brand/Org}}$	0.16	0.29	0.22	0.00	0.07	0.04	0.10	0.08	0.06

C Appendix for Section 3

C.1 Validation details

This section lists the exact data moments cited in Table 7 and the corresponding model experiment specifications.

Table A4: Data moments cited in Table 7

Paper	Moment	Source	Value	SE
Goolsbee (1998b)	supply elasticity	Table VII, col. 1	1.14	0.51
	price response	Table II, col. 2	-0.17	0.03
	producer wage premium response	Table III, col. 3	-0.16	0.03
	implied wage-based elasticity	Table VII, col. 1; Table II, col. 2; Table III, col. 3	1.21	–
House and Shapiro (2008)	structural supply elasticity ξ	Table 4, Panel A, row 1 (WLS baseline)	7.68	1.85
House et al. (2022)	production response	Table 3, col. 1	1.12	0.36
	equipment purchases response	Table 3, col. 1	1.94	0.44
	purchases minus production	Table 3, col. 1	0.82	0.15
	equipment producer price response	Table 4, col. 1	-0.12	0.10
	equipment employment/hours response	Table 4, col. 1	0.77	0.45
	equipment producer wage response	Table 4, col. 1	1.28	0.91
	structure production response	Table 4, col. 1	0.87	0.32
	structure price response	Table 4, col. 1	0.54	0.12
Antonova et al. (2025)	Military multiplier (Cold War)	Figure 4	0.86	—
	Military multiplier (post-Cold War)	Figure 4	0.41	—
	Mfg PPI response (Cold War)	Figure 3, Panel A	0.2%	—
	Mfg PPI response (post-Cold War)	Figure 3, Panel B	0.4%	—
Topel and Rosen (1988)	housing start response	Table 3, Panel B, Model 2	1.68	—
	housing start response after 1y	Table 3, Panel B, Model 2	2.69	—
	housing start response after 2y	Table 3, Panel B, Model 2	2.76	—

Data moments.

Current model experiment. For Goolsbee (1998b), the model counterpart is an equipment-demand shock in the open-economy/fixed-rate quarterly calibration. We export two calibration variants for each validation exercise. The main-text numbers use the sample-specific TS calibration averaged over 1963–1988, which is the overlap between Goolsbee's 1959–1988 sample and the available open TS data. We also export a 2017–2023 end-sample robustness calibration. In both cases, the policy is announced four quarters ahead and first affects cash flows at impact quarter $t = 0$. The main elasticity comparison uses a permanent shock, and the reported year-1 and year-2 moments average quarterly model responses over quarters 0–3 and 4–7, respectively. The companion duration diagnostic uses a one-year pulse: the subsidy is announced at $t = -4$, turns on at $t = 0$, and is announced to expire one year later at $t = 4$. For the producer-wage-premium row, we use the paper's wage-premium coefficient from Table III, col. 3 together with the one-year quantity response implied by Table VII, col. 1 and Table II, col. 2: $1.14 \times (-0.17)/(-0.16) = 1.21$. The model counterpart uses the same year-1 annualized equipment quantity response divided by the year-1 equipment wage premium.

For House and Shapiro (2008), the model counterpart is just an anticipated temporary shock to τ_{equip} in

the open-economy/fixed-rate quarterly calibration. We export both a sample-specific calibration averaged over 2000–2006 and a 2017–2023 end-sample robustness calibration. The subsidy path follows the paper's temporary timing profile: it is zero in 2001:I–2001:III, turns on in 2001:IV–2003:II, steps up in 2003:III–2004:IV, and returns to zero in 2005:I–2006:IV. For the model experiment, we normalize the initial active phase to a unit shock and preserve the paper's relative step-up, so the path is 1 in the first active phase and 1.77/1.06 in the second. The main-text model entry is the ratio of the average equipment quantity response to the average equipment producer-price response over the active treatment quarters 2001:IV–2004:IV.

For House et al. (2022), we match the reduced-form column 1 estimates in Tables 3 and 4. The model counterpart uses the open-economy/fixed-rate quarterly calibration, and we export both a sample-specific calibration and a 2017–2023 end-sample robustness calibration. The sample-specific model averages the open TS calibration over 1963–2009, which is the overlap between the paper's 1959:1–2009:4 sample and the available TS data. The two equipment rows use a permanent equipment-demand shock announced four quarters ahead and realized from impact quarter $t = 0$ onward. The structures row uses the analogous permanent structures-demand shock with the same timing. Table 7 reports three impact-quarter elasticities: purchases relative to producer prices, purchases relative to producer wages, and structures expenditures relative to producer prices. In the data, these are formed as the ratios $1.94/(-0.12) = -16.2$, $1.94/1.28 = 1.5$, and $0.87/0.54 = 1.6$. In the model, purchases are mapped to the relevant investment quantity, producer prices and wages are compared to the corresponding sectoral price and wage responses, and the structures row uses the model's structures expenditure and producer-price responses under the structures-demand experiment.

For Topel and Rosen (1988), the model counterpart is a permanent structures-demand shock in the open-economy/fixed-rate quarterly calibration. The sample-specific calibration averages available open TS data overlapping Topel and Rosen's 1947–1984 sample. Housing starts map to the model's structures investment quantity I_{struct}. Housing valuations map to a composite housing price: $(1-\omega)\ \mathrm{d}\log p_{K,\text{struct}} + \omega\ \mathrm{d}\log p_{L,\text{land}}$, where ω is the model-implied land share of housing wealth, computed from the consumption-sector cost shares and user costs. Year-1, 2, and 3 elasticities average quarterly responses over quarters 0–3, 4–7, and 8–11, respectively.

For Antonova et al. (2025), the model counterpart is a permanent equipment-demand shock in the open-economy/fixed-rate quarterly calibration. The sample-specific calibration averages available open TS data overlapping the 1947–1990 Cold War period. Military output maps to equipment investment I_{equip} and manufacturing PPI maps to the equipment producer price $p_{I,\text{equip}}$. The year-1 elasticity averages quarterly responses over quarters 0–3.

C.2 Time-Series Calibration

OLG-spender demand-side. In the baseline representative-agent demand block, we set $\psi = 1$. For the OLG-spender extension, we keep $\psi = 1$ and fix $(\rho, \omega) = (2/3, 1/3)$. The functional form in Appendix A is chosen so that it numerically nests the representative-agent benchmark under $\omega = 0$ and $\rho = \frac{1}{1+r}$.

The OLG-spender extension reduces the long-run price response from 0.45 (RA baseline) to 0.26. The mechanism works through two reinforcing channels. First, the hand-to-mouth share ($\omega = 0.257$) raises the aggregate marginal propensity to consume, so a smaller fraction of the tax windfall is saved. Second, the perpetual-youth discount ($\rho = 0.685$ annually) shortens the effective planning horizon of the saving cohort, dampening the wealth effect of rising asset values on desired capital accumulation. Together, these reduce the savings response to the demand shock, so less capital accumulates in the long run and the price effect is

correspondingly smaller. The parameter ω is identified from the share of hand-to-mouth households in the SCF ($\approx 25\%$), and ρ is pinned down by the decay of intertemporal MPCs in **?**: $M_{0,0} = 0.52$ and $M_{1,0} = 0.18$.

Adjustment cost. We use BEA Fixed Assets data for depreciation rates by type, averaged over the 1990–2023 period, and set $\delta_{\text{org}} = \delta_{\text{ipp}}$. For type-specific producer prices p_{I_n}, we use the producer prices from the same BEA commodity data. Specifically, for each capital type n, we compute the investment-weighted average of commodity-level gross-output price growth factors (propagated through the Leontief inverse), cumulate these into a price level, and normalize to 2017. We construct the consumption price index p_C analogously using private consumption weights. The relative price growth $\pi_{I_n,t}$ is the growth rate of type-n producer price relative to consumption producer price.

We then use a mapping between θ and external evidence on investment-q responses. The quadratic adjustment cost model predicts the following investment semi-elasticity

$$\mathrm{d}\left(\frac{p_{I_n,t} I_{n,t}}{K_{n,t}}\right) = \Phi\, \mathrm{d}\left(\frac{p_{K_n,t}}{p_{I_n,t}}\right)$$

Evaluating in partial equilibrium (i.e., holding p_I constant) and re-arranging we obtain the elasticity

$$\frac{\mathrm{d}\log I_{n,t}}{\mathrm{d}\log p_{K_n,t}} = \Phi \frac{p_{K_n,t} K_{n,t}}{p_{I_n,t} I_{n,t}} \implies \theta = \frac{1}{\phi} \frac{p_{I_n,t} I_{n,t}}{p_{K_n,t} K_{n,t}}$$

We can further reduce the system using the steady-state q-theory equation $p_I I = (1-\theta) p_K K$ and we obtain

$$\theta_n = \frac{\delta_n / \Phi}{1 + \delta_n / \Phi}.$$

User cost of capital and capital shares. Now, we want to construct an expression for the user cost of capital $\text{uc}_n \equiv r + (1-\theta_n)\delta_n - \pi_{I_n}$. This is necessary in order to identify the sector-specific cost shares by capital type, which can not be obtained directly from the BEA data since it requires assumptions about sector-specific capital stocks.

We run the perpetual inventory in quantity units (nominal investment deflated by the type-specific producer price) and accumulate

$$K_{sn,t+1} = (1-\delta_n) K_{sn,t} + K_{sn,t}^{\theta_n} I_{sn,t}^{1-\theta_n}$$

We can then express the capital stock at current prices (relative to consumption) via $K_{sn,t} \cdot p_{I_n,t} / p_{C,t}$. This matches BEA's methodology of building a real quantity index then marking to market. For the initial condition, we use BEA fixed asset stocks (and for ORG, we initialize with the IPP stock). Over 1963–2022, the resulting stocks track BEA closely: the PIM-to-BEA ratio is 0.89 for structures, 1.35 for equipment, and 1.25 for IPP.

Along a balanced growth path we have that the aggregate discount rate is given by

$$r_{t+1} = \frac{r_{K,t+1} K_{t+1} - p_{I,t+1} I_{t+1}}{V_t} + \pi_{I,t+1}$$

That is, r equals the aggregate capital income yield, net of depreciation, relative price changes. For the investment price deflator and the depreciation term, we use time-varying weighted average across capital

types, weighted by the consumption-equivalent value of each capital type.

For the capital income yield, we use IMA data. We construct the aggregate net enterprise value V_t from the IMA by consolidating the business sector: equity at market value plus debt and loans for the corporate sectors (nonfinancial and financial), proprietors' equity for the noncorporate sector, and real estate at market value for the household sector. We remove a share 20% of household sector housing wealth to net out the value of land, which is not reproducible capital.

Capital income is gross operating surplus (GOS) from the IMA in the consolidated business sector, with two adjustments. First, for the noncorporate sector, GOS includes mixed income (proprietors' income that combines labor and capital returns); we attribute one-third to capital and two-thirds to labor, following the standard NIPA convention. Second, we net out land rents by removing 20% of household GOS.

We construct the time-varying sector-specific capital shares as

$$\alpha_{sK_n,t} = \frac{u_{K_n,t}K_{sn,t}}{\sum_n u_{K_n,t}K_{sn,t}}\alpha_{sK,t}, \qquad \lambda_{K_n,t} = \sum_s \alpha_{sK_n,t}\lambda_s,$$

which are the shares that rationalizes the sectoral paths of capital accumulation, conditional on the user cost. The values $\alpha_{sK,t}$ are observed directly in the BEA (nonlabor share), and we created the sectoral breakdown by capital type $\alpha_{sK_n,t}$ by using the user cost estimates. The second part of the equation also requires only BEA data (sectoral shares λ_s). Moreover, by construction, we have $\sum_n \lambda_{K_n,t} = \lambda_{K,t}$, where the aggregate labor share is entirely determined by EBA data.

C.3 Derivation of the long-run decomposition

We generalize Proposition 1.1 to N capital types and F labor types. In the long run, $\mathrm{d}\log I_n = \mathrm{d}\log K_n$ (accumulation) and $\mathrm{d}\log p_{K_n} = \mathrm{d}\log p_{I_n}$ (investment-q), so the valuation equation gives $\mathrm{d}\log r_{K_n} = \mathrm{d}\log p_{I_n} - \mathrm{d}\log(1-\tau)$. Substituting into the $I_{n'}$ pricing equation:

$$(\mathbf{I} - \boldsymbol{\alpha}_{IK})\,\mathbf{p}_I = -\boldsymbol{\alpha}_{IK}\mathbf{1}\ \mathrm{d}\log(1-\tau) + \boldsymbol{\omega}_I, \tag{C.1}$$

where $\boldsymbol{\alpha}_{IK}$ is the $N \times N$ matrix with entry $\alpha_{I_{n'}K_m}$ and $\omega_{I_{n'}} \equiv \sum_f \alpha_{I_{n'}L_f}\,\mathrm{d}\log w_{I_{n'}f}$. Since $\mathrm{d}\log p_{K_n} = \mathrm{d}\log p_{I_n}$, the aggregate is $\mathrm{d}\log p_K = \sum_n(\lambda_{K_n}/\lambda_K)\,\mathrm{d}\log p_{I_n}$.

With $F = 1$ and $\phi = \infty$, wages equalize across sectors, so $\omega_{I_{n'}} = (1 - \alpha_{I_{n'}K})\,\mathrm{d}\log w$. Eliminating $\mathrm{d}\log w$ via the C pricing equation and solving (C.1):

$$\frac{\mathrm{d}\log p_K}{\mathrm{d}\log(1-\tau)} = 1 + \sum_{n=1}^{N}\left(\alpha_{CK_n} - \frac{\lambda_{K_n}}{\lambda_K}\right)\left((\mathbf{I} - \boldsymbol{\alpha}_{IK})^{-1}\mathbf{1}\right)_n. \tag{C.2}$$

When $N = 1$, $\left((\mathbf{I} - \boldsymbol{\alpha}_{IK})^{-1}\mathbf{1}\right)_1 = 1/(1-\alpha_{IK})$ and (C.2) reduces to the technology asymmetry term $(\alpha_{CK} - \alpha_{IK})/(1-\alpha_{IK})$ from Proposition 1.1. Subtracting gives

$$\mathrm{Cov}_K = \sum_{n=1}^{N}\left(\alpha_{CK_n} - \frac{\lambda_{K_n}}{\lambda_K}\right)\left(\left((\mathbf{I} - \boldsymbol{\alpha}_{IK})^{-1}\mathbf{1}\right)_n - \frac{1}{1-\alpha_{IK}}\right). \tag{C.3}$$

Turning to Cov_L, consider $N = 1$ with F labor types. The labor mobility condition $\mathrm{d}\log w_{If} - \mathrm{d}\log w_{Cf} =$

$\frac{1}{1+\phi_f}(\mathrm{d}\log(p_K I) - \mathrm{d}\log C)$ can be substituted into the pricing equations. Solving yields

$$\frac{\mathrm{d}\log p_K}{\mathrm{d}\log(1-\tau)} = \frac{\alpha_{CK} - \alpha_{IK}}{1-\alpha_{IK}} + \frac{(1-\alpha_{CK})\lambda_K}{\alpha_{CK}\lambda_C}\left(1 - \frac{1-\lambda_K}{(1-\alpha_{CK})(1-\alpha_{IK})}\sum_{f=1}^{F}\frac{\phi_f}{1+\phi_f}\frac{\alpha_{CL_f}\alpha_{IL_f}}{\lambda_{L_f}}\right). \tag{C.4}$$

Setting $F = 1$ (so $\phi_f = \phi$ for all f), the second term reduces to the reallocation friction $\frac{1}{1+\phi}\frac{(1-\alpha_{CK})\lambda_K}{\alpha_{CK}\lambda_C}$. The difference is

$$\mathrm{Cov}_L = \frac{(1-\alpha_{CK})\lambda_K}{\alpha_{CK}\lambda_C}\left(\frac{\phi}{1+\phi} - \frac{1-\lambda_K}{(1-\alpha_{CK})(1-\alpha_{IK})}\sum_{f=1}^{F}\frac{\phi_f}{1+\phi_f}\frac{\alpha_{CL_f}\alpha_{IL_f}}{\lambda_{L_f}}\right). \tag{C.5}$$