Technology Adoption, Innovation, and Inequality in a Global World*

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First version: September 22, 2022
Current version: March 26, 2026
[most recent version here]

Abstract

I develop a tractable multi-country endogenous growth model with a technology adoption margin. Innovation and adoption are skill-intensive activities, and a trade-off arises whether skilled labor is used to push out the technological frontier or to adopt existing technology. I use the theory to revisit the effect of market integration on growth among asymmetric countries with large differences in innovative capacity. Transitional dynamics and long-run effects implied by the model differ substantially from benchmark models and jointly explain stellar per capita growth in emerging markets and the disappointing growth performance of advanced economies after rising global market integration since the 1990s.

*E-mail: florian.trouvain@economics.ox.ac.uk. This study uses the weakly anonymous Establishment History Panel 1975-2019 data of the German Federal Employment Agency (BA) at the Institute for Employment Research (IAB) in Nuremberg. Data access was provided via on-site use at the Research Data Centre (FDZ) of the IAB and remote data access. I thank Sandra Dummert, Heiner Frank, Lisa Schmidtlein, and Philipp vom Berge for expert research support from the IAB. For generous advising I am extremely grateful to Dominick Bartelme, John Leahy, Dmitriy Stolyarov, and Linda Tesar. For their insights I thank Mark Aguiar, Andres Blanco, Charlie Brown, John Bound, Mike Blank, Paco Buera, Antonio Ciccone, Maarten De Ridder, Max Dvorkin, Jonathan Eaton, Hartmut Egger, John Fernald, Cecilia Fieler, Carlos Garriga, Josh Hausman, Elhanan Helpman, Rishabh Kirpalani, Sam Kortum, John Laitner, Moritz Lenel, Paolo Martellini, Josh Martin, Eduardo Morales, Emir Murathanoglu, Ezra Oberfield, Pablo Ottonello, Stephen Parente, Michael Peters, Lukasz Rachel, B. Ravikumar, Steve Redding, Paulina Restrepo-Echavarria, Richard Rogerson, Hannah Rubinton, Juan Sanchez, Ana Maria Santacreu, Karthik Sastry, Brit Sharoni, Yongs Shin, Sebastian Sotelo, Jagadeesh Sivadasan, Gianluca Violante, Mark Wright, and Fabrizio Zilibotti. I thank the St. Louis Fed and the International Economic Section at Princeton University for their hospitality, and the German Academic Scholarship Foundation for financial support.

1 Introduction

Over the past three decades, globalization has dramatically expanded markets for innovation, driving record levels of patenting, R&D spending, and technological progress. Yet in many advanced economies — including the United States and Western Europe – productivity growth has slowed, wage growth has stagnated, and inequality has surged. This paradox lies at the heart of modern globalization: how can innovation be accelerating while incomes for many are falling behind? In this paper, I develop a new general equilibrium growth model that explains why globalization has been accompanied by increasing inequality and unexpectedly weak growth in the very countries driving the world's technological frontier.

The key step is to embed a skill-intensive technology adoption margin into an otherwise standard model of endogenous long-run growth, which gives rise to a novel innovation-adoption trade-off. By adoption I mean a costly and purposeful effort to use a specific technology, which is different from diffusion that happens automatically, or imitation related to the question of the ownership of technology. Adoption here concerns the mundane but essential activity of incorporating new technologies into a firm's production process. While often an afterthought, it is clear that frontier innovation is inconsequential without adoption. The main assumption in the model is that both the invention of new ideas and the adoption of existing ones requires skilled labor.

A trade-off between adopting technology vis-a-vis pushing out the technological frontier emerges as both activities ultimately have to draw on the same scarce factor, skilled labor. This tension becomes particularly acute during an episode of market integration between advanced economies and emerging markets. In the integrated equilibrium, emerging markets adopt advanced economies' technology. Foreign technology adoption raises the returns to innovation, and induces a reallocation of skilled labor within advanced economies from domestic adoption toward innovation for a larger global market. Since aggregate growth depends on both the technological frontier and the degree of technology adoption, positive innovation effects can be entirely undone by weak technology adoption, explaining uneven and sluggish growth in advanced economies as emerging markets join the world economy. This innovation-adoption trade-off is distinct from the more commonly studied trade-off between innovation vs. production because production is easily outsourced to emerging markets while local technology adoption is not. Formalizing and exploring this mechanism by means of a tractable general equilibrium model is the main contribution of the paper.

My analysis proceeds in three steps. In the first step, I develop a closed economy version of Romer (1990)'s benchmark two-sector growth model with endogenous technology adoption. Firms in the research sector invent new technology using skilled labor, while firms in the production sector combine capital goods with production labor to produce output. My key departure is that production sector firms solve a dynamic adoption problem: they decide how much skilled labor they devote to adopting new capital goods, which embody frontier technology. The solution pins down an endogenous

technology adoption gap.

The closed economy setup highlights two countervailing forces that determine the decentralized equilibrium. On the one hand, the model features a complementarity between adoption and innovation on the market for ideas. Ideas only become profitable after they are adopted so that higher adoption effort in the production sector pushes up the net present value of developing an idea where I use the term idea and innovation interchangeably. Intuitively, innovators would like their ideas to penetrate the market as quickly as possible. On the other hand, by virtue of modeling both innovation and adoption as skill-intensive activities, a factor market rivalry arises as innovators and adopters compete for the same scarce resource, skilled labor. If one activity expands, it does so at the expense of the other in this general equilibrium setting.

Despite the fact that innovation and technology adoption are forward-looking interdependent activities, the steady state is surprisingly tractable and the equilibrium adoption gap is a negative power function of the skill premium. This simple relationship unlocks all key results in the paper, and captures the intuition that when skill becomes relatively more scarce, production sector firms endogenously adopt less technology.

In the second step, I generalize the model to a multi-country setting to study economic growth in an integrated world. I embed the model into a semi-endogenous growth framework à la Jones (1995) to allow for a realistic degree of scale effects. The model is consistent with key stylized facts of economic growth in open economies. First, the long run equilibrium is characterized by a stationary cross-country income distribution where all countries grow at the same rate. Yet, growth spurts are possible when countries join the global economy and gain access to the world technological frontier. Both aspects follow from a model with endogenous technology adoption that features an advantage of backwardness. Second, countries with higher research productivity, larger populations, or higher skilled labor shares contribute more to the global technological frontier. Third, while the technological frontier exhibits scale effects and depends on the total size of the global research force, a country's individual productivity is unrelated to its size as it adopts technologies invented elsewhere. Incorporating these features is necessary for any coherent treatment of technological change in a globalized world. This is a difficult task in models that only allow for innovation, but becomes trivial once innovation and adoption are modeled jointly.

I next establish a sharp theoretical distinction between symmetric market integration among similar countries—such as the formation of the European Common Market in the late 20th century—and asymmetric integration between emerging markets and advanced economies. Symmetric integration amongst advanced economies has no adverse effects on inequality or technology adoption and delivers the standard gains from integration. The reason is that foreign adoption, which raises the incentive to innovate, and foreign innovation, which reduces it, exactly cancel. Underlying this result is a classic Ricardo-Heckscher-Ohlin-Krugman type logic where for identical technology and factor endowments prices remain unchanged, while gains from integration are driven by standard scale

effects as each country uses the other country's technology.

This contrasts with the globalization shock of the 1990s and 2000s, marked by a pronounced asymmetry between advanced economies ("West") and emerging markets ("East") ushering in a period of unprecedented market integration. Consider the following thought experiment: each economy is on a balanced growth path, and an unanticipated liberalization in the East moves each country from autarky to free trade in final goods and technology, reflecting the fall of the Iron Curtain or China's ascension to the World Trade Organization. Assuming that the West develops frontier technology and can partially protect its intellectual property, fast technology adoption of emerging markets in the integrated equilibrium raises the returns to innovation in the West. This leads to heightened research effort in the West and temporarily faster growth of the technological frontier, which is the standard mechanism emphasized in the literature. While this effect is present in my framework, there is an important twist to this logic: productivity not only depends on the technological frontier but also on its adoption.

An expanding research sector induces a rise in the skill premium, causes a reallocation of skilled labor away from the production sector, and ultimately hurts technology adoption. The chain of events is most easily understood through the lens of the Heckscher-Ohlin model where skill becomes scarce in advanced economies due to heightened specialization in innovation in the integrated equilibrium. This skill scarcity coincides with a rising skill premium and a widening technology adoption gap.

Since aggregate growth in the West depends on both the evolution of the technological frontier and the degree of technology adoption, the impact of an asymmetric globalization shock is ex-ante unclear. Whether positive or negative effects dominate depends on externalities in innovation and technology adoption. I explore this by contrasting the efficient allocation with the decentralized equilibrium. I find that when ideas are harder to find (Bloom et al., 2020) and there are mild learning spillovers in adoption, underinvestment in adoption relative to innovation occurs. In that case, integration with a foreign economy that adopts technology aggressively but contributes little frontier technology themselves induces a growth slowdown in advanced economies. Note that sluggish growth in the West is consistent with exceptional growth in the East, which is often ignored in the rich-country focused debate around the productivity slowdown. In fact, the mechanism relies on it as key driving force since technology adoption in the East induces the bias in favor of global innovation vs. local adoption in the West.

In the third step, I combine cross-country data supplemented with worker and establishment-level micro data to quantify the impact of globalization on growth and inequality. Solving for the entire transition path and focusing on the West, I find that the asymmetric globalization shock generates a sizable increase in innovation and employment of skilled labor in the research sector. The skill premium increases by 20%, which leads production sector firms to cut down on skilled labor for adoption purposes, ultimately reducing technology adoption by 17%. The calibration predicts a cumulative drop in real wages of production workers of 10%, relative to the counterfactual balanced

growth path in autarky. While skilled labor gains in real terms, after aggregating up worker incomes, I arrive at a cumulative growth drag of about7% for the aggregate wage bill. The impact on total per capita income is around minus 3%, and the difference is accounted for by the role of asset accumulation and royalties earned abroad. High innovative activity and an increase in the valuation of technology coincides with sluggish per capita growth as global innovation crowds out domestic technology adoption.

Transition dynamics are crucial to understanding these results. While productivity growth in the very long run tracks growth of the technological frontier, along the transition path, they are divorced. A widening adoption gap exerts downward pressure on realized productivity growth rates despite elevated growth of the technological frontier.

In contrast to this bleak scenario, full convergence of the East in terms of skill endowments and innovative capacity would restore the powerful pro-growth effects of market integration while simultaneously reducing inequality in the West, consistent with the theoretical results on symmetric integration. The model thus suggests that the rise of India or China as research powerhouses would be extremely desirable from the point of view of unskilled workers in the West, which contrasts with popular fears about emerging markets' abilities to compete in high-tech.

I conclude by citing additional evidence in favor of the main mechanism and discussing related issues such as skill biased technological change and automation. Overall, little attention has been paid to the role of technology adoption in a globalized world,[1] perhaps in part due to the complexity of incorporating both innovation and adoption at once in a multi-country setting. I provide a tractable framework –directly nesting benchmark growth models– to study the intricate interplay between innovation and technology adoption that should be relevant to a host of issues at the nexus of growth and inequality.

Relationship to the literature. I build on the literature on endogenous growth (Romer, 1990; Grossman and Helpman, 1991b; Aghion and Howitt, 1990), and incorporate a technology adoption margin into Jones (1995). A large literature uses variants of these models to study the aggregate productivity slowdown,[2] focusing on falling population growth and declining business dynamics as drivers.[3] Relative to this literature, I focus on the distinction between innovation and technology adoption, and emphasize the latter to understand patterns of global growth.[4]

[1] Jovanovic (1997) argues that costs of technology adoption as a share of GDP exceed costs associated with innovation by an order of magnitude.

[2] The productivity slowdown is a robust feature of the data, although its onset differs somewhat across countries. Fernald (2015) and Cette, Fernald, and Mojon (2016) point out that this slowdown started before the financial crisis.

[3] See Peters and Walsh (2021), Jones (2020), Engbom et al. (2019), and Hopenhayn, Neira, and Singhania (2018) on works that highlight the role of population growth on productivity and business dynamics. An alternative explanation for the productivity slowdown focuses on firm dynamics and biased technology shocks, see De Ridder (2019), Olmstead-Rumsey (2019), Rempel (2021), Akcigit and Ates (2019), and Aghion et al. (2019), building on Klette and Kortum (2004). Akcigit, Ates, and Impullitti (2018) consider a two-country model, and explore the impact of trade policy on business dynamics and growth. Incorporating an endogenous technology adoption margin along the lines proposed here is complementary to the overall agenda in this strand of the literature.

[4] The theory shares some predictions with models of directed technological change (Acemoglu, 2002; Acemoglu, 2003;

Second, a large literature studies the role of technology adoption for cross-country income differences, see Parente and Prescott (1994), Lucas (2009a), Comin and Hobijn (2010a), Lucas and Moll (2014), or Comin and Mestieri (2014; 2018). Barro and Sala-i-Martin (1997) and Acemoglu, Aghion, and Zilibotti (2006) study developing economies' choice between adoption and innovation.

A small number of papers has modeled innovation and adoption jointly. Konig et al. (2021), building on König, Lorenz, and Zilibotti (2016), as well as Benhabib, Perla, and Tonetti (2021), Hopenhayn and Shi (2020), and Sampson (2023) develop heterogeneous firm models where high productivity firms innovate, while laggard firms imitate high productivity firms. Comin and Gertler (2006) and Anzoategui et al. (2019) study business cycle fluctuations in a model with both innovation and adoption. The complementarity between innovation and adoption on the market for ideas is related to Comin and Hobijn (2007), which focuses on initial implementation of new technologies. Relative to these works, I draw out an innovation-adoption trade-off embedded in an otherwise standard semi-endogenous growth model with realistic scale effects.[5]

Third, this paper relates to the literature on trade and growth. Rivera-Batiz and Romer (1991) emphasize the pro-growth effects of market integration. Grossman and Helpman (1991a) provide the classic treatment, and Melitz and Redding (2021) survey the subsequent literature. Eaton and Kortum (1999) develop a model of global growth with exogenous diffusion of ideas and highlight the role of global diffusion gaps in shaping innovation incentives.[6] In contrast, I make diffusion endogenous, so that these gaps reflect adoption choices. Once adoption is modeled as a choice, asymmetric market integration gives rise to a novel innovation-adoption trade-off that alters the impact of globalization on growth and inequality.

Sampson (2016; 2023) and Perla, Tonetti, and Waugh (2021) highlight the role of learning in heterogeneous firm settings, similar to Buera and Oberfield (2020)'s model of international technology diffusion. The dynamic gains from trade are large in much of the literature and hard to square with the growth slowdown. Modeling both innovation and adoption, and recognizing the inherent tension this generates in a general equilibrium model, can break the strong pro-growth effects in the case of

Acemoglu, Gancia, and Zilibotti, 2015), models of appropriate technology (Atkinson and Stiglitz, 1969; Basu and Weil, 1998; Acemoglu and Zilibotti, 2001; Caselli and Coleman, 2006), and models of technological revolutions (Galor and Tsiddon, 1997; Greenwood and Yorukoglu, 1997; Caselli, 1999).

[5]The innovation-adoption trade-off contrasts with the previous literature, which focuses on heterogeneous firm setups where leading firms innovate and less productive firms learn from leading firms. Since innovators don't internalize the learning spillover, innovation is undersupplied, see König, Lorenz, and Zilibotti (2016), Benhabib, Perla, and Tonetti (2021), and Sampson (2023). In such settings, globalization has strong pro-growth effects as it favors larger innovative companies that can overcome the fixed cost of exporting (Melitz, 2003), and partially corrects for the market failure (Sampson, 2016; Perla, Tonetti, and Waugh, 2021).

[6]A quantitatively focused trade literature builds on this line of work, see Lind and Ramondo (2022), Cai, Li, and Santacreu (2022), and Somale (2021). Relative to this literature, I keep the model simple to draw out a novel innovation-adoption trade-off in the presence of an asymmetric market integration shock. However, in steady state, the framework developed here is as tractable as quantitative trade models so one could incorporate many countries and bilateral trade cost. Transition dynamics are important and the quantitative section is focused on dynamics. A related literature in international trade has studied the distributional consequences of globalization, which appear modest, see Helpman (2016). In this literature, the closest paper is Arkolakis et al. (2018), which embeds export platforms into a static multi-country Melitz model to study specialization in innovation and production.

asymmetric integration whilst preserving the positive link between market size and innovation.

The rest of the paper proceeds as follows. Section 2 presents a model of innovation and adoption. Section 3 models the open economy. Section 4 quantifies the model and studies transition dynamics. Lastly, I discuss empirical evidence consistent with the key mechanism and conclude.

2 Closed Economy

2.1 Environment

I outline the economic environment next, which is cast in continuous time indexed by t. The model is a two-sector model of long-run growth. I first discuss the problem of a representative household, and then develop the production sector and research sector separately. All derivations and proofs are deferred to the appendix.

Households

A representative household supplies their labor inelastically, which leads to an economy wide endowment of L_t units of production labor and H_t units of high skilled labor. The household's labor endowment grows at constant rate $g_H = g_L > 0$. Factors earn income at wage rates $w_{L,t}$ and $w_{H,t}$, and the skill premium is defined as $s_t := \frac{w_{H,t}}{w_{L,t}}$. The household solves a consumption-saving problem

$$\begin{aligned} U &= \max_{\{c_t, B_t\}_{t\geq 0}} \int_0^\infty e^{-(\rho - g_L)t} \log c_t \, dt \\ \text{s.t.} & \\ \dot{B}_t &= r_t B_t + w_{H,t} H_t + w_{L,t} L_t - C_t, \end{aligned} \tag{1}$$

with the usual transversality condition in place. Total assets in the economy are denoted by B_t, changes in total assets $\dot{B}_t$ represent net savings and r_t is the return on assets.

Production Sector

Final Goods Production. A competitive final goods producer combines differentiated intermediate goods using a Benassy (1996)-CES aggregator

$$Y_t = M_t^{-\frac{1}{\sigma-1}} \left(\int_0^{M_t} y_{i,t}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}, \tag{2}$$

where M_t denotes the mass of available intermediate goods, indexed on the interval $i \in [0, M_t]$.[7] Final output can be consumed or turned into an investment good, $Y_t = C_t + I_t$, where C_t and I_t are

[7] Using the productivity shifter $M_t^{-\frac{1}{\sigma-1}}$ ensures that there are no scale effects in the production sector. An alternative setup building on Hopenhayn (1992) is presented in the appendix where no such assumption is needed to maintain a production sector that is constant-returns-to-scale. The payoff here is that constant-returns-to-scale production on the firm level makes the exposition easier. Scale effects are present but confined to the research sector.

aggregate consumption and investment, and the final good serves as numeraire. Physical capital depreciates at rate δ_k giving rise to the standard neoclassical capital accumulation, $\dot{K} = I_t - \delta_k K_t$. I refer to intermediate goods producers $i \in [0, M_t]$ as "firms", in contrast to firms in the research sector referred to as "innovators".

Firms. Infinitesimal intermediate goods firms compete monopolistically and solve a static profit maximization problem and a dynamic technology adoption problem. The static problem involves setting a monopoly price, hiring labor, and renting capital goods. The dynamic problem concerns the adoption of new technology, which is costly.

Static Firm Problem. Firms produce differentiated varieties using production labor $l_{i,t}$ and a continuum of symmetric capital goods $x_{ij,t} \in [0, A_{i,t}]$. At time t, $A_{i,t}$ describes the set of capital varieties available to firm i; conditional on this set, the firm then chooses how much of each available capital good $x_{ij,t}$ to rent. Each capital good corresponds to a distinct technology, raising firm productivity through a variety effect as in the original Romer (1990) model. In this sense, $A_{i,t}$ captures firm level productivity. Firm output is given by

$$y_{i,t} = \left(\int_0^{A_{i,t}} \left(\frac{x_{ij,t}}{\alpha} \right)^{\alpha} dj \right) \left(\frac{l_{i,t}}{1-\alpha} \right)^{1-\alpha}. \tag{3}$$

Production labor is hired at rate $w_{L,t}$, and all capital goods are rented at common rate $p_{x_{j,t}} = p_{x_t}$.

The static problem leads to operating profits that are log-linear in aggregate output Y_t, factor prices, and firm productivity

$$\Pi_{i,t} = \frac{Y_t}{\sigma} \left(\frac{\sigma}{\sigma - 1} p_{x,t}^{\alpha} w_{L,t}^{1-\alpha} \right)^{1-\sigma} A_{i,t}^{(\sigma-1)(1-\alpha)}. \tag{4}$$

Productivity $A_{i,t}$ is taken as given in the static problem but endogenized through a dynamic technology-adoption decision, described next.

Technology Adoption. Technology adoption is equivalent to expanding the set of capital goods available to the firm. Relative to the set $[0, A_{i,t}]$, the superset $[0, A_{\mathrm{F},t}]$ contains all existing capital goods including recent inventions not adopted yet. This technological frontier $A_{\mathrm{F},t}$ is common to all firms, and $A_{i,t} \leq A_{\mathrm{F},t}$. Formally, firm i makes a costly choice to add the flow, $\dot{A}_{i,t}$, of new technology.

Following Nelson and Phelps (1966), I make two crucial assumptions on the adoption process. First, technology adoption requires skilled labor. Second, technology adoption features an "advantage of backwardness". Equation (5) formalizes these concepts

$$\dot{A}_{i,t} = \nu A_{\mathrm{F},t}^{1-\theta} A_{i,t}^{\theta} \left(\iota_{i,t} \right)^{\beta}, \tag{5}$$

where $\iota_{i,t}$ denotes skilled labor devoted to adoption in firm i, $1 - \theta \geq 0$ governs the advantage of backwardness, and $\beta \in (0, 1)$ induces diminishing returns to technology adoption at a point in time.

The parameter $\nu > 0$ is a constant assumed sufficiently small to avoid a corner solution at $A_{i,t} = A_{\mathrm{F},t}$.[8] The setup gives rise to a dynamic trade-off between the cost of skilled labor today, and the future benefit of adopting technology.

It is convenient to use a normalized level of technology relative to the technological frontier,

$$z_{i,t} := \frac{A_{i,t}}{A_{\mathrm{F},t}} \in (0, 1].$$

After recognizing that firms differ only in their normalized productivity z, I drop the firm subscript i and keep track of z instead. Finally, I introduce an exogenous exit shock arriving at rate δ_{X}.

The firm's dynamic adoption problem can be characterized by the Hamilton-Jacobi-Bellman equation

$$\begin{aligned} V_t\left(r_t + \delta_{\mathrm{X}}\right) &= \max_{h_t} \Pi_t\left(z\right) - w_{H,t}\iota_t + \dot{z}_t \partial_z V_t + \dot{V}_t \\ \text{s.t.} & \\ \dot{z}_t &= \nu z_t^{\theta} \iota_t^{\beta} - g_{\mathrm{F},t} z_t, \end{aligned} \tag{6}$$

where $g_{w_H,t}, \partial_z V_t, \dot{V}_t, g_{\mathrm{F},t}$ represent skilled wage growth, derivatives with respect to z and t of the value function, and the growth rate of the technological frontier.

An interior solution satisfies the first-order condition

$$\iota_t = \left(\frac{\beta \nu z_t^{\theta} \partial_z V_t}{w_{H,t}}\right)^{\frac{1}{1-\beta}}, \tag{7}$$

which is the technology adoption problem at the heart of the paper. The high-skilled wage represents the cost of technology adoption, while the term $\beta \nu z_t^{\theta} \partial_z V_t$ captures the marginal benefit of hiring skilled labor.

Using standard optimal control techniques, I show in the appendix that the firm's optimal adoption effort obeys

$$\frac{\dot{\iota}_t}{\iota_t}\left(1-\beta\right) = r_t - g_{w_{H,t}} + \delta_{\mathrm{X}} + \left(1-\theta\right) g_{\mathrm{F},t} - \frac{\beta \iota_t^{\beta-1} \nu z_t^{\theta}}{w_{H,t}} \underbrace{\frac{\Pi_t\left(\sigma-1\right)\left(1-\alpha\right)}{z_t}}_{\partial_z \Pi_t}, \tag{8}$$

where $\frac{\Pi_t(1-\alpha)(\sigma-1)}{z_t} = \partial_z \Pi_t$ is the partial effect of normalized productivity on profits. The term $\frac{\beta \iota_t^{\beta-1} \nu z_t^{\theta}}{w_{H,t}} \frac{\Pi_t(\sigma-1)(1-\alpha)}{z_t}$ in (8) captures the ratio of the marginal benefit ($\Pi_t\left(\sigma-1\right)\left(1-\alpha\right)/z_t$) to the marginal cost ($w_{H,t}/\beta \iota_t^{\beta-1} \nu z_t^{\theta}$). The remaining terms, $r_t - g_{w_{H,t}} + \delta_{\mathrm{X}} + \left(1-\theta\right) g_{\mathrm{F},t}$, form an effective

[8] An alternative is to use the original Nelson-Phelps specification $\dot{A}_{i,t} = \left(A_{\mathrm{F},t} - A_{i,t}\right) g\left(h_{i,t}\right)$ for some monotone function g. This doesn't alter any qualitative insights of the model but ensures that the firm can never hit a corner solution. My specification has an additional degree of freedom in θ which allows me to match the speed of convergence across countries. See Lucas (1993), Parente and Prescott (1994), and Sampson (2023) for similar formulations.

discount rate. In addition to the usual components –interest rates, wage growth, and death shock– the discount rate is affected by two non-standard forces: the advantage of backwardness, $(1-\theta)\, g_{\mathrm{F},t}$, and skilled wage growth. Intuitively, given an advantage of backwardness is present ($\theta < 1$), delaying investment in technology adoption is beneficial as waiting allows firms to adopt new technologies at lower cost. Conversely, faster growth in skilled wages raises the incentive to adopt early.

Adoption effort behaves like investment in an intangible capital stock with adjustment frictions. The exponent θ makes past adoption decisions more or less important for the firm's ability to adopt future technology. Similarly, a more linear ι^{β} (high β) makes firm adoption choices relatively more sensitive to the cost of adoption.

Entry and Adoption Spillover. I close the production sector by assuming free entry after paying a fixed cost f_{E} in terms of unskilled labor. Entrants draw their productivity from the distribution of incumbents so

$$V_{\mathrm{E},t} = \int V_t(z)\, dF_t(z), \tag{9}$$

where $V_{\mathrm{E},t}$ is the value of an entrant and dF_t is the incumbent distribution over z. The expected present discounted value of entry must not exceed the entry cost, $V_{\mathrm{E},t} \leq f_{\mathrm{E}} w_{L,t}$. This free entry condition pins down firm entry, $\dot{M}_t = \frac{L_{\mathrm{E},t}}{f_{\mathrm{E}}} - \delta_{\mathrm{X}} M_t$, where $L_{\mathrm{E},t}$ and $L_{\mathrm{P},t}$ denote unskilled labor devoted to entry and production of intermediates.

Equation (9) embodies the key spillover in technology adoption, which is that entrants learn from incumbents. This assumption features prominently in the idea flow literature, especially Luttmer (2007), and is even more natural in the context of technology adoption. A spillover is necessary in any growth model where private firms' technology adoption is a key ingredient of long-run growth whenever there is positive firm entry. To see why, imagine that incumbents improve their productivity at a constant rate by adopting new technology, while entrants enter at some fixed productivity level. In that case the firm size distribution would be ever diverging.[9]

A convenient implication of the spillover embedded in the entry technology in (9) is that the model collapses to a homogeneous firm model. Since entrants are, on average, as productive as incumbents, and technology adoption in (5) has no exogenous firm-specific parameters, the only fixed point is one where firms make identical choices.[10] This simplification allows me to express normalized firm profits in closed form whenever the free entry condition binds,

[9] In contrast to learning spillovers at entry, Lucas (2009b), Lucas and Moll (2014), and Perla and Tonetti (2014) highlight knowledge spillovers among incumbents. Two comments are in order. First, note that the spillovers in Perla and Tonetti (2014) could be re-interpreted as spillovers to entrants if the lowest productive firms, which are the adopters in their setting, were to be replaced by new entrants. The framework of Lucas (2009b) and Lucas and Moll (2014) are different as all agents continuously learn from another.

[10] I show in the appendix how to allow for a weaker spillover at entry, for example where the entry distribution $F_t(z|\mathrm{E})$ is modeled as a linear function of average incumbent productivity, $z_{\mathrm{E}} = \lambda_{\mathrm{E}} \int z dF_t(z)$, with $\lambda_{\mathrm{E}} \in (0,1)$. This gives rise to a heterogeneous firm model where entrants have a lower productivity than incumbents, and productivity is increasing in firm age as older firms have adopted more technology.

$$V_t = \frac{\Pi_t - w_{H,t}\iota_t}{r_t - g_{w_{L,t}} + \delta_{\mathrm{X}}}. \tag{10}$$

The value function in (10) is standard except for the fact that the cost of technology adoption, $w_{H,t}\iota_t$, need to be subtracted from operating profits. It is because of this that I deviate from an assumption of perfect competition in the production sector. Since technology adoption is costly and firms produce according to a constant-returns-to-scale technology, I need profits to compensate for the cost of adoption.[11]

Research Sector

I build on Romer (1990) where innovators pay a fixed cost in skilled labor to create new ideas. These new ideas then enter the economy in the form of differentiated capital goods. The key difference to Romer (1990) is that innovators can only monetize their innovation if production firms have adopted their technology. Consequently, forward-looking innovators internalize the timing of adoption, which generates a novel feedback between innovation and adoption absent from the benchmark model.[12]

Value of Innovation and Technology Adoption. For simplicity, I assume that technologies are adopted in the order in which they are invented.[13] Let $\tau_t \in R^+$ denote the waiting time between invention at calendar time t and adoption. Denote by $\pi_{\mathrm{I},u}$ the flow profits generated by an adopted innovation at time u. The present discounted value of an innovation created at time t is

$$V_{\mathrm{I},t} \quad = \quad \int_{t+\tau_t}^{\infty} \exp\left(-\int_t^u r_v dv\right) \Pi_{\mathrm{I},u} du. \tag{11}$$

Note that discounting begins at the time of invention t, while profits accrue only after adoption $t+\tau_t$.

Adoption in the order of invention induces a simple queue, which allows me to pin down the waiting time in closed form. At time t, a new cohort of inventors enters the economy. The measure of technologies that stand between the cohort entering at t, and currently adopted technologies, is $A_{\mathrm{F},t} - A_t$. Since new ideas invented after time t enter the queue behind this cohort, the waiting time τ_t depends only on how quickly firms adopt existing technology, which is captured in the adoption flow

$$\frac{d}{dv}\left(A_{\mathrm{F},t} - A_v\right) = -\dot{A}_v = -g_{A,v}A_v, \quad v \in [t, t+\tau_t]. \tag{12}$$

This differential equation, together with boundary conditions $A_{\mathrm{F},t} - A_t$ at invention, and zero at adop-

[11] This is, of course, the same argument put forth in Schumpeter (1942) and Romer (1990) as to why constant-returns-to-scale competitive economies are not consistent with theories of endogenous growth. A competitive model of adoption is feasible when following the Lucas (1978) and Hopenhayn (1992) class of models with diminishing returns on the firm level as detailed in the appendix.

[12] This complementarity seems related to the the influential work of Comin and Gertler (2006), although the concept does not explicitly appear their paper, which focuses on short-run amplification of cyclical shocks. Comin and Hobijn (2007) is the less known but more relevant precursor, which focuses on complementarities in initial implementation of technology in a model where innovators race for patents, see Reinganum (1989).

[13] Implicit in this model is that all firms in the production sector adopt technology in the same order. I develop a version with stochastic adoption in the appendix, which relaxes this assumption without changing any of the main insights of the model, as long as the expected waiting time is a function of adoption effort in the production sector.

tion ($A_{\mathrm{F},t} - A_{t+\tau} = 0$), delivers a tractable expression of the waiting time

$$\tau_t = -\frac{\log z_t}{\frac{\int_t^{t+\tau_t} g_{A,v} dv}{\tau_t}}. \tag{13}$$

This waiting time is fully determined by production sector firms' adoption effort. Past adoption effort pins down the current frontier gap defined as $-\log z_t$. Future adoption effort pins down the average speed at which new technology is adopted, $\int_t^{t+\tau_t} g_{A,v} dv/\tau_t$. Intuitively, dividing this frontier gap by average speed then yields the waiting time, which is a statistic measured in units of time τ_t.

Innovation Spillovers and Free Entry. The rest of the research sector is standard and follows Jones (1995) closely. Innovators produce new ideas with skilled labor according to

$$\dot{A}_{\mathrm{F},t} = \frac{H_{\mathrm{I},t}}{f_{\mathrm{I},t}} = \gamma A_{\mathrm{F},t}^{\phi} \left(\frac{H_{\mathrm{I},t}}{L_t}\right)^{\lambda-1} H_{\mathrm{I},t},$$

where $H_{\mathrm{I},t}$ denotes the total amount of skilled labor devoted to innovation. Infinitesimal inventors takes the fixed cost $f_{\mathrm{I},t} = \left(\frac{H_{\mathrm{I},t}}{L_t}\right)^{1-\lambda} / \gamma A_{\mathrm{F},t}^{\phi}$ as given. This fixed cost depends on an exogenous fundamental research productivity γ, and the familiar spillovers: a dynamic knowledge spillover $A_{\mathrm{F},t}^{\phi}$ where $\phi < 1$ governs the sign and strength of the spillover, and a congestion externality $\left(\frac{H_{\mathrm{I},t}}{L_t}\right)^{1-\lambda}$ parameterized by $\lambda \in (0, 1]$.[14]

After invention, each innovator holds an infinitely lived patent. Innovators rent capital K_t from households to produce differentiated capital goods $X_{j,t}$ that embody their ideas. Supplying one unit of differentiated capital requires one unit of capital, so the marginal cost of production equals the user cost of capital, $r_t + \delta_k$. Firms in the production sector combine capital varieties using the CES aggregator embedded in equation (3). Monopolistic competition among innovators therefore implies a constant markup over marginal cost. As a result, the rental price of capital goods equals $p_{x,t} = \frac{1}{\alpha}(r_t + \delta_k)$.

The equilibrium amount of skilled labor devoted to innovation $H_{\mathrm{I},t}$ is pinned down by a free entry condition, $V_{\mathrm{I},t} \leq f_{\mathrm{I},t} w_{H,t}$. Whenever the free entry condition binds, the inventor value function takes the closed-form

$$V_{\mathrm{I},t} = \frac{\exp\left(-\int_t^{t+\tau} r_v dv\right) \Pi_{\mathrm{I},t+\tau} \cdot [1 + \dot{\tau}_t]}{r_t - g_{w_{H,t}} - (1-\lambda)\left(g_{H_{\mathrm{I},t}} - g_{L,t}\right) + \phi g_{\mathrm{F},t}}, \tag{14}$$

where a standard discount rate $r_t - g_{w_{H,t}} - (1-\lambda)\left(g_{H_{\mathrm{I},t}} - g_{L,t}\right) + \phi g_{\mathrm{F},t}$ accounts for firm death, time discounting, and appreciation of the value of an innovation. Non-standard is the term $\exp\left(-\int_t^{t+\tau} r_v dv\right) \Pi_{\mathrm{I},t+\tau} \cdot [1 + \dot{\tau}_t]$, which accounts for the fact that profits arrive with a delay, and the delay itself could change

[14] The congestion force is slightly different from the one in Jones (1995), and chosen with an eye toward the open economy to guarantee a well-behaved interior equilibrium with every country active in each sector. A micro-foundation can be provided using research skill heterogeneity as in Phelps (1966), see Eaton and Kortum (1999).

over time, $\dot{\tau}_t$.

Normalized System

It is convenient to study the equivalent normalized system, where I choose the normalizing factor such that the variables become stationary along a balanced growth path. For the production sector, I use the following normalizations, $v_t := \frac{V_t}{w_{L,t}}$, $\pi_t := \frac{\Pi_t}{w_{L,t}}$, $\frac{L_{\mathrm{P},t}}{L_t} := l_{\mathrm{P},t}$, $\frac{L_{\mathrm{E},t}}{L_t} := l_{\mathrm{E},t}$, $m_t := \frac{M_t}{L_t}$, which give rise to the following equations

$$v_t = f_{\mathrm{E}}, \tag{15}$$

$$v_t = \frac{\pi_t - s_t \iota_t}{r_t - g_{w_L,t} + \delta_{\mathrm{X}}}, \tag{16}$$

$$\dot{m} = \frac{l_{\mathrm{E}}}{f_{\mathrm{E}}} - (g_L + \delta_{\mathrm{X}})\, m. \tag{17}$$

Total skilled labor devoted to technology adoption equals $H_{\mathrm{A}} = \int \iota_t di = M_t \iota_t$. The normalized amount of skilled labor devoted to technology adoption is defined as $h_{\mathrm{A},t} := \frac{H_{\mathrm{A},t}}{L_t}$, while the normalized amount of skilled labor devoted to innovation is defined as $h_{\mathrm{I},t} = \frac{H_{\mathrm{I},t}}{L_t}$. Total normalized supply of skilled labor equals $h_t = \frac{H_t}{L_t}$.

Similarly, I normalize variables pertaining to the research sector, $v_{\mathrm{I},t} := \frac{V_{\mathrm{I},t}}{A_{\mathrm{F},t}^{-\phi} w_{L,t}}$, $\pi_{\mathrm{I},t} := \frac{\Pi_{\mathrm{I},t}}{A_{\mathrm{F},t}^{-\phi} w_{L,t}}$, and $a_{\mathrm{F},t} := \frac{A_{\mathrm{F},t}^{1-\phi}}{L_t}$. Together with binding free entry, the normalized equations read

$$v_{\mathrm{I},t} = \frac{s_t h_{\mathrm{I},t}^{1-\lambda}}{\gamma}, \tag{18}$$

$$v_{\mathrm{I},t} = \frac{\exp\left(-\int_t^{t+\tau} r_v dv\right) \pi_{\mathrm{I},t+\tau} \cdot [1 + \dot{\tau}_t]}{r_t - g_{w_H,t} - (1-\lambda)\left(g_{H_{\mathrm{I},t}} - g_L\right) + \phi g_{\mathrm{F},t}}, \tag{19}$$

$$\dot{a}_{\mathrm{F},t} = (1-\phi)\left(\gamma h_{\mathrm{I},t}^{\lambda} - a_{\mathrm{F},t} \frac{g_L}{1-\phi}\right). \tag{20}$$

Table 1 summarizes the key features of the economic environment. In this table, I use three additional results that reduce the complexity of the system substantially, at the cost of blurring assumptions (confined to the text before this subparagraph) with equilibrium outcome. First, since firms in the production sector are symmetric, profits across firms are identical and take the simple form $\Pi_t = \frac{Y_t}{M_t} \frac{1}{\sigma}$. In normalized form the expression simplifies to $\pi_t = \frac{l_{\mathrm{P},t}}{(\sigma-1)(1-\alpha)} \frac{1}{m_t}$. Next, define average normalized productivity as $\tilde{z}_t = \left(\int z^{\sigma-1} dF_t(z)\right)^{\frac{1}{\sigma-1}}$ as in Melitz (2003), which every individual firm treats parametrically. This allows me to express profits as $\Pi_t = \frac{Y_t}{M_t} \frac{1}{\sigma} \left(\frac{z}{\tilde{z}}\right)^{(\sigma-1)(1-\alpha)}$, which is the correct object to compute the partial effect of productivity on profits from the point of view of an individual firm.[15] Second, since the equilibrium in the research sector also features symmetric

[15] In a symmetric equilibrium, it must be that $z_{i,t} = \tilde{z}_t \, \forall i$. Yet, the partial derivative is still we defined and given by $\frac{\partial \pi_t}{\partial z}$ evaluated at $z_{i,t} = \tilde{z}_t$.

innovators, I have $\Pi_{\mathrm{I},t} = \frac{\alpha(1-\alpha)Y\frac{\sigma-1}{\sigma}}{A_t} = \frac{\alpha L_{\mathrm{P},t} w_{L,t}}{z_t A_{\mathrm{F},t}}$, which, in normalized form, equals $\pi_{\mathrm{I},t} = \frac{\alpha l_{\mathrm{P},t}}{a_{\mathrm{F},t} z_t}$. Lastly, note that I can express aggregate output as $Y_t = \left(\frac{X_t}{\alpha}\right)^{\alpha} \left(\frac{A_t L_{\mathrm{P},t}}{1-\alpha}\right)^{1-\alpha}$ after aggregating over all intermediate goods.

Table 1. Economic Environment

Agg. Production	
Output, capital accumulation	$Y_t = \left(\frac{X_t}{\alpha}\right)^{\alpha} \left(\frac{A_t L_{\mathrm{P},t}}{1-\alpha}\right)^{1-\alpha}, \quad \dot{K}_t = Y_t - C_t - \delta_k K_t$
Euler equation	
Inter-temporal consumption	$\frac{\dot{c}_t}{c_t} = r_t - \rho$
Adoption & Firms	
Adoption technology	$\dot{z}_t = \nu \iota_t^{\beta} z_t^{\theta} - g_{\mathrm{F},t} z_t$
Adoption effort	$\frac{\dot{\iota}_t}{\iota_t} = \frac{r_t - g_{w_{H,t}} + \delta_{\mathrm{X}} + (1-\theta) g_{\mathrm{F},t} - \frac{1}{s_t} \frac{\partial \dot{z}_t}{\partial \iota_t} \frac{\partial \pi_t}{\partial z}}{1-\beta}$
Firm profits	$\pi_t = \frac{Y_t}{M_t w_{L,t}} \frac{1}{\sigma} \left(\frac{z_t}{\tilde{z}_t}\right)^{(\sigma-1)(1-\alpha)} = \frac{l_{\mathrm{P},t}}{(\sigma-1)(1-\alpha)} \frac{1}{m_t}$
Labor/capital demand	$L_{\mathrm{P},t} w_{L,t} = \frac{\sigma-1}{\sigma} (1-\alpha) Y_t, \quad X_t p_{x,t} = \frac{\sigma-1}{\sigma} (\alpha) Y_t$
Free entry	$\frac{\pi_t - s_t \iota_t}{r_t - g_{w_{L,t}} + \delta_{\mathrm{X}}} = f_{\mathrm{E}}$
Firm creation	$\dot{m}_t = \frac{l_{\mathrm{E},t}}{f_{\mathrm{E}}} - (\delta_{\mathrm{X}} + g_L) m_t$
Innovation	
Mark-up	$p_{x,t} = \frac{r_t + \delta_k}{\alpha}$
Free entry	$\frac{\exp\left(-\int_t^{t+\tau} r_v - g_{w_{L,v}} + \phi g_{\mathrm{F},v} dv\right)}{r_t - g_{w_{H,t}} - (1-\lambda)\left(g_{H_{\mathrm{I},t}} - g_L\right) + \phi g_{\mathrm{F},t}} \frac{\alpha l_{\mathrm{P},t+\tau} [1 + \dot{\tau}_t]}{a_{\mathrm{F},t+\tau} z_{t+\tau}} = \frac{s_t h_{\mathrm{I},t}^{1-\lambda}}{\gamma}$
Idea creation	$\dot{a}_{\mathrm{F},t} = (1-\phi) \left(\gamma h_{\mathrm{I},t}^{\lambda} - a_{\mathrm{F},t} \frac{g_L}{1-\phi}\right)$
Market clearing	
Skilled labor	$h_{\mathrm{A},t} + h_{\mathrm{I},t} = h$
Unskilled labor	$l_{\mathrm{E},t} + l_{\mathrm{P},t} = 1$
Capital goods	$X_t = K_t$

2.2 Balanced Growth Path

I first focus on a balanced growth path (BGP) to illustrate two countervailing forces that are not present in the benchmark model. On the one hand, adoption and innovation compete for skilled labor, which gives rise to a factor market rivalry. On the other hand, innovation and adoption are complements on the market for ideas/capital goods. I isolate each channel, and show how they jointly pin down the decentralized allocation. I then contrast this allocation with the constrained social optimum. I end this section by discussing model extensions and how the model compares to the previous literature.

The definition of a BGP is almost identical to Jones (1995), with the exception that equilibrium in the production sector gives rise to a constant technology adoption gap defined as $-\log z$. To ensure the BGP is well behaved with an interior adoption gap, I impose the following parameter restriction, $\rho + \delta_{\mathrm{X}} + (1-\theta)\, g_{\mathrm{F}} > \beta\,(\sigma-1)\,(1-\alpha)\, g_{\mathrm{F}}$.[16]

Proposition 1. *The model admits a unique balanced growth path with a constant and positive adoption gap, $-\log z$, where population grows at exogenous rate $g_L = g_H > 0$, wages, per capita consumption, productivity, and the technological frontier grow at constant rate $g_{w_L} = g_{w_H} = g_c = g_A = g_F = \frac{g_L}{1-\phi}$.* The endogenous variables $\{s, a_F, m, l_E, l_P, \iota, h_A, h_I\}$ *are constant, the interest rate equals $r = \rho + g_F$, and free entry binds in each sector.*

Skill Premium and Technology Adoption. In general, the firm-level demand of skilled labor devoted to adoption is such that the marginal benefit of an additional unit of skill matches the effective discount term. On the BGP with $\dot{z} = 0$ and $\dot{\iota} = 0$, this condition implies a closed-form solution for skilled labor demand on the firm-level that depends on the skill premium and firm profits according to

$$\iota = \frac{1}{s}\frac{\beta\,(\sigma-1)\,(1-\alpha)\, g_{\mathrm{F}}}{\rho + \delta_{\mathrm{X}} + (1-\theta)\, g_{\mathrm{F}}}\pi = \frac{1}{s}\kappa_\iota \pi, \tag{21}$$

where $\kappa_\iota := \frac{\beta(\sigma-1)(1-\alpha)g_{\mathrm{F}}}{\rho+\delta_{\mathrm{X}}+(1-\theta)g_{\mathrm{F}}}$ collects parameters that are exogenous on the BGP. Equation (21) shows that firm-level demand for skilled labor depends on normalized profits,[17] and the skill premium, which serves as key summary statistic of the cost of technology adoption. The reason the skill premium emerges as the correct summary statistic of the cost of technology adoption depends on two assumptions. The first one is that technology adoption requires skilled labor. This, however, is not enough. The other key assumption is that firm entry requires unskilled labor, or at least less skilled labor than adoption.[18] Once entry costs are denominated in unskilled labor, one can show that along a BGP, profits have to be proportional to unskilled wages. Furthermore, note that the partial effect of productivity on profits is proportional to profits, $\frac{\partial \Pi}{\partial z} = \frac{(\sigma-1)(1-\alpha)\Pi}{z}$, which is proportional to unskilled wages, while the cost of adoption depend on high-skilled wages. Consequently, the ratio of the two emerges as the correct summary statistic that pins down technology adoption.[19]

[16] The economics contained in this restriction is that adoption cannot be too attractive, otherwise firms will hit the boundary and the first-order condition doesn't apply. What makes adoption attractive are weak diminishing returns to adoption (β large), and a large effect of productivity on profits ($(\sigma-1)\,(1-\alpha)$). In contrast, high effective discounting due to impatience, death shock, and advantage of backwardness reduces the incentive to invest in technology adoption.

[17] The reader can see now that $\rho+\delta_{\mathrm{X}}+(1-\theta)\, g_{\mathrm{F}} > \beta\,(\sigma-1)\,(1-\alpha)\, g_{\mathrm{F}}$ ensures spending on adoption is less than operating profits, $\iota s < \pi$, which is needed to sustain net profits.

[18] I generalize and explain at the end of this section that the logic of the paper goes through as long as entry is less skill-intensive than technology adoption.

[19] Formally, one needs to show that normalized profits only depend on exogenous parameters, at which point equation (21) is justified in the sense that π is fixed and the skill premium is the correct summary statistic of the cost of adoption relative to its benefits. Note that $V_t = \frac{\Pi_t(1-\kappa_\iota)}{\rho+\delta_{\mathrm{X}}} = f_{\mathrm{E}} w_{L,t} \Leftrightarrow \pi = \frac{f_{\mathrm{E}}(\rho+\delta_{\mathrm{X}})}{1-\kappa_\iota}$, which establishes that normalized profits don't depend on the skill premium.

Having established that equilibrium adoption effort is inversely related to the skill premium, and normalized profits are constant, I turn to a key results of the paper that is essential in creating a trade-off between innovation and adoption.

Proposition 2. *An increase in the skill premium, ceteris paribus, leads to an increase in the technology adoption gap*

$$-\frac{\partial \log z}{\partial \log s} = \frac{\beta}{1-\theta}.$$

Proposition 2 states that a one percent increase in the skill premium leads to a drop in normalized productivity by $\frac{\beta}{1-\theta}$ percent, and thus a widening adoption gap. This partial equilibrium result is general in the sense that any shock that leads to an increase in the skill premium, which does not directly impact the adoption technology or normalized profits, causes a widening gap between the technological frontier and current productivity.

To understand the proposition, take account of the fact that on a BGP, $z_t = z$, the adoption gap is proportional to adoption effort. Adoption effort, in turn, depends on the skill premium. Solving for z using $\dot{z} = 0$ and $\iota = \frac{1}{s}\kappa_\iota \pi$, the normalized productivity level equals

$$z = \left(\frac{\nu \left(\frac{1}{s}\kappa_\iota \pi \right)^\beta}{g_F} \right)^{\frac{1}{1-\theta}}. \tag{22}$$

Despite the fact that innovation and technology adoption are forward-looking interdependent activities, the steady state is surprisingly tractable and the equilibrium adoption gap is a negative power function of the skill premium, $z \propto s^{-\frac{\beta}{1-\theta}}$. This simple relationship unlocks all key results in the paper, and captures the intuition that when skill becomes scarce signaled by a rising skill premium, production sector firms endogenously adopt less technology. A market integration shock, which I will analyze in the open economy section, can precisely cause such an increase in the skill premium due to specialization in innovation.

Complementarity between Innovation and Adoption. Having understood how equilibrium adoption is pinned down on the BGP, I zoom in on the complementarity between innovation and adoption on the market for ideas. The two are tied together through the endogenous waiting time. Given $g_{A,t} = g_A$, the endogenous waiting time in (13) simplifies to

$$\tau = -\frac{\log z}{g_A}.$$

With this constant waiting time,[20] the normalized present discounted value of an innovation in (18)

[20] The expression looks identical to the endogenous waiting time in Perla, Tonetti, and Waugh (2021) but has a completely different interpretation. In their work, z represents the gap between a firm's productivity and the lower bound of the firm productivity distribution.

simplifies to

$$v_{\mathrm{I}} = \frac{1}{\tilde{\rho} + g_{\mathrm{F}}} \frac{\alpha l_{\mathrm{P}}}{a_{\mathrm{F}}} z^{\frac{\tilde{\rho}}{g_A}}, \tag{23}$$

where the effective household discount factor defined as $\tilde{\rho} := \rho - g_L$ depends on impatience and population growth. Inspecting equation (23), the reader might notice that the value function looks almost identical to the one in Romer (1990) and Jones (1995), save for the term $z^{\frac{\tilde{\rho}}{g_A}}$. This term captures the fact that the technology adoption gap reduces the present discounted value of an innovation. Intuitively, innovators would like their ideas to penetrate the market rather sooner than later. The extent to which the value of innovation responds to the adoption gap depends on how costly waiting is, captured in the effective household discount factor $\tilde{\rho}$, relative to the speed at which the "adoption queue" is moving, g_A. In the limit for $z \to 1$, the expression converges to the present discounted value in the benchmark Romer-Jones model.

Equation (23) shows that the value of innovation is increasing in z, and thus increasing in firm-level adoption effort ι, see (22). Yet, given free entry, an increase in the value of innovation is arbitraged away by new innovators. Therefore, the effect of a falling adoption gap shows up in the form of more skilled labor devoted to research, h_{I}, and an expansion of the technological frontier A_{F}. This complementarity between innovation and adoption on the market for capital goods is captured in the following two equations[21]

$$h_{\mathrm{I}} = \frac{1}{s} \frac{g_{\mathrm{F}}}{\tilde{\rho} + g_{\mathrm{F}}} \alpha l_{\mathrm{P}} z^{\frac{\tilde{\rho}}{g_A}}, \tag{24}$$

$$A_{\mathrm{F},t} = \left(\frac{\gamma h_{\mathrm{I}}^{\lambda} L_t}{g_{\mathrm{F}}} \right)^{\frac{1}{1-\phi}}. \tag{25}$$

Combining equation (24) and (25), a one percent increase in z, ceteris paribus, shifts out the technological frontier

$$\frac{\partial \log A_{\mathrm{F},t}}{\partial \log z} = \frac{\tilde{\rho}}{g_A} \times \frac{\lambda}{1-\phi},$$

where the first term, $\tilde{\rho}/g_A$, captures how a narrower adoption gap stimulates additional research, while the second term, $\lambda/(1-\phi)$, captures how additional research translates into new technology.[22] As before, this complementarity is a partial equilibrium result, and I turn to general equilibrium next.

General Equilibrium. The interaction of innovation and adoption in general equilibrium pins down

[21] Since $\dot{a}_{\mathrm{F}} = 0$ along a balanced growth path, the resource constraint implies $\frac{\gamma h_{\mathrm{I}}^{\lambda}}{g_{\mathrm{F}}} = a_{\mathrm{F}}$. Inserting this into the free entry condition, $v_{\mathrm{I}} = \frac{s h_{\mathrm{I}}^{1-\lambda}}{\gamma}$, yields normalized demand for skilled labor in the research sector, h_{I}. To obtain (25), recall that $a_{\mathrm{F}} := \frac{A_{\mathrm{F},t}^{1-\phi}}{L_t}$.

[22] This is to be understood as a long-run level effect relative to some alternative balanced growth with lower research activity. There model setup could be couched in the fully-endogenous style of the original Romer (1990) model with constant population. In that case, changes in the adoption gap would permanently shift the growth rate.

the skill premium, which is the key endogenous object. Once the skill premium is known, research effort and adoption effort follow immediately. I defer other equilibrium outcomes to the appendix to save space to focus on the key market clearing condition for skilled labor given by

$$h_{\mathrm{I}} + h_{\mathrm{A}} = h. \tag{26}$$

To obtain aggregate adoption effort, I sum over all firms' adoption effort, $h_{\mathrm{A}} = \iota m = \frac{1}{s}\kappa_{\iota}\pi m$, where πm simplifies conveniently to $\pi m = \frac{l_{\mathrm{P}}}{(\sigma-1)(1-\alpha)}$.[23] Combining this with normalized skilled labor demand from the research sector form (24), the market clearing condition becomes a simple function of the skill premium, the adoption gap, and exogenous parameters

$$\underbrace{\frac{1}{s}\frac{g_{\mathrm{F}}}{\tilde{\rho}+g_{\mathrm{F}}}\alpha l_{\mathrm{P}} z^{\frac{\tilde{\rho}}{g_A}}}_{h_{\mathrm{I}}} + \underbrace{\frac{1}{s}\frac{\kappa_{\iota}}{(\sigma-1)(1-\alpha)}l_{\mathrm{P}}}_{h_{\mathrm{A}}} = h. \tag{27}$$

Note that l_{P}, the share of unskilled labor devoted to production, is independent of the skill premium, research effort, or adoption effort, and can thus be treated like an exogenous variable for all practical purposes.[24] Both demand for skilled labor in innovation and adoption are downward-sloping in the skill premium, as indicated in the first and second panel in figure 1. Skilled labor is mobile across sectors, and equilibrium is reached when the total relative demand for skilled labor matches aggregate supply. Note that (27) is a non-linear equation because z itself is a function of s, which is the only aspect of the closed-economy model that requires a numerical solution.

The market-clearing skill premium immediately pins down the technological frontier and the adoption gap, which, in turn, determines wages

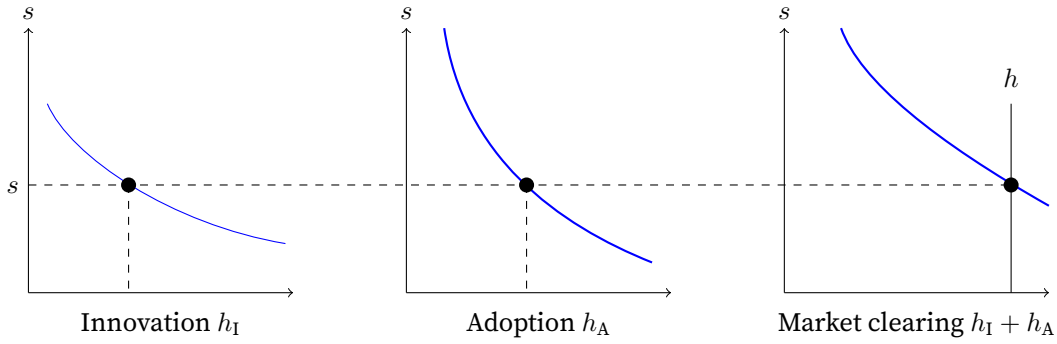
$$\begin{aligned} w_{L,t} &= \left(\frac{\sigma-1}{\sigma}p_x^{-\alpha}\right)^{\frac{1}{1-\alpha}} A_{\mathrm{F},t} z, \quad p_x = \frac{\rho+g_{\mathrm{F}}+\delta_k}{\alpha}, \\ w_{H,t} &= w_{L,t} s. \end{aligned} \tag{28}$$

Equation (28) highlights that stable long-run growth must ultimately originate at the technological frontier. At the same time, adoption gaps generate powerful level effects, which have long been recognized as important drivers of cross-country inequality (Nelson and Phelps, 1966), which I incor-

[23] I use the fact that $Y_t \frac{\sigma-1}{\sigma}(1-\alpha) = L_{\mathrm{P},t} w_{L,t}$, which follows from Cobb-Douglas production in a monopolistically competitive setting, and $\pi m = \frac{Y}{w_{L,t}L_t}\frac{1}{\sigma}$.

[24] The equilibrium value for l_{P} is found by combining the free entry condition, $\pi(1-\kappa_{\iota}) = (\rho+\delta_{\mathrm{X}})f_{\mathrm{E}}$ with normalized equilibrium profits, $\pi = \frac{Y}{Mw}\frac{1}{\sigma} = \frac{l_{\mathrm{P}}}{(\sigma-1)(1-\alpha)}\frac{1}{m}$, and the resource constraint for firm creation on a BGP, $m = \frac{1-l_{\mathrm{P}}}{f_{\mathrm{E}}(\delta_{\mathrm{X}}+g_L)}$. Firm entry in the production sector, and hence the share of the unskilled workforce devoted to entry is independent of the skill premium because entry only requires unskilled labor. I generalize the model at the end of this section by assuming entry requires both types of labor, and I also discuss other important extensions like skilled labor in production and endogenous skilled labor supply.

Figure 1. Market Clearing for Skilled Labor



porate in the open economy setting with country-specific adoption gaps. The same equation also underscores the role of transition dynamics: changes in the frontier and adoption gaps along the adjustment path can shape growth and inequality within advanced economies, much as human capital accumulation and rising research effort contributed to elevated growth over the twentieth century (Jones, 2002; Fernald and Jones, 2014).[25] This is to say: a priori, there is no reason to think the changes in z are less important than changes in research effort, even in frontier economies.

To better understand how innovation and adoption interact in the decentralized general equilibrium, I compute the ratio of the two

$$\frac{h_A}{h_I} = \frac{\beta}{1-\theta}\frac{1}{\alpha}\left[\frac{\tilde{\rho}}{g_F}+1\right]\frac{z^{-\frac{\tilde{\rho}}{g_A}}}{1+\frac{\tilde{\rho}+\delta_X+g_L}{g_F(1-\theta)}}. \tag{29}$$

Intuitively, a high β shifts skilled labor into technology adoption, as adoption becomes relatively more useful, while a strong advantage of backwardness $(1-\theta)$ pushes labor away from adoption. Moreover, the ratio of skilled labor in adoption vs. innovation is unrelated to research productivity γ. Consequently, neither the allocation of skilled labor across sectors nor the skill premium respond to changes in research productivity in the long run.[26] Similarly, an increase in adoption efficiency in the form of a rising ν would raise innovation and lower the adoption gap (recall $\dot{z}_t = \nu z_t^{\theta}\iota_t^{\beta} - g_{F,t}z$). That is to say, neither research nor adoption productivity is able to break the complementarity between innovation and adoption in general equilibrium in the closed economy.[27]

[25] Interestingly, in the growth accounting exercises of Jones (2002) and Fernald and Jones (2014) there is no technological frontier gap. An assumption that is relaxed in Peters and Trouvain (2026), which leads to a revision of the role of technological frontier growth over the 20th century.

[26] Changes in fundamental research productivity thus do not lead to a reallocation of skill across sectors. This result will no longer be true in the open economy where differences in fundamental research productivity generate comparative advantage, and hence specialization.

[27] This is true even though skilled labor devoted to adoption falls as ν rises, which is true by virtue of the factor market

Equation (29) also shows that an expansion in the supply of skilled labor would boost both innovation and adoption through a fall in the skill premium, but the effect on the research sector would be larger due to second round effects: a fall in s first makes the key input, skilled labor, cheaper. Furthermore, a falling technology gap boosts innovation through a second round effect, and (29) shows that the research share would increase as skilled labor expands and the adoption gap falls. This prediction rings true for the US economy after WW2, where an expansion in the skilled labor share and a decline in the skill premium (Katz and Murphy, 1992) coincide with overall fast growth and rising research effort (Jones, 2002), at least up until the 1990s.[28] The important takeaway, however, is that innovation and adoption rise in lock-step as skilled labor supply expands over the 20th century. In this sense, the complementarity dominates in the closed economy even in general equilibrium.

In the open economy section, I show that a globalization shock among asymmetric countries is exactly the kind of shock that can break this complementarity. Fast technology adoption of emerging markets in the integrated equilibrium raises the returns to innovation in the West, at which point the domestic factor market rivalry becomes the dominant force characterizing the domestic interaction between innovation and adoption, allowing the research sector and skilled labor to outgrow the rest of the economy. Before moving to the open economy, it is helpful to analyze the efficiency properties of the model, which ultimately determines the impact of shocks, including a market integration shock (Atkeson and Burstein, 2019; Perla, Tonetti, and Waugh, 2021).

Efficiency. The allocation of skilled labor is generically inefficient, and depends on externalities in both innovation and technology adoption. In the appendix, I derive the socially optimal allocation where the social planner decides on the allocation of skilled labor across sectors, taking endogenous firm entry in the production sector as given.[29] To provide the key intuition, I focus on the case with low effective discount rate, $\tilde{\rho} \to 0$, which pins down the decentralized allocation in closed form. The superscripts SP and DC denote the social planner and decentralized solution, and compute the ratio

clearing condition.

[28] While many case studies have been performed, I am not aware of a study that systematically documents changing adoption gaps within the US (Comin and Hobijn (2010b) focus on the cross-country context which also lines up with the prediction as skilled labor expanded globally). An instructive graph documenting falling adoption gaps for different technologies in the US can be found here: https://ourworldindata.org/grapher/technology-infrastructure-diffusion-us. In current work in progress, we explore this issue more carefully (Peters and Trouvain, 2026).

[29] The model is not well-suited to study optimal policy with respect to endogenous firm entry in the production sector. For one, the fact that adoption has to happen on the firm level means there is a strange incentive to curtail firm entry. Moreover, I took out scale effects in the production sector using the productivity shifter $M_t^{-\frac{1}{\sigma-1}}$, which introduces inefficiencies unrelated to the key trade-off concerning the allocation of skilled labor to innovation or adoption. One can rid the model of this inefficiency by resorting to a Hopenhayn version with diminishing returns on the firm level, or by assuming that the number of firms is always proportional to the number of workers, $M_t = L_t$, similar to Jones (2022). I emphasize that this inefficiency does not interact with the misallocation of skill across sectors.

of skilled labor devoted to adoption vis-a-vis innovation

$$\left(\frac{h_{\mathrm{A}}}{h_{\mathrm{I}}}\right)^{\mathrm{SP}} = \frac{\beta}{1-\theta}\frac{1}{\lambda}\left(1-\phi\right), \tag{30}$$

$$\left(\frac{h_{\mathrm{A}}}{h_{\mathrm{I}}}\right)^{\mathrm{DC}} = \frac{\beta}{1-\theta}\frac{1}{\alpha}\left(1\right)\frac{1}{1+\frac{\delta_{\mathrm{X}}+g_L}{(1-\theta)g_{\mathrm{F}}}}. \tag{31}$$

Private and planner allocation differ in four ways. Three of which are standard and arise due to inefficiencies in the research sector, as elaborated in Jones (1995): a markup distortion for $\alpha < 1$, a negative dynamic knowledge externalities for $\phi < 0$, and a congestion externality for $\lambda < 1$.[30]

The novel feature is the externality associated with technology adoption, whereby spillovers at entry lead to too little adoption. The reason is that incumbents do not internalize that entrants benefit from their adoption efforts. This mechanism is captured in (31) by the term $\frac{\delta_{\mathrm{X}}+g_L}{(1-\theta)g_{\mathrm{F}}} > 0$. The inefficiency disappears when $\delta_{\mathrm{X}} = -g_L$, which can be interpreted as incumbents creating additional spin-off establishments at a rate that matches the population growth rate so gross entry is zero. If so, firms fully internalize the benefits of adoption.[31]

Inefficiencies in the closed economy shape how market integration affects growth, see Perla, Tonetti, and Waugh (2021). With underinvestment in adoption—exacerbated by specialization in the open economy—market integration can lead to sluggish and uneven growth.

Extensions. The appendix considers several extensions, including heterogeneous firms with imperfect knowledge spillovers, complementary public investments that promote technology adoption, immigration, and skilled labor in entry and production. The core mechanism is robust across these environments. A central insight is that the negative relationship between the skill premium and technology adoption remains intact as long as firm entry is less skill-intensive than technology adoption–a mild and empirically plausible restriction. In fact, if entry is modeled as a Cobb-Douglas labor aggregator with skill share μ so the entry cost equal $f_{\mathrm{E}} w_{L,t}^{1-\mu} w_{H,t}^{\mu}$, then the passthrough from skill premium to technology adoption changes to

$$-\frac{\partial \log z}{\partial \log s} = (1-\mu)\frac{\beta}{1-\theta}.$$

For any $\mu < 1$, the key insight survives. Moreover, one can rescale $\beta/1-\theta$ to generate the same reduced form link between adoption and technology, $-\partial \log z/\partial \log s$, for varying degrees of skill-intensity. Perhaps surprisingly, none of this depends on how skilled labor enters in production as I

[30] Note that Jones (1995) finds that for reasonable parameterizations the decentralized allocation features too little innovation. Jones' conclusion is partly driven by the assumption that the production sector is efficient. In a model with technology adoption, this trade-off becomes more interesting since technology adoption features externalities as well.

[31] The influential model of technology adoption of Parente and Prescott (1994) can be understood as an efficient knife edge with $\delta_{\mathrm{X}} = g_L = 0$ and exogenous frontier growth. There is one additional externality related to search-and-matching frictions that arise from the endogenous adoption margin. This friction is ambiguous in the sense that there can be too much or too little adoption, and disappears when $\tilde{\rho} \to 0$.

show in the appendix, and the relevant margins are confined to the cost of entry relative to the cost of technology adoption.

I stress that the key assumption, namely that technology adoption is modeled as a relatively skill-intensive activity finds support in recent empirical work. Bijnens, Humlum, and Vanormelingen (2025) use detailed firm-to-firm trade data to show that the adoption of new capital goods requires directly related overhead skilled labor services. Moreover, the assumption that adopted technology complements unskilled labor finds direct empirical support in Bloom et al. (2025), which documents that new technologies diffuse primarily through their adoption in lower-skilled jobs, consistent with the complementarity between adoption and low-skilled wages in the model.

Another important extension endogenizes the supply of skilled labor. Since the mechanism operates through movements in the skill premium, any force that dampens its response, like endogenous labor supply, weakens the innovation-adoption trade-off. At the same time, changes in the skill premium together with the elasticity $\beta/1-\theta$ form a sufficient statistic for the impact on technology adoption in the baseline model. Conditional on a change in the skill premium, the elasticity of skilled labor supply becomes irrelevant, as it is already embodied in that change. In practice, whatever the elasticity of skilled labor supply, it was not large enough to prevent the substantial rise in the skill premium observed in recent decades. More broadly, the framework is generally robust to extensions that leave the technologies governing entry and adoption unchanged and generate the same movement in the skill premium. Lastly, Appendix A.10 contains a more detailed discussion of how the model differs from the recent idea flow literature, and models that incorporate both innovation and technology adoption.

3 Open Economy

In this section, I generalize the setup to a multi-country open economy model. Table 2 provides an overview of the economic environment in the open economy, and I develop the results on the BGP in the main text below.

Table 2. Economic Environment

Countries	
Index	$n \in \{1, ..., N\} := \mathcal{N}$
Country-heterogeneity	$\gamma = \{\gamma_n\}_{n\in\mathcal{N}}$, $\mathbf{h} = \{h_n\}_{n\in\mathcal{N}}$, $L_t = L_{n,t}\ \forall n \in \mathcal{N}$
Net exports	$\text{NX}_{nk,t}$:= Final good net exports from $n \in \mathcal{N}$ to $k \in \mathcal{N}$.
Agg. Production	
Output, capital accumulation	$Y_{n,t} = \left(\frac{X_{n,t}}{\alpha}\right)^{\alpha} \left(\frac{A_{n,t} L_{\text{P},n,t}}{1-\alpha}\right)^{1-\alpha}, \quad \dot{K}_{n,t} = Y_{n,t} - C_{n,t} - \delta_k K_{n,t} - \sum_k \text{NX}_{nk,t}$
Euler equation	
Inter-temporal consumption	$\frac{\dot{c}_{n,t}}{c_{n,t}} = r_{n,t} - \rho$
Adoption & Firms	
Adoption technology	$\dot{z}_{n,t} = \nu \iota_{n,t}^{\beta} z_{n,t}^{\theta} - g_{\text{F},t} z_{n,t}$
Adoption effort	$\frac{\dot{\iota}_{n,t}}{\iota_{n,t}} = \frac{r_{n,t} - g_{w_H,n,t} + \delta_{\text{X}} + (1-\theta) g_{\text{F},t} - \frac{1}{s_{n,t}} \frac{\partial \dot{z}_{n,t}}{\partial \iota_n} \frac{\partial \pi_{n,t}}{\partial z_n}}{1-\beta}$
Firm profits	$\pi_{n,t} = \frac{Y_{t,n}}{M_{t,n} w_{L,n,t}} \frac{1}{\sigma} \left(\frac{z_{n,t}}{\tilde{z}_{n,t}}\right)^{(\sigma-1)(1-\alpha)} = \frac{l_{\text{P},n,t}}{(\sigma-1)(1-\alpha)} \frac{1}{m_{n,t}}$
Labor/capital demand	$L_{\text{P},n,t} w_{L,n,t} = \frac{\sigma-1}{\sigma}(1-\alpha) Y_{n,t}, \quad X_{n,t} p_{x,n,t} = \frac{\sigma-1}{\sigma}(\alpha) Y_{n,t}$
Free entry	$\frac{\pi_{n,t} - s_{n,t} \iota_{n,t}}{r_{n,t} - g_{w_L,n,t} + \delta_{\text{X}}} = f_{\text{E}}$
Firm creation	$\dot{m}_{n,t} = \frac{l_{\text{E},n,t}}{f_{\text{E}}} - (\delta_{\text{X}} + g_L) m_{n,t}$
Innovation	
Mark-up	$p_{x,n,t} = \frac{r_{n,t} + \delta_k}{\alpha}$
Free entry	$\sum_n \frac{\exp\left(-\int_t^{t+\tau_n} r_{k,v} - g_{w_L,n,v} + \phi g_{\text{F},v} dv\right)}{r_{k,t} - g_{w_H,k,t} - (1-\lambda)\left(g_{H_{\text{I},k,t}} - g_L\right) + \phi g_{\text{F},t}} \frac{\alpha l_{\text{P},t+\tau_n} [1 + \dot{\tau}_{n,t}]}{a_{\text{F},t+\tau_n} z_{n,t+\tau_n}} = \frac{s_{k,t} h_{\text{I},k,t}^{1-\lambda}}{\gamma_k}$
Idea creation	$\dot{a}_{\text{F},t}^{\text{W}} = (1-\phi)\left(\sum_n \gamma_n h_{\text{I},n,t}^{\lambda} - a_{\text{F},t} \frac{g_L}{1-\phi}\right), \quad a_{\text{F},t}^{\text{W}} := \left(\sum_n A_{\text{F},n,t}\right)^{1-\phi} / L_t$
Market clearing	
Skilled labor	$h_{\text{A},n,t} + h_{\text{I},n,t} = h_n$
Unskilled labor	$l_{\text{E},n,t} + l_{\text{P},n,t} = 1$
Capital goods	$X_{n,t} = K_{n,t}$
Final goods	$\sum_n \sum_k \text{NX}_{nk,t} = 0$

Country Differences. Countries differ along two dimensions only: research productivity γ_n, and relative skill endowment h_n, where $n \in \{1, ..., N\} := \mathcal{N}$ is a country index of a total of N countries. Asymmetric integration refers to integration across countries that differ in at least one of these dimensions. All other features of the environment, including the technology governing adoption, are identical across countries. Despite its parsimony, this structure is sufficient to generate a realistic cross-country income distribution driven entirely by endogenous productivity differences that emerge from research productivity and skill endowments.

International Trade. Countries frictionlessly trade a homogenous final good. Capital goods are produced locally to abstract from offshoring. Importantly, even if a domestic idea is embodied in foreign

capital abroad, the domestic inventor still receives profits coming from the markup applied to the capital good. Trade thus occurs to compensate foreign technology owners for access to their intellectual property as in Burstein and Monge-Naranjo (2009) and McGrattan and Prescott (2010).

Adoption in the Open Economy. Relative to the closed economy, firms now adopt technology from the world technological frontier, which is the sum of all ideas in each country,

$$A_{\mathrm{F},t}^{\mathrm{W}} = \sum_{n} A_{\mathrm{F},n,t}.$$

Conditional on prices, the firms adoption problem is otherwise unchanged. Consequently, along a BGP, firms spend a constant fraction of their operating profits on technology adoption, so that

$$\iota_n = \frac{1}{s_n} \kappa_\iota \pi$$

still holds. This makes the model very tractable as all local feedback effects from innovation are captured in the relative price of skill, s_n. Moreover, aggregate normalized demand for skilled labor in adoption remains unchanged relative to the closed economy, and still equals $h_{\mathrm{A},n} = \frac{1}{s} \frac{\kappa_\iota}{(\sigma-1)(1-\alpha)} l_{\mathrm{P}}$.

Relative productivity across countries is pinned down by the relative amount of skilled labor devoted to technology adoption, and the overall importance of skilled labor in adopting technology parametrized by the advantage of backwardness $(1-\theta)$ and static diminishing returns to adoption (β)

$$\left(\frac{\iota_k}{\iota_n}\right)^{\frac{\beta}{1-\theta}} = \left(\frac{h_{\mathrm{A},k}}{h_{\mathrm{A},n}}\right)^{\frac{\beta}{1-\theta}} = \frac{z_k}{z_n} = \frac{w_{L,k}}{w_{L,n}} \quad \forall\, k, n \in \mathcal{N}. \tag{32}$$

The result follows from noting that the normalized measure of firms, m, will be identical across countries.[32]

Innovation. I assume that the knowledge spillover $\left(A_{\mathrm{F},t}^{\mathrm{W}}\right)^{\phi}$ is global in the integrated equilibrium,[33] but otherwise the same law of motion as in the closed economy applies, $\dot{A}_{\mathrm{F},n,t} = \frac{H_{\mathrm{I},n,t}}{f_{\mathrm{I},n,t}} = \gamma_n \left(A_{\mathrm{F},n,t}^{\mathrm{W}}\right)^{\phi} H_{\mathrm{I},n,t}^{\lambda} L_{n,t}^{1-\lambda}$. Using the normalized variables $a_{\mathrm{F}}^{\mathrm{W}} := \left(\frac{A_{\mathrm{F}}^{\mathrm{W}}}{L_t}\right)^{1-\phi}$ and $h_{\mathrm{I},n} := \frac{H_{\mathrm{I},n,t}}{L_t}$, the share of ideas developed in country equals

$$\chi_n = \frac{\gamma_n h_{\mathrm{I},n}^{\lambda}}{g_{\mathrm{F}} a_{\mathrm{F}}^{\mathrm{W}}}. \tag{33}$$

Equation (33) says that countries with higher research productivity or greater research effort therefore contribute disproportionately to the global frontier.

In the absence of trade cost, the value of an innovation is unrelated to where the idea was invented.

[32] Productivity here refers to the rate at which production sector firms turn physical inputs into physical output, which is a simple metric that also determines unskilled worker wages. It need not coincide with an aggregate measure of productivity that aims to incorporate gains from trade, see Burstein and Cravino (2015) for an in-depth discussion.

[33] See Grossman and Helpman (1991a) for an in-depth discussion of this issue. Global knowledge spillovers seem a natural assumption in a model of long-run growth. The knowledge spillover is assumed to be local in autarky.

Consequently, free entry ties the cost of innovation across countries together, and delivers countries' relative research effort as a function of skilled worker wages and research productivities,

$$\left(\frac{h_{\mathrm{I},k}}{h_{\mathrm{I},n}}\right)^{1-\lambda} = \frac{\gamma_k}{\gamma_n}\left(\frac{w_{H,k}}{w_{H,n}}\right)^{-1} = \frac{\gamma_k}{\gamma_n}\left(\frac{s_k}{s_n}\right)^{-\frac{1-\theta-\beta}{1-\theta}} \quad \forall n, \tag{34}$$

where the second equality uses the link between skill premia and skilled wages. Because congestion in research is country-specific ($\lambda < 1$), the free entry condition always binds.

Combining equation (33) and (34) determines the share of ideas produced in country k

$$\chi_k = \frac{\gamma_k^{\frac{1}{1-\lambda}} s_k^{-\frac{1-\beta-\theta}{1-\theta}\frac{\lambda}{1-\lambda}}}{\sum_n \gamma_n^{\frac{1}{1-\lambda}} s_n^{-\frac{1-\beta-\theta}{1-\theta}\frac{\lambda}{1-\lambda}}}. \tag{35}$$

This share captures comparative advantage in research, and depends on the two key determinants of innovation: differences in skill premia related to skill endowments, and differences in fundamental research productivities. The former generates trade patterns driven by factor endowments in the spirit of Heckscher-Ohlin, while the latter reflects Ricardian technology differences. To guarantee a well-behaved open economy equilibrium, I impose $\beta < 1 - \theta$, which guarantees that higher skill premia in a country reduce its research share. Intuitively, this condition ensures that an increase in the skill premium also implies an increase in skilled worker wages.

To characterize aggregate innovation effort, I focus on country k and derive the normalized value of innovation, $v_{\mathrm{I},k} := \frac{V_{\mathrm{I},t}}{\left(A^{\mathrm{W}}_{\mathrm{F},t}\right)^{-\phi} w_{L,k,t}}$,

$$v_{\mathrm{I},k} = \frac{1}{\tilde{\rho} + g_{\mathrm{F}}} \sum_n \frac{\alpha l_{\mathrm{P}}}{a_{\mathrm{F}}} \frac{w_{L,n,t}}{w_{L,k,t}} z_n^{\frac{\tilde{\rho}}{g_A}} = \frac{1}{\tilde{\rho} + g_{\mathrm{F}}} \frac{\alpha l_{\mathrm{P}}}{a_{\mathrm{F}} z_k} \sum_n z_n^{1+\frac{\tilde{\rho}}{g_A}}, \tag{36}$$

where I used the fact that labor devoted to production is identical across countries. Equation (36) highlights the role of cross-country adoption gaps in determining the value of innovation. Holding country k's adoption gap fixed, rising foreign technology adoption affects innovation through two channels. First, a declining adoption gap leads to a falling waiting time, $z_n^{\frac{\tilde{\rho}}{g_A}}$. Second, higher foreign adoption raises foreign real wages, expanding global market size. The closed-economy expression is nested by setting $z_n = 0$ for all $n \neq k$.

Market Clearing. Combining (33), (35), and(36) yields the relative share of skilled labor in innovation in country k consistent with free entry

$$h_{\mathrm{I},k} = \underbrace{\frac{1}{s_k} \frac{g_{\mathrm{F}}}{\tilde{\rho} + g_{\mathrm{F}}} \alpha l_{\mathrm{P}} z_k^{\frac{\tilde{\rho}}{g_A}}}_{\text{same as autarky}} \underbrace{\chi_k(\gamma, \mathbf{s})}_{\text{specialization}} \underbrace{\sum_n \left(\frac{z_n}{z_k}\right)^{1+\frac{\tilde{\rho}}{g_A}}}_{\text{rel. market size}} \tag{37}$$

where I highlight the connection to the closed-economy expression, which is extended by two terms. The first additional term captures specialization χ which is a function of the whole vector of exogenous research productivity and endogenous skill premia ($\gamma = \{\gamma_n\}, \mathbf{s} = \{s_n\}$ for $n \in \mathcal{N}$). The second term captures the size of the world economy relative to the domestic economy. Since the adoption gap remains a function of the skill premium, $z_n \propto s_n^{-\frac{\beta}{1-\theta}}$, the allocation of skilled labor to innovation within each country is fully pinned down by exogenous parameters and skill premia.

Market clearing in the open economy thus boils down to finding a set of skill premia consistent with skilled labor market clearing in each country,

$$h_{\mathrm{I},n}(\gamma, \mathbf{s}) + h_{\mathrm{A},n} = h_n \quad \forall n \in \mathcal{N}. \tag{38}$$

Given a set of skill premia solving (38), each countries' research share and technology adoption gap follows directly $\{\chi_n, z_n\}_{n\in\mathcal{N}}$, as well as the normalized world technological frontier $a_{\mathrm{F}}^{\mathrm{W}} = \frac{\sum_n \gamma_n h_{\mathrm{I},n}^{\lambda}}{g_{\mathrm{F}}}$.

3.1 Discussion of the Open Economy Model

Patterns of Trade. Final goods trade is unbalanced. Countries more specialized in innovation earn royalties abroad, matched by final goods flows from the net importers of technology

$$\underbrace{\sum_n \mathrm{NX}_{nk,t}}_{\text{net trade inflow into } k} = \frac{\sigma-1}{\sigma}\alpha(1-\alpha)Y_t^{\mathrm{W}}\left[\chi_k - \frac{Y_{k,t}}{Y_t^{\mathrm{W}}}\right],$$

where $Y_t^{\mathrm{W}} = \sum_n Y_{n,t}$ is world GDP. In an equilibrium where technology is produced in the West due to higher research productivity or greater skill endowments, final goods flow from East to West as compensation for technology usage. Under correct measurement, the goods trade deficit would be offset by a services surplus, leaving the current account balanced.

However, a growing body of evidence shows that multinational profit shifting and the relocation of intangible assets distort this accounting. Guvenen et al. (2022) document that a substantial fraction of income attributed to U.S. foreign affiliates reflects profits generated by intangible capital developed domestically but booked abroad. As a result, royalty income may not be fully recorded as service exports and instead appears as retained earnings abroad, mechanically widening the measured trade deficit. Large unbalanced inflows of the tradable final good into advanced economies from emerging markets have attracted much attention, especially in the US context. The model highlights that this is a natural implication of the invention of frontier technology being relatively concentrated in a few rich countries in an integrated world.[34]

[34]This lines up with the observation that the trade balance tends to be more negative than the current account balance, particularly so when focusing on the USA during the 2000s. This observation becomes even more striking when valuation effects on foreign assets are included, see Gourinchas and Rey (2007).

Cross-Country Productivity Differences, Scale Effects, and Technology Adoption. The model is consistent with key stylized facts of economic growth in open economies.

First, the long run equilibrium is characterized by a stationary cross-country income distribution where all countries grow at the same rate. Yet, growth spurts are possible when countries join the global economy and gain access to the world technological frontier. Both aspects follow from a model with endogenous technology adoption that features an advantage of backwardness. Related to this is the elevated role of skill in technology adoption, which is consistent both with causal micro evidence (Foster and Rosenzweig, 1996; Foster and Rosenzweig, 2010) and the aggregate relationship between skill, GDP per capita, and technology adoption gaps as documented in Caselli and Coleman (2001) and Comin and Hobijn (2004; 2010). The setup thus reconciles the findings Mankiw, Romer, and Weil (1992), which focuses on human capital differences across countries, with Parente and Prescott (1994) emphasizing technology adoption and TFP differences as the key driver of cross-country inequality.

One caveat is in order. If skill is key, and skill is scarce in poor countries, it ought to be true that the price of skill is high in poorer countries. While skill premia tend to be higher in poor countries, they are not as high as a standard model of neoclassical production with two types of labor like Mankiw, Romer, and Weil (1992) would suggest. Caselli and Coleman (2006) make this point explicitly. Relative to this, I note that endogenous specialization in innovation mitigates this issue: skill-rich countries specialize in invention, which pushes toward factor-price equalization and narrows skill premia. This is, of course, an instance of the classic factor price equalization theorem of Stolper and Samuelson (1941).

Second, countries with higher research productivity, larger populations (introduced in the next section), or higher skilled labor shares contribute more to the global technological frontier. I stress that relative to other studies, which take out scale effects in open economy settings,[35] I maintain the non-rivalry of ideas and the important link between skilled labor endowment and idea production. These types of scale effects are extremely natural, and find strong support in the data.

Third, while the technological frontier exhibits scale effects and depends on the total size of the global research force, a country's individual productivity is unrelated to its size as it adopts technologies invented elsewhere.[36] This is why the technological frontier has to be modeled as a global one: If countries were to adopt only their own local innovations, a skill-rich but small country like Denmark would be a poor place.

Technically, the key to generating realistic scale effects is that endogenous firm entry ensures that a country twice as large requires twice as many skilled workers to keep the amount of skilled labor

[35] See for instance Sampson (2023), Sampson (2016), and Perla, Tonetti, and Waugh (2021)

[36] The idea that scale effects disappear in the cross-section of countries once technology diffuses is already implicit in Nelson and Phelps (1966), and explicit in Benhabib and Spiegel (1994) and especially Howitt (2000). The contribution here is to provide an explicit micro-foundation that tractably embeds this into a model where the evolution of the frontier is endogenous, and responds to technology adoption choices. See also Ramondo, Rodríguez-Clare, and Saborío-Rodríguez (2016) for a view from the trade literature, and Gross and Klein (2024), which differentiates between local and global ideas to develop a model with realistic cross-country scale effects.

devoted to technology adoption within each firm constant.[37] It seems a natural assumption in the context of adoption since any one idea is adopted many times highlighting the rivalrous nature of adoption.

The setup again generalizes by incorporating skilled labor in entry and production, and the central assumption is that technology adoption is more skill intensive than entry. This is a good point to consider what happens if one were to assume instead that entry is more skill-intensive than adoption. If so, skill-scarce poor countries would have relatively few firms, and the few firms would spend a relatively high amount of skilled labor on technology adoption. This is, of course, contrary to a large body of evidence that shows that relatively many firms with low levels of technology adoption are the predominant mode of production in poor skill-scarce countries, see Cirera, Comin, and Cruz (2022) for detailed evidence.

Incorporating these features is necessary for any coherent treatment of technological change in a globalized world. This is a difficult task in models that only allow for innovation, but becomes trivial once innovation and adoption are modeled jointly. I will show next that such a richer model offers new insights into the impact of market integration on growth.

3.2 Globalization, Growth, and Inequality

Globalization matters in this model because it provides access to the global technological frontier and allows for gains from trade in innovation. While the trade elasticity of the final good is infinite –shutting down the usual gains from goods trade– trade in the homogeneous good remains essential. It allows countries to access foreign technology by compensating its owners. As a result, gains from market integration can be large even when observed trade flows are small, since access to ideas does not require shipping entire capital goods across countries.[38]

The model admits a simple summary statistic for the long-run wage effects of market integration, similar to Arkolakis, Costinot, and Rodríguez-Clare (2012) (ACR).

Proposition 3. *The ratio of unskilled worker wages in the open economy vs. closed economy for some country k reads*

$$\frac{w_{L,k,t}^{open}}{w_{L,k,t}^{closed}} = \underbrace{\left(\frac{h_{I,k}^{open}}{h_{I,k}^{closed}}\right)^{\frac{\lambda}{1-\phi}} \left(\frac{1}{\chi_k}\right)^{\frac{1}{1-\phi}}}_{\textit{Innovation margin}} \underbrace{\left(\frac{s_k^{open}}{s_k^{closed}}\right)^{-\frac{\beta}{1-\theta}}}_{\textit{Adoption margin}}. \tag{39}$$

Skilled wages follow immediately from $w_{H,k,t} = s_{k,t} w_{L,k,t}$. The key distinction relative to ACR is that changes in the skill premium enter the statistic through the adoption margin. As a result,

[37] The role of endogenous entry in dealing with scale effects is reminiscent of Peretto (1998) and Young (1998). Their focus rests on constructing a fully-endogenous growth model where long-run growth is unrelated to the level of the population.

[38] A country in my baseline calibration would have to export 14% of its final output when its entire idea stock is held by foreigners. In contrast, in a symmetric equilibrium where every country holds an even share of global technology with sufficiently rich financial markets, no goods would have to be shipped whatsoever as net trade flows are zero and trade in financial asset would render trade in final goods redundant.

equation (39) is not a sufficient statistic in the ACR sense, since autarky skill premia and research effort are not directly observable.[39]

The impact of market integration on the technological frontier is captured by two forces: increased innovative effort in the home economy, $\left(h_{\mathrm{I},k}^{open}/h_{\mathrm{I},k}^{closed}\right)^{\frac{\lambda}{1-\phi}}$, and gains from ideas developed abroad, $(1/\chi_k)^{\frac{1}{1-\phi}}$, both of which depend on a constant scale elasticity $1/1-\phi$. When the share of ideas developed in the home economy is small, the gains from integration are large. The logic mirrors ACR, where a large import share signals substantial losses from reverting to autarky.

The novel feature in equation (39) is the endogenous adoption margin, which enters through the skill price ratio $\left(s^{open}/s^{closed}\right)^{-\frac{\beta}{1-\theta}}$. An increase in the skill premium, ceteris paribus, reduces unskilled wages by lowering domestic technology adoption. This channel changes how market integration affects growth.

I next establish a sharp theoretical distinction between symmetric market integration among similar countries—such as the formation of the European Common Market in the late 20th century—and asymmetric integration between emerging markets and advanced economies. Symmetric integration has no adverse effects on inequality or technology adoption and delivers the standard gains from integration. The reason is that foreign adoption, which raises the incentive to innovate, and foreign innovation, which reduces it, exactly cancel. Underlying this result is a classic Ricardo-Heckscher-Ohlin-Krugman type logic where for identical technology and factor endowments, prices remain unchanged, while gains from integration are driven by standard scale effects as each country gains access to the other's technology.

Proposition 4. *Symmetric integration with $\gamma_n = \gamma$, $h_n = h\ \forall n \in \mathcal{N}$ leaves the skill premium and the technology adoption gap unchanged relative to autarky, and raises income by a factor of $N^{\frac{1}{1-\phi}}$.*

The proposition follows from noting that for symmetric countries the open economy market clearing condition in (38) collapses to the closed economy. Consequently, skill premium and adoption gap don't respond in the long run. By symmetry, every country produces a fraction $\frac{1}{N}$ of ideas. Plugging this into (39) completes the proof.

In the case of symmetric integration, real wage gains accrue to all workers without adverse inequality effects. The strength of this response depends on ϕ, which governs how difficult it is to discover new ideas. Even when ideas are harder to find, gains from market integration are generally very large, since any one country tends to hold only a small share of the global technological frontier.[40]

These pro-growth effects of globalization potentially disappear in the case of asymmetric integration, a central element of the globalization shock of the 1990s and 2000s. Suppose that the West

[39] I thank a referee for pointing this out.

[40] To illustrate this point note that in 2022 the USA accounts for about 20% of global patents. Using global patents as a crude proxy for a country's share in the global frontier, and assuming there are strong diminishing returns to innovation with $\phi = -1$, the gains in the open economy for the US will be of the order of $5^{\frac{1}{1-(-1)}} - 1 \approx 130\%$. For a small open economy like the UK, the gains would be around 500%.

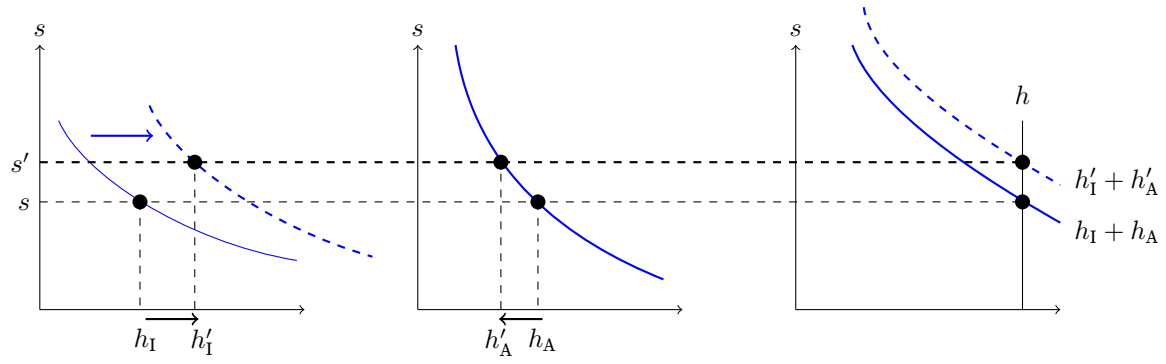
develops frontier technology and can partially protect its intellectual property. Technology adoption of emerging markets in the integrated equilibrium raises the returns to innovation in the West, increasing research effort and temporarily accelerating frontier growth – the standard mechanism emphasized in the literature. To see this most starkly, consider the case where comparative advantage in research is very large for country k, so $\chi_k \approx 1$. In this case,

$$h_{\mathrm{I},k} = \frac{1}{s_k} \frac{g_{\mathrm{F}}}{\tilde{\rho} + g_{\mathrm{F}}} \alpha l_{\mathrm{P}} z_k^{\frac{\tilde{\rho}}{g_A}} \sum_n \left(\frac{z_n}{z_k} \right)^{1+\frac{\tilde{\rho}}{g_A}}.$$

Relative to autarky, where $z_n = 0 \; \forall n \neq k$, integration exerts additional pull on innovation through the term $\sum_n (z_n/z_k)^{1+\frac{\tilde{\rho}}{g_A}}$. When wage growth and technology adoption accelerate in emerging markets, this pull is amplified, reflecting the complementarity between innovation and adoption.

The key twist in the model is that the expansion of research effort comes at the expense of domestic technology adoption as depicted in figure 2. Technology adoption in emerging markets raises the returns to innovation in the West. This leads to heightened research effort and temporarily faster growth of the technological frontier. At the same time, this reallocation requires a reallocation of skilled labor away from the production sector. This reallocation is brought about by an increase in the skill premium, and ultimately hurts technology adoption in the West. Note that the complementarity between innovation and adoption doesn't vanish but plays out between domestic innovators and foreign adopters, at which point the domestic factor market rivalry becomes the dominant force characterizing the domestic interaction between innovation and adoption.

Figure 2. Open Economy Market Clearing for Skilled Labor



While skilled workers always gain, the impact on unskilled wages and aggregate GDP per capita is unclear, and ultimately tied to quantitative question of whether positive innovation or negative adoption effects dominate. Moreover, the previous discussion ignored transition dynamics, which are cru-

cial. While productivity growth in the very long run tracks growth of the technological frontier, along the transition path, they are divorced. A widening adoption gap can exert downward pressure on realized productivity growth rates despite elevated growth of the technological frontier as improvements in frontier technology. In the final section of the paper, I quantify a more general version of the model to explore these questions in a two-country model.

Before turning to the quantitative analysis, I clarify one important issue. In the open economy, technology adoption –conditional on the skill premium– does not respond directly to market integration. The horizontal blue arrow in the first panel embodies the global demand shock for ideas, which is the key driving force in my model. In principle, globalization may also alter the demand for adoption in the production sector and shift the demand curve for adoption in the second panel in figure 2. I highlight two points here and defer formal details to Appendix A.8.

First, there is no long-run effect on demand for technology adoption in my baseline model. To see why this is the case, note that aggregate spending on skilled labor for adoption purposes is a constant share of aggregate profits, $w_{H,t}H_{\mathrm{A},t} = \frac{Y_t \kappa_\iota}{\sigma}$. Globalization matters but its impact on adoption is fully pinned down by movements in the skill premium since output is proportional to the unskilled wage bill.

This neutrality result survives several extensions, including intermediate-goods trade a la Krugman (1980) or environments with country-specific varieties and iceberg trade costs as in Armington (1969). The deeper reason neither extension impacts adoption directly is that in homogeneous firm models with monopolistic competition, firm size measured in terms of employment is unrelated to market size so the only way in which globalization impacts adoption is through the relative price of skill. Second, Melitz-style selection effects would break this neutrality result. A change in the firm size distribution generically changes adoption incentives. I develop such a heterogeneous-firm extension in the appendix where I show that the basic link between skill premium and weak adoption remains intact.

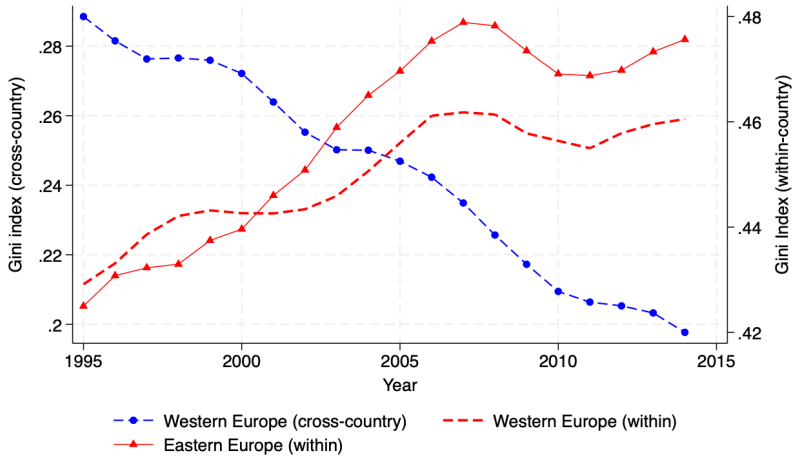
4 Quantification

I quantify a version of the model to explain three striking features of global growth since the mid 90s, two of which are depicted in figure 3.

First, strong growth in emerging markets, especially Eastern Europe and China, have lead to a decline in global inequality measured as dispersion in income per capita across countries. Using PPP-adjusted GDP per capita form the PWT, the cross-country Gini index declines by around seven pp. from 1995 to 2015 when focusing on a set of West and East European economies.[41] Second, within-

[41] Convergence is not uniform, and many countries, especially in Africa, have seen much less catchup growth, if at all. See Milanovic (2016) for an in-depth discussion of global convergence. The focus on Europe is simply due to easily comparable

Figure 3. Cross-Country Convergence and Within-Country Divergence



The data is based on the World Inequality Database, see Alvaredo et al. (2020). The gini index is computed over the whole population and uses pre-tax income, split concept. Aggregates are employment-weighted averages within each country group, where I compute the within-country Gini index using a 3-year moving average. Country income is based on ppp-adjusted output per capita using PWT V10. Country list for Western Europe: Austria, Belgium, Denmark, Finland, France, Germany, Iceland, Luxembourg, Netherlands, Norway, Sweden, Switzerland, United Kingdom; Country list for Eastern Europe: Albania, Bosnia and Herzegovina, Bulgaria, Croatia, Czech Republic, Estonia, Hungary, Latvia, Lithuania, Montenegro, Poland, Romania, Serbia, Slovakia, Slovenia. I do not consider Southern European countries.

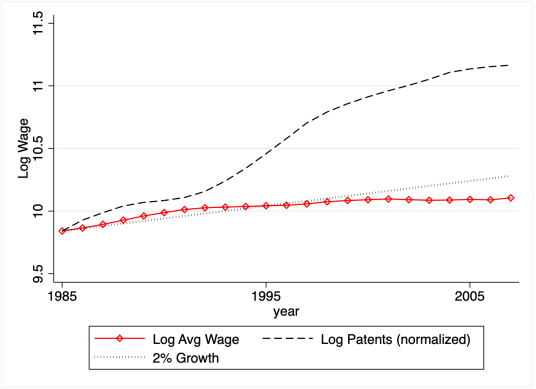
country income inequality went up substantially. When measured in terms of pre-tax per capita income the rise in within-country inequality matches the decline in cross-country inequality.

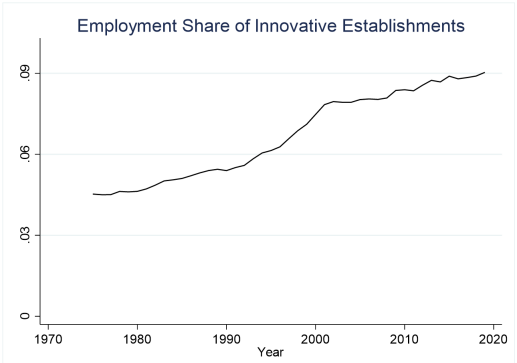
Third, while per capita growth was exceptional in Eastern Europe and some parts of Asia, per capita growth in the West was abysmal during this episode of rising market integration. This is particular puzzling against the backdrop of rising research effort and strong patent growth, a common proxy for innovation. I showcase this for the German economy, which has long-run administrative matched employer-employee data including information on worker education, and is a major producer of frontier technology. The left panel in figure 4 contrasts strong patent growth against weak wage growth. While wage growth and patent growth moved in lock-step at the beginning of the sample, they diverge since the 1990s when patenting takes off and wage growth begins to slow down. The right panel in figure 4 documents the increasing total employment share in the research sector, here broadly defined including headquarter services, IT, and Finance. The model developed can account account qualitatively for all these patterns using one simple framework that takes into account the interplay of innovation and technology adoption in a globalized world.

Generalization. I next explore how far the model can go in rationalizing these empirical patterns. To

data series.

Figure 4. Growth, Patents, and Inequality in Germany





Data for patents comes from the Crios Patstata database, see Coffano and Tarasconi (2014). Wage data is computed based on the PWT version 09, combining real national gdp (not PPP) with their measure of the labor share and dividing thorough by the total population. Patents are normalized so that the wage level and patent level coincide in 1984. GDP per capita growth does better than wages, but still grows substantially below trend, leading to an overall growth slowdown. Employment shares in production and research sector are based on IAB data for West Germany with additional details in the appendix. This classification can be carried out using different sectoral codes (broad vs. narrow where the narrower codes) and a question arises whether IT and finance belong into the research or production sector. I use a broad definition here, and compute alternative definitions in the appendix. While the overall level of research employment falls when excluding IT and finance, growth in research employment actually increases.

do so, I generalize the model along three dimensions. First, I introduce exogenous innovator death shocks that arrive at rate δ_{I}. These shocks hit both adopted and unadopted ideas, which innovators take into account when entering the research sector. The waiting time thus generalizes to

$$\tau_t = \frac{\log z_t}{\frac{\int_t^{t+\tau_t} g_{A,v} + \delta_{\mathrm{I}} dv}{\tau_t}}.$$

Idea death shocks lead to more realistic convergence dynamics, as well as entry and exit dynamics consistent with the data. Second, I now allow for differences in country size L_n. Third, I introduce a technology adoption friction $\mu_{\mathrm{D}} < 1$ operating in the emerging market in autarky similar to Parente and Prescott (1994) to avoid a counterfactually large autarky skill premium in the emerging market explained below.

Quantification. To quantify the aggregate implications of the theory I have to pin down a set of parameters $\Theta = \{\rho, \alpha, \delta_k, \sigma, f_{\mathrm{E}}, \delta_{\mathrm{X}}, \{L_n\}, \{\mu_n\}, \{\gamma_n\}, \delta_{\mathrm{I}}, \nu, \beta, \theta, \lambda, \{h_n\}, g_L\}$ where the inner parentheses and subscript n indicate that the parameters are set separately for each country.

$\{\rho, \alpha, \delta_k, \sigma, g_{\mathrm{F}}\}$**:** I set the capital share equal to .4, capital depreciation equals 5%, and the discount rate is 5%. Together with income growth of one percent, $g_{\mathrm{F}} = 1\%$, this implies a long-run real rate of 6% consistent with stock market returns. I set the elasticity of substitution across intermediate goods varieties equal to 2.5, consistent with evidence from Broda and Weinstein (2006).

$\{\delta_{\mathrm{X}}, \delta_{\mathrm{I}}\}$**:** I use the IAB establishment panel data to compute annual firm exit rates in the production and research sector, which equa5 in each sector. Recall that the higher the arrival rate of death shocks (δ_{X}) in the production sector, the stronger is the adoption externality. I choose values that are similar or lower than the empirical estimates from Garcia-Macia, Hsieh, and Klenow (2019) (ranging from

8% to 4% depending on firm age and time period), Peters and Walsh (2019) (5.4%), or Sampson (2023) (10%). Using such higher values would amplify the main mechanism.

$\{\theta, \beta, \phi, \lambda\}$: The implications of the theory hinge on the importance of technology adoption and innovation, which directly relates to the ratio $\frac{\beta}{1-\theta}$ and $1-\phi$. I start by calibrating θ such that the model is consistent with catch-up growth convergence patterns across countries. Barro's "Iron law" (Barro, 1991) suggests countries converge at a rate of 2%, i.e., the coefficient in the cross-country convergence regression, after controlling for additional covariates such as human capital, is close to $-.02$. I linearize the law of motion of z to show that the advantage of backwardness $1-\theta$ is the key parameter governing this speed of convergence

$$\frac{\dot{z}}{z} \approx \underbrace{(1-\theta)\left(g_{\mathrm{F}}+\delta_{\mathrm{I}}\right)}_{=\hat{\beta}_B}\left(\log z_{ss}-\log z_t\right)+\beta\left(g_{\mathrm{F}}+\delta_{\mathrm{I}}\right)\left(\log h_{ss}-\log h_t\right).$$

If $\delta_{\mathrm{I}}+g_{\mathrm{F}}=5\%$, a reasonable estimate for θ is 0.55 which ensures that $\hat{\beta}_B \approx -.02$. This value is consistent with Lucas (2009a)'s calibration of the same advantage of backwardness parameter.

Given θ, cross-country inequality disciplines β. In steady state,

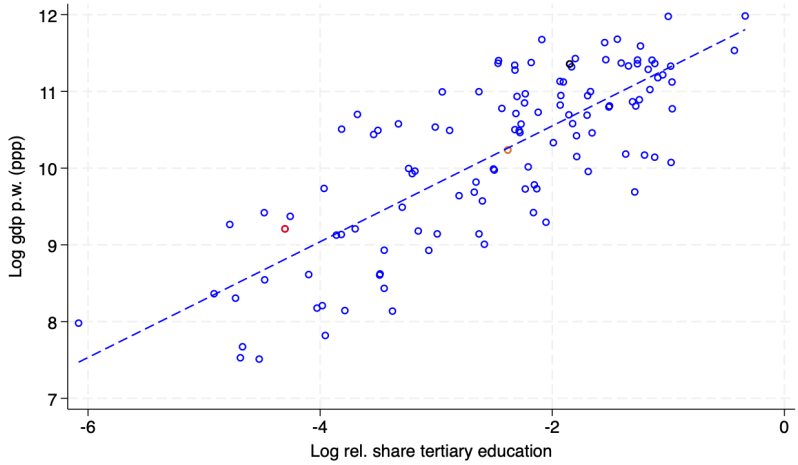
$$\frac{w_{L,n,t}}{w_{L,k,t}} = \left(\frac{h_{\mathrm{A},n,t}}{h_{\mathrm{A},k,t}}\right)^{\frac{\beta}{1-\theta}}. \tag{40}$$

Conditional on a distribution of the relative amount of skilled labor devoted to adoption across countries $\{h_{\mathrm{A},n}\}$, and conditional on θ, the parameter β maps this distribution into observed cross country unskilled wage inequality. Setting $\beta = .35$ implies $\beta/1-\theta \approx .78$, consistent with the cross-sectional relationship between relative tertiary education shares and output depicted in figure 5. While $h_{\mathrm{A},n,t}$ is in general not observed, it can be proxied using country skill endowments as long as the country does not innovate, which is a reasonable assumption for middle and low income countries. The resulting slope coefficient when dropping the top 10% of richest countries equals 0.74 , which lines up nicely with the calibrated parameter.[42]

While the cross-country relationship between skill shares and income is consistent with the model, the direction of causality is not unidirectional. Human capital accumulation is itself an endogenous outcome shaped by institutions, culture, and historical conditions (Sokoloff and Engerman, 2000). In that sense, the regression should not be interpreted as causal. That said, the focus here is narrower. The model emphasizes the role of skill in technology adoption. Conditional on access to a common global frontier, higher skilled labor endowments increase adoption and raise productivity. This mechanism is supported by both macro and micro evidence. Comin and Hobijn (2004) document a strong link between tertiary education and technology adoption across countries. At the micro level, exoge-

[42] In closed economies the approach would not work since country-specific technological frontiers would confound the link between relative skill endowments and real wages, hence I run this cross-sectional regression in 2015 to capture the post-integration state.

Figure 5. Relative High-Skill Share and Output p.c.



The plot combines ppp-adjusted output per worker from PWT 9.1 with the Barro and Lee (2013) dataset on schooling for a cross-section of countries in 2015. I equate high-skilled labor with the share of completed tertiary education of the population, and low-skilled labor is its complement. The colored dots refer to Congo, Brazil, and Germany in this order from left to right. When implementing the linear regression I use lagged skill ratios as instrument to deal with measurement error in a two-stage least square regression.

nous variation in local skill supply induces technology adoption: Lewis (2011) and Beaudry, Doms, and Lewis (2010) for the United States; Foster and Rosenzweig (1996) and Imbert et al. (2022) for India and China. The direction runs from skill composition to technology adoption.

Turning to innovation, note that the long-run frontier growth rate is endogenous, and equals $g_L/1-\phi$. To keep the model simple, I will assume a constant long-run labor force growth rate $g_L = 2\%$ with fixed high-skill vs. low-skill shares.[43]

I set the dynamic knowledge externality to $\phi = -1$ in the baseline calibration. This parameterization is more optimistic than what the evidence in Bloom et al. (2020) suggests, which is closer to $\phi = -1.5$. Two considerations motivate this choice. First, with an endogenous technology adoption gap, the mapping between research effort and productivity is more complicated than in Bloom et al. (2020): the framework here highlights that ideas may appear harder to find, in part, because less ideas are adopted. Second, the muted response of advanced economies' GDP growth to globalization makes it unlikely that dynamic knowledge spillovers are positive. Calibrating the model using a more optimistic dynamic externality leads to counterfactually strong growth for all countries.[44] I offer ro-

[43] Slowing population growth and rising educational attainment represent two confounding factors that push in opposite direction: slowing population growth reduces long-run growth in idea-based growth models (Jones, 1995) while rising educational attainment is pushing the other way. The model developed here could in principled be used to study this trade-off, but one would have to think harder about selection into schooling, and the substitutability between skilled and unskilled workers in production vs. technology adoption vs. innovation. These are important issues for future work.

[44] I caution against interpolating empirical estimates based on regional or sectoral variation. Strong local agglomeration

bustness exercises with respect to β and ϕ to show how sensitive the results are to either parameter in appendix B.4.

In the canonical Romer–Jones framework, the capital share and the markup are jointly determined by α, which implies relatively high markups. I maintain this structure to keep the calibration directly comparable to estimates of ϕ in Jones (1995; 2002), Fernald and Jones (2014), and Bloom et al. (2020), all of which are grounded in this specification. Changing the elasticity of substitution across capital goods necessitates a reinterpretation of ϕ. This goes back to the observation that it is difficult to distinguish empirically between ideas becoming harder to find (a low ϕ) and ideas becoming less valuable (a high elasticity of substitution). What ultimately matters for the quantitative implications of the model is whether adoption is inefficiently low in autarky. Lower markups in innovation make this more likely, but recalibrating ϕ to match long-run growth offsets that effect.

To calibrate the congestion force λ,[45] I note that cross-country specialization in innovation should contain information on this parameter. Under identical research productivity among innovating countries ($\gamma_n = \gamma$), the model implies a log-linear relationship between the share of ideas, the total number of researchers, and country size

$$\log \chi_{n,t} = \alpha_t + \lambda \log H_{\mathrm{I},n,t} + (1-\lambda) \log L_{n,t}. \tag{41}$$

This relationship provides a natural moment to discipline λ.

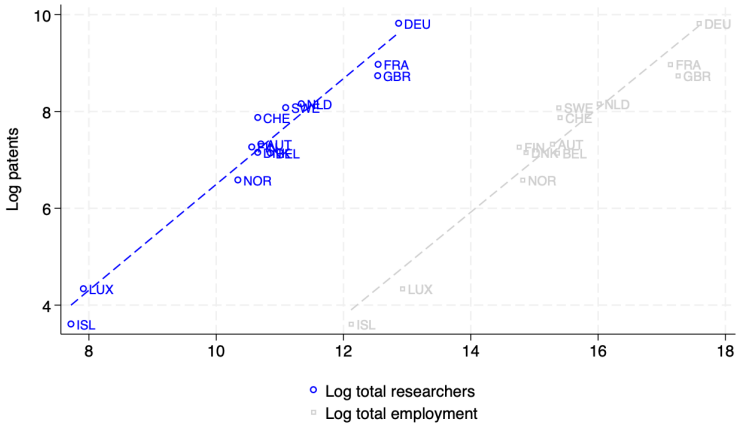
I approximate χ_n using internationally protected patent flows over 2011–2019, which proxy for idea shares in steady state. Data on researchers and employment are drawn from the OECD Science, Technology, and Innovation database. Among advanced economies, the elasticity of patenting with respect to the number of researchers is close to unity, with a correlation of 0.97 (Figure 6). This supports the assumption that research productivity is broadly similar across rich countries.[46] Moreover, the fact that global patents are tightly related to a country's number of researchers, which, among rich countries, is tightly correlated with country size, highlights once more why a multi-country semi-endogenous growth model is the right model to study long-run growth. Scale matters, and scale differs across countries.

effects can coexist with large diminishing returns to research for the world as a whole.

[45] My congestion externality differs slightly from Jones (1995)'s original setting with $f_{\mathrm{I},t} \propto H_{\mathrm{I},t}^{1-\lambda} A_{\mathrm{F},t}^{\phi}$, where the frontier growth rate on the BGP equals $g_{\mathrm{F}} = \lambda g_L / 1 - \phi$. If countries have different population sizes and the static congestion is local (which is what one would like for a well-behaved open economy equilibrium), it is preferable to use the ratio of researchers to unskilled labor $(H_{\mathrm{I},t}/L_t)^{1-\lambda}$. Otherwise, a strong counterfactual bias toward innovation in small countries would emerge since the total number of researchers of a small country is generically smaller.

[46] While Sampson (2023) provides evidence that sectoral research productivity differs across countries, he finds that differences are much larger across advanced vs. emerging economies than within each country group. Moreover, sectoral heterogeneity likely overstate differences in aggregate research productivity on the country level, which is the focus of Sampson (2023). I thus view the findings in Sampson (2023) as broadly consistent with my calibration.

Figure 6. Specialization in Innovation



The plot uses data from the OECD "science, technology, and innovation" database. I plot patents flied under the PCT (global patents) against and the total number of researchers similar to Bloom et al. (2020). The set of countries is the same as in the previous plot for the West. Each dot represents a simple average within each country over the period 2011-2019 to avoid the confounding influence of the financial crisis and the pandemic.

Estimating equation (41) directly is complicated by multicollinearity between total employment and researcher counts, as well as by endogeneity concerns. A baseline cross-sectional regression nevertheless yields $\lambda = .98$. Since small differences in research productivity and skill endowments are likely to induce an upward bias, I make a back of the envelope adjustment and use $\lambda = .9$ as my baseline value.

I am left with the following parameters $\{f_{\mathrm{E}}, \{L_n\}, \{\mu_n\}, \{\gamma_n\}, \nu, \{h_n\}\}$. I assume that h in the advanced economy is .15, roughly consistent with data from Barro and Lee (2013) over the period 1980 – 2015, and similar to the high skill-low-skill ratio in Acemoglu et al. (2018). I target a skill premium of $s = 1.5$ in the initial equilibrium for the advanced economy close to Buera et al. (2022), and I target a relative productivity level of $z = .75$ in autarky, which means that the waiting time for an idea to be adopted equals 5 years. Bloom et al. (2025) show that even within advanced economies, diffusion gaps can persist for decades. Since I don't model firm heterogeneity, the reader should think of this waiting time as the moment when the product has reached substantial market penetration.[47] I then use z and s to pin down the fixed cost of entry in the production sector as well as the shifter in the adoption technology $\{f_{\mathrm{E}}, \nu\} = \{1.25, .21\}$. I normalize unskilled labor in the advanced economy to unity, and assume that the foreign economy is of relative size $\frac{L^*}{L} = .66$, consistent with the relative size of Eastern Europe. Lastly, I normalize research productivity in the West to unity without loss of generality.

[47] Golder and Tellis (1997) finds that it takes on average six years for a new consumer durables to achieve substantial market penetration.

I will consider different values for γ^* and h^* in the quantitative exercise. For the case of asymmetric integration, I assume $\gamma^* \approx 0$, $h^* = .05$, which implies that all innovation is produced in the West. This captures a salient feature of market integration in the 1990s and 2000s, where a large increase in patenting activity in Europe coincides with virtually no patenting in Eastern Europe, see Khan and Dernis (2006) for supporting evidence.[48]

In an alternative scenario, I assume that skill endowments and research productivity in the East gradually converge to Western levels. Proposition 4 implies that, under symmetric integration, long-run inequality remains unchanged while growth increases. I model convergence as occurring over roughly 30 years (see Figure 20 in the appendix). During the transition, the rich country experiences weak adoption and skill scarcity. In the long run, however, convergence reduces comparative advantage in innovation and eliminates these imbalances. Eventually, an expansion of the technological frontier raises wages for all. Before I compute these transition dynamics, I have to take a stance on the initial conditions.

Initial Conditions. I first solve the transition dynamics in autarky in the West by introducing an expansion in relative skilled labor endowments, from 0.1 to 0.15, which delivers elevated growth, falling inequality, and a rising research share along the transition path, all of which are key features of advanced economies' growth trajectory since WW2.[49] The model is thus consistent with the observation in Fernald and Jones (2014) that temporary forces pushed up the growth rate over the 20th century.

I then introduce a globalization shock by adding another emerging economy to the world economy in the mid 90s. I thus have to take a stance on initial conditions in the East, which again must differ from their long-run steady state since these economies grow at an elevated rate. I first ensure that the model is consistent with the initial income gap across East and West in the mid 90s. Having close to zero research productivity in the East in autarky, however, would imply a counterfactually large wage gap. I fix this by assuming that the East's innovation in autarky is easier as it copies technology already invented in the West.[50] Moreover, since skill is relatively scarce in the East, ceteris paribus, the skill premium should be relatively high. In the data, this is not true and I calibrate the model such that the skill premium in East and West are the same in autarky. This can be done by imposing a simple technology adoption wedge similar to Parente and Prescott (1994). Specifically, suppose that there is a market-share reallocation friction μ^*_{closed} in autarky such that the impact of technology adoption on

[48] For more recent years, this assumption may be less appropriate, especially with respect to other emerging markets such like China. I explore the implications of this shift in the symmetric integration scenario later on.

[49] See appendix B.1.6 for more details.

[50] The problem here is that the emerging market block I consider is small with a relative size of .39, and the initial income gap, while large, is not nearly large enough. The model could be applied without this ad-hoc assumption to China, which is a much larger economy starting out from a lower income level. Because the allocation of skilled labor across sectors is unrelated to research productivity in autarky, I simply solve for the allocation in the emerging market using the standard solution routine, and then scale wages by some constant factor, which is meant to capture that copying is easier than innovating. Note that no matter how easy copying is, as long there is some cost, there is no incentive to do so in the integrated equilibrium since Bertrand competition would drive profits to zero.

profits is suppressed

$$\partial_z \pi^*_{\text{autarky}} = \underbrace{\mu^*_{closed}}_{<1} (\sigma - 1)(1 - \alpha) \frac{\pi^*_{\text{autarky}}}{z^*},$$

where $\partial_z \pi = (\sigma - 1)(1 - \alpha)\pi/z$ is the frictionless benchmark always operating in the West, see Hsieh and Klenow (2014) for evidence in favor of such reallocation frictions. In a model with endogenous technology adoption, such frictions reduce the incentive to adopt technology. This suppresses the demand for skilled labor twofold. Fist, directly through lower demand for skilled labor in the production sector, and then indirectly through the equilibrium effect of weak technology adoption on the present discounted value of innovation. Consequently, the skill premium can be arbitrarily depressed.

While this is not the focus of the paper, a skill intensive technology adoption margin provides a promising candidate explanation for low productivity and the weak link between skill scarcity and skill premium along the development spectrum, see Caselli and Coleman (2006). Regarding the Soviet Union, the lack of incentive to improve productivity due to central planning has been widely acknowledge, see Kornai (1992). Consistent with this are the low low levels of inequality within and across worker groups, see Brainerd (1998), despite skill being more scarce than in West. I loosely follow this evidence when calibrating the technology adoption wedge in the East so as to match the skill premium in autarky of 1.5. Once markets integrate, this wedge disappears, i.e., $\mu^*_{open} = 1$, reflecting reforms in emerging markets.

Lastly, in a world where ideas are harder to find, it is natural to assume that countries develop the same technology in autarky. Although integration eliminates the incentive to duplicate innovation going forward, some overlap in varieties may exist at the moment of integration. It is therefore necessary to specify the initial allocation of patent rights across countries. I assume that the West holds a share ζ of the existing technology stock and the East the remaining $1 - \zeta$. Appendix B.3 provides a micro-foundation based on small quality differences across varieties. To discipline this initial reallocation of ownership, I match the sharp decline in income observed in Eastern Europe following the fall of the Iron Curtain. This calibration implies $\zeta = .99$.[51] Table 3 summarizes parameters and key moments used for the calibration.

Long-Run Effects. I first report the long-run effects of market integration before I turn attention to transition dynamics. Table 7 reports the results in the form of the cumulative effect, i.e., long-run level effect relative to a balanced growth path in autarky.

In the case of asymmetric integration where all innovation is produced in the West, wages of unskilled workers in advanced economies fall by about 10% in real terms. This contrasts with wage gains for skilled labor of around 12%. Even though the technological frontier increases by 7%, the

[51] Note that the instantaneous change in GDP in the East at time zero equals approximately $\frac{\Delta Y^*}{Y^*} = -\frac{\sigma-1}{\sigma}\alpha(1-\alpha)$ for ζ close to one, which is the share of output paid to the owners of technology in autarky. For the baseline calibration this implies a drop of 14.4%.

Table 3. Parameterization

Parameter		Value	Target/Source
Household			
ρ	discount factor	0.05	standard value
g_L	pop. growth	0.02	standard value
h	rel. skill share	0.15	Barro/Lee (2013)
h^*	rel. skill share	0.05	see text
$\frac{L^*}{L}$	rel. size	0.75	see text
$\frac{w_L}{w_L^*}$	init. wage gap	4.5	see text
Production/Adoption			
α	capital share	0.4	standard value
δ_k	capital deprecication	0.05	standard value
δ_{X}	exit	0.05	IAB data
σ	substitution	2.5	Broda/Weinstein (2006)
β	static curvature	0.35	see text
θ	adv. backwardness	0.55	see text
f_{E}	entry cost	1.25	skill premium/waiting time
ν	adoption tech.	0.21	skill premium/waiting time
Innovation			
ϕ	dyn. externality	-1.0	see text
λ	congestion	0.9	see text
δ_{I}	exit innovation	0.06	IAB data
γ	research prod.	1	normalization
γ^*	research prod.	0.01	see text
ζ	initial idea share	0.99	initial output drop East

Note: The table provides the parameters used in the baseline calibration, which refers to the case of asymmetric integration.

adoption gap widens by 17% explaining weak wage growth for unskilled labor in advanced economies. I also provide a measure of changes in GDP per capita y.[52] The negative effect of integration is still there, but somewhat weaker than the wage effects, consistent with GDP growth performing better than wage growth data over the past couple of decades, see for instance Autor et al. (2020). The role of intellectual property accumulation and asset income in the open economy explains this difference.

These negative findings contrast with the growth experience in the East: all workers gain massively albeit skilled labor gains relatively more. The gains are entirely accounted for by the adoption

[52] Measuring GDP poses conceptual challenges. An issue arises as to where the value of ideas appears when computing GDP per capita, which itself might depend on firm's profit shifting. I propose the following measure, which corresponds to household consumption per capita. Note that with log utility, households will consume a fraction $\tilde{\rho}$ of their assets. I thus approximate real GDP per capita as $y := \frac{w_L L + w_H H + \tilde{\rho} B}{L+H}$ where B is the value of total household assets. This total value of assets does not change much because an increase in intellectual property roughly cancels with the relative fall in assets held in physical capital and production sector firms. However, because capital income is a substantial share of household income, including this term weakens the overall negative effect.

Figure 7. Long-Run Level Effects

Asymmetric Integration			Symmetric Integration		
	West	East		West	East
χ	1.0	0.0	χ	0.571	0.429
$\Delta \log w_L$	-0.091	1.174	$\Delta \log w_L$	0.28	1.666
$\Delta \log w_H$	0.119	1.576	$\Delta \log w_H$	0.28	1.702
$\Delta \log s$	0.21	0.402	$\Delta \log s$	0.0	0.036
$\Delta \log (y)$	-0.029	1.148	$\Delta \log (y)$	0.28	1.861
$\Delta \log z$	-0.163	1.101	$\Delta \log z$	-0.0	1.386
$\Delta \log A_{\mathrm{F}}^{\mathrm{W}}$	0.073	0.073	$\Delta \log A_{\mathrm{F}}^{\mathrm{W}}$	0.28	0.28

Note. The table summarizes the long-run effect of market integration on income and inequality. The left panel uses the baseline calibration, while the right panel assume that skilled labor endowment and research productivity in the East coincide with the West. Note that the log change in the value of patents is infinite in the case of no innovation in the East in the integrated equilibrium. The adoption gap is defined relative to the global technological frontier $\log z_n = A_n/A_{\mathrm{F}}^{\mathrm{W}}$. For things to add up, I thus defined frontier technological growth as $\Delta \log A_{\mathrm{F}}^{\mathrm{W}} = \log A_{\mathrm{F}}^{\mathrm{W}} - \log A_{\mathrm{F}}$ where A_{F} refers to the frontier in autarky, which resides in the West. Note that I net out exogenous long-run growth $(1-\phi)^{-1} \Delta \log L$ for non-stationary variables.

of frontier technology. Because adoption is a skill-intensive activity, there is pressure on the skill premium in emerging markets in spite of virtually zero research effort. Importantly, fast technology adoption in the East has a feedback effect on innovation in the West in the integrated equilibrium. This feedback effect induces increasing innovation at the cost of lower technology adoption, with overall negative long-run effects for the West as a whole.

To understand this long-run effects, note that in the baseline calibration the autarky equilibrium in the West is characterized by insufficient adoption. The share of skilled labor devoted to adoption vis-a-vis innovation in the decentralized autarky equilibrium is roughly 40%, whilst a planner would allocate 2/3rds of the skilled work force to technology adoption.[53] This inefficiency is amplified in the open economy explaining the underwhelming impact of globalization on aggregate growth. What is crucial for this outcome is that i) there are learning externalities in adoption, and ii) dynamic knowledge externalities are sufficiently weak. Intuitively, if ideas are harder to find, shifting skilled labor into innovation has only modest effects on frontier growth, which are dominated by the negative impact of reduced technology adoption. An implication of this is that subsidizing innovation in this model would be counterproductive for unskilled workers in advanced economies: the skill premium would widen further, and more labor skilled labor would be diverted away from domestic technology adoption.[54]

[53] An important assumption underlying this welfare analysis hinges on the planner taking a national perspective. A planner that cares about world output instead faces a different trade-off. Intuitively, pushing out the frontier helps both the domestic and foreign economies, so more labor is devoted to producing frontier technology. Any welfare statement thus crucially depends on whether the scope of the analysis is global or national.

[54] I find that at for $\phi \approx 0$, all workers gain, and skilled workers gain a lot resurrecting the strong pro-growth effect of integration. Again, the predictions of the theory are directly tied to how much harder ideas are to find. Could it be, then, that all that is needed to reconcile the effects of the recent globalization wave on growth is a model where ideas are harder to find? Not quite. Using Jones (1995)'s model the impact of integration on long-run wages are a log-linear function of the increase in the size of

This bleak scenario contrasts with the case of symmetric integration where the gains from market integration are positive for all worker groups across all countries. Specifically, wages in advanced economies go up by 30%. This is, of course, just an instance of proposition 4: symmetric integration leaves the allocation of skill across sectors unchanged but delivers desirable scale effects as more skilled workers are engaged in idea production. The pro-growth effects in the case of symmetric integration without any adverse inequality effects illustrate the more complex link between growth and globalization. The model is consistent with the beneficial role of increasing integration across advanced economies in driving strong post-WW2 growth, while simultaneously allowing for the possibility of uneven and sluggish growth in the aftermath of market integration among asymmetric countries. The analysis so far has been restricted to long-run effects and I turn to transition dynamics next.

Transition dynamics. To simplify the transition dynamics, I make two additional assumptions. First, suppose households reinvest a constant fraction χ_{sav} of the final output good in each country where $\chi_{\text{sav}} = \left(\frac{\sigma-1}{\sigma}\alpha\right) \cdot \alpha \frac{g_L+g_F+\delta_k}{\rho+g_F+\delta_k}$ is set such that the saving rate is consistent with the long-run supply of savings from the household sector.[55] The second simplification is that I keep the normalized measure of firms in the production sector fixed at its long-run level m, which is the same in closed and open economy and across rich and poor countries.[56]

Figure 8 plots the transition dynamics of wages in the case of asymmetric integration on the left panel, and the skill premium and value of innovation on the right panel. The model generates strong convergence dynamics in the sense that the income gap between East and West is cut in half over the period from 1995 to 2020.

The right panel shows the dynamics of the skill premium and the normalized value of innovation. Both skill premium and the value of innovation jump up upon integration. This reflects the immediate impact of the demand for technology from the East. Investment in innovation happens immediately, which drives up the skill premium on impact. Furthermore, elevated technology adoption in the East raises the value of innovation both because foreign wages increase ($w \propto z$) and because adoption times ($\tau = -\log z/(\tilde{g}_A + \delta_I)$) fall. Unsurprisingly, since the free entry condition in the research sector ties the value of innovation to the skill premium, the increase in the value of innovation closely tracks the increase in the skill premium.

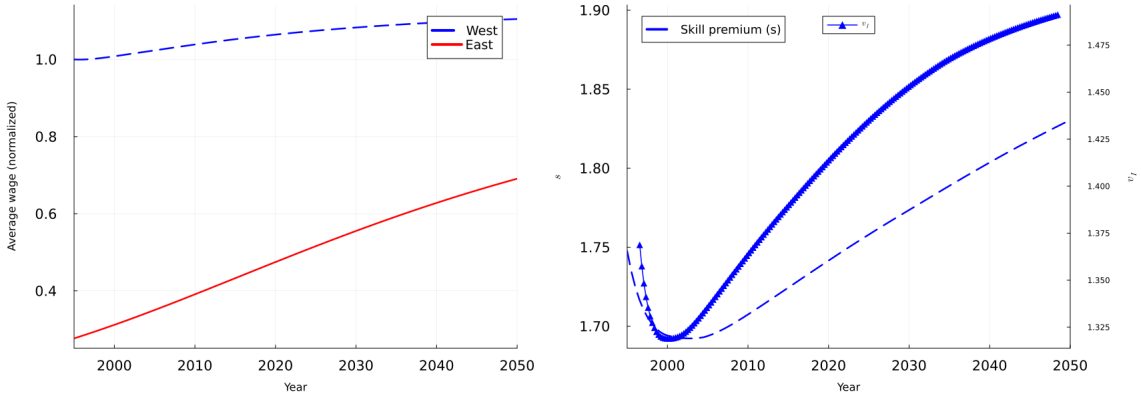
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the population. In the integrated equilibrium with only the advanced economy specializing in innovation, market integration delivers an increase in productivity and wages by a factor of $\left(1+\frac{L^*}{L}\right)^{\frac{\lambda}{1-\phi}}$, which is strictly positive. The assumption that ideas are harder to find is necessary but not sufficient, and needs to be combined with a model where technology adoption complements frontier innovation.

[55] In principal, one can solve a model with forward-looking household consumption, which would introduce consumption smoothing and inter-temporal trade. I prefer to simplify, which gives rise to a subtle issue: foreigners now may hold domestic physical capital.

[56] The response of endogenous entry in the production sector is both computationally taxing and matters little quantitatively. I plot the value of firms in the production sector, v, in the appendix, which does not differ by more than 3% compared to its long-run value suggesting the channel is unimportant.

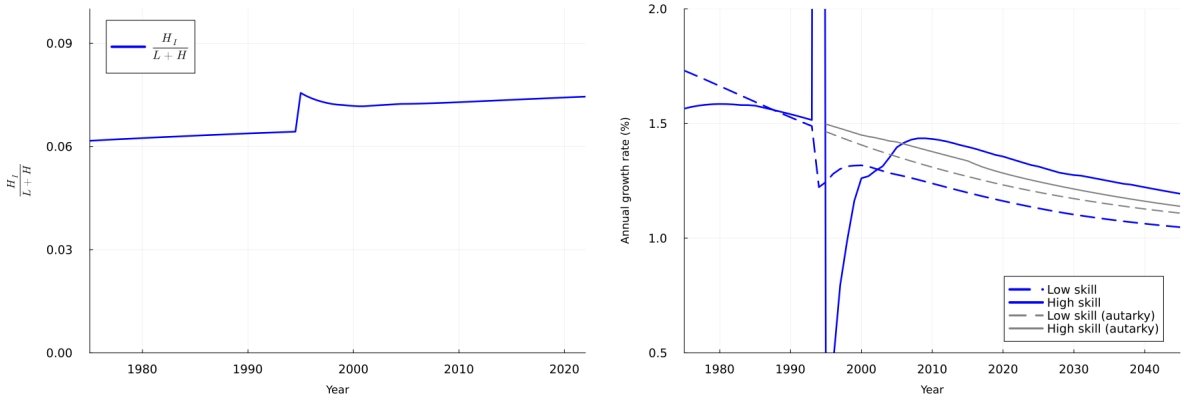
Figure 8. Wages & Technology along the Transition Path



Note. The average wage is defined as $\tilde{w}_t = \frac{w_{L,t}(1+h_t s_t)}{1+h_t}$, and all wages are normalized by the initial wage in the West in the integrated equilibrium.

A key statistic in the model is the relative share of labor devoted to research, defined as $\frac{H_F}{H+L}$, which is plotted in figure 9. I plot the closed economy dynamics for 20 years running up the the globalization shock introduced in 1995. The share of labor devoted to innovation increases gently in that pre-globalization period, see figure 4, driven by a falling adoption gap which ultimately originated from a skilled labor expansion, see appendix B.1.6 for more details. Upon integration, innovation jumps up by about 1.5 p.p., consistent with the discrete increase in market size and the forward-looking nature of R&D. The model predicts an increase of research employment by roughly one percentage point. The magnitude is in the right ballpark but understates somewhat the large increase in research employment in Germany plotted in figure 4. A richer model with endogenous skill-accumulation would certainly be able to generate larger adjustments. Similarly, the right kind of sectoral adjustment costs would generate a monotone increase more similar to the data.

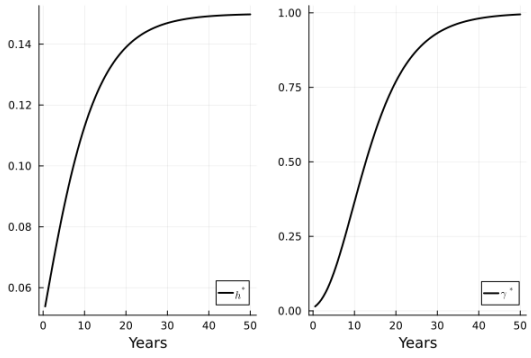
Figure 9. Innovation and Wage Growth (West)



The overall weak wage growth hides a great deal of heterogeneity across worker types with a growth drag for unskilled workers as indicated by the gap between the dashed grey and dashed blue line in figure 9. Because the skill premium jumps, skilled wage growth is not smooth, but the overall message is clear: skilled wages grow substantially faster than unskilled ones after an asymmetric integration shock.[57] Overall, the model succeeds in generating uneven wage growth patterns across groups, and the negative drag on skilled worker wages eventually dominates the aggregate long-run effect. Figure 25 in the appendix shows the evolution of the skill premium and the Gini Index in Germany, both of which shoot up in the mid 1990s, consistent with the model and the impact of market integration on the returns to innovation. This pattern of robust innovative activity, weak productivity growth, and stagnation for unskilled worker wages holds broadly across advanced economies and poses a puzzle for benchmark models of endogenous growth. A model that acknowledges the importance of technology adoption, and considers the impact on globalization on the returns to "local adoption" vis-a-vis "global innovation" naturally accounts for the decoupling of innovation and unskilled worker wage growth. Similarly, fast growth in emerging markets, and rising profits and stock market valuations of multinational companies in advanced economies are perfectly consistent with the increasing value of innovation in a globalization world.

Symmetric Integration. This bleak scenario contrasts with the benevolent implication of symmetric market integration as I have argued before. I next plot the transition dynamics for the case when emerging markets converge in terms of fundamental research productivity and skill endowment to the level of advanced economies. I assume convergence in fundamentals takes 30 years, see figure 10.

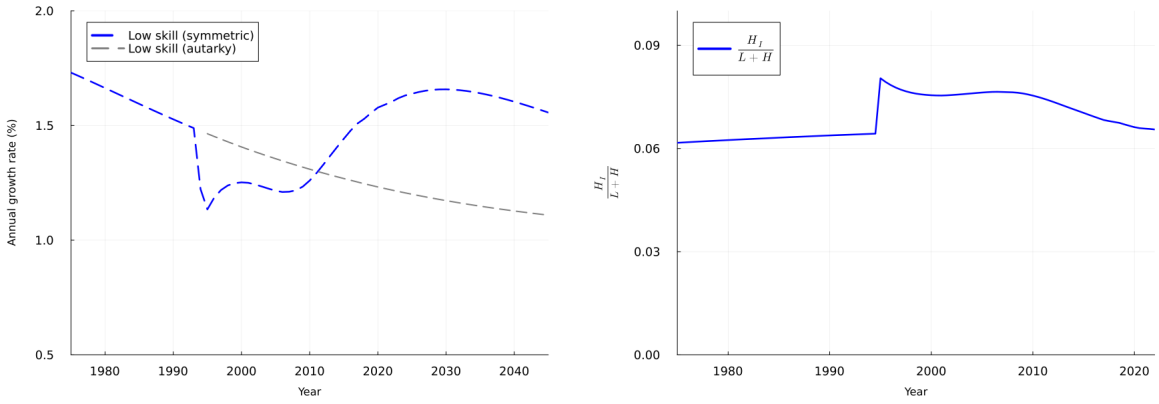
Figure 10. Full Convergence in East



[57] Interestingly, even in the absence of globalization, skilled wages would have grown somewhat faster (gap between solid lines) since the falling adoption gap raises the returns to innovation. The pull of adoption on innovation becomes the dominant dynamic force after a skilled labor supply shock because adoption gaps are slow-moving relative to other state variables, and hence determine long-run dynamics. This point is more carefully explored in the appendix.

Surprisingly, symmetric and asymmetric integration do not look very different initially, and if anything, the impact on unskilled wages initial is more negative. The reason is that it takes time for the emerging market to innovate, and the dynamics early on are dominated by even stronger technology adoption in the East, which is sped up by skilled labor expansion. The initial dynamics are thus an amplified version of the asymmetric case, and technology adoption and unskilled wages in advanced economies stagnate. Consistent with this is the even larger initial increase in research labor plotted on the right panel. However, continued convergence in emerging markets means that they eventually contribute to the technological frontier. At that point, the negative impact is fully reversed and elevated productivity growth and falling inequality coincide with fast aggregate GDP growth.[58]

Figure 11. Wage Growth Unskilled West - Symmetric Integration



5 Discussion & Conclusion

The model is consistent with a number of empirical facts that have so far only been considered in isolation: rising innovative effort and an increasing skill premium against the backdrop of sluggish productivity growth and overall wage stagnation in advanced economies. The key mechanism that accounts for the puzzling evolution of these secular trends is that innovation in a globalized world potentially comes at the cost of weakened local technology adoption. While a clean causal identification strategy for this theory of global growth is beyond the scope of this paper, there are a number of distinct predictions of the model that are borne out in the data, which buttress the model's plausibility.

Macro Evidence. The model makes distinct prediction about the growth trajectory of advanced economies and emerging markets. Paradoxically, elevated research intensity coexist with weak aggregate growth

[58] I note that even skilled workers in the advanced economy are better off in this scenario as the fall in the skill premium is dominated by overall technological growth.

in advanced economies, while emerging markets grow fast due to technology adoption. Technology is embodied in capital goods in my framework so both adoption and tangible investment should be relatively high in emerging markets.

I next compile data from KLEMS to measure growth and investment among Western and Eastern European economies after the fall of the Iron Curtain, where I use measures that explicitly account for intangible investment, which contains R&D investment and other types of intangible investment that arguably represent a small share of overall technology adoption effort.

I use the EUKLEMS–INTANProd analytical module (Bontadini et al., 2023). Total investment equals tangible plus intangible investment is expressed relative to GDP. Tangible investment comprises machinery, equipment, ICT hardware, and structures. Intangible investment includes research and development (R&D), software and databases, organizational capital, firm-specific training, brand, design, other intellectual property products, and new financial products.

I interpret R&D as frontier innovation. Adoption investment refers to complementary intangible assets that facilitate the diffusion and implementation of existing technologies, measured primarily by organizational capital and firm-specific training. This is an extremely narrow definition that leaves out most of what constitutes technology adoption efforts concerning time spend by ordinary employees within the firm as part of their usual job.[59]

Table 12 shows three facts consistent with the theory. First, growth is slow in advanced economies and fast in emerging markets. Second, research effort is high in advanced economies relative to emerging markets.[60] Third, adoption spending appears relatively similar. Perhaps surprisingly, this last fact is consistent with the long-run predictions of the model where firms spend a constant share of their profits on adoption. A rising skill premium of course means that adoption effort is falling in real terms. Moreover, note that tangible investment rate are relatively high in Eastern Europe, and the gap would be much larger if one were to use net investment rates, which is consistent with elevated technology adoption since technology is embodied in capital goods.

In figure 13 I relate the share of adoption and innovation within intangible investment to overall growth from 1996 to 2023. This helps isolate the role of each because weak adoption simultaneously implies weak tangible investment rates. Consistent with the previous findings, the first panel shows that countries that can focus on technology adoption in the open economy experience a growth spurt. The second panel produces the mirror image by focusing on innovation as a share of intangible, which is unimportant for emerging markets and consistent with the previous literature (Barro

[59] In a broader definition, I also include software, which increase the adoption share of advanced economies somewhat but does not overturn the main result that R&D is relatively high in advanced economies, and tangible investment together with adoption dominates in emerging markets. Note that software represents a relatively recent general-purpose technology that is disproportionately accumulated in advanced economies consistent with my model. Including software in the adoption measure may therefore mechanically raise measured "adoption" in Western Europe, since older technology waves do not have separate intangible categories in the data.

[60] It is also true that research effort in advanced economies is high relative to the previous decades, see Bloom et al. (2020).

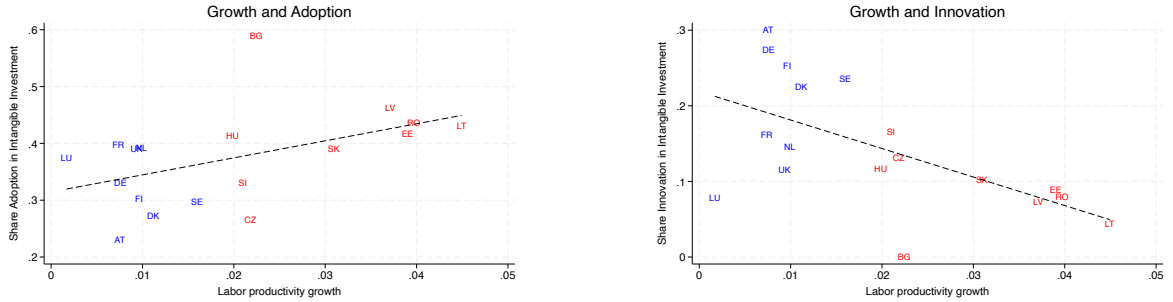
Figure 12. Investment and Growth Across Europe

	West	East	definition
g_y	0.89	3.01	output per worker growth (%)
I_{I}/Y	2.23	0.83	r&d gross inv. rel. to output (%)
I_{A}/Y	3.79	3.41	adoption gross inv. rel. to output (%)
I_{T}/Y	18.10	21.96	tangible gross inv. rel. to output (%)

Note. I use the EUKLEMS–INTANProd analytical module (Bontadini et al., 2023) from 1996 – 2023 and focus on total market industries (TOT_IND). Western Europe includes Austria, Belgium, Denmark, Finland, France, Germany, Luxembourg, the Netherlands, Sweden, and the United Kingdom; Eastern Europe includes Bulgaria, Croatia, the Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland (with limited intangible coverage), Romania, Slovakia, and Slovenia. Total investment equals tangible plus intangible investment (I_Tang + I_Intang) and is expressed relative to adjusted value added (VAadj). Tangible investment comprises machinery, equipment, ICT hardware, and structures. Intangible investment includes research and development (R&D), software and databases, organizational capital, firm-specific training, brand, design, other intellectual property products, and new financial products. Real output per worker is computed as real adjusted value added (VAadj_q) divided by employment (EMP), and growth is measured as log differences. Regional statistics are simple (unweighted) country averages.

and Martin, 1997; Acemoglu, Aghion, and Zilibotti, 2006). Within advanced economies, research effort appears positively related with growth. The model is consistent with this fact, too. Relatively skill-intensive innovative regions within a group of integrated economies do better as the demand for skill increases in a globalized world. The losers amongst advanced economies are the ones more specialized in production, think for instance Italy. This logic can be taken to the regional level within-country level whereby innovative regions in rich countries grow away from the production-focused rest of the economy, which I explore next.

Figure 13. Adoption, Innovation, and Growth in Europe

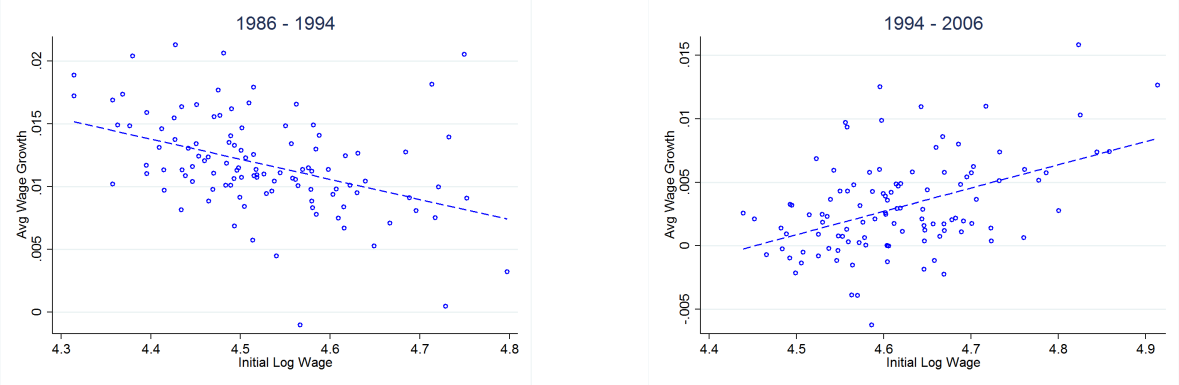


I compute simple averages from 1996 to 2023 using data from Bontadini et al. (2023), see table 12 for details. The share variable on the Y-axis is the share of adoption or research in total intangible investment.

Micro Evidence. The model has distinct implications for regional growth patterns within advanced economies based on the logic that specialization in innovation vis-a-vis production is extremely uneven across space: if high-income regions have a persistent advantage in innovation, then market

integration ought to further raise growth in innovative regions within advanced economies. This effect would be accompanied by a reallocation of skilled labor away from relatively poorer regions and coincide with weak growth in production-focused regions as technology stalls. Figure 14 plots average wage growth, defined as the total wage bill of full-time employees over total full-time employment, against the log of the initial average real wage for a local labor market in West Germany.[61] Wage growth in the early period from 1986 – 1994 was, on average, higher for laggard regions. These growth patterns are turned upside down in the 2000s, where high-income places grew relatively fast while laggard regions stagnated. Since high income regions are more specialized in innovation, and the changing growth patterns are naturally explained by the rising returns to innovation in the aftermath of global market integration.

Figure 14. Regional Convergence in Germany



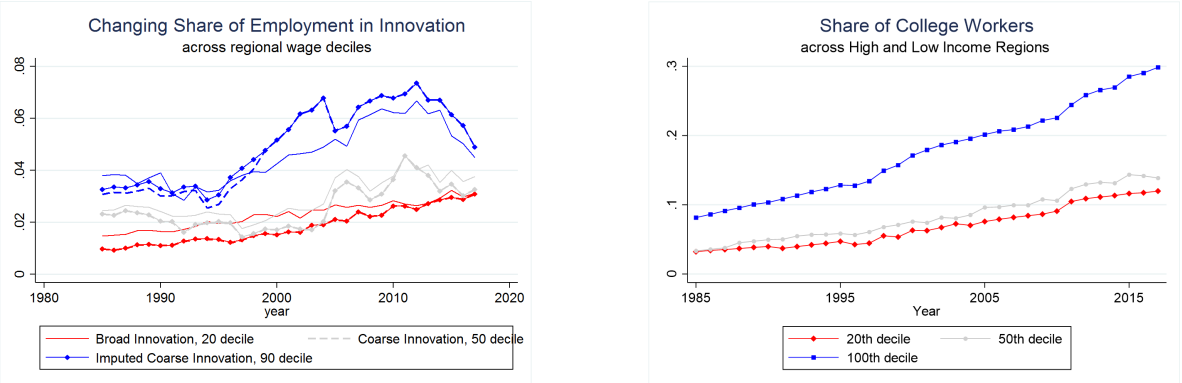
Using data from the BHP establishment sample, the figure plots average wage growth against initial the initial average wage in real terms. The plot shows how growth pre 1994 was biased towards lagging regions, while from 1994 onwards growth was biased towards high income regions. I stop short of the financial crisis, but have looked at convergence patterns from 2006 - 2015 as well which are mostly neutral with a regression coefficient statistically indistinguishable from zero at standard levels of significance. See the appendix for plots for high, middle, and low skilled wages separately. A common concern is that international trade, and in particular import exposure following Autor, Dorn, and Hanson (2013), fully explains weak growth in laggard regions. To consider the effect of import exposure on wage growth, I run a convergence regression with an additional import exposure variable as control. Import competition accounts for virtually none of the stagnation in laggard regions.

I next document the higher specialization of high income regions in innovative activity in figure 15. Moreover, there appears to be further divergence unfolding in the 2000s in terms of innovative activity across high and low income regions, precisely when the mode predicts that the returns to innovation take off due to adoption in emerging markets. The right panel in figure 15 then highlights that increase in the share of college workers in innovative high-income regions outpaced the rest of the economy. The theory suggests that this expansion of skilled labor in innovative regions comes at the cost of technology adoption. Consistent with this interpretation is that growth in the production-

[61] Clearly, German integration poses econometric challenges. However, note that internal integration, at least through the lens of the model, does not generate a bias towards innovation as skilled labor would earn a high return in Eastern Germany to help adopt technology. This appears consistent with the timing of both the rise in inequality and the productivity slowdown that takes hold in the mid 90s and early 2000s.

focused hinterlands disappears entirely.[62] These changing regional convergence patterns are more broadly true across advanced economies including the USA (Berry and Glaeser, 2005; Ganong and Shoag, 2017; Giannone, 2017; Rubinton, 2020), and should be uncontroversial. The general equilibrium structure of the model makes clear that the acceleration in skill growth in innovative centers comes at the cost of production-focused regions, which could appropriately be described as brain drain.

Figure 15. Share of College Workers across Regions



These plots compute skill share and employment in innovation across high and low income regions in Germany by grouping regions into wage deciles and computing simple averages. The plots are purely cross-sectional in the sense that I assign labor markets into bins each year so that for example the set of places in the top bin can change every year. In practice, whether one fixed the income ranking in 1994 instead does not change the broad patterns. There is substantial sampling variation within each region, however, and the cross sectional plots is smoother, which is why I prefer it.

A final piece of empirical evidence that lines up with the spirit of the model is Andrews, Criscuolo, and Gal (2015) and Andrews, Criscuolo, and Gal (2016), which documents that the growth slowdown appears to be driven by weak growth of laggard companies. If one relabels firms in the research sector as "leaders" and firms in the production sector as laggards, then the predictions of the model line up exactly with their findings. Innovative leaders can market their innovations globally, which drains domestically-focused followers from skilled labor, leading to overall weak productivity growth and stagnant wages.

Alternative Hypotheses. Skill-biased technological change is perhaps the most influential explanation for the rise in the skill premium (Katz and Murphy, 1992; Bound and Johnson, 1992). There is little doubt that recent technological change has been skill-biased. However, skill bias alone does not explain sluggish aggregate growth or the stagnation of unskilled wages (Acemoglu and Autor, 2011; Autor, 2019). The same applies to models of automation. While these frameworks can generate adverse distributional effects for particular worker groups (Autor, Levy, and Murnane, 2003; Acemoglu and Restrepo, 2018), they tend to raise productivity growth (see also Jones and Tonetti (2026)). In

[62] A common concern is that international trade, and in particular import exposure following Autor, Dorn, and Hanson (2013), fully explains weak growth in laggard regions. To consider the effect of import exposure on wage growth, I run a convergence regression with an additional import exposure variable as control. Import competition accounts for virtually none of the stagnation in laggard regions.

contrast, a model with endogenous, skill-intensive technology adoption links a rising skill premium directly to weak productivity growth.

Turning to the growth slowdown, semi-endogenous growth models imply a positive link between population growth and productivity growth. A decline in population growth therefore provides a natural explanation for weaker aggregate performance. Yet, through the lens of models in which ideas become harder to find (Jones, 1995), slower population growth should be accompanied by a declining share of labor devoted to innovation. Empirically, however, innovative effort has continued to rise. The fact that ever more resources are devoted to an activity that appears to be getting harder is puzzling. Taking globalization seriously helps resolve this tension: technology adoption in emerging markets sustains innovation incentives in advanced economies, even as domestic growth remains subdued.

Conclusion. In my calibration, weak domestic technology adoption can entirely offset the gains from additional innovation following market integration between advanced economies and emerging markets. The mechanism generates sizable real wage losses for unskilled workers in rich countries and reconciles weak aggregate growth with rising innovative effort. All of which is an intuitive consequence of an asymmetric globalization shock – a defining feature of globalization after the fall of the Iron Curtain.

I show theoretically that innovation and adoption are complementary since innovators rely on successful technology adoption in the production sector to be profitable. Asymmetric globalization and foreign technology adoption can break this complementarity in the domestic economy in the sense that the research sector expands while domestic technology adoption and unskilled wage growth falls. Growth is uneven and sluggish, not because the technological frontier stops advancing, but because the link between frontier growth and domestic adoption is weakened.

At the same time, the framework preserves the central role of globalization in sustaining long-run growth through the non-rivalry of ideas, and the ensuing scale effects. First, even in the case of asymmetric integration, the technological frontier expands and there are massive gains in emerging markets. Second, when integration become more symmetric, i.e., when emerging markets converge fully and contribute to the technological frontier, then the the pro-growth effects of market integration –even in advanced economies– are resurrected.

Concerns about the adverse effects of the ability of emerging markets to compete in the market for ideas may thus be misplaced. Innovation in emerging markets not only pushes out the global technological frontier, but also induces a reallocation of skilled labor toward adoption activities within advanced economies. This reallocation strengthens technology adoption, reduces inequality, and lifts up aggregate growth.

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A Theory Appendix

A.1 Production Firm

A.1.1 Static minimization problem of firm in production sector

Without loss of generality, one can think of the capital goods $x_{j,t}$ as contained in the interval $[0, A_t]$ where $\int_0^{A_t} dj = A_t$. Given total expenditure on capital goods $\int_0^{A_t} p_{x,t} x_{j,t} dj = p_{x,t} \tilde{x}_{j,t}$ where $\int x_{j,t} dj = \tilde{x}_t$, and I used the fact that prices across capital goods are identical. This is consistent with the model of horizontal innovation introduced later. I focus on the problem of an individual firm $i \in [0, M_t]$ but omit firm subscript i since firms are homogeneous. The firm solves

$$\begin{array}{ll} \max & \int_0^{A_t} \left(\frac{x_{j,t}}{\alpha}\right)^{\alpha} dj \\ s.t. & \int p_{j,t} x_{j,t} dj \leq I. \end{array} \tag{42}$$

This well-known problem (Dixit and Stiglitz, 1977) leads to the following first order condition

$$\frac{x_{j,t}}{x_{l,t}} = \left(\frac{p_{X,t}}{p_{X,t}}\right)^{-\frac{1}{1-\alpha}} = 1 \ ,$$

and since the capital goods are homogeneous it follows that $x_j = x_l \, \forall j, l$. As a consequence, the total quantity of each individual capital good variety must read $p_{x,t} x_{j,t} = \frac{p_{x,t} \tilde{x}_t}{A_t}$ where the last equality holds because of the symmetry assumption.

Now I can substitute this into the firm production function and find the minimal cost of producing one unity of output, given factor prices. This leads to the following cost minimization problem

$$\begin{array}{ll} \min & w_{L,t} l_t + p_{x,t} \tilde{x}_t \\ s.t. & \left(\int_0^{A_t} \left(\frac{\tilde{x}_t}{\alpha} \frac{1}{A_t}\right)^{\alpha} dj\right) \left(\frac{l_t}{1-\alpha}\right)^{1-\alpha} \geq 1 \end{array}$$

The problem further simplifies to

$$\begin{array}{ll} \min & w_{L,t} l_t + p_{x,t} \tilde{x}_t \\ s.t. & \left(\frac{\tilde{x}_t}{\alpha}\right)^{\alpha} \left(\frac{A_t l_t}{1-\alpha}\right)^{1-\alpha} \geq 1. \end{array}$$

which has the convenient Cobb-Douglas structure with labor-augmenting technological change. The first order conditions lead to the constant ratio of expenditure shares on labor and capital

$$\frac{p_{x_t} \tilde{x}_t}{w_{L,t} l_t} = \frac{\alpha}{1-\alpha}.$$

Together with the binding constraint, $\left(\frac{\tilde{x}_t}{\alpha}\right)^{\alpha} \left(\frac{A_t l_t}{1-\alpha}\right)^{1-\alpha} = 1$, the cost-minimizing bundle of labor and

capital leads to a marginal (and average) unit cost of

$$mc_t = (p_{x,t})^{\alpha} \left(\frac{w_{L,t}}{A_t}\right)^{1-\alpha}.$$

Average and marginal cost coincide since the production function features constant returns in capital and labor, conditional on A_t.

This constant-marginal cost results is important as it simplifies the firm's price setting problem, taking aggregate variables as given. Formally, the problem reads

$$\max_p \quad Y_t p^{-\sigma} [p - mc_t]$$

which leads to the well-known constant markup over marginal cost,

$$p_t = \frac{\sigma}{\sigma - 1} mc.$$

This constitutes a solution to the static firm problem. Since profits are strictly decreasing in marginal cost, it is indeed optimal to achieve lowest cost and then charge a constant markup over marginal cost.

A.1.2 Dynamic Firm Problem and Adoption Gap

The firms' adoption problem is summarized in the following HJB equation

$$\begin{aligned} V_t (r_t + \delta_{\mathrm{X}}) &= \max_{h_{i,t}} \Pi_t (A_{i,t}) - w_{H,t} \iota_{i,t} + \dot{A}_{i,t} \partial_A V_t + \dot{V}_t \\ &\text{s.t.} \\ \dot{A}_{i,t} &= \nu A_{\mathrm{F},t}^{1-\theta} A_{i,t}^{\theta} h_{i,t}^{\beta} - \delta_{\mathrm{I}} A_{i,t}, \end{aligned} \tag{43}$$

where δ_{X} is an exogenous firm death shock and $\partial_A V := \frac{\partial V}{\partial A}$ denotes the partial derivative. For clarity, I keep the firm index i to show over which productivity the firm has control over. I now drop this subscript. I also focus on the case with idea death, $\delta_{\mathrm{I}} > 0$. The reader can set this parameter to zero to obtain all the results of the simple model in the first part of the paper.

To solve the production firm's adoption problem, it is useful to rewrite the problem using a normalized value function $v_t = \frac{V_t}{w_{L,t}}$, as well as normalizing the state variable A_t by $A_{\mathrm{F},t}$, i.e. the state becomes z_t. With these assumptions, I obtain a system that is stationary in the steady state. In the log utility case with $r = \rho + g_{\mathrm{F}}$ on the BGP, this leads to the following recursive formulation of the firm adoption problem,

$$\begin{aligned} v_t (\rho + \delta_{\mathrm{X}}) &= \max_{\iota} \frac{\Pi_t(z)}{w_{L,t}} - s_t \iota_t + \dot{z}_t \partial_z v_t + \dot{v}_t \\ &s.t. \\ \dot{z}_t &= \nu z_t^{\theta} \iota_t^{\beta} - (g_{\mathrm{F},t} + \delta_{\mathrm{I}}) z_t. \end{aligned} \tag{44}$$

A solution to the program (44) needs to satisfy the following first order condition

$$\left\{\frac{\beta\nu z_t^{\theta}\partial_z v_t}{s_t}\right\}^{\frac{1}{1-\beta}} = \iota \ . \tag{45}$$

Equation (45) captures the trade-off of the effect on firm value of a marginal increase in ι relative to its cost s_t. In anticipation of the solution, I derive the derivative of ι with respect to z and t,

$$\begin{aligned}\frac{\partial_{zz} v_t}{\partial_z v_t} + \frac{\theta}{z_t} &= (1-\beta)\frac{\partial_z h_t}{h_t} \\ \frac{\partial_z \dot{v}_t}{\partial_z v_t} - \frac{\dot{s}_t}{s_t} &= (1-\beta)\frac{\dot{\iota}_t}{\iota_t}.\end{aligned}$$

Next, derive the envelope condition of the HJB equation to get

$$\begin{aligned}(\partial_z v_t)\left(r_t - g_{w_{L,t}} + \delta_{\mathrm{X}}\right) &= \frac{\partial_z \Pi_t}{w_{L,t}} + \dot{z}_t\left(\partial_{zz} v_t\right) + \left(\theta z_t^{\theta-1}\nu\iota_t^{\beta} - \left(g_{\mathrm{F},t} + \delta_{\mathrm{I}}\right)\right)\left(\partial_z v_t\right) + \partial_z \dot{v}_t \\ (\partial_z v_t)\left(r_t - g_{w_{L,t}} + \delta_{\mathrm{X}} + (1-\theta)\left(g_{\mathrm{F},t} + \delta_{\mathrm{I}}\right)\right) &= \frac{\partial_z \Pi_t}{w_{L,t}} + \dot{z}_t\left(\partial_{zz} v_t\right) + \left(\partial_z v_t\right)\frac{\theta}{z_t}\left(z_t^{\theta}\nu\iota_t^{\beta} - z_t\left(g_{\mathrm{F},t} + \delta_{\mathrm{I}}\right)\right) + \left(\partial_z v_t\right)\left\{(1-\beta)\frac{\dot{\iota}_t}{\iota_t}\right. \\ (\partial_z v_t)\left(r_t - g_{w_{L,t}} + \delta_{\mathrm{X}} + (1-\theta)\left(g_{\mathrm{F},t} + \delta_{\mathrm{I}}\right)\right) &= \frac{\partial_z \Pi_t}{w_{L,t}} + \left(\partial_z v_t\right)\left(\frac{\partial_{zz} v_t}{\partial_z v_t} + \frac{\theta}{z_t}\right)\dot{z}_t + \left(\partial_z v_t\right)\left\{(1-\beta)\frac{\dot{\iota}_t}{\iota_t} + \frac{\dot{s}_t}{s_t}\right\} \\ r_t - g_{w_{L,t}} + \delta_{\mathrm{X}} + (1-\theta)\left(g_{\mathrm{F},t} + \delta_{\mathrm{I}}\right) &= \frac{\partial_z \Pi_t}{w_{L,t}}\frac{1}{\partial_z v_t} + (1-\beta)\frac{\dot{\iota}_t}{\iota_t} + \frac{\dot{s}_t}{s_t}.\end{aligned}$$

Now I can substitute in the first order condition and use the fact that I know the derivative of the profit function to get

$$\begin{aligned}\frac{\dot{\iota}_t}{\iota_t} &= \frac{1}{1-\beta}\left\{\left(r_t - g_{w_{L,t}} + \delta_{\mathrm{X}} + (1-\theta)\left(g_{\mathrm{F},t} + \delta_{\mathrm{I}}\right)\right) - \frac{1}{\partial_z v_t}\left[\frac{\Pi_t}{w_{L,t}}\frac{(\sigma-1)(1-\alpha)}{z_t}\right] - \frac{\dot{s}_t}{s_t}\right\} \\ \frac{\dot{\iota}_t}{\iota_t} &= \frac{1}{1-\beta}\left\{\left(r_t - g_{w_{L,t}} + \delta_{\mathrm{X}} + (1-\theta)\left(g_{\mathrm{F},t} + \delta_{\mathrm{I}}\right)\right) - \frac{\beta\nu z_t^{\theta}\iota_t^{\beta-1}}{s_t}\left[\frac{\Pi_t}{w_{L,t}}\frac{(\sigma-1)(1-\alpha)}{z_t}\right] - \frac{\dot{s}_t}{s_t}\right\}\end{aligned}$$

Moreover, recall that the law of motion of relative technology reads

$$\frac{\dot{z}_t}{z_t} = \nu z_t^{\theta-1}\iota_t^{\beta} - \left(g_{\mathrm{F},t} + \delta_{\mathrm{I}}\right).$$

In the steady state, we have two loci given by

$$\iota^{1-\beta} = \frac{1}{s}\frac{\beta(\sigma-1)(1-\alpha)(g_{\mathrm{F}}+\delta_{\mathrm{I}})}{\rho+\delta_{\mathrm{X}}+(1-\theta)(g_{\mathrm{F}}+\delta_{\mathrm{I}})}\left[\frac{\Pi_t}{w_{L,t}}\right]\frac{\nu z_t^{\theta-1}}{(g_{\mathrm{F}}+\delta_{\mathrm{I}})}\ , \tag{46}$$

$$z^{1-\theta} = \frac{\nu\iota^{\beta}}{g_{\mathrm{F}}+\delta_{\mathrm{I}}}. \tag{47}$$

If we combine these two equations one can see that a constant spending on learning activity follows

$$\iota s = \frac{\beta(\sigma-1)(1-\alpha)(g_{\mathrm{F}}+\delta_{\mathrm{I}})}{\rho+\delta_{\mathrm{X}}+(1-\theta)(g_{\mathrm{F}}+\delta_{\mathrm{I}})}\left[\frac{\Pi_t}{w_{L,t}}\right].$$

This leads to an inequality that needs to be satisfied for the equilibrium to be well-defined, namely

$$\beta\left(\sigma-1\right)\left(1-\alpha\right)\left(g_{\mathrm{F}}+\delta_{\mathrm{I}}\right) \quad < \quad \rho+\delta_{\mathrm{X}}+\left(1-\theta\right)\left(g_{\mathrm{F}}+\delta_{\mathrm{I}}\right).$$

The left hand side represents the additional benefit of improving your productivity, which combines the diminishing returns in learning (β) with the elasticity of the profit function ($(\sigma-1)(1-\alpha)$). The right hand side consist of effective costs in steady state, which is related to effective discounting as well as the advantage of backwardness. The firm needs to take into account that as it climbs up the technological ladder, the pull force introduced through the advantage of backwardness diminishes. This gives rise to an endogenous adoption gap as a function of the relative price of skill. Moreover, climbing up the ladder is costly when discounting is high since the benefits only accrue in the future.

Note that the size of the fixed cost of entering the production sector impacts technology adoption since ι is proportional to firm profits. However, h_{A}, i.e., aggregate normalized demand for skilled labor in the production sector is unrelated to f_{E}. One could break the link between z and f_{E} by assuming that the parameter ν is proportional to the fixed cost $\nu=\nu_0\cdot f_{\mathrm{E}}^{-\beta}$.

A.1.3 Q-Theory of Investment

A complementary approach to derive the firm problem is to set up a standard current-value Hamiltonian. Written in these terms, an obvious connection to the large literature on investment dynamics in the presence of adjustment frictions emerges.

The current-value Hamiltonian reads

$$H=\pi_t-s_t\iota_t+q_t\left[\nu z_t^{\theta}\iota_t^{\beta}-\left(g_{\mathrm{F},t}+\delta_{\mathrm{I}}\right)z_t\right]$$

with optimality conditions

$$\begin{aligned}
H_h=0 &\Rightarrow\\
s_t=q_t\beta\nu z_t^{\theta}\iota_t^{\beta-1}&
\end{aligned}$$

$$\begin{aligned}
H_z=-\dot{q}_t+\left(r_t-g_{w_{L,t}}+\delta_{\mathrm{X}}\right)q_t &\Rightarrow\\
\partial_z\pi_t+q_t\left[\nu\theta z_t^{\theta-1}\iota_t^{\beta}-\left(g_{\mathrm{F},t}+\delta_{\mathrm{I}}\right)\right]=-\dot{q}_t+\left(r_t-g_{w_{L,t}}+\delta_{\mathrm{X}}\right)q_t &\Rightarrow\\
\partial_z\pi_t+q_t\left[\theta\frac{\dot{z}_t}{z_t}-\left(\left(r_t-g_{w_{L,t}}+\delta_{\mathrm{X}}\right)+\left(1-\theta\right)\left(g_{\mathrm{F},t}+\delta_{\mathrm{I}}\right)\right)\right]=-\dot{q}_t.&
\end{aligned}$$

Letting $q_t=\partial_z v_t$, it is immediate that this approach coincides with the envelope conditions used before.

Next, define the integrating factor $\zeta_t=\theta\log z_t-\left[r_t-g_{w_{L,t}}+\delta_{\mathrm{X}}+\left(1-\theta\right)\left(g_{\mathrm{F},t}+\delta_{\mathrm{I}}\right)\right]t$ where the

time derivative equals $\dot{\zeta}_t = \theta \frac{\dot{z}_t}{z_t} - \left(\left(r_t - g_{w_{L,t}} + \delta_{\mathrm{X}} \right) + (1-\theta) \left(g_{\mathrm{F},t} + \delta_{\mathrm{I}} \right) \right)$. Using this integrating factor, I get

$$\begin{aligned}
\dot{q}_t e^{\zeta_t} + e^{\zeta_t} q_t \dot{\zeta}_t &= e^{\zeta_t} \partial_z \pi_t \\
\partial_t q_t e^{\zeta_t} &= e^{\zeta_t} \partial_z \pi_t \Rightarrow \\
\int_{t_0}^{\infty} \partial_t q_t e^{\zeta_t} dt &= \int_{t_0}^{\infty} e^{\zeta_t} \partial_z \pi_t dt \Rightarrow \\
q_{t_0} e^{\zeta_{t_0}} - \underbrace{q_{\infty} e^{\zeta_{\infty}}}_{=0} &= \int_{t_0}^{\infty} e^{\zeta_t} \partial_z \pi_t dt
\end{aligned}$$

where the value $q_{\infty} e^{\zeta_{\infty}}$ converges to zero since $\lim_{t \to \infty} \zeta_t = -\infty$. Consequently, the proposition in the main body of the paper follows

$$\begin{aligned}
q_t &= \int_t^{\infty} e^{\zeta_s - \zeta_t} \partial_z \pi_s ds \\
&= \int_t^{\infty} e^{-\left[r_t - g_{w_{L,t}} + \delta_{\mathrm{X}} + (1-\theta)(g_{\mathrm{F},t} + \delta_{\mathrm{I}}) \right](s-t)} \left(\frac{z_s}{z_t} \right)^{\theta} \partial_z \pi_s ds.
\end{aligned}$$

The next proposition summarizes the result and discusses how it relates to standard models with adjustment costs (q-theory of investment).

Proposition 5. *The value of an increase in normalized productivity equals*

$$\partial_z v = \int_t^{\infty} e^{-\left[r - g_{w_L} + \delta_X + (1-\theta) g_F \right](u-t)} \left(\frac{z_u}{z_t} \right)^{\theta} \partial_z \pi du \tag{48}$$

where $r - g_{w_L} + \delta_X + (1-\theta) g_F$ is an effective discount factor that combines standard inter-temporal discounting and firm death $r - g_{w_L} + \delta_X$ with new terms that depend on the advantage of backwardness $(1-\theta) g_F$. Moreover, the term $\left(\frac{z_u}{z_t} \right)^{\theta}$ captures that for $\theta > 0$ past adoption makes future adoption easier. The term $\partial_z \pi$ captures how productivity improvements raise profits.

Relative to standard q-theory, a new term emerges that accounts for the advantage of backwardness $(1-\theta) g_{\mathrm{F}}$. Intuitively, if the advantage of backwardness is strong, delaying investment in technology adoption is beneficial, which appears as a higher effective discount factor. Similarly, if there is an anticipated increase in the relative price of skilled labor $\frac{\dot{s}}{s}$, investing in technology adoption is especially advantageous today relative to tomorrow, pushing down the effective discount factor. Lastly, θ is inversely related to the advantage of backwardness, and captures the idea that adopting technology is easier for firms that already use sophisticated technology. This parameter will be key in generating realistic TFP convergence dynamics across countries.

Second, and in contrast to standard q-theory, note that the mapping from skilled labor to technol-

ogy adoption is non-linear, even in the long run since $\beta < 1$. This ensures that production sector firms always hire a positive but finite amount of skilled labor for technology adoption. An implication is that changes in the skill premium induce finite adjustments on technology adoption and skilled labor demand, in contrast to the standard q-theory where investment is perfectly elastic in the long-run.

A.1.4 Firm value function off and on the balanced growth path

Suppose that free entry into innovation and production holds. In that case, it must be that $f_{\mathrm{E}} = v(t,z)$. Now the value function solves the HJB

$$\left(r_t + \delta_{\mathrm{X}} - g_{w_{L,t}}\right) v_t \quad = \quad \max_{\iota} \pi_t - s_t \iota + \dot{z}_t \left(\partial_z v_t\right) + \dot{v}_t.$$

This dynamic HJB equation is tied to the free entry condition in a useful way, see the discussion in Peters and Walsh (2019), which I leverage in the next steps. Note that totally differentiating $f_{\mathrm{E}} = v(z,t)$ with respect to time t implies $\dot{z}\left(\partial_z v\right) = -\dot{v}$. I use this relationship to simplify the HJB equation where it must be understood that h solves the dynamic adoption problem. Rearranging yields

$$v \quad = \quad \frac{\pi_t - s_t \iota_t}{r_t + \delta_{\mathrm{X}} - g_{w_{L,t}}}$$

where I did not assume anything about the stationarity of any of the variables.

Care must be taken for the case when the free entry condition does not hold. In that case, I can compute the firm value by piecing together the part of the problem where no entry occurs (so I know exactly what the measure of firms is and hence can back out profits and the optimal adoption decision) plus the value when free entry is again binding.

A.2 Research Sector

Long-run growth rate. To see why the long-run growth rate equals $\frac{g_L}{1-\phi}$, rearrange the resource constraint in innovation

$$\begin{aligned}
\dot{A}_{\mathrm{F},t} = \gamma A_{\mathrm{F},t}^{\phi} \left(\frac{H_{\mathrm{I},t}}{L_t}\right)^{\lambda-1} H_{\mathrm{I},t} - \delta_{\mathrm{I}} A_{\mathrm{F},t} \Rightarrow \\
\left(g_{\mathrm{F}} + \delta_{\mathrm{I}}\right) = \gamma A_{\mathrm{F},t}^{\phi-1} h_{\mathrm{I}}^{\lambda-1} H_{\mathrm{I},t} \\
\left(g_{\mathrm{F}} + \delta_{\mathrm{I}}\right) = \gamma \frac{L_t}{A_{\mathrm{F},t}^{1-\phi}} h_{\mathrm{I}}^{\lambda-1} \frac{H_{\mathrm{I},t}}{L_t} \Rightarrow \\
a_{\mathrm{F}} = \frac{\gamma h_{\mathrm{I}}^{\lambda}}{g_{\mathrm{F}} + \delta_{\mathrm{I}}}
\end{aligned}$$

where I used the definitions $a_{\text{F}} = \frac{A_{\text{F},t}^{1-\phi}}{L_t}$ and $h_{\text{I}} = \frac{H_{\text{I},t}}{L_t}$, and there are no t subscripts since these objects are constant on a BGP, which is what pins down the long-run growth rate. It is easy to see now that along a BGP with positive population growth and $\phi < 1$, it must be that $g_{\text{F}} = \frac{g_L}{1-\phi}$.

Note that if I had chosen the congestion force using $f_{\text{I}} = H_{\text{I},t}^{1-\lambda}$, the link between aggregate per capita growth and population growth would be $g_{\text{F}} = \frac{\lambda}{1-\phi} g_L$. In an earlier version, I used this exact formulation, which works well in the closed economy. In the open economy, small countries would have a large incentive to specialize in innovation to the extent that this congestion externality is "local" as opposed to global.

Similarly, the law of motion of normalized ideas follows from noting that by definition

$$\frac{\dot{a}_{\text{F},t}}{a_{\text{F},t}} = (1-\phi)\, g_{\text{F},t} - g_L$$

and after substituting out $g_{\text{F},t} = \gamma A_{\text{F},t}^{\phi-1} \left(\frac{H_{\text{I},t}}{L_t}\right)^{\lambda-1} H_{\text{I},t} - \delta_{\text{I}}$,

$$\begin{aligned}
\frac{\dot{a}_{\text{F},t}}{a_{\text{F},t}} &= (1-\phi)\left[\gamma A_{\text{F},t}^{\phi-1}\left(\frac{H_{\text{I},t}}{L_t}\right)^{\lambda-1}\frac{H_{\text{I},t}}{L_t}L_t - \delta_{\text{I}}\right] - g_L \\
&= (1-\phi)\left[\frac{\gamma h_{\text{F},t}^{\lambda}}{a_{\text{F},t}} - \delta_{\text{I}}\right] - g_L \\
&= (1-\phi)\left[\frac{\gamma h_{\text{F},t}^{\lambda}}{a_{\text{F},t}} - \left(\delta_{\text{I}} + \frac{g_L}{1-\phi}\right)\right],
\end{aligned}$$

which implies

$$\dot{a}_{\text{F},t} = (1-\phi)\left[\gamma h_{\text{F},t}^{\lambda} - a_{\text{F},t}\left(\delta_{\text{I}} + \frac{g_L}{1-\phi}\right)\right]$$

as in the main text.

Decentralized equilibrium. Assume free entry holds

$$V_{\text{I},t} = \frac{f_{\text{I},t} w_{H,t}}{A_{\text{F},t}^{\phi}}. \tag{49}$$

Totally differentiating (49) with respect to time t yields

$$\dot{V}_{\text{I},t} = V_{\text{I},t}\left((1-\lambda)\left(g_{H_{\text{I},t}} - g_L\right) + g_{w_{L,t}} + g_{s_t} - \phi g_{\text{F},t}\right) \tag{50}$$

where I used the fact that $f_{\text{I},t} = \frac{\left(\frac{H_{\text{I},t}}{L_t}\right)^{1-\lambda}}{\gamma}$.

Recall the expression for the net present value of an idea

$$V_{\mathrm{I},t} = \int_{t+\tau_t}^{\infty} \exp\left(-\int_t^u (r_v + \delta_{\mathrm{I}})\, dv\right) \Pi_{\mathrm{I},u} du \ . \tag{51}$$

Totally differentiating (51) with respect to time yields

$$\dot{V}_{\mathrm{I},t} = -\exp\left(-\int_t^{t+\tau_t} (r_v + \delta_{\mathrm{I}})\, dv\right) \Pi_{\mathrm{I},t+\tau} \cdot [1 + \dot{\tau}_t] + (r_t + \delta_{\mathrm{I}})\, V_{\mathrm{I},t}, \tag{52}$$

which expresses the value of an innovation in terms of its properly discounted flow profits, $\exp\left(-\int_t^{t+\tau_t} (r_v + \delta_I)\, dv\right) \Pi_{\mathrm{I},t+}$ $[1 + \dot{\tau}_t]$, taking into account changes in the waiting time $\dot{\tau}_t$, as well as appreciation $\dot{V}_{\mathrm{I},t}$. Combining (50) and (52) yields

$$V_{\mathrm{I},t} = \frac{\exp\left(-\int_t^{t+\tau_t} (r_v + \delta_{\mathrm{I}})\, dv\right) \Pi_{\mathrm{I},t+\tau} \cdot [1 + \dot{\tau}_t]}{r_t - g_{w_{L,t}} - g_{s_t} + \delta_{\mathrm{I}} - (1-\lambda)\left(g_{H_{\mathrm{I},t}} - g_L\right) + \phi g_{\mathrm{F},t}}$$

which is identical to the expression in the paper.

In anticipation of solving for a balanced growth path equilibrium, I normalize the value function using $\frac{w_{L,t}}{A_{\mathrm{F},t}^{\phi}}$ as normalizing factor so that $v_{\mathrm{I},t} := \frac{V_{\mathrm{I},t}}{w_{L,t}} A_{\mathrm{F},t}^{\phi}$, and by free entry

$$v_{\mathrm{I},t} = \frac{s_t}{\gamma} h_{\mathrm{F},t}^{1-\lambda}.$$

Using this normalization

$$\begin{aligned}
v_{\mathrm{I},t} &= \frac{A_{\mathrm{F},t}^{\phi}}{w_{L,t}} \frac{\exp\left(-\int_t^{t+\tau} (r_v + \delta_{\mathrm{I}})\, dv\right)}{r_t - g_{w_{L,t}} - g_{s_t} + \delta_{\mathrm{I}} - (1-\lambda)\left(g_{H_{\mathrm{I},t}} - g_L\right) + \phi g_{\mathrm{F},t}} \Pi_{\mathrm{I},t+\tau} \left(1 + \dot{\tau}_t\right) \\
v_{\mathrm{I},t} &= \frac{A_{\mathrm{F},t}^{\phi}}{w_{L,t}} \frac{\exp\left(-\int_t^{t+\tau} (r_v + \delta_{\mathrm{I}})\, dv\right)}{r_t - g_{w_{L,t}} - g_{s_t} + \delta_{\mathrm{I}} - (1-\lambda)\left(g_{H_{\mathrm{I},t}} - g_L\right) + \phi g_{\mathrm{F},t}} \frac{\alpha w_{L,t+\tau} L_{\mathrm{P},t+\tau}}{A_{\mathrm{F},t+\tau} z_{t+\tau}} \left(1 + \dot{\tau}_t\right) \\
v_{\mathrm{I},t} &= \frac{\exp\left(-\int_t^{t+\tau} (r_v + \delta_{\mathrm{I}} - g_{w_L} + \phi g_{\mathrm{F}})\, dv\right)}{r_t - g_{w_{L,t}} - g_{s_t} + \delta_{\mathrm{I}} - (1-\lambda)\left(g_{H_{\mathrm{I},t}} - g_L\right) + \phi g_{\mathrm{F},t}} \frac{\alpha L_{\mathrm{P},t+\tau}}{A_{\mathrm{F},t+\tau}^{1-\phi} z_{t+\tau}} \left(1 + \dot{\tau}_t\right) \\
v_{\mathrm{I},t} &= \frac{\exp\left(-\int_t^{t+\tau} (r_v + \delta_{\mathrm{I}} - g_{w_L} + \phi g_{\mathrm{F}})\, dv\right)}{r_t - g_{w_{L,t}} - g_{s_t} + \delta_{\mathrm{I}} - (1-\lambda)\left(g_{H_{\mathrm{I},t}} - g_L\right) + \phi g_{\mathrm{F},t}} \frac{\alpha l_{\mathrm{P},t+\tau}}{a_{\mathrm{F},t+\tau} z_{t+\tau}} \left(1 + \dot{\tau}_t\right)
\end{aligned}$$

where $h_{\mathrm{I},t} := \frac{H_{\mathrm{I},t}}{L_t}$ and $a_{\mathrm{F},t} = \frac{A_{\mathrm{F},t}^{1-\phi}}{L_t}$ are normalized variables that are constant along a BGP.

Given log utility, the real rate equals $r = \rho + g_{\mathrm{F}}$. Along a BGP, v_I simplifies to

$$\begin{aligned} v_{\mathrm{I}} &= \frac{\exp\left(-\int_t^{t+\tau}\left(\tilde{\rho}+g_{\mathrm{F}}+\delta_{\mathrm{I}}\right)dv\right)}{\tilde{\rho}+g_{\mathrm{F}}+\delta_{\mathrm{I}}}\frac{\alpha l_{\mathrm{P}}}{a_{\mathrm{F}}z} \\ &= \frac{\exp\left(\frac{\tilde{\rho}+g_{\mathrm{F}}+\delta_{\mathrm{I}}}{g_{\mathrm{F}}+\delta_{\mathrm{I}}}\log z\right)}{\tilde{\rho}+g_{\mathrm{F}}+\delta_{\mathrm{I}}}\frac{\alpha l_{\mathrm{P}}}{a_{\mathrm{F}}z} \\ &= \frac{1}{\tilde{\rho}+g_{\mathrm{F}}+\delta_{\mathrm{I}}}\frac{\alpha l_{\mathrm{P}} z^{\frac{\tilde{\rho}}{g_{\mathrm{F}}+\delta_{\mathrm{I}}}}}{a_{\mathrm{F}}}. \end{aligned}$$

Using free entry and in particular $\frac{s}{\gamma}h_{\mathrm{I}}^{1-\lambda} = v_{\mathrm{I}}$ I can derive normalized equilibrium demand for skilled labor

$$h_{\mathrm{I}} = \left\{\frac{\gamma}{s}\frac{1}{\tilde{\rho}+g_{\mathrm{F}}+\delta_{\mathrm{I}}}\frac{\alpha l_{\mathrm{P}} z^{\frac{\tilde{\rho}}{g_{\mathrm{F}}+\delta_{\mathrm{I}}}}}{a_{\mathrm{F}}}\right\}^{\frac{1}{1-\lambda}}.$$

Combining this with the resource constraint for ideas in the steady state, $a_{\mathrm{F}} = \frac{\gamma h_{\mathrm{I}}^{\lambda}}{g_{\mathrm{F}}+\delta_{\mathrm{I}}}$, leads to

$$h_{\mathrm{I}} = \frac{1}{s}\frac{g_{\mathrm{F}}+\delta_{\mathrm{I}}}{\tilde{\rho}+g_{\mathrm{F}}+\delta_{\mathrm{I}}}\alpha l_{\mathrm{P}} z^{\frac{\tilde{\rho}}{g_{\mathrm{F}}+\delta_{\mathrm{I}}}},$$

which is the same expression as in the main part of the paper.

A.2.1 Waiting time for innovator

The waiting time τ for an innovator's idea to be adopted can be derived as follows. Recall equation (12). Use an integrating factor and note that on the balanced growth path with a constant adoption gap, $g_A = g_{\mathrm{F}}$. Normalizing the time of entry to zero ($t_{\mathrm{E}} = 0$) so that calendar time t coincides with waiting time, I can derive the waiting time as a solution to the following differential equation

$$\dot{W}_t = -\delta_{\mathrm{I}} W_t - A_t\left(g_{A,t}+\delta_{\mathrm{I}}\right).$$

Using an integrating factor $e^{\delta_{\mathrm{I}} t}$

$$\begin{aligned}
\int_0^t \frac{\partial \exp(\delta_{\mathrm{I}} u) W_u}{\partial u} &= -\int_0^t \exp(\delta_{\mathrm{I}} t) A_u (\delta_{\mathrm{I}} + g_A) du \\
\exp(\delta_{\mathrm{I}} t) W_t - W_0 &= -A_0 \int_0^t e^{\int_0^u g_A(x) + \delta_{\mathrm{I}} dx} (\delta_{\mathrm{I}} + g_A) du \\
\exp(\delta_{\mathrm{I}} t) W_t - W_0 &= -A_0 \int_0^t e^{\int_0^u \zeta_x dx} (\zeta_u) du \\
\exp(\delta_{\mathrm{I}} t) W_t - W_0 &= -A_0 \left(e^{\int_0^t g_A(x) + \delta_{\mathrm{I}} dx} - 1 \right) \\
W_t - W_0 \exp(-\delta_{\mathrm{I}} t) &= A_0 \exp(-\delta_{\mathrm{I}} t) - A_t \\
W_t - (A_{\mathrm{F},0} - A_0) \exp(-\delta_{\mathrm{I}} t) &= A_0 \exp(-\delta_{\mathrm{I}} t) - A_t \\
W_t - (A_{\mathrm{F},0}) \exp(-\delta_{\mathrm{I}} t) &= -A_t \\
(A_{\mathrm{F},0}) \exp(-\delta_{\mathrm{I}} t) - A_0 e^{\int_0^t g_A(u) du} &= W_t \\
(A_{\mathrm{F},0}) \exp(-\delta_{\mathrm{I}} t) \left(1 - z_0 e^{\int_0^t g_A(u) + \delta_{\mathrm{I}} du} \right) &= W_t.
\end{aligned}$$

Now set $W(0,t) = 0$, which is the point in time when the waiting time is zero. From the previous derivations it is clear that this happens when

$$1 = z_0 e^{\int_0^t g_A(u) + \delta_{\mathrm{I}} du}. \tag{53}$$

The waiting time is thus implicitly defined by (53). Taking logs and rearranging yields

$$-\frac{\log z_0}{\frac{\int_0^t g_A(u) + \delta_{\mathrm{I}} du}{t}} = t.$$

In general, instead of starting at time zero, entering cohorts start in calendar time t and hit the market at $t + \tau_t$, so the expression generalizes to

$$\tau = -\frac{\log z_t}{\frac{\int_t^{t+\tau} g_A(u) + \delta_{\mathrm{I}} du}{\tau}}, \tag{54}$$

in line with the claim in the main text.

This waiting time is an endogenous object that depends on inventors' and adopters' choices, not just in t but also in periods going forward. In the steady state, however, the expression collapses to a simple statistic

$$\tau = -\frac{\log z}{g_A + \delta_{\mathrm{I}}}.$$

Next, I derive the time derivative $\dot{\tau}$ which is important to compute transition dynamics. Note that (54) implies

$$\tau \delta_{\mathrm{I}} \quad = \log A_{\mathrm{F},t} - \log A_{t+\tau}$$

I totally differentiate this expression to obtain

$$\begin{aligned} d\tau\delta_{\mathrm{I}} &= g_{\mathrm{F}}(t)\,dt - g_A(t+\tau)\,(dt+d\tau) \Rightarrow \\ \frac{d\tau}{dt} &= \frac{g_{\mathrm{F}}(t) - g_A(t+\tau)}{g_A(t+\tau)+\delta_{\mathrm{I}}}. \end{aligned}$$

One might be concerned whether $1+\tau' > 0$, i.e., if $\tau' > -1$. To see that this concern is immaterial take account of the following fact. Because $\beta < 1$, the marginal benefit of adopting technology are infinite so that I can safely assume $g_A > -\delta_{\mathrm{I}}$ (if there was no adoption the two would be equal). Then, I prove by contradiction that $\tau' < -1$ cannot be the case. Suppose $\tau' < -1$. Then,

$$\begin{aligned} \frac{g_{\mathrm{F}} - g_A(t+\tau)}{\delta_{\mathrm{I}} + g_A(t+\tau)} < -1 \qquad & \Rightarrow \\ g_{\mathrm{F}} - g_A(t+\tau) < -\delta_{\mathrm{I}} - g_A(t+\tau) \qquad & \Rightarrow \\ g_{\mathrm{F}} < -\delta_{\mathrm{I}}. \end{aligned}$$

However, even if there is no research effort whatsoever, the worst frontier growth rate must be at least as high as $-\delta_{\mathrm{I}}$. Thus by contradiction it must be that $\dot{\tau} > -1$.

A.3 Constrained Planner Problem

The allocation of skilled labor will be generically inefficient, and the inefficiency is central to understand the response of the economy to globalization. I derive the socially optimal allocation of skilled labor where the only margin the social planner decides on is the allocation of skilled labor between innovation and adoption taking endogenous firm entry in the production sector as given.[63]

The planner solves

$$\max_{\{H_{\mathrm{F},t},c_t\}_{t\geq 0}} \int_0^\infty e^{-\tilde{\rho}t} \log(c_t)\,dt,$$

subject to the following constraints

$$\begin{aligned} Y &= \left(\tfrac{AL_{\mathrm{P}}}{1-\alpha}\right)^{1-\alpha} \left(\tfrac{K}{\alpha}\right)^{\alpha} \\ \dot{K} &= Y - C - \delta_k K \\ \dot{A}_{\mathrm{F}} &= \gamma A_{\mathrm{F}}^{\phi} L^{1-\lambda} H_{\mathrm{F}}^{\lambda} \\ \dot{A} &= \nu A^{\theta} A_{\mathrm{F}}^{1-\theta} \left(\tfrac{H-H_{\mathrm{F}}}{M}\right)^{\beta}, \end{aligned}$$

[63] The model is not well-suited to study this margin. For one, the fact that adoption has to happen on the firm level means there is a strange incentive to curtail firm entry. Moreover, I took out scale effects in the production sector using the productivity shifter $M_t^{-\frac{1}{\sigma-1}}$, which introduces inefficiencies unrelated to the key trade-off of allocating skilled labor to innovation or adoption.

and $\frac{\dot{L}}{L} = \frac{\dot{H}}{H} = g_L$.

The constrained efficient allocation of skilled labor across innovation and adoption equals

$$\left(\frac{h_{\mathrm{A}}}{h_{\mathrm{I}}}\right)^{\mathrm{SP}} = \frac{\beta}{1-\theta}\frac{1}{\lambda}\left[\frac{\tilde{\rho}}{g_{\mathrm{F}}} + (1-\phi)\right], \tag{55}$$

where SP stands for social planner. Recall the decentralized allocation (DC)

$$\left(\frac{h_{\mathrm{A}}}{h_{\mathrm{I}}}\right)^{\mathrm{DC}} = \frac{\beta}{1-\theta}\frac{1}{\alpha}\left[\frac{\tilde{\rho}}{g_{\mathrm{F}}} + 1\right]\frac{z^{-\frac{\tilde{\rho}}{g_A}}}{1+\frac{\tilde{\rho}+\delta_{\mathrm{X}}+g_L}{(1-\theta)g_{\mathrm{F}}}}, \tag{56}$$

which generically differs from the planner solution implying that the allocation of skilled labor is generically inefficient.

I provide a comprehensive discussion of the discrepancy between planner and decentralized allocation next where I isolate the distortions induced by the innovation and adoption margin separately. To do so, I need to generalize the baseline model to allow for an additional use of skilled labor in production, beyond its role in technology adoption. If there is only innovation and adoption, shutting down one margin makes the allocation trivially efficient. If, on the other hand, there is skilled labor in innovation, adoption, and production, I can shut down the innovation and adoption margin in turn, and study under what conditions the decentralized equilibrium coincides with the planner solution.

A.3.1 Solve Planner Problem

The planner takes M and L_{P} as given, and I can solve for the optimal allocation using a Hamiltonian.

$$H = \max_{H_{\mathrm{I}},C} e^{-(\rho-g_L)t}\log(c) + \mu_K\left[(AL_{\mathrm{P}})^{1-\alpha}K^{\alpha} - C - \delta_k K\right] + \mu_A\left[\nu A^{\theta}A_{\mathrm{F}}^{1-\theta}\left(\frac{h_{\mathrm{A}}}{m}\right)^{\beta} - \delta_{\mathrm{I}}A\right] + \mu_{A_{\mathrm{F}}}\left[\gamma A_{\mathrm{F}}^{\phi}Lh_{\mathrm{I}}^{\lambda} - \delta_{\mathrm{I}}A_{\mathrm{F}}\right]$$

$$
\begin{aligned}
dC: \quad & \mu_K L = e^{-(\rho - g_L)t}\frac{1}{c} \\
dk: \quad & -\frac{\dot{\mu}_K}{\mu_K} = \alpha\left(\frac{AL_{\mathrm{P}}}{K}\right)^{1-\alpha} - \delta_k \\
dh_{\mathrm{F}}: \quad & \frac{\mu_A}{\mu_{A_{\mathrm{F}}}} = \frac{\lambda\gamma A_{\mathrm{F}}^{\phi} L h_{\mathrm{I}}^{\lambda}}{\beta\nu A^{\theta} A_{\mathrm{F}}^{1-\theta}\left(\frac{h_{\mathrm{A}}}{m}\right)^{\beta}\frac{h_{\mathrm{I}}}{h_{\mathrm{A}}}} \\
dA: \quad & -\frac{\dot{\mu}_A}{\mu_A} = \frac{\mu_K}{\mu_A}\left(\frac{1-\alpha}{A}\right)(AL_{\mathrm{P}})^{1-\alpha}K^{\alpha} + \left[\theta\nu A^{\theta-1}A_{\mathrm{F}}^{1-\theta}\left(\frac{h_{\mathrm{A}}}{m}\right)^{\beta} - \delta_{\mathrm{I}}\right] \\
dA_{\mathrm{F}}: \quad & -\frac{\dot{\mu}_{A_{\mathrm{F}}}}{\mu_{A_{\mathrm{F}}}} = \frac{\mu_A}{\mu_{A_{\mathrm{F}}}}\left[(1-\theta)\,\nu A^{\theta}A_{\mathrm{F}}^{-\theta}\left(\frac{h_{\mathrm{A}}}{m}\right)^{\beta}\right] + \left[\phi\gamma A_{\mathrm{F}}^{\phi-1}Lh_{\mathrm{I}}^{\lambda} - \delta_{\mathrm{I}}\right]
\end{aligned}
$$

where the equation (dA) implies a link between the marginal utility of an extra unit of capital (μ_K) and the multiplier (μ_A), and after rearranging

$$
-\frac{\dot{\mu}_A}{\mu_A} = \frac{\mu_K L_{\mathrm{P}}}{\mu_A}(1-\alpha)\left(\frac{K}{AL_{\mathrm{P}}}\right)^{\alpha} + \left[\theta\nu A^{\theta-1}A_{\mathrm{F}}^{1-\theta}\left(\frac{h_{\mathrm{A}}}{m}\right)^{\beta} - \delta_{\mathrm{I}}\right]
$$

it becomes clear that in a steady state $\frac{\dot{\mu}_A}{\mu_A} = \frac{\dot{\mu}_K}{\mu_K} + g_L$ since $\left(\frac{K}{AL_{\mathrm{P}}}\right)^{\alpha} + \left[\theta\nu A^{\theta-1}A_{\mathrm{F}}^{1-\theta}\left(\frac{h_{\mathrm{A}}}{m}\right)^{\beta} - \delta_{\mathrm{I}}\right]$ is stationary, and $\frac{\dot{L}_{\mathrm{P}}}{L_{\mathrm{P}}} = g_L$ as the relative share of labor devoted to production is constant.

The usual link between marginal product of capital, discounting, and per capita consumption growth emerges, which pins down the ratio of physical capital to effective labor used for production (as opposed to entry)

$$
\frac{K}{AL_{\mathrm{P}}} = \left(\frac{\alpha}{\rho + g_{\mathrm{F}} + \delta_k}\right)^{\frac{1}{1-\alpha}}.
$$

Note that the planner accumulates more capital than households in the decentralized equilibrium due to the markup externality. Next, note that $\frac{\dot{\mu}_A}{\mu_A} = \frac{\dot{\mu}_{A_{\mathrm{F}}}}{\mu_{A_{\mathrm{F}}}}$. Combining (dA_F) with (dh_{F}), and recall

$\tilde{\rho} = \rho - g_L$, yields

$$
\begin{aligned}
-\frac{\dot{\mu}_{A_{\mathrm{F}}}}{\mu_{A_{\mathrm{F}}}} &= \frac{\mu_A}{\mu_{A_{\mathrm{F}}}} \left[(1-\theta)\, \nu A^{\theta} A_{\mathrm{F}}^{-\theta} \left(\frac{h_{\mathrm{A}}}{m} \right)^{\beta} \right] + \left[\phi \gamma A_{\mathrm{F}}^{\phi-1} L h_{\mathrm{I}}^{\lambda} - \delta_{\mathrm{I}} \right] \\
\rho - g_L + g_{\mathrm{F}} &= \frac{\mu_A}{\mu_{A_{\mathrm{F}}}} \left[(1-\theta)\, \nu A^{\theta} A_{\mathrm{F}}^{-\theta} \left(\frac{h_{\mathrm{A}}}{m} \right)^{\beta} \right] + \left[\phi \gamma A_{\mathrm{F}}^{\phi-1} L h_{\mathrm{I}}^{\lambda} - \delta_{\mathrm{I}} \right] \\
\tilde{\rho} + g_{\mathrm{F}} &= \frac{\lambda \gamma A_{\mathrm{F}}^{\phi} L h_{\mathrm{F}}^{\lambda}}{\beta \nu A^{\theta} A_{\mathrm{F}}^{1-\theta} \left(\frac{h_{\mathrm{A}}}{m} \right)^{\beta} \frac{h_{\mathrm{I}}}{h_{\mathrm{A}}}} \left[(1-\theta)\, \nu A^{\theta} A_{\mathrm{F}}^{-\theta} \left(\frac{h_{\mathrm{A}}}{m} \right)^{\beta} \right] + \left[\phi \gamma A_{\mathrm{F}}^{\phi-1} L h_{\mathrm{I}}^{\lambda} - \delta_{\mathrm{I}} \right] \\
\tilde{\rho} + g_{\mathrm{F}} &= \frac{\lambda \gamma A_{\mathrm{F}}^{\phi} L h_{\mathrm{F}}^{\lambda}}{\beta \nu A^{\theta} A_{\mathrm{F}}^{1-\theta} \left(\frac{h_{\mathrm{A}}}{m} \right)^{\beta} \frac{h_{\mathrm{I}}}{h_{\mathrm{A}}}} \left[(1-\theta)\, \nu A^{\theta} A_{\mathrm{F}}^{-\theta} \left(\frac{h_{\mathrm{A}}}{m} \right)^{\beta} \right] + \left[\phi \gamma A_{\mathrm{F}}^{\phi-1} L h_{\mathrm{I}}^{\lambda} - \delta_{\mathrm{I}} \right] \\
\tilde{\rho} + g_{\mathrm{F}} &= \frac{\lambda \gamma A_{\mathrm{F}}^{\phi-1} L h_{\mathrm{I}}^{\lambda}}{\beta \frac{h_{\mathrm{I}}}{h_{\mathrm{A}}}} (1-\theta) + \left[\phi \gamma A_{\mathrm{F}}^{\phi-1} L h_{\mathrm{I}}^{\lambda} - \delta_{\mathrm{I}} \right].
\end{aligned}
$$

Using $a_{\mathrm{F}} = \frac{A_{\mathrm{F}}^{1-\phi}}{L}$ and in the steady state $a_{\mathrm{F}} = \frac{\gamma h_{\mathrm{I}}^{\lambda}}{g_{\mathrm{F}} + \delta_{\mathrm{I}}}$, and $g_{\mathrm{F}} = \frac{1}{1-\phi} g_L$,

$$
\tilde{\rho} + g_{\mathrm{F}} = \frac{\lambda \left(g_{\mathrm{F}} + \delta_{\mathrm{I}} \right)}{\beta \frac{h_{\mathrm{I}}}{h_{\mathrm{A}}}} (1-\theta) + \left[\phi \left(g_{\mathrm{F}} + \delta_{\mathrm{I}} \right) - \delta_{\mathrm{I}} \right].
$$

Rearranging yields the final result

$$
\frac{h_{\mathrm{A}}}{h_{\mathrm{I}}} = \frac{\beta}{1-\theta} \cdot \frac{1}{\lambda} \left[\frac{\tilde{\rho}}{g_{\mathrm{F}} + \delta_{\mathrm{I}}} + (1-\phi) \right].
$$

A.3.2 Planner Problem with Skilled Labor in Production

To study the distinct inefficiencies induced by externalities and markups in each sector, I generalize the baseline mode and assume that the production of intermediate goods also requires skilled labor as a production factor

$$
y_i = \left(\int_{j \in \Omega_{A_i}} \left(\frac{x_{ij}}{\alpha} \right)^{\alpha} dj \right) \left(\frac{l_i^{1-\eta} h_{\mathrm{p},i}^{\eta}}{1-\alpha} \right)^{1-\alpha}
$$

with $\eta \in [0, 1)$, and $h_{\mathrm{P}} := \frac{\int h_{\mathrm{p},i} di}{L}$ analogous to the definition of h_{I} and h_{A}.

The planner takes M and L_{P} as given, and chooses the allocation of skilled labor across sectors, and consumption to take account of the inter-temporal dimension of the problem. The difference to the previous setup is that skilled labor is also used in production so setting the skilled labor share in production equal to zero restores the original. I set up the present value Hamiltonian and solve the

following program

$$\begin{aligned} H \quad = \max_{H_{\mathrm{I}},C} e^{-\tilde{\rho}t}\log(c) \quad & +\mu_K\left[\left(\frac{l_{\mathrm{P}}^{1-\eta}h_{\mathrm{P}}^{\eta}}{1-\alpha}\right)^{1-\alpha}\left(\frac{K}{AL}\right)^{\alpha}AL - c\cdot L - \delta_k K\right] \\ & +\mu_A\left[\nu A^{\theta}A_{\mathrm{F}}^{1-\theta}\left(\frac{h_{\mathrm{A}}}{m}\right)^{\beta} - \delta_{\mathrm{I}}A\right] \\ & +\mu_{A_{\mathrm{F}}}\left[\gamma A_{\mathrm{F}}^{\phi}Lh_{\mathrm{I}}^{\lambda} - \delta_{\mathrm{I}}A_{\mathrm{F}}\right]. \end{aligned}$$

The first order conditions read

$$\begin{aligned} dc: \quad & \mu_K L = e^{-\tilde{\rho}t}\frac{1}{c} \\ dh: \quad & \mu_k\frac{\eta(1-\alpha)}{h_{\mathrm{p}}}\left(\frac{l_{\mathrm{P}}^{1-\eta}h_{\mathrm{P}}^{\eta}}{1-\alpha}\right)^{1-\alpha}\left(\frac{K}{AL}\right)^{\alpha}AL = \mu_A\frac{\beta\nu A^{\theta}A_{\mathrm{F}}^{1-\theta}}{h_{\mathrm{A}}}\left(\frac{h_{\mathrm{A}}}{m}\right)^{\beta} = \mu_{A_{\mathrm{F}}}\frac{\lambda\gamma A_{\mathrm{F}}^{\phi}Lh_{\mathrm{I}}^{\lambda}}{h_{\mathrm{I}}} \end{aligned}$$

and the optimality conditions associated with the co-state variables read

$$\begin{aligned} dC: \quad & \mu_K L = e^{-\tilde{\rho}t}\frac{1}{c} \\ dh: \quad & \mu_K\frac{\eta(1-\alpha)}{h_{\mathrm{p}}}\left(\frac{l_{\mathrm{P}}^{1-\eta}h_{\mathrm{P}}^{\eta}}{1-\alpha}\right)^{1-\alpha}\left(\frac{K}{AL}\right)^{\alpha}AL = \mu_A\frac{\beta\nu A^{\theta}A_{\mathrm{F}}^{1-\theta}}{h_{\mathrm{A}}}\left(\frac{h_{\mathrm{A}}}{m}\right)^{\beta} = \mu_{A_{\mathrm{F}}}\frac{\lambda\gamma A_{\mathrm{F}}^{\phi}Lh_{\mathrm{I}}^{\lambda}}{h_{\mathrm{F}}} \\ dk: \quad & -\dot{\mu}_K = \mu_K\left[\alpha\left(\frac{l_{\mathrm{P}}^{1-\eta}h_{\mathrm{P}}^{\eta}}{1-\alpha}\right)^{1-\alpha}k^{\alpha-1} - \delta_k\right] \text{ (with } k := \frac{K}{AL}\text{)} \\ dA: \quad & -\dot{\mu}_A = \mu_K\left(\frac{1-\alpha}{A}\right)\left(\frac{l_{\mathrm{P}}^{1-\eta}h_{\mathrm{P}}^{\eta}}{1-\alpha}\right)^{1-\alpha}\left(\frac{K}{AL}\right)^{\alpha}AL + \mu_A\left[\theta\nu A^{\theta-1}A_{\mathrm{F}}^{1-\theta}\left(\frac{h_{\mathrm{A}}}{m}\right)^{\beta} - \delta_{\mathrm{I}}\right] \\ dA_{\mathrm{F}}: \quad & -\dot{\mu}_{A_{\mathrm{F}}} = \mu_A\left[(1-\theta)\nu A^{\theta}A_{\mathrm{F}}^{-\theta}\left(\frac{h_{\mathrm{D}}}{m}\right)^{\beta}\right] + \mu_{A_{\mathrm{F}}}\left[\phi\gamma A_{\mathrm{F}}^{\phi-1}Lh_{\mathrm{I}}^{\lambda} - \delta_{\mathrm{I}}\right]. \end{aligned}$$

Equation (dA) implies a link between the marginal utility of an extra unit of capital (μ_K) and the

multiplier (μ_A), and after rearranging

$$-\frac{\dot{\mu}_K}{\mu_K} = \alpha \left(\frac{l_{\mathrm{P}}^{1-\eta} h_{\mathrm{P}}^{\eta}}{1-\alpha} \right)^{1-\alpha} k^{\alpha-1} - \delta_k \tag{57}$$

$$-\frac{\dot{\mu}_A}{\mu_A} = \frac{\mu_K}{\mu_A} (1-\alpha) \left(\frac{l_{\mathrm{P}}^{1-\eta} h_{\mathrm{P}}^{\eta}}{1-\alpha} \right)^{1-\alpha} \left(\frac{K}{AL} \right)^{\alpha} L + \left[\theta \nu A^{\theta-1} A_{\mathrm{F}}^{1-\theta} \left(\frac{h_{\mathrm{A}}}{m} \right)^{\beta} - \delta_{\mathrm{I}} \right] \tag{58}$$

$$-\frac{\dot{\mu}_{A_{\mathrm{F}}}}{\mu_{A_{\mathrm{F}}}} = \frac{\mu_A}{\mu_{A_{\mathrm{F}}}} \left[(1-\theta) \nu A^{\theta} A_{\mathrm{F}}^{-\theta} \left(\frac{h_{\mathrm{A}}}{m} \right)^{\beta} \right] + \left[\phi \gamma A_{\mathrm{F}}^{\phi-1} L h_{\mathrm{I}}^{\lambda} - \delta_{\mathrm{I}} \right]. \tag{59}$$

Inefficient Adoption. Focus first on the case with exogenous innovation, so I can ignore all terms that involve the co-state variable $\mu_{A_{\mathrm{F}}}$. Note that from (dc) I have $-\frac{\dot{\mu}_K}{\mu_K} = \rho + g_{\mathrm{F}}$ along a BGP. Next, note that the only way for $\frac{\dot{\mu}_A}{\mu_A}$ in (58) to be constant is for $\frac{\dot{\mu}_A}{\mu_A} = \frac{\dot{\mu}_K}{\mu_K} + g_L$ to be true. This implies

$$\begin{aligned}
\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}} &= \frac{\mu_K}{\mu_A} (1-\alpha) \left(\frac{l_{\mathrm{P}}^{1-\eta} h_{\mathrm{P}}^{\eta}}{1-\alpha} \right)^{1-\alpha} \left(\frac{K}{AL} \right)^{\alpha} L + \theta \nu z^{\theta-1} \left(\frac{h_{\mathrm{A}}}{m} \right)^{\beta} \\
\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}} &= \frac{\mu_K}{\mu_A} (1-\alpha) \left(\frac{l_{\mathrm{P}}^{1-\eta} h_{\mathrm{P}}^{\eta}}{1-\alpha} \right)^{1-\alpha} \left(\frac{K}{AL} \right)^{\alpha} L + \theta \left(g_{\mathrm{F}} + \delta_{\mathrm{I}} \right) \\
\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}} &= \frac{\mu_K}{\mu_A} (1-\alpha) \left(\frac{l_{\mathrm{P}}^{1-\eta} h_{\mathrm{P}}^{\eta}}{1-\alpha} \right)^{1-\alpha} \left(\frac{K}{AL} \right)^{\alpha} L + \theta \left(g_{\mathrm{F}} + \delta_{\mathrm{I}} \right)
\end{aligned} \tag{60}$$

where the second lines uses the link between h_{A} and z in the steady state. Next, note

$$\frac{\mu_k}{\mu_A} \frac{\eta (1-\alpha)}{h_{\mathrm{p}}} \left(\frac{l_{\mathrm{P}}^{1-\eta} h_{\mathrm{P}}^{\eta}}{1-\alpha} \right)^{1-\alpha} \left(\frac{K}{AL} \right)^{\alpha} L = \frac{\beta \nu z^{\theta-1}}{h_{\mathrm{D}}} \left(\frac{h_{\mathrm{D}}}{m} \right)^{\beta}$$

$$\frac{\mu_k}{\mu_A} = \frac{h_{\mathrm{P}}}{h_{\mathrm{D}}} \frac{\beta \left(g_{\mathrm{F}} + \delta_{\mathrm{I}} \right)}{\eta (1-\alpha) \left(\frac{l_{\mathrm{P}}^{1-\eta} h_{\mathrm{P}}^{\eta}}{1-\alpha} \right)^{1-\alpha} \left(\frac{K}{AL} \right)^{\alpha} L}$$

which can be used in (60) to get

$$\frac{h_{\mathrm{A}}}{h_{\mathrm{P}}} = \frac{\beta}{1-\theta} \frac{1}{\eta} \left\{ \frac{1}{1 + \frac{\tilde{\rho}}{(1-\theta)(g_{\mathrm{F}} + \delta_{\mathrm{I}})}} \right\}$$

which characterizes the efficient allocation of skilled labor in the case without innovation but with skilled labor as a factor of production in the intermediates good sector.

To obtain the decentralized solution, note that a fraction of total output is spend on adoption and

a constant fraction is spent on skilled labor in production

$$H_{\mathrm{A}} w_H = \frac{Y}{\sigma} \frac{\beta (\sigma - 1)(1-\alpha)(g_{\mathrm{F}} + \delta_{\mathrm{I}})}{\rho + \delta_{\mathrm{X}} + (1-\theta)(g_{\mathrm{F}} + \delta_{\mathrm{I}})}$$
$$H_{\mathrm{P}} w_H = Y \frac{\sigma - 1}{\sigma} (1-\alpha)\eta$$

and the ratio equals

$$\left(\frac{h_{\mathrm{A}}}{h_{\mathrm{P}}}\right)^{\mathrm{DC}} = \frac{\beta}{1-\theta}\frac{1}{\eta}\left\{\frac{1}{1 + \frac{\tilde{\rho} + \delta_{\mathrm{X}} + g_L}{(1-\theta) g_{\mathrm{F}}}}\right\}. \tag{61}$$

This setup is very close to the one in Parente and Prescott (1994). This expression contrasts with the solution to the constrained planner problem in (62)

$$\left(\frac{h_{\mathrm{A}}}{h_{\mathrm{P}}}\right)^{\mathrm{SP}} = \frac{\beta}{1-\theta}\frac{1}{\eta}\left\{\frac{1}{1 + \frac{\tilde{\rho}}{(1-\theta) g_{\mathrm{F}}}}\right\}. \tag{62}$$

Since $\delta_{\mathrm{X}} + g_L > 0$, the decentralized allocation suffers from underinvestment in technology adoption. As argued before, this inefficiency is a generic feature of models where incumbent firms make costly adoption choices, and entrants learn from incumbents. Note that I abstract away from any learning spillovers among incumbents, which would amplify this inefficiency.[64] Moreover, the result highlights how the efficiency result in Parente and Prescott (1994) constitutes a knife-edge case hinging on zero population growth ($g_L = 0$) and no churn among incumbents ($\delta_{\mathrm{X}} = 0$), in which case (61) and (62) agree. If there was firm entry in Parente and Prescott (1994), one would have to assume some form of spillover to tie together the productivity of incumbents and entrants so that a stationary firm size distribution emerges.

Inefficient Innovation. Next, suppose firms are automatically at the technological frontier, so let's set μ_A to zero while $z = 1$. This scenario is almost identical to Jones (1995). In that case,

$$dh: \qquad \mu_{A_{\mathrm{F}}} \frac{\lambda \gamma A_{\mathrm{F}}^{\phi} L h_{\mathrm{I}}^{\lambda}}{h_{\mathrm{I}}} = \mu_k \frac{\eta(1-\alpha)}{h_{\mathrm{p}}} \left(\frac{l_{\mathrm{P}}^{1-\eta} h_{\mathrm{P}}^{\eta}}{1-\alpha}\right)^{1-\alpha} \left(\frac{K}{AL}\right)^{\alpha} AL$$
$$dA_F: \qquad -\frac{\dot{\mu}_{A_{\mathrm{F}}}}{\mu_{A_{\mathrm{F}}}} = \frac{\mu_K}{\mu_{A_{\mathrm{F}}}} (1-\alpha) \left(\frac{l_{\mathrm{P}}^{1-\eta} h_{\mathrm{P}}^{\eta}}{1-\alpha}\right)^{1-\alpha} \left(\frac{K}{A_{\mathrm{F}} L}\right)^{\alpha} L + \left[\phi \gamma A_{\mathrm{F}}^{\phi-1} L h_{\mathrm{I}}^{\lambda} - \delta_{\mathrm{I}}\right].$$

[64] There is a large literature suggesting that technology adoption effort may be inefficiently low, see for example Foster and Rosenzweig (2010) in the context of agricultural production in developing economies.

The same logic as before implies $\frac{\dot{\mu}_{A_{\text{F}}}}{\mu_{A_{\text{F}}}} = \frac{\dot{\mu}_K}{\mu_k} + g_L$ so

$$\tilde{\rho} + g_{\text{F}} + \delta_{\text{I}} = \frac{\mu_K}{\mu_{A_{\text{F}}}} (1-\alpha) \left(\frac{l_{\text{P}}^{1-\eta} h_{\text{P}}^{\eta}}{1-\alpha} \right)^{1-\alpha} \left(\frac{K}{A_{\text{F}} L} \right)^{\alpha} L + \left[\phi \gamma A_{\text{F}}^{\phi-1} L h_{\text{I}}^{\lambda} - \delta_{\text{I}} \right]$$

$$\tilde{\rho} + g_{\text{F}} + \delta_{\text{I}} = \frac{\gamma L}{A_{\text{F}}^{1-\phi}} h_{\text{I}}^{\lambda} \left(\lambda \frac{h_{\text{P}}}{h_{\text{I}}} \frac{1}{\eta} + \phi \right).$$

Now using $a_{\text{F}} = \frac{A_{\text{F}}^{1-\phi}}{L}$ and $\gamma h_{\text{I}}^{\lambda} = a_{\text{F}} (g_{\text{F}} + \delta_{\text{I}})$ I get the ratio of skilled labor in production vis-a-vis innovation.

$$\frac{h_{\text{P}}}{h_{\text{I}}} = \frac{\eta}{\lambda} \left\{ \frac{\tilde{\rho}}{g_{\text{F}} + \delta_{\text{I}}} + 1 - \phi \right\}.$$

In contrast, the decentralized allocation in the model without adoption equals

$$\frac{h_{\text{P}}}{h_{\text{I}}} = \frac{\eta}{\alpha} \left(\frac{\tilde{\rho}}{g_{\text{F}} + \delta_{\text{I}}} + 1 \right), \tag{63}$$

which can be derived following the step in Jones (1995), and, trivially, features all the same externalities already present in Jones (1995): markup distortions in α leading to too little innovation, negative dynamic knowledge externalities for the case of $\phi < 0$ leading to too much innovation, and a congestion externality for $\lambda < 1$ that also leads to too much innovation, all else equal. For the innovation-production trade-off considered in (63), Jones concludes that for reasonable parameter values, the economy is characterized by insufficient innovation even when there are negative research externalities with $\phi < 0$ and $\lambda < 1$.

Jones' conclusion is partly driven by the assumption that the production sector is efficient. In a model with technology adoption, as I have shown before, this trade-off becomes more interesting since technology adoption features externalities as well. It is ex ante unclear weather externalities in innovation or technology adoption dominate.

A final remark before I conclude this welfare analysis relates to the fact that the full model with both technology adoption and innovation features externalities that are absent when considering the simplified model where I isolated each margin separately. To see this, assume $\alpha = \lambda$, $\phi = 0$, and $\delta_{\text{X}} = -g_L$, which renders innovation and adoption efficient when considered in isolation. In that case, it is still true that planner and decentralized allocation disagree, which can be seen by computing the ratio of the planner and the decentralized allocation

$$\frac{\left(\frac{h_{\text{A}}}{h_{\text{I}}} \right)^{\text{SP}}}{\left(\frac{h_{\text{A}}}{h_{\text{I}}} \right)^{\text{DC}}} = z^{\frac{\tilde{\rho}}{g_A + \delta_{\text{I}}}} \left(1 + \frac{\tilde{\rho}}{(1-\theta)(g_{\text{F}} + \delta_{\text{I}})} \right), \tag{64}$$

and since $z < 1$ and $1 + \frac{\tilde{\rho}}{(1-\theta)(g_{\text{F}}+\delta_{\text{I}})} > 1$ it is unclear if the decentralized allocation features too little

or too much adoption. My interpretation of this result is that the advantage of backwardness on the one hand, and the wait-in-line assumption and congestion in bringing ideas to market on the other, induce externalities similar to the search literature. Innovators don't take into account that they make adoption easier. And adopters don't internalize their impact on the waiting time.

A.4 Open Economy

Derivation of the share of ideas originating from country k.
You start with the free entry condition into research, and because the value of innovation is the same no matter where you innovate (because ideas are sold to the same world market frictionlessly), and assuming all countries innovate (which is always true for $\lambda < 1$), I have

$$\begin{aligned} \frac{\left(h_{\mathrm{I},n}\right)^{1-\lambda} w_{H,n}}{\gamma_n \left(A_{\mathrm{F}}^{\mathrm{W}}\right)^{\phi}} &= \frac{\left(h_{\mathrm{I},k}\right)^{1-\lambda} w_{H,k}}{\gamma_k \left(A_{\mathrm{F}}^{\mathrm{W}}\right)^{\phi}} \Rightarrow \\ \frac{\left(h_{\mathrm{I},n}\right)^{1-\lambda} w_{H,n}}{\gamma_n} &= \frac{\left(h_{\mathrm{I},k}\right)^{1-\lambda} w_{H,k}}{\gamma_k}. \end{aligned} \tag{65}$$

Next, use the fact that by the resource constraint, a link between share of ideas and skilled labor devoted to innovation emerges

$$\left(g_{\mathrm{F}} + \delta_{\mathrm{I}}\right) \chi_n a_{\mathrm{F}}^{\mathrm{W}} = \gamma_n h_{\mathrm{I},n}^{\lambda}.$$

Substituting this in (65) and rearranging yields

$$\begin{aligned} \frac{\left(\frac{\chi_n}{\gamma_n}\right)^{\frac{1-\lambda}{\lambda}} w_{H,n}}{\gamma_n} &= \frac{\left(\frac{\chi_k}{\gamma_k}\right)^{\frac{1-\lambda}{\lambda}} w_{H,k}}{\gamma_k} \Rightarrow \\ \frac{\chi_n}{\chi_k} &= \left(\frac{\gamma_n}{\gamma_k}\right)^{\frac{1}{1-\lambda}} \left(\frac{w_{H,n}}{w_{H,k}}\right)^{-\frac{\lambda}{1-\lambda}}. \end{aligned}$$

Next, note that the high-skilled wage ratio is a function of the skill premium alone $\frac{w_{H,k}}{w_{H,n}} = \left(\frac{s_k}{s_n}\right)^{\frac{1-(\beta+\theta)}{1-\theta}}$. Summing over all countries implies

$$\sum_n \frac{\chi_n}{\chi_k} = \sum_n \left(\frac{\gamma_n}{\gamma_k}\right)^{\frac{1}{1-\lambda}} \left(\frac{w_{H,n}}{w_{H,k}}\right)^{-\frac{\lambda}{1-\lambda}}$$

$$\sum_n \frac{\chi_n}{\chi_k} = \sum_n \left(\frac{\gamma_n}{\gamma_k}\right)^{\frac{1}{1-\lambda}} \left(\left(\frac{s_n}{s_k}\right)^{\frac{1-(\beta+\theta)}{1-\theta}}\right)^{-\frac{\lambda}{1-\lambda}}$$

$$\frac{1}{\chi_k} = \frac{\sum_n \gamma_n^{\frac{1}{1-\lambda}} \left(s_n^{\frac{1-(\beta+\theta)}{1-\theta}}\right)^{-\frac{\lambda}{1-\lambda}}}{\gamma_k^{\frac{1}{1-\lambda}} \left(s_k^{\frac{1-(\beta+\theta)}{1-\theta}}\right)^{-\frac{\lambda}{1-\lambda}}} \Rightarrow$$

$$\chi_k = \frac{\gamma_k^{\frac{1}{1-\lambda}} \left(s_k^{\frac{1-(\beta+\theta)}{1-\theta}}\right)^{-\frac{\lambda}{1-\lambda}}}{\sum_n \gamma_n^{\frac{1}{1-\lambda}} \left(s_n^{\frac{1-(\beta+\theta)}{1-\theta}}\right)^{-\frac{\lambda}{1-\lambda}}}.$$

This establishes the result in the main part of the paper.

Derivation of the key market clearing condition in the open economy.

$$\frac{w_{H,k}}{\gamma_k} h_{\mathrm{I},k}^{1-\lambda} = \frac{1}{\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \frac{\alpha L_{\mathrm{P}}}{\left(A_{\mathrm{F}}^{\mathrm{W}}\right)^{1-\phi}} \sum_n w_n z_n^{\frac{\tilde{\rho}}{g_A + \delta_{\mathrm{I}}}}$$

$$\frac{w_{H,k}}{\gamma_k} h_{\mathrm{I},k}^{1-\lambda} = \frac{1}{\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \frac{\alpha l_{\mathrm{P}}}{\left(a_{\mathrm{F}}^{\mathrm{W}}\right)} \sum_n w_n z_n^{\frac{\tilde{\rho}}{g_A + \delta_{\mathrm{I}}}}$$

$$\frac{w_{H,k}}{\gamma_k} h_{\mathrm{I},k}^{1-\lambda} = \frac{g_{\mathrm{F}} + \delta_{\mathrm{I}}}{\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \frac{\alpha l_{\mathrm{P}}}{\gamma_k h_{\mathrm{I},k}^{\lambda}} \chi_k \sum_n w_n z_n^{\frac{\tilde{\rho}}{g_A + \delta_{\mathrm{I}}}}$$

$$z_k s_k h_{\mathrm{I}} = \frac{g_{\mathrm{F}} + \delta_{\mathrm{I}}}{\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \alpha l_{\mathrm{P}} \chi_k \sum_n z_n^{1 + \frac{\tilde{\rho}}{g_A + \delta_{\mathrm{I}}}}$$

$$z_k s_k h_{\mathrm{I},k} = \frac{g_{\mathrm{F}} + \delta_{\mathrm{I}}}{\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \frac{\gamma_k^{\frac{1}{1-\lambda}} s_k^{-\frac{1-\beta-\theta}{1-\theta}\frac{\lambda}{1-\lambda}}}{\sum_n \gamma_n^{\frac{1}{1-\lambda}} s_n^{-\frac{1-\beta-\theta}{1-\theta}\frac{\lambda}{1-\lambda}}} \alpha l_{\mathrm{P}} \sum_n z_n^{1 + \frac{\tilde{\rho}}{g_A + \delta_{\mathrm{I}}}}.$$

Note that $z = s^{-\frac{\beta}{1-\theta}} \kappa_z$, same as in the closed economy, and collect terms that are constant along a balanced growth path so

$$h_{\mathrm{I},k} = \Lambda_{\mathrm{FO}} \frac{\gamma_k^{\frac{1}{1-\lambda}} s_k^{-\frac{1-\beta-\theta}{1-\theta}\frac{1}{1-\lambda}}}{\sum_n \gamma_n^{\frac{1}{1-\lambda}} s_n^{-\frac{1-\beta-\theta}{1-\theta}\frac{\lambda}{1-\lambda}}} \sum_n z_n^{1 + \frac{\tilde{\rho}}{g_A + \delta_{\mathrm{I}}}}.$$

Combine this with the market clearing condition

$$h_k = \frac{\Lambda_{\mathrm{D}}}{s_k} + \Lambda_{\mathrm{FO}} \frac{\gamma_k^{\frac{1}{1-\lambda}} s_k^{-\frac{1-\beta-\theta}{1-\theta}\frac{1}{1-\lambda}}}{\sum_n \gamma_n^{\frac{1}{1-\lambda}} s_n^{-\frac{1-\beta-\theta}{1-\theta}\frac{\lambda}{1-\lambda}}} \sum_n z_n^{1+\frac{\tilde{\rho}}{g_A+\delta_{\mathrm{I}}}}.$$

Generalized ACR Formula. Recall the expression for real wages in the open economy relative to the closed economy reads

$$\frac{w_k^{open}}{w_k^{closed}} = \left(\frac{h_{\mathrm{I},k}^{open}}{h_{\mathrm{I},k}^{closed}}\right)^{\frac{\lambda}{1-\phi}} \left(\frac{1}{\chi_k}\right)^{\frac{1}{1-\phi}} \cdot \left(\frac{s_k^{open}}{s_k^{closed}}\right)^{-\frac{\beta}{1-\theta}}$$

and

$$\frac{w_H^{open}}{w_H^{closed}} = \left(\frac{h_{\mathrm{I},k}^{open}}{h_{\mathrm{I},k}^{closed}}\right)^{\frac{\lambda}{1-\phi}} \left(\frac{1}{\chi_k}\right)^{\frac{1}{1-\phi}} \cdot \left(\frac{s_k^{open}}{s_k^{closed}}\right)^{\frac{1-(\beta+\theta)}{1-\theta}}.$$

To derive this expression, start with the resource constraint which implies a link between equilibrium research effort and the share of ideas invented by country k

$$a_{\mathrm{F}}^{\mathrm{W}} = \frac{\gamma_k h_{\mathrm{I},k}^{\lambda}}{(g_{\mathrm{F}} + \delta_{\mathrm{I}})\,\chi_k}.$$

Now the ratio of frontier technology in the open and closed economy is given by $\frac{A_{\mathrm{F}}^{\mathrm{W},open}}{A_{\mathrm{F}}^{\mathrm{W},closed}} = \left(\frac{a_{\mathrm{F}}^{\mathrm{W},open}}{a_{\mathrm{F}}^{\mathrm{W},closed}}\right)^{\frac{1}{1-\phi}} = \left(\frac{h_{\mathrm{I},k}^{open}}{h_{\mathrm{I},k}^{closed}}\right)^{\frac{\lambda}{1-\phi}} \left(\frac{1}{\chi_k}\right)^{\frac{1}{1-\phi}}$ where I used the fact that $\chi_k^{closed} = 1$.

To study the real wage effects I have to account for the adoption margin since $\frac{w_k^{open}}{w_k^{closed}} = \frac{A_{\mathrm{F}}^{\mathrm{W},open}}{A_{\mathrm{F}}^{\mathrm{W},closed}} \frac{z_k^{open}}{z_k^{closed}}$. Note that $\frac{z_k^{open}}{z_k^{closed}} = \left(\frac{s_k^{open}}{s_k^{closed}}\right)^{-\frac{\beta}{1-\theta}}$, which delivers the result. For skilled wages the skill premium needs to be added, $\frac{w_H^{open}}{w_H^{closed}} = \frac{A_{\mathrm{F}}^{\mathrm{W},open}}{A_{\mathrm{F}}^{\mathrm{W},closed}} \frac{z^{open}}{z^{closed}} \frac{s^{open}}{s^{closed}}$.

These wage ratios reflect the long-run differences in wages after all temporary adjustments have taken place. In particular, since the long-run supply of capital is perfectly elastic, the capital-effective labor ratio is the same in the open and closed economy and is thus netted out in the ratio. This concludes the derivation.

Open Economy with country size differences. I next generalize the framework to allow for countries of heterogeneous sizes, which helps me to take the framework to the data. I use the "home economy" aka the West as baseline for the normalizations. This means that the definition of the normalized world technological frontier is unchanged. Define $b_n = \frac{L_n}{L} \in R^+$ as a weight attached to country c relative to the home economy. You could as well pick $L_{\mathrm{W}} = \sum_n L_n$ or any other normalizing factor that grows as the same rate as the labor force.

The growth of the technological frontier now reads

$$\begin{aligned}
\frac{\dot{A}_{\mathrm{F}}^{\mathrm{W}}}{A_{\mathrm{F}}^{\mathrm{W}}} &= \sum_n \frac{\dot{A}_{\mathrm{F},n}}{A_{\mathrm{F}}^{\mathrm{W}}} \\
&= \sum_n \chi_n \frac{\dot{A}_{\mathrm{F},n}}{A_{\mathrm{F},n}} \\
&= \sum_n \chi_n \left(\frac{\gamma_n}{\chi_n} \left(A_{\mathrm{F}}^{\mathrm{W}}\right)^{\phi-1} L_n h_{\mathrm{I},n}^{\lambda} - \delta_{\mathrm{I}} \right) \\
&= \sum_n \chi_n \left(\frac{\gamma_n}{\chi_n} \frac{1}{a_{\mathrm{F}}^{\mathrm{W}}} h_{\mathrm{I},n}^{\lambda} b_n - \delta_{\mathrm{I}} \right) \\
&= \sum_n \chi_n \left(\frac{\gamma_n}{\chi_n} \frac{h_{\mathrm{I},n}^{\lambda} b_n}{a_{\mathrm{F}}^{\mathrm{W}}} - \delta_{\mathrm{I}} \right) \\
&= \sum_n \frac{\gamma_n h_{\mathrm{I},n}^{\lambda} b_n}{a_{\mathrm{F}}^{\mathrm{W}}} - \delta_{\mathrm{I}}.
\end{aligned}$$

This result of course coincides with the previous result when countries are equal-sized and $b_n = 1$. Consequently,

$$g_{a_{\mathrm{F}}^{\mathrm{W}}} = (1-\phi) \left\{ \sum_n \frac{\gamma_n h_{\mathrm{I},n}^{\lambda} b_n}{a_{\mathrm{F}}^{\mathrm{W}}} - \left(\delta_{\mathrm{I}} + \frac{1}{1-\phi} g_L \right) \right\}$$

and in the steady state

$$a_{\mathrm{F}}^{\mathrm{W}} = \frac{\sum_n \gamma_n h_{\mathrm{I},n}^{\lambda} b_n}{g_{\mathrm{F}} + \delta_{\mathrm{I}}}.$$

By free entry, I have

$$\frac{h_{\mathrm{I},n}^{1-\lambda} w_{H,n}}{\gamma_n} = \frac{h_{\mathrm{I},k}^{1-\lambda} w_{H,k}}{\gamma_k}$$

and by the resource constraint I have

$$\frac{\chi_n}{\chi_k} = \frac{\gamma_n h_{\mathrm{I},n}^{\lambda} b_n}{\gamma_k h_{\mathrm{I},k}^{\lambda} b_k}.$$

Combining free entry and resource constraint delivers

$$\chi_k = \frac{\gamma_k^{\frac{1}{1-\lambda}} s_k^{-\frac{\lambda}{1-\lambda}\frac{1-\theta-\beta}{1-\theta}} b_k}{\sum_n \gamma_n^{\frac{1}{1-\lambda}} s_n^{-\frac{\lambda}{1-\lambda}\frac{1-\theta-\beta}{1-\theta}} b_n}$$

where I used $w_{H,k} \propto A_{\mathrm{F}}^{\mathrm{W}} s^{\frac{1-\theta-\beta}{1-\theta}}$.

I next derive the demand for skilled labor as before, using

$$\begin{aligned}
\frac{s_k}{\gamma_k} h_{\mathrm{I},k}^{1-\lambda} &= \frac{1}{w_k} \frac{1}{\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \frac{\alpha}{\left(A_{\mathrm{F}}^{\mathrm{W}}\right)^{1-\phi}} \sum_n L_{\mathrm{P},n} w_n z_n^{\frac{\tilde{\rho}}{g_A + \delta_{\mathrm{I}}}} \\
\frac{s_k}{\gamma_k} h_{\mathrm{I},k}^{1-\lambda} &= \frac{1}{w_k} \frac{1}{\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \frac{\alpha l_{\mathrm{P}}}{a_{\mathrm{F}}^{\mathrm{W}}} \sum_n w_n z_n^{\frac{\tilde{\rho}}{g_A + \delta_{\mathrm{I}}}} \frac{L_n}{L} \\
\frac{s_k z_k}{\gamma_k} h_{\mathrm{I},k}^{1-\lambda} &= \frac{1}{\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \frac{\alpha l_{\mathrm{P}}}{a_{\mathrm{F}}^{\mathrm{W}}} \sum_n z_n^{1 + \frac{\tilde{\rho}}{g_A + \delta_{\mathrm{I}}}} b_n.
\end{aligned}$$

Next, use $\chi_k = \frac{\gamma_k h_{\mathrm{I},k}^{\lambda} b_k}{a_{\mathrm{F}}^{\mathrm{W}} (g_{\mathrm{F}} + \delta_{\mathrm{I}})}$ to obtain

$$\begin{aligned}
\frac{s_k z_k}{\gamma_k} h_{\mathrm{I},k}^{1-\lambda} &= \frac{1}{\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \frac{\alpha l_{\mathrm{P}}}{a_{\mathrm{F}}^{\mathrm{W}}} \sum_n z_n^{1 + \frac{\tilde{\rho}}{g_A + \delta_{\mathrm{I}}}} b_n \\
\frac{s_k z_k}{\gamma_k} h_{\mathrm{I},k}^{1-\lambda} &= \frac{g_{\mathrm{F}} + \delta_{\mathrm{I}}}{\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \frac{\chi_k \alpha l_{\mathrm{P}}}{\gamma_k h_{\mathrm{I},k}^{\lambda} b_k} \sum_n z_n^{1 + \frac{\tilde{\rho}}{g_A + \delta_{\mathrm{I}}}} b_n \\
s_k z_k h_{\mathrm{I},k}^{1-\lambda} &= \frac{g_{\mathrm{F}} + \delta_{\mathrm{I}}}{\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \frac{\chi_k \alpha l_{\mathrm{P}}}{h_{\mathrm{I},k}^{\lambda} b_k} \sum_n z_n^{1 + \frac{\tilde{\rho}}{g_A + \delta_{\mathrm{I}}}} b_n \\
s_k z_k h_{\mathrm{I},k} &= \frac{g_{\mathrm{F}} + \delta_{\mathrm{I}}}{\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \frac{\gamma_k^{\frac{1}{1-\lambda}} s_k^{-\frac{\lambda}{1-\lambda} \frac{1-\theta-\beta}{1-\theta}} b_k}{\sum_n \gamma_n^{\frac{1}{1-\lambda}} s_n^{-\frac{\lambda}{1-\lambda} \frac{1-\theta-\beta}{1-\theta}} b_n} \alpha l_{\mathrm{P}} \frac{1}{b_k} \sum_n z_n^{1 + \frac{\tilde{\rho}}{g_A + \delta_{\mathrm{I}}}} b_n \\
s_k z_k h_{\mathrm{I},k} &= \frac{g_{\mathrm{F}} + \delta_{\mathrm{I}}}{\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \frac{\gamma_k^{\frac{1}{1-\lambda}} s_k^{-\frac{\lambda}{1-\lambda} \frac{1-\theta-\beta}{1-\theta}}}{\sum_n \gamma_n^{\frac{1}{1-\lambda}} s_n^{-\frac{\lambda}{1-\lambda} \frac{1-\theta-\beta}{1-\theta}} b_n} \alpha l_{\mathrm{P}} \sum_n z_n^{1 + \frac{\tilde{\rho}}{g_A + \delta_{\mathrm{I}}}} b_n \\
\kappa_z s_k^{\frac{1-\theta-\beta}{1-\theta}} h_{\mathrm{I},k} &= \frac{g_{\mathrm{F}} + \delta_{\mathrm{I}}}{\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \frac{\gamma_k^{\frac{1}{1-\lambda}} s_k^{-\frac{\lambda}{1-\lambda} \frac{1-\theta-\beta}{1-\theta}}}{\sum_n \gamma_n^{\frac{1}{1-\lambda}} s_n^{-\frac{\lambda}{1-\lambda} \frac{1-\theta-\beta}{1-\theta}} b_n} \alpha l_{\mathrm{P}} \sum_n z_n^{1 + \frac{\tilde{\rho}}{g_A + \delta_{\mathrm{I}}}} b_n \\
h_{\mathrm{I},k} &= \Lambda_{\mathrm{FO}} \frac{g_{\mathrm{F}} + \delta_{\mathrm{I}}}{\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \frac{\gamma_k^{\frac{1}{1-\lambda}} s_k^{-\frac{1}{1-\lambda} \frac{1-\theta-\beta}{1-\theta}}}{\sum_n \gamma_n^{\frac{1}{1-\lambda}} s_n^{-\frac{\lambda}{1-\lambda} \frac{1-\theta-\beta}{1-\theta}} b_n} \alpha l_{\mathrm{P}} \sum_n z_n^{1 + \frac{\tilde{\rho}}{g_A + \delta_{\mathrm{I}}}} b_n.
\end{aligned}$$

Note that for the symmetric country case with all endogenous variables and exogenous productivity shifters identical across countries, the expression would coincide with the solution to the closed economy setup since the term $\sum_n b_n$ appears in both nominator and denominator.

Inserting this expression into the market clearing condition and iterating over a set of skill premia across countries delivers the long-run steady state allocation

$$\frac{\Lambda_{\mathrm{D}}}{s_k} + \Lambda_{\mathrm{FO}} \frac{g_{\mathrm{F}} + \delta_{\mathrm{I}}}{\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \frac{\gamma_k^{\frac{1}{1-\lambda}} s_k^{-\frac{1}{1-\lambda} \frac{1-\theta-\beta}{1-\theta}}}{\sum_n \gamma_n^{\frac{1}{1-\lambda}} s_n^{-\frac{\lambda}{1-\lambda} \frac{1-\theta-\beta}{1-\theta}} b_n} \alpha l_{\mathrm{P}} \sum_n z_n^{1 + \frac{\tilde{\rho}}{g_A + \delta_{\mathrm{I}}}} b_n = h_k.$$

A.4.1 Trade cost and bargaining in open economy

I will next show how to extend the open economy model to include trade cost and bargaining. These extensions are potentially important in taming the impact of the East on innovation in the West. In a model where innovation happens in the West, and the East is a relatively large country that adopts Western technology, the pull force can become so strong that the skill premium in the West shoots up so much that Western workers are left impoverished. Two reasonable assumptions to avoid this outcome relate to the role of trade cost, and bargaining over innovator rents, which I consider in turn.

Trade cost. Recall the flow profits of innovation in the steady state are proportional to

$$\pi_{\mathrm{I}} \propto \sum_n \frac{\alpha L_n^P w_n z^{\frac{\tilde{\rho}}{g_A+\delta_{\mathrm{I}}}}}{A_F z_n}.$$

One could easily introduce ice berg trade cost from innovator i to technology user c

$$\pi_{\mathrm{I}} \propto \sum_n \frac{1}{\tau_{in}} \frac{\alpha L_n^P w_n z^{\frac{\tilde{\rho}}{g_A+\delta_{\mathrm{I}}}}}{A_F z_n}$$

where $\tau_{in} \geq 1$ and $\tau_{ii} = 1$.

The trade cost is somewhat non standard because the model differs from, for instance, from Krugman (1980) where the trade elasticity $\sigma - 1$ shapes the importance of trade cost. The reason is that the only good that is traded is the royalty associated with innovator profits. The locally optimal markup is applied onto domestic cost, which maximizes total domestic revenues, and there is no way to further raise profits. However, the profits need to be shipped back to the holder of the patents, and trade costs apply so only a fraction $\frac{1}{\tau_{ij}}$ of the value of the idea arrive at home.

One could envision that financial markets can help overcome this trade cost when both countries hold ideas. That is, instead of shipping the royalty back one could simply trade it for the royalty that accrues to foreign firms in the domestic economy. The extent to which this is possible depends on the degree of specialization and would not work if all ideas are held in the West.

Bargaining over royalties. An alternative interpretation for the wedge parameter τ could be microfounded by considering explicit bargaining over innovator profits. For example, one could imagine that the Chinese government could try to extract rents from foreign innovators' ideas. This would help reduce the impact of foreign adoption on innovation in the West. If the (corrupt) government wastes this income, the model coincides exactly with the previous extension. Alternatively, one could rebate the profits to households.

A.4.2 Total Asset Stock in Open Economy

I derive the total value of assets in the open economy, which depends on the value of domestic capital, domestic production sector firms, and global intellectual property. Note that ideas are adopted at different points in time in different countries, which I need to keep track off. I focus on the two-country case, and I focus on the value of an idea, which is the complicated object when constructing the total value of assets held by the home economy. I focus on the case where all intellectual property is held in the home economy (West).

It is easy to see that the value of an innovation $V_{\mathrm{I}}^{\mathrm{W}}$ can be split into home and foreign component

$$V_{\mathrm{I}}^{\mathrm{W}} = V_{\mathrm{I}} + V_{\mathrm{I}}^{*}.$$

Next, note that along a BGP a simple expression for the value of an innovation of ideas that are already adopted, denoted by V_A, obtains

$$\begin{aligned} V_A &= \frac{1}{\rho - g_L + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \frac{\alpha L_{\mathrm{P}} w_t}{A} \\ V_A^{*} &= \frac{1}{\rho - g_L + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \frac{\alpha L_{\mathrm{P}}^{*} w_t^{*}}{A^{*}}. \end{aligned}$$

Conveniently, changes in the real wage for a fixed long-run interest rate have no bearing on the value of an innovation since they directly cancel with the total numbers of adopted ideas in the denominator (recall $w \propto A$).

The forces that do matter are i) changes in the waiting time, and ii) an extensive margin effect in the form of another country using ideas. I focus on the first issue first, and introduce the following notation. Let $V_{\mathrm{I},t}(k)$ denote the value of an innovation at time t that is k years away from adoption. Clearly, this asset has value, but its value ought to be below ideas that are already in use at time t. Using the HJB equation $(r + \delta_{\mathrm{I}}) V_{\mathrm{I}} = 0 + \dot{V}_{\mathrm{I}}$, and noting $V_{\mathrm{I},t+k}(0) = V_{A,t+k}$, I have

$$\begin{aligned} V_{\mathrm{I},t}(k) &= V_{\mathrm{I},t+k}(0) e^{-(r+\delta_{\mathrm{I}})k} \\ &= V_{A,t+k} e^{-(r+\delta_{\mathrm{I}})k} \\ &= V_{A,t} e^{-(r+\delta_{\mathrm{I}}-g_L)k} \end{aligned}$$

where the last line follows from using $\frac{V_{A,t+k}}{V_{A,t}} = e^{g_L k}$.

Next, note that if we knew the distribution of k, i.e., the waiting time of non-adopted ideas, we could already compute the total value of intellectual property using

$$A_t \cdot V_{A,t} + (A_{\mathrm{F},t} - A_t) \cdot \int_0^{\tau} V_{\mathrm{I},t}(k)\, dF(k) + A_t^{*} \cdot V_{A,t}^{*} + (A_{\mathrm{F},t} - A_t^{*}) \cdot \int_0^{\tau^{*}} V_{\mathrm{I},t}(k^{*})\, dF(k^{*})$$

where $dF(k)$ is the appropriate marginal distribution over wait times ranging from zero to τ.

The distribution of k can be derived as follows. Note that the share of ideas close to adoption among all unadopted ideas depends on how much entry there is, which is directly proportional to the gross entry rate $g_{\mathrm{F}} + \delta_{\mathrm{I}}$. In fact, the distribution k follows a truncated exponential distribution

$$f(k) = \frac{(g_{\mathrm{F}} + \delta_{\mathrm{I}})\, e^{(g_{\mathrm{F}}+\delta_{\mathrm{I}})k}}{e^{(g_{\mathrm{F}}+\delta_{\mathrm{I}})\tau} - 1}, k \in [0, \tau],$$

where the distribution in the foreign economy features a different τ. Putting the pieces together, and focusing on the value of innovation in the advanced economy, yields

$$\begin{aligned}
A_t \cdot V_{A,t} + (A_{\mathrm{F},t} - A_t) \cdot \int_0^\tau V_{\mathrm{I},t}(k)\, dF(k) &= A_t \cdot V_{A,t} \left(1 + \left(\frac{A_{\mathrm{F},t}}{A_t} - 1\right) \cdot \int_0^\tau \frac{V_{\mathrm{I},t}(k)}{V_{A,t}} dF(k)\right) \\
&= A_t \cdot V_{A,t} \left(1 + \left(\frac{1-z}{z}\right) \cdot \int_0^\tau \frac{V_{\mathrm{I},t}(k)}{V_{A,t}} dF(k)\right) \\
&= A_t \cdot V_{A,t} \left(1 + \left(\frac{1-z}{z}\right) \cdot \int_0^\tau e^{-(r+\delta_{\mathrm{I}}-g_L)k} dF(k)\right) \\
&= A_t \cdot V_{A,t} \left(1 + \left(\frac{1-z}{z}\right) \cdot \int_0^\tau e^{-(r+\delta_{\mathrm{I}}-g_L)k} \frac{(g_{\mathrm{F}} + \delta_{\mathrm{I}})\, e^{(g_{\mathrm{F}}+\delta_{\mathrm{I}})k}}{e^{(g_{\mathrm{F}}+\delta_{\mathrm{I}})\tau} - 1} dk\right) \\
&= A_t \cdot V_{A,t} \left(1 + \left(\frac{1-z}{z}\right) \frac{g_{\mathrm{F}} + \delta_{\mathrm{I}}}{e^{(g_{\mathrm{F}}+\delta_{\mathrm{I}})\tau} - 1} \cdot \int_0^\tau e^{-(r-g_{\mathrm{F}}-g_L)k} dk\right) \\
&= A_t \cdot V_{A,t} \left(1 + \left(\frac{1-z}{z}\right) \frac{g_{\mathrm{F}} + \delta_{\mathrm{I}}}{e^{(g_{\mathrm{F}}+\delta_{\mathrm{I}})\tau} - 1} \cdot \int_0^\tau e^{-(\rho-g_L)k} dk\right) \\
&= A_t \cdot V_{A,t} \left(1 + \left(\frac{1-z}{z}\right) \frac{g_{\mathrm{F}} + \delta_{\mathrm{I}}}{e^{(g_{\mathrm{F}}+\delta_{\mathrm{I}})\tau} - 1} \cdot (\rho - g_L) \left[e^{-(\rho-g_L)k}\right]_\tau^0\right) \\
&= A_t \cdot V_{A,t} \left(1 + \left(\frac{1-z}{z}\right) \frac{1 - e^{-(\rho-g_L)\tau}}{e^{(g_{\mathrm{F}}+\delta_{\mathrm{I}})\tau} - 1} \cdot (g_{\mathrm{F}} + \delta_{\mathrm{I}})(\rho - g_L)\right) \\
&= A_t \cdot V_{A,t} \left(1 + \left(\frac{1-z}{z}\right) \frac{1 - z^{\frac{\rho-g_L}{g_{\mathrm{F}}+\delta_{\mathrm{I}}}}}{\frac{1-z}{z}} \cdot (g_{\mathrm{F}} + \delta_{\mathrm{I}})(\rho - g_L)\right) \\
&= A_t \cdot V_{A,t} \left(1 + \left(1 - z^{\frac{\rho-g_L}{g_{\mathrm{F}}+\delta_{\mathrm{I}}}}\right) \cdot (g_{\mathrm{F}} + \delta_{\mathrm{I}})(\rho - g_L)\right)
\end{aligned}$$

where I substituted out $\tau = -\frac{\log z}{g_{\mathrm{F}}+\delta_{\mathrm{I}}}$. Now adding the value of intellectual property across both countries yields

$$Tot_pat_value = \frac{\alpha L_{\mathrm{P}} w}{\tilde{\rho} + g_{\mathrm{F}} + \delta_{\mathrm{I}}} \left\{ \left(1 + \left(1 - z^{\frac{\rho-g_L}{g_{\mathrm{F}}+\delta_{\mathrm{I}}}}\right) \cdot (g_{\mathrm{F}} + \delta_{\mathrm{I}})(\rho - g_L)\right) + \underbrace{\frac{z^* b^*}{z} \left(1 + \left(1 - z^{*\frac{\rho-g_L}{g_{\mathrm{F}}+\delta_{\mathrm{I}}}}\right) \cdot (g_{\mathrm{F}} + \delta_{\mathrm{I}})(\rho - g_L)\right)}_{\text{value of patents abroad}} \right\}$$

where b^* keeps track of the relative country size $b^* = \frac{L_{\mathrm{P}}^*}{L_{\mathrm{P}}}$. Using this expression it is easy to compare the total value of ideas to the counterfactual autarky equilibrium. In the counterfactual autarky equilibrium, the value of patents abroad would be gone, and I evaluate adoption gap and wages at the counterfactual autarky level.

Next, I add up all assets in the economy and normalize by L to get

$$\begin{aligned} Assets &= \frac{Tot_pat_value}{L} + \frac{K}{L} + mV_M \\ &= w\left(\frac{Tot_pat_value}{Lw} + \frac{\alpha^2}{1-\alpha}\frac{l_P}{\rho + g_{\mathrm{F}} + \delta_K} + m \cdot f_{\mathrm{E}}\right). \end{aligned}$$

I can construct this expression and compare it conveniently across different counterfactual scenarios where all elements inside the parentheses are stationary. Lastly, note that one can simply apply the factor χ to the value of patents when both countries are innovation, and one can use the wage of the emerging market as normalizing factor to perform the same analysis.

A.5 Hopenhayn Version

A.5.1 Static Problem

I solve the static problem of the firm first. I ignore the overhead adoption cost for now but it will show up in the dynamic problem, of course. Recall the production function

$$y_i = \left(\left(\int_0^{A_i} \left(\frac{x_j}{\alpha}\right)^{\alpha} dj\right)\left(\frac{l_i^{\zeta}}{\zeta\left(1-\alpha\right)}\right)^{1-\alpha}\right)$$

where the reader should take account of the curvature induced by $\zeta < 1$.

The firm solves the static problem –abstracting away from the technology adoption margin for now– where the price of the final good is normalized to unity, $P_y := 1$.

$$\max_{x_j, l_i} y_i - w_L l_i + \int p_x x_j dj \tag{66}$$

I solve this problem in two stages. First, consider a given expenditure on intermediate goods $p_x \int x_j dj := R_X$ and solve for the optimal demand for each intermediate good

$$x_j^D = \frac{R_X}{P_X^{\frac{\alpha}{\alpha-1}}} p_{x,j}^{\frac{1}{\alpha-1}}$$

where the ideal price index reads $P_X = \left(\int p_{x,j}^{-\frac{\alpha}{1-\alpha}} dj\right)^{-\frac{1-\alpha}{\alpha}}$. Given symmetry, all prices are identical

so the firms optimally spreads total capital expenditure evenly across intermediates

$$x_j = \frac{\tilde{x}}{A_i} \ \forall j.$$

Now I plug this solution back into (66), and solve the following static maximization problem

$$\max_{X_i, l_i} \left(\left(\frac{\tilde{x}_i}{\alpha} \right)^{\alpha} \left(A_i \left(\frac{l_i}{\zeta (1-\alpha)} \right)^{\zeta} \right)^{1-\alpha} \right) - w_L l_i - p_x X_i$$

which leads to the following first-order conditions

$$\begin{aligned} y_i \alpha &= X_i p_x \\ y_i \zeta (1-\alpha) &= l_i w_L. \end{aligned}$$

Factor payments are constant shares of revenue, due to the Cobb-Douglas assumption, so firm operating profits equal $\Pi = y - l_i w - X_i p_x = (1-\zeta)(1-\alpha) y$. Moreover, after inverting the first order conditions and plugging them back into the production function, output appears as a function of of input prices, and importantly, of the productivity of the firm A_i

$$\begin{aligned} y_i &= \left(\frac{X_i}{\alpha} \right)^{\alpha} \left(A_i \left(\frac{l_i}{\zeta (1-\alpha)} \right)^{\zeta} \right)^{1-\alpha} \\ &= \left(\frac{y_i \alpha}{p_x \alpha} \right)^{\alpha} \left(A_i \left(\frac{\left(\frac{y_i \zeta (1-\alpha)}{w} \right)}{\zeta (1-\alpha)} \right)^{\zeta} \right)^{1-\alpha} \\ &= \left(\frac{y_i}{p_x} \right)^{\alpha} \left(A_i \left(\frac{y_i}{w} \right)^{\zeta} \right)^{1-\alpha} \\ &= y_i^{\alpha + \zeta(1-\alpha)} (p_x)^{-\alpha} \left(\frac{w^{\zeta}}{A_i} \right)^{-(1-\alpha)} \\ &= y_i^{\alpha + \zeta(1-\alpha)} (p_x)^{-\alpha} \left(\frac{w^{\zeta}}{A_i} \right)^{-(1-\alpha)} \\ &\Leftrightarrow \\ y_i &= \left\{ (p_x)^{-\alpha} \left(\frac{w^{\zeta}}{A_i} \right)^{-(1-\alpha)} \right\}^{\frac{1}{(1-\alpha)(1-\zeta)}}. \end{aligned}$$

Take account of the partial derivative of output with respect to productivity

$$\frac{\partial \log y}{\partial \log A} = \frac{1}{1-\zeta}, \tag{67}$$

which will become relevant for the dynamic problem considered next. Since $\pi = (1-\zeta)(1-\alpha)y$, it follows that $\frac{\partial \pi}{\partial A} = (1-\alpha)\frac{y}{A} = \frac{1}{1-\zeta}\frac{\pi}{A}$.

A.5.2 Dynamic Adoption Problem

I first solve the problem of an incumbent firm, i.e., the technology adoption problem, before I turn attention to the free entry condition. The dynamic problem reads

$$\begin{aligned}(r+\delta_X)V &= \max_{\iota} \Pi - w_H \iota + \dot{V} + V_A \dot{A} \\ &\text{s.t.} \\ \dot{A} &= \nu A^{\theta} A_{\mathrm{F}}^{1-\theta} \iota^{\beta} - \delta_{\mathrm{I}} A.\end{aligned}$$

To make the problem stationary, I normalize the value function by the real wage of unskilled labor, and I focus on the state variable $z := \frac{A}{A_{\mathrm{F}}}$, which will be constant in equilibrium while A and A_{F} will be continuously growing. Define $v := \frac{V}{w}$, and $z := \frac{A}{A_{\mathrm{F}}}$, and note that this change of variable has no effect on the underlying economics, which can now be studied in the stationary system

$$\begin{aligned}(r - g_w + \delta_X)v &= \pi - s\iota + \dot{v} + v_z \dot{z} \\ &\text{s.t.} \\ \dot{z} &= \nu z^{\theta} h^{\beta} - (\delta_I + g_F) z.\end{aligned} \tag{68}$$

An interior solution to the system (68) satisfies the first-order condition

$$\iota = \left\{ \frac{v_z \beta \nu z^{\theta}}{s} \right\}^{\frac{1}{1-\beta}}, \tag{69}$$

which captures the trade-off between the cost of adoption in terms of the price of skilled labor, and the benefit of improving the firm's productivity.

To derive the differential equation that governs optimal skilled labor investment choices, I differ-

entiate (68) with respect to z, and after using the Envelope theorem, I arrive at

$$(r - g_w + \delta_X) = \frac{\pi}{v_z} + \frac{\dot{v}_z}{v_z} + \frac{v_{zz}\dot{z}}{v_z} + \nu\theta z^{\theta-1}\iota^{\beta} - (\delta_I + g_F)\,. \tag{70}$$

Now taking logs of (69) and differentiating with respect to time yields

$$\frac{\dot{\iota}}{\iota} = \frac{1}{1-\beta}\left\{\frac{\dot{v}_z}{v_z} + \frac{v_{zz}}{v_z}\dot{z} + \theta\frac{\dot{z}}{z} - \frac{\dot{s}}{s}\right\}. \tag{71}$$

Now plug (70) into (71) to get

$$\begin{aligned}
\frac{\dot{\iota}}{\iota} &= \frac{1}{1-\beta}\left\{(r - g_w + \delta_X) - \frac{\pi_z}{v_z} - \theta\nu z^{\theta-1}\iota^{\beta} + (\delta_{\mathrm{I}} + g_{\mathrm{F}}) + \theta\frac{\dot{z}}{z} - \frac{\dot{s}}{s}\right\} \\
\frac{\dot{\iota}}{\iota} &= \frac{1}{1-\beta}\left\{(r - g_w + \delta_X) - \frac{\pi_z}{v_z} - \theta\nu z^{\theta-1}\iota^{\beta} + (\delta_{\mathrm{I}} + g_{\mathrm{F}}) + \theta\left[\nu z^{\theta-1}\iota^{\beta} - (\delta_A + g_F)\right] - \frac{\dot{s}}{s}\right\} \\
\frac{\dot{\iota}}{\iota} &= \frac{1}{1-\beta}\left\{r - g_w + \delta_X + (1-\theta)(\delta_{\mathrm{I}} + g_{\mathrm{F}}) - \frac{\dot{s}}{s} - \frac{\pi_z}{v_z}\right\} \\
\frac{\dot{\iota}}{\iota} &= \frac{1}{1-\beta}\left\{r - g_w + \delta_X + (1-\theta)(\delta_{\mathrm{I}} + g_{\mathrm{F}}) - \frac{\dot{s}}{s} - \frac{\beta\iota^{\beta-1}\nu z^{\theta-1}}{s}\pi_z z\right\}.
\end{aligned}$$

The final piece to derive the dynamic investment equation involves deriving the partial elasticity of profits with respect to productivity, which is $\frac{1}{1-\zeta}$ as can be seen in equation (67), which yields

$$\frac{\dot{\iota}}{\iota} = \frac{1}{1-\beta}\left\{r - g_w + \delta_X + (1-\theta)(\delta_{\mathrm{I}} + g_{\mathrm{F}}) - \frac{\dot{s}}{s} - \frac{\beta\nu z^{\theta}\iota^{\beta-1}}{s}\left[\frac{1}{1-\zeta}\frac{\pi}{z}\right]\right\}. \tag{72}$$

For now, assume that a well-defined steady state exists, which I will prove later, together with stability and uniqueness properties, and assume that in such a steady state the adoption gap is constant and firms higher a constant number of skilled workers to adopt technology, i.e., $\dot{z} = 0$ and $\dot{\iota} = 0$ so that

$$z = \left(\frac{\nu}{\delta_{\mathrm{I}} + g_{\mathrm{F}}}\right)^{\frac{1}{1-\theta}}(\iota)^{\frac{\beta}{1-\theta}} \quad \text{(from resource constraint } \dot{z}\text{)}$$

Using this link, one can further simplify (72) to derive h_* in the steady state, which is inversely related to the skill premium,

$$r - g_w + \delta_X + (1-\theta)(\delta_I + g_F) = \frac{(\delta_I + g_F)}{\iota s}\frac{\beta}{1-\zeta}\pi$$

$$\Leftrightarrow$$

$$\iota = \frac{1}{s}\frac{(\delta_{\mathrm{I}} + g_{\mathrm{F}})\frac{\beta}{1-\zeta}}{r - g_w + \delta_X + (1-\theta)(\delta_{\mathrm{I}} + g_{\mathrm{F}})}\pi. \tag{73}$$

One can verify that the following inequality is necessary for a well-defined balanced growth path with positive adoption gap

$$\frac{\beta}{1-\zeta} < \frac{r - g_w + \delta_X}{\delta_{\mathrm{I}} + g_{\mathrm{F}}} + (1-\theta). \tag{74}$$

Proof. By contradiction, suppose that $\frac{\beta}{1-\zeta} \geq \frac{r-g_w+\delta_X}{\delta_{\mathrm{I}}+g_{\mathrm{F}}} + (1-\theta)$. In that case, spending on learning in the steady state, ιs, is weakly larger than total operating profits π, so that the net profits $\frac{\pi}{w} - s\iota$ are non-positive. However, firms need to make positive profits to cover the initial fixed cost of entry. Thus, any well-defined balanced growth path requires (74) to hold. □

A.5.3 Free Entry

To close the model, Hopenhayn (1992) introduces a free entry condition such that the cost of entry, paid in labor, equals the present discounted value of the firm

$$f_{\mathrm{E}} w_{L,t} = \int V dF_t(A). \tag{75}$$

From here on out, the steps are virtually identical to the baseline model.

In the main part of the paper I assume strong spillovers in the form of entrants drawing from the incumbent distribution. I next show how to make this entry spillover weaker, which gives rise to a heterogeneous firm model.

A.6 Heterogeneous Firms with Partial Knowledge Spillovers

A simplifying assumption in the paper is the complete knowledge spillover from incumbents to entrants. An alternative specification is one where the entrant only obtains a fraction $\lambda_{\mathrm{E}}\mathbb{E}[z]$ where $\lambda_{\mathrm{E}} < 1$. This tweak turns the setting into a heterogeneous firm model where entrants learns from the most sophisticated incumbent, but imperfectly so. A well-defined equilibrium is characterized by a distribution $f(z)$ with support $z \in [\lambda_{\mathrm{E}}\mathbb{E}[z], z_{\max}]$. This leads to a normalized free entry condition

$$f_{\mathrm{E}} = v(\lambda_{\mathrm{E}}\mathbb{E}[z]).$$

Building on Hopenhayn (1992) and Melitz (2003), the profit ratio of any two firms can be expressed as $\frac{\pi_i}{\pi_j} = \left(\frac{z_i}{z_j}\right)^{(1-\alpha)(\sigma-1)}$, and normalized profits for firm i are given by $\pi(z_i) = \frac{(z_i)^{(1-\alpha)(\sigma-1)}}{\mathbb{E}\left[z^{(1-\alpha)(\sigma-1)}\right]} \frac{l_P}{m(\sigma-1)(1-\alpha)}$. Instead of using i subscript, I now index firms by z.

Next, consider the problem of some firm with productivity z in the steady state (so $\dot{v} = 0$ and $r - g_{w_L} = \rho$)

$$(\rho + \delta_{\mathrm{X}})\, v(z) = \max_{\iota} \pi(z) - s\iota + (\partial_z v) \cdot \left[\nu z^{\theta} \iota^{\beta} - (g_{\mathrm{F}} + \delta_{\mathrm{I}})\, z\right].$$

The first order condition still reads

$$\iota(z) = \left\{ \frac{(\partial_z v)\, \beta \nu z^{\theta}}{s} \right\}^{\frac{1}{1-\beta}}.$$

I assume for simplicity that $\dot{s} = 0$ and derive a similar dynamic investment equation as for the homogeneous firm case

$$\frac{\dot{\iota}}{\iota}(1-\beta) = (\rho + \delta_{\mathrm{X}} + (1-\theta)(g_{\mathrm{F}} + \delta_{\mathrm{I}})) - \frac{\beta \nu z^{\theta} \iota^{\beta-1}}{s} \left[\pi \frac{(1-\alpha)(\sigma-1)}{z}\right].$$

It is useful to rewrite this expression relative to the firms with the maximum productivity

$$\begin{aligned}\frac{\dot{\iota}}{\iota}(1-\beta) = {} & (\rho + \delta_{\mathrm{X}} + (1-\theta)(g_{\mathrm{F}} + \delta_{\mathrm{I}})) \\ & - \frac{\beta(\sigma-1)(1-\alpha)(z_{\max})^{\theta-1} h_{\max}^{\beta-1} \nu}{s} \pi_{\max} \left(\frac{z}{z_{\max}}\right)^{\theta-1+(1-\alpha)(\sigma-1)} \left(\frac{\iota_{\max}}{\iota}\right)^{1-\beta}\end{aligned}$$

which helps to pin down the equilibrium dynamics. By construction, the most productive firm hires a constant amount of skilled labor with the only difference to the homogenous firm model being that the steady state profits are larger. This is a direct consequence of starting out with an initially lower productivity. Higher long-run profits have to make up for low profits after the firm just entered, since the entry cost are the same in both cases.

For this equilibrium to be well-defined, I need entering firms to improve their productivity so that they converge to the more profitable incumbents in the long-run. This is not trivial. One way to see this is to sue a phase-diagram in the h-z space assuming a stationary equilibrium exist for some $z_{\max}$. Since there is no risk the differential equation I have derived beforehand applies. Slightly rewriting and ignoring a a scalar $1-\beta$ (which does not matter for the issue of existence but surely matters for the speed of convergence) leads to

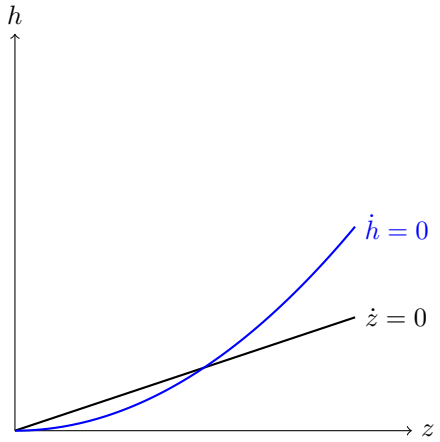
$$\frac{\dot{\iota}}{\iota} = \kappa_1 \left(1 - \left(\frac{z}{z_{\max}}\right)^{\theta-1+(1-\alpha)(\sigma-1)} \left(\frac{\iota_{\max}}{\iota}\right)^{1-\beta}\right).$$

Assuming $\dot{\iota} = 0$ implies $\iota = \left(\frac{z}{z_{\max}}\right)^{\frac{\theta-1+(1-\alpha)(\sigma-1)}{1-\beta}} \iota_{\max}$. Moreover, assume $\frac{\theta-1+(1-\alpha)(\sigma-1)}{1-\beta} > 0$ so the ι-locus is upward-sloping in z. In addition, note that the z-locus implies a positive link between ι and z as well, $\iota = \left(\frac{g_{\mathrm{F}}+\delta_{\mathrm{I}}}{\nu} z^{1-\theta}\right)^{\frac{1}{\beta}}$. When

$$\frac{\theta - 1 + (1-\alpha)(\sigma-1)}{1-\beta} < \frac{1-\theta}{\beta},$$

one can show that the z-locus cuts the h-locus once from below. The associated stability analysis suggests convergence from below, see figure 16.

Figure 16. Phase Diagram



This structure gives rise to a meaningful stationary distribution whereby firms start out small and improve their productivity over time. Several features are noteworthy. First, the firm size distribution is independent of the relative price of skill s. What this suggests is that a new stationary equilibrium with a higher price of skill produces an identical wave but shifted to the left, i.e., a permanently lower level of adoption across all firms. This traveling wave property is not surprising in light of recent work on heterogeneous firms, see Luttmer (2007), König, Lorenz, and Zilibotti (2016), Sampson (2016), Benhabib, Perla, and Tonetti (2021) and Perla and Tonetti (2014). Crucially, the partial equilibrium elasticity $\frac{\partial \log \mathbb{E}[z]}{\partial \log s} = -\frac{\beta}{1-\theta}$ computed in the main text still applies. Second, demand for skilled labor in the production sector can be derived by integrating over all productivity levels

$$h_{\mathrm{D}} = m \int_{\lambda_{\mathrm{E}} \mathbb{E}[z]}^{z_{\max}} f(z)\, h(z)\, dz.$$

Third, the innovator problem in the steady state needs to be updated as follows

$$V = \mathbb{E}_z \left[V(z) \right] \tag{76}$$

where $V(z) = \int_{t+\tau(z)}^{\infty} \exp\left(- \int_t^s \left[r_u + \delta_{\mathrm{X}} \right] du \right) \Pi_{\mathrm{I}}(z,s)\, ds$ is a function of the firm-specific z-level which matters both in terms of firm size and how long it takes for an idea to be adopted by a firm of type z. The problem is conceptually the same as before except now one needs to keep track of the distribution of firm-specific adoption gaps. Of course, equation (76) is also conceptually very close to the value function of an innovator in the open economy in the main text, which is a discretized version of (76) where z's pertain to different countries.

One desirable feature of the heterogeneous firm model is that it can speak to the distinction between the extensive and intensive margin of technology adoption, i.e., whether a specific technology is used in a country, and whether all firms in a country use this specific technology, see Comin and Hobijn (2004) and Banerjee and Duflo (2005) for empirical evidence on the importance of the intensive margin, i.e., differences across firms within a country.

A.7 Generalization of Model with Skilled Labor in Entry & Production

Skilled Labor in Entry and Production. I next generalize the model with skilled labor in entry and production. I focus on steady states and drop time subscripts. Suppose it takes

$$\left(\frac{l}{1-\mu} \right)^{1-\mu} \left(\frac{\iota_{\mathrm{E}}}{\mu} \right)^{\mu} / f_{\mathrm{E}}$$

units of unskilled and skilled labor to produce a unit flow of new entrants. Similarly, suppose production also requires a mix of both skill types where I use the same functional form with skill share μ. I hold the labor share fixed at $1-\alpha$ but now use the skill-aggregator E

$$E = \left(\frac{L_{\mathrm{P}}}{1-\mu} \right)^{1-\mu} \left(\frac{H_{\mathrm{P}}}{\mu} \right)^{\mu}.$$

Focusing on entry first, note that unskilled and skilled labor devoted to entry in normalized form ($l_{\mathrm{E}} = \frac{L_{\mathrm{E}}}{L}$, $h_{\mathrm{E}} = \frac{H_{\mathrm{E}}}{L}$) equals

$$\begin{aligned} l_{\mathrm{E}} &= (1-\mu)\, m \left(\delta_{\mathrm{X}} + g_L \right) s^{\mu}, \\ h_{\mathrm{E}} &= \mu m \left(\delta_{\mathrm{X}} + g_L \right) s^{-(1-\mu)}, \\ h_{\mathrm{E}} &= \frac{\mu}{1-\mu} \frac{1}{s} l_{\mathrm{E}}. \end{aligned}$$

Moreover, using free entry I can pin down normalized equilibrium profits.

$$\begin{aligned}\pi - s\iota &= (\rho + \delta_{\mathrm{X}}) f_{\mathrm{E}} s^{\mu} \\ \pi (1 - \kappa_{\pi}) &= (\rho + \delta_{\mathrm{X}}) f_{\mathrm{E}} s^{\mu} \\ &\Rightarrow \\ \pi &= \frac{\rho + \delta_{\mathrm{X}}}{1 - \kappa_{\iota}} f_{\mathrm{E}} s^{\mu}.\end{aligned}$$

Note that equilibrium demand for skilled labor on the firm level thus equals

$$\begin{aligned}\iota &= \frac{1}{s} \kappa_{\iota} \pi \\ \iota &= s^{-(1-\mu)} \frac{\kappa_{\iota}}{1 - \kappa_{\iota}} (\rho + \delta_{\mathrm{X}}) f_{\mathrm{E}}. \end{aligned} \tag{77}$$

Now one can use this result to compute

$$\frac{\partial \log z}{\partial \log s} = - (1 - \mu) \frac{\beta}{1 - \theta},$$

which generalizes the main result in the paper.

Moreover, the free entry condition also pins down unskilled labor devoted to entry

$$\begin{aligned}\pi - s\iota &= (\rho + \delta_{\mathrm{X}}) f_{\mathrm{E}} s^{\mu} \\ \frac{l_{\mathrm{P}}}{m} \frac{1 - \kappa_{\pi}}{(\sigma - 1)(1 - \alpha)(1 - \mu)} &= (\rho + \delta_{\mathrm{X}}) f_{\mathrm{E}} s^{\mu} \\ \frac{1 - l_{\mathrm{E}}}{m} \frac{1 - \kappa_{\pi}}{(\sigma - 1)(1 - \alpha)(1 - \mu)} &= (\rho + \delta_{\mathrm{X}}) f_{\mathrm{E}} s^{\mu} \\ \frac{1 - l_{\mathrm{E}}}{l_{\mathrm{E}}} \frac{(1 - \kappa_{\pi})((1 - \mu)(\delta_{\mathrm{X}} + g_L) s^{\mu})}{(\sigma - 1)(1 - \alpha)(1 - \mu)} &= (\rho + \delta_{\mathrm{X}}) f_{\mathrm{E}} s^{\mu} \\ &\Rightarrow \\ l_{\mathrm{P}} (1 - \kappa_{\pi}) (\delta_{\mathrm{X}} + g_L) &= l_{\mathrm{E}} (\sigma - 1)(1 - \alpha)(\rho + \delta_{\mathrm{X}}) f_{\mathrm{E}}\end{aligned}$$

This implies unskilled labor devoted to entry and production equals

$$\begin{aligned}l_{\mathrm{E}} &= \frac{(\delta_{\mathrm{X}} + g_L)(1 - \kappa_{\pi})}{(\sigma - 1)(1 - \alpha)(\rho + \delta_{\mathrm{X}}) f_{\mathrm{E}} + (\delta_{\mathrm{X}} + g_L)(1 - \kappa_{\pi})}, \\ l_{\mathrm{P}} &= \frac{(\sigma - 1)(1 - \alpha)(\rho + \delta_{\mathrm{X}}) f_{\mathrm{E}}}{(\sigma - 1)(1 - \alpha)(\rho + \delta_{\mathrm{X}}) f_{\mathrm{E}} + (\delta_{\mathrm{X}} + g_L)(1 - \kappa_{\pi})}.\end{aligned} \tag{78}$$

High skilled labor used for entry depends on the skill premium and reads

$$\begin{aligned} h_{\mathrm{E}} &= \frac{\mu}{1-\mu}\frac{1}{s}l_{\mathrm{E}}, \\ &= \frac{\mu}{1-\mu}\frac{1}{s}\frac{\left(\delta_{\mathrm{X}}+g_L\right)\left(1-\kappa_\pi\right)}{\left(\sigma-1\right)\left(1-\alpha\right)\left(\rho+\delta_{\mathrm{X}}\right)f_{\mathrm{E}}}l_{\mathrm{P}}. \end{aligned}$$

Next, using the free entry condition, $\pi - s\iota = \left(\rho+\delta_{\mathrm{X}}\right)f_{\mathrm{E}}s^{\mu}$, I can solve for the equilibrium measure of firms, which now becomes a function of the skill premium as well

$$\begin{aligned} \pi - s\iota &= \left(\rho+\delta_{\mathrm{X}}\right)f_{\mathrm{E}}s^{\mu} \\ \frac{L_{\mathrm{P}}w_L\frac{\sigma}{\sigma-1}\frac{1}{(1-\alpha)(1-\mu)}}{Mw_L}\frac{\left(1-\kappa_\iota\right)}{\sigma} &= \left(\rho+\delta_{\mathrm{X}}\right)f_{\mathrm{E}}s^{\mu} \\ \frac{l_{\mathrm{P}}}{m}\frac{\left(1-\kappa_\iota\right)}{\left(\sigma-1\right)\left(1-\alpha\right)\left(1-\mu\right)} &= \left(\rho+\delta_{\mathrm{X}}\right)f_{\mathrm{E}}s^{\mu}. \end{aligned}$$

This implies that the normalized measure of firms is falling in the skill premium,

$$m = s^{-\mu}\frac{l_{\mathrm{P}}}{\left(\rho+\delta_{\mathrm{X}}\right)f_{\mathrm{E}}}\frac{1-\kappa_\pi}{\left(\sigma-1\right)\left(1-\alpha\right)\left(1-\mu\right)}.$$

Next, I turn attention to skilled labor in production. Cost minimization implies that the price of this labor bundle equals $w_E = w_L^{1-\mu}w_H^{\mu}$. Total spending on unskilled workers in production now equals $L_{\mathrm{P}}w_L = \left(1-\mu\right)\left(1-\alpha\right)\frac{\sigma-1}{\sigma}Y_t$. Skilled labor in production can be written as a function of unskilled labor and the skill premium, $H_{\mathrm{P}} = \frac{\mu}{1-\mu}L_{\mathrm{P}}\frac{1}{s}$. This also highlights the labor devoted to innovation is largely unchanged, $H_{\mathrm{I}} = \frac{1}{s}\frac{g_{\mathrm{F}}}{\tilde{\rho}+g_{\mathrm{F}}}\frac{\alpha L_{\mathrm{P}}}{1-\mu}z^{\frac{\tilde{\rho}}{g_A}}$. Lastly, consider aggregate skilled labor devoted to adoption

$$\begin{aligned} \iota M &= \frac{1}{s}\frac{\beta\left(1-\alpha\right)\left(\sigma-1\right)g_{\mathrm{F}}}{\rho+\delta_{\mathrm{X}}+\left(1-\theta\right)g_{\mathrm{F}}}\pi M \\ &= \frac{1}{s}\frac{\beta\left(1-\alpha\right)\left(\sigma-1\right)g_{\mathrm{F}}}{\rho+\delta_{\mathrm{X}}+\left(1-\theta\right)g_{\mathrm{F}}}\frac{Y}{Mw_L}\frac{M}{\sigma} \\ &= \frac{1}{s}\frac{\beta\left(1-\alpha\right)\left(\sigma-1\right)g_{\mathrm{F}}}{\rho+\delta_{\mathrm{X}}+\left(1-\theta\right)g_{\mathrm{F}}}\frac{L_{\mathrm{P}}}{\left(1-\mu\right)\left(\sigma-1\right)\left(1-\alpha\right)}. \end{aligned}$$

Surprisingly, aggregate adoption effort is not changing as the rising individual adoption effort is exactly offset by the decline in the measure of firms.

Adding up yields the market clearing condition

$$\underbrace{\frac{1}{s}\frac{\mu}{1-\mu}\frac{\left(\delta_{\mathrm{X}}+g_L\right)\left(1-\kappa_\pi\right)}{\left(\sigma-1\right)\left(1-\alpha\right)\left(\rho+\delta_{\mathrm{X}}\right)f_{\mathrm{E}}}l_{\mathrm{P}}}_{h_{\mathrm{E}}} + \underbrace{\frac{1}{s}\frac{\mu}{1-\mu}l_{\mathrm{P}}}_{h_{\mathrm{P}}} + \underbrace{\frac{\kappa_\pi}{s}\frac{1}{\left(1-\mu\right)\left(\sigma-1\right)\left(1-\alpha\right)}l_{\mathrm{P}}}_{h_{\mathrm{A}}} + \underbrace{\frac{1}{s}\frac{g_{\mathrm{F}}}{\tilde{\rho}+g_{\mathrm{F}}}z\left(s\right)^{\frac{\tilde{\rho}}{g_A}}\frac{\alpha l_{\mathrm{P}}}{1-\mu}}_{h_{\mathrm{I}}} = h,$$

and it is easy to see how I nest the expression in the main text for $\mu = 0$.

Consider a special case with no innovation in the laggard country ($\gamma \to 0$) in an open economy equilibrium. This special case is to show that aggregate skill endowments continue to be the key determinant of adoption gaps across countries that do not innovate. Thus, dropping labor devoted to innovation from the market clearing condition yields

$$\underbrace{\frac{1}{s}\frac{\mu}{1-\mu}\frac{(\delta_X + g_L)(1-\kappa_\pi)}{(\sigma-1)(1-\alpha)(\rho+\delta_X)f_E}l_P}_{h_E} + \underbrace{\frac{1}{s}\frac{\mu}{1-\mu}l_P}_{h_P} + \underbrace{\frac{\kappa_\pi}{s}\frac{1}{(\sigma-1)(1-\alpha)}l_P}_{h_A} = h. \tag{79}$$

$$\frac{1}{s}l_P\left(\frac{\mu}{1-\mu}\frac{(\delta_X + g_L)(1-\kappa_\pi)}{(\sigma-1)(1-\alpha)(\rho+\delta_X)f_E} + \frac{\mu}{1-\mu} + \kappa_\pi\frac{1}{(\sigma-1)(1-\alpha)}\right) = h \tag{80}$$

Equation (79) highlights that the skill premium is directly proportional to normalized labor endowment. This is the case because l_P is independent of the skill premium, see equation (78), and all other parameters are identical across countries and simply scale the market clearing condition. Consequently, in equilibrium, we have

$$\frac{1}{s} \propto h.$$

Using this, together with (77), it follows immediately that the adoption gap is proportional to

$$z \propto s^{-(1-\mu)\frac{\beta}{1-\theta}} \propto h^{(1-\mu)\frac{\beta}{1-\theta}}.$$

In this generalized version, the link between skill premium and adoption gap changes to

$$-\frac{\partial \log z}{\partial \log s} = (1-\mu)\frac{\beta}{1-\theta}. \tag{81}$$

This adjustment is entirely due to the role of skilled labor in entry and unrelated to skilled labor in production. The intuition for this result is that the benefit of technology adoption is tied to firm profits, which are tied to the entry cost. In the limiting case where entry only requires skilled labor, an increase in the price of skill, which raises the cost of adoption, is exactly offset by an increase in firm profits. Any intermediate case in which entry is less skill-intensive than adoption preserves the link between the skill premium and technology adoption. Importantly, there is a degree of freedom in the parameter $\beta/(1-\theta)$: for any $\mu < 1$, one can rescale $\beta/(1-\theta)$ to match the same reduced-form passthrough from skill premium to technology adoption, $-\frac{\partial \log z}{\partial \log s}$. In this sense, allowing skilled labor in entry or production does not materially alter the model's predictions, nor does it break the central link between skill premia and technology adoption emphasized in the paper.

So why not incorporating skilled labor in entry? When entry costs increase with the skill premium, so does equilibrium firm size. More formally, normalized profits are proportional to the skill premium, $\pi \propto s^\mu$, and the opposite is true for the equilibrium measure of firms, $m \propto s^{-\mu}$. Hence,

poor countries with high skill premia would be predicted to have relative large and relatively few firms compared to advanced economies, inconsistent with a large body of evidence.[65] The baseline model avoids this implication, which implicitly assumes that entrepreneurs are available in both poor and rich economies.

Endogenous Skilled Labor Supply. A more elastic supply of skills attenuates the innovation–adoption trade-off in general equilibrium by dampening movements in the skill premium. In fact, in the limit of perfectly elastic skilled labor supply, adoption gaps are constant. I largely abstract from endogenous skill supply, consistent with the empirical observation that the expansion of skilled labor has slowed substantially while inequality has continued to rise.

To develop this argument formally, suppose that an exogenously growing population of workers split between unskilled and skilled labor $N = L + H$ where $\psi(s)$ is the share of workers that are skilled, which is now an upward sloping function of the skill premium. I drop labor skilled labor in entry and production for ease of exposition, and derive the updated market clearing condition for skilled labor

$$\frac{1}{s}\frac{g_{\mathrm{F}}}{\tilde{\rho}+g_{\mathrm{F}}}z(s)^{\frac{\tilde{\rho}}{g_A}}\alpha l_{\mathrm{P}}+\frac{\kappa_\pi}{s}\frac{1}{(\sigma-1)(1-\alpha)}l_{\mathrm{P}}=\frac{\psi(s)}{1-\psi(s)}.$$

The key difference is that the right hand side is upward-sloping in the skill premium, capturing the idea that as skill becomes scarce, households supply more skilled labor. I impose the following functional form $\frac{\psi(s)}{1-\psi(s)} = \psi_0 (s-\kappa)^\eta$ where the baseline case refers to $\eta \to 0$. As $\eta \to \infty$ and skilled labor supply becomes perfectly elastic,[66] the skill premium is exogenously fixed at $s = \kappa > 1$. In that scenario, the adoption margin is mute, and the implications of the theory with respect to a market integration shock are extremely similar to the simpler model of Jones (1995) without adoption margin and one type of labor. I highlight the role of skill scarcity in driving weak technology adoption. Surely skilled labor supply is somewhat elastic, but not nearly enough to counteract the substantial increase in the skill premium observed over the past couple of decades. If the reader wants to pursue this specification, it helps to chose a different normalization based on N_t,since L_t is now endogenous, i.e., $a_{\mathrm{F}} = \frac{A_{\mathrm{F}}^{1-\phi}}{N_t}$ etc., but the basic structure of the theory is otherwise unchanged.

This extension with endogenous labor supply also approximates another natural extension, which is to allow for unskilled labor to be used in the research sector as well. For concreteness, suppose that the entry cost combines each labor type according to a Cobb-Douglas production function with skilled labor share μ_{I}, so the entry cost becomes $f_{\mathrm{I}}(w_H)^{\mu_{\mathrm{I}}}(w_L)^{1-\mu_{\mathrm{I}}}$. In that case, an expansion of the research sector will simultaneously raise demand for unskilled labor.[67] It will still be true, though, that the impact on unskilled worker wages can be characterized by the evolution of frontier technology A_{F} and the relative technology level z alone. Instead of making additional assumptions on the

[65] See, for instance, Poschke (2018) for a systematic study of firm size along the development spectrum which finds that firm size is increasing in development.

[66] I do not model the actual investment cost, see for instance Acemoglu (2009) for a model with explicit schooling choice.

[67] See Helpman (2016) for a summary of the large literature on factor-biased trade and inequality.

factor intensity in each sector, one can capture these considerations by allowing the supply of skilled labor to be upward sloping in the skill premium.

A.8 Adoption and Innovation in Models with Intermediate Goods Trade and Armington Differentiation

clarify the result that technology adoption, conditional on the skill premium, does not respond to market integration, and investigate whether this finding survives in richer models of international trade. In particular, I ask whether allowing for intermediate goods trade a la Krugman (1980), or Armington-style differentiation of final goods, overturns this invariance result.

It does not. Firm-level adoption effort remains unrelated to globalization even when the framework allows for intermediate goods trade with iceberg trade costs for shipping goods from origin k to destination n: $\tau_{kn} \geq 1$; $\tau_{kk} = 1$. This holds both in the standard Krugman model with love-of-variety effects and in a version where these effects are neutralized via the normalization $M_t^{-\frac{1}{\sigma-1}}$. In the latter case, the model collapses to an Armington structure in which the measure of firms is irrelevant, but countries still produce differentiated goods.

This can be seen formally from the free-entry condition. In the open economy, firm profits are

$$\Pi_{k,t}^{\text{open}} = \sum_k \Pi_{kn,t}^{\text{open}} = \sum_n p_{kn,t} \left(\frac{1}{\sigma}\right) y_{kn} = \frac{w_{L,k,t}}{A_{k,t} k^{\alpha}} \frac{1}{\sigma - 1} \sum_n \tau_{kn} y_{kn} \quad .$$

where $k := \frac{w_{L,k,t}}{p_x A_{k,t}} = \frac{\alpha}{1-\alpha} \frac{K_{k,t}}{A_{k,t} L_{\text{P},k,t}}$, and producing $y_{kn,t}$ units of output requires $\frac{\tau_{kn} y_{kn}}{A_k k^{\alpha}}$ units of unskilled labor.

Let l_{kn} denote labor devoted to destination n. Then profits can be written as

$$\Pi_{k,t}^{\text{open}} = \frac{w_{L,k,t}}{\sigma - 1} \sum_n l_{kn}.$$

Free entry implies

$$\frac{w_{L,k,t} f_{\text{E}}}{1 - \kappa_\iota} = \frac{w_{L,k,t}}{\sigma - 1} \sum_n l_{kn},$$

which implies

$$\sum_n l_{kn} = l_k^{closed}.$$

Thus, opening to trade changes the composition of production across markets but not total firm employment. Since adoption incentives depend on firm size rather than destination-specific sales, globalization has no direct effect on technology adoption.

Of course, wages now depend on foreign demand for domestic goods, and are generally higher in an open economy. However, wages cancel out of the free-entry condition, so entry and adoption

incentives are unchanged. In particular,

$$\frac{\Pi_{k,t}^{\text{open}}}{w_{L,k,t}^{\text{open}}}\left(1-\kappa_{\iota}^{open}\right)=\frac{\Pi_{k,t}^{\text{closed}}}{w_{L,k,t}^{\text{closed}}}\left(1-\kappa_{\iota}^{closed}\right)$$

For this to be true, I need to show that $\kappa_{\iota}^{open}=\kappa_{\iota}^{closed}$. For this to hold, it is sufficient that firms devote a constant fraction of normalized profits to technology adoption. This continues to be true because profits are log-linear in productivity and additive across destinations:

$$\partial_z \Pi_{k,t}^{\text{open}}=\sum_n \frac{(\sigma-1)(1-\alpha)}{z}\Pi_{kn,t}=\frac{(\sigma-1)(1-\alpha)}{z}\sum_n \Pi_{kn,t}.$$

Interestingly, a very similar result holds for innovation, where, conditional on wages, the value of an innovator remains unchanged in the open economy. Recall

$$v_{\text{I},k}=\frac{1}{\tilde{\rho}+g_{\text{F}}}\frac{\alpha l_{\text{P}}}{a_{\text{F}}}\sum_n \frac{w_{L,n,t}}{w_{L,k,t}}z_n^{\frac{\tilde{\rho}}{g_A}}, \tag{82}$$

which is the appropriate expression for the value of innovation, and is fully pinned down by wages. Again, these will be different in a richer model of trade, but the impact on innovation will still be fully summarized by (82). Taking the role of trade costs more seriously and allowing for costly repatriation of profits yields

$$v_{\text{I},k}=\frac{1}{\tilde{\rho}+g_{\text{F}}}\frac{\alpha l_{\text{P}}}{a_{\text{F}}}\sum_n \frac{\frac{w_{L,n,t}}{w_{L,k,t}}z_n^{\frac{\tilde{\rho}}{g_A}}}{\tau_{nk}}. \tag{83}$$

In either case, normalized innovation incentives depend only on relative wages, not on market size per se. An equal increase in unskilled wages across countries therefore leaves domestic innovation unchanged, which is necessary for balanced growth. If foreign unskilled wages rise faster than domestic wages –as appears plausible for the 2000s– the incentive to innovate in the West becomes even stronger. From this perspective, the specialization effect of globalization on innovation is likely understated, while the case for large real-income losses among unskilled workers in advanced economies becomes weaker.

Finally, this invariance result can be broken by introducing Melitz-style selection. When trade affects which firms operate and export, average firm size and the firm size distribution change, and adoption incentives respond. I develop such a heterogeneous-firm extension in the appendix. One result that survives is that, all else equal, a higher skill premium still reduces technology adoption. The aggregate growth effects of this extension depend critically on the nature of spillovers across firms. A useful comparison is Atkeson and Burstein (2010), where endogenous innovation matters little for

the impact of globalization on growth because the model is efficient; see the insightful discussion in Perla, Tonetti, and Waugh (2021).

A.9 Other Model Extensions

Stochastic Adoption. Since asset markets are complete and there are no stochastic shocks, risk plays no role when potential innovators consider entry into innovation. It is thus not surprising that stochastic adoption does not change any of the results qualitatively.

For example, a different version that I have experimented with is to let un-adopted ideas to be uniformly sampled at Poisson rate $\frac{A(g_A+\delta_I)dt}{A_F-A} = \frac{z}{1-z}(g_A+\delta_I)$ where $\frac{1}{A_F-A}$ is the uniform density and $A(g_A+\delta_I)dt$ is the flow of new ideas that are adopted at each instant. The probability density is then simply the product of the two, given statistical independence. On a balanced growth path with constant relative technology level z, it is again true that a z close to unity makes the adoption friction vanish. In contrast, as z approaches zero, the net present value of an innovation falls to zero as well since the adoption probability converges to zero as well.

Using this alternative functional form, one can follow the same steps as in the main text and compute the expected present discounted value of a patent. The insight that adoption and innovation are complementary on the market for ideas are robust to this alternative functional form. While stochastic adoption is more realistic in the sense that most innovators do not know when, if ever, their idea becomes profitable, this version of the model would be slightly less tractable regarding the market clearing condition for skilled labor.

The value function of an innovator would now look as follows

$$(r - g_V + \delta_I) V_I = 0 + \lambda(z)\left(V_I^{\text{adopted}} - V_I\right) \tag{84}$$

where $g_V = \frac{\dot{V}}{V}$ is pinned down by the free entry condition, $\lambda(z) = \frac{z}{1-z}(g_A+\delta_I)$, and $V_I^{\text{adopted}} = \int_t^\infty e^{-\int_t^u r_v+\delta_I dv}\Pi_I(u)\,du$. The zero in (84) is meant to highlight that there are no flow profits up until the idea is adopted.

Government & Complementary Infrastructure. A classic theme in the growth literature is government capacity, and the important role of complementary public infrastructure investment. For example, the adoption of motorized vehicles will not occur unless the government builds roads. An easy way to incorporate this aspect into the model is to generalize the adoption technology with a notion of public infrastructure $A_{G,t}$ so that the net flow of ideas reads $\dot{A}_i = A_F^{1-\theta}A_G^{(1-\phi)\theta}A_i^{\phi\theta} - \delta_I A_i$. A normalized version simplifies to $\dot{z}_i = \nu z_G^{(1-\phi)\theta} z_i^{\phi\theta}\iota_i^\beta - (\delta_I + g_F) z_i$ where the firm takes z_G as given. Without complementary government investment z_G will be zero and the returns to adoption are zero as well. If the government invests in $A_G^{(1-\phi)\theta}$ at a rate consistent with a balanced growth path, then all of the previous derivations go through after updating the term θ by a factor of ϕ. Note that capable

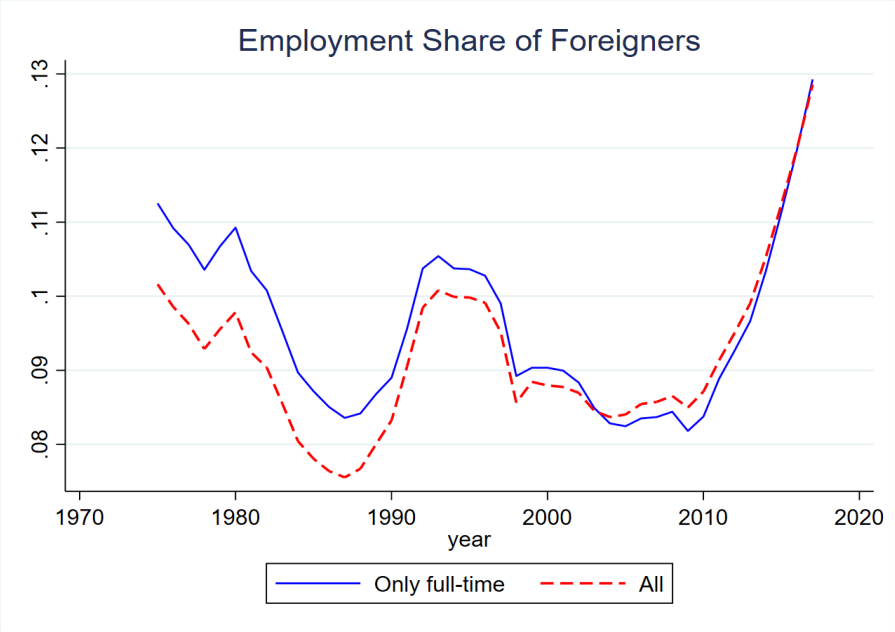
government employees are likely to be instrumental in building up complementary public infrastructure. The role of skilled labor remains pivotal.

Immigration. In the baseline model, unskilled immigration into the advanced economy depresses unskilled wages through two channels. First, there is a direct effect: an increase in the supply of unskilled labor lowers its relative price. Second, a larger unskilled workforce raises the demand for skilled labor devoted to technology adoption, which crowds out innovation effort and slows frontier growth.

In contrast, inflows of skilled labor are unambiguously beneficial for unskilled workers. By expanding the supply of skills, skilled immigration raises both technology adoption and innovation, thereby increasing productivity and unskilled wages. The effect on skilled workers themselves is ambiguous but likely positive: although the relative skilled wage declines, higher adoption rates and faster frontier growth can raise skilled wages in absolute terms.

Figure 17 evaluates the quantitative relevance of immigration for wage stagnation in Germany. The foreign employment share has declined since the mid-1990s, reaching an all-time low in the 2000s. This pattern suggests that immigration is unlikely to be a primary driver of weak wage growth in Germany at the turn of the century. More broadly, the dominant shift during this period appears to have occurred in trade integration rather than labor mobility. This does not imply that immigration is unimportant. It is difficult to account for technological frontier growth—particularly in the United States—without the contribution of foreign researchers and entrepreneurs. The framework developed here could, in principle, be extended to study brain drain, innovation gains, and global equilibrium effects under free migration with realistic scale effects. I leave such an extension to future work.

Figure 17. Immigration



The figure plots the share of foreign workers in West Germany, using the BHP of the IAB.

A.10 Relation to the Literature

The framework emphasizes a novel trade-off between innovation and adoption that distinguishes it from the existing literature. In contrast to to Perla and Tonetti (2014), Sampson (2016), König, Lorenz, and Zilibotti (2016), Perla, Tonetti, and Waugh (2021), Benhabib, Perla, and Tonetti (2021), and Sampson (2023), my model features an explicit tension between resources devoted to innovation and to adoption. In much of the earlier work, adopters primarily learn from innovators, so this trade-off is muted and the dominant friction is the inability of innovative firms to internalize the positive spillovers they generate for laggards. In such environments, market integration and Melitz (2003)-style selection –by reallocating market share toward larger, more productive firms– tend to generate strong pro-growth effects. These predictions contrast with the experience of advanced economies, where an unprecedented increase in market integration has been accompanied by a growth slowdown.[68] Another crucial difference is to embed adoption spillovers into a model with a realistic degrees of scale effects where ideas are harder to find, consistent with a growing body of evidence.[69]

[68] It seems there are two views. First, globalization may be relatively unimportant for long-run growth. In that case, the vast literature on the pro-growth effects of globalization may be correct, but quantitatively unimportant. Second, globalization may play an important role in shaping growth and inequality, especially in the recent wave of globalization. If so, we must be missing an important aspect. I aim to provide an answer to this second view.

[69] See Jones (1995), Kortum (1997), Jones (2009), Gordon (2016), and Bloom et al. (2020).

The comparison between planner and decentralized solution anticipates that spillovers in adoption and the extent to which ideas are harder to find, ϕ, will be a key parameters shaping the impact of market integration on growth.

Second, the model relates to Comin and Gertler (2006), who study endogenous innovation and adoption in a business-cycle framework. While both models feature an adoption margin, the underlying mechanisms differ in fundamental ways. The present framework nests the benchmark Romer-Jones model exactly, which allows key equilibrium objects—most notably the technology adoption gap—to be characterized in closed form. This allows for a transparent analysis of the efficiency properties of the model, which can be directly compared to the benchmark model, and differ fundamentally from Comin and Gertler (2006).[70] Conceptualizing these spillover in a tractable framework is an important contribution of the model, as these spillovers shape the qualitative and quantitative implications of market integration on growth.[71] Moreover, the model introduces a factor-market rivalry between innovation and adoption by making adoption relatively skill intensive. This generates a novel innovation-adoption trade-off that responds sharply to globalization. Such a trade-off is absent in Comin and Gertler (2006), where both innovation and adoption are financed in final goods rather than competing for skilled labor. This is more generally true for lab-equipment models (Romer, 1987) where innovation and adoption only require final output. In fact, my current model can be turned into a lab-equipment model by assuming firm entry, technology adoption, and innovation only require final output.[72] In that case, the BGP adoption gap is unrelated to the allocation of resources between innovation and adoption. Anzoategui et al. (2019), which builds on Comin and Gertler (2006), assumes skill-intensive innovation and adoption. Yet, the trade-off between innovation and adoption still plays no role for three reasons. First, the model's focus is on short-run fluctuations in the closed economy so the complementarity between the two activities dominates. Second, they impose a very strong spillover from innovation to adoption, which make underinvestment in adoption extremely unlikely. Third, and related, the model is fully endogenous, so pushing more labor into innovation implies forever faster growth, taking the bite out of the innovation-adoption trade-off.

Comin and Hobijn (2007) is the less known but perhaps most relevant precursor to the complementarity between innovation and adoption I derived. They focus on complementarities in initial implementation of technology in a model where innovators race for patents, see Reinganum (1989). The models differ both in how adoption and innovation are set-up, and what spillovers drive the results. However, they point out that more successful implementation can fuel innovation, and in that sense, I view them as closely related. Having said that, my theory nests benchmark frameworks, generalizes effortlessly to an open economy setting, and then shows how globalization can break the

[70] Note that I nest a version of the canonical Parente and Prescott (1994)-model of technology adoption. This not only allows me to take the framework effortlessly to the cross-country setting, but also provides a micro-foundation for the adoption spillover building on the recent idea flow literature that depends on firm entry and exit, which has a direct counterpart in the micro data.

[71] See Atkeson and Burstein (2019) on how inefficiencies shape counterfactuals in endogenous growth models.

[72] With positive population growth, I need $\phi < 0$.

complementarity between innovation and adoption. So the two papers end up being quite different.

B Computational Appendix

Here I show how to compute the solution and solve for transition dynamics based on the finite difference method in Achdou et al. (2022).

B.1 Closed Economy Computations

B.1.1 Technology Adoption (with Distribution) – Steady State

The normalized firm problem reads

$$\left(r_t + \delta_{\mathrm{X}} - g_{w_{L,t}}\right) v_t\left(z\right) = \max \pi_t\left(z\right) - s_t \iota_t + \partial_z v_t \cdot \left[\nu z_t^{\theta} \iota_t^{\beta} - \left(\delta_{\mathrm{I}} + g_{\mathrm{F},t}\right) z\right] + \dot{v}_t$$

subject to the law of motion for z and the evolution of aggregate state variables captured in the time index t.

The first-order condition reads

$$\left\{\frac{\left(\partial_z v_t\right) \beta \nu z_t^{\theta}}{s_t}\right\}^{\frac{1}{1-\beta}} = \iota_t.$$

I use the following boundary conditions for the finite difference scheme where v^{f} and v^{b} denote forward and backward difference, respectively,

$$\left(\partial_z v^{\mathrm{f}}\right) = \frac{s}{\beta \nu \overline{z}^{\theta}} \left\{\frac{\left(\delta_{\mathrm{I}} + g_{\mathrm{F},t}\right) \overline{z}^{1-\theta}}{\nu}\right\}^{\frac{1-\beta}{\beta}},$$

$$\left(\partial_z v^{\mathrm{b}}\right) = \frac{s}{\beta \nu \underline{z}^{\theta}} \left\{\frac{\left(\delta_{\mathrm{I}} + g_{\mathrm{F},t}\right) \underline{z}^{1-\theta}}{\nu}\right\}^{\frac{1-\beta}{\beta}}.$$

The bounds $\overline{z}$ and $\underline{z}$ represent the highest and lowest value of z on the grid space $z_grid = [\underline{z}, z_2, z_3, ..., \overline{z}]$.

Rewrite the problem as follows

$$\left(r_t + \delta_{\mathrm{X}} - g_{w_{L,t}}\right) v_t\left(z\right) = B_t z^{(\sigma-1)(1-\alpha)} - s_t \iota_t + \partial_z v_t \cdot \left[\nu z_t^{\theta} \iota_t^{\beta} - \left(\delta_{\mathrm{I}} + g_{\mathrm{F},t}\right) z\right] + \dot{v}_t, \tag{85}$$

whereB_t captures, among other things, the equilibrium measure of firms as well as the properly weighted average productivity level $\tilde{z} = \left\{\mathbb{E}\left[z^{(\sigma-1)(1-\alpha)}\right]\right\}^{\frac{1}{(\sigma-1)(1-\alpha)}}$. The heterogeneity arises only for the case of imperfect spillover at entry, $\lambda_{\mathrm{E}} < 1$, which is nested by the algorithm but can be shut down by setting $\lambda_{\mathrm{E}} = 1$.

Note that $\int_0^{M_t} B_t z_i^{(\sigma-1)(1-\alpha)} di = B_t M_t \tilde{z}^{(\sigma-1)(1-\alpha)} = \frac{Y_t}{\sigma}$. This can be rearranged to

$$B_t = \frac{l_{\mathrm{P},t}}{(\sigma-1)(1-\alpha) m_t \tilde{z}_t^{(1-\alpha)(\sigma-1)}}.$$

Moreover, in the steady state $m = \frac{1-l_{\mathrm{P}}}{(g_L+\delta_{\mathrm{X}})f_{\mathrm{E}}}$. So for a value of B_t, and a solution to (85), I can infer all endogenous variables.

Steady State. To find B in the steady state, I solve (85) for a guess of B and check if the free entry condition holds

$$f_{\mathrm{E}} = v(z_{\mathrm{E}}).$$

A slightly similar free entry condition that delivers an identical steady state but computationally smoother is

$$l_{\mathrm{E},t} = l_{\mathrm{E}} e^{\beta_{\mathrm{E}}(v(z_{\mathrm{E},t})-f_{\mathrm{E}})},$$

where l_{E} is labor devoted to entry in the steady state .

The density of incumbents g_z and entrants $g_{z_{\mathrm{E}}}$ follows from the KFE equation

$$\dot{g}_z = \mathcal{A}^T(g_z) g_z$$

where I make clear that the infinitesimal generator $\mathcal{A}^T(g_z)$ is itself a function of the stationary density since entrants' productivity is assumed to be a function of the average

$$z_{\mathrm{E}} = \lambda_{\mathrm{E}} \mathbb{E}[z].$$

A flow of $g_L + \delta_{\mathrm{X}}$ firms enters at productivity z_{E}, accounting for both net entry rate (which is the same as long run population growth) and exogenous firm death. In matrix form, this means I have to add $-(g_L+\delta_{\mathrm{X}})$ on the diagonal, and $(g_L+\delta_{\mathrm{X}})\mathbb{1}_{z=z_{\mathrm{E}}}$in each column of $\mathcal{A}^T$. If z_{E} falls between two grid points, I turn the entry draw into a probabilistic one using a linear interpolation.

I can now iterate on guesses for B, which implies a stationary distribution g_z, values for l_{P} and m follow

$$l_{\mathrm{P}} = \frac{B\tilde{z}^{(1-\alpha)(\sigma-1)}}{\frac{(g_L+\delta_{\mathrm{X}})f_{\mathrm{E}}}{(1-\alpha)(\sigma-1)} + B\tilde{z}^{(1-\alpha)(\sigma-1)}}$$
$$m = \frac{1-l_{\mathrm{P}}}{(g_L+\delta_{\mathrm{X}})f_{\mathrm{E}}}$$

which in turn imply aggregate demand for skilled labor from the production sector

$$h_{\mathrm{A}} = m \cdot \int \iota(z) g_z(z) dz.$$

B.1.2 Transitions

To solve for technology adoption choices along a transition path, I can again use a finite difference scheme that now explicitly takes account of a changing time derivative, $\dot{v}_t$. From the point of view of an infinitesimal production sector firms, the demand shifter B, skill premia, unskilled wage growth, interest rates, and evolution of the technological frontier are exogenous, and captured in the sequences $\{B_t, s_t, g_{w_{L,t}}, r_t, g_{\mathrm{F},t}\}_{t\in[0,T]}$ where $t = 0$ represents the initial period, and T is sufficiently large that the system arrived at the new steady state.

There are two control variables

1. ι_t (tech. adoption),
2. $l_{\mathrm{E},t}$ (firm entry).

There is on idiosyncratic endogenous state variable

1. $z_{i,t}$.

And there are two aggregate endogenous state variables that, in turn, determine B_t and demand for skilled labor from the production sector, h_{A}:

1. $\tilde{z}_t$ (avg. normalized productivity), the entire distribution $g_{z,t}(z)$, implied by individual choices, pins down this aggregate state,
2. m_t (normalized firm measure), which in turn depends on the entire sequence of firm entry $l_{\mathrm{E},t}$.

I next describe production and innovation sector in turn along a transition path, and then integrate them in general equilibrium using the market clearing condition for skilled labor.

B.1.3 Production Sector

A **partial equilibrium in the production sector** thus consists of finding an optimal sequence of adoption and entry choices consistent with the aggregation constraints implied by the aggregate state variables. Here is an algorithm for this subproblem.

1. Given a sequence $\{a_{\mathrm{F},t}, s_t\}$, guess variable $\{B_t, g_{w_{L,t}}, r_t, v_{\mathrm{E},t}\}$.
2. Solve incumbent firm HJB holding $\{B_t, g_{w_{L,t}}, r_t, v_{\mathrm{E},t}\}$ fixed.

(a) The first order condition pins down adoption effort

$$\left\{\frac{\left(\partial_z v_{t+\Delta}\right)\beta\nu z_{t+\Delta}^{\theta}}{s_t}\right\}^{\frac{1}{1-\beta}} = \iota_t,$$

where each t is a step in time.

The value function v_t follows recursively

$$\begin{aligned}\left(r_t - g_{w,t} + \delta_{\mathrm{X}}\right) v_t &= \pi_t - \iota_t s_t + A_t v_t + \frac{v_{t+\Delta} - v_t}{\Delta} \\ &\Rightarrow \\ v_t &= \left[\left(\frac{1}{\Delta} + r_t - g_{w,t} + \delta_{\mathrm{X}}\right) I - A_t\right]^{-1}\left(\pi_t - \iota_t s_t + \frac{1}{\Delta} v_{t+\Delta}\right)\end{aligned}$$

where it should be clear that profits and adoption effort are a function of z and I is the identity matrix.

This pins down the vector of adoption effort as a function of z, i.e., $\{\iota_t(z)\}$.

(b) Now iterate forward on the KFE, which pins down new values for $\{B_t, g_{w_{L,t}}, r_t, v_{\mathrm{E},t}\}$.

i. $\tilde{z}_t^{(1-\sigma)(1-\alpha)} = \mathbb{E} z^{(\sigma-1)(1-\alpha)}$

ii. $l_{\mathrm{E},n} = l_{ss} e^{\beta_{\mathrm{E}}(v_n - v_{ss})}$

iii. The evolution of m is implied by $g_{m,n} = \frac{l_{\mathrm{E},n}}{f_{\mathrm{E}} m_n} - \left(g_{L,n} + \delta_{\mathrm{X}}\right)$.

iv. The evolution of $\tilde{z}$ is implied by i) initial z_0, and the KFE using an implicit scheme

$$g_{n+1}(z) = \left(I - \Delta t \cdot A_n^T\right)^{-1} g_n(z),$$

where A_n^T is the transpose of the solution of the incumbent firm HJB and firm entry implied by $l_{\mathrm{E},n}$.

v. Go back to i) until you have reached T

vi. The law of motion of physical normalized capital $k_t = \frac{K_t}{A_t L_{\mathrm{P},t}}$ follows

$$\frac{\dot{k}}{k} = \chi_s \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} k^{\alpha-1} - \left(g_L + g_{l_{\mathrm{P}}} + g_{\mathrm{F}} + g_z + \delta_k\right),$$

where χ_s is the constant fraction of output invested in physical capital. In discretized form, I have

$$k_{t+\Delta} = k_t \cdot e^{\Delta\left[\chi_s \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} k^{\alpha-1} - \left(g_L + g_{l_{\mathrm{P}}} + g_{\mathrm{F}} + g_z + \delta_k\right)\right]}$$

Note that initial k_0 jumps whenever $l_{\mathrm{E},0}$ jumps. After that jump, l_{E} evolves smoothly.

The initial jump equals $k_0 = k_{old_ss} * l_{\mathrm{P},old_ss}/l_{\mathrm{P},0}$. Note that we have everything to compute the evolution of $\{k_t\}$.
This then pins down a new interest rate sequence $\{r_t\}$

$$r_t = \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \frac{\sigma-1}{\sigma}\alpha k_t^{\alpha-1} - \delta_k.$$

Given this law of motion of capital, together with optimal firm entry, the equilibrium interest rate follows

$$r_t = \left(\frac{\sigma-1}{\sigma}\alpha\right)\alpha^{-\alpha}\left(1-\alpha\right)^{-(1-\alpha)}\alpha k_t^{\alpha-1} - \delta_k$$

where the normalized capital stock in steady state reads

$$k_{\mathrm{ss}} = \left\{\frac{\chi_s}{g_L + g_{\mathrm{F}} + \delta_k}\right\}^{\frac{1}{1-\alpha}} \alpha^{-\frac{\alpha}{1-\alpha}}\left(1-\alpha\right)^{-1}.$$

The saving rate, which is kept constant along the transition path, equals $\chi_s = \left(\frac{\sigma-1}{\sigma}\alpha\right) \cdot \alpha\frac{g_L+g_{\mathrm{F}}+\delta_k}{\rho+g_{\mathrm{F}}+\delta_k}$. This savings rate induces a steady state normalized capital stock, $k_{\mathrm{ss}} = \left\{\left(\frac{\sigma-1}{\sigma}\alpha\right) \cdot \frac{\alpha}{\rho+g_{\mathrm{F}}+\delta_k}\right\}^{\frac{1}{1-\alpha}} \alpha^{-\frac{\alpha}{1-\alpha}}\left(1-\alpha\right)^{-1}$, which coincides with the normalized capital stock in the steady state when the saving rate is endogenous and based on log preferences.
With this sequence of $\{k_n, \tilde{z}_n\}$, we can compute wages and wage growth

$$w_{L,t} = \left(\frac{1-\alpha}{\alpha}\right)^{\alpha} \frac{\sigma-1}{\sigma} A_t k_t^{\alpha}$$

where $A_0 = 1$ without any loss of generality,[73] and $g_A = g_{\mathrm{F}} + g_z$ and $g_{w_L} = g_A + \alpha g_k$. Lastly, compute an new vector $\{B_n\}$ using $B_n = \frac{l_{\mathrm{P},n}}{m_n(\sigma-1)(1-\alpha)} \frac{1}{\tilde{z}_n^{(1-\sigma)(1-\alpha)}}$.

(c) Update $\{B_t, g_{w_{L,t}}, r_t, v_{\mathrm{E},t}\}$ gently using a relaxation scheme $\tilde{z} = relax \cdot \tilde{z}_{old} + (1 - relax) \cdot \tilde{z}_{new}$ for $relax$ sufficiently close to one.

3. Iterate on a), b), c) until convergence in $\{\tilde{z}_n, m_n, l_{\mathrm{P},n}, B_n\}$. Note this also implies convergence in wage growth and interest rates. If you make entry more responsive ($\beta_{\mathrm{E}} \uparrow$), you will want to pick a more gentle relaxation parameter to avoid ossiciliations.

[73] Since everything is growing, the initial productivity level is arbitrary.

B.1.4 Innovation

Using the normalized value of innovation,

$$v_{\mathrm{I},t} := \frac{V_{\mathrm{I},t}}{w_{L,t} A_{\mathrm{F},t}^{-\phi}},$$

leads to the following recursion

$$v_{\mathrm{I},t}\left(r_t - g_{w_{L,t}} + \delta_{\mathrm{I}} + \phi g_{\mathrm{F},t}\right) = e^{-\int_t^{t+\tau_t}\left(r_t - g_{w_{L,t}} + \delta_{\mathrm{I}} + \phi g_{\mathrm{F},t}\right)dv} \frac{\alpha l_{\mathrm{P},t+\tau}}{z_{t+\tau} a_{\mathrm{F},t+\tau}} \cdot \left[1 + \dot{\tau}_t\right] + \dot{v}_{\mathrm{I}}.$$

I focus on the homogeneous firm case so that $\tilde{z} = z$,but one can generalize the model to the heterogeneous firm case.[74]

In discretized form the normalized HJB reads

$$\begin{aligned} v_{\mathrm{I},t}\left(r_{t+\Delta} - g_{w_{L,t+\Delta}} + \delta_{\mathrm{I}} + \phi g_{\mathrm{F},t+\Delta}\right) = e^{-\int_t^{t+\tau}\left(r_t - g_{w_{L,t}} + \delta_{\mathrm{I}} + \phi g_{\mathrm{F},t}\right)dv} \frac{\alpha l_{\mathrm{P},t+\tau}}{z_{t+\tau} a_{\mathrm{F},t+\tau}} \cdot \left[1 + \dot{\tau}_t\right] \\ + \frac{v_{\mathrm{I},t+\Delta} - v_{\mathrm{I},t}}{\Delta}. \end{aligned}$$

The free entry condition maps the value of innovation into entry according to

$$\left(\frac{\gamma v_{\mathrm{I},t}}{s_t}\right)^{\frac{1}{1-\lambda}} = h_{\mathrm{F},t}.$$

Note that this free entry condition always binds for the case of $\lambda < 1$.

To compute the value of innovation, innovators have to correctly forecast the waiting time, τ. Note that τ_0 jump at time zero, and the size of the jump is implicitly defined by

$$\tau_0 = -\frac{\log z_0}{\frac{\int_0^{\tau_0} g_A(x) + \delta_{\mathrm{I}} dx}{\tau_0}}.$$

For a given evolution of $\{g_{A,t}\} = \{g_{z,t} + g_{\mathrm{F},t}\}$, one can use a simple root finding method to get τ_0. Once the initial wait time is determined, one can use the law of motion of the waiting time

$$\dot{\tau}_t = \frac{g_{\mathrm{F},t} - g_{A,t+\tau}}{\delta_{\mathrm{I}} + g_{A,t+\tau}},$$

[74] In that case, flow profits would be computed against the entire distribution $V_{\mathrm{I},t} = \int \int_{t+\tau_z}^{\infty} e^{-\int_t^s r_v dv} \Pi_{\mathrm{I},s}(z)\, ds f_t(z)\, dz$, and where changes in $\dot{\tau}_t$ would be captured in the distributional dynamics.

which in discretized form can be rewritten to

$$\tau_{t+\Delta} = \tau_t + \Delta \cdot \frac{g_{\mathrm{F},t} - g_{A,t+\tau}}{\delta_{\mathrm{I}} + g_{A,t+\tau}}, \qquad \tau_{t+\Delta} \geq 0.$$

This recursive representation can be used to compute equilibrium in the research sector off the balanced growth path.

The following algorithm can be used to compute equilibrium in the research sector as a function of the skill premium and production sector $\{s_t, z_t, l_{\mathrm{P},t}\}$.

1. Take $\{s_t, z_t, l_{\mathrm{P},t}, r_t\}$ as given
2. Guess a sequence $\{g_{\mathrm{F},t}\}$.
3. The guess implies $\{\tau_t\}$. This allows me to solve the innovator problem backwards (there is not much to solve here other than computing the value of an idea recursively). Given $\{s_t, r_t, g_{\mathrm{F},t}, \tau_t\}$, get a sequence $\{v_{\mathrm{I},t}\}$.
4. Given a sequence of $\{s_t\}$ and $\{v_{\mathrm{I},t}\}$, derive skilled labor devoted to entry $h_{\mathrm{I},t} = \left\{\frac{\gamma v_{\mathrm{I},t}}{s_t}\right\}^{\frac{1}{1-\lambda}}$
   (a) Use $\{h_{\mathrm{I}}\}$ to derive new sequence $\{a_{\mathrm{F}}^{\mathrm{new}}\}$ using the resource constraint related to the creation of new ideas

$$g_{\mathrm{F},t} = \frac{\gamma h_{\mathrm{I},t}^{\lambda}}{a_{\mathrm{F},t}} - \delta_{\mathrm{I}},$$
$$g_{a_{\mathrm{F},t}} = (1-\phi)\left(g_{\mathrm{F},t} - \frac{g_L}{1-\phi}\right),$$

   which gives rise to the discretized law of motion

$$a_{\mathrm{F},t+\Delta} = a_{\mathrm{F},t} e^{\Delta g_{a_{\mathrm{F},t}}}.$$

   i. Update $a_{\mathrm{F},t,n} = relax * \{a_{\mathrm{F},t,n-1}\} + (1 - relax) * \{a_{\mathrm{F},t}^{\mathrm{new}}\}$
   ii. Go back to 3. until convergence occurs, $a_{\mathrm{F},n} \approx a_{\mathrm{F},n+1}^{\mathrm{new}}, \tau_n \approx \tau_{n+1}$.
5. The outcome is twofold: you get a new solution for $\{g_{\mathrm{F},t}\}$ and you get a new sequence for skilled labor devoted to innovation $\{h_{\mathrm{I},t}\}$.

B.1.5 General equilibrium

Step 1. Iterate on $\{a_{\mathrm{F},t}\}$ up until innovation and production sector have converged.
Step 2. After both sectors agree on $\{a_{\mathrm{F},t}\}$, compute implied demand for skilled labor in each sector,

$\{h_{\mathrm{A}}, h_{\mathrm{I}}\}$. Update the skill premium gently upward whenever $h_{\mathrm{A}} + h_{\mathrm{I}} > h$, and downward otherwise,

$$s_{t,n+1} = \min\{\max\{s_{t,n} \times^{(1-relax)(h_{\mathrm{A},t,n}+h_{\mathrm{I},t,n}-h)}, 0.95 * s_{t,n}\}, 1.05 * s_{t,n}\}.$$

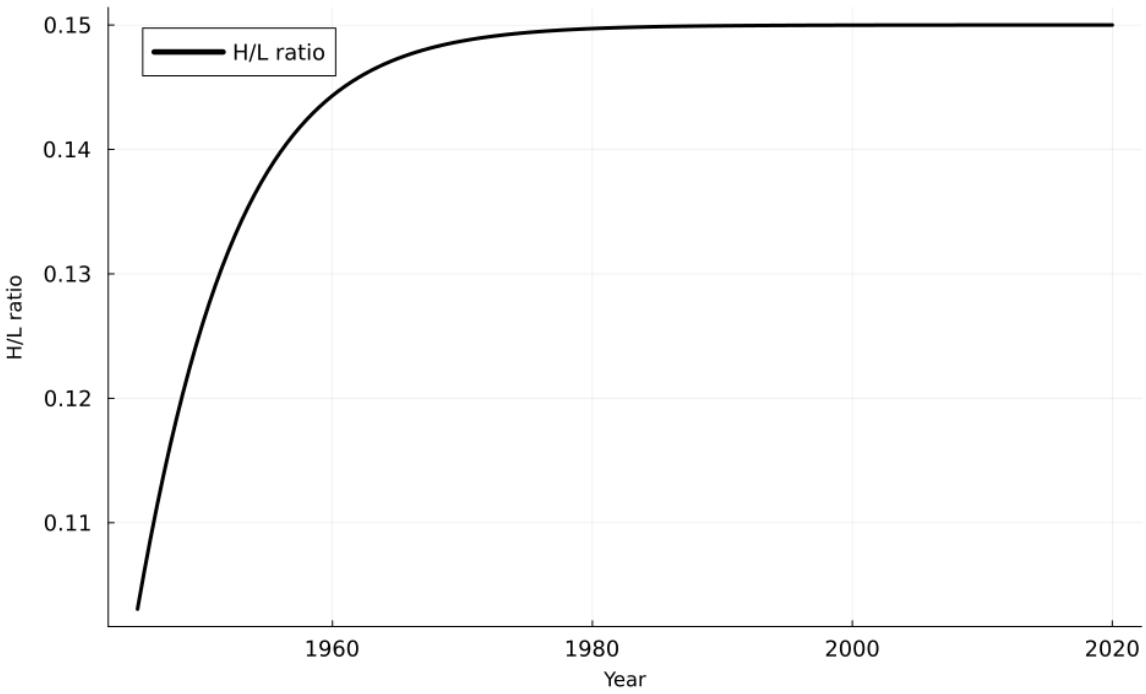
For sufficiently gently updating, this algorithm computes the transition dynamics of the model in the closed economy.

B.1.6 Autarky Skilled Labor Supply Shock

The figure below computes transition dynamics for a closed economy that is calibrated as in the main text. I shock the relative skilled labor supply from .10 to .15 as in figure 18. The transitions are then computed using the previous algorithm under perfect foresight. Figure 19 plots the results.

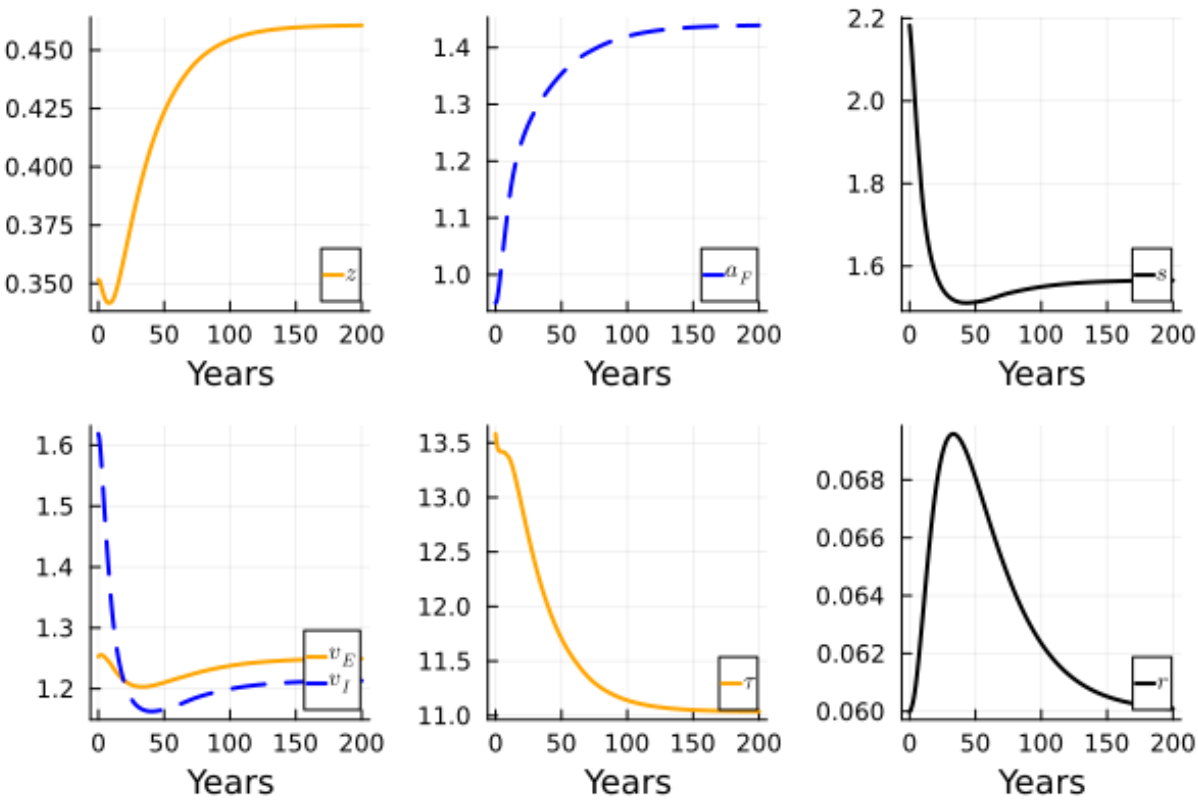
Three points are noteworthy. First, note that innovation responds quicker than technology adoption in the sense that convergence to its long-run steady state is faster. This is explained by the advantage of backwardness, which naturally leads to a lagged response of technology adoption. Second, the dynamics of the skill premium are non-monotone, and a fast initial fall in the price of skill is followed by slow but small rise in the price of skill later on. The increase in the price of skill, which ultimately is a function of the demand for skilled labor in this general equilibrium model, is directly related to increasing and lagged technology adoption effort. Third, the value of entry in the production sector is hardly moving, and deviates from its long-run value by less than 1% highlighting that the simplification is unlikely to matter quantitatively for the transition dynamics. The evolution of the value of entry into the research sector roughly follows the skill premium, and the difference in the two series is due to congestion whenever $\lambda < 1$.

Figure 18. Skilled Labor Expansion in Autarky



In this exercise I increase the relative skilled labor supply from .10 to .15.

Figure 19. Skilled Labor Expansion in Autarky – Results



In this exercise I increase the relative skilled labor supply from .10 to .15.

B.2 Open Economy

To solve for transition dynamics in the open economy, I follow similar steps. Note that the problem of production sector firms is unchanged. Firms in the research sector now need to take into account both opportunities abroad and competition at home when they consider whether to pay the fixed cost of producing an idea.

B.2.1 Innovation

The value of an innovation generalizes as follows in the open economy where I focus on the case of differences in size captured by country-specific unskilled labor endowments L_c. I consider a version where I use L from the home economy as the normalizing factor so $b_k = \frac{L_k}{L}$ relative to the home

economy. The value of an idea reads

$$\begin{aligned} V_{\mathrm{I}}\left(r+\delta_{\mathrm{I}}\right) &= \sum_{c} e^{-\int_{t}^{t+\tau_c}\left(r_v+\delta_{\mathrm{I}}\right)dv}\pi_{\mathrm{I},c,t+\tau}\cdot\left[1+\dot{\tau}_{c,t}\right]+\dot{V}_{\mathrm{I}} \\ &= \sum_{c} e^{-\int_{t}^{t+\tau_c}\left(r_v+\delta_{\mathrm{I}}\right)dv}\frac{\alpha L_{\mathrm{P},c,t+\tau_c}w_{L,c,t+\tau_c}}{A^{\mathrm{W}}_{\mathrm{F},t+\tau_c}z_{c,t+\tau_c}}\cdot\left[1+\dot{\tau}_{c,t}\right]+\dot{V}_{\mathrm{I}} \\ &= \sum_{c} e^{-\int_{t}^{t+\tau_c}\left(r_v+\delta_{\mathrm{I}}\right)dv}\frac{\alpha l_{\mathrm{P}}w_{L,c,t+\tau_c}b_c}{A^{\mathrm{W}}_{\mathrm{F},t+\tau_c}z_{c,t+\tau_c}}L\cdot\left[1+\dot{\tau}_{c,t}\right]+\dot{V}_{\mathrm{I}}, \end{aligned}$$

where I used the fact that the allocation of unskilled labor in production vis-a-vis entry is kept constant.

Next, use the normalization $v_{\mathrm{I},k,t}:=\frac{V_{\mathrm{I}}}{w_{L,k}\left(A^{\mathrm{W}}_{\mathrm{F}}\right)^{-\phi}}$. Note that the normalization is country-specific because wages are. The normalized value of an idea reads

$$\begin{aligned} v_{\mathrm{I},k,t}\left(r_k-g_{w_{L,k}}+\delta_{\mathrm{I}}+\phi g_{\mathrm{F}}\right) &= \sum_{c}\frac{e^{-\int_{t}^{t+\tau_c}\left(r_k+\delta_{\mathrm{I}}\right)dv}}{w_{L,k,t}}\frac{\left(A^{\mathrm{W}}_{\mathrm{F},t}\right)^{\phi}}{A^{\mathrm{W}}_{\mathrm{F},t+\tau_c}z_{c,t+\tau_c}}\alpha L_{\mathrm{P},c,t+\tau_c}w_{L,c,t+\tau_c}\cdot\left[1+\dot{\tau}_{c,t}\right]+\dot{\tilde{v}}_{\mathrm{I}} \\ &= \sum_{c}\frac{e^{-\int_{t}^{t+\tau_c}\left(r_k+\delta_{\mathrm{I}}\right)dv}}{w_{L,k,t}}\left(\frac{A^{\mathrm{W}}_{\mathrm{F},t}}{A^{\mathrm{W}}_{\mathrm{F},t+\tau_c}}\right)^{\phi}\frac{L_{t+\tau_c}\alpha l_{\mathrm{P}}w_{L,c,t+\tau_c}}{\left(A^{\mathrm{W}}_{\mathrm{F},t+\tau_c}\right)^{1-\phi}z_{c,t+\tau_c}}b_c\cdot\left[1+\dot{\tau}_{c,t}\right]+\dot{\tilde{v}}_{\mathrm{I}} \\ &= \sum_{c} e^{-\int_{t}^{t+\tau_c}\left(r_k-g_{w_L}+\delta_{\mathrm{I}}+\phi g_{\mathrm{F}}\right)dv}\left(\frac{\alpha l_{\mathrm{P}}}{a^{\mathrm{W}}_{\mathrm{F},t+\tau_c}z_{c,t+\tau_c}}\frac{w_{L,c,t+\tau_c}}{w_{L,k,t+\tau_c}}\right)\cdot b_c\cdot\left[1+\dot{\tau}_{c,t}\right]+\dot{\tilde{v}}_{\mathrm{I}} \end{aligned}$$

where the second line uses the fact that I assumed that firm entry in production stays at its long-run steady state value, i.e., $l_{\mathrm{P},t}=l_{\mathrm{P}}$.

The reader will note that the expression is almost identical to the closed economy one, except I have to sum over all countries and take into account that wages w_c, waiting times τ_c, adoption gaps z_c, and weights b_c are country specific.

A country-specific free entry condition holds

$$\tilde{v}_{\mathrm{I},k,t}=\frac{1}{\gamma_k}h^{1-\lambda}_{\mathrm{I},k,t}s_{k,t},$$

where a binding inequality is guaranteed as long as $\lambda<1$.

The evolution of the technological frontier follows from the free entry equilibrium and the entry

technology

$$\begin{aligned}\frac{\dot{A}_{\mathrm{F}}^{\mathrm{W}}}{A_{\mathrm{F}}^{\mathrm{W}}} &= \sum_c \frac{A_{\mathrm{F},c}}{A_{\mathrm{F}}^{\mathrm{W}}} \frac{\dot{A}_{\mathrm{F},c}}{A_{\mathrm{F},c}} \\ &= \sum_c \chi_c \left\{ \frac{\gamma_c \left(A_{\mathrm{F}}^{\mathrm{W}}\right)^{\phi} L h_{\mathrm{I},c}^{\lambda} b_c}{A_{\mathrm{F},c}} - \delta_{\mathrm{I}} \right\} \\ g_{\mathrm{F}}^{\mathrm{W}} &= \sum_c \frac{\gamma_c h_{\mathrm{I},c}^{\lambda} b_c}{a_{\mathrm{F}}^{\mathrm{W}}} - \delta_{\mathrm{I}}.\end{aligned}$$

The normalized technological frontier $a_{\mathrm{F}}^{\mathrm{W}} := \left(\frac{A_{\mathrm{F}}^{\mathrm{W}}}{L^{\frac{1}{1-\phi}}} \right)^{1-\phi}$ thus evolves according to

$$\begin{aligned}g_{a_{\mathrm{F}}^{\mathrm{W}}} &= (1-\phi)\, g_{\mathrm{F}}^{\mathrm{W}} - g_L \\ &= (1-\phi) \left\{ \frac{\sum_c \gamma_c h_{\mathrm{I},c}^{\lambda} b_c}{a_{\mathrm{F}}^{\mathrm{W}}} - \left(\delta_{\mathrm{I}} + \frac{g_L}{1-\phi} \right) \right\}.\end{aligned}$$

Note that this law of motion in the open economy is independent of the split of ownership of patents assumed at time zero, i.e., the share ζ does not show up.

Initial Conditions and Intellectual Property Ownership. I assume that the emerging market copies ideas from the advanced economy, which means that their innovation productivity is artificially high in autarky, but falls in the integrated equilibrium $\gamma^{*,copy} > \gamma^*$. The reason for this is that the emerging market is relatively small in my calibration, and the lack of scale would require an artificially high research productivity. Practically, I can simply solve the model for the low research productivity γ^* because the allocation of skilled labor to each sector is unrelated to research productivity γ^* in autarky. I then scale the frontier technology and real wage by the appropriate factor to obtain a wage gap ω, which implicitly pins down$\gamma^{*,copy}$. In the open economy, there is no incentive to copy technology since Bertrand competition would drive profits to zero.

Having clarified this, note that $A_{\mathrm{F}}^* \subset A_{\mathrm{F}}$, since emerging markets copy technology. More generally, even if technology wasn't copied, it is natural to assume that because ideas are harder to find, the same ideas are invented in each country. I thus have to take a stance of who owns what intellectual property at the time of integration. This is parameterized by ζ, the share of initial patents held in the advanced economy.

Consequently, at time $t \to 0$ where 0 demarcates the time when markets become integrated, the

normalized frontier technology level in the West reads

$$\left(\frac{A_{\mathrm{F},t,0+\Delta}}{L_t^{\frac{1}{1-\phi}}}\right)^{1-\phi} = \left(\frac{\zeta A_{\mathrm{F},0-\Delta}}{L^{\frac{1}{1-\phi}}}\right)^{1-\phi}$$
$$= (\zeta)^{1-\phi} a_{\mathrm{F},0-\Delta},$$

and similarly the normalized level of frontier technology in the East reads $(1-\zeta)^{1-\phi} a_{\mathrm{F},0-\Delta}$ for $\Delta \to 0$. Both countries lose some of their ideas due to duplication, and I provide a micro-foundation for the ζ in appendix B.3, where small quality differences pin down which variety is produced where.

Since the frontier jumps from the point of view of the emerging market, note that the frontier gap, which is defined relative to the frontier, jumps as well. Formally,

$$z^*_{0+\Delta} = \frac{z^*_{0-\Delta} A^*_{\mathrm{F},0-\Delta}}{A^{\mathrm{W}}_{\mathrm{F},0+\Delta}},$$

where $A^{\mathrm{W}}_{\mathrm{F},0+\Delta} = A_{\mathrm{F},0+\Delta}$ as the frontier at time zero is defined by the West (recall $A^*_{\mathrm{F},t} \subset A_{\mathrm{F},t}$ for $t < 0$).

The evolution of $z_{c,t}$ then follows from domestic firms' adoption choices. The evolution of the normalized technology level within each country now changes to

$$\frac{\dot{A}_{\mathrm{F},c,t}}{A_{\mathrm{F},c,t}} = \frac{\gamma_{c,t} h^{\lambda}_{\mathrm{I},c,t} b_c}{\chi_{c,t} a^{\mathrm{W}}_{\mathrm{F},t}} - \delta_{\mathrm{I}},$$

and in terms of normalized frontier growth rates, I have

$$g_{a_{\mathrm{F},c,t}} = (1-\phi)\left[\frac{\gamma_{c,t}}{\chi_{c,t}} \frac{h^{\lambda}_{\mathrm{I},c,t} b_c}{a^{\mathrm{W}}_{\mathrm{F},t}} - \left(\delta_{\mathrm{I}} + \frac{g_L}{1-\phi}\right)\right].$$

Capital Accumulation. As in the closed economy, the problem is simplified by assuming that a share of domestic final goods production is re-invested at a constant rate χ_s, which is the same across countries and consistent with the long-run equilibrium interest rate. This means that $1-\chi_s$ is the share of final output consumed.

Note that this share is sensitive to the initial allocation of patent ownership in the integrated equilibrium, ζ. At time zero, a share $Y^* \frac{\sigma-1}{\sigma} \alpha (1-\alpha) \zeta (1-\chi_s)$ of output in the East is consumed by the West, and vice versa a share $Y \frac{\sigma-1}{\sigma} \alpha (1-\alpha) \zeta (1-\chi_s)$ of output in the West is consumed by the East. The reason is that households re-invest a constant fraction of income, and if income is earned abroad, then a fraction of that foreign income is re-invested abroad. An implication is that some domestically used capital is now held by foreigners. However, for the equilibrium dynamics of innovation and

technology adoption this split is irrelevant. After understanding this simplification, the following algorithm applies.

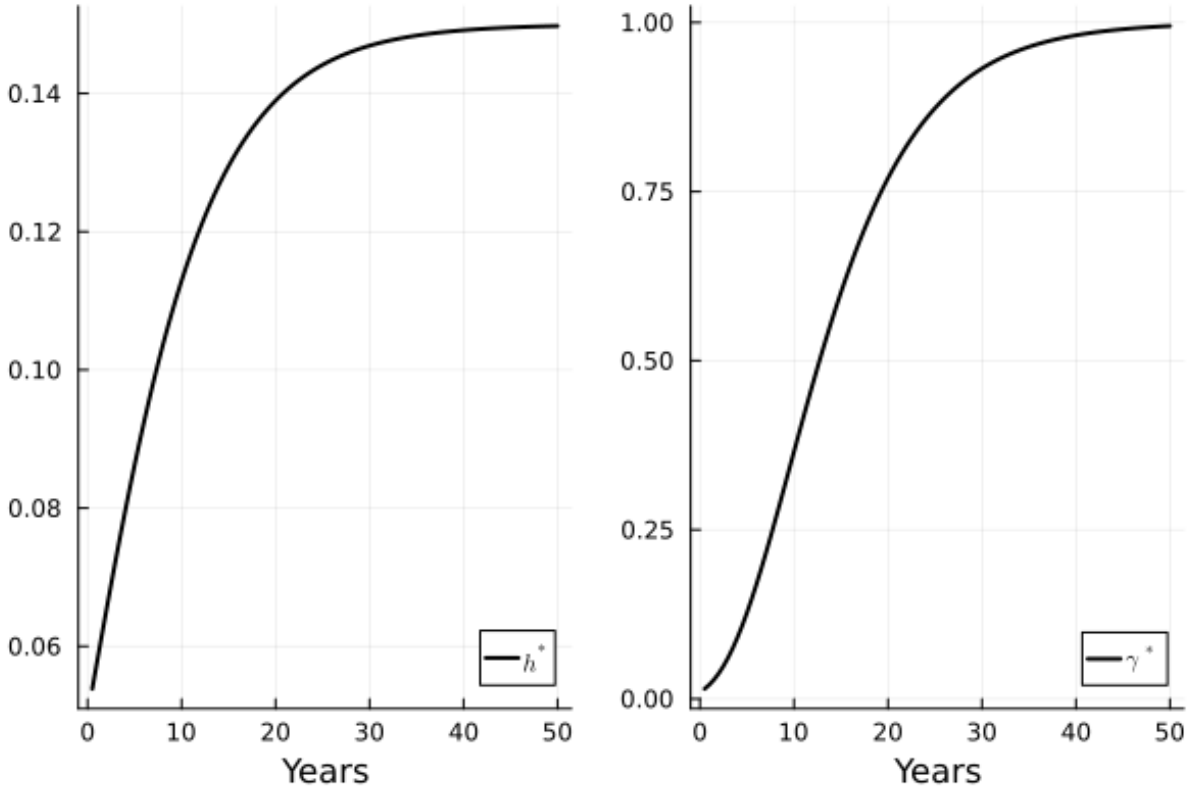
Algorithm.

1. Guess $\{s_{c,t}\}$ for each country, holding $\{a_{\mathrm{F},t}^{\mathrm{W}}\}$ fixed.
2. Solve optimal adoption choice backward, and $\{\{\tilde{z}_{c,t}\},\{r_{c,t}\}\}$ forward. Iterate until convergence for both countries.
   (a) Compute $\{\tau,\tau^*\}$ and $\{w,w^*\}$ taking $\{z,z^*,k,k^*\}$ as given for each country.
   (b) Solve innovator's problem
      i. Compute the value of innovation in each country recursively $\{v_{\mathrm{I}},v_{\mathrm{I}}^*\}$
      ii. Implies equilibrium research effort based on free entry into research in each country $\{h_{\mathrm{I}},h_{\mathrm{I}}^*\}$
      iii. Compute evolution of aggregate technological frontier using $\{h_{\mathrm{I}},h_{\mathrm{I}}^*\}$.
      iv. Go back to i) and iterate up until $\left(a_{\mathrm{F}}^{\mathrm{W}}\right)^{n+1}\approx\left(a_{\mathrm{F}}^{\mathrm{W}}\right)^{n}$ where n stands for n^{th} iteration. Update the new guess gently $\left(a_{\mathrm{F}}^{\mathrm{W}}\right)^{next}=relax*\left(a_{\mathrm{F}}^{\mathrm{W}}\right)^{old}+(1-relax)*\left(a_{\mathrm{F}}^{\mathrm{W}}\right)^{new}$.
   (c) Go back to a) and update $\{\tau,\tau^*\}$ and $\{w,w^*\}$ gently. Note that while I treat $\{z,z^*,k,k^*\}$ as fixed for now, waiting times and wages change since they also depend on the evolution of the technological frontier. Iterate till convergence. This always involves the inner loop i) – iv) and the outer loop a) – c).
3. Given a new solution for the evolution of the technological frontier, return to 2. and iterate on all previous loops up until convergence in $\{z,z^*,k,k^*,a_{\mathrm{F}}^{\mathrm{W}}\}$.
4. Finally check skilled labor market clearing in each country, and use the excess demand function to update the skill premium in the right direction. Iterate over all loops up until the sequence of skill premia has converged in each country.

Full convergence. In the case of full convergence I let the research productivity and skill endowment of the poor country converge to the level of the rich country, i.e., ,$\gamma^*\to\gamma,h^*\to h$. I assume that the emerging market fully converges to the fundamentals of the advanced economy within 30 years. I impose a process that mimics convergence in the Solow model, and uses the formula $h_t^*=\left\{h^{1-\alpha_{\text{conv}}}\left(1-e^{-(1-\alpha_{\text{conv}})t}\right)+\left(h_0^*\right)^{1-\alpha_{\text{conv}}}e^{-(1-\alpha_{\text{conv}})t}\right\}^{\frac{1}{1-\alpha_{\text{conv}}}}$ where α_{conv} governs the speed of convergence, and an analogous expression applies to γ_t^*. I assume $\alpha_{\text{conv}}=.87$, which induces convergence patters as depicted in figure 20.

The previous computational algorithm applies almost unchanged. Changes in skilled labor endowments shows up in the market clearing condition. And the free entry condition into research

Figure 20. Full Convergence in East



needs to be updated slightly $\tilde{v}_{\mathrm{I},k,t} = \frac{1}{\gamma_{k,t}} h_{\mathrm{I},k,t}^{1-\lambda} s_{k,t}$ where the exogenous research productivity is now time varying for the emerging market. Laws of motion of idea creation also need to be updated to take account of the changing research productivity.

B.3 Initial Idea-Ownership in Integrated Equilibrium

The issue here is that in autarky countries may invent identical technology, which begs the question: who owns the technology in the initial integrated equilibrium. I say initial because in the long-run the initial allocation doesn't matter and the share of ideas is fully pinned down by differences in research specialization and country size. I next offer a micro-foundation that pins down this initial ownership allocation.

Suppose all ideas only invented by the West are held by the West. Ideas that overlap in the interval $[0, A_{\mathrm{F}}^*]$ are split across East and West as follows. Suppose that the same idea across countries differs by some quality-metric $q > 0$ such that the effective quality is $q_c = 1 + \epsilon_c + u_c$. The parameter $\epsilon_c > 0$

is a country-specific quality shifter. Let the shock u be bounded between $[-1, 1]$, mean-zero, and with a vanishingly small variance. Further, suppose that an innovator needs to pay a fixed cost f_o for as long as the idea exists. As argued in Acemoglu et al. (2018), such a setup converges in the limit to the simple baseline monopoly pricing when $f_o \to 0, \mathbb{V}[u] \to 0$, and $\epsilon_c \to 0 \, \forall c$, but the ratio $\frac{\epsilon_c}{\epsilon_k} \to b > 0$ converges to something bounded away from zero. This sustains an ownership structure at time zero where the advanced economy's share of global patents equals ζ, with ζ being a function of b and the initial gap $\frac{A_F}{A_F^*}$.

B.4 Additional Results

I next report the long-run wage effects for different values of ϕ and β. I note that the negative impact on globalization becomes more pronounced for more negative values of ϕ. Skilled labor always gains, no matter how negative the knowledge externality is.

Figure 21. Robustness ϕ and β

	$(\Delta \log w_L, \Delta \log w_H)$		
$(\Delta \log w_L, \Delta \log w_H)$	β =0.2	β =0.3	β =0.4
ϕ =0.5	(0.109 ,0.487)	(0.128 ,0.393	(0.127 ,0.304)
ϕ =-0.2	(-0.062 ,0.324)	(-0.057 ,0.207)	(-0.045 ,0.127)
ϕ =-1.5	(-0.12 ,0.266)	(-0.118 ,0.141)	(-0.097 ,0.067)

Note. The table computes long-run wage effects holding all other deep parameters of the model fixed other than ν, which is set to deliver $z = .6$ consistent with the baseline calibration. Each cell reports a long-run wage effect where the first entry is unskilled wages and the second entry is skilled wages: $(\Delta \log w_L, \Delta \log w_H)$.

C Empirical Appendix

C.1 Micro Evidence on the Link between Adoption and Skill

Finally, the key mechanism is consistent with and provides a micro-foundation for a number of recent studies: using micro data and a causal estimation design based on cross-regional variation, Lewis (2011), Beaudry, Doms, and Lewis (2010), and Imbert et al. (2022), provide compelling evidence that a change in the local skill mix towards less skilled workers reduces a local labor market's adoption of new technologies. Recent work by Bijnens, Humlum, and Vanormelingen (2025) and Bloom et al. (2025) also lines up well with the key assumption. The first study uses detailed firm-to-firm trade data to show that skilled labor plays a central role in technology adoption, consistent with the model's emphasis on overhead skilled labor services required to adopt capital goods; the second study by Bloom et al. (2025) documents that new technologies diffuse primarily through their adoption in lower-skilled jobs, consistent with the complementarity between adoption and low-skilled wages in the model. Slowing diffusion then immediately rationalizes weak unskilled worker wage growth.

Voth, Caprettini, and Trew (2023).

C.2 Calibrating λ

To calibrate the model I have to pin down λ,which does not have a direct antecedent in the literature. As explained in the main body of the paper the parameter calibrated in Jones (1995) is slightly different. In principal, cross-country research specialization among a set of countries with similar research productivities γ allows me to identify λ since the theory implies that the steady state share of ideas is a function of specialization in research, and total country size measured in terms of unskilled labor

$$\log \chi_c = \alpha_0 + \lambda \log H_{\mathrm{I}} + (1-\lambda) \log L.$$

I use data from the OECD, and combine it with Barro and Lee's skill measure. I proxy the steady state patent share (a stock) with average patents over the period from 2011-2019 (a flow). Note that patents are an imperfect proxy that likely understate innovative activity and technological change. As long as the paten share is a linear function of the larger share of actual ideas develop in some country, the approach works nonetheless.

The measure of patents is based on patents captured by the Patent Cooperation Treaty (PIC), which are global patents that simultaneously protect intellectual property among several countries. This selection helps make patents comparable across countries, and is closest to the notion of "global ideas" in the paper. Clearly, patents are a crude measure of innovative activity, and there is no reason to believe the economy is exactly in steady state. Nor should we think that there is no heterogeneity in innovative productivity among rich countries in Europe. Overall, the approach is imperfect.

Perhaps surprisingly, the empirical results are extremely sensible without any massaging by using just a simple simple linear regression. After making an ad-hoc adjustment for the bias induced by unobserved heterogeneity in patenting productivity,[75] I arrive at an estimate $\lambda = .9$ which is a reasonable value for a first pass.

Figure 22. Regression for λ

	log patents	log patents	log patents	log patents	log patents
log researchers	1.093***		0.980**		0.878**
	(0.0644)		(0.335)		(0.390)
log employment		1.070***	0.113	1.093***	0.122
		(0.0745)	(0.353)	(0.0657)	(0.390)
log (research/employment)				0.980**	
				(0.335)	
N	13	13	13	13	13
R^2	0.961	0.944	0.961	0.961	

The outcome variable is log patents. Employment is defined as total unskilled labor L consistent with the theory. Robust standard error are computed, * p<0.10, ** p<0.05, *** p<0.01. The last column runs a constrained regression enforcing that the slope coefficients have to add up to one.

I provide robustness results next where I use total employment instead of unskilled labor but the estimation is otherwise unchanged.

Table 4. Regression for λ robustness

	log patents	log patents	log patents	log patents
log researchers	1.093***		0.942**	
	(0.0644)		(0.360)	
log employment		1.074***	0.152	1.093***
		(0.0746)	(0.383)	(0.0654)
log (research/employment)				0.942**
				(0.360)
N	13	13	13	13
R^2	0.961	0.945	0.961	0.961

The outcome variable is log patents. The main difference to the previous table is that employment is now total employment. Robust standard error are computed, * p<0.10, ** p<0.05, *** p<0.01. The last column runs a constrained regression enforcing that the slope coefficients have to add up to one.

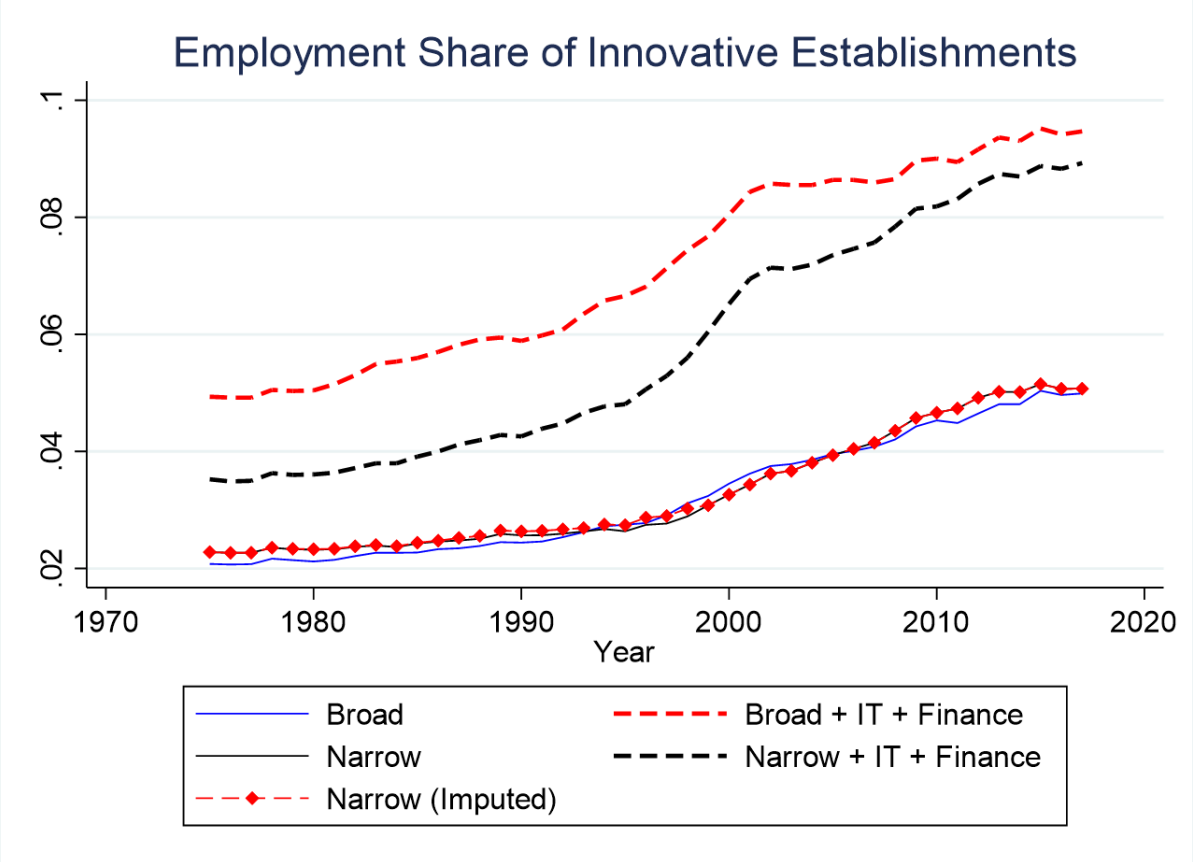
[75] Sampson (2023) argues heterogeneity in innovative productivity is a lot smaller among advanced economies relative to the gap between advanced economies and emerging markets.

C.3 IAB Data

The IAB data is provided by the Institute of Employment Research in Nuremberg. While data access is restricted, the application process is fairly straightforward and open to researchers. The dataset combines worker level data including wage, skill, and demographics with establishment level data including total employment and sectoral classification.

Sectoral Classification. The model features a two-sector structure so I classify establishments into innovative and production establishments. I use different definitions for robustness. Figure 23 plots the evolution of the aggregate employment share of innovative establishments.

Figure 23. Employment Share in West-Germany



Employment shares in production and research sector are based on IAB data for West Germany with additional details in the appendix. This classification can be carried out using different sectoral codes (broad vs. narrow where the narrower codes are unavailable for earlier periods) and a question arises whether IT and finance belong into the research or production sector.

The sectoral classifications are defined in the do-file changing_innovation_activity.do. Below is a short summary of the sectors that appear in the definition.

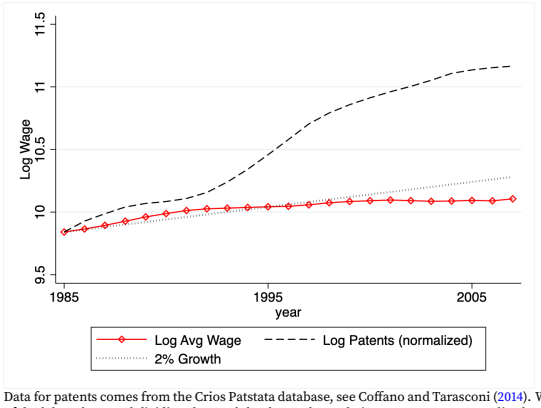
An important statistic omitted in the main part of the paper is the increase in the skill premium plotted in figure 25. Note that the IAB data is not suitable to document this increase due to heavy top coding. The skill premium appears stable in the beginning of the sample period and then widens substantially since the mid 90s.

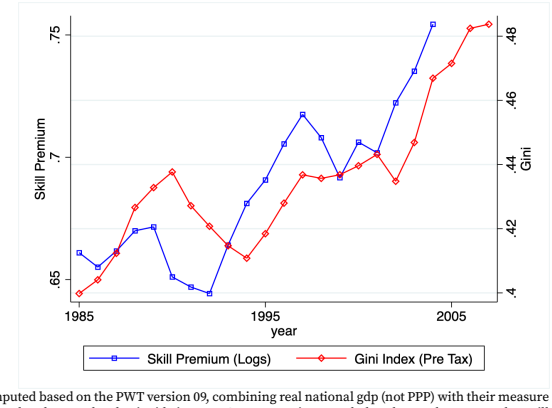
Figure 24. Sectoral classification

Industry Digit	variable_na	sector code	label	baseline	baseline plus Finance and IT
5-Steller	(w93_5)	65110	Zentralbanken	0	1
5-Steller	(w93_5)	65121	Kreditbanken einschliesslich Zw	0	1
5-Steller	(w93_5)	65124	Genossenschaftliche Zentralban	0	1
5-Steller	(w93_5)	65126	Realkreditinstitute	0	1
5-Steller	(w93_5)	65127	Kreditinstitute mit Sonderaufg	0	1
5-Steller	(w93_5)	65210	Institutionen fuer Finanzierung	0	1
5-Steller	(w93_5)	65220	Spezialkreditinstitute	0	1
5-Steller	(w93_5)	65231	Kapitalanlagegesellschaften	0	1
5-Steller	(w93_5)	65233	Sonstige Finanzierungsinstitut	0	1
5-Steller	(w93_5)	67110	Effekten- und Warenterminboerse	0	1
5-Steller	(w93_5)	67120	Effektenverm und	0	1
5-Steller	(w93_5)	67130	Sonstige mit dem Kreditgewerbe	0	0
5-Steller	(w93_5)	72201	Softwareberatung	0	1
5-Steller	(w93_5)	72202	Softwareentwicklung	0	1
5-Steller	(w93_5)	72301	Datenerfassungsdienste	0	1
5-Steller	(w93_5)	72302	Datenverarbeitungs- und Tabell	0	1
5-Steller	(w93_5)	72303	Bereitstellungsdienste fuer Tei	0	1
5-Steller	(w93_5)	72304	Sonstige Datenverarbeitungsdie	0	1
5-Steller	(w93_5)	72400	Datenbanken	0	1
5-Steller	(w93_5)	72500	Instandhaltung und Reparatur v	0	0
5-Steller	(w93_5)	72601	Informationsvermittlung	0	1
5-Steller	(w93_5)	72602	Mit der Datenverarbeitung verb	0	1
5-Steller	(w93_5)	73101	Forschung und Entwicklung im	1	1
5-Steller	(w93_5)	73102	Forschung und Entwicklung im	1	1
5-Steller	(w93_5)	73103	Forschung und Entwicklung im	1	1
5-Steller	(w93_5)	73104	Forschung und Entwicklung im	1	1
5-Steller	(w93_5)	73105	Forschung und Entwicklung im	1	1
5-Steller	(w93_5)	73201	Forschung und Entwicklung im	1	1
5-Steller	(w93_5)	73202	Forschung und Entwicklung im	1	1
5-Steller	(w93_5)	74111	Rechtsanwaltskanzleien mit Not	0	0
5-Steller	(w93_5)	74112	Rechtsanwaltskanzleien ohne No	0	0
5-Steller	(w93_5)	74113	Notariat	0	0
5-Steller	(w93_5)	74114	Patentanwaltskanzleien	1	1
5-Steller	(w93_5)	74115	Sonstige Rechtsberatung	0	0
5-Steller	(w93_5)	74121	Praxen von Wirtschaftspruefern,	0	0
5-Steller	(w93_5)	74122	Praxen von vereidigten Buchprue	0	0
5-Steller	(w93_5)	74131	Marktforschung	1	1
5-Steller	(w93_5)	74132	Meinungsforschung	0	0
5-Steller	(w93_5)	74141	Unternehmensberatung	1	1
5-Steller	(w93_5)	74142	Public-Relations-Beratung	1	1
5-Steller	(w93_5)	74151	Beteiligungsgesellschaften mit	0	1
5-Steller	(w93_5)	74152	Sonstige Beteiligungsgesellsch	0	1
5-Steller	(w93_5)	74153	Geschlossene Immobilienfonds m	0	0
5-Steller	(w93_5)	74154	Geschlossene Immobilienfonds m	0	0
5-Steller	(w93_5)	74155	Komplementaergesellschaften	0	1
5-Steller	(w93_5)	74156	Verwaltung und Fuehrung von	1	1
5-Steller	(w03_5)	65110	Zentralbanken	0	1
5-Steller	(w03_5)	65121	Kreditbanken einschliesslich Zwei	0	1
5-Steller	(w03_5)			0	
5-Steller	(w03_5)	65124	Genossenschaftliche Zentralbanken	0	1
5-Steller	(w03_5)	65126	Realkreditinstitute	0	1
5-Steller	(w03_5)	65127	Kreditinstitute mit Sonderaufgabe	0	1
5-Steller	(w03_5)	65210	Institutionen fuer Finanzierungsl	0	1
5-Steller	(w03_5)	65220	Spezialkreditinstitute	0	1
5-Steller	(w03_5)	65231	Kapitalanlagegesellschaften	0	1
5-Steller	(w03_5)	65233	Sonstige Finanzierungsinstitution	0	1
5-Steller	(w03_5)	67110	Effekten- und Warenboersen	0	1
5-Steller	(w03_5)	67120	Effektenvermittlung und	0	1
5-Steller	(w03_5)			0	
5-Steller	(w03_5)	72210	Verlegen von Software	0	1
5-Steller	(w03_5)	72221	Softwareberatung	0	1
5-Steller	(w03_5)	72222	Entwicklung und Programmierung vo	0	1
5-Steller	(w03_5)	72223	Sonstige Softwareentwicklung	0	1
5-Steller	(w03_5)	72301	Datenerfassungsdienste	0	1
5-Steller	(w03_5)	72303	Bereitstellungsdienste fuer Teiln	0	1
5-Steller	(w03_5)	72305	Sonstige Datenverarbeitungsdienst	0	1
5-Steller	(w03_5)	72400	Datenbanken	0	1
5-Steller	(w03_5)	72601	Informationsvermittlung	0	1
5-Steller	(w03_5)	72602	Mit der Datenverarbeitung verbund	0	1
5-Steller	(w03_5)	73101	Forschung und Entwicklung im	1	1
5-Steller	(w03_5)	73102	Forschung und Entwicklung im	1	1
5-Steller	(w03_5)	73103	Forschung und Entwicklung im	1	1
5-Steller	(w03_5)	73104	Forschung und Entwicklung im	1	1
5-Steller	(w03_5)	73105	Forschung und Entwicklung im	1	1
5-Steller	(w03_5)	73201	Forschung und Entwicklung im	1	1
5-Steller	(w03_5)	73202	Forschung und Entwicklung im	1	1
5-Steller	(w03_5)	74111	Rechtsanwaltskanzleien mit Notari	0	0
5-Steller	(w03_5)	74112	Rechtsanwaltskanzleien ohne Notar	0	0
5-Steller	(w03_5)	74113	Notariate	0	0
5-Steller	(w03_5)	74114	Patentanwaltskanzleien	1	1
5-Steller	(w03_5)	74115	Sonstige Rechtsberatung	0	0
5-Steller	(w03_5)	74121	Praxen von Wirtschaftsprueferinne	0	0
5-Steller	(w03_5)	74122	Praxen von vereidigten Buchpruefe	0	0
5-Steller	(w03_5)	74123	Praxen von Steuerberaterinnen und	0	0
5-Steller	(w03_5)	74124	Praxen von Steuerbevollmaechtigte	0	0
5-Steller	(w03_5)	74125	Buchfuehrung (ohne Datenverarbeit	0	0

Industry Di	variable_na	sector code	label	baseline	baseline plus Finance and IT
3-Steller	(w93_3)	651	Zentralbanken u. Kreditinst.	0	1
3-Steller	(w93_3)	652	So. Finanzierungsinstitute	0	1
3-Steller	(w93_3)	722	Softwarehaeuser	0	1
3-Steller	(w93_3)	723	Datenverarbeitungsdienste	0	0
3-Steller	(w93_3)	724	Datenbanken	0	0
3-Steller	(w93_3)	726	Verb Ttg. der Datenverarb.	0	0
3-Steller	(w93_3)	731	F&E Naturwissenschaft	1	1
3-Steller	(w93_3)	732	F&E Recht, Wirtschaft usw.	1	1
3-Steller	(w93_3)	741	Beratungsunternehmen	1	1
3-Steller	(w93_3)	742	Architektur- u. Ingenieurbuero	0	0
3-Steller	(w93_3)	743	Tech., physik. u. chem.	0	0
3-Steller	(w93_3)	744	Werbung	0	0
5-Steller	(w03_5)	74131	Marktforschung	1	1
5-Steller	(w03_5)	74132	Meinungsforschung	0	0
5-Steller	(w03_5)	74141	Unternehmensberatung	1	1
5-Steller	(w03_5)	74142	Public-Relations-Beratung	1	1
5-Steller	(w03_5)	74151	Managementtaetigkeiten von Hold	1	1
5-Steller	(w03_5)	74152	Managementtaetigkeiten von sonst	1	1
5-Steller	(w03_5)	74153	Geschlossene Immobilienfonds mit	0	0
5-Steller	(w03_5)	74154	Geschlossene Immobilienfonds mit	0	0
5-Steller	(w03_5)	74155	Komplementaergesellschaften	0	1
5-Steller	(w03_5)	74156	Verwaltung und Fuehrung von	1	1
5-Steller	(w08_5)	62011	Entwicklung und Programmierung	0	1
5-Steller	(w08_5)	62019	Sonstige Softwareentwicklung	0	1
5-Steller	(w08_5)	62020	Erbringung von Beratungsleistunge	0	1
5-Steller	(w08_5)	62030	Betrieb von Datenverarbeitungsein	0	1
5-Steller	(w08_5)	62090	Erbringung von sonstigen Dienstle	0	1
5-Steller	(w08_5)	63110	Datenverarbeitung, Hosting und da	0	1
5-Steller	(w08_5)	63120	Webportale	0	0
5-Steller	(w08_5)	63910	Korrespondenz- und Nachrichtenbü	0	0
5-Steller	(w08_5)	63990	Erbringung von sonstigen Informat	0	0
5-Steller	(w08_5)	64110	Zentralbanken	0	1
5-Steller	(w08_5)	64191	Kreditbanken einschliesslich Zwei	0	0
5-Steller	(w08_5)	64192	Kreditinstitute des Sparkassensek	0	0
5-Steller	(w08_5)	64193	Kreditinstitute des Genossenschaf	0	0
5-Steller	(w08_5)	64194	Realkreditinstitute	0	1
5-Steller	(w08_5)	64195	Kreditinstitute mit Sonderaufgabe	0	1
5-Steller	(w08_5)	64196	Bausparkassen	0	0
5-Steller	(w08_5)	64200	Beteiligungsgesellschaften	0	1
5-Steller	(w08_5)	64300	Treuhand- und sonstige Fonds	0	1
5-Steller	(w08_5)	64910	Institutionen fuer Finanzierungsl	0	1
5-Steller	(w08_5)	64921	Spezialkreditinstitute (ohne Pfan	0	1
5-Steller	(w08_5)	64922	Leihhaeuser	0	0
5-Steller	(w08_5)	64991	Investmentaktiengesellschaften un	0	1
5-Steller	(w08_5)	64999	Sonstige Finanzierungsinstitution	0	1
5-Steller	(w08_5)	66110	Effekten- und Warenboersen	0	1
5-Steller	(w08_5)	66120	Effekten- und Warenhandel	0	1
5-Steller	(w08_5)	66190	Sonstige mit Finanzdienstleistung	0	0
5-Steller	(w08_5)	66210	Risiko- und Schadensbewertung	0	0
5-Steller	(w08_5)	66220	Taetigkeit von Versicherungsmakle	0	0
5-Steller	(w08_5)	66290	Sonstige mit Versicherungsdienstl	0	0
5-Steller	(w08_5)	66300	Fondsmanagement	0	1
5-Steller	(w08_5)	69101	Rechtsanwaltskanzleien mit Notari	0	0
5-Steller	(w08_5)	69102	Rechtsanwaltskanzleien ohne Nota	0	0
5-Steller	(w08_5)	69103	Notariate		0
5-Steller	(w08_5)	69104	Patentanwaltskanzleien	1	1
5-Steller	(w08_5)	69109	Erbringung sonstiger juristischer	0	0
5-Steller	(w08_5)	69201	Praxen von Wirtschaftspruefernne	0	0
5-Steller	(w08_5)	69202	Praxen von vereidigten Buchpruefe	0	0
5-Steller	(w08_5)	69203	Praxen von Steuerbevollmaechtigte	0	0
5-Steller	(w08_5)	69204	Buchfuehrung (ohne Datenverarbei	0	0
5-Steller	(w08_5)	70101	Managementtaetigkeiten von Hold	1	1
5-Steller	(w08_5)	70109	Sonstige Verwaltung und Fuehrung	1	1
5-Steller	(w08_5)	70210	Public-Relations-Beratung	1	1
5-Steller	(w08_5)	70220	Unternehmensberatung	1	1
5-Steller	(w08_5)	72110	Forschung und Entwicklung im	1	1
5-Steller	(w08_5)	72190	Sonstige Forschung und Entwicklu	1	1
5-Steller	(w08_5)	72200	Forschung und Entwicklung im	1	1
5-Steller	(w08_5)	73110	Werbeagenturen	0	0
5-Steller	(w08_5)	73120	Vermarktung und Vermittlung von	0	0
5-Steller	(w08_5)	73200	Markt- und Meinungsforschung	1	1

Figure 25. Growth, Patents, and Inequality in Germany





Data for patents comes from the Crios Patstata database, see Coffano and Tarasconi (2014). Wage data is computed based on the PWT version 09, combining real national gdp (not PPP) with their measure of the labor share and dividing thorough by the total population. Patents are normalized so that the wage level and patent level coincide in 1984. GDP per capita growth does better than wages, but still grows substantially below trend, leading to an overall growth slowdown. Data for the skill premium, denoted as $\log\left(\frac{w_H}{w_L}\right)$ where the wage rates are the price of one hour of skilled or unskilled labor, comes from the KLEMS data version 07. Skill here refers to college-educated workers, group 3 in the Klems data. I do not make additional adjustments for efficiency units within skill group, which does not change the broad pattern. See the discussion and adjustments made in Buera et al. (2022) who also use the Klems data.The Gini index is pre tax and taken from the World Inequality Database of Alvaredo et al. (2020).