Innovation Spillovers Along the Supply Chain

(Link to Most Recent Version)

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Abstract

This paper develops a framework to study how innovation propagates through dynamic supply chain relationships. I document that supply chain links shape R&D investment and that innovation influence relationship formation. I build a dynamic heterogeneous-firm model with endogenous many-to-many supply chain relationships and innovation in general equilibrium. Complementarity in input quality generates positive sorting and spillovers that increase innovation incentives and widen disparities between high- and low-quality firms. Using quasi-exogenous variation from R&D tax credits, I show that partner innovation predicts increases in R&D, firm value, and returns to R&D. In the calibrated model, bidirectional spillovers across endogenous supply chains account for one-fifth of aggregate output.

Keywords: Innovation, supply chains, product complementarity, spillover effect, AI

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1 Introduction

Understanding the drivers of economic growth and innovation is of significant interest for academic researchers and policymakers. A main takeaway from the innovation literature is that technology spillovers across firms are critical for understanding innovation and firm outcomes. The insight that innovation spillovers propagate as firms interact and learn from each other has led to a large focus on how horizontal strategic interactions and innovation spillovers from learning across competitors impact innovation and rents (Romer (1986); Grossman and Helpman (1993); Aghion and Howitt (1992); Klette and Kortum (2004); Aghion et al. (2005)). In contrast, much less attention has been paid to how innovation transmits *vertically* through dynamic complementary relationships across the production network and how innovation impacts the development and persistence of these relationships.

In this paper, I fill this void by developing a novel theoretical framework to study innovation spillovers across the supply chain which emphasizes the dual importance of dynamic supply chain relationships for determining firm value and facilitating innovation spillovers and of innovation itself in shaping supply chain formation in general equilibrium. In the model, demand for products increase with the quality of inputs along the supply chain.Hence, firm valuation depends not only on a firm's own innovation but also on the intangible capital embodied in its strategic supply chain relationships. Complementarity in quality generates positive sorting between supply chain partners. Innovation spillovers arise because when firms innovate, quality improvements of inputs increase industry profits and valuations in sectors connected along the supply chain by increasing demand for inputs, particularly high quality inputs, but widen the difference in profits and valuations between high and low quality firms. This induces both supply chain partners and competitors to innovate further in order to improve product quality.

The model not only rationalizes vertical spillovers accounts for an important fraction of aggregate R&D and output.

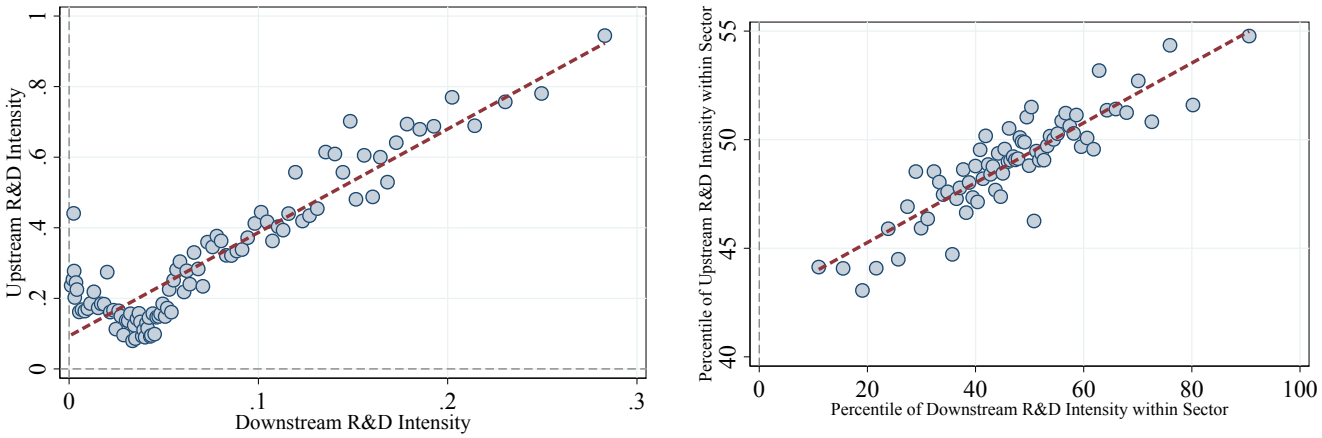
The reliance of modern production on interactions between supply chain partners underscores the importance of understanding innovation spillovers across these complementary relationships. Products often contain innovative inputs from multiple firms and thus embody innovation across several interconnected firms. Indeed, many firms at the technology frontier stress that their competitive advantage depends, in part, on their supply chain relationships.[1] The importance of innovation spillovers along the supply chain as illustrated by my model is evident when we consider the example of recent technological advances in artificial intelligence (AI). Because training AI models requires large amounts of computer processing power, firms such as Open AI, Google, and others increased demand for high-performance computer processors, propelling Nvidia's outsized success in the semiconductor sector.[2] In the other direction, earlier innovations in computer processing by Nvidia and its peers accelerated AI innovation and ultimately contributed to the commercial success of AI firms, highlighting the importance of strategic complementarities in innovation along the supply chain.[3]

Similar patterns underscore the importance of supply chains for innovation and firm value across a broad range of industries. Using over 80,000 annual observations on supplier-customer relationships among public U.S. firms spanning from 1980 to 2018, I document that firms' innovation closely co-moves with the innovation of their supply chain partners (left panel of Figure 1) Moreover, firms exhibit positive sorting in innovation intensity between supply chain partners. Firms with higher innovation intensity, relative to their peers, tend to match with more innovative partners, suggesting

[1]" Apple will partner with TSMC, Samsung, Corning & Texas Instruments in ambitious US$600bn American Manufacturing Program spanning iPhone chips to AI servers " (Apple Inc. (2025); Darley (2025))

[2]" Nvidia's profit jumps 80% as company rides tech's A.I. boom " (Mickle (2025))

[3]" Nvidia chips make gains in training largest AI systems " (Nellis (2025))



(A) Upstream/Downstream R&D Intensity (B) Upstream/Downstream Within-Sector Percentile

FIGURE 1 – Relationship between Upstream and Downstream Innovation

Note: Using annual firm-level data on customer-supplier relationships and financials collected from Compustat and FactSet (1980–2018), Panel A compares firm R&D intensity (R&D expenditure / lagged sales) with the average R&D intensity of downstream supply chain partners. Firm-year observations are binned into 100 groups based on the R&D intensity of downstream partners. Panel B compares the percentile of R&D intensity within the SIC3 sector in each year for downstream and upstream partners. Supply chain relationships observed annually are binned into 100 groups based on the percentile rank of the downstream firm.

that not only do supply chains impact innovation, but that, conversely, innovation plays a role in supply chain formation (right panel of Figure 1).

Motivated by this, I develop a tractable, dynamic heterogeneous-firm model in general equilibrium that endogenizes both the intensity of supply chain relationships and firms' innovation decisions. The model builds on standard ingredients from classic innovation models (Grossman and Helpman (1991); Aghion and Howitt (1992)): firms compete monopolistically, producing differentiated goods with linear technology and innovating to improve expected product quality and increase profits. To capture the role of endogenous supply chains in shaping innovation spillover, I add three new ingredients. First, firms compete monopolistically in either the upstream or downstream sector. Downstream firms produce differentiated products using their own blueprint and a basket of upstream inputs purchased from the upstream sector. Critically, demand of products ultimately depends on the *quality* of upstream and downstream inputs. This is different from standard production network models which aggregate quantities.

Second, to tractably model many-to-many supply chain relationships and upstream market power, downstream firms in the model optimally allocate across many imperfectly substitutable upstream inputs, where increasing their allocations to higher quality upstream inputs increases the overall quality of their products. Hence, the *intensity* of each supply chain relationship varies endogenously with the quality of both the upstream and downstream firms and the price of the upstream input. Third, to capture the idea that product quality is shaped by the quality of all inputs used in production, final demand for downstream products increases not only with the downstream firm's own quality, but also with the quality of the bundle of upstream inputs it embeds into its product, and these qualities also combine with complementarity.

In the model, innovation by upstream and downstream firms propagate through two primary mechanisms. First, the endogeneity of the supply chain relationships gives rise to *sorting*. In particular, complementarity in the quality of inputs within a product (across sectors) and substitutability in competing inputs (within sectors) are sufficient to induce positive sorting: higher quality downstream firms to optimally choose higher quality upstream inputs. Expanding the range in upstream quality from which downstream firms choose increases downstream profits (particularly for firms at the extremes of the quality distribution) and downstream innovation overall, but at the cost of increased upstream markups. Second, relationships along the supply chain generate *spillovers* since quality improvements by supply chain partners increase overall demand and returns from innovation. These two mechanisms—spillover and sorting—interact to generate heterogeneity in innovation spillover across the firm quality distribution and create a feedback effect from bidirectional spillovers across sectors.

Intuitively, from the perspective of downstream firms, improvements to upstream quality have two *spillover* effects: Higher upstream quality increases demand and average downstream profits, but inputs on average become more costly. If inputs are complementary in quality, high quality

downstream firms experience greater increases in demand from upstream quality improvements compared to low quality downstream firms. For low quality firms, smaller increases in demand may not offset increased costs, counterintuitively reducing profits.

Sorting exacerbates the extent which quality improvements upstream *heterogeneously* benefit higher quality downstream firms. Increasing the proportion of high quality upstream firms offers more choice across high quality inputs, effectively reducing the prices of inputs chosen by high quality downstream firms. In contrast, upstream bundles by lower quality downstream firms become more costly, so these firms may optimally reduce the quality of their upstream bundles, further reducing profits. Moreover, upstream quality improvements incentivize *downstream innovation*, both in the intensive margin, where currently innovating firms innovate more, and in the extensive margin, where higher quality downstream products now become commercially viable.

In the other direction, from the perspective of upstream firms, an increase in downstream quality improves demand for downstream products and thus raises overall demand for upstream inputs, increasing *upstream innovation*. In the model, sorting is crucial for explaining *heterogeneity* in these innovation spillovers: as downstream firms improve, they reallocate demand away from low quality inputs toward high quality upstream inputs, increasing prices for high quality inputs and depressing prices for low quality inputs. Consequently, downstream quality improvements benefit high quality upstream firms but may erode profits for low quality upstream firms.

Taken together, these bidirectional spillovers generate a *feedback effect*: upstream quality improvements induce downstream firms to innovate more, increasing downstream quality, which in turn incentivizes upstream firms to further innovate, amplifying the effect of a single upstream or downstream innovation shock. In general equilibrium, the upstream and downstream quality distributions together determine how profits vary with quality, and hence expectations over both distributions across future periods determine optimal innovation decisions by upstream and down-

stream firms. These innovation policies, in turn, endogenously dictate how the distributions of upstream and downstream quality evolve.

This feedback effect from vertical innovation spillovers also offers new insights into horizontal innovation spillovers. In the short run, quality improvements by competitors erode profits not only by decreasing demand but also by increasing input costs. In the long run, these quality improvements induce innovation by supply chain partners, which then feeds back to the original sector, increasing innovation and widening the gap between high and low quality firms in both sectors. In this way, even without knowledge spillovers from learning across competitors, the model generates strategic complementarities in innovation between competitors because high quality firms benefit from innovations by competitors that encourage shared supply chain partners to innovate.

I corroborate the main predictions of the model using firm-level evidence from U.S. public firms. Using exogenous variation in R&D expenditure arising from firm-level exposure to state and federal R&D tax credits, following Wilson (2009); Bloom et al. (2013); Arque-Castells and Spulber (2022), to account for correlated shocks unrelated to the supply chain relationship, I show that innovation spillovers propagate across supply chains, increasing innovation, firm value, and returns to R&D. To disentangle spillovers through the supply chain network that are attributable to trade relationships from those arising through knowledge spillovers, I utilize the fact that within an individual supply chain relationship, the intensity of the trade relationship may differ from the technological relationship, as proxied for using the patent citation network.

I find that trade relationships are critical for innovation spillovers, even after controlling for knowledge spillovers. A 1 p.p. increase in innovation from a firm's downstream network corresponds to a 1.26 p.p. increase in the firm's R&D intensity attributable to trade relationships and a 0.27 p.p. increase attributable to knowledge spillovers. Innovation spillovers from supply chain partners also predict increases in market valuation, as measured by Tobin's Q, and returns to R&D.

Consistent with supply chain relationships being endogenous and innovation being forward looking, I find that innovation propagates not only from current partners but also from potential partners. Each percentage point by which the innovation intensity of a firm's current downstream partners exceeds that of its downstream network corresponds to an additional 0.85 p.p. and 0.18 p.p. increase due to trade relationships and knowledge spillovers, respectively. Innovation spillovers also transmit heterogeneously: firms that do not innovate do not experience increases in market value from innovations by their supply chain partners. I further validate the robustness of these results by examining the effects of realized R&D outcomes as measured through patent issuances and corresponding market valuations, following Kogan et al. (2017), and find evidence that innovation spillovers by supply chain partners predict returns.

Finally, I use the model to quantify the importance of sorting and feedback arising from bidirectional spillovers accounts for about one-fifth of aggregate output and two-thirds of aggregate R&D.

The two channels interact to amplify spillovers along the supply chain. Without feedback in innovation across the supply chain, sorting reduces output because higher quality upstream firms can extract greater rents. Sorting improves the allocation of R&D investment. Without sorting, feedback along the supply chain is less effective because higher quality firms are not matching more with higher quality partners. Finally, I show innovation spillovers along the supply chain influence the propagation of fiscal and monetary policy: feedback between supply chain partners increases the impact of R&D tax credits for output growth five-fold and doubles the impact of a discount rate shock on output.

The paper proceeds as follows: Section 2 introduces the model. Section 3 describes the model results. Section 4 validates the main predictions of the model empirically. Section 5 describes the quantitative exercises. Section 6 concludes.

Related Literature.

This paper contributes to research on spillovers along supply chains. A large literature studies how shocks propagate and are amplified across interconnected sectors (Long and Plosser (1983); Horvath (2000); Cohen and Frazzini (2008); Atalay et al. (2011); Foerster et al. (2011); Acemoglu et al. (2012, 2016a); Atalay (2017); Herskovic (2018); Oberfield (2018); Bigio and La'O (2020)) and across individual supply chain relationships (Javorcik (2004); Cohen and Frazzini (2008); Barrot and Sauvagnat (2016); Dou et al. (2022); Grieser et al. (2024)). I add to literature to show that innovation shocks, in particular, propagate along the production network both at the sector level and through individual supply chain relationships, generating sizable spillovers. My mechanism links micro-level firm relationships to sector-level propagation of innovation shocks, providing a cohesive way to consider spillovers across both horizontal competitive relationships and vertical supply chain relationships.

While prior work generally finds spillovers between supply partners to be positive, I document that innovation spillovers along the supply chain propagate heterogeneously, only benefitting innovative, high quality firms. I provide a mechanism by which innovations by supply chain partners can instead erode firm profits. Moreover, consistent with relationships being endogenous, I show that spillovers propagate not only along existing trade links but also potential relationships.

As in Boehm et al. (2019); Titman (2021); Elliott et al. (2022), and Aral et al. (2025), I find supply chains are an important source of risk to firms. I show that these risks arise not only because supply chain partners produce specialized necessary inputs, but also because innovations by supply chain partners incentivize innovation among competitors and erode profits for firm producing lower quality inputs, hence not only increasing the returns but raising the stakes from innovation.

The paper contributes to the large literature on innovation spillovers and technology diffusion (Ellison et al. (2010); Bloom et al. (2013); Acemoglu et al. (2016b); Zacchia (2020); Han et al. (2024);

Herkenhoff et al. (2024)). Consistent with this work, I use patent citations to measure technological proximity and exploit variation from R&D tax incentives to identify causal effects as in Wilson (2009); Arque-Castells and Spulber (2022), and others. However, I highlight a mechanism distinct from knowledge spillovers: innovation by one firm changes the innovation incentives of its trading partners. My paper implies that some of the knowledge spillovers between competitors documented in earlier work may be attributable to overlapping shared suppliers or customers rather than by knowledge flows alone.

This paper contributes to a growing literature connecting innovation and production networks. Chu et al. (2019) document that innovation by one firm affects directly connected supply-chain partners; I similarly find firm-level spillovers, but I identify trade relationships and production coordination as the central channels through which innovation incentives shift. Relatedly, recent work using sector-level production network linkages (Acemoglu et al. (2023); Liu and Ma (2023)) shows that innovation shocks propagate across industries. I complement these papers by focusing on the endogenous innovation decision and jointly examining firm-level and sector-level spillovers, showing how heterogeneity in firms' innovation decisions and supply-chain positions shapes the distribution and magnitude of innovation spillovers in equilibrium. Dasaratha (2023) studies innovation when firms trade off gains from collaboration with benefits from secrecy. Casal (2025) considers a model where supplier firms can invest in either lock-in or productive technologies which reduce costs. I focus on innovations which generate quality improvements. In my model, higher quality inputs are less substitutable with lower quality inputs, and hence suppliers can differentiate themselves and generate persistent relationships through by improving the quality of their profit even without supply chain frictions.

Furthermore, the paper contributes to research on innovation, firm heterogeneity, and dynamic industry structure (Klette and Kortum (2004); Aghion et al. (2005); Acemoglu and Akcigit (2012);

Acemoglu et al. (2018)). I show that innovation shocks in one sector exacerbate firm concentration and innovation intensity in related sectors. In the model, firm growth endogenously slows with age as firms become more constrained by their supply chain partners.

I contribute to the large literature on heterogenous markups and product market competition (**?????**). In my paper, prices for upstream inputs arise endogenously and are increasing as downstream quality surpasses upstream quality. In addition, I provide a mechanism by which inputs of similar quality are more substitutable.

I contribute to the large literature on Schumpeterian growth, product market competition, and creative destruction (Schumpeter (1939); Aghion and Howitt (1992); Bustamante and Donangelo (2017); Garcia-Macia et al. (2019); Kakhbod et al. (2024)). I show innovation by supply chain partners intensify horizontal competition, by increasing the risk of creative destruction and technology obsolescence for producers of lower quality inputs and by increasing incentives for competitors to innovate.

My paper relates to a growing literature on intangible capital (Eisfeldt and Papanikolaou (2013); Crouzet and Eberly (2021); Ayyagari (2024); Jagannathan et al. (2025); Chen et al. (2025)). I find that supply chain relationships are an important source intangible capital to firms. Innovations by supply chain partners increase returns to investment in innovation and increase the value of capital embodied in past R&D expenditures.

My paper contributes to a large literature on specialized investments and hold-up problems between supply chain partners (Anjos and Fracassi (2018); Frésard et al. (2020); Bena et al. (2023)). Consistent with these and many other papers, firm investments in innovation do not fully internalize positive spillovers to their supply chain partners. I show theoretically that firms may stop innovating if the quality of their supply chain partners lags too far behind. In addition, my paper offers a useful setting to consider the benefits and costs to vertical integration and M&A and frictions on

contracting.

I contribute to the literature on endogenous supply chains (Acemoglu and Azar (2020); Kopytov et al. (2024)). I show innovation drives firm supply chain formation decisions and that evolving supply chain relationships amplify innovation shocks. Finally, my paper contributes to our larger understanding of endogenous sorting and feedback effects in economic relationships. A key insight from the matching literature is that relationships between agents are endogenous, creating identification challenges when estimating causal effects. Sørensen (2007) addresses this in venture capital markets by developing a two-sided matching model to separate the influence of VC expertise from sorting effects, while Gabaix and Landier (2008) use an assignment model to show how CEO-firm matching on complementary characteristics drives equilibrium pay. I build on this tradition by developing a parsimonious two-sided matching model to disentangle the impact of sorting from firm quality within the context of supply chains. A critical distinction in my framework is that firm quality itself is an endogenous outcome of innovation decisions, creating a dynamic feedback loop: innovation by supply chain partners impact sorting and incentivizes firm innovation. This relationship between sorting and feedback distinguishes my work from prior matching models where agent characteristics are treated as fixed.

2 Model

To characterize and quantify the mechanism by which innovations propagate along the supply chain, in this section I introduce a novel dynamic, heterogeneous-firm model which endogenizes both many-to-many supply chain relationships and firm innovation decisions in general equilibrium.

To illustrate how innovations spillover between firms in sectors connected through the supply chain, I introduce a simple supply chain structure, with two intermediate sectors, an upstream and downstream sector (e.g. a sector producing computer processors or "chips" and a sector producing

computers). Firms in each sector produce differentiated goods which vary in quality and firms can innovate to improve quality in expectation. In the model, downstream firms produce intermediate inputs along the supply chain is complementary in quality, higher quality downstream firms optimally choose a higher quality bundle of upstream inputs and hence tilt their allocation towards higher quality upstream inputs, endogenously generating positive assortative matching in quality.

These relationships are critical to understanding the spillover of innovation by supply chain partners to profits: due to the complementarity along the supply chain, innovation by supply chain partners largely benefits firms producing higher quality products. Moreover, quality improvements by supply chain partners not only impact current profits, but change firm incentives to innovate, resulting in strategic complementarity in innovation between upstream and downstream firms. Endogenous sorting between the supply chain partners exacerbates the impact of innovation spillovers. Feedback between sectors and sorting across firms incentivizes higher levels of innovation and amplifies the effect of aggregate shocks.

2.1 Agents and Technology

The model is a discrete time infinite horizon model, with two monopolistically competitive intermediate sectors and a perfectly competitive final good sector oriented vertically along the production network.

Sectors.

1. The upstream (chip) sector contains atomistic, monopolistically competitive firms indexed by $k \in K = [0, 1]$ which supply differentiated inputs of quantity x_{kt}^S to the downstream sector and vary in quality z_{kt}^S, where $\left\{z_{kt}^S\right\}_{k \in K} = \Phi_t^S$ is the distribution of quality of upstream firms in time t. Each unit of upstream input requires $\psi^S z_{kt}^S$ of the final good.

2. The downstream (computer) sector contains atomistic, monopolistically competitive firms

indexed by $i \in I = [0,1]$ which supply differentiated goods of quantity x_{it} to the final goods sector and vary in quality z_{it}^C, where $\left\{z_{it}^C\right\}_{i\in I} = \Phi_t^C$ is the distribution of quality of downstream firms in time t. Each unit of downstream intermediate good requires $\psi^C z_{it}^C$ of the final good and measure 1 units of the upstream input.

3. The final goods sector contains a continuum of firms. Firms produce using downstream intermediate goods and labor and operate perfectly competitively to produce the final good.

Representative Household. Given wages w_t and dividends d_{it}^C and d_{kt}^S from downstream and upstream firms, a representative household which owns the firms supplies labor L inelastically and consumes the final good where

$$C_t \leq w_t L + \int_0^1 d_{it}^C di + \int_0^1 d_{kt}^S dk$$

and have discount factor $\beta \in (0,1)$.

Final Goods Sector. Final goods, Y_t, are produced by perfectly competitive firms with Cobb Douglas production, using labor L_t and a basket of intermediate downstream goods $\{x_{it}\}_{i\in I}$ where

$$Y_t = \frac{\epsilon}{\epsilon - 1}\left(\int_0^1 z_{it}\left(x_{it}\right)^{\frac{\epsilon-1}{\epsilon}} di\right) L_t^{\frac{1}{\epsilon}}.$$

Intermediate goods produced by the downstream sector are assumed to be substitutes with elasticity $\epsilon > 1$. Given wages w_t and downstream prices $\{p_i\}_{i\in I}$, the final good producers choose downstream intermediate goods and labor to solve a static profit maximization problem,

$$\max_{L_t, \{x_{it}\}_{i\in[0,1]}} Y_t - \int_0^1 p_i x_{it} di - w_t L_t.$$

Downstream Sector. Firms in the downstream (computer) sector indexed by i vary in quality z_{it}^C and supply differentiated intermediate goods to the final good sector. Producing one unit of the intermediate good requires $\psi^C z_{it}^C$ of the final good, consistent with higher quality blueprints being more costly to produce, and a measure 1 of the upstream input. Downstream production requires upstream units where quantity x_{it} is produced according to production function,

$$x_{it} = \min\left\{\frac{y_{it}^C}{\psi^C z_{it}^C}, x_{it}^S\right\},$$

where y_{it}^C is the quantity of final good used, $x_{it}^S = \int_0^1 x_{ikt}^S dk$ is the total quantity sourced from upstream firms, and x_{ikt}^S is the quantity demanded by downstream firm i of firm k's upstream input. Upstream (chip) inputs are perfectly substitutable in quantity, but will vary in quality and be imperfectly substitutable.

The downstream firm faces demand $x_{it} = \left(\frac{z_{it}}{p_{it}}\right)^{\epsilon} L_t$ from final good producers. Critically, demand for the downstream product increases with overall product quality,

$$z_{it} = \left(\kappa_C \left(z_{it}^C\right)^{\frac{\nu-1}{\nu}} + \kappa_S \left(\bar{z}_{it}^S\right)^{\frac{\nu-1}{\nu}}\right)^{\frac{\nu}{\nu-1}},$$

which depends on the firm's own quality z_{it}^C and the quality of its chosen bundle of inputs $\bar{z}_{it}^S$.

If the quality of inputs along the supply chain is complementary ($\nu < 1$), then higher quality upstream bundles will increase demand more for when paired with higher quality downstream blueprints than with lower quality downstream blueprints. Intuitively, this reflects the idea that a superior screen to a phone is most appreciated with a fast processor and an efficient battery.

The quality of the bundle of upstream inputs, $\bar{z}_{it}^S$, is a CES aggregate of the quality of inputs

sourced from the upstream sector,

$$\bar{z}_{it}^{S} = \left[\int_0^1 \left(w_{ikt} z_{kt}^{S} \right)^{\frac{\zeta-1}{\zeta}} dk \right]^{\frac{\zeta}{\zeta-1}},$$

where z_{kt}^{S} is the quality of upstream firm k in time t and $\zeta > 1$, reflecting a preference for variety over substitutable upstream inputs, and weight $w_{ikt} = x_{ikt}^{S}/x_{it}^{S}$ is the share of the basket allocated to inputs from firm k chosen by downstream firm i, so that $\int_0^1 w_{ikt} dk = 1$.

The downstream firm indexed by $i \in I$ takes upstream prices $\left\{p_{kt}^{S}\right\}_{k\in K}$ and qualities $\left\{z_{kt}^{S}\right\}_{k\in K}$ as given and chooses unit price p_{it}, share of final goods y_{it}^{C}, and upstream inputs $\left\{x_{ikt}^{S}\right\}_{k\in K}$ to maximize profits,

$$\pi^{C}\left(z_{it}^{C} \mid \Phi_t^{C}, \Phi_t^{S}\right) = \max_{p_{it}, y_{it}^{C}, \left\{x_{ikt}^{S}\right\}_{k\in K}} p_{it} x_{it} - y_{it}^{C} - \int_0^1 p_{kt}^{S} x_{ikt}^{S} dk, \tag{2.1}$$

Given the production function, the profit maximization problem of the downstream profits can instead be expressed as a choice over unit price p_i and upstream weights $\{w_{ikt}\}_{k\in K}$, such that

$$\pi^{C}\left(z_{it}^{C} \mid \Phi_t^{C}, \Phi_t^{S}\right) = \max_{p_{it}, \{w_{ikt}\}_{k\in K}} x_{it}\left(p_{it} - \psi^{C} z_{it}^{C} - \bar{p}_{it}^{S}\right) \text{ st } \int_0^1 w_{ikt} dk = 1, \tag{2.2}$$

where $w_{ikt} = x_{ikt}^{S}/x_{it} \in [0,1]$ and $\bar{p}_{it}^{S} = \int_0^1 w_{ikt} p_{kt}^{S} dk$.

Upstream Firms. Upstream firms indexed by $k \in K$ vary in quality z_{kt}^{S} and produce using linear production at unit cost $\psi^{S} z_{kt}^{S}$ (again, consistent with high quality products being more costly to produce). Quantity x_{kt}^{S} is produced with linear production function, $x_{kt}^{S} = \frac{y_{kt}^{S}}{\psi^{S} z_{kt}^{S}}$. where y_{kt}^{S} is the quantity of the final good used in production. Upstream firms similarly compete monopolistically and choose single price p_{kt}^{S} to maximize profits in period t given downstream demand $x_{kt}^{S} = \int_0^1 x_{ikt}^{S} di$.

Hence, they solve profit maximization problem

$$\pi^S\left(z_{kt}^S \mid \Phi_t^C, \Phi_t^S\right) = \max_{p_{kt}^S}\left(p_{kt}^S - \psi^S z_{kt}^S\right) x_{kt}^S \tag{2.3}$$

Critically, downstream firms will vary in their price elasticity of demand for upstream inputs and hence the price elasticity of demand faced upstream firms will depend on both the upstream and downstream quality distribution.

Innovation. Firms operate over multiple periods and discount the future given discount factor, $\beta \in (0,1)$, and exogenous probability of death υ. Intertemporally, downstream firm's quality z_{it}^C is endogenous. A downstream firm in period t with quality z_{it}^C realizes period $t+1$ potential quality, $\widehat{z}_{it+1}^C = z_{it}^C + z_{it}^C d\Theta_{it}^C$, where $d\Theta_{it}^C$ is drawn from Poisson distribution with mean $\alpha_i\left(\frac{\eta_{it}^C}{z_{it}^C}\right)^{\xi}$, where η_{it}^C is innovation spending by the firm in units of final good, and $\xi \in (0,1)$, reflecting decreasing returns to innovation. Note that both upstream and downstream profits depend not only on their own quality, but also jointly on the upstream and downstream distribution Φ_t^S and Φ_t^C. Hence, the value function of downstream firm i in period t can be denoted as $V^C\left(z_{it}^C \mid \Phi_t^C, \Phi_t^S\right)$ and similarly the upstream firm k in period t denoted as $V^S\left(z_{kt}^S \mid \Phi_t^C, \Phi_t^S\right)$.

In the model, downstream profits depend on upstream quality and may in fact not be monotonic in downstream quality, z_{it}^C. Hence, downstream value function $V^C\left(z_{it+1}^C \mid \Phi_{t+1}^C, \Phi_{t+1}^S\right)$ may also be non-monotonic in quality. To ensure that future value $V^C\left(z_{it+1}^C \mid \Phi_{t+1}^C, \Phi_{t+1}^S\right)$ is increasing in innovation η_{it}^C, firms can choose next period's quality $z_{it+1}^C \in \left[z_{it}^C, \hat{z}_{it+1}^C\right]$ to maximize firm valuation such that

$$z_{it+1}^C \in \arg\max_{z_{it+1} \in \left[z_{it}^C, \hat{z}_{it+1}^C\right]} V^C\left(z_{it+1}^C \mid \Phi_{t+1}^C, \Phi_{t+1}^S\right).$$

Given the probability of survival $1-\upsilon$, the downstream value function can be expressed as

$$V^C\left(z_{it}^C \mid \Phi_t^C, \Phi_t^S\right) = \max_{i_{it}} \pi^C\left(z_{it}^C \mid \Phi_t^C, \Phi_t^S\right) - i_{it}^C z_{it}^C + E_t\left[(1-\upsilon)\,\beta V^C\left(z_{it+1}^C \mid \Phi_{t+1}^C, \Phi_{t+1}^S\right)\right]$$

where β is the discount factor, and $i_{it}^C = \eta_{it}^C / q_{it}^C$ is the innovation intensity of the downstream firm. Upstream firms with quality z_{kt}^S can similarly invest η_{kt}^S in quality-improving innovation and solve the following value function

$$V^S\left(z_{kt}^S \mid \Phi_t^C, \Phi_t^S\right) = \max_{i_{kt}^S} \pi^S\left(z_{kt}^S \mid \Phi_t^C, \Phi_t^S\right) - i_{kt}^S z_{kt}^S + E_t\left[(1-\upsilon)\,\beta V^S\left(z_{kt+1}^S \mid \Phi_{t+1}^C, \Phi_{t+1}^S\right)\right]$$

where $i_{kt}^S = \eta_{kt}^S / q_{kt}^S$ is the innovation intensity of the upstream firm.

Aggregation. In each period, t, the following markets clear:

1. Final good market

$$C_t + \int_0^1 \psi^C z_{it}^C di + \int_0^1 \psi^S z_{kt}^S dk + \int_0^1 i_{it}^C z_{it}^C di + \int_0^1 i_{kt}^S z_{kt}^S dk = Y_t$$

2. Labor market: $L_t = L$

3. Market for intermediate goods produced by downstream firms:

   For each downstream i in period t, the downstream firm i produces x_{it} which is used to produce the final good.

4. Market for inputs produced by upstream firms: $\int_0^1 x_{ikt}^S di = x_{kt}^S,\ \forall k \in K$

Stationary equilibrium and exogenous entry and exit. Every period, a fraction υ of firms in each sector exit with residual value 0 and are replaced with an equal measure of upstream and

downstream firms which begin with quality z^C_{min}or z^S_{min}, respectively. The stationary equilibrium can be characterized by distributions Φ^C and Φ^S, quantities $\{x_{it}\}_{i\in I}, \{x^S_{ikt}\}_{i\in I,k\in K}, \{x^S_{kt}\}_{k\in K}$, prices $\{p_{it}\}_{i\in I}, \{p^S_{kt}\}_{k\in K}$, and innovation intensities $\{i^C_{it}\}_{i\in I}, \{i^S_{kt}\}_{k\in K}$ such that

1. Each downstream firm $i \in I$ with productivity z^C_{it} chooses price p_{it}, quantity x_{it}, and upstream allocations$\{x^S_{ikt}\}_{k\in K}$ to solve its static profit maximization problem.

2. Each upstream firm $k \in K$ with productivity z^S_{kt} chooses price p^S_{kt} and quantity x^S_{kt} to solve its static profit maximization problem.

3. Innovation policy functions $i^C\left(z^C_{it} \mid \Phi^C, \Phi^S\right)$ and $i^S\left(z^S_{kt} \mid \Phi^C, \Phi^S\right)$ solve upstream and downstream value functions, respectively.

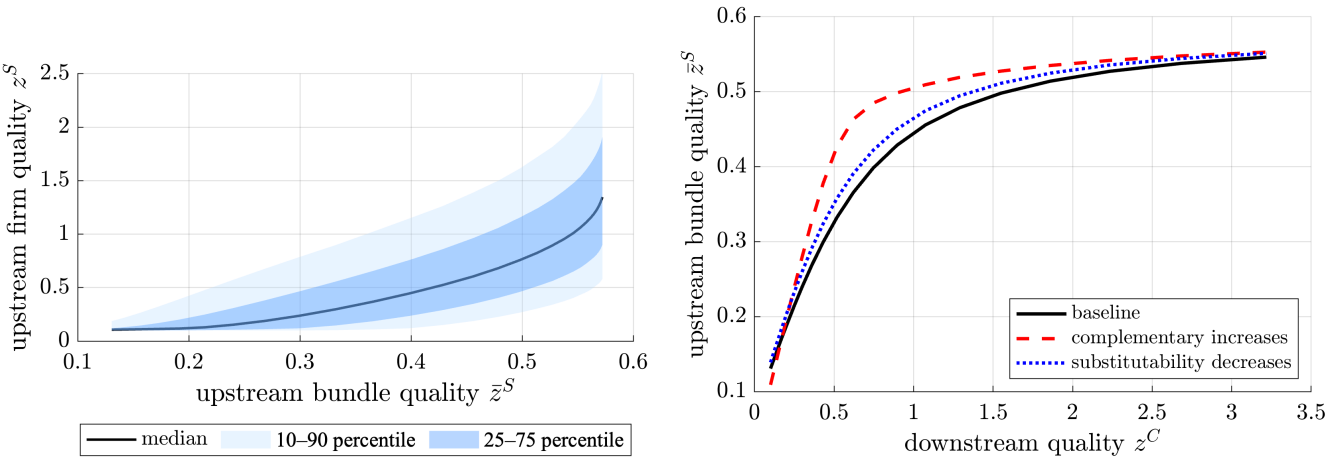
4. Distributions, Φ^C and Φ^S, evolve according to innovation policy functions, $i^{C*}\left(z^C_{it} \mid \Phi^C, \Phi^S\right)$ and $i^{S*}\left(z^S_{kt} \mid \Phi^C, \Phi^S\right)$, and given exogenous probability of exit υ. Consistent with some firms not innovating in the economy, $\alpha_i \in \{\alpha^L, \alpha^H\}$. A fraction of firms begin the period with $\alpha^L = 0$ and the remaining have skill $\alpha^H > 0$.

5. Markets for upstream and downstream goods clear.

3 Model Results

3.1 Static Equilibrium and Positive Assortative Matching

The downstream static profit maximization problem in Equation 2.2 provides insight into how demand for upstream inputs vary endogenously with both downstream quality, z^C_i, and upstream quality, z^S_k. For this subsection, I drop the period t subscript since all choices and state variables in the static problem occur in the same period. This is best illustrated in two steps: (1) Given target bundle quality, $\bar{z}^S$, how does a downstream firm choose weights $\{w_k\}_{k\in K}$ across upstream inputs



(A) Allocation Across Upstream Inputs (B) Downstream Optimal Upstream Quality

FIGURE 2 – Downstream Allocation

Note: For each desired quality bundle quality $\bar{z}^S$ on the x-axis, Panel A plots the median quality of the upstream inputs chosen by the downstream firm and the 10–90 and 25–75 quality percentiles. For each downstream quality z_i^C, Panel B plots the quality of its optimal bundle $\bar{z}^S$. The blue and green dotted lines correspond to increased complementarity and decreased downstream substitutability, respectively.

chosen and what is the resulting per unit price $\bar{p}^S$? (2) Given current downstream quality z_i^C, and a price schedule across potential upstream qualities, what is the firm's optimal bundle quality $\bar{z}_i^S$ and the resulting profits $\pi^C\left(z_i^C\right)$?

Definition 3.1. [Price Minimization Problem] Given upstream prices $\left\{p_k^S\right\}_{k\in K}$ and qualities $\left\{z_k^S\right\}_{k\in K}$, function g characterizes the minimum price needed to achieve target upstream bundle quality $\bar{z}^S$

$$g\left(\bar{z}^S\right) = \min_{\{w_k\}_{k\in K}} \int_0^1 w_k p_k^S dk \text{ st } \left[\int_0^1 \left(w_k z_k^S\right)^{\frac{\zeta-1}{\zeta}} dk\right]^{\frac{\zeta}{\zeta-1}} = \bar{z}^S \text{ and } \int_0^1 w_k dk = 1$$

where w_k is the relative weight allocated to upstream firm k's product and μ is the multiplier on constraint $\int_0^1 w_k dk = 1$.

In step one, definition 3.1, formally characterizes the price minimization problem of the downstream firm which chooses weights $\{w_k\}_{k\in K}$ to minimize the bundle price $\bar{p}^S = g\left(\bar{z}^S\right)$ for targeted quality $\bar{z}^S$ for a given upstream distribution and associated upstream prices. Critically, because downstream firms require measure one of the upstream input to produce a single unit, downstream

firms face the capacity restriction, $\int_0^1 w_k dk = 1$, which has shadow price μ. The restriction is consistent with the idea that a firm cannot simply add more inputs in a single unit to generate a higher quality bundle of inputs $\bar{z}^S$ (nor can it use less than a single unit of the upstream input in its product), and thus the maximum quality the downstream firm can attain is constrained by the upstream quality distribution. For example the production technology of automobiles requires only a single engine and steering wheel. An automobile company who wants to improve the quality of their product will need to include a better engine, not multiple engines. Without it, the maximum achievable $\bar{z}^S$is unbounded.

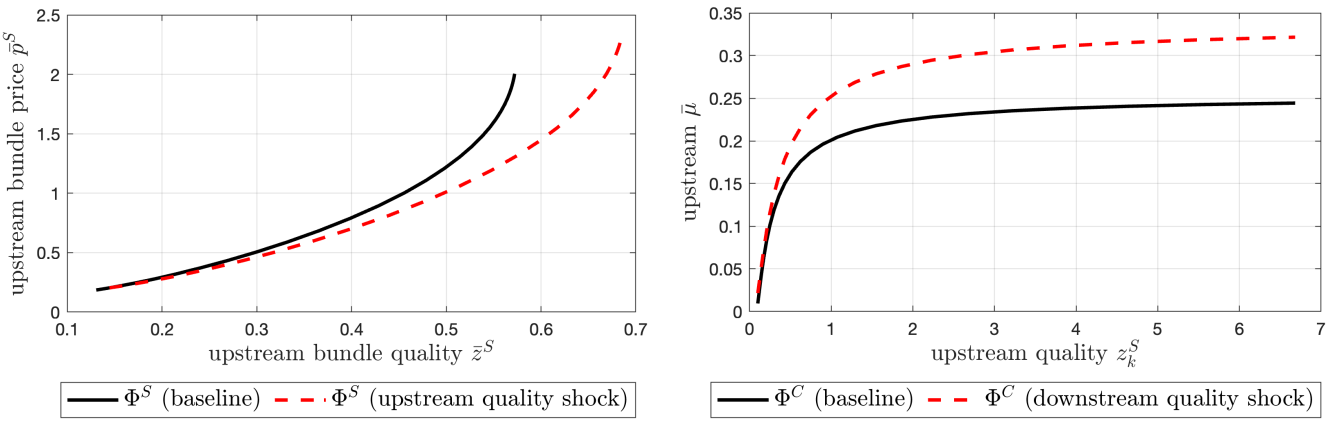
Proposition 3.2. *[Optimal Allocation of Upstream Inputs] Given upstream prices and qualities, downstream firm i will choose optimal weights such that*

$$\frac{w_{ik}}{w_{ij}} = \left(\frac{p_k^S + \mu_i}{p_j^S + \mu_i}\right)^{-\zeta} \left(\frac{z_k^S}{z_j^S}\right)^{\zeta - 1}$$

where μ_i is the shadow price of capacity constraint $\int_0^1 w_{ik} = 1$ associated with its desired upstream bundle quality, $\bar{z}^S$.

Proposition 3.3. *If $\zeta > 1$, then μ_i is increasing in $\bar{z}_i^S$. If upstream prices are increasing in quality, then w_{ik}/w_{ij} is increasing in (and hence in $\bar{z}_i^S$).*

By taking first order conditions of the price minimization problem, for downstream firm i and upstream firms $j, k \in K$, Proposition 3.2 characterizes relative weights w_{ik}/w_{ij}. Critically, if upstream inputs are imperfectly substitutable, $\zeta > 1$, and $p_k^S > p_j^S$, then relative allocation w_{ik}/w_{ij} is increasing in the multiplier on the capacity constraint, μ_i. Notably, μ_i is increasing in $\bar{z}_i^S$. As shown in Figure 2, when the downstream firm wants to achieve a higher quality bundle, the shadow price on the capacity restriction increases. If upstream prices are increasing in quality, this induces the firm to allocate greater weight to the higher quality input $z_k^S > z_j^S$. The capacity restriction



(A) Price of Upstream Bundle $g\left(\bar{z}^S\right)$ (B) $\bar{\mu}_k$ Faced by Upstream Firms

FIGURE 3 – Upstream Input Prices

Note: Panel A plots the unit price $\bar{p}^S = g\left(\bar{z}^S\right)$ for given upstream bundle quality $\bar{z}^S$ for the baseline distribution (black) and after an improvement to upstream quality (red). Panel B plots sufficient statistic $\bar{\mu}$ as a function of downstream quality (black) and after an improvement to downstream quality (red) where optimal price is $p_k^{S*} = \frac{\zeta}{\zeta-1}\psi^S z_k^S + \frac{1}{\zeta-1}\bar{\mu}_k$.

is critical to creating allocations which vary with $\bar{z}_i^S$, and as shown later, creating positive sorting under complementarity. Without the constraint, if $\mu_i = 0$, then all downstream firms would choose the same relative weights invariant to $\bar{z}_i^S$, and there would be no positive sorting. Product substitutability across upstream inputs also plays an important role: as ζ decreases and upstream inputs becomes less substitutable, relative weights w_{ik}/w_{ij} increase and sorting becomes more pronounced.

Proposition 3.4. *[Convexity of Upstream Price Schedule] If $\zeta > 1$, then $g', g'' > 0$.*

To understand firm i's optimal choice of upstream quality $\bar{z}_i^S$, given downstream quality z_i^C, I first establish Proposition 3.4—when upstream inputs are imperfectly substitutable, $\zeta > 1$, the price function g is both increasing and convex in $\bar{z}^S$, because, to achieve a higher quality bundle, $\bar{z}_i^S$, the downstream firm must allocate greater weight to higher quality inputs, but this comes with diminishing returns. Increases to the upstream quality distribution, decreases $\mu\left(\bar{z}^S\right)$ and flattens the price function $g\left(\bar{z}^S\right)$. Intuitively, achieving the same higher quality bundle is less expensive because the downstream firm suffers less from decreasing returns, and does not need to reach as much for quality since its optimal allocation is closer to the upstream distribution.

Proposition 3.5. *[Optimal Upstream Bundle Quality] Suppose $\epsilon > 1$, $\nu < 1$, and $g', g'' > 0$. Then firms choose $\bar{z}_i^S$ such that $MR\left(\bar{z}_i^S\right) = g'\left(\bar{z}_i^S\right)$ where $\Psi_i = \psi^C z_i^C + g\left(\bar{z}_i^S\right)$ is defined as the per unit cost and $MR\left(\bar{z}_i^S\right) = \frac{\epsilon}{\epsilon-1}\Psi_i \kappa_S \left(\bar{z}_i^S\right)^{-\frac{1}{\nu}} (z_i)^{\frac{1-\nu}{\nu}}$ is the per unit marginal return to increasing $\bar{z}_i^S$ and z_i^C. If $z_i^C > z_m^C$, then $\bar{z}_i^S > \bar{z}_m^S$.*

In the second step, given price schedule $g\left(\bar{z}_i^S\right)$, I consider firm i's optimal choice of upstream bundle $\bar{z}_i^S$ given its own quality z_i^C. Defining unit costs as $\Psi_i = \psi^C z_i^C + g\left(\bar{z}_i^S\right)$, and substituting optimal price $p_i = \frac{\epsilon}{\epsilon-1}\Psi_i$, the downstream profit function $\pi^C\left(z_i^C\right)$ can be expressed as

$$\pi^C\left(z_i^C\right) = \max_{\bar{z}_i^S} \left(\frac{\epsilon-1}{\epsilon}\right)^{\epsilon} \frac{1}{\epsilon-1} L\left(z_i\right)^{\epsilon} \left(\Psi_i\right)^{1-\epsilon}. \tag{3.1}$$

Per Proposition 3.5, the downstream firm will choose upstream quality $\bar{z}_i^S$ by setting the marginal return from increasing $\bar{z}_i^S$, $MR\left(\bar{z}_i^S\right)$ equal to marginal cost $g'\left(\bar{z}_i^S\right) > 0$,

$$MR\left(\bar{z}_i^S\right) = g'\left(\bar{z}_i^S\right).$$

Intuitively, after an increase in upstream quality $\bar{z}_i^S$, the downstream firm faces a trade off: higher quality inputs increase joint quality z_i and hence increase demand from the final goods sector, but higher quality inputs are increasingly more expensive (since g is convex). Due to complementarity, $(\nu < 1)$ between upstream and downstream inputs in joint quality z_i , the marginal return to increasing upstream quality $\bar{z}_i^S$, $MR\left(\bar{z}_i^S\right)$, is greater when downstream quality z_i^C is higher.

$$\frac{dMR\left(\bar{z}_i^S\right)}{dz_i^C} = \frac{\epsilon}{\epsilon-1} z_i^{-1} \kappa_S \left(\frac{z_i^S}{z_i}\right)^{-\frac{1}{\nu}} \psi^C + \frac{1-\nu}{\nu} p_i z_i^{-2} \kappa_S \kappa_C \left(\frac{z_i^S}{z_i}\right)^{-\frac{1}{\nu}} \left(\frac{z_i^C}{z_i}\right)^{-\frac{1}{\nu}} > 0.$$

Hence, provided that $\epsilon > 1$ and $\nu < 1$, as shown in Figure 2, higher quality downstream firms choose higher quality upstream bundles. This result is most pronounced when downstream products

become less substitutable (ϵ declines) or if complementarity increases (ν falls). If $\zeta > 1$ and upstream prices are increasing in upstream quality, this is sufficient to prove positive assortative matching—higher quality downstream firms allocate create unit weights to products by higher quality upstream firms.

Proposition 3.6. *[Upstream Optimal Price] Suppose $\zeta > 1$.*

Proposition 3.7. *(1) Upstream firm will choose optimal price $p_k^S = \frac{1}{\zeta-1}\bar{\mu}_k + \frac{\zeta}{\zeta-1}\psi^S z_k^S$ where sufficient statistic $\bar{\mu}_k$ reflects the shadow price on the capacity restriction $\{\mu_i\}_{i\in I}$ faced the firm's downstream customers,*

$$1 = \int_0^1 \frac{x_i w_{ik}}{x_k^S}\left[\frac{1}{\zeta} + \left(\frac{\zeta-1}{\zeta}\right)\left(\frac{\psi^S z_k^S + \mu_i}{\psi^S z_k^S + \bar{\mu}_k}\right)\right]^{-1} di$$

Proposition 3.8. *(2) p_k^S is increasing in z_k^S.*

Proposition 3.9. *(3) If all upstream firms are homogeneous in quality z_k^S, then $\mu_i = 0$ for all i and $p_k^S = \frac{\zeta}{\zeta-1}\psi^S z_k^S$.*

Next, I consider the static upstream profit maximization problem and the optimal characterized in Proposition 3.2. Upstream firm k which produces differentiated product with quality z_k^S, chooses prices p_k^S to maximize profits subject to demand $x_k^S = \int_0^1 x_{ik}^S dk$. Using 3.2 it is evident that because downstream firms face a capacity restriction, the elasticity of demand varies with quality z_i^C—higher quality downstream firms are less elastic,

$$-\frac{dx_{ik}^S/x_{ik}^S}{dp_k^S/p_k^S} = \zeta\frac{p_k^S}{p_k^S + \mu_i}.$$

Indeed, firm k's overall demand elasticity can be characterized as a quantity-weighted average of demand elasticities from all downstream firms: $\frac{dx_k^S/x_k^S}{dp_k^S/p_k^S} = \int_0^1 \frac{x_{ik}^S}{x_k^S}\left(\frac{dx_{ik}^S/x_{ik}^S}{dp_k^S/p_k^S}\right) di$. The upstream firm

which solves the profit maximization problem in Equation 2.3 will choose optimal price p_k^S such that

$$\frac{1}{\zeta} = \int_0^1 \frac{x_{ik}^S}{x_k^S} \left(\frac{p_k^S - \psi^S z_k^S}{p_k^S + \mu_i} \right) di \tag{3.2}$$

where p_k^S can be characterized as

$$p_k^S = \frac{1}{\zeta - 1} \bar{\mu}_k + \frac{\zeta}{\zeta - 1} \psi^S z_k^S \tag{3.3}$$

which is a function of sufficient statistic $\bar{\mu}_k$, which per Proposition 3.6, reflects an aggregate of shadow price capacity restriction $\{\mu_i\}_{i \in I}$ faced by the firm's customers. Recall that the shadow price μ_i reflects the extent the downstream firm is constrained by the upstream quality distribution. If overall, the upstream distribution lags in quality and downstream firms are more constrained, this will lead to greater prices, p_k^S, and markups, particularly for high quality upstream firms to which the downstream firms will target their demand. Intuitively, we can think of prices and profits being greatest when the upstream firm is of higher quality relative to their peers and there is greater demand from an even higher quality downstream distribution (consistent with Nvidia's performance following advances to AI technology). Observe that if $\mu_i = 0$ for all firms, either because there is no capacity restriction or because all upstream firms have equal quality, we return to the standard result: $p_k^S = \frac{\zeta}{\zeta - 1} \psi^S z_k^S$. Finally, for $\zeta > 1$, as it follows from equation 3.2 that upstream prices p_k^S are increasing in quality z_k^S.

Corollary 3.10. *[Positive Assortative Matching] Suppose $\epsilon > 1$, $\zeta > 1$ and $\nu < 1$. If $z_m^C > z_i^C$ and $z_k^S > z_j^S$, then $w_{ik}/w_{ij} > w_{mk}/w_{mj}$.*

This leads to a key result: the minimal assumptions of substitutability between firms within the same sector, $\{\epsilon, \zeta\} > 1$, and complementarity in input quality across sectors, $\nu < 1$, can produce

positive assortative matching. Specifically, higher quality downstream firms will demand relatively greater quantities from higher quality upstream firms, and analogously higher quality upstream firms will sell more to higher quality downstream firms, consistent with the positive assortative matching observed in the data. Moreover, as discussed previously, positive sorting hinges on the capacity constraint binding implied by the production technology, and becomes most pronounced when complementarity between inputs across sectors increases (ν declines) and when substitutability within sectors declines (ϵ and ζ fall).

In sum, upstream and downstream prices, quantities, and profits are jointly determined by both the upstream and downstream quality distribution. When the upstream distribution is not degenerate and lags behind the downstream distribution, this increases returns from sorting by downstream firms but enables higher quality upstream firms to extract greater rents.

3.2 Downstream Profits

This section first considers how downstream profits $\pi^C\left(z_i^C \mid \Phi^C, \Phi^S\right)$ vary with its own quality, z_i^C, and then examines how changes to the upstream quality impact downstream profits.

Definition 3.11. Given downstream quality z_i^C, upstream quality $\bar{z}_i^S$, and price schedule $g\left(\bar{z}_i^S\right)$, I define the elasticity of downstream profits to its own quality as $\varphi^C\left(z_i^C\right) = \frac{d\pi_i^C/\pi_i^C}{dz_i^C/z_i^C}$ and the cross elasticity of downstream profits to upstream quality as $\hat{\varphi}^C\left(z_i^C\right) = \frac{d\pi_i^C/\pi_i^C}{d\bar{z}_i^S/\bar{z}_i^S}$.

Proposition 3.12. *[Downstream Profits] Suppose $\epsilon > 1$, $\nu < 1$, and $g', g'' > 0$. For given upstream prices $\left\{p_k^S\right\}_{k\in K}$ and upstream quality distribution $\left\{z_k^S\right\}_{k\in K} = \Phi^S$, there exists threshold $z^{C*} < z^{C**}$ where $\varphi\left(z^{C*}\right) = 1$ and $\varphi\left(z^{C**}\right) = 0$ such that*

Proposition 3.13. *(1) Elasticity φ^C is the sum of a positive demand effect and a negative cost*

effect

$$\varphi^C\left(z_i^C\right) = \underbrace{\epsilon \frac{dz_i/z_i}{dz_i^C/z_i^C}}_{demand} + \underbrace{(1-\epsilon)\frac{d\Psi_i/\Psi_i}{dz_i^C/z_i^C}}_{cost}$$

where $\frac{dz_i/z_i}{dz_i^C/z_i^C} = \kappa_C \left(\frac{z_i^C}{z_i}\right)^{\frac{\nu-1}{\nu}}$ *and* $\frac{d\Psi_i/\Psi_i}{dz_i^C/z_i^C} = \psi^C \frac{z_i^C}{\Psi_i}$.

Proposition 3.14. *(2) For* $z_i^C < z^{C*}$*, such that* $\varphi^C\left(z_i^C\right) > 1$*, downstream profits* $\pi^C\left(z_i^C\right)$ *are increasing and convex in* z_i^C

Proposition 3.15. *(3) For* $z^{C*} < z_i^C < z^{C**}$*, such that* $0 < \varphi^C\left(z_i^C\right) < 1$*, downstream profits are increasing and concave in* z_i^C.

Proposition 3.16. *(4) For* $z_i^C > z^{C**}$*, such that* $\varphi^C\left(z_i^C\right) < 0$*, downstream profits are decreasing* z_i^C.

First, from the downstream profit function characterized in equation 3.1, we can see how downstream profits evolve with its own quality for a given upstream distribution Φ^S and corresponding prices $\left\{p_k^S\right\}_{k\in K}$. Contrary to standard models, per Proposition 3.12, downstream profits are only monotonically increasing in quality z_i^C up to maximum z^{C**}. Indeed, the elasticity of downstream profits to downstream quality, φ^C, can be expressed as the sum of two forces. First, increasing the downstream firm's own quality z_i^C increases demand, but has diminishing returns due to complementarity between z_i^C and the chosen upstream bundle. Second, due to linear input costs $\psi^C z_i^C$, increasing downstream quality z_i^C increases unit costs Ψ_i at least linearly (since g is convex). For z_i^C small, the first dominates and φ^C and profits are increasing. However, as z_i^C increases, the second dominates, and downstream profits level out and then decline. In this region, since the upstream distribution is fixed, the increased demand from greater joint quality, z_i, is not sufficient to offset the increased costs of production. Consequently, if downstream firms take the upstream distribution and prices as fixed, they will not innovate beyond z^{C**} where $\varphi^C\left(z^{C**}\right) = 0$.

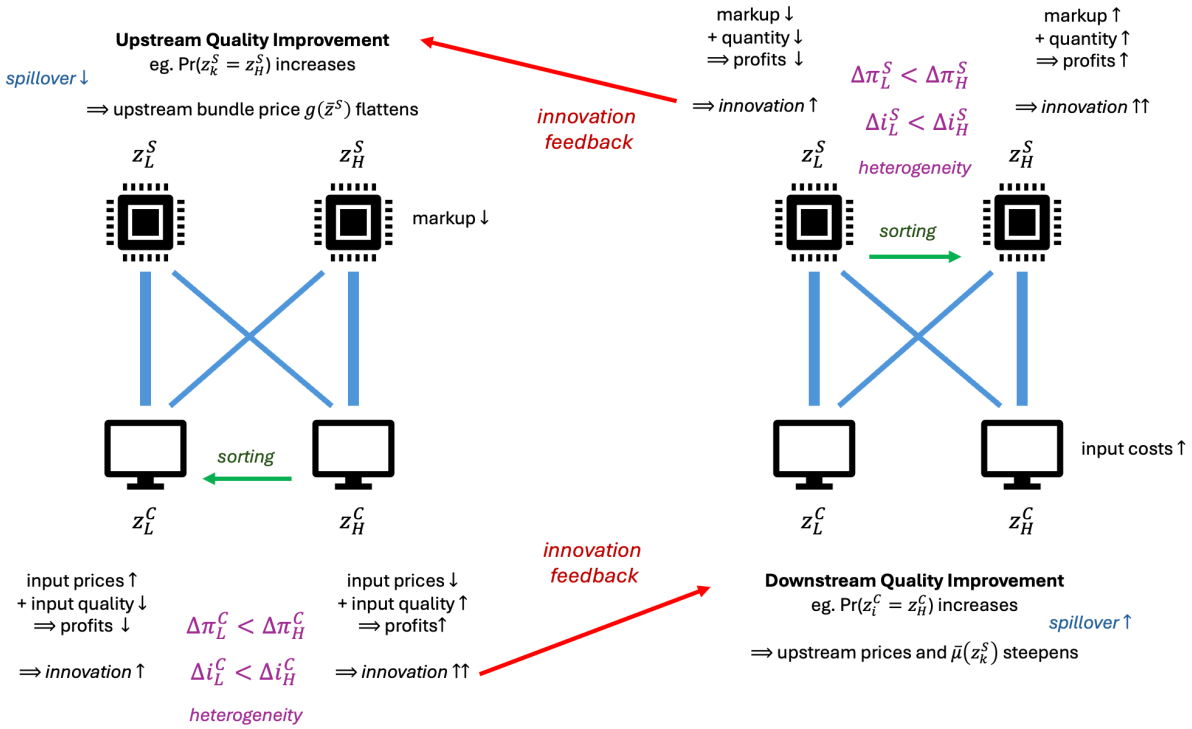


FIGURE 4 – Diagram of Model Mechanism

Moreover, by examining $d\varphi^C/dz_i^C$, one can see that as firms approach threshold z^{C**}, downstream profits become concave, suggesting declining returns to quality-improving innovation. Indeed, downstream profits are concave as long as $\varphi^C\left(z_i^C\right) < 1$. However, when downstream quality is sufficiently small such that $z_i^C < z^{C*}$, where $\varphi\left(z^{C*}\right) = 1$, profits are convex in quality. In this region, the downstream firm's quality lags sufficiently behind the upstream quality distribution to the extent that the higher input costs associated with higher upstream quality diminish downstream profits, thereby discouraging innovation.[4]

Proposition 3.17. *[Upstream Technology Spillover] Suppose $\epsilon > 1$, $\nu < 1$, and g convex and all firms have upstream quality $z_k^S = \bar{z}^S$, then*

Proposition 3.18. *(1) Cross elasticity $\hat{\varphi}$ is the sum of a positive demand effect and a negative cost*

[4]A policy which introduced lower quality upstream inputs would increase innovation for these firms. While some downstream firms may lag behind, as discussed in the subsequent section, upstream firms would not innovate such that upstream quality outstrips all downstream firms.

effect

$$\hat{\varphi}^C\left(z_i^C\right) = \underbrace{\epsilon \frac{dz_i/z_i}{d\bar{z}^S/\bar{z}^S}}_{demand} + \underbrace{(1-\epsilon)\frac{d\Psi_i/\Psi_i}{d\bar{z}^S/\bar{z}^S}}_{cost}$$

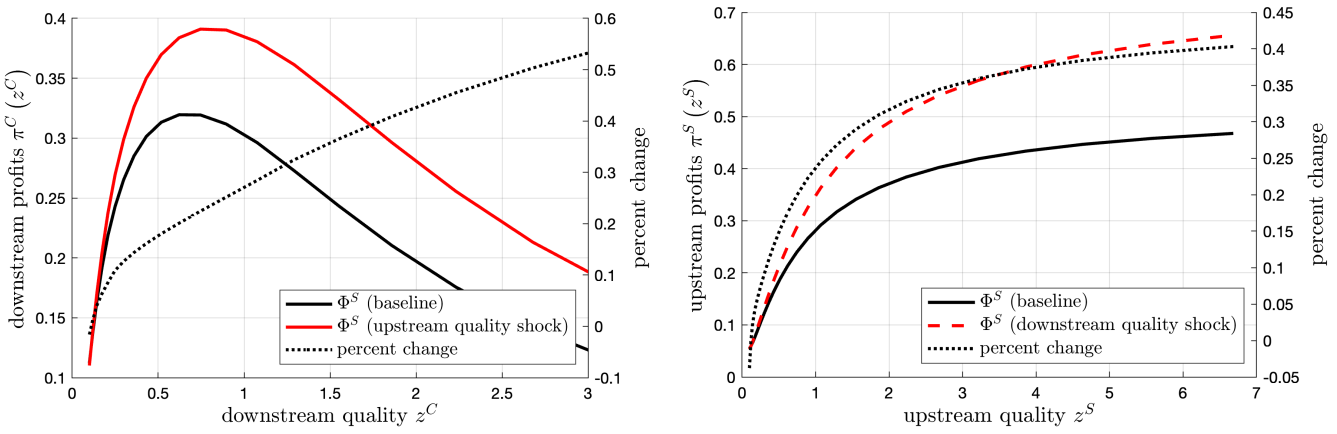
where $\frac{dz_i/z_i}{d\bar{z}^S/\bar{z}^S} = \kappa_S \left(\frac{\bar{z}^S}{z_i}\right)^{\frac{\nu-1}{\nu}}$ *and* $\frac{d\Psi_i/\Psi_i}{d\bar{z}^S/\bar{z}^S} = g'\left(\bar{z}^S\right)\frac{\bar{z}^S}{\Psi_i}$.

Proposition 3.19. *(2) Threshold* z^{C**} *increases with upstream quality* $\bar{z}^S$.

Proposition 3.20. *(3) Elasticity* $\varphi^C\left(z_i^C\right)$ *increases with* $\bar{z}^S$ *and cross elasticity* $\hat{\varphi}^C\left(z_i^C\right)$ *increases with* z_i^C.

Proposition 3.21. *(4) There exists threshold* z^{C***} *such that for* $z_i^C > z^{C***}$*, profits are increasing in upstream quality ,*$\hat{\varphi}^C\left(z_i^C\right) > 0$*, and decreasing otherwise.*

Second, I consider how upstream quality improvements impact downstream profits. For this section, I consider downstream profits under the special case where all upstream firms have homogeneous quality z_k^S, and thus the aggregate bundle quality and price is $\bar{z}^S = z_k^S$ and $\bar{p}^S = \frac{\zeta}{\zeta-1}\psi^S\bar{z}^S$. Three observations described in Proposition 3.17 become apparent: First, for $\nu < 1$ and $\epsilon > 1$, as upstream quality $\bar{z}^S$ increases, $\frac{dz_i/z_i}{dz_i^C/z_i^C}$ increase and $\frac{d\Psi_i/\Psi_i}{dz_i^C/z_i^C}$ decrease, increasing $\varphi\left(z_i^C\right)$ and threshold, z^{C**}, at which downstream profits are maximized: upstream quality constrains the set of economically feasible downstream z_i^C. Secondly, upstream quality impacts downstream profits. Specifically, $\hat{\varphi}^C$ is increasing in z_i^C , meaning upstream technology spillover is heterogeneous, predominantly benefiting higher quality downstream firms. The cross elasticity of downstream profits to upstream quality, $\hat{\varphi}^C$, can be expressed as the sum of the positive demand effect, where increases to upstream quality increase joint quality z_i and hence demand, and the negative cost effect, since higher quality inputs are more expensive. Indeed, for lower quality downstream firms the second force dominates, leading to the third observation: for $z_i^C < z^{C***}$ where $\hat{\varphi}^C\left(z^{C***}\right) = 0$, upstream technology improvements reduce profits, and increase profits otherwise.



(A) Downstream Profits (B) Upstream Profits

FIGURE 5 – Upstream and Downstream Profits

Note: For a given upstream distribution, the black line of Panel A plots downstream profits as a function of its own quality and the red line plots downstream profits after an improvement to upstream quality. The dotted line plots the percent change from the upstream quality improvement. Similarly, the black line of Panel B plots upstream profits as a function of upstream quality with the red line corresponding to an improvement to downstream quality and the dotted line the percent change from the downstream quality improvement.

While the above analysis simplifies the problem to the case where all upstream firms have the same quality, the results are qualitatively similar when the upstream distribution is not degenerate and downstream firms have a choice over upstream basket quality $\bar{z}_i^S$. The appendix considers the case where the firms in the upstream distribution can be of either high or low quality. Intuitively, when the proportion over the higher quality upstream input increases, the maximum achievable $\bar{z}_i^S$ increases, and the price function $g\left(\bar{z}_i^S\right)$ flattens. Higher quality downstream firms who had previously been reaching for quality, benefit from lower prices and optimally choose a higher quality basket $\bar{z}_i^S$. In contrast, lower quality firms not only face higher input prices for their previously chosen basket, but also optimally shift to a lower quality basket and thus face lower demand.

3.3 Upstream Profits

This section first examines how upstream profits $\pi^S\left(z_k^S \mid \Phi^C, \Phi^S\right)$ vary with its own upstream quality z_k^S and second examines how changes to the downstream quality impact upstream profits.

Definition 3.22. I define the elasticity of upstream profits to its own quality as $\varphi^S\left(z_k^S\right) = \frac{d\pi_k^S/\pi_k^S}{dz_k^S/z_k^S}$.

If all downstream firms have homogenous quality $z_i^C = z^C$, then I define the cross elasticity of downstream profits to upstream quality $\hat{\varphi}^S\left(z_k^S\right) = \frac{d\pi_k^S/\pi_k^S}{dz^C/z^C}$.

Proposition 3.23. *[Upstream Profits] Suppose $\epsilon, \zeta > 1$ and $\nu < 1$ and all downstream firms have homogeneous quality $z_i^C = z^C$ with shadow price $\mu_i = \mu$, quantity $x_i = x^C$, and aggregate $\mathcal{A}_{i0} = \mathcal{A}_0$.*

Proposition 3.24. *(1) Elasticity φ^S can be expressed as follows*

$$\varphi^S\left(z_k^S\right) = (\zeta - 1)\left[1 + \frac{\psi^S z_k^S}{\mu}\right]^{-1}$$

Proposition 3.25. *(2) Profits are increasing in quality, if $1 + \frac{\psi^S z_k^S}{\mu} > 0$ and decreasing otherwise. If $\mu < 0$, then profits are increasing provided $z_k^S < z^{S**} = -\frac{\mu}{\psi^S}$.*

Proposition 3.26. *(3) As $z_k^S \to z^{S**}$, $\pi^S\left(z_k^S\right)$ is concave.*

First, I consider how upstream profits vary with its own quality, z_k^S, in the special case where all downstream firms have homogeneous quality $z_i^C = z^C$ and hence identical shadow price $\mu_i = \mu$. Substituting optimal price p_k^S it is straight forward to characterize upstream profits as a function of its own quality z_k^S and downstream quantity x^C, $\mathcal{A}_0$, and μ.

$$\pi^S\left(z_k^S\right) = \frac{1}{\zeta - 1}\left(\frac{\zeta}{\zeta - 1}\right)^{-\zeta}\left(\frac{x^C}{\mathcal{A}_0}\right)\left(\frac{z_k^S}{\mu + \psi^S z_k^S}\right)^{\zeta - 1} \tag{3.4}$$

Examining elasticity $\varphi^S\left(z_k^S\right)$ as characterized in Proposition 3.23 provides insight into how upstream profits and therefore innovation evolves with upstream quality, z_k^S. As before, now the downstream distribution acts as a limit on the range of upstream quality which is economically feasible. Observe that upstream profits are increasing in quality, $\varphi^S > 0$, provided μ is positive, meaning that downstream firms would prefer higher quality inputs, or upstream quality is sufficiently small such that $\psi^S z_k^S < -\mu$. Indeed, if the downstream distribution is fixed, upstream firms will not innovate

beyond z^{S**} where $z^{S**} = -\frac{\mu}{\psi^S}$. Furthermore, $d\varphi^S/dz_k^S < 0$, suggesting that for $\mu > 0$ or z_k^S approaching z^{S**}, upstream profits are concave in quality. As with downstream firms, as upstream firm quality increases, returns from innovation decline.

Proposition 3.27. *[Downstream Technology Spillover] Suppose $\epsilon, \zeta > 1$ and $\nu < 1$ and all downstream firms have homogenous quality $z_i^C = z^C$. Then*

Proposition 3.28. *(1) Cross elasticity $\hat{\varphi}^S$ can be decomposed into a aggregate demand effect and a reallocation effect*

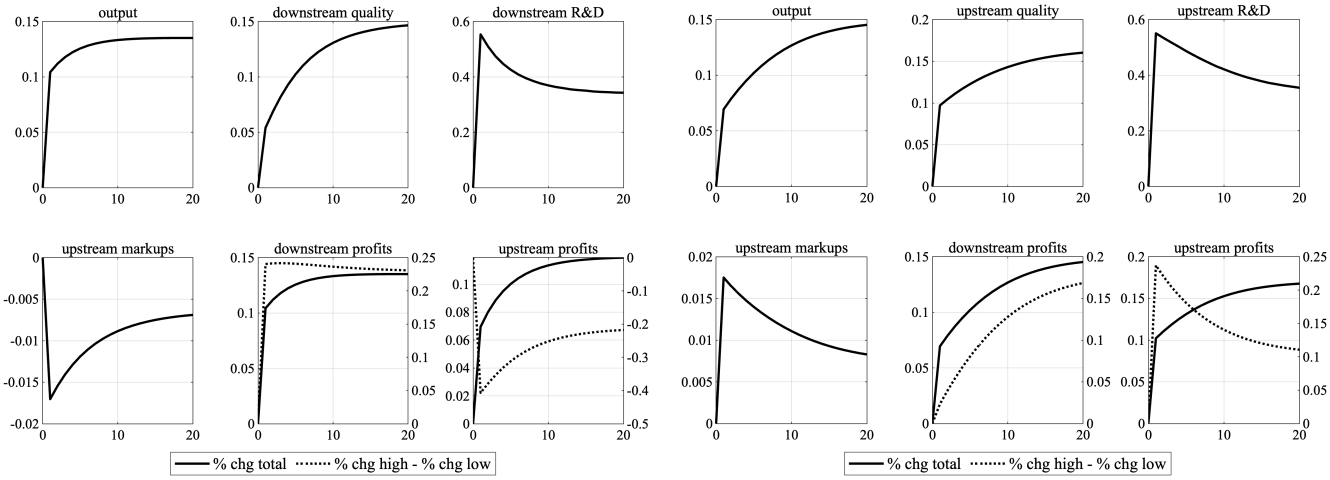
$$\hat{\varphi}^S\left(z_k^S\right) = \underbrace{\frac{d\left(\frac{x^C}{\mathcal{A}_0}\right)}{dz^C}\frac{z^C}{\left(\frac{x^C}{\mathcal{A}_0}\right)}}_{demand} - \underbrace{\left(\zeta - 1\right)\frac{z^C}{\mu + \psi^S z_k^S}\frac{d\mu}{dz^C}}_{reallocation}$$

Proposition 3.29. *(2) Threshold z^{S**} increases with downstream quality z_i^C.*

Proposition 3.30. *(3) Elasticity $\varphi^S\left(z_k^S\right)$ increases with z^C and cross elasticity $\hat{\varphi}^S\left(z_k^S\right)$ increases with z_k^S*

Proposition 3.31. *(4) There exists threshold z^{S***} such that for $z_i^S > z^{C***}$, profits are increasing in downstream quality ,$\hat{\varphi}^S\left(z_k^S\right) > 0$, and decreasing otherwise.*

Second, I consider how upstream profits vary with downstream quality z^C, again considering the special case where all downstream firms have homogeneous quality as discussed in Proposition 3.27. This delivers three key observations. First, the range of economically feasible upstream qualities, for which upstream profits are increasing in upstream quality, is increasing in downstream quality z^C. Recall that μ reflects the extent the downstream firms are constrained. Hence as z^C increases so does μ. If μ is negative, then threshold z^{S**} also increases. Second, technology spillover from downstream firms is heterogeneous, with profits increasing more for higher quality upstream firms. Indeed, cross elasticity $\hat{\varphi}^S$ of upstream profits to downstream quality can be decomposed into a positive aggregate



(A) Upstream Quality Shock (B) Downstream Quality Shock

FIGURE 6 – Transition Path After Permanent Quality Improvement Without Feedback

Note: Panel A considers the impact on aggregate moments of a permanent 10 percent increase in the quality of all upstream firms (compared to the stationary distribution) if downstream firms can respond and innovate, but the upstream distribution is held fixed. Panel B considers the impact on aggregate moments of a permanent 10 percent increase in the quality of all downstream firms (compared to the stationary distribution) if upstream firms can respond and innovate, but the downstream distribution is held fixed.

demand effect, where downstream innovation increases demand for inputs equally, and a reallocation effect. Under the reallocation effect, when downstream quality improves, complementarity means that downstream firms shift relative demand to higher quality inputs, predominantly benefitting higher quality upstream firms, and leading to the impact of downstream spillover as characterized by cross elasticity $\hat{\varphi}^S$ to be increasing in upstream quality. Indeed, thirdly, for sufficiently low quality downstream firms, the negative effect of reallocation dominates the positive demand effect. Consequently for these firms, downstream technology improvement propagates negatively, reducing profits.

3.4 Transition Path

To understand not only the partial equilibrium effect but also the industry equilibrium effect of sorting, I consider the transition path of outcomes in the economy to observe the short- and long-run effects of innovation spillovers without two-way feedback

In the left panel of Figure 6, I consider the impact of a permanent improvement in upstream

quality when downstream firms respond but the upstream distribution is held fixed. In the short run, the upstream quality improvements increase average quality of the final production, immediately increasing output and average downstream profits. Due to product complementarity, higher quality downstream firms benefit more from upstream quality improvements even as average upstream input prices increases, while for low quality downstream firms, the increased input costs may offset any increases to demand, reducing profits. Endogenous sorting exacerbates this. The upstream quality improvement increases choice for higher quality upstream inputs and flattens prices of the upstream bundle as a function of quality, decreasing average upstream markups. This further benefits higher quality downstream firms whose previous upstream bundle is now cheaper, while lower quality downstream firms previous upstream bundle may be more expensive, leading them to optimally substitute to lower quality upstream firms, decreasing profits further. Downstream firms are incentivized to innovate, leading to increased quality improvement and output growth following the initial shock. This increases upstream profits, predominantly benefitting higher quality upstream firms as downstream firms reallocate towards higher quality upstream inputs.

Second, in the right panel of Figure 6, I consider the impact of a permanent improvement in downstream quality when upstream firms respond but the downstream distribution is held fixed. Reallocation arising from endogenous sorting is key to understanding innovation spillovers from downstream quality improvements. Downstream quality improvements increase overall demand for upstream inputs, increasing average upstream profits. However, since supply chains are endogenous, these higher quality downstream firms optimally increase demand for high quality upstream inputs, steepening upstream prices as a function of upstream quality and increasing markups. This overwhelmingly benefits higher quality upstream firms which experience greater profit increases than lower quality downstream firms. The downstream quality improvement increases upstream incentives to innovate, both in aggregate, and by widening the disparities in profits between high

and low quality upstream firms. This leads to increases to downstream profits and further increases to output over subsequent periods, as the upstream distribution evolves.

4 Empirical Evidence

4.1 R&D Spillover

To corroborate the main predictions of the model and to provide evidence of innovation spillover along the supply chain, I use firm-level data on U.S. public firms from 1980 to 2018. I show that innovation spillovers along the supply chain are quantitatively important, affecting innovation, firm value, and returns to R&D. Moreover, the economic trade relationships between supply chain partners drive much of this spillover, even after accounting for spillovers due to knowledge diffusion.

Data and Sample Selection. I merge annual data on over 80,000 supply chain relationships collected from Compustat and Factset with annual financial data from Compustat to identify innovation spillover along the supply chain. These relationships correspond to over 46,532 supplier-year observations reporting an average of 5.6 customers and 24,657 customer-year observations reporting an average of 64.7 suppliers. In three-quarters of these relationships one or both parties report positive R&D expenditure and in over half both parties report positive R&D expenditure.

Challenges to Identification: Endogenous Supply Changes. Quantifying innovation spillovers across supply chains poses several challenges. First, firms connected along the supply chain may innovate in tandem for reasons unrelated to trade relationships and thus observed regression coefficients may reflect the impact of correlated shocks. For example, new research opportunities that affect all sectors—e.g. electrification, computers, etc.—may induce connected firms to innovate more in tandem. To address this, I instrument for spillovers using exogenous variation in innovation arising from firm-level variation in exposure to state and federal R&D tax credits, and

instrument firm R&D expenditure on fluctuations in user costs, following Wilson (2009); Bloom et al. (2013); Arque-Castells and Spulber (2022), among others to calculate $\widehat{RdIntensity}_{jt}$, the R&D intensity which correspond to a firm's exposure to quasi-exogenous variation in R&D tax credits. The Appendix includes the first stage and provides additional detail on the construction of R&D user costs.

Second, since supply chains may evolve, innovation may spill over from not only current supply chain partners but also from potential future partners. Hence, forward looking R&D investment may respond to innovation by both existing partners and by potential partners in the anticipation of future supply chain relationships. Several previous efforts to quantify spillovers across supply chain partners focus on current supply chain relationships or implicitly assume that observed links are static, at least in the short run, or occur for reasons unrelated to innovation, such as geographic proximity. This assumption overlooks the possibility that innovation spillovers may also arise from potential partners, either through expectations of future relationships or through indirect spillovers from current partners. Ignoring this channel may lead researchers to overestimate the role of switching costs in explaining relationship persistence.

To address this issue, I take a simple approach to identifying potential partners. For each firm, I define a firm's downstream network for that year to include not only current or observed downstream partners, but to also potential downstream partners, which I define as competitors of current downstream partners and the current downstream partners of competitors, where competitors are defined as firms operating in the same 3-digit SIC sector. I similarly define a firm's upstream network as not only a firm's current upstream partners, but also as any competitors to their upstream partners or upstream partners of the firm's competitors.

Then, to identify the innovation spillover from a firm's downstream network and current downstream partners, I first define $TradeSpill_{it}$ and $TradeSpill0_{it}$ as the sales-weighted average R&D

intensity of firm i's downstream network J'_{it} and current downstream partners $J_{it} \subseteq J'_{it}$, respectively. Because I am interested in spillover due to the economic relationship, I weigh the R&D intensity of each downstream firm by the average of their sales over the previous 5 years,

$$TradeSpill_{it} = \frac{\sum_{j \in J'_{it}} Sales_{jt} \widehat{RdIntensity}_{jt}}{\sum_{j \in J'_{it}} Sales_{jt}}, \qquad TradeSpill0_{it} = \frac{\sum_{j \in J_{it}} Sales_{jt} \widehat{RdIntensity}_{jt}}{\sum_{j \in J_{it}} Sale_{jt}}.$$

Next, to isolate the innovation spillover from firms' current relationships specifically, I similarly define $TradeSpillCurrent_{kt}$ to reflect the extent innovation intensity by of a firm's current downstream partners exceeds the innovation intensity of its downstream network, the wider set of both current and potential downstream partners

$$TradeSpillCurrent_{it} = TradeSpill0_{it} - TradeSpill_{it}.$$

To estimate the impact of downstream innovation on firm R&D intensity, I regress firm R&D intensity, $RdIntensity_{it}$, on contemporaneous spillover from i's downstream network and current partners, $TradeSpill_{it}$ and $TradeSpillCurrent_{it}$, respectively, controlling for firm and year fixed effects, δ_i and η_t, respectively, and vector of firm controls Γ.

$$RdIntensity_{it} = \beta_0 + \beta_1 TradeSpill_{it} + \beta_2 TradeSpillCurrent_{it} + \Gamma X_{it} + \delta_i + \eta_t + \epsilon_{it} \qquad (4.1)$$

where β_1 captures the response of firm R&D intensity to innovation by its supply chain network.

As shown in Column 1 of Table I, a 1 p.p. increase in innovation from current downstream partners alone corresponds to a 45 basis points increase in firm R&D intensity. Column 2 of Table I presents the regression results of specification 4.1. Here, a 1 p.p. increase in innovation from a firm's downstream network corresponds to a 35 b.p. increase in firm R&D intensity. However, individual

	(1) RdIntensity	(2) RdIntensity	(3) RdIntensity	(4) RdIntensity	(5) RdIntensity	(6) Patents/RD	(7) PatVal/RD	(8) Tobins Q	(9) Tobins Q
TradeSpill0	0.45***								
	(12.18)								
TradeSpill		0.35***	0.34***	1.28***	1.26***	2.16***	2.28**	1.20**	-0.95
		(7.27)	(6.94)	(5.19)	(5.12)	(3.40)	(2.35)	(2.23)	(-1.20)
CurrentTradeSpill		0.46***	0.45***	0.85***	0.85***	1.33**	0.76	-0.00	-1.11
		(12.38)	(12.02)	(3.62)	(3.59)	(2.09)	(0.82)	(-0.00)	(-1.43)
CitesSpill			0.22***		0.27***	-0.16	-0.47*	-0.32**	
			(3.48)		(2.88)	(-1.17)	(-1.67)	(-2.21)	
CurrentCitesSpill			0.08		0.18**	-0.02	-0.06	-0.25*	
			(1.34)		(2.07)	(-0.12)	(-0.28)	(-1.85)	
N	19276	19276	19276	19276	19276	4285	4285	10406	6688
F(trade)	148.35	79.92	76.08	22.36	21.42	18.62	12.78	29.55	1.22
F(cites)			7.60		4.15	1.24	1.81	2.52	
F(current)		153.16	77.72	13.08	8.54	2.19	0.36	1.71	2.05
Sample	RandD>0	RandD>0	RandD>0	RandD>0	RandD>0	RandD>0	RandD>0	RandD>0	RandD=0
FE	Firm, Year	Firm, Year	Firm, Year	Firm, Year	Firm, Year	Firm, Year	Firm, Year	Firm, Year	Firm, Year
IV	No	No	No	Yes	Yes	Yes	Yes	Yes	Yes

t statistics in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

TABLE I – Impact of Downstream R&D on Firm R&D Intensity, Returns to R&D, and Value

Note: The table presents the regression results for specification 4.1 and 4.2 using relationship and supply chain data on US public firms from 1980 to 2018. Column 1 reports the coefficient of upstream R&D intensity regressed on downstream R&D intensity of existing supply chain partners. Columns 3 and 4 presents the uninstrumented impact of downstream spillovers on firm R&D intensity. Column 5 and 6 presents the impact on R&D intensity where downstream spillovers are instrumented using R&D tax credits. Columns 5 and 6 present the impact of downstream spillovers on returns to R&D as measured by Patents per dollar of R&D spending and Patent Value per dollar of R&D spending, and 9. Column 8 and 9 present the impact of downstream spillovers on Tobin's Q for firms which invest in R&D and firms which do not invest in R&D.

relationships are also important—each percentage point by which innovation from a firm's current downstream partners exceed its wider network corresponds to an additional 46 p.p. increase in a firm's innovation.

Challenges to Identification: Knowledge Spillovers. A second challenge to identification is that technology spillover may occur across firms connected along the supply chain not only because these firms are economically linked through the production network but because firms learn from their supply chain partners—knowledge spillovers across the supply chain. To disentangle innovation spillover from supply chain partners due to economic links versus knowledge spillovers, I exploit the fact that while production networks and citation networks, which are often used as proxies for knowledge spillovers, may coincide, the intensity of a firm's connection to their supply chain partners through the citation network and its connection through the production network may differ. I define the knowledge network using the citation network sourced from patent filings. Specifically, for firms who filed patents in over the past 5 years, I similarly define downstream spillover from knowledge spillovers, $CitesSpill0_{it}$ and $CitesSpill_{it}$, as the citation weighted average of R&D intensity from current supply chain partners and potential supply chain partners, respectively, where $Cites_{kit}$ is the 5-year average number of citations by firm i of patents produced by firm i and similarly define $CitesSpillCurrent_{it}$ as the difference between $DownSpillCites0_{it}$ and $CitesSpill_{it}$.

$$CitesSpill_{it} = \frac{\sum_{j \in J'_{it}} Cites_{jit} \widehat{RdIntensity}_{jt}}{\sum_{j \in J'_{it}} Cites_{jit}}, \qquad CitesSpill0_{it} = \frac{\sum_{j \in J_{it}} Cites_{jit} \widehat{RdIntensity}_{jt}}{\sum_{j \in J_{it}} Cites_{jit}}$$

Column 3 of Table I presents the regression results for the following equation,

$$RdIntensity_{it} = \beta_0 + \beta_1 TradeSpill_{it} + \beta_2 TradeSpillCurrent_{it} + \beta_3 CitesSpill_{it} \tag{4.2}$$
$$+ \beta_4 CitesSpillCurrent_{it} + \Gamma X_{it} + \delta_i + \eta_t + \epsilon_{it}$$

After accounting for knowledge spillovers, a 1 p.p. increase in innovation from a firm's downstream network corresponds to 34 b.p. increase in R&D intensity from economic links and 22 b.p. from knowledge spillovers. Each b.p. the firm's current downstream partners exceeds its downstream network corresponds to an additional 45 b.p. increase in R&D intensity from economic links and 7 b.p. from knowledge spillovers from current downstream partners.

Columns 4 and 5 of Table I repeat the analysis in Column 2 and 3 for spillovers from economic links alone and including knowledge spillovers, respectively, instrumenting downstream R&D intensity with exogenous variation in R&D user costs and including controls for a firm's own exposure to R&D user costs. As shown in Column 5, a 1 p.p. increase in innovation by a firm's downstream network results in a 1.26 p.p. and 0.27 p.p. increase in R&D intensity due to economic links and knowledge spillovers, respectively. Each p.p. of innovation by a firm's current downstream partners which exceeds its network, results in a 0.84 p.p. and 0.18 p.p. increase in R&D intensity due to economic links and knowledge spillovers, respectively.

Not only does downstream spillover through the supply chain increase firm innovation, but we can evaluate whether, consistent with the model, downstream innovation contributes to R&D productivity and firm valuation. I proxy for R&D productivity in two ways. First, I define R&D productivity, $PatentsPer\$_{it}$, as the average number of patents filed over the subsequent three years, $t+1$ to $t+3$, divided by R&D expenditure in year t. Column 6 of Table I present the impact of downstream innovation on $PatentsPer\$_{it}$, where downstream innovation is instrumented using exogenous variation in R&D tax credits as before. The regression results suggest that while downstream innovation increases R&D productivity, this is largely attributable to the economic relationship rather than due to knowledge spillovers. A 1 p.p. increase in innovation by a firm's downstream network contributes to a 2.1 percent increase in patents per dollar of R&D spent. Each p.p. of innovation by a firm's current downstream partners which exceeds its downstream network,

contributes an additional 1.3 percent.

In contrast, downstream spillover due knowledge spillovers does not significantly impact R&D productivity. Since patents might vary in value, in Column 7, I instead define R&D productivity, $PatentValPer\$_{it}$, as the total value of patents filed over the subsequent three years, $t+1$ to $t+3$, divided by R&D expenditure in year t, using patent valuations estimated by Kogan et al. (2017). As before, a 1 p.p. increase in innovation by a firm's downstream network corresponds to a 2.3 p.p. increase in R&D productivity. However, the effect of a firm's current downstream partners on R&D productivity, as defined using patent valuations, is no longer statistically significant.

Next, in Columns 8 and 9 of Table I, I evaluate the impact of downstream innovation on Tobin's Q for firms which invest in R&D expenditure, replacing the dependent variable with $logTobinQ_{it}$. While a 1 p.p. increase in innovation by a firm's downstream network corresponds to a 1.2 percent increase in Tobin's Q, this is largely attributable to spillover from economic relationships rather than knowledge spillovers. Moreover, the impact of innovation by firm's current downstream partners alone on Tobin's Q is insignificant. Taken together this suggests that for innovative firms, variation in firm valuation from downstream relationship arises, not due to knowledge spillovers, but economic relationships with the downstream network as a whole, not only existing partners. Finally, I evaluate the impact of downstream innovation on Tobin's Q for non-innovative firms instead, defined as firms which do not invest in R&D, focusing only on the impact of economic links, since the firms are unlikely to produce patents. For these firms, downstream innovation, by either its wider downstream network or current downstream partners, has no significant impact on valuation, suggesting that a firm's own innovation activity impacts the transmission of innovation shocks from downstream partners.

Robustness. These results are robust to defining $TradeSpill_{it}$ and $TradeSpillCurrent_{it}$ using equal-weighted rather than sales-weighted R&D intensity and are robust to bootstrapping

	(1) LogRandD(t+1)	(2) LogRandD(t+1)	(3) InnovShock	(4) InnovShock	(5) ChgTobins Q	(6) ChgTobins Q
TradeSpill	0.990***	0.909**	0.238***	0.200***	1.751***	1.722***
	(0.354)	(0.354)	(0.070)	(0.069)	(0.463)	(0.464)
TradeSpillCurrent	0.073	0.039	0.014	-0.002	-0.002	-0.016
	(0.051)	(0.052)	(0.010)	(0.010)	(0.061)	(0.063)
CitesSpill		0.416***		0.235***		-0.090
		(0.065)		(0.013)		(0.085)
CitesSpillCurrent		0.124**		0.056***		0.081
		(0.056)		(0.011)		(0.069)
InnovShock					0.663***	0.677***
					(0.045)	(0.046)
r2	0.975	0.975	0.698	0.706	0.546	0.546
n	16356	16356	17499	17499	7636	7636
F	3887	2930	516	458	477	383
Sample	RandD>0	RandD>0	RandD>0	RandD>0	RandD>0	RandD>0
FE	Firm, Year	Firm, Year	Firm, Year	Firm, Year	Firm, Year	Firm, Year

Standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

TABLE II – Impact of Downstream Technology Shocks on Firm R&D and Valuation
Note: This table presents the regression results for table 4.3 . Column 1 show the impact of innovation shocks from trade relationships firm's downstream network and current downstream partners. Column 2 adds innovation shocks from knowledge spillovers. Columns 3 and 4 present the effect of downstream innovation shocks on upstream innovation shocks. Columns 5 and 6 show the effect of the same downstream innovation shocks on upstream changes to firm valuation as measured by Tobin's Q.

standard errors. Finally, we could proceed in a similar way to study the impact of innovation spillover from upstream firms: I similarly define $TradeSpill_{it}$, $TradeSpillCurrent_{it}$, $CitesSpill_{it}$, and $CitesSpillCurrent_{it}$ to reflect the impact of innovation shocks through the supply chain and through knowledge spillovers by firms upstream to firm i in year t. Table A.2 in the AppendixB presents the analogous results for the impact of upstream spillover.

4.2 Patent Valuations

In this section, instead of looking at variation in R&D investment, I focus on variation in the realized outcomes from past R&D spending, measured by the value of patents issued to firms. For this, I lean on the data of patent valuations from Kogan et al. (2017) which uses daily firm-level increases to equity prices upon the issuance of new patents to estimate the market value of each individual patents. A key assumption is that each patent issuance reflects new information on the

realization of firm innovation not previously reflected in the firm's market value. For each firm i, I define a firm's innovation shock, $InnovShock_{it}$, as the total market value of all patents issued to firm i in period t, $PatentValue_{it}$, divided by the market value of firm i as of the beginning of the period, $MarketValue_{it-1}$, winsorized at the 1 percent level.

$$InnovShock_{it} = \frac{PatentValue_{it}}{MarketValue_{it-1}}$$

On average these firms experience a 1.7 percent increase to market value attributable to issued patents each month. The distribution is naturally right skewed with the firm's innovation shock reflecting the change in the firm's market value attributed to patent issuances.

I define firm i's exposure to downstream innovation shocks by current partners, $TradeSpill0_{it}$ as the sales-weighted average innovation shock experienced in year t by its current downstream firms $j \in J_t$, where J_t is these set of firms which invest in R&D known to be downstream of firm i as of the beginning of year t. I similarly define $TradeSpill_{it}$ as the sales-weighted average of innovation shocks experienced by all firms in $i's$ downstream network $J'_t \supset J_t$ where J'_t is defined as in the previous section to include both current and potential downstream partners. As before, I define $TradeSpillCurrent_{it}$ to measure the extent which innovation shocks by a firm's current partners exceed that of its downstream network.

$$TradeSpill_{it} = \frac{\sum_{j \in J'_t} Sales_{jt} InnovShock_{jt}}{\sum_{j \in J'_t} Sales_{jt}}, \qquad TradeSpill0_{it} = \frac{\sum_{j \in J_t} Sales_{jt} InnovShock_{jt}}{\sum_{j \in J_t} Sales_{jt}}$$

$$TradeSpillCurrent_{it} = TradeSpill0_{it} - TradeSpill_{it}$$

As in the previous section, to define spillover from downstream firms $j \in J_{it}$ to firm i due to knowledge spillovers, I weigh downstream innovation shocks by i's dependence on each firm j as a

source of technology spillover using the 5-year average of citations of patents filed by firm i of patents filed by firm j. I similarly define $DownSpillCitesCurrent_{it}$ as the difference between $CitesSpill0_{it}$ and $CitesSpill_{it}$.

$$CitesSpill_{it} = \frac{\sum_{j \in J'_{it}} Cites_{jit} InnovShock_{jt}}{\sum_{j \in J'_{it}} Cites_{jit}}, \qquad CitesSpill0_{it} = \frac{\sum_{j \in J_{it}} Cites_{jit} InnovShock_{jt}}{\sum_{j \in J_{it}} Cites_{jit}}$$

To understand how technology shocks from downstream supply chain partners due to economic links and knowledge spillovers impact future R&D, I regress future log R&D expenditure, $LogR\&D_{it+1}$ on $TradeSpill_{it}$ and $TradeSpillCurrent_{it}$ to reflect spillover through economic links and $CitesSpill_{it}$ and $CitesSpillCurrent_{it}$ to reflect spillover from knowledge spillovers. I include firm and year fixed effects, δ_i and η_t, respectively, and include vector of controls, Γ, for period t and $t-1$ log R&D and log sales.

$$\begin{aligned} LogR\&D_{it+1} =& \beta_0 + \beta_1 TradeSpill_{it} + \beta_2 TradeSpillCurrent_{it} + \beta_3 CitesSpill_{it} \\ &+ \beta_4 CitesSpillCurrent_{it} + \Gamma X_{it} + \delta_i + \eta_t + \epsilon_{it} \end{aligned} \tag{4.3}$$

Columns 1 and 2 of Table II present the regression results. A 1 p.p. increase in the average firm value of its downstream network due to innovation, correspond to a 0.91 percent increase in R&D from the economic relationship and a 0.42 percent increase from knowledge spillovers. Each additional p.p. increase to firm value from current partners corresponded to a 0.12 percent increase in R&D due to knowledge spillovers. Columns 3 and 4 consider whether returns to innovation increase with productivity improvements by supply chain partners, by replacing the dependent variable of Equation 4.3 with, $InnovShock_{it}$, the increase to firm i's market value attributable to patents in period t. A 1 p.p. increase in average value of a firm's downstream network attributable to

innovation corresponds to a 0.20 p.p. increase in firm value attributable to patents issued in period t due to economic links and a 0.23 p.p. increase from knowledge spillovers. Finally, columns 5 and 6 consider the impact on Tobin's Q. Spillover from the downstream network through economic links is both economically and statistically significant. Each p.p. increase in downstream value attributable to patents corresponds to a 1.72 percent increase in firm value. In contrast, each p.p. increase in a firm's own valuation attributable to patents corresponds to only 0.68 percent increase to firm value. Table A.3 presents the analogous results for spillover from upstream partners, with qualitatively similar results.

While the previous results demonstrate that economic links are critical to technology spillover from supply chain partners, the following results suggest a firm's own innovation decisions also impact spillover. Table III interacts $TradeSpill_{it}$ and $TradeSpillCurrent_{it}$ with a dummy $PositiveR\&D_{it}$ if the firm itself invests in R&D and considers the impact of technology spillover on changes to firm value over period t, $\Delta TobinsQ_{it}$, log sales in period $t+1$, $\log Sales_{it+1}$, and capital expenditure in period $t+1$, $\log CapX_{it+1}$

$$\begin{aligned} Dependent_{it} =& \beta_0 + \beta_1 TradeSpill_{it} + \beta_2 TradeSpillCurrent_{it} \\ &+ \beta_3 TradeSpill_{it} \times PositiveRD_{it} + \beta_4 TradeSpillCurrent_{it} \times PositiveR\&D_{it} \\ &+ \beta_5 PositiveR\&D_{it} + \beta_6 InnovShock_{it} + \Gamma X_{it} + \delta_i + \eta_t + \epsilon_{it}. \end{aligned} \tag{4.4}$$

Columns 1 and 2 suggest that the positive impact of technology spillover accrue only to firms which themselves innovation. In fact, downstream innovation shocks negatively predict future capital expenditure for firms which do not innovate.

	(1) ChgTobins Q	(2) LogSale(t+1)	(3) LogCapX(t+1)	(4) ChgTobins Q	(5) LogSale(t+1)	(6) LogCapX(t+1)
Spill	0.157	-0.528	-1.753***	0.320	0.155	-0.503
	(0.567)	(0.539)	(0.668)	(0.241)	(0.183)	(0.470)
SpillCurrent	0.129	-0.050	-0.082	-0.080	0.106***	0.006
	(0.116)	(0.111)	(0.138)	(0.052)	(0.035)	(0.090)
Spill X Positive R&D	1.952***	1.003*	3.057***	0.019	-0.229	0.500
	(0.588)	(0.577)	(0.715)	(0.267)	(0.191)	(0.490)
SpillCurrent X Positive R&D	-0.132	-0.022	0.139	0.124**	-0.008	0.003
	(0.128)	(0.128)	(0.158)	(0.058)	(0.039)	(0.101)
InnovShock	0.671***	0.038	-0.013	0.409***	-0.016	0.087
	(0.042)	(0.049)	(0.061)	(0.039)	(0.027)	(0.070)
Positive R&D	0.078**	-0.113***	-0.198***	0.257***	-0.085***	0.026
	(0.033)	(0.028)	(0.035)	(0.049)	(0.027)	(0.070)
r2	0.530	0.975	0.946	0.493	0.991	0.950
n	10571	24899	24899	5014	12488	12488
F	468	2780	1009	174	3776	992
Partner	Downstream	Downstream	Downstream	Upstream	Upstream	Upstream
Sample	All	All	All	All	All	All
FE	Firm, Year	Firm, Year	Firm, Year	Firm, Year	Firm, Year	Firm, Year

Standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

TABLE III – Heterogenous Impact of Downstream and Upstream Technology Shocks
Note: This table presents the results of equation 4.2 for downstream and upstream firms. Column 1 evaluate the impact innovation shocks by downstream partners interacted with a dummy corresponding to whether the firm innovates on the contemporaneous change in firm valuation as measured by Tobin's Q. Columns 2 and 3 evaluate the impact on next year sales and capital expenditure. Columns 4 to 6 evaluate the impact of innovation shocks by upstream partners.

5 Quantitative Exercise

In this section, I use the model to understand the macroeconomic importance of sorting and feedback. Subsection 5.1 discusses the calibration of the model. Subsection 5.2 considers the impact of sorting—the ability of an individual downstream firm to choose its optimal bundle—on the downstream firm's value and innovation decision. Subsection 5.3 considers the impact of sorting and feedback separately and together on output and R&D. Subsection 5.3 considers the impact of feedback on the propagation of an innovation subsidy to firms in the downstream and upstream sector, separately and together. Finally, subsection 5.5 considers the impact of a discount rate shock to innovation and output.

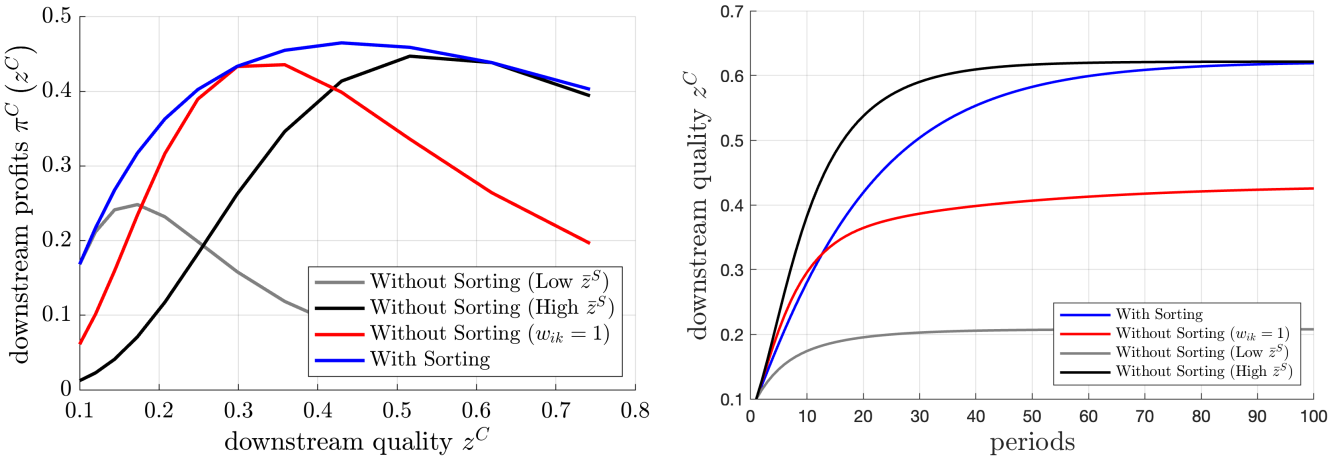
		Parameter	Source//Target
ν	.3	Across-Sector Elasticity of Substitution	Beta of Upstream to Downstream R&D Intensity = .45
$\kappa_S, 1-\kappa_C$	.35	Coefficient on Upstream Quality	Upstream R&D/Total R&D = 40%
α^H	.17	Coefficient on Innovation Production Fn	Total R&D/GDP = 2.7%
$\psi^C/L, \psi^S/L$	0.004	Linear Cost of Production	% of Profits of Top Decile within Sector = 39%
η, ϵ	4	Within-Sector Elasticity of Substitution	Atkeson and Burstein (2008)
ξ	.5	Concavity of Innovation Production Fn	Blundell, Griffith, and Windmeijer (2002)
β	.97	Discount Factor	Acemoglu et al (2013) and others
υ	.075	Probability of Exit	% of Firms that Exit
$pr\left(\alpha^H\right)$	.75	Probability New Firm Innovates	% of New Firms which Innovate

FIGURE 7 – Calibration

5.1 Calibration

I calibrate the model using a combination of parameter estimates common in the literature and estimates derived from targeted moments as shown in Figure 7. I set an annual discount factor of $\beta = 0.97$. I follow the innovation literature and set the elasticity of successful innovation as $\xi = .5$, consistent with elasticities estimated by Blundell et al. (2002) and Hall and Ziedonis (2001). Consistent with my Compustat sample, I set the probability a firm exits as 7.5 percent and the probability a new firm is innovative as 75 percent. Following Atkeson and Burstein (2008) and others, I use a within-sector elasticity of substitution for both sectors of $\eta = \epsilon = 4$.

I target four parameters in the data. First, I target the coefficient on the innovation production function of innovative upstream and downstream firms α^H to match an average R&D to GDP over the sample period of 2.7%. I target the normalized unit cost of production to match the percent of profits within a period attributable to the top ten percent of firms within a sector of 39 percent. I target the across-sector elasticity of substitution to match the regression coefficient of 0.45 as estimated in Table I to reflect the coefficient on average R&D intensity of downstream partners as it predicts upstream R&D intensity. Finally, financial and supply chain data from Compustat suggest that about 40 percent of R&D and patents are produced by firms which list at least one firm as an important downstream partner. I use this moment to target the coefficient on upstream quality $\kappa_S = 1 - \kappa_C$ to reflect R&D invested upstream as a fraction of total R&D.



(A) Downstream Profits (B) Downstream Quality Over Time

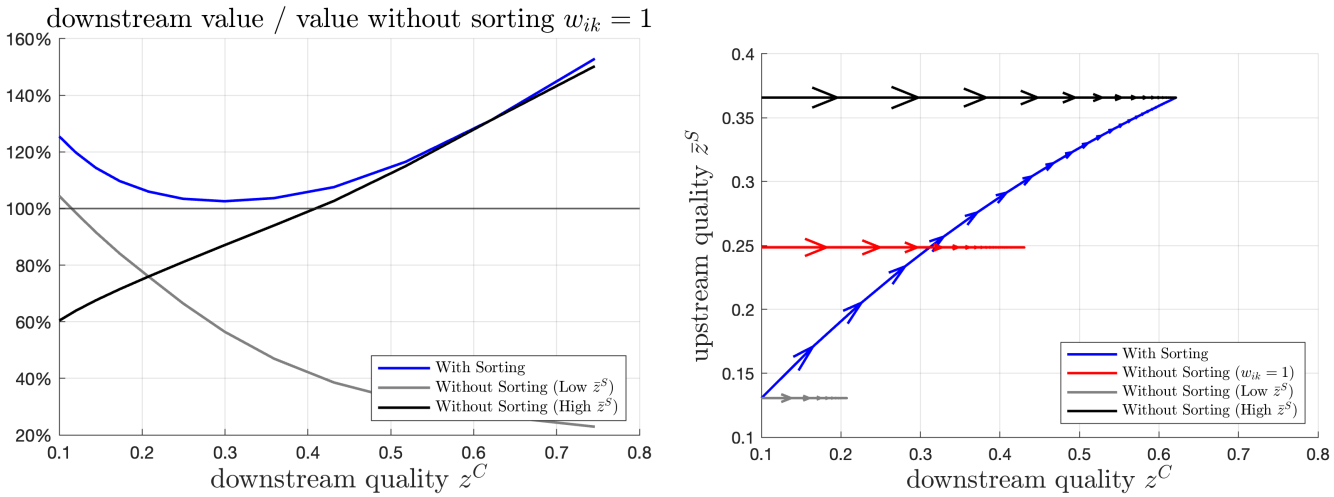
FIGURE 8 – Impact of Sorting on Downstream Profits and Innovation

Note: Panel A compares downstream profits under four scenarios: (i) the downstream firm only has access to the bundle used by the lowest quality downstream firm, (ii) the downstream firm only has access to the bundle used by the highest quality downstream firm, (iii) the downstream firm uses an equal share of all upstream firms $w_k = 1$, and (iv) the downstream firm can flexibly choose its optimal bundle given the upstream distribution. Panel B plots average quality z^C over time for a downstream firm starting at $z^C_{\min}$.

5.2 Impact of Sorting for Downstream Firms

In the baseline model, downstream firms choose their optimal upstream quality across a range of upstream qualities determined by the upstream distribution. This decision depends on both the price schedule of the upstream bundle $g\left(\bar{z}^S\right)$ and, critically, the quality of the downstream firm's own blueprint. Because supply chain relationships are not random—higher quality downstream firms endogenously choose higher quality suppliers—this poses a challenge understanding the relative value of matching with higher quality suppliers. Within the context of the model, I can shut down sorting—the ability of firms to choose their partners—and compare downstream value and innovation to counterfactuals in which they are permanently matched to different partners.

To understand the partial equilibrium value of the supply chain relationship and the ability to endogenously choose their partners from the perspective of individual downstream firms, in the left panel of Figure 8, I compare profits of a downstream firm in the baseline economy (blue) which can endogenously sort with three counterfactuals where the downstream firm faces the same upstream



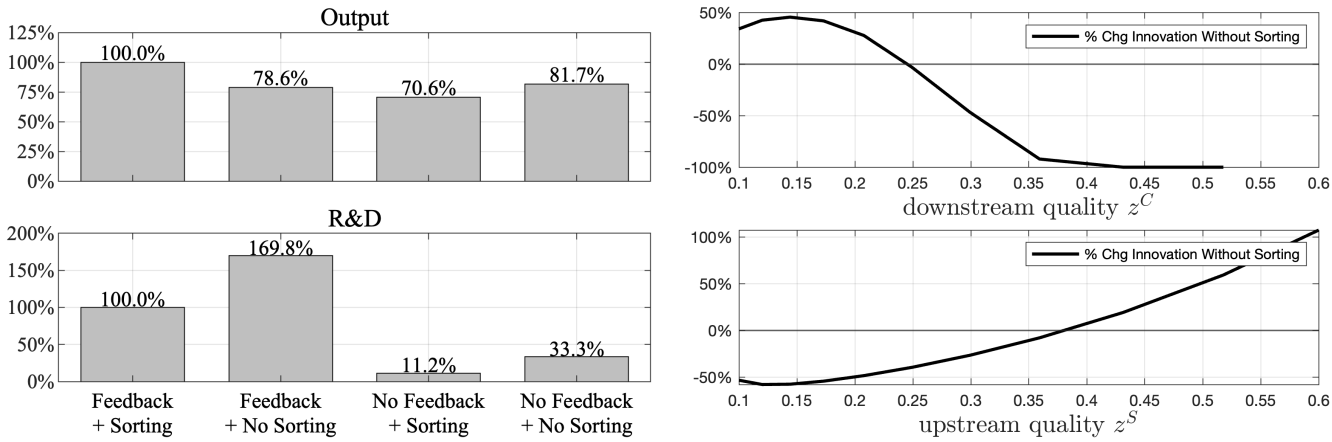
(A) Downstream Value (B) Upstream Allocation

FIGURE 9 – Impact of Sorting on Downstream Value and Upstream Allocation

Note: Panel A compares the value of a downstream firm for different downstream qualities under three scenarios: (i) if it is matched to the upstream bundle chosen by the worst quality downstream firm, (ii) if it is matched to the upstream bundle chosen by the best quality downstream firm, and (iii) if it is able to choose its allocation across all upstream bundles. Values are expressed as a percent of the value of the firm without sorting, if it must choose equal weights across all upstream firms. Panel B plots the trajectory of a downstream firm which starts at $z^C_{\min}$, again comparing (i) if it is matched to the upstream bundle chosen by the worst quality downstream firm, (ii) if it is matched to the upstream bundle chosen by the best quality downstream firm, (iii) if it is able to choose its allocation across all upstream bundles, and (iv) if it must choose equal weights across upstream firms. The arrows correspond to the average movement across three periods.

quality distribution and price schedule but must choose (i) the bundle chosen by the lower quality downstream firm (gray), (ii) the bundle chosen by the highest quality downstream firm (black), and (iii) an equal weighted bundle of all upstream firms (red). The right panel of Figure 8 plots across time the average quality of a downstream firm which begins at the minimum quality, reflecting the innovation decisions of the downstream firm.

As shown in the right panel of Figure 8, downstream firms which are permanently matched with the higher quality upstream bundle innovate more and grow faster. Compared to being matched with either the low quality bundle or the equal-weighted bundle, they achieve a higher maximum quality. However, this comes at the cost of earlier profits, which are lower than if the firm were matched with the equal-weighted or lower quality bundle. Endogenous sorting (blue) enables all downstream firms to achieve weakly higher profits than compared to the three counterfactuals, with benefits from choice accruing most to firms at the extremes of the quality distribution. However,



(A) Output and R&D (B) Change in R&D Without Sorting

FIGURE 10 – Impact of Sorting and Feedback

Note: Panel A compares baseline output and R&D with both feedback and sorting to three counterfactual scenarios: (i) with feedback but without sorting, (ii) without feedback but with sorting, and (iii) without feedback or sorting. Amounts are expressed as a percent of the baseline. Panel B compares innovation in the economy with feedback but without sorting to the baseline for different levels of downstream and upstream quality. Without sorting, R&D increases for lower quality downstream firms and higher quality upstream firms.

as shown in the right panel of 8, lower quality downstream firms initially grow slower.

The left panel of Figure 9 shows the value of sorting, comparing the value of the downstream firm as a percent of firm value when matched with the equal-weighted bundle (i) with endogenous sorting, (ii) if permanently matched to the low quality bundle, and (iii) if permanently matched to the high quality bundle. As before, compared to the equal-weighted bundle, sorting benefits firms at the extremes of the downstream quality distribution.

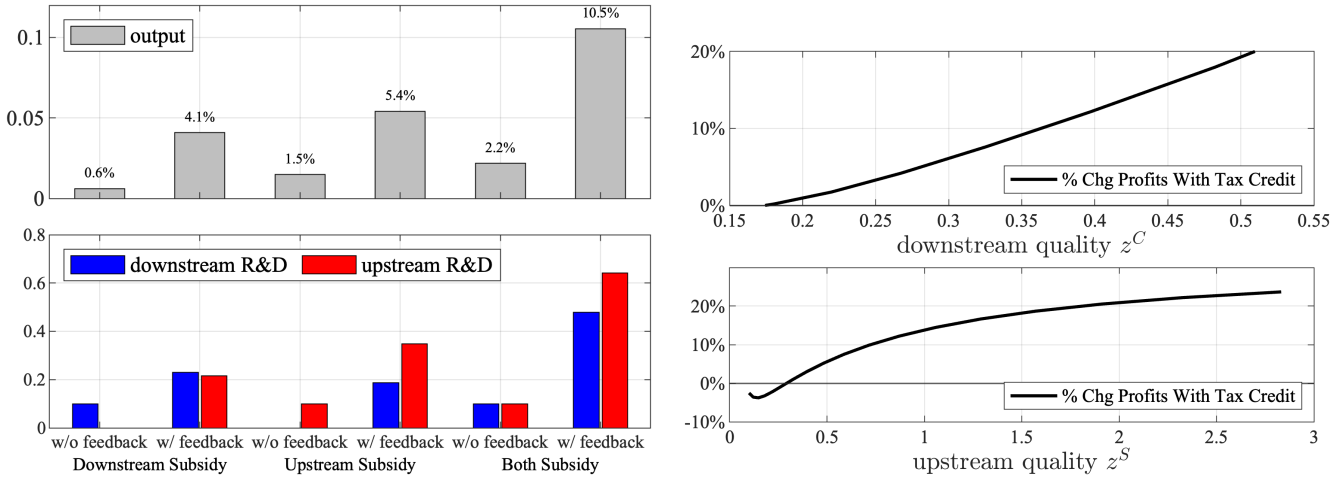
5.3 Impact of Sorting and Feedback on Output

In this subsection, I evaluate the impact of sorting and feedback on aggregate output in the economy. Given that firms are atomistic, the broader economy is insulated from the outcomes of any single firm. Therefore, to understand the iterative feedback between supply chain partners—specifically how innovation shifts profits and induces further development—a general equilibrium perspective is critical. Hence in this section, I construct two counterfactual economies where I allow either the downstream sector can innovate or the upstream sector can innovate separately.

This gives me R&D investment and distributions of the downstream and upstream sector, respectively, without feedback between the sectors. I then construct a third counterfactual economy—the economy without feedback—for which aggregate output is calculated given the distributions generated by these counterfactual economies. The third column of the left panel of Figure 10 presents output in the counterfactual economy without feedback but with sorting as a percent of the baseline economy with both sorting and feedback. Compared the the baseline economy, overall R&D is much lower and output is 29 percent lower, suggesting the amplification from feedback along the supply chain is quantitatively important to understanding innovation and aggregate output.

In the previous subsection, I considered the partial equilibrium effect of sorting to an individual downstream firm. However, if all downstream firms cannot endogenously choose their upstream suppliers, but must instead source equal weights from all upstream firms, this will change the elasticity of demand faced by upstream firms and hence optimal prices—upstream firms will instead set $p_k^S = \frac{\zeta}{\zeta-1}\psi^S z_k^S$ as if $\bar{\mu}_k = 0$ for all $k \in K$. Sorting has two effects: it increases allocative efficiency from the complementarity across inputs, but may result in increased upstream markups. Indeed, as shown in Figure 10, without sorting, output is 21 percent lower, but R&D is higher. The right panel of Figure 10 compares upstream and downstream R&D at different quality levels with and without sorting. Without sorting, lower quality downstream firms and higher quality upstream firms innovate more.

Taken together, my results indicate that firm sorting and sectoral feedback account for approximately one-fifth of output and they interact to amplify the impact of spillovers. Without feedback, perhaps counterintuitively, enabling downstream firms to endogenously choose their upstream partners, results in lower aggregate output because these upstream partners can command higher prices. Without sorting, however, feedback is less effective, since sorting means that higher quality firms, particularly downstream firms, are responding to higher quality partners, and thus innovating more.



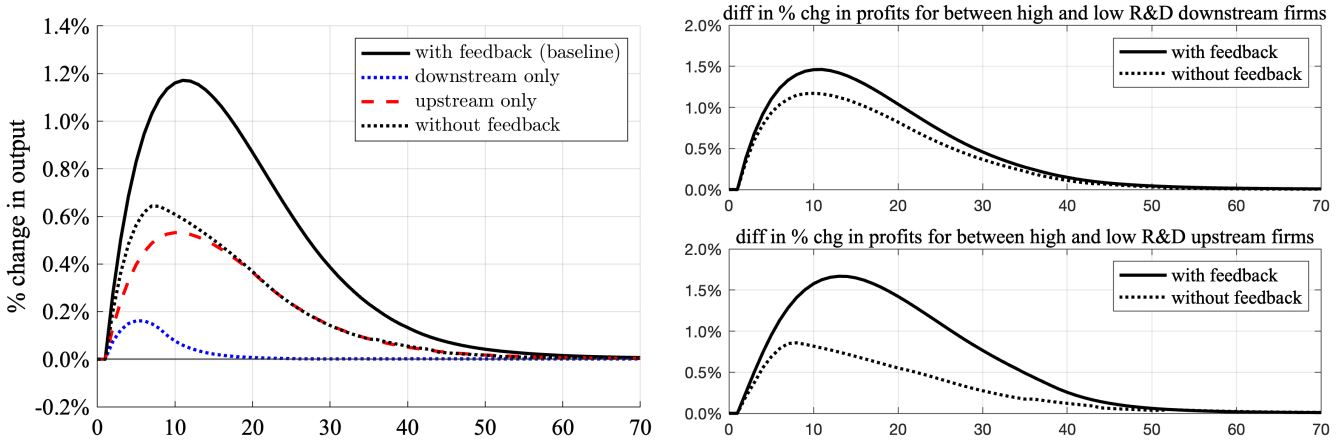
(A) Output and R&D (B) Change in Upstream and Downstream Profits

FIGURE 11 – Impact of R&D Tax Credits

Note: Panel A plots the impact of (i) a 10 percent downstream R&D subsidy, (ii) a 10 percent upstream R&D subsidy, and (iii) a 10 percent R&D subsidy to both upstream and downstream firms, with and without feedback. Panel B plots the change in profits after the R&D subsidy for downstream and upstream firms when there is innovation feedback.

5.4 Feedback on Propagation of R&D Tax Credits

In this subsection, I consider the impact of feedback on output and R&D for a 10 percent subsidy to R&D expenditure by (i) downstream firms, (ii) upstream firms, and (iii) both upstream and downstream firms. As in the previous subsection, to understand the impact of feedback, I consider how each distribution evolves when the other distribution is held fixed. As shown in the left panel of Figure **??**, feedback serves to amplify the impact of the downstream R&D subsidy, increasing the impact on output from 0.6 percent to 4.1 percent. When upstream firms also innovate, downstream firms respond by innovating more, amplifying the effect of the initial tax credit. Furthermore, feedback amplifies the effect of a R&D subsidy to both sectors, increasing the impact on output from 2.2 percent without feedback to 10.5 percent with feedback. Finally, as shown in the left panel of Figure **??**, the uniform R&D tax credit has a heterogeneous effect on quality: profits increase more for higher quality upstream and and downstream firms. For lower quality upstream firms, the negative effect from reallocation dominates, resulting in lower profits.



(A) Impact of Feedback on Output

(B) Difference in Change in Profits for High and Low R&D Firms

FIGURE 12 – Impact of Discount Rate Shock

Note: Panel A plots output after a transitory discount rate shock under the baseline economy with feedback compared to output if (i) only downstream firms innovated, (ii) only upstream firms innovated, and (iii) without feedback. Panel B plots the difference between the change in profits for high R&D and low R&D upstream and downstream firms with and without feedback.

5.5 Feedback on Propagation of Discount Rate Shock

The previous subsection demonstrated that feedback along the supply chain plays an important role in the propagation of R&D subsidies. Moreover, due to the heterogeneous impact of innovation spillovers, R&D subsidies which are uniform across firms, can differentially benefit higher quality, more innovative firms. In this subsection, I consider the impact of a transitory shock with perfect foresight to the discount factor, increasing the discount factor by 1 p.p. with a decay rate of 0.9, on aggregate output. To understand the impact of feedback, in Figure 12 I consider the change to output under the baseline economy with feedback where both sectors innovate and evolve to three counterfactuals: (i) the downstream sector innovates and the upstream distribution does not change (blue), (ii) the upstream sector innovates and the downstream distribution does not change (red), and (iii) without feedback (dotted line) where the downstream sector and upstream sector evolve as in (i) and (ii). If only the downstream sector innovates, the increased patience from the discount rate shock has a muted effect because returns to innovation by downstream firms are

still constrained by the upstream distribution. Upstream firms similarly innovate less when the downstream sector does not evolve.

Comparing the baseline with feedback to the counterfactual without feedback, we see that feedback along the supply chain amplifies and prolongs the impact of the discount rate shock, since the upstream and downstream distributions are slow moving, reinforcing an increased level of innovation even after the shock has dissipated. Furthermore, as shown in the right panel, profits of higher quality, innovative firms, benefit more from the discount rate shock, and this is due in part to the feedback effect along the supply chain.

Finally, consistent with feedback being important for the propagation of discount rate shocks, I consider whether the value of firms with more innovative supply chain respond more fluctuations in the discount rate. For each firm, I estimate the beta of the firm's log returns over a 5 year window on changes to the discount rate as measured by the inverse to the CAPE-Shiller index. Figure 13 plots a binscatter of this beta against the average R&D intensity of each firm's downstream partners over the same period controlling for the firm's own R&D intensity. Consistent with the model, valuations of firms with more innovative downstream partners respond to discount rate rate fluctuations.

6 Conclusion

This paper advances our understanding of both innovation and production networks by introducing a novel framework to understand innovation spillover along endogenous supply chains, offering insights into how supply chains impact firm value and how innovation spillovers propagate both at the sector level and at the firm-level.

Given motivating evidence of innovation spillovers along the supply chain and positive assortative matching in innovation by supply chain partners from U.S. public firms, I introduce a tractable,

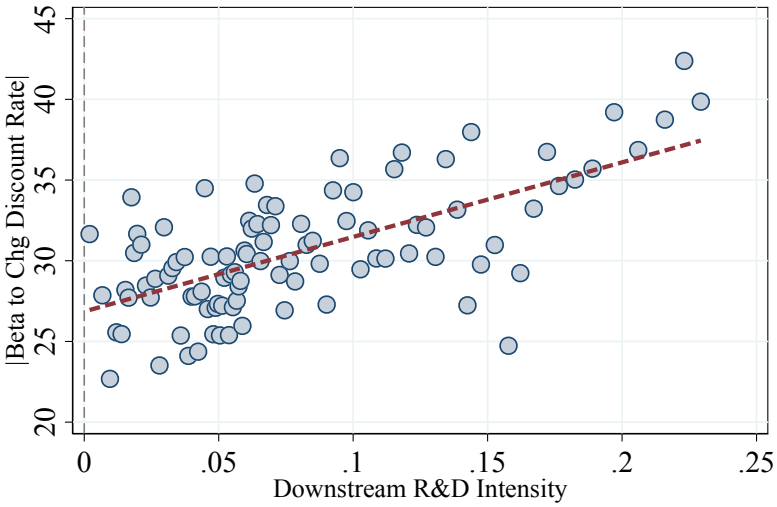


FIGURE 13 – Impact of Partner Innovation on Response to Discount Rate Changes

Note: For each firm-year, monthly log returns for the subsequent 60 months are regressed on changes to discount rates as measured by the inverse of the CAPE-Shiller index. Then firm-years are sorted into 100 bins based on downstream R&D intensity. Each circle reflects the average coefficient after controlling for the firm's own R&D intensity.

dynamic, heterogeneous-firm model which endogenizes both the intensity of the supply chain relationships and the innovation decisions of firms in general equilibrium. Firm value depends not only on the firm's own decisions and intellectual capital but also on the intangible capital embodied in their supply chain relationships. Innovation along the supply chain impacts firm returns and innovation. Complementarity of product inputs in two dimensions along the supply chain—quantity and quality—interact to generate both positive sorting in supply chain relationships and innovation spillover. This complementarity which creates innovation spillovers as quality improvements by supply chain partners increase returns to innovation, also results in decreasing returns to improving firms' own quality, since the firms are constrained by the quality of firms connected along the supply chain, meaning firm growth endogenously slows with age.

The paper underlines the importance of supply chain choice for input prices, firm value, and innovation propagation. Downstream production requires upstream inputs which are perfectly substitutable in quantities but imperfectly in quality. This provides a tractable means of modeling many-to-many supply chain relationships which endogenously vary with upstream and downstream

quality. In the model, observed elasticity of substitutions between upstream inputs depends on both upstream and downstream quality, since upstream firms which are similar in quality are effectively more substitutable. Because complementarity means that higher quality downstream firms are less price elastic, the model endogenously generates variation in upstream markups, where markups increase with the difference between upstream and downstream quality—price increases reflect downstream firms reaching for quality which enable high quality upstream firms to extract greater rents. Counterintuitively, increased choice in quality by downstream firms across upstream inputs increases upstream prices and markups.

Endogenous supply chain relationships combined with product complementarity provide an important mechanism by which innovation spillovers induce reallocation along supply chain relationships and change input prices as a function of quality, impacting firm valuation and returns to innovation. Downstream (upstream) quality improvements steepen (flatten) this price schedule. Both widen the distance in profits and valuations between high and low quality firms and further increase incentives to innovate.

Empirically, the paper validates the main predictions of the model and provides evidence that trade relationships are important to innovation spillovers along the supply chain. Using firm-level variation in R&D tax credit, I provide evidence that innovations by supply chain partners propagate through trade relationships to increase firm innovation, valuations, and returns to innovation, and that these results are robust to controlling for knowledge spillovers. Consistent with supply chain relationships being endogenous and innovation being forward looking, I show that innovation spillovers can propagate from both current and potential supply chain relationships and that spillovers propagate heterogeneously, improving firm value only when the firm itself innovates.

The model also provides a vehicle for understanding the importance of endogenous relationships for firm-level outcomes and the importance of sorting and feedback for aggregate outcomes in general

equilibrium. I show that when downstream firms are able to endogenously choose their partners, they achieve higher profits, and innovate further, but may innovate less at lower quality levels. Sorting ultimately benefits firms at the extremes of the quality distribution. The paper offers novel insights into how sorting and feedback together are crucial to understanding aggregate outcomes. In the model, both quality distributions determine profits which influence innovation decisions which in turn endogenously shape quality distributions. Without feedback, sorting might reduce output since upstream firms can command greater rents. Without sorting, feedback is less effective because high quality downstream firms cannot match and respond more to high quality upstream firms. These mechanisms have important implications to industrial policy and anti-trust, fiscal and monetary policy, and asset pricing. Supply chain relationships are a valuable source of intangible capital and firms benefit from coordination along the supply chain, providing incentives for vertical integration. Spillovers along these relationships amplify efforts from industrial and fiscal policy to increase growth through innovation subsidies. Endogenous sorting and feedback along the supply chain impact the propagation discount rates shocks, with aggregate and cross-sectional implications relevant to both monetary policy and asset pricing.

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Supplemental Online Appendix

NOT FOR PUBLICATION

A Model

A.1 Price minimization problem g

A.1.1 Solution to price minimization problem

The price minimization problem is

$$g\left(\bar{z}_i^S\right) = \min_{\{w_{ik}\}_{k\in K}} \underbrace{\int_0^1 w_{ik} p_k^S dk}_{\bar{p}_i^S} \text{ st } \left[\int_0^1 \left(w_{ik} z_k^S\right)^{\frac{\zeta-1}{\zeta}} dk\right]^{\frac{\zeta}{\zeta-1}} = \bar{z}_i^S \text{ and } \int_0^1 w_{ik} = 1$$

Let λ_i and μ_i be the multipliers on each constraint. Then

$$p_k^S + \mu_i = \lambda_i \left(\bar{z}_i^S\right)^{\frac{1}{\zeta}} \left(z_k^S\right)^{\frac{\zeta-1}{\zeta}} \left(w_{ik}\right)^{-\frac{1}{\zeta}} \iff w_{ik} = \lambda_i^{\zeta} \left(\bar{z}_i^S\right) \left(z_k^S\right)^{\zeta-1} \left(p_k^S + \mu_i\right)^{-\zeta}$$

Summing across k, gives us the following relationships between $\bar{p}_i^S$, μ_i, and λ_i:

$$\bar{p}_i^S + \mu_i = \lambda_i \bar{z}_i^S$$

$$1 = \lambda_i^{\zeta} \bar{z}_i^S \mathcal{A}_{0i}$$

$$\bar{p}_i^S = \left(\mathcal{A}_{0i}\right)^{-1} \mathcal{A}_{1i}$$

where $\mathcal{A}_{0i} = \int_0^1 \left(z_k^S\right)^{\zeta-1} \left(p_k^S + \mu_i\right)^{-\zeta} dk$ and $\mathcal{A}_{1i} = \int_0^1 p_k^S \left(z_k^S\right)^{\zeta-1} \left(p_k^S + \mu_i\right)^{-\zeta} dk$ are functions of μ_i. Thus the weight to firm k can be expressed as

$$w_{ik} = \left(\mathcal{A}_{0i}^{-1}\right) \left(z_k^S\right)^{\zeta-1} \left(p_k^S + \mu_i\right)^{-\zeta}$$

Thus for any $\bar{z}_i^S$ we can find μ_i which solves

$$\bar{z}_i^S = \left(\mathcal{A}_{i0}\right)^{\frac{1}{\zeta-1}} \left[\tilde{p}^S + \mu_i\right]^{\frac{\zeta}{\zeta-1}} = \left[\left(\mathcal{A}_{i0}\right)^{\frac{1}{\zeta}} \mu_i + \left(\mathcal{A}_{i0}\right)^{\frac{1}{\zeta}-1} \mathcal{A}_{i1}\right]^{\frac{\zeta}{\zeta-1}}$$

The multiplier on the constraint takes values $\mu_{i0} \in \left(-\bar{p}_{\min}^S, \infty\right)$ where $\bar{p}_{\min}^S = \min_{k\in K} p_k^S$. $\bar{z}_i^S$ and $\bar{p}_i^S$ are monotonically increasing in μ_{i0} (proven later). Note, if Φ^S is discrete with probability weights $\left\{\phi_k^S\right\}_{k\in K}$ for qualities $\left\{z_k^S\right\}_{k\in K}$, then $w_{ik} = \omega_{ik}/\phi_k^S$ where ω_{ik}^S is the allocation to type k.

A.1.2 Domain of g

Observe that g has domain $\bar{z}^S \in \left[0, \bar{z}_{\max}^S\right]$ where

$$\bar{z}_{\max}^S = \max_{\{w_{ik}\}_{k\in K}} \left[\int_0^1 \left(w_{ik} z_k^S\right)^{\frac{\zeta-1}{\zeta}} dk\right]^{\frac{\zeta}{\zeta-1}} \text{ st } \int_0^1 w_{ik} dk = 1$$

which has solution

$$\bar{z}_{\max}^S = \left[\int_0^1 \left(z_k^S\right)^{\zeta-1} dk\right]^{\frac{1}{\zeta-1}} \qquad w_{k,\max} = \left(\bar{z}_{\max}^S\right)^{1-\zeta} \left(z_k^S\right)^{\zeta-1}$$

Since the downstream firm must choose measure 1 of the upstream input, and firms are atomistic, $\bar{z}^S \to 0$ if the firm allocates $w_{ik} = 1$ to goods from a single firm. However, with the minimum per unit price $\bar{p}_{\min}^S = \min\left\{p_k^S\right\}_{k\in K}$. At that price, if the set of firms which offer the minimum prices is not atomistic, the firm might be able to achieve a higher $\bar{z}^S$, which it would strictly prefer. Indeed, the relevant domain of g

is $\bar{z}^S \in \left[\bar{z}^S_{\min}, \bar{z}^S_{\max}\right]$ where

$$\bar{z}^S_{\min} = \max_{\{w_{ik}\}_{k\in K}} \left[\int_0^1 \left(w_{ik} z_k^S\right)^{\frac{\zeta-1}{\zeta}} dk\right]^{\frac{\zeta}{\zeta-1}} \text{ st } \int_0^1 w_{ik} p_k^S dk = \bar{p}^S_{\min} \text{ and } \int_0^1 w_{ik} dk = 1$$

This has solution

$$\bar{p}^S_{\min} = \left(\mathcal{A}_{0i}\right)^{-1} \mathcal{A}_{1i}$$

where $\mathcal{A}_{0i} = \int_0^1 \left(z_k^S\right)^{\zeta-1} \left(p_k^S + \mu_i\right)^{-\zeta} dk$ and $\mathcal{A}_{1i} = \int_0^1 p_k^S \left(z_k^S\right)^{\zeta-1} \left(p_k^S + \mu_i\right)^{-\zeta} dk$ and $\bar{z}^S_{\min} = \left(\bar{p}_i^S + \mu_i\right)^{\frac{\zeta}{\zeta-1}} \left(\mathcal{A}_{0i}\right)^{\frac{1}{\zeta-1}}$.

A.1.3 g is convex, $\bar{p}^S$ and $\bar{z}^S$ increasing in μ

Define price minimization function g as

$$g\left(\bar{z}_i^S\right) = \min_{\{w_{ik}\}_{k\in K}} \underbrace{\int_0^1 w_{ik} p_k^S dk}_{\bar{p}_i^S} st \left[\int_0^1 \left(w_{ik} z_k^S\right)^{\frac{\zeta-1}{\zeta}} dk\right]^{\frac{\zeta}{\zeta-1}} = \bar{z}_i^S \text{ and } \int_0^1 w_{ik} dk = 1$$

If $\zeta > 1$, then g is convex.

Proof. Define $\bar{z}^S\left(w_i\right)$ and $\bar{p}^S\left(w_i\right)$as the aggregate quality and price achieved given allocation $w_i = \{w_{ik}\}_{k\in K}$.

$$\bar{z}^S\left(w_i\right) = \left[\int_0^1 \left(w_{ik} z_k^S\right)^{\frac{\zeta-1}{\zeta}} dk\right]^{\frac{\zeta}{\zeta-1}} \qquad \bar{p}^S\left(w_i\right) = \int_0^1 w_{ik} p_k^S dk$$

Let $m\left(w_i\right) = \int_0^1 \left(w_{ik} z_k^S\right)^{\frac{\zeta-1}{\zeta}} dk$. The Hessian matrix of m is diagonal matrix with elements

$$\frac{1-\zeta}{\zeta^2} \left(w_{ik}\right)^{\frac{\zeta-1}{\zeta}-2} \left(z_k^S\right)^{\frac{\zeta-1}{\zeta}}$$

Since for $\zeta > 1$, all elements are non positive, m is quasi-concave. Hence monotonic increasing transformation $\bar{z}^S\left(w_i\right)$=$\left[m\left(w_i\right)\right]^{\frac{\zeta}{\zeta-1}}$ is quasi concave. Since $\bar{z}^S\left(w_i\right)$ is homogeneous of degree 1 and quasi concave, it is also concave. The inverse of the downstream firm's price minimization problem can be characterized by quality maximization problem $h = g^{-1}$

$$h\left(\bar{z}_i^S\right) = \max_{\{w_{ik}\}_{k\in K}} \left[\int_0^1 \left(w_{ik} z_k^S\right)^{\frac{\zeta-1}{\zeta}} dk\right]^{\frac{\zeta}{\zeta-1}} \text{ st } \int_0^1 w_{ik} p_k^S dk = \bar{p}_i^S \text{ and } \int_0^1 w_{ik} = 1$$

It is straightforward to show h is also concave. Consider $\bar{z}_0^S = h\left(\bar{p}_0^S\right)$ and $\bar{z}_1^S = h\left(\bar{p}_1^S\right)$ where $\bar{p}_1^S > \bar{p}_0^S$ with optimal allocations w_0 and w_1, respectively. Trivially, $\bar{z}_1^S > \bar{z}_0^S$. Consider $\lambda \in (0,1)$ and allocation $w_\lambda = (1-\lambda) w_0 + \lambda w_1$. Since $\bar{z}^S$ is concave

$$\bar{z}^S > (1-\lambda) \bar{z}_0^S + \lambda \bar{z}_1^S$$

We can also verify

$$\sum_{k\in K} w_{\lambda k} = (1-\lambda) \sum_{k\in K} w_{0k} + \lambda \sum_{k\in K} w_{1k} = 1$$

$$\bar{p}_\lambda^S = \sum_{k\in K} \lambda p_k w_{\lambda k} = (1-\lambda) \sum_{k\in K} p_k w_{0k} + \lambda \sum_{k\in K} p_k w_{1k} = (1-\lambda) \bar{p}_0^S + \lambda \bar{p}_1^S$$

Hence h is concave. Since $g = h^{-1}$, g is convex. ■

On domain $\bar{z}^S \in \left[\bar{z}^S_{\min}, \bar{z}^S_{\max}\right]$, price function $\bar{p}^S = g\left(\bar{z}^S\right)$ is monotonically increasing and convex.

Proof. A convex function achieves at most 1 minimum. Since this occurs at $\bar{z}^S_{\min}$, g is monotonically

increasing and convex on domain $\bar{z}^S \in \left[\bar{z}^S_{\min}, \bar{z}^S_{\max}\right]$ ■

If $\zeta > 1$, then $\frac{d\bar{p}^S}{d\mu} > 0$

Proof. Recall that $\bar{p}^S = (\mathcal{A}_0)^{-1} \mathcal{A}_1$. Thus

$$\frac{d\bar{p}^S}{d\mu} = \zeta (\mathcal{A}_0)^{-1} \left[\bar{p}^S (\mathcal{A}_2) - (\mathcal{A}_3)\right]$$

where $\mathcal{A}_0 = \int_0^1 \left(z_k^S\right)^{\zeta-1} \left(p_k^S + \mu\right)^{-\zeta} dk$ and $\mathcal{A}_2 = \int_0^1 \left(z_k^S\right)^{\zeta-1} \left(p_k^S + \mu\right)^{-\zeta-1} dk$ and $\mathcal{A}_3 = \int_0^1 p_k^S \left(z_k^S\right)^{\zeta-1} \left(p_k^S + \mu\right)^{-\zeta-1} dk$. Since $\zeta > 1$, and $\mathcal{A}_0 > 0$, it is sufficient to show that $\bar{p}^S (\mathcal{A}_2) - (\mathcal{A}_3) > 0$. Define

$$\tilde{W} = \int_0^1 \left(z_k^S\right)^{\zeta-1} \left(p_k^S + \mu\right)^{-\zeta-1} dk$$

$$\tilde{w}_{ik} = \tilde{W} \left(z_k^S\right)^{\zeta-1} \left(p_k^S + \mu\right)^{-\zeta-1}$$

Then

$$\bar{p}^S (\mathcal{A}_2) - (\mathcal{A}_3) = \int_0^1 \left(\bar{p}^S - p_k^S\right) \left(z_k^S\right)^{\zeta-1} \left(p_k^S + \mu\right)^{-\zeta-1} dk = \tilde{W} \int_0^1 \left(\bar{p}^S - p_k^S\right) \tilde{w}_{ik} dk$$

Observe that since $\zeta > 1$

$$\bar{p}^S = \int_0^1 p_k^S w_{ik} dk > \int_0^1 p_k^S \tilde{w}_{ik} dk$$

Thus $\bar{p}^S (\mathcal{A}_2) - (\mathcal{A}_3) > 0$ ■

If $\zeta > 1$, then $\frac{d\bar{z}^S}{d\mu} > 0$

Proof. Note that $\bar{z}^S = (\mathcal{A}_0)^{\frac{1}{\zeta-1}} \left[(\mu + \bar{p})\right]^{\frac{\zeta}{\zeta-1}}$ where $\mathcal{A}_0 = \int_0^1 \left(z_k^S\right)^{\zeta-1} \left(p_k^S + \mu\right)^{-\zeta} dk$. As $\zeta > 1$, and $\mathcal{A}_0 > 0$, it is sufficient to show that $\mu + \bar{p}$ is increasing in μ. Since $\frac{d\bar{p}^S}{d\mu}$, it follows that $\frac{d\bar{z}^S}{d\mu}$. ■

A.2 Positive Assortative Matching

If $\zeta > 1$, $p_k^S > p_j^S$, and $\bar{z}_m^S > \bar{z}_i^S$, then $w_{ik}/w_{ij} > w_{mk}/w_{mj}$

Proof. The first order condition of the price minimization problem, given target $\bar{z}_i^S$, gives

$$p_k^S + \mu_i = \lambda_i \left(\bar{z}_i^S\right)^{\frac{1}{\zeta}} \left(z_k^S\right)^{\frac{\zeta-1}{\zeta}} \left(w_{ik}\right)^{\frac{-1}{\zeta}}$$

Thus for firms $k, j \in K$

$$\frac{w_{ik}}{w_{ij}} = \left(\frac{z_k^S}{z_j^S}\right)^{\zeta-1} \left(\frac{p_k^S + \mu_i}{p_j^S + \mu_i}\right)^{-\zeta}$$

If $p_k^S > p_j^S$, then $\frac{w_{ik}}{w_{ij}}$ increasing in μ_i. Since $\bar{z}_i^S$ increasing in μ_i, then $\frac{w_{ik}}{w_{ij}}$ increasing in $\bar{z}_i^S$. ■

Suppose $\epsilon > 1$ and $\nu < 1$, and price function $g'\left(\bar{z}^S\right) > 0$ and $g''\left(\bar{z}^S\right) > 0$. If $z_m^C > z_i^C$, then $\bar{z}_m^{S*} > \bar{z}_i^{S*}$. (The optimal upstream basket is increasing in downstream quality).

Proof. The downstream firm solve profit function

$$\pi^C \left(z_i^C\right) = \max_{\bar{z}_i^S} \left(\frac{\epsilon - 1}{\epsilon}\right)^{\epsilon} \frac{1}{\epsilon - 1} L (z_i)^{\epsilon} (\Psi_i)^{1-\epsilon}$$

where $z_i = \left(\kappa_C \left(z_i^C\right)^{\frac{\nu-1}{\nu}} + \kappa_S \left(\bar{z}_i^S\right)^{\frac{\nu-1}{\nu}}\right)^{\frac{\nu}{\nu-1}}$ and $\Psi_i = \psi^C z_i^C + g\left(\bar{z}_i^S\right)$. Optimal basket choice $\bar{z}_i^S$ satisfies

$$\epsilon \frac{\bar{z}_i^S}{z_i} \frac{dz_i}{d\bar{z}_i^S} + (1 - \epsilon) \frac{\bar{z}_i^S}{\Psi_i} \frac{d\Psi_i}{d\bar{z}_i^S} = 0$$

This equation can be rewritten as $F = \epsilon F_1 + (1-\epsilon) F_2 = 0$ where

$$F_1 = \frac{\bar{z}_i^S}{z_i}\frac{dz_i}{d\bar{z}_i^S} = \kappa_S \left(\frac{\bar{z}_i^S}{z_i}\right)^{\frac{\nu-1}{\nu}} \qquad \text{and}_2 = \frac{\bar{z}_i^S}{\Psi_i}\frac{d\Psi_i}{d\bar{z}_i^S} = \frac{\bar{z}_i^S}{\psi^C z_i^C + g\left(\bar{z}_i^S\right)} g'\left(\bar{z}_i^S\right)$$

Since $\nu < 1$, $\frac{dF_1}{d\bar{z}_i^S} < 0$ and $\frac{dF_1}{dz_i^C} > 0$. In addition, $\frac{dF_2}{d\bar{z}_i^S} > 0$, since $g', g'' > 0$[5], and $\frac{dF_2}{dz_i^C} < 0$. Recall $\epsilon > 1$. Hence $\frac{dF}{d\bar{z}_i^S} < 0$ and $\frac{dF}{dz_i^C} > 0$. Thus by the Implicit Function Theorem, $\frac{d\bar{z}_i^S}{dz_i^C} = -\frac{dF/dz_i^C}{dF/d\bar{z}_i^S} > 0$. ■

Upstream firms choose $p_k^S = \frac{1}{\zeta-1}\bar{\mu}_k + \frac{\zeta}{\zeta-1}\psi^S z_k^S$ where $\bar{\mu}_k$ solves

$$1 = \int_0^1 \frac{x_i w_{ik}}{x_k^S}\left[\frac{1}{\zeta} + \left(\frac{\zeta-1}{\zeta}\right)\left(\frac{\psi^S z_k^S + \mu_i}{\psi^S z_k^S + \bar{\mu}_k}\right)\right]^{-1} di$$

Proof. Upstream firm k with quality z_k^S faces demand $x_k^S = \int_0^1 x_{ik}^S di$ where $x_{ik}^S = w_{ik}x_i$ and

$$x_{ik}^S = x_i\left(z_k^S\right)^{\zeta-1}\left(\mathcal{A}_{i0}\right)^{-1}\left(p_k^S + \mu_i\right)^{-\zeta}$$

Upstream firms take $x_i, \mathcal{A}_{i0}$, and μ_i as given for all $i \in I$ and set the ratio of revenue to gross margin equal to elasticity of demand

$$\frac{p_k^S}{p_k^S - \psi^S z_k^S} = -\frac{dx_k^S/x_k^S}{dp_k^S/p_k^S}$$

The price elasticity it faces is a quantity-weighted average of downstream price elasticity of demand

$$\frac{dx_k^S/x_k^S}{dp_k^S/p_k^S} = \int_0^1 \frac{x_{ik}^S}{x_k^S}\left(\frac{dx_{ik}^S/x_{ik}^S}{dp_k^S/p_k^S}\right) di$$

where elasticity of demand of i for firm k's input is increasing in p_k^S and decreasing in μ_i. That is, demand by higher quality downstream firms is less responsive to price.

$$-\frac{dx_{ik}^S/x_{ik}^S}{dp_k^S/p_k^S} = \zeta\frac{p_k^S}{p_k^S + \mu_i}$$

Hence the optimal price satisfies

$$\frac{1}{\zeta} = \int_0^1 \frac{x_{ik}^S}{x_k^S}\left(\frac{p_k^S - \psi^S z_k^S}{p_k^S + \mu_i}\right) di$$

Substituting $p_k^S = \frac{1}{\zeta-1}\bar{\mu}_k + \frac{\zeta}{\zeta-1}\psi^S z_k^S$, then

$$1 = \int_0^1 \frac{x_{ik}^S}{x_k^S}\left(\frac{1}{\zeta} + \left(\frac{\zeta-1}{\zeta}\right)\frac{\mu_i + \psi^S z_k^S}{\bar{\mu}_k + \psi^S z_k^S}\right)^{-1} di$$

■

p_k^S is increasing in z_k^S.

Proof. Recall that upstream firms choose p_k^S such that

$$\frac{1}{\zeta} = \int_0^1 \frac{x_{ik}^S}{x_k^S}\left(\frac{p_k^S - \psi^S z_k^S}{p_k^S + \mu_i}\right) di$$

[5]Since g convex, elasticity of price with respect to upstream quality $\frac{dg\left(\bar{z}_i^S\right)/g\left(\bar{z}_i^S\right)}{d\bar{z}_i^S/\bar{z}_i^S}$ is increasing in upstream quality $\bar{z}_i^S$

Note $x_k^S = \int_0^1 x_{ik} di$ where $x_{ik} = w_{ik} x_i$. Thus the previous equation can be rewritten as follows where $F = 0$.

$$F = \int_0^1 x_i w_{ik}^C \left[\frac{1}{\zeta} \left(p_k^S - \psi^S z_k^S \right)^{-1} - \left(p_k^S + \mu_i \right)^{-1} \right] di$$

Recall that $w_{ik} = \left(z_k^S \right)^{\zeta - 1} \left(\mathcal{A}_{i0} \right)^{-1} \left(p_k^S + \mu_i \right)^{-\zeta}$. Substituting $\frac{dw_{ik}}{dz_{ik}^S} = (\zeta - 1) \frac{w_{ik}}{z_k^S}$, $\frac{dw_{ik}}{dp_k^S} = -\zeta \frac{w_{ik}}{p_k^S + \mu_i}$, and $F = 0$, we can evaluate $\frac{dF}{dz_k^S}$ and $\frac{dF}{dp_k^S}$.

$$\frac{dF}{dz_k^S} = \int_0^1 x_i w_{ik} \frac{\psi^S}{\zeta} \left(p_k^S - \psi^S z_k^S \right)^{-2} di$$

Since $p_k^S - \psi^S z_k^S > 0$, $\frac{dF}{dz_k^S} > 0$. Next, $\frac{dF}{dp_k^S} = -\mathcal{I}_1 - \zeta \mathcal{I}_2$ where

$$\mathcal{I}_1 = \int_0^1 x_i w_{ik} \left[\frac{1}{\zeta} \left(p_k^S - \psi^S z_k^S \right)^{-2} - \left(p_k^S + \mu_i \right)^{-2} \right] di$$

$$\mathcal{I}_2 = \int_0^1 x_i w_{ik} \left[\frac{1}{\zeta} \left(p_k^S - \psi^S z_k^S \right)^{-1} \left(p_k^S + \mu_i \right)^{-1} - \left(p_k^S + \mu_i \right)^{-2} \right] di$$

By Jensen's Inequality $\mathcal{I}_1 > 0$. Then $\mathcal{I}_2 > \mathcal{I}_2 - \zeta \mathcal{I}_1$ where

$$\mathcal{I}_2 - \zeta \mathcal{I}_1 = \left(p_k^S - \psi^S z_k^S \right) F + (\zeta - 1) \int_0^1 x_i w_{ik} \left(p_k^S + \mu_i \right)^{-2} di > 0$$

Note $F = 0$ and $p_k^S + \mu_i > 0$ and $\zeta > 1$. Hence $\frac{dF}{dp_k^S} < 0$. Thus, by Implicit Function Theorem, $\frac{dp_k^S}{dz_k^S} = -\frac{dF/dz_k^S}{dF/dp_k^S} > 0$ ■

[Positive Assortative Matching] Suppose $\epsilon, \zeta > 1$ and $\nu < 1$. If $z_m^C > z_i^C$ and $z_k^S > z_j^S$, then $w_{ik}/w_{ij} > w_{mk}/w_{mj}$.

Proof. Since $\zeta > 1$, $p_k^S > p_j^S$. Since $\epsilon > 1$ and $\nu < 1$, if $z_m^C > z_i^C$ then optimal basket $\bar{z}_m^S > \bar{z}_i^S$ so $\mu_m > \mu_i$. Thus $w_{ik}/w_{ij} > w_{mk}/w_{mj}$. ■

A.3 Downstream Profits

Given downstream quality z_i^C, chosen upstream quality $\bar{z}_i^S$, and price schedule $g\left(\bar{z}_i^S\right)$, define unit costs faced by the downstream firm as

$$\Psi_i = \psi^C z_i^C + g\left(\bar{z}_i^S\right)$$

Substituting demand $x_i = \left(\frac{z_i}{p_i} \right)^{\epsilon} L$ and optimal price $p_i = \frac{\epsilon}{\epsilon - 1} \Psi_i$, profits for a downstream firm with blueprint quality z_i^C can be expressed as

$$\pi^C\left(z_i^C\right) = \left(\frac{\epsilon - 1}{\epsilon} \right)^{\epsilon} \frac{1}{\epsilon - 1} L \left(z_i \right)^{\epsilon} \left(\Psi_i \right)^{1 - \epsilon}$$

The elasticity of downstream profits to its own quality, $\varphi^C\left(z_i^C\right) = \frac{d\pi_i^C / \pi_i^C}{dz_i^C / z_i^C}$, can be expressed as

$$\varphi^C\left(z_i^C\right) = \epsilon \frac{dz_i / z_i}{dz_i^C / z_i^C} + (1 - \epsilon) \frac{d\Psi_i / \Psi_i}{dz_i^C / z_i^C}$$

where $\frac{dz_i / z_i}{dz_i^C / z_i^C} = \kappa_C \left(\frac{z_i^C}{z_i} \right)^{\frac{\nu - 1}{\nu}}$ and $\frac{d\Psi_i / \Psi_i}{dz_i^C / z_i^C} = \psi^C \frac{z_i^C}{\Psi_i}$.

The cross elasticity of downstream profits to upstream quality, $\hat{\varphi}^C\left(z_i^C\right) = \frac{d\pi_i^C/\pi_i^C}{d\bar{z}_i^S/\bar{z}_i^S}$, can be expressed as

$$\hat{\varphi}^C\left(z_i^C\right) = \epsilon \frac{dz_i/z_i}{d\bar{z}_i^S/\bar{z}_i^S} + (1-\epsilon)\frac{d\Psi_i/\Psi_i}{d\bar{z}_i^S/\bar{z}_i^S}$$

where $\frac{dz_i/z_i}{d\bar{z}_i^S/\bar{z}_i^S} = \kappa_S\left(\frac{\bar{z}_i^S}{z_i}\right)^{\frac{\nu-1}{\nu}}$ and $\frac{d\Psi_i/\Psi_i}{d\bar{z}_i^S/\bar{z}_i^S} = g'\left(\bar{z}_i^S\right)\frac{\bar{z}_i^S}{\Psi_i}$.

If all downstream firms choose $\bar{z}_i^S = \bar{z}^S$ with price $g\left(\bar{z}^S\right) = \frac{\zeta}{\zeta-1}\psi^S\bar{z}^S$, then $\frac{d\Psi_i/\Psi_i}{d\bar{z}_i^S/\bar{z}_i^S} = \frac{\frac{\zeta}{\zeta-1}\psi^S\bar{z}^S}{\Psi_i}$.

Suppose $\epsilon > 1$, $\nu < 1$, g convex, and $\epsilon + (1-\epsilon)\psi^C < 0$. For given upstream prices $\left\{p_k^S\right\}_{k\in K}$ and upstream distribution Φ^S, there exists threshold $z^{C*} < z^{C**}$ such that

1. For $z_i^C < z^{C*}$, downstream profits $\pi^C\left(z_i^C\right)$ are increasing and convex in z_i^C
2. For $z^{C*} < z_i^C < z^{C**}$, downstream profits are increasing and concave in z_i^C.
3. For $z_i^C > z^{C**}$, downstream profits are decreasing z_i^C.

Proof. As $z_i^C \to \infty$, $\frac{dz_i/z_i}{d\bar{z}_i^S/\bar{z}_i^S} \to 0$, $\frac{d\Psi_i/\Psi_i}{d\bar{z}_i^S/\bar{z}_i^S} \to 1$, and $\varphi^C\left(z_i^C\right) \to (1-\epsilon) < 0$. Thus there exists z^{C**} where $\varphi^C\left(z^{C**}\right) = 0$. Moreover, for $z_i^C < z^{C**}$, $\pi^C\left(z_i^C\right)$ are increasing in z_i^C, and decreasing otherwise. The second derivative of profits can be expressed as

$$\frac{d^2\pi^C}{d\left(z_i^C\right)^2} = \frac{\pi^C}{z_i^C}\left[\frac{\varphi^C\left(z_i^C\right)}{z_i^C}\left[\varphi\left(z_i^C\right)-1\right] + \frac{d\varphi^C\left(z_i^C\right)}{dz_i^C}\right]$$

where

$$\frac{d\varphi^C\left(z_i^C\right)}{dz_i^C} = \epsilon\kappa_C\frac{\nu-1}{\nu}\left(\frac{z_i^C}{z_i}\right)^{-\frac{1}{\nu}} + (1-\epsilon)\frac{\frac{\psi^C z_i^C}{g(\bar{z}^S)}}{\left(1+\frac{\psi^C z_i^C}{g(\bar{z}^S)}\right)^2}$$

Since $\epsilon > 1$ and $\nu < 1$, $\frac{d\varphi^C\left(z_i^C\right)}{dz_i^C} < 0$. As $z_i^C \to z^{C**}$, $\varphi\left(z_i^C\right) \to 0$. Hence $\frac{d^2\pi^C}{d\left(z_i^C\right)^2} < 0$. Observe that for $\varphi\left(z_i^C\right)$ sufficiently large, such that $\frac{d^2\pi^C}{d\left(z_i^C\right)^2} > 0$, establishing lower threshold z^{C*}. Intuitively, when Φ^S is held constant, there are decreasing returns to increasing z_i^C. However, increasing z_i^C is accompanied by linearly increasing blueprint cost. For $z_i^C > z^{C**}$, the latter dominates the former. ■

Suppose $\epsilon > 1$, $\nu < 1$, and g convex and all firms use bundle quality $\bar{z}^S$ with price $g\left(\bar{z}^S\right) = \frac{\zeta}{\zeta-1}\psi^S\bar{z}^S$

1. Threshold z^{C**} increases with upstream quality $\bar{z}^S$.
2. Elasticity $\varphi^C\left(z_i^C\right)$ increases with $\bar{z}^S$ and cross elasticity $\hat{\varphi}^C\left(z_i^C\right)$ increases with z_i^C.
3. There exists threshold z^{C***} such that for $z_i^C > z^{C***}$, profits are increasing in upstream quality ,$\hat{\varphi}^C\left(z_i^C\right) > 0$, and decreasing otherwise.

Proof. (1) If the upstream distribution is degenerate then, all downstream firms use bundle $\bar{z}^S$ with price $\bar{p}^S = \frac{\zeta}{\zeta-1}\psi^S\bar{z}^S$. Since $\nu < 1$, $\frac{dz_i/z_i}{d\bar{z}_i^S/\bar{z}_i^S}$ increases with $\bar{z}^S$ and $\frac{dz_i/z_i}{d\bar{z}_i^S/\bar{z}_i^S}$ decrease with $\bar{z}^S$. Since $\epsilon > 1$, threshold z^{C**} such that $\varphi^C\left(z^{C**}\right) = 0$ must increase. ■

Proof. (3) Note that

$$\frac{d\varphi^C\left(z_i^C\right)}{d\bar{z}^S} = \epsilon\frac{1-\nu}{\nu}\kappa_C\kappa_S\left(\frac{z_i^C}{z_i}\right)^{\frac{\nu-1}{\nu}}\left(\frac{\bar{z}^S}{z_i}\right)^{\frac{\nu-1}{\nu}}\frac{1}{\bar{z}^S} + (\epsilon-1)\frac{\frac{\zeta}{\zeta-1}\psi^C\psi^S z_i^C}{\left(\Psi_i\right)^2} > 0$$

$$\frac{d\hat{\varphi}^C\left(z_i^C\right)}{dz_i^C}=\epsilon\frac{1-\nu}{\nu}\kappa_C\kappa_S\left(\frac{z_i^C}{z_i}\right)^{\frac{\nu-1}{\nu}}\left(\frac{\bar{z}^S}{z_i}\right)^{\frac{\nu-1}{\nu}}\frac{1}{z_i^C}+(\epsilon-1)\frac{\frac{\zeta}{\zeta-1}\psi^C\psi^S\bar{z}^S}{\left(\Psi_i\right)^2}>0$$

(2) As $z^C\to 0$, $\hat{\varphi}^C\to 1-\epsilon<0$. Since $\hat{\varphi}$ is continuous and increasing in z_i^C, there exists z^{C***} such that for $z_i^C>z^{C***}$, $\hat{\varphi}^C\left(z_i^C\right)\geq 0$, and $\hat{\varphi}^C\left(z_i^C\right)>0$ otherwise. ■

A.4 Upstream Profits

Define the elasticity of upstream profits to its own quality as $\varphi^S\left(z_k^S\right)=\frac{d\pi_k^S/\pi_k^S}{dz_k^S/z_k^S}$. If all downstream firms have homogenous quality $z_i^C=z^C$, then define the cross elasticity of downstream profits to upstream quality $\hat{\varphi}^S\left(z_k^S\right)=\frac{d\pi_k^S/\pi_k^S}{dz^C/z^C}$.

Suppose $\epsilon,\zeta>1$ and $\nu<1$ and all downstream firms have homogenous quality $z_i^C=z^C$ with shadow price $\mu_i=\mu$, quantity $x_i=x^C$, and aggregate $\mathcal{A}_{i0}=\mathcal{A}_0$.

(1) Profits are increasing in quality, if $1+\frac{\psi^S z_k^S}{\mu}>0$ and decreasing otherwise. Define z^{S**} as this threshold.

Proposition A.1. *(2) As $z_k^S\to z^{S**}$, $\pi^S\left(z_k^S\right)$ is concave.*

Proof. Since all downstream are identical, upstream firm k with quality z_k^S optimally chooses $p_k^S=\frac{1}{\zeta-1}\mu+\frac{\zeta}{\zeta-1}\psi^S z_k^S$ and face demand

$$x_k=\left(z_k^S\right)^{\zeta-1}x^C\left(\mathcal{A}_0\right)^{-1}\left(\frac{\zeta}{\zeta-1}\left[\mu+\psi^S z_k^S\right]\right)^{-\zeta}$$

Upstream profits can be expressed as a function of its own quality z_k^S and downstream moments x^C, $\mathcal{A}_0$, and μ.

$$\pi^S\left(z_k^S\right)=x_k\left(p_k^S-\psi^S z_k^S\right)=\frac{1}{\zeta-1}\left(\frac{\zeta}{\zeta-1}\right)^{-\zeta}\left(\frac{x^C}{\mathcal{A}_0}\right)\left(\frac{z_k^S}{\mu+\psi^S z_k^S}\right)^{\zeta-1}$$

Thus elasticity of profits to upstream quality $\varphi^S\left(z_k^S\right)=(\zeta-1)\left[1+\frac{\psi^S z_k^S}{\mu}\right]^{-1}$ is positive provided $1+\frac{\psi^S z_k^S}{\mu}>0$. Since

$$\frac{d\varphi^S\left(z_k^S\right)}{dz_k^S}=-1\left(\zeta-1\right)\left[1+\frac{\psi^S z_k^S}{\mu}\right]^{-2}\psi^S$$

and $\zeta>1$, for $1+\frac{\psi^S z_k^S}{\mu}>0$, $\frac{d\varphi^S\left(z_k^S\right)}{dz_k^S}<0$ and the profit function is concave in quality. ■

If $\zeta>1$ and $\mu_i>\psi^S z_k^S$, then elasticity $\frac{d\left(\frac{x_i}{\mathcal{A}_{i0}}\right)/\left(\frac{x_i}{\mathcal{A}_{i0}}\right)}{dz_i^C/z_i^C}>0$

Proof. The demand effect can be expressed as the difference between $\frac{dx_i/x_i}{dz_i^C/z_i^C}$ and $\frac{d\mathcal{A}_{i0}/\mathcal{A}_{i0}}{dz_i^C/z_i^C}$

$$\frac{d\left(\frac{x_i}{\mathcal{A}_{i0}}\right)/\left(\frac{x_i}{\mathcal{A}_{i0}}\right)}{dz_i^C/z_i^C}=\frac{dx_i/x_i}{dz_i^C/z_i^C}-\frac{d\mathcal{A}_{i0}/\mathcal{A}_{i0}}{dz_i^C/z_i^C}$$

Each term can be expressed as

$$\frac{dx_i/x_i}{dz_i^C/z_i^C}=\epsilon\left[\frac{dz_i/z_i}{dz_i^C/z_i^C}-\frac{dp_i/p_i}{dz_i^C/z_i^C}\right]=\epsilon\left[\frac{\kappa_C\left(z_i^C\right)^{\frac{\nu-1}{\nu}}}{\kappa_C\left(z_i^C\right)^{\frac{\nu-1}{\nu}}+\kappa_S\left(\bar{z}_i^S\right)^{\frac{\nu-1}{\nu}}}-\frac{\psi^C z_i^C}{\psi^C z_i^C+\bar{p}_i^S}\right]$$

$$\frac{d\mathcal{A}_{i0}/\mathcal{A}_{i0}}{dz_i^C/z_i^C}=-\zeta\left(\int_0^1\left(z_k^S\right)^{\zeta-1}\left(\frac{\zeta}{\zeta-1}\right)^{-\zeta}\left(\mu_i+\psi^S z_k^S\right)^{-\zeta-1}dk\right)\left(\frac{d\mu_i}{dz_i^C}\right)\left(\frac{z_i^C}{A_{i0}}\right)$$

Note $\frac{dx_i/x_i}{dz_i^C/z_i^C}$ always positive. If $\mu_i > \psi^S z_k^S$ then $\frac{d\mathcal{A}_{i0}/\mathcal{A}_{i0}}{dz_i^C/z_i^C} < 0$. Thus $\frac{d\left(\frac{x_i}{\mathcal{A}_{i0}}\right)/\left(\frac{x_i}{\mathcal{A}_{i0}}\right)}{dz_i^C/z_i^C} > 0$. ■

Suppose $\epsilon, \zeta > 1$ and $\nu < 1$ and all downstream firms have homogenous quality $z_i^C = z^C$. Then

1. Threshold z^{S**} increases with downstream quality z_i^C.
2. Elasticity $\varphi^S\left(z_k^S\right)$ increases with z^C and cross elasticity $\hat{\varphi}^S\left(z_k^S\right)$ increases with z_k^S
3. There exists threshold z^{S***} such that for $z_i^S > z^{C***}$, profits are increasing in downstream quality ,$\hat{\varphi}^S\left(z_k^S\right) > 0$, and decreasing otherwise.

Proof. Recall that since $\epsilon, \zeta > 1$ and $\nu < 1$, z^C is increasing in both $\bar{z}^S$ and μ. Hence z^{S**} increases. ■

Proof. Cross elasticity of upstream profits to downstream quality can be characterized as the sum the demand effect which impacts all upstream firms equally and the reallocation effect which benefits higher quality firms.

$$\hat{\varphi}^S\left(z_k^S\right) = \underbrace{\frac{d\left(\frac{x^C}{\mathcal{A}_0}\right)}{dz^C}\frac{z^C}{\left(\frac{x^C}{\mathcal{A}_0}\right)}}_{demand} - \underbrace{\left(\zeta - 1\right)\frac{z^C}{\mu + \psi^S z_k^S}\frac{d\mu}{dz^C}}_{reallocation}$$

Since $\frac{d\mu}{dz^C} > 0$, $\frac{d\varphi^S\left(z_k^S\right)}{dz^C} > 0$ and $\frac{d\hat{\varphi}^S\left(z_k^S\right)}{dz_k^S} > 0$

$$\frac{d\varphi^S\left(z_k^S\right)}{dz^C} = \left(\zeta - 1\right)\left[1 + \frac{\psi^S z_k^S}{\mu}\right]^{-2}\frac{\psi^S z_k^S}{\left(\mu\right)^2}\frac{d\mu}{dz^C} > 0$$

$$\frac{d\hat{\varphi}^S\left(z_k^S\right)}{dz_k^S} = \left(\zeta - 1\right)\psi^S\frac{z^C}{\left(\mu + \psi^S z_k^S\right)^2}\frac{d\mu}{dz^C} > 0$$

Observe that for $1 + \frac{\psi^S z_k^S}{\mu} > 0$, the demand effect is positive, and for sufficiently high z_k^S, $\hat{\varphi}^S\left(z_k^S\right) > 0$. Hence for $z_k^S > z^{S***}$, where $\hat{\varphi}^S\left(z^{S***}\right) = 0$, upstream profits are increasing in downstream quality. ■

A.5 Upstream firm quality $z_k^S \in \left\{z_0^S, z_1^S\right\}$

Suppose the upstream distribution is discrete with K qualities. Given allocation ω where $\omega_k = \phi_k w_k$, upstream basket quality and price can be expressed as

$$\bar{z}^S\left(\omega\right) = \left[\sum_{k\in K}\phi_k^S\left(z_k^S\frac{\omega_k}{\phi_k^S}\right)^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}}, \qquad \bar{p}^S\left(\omega\right) = \sum_{k\in K}\omega_k p_k^S$$

Consider the special case where upstream firms can be either high or low quality: $z_k^S \in \left\{z_0^S, z_1^S\right\}$, with fraction ϕ^S of upstream firms having higher quality $z_1^S > z_0^S$. Then

$$\bar{z}^S\left(\omega\right) = \left[\left(\phi_0^S\right)^{\frac{1}{\zeta}}\left(\omega_0 z_0^S\right)^{\frac{\zeta-1}{\zeta}} + \left(\phi_1^S\right)^{\frac{1}{\zeta}}\left(\omega_1 z_1^S\right)^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}}$$

$$\bar{p}^S\left(\omega\right) = \omega_0 p_0^S + \omega_1 p_1^S$$

Define $m\left(\omega\right) = \left(\phi_0^S\right)^{\frac{1}{\zeta}}\left(\omega_0 z_0^S\right)^{\frac{\zeta-1}{\zeta}} + \left(\phi_1^S\right)^{\frac{1}{\zeta}}\left(\omega_1 z_1^S\right)^{\frac{\zeta-1}{\zeta}}$. Then

$$\frac{dm\left(\omega\right)}{d\phi_1} = -\frac{1}{\zeta}\left(\frac{\omega_0 z_0^S}{\phi_0^S}\right)^{\frac{\zeta-1}{\zeta}} + \frac{1}{\zeta}\left(\frac{\omega_1 z_1^S}{\phi_1^S}\right)^{\frac{\zeta-1}{\zeta}} > 0 \iff \frac{\omega_1 z_1^S}{\phi_1^S} > \frac{\omega_0 z_0^S}{\phi_0^S}$$

Hence there exists threshold $\omega_1^{**} = \frac{z_0^S}{z_0^S + z_1^S \frac{\phi_0^S}{\phi_1^S}}$ such that for $\omega_1 > \omega_1^{**}$, $\frac{dm(\omega)}{d\phi_1} > 0$. Since $\zeta > 1$, $\frac{d\bar{z}^S(\omega)}{d\phi_1} > 0$.

If prices do not change with ϕ_1^S, then neither does $\bar{p}^S(\omega)$. Note that $\bar{z}^S$ is monotonic in ω_1. Thus as ϕ_1^S increases, there exists threshold $\bar{z}^{S**} = \bar{z}^S([1-\omega_1^{**}, \omega_1^{**}])$ such basket $\bar{z}_i^S$ becomes less expensive if $\bar{z}_i^S > \bar{z}^{S**}$ and vice versa. Since z_i^C is increasing in optimal basket $\bar{z}_i^S$, there exists threshold z^{C**} such that for $z_i^C > z^{C**}$. Since downstream firm i can achieve the original basket $\bar{z}_i^S$ at lower cost profits under its original ω serve as a lower bound, and any changes to ω must improve profits. For these firms, profits are increasing in ϕ_1^S: $\frac{d\pi^C(z_i^C)}{d\phi_1^C} \geq 0$. Note that

$$\frac{d\log\pi^C}{d\log\phi_1^S} = \epsilon\kappa_S \left(\frac{\bar{z}_i^S}{z_i}\right)^{\frac{\nu-1}{\nu}} \frac{d\bar{z}_i^S}{d\phi_1^C}\frac{\phi_1^S}{\bar{z}_i^S} + (1-\epsilon)\frac{\bar{p}_i^S}{\Psi_i}\frac{d\bar{p}_i^S}{d\phi_1^S}\frac{\phi_1^S}{\bar{p}_i^S}$$

Downstream profits are increasing in ϕ_1^S provided

$$\frac{d\bar{z}_i^S}{d\bar{p}_i^S} = \frac{\frac{d\bar{z}_i^S}{d\phi_1^S}}{\frac{d\bar{p}_i^S}{d\phi_1^S}} > \left(\frac{\epsilon-1}{\epsilon}\right)\frac{z_i}{\Psi_i\kappa_S}\left(\frac{\bar{z}_i^S}{z_i}\right)^{\frac{1}{\nu}} > 0$$

where

$$\frac{d\bar{z}_i^S}{d\phi_1^C} = \frac{1}{\zeta-1}\left[\bar{z}_i^S\right]^{\frac{1}{\zeta}}\left[\left(\frac{\omega_1 z_1^S}{\phi_1^S}\right)^{\frac{\zeta-1}{\zeta}} - \left(\frac{\omega_0 z_0^S}{\phi_0^S}\right)^{\frac{\zeta-1}{\zeta}}\right]$$

$$\frac{d\bar{p}_i^S}{d\phi_1^S} = \omega_0\frac{dp_0^S}{d\phi_1^S} + \omega_1\frac{dp_1^S}{d\phi_1^S}$$

If $\omega_1 < \omega_1^{**}$, then $\frac{d\bar{z}_i^S}{d\phi_1^S} < 0$: as ϕ_1^S increase firm i chooses lower basket quality $\bar{z}_i^S$. Note that $\frac{dp_1^S}{d\phi_1^S} < 0$ and $\frac{dp_0^S}{d\phi_1^S} > 0$: as the share of high quality upstream firms increase prices of competing high quality firms fall. In this region, if $\frac{d\bar{p}_i^S}{d\phi_1^S} > 0$ then $\frac{d\bar{z}_i^S}{d\bar{p}_i^S} < 0$ so $\frac{d\pi^C(z_i^C)}{d\phi_1^S} < 0$.

A.6 Sorting

Measuring Relationships Persistence

Defining Δ_{ik} as the expected percent of the relationship that is lost from period t to $t+1$

$$\Delta_{ikt} = E\left[\max\left(\frac{w_{ikt} - w_{ikt+1}}{w_{ikt}}, 0\right)\right]$$

If neither firms innovate, if the economy is at its stationary equilibrium, then

$$\Delta_{ikt} = 1 - \left(1-\phi^S\right)\left(1-\phi^C\right)$$

where ϕ^S and ϕ^C is the exogenous probability the firm exists. If the firm innovates, then Δ_{ik} increases. Then

$$\Delta_{ikt} = \underbrace{\left[1-\left(1-\phi^S\right)\left(1-\phi^C\right)\right]}_{1-bothsurvive} + \underbrace{\left(1-\phi^C\right)\left(1-\phi^S\right)}_{bothsurvive}\sum_{z_{it+1}^C}\sum_{z_{kt+1}^S}\pi^C\left(z_{it+1}^C \mid z_{it}^C\right)\pi^S\left(z_{it+1}^S \mid z_{it}^S\right)\max\left(\frac{w_{ikt}-w_{ikt+1}}{w_{ikt}}, 0\right)$$

This measure can be aggregated to the firm and sector level

$$\Delta_{it}^C = \int_0^1 w_{ikt}\Delta_{ikt}dk$$

$$\Delta_t^C = \int_0^1 \Delta_{it}^C di$$

Limits to Sorting

Alternatively, suppose instead of being fully flexible, downstream firms cannot choose fraction τ of their suppliers. Then

$$\tilde{w}_{ik} = \tau + (1-\tau) w_{ik}$$

Suppose firms act disjointly and choose w_{ik} as if solving the original problem. Then

$$\tilde{z}^S(w_k) = \left[\int_0^1 (\tau + (1-\tau) w_k)^{\frac{\zeta-1}{\zeta}} \left(z_k^S\right)^{\frac{\zeta-1}{\zeta}} dk\right]^{\frac{\zeta}{\zeta-1}}$$

$$\tilde{p}^S(w_k) = \tau \int_0^1 p_k^S dk + (1-\tau) \int_0^1 w_k p_k^S dk$$

Recall $w_{ik} = \left(\mathcal{A}_{0i}^{-1}\right)\left(z_k^S\right)^{\zeta-1}\left(p_k^S + \mu_i\right)^{-\zeta}$. Then

$$\tilde{w}_{ik}\left(p_k^S\right) = \tau \underbrace{\left(z_k^S\right)^{\zeta-1}\left(p_k^S\right)^{-\zeta}} + (1-\tau) \underbrace{\left(\mathcal{A}_{0i}^{-1}\right)\left(z_k^S\right)^{\zeta-1}\left(p_k^S + \mu_i\right)^{-\zeta}}$$

Hence demand can be expressed as

$$x_k = \int_0^1 x_{ik} di = \int_0^1 \tilde{w}_k x_i di$$

$$x_k^S = \left(z_k^S\right)^{\zeta-1} \tau \left(p_k^S\right)^{-\zeta} \int_0^1 x_i di + \left(z_k^S\right)^{\zeta-1} (1-\tau) \int_0^1 \left(\mathcal{A}_{0i}^{-1}\right) x_i \left(p_k^S + \mu_i\right)^{-\zeta} di$$

and upstream firms solve

$$\max x_k^S \left(p_k^S - \psi^S z_k^S\right)$$

$$x_k^S + \frac{dx_k^S}{dp_k^S}\left(p_k^S - \psi^S z_k^S\right) = 0$$

$$\frac{dx_k^S}{dp_k^S} = -\zeta\tau \left(z_k^S\right)^{\zeta-1}\left(p_k^S\right)^{-\zeta-1} \int_0^1 x_i di - \zeta(1-\tau)\left(z_k^S\right)^{\zeta-1} \int_0^1 \left(\mathcal{A}_{0i}^{-1}\right) x_i \left(p_k^S + \mu_i\right)^{-\zeta-1} di$$

Then p_k^S solves

$$\tau\left(p_k^S\right)^{-\zeta} \int_0^1 x_i di + (1-\tau) \int_0^1 \left(\mathcal{A}_{0i}^{-1}\right) x_i \left(p_k^S + \mu_i\right)^{-\zeta} di$$

$$-\zeta\left(p_k^S - \psi^S z_k^S\right)\left[\tau\left(p_k^S\right)^{-\zeta-1} \int_0^1 x_i di + (1-\tau) \int_0^1 \left(\mathcal{A}_{0i}^{-1}\right) x_i \left(p_k^S + \mu_i\right)^{-\zeta-1} di\right] = 0$$

Recall that previously

$$\bar{z}_{\max}^S = \left[\int_0^1 \left(z_k^S\right)^{\zeta-1} dk\right]^{\frac{1}{\zeta-1}} \qquad w_{k,\max} = \left(\bar{z}_{\max}^S\right)^{1-\zeta}\left(z_k^S\right)^{\zeta-1}$$

and $w_{k,\min} = \frac{1}{\phi_S}\mathbf{1}\left\{p_k^S = \min\left\{p^S\right\}\right\}$.

B Empirical Evidence

B.1 Construction of R&D tax credits

Since supplier and customer firms R&D investment decisions may coincide for many reasons, including changes to the technology space, this poses an identification challenge. In order to identify the spillover from the existing supplier customer relationship, I exploit plausibly exogenous firm-level to the user cost of R&D investment arising from differential firm exposure to state and federal R&D tax credits, following Wilson (2009) Bloom et al. (2013), and others. I use a two stage approach, where I first instrument each firm's R&D intensity on its state and federal user cost of R&D, and then regress the firm's R&D intensity on its customers' predicted R&D intensity.

Firms are differentially exposed to state R&D tax credits due to differences in state-level corporate income taxes and R&D subsidies rates in conjunction with firm-level differences in exposure to each state. I follow Bloom, et. al. 2013, and assume each firm's state user cost of R&D , ρ_{sit}, is proportional to a weighted average of the state-level user cost of capital, $\tilde{\rho}_{st}$, weighted by, θ_{kst}, the firm's R&D exposure to state k. To measure θ_{kst}, I use the the proportion of unique inventors located in state k corresponding to patents produced by the firm i in the previous 5 years. That is, the firm's state user cost of R&D, ρ^s_{it}, is the weighted

$$\rho^s_{it} = \log\left(\sum_{k=1}^{50} \theta_{kst}\tilde{\rho}_{st}\right)$$

While federal corporate tax rates and subsidies vary across time, firms also have different federal R&D user costs cross sectionally due to non-linearities in the percentage of R&D which qualifies for the credit. Since the goal of this tax credit was to induce additional R&D investment, the subsidy was applied to R&D only above the firm's tax applicable base, which at different points in time was a function of either past R&D or past sales. Moreover, the subsidy was not available to R&D exceeding twice the base. These rules, among others, induced differences in a firm's user cost of R&D arising from federal R&D tax credits, ρ^f_{it}, due the firm's past sales and R&D differences, as well as changes to the tax code over time. I largely follow Bloom et al. (2013) and Arque-Castells and Spulber (2022) in their estimation of firms' R&D user cost from federal R&D tax credits. I use Wilson (2009) and Arque-Castells and Spulber (2022) estimates of state and federal level corporate tax rates and R&D subsidy rates. With the state and federal user costs in hand, in the first stage, I project each firm's R&D expenditure and R&D intensity as predicted by the state and federal user costs.

$$lxrd_{it} = \gamma_0 + \gamma_{1t}\rho^s_{it} + \gamma_{2t}\rho^f_{it} + \eta_{it} \Rightarrow \widehat{lxrd_{it}}$$

$$\widehat{lrdintensity_{it}} = \frac{\widehat{lxrd_{it}}}{lsale_{it}}$$

As before, I define the firm's instrumented customer R&D intensity as the average predicted R&D intensity of its customers $k = 1, ..., N$

$$\widehat{customer_lrdintensity_{it}} = \frac{1}{N}\sum_{k=1}^{N} \widehat{lrdintensity_{kt}}$$

In the second stage, to estimate the role of customer R&D on a supplier's R&D, I regress a firm's R&D intensity on the predicted customer R&D intensity. I control for the firm's own exposure to state and federal R&D user costs and, to capture changes to the industry environment which may impact innovation, I include for industry (SIC3) by year fixed effects δ_{it}. Standard errors are bootstrapped.

$$lrdintensity_{it} = \beta_0 + \beta_1 \widehat{customer_lrdintensity_{it}} + \beta_2\rho^s_{it} + \beta_3\rho^f_{it} + \delta_{it} + \epsilon_{it}$$

B.2 First Stage

	(1) LogR&D		(2) LogR&D		(3) LogR&D	
LogUserCostState	-2.89***	(0.19)	-3.04***	(0.18)		
LogUserCostFederal	-1.03***	(0.02)			-1.28***	(0.02)
r2	0.93		0.90		0.92	
n	49599		65804		145956	
F	1102		270		6478	
Fixed Effects	Firm Year		Firm Year		Firm Year	

Standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

TABLE A.1 – Impact of State and Federal Variation in R&D User Cost on R&D Expenditure

B.3 Impact of Upstream R&D Spillover

	(1) RdIntensity	(2) RdIntensity	(3) RdIntensity	(4) RdIntensity	(5) RdIntensity	(6) Patents/RD	(7) PatVal/RD	(8) Tobins Q	(9) Tobins Q
TradeSpill0	0.02***								
	(6.20)								
TradeSpill		0.02**	0.02**	0.01**	0.01**	0.04***	0.31***	0.25***	0.11***
		(2.27)	(2.19)	(2.54)	(2.50)	(3.64)	(9.61)	(8.13)	(4.57)
CurrentTradeSpill		0.02***	0.02***	0.00**	0.00**	0.00	0.00	0.04***	0.02*
		(6.13)	(5.99)	(2.51)	(2.45)	(0.08)	(0.54)	(4.64)	(1.89)
CitesSpill			0.02		0.02	-0.01	-0.29**	0.12	
			(1.02)		(0.73)	(-0.15)	(-2.08)	(1.48)	
CurrentCitesSpill			0.01		0.01	-0.04*	-0.26***	0.05	
			(0.71)		(0.85)	(-1.68)	(-3.47)	(0.86)	
N	9114	9114	9114	9114	9114	4257	4257	3655	4440
F(trade)	38.39	19.21	18.28	4.90	4.64	6.66	46.54	41.82	10.47
F(cites)			0.56		0.41	1.44	6.54	1.10	
F(current)		37.64	18.68	6.29	3.17	1.42	6.21	11.45	3.58
Sample	RandD>0	RandD>0	RandD>0	RandD>0	RandD>0	RandD>0	RandD>0	RandD>0	RandD=0
FE	Firm, Year	Firm, Year	Firm, Year	Firm, Year	Firm, Year	Firm, Year	Firm, Year	Firm, Year	Firm, Year
IV	No	No	No	Yes	Yes	Yes	Yes	Yes	Yes

t statistics in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

TABLE A.2 – Impact of Upstream R&D on Firm R&D Intensity

	(1)	(2)	(3)	(4)	(5)	(6)
	LogRandD(t+1)	LogRandD(t+1)	InnovShock	InnovShock	ChgTobins Q	ChgTobins Q
TradeSpill	0.327*	0.144	0.227***	0.206***	0.437**	0.437**
	(0.197)	(0.201)	(0.045)	(0.046)	(0.205)	(0.206)
TradeSpillCurrent	0.016	-0.025	0.015**	0.017*	0.039	0.013
	(0.030)	(0.039)	(0.007)	(0.009)	(0.033)	(0.042)
CitesSpill		0.215***		0.023*		-0.019
		(0.052)		(0.012)		(0.056)
CitesSpillCurrent		0.015		-0.013		0.057
		(0.047)		(0.011)		(0.047)
InnovShock					0.463***	0.468***
					(0.047)	(0.047)
r2	0.978	0.978	0.809	0.809	0.525	0.525
n	8238	8238	8551	8551	3051	3051
F	2391	1803	389	305	162	130
Sample	RandD>0	RandD>0	RandD>0	RandD>0	RandD>0	RandD>0
FE	Firm, Year	Firm, Year	Firm, Year	Firm, Year	Firm, Year	Firm, Year

Standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

TABLE A.3 – Impact of Upstream Technology Shocks on Firm R&D and Valuation