Human Capital Tilting: From Financial Returns to the Wage Distribution*

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Abstract

We explore the implications of a new fact: expected financial returns are inversely correlated to the rich-poor gap in parents' expenditure share on children's human capital. A model with endogenous human capital accumulation and credit constraints naturally generates this pattern and connects low expected financial returns to rising wage inequality, falling intergenerational mobility, and a rising high-earner labor share. In the calibrated model, 3% lower financial returns generates a 17% rise in cross-sectional labor income variance and a 7% higher parent-child labor income intergenerational elasticity. We use our model to study additional implications, including the effects of wealth taxation.

JEL Codes: E21, G11, D31, E24, J24

Keywords: Human capital, financial returns, borrowing constraints, income inequality, intergenerational mobility, working rich.

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> The professional classes especially, while generally eager to save some capital *for* their children, are even more on the alert for opportunities of investing it *in* them.
>
> —Alfred Marshall, *Principles of Economics*, 1890

How is the labor income distribution affected by a low financial return environment? Over the past 40-50 years, interest rates in advanced economies have been declining and are projected to remain low (Laubach and Williams, 2003; Holston et al., 2017), an important driver of low overall expected rates of return-on-capital. During a similar timeframe, and within similar advanced economies, labor income dispersion has risen (Piketty and Saez, 2003, 2006; Piketty et al., 2018). This paper offers theory and evidence on the effect of low expected financial returns on high labor income inequality, via uneven human capital accumulation.

We start with a novel motivating fact: the rich-poor gap in parent-on-child educational investment has grown since 1980 in the US, a period of falling real interest rates and other expected financial returns. In the 20 years prior to 1980, rates fell and then rose, and the rich-poor educational investment gap mirrored this pattern, growing in the 1960s and falling in the 1970s. We refer to this phenomenon as *human capital tilting*: low rates coincide with a larger human capital investment gap, and vice versa.

To understand the causes and consequences of human capital tilting, we present a model of the family that consumes and invests in both financial and human capital, subject to borrowing constraints. The key question for our model is what happens when expected future financial returns fall.

Like any real investment activity, human capital investment should rise as interest rates and other expected financial returns fall. Intuitively, the "discount rate" on human capital declines with expected return-on-capital. At the same time, the "cash flows" of human capital are future wages that do not offset the discount rate effect—in fact, under standard assumptions, wages rise as the return-on-capital falls, due to a capital deepening effect. By contrast, low rates induce a fall in both the discount rate and future cash flows associated to financial assets. The net effect is an increase in the value of human capital, relative to financial assets, and thus a greater incentive to invest in education. One can also think of this as a pure portfolio choice effect: a falling expected return to capital incentivizes substitution into human capital.

With credit constraints, human capital investment rises less for poorer households, because they are reluctant to borrow. Said differently, poorer households' effective discount rate also includes the shadow value of binding financial constraints, which is rel-

atively insensitive to the return-on-capital. The result of a low-rate environment is thus a disproportionately large increase in educational investment for wealthier households. That is, *human capital tilting* occurs naturally in our framework.

Human capital tilting is the key driving mechanism behind a series of other theoretical results. We prove that falling rates cause greater labor income dispersion (Piketty et al., 2018; Blanchet et al., 2021), the emergence of the working rich (Piketty and Saez, 2003; Kaplan and Rauh, 2013; Smith et al., 2019; Eisfeldt et al., 2021), and lower intergenerational income mobility (Corak, 2013; Chetty et al., 2014; Davis and Mazumder, 2017; Chetty et al., 2017). In this sense, a low *financial* return environment can unite a number of emerging patterns in *labor* income dynamics. In a calibrated version of the model, a 3% reduction in real rates generates a 17% rise in cross-sectional log labor income variance and a 7% higher parent-child labor income inter-generational elasticity. In addition to these effects, our model connects falling rates to a rising education price, consistent with the data, which can crowd out the investment demand of borrowing-constrained parents.

We then show how, from a normative perspective, human capital tilting and the associated changes in inequality and mobility are inefficient because human capital investment becomes less aligned with talent and more aligned with wealth. We also argue that using capital taxes to combat inequality is partially self-defeating, since capital taxes reduce financial returns, thereby amplifying human capital tilting and offsetting some of the inequality reduction. Many papers have studied welfare and policy in the context of constrained human capital investment Heckman et al. (1998); Benabou (2002); Krueger and Ludwig (2013); Heathcote et al. (2017). Our novelty is the introduction of the portfolio choice margin between financial and human bequests, which most of the literature abstracts from, and therefore the analysis of how financial returns affect the efficiency of human capital investment.

Although the mechanisms we espouse are relatively neoclassical, our paper is unusual in connecting discount rates to labor income distributions. By and large, the literature on wage inequality has focused on forces that directly affect wages, e.g., skill-biased technical change (Katz and Murphy, 1992; Katz and Autor, 1999; Krusell et al., 2000; Acemoglu, 2002; Autor et al., 2003). In that context, Murphy and Topel (2016) show how human capital accumulation and inequality respond to changes in the skill premium. A recent literature on educational attainment emphasizes the importance of credit constraints (Lochner and Monge-Naranjo, 2011; Hai and Heckman, 2017; Caucutt and Lochner, 2020) but does not study the resulting income distribution nor the role of discount rates.

On the other hand, interest rates and other rates of return lie at the core of a fast-growing literature on wealth inequality (Saez and Zucman, 2016; Benhabib and Bisin, 2018; Benhabib et al., 2019; Fagereng et al., 2020, 2021; Greenwald et al., 2021; Hubmer et al., 2021). Universally, this literature has abstracted from the endogenous determination of labor income and skills. For example, the recent study by Greenwald et al. (2021) argues that human wealth increases as interest rates fall, moreso for the poor and young, which tends to mitigate the effects of rising financial wealth inequality. The opposite conclusion holds once you allow for human capital *investments* under low interest rates: the presence of human capital amplifies financial inequalities.

Many theoretical studies of inequality zoom in on the extreme right tail (Jones, 2015; Gabaix et al., 2016; Jones and Kim, 2018; Gomez, 2025), partly because the tail has changed the most and partly because the economic mechanisms plausibly differ from those affecting the rest of the distribution. By contrast, our human capital forces are more applicable to the entire distribution (and perhaps least applicable to the tail).

A recent literature studies how monetary policy—and interest rates more generally—affects investment across the distribution of firms under financial frictions. This literature presents somewhat disparate results depending on the setting and details. For example, Ottonello and Winberry (2020) argue empirically that more financially-constrained firms are less sensitive to interest rates, whereas Jeenas (2019) argues the opposite using similar data but a different empirical specification. Gopinath et al. (2017) argues that both productivity and financial constraints matter for the heterogeneity of investment responses, and Asriyan et al. (2025) argues that general equilibrium forces can reverse partial equilibrium responses in this context. Unlike these papers, we are focused on human capital investments and household income inequality, rather than firms.

Two other related papers, which may have consequences for inequality, are Liu et al. (2022) and Gomez and Gouin-Bonenfant (2024). Like us, both papers study how low interest rates amplify investment incentives (into market power and entrepreneurship, respectively) moreso for one subgroup (industry leaders and wealthy business-owners, respectively). Unlike these papers, we focus on human capital investment and the role of credit constraints (these papers study frictionless mechanisms).

Finally, Auclert and Rognlie (2020) and Mian et al. (2021) have argued theoretically that income inequality can depress interest rates, through demand for savings. Together with our paper—which establishes the reverse causality—this suggests a potential two-way feedback between inequality and interest rates that may be worthy of investigation.

The remainder of the paper is organized as follows. Section 1 documents stylized facts on human capital tilting in the data. Section 3 sets up the model and develops all

of the theoretical insights. Section 6 quantifies the size of the model effects. Section 7 contains concluding remarks.

1 Motivational facts: human capital tilting in the data

We first document that the educational expenditure Engel curve steepens as expected returns on financial assets fall. Some existing research explores related patterns. Corak (2013) and Schneider et al. (2018) have shown that the gap in the amounts spent on education has widened since the 1970s between high- and low-income households. Kornrich and Furstenberg (2013) have shown that the ratio of educational spending to income has risen more for high-income households. The relative increase in monetary spending on human-capital investment by richer households is not a substitution away from time input into monetary input, possibly in response to a divergence in the market cost of time; Ramey and Ramey (2010) have shown that time spent on children has increased more for high-income parents since the mid-1990s.

1.1 Data

We use the Consumer Expenditure Survey (CEX), for years 1960-1961, 1972-1973 and 1980-2015, to estimate the elasticity of human capital investment with respect to permanent income. The CEX allows us to estimate a proxy for the consumption share of human capital investment for households of different income levels, going back 55 years. The long time series is important, as we will examine how the elasticity gradient correlates with financial expected returns, which are slow-moving.

The main expenditure variable includes tuition, fees, books, supplies and equipment for day care centers, nursery school, other schools and college, as well as rental of books and equipment and other school-related expenses. While this measure has the benefit of being straightforward and consistent across the various CEX waves, it is only a small fraction of parents' monetary investments into children's human capital, not to mention the time investment. (Conceptually, many types of unmeasured monetary investments can affect and augment children's human capital: they can include expenditures on formative experiences, cognitive and emotional growth, and social networks, nutrition, health and physical fitness, to name a few.)

The main consumption variable is total expenditure. Expenditure need not equal consumption, especially given difficulties in allocating durable service flows over time. We focus on expenditure mainly for consistency across years. For example, the older

waves (1960-1961 and 1972-1973) do not have enough information to impute flow of vehicle consumption.

Permanent income is estimated by instrumenting for total income with consumption, following an extensive literature (Modigliani and Brumberg, 1954; Friedman, 1957; Battistin et al., 2009; Charles et al., 2009). Our model is a generational model that abstracts from higher-frequency income shocks, and hence calls for sorting households by their permanent income.

Finally, to make amounts comparable across years, we deflate using the Consumer Price Index (CPI) from BLS, so that all variables are in 2015 dollars.

1.2 Education expenditure by permanent income

Our goal is to estimate the elasticity of educational expenditure to permanent income and to estimate its change over time. Figure 1a plots the ratio of educational expenditure to total expenditure against log total expenditure, for quintile groups of households sorted by the log total expenditure. In this figure, total expenditure is used directly as a proxy for permanent income. Because families' human capital investment into children depends strongly on where the children are in their life cycles, we control for this by residualizing both the ratios and the log total expenditures with age fixed effects using the age of household heads.[1] The six different lines along with their markers plot the ratios for six broad groups of years, with darker colored lines denoting later cohorts.

The first noticeable feature of Figure 1a is that education-to-consumption ratios are increasing in permanent income for all years, i.e., human capital investment is a luxury. The level of educational expenditure share may seem low, ranging from roughly 0.5% to 3.5%. This is for two reasons: (i) we only capture a small fraction of the total monetary human capital investment, and (ii) the ratios are formed across all households regardless of the presence of school-age children. The cross-sectional spread, especially in recent years, is large. For example, in the latest years (2011-2015), the top consumption quintile households spent 3.5% of their expenditure on education, whereas the ratio is less than 0.5% for the bottom consumption quintile households.

Second, the slope is increasing over time. For example, the bottom quintile ratios across multiple years are mostly clustered around 0.5%, but the top quintile ratios display a large increase over time. An alternative way to visualize the change over time is to plot the same average educational expenditure share over time for each consumption

[1] We first calculate education-to-total expenditure ratios for each household (excluding households that report zero total expenditure), then residualize both that ratio and log consumption expenditure by age, then sort by the residualized log consumption.

quintile group—see Figure 2a. Each line with its markers now represents a consumption quintile group. The bottom three quintile groups' ratios are stagnant, with a slight hump-shaped pattern over time. By contrast, the top quintile group's education-to-consumption ratio increased monotonically over time, from a little above 1% in 1960 to almost 3.5% in 2011-2015.

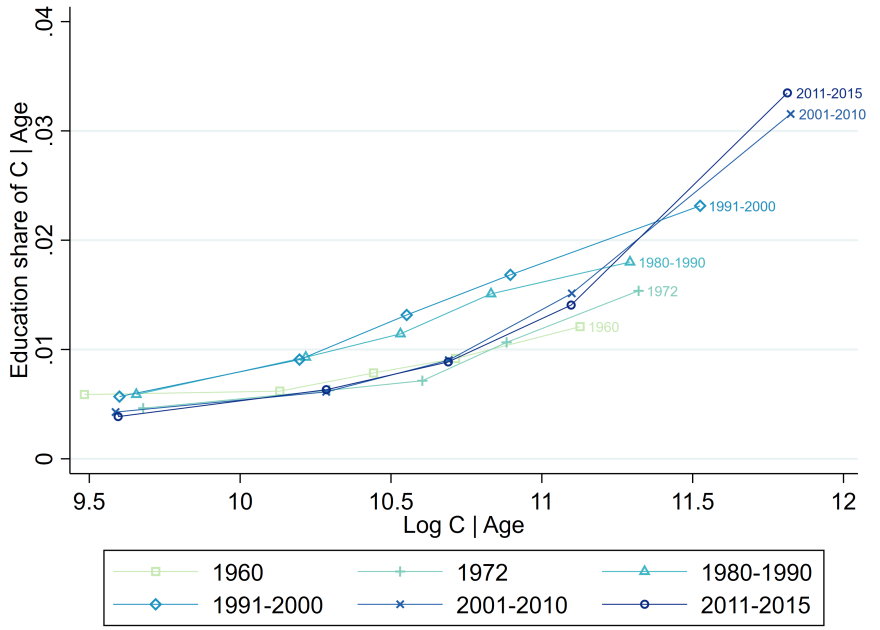
Figures 1b and 2b instead plot ratios of educational expenditure to income, rather than total expenditure, on the y-axis. The households are still sorted into quintiles by total expenditures; this roughly corresponds to a permanent income regression where income is instrumented by consumption. We again find similar patterns.

Our theory predicts human capital tilting: the slope of educational investment, as a function of permanent income, rises when real rates fall. In Figure 3, we therefore plot the estimated slopes and real rates together. We summarize the relationship between educational-spending share and permanent income using a semi-elasticity estimated by regressing consumption share of education on log income, instrumented using log consumption expenditure, with age and year fixed effects, for a group of years. Since the predicted relationship is negative (i.e., low R leads to greater tilting), we plot $(-1)\times$ estimated slopes in Figure 3 to facilitate the comparison.

For real rates, the ideal variable is an expected rate of return across all financial assets with similar duration to human capital investments, adjusted by the inflation expectation over the same horizon. Such estimates are not readily available, so we instead plot three imperfect alternatives. The first series is the 10-year real rate, plotted in solid blue. For a risk-free financial investment, this time series is close to ideal, yet it only goes back to 1982. The second series is a 1-year real rate, plotted in solid red, which goes back further in time but is less ideal in terms of its short horizon. As expected, the real 10-year and 1-year rates exhibit similar time series movements in the years that they overlap since 1982. The third series is the earnings-to-price ratio of the US equity market, plotted in solid green. The earnings-to-price ratio is a common proxy of expected returns on the stock market and hence complements the other two risk-free series.

Figure 3 presents a critical takeaway. The (negative of) estimated slopes of the human capital Engel curve tracks the real-rate proxies, decreasing first from 1960 to the 1970s, then increasing to early 1980s, and then declining in the years afterwards. The link between the human capital Engel curve and the earnings-to-price ratio of the stock market also exhibits qualitatively similar patterns. To the best of our knowledge, this time-series correlation is new to the literature.

(a) Educational expenditure as a fraction of total consumption expenditure



(b) Educational expenditure as a fraction of total income

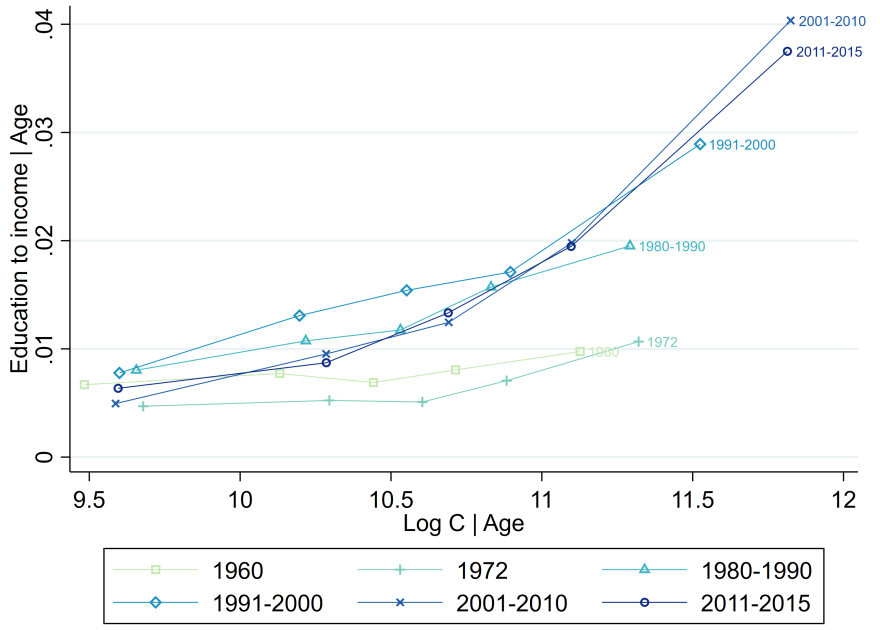
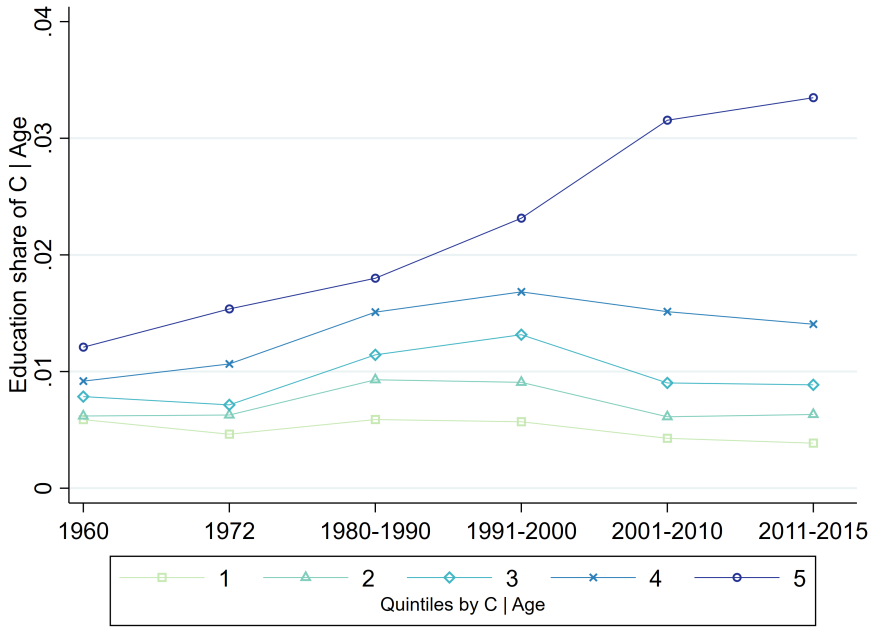


Figure 1: Education expenditure (y-axis) vs. Consumption expenditure (x-axis) for 5 consumption quintiles in the US. The top panel plots education expenditures as a fraction of total consumption expenditures. The bottom panel plots education expenditures as a fraction of income. Both the education expenditure share variables in the y-axes and the consumption variable on the x-axis are residualized with respect to age.

(a) Educational expenditure as a fraction of total consumption expenditure



(b) Educational expenditure as a fraction of total income

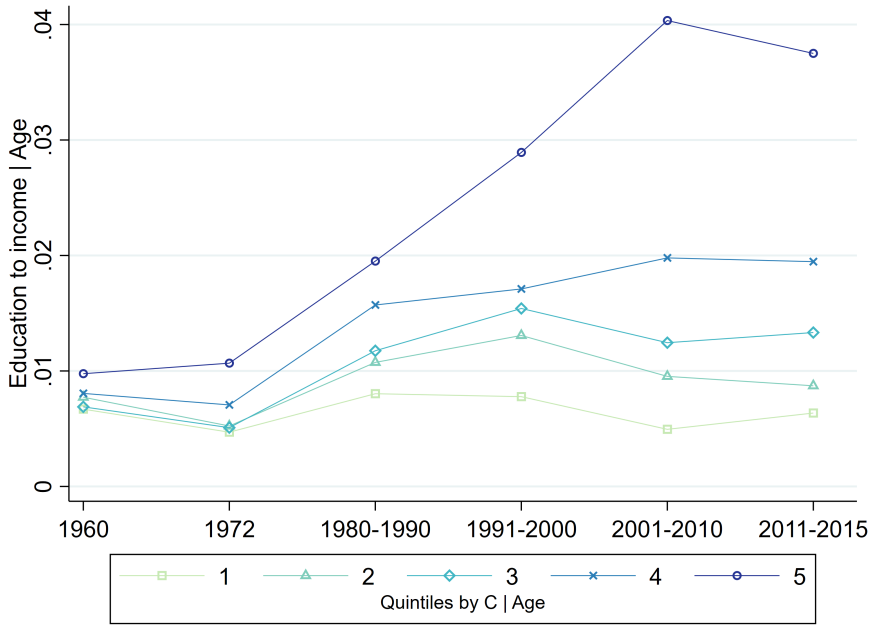


Figure 2: Education expenditure share over time. The top panel plots education expenditures as a fraction of total consumption expenditures. The bottom panel plots education expenditures as a fraction of income. Both the education expenditure share variables in the y-axes and the consumption variable used in forming the quintiles are residualized with respect to age.

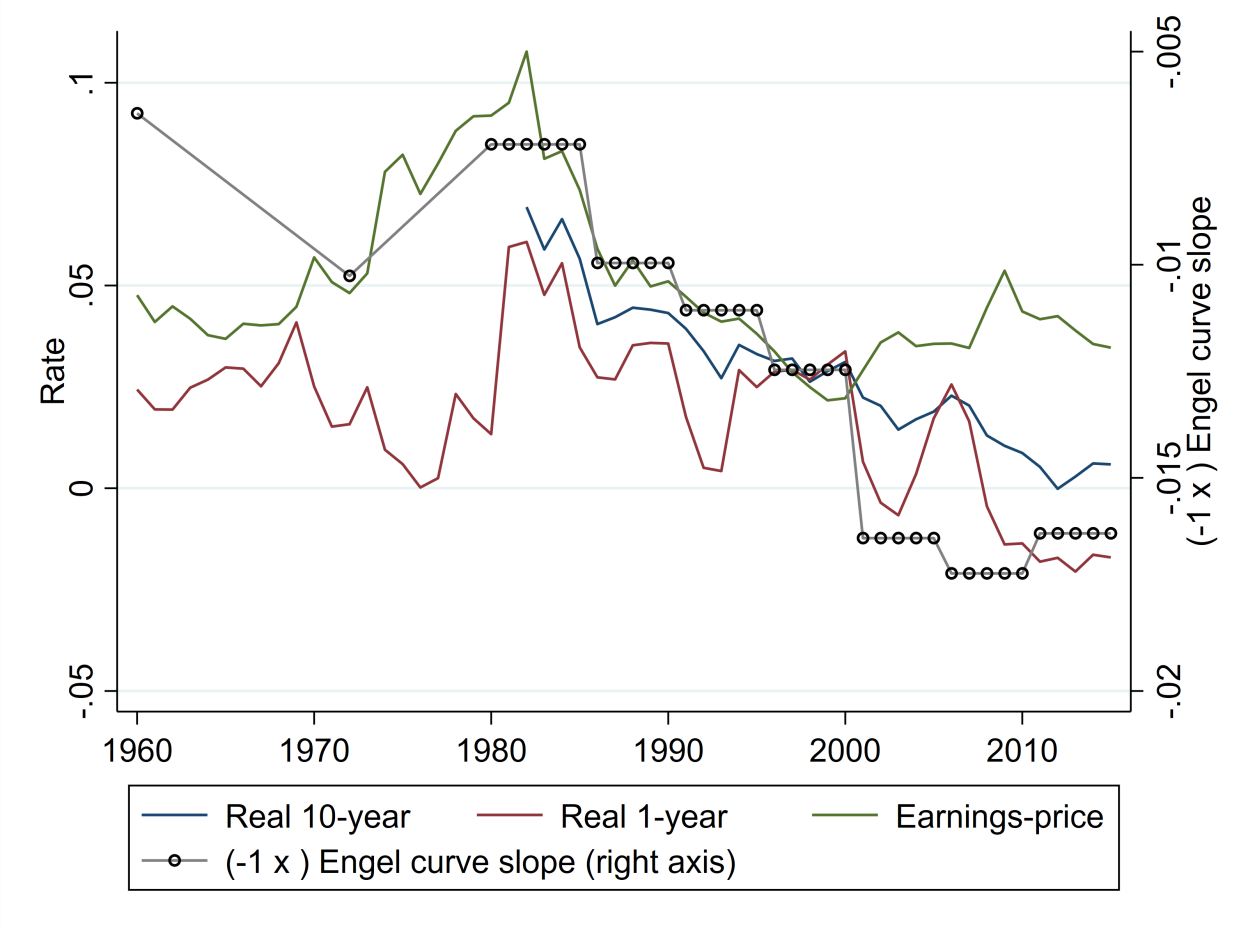


Figure 3: Education Engel slope (hollow circles, right axis) and real rates of return (solid lines, left axis). The "Engel curve slope", for a group of years, is defined as the regression coefficient from a 2-stage least squares regression of the education expenditure share on log income, with log income instrumented by log total expenditure, and including with age and year fixed effects. The figure plots the negative of this 2SLS regression coefficient. The "Real 10-year" (solid blue line) corresponds to the 10-year real Treasury yield from the Federal Reserve Bank of Cleveland, retrieved from FRED. The "Real 1-year" (solid red line) corresponds to the difference between the yield on 1-year Treasury bills, retrieved from the Macrohistory database (https://www.macrohistory.net/database/) constructed in Jordà et al. (2019), and the contemporaneous 1-year inflation expectation, retrieved from the Ludvigson Surveys. The "Earnings-price" (solid green line) corresponds to the earnings-to-price ratio on the S&P500 stock index, retrieved from the database (http://www.econ.yale.edu/~shiller/data.htm) constructed in Shiller (2000). Earnings are the sum of the last four quarters of earnings, summed over all firms in the index.

2 Human capital tilting: a simple graphical example

Our main question concerns the impact of a change in expected future financial returns, via parental incentives to invest in their child's human capital. More specifically, recall that we refer to human capital "tilting" as the disproportionate increase in human capital investment by rich parents. This section uses a simple example to show that human capital tilting is the natural outcome of parents' *portfolio choice* between leaving a financial bequest and educating their children.

Before explaining with our full model, we provide a graphical analysis for a simpler example. Imagine a parent trying to maximize their child's total endowment, $R'a' + y'$, which consists of financial assets $R'a'$ and human capital y'. Human capital is produced by parental investment x, the key choice in this setting, according to a concave production function $y' = g(x)$. Parents also have financial resources n, meaning that they are able to save $a' = n - x$ for their child. These savings earn a return R' which is then passed on to the child as $R'a'$. Parents are also subject to an intergenerational borrowing constraint, $a' \geq 0$. Figures 4-6 plot the solution and convey the key forces involved in a change to expected future returns R'.

Figure 4 plots the investment optimality condition in this simpler model. The marginal rate of transformation between a child's financial assets $R'a' = R'(n - x)$ and his labor income $y' = g(x)$ is captured via the concave curves (the production possibilities frontier) that have slopes $-R'/g'(x)$. However, $R'a'$ and y' are equally valuable to the parent—the $R'a' + y'$ isoquants, which have slope -1 in $(R'a', y')$-space, represent indifference curves for the parent. The tangency between the possibility frontier and these indifference curves is therefore such that $g'(x) = R'$. The left panel plots the optimality condition for an unconstrained parent, which sits at this tangency point, while the right panel plots the optimality condition of a constrained parent with $a' = 0$. It is clear that the constrained parent underinvests in education.

Figure 5 displays the tilting mechanism. If R' falls, the production possibility frontier rotates counterclockwise around the point $(g(n), 0)$, but the indifference curves and borrowing constraint are unaffected. The result is an increase in human capital for the children of unconstrained parents, and no change for constrained parents.

Figure 6 presents an alternative visualization via investment demand curves, treating R' as the price and x as the quantity. Unconstrained parents invest until their demand $g'(x)$ meets their marginal opportunity cost R'. Constrained parents are on the vertical (inelastic) part of their demand curve and underinvest. When R' falls (right panel), it is clear that the investment gap widens between unconstrained and constrained parents.

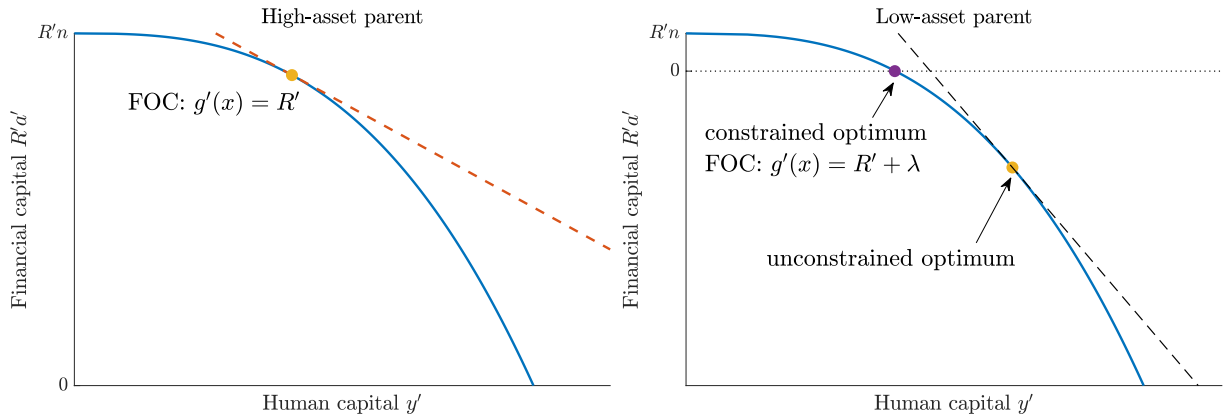


Figure 4: Optimal human capital investment in the simplified model described in the text. The dashed lines, which have a slope of -1, represent parental indifference curves. The solid curves, which have a slope of $-R'/g'(x)$, represent the production possibility frontier. The left panel captures an unconstrained parent. The right panel captures a constrained parent.

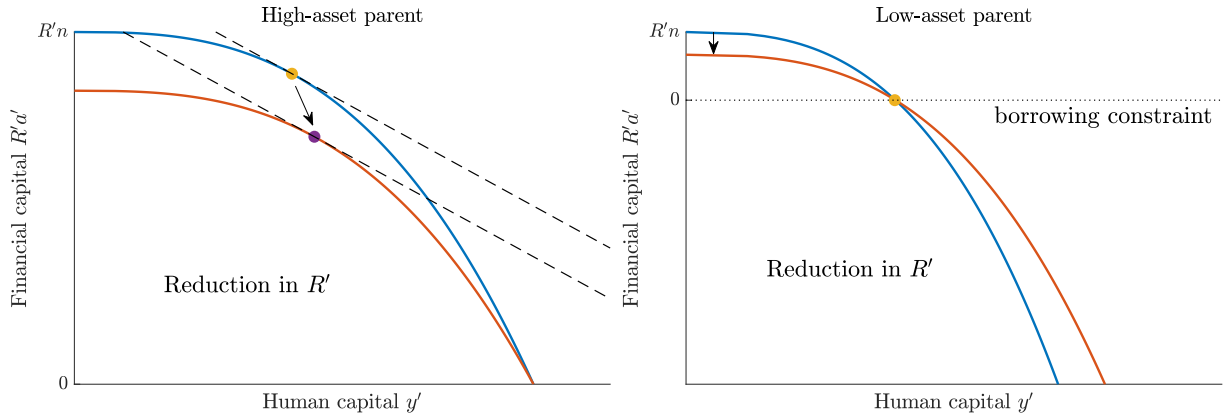


Figure 5: Tilting mechanism in the simplified model described in the text. The left panel captures an unconstrained parent. The right panel captures a constrained parent.

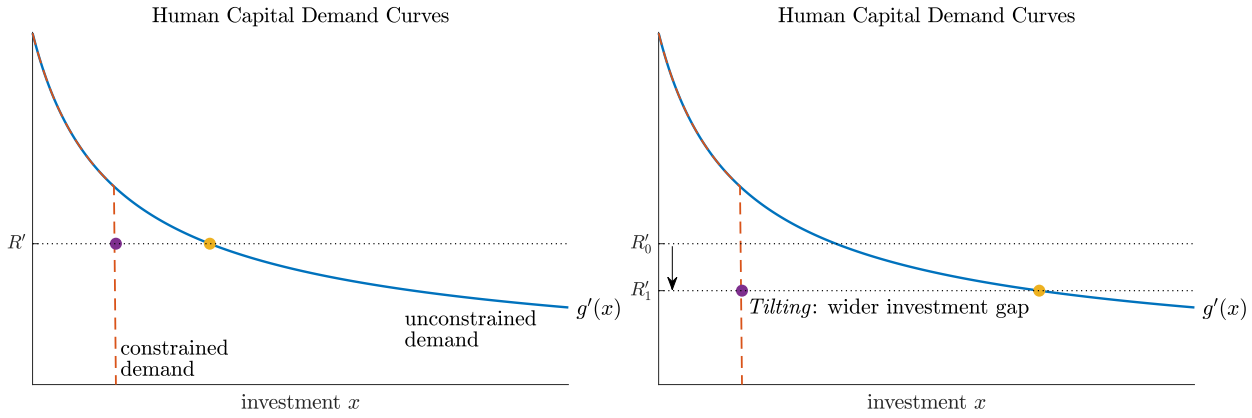


Figure 6: Investment demand and tilting in the simplified model described in the text.

3 A model of human capital tilting

The following small, open economy is composed of overlapping generations of families. Each agent lives for two dates: when young, they are a "child"; when old, we call them a "parent" that is associated with a newborn child. Parents can differ in their assets and skills, earn wages from work, consume, leave financial bequests, and make educational investments in their children (human capital). Parents are also subject to a borrowing constraint. Parents' decisions, plus additional random shocks, lead to the children's distribution of assets and skills. The wage rate w and price of human capital investments p are equilibrium objects. The marginal product of capital R is exogenous.

Individual family problem. Consider a parent with financial assets a, human capital h, talent z, and luck ε. She earns labor income

$$y := w\varepsilon z h.$$

Define the net wealth of a parent after earning labor and capital income, but before making consumption decisions and before investing in their child's human capital:

$$n := Ra + y. \tag{1}$$

With wealth n, the parent makes consumption and investment decisions that result in the child's assets a' and human capital h', determined as

$$a' = n - c - px \tag{2}$$

$$h' := \theta(h)g(x). \tag{3}$$

Equation (2) is a standard budget constraint: the parent consumes c and spends px on child education. Equation (3) is the human capital production function: parents combine some financial investment x along with their own skills h to leave the child with h'. We include an explicit role for parental human capital h in the productivity $\theta(h)$ of the child's skill formation, reflecting the complementarities discussed in Cunha and Heckman (2007) and Becker et al. (2018). This feature is not critical for our theoretical results but helps our model quantitatively.

The child also inherits talent z', which is an exogenous combination of parental talent

z and some random luck ν, according to

$$\log z' = (1-\psi)\log \bar{z} + \psi \log z + \sigma_\nu \nu. \tag{4}$$

The talent luck is an iid idiosyncratic shock drawn from a standard Normal distribution, i.e., $\nu \sim N(0,1)$. The child also experiences labor market luck ε'. We assume that ε and ε' are iid idiosyncratic shocks both drawn from distribution Ξ with mean 1. (The only purpose of $\varepsilon, \varepsilon'$ are to enrich income distributions in quantitative exercises.)

Parents make all the decisions in this economy. At the time of her decisions, the parent knows their net worth n, the investment price p, future returns (R', w'), and her own child's talent level z'.

The parent utility function is given by

$$u(c) + \beta u(\mathbb{E}[n']), \tag{5}$$

where u is the flow utility function applied to both parental consumption c and the "warm glow" from the expected bequest $\mathbb{E}[n'] = R'a' + \mathbb{E}[y']$ to the child (this expectation is conditional on the parent's information set, but we simply write it as $\mathbb{E}[\cdot]$ when this conditioning is clear from the context). Herein lies a crucial aspect of the model: the expected bequest is in *market-value units*, in the sense that it includes both bequeathed financial assets a' and human skills h' left to the child, *both after earning their expected returns*. Insofar as parents care about the total resources their child will have, n', our utility assumption thus embeds a degree of rationality into the warm glow decision.

Two implicit assumptions are worth discussing here. First, the assumption that parents only care about the *expectation* of the bequest (i.e., $\mathbb{E}[n']$ rather than n') is for simplicity—this shortcut is used in Becker et al. (2018) as well. Such an assumption could alternatively be justified as a first-order approximation to an expected utility model for small labor income shocks ε' (additionally, if there were capital return shocks, then our assumption approximates those to be a small factor in bequest motives). Under our assumption, parents do not make precautionary decisions due to the child's income risk, which will allow for clean analytical results and also illustrates that our mechanisms are completely distinct from inter-generational insurance motives. Second, the implicit assumption that the bequest includes the market value of human capital $\mathbb{E}[y']$ is motivated by the logic of conventional frictionless family models that imply a fungibility between time and financial assets (e.g., leisure time comes at the opportunity cost of labor income). Note that we also abstract from a labor supply decision in this model.

Finally, parents additionally face the borrowing constraint

$$a' \geq \underline{a}, \tag{6}$$

where $\underline{a} \leq 0$. Our theoretical propositions often assume $\underline{a} = 0$, which denotes no intergenerational borrowing and is also thus the natural case to consider.

Parents maximize (5) subject to their intergenerational budget constraint (2) and their borrowing constraint (6). We assume the following functional forms for utility and human capital production:

$$\begin{aligned} u(c) &= \frac{c^{1-\gamma} - 1}{1-\gamma}, \quad \gamma > 0, \\ \theta(h) &= \bar{\theta} h^{\zeta}, \\ g(x) &= x^{\alpha}, \quad \alpha + \zeta < 1. \end{aligned}$$

These commonly-adopted functional forms have two properties which are convenient. Both satisfy an Inada condition at 0, which delivers interior optima, and both feature constant elasticity and curvature, so that none of our results emanate from exogenous cross-sectional heterogeneity in consumption or investment propensities.

Equilibrium. Let $F_t(a, h, z)$ denote the time-t endogenous distribution of parents' assets, human capital, and heritable talent. The next-period distribution $F_{t+1}(a, h, z)$ is determined from F_t and the evolution equations for a_{t+1}, h_{t+1}, and z_{t+1}, given in (2), (3), and (4). Recall also that Ξ denotes the exogenous distribution of labor market luck ε, while the talent shock ν is a standard Normal whose CDF we denote by Φ. To economize on notation, we will often write $M_t = F_t \times \Xi \times \Phi$ for the product measure, which is the key object for integrating over individual parents.

We assume an aggregate production function for consumption goods, the numéraire, given by a Cobb-Douglas aggregator of capital K and effective units of labor L,

$$Y_t = K_t^{\rho} L_t^{1-\rho}. \tag{7}$$

The representative firm rents capital and labor at prices R_t and w_t. So

$$R_t = \rho Y_t / K_t \tag{8}$$

$$w_t = (1-\rho) Y_t / L_t \tag{9}$$

Recall that this is a small open economy, so K_t adjusts elastically to R_t, which is exoge-

nous. By contrast, w_t is determined endogenously.

Total education goods are produced via an investment of consumption goods (investment I_t) according to the following production function

$$X_t = I_t^{\eta}. \tag{10}$$

The relative price of education goods will be denoted by p_t. Maximizing profits, education producers set $\eta p_t I_t^{\eta-1} = 1$. Substituting this back into (10), we have

$$X_t = (\eta p_t)^{\frac{\eta}{1-\eta}} \tag{11}$$

which can be thought of as the education supply curve.

The market clearing conditions are as follows. As a consequence of the open capital market, there is neither a goods nor capital market clearing condition. The two market clearing conditions pertain to labor and education investment. Aggregate effective units of labor are given by summing all workers' talents and skills,

$$L_t = \int zh dM_t(a, h, z, \varepsilon, \nu). \tag{12}$$

Aggregate education investment comes from the cumulation of family investment decisions,

$$X_t = \int x(a, h, z, z', \varepsilon) dM_t(a, h, z, \varepsilon, \nu), \tag{13}$$

where $x(a, h, z, z', \varepsilon)$ is the parents' investment policy function, and where $z' = (1 - \psi)\bar{z} + \psi z + \sigma_\nu \nu$ from equation (4).

Special case: No-Borrowing Log (NBL). In our model, we will often want to clarify the mechanics by simplifying to the special case with no borrowing ($\underline{a} = 0$) and log utility ($\gamma = 1$). The economics will be particularly clear with these parameters.

Definition 1. The *No-Borrowing-Log (NBL)* economy is defined by $\underline{a} = 0$ and $\gamma = 1$.

3.1 Partial equilibrium analysis

We now provide the details of the partial equilibrium for our full model. That is, taking prices as exogenous, we illustrate the parental incentives to make human capital

investments. To keep notation compact, define the following composite variable:

$$\tilde{w}' := w'z'\theta(h), \tag{14}$$

which captures the family-specific payoff to a child's human capital.

Let $\lambda\beta u'(\mathbb{E}[n'])$ denote the Lagrange multiplier on the borrowing constraint. Optimality of parental consumption (c) and investment (x) imply

$$\begin{aligned} u'(c) &= \beta R' u'(\mathbb{E}[n']) + \lambda\beta u'(\mathbb{E}[n']) \\ \beta\tilde{w}' g'(x) u'(\mathbb{E}[n']) &= \beta R' p u'(\mathbb{E}[n']) + \lambda\beta u'(\mathbb{E}[n'])p. \end{aligned}$$

The consumption-savings decision is intertemporal, trading off parental utility against bequest utility. The investment decision, by contrast, is a static trade-off between financial returns and the returns to human capital, both in terms of the child's utility. Manipulating the investment FOC illuminates the economics; let us divide by $\beta u'(\mathbb{E}[n'])p$ to obtain

$$\frac{\tilde{w}' g'(x)}{p} = R' + \lambda. \tag{15}$$

This says that the marginal benefit of human capital investment is optimally linked to an individual's "discount rate," where the discount rate not only includes the exogenous expected rate of return R' but also the shadow cost of borrowing at the constraint λ, denominated in units of child marginal utility. Consequently, a constrained parent has a higher discount rate they apply to investment decisions.

Using the functional forms for u and g, unconstrained parents (with $\lambda = 0$) choose

$$x^* := \left(\frac{\alpha\tilde{w}'}{R'p}\right)^{\frac{1}{1-\alpha}} \tag{16}$$

$$c^* := \frac{R'(n - px^*) + \tilde{w}' g(x^*)}{R' + (\beta R')^{1/\gamma}}. \tag{17}$$

Before moving on, we use these unconstrained optima to emphasize that human capital tilting *requires borrowing constraints*. Indeed, in a benchmark unconstrained model ($\underline{a} = -\infty$), there are no cross-sectional differences in investment responses to financial rates of return. For instance, while low future financial returns can induce human capital investment, this occurs proportionally across the income and wealth distribution.

Lemma 1. *If $\underline{a} = -\infty$, then all parents are unconstrained and their investment all responds*

proportionally to R':

$$\frac{\partial^2 \log x^*}{\partial a \partial R'} = \frac{\partial^2 \log x^*}{\partial z \partial R'} = \frac{\partial^2 \log x^*}{\partial h \partial R'} = \frac{\partial^2 \log x^*}{\partial \varepsilon \partial R'} = 0.$$

As constraints are crucial to our mechanism, we define the set of borrowing-constrained parents,

$$\mathcal{B} := \left\{ (n, \tilde{w}') : c^* + px^* + \underline{a} \geq n \right\}. \tag{18}$$

Its complement $\mathcal{U} := \mathcal{B}^c$ represents the unconstrained parents. Intuitively, borrowing-constrained parents are those with low n and high $\tilde{w}'$.

For constrained parents, we may combine the FOCs to eliminate λ. We obtain a condition equating the marginal utility of consumption to the marginal value of human capital investment:

$$\frac{\beta \tilde{w}' g'(x)}{p} = \frac{u'(c)}{u'(\mathbb{E}[n'])}. \tag{19}$$

Using the functional forms and the borrowing constraint, a single equation determines investment x for constrained parents, with c then determined from the constraint:

$$x_{\mathcal{B}} := \left\{ x : \left(\frac{\alpha \beta \tilde{w}'}{p} \right)^{\frac{1}{\gamma}} (n - px - \underline{a}) = \tilde{w}' x^{\frac{1-\alpha+\alpha\gamma}{\gamma}} + R' \underline{a} x^{\frac{1-\alpha}{\gamma}} \right\} \tag{20}$$

$$c_{\mathcal{B}} := n - p x_{\mathcal{B}} - \underline{a}. \tag{21}$$

One can show that $x_{\mathcal{B}}$ is determined uniquely.

We often study the NBL economy, which has the following closed-form solution.

Lemma 2. *Consider the NBL economy ($\underline{a} = 0$, $\gamma = 1$). Then,*

$$x^* = \left(\frac{\alpha \tilde{w}'}{R' p} \right)^{\frac{1}{1-\alpha}} \quad \text{and} \quad x_{\mathcal{B}} = \frac{\alpha \beta}{1 + \alpha \beta} \frac{n}{p}$$

$$c^* = \frac{n}{1+\beta} + \frac{1-\alpha}{\alpha} \frac{p x^*}{1+\beta} \quad \text{and} \quad c_{\mathcal{B}} = \frac{1}{1+\alpha\beta} n.$$

Furthermore, constrained parents underinvest in human capital, in the sense that $x_{\mathcal{B}} < x^*$ for all parents with $(n, \tilde{w}') \in int(\mathcal{B})$.

Constrained parents' behavior is particularly simple in Lemma 2: they consume and invest constant fractions of liquid net wealth n. A reduction in R' has no partial-equilibrium effect on behavior because constrained parents cannot borrow more than they already do, exactly as suggested by Figures 4-6.

The effect of R' on unconstrained investment x^* is a pure portfolio choice effect: lower R' decreases the value of financial investment relative to human investment, so x^* unambiguously rises. In addition, there is an impact of R' on c^*, entirely through wealth effects: lower R' increases the net-present-value of child income through the term $\frac{\tilde{w}'g(x^*)-R'px^*}{R'} = \frac{1-\alpha}{\alpha}px^*$, so c^* also unambiguously rises. In passing, notice how the presence of real assets creates an upward-bias for traditional inference about the elasticity of intertemporal substitution, or EIS (i.e., despite unitary EIS, human capital implies an inverse relationship between c and R').

The conventional economics of borrowing constraints—that financial assets matter for constrained folks—are particularly transparent in Lemma 2. Whereas higher assets n allow an increase in both consumption and investment $c_{\mathcal{B}}$ and $x_{\mathcal{B}}$, assets n have no effect on unconstrained investment x^*. The reason is that, unlike financial investment, human capital investment has decreasing returns to scale, implying an optimal amount of investment in children; unconstrained parents can always borrow enough to achieve this optimal investment, so greater financial wealth has no impact.

Appendix B moves beyond the NBL case and shows that human capital tilting is more general. Namely, we always have $\frac{\partial}{\partial R'}x^* < 0 \leq \frac{\partial}{\partial R'}x_{\mathcal{B}}$, so a decline in R' leads unconstrained parents to invest more in their children while constrained parents invest less. A gap opens between skills of children, depending on whether their parents happened to be constrained or not.

We conclude with two remarks connecting our data to the model.

Remark 1. *Section 1 examines the investment-consumption ratio (x/c), because this measure purges unmodeled effects that apply to all types of expenditures. Our model is consistent with the empirical patterns. Indeed, a corollary of Proposition B.1 is that x^*/c^* and $x_{\mathcal{B}}/c_{\mathcal{B}}$ move in opposite directions, in response to R' changes (as long as γ is not too small). Assuming parental consumption c is negatively associated with financial constraints, we predict a steeper slope in the association between x/c and c during low-R' regimes, which is what the data shows.*

Remark 2. *When we examine educational investment empirically, it looks like a "luxury good" in the sense that x/c is increasing in c in all surveys. Our model does not make a clear qualitative prediction that this slope needs to be positive. We explore this issue in Appendix C and show that large ζ (role of parental h in producing child h') and large ψ (influence of parental talent z in producing child talent z') lead naturally to a positive slope between x/c and c.*

3.2 Pricing and tilting in equilibrium

In this section, we address the impact of a low-return environment on market prices (wages and education prices) and then the equilibrium feedback of these prices into human capital investment.

Determining the response of wages is trivial. Due to the CRS production, the wage w' is determined purely as a function of the capital return R'. In particular, solving equations (8)-(9), the equilibrium wage is

$$w = (1-\rho)\left(\frac{\rho}{R}\right)^{\frac{\rho}{1-\rho}} \tag{22}$$

That is, a lower return R' is associated with more capital ("capital deepening") which raises the marginal product of labor, hence raises w'.

Next, we analyze the education price. Differentiate the market clearing condition (13) to obtain

$$\frac{d\log p}{d\log R'} = \left(\frac{\eta}{1-\eta} - \frac{\overline{\mathbb{E}}[x\frac{\partial \log x}{\partial \log p}]}{\overline{\mathbb{E}}[x]}\right)^{-1} \frac{\overline{\mathbb{E}}[x(\frac{\partial \log x}{\partial \log R'} + \frac{\partial \log x}{\partial \log w'}\frac{d\log w'}{d\log R'})]}{\overline{\mathbb{E}}[x]}, \tag{23}$$

where $\overline{\mathbb{E}}$ denotes cross-sectional average, e.g., $\overline{\mathbb{E}}[f] := \int f dM$. According to expression (23), education prices are driven by two factors: the weighted-average investment responses to (i) financial returns (i.e., $\frac{\partial \log x}{\partial \log R'}$ and (ii) wages (i.e., $\frac{\partial \log x}{\partial \log w'}\frac{d\log w'}{d\log R'}$). On average, low financial returns induce human capital investment demand, so the first factor is positive. Low financial returns also raise wages, which induces a further desire to invest in human capital via the second factor. Given a rising demand for human capital investment, its price increases.

Lemma 3. *Consider the NBL economy ($\underline{a} = 0$, $\gamma = 1$). A fall in the rate R' induces a higher wage and higher education prices, with the following expressions:*

$$\frac{d\log w'}{d\log R'} = -\frac{\rho}{1-\rho}$$
$$\frac{d\log p}{d\log R'} = -\frac{\kappa}{1-\rho}, \quad \textit{where} \quad \kappa := \frac{\overline{\mathbb{E}}[\mathbf{1}_{\mathcal{U}}x]}{\overline{\mathbb{E}}[\mathbf{1}_{\mathcal{U}}x] + \frac{\eta(1-\alpha)}{1-\eta}\overline{\mathbb{E}}[x] + (1-\alpha)\overline{\mathbb{E}}[\mathbf{1}_{\mathcal{B}}x]}$$

Given the response of wages and education prices, our previous tilting results are slightly modified as follows. First, higher future wages further amplifies the investment demand of unconstrained parents, whereas it does not impact constraint parents' invest-

ment, thus amplifying the tilting mechanism. Second, higher education prices causes constrained parents' investments to fall on net, where previously there was no effect. This latter "crowding out" implication is important in applications, because it can potentially help resolve the puzzle that median labor earnings have not increased over time, even as financial returns have fallen and skill premia have risen. More subtly, as we will see, the rise in p creates a pecuniary externality that partly manifests in the reduction in constrained parents' investment.

Proposition 1. *Consider the NBL economy ($\underline{a} = 0$, $\gamma = 1$). In equilibrium, changes in the rate R' induce the following human capital investment responses:*

$$\frac{d\log x^*}{d\log R'} = -\frac{1}{1-\alpha}\frac{1-\kappa}{1-\rho}$$
$$\frac{d\log x_{\mathcal{B}}}{d\log R'} = \frac{\kappa}{1-\rho}, \quad \textit{where} \quad \kappa := \frac{\overline{\mathbb{E}}[\mathbf{1}_{\mathcal{U}} x]}{\overline{\mathbb{E}}[\mathbf{1}_{\mathcal{U}} x] + \frac{\eta(1-\alpha)}{1-\eta}\overline{\mathbb{E}}[x] + (1-\alpha)\overline{\mathbb{E}}[\mathbf{1}_{\mathcal{B}} x]} \in (0,1)$$

In particular, a fall in R' leads to more unconstrained investment and less constrained investment.

Now that we have understood how human capital tilting can emerge in equilibrium, we explore several of its implications. To that end, it is useful to define the following simple tilting statistic that sits at the core of many subsequent results.

Definition 2. For an individual parent, we define the *degree of human capital tilting* by the excess propensity to respond to R' when unconstrained versus constrained:

$$\chi := \frac{d\log x_{\mathcal{B}}}{d\log R'} - \frac{d\log x^*}{d\log R'} \tag{24}$$

Note that in the NBL economy, χ is constant across all parents.[2]

3.3 Labor income distribution

We now turn to one of the central questions of our analysis: What are the implications of finance-driven human capital tilting on the labor income distribution and its dynamics? We show below that tilting implies a rise in labor income inequality, a decrease in intergenerational income mobility, and an increase in labor's share of income for relatively richer households (the so-called "working rich" phenomenon). All of these patterns are in line with the US experience over the last 50 years.

[2] In the NBL economy, $\frac{d\log x^*}{d\log R'}$ and $\frac{d\log x_{\mathcal{B}}}{d\log R'}$ are functions only of aggregate variables.

Starting with *labor income inequality*, we have the following proxy:

$$\mathcal{I} := \overline{\text{Var}}\Big[\log(y')\Big],$$

where $\overline{\text{Var}}$ denotes cross-sectional variance. Income variance is the simplest comprehensive inequality measure that also emphasizes how our model and mechanism are less about extreme income concentration and more about income inequality throughout the distribution.[3] When R' falls, high-income and high-wealth parents invest even more in their kids, whereas low-income, low-wealth parents cannot—this raises inequality in the next generation. The following result establishes that this mapping between human capital tilting and inequality is exact in our model.

Proposition 2. *Consider the NBL economy ($\underline{a} = 0$, $\gamma = 1$). Holding fixed the set $\mathcal{B}$, we have the following mapping between human capital tilting χ and labor income inequality $\mathcal{I}$:*

$$\frac{d\mathcal{I}}{d\log R'} = -\overline{\mathbb{E}}\mathbf{1}_{\mathcal{B}}(1-\overline{\mathbb{E}}\mathbf{1}_{\mathcal{B}}) \times \chi$$

Consequently, when R' falls, inequality rises if and only if human capital tilting prevails.

Next, we consider the following proxy for *immobility*:

$$\mathcal{M} := \frac{\overline{\text{Cov}}[\log y', \log y]}{\overline{\text{Var}}[\log y]},$$

where $\overline{\text{Var}}$ are again $\overline{\text{Cov}}$ cross-sectional operators. Our immobility measure, commonly called the "intergenerational elasticity of income" (IGE), is a regression coefficient of log child labor income on log parent labor income. Such a regression captures how much a child's labor income depends on her parent's. Human capital tilting is responsible for a decrease in mobility. High-income parents disproportionately invest in their kids, thereby creating kids whose incomes are more similar to their own; low-income parents don't invest more, which drags down their children's income toward their own. Again, our model features an exact one-to-one mapping between tilting and immobility.

Proposition 3. *Consider the NBL economy ($\underline{a} = 0$, $\gamma = 1$). Holding fixed the set $\mathcal{B}$, we have the following mapping between human capital tilting χ and intergenerational immobility $\mathcal{M}$:*

$$\frac{d\mathcal{M}}{d\log R'} = -\Big(\frac{\overline{\mathbb{E}}[\log y \mid \mathcal{U}] - \overline{\mathbb{E}}[\log y \mid \mathcal{B}]}{\overline{Var}[\log y]}\Big)\overline{\mathbb{E}}\mathbf{1}_{\mathcal{B}}(1-\overline{\mathbb{E}}\mathbf{1}_{\mathcal{B}}) \times \chi$$

[3] Our model is not well-suited to explain rising top percentile income shares. Indeed, one can easily show that the tail of the income distribution is independent of R'.

Consequently, when R' falls, immobility rises if and only if human capital tilting prevails.

Finally, we examine the following *labor share slope*:

$$\mathcal{L} := \lim_{\bar{a}\to\infty} \overline{\mathbb{E}}\Big[\log(\frac{y'}{y'+R'a'}) \mid a' > \bar{a}\Big] - \lim_{\bar{a}\to 0} \overline{\mathbb{E}}\Big[\log(\frac{y'}{y'+R'a'}) \mid a' \leq \bar{a}\Big].$$

The labor share slope measure looks at children's ratio of labor income to total income, comparing high-asset children to low-asset children. This slope is negative almost by construction. However, the labor share slope rises when R' falls. In other words, as R' falls richer households are increasingly likely to be high-skilled laborers than capitalists or rentiers. Here, the result is not exclusively due to human capital tilting, but that is an important contributor, in the sense that $\frac{d\mathcal{L}}{d\log R'}$ is more negative when χ is higher. We make this last point precise, and link it concretely to financial frictions, by introducing an alternative labor share slope that is the same in every way except it conditions on the low-asset parent being unconstrained:

$$\mathcal{L}^{alt} := \lim_{\bar{a}\to\infty} \overline{\mathbb{E}}\Big[\log(\frac{y'}{y'+R'a'}) \mid a' > \bar{a}\Big] - \lim_{\bar{a}\to 0} \overline{\mathbb{E}}\Big[\log(\frac{y'}{y'+R'a'}) \mid a' \leq \bar{a}, \mathcal{U}\Big].$$

The change in the truly observed labor share slope, net of this alternative measure that ignores financial frictions, is precisely related to the tilting statistic χ.

Proposition 4. *Consider the NBL economy ($\underline{a} = 0$, $\gamma = 1$). Holding fixed the set $\mathcal{B}$, a decrease in R' raises the labor share slope: $\frac{d\mathcal{L}}{d\log R'} < 0$. In addition, we have the following mapping between human capital tilting χ and the excess labor share slope:*

$$\frac{d}{d\log R'}\mathcal{L} - \frac{d}{d\log R'}\mathcal{L}^{alt} = -\frac{1+\alpha\beta}{1+\beta}\chi.$$

The derivations in Propositions 2-4 hold fixed the set $\mathcal{B}$ of borrowing-constrained parents. But the set $\mathcal{B}$ will also respond: a fall in R' induces more parental consumption and investment, which leads to an equilibrium in which a greater fraction of parents are borrowing constrained. This provides an amplifying force on inequality and immobility, because borrowing-constrained parents underinvest in their children's human capital. Later, when we discuss efficiency, the fact that low rates induce more borrowing constraints to bind also becomes relevant.

Proposition 5. *Consider the NBL economy ($\underline{a} = 0$, $\gamma = 1$). We have $\frac{d}{dR'}\overline{\mathbb{E}}\mathbf{1}_{\mathcal{B}} < 0$.*

3.4 Aggregate labor income

In addition to the labor income distribution, we investigate *aggregate* labor income. In our model, a reduction in R' is driven by capital inflows, and so the conventional effect is that this raises labor productivity and hence labor income. This static force is further amplified by the dynamic investment incentive that raises overall human capital. But with financial constraints, the investment effect is dampened through two channels: (i) the aggregate investment is less with financial constraints; (ii) the parents who increase investment are not necessarily the ones with talented kids.

Define the labor income of a single generation of children:

$$\mathcal{Y}' := \overline{\mathbb{E}}[y']. \tag{25}$$

Analogously, let $\mathcal{Y}'_{FB}$ denote the same generation's labor income in the frictionless "first-best" economy with $\underline{a} = -\infty$. We have the following result.

Proposition 6. *The effect of R' on aggregate labor income is*

$$\frac{d\log \mathcal{Y}'}{d\log R'} = \frac{d\log w'}{d\log R'} + \alpha\left[\frac{d\log x^*}{d\log R'} + \frac{\overline{\mathbb{E}}[\mathbf{1}_{\mathcal{B}} y' \chi]}{\mathcal{Y}}\right]. \tag{26}$$

Consider the NBL economy ($\underline{a} = 0$, $\gamma = 1$) in which χ is a constant. Assuming that parameters are such that $\frac{d\log p_{FB}}{d\log R'} - \frac{d\log p}{d\log R'}$ is close enough to zero,[4] we have

$$\frac{d\log \mathcal{Y}'}{d\log R'} - \frac{d\log \mathcal{Y}'_{FB}}{d\log R'} > 0 \Leftrightarrow \chi > 0.$$

That is, a decline in R' raises children's aggregate labor income by less in the constrained economy if and only if the constrained economy features human capital tilting.

Equation (26) helps decompose the forces involved. As R' falls, the wage w' rises, which is a force to increase labor income. But the effect $\frac{d\log w'}{d\log R'}$ operates solely through the production function and is thus equally strong with and without borrowing constraints. Second, unconstrained parents increase their investment demand when R' falls, but $\frac{d\log x^*}{d\log R'}$ is also equal in the constrained and unconstrained economies, provided $\frac{d\log p}{d\log R'}$

[4]This requires picking η differently in the constrained and unconstrained economies, as the proof shows. Note that it is always possible to equate $d\log p / d\log R'$ in the two economies: for instance, by taking $\eta \to 1$, we have $d\log p = 0$ for any perturbation.

is held fixed in the two economies. Indeed, the total derivative of x^* is

$$\frac{d\log x^*}{d\log R'} = \frac{\partial \log x^*}{\partial \log R'} + \frac{\partial \log x^*}{\partial \log w'}\frac{d\log w'}{d\log R'} + \frac{\partial \log x^*}{\partial \log p}\frac{d\log p}{d\log R'}$$

The direct substitution effect (first term) and effect through equilibrium wages (second term) are independent of the borrowing constraint $\underline{a}$. The impact of education prices (third term) does potentially depend on $\underline{a}$, but the proposition holds fixed this effect. Thus, the only term in (26) that is unique to the constrained economy is the human capital tilting term $\frac{\overline{\mathbb{E}}[\mathbf{1}_{\mathcal{B}} y' \times \chi]}{\mathcal{Y}}$. Tilting $\chi > 0$ dampens the impact of low rates and financial returns on aggregate income.

3.5 Kaldor-Hicks efficiency

While we have examined several consequences for aggregate labor income and its distribution, it is also instructive to examine welfare efficiency. Indeed, our core mechanism—human capital tilting—arises due to borrowing constraints, which are also the source of all inefficiencies in this economy.

A hallmark measure of welfare is so-called Kaldor-Hicks efficiency (studied extensively in Dávila and Schaab, 2025). This measure evaluates the aggregate willingness-to-pay, in consumption units, for some policy or, more generally, some perturbation in the environment. If such an aggregate willingness-to-pay is positive, then the perturbation is "potentially Pareto optimal," in the sense that everybody is better off if the perturbation is bundled with transfers whereby the winners compensate the losers.

We start by computing this measure for a single generation. Later, we account for the overlapping generations structure. Welfare of one parent's generation is given by

$$\mathcal{W} := \overline{\mathbb{E}}\left[u(c) + \beta u(\mathbb{E}[n'])\right]. \tag{27}$$

The money-metric change in welfare scales the welfare perturbation by some marginal utility, to convert utils into consumption units.[5] For convenience, we choose the marginal utility of bequest as the scaling factor and examine the following weighted derivative for any perturbation ι:

$$\frac{\delta \mathcal{W}}{\delta \iota} := \int \frac{1}{\beta u'(\mathbb{E}[n'])}\frac{d}{d\iota}\left[u(c) + \beta u(R'(n-c-px) + w'z'h')\right] dM \tag{28}$$

[5]Because our model abstracts from precautionary savings and risk effects, money-metric welfare closely tracks with consumption-equivalent measures of social welfare as in Benabou (2002) and Heathcote et al. (2017), if we focus solely on the children's generation.

We are interested in the perturbation $\delta\iota = \delta \log R'$. We contrast the result for equilibrium welfare $\mathcal{W}$ against $\mathcal{W}_{FB}$, defined as the analogous welfare in the first-best economy with no borrowing frictions ($\underline{a} = -\infty$). For simplicity, the following result uses the limit $\eta \to 1$ to shut down the impact of education prices (when $\eta \to 1$, we have $p \to 1$).

Proposition 7. *Consider the economy with $\eta \to 1$. The effect of R' on Kaldor-Hicks efficiency, relative to first-best, is*

$$\frac{\delta \mathcal{W}}{\delta \log R'} - \frac{\delta \mathcal{W}_{FB}}{\delta \log R'} = \mathcal{K}' - \mathcal{K}'_{FB} + (\mathcal{Y}' - \mathcal{Y}'_{FB}) \frac{d \log w'}{d \log R'},$$

where $\mathcal{K}' := R' \overline{\mathbb{E}}[a']$ is aggregate capital income, and $\mathcal{K}'_{FB}$ is aggregate capital income in the frictionless economy. In the NBL economy ($\underline{a} = 0$, $\gamma = 1$), we have $\mathcal{K}' - \mathcal{K}'_{FB} > 0 > \mathcal{Y}' - \mathcal{Y}'_{FB}$, and so a decline in R' increases the wedge between equilibrium efficiency and first-best efficiency.

Proposition 7 isolates two critical drivers of efficiency that both work in the same direction. With borrowing constraints, parents save excessively and invest too little in human capital, i.e., $\mathcal{K}' - \mathcal{K}'_{FB} > 0$ and $\mathcal{Y}' - \mathcal{Y}'_{FB} < 0$, respectively. As a result of excessive financial savings, a decrease in R' reduces capital income excessively (or restated in terms of borrowing, a decrease in R' does not reduce borrowing costs as much as it optimally should). As a result of deficient human capital investment, a decrease in R' that raises wages then raises incomes too little.

4 Long-run equilibrium

Up until now, we have effectively focused solely on a single generation. That is, a decline in R' affects the incentives of one generation of parents, which filters through to their children's labor and capital income. Restricting our focus in this way allows for several clean analytical results.

We now address the question of what happens in the steady state of the model, given a permanent shift in R'. How does labor income inequality and its increased persistence feed into subsequent generations' human capital and long-run wealth inequality? Does the economy feature excessive savings and suboptimal human capital accumulation even in the long run, as the static welfare analysis in Section 3.5 suggests?

5 Implications for tax policy

We use our framework to highlight a central but understudied trade-off in tax policy. Specifically, we are interested in how the potential for human capital tilting affects the trade-off between labor and capital taxes. Of course, there are numerous mechanisms at play when comparing labor and capital taxes, most of which we abstract from. Our aim is to zoom in on the novel contribution of human capital tilting.

Since our model's competitive equilibrium features too little human capital and too much financial capital, it seems natural to conjecture social benefits from capital taxation in lieu of labor income taxation. On the one hand, a lower labor income tax incentivizes human capital accumulation, a thoroughly explored and well-established force (Heckman et al., 1999; Benabou, 2002; Krueger and Ludwig, 2013; Heathcote et al., 2017; Stantcheva, 2017). At the same time, higher capital gains and wealth taxes depress financial returns, incentivizing a shift away from financial capital and towards human capital. These arguments both work in favor of a bias toward taxing financial capital.

On the other hand, capital and wealth taxes induce human capital tilting. This mechanism is identical to the effect we have unveiled that operates through R': a higher capital tax disproportionately incentivizes rich parents to invest in human capital, even if their child is less talented, creating a misallocation of human capital that depresses the aggregate level of labor income. Thus, higher capital taxes can have a downside. Which effect dominates, and what is optimal tax policy in our economy?

5.1 Setup

Let us formalize this discussion and provide some answers. First, we introduce proportional tax on capital and labor, at rates τ_k and τ_y, respectively. These tax rates only apply to domestic agents. The child's after-tax labor and capital income is now $(1-\tau_y)y' = (1-\tau_y)w'z'h'$ and $(1-\tau_k)R'a'$. Next, we would like to avoid welfare implications to arise directly from redistribution, which matters in a world with financial frictions. For this reason, we assume taxes raised lead to government spending that does not enter into agents' utility functions. The government must meet a spending goal, enforced by the following budget constraint:

$$G = \tau_k \mathcal{K}' + \tau_y \mathcal{Y}'. \tag{29}$$

where G is a parameter, and $\mathcal{K}'$ and $\mathcal{Y}'$ are pre-tax capital and labor income, defined as

$$\mathcal{K}' := \int R' a' dM \quad \text{and} \quad \mathcal{Y}' := \int w' z' h' dM$$

Constraint (29) implies a negative mapping between capital and labor taxes.

The parental optimality conditions are slightly modified in this environment: R' must be replaced by $(1-\tau_k)R'$ everywhere, and w' must be replaced by $(1-\tau_y)w'$ everywhere. For example, the key optimality condition for human capital investment x now reads

$$\frac{(1-\tau_y)\tilde{w}' g'(x)}{p} = (1-\tau_k)R' + \lambda, \tag{30}$$

which makes clear that a higher labor tax disincentivizes investment, while a higher capital tax incentivizes it. Overall, optimal consumption and investment are identical to Lemma 2, if we reinterpret R' and w' as after-tax returns, i.e.,

Lemma 4. *Consider the NBL economy ($\underline{a} = 0$, $\gamma = 1$). Then,*

$$x^* = \left(\frac{\alpha(1-\tau_y)\tilde{w}'}{(1-\tau_k)R'p}\right)^{\frac{1}{1-\alpha}} \quad \textit{and} \quad x_{\mathcal{B}} = \frac{\alpha\beta}{1+\alpha\beta}\frac{n}{p}$$
$$c^* = \frac{n}{1+\beta} + \frac{1-\alpha}{\alpha}\frac{px^*}{1+\beta} \quad \textit{and} \quad c_{\mathcal{B}} = \frac{1}{1+\alpha\beta}n.$$

The conditions for the representative firm and the market clearing conditions are identical to the baseline model. One important implication is that w' and R' are unaffected by taxes. Indeed, the relationship between w' and R' is still given by equation (22). Since capital taxes only apply to domestic agents, the external return-on-capital is still R'. Due to the small open economy assumption, R' is unaffected by taxes, which using (22) implies that w' is also unaffected by taxes.

By contrast, education price p is affected by taxes as follows. Differentiate the market clearing condition (13), using the fact that investment x is a function of taxes only through the after-tax returns $(1-\tau_y)w'$ and $(1-\tau_k)R'$, as well as the impact of taxes on education's price, to obtain

$$\frac{d\log p}{d\tau_k} = \left(\frac{\eta}{1-\eta} - \frac{\overline{\mathbb{E}}[x\frac{\partial \log x}{\partial \log p}]}{\overline{\mathbb{E}}[x]}\right)^{-1} \frac{\overline{\mathbb{E}}[x(\frac{\partial \log x}{\partial \log(1-\tau_k)R'}\frac{d\log(1-\tau_k)R'}{d\tau_k} + \frac{\partial \log x}{\partial \log(1-\tau_y)w'}\frac{d\log(1-\tau_y)w'}{d\tau_k})]}{\overline{\mathbb{E}}[x]}.$$

From here, we can use Lemma 4 and equation (29) to obtain

Lemma 5. *Consider the NBL economy ($\underline{a} = 0$, $\gamma = 1$). We have $\frac{d \log R'}{d\tau_k} = \frac{d \log w'}{d\tau_k} = 0$, and*

$$\frac{d \log p}{d\tau_k} = -\frac{1}{1-\tau_y}\Big(\frac{d\tau_y}{d\tau_k} - \frac{1-\tau_y}{1-\tau_k}\Big)\kappa, \quad \textit{where} \quad \kappa := \frac{\overline{\mathbb{E}}[\mathbf{1}_{\mathcal{U}} x]}{\overline{\mathbb{E}}[\mathbf{1}_{\mathcal{U}} x] + \frac{\eta(1-\alpha)}{1-\eta}\overline{\mathbb{E}}[x] + (1-\alpha)\overline{\mathbb{E}}[\mathbf{1}_{\mathcal{B}} x]},$$

and where $\frac{d\tau_y}{d\tau_k}$ captures the implicit mapping between τ_y and τ_k from constraint (29). In particular, if $G = \tau_k = \tau_y = 0$, we have $\frac{d \log p}{d\tau_k} = (1 + \mathcal{K}'/\mathcal{Y}')\kappa > 0$.

To evaluate tax policy, we use these results along with our notion of Kaldor-Hicks efficiency discussed in Section 3.5.

Proposition 8. *The effect of taxes on Kaldor-Hicks efficiency, relative to first-best, is*

$$\frac{\delta \mathcal{W}}{\delta \tau_k} - \frac{\delta \mathcal{W}_{FB}}{\delta \tau_k} =,$$

where $\mathcal{K}' := R'\overline{\mathbb{E}}[a']$ is aggregate pre-tax capital income, and $\mathcal{K}'_{FB}$ is aggregate pre-tax capital income in the frictionless economy. In the NBL economy ($\underline{a} = 0$, $\gamma = 1$), we have $\mathcal{K}' - \mathcal{K}'_{FB} > 0 > \mathcal{Y}' - \mathcal{Y}'_{FB}$, and so a decline .

5.2 Static tax trade-offs

We start from the perspective of a single generation. Suppose taxes are unexpectedly announced at time t, imposed on the returns accruing to children in generation $t+1$, and then repealed immediately afterward. That is, $\tau_k = \tau_y = 0$ for all periods except for one.

5.3 Long-run taxation

Higher capital taxes reduce financial income, inducing a substitution towards human capital investment. In a representative-agent model, the effects of this tax-induced substitution would be unclear.[6] In our heterogeneous-agent model, the substitution towards human capital is unequal: rich, unconstrained parents accumulate human capital, whereas poor, constrained parents decumulate.

[6]The typical intuition is to avoid taxing accumulable factors (Judd, 1985; Chamley, 1986), which suggests taxing labor instead of capital. But in the real world, effective units of labor are produced by accumulable skills. Furthermore, human capital may create positive externalities on growth that physical capital does not.

6 Quantitative magnitudes

Although our model is purposefully simple, we propose a numerical illustration to get a sense of magnitudes. In particular, we will first consider a calibration with a high interest rate that roughly matches some data from around 1980. We refer to this as the baseline calibration. Then, holding fixed the rest of the calibration, we will consider a lower interest rate and estimate the effects on human capital tilting, labor income inequality, intergenerational mobility, and the prevalence of the working rich. Conceptually, this exercise estimates how much the changes in tilting, inequality, immobility, and top labor shares can be attributed to a fall in rates between 1980 and 2010.

6.1 Calibration

Let us discuss the baseline calibration. First, parental and child labor income are given by the expressions $y = wzh\varepsilon$ and $y' = wz'h'\varepsilon'$, respectively. Here, we are allowing for an explicit wage rate w, which is chosen in order to obtain an average parental income of 40,000 per year.

The parental skill variables (z, h, ε) are drawn from a multivariate lognormal distribution, with unit means, zero correlations, and standard deviations (of the logs of these skill variables) of 0.22, 0.44, and 0.49. These standard deviations are chosen so that idiosyncratic shock ε explains half the variance of parental income, h explains 40%, z explains 10%, and so that the parental labor income Gini coefficient is 0.375 (this is approximately the value for the US in 1980).

Recall the child human capital is formed according to the production function $h' = \theta h^{\zeta} x^{\alpha}$. We calibrate $\theta = 0.07$, $\alpha = 0.35$, and $\zeta = 0.30$ in order to obtain the following in the baseline: a median child income of approximately 50,000; a top 10 percent child share of total income of around 35% (the US value in 1980, see Piketty and Saez, 2003); and a log-log IGE of approximately 0.3 (approximately the US value for cohorts born in 1955, see Davis and Mazumder, 2017). The child exogenous skill is inherited from the parents' according to $z' = z^{\psi}$, with $\psi = 0.10$ picked to reflect the relatively small estimates of genetic heritability of income-relevant talents (Bowles and Gintis, 2002). The child idiosyncratic shock ε' is drawn from the same distribution as the parent ε.

Parental financial assets a are drawn from a lognormal distribution that has a mean of 100,000, a standard deviation calibrated to match a wealth Gini of 0.5 (somewhat lower than the 1980 US wealth Gini of 0.75), and correlations of $\text{corr}[\log(a), \log(z)] = 0.1$ and $\text{corr}[\log(a), \log(h)] = 0.4$. The somewhat lower dispersion in wealth calibrated here is meant to capture the fact that assets in our model are liquid while many assets in the

real world, especially for the richest families, are not. (That said, most of our results are robust to having a much wider dispersion of parental assets, with the exception that we struggle to match the flat 1970s slope of x/c against c in such a calibration.) The choice of correlations is so that $\log(z), \log(h)$ have the same correlations with $\log(y)$ as with $\log(a)$. For simplicity, we do not allow net borrowing, $b = 0$.

The annual discount factor is set to the relatively standard value of $\beta_{\text{annual}} = 0.95$. The baseline EIS is set to $\gamma^{-1} = 0.2$, a relatively low value on which we will perform some sensitivity analysis. The baseline annual real interest rate is set to $R_{\text{annual}} = 1.04$, in line with the "rstar" estimated by Laubach and Williams (2003); in the late 1970s and early 1980s, the long term real rate was arguably much higher, but we take a conservative approach given the difficulty in measuring inflation expectations.

Finally, in interpreting our model, we must recognize that a period our model is a "generation." Thus, we compound our discount factor and interest rate to reflect a generation of 20 years: $\beta = \beta_{\text{annual}}^{20} = 0.3585$ and $R = R_{\text{annual}}^{20} = 2.19$. A summary of our parameter choices are contained in Table 1 below.

Parameter	Description	Value
γ^{-1}	elasticity of intertemporal substitution	0.20
β_{annual}	Annual discount factor	0.95
α	HC prod. elasticity to parent investment	0.35
ψ	Exogenous skill persistence	0.10
ζ	HC prod. elasticity to parent HC	0.30
θ	HC prod. TFP	0.07
w	Wage rate	$\approx$ 40,000
b	Borrowing limit	0
R_{annual}	Annual real interest rate	1.04

Table 1: Key parameter values for the model.

6.2 Baseline results

To compute aggregate and distributional statistics, we simulate 1 million households from our model. Table 2 shows some model-implied statistics for human capital tilting, labor income inequality, intergenerational income mobility, and working-rich prevalence. Overall, the model does a reasonable job of generating relatively realistic child income distribution (panel B), levels of intergenerational mobility (panel C), and labor share of income across the distribution (panel D). Moreover, the investment-to-consumption ratio is relatively flat cross the parental income distribution (panel A), in line with the data on educational expenditure shares in 1980.

Statistic	Value ($R_{\text{annual}} = 1.04$)	Value ($R_{\text{annual}} = 1.01$)
Fraction constrained	39.2%	70.0%
A. Tilting Measures: x/c ratios at various parental labor income quantiles		
$\mathbb{E}[\frac{x}{c} \mid y = y^{\text{quantile}}_{0.10}]$	0.10	0.13
$\mathbb{E}[\frac{x}{c} \mid y = y^{\text{quantile}}_{0.25}]$	0.10	0.14
$\mathbb{E}[\frac{x}{c} \mid y = y^{\text{quantile}}_{0.50}]$	0.11	0.16
$\mathbb{E}[\frac{x}{c} \mid y = y^{\text{quantile}}_{0.75}]$	0.11	0.17
$\mathbb{E}[\frac{x}{c} \mid y = y^{\text{quantile}}_{0.90}]$	0.10	0.18
B. Inequality Measures: distribution of child labor income		
$(y')^{\text{quantile}}_{0.10}$	26,810	28,570
$(y')^{\text{quantile}}_{0.25}$	38,839	43,104
$(y')^{\text{quantile}}_{0.50}$	57,601	66,856
$(y')^{\text{quantile}}_{0.75}$	84,587	101,761
$(y')^{\text{quantile}}_{0.90}$	118,542	146,413
$\log(\frac{\text{Var}[\log y']}{\text{Var}[\log y]})$	-0.34	-0.16
Top 10pct share: $\frac{\mathbb{E}[y'\mathbf{1}\{y' \geq (y')^{\text{quantile}}_{0.90}\}]}{\mathbb{E}[y']}$	32.2%	35.5%
C. Mobility Measures: relative and absolute intergenerational dependence		
log-log IGE	0.324	0.401
$\mathbb{E}[\log(y') \mid y = y^{\text{quantile}}_{0.75}] - \mathbb{E}[\log(y') \mid y = y^{\text{quantile}}_{0.25}]$	0.33	0.39
$\mathbb{E}[\log(y') \mid y = y^{\text{quantile}}_{0.90}] - \mathbb{E}[\log(y') \mid y = y^{\text{quantile}}_{0.10}]$	0.58	0.73
D. Working-Rich Measures: labor share at various labor income quantiles		
$\mathbb{E}[\frac{y'}{y'+R_{\text{annual}}a'} \mid y' = (y')^{\text{quantile}}_{0.10}]$	0.979	0.998
$\mathbb{E}[\frac{y'}{y'+R_{\text{annual}}a'} \mid y' = (y')^{\text{quantile}}_{0.25}]$	0.925	0.990
$\mathbb{E}[\frac{y'}{y'+R_{\text{annual}}a'} \mid y' = (y')^{\text{quantile}}_{0.50}]$	0.789	0.957
$\mathbb{E}[\frac{y'}{y'+R_{\text{annual}}a'} \mid y' = (y')^{\text{quantile}}_{0.75}]$	0.638	0.868
$\mathbb{E}[\frac{y'}{y'+R_{\text{annual}}a'} \mid y' = (y')^{\text{quantile}}_{0.90}]$	0.499	0.748

Table 2: Key statistics for the baseline model calibration with $R_{\text{annual}} = 1.04$ and the counterfactual experiment with $R_{\text{annual}} = 1.01$. Other parameters are in Table 1.

Now, we do an experiment where we reduce the interest rate from $R_{\text{annual}} = 1.04$ to $R_{\text{annual}} = 1.01$. The results are in the final column of Table 2. With the lower real interest rate, there is a significant amount of human capital tilting: investment-to-consumption

x/c increases from 0.13 at the 10th percentile of parent labor income to 0.18 at the 90th percentile (panel A). This tilting drives significant inequality in terms of income distribution variance, although only a moderate change in the top 10th percentile of labor income (panel B). (The latter is somewhat to be expected, given the result of Proposition **??** in the appendix, which shows that the Pareto tail of income in our model is unaffected by R.) All measures of intergenerational persistence are significantly higher (panel C). And finally, the labor share increases across the distribution, particularly at higher incomes, capturing the working rich phenomenon (panel D).

Relative to the data, our counterfactual exercise can account for a large degree of changes in inequality and mobility. For inequality, the cross-sectional variance in log labor incomes rises by 18% due to our mechanism: if we compare the economy with low versus high rates, the log change in child labor income variance is $\log[\mathcal{I}(1.01)] - \log[\mathcal{I}(1.04)] = 0.18$. For mobility, the log-log inter-generational elasticity (IGE) for labor incomes rises by 7.7%: if we compare the low- and high-rate economies, the change in child-on-parent log income regression coefficients is $\log[\mathcal{M}(1.01)] - \log[\mathcal{M}(1.04)] = 0.077$.

Let us mention two obvious limitations of the current model, which can be seen in the quantitative results above. First, Table 2 makes it seem like low rates cause the labor share to increase in aggregate, in contrast to the time series data in the US. However, this effect is due to the fact that we are essentially holding aggregate physical capital fixed. In reality, as interest rates have fallen, there has been a massive inflow of foreign capital into the US (e.g., "global savings glut"). Appendix **??** shows that a fall in R, in an environment with a neoclassical production function featuring gross substitutability between labor and capital, necessarily generates a fall in the aggregate labor share, driven by capital deepening.

Second, Table 2 makes it seem like low rates cause labor incomes to rise across the board, in contrast to evidence in the US that the median wage has fallen between 1980 and 2010. This is an artifact of our assumption that the price of human capital investments is constant. In reality, the price of college and other educational goods have risen dramatically in this time period. Appendix **??** endogenizes the supply of human capital investment goods, so that the increase in investment demand due to a fall in R generate a *decrease* in investments for poor households.

In summary, one should not take literally the result that child labor incomes and labor share rise across the board. Simple extensions of the baseline model can produce stagnancy or even declines in both average child labor incomes and the aggregate labor share. Thus, the results of Table 2 speak more to changes in cross-sectional dispersion

and intergenerational persistence. Also, due to the caveats just mentioned, it is more difficult to get a trustworthy quantitative assessment on how our model performs on the rise of working rich.

6.3 The role of the EIS

One interesting aspect of our results is that they hold despite a relatively low value for the EIS ($\gamma^{-1} = 0.20$). A natural conjecture is that changes in R would produce more dramatic counterfactuals under a higher EIS, but this turns out to be false. Indeed, recall the discussion surrounding equation (15), where we argued that human capital investment is largely a portfolio choice decision: parents compare the marginal rate of return on investing in their child's human capital with the rate of return they get on financial investments. By contrast, the EIS governs intertemporal decisions, i.e., the tradeoff between consumption and all types of investment (including both financial and human). In the model, we rerun our counterfactual experiment comparing a low-R to a high-R economy, at different values of the EIS γ^{-1}. The results are in Table 3, which show that higher EIS actually reduces the effect of low R on tilting, inequality, and mobility. A high EIS increases the sensitivity of the working rich to interest rates, but not because of the human capital investment margin. Rather, a high EIS exacerbates the financial bequest margin: low R causes significantly more consumption and less bequests with high EIS, which means that child income is moreso comprised of labor income than capital income.

	Change from $R_{\text{annual}} = 1.04$ to $R_{\text{annual}} = 1.01$			
Statistic \ EIS	$\frac{1}{\gamma} = 0.10$	$\frac{1}{\gamma} =$ **0.20**	$\frac{1}{\gamma} = 0.50$	$\frac{1}{\gamma} = 1.00$
A. Tilting: $\mathbb{E}[\frac{x}{c} \mid y = y^{\text{quantile}}_{0.90}] - \mathbb{E}[\frac{x}{c} \mid y = y^{\text{quantile}}_{0.10}]$	0.06	**0.05**	0.03	0.02
B. Inequality: $\log(\text{Var}[\log y'])$	0.19	**0.18**	0.15	0.12
C. Mobility: log-log IGE	0.083	**0.077**	0.070	0.053
D. Working-Rich: $\mathbb{E}[\frac{y'}{y'+R_{\text{annual}}a'} \mid y' = (y')^{\text{quantile}}_{0.90}] - \mathbb{E}[\frac{y'}{y'+R_{\text{annual}}a'}]$	0.095	**0.105**	0.141	0.207

Table 3: Comparison of counterfactual experiments under different EIS calibrations (γ^{-1}). The table shows changes in some key statistics for the model with $R_{\text{annual}} = 1.01$ relative to the model with $R_{\text{annual}} = 1.04$. Other parameters are in Table 1. The second column is our benchmark calibration.

7 Conclusion

This paper begins from a simple fact that we call *human capital tilting*: the gap between rich and poor families' educational expenditure shares has increased when expected future financial returns have fallen, and vice versa when those expected returns rose. Human capital tilting emerges naturally in a model with intergenerational bequests of both financial and human capital, provided financial frictions are present. The tilting phenomenon leads to several equilibrium consequences: higher labor income inequality, lower intergenerational mobility, an increased prevalence of the working rich, and rising education prices. All of these have been observed in the data. Whereas they are often treated in isolation, our framework opens the door to connecting financial factors (like asset returns and borrowing constraints) to labor market outcomes (like wages and inequality).

References

Acemoglu, Daron, "Technical change, inequality, and the labor market," *Journal of Economic Literature*, 2002, *40* (1), 7–72.

Asriyan, Vladimir, Luc Laeven, Alberto Martin, Alejandro Van der Ghote, and Victoria Vanasco, "Falling interest rates and credit reallocation: lessons from general equilibrium," *Review of Economic Studies*, 2025, *92* (4), 2197–2227.

Auclert, Adrien and Matthew Rognlie, "Inequality and aggregate demand," 2020. NBER working paper.

Autor, David H, Frank Levy, and Richard J Murnane, "The skill content of recent technological change: An empirical exploration," *The Quarterly Journal of Economics*, 2003, *118* (4), 1279–1333.

Battistin, Erich, Richard Blundell, and Arthur Lewbel, "Why is consumption more log normal than income? Gibrat's law revisited," *Journal of Political Economy*, 2009, *117* (6), 1140–1154.

Becker, Gary S, Scott Duke Kominers, Kevin M Murphy, and Jörg L Spenkuch, "A theory of intergenerational mobility," *Journal of Political Economy*, 2018, *126* (S1), S7–S25.

Benabou, Roland, "Tax and education policy in a heterogeneous-agent economy: What levels of redistribution maximize growth and efficiency?," *Econometrica*, 2002, *70* (2), 481–517.

Benhabib, Jess, Alberto Bisin, and Mi Luo, "Wealth distribution and social mobility in the US: A quantitative approach," *American Economic Review*, 2019, *109* (5), 1623–47.

__ **and** __ , "Skewed wealth distributions: Theory and empirics," *Journal of Economic Literature*, 2018, *56* (4), 1261–91.

Blanchet, Thomas, Lucas Chancel, Amory Gethin et al., "How unequal is Europe? Evidence from distributional national accounts, 1980-2017," *American Economic Journal: Applied Economics (forthcoming)*, 2021.

Bowles, Samuel and Herbert Gintis, "The inheritance of inequality," *Journal of Economic Perspectives*, 2002, *16* (3), 3–30.

Caucutt, Elizabeth M and Lance Lochner, "Early and late human capital investments, borrowing constraints, and the family," *Journal of Political Economy*, 2020, *128* (3), 1065–1147.

Chamley, Christophe, "Optimal taxation of capital income in general equilibrium with infinite lives," *Econometrica: Journal of the Econometric Society*, 1986, pp. 607–622.

Charles, Kerwin Kofi, Erik Hurst, and Nikolai Roussanov, "Conspicuous consumption and race," *The Quarterly Journal of Economics*, 2009, *124* (2), 425–467.

Chetty, Raj, David Grusky, Maximilian Hell, Nathaniel Hendren, Robert Manduca, and Jimmy Narang, "The fading American dream: Trends in absolute income mobility since 1940," *Science*, 2017, *356* (6336), 398–406.

_ , Nathaniel Hendren, Patrick Kline, Emmanuel Saez, and Nicholas Turner, "Is the United States still a land of opportunity? Recent trends in intergenerational mobility," *American Economic Review*, 2014, *104* (5), 141–47.

Corak, Miles, "Income inequality, equality of opportunity, and intergenerational mobility," *Journal of Economic Perspectives*, 2013, *27* (3), 79–102.

Cunha, Flavio and James Heckman, "The technology of skill formation," *American Economic Review*, 2007, *97* (2), 31–47.

Dávila, Eduardo and Andreas Schaab, "Welfare assessments with heterogeneous individuals," *Journal of Political Economy*, 2025, *133* (9), 2918–2961.

Davis, Jonathan and Bhashkar Mazumder, "The decline in intergenerational mobility after 1980," 2017. FRB of Chicago Working Paper No. WP-2017-5.

Eisfeldt, Andrea L, Antonio Falato, and Mindy Z Xiaolan, "Human capitalists," 2021. NBER working paper.

Fagereng, Andreas, Luigi Guiso, Davide Malacrino, and Luigi Pistaferri, "Heterogeneity and persistence in returns to wealth," *Econometrica*, 2020, *88* (1), 115–170.

_ , Martin Blomhoff Holm, Benjamin Moll, and Gisle Natvik, "Saving Behavior Across the Wealth Distribution: The Importance of Capital Gains," 2021. Unpublished working paper.

Friedman, Milton, *A theory of the consumption function*, Princeton University Press, 1957.

Gabaix, Xavier, Jean-Michel Lasry, Pierre-Louis Lions, and Benjamin Moll, "The dynamics of inequality," *Econometrica*, 2016, *84* (6), 2071–2111.

Gomez, Matthieu, "Wealth inequality and asset prices," *Review of Economic Studies*, 2025, *92* (6), 3924–3967.

_ and Émilien Gouin-Bonenfant, "Wealth inequality in a low rate environment," *Econometrica*, 2024, *92* (1), 201–246.

Gopinath, Gita, Şebnem Kalemli-Özcan, Loukas Karabarbounis, and Carolina Villegas-Sanchez, "Capital allocation and productivity in South Europe," *The Quarterly Journal of Economics*, 2017, *132* (4), 1915–1967.

Greenwald, Daniel L, Matteo Leombroni, Hanno Lustig, and Stijn Van Nieuwerburgh, "Financial and Total Wealth Inequality with Declining Interest Rates," Technical Report, National Bureau of Economic Research 2021.

Hai, Rong and James J Heckman, "Inequality in human capital and endogenous credit constraints," *Review of Economic Dynamics*, 2017, *25*, 4–36.

Heathcote, Jonathan, Kjetil Storesletten, and Giovanni L Violante, "Optimal tax progressivity: An analytical framework," *The Quarterly Journal of Economics*, 2017, *132* (4), 1693–1754.

Heckman, James J, Lance Lochner, and Christopher Taber, "Tax Policy and Human-Capital Formation," *American Economic Review Papers and Proceedings*, 1998, *88* (2), 293–297.

_ , _ , and _ , "Human capital formation and general equilibrium treatment effects: a study of tax and tuition policy," *Fiscal Studies*, 1999, *20* (1), 25–40.

Holston, Kathryn, Thomas Laubach, and John C Williams, "Measuring the natural rate of interest: International trends and determinants," *Journal of International Economics*, 2017, *108*, S59–S75.

Hubmer, Joachim, Per Krusell, and Anthony A Smith Jr, "Sources of US wealth inequality: Past, present, and future," *NBER Macroeconomics Annual*, 2021, *35* (1), 391–455.

Jeenas, Priit, "Firm balance sheet liquidity, monetary policy shocks, and investment dynamics," 2019. Unpublished working paper.

Jones, Charles I, "Pareto and Piketty: The Macroeconomics of Top Income and Wealth Inequality," *The Journal of Economic Perspectives*, 2015, *29* (1), 29–46.

_ and Jihee Kim, "A Schumpeterian model of top income inequality," *Journal of Political Economy*, 2018, *126* (5), 1785–1826.

Jordà, Òscar, Katharina Knoll, Dmitry Kuvshinov, Moritz Schularick, and Alan M Taylor, "The rate of return on everything, 1870–2015," *The Quarterly Journal of Economics*, 2019, *134* (3), 1225–1298.

Judd, Kenneth L, "Redistributive taxation in a simple perfect foresight model," *Journal of Public Economics*, 1985, *28* (1), 59–83.

Kaplan, Steven N and Joshua Rauh, "It's the market: The broad-based rise in the return to top talent," *Journal of Economic Perspectives*, 2013, *27* (3), 35–56.

Katz, Lawrence F and David Autor, "Handbook of Labor Economics," *Handbook of Labor Economics*, 1999, *3*.

__ **and Kevin M Murphy**, "Changes in relative wages, 1963–1987: supply and demand factors," *The Quarterly Journal of Economics*, 1992, *107* (1), 35–78.

Kornrich, Sabino and Frank Furstenberg, "Investing in children: Changes in parental spending on children, 1972–2007," *Demography*, 2013, *50* (1), 1–23.

Krueger, Dirk and Alexander Ludwig, "Optimal progressive labor income taxation and education subsidies when education decisions and intergenerational transfers are endogenous," *American Economic Review Papers and Proceedings*, 2013, *103* (3), 496–501.

Krusell, Per, Lee E Ohanian, José-Víctor Ríos-Rull, and Giovanni L Violante, "Capital-skill complementarity and inequality: A macroeconomic analysis," *Econometrica*, 2000, *68* (5), 1029–1053.

Laubach, Thomas and John C Williams, "Measuring the natural rate of interest," *Review of Economics and Statistics*, 2003, *85* (4), 1063–1070.

Liu, Ernest, Atif Mian, and Amir Sufi, "Low interest rates, market power, and productivity growth," *Econometrica*, 2022, *90* (1), 193–221.

Lochner, Lance J and Alexander Monge-Naranjo, "The nature of credit constraints and human capital," *American Economic Review*, 2011, *101* (6), 2487–2529.

Mian, Atif, Ludwig Straub, and Amir Sufi, "Indebted demand," *The Quarterly Journal of Economics*, 2021, *136* (4), 2243–2307.

Modigliani, Franco and Richard Brumberg, "Utility analysis and the consumption function: An interpretation of cross-section data," *Franco Modigliani*, 1954, *1* (1), 388–436.

Murphy, Kevin M and Robert H Topel, "Human capital investment, inequality, and economic growth," *Journal of Labor Economics*, 2016, *34* (S2), S99–S127.

Ottonello, Pablo and Thomas Winberry, "Financial heterogeneity and the investment channel of monetary policy," *Econometrica*, 2020, *88* (6), 2473–2502.

Piketty, Thomas and Emmanuel Saez, "Income Inequality in the United States, 1913-1998.," *Quarterly Journal of Economics*, 2003, *118* (1).

__ **and** __ , "The evolution of top incomes: a historical and international perspective," *American Economic Review*, 2006, *96* (2), 200–205.

__ **,** __ **, and Gabriel Zucman**, "Distributional national accounts: methods and estimates for the United States," *The Quarterly Journal of Economics*, 2018, *133* (2), 553–609.

Ramey, Garey and Valerie A Ramey, "The Rug Rat Race," *Brookings Papers on Economic Activity*, 2010, *2010* (1), 129–176.

Saez, Emmanuel and Gabriel Zucman, "Wealth inequality in the United States since 1913: Evidence from capitalized income tax data," *The Quarterly Journal of Economics*, 2016, *131* (2), 519–578.

Schneider, Daniel, Orestes P Hastings, and Joe LaBriola, "Income inequality and class divides in parental investments," *American Sociological Review*, 2018, *83* (3), 475–507.

Shiller, Robert, "Irrational exuberance," *Princeton, New Jersey, Princeton University*, 2000.

Smith, Matthew, Danny Yagan, Owen Zidar, and Eric Zwick, "Capitalists in the Twenty-first Century," *The Quarterly Journal of Economics*, 2019, *134* (4), 1675–1745.

Stantcheva, Stefanie, "Optimal taxation and human capital policies over the life cycle," *Journal of Political Economy*, 2017, *125* (6), 1931–1990.

Online Appendix to Human Capital Tilting: From Financial Returns to the Wage Distribution*

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*The views expressed in this paper are those of the authors and do not reflect those of the Federal Reserve Bank of Chicago or the Federal Reserve System.

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A Proofs

Proof of Lemma 1. Takes logs of (16) and differentiate as stated. □

Proof of Lemma 2. Substitute $\underline{a} = 0$, $\gamma = 1$ into (16), (17), (20), and (21). To demonstrate the claim about underinvestment, $x^* > x_{\mathcal{B}}$, we first use the formulas for x^* and $x_{\mathcal{B}}$ to show that

$$x^* > x_{\mathcal{B}} \Leftrightarrow n < \frac{1+\alpha\beta}{\alpha\beta}\left(\frac{\alpha\tilde{w}'}{R'}\right)^{\frac{1}{1-\alpha}}\left(\frac{1}{p}\right)^{\frac{\alpha}{1-\alpha}}.$$

At the same time, a constrained parent in $\text{int}(\mathcal{B})$ has $c^* + px^* > n$. Using the formulas for c^* and x^*, this is equivalent to

$$c^* + px^* > n \Leftrightarrow n < \frac{1+\alpha\beta}{\alpha\beta}\left(\frac{\alpha\tilde{w}'}{R'}\right)^{\frac{1}{1-\alpha}}\left(\frac{1}{p}\right)^{\frac{\alpha}{1-\alpha}}.$$

Thus, a constrained parent has $x^* > x_{\mathcal{B}}$. □

Proof of Lemma 3. Taking logs of (22) and differentiating leads to the result for w.

For the derivative of p, first calculate the derivatives of x from Lemma 2. Then, substitute these derivatives, as well as $\frac{d\log w'}{d\log R'} = -\frac{\rho}{1-\rho}$, into equation (23). □

Proof of Proposition 1. Compute the total derivatives of x in Lemma 2 as

$$\begin{aligned}\frac{d\log x^*}{d\log R'} &= -\frac{1}{1-\alpha}\left(1 + \frac{d\log p}{d\log R'} - \frac{d\log w'}{d\log R'}\right)\\ \frac{d\log x_{\mathcal{B}}}{d\log R'} &= -\frac{d\log p}{d\log R'}.\end{aligned}$$

Substituting the responses of w' and p from Lemma 3, and then simplifying, we obtain the final result. □

Lemma A.1. *Define the fraction of borrowing-constrained parents $\varphi := \mathbb{E}[\mathbf{1}_{\mathcal{B}}]$, the following between-group difference of an arbitrary variable v,*

$$\Delta_v := \mathbb{E}[\log(v) \mid \mathcal{U}] - \mathbb{E}[\log(v) \mid \mathcal{B}], \tag{A.1}$$

and the within-group regression coefficients $\beta^{\mathcal{G}}_{v_1,v_2} := \frac{Cov[\log(v_1),\log(v_2)|\mathcal{G}]}{Var[\log(v_2)|\mathcal{G}]}$*, for group* $\mathcal{G} \in \{\mathcal{B},\mathcal{U}\}$*. The inequality and mobility measures obey the following identities:*

$$\mathcal{I} = (1-\varphi)Var[\log(y') \mid \mathcal{U}] + \varphi Var[\log(y') \mid \mathcal{B}] + \varphi(1-\varphi)\Delta_{y'}$$
$$\mathcal{M} = \Big((1-\varphi)\frac{Var[\log(y) \mid \mathcal{U}]}{Var[\log(y)]}\Big)\beta^{\mathcal{U}}_{y',y} + \Big(\varphi\frac{Var[\log(y) \mid \mathcal{B}]}{Var[\log(y)]}\Big)\beta^{\mathcal{B}}_{y',y} + \Big(\varphi(1-\varphi)\frac{(\Delta_y)^2}{Var[\log(y)]}\Big)\frac{\Delta_{y'}}{\Delta_y}.$$

Proof of Lemma A.1. First, we do a general covariance decomposition with two groups $\mathcal{G}_1$ and $\mathcal{G}_2$ and two variables X and Y. We let $\text{Cov}_j[X,Y] := \text{Cov}[X,Y \mid \mathcal{G}_j]$ and $\mathbb{E}_j[X] := \mathbb{E}[X \mid \mathcal{G}_j]$ denote the conditional covariance and mean for group j, and $q_j := \mathbb{E}[\mathbf{1}_{\mathcal{G}_j}]$ denote the population share (probability) of group j (note $q_1 + q_2 = 1$). Then,

$$\begin{aligned}
\text{Cov}[X,Y] &= q_1\mathbb{E}_1[XY] + q_2\mathbb{E}_2[XY] - (q_1\mathbb{E}_1[X] + q_2\mathbb{E}_2[X])(q_1\mathbb{E}_1[Y] + q_2\mathbb{E}_2[Y]) \\
&= q_1\text{Cov}_1[X,Y] + q_2\text{Cov}_2[X,Y] + q_1\mathbb{E}_1[X]\mathbb{E}_1[Y] + q_2\mathbb{E}_2[X]\mathbb{E}_2[Y] \\
&\quad - (q_1\mathbb{E}_1[X] + q_2\mathbb{E}_2[X])(q_1\mathbb{E}_1[Y] + q_2\mathbb{E}_2[Y]) \\
&= q_1\text{Cov}_1[X,Y] + q_2\text{Cov}_2[X,Y] + q_1q_2(\mathbb{E}_1[X] - \mathbb{E}_2[X])(\mathbb{E}_1[Y] - \mathbb{E}_2[Y]).
\end{aligned}$$

As a special case, we have $\text{Var}[X] = q_1\text{Var}_1[X] + q_2\text{Var}_2[X] + q_1q_2(\mathbb{E}_1[X] - \mathbb{E}_1[X])^2$. Applying this variance result with X set to $\log(y')$ and $\mathcal{G}_1 = \mathcal{U}$ and $\mathcal{G}_2 = \mathcal{B}$ delivers the decomposition for $\mathcal{I}$. Then, the regression betas are obtained by doing some basic manipulations to obtain

$$\frac{\text{Cov}[X,Y]}{\text{Var}[X]} = \Big(q_1\frac{\text{Var}_1[X]}{\text{Var}[X]}\Big)\beta_1 + \Big(q_2\frac{\text{Var}_2[X]}{\text{Var}[X]}\Big)\beta_2 + \Big(q_1q_2\frac{(\mathbb{E}_1[X] - \mathbb{E}_2[X])^2}{\text{Var}[X]}\Big)\frac{\mathbb{E}_1[Y] - \mathbb{E}_2[Y]}{\mathbb{E}_1[X] - \mathbb{E}_2[X]},$$

where $\beta_j := \frac{\text{Cov}_j[X,Y]}{\text{Var}_j[X]}$ is the within-group regression coefficient. One can verify, through straightforward calculation, that the weights in the β decomposition (i.e., the objects in the large parentheses) sum up to 1. Applying this β decomposition to $X = \log(y)$ and $Y = \log(y')$ yields the result on $\mathcal{M}$. □

Proof of Proposition 2. As stated in the proposition, the entire proof will hold fixed the set $\mathcal{B}$ (and consequently its complement $\mathcal{U}$). In terms of the notation of Lemma A.1, we may hold fixed φ.

Note that any child's labor income $y' = w'\varepsilon'z'\theta(h)g(x)$ can be written in log form as

$$\log(y') = \log w' + \log\varepsilon' + \log z' + \log\theta(h) + \alpha\log x. \tag{A.2}$$

There is only one endogenous object which is both responsive to R' and has cross-sectional heterogeneity, namely x. Using Lemma 2, we see that the determinants of x^*

and $x_{\mathcal{B}}$ feature no variables which are again both responsive to R' and possess cross-sectional heterogeneity. Thus, we have

$$\frac{d}{dR'}\overline{\text{Var}}[\log(y') \mid \mathcal{U}] = \alpha \frac{d}{dR'}\overline{\text{Var}}[\log(x^*) \mid \mathcal{U}] = 0$$
$$\frac{d}{dR'}\overline{\text{Var}}[\log(y') \mid \mathcal{B}] = \alpha \frac{d}{dR'}\overline{\text{Var}}[\log(x_{\mathcal{B}}) \mid \mathcal{B}] = 0.$$

On the other hand, we calculate the between-group difference $\Delta_{y'}$ that is defined above by (A.1). Using (A.2) and the expressions in Proposition 1, this between-group difference is equal to

$$\begin{aligned}\frac{d}{dR'}\Delta_{y'} &= \alpha \frac{d}{dR'}\Big\{\overline{\mathbb{E}}[\log(x^*) \mid \mathcal{U}] - \overline{\mathbb{E}}[\log(x_{\mathcal{B}}) \mid \mathcal{B}]\Big\} \\ &= \alpha \underbrace{\frac{d}{dR'}\big\{\log(x^*) - \log(x_{\mathcal{B}})\big\}}_{=-\chi/R'} = -\frac{1}{R'}\frac{\alpha}{1-\rho}\Big(\frac{1-\kappa}{1-\alpha} + \kappa\Big),\end{aligned}$$

where recall $\kappa \in (0,1)$ is defined in Proposition 1. Thus, using the decomposition of Lemma A.1 and holding fixed $\mathcal{B}$ (hence also φ), we have

$$\frac{d}{dR'}\mathcal{I} = \varphi(1-\varphi)\frac{d}{dR'}\Delta_{y'} < 0.$$

In this process, notice that we have established the mapping between χ and $\frac{d}{dR'}\mathcal{I}$. □

Proof of Proposition 3. As stated in the proposition, the entire proof will hold fixed the set $\mathcal{B}$ (and consequently its complement $\mathcal{U}$). In terms of the notation of Lemma A.1, we may hold fixed φ.

Again use equation (A.2), and the similar formula for parental income $\log(y) = \log(w) + \log(\varepsilon) + \log(z) + \log(h)$, to compute

$$\frac{d}{dR'}\beta^{\mathcal{U}}_{y',y} \propto \frac{d}{dR'}\overline{\text{Cov}}[\log(y'), \log(y) \mid \mathcal{U}] = \alpha \frac{d}{dR'}\overline{\text{Cov}}[\log(x^*), \log(y) \mid \mathcal{U}] = 0$$
$$\frac{d}{dR'}\beta^{\mathcal{B}}_{y',y} \propto \frac{d}{dR'}\overline{\text{Cov}}[\log(y'), \log(y) \mid \mathcal{B}] = \alpha \frac{d}{dR'}\overline{\text{Cov}}[\log(x_{\mathcal{B}}), \log(y) \mid \mathcal{B}] = 0.$$

Using the decomposition of Lemma A.1 and holding fixed $\mathcal{B}$ (hence also φ), we obtain

$$\frac{d}{dR'}\mathcal{M} = \varphi(1-\varphi)\frac{\Delta_y}{\overline{\text{Var}}[\log(y)]}\frac{d}{dR'}\Delta_{y'} = \frac{\Delta_y}{\overline{\text{Var}}[\log(y)]}\Big(\frac{d}{dR'}\mathcal{I}\Big) < 0.$$

In this process, notice that we have established the mapping between χ and $\frac{d}{dR'}\mathcal{M}$. □

Proof of Proposition 4. Define the pre-tax labor share for any individual:

$$\ell := \frac{y'}{y' + R'a'} = \frac{w'\varepsilon' z'\theta(h)g(x)}{w'\varepsilon' z'\theta(h)g(x) + R'[n - c - px]}.$$

Differentiating with respect to R', we obtain the general formula

$$\frac{d\log \ell}{d\log R'} = (1-\ell)\Big[-1 + \frac{d\log w'}{d\log R'} + \frac{px}{a'}\frac{d\log p}{d\log R'} + \frac{c}{a'}\frac{d\log c}{d\log R'} + \Big(\alpha + \frac{px}{a'}\Big)\frac{d\log x}{d\log R'}\Big] \quad \text{(A.3)}$$

First, we consider a borrowing-constrained parent. Since $\underline{a} = 0$, this parent plans to leave $a' = 0$, which implies $\ell = 1$. Hence, expression (A.3) becomes

$$\frac{d\log \ell}{d\log R'}\Big|_{\mathcal{B}} = \Big(\frac{R'px_{\mathcal{B}}}{y'}\Big)\frac{d\log p}{d\log R'} + \Big(\frac{R'c_{\mathcal{B}}}{y'}\Big)\frac{d\log c_{\mathcal{B}}}{d\log R'} + \Big(\frac{R'px_{\mathcal{B}}}{y'}\Big)\frac{d\log x_{\mathcal{B}}}{d\log R'}.$$

Using Lemma 2, we have $\frac{d\log c_{\mathcal{B}}}{d\log R'} = 0$. Using Lemma 3 and Proposition 1, we have $\frac{d\log x_{\mathcal{B}}}{d\log R'} + \frac{d\log p}{d\log R'} = 0$. Thus,

$$\frac{d\log \ell}{d\log R'}\Big|_{\mathcal{B}} = 0$$

Next, we consider an unconstrained parent. We may directly compute, using the expression for c^* in Lemma 2, that

$$\frac{d\log c^*}{d\log R'} = \frac{1}{c^*}\frac{1-\alpha}{\alpha}\frac{px^*}{1+\beta}\Big[\frac{d\log x^*}{d\log R'} + \frac{d\log p}{d\log R'}\Big] \quad \text{(A.4)}$$

Substituting this into (A.3), we have

$$\begin{aligned}\frac{d\log \ell}{d\log R'}\Big|_{\mathcal{U}} &= (1-\ell)\Big(\frac{d\log w'}{d\log R'} - 1\Big) \\ &\quad + (1-\ell)\frac{px^*}{a'}\frac{1-\alpha+\alpha(1+\beta)}{\alpha(1+\beta)}\Big[\frac{d\log p}{d\log R'} + \frac{d\log x^*}{d\log R'}\Big] + (1-\ell)\alpha\frac{d\log x^*}{d\log R'}\end{aligned}$$

Using Lemma 3 and Proposition 1, we have that $\frac{d\log w'}{d\log R'} < 0$, $\frac{d\log p}{d\log R'} < 0$, and $\frac{d\log x^*}{d\log R'} < 0$. Therefore,

$$\frac{d\log \ell}{d\log R'}\Big|_{\mathcal{U}} < 0.$$

We may also take the limit $a' \to 0$ (hence, $\ell \to 1$) in the expression for $\frac{d\log \ell}{d\log R'}\big|_{\mathcal{U}}$ to get

$$\lim_{a'\to 0} \frac{d\log \ell}{d\log R'}\Big|_{\mathcal{U}} = \overbrace{\Big(\frac{R'px^*}{y'}\Big)}^{=\alpha} \frac{1+\alpha\beta}{\alpha(1+\beta)} \overbrace{\Big[\frac{d\log p}{d\log R'} + \frac{d\log x^*}{d\log R'}\Big]}^{=-\chi \text{ (tilting)}}$$

On the other hand, also take $a' \to \infty$ (hence, $\ell \to 0$) in the expression for $\frac{d\log \ell}{d\log R'}\big|_{\mathcal{U}}$ to get

$$\lim_{a'\to \infty} \frac{d\log \ell}{d\log R'}\Big|_{\mathcal{U}} = \Big(\frac{d\log w'}{d\log R'} - 1 + \alpha\frac{d\log x^*}{d\log R'}\Big)$$

Finally, we apply these calculations to the quantity in question. Note that as child assets increase without bound ($a' \to \infty$), the probability that his/her parent is unconstrained rises to 1. By contrast, as child assets decrease to zero ($a' \to \underline{a} = 0$), the probability his/her parent is constraint falls rise to 1. Thus, we obtain after interchanging the order of limits,

$$\frac{d}{d\log R'}\mathcal{L} = \lim_{a'\to\infty} \overline{\mathbb{E}}\Big[\frac{d\log \ell}{d\log R'}\Big] - \lim_{a'\to 0} \overline{\mathbb{E}}\Big[\frac{d\log \ell}{d\log R'}\Big] = \lim_{a'\to\infty} \underbrace{\overline{\mathbb{E}}\Big[\frac{d\log \ell}{d\log R'}\Big|_{\mathcal{U}}\Big]}_{<0 \text{ for any } a'} - \underbrace{\overline{\mathbb{E}}\Big[\frac{d\log \ell}{d\log R'}\Big|_{\mathcal{B}}\Big]}_{=0} < 0,$$

which proves the first result. In addition, since $\lim_{a'\to 0} \frac{d\log \ell}{d\log R'}|_{\mathcal{U}} = -\frac{1+\alpha\beta}{1+\beta}\chi$, we have that

$$\frac{d}{d\log R'}\mathcal{L} - \frac{d}{d\log R'}\mathcal{L}^{alt} = \lim_{a'\to 0} \overline{\mathbb{E}}\Big[\frac{d\log \ell}{d\log R'}\Big|_{\mathcal{U}}\Big] - \lim_{a'\to 0} \overline{\mathbb{E}}\Big[\frac{d\log \ell}{d\log R'}\Big] = -\frac{1+\alpha\beta}{1+\beta}\chi$$

as desired. □

Proof of Proposition 5. Compute the following, noting that the distributions of (a, z, h, ε) are unaffected by R':

$$\begin{aligned}\frac{d}{dR'}\overline{\mathbb{E}}[\mathbf{1}_{\mathcal{B}}] &= \overline{\mathbb{E}}[\frac{d}{dR'}\mathbf{1}_{\mathcal{B}}] \\ &= \overline{\mathbb{E}}\Big[\Big(\frac{d}{d(c^*+px^*)}\mathbf{1}_{\mathcal{B}}\Big)\frac{d(c^*+px^*)}{dR'}\Big] = \frac{d(c^*+px^*)}{dR'}\Big|_{\partial\mathcal{B}}\end{aligned}$$

using the fact that the "derivative" of the indicator function $\mathbf{1}_{\mathcal{B}}$ is the Dirac delta function evaluated at the boundary of the constraint set, i.e., $\partial\mathcal{B} = \{c^* + px^* = n\}$. Next, use

Lemma 2 to compute

$$\frac{d(c^* + px^*)}{d\log R'} = \frac{1-\alpha+\alpha(1+\beta)}{\alpha(1+\beta)} px^* \left[\frac{d\log x^*}{d\log R'} + \frac{d\log p}{d\log R'}\right]$$

Finally, we use the expressions in Lemma 3 and Proposition 1 to obtain

$$\frac{d(c^* + px^*)}{d\log R'} = -\frac{1+\alpha\beta}{\alpha(1+\beta)} \frac{px^*}{1-\rho} \frac{1-\alpha\kappa}{1-\alpha}$$

Since this is negative, this proves that $\frac{d}{dR'}\overline{\mathbb{E}}[\mathbf{1}_{\mathcal{B}}] < 0$. □

Proof of Proposition 6. Taking the cross-sectional average over labor market luck ε', we may write the labor income measure as:

$$\mathcal{Y}' = \int w'z'h'dM,$$

where recall that $h' = \theta(h)g(x) = \theta(h)x^{\alpha}$. Differentiating with respect to R', we have

$$\frac{d\mathcal{Y}'}{d\log R'} = \int w'z'h' \left[\frac{d\log w'}{d\log R'} + \alpha\frac{d\log x}{d\log R'}\right] dM$$

Now, split the integral into constrained and unconstrained parents, and use the fact that $\frac{d\log x^*}{d\log R'}$ is only a function of aggregate states to obtain

$$\frac{d\mathcal{Y}'}{d\log R'} = \mathcal{Y}'\frac{d\log w'}{d\log R'} + \alpha\mathcal{Y}'\frac{d\log x^*}{d\log R'} + \alpha\int_{\mathcal{B}} w'z'h'\chi dM,$$

which directly implies equation (26).

Now, consider the NBL economy ($\underline{a} = 0$, $\gamma = 1$) in which χ is a constant. Compare an identical economy without frictions ($\underline{a} = -\infty$). We have

$$\begin{aligned}
\frac{d\log \mathcal{Y}'}{d\log R'} &= \frac{d\log w'}{d\log R'} + \alpha\left[\frac{d\log x^*}{d\log R'} + \chi\frac{\overline{\mathbb{E}}[\mathbf{1}_{\mathcal{B}}y']}{\mathcal{Y}}\right] \\
\frac{d\log \mathcal{Y}'_{FB}}{d\log R'} &= \frac{d\log w'}{d\log R'} + \alpha\frac{d\log x^*_{FB}}{d\log R'} \\
\Rightarrow \frac{d\log \mathcal{Y}'}{d\log R'} - \frac{d\log \mathcal{Y}'_{FB}}{d\log R'} &= \alpha\left[\frac{d\log x^*}{d\log R'} - \frac{d\log x^*_{FB}}{d\log R'} + \chi\frac{\overline{\mathbb{E}}[\mathbf{1}_{\mathcal{B}}y']}{\mathcal{Y}}\right]
\end{aligned}$$

where $x^*_{FB} = (\alpha w'_{FB}z'\theta(h)/R'p_{FB})^{1/(1-\alpha)}$ is the unconstrained investment in the frictionless economy. Note that x^*_{FB} and x^* are identical functions of prices, and furthermore

that $w' = w'_{FB}$. Therefore,

$$\frac{d\log x^*}{d\log R'} - \frac{d\log x^*_{FB}}{d\log R'} = \frac{\partial \log x^*}{\partial \log p}\frac{d\log p}{d\log R'} - \frac{\partial \log x^*_{FB}}{\partial \log p_{FB}}\frac{d\log p_{FB}}{d\log R'}.$$

Thus, if we fix $\frac{d\log p}{d\log R'} = \frac{d\log p_{FB}}{d\log R'}$, we obtain the result that $\frac{d\log \mathcal{Y}'}{d\log R'} - \frac{d\log \mathcal{Y}'_{FB}}{d\log R'} > 0$ if and only if $\chi > 0$. □

Proof of Proposition 7. By the envelope theorem, and using the fact that the Lagrange multiplier on the borrowing constraint has been defined as $\beta\lambda u'(\mathbb{E}[n'])$, we have that

$$\begin{aligned}\frac{\delta}{\delta \log R'}\mathcal{W} &= \int \Big[R'a' + w'z'h'\frac{d\log w'}{d\log R'} - (R'+\lambda)px\frac{d\log p}{d\log R'}\Big]dM \\ &= \int \Big[R'a' + w'z'h'\frac{d\log w'}{d\log R'} - \alpha w'z'h'\frac{d\log p}{d\log R'}\Big]dM \\ &= \mathcal{K}' + \mathcal{Y}'\Big(\frac{d\log w'}{d\log R'} - \alpha\frac{d\log p}{d\log R'}\Big)\end{aligned}$$

where the second line uses the FOC (15) to substitute λ, and the third line defines total capital income $\mathcal{K}' := R'\overline{\mathbb{E}}[a'] = \overline{\mathbb{E}}[n' - y']$. Repeating the same algebra in the first-best economy, we have

$$\frac{\delta}{\delta \log R'}\mathcal{W}_{FB} = \mathcal{K}'_{FB} + \mathcal{Y}'_{FB}\Big(\frac{d\log w'_{FB}}{d\log R'} - \alpha\frac{d\log p_{FB}}{d\log R'}\Big)$$

Since $w' = w'_{FB}$, we have

$$\begin{aligned}\frac{\delta\mathcal{W}}{\delta \log R'} - \frac{\delta\mathcal{W}_{FB}}{\delta \log R'} = \mathcal{K}' - \mathcal{K}'_{FB} + (\mathcal{Y}' - \mathcal{Y}'_{FB})\Big(\frac{d\log w'}{d\log R'} - \alpha\frac{d\log p}{d\log R'}\Big) \\ + \alpha\mathcal{Y}'_{FB}\Big(\frac{d\log p_{FB}}{d\log R'} - \frac{d\log p}{d\log R'}\Big).\end{aligned}$$

Finally, write the expressions for the wedges between labor and capital income:

$$\begin{aligned}\mathcal{Y}'_{FB} - \mathcal{Y}' &= \int \tilde{w}'\Big[(x^*_{FB})^\alpha - (x^*)^\alpha\Big]dM + \int_{\mathcal{B}} \tilde{w}'\Big[(x^*)^\alpha - x_{\mathcal{B}}^\alpha\Big]dM \\ \mathcal{K}'_{FB} - \mathcal{K}' &= R'\Big(\int [c^* + px^* - c^*_{FB} - p_{FB}x^*_{FB}]dM + \int_{\mathcal{B}}[c_{\mathcal{B}} + px_{\mathcal{B}} - c^* - px^*]dM\Big).\end{aligned}$$

These are the general expressions we use for the proof.

Now, let us consider the NBL economy ($\underline{a} = 0$, $\gamma = 1$). Furthermore, we take the limit $\eta \to 1$, so that $p_{FB} - p \to 0$, which then implies that $x^*_{FB} - x^* \to 0$ and $c^*_{FB} - c^* \to 0$. In

that case, since $x^* > x_{\mathcal{B}}$ by Lemma 2, we have

$$\mathcal{Y}'_{FB} - \mathcal{Y}' > \int \tilde{w}' \Big[(x^*_{FB})^\alpha - (x^*)^\alpha \Big] dM \underset{\eta \to 1}{\longrightarrow} 0$$

Similarly, since the definition of a constrained parent is $c^* + px^* > c_{\mathcal{B}} + px_{\mathcal{B}}$, we have

$$\mathcal{K}'_{FB} - \mathcal{K}' < R' \int [c^* + px^* - c^*_{FB} - p_{FB} x^*_{FB}] dM \underset{\eta \to 1}{\longrightarrow} 0$$

Therefore, substituting $\frac{d \log w'}{d \log R'} = -\frac{\rho}{1-\rho}$ and $\frac{d \log p}{d \log R'} = 0$ (since $\eta \to 1$),

$$\lim_{\eta \to 1} \left(\frac{\delta \mathcal{W}}{\delta \log R'} - \frac{\delta \mathcal{W}_{FB}}{\delta \log R'} \right) = \underbrace{\lim_{\eta \to 1} (\mathcal{K}' - \mathcal{K}'_{FB})}_{>0} - \frac{\rho}{1-\rho} \underbrace{\lim_{\eta \to 1} (\mathcal{Y}' - \mathcal{Y}'_{FB})}_{<0} > 0.$$

This proves that a reduction in R' raises first-best (money-metric) welfare moreso than equilibrium (money-metric) welfare. □

Proof of Lemma 4. Identical to Lemma 2. □

Proof of Lemma 5. The fact that R' and w' are independent of τ_k is discussed in the text, since R' is exogenous and w' obeys equation (22).

Next, we start from the expression for $\frac{d \log p}{d \tau_k}$ in the text, substitute $\frac{d \log w'}{d \tau_k} = \frac{d \log R'}{d \tau_k} = 0$, and take the derivatives of $(1 - \tau_k)$ and $(1 - \tau_y)$ to obtain

$$\frac{d \log p}{d \tau_k} = -\left(\frac{\eta}{1-\eta} - \frac{\overline{\mathbb{E}}[x \frac{\partial \log x}{\partial \log p}]}{\overline{\mathbb{E}}[x]} \right)^{-1} \frac{\overline{\mathbb{E}}[x (\frac{\partial \log x}{\partial \log (1-\tau_k) R'} \frac{1}{1-\tau_k} + \frac{\partial \log x}{\partial \log (1-\tau_y) w'} \frac{1}{1-\tau_y} \frac{d \tau_y}{d \tau_k})]}{\overline{\mathbb{E}}[x]} \tag{A.5}$$

In expression (A.5), the derivative $\frac{d \tau_y}{d \tau_k}$ captures the implicit mapping from τ_k to τ_y in (A.6). In particular, we may differentiate (29) to get

$$\frac{d \tau_y}{d \tau_k} = -\frac{\mathcal{K}'}{\mathcal{Y}'} - \frac{\tau_k}{\mathcal{Y}'} \frac{d \mathcal{K}'}{d \tau_k} - \frac{\tau_y}{\mathcal{Y}'} \frac{d \mathcal{Y}'}{d \tau_k} \tag{A.6}$$

Finally, we use the results of Lemma 4 to substitute the partial derivatives of x. Simplifying the expression yields the final result. □

Proof of Proposition 8. By the envelope theorem, using the fact that the Lagrange multiplier on the borrowing constraint has been defined as $\beta\lambda u'(\mathbb{E}[n'])$, and the facts that R' and w' are independent of taxes, we have that

$$\begin{aligned}
\frac{\delta}{\delta\tau_k}\mathcal{W} &= \int \Big[- R'a' - w'z'h'\frac{d\tau_y}{d\tau_k} - ((1-\tau_k)R' + \lambda)px\frac{d\log p}{d\tau_k}\Big]dM \\
&= \int \Big[- R'a' - w'z'h'\frac{d\tau_y}{d\tau_k} - \alpha(1-\tau_y)w'z'h'\frac{d\log p}{d\tau_k}\Big]dM \\
&= -\mathcal{K}' - \mathcal{Y}'\frac{d\tau_y}{d\tau_k} - \alpha(1-\tau_y)\mathcal{Y}'\frac{d\log p}{d\tau_k} \\
&= \tau_k\frac{d\mathcal{K}'}{d\tau_k} + \tau_y\frac{d\mathcal{Y}'}{d\tau_k} - \alpha(1-\tau_y)\mathcal{Y}'\frac{d\log p}{d\tau_k}
\end{aligned}$$

The second line uses the FOC (30) to substitute λ. The third line uses the definitions of aggregate (pre-tax) capital and labor income $\mathcal{K}'$ and $\mathcal{Y}'$. The fourth line uses equation (A.6) to substitute $\frac{d\tau_y}{d\tau_k}$. Repeating the same algebra in the frictionless economy, we have

$$\frac{\delta}{\delta\tau_k}\mathcal{W}_{FB} = \tau_k\frac{d\mathcal{K}'_{FB}}{d\tau_k} + \tau_y\frac{d\mathcal{Y}'_{FB}}{d\tau_k} - \alpha(1-\tau_y)\mathcal{Y}'_{FB}\frac{d\log p_{FB}}{d\tau_k}$$

Using the assumption of the NBL economy ($\gamma = 1$, $\underline{a} = 0$), Lemma 5 and equation (A.6) implies

$$\begin{aligned}
\frac{d\log p}{d\tau_k} &= \Big(1 + \frac{\mathcal{K}'}{\mathcal{Y}'} + \frac{\tau_k}{\mathcal{Y}'}\frac{d\mathcal{K}'}{d\tau_k} + \frac{\tau_y}{\mathcal{Y}'}\frac{d\mathcal{Y}'}{d\tau_k}\Big)\kappa \\
\frac{d\log p_{FB}}{d\tau_k} &= \Big(1 + \frac{\mathcal{K}'_{FB}}{\mathcal{Y}'_{FB}} + \frac{\tau_k}{\mathcal{Y}'_{FB}}\frac{d\mathcal{K}'_{FB}}{d\tau_k} + \frac{\tau_y}{\mathcal{Y}'_{FB}}\frac{d\mathcal{Y}'_{FB}}{d\tau_k}\Big)\kappa_{FB},
\end{aligned}$$

where $\kappa_{FB} = \frac{1-\eta}{1-\alpha\eta} > \kappa$. Substituting these results, we have

$$\begin{aligned}
\frac{\delta}{\delta\tau_k}\mathcal{W} &= \big(1-\alpha(1-\tau_y)\kappa\big)\Big(\tau_k\frac{d\mathcal{K}'}{d\tau_k} + \tau_y\frac{d\mathcal{Y}'}{d\tau_k}\Big) - \alpha(1-\tau_y)\Big(\mathcal{K}' + \mathcal{Y}'\Big)\kappa \\
\frac{\delta}{\delta\tau_k}\mathcal{W}_{FB} &= \big(1-\alpha(1-\tau_y)\kappa_{FB}\big)\Big(\tau_k\frac{d\mathcal{K}'_{FB}}{d\tau_k} + \tau_y\frac{d\mathcal{Y}'_{FB}}{d\tau_k}\Big) - \alpha(1-\tau_y)\Big(\mathcal{K}'_{FB} + \mathcal{Y}'_{FB}\Big)\kappa_{FB}
\end{aligned}$$

We compute the impact of taxes on aggregate capital and labor income. By differentiating their definitions, and using $a' = \underline{a}$ for constrained parents, we have

$$\begin{aligned}
\frac{d\mathcal{K}'}{d\tau_k} &= -R'\int_{\mathcal{U}}\Big[c^*\frac{d\log c^*}{d\tau_k} - px^*\Big(\frac{d\log p}{d\tau_k} + \frac{d\log x^*}{d\tau_k}\Big)\Big]dM \\
\frac{d\mathcal{Y}'}{d\tau_k} &= \alpha\int_{\mathcal{U}} y'\frac{d\log x^*}{d\tau_k}dM + \alpha\int_{\mathcal{B}} y'\frac{d\log x_{\mathcal{B}}}{d\tau_k}dM
\end{aligned}$$

Using the results of Lemma 4 to compute $\frac{d\log c^*}{d\tau_k}$, $\frac{d\log x^*}{d\tau_k}$, and $\frac{d\log x_{\mathcal{B}}}{d\tau_k}$, using the result of Lemma 5 to substitute $\frac{d\log p}{d\tau_k}$, using the relationship $R'px^* = \alpha y'$ for unconstrained agents, and finally using the tax constraint (A.6), we have

$$\frac{d\mathcal{K}'}{d\tau_k} = -\frac{(1+\alpha\beta)(1-\alpha\kappa)}{(1-\alpha)(1+\beta)}\Big(\int_{\mathcal{U}} y' dM\Big)\Big[\frac{1}{1-\tau_k} + \frac{1}{1-\tau_y}\Big(\frac{\mathcal{K}'}{\mathcal{Y}'} + \frac{\tau_k}{\mathcal{Y}'}\frac{d\mathcal{K}'}{d\tau_k} + \frac{\tau_y}{\mathcal{Y}'}\frac{d\mathcal{Y}'}{d\tau_k}\Big)\Big]$$
$$\frac{d\mathcal{Y}'}{d\tau_k} = \alpha\Big(\frac{1-\kappa}{1-\alpha}\int_{\mathcal{U}} y' dM - \kappa\int_{\mathcal{B}} y' dM\Big)\Big[\frac{1}{1-\tau_k} + \frac{1}{1-\tau_y}\Big(\frac{\mathcal{K}'}{\mathcal{Y}'} + \frac{\tau_k}{\mathcal{Y}'}\frac{d\mathcal{K}'}{d\tau_k} + \frac{\tau_y}{\mathcal{Y}'}\frac{d\mathcal{Y}'}{d\tau_k}\Big)\Big]$$

This constitutes two equations and two unknowns. The solution is

$$\frac{d\mathcal{K}'}{d\tau_k} = \frac{b_k}{1-\tau_k}\left(\frac{(1-\tau_y)\mathcal{Y}' + (1-\tau_k)\mathcal{K}'}{(1-\tau_y)\mathcal{Y}' - \tau_y b_y - \tau_k b_k}\right)$$
$$\frac{d\mathcal{Y}'}{d\tau_k} = \frac{b_y}{1-\tau_k}\left(\frac{(1-\tau_y)\mathcal{Y}' + (1-\tau_k)\mathcal{K}'}{(1-\tau_y)\mathcal{Y}' - \tau_y b_y - \tau_k b_k}\right)$$

where

$$b_k := -\frac{(1+\alpha\beta)(1-\alpha\kappa)}{(1-\alpha)(1+\beta)}\int_{\mathcal{U}} y' dM$$
$$b_y := \alpha\Big(\frac{1-\kappa}{1-\alpha}\int_{\mathcal{U}} y' dM - \kappa\int_{\mathcal{B}} y' dM\Big)$$

We use these results to compute the optimal tax, which sets $\frac{\delta\mathcal{W}}{\delta\tau_k} = 0$:

$$\frac{\alpha(1-\tau_y)\kappa}{1-\alpha(1-\tau_y)\kappa} = \frac{1}{1-\tau_k}\Big(\frac{\tau_k b_k + \tau_y b_y}{\mathcal{K}' + \mathcal{Y}'}\Big)\left(\frac{(1-\tau_y)\mathcal{Y}' + (1-\tau_k)\mathcal{K}'}{(1-\tau_y)\mathcal{Y}' - \tau_y b_y - \tau_k b_k}\right)$$

Using the government budget constraint (29) to eliminate τ_y, and then simplifying, we obtain

$$\frac{\alpha\big(1+\tau_k\frac{\mathcal{K}'}{\mathcal{Y}'} - \frac{G}{\mathcal{Y}'}\big)\kappa}{1-\alpha\big(1+\tau_k\frac{\mathcal{K}'}{\mathcal{Y}'} - \frac{G}{\mathcal{Y}'}\big)\kappa} = \frac{1}{1-\tau_k}\left(\frac{1+\frac{\mathcal{K}'}{\mathcal{Y}'} - \frac{G}{\mathcal{Y}'}}{1+\frac{\mathcal{K}'}{\mathcal{Y}'}}\right)\left(\frac{\tau_k\frac{\mathcal{K}'}{\mathcal{Y}'}\big(\frac{b_k}{\mathcal{K}'} - \frac{b_y}{\mathcal{Y}'}\big) + \frac{G}{\mathcal{Y}'}\frac{b_y}{\mathcal{Y}'}}{1+\tau_k\frac{\mathcal{K}'}{\mathcal{Y}'}\big(1-\frac{b_k}{\mathcal{K}'}+\frac{b_y}{\mathcal{Y}'}\big) - \frac{G}{\mathcal{Y}'}\big(1+\frac{b_y}{\mathcal{Y}'}\big)}\right) \tag{A.7}$$

This equation is a single equation in the unknown optimal tax τ_k, which can be computed given the distribution M.

We now compare these expressions locally at $\tau_k = \tau_y = 0$ (which clearly requires the government spending target to be $G = 0$). At $\tau_k = \tau_y = 0$, the welfare expressions

become

$$\frac{\delta}{\delta\tau_k}\mathcal{W} = -\alpha\Big(\mathcal{K}' + \mathcal{Y}'\Big)\kappa$$
$$\frac{\delta}{\delta\tau_k}\mathcal{W}_{FB} = -\alpha\Big(\mathcal{K}'_{FB} + \mathcal{Y}'_{FB}\Big)\kappa_{FB}$$

Next, we may compute

$$\mathcal{K}'_{FB} + \mathcal{Y}'_{FB} - (\mathcal{K}' + \mathcal{Y}') = R' \int_{\mathcal{U}} [c^* + px^* - c^*_{FB} - p_{FB}x^*_{FB}] + w' \int z'\theta(h)[(x^*_{FB})^\alpha - (x^*)^\alpha]dM + w' \int_{\mathcal{B}} z'\theta(h)[(x^*)^\alpha - (x_{\mathcal{B}})^\alpha]dM$$

The only difference in the unconstrained investment and consumption in equilibrium versus the frictionless model is the difference in education prices, p versus p_{FB}. As $\eta \to 1$, we have $p - p_{FB} \to 0$, and therefore $x^* - x^*_{FB} \to 0$ and $c^* - c^*_{FB} \to 0$. In that case, we have

$$\mathcal{K}'_{FB} + \mathcal{Y}'_{FB} - (\mathcal{K}' + \mathcal{Y}') \underset{\eta\to 1}{\longrightarrow} w' \int_{\mathcal{B}} z'\theta(h)[(x^*)^\alpha - (x_{\mathcal{B}})^\alpha]dM > 0.$$

Thus, $\mathcal{K}'_{FB} + \mathcal{Y}'_{FB} > \mathcal{K}' + \mathcal{Y}'$ for all η close enough to 1. Going back to the earlier welfare expressions, and using $\kappa_{FB} > \kappa$, this implies that, locally at $\tau_k = \tau_y = 0$,

$$\frac{\delta}{\delta\tau_k}\mathcal{W}_{FB} < \frac{\delta}{\delta\tau_k}\mathcal{W} < 0$$

for all $\eta < 1$ close enough to 1. Hence, wealth taxes are suboptimal, but less so with financial constraints. □

B Comparative statics outside the NBL case

Moving beyond the NBL case ($\gamma = 1$ and $\underline{a} = 0$), we derive the following useful comparative statics that generalize the example insights discussed in the main text.

Proposition B.1. *For a borrowing-constrained parent—i.e., with $(a, z, h, \varepsilon) \in int(\mathcal{B})$—consumption*

and investment respond to R' and n as follows, all else equal:

$$\frac{\partial c_{\mathcal{B}}}{\partial R'} = \frac{\underline{a} x_{\mathcal{B}}^{\frac{1-\alpha}{\alpha}}}{(\alpha\beta\tilde{w}')^{\frac{1}{\gamma}}(1+\frac{1-\alpha}{\gamma}\frac{c_{\mathcal{B}}}{x_{\mathcal{B}}}) + \alpha\tilde{w}' x_{\mathcal{B}}^{\frac{(1-\gamma)(1-\alpha)}{\gamma}}} \quad \textit{and} \quad \frac{\partial x_{\mathcal{B}}}{\partial R'} = -\frac{1}{p}\frac{\partial c_{\mathcal{B}}}{\partial R'}$$

$$\frac{\partial c_{\mathcal{B}}}{\partial n} = \frac{(\alpha\beta\tilde{w}')^{\frac{1}{\gamma}}}{(\alpha\beta\tilde{w}')^{\frac{1}{\gamma}}(1+\frac{1-\alpha}{\gamma}\frac{c_{\mathcal{B}}}{x_{\mathcal{B}}}) + \alpha\tilde{w}' x_{\mathcal{B}}^{\frac{(1-\gamma)(1-\alpha)}{\gamma}}} \quad \textit{and} \quad \frac{\partial x_{\mathcal{B}}}{\partial n} = \frac{1}{p}\Big(1 - \frac{\partial c_{\mathcal{B}}}{\partial n}\Big).$$

Conversely, the comparative statics for an unconstrained parent are

$$\frac{\partial \log c^*}{\partial \log R'} = \frac{(\beta R')^{1/\gamma}}{R' + (\beta R')^{1/\gamma}}(1-\gamma^{-1}) - \frac{\tilde{w}' g(x^*)/c^*}{R' + (\beta R')^{1/\gamma}} \quad \textit{and} \quad \frac{\partial \log x^*}{\partial \log R'} = -\frac{1}{1-\alpha}$$

$$\frac{\partial c^*}{\partial n} = \frac{R'}{R' + (\beta R')^{1/\gamma}} \quad \textit{and} \quad \frac{\partial x^*}{\partial n} = 0.$$

Proof of Proposition B.1. To obtain the results for borrowing constrained parents, differentiate expression (20) using the implicit function theorem, and then use (21). To obtain the results for unconstrained parents, differentiate expressions (16)-(17). □

Let us discuss the additional economic forces arising due to $\gamma \neq 1$ and $\underline{a} < 0$. For unconstrained parents, higher returns R' incentivize savings (substitution effect) but also increase future income from existing savings, which then incentivizes consumption (income effect). The balance of these effects is captured by the term $\frac{(\beta R')^{1/\gamma}}{R' + (\beta R')^{1/\gamma}}(1-\gamma^{-1})$, which clearly vanishes when $\gamma = 1$. In addition to this, there is still the previous log utility effect that higher R' reduces the present value of child income, which reduces parental consumption through the term $-\frac{\tilde{w}' g(x^*)/c^*}{R' + (\beta R')^{1/\gamma}}$.

For constrained parents, who have $c_{\mathcal{B}} + p x_{\mathcal{B}} = n - \underline{a}$, higher rates R' do not move the borrowing limit, nor labor income in partial equilibrium. The only possible effect is to shift resources from consumption to investment, or vice versa (i.e., $\frac{\partial (c_{\mathcal{B}} + p x_{\mathcal{B}})}{\partial R'} = 0$). But here with $\underline{a} < 0$, $c_{\mathcal{B}}$ falls and $x_{\mathcal{B}}$ rises due to a balance-sheet effect: higher R' makes borrowers' plan more expensive, which reduces expected child consumption; parents try to offset this ex-ante by consuming less and investing more. The effects of R' on $c_{\mathcal{B}}$ and $x_{\mathcal{B}}$ are both of order $O(\underline{a})$ because the size of the balance-sheet effect of R' is proportional to the amount of borrowing, namely $-\underline{a}$.

C Investment-to-consumption gradient

This section is dedicated to exploring the conditions under which an unconstrained household will exhibit an x/c that is increasing in c, as in the data. Initially, one may

think this is difficult to obtain, because Proposition B.1 shows that $\frac{\partial \log(x/c)}{\partial a} < 0$ for unconstrained parents. But if other factors besides financial wealth are the primary drivers of c, then it may be possible to obtain x/c which is increasing in c. For this section, suppose there is no borrowing constraint (i.e., $\underline{a} = \infty$).

By inspection of the various optimality conditions, we see that heterogeneity in parental consumption is governed by heterogeneity in (n, z, h, ε). By differentiating expressions (16) and (17), we obtain the following set of comparative statics:

$$\begin{aligned}
\frac{\partial \log c}{\partial a} &= \frac{R'}{R'[n-px]+\tilde{w}'g(x)} & \frac{\partial \log x}{\partial a} &= 0 \\
\frac{\partial \log c}{\partial \log \varepsilon} &= \frac{R'y}{R'[n-px]+\tilde{w}'g(x)} & \frac{\partial \log x}{\partial \log \varepsilon} &= 0 \\
\frac{\partial \log c}{\partial \log z} &= \frac{R'y+\psi\tilde{w}'g(x)}{R'[n-px]+\tilde{w}'g(x)} & \frac{\partial \log x}{\partial \log z} &= \frac{\psi}{1-\alpha} \\
\frac{\partial \log c}{\partial \log h} &= \frac{R'y+\zeta\tilde{w}'g(x)}{R'[n-px]+\tilde{w}'g(x)} & \frac{\partial \log x}{\partial \log h} &= \frac{\zeta}{1-\alpha}
\end{aligned}$$

Combining these results, we obtain the following x/c partial slopes against the four variables that increase parental consumption:

$$\begin{aligned}
\frac{\partial \log(x/c)}{\partial a} &= -\frac{R'}{R'[n-px]+\tilde{w}'g(x)} \\
\frac{\partial \log(x/c)}{\partial \log \varepsilon} &= -\frac{R'y}{R'[n-px]+\tilde{w}'g(x)} \\
\frac{\partial \log(x/c)}{\partial \log z} &= \frac{\psi}{1-\alpha} - \frac{R'y+\psi\tilde{w}'g(x)}{R'[n-px]+\tilde{w}'g(x)} \\
\frac{\partial \log(x/c)}{\partial \log h} &= \frac{\zeta}{1-\alpha} - \frac{R'y+\zeta\tilde{w}'g(x)}{R'[n-px]+\tilde{w}'g(x)}
\end{aligned}$$

Notice that the first and second expressions are unambiguously negative. Thus, for this model to replicate the empirical result that x/c is increasing in c, parental consumption cannot be significantly determined by wealth a and transitory luck ε.

Supposing this to be true, the third and fourth expressions above become relevant. These expressions will be positive under the following conditions. The third expression is only positive if talent heritability ψ and human investment returns-to-scale α are large. Whereas returns-to-scale α are plausibly non-trivial, the genetic heritability of income-relevant talents explains only a small fraction of the persistence of income, education, and wealth (Bowles and Gintis, 2002), which suggests a smaller value of ψ. By contrast, the fourth expression is positive if the complementarities between parent and child human capital, measured by ζ, are sufficiently large. A recent literature emphasizes the quantitative importance of these parent-child complementarities (Cunha and Heckman, 2007; Caucutt and Lochner, 2020).

Finally, one can also see that for sufficiently rich parents (i.e., $a \to \infty$, which implies

that $n \to \infty$ and hence $c \to \infty$), both $\frac{\partial \log(x/c)}{\partial \log z}$ and $\frac{\partial \log(x/c)}{\partial \log h}$ necessarily become positive, whereas $\frac{\partial \log(x/c)}{\partial a}$ and $\frac{\partial \log(x/c)}{\partial \log \varepsilon}$ necessarily become negligible. Consequently, for sufficiently rich parents, it must be the case that x/c is increasing in c.