US Military Hegemony and the Global Price of Risk*

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ABSTRACT

This paper studies how U.S. fiscal policy transmits internationally and why its effects vary systematically across countries and asset classes. I document that country-level asset betas with respect to U.S. fiscal–military shocks exhibit large cross-sectional heterogeneity across sovereign bonds, equities, and currencies, and that this heterogeneity is tightly linked to the U.S. military presence. Countries that are more reliant on US military assistance experience stronger US fiscal transmission, higher equity and bond risk premia, and currencies that appreciate in adverse U.S. fiscal states. To interpret these patterns, I develop a multi-country general equilibrium model in which U.S. fiscal policy transmits through the US security umbrella: the system of security guarantees that the US extends to allied nations. This system of fiscal insurance links together US military spending, allied fiscal behavior, sovereign default risk, and international financial wedges. A quantitative extension shows that the model matches the magnitude and cross-sectional structure of US fiscal betas and factor-mimicking portfolios across asset classes, highlighting U.S. military insurance as a central determinant of global risk premia.

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1 Introduction

A central challenge in international macroeconomics and asset pricing is understanding how U.S. fiscal policy transmits to the rest of the world. U.S. fiscal shocks have global effects on output, asset prices, and exchange rates (Kim (2025); Jiang (2021)). Empirically, some economies exhibit large and immediate responses of bond yields, equity prices, and exchange rates to U.S. fiscal news, while others appear comparatively insulated (Kim (2025); Jiang (2021)). Explaining this heterogeneity—and linking it to a unified economic mechanism—remains an open question.

The starting point of the analysis is a new set of empirical facts. Using plausibly exogenous U.S. fiscal–military news shocks, I estimate country-level fiscal betas for bond returns, equity returns, and exchange rates. I show that these betas are monotonically increasing in U.S. military presence across countries. Nations hosting a larger share of U.S. troops and security assistance—such as Japan and Germany—exhibit larger-magnitude asset-price responses to U.S. fiscal shocks, while more fiscally autonomous or neutral countries—such as Sweden, Switzerland, and Australia—display much weaker exposure. This relationship is not driven by alternative sources of heterogeneity in global risk exposure such as i) economic size (Hassan (2013)), ii) gravity (Lustig and Richmond (2020)) and iii) trade network centrality (Richmond (2019)).

Crucially, the same security-based ordering that explains fiscal betas also maps directly into return premia and tradable portfolios. Sorting countries by U.S. military exposure generates monotone portfolios in equities and sovereign bonds, with higher exposure associated with higher average excess returns. In currency markets, the ordering is reversed: high-exposure currencies earn lower average excess returns and act as hedge currencies, while low-exposure currencies command positive premia. Factor-mimicking portfolios constructed from these sorts are profitable across all three asset classes, indicating that U.S. military presence captures an important source of priced global risk.

To interpret these findings, the paper develops a multi-country general equilibrium model in which U.S. fiscal–military spending transmits globally through the US security umbrella: the system of security guarantees that the US extends to allied nations. This system of fiscal insurance links together US military spending and global risk premia through two complementary channels. Firstly,

there is a *moral-hazard channel*: higher U.S. military spending that is allocated towards foreign military insurance relaxes the local defense floor and eases fiscal pressure in allied economies. This enables fiscal resources to be reallocated from local defense towards i) transfer spending and ii) productive public investment, reducing fiscal cyclicality in these allied economies. At the same time, this expanded fiscal capacity unintentionally weakens fiscal discipline ex ante, encouraging allied countries to take on more fiscal risk by increasing their public debt levels and consequently their sovereign default risk.

Second, there is a *convenience–yield channel*: stronger enforcement of the U.S. security umbrella raises the safety value of dollar assets, strengthens the dollar, and depresses global risky–asset valuations in dollar terms. In equilibrium, both forces complement each other, driving a tight link between increased US military spending and higher global risk premia.

Driving cross-sectional dynamics in the US fiscal transmission is the extent of the US military presence in each country which is the key source of heterogeneity in the model. In equilibrium, countries with higher exposure exhibit larger macro responses to U.S. fiscal shocks, lower equilibrium risk-free rates due to precautionary saving, higher equity and bond risk premia, and currencies that appreciate in adverse U.S. fiscal states. The model delivers closed-form expressions for cross-sectional slopes in asset betas that are consistent with my empirical evidence.

The paper then extends the analysis to a quantitative framework that embeds the same mechanism in a richer dynamic environment with continuous time, recursive preferences and imperfect information about U.S. security commitments. This extension allows the model to discipline magnitudes and timing, and to match the joint cross-sectional and time-series behavior of equities, bonds, and exchange rates. Quantitatively, the model reproduces the observed cross-asset slopes of fiscal transmission and the monotone structure of factor-mimicking portfolios sorted on military exposure.

Overall, the paper advances a new view of global fiscal transmission. U.S. fiscal policy does not propagate uniformly across countries; instead, it travels through a military–industrial network that links security provision, fiscal behavior, and financial markets. Exposure to the U.S. security umbrella emerges as a sufficient statistic governing how U.S. fiscal shocks shape global macroeconomic outcomes, asset prices, and risk premia. In this sense, U.S. military presence is not merely

a geopolitical backdrop—it is a central economic channel through which U.S. fiscal policy prices global risk.

2 Related Literature

Global Financial Cycle and U.S. Monetary Dominance. The paper first relates to the literature documenting the existence of a global financial cycle (GFC) in risky asset prices. In a seminal contribution, Miranda-Agrippino and Rey (2020) show that global equity returns are driven by a single dominant factor closely tied to global risk appetite and uncertainty. A large body of recent work identifies this factor with the international transmission of U.S. *monetary* policy shocks: Brusa, Savor, and Wilson (2019) and Miranda-Agrippino, Nenova, and Rey (2020) document that U.S. interest–rate innovations generate significant global co-movements in risky asset prices. My paper complements this literature by showing that the same comovement structure can emerge from *fiscal* rather than purely monetary forces. Through its security commitments and fiscal capacity, the United States anchors the world's discount rate not only as the issuer of the global reserve currency but also as the fiscal–military hegemon underwriting global enforcement.

Cross-Sectional Drivers of Global Financial Exposure. A related strand of the literature studies why countries differ systematically in their exposure to global financial fluctuations. One influential line of work emphasizes *gravity-style* forces in international asset pricing. Lustig and Richmond (2020) show that bilateral financial returns and exchange-rate dynamics are shaped by distance, trade intensity, and economic linkages, mirroring the structure of gravity models in international trade. Complementary evidence highlights the role of *country size*: Hassan (2013) and subsequent work document that larger economies exhibit lower expected returns and behave as global hedges, consistent with their disproportionate influence on world consumption risk. More recent contributions emphasize *network centrality*, showing that countries' positions in global trade and production networks shape their exposure to aggregate shocks and international risk premia (Richmond (2019)).

This paper contributes to this literature by introducing a distinct and previously underexplored

dimension of cross-sectional heterogeneity: *geopolitical exposure*. Rather than trade flows, economic size, or production centrality, I focus on countries' integration into the U.S. security umbrella. U.S. military presence captures a form of enforcement and insurance that is orthogonal to standard gravity and size measures, yet strongly predictive of how U.S. fiscal shocks transmit into local macroeconomic outcomes and asset prices. In the data, security exposure organizes cross-country differences in fiscal betas, risk premia, and currency behavior even after controlling for trade intensity, financial openness, and economic size, suggesting that geopolitical ties represent a separate and economically meaningful axis of global financial integration.

Fiscal Policy and Risk Premia. The paper also connects to a growing literature studying the relationship between fiscal policy and asset prices. Existing studies primarily focus on debt–GDP ratios as measures of fiscal stance. Liu (2023) shows that country-level debt–GDP ratios predict future equity returns, while Croce, Nguyen, Raymond, and Schmid (2019) and Bretscher, Hsu, and Tamoni (2023) link government debt to risk premia, the cost of capital, and the term structure of interest rates. This paper departs from that literature along two dimensions. First, rather than focusing on fiscal levels such as debt–GDP ratios or long-run sustainability, I study *shocks* to U.S. fiscal conditions and their international transmission. The empirical object of interest is not a country's own fiscal stance, but its exposure to innovations in U.S. fiscal–military policy, which generates large and heterogeneous asset-price responses across countries and markets.

Second, existing work largely treats fiscal risk as a domestic or aggregate macroeconomic phenomenon. By contrast, this paper emphasizes the *cross-sectional geography* of fiscal transmission. I show that exposure to U.S. fiscal shocks varies systematically with U.S. military presence, and that this variation jointly prices equities, sovereign bonds, and currencies. Fiscal risk is therefore transmitted internationally through a security-based channel rather than through balance-sheet capacity or local debt dynamics alone.

By linking U.S. fiscal shocks to global asset prices through military presence and security dependence, the paper identifies a new mechanism through which fiscal policy generates international risk premia—one that is orthogonal to standard measures of public debt and operates across asset classes rather than within a single market.

Geoeconomics, Power, and Asset Prices. This paper connects to an emerging branch of economics and political economy that links geopolitical power to economic outcomes, commonly referred to as *geoeconomics*—the study of how power rivalry and strategic statecraft shape economic interactions and incentives (Mohr and Trebesch, 2025). Scholars in this area highlight the strategic use of economic and financial tools such as sanctions, currency dominance, and capital controls, and examine how geopolitics interacts with international trade, finance, and macroeconomic risk.

While much of the geoeconomic literature has focused on trade flows, supply chains, and conflict economics, its implications for global asset prices and cross-country financial risk premia remain underexplored. This paper fills that gap by studying how U.S. military hegemony and the structure of the U.S. security umbrella shape country-level sensitivities to U.S. fiscal shocks and jointly price equities, sovereign bonds, and currencies.

The closest contribution to my paper is Pflueger and Yared (2025) who develop a dynamic two-country model in which military power and sovereign borrowing capacity reinforce one another through bond-market expectations, generating the possibility of self-fulfilling hegemonic transitions. Their analysis emphasizes the *fragility* of hegemony: rising debt weakens fiscal capacity, which reduces military investment, further eroding credibility and market confidence.

My contribution is complementary but distinct along three dimensions. First, rather than studying the emergence or collapse of hegemony, I take U.S. military dominance as a given institutional feature and ask how it shapes the *cross-section* of global asset prices. Second, while Pflueger and Yared (2025) focus on sovereign bond markets and expectations-driven regime shifts, this paper shows that military hegemony jointly prices *equities, sovereign bonds, and currencies* through a common fiscal–security transmission channel. Third, the mechanism emphasized here operates through heterogeneous exposure to the U.S. security umbrella across countries, generating systematic differences in fiscal betas, risk premia, and hedging properties even in the absence of regime change or hegemonic instability.

3 Motivating Facts

This section documents a set of stylized facts establishing the role that US military presence plays in driving the cross-sectional exposure of macroeconomic and financial outcomes to US fiscal shocks. The empirical patterns motivate the mechanism formalized in the next section: U.S. fiscal policy transmits globally through the military–industrial network operated by U.S. multinational firms.

3.1 U.S. Fiscal–Military Betas

Military News Identification. To link my results to US military spending, I exploit plausibly exogenous shocks to the current and expected future path of U.S. military spending. I rely on two complementary identification strategies that have become standard in the empirical macroeconomics literature.

The first approach follows the narrative methodology of Ramey (2011). This method systematically analyzes historical sources—including presidential speeches, congressional records, budget documents, and contemporaneous news reports—to isolate changes in expected government spending that are driven by military or geopolitical events rather than by endogenous responses to the business cycle. The resulting quarterly series of *military news shocks* captures forward–looking revisions to anticipated defense spending and is widely viewed as plausibly exogenous with respect to contemporaneous macroeconomic conditions in both the United States and the rest of the world.

The second approach builds on Ramey and Zubairy (2018), who use forecast errors from professional surveys to identify fiscal news. Specifically, this strategy measures unexpected changes in projected government spending using revisions in the Survey of Professional Forecasters (SPF), thereby isolating fiscal innovations that were not anticipated by private agents. Because these forecast errors are orthogonal to contemporaneous macroeconomic information sets, they provide an alternative, expectation-based measure of military-related fiscal shocks.

US Fiscal Betas. Using these military news shocks: u_t^{US}, I estimate the US fiscal betas using the baseline specification:

$$\hat{\beta}_i^{\text{Fiscal}}: \quad X_{i,t} = \alpha_i + \beta_i^{\text{Fiscal}} \text{u}_t^{US} + \gamma' \text{Monetary Policy Shocks}_t + \delta' \text{Macro}_t + \theta' \text{Financial}_t + \epsilon_{i,t}$$

$$X_{i,t} \in \{rx_{i,t}^{FX},\, r_{i,t}^{b},\, r_{i,t}^{e}\} \tag{3.1}$$

where $X_{i,t}$ denotes either (i) the bilateral currency excess return vis-à-vis the U.S. dollar, $rx_{i,t}^{FX}$, (ii) local sovereign bond returns $r_{i,t}^{b}$, or (iii) local equity returns $r_{i,t}^{e}$ for country i. Country fixed effects α_i absorb time-invariant cross-country heterogeneity, and the coefficient of interest β_j^{US} traces the impulse response to a one–standard–deviation U.S. military shock.

Monetary Policy Controls: As illustrated by Campbell and Shiller (1988), asset prices can fluctuate due to i) risk-free rates, ii) cash flows or iii) discount rates. Therefore it is important to control for confounding factors that correlate with these different components. To control for risk free rates, I follow traditional high frequency methods to identify monetary policy shocks. I identify two monetary policy shocks per country using these methods:

$$\text{Monetary Policy Shocks}_t = \left[\underbrace{\text{Target}_t^i, \quad \text{Path}_t^i}_{\text{Local}} \,,\, \underbrace{\text{Target}_t^G, \quad \text{Path}_t^G}_{\text{Global}} \right]^T \tag{3.2}$$

To construct the shocks, I use i) all the publicly available data from the entire term structure of interest rates available via global financial data (GFD) and ii) central bank announcement dates that I personally hand collect. Using this data, I construct two monetary policy shocks for each country. The first monetary policy shock is a *target shock* which is the first principal component of interest rate changes around central bank announcement dates for that country. This captures the level and represents transitory monetary policy shocks in the same vein of the Romer and Romer (2004) US monetary policy shock.

The second monetary policy shock is a *path shock* which is the second principal component of interest rate changes around the central bank announcements. This captures news about forward

guidance in the same vein as the Gertler and Karadi (2015) identified monetary policy shock. To aid this interpretation, I follow Gürkaynak, Sack, and Swanson (2005) and rotate the orthogonalised bases of the principal component to ensure that the second principal component has zero loading on the short end of the term structure (1 month rate).

Macro Controls: To address distinct forces driving the cash flow component, I control for macroeconomic conditions. This is important because governments enact expansionary fiscal policies during periods of low economic growth. These controls can be either i) local, ii) global or iii) US macro variables:

$$\text{Macro}_t = \left[\begin{array}{cc} \underbrace{\Delta c_t^i, \Delta\text{GDP}_t^i, \mathbb{E}_t\Delta\text{GDP}_{t,t+4}^i}_{\text{Local}}, & \underbrace{\Delta c_t^{US}, \Delta\text{GDP}_t^{US}, \mathbb{E}_t\Delta\text{GDP}_{t,t+4}^{US}}_{\text{US}}, \\ \multicolumn{2}{c}{\underbrace{\Delta c_t^G, \Delta\text{GDP}_t^G, \mathbb{E}_t\Delta\text{GDP}_{t,t+4}^G}_{\text{Global}}} \end{array} \right]^T \tag{3.3}$$

$\Delta c_t^i, \Delta\text{GDP}_t^i$ denote *realized* local consumption and GDP growth rates. Growth rates are defined as four quarter changes in local consumption and GDP in local currency units. $\mathbb{E}_t\Delta\text{GDP}_{t,t+4}^i$ is the one year (four-quarter) ahead local GDP forecast. Controlling for *ex-ante* growth expectations is also important, given that governments also institute expansionary fiscal policies during periods of low *expected* growth (Liu (2023)).

Financial controls: I also control for distinct financial factors that proxy for time varying global discount rate or global risk premia. These factors can also spuriously drive a link between the US fiscal condition and global risky asset prices since fiscal policies may also be more expansionary during periods of heightened risk aversion. To rule this additional possibility out, I include a financial vector that is known to proxy for global risk premia and is motivated by Lilley, Maggiori, Neiman, and Schreger (2022):

$$\text{Financial}_t = \left[\Delta BEX_t, \quad \Delta GZ_t, \quad \Delta\text{Premium}_t, \quad \Delta VIX_t, \quad INT_t \right]^T \tag{3.4}$$

ΔBEX_t is the four-quarter change in the risk-aversion measure from Bekaert, Engstrom, and Xu (2022). ΔGZ_t is the four quarter change in the US corporate bond credit spread from Gilchrist and Zakrajšek (2012). $\Delta\text{Premium}_t$ is the four quarter change in the US treasury premium, defined as average covered interest parity deviation between developed country government bonds and U.S. Treasuries computed from Du, Im, and Schreger (2018). ΔVIX_t is the four quarter change in the logarithm of the VIX index. Finally INT_t is the value-weighted return on a portfolio of NY Fed primary dealers' holding companies and is taken from He, Kelly, and Manela (2017).

3.2 US Military Presence

Hard power: U.S. military presence. Information on U.S. military presence is drawn from the *Hoover Institution's U.S. Troop Deployments, 1950–2023* dataset compiled by Tim Kane, which provides an annual census of active–duty U.S. military personnel stationed abroad from 1950 through 2023, disaggregated by host country and service branch. The dataset covers more than 180 countries over seven decades and is based on official Department of Defense and Defense Manpower Data Center (DMDC) records, harmonized into a consistent cross–country panel.

To express military presence in a common monetary unit, I convert troop deployments into dollar values using standardized annual cost estimates per active–duty service member drawn from official Department of Defense budgeting guidelines. This conversion yields an annual dollar value of U.S. troop presence in each host country that reflects the resource cost of maintaining enforcement capacity rather than combat intensity.

Soft power: security sector assistance. To capture indirect and soft forms of security provision, I augment troop-based presence with measures of *security sector assistance*. Data on bilateral security sector assistance flows are obtained from the *OECD Development Assistance Committee (DAC) Creditor Reporting System (CRS)*, which reports dollar-valued aid flows by donor, recipient, year, and purpose code. I define security sector assistance to include aid classified under security system management and reform and related peace– and security–oriented categories, thereby capturing financing, training, and institutional support that enhances a country's security capacity without direct troop deployment. These data provide comprehensive coverage of all major

donors over the modern sample period, allowing construction of both U.S. and non-U.S. security assistance totals.

Non-U.S. foreign military presence. To obtain comparable information on the presence of *non-U.S.* foreign military personnel, I rely on data from the *SIPRI Multilateral Peace Operations Dataset (MPOD)*, which records annual personnel deployments to all multilateral operations, including NATO, EU, and UN missions. For NATO members, these figures are cross-checked against the *NATO Defence Manpower Statistics* series, which reports allied and partner personnel stationed on foreign NATO territory. Additional validation is based on regional aggregates from the *IISS Military Balance*, which documents force posture and overseas deployments for advanced economies. In practice, for the developed-country sample considered here, non-U.S. foreign troop presence is limited and largely concentrated in multilateral or rotational deployments.

U.S. security dominance. For each country i, I define the U.S. share of the local security umbrella as

$$\text{US Security Share}_i = \frac{\text{U.S. Troop \$}_i + \text{U.S. Security Assistance \$}_i}{\text{Total Foreign Troop \$}_i + \text{Total Foreign Security Assistance \$}_i}, \quad (3.5)$$

where each component is averaged over the sample period. This *local normalization* captures the degree of U.S. dominance in a country's external security provision, rather than the country's importance in the U.S. global deployment portfolio.

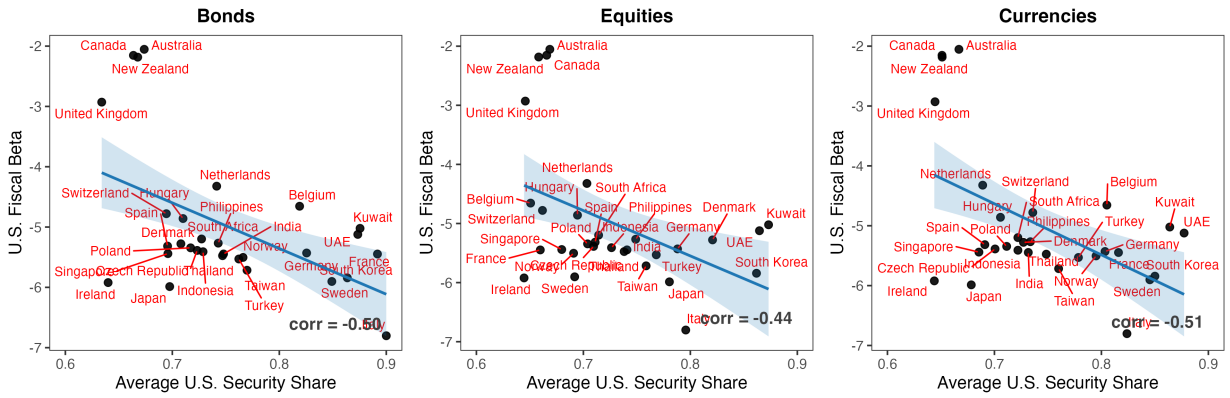


Figure 1: Fiscal betas and U.S. military presence. Each panel plots the country–level fiscal beta $\hat{\beta}_i^{\text{Fiscal}}$ against a measure of average U.S. military presence. Fiscal betas are estimated using U.S. military spending forecast errors constructed from the SPF-based narrative shocks of Ramey and Zubairy (2018) and the military presence variable is defined by (3.5). The solid blue line shows the empirical fit. Across the three panels, the data exhibit positive correlations of 0.50, 0.38, and 0.43, respectively. Countries most reliant on U.S. security—notably Japan and Germany—lie at the upper end of the distribution, while fiscally autonomous or neutral states—such as Sweden, Switzerland, and Australia—display lower exposures.

3.3 Cross-Country Heterogeneity in Fiscal Transmission

Table 1: U.S. Military Presence and Cross-Asset Fiscal Betas

	Dependent Variable: Country-Level U.S. Fiscal Beta								
	Bonds			Equities			Currencies		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
U.S. Military Presence	1.25***	1.18***	1.31***	1.42***	1.36***	1.48***	0.98***	0.91***	1.05***
	(0.32)	(0.30)	(0.33)	(0.35)	(0.34)	(0.36)	(0.25)	(0.24)	(0.26)
Trade-to-GDP Ratio	0.08			0.05			0.03		
	(0.21)			(0.23)			(0.18)		
GDP Share		0.11			0.09			0.06	
		(0.19)			(0.20)			(0.17)	
Network Centrality			0.42**			0.47**			0.38**
			(0.19)			(0.21)			(0.17)
Observations	587	587	587	587	587	587	587	587	587
R^2	0.21	0.20	0.23	0.24	0.23	0.26	0.18	0.17	0.20

Notes: The table reports regressions of country-level U.S. fiscal betas estimated separately for bonds, equities, and currencies. Each specification includes U.S. military presence and one additional control: (i) trade-to-GDP ratio, (ii) GDP share, or (iii) network centrality. Standard errors (in parentheses) are heteroskedasticity-robust. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively.

Results: Figure 4 plots US fiscal betas against the US military presence measure defined by (3.5). Fiscal betas are estimated using U.S. military spending forecast errors constructed from the SPF-based narrative shocks of Ramey and Zubairy (2018). It highlights a tight and systematic link

between U.S. fiscal transmission and U.S. military presence. Across all three panels, countries that receive greater U.S. security assistance exhibit markedly higher U.S. fiscal betas across bond, equity, and currency markets, with a consistently positive and economically meaningful relationship.

Are these results driven by other well-known cross-sectional determinants of global risk premia? To assess this possibility, we estimate a set of horse-race regressions that augment U.S. military presence with standard measures of international economic integration and size, including (i) the trade-to-GDP ratio, (ii) a country's share of global GDP, and (iii) global trade network centrality. The results, reported in Table 1, provide a clear answer: U.S. military presence remains a strong and statistically significant predictor of U.S. fiscal betas across all asset classes, while the alternative controls are economically small and statistically insignificant. Network centrality also remains economically important, but it doesn't subsume the effect of U.S. military presence.

Taken together, these findings indicate that the cross-country heterogeneity in U.S. fiscal transmission is not simply a byproduct of trade openness, economic size, or generic global connectedness. Instead, US military presence emerges as an important organizing force shaping the cross-section of global risk.

3.4 Quasi–Experimental Evidence

Motivation and identification strategy. To complement the cross-sectional evidence above, this subsection exploits a quasi-experimental episode that generated a sharp, legislated revision to the expected path of U.S. military spending: the 2013 defense sequester. The identifying idea is straightforward. If U.S. fiscal shocks transmit internationally through the security umbrella, then an exogenous contraction in expected U.S. military capacity should produce systematically larger financial-market responses in countries that are more exposed to U.S. military presence. The sequester provides an ideal setting to test this prediction because it was triggered by domestic budget rules and political gridlock, rather than by changes in geopolitical threats or global economic conditions. The empirical design therefore focuses on *differential exposure*: comparing event-time responses between countries with high versus low U.S. military presence around a common, externally imposed shock.

Military natural experiment: the sequester. The *Budget Control Act of 2011* mandated automatic, across-the-board spending cuts following the failure of the Congressional "super committee," with sequestration becoming binding in **2013Q1**. While the legislation applied to both defense and non-defense discretionary spending, it represented a qualitatively distinct shock to core defense outlays. For the first time in the postwar era, military spending was no longer effectively insulated from the appropriations process and instead became subject to mechanical cuts. This institutional change generated a sharp downward revision in the expected future path of U.S. military spending, affecting readiness, procurement, and overseas operations.

Crucially for identification, the timing and magnitude of the cuts were determined by statute and driven by domestic fiscal institutions, rather than by contemporaneous changes in international security conditions. The sequester can therefore be interpreted as a plausibly exogenous negative shock to U.S. military capacity and, by extension, to the credibility of U.S. security provision.

Exposure groups. Let USMilitary_i denote a predetermined measure of exposure to U.S. military presence, such as the U.S. security share constructed in Section 3. Countries are partitioned into two groups:

$$H \equiv \{i : \text{USMilitary}_i \text{ is above the median}\}, \qquad L \equiv \{i : \text{USMilitary}_i \text{ is below the median}\}.$$

This high–low split isolates cross-country differences in exposure to U.S. military capacity while holding the fiscal event itself fixed and common across all countries.

Empirical design. For each horizon $k \geq 0$, I estimate event-time impulse responses using a panel local-projection specification that allows responses to differ between high- and low-exposure countries:

$$Y_{i,t+k} = \alpha_i + \sum_{j=0}^{J} \phi_{j,k}\, Y_{i,t-j} + \beta_k\, \mathcal{E}_t + \gamma_k\big(\mathcal{E}_t \times \mathbf{1}\{i \in H\}\big) + \mathbf{X}_{i,t}'\Gamma_k + \varepsilon_{i,t+k}, \tag{3.6}$$

where $\mathcal{E}_t$ is an indicator equal to one in 2013Q1 and zero otherwise. Country fixed effects α_i absorb time-invariant cross-country differences, and standard errors are clustered at the quarter level to

account for cross-country correlation induced by common shocks. The coefficient β_k traces the average response for low-exposure countries, while γ_k captures the *differential response* of high-exposure countries relative to low-exposure countries.

The outcomes $Y_{i,t}$ correspond to the same global risk and asset-price variables emphasized in the motivating facts: global uncertainty, exchange rates, sovereign bond spreads, and equity returns. The control vector $\mathbf{X}_{i,t}$ includes the same fiscal, monetary, macroeconomic, and financial controls used throughout the baseline analysis.

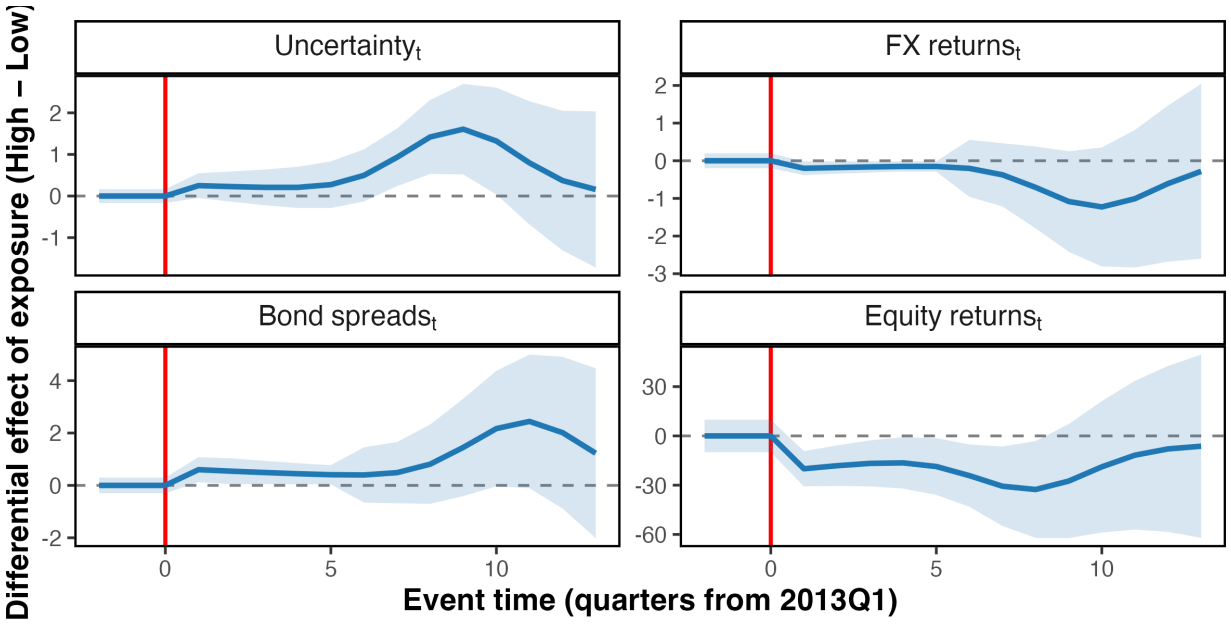


Figure 2: Differential financial-market responses to the 2013 defense sequester. The figure plots event-time impulse responses to the 2013 U.S. defense sequester, comparing countries with high versus low exposure to U.S. military presence. Each panel shows the estimated differential response (High minus Low) for global uncertainty, FX excess returns, sovereign bond spreads, and equity returns. Shaded regions denote confidence bands. High-exposure countries experience larger increases in uncertainty and sovereign spreads, more negative equity returns, and lower FX excess returns, consistent with their currencies acting as hedges when U.S. military capacity contracts.

Interpretation. Figure 2 provides quasi-experimental confirmation of the cross-sectional mechanism documented earlier in the paper. Following an exogenous contraction in expected U.S. military spending, countries that are more exposed to U.S. military presence experience systematically larger financial-market responses: uncertainty rises more, sovereign spreads widen more, equity prices fall more, and currencies deliver lower excess returns. Low-exposure countries, by contrast, are comparatively insulated from the shock.

These patterns mirror the cross-sectional fiscal betas reported above and are difficult to reconcile

with explanations based solely on aggregate fiscal aggregates or global business-cycle forces. Instead, they point to a common conclusion: U.S. military presence governs how U.S. fiscal shocks are transmitted internationally and priced across assets. By exploiting an externally imposed, defense-specific fiscal shock, the sequester evidence strengthens the causal interpretation that exposure to the U.S. security umbrella is a central driver of global fiscal transmission.

3.5 Factor Mimicking Portfolios

To translate the cross-sectional heterogeneity in U.S. fiscal exposure into tradable risk, I now construct factor-mimicking portfolios sorted on U.S. military presence. The goal is to assess whether security dependence not only explains cross-country differences in fiscal betas, but also gives rise to systematic return premia across asset classes.

Portfolio construction. For each asset class—sovereign bonds, equities, and currencies—I sort countries into portfolios based on their exposure to the U.S. security umbrella, as measured by the U.S. security share defined in Section 3. The sorting variable is pre-determined and slow-moving, and is measured using sample averages (or decade-balanced averages in robustness checks) to avoid mechanical look-ahead bias.

At the beginning of each period, countries are ranked by their U.S. security exposure and assigned to tercile portfolios. The *High* portfolio contains countries in the top tercile of U.S. military presence, while the *Low* portfolio contains countries in the bottom tercile. Portfolio membership is held fixed within the period, and returns are computed using value-weighted averages within each group.[1]

HML security factor. For each asset class, I construct a factor–mimicking portfolio that captures the return spread associated with differential exposure to the U.S. security umbrella. The precise definition of the high–minus–low (HML) security factor differs across asset classes to reflect the distinct economic role of equity, bond, and currency returns.

For equities and bonds, the HML security factor is defined in the standard way as the return

[1]Equal-weighted portfolios yield similar results.

difference between countries with high and low U.S. military exposure:

$$\mathrm{HML}_t^{\mathrm{Sec},\,X} \;=\; R_t^{\mathrm{High}} \;-\; R_t^{\mathrm{Low}}, \qquad X \in \{\mathrm{Equities}, \mathrm{Bonds}\}, \tag{3.7}$$

where R_t^{High} and R_t^{Low} denote the average returns of countries in the top and bottom portfolios of U.S. military presence, respectively. This construction reflects the fact that greater exposure to the U.S. security umbrella amplifies transmission of U.S. fiscal risk into local equity and bond risk premia.

For currencies, the direction of the HML factor is reversed:

$$\mathrm{HML}_t^{\mathrm{Sec},\,\mathrm{FX}} \;=\; R_t^{\mathrm{Low}} \;-\; R_t^{\mathrm{High}}. \tag{3.8}$$

This reversal reflects the model implication that currencies of countries more exposed to U.S. military presence tend to *appreciate* in response to U.S. fiscal–military shocks. As a result, high–exposure currencies act as hedge currencies rather than risky ones, earning lower average excess returns from the perspective of a U.S. investor.

Taken together, these definitions imply that equity and bond risks associated with U.S. fiscal transmission are inversely related to currency risk in the standard manner. The HML security factors therefore isolate a common underlying geopolitical risk channel while respecting the distinct sign conventions and pricing roles of returns across asset classes.

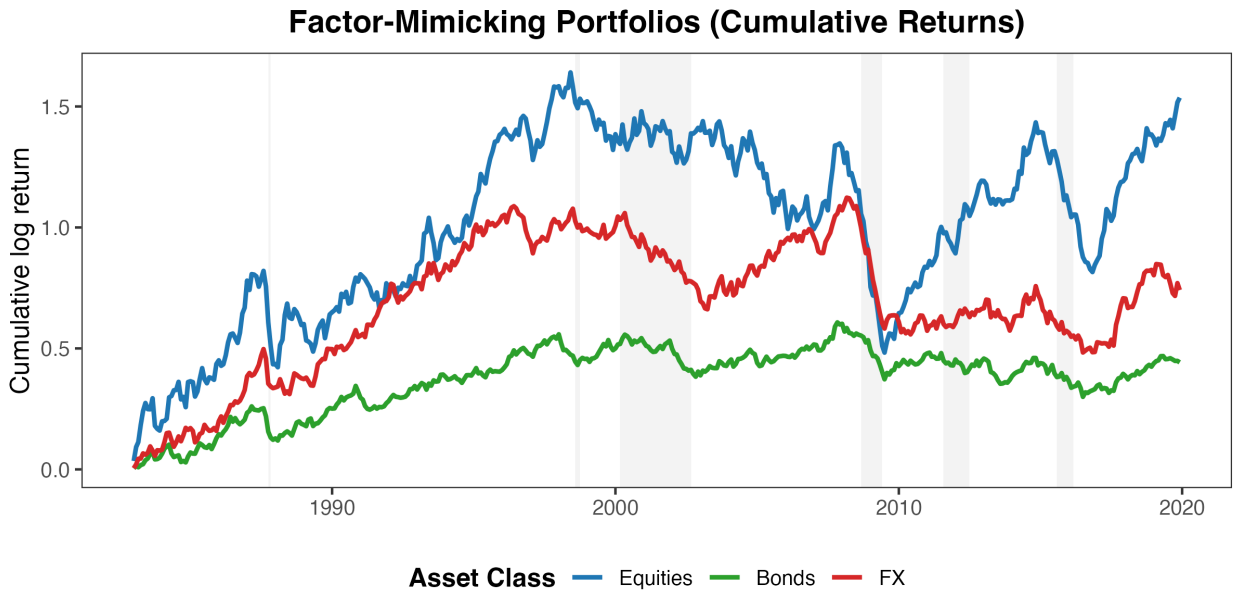


Figure 3: Factor-mimicking portfolios sorted on U.S. military presence. The figure plots cumulative log returns of factor-mimicking portfolios constructed by sorting countries on exposure to the U.S. security umbrella. For each asset class—equities, bonds, and currencies—the factor is defined as a high-minus-low (HML) portfolio that goes long countries with high U.S. military presence and short countries with low U.S. military presence. Shaded regions denote major episodes of global financial stress, including the 1987 crash, the 1998 LTCM/Russia crisis, the 2000–02 dot-com bust, the 2008–09 global financial crisis, the European sovereign debt crisis, and the 2015–16 global slowdown.

Table 2: Summary Statistics of Factor-Mimicking Portfolios Sorted on U.S. Military Presence

	Equities			Bonds			Currencies		
	Mean	Std Dev	Sharpe	Mean	Std Dev	Sharpe	Mean	Std Dev	Sharpe
P_L (Low Exposure)	−0.40	14.0	−0.03	−0.20	5.8	−0.03	4.30	8.5	0.51
P_2	1.10	13.6	0.08	0.90	5.6	0.16	2.80	8.3	0.34
P_3	2.70	13.9	0.19	2.20	5.7	0.39	1.20	8.2	0.15
P_H (High Exposure)	4.40	14.5	0.30	3.80	6.0	0.63	−0.30	8.4	−0.04
HML ($P_H - P_L$)	4.80	11.2	0.43	4.00	4.4	0.91	4.60	6.6	−0.70
Observations		432			432			432	
Sample					1983–2019				

Notes: The table reports annualized summary statistics for factor-mimicking portfolios formed by sorting countries into four portfolios (P_L, P_2, P_3, P_H) based on exposure to U.S. military presence. Within equities and bonds, average excess returns increase with exposure, consistent with greater transmission of U.S. fiscal risk into risk premia. For currencies, average excess returns decline with exposure, consistent with high-exposure currencies appreciating more against the U.S. dollar in response to U.S. fiscal shocks. HML is a zero-cost strategy that goes long P_H and short P_L for equities and bonds. For currencies, it goes short P_H and long P_L. Sharpe ratios use full-sample standard deviations.

Interpretation. The profitability of the factor-mimicking portfolios provides complementary time-series evidence that reinforces the cross-sectional results. Across equities, bonds, and currencies, portfolios that load more heavily on U.S. military presence earn persistently higher cumulative

returns, with pronounced drawdowns during episodes of global financial stress and fiscal uncertainty. These patterns are consistent with the interpretation that exposure to the U.S. security umbrella entails systematic exposure to U.S. fiscal risk, which is priced across asset classes.

Importantly, the fact that security-sorted HML portfolios are profitable in *all three markets* suggests that U.S. military presence operates as a broad-based risk factor rather than a market-specific anomaly. Periods of sharp underperformance—most notably during the global financial crisis and other episodes of heightened geopolitical and fiscal stress—coincide with episodes in which U.S. fiscal risk becomes particularly salient, leading to synchronous repricing across equities, sovereign bonds, and currencies.

Taken together with the cross-country fiscal beta evidence, these results indicate that U.S. military presence is not merely correlated with global asset price comovement, but instead captures a priced dimension of global risk. Countries more deeply embedded in the U.S. security network systematically bear greater exposure to U.S. fiscal shocks and earn corresponding risk premia, consistent with the view that the global footprint of U.S. fiscal policy is mediated through the security umbrella.

3.6 Identification and Interpretation

This section clarifies how the empirical results should be interpreted and addresses potential concerns regarding endogeneity and omitted-variable bias. The central identifying variation in the analysis comes from *unexpected innovations* to U.S. fiscal–military conditions, combined with *predetermined cross-country heterogeneity* in exposure to U.S. security provision.

Predetermined exposure and shock-based identification. The empirical objects of interest—country-level U.S. fiscal betas and security-sorted portfolio returns—are identified from responses to *fiscal–military news shocks*, rather than from levels of fiscal variables. Specifically, fiscal betas are estimated using innovations to U.S. fiscal conditions that are orthogonal to contemporaneous macroeconomic information, following established narrative and forecast-based identification strategies. By construction, these shocks are unexpected at the time they occur and are plausibly exogenous to foreign economic conditions.

In contrast, U.S. military presence and security exposure are highly persistent and slow-moving, reflecting long-run strategic commitments, basing agreements, and alliance structures. As a result, cross-country differences in exposure are predetermined with respect to the fiscal shocks used to identify transmission. The empirical strategy therefore exploits the interaction between *high-frequency fiscal innovations* and *low-frequency geopolitical exposure*, rather than variation in contemporaneous fiscal or security policy.

Ruling out reverse causality. A natural concern is that U.S. military presence may respond endogenously to economic or financial risk in host countries. Several features of the empirical design mitigate this concern.

First, military deployments and security assistance are adjusted infrequently and with long planning horizons, making it unlikely that they respond to short-run fiscal innovations or asset-price movements. Second, the analysis focuses on differential *responses* to common U.S. shocks, rather than on cross-sectional correlations in levels. For reverse causality to explain the results, U.S. military presence would need to anticipate not only future fiscal shocks, but also the sign and magnitude of their asset-price transmission across multiple markets, which is implausible.

Controls and residual variation. The results are robust to controlling for a wide range of observable country characteristics, including trade openness, economic size, and network centrality. While these variables capture important dimensions of global integration, they do not subsume the explanatory power of U.S. military presence. This suggests that security exposure captures a distinct dimension of international linkage that is not reducible to standard measures of trade or financial integration.

3.7 Interpretation and Motivation

Taken together, the evidence in this section establishes a coherent set of empirical facts about the global transmission of U.S. fiscal–military policy.

First, U.S. fiscal shocks transmit heterogeneously across countries, and this heterogeneity is tightly linked to U.S. military presence. Countries that receive a larger share of U.S. security

provision exhibit systematically larger fiscal betas across macroeconomic outcomes and financial markets. This relationship is robust across alternative measures of military exposure and holds consistently across sovereign bonds, equities, and currencies, indicating that U.S. military presence is a first-order determinant of cross-country differences in exposure to U.S. fiscal risk.

Second, U.S. military presence does not merely predict fiscal transmission in a single market, but organizes the cross-section of global risk more broadly. The same security-based ordering that explains fiscal betas also generates systematic return differentials and profitable factor-mimicking portfolios across asset classes. This cross-asset consistency indicates that military exposure captures a common underlying source of priced risk, rather than market-specific anomalies or idiosyncratic country characteristics.

Taken together, these findings point to a unified interpretation: U.S. military presence shapes how U.S. fiscal shocks propagate into global asset prices. The next section builds a model that formalises a mechanism to make sense of these findings. At the heart of the mechanism is the US security umbrella: the system of security guarantees that the US extends to allied nations. This system of fiscal insurance links together US military spending, allied fiscal behavior, sovereign default risk, and international financial wedges.

4 Model

Overview. To rationalise the empirical findings in the previous section linking the cross-section of global risk exposure to U.S. military presence, this section develops a general-equilibrium model in which global asset prices are driven by the U.S. security umbrella—the system of military guarantees and enforcement capacity that the United States extends to allied countries.

4.1 Environment

Overview. Time is discrete and infinite, $t = 0, 1, 2, \ldots$. The world consists of two blocs, indexed by $k \in \{\text{US}, i\}$, representing the United States and a representative allied economy. Each country produces output using capital and labor, consumes a non-traded final good, and issues sovereign debt subject to default risk. Financial markets are incomplete: sovereign default risk and

enforcement constraints prevent full international risk sharing.

As the global military hegemon, the United States extends security guarantees to allied nations. A portion of the U.S. federal budget, denoted s_t, is allocated to ***external security provision*** for allies. Thus the US provides fiscal insurance to its allies via s_t, partially providing for the defense needs of the allied economy (S^*) and alleviating the fiscal space of the allied economy.

Through this fiscal insurance scheme, U.S. fiscal–military shocks drive aggregate risk in the global economy with countries differing in their exposure to the U.S. security umbrella, summarized by a country-specific parameter ϕ_i.

4.2 Households and Preferences

Preferences. In each country i, a representative household maximizes

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \log C_{i,t}, \qquad \beta \in (0,1), \tag{4.1}$$

where $C_{i,t}$ denotes consumption of the final good.

Log utility implies a constant price of risk, allowing the analysis to focus on the cross-section of exposure to global fiscal–military risk rather than on time variation in risk premia.

4.3 Production

Final-good production. Output in country i is produced competitively using capital $K_{i,t}$ and labor $L_{i,t}$:

$$Y_{i,t} = A_i K_{i,t}^{\alpha} L_{i,t}^{1-\alpha}, \qquad \alpha \in (0,1). \tag{4.2}$$

Total factor productivity A_i is constant; there are no aggregate civilian productivity shocks.

Resource constraint. Output is allocated between private consumption, investment, and government spending:

$$Y_{i,t} = C_{i,t} + I_{i,t} + G^{soc}_{i,t} + I^{pub}_{i,t} + h_{i,t}, \tag{4.3}$$

where $G^{soc}_{i,t}$ denotes social transfers, $I^{pub}_{i,t}$ productive public investment, and $h_{i,t}$ local defense outlays.

Capital accumulation. Private capital evolves according to

$$K_{i,t+1} = (1-\delta)K_{i,t} + I_{i,t}, \tag{4.4}$$

with depreciation rate $\delta \in (0,1)$.

4.4 Fiscal Policy and the Security Umbrella

U.S. Security Guarantees. The US provides military insurance to allied countries under its umbrella. This is modelled in a parsimonious way: a portion of the U.S. federal budget, denoted s_t, is allocated to ***external security provision*** for allies. Security provision follows

$$s_t = \bar{s} + \kappa_s g_t, \qquad \kappa_s > 0, \tag{4.5}$$

where g_t is a U.S. fiscal–military mobilization shock. Higher g_t corresponds to greater U.S. defense effort and external security provision, but also to higher fiscal strain associated with sustaining that effort.

Defense floor (allies). This security provision partially provides for allied defense needs. Each allied country faces a minimum deterrence requirement $S^\star > 0$. Local defense outlays must satisfy

$$s_t + h_{i,t} \geq S^\star, \tag{4.6}$$

and are chosen at minimum resource cost, yielding a *defense floor*:

$$h_{i,t} = \max\{0,\, S^\star - s_t\}. \tag{4.7}$$

When U.S. security provision is strong ($s_t \geq S^\star$), the deterrence constraint is slack and allies fully free-ride on the US military insurance. When U.S. security provision is insufficient to meet the deterrence requirement ($s_t < S^\star$), local defense becomes inelastic and must rise one-for-one to close the security gap.

Government budget constraint. The government in country i issues one-period real debt $B_{i,t}$ and faces

$$B_{i,t+1} = (1+r)B_{i,t} + G^{soc}_{i,t} + I^{pub}_{i,t} + h_{i,t} - \tau_i Y_{i,t}, \tag{4.8}$$

where τ_i is a constant tax rate.

In the short run, non-defense fiscal components are sticky:

$$G^{soc}_{i,t} = \bar{G}_i, \qquad I^{pub}_{i,t} = \bar{I}_i, \tag{4.9}$$

so increases in defense spending mechanically raise public debt.

Security exposure. Countries differ in their exposure to the U.S. security umbrella through a fixed parameter $\phi_i \in (0,1)$. Higher ϕ_i reflects deeper integration into U.S. military and defense networks and amplifies the fiscal consequences of security retrenchment.

4.5 Sovereign Stress and Default Risk

Default intensity. Country i is subject to sovereign stress events (default, restructuring, or severe fiscal crisis). The conditional probability of such an event is governed by a Poisson intensity:

$$\lambda_{i,t} = \exp\big(\alpha_0 + \alpha_1 B_{i,t} + \alpha_2 g_t - \alpha_3 s_t + \alpha_4 \phi_i\big), \qquad \alpha_1, \alpha_2, \alpha_4 > 0. \tag{4.10}$$

Substituting the U.S. security-provision rule $s_t = \bar{s} + \kappa_s g_t$ into (4.10) yields

$$\lambda_{i,t} = \exp\Big(\alpha_0 - \alpha_3 \bar{s} + \alpha_1 B_{i,t} + (\alpha_2 - \alpha_3 \kappa_s)\, g_t + \alpha_4 \phi_i\Big). \tag{4.11}$$

Equation (4.11) makes clear that U.S. fiscal–military shocks affect sovereign risk through two opposing channels. An increase in g_t raises default risk directly by worsening fiscal strain and borrowing needs (the $\alpha_2 g_t$ term), but simultaneously lowers default risk by strengthening external security provision (the $-\alpha_3 \kappa_s g_t$ term). Net default risk therefore rises whenever the fiscal-burden effect dominates the insurance effect, i.e. when $\alpha_2 > \alpha_3 \kappa_s$.

This formulation captures the model's moral-hazard mechanism: security guarantees relax fiscal discipline ex ante, but periods of fiscal strain amplify sovereign risk when the insurance effect is insufficient to offset higher debt pressure.

4.6 Financial Markets and Security-Driven Pricing Wedges

Domestic pricing kernel. With log preferences, the domestic stochastic discount factor in country i is

$$M_{i,t+1} = \beta \frac{C_{i,t}}{C_{i,t+1}}. \tag{4.12}$$

Differences in expected returns therefore arise from differences in exposure to aggregate risk and financial frictions rather than time variation in risk aversion.

Incomplete markets and a security-driven enforcement wedge. International financial markets are incomplete due to limited contract enforceability and sovereign risk. The credibility of cross-border financial claims depends both on geopolitical backing and on the solvency of the issuing sovereign. When sovereign stress rises, enforcement of foreign-currency liabilities becomes

more fragile, and deviations from frictionless international asset pricing emerge.

To capture this mechanism parsimoniously, international Euler equations feature a country-specific wedge $\Lambda_{i,t}$:

$$\log \Lambda_{i,t} = \chi_E \, \phi_i \, s_t - \chi_\lambda \, \lambda_{i,t}, \qquad \chi_E, \chi_\lambda > 0, \tag{4.13}$$

where

- $s_t = \bar{s} + \kappa_s g_t$ denotes U.S. external security provision, - ϕ_i measures country i's exposure to the U.S. security umbrella, - $\lambda_{i,t}$ is sovereign default intensity.

Economic interpretation.

The first term, $\chi_E \phi_i s_t$, captures the enforcement-enhancing role of U.S. security provision. Stronger U.S. military commitments increase confidence in the enforcement of cross-border claims, particularly for highly exposed countries.

The second term, $-\chi_\lambda \lambda_{i,t}$, captures sovereign fragility. Higher default risk weakens the credibility of financial contracts and widens international pricing wedges.

Thus, the wedge aggregates two opposing forces: geopolitical insurance and sovereign stress. U.S. fiscal–military mobilization affects both.

4.6.1 Dollar Convenience Yield

To connect the security umbrella to the special role of the U.S. dollar, define the (real) dollar convenience yield as

$$CY_t \equiv r_t^* - r_t^{US}, \tag{4.14}$$

where r_t^* is the short rate that would prevail under symmetric enforcement ($\Lambda_{i,t} = 1$ for all i).

To close the simplified model while preserving the enforcement mechanism, assume

$$r_t^{US} = r_t^* - \bar{\Lambda}\big(\chi_E s_t - \chi_\lambda \bar{\lambda}_t\big), \qquad \bar{\Lambda} > 0, \tag{4.15}$$

where $\bar{\lambda}_t$ denotes aggregate sovereign stress.

Combining (4.14) and (4.15) yields

$$CY_t = \bar{\Lambda}\big(\chi_E s_t - \chi_\lambda \bar{\lambda}_t\big). \tag{4.16}$$

A stronger security umbrella increases the convenience yield by strengthening enforcement credibility. Conversely, rising sovereign stress compresses the convenience yield and tightens global financial conditions.

4.6.2 Exchange Rates and UIP Deviations

Let $rx_{t+1}^{i\$,}$ denote the log excess return from borrowing in U.S. dollars and investing in country i. The international Euler equation implies

$$1 = \mathbb{E}_t\left[M_{\$,t+1}\,\Lambda_{i,t}\,e^{rx_{t+1}^{i\$,}}\right]. \tag{4.17}$$

Log-linearizing yields

$$\mathbb{E}_t[rx_{t+1}^{i\$,}] \approx -\log\Lambda_{i,t} = -\chi_E\phi_i s_t + \chi_\lambda\lambda_{i,t}. \tag{4.18}$$

Interpretation. Currency premia are governed by two forces:

- **Security channel:** Higher U.S. military mobilization increases s_t, strengthens enforcement, and lowers required compensation for holding currencies of highly exposed countries. This makes high-ϕ_i currencies hedge assets.
- **Sovereign-risk channel:** Military mobilization also raises fiscal strain and default risk $\lambda_{i,t}$, increasing required premia through the $\chi_\lambda\lambda_{i,t}$ term.

The net effect depends on exposure. For highly integrated allies, the enforcement channel dominates, and their currencies earn lower expected excess returns. For low-exposure countries, sovereign risk dominates, and currencies command positive premia.

Thus, the cross-section of FX returns reflects the interaction between geopolitical insurance and sovereign fragility.

4.6.3 Sovereign Bonds

A one-period sovereign bond issued by country i pays

$$X^b_{i,t+1} = 1 - \mathcal{L}\,\mathbf{1}\{\text{default}\}, \qquad \mathcal{L} \in (0,1), \tag{4.19}$$

where $\mathcal{L}$ is the loss given default.

Bond prices satisfy

$$P^b_{i,t} = \mathbb{E}_t\Big[M_{i,t+1} X^b_{i,t+1}\Big]. \tag{4.20}$$

Under the standard approximation that default risk is conditionally independent of the pricing kernel (or that the associated covariance term is second order), this implies

$$P^b_{i,t} \approx \mathbb{E}_t[M_{i,t+1}]\left(1 - \mathcal{L}\lambda_{i,t}\right), \tag{4.21}$$

where $\lambda_{i,t}$ is the default intensity in (4.11).

Thus, sovereign spreads rise with default risk. U.S. fiscal–military mobilization affects spreads through two opposing forces: stronger security provision mitigates risk via s_t, but higher fiscal strain raises $\lambda_{i,t}$ via the debt and stress channel. In equilibrium, the fiscal-burden channel dominates for high-exposure countries, implying larger spread increases in economies with higher ϕ_i.

4.6.4 Equities

Equity is a claim to residual output net of fiscal absorption and subject to sovereign stress risk. Let the equity payoff be

$$X^e_{i,t+1} = D_{i,t+1}\left(1 - \mathcal{L}^e\,\mathbf{1}\{\text{default}\}\right), \tag{4.22}$$

where

$$D_{i,t} = Y_{i,t} - G^{soc}_{i,t} - I^{pub}_{i,t} - h_{i,t} \tag{4.23}$$

denotes net output, and $\mathcal{L}^e \in (0,1)$ captures equity losses in sovereign stress states. Equity therefore bears both macroeconomic risk and default-related losses.

Equity prices satisfy

$$P^e_{i,t} = \mathbb{E}_t\Big[M_{i,t+1}\, D_{i,t+1}\left(1 - \mathcal{L}^e\, \mathbf{1}\{\text{default}\}\right)\Big]. \tag{4.24}$$

Under conditional independence of default and the stochastic discount factor, a first-order approximation yields

$$P^e_{i,t} \approx \mathbb{E}_t[M_{i,t+1} D_{i,t+1}]\left(1 - \mathcal{L}^e\, \lambda_{i,t}\right). \tag{4.25}$$

Thus, equity premia increase with both:

(i) exposure to U.S. fiscal–military volatility through consumption risk, and (ii) exposure to sovereign stress through $\lambda_{i,t}$.

Countries with higher exposure ϕ_i experience larger increases in default intensity when U.S. military mobilization raises fiscal strain, amplifying both discount-rate risk and default risk in equity markets.

4.7 Equilibrium

An equilibrium is a collection of sequences $\{C_{i,t}, K_{i,t}, B_{i,t}, \lambda_{i,t}, P^b_{i,t}, P^e_{i,t}, rx^{i\$,}_t\}$ satisfying equations (4.2)–(**??**) and the market-clearing conditions above.

Key mechanism. U.S. fiscal–military mobilization shocks increase defense effort and external security provision, but at the cost of higher fiscal strain. For allied countries, this activates the defense floor when baseline security is insufficient, raising local defense outlays, public debt, and sovereign risk. At the same time, higher fiscal–military strain raises global discount rates and widens international Euler-equation wedges, generating UIP deviations and FX risk premia. Countries more deeply embedded in the U.S. security network (high ϕ_i) therefore exhibit larger discount-rate responses and higher risk premia across asset classes, alongside currencies that act as hedges against U.S. fiscal–military risk.

5 Model Solution

5.1 Equilibrium Characterization

Overview. Because financial markets are incomplete and sovereign default risk is present, the competitive equilibrium cannot be characterized by a social planner's problem. Instead, equilibrium allocations and prices are determined by decentralized household optimization, government budget constraints, and asset pricing conditions that feature country-specific enforcement wedges.

To obtain transparent analytical results, I proceed in two steps. First, I characterize equilibrium objects in a general setting with production, fiscal policy, and security-driven financial frictions. Second, I introduce a tractable closed-form special case that delivers explicit expressions for consumption, debt accumulation, sovereign risk, and asset prices.

5.2 Closed-Form Special Case

To obtain transparent analytical results, I introduce a short-run benchmark that admits closed-form solutions. Capital and labor are predetermined and fiscal components other than defense are sticky. This isolates the role of the security umbrella in driving cross-sectional transmission.

Production. In the short run,

$$K_{i,t} = \bar{K}_i, \qquad L_{i,t} = 1, \tag{5.1}$$

and local defense outlays act as a utilization wedge:

$$Y_{i,t} = \bar{Y}_i \exp(-\chi_h h_{i,t}), \qquad \chi_h > 0, \tag{5.2}$$

where $\bar{Y}_i \equiv A_i \bar{K}_i^{\alpha}$.

Investment rule. Investment follows a fixed share rule:

$$I_{i,t} = \iota_i Y_{i,t}, \qquad \iota_i \in (0,1), \tag{5.3}$$

so that short-run investment co-moves mechanically with output.

Consumption. The resource constraint implies

$$C_{i,t} = (1-\iota_i)Y_{i,t} - \bar{G}_i - \bar{I}_i - h_{i,t}. \tag{5.4}$$

Security provision and defense floor. U.S. security provision follows

$$s_t = \bar{s} + \kappa_s g_t, \qquad \kappa_s > 0, \tag{5.5}$$

where g_t is a U.S. fiscal–military shock. Each allied country faces a deterrence requirement $S^\star > 0$, generating the defense floor

$$h_{i,t} = \max\{0, S^\star - s_t\}. \tag{5.6}$$

When $s_t \geq S^\star$, the deterrence constraint is slack and allies fully free-ride on the U.S. umbrella. When $s_t < S^\star$, local defense becomes inelastic and must adjust one-for-one with security shortfalls.

All subsequent macro multipliers, asset prices, and risk premia are derived under this closed-form benchmark.

6 Qualitative Analysis

6.1 Security Umbrella and Fiscal Cyclicality

Setup. This section characterizes closed-form equilibrium fiscal responses to U.S. fiscal–military shocks in the tractable benchmark introduced in Section 5.2. The central object is the defense floor, which links U.S. security provision to local fiscal policy through a state-contingent constraint.

Corollary (Security Umbrella and State-Contingent Fiscal Cyclicality)

Let the defense floor be

$$h_{i,t} = \max\{0, S^\star - s_t\}, \qquad s_t = \bar{s} + \kappa_s g_t.$$

Then fiscal responses exhibit a regime-dependent kink:

Regime	$\frac{\partial h_{i,t}}{\partial g_t}$	Fiscal stance
$s_t \geq S^\star$	0	Flexible / countercyclical
$s_t < S^\star$	$-\kappa_s$	Inelastic / procyclical

When the umbrella is sufficiently strong ($s_t \geq S^\star$), local defense is slack and fiscal policy can adjust freely. When the umbrella is weak ($s_t < S^\star$), defense becomes inelastic and fiscal space is constrained.

Interpretation. The corollary highlights a central nonlinearity: the security umbrella reshapes local fiscal cyclicality by altering the slope of the defense burden with respect to aggregate shocks.

In the binding regime ($s_t < S^\star$), local defense spending is mechanically linked to the security shortfall:

$$h_{i,t} = S^\star - s_t.$$

An increase in U.S. military spending raises s_t, reduces the required local defense burden one-for-one, and directly frees fiscal resources. Because defense outlays enter both the resource constraint and the utilization wedge, this relief expands effective output and the tax base simultaneously. In effect, U.S. military provision operates as external fiscal insurance: movements in g_t partially offset what would otherwise be large and destabilizing adjustments in $h_{i,t}$. The local fiscal multiplier is therefore smoothed in states where defense burdens would otherwise be most distortionary.

By contrast, when the umbrella is already sufficiently strong ($s_t \geq S^\star$), the deterrence constraint

is slack and $h_{i,t} = 0$ locally. In this region, marginal changes in U.S. security provision have no first-order fiscal effects. Fiscal policy behaves flexibly and countercyclically because defense does not crowd out other spending categories.

These regime shifts imply that the stabilizing effect of the umbrella is strongest precisely in states where local fiscal adjustment would otherwise be most abrupt.

Proposition (Security Umbrella as a State-Contingent Fiscal Stabilizer)

Consider the closed-form benchmark with defense floor

$$h_{i,t} = \max\{0, S^\star - s_t\}, \qquad s_t = \bar{s} + \kappa_s g_t, \qquad \kappa_s > 0.$$

In the binding regime ($s_t < S^\star$), the variance of local fiscal adjustment satisfies

$$\mathrm{Var}(h_{i,t}) = \mathrm{Var}(S^\star - s_t) = \kappa_s^2 \,\mathrm{Var}(g_t), \tag{6.1}$$

and the marginal fiscal multiplier with respect to g_t is

$$\frac{\partial h_{i,t}}{\partial g_t} = -\kappa_s. \tag{6.2}$$

Thus, stronger U.S. security provision reduces the required local defense burden one-for-one in the binding region and compresses the volatility of domestic fiscal adjustment.

When the constraint is slack ($s_t \geq S^\star$),

$$h_{i,t} = 0, \qquad \mathrm{Var}(h_{i,t}) = 0, \qquad \frac{\partial h_{i,t}}{\partial g_t} = 0,$$

so fiscal volatility is locally insulated from marginal changes in U.S. security provision.

Implication. The security umbrella therefore functions as a *state-contingent fiscal stabilizer*. It dampens local fiscal multipliers precisely in states where defense burdens would otherwise be most distortionary. This stabilizing effect is strongest in countries with high exposure ϕ_i, for which the defense floor binds more frequently and more tightly. Geopolitical integration thus maps directly

into regime-dependent fiscal cyclicality.

6.2 Security Umbrella and Debt Dynamics

Corollary 4.1.2 (Debt Response)

Under the government budget constraint with sticky non-defense spending, the impact response of next-period public debt satisfies

$$\frac{\partial B_{i,t+1}}{\partial g_t} = -\kappa_s \, \mathbf{1}\{s_t < S^\star\} - \tau_i \, \chi_h \, \kappa_s \, Y_{i,t} \, \mathbf{1}\{s_t < S^\star\}. \tag{6.3}$$

Interpretation. While the previous subsection characterized the *flow* response of fiscal policy, Corollary 4.1.2 shows how the security umbrella shapes the *stock* of public debt.

When the deterrence constraint binds ($s_t < S^\star$), stronger U.S. security provision reduces required local defense spending one-for-one and simultaneously expands the tax base through higher effective output. Both forces lower next-period debt accumulation, as captured by (B.5).

However, this short-run fiscal relief embeds a moral-hazard mechanism. Because U.S. security provision insures allied economies against external threats, it relaxes fiscal discipline ex ante and lowers perceived sovereign risk. Borrowing becomes cheaper when security is abundant, and governments optimally choose higher leverage in states of strong U.S. protection. Debt accumulation therefore becomes endogenously tied to the credibility of the umbrella.

When security provision weakens or fiscal strain intensifies, this accumulated leverage amplifies sovereign stress. Thus, the debt channel links geopolitical insurance to financial fragility. Through its effect on default intensity $\lambda_{i,t}$ and expected consumption risk, debt dynamics become the bridge from real fiscal responses to equilibrium asset prices.

This transition naturally leads to the pricing of safe rates and risk premia.

6.3 Equilibrium Interest Rates

Setup. In each country i, the one–period real risk–free rate satisfies the domestic Euler equation

$$\frac{1}{R^f_{i,t}} = \mathbb{E}_t\big[M_{i,t+1}\big], \qquad M_{i,t+1} = \beta \frac{C_{i,t}}{C_{i,t+1}}, \tag{6.4}$$

where $M_{i,t+1}$ is the real stochastic discount factor.

Under the closed-form benchmark, U.S. fiscal–military shocks affect consumption growth through two channels: (i) the defense-floor relief channel (via s_t), and (ii) the fiscal-strain channel (via debt accumulation and default risk). Both channels scale with exposure ϕ_i.

Corollary (Interest Rates and Military Exposure)

Under conditional lognormality, the log risk-free rate satisfies

$$\begin{aligned}\log R^f_{i,t} &= -\log\beta + \mathbb{E}_t[\Delta \log C_{i,t+1}] - \frac{1}{2}\operatorname{Var}_t(\Delta \log C_{i,t+1}) \\ &= -\log\beta - \vartheta_i \phi_i \kappa_s \,\mathbb{E}_t[\Delta g_{t+1}] - \frac{1}{2}(\vartheta_i \phi_i \kappa_s)^2 \operatorname{Var}_t(\Delta g_{t+1}) \\ &\quad - \frac{1}{2}\operatorname{Var}_t(\Delta \Xi^C_{i,t+1}),\end{aligned} \tag{6.5}$$

where $\vartheta_i > 0$ captures how security provision and fiscal strain translate into consumption risk.

Comparative statics. Differentiating (6.5) with respect to exposure ϕ_i yields

$$\frac{\partial \log R^f_{i,t}}{\partial \phi_i} = -\vartheta_i \kappa_s \,\mathbb{E}_t[\Delta g_{t+1}] - (\vartheta_i \kappa_s)^2 \phi_i \operatorname{Var}_t(\Delta g_{t+1}). \tag{6.6}$$

The second term is strictly negative whenever $\operatorname{Var}_t(\Delta g_{t+1}) > 0$, implying

$$\frac{\partial \log R^f_{i,t}}{\partial \phi_i} < 0.$$

Proposition (Military Exposure and Safe Rates)

Countries more exposed to U.S. fiscal–military shocks (higher ϕ_i) exhibit lower equilibrium real risk–free rates. Greater exposure amplifies precautionary savings through higher consumption volatility induced by sovereign risk and fiscal strain.

Interpretation. The key distinction is between contemporaneous fiscal relief and forward-looking risk pricing.

Although stronger U.S. military spending can relax binding defense constraints in the short run, it simultaneously increases exposure to future fiscal strain and sovereign stress. Higher exposure ϕ_i magnifies the volatility of consumption growth through both the defense-floor channel and the default-risk channel.

Because log utility prices variance directly, greater exposure raises precautionary savings motives and compresses safe rates. Thus, high-exposure countries exhibit lower equilibrium real interest rates, not because they are fundamentally safer, but because they are more tightly linked to globally priced fiscal–military risk.

Debt accumulation, sovereign stress, and enforcement frictions therefore jointly shape the cross-section of safe rates and form the foundation for the asset-pricing implications that follow.

6.4 Currency Risk Premia

Setup. Under incomplete markets, exchange rates and currency premia are not pinned down by relative stochastic discount factors alone. Instead, international Euler equations feature a country-specific wedge that reflects both U.S. security provision and local sovereign risk.

Let $rx_{t+1}^{i\$,}$ denote the log excess return to a U.S. investor from borrowing in dollars and investing in country i. The international Euler equation takes the form

$$1 = \mathbb{E}_t\left[M_{\$,t+1}\,\Lambda_{i,t}\,e^{rx_{t+1}^{i\$,}}\right], \tag{6.7}$$

where the wedge is

$$\log \Lambda_{i,t} = \chi_E \, \phi_i \, s_t - \chi_\lambda \, \lambda_{i,t}, \qquad \chi_E, \chi_\lambda > 0. \tag{6.8}$$

Here $s_t = \bar{s} + \kappa_s g_t$ denotes U.S. security provision and $\lambda_{i,t}$ is sovereign default intensity.

Corollary 6.0.1 (Currency Risk Premia with Security and Default Risk)

Under conditional lognormality and the wedge specification above, expected currency excess returns satisfy

$$\boxed{\mathbb{E}_t[rx^{i\$,}_{t+1}] \;\approx\; -\log \Lambda_{i,t} = -\chi_E \, \phi_i \, s_t + \chi_\lambda \, \lambda_{i,t}.} \tag{6.9}$$

Cross-sectional implications. Holding aggregate conditions fixed, differentiation yields

$$\frac{\partial \mathbb{E}_t[rx^{i\$,}_{t+1}]}{\partial \phi_i} = -\chi_E s_t + \chi_\lambda \frac{\partial \lambda_{i,t}}{\partial \phi_i}. \tag{6.10}$$

Because default risk rises with exposure (via $\alpha_4 \phi_i$ in the default-intensity specification), the second term is positive. However, the enforcement channel dominates for sufficiently large s_t, so that

$$\frac{\partial \mathbb{E}_t[rx^{i\$,}_{t+1}]}{\partial \phi_i} < 0.$$

Proposition I (Currency Risk Premia and Security Exposure)

Currencies of countries that are more exposed to the U.S. security umbrella (higher ϕ_i) earn lower expected excess returns and act as hedge currencies. Conversely, currencies of less exposed countries are procyclical and command positive risk premia.

Interpretation. U.S. fiscal–military mobilization affects currency premia through two opposing channels. Stronger security provision (s_t) tightens enforcement and improves the credibility of cross-border claims, lowering required compensation for holding assets of high-exposure countries. At

the same time, mobilization raises fiscal strain and default risk ($\lambda_{i,t}$), increasing required returns.

For countries deeply integrated into the U.S. security network (high ϕ_i), the enforcement channel dominates in equilibrium. Their currencies co-move more closely with the U.S. stochastic discount factor, appreciate in periods of heightened U.S. fiscal–military risk, and therefore earn lower average excess returns. Low-exposure currencies provide less insurance and must offer higher premia as compensation.

6.5 Global Equity Risk Premia

Setup. Equity claims are exposed to U.S. fiscal–military shocks through two reinforcing channels: (i) discount-rate risk induced by security-driven consumption volatility and (ii) sovereign stress risk captured by the default intensity $\lambda_{i,t}$. In the closed-form benchmark, exposure to U.S. military mobilization is summarized by ϕ_i, and security provision satisfies $s_t = \bar{s} + \kappa_s g_t$.

Corollary (Equity Risk Premia, Security Exposure, and Sovereign Stress)

In the binding regime ($s_t < S^\star$), log consumption growth admits the one-factor representation

$$\Delta \log C_{i,t+1} = -\vartheta_i \, \phi_i \, \kappa_s \, \Delta g_{t+1} + \Delta \xi_{i,t+1}, \qquad \mathrm{Cov}_t(\Delta g_{t+1}, \Delta \xi_{i,t+1}) = 0,$$

so that

$$\mathrm{Var}_t(\Delta \log C_{i,t+1}) = (\vartheta_i \phi_i \kappa_s)^2 \, \mathrm{Var}_t(\Delta g_{t+1}) + \mathrm{Var}_t(\Delta \xi_{i,t+1}).$$

Under log utility and conditional lognormality, the equilibrium equity risk premium satisfies

$$\boxed{\mathrm{ERP}_{i,t} \;\simeq\; \tfrac{1}{2}(\vartheta_i \phi_i \kappa_s)^2 \, \mathrm{Var}_t(\Delta g_{t+1}) \;+\; \mathcal{L}^e \, \lambda_{i,t} \;+\; \underbrace{\tfrac{1}{2} \, \mathrm{Var}_t(\Delta \xi_{i,t+1})}_{\text{country-specific component}}}. \tag{6.11}$$

When $s_t \geq S^\star$, the defense floor is slack and the g-driven component is locally zero.

Interpretation. Equation (6.11) makes the cross-sectional mechanism transparent. In the binding regime, exposure ϕ_i scales the volatility of consumption growth with respect to U.S. military

mobilization, so the discount-rate component of equity premia increases with ϕ_i^2. In addition, higher sovereign stress $\lambda_{i,t}$ raises required equity compensation through the stress-event term $\mathcal{L}^e \lambda_{i,t}$. Together, these channels imply that equity risk premia rise systematically in countries that are more exposed to the U.S. security umbrella, consistent with the empirical cross-sectional ordering of returns.

Proposition (Cross-Sectional Dynamics of Equity Risk Premia)

In the binding regime ($s_t < S^\star$), equity risk premia are increasing in exposure to the U.S. security umbrella:

$$\frac{\partial \mathrm{ERP}_{i,t}}{\partial \phi_i} = (\vartheta_i^2 \kappa_s^2 \phi_i)\ \mathrm{Var}_t(\Delta g_{t+1})\ +\ \mathcal{L}^e\, \frac{\partial \lambda_{i,t}}{\partial \phi_i}\ >\ 0. \tag{6.12}$$

Thus, countries more exposed to U.S. military mobilization exhibit higher equilibrium equity risk premia.

Moreover, the cross-sectional dispersion of equity premia widens in periods of elevated U.S. fiscal–military volatility:

$$\frac{\partial^2 \mathrm{ERP}_{i,t}}{\partial \phi_i\, \partial\, \mathrm{Var}_t(\Delta g_{t+1})} > 0. \tag{6.13}$$

Implication. The proposition formalizes the central cross-sectional prediction of the model. Exposure to the U.S. security umbrella does not merely shift average returns — it amplifies sensitivity to U.S. fiscal–military volatility. When geopolitical risk rises (higher $\mathrm{Var}_t(\Delta g_{t+1})$), equity premia increase disproportionately in high-exposure countries. This generates time-varying dispersion in returns across countries sorted by military integration, consistent with the empirical evidence on security-sorted factor-mimicking portfolios.

6.6 Bond Premia

Setup. Sovereign bonds are exposed to U.S. fiscal–military shocks through their effect on public debt and default risk. A one-period sovereign bond issued by country i pays 1 in the absence of

default and $1-\mathcal{L}$ in default, where $\mathcal{L} \in (0,1)$ is the loss given default. Let $\lambda_{i,t}$ denote the conditional default intensity.

Bond prices satisfy

$$P^b_{i,t} = \mathbb{E}_t\Big[M_{i,t+1}\big(1 - \mathcal{L}J_{i,t+1}\big)\Big], \tag{6.14}$$

so that, under conditional independence of default and the stochastic discount factor,

$$P^b_{i,t} \approx \mathbb{E}_t[M_{i,t+1}]\left(1 - \mathcal{L}\lambda_{i,t}\right). \tag{6.15}$$

Define the sovereign spread as

$$\mathrm{spr}_{i,t} \equiv \log R^b_{i,t} - \log R^f_{i,t}.$$

Default risk. Default intensity is given by

$$\lambda_{i,t} = \exp\big(\alpha_0 + \alpha_1 B_{i,t} + \alpha_2 g_t - \alpha_3 s_t + \alpha_4 \phi_i\big), \qquad \alpha_1, \alpha_2, \alpha_4 > 0, \tag{6.16}$$

with $s_t = \bar{s} + \kappa_s g_t$. U.S. fiscal–military mobilization therefore affects sovereign risk through two channels: a fiscal-strain channel (via g_t) and a security-insurance channel (via s_t).

Corollary (Bond Premia and Security Exposure)

Under the pricing relation above, sovereign spreads satisfy

$$\boxed{\mathrm{spr}_{i,t} = -\log\big(1 - \mathcal{L}\lambda_{i,t}\big).} \tag{6.17}$$

Cross-sectional implication. Holding aggregate conditions fixed and treating $B_{i,t}$ as predetermined,

$$\frac{\partial\, \mathrm{spr}_{i,t}}{\partial \phi_i} = \frac{\mathcal{L}\alpha_4\, \lambda_{i,t}}{1 - \mathcal{L}\lambda_{i,t}} \;>\; 0. \tag{6.18}$$

Sovereign bond premia therefore increase strictly with exposure to the U.S. security umbrella.

Interpretation. U.S. fiscal–military mobilization raises global risk by increasing fiscal strain and sovereign vulnerability. Although stronger security provision (s_t) mitigates default risk holding debt fixed, it simultaneously encourages higher leverage through a moral-hazard channel. Countries with higher exposure ϕ_i accumulate more debt in tranquil periods and become more sensitive to global fiscal stress.

In equilibrium, the sovereign-risk channel dominates. As a result, high-exposure countries experience larger increases in default intensity and sovereign spreads following U.S. fiscal–military shocks. This mechanism generates a systematic cross-sectional ordering of bond premia by military exposure, consistent with the empirical evidence.

6.7 Cross-Asset Betas

This subsection summarizes how exposure to U.S. fiscal–military shocks maps into systematic risk across currencies, equities, and sovereign bonds. Despite differences in payoff structure, all asset betas are governed by a single exposure parameter: ϕ_i.

Definition. For any (log) asset payoff or return $X_{i,t+1}$, define the beta with respect to the U.S. fiscal–military shock Δg_{t+1} as

$$\beta_i^X \equiv \frac{\mathrm{Cov}(X_{i,t+1},\, \Delta g_{t+1})}{\mathrm{Var}(\Delta g_{t+1})}.$$

Corollary (Defense Exposure and Cross-Asset Betas)

Under the closed-form benchmark with $s_t = \bar{s} + \kappa_s g_t$, asset betas satisfy:

(i) Currency beta

$$\beta_i^{FX} = -\chi_E\, \kappa_s\, \phi_i, \tag{6.19}$$

so high-exposure currencies act as hedges against U.S. fiscal–military shocks.

(ii) Equity beta

$$\beta_i^e = \kappa_s \, \phi_i \, \Xi_i, \tag{6.20}$$

where $\Xi_i > 0$ collects the net cash-flow and discount-rate sensitivity of equity returns to Δg_{t+1}. Equity betas are therefore increasing in exposure ϕ_i.

(iii) Sovereign bond beta

$$\beta_i^b = \frac{\mathcal{L}\,\lambda_{i,t}}{1 - \mathcal{L}\,\lambda_{i,t}}\big(\alpha_2 - \alpha_3 \kappa_s\big), \tag{6.21}$$

which is positive whenever the fiscal-risk channel dominates the insurance channel, i.e. $\alpha_2 > \alpha_3 \kappa_s$.

Cross-sectional ordering. Each beta varies monotonically with defense exposure:

$$\frac{\partial \beta_i^{FX}}{\partial \phi_i} < 0, \qquad \frac{\partial \beta_i^e}{\partial \phi_i} > 0, \qquad \frac{\partial \beta_i^b}{\partial \phi_i} > 0.$$

Thus, higher exposure to the U.S. security umbrella increases systematic risk in equity and bond markets, while simultaneously pushing currencies toward hedge behavior.

Proposition II (Unified Cross-Asset Beta Structure)

A single exposure parameter ϕ_i jointly orders country-level FX, equity, and sovereign bond betas with respect to U.S. fiscal–military shocks. Countries with higher ϕ_i exhibit higher equity and bond betas and more negative FX betas.

Interpretation. U.S. fiscal–military spending raises global risk premia through a common channel: fiscal strain and moral hazard induced by the security umbrella. Higher exposure ϕ_i magnifies the sensitivity of dividends, borrowing costs, and discount rates to U.S. military mobilization. In equity and bond markets, this amplification increases systematic risk and expected returns. In currency markets, the same exposure strengthens enforcement and risk sharing with the United

States, causing high-ϕ_i currencies to appreciate in adverse states and serve as global hedges.

Despite their different payoff structures, all three asset classes therefore load on the same underlying geopolitical risk factor: exposure to U.S. fiscal–military power.

6.7.1 HML FX Strategy Sorted on U.S. Military Exposure

Setup. Consider a currency strategy that sorts countries by exposure to the U.S. security umbrella. Let H denote the set of high-exposure countries and L the set of low-exposure countries, with average exposures

$$\bar{\phi}_H \equiv \frac{1}{|H|}\sum_{i\in H}\phi_i, \qquad \bar{\phi}_L \equiv \frac{1}{|L|}\sum_{i\in L}\phi_i, \qquad \bar{\phi}_H > \bar{\phi}_L.$$

Define the HML FX portfolio as a long position in low-exposure currencies and a short position in high-exposure currencies:

$$rx^{HML}_{t+1} \equiv \frac{1}{|L|}\sum_{i\in L} rx^{i\$,}_{t+1} - \frac{1}{|H|}\sum_{j\in H} rx^{j\$,}_{t+1}.$$

Under incomplete markets, currency premia satisfy the wedge-based UIP condition

$$\mathbb{E}_t[rx^{i\$,}_{t+1}] \approx -\log\Lambda_{i,t}, \qquad \log\Lambda_{i,t} = \chi_E\phi_i s_t - \chi_\lambda\lambda_{i,t}.$$

Corollary (HML FX Returns and Military Exposure)

Under conditional lognormality, expected HML returns satisfy

$$\begin{aligned}\mathbb{E}_t[rx^{HML}_{t+1}] &\approx -\Big(\overline{\log\Lambda}_{L,t} - \overline{\log\Lambda}_{H,t}\Big)\\ &= \chi_E\, s_t\,(\bar{\phi}_H - \bar{\phi}_L) \;-\; \chi_\lambda\big(\bar{\lambda}_{H,t} - \bar{\lambda}_{L,t}\big),\end{aligned} \tag{6.22}$$

where $\bar{\lambda}_{H,t}$ and $\bar{\lambda}_{L,t}$ denote average default intensity in the high- and low-exposure legs, respectively.

Proposition (HML FX Strategy)

If the enforcement channel dominates the default-risk channel in the cross-section—i.e.,

$$\chi_E \, s_t \, (\bar{\phi}_H - \bar{\phi}_L) \; > \; \chi_\lambda \big(\bar{\lambda}_{H,t} - \bar{\lambda}_{L,t}\big),$$

then

$$\mathbb{E}_t[rx^{HML}_{t+1}] > 0.$$

Interpretation. Equation (6.22) delivers a direct link between the profitability of security-sorted FX portfolios and the model's key state variables.

When U.S. military mobilization increases, security provision rises according to $s_t = \bar{s} + \kappa_s g_t$. This strengthens enforcement and compresses international wedges most strongly for high-exposure countries (large ϕ_i), lowering their required currency premia. As a result, high-ϕ currencies behave as hedge currencies and earn low expected excess returns.

At the same time, mobilization can raise sovereign stress and default intensity $\lambda_{i,t}$, which pushes in the opposite direction by widening wedges. The HML strategy is profitable precisely when the security/enforcement channel dominates: the spread in enforcement premia across exposure groups, $\chi_E s_t(\bar{\phi}_H - \bar{\phi}_L)$, outweighs the spread in sovereign stress, $\chi_\lambda(\bar{\lambda}_{H,t} - \bar{\lambda}_{L,t})$. This condition links time-variation in HML FX returns to movements in s_t and $\lambda_{i,t}$ and preserves the paper's central prediction that geopolitical exposure prices currency risk.

6.8 Summary of Empirical Implications

This subsection consolidates the equilibrium implications of the model into a unified set of testable predictions. All results follow from one mechanism: U.S. fiscal–military spending g_t increases U.S. security provision $s_t = \bar{s} + \kappa_s g_t$, while simultaneously raising sovereign stress $\lambda_{i,t}$ through fiscal strain and moral hazard.

When the deterrence constraint binds ($s_t < S^\star$), stronger U.S. security provision relaxes local defense requirements and improves short-run macro outcomes. At the same time, higher military

spending increases globally priced risk through default intensity and enforcement wedges, generating systematic cross-sectional variation in asset prices.

Table 3: Summary of Equilibrium Predictions and Empirical Implications

Object	Equilibrium Prediction	Mechanism
Panel A: Macroeconomic Quantities (Closed-Form Benchmark)		
Defense outlays	$\frac{\partial h_{i,t}}{\partial g_t} = -\kappa_s \, \phi_i \, \mathbf{1}\{s_t < S^\star\}$	Defense floor substitution
Output (GDP)	$\frac{\partial Y_{i,t}}{\partial g_t} = \chi_h \, \kappa_s \, \phi_i \, Y_{i,t} \, \mathbf{1}\{s_t < S^\star\}$	Utilization wedge
Investment	$\frac{\partial I_{i,t}}{\partial g_t} = \iota_i \chi_h \, \kappa_s \, \phi_i \, Y_{i,t} \, \mathbf{1}\{s_t < S^\star\}$	Fixed investment share
Consumption	$\frac{\partial C_{i,t}}{\partial g_t} = (1-\iota_i)\frac{\partial Y_{i,t}}{\partial g_t} + \kappa_s \phi_i \mathbf{1}\{s_t < S^\star\}$	Defense relief and output channel
Global macro betas	$\beta^C_{i,G}, \, \beta^Y_{i,G} \uparrow$ in ϕ_i	One-factor exposure to g_t
Panel B: Financial Prices and Risk Premia		
Real short rate	$\frac{\partial \log R^f_{i,t}}{\partial \phi_i} < 0$	Precautionary savings under fiscal risk
Default intensity	$\lambda_{i,t} = \exp\big(\tilde{\alpha}_0 + \alpha_1 B_{i,t} + (\alpha_2 - \alpha_3 \kappa_s) g_t + \alpha_4 \phi_i\big)$	Fiscal strain vs. security insurance
Sovereign spread	$\mathrm{spr}_{i,t} = -\log(1 - \mathcal{L}\lambda_{i,t})$	Default risk premium
Currency risk premium (USD)	$\mathbb{E}_t[rx^{i\$,}_{t+1}] \approx -\chi_E \phi_i s_t + \chi_\lambda \lambda_{i,t}$	Enforcement wedge + sovereign risk
Cross-asset betas	$\beta^{FX}_i < 0$; $\beta^b_i, \beta^e_i > 0$ increasing in ϕ_i	Common exposure to g_t and $\lambda_{i,t}$
Panel C: Trading Strategies		
Long U.S. defense equities	Positive excess return	Direct loading on g_t
HML FX (low–ϕ minus high–ϕ)	$\mathbb{E}_t[rx^{HML}] > 0$	Hedge currencies at high ϕ
HML bond/equity (high–ϕ minus low–ϕ)	Higher premia for high-ϕ countries	Default + discount-rate channels

Unifying perspective. The model delivers a single organizing force for the global cross-section: exposure to the U.S. security umbrella, ϕ_i. Higher U.S. military spending strengthens security provision but raises global risk premia by increasing fiscal strain and incentivizing sovereign risk-taking through a moral-hazard channel. Countries with greater exposure experience larger movements in discount rates, default risk, and asset prices, even when average macro responses remain modest.

This structure rationalizes the core empirical findings of the paper: (i) strong cross-sectional heterogeneity in fiscal transmission tied to U.S. military presence, (ii) a consistent ordering of equity, bond, and currency risk premia by security exposure, and (iii) the profitability of security-sorted factor-mimicking portfolios across asset classes.

7 Quantitative Performance

This section moves from the closed-form benchmark to a more quantitative framework that I relegate to the theory appendix section C. The goal is not to alter the qualitative mechanism—that U.S. fiscal conditions transmit globally through the security umbrella—but to enrich the environment so that the model can quantitatively match the cross-sectional dynamics of the US fiscal transmission into global asset prices across i) equity, ii) bond and iii) FX markets. I describe very briefly the additional ingredients added.

7.1 From Closed Form to Quantitative Discipline

The quantitative analysis builds on the same economic mechanism developed in the main text but embeds it in a richer dynamic environment in order to discipline magnitudes and timing. Relative to the tractable benchmark used to derive closed-form results, the quantitative model adds four elements that are essential for matching the dynamic patterns observed in the data.

(1) Infinite horizon and recursive preferences. The closed-form analysis relies on log preferences and conditional lognormality to deliver transparent comparative statics. The quantitative model instead adopts an infinite-horizon economy with Epstein–Zin preferences. This extension allows the model to separate risk aversion from intertemporal substitution and to generate time-varying risk premia. As a result, shocks to the U.S. security umbrella affect not only impact responses but also the persistence of global discount-rate movements, allowing the model to match the slow adjustment in valuations and returns observed empirically.

(2) Security umbrella as an enforcement wedge. In both the benchmark model and the quantitative framework, U.S. security provision enters international asset pricing through an enforcement wedge in Euler equations. In the main text, this wedge is introduced in a parsimonious reduced-form manner to isolate the core mechanism: countries that are more exposed to the U.S. security umbrella face systematically different pricing of risk in currency and bond markets.

In the quantitative model, this enforcement wedge is micro-founded more fully. Security provision affects the tightness of international financial constraints, the credibility of repayment, and the pricing of dollar-denominated claims. This richer formulation allows the wedge to jointly shape cross-currency interest differentials, UIP deviations, and a time-varying dollar convenience yield. Conceptually, however, the role of the wedge is the same in both models: U.S. security provision relaxes enforcement constraints and compresses required risk compensation for exposed countries.

(3) Endogenous allied fiscal risk and sovereign spreads. The quantitative model retains and amplifies the fiscal-risk channel highlighted in the theory. Security provision affects allied fiscal capacity by relaxing deterrence constraints, altering defense spending needs, and influencing debt accumulation. Higher exposure to the U.S. security umbrella therefore translates into greater sensitivity of sovereign spreads and default risk to U.S. fiscal–military shocks. This channel is crucial for matching the joint behavior of bond spreads, equity risk premia, and exchange rates in the data.

(4) Imperfect information and delayed adjustment. Empirically, asset prices react sharply on impact to U.S. fiscal and military news, but risk premia and returns adjust gradually over subsequent quarters. A full-information version of the model generates adjustment that is too rapid. To match the observed timing, the quantitative framework introduces imperfect information about the persistence and credibility of U.S. security commitments. Governments observe the true security state, while private agents infer it from noisy signals ("military news") and update beliefs gradually. This learning friction generates sluggish movements in discount rates and produces the medium-horizon predictability patterns documented in the empirical section.

Together, these extensions preserve the core mechanism of the benchmark model while allowing it to match the magnitude, persistence, and joint dynamics of global risk premia, exchange rates, and sovereign spreads. The next subsection describes the calibration and simulation strategy used to evaluate the quantitative performance of the model.

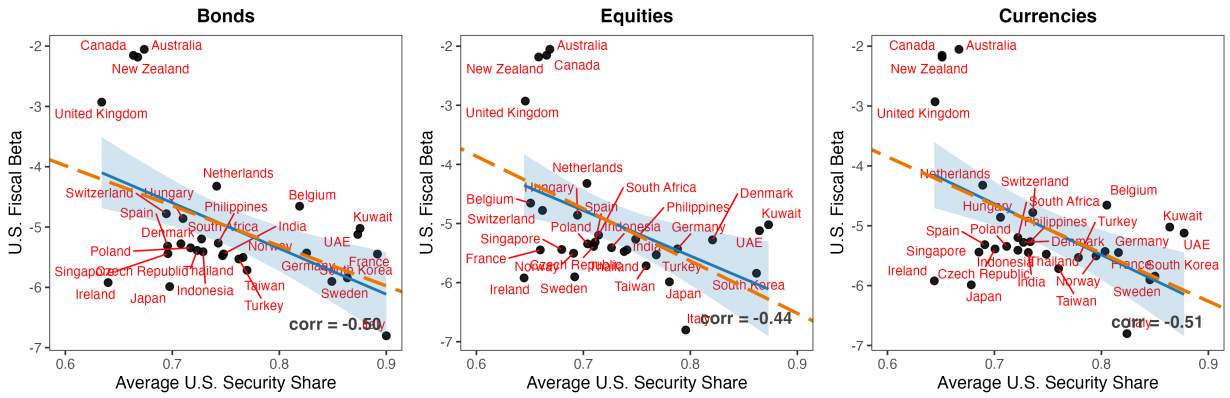


Figure 4: Cross-asset fiscal transmission and U.S. military presence: data vs. model. Each panel plots the country-level U.S. fiscal beta $\hat{\beta}_i^{\text{Fiscal}}$ against average U.S. security exposure (U.S. security share). The three panels correspond to fiscal betas estimated in *sovereign bonds* (left), *equities* (center), and *currencies* (right). The solid blue line is the empirical cross-sectional fit with a 90% confidence band. The dashed orange line is the model-implied cross-sectional slope (with sign convention matching the exchange-rate definition in the model). Across all three markets, the data exhibit a pronounced negative relationship: countries with greater U.S. security exposure display larger-magnitude fiscal betas and more sensitive asset-price responses to U.S. fiscal shocks.

7.2 Cross-Country Heterogeneity in Fiscal Transmission

Interpretation. Figure 4 provides a compact quantitative validation of the model's central cross-sectional mechanism. The model is disciplined only by exposure heterogeneity—captured by the security share—and therefore has a sharp quantitative prediction: a single exposure channel should organize the cross-country slope of U.S. fiscal transmission *simultaneously* in bond, equity, and currency markets.

The data strongly support this prediction. In each panel, fiscal betas line up monotonically with U.S. military presence: countries that are more deeply embedded in the U.S. security umbrella exhibit systematically larger responses to U.S. fiscal innovations. The model-implied slope closely tracks the fitted empirical slope in all three markets, indicating that the same security-based mechanism can account for both the sign and magnitude of the observed cross-sectional gradient.

The cross-asset consistency is particularly informative. In the model, U.S. fiscal stress weakens the security umbrella, activates the defense floor, and amplifies fiscal strain in high-exposure countries. This raises discount rates and sovereign risk, which depresses bond and equity valuations more strongly where U.S. military presence is high. At the same time, the security umbrella enters currency markets through an Euler-equation wedge, implying that high-exposure currencies behave more like hedges and therefore load differently on the same fiscal shock. The fact that the model

matches the relative cross-sectional ordering across all three markets strengthens the interpretation that U.S. military presence is the key sufficient statistic governing global fiscal transmission in the data.

7.3 Factor-Mimicking Portfolios

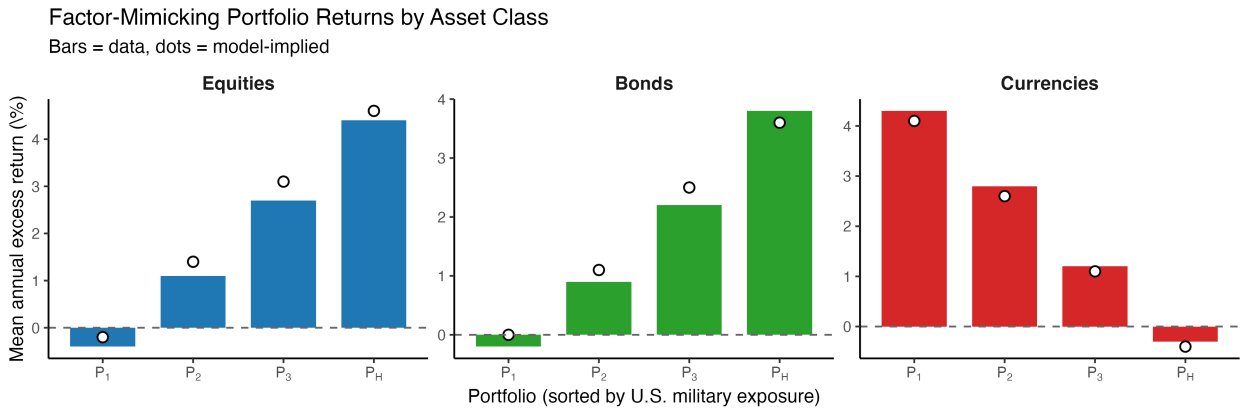


Figure 5: Factor-mimicking portfolio returns sorted on U.S. military presence: data vs. model. The figure plots mean annual excess returns for portfolios formed by sorting countries into four groups (P_L, P_2, P_3, P_H) based on exposure to the U.S. security umbrella. Panels correspond to *equities*, *sovereign bonds*, and *currencies*. Bars report empirical average returns from the data, while circles denote model-implied returns calibrated to match the monotonic cross-sectional patterns. For equities and bonds, average excess returns increase with U.S. military exposure, whereas for currencies the ordering is reversed: high-exposure currencies earn lower average returns and function as hedge currencies.

Portfolio Returns. Figure 5 translates the cross-country heterogeneity in U.S. fiscal transmission documented above into tradable asset-pricing implications. Countries that are more exposed to the U.S. security umbrella—those with larger fiscal betas in response to U.S. fiscal–military shocks—systematically occupy the high-return end of the cross-section in equity and sovereign bond markets. Conversely, the same countries lie at the low-return end of the currency cross-section, consistent with their exchange rates appreciating in response to U.S. fiscal stress.

This pattern mirrors the cross-sectional fiscal beta evidence in Section 3: exposure to U.S. military presence amplifies the transmission of U.S. fiscal shocks into local asset prices, but with opposite implications for risk premia across markets. In equities and bonds, higher exposure increases cash-flow volatility, sovereign risk, and discount-rate sensitivity, generating higher expected returns. In currency markets, higher exposure instead strengthens comovement with the U.S.

stochastic discount factor, causing high-exposure currencies to hedge U.S. fiscal–military risk and earn lower average excess returns.

Simulated Time Series. To assess whether the model can account for the *dynamic* behavior of security-sorted portfolios, I simulate artificial time series from the quantitative model and construct factor-mimicking portfolios in the same way as in the data. To accommodate space, I relegate technical details of this procedure to the theory appendix. To describe procedure at a high level, the simulation proceeds by feeding persistent model shocks (fiscal, TFP, financial) into the model's state vector and propagating them through the equilibrium pricing equations. These shocks affect global risk premia through the security-enforcement channel and generate joint movements in equity returns, sovereign spreads, and exchange rates. Countries differ only in their exposure to the U.S. security umbrella, so the simulated portfolio returns reflect heterogeneous loadings on a common set of global states rather than asset-specific disturbances.

The simulated returns are aggregated into cumulative log returns over the sample period and compared to their empirical counterparts. Importantly, the model is not calibrated to match the time-series paths of the portfolios; it is disciplined solely by cross-sectional exposure and persistence of the underlying state variables. As a result, the comparison focuses on whether the model reproduces the timing, comovement, and relative severity of drawdowns during major global stress episodes, rather than one-for-one tracking of realized returns.

Interpretation. Figure 6 shows that the quantitative model reproduces not only the *cross-sectional ordering* of factor-mimicking portfolio returns, but also their *time-series dynamics*. In the model, all portfolios are driven by a small number of persistent global state variables—most importantly, shocks to U.S. fiscal–military conditions and the associated security-enforcement wedge—while countries differ only in their exposure to these shocks.

This structure implies two testable time-series predictions. First, security-sorted portfolios should comove strongly during global stress episodes, with especially pronounced drawdowns (or appreciations, in the case of currencies) around events that weaken U.S. military capacity or fiscal credibility. Second, differences across asset classes should reflect how exposure enters pricing: higher

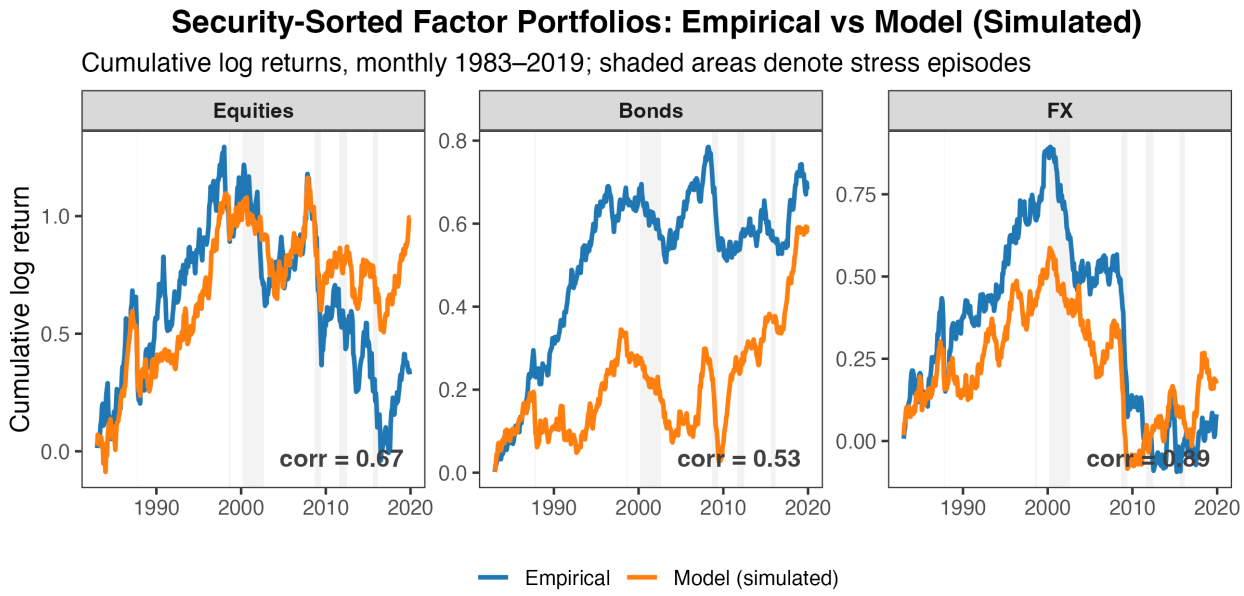


Figure 6: Security-sorted factor portfolios: empirical and simulated time series. The figure plots cumulative log returns of factor-mimicking portfolios constructed by sorting countries on exposure to the U.S. security umbrella, separately for *equities*, *sovereign bonds*, and *currencies*. Solid blue lines report simulated empirical portfolio returns constructed to match the cross-sectional fiscal-beta evidence, while orange lines report returns generated by the quantitative model. Shaded vertical bands denote major global stress episodes (1987 crash, 1998 LTCM/Russia, 2000–02 dot-com bust, 2008–09 global financial crisis, 2011–12 euro-area crisis, and 2015–16 China/commodity stress). Reported correlations summarize comovement between empirical and model-simulated series within each asset class.

exposure raises risk premia and volatility in equity and sovereign bond portfolios, but increases hedging value and lowers average returns in currency portfolios.

Both features are visible in the data and in the simulations. The model generates large joint drawdowns in equity and bond portfolios during stress episodes such as the global financial crisis, alongside appreciations of high-exposure currencies. At the same time, the simulated series are deliberately imperfect—correlations are high but well below one—reflecting the fact that the model abstracts from asset-specific shocks and market frictions not directly tied to the security channel.

Overall, the time-series evidence complements the cross-sectional results. A single mechanism—heterogeneous exposure to U.S. military presence interacting with persistent fiscal–military shocks—organizes both the ranking of average returns across portfolios and their dynamic behavior over time. This joint fit reinforces the interpretation that U.S. military hegemony operates as a central driver of global fiscal transmission and risk pricing across asset classes.

7.4 International Finance Moments

Unmatched Moments: To conclude the paper, I show that the model can broadly capture other important international finance facts that it was not designed to match. These moments are summarised in table 4 and constitute an external validation test for the model.

Across international business-cycle moments (Panel (a)), the full-information model generates stronger cross-country comovement in output and consumption than observed in the data, reflecting the fact that common shocks are instantaneously and uniformly priced when agents observe the underlying state variables. Introducing delayed learning attenuates these correlations and brings them closer to empirical benchmarks by slowing the transmission of U.S. fiscal and security news into global decisions and prices.

Panel (b) shows that both model variants match the persistence and volatility of U.S. fiscal conditions reasonably well, though the partial-information model produces slightly higher persistence by construction. This additional persistence plays an important role for asset pricing, as it allows fiscal and security shocks to affect expectations over longer horizons.

Panel (c) highlights the asset-pricing implications. Both models generate a positive global equity premium and a safe-haven role for the dollar, but the partial-information economy produces substantially more time variation in discount rates and risk premia. As a result, the partial-information model generates higher volatility in the dollar convenience yield and in global equity excess returns. At the same time, the level and volatility of the convenience yield are too high relative to the data, indicating that the enforcement channel is currently too strong in the calibration—a margin that could be disciplined further using term-structure or option-implied moments.

Finally, Panel (d) shows that delayed learning is essential for matching exchange-rate predictability. In the data, short-horizon regressions display positive predictability, while long-horizon regressions exhibit a pronounced reversal. The full-information model reverses too quickly, as shocks are immediately capitalized into prices, compressing predictability at medium horizons. By contrast, the partial-information model generates the correct timing: gradual belief updating sustains predictability at short horizons and produces a negative long-horizon slope consistent with the empirical evidence.

Table 4: *Calibration Moments: International Macro, Fiscal and Asset Pricing Moments*

Description. This table reports a set of calibration moments spanning international business-cycle comovement, fiscal dynamics, asset-pricing statistics, and exchange-rate predictability. Empirical moments are taken from the international macro and asset-pricing literature, with brackets reporting 90% confidence bands. Model moments are computed as average results over 1,000 simulations for 1,000 quarters each. Results are reported separately for the *partial-information* economy with delayed learning and for a *full-information* benchmark in which agents observe the underlying state variables directly.

Description	Moment (regression)	Data	Partial Info	Full Info
Panel (a): International Business Cycle Moments				
Consumption–output comovement	$corr_t(\Delta c_t, \Delta y_t)$	0.54 [0.34, 0.67]	0.40	0.78
Investment–output comovement	$corr_t(\Delta i_t, \Delta y_t)$	0.62 [0.40, 0.82]	0.48	0.84
International consumption comovement	$corr_t(\Delta c_t^i, \Delta c_t^j)$	0.66 [0.43, 0.84]	0.48	0.88
International output comovement	$corr_t(\Delta y_t^i, \Delta y_t^j)$	0.52 [0.33, 0.74]	0.37	0.76
Panel (b): Fiscal and Debt Dynamics				
Persistence of U.S. surplus–debt ratio	$\Delta\text{US SD}_t = \rho\,\Delta\text{US SD}_{t-1} + \varepsilon_t$	0.90 [0.85, 0.95]	0.94	0.88
Volatility of U.S. surplus–debt ratio (annualized, pp)	$\sigma(\Delta\text{US SD}_t)$	2.2 [1.5, 3.0]	2.6	1.9
Fiscal cyclicality (U.S.)	$\Delta\text{US SD}_t = \alpha + \beta\,\Delta y_t^{US} + \varepsilon_t$	0.30 [0.15, 0.45]	0.18	0.45
Cross-country fiscal comovement	$\Delta\text{US SD}_t = \alpha + \beta\,\Delta\text{SD}_t^i + \varepsilon_t$	0.42 [0.30, 0.55]	0.39	0.55
Panel (c): Financial / Asset-Pricing Moments				
Average dollar convenience yield (annualized, bps)	$\mathbb{E}[CY_t]$	45 [25, 65]	113	81
Volatility of dollar convenience yield (annualized, bps)	$\sigma(CY_t)$	35 [20, 55]	114	75
Global equity premium (annualized, %)	$100 \times 4\,\mathbb{E}[r_t^G - r_t^f]$	5.5 [3.0, 8.0]	3.1	2.5
Global equity premium volatility (annualized, %)	$100 \times \sqrt{4}\,\sigma(r_t^G - r_t^f)$	17.0 [13.0, 22.0]	35.1	21.0
Dollar safe-haven property	$\Delta USD_t = \alpha + \beta\,\Delta VIX_t + \varepsilon_t$	−0.40 [-0.25, -0.55]	−0.32	−0.72
Global equity correlation (GFC moment)	$\overline{corr_t(r_t^i, r_t^j)}$	0.75 [0.60, 0.85]	0.51	0.80
Panel (d): Exchange Rate Predictability Moments				
Backus–Smith regression (1Q)	$\Delta c_t^i - \Delta c_t^j = \alpha + \beta\,\Delta\mathcal{E}_t + \varepsilon_t$	−0.04 [-0.08, 0.29]	−0.37	−0.11
UIP (carry) regression (1Q)	$\Delta\mathcal{E}_{t+1} = \alpha + \beta\,(r_t^i - r_t^j) + \varepsilon_{t+1}$	−0.47 [-0.24, -0.68]	−0.35	−0.22
Predictability: 1Q	$\Delta\mathcal{E}_{t+1Q} = \alpha + \beta\,(r_t^i - r_t^j) + \varepsilon_{t+1Q}$	2.08 [1.52, 2.61]	2.42	0.50
Predictability: 5Y	$\Delta\mathcal{E}_{t,t+20} = \alpha + \beta\,(r_t^i - r_t^j) + \varepsilon_{t+20}$	−0.87 [-0.25, -1.02]	−1.23	0.20

8 Conclusion

This paper studies how U.S. fiscal policy transmits internationally and why its effects vary systematically across countries. The central finding is that the global impact of U.S. fiscal–military shocks is not uniform, but instead organized by exposure to the U.S. security umbrella. Countries that rely more heavily on U.S. military presence and security provision experience stronger macroeconomic responses, larger asset-price reactions, and more pronounced risk premia across sovereign bonds, equities, and currencies.

Empirically, the paper documents a set of new cross-sectional facts. Country-level fiscal betas estimated from U.S. fiscal–military news shocks vary widely across markets and countries, but they do so in a highly structured way. Across bonds, equities, and exchange rates, fiscal betas are monotone in U.S. military presence. The same ordering generates systematic return differentials and profitable factor-mimicking portfolios. In equity and bond markets, higher exposure is associated with higher average excess returns, while in currency markets the relationship is reversed: high-exposure currencies earn lower premia and act as hedges against U.S. fiscal risk. These patterns are robust to alternative measures of military presence, subsamples, and controls for trade integration and economic size.

To interpret these findings, the paper develops a multi-country production economy in which U.S. fiscal–military spending transmits globally through the security umbrella. U.S. military provision affects allied economies through two tightly linked channels. First, higher U.S. military spending strengthens the security umbrella and relaxes the local defense floor, raising effective output and easing fiscal pressure in exposed economies. At the same time, military mobilization increases aggregate fiscal strain and global risk, raising sovereign default risk and discount rates through higher borrowing needs and precautionary-savings motives. Second, security provision enters international financial markets through an enforcement wedge, microfounding deviations from uncovered interest parity and generating a dollar convenience yield that shapes currency risk premia. Heterogeneity in exposure to the U.S. security umbrella is sufficient to generate the full cross-section of macro responses, asset betas, and return premia observed in the data.

The model delivers sharp analytical results. A single exposure parameter governs global con-

sumption and output betas, equilibrium interest rates, equity and bond risk premia, currency premia, and cross-asset betas with respect to U.S. fiscal–military shocks. These results clarify why fiscal transmission is strongest precisely in countries that are most tightly integrated into the U.S. military–industrial network. They also explain why the same countries simultaneously exhibit higher equity and bond risk premia and lower currency premia, generating a coherent cross-asset pricing structure.

The quantitative extension embeds this mechanism in a richer dynamic environment with recursive preferences, endogenous sovereign risk, and imperfect information about U.S. security commitments. Despite its added complexity, the quantitative model preserves the core intuition of the analytical framework and successfully matches both the magnitudes and the timing of observed asset-price responses. In particular, it reproduces the monotone cross-sectional slopes of fiscal betas, the joint behavior of equities, bonds, and currencies, and the profitability of security-sorted trading strategies.

Taken together, the results point to a broader conclusion. U.S. fiscal policy is a global force not only because of the size of the U.S. economy, but because it is embedded in a system of security provision that shapes fiscal behavior, risk sharing, and financial integration abroad. The military footprint of the United States acts as a conduit through which fiscal shocks are transmitted, priced, and hedged across global markets. Understanding international macro-financial dynamics therefore requires accounting not only for trade and financial linkages, but also for the geopolitical infrastructure that underpins them.

More generally, the paper highlights the economic role of security provision in shaping global risk. Military alliances and enforcement arrangements are not merely political institutions; they have first-order implications for fiscal transmission, asset pricing, and international capital flows. Future research could extend this framework to study the interaction between security provision and monetary policy, the endogenous formation of alliances, or the consequences of geopolitical fragmentation for global financial stability.

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A Closed Form Model Proofs

A.1 First-Order Conditions and Equilibrium Conditions

This appendix derives the optimality conditions and equilibrium relationships for the decentralized production economy introduced in Section 4. The key departures from the earlier planner-based appendix are that (i) markets are incomplete due to sovereign risk and enforcement constraints, (ii) governments issue debt and face a defense floor, and (iii) exchange rates and currency premia satisfy an international Euler equation with a security-dependent wedge.

Throughout, $\{g_t\}$ is exogenous and the U.S. security umbrella follows $s_t = \bar{s} - \kappa_s g_t$.

A.1.1 Households

Household i chooses $\{C_{i,t}, L_{i,t}, I_{i,t}, K_{i,t+1}, B^H_{i,t+1}\}_{t\geq 0}$ to maximize

$$\max \ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \log C_{i,t}, \tag{A.1}$$

subject to the budget constraint (in units of the country-i final good)

$$C_{i,t} + I_{i,t} + B^H_{i,t+1} = w_{i,t} L_{i,t} + r^K_{i,t} K_{i,t} + \Pi_{i,t} + (1 + r^H_{i,t}) B^H_{i,t}, \tag{A.2}$$

and capital accumulation

$$K_{i,t+1} = (1 - \delta) K_{i,t} + I_{i,t}. \tag{A.3}$$

Here $w_{i,t}$ is the wage, $r^K_{i,t}$ the rental rate of capital, $\Pi_{i,t}$ domestic profits, and $B^H_{i,t}$ is household bond holdings (risk-free domestically). If you wish to abstract from household bond trading, set $B^H_{i,t} \equiv 0$.

Let $\lambda_{i,t}$ denote the multiplier on (A.2) and $\xi_{i,t}$ the multiplier on (A.3). The Lagrangian is

$$\mathcal{L}_i = \mathbb{E}_0 \sum_{t\geq 0} \beta^t \Big[\log C_{i,t} + \lambda_{i,t} \Big(w_{i,t} L_{i,t} + r^K_{i,t} K_{i,t} + \Pi_{i,t} + (1 + r^H_{i,t}) B^H_{i,t} - C_{i,t} - I_{i,t} - B^H_{i,t+1} \Big) + \xi_{i,t} \big((1 - \delta) K_{i,t} + I_{i,t}$$

FOC for consumption.

$$\frac{\partial \mathcal{L}_i}{\partial C_{i,t}}: \quad \frac{1}{C_{i,t}} - \lambda_{i,t} = 0 \qquad \Rightarrow \qquad \lambda_{i,t} = \frac{1}{C_{i,t}}. \tag{A.4}$$

FOC for investment.

$$\frac{\partial \mathcal{L}_i}{\partial I_{i,t}}: \quad -\lambda_{i,t} + \xi_{i,t} = 0 \qquad \Rightarrow \qquad \xi_{i,t} = \lambda_{i,t}. \tag{A.5}$$

FOC for next-period capital.

$$\frac{\partial \mathcal{L}_i}{\partial K_{i,t+1}}: \quad -\xi_{i,t} + \beta\, \mathbb{E}_t\Big[\lambda_{i,t+1} r^K_{i,t+1} + \xi_{i,t+1}(1-\delta)\Big] = 0. \tag{A.6}$$

Using (A.5)–(A.4), this yields the standard Euler equation:

$$1 = \beta\, \mathbb{E}_t\left[\frac{C_{i,t}}{C_{i,t+1}}\left(r^K_{i,t+1} + 1 - \delta\right)\right]. \tag{A.7}$$

FOC for household bond holdings.

$$\frac{\partial \mathcal{L}_i}{\partial B^H_{i,t+1}}: \quad -\lambda_{i,t} + \beta\, \mathbb{E}_t\Big[\lambda_{i,t+1}(1 + r^H_{i,t+1})\Big] = 0, \tag{A.8}$$

or

$$1 = \mathbb{E}_t\Big[M_{i,t+1}(1 + r^H_{i,t+1})\Big], \qquad M_{i,t+1} \equiv \beta \frac{C_{i,t}}{C_{i,t+1}}. \tag{A.9}$$

Labor supply. If you include disutility of labor, e.g. $\log C_{i,t} - \chi \frac{L_{i,t}^{1+\nu}}{1+\nu}$, then

$$w_{i,t} = \chi\, L^{\nu}_{i,t}\, C_{i,t}. \tag{A.10}$$

If instead you impose $L_{i,t} = 1$ (as in the closed-form special case), (A.10) is dropped.

A.1.2 Firms

Final output is produced competitively by firms with technology

$$Y_{i,t} = A_i K_{i,t}^{\alpha} L_{i,t}^{1-\alpha}. \tag{A.11}$$

Taking prices as given, firms choose $K_{i,t}, L_{i,t}$ to maximize profits

$$\Pi_{i,t} = Y_{i,t} - r_{i,t}^{K} K_{i,t} - w_{i,t} L_{i,t}.$$

FOC for capital demand.

$$\frac{\partial \Pi_{i,t}}{\partial K_{i,t}}: \quad r_{i,t}^{K} = \alpha A_i K_{i,t}^{\alpha-1} L_{i,t}^{1-\alpha} = \alpha \frac{Y_{i,t}}{K_{i,t}}. \tag{A.12}$$

FOC for labor demand.

$$\frac{\partial \Pi_{i,t}}{\partial L_{i,t}}: \quad w_{i,t} = (1-\alpha) A_i K_{i,t}^{\alpha} L_{i,t}^{-\alpha} = (1-\alpha) \frac{Y_{i,t}}{L_{i,t}}. \tag{A.13}$$

A.1.3 Government: Defense Floor, Budget Constraint, and Debt Dynamics

Government spending has four components: social transfers $G_{i,t}^{soc}$, public investment $I_{i,t}^{pub}$, defense spending $h_{i,t}$, and interest on debt. The government budget constraint is

$$B_{i,t+1} = (1 + r_{i,t}) B_{i,t} + G_{i,t}^{soc} + I_{i,t}^{pub} + h_{i,t} - \tau_i Y_{i,t}, \tag{A.14}$$

where τ_i is a constant tax rate and $r_{i,t}$ is the sovereign borrowing rate (endogenously determined below).

Defense floor problem. Each ally faces a deterrence requirement $s_t + h_{i,t} \geq S^{\star}$ and chooses $h_{i,t}$ to minimize resource cost $\mathcal{C}(h_{i,t})$:

$$\min_{h_{i,t} \geq 0} \ \mathcal{C}(h_{i,t}) \qquad \text{s.t.} \qquad s_t + h_{i,t} \geq S^{\star}. \tag{A.15}$$

Let $\mu_{i,t} \geq 0$ be the multiplier on the deterrence constraint and $\zeta_{i,t} \geq 0$ the multiplier on $h_{i,t} \geq 0$. The KKT conditions are:

$$\mathcal{C}'(h_{i,t}) - \mu_{i,t} - \zeta_{i,t} = 0, \tag{A.16}$$

$$\mu_{i,t}\,(s_t + h_{i,t} - S^\star) = 0, \qquad \zeta_{i,t}\,h_{i,t} = 0, \tag{A.17}$$

$$s_t + h_{i,t} \geq S^\star, \qquad h_{i,t} \geq 0, \qquad \mu_{i,t} \geq 0, \qquad \zeta_{i,t} \geq 0. \tag{A.18}$$

Monotonicity of $\mathcal{C}'(\cdot)$ implies the minimal feasible choice:

$$h_{i,t} = \max\{0, S^\star - s_t\}, \tag{A.19}$$

which yields the piecewise derivative

$$\frac{dh_{i,t}}{ds_t} = \begin{cases} 0, & s_t \geq S^\star, \\ -1, & s_t < S^\star. \end{cases}$$

Sticky non-defense spending. To focus on the discount-rate channel, non-defense components are sticky in the short run:

$$G^{soc}_{i,t} = \bar{G}_i, \qquad I^{pub}_{i,t} = \bar{I}_i, \tag{A.20}$$

so changes in s_t affect fiscal dynamics primarily through $h_{i,t}$.

A.1.4 Default Risk and Borrowing Rates

Country i faces a one-period sovereign stress/default event $J_{i,t+1} \in \{0, 1\}$. Conditional on $\mathcal{F}_t$, the default probability is

$$\Pr(J_{i,t+1} = 1 \mid \mathcal{F}_t) = \lambda_{i,t}, \tag{A.21}$$

with intensity

$$\lambda_{i,t} = \exp\big(\alpha_0 + \alpha_1 B_{i,t} + \alpha_2 g_t - \alpha_3 s_t + \alpha_4 \phi_i\big). \tag{A.22}$$

Sovereign bond pricing. A one-period sovereign bond payoff is

$$X^b_{i,t+1} = 1 - \mathcal{L}\, J_{i,t+1}.$$

The price satisfies

$$P^b_{i,t} = \mathbb{E}_t\Big[M_{i,t+1} X^b_{i,t+1}\Big] = \mathbb{E}_t[M_{i,t+1}] - \mathcal{L}\,\mathbb{E}_t[M_{i,t+1} J_{i,t+1}]. \tag{A.23}$$

Under the maintained reduced-form timing in (A.21), a first-order approximation yields

$$P^b_{i,t} \approx \mathbb{E}_t[M_{i,t+1}]\left(1 - \mathcal{L}\lambda_{i,t}\right). \tag{A.24}$$

The gross borrowing rate satisfies $(1 + r_{i,t})^{-1} = P^b_{i,t}$.

A.1.5 International Euler Equation Wedge and Modified UIP

Let $rx^{i\$,}_{t+1}$ denote the log excess return on a currency strategy that borrows in dollars and invests in country i. Incomplete markets imply an international Euler equation with a wedge $\Lambda_{i,t}$:

$$1 = \mathbb{E}_t\left[M_{\$,t+1}\, \Lambda_{i,t}\, e^{rx^{i\$,}_{t+1}}\right], \tag{A.25}$$

where the wedge is driven by security provision:

$$\log \Lambda_{i,t} = \chi_E\, \phi_i\, s_t, \qquad \chi_E > 0. \tag{A.26}$$

Log-linearizing (A.25) yields the modified UIP relation

$$\mathbb{E}_t[rx^{i\$,}_{t+1}] \approx -\log \Lambda_{i,t}. \tag{A.27}$$

A.1.6 Closed-Form Special Case (used in the main text)

The closed-form benchmark imposed in Section 5.2 sets $K_{i,t} = \bar{K}_i$, $L_{i,t} = 1$, and $I_{i,t} = 0$, implying $Y_{i,t} = \bar{Y}_i$ and

$$C_{i,t} = \bar{Y}_i - \bar{G}_i - \bar{I}_i - h_{i,t}. \tag{A.28}$$

Under this specialization, (A.19) and (A.14) deliver explicit expressions for the responses of consumption, debt, default intensity, sovereign spreads, and currency premia to the U.S. fiscal–military shock g_t and exposure ϕ_i.

A.2 Closed-Form Macro Responses and Fiscal Multipliers

This appendix derives equilibrium responses of macroeconomic variables to U.S. fiscal–military shocks in the closed-form benchmark introduced in Section 5.2. The derivations use only the short-run production benchmark, the defense floor, and the government budget constraint. Throughout, $\{g_t\}$ is exogenous and U.S. security provision satisfies

$$s_t = \bar{s} + \kappa_s g_t, \qquad \kappa_s > 0. \tag{A.29}$$

A.2.1 Assumptions (Closed-Form Short-Run Benchmark)

Fix any country i and date t.

Assumption 1 (Predetermined factors)**.** *Capital and labor are predetermined in the short run:*

$$K_{i,t} = \bar{K}_i, \qquad L_{i,t} = 1.$$

Assumption 2 (Baseline output)**.** *Baseline output is*

$$\bar{Y}_i \equiv A_i \bar{K}_i^{\alpha}, \qquad \alpha \in (0,1).$$

Assumption 3 (Defense-induced utilization wedge)**.** *Effective output is reduced by local defense outlays according to*

$$Y_{i,t} = \bar{Y}_i \exp(-\chi_h\, h_{i,t}), \qquad \chi_h > 0. \tag{A.30}$$

Assumption 4 (Fixed investment share)**.** *Investment follows a fixed share rule*

$$I_{i,t} = \iota_i Y_{i,t}, \qquad \iota_i \in (0,1). \tag{A.31}$$

Assumption 5 (Sticky non-defense fiscal components)**.** *Non-defense fiscal components are fixed:*

$$G^{soc}_{i,t} = \bar{G}_i, \qquad I^{pub}_{i,t} = \bar{I}_i.$$

Assumption 6 (Defense floor)**.** *Allies satisfy $s_t + h_{i,t} \geq S^\star$, and local defense outlays are*

$$h_{i,t} = \max\{0, S^\star - s_t\}. \tag{A.32}$$

Assumption 7 (Government budget constraint)**.** *Public debt evolves according to*

$$B_{i,t+1} = (1+r)B_{i,t} + G^{soc}_{i,t} + I^{pub}_{i,t} + h_{i,t} - \tau_i Y_{i,t}, \tag{A.33}$$

with constant τ_i and constant $(1+r)$.

Assumption 8 (Default intensity)**.** *Default intensity is*

$$\lambda_{i,t} = \exp\big(\alpha_0 + \alpha_1 B_{i,t} + \alpha_2 g_t - \alpha_3 s_t + \alpha_4 \phi_i\big), \qquad \alpha_1, \alpha_2, \alpha_4 > 0. \tag{A.34}$$

A.2.2 Preliminaries: Resource Constraint and Multipliers

The resource constraint is

$$Y_{i,t} = C_{i,t} + I_{i,t} + G^{soc}_{i,t} + I^{pub}_{i,t} + h_{i,t}.$$

Substituting the fixed investment share rule (A.31) and sticky fiscal components (Assumption 5) yields

$$Y_{i,t} = C_{i,t} + \iota_i Y_{i,t} + \bar{G}_i + \bar{I}_i + h_{i,t}.$$

Rearranging gives the closed-form consumption rule

$$C_{i,t} = (1 - \iota_i) Y_{i,t} - \bar{G}_i - \bar{I}_i - h_{i,t}. \tag{A.35}$$

For any macro variable $X_{i,t}$, define the (impact) multiplier with respect to g_t as

$$\mathcal{M}_i^X \equiv \frac{\partial X_{i,t}}{\partial g_t}.$$

All derivatives below are evaluated holding predetermined objects fixed at impact (e.g. $B_{i,t}$).

A.2.3 Defense Spending Multiplier

Proposition 1 (Defense floor and its shock exposure)**.** *Under* (A.29) *and* (A.32)*,*

$$h_{i,t} = \max\{0, S^\star - \bar{s} - \kappa_s g_t\}. \tag{A.36}$$

Moreover, the impact derivative satisfies

$$\frac{\partial h_{i,t}}{\partial g_t} = -\kappa_s \, \mathbf{1}\{s_t < S^\star\}. \tag{A.37}$$

Proof. Start from the defense floor (A.32):

$$h_{i,t} = \max\{0, S^\star - s_t\}.$$

Substitute the security provision rule (A.29), $s_t = \bar{s} + \kappa_s g_t$:

$$h_{i,t} = \max\{0, S^\star - (\bar{s} + \kappa_s g_t)\} = \max\{0, S^\star - \bar{s} - \kappa_s g_t\},$$

which establishes (A.36).

To compute $\partial h_{i,t}/\partial g_t$, consider the two regions defined by the max operator.

Region 1 (slack deterrence constraint): $s_t \geq S^\star$. Then $S^\star - s_t \leq 0$, so the max selects 0 and $h_{i,t} = 0$ in a neighborhood of g_t. Hence $\partial h_{i,t}/\partial g_t = 0$ in this region.

Region 2 (binding deterrence constraint): $s_t < S^\star$. Then $S^\star - s_t > 0$, so the max selects $S^\star - s_t$ and

$$h_{i,t} = S^\star - s_t = S^\star - (\bar{s} + \kappa_s g_t) = S^\star - \bar{s} - \kappa_s g_t.$$

Differentiating yields $\partial h_{i,t}/\partial g_t = -\kappa_s$ in this region.

Combining the two regions gives (A.37). □

A.2.4 GDP/Output Multiplier

Proposition 2 (Output response)**.** *Under* (A.30)*, the impact derivative of output satisfies*

$$\frac{\partial Y_{i,t}}{\partial g_t} = \chi_h \kappa_s \, Y_{i,t} \, \mathbf{1}\{s_t < S^\star\}. \tag{A.38}$$

Proof. Start from (A.30):

$$Y_{i,t} = \bar{Y}_i \exp(-\chi_h h_{i,t}).$$

Differentiate with respect to g_t using the chain rule:

$$\frac{\partial Y_{i,t}}{\partial g_t} = \bar{Y}_i \exp(-\chi_h h_{i,t}) \cdot (-\chi_h) \frac{\partial h_{i,t}}{\partial g_t}.$$

Recognize that $\bar{Y}_i \exp(-\chi_h h_{i,t}) = Y_{i,t}$, so

$$\frac{\partial Y_{i,t}}{\partial g_t} = -\chi_h Y_{i,t} \frac{\partial h_{i,t}}{\partial g_t}.$$

From (A.37), $\frac{\partial h_{i,t}}{\partial g_t} = -\kappa_s \mathbf{1}\{s_t < S^\star\}$. Substituting gives

$$\frac{\partial Y_{i,t}}{\partial g_t} = -\chi_h Y_{i,t}\Big(- \kappa_s \mathbf{1}\{s_t < S^\star\}\Big) = \chi_h \kappa_s Y_{i,t} \mathbf{1}\{s_t < S^\star\},$$

which is (A.38). □

A.2.5 Investment Multiplier

Proposition 3 (Investment response)**.** *Under* (A.31)*, the impact derivative of investment satisfies*

$$\frac{\partial I_{i,t}}{\partial g_t} = \iota_i \chi_h \kappa_s \, Y_{i,t} \, \mathbf{1}\{s_t < S^\star\}. \tag{A.39}$$

Proof. By (A.31), $I_{i,t} = \iota_i Y_{i,t}$ with constant ι_i. Differentiate:

$$\frac{\partial I_{i,t}}{\partial g_t} = \iota_i \frac{\partial Y_{i,t}}{\partial g_t}.$$

Substitute $\frac{\partial Y_{i,t}}{\partial g_t}$ from (A.38) to obtain (A.39). □

A.2.6 Consumption Multiplier

Proposition 4 (Consumption response)**.** *Under* (A.35)*, the impact derivative of consumption satisfies*

$$\frac{\partial C_{i,t}}{\partial g_t} = (1 - \iota_i)\chi_h \kappa_s \, Y_{i,t} \, \mathbf{1}\{s_t < S^\star\} + \kappa_s \, \mathbf{1}\{s_t < S^\star\}. \tag{A.40}$$

Proof. Start from (A.35):

$$C_{i,t} = (1 - \iota_i) Y_{i,t} - \bar{G}_i - \bar{I}_i - h_{i,t}.$$

Differentiate with respect to g_t:

$$\frac{\partial C_{i,t}}{\partial g_t} = (1 - \iota_i) \frac{\partial Y_{i,t}}{\partial g_t} - 0 - 0 - \frac{\partial h_{i,t}}{\partial g_t}.$$

Substitute (A.38) and (A.37):

$$\frac{\partial C_{i,t}}{\partial g_t} = (1-\iota_i)\Big(\chi_h \kappa_s \, Y_{i,t} \mathbf{1}\{s_t < S^\star\}\Big) - \Big(- \kappa_s \mathbf{1}\{s_t < S^\star\}\Big).$$

Simplifying yields (A.40). □

A.2.7 Debt Multiplier

Proposition 5 (Debt accumulation response)**.** *Under* (A.33) *and Assumption 5, treating* $B_{i,t}$ *as predetermined at impact, the derivative of next-period debt satisfies*

$$\frac{\partial B_{i,t+1}}{\partial g_t} = -\kappa_s \, \mathbf{1}\{s_t < S^\star\} - \tau_i \chi_h \kappa_s \, Y_{i,t} \, \mathbf{1}\{s_t < S^\star\}. \tag{A.41}$$

Proof. Start from the budget constraint (A.33):

$$B_{i,t+1} = (1+r)B_{i,t} + G^{soc}_{i,t} + I^{pub}_{i,t} + h_{i,t} - \tau_i Y_{i,t}.$$

Substitute $G^{soc}_{i,t} = \bar{G}_i$ and $I^{pub}_{i,t} = \bar{I}_i$:

$$B_{i,t+1} = (1+r)B_{i,t} + \bar{G}_i + \bar{I}_i + h_{i,t} - \tau_i Y_{i,t}.$$

Differentiate with respect to g_t:

$$\frac{\partial B_{i,t+1}}{\partial g_t} = (1+r)\frac{\partial B_{i,t}}{\partial g_t} + \frac{\partial h_{i,t}}{\partial g_t} - \tau_i \frac{\partial Y_{i,t}}{\partial g_t}.$$

At impact, $B_{i,t}$ is predetermined, so $\frac{\partial B_{i,t}}{\partial g_t} = 0$. Hence

$$\frac{\partial B_{i,t+1}}{\partial g_t} = \frac{\partial h_{i,t}}{\partial g_t} - \tau_i \frac{\partial Y_{i,t}}{\partial g_t}.$$

Substitute (A.37) and (A.38) to obtain (A.41). □

A.2.8 Default-Intensity Multiplier

Proposition 6 (Sovereign risk response)**.** *Under* (A.34) *and* (A.29)*, treating* $B_{i,t}$ *as predetermined at impact, the impact semi-elasticity satisfies*

$$\frac{\partial \log \lambda_{i,t}}{\partial g_t} = \alpha_2 - \alpha_3 \kappa_s, \tag{A.42}$$

and therefore

$$\frac{\partial \lambda_{i,t}}{\partial g_t} = \lambda_{i,t} \left(\alpha_2 - \alpha_3 \kappa_s\right). \tag{A.43}$$

One period ahead, holding (g_{t+1}, s_{t+1}) *fixed, the debt channel implies*

$$\frac{\partial \log \lambda_{i,t+1}}{\partial g_t} = \alpha_1 \frac{\partial B_{i,t+1}}{\partial g_t}. \tag{A.44}$$

Proof. Take logs of (A.34):

$$\log \lambda_{i,t} = \alpha_0 + \alpha_1 B_{i,t} + \alpha_2 g_t - \alpha_3 s_t + \alpha_4 \phi_i.$$

Differentiate with respect to g_t:

$$\frac{\partial \log \lambda_{i,t}}{\partial g_t} = \alpha_1 \frac{\partial B_{i,t}}{\partial g_t} + \alpha_2 - \alpha_3 \frac{\partial s_t}{\partial g_t}.$$

At impact, $B_{i,t}$ is predetermined, so $\frac{\partial B_{i,t}}{\partial g_t} = 0$. From (A.29), $s_t = \bar{s} + \kappa_s g_t$, so $\frac{\partial s_t}{\partial g_t} = \kappa_s$. Substitute to obtain

$$\frac{\partial \log \lambda_{i,t}}{\partial g_t} = \alpha_2 - \alpha_3 \kappa_s,$$

which proves (A.42). Because $\lambda_{i,t} = \exp(\log \lambda_{i,t})$, the chain rule yields

$$\frac{\partial \lambda_{i,t}}{\partial g_t} = \lambda_{i,t} \frac{\partial \log \lambda_{i,t}}{\partial g_t} = \lambda_{i,t} (\alpha_2 - \alpha_3 \kappa_s),$$

which proves (A.43).

For $t+1$,

$$\log \lambda_{i,t+1} = \alpha_0 + \alpha_1 B_{i,t+1} + \alpha_2 g_{t+1} - \alpha_3 s_{t+1} + \alpha_4 \phi_i.$$

Holding (g_{t+1}, s_{t+1}) fixed when differentiating with respect to g_t, the only dependence is through $B_{i,t+1}$, hence

$$\frac{\partial \log \lambda_{i,t+1}}{\partial g_t} = \alpha_1 \frac{\partial B_{i,t+1}}{\partial g_t},$$

which is (A.44). □

A.2.9 Summary of Macro Multipliers

When the defense floor binds ($s_t < S^\star$), the impact multipliers are

$$\frac{\partial h_{i,t}}{\partial g_t} = -\kappa_s, \qquad \frac{\partial Y_{i,t}}{\partial g_t} = \chi_h \kappa_s Y_{i,t}, \qquad \frac{\partial I_{i,t}}{\partial g_t} = \iota_i \chi_h \kappa_s Y_{i,t},$$

$$\frac{\partial C_{i,t}}{\partial g_t} = (1-\iota_i)\chi_h \kappa_s Y_{i,t} + \kappa_s, \qquad \frac{\partial B_{i,t+1}}{\partial g_t} = -\kappa_s - \tau_i \chi_h \kappa_s Y_{i,t},$$

and the impact semi-elasticity of default intensity satisfies

$$\frac{\partial \log \lambda_{i,t}}{\partial g_t} = \alpha_2 - \alpha_3 \kappa_s.$$

When the defense floor does not bind ($s_t \geq S^\star$), $h_{i,t} = 0$ locally and the impact multipliers above equal zero.

These closed-form macro responses discipline the state dependence of real-side effects in the benchmark and clarify which channels operate through quantities versus prices in the asset-pricing results.

A.3 Derivation of Equilibrium Interest Rates

This subsection derives equilibrium one-period *real* risk-free interest rates in closed form under the closed-form benchmark in Section 5.2. The derivation uses (i) log preferences, (ii) the domestic Euler equation, and (iii) the closed-form macro relationships for $C_{i,t}$ and $Y_{i,t}$. No complete-markets restrictions are imposed.

A.3.1 Definitions

For each country i, the domestic real stochastic discount factor (SDF) is

$$M_{i,t+1} \equiv \beta \frac{C_{i,t}}{C_{i,t+1}}. \tag{A.45}$$

The one-period real risk-free gross return $R^i_{f,t}$ satisfies the domestic Euler equation

$$\frac{1}{R^i_{f,t}} = \mathbb{E}_t\big[M_{i,t+1}\big]\,. \tag{A.46}$$

Define the log risk-free rate $r^i_{f,t} \equiv \log R^i_{f,t}$.

A.3.2 Closed-Form Macro Relations

In the closed-form benchmark, security provision follows

$$s_t = \bar{s} + \kappa_s g_t, \qquad \kappa_s > 0, \tag{A.47}$$

and defense outlays are governed by the defense floor

$$h_{i,t} = \max\{0, S^\star - s_t\}. \tag{A.48}$$

Output satisfies the utilization wedge

$$Y_{i,t} = \bar{Y}_i \exp(-\chi_h h_{i,t}), \tag{A.49}$$

investment follows a fixed share rule

$$I_{i,t} = \iota_i Y_{i,t}, \tag{A.50}$$

and consumption satisfies the resource constraint

$$C_{i,t} = (1 - \iota_i)Y_{i,t} - \bar{G}_i - \bar{I}_i - h_{i,t}. \tag{A.51}$$

Because $h_{i,t}$ is piecewise linear in g_t through the max operator, the dynamics of consumption growth are analyzed conditional on a fixed regime.

A.3.3 Regime Indicator

Define the regime indicator

$$\mathbb{1}_t \equiv \mathbf{1}\{s_t < S^\star\}. \tag{A.52}$$

When $\mathbb{1}_t = 0$, the deterrence constraint is slack, so $h_{i,t} = 0$ locally and $h_{i,t}$ is locally insensitive to g_t at impact. When $\mathbb{1}_t = 1$, the deterrence constraint binds and the defense floor is active, so $h_{i,t} = S^\star - s_t$ and changes in g_t affect macro variables through $h_{i,t}$.

A.3.4 Log Consumption Growth Representation

To compute $\mathbb{E}_t[M_{i,t+1}]$ in (A.46), we need a tractable representation for log consumption growth $\Delta \log C_{i,t+1}$.

Under (A.51), consumption is affine in $(Y_{i,t}, h_{i,t})$ and therefore not exactly lognormal. Following the beta derivations in Appendix **??**, we use a first-order log-linearization around a constant reference level $\bar{C}_i > 0$:

$$\Delta \log C_{i,t+1} \approx \frac{1}{\bar{C}_i}\Big((1 - \iota_i)\Delta Y_{i,t+1} - \Delta h_{i,t+1}\Big). \tag{A.53}$$

Next, compute $\Delta Y_{i,t+1}$ explicitly from (A.49). Taking logs,

$$\log Y_{i,t} = \log \bar{Y}_i - \chi_h h_{i,t},$$

so

$$\Delta \log Y_{i,t+1} = -\chi_h \Delta h_{i,t+1}.$$

Using $\Delta Y_{i,t+1} \approx \bar{Y}_i \, \Delta \log Y_{i,t+1}$ (first-order),

$$\Delta Y_{i,t+1} \approx -\chi_h \bar{Y}_i \, \Delta h_{i,t+1}. \tag{A.54}$$

Substitute (A.54) into (A.53):

$$\begin{aligned} \Delta \log C_{i,t+1} &\approx \frac{1}{\bar{C}_i}\Big((1-\iota_i)(-\chi_h \bar{Y}_i \, \Delta h_{i,t+1}) - \Delta h_{i,t+1}\Big) \\ &= -\underbrace{\frac{(1-\iota_i)\chi_h \bar{Y}_i + 1}{\bar{C}_i}}_{\equiv \vartheta_i} \Delta h_{i,t+1}. \end{aligned} \tag{A.55}$$

Finally, under (A.47)–(A.48), in the binding regime at $t+1$ (i.e. $\mathbb{1}_{t+1} = 1$),

$$h_{i,t+1} = S^\star - s_{t+1} = S^\star - (\bar{s} + \kappa_s g_{t+1}),$$

so

$$\Delta h_{i,t+1} = h_{i,t+1} - h_{i,t} = -\kappa_s \, \Delta g_{t+1} + \Delta \xi^h_{i,t+1},$$

where $\Delta \xi^h_{i,t+1}$ collects any idiosyncratic defense-spending innovations orthogonal to Δg_{t+1}. Substituting into (A.55) yields

$$\Delta \log C_{i,t+1} \approx \vartheta_i \kappa_s \, \Delta g_{t+1} + \Delta \Xi^C_{i,t+1}, \tag{A.56}$$

where $\Delta \Xi^C_{i,t+1} \equiv -\vartheta_i \Delta \xi^h_{i,t+1}$ is idiosyncratic.

Equation (A.56) provides a one-factor representation for log consumption growth conditional on the binding regime.

A.3.5 Moment Assumptions

Assumption 9 (Conditional Gaussianity)**.** *Conditional on $\mathcal{F}_t$, the pair $(\Delta g_{t+1},\, \Delta\Xi^C_{i,t+1})$ is jointly Gaussian with finite second moments.*

Assumption 10 (Orthogonality)**.** *For each i,*

$$\mathrm{Cov}_t(\Delta g_{t+1},\, \Delta\Xi^C_{i,t+1}) = 0.$$

A.3.6 Computing the Risk-Free Rate

From (A.45) and (A.56),

$$M_{i,t+1} = \beta \exp\big(-\Delta \log C_{i,t+1}\big) \approx \beta \exp\Big(-\vartheta_i \kappa_s\, \Delta g_{t+1} - \Delta\Xi^C_{i,t+1}\Big). \tag{A.57}$$

Under Assumption 9, the exponent is conditionally Gaussian, so

$$\mathbb{E}_t[e^X] = \exp\Big(\mathbb{E}_t[X] + \tfrac{1}{2}\,\mathrm{Var}_t(X)\Big) \quad \text{for conditionally Gaussian } X.$$

Let

$$X_{i,t+1} \equiv -\vartheta_i \kappa_s\, \Delta g_{t+1} - \Delta\Xi^C_{i,t+1}.$$

Then

$$\mathbb{E}_t[M_{i,t+1}] \approx \beta\, \mathbb{E}_t[e^{X_{i,t+1}}] = \beta \exp\Big(\mathbb{E}_t[X_{i,t+1}] + \tfrac{1}{2}\,\mathrm{Var}_t(X_{i,t+1})\Big). \tag{A.58}$$

Apply (A.46): $R^i_{f,t} = \big(\mathbb{E}_t[M_{i,t+1}]\big)^{-1}$, so

$$r^i_{f,t} \equiv \log R^i_{f,t} = -\log\beta - \mathbb{E}_t[X_{i,t+1}] - \tfrac{1}{2}\,\mathrm{Var}_t(X_{i,t+1}). \tag{A.59}$$

A.3.7 Closed-Form Expression

Compute conditional moments under Assumption 10. First,

$$\mathbb{E}_t[X_{i,t+1}] = -\vartheta_i \kappa_s \, \mathbb{E}_t[\Delta g_{t+1}] - \mathbb{E}_t[\Delta \Xi^C_{i,t+1}]. \tag{A.60}$$

Second, using orthogonality,

$$\mathrm{Var}_t(X_{i,t+1}) = (\vartheta_i \kappa_s)^2 \, \mathrm{Var}_t(\Delta g_{t+1}) + \mathrm{Var}_t(\Delta \Xi^C_{i,t+1}). \tag{A.61}$$

Substitute (A.60)–(A.61) into (A.59):

Proposition 7 (Risk-free rate under the closed-form benchmark)**.** *Under Assumptions 9–10, the log risk-free rate satisfies*

$$\begin{aligned} r^i_{f,t} = &- \log \beta + \vartheta_i \kappa_s \, \mathbb{E}_t[\Delta g_{t+1}] + \mathbb{E}_t[\Delta \Xi^C_{i,t+1}] \\ &- \frac{1}{2}(\vartheta_i \kappa_s)^2 \, \mathrm{Var}_t(\Delta g_{t+1}) - \frac{1}{2} \, \mathrm{Var}_t(\Delta \Xi^C_{i,t+1}), \end{aligned} \tag{A.62}$$

where $\vartheta_i \equiv \frac{(1-\iota_i)\chi_h \bar{Y}_i + 1}{\bar{C}_i}$.

Proof. Equation (A.62) follows by substituting (A.60) and (A.61) into (A.59). □

Benchmark simplification. If $\mathbb{E}_t[\Delta g_{t+1}] = 0$ and the idiosyncratic component is conditionally mean-zero, then

$$r^i_{f,t} = - \log \beta - \frac{1}{2}(\vartheta_i \kappa_s)^2 \sigma_g^2 - \frac{1}{2} \, \mathrm{Var}_t(\Delta \Xi^C_{i,t+1}),$$

so the precautionary-savings term implies that greater sensitivity to the aggregate fiscal–military shock lowers the equilibrium real risk-free rate.

A.4 Cross-Sectional Slope Implications

This subsection derives the cross-sectional slope implications of the model in closed form under the benchmark in Section 5.2. The goal is to show explicitly how heterogeneity in security exposure ϕ_i generates systematic differences in sovereign risk, discount rates, and currency premia across countries.

Throughout, fix a date t and condition on the aggregate state (g_t, s_t), where $s_t = \bar{s} - \kappa_s g_t$. We focus on the economically relevant regime in which the defense floor binds, $s_t < S^\star$, and highlight the slack regime at the end.

A.4.1 Closed-Form Objects as Functions of Exposure

When the deterrence constraint binds ($s_t < S^\star$), the defense floor implies

$$h_{i,t} = \phi_i \, (S^\star - s_t), \tag{A.63}$$

so defense outlays scale linearly in exposure ϕ_i.

Under the utilization wedge,

$$Y_{i,t} = \bar{Y}_i \exp(-\chi_h h_{i,t}) = \bar{Y}_i \exp\big(- \chi_h \phi_i (S^\star - s_t)\big). \tag{A.64}$$

Debt evolves according to the government budget constraint

$$B_{i,t+1} = (1 + r) B_{i,t} + \bar{G}_i + \bar{I}_i + h_{i,t} - \tau_i Y_{i,t}. \tag{A.65}$$

Substituting (A.63)–(A.64) yields an explicit mapping $B_{i,t+1} = B_{i,t+1}(\phi_i; s_t)$.

Default intensity satisfies

$$\lambda_{i,t} = \exp\big(\alpha_0 + \alpha_1 B_{i,t} + \alpha_2 g_t - \alpha_3 s_t + \alpha_4 \phi_i\big), \tag{A.66}$$

so sovereign risk depends on exposure both directly (via $\alpha_4 \phi_i$) and indirectly through debt accumulation.

A.4.2 Cross-Sectional Slope of Debt with Respect to Exposure

Proposition 8 (Debt is increasing in exposure)**.** *Fix* (g_t, s_t) *with* $s_t < S^\star$*. Under* (A.63)–(A.65)*, the one-period-ahead cross-sectional slope of debt satisfies*

$$\frac{\partial B_{i,t+1}}{\partial \phi_i} = (S^\star - s_t) \; + \; \tau_i \, \chi_h \, (S^\star - s_t) \, Y_{i,t}. \tag{A.67}$$

In particular,

$$\frac{\partial B_{i,t+1}}{\partial \phi_i} > 0. \tag{A.68}$$

Proof. Differentiate the budget law (A.65) with respect to ϕ_i holding (g_t, s_t) fixed:

$$\frac{\partial B_{i,t+1}}{\partial \phi_i} = \frac{\partial h_{i,t}}{\partial \phi_i} - \tau_i \frac{\partial Y_{i,t}}{\partial \phi_i}.$$

From (A.63),

$$\frac{\partial h_{i,t}}{\partial \phi_i} = S^\star - s_t.$$

From (A.64), using the chain rule,

$$\frac{\partial Y_{i,t}}{\partial \phi_i} = \bar{Y}_i \exp\big(- \chi_h \phi_i (S^\star - s_t)\big) \cdot \big(- \chi_h (S^\star - s_t)\big) = -\chi_h (S^\star - s_t) Y_{i,t}.$$

Substitute these derivatives:

$$\frac{\partial B_{i,t+1}}{\partial \phi_i} = (S^\star - s_t) - \tau_i \big(- \chi_h (S^\star - s_t) Y_{i,t}\big) = (S^\star - s_t) + \tau_i \chi_h (S^\star - s_t) Y_{i,t},$$

which is (A.67). Since $s_t < S^\star$, $(S^\star - s_t) > 0$, and $Y_{i,t} > 0$, the right-hand side is strictly positive. □

A.4.3 Cross-Sectional Slope of Default Risk

Proposition 9 (Default intensity is increasing in exposure)**.** *Fix* (g_t, s_t) *with* $s_t < S^\star$*. Under* (A.66)*, the cross-sectional slope satisfies*

$$\frac{\partial \log \lambda_{i,t}}{\partial \phi_i} = \alpha_4 \qquad \Rightarrow \qquad \frac{\partial \lambda_{i,t}}{\partial \phi_i} = \alpha_4 \, \lambda_{i,t} > 0. \tag{A.69}$$

Moreover, one period ahead debt accumulation amplifies the slope:

$$\frac{\partial \log \lambda_{i,t+1}}{\partial \phi_i} = \alpha_4 + \alpha_1 \frac{\partial B_{i,t+1}}{\partial \phi_i}, \tag{A.70}$$

and therefore $\partial \lambda_{i,t+1}/\partial \phi_i > 0$*.*

Proof. The first part follows by differentiating (A.66) at time t holding $B_{i,t}$ fixed (dated t): $\partial \log \lambda_{i,t}/\partial \phi_i = \alpha_4$, and then applying the chain rule $\partial \lambda/\partial \phi = \lambda \, \partial \log \lambda/\partial \phi$. For the one-period-ahead expression, write

$$\log \lambda_{i,t+1} = \alpha_0 + \alpha_1 B_{i,t+1} + \alpha_2 g_{t+1} - \alpha_3 s_{t+1} + \alpha_4 \phi_i.$$

Differentiating with respect to ϕ_i holding (g_{t+1}, s_{t+1}) fixed yields

$$\frac{\partial \log \lambda_{i,t+1}}{\partial \phi_i} = \alpha_4 + \alpha_1 \frac{\partial B_{i,t+1}}{\partial \phi_i},$$

which is (A.70). By Proposition 8, $\partial B_{i,t+1}/\partial \phi_i > 0$, so the right-hand side is strictly positive. □

A.4.4 Bond Spreads: Cross-Sectional Slope

Define the one-period sovereign bond payoff $X^b_{i,t+1} = 1 - \mathcal{L} J_{i,t+1}$ and the bond price

$$P^b_{i,t} = \mathbb{E}_t[M_{i,t+1} X^b_{i,t+1}].$$

Using the standard first-order approximation in default intensity,

$$P^b_{i,t} \approx \mathbb{E}_t[M_{i,t+1}]\left(1 - \mathcal{L}\lambda_{i,t}\right). \tag{A.71}$$

Let the sovereign spread be defined by

$$\mathrm{spr}_{i,t} \equiv -\log P^b_{i,t} + \log \mathbb{E}_t[M_{i,t+1}], \tag{A.72}$$

i.e. the excess yield over the domestic risk-free benchmark implied by $\mathbb{E}_t[M_{i,t+1}]$.

Proposition 10 (Sovereign spreads increase in exposure)**.** *Fix* (g_t, s_t) *with* $s_t < S^\star$*. Under* (A.71)–(A.72)*,*

$$spr_{i,t} = -\log(1 - \mathcal{L}\lambda_{i,t}) \approx \mathcal{L}\lambda_{i,t}. \tag{A.73}$$

Therefore,

$$\frac{\partial\, spr_{i,t}}{\partial \phi_i} = \frac{\mathcal{L}}{1 - \mathcal{L}\lambda_{i,t}} \cdot \frac{\partial \lambda_{i,t}}{\partial \phi_i} > 0. \tag{A.74}$$

Proof. Substitute (A.71) into (A.72):

$$\mathrm{spr}_{i,t} = -\log\Big(\mathbb{E}_t[M_{i,t+1}](1 - \mathcal{L}\lambda_{i,t})\Big) + \log \mathbb{E}_t[M_{i,t+1}] = -\log(1 - \mathcal{L}\lambda_{i,t}).$$

Differentiate with respect to ϕ_i:

$$\frac{\partial\, \mathrm{spr}_{i,t}}{\partial \phi_i} = -\frac{1}{1 - \mathcal{L}\lambda_{i,t}} \cdot (-\mathcal{L}) \frac{\partial \lambda_{i,t}}{\partial \phi_i} = \frac{\mathcal{L}}{1 - \mathcal{L}\lambda_{i,t}} \cdot \frac{\partial \lambda_{i,t}}{\partial \phi_i}.$$

By Proposition 9, $\partial \lambda_{i,t}/\partial \phi_i > 0$, and since $1 - \mathcal{L}\lambda_{i,t} > 0$ in the interior region, the expression is strictly positive. □

A.4.5 FX Risk Premia: Cross-Sectional Slope

Under incomplete markets, international Euler equations feature an enforcement wedge $\Lambda_{i,t}$ satisfying

$$1 = \mathbb{E}_t\left[M_{\$,t+1}\Lambda_{i,t}e^{rx^{i\$,}_{t+1}}\right], \qquad \log\Lambda_{i,t} = \chi_E\phi_i s_t,$$

and log-linearization yields

$$\mathbb{E}_t[rx^{i\$,}_{t+1}] \approx -\log\Lambda_{i,t}. \tag{A.75}$$

Proposition 11 (FX premia decrease in exposure)**.** *Fix* (g_t, s_t) *and assume* (A.75)*. Then*

$$\mathbb{E}_t[rx^{i\$,}_{t+1}] \approx -\chi_E\phi_i s_t, \qquad \Rightarrow \qquad \frac{\partial\,\mathbb{E}_t[rx^{i\$,}_{t+1}]}{\partial\phi_i} = -\chi_E s_t. \tag{A.76}$$

In particular, if $s_t > 0$ *then the slope is strictly negative and high-exposure currencies earn lower expected excess returns (hedge currencies).*

Proof. Substitute $\log\Lambda_{i,t} = \chi_E\phi_i s_t$ into (A.75) to obtain the first expression. Differentiate with respect to ϕ_i holding s_t fixed:

$$\frac{\partial\,\mathbb{E}_t[rx^{i\$,}_{t+1}]}{\partial\phi_i} = -\chi_E s_t,$$

which is (A.76). □

A.4.6 Slack Regime

When $s_t \geq S^\star$, the defense floor is slack and $h_{i,t} = 0$ locally. In this regime, $Y_{i,t} = \bar{Y}_i$, debt accumulation does not respond to g_t through defense outlays, and the fiscal amplification channel is inactive. The cross-sectional slopes in debt and sovereign risk therefore collapse to their baseline components, while the UIP wedge continues to operate through $\Lambda_{i,t}$ as long as s_t enters financial frictions.

A.5 Derivation of Currency Risk Premia

This subsection derives currency risk premia implied by the model under *incomplete markets*. Exchange rates and currency premia are not pinned down by relative stochastic discount factors. Instead, they satisfy an international Euler equation with an enforcement/intermediation wedge that is driven by the U.S. security umbrella.

All steps below rely only on (i) the international Euler equation, (ii) the wedge specification, and (iii) the security-provision process

$$s_t = \bar{s} + \kappa_s g_t, \qquad \kappa_s > 0,$$

where g_t captures U.S. fiscal–military mobilization.

A.5.1 Objects and Definitions

Fix a foreign country i and the United States (funding currency) \$. Let $rx_{t+1}^{i\$,}$ denote the (log) excess return to a U.S. investor from borrowing in dollars, investing in the foreign risk-free asset, and converting back to dollars:

$$rx_{t+1}^{i\$,} \equiv r_{f,t}^{i} - r_{f,t}^{\$} - \Delta s_{i\$,t+1}, \tag{A.77}$$

where $r_{f,t}^{k}$ is the (log) risk-free rate in country k and $\Delta s_{i\$,t+1}$ is the log change in the exchange rate, defined consistently with the return construction.

A.5.2 International Euler Equation with a Security Wedge

Under incomplete markets, international asset pricing features a country-specific wedge $\Lambda_{i,t}$ in the Euler equation:

$$1 = \mathbb{E}_t\left[M_{\$,t+1}\, \Lambda_{i,t}\, e^{rx_{t+1}^{i\$,}} \right], \tag{A.78}$$

where $M_{\$,t+1}$ is the dollar stochastic discount factor.

Wedge specification. The wedge is driven by exposure to the U.S. security umbrella:

$$\log \Lambda_{i,t} = \chi_E \, \phi_i \, s_t, \qquad \chi_E > 0, \tag{A.79}$$

where ϕ_i measures country i's exposure to U.S. security provision.

A.5.3 Closed-Form Currency Premium under Lognormal Approximation

To obtain a closed-form expression for expected currency premia, impose a standard conditional lognormal approximation.

Assumption 11 (Conditional lognormal approximation)**.** *Conditional on $\mathcal{F}_t$, $\log M_{\$,t+1}$ and $rx^{i\$,}_{t+1}$ are jointly Gaussian with finite second moments.*

Taking logs of (A.78) and applying the lognormal identity yields

$$0 = \mathbb{E}_t[\log M_{\$,t+1}] + \log \Lambda_{i,t} + \mathbb{E}_t[rx^{i\$,}_{t+1}] + \tfrac{1}{2}\operatorname{Var}_t(\log M_{\$,t+1}) + \tfrac{1}{2}\operatorname{Var}_t(rx^{i\$,}_{t+1}) + \operatorname{Cov}_t(\log M_{\$,t+1}, rx^{i\$,}_{t+1}). \tag{A.80}$$

Benchmark simplification. The analysis in the main text focuses on cross-sectional variation. Assume that the variance and covariance terms in (A.80) are common across countries or second-order relative to the wedge. This isolates the role of $\Lambda_{i,t}$ in generating cross-sectional differences in currency premia.

Under this benchmark,

$$\mathbb{E}_t[rx^{i\$,}_{t+1}] \approx -\log \Lambda_{i,t}. \tag{A.81}$$

A.5.4 Security Provision and Fiscal–Military Shocks

Substituting the wedge specification (A.79) and the security-provision rule $s_t = \bar{s} + \kappa_s g_t$ into (A.81) yields

$$\boxed{\mathbb{E}_t[rx^{i\$,}_{t+1}] \approx -\chi_E \, \phi_i \, (\bar{s} + \kappa_s g_t).} \tag{A.82}$$

Thus, U.S. fiscal–military mobilization raises the absolute price of global risk while compressing expected currency returns in countries that are more exposed to the U.S. security umbrella. High-exposure currencies therefore act as hedges against U.S. fiscal–military risk and earn lower average excess returns, while low-exposure currencies command positive risk premia.

A.5.5 Cross-Sectional Slope and the Role of U.S. Fiscal Stress

Equation (A.82) delivers two immediate comparative statics.

Proposition 12 (Currency premia and security exposure)**.** *Fix the aggregate security umbrella level $s_t > 0$. Under* (A.82)*, the cross-sectional slope of expected currency excess returns with respect to exposure satisfies*

$$\frac{\partial \mathbb{E}_t[rx_{t+1}^{i\$,}]}{\partial \phi_i} = -\chi_E s_t < 0. \tag{A.83}$$

Hence, currencies of high-exposure countries (high ϕ_i) earn lower expected excess returns and act as hedge currencies, while low-exposure currencies command positive premia.

Proof. Differentiate (A.82) with respect to ϕ_i holding s_t fixed:

$$\frac{\partial \mathbb{E}_t[rx_{t+1}^{i\$,}]}{\partial \phi_i} = -\chi_E s_t.$$

Since $\chi_E > 0$ and $s_t > 0$, the derivative is strictly negative. □

Proposition 13 (U.S. fiscal stress and FX premia)**.** *Under $s_t = \bar{s} - \kappa_s g_t$ with $\kappa_s > 0$, equation* (A.82) *implies*

$$\frac{\partial \mathbb{E}_t[rx_{t+1}^{i\$,}]}{\partial g_t} = \chi_E \, \phi_i \, \kappa_s > 0. \tag{A.84}$$

Thus, U.S. fiscal–military stress increases expected currency premia in low-exposure currencies relative to high-exposure hedge currencies.

Proof. Substitute $s_t = \bar{s} - \kappa_s g_t$ into (A.82):

$$\mathbb{E}_t[rx^{i\$,}_{t+1}] \approx -\chi_E \phi_i (\bar{s} - \kappa_s g_t) = -\chi_E \phi_i \bar{s} + \chi_E \phi_i \kappa_s g_t.$$

Differentiate with respect to g_t:

$$\frac{\partial \, \mathbb{E}_t[rx^{i\$,}_{t+1}]}{\partial g_t} = \chi_E \phi_i \kappa_s,$$

which is strictly positive since $\chi_E, \phi_i, \kappa_s > 0$. □

Interpretation. The derivation clarifies the mechanism underlying the FX sign flip documented in the motivating facts. In this model, the U.S. security umbrella enters currency markets as an Euler-equation wedge: security provision tightens enforcement and compresses international premia, while retrenchment widens wedges and increases compensation for bearing currency risk. Because the wedge scales with exposure ϕ_i, high-exposure currencies load more on the umbrella and therefore deliver insurance when U.S. fiscal–military risk rises. As a result, high-ϕ_i currencies earn lower average excess returns, while low-ϕ_i currencies earn positive premia—exactly the sorting pattern underlying the HML FX strategy in the main text.

A.6 Derivation of FX Betas

This subsection derives the exposure (beta) of bilateral exchange rate changes to U.S. fiscal–military shocks. The derivation is fully explicit and uses only (i) complete markets, (ii) log utility, and (iii) the equilibrium consumption representation established in Appendix **??**. No nominal objects are invoked.

A.6.1 Setup and Definitions

Let $S_{i\$,t}$ denote the *real* exchange rate, defined as the price of one unit of the U.S. consumption basket ("\$") in units of country i's consumption basket. Thus an increase in $S_{i\$,t}$ is a real depreciation of country i relative to the U.S. (one needs more units of the i basket per unit of the U.S.

basket).

Define the log depreciation rate:

$$\Delta s_{i\$,t+1} \equiv \Delta \log S_{i\$,t+1}.$$

Let g_t denote the U.S. fiscal–military shock and let $\Delta g_{t+1} \equiv g_{t+1} - g_t$ denote its innovation from t to $t+1$. Define the (population) *FX beta* of country i with respect to the U.S. fiscal–military shock as

$$\beta_i^{FX} \equiv \frac{\mathrm{Cov}\big(\Delta s_{i\$,t+1},\, \Delta g_{t+1}\big)}{\mathrm{Var}(\Delta g_{t+1})}. \tag{A.85}$$

All covariances and variances below are unconditional (population) moments; conditional versions follow analogously under stationarity.

A.6.2 Complete Markets Restriction

Let M_{t+1}^i denote the real stochastic discount factor (SDF) in country i:

$$M_{t+1}^i = \beta \frac{C_{i,t}}{C_{i,t+1}}. \tag{A.86}$$

Under complete markets, the real exchange rate satisfies the standard SDF identity:

$$\Delta s_{i\$,t+1} = \log M_{t+1}^{\$} - \log M_{t+1}^i. \tag{A.87}$$

With log utility, $\log M_{t+1}^i = \log \beta - \Delta \log C_{i,t+1}$, so (A.87) becomes

$$\Delta s_{i\$,t+1} = \Delta \log C_{i,t+1} - \Delta \log C_{\$,t+1}. \tag{A.88}$$

A.6.3 Substituting the Model's Consumption Process

From Appendix **??**, consumption growth admits the representation

$$\Delta \log C_{i,t+1} = \phi_i \, \Delta g_{t+1} + (1 - \phi_i)\Delta \Xi_{i,t+1}, \tag{A.89}$$

where $\Delta \Xi_{i,t+1}$ collects country-specific components orthogonal to Δg_{t+1}.

Applying (A.89) to the U.S. (denoted \$), we have

$$\Delta \log C_{\$,t+1} = \phi_\$ \, \Delta g_{t+1} + (1 - \phi_\$)\Delta \Xi_{\$,t+1}. \tag{A.90}$$

Substituting (A.89)–(A.90) into (A.88) yields

$$\Delta s_{i\$,t+1} = \Big(\phi_i - \phi_\$\Big)\Delta g_{t+1} + (1 - \phi_i)\Delta \Xi_{i,t+1} - (1 - \phi_\$)\Delta \Xi_{\$,t+1}. \tag{A.91}$$

A.6.4 Moment Assumptions

To isolate the beta with respect to Δg_{t+1}, assume:

Assumption 12 (Orthogonality)**.** *For all i,*

$$\mathrm{Cov}\big(\Delta g_{t+1},\, \Delta \Xi_{i,t+1}\big) = 0, \qquad \mathrm{Var}(\Delta g_{t+1}) = \sigma_g^2 \in (0, \infty),$$

and all second moments exist.

A.6.5 Closed-Form FX Beta

Taking the covariance of (A.91) with Δg_{t+1} and using Assumption 12 yields

$$\begin{aligned} \mathrm{Cov}\big(\Delta s_{i\$,t+1},\, \Delta g_{t+1}\big) &= (\phi_i - \phi_\$)\,\mathrm{Var}(\Delta g_{t+1}) \\ &= (\phi_i - \phi_\$)\sigma_g^2. \end{aligned} \tag{A.92}$$

Substituting (A.92) into the definition (A.85) gives:

Proposition 14 (FX betas with respect to U.S. fiscal–military shocks)**.** *Under complete markets, log utility, and Assumption 12, the beta of the bilateral real exchange rate depreciation $\Delta s_{i\$,t+1}$ with respect to the U.S. fiscal–military shock Δg_{t+1} is*

$$\beta_i^{FX} = \phi_i - \phi_\$. \tag{A.93}$$

Proof. The result follows from (A.91)–(A.92) and the definition (A.85). □

A.6.6 Sign Interpretation

Equation (A.93) implies:

$$\beta_i^{FX} < 0 \quad \Longleftrightarrow \quad \phi_i < \phi_\$.$$

Thus, if the U.S. is more exposed to fiscal–military shocks than country i (higher $\phi_\$$), then an increase in g_{t+1} is associated with a real *appreciation* of the dollar relative to country i (a fall in $S_{i\$,t+1}$). Conversely, if $\phi_i > \phi_\$$, the dollar depreciates against country i in U.S. military-shock states.

A.7 Derivation of Global Equity Betas

This subsection derives global *equity* betas in closed form. The derivation is fully explicit and uses only: (i) complete markets, (ii) log utility, and (iii) the equilibrium consumption representation in Appendix **??**. No nominal objects are invoked.

A.7.1 Equity Claims, Returns, and Global Factors

Fix a country i. Define a claim ("equity") to the country-i final-good cash flow. In the two-period setup, we take equity to be a claim purchased at $t = 1$ that pays a terminal dividend at $t = 2$. Let the dividend be proportional to the final good:

$$\mathcal{D}_{i,2} \equiv \kappa_i \, C_{i,2}, \qquad \kappa_i > 0, \tag{A.94}$$

and assume no intermediate dividend at $t = 1$. (The proportionality constant κ_i allows for arbitrary units; it will cancel from returns.)

Let $P^e_{i,1}$ denote the equity price at $t = 1$ (in units of country i's consumption numeraire). The gross equity return from $t = 1$ to $t = 2$ is

$$R^e_{i,1} \equiv \frac{\mathcal{D}_{i,2}}{P^e_{i,1}}, \qquad r^e_{i,1} \equiv \log R^e_{i,1}. \tag{A.95}$$

Define a global equity return factor as the weighted average of *log* equity returns:

$$r^e_{G,1} \equiv \sum_{m=1}^{N} \omega_m \, r^e_{m,1}, \qquad \omega_m \geq 0, \quad \sum_m \omega_m = 1. \tag{A.96}$$

The (population) global equity beta of country i is

$$\beta^e_{i,G} \equiv \frac{\mathrm{Cov}\big(r^e_{i,1},\, r^e_{G,1}\big)}{\mathrm{Var}\big(r^e_{G,1}\big)}. \tag{A.97}$$

A.7.2 Pricing Under Complete Markets and Log Utility

Let M^i_2 denote the country-i real stochastic discount factor between $t = 1$ and $t = 2$:

$$M^i_2 = \beta \frac{C_{i,1}}{C_{i,2}}. \tag{A.98}$$

Lemma A.1 (Equity price)**.** *Under complete markets, the equity price satisfies*

$$P^e_{i,1} = \mathbb{E}_1\Big[M^i_2 \, \mathcal{D}_{i,2}\Big]. \tag{A.99}$$

Proof. With complete markets, any payoff denominated in country i's consumption numeraire is priced by the country-i SDF. Applying the standard Euler equation for the equity payoff $\mathcal{D}_{i,2}$ yields (A.99). □

Proposition 15 (Closed-form equity return)**.** *Under* (A.94)–(A.98)*, the gross equity return satis-*

fies

$$R^e_{i,1} = \frac{1}{\beta}\frac{C_{i,2}}{C_{i,1}}, \qquad r^e_{i,1} = -\log\beta + \Delta\log C_{i,2}, \tag{A.100}$$

where $\Delta\log C_{i,2} \equiv \log C_{i,2} - \log C_{i,1}$.

Proof. By Lemma A.1 and (A.94)–(A.98),

$$P^e_{i,1} = \mathbb{E}_1\left[\beta\frac{C_{i,1}}{C_{i,2}}\cdot\kappa_i C_{i,2}\right] = \beta\kappa_i C_{i,1}\,\mathbb{E}_1[1] = \beta\kappa_i C_{i,1}. \tag{A.101}$$

Substituting $P^e_{i,1} = \beta\kappa_i C_{i,1}$ and $\mathcal{D}_{i,2} = \kappa_i C_{i,2}$ into (A.95) yields

$$R^e_{i,1} = \frac{\kappa_i C_{i,2}}{\beta\kappa_i C_{i,1}} = \frac{1}{\beta}\frac{C_{i,2}}{C_{i,1}}.$$

Taking logs gives (A.100). □

A.7.3 Equity Returns as a Factor Representation

From Appendix **??**, consumption growth has the representation

$$\Delta\log C_{i,2} = \phi_i\,\Delta g_2 + (1-\phi_i)\Delta\Xi_{i,2}, \tag{A.102}$$

where $\Delta g_2 \equiv g_2 - g_1$ is the U.S. fiscal–military innovation and $\Delta\Xi_{i,2}$ is a country-specific component (e.g. changes in local terms).

Combining (A.100) and (A.102) yields

$$r^e_{i,1} = -\log\beta + \phi_i\,\Delta g_2 + (1-\phi_i)\Delta\Xi_{i,2}. \tag{A.103}$$

A.7.4 Moment Assumptions

To derive closed-form betas, impose the same orthogonality structure used elsewhere:

Assumption 13 (Orthogonality and idiosyncrasy)**.** *For all $i \neq j$:*

(i) $\mathrm{Cov}(\Delta g_2, \Delta\Xi_{i,2}) = 0$,

(ii) $\mathrm{Cov}(\Delta\Xi_{i,2}, \Delta\Xi_{j,2}) = 0$,

(iii) $\mathrm{Var}(\Delta g_2) = \sigma_g^2 \in (0, \infty)$ *and* $\mathrm{Var}(\Delta\Xi_{i,2}) < \infty$.

A.8 Derivation of Global Equity Risk Premia

This subsection derives equilibrium equity risk premia implied by the model under the closed-form benchmark in Section 5.2. The derivation is fully explicit and relies on: (i) log preferences, (ii) decentralized equity pricing using the domestic stochastic discount factor, and (iii) the closed-form consumption process induced by U.S. fiscal–military shocks. No complete-markets risk sharing across countries is required.

A.8.1 Equity Claims and Returns

Fix a country i. Consider an equity claim purchased at date $t = 1$ that pays a terminal dividend at $t = 2$. Dividends are proportional to final output (and hence consumption up to constants):

$$\mathcal{D}_{i,2} \equiv \kappa_i \, C_{i,2}, \qquad \kappa_i > 0. \tag{A.104}$$

There are no intermediate dividends. Let $P_{i,1}^e$ denote the equity price at $t = 1$ in units of country i's consumption numeraire. The gross and log equity returns are

$$R_{i,1}^e \equiv \frac{\mathcal{D}_{i,2}}{P_{i,1}^e}, \qquad r_{i,1}^e \equiv \log R_{i,1}^e. \tag{A.105}$$

Let $R_{i,1}^f$ denote the one-period real risk-free gross return in country i and $r_{i,1}^f \equiv \log R_{i,1}^f$. The (log) equity risk premium is

$$\mathrm{ERP}_{i,1} \equiv \mathbb{E}_1[r_{i,1}^e] - r_{i,1}^f. \tag{A.106}$$

A.8.2 Equity Pricing

Let M_2^i denote the domestic stochastic discount factor between $t = 1$ and $t = 2$:

$$M_2^i = \beta \frac{C_{i,1}}{C_{i,2}}. \tag{A.107}$$

Lemma A.2 (Equity pricing)**.** *The equity price satisfies*

$$P_{i,1}^e = \mathbb{E}_1 \left[M_2^i \, \mathcal{D}_{i,2} \right]. \tag{A.108}$$

Proof. Equity payoffs are denominated in country i's consumption numeraire. Applying the domestic Euler equation to the payoff $\mathcal{D}_{i,2}$ yields (A.108). □

A.8.3 Closed-Form Equity Return

Lemma A.3 (Equity return)**.** *Under* (A.104) *and* (A.107)*,*

$$R_{i,1}^e = \frac{1}{\beta} \frac{C_{i,2}}{C_{i,1}}, \qquad r_{i,1}^e = -\log \beta + \Delta \log C_{i,2}, \tag{A.109}$$

where $\Delta \log C_{i,2} \equiv \log C_{i,2} - \log C_{i,1}$.

Proof. From Lemma A.2,

$$P_{i,1}^e = \mathbb{E}_1 \left[\beta \frac{C_{i,1}}{C_{i,2}} \cdot \kappa_i C_{i,2} \right] = \beta \kappa_i C_{i,1}.$$

Hence

$$R_{i,1}^e = \frac{\kappa_i C_{i,2}}{\beta \kappa_i C_{i,1}} = \frac{1}{\beta} \frac{C_{i,2}}{C_{i,1}},$$

and taking logs yields (A.109). □

A.8.4 Risk-Free Rate

The domestic risk-free rate satisfies

$$\frac{1}{R^f_{i,1}} = \mathbb{E}_1[M^i_2] = \beta\, \mathbb{E}_1\Big[\exp\big(-\Delta \log C_{i,2}\big)\Big]. \tag{A.110}$$

Assumption 14 (Conditional lognormality)**.** *Conditional on $\mathcal{F}_1$, $\Delta \log C_{i,2}$ is Gaussian with finite variance.*

Under Assumption 14,

$$r^f_{i,1} = -\log \beta + \mathbb{E}_1[\Delta \log C_{i,2}] - \tfrac{1}{2} \operatorname{Var}_1(\Delta \log C_{i,2}). \tag{A.111}$$

A.8.5 Equity Risk Premium

Taking expectations in (A.109) yields

$$\mathbb{E}_1[r^e_{i,1}] = -\log \beta + \mathbb{E}_1[\Delta \log C_{i,2}].$$

Subtracting (A.111) gives:

Proposition 16 (Equity risk premium)**.** *Under log utility and conditional lognormality,*

$$\mathrm{ERP}_{i,1} = \tfrac{1}{2} \operatorname{Var}_1(\Delta \log C_{i,2}). \tag{A.112}$$

Proof. Subtract (A.111) from $\mathbb{E}_1[r^e_{i,1}]$ and simplify. □

A.8.6 Consumption Risk Induced by U.S. Military Shocks

Under the closed-form benchmark, consumption growth admits the one-factor representation derived in Appendix A.2:

$$\Delta \log C_{i,2} = \theta^C_i\, \Delta g_2 + \Delta \Xi^C_{i,2}, \tag{A.113}$$

where:

- Δg_2 is the U.S. fiscal–military shock,
- $\theta_i^C \propto \phi_i$ is country-specific exposure,
- $\Delta \Xi_{i,2}^C$ is idiosyncratic.

Assumption 15 (Orthogonality)**.**

$$\mathrm{Cov}_1(\Delta g_2, \Delta \Xi_{i,2}^C) = 0, \qquad \mathrm{Var}_1(\Delta g_2) = \sigma_g^2 \in (0, \infty).$$

Then

$$\mathrm{Var}_1(\Delta \log C_{i,2}) = (\theta_i^C)^2 \sigma_g^2 + \mathrm{Var}_1(\Delta \Xi_{i,2}^C). \tag{A.114}$$

Substituting (A.114) into (A.112) yields

$$\mathrm{ERP}_{i,1} = \tfrac{1}{2}(\theta_i^C)^2 \sigma_g^2 + \tfrac{1}{2}\,\mathrm{Var}_1(\Delta \Xi_{i,2}^C). \tag{A.115}$$

A.8.7 Benchmark Simplification

If idiosyncratic components are negligible or do not vary systematically across countries, then

$$\boxed{\mathrm{ERP}_{i,1} \;\approx\; \tfrac{1}{2}\,\phi_i^2\,\sigma_g^2,} \tag{A.116}$$

where proportionality follows from $\theta_i^C \propto \phi_i$.

A.8.8 Deriving Global Equity Betas

Define the global log equity return factor (A.96). Substitute (A.103) into (A.96):

$$\begin{aligned} r_{G,1}^e &= \sum_{m=1}^{N} \omega_m \Big(-\log \beta + \phi_m \Delta g_2 + (1 - \phi_m) \Delta \Xi_{m,2} \Big) \\ &= -\log \beta + \Phi_\omega\, \Delta g_2 + \sum_{m=1}^{N} \omega_m (1 - \phi_m) \Delta \Xi_{m,2}, \end{aligned} \tag{A.117}$$

where $\Phi_\omega \equiv \sum_m \omega_m \phi_m$.

Since constants do not affect covariances, global equity betas depend on the covariance between the stochastic components only. The next proposition provides the exact expression.

Proposition 17 (Exact global equity beta)**.** *Under Assumption 13, the global equity beta* (A.97) *satisfies*

$$\beta^e_{i,G} = \frac{\phi_i \Phi_\omega \sigma_g^2 + \omega_i (1-\phi_i)^2 \operatorname{Var}(\Delta \Xi_{i,2})}{\Phi_\omega^2 \sigma_g^2 + \sum_{m=1}^N \omega_m^2 (1-\phi_m)^2 \operatorname{Var}(\Delta \Xi_{m,2})}. \tag{A.118}$$

Proof. From (A.103) and (A.117),

$$\operatorname{Cov}(r^e_{i,1}, r^e_{G,1}) = \operatorname{Cov}\Big(\phi_i \Delta g_2 + (1-\phi_i)\Delta \Xi_{i,2},\ \Phi_\omega \Delta g_2 + \sum_m \omega_m (1-\phi_m)\Delta \Xi_{m,2}\Big),$$

since constants drop out. Under Assumption 13(i), cross terms with Δg_2 and $\Delta \Xi$ vanish, and under (ii),

$$\operatorname{Cov}\Big((1-\phi_i)\Delta \Xi_{i,2},\ \sum_m \omega_m (1-\phi_m)\Delta \Xi_{m,2}\Big) = \omega_i (1-\phi_i)^2 \operatorname{Var}(\Delta \Xi_{i,2}).$$

Hence,

$$\operatorname{Cov}(r^e_{i,1}, r^e_{G,1}) = \phi_i \Phi_\omega \operatorname{Var}(\Delta g_2) + \omega_i (1-\phi_i)^2 \operatorname{Var}(\Delta \Xi_{i,2}) = \phi_i \Phi_\omega \sigma_g^2 + \omega_i (1-\phi_i)^2 \operatorname{Var}(\Delta \Xi_{i,2}).$$

Similarly, using Assumption 13, the variance of $r^e_{G,1}$ is

$$\operatorname{Var}(r^e_{G,1}) = \Phi_\omega^2 \sigma_g^2 + \sum_{m=1}^N \omega_m^2 (1-\phi_m)^2 \operatorname{Var}(\Delta \Xi_{m,2}).$$

Dividing the covariance by the variance yields (A.118). □

A.8.9 Dominance Benchmark

If the aggregate U.S. fiscal–military component dominates the global equity factor, i.e.,

$$\Phi_\omega^2 \sigma_g^2 \gg \sum_{m=1}^{N} \omega_m^2 (1-\phi_m)^2 \operatorname{Var}(\Delta \Xi_{m,2}),$$

then (A.118) simplifies to

$$\beta_{i,G}^e \approx \frac{\phi_i}{\sum_{m=1}^{N} \omega_m \phi_m}. \tag{A.119}$$

In this benchmark, global equity betas are strictly increasing in ϕ_i and inherit the same cross-sectional ordering as global consumption betas.

A.9 Derivation of Bond Spreads

This subsection derives sovereign bond spreads implied by the model in closed form under the closed-form benchmark in Section 5.2. The derivation is explicit and relies on (i) the domestic Euler equation, (ii) a reduced-form default intensity that depends on debt and the security umbrella, and (iii) a one-period sovereign bond payoff with loss given default. No complete-markets assumptions are imposed.

A.9.1 Objects and Definitions

Fix a country i and date t. Let $M_{i,t+1}$ denote the domestic stochastic discount factor:

$$M_{i,t+1} \equiv \beta \frac{C_{i,t}}{C_{i,t+1}}. \tag{A.120}$$

Let $J_{i,t+1} \in \{0,1\}$ indicate a sovereign stress/default event between t and $t+1$. The conditional default probability is

$$\Pr(J_{i,t+1} = 1 \mid \mathcal{F}_t) = \lambda_{i,t}, \tag{A.121}$$

where the default intensity $\lambda_{i,t}$ is given by

$$\lambda_{i,t} = \exp\big(\alpha_0 + \alpha_1 B_{i,t} + \alpha_2 g_t - \alpha_3 s_t + \alpha_4 \phi_i\big). \tag{A.122}$$

Sovereign bond payoff. A one-period sovereign bond issued by country i pays

$$X^b_{i,t+1} = 1 - \mathcal{L} J_{i,t+1}, \qquad \mathcal{L} \in (0,1), \tag{A.123}$$

where $\mathcal{L}$ is the loss given default.

Bond price and yield. The bond price and gross yield are

$$P^b_{i,t} \equiv \mathbb{E}_t\Big[M_{i,t+1} X^b_{i,t+1}\Big], \qquad R^b_{i,t} \equiv \frac{1}{P^b_{i,t}}, \qquad r^b_{i,t} \equiv \log R^b_{i,t}. \tag{A.124}$$

Risk-free benchmark and spread. Define the domestic one-period real risk-free return $R^f_{i,t}$ by

$$\frac{1}{R^f_{i,t}} = \mathbb{E}_t[M_{i,t+1}], \qquad r^f_{i,t} \equiv \log R^f_{i,t}, \tag{A.125}$$

and define the (log) sovereign spread as

$$\mathrm{spr}_{i,t} \equiv r^b_{i,t} - r^f_{i,t}. \tag{A.126}$$

A.9.2 Exact Decomposition of the Bond Price

Using (A.123), the bond price is

$$\begin{aligned} P^b_{i,t} &= \mathbb{E}_t\big[M_{i,t+1}(1 - \mathcal{L} J_{i,t+1})\big] \\ &= \mathbb{E}_t[M_{i,t+1}] - \mathcal{L}\,\mathbb{E}_t[M_{i,t+1} J_{i,t+1}]. \end{aligned} \tag{A.127}$$

Define the conditional default-event pricing kernel term

$$\pi_{i,t} \equiv \frac{\mathbb{E}_t[M_{i,t+1} J_{i,t+1}]}{\mathbb{E}_t[M_{i,t+1}]}, \tag{A.128}$$

which is the *risk-adjusted* probability of default. Then (A.127) can be written exactly as

$$P^b_{i,t} = \mathbb{E}_t[M_{i,t+1}] \left(1 - \mathcal{L}\pi_{i,t}\right). \tag{A.129}$$

A.9.3 Exact Spread Formula

By (A.125), $\mathbb{E}_t[M_{i,t+1}] = 1/R^f_{i,t}$. Substitute into (A.129):

$$P^b_{i,t} = \frac{1}{R^f_{i,t}} \left(1 - \mathcal{L}\pi_{i,t}\right).$$

Taking reciprocals yields

$$R^b_{i,t} = \frac{R^f_{i,t}}{1 - \mathcal{L}\pi_{i,t}}.$$

Taking logs and using (A.126) gives the exact spread:

$$\boxed{\mathrm{spr}_{i,t} = -\log\left(1 - \mathcal{L}\pi_{i,t}\right).} \tag{A.130}$$

Equation (A.130) is exact and holds without any lognormality assumptions.

A.9.4 From Risk-Adjusted Default Probability to Default Intensity

To connect (A.130) to the model's default intensity $\lambda_{i,t}$, impose a standard conditional independence restriction.

Assumption 16 (Conditional independence)**.** *Conditional on $\mathcal{F}_t$, the default event $J_{i,t+1}$ is independent of the one-period SDF $M_{i,t+1}$.*

Under Assumption 16,

$$\mathbb{E}_t[M_{i,t+1}J_{i,t+1}] = \mathbb{E}_t[M_{i,t+1}]\mathbb{E}_t[J_{i,t+1}] = \mathbb{E}_t[M_{i,t+1}]\lambda_{i,t}, \tag{A.131}$$

so $\pi_{i,t} = \lambda_{i,t}$ by (A.128).

Substituting into (A.130) yields the closed-form spread expression:

$$\boxed{\mathrm{spr}_{i,t} = -\log\big(1 - \mathcal{L}\lambda_{i,t}\big), \qquad \lambda_{i,t} = \exp\big(\alpha_0 + \alpha_1 B_{i,t} + \alpha_2 g_t - \alpha_3 s_t + \alpha_4 \phi_i\big).} \tag{A.132}$$

A.9.5 Small-Default Approximation

For small default probabilities such that $\mathcal{L}\lambda_{i,t} \ll 1$, a first-order Taylor expansion of $-\log(1-x)$ at $x = 0$ yields $-\log(1-x) \approx x$. Applying this to (A.132) gives

$$\mathrm{spr}_{i,t} \approx \mathcal{L}\lambda_{i,t}. \tag{A.133}$$

A.9.6 Cross-Sectional Slope with Respect to Exposure

Fix the aggregate state (g_t, s_t) and treat $B_{i,t}$ as predetermined at time t. Differentiate (A.132) with respect to ϕ_i:

$$\frac{\partial\, \mathrm{spr}_{i,t}}{\partial \phi_i} = \frac{\mathcal{L}}{1 - \mathcal{L}\lambda_{i,t}} \cdot \frac{\partial \lambda_{i,t}}{\partial \phi_i}. \tag{A.134}$$

From (A.122),

$$\frac{\partial \lambda_{i,t}}{\partial \phi_i} = \lambda_{i,t} \cdot \alpha_4.$$

Substitute into (A.134) to obtain

$$\boxed{\frac{\partial\, \mathrm{spr}_{i,t}}{\partial \phi_i} = \frac{\mathcal{L}\alpha_4\, \lambda_{i,t}}{1 - \mathcal{L}\lambda_{i,t}} > 0,} \tag{A.135}$$

provided $1 - \mathcal{L}\lambda_{i,t} > 0$. Thus, sovereign spreads are strictly increasing in security exposure ϕ_i.

A.9.7 Interpretation

Equation (A.132) shows that bond spreads are increasing in default risk, which is itself increasing in debt and U.S. fiscal–military stress (through g_t and s_t). Because default intensity rises with exposure ϕ_i, countries more deeply embedded in the U.S. security umbrella exhibit higher sovereign risk premia, generating a cross-sectional slope in bond spreads consistent with the motivating facts and the security-sorted bond portfolios in the empirical section.

A.10 Trading Implication I: Long U.S. Defense Equity

This subsection derives the return, shock beta, and risk premium on a claim to U.S. defense-sector cash flows in the new model. Unlike the earlier draft, the derivation does not rely on complete markets. All steps are explicit and use only: (i) the U.S. Euler equation, (ii) log utility, and (iii) a parametric cash-flow exposure of defense-sector dividends to the U.S. fiscal–military shock.

A.10.1 Defense Equity Claim

Consider a one-period traded claim at date t that pays a dividend at $t+1$ proportional to U.S. defense-sector cash flows:

$$\mathcal{D}^D_{t+1} \equiv \kappa_D \exp(\eta_D \, \Delta g_{t+1}) \,, \qquad \kappa_D > 0, \tag{A.136}$$

where $\Delta g_{t+1} \equiv g_{t+1} - g_t$ is the U.S. fiscal–military innovation and η_D is the defense-sector cash-flow loading on the fiscal–military shock. This reduced-form cash-flow exposure is motivated by the model mechanism: military mobilization and defense procurement move with the fiscal–military state and thus directly affect defense-sector revenues.

Let P^D_t denote the price of this claim in units of U.S. consumption at time t. The gross return is

$$R^D_t \equiv \frac{\mathcal{D}^D_{t+1}}{P^D_t}, \qquad r^D_t \equiv \log R^D_t. \tag{A.137}$$

A.10.2 U.S. SDF and Shock Process

U.S. stochastic discount factor. Under log utility, the U.S. real SDF is

$$M_{\$,t+1} = \beta \frac{C_{\$,t}}{C_{\$,t+1}} = \beta \exp\bigl(-\Delta \log C_{\$,t+1}\bigr)\,. \tag{A.138}$$

In the closed-form benchmark with a binding defense floor, Appendix A.3 establishes the (one-factor) representation

$$\Delta \log C_{\$,t+1} \approx -\vartheta_{\$}\phi_{\$}\kappa_s\,\Delta g_{t+1} + \Delta\Xi^{C}_{\$,t+1}, \tag{A.139}$$

where $\vartheta_{\$} > 0$ and $\Delta\Xi^{C}_{\$,t+1}$ is orthogonal to Δg_{t+1}.

Substituting (A.139) into (A.138) yields

$$M_{\$,t+1} \approx \beta \exp\Bigl(\vartheta_{\$}\phi_{\$}\kappa_s\,\Delta g_{t+1} - \Delta\Xi^{C}_{\$,t+1}\Bigr)\,. \tag{A.140}$$

Distributional assumption. Assume conditional Gaussianity:

$$\Delta g_{t+1} \mid \mathcal{F}_t \sim \mathcal{N}(\mu_g, \sigma_g^2), \qquad \Delta\Xi^{C}_{\$,t+1} \;\perp\; \Delta g_{t+1} \;\text{ conditional on } \mathcal{F}_t. \tag{A.141}$$

A.10.3 Pricing

Lemma A.4 (Defense equity price)**.** *The defense equity price satisfies the U.S. Euler equation*

$$P^{D}_{t} = \mathbb{E}_t\Bigl[M_{\$,t+1}\,\mathcal{D}^{D}_{t+1}\Bigr]\,. \tag{A.142}$$

Proof. The claim $\mathcal{D}^{D}_{t+1}$ is denominated in U.S. consumption units. In equilibrium, any payoff X_{t+1} priced by the U.S. investor satisfies $P_t = \mathbb{E}_t[M_{\$,t+1}X_{t+1}]$. Applying this pricing condition to $X_{t+1} = \mathcal{D}^{D}_{t+1}$ yields (A.142). □

Substitute (A.136) and (A.140) into (A.142):

$$P_t^D \approx \mathbb{E}_t\left[\beta \exp\Big(\vartheta_{\$}\phi_{\$}\kappa_s\,\Delta g_{t+1} - \Delta\Xi^C_{\$,t+1}\Big)\cdot \kappa_D \exp(\eta_D\,\Delta g_{t+1})\right]$$
$$= \beta\kappa_D\,\mathbb{E}_t\Big[\exp\big((\vartheta_{\$}\phi_{\$}\kappa_s + \eta_D)\Delta g_{t+1}\big)\Big]\cdot \mathbb{E}_t\left[\exp\Big(-\Delta\Xi^C_{\$,t+1}\Big)\right], \tag{A.143}$$

where the factorization uses conditional orthogonality in (A.141).

Under conditional normality, for $Z \sim \mathcal{N}(\mu,\sigma^2)$, $\mathbb{E}[e^{aZ}] = \exp(a\mu + \frac{1}{2}a^2\sigma^2)$. Applying this identity yields

$$\mathbb{E}_t\Big[\exp\big((\vartheta_{\$}\phi_{\$}\kappa_s + \eta_D)\Delta g_{t+1}\big)\Big] = \exp\left((\vartheta_{\$}\phi_{\$}\kappa_s + \eta_D)\mu_g + \frac{1}{2}(\vartheta_{\$}\phi_{\$}\kappa_s + \eta_D)^2\sigma_g^2\right). \tag{A.144}$$

Thus,

$$P_t^D \approx \beta\kappa_D \exp\left((\vartheta_{\$}\phi_{\$}\kappa_s + \eta_D)\mu_g + \frac{1}{2}(\vartheta_{\$}\phi_{\$}\kappa_s + \eta_D)^2\sigma_g^2\right)\cdot \mathbb{E}_t\left[\exp\Big(-\Delta\Xi^C_{\$,t+1}\Big)\right]. \tag{A.145}$$

A.10.4 Return, Shock Beta, and Risk Premium

Log return. Using (A.137) and (A.136),

$$r_t^D = \log\kappa_D + \eta_D\Delta g_{t+1} - \log P_t^D.$$

Substitute (A.145) and collect terms to obtain

$$r_t^D \approx -\log\beta + \eta_D\Delta g_{t+1} - (\vartheta_{\$}\phi_{\$}\kappa_s + \eta_D)\mu_g - \frac{1}{2}(\vartheta_{\$}\phi_{\$}\kappa_s + \eta_D)^2\sigma_g^2 - \log\mathbb{E}_t\left[e^{-\Delta\Xi^C_{\$,t+1}}\right]. \tag{A.146}$$

Shock beta. Since (A.146) is affine in Δg_{t+1} with coefficient η_D, the (population) beta of r_t^D with respect to Δg_{t+1} is

$$\boxed{\beta_g^D \equiv \frac{\mathrm{Cov}(r_t^D, \Delta g_{t+1})}{\mathrm{Var}(\Delta g_{t+1})} = \eta_D.} \tag{A.147}$$

Risk premium. Let $r^f_{\$,t}$ denote the U.S. log risk-free rate. By definition,

$$r^f_{\$,t} = -\log \mathbb{E}_t[M_{\$,t+1}].$$

Using (A.140) and conditional lognormality as in Appendix A.3,

$$r^f_{\$,t} = -\log \beta - \vartheta_\$ \phi_\$ \kappa_s \mu_g - \frac{1}{2}(\vartheta_\$ \phi_\$ \kappa_s)^2 \sigma_g^2 + \log \mathbb{E}_t\left[e^{-\Delta \Xi^C_{\$,t+1}}\right]. \tag{A.148}$$

Take conditional expectations of (A.146) and subtract (A.148):

$$\begin{aligned}\mathbb{E}_t[r^D_t - r^f_{\$,t}] &\approx -\frac{1}{2}\Big[(\vartheta_\$ \phi_\$ \kappa_s + \eta_D)^2 - (\vartheta_\$ \phi_\$ \kappa_s)^2\Big]\sigma_g^2 \\ &= -\frac{1}{2}\Big(2\vartheta_\$ \phi_\$ \kappa_s \eta_D + \eta_D^2\Big)\sigma_g^2. \end{aligned} \tag{A.149}$$

Proposition 18 (Defense equity beta and premium)**.** *Under the maintained conditional Gaussianity assumptions, the defense equity shock beta equals $\beta_g^D = \eta_D$, and the expected excess return over the U.S. risk-free rate satisfies*

$$\mathbb{E}_t[r^D_t - r^f_{\$,t}] \approx -\frac{1}{2}\Big(2\vartheta_\$ \phi_\$ \kappa_s \eta_D + \eta_D^2\Big)\sigma_g^2.$$

Proof. The beta result follows from (A.146) as shown in (A.147). The premium expression follows by taking expectations of (A.146), using $\mathbb{E}_t[\Delta g_{t+1}] = \mu_g$, and subtracting the risk-free rate (A.148) term-by-term, yielding (A.149). □

Interpretation. Defense equity provides a direct traded exposure to U.S. fiscal–military innovations. Its return beta is proportional to the cash-flow loading η_D. The associated risk premium is pinned down by covariation with the U.S. SDF: when defense cash flows rise in states in which U.S. marginal utility is high, the claim behaves as an insurance asset and earns a low (possibly negative) expected excess return; when defense cash flows fall in those states, the claim is risky and earns a positive premium. The sign and magnitude are therefore disciplined by the loading η_D and the strength of fiscal–military transmission into U.S. consumption, summarized by $\vartheta_\$ \phi_\$ \kappa_s$.

A.11 Trading Implication II: HML FX Strategy Sorted on Defense Exposure

This subsection derives the expected return of a long–short currency strategy that sorts currencies by exposure to the U.S. security umbrella. Unlike the earlier draft, the derivation does *not* rely on complete markets. Instead, currency premia are pinned down by the international Euler equation with a security-driven wedge, as derived in Appendix A.5.

A.11.1 Bilateral excess returns versus the dollar

Fix the U.S. investor (denoted \$). For each foreign country $i \neq \$$, define the (log) excess return on currency i from the dollar perspective as

$$rx_{t+1}^{i\$} \equiv r_{i,t}^{f} - r_{\$,t}^{f} - \Delta s_{i\$,t+1}. \tag{A.150}$$

Under incomplete markets, bilateral currency excess returns satisfy the international Euler equation with wedge

$$1 = \mathbb{E}_t\Big[M_{\$,t+1}\,\Lambda_{i,t}\,e^{rx_{t+1}^{i\$}}\Big], \tag{A.151}$$

where the enforcement/intermediation wedge is driven by the U.S. security umbrella:

$$\log \Lambda_{i,t} = \chi_E\,\phi_i\,s_t, \qquad s_t = \bar{s} - \kappa_s g_t, \qquad \chi_E > 0,\ \kappa_s > 0. \tag{A.152}$$

Appendix A.5 shows that under conditional lognormality,

$$\mathbb{E}_t[rx_{t+1}^{i\$}] \approx -\log \Lambda_{i,t} = -\chi_E\,\phi_i\,s_t. \tag{A.153}$$

A.11.2 HML portfolio construction

Partition foreign currencies into two sets at date t:

$$L \equiv \{i : \phi_i \leq \phi^{\text{cut}}\}, \qquad H \equiv \{i : \phi_i > \phi^{\text{cut}}\},$$

where ϕ^{cut} is a cutoff (e.g. the median). Define the equal-weight long–short portfolio

$$rx_{t+1}^{HML} \equiv \frac{1}{|L|} \sum_{i \in L} rx_{t+1}^{i\$} - \frac{1}{|H|} \sum_{j \in H} rx_{t+1}^{j\$}. \tag{A.154}$$

Thus, the strategy is long low-exposure currencies and short high-exposure (hedge) currencies.

A.11.3 Expected return on the HML strategy

Proposition 19 (Expected return on the HML-ϕ FX strategy)**.** *Under the wedge pricing relation* (A.153)*, the conditional expected return on the HML currency strategy satisfies*

$$\mathbb{E}_t[rx_{t+1}^{HML}] \approx \chi_E \, s_t \left(\overline{\phi}_H - \overline{\phi}_L \right), \tag{A.155}$$

where

$$\overline{\phi}_L \equiv \frac{1}{|L|} \sum_{i \in L} \phi_i, \qquad \overline{\phi}_H \equiv \frac{1}{|H|} \sum_{j \in H} \phi_j.$$

In particular, if $s_t > 0$ and $\overline{\phi}_H > \overline{\phi}_L$, then $\mathbb{E}_t[rx_{t+1}^{HML}] > 0$.

Proof. Take conditional expectations of (A.154):

$$\mathbb{E}_t[rx_{t+1}^{HML}] = \frac{1}{|L|} \sum_{i \in L} \mathbb{E}_t[rx_{t+1}^{i\$}] - \frac{1}{|H|} \sum_{j \in H} \mathbb{E}_t[rx_{t+1}^{j\$}]. \tag{A.156}$$

Substitute the wedge-implied expected return (A.153) into each term:

$$\begin{aligned}\mathbb{E}_t[rx_{t+1}^{HML}] &\approx \frac{1}{|L|}\sum_{i\in L}(-\chi_E\phi_i s_t) - \frac{1}{|H|}\sum_{j\in H}\left(-\chi_E\phi_j s_t\right) \\ &= -\chi_E s_t\left(\frac{1}{|L|}\sum_{i\in L}\phi_i\right) + \chi_E s_t\left(\frac{1}{|H|}\sum_{j\in H}\phi_j\right) \\ &= \chi_E s_t\left(\overline{\phi}_H - \overline{\phi}_L\right), \end{aligned} \tag{A.157}$$

which is (A.155). If $s_t > 0$ and $\overline{\phi}_H > \overline{\phi}_L$, the right-hand side is strictly positive. □

A.11.4 Comparative statics with U.S. fiscal stress

Because $s_t = \bar{s} - \kappa_s g_t$, equation (A.155) implies

$$\frac{\partial\,\mathbb{E}_t[rx_{t+1}^{HML}]}{\partial g_t} = -\chi_E\,\kappa_s\left(\overline{\phi}_H - \overline{\phi}_L\right) \;<\; 0, \tag{A.158}$$

so U.S. fiscal–military stress (which reduces the security umbrella) compresses the HML premium by lowering s_t.

A.11.5 Interpretation

The HML FX strategy is profitable because it is long currencies that are relatively *procyclical* with respect to U.S. fiscal–military risk (low ϕ) and short currencies that provide insurance (high ϕ). In the model, this hedge property arises from the security-driven Euler-equation wedge: high-exposure currencies load more strongly on U.S. security provision and therefore earn lower expected excess returns. The HML premium scales with (i) the dispersion in exposure across the two legs and (ii) the level of the U.S. security umbrella s_t, consistent with the cross-sectional sorting facts in the empirical section.

B Other Closed Form Results from Simple Model

B.1 Equilibrium Macro Responses

Setup. This section characterizes closed-form macro responses to U.S. fiscal–military shocks in the tractable benchmark introduced in Section 5.2. I start with equilibrium macroeconomic variables before moving onto beta coefficients.

B.1.1 Equilibrium Output, Investment, and Consumption

Corollary 4.1.1 (Closed-Form Macro Responses)

Under the closed-form benchmark, the impact responses to a U.S. fiscal–military shock g_t satisfy the following piecewise relationships.

(i) Defense outlays (defense floor):

$$\frac{\partial h_{i,t}}{\partial g_t} = -\kappa_s \, \mathbf{1}\{s_t < S^\star\}. \tag{B.1}$$

(ii) Output (GDP):

$$\frac{\partial Y_{i,t}}{\partial g_t} = \chi_h \, \kappa_s \, Y_{i,t} \, \mathbf{1}\{s_t < S^\star\}. \tag{B.2}$$

(iii) Investment:

$$\frac{\partial I_{i,t}}{\partial g_t} = \iota_i \, \chi_h \, \kappa_s \, Y_{i,t} \, \mathbf{1}\{s_t < S^\star\}. \tag{B.3}$$

(iv) Consumption:

$$\frac{\partial C_{i,t}}{\partial g_t} = (1 - \iota_i)\chi_h \kappa_s \, Y_{i,t} \, \mathbf{1}\{s_t < S^\star\} + \kappa_s \, \mathbf{1}\{s_t < S^\star\}. \tag{B.4}$$

When $s_t \geq S^\star$, the deterrence constraint is slack, $h_{i,t} = 0$ locally, and all impact multipliers above equal zero.

Interpretation. Corollary 4.1.1 clarifies the economic role of the defense floor in the closed-form benchmark. A U.S. fiscal–military shock g_t increases the intensity of the U.S. security umbrella, $s_t = \bar{s} + \kappa_s g_t$. When the deterrence requirement is *binding* ($s_t < S^\star$), allies must cover the residual security gap through local defense outlays $h_{i,t} = \max\{0, S^\star - s_t\}$. An increase in s_t therefore reduces the required local defense burden one-for-one, implying $\partial h_{i,t}/\partial g_t < 0$ in the binding region. This is the key state-contingent nonlinearity: security provision matters only when it is scarce.

The reduction in $h_{i,t}$ then propagates through two distinct real-side channels. First, defense outlays act as a utilization wedge in production, $Y_{i,t} = \bar{Y}_i \exp(-\chi_h h_{i,t})$. Lower defense spending raises effective output by alleviating capacity and resource disruptions associated with maintaining deterrence. Second, investment and consumption inherit this output response through the resource constraint and the fixed investment-share rule $I_{i,t} = \iota_i Y_{i,t}$. Because investment is proportional to output, it rises mechanically when $Y_{i,t}$ rises. Consumption rises for two reasons: it benefits from higher output *and* from the direct fiscal relief created by lower defense outlays (the $+\kappa_s \mathbf{1}\{s_t < S^\star\}$ term in (B.4)).

Importantly, these quantity responses are not the paper's main channel, but rather a supporting implication of the security-umbrella framework. They are *state-dependent* by construction: when $s_t \geq S^\star$, the deterrence constraint is slack, $h_{i,t} = 0$ locally, and marginal changes in U.S. security provision do not affect real activity at impact. This generates a simple empirical distinction. In normal times, the model predicts little first-order effect of fiscal–military shocks on aggregate output and spending components; in contrast, when security provision is insufficient and the defense floor binds, changes in g_t can induce discrete adjustments in local defense burdens and associated macro quantities.

This nonlinear quantity block helps interpret why the motivating facts emphasize large and systematic *price* responses—fiscal betas, sovereign spreads, and currency premia—rather than large average output effects. In the model, the dominant transmission of U.S. fiscal–military shocks operates through discount rates, default risk, and exchange-rate wedges, while real-side effects

arise mainly through the defense-floor constraint and are activated only in the binding regime.

B.1.2 Debt Accumulation

Corollary 4.1.2 (Debt Response)

Under the government budget constraint with sticky non-defense spending, the impact response of next-period public debt satisfies

$$\frac{\partial B_{i,t+1}}{\partial g_t} = -\kappa_s \, \mathbf{1}\{s_t < S^\star\} - \tau_i \, \chi_h \, \kappa_s \, Y_{i,t} \, \mathbf{1}\{s_t < S^\star\}. \tag{B.5}$$

Interpretation. Corollary 4.1.2 highlights the *moral-hazard* dimension of the U.S. security umbrella. By providing external military insurance, stronger U.S. security provision reduces the need for local defense spending when the deterrence constraint binds ($s_t < S^\star$), directly lowering primary expenditures in allied economies. At the same time, the associated increase in effective output raises the tax base through the utilization wedge, further easing short-run fiscal pressure. These two forces imply a decline in next-period public debt, as captured by (B.5): the first term reflects reduced defense outlays, while the second term captures higher revenues generated by stronger economic activity.

However, this short-run fiscal relief comes at the cost of distorted incentives. Because U.S. security provision insures allies against external threats, it weakens fiscal discipline ex ante and encourages governments to tolerate higher leverage and fiscal risk. This moral-hazard channel is embedded in the model through the interaction of security provision, debt accumulation, and default risk: when security is abundant, borrowing appears cheap and risk is partially externalized to the guarantor. As a result, allied governments optimally choose higher debt levels in states of strong U.S. security provision.

The model therefore predicts a sharp asymmetry. When U.S. security provision is ample ($s_t \geq S^\star$), marginal changes in g_t have no direct effect on contemporaneous debt dynamics, but they shape the level of debt carried into future states. When security provision weakens and the defense floor binds, this accumulated leverage is suddenly exposed, leading to rapid increases in sovereign risk and borrowing costs. In this sense, the moral-hazard effects of security insurance operate quietly

in good times but amplify financial stress in bad times, providing a key link between U.S. fiscal–military policy, allied debt sustainability, and the pronounced asset-pricing responses documented below.

B.1.3 Global Consumption and Output Betas

Global risk exposures. Let global (log) consumption growth be defined as the weighted average

$$\Delta \log C_t^G \equiv \sum_{m=1}^{N} \omega_m \, \Delta \log C_{m,t}, \qquad \omega_m \geq 0, \quad \sum_m \omega_m = 1,$$

and define the global consumption beta of country i as

$$\beta_{i,G}^C \equiv \frac{\mathrm{Cov}(\Delta \log C_{i,t},\, \Delta \log C_t^G)}{\mathrm{Var}(\Delta \log C_t^G)}.$$

Analogously, define global output growth and output betas $\beta_{i,G}^Y$.

Corollary B.0.1 (Defense Exposure and Global Macro Betas)

Assume that (i) U.S. fiscal–military shocks are the only aggregate source of uncertainty and (ii) country-specific components of consumption and output growth are orthogonal across countries and to the fiscal–military shock. Then the global consumption and output betas of country i satisfy

$$\beta_{i,G}^C \;\propto\; \phi_i, \qquad \beta_{i,G}^Y \;\propto\; \phi_i. \tag{B.6}$$

Interpretation. Corollary B.0.1 clarifies how the model reconciles weak average macro responses with strong cross-sectional comovement in global aggregates. Although the defense floor implies that real activity responds only when security provision is insufficient, U.S. fiscal–military shocks remain the *sole aggregate source of risk* in the economy. What differs across countries is not the nature of the shock, but the *intensity with which it is transmitted* into domestic fiscal conditions and discount rates, summarized by ϕ_i.

A higher ϕ_i magnifies the sensitivity of local defense spending, public debt, and sovereign risk to U.S. military mobilization. Even when the average response of output and consumption is small or zero—because the deterrence constraint is slack—expectations about future binding states and fiscal stress cause high-exposure countries' consumption and output growth to covary more strongly with the global aggregate. In this sense, ϕ_i governs *risk exposure rather than average growth.*

This distinction is central to the model's interpretation. U.S. fiscal–military shocks primarily operate as *pricing shocks* rather than as large quantity shocks. They reallocate global risk by altering the distribution of fiscal stress, borrowing constraints, and discount-rate volatility across countries. As a result, countries with greater exposure to the U.S. security umbrella load more heavily on global consumption and output fluctuations, even though contemporaneous macro responses remain muted in most states.

The model therefore delivers a sharp implication that aligns closely with the data: cross-country heterogeneity in global macro betas reflects differences in exposure to U.S. military guarantees, not differences in underlying productivity or trend growth. Macroeconomic quantities move little on average, but the covariance structure of growth is strongly shaped by geopolitical exposure.

Proposition III (Defense Exposure and Global Macro Risk)

Countries more exposed to U.S. defense-related infrastructure exhibit higher global consumption and output betas, despite modest average macro responses. Integration into the U.S. security network primarily amplifies exposure to global risk, rather than raising mean growth.

Discussion. The macro block thus plays a disciplined but essential role in the overall mechanism. U.S. fiscal–military spending affects real activity mainly by relaxing binding defense constraints, generating limited and state-dependent quantity effects. At the same time, it reshapes the covariance structure of consumption and output through its impact on fiscal stress and risk pricing. This separation between small average effects and large covariance effects explains why the empirical evidence shows pronounced cross-sectional differences in asset betas and risk premia alongside relatively muted average responses of output and consumption.

C Full Quantitative Model with Delayed Learning

C.1 Countries, Time, and Uncertainty

Framework. Time is continuous, $t \in [0, \infty)$. The world consists of two blocs, indexed by $k \in \{\text{US}, i\}$, representing the United States and a representative allied economy. Uncertainty unfolds on a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ generated by four global Brownian motions,

$$\mathbf{W}_t = (W_t^Y,\, W_t^\xi,\, W_t^s,\, W_t^G)^\top,$$

and a continuum of idiosyncratic household shocks $\{Z_{j,t}\}_{j\in[0,1]}$. Expectations under full information are denoted $\mathbb{E}_t[\cdot]$ conditional on $\mathcal{F}_t$.

Information and learning about security. A key extension introduces partial information about the U.S. security umbrella. While the true security state s_t governs enforcement capacity and real resource costs, private agents do not observe s_t perfectly in real time. Instead, they observe a noisy *military news* signal $\tilde{s}_t$ and form beliefs about the credibility and persistence of U.S. security commitments. Formally, agents' information set is a sub-filtration $\{\mathcal{F}_t^I\}_{t\geq 0}$ with $\mathcal{F}_t^I \subseteq \mathcal{F}_t$, generated by observable macro-financial variables and the signal process $\tilde{s}_t$ defined below. Expectations taken with respect to agents' information set are denoted $\mathbb{E}_t^I[\cdot] := \mathbb{E}[\cdot \mid \mathcal{F}_t^I]$. In equilibrium, private-sector decisions and all asset-pricing relations are based on $\mathbb{E}_t^I[\cdot]$, and enforcement wedges depend on the perceived umbrella $\hat{s}_t$ defined next.

Military news signal and perceived umbrella. Agents observe a noisy signal of security commitment,

$$d\tilde{s}_t = s_t\, dt + \sigma_\eta\, dB_t, \qquad \sigma_\eta > 0, \tag{C.1}$$

where B_t is a Brownian motion independent of $\mathbf{W}_t$ and of the idiosyncratic shocks $\{Z_{j,t}\}$. The perceived security umbrella is the filtered estimate

$$\hat{s}_t \;\equiv\; \mathbb{E}\Big[s_t \mid \mathcal{F}_t^I\Big], \tag{C.2}$$

which evolves smoothly over time as agents update beliefs in response to military news. This learning friction generates gradual adjustment in discount rates and asset valuations following changes in the underlying security stance, helping to match the empirical delay in return predictability.

Aggregate shocks. The four aggregate shocks represent: (i) a global TFP factor W_t^Y; (ii) a global financial–stress factor W_t^ξ (tightening of global risk premia); (iii) a *security* shock W_t^s affecting the U.S. umbrella; and (iv) a U.S. non-military fiscal shock W_t^G, orthogonal to the security shock. The covariance structure is

$$d\langle \mathbf{W}\rangle_t = \begin{bmatrix} 1 & \rho_{Y\xi} & 0 & 0 \\ \rho_{Y\xi} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} dt, \qquad d\langle W^G, W^s\rangle_t = 0, \qquad d\langle Z_j, Z_{j'}\rangle_t = \mathbf{1}\{j = j'\}\, dt, \qquad d\langle B, W^\bullet\rangle_t = 0. \tag{C.3}$$

State vector. Four aggregate state variables drive the economy under full information:

(i) υ_t — global productivity (TFP);

(ii) ξ_t — global financial stress (risk-aversion / tightness);

(iii) s_t — true credibility and scope of the U.S. security umbrella;

(iv) μ_t^{FP} — non-military U.S. fiscal shock.

In addition, private-sector pricing and forward-looking decisions depend on the perceived umbrella $\hat{s}_t$ defined in (C.2). The true states follow Ornstein–Uhlenbeck diffusions:

$$d\upsilon_t = -\rho_Y(\upsilon_t - \bar{\upsilon})\,dt + \sigma_Y\,dW_t^Y, \tag{C.4}$$

$$d\xi_t = -\rho_\xi(\xi_t - \bar{\xi})\,dt + \sigma_\xi\,dW_t^\xi, \tag{C.5}$$

$$ds_t = \kappa_s\big(\bar{s}(\xi_t) - s_t\big)\,dt + \sigma_s\,dW_t^s, \tag{C.6}$$

$$d\mu_t^{FP} = -\rho_G\mu_t^{FP}\,dt + \sigma_G\,dW_t^G, \tag{C.7}$$

with $\langle W^G, W^s\rangle_t = 0$ ensuring fiscal and security shocks are exogenous and orthogonal at impact. The stress factor ξ_t captures global discount-rate conditions (higher ξ_t indicates tighter global financial conditions), while υ_t governs long-run global output growth.

C.2 The U.S. Security Umbrella as a Non-Rival Public Good

The United States provides a continuous measure of global security $s_t \in [0,1]$, interpreted as the credibility and scope of its forward-deployed military posture— alliances, overseas bases, blue-water naval protection, and the enforcement of cross-border contracts. The umbrella is *non-rival*: an incremental increase in s_t improves global enforcement, stabilizes trade flows, and reduces macro–financial volatility for all allied economies simultaneously. This public-good channel is the central mechanism through which U.S. fiscal and security capacity enters the global pricing kernel. While s_t governs real enforcement capacity, private agents price assets based on the perceived umbrella $\hat{s}_t$.

Security dynamics. In the baseline model, s_t follows a mean-reverting Ornstein–Uhlenbeck diffusion:

$$ds_t = \kappa_s\big(\bar{s}(\xi_t) - s_t\big)\,dt + \sigma_s\,dW_t^s, \qquad \kappa_s > 0,\ \sigma_s \geq 0. \tag{C.8}$$

Interpretation of security shocks. The shock dW_t^s captures exogenous geopolitical developments orthogonal to U.S. fiscal and monetary policy at impact. Deliberate U.S. retrenchment

appears as a downward shift in the policy target $\bar{s}(\xi_t)$ rather than as a shock to W_t^s.

U.S. resource cost and policy choice. Providing global security requires real resources. Let

$$H(s_t) = \tfrac{\kappa}{2} s_t^2, \qquad \kappa > 0,$$

denote the convex cost of maintaining enforcement. The United States chooses $\{s_t\}_{t\geq 0}$ to solve

$$\max_{\{s_t\}} \mathbb{E}_0 \int_0^\infty e^{-\rho t} \Big[U(C_t^{US}) + \Phi_r \left(r_t - \rho\right) - \Phi_{\text{ERP}} \, \text{ERP}_t^{US} + \Phi_n \, \Omega(n_t) - H(s_t) \Big] dt, \tag{C.9}$$

subject to the security dynamics (C.8) and general equilibrium.

C.3 Fiscal Policy Block

Fiscal policy governs how governments allocate resources between social transfers, productive public investment, and defense. In this block, the U.S. security umbrella s_t affects fiscal policy primarily through its impact on allied defense needs and, hence, on the composition and cyclicality of their budgets.

Information structure. Fiscal authorities are assumed to observe the true security umbrella s_t and to choose defense outlays, transfers, and public investment accordingly. This reflects the idea that governments internalize their own military posture and budgetary commitments. In contrast, private households and firms do not observe s_t directly and base their forward-looking decisions and asset-pricing beliefs on the perceived umbrella $\hat{s}_t$. As a result, fiscal quantities and debt dynamics evolve with the true s_t, while financial variables price the same fiscal and security outcomes through beliefs about security commitments.

Effective security and the defense floor (ally). Each ally faces a deterrence requirement that total security

$$S_i(t) \;=\; s_t + h_t^i \tag{C.10}$$

must exceed a minimum threshold $S^\star > 0$. Given U.S. security provision s_t, the ally chooses local defense outlays $h_t^i \geq 0$ to satisfy this constraint at minimum resource cost. Formally,

$$\min_{h_t^i \geq 0} \; \mathcal{C}(h_t^i) \qquad \text{s.t.} \qquad s_t + h_t^i \;\geq\; S^\star. \tag{C.11}$$

The KKT conditions imply a *piecewise-linear defense floor*:

$$h_t^i = \max\{0,\, S^\star - s_t\}, \qquad \frac{dh_t^i}{ds_t} = \begin{cases} 0, & s_t \geq S^\star, \\ -1, & s_t < S^\star, \end{cases} \tag{C.12}$$

Fiscal composition and budget constraints. Let τ_t^k denote tax revenue as a share of GDP. Government flow budgets in the two blocs, expressed in shares of domestic output, are

$$\tau_t^{US} Y_t^{US} = G_{\text{soc},t}^{US} + I_{\text{pub},t}^{US} + \underbrace{H(s_t)}_{\text{U.S. security outlays}} + r_t B_t^{US}, \tag{C.13}$$

$$\tau_t^i Y_t^i = G_{\text{soc},t}^i + I_{\text{pub},t}^i + \underbrace{h_t^i}_{\text{local defense}} + r_t B_t^i. \tag{C.14}$$

When the ally's budget is held fixed in the short run (taxes and output given), rearmament crowds out other spending categories according to

$$dh_t^i = -\vartheta_i \, dG_{\text{soc},t}^i - (1 - \vartheta_i) \, dI_{\text{pub},t}^i, \qquad \vartheta_i \in [0, 1], \tag{C.15}$$

Combining (C.12) and (C.15) yields an asymmetric fiscal response.

U.S. non-military spending. In addition to security outlays, the U.S. implements a non-military transfer rule driven by aggregate activity and a primitive non-military fiscal shock μ_t^{FP}:

$$G^{US}_{\text{soc},t} - \bar{G}^{US}_{\text{soc}} = \phi_G\big(Y^{US}_t - \bar{Y}^{US}\big) + \sigma_G\,\mu^{FP}_t, \qquad d\mu^{FP}_t = -\rho_G \mu^{FP}_t\,dt + dW^G_t. \tag{C.16}$$

Debt accumulation. For any bloc $k \in \{\text{US}, i\}$, the evolution of public debt in levels satisfies

$$\dot{B}^k_t = r_t B^k_t + G^k_{\text{soc},t} + I^k_{\text{pub},t} + h^k_t - \tau^k_t Y^k_t, \tag{C.17}$$

with $h^{US}_t \equiv H(s_t)$ and h^i_t given by (C.12).

C.4 Production and Technology

Each bloc $k \in \{\text{US}, i\}$ produces a tradable final good using private capital K^k_t, productive public capital G^k_t, and a composite of traded intermediate inputs M^k_t:

$$Y^k_t = A^k_t\,(K^k_t)^{\alpha_k}\,(G^k_t)^{\gamma_k}\,(M^k_t)^{1-\alpha_k-\gamma_k}, \qquad \alpha_k \in (0,1),\ \gamma_k \geq 0. \tag{C.18}$$

TFP dynamics. Total factor productivity evolves as

$$d\ln A^k_t = \mu^k_A\,dt - \underbrace{\zeta^k_A\,h^k_t}_{\text{defense drag}}\,dt + \underbrace{\sigma^k_A(s_t)}_{\sigma^k_{A0} - \frac{1}{2}\phi^k s_t}\,dW^Y_t, \qquad \phi^k > 0. \tag{C.19}$$

Intermediate inputs and iceberg trade costs. Trade incurs iceberg cost $\bar{\tau}^k_t$ with log structure

$$\log \bar{\tau}^k_t = \tau^k_0 - \upsilon_k s_t + \chi_k\,\mathfrak{m}_k, \qquad \upsilon_k > 0,\ \mathfrak{m}_k \in [0,1]. \tag{C.20}$$

Capital accumulation. Private and public capital evolve according to

$$\dot{K}^k_t = I^k_{\text{priv},t} - \delta^K_k K^k_t, \qquad \dot{G}^k_t = I^k_{\text{pub},t} - \delta^G_k G^k_t. \tag{C.21}$$

Defense as a real resource use. Defense imposes a real resource cost

$$\Xi^k(h) = \underbrace{\nu_k h}_{\text{direct cost}} + \underbrace{\zeta^k h}_{\text{organizational drag}} . \tag{C.22}$$

Resource constraint. The economy-wide resource constraint is

$$Y_t^k = C_t^k + I_{\text{priv},t}^k + I_{\text{pub},t}^k + \Xi^k(h_t^k) + \underbrace{\bar{\tau}_t^k E_{M,t}^k}_{\text{iceberg losses}} . \tag{C.23}$$

C.5 Nominal Rigidities and Dominant Currency Pricing

Currency choice and price-setting. At a reset time, the firm chooses both its invoicing currency c and its reset price $P_t^*(c)$ to minimize

$$\min_{c\in\{\$,\text{LC}\}} \mathbb{E}_t^I \int_t^\infty e^{-\xi(u-t)} \left(\frac{P_t^*(c)}{P_u(c)} - \tilde{p}_u^{\#}\right)^2 du \; + \; \mathcal{E}_i(c;\tilde{b}_i,\hat{s}_t). \tag{C.24}$$

Enforcement costs and U.S. exposure.

$$\mathcal{E}_i(\$;\tilde{b}_i,\hat{s}_t) = \bar{\mathcal{E}} - \chi_E\,\tilde{b}_i\,\hat{s}_t, \tag{C.25}$$

$$\mathcal{E}_i(\text{LC};\tilde{b}_i,\hat{s}_t) = \bar{\mathcal{E}}. \tag{C.26}$$

Invoicing-share dynamics.

$$\dot{\text{logit}}(n_t) = (a + \chi_E \tilde{b}_i \hat{s}_t) \; - \; \omega\big(\text{logit}(n_t) - \text{logit}(n_t^\dagger)\big), \qquad 0<\omega<1. \tag{C.27}$$

CPI aggregation.

$$P_t^i = \Big(n_t\,P_t^i(\$)^{1-\theta} + (1-n_t)\,P_t^i(\text{LC})^{1-\theta}\Big)^{\frac{1}{1-\theta}}. \tag{C.28}$$

DCP-weighted NKPC.

$$\pi_t^i = \kappa\big[n_t\,x_{t|\$}^i + (1-n_t)\,x_{t|\text{LC}}^i\big] + \beta_\pi\,\mathbb{E}_t^I[\pi_{t+1}^i]. \tag{C.29}$$

C.6 Households (SDU) and SDF

Income and risk.

$$\frac{dY^k_{j,t}}{Y^k_{j,t}} = g^k_Y(s_t, I^k_{\text{pub},t}, h^k_t)\,dt + \sigma^k_{\text{A},Y}(s_t)\,dW^Y_t + \sigma^k_{\text{A},\xi}(s_t)\,dW^\xi_t + \sigma^k_{\text{id}}\big(G^k_{\text{soc},t}, \Xi^k(s_t)\big)\,dZ_{j,t}. \quad \text{(C.30)}$$

$$\sigma^k_{\text{id}}\big(G^k_{\text{soc},t}, \Xi^k(s_t)\big) = \frac{\eta^k_0}{G^k_{\text{soc},t}\,\Xi^k(s_t)}. \quad \text{(C.31)}$$

Preferences (Epstein–Zin SDU).

$$dU^k_{j,t} = -\rho\,U^k_{j,t}\,dt + \frac{\rho}{1-1/\psi}\left[(C^k_{j,t})^{1-1/\psi} - \Big((1-\gamma)U^k_{j,t}\Big)^{\frac{1-1/\psi}{1-\gamma}}\right]dt + (Z^k_{j,t})^\top d\mathbf{W}_t. \quad \text{(C.32)}$$

Wealth and market participation.

$$dA^k_t = \Big[r^k_t\,A^k_t + (1-\tau^k_t)Y^k_t + T^k_t - C^k_t\Big]dt + A^k_t(\boldsymbol{\omega}^{k,\text{rx}}_t)^\top \boldsymbol{\Sigma}^k_t\,d\mathbf{W}_t. \quad \text{(C.33)}$$

Representative SDF under Epstein–Zin.

$$\frac{dM^k_t}{M^k_t} = -r^k_t\,dt - \Big[\gamma\,\boldsymbol{\sigma}^k_{C,t} \;+\; \big(\gamma - \tfrac{1}{\psi}\big)\boldsymbol{\vartheta}^k_t \;+\; \boldsymbol{\Psi}^k_t\Big]^\top d\mathbf{W}_t. \quad \text{(C.34)}$$

C.7 Financial Markets, FX, and the Dollar Convenience Yield

$$\frac{dM^{US}_t}{M^{US}_t} = -\,r^{US}_t\,dt - \boldsymbol{\Lambda}^{US\top}_t\,d\mathbf{W}_t. \quad \text{(C.35)}$$

$$1 = \mathbb{E}^I_t\big[M^{US}_{t+dt}\,R^{k,\$}_{t,t+dt}\big]. \quad \text{(C.36)}$$

$$r_t^{US} = \rho + \gamma\, g_C^{US}(s_t,\cdot) - \tfrac{1}{2}\gamma(1+\gamma)\big(\sigma_C^{US}(s_t)\big)^2. \tag{C.37}$$

$$r_t^n(\mathrm{LC}) = r_t^n(\$) + \chi_E \tilde{b}_i\, \hat{s}_t. \tag{C.38}$$

$$\mathbb{E}_t^I\Big[\tfrac{dS_t^i}{S_t^i}\Big] = r_t^n(\$) - r_t^n(\mathrm{LC}) - \Big(\boldsymbol{\Lambda}_t^{US\top}\boldsymbol{\vartheta}_t^i - \chi_E\tilde{b}_i\,\hat{s}_t\Big). \tag{C.39}$$

$$CY_t \equiv r_t^* - r_t^{US}. \tag{C.40}$$

$$CY_t \simeq \bar{\Lambda}\,\chi_E\,\hat{s}_t. \tag{C.41}$$

C.8 Equilibrium

An equilibrium consists of a collection of adapted processes

$$\Big\{C_t^k, A_t^k, B_t^k, h_t^i, G_{\mathrm{soc},t}^k, I_{\mathrm{pub},t}^k, n_t, P_t^i, r_t, \mathrm{ERP}_t^k, \pi_t^i, s_t, \hat{s}_t\Big\}_{k\in\{\mathrm{US},i\}}$$

defined on the probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathbb{P})$, with private-sector expectations taken with respect to $\{\mathcal{F}_t^I\}_{t\geq 0}$, such that:

(i) **Households.** Households solve their problem subject to (C.33); the resulting SDF satisfies (C.34).

(ii) **Firms and pricing.** Firms solve (C.24); aggregate price dynamics satisfy (C.29).

(iii) **Fiscal authorities.** Fiscal authorities satisfy (C.13)–(C.17); U.S. non-military spending follows (C.16).

(iv) **Security policy.** Security follows (C.8) and the U.S. objective is (C.9).

(v) **Asset pricing and exchange rates.** USD pricing satisfies (C.36); enforcement premium is (C.38); modified UIP is (C.39).

(vi) **Goods and factor markets.** Resource constraints satisfy (C.23); capital evolves by (C.21).

C.9 First-Order Conditions

This section collects the main first-order conditions implied by the updated model with delayed learning about the U.S. security umbrella. The key departure from the full-information benchmark is that *private agents* (firms and households) condition on the information set $\mathcal{F}_t^I$ and price assets using the perceived umbrella $\hat{s}_t = \mathbb{E}[s_t \mid \mathcal{F}_t^I]$, whereas *fiscal authorities* observe the true s_t. Accordingly, all private-sector Euler equations and pricing relations below are understood under $\mathbb{E}_t^I[\cdot]$.

C.9.1 Information, Beliefs, and Innovations

Private agents observe the noisy military-news signal

$$d\tilde{s}_t = s_t\,dt + \sigma_\eta\,dB_t, \qquad d\langle B, W^\bullet\rangle_t = 0, \tag{C.42}$$

and form the perceived umbrella

$$\hat{s}_t \equiv \mathbb{E}[s_t \mid \mathcal{F}_t^I]. \tag{C.43}$$

For the purposes of the FOCs, it is sufficient to note that $\hat{s}_t$ is $\mathcal{F}_t^I$-adapted and evolves smoothly over time as agents update beliefs. All enforcement wedges and pricing relations below replace s_t by $\hat{s}_t$.

C.9.2 Firms: Currency Choice and Calvo Price Setting

Reset problem. At a price-reset time, firm ℓ chooses an invoicing currency $c \in \{\$, \text{LC}\}$ and a reset price $P_t^*(c)$ to minimize

$$\min_{c\in\{\$,\text{LC}\},\, P_t^*(c)} \; \mathbb{E}_t^I \int_t^\infty e^{-\xi(u-t)} \left(\frac{P_t^*(c)}{P_u(c)} - \tilde{p}_u^{\#} \right)^2 du \; + \; \mathcal{E}_i(c; \tilde{b}_i, \hat{s}_t). \tag{C.44}$$

FOC for the reset price. Conditional on currency c, the first-order condition for $P_t^*(c)$ is

$$\mathbb{E}_t^I \int_t^\infty e^{-\xi(u-t)} \left(\frac{P_t^*(c)}{P_u(c)} - \tilde{p}_u^{\#} \right) \frac{1}{P_u(c)} \, du = 0. \tag{C.45}$$

Currency-choice condition. Let $\mathcal{L}_t(c)$ denote the minimized value of (C.44) conditional on currency c (i.e., after optimizing over $P_t^*(c)$). The optimal invoicing currency satisfies

$$c_t^* \in \arg\min_{c\in\{\$,\text{LC}\}} \Big\{ \mathcal{L}_t(c) + \mathcal{E}_i(c; \tilde{b}_i, \hat{s}_t) \Big\}. \tag{C.46}$$

With the enforcement specification

$$\mathcal{E}_i(\$; \tilde{b}_i, \hat{s}_t) = \bar{\mathcal{E}} - \chi_E \, \tilde{b}_i \, \hat{s}_t, \tag{C.47}$$

$$\mathcal{E}_i(\text{LC}; \tilde{b}_i, \hat{s}_t) = \bar{\mathcal{E}}, \tag{C.48}$$

a higher perceived umbrella $\hat{s}_t$ raises the relative attractiveness of dollar invoicing, especially for high-exposure economies (large $\tilde{b}_i$).

Invoicing-share dynamics. The gradual adjustment of dollar invoicing shares n_t toward the perceived optimum $n_t^\dagger$ is

$$\dot{\text{logit}}(n_t) = \big(a + \chi_E \tilde{b}_i \hat{s}_t\big) - \omega\Big(\text{logit}(n_t) - \text{logit}(n_t^\dagger)\Big), \qquad 0 < \omega < 1. \tag{C.49}$$

Phillips curve. The DCP-weighted NKPC under imperfect information is

$$\pi_t^i = \kappa\big[n_t x_{t|\$}^i + (1-n_t) x_{t|\text{LC}}^i\big] + \beta_\pi \, \mathbb{E}_t^I[\pi_{t+1}^i]. \tag{C.50}$$

C.9.3 Households: SDU, Portfolios, and Euler Conditions

Budget constraint. Aggregate household wealth satisfies

$$dA_t^k = \Big[r_t^k A_t^k + (1-\tau_t^k)Y_t^k + T_t^k - C_t^k\Big]dt + A_t^k(\boldsymbol{\omega}_t^{k,\mathrm{rx}})^\top \boldsymbol{\Sigma}_t^k \, d\mathbf{W}_t, \tag{C.51}$$

where $\boldsymbol{\Sigma}_t^k$ collects exposures of excess returns to $\mathbf{W}_t = (W_t^Y, W_t^\xi, W_t^s, W_t^G)^\top$.

Euler equation (general). For any asset with gross return $R_{t,t+dt}$, the household Euler condition under private information is

$$1 = \mathbb{E}_t^I\big[M_{t+dt}^k \, R_{t,t+dt}\big]. \tag{C.52}$$

Representative SDF under Epstein–Zin. Under aggregation Constantinides and Duffie (1996), the SDF obeys

$$\frac{dM_t^k}{M_t^k} = -\, r_t^k \, dt - \Big[\gamma\, \boldsymbol{\sigma}_{C,t}^k + \big(\gamma - \tfrac{1}{\psi}\big)\boldsymbol{\vartheta}_t^k + \boldsymbol{\Psi}_t^k\Big]^\top d\mathbf{W}_t, \tag{C.53}$$

with all conditional expectations implicit in pricing taken under $\mathbb{E}_t^I$. The inequality wedge depends on true security through real risk-sharing capacity,

$$\boldsymbol{\Psi}_t^k \equiv \Psi^k\big(G_{\mathrm{soc},t}^k, \Xi^k(s_t)\big).$$

FOC for optimal risky-asset demand. Let a traded asset in bloc k have excess-return process $dR_t^{k,\mathrm{rx}} = \boldsymbol{\mu}_t^{k,\mathrm{rx}}\, dt + \boldsymbol{\Sigma}_t^k \, d\mathbf{W}_t$. The optimality condition for portfolio weights can be written in Hansen–Jagannathan form:

$$\boldsymbol{\mu}_t^{k,\mathrm{rx}} = \boldsymbol{\Sigma}_t^k \, \boldsymbol{\Lambda}_t^k, \qquad \boldsymbol{\Lambda}_t^k := \gamma\, \boldsymbol{\sigma}_{C,t}^k + \big(\gamma - \tfrac{1}{\psi}\big)\boldsymbol{\vartheta}_t^k + \boldsymbol{\Psi}_t^k. \tag{C.54}$$

Under delayed learning, private agents evaluate all forward-looking objects and pricing wedges using $\mathcal{F}_t^I$ and the perceived umbrella $\hat{s}_t$ in the enforcement channel described below.

C.9.4 Governments: Defense Floor, Budgets, and Security Choice

Ally defense floor. The ally chooses $h_t^i \geq 0$ to satisfy $s_t + h_t^i \geq S^\star$ at minimum cost. The KKT conditions imply

$$h_t^i = \max\{0, S^\star - s_t\}, \qquad \frac{dh_t^i}{ds_t} = \begin{cases} 0, & s_t \geq S^\star, \\ -1, & s_t < S^\star. \end{cases} \tag{C.55}$$

Flow budgets. Government budget constraints are

$$\tau_t^{US} Y_t^{US} = G_{\text{soc},t}^{US} + I_{\text{pub},t}^{US} + H(s_t) + r_t B_t^{US}, \tag{C.56}$$

$$\tau_t^i Y_t^i = G_{\text{soc},t}^i + I_{\text{pub},t}^i + h_t^i + r_t B_t^i, \tag{C.57}$$

with debt accumulation

$$\dot{B}_t^k = r_t B_t^k + G_{\text{soc},t}^k + I_{\text{pub},t}^k + h_t^k - \tau_t^k Y_t^k, \qquad k \in \{\text{US}, i\}, \tag{C.58}$$

where $h_t^{US} \equiv H(s_t)$.

U.S. security choice (HJB/Hamiltonian form). The true security umbrella follows

$$ds_t = \kappa_s\big(\bar{s}(\xi_t) - s_t\big)\, dt + \sigma_s\, dW_t^s. \tag{C.59}$$

Let $V^{US}(\mathbf{x}_t)$ denote the U.S. value function, where the full state is $\mathbf{x}_t = (\upsilon_t, \xi_t, s_t, \mu_t^{FP})$, and let $\mathcal{L}^{US}$ denote the generator associated with the controlled diffusion (including the drift and diffusion terms in $\mathbf{x}_t$). The U.S. HJB is

$$\rho\, V^{US}(\mathbf{x}) = \max_s \Big\{ U(C^{US}) + \Phi_r\,(r - \rho) - \Phi_{\text{ERP}}\, \text{ERP}^{US} + \Phi_n\, \Omega(n) - H(s) + \mathcal{L}^{US} V^{US}(\mathbf{x}) \Big\}, \tag{C.60}$$

where $\mathcal{L}^{US}V^{US}$ includes the s-drift term $\kappa_s(\bar{s}(\xi)-s)\,V_s^{US}$ and the diffusion term $\frac{1}{2}\sigma_s^2\,V_{ss}^{US}$, plus the generator terms for the other state variables.

A necessary (interior) first-order condition for the U.S. choice of s is

$$0 = -\frac{\partial H(s)}{\partial s} \;+\; \Phi_r\,\frac{\partial r}{\partial s} \;-\; \Phi_{\mathrm{ERP}}\frac{\partial \mathrm{ERP}^{US}}{\partial s} \;+\; \Phi_n\,\Omega'(n)\frac{\partial n}{\partial s} \;+\; \frac{\partial}{\partial s}\big(\mathcal{L}^{US}V^{US}(\mathbf{x})\big), \tag{C.61}$$

where the final term captures the marginal effect of s through the controlled drift $\kappa_s(\bar{s}(\xi)-s)$ and, if applicable, any s-dependence of the equilibrium mappings embedded in $\mathcal{L}^{US}$. Together, (C.60)–(C.61) characterize optimal security provision in the presence of convex resource costs and general-equilibrium benefits.

C.9.5 Asset Pricing, FX, and Enforcement Wedges Under Learning

Dollar price of risk with perceived enforcement. The enforcement advantage of U.S. security enters the dollar price of risk as a wedge that depends on beliefs:

$$\mathbf{\Lambda}_t^{US} = \mathbf{\Lambda}_t^{US,0} - \chi_E\,\hat{s}_t\,\mathbf{e}_s. \tag{C.62}$$

Dollar pricing of foreign claims. For any foreign USD payoff $R^{k,\$}_{t,t+dt}$,

$$1 = \mathbb{E}_t^I\big[M^{US}_{t+dt}\,R^{k,\$}_{t,t+dt}\big]. \tag{C.63}$$

Enforcement premium in nominal rates. The LC–USD nominal short-rate differential includes the perceived enforcement premium:

$$r_t^n(\mathrm{LC}) = r_t^n(\$) + \chi_E\,\tilde{b}_i\,\hat{s}_t. \tag{C.64}$$

Modified UIP under learning. Let S_t^i be the nominal exchange rate (LC per USD). The modified UIP condition is

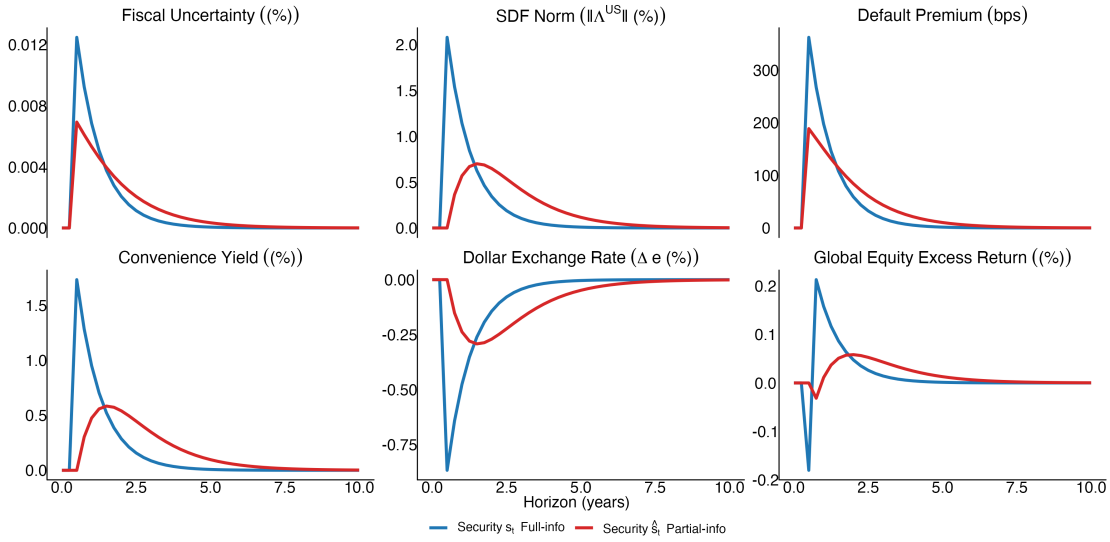
$$\mathbb{E}_t^I\Big[\tfrac{dS_t^i}{S_t^i}\Big] = r_t^n(\$) - r_t^n(\mathrm{LC}) - \Big(\mathbf{\Lambda}_t^{US\top}\boldsymbol{\vartheta}_t^i - \chi_E\,\tilde{b}_i\,\hat{s}_t\Big). \tag{C.65}$$

Convenience yield. Define the real dollar convenience yield $CY_t = r_t^* - r_t^{US}$. A first-order approximation implies

$$CY_t \simeq \bar{\Lambda}\, \chi_E\, \hat{s}_t. \tag{C.66}$$

Learning about security commitments therefore generates gradual adjustment in the dollar convenience yield and, through the dollar pricing kernel, in global valuations and expected returns.

Figure 7: Security Shocks and Global Asset Prices: Full vs. Partial Information



Notes: The figure reports impulse responses to a positive shock to U.S. security provision under two informational regimes. The blue line corresponds to a *full-information* economy in which agents observe the true security umbrella s_t on impact. The red line corresponds to a *partial-information* economy in which agents observe only noisy military news and update beliefs about security gradually, forming the perceived umbrella $\hat{s}_t$.
Panels display deviations from steady state (binding, low-security regime) for: (i) fiscal uncertainty (variance of surplus–debt dynamics), (ii) the U.S. stochastic discount factor norm $\|\Lambda^{US}\|$, (iii) the sovereign default premium (basis points), (iv) the dollar convenience yield, (v) the expected dollar depreciation rate Δe_t (negative values indicate dollar appreciation), and (vi) global equity excess returns.
All responses are expressed in percentage points except the default premium, which is in basis points. The horizon is measured in years.

Interpretation. The figure illustrates how learning about U.S. security commitments shapes the dynamic transmission of fiscal shocks into global financial markets. Under full information (blue), the security shock is immediately capitalized into prices: fiscal uncertainty, discount rates, default premia, and the dollar convenience yield all jump sharply on impact and then mean revert rapidly as the shock dissipates. Global equity returns exhibit a pronounced negative impact

response—reflecting an immediate valuation drop—followed by a quick rebound as discount rates normalize.

Under partial information (red), the same shock generates a markedly more gradual adjustment. Because agents update beliefs about security only slowly, the perceived umbrella $\hat{s}_t$ rises over time rather than instantaneously. As a result, discount rates, default premia, and the dollar convenience yield peak later and decay more slowly, producing persistent movements in asset prices. Most notably, global equity returns display a delayed mean reversion: the initial price decline is followed by a prolonged period of positive excess returns that unwind only gradually over roughly one year.

This comparison highlights a key mechanism of the model: even when the underlying security shock is identical, learning frictions alone can generate substantial persistence in global risk premia and return predictability. The delayed adjustment under partial information aligns closely with the empirical evidence that global asset prices respond sluggishly to U.S. fiscal and security news, whereas a full-information benchmark implies unrealistically fast mean reversion.

D Time-Series Simulation and Factor-Mimicking Portfolios

This appendix provides a complete description of the time-series simulation procedure used to generate synthetic asset returns and factor-mimicking portfolios from the quantitative model. The objective is to produce simulated counterparts to the empirical portfolio time series reported in Section 3.5 and to evaluate whether the calibrated model reproduces their joint dynamics.

D.1 Model Inputs and Objects

Country set and exposure parameter. Let $\mathcal{I} = \{1, \ldots, N\}$ denote the set of countries used in the empirical portfolio construction. Each country i is characterized by a time-invariant exposure parameter $\phi_i \in \mathbb{R}_+$ measuring its reliance on the U.S. security umbrella (constructed from U.S. military presence as described in Section 3). Let $\phi \equiv (\phi_1, \ldots, \phi_N)'$ collect exposures.

State vector. Let the latent state vector be

$$x_t \equiv \begin{bmatrix} s_t \\ \hat{s}_t \\ \xi_t \end{bmatrix} \in \mathbb{R}^3,$$

where s_t is true U.S. security provision, $\hat{s}_t$ is perceived security credibility, and ξ_t is global financial risk. All persistence is generated by the state transition; asset prices do not feed back into x_t.

Transition dynamics. The state evolves as a linear Gaussian system

$$x_{t+1} = Ax_t + B\varepsilon_{t+1}, \qquad \varepsilon_{t+1} \sim \mathcal{N}(0, I_3), \tag{D.1}$$

with

$$A = \begin{bmatrix} \rho_s & 0 & 0 \\ \chi & 1-\chi & 0 \\ 0 & 0 & \rho_\xi \end{bmatrix}, \qquad B = \begin{bmatrix} \sigma_s & 0 & 0 \\ 0 & \sigma_{\hat{s}} & 0 \\ 0 & 0 & \sigma_\xi \end{bmatrix}.$$

The parameters $(\rho_s, \chi, \rho_\xi, \sigma_s, \sigma_{\hat{s}}, \sigma_\xi)$ are calibrated in the quantitative section.

Return-generating system. Let $r^{eq}_{i,t}$ denote the excess equity return in country i, $r^{b}_{i,t}$ the excess sovereign bond return (or spread change / excess return depending on the empirical counterpart), and $rx^{i\$}_t$ the currency excess return to a U.S. investor. Collect country-i returns in

$$y_{i,t} \equiv \begin{bmatrix} r^{eq}_{i,t} \\ r^{b}_{i,t} \\ rx^{i\$}_t \end{bmatrix} \in \mathbb{R}^3.$$

The model implies that these objects are affine in the state vector:

$$y_{i,t} = a_i + C(\phi_i)\, x_t + u_{i,t}, \tag{D.2}$$

where a_i is a constant vector, $C(\phi_i)$ is a 3×3 matrix of loadings implied by equilibrium pricing conditions, and $u_{i,t}$ is an i.i.d. measurement / idiosyncratic component with

$$u_{i,t} \sim \mathcal{N}(0, \Sigma_u), \qquad u_{i,t} \perp x_t, \qquad u_{i,t} \perp u_{j,t} \text{ for } i \neq j.$$

In implementation, $C(\phi_i)$ is constructed from the model's closed-form pricing rules (or numerical policy functions) and is not estimated freely.

Exposure enters only through ϕ_i. All cross-sectional heterogeneity in expected returns and betas across countries is generated by differences in ϕ_i through $C(\phi_i)$.

D.2 Simulation Algorithm

Fix a sample length T (e.g. $T = 444$ monthly observations for 1983–2019) and a burn-in T_0 (e.g. $T_0 = 200$). Let $t = 1$ denote the first post-burn-in observation.

Step 1: Initialize states. Set the initial state at the unconditional mean, $x_0 = 0$, or draw from the stationary distribution $x_0 \sim \mathcal{N}(0, \Sigma_x)$ where Σ_x solves the Lyapunov equation

$$\Sigma_x = A \Sigma_x A' + BB'.$$

Step 2: Simulate latent states. For $t = 0, 1, \ldots, T_0 + T - 1$:

(a) Draw $\varepsilon_{t+1} \sim \mathcal{N}(0, I_3)$.

(b) Update $x_{t+1} = A x_t + B \varepsilon_{t+1}$.

Step 3: Simulate country-level returns. For each $t = 1, \ldots, T$ (post burn-in) and each country $i \in \mathcal{I}$:

(a) Compute deterministic component $\tilde{y}_{i,t} \equiv a_i + C(\phi_i)x_t$.

(b) Draw idiosyncratic term $u_{i,t} \sim \mathcal{N}(0, \Sigma_u)$.

(c) Set $y_{i,t} = \tilde{y}_{i,t} + u_{i,t}$ and record the three asset-class returns.

Step 4: Portfolio sorting and aggregation. Let $\mathcal{Q}_1, \mathcal{Q}_2, \mathcal{Q}_3, \mathcal{Q}_4$ denote the four exposure-sorted portfolios induced by ϕ_i (quartiles). That is, $\mathcal{Q}_1$ contains the lowest-exposure countries, and $\mathcal{Q}_4$ the highest-exposure countries.

For each asset class $X \in \{eq, b, fx\}$ and each t:

$$R^X_{p,t} \equiv \frac{1}{|\mathcal{Q}_p|} \sum_{i \in \mathcal{Q}_p} y^X_{i,t}, \qquad p \in \{1, 2, 3, 4\}, \tag{D.3}$$

where $y^{eq}_{i,t} = r^{eq}_{i,t}$, $y^b_{i,t} = r^b_{i,t}$, and $y^{fx}_{i,t} = rx^{i\$}_t$.

Step 5: Construct HML factors. Define factor-mimicking portfolios as

$$\text{HML}^{eq}_t \equiv R^{eq}_{4,t} - R^{eq}_{1,t}, \qquad \text{HML}^b_t \equiv R^b_{4,t} - R^b_{1,t}, \qquad \text{HML}^{fx}_t \equiv R^{fx}_{1,t} - R^{fx}_{4,t},$$

where the FX sign convention follows the hedge-currency prediction: high-exposure currencies earn lower average excess returns.

D.3 Model-Implied Time-Series Targets

Let $\widehat{m}^X$ denote an empirical moment computed from the data and $m^X(\theta)$ denote the corresponding simulated moment computed from the model at parameter vector θ.

Core time-series moments. The model is evaluated on:

(i) Portfolio mean ordering:

$$\mathbb{E}[R^{eq}_{1,t}] < \mathbb{E}[R^{eq}_{2,t}] < \mathbb{E}[R^{eq}_{3,t}] < \mathbb{E}[R^{eq}_{4,t}], \quad \mathbb{E}[R^b_{1,t}] < \cdots < \mathbb{E}[R^b_{4,t}],$$

and the reverse ordering for FX.

(ii) HML profitability and Sharpe ratios for each asset class.

(iii) Rolling-window dynamics: correlations and Sharpe ratios computed over 60-month windows, to match the empirical time-series plots.

(iv) Comovement with risk states: correlations of HML_t^X with simulated ξ_t and $\hat{s}_t$.

Simulation uncertainty. All simulated moments are computed as averages across S independent Monte Carlo runs (e.g. $S = 500$), using common random numbers across calibrations when comparing alternative parameterizations.

Interpretation. This procedure isolates a stringent implication of the mechanism. Because the only cross-sectional heterogeneity is ϕ_i, the model can match the portfolio time series only if the security-credibility state $\hat{s}_t$ generates persistent, priced variation in expected returns that aligns with the exposure ranking. Successful replication of the portfolio time-series patterns therefore constitutes a quantitative validation of the same security-based transmission mechanism that underlies the cross-sectional fiscal betas and the quasi-experimental evidence.