The Good, the Bad and the Ugly

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ABSTRACT:

If every activity can be taxed or regulated, the first-best tax rate on each activity is equal its marginal external damage (Pigou, 1920). This paper builds and employs a new analytical general equilibrium model to find a closed-form solution for the second-best optimal (SBO) tax rate on a relatively "good" clean production or disposal method, and the SBO tax rate on a "bad" polluting production or disposal method, in the case where no tax can be imposed or enforced on the most polluting "ugly" production or disposal method (*e.g.*, illegal dumping). This model can be applied to any environmental pollution problem, including disposal into the air, water, or land. We use intuition from Corlett and Hague (1953) to show that the tax on bad disposal might be *lower* than the tax on good disposal, or even lower than zero, if ugly disposal is enough better as a substitute for bad disposal than it is for good disposal. Our model captures not only differences in how the partial equilibrium cross-price elasticities help determine general equilibrium effects of each available tax on emissions or disposal, but also how all own-price and cross-price elasticities and share parameters help determine those general equilibrium outcomes – including the amount of untaxed ugly disposal. And this general equilibrium change in each of the three disposal amounts is what matters for setting the two available SBO tax rates. We also show how these elasticities combine with each disposal method's marginal external damage parameter to determine the two SBO tax rates on good and bad disposal.

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Government policy can help guide firms and households that face a variety of options to reduce environmental damages from emissions into waterways, into the air, or disposal of solid waste. We study this problem in a new analytical general equilibrium (AGE) model that lumps waste options into three categories. For local air pollutants and carbon emissions, electricity generation with the lowest marginal external damage (MED) is from "good" renewable sources like wind turbines or solar panels. The second category with "bad" external damage is from fossil fuels such as natural gas, while a third more "ugly" category is the use of coal – with even higher carbon content per kilowatt hour. Liquid waste might be treated by good technology or partially treated by a bad technology before release. The most ugly category with highest MED is the release of untreated liquid waste. Solid waste can be recycled (low MED), placed in garbage for the sanitary landfill (middle MED), or dumped in the woods or vacant lots (highest MED).

In the solution of Pigou (1920), a welfare-maximizing planner would impose a first-best optimal (FBO) tax on each form of production or waste disposal at a rate equal to its own MED. Unfortunately, however, the tax on ugly waste might be unavailable. A tax on coal-fired generation has been entirely blocked in the U.S. and other countries for political reasons, perhaps related to distributional effects of raising electricity prices for low-income families. Or, an air pollution problem might involve illegal emissions that are hidden and thus uncontrolled. Also, tax on ugly water releases or solid waste may be infeasible because of problems with monitoring, enforcement, or administration. Dumping can be strictly illegal, of course, but monitoring is difficult and costly. Enforcement is generally poor (see *e.g.* Slemrod, 2019).

Used motor oil can be dumped into storm drains or along railroad tracks. Sigman (1998) finds illegal dumping of used oil responds positively to costs of legal waste disposal or to costs of re-refining for reuse. In another example, renewable fuels are expensive for informal sector brick ovens in Mexico, but a policy to encourage propane can avoid use of "cheap, highly polluting fuels such as used tires and scrap wood" (Blackman and Bannister, 1998, p.1). Illegal dumping is further discussed in Choe and Fraser (1999), Shinkuma (2003), and Ino (2011).

In the case where a tax on ugly disposal is not available, the question we ask is how best to tax the other two options. The FBO Pigovian solution would tax natural gas at its MED, but the second-best optimal (SBO) tax might turn into a subsidy if natural gas is better than renewables as a substitute for coal.[1] Similarly, the SBO tax on bad wastewater treatment might

[1] Holland *et al.* (2026) find that existing natural gas plants have sufficient spare capacity for immediate replacement of most U.S. coal generation, which might suggest easier substitution into natural gas than into renewables.

be positive or negative, depending not only on the MED of each treatment technology but also on the cross-price elasticities between good, bad, and ugly disposal. Also, many communities impose a positive tax per bag or can of garbage to conserve scarce space in landfills and to reduce leachate and methane emissions, but the SBO tax per bag of garbage might be zero or negative if that option best reduces the highest damages from dumping.[2]

A major part of this problem was captured over seventy years ago in a simple model with no environmental externalities. In that case, if labor supply is fixed, then a FBO policy can collect any necessary revenue by taxing all commodities at the same rate. But Corlett and Hague (CH, 1953) consider endogenous labor supply with two taxable commodities and untaxable leisure in utility. They show that uniform taxation of commodities is no longer optimal. As an indirect way to tax leisure, the SBO tax rate is lower on the good that is a better substitute for leisure in utility and higher on a good that is a complement (or less of a substitute) for leisure.

Subsequent optimal tax papers elaborated on this point by interpreting the social planner's first-order conditions.[3] None of those papers apply this idea to waste disposal options. In summary, then, our paper makes the following four new contributions.

First, we show how this CH result can be applied to the externality policy problem where one type of disposal cannot be taxed.

Second, we build a new AGE model not just to compare first-order conditions but to find a closed-form solution for each SBO tax rate. We differentiate utility, all production functions, and all resource constraints to find 19 linear equations. We can then find a closed-form solution for equilibrium effects of a small change in a tax or subsidy rate on any of the 19 unknown outcomes (commodity prices, disposal prices, and all quantities). This problem could be studied for a vector of goods in household utility, but the policy problem for waste in production or

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[2] Fullerton and Kinnaman (1995) derive a FBO deposit-refund system (DRS) in a model with garbage, recycling, and illicit dumping. Each purchase of an output gets a tax equal to the MED of dumping it; recycling gets a tax equal to its MED minus a refund of the output tax; and garbage gets a tax equal to its MED minus refund of the output tax. This combination leaves a tax equal to MED of dumping on anything presumed dumped (*i.e.*, no disposal through landfill or recycling). Thus, a FBO DRS might circumvent the need for SBO tax rates. For a case that necessitates the second-best policies in this paper, it must be that the FBO DRS is not available. For example, politicians may like a subsidy to recycling but abhor a tax on output. Or perhaps coordination is too difficult when a regional level of government controls output taxes and local government controls recycling policy.

[3] Google Scholar finds 951 citations to CH (1953), the huge majority of which elaborate on second-best optimal taxation when leisure is not taxable. Environmental papers discuss SBO taxation of a polluting good when leisure is not taxable (*e.g.*, Bovenberg and Goulder, 1996, or West and Williams, 2007). Also relevant are SBO commodity tax papers by Green and Sheshinski (1976), Diamond (1973), and Atkinson and Stiglitz (1976). These optimal tax papers are reviewed and summarized in *e.g.*, Sandmo (1976) and Auerbach and Hines (2002). As far as we can tell, none of the 951 papers that cite Corlett and Hague (1953) replace leisure as the untaxed good – as done here.

disposal of any one of those commodities is basically the same as for our AGE model's one commodity that generates waste. For tractability, the many identical households in our model get utility from consumption of one commodity that creates waste and one that does not.

Third, we interpret analytical results to show how each outcome depends on parameters such as the elasticity of substitution between the two goods in utility, all cross-price elasticities across waste options, and all initial input shares in production as well as the initial shares of total primary factors employed in each output or disposal sector. The change in each waste has an ambiguous sign. Indeed, any change in tax on bad waste might lead to an increase or decrease in either good waste or ugly waste. But our solutions include terms that can be interpreted. Only one of these terms is the "CH effect", so we demonstrate other effects within our AGE solutions.

Fourth, we calibrate the model for a numerical example. Our analytics provide intuition but not specific policy predictions, so we use this calibration only to illustrate the concepts in our discussion of results. For example, we explore numerically the parameter space that yields the outcome where a tax on bad disposal is less than the tax on good disposal, and even where the tax on bad disposal becomes a subsidy (to avoid high damages of ugly disposal). This result may be counter-intuitive, or at least interesting, but not be surprising. Residential garbage collection often has a marginal price of zero – specifically to avoid illegal dumping. Thus, our model and derivations formalize what good policymakers may already know.

For several reasons, our calibration is constructed to represent a general approximation of any pollution problem (in the air, land, or water). First, our AGE model cannot represent details of any one actual pollution problem. Tractability limits our model to only one sector requiring pollution or disposal, with only three choices for method.[4] Second, accounting data can show primary factors used in production of physical goods vs. services, but not resources used for clean treatment of waste vs. untreated disposal, and certainly not illegal disposal. Third, many elasticities used here have never been estimated – even the own-price Allen elasticities but especially cross-price elasticities. Our results are highly sensitive to elasticities, which can vary over a wide range from negative to large positive values. Fourth, marginal damages from greenhouse gases are well measured by the "social cost of carbon" (*e.g.*, Rode *et al.* 2021), but a proper comparison of damages from each type of electricity generation would also include local pollutants that depend on the size of local populations and other local circumstances. The MED

[4] Our three methods of disposal requires a 3x3 matrix of own-price and cross-price elasticities, but it provides all the same intuition as a more complicated model with four or more methods of disposal and a 4x4 or larger matrix.

from water pollutants are not well measured and must depend on how the three types are categorized. Measures of MED for recycling and for landfills are not well measured, and the MED from dumping is not available (see *e.g.*, Kinnaman 2014).

Thus, existing estimates of parameters are not yet sufficient to undertake a meaningful calibration that adequately represents any particular problem. Our generic calibration, explained below, provides useful guidance for how to think about any pollution problem. We show how a bad pollutant might optimally be taxed more than – or less than – a low-polluting option. This generality may appear to say that "any result is possible", but our results show the *circumstances* under which each such result pertains. Readers can apply the concepts in this paper to any pollution problem (*e.g.*, if switching from coal to natural gas is easier than to renewables).[5] We show the sensitivity of each SBO tax or subsidy rate to each parameter, and we show which parameters are most important to estimate in future empirical work.

How do our results relate to prior results? With leisure untaxed, Bovenberg and Goulder (1996) calculate that the SBO tax on a polluting good can be higher or lower than its MED, depending on circumstances. They assume leisure is separable in utility, so all goods are equally substitutable for leisure. Following CH (1953), West and Williams (2007) estimate a cross-price elasticity between gasoline and untaxed leisure. They "find that gasoline is a leisure complement, and thus that the optimal [SBO] gas tax exceeds the marginal external damage associated with gas use by roughly 35%" (p. 594). In both of those papers, the pollution tax rate is second-best because the labor-leisure choice is distorted. Here, we have no leisure at all and instead use ugly disposal as the untaxed choice. In our numerical example below, we show that the SBO tax on a bad form of disposal can be twice as high as its MED or less than zero, depending on cross-price elasticities. Illegal dumping can be a very good substitute for a taxable form of polluting disposal – one that is less ugly but still bad.

Across the three disposal methods, our primary calibration below employs the same share parameters and elasticity parameters, but different marginal damage parameters. In this generic example, as expected, the SBO tax on bad disposal exceeds the SBO tax on good disposal. But if the Allen cross-price elasticity of ugly disposal with respect to the price of bad disposal is doubled from 0.5 to 1.0 (or higher) and the MED from ugly disposal is about three times the

[5] Our model compares two long-run equilibria, with and without a particular tax change, though a dynamic model could capture how a carbon tax induces more short-run shift into natural gas and long-run shift to renewables. Still, our model can be used to compare outcomes assuming high or low cross-price elasticities.

MED from bad disposal (or higher), then the SBO tax on bad disposal is less than the tax on good disposal. And if that elasticity is 1.5 (or higher) and the MED from ugly is four times the MED from bad (or higher), then the SBO tax on bad pollution is negative. Thus, optimal policy could subsidize bad behavior when even-worse ugly disposal (a) is highly damaging, (b) cannot be reduced directly, and (c) can be reduced by subsidizing an alternative.

The first section below presents our analytical general equilibrium model, its basic equations, and the linearization process. The second section finds closed-form solutions for the effects of a small change in tax rate on various key outcomes – including changes in each of the three types of disposal. Section 3 provides welfare analysis and finds closed-form solutions for the three FBO Pigovian tax rates and then the two SBO tax or subsidy rates. Section 4 explains our calibration, and section 5 provides numerical results. Section 6 concludes.

1. Analytical General Equilibrium Model

We model a closed economy in long-run equilibrium with two outputs, where X represents pollution-intensive goods and Y represents services – with no environmental damages. Without yet accounting for pollution or the cost of pollution, the two production functions can be written in general form as $X = X(K_X)$ and $Y = Y(K_Y)$, where each uses a single primary input K that is a composite of capital, labor, and any other primary input. This factor is perfectly mobile, has a single price p_K, and is in fixed total supply $\bar{K}$. Both sectors are perfectly competitive, with production that is constant returns to scale (CRTS). Hence, we can scale the entire economy so that all units are defined per household. We assume n is a large number of identical households, so n times each input or output is the aggregate amount. Also, CRTS means production functions can be written as $X = \frac{K_X}{c_X}$ and $Y = \frac{K_Y}{c_Y}$, where c_X and c_Y are each a fixed technology parameter that reflects necessary use of the primary factor per unit of output. The zero-profit conditions (ZPC) for the two sectors are $p_X X = p_K K_X$ and $p_Y Y = p_K K_Y$.

Each basic equation is also totally differentiated and re-arranged to yield (⇒) a linear equation that uses a "hat" to denote any proportional change (*e.g.*, $\hat{X} \equiv \frac{dX}{X}$). Derivations of these linear equations are in the online Appendix A. The key equations are numbered, for the later use of N independent linear equations to solve for N unknown changes in prices and quantities:

$$X = K_X/c_X \qquad \Rightarrow \qquad \hat{X} = \widehat{K_X} \tag{1}$$

$$p_X X = p_K K_X \qquad \Rightarrow \qquad \widehat{p_X} + \hat{X} = \widehat{p_K} + \widehat{K_X} \tag{2}$$

$$Y = K_Y/c_Y \qquad \Rightarrow \qquad \hat{Y} = \widehat{K_Y} \tag{3}$$

$$p_Y Y = p_K K_Y \qquad \Rightarrow \qquad \widehat{p_Y} + \hat{Y} = \widehat{p_K} + \widehat{K_Y} \tag{4}$$

Insert equation (1) into equation (2) to get $p_X = p_K c_X$, since only K is used to produce X. The implied linear equation is $\widehat{p_X} = \widehat{p_K}$ (which is not a numbered independent linear equation because it is derived from the first two). Analogously, equations (3) and (4) imply $\widehat{p_Y} = \widehat{p_K}$.

So far X is output, but p_X accounts only for the cost of primary inputs in its production. We next model the three polluting inputs to production, or equivalently the three forms of disposal after consumption. The consumer purchase decision depends on the full cost of purchase and required disposal, so our model does not distinguish whether this extra cost of pollution is paid by the firm and reflected in the sales price, or instead later paid by the consumer. Either way, p_X^F is the full price paid by consumers for production plus additional resources necessary for pollution disposal. To allow for substitution across the polluting inputs or disposal methods, we assume that feasible combinations are given by the disposal technology equation:

$$X = W(G_X, B_X, U_X)$$

where each unit of waste W is defined per unit of the sector's output.[6] This waste equation is the part of the production function for the firm's choices across waste disposal methods or the household's choice of disposal methods, where G_X is good disposal, B_X is bad disposal, and U_X is ugly disposal. The firm or the household chooses the amount of each kind of disposal to minimize the total cost of disposal.

The three kinds of disposal for X could represent air emissions from three different fossil fuel resources, three ways to treat water pollution, or three ways to dispose of consumer waste (*e.g.*, recycling, garbage, and dumping). Each has its own MED defined as μ_i $(i = G, B, U)$. We assume $0 < \mu_G < \mu_B < \mu_U$, to fit our definitions.[7] Households face a private cost for each type of disposal that might include the cost of their own garbage and recycling bins, the space in their house or garage to store them, and the time they take for daily disposal and weekly transport to the curb. We define private costs by fixed parameters c_G per unit of good disposal, c_B per unit of

[6] This equation and unit convention together are equivalent to assuming the sector has a production function $X = X[K_X, W(G_X, B_X, U_X)]$ with a fixed ratio between K_X and W.

[7] Even good disposal such as recycling is not harmless and has associated environmental damages from collection, transportation, and processing. Analogously, even the "Good" portrayed by Clint Eastwood in the 1966 film causes damages despite his name.

bad disposal, and c_U per unit of ugly disposal. Then:

$$K_G = c_G G_X \quad \Rightarrow \quad \widehat{K_G} = \widehat{G_X} \tag{5}$$

$$K_B = c_B B_X \quad \Rightarrow \quad \widehat{K_B} = \widehat{B_X} \tag{6}$$

$$K_U = c_U U_X \quad \Rightarrow \quad \widehat{K_U} = \widehat{U_X} \tag{7}$$

$$\bar{K} = K_X + K_Y + K_G + K_B + K_U \quad \Rightarrow \tag{8}$$

$$0 = \widehat{K_X}\theta_{KX} + \widehat{K_Y}\theta_{KY} + \widehat{K_G}\theta_{KG} + \widehat{K_B}\theta_{KB} + \widehat{K_U}\theta_{KU}$$

Equation (5) implies that the per-unit cost of producing G_X is $p_K c_G$. Also, equations (6) and (7) imply that $p_K c_B$ and $p_K c_U$ are the per unit cost of B_X and U_X (respectively).[8] Equation (8) is the economy's resource constraint (per household). Other equations below require similar ratio or share parameters, so linearization uses θ_{ji} to represent the fraction of the first subscript (j) attributable to the second subscript (i). Here, for example, $\theta_{Ki} \equiv K_i/\bar{K}$ is the fraction of $\bar{K}$ attributable to K_i (for $i = X, Y, G, B, U$), and $\theta_{KX} + \theta_{KY} + \theta_{KG} + \theta_{KB} + \theta_{KU} = 1$.

The relevant policy variables are positive or negative tax rates: τ_G per unit of good disposal, τ_B per unit of bad disposal, and τ_U per unit of ugly disposal (at least for the simple case where we show that our model matches Pigou's FBO solution).[9] The household's full cost of good disposal ($p_G G_X$) equals the sum of private cost ($p_K c_G G_X$) and tax cost ($\tau_G G_X$). We then define $\theta_{Gp} \equiv p_K c_G/(p_K c_G + \tau_G)$ for the private share of the cost and $\theta_{G\tau} \equiv \tau_G/(p_K c_G + \tau_G)$ for the tax share (where $\theta_{Gp} + \theta_{G\tau} = 1$). With similar definitions for the other inputs, we have:

$$p_G G_X = p_K c_G G_X + \tau_G G_X \quad \Rightarrow \quad \widehat{p_G} + \widehat{G_X} = \theta_{Gp}(\widehat{p_K} + \widehat{G_X}) + \theta_{G\tau}(\widehat{\tau_G} + \widehat{G_X}) \tag{9}$$

$$p_B B_X = p_K c_B B_X + \tau_B B_X \quad \Rightarrow \quad \widehat{p_B} + \widehat{B_X} = \theta_{Bp}(\widehat{p_K} + \widehat{B_X}) + \theta_{B\tau}(\widehat{\tau_B} + \widehat{B_X}) \tag{10}$$

$$p_U U_X = p_K c_U U_X + \tau_U U_X \quad \Rightarrow \quad \widehat{p_U} + \widehat{U_X} = \theta_{Up}(\widehat{p_K} + \widehat{U_X}) + \theta_{U\tau}(\widehat{\tau_U} + \widehat{U_X}) \tag{11a}$$

$$\text{or} \quad p_U U_X = p_K c_U U_X \quad \Rightarrow \quad \widehat{p_U} + \widehat{U_X} = \widehat{p_K} + \widehat{U_X} \quad \text{with fixed } \tau_U = 0 \tag{11b}$$

$$p_W X = p_G G_X + p_B B_X + p_U U_X \quad \Rightarrow \quad \hat{X} + \widehat{p_W} = \tag{12}$$

$$\theta_{XG}\left(\widehat{p_G} + \widehat{G_X}\right) + \theta_{XB}\left(\widehat{p_B} + \widehat{B_X}\right) + \theta_{XU}\left(\widehat{p_U} + \widehat{U_X}\right)$$

$$p_X^F X = p_X X + p_W \mathrm{X} \quad \Rightarrow \quad \widehat{p_X^F} + \hat{X} = \theta_{pX}(\widehat{p_X} + \hat{X}) + \theta_{pW}(\widehat{p_W} + \hat{X}) \tag{13}$$

[8] An alternative interpretation of the same equations is that the household pays haulers to collect G_X and B_X, firms with CRTS production functions $G_X = K_G/c_G$ and $B_X = K_B/c_B$. Competitive prices would match their per-unit costs of production, $p_K c_G$ and $p_K c_B$. Thus, in this interpretation, consumer costs are the same as in the text.

[9] Optimal tax theory is focused on tax rates, but our intuition applies equally to quantity regulation of pollutants, where our tax rate is essentially the shadow price on a quantity constraint (when one pollutant cannot be limited).

For bad disposal, equation (10) is analogous to (9). For ugly disposal, (11a) assumes $\tau_U \neq 0$ and is similar to (9) and (10), while (11b) is the case with fixed $\tau_U = 0$ (for SBO solutions).[10] In that case, $\widehat{p_U} = \widehat{p_K}$. Then, because the quantity of X must match the quantity of waste W, equation (12) defines the overall cost of waste disposal p_W per unit of X. To arrive at equation (12), add the cost of good disposal, $p_G G_X$, bad disposal, $p_B B_X$, and ugly disposal, $p_U U_X$ (*e.g.* sorting cost, separating materials, taking bins to curbs, or transporting waste to the woods for dumping). In the linearization of equation (12), we define $\theta_{XG} \equiv \frac{p_G G_X}{p_W X}$, $\theta_{XB} \equiv \frac{p_B B_X}{p_W X}$, and $\theta_{XU} \equiv \frac{p_U U_X}{p_W X}$ (where all three sum to one). Then equation (13) defines a household's *full* spending on X as $p_X^F X \equiv p_X X + p_W X$, the sum of purchase costs and necessary disposal costs. Divide both sides by X to get the basic equation for the full price p_X^F on the left side of (13). The linearized equation (13) defines $\theta_{pX} \equiv \frac{p_X}{p_X^F}$ and $\theta_{pW} \equiv \frac{p_W}{p_X^F}$, where $\theta_{pX} + \theta_{pW} = 1$.

In other words, the full cost p_X^F to consume one unit of good X is a combination of purchase and disposal costs. Given how this full price varies with disposal costs, the consumer can substitute from X into Y if an increase in tax τ_G raises p_G (which raises p_X^F by an amount that depends on the good disposal share θ_{XG}). Similarly, the household can consume more of good X and less of Y in response to a larger subsidy τ_B (depending on the bad disposal share θ_{XB}).

Differentiation of the waste equation yields the linear equation (14) where $\theta_{XG} \equiv \frac{p_G G_X}{P_W X}$, $\theta_{XB} \equiv \frac{p_B B_X}{P_W X}$, and $\theta_{XU} \equiv \frac{p_U U_X}{P_W X}$ are the expenditure shares in the waste function for the good, the bad and the ugly (where $\theta_{XG} + \theta_{XB} + \theta_{XU} = 1$).

$$X = W(G_X, B_X, U_X) \qquad \Rightarrow \qquad \hat{X} = \theta_{XG}\widehat{G_X} + \theta_{XB}\widehat{B_X} + \theta_{XU}\widehat{U_X} \tag{14}$$

$$\widehat{G_X} - \widehat{U_X} = \theta_{XG}(e_{GG} - e_{UG})\widehat{p_G} + \theta_{XB}(e_{GB} - e_{UB})\widehat{p_B} + \theta_{XU}(e_{GU} - e_{UU})\widehat{p_U} \tag{15}$$

$$\widehat{B_X} - \widehat{U_X} = \theta_{XG}(e_{BG} - e_{UG})\widehat{p_G} + \theta_{XB}(e_{BB} - e_{UB})\widehat{p_B} + \theta_{XU}(e_{BU} - e_{UU})\widehat{p_U} \tag{16}$$

To model the choices among the three inputs in equations (15) and (16), we follow Mieszkowski (1972) and Fullerton and Heutel (2007). The Allen elasticity of substitution, e_{ij}, between two inputs i and j can be interpreted as the percent change in the ratio of inputs (i/j) per one percent

[10] We could easily weaken the policy constraint to say that τ_U is not fixed at zero but at a sub-optimally positive level. That generalization would have little effect on our results below about how SBO tax rates are determined by elasticities, MED parameters, and share parameters. The main effect would be to complicate aesthetics of solutions by adding more terms, because total revenue $T = \tau_G G_X + \tau_B B_X + \tau_U U_X$ would be affected by changes in U_X. Below, we assume total tax revenue is fixed, so a small change in τ_B is offset by an opposite change in τ_G.

change in the price ratio (p_j/p_i). These e_{ij} are partial equilibrium (PE) elasticities in the sense that "nothing else changes" (only the ratios of prices and quantities). Differentiation of that definition yields (15) and (16). In production, or after consumption, the Allen elasticities capture flexibility to switch disposal methods. As demonstrated by Allen (1938), this elasticity is positive when the two inputs are substitutes and is negative when they are complements. He shows that $e_{ij} = e_{ji}$, that $e_{ii} < 0$, and that at most one of the three cross-price elasticities can be negative. The Allen adding-up constraint says $a_{iG} + a_{iB} + a_{iU} = 0$, where $a_{ij} = e_{ij}\theta_{Xj}$

Pre-existing disposal tax rates yield tax revenue T per household in equation (17), where differentiation uses shares $\theta_{TG} \equiv \tau_G G_X/T$ and $\theta_{TB} \equiv \tau_B B_X/T$ (which sum to one). In the case where ugly disposal cannot be taxed, we have:

$$T = \tau_G G_X + \tau_B B_X \qquad \Rightarrow \qquad \hat{T} = \theta_{TG}\left(\widehat{\tau_G} + \widehat{G_X}\right) + \theta_{TB}\left(\widehat{\tau_B} + \widehat{B_X}\right) \tag{17}$$

$$I = p_K \bar{K} + T \qquad \Rightarrow \qquad \hat{I} = \frac{T}{I}\hat{T} \tag{18}$$

We set $\hat{T} = 0$, so (17) implies a fixed revenue requirement, as if a state or local waste disposal agency must pay for given disposal services by use only of disposal taxes. An increase in τ_B must be accompanied by a change in τ_G. This assumption allows us to address our key question about the two SBO tax rates relative to each other, but it also avoids conflating the economic effects of a disposal tax rate increase with other economic effects from the use of extra revenue. Thus, we avoid a change in public spending that would have its own economic effects; we avoid a change in another distorting tax that would greatly complicate our second-best analysis; and we avoid a change in a pre-existing lump-sum tax that would have its own welfare effects.

As shown in equation (18), the fixed T is also a fixed transfer in the household's income, I. We assume the primary factor is numeraire ($\widehat{p_K} = 0$), so factor income $p_K \bar{K}$ is also unchanged. Together, these assumptions imply that income is unchanged.

Finally, the household utility function in equation (19) is continuous, quasi-concave, and twice differentiable. It is increasing in consumption goods X and Y, and decreasing in each of the three total disposal amounts, that is, public bads that are separable in utility. Because n is a large number, each household disregards its own contribution to the total public externalities. Then the linear equation defines σ_Ψ as the elasticity of substitution between X and Y in utility.

$$\Psi = \Psi(X, Y;\, nG_X, nB_X, nU_X) \qquad \Rightarrow \qquad \hat{X} - \hat{Y} = \sigma_\Psi\left(\widehat{p_Y} - \widehat{p_X^F}\right) \tag{19}$$

The household maximizes utility subject to its budget. If the household chooses disposal

methods, then the technological disposal function, $X = W(G_X, B_X, U_X)$ can be substituted into the utility function. Then each household effectively chooses only G_X, B_X, U_X, and Y.

2. General Equilibrium Outcomes with Three Polluting Inputs

We have ten unknown quantities ($\hat{X}$, $\widehat{K_X}$, $\hat{Y}$, $\widehat{K_Y}$, $\widehat{B_X}$, $\widehat{K_B}$, $\widehat{G_X}$, $\widehat{K_G}$, $\widehat{U_X}$, and $\widehat{K_U}$), eight prices ($\widehat{p_K}$, $\widehat{p_Y}$, $\widehat{p_X}$, $\widehat{p_B}$, $\widehat{p_G}$, $\widehat{p_U}$, $\widehat{p_W}$, and $\widehat{p_X^F}$), two tax rates ($\widehat{\tau_B}$ and $\widehat{\tau_G}$), and two other values ($\hat{T}$ and $\hat{I}$). Of those 22 variables, we set one numeraire ($\widehat{p_K} = 0$), and one exogenous policy shock (*e.g.*, $\widehat{\tau_G} > 0$ or $\widehat{\tau_B} > 0$). We assume the government budget to be a fixed amount ($\hat{T} = 0$) and that both tax rates are on the upward-sloping (normal) part of their Laffer curve. Thus, an exogenous policy shock $\widehat{\tau_B} > 0$ raises revenue that can be used to reduce the other tax rate ($\widehat{\tau_G} < 0$). These assumptions reduce the number of unknowns from 22 to 19, so we can solve for outcomes using the 19 numbered equations above.

2.1 Solutions for Changes in the Good, the Bad and the Ugly

Our closed-form solutions for equilibrium outcomes involve many terms that include share parameters, elasticities, and tax rate changes. Full derivations are shown in Appendix B. A later section solves for SBO tax rates. This section finds the change in each disposal quantity from an exogenous increase in the tax on bad disposal ($\widehat{\tau_B}$), but we start with an intermediate equation that is not yet a closed-form solution because it includes an endogenous change in $\widehat{\tau_G}$:

$$\widehat{U_X} = \varepsilon_{UB}\widehat{\tau_B} + \varepsilon_{UG}\widehat{\tau_G} \tag{20}$$

where $\varepsilon_{UB} \equiv \left[e_{UB}(1-\theta_{KU}) - e_{BB}(\theta_{KB}) - e_{GB}(\theta_{KG}) - \sigma_\Psi \theta_{pW}\theta_{KY}\right]\theta_{XB}\theta_{B\tau}$ and $\varepsilon_{UG} \equiv \left[e_{UG}(1-\theta_{KU}) - e_{GG}(\theta_{KG}) - e_{BG}(\,\theta_{KB}) - \sigma_\Psi \theta_{pW}\theta_{KY}\right]\theta_{XG}\theta_{G\tau}$.

The Allen elasticity e_{ij} is the PE effect on i for a change in price j (with no other price or quantity changes). Our full general equilibrium (GE) calculations below use E_{ij} to denote the full GE effect on i for a change in price j, but elasticities ε_{ij} in equation (20) are neither PE nor GE. In this intermediate equation, the first term uses ε_{UB} to denote the percent change in U_X from a one percent exogenous change in τ_B, but without accounting yet for the endogenous $\widehat{\tau_G}$ in the second term. Analogously, the second term uses ε_{UG} as the percent change in U_X from a one percent change only in τ_G without accounting for any change in τ_B. Both are composite elasticities that depend upon several PE elasticity parameters e_{ij} and cost shares (the θ's). We now explain equation (20) in more detail, starting with the PE cross-price elasticities of interest

to Corlett and Hague (CH, 1953). Their logic would compare e_{UB} and e_{UG} to see whether ugly disposal responds more to the tax on bad disposal or to the tax on good disposal.

In our case, the cross-price elasticity e_{UB} appears in the first term of ε_{UB}, weighted by $(1-\theta_{KU})$ and multiplied by the change $\widehat{\tau_B}$ to get the direct effect on dumping from the change in tax on bad disposal. Then, because of the revenue constraint, $\widehat{\tau_B} > 0$ implies $\widehat{\tau_G} < 0$. The effect on dumping (U_X) from this reduced tax on good disposal is in ε_{UG}, where the cross-price elasticity e_{UG} is also weighted by $(1-\theta_{KU})$ but multiplied by $\widehat{\tau_G}$. Thus, in our AGE model, the effect on dumping does not just compare e_{UB} vs. e_{UG}. It also depends on expenditure shares and on the size of each tax rate change (given fixed total tax revenue).

Next, look at the second terms in ε_{UB} and ε_{UG}. Within ε_{UB}, the PE elasticity e_{BB} is weighted by the share θ_{KB} and multiplied by $\widehat{\tau_B} > 0$. This term captures the fact that $\widehat{\tau_B} > 0$ also affects the quantity of B_X, which itself has an indirect effect on the quantity of U_X (because it changes labor and capital resources available for other uses). Within ε_{UG}, the elasticity e_{GG} is weighted by θ_{KG} and then multiplied by $\widehat{\tau_G} < 0$, to capture the fact that $\widehat{\tau_G} < 0$ affects the quantity of G_X, which also affects primary factors available for other uses like ugly disposal.

The third terms in ε_{UB} and ε_{UG} are $-e_{GB}(\theta_{KG})$ and $-e_{BG}(\theta_{KB})$, respectively, where the change in each tax rate also has an indirect effect on the other taxable form of disposal. Those terms also reflect GE changes in resources available for other uses.

The fourth term in both ε_{UB} and ε_{UG} is $-\sigma_\Psi \theta_{pW} \theta_{KY}$, to reflect the fact that an increase in either tax rate raises overall costs of disposal, p_W, and thus raises the full price to buy more of X. The elasticity σ_Ψ indicates how households substitute out of X into Y. That substitution reduces X, so it reduces all forms of disposal (including dumping, via this fourth term).

Our expectation is that the direct effects on ugly U_X in the first terms containing e_{UB} and e_{UG} outweigh other indirect effects, but perverse outcomes are possible. Later calibration and parameterization of our GE model will indicate the relative size of each component effect. As shown by Allen (1938), all PE cross-price elasticities can be positive, or at most one cross-price elasticity can be negative. For brevity here, we limit discussion to the simpler case where all three disposal methods are substitutes for each other (e_{GB}, e_{GU}, and e_{BU} are all positive).

Having carefully discussed intuition for the interpretation of each term within equation (20) for the change in dumping, we now show the other disposal solutions with less discussion.

$$\widehat{B_X} = \varepsilon_{BB}\widehat{\tau_B} + \varepsilon_{BG}\widehat{\tau_G} \tag{21}$$

where $\varepsilon_{BB} \equiv \left[e_{UB}(-\theta_{KU}) + e_{BB}(1-\theta_{KB}) - e_{GB}(\theta_{KG}) - \sigma_\Psi \theta_{pW}\theta_{KY}\right]\theta_{XB}\theta_{B\tau}$ and

$\varepsilon_{BG} \equiv \left[e_{UG}(-\theta_{KU}) - e_{GG}(\theta_{KG}) + e_{BG}(1-\theta_{KB}) - \sigma_\Psi \theta_{pW}\theta_{KY}\right]\theta_{XG}\theta_{G\tau}$.

$$\widehat{G_X} = \varepsilon_{GB}\widehat{\tau_B} + \varepsilon_{GG}\widehat{\tau_G} \tag{22}$$

where $\varepsilon_{GB} \equiv \left[e_{UB}(-\theta_{KU}) - e_{BB}(\theta_{KB}) + e_{GB}(1-\theta_{KG}) - \sigma_\Psi \theta_{pW}\theta_{KY}\right]\theta_{XB}\theta_{B\tau}$ and

$\varepsilon_{GG} \equiv \left[e_{UG}(-\theta_{KU}) + e_{GG}(1-\theta_{KG}) - e_{BG}(\theta_{KB}) - \sigma_\Psi \theta_{pW}\theta_{KY}\right]\theta_{XG}\theta_{G\tau}$.

In Equation (21), we use ε_{BB} to denote an intermediate elasticity for the percentage change in the amount of bad disposal from a one percent change only in the tax on bad disposal, and ε_{BG} as the percent change in the amount of B_X from a one percent change only in the tax on good disposal. In equation (22), ε_{GB} and ε_{GG} are analogous elasticities for the change in good disposal. All four of these elasticities contain direct and indirect PE elasticity parameters and expenditure shares analogous to those described earlier for how tax rates affect U_X.

2.2 Solving for $\widehat{\tau_G}$ in Terms of Fixed Parameters and an Exogenous $\widehat{\tau_B}$

Assuming an exogenous shock $\widehat{\tau_B} > 0$ and using the government revenue constraint in equation (17) as well as the changes to each tax base in (21) and (22), we obtain:

$$E_{TG}\widehat{\tau_G} = -E_{TB}\widehat{\tau_B} \tag{23}$$

where $E_{TG} \equiv \theta_{TG} + \theta_{TG}\varepsilon_{GG} + \theta_{TB}\varepsilon_{BG}$ and $E_{TB} \equiv \theta_{TB} + \theta_{TB}\varepsilon_{BB} + \theta_{TG}\varepsilon_{GB}$.

Online Appendix C uses (23) to solve for $\widehat{\tau_G}$, and it shows that E_{TG} is the GE elasticity of total tax revenue T for a change in the tax rate on good disposal τ_G. Analogously, E_{TB} is the GE elasticity for the percent change in T from a one percent change in τ_B. Equation (23) requires that the revenue increase ($E_{TB}\widehat{\tau_B}$) matches the revenue loss from the tax cut ($E_{TG}\widehat{\tau_G}$).

The first term of E_{TB} is θ_{TB}, the initial share of tax revenue from τ_B. When multiplied by $\widehat{\tau_B} > 0$, this first term is the increase in revenue from raising the tax rate on the existing tax base (B_X). The second term within E_{TB} is $\theta_{TB}\varepsilon_{BB}$, which captures the fact that $\widehat{\tau_B} > 0$ reduces the tax base B_X (a negative effect on revenue). Finally, the third term of E_{TB} includes ε_{GB}, the percent change in G_X from a one percent change in τ_B.[11] This third term is positive when ε_{GB} positive, because it is multiplied by $\widehat{\tau_B} > 0$ and by the share of revenue from the other tax base ($\theta_{TG} > 0$).

[11] Within ε_{GB} of equation (22), an exogenous $\widehat{\tau_B} > 0$ affects G_X most directly through a positive cross-price term $e_{GB}(1-\theta_{KG})$, thereby expanding tax revenue collected from G_X. Two other terms within ε_{GB} are $e_{UB}(-\theta_{KU})$ and $-\sigma_\Psi \theta_{pW}\theta_{KY}$. These terms are negative, so the effect on G_X is not unambiguously positive.

In summary, the first term within E_{TB} represents the increase in revenue from the existing tax base due to a higher tax rate. The second term represents the decrease in tax revenue from pricing out some consumers, and the third term represents the expected indirect increase in total revenue from substitution to more good disposal (which is also taxed).

Parameters within E_{TG} are analogous. On the normal sides of both Laffer curves, both GE revenue elasticities E_{TG} and E_{TB} are positive. If $E_{TB} > E_{TG}$, then total tax revenue increases more in response to a one percent change in τ_B than to a one percent change in τ_G.

2.3 Closed-form Solutions for Changes in Amounts of the Good, the Bad and the Ugly

Equations (20)–(22) show changes in the three kinds of disposal, but each is a function not just of exogenous parameters and tax shock ($\widehat{\tau_B} > 0$). Each also depends on the change in the other tax rate necessary to balance the budget. Here, we solve for these outcomes in terms only of exogenous parameters and the exogenous shock. Full derivations are in online Appendix D.

For closed-form solutions, substitute (23) into equations (20), (21), and (22) to get:

$$\widehat{G_X} = E_{GB}\widehat{\tau_B} \quad \text{where} \quad E_{GB} \equiv \varepsilon_{GB} - \varepsilon_{GG}\frac{E_{TB}}{E_{TG}} \tag{24}$$

$$\widehat{B_X} = E_{BB}\widehat{\tau_B} \quad \text{where} \quad E_{BB} \equiv \varepsilon_{BB} - \varepsilon_{BG}\frac{E_{TB}}{E_{TG}} \tag{25}$$

$$\widehat{U_X} = E_{UB}\widehat{\tau_B} \quad \text{where} \quad E_{UB} \equiv \varepsilon_{UB} - \varepsilon_{UG}\frac{E_{TB}}{E_{TG}} \tag{26}$$

Here, we define E_{iB} as the full GE elasticity for the percentage change in disposal i from a one percent change in the tax on the bad disposal τ_B.

In our discussion above, the logic of the CH effect compares the two key PE cross-price elasticities e_{UB} and e_{UG}. In the closed-form solution of equation (26), the single coefficient (E_{UB}) on the exogenous shock ($\widehat{\tau_B} > 0$) makes exactly that comparison of elasticities. The definition of E_{UB} starts with ε_{UB}, defined at equation (20) to include e_{UB} times (1– θ_{KU}), and it subtracts ε_{UG}, defined to include e_{UG} times the same (1– θ_{KU}). Intuition for the appearance of other elasticities in these equations is also explained above.

Next, we use these changes in each kind of disposal to solve for FBO and SBO tax rates.

3. Welfare Analysis

Before turning to the main analysis about second-best tax rates when ugly disposal cannot be taxed, we first discuss welfare analysis generally in our model, and we show that results reduce to the simple FBO Pigovian solution if all three types of disposal can be taxed.

3.1 The FBO Pigovian Solution

To proceed, we first substitute the waste function $X = W(G_X, B_X, U_X)$ into the household utility function in equation (19), so that the household can maximize the following Lagrangean by choice of disposal methods G_X, B_X, U_X, and consumption of services, Y:

$$\mathcal{L} = \Psi[W(G_X, B_X, U_X), Y;\ nG_X, nB_X, nU_X] + \lambda[I - p_X^F X - p_Y Y] \tag{27}$$

The household's full cost of X is p_X^F, composed of the purchase costs and all disposal costs (with taxes included in those disposal costs). Assuming n is sufficiently large, the household ignores its own contribution to the total public externalities (nG_X, nB_X, nU_X).

We totally differentiate the utility function and substitute in the household's first-order conditions and the economy's resource constraint. The full derivation is in Appendix E. Then we divide by λ, the marginal utility of income, to arrive at the dollar value of the change in utility ($d\Psi/\lambda$). Afterward, divide both sides by income I, to express the cash-equivalent change as a fraction of income. Then our measure of the relative change in welfare is:

$$\frac{d\Psi}{\lambda I} = (\tau_G - \mu_G)\frac{G_X}{I}\widehat{G_X} + (\tau_B - \mu_B)\frac{B_X}{I}\widehat{B_X} + (\tau_U - \mu_U)\frac{U_X}{I}\widehat{U_X} \tag{28}$$

where the marginal damages per unit of each kind of disposal are defined as: $\mu_G \equiv -\frac{n}{\lambda}\frac{\partial \Psi}{\partial nG_X}$, $\mu_B \equiv -\frac{n}{\lambda}\frac{\partial \Psi}{\partial nB_X}$, and $\mu_U \equiv -\frac{n}{\lambda}\frac{\partial \Psi}{\partial nU_X}$. To define damages as positive, each negative effect on utility has a minus sign out front.[12] We assume that second-order conditions hold, so that welfare is at a maximum when $d\Psi = 0$.

Our linearization above initially showed how a small change in tax leads to small changes in outcomes. But next, we solve for optimal tax rates in levels, not changes. That is, we solve equation (28) not for effects of a change such as $\widehat{\tau_B}$, but for the levels of rates τ_B and τ_G appearing in (28) such that no change in any tax rate can raise welfare (*i.e.*, $d\Psi = 0$).

When taxes are available for *all* disposal methods, our Appendix E shows that the FBO rates per unit of disposal are $\tau_G = \mu_G$, $\tau_B = \mu_B$, and $\tau_U = \mu_U$. Indeed, substitution of those tax rates into equation (28) shows that welfare is maximized ($\frac{d\Psi}{\lambda I} = 0$).

[12] This procedure does not assume marginal damages are constant. The MED from additional disposal of each type might usually rise with the total amount of it.

3.2 Welfare Analysis with No Tax on Ugly Disposal

When ugly disposal cannot be taxed ($\tau_U = 0$), equation (28) becomes:

$$\frac{d\Psi}{\lambda I} = (\tau_G - \mu_G)\frac{G_X}{I}\widehat{G_X} + (\tau_B - \mu_B)\frac{B_X}{I}\widehat{B_X} - \mu_U\frac{U_X}{I}\widehat{U_X} \tag{28'}$$

We first solve for the SBO tax rate τ_B^* with two policy constraints – where τ_U is fixed at zero and where τ_G is arbitrarily fixed at some non-optimal level. To do so, we set the change in utility in (28′) to zero to solve for the planner's τ_B that maximizes welfare. That is, we reorganize terms so that τ_B is by itself on the left-hand side. Then substitute equations (24), (25), and (26) for $\widehat{G_X}$, $\widehat{B_X}$, and $\widehat{U_X}$ to arrive at a closed-form solution for τ_B^*:

$$\tau_B^* = \mu_B + (\mu_G - \tau_G)\frac{G_X}{B_X}\frac{\widehat{G_X}}{\widehat{B_X}} + \mu_U\frac{U_X}{B_X}\frac{\widehat{U_X}}{\widehat{B_X}} = \mu_B + \underbrace{(\mu_G - \tau_G)\frac{G_X}{B_X}\left(\frac{E_{GB}}{E_{BB}}\right)}_{Cross-Price\ Term} + \underbrace{\mu_U\frac{U_X}{B_X}\left(\frac{E_{UB}}{E_{BB}}\right)}_{Corlett-Hague\ Term} \tag{29}$$

This SBO tax τ_B^* is comprised of three MED parameters (μ_G, μ_B, and μ_U), two ratios of pre-existing polluting inputs ($\frac{G_X}{B_X}$ and $\frac{U_X}{B_X}$), the fixed existing τ_G, and five GE elasticity parameters (E_{GB}, E_{BB}, E_{UB}, E_{TB}, and E_{TG}).[13]

The terms in this equation tell the social planner how much the SBO tax rate τ_B^* deviates from the FBO solution where $\tau_B = \mu_B$. The "cross-price term" values the welfare effect of how τ_B affects the GE amount of good disposal – valued by the net uncorrected portion of the externality from good disposal ($\mu_G - \tau_G$). We said above that the Allen cross-price elasticity e_{GB} could have either sign, but our discussion would focus on the "normal" case where any two types of disposal are substitutes (so e_{GB} is positive). Similarly, to avoid discussing both possible signs here, we discuss only the case where the GE elasticity for good disposal with respect to the price of bad disposal is positive.[14] In the case of GE substitutes, where $\widehat{\tau_B} > 0$ leads to positive $\widehat{G_X}$ and a negative $\widehat{B_X}$, then the ratio E_{GB}/E_{BB} in the cross-price term is negative. However, the sign of that cross-price term may be changed by ($\mu_G - \tau_G$), the net distortion per additional unit of G_X,

[13] Each GE elasticity is a calculated combination of other intermediate elasticities (ε_{GG}, ε_{GB}, ε_{BG}, ε_{BB}, ε_{UG}, and ε_{UB}), each of which is a calculated combination of fixed PE elasticity parameters (all of the e_{ij}).

[14] Interestingly, equations (20)–(22) show how each GE cross-price elasticity depends on all Allen own-price and cross-price elasticities e_{ij} and share parameters θ. For example, the GE elasticity of good disposal with respect to the price of bad disposal depends on all e_{ij}, because the tax on bad disposal also leads to GE changes in bad disposal and ugly disposal, which have GE feedback effects on the amount of good disposal.

depending on whether it is overtaxed or undertaxed.

The last term in (29) is the "Corlett-Hague term", including a ratio of GE elasticities for ugly disposal and bad disposal. The GE change in ugly disposal is multiplied by μ_U to get the social cost per unit of U_X. Without any tax on ugly disposal, a negative CH-term in (29) would be a reduction in the planner's τ_B^* below μ_B (to encourage switching from ugly to bad). We next explain each of these terms more fully.

3.3 Cross-Price Term

Relative to the FBO tax rate ($\tau_B = \mu_B$), the cross-price term adjusts the tax on bad disposal to account for uncorrected externalities from substitution into good disposal. The term ($\mu_G - \tau_G$) is the portion of the MED from good disposal that is not internalized by τ_G. If the fixed tax τ_G happens to equal μ_G, then any effect on good disposal is irrelevant, because it would have no effect on social welfare (so the planner does not modify τ_B^*). If the arbitrary τ_G is too high ($\tau_G > \mu_G$), resulting in suboptimal low levels of good disposal, the planner adjusts τ_B^* upward to encourage substitution into the over-taxed G_X. But if the tax on good disposal is too low ($\tau_G < \mu_G$), then G_X is too high. But τ_G is fixed, so the planner's only option is to modify τ_B. The cross-price term tells the planner how to set τ_B^* to achieve the best tradeoff between increased damages from bad disposal and the uncorrected damages from excess good disposal.

The magnitude of the cross-price term is determined by three factors. First, a larger positive net distortion ($\mu_G - \tau_G$) raises the planner's τ_B^* further above μ_B. Second, the ratio of polluting inputs (G_X/B_X) gives more weight to that correction when the amount of good disposal is larger. Third, the sign and magnitude depends on the ratio E_{GB}/E_{BB}. If the good and bad disposal methods are stronger GE substitutes for each other (larger $E_{GB} > 0$), then this ratio is a larger negative number that yields a larger cross-price term. Then if $\mu_G - \tau_G > 0$, the planner reduces τ_B^* further (perhaps below μ_B or even below zero).

3.4 Corlett-Hague Term

Next, the SBO tax rate τ_B^* in (29) is adjusted by the Corlett-Hague term to internalize externalities from the ugly disposal. Because the tax on ugly disposal is constrained at zero, the planner uses the tax on bad disposal as an indirect way to manage the ugly externality.

If bad and ugly disposal are GE substitutes, then E_{UB}/E_{BB} is negative. It is multiplied by the positive MED from ugly disposal, μ_U, so the whole CH term would then show how much the

planner would reduce the SBO tax on bad disposal τ_B^* to avoid ugly damages.

The magnitude of the CH term is determined by three factors. First, it is proportional to the size of the untaxed distortion. A larger μ_U means more downward adjustment in the tax on bad disposal. Second, the ratio of polluting inputs (U_X/B_X) puts more weight on this correction when ugly disposal is larger. Third, the magnitude depends on the size of the ratio E_{UB}/E_{BB} that indicates the rate of substitution into ugly disposal from taxing bad disposal.

3.5 Solutions for Both SBO Tax Rates Simultaneously

The previous sections solve the 19 linear equations of our AGE model for the effects of an exogenous change $\widehat{\tau_B} > 0$, and set the differentiated welfare equation to zero to show how the SBO tax rate τ_B^* depends on fixed parameters and an arbitrary τ_G equation (29) above. For convenience, that equation is repeated below. Then we undertake the same procedure to solve for the effects of an exogenous change $\widehat{\tau_G} > 0$ and again set the differentiated welfare equation to zero to show how the SBO tax rate τ_G^* depends on fixed parameters and an arbitrary τ_B:

$$\tau_B^* = \mu_B + \underbrace{(\mu_G - \tau_G)\frac{G_X}{B_X}\left(\frac{E_{GB}}{E_{BB}}\right)}_{Cross-Price\ Term} + \underbrace{\mu_U \frac{U_X}{B_X}\left(\frac{E_{UB}}{E_{BB}}\right)}_{Corlett-Hague\ Term} \tag{29}$$

$$\tau_G^* = \mu_G + \underbrace{(\mu_B - \tau_B)\frac{B_X}{G_X}\left(\frac{E_{BG}}{E_{GG}}\right)}_{Cross-Price\ Term} + \underbrace{\mu_U \frac{U_X}{G_X}\left(\frac{E_{UG}}{E_{GG}}\right)}_{Corlett-Hague\ Term} \tag{30}$$

where $E_{GG} \equiv \varepsilon_{GG} - \varepsilon_{GB}\frac{E_{TG}}{E_{TB}}$, $E_{BG} \equiv \varepsilon_{BG} - \varepsilon_{BB}\frac{E_{TG}}{E_{TB}}$, and $E_{UG} \equiv \varepsilon_{UG} - \varepsilon_{UB}\frac{E_{TG}}{E_{TB}}$. We define E_{iG} as the full GE elasticity for the percentage change in disposal i from a one percent change in the tax on the good disposal τ_G. Full derivations are found in the online Appendix F.

Equations (29) and (30) are not consistent with each other, however, because each has an arbitrary value for the other tax rate. Therefore, we next solve simultaneously for the two tax rates that can lead the economy to a SBO general equilibrium. These two equations alone are not sufficient to solve for the two SBO tax rates, however, because they are co-linear. Our model has a required total tax revenue T, and nothing in those two equations determines the two tax rates together that would collect the required revenue. Because equation (17) says $T = \tau_G G_X + \tau_B B_X$, we can solve it for τ_G and plug it into (29). We also solve (17) for τ_B and plug it into (30). The resulting two equations for the two SBO tax rates simultaneously are:

$$\tau_B^* = \frac{\mu_B \quad + \quad \underbrace{(\mu_G - \frac{T}{G_X})\frac{G_X}{B_X}\left(\frac{E_{GB}}{E_{BB}}\right)}_{Cross-Price\ Term} \quad + \quad \underbrace{\mu_U \frac{U_X}{B_X}\left(\frac{E_{UB}}{E_{BB}}\right)}_{Corlett-Hague\ Term}}{1 - E_{GB}/E_{BB}} \tag{31}$$

$$\tau_G^* = \frac{\mu_G \quad + \quad \underbrace{(\mu_B - \frac{T}{B_X})\frac{B_X}{G_X}\left(\frac{E_{BG}}{E_{GG}}\right)}_{Cross-Price\ Term} \quad + \quad \underbrace{\mu_U \frac{U_X}{G_X}\left(\frac{E_{UG}}{E_{GG}}\right)}_{Corlett-Hague\ Term}}{1 - E_{BG}/E_{GG}} \tag{32}$$

The intuition for each of the three main terms in the numerator of each of these two equations matches the intuition provided above for these terms in equations (29) and (30) and so will not be repeated here. Four additional points are relevant.

First, all three terms in the numerator of the simultaneous equations (31) and (32) now have a denominator. Each denominator is one minus the ratio of GE elasticities from the cross-price term. Thus, if E_{BG} and E_{GB} are negative, then the denominator is positive and larger than one. It scales down every term in the numerator but does not change any sign.

Second, the cross-price term in the numerator of τ_B^* in (29) includes $(\mu_G - \tau_G)$, using an arbitrary non-optimal τ_G, but in (31) it is replaced by $(\mu_G - \frac{T}{G_X})$. Perhaps surprisingly, this change replaces the arbitrary tax per unit τ_G with a calculated maximum tax per unit that could collect *all* needed revenue T. The rate $\frac{T}{G_X}$ is the maximum possible arbitrary tax rate τ_G. So, assuming this maximum tax is larger than the damage per unit of good disposal, μ_G, then this difference $\mu_G - \frac{T}{G_X}$ is negative. If it is multiplied by a negative ratio E_{GB}/E_{BB}, then the whole term is positive. This entire cross-price term adds a positive term to the SBO tax rate τ_B^*. In a sense, the earlier calculation in (29) gets a term that may add or subtract from MED μ_B to get the SBO tax on bad disposal τ_B^*, but equation (31) merely resets the point of comparison away from an arbitrary τ_G to the maximum possible arbitrary τ_G to get a term that shows how much to *add* to the SBO tax on bad disposal τ_B^*.

Third, the intuition for comparing τ_G^* equations (30) and (32) are perfectly analogous to the comparison of equations for τ_B^* in equations (29) and (31), so will not be repeated here.

Fourth, in the SBO tax rates of equations (31) and (32), the tax revenue requirement T is still arbitrary. This T itself enters negatively. If it is multiplied by a negative elasticity ratio in both equations, then higher revenue T requires higher values of both SBO tax rates.

To step back and discuss what it all means, we note that the original SBO tax literature

based on Corlett and Hague (1953) suggests that the tax on a first taxable good be higher than the tax on the other taxable good if the PE cross-price elasticity between leisure and the first taxable good is smaller than the PE cross-price elasticity between leisure and the other taxable good. Then a tax on the first good would increase leisure by less than would a tax on the other good. That PE logic is not applied directly in our case. In our model, the relationship between the two tax rates and the two elasticities is seen by comparing the third CH-term of τ_B^* in (31) to the third CH-term of τ_G^* in (32). But the comparison of rates is complicated by five factors.

First, the relevant cross-price elasticities in our model are not the PE elasticities but the GE elasticities, *i.e.*, the effect of a disposal tax that takes into account all other GE effects on ugly disposal (*not* holding all else constant).

Second, the size of that Corlett-Hague effect depends importantly on the marginal external damages from ugly disposal, μ_U (especially as compared to the other MED).

Third, the relative size of each tax rate also depends on the MED of the other taxable method of disposal and the GE cross-price elasticity between the two taxable methods of disposal (*i.e.*, comparing the two "cross-price terms" in the two SBO tax rate solutions).

Fourth, all of those factors just listed do not determine directly the two tax rates relative to each other, but only the extent to which each tax rate is raised or reduced from the MED of the disposal method being taxed.

Fifth, analytical work could take the derivative of each tax rate with respect to each key parameter. That procedure might appear to show exactly how τ_B^* or τ_G^* is affected by the MED of ugly damages, since μ_U enters each tax rate negatively and only once in the numerator. But that observation implies that a higher μ_U would reduce both SBO tax rates (*i.e.*, to shift away from ugly disposal by reducing both tax rates). But our AGE model builds in a revenue constraint, so an increase in μ_U cannot reduce both tax rates. This constraint is built into the GE elasticities, so the GE effect of μ_U cannot be determined so easily. All parameters matter for GE outcomes. We cannot take a derivative of (31) or (32) with respect to any parameter, because each parameter appears many times in the numerator and in the denominator of each tax rate. For these reasons, we turn to calibration and sensitivity analysis for the effect of each parameter on each tax rate.

4. Calibration

Our choices for "primary" parameter values are shown in **Table 1** (so that later we can vary key parameters in sensitivity analysis). For this hypothetical closed economy, total income

is 1.0. For example, if this 1.0 refers to one trillion dollars, then the first column says that $350 billion of primary factors are used to produce X in the initial equilibrium (before any tax shock). Then $350 billion is the cost to produce Y, and $100 billion is used to undertake each kind of disposal (each of which is one-third of total waste costs). If we normalize all pre-tax prices to 1, then all three disposal quantities are 0.1 (*i.e.*, G_X, B_X, and U_X are each 100 billion units).

Table 1: Initial Equilibrium Parameter Values

Parameter	Value	Parameter	Value	Parameter	Value
θ_{KX}	0.35	θ_{pX}	0.538	μ_G	0.05
θ_{KY}	0.35	θ_{pW}	0.462	μ_B	0.15
θ_{KG}	0.1	$\theta_{G\tau}$	0.1	μ_U	0.5
θ_{KB}	0.1	$\theta_{B\tau}$	0.1	e_{UG}	0.5
θ_{KU}	0.1	θ_{TG}	0.5	e_{UB}	0.5
θ_{XG}	0.333	θ_{TB}	0.5	e_{GB}, e_{BG}	0.5
θ_{XB}	0.333	T	0.022	e_{GG}, e_{BB}, e_{UU}	–1
θ_{XU}	0.333	σ_Ψ	1		

Note: All parameter values in this table are assumed values except for T, θ_{pX}, and θ_{pW}. These three values are calculated from the other assumed parameter values.

In the second column, the initial tax share is 10% of disposal cost for the two taxable disposal methods. The two tax rates are initially the same as each other, and the quantities match, so each disposal tax revenue is half of total tax revenue. The household elasticity of substitution between X and Y in utility is one. In the third column, values for the MED are chosen such that $\mu_G < \mu_B < \mu_U$. Finally, all cross-price elasticities are 0.5, and each own-price elasticity is –1.

In sensitivity analysis, each MED can be varied independently. That is, we can alter the external cost of ugly disposal μ_U without a change in any other parameter value. In addition, we want to vary the key cross-price elasticity e_{UB}, but some other parameter must change for the symmetrical Allen elasticity matrix to satisfy the adding up constraint $a_{iG} + a_{iB} + a_{iU} = 0$ (where $a_{ij} = e_{ij}\theta_{Xj}$ for each row of the matrix, with $i = G, B, U$). We want to show results for different values of e_{UB}, but without changes in other cross-price elasticities. Therefore, as our program varies e_{UB}, we have it calculate the necessary change in the own-price elasticities that are necessary to satisfy the adding-up constraint.

Next, **Table 2** maps the primary Allen Elasticities e_{ij} into the full GE elasticities E_{ij}, using definitions in equations (20-26) and the parameter values found in Table 1. This table shows that the GE elasticities can be quite different from the PE Allen elasticities, which points

toward the importance of using a GE model. Our definitions of the GE elasticities in equations (20-26) incorporate all assumptions of the model, including the fixed revenue requirement.

Table 2: Mapping Primary Allen PE Elasticities to Full GE Elasticities

Allen PE Elasticity	Assumed Value	Intermediate Elasticity	Calculated Value	Full GE Elasticity	Calculated Value
e_{UG}	0.5	ε_{UG}	0.011	E_{UG}	0
e_{UB}	0.5	ε_{UB}	0.011	E_{UB}	0
e_{GB}	0.5	ε_{GB}	0.011	E_{GB}	0.050
e_{BG}	0.5	ε_{BG}	0.011	E_{BG}	0.050
e_{GG}	–1	ε_{GG}	–0.039	E_{GG}	–0.050
e_{BB}	–1	ε_{BB}	–0.039	E_{BB}	–0.050
e_{UU}	–1			E_{TG}	0.486
				E_{TB}	0.486

The revenue requirement means that an increase in one tax rate implies a simultaneous decrease in the other tax rate, which is part of what makes the GE elasticities for this model quite different from PE elasticities. In the first rows of Table 2 for the cross-price elasticities, our primary assumption is that all three e_{ij} are 0.5 (when $i \neq j$). However, the table shows that GE elasticities E_{UG} and E_{UB} are zero (and other GE elasticities are near zero). In fact, ugly disposal can only be affected indirectly, through taxes on good and bad disposal, so the GE outcomes for ugly disposal are determined entirely by these indirect price changes. Specifically, in definitions at equations (20)-(26), each E_{ij} is a weighted average of *all* PE elasticities ($e_{ij} = 0.5$, $e_{ii} = -1$). In this way, the E_{ij} capture all the ways the PE responses are restricted in the GE model by other equations – resource constraints, budget constraints, and factor shares.

Table 3: PE Substitutes to GE Complements

Allen PE Elasticity	Assumed Value	Intermediate Elasticity	Calculated Value	Full GE Elasticity	Calculated Value
e_{UG}	0.5	ε_{UG}	0.011	E_{UG}	–0.003
e_{UB}	0.6	ε_{UB}	0.015	E_{UB}	0.003
e_{GB}	0.5	ε_{GB}	0.011	E_{GB}	0.050
e_{BG}	0.5	ε_{BG}	0.011	E_{BG}	0.053
e_{GG}	–1	ε_{GG}	–0.039	E_{GG}	–0.050
e_{BB}	–1.1	ε_{BB}	–0.042	E_{BB}	–0.053
e_{UU}	–1.1			E_{TG}	0.486
				E_{TB}	0.485

To see the analogous mapping for a different case in our sensitivity analysis, **Table 3** increases e_{UB} from 0.5 to 0.6 (and adjusts e_{BB} and e_{UU} to satisfy the adding-up constraint). The table shows that this change in e_{UB} from 0.5 to 0.6 makes the GE elasticity E_{UG} slightly negative. This result may seem odd, but again the logic is that this GE elasticity is a weighted average of positive and negative PE elasticities. Given the assumed symmetry in our initial assumptions, the disposal method that is changed to a stronger PE substitute for ugly disposal becomes a GE substitute, and the weaker PE substitute becomes a GE complement.

5. Numerical Results

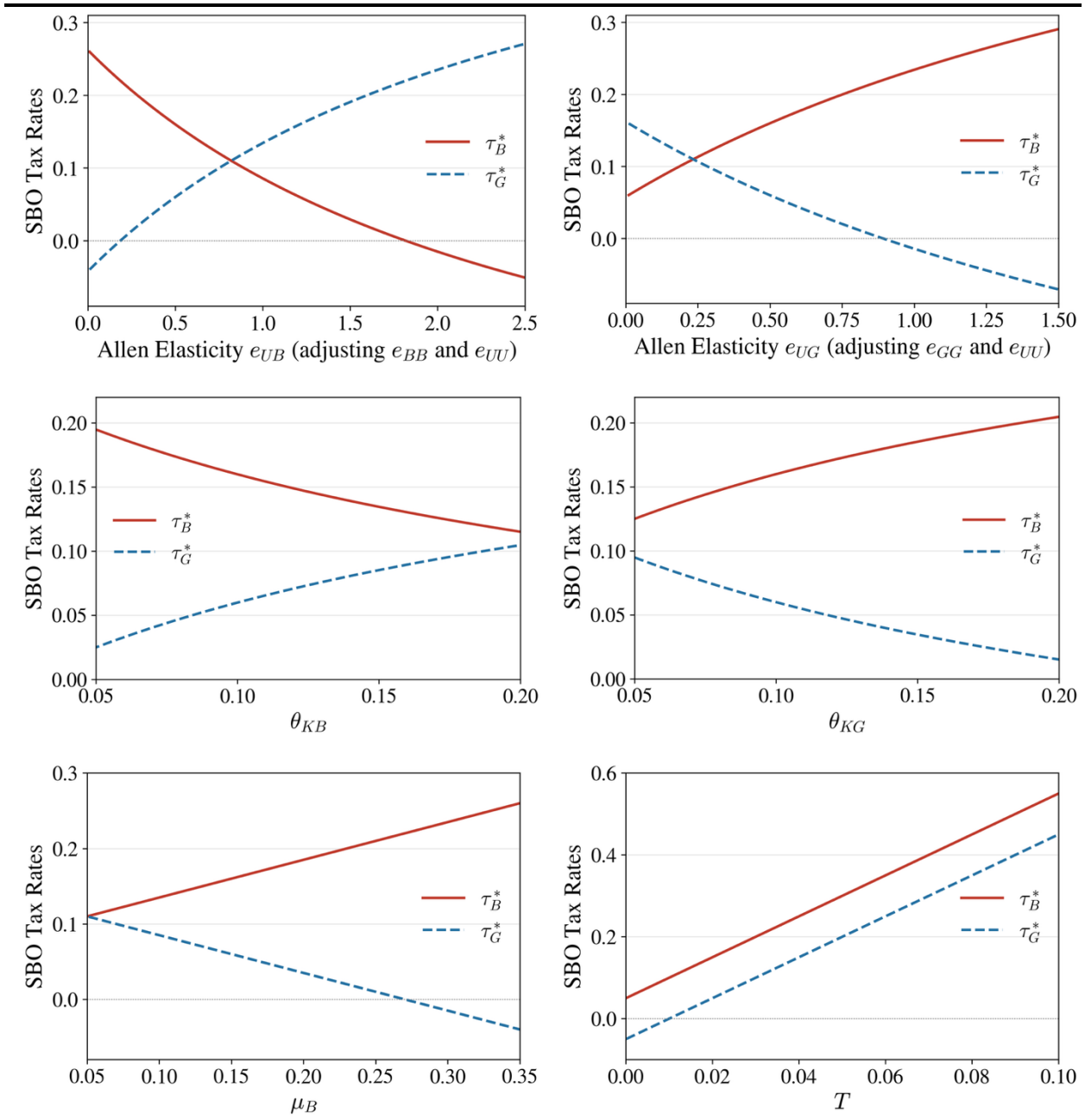
To show the effects of parameters on the two SBO tax rates, **Figure 1** varies one key parameter at a time. The parameter most discussed above is e_{UB}, the Allen elasticity of ugly disposal with respect to the price of bad disposal. If U_X and B_X are better substitutes (higher e_{UB}), we expect a lower SBO tax on B_X. As expected, the **upper left** figure shows how a larger e_{UB} leads to a smaller τ_B^*. If e_{UB} is high enough, τ_B^* can be less than τ_G^* or even negative. Even when landfill (or natural gas) has substantial external costs, for example, a tax on it encourages enough ugly pollution that the SBO policy is a subsidy to bad pollution. The figure also shows effects of a revenue requirement: a lower τ_B^* requires a higher tax on good disposal τ_G^*.

The **upper right** figure shows analogous results for e_{UG}, the Allen elasticity of ugly with respect to the price of good. As U_X and G_X become weaker substitutes (leftward on its horizontal axis), the tax on good disposal may again become larger than the tax on bad disposal. In the other direction, if e_{UG} is high enough, then τ_G^* turns negative. The SBO policy toward good recycling can be a subsidy, in order to get revenue from bad disposal and avoid ugly disposal.

Primary parameters start with equal weights on the three wastes (θ_{KG}= θ_{KB}= θ_{KU}= 0.1). The **center left** graph of Figure 1 varies the weight on bad disposal θ_{KB} from half to twice that value (from 0.05 to 0.20). All other shares are changed proportionately so that all shares add to one.[15] Perhaps surprisingly, the figure shows that a larger bad disposal share reduces the SBO tax on bad disposal. The explanation is that a higher share for bad disposal means more revenue from any particular tax rate, so a lower τ_B^* raises more revenue than otherwise. Analogously, the **center right** figure shows that a larger share for good disposal θ_{KG} reduces the SBO tax on good disposal. Then, given the fixed total revenue, a lower τ_G^* implies a higher τ_B^*.

[15] For example, increasing θ_{KB} from 0.10 to 0.19 means that we reduce both θ_{KX} and θ_{KY} by 10% (from 0.35 to 0.315) and both θ_{KG} and θ_{KU} by 10% (from 0.10 to 0.09).

Figure 1: Effects of Varying Key Parameters on the Two SBO Tax Rates



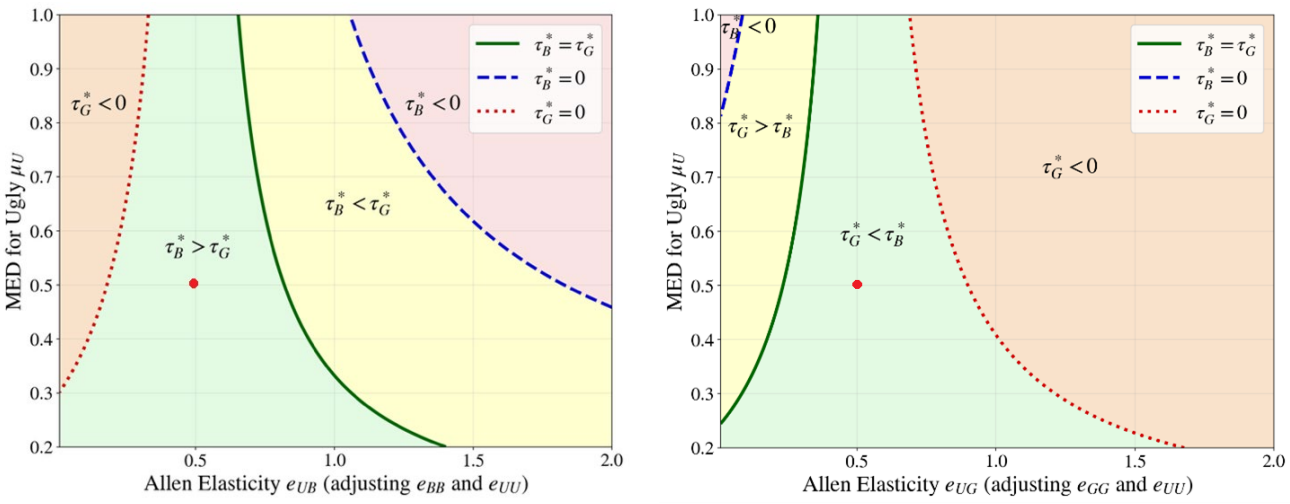
Next, we vary the assumed marginal external damages. Initial values are $\mu_G = 0.05$, $\mu_B = 0.15$, and $\mu_U = 0.5$. Intuitively, one might expect a higher μ_U to reduce the SBO tax rate on whichever alternative is a better substitute for ugly disposal. When other parameters remain at their initial values, however, the two taxable alternatives are equally good substitutes for ugly disposal – and the CH terms in Equations (31) and (32) reduce to zero. Then E_{UG} and E_{UB} in Table 2 equal zero. In that case, neither SBO tax rate varies with μ_U, but instead τ_B^* is always

higher than τ_G^* (because μ_B is higher than μ_G). When the social planner cannot act directly *or indirectly* to reduce costly ugly disposal, the next best step is simply to control damages from bad disposal. We do not show the figure with two flat lines as μ_U varies.

Instead, the **bottom left** of Figure 1 increases μ_B and shows that τ_B^* rises. Then, because of the fixed revenue requirement, τ_G^* can become negative with a higher μ_B.

Table 1 shows that $T = 0.022$ with all assumed primary parameter values, but the **lower right** corner of Figure 1 shows that increases in the fixed revenue requirement raises both SBO tax rates. At a revenue requirement of zero, the curve for τ_B^* starts out positive and τ_G^* is negative (to capture the fact that bad disposal is more damaging than good disposal). The difference remains constant, as both tax rates rise with the revenue requirement.

Figure 2: Relative SBO Tax Rates for Varying MED and Allen Elasticities



Next, **Figure 2** varies two parameters in a single diagram. In the left panel, the horizontal axis varies e_{UB} (0.5 in our primary parameters), and the vertical axis varies the MED per unit of ugly disposal, μ_U (also 0.5 in our primary parameters). With all primary parameters, the red dot in the figure shows that the SBO tax on bad disposal exceeds the SBO tax on good disposal, as expected. If we keep μ_U fixed at 0.5 and move directly westward, then reducing e_{UB} to 0.18 or below makes τ_G^* turn negative (a subsidy on good disposal). If we instead move directly eastward, increasing e_{UB} to 0.82 or above raises the SBO tax on good disposal above the SBO tax on bad disposal. Finally, still keeping μ_U at 0.5, then increasing e_{UB} to 1.83 or above turns τ_B^* negative. In general, τ_B^* is more likely negative with a high e_{UB}, a high μ_U, or both.

In other words, the optimal policy could subsidize badly polluting behavior if the even-

worse ugly disposal is highly damaging and cannot be reduced directly but can be reduced enough by subsidizing a less-polluting alternative.

The right panel of Figure 2 varies e_{UG} on the horizontal axis, while the vertical axis again varies μ_U. The red dot at same starting point (0.5, 0.5) still shows $\tau_B^* > \tau_G^*$, as expected. Again moving directly westward, reducing e_{UG} to 0.23 or below raises τ_G^* above τ_B^* (a higher tax rate on the less damaging disposal method). If we instead move directly eastward and increase e_{UG} to 0.90 or above, then τ_G^* becomes negative (a subsidy on good disposal). In general, τ_G^* is more likely negative with a high e_{UG}, a high μ_U, or both.

6. Conclusions

This paper builds and employs a new analytical general equilibrium model to find a closed-form solution for the second-best optimal (SBO) tax on a relatively "good" clean production or disposal method, and the SBO tax rate on a "bad" polluting production or disposal method, in the case where no tax can be imposed or enforced on the most polluting "ugly" production method (*e.g.* using coal) or disposal method (*e.g.*, illegal dumping). This model can be applied to any environmental pollution problem, including disposal into the air, water, or land. We show that the tax on bad disposal might be *lower* than the tax on good disposal, if ugly disposal is enough better as a substitute for bad disposal than it is for good disposal. Our model captures not only differences in how the partial equilibrium cross-price elasticities help determine general equilibrium effects of each available disposal tax, but also how all own-price and cross-price elasticities and share parameters help determine those general equilibrium outcomes, including the amount of untaxed ugly disposal. And this general equilibrium change in each disposal amount is what matters for setting the two SBO tax rates. We also show how these elasticities combine with each disposal method's marginal external damage parameter to determine the two SBO tax rates on good and bad disposal.

Available data and estimates are not sufficiently reliable to calibrate this model for a particular problem with three types of pollution, three own-price elasticities, three cross-price elasticities, and multiple share parameters. Illegal dumping is particularly difficult to observe. But our model provides clear intuition about how to set second-best policy for any pollution problem, and our results clearly show which parameters need to be estimated to be able to set optimal policy for any two types of pollution when a third type cannot be directly controlled.

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