Strategic Influence in Geopolitical Networks*

Ernest Liu†
Stephen J. Redding‡
Bennett Smith-Worthington§
David Y. Yang¶

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Abstract

We develop a tractable theoretical model of geopolitics as a network coordination game with strategic influence. Countries choose policies that trade off domestic interests against incentives to align with neighbors in a geopolitical network. Before policies are chosen, countries can make costly strategic investments to influence one another's policies. Since influence propagates throughout the network, bilateral influence has cascading effects, such that one country can influence another indirectly through third parties. We implement our model quantitatively using data on countries' domestic political interests (as measured by democratic orientation) and geopolitical actions (as measured by UN General Assembly voting). We use the assumption that the cross-country covariance matrices of actions and interests admit a joint diagonalization to identify the unobserved geopolitical network using its eigencomponents. We validate our inferred geopolitical network using a range of empirical proxies, and show that the estimated eigencomponents capture interpretable geopolitical cleavages, such as the Cold War division between the Western and Soviet blocs.

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†Princeton University, JRR Building, Princeton, NJ 08544 and NBER. Email: ernestliu@princeton.edu.

‡Stanford University, 579 Jane Stanford Way, Stanford, CA 94305, NBER and CEPR. Email: reddings@stanford.edu.

§Princeton University, JRR Building, Princeton, NJ 08544. Email: bennettsw@princeton.edu.

¶Littauer Center M-31, 1805 Cambridge Street, Cambridge, MA 02138 and NBER. Email: davidyang@fas.harvard.edu.

1 Introduction

Geopolitics is inherently a network phenomenon. Power is rarely exercised in a purely dyadic way, with one country acting on another in isolation. Instead, states are embedded in webs of alliances, trade and financial ties, ideological affinities, and security relationships. Because of these interdependencies, the effect of any geopolitical action depends not only on the two countries directly involved, but also on how third countries respond. Even the decisions of a hegemon are often shaped by the strategic positioning of its allies and its potential rivals. A sanction is effective only if others enforce it; a military threat is credible only if allies coordinate behind it; and a diplomatic initiative succeeds or fails depending on whether pivotal states align, hedge, or defect.

Recent events make this clear. Russia's invasion of Ukraine was not simply a bilateral conflict between Russia and Ukraine. It quickly became a problem of coalition formation and coordination across Europe, North America, and beyond, involving sanctions, military assistance, energy reallocation, and diplomatic positioning. The same logic extends well beyond war. US export controls on advanced semiconductors matter because key supplier states coordinate complementary restrictions. Maritime disputes in the South China Sea turn on whether regional states coordinate their positions or fragment under competing economic and security ties. And during the Cold War, confrontations such as the Berlin Airlift, the Korean and Vietnam Wars, and the Cuban Missile Crisis were not isolated flashpoints, but episodes embedded in a broader geopolitical network linking the Western and Soviet blocs. More generally, events in one region often reverberate elsewhere because influence propagates through networks of aligned and competing states.

This paper develops a tractable — both theoretically and empirically — framework for analyzing geopolitics from that network perspective. Viewing geopolitics through the lens of networks changes how we think about power. In a purely dyadic world, power is direct leverage over another state. In a networked world, power is also positional. Countries can become influential because they occupy central locations in coalitions, broker between blocs, control chokepoints, or act as pivotal swing states. Our network perspective also changes how we think about polarity. Rather than defining unipolarity or multipolarity solely as rankings of national capabilities, it treats polarity as a property of the system: a question of how influence, coalition formation, and coordination are organized across the network. Finally, it offers a new way to understand why some issues generate broad international convergence while others produce sharp polarization. Polarization is especially likely when disagreement aligns with major geopolitical cleavages, so that connected countries reinforce one another's positions and cross-bloc divisions become amplified.

We make two contributions. First, we develop a tractable theoretical model of strategic investments in geopolitical networks. We allow all countries to make these strategic investments and our network structure encompasses unipolar, multipolar or nonpolar worlds. Countries choose poli-

cies that trade off domestic interests against incentives to align with the interests of neighbors in a geopolitical network. Before policies are chosen, countries can make costly bilateral strategic investments to influence one another's policy choices. Since influence propagates through the network, bilateral influence has cascading effects: shifting one country's policy changes equilibrium policies throughout the system, so a country may affect another indirectly by influencing third parties. Second, we implement our model quantitatively using data on countries' political interests (as measured by democratic orientation) and geopolitical actions (as measured by UN General Assembly voting). We use the assumption that the cross-country covariance matrices of actions and interests admit a joint diagonalization to identify the unobserved geopolitical network using its underlying eigencomponents. We validate our inferred geopolitical network using a range of empirical proxies, and show that the estimated eigencomponents capture interpretable geopolitical cleavages, such as the Cold War division between the Western and Soviet Blocs.

Developing a tractable quantitative model of geopolitical networks with strategic investments raises both theoretical and empirical challenges. From a theoretical perspective, games with many asymmetric agents, in which each agent strategically influences all other agents, feature high-dimensional state spaces. If each of the N agents can strategically influence all other agents, the state space grows at rate $N \times N$. If agents are connected by rich networks of economic and political linkages, the payoffs from bilateral interactions between a pair of agents depend on their interactions with all other agents, which makes it hard to obtain sharp analytical results for how the network structure affects outcomes. For this reason, existing research often makes strong assumptions about the structure of geopolitical networks, such as the distinction between a single hegemon, or two competing hegemons, and other satellite countries.

From an empirical perspective, geopolitical networks are typically not directly observed. Instead, we observe a range of empirical proxies for geopolitical linkages between countries, such as geographical distance, international trade, colonial relationships or cultural linkages. The mapping from these observed empirical proxies to the unobserved geopolitical linkage, and the relative importance of each of these different empirical proxies, can be hard to discern. Although we observe a range of geopolitical actions taken by countries towards one another (including for example votes in the United Nations General Assembly), it is challenging to quantify the extent to which these geopolitical actions reflect the domestic interests of countries (e.g., common shared values or ideologies) versus the strategic influence of countries on one another through the network of geopolitical linkages between countries. Even if the geopolitical actions and domestic interests of countries can both be observed, it is unclear whether this information is sufficient to infer the underlying structure of the geopolitical network. As a result, existing research again makes strong assumptions about the structure of the geopolitical network, such as the assumption that geopolitical linkages can be fully captured by geography or international trade.

We begin by describing the setup of our framework. Our theoretical model considers a world that consists of N countries connected by a weighted graph or geopolitical network $\boldsymbol{G}$. This geopolitical network captures bilateral considerations for coordination in foreign policy, which may reflect alliances, economic dependence, or other strategic linkages. For a given international issue, each country i has a domestic interest θ_i and chooses a geopolitical action a_i. The geopolitical action is the outcome of a network "beauty-contest" coordination game: countries balance alignment with their own domestic interest and alignment with the actions of their network neighbors. Prior to the coordination game, each country engages in costly bilateral strategic influence to shift other countries' incentives for geopolitical actions. Country i may strategically influence j not because i wants to coordinate with j, but because doing so indirectly influences a third country k—who coordinates with j either directly or indirectly through the network—with whom i seeks alignment. The global equilibrium is a fixed point of these bilateral choices, where each country internalizes the full cascading impact of its influence on geopolitical actions across the geopolitical network.

We provide a closed-form characterization of the equilibrium profile of geopolitical actions and bilateral influence. The model admits a unique equilibrium, in which each country's equilibrium action is a convex combination of all countries' domestic interests. This equilibrium is "conservative" in the sense that, across countries, the average geopolitical actions and average domestic interests coincide; and net geopolitical influence sums to zero across countries. These properties formalize a basic feature of multilateral influence: strategic pressure reallocates incentives across countries but, in aggregate, does not create net persuasion "out of thin air."

We next theoretically characterize the structural features of international issues that drive either polarization or convergence in geopolitical actions across countries. We show that the equilibrium mapping from heterogeneous domestic interests to geopolitical actions is a spectral function of the geopolitical network. This implies a property of "spectral decoupling": when domestic interests are decomposed along the network's eigenvectors—which we interpret as latent geopolitical cleavages—the equilibrium scales each dimension of conflict independently. Issues that align with large eigenvalues are those where disagreements are small across network neighbors but large across blocs. We show that these issues generate a geopolitical echo-chamber effect, amplifying cross-bloc frictions into stark policy polarization. Conversely, issues associated with small eigenvalues are neutralized by local coordination motives, leading to highly convergent policy outcomes.

We characterize the geometry of strategic influence by introducing a "contestability index," which is issue-dependent and measures the degree of misalignment in the foreign influence directed at any given country. At one extreme, a country may face perfectly monolithic foreign pressure, where all foreign actors push its policy in a unified direction. At the other extreme, a country may face a geopolitical tug-of-war, where foreign actors exert competing, contradictory pressures that partially cancel each other out. Our index formally identifies these highly contested "swing states" and

shows how a country's status as a battleground for influence depends on the interaction between its network position and the specific international issue at hand.

We transition from the analysis of a single international issue to an environment in which countries repeatedly play the coordination and strategic influence game over a fixed geopolitical network $\boldsymbol{G}$, with domestic interests $\boldsymbol{\theta}$ for each international issue drawn from a fixed distribution of global geopolitical shocks. This stochastic arrival generates variation in both domestic interests and equilibrium geopolitical actions $\boldsymbol{a}$. We establish a structural identification result that the eigenvalues and eigenvectors of the geopolitical network (and hence the entire geopolitical network) are recoverable from these observables. We make a graph stationarity assumption that the set of latent issues that determine the strength of geopolitical connections between countries is the same as those that generate differences in domestic interests across countries. Under this assumption, the cross-country covariance matrices of domestic interests, $\boldsymbol{\Sigma}_\theta$, and geopolitical actions, $\boldsymbol{\Sigma}_{\boldsymbol{a}}$, both share the same eigenbasis as the geopolitical network ($\boldsymbol{G}$). Within the structure of the model, these two cross-country covariance matrices contain sufficient information to identify the eigenvectors and eigenvalues for the direction and strength of the geopolitical cleavages of the network, as long as there is variation in domestic interests along each of these cleavages.

Our theoretical identification result motivates an empirical implementation based on the approximate joint diagonalization of the covariance matrices of $\boldsymbol{\theta}$ and $\boldsymbol{a}$. We use two widely used sources of cross-country political data. We measure cross-country covariance in domestic interests $\boldsymbol{\Sigma}_\theta$ using bilateral similarities in the country-year indicators from V-Dem, aggregating 35 variables that measure domestic interests and political institutions. We proxy geopolitical actions $\boldsymbol{a}$ using UN General Assembly voting ideal points from Bailey et al. (2017). We treat $\boldsymbol{G}$ as evolving slowly and implement the joint diagonalization separately by decade. The resulting decomposition is strongly low-dimensional: using only the top five eigencomponents, the reconstructed covariance matrices achieve R^2 above 80% across the sample and around 95% in recent decades. The first eigenvector is a constant that does not affect policy convergence or divergence across countries. The second eigenvector—the leading nontrivial cleavage—tracks well-known geopolitical realignments. Prior to 1989 it aligns with the Cold War division between the West and the Soviet bloc; in the 1990s it shifts sharply with the collapse of the Soviet Union; and after 2000 it is consistent with a cleavage in which the US and Western Europe occupy one pole while China occupies the opposite, with many countries in the Global South lying in between.

Finally, we use the estimated eigenvectors and eigenvalues to construct an inferred geopolitical network for each decade. We validate the inferred dyadic links against external covariates: inferred edges correlate with dyadic measures of foreign bilateral influence capacity (FBIC "bandwidth" measures) and with standard cultural and historical ties (common language, religion, and colonial links); they also covary with bilateral differences in Polity scores and gravity-style variables such as distance

and trade flows where available.

Taken together, we provide a tractable way to study geopolitics as a system of strategic influence in networks. Our framework shows how indirect influence, coalition structure, and latent geopolitical cleavages shape the translation of domestic interests into international action, and it offers an empirical strategy for recovering those structures from observed data. More broadly, our framework is relevant not only for understanding military rivalry and spheres of influence, but also for analyzing geoeconomics, technology competition, sanctions, and diplomatic coordination in an increasingly interconnected global economy.

Our paper is related to several strands existing research. First, we connect with recent research on geoeconomics, including Becko and O'Connor (2024), Clayton et al. (2024), Broner et al. (2024), Kleinman et al. (2024), Liu and Yang (2025), and Clayton et al. (2026).[1] Much of this research assumes a distinction between a hegemon and other individual nations. In contrast, we allow theoretically and empirically for a rich geopolitical network, in which whether or not a hegemon emerges is an endogenous outcome, and countries can be connected by alliances.

Second, we contribute to research on networks in economics, including Wolinsky and Jackson (1996), Jackson (2008), Acemoglu et al. (2012), Chaney (2014), Oberfield (2018), Goyal (2023), Elliott et al. (2022), Galeotti et al. (2024), Taschereau-Dumouchel (2024), and Cerreia-Vioglio et al. (2026). Our model builds on the theory literature on network games and beauty contests, including Morris and Shin (2002), Galeotti et al. (2010), and Allen et al. (2006). We apply these theoretical techniques to geopolitics. We extend a network coordination game to include an an influence stage of endogenous strategic investments that distort coordination incentives.

Third, our empirical approach contributes to the econometric literature on the structural identification of networks, including Graham (2017), Mele (2017), Paula (2020), and Fernández-Val and Chen (2021). We provide conditions under which the cross-country covariances of domestic interests and geopolitical actions identify the unobserved geopolitical network.

Fourth, our work builds on the existing literature on geopolitics. This literature includes approaches emphasizing central locations (following Mackinder 1904); rimland locations (after Spykman 1944); hegemonic power and imperial overstretch (e.g., Mahan 1890 and Kennedy 1987); competition between competing ideologies (as during the Cold War in Gaddis 2005); armed conflict (e.g., Fearon 1995); chokepoints (e.g., Farrell and Newman 2023 and Fishman 2025); resource competition (e.g., Miller 2022); and spheres of influence (e.g., Allison 2020).

The remainder of the paper is structured as follows. Section 2 sets up the model and derives our main theoretical results regarding equilibrium influence, spectral decoupling, and contestability. Section 3 introduces our data and empirical strategy for recovering the geopolitical network. Section 6 summarizes our conclusions.

[1] For recent reviews, see Clayton et al. (2025) and Mohr and Trebesch (2025).

2 Model

We consider a world that consists of N countries indexed by $i, j \in \{1, \ldots, N\}$. The countries are connected by a weighted graph that can be represented by the matrix $\boldsymbol{G}$, where we use boldface throughout the paper to denote vectors (lowercase) and matrices (uppercase). The elements of this matrix $G_{ij} \in \mathbb{R}_{\geq 0}$ capture the magnitude of the political connections between countries i and j. The representative domestic policy-maker in each country (the national government) has a domestic interest $\theta_i \in \mathbb{R}$ and can take a geopolitical action $a_i \in \mathbb{R}$.

We interpret $\boldsymbol{G}$ as the geopolitical network. It captures bilateral considerations for coordination in foreign policy. It could reflect strategic alliances (e.g., G_{ij} is large between allies), economic dependence (e.g., G_{ij} is increasing in bilateral trade), or other considerations for coordination (e.g., US and Russia deliberating over a nuclear deal). We view $\boldsymbol{G}$ as reflecting medium to long-run geopolitical relationships between countries.

We interpret $\boldsymbol{\theta}$ as the heterogeneous domestic interests of countries over a specific international issue. For example, countries around the world may react differently to capturing of President Maduro of Venezuela or President Trump's threat of invading Greenland. We begin by analyzing the equilibrium for a given $\boldsymbol{\theta}$. Later we allow for multiple international issues, each of which corresponds to different $\boldsymbol{\theta}$, by assuming that $\boldsymbol{\theta}$ is drawn from a distribution across issues.

We interpret $\boldsymbol{a}$ as the geopolitical actions that countries adopt in response to the international issue. These geopolitical actions are influenced not only by domestic interests but also by global considerations of coordinating geopolitical actions with other nations. We model the choice of geopolitical actions $\boldsymbol{a}$ as the solution of a beauty-contest coordination game over the network, where each country i chooses its geopolitical action (a_i) to balance its alignment with its own domestic interests (θ_i) and the geopolitical actions of its network neighbors (a_j for $j \neq i$).

Prior to the coordination game, each country i can engage in costly strategic influence (ω_{ij}) to alter any other country j's incentives to choose geopolitical actions. The game proceeds in two stages for each issue $\boldsymbol{\theta}$. In the influence stage, countries observe $\boldsymbol{\theta}$ and choose bilateral strategic influence. In the coordination stage, countries choose geopolitical actions, taking bilateral strategic influence as given. Our main interest is to analyze how the matrix of bilateral strategic influence ($\boldsymbol{\Omega}$) depends on the geopolitical network $\boldsymbol{G}$ and domestic interests $\boldsymbol{\theta}$. The game has perfect information: each country knows the domestic interests ($\boldsymbol{\theta}$) and strategic influence ($\boldsymbol{\Omega}$) of all countries.

In what follows, we first introduce a special case of the model with only the coordination stage without any bilateral strategic influence. We then develop the full model with both the influence and coordination stages. In our baseline model, we assume for expositional simplicity that the geopolitical network is undirected ($\boldsymbol{G}$ is symmetric) and regular (every row of $\boldsymbol{G}$ sums to the same value). We denote the common row sum of $\boldsymbol{G}$ by $1 - \alpha$. In a later subsection, we extend our baseline model

to introduce asymmetries in the geopolitical network.

2.1 Model without Strategic Influence

In the special case of the model without strategic influence, countries play a beauty-contest coordination game over the geopolitical network. The representative policy-maker in each country i chooses its geopolitical action a_i to minimize its misalignment with its domestic interest (θ_i) and the geopolitical actions of its network neighbors (a_j), taking the geopolitical actions chosen by other countries as given. We assume that country i solves the following problem:

$$\max_{a_i} \left\{ - \left[\alpha \left(a_i - \theta_i \right)^2 + \sum_{j=1}^{N} G_{ij} \left(a_i - a_j \right)^2 \right] \right\}, \tag{1}$$

where the bilateral geopolitical links (G_{ij}) capture the extent to which countries i and j want their geopolitical actions to align.[2]

Taking the first-order condition with respect to a_i, we obtain country i's best-response function:

$$a_i = \alpha \theta_i + \sum_{j=1}^{N} G_{ij} a_j. \tag{2}$$

Noting that $\alpha + \sum_j G_{ij} = 1$, each country's best response is a weighted average of its own domestic interest (θ_i) and the geopolitical actions of its network neighbors (a_j). In vector form, this best-response function can be written as:

$$\boldsymbol{a} = \alpha \boldsymbol{\theta} + \boldsymbol{G} \boldsymbol{a}. \tag{3}$$

The equilibrium in geopolitical actions corresponds to the fixed point in which each country's best-response is consistent with the best-responses of all other countries:

$$\boldsymbol{a} \equiv \alpha \left(\boldsymbol{I} - \boldsymbol{G} \right)^{-1} \boldsymbol{\theta}. \tag{4}$$

2.2 Model with Strategic Influence

The full model includes both the influence and coordination stages of the game. In the influence stage, each country i can choose to engage in costly bilateral strategic influence to change the incentives any country j to choose geopolitical actions. After strategic influence has occurred, the game moves on to the coordination stage, in which each country i chooses its geopolitical action a_i, taking as given the geopolitical actions of its network neighbors (a_j) and bilateral strategic influence (ω_{ij}).

[2] Although the summation in the second term includes the own country $i = j$, the own-country entries in this second term are necessary equal to zero (since $a_i = a_j$ for $i = j$).

We assume that country i's payoff takes the following form:

$$\begin{aligned} U_i(\boldsymbol{a}, \boldsymbol{\Omega}) &= -\left[\alpha (a_i - \theta_i)^2 + \sum_{j=1}^{N} G_{ij} (a_i - a_j)^2\right] \\ &\quad -\kappa \sum_{k=1}^{N} \omega_{ki}^2 \\ &\quad +2 \sum_{m=1}^{N} \omega_{im} (a_i - \theta_i), \end{aligned} \tag{5}$$

where $\omega_{ki} \in \mathbb{R}$ is the influence exerted by country i on country k—the first subscript indicates the country receiving influence and the second subscript indicates country exerting influence—and $\boldsymbol{\Omega} \equiv [\omega_{ki}]$ is the matrix of bilateral influence.

The first line in (5) is the country i's payoff from the network coordination game. The second line is country i's convex cost of exerting strategic influence. The parameter $\kappa > 1/\alpha$ is a cost shifter for strategic influence. Strategic influence can be either positive or negative. In both cases, the costs of strategic influence increase quadratically with its distance from zero. The third line is the payoff from receiving foreign influence ω_{im} exerted by each country m. Receiving positive net strategic influence ($\sum_{=m=1}^{N} \omega_{im} > 0$) raises the marginal benefit of choosing an action a_i greater than domestic interests θ_i. Conversely, receiving negative net strategic influence ($\sum_{m=1}^{N} \omega_{im} < 0$) increases the incentive to choose an action a_i lower than domestic interests θ_i.

Therefore, strategic influence is valuable to country i insofar as it shifts its actions away from what it would choose based purely on domestic interests. This formulation of the impact of strategic influence on actions using domestic interests as the benchmark has several convenient properties. First, it implies translation invariance in $\boldsymbol{\theta}$: adding a constant c to every country's domestic interest θ_i shifts every equilibrium action by c without changing welfare, since the deviation $a_i - \theta_i$ is invariant to uniform translations. Second, this specification permits self-influence—country i can exert influence on itself ($\omega_{ii} > 0$)—which captures the idea that a country may invest resources to entrench its own policy stance. Our concavity assumption $\kappa > 1/\alpha$ ensures that the returns to self-influence are bounded. Incorporating self-influence allows us to apply linear algebra tools without separately dealing with the diagonals $\{\omega_{ii}\}$ of the influence matrix $\boldsymbol{\Omega}$. Finally, because θ_i is exogenous to country i, the term in $\sum_{m=1}^{N} \omega_{im}\theta_i$ in the third line does not affect country i's best-response choice of actions in the coordination stage, nor does not enter the payoff of any influencing country m. Therefore, it only affects the equilibrium through country i's choice of self-influence ω_{ii}.

To illustrate the impact of strategic influence on countries' objectives, we can re-write country i's payoff in equation (5) in the following form:

Lemma 1. *The payoff of country i in equation (5) can be rewritten as:*

$$U_i(\boldsymbol{a}, \boldsymbol{\Omega}) = -\left[\alpha\left(a_i - \left(\theta_i + \frac{\sum_{m=1}^{N}\omega_{im}}{\alpha}\right)\right)^2 + \sum_{j=1}^{N} G_{ij}\left(a_i - a_j\right)^2\right] + \frac{\left(\sum_{m=1}^{N}\omega_{im}\right)^2}{\alpha} - \kappa\sum_{k=1}^{N}\omega_{ki}^2. \quad (6)$$

Comparing equations (6) and (1), the net strategic influence received by country i acts like a shift in its domestic interests from θ_i to $\theta_i + \sum_{m=1}^{N}\omega_{im}/\alpha$, with net transfers of $\left(\sum_{m=1}^{N}\omega_{im}\right)^2/\alpha$.

2.3 Equilibrium

We solve the game through backwards induction. We first solve for equilibrium geopolitical actions in the second stage of the game taking bilateral influence as given. We then solve for equilibrium bilateral strategic influence in the first-stage of the game.

2.3.1 Equilibrium at the Coordination Stage

In the coordination stage, country i chooses its geopolitical action (a_i) to maximize its payoff (6), taking as given bilateral strategic influence ($\boldsymbol{\Omega}$) and the geopolitical actions of all other countries (a_j). The first-order condition or best-response function for country i takes a similar form as in equation (2), but modified by the impact of net strategic influence:

$$a_i = \alpha\theta_i + \sum_{j=1}^{N} G_{ij} + \sum_{m=1}^{N}\omega_{im}. \quad (7)$$

In vector form, this best-response function can be written as:

$$\boldsymbol{a} = \alpha\boldsymbol{\theta} + \boldsymbol{G}\boldsymbol{a} + \boldsymbol{\Omega}\mathbf{1}. \quad (8)$$

Therefore, relative to the pure beauty-contest game in equation (3), country m's strategic influence shifts the target country i's geopolitical action by ω_{im}, taking the actions of all other countries as given. Moving $\boldsymbol{G}\boldsymbol{a}$ to the left-hand side, and multiplying both sides by $(\boldsymbol{I} - \boldsymbol{G})^{-1}$, we can write equilibrium actions in the coordination stage as an explicit function of domestic interests and influence:

$$\boldsymbol{a} = (\boldsymbol{I} - \boldsymbol{G})^{-1}(\alpha\boldsymbol{\theta} + \boldsymbol{\Omega}\mathbf{1}). \quad (9)$$

The ik-th entry of the matrix $(\boldsymbol{I} - \boldsymbol{G})^{-1}$ captures how country k's domestic interest (θ_k) and net strategic influence received ($[\boldsymbol{\Omega}\mathbf{1}]_k = \sum_{j=1}^{N}\omega_{kj}$) affect the equilibrium geopolitical actions chosen by country i. Note that this matrix inverse has the following Neumann series representation: $(\boldsymbol{I} - \boldsymbol{G})^{-1} = \boldsymbol{I} + \boldsymbol{G} + \boldsymbol{G}^2 + \cdots$. In this representation, each successive power of the matrix $\boldsymbol{G}$ captures a higher-order network connection: $\boldsymbol{G}_{ik}$ is the connection between i and k, $\boldsymbol{G}_{ik}^2 = \sum_{j=1}^{N} G_{ij}G_{jk}$ is the indirect connection between i and k through a common neighbor j, and so on. Therefore, foreign influence on country k affects country i's action if they are either directly or indirectly connected through the network.

2.3.2 Equilibrium at the Influence Stage

In the influence stage, country i chooses its bilateral strategic influence $\{\omega_{ki}\}_k$ to maximize its objective $U_i\left(\boldsymbol{a}\left(\boldsymbol{\Omega}\right), \boldsymbol{\Omega}\right)$. In the first-order condition for country i's optimal choice of strategic influence, the total derivative of its objective with respect to its bilateral strategic influence (ω_{ki}) can be decomposed into the following three terms:

$$0 = \frac{\mathrm{d}U_i}{\mathrm{d}\omega_{ki}} = \underbrace{\frac{\partial U_i}{\partial a_i}\frac{\partial a_i}{\partial \omega_{ki}}}_{\text{effect through } a_i} + \underbrace{\sum_{j\neq i}\frac{\partial U_i}{\partial a_j}\frac{\partial a_j}{\partial \omega_{ki}}}_{\text{effect through } a_j,\, j\neq i} + \underbrace{\left.\frac{\partial U_i}{\partial \omega_{ki}}\right|_{\boldsymbol{a}}}_{\text{direct effect}} \tag{10}$$

The first term on the right-hand side is the indirect effect of ω_{ki} on country i's payoff through the change in i's own action a_i. Because a_i is chosen optimally at the coordination stage, this term vanishes by the envelope theorem. The second term is the indirect effect through changes in other countries' actions a_j ($j \neq i$). Since country i does not control these actions, the envelope theorem does not apply, and this second term does not zero out. The third term is the direct effect of ω_{ki} on country i's payoff holding all actions fixed, which consists of the marginal cost of strategic influence ($-2\kappa\,\omega_{ki}$), and the marginal benefit of self-influence $2\left(a_i - \theta_i\right)$ for $k = i$.

Evaluating the non-zero terms in the first-order condition (10) yields the best-response function for country i's strategic influence towards country k:

$$\kappa\,\omega_{ki} = \sum_{j=1}^{N}\left[\left(\boldsymbol{I} - \boldsymbol{G}\right)^{-1}\right]_{jk} G_{ij}\left(a_i - a_j\right) + \mathbf{1}_{k=i}\left(a_i - \theta_i\right). \tag{11}$$

The left-hand side is the marginal cost of the strategic influence exerted by country i on country k. The right-hand side is the marginal benefit of this strategic influence. The first term on the right-hand side captures country i's incentive to influence country k to take a higher action, taking into account the equilibrium propagation of higher a_k through the network. Country i's payoff is higher when its neighbors' actions are closer to its own, as captured by the gap $(a_i - a_j)$. If this gap is positive, country i benefits from a higher a_j, and vice versa. The object $\left[\left(\boldsymbol{I} - \boldsymbol{G}\right)^{-1}\right]_{jk}$ in this first term captures how influencing country k raises country j's action in the coordination equilibrium, accounting for both the direct effect on k and all indirect network propagation. The object G_{ij} in this first-term captures how important country j's action is for country i's payoff. Summing across countries j, we obtain country i's overall incentive to influence country k, taking into account the full network of policy coordination.

The second term on the right-hand side arises only for self-influence ($k = i$) and reflects the direct benefit of shifting country i's own action further from its domestic interest θ_i. Intuitively, when $a_i > \theta_i$—that is, when the geopolitical coordination environment pushes i to act above its domestic interest—additional self-influence raises a_i further, amplifying the deviation from θ_i with

marginal benefit $(a_i - \theta_i)$. This second term is absent for $k \neq i$, since the influence benefit of $\sum_{m=1}^{N} \omega_{im} (a_i - \theta_i)$ in the third line of country i's payoff in equation (5) depends on ω_{ki} only through ω_{ii}, as discussed earlier.

Summing across i in the best-response function (11), and noting that $\sum_{i=1}^{M} \mathbf{1}_{k=i} (a_i - \theta_i) = a_k - \theta_k$, we can write the net strategic influence received by country k as:

$$\kappa \sum_{i=1}^{N} \omega_{ki} = \sum_{i=1}^{N} \sum_{j=1}^{N} G_{ij} (a_i - a_j) \left[(\boldsymbol{I} - \boldsymbol{G})^{-1} \right]_{jk} + a_k - \theta_k. \tag{12}$$

Since $\boldsymbol{G}$ is symmetric with a constant row sum of $1 - \alpha$, we have $\sum_{i=1}^{N} G_{ij} (a_i - a_j) = [\boldsymbol{G}\boldsymbol{a}]_j - (1 - \alpha) a_j$, which enables us to re-write this relationship as:

$$\begin{aligned} \kappa \boldsymbol{\Omega} \mathbf{1} &= \boldsymbol{a} - \boldsymbol{\theta} + (\boldsymbol{I} - \boldsymbol{G})^{-1} [\boldsymbol{G}\boldsymbol{a} - (1 - \alpha) \boldsymbol{a}], \\ &= \alpha (\boldsymbol{I} - \boldsymbol{G})^{-1} \boldsymbol{a} - \boldsymbol{\theta}. \end{aligned} \tag{13}$$

Therefore, the net strategic influence received by country k is proportional to the gap between the network-weighted sum of equilibrium actions, $\alpha \left[(\boldsymbol{I} - \boldsymbol{G})^{-1} \boldsymbol{a} \right]_k = \alpha (a_k + [\boldsymbol{G}\boldsymbol{a}]_k + [\boldsymbol{G}^2 \boldsymbol{a}]_k + \cdots)$, and its domestic interest θ_k. Country k receives positive net influence whenever the average equilibrium action of itself, its neighbors, its neighbors' neighbors, and so on collectively exceeds θ_k / α. Therefore, the ability of a country to obtain transfers through strategic influence depends on its domestic interests relative to equilibrium actions in the network. Country k receives net influence to take a higher action if its domestic interest is below the equilibrium actions of itself, its neighbors, its neighbors' neighbors, and so on.

Substituting for the equilibrium actions from the coordination game $\boldsymbol{a}(\boldsymbol{\Omega})$ in equation (9), we obtain the following closed-form solutions for equilibrium net influence and geopolitical actions in terms of domestic interests (θ) and the geopolitical network ($\boldsymbol{G}$).

Proposition 1. *The equilibrium vectors of net influence and geopolitical actions satisfy:*

$$\boldsymbol{\Omega} \mathbf{1} = - \left(\kappa (\boldsymbol{I} - \boldsymbol{G})^2 - \alpha \boldsymbol{I} \right)^{-1} \left((\boldsymbol{I} - \boldsymbol{G})^2 - \alpha^2 \boldsymbol{I} \right) \boldsymbol{\theta}$$

$$\boldsymbol{a} = \left(\kappa (\boldsymbol{I} - \boldsymbol{G})^2 - \alpha \boldsymbol{I} \right)^{-1} (\boldsymbol{I} - \boldsymbol{G}) (\kappa \alpha - 1) \boldsymbol{\theta}$$

Moreover, as $\kappa \to \infty$, $\boldsymbol{\Omega} \mathbf{1} \to \mathbf{0}$ *and* $\boldsymbol{a} \to \alpha (\boldsymbol{I} - \boldsymbol{G})^{-1} \boldsymbol{\theta}$*; and as* $\kappa \downarrow 1/\alpha$*,*

$$\boldsymbol{a} \to \bar{\theta} \mathbf{1}, \qquad \boldsymbol{\Omega} \mathbf{1} \to \alpha \left(\bar{\theta} \mathbf{1} - \boldsymbol{\theta} \right), \tag{14}$$

where $\bar{\theta} \equiv \frac{1}{n} \sum_i \theta_i$.

To interpret Proposition 1, recall that $\kappa > 1/\alpha$ is the cost shifter of influence. Note also that the expression for equilibrium actions in this proposition admits a power-series representation:

$$\boldsymbol{a} = (\kappa \alpha - 1) \sum_{n=0}^{\infty} c_n \left(\frac{\boldsymbol{G}}{1 - \alpha} \right)^n \boldsymbol{\theta}, \tag{15}$$

where c_n is a weakly increasing sequence defined by the recursion

$$c_0 = \frac{1}{\kappa - \alpha}, \qquad c_1 = \frac{\kappa + \alpha}{(\kappa - \alpha)^2}, \qquad c_n = \frac{1}{\kappa - \alpha}\left(2\kappa\, c_{n-1} - \kappa\, c_{n-2}\right) \quad \forall\, n \geq 2. \tag{16}$$

In this power-series representation, $\boldsymbol{G}/(1-\alpha)$ is row-stochastic, and $(\boldsymbol{G}/(1-\alpha))^n \boldsymbol{\theta}$ is the weighted average of the domestic interests of country i's n-step network neighbors. The coefficient c_n determines the weight placed on network distance n. This weight is decreasing in network distance for κ in equation (16), so that closer neighbors receive greater weight.

To gain further understanding for Proposition 1, it is useful to consider the two limits of the influence cost shifter (κ). As influence becomes infinitely costly ($\kappa \to \infty$), net influence vanishes and the game converges back to the beauty contest without influence. As influence becomes as cheap as feasible—κ converges to $1/\alpha$ from above ($\kappa \downarrow 1/\alpha$)—equilibrium actions converge to the unweighted global average $\bar{\theta}$. Intuitively, when influence is cheap, countries can push each other toward a common action, which is pinned down by the equal-weighted average of domestic interests.[3] Net influence acts as a corrective force: countries with domestic interests lie above the global average ($\theta_k > \bar{\theta}$) receive negative net influence that pulls their actions down, while those with domestic interests below the global average receive positive net influence that pushes their actions up.

For any interior value of influence costs ($\kappa \in (1/\alpha, \infty)$), the two equilibrium forces of beauty-contest coordination and strategic influence are both active, and the equilibrium of the game smoothly interpolates between these two limits. Although countries exert strategic influence, its magnitude is strictly attenuated relative to that in the lower limit. A reduction in the cost of influence (a lower κ) reduces the differences in equilibrium geopolitical actions across countries. As strategic influence becomes cheaper, the resulting web of bilateral influence pushes countries further away from their idiosyncratic domestic interests and toward the network consensus $\bar{\theta}$. To formally establish this comparative static, and precisely characterize how strategic influence reshapes the cross-country dispersion of actions, we use a spectral decomposition of the geopolitical network, as developed in Section 2.4 below.

2.3.3 Properties of the Equilibrium

We now summarize a number of convenient properties of the equilibrium in Proposition 1.

Proposition 2. *The equilibrium has the following properties.*

1. *The equilibrium is unique.*

2. *Each country's equilibrium action is a convex combination over the domestic interests of all countries.*

[3] In this limit, equilibrium actions are disproportionately determined by distant linkages: the coefficient c_n grows geometrically at rate $\left(\frac{1}{1-\alpha}\right)^n$ for large n.

3. *The equilibrium is translational invariant to $\boldsymbol{\theta}$: adding a constant c to all θ_i's results in a new equilibrium in which all actions are also shifted by c, without any change in payoffs.*

4. *The equilibrium is conservative: the average geopolitical action and the average domestic interest across countries coincide:* $\mathbf{1}'\boldsymbol{a} = \mathbf{1}'\boldsymbol{\theta}$.

5. *The equilibrium strategic influence nets out to zero:* $\mathbf{1}'\boldsymbol{\Omega}\mathbf{1} = 0$.

Our model features a unique equilibrium despite the rich network interactions between countries. The linearity of the best-response function (11) in domestic interests implies that each country's equilibrium geopolitical action is a convex combination of the domestic interests of all countries. Our assumption that the return to strategic influence depends on the gap between geopolitical actions and domestic interests ensures that the equilibrium is translational invariant to domestic interests. Therefore, adding a constant to all domestic interests simply results in a new equilibrium, in which all actions are also shifted by c, without any change in payoffs. Equilibrium geopolitical actions are also conservative, in the sense that the average geopolitical action and average domestic interest coincide. Finally, equilibrium strategic influence is positive for some bilateral pairs and negative for others, but averages out to zero across all bilateral pairs.

2.4 Spectral Decomposition and Geopolitical Cleavage

In reality, some international issues can lead to a stark divergence in geopolitical actions across countries. An example is Russia's invasion of Ukraine. A western coalition led by NATO unified to provide military aid to Kyiv and formally condemned Moscow at the UN. In contrast, Russia's strategic allies—including China, Iran, and Belarus—absorbed western pressure and continued to supply Russia with critical economic and military support. Other international issues can induce aligned policy responses. Examples include coordinated disaster relief (e.g., the 2023 Turkey–Syria earthquake) or localized peacekeeping efforts (e.g., anti-piracy operations in the Gulf of Aden).

2.4.1 Policy Divergence and Convergence

In our model, equilibrium geopolitical actions in Proposition 1 depend on bilateral influence (in the influence stage) and global coordination (in the coordination stage), as determined by the domestic interests for a specific international issue ($\boldsymbol{\theta}$) and the geopolitical network ($\boldsymbol{G}$). We now formally investigate the underlying characteristics of domestic interests ($\boldsymbol{\theta}$) that determine whether there is policy divergence or convergence across countries. We use a spectral decomposition of the geopolitical network $\boldsymbol{G}$ to provide this characterization:

$$\boldsymbol{G} = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^{\top} \qquad (\text{equivalently, } \boldsymbol{G} = \sum_{k=1}^{N} \lambda_k \boldsymbol{u}_k \boldsymbol{u}_k^{\top}). \tag{17}$$

where $\boldsymbol{\Lambda}$ is the diagonal matrix of $\boldsymbol{G}$'s eigenvalues $\lambda_k \in \mathbb{R}$, and each column of $\boldsymbol{U}$ is an associated eigenvector $\boldsymbol{u}_k$. Since $\boldsymbol{G}$ is real and symmetric, this spectral decomposition necessarily exists. Furthermore, the symmetry of $\boldsymbol{G}$ implies that the vectors are orthonormal, $\boldsymbol{U}^{-1} = \boldsymbol{U}^\top$, and all eigenvalues are real. We rank these eigenvalues in decreasing order of absolute value.

We next note that any vector of domestic interests ($\boldsymbol{\theta}$) can be projected onto $\boldsymbol{G}$'s eigenbasis:

$$\boldsymbol{\theta} = \sum_{k=1}^{N} \widetilde{\theta}_k \boldsymbol{u}_k, \qquad \widetilde{\boldsymbol{\theta}} \equiv \boldsymbol{U}^\top \boldsymbol{\theta}. \tag{18}$$

That is, any vector $\boldsymbol{\theta}$ of domestic interests across countries can be written as a linear combination of $\boldsymbol{G}$'s eigenvectors $\{\boldsymbol{u}_k\}$, where $\widetilde{\theta}_k$ are the coordinates after the projection.

Each eigenvector captures a particular axis of polarization, which can be interpreted as a latent geopolitical cleavage—a specific arrangement of domestic interests across countries. We now show that each latent geopolitical cleavage translates into a natural notion of disagreements among network neighbors. We define the following network disagreement measure:

$$\mathcal{S}(\theta) \equiv \frac{1}{2} \sum_{i,j} G_{ij} (\theta_i - \theta_j)^2,$$

which captures how dissimilar domestic interests are averaged across network neighbors.

Proposition 3. *The network disagreement of an eigenvector $\boldsymbol{u}_k$ is $\mathcal{S}(\boldsymbol{u}_k) = 1 - \alpha - \lambda_k$. Moreover,*

$$\boldsymbol{u}_1 = \arg\max_{\|\boldsymbol{\theta}\|=1} \mathcal{S}(\boldsymbol{\theta}) \propto \mathbf{1}, \qquad \lambda_1 = 1 - \alpha.$$

$$\boldsymbol{u}_k = \arg\max_{\boldsymbol{\theta} \perp \{\boldsymbol{u}_j\}_{j<k}, \|\boldsymbol{\theta}\|=1} \mathcal{S}(\boldsymbol{\theta}) \text{ for all } k > 1.$$

We use Proposition 3 to characterize the extent of policy divergence or convergence given domestic interests ($\boldsymbol{\theta}$). First, the network disagreement of an eigenvector $\boldsymbol{u}_k$ is directly and negatively tied to its associated eigenvalue λ_k. Second, the eigenvector with the largest eigenvalue is a constant vector, and each subsequent eigenvector $\boldsymbol{u}_k$ is the vector of domestic interests, among those orthogonal to all prior eigenvectors, that minimizes network disagreement.

Third, because the eigenvectors are pairwise orthogonal, each eigenvector $\boldsymbol{u}_k$ averages to zero across countries for all $k > 1$ (since $\mathbf{1}'\boldsymbol{u}_k = 0$). Therefore, each eigenvector captures a geopolitical cleavage—a particular configuration of disagreements in domestic interests across countries. Those with positive entries in $\boldsymbol{u}_k$ are on one side of the cleavage while those with negative entries are on the other side of the cleavage.

Fourth, for all $k > 1$, Proposition 3 shows that the geopolitical cleavages with large eigenvalues capture international issues where neighbors in tightly-connected geopolitical blocs (with high G_{ij}) tend to have similar domestic interests, whereas the disagreements across blocs (with low G_{ij}) can

be large. Conversely, the cleavages with small or negative eigenvalues capture issues where neighbors in tightly-connected geopolitical blocs (with high G_{ij}) tend to have dissimilar domestic interests, whereas the disagreements across blocs (with low G_{ij}) can be small.

We now show that geopolitical cleavages with large eigenvalues tend to translate into divergent policy outcomes, whereas those with small eigenvalues tend to lead to policy convergence. We use the result from Proposition 1 that the equilibrium geopolitical action profile $\boldsymbol{a}$ can be characterized as a linear transformation of the domestic interests $\boldsymbol{\theta}$: $\boldsymbol{a} = \mathcal{L}(\boldsymbol{G})\boldsymbol{\theta}$, where $\mathcal{L}(\boldsymbol{G}) \equiv \left(\kappa(\boldsymbol{I}-\boldsymbol{G})^2 - \alpha\boldsymbol{I}\right)^{-1}(\boldsymbol{I}-\boldsymbol{G})(\kappa\alpha-1)$.

Proposition 4. *The operator $\mathcal{L}(\boldsymbol{G})$, mapping from domestic interests to geopolitical actions, is a spectral function of $\boldsymbol{G}$: it shares the same eigenbasis as $\boldsymbol{G}$. This implies spectral decoupling: when domestic interests are decomposed along $\boldsymbol{G}$'s eigenvectors, the equilibrium mapping scales each dimension of cleavages independently without spilling over into others:*

$$\boldsymbol{a} = \sum_{\ell} L(\lambda_\ell)\,\widetilde{\theta}_\ell \boldsymbol{u}_\ell, \qquad L(\lambda) \equiv \frac{(1-\lambda)(\kappa\alpha-1)}{\kappa(1-\lambda)^2 - \alpha};$$

or, equivalently, projecting the equilibrium action profile onto $\boldsymbol{G}$'s eigenbasis $\widetilde{\boldsymbol{a}} \equiv \boldsymbol{U}^\top \boldsymbol{a}$, we have:

$$\widetilde{a}_\ell = \sum_{\ell} L(\lambda_\ell)\,\widetilde{\theta}_\ell.$$

The scaling factor $L(\cdot)$ is non-negative, bounded above by one ($0 \leq L(\cdot) \leq 1$), and is strictly increasing in the eigenvalue λ_ℓ ($L'(\cdot) > 0$), implying that eigenvectors with larger eigenvalues translate into larger differences in geopolitical actions. Finally, the scaling factor $L(\cdot)$ is increasing in κ.

Proposition 4 establishes that the complex interplay of strategic influence and network coordination resolves into a tractable linear operator $\mathcal{L}(\boldsymbol{G})$ that shares the eigenbasis of $\boldsymbol{G}$ and is diagonalizable by the same latent cleavages. A key implication of this result is that equilibrium geopolitical actions exhibit a spectral decoupling along the network's axes of geopolitical cleavage. When domestic interests are decomposed along these eigenvectors, the equilibrium mapping scales each cleavage independently, without spilling over to the other cleavages.

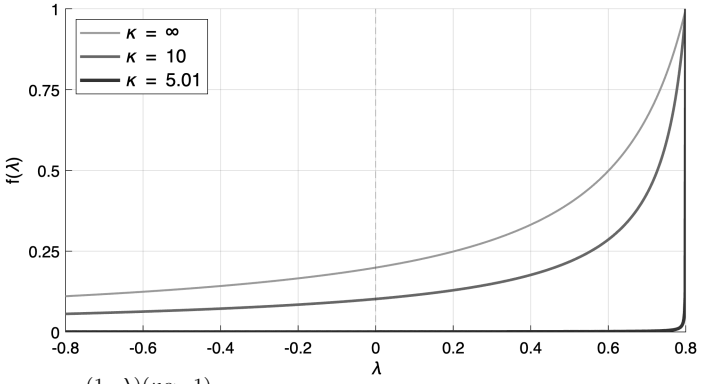
Proposition 4 also shows that the scaling factor, which translates domestic interests into geopolitical actions, depends only on the eigenvalue of the geopolitical cleavage. International disagreements associated with large eigenvalues (where disagreements are small between geopolitically-close countries and large between geopolitically-distant countries) tend to translate into large differences in geopolitical actions. This divergence is driven by a geopolitical echo-chamber effect: because the coordination game incentivizes alignment with immediate neighbors, connected countries mutually reinforce each other's stances, amplifying cross-bloc frictions into stark policy polarization. Conversely, disagreements associated with small eigenvalues (where disagreements are large

between geopolitically-close countries and small between geopolitically-distant countries) are neutralized by local coordination incentives, leading to convergent policy outcomes.

Finally, Proposition 4 also demonstrates that a lower cost of influence systematically lowers polarization in equilibrium geopolitical actions. As strategic influence becomes cheaper, the resulting web of bilateral pressure pushes countries further away from their idiosyncratic domestic interests and toward the network consensus.

In Figure 1, we illustrate these results numerically, using the property that the scaling factor from domestic interests to geopolitical actions ($L(\lambda)$) depends only on the network row sum (α) and influence costs (κ). We plot the value of this scaling factor as a function of the geopolitical cleavage's eigenvalue (λ) for $\alpha = 0.2$ and three different values of κ. As $\kappa \to \infty$, $L(\lambda) = \frac{\alpha}{1-\lambda}$, which is the scaling factor of the beauty-contest game without strategic influence: larger eigenvalues map into larger policy divergence, but all cleavages contribute. As κ decreases, geopolitical cleavages associated with small eigenvalues become disproportionately attenuated in their contribution to geopolitical actions, while those with large eigenvalues remain largely intact. As κ continues to decline towards $1/\alpha$, all cleavages but the Perron vector (i.e., the eigenvector with eigenvalue $1-\alpha$) cease to matter: $L(\lambda) \to 0$ for every $\lambda < 1-\alpha$, while $L(1-\alpha) \to 1$. At the lower limit of influence costs ($\kappa \downarrow 1/\alpha$), only the network-wide consensus survives, and equilibrium actions collapse to $\bar{\theta}\mathbf{1}$.

Figure 1: Scaling Factor ($L(\lambda)$) Mapping Eigenvalues (λ) to Geopolitical Actions ($\boldsymbol{a}$)



Note: the scaling factor ($L(\lambda) \equiv \frac{(1-\lambda)(\kappa\alpha-1)}{\kappa(1-\lambda)^2-\alpha}$) that maps the eigenvalues of a geopolitical cleavage (λ) to geopolitical actions ($\boldsymbol{a}$) for a network row sum of $\alpha = 0.2$ and three values of influence costs: $\kappa = 5.01$, $\kappa = 10$, and $\kappa = \infty$.

2.4.2 Strategic Influence and Domestic Interests

Equilibrium geopolitical actions ($\boldsymbol{a}$) in the previous subsection are shaped by the strategic influence ($\boldsymbol{\Omega}(\boldsymbol{\theta})$) induced by domestic interests ($\boldsymbol{\theta}$) given the geopolitical network ($\boldsymbol{G}$). To systematically characterize the role of strategic influence, we introduce a *contestability index* for each country i, which measures the degree of misalignment in the foreign influence directed at that country. Formally, we define the contestability index ($\chi_i(\boldsymbol{\theta}) \in [0,1]$) for each country i as one minus the cosine

similarity between the foreign influence that i receives and a constant vector of ones ($\mathbf{1}$):

$$\chi_i(\boldsymbol{\theta}) \equiv= 1 - \frac{\sum_{m \neq i} \omega_{im}(\boldsymbol{\theta})}{\sqrt{(N-1)\sum_{m \neq i} \omega_{im}(\boldsymbol{\theta})^2}},$$

where we leave out self influence to focus on contestability from foreign influence. The cosine similarity with $\mathbf{1}$ measures how close foreign influence is to being uniform, and hence one minus cosine similarity captures the degree of misalignment in foreign influence.

At the lower bound of the contestability index ($\chi_i = 0$), foreign influence is perfectly monolithic: every foreign nation pushes country i's policy by the same amount in a unified direction. Therefore, a low value for the contestability index (χ_i) indicates a strong consensus of foreign pressure, which pushes country i's action (a_i) in that unified direction. In contrast, at the upper bound of the contestability index ($\chi_i = 1$), the net influence on country i sums exactly to zero despite active foreign influence. Hence, a high value for the contestability index (χ_i) reflects a geopolitical tug-of-war, in which country i is a highly-contested swing state or a battleground state for foreign influence. In this case, foreign actors exert competing, contradictory pressures, which partially cancel each other out, and hence have less effect on country i's action (a_i).

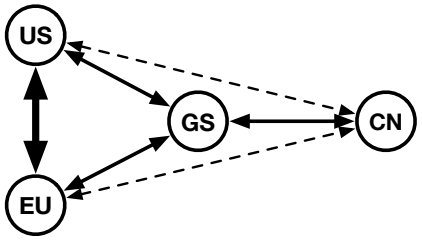
2.4.3 Example Network

To illustrate these analytical results and provide further intuition, we consider an example network ($\boldsymbol{G}$) of four countries, as illustrated in the top panel of Figure 2. Countries 1 and 2 (we refer to them as the US and the EU respectively) are strongly connected; country 3 is equally connected to everyone (the Global South); and country 4 (China) is only weakly connected to countries 1 and 2.

We report the assumed strength of the network connections (G_{ij}) and the implied eigenvalues (u_ℓ) and eigenvectors (λ_ℓ) in the lower panel of Figure 2. The first eigenvector $\boldsymbol{u}_1$ is a constant vector. The second eigenvector $\boldsymbol{u}_2$ captures a primary geopolitical cleavage: international issues where the US and the EU take one side; China takes the polar opposite; and the Global South occupies the middle. When domestic interests align with this axis, the Global South's contestability index reaches its upper bound ($\chi = 1$). Because the Western bloc and China attempt to pull the Global South in exactly opposite directions, their competing strategic influence cancel out, thereby rendering it a highly-contestable swing state for such international issues.

The third eigenvector $\boldsymbol{u}_3$ captures a different structural divide—one where China aligns with the US and the EU, but all three disagree with the Global South. In this scenario, the Global South's contestability index drops to its lower bound ($\chi = 0$). Rather than facing a tug-of-war, the Global South is subjected to monolithic foreign pressure, as all major global powers exert influence to shift its geopolitical action in the exact same direction.

Figure 2: Example of a Geopolitical Network



$$\boldsymbol{G} = \begin{bmatrix} 0 & .5 & .3 & .1 \\ .5 & 0 & .3 & .1 \\ .3 & .3 & 0 & .3 \\ .1 & .1 & .3 & .4 \end{bmatrix} \qquad \begin{array}{ll} \boldsymbol{u}_\ell \propto & \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 1 \\ 0 \\ -2 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 1 \\ -3 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} \\ \lambda_\ell = & 0.9, \qquad 0.3, \qquad -0.3, \qquad -0.5 \end{array}$$

Note: Countries 1 and 2 (the U.S. and the E.U., respectively) are strongly connected; country 3 is equally connected to everyone (the Global South (GS)); and country 4 (China (CN)) is only weakly connected to countries 1 and 2. Top panel shows a network graph with thick solid arrows denoting the strongest connections; medium solid arrows denoting intermediate connections; and medium dashed arrows showing the weakest connections. Lower panel shows the network matrix $\boldsymbol{G}$, with elements G_{ij} denoting the strength of network connections between countries i and j, and the implied eigenvalues (u_ℓ) and eigenvectors (λ_ℓ).

Finally, the eigenvector associated with the smallest eigenvalue captures localized disagreements, such as a sharp policy fracture strictly between the US and the EU. Because the US and the EU exert immense pressure to pull the rest of the network in diametrically opposite directions, their efforts effectively neutralize each other abroad. As a result, both China and the Global South absorb perfectly offsetting foreign influence, pushing their contestability indices to exactly one ($\chi = 1$), and turning the rest of the globe into a battleground for an intra-Western schism.

2.5 Stochastic International Issues and Identification

We have so far focused on geopolitical actions in response to a single international issue, treating the vector of domestic interests for that international issue ($\boldsymbol{\theta}$) as fixed. In this section, we extend our theoretical analysis to a setting in which international issues arrive stochastically, and the domestic interests for each international issue ($\boldsymbol{\theta}$) are drawn from a fixed distribution of geopolitical shocks. We interpret these stochastic draws for domestic interests as capturing the shifting landscape of international crises, such as the Berlin Airlift, the Cuban Missile Crisis, the U.S. proposal to purchase Greenland, or Chinese military exercises in the Taiwan Strait. We use this stochastic specification to provide conditions under which the unobserved geopolitical network ($\boldsymbol{G}$) can be identified from observed geopolitical actions ($\boldsymbol{a}$) and domestic interests ($\boldsymbol{\theta}$).

To provide a sharp analytical characterization of geopolitical actions with stochastic international issues, we make the following two assumptions. First, we are concerned with the cross-country variance and covariance of domestic interests across diverse international issues, which de-

pends on relative domestic interests. Therefore, we normalize the expected value of domestic interests to zero ($\mathbb{E}[\boldsymbol{\theta}] = \mathbf{0}$), without loss of generality. Second, we assume that domestic interests ($\boldsymbol{\theta}$) and the geopolitical network ($\boldsymbol{G}$) inhabit the same underlying space of geopolitical issues, in the sense that the cross-country variance-covariance matrix of domestic interests ($\boldsymbol{\theta}$) shares the same eigenbasis as the geopolitical network ($\boldsymbol{G}$).

Assumption 1. ***(Graph Stationarity)*** *$\boldsymbol{\Sigma}_\theta$ is a spectral function of $\boldsymbol{G}$: it shares the same eigenbasis as $\boldsymbol{G}$.*

Assumption 1 implies that the same eigenvectors that describe the geometry of $\boldsymbol{G}$—the geopolitical cleavages—are the exact same axes along which domestic interests naturally covary across countries. It implies that $\widetilde{\boldsymbol{\Sigma}}_\theta \equiv \boldsymbol{U}^\top \boldsymbol{\Sigma}_\theta \boldsymbol{U}$ is a diagonal matrix:

$$\widetilde{\boldsymbol{\Sigma}}_\theta \equiv \boldsymbol{U}^\top \boldsymbol{\Sigma}_\theta \boldsymbol{U} = \begin{bmatrix} \sigma_1^2 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_N^2 \end{bmatrix}. \tag{19}$$

To interpret this assumption, note that the vector of domestic interests associated with each international issue can be written as a linear combination over latent cleavages:

$$\boldsymbol{\theta} = \widetilde{\theta}_1 \boldsymbol{u}_1 + \widetilde{\theta}_2 \boldsymbol{u}_2 + \cdots \widetilde{\theta}_N \boldsymbol{u}_N. \tag{20}$$

Therefore, Assumption 1 implies that the coordinates of $\boldsymbol{\theta}$ are in the eigenbasis of $\boldsymbol{G}$, such that the weights on the eigencomponents ($\widetilde{\theta}_\ell$) are drawn independently with variance σ_ℓ^2 for each eigencomponent ℓ. Intuitively, we assume that the set of latent issues that determine the strength of geopolitical connections between countries are the same as those that generate differences in domestic interests across countries.[4] We provide evidence on the extent to which this assumption provides a good approximation to the data in empirical specification checks below.

Combining Assumption 1 with the mapping from domestic interests to geopolitical actions in Proposition 4, we obtain the following result that the cross-country covariance matrix of geopolitical actions is also a spectral function of $\boldsymbol{G}$ and shares its eigenbasis.

Lemma 2. *Under Assumption 1, the cross-country covariance matrix of geopolitical actions, $\boldsymbol{\Sigma}_a$, is also a spectral function of $\boldsymbol{G}$ and shares its eigenbasis.*

Proof. Recall that $\boldsymbol{a} = \mathcal{L}(\boldsymbol{G})\boldsymbol{\theta}$. By Proposition 2, $\mathbb{E}[\boldsymbol{\theta}] = \mathbf{0}$ implies that $\mathbb{E}[\boldsymbol{a}] = \mathbf{0}$. The covariance matrix of geopolitical actions is thus

$$\boldsymbol{\Sigma}_a \equiv Cov[\boldsymbol{a}] = \mathbb{E}\left[\boldsymbol{a}\boldsymbol{a}^\top\right] = \mathcal{L}(\boldsymbol{G})\,\mathbb{E}\left[\boldsymbol{\theta}\boldsymbol{\theta}^\top\right]\mathcal{L}(\boldsymbol{G})^\top = \mathcal{L}(\boldsymbol{G})\,\boldsymbol{\Sigma}_\theta \mathcal{L}(\boldsymbol{G})^\top.$$

The result follows immediately from Assumption 1, as $\mathcal{L}(\boldsymbol{G})$ and $\boldsymbol{\Sigma}_\theta$ are both strictly diagonalizable by the shared eigenbasis $\boldsymbol{U}$ of $\boldsymbol{G}$. □

[4] An analogous assumption is common in network research in signal processing (e.g., Marques et al. 2017).

We now use these properties that the covariance matrixes of domestic interests ($\boldsymbol{\Sigma}_\theta$) and geopolitical actions ($\boldsymbol{\Sigma}_a$) have the same eigenbasis as the geopolitical network ($\boldsymbol{G}$) to identify the eigenvectors (u_ℓ) and eigenvalues (λ_ℓ) of this network, which correspond to underlying geopolitical cleavages, and from which we can recover the entire geopolitical network.

Proposition 5. ***(Identification)*** *Let $\widetilde{\boldsymbol{\Sigma}}_a \equiv \boldsymbol{U}^\top \boldsymbol{\Sigma}_a \boldsymbol{U}$, $\widetilde{\boldsymbol{\Sigma}}_\theta \equiv \boldsymbol{U}^\top \boldsymbol{\Sigma}_\theta \boldsymbol{U}$ be the projection of $\boldsymbol{\Sigma}_a$ and $\boldsymbol{\Sigma}_\theta$ onto the eigenbasis $\boldsymbol{U}$. The eigenvalues (λ_ℓ) and eigenvectors ($\boldsymbol{u}_\ell$) of $\boldsymbol{G}$ are identified if the ℓ-th diagonal element of $\widetilde{\boldsymbol{\Sigma}}_\theta$ is non-zero, with*

$$\lambda_\ell = L^{-1}\left(\sqrt{\left(\widetilde{\boldsymbol{\Sigma}}_a\right)_{\ell\ell} \Big/ \sigma_\ell^2}\right), \tag{21}$$

where $L(\cdot)$ is defined in Proposition 4; σ_ℓ^2 is the ℓ-th diagonal entry of $\widetilde{\boldsymbol{\Sigma}}_\theta$; and we recover $\boldsymbol{u}_\ell$ from equation (19) using this solution for σ_ℓ^2.

In this identification result, we use the property from Proposition 4 that the scaling function ($L(\lambda_\ell)$) mapping domestic interests to geopolitical actions is strictly monotonic in the eigenvalues (λ_ℓ), which implies that it can be inverted. Given the observed covariance matrices of geopolitical actions ($\boldsymbol{\Sigma}_a$) and domestic interests ($\boldsymbol{\Sigma}_\theta$), we can use Proposition 4 to identify the eigenvectors (u_ℓ) and eigenvalues (λ_ℓ) for each geopolitical cleavage ℓ, as long as there is variation in domestic interests along that geopolitical cleavage ($\sigma_\ell^2 > 0$). If instead $\sigma_\ell^2 = 0$, the global network never experiences a shock for which domestic interests lie along that specific cleavage, rendering the data uninformative about network behavior in that direction. Consequently, $\boldsymbol{G}$ is only identified on the subspace spanned by the eigenvectors for which domestic interests actually vary in the data. On the orthogonal complement, λ_ℓ remains unrestricted by $\boldsymbol{\Sigma}_\theta$ and $\boldsymbol{\Sigma}_a$.

Intuitively, graph stationarity in Assumption 1 implies that the observed cross-country covariance matrix for domestic interests contains information about cleavages in the geopolitical network. Additionally, Proposition 4 maps the cross-country covariance matrix for geopolitical actions to the same cleavages in the geopolitical network. Within the structure of the model, these two cross-country covariance matrices contain sufficient information to identify the eigenvectors and eigenvalues for the direction and strength of the geopolitical cleavages of the network, as long as there is variation in domestic interests along each of these cleavages.

From these N eigenvectors and eigenvalues, we can reconstruct the implied unobserved geopolitical network:

$$\boldsymbol{G} = \sum_\ell \lambda_\ell \boldsymbol{u}_\ell \boldsymbol{u}_\ell^\top,$$

where we only sum over the geopolitical cleavages that can be identified from the observed variation in the data. As a result of the property of spectral decoupling, the unobserved dimensions of $\boldsymbol{G}$ represent latent geopolitical fault lines that are never activated by historical shocks, and hence never captured in the observed variation in domestic interests and geopolitical actions. Because these silent

dimensions do not interact with or spill over into the observable dimensions, it is without loss of generality to omit them from our subsequent empirical analysis.

2.6 *Ex Ante* Welfare

With stochastic international issues, we measure each country's *ex ante* welfare as its expected payoff ($\mathbb{E}_{\boldsymbol{\theta}}[U_i]$) over the distribution of the draws of domestic interests (θ). We now show that the *ex ante* welfare of each country depends on the network structure as well as the set of international issues that arise. We express each country i's *ex ante* welfare as a function of the eigenvectors and eigenvalues of $\boldsymbol{G}$, as well as of the matrix $\boldsymbol{\Sigma}_\theta$ capturing the covariance of domestic interests.

Proposition 6. ***(Ex Ante Welfare)*** *Under Assumption 1, ex ante welfare for country i can be written as:*

$$\mathbb{E}_\theta[U_i] = \sum_\ell \sigma_\ell^2 \times \Gamma_{i\ell}\left(\{u_k\}, \{\lambda_k\}\right),$$

where $\Gamma_{i\ell}\left(\{u_k\}, \{\lambda_k\}\right)$ is country i's payoff along geopolitical cleavage ℓ, defined as:

$$\begin{aligned}\Gamma_{im} &= \left(L(\lambda_m) - 1\right)\left[2\frac{\alpha^2 - (1-\lambda_m)^2}{\kappa(1-\lambda_m)^2 - \alpha} - \alpha\left(L(\lambda_m) - 1\right)\right]u_{im}^2 \\ &\quad - L(\lambda_m)^2 \sum_q C_{mmq}\left(1 - \alpha - 2\lambda_m + \lambda_q\right)u_{iq} \\ &\quad - \frac{1}{\kappa}\sum_p\left(\sum_q C_{mpq}\left[\frac{L(\lambda_m)(1-\lambda_q)}{1-\lambda_p} - 1\right]u_{iq}\right)^2,\end{aligned}$$

where $L(\cdot)$ is as defined in Proposition 4, and $C_{mpq} \equiv \sum_{k=1}^N u_{km}u_{kp}u_{kq} = \langle \boldsymbol{u}_m \circ \boldsymbol{u}_p, \boldsymbol{u}_q\rangle$ is the spectral interaction tensor that captures how eigenmodes interact through the nonlinearity of the country-level products of the eigenvectors $u_{km}u_{kp}u_{kq}$.

Proposition 6 shows that the *ex ante* welfare of country i depends on the network structure (as encoded in the eigenvectors $\{u_k\}$ and eigenvalues $\{\lambda_k\}$ of $\boldsymbol{G}$), the equilibrium interactions between countries through coordination and influence (as captured by the function $\Gamma_{i\ell}(\cdot)$), and the distribution of international issues (as captured by the variance σ_ℓ^2 with which each latent cleavage is drawn).

Ex ante welfare is additive across each of the geopolitical cleavages, because of spectral decoupling. Each of these geopolitical cleavages ℓ captures a latent axis of disagreement among countries. The function $\Gamma_{i\ell}$ measures country i's equilibrium payoff exposure to cleavage ℓ—how much country i stands to gain or lose when an international issue loads on that cleavage. This exposure depends on i's position in the network: a country can be highly exposed to one cleavage but insulated from

another. For instance, the United States may be centrally positioned along the axis of US-Soviet rivalry, but peripheral to a localized conflict between India and Pakistan. How influential a country is for other nations' policy choices, how contested its policy choices are through strategic influence, and how much policy coordination it engages in are all contingent on the specific cleavage.

The variance σ_ℓ^2 determines how prominently geopolitical cleavage ℓ features in the distribution of international issues. A large variance σ_ℓ^2 implies that international issues are more likely to load on cleavage ℓ, increasing its weight in every country's *ex ante* welfare. The additive structure for *ex ante* welfare implies that one can evaluate the welfare consequences of each cleavage independently, and then aggregate across the full spectrum of cleavages.

2.7 Introducing Network Asymmetry

For expositional simplicity, we have so far assumed that the geopolitical network is symmetric. We now relax this assumption to allow the geopolitical network to be directed ($G_{ij} \neq G_{ji}$), such that the effect of country i on policy in country j can differ from that of country j on policy in country i. We thus incorporate asymmetries in the geopolitical network, reflecting differences in economic dependence, military capacity, or diplomatic reach.

We incorporate these network asymmetries by allowing for a geopolitical shifter for each country (π_i) that captures its structural importance in the network. Formally, let $\boldsymbol{H}$ be a symmetric, nonnegative matrix with the Perron (i.e., largest) eigenvalue $1-\alpha$ and spectral decomposition $\boldsymbol{H} = \sum_m \lambda_m \boldsymbol{v}_m \boldsymbol{v}_m^\top$, where $\{\boldsymbol{v}_m\}$ are orthonormal eigenvectors. Let $\boldsymbol{\pi}$ denote the Perron eigenvector of $\boldsymbol{H}$, normalized so that $\boldsymbol{\pi} > \mathbf{0}$ and $\|\boldsymbol{\pi}\| = 1$, and let $\boldsymbol{\Pi} = \mathrm{Diag}(\boldsymbol{\pi})$. The matrix $\boldsymbol{H}$ does not have constant row sums in general: the row sums $\sum_{j=1}^N H_{ij}$ vary across countries, reflecting heterogeneous latent geopolitical connectivity. We construct the directed network as:

$$\boldsymbol{G} = \boldsymbol{\Pi}^{-1} \boldsymbol{H} \boldsymbol{\Pi}, \tag{22}$$

so that $G_{ij} = (\pi_j/\pi_i)\, H_{ij}$. That is, country i is connected geopolitically to country j in proportion to j's structural importance (π_j) relative to its own structural importance (π_i). In other words, countries that are structurally important have a disproportionate impact on their neighbors, while being less sensitive to any single neighbor themselves.

The Perron condition $\boldsymbol{H}\boldsymbol{\pi} = (1-\alpha)\,\boldsymbol{\pi}$ ensures that $\boldsymbol{G}$ has constant row sums $1-\alpha$, even though $\boldsymbol{H}$ does not: $\boldsymbol{G}\mathbf{1} = \boldsymbol{\Pi}^{-1}\boldsymbol{H}\boldsymbol{\pi} = (1-\alpha)\,\mathbf{1}$. Since $\boldsymbol{G}$ and $\boldsymbol{H}$ are related by a similarity transform, they share the same eigenvalues $\{\lambda_m\}$. The right eigenvectors of $\boldsymbol{G}$ are $\boldsymbol{u}_m^R = \boldsymbol{\Pi}^{-1}\boldsymbol{v}_m$, and the left eigenvectors are $\boldsymbol{u}_m^L = \boldsymbol{\Pi}\boldsymbol{v}_m$, which satisfies biorthonormality: $\left(\boldsymbol{u}_m^L\right)^\top \boldsymbol{u}_p^R = \mathbf{1}_{m=p}$.

We scale the costs of strategic influence by structural importance, such that the cost of country i exerting influence on country k is given by $\kappa\,(\pi_k^2/\pi_i^2)\,\omega_{ki}^2$. We thus capture the idea that influencing a

structurally important country (large π_k) is expensive, because its actions are anchored by dense network connections. Additionally, projecting influence from a peripheral country (small π_i) is costly, reflecting its limited diplomatic and economic reach.

Combining these assumptions, country i's payoff is now given by:

$$U_i(\boldsymbol{a}, \boldsymbol{\Omega}) = -\left[\alpha(a_i - \theta_i)^2 + \sum_j G_{ij}(a_i - a_j)^2\right] - \kappa \sum_k \left(\frac{\pi_k \omega_{ki}}{\pi_i}\right)^2 + 2\sum_m \omega_{im}(a_i - \theta_i),$$

where a symmetric network is the special case in which $\boldsymbol{H}$ has constant row sums, which implies that the Perron vector is constant and $\pi_i = \pi_k$ for all i, k.

We now show that our main analytical results for the properties of the policy coordination and strategic influence game extend to this case of asymmetric networks.

Proposition 7. *Suppose* $\boldsymbol{G} = \boldsymbol{\Pi}^{-1}\boldsymbol{H}\boldsymbol{\Pi}$ *with influence costs* $\kappa(\pi_k^2/\pi_i^2)\omega_{ki}^2$. *Then:*

1. Propositions 1 and 2 continue to hold. In particular:

   (a) Equilibrium net influence and geopolitical actions satisfy:

   $$\boldsymbol{\Omega 1} = -\left(\kappa(\boldsymbol{I} - \boldsymbol{G})^2 - \alpha \boldsymbol{I}\right)^{-1}\left((\boldsymbol{I} - \boldsymbol{G})^2 - \alpha^2 \boldsymbol{I}\right)\boldsymbol{\theta},$$

   $$\boldsymbol{a} = \left(\kappa(\boldsymbol{I} - \boldsymbol{G})^2 - \alpha \boldsymbol{I}\right)^{-1}(\boldsymbol{I} - \boldsymbol{G})(\kappa\alpha - 1)\boldsymbol{\theta}.$$

   (b) The equilibrium inherits uniqueness, the convex combination property, $\boldsymbol{\Pi}^2$-weighted conservativeness ($\sum_i \pi_i^2 a_i = \sum_i \pi_i^2 \theta_i$ and $\sum_i \pi_i^2 \sum_m \omega_{im} = 0$), and the two limiting benchmarks ($\kappa \to \infty$ and $\kappa \downarrow 1/\alpha$) from the symmetric baseline.

2. The equilibrium continues to admit a spectral representation:

   $$a_i = \sum_m L(\lambda_m)\left[\boldsymbol{u}_m^R\right]_i \left(\boldsymbol{u}_m^L\right)^\top \boldsymbol{\theta}, \tag{23}$$

   where the scaling function $L(\lambda) \equiv (1 - \lambda)(\kappa\alpha - 1) / \left(\kappa(1 - \lambda)^2 - \alpha\right)$ is identical to that under a symmetric network.

Proposition 7 establishes that the equilibrium vectors of net influence and geopolitical actions satisfy the same relationships as for a symmetric network in Proposition 1. The properties of the equilibrium are also similar to those with a symmetric network in Proposition 2. Finally, the spectral decomposition of geopolitical actions in terms of domestic interests in Proposition 4 extends to the asymmetric network by working in $\boldsymbol{\Pi}$-weighted coordinates. Specifically, we define adjusted domestic interests

and geopolitical actions as $\tilde{\theta}_m \equiv \boldsymbol{v}_m^\top \boldsymbol{\Pi}\boldsymbol{\theta}$ and $\tilde{a}_m \equiv \boldsymbol{v}_m^\top \boldsymbol{\Pi}\boldsymbol{a}$, with corresponding vector representations of $\tilde{\boldsymbol{\theta}} = \boldsymbol{V}^\top \boldsymbol{\Pi}\boldsymbol{\theta}$ and $\tilde{\boldsymbol{a}} = \boldsymbol{V}^\top \boldsymbol{\Pi}\boldsymbol{a}$. These adjusted variables with a tilde are projections of domestic interests and equilibrium actions onto the $\boldsymbol{\Pi}$-weighted eigenbasis of $\boldsymbol{H}$.

The equilibrium using these projected coordinates is structurally identical to that for a symmetric network, with $\boldsymbol{H}$ replacing $\boldsymbol{G}$. The closed-form expressions, the net influence equation, and the spectral scaling function $L(\lambda)$ all carry over unchanged. For each eigenmode, we have:

$$\tilde{a}_m = L(\lambda_m)\,\tilde{\theta}_m, \qquad L(\lambda) \equiv \frac{(1-\lambda)(\kappa\alpha - 1)}{\kappa(1-\lambda)^2 - \alpha}, \tag{24}$$

In the original coordinates, the equilibrium action of country i is:

$$a_i = \sum_m L(\lambda_m) \left[\boldsymbol{u}_m^R\right]_i \left(\boldsymbol{u}_m^L\right)^\top \boldsymbol{\theta}. \tag{25}$$

Note that the scaling function $L(\cdot)$ is identical to the symmetric network case. Therefore, all the properties of the scaling function (non-negativity, boundedness, monotonicity in λ and κ, the two limiting benchmarks) carry over to the asymmetric network case. What changes is the geometry of geopolitical cleavages. In scalar form, the equilibrium action of country i can be written as:

$$a_i = \sum_m L(\lambda_m) \frac{[\boldsymbol{v}_m]_i}{\pi_i} \sum_J [\boldsymbol{v}_m]_j \pi_j \theta_j.$$

With a symmetric network, the loading of country i's action on geopolitical cleavage m is $[\boldsymbol{v}_m]_i$, and the importance of country j's domestic interest in cleavage m is also $[\boldsymbol{v}_m]_j$. In contrast, with the asymmetric network, the loading of country i's action becomes $\left[\boldsymbol{u}_m^R\right]_i = v_{mi}/\pi_i$, and the importance of country j's domestic interest becomes $v_{mj}\pi_j$. As a result, structurally-important countries (large π_i) have attenuated action loadings (v_{mi}/π_i) and magnified interest weights ($v_{mi}\pi_i$) along every geopolitical cleavage. For these structurally-important countries, their own actions are anchored by dense network connections, while their domestic interests carry disproportionate weight in shaping other countries' policies. Peripheral countries display the reverse pattern—amplified action loadings but dampened interest weights—making them highly responsive to each geopolitical issue yet less consequential in determining others' actions.

We next extend our network identification result to this case of an asymmetric network, showing that we can recover the spectral properties of the network from the observed co-variance matrices for domestic interests ($\boldsymbol{\Sigma_\theta}$) and geopolitical actions ($\boldsymbol{\Sigma_a}$). We again assume graph stationarity, such that the set of latent issues that determine the strength of geopolitical connections between countries are the same as those that generate differences in domestic interests across countries.

Assumption 2. ***(Graph Stationarity for Directed Networks)*** *The $\boldsymbol{\Pi}$-weighted covariance matrix $\widetilde{\boldsymbol{\Sigma}}_\theta \equiv \boldsymbol{\Pi}\boldsymbol{\Sigma}_\theta\boldsymbol{\Pi}$ shares the eigenbasis of $\boldsymbol{H}$: $\widetilde{\boldsymbol{\Sigma}}_\theta = \sum_m \sigma_m^2 \boldsymbol{v}_m \boldsymbol{v}_m^\top$.*

In the special case in which the network is symmetric ($\boldsymbol{G} = \boldsymbol{H}$, $\Pi \propto I$), Assumption 2 reduces to Assumption 1. More generally, when the network is asymmetric, the Π-weighting captures the fact that structurally-important countries can exhibit systematically larger variation in domestic interests, reflecting their dense connections to the geopolitical network. Under Assumption 2, the cross-country covariance matrices for domestic interests ($\boldsymbol{\Sigma_\theta}$) and geopolitical actions ($\boldsymbol{\Sigma_a}$) in their original coordinates are given by:

$$\boldsymbol{\Sigma}_\theta = \sum_m \sigma_m^2 \, \boldsymbol{u}_m^R (\boldsymbol{u}_m^R)^\top, \qquad \boldsymbol{\Sigma}_a = \sum_m L(\lambda_m)^2 \sigma_m^2 \, \boldsymbol{u}_m^R (\boldsymbol{u}_m^R)^\top. \tag{26}$$

Combining Assumption 2 with the mapping from domestic interests to geopolitical actions from Proposition 7, we are again able to identify the eigenvalues ($\boldsymbol{u}_\ell$) and eigenvectors (λ_ℓ) of the network, and hence recover the entire geopolitical network

Proposition 8. ***(Identification)*** *Under Assumption 2, the eigenvalue λ_m and eigenvector $\boldsymbol{u}_\ell$ are identified whenever $\sigma_m^2 > 0$, via:*

$$\lambda_m = L^{-1}\left(\sqrt{\frac{[\tilde{\Sigma}_a]_{mm}}{[\tilde{\Sigma}_\theta]_{mm}}} \right),$$

where the diagonal entries are taken after projection onto the shared eigenbasis.

Proposition 5 identifies the eigenvalues λ_m and eigenvectors $\boldsymbol{u}_\ell$ (and hence recovers the entire geopolitical network ($\boldsymbol{H}$)) given the eigenbasis of $\boldsymbol{H}$. For the special case of a symmetric network, this eigenbasis is pinned down by graph stationarity alone. More generally, when the network is asymmetric, there is an additional unknown—the Perron vector $\boldsymbol{\pi}$—since the eigenbasis of $\boldsymbol{H}$ is only revealed after $\boldsymbol{\Pi}$-weighting the covariance matrices. If the structural importance of a country (π_ℓ) can be directly measured empirically (e.g., using GDP shares), the Perron vector $\boldsymbol{\pi}$ can be directly constructed using these empirical weights. Alternatively, since $\boldsymbol{\pi}$ is the Perron eigenvector of $\boldsymbol{H}$, estimation can proceed via an iterative indirect inference procedure. First, given an initial guess for $\boldsymbol{\pi}$, one can implement the $\boldsymbol{\Pi}$-weighted diagonalization, compute the eigenvalues (λ_ℓ) and eigenvectors (u_ℓ), and recover the geopolitical network ($\boldsymbol{H}$). Second, given empirical moments for the properties of the geopolitical network ($\boldsymbol{H}$), one can compare the value of these moments in the model and data, and update the initial guess for $\boldsymbol{\pi}$ to target the empirical values of the moments in the model. We describe this iterative procedure in the Appendix B.

Therefore, the methods that we developed in previous subsections for a symmetric network carry over to the more general case of an asymmetric network considered in this subsection. The asymmetric network extension introduces a new object, the Perron vector $\boldsymbol{\pi}$, which governs the heterogeneous structural importance of countries in the network. This heterogeneity in structural importance governs asymmetries in countries' incentives to coordinate policies with others and engage in strategic influence towards others. The transformation $\boldsymbol{G} = \boldsymbol{\Pi}^{-1}\boldsymbol{H}\boldsymbol{\Pi}$ preserves the full analytical

tractability of the symmetric network case: the properties of the equilibrium, spectral decoupling, the scaling function $L(\lambda)$, and the identification of the eigenvalues and eigenvectors of the network all carry over unchanged in $\mathbf{\Pi}$-weighted coordinates. What changes is the geometry of geopolitical cleavages: the geopolitical actions of structurally-important countries respond less to disagreement with their neighbors, whereas their interests carry disproportionate weight in shaping the geopolitical actions of those neighbors. Peripheral countries display the opposite pattern—their geopolitical actions are highly responsive to each international issue, but their interests are less consequential in determining the geopolitical actions of other countries.

3 Empirical Implementation

3.1 Data

To estimate the structure of the geopolitical network and test the predictions of our theoretical model, we use two comprehensive global datasets that map directly to the model's two core variables: domestic interests ($\boldsymbol{\theta}$) and geopolitical actions ($\boldsymbol{a}$).

3.1.1 Measuring Geopolitical Actions (a): UN General Assembly Voting

We measure countries' geopolitical actions ($\boldsymbol{a}$) using United Nations General Assembly (UNGA) voting data. Roll call votes in the UNGA are recorded as "no" (coded 1), "abstain" (coded 2), or "yes" (coded 3). Bailey et al. (2017) control for heterogeneity in the resolutions being voted on by using a dynamic spatial voting model to estimate each country's distance from an "ideal point" on a global policy spectrum. We use these ideal point distances as our measure of geopolitical actions. They capture a state's revealed diplomatic stance on the international stage after foreign lobbying and alignment. As such, they correspond closely to the notion of geopolitical actions in the model, which are undertaken after strategic influence and coordination.

To capture equilibrium geopolitical actions in response to long-run geopolitical alignment, we use decadal averages of countries's ideal point distances. Let s_{ikt} denote country i's estimated UNGA ideal point distance from country k in year t. We construct the empirical cross-country covariance matrix of geopolitical actions, $\mathbf{\Sigma}_a$, by calculating the covariance of country i and j's voting distances relative to all other countries k over each ten-year period:

$$\left(\mathbf{\Sigma}_{a,t}\right)_{ij} = \sum_{t=1}^{10}\sum_{k=1}^{N}\left(s_{ikt} - \bar{s}_{it}\right)\left(s_{jkt} - \bar{s}_{jt}\right), \qquad \bar{s}_{it} \equiv \sum_{k=1}^{N} s_{ikt}/N. \tag{27}$$

3.1.2 Measuring Domestic Interests (θ): Varieties of Democracy (V-Dem)

We measure countries' domestic interests ($\boldsymbol{\theta}$) using orientation towards democracy or autocracy, as a fundamental institutional characteristic of countries that affects their international positions. Specifically, we use the Varieties of Democracy (V-Dem) dataset, which aggregates expert evaluations to score countries across hundreds of disaggregated domestic political and social dimensions, such as freedom of the press, gender equality, civil liberties, and judicial independence. These metrics reflect the internal political architecture and ideological priorities of a state independently of foreign lobbying and alignment. As such, they correspond closely to the notion of domestic interests in the model, which are defined separately from strategic influence and coordination.

The V-Dem dataset contains over one thousand variables that record features of different dimensions of political institutions. These variables are aggregated into 48 separate index variables, each of which summarize a country's political institutions relating to a given topic, such as freedom of expression or electoral democracy. However, it is a priori unclear which of these 48 indices are relevant to geopolitical actions taken on the international stage. In order to address this, we use LASSO to select which differences in these indices predict differences in countries' UNGA ideal points in the UNGA voting data.[5] This model selection procedure results in 35 variables being selected, the names of which are listed in Table 1 below.

Let x_{imt} denote country i's V-Dem score on domestic issue m in year t. We construct the cross-country covariance matrix of domestic interests, $\boldsymbol{\Sigma}_\theta$, by computing the covariance between the interests vectors of countries i and j across 35 distinct domestic issues over each ten-year period:

$$\left(\boldsymbol{\Sigma}_{\theta,t}\right)_{ij} = \sum_{t=1}^{10}\sum_{m=1}^{35}\left(x_{imt} - \bar{x}_{it}\right)\left(x_{jmt} - \bar{x}_{jt}\right), \qquad \bar{x}_{it} \equiv \sum_{m=1}^{35} x_{imt}/N.$$

3.1.3 Measuring Structural Importance (π): GDP

We estimate an asymmetric geopolitical network using a measure of the structural importance of countries ($\boldsymbol{\Pi}$). We assume that countries structural importance in the network is well-proxied by their GDP. Therefore, we use the log of the decadal average of GDP to measure structural importance in each decade. Using logarithms smooths out differences in size across countries and avoids the $\boldsymbol{\Pi}$-weighted matrices being heavily dominated by a few large countries. As a robustness check, we also use a measure of aggregate outward bilateral influence from the FBIC data as an alternative measure countries' structural importance in the network.

[5] To be more precise, we run a penalized regression of the form $|a_{it} - a_{jt}| = \sum_{k=1}^{48} \beta_k |v_{it} - v_{jt}| + \epsilon_{ijt}$. The variable a_{it} denotes the UNGA ideal point of country i in year t, and so the outcome is the absolute difference in countries i and j 's UN voting behavior in year t. We relate this outcome variable to the difference in country i and j in year t of the 48 indices of political institutions that V-DEM provides (denoted $|v_{it} - v_{jt}|$); LASSO selects the dimensions k which are predictive of the outcome.

Table 1: Varieties of Democracy Variables used for Domestic Interests

Variable	Description
v2x_EDcomp_thick	Electoral component index
v2x_api	Additive polyarchy index
v2x_civlib	Civil liberties index
v2x_clphy	Physical violence index
v2x_clpol	Political civil liberties index
v2x_clpriv	Private civil liberties index
v2x_corr	Political corruption index
v2x_delibdem	Deliberative democracy index
v2x_diagacc	Diagonal accountability index
v2x_divparctrl	Divided party control index
v2x_egaldem	Egalitarian democracy index
v2x_elecoff	Elected officials index
v2x_elecreg	Electoral regime index
v2x_ex_confidence	Confidence dimension index
v2x_ex_direlect	Direct election dimension index
v2x_ex_hereditary	Hereditary dimension index
v2x_ex_military	Military dimension index
v2x_ex_party	Ruling party dimension index
v2x_feduni	Division of power index
v2x_frassoc_thick	Freedom of association thick index
v2x_gencl	Women civil liberties index
v2x_gender	Women political empowerment index
v2x_genpp	Women political participation index
v2x_hosabort	Presidential election aborted
v2x_hosinter	Chief executive no longer elected
v2x_jucon	Judicial constraints on the executive index
v2x_libdem	Liberal democracy index
v2x_liberal	Liberal component index
v2x_partipdem	Participatory democracy index
v2x_pubcorr	Public sector corruption index
v2x_regime	Regimes of the world—the RoW measure
v2x_regime_amb	Regimes of the world—the RoW measure (with ambiguous case categories)
v2x_suffr	Share of population with suffrage

Note: 35 variables from the Varieties of Democracy (V-Dem) dataset selected by LASSO as predicting differences in the UNGA ideal points as estimated by Bailey et al. (2017).

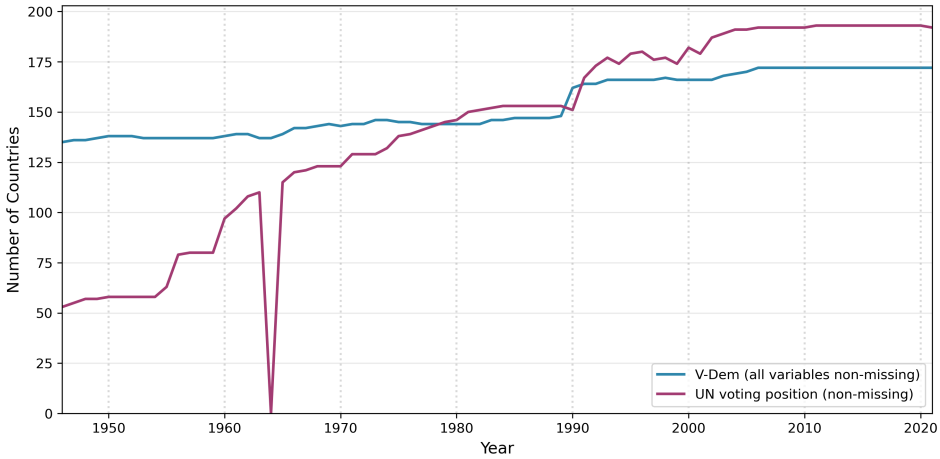
3.1.4 Other Data Sources and Coverage

In empirical validation checks of our inferred geopolitical network ($\boldsymbol{G}$), we use a range of empirical proxies for bilateral geopolitical connections between countries. We use variables from the CEPII Gravity dataset (bilateral trade, distance and indicators for cultural and historical linkages from Conte et al. 2022), the Polity Project (the difference in polity scores between countries), and the Foreign Bilateral Influence Capacity (FBIC) data.

The number of countries used to recover the network $\boldsymbol{G}$ varies across decades, because of the emergence of new states and missing data for some years. As plotted in Figure 1, the coverage of V-Dem and UN voting data grows over time, ultimately converging to roughly 175 countries with non-

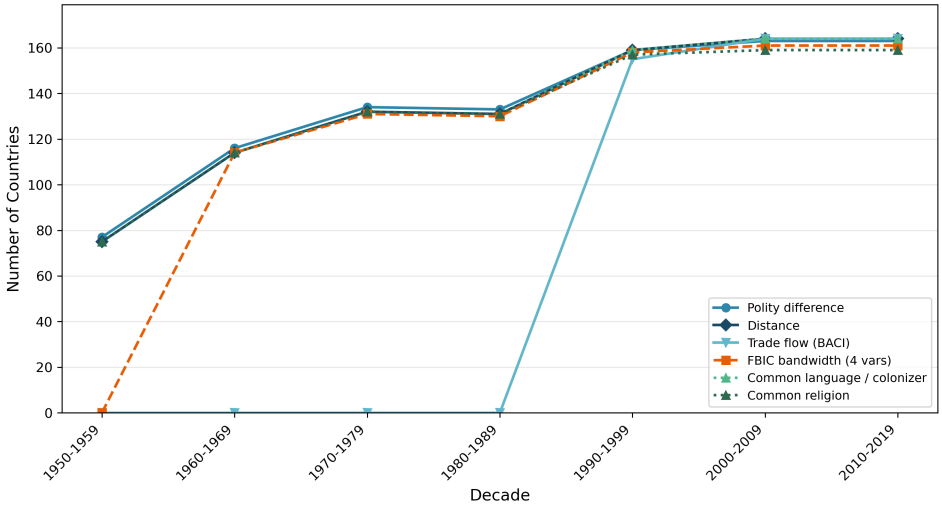
missing data on both domestic interests and geopolitical actions. Figure 2 reports country coverage for the variables used in empirical validation checks of our inferred geopolitical network ($\boldsymbol{G}$). In general, we have high coverage throughout our sample period, with the exception that we currently only use bilateral trade flow data from 1990 onwards.

Figure 3: Country Coverage in the Varieties of Democracy (V-Dem) and UNGA Datasets



Note: Number of countries with non-missing data in the Varieties of Democracy (V-Dem) and UNGA datasets.

Figure 4: Country Coverage for Bilateral Proxies for Geopolitical Links



Note: Number of countries with non-missing data for the bilateral proxies for geopolitical links.

3.2 Estimating Geopolitical Cleavages

We implement our empirical analysis separately by decade, recognizing that $\boldsymbol{G}$ is slow moving and stable within a decade. To estimate the eigenvectors and eigenvalues of the geopolitical network, we begin by calibrating the parameters of the scaling function ($L(\lambda)$). In our baseline specification,

we set $\alpha = 0.4$ and $\kappa = 5$, which ensures that the parameter restriction for the costs of strategic influence to be sufficiently convex ($\kappa > 1/\alpha$) is satisfied. In robustness checks, we examine the sensitivity of our empirical results to alternative values of these parameters.

We estimate our asymmetric network specification by weight the covariance matrices of geopolitical actions ($\boldsymbol{\Sigma_a}$) and domestic interests ($\boldsymbol{\theta}$) using the structural importance matrix ($\boldsymbol{\Pi}$), which yields $\tilde{\boldsymbol{\Sigma}}_{\theta,t} = \boldsymbol{\Pi\Sigma_{\theta,t}\Pi}$ and $\tilde{\boldsymbol{\Sigma}}_{a,t} = \boldsymbol{\Pi\Sigma_{a,t}\Pi}$. Even if the true covariance matrixes for geopolitical actions ($\boldsymbol{\Sigma_a}$) and domestic interests ($\boldsymbol{\Sigma_\theta}$) share the same eigenbasis, their empirical counterparts need not because of sampling variation and measurement error. Therefore, we estimate the shared eigenbasis of these matrixes using an an approximate joint-diagonalization procedure. For each decade, we search for an orthonormal basis $\boldsymbol{U}_t$ that minimizes the off-diagonals of $\boldsymbol{U}_t'\boldsymbol{\Sigma}_{\theta,t}\boldsymbol{U}_t$ and $\boldsymbol{U}_t'\boldsymbol{\Sigma}_{a,t}\boldsymbol{U}_t$:

$$\min_{\boldsymbol{U}_t} \sum_i \sum_{j \neq i} \left\{ \left[\boldsymbol{U}_t' \tilde{\boldsymbol{\Sigma}}_{\theta,t} \boldsymbol{U}_t \right]_{ij}^2 + \left[\boldsymbol{U}_t' \tilde{\boldsymbol{\Sigma}}_{a,t} \boldsymbol{U}_t \right]_{ij}^2 \right\}.$$

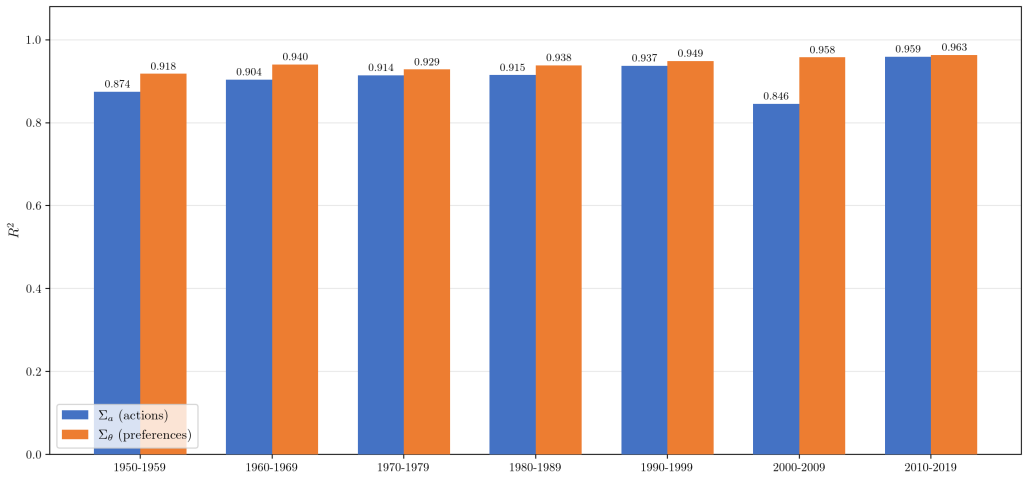
We examine the empirical success of this approximate joint-diagonalization procedure using the analog of an R^2: the share of total variance in the empirical matrices explained by the matrices that are reconstructed using the approximate joint basis.[6] Figure 5 shows that the reconstruction quality is high, with an R^2 of around 85% in recent decades and above 75% across the entire sample, using only the top-5 principal components from the approximate joint-diagonalization (out of the total number of components equal to the number of countries in our sample for each decade).[7]

Mechanically, adding more eigenvectors necessarily improves the fit of the joint-diagonalization. But we find that the top-five eigenvectors explain the vast majority of the observed cross-country covariance in political actions ($\boldsymbol{\Sigma_a}$) and domestic interests ($\boldsymbol{\Sigma_\theta}$). This pattern reflects a core feature of geopolitical networks: they are dominated by a few leading geopolitical cleavages, as captured by the top few eigenvectors, such that most other potential geopolitical cleavages do not matter in practice. The discarded geopolitical cleavages not only are characterized by low eigenvalues—meaning the network naturally neutralizes divergence along them—but they also represent obscure geopolitical fault lines that rarely manifest in actual international shocks.

[6] This statistic is closely related to the least squares criterion (Frobenius Norm), which equals the sum of the squares of all the off-diagonals of $\boldsymbol{U}_t'\boldsymbol{\Sigma}_{\theta,t}\boldsymbol{U}_t$ and $\boldsymbol{U}_t'\boldsymbol{\Sigma}_{a,t}\boldsymbol{U}_t$. We obtain similar results using other commonly-used metrics for the empirical success of the joint diagonalization, such as the Amari Index or the Kullback-Leibler Divergence.

[7] Specifically, we project $\boldsymbol{\Sigma}_{\theta,t}$ and $\boldsymbol{\Sigma}_{a,t}$ onto the orthonormal basis $\boldsymbol{U}_t$. We use the eigenvalues of the geopolitical action matrix to rank models and keep the top-five eigenvalues. We then project back to the original space, and examine how much of the variation in the original matrix can be explained by these approximately joint-diagonalized matrices $\boldsymbol{U}\,\mathrm{Diag}\left(\boldsymbol{U}_t'\boldsymbol{\Sigma}_{\theta,t}\boldsymbol{U}_t\right)_{top5}\boldsymbol{U}'$ and $\boldsymbol{U}\,\mathrm{Diag}\left(\boldsymbol{U}_t'\boldsymbol{\Sigma}_{a,t}\boldsymbol{U}_t\right)_{top5}\boldsymbol{U}'$, as measured by the R^2.

Figure 5: Reconstruction Quality for the Joint Diagonalization of $\Sigma_{\theta,t}$ and $\Sigma_{a,t}$



Note: Reconstruction quality for the joint diagonalization of $\Sigma_{\theta,t}$ and $\Sigma_{a,t}$ as measured by the share of the total variance in the empirical matrices for domestic interests and geopolitical actions explained by a reconstructed matrix.

3.3 Validation of the Inferred Geopolitical Network

Using the top-five geopolitical cleavages from our joint diagonalization in the previous subsection, we can recover the unobserved geopolitical network each decade ($\boldsymbol{G_t} = \boldsymbol{U_t}\boldsymbol{\Lambda_t}\boldsymbol{U_t}^\top$) from its top-five eigenvectors ($\boldsymbol{u}_{k,t}$) and eigenvalues ($\lambda_{k,t}$). We now report a number of validation checks on this inferred geopolitical network.

3.3.1 Correlation with Cross-country Dependence

We begin by presenting evidence that our recovered decade-specific asymmetric geopolitical network $\boldsymbol{G_t}$ exhibits empirically meaningful variation in geopolitical relations. To the extent that $\boldsymbol{G_t}$ captures the preference weights that countries place on others in action alignment, we can validate the estimates of $\boldsymbol{G_t}$ matrix by examining whether it aligns with countries' diplomatic resources allocation that may be intended to foster such action alignment. One such measure is Formal Bilateral Influence Capacity (FBIC), which measures state-sanctioned or state-sponsored and publicly acknowledged influence leverage across countries. They include diplomatic exchange, arm transfers, and goods trade. We view this as a sanity check if strong correlation exists between the two.

Specifically, we run a regression relating decade-average FBIC dependence measures to the edges of our geopolitical network G_{ijt}:

$$G_{ijt} = \beta x_{ijt} + \alpha_i + \gamma_j + \epsilon_{ijt}, \tag{28}$$

where $x_{ijt}$is an FBIC measure of how dependent country i is on country$j$ in decade t, just as the outcome G_{ijt} measures how much country i weighs country j's actions in their payoff in decade

t. The variables $\alpha_i$and γ_j are country i and j fixed effects, and standard errors are clustered on the (unordered) country pair level.

Before presenting the regression results demonstrating that the identified network features strongly relate to external measures of countries' political ties, it is important to note why these results are not mechanical. One may expect that, given that our covariance matrices reflect how similar countries' UN voting and domestic political institutions are that these similarities should naturally predict external measures of political ties between countries. However, we note that the results here correspond not to direct features of the covariance matrices, but of the geopolitical network $\boldsymbol{G_t}$, with edges $G_{ijt} = \frac{\pi_{jt}}{\pi_{it}} \sum_{m=1} \lambda_{mt} [\boldsymbol{v}_{mt}]_i [\boldsymbol{v}_{mt}]_j = \frac{\pi_{jt}}{\pi_{it}} \sum_{m=1} L^{-1}\left(\sqrt{\frac{[\tilde{\Sigma}_{at}]_{mm}}{[\tilde{\Sigma}_{\theta t}]_{mm}}}\right) [\boldsymbol{v}_{mt}]_i [\boldsymbol{v}_{mt}]_j$. That is, the edges of the network are given by a linear combination of i) structural importance weights ii) the eigenvalues of the geopolitical network and iii) entries of the basis vectors.

Table 2: Geopolitical Network Edges and FBIC Dependence

	Outcome: G_{ijt}		
	Dependence	Economic dependence	Security dependence
	(1)	(2)	(3)
x_{ijt}	0.165^{***}	0.146^{***}	0.0783^{***}
	(0.0135)	(0.0135)	(0.00878)
Observations	111333	111356	111415
Country i FE:	Yes	Yes	Yes
Country j FE:	Yes	Yes	Yes

Note: This table regresses the ij-th entry of the $\boldsymbol{G}_t$ network for decade t on decade-level averages of ordered dyadic variables. These variables come from the Foreign Bilateral Influence Capacity (FBIC) dataset, which measures the degree of dependence of country i on country j along different dimensions. The first column is the composite/overall FBIC dependence, whereas the latter columns display coefficients for each dimension separately. FBIC data is available starting in 1960. Standard errors are clustered at the pair level, with significance levels denoted * $p < 0.1$, ** $p < .05$, *** $p < 0.01$.

The results of the regression in Equation (28) are displayed in Table 2. We observe a strong correlation between the estimated geopolitical network and the FBIC dependence. Because there are no natural units in which to express the values of the edges G_{ijt}, we standardize both the outcome

variable and covariates within-decade. Therefore the interpretation of the coefficients in the above table are that a one standard deviation increase in FBIC dependence is associated with a $.16$ standard deviation increase in the edge weight G_{ijt}.

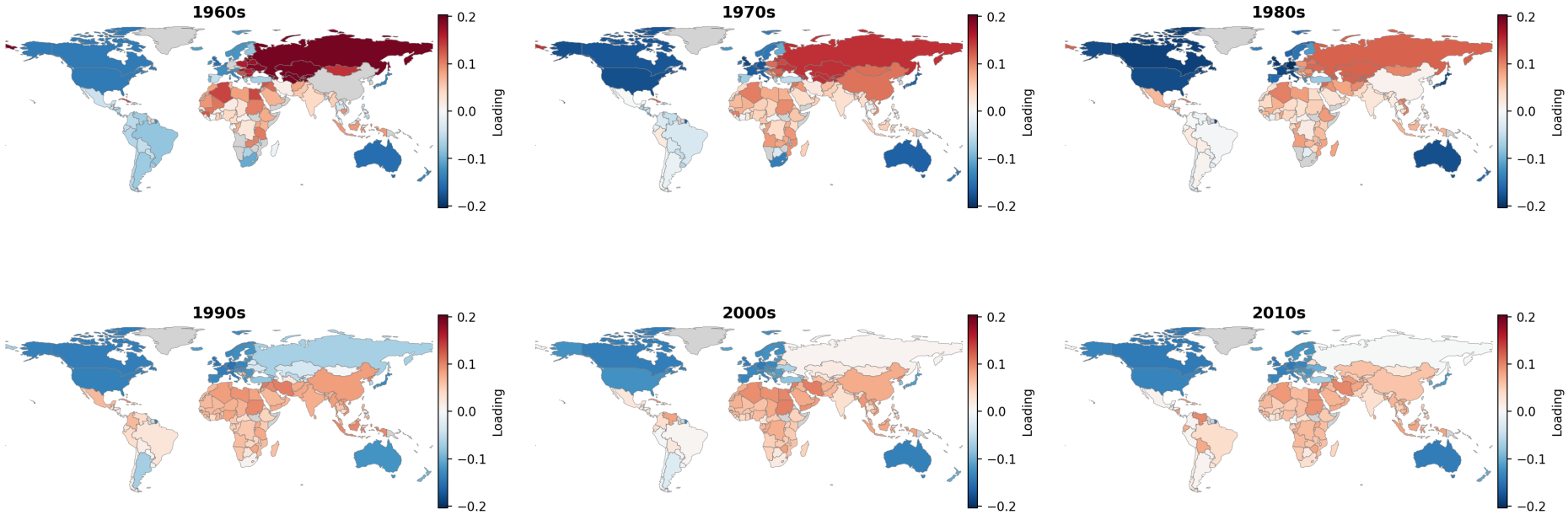
Although one might expect the eigenvectors recovered from the approximate joint basis of the covariance matrices to relate to external measures of political ties, the entries of said eigenvectors are also weighted by the eigenvalues of $\boldsymbol{G}_t$ which depend on the ratio of the diagonal entries of the covariance matrices. It is not a priori obvious that such a non-trivial transformation of features of the covariance matrices should return an intuitive geopolitical network. As a result, the fact that the edges of the geopolitical network relate to other measures of political relations is not a mechanical feature of the data, but instead provides evidence in support of the validity of the recovered network.

3.3.2 Evolution of Geopolitical Cleavages

Yet another way to assess the validity of the estimated geopolitical network is to examine its eigenvectors for leading geopolitical cleavages and their evolution over time. Figure 6 visualizes $\boldsymbol{u}_2$, the eigenvector associated with the second-largest eigenvalue, across our sample period. Recall that this principal eigenvector captures the network's deepest geopolitical cleavage—international issues where tightly connected blocs exhibit high internal consensus but stark cross-bloc polarization. In the figure, each panel represents a distinct decade, with the heatmap diverging from red to blue to denote polar opposite domestic interests along this structural axis.

During the three decades prior to 1989, this eigenvector cleanly captures the Cold War division, highlighting the stark ideological divide between the West and the Soviet bloc. Interestingly, following the fall of the Iron Curtain in the 1990–1999 panel, the spatial distribution shifts dramatically: the former Soviet states realign away from their historical extreme, turning blue to anchor the opposite end of the spectrum relative to China and much of Asia and Latin America. Finally, over the two decades post-2000, the heatmap reflects the structural reorganization of the international network, with China emerging as the central pole of a geopolitical bloc standing in direct opposition to the United States and Western Europe.

Figure 6: Second Dominant Eigenvector by Decade



Note: Figure shows the second dominant eigenvector, which captures the network's deepest geopolitical cleavage, for which tightly connected blocs exhibit high internal consensus but stark cross-bloc polarization; second dominant eigenvector displayed by decade; heatmap from red to blue, where the pair of countries with the darkest red and the darkest blue are those with the greatest geopolitical cleavage.

4 Structure of the Geopolitical Network

Having validated our geopolitical network, we now present descriptive evidence on the key empirical features of this geopolitical network.

4.1 Country Ties and the Geopolitical Network

First, we relate the bilateral connections in our geopolitical network to a range of empirical proxies for geopolitical connections between countries used in the existing economics and political science literature. In regressions analogous to that in Equation 28, we relate network edges to differences in countries' polity scores, their trade flow, geographical distance, and indicators of common religion, language, and historical colonizers. The results in Table 3 illustrate that the network reflects intuitive empirical patterns, namely that countries with more similar political institutions (column 1) and more cultural/historical ties (columns 4-6) have larger network edge values in our recovered network, whereas countries that are more politically dissimiliar (column 1) or more geographically distant (column 3) have lower network edge values.

Note that the covariates in the first three columns — polity score distance, trade flow, and geographical distance — are all standardized on the decade level, whereas the latter three covariates are simply indicator variables. Of the continuous covariates, polity score distance has the largest (in magnitude) association with the network edges, whereas two countries' populations sharing a religion is the largest predictor of network edges for the indicator variables. The results in Tables 2 and 3 provide evidence that our recovered geopolitical network exhibits sensible cross-sectional variation that qualitatively matches both external measures of dependence and of more general economic and cultural interactions. Given the robustness of these exercises, we next provide descriptives regarding features of the estimated network itself, rather than its relation to external measures.

Table 3: Geopolitical Network Edges and Measures of Countries' Ties

	Outcome: G_{ijt}					
	$\lvert\text{Polity}_{it} - \text{Polity}_{jt}\rvert$	Trade flow_{ijt}	Distance_{ij}	$\text{Common religion}_{ij}$	$\text{Common language}_{ij}$	$\text{Common colonizer}_{ij}$
	(1)	(2)	(3)	(4)	(5)	(6)
x_{ijt}	-0.411***	0.109***	-0.257***	0.851***	0.381***	0.524***
	(0.00713)	(0.0307)	(0.00977)	(0.0320)	(0.0262)	(0.0255)
Observations	120766	61147	119622	117486	119622	119622
Country i FE:	Yes	Yes	Yes	Yes	Yes	Yes
Country j FE:	Yes	Yes	Yes	Yes	Yes	Yes

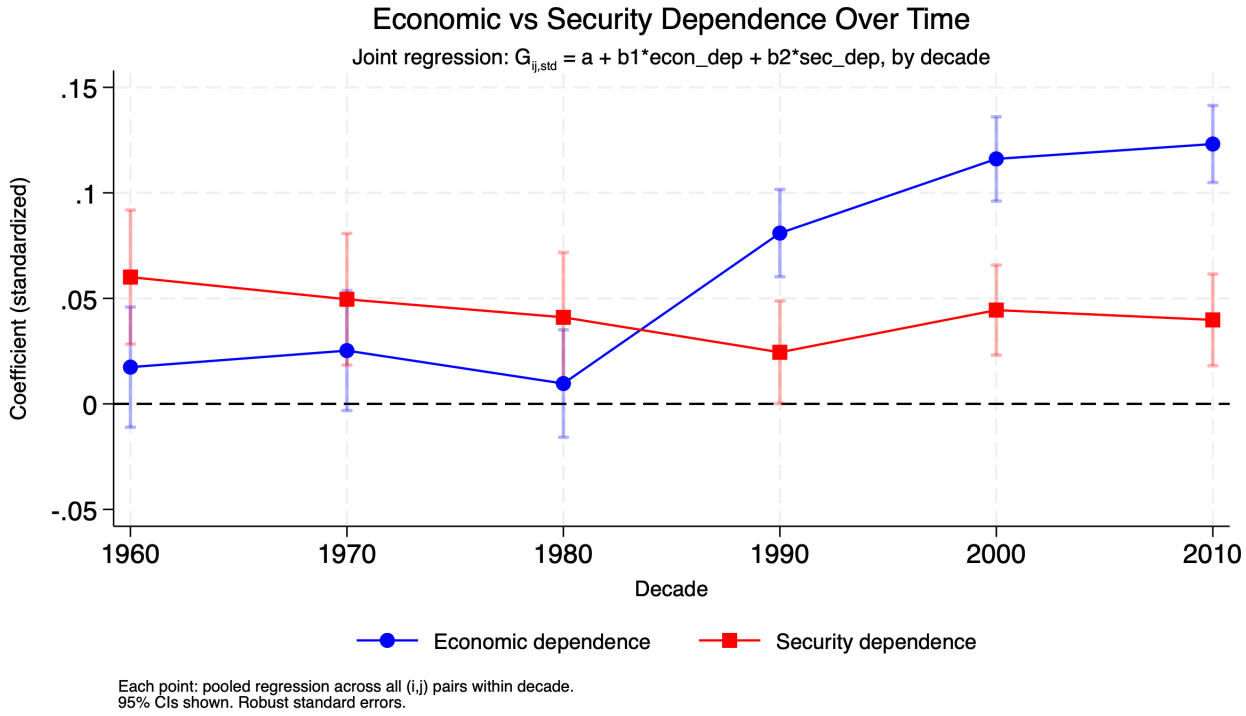
Note: This table regresses the ij-th entry of the G network for decade t on decade-level averages of dyadic variables. Column 1's outcome variable is the distance between country i and country j's average polity scores in each decade t. Column 2's outcome variable is the average trade flow between country country i and country j in decade t. Column 3 is the geographical distance between country i and country j. Columns 3-6 have binary outcomes denoting whether or not countries i and j share a common religion, common language, or common colonizer. CEPII trade flow data (Column 2) goes back to 1990, whereas polity data (Column 1) covers the entire panel. Standard errors are clustered on the unordered pair level, with significance levels denoted * $p < 0.1$, ** $p < .05$, *** $p < 0.01$.

4.2 Economic Versus Security Ties

Second, we examine the relative importance of economic and security relationships in the FBIC data in predicting the strength of connections between countries in our inferred geopolitical network. In Table 2, we find that when the FBIC dependence measure is separated into its economic and security subparts, the relation of economic dependence to the network edge is roughly twice as large as the relation of security dependence to the network edge. This can be interpreted as suggestive evidence that our geopolitical network reflects economic ties more than security ties, but nonetheless robustly reflects both features.

Does such relationship change over time? To this end, we run regress the network edge values on security or economic dependence within each decade separately and plot the coefficients and 95% confidence intervals in Figure 7:

Figure 7: Economic vs. Security Dependence and Geopolitical Ties Over Time



Note: Estimated coefficients from a regression of inferred geopolitical connections on both economic dependence and security dependence; regression estimated separately by decade; colored markers correspond to the point estimates; vertical bars correspond to the 95 percent confidence intervals.

During the Cold War decades of the 1960s to 1980s, security dependence is slightly more predictive than economic dependence of the geopolitical network. However, the fall of the USSR causes a dramatic reversal to this relationship: economic dependence becomes a stronger predictor of geopolitical relations than does security, and the gap is larger than during even the height of the Cold War. As the world has become more globalized and trade relations more integrated, this gap has continued to widen, resulting in economic ties being statistically distinguishable from security ties in its relative importance since the 2000s. This illustrates that what constitutes geopolitical ties has changed over time, as economic relationships play a more important part in an increasingly interconnected world economic system.

What constitutes geopolitical ties between two countries intuitively depends on the political institutions of those countries. The leaders of democratic and autocratic states face different political incentives, and so the forces that cause them to align their policies with other countries may vary. For example, if autocratic states tend to have economies that rely on the export of a small number of goods, then they may have especially large reasons to align with countries that import those goods. Alternatively, democratic countries may have more diversified economic portfolios, and as a result trade is less related to their geopolitical alignment. Importantly, the asymmetric nature of the net-

work makes it so that one can differentiate between the forces that cause a nation of interest 'receive' more weight from another country vs. 'send' more weight to another country. To shed light on this, we run a regression of G_{ijt} on the economic and security dependence of i on j, where we split the sample by democratic j and autocratic j (sender), as well as democratic i and autocratic i (receiver). The regression results of this split-sample analysis are displayed in Table 4:

Table 4: Relation of Economic and Security Dependence to G_{ijt} by regime type

	Outcome: G_{ijt}			
	Split by sender j		Split by receiver i	
	Autocratic sender	Democratic sender	Autocratic receiver	Democratic receiver
	(1)	(2)	(3)	(4)
Economic dependence	0.263***	0.0654***	0.149***	0.140***
	(0.0248)	(0.0115)	(0.0173)	(0.0197)
Security dependence	0.00174	0.0451***	-0.0169	0.0703***
	(0.0144)	(0.00845)	(0.0112)	(0.0140)
Observations	29569	44968	29561	44952
Receiver i FE:	Yes	Yes	Yes	Yes
Sender j FE:	Yes	Yes	Yes	Yes

Note: This table reports coefficients from regressing the standardized G_{ijt} network entry on standardized economic and security dependence of country i on country j (from FBIC), estimated jointly. Columns (1)–(2) split the sample by whether country j (the sender) is an autocracy (Polity ≤ -6) or democracy (Polity ≥ 6). Columns (3)–(4) split by whether country i (the receiver) is an autocracy or democracy. All regressions include country i and country j fixed effects. Standard errors are clustered at the unordered pair level, with significance levels denoted * $p < 0.1$, ** $p < .05$, *** $p < 0.01$.

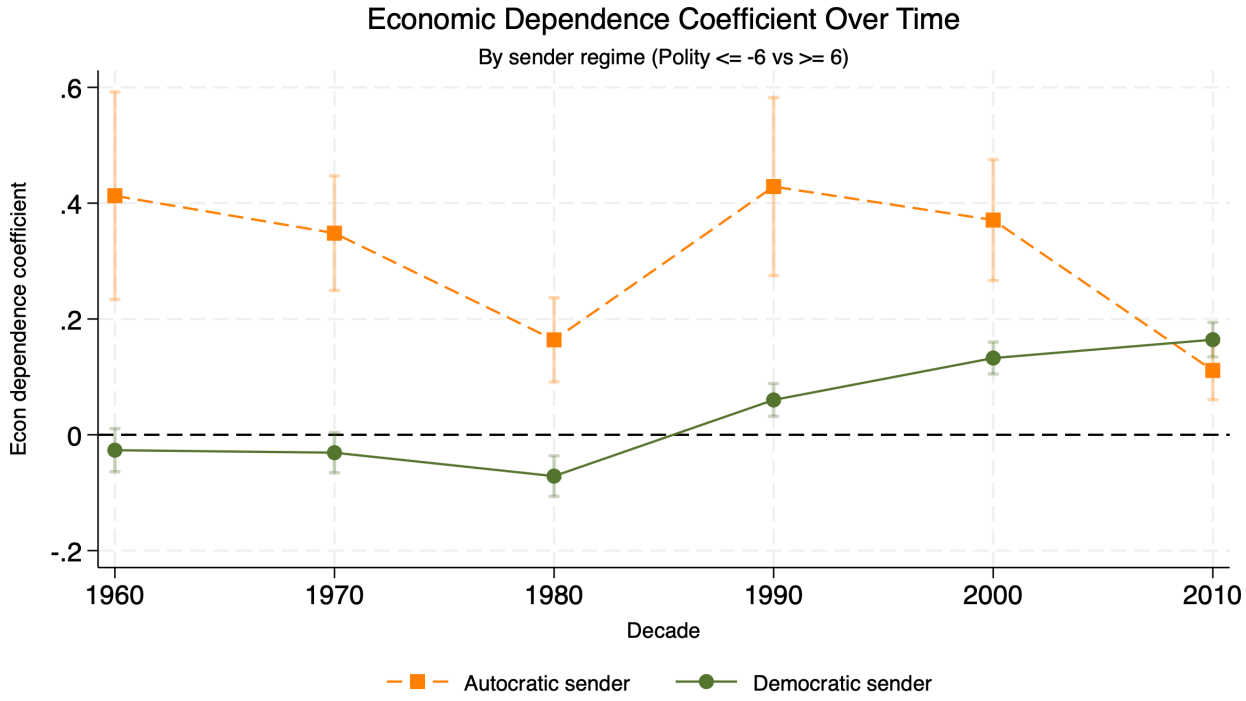
Columns 1 and 2 compare the relation of economic/security dependence of country j on i to G_{ijt} by regime type, meaning that these regressions analyze how countries' geopolitical considerations vary by the regime type of the country on which they depend. The results are striking: the geopolitical weights of receiver countries i are roughly 4 times more responsive to economic dependence on sender autocratic nations than of democratic nations. In addition, the geopolitical weights of receiver countries i are statistically unrelated to security dependence on autocratic nations, whereas they are related to security dependence on democratic nations. This means that countries who are economically dependent on autocrats also weigh those autocrats' actions heavily, while countries that are economically reliant on democrats weigh them far less (but still positively). In contrast, security dependence on autocrats is not associated with more weight on their actions, whereas coun-

tries that rely for security on democratic nations do weigh their actions.

Next, we consider how the regime type of the receiver country i determines the relationship of types of dependence to action weights G_{ijt}. Interestingly, both democratic and autocratic countries i put very similar weight on the actions of countries j on which they rely economically. However, only democratic countries i weigh the political actions of a partner country j based on security dependence. The findings in columns 1-4 are consistent with a world in which democratic nations enter into security agreements with each other and align their geopolitical actions. In addition, columns 3 and 4 suggest a commonality between the situations of both democratic and autocratic receiver countries i: if they rely economically on another country j, then they align their actions with that country. Finally, columns 1 and 2 present evidence that although countries i weigh the actions of sender countries j based off of economic dependence, they do so far more when the sender j is autocratic rather than democratic.

Finally, we can demonstrate that the regime-type heterogeneity evolves over time. We present four figures, which directly compare the importance of a type of dependence based on the regime type of the sender/receiver. In the main text, we focus on the most stark finding from the previous table: economic dependence on autocrats is far more predictive of geopolitical consideration than is economic dependence on democracies. We present the analogous results for other specifications (economic vs. security dependence, sender vs. receiver) in Figures 15, 16, and 17. As shown in Figure 8, the last 60 years have exhibited remarkable dampening of this regime-type heterogeneity. In the 1960s, there was a gap between the coefficient on economic dependence on autocracies vs. democracies of roughly $.4$, whereas now there is no significant gap:

Figure 8: Economic Dependence and G_{ijt} by Regime Type Over Time



Although the convergence itself is striking, what is arguably more surprising is the timing: the convergence has taken place almost entirely in the last two decades. The gap between coefficients drops from the 1990s to 2000s, but then drops massively into the 2010s such that the two coefficient are statistically indistinguishable from each other. This may follow from the growth in interconnectedness of the world's economies so that economic leverage is increasingly available to both democratic and autocratic countries, and critically, that they are both willing to use it.[8]

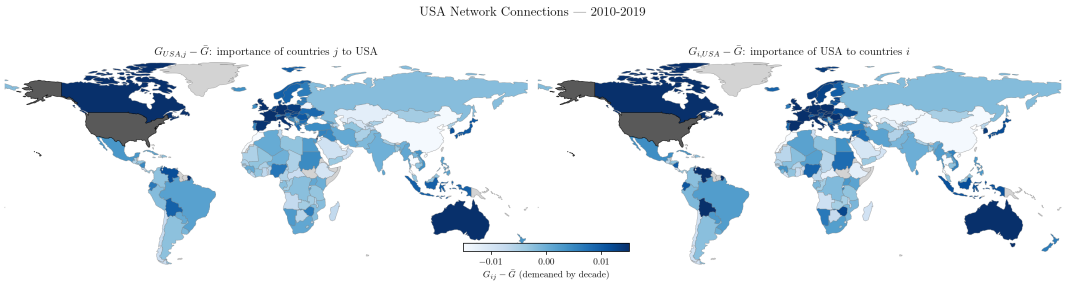
4.3 Individual Countries' Network Connections

The next two maps show the strength of geopolitical connections for the USA (Figure 9) and China (Figure 10) for the decade 2010-2019. In both figures, the left panel shows the importance of other countries to either the USA or China, whereas the right panel shows the importance of the USA or China to other countries.

The above maps illustrate the divergence in the USA and China's geopolitical priorities in the 2010s. The USA weighs the actions of nations in Western Europe heavily, whereas China weighs the

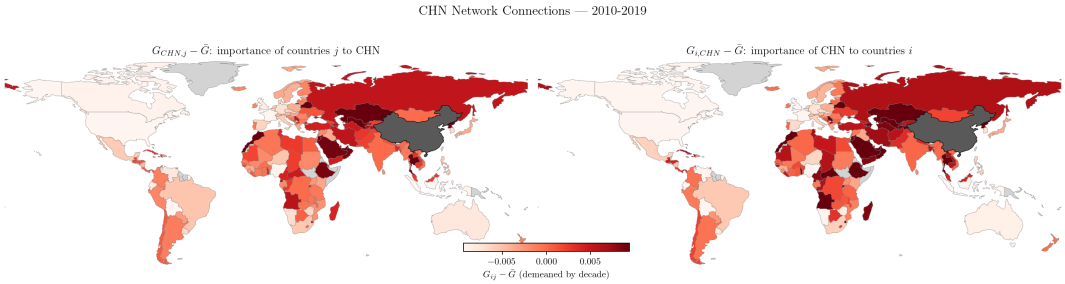
[8] We note that the results in Figure 8, which show a decrease in the importance of economic dependence for autocracies and an increase in the importance for democracies, are consistent with the results in Figure 7 because the number of democracies has grown from 22 in the 1960s to 83 in the 2010s, whereas the number of autocracies has stayed relatively stable (23 in 1960s vs 20 in 2010s).

Figure 9: Country Geopolitical Connections to the United States



Note: Heatmap of country geopolitical connections to the United States from 2010-19, where dark blue denotes most-strongly connected, and light blue denotes least-strongly connected.

Figure 10: Country Geopolitical Connections to China

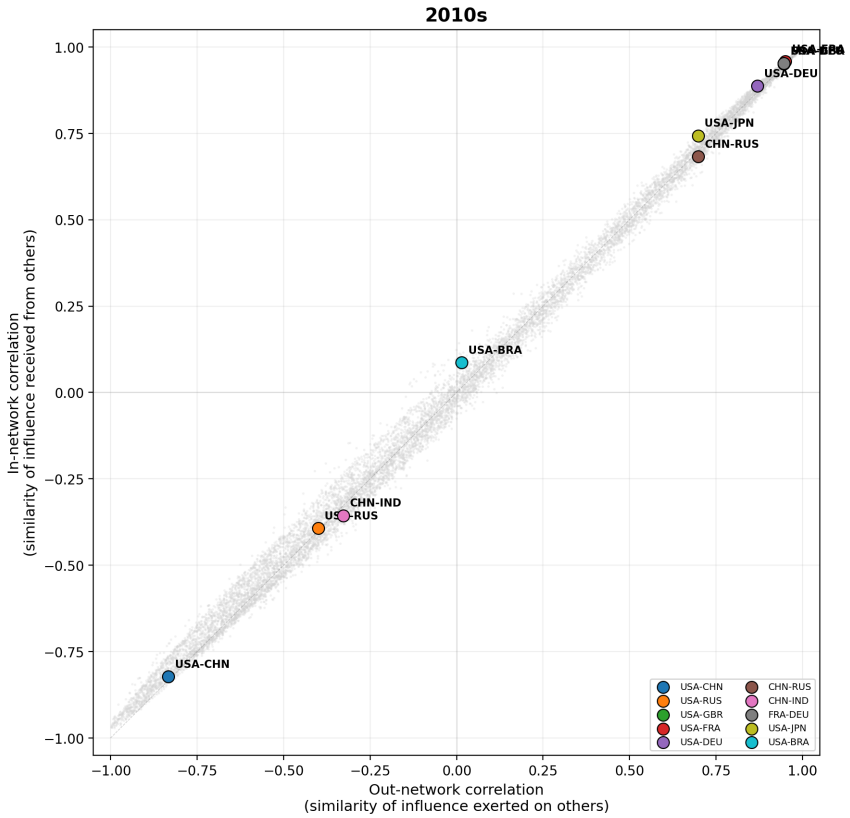


Note: Heatmap of country geopolitical connections to China from 2010-19, where dark red denotes most-strongly connected, and light red denotes least-strongly connected.

actions of South Asian (Pakistan, India) and South American (Brazil and Argentina) the most. In this decade, the country whose actions the USA weighed the most is Israel, whereas China weighs the most the actions of Pakistan. These figures also demonstrate the importance of asymmetry in our network: although the USA puts a lot of weight on the actions of Western European countries, those same countries put even more weight on the USA's actions. The same asymmetry for China is most prominently seen in West Africa, where countries like Ghana and Senegal put far more weight on China's actions than China does on theirs.

The difference in the USA and China's networks begs the question of how correlated are any two country pairs' geopolitical networks; although China and the USA are geopolitical rivals, is it possible that there are many other country pairs whose respective networks exhibit differences of a similar magnitude. To investigate this more fully, we correlate the in and out network of each country pair in the 2010s and plot them in Figure 11.

Figure 11: Correlation of Country Pairs' In/Out-Networks in 2010s



Note: For each pair of countries in our sample (e.g., USA-CHN) for the decade 2010-19, the figure displays the correlation between the two countries in-network connections (vertical axis) against the correlation between the two countries out-network connections (horizontal axis); each gray dot shows a different country pair; selected country pairs highlighted using color as noted in the legend; a perfectly symmetric network would feature all country pairs along the diagonal; in-network connections (vertical axis) measure influence received from others; out-network connection (horizontal axis) measure influenced exerted on others.

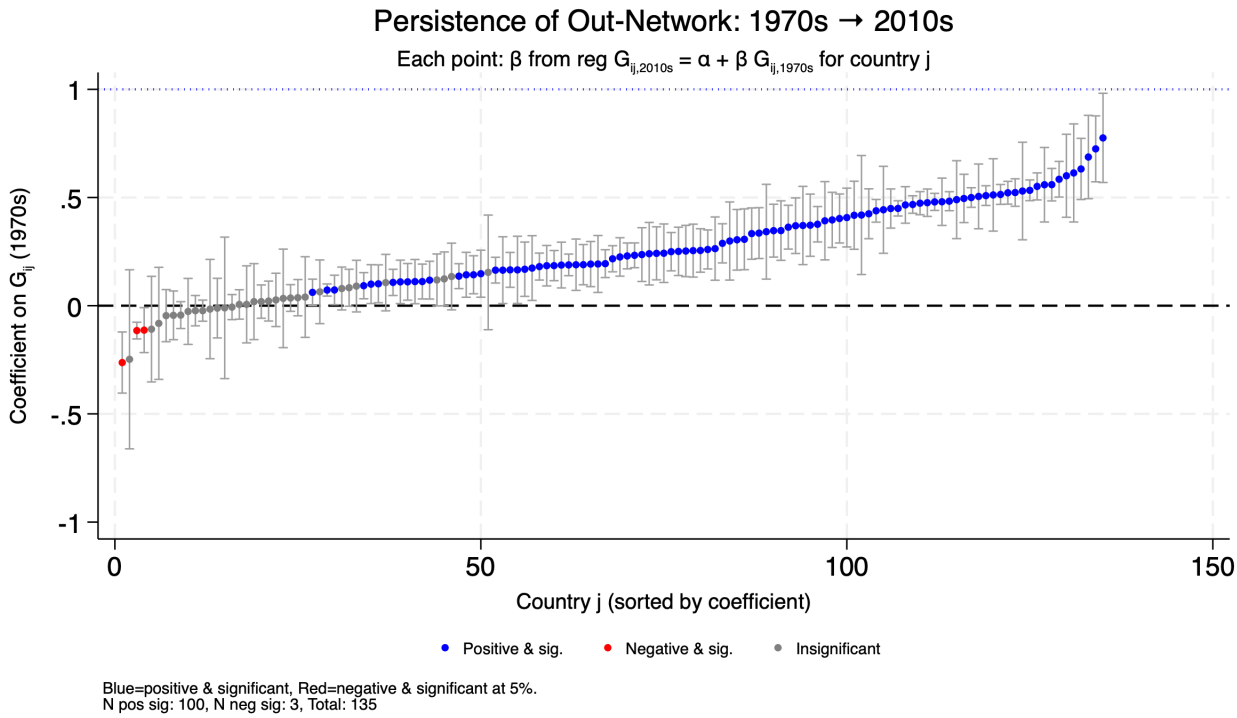
This figure underscores two features of the network: approximate symmetry and substantial heterogeneity. All country pairs are quite close to the 45 degree line, indicating that if two countries have a certain level of correlation in their out-network, a similar correlation exists for their in-network. Given that the estimated asymmetric network exhibits a large degree of symmetry, this finding follows naturally. Despite the relative symmetry of the estimated network, we nonetheless see significant differences in countries' geopolitical network. As noted in the lower left of the figure, the USA and China have among the most negatively correlated geopolitical networks of any country pairs in the world, whereas the geopolitical ties of USA and Japan or China and Russia are remarkably similar.

4.4 Temporal Stability of the Geopolitical Network

The changes in the predictors of geopolitical ties suggests that there may be significant variation in countries' geopolitical networks over time, whether this is shaped by regime change, shifts in inter-

national political stature, and more. To investigate the degree to which countries' political ties are consistent over time, we regression each country's out-network connections in 2010 on its connections in 1970. A positive coefficient indicates persistence of geopolitical networks over time, whereas a negative coefficient represents reversals in political ties. We plot each country's coefficient from this regression in Figure 12.

Figure 12: Persistence of Geopolitical Networks: 1970s vs. 2010s



Note: For each country (e.g., USA), the figure displays the correlation between its out-network connections (influence exerted on others) in 1970-79 and those in 2010-19; countries are sorted along the horizontal axis by the size of the estimated correlation; solid circles show the point estimates; vertical bars show the 95 percent confidence intervals; red indicates negative and significant at the 5 percent level; blue indicates positive and significant at the 5 percent level.

The results provide evidence of a high level of heterogeneity in the persistence of countries' geopolitical networks from the 1970s to 2010s. Such variation is not entirely surprising given the degree to which many nations' political institutions and roles in international politics have shifted in the last half-century. We note that India, Iran, and Venezuela are among the countries with the most negative coefficients, suggesting that these countries' political trajectories were among the most stark reversals in the world. On the other extreme, Canada, France, and the United Kingdom are among the most stable, which reflects the stability of the Western coalition over the last fifty years.

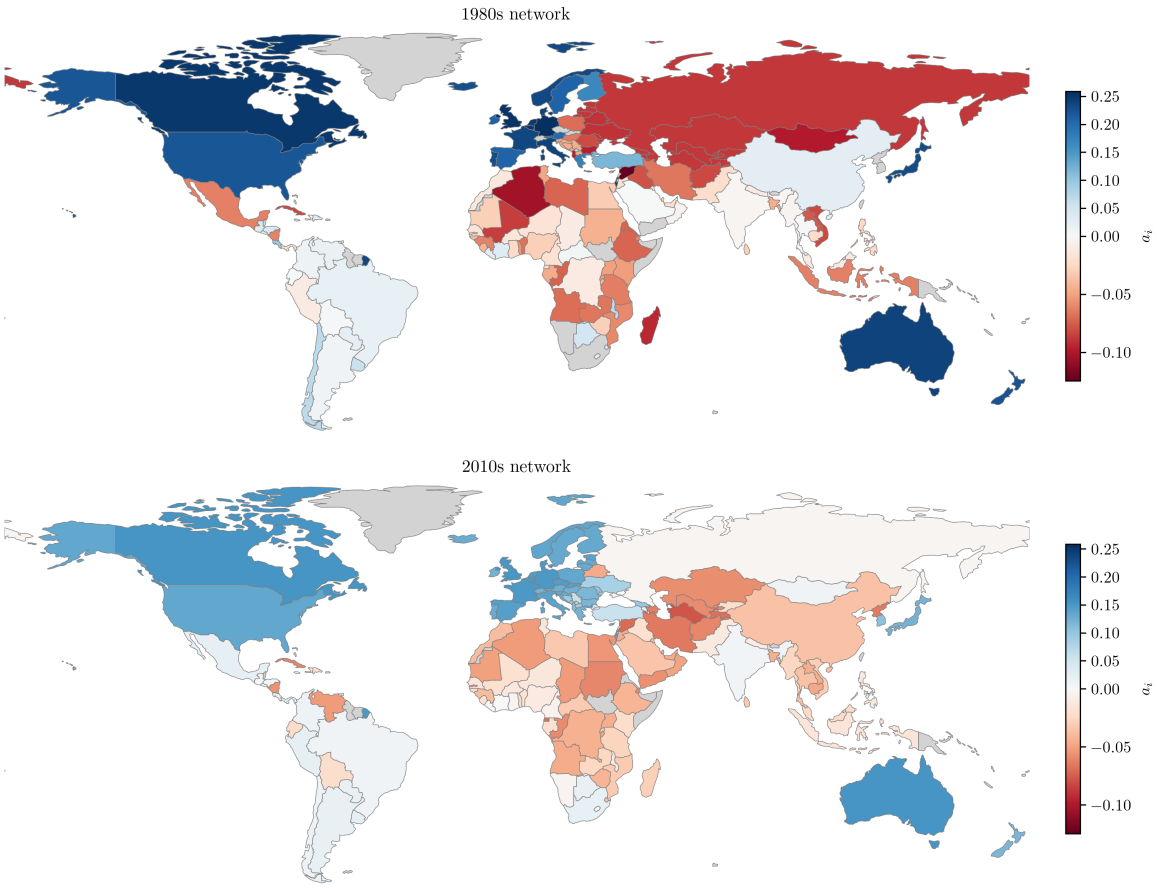
5 Geopolitical Simulations

Having described the features of our inferred geopolitical network, we now show how it can be used to simulate geopolitical actions in response to actual or counterfactual international issues. To do so, we investigate equilibrium actions across different decades' networks for a given international issue.

5.1 Sanctions Against Iran in Different Decades

We next examine the types of geopolitical environments that enable sanctions to succeed? Intuitively, sanctions are likely less ineffective when the target is able to turn to other countries from which they can seek relief, and more effective when few such alternative sources of support exist. To provide evidence on the role of the geopolitical environment in determining the success of sanctions, we evaluate the impact of the United States imposing extensive economic sanctions on Iran in the 1980s versus the 2010s? As can be seen in Appendix Figure 19, we treat this issue as one that falls roughly along Cold War lines, with countries like the USA and France on one side of the loading and the USSR and Soviet-bloc states falling weakly on the other side of the loading. Despite the fact that the USA falls very heavily against Iran and the USSR is only slightly with Iran, Figure 13 shows that international support for sanctions is vastly different between the 1980s and 2010s:

Figure 13: Support for Sanctions Against Iran: 1980s vs. 2010s



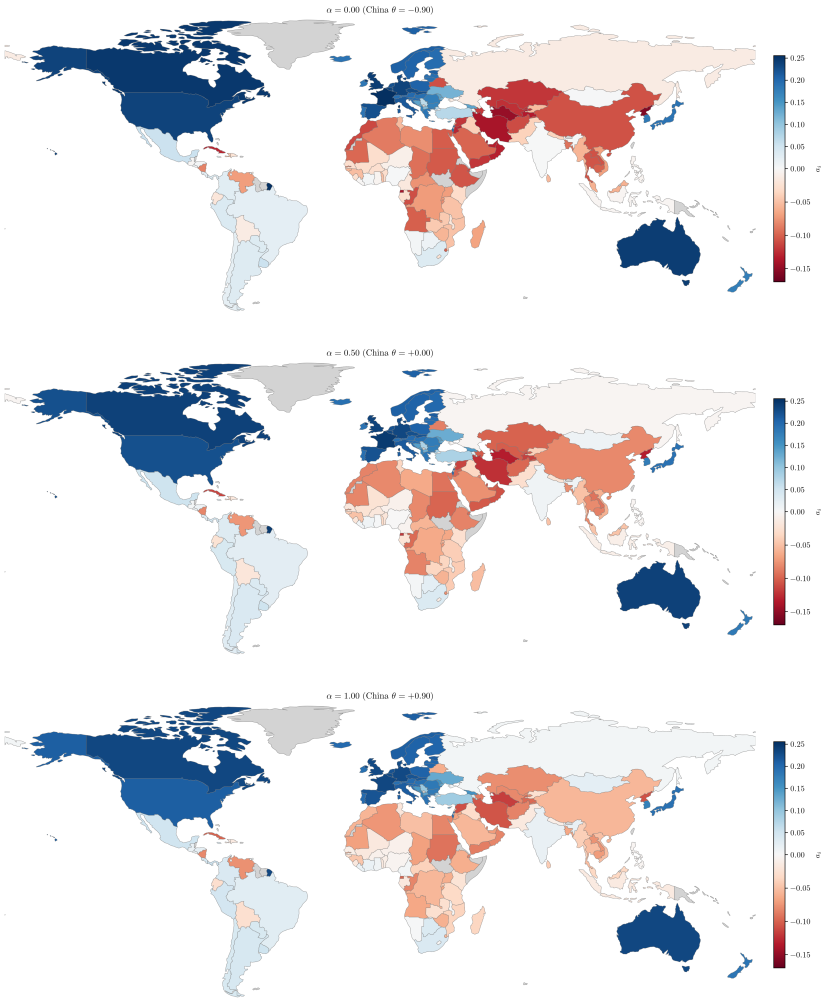
Note: geopolitical simulations for economic sanctions on Iran; simulations assume domestic interests and solve for equilibrium political actions given our inferred geopolitical network for 1980-9 (top panel) and 2010-9 (bottom panel).

This difference in international support reflects differences in the geopolitical network between the two time periods. Although the USA's push for sanctions against Iran is the same in the two time periods, the responses of Russia/Soviet bloc countries and China are notably different. In the 1980s, despite having little preference against Iranian sanctions, the Soviet bloc mobilizes to move harshly against the USA's move to sanction Iran. Interestingly, China mildly supports the sanctions in the 1980s, but not in the 2010s. These results qualitatively reflect some long-standing tensions that were at play in the UN Security Council vote on Iran sanctions in 2010 (Resolution 1929), as it matches the behavior of the Western bloc. The countries that behave differently in the simulation in comparison to the true votes is because of historical specificities: Turkey, Brazil, and Iran had just signed an alternative deal weeks prior that was meant to allay concerns of Iranian nuclear enrichment. In addition, Russia's then-president Dmitry Medvedev (who served only three years of the relevant decade) had very different policy positions than did Russia's president for the rest of the decade, Vladimir Putin.

5.2 A China with American Preferences

Finally, we examine the degree to which new leaders have the capacity to change the course of their country's geopolitical strategy. Suppose that the next leader of China had preferences far more similar to the United States than previous Chinese leaders. Would this new China necessarily take actions that align with the United States on international issues? Because of long-term trade deals, contracts, military agreements, or other irreversible political investments, China's new leader would still be constrained by the same geopolitical network. To illustrate the limitations that individual country leaders face in the short term, we simulate a $\boldsymbol{\theta}$ in which the USA and its Western allies fall positively on some issue (e.g. sanctions on North Korea), whereas China, Russia, Venezuela, and others load negatively on this issue (see Appendix Figure 18). Then, we generate two alternative Chinese preferences; one where its preference is the midpoint between the USA and the original Chinese preferences, and another where China has the same preferences to the USA. We only vary the preferences of China and leave fixed the elements of $\boldsymbol{\theta}$ relating to other countries' preferences. We then simulate the equilibrium actions under the three Chinese preferences. The results are shown in Figure 14:

Figure 14: Equilibrium Actions Under Different Chinese Preferences in 2010s



This figure displays the equilibrium actions of all countries in the 2010s $\boldsymbol{G}$ network, where each row varies the preferences of China. The top row is the scenario in which the USA and China have opposite preferences: $\theta_{USA} = .9$ and $\theta_{CHN} = -.9$; the second panel shows actions when $\theta_{CHN} = 0$ (midpoint of USA and China's original preference); the last panel shows actions when China and the USA have the same preferences. China's actions shift mildly towards the USA, but even in the case where the USA and China have the exact same preferences, they still load on different sides of the issue. This is because China's allies have not changed their preferences, and so Chinese actions are still anchored towards their allies. The geopolitical network makes it so that there is not full pass-through of Chinese preferences to actions; the network is a source of political rigidity.

6 Conclusions

Understanding geopolitics inherently involves thinking about networks. A country not only has direct influence over another nation, but also has indirect influence through other third nations. The power of any one nation depends on the characteristics and network connections of all other nations: whether the world is unipolar with a single hegemon, multipolar with competing hegemons, or nonpolar with no clear hegemon. Even if nations individually exert little political power, they can exert considerable power collectively, if they are connected by historical links or alliances.

We develop a tractable theoretical model of geopolitical networks that can be implemented quantitatively. We model geopolitics as a network coordination game with strategic influence. Countries choose policies that trade off domestic interests against incentives to align with neighbors in a geopolitical network. Prior to policy choice, countries can make costly investments to influence other countries' policies. Because influence propagates through the network, bilateral influence has cascading effects: shifting one country's policy changes equilibrium policies throughout the system, so a country may affect another indirectly by influencing third parties.

We show that the equilibrium mapping from countries' heterogeneous domestic interests to their geopolitical actions is a spectral function of the geopolitical network. This implies a property of "spectral decoupling": when domestic interests are decomposed along the network's eigenvectors—which we interpret as latent geopolitical cleavages—the equilibrium scales each dimension of conflict independently. Issues that align with large eigenvalues are those where disagreements are small across network neighbors but large across blocs. We show that these issues generate a geopolitical echo-chamber effect, amplifying cross-bloc frictions into stark policy polarization. Conversely, issues associated with small eigenvalues are neutralized by local coordination motives, leading to highly convergent policy outcomes.

We interpret countries as choosing geopolitical actions and strategic influence in response to the stochastic arrival of international issues and a fixed geopolitical network. Domestic interests for each international issue are drawn from a fixed distribution of geopolitical shocks. We assume graph stationarity such that the set of latent issues that determine the strength of geopolitical connections between countries is the same as those that generate differences in domestic interests across countries. Using this assumption, we establish a structural identification result that the eigenvalues and eigenvectors of the geopolitical network (and hence the entire geopolitical network) can be recovered from the cross-country covariances of domestic interests and geopolitical actions.

We implement our model quantitatively using data on countries' domestic interests (as measured by democratic orientation) and geopolitical actions (as measured by UN General Assembly voting). We treat the geopolitical network as evolving slowly and implement our analysis by decade. We find that geopolitical network has a low-dimensional representation in terms of a small number

of interpretable eigencomponents. Using the top-five eigencomponents, the reconstructed cross-country covariance matrices for domestic interests and geopolitical actions achieve R^2 above 80% across the sample and around 95% in recent decades. The leading nontrivial cleavage tracks well-known geopolitical realignments. Prior to 1989 it aligns with the Cold War division between the West and the Soviet bloc; in the 1990s it shifts sharply with the collapse of the Soviet Union; and after 2000 it is consistent with a cleavage in which the US and Western Europe occupy one pole while China occupies the opposite, with many countries in the Global South lying in between.

Taken together, our findings highlight the insights from a network-based approach for understanding the geopolitical landscape.

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Online Appendix for "Strategic Influence in Geopolitical Networks" (Not for Publication)[*]

Ernest Liu[†]
Princeton University and NBER

Stephen J. Redding[‡]
Stanford University, NBER and CEPR

Bennett Smith-Worthington[§]
Princeton University

David Yang[¶]
Harvard University, NBER and CEPR

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[†]Princeton University, JRR Building, Princeton, NJ 08544 and NBER. Email: ernestliu@princeton.edu.

[‡]Stanford University, 579 Jane Stanford Way, Stanford, CA 94305, NBER, and CEPR. Email: reddings@stanford.edu.

[§]Princeton University, JRR Building, Princeton, NJ 08544. Email: bennettsw@princeton.edu.

[¶]Littauer Center M-31, 1805 Cambridge Street, Cambridge, MA 02138. Email: davidyang@fas.harvard.edu.

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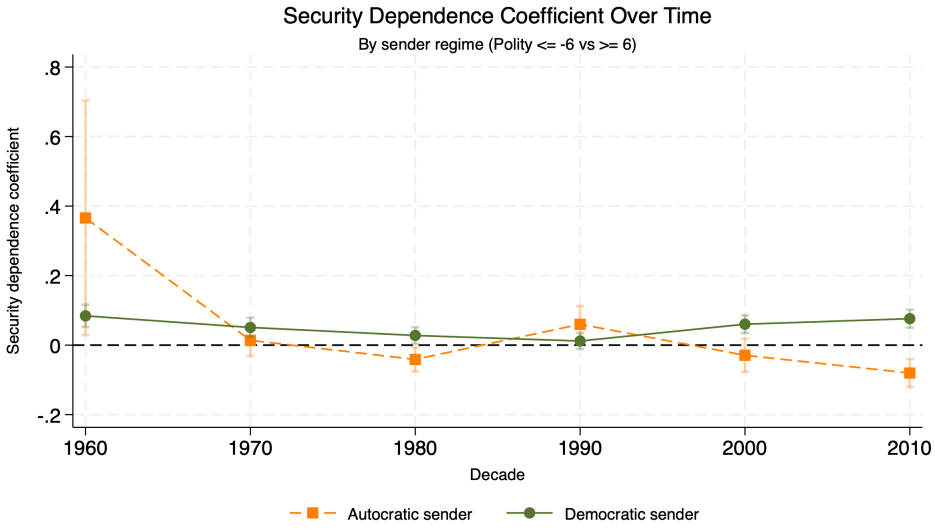


Figure 16: Evolution of Economic Dependence by Receiver Regime Type Over Time

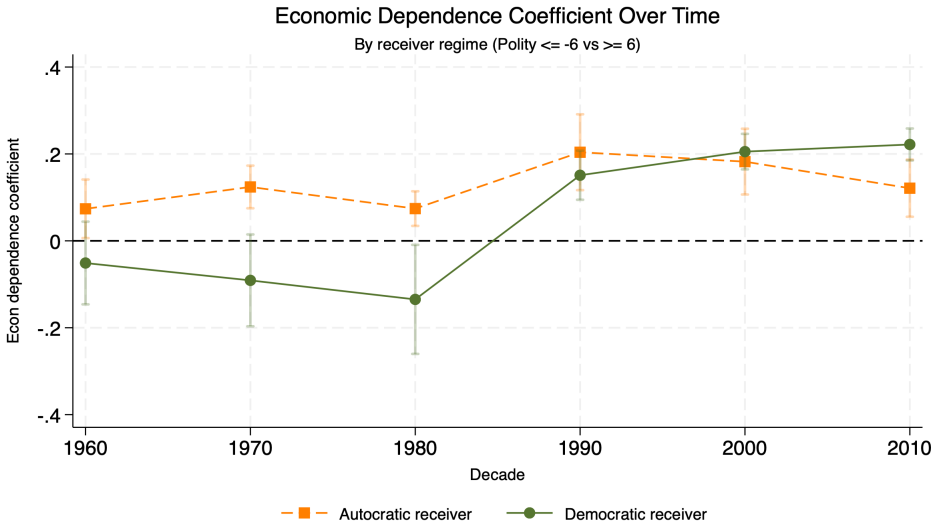


Figure 17: Evolution of Security Dependence by Receiver Regime Type Over Time

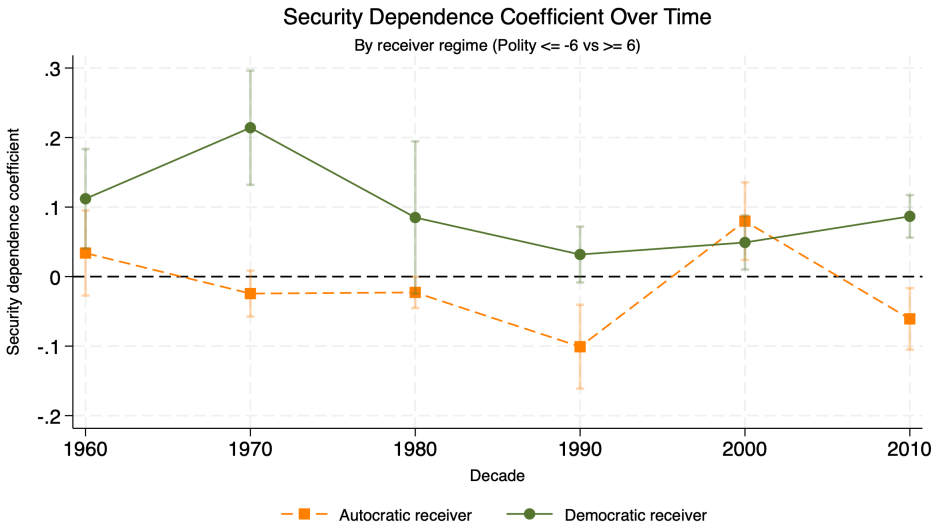


Figure 18: Simulated Preferences for USA-China Dividing Issue

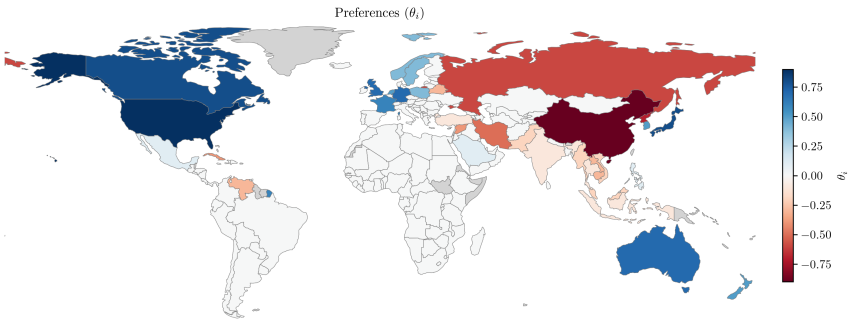
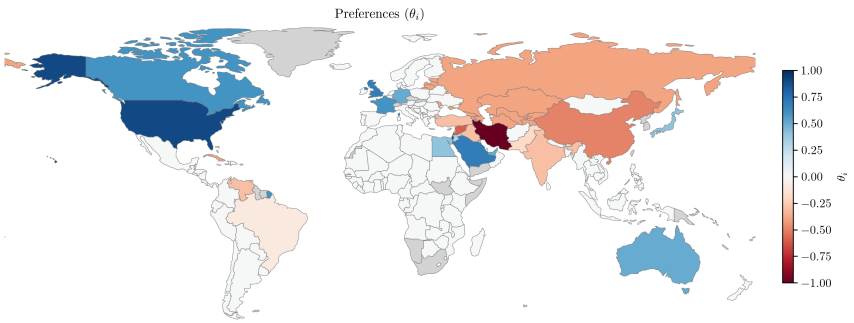


Figure 19: Simulated Preferences for Sanctions on Iran



B Estimation of the Directed Network

Since the eigenbasis of $\boldsymbol{H}$ is revealed only after Π-weighting, estimation of the directed network requires jointly recovering $\boldsymbol{\pi}$ and $\boldsymbol{H}$. We propose an iterated procedure that exploits the fixed-point structure of the model: because $\boldsymbol{\pi}$ is the Perron eigenvector of $\boldsymbol{H}$, any candidate $\boldsymbol{H}$ pins down the correct $\boldsymbol{\pi}$, which in turn determines the Π-weighting needed to estimate $\boldsymbol{H}$.

Given estimated covariance matrices $\hat{\Sigma}_\theta$ and $\hat{\Sigma}_a$:

1. *Initialize.* Set $\boldsymbol{\pi}^{(0)}$ from auxiliary data (e.g., the FBIC bandwidth measure) or from $N^{-1/2}\mathbf{1}$.
2. Π*-weighted diagonalization.* Given $\boldsymbol{\pi}^{(t)}$, form $\hat{\tilde{\Sigma}}_\theta^{(t)} = \Pi^{(t)}\hat{\Sigma}_\theta\Pi^{(t)}$ and $\hat{\tilde{\Sigma}}_a^{(t)} = \Pi^{(t)}\hat{\Sigma}_a\Pi^{(t)}$. Jointly diagonalize these two symmetric matrices to obtain an approximate eigenbasis $\left\{\hat{\boldsymbol{v}}_m^{(t)}\right\}$ and eigenvalue ratios $\hat{\ell}_m^{(t)\,2} = \left[\hat{\tilde{\Sigma}}_a^{(t)}\right]_{mm} / \left[\hat{\tilde{\Sigma}}_\theta^{(t)}\right]_{mm}$.
3. *Recover* $\boldsymbol{H}$. Invert the scaling function to obtain $\hat{\lambda}_m^{(t)} = L^{-1}\left(\hat{\ell}_m^{(t)}\right)$ and reconstruct $\hat{\boldsymbol{H}}^{(t)} = \sum_m \hat{\lambda}_m^{(t)}\,\hat{\boldsymbol{v}}_m^{(t)}\left(\hat{\boldsymbol{v}}_m^{(t)}\right)^\top$.
4. *Update* $\boldsymbol{\pi}$. Set $\boldsymbol{\pi}^{(t+1)}$ equal to the Perron eigenvector of $\hat{\boldsymbol{H}}^{(t)}$, normalized so that $\left\|\boldsymbol{\pi}^{(t+1)}\right\| = 1$.
5. *Iterate* steps 2–4 until $\left\|\boldsymbol{\pi}^{(t+1)} - \boldsymbol{\pi}^{(t)}\right\| < \varepsilon$.

The true $\boldsymbol{\pi}$ is a fixed point of this iteration: when $\boldsymbol{\pi}^{(t)} = \boldsymbol{\pi}$, the Π-weighted covariances share the eigenbasis of $\boldsymbol{H}$ exactly, so the joint diagonalization recovers $\boldsymbol{H}$ and the Perron eigenvector returns $\boldsymbol{\pi}^{(t+1)} = \boldsymbol{\pi}$. Intuitively, the procedure converts the estimation of a directed network—which would require decomposing a non-symmetric matrix—into a sequence of symmetric eigendecompositions, each conditioned on the current estimate of structural importance. Given the final $\hat{\boldsymbol{\pi}}$ and $\hat{\boldsymbol{H}}$, the directed network is $\hat{\boldsymbol{G}} = \hat{\Pi}^{-1}\hat{\boldsymbol{H}}\hat{\Pi}$.

Remark 1. A diagnostic for the quality of $\hat{\boldsymbol{\pi}}$ is the relative off-diagonal norm $\left\|\mathrm{off}\left(\hat{\Pi}\hat{\Sigma}_\theta\hat{\Pi}\right)\right\| / \left\|\mathrm{Diag}\left(\hat{\Pi}\hat{\Sigma}_\theta\hat{\Pi}\right)\right\|$, where $\mathrm{off}(\cdot)$ zeros out the diagonal. Under graph stationarity, $\Pi\Sigma_\theta\Pi$ shares the eigenbasis of $\boldsymbol{H}$, so this quantity vanishes at the true $\boldsymbol{\pi}$ when evaluated in that eigenbasis. In finite samples, the off-diagonal residual provides a goodness-of-fit test for both the graph stationarity assumption and the directed network specification $\boldsymbol{G} = \Pi^{-1}\boldsymbol{H}\Pi$.

C Proof of Proposition 6

We derive the expected payoff $\mathbb{E}[U_i]$ when $\boldsymbol{\theta}$ is drawn from a distribution whose covariance $\boldsymbol{\Sigma}_\theta$ is diagonal in $\boldsymbol{G}$'s eigenbasis. The payoff of country i is

$$U_i = \underbrace{-\left[\alpha\,(a_i-\theta_i)^2 + \sum_j G_{ij}\,(a_i-a_j)^2\right]}_{\text{First term (beauty contest)}} + \underbrace{2\left(\sum_j \omega_{ij}\right)(a_i-\theta_i)}_{\text{Second term (influence received)}} - \underbrace{\kappa\sum_k \omega_{ki}^2}_{\text{Third term (influence exerted)}}. \tag{29}$$

Preliminaries. Let $\boldsymbol{G} = \sum_m \lambda_m \boldsymbol{u}_m \boldsymbol{u}_m^\top$ with $\{\boldsymbol{u}_m\}$ orthonormal. Define $\tilde{\theta}_m = \boldsymbol{u}_m^\top \boldsymbol{\theta}$ and $\widetilde{\boldsymbol{\Sigma}}_\theta = \mathrm{Diag}(\sigma_1^2, \ldots, \sigma_N^2)$, so that $\mathbb{E}\left[\tilde{\theta}_m \tilde{\theta}_p\right] = \sigma_m^2\,\mathbf{1}_{m=p}$. The equilibrium action in mode m is $\tilde{a}_m = L_m\,\tilde{\theta}_m$ where

$$L(\lambda) \equiv \frac{(\kappa\alpha - 1)(1-\lambda)}{\kappa(1-\lambda)^2 - \alpha}. \tag{30}$$

The equilibrium net influence received, $\sum_j \omega_{ij}$, has mode-m projection $\Phi_m\,\tilde{\theta}_m$ where

$$\Phi(\lambda) \equiv \frac{\alpha^2 - (1-\lambda)^2}{\kappa(1-\lambda)^2 - \alpha} = (1-\lambda)\,L(\lambda) - \alpha, \tag{31}$$

the second equality following from $\sum_j \omega_{ij} = [(\boldsymbol{I} - \boldsymbol{G})\,\boldsymbol{a} - \alpha\boldsymbol{\theta}]_i$. In country coordinates:

$$a_i - \theta_i = \sum_m (L_m - 1)\,\tilde{\theta}_m\,u_{im}, \tag{32}$$

$$\sum_j \omega_{ij} = \sum_m \Phi_m\,\tilde{\theta}_m\,u_{im}. \tag{33}$$

Overlap tensor. Define

$$C_{mpq} \equiv \sum_{k=1}^{N} u_{km}\,u_{kp}\,u_{kq}. \tag{34}$$

The expansion identity $u_{im}\,u_{ip} = \sum_q C_{mpq}\,u_{iq}$ and the $(\boldsymbol{I} - \boldsymbol{G})$-identity

$$u_{im}\,u_{ip} - \sum_j G_{ij}\,u_{jm}\,u_{jp} = \sum_q C_{mpq}\,(1-\lambda_q)\,u_{iq} \tag{35}$$

will be used throughout.

First term (beauty contest). From (32):

$$\mathbb{E}\left[\alpha\,(a_i-\theta_i)^2\right] = \alpha\sum_m \sigma_m^2\,(L_m-1)^2\,u_{im}^2.$$

For the coordination loss, $a_i - a_j = \sum_m L_m\tilde{\theta}_m\,(u_{im} - u_{jm})$, so

$$\mathbb{E}\left[\sum_j G_{ij}\,(a_i-a_j)^2\right] = \sum_m \sigma_m^2 L_m^2 \sum_j G_{ij}\,(u_{im}-u_{jm})^2.$$

Expanding $\sum_j G_{ij}\left(u_{im}-u_{jm}\right)^2$ and applying $\sum_j G_{ij}u_{jm}^2=\sum_q C_{mmq}\lambda_q u_{iq}$:

$$\sum_j G_{ij}\left(u_{im}-u_{jm}\right)^2=\left(1-\alpha-2\lambda_m\right)u_{im}^2+\sum_q C_{mmq}\lambda_q\, u_{iq}. \tag{36}$$

Second term (influence received). From (32) and (33), using $\mathbb{E}\left[\tilde{\theta}_m\tilde{\theta}_p\right]=\sigma_m^2\,\mathbf{1}_{m=p}$:

$$\mathbb{E}\left[2\left(\sum_j \omega_{ij}\right)(a_i-\theta_i)\right]=2\sum_m \sigma_m^2\,\Phi_m\left(L_m-1\right)u_{im}^2. \tag{37}$$

Third term (influence exerted). The bilateral FOC for the optimal influence ω_{ki} exerted by i on k is

$$\kappa\omega_{ki}=\sum_j G_{ij}\left(a_i-a_j\right)\left[\left(\boldsymbol{I}-\boldsymbol{G}\right)^{-1}\right]_{jk}+\mathbf{1}_{k=i}\left(a_i-\theta_i\right).$$

Define $B_{ip}\equiv\kappa\boldsymbol{\omega}_{i\cdot}^{\top}\boldsymbol{u}_p$. By Parseval, $\kappa^2\sum_k\omega_{ki}^2=\sum_p B_{ip}^2$. Projecting the FOC onto $\boldsymbol{u}_p$, substituting the spectral expansions, and applying (35):

$$B_{ip}=\sum_m\tilde{\theta}_m\sum_q C_{mpq}\left[\frac{L_m\left(1-\lambda_q\right)}{1-\lambda_p}-1\right]u_{iq}. \tag{38}$$

Taking expectations:

$$\mathbb{E}\left[\kappa\sum_k\omega_{ki}^2\right]=\frac{1}{\kappa}\sum_m\sigma_m^2\sum_p\left(\sum_q C_{mpq}\left[\frac{L_m\left(1-\lambda_q\right)}{1-\lambda_p}-1\right]u_{iq}\right)^2. \tag{39}$$

Combining. $\mathbb{E}\left[U_i\right]=\sum_m\sigma_m^2\,\Gamma_{im}$ where

$$\begin{aligned}\Gamma_{im}=\;&\underbrace{-\alpha\left(L_m-1\right)^2u_{im}^2-L_m^2\left[\left(1-\alpha-2\lambda_m\right)u_{im}^2+\sum_q C_{mmq}\lambda_q\,u_{iq}\right]}_{\text{First term (beauty contest)}}\\&+\;\underbrace{2\,\Phi_m\left(L_m-1\right)u_{im}^2}_{\text{Second term (influence received)}}\\&\underbrace{-\frac{1}{\kappa}\sum_p\left(\sum_q C_{mpq}\left[\frac{L_m\left(1-\lambda_q\right)}{1-\lambda_p}-1\right]u_{iq}\right)^2}_{\text{Third term (influence exerted)}}.\end{aligned} \tag{40}$$

Interpretation. The first term captures the beauty-contest payoff: both the cost of deviating from national interest ($\alpha\left(L_m-1\right)^2$) and the coordination loss with neighbors. The coordination loss

couples eigenmodes through the overlap tensor C_{mmq}, reflecting how country i's squared loading on cleavage m relates to its neighbors' loadings on other cleavages. The second term captures the benefit of influence received: $\Phi_m > 0$ when $(1 - \lambda_m) < \alpha$ (modes where coordination pressure is strong), so influence receipt partially offsets the beauty-contest distortion. The third term is the cost of influence exerted, which couples all eigenmodes through the products C_{mpq}—even though equilibrium actions decouple mode by mode, the quadratic cost of bilateral influence introduces cross-mode interactions.