ESG and Mutual Fund Competition*

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Abstract

We model competition among mutual funds with diverse Environmental, Social, and Governance (ESG) mandates and conventional funds, when investors have heterogeneous sustainability preferences. In a baseline model, we characterize an equilibrium with segmentation, in which ESG funds cater exclusively to sustainability-minded investors, while conventional funds target only ESG-neutral investors. The model predicts that conventional funds survive only if they are of high quality. In contrast, underperforming ESG funds can persist due to the presence of investors with heterogeneous ESG preferences. Moreover, we show that competition from conventional funds is essential to curb the potential for ESG funds to set high fees and reduce investors' welfare. By extending the model, we show that a form of market integration arises in equilibrium in which all investors invest in ESG funds is possible only if ESG funds can outperform conventional funds before fees. Finally, when ESG investors have less rigid preferences for sustainable investing, competition from conventional funds lowers equilibrium ESG fund fees relative to the baseline model. Overall, our findings show that investors' heterogeneous sustainability preferences together with before-fee performance differences across funds jointly determine whether markets are segmented or integrated.

JEL codes: G11; G20; G23.

Keywords: ESG investors' preferences; ESG mutual funds; competition; fund fees

1. Introduction

Socially responsible investing (SRI) emerged in the 1970s as an investing strategy in which investors seek financial returns but screen out companies using ethical and moral criteria (such as tobacco, alcohol, weapons, etc.). ESG (Environmental, Social and Governance) investing builds on SRI and incorporates ESG criteria into the decision-making process. The asset management industry's embrace of ESG investment principles was initially relatively slow but has accelerated in recent years due to increasing firm and investor concerns about the impact of corporate activities on stakeholders and society more broadly. This interest has made ESG investing one of the fastest growing investment trends, with more than $30 trillion Global ESG assets in 2022 and expected to reach $40 trillion assets by 2030.[1] This trend has led to heightened competition among funds in the ESG space, but also between ESG funds and conventional funds. In this paper, we investigate how the coexistence of ESG funds with conventional funds affects the nature of mutual fund competition. In particular, we ask: How do ESG funds compete for investors with heterogenous ESG preferences? How do competition in the ESG space and in the conventional space affect each other? And, what is the impact on investors' welfare?

To achieve our goal, we build a model in which investors have heterogeneous preferences for sustainability and mutual funds compete to attract investors' money. In our model there are two types of funds: ESG funds, which are managed according to ESG and financial criteria;

*We are grateful for comments from Judith Bohnenkamp, Julio Crego, Xander Hut, Sonia Falconieri, Inés Macho Stadler, Carolina Manzano, Ricardo Ribeiro, Fernando Zapatero and participants at the 2024 Finance Theory Group Summer Conference, 31st Finance Forum, 2024 Jornadas de Economía Industrial, 21st Corporate Finance Days, KU Leuven, 5th Annual Boca-ECGI Corporate Finance and Governance Conference, the 8th GRASFI Annual Conference, ESMA 2025 Research Conference, 2025 Symposium of Economic Analysis, NOVA School of Business and Economics and Católica-Lisbon School of Economics and Business seminars. Javier Gil-Bazo acknowledges financial support from the Severo Ochoa Programme for Centres of Excellence in R&D (Barcelona School of Economics CEX2024-001476-S), funded by MCIN/AEI/10.13039/501100011033, and the Spanish Ministry of Science and Innovation (research grant PID2023-153288NB-I00). Ariadna Dumitrescu acknowledges financial support from the Spanish Government, Ministry of Science and Innovation's grant PID2021-123748NB-I00 and PID2024-159188NB-I00, 2025-URL-090, and Banc Sabadell.

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[1] See Bloomberg Intelligence report, "Global ESG assets predicted to hit $40 trillion by 2030, despite challenging environment," February 08, 2024, available at https://www.bloomberg.com/company/press/global-esg-assets-predicted-to-hit-40-trillion-by-2030-despite-challenging-environment-forecasts-bloomberg-intelligence/.

and conventional funds, which invest according to financial criteria exclusively. Both ESG and conventional funds are actively managed and can be of either high or low quality in terms of the asset manager's ability to deliver risk-adjusted returns (alpha). In the baseline model, the high-quality ESG fund cannot outperform the high-quality conventional fund before fees. However, we relax this assumption in an extension. Funds with ESG mandates cater to investors with sustainability preferences by offering a range of approaches that align with investors' environmental, social, and governance values. In our model, investors' preferences for sustainability are heterogeneous in two different ways. First, we assume that while some investors, which we refer to as neutral investors, derive utility exclusively from alpha, other investors, which we refer to as ESG investors, derive both pecuniary utility from alpha and non-pecuniary utility from investing according to sustainability principles. In our baseline model, we assume that ESG investors' preference for sustainability is so strong that they shun conventional funds altogether, but we also explore the case in which ESG investors have finite disutility from investing in conventional funds.[2] Second, ESG investors also differ from each other in that they value different ESG objectives differently. For instance, some ESG investors may be primarily motivated by environmental concerns, while others may place more importance on social issues. Consequently, ESG investors derive more non-pecuniary utility from ESG funds whose mandates include objectives that are more closely aligned with their own personal preferences. We further assume that investors can invest in risky assets only through mutual funds as in Gennaioli et al. (2015), so their utility of not investing is zero.

Our baseline model delivers several predictions. First, in equilibrium low-quality conventional funds are driven out of the market by high-quality conventional funds. Second, there is segmentation in that ESG funds cater to ESG investors and refuse to compete for neutral investors, who invest only in high-quality conventional funds. Third, we derive the equilibrium fees and assets under management for all types of funds. We show that if the differentiation among ESG funds is important enough for ESG investors relative to performance

[2]This extension of the model introduces endogenous no-leakage constraints on ESG fees and generates richer market structures, including partial integration, endogenous fee caps, and regimes in which ESG funds cannot profitably operate.

differences, ESG funds of different quality will coexist in the ESG segment of the market and the low-quality funds will do so despite offering lower after-fee expected alpha. Therefore, the existence of investors' ESG preferences can explain the survival of ESG funds that are expected to underperform. This prediction is consistent with the empirical evidence that investors in ESG funds are less sensitive to fund underperformance (Bollen, 2007; Renneboog et al., 2011). Fourth, our model delivers a novel prediction related to cross sectional differences in performance: We should expect differences in performance among ESG funds but not among conventional funds. This is a natural consequence of segmentation. Competition is fierce in the conventional segment of the market, but is relaxed by investors' heterogeneous preferences in the ESG segment. To the extent that managerial skill persists, the model predicts that observed differences in performance among ESG funds also persist through time. In contrast, any performance differences among conventional funds are the consequence of luck and short-lived.

Finally, the model yields a nuanced prediction regarding strategic fee setting. Under most parameter configurations, ESG funds charge higher average fees (and obtain higher mark-ups) than conventional funds because investors' sustainability preferences weaken price competition among ESG fund managers. However, the model also uncovers an interesting case: when the mass of neutral investors is small and differentiation between ESG funds is not too large, the fee of the high-quality conventional fund is lower than the average fee of the two ESG funds. Intuitively, when differentiation is not too high, the high-quality ESG fund cannot raise its fee too much. At the same time, if the mass of neutral investors is small, the high-quality conventional fund can raise its fee without fear that the high-quality ESG fund will attempt to compete for neutral investors. Therefore, our model can reconcile conflicting empirical evidence regarding differences in fees between ESG and conventional funds. While Gil-Bazo et al. (2010) find no significant differences in fees between SRI and conventional U.S. equity funds, more recent studies by Raghunandan and Rajgopal (2022), Baker et al. (2022), and Huij et al. (2023) document that ESG funds charge higher fees than conventional funds. In contrast, Henke (2016), ESMA (2022), Gibbon et al. (2025), and Black and Kölbel (2025)

document lower fees for ESG funds than for conventional funds.

We then extend the baseline model along two dimensions. First, we let the high-quality ESG fund outperform the high-quality conventional fund before fees. Superior ESG performance weakens the conventional fund's role as an outside option and at the same time induces ESG funds to lower fees and attract neutral investors, leading to market integration. Second, we relax the assumption that ESG investors experience infinite disutility from investing in conventional funds. We show that when sustainability preferences are less rigid, conventional funds discipline ESG pricing through endogenous fee caps and may prevent ESG funds from operating. These extensions show that ESG-driven segmentation depends on the interaction between relative performance and the strength of investors' non-pecuniary motives.

Our results carry important implications. While ESG mandates have become increasingly popular due to investors' growing interest in sustainabilty and several initiatives, such as the United Nations Principles for Responsible Investment Pledge and the Net Zero Asset Managers Initiative, investors and policy makers must be aware that ESG preferences can weaken competition. As a consequence, ESG-minded investors may end up holding underperforming funds and paying excessive fees. Also, the availability of conventional funds that are managed according to purely financial criteria benefits ESG investors. By providing an alternative for neutral investors, conventional funds limit the market power of ESG funds, leading to lower fees and improved investor welfare in the ESG segment of the market.

Our paper is related to the theoretical literature concerned with the impact of sustainability preferences on financial investment and asset prices.[3] Heinkel et al. (2001) show that polluting firms are pushed to reform because exclusionary screening negatively impacts their valuations. Pedersen et al. (2021) model ESG preferences and characterize an ESG-efficient frontier when the market is populated by ESG-motivated, ESG-aware and ESG-unaware investors. Pástor et al. (2021) show that the presence of investors who derive non-pecuniary utility from sustainable investing increases the prices of green stocks and decreases those of brown assets relative to the standard model with no preferences for sustainability. In a model

[3] A large empirical literature has tested the predictions of the theory. See, for instance, Hong and Kacperczyk (2009), Luo and Balvers (2017), Zerbib (2022), Pástor et al. (2022).

with partial segmentation, Zerbib (2022) shows that the expected returns of the assets are affected by the tastes and exclusion strategies of heterogeneous investors. Goldstein et al. (2022) analyze how investors' ESG preferences and the use of differential information alter the process of information aggregation in prices.

While these studies investigate price formation in the context of asset pricing models, we study for the first time how the presence of investors with sustainability preferences shapes the nature of competition in the market for mutual funds. Moreover, whereas the theoretical literature on sustainable finance predominantly treats ESG-minded investors as a homogeneous group, we demonstrate that heterogeneous preferences for sustainability crucially determine market outcomes. The assumption of heterogenous preferences is consistent with a growing body of empirical evidence. Giglio et al. (2025), Degryse et al. (2023), using survey data, and Siemroth and Hornuf (2023), using experimental data from a crowdfunding platform, show that there is also substantial heterogeneity across investors' preferences for sustainable investments. Thus, Giglio et al. (2025) show that 25% of the survey respondents (Vanguard clients) are primarily motivated by ethical considerations, while 22% are driven by climate hedging motives. Degryse et al. (2023), using a household survey of Dutch investors, show that 75% of sustainable investors are concerned with environmental issues, while only 17.1% are concerned with social issues. Finally, Siemroth and Hornuf (2023), using data from a crowdfunding platform, show that sustainable investments are mainly driven by valuing environmental impact rather than social impact.[4]

Our study is also related to the theoretical literature that investigates the influence of ESG investing on corporate behavior. Chowdhry et al. (2019) studies firms' optimal financing in the presence of externalities, highlighting the impact arising from the coexistence of profit and socially motivated investors on firms' profitability. Similarly, Oehmke and Opp (2024) study responsible investing in the presence of moral hazard, emphasizing the intricate interplay between production externalities and corporate financing constraints.Landier and Lovo (2025) build a general equilibrium model where sustainable investors yield equivalent returns to

[4]Note, however, that Lopez-de Silanes et al. (2024) show that institutional investors regard the governance dimension as the most critical factor in selecting portfolio companies.

regular investors in a market subject to a search friction. Their analysis focuses on the strategy of a unique ESG fund that maximizes social welfare and forces companies to partially internalize externalities and improve social welfare despite the selfishness of all agents. Finally, Green and Roth (2021) show that value-aligned investors who own socially valuable firms, compete against each other and push the price of firms upwards. Our paper contributes to this literature by showing how investors' preferences for sustainability influence the strategic decisions of mutual funds.

Finally, we contribute to a line of theoretical research on mutual funds that departs from the perfectly competitive setting of Berk and Green (2004). These models study the impact of heterogeneous consumer tastes (Metrick and Zeckhauser, 1998, Massa; 2003; Hortaçsu and Syverson, 2004; Wahal and Wang, 2011; Khorana and Servaes, 2012; Hoberg et al., 2018; Kostovetsky and Warner, 2020), asymmetric information and different sensitivities to fees (Gil-Bazo and Ruiz-Verdú, 2008), heterogeneous liquidity needs (Nanda et al., 2000) and frictions (Dumitrescu and Gil-Bazo, 2018; Gârleanu and Pedersen, 2018; Roussanov et al., 2021). Like those studies, ours explains why strategically set fees may not offset differences in quality. The novelty of our model is that we introduce a new dimension in which investors differ: preferences for sustainable investments. Our paper shows that mutual funds that develop strategies in order to cater for heterogeneous preferences in terms of sustainability create product differentiation, and this has important implications for fee setting, gross returns, and asset allocation.

The rest of the paper is organized as follows. In section 2, we present the theoretical framework of our analysis. In section 3, we present the equilibrium in three different markets with and without ESG and conventional funds. Section 4 compares these markets and discusses how the coexistence of ESG and conventional funds impacts competition and investor welfare. Section 5 extends the baseline framework allowing for positive greenium and ESG investors participation in the conventional market. Finally, section 6 concludes. All proofs can be found in the Appendix.

2. The Model

Actively managed funds compete for investors' money and they vary accross two dimensions: sustainability and financial performance. Funds can invest considering ESG issues (ESG funds) or they can ignore them (conventional funds). In terms of financial performance funds may be of either high quality or low quality, reflecting the manager's skill to earn positive risk-adjusted returns, or alpha. In our economy there are two ESG funds, denoted by HS and LS, that have expected risk-adjusted return of high-quality, R_{HS}, and low-quality R_{LS}, respectively, with $R_{HS} > R_{LS}$. In addition, there are two conventional funds, denoted by HC and LC, and these funds are also of high and low quality, respectively, R_{HC} and R_{LC}, with $R_{HC} > R_{LC}$. For simplicity, we refer to expected risk-adjusted return as return. A fund's type is common knowledge.

We consider two continuums of investors: investors with ESG preferences, with mass λ_S, and neutral investors, without ESG preferences, with mass λ_N. ESG investors prefer ESG funds over conventional funds, and among ESG funds, they prefer funds with an investment objective that is more aligned with their personal preferences.[5] To model ESG investors' preference for a particular ESG fund, we use Hotelling's (1929) model of horizontal differentiation and assume that ESG investors are uniformly distributed along a line of length λ_S with the two ESG funds located at the extremes of the line. The proximity of a fund to an investor measures the alignment between the fund's ESG objective and the investor's preference. The shorter the distance, the closer the alignment, and the higher the non-pecuniary utility derived by the investor from investing in that fund.[6] The HS fund is located at $x = 0$, while the LS fund is located at $x = \lambda_S$.

We assume that an ESG investor's non-pecuniary utility of investing in an ESG fund that is a linear function of the investor's proximity to the fund.[7] That is, an investor located at x

[5]The empirical literature on sustainability preferences indicates that investors value sustainability and are willing to sacrifice returns to participate in ESG investing (Hartzmark and Sussman, 2019; Barber et al., 2021; Bauer et al., 2021; Ceccarelli et al., 2024; Heeb et al., 2023).

[6]In general the investors assess a fund across many ESG dimensions (indicators). Their distance from a certain fund objective can be calculated using a normalized Euclidean Distance resulting in a single scalar metric that captures the overall magnitude of deviation from the ESG fund's objective.

[7]Note that Wu and Zechner (2023) also model non-pecuniary payoffs that decrease linearly with the distance between a firm's political stance and the investor's own political preference.

derives non-pecuniary utility equal to $u_0 - kx$ by investing in the HS fund and non-pecuniary utility of $u_0 - k(\lambda_S - x)$ from investing in the LS fund, with $u_0 > 0$. Notice that when the investor's preferences are perfectly aligned with the fund's objective, the misalignment cost is 0. This is the case for the investor located at $x = 0$ investing in fund HS and for the investor located at $x = \lambda_S$ investing in fund LS. The parameter k can be interpreted as the intensity of the investor's preference for ESG for a specific ESG dimension. When $k = 0$, ESG investors derive the same non-pecuniary utility from investing in either fund.[8] We assume that the misalignment between ESG investors' preferences and conventional funds is so large that ESG investors suffer a high disutility from investing in those funds and avoid them altogether.[9] This implies that ESG investors are restricted to investing in one of the two ESG funds. Neutral investors may invest in any fund and their utility depends only on the net return of the fund they invest in. Both neutral and ESG investors have a reservation utility of zero, corresponding to the utility of not investing in any fund.[10]

2.1. The investor's problem

Each investor is endowed with one dollar and chooses one fund. The fund charges a fee, f, per dollar invested. Neutral investors derive a utility equal to $U_{\varphi}^{N} = R_{\varphi} - f_{\varphi}$, for investing in the active mutual fund of type $\varphi \in \{HS, LS, HC, LC\}$, where HS and LS denote ESG funds with different ESG integration strategy, HC a high quality conventional fund and LC a low quality conventional fund. The ESG investor i is willing to invest only in ESG funds and derives a utility equal to $U_{i,\varphi}^{ESG} = R_{\varphi} - f_{\varphi} + (u_0 - kd_{i,\varphi})$, where $d_{i,\varphi}$ denotes the distance between the investor and the fund. An investor decides to invest with fund φ as long as the utility from investing with that fund is higher than the utility from investing with any other fund and higher than her reservation utility.

The demand of neutral investors is split equally across all active funds offering the highest

[8] Similarly to the standard interpretation of the Hotelling model, the "distance" between the two funds can represent the difference in investment objectives between the two funds, not geographical distance.

[9] Note that a sufficient condition for this to hold is to assume that disutility from investing in the conventional funds higher or equal than $R_{HS} - R_{HC} - u_0$

[10] As in the model of Gennaioli et al. (2015), we assume that investors cannot directly invest in stocks, which can be explained if investors cannot take risks without the advice of asset managers.

positive net-of-fee return and is zero for all the other funds.

The demand of ESG investors for conventional funds is zero by assumption. Each ESG investor chooses to invest in the ESG fund offering the highest utility given the fund's return, fee, and the proximity between the fund and the investor, as long as this utility is positive, otherwise she decides not to invest. In case that both funds offer the same positive utility, her wealth is split equally between them.

2.2. The fund manager's problem

Fund managers choose the fees that maximize their profits given investors' demand functions and the other managers' strategies. Without loss of generality, we assume that the marginal cost to the manager of operating the fund is zero. Therefore, the manager's problem becomes:

$$\max_{f_\varphi} \Pi_\varphi = f_\varphi \left(q_{S,\varphi} + q_{N,\varphi} \right),$$

where $q_{S,\varphi}$, $q_{N,\varphi}$ denote the total demand for the fund from ESG and neutral investors, respectively in fund $\varphi \in \{HS, LS, HC, LC\}$.

3. Equilibrium

3.1. Market with only conventional funds

We consider first the case when only conventional funds exists. Thus, there exists a high quality conventional fund that has a return R_{HC} and a low quality conventional fund that has a return R_{LC}. The ESG investors are not willing to invest in any conventional fund and their utility is therefore equal to 0. As a result, the conventional funds compete only for neutral investors. Since the fund's type and return are common knowledge, the two funds compete à la Bertrand. Since the returns are such that $R_{HC} > R_{LC}$, the HC fund can drive the LC fund out of the market simply by setting a fee such that $R_{HC} - f_{HC} \geq R_{LC}$. In that case, LC cannot set a positive fee and stay in business. Therefore, the HC fund gains all the market of neutral investors and the LC fund does not operate. The high quality fund will

choose $f_{HC} = \Delta_C$, where $\Delta_C \equiv R_{HC} - R_{LC}$. All neutral investors invest with the high-quality conventional fund and their utility is equal to $U^N_{HC} = R_{HC} - f_{HC} = R_{HC} - \Delta_C = R_{LC}$.

3.2. Market with both ESG and conventional funds

The two ESG funds, HS and LS, are horizontally differentiated products for ESG investors due to their different ESG strategy. The two conventional funds, HC and LC, on the other hand, are not differentiated in that they are perfect substitutes for neutral investors. Let us define the *greenium* as $g \equiv R_{HC} - R_{HS}$. In the baseline case we focus on the case of negative or absent greenium, $g \leq 0$ (conventional fund return dominance). Just as in the previous case, conventional funds engage in Bertrand-like competition. The HC fund sets a fee $f_{HC} \leq \Delta_C$, and the LC is driven out of the market.

Lemma 1. *In equilibrium, no investors choose to invest in the low quality conventional (LC) fund.*

In next lemma, we show that neutral investors do not invest in ESG funds if the return of high-quality ESG funds is lower than the return of high-quality conventional funds, $R_{HS} > R_{HC}$.

Lemma 2. *In equilibrium, if $R_{HS} \leq R_{HC}$, ESG funds cater to ESG investors and conventional funds cater to neutral investors.*

To see how this form of segmentation arises in equilibrium in the case when $R_{HS} \leq R_{HC}$, assume that ESG funds decide to compete against conventional funds for neutral investors. If the two high-quality funds, HS and HC, engage in price competition, since $R_{HS} \leq R_{HC}$ they have incentive to undercut its fee until the fee of high-quality ESG fund HS equals zero, i.e., $f_{HS} = 0$. At that fee, the HS fund makes zero profits. However, the HS fund can always charge an arbitrarily small but positive fee and make positive profits by operating only with ESG investors. More specifically, it is sufficient for the HS fund to charge a fee lower than $\Delta \equiv R_{HS} - R_{LS}$ to dominate the LS fund for investors who have a sufficiently high preference for HS regardless of the fee charged by the LS fund. This ensures that those investors closest to HS are willing to invest in it and therefore HS makes a positive profit. Consequently, in

equilibrium HS does not sell to neutral investors. For the same reasons that LC is driven out of the market, and since $R_{LS} \leq R_{HS}$, LS does not attract any neutral investors.

The two ESG funds, HS and LS, compete against each other only for ESG investors. When all ESG investors purchase ESG funds, the market is covered. When some ESG investors decide not to invest, the market is not covered and the two ESG funds act as local monopolies. Depending on the values of the model parameters, we have three possible situations. First, the market is covered and all ESG investors invest in the HS fund. This happens when k is small enough, i.e., when investors exhibit less intense preferences for a particular ESG strategy so the two ESG funds are closer substitutes. Second, the market is covered and the two ESG funds share the market of ESG investors, each catering to the set of investors with more aligned preferences. This happens for an intermediate value of k. Third, when k is very high, the market is not covered. Some investors leave the market, funds act as local monopolies and set fees accordingly. Consequently, heterogeneity in investors' preferences for ESG allow both HS and LS funds to coexist in equilibrium. It is also necessary that the difference in performance is not too large, as large differences in performance lead all investors to concentrate in the best performing fund. Let us define the thresholds $k_1 \equiv \dfrac{\Delta}{3\lambda_S}, k_2 \equiv \dfrac{2u_0 + R_{HS} + R_{LS}}{3\lambda_S}$, which determine three distinct equilibrium regions:

- An *exclusionary ESG duopoly* is an equilibrium in which both ESG funds are potential competitors, but the low-quality ESG fund is endogenously driven to zero demand by the pricing of the high-quality fund.
- A *covered ESG duopoly* is an equilibrium in which both ESG funds are active and the ESG market is fully covered.
- An *uncovered ESG market* is an equilibrium in which both ESG funds operate as local monopolies and some ESG investors optimally choose not to invest.

Proposition 1 (Equilibrium fees and market shares)**.** *There exists an equilibrium that is segmented, i.e. ESG investors invest only in ESG funds and neutral investors invest only in HC. There are three regions depending on k with the following equilibrium structure:*

A. *Exclusionary ESG duopoly* ($k \leq k_1$). *The ESG fund HS serves all ESG investors and LS is inactive. Neutral investors are served by HC, while LC is inactive.*

B. *Covered ESG duopoly* ($k_1 < k < k_2$). *Both ESG funds are active and the ESG market is covered, HS and LS split ESG investors. Neutral investors are served by HC, while LC is inactive.*

C. *Uncovered ESG market* ($k \geq k_2$). *Both ESG funds are active as local monopolies and some ESG investors do not invest. Neutral investors are served by HC, while LC is inactive.*

Equilibrium fees and market shares for each region are reported in Table 1.

Table 1: Proposition 1 Equilibrium fees and market shares

Region	**Fund**	**Fee**	**ESG Market Share**	**Neutral Market Share**
Exclusionary ESG duopoly	HS	$\Delta - k\lambda_S$	λ_S	0
	LS	0	0	0
	HC	$\min\{\Delta_C, f_e^A\}$	0	λ_N
	LC	0	0	0
Covered ESG duopoly	HS	$k\lambda_S + \frac{1}{3}\Delta$	$\frac{1}{2k}\left(k\lambda_S + \frac{1}{3}\Delta\right)$	0
	LS	$k\lambda_S - \frac{1}{3}\Delta$	$\frac{1}{2k}\left(k\lambda_S - \frac{1}{3}\Delta\right)$	0
	HC	$\min\{\Delta_C, f_e^B\}$	0	λ_N
	LC	0	0	0
Uncovered ESG market	HS	$\frac{1}{2}(R_{HS} + u_0)$	$\frac{1}{2k}(R_{HS} + u_0)$	0
	LS	$\frac{1}{2}(R_{LS} + u_0)$	$\frac{1}{2k}(R_{LS} + u_0)$	0
	HC	$\min\{\Delta_C, f_e^C\}$	0	λ_N
	LC	0	0	0

Notes. The deviation-prevention fees f_e^A, f_e^B, and f_e^C are defined in the proof of Proposition 1 Appendix.

In the cases in which the LS fund survives, i.e., when $k > k_1$, it charges a lower fee than the HS fund. Moreover, when $k_1 < k < k_2$, the difference in fees between both funds equals $\frac{2}{3}\Delta = \frac{2}{3}(R_{HS} - R_{LS})$. When $k \geq k_2$, the difference is $\frac{1}{2}(R_{HS} - R_{LS})$. In either case, the difference in fees is not enough to fully offset the difference in before-fee performance. Therefore, when the LS fund survives in the market, it does so despite offering lower net performance than the high quality ESG fund, HS. In other words, in equilibrium, we expect differences in after-fee returns between ESG funds. In contrast, conventional funds are not expected to exhibit any differences in performance.

Notice that our model is static. However, since there is no asymmetric information and therefore, no role for learning about performance, a dynamic model built as a succession of static models would yield an identical outcome provided that pre-fee performance does not change over time. This observation helps understand why differences in net performance can persist. Performance persistence in our model is the consequence of variety of ESG styles and preferences in the ESG segment of the market. If funds were identical in the ESG dimension or if ESG investors all valued the same ESG attributes ($k = 0$), competition between ESG funds would drive the lower quality one out of the market, just like in the conventional segment of the market.

Figure 1 shows the equilibrium fees of the high-quality ESG fund and the high-quality conventional fund as a function of k. In order to ensure that the HS fund does not want to deviate from the equilibrium with segmentation, the HC fund always sets a fee lower than that charged by HS so that the profit obtained by HS from serving only ESG investors is higher than the profit obtained by lowering the fee in order to serve also the entire market of neutral investors. Therefore, although the markets are segmented, the fees in the conventional segment are influenced by competition from ESG funds.

When the strength of ESG investors' preferences is weak enough (i.e., $k \leq k_1$), k is small relative to the difference in returns between the LS and HS funds, and fees decrease with k. In this case, in order to keep the LS fund out of the market, the HS fund is forced to lower the fee as k increases. Otherwise, some investors will switch to the LS fund.

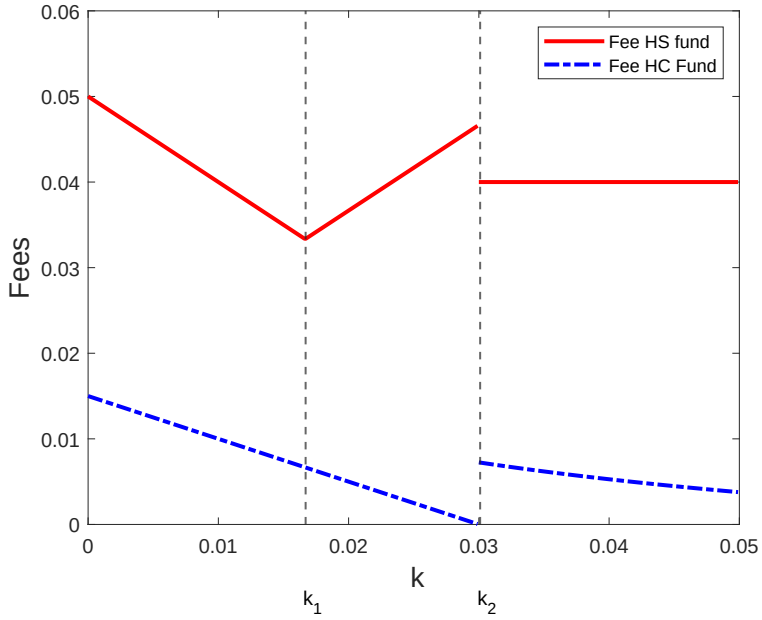


Figure 1: Fees of the HS fund and the HC fund. Parameter values: $R_{HC} = R_{HS} = 6\%$, $R_{LC} = R_{LS} = 1\%$, $\lambda_S = 1$, $\lambda_N = 1$ and $u_0 = 0.02$.

When k is high enough, $k_1 < k < k_2$, the market is covered by the two funds. In this case, the horizontal differentiation between the two funds given investors' preferences, is higher than the cost of investing with a less-than perfectly aligned fund. As the value of k increases, the two funds compete less intensely for the same group of investors, that is, the market power effect begins to outweigh the effect of direct competition. Consequently, investors located closer to a particular fund become effectively "captive," granting that fund greater monopoly power and enabling it to raise its fee. When k becomes sufficiently large (i.e., $k > k_2$), some investors, whose preferences are poorly aligned with either ESG fund, choose to exit the market altogether. As a result, demand for both ESG funds declines, and the market power effect becomes the dominant remaining force.

For a wide range of parameter values, the average fees are higher in the ESG segment than in the conventional segment because investors' sustainability preferences weaken competition

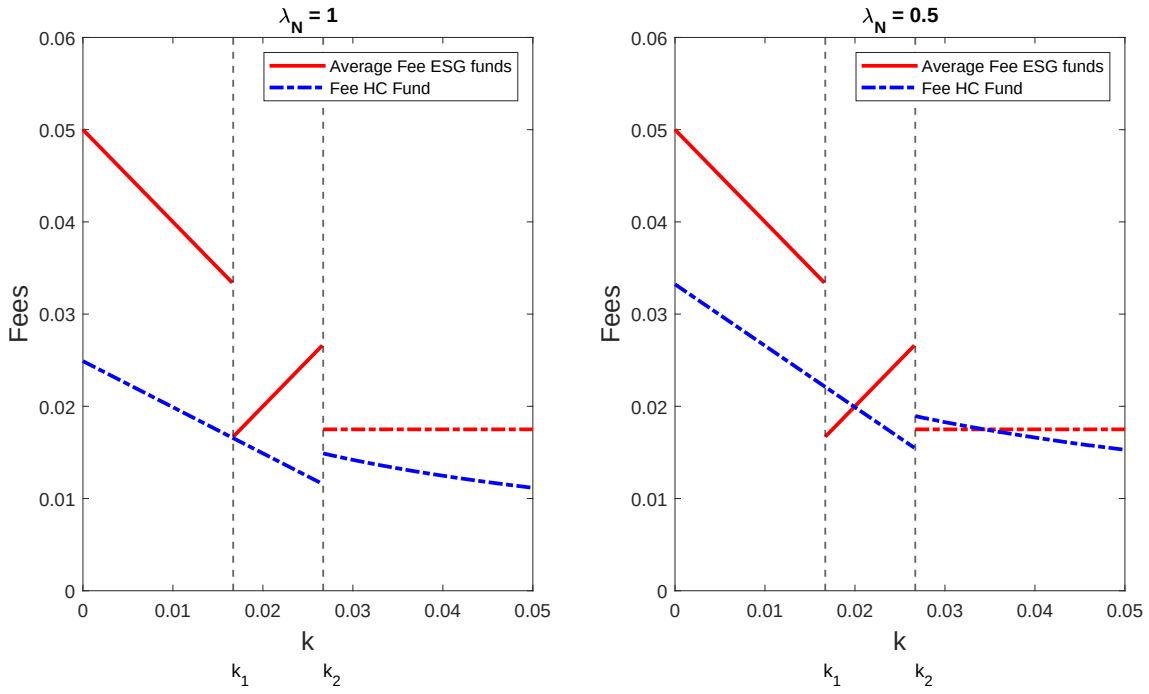


Figure 2: Comparison of fees in the ESG and conventional segment. Parameter values: $R_{HC} = R_{HS} = 6\%$, $R_{LC} = R_{LS} = 1\%$, $\lambda_S = 1$ and $u_0 = 0.01$.

among ESG funds and allow managers to extract higher markups. In contrast, conventional funds face fierce Bertrand-like price competition because neutral investors care only about net returns. However, when competition from the conventional segment is limited because the mass of neutral investors is small (small λ_N), the high-quality conventional fund (HC) can charge a higher fee than the average ESG fund for some parameter configurations. This case is illustrated in Figure 2. The equilibrium fee of the HC fund decreases as the differentiation parameter k increases. The HC fund, which serves the entire mass of neutral investors, possesses residual market power in the conventional segment once the low-quality conventional fund exits. However, ESG funds compete against each other despite investors' heterogeneous sustainability preferences, which determines their pricing when both ESG funds are active. As the two ESG funds become more differentiated ($k_1 < k < k_2$), the competition between them relaxes, allowing the HS fund to increase its fee. However, when HS still faces relatively strong competition from LS and the size of the neutral segment (λ_N) is small, the potential

gain from deviating in order to attract neutral investors is small for the HS fund. In this case, HC can raise its fee without fear of losing all the demand from neutral investors. As differentiation increases, competition in the ESG segment relaxes, HS increases its fee and the average ESG fee becomes again larger than the fee of the HC fund. A similar effect can be seen when $k > k_2$, when both HS and LS funds act as local monopolies. Since the market for neutral investors is small, the HS fund does not have a strong incentive to deviate and prefers to cater to the investors with preferences close to it. Therefore, in equilibrium, the HC fund may charge a higher fee than the average ESG fund, despite operating in a segment without sustainability differentiation. This apparently counterintuitive result highlights that limited competition from conventional funds can lead to higher fees for HC, even when ESG investors pay a "greenium" in other parts of the market. This result may explain the empirical evidence in Henke (2016), ESMA (2022), Gibbon et al. (2025), and Black and Kölbel (2025) that ESG funds charge lower fees.

3.3. Market with only ESG funds

We now study a market in which only ESG funds are offered to all types of investors. As mentioned above, the two ESG funds, HS and LS, are differentiated products from the point of view of ESG investors because of their particular ESG objectives. Let us first consider the case where there are no neutral investors. In this case, an equilibrium always exists and it is characterized in the following lemma.

Lemma 3 (ESG-only market equilibrium without neutral investors)**.** *Suppose that only ESG funds are present in the market. Then there exist three regions in k with the following equilibrium structure:*

A. Exclusionary ESG duopoly *($k \leq k_1$).* *The high-quality ESG fund HS serves all ESG investors, while the low-quality ESG fund LS is inactive.*

B. Covered ESG duopoly *($k_1 < k < k_2$).* *Both ESG funds are active and the ESG market is fully covered, HS and LS split ESG investors.*

C. Uncovered ESG market *($k \geq k_2$).* *Both ESG funds are active as local monopolies and some ESG investors do not invest.*

Equilibrium fees and market shares in each region are reported in Table 2.

Table 2: Equilibrium fees and ESG market shares in an ESG-only market without neutral investors

Region	**Fund**	**Fee**	**ESG Market Share**
Exclusionary ESG duopoly	HS	$\Delta - k\lambda_S$	λ_S
	LS	0	0
Covered ESG duopoly	HS	$k\lambda_S + \frac{1}{3}\Delta$	$\frac{1}{2}\lambda_S + \frac{1}{6k}\Delta$
	LS	$k\lambda_S - \frac{1}{3}\Delta$	$\frac{1}{2}\lambda_S - \frac{1}{6k}\Delta$
Uncovered ESG market	HS	$\frac{1}{2}(R_{HS} + u_0)$	$\frac{1}{2k}(R_{HS} + u_0)$
	LS	$\frac{1}{2}(R_{LS} + u_0)$	$\frac{1}{2k}(R_{LS} + u_0)$

All ESG investors choose to invest only with the high quality fund when the intensity of ESG preferences, k, is low. Similarly to the case when there are only conventional funds, the HS fund is able to drive out of the market the LS fund. When k takes an intermediate value, the two funds share the market. Finally, when k is high, the market is not covered and both funds act as local monopolies. The equilibrium fees and quantities for the ESG funds in this case are exactly the same as the ones when both ESG and conventional funds exists. In that case, due to segmentation of investors, ESG funds were competing only for attracting the ESG investors.

The existence of a positive mass of neutral investors changes the equilibrium in the market for ESG funds in a fundamental way, as funds also compete this mass of investors. Since no conventional funds are available, the neutral investors choose among the ESG funds the one that offers them the highest possible net return.

Let us define the thresholds $k_1^O \equiv \dfrac{\Delta}{2\lambda_N + 3\lambda_S}$, and $k_2^O \equiv \dfrac{2u_0 + R_{HS} + R_{LS}}{2\lambda_S + \lambda_N}$.

Proposition 2 (ESG-only funds with neutral investors)**.** *If the mass of neutral investors is high,* $\left(\lambda_N \geq \frac{\Delta}{k}\right)$ *no equilibrium exists. If the mass of neutral investors is low,* $\left(\lambda_N < \frac{\Delta}{k}\right)$ *then there exist three equilibrium regions in* k*:*

A. Exclusionary ESG duopoly. *For* $k \leq k_1^O$*, the high-quality ESG fund serves all investors, while the low-quality ESG fund is inactive.*

B. Covered ESG duopoly. *For* $k_1^O < k \leq \min\left\{k_2^O, \frac{\Delta}{2\lambda_N}\right\}$*, both ESG funds are active and the market is covered.*

C. Uncovered ESG market. *For* $k_2^O < k \leq \max\left\{k_2^O, \frac{\Delta}{\lambda_N}\right\}$*, both ESG funds act as local monopolies and some investors do not invest.*

No equilibrium exists for $\min\left\{k_2^O, \frac{\Delta}{2\lambda_N}\right\} < k \leq k_2^O$ *or* $k > \max\left\{k_2^O, \frac{\Delta}{\lambda_N}\right\}$*.*

Equilibrium fees and market shares in each region are reported in Table 3.

Table 3: Equilibrium fees and ESG market shares in an ESG-only market with neutral investors

Region	**Fund**	**Fee**	**ESG Market Share**	**Neutral Market Share**
Exclusionary	HS	$\Delta - k\lambda_S$	λ_S	λ_N
ESG duopoly	LS	0	0	0
Covered	HS	$k\lambda_S + \frac{4}{3}k\lambda_N + \frac{1}{3}\Delta$	$\frac{1}{2}\lambda_S + \frac{2}{3}\lambda_N + \frac{1}{6k}\Delta$	0
ESG duopoly	LS	$k\lambda_S + \frac{2}{3}k\lambda_N - \frac{1}{3}\Delta$	$\frac{1}{2}\lambda_S + \frac{1}{3}\lambda_N - \frac{1}{6k}\Delta$	0
Uncovered	HS	$\frac{1}{2}(R_{HS} + u_0 + k\lambda_N)$	$\frac{1}{2k}(R_{HS} + u_0 + k\lambda_N)$	0
ESG market	LS	$\frac{1}{2}(R_{LS} + u_0)$	$\frac{1}{2k}(R_{LS} + u_0)$	0

When the mass of neutral investors is large, there is no equilibrium. Both funds have incentives to undercut each other's fee and get the whole market of neutral investors. In the absence of ESG investors, this would drive the LS fund out of the market. However, LS can always charge a positive fee to ESG investors who are the most aligned with its ESG objective. But if it did, then the HS would raise its fee, which would give incentives to LS to deviate,

lower its fee, and gain the whole neutral segment of the market. Therefore, when there are many neutral investors, there is no pair of fees, f_{HS}, f_{LS}, that is incentive compatible.

The fact that $k_1^O \leq k_1$ and $k_2^O \leq k_2$ shows that competition in the case only ESG funds exist is relaxed and HS fund can charge higher fees as more investors are willing to invest with it.

4. Discussion

The asset management industry currently faces an important transformation as environmental and social issues gain prominence, leading to the integration of ESG factors into the funds' investment processes and strategies. In response to the increasing demand for ESG investments, asset managers have launched new funds. According to Morningstar, the number of funds with sustainable mandates that were launched in the U.S. hit a record level (119 funds in 2021, 103 funds in 2022 and 67 funds in 2023).[11] Globally, ESG assets are expected to exceed $53 trillion by 2025, representing more than a third of the $140.5 trillion in projected total assets under management.[12] Given this context, it seems pertinent to consider the potential consequences of a market structure where only funds with an ESG mandate exist. How would this impact mutual fund competition and investor welfare?

In order to shed light on this question, we use our model to contrast the equilibrium fees and investors' welfare between a scenario featuring both conventional funds and ESG funds and one where only ESG funds are available. Notice first that the absence of conventional funds can lead to situations when the equilibrium fails to exist. As explained above, if there are only ESG funds in the market and the mass of neutral investors is high relative to differences in performance of the two funds (for example), the market break downs. The existence of conventional funds in the market in this situation solves this problem and, therefore, improves the welfare of all market participants.

[11] "Global Sustainable Fund Flows: Q1 2024 in Review," report available at https://www.morningstar.com/lp/global-esg-flows.

[12] See https://www.bloomberg.com/professional/blog/esg-assets-may-hit-53-trillion-by-2025-a-third-of-global-aum/

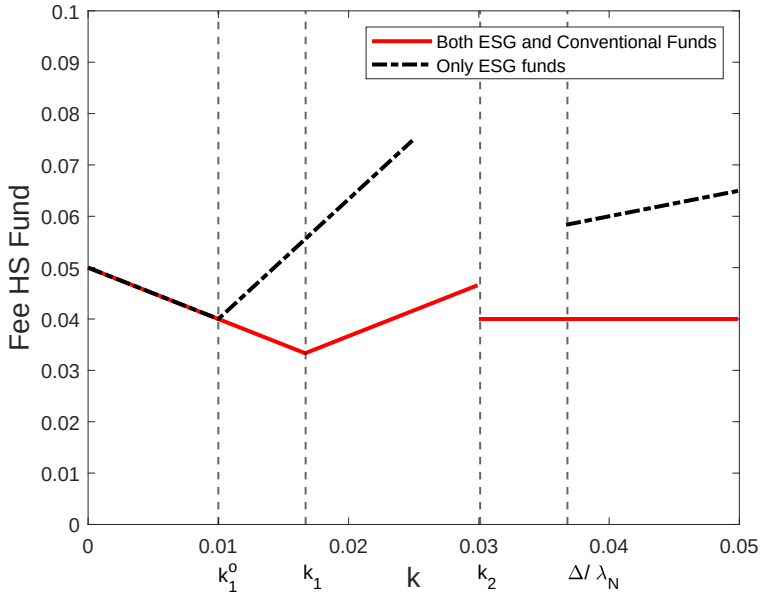


Figure 3: Fees of the HS fund in an economy where both ESG and conventional funds exist and in an economy where only ESG funds exist. Parameter values: $R_{HC} = R_{HS} = 6\%$, $R_{LC} = R_{LS} = 1\%$, $\lambda_S = 1$, $\lambda_N = 1$ and $u_0 = 0.02$.

Let us next consider the equilibrium with only ESG funds catering to ESG and neutral investors and compare it to the equilibrium where both ESG and conventional funds compete.

Proposition 3. *The fee of the high quality ESG fund (HS fund) in an economy where only ESG funds exists is higher than in the case where conventional and ESG funds coexist.*

We have shown that $f_{HS}^O \geq f_{HS}$ and that $f_{HS} > f_{HC}$. As a result, the fee charged to neutral investors where there are only ESG funds is higher than or equal to the fee when both ESG and conventional funds are available (see Figure 3).

Proposition 4. *Investors' welfare in an economy where only ESG funds exists is lower than in the case when both conventional and ESG funds coexist.*

Notice that the welfare of the neutral investors is always strictly lower in the case where only ESG funds exists (see Figure 4). When the misalignment cost is low ($k < k_1$) the fees and

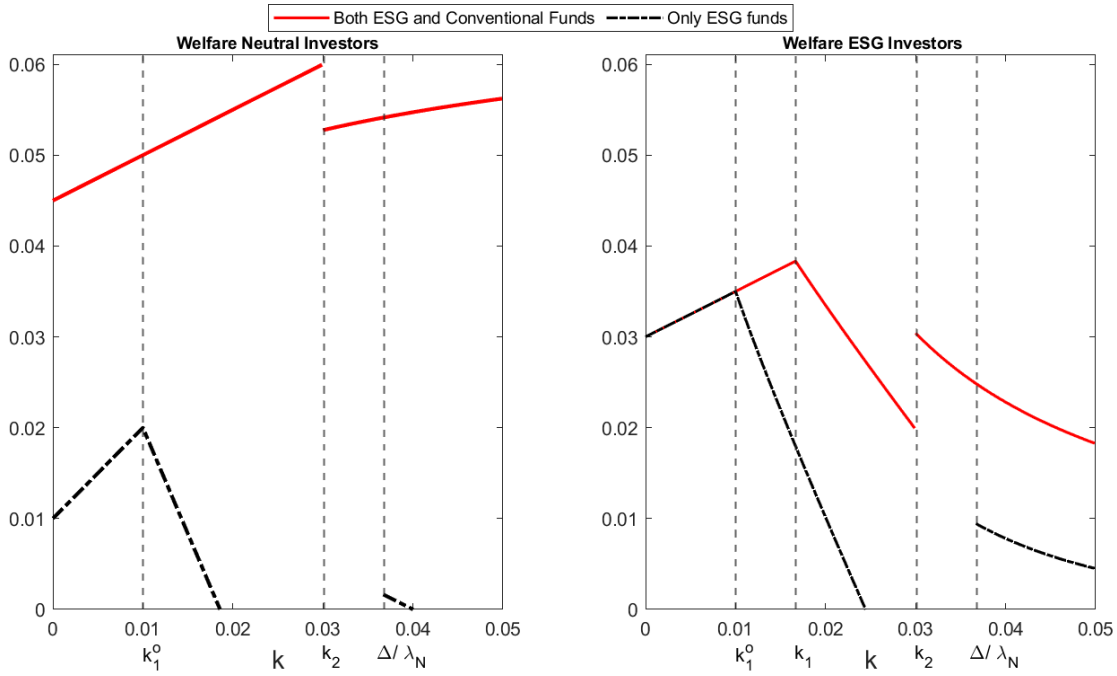


Figure 4: Investors' welfare in an economy where both ESG and conventional funds exist and in an economy where only ESG funds exist. Parameter values: $R_{HC} = R_{HS} = 6\%$, $R_{LC} = R_{LS} = 1\%$, $\lambda_S = 1$, $\lambda_N = 1$ and $u_0 = 0.02$.

the quantities are equal in both cases and therefore the welfare of ESG investors is exactly the same. In all the other cases, the welfare of ESG investors is higher when ESG and conventional funds coexist in the market.

Consequently, competition from conventional funds not only improves the welfare of investors without a preference for sustainability, but it also helps ESG investors, as it reduces the equilibrium fees of ESG funds.

5. Extensions of the Baseline Model

In this section, we generalize our results in the baseline model along two dimensions. First, we allow for difference in before-fee performance between ESG and conventional funds to be positive. Second, we relax the assumption that ESG investors experience infinite disutility from investing in conventional funds.

5.1. Equilibrium Under ESG Return Dominance $R_{HS} > R_{HC}$

The baseline analysis has focused on the case in which the greenium is positive or absent, so that ESG pre-fee returns are dominated by conventional returns. Neutral investors compare net returns and market outcomes are primarily shaped by investor preferences and horizontal differentiation. We now turn to the case of a negative greenium, $g < 0$ (ESG fund return dominance) in which the high-quality ESG fund uverperforms its conventional counterpart in pre-fee terms. We preserve the assumption $K = \infty$ from the baseline model. In this case the conventional fund constitutes a weaker outside option for investors, relaxing the competitive constraints faced by ESG funds. As a result, ESG funds can sustain higher fees, are more likely to attract neutral investors, and market segmentation becomes less likely. The analysis that follows characterizes equilibrium fees, market shares, and participation patterns under ESG return dominance, highlighting how the weakening of the outside option reshapes the investment allocations in the baseline model.

We define an integration equilibrium as an equilibrium in which market segmentation breaks down: either ESG funds attract ESG-neutral investors, or ESG investors invest in conventional funds, so that investor types are no longer served exclusively by one class of funds. This contrasts with the segmented equilibrium, in which ESG funds serve only ESG investors and conventional funds serve only neutral investors.

Let $f^0_\varphi(k)$ and $q^0_{S,\varphi}(k)$, with $\varphi \in \{HS, LS\}$ denote the equilibrium ESG fees and market shares in the ESG segment in baseline model (Proposition 1) and $\bar{f} \equiv R_{HS} - R_{HC} > 0$ denote the largest fee that fund HS can charge while still weakly dominating HC in net return even if HC sets $f_{HC} = 0$. We define the fees $f^I_\varphi(k)$ and demands $q^I_{S,\varphi}(k)$ with $\varphi \in \{HS, LS\}$ in an integration equilibrium in which the high-quality ESG fund HS serves both ESG and neutral investors.

Proposition 5 (ESG return dominance)**.** *The equilibrium allocation is characterized as follows:*

(i) If $f^0_{HS}(k) \leq \bar{f}$, there exists an equilibrium with integration in which HS and LS set the baseline ESG fees $(f^0_{HS}(k), f^0_{LS}(k))$, all ESG investors are allocated exactly as in

Proposition 1, and all neutral investors invest in HS. In this equilibrium HC and LC are inactive.

(ii) If $f^0_{HS}(k) > \bar{f}$, there exists a threshold $\bar{\lambda}_N(k) \geq 0$ such that:

- *If $\lambda_N \leq \bar{\lambda}_N(k)$, there exists an equilibrium with* segmentation. *Neutral investors invest in HC and ESG investors are allocated as in Proposition 1.*
- *If $\lambda_N > \bar{\lambda}_N(k)$, there exists an equilibrium with integration, and all neutral investors invest in HS. HS fund sets $f^I_{HS}(k) = \bar{f}$ and attracts all neutral investors, $f^I_{LS}(k) \in \arg\max_{f \geq 0} f\, q_{S,LS}(f, \bar{f}, k)$ and HC and LC are inactive,*

where

$$\bar{\lambda}_N(k) \equiv \frac{f^0_{HS}(k)\, q^0_{S,HS}(k) - \bar{f}\, q^I_{S,HS}(k)}{\bar{f}}.$$

When $K = \infty$, ESG investors never invest in conventional funds, so demand fully segments by investor type. As a result, the competition between HS and LS for ESG investors is identical to the baseline ESG-only economy analyzed in Proposition 1. ESG return dominance ($R_{HS} > R_{HC}$) gives fund HS an advantage on the neutral segment, but this advantage is constrained by its optimal pricing on the ESG segment. If the ESG-optimal fee $f^0_{HS}(k)$ from Proposition 1 satisfies $f^0_{HS}(k) \leq \bar{f} \equiv R_{HS} - R_{HC}$, then HS can charge the same fee to ESG investors and still offer neutral investors a weakly higher net return than HC, thereby attracting the entire neutral segment without affecting the ESG equilibrium.[13] If instead $f^0_{HS}(k) > \bar{f}$, HS faces a trade-off between maintaining high ESG rents and capturing neutral investors. For small λ_N, HS prefers to preserve the segmented equilibrium characterized in Proposition 1 and concedes neutral investors to HC. For sufficiently large λ_N, HS optimally lowers its fee to $\bar{f}$, distorts the ESG equilibrium relative to Proposition 1, and captures the entire neutral segment, leading to the exit of HC. Relative to the $K = \infty$ case with negative greenium ($R_{HS} < R_{HC}$), where HS cannot profitably attract neutral investors at any fee and market segmentation is unavoidable, positive greenium allows HS to internalize the neutral segment provided the loss in ESG rents is sufficiently small.

[13] Note that that this fee is the opposite of the greenium $\bar{f} = -g$.

Note that in the integrated equilbrium (case (i) or (ii) when $\lambda_N > \bar{\lambda}_N(k)$), the mechanism is identical to that in Section 3.3 in the sense that ESG investors are fully segmented and the HS–LS subgame coincides with Proposition 1, with the same three regions $k \leq k_1$, $k_1 < k < k_2$, and $k \geq k_2$. In both cases, the only source of market integration is the potential migration of neutral investors between HS and HC, while conventional funds compete exclusively for neutral investors and LC is inactive. The difference is entirely driven by the sign of the greenium. With negative greenium ($R_{HS} < R_{HC}$), HS cannot profitably attract neutral investors at any fee, so market segmentation is unavoidable, while with positive greenium ($R_{HS} > R_{HC}$), HS may optimally reduce its fee to attract the neutral investors, trading off rents from neutral investors against those from ESG investors.

When ESG funds deliver a greepnium, the model predicts that higher performant ESG funds may optimally internalize the neutral investor segment by reducing fees, an outcome that should be empirically observable as coordinated fee reductions accompanied by inflows from neutral (non-ESG) accounts. The neutral investors migration to ESG funds arises only when the mass of neutral investors is sufficiently large and the required fee concession does not create excessive losses in ESG rents. Conversely, the persistence of market segmentation characterized by continued absence of neutral inflows despite ESG outperformance, signals either a small neutral investor base or the presence of frictions that impede neutral migration.

5.2. Finite Leakage Costs and Endogenous Fee Caps

Similarly to the baseline model we assume that ESG investors are uniformly distributed on $[0, \lambda_S]$ and have preferences for the ESG funds, and neutral investors have mass λ_N and no ESG preferences. We relax here however the assumption that the misalignment between ESG investors' preferences and conventional funds is so large that ESG investors suffer a high disutility from investing in those funds and avoid them altogether. We therefore, consider the following utility for the ESG investors when they invest in the conventional funds

$$U_{i,\varphi}^{ESG} = R_\varphi - f_\varphi - K,$$

with $\varphi \in \{HC, LC\}$ and $K \geq 0$ and finite.

Allowing a finite K turns the conventional fund (HC) into a credible outside option for ESG investors. The parameter K captures the strength of investors' disutility from violating their sustainability preferences. A small K reflects more tolerant or less committed ESG investors, while a large K reflects stricter ESG preferences. When K is small, ESG investors are less penalized for investing in conventional funds, so ESG managers must reduce their fees to prevent leakage. Let us define for any fund $\varphi \in \{HS, LS\}$ the unconstrained optimal fee f_φ (the solution to the first-order condition of its profit maximization ignoring the HC no-leakage constraint). Define the HC no-leakage cap $\overline{f}_\varphi(K)$ as the maximal fee ESG fund φ can charge such that an ESG investor indifferent between fund φ and HC exists given K,

$$\begin{aligned} \overline{f}_{HS}(f_{HC}, K) &\equiv (R_{HS} - R_{HC}) + f_{HC} + K, \\ \overline{f}_{LS}(f_{HC}, K, k) &\equiv (R_{LS} - R_{HC}) + f_{HC} + K + k\lambda_S. \end{aligned} \tag{1}$$

Similarly to Proposition 1 we define $k_1(K)$ as the smallest value of k such that, given the HC no-leakage cap, the low-quality ESG fund LS can operate with positive demand and non-negative profit, and $k_2(K)$ as the smallest value of k such that, under the leakage-constrained fees, some ESG investors optimally exit (i.e., the ESG market becomes uncovered).

Proposition 6 (Finite-K equilibrium)**.** *There exist thresholds $k_1(K) \in (0, \infty)$ and $k_2(K) \in \big(k_1(K), \infty\big)$ such that there are three equilibrium regions depending on k, with nonnegative equilibrium fees $f_\varphi \geq 0$:*

A. Exclusionary ESG duopoly *($k \leq k_1(K)$)**.*** *The high-quality ESG fund HS is active while the low-quality ESG fund LS is excluded from the market. The ESG fee of HS is capped by the non-leakage constraint induced by K. The conventional fund HC sets its fee to deter deviations by ESG funds.*

B. Covered ESG duopoly *($k_1(K) < k < k_2(K)$)**.*** *Both ESG funds are active and the ESG market is fully covered. ESG fees coincide with the interior duopoly solutions subject to the non-leakage caps imposed by K. The conventional fund HC sets its fee to*

deter deviations by ESG funds.

C. Uncovered ESG market *($k \geq k_2(K)$).* *Both ESG funds act as local monopolies subject to the non-leakage constraint, and some ESG investors optimally choose not to invest. The conventional fund HC sets its fee to deter deviations by ESG funds.*

In all regions, the low-quality conventional fund is inactive. Closed-form expressions for equilibrium fees and market shares, as well as the definitions of the leakage caps and deviation-prevention fees, are presented in the Appendix and summarized in Table 4.

Table 4: Proposition 6: Finite-K equilibrium fees and market shares

Region	**Fund**	**Fee**	**ESG Market Share**	**Neutral Market Share**
Exclusionary ESG duopoly	HS	$\min\{f_{HS}, \overline{f}_{HS}(K)\}$	$q^*_{S,HS}(k,K)$	$q^*_{N,HS}(k,K)$
	LS	0	0	0
	HC	$\min\{\Delta_C, f^A_K\}$	0	$q^*_{N,HC}(k,K)$
	LC	0	0	0
Covered ESG duopoly	HS	$\min\{f_{HS}, \overline{f}_{HS}(K)\}$	$q^*_{S,HS}(k,K)$	$q^*_{N,HS}(k,K)$
	LS	$\min\{f_{LS}, \overline{f}_{LS}(K,k)\}$	$q^*_{S,LS}(k,K)$	$q^*_{N,LS}(k,K)$
	HC	$\min\{\Delta_C, f^B_K\}$	0	λ_N
	LC	0	0	0
Uncovered ESG market	HS	$\min\{f_{HS}, \overline{f}_{HS}(K)\}$	$q^*_{S,HS}(k,K)$	$q^*_{N,HS}(k,K)$
	LS	$\min\{f_{LS}, \overline{f}_{LS}(K,k)\}$	$q^*_{S,LS}(k,K)$	$q^*_{N,LS}(k,K)$
	HC	$\min\{\Delta_C, f^C_K\}$	0	λ_N
	LC	0	0	0

Notes. Fees f_{HS} and f_{LS} are fees in the unrestricted case, $\overline{f}_{HS}(K)$ and $\overline{f}_{LS}(K,k)$ are leakage caps defined in (1). Fees f^A_K, f^B_K, and f^C_K are deviation-prevention fees defined in the Appendix. $q^*_{S,HS}(k,K)$, $q^*_{N,HS}(k,K)$, $q^*_{S,LS}(k,K)$, $q^*_{N,LS}(k,K)$ denote equilibrium quantities.

Note that $k_1(K)$ is decreasing in K as higher K relaxes leakage, making it easier for LS to survive. Moreover, $k_2(K)$ is increasing in K as a larger penalty K weakens HC as an outside option, allowing ESG funds to sustain coverage for larger k. As $K \to \infty$ the

original Proposition 1 is recovered. Note also that for the following analysis we analyze the case $R_{HC} \geq R_{HS}$. For the opposite case $K_1 < 0$ and the neutral investors might go to HS fund (depends on the fees).

The parameter K determines the attractiveness of the conventional fund HC as an outside option for ESG investors and therefore determines whether ESG funds can profitably operate and how the market is structured. When K is small, the utility loss from investing in HC is limited, so HC is very competitive. Any ESG fund attempting to attract investors must then set a fee sufficiently low to satisfy the no-leakage condition relative to HC fund. For sufficiently small K, this constraint is so tight that the constrained fee lies below the profit-maximizing level and does not generate positive demand. As a result, ESG funds cannot profitably operate and the conventional fund serves the entire market.

As K increases, the relative attractiveness of HC for ESG investors declines, relaxing the no-leakage constraint and allowing ESG funds to become active. However, for intermediate values of K, the constraint remains binding for at least one ESG fund. In this range, ESG funds operate under constrained fees and compete directly with HC for ESG investors. Only those investors with the strongest sustainability preferences remain with ESG funds, while others optimally migrate to the high quality conventional fund. The resulting equilibrium features partial market coverage and integration between ESG and conventional funds, with endogenous investor migration across fund types.

When K is sufficiently large, HC is not anymore a relevant outside option for ESG investors. The no-leakage constraint no longer binds, and ESG funds can charge their unconstrained profit-maximizing fees without losing investors to HC. In equilibrium, all ESG investors are served by ESG funds, while the conventional fund caters exclusively to neutral investors. This case corresponds to full market segmentation and nests the outcome of the baseline model as a limiting case in which ESG investors never consider the conventional fund.

Observe that $f_\varphi(K)$ is continuous in K and f_φ is independent of K. Hence there exists a threshold K_1 such that for $K < K_1$, $f_\varphi(K)$ becomes negative, so the ESG funds φ does not operate. Note that this threshold varies for each k-region $\{A, B, C\}$ defined in Proposition 6.

We also define the demand mappings $\Phi_\varphi(f_\varphi)$ and $\Psi_\varphi(f_\varphi)$ in case the ESG fund φ competes with the HC fund (integration) or not (segmentation), respectively.

Proposition 7. *For each k-region $X \in \{A, B, C\}$ there exists a threshold $K_1 > 0$ such that the equilibrium allocation is characterized as follows.*

1. ***Conventional-funds dominance ($K < K_1$).*** *All ESG funds are inactive and market shares are*

$$q^*_{S,HS}(k, K) = q^*_{S,LS}(k, K) = 0, \qquad q^*_{N,HC}(k, K) = \lambda_N + \lambda_S.$$

2. ***Integration ($K \geq K_1$ and at least one cap binds).*** *If $K \geq K_1$ and $f_\varphi \geq \overline{f}_\varphi(K)$ for at least one $\varphi \in \{HS, LS\}$, then $f^{\text{eq}}_\varphi = \min\{f_\varphi, \overline{f}_\varphi(K)\}$ and market shares are*

$$q^*_{S,\varphi}(k, K) = \Phi_\varphi\left(f^{\text{eq}}_\varphi\right), \qquad q^*_{N,HC}(k, K) = \lambda_N + \lambda_S - \sum_\varphi q^K_\varphi.$$

3. ***Segmentation ($K \geq K_1$ and no cap binds).*** *If $K \geq K_1$ and $f_\varphi < f_\varphi(K)$ for all active ESG funds φ, then $f^{\text{eq}}_\varphi = f_\varphi$, and market shares are*

$$q^*_{S,\varphi}(k, K) = \Psi_\varphi(f_\varphi), \qquad q^*_{N,HC}(k, K) = \lambda_N, \qquad \textit{with} \quad \sum_\varphi q^K_{S,\varphi} \leq \lambda_S.$$

The model predicts that ESG-conventional fee differences, market segmentation, and the persistence of low-quality ESG funds all increase with K. Conversely, as ESG preferences become less rigid (smaller K), ESG and conventional funds compete more directly, leading to lower fees, higher overall welfare, and a reduced role for ESG-based market power.

6. Conclusions

In this paper, we study how competition among mutual funds is shaped by the presence of investors with sustainability preferences and mutual funds that cater to those preferences. To achieve our goal, we construct a model of the mutual fund market where active funds with

different ESG objectives and different quality compete for investors' money. Investors have heterogenous preferences for ESG. Not all investors care for sustainability, and among those who do, they value different ESG objectives differently. The model predicts that in equilibrium the market is segmented: neutral investors (those with no preference for ESG) invest only in conventional funds and ESG investors invest only in ESG funds. While competition is fierce in the conventional segment of the market and only the best funds survive, it is relaxed by investors' ESG preferences in the ESG segment of the market. If the intensity of ESG investors' preferences is sufficiently high, ESG funds of lower quality will be able to survive.

Since there is no asymmetric information in our model, there is no role for learning either, so the outcome of a multi-period version of the model is identical to that of the static version presented in the paper. This implies that, to the extent that managerial skill persists, performance differences among ESG funds will also persist through time. Among conventional funds, however, competition ensures the homogeneity of operating funds, so one would not expect to observe differences in performance.

Our results in the baseline model unveil a form of segmentation that arises as a consequence of both investor heterogeneity in their ESG preferences and product differentiation along the ESG dimension. ESG acts as a friction that relaxes competition between mutual funds in terms of performance and fees. The consequences are important. First, investors may end up investing in underperforming fund and paying excessive fees. Second, the availability of conventional funds with purely financial objectives benefits ESG investors. By providing an alternative for neutral investors, conventional funds limit the market power of ESG funds, leading to lower fees and improved investor welfare in the ESG segment of the market. Therefore, our analysis suggests that public and private initiatives aimed at making ESG considerations a common objective in delegated asset management has potentially important costs for investors that cannot be excluded from the ongoing debate.

Our extensions highlight that the degree of market segmentation hinges critically on two forces: the relative performance of ESG funds and the rigidity of investors' sustainability preferences. When ESG funds outperform conventional funds, they may optimally reduce fees to

attract neutral investors, leading to market integration and the exit of conventional funds. Conversely, when ESG investors face only a finite disutility from investing conventionally, conventional funds discipline ESG pricing through endogenous fee caps, and sufficiently weak sustainability preferences can eliminate ESG funds altogether. Together, these extensions show that ESG-driven segmentation is not mechanical, but instead emerges from the interaction between performance differences and the strength of investors' non-pecuniary motives.

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Appendix

Proof of Proposition 1. The manager of each fund φ, with $\varphi \in \{HS, LS, HC, GB\}$ solves the following maximization problem

$$\max_{f_\varphi \geq 0} \Pi_\varphi = f_\varphi \left(q_{P,\varphi} + q_{N,\varphi} \right).$$

We have shown that there is segmentation: ESG investors do not invest in conventional funds, and therefore $q_{S,\varphi} = 0$, for $\varphi \in \{HC, LC\}$.

Therefore, we solve for the optimal strategy of HS and LS funds under segmentation. The ESG investor i decides to invest in fund HS if her participation constraint is satisfied, i.e. $U_{i,HS}^{ESG} \equiv U_{HS} \geq 0$. We denote by x the demand of investors for fund HS. Since investors have each \$1 to invest, x is also the location of the investor that is indifferent between investing or not with the fund. Similarly, an investor j decides to invest in fund LS if her participation constraint is satisfied, i.e. $U_{j,LS}^{ESG} \equiv U_{LS} \geq 0$. We denote by y the demand of investors for fund LS, and the location of the last investor willing to invest in fund LS is $y = \lambda_S - x$. The manager of fund HS solves the following maximization problem

$$\begin{aligned} \max_{f_{HS}} \Pi_{HS} &= f_{HS} x \\ s.t.\ U_{HS} &= R_{HS} - f_{HS} + (u_0 - kx) \geq 0 \\ x + y &\leq \lambda_S, \\ f_{HS} &\geq 0, \end{aligned}$$

and the manager of fund LS solves

$$\begin{aligned} \max_{f_{LS}} \Pi_{LS} &= f_{LS} y \\ s.t.\ U_{LS} &= R_{LS} - f_{LS} + (u_0 - ky) \geq 0 \\ x + y &\leq \lambda_S, \\ f_{LS} &\geq 0. \end{aligned}$$

The Lagrangean for the problem of the manager of the HS fund is

$$\mathcal{L}_{HS} = f_{HS}x - \mu_{HS}\left(-\left(u_0 - kx\right) - R_{HS} + f_{HS}\right) - \eta\left(x + y - \lambda_S\right),$$

and the one for the problem of the manager of the LC fund is

$$\mathcal{L}_{LS} = f_{LS}y - \mu_{LS}\left(-\left(u_0 - ky\right) - R_{LS} - \delta_2 + f_{LS}\right) - \eta\left(x + y - \lambda_S\right).$$

The Kuhn-Tucker conditions for these problems are

$$\begin{array}{rcl} \dfrac{\partial \mathcal{L}_\varphi}{\partial f_\varphi} & \leq & 0, \quad f_\varphi \geq 0, \quad f_\varphi \dfrac{\partial \mathcal{L}_\varphi}{\partial f_\varphi} = 0, \\ U_\varphi & = & R_\varphi - \ f_\varphi + \left(u_0 - kx\right) \geq 0, \quad \mu_\varphi \geq 0, \ \mu_\varphi\left(-\left(u_0 - kx\right) - R_\varphi + f_\varphi\right) = 0, \\ x + y & \leq & \lambda_S, \ \eta \geq 0, \ \eta\left(x + y - \lambda_S\right) = 0\,, \end{array} \tag{2}$$

for $\varphi \in \{HS, LS\}$.

Since the manager's profit in case $f_\varphi = 0$ equals 0, we consider only the cases when $f_\varphi > 0$. Therefore, we have

$$\begin{array}{rcl} \dfrac{\partial \mathcal{L}_{HS}}{\partial f_{HS}} & = & x + f_{HS}\dfrac{\partial x}{\partial f_{HS}} - \mu_{HS}\left(k\dfrac{\partial x}{\partial f_{HS}} + 1\right) - \eta\dfrac{\partial x}{\partial f_{HS}} = 0, \\ \dfrac{\partial \mathcal{L}_{LS}}{\partial f_{LS}} & = & y + f_{LS}\dfrac{\partial y}{\partial f_{LS}} - \mu_{LS}\left(k\dfrac{\partial y}{\partial f_{LS}} + 1\right) - \eta\dfrac{\partial y}{\partial f_{LS}} = 0. \end{array} \tag{3}$$

Case I $\mu_{HS} = \mu_{LS} = \eta = 0$

The Kuhn-Tucker conditions (2) imply that in this case

$$\begin{array}{rcl} x + f_{HS}\dfrac{\partial x}{\partial f_{HS}} & = & 0, \\ y + f_{LS}\dfrac{\partial y}{\partial f_{LS}} & = & 0, \\ U_{HS} & \geq & 0, \ U_{LS} \geq 0 \text{ and } x + y \leq \lambda_S. \end{array}$$

Case I.1 If $x + y < \lambda_S$ then $U_{HS} = U_{LS} = 0$ and from here combined with the conditions

(3) we obtain that both funds act as local monopolies and therefore

$$\begin{aligned} f_{HS} &= \frac{R_{HS}+u_0}{2}, \\ f_{LS} &= \frac{R_{LS}+u_0}{2}, \\ x &= \frac{R_{HS}+u_0}{2k}, \\ y &= \frac{R_{LS}+u_0}{2k}. \end{aligned}$$

Since $x+y<\lambda_S$, but this is possible if and only if $\frac{R_{HS}+u_0}{2k}+\frac{R_{LS}+u_0}{2k}<\lambda_S$ or equivalently $k>\frac{1}{2\lambda_S}\left(2u_0+R_{HS}+R_{LS}\right)$.

Case I.2 If $x+y=\lambda_S$ the ESG investors decide whether to invest in the HS fund or the LS fund depending on their location. In this case the market is covered and the two funds compete for attracting the investors. Thus, we have that the marginal investor's location, x, satisfies

$$R_{HS}-f_{HS}+(u_0-kx)=R_{LS}-f_{LS}+\left(u_0-k\left(\lambda_S-x\right)\right).$$

Therefore the marginal investor is located at

$$x=\frac{\lambda_S}{2}+\frac{R_{HS}-f_{HS}-\left(R_{LS}-f_{LS}\right)}{2k}=\frac{\lambda_S}{2}+\frac{r_{HS}-r_{LS}}{2k}, \tag{4}$$

where $r_{HS}\equiv R_{HS}-f_{HS}$ and $r_{LS}\equiv R_{LS}-f_{LS}$.

It follows

$$\begin{aligned} q_{HS} &= \frac{\lambda_S}{2}+\frac{r_{HS}-r_{LS}}{2k}, \\ q_{LS} &= \frac{\lambda_S}{2}-\frac{r_{HS}-r_{LS}}{2k}. \end{aligned}$$

Therefore the conditions (3) become

$$\begin{aligned} \frac{\lambda_S}{2}+\frac{r_{HS}-r_{LS}}{2k}+\left(-\frac{1}{2k}\right)f_{HS} &= 0, \\ \frac{\lambda_S}{2}-\frac{r_{HS}-r_{LS}}{2k}+\left(-\frac{1}{2k}\right)f_{LS} &= 0. \end{aligned}$$

Solving for f_{HS} and f_{LS} and imposing that fees are positive we find that in equilibrium

$$f_{HS} = \begin{cases} k\lambda_S + \frac{1}{3}\Delta & \text{if} \quad k\lambda_S - \frac{1}{3}\Delta > 0, \\ \Delta - k\lambda_S & otherwise, \end{cases}$$

$$f_{LS} = \begin{cases} k\lambda_S - \frac{1}{3}\Delta & \text{if} \quad k\lambda_S - \frac{1}{3}\Delta > 0, \\ 0 & otherwise, \end{cases}$$

where $\Delta = R_{HS} - R_{LS}$. The optimal quantities invested in each fund are therefore equal to

$$q^*_{HS} = \begin{cases} \frac{1}{2}\lambda_S + \frac{1}{6k}\Delta & \text{if} \quad k\lambda_S - \frac{1}{3}\Delta > 0, \\ \lambda_S & otherwise, \end{cases}$$

$$q^*_{LS} = \begin{cases} \frac{1}{2}\lambda_S - \frac{1}{6k}\Delta & \text{if} \quad k\lambda_S - \frac{1}{3}\Delta > 0, \\ 0 & otherwise. \end{cases}$$

Notice that for this to be an equilibrium we need to have that the utility of the marginal investor x is higher or equal to zero, i.e. $U_\varphi = R_\varphi - f_\varphi + (u_0 - kx) \geq 0$, for $\varphi \in \{HS, LS\}$. We therefore need $k \leq \frac{1}{3\lambda_S}(R_{HS} + R_{LS} + 2u_0)$ in order for this to be a possible solution.

Case II $\mu_{HS} = \mu_{LS} = 0$, $\eta \neq 0$

The Kuhn-Tucker conditions (2) imply that $x + y = \lambda_S$, $\eta > 0$, $U_{HS} \geq 0, U_{LS} \geq 0$ and

$$\begin{aligned} x + \frac{\partial x}{\partial f_G}(f_{HS} - \eta) &= 0, \\ y + \frac{\partial y}{\partial f_B}(f_{LS} - \eta) &= 0. \end{aligned}$$

Since $x + y = \lambda_S$, it implies that $U_{HS} = U_{LS}$ and therefore x is as defined in (4) and therefore

$$\begin{aligned} x + \left(-\frac{1}{2k}\right)(f_{HS} - \eta) &= 0, \\ y + \left(-\frac{1}{2k}\right)(f_{LS} - \eta) &= 0. \end{aligned}$$

Consequently, we have that

$$\begin{aligned} x &= \frac{1}{2k}\left(f_{HS}-\eta\right), \\ y &= -\frac{1}{2k}\left(f_{LS}-\eta\right). \end{aligned}$$

Since $\eta > 0$ the profit of the HS fund manager is $f_{HS}x = \frac{f_{HS}}{2k}\left(f_{HS}-\eta\right) < \frac{\left(f_{HS}\right)^2}{2k}$ and in this case we do not obtain a global optimum.

Case III $\mu_{HS} \neq 0$, $\mu_{LS} = 0$, $\eta \neq 0$

This implies that $x + y = \lambda_S,\ \eta > 0$, $U_{HS} = 0, \mu_{HS} > 0$ and $U_{LS} \geq 0$. However, since $U_{HS} = 0$ and $x + y = \lambda_S$, we have that

$$x = \frac{R_{HS} - f_{HS} + u_0}{k},$$

and

$$\begin{aligned} \frac{\partial \mathcal{L}_{HS}}{\partial f_{HS}} &= x + f_{HS}\frac{\partial x}{\partial f_{HS}} - \mu_{HS}\left(k\frac{\partial x}{\partial f_{HS}} + 1\right) - \eta\frac{\partial x}{\partial f_{HS}} = 0, \\ \frac{\partial \mathcal{L}_{LS}}{\partial f_{LS}} &= y + f_{LS}\frac{\partial y}{\partial f_{LS}} - \eta\frac{\partial y}{\partial f_{LS}} = 0. \end{aligned}$$

Since $y = \lambda_S - x$, and $\frac{\partial x}{\partial f_{LS}} = 0$ we can rewrite $\frac{\partial \mathcal{L}_{LS}}{\partial f_{LS}} = \lambda_S - x - f_{LS}\frac{\partial x}{\partial f_{LS}} + \eta\frac{\partial x}{\partial f_{LS}} = \lambda_S - x$. This implies that $x = \lambda_S,\ y = 0$ and $f_{HS} = R_{HS} + u_0 - k\lambda_S$. Finally, we have that $x + \left(f_{HS} - \eta\right)\frac{\partial x}{\partial f_{HS}} = 0 \Leftrightarrow \lambda_S + \left(f_{HS} - \eta\right)\left(-\frac{1}{k}\right) = 0 \Leftrightarrow k\lambda_S + \eta = f_{HS} \Leftrightarrow \eta = f_{HS} - k\lambda_S$. Therefore $\eta = R_{HS} + u_0 - k\lambda_S - k\lambda_S = R_{HS} + u_0 - 2k\lambda_S > 0$ if $2k\lambda_S < R_{HS} + u_0$. In this case the market is covered by the HS fund.

Case IV $\mu_{HS} = 0$, $\mu_{LS} \neq 0$, $\eta \neq 0$

This implies that $x + y = \lambda_S,\ \eta > 0$, $U_{LS} = 0, \mu_{LS} > 0$ and $U_{HS} \geq 0$. However, since $x + y = \lambda_S$ it implies also $U_{HS} = 0$. So, we have that

$$\begin{aligned} x &= \frac{R_{HS} - f_{HS} + u_0}{k}, \\ y &= \frac{R_{LS} - f_{LS} + u_0}{k}, \end{aligned}$$

and the conditions (3) become in this case

$$\begin{aligned}\frac{\partial \mathcal{L}_{HS}}{\partial f_{HS}} &= x + f_{HS}\frac{\partial x}{\partial f_{HS}} - \eta\frac{\partial x}{\partial f_{HS}} = 0,\\ \frac{\partial \mathcal{L}_{LS}}{\partial f_{LS}} &= y + f_{LS}\frac{\partial y}{\partial f_{LS}} - \mu_{LS}\left(k\frac{\partial y}{\partial f_{LS}} + 1\right) - \eta\frac{\partial y}{\partial f_{LS}} = 0.\end{aligned}$$

Notice that $k\frac{\partial y}{\partial f_{LS}} + 1 = k\left(-\frac{1}{k}\right) + 1 = 0$, and therefore this case is reduced to Case II, where we have proved that the solution obtained is not a global optimum.

Case V $\mu_{HS} = 0$, $\mu_{LS} \neq 0$, $\eta = 0$

This implies that $x + y \leq \lambda_S$, $U_{LS} = 0, \mu_{LS} > 0$ and $U_{HS} \geq 0$. Since $x + y \leq \lambda_S$, $U_{LS} = 0$ it implies also $U_{HS} = 0,$ so we have

$$\begin{aligned}x &= \frac{R_{HS} - f_{HS} + u_0}{k},\\ y &= \frac{R_{LS} - f_{LS} + u_0}{k},\end{aligned}$$

and therefore the conditions (3) become

$$\begin{aligned}\frac{\partial \mathcal{L}_{HS}}{\partial f_{HS}} &= x + f_{HS}\frac{\partial x}{\partial f_{HS}} = 0,\\ \frac{\partial \mathcal{L}_{LS}}{\partial f_{LS}} &= y + f_{LS}\frac{\partial y}{\partial f_{LS}} = 0,\end{aligned}$$

since $\left(k\frac{\partial y}{\partial f_{LS}} + 1\right) = k\left(-\frac{1}{k}\right) + 1 = 0$. Consequently, we have the same solution as in Case I.1.

Case VI $\mu_{HS} \neq 0$, $\mu_{LS} = 0$, $\eta = 0$

This implies that $x + y \leq \lambda_S$, $U_{HS} = 0, \mu_{HS} > 0$ and $U_{LS} \geq 0$. Since $x + y \leq \lambda_S$, $U_{HS} = 0$ it implies also $U_{LS} = 0,$ so we have again that

$$\begin{aligned}x &= \frac{R_{HS} - f_{HS} + u_0}{k},\\ y &= \frac{R_{LS} - f_{LS} + u_0}{k}.\end{aligned}$$

Therefore the conditions (3) become

$$\begin{aligned}\frac{\partial \mathcal{L}_{HS}}{\partial f_{HS}} &= x + f_{HS}\frac{\partial x}{\partial f_{HS}} = 0,\\ \frac{\partial \mathcal{L}_{LS}}{\partial f_{LS}} &= y + f_{LS}\frac{\partial y}{\partial f_{LS}} = 0,\end{aligned}$$

since again $k\dfrac{\partial x}{\partial f_{HS}} + 1 = k\left(-\dfrac{1}{k}\right) + 1 = 0$. Consequently, we have again the same solution as in Case I.1.

Case VII $\mu_{HS} \neq 0$, $\mu_{LS} \neq 0$, $\eta = 0$

This implies that $x + y \leq \lambda_S$, $U_{LS} = 0, \mu_{LS} > 0$ and $U_{HS} = 0\,, \mu_{LS} > 0$. Similarly to the Cases V and VI we obtain the same solution as in Case I.1.

Case VIII $\mu_{HS} \neq 0$, $\mu_{LS} \neq 0$, $\eta \neq 0$

This implies that $x + y = \lambda_S$, $U_{LS} = 0, \mu_{LS} > 0$ and $U_{HS} = 0\,, \mu_{LS} > 0$ and again we have the same solution as in Case I.1 if $\lambda_S = \dfrac{1}{2k}\left(2u_0 + R_{HS} + R_{LS}\right)$.

We compare the possible equilibria to find the global optimum. The solution in Case I.1 is obtained if and only if $k \geq \dfrac{1}{2\lambda_S}\left(2u_0 + R_{HS} + R_{LS}\right)$, while the solution in Case I.2 is obtained if and only if $k \leq \dfrac{1}{3\lambda_S}\left(2u_0 + R_{HS} + R_{LS}\right)$. This implies that in the interval $k \in \left[\dfrac{1}{2\lambda_S}\left(2u_0 + R_{HS} + R_{LS}\right), \dfrac{1}{3\lambda_S}\left(2u_0 + R_{HS} + R_{LS}\right)\right]$ we have two candidates to a global optimum. To see which one is the global optimum let us compare the two profits. In case the solution is the one obtained in Case I.1, the profit of fund HS is the monopoly profit $\dfrac{1}{4k}\left(R_{HS} + u_0\right)^2$ while in the case Case I.2 is equal to $\dfrac{1}{2k}\left(k\lambda_S + \dfrac{1}{3}\Delta\right)^2$. Note that $\dfrac{1}{4k}\left(R_{HS} + u_0\right)^2 - \left(k\lambda_S + \dfrac{1}{3}\Delta\right)\left(\dfrac{1}{2}\lambda_S + \dfrac{1}{6k}\Delta\right) = -\dfrac{1}{36}\dfrac{18k^2\lambda_S^2 + 2\Delta^2 - 9\left(R_{HS} + u_0\right)^2 + 12k\Delta\lambda_S}{k}$ is positive for all $k \in [0, k^*]$ where $k^* = -\dfrac{1}{6\lambda_S}\left(2\Delta - 3\sqrt{2}\left(R_{HS} + u_0\right)\right)$.

Since $\dfrac{1}{3\lambda_S}\left(2u_0 + R_{HC} + R_{LC}\right) < -\dfrac{1}{6\lambda_S}\left(2\Delta - 3\sqrt{2}\left(R_{HS} + u_0\right)\right) = k^*$ this implies that for all $k \in \left[\dfrac{1}{2\lambda_S}\left(2u_0 + R_{HS} + R_{LS}\right), \dfrac{1}{3\lambda_S}\left(2u_0 + R_{HS} + R_{LS}\right)\right]$ the local monopoly is the global equilibrium. Therefore, if $\lambda_S \geq \dfrac{1}{2k}\left(2u_0 + R_{HS} + R_{LS}\right)$ the solution is the one obtained in Case I.1, otherwise the solution is the one from Case I.2. We have defined $k_1 \equiv \dfrac{\Delta}{3\lambda_S}$ and

$k_2 \equiv \frac{1}{2\lambda_S}(2u_0 + R_{HS} + R_{LS})$. With these notations, the equilibrium fees are

$$f^*_{HS} = \begin{cases} \Delta - k\lambda_S & if \quad k \leq k_1, \\ k\lambda_S + \frac{1}{3}\Delta & if \quad k_1 < k < k_2, \\ \frac{1}{2}(R_{HS} + u_0) & if \quad k \geq k_2, \end{cases} \quad \text{and}$$

$$f^*_{LS} = \begin{cases} 0 & if \quad k \leq k_1, \\ k\lambda_S - \frac{1}{3}\Delta & if \quad k_1 < k < k_2, \\ \frac{1}{2}(R_{LS} + u_0) & if \quad k \geq k_2. \end{cases}$$

And the optimal quantities invested in each ESG fund are therefore equal to

$$q^*_{HS} = \begin{cases} \lambda_S & if \quad k \leq k_1, \\ \frac{1}{2}\lambda_S + \frac{1}{6k}\Delta & if \quad k_1 < k < k_2, \\ \frac{1}{2k}(R_{HS} + u_0) & if \quad k \geq k_2, \end{cases}$$

$$q^*_{LS} = \begin{cases} 0 & if \quad k \leq k_1, \\ \frac{1}{2}\lambda_S + \frac{1}{6k}\Delta & if \quad k_1 < k < k_2, \\ \frac{1}{2k}(R_{LS} + u_0) & if \quad k \geq k_2. \end{cases}$$

Finally, we have to determine the optimal fee for the HC fund. The optimal fee is determined in such a way that the HS fund does not want to mimic the fee of the HC fund. We find next the fee f_{HC} in each of the three cases above, which are determined by the value of k.

Case A. Exclusionary ESG duopoly First we considered the case when $k \leq k_1$, when the fund HS covers all the market of ESG investors and fund LS does not participate in the market (sets a fee $f_{LS} = 0$).

The fee f that makes the manager of the HS fund to be indifferent between operating

only with the ESG investors and operating with both ESG and neutral investors is such that

$$f_{HS}q_{HS} = f\left(q_{HS}\left(f, f_{LS}\right) + \lambda_N\right). \tag{5}$$

Replacing the optimal values for f_{HS}, f_{LS} and q_{HS} in this case we obtain that (5) is equivalent to

$$\left(\Delta - k\lambda_S\right)\lambda_S = f(\lambda_S + \lambda_N).$$

Thus, we define as $f_e^A = \dfrac{\lambda_S}{\lambda_N + \lambda_S}\left(\Delta - k\lambda_S\right)$, the fee that fund HC sets to deter deviation by ESG funds. Therefore the manager of the HC fund chooses

$$f_{HC}^A = \min\left\{\Delta_C, f_e^A\right\}, \tag{6}$$

and at this fee none of the funds have incentives to deviate, and the HC funds keeps the LC fund out of the market.

Case B. Covered ESG duopoly In the second case we consider values for k such that $k_1 \leq k \leq k_2$. In this case the market is covered by the two funds. The fee f that makes the manager of the HC fund to be indifferent between operating only with the ESG investors and operating with both ESG and neutral investors is such that

$$f_{HS}q_{HS} = f\left(q_{HS}\left(f, f_{LS}\right) + \lambda_N\right). \tag{7}$$

We define $T \equiv f_{HS} = k\lambda_S + \dfrac{1}{3}\Delta$. With these notations the equation (7) can be rewritten as

$$\frac{T^2}{2k} = f\left(\frac{T}{k} - \frac{f}{2k} + \lambda_N\right).$$

Notice that in order to be able to serve all the market of neutral investors, the manager of the HS fund should deviate to a fee $f < f_{HS} = T$.

We define

$$G\left(f\right) \equiv \frac{T^2}{2k} - f\left(\frac{T}{k} - \frac{f}{2k} + \lambda_N\right)$$

and it results that

$$G(f) = \frac{1}{18}\frac{9k^2\lambda_S^2 - 6f\Delta + \Delta^2 + 9f^2 - 18fk\lambda_N - 18fk\lambda_S + 6k\Delta\lambda_S}{k}.$$

Notice that $G'(f) = 0$ has a solution $\widetilde{f} = \frac{1}{3}\Delta + k\lambda_N + k\lambda_S > k\lambda_S + \frac{1}{3}\Delta$ and this implies that for all $f \in \left(0, \widetilde{f}\right)$ the function $G(f)$ is decreasing. Moreover, since $G(0) = \frac{1}{18}\frac{(\Delta + 3k\lambda_S)^2}{k} > 0$ and $G\left(k\lambda_S + \frac{1}{3}\Delta\right) = -\frac{1}{3}\lambda_N(\Delta + 3k\lambda_S) < 0$ it implies that it exists a fee $f^* \in \left(0, k\lambda_S + \frac{1}{3}\Delta\right)$ such that $G(f) = 0$. We define as $f_e^B = f^*$. Therefore, the manager of the HC fund chooses $f_{HC}^* = \min\left\{\Delta_C, f_e^B\right\}$ and at this fee none of the funds have incentives to deviate, and the HC funds keeps the LC fund out of the market.

Case C. Uncovered ESG market Finally, we consider the case when the market is not covered, i.e. when $k \geq k_2$. As explained above, in this case the fund HS acts as a monopoly.

The fee f that makes the manager of the HS fund to be indifferent between operating only with the ESG investors and operating with both ESG and neutral investors is such that

$$f_{HS}q_{HS} = f\left(q_{HS}(f) + \lambda_N\right), \tag{8}$$

where $f_{HS} = \frac{1}{2}(R_{HS} + u_0)$ and $q_{HS} = \frac{(R_{HS} + u_0)}{2k}$. Replacing these values (8) becomes

$$\frac{(R_{HS} + u_0)^2}{4k} = f\left(\frac{R_{HS} - f + u_0}{k} + \lambda_N\right).$$

We define

$$H(f) \equiv -4f(R_{HS} + u_0) + (R_{HS} + u_0)^2 + 4f^2 - 4fk\lambda_N.$$

Notice that the equation $H'(f) = 0$ has a solution $\widehat{f} = \frac{1}{2}(R_{HS} + u_0) + \frac{1}{2}k\lambda_N$. This implies that for all fees $0 < f < \widehat{f}$ we have that $H'(f) < 0$. Since $H(0) = (R_{HS} + u_0)^2 > 0$ and $H\left(\frac{1}{2}(R_{HS} + u_0)\right) = -\frac{1}{2}(R_{HS} + u_0)(5(R_{HS} + u_0) + 9k\lambda_N) < 0$, there exists a fee $f^{**} \in \left(0, \frac{1}{2}(R_{HS} + u_0)\right)$ such that $H(f^{**}) = 0$. We define as $f_e^C = f^{**}$. As a result, the manager of the HCfund chooses $f_{HC}^* = \min\left\{\Delta_C, f_e^C\right\}$, and at this fee none of the funds have

incentives to deviate, and the HC funds keeps the LC fund out of the market.

Notice that, in equilibrium, in order for fund HS not to be able to mimic fund HC we have $f^*_{HC} < f_{HS}$ always. □

Proof of Proposition 2. Similarly to the case when both conventional funds and ESG exist in the market, we write the maximization problems for fund HS and LS :

$$\begin{aligned} \max_{f_{HS}} \Pi_S &= f_{HS} q_{HS} \\ s.t.\ U_{HS} &= R_{HS} - f_{HS} + u_0 - kx \geq 0 \\ x + y &\leq \lambda_{LS} \\ f_{HS} &\geq 0 \\ x &\geq 0 \end{aligned}$$

and

$$\begin{aligned} \max_{f_{LS}} \Pi_{HS} &= f_{LS} q_{LS} \\ s.t.\ U_{LS} &= R_{LS} - f_{LS} + u_0 - ky \geq 0 \\ x + y &\leq \lambda_{LS} \\ f_{HS} &\geq 0 \\ y &\geq 0. \end{aligned}$$

The only difference is that now the neutral investors may invest with any of the funds HS and LS depending on which one gives a higher net return. Thus, the demand for the fund HS is

$$q_{HS}\left(f_{HS}, f_{LS}\right) = \begin{cases} x + \lambda_N & \text{if} \quad f_{HS} < \Delta + f_{LS}, \\ x + \dfrac{\lambda_N}{2} & \text{if} \quad f_{HS} = \Delta + f_{LS}, \\ x & \text{if} \quad f_{HS} > \Delta + f_{LS}, \end{cases}$$

and the demand for fund LS is

$$q_{LS}\left(f_{HS}, f_{LS}\right) = \begin{cases} \lambda_S - x & \text{if} \quad f_{HS} < \Delta + f_{LS}, \\ \lambda_S - x + \dfrac{\lambda_N}{2} & \text{if} \quad f_{HS} = \Delta + f_{LS}, \\ \lambda_S - x + \lambda_N & \text{if} \quad f_{HS} > \Delta + f_{LS}. \end{cases}$$

The Lagrangean for the problem of the manager of the HS fund is

$$\mathcal{L}_{HS} = f_{HS} q_{HS} - \mu_{HS}\left(-u_0 + kx - R_{HS} + f_{HS}\right) - \eta\left(x + y - \lambda_S\right) + \gamma x,$$

and the one for the problem of the manager of the LS fund is

$$\mathcal{L}_{LS} = f_{LS} q_{LS} - \mu_{LS}\left(-u_0 + ky - R_{LS} + f_{LS}\right) - \eta\left(x + y - \lambda_S\right) + \delta y.$$

The Kuhn-Tucker conditions for these problems are

$$\begin{aligned} \frac{\partial \mathcal{L}_\varphi}{\partial f_\varphi} &\leq 0, \quad f_\varphi \geq 0, \; f_\varphi \frac{\partial \mathcal{L}_\varphi}{\partial f_\varphi} = 0, \\ U_\varphi &\geq 0, \; \mu_\varphi \geq 0 \; , \; \mu_\varphi U_\varphi = 0, \\ x + y &\leq \lambda_U, \; \eta \geq 0, \; \eta\left(x + y - \lambda_U\right) = 0 \end{aligned} \tag{9}$$

for $\varphi \in \{HS, LS\}$. Using the Kuhn-Tucker conditions and similarly to the case when both ESG funds and conventional funds co-existing in the market, we have three candidates to the global optimum: when both funds act as local monopolies, when they cover the market of ESG investors and when the market of ESG investors is covered by the HS fund and the fund LS is driven out of the market. Moreover, the existence of a positive mass of neutral investors in search for the highest possible net return creates three cases depending on the relationship of the net returns of the funds HS and LS.

When the firms act as local monopolies the revenue for the funds HS and LS equal to

$$\Pi_{HS}^{M}(f_{HS}, f_{LS}) = \begin{cases} \left(\frac{R_{HS} - f_{HS} + u_0}{k} + \lambda_N\right) f_{HS} & \text{if} \quad R_{HS} - f_{HS} > R_{LS} - f_{LS}, \\ \left(\frac{R_{HS} - f_{HS} + u_0}{k} + \frac{\lambda_N}{2}\right) f_{HS} & \text{if} \quad R_{HS} - f_{HS} = R_{LS} - f_{LS}, \\ \frac{R_{HS} - f_{HS} + u_0}{k} f_{HS} & if \quad R_{HS} - f_{HS} < R_{LS} - f_{LS}, \end{cases}$$

$$\Pi_{LS}^{M}(f_{HS}, f_{LS}) = \begin{cases} \frac{R_{LS} - f_{LS} + u_0}{k} f_{LS} & \text{if} \quad R_{HS} - f_{HS} > R_{LS} - f_{LS}, \\ \left(\frac{R_{LS} - f_{LS} + u_0}{k} + \frac{\lambda_N}{2}\right) f_{LS} & \text{if} \quad R_{HS} - f_{HS} = R_{LS} - f_{LS}, \\ \left(\frac{R_{LS} - f_{LS} + u_0}{k} + \lambda_N\right) f_{LS} & if \quad R_{HS} - f_{HS} < R_{LS} - f_{LS}, \end{cases}$$

respectively.

Case 1 $R_{HS} - f_{HS} > R_{LS} - f_{LS}$

In this case the demand for fund HS equals $\frac{R_{HS} - f_{HS} + u_0}{k} + \lambda_N$ and the demand for the fund LS equals $\frac{R_{LS} - f_{LS} + u_0}{k}$. Therefore the best responses functions are

$$\begin{aligned} \phi_{HS}^{M1}(f_{LS}) &= \frac{1}{2}(R_{HS} + u_0 + k\lambda_N), \\ \phi_{LS}^{M1}(f_{HS}) &= \frac{1}{2}(R_{LS} + u_0). \end{aligned}$$

Case 2 $R_{HS} - f_{HS} = R_{LS} - f_{LS}$

In this case the demand for fund HS equals $\frac{R_{HS} - f_{HS} + u_0}{k} + \frac{\lambda_N}{2}$ and the demand for the fund LS equals $\frac{R_{LS} - f_{LS}}{k} + \frac{\lambda_N}{2}$, and therefore the best responses are

$$\begin{aligned} \phi_{HS}^{M2}(f_{LS}) &= \frac{1}{2}\left(R_{HS} + u_0 + \frac{1}{2}k\lambda_N\right) \\ \phi_{LS}^{M2}(f_{HS}) &= \frac{1}{2}\left(R_{LS} + u_0 + \frac{1}{2}k\lambda_N\right). \end{aligned}$$

In this case $R_{HS} - f_{HS} = \frac{1}{2}R_{HS} - \frac{1}{2}u_0 - \frac{1}{4}k\lambda_N > R_{LS} - f_{LS} = \frac{1}{2}R_{LS} - \frac{1}{2}u_0 - \frac{1}{4}k\lambda_N$ since $R_{HS} > R_{LS}$ and therefore, in this case we do not have an equilibrium.

Case 3 $R_{HS} - f_{HS} < R_{LS} - f_{LS}$

In this case the demand for fund HS equals $\dfrac{R_{HS}-f_{HS}+u_0}{k}$ and the demand for the fund LS equals $\dfrac{R_{LS}-f_{LS}+u_0}{k}+\lambda_N$, and therefore the best responses are

$$\begin{aligned}\phi_{HS}^{M3}(f_{LS}) &= \frac{1}{2}(R_{HS}+u_0),\\ \phi_{LS}^{M3}(f_{HS}) &= \frac{1}{2}(R_{HS}+u_0+k\lambda_N).\end{aligned}$$

As a result, the best responses in case the market is not covered are:

$$\phi_{HS}^{M}(f_{LS})=\begin{cases}\phi_{HS}^{M1}(f_{LS})=\dfrac{1}{2}(R_{HS}+u_0+k\lambda_N) & \text{if} \quad R_{HS}-f_{HS}>R_{LS}-f_{LS},\\ \phi_{HS}^{M2}(f_{LS})=\dfrac{1}{2}(R_{HS}+u_0) & \text{if} \quad R_{HS}-f_{HS}<R_{LS}-f_{LS},\end{cases}$$

$$\phi_{LS}^{M}(f_{HS})=\begin{cases}\phi_{LS}^{M1}(f_{HS})=\dfrac{1}{2}(R_{LS}+u_0) & \text{if} \quad R_{HS}-f_{HS}>R_{LS}-f_{LS},\\ \phi_{LS}^{M2}(f_{HS})=\dfrac{1}{2}(R_{LS}+u_0+k\lambda_N) & \text{if} \quad R_{HS}-f_{HS}<R_{LS}-f_{LS}.\end{cases}$$

The equilibrium quantities in this case are

$$x(f_{HS},f_{LS})=\begin{cases}\dfrac{1}{2k}(R_{HS}+u_0-k\lambda_N) & \text{if} \quad R_{HS}-f_{HS}>R_{LS}-f_{LS},\\ \dfrac{1}{2k}(R_{HS}+u_0) & \text{if} \quad R_{HS}-f_{HS}<R_{LS}-f_{LS},\end{cases}$$

and

$$y(f_{HS},f_{LS})=\begin{cases}\dfrac{1}{2k}(R_{LS}+u_0) & \text{if} \quad R_{HS}-f_{HS}>R_{LS}-f_{LS},\\ \dfrac{1}{2k}(R_{LS}+u_0-k\lambda_N) & \text{if} \quad R_{HS}-f_{HS}<R_{LS}-f_{LS}.\end{cases}$$

Therefore, we have two candidates to equilibrium:
$(f_{HS}^*,f_{LS}^*)\in\left\{\left(\frac{1}{2}(R_{HS}+u_0+k\lambda_N),\frac{1}{2}(R_{LS}+u_0)\right),\qquad\left(\frac{1}{2}(R_{HS}+u_0),\frac{1}{2}(R_{LS}+u_0+k\lambda_N\right.\right.$
Let us consider first $(f_{HS}^*,f_{LS}^*)=\left(\frac{1}{2}(R_{HS}+u_0+k\lambda_N),\frac{1}{2}(R_{LS}+u_0)\right)$.

Since $\phi_{HS}^{M1b}(f_{LS})=\dfrac{1}{2}(R_{HS}+u_0+k\lambda_N)>\phi_{HS}^{M2}(f_{LS})=\dfrac{1}{2}(R_{HS}+u_0)$ whenever $\lambda_N>0$, the fund HS maximizes the revenue. Notice that this equilibrium satisfies the condition $R_{HS}-f_{HS}>R_{LS}-f_{LS}$.

We need to check whether the fund LS is willing to deviate from setting $\dfrac{1}{2}(R_{LS}+u_0)$.

As explained, the only option to deviate would be to set $f_{LS} = \frac{1}{2}(R_{LS} + u_0 + k\lambda_N)$. The fund LS' s revenue is in this case equal to

$$\Pi_{LS}^{M1}\left(\phi_{LS}^{M1}(f_{HS}), f_{HS}\right) = \frac{1}{4k}(R_{LS} + u_0)^2.$$

And if it deviates to $\phi_{LS}^{M2}(f_{HS}) = \frac{1}{2}(R_{LS} + u_0 + k\lambda_N)$ it equals

$$\Pi_{LS}^{M1}\left(\phi_{LS}^{M2}(f_{HS}), f_{HS}\right) = \left(\frac{R_{LS} + u_0 - \frac{1}{2}(R_{LS} + u_0 + k\lambda_N)}{k}\right)\frac{1}{2}(R_{LS} + u_0 + k\lambda_N) < \frac{1}{4k}(R_{LS} + u_0)^2.$$

Notice that in this case we have that $R_{HS} - f_{HS} = \frac{1}{2}(R_{HS} - u_0 - k\lambda_N) > \frac{1}{2}(R_{LS} - u_0 - k\lambda_N) = R_{LS} - \phi_{LS}^{M2}(f_{HS})$ so despite deviating the fund LS does not manage to attract the neutral investors and therefore is not an equilibrium.

The equilibrium $(f_{HS}^*, f_{LS}^*) = \left(\frac{1}{2}(R_{HS} + u_0 + k\lambda_N), \frac{1}{2}(R_{LS} + u_0)\right)$ satisfies the condition $R_{HS} - f_{HS} > R_{LS} - f_{LS}$ if and only if $\frac{1}{2}(R_{HS} - u_0 - k\lambda_N) > \frac{1}{2}(R_{LS} - u_0)$ or equivalently $k < \frac{\Delta}{\lambda_N}$. Moreover, this equilibrium with local monopolies exists if and only if $x + y < \lambda_S$, which is equivalent to $\frac{1}{2k}(R_{HS} + u_0 - k\lambda_N) + \frac{1}{2k}(R_{LS} + u_0) < \lambda_S$ or $\frac{2u_0 + R_{HS} + R_{LS}}{2\lambda_S + \lambda_N} < k$. We define k_2^O as $k_2^O \equiv \frac{2u_0 + R_{HS} + R_{LS}}{2\lambda_S + \lambda_N}$.

When $k > \frac{\Delta}{\lambda_N}$ we have that $R_{HS} - f_{HS} = \frac{1}{2}(R_{HS} - u_0 - k\lambda_N) < \frac{1}{2}(R_{HS} - u_0 - \Delta) = \frac{1}{2}(R_{LS} - u_0) = R_{LS} - f_{LS}$, so this cannot be an equilibrium. In this case, in order to attract the neutral investors, the funds start to undercut each other until the fund LS is driven out of the market $f_{LS} = 0$. However, the fund LS would prefer to charge a higher fee $f_{LS} = \frac{1}{2}(R_{LS} + u_0)$ and make a positive profits serving the ESG investors close to him. Therefore, no equilibrium exists in this case.

The other candidate equilibrium is $(f_{HS}^*, f_{LS}^*) = \left(\frac{1}{2}(R_{HS} + u_0), \frac{1}{2}(R_{LS} + u_0 + k\lambda_N)\right)$ when $R_{HS} - f_{HS} < R_{LS} - f_{LS}$. However, notice that $R_{HS} - f_{HS} = \frac{1}{2}(R_{HS} - u_0) > \frac{1}{2}(R_{LS} - u_0) > \frac{1}{2}(R_{LS} - u_0 - k\lambda_N) = R_{LS} - f_{LS}$, and therefore this is never an equilibrium.

Consequently, the equilibrium $(f_{HS}^*, f_{LS}^*) = \left(\frac{1}{2}R_{HS} + \frac{1}{2}k\lambda_N, \frac{1}{2}R_{LS}\right)$ exists if $k < \frac{\Delta}{\lambda_N}$

and is unique.

Let us consider next the case when $k < \dfrac{2u_0 + R_{HS} + R_{LS}}{2\lambda_S + \lambda_N}$. In this case the market is covered. Similarly, to the case where both conventional and ESG funds coexist, the location of the marginal ESG investor is

$$x = \frac{\lambda_S}{2} + \frac{R_{HS} - f_{HS} - R_{LS} + f_{LS}}{2k} = \frac{\lambda_S}{2} + \frac{r_{HS} - r_{LS}}{2k}.$$

It follows that

$$\begin{aligned} x &= \frac{\lambda_S}{2} + \frac{r_{HS} - r_{LS}}{2k}, \\ y &= \frac{\lambda_S}{2} - \frac{r_{HS} - r_{LS}}{2k}. \end{aligned}$$

The optimal demand for fund HS is

$$q_{HS}\left(f_{HS}, f_{LS}\right) = \begin{cases} \dfrac{\lambda_S}{2} + \dfrac{R_{HS} - f_{HS} - R_{LS} + f_{LS}}{2k} + \lambda_N & \text{if} \quad R_{HS} - f_{HS} > R_{LS} - f_{LS}, \\ \dfrac{\lambda_S}{2} + \dfrac{R_{HS} - f_{HS} - R_{LS} + f_{LS}}{2k} + \dfrac{\lambda_N}{2} & \text{if} \quad R_{HS} - f_{HS} = R_{LS} - f_{LS}, \\ \dfrac{\lambda_S}{2} + \dfrac{R_{HS} - f_{HS} - R_{LS} + f_{LS}}{2k} & \text{if} \quad R_{HS} - f_{HS} < R_{LS} - f_{LS}. \end{cases}$$

Similarly, the optimal demand for fund LS is

$$d_{LS}\left(f_{HS}, f_{LS}\right) = \begin{cases} \dfrac{\lambda_S}{2} - \dfrac{R_{HS} - f_{HS} - R_{LS} + f_{LS}}{2k} & \text{if} \quad R_{HS} - f_{HS} > R_{LS} - f_{LS}, \\ \dfrac{\lambda_S}{2} - \dfrac{R_{HS} - f_{HS} - R_{LS} + f_{LS}}{2k} + \dfrac{\lambda_N}{2} & \text{if} \quad R_{HS} - f_{HS} = R_{LS} - f_{LS}, \\ \dfrac{\lambda_S}{2} + \dfrac{R_{HS} - f_{HS} - R_{LS} + f_{LS}}{2k} + \lambda_N & \text{if} \quad R_{HS} - f_{HS} < R_{LS} - f_{LS}, \end{cases}$$

and the revenues are

$$\Pi_{HS}\left(f_{HS}, f_{LS}\right) = \begin{cases} \left(\dfrac{\lambda_S}{2} + \dfrac{R_{HS} - f_{HS} - R_{LS} + f_{LS}}{2k} + \lambda_N\right) f_{HS} & \text{if} \quad R_{HS} - f_{HS} > R_{LS} - f_{LS}, \\ \left(\dfrac{\lambda_S}{2} + \dfrac{R_{HS} - f_{HS} - R_{LS} + f_{LS}}{2k} + \dfrac{\lambda_N}{2}\right) f_{HS} & \text{if} \quad R_{HS} - f_{HS} = R_{LS} - f_{LS}, \\ \left(\dfrac{\lambda_S}{2} + \dfrac{R_{HS} - f_{HS} - R_{LS} + f_{LS}}{2k}\right) f_{HS} & \text{if} \quad R_{HS} - f_{HS} < R_{LS} - f_{LS}, \end{cases}$$

$$\Pi_{LS}(f_{HS}, f_{LS}) = \begin{cases} \left(\frac{\lambda_S}{2} - \frac{R_{HS} - f_{HS} - R_{LS} + f_{LS}}{2k}\right) f_{LS} & \text{if} \quad R_{HS} - f_{HS} > R_{LS} - f_{LS}, \\ \left(\frac{\lambda_S}{2} - \frac{R_{HS} - f_{HS} - R_{LS} + f_{LS}}{2k} + \frac{\lambda_N}{2}\right) f_{LS} & \text{if} \quad R_{HS} - f_{HS} = R_{LS} - f_{LS}, \\ \left(\frac{\lambda_S}{2} - \frac{R_{HS} - f_{HS} - R_{LS} + f_{LS}}{2k} + \lambda_N\right) f_{LS} & \text{if} \quad R_{HS} - f_{HS} < R_{LS} - f_{LS}, \end{cases}$$

respectively.

Case 1 $R_{HS} - f_{HS} > R_{LS} - f_{LS}$

The revenue functions in this case are the following

$$\begin{aligned} \Pi^1_{HS}(f_{HS}, f_{LS}) &= \frac{1}{2k}(k\lambda_S + 2k\lambda_N + \Delta + f_{LS} - f_{HS}) f_{HS}, \\ \Pi^1_{LS}(f_{HS}, f_{LS}) &= \frac{1}{2k}(k\lambda_S - (\Delta + f_{LS} - f_{HS})) f_{LS}. \end{aligned}$$

From the first order conditions for the maximization of the revenue for each fund we obtain each fund's best reply in this case

$$\begin{aligned} \phi^1_{HS}(f_{LS}) &= \frac{1}{2}(k\lambda_S + 2k\lambda_N + \Delta + f_{LS}), \\ \phi^1_{LS}(f_{HS}) &= \frac{1}{2}(k\lambda_S - \Delta + f_{HS}). \end{aligned}$$

The equilibrium in this case is

$$\begin{aligned} f_{HS} &= k\lambda_S + \frac{4}{3}k\lambda_N + \frac{1}{3}\Delta, \\ f_{LS} &= k\lambda_S + \frac{2}{3}k\lambda_N - \frac{1}{3}\Delta. \end{aligned}$$

Notice that $f_{HS} > 0$, while $f_{LS} = k\lambda_S + \frac{2}{3}k\lambda_N - \frac{1}{3}\Delta > 0$ if and only if $k > \frac{\Delta}{2\lambda_N + 3\lambda_S}$. Otherwise $f_{LS} = 0$ and the fund HS covers the entire market $R_{HS} - f_{HS} + u_0 - k\lambda_S = R_{LS} + u_0$, so $f_{HS} = \Delta - k\lambda_S$.

Case 2 $R_{HS} - f_{HS} = R_{LS} - f_{LS}$

Notice that in this case the first order conditions are

$$\begin{aligned}\frac{1}{2k}\left(k\lambda_S+\Delta+f_{LS}-f_{HS}+k\lambda_N\right)-\frac{1}{2k}f_{HS} &= 0,\\ \frac{1}{2k}\left(k\lambda_S-\left(\Delta+f_{LS}-f_{HS}\right)+k\lambda_N\right)-\frac{1}{2k}f_{LS} &= 0\end{aligned}$$

and the solution of this system is

$$\begin{aligned}f_{HS} &= k\lambda_S+k\lambda_N+\frac{1}{3}\Delta,\\ f_{LS} &= k\lambda_S+k\lambda_N-\frac{1}{3}\Delta.\end{aligned}$$

However, since $f_{HS}=\Delta+f_{LS}$ this can be a solution only if $\Delta=0$ (as $2k\lambda_S+k\lambda_N+\frac{1}{3}\Delta=\Delta+2k\lambda_S+k\lambda_N-\frac{1}{3}\Delta$). Moreover, notice that this cannot be an equilibrium because by undercutting by an ε any fund can be better off by increasing her demand with the entire mass of neutral investors, λ_N.

Case 3 $R_{HS}-f_{HS}<R_{LS}-f_{LS}$

The revenue functions in this case are the following

$$\begin{aligned}\Pi^3_{HS}\left(f_{HS},f_{LS}\right) &= \frac{1}{2k}\left(\Delta+f_{LS}-f_{HS}+k\lambda_S\right)f_{HS},\\ \Pi^3_{LS}\left(f_{HS},f_{LS}\right) &= \frac{1}{2k}\left(k\lambda_S-\left(\Delta+f_{LS}-f_{HS}\right)+2k\lambda_N\right)f_{LS}.\end{aligned}$$

From the first order conditions for the maximization of the revenue for each fund we obtain each fund's best reply in this case equal to

$$\begin{aligned}\phi^3_{HS}\left(f_{LS}\right) &= \frac{1}{2}\left(k\lambda_S+\Delta+f_{LS}\right),\\ \phi^3_{LS}\left(f_{HS}\right) &= \frac{1}{2}\left(k\lambda_S+2k\lambda_N-\Delta+f_{HS}\right),\end{aligned}$$

and the equilibrium fees are:

$$\begin{aligned}f_{HS} &= k\lambda_S+\frac{2}{3}k\lambda_N+\frac{1}{3}\Delta,\\ f_{LS} &= k\lambda_S+\frac{4}{3}k\lambda_N-\frac{1}{3}\Delta.\end{aligned}$$

As a result, the best responses in case the market is covered are:

$$\phi_{HS}(f_{LS}) = \begin{cases} \phi^1_{HS}(f_{LS}) = \frac{1}{2}(k\lambda_S + 2k\lambda_N + \Delta + f_{LS}) & \text{if} \quad R_{HS} - f_{HS} > R_{LS} - f_{LS}, \\ \phi^2_{HS}(f_{LS}) = \frac{1}{2}(k\lambda_S + \Delta + f_{LS}) & \text{if} \quad R_{HS} - f_{HS} < R_{LS} - f_{LS}, \end{cases}$$

$$\phi_{LS}(f_{HS}) = \begin{cases} \phi^1_{LS}(f_{HS}) = \frac{1}{2}(k\lambda_S - \Delta + f_{HS}) & \text{if} \quad R_{HS} - f_{HS} > R_{LS} - f_{LS}, \\ \phi^2_{LS}(f_{HS}) = \frac{1}{2}(2k\lambda_N + k\lambda_S - \Delta + f_{HS}) & \text{if} \quad R_{HS} - f_{HS} < R_{LS} - f_{LS}. \end{cases}$$

Notice that when $2k\lambda_N = 0$ then $\phi^1_{HS}(f_{LS}) = \phi^2_{HS}(f_E)$, so the best response is linear. When $2k\lambda_N > 0$ there is a jump in the best responses of the HS abd LS fund. Notice also $2k\lambda_N \geq 0$, it follows that $\phi^1_{HS}(f_{LS}) \geq \phi^2_{HS}(f_{LS})$ and $\phi^2_{LS}(f_{HS}) \geq \phi^1_{LS}(f_{HS})$.

Therefore, we have two candidates to equilibrium: $(f_{HS}, f_{LS}) \in \left\{ \left(k\lambda_S + \frac{4}{3}k\lambda_N + \frac{1}{3}\Delta, \right.\right.$ $\left.\left. k\lambda_S + \frac{2}{3}k\lambda_N - \frac{1}{3}\Delta \right), \left(k\lambda_S + \frac{2}{3}k\lambda_N + \frac{1}{3}\Delta, k\lambda_S + \frac{4}{3}k\lambda_N - \frac{1}{3}\Delta \right) \right.$.

Let us consider first $(f_{HS}, f_{LS}) = \left(k\lambda_S + \frac{4}{3}k\lambda_N + \frac{1}{3}\Delta,\ k\lambda_S + \frac{2}{3}k\lambda_N - \frac{1}{3}\Delta \right)$. Since $\phi^1_{HS}(f_{LS}) \geq \phi^2_{HS} f_{LS}$, the fund HS maximizes his revenue. Notice that this equilibrium satisfies the condition $R_{HS} - f_{HS} > R_{LS} - f_{LS}$ if and only if $\Delta > 2k\lambda_N$.

We need to check whether the fund LS is willing to deviate from setting $f_{LS} = k\lambda_S + \frac{2}{3}k\lambda_N - \frac{1}{3}\Delta$ when the fund HS sets $k\lambda_S + \frac{4}{3}k\lambda_N + \frac{1}{3}\Delta$. As explained, the only option to deviate would be to set $f_{LS} = k\lambda_S + \frac{4}{3}k\lambda_N - \frac{1}{3}\Delta$.

Let us check whether this is an equilibrium. The revenue functions in this case are:

$$\begin{aligned} \Pi^1_{LS}\left(f_{HS}, \phi^1_{LS}(f_{HS})\right) &= \left(\frac{\lambda_S}{2} - \frac{R_{HS} - f_{HS} - R_{LS} + f_{LS}}{2k} \right) f_{LS} \\ &= \frac{1}{2k}\left(k\lambda_S + \frac{2}{3}k\lambda_N - \frac{1}{3}\Delta \right)^2, \\ \Pi^1_{HS}\left(f_{HS}, \phi^2_{LS}(f_{HS})\right) &= \left(\frac{\lambda_S}{2} - \frac{1}{6}\frac{\Delta}{k} \right)\left(k\lambda_S + \frac{4}{3}k\lambda_N - \frac{1}{3}\Delta \right). \end{aligned}$$

Since $\Pi^1_{HS}\left(f_{HS}, \phi^1_{LS}(f_{HS})\right) - \Pi^1_{HS}\left(f_{HS}, \phi^2_{LS}(f_{HS})\right) = \frac{2}{9}k\lambda_N^2 > 0$ the LS fund does not

have any incentive to deviate and

$$(f_{HS}, f_{LS}) = \left(k\lambda_S + \frac{4}{3}k\lambda_N + \frac{1}{3}\Delta,\ k\lambda_S + \frac{2}{3}k\lambda_N - \frac{1}{3}\Delta \right)$$

is an equilibrium if $\Delta > 2k\lambda_N$.

Let us consider next $(f_{HS}, f_{LS}) = \left(k\lambda_S + \frac{2}{3}k\lambda_N + \frac{1}{3}\Delta, k\lambda_S + \frac{4}{3}k\lambda_N - \frac{1}{3}\Delta \right)$. This is an equilibrium if $R_{HS} - f_{HS} < R_{LS} - f_{LS}$. However $R_{HS} - f_{HS} - R_{LS} + f_{LS} = \frac{1}{3}(\Delta + 2k\lambda_N) > 0$, so this cannot be an equilibrium.

As a result the only equilibrium is

$$\begin{aligned} f_{HS} &= k\lambda_S + \frac{4}{3}k\lambda_N + \frac{1}{3}\Delta, \\ f_{LS} &= k\lambda_S + \frac{2}{3}k\lambda_N - \frac{1}{3}\Delta, \end{aligned}$$

and $f_{LS} \geq 0$ implies $k \geq \dfrac{\Delta}{2\lambda_N + 3\lambda_S}$. We denote by $k_1^O \equiv \dfrac{\Delta}{2\lambda_N + 3\lambda_S}$, and $k_2^O \equiv \dfrac{2u_0 + R_{HS} + R_{LS}}{\lambda_N + 2\lambda_S}$.

Summarizing, if $\Delta > 2k\lambda_N$ the equilibrium exists except if $k \in \left(\min\left\{ k_2^O, \dfrac{\Delta}{2\lambda_N} \right\}, k_2^O \right)$, and the equilibrium fees and quantities are:

$$f_{HS}^{O*} = \begin{cases} \Delta - k\lambda_S & \text{if} \quad k \leq k_1^O, \\ k\lambda_S + \frac{4}{3}k\lambda_N + \frac{1}{3}\Delta & \text{if} \quad k_1^O < k \leq \min\left\{ k_2^O, \dfrac{\Delta}{2\lambda_N} \right\}, \\ \frac{1}{2}(R_{HS} + u_0 + k\lambda_N) & \text{if} \quad k_2^O < k \leq \max\left\{ k_2^O, \dfrac{\Delta}{\lambda_N} \right\}, \end{cases}$$

$$f_{LS}^{O*} = \begin{cases} 0 & \text{if} \quad k \leq k_1^O, \\ k\lambda_S + \frac{2}{3}k\lambda_N - \frac{1}{3}\Delta & \text{if} \quad k_1^O \leq k \leq \min\left\{ \dfrac{\Delta}{2\lambda_N}, k_2^O \right\}, \\ \frac{1}{2}(R_{LS} + u_0) & \text{if} \quad k_2^O < k \leq \max\left\{ k_2^O, \dfrac{\Delta}{\lambda_N} \right\}, \end{cases}$$

$$
q_{HS}^{O*} = \begin{cases} \lambda_S + \lambda_N & \text{if} \quad k \leq k_1^O, \\ \frac{1}{2}\lambda_S + \frac{2}{3}\lambda_N + \frac{1}{6k}\Delta & \text{if} \quad k_1^O < k \leq \min\left\{k_2^O, \frac{\Delta}{2\lambda_N}\right\}, \\ \frac{1}{2k}\left(R_{HS} + u_0 + k\lambda_N\right) & \text{if} \quad k_2^O < k \leq \max\left\{k_2^O, \frac{\Delta}{\lambda_N}\right\}, \end{cases}
$$

$$
q_{LS}^{O*} = \begin{cases} 0 & \text{if} \quad k \leq k_1^O, \\ \frac{1}{2}\lambda_S + \frac{1}{3}\lambda_N - \frac{1}{6k}\Delta & \text{if} \quad k_1^O < k \leq \min\left\{k_2^O, \frac{\Delta}{2\lambda_N}\right\}, \\ \frac{1}{2k}\left(R_{LS} + u_0\right) & \text{if} \quad k_2^O < k \leq \max\left\{k_2^O, \frac{\Delta}{\lambda_N}\right\}. \end{cases}
$$

If $2k\lambda_N \geq \Delta$, the net return of fund HS becomes smaller than the one from fund LS, so all the neutral investors decide to invest with the fund LS.

However, the fund HS might try to undercut fund LS such that all neutral investors invest with it. To do this chooses $f_{HS} = \Delta - \varepsilon$ such that it forces fund LS out of the market, $f_{LS} = 0$. However, the fund LS can always deviate from this equilibrium and choose a positive fee and serve locally the ESG investors with preferences similar to her investment objective. So in this case no equilibrium exists. □

Proof of Proposition 3. We compare the fees all funds charge in equilibrium. Since

$$
\begin{aligned} k_1 &\equiv \frac{\Delta}{3\lambda_S} > k_1^O = \frac{\Delta}{2\lambda_N + 3\lambda_S} \text{ and} \\ k_2 &\equiv \frac{2u_0 + R_{HS} + R_{LS}}{2\lambda_S} > k_2^O = \frac{2u_0 + R_{HS} + R_{LS}}{\lambda_N + 2\lambda_S} \end{aligned}
$$

We consider the following cases:

Case 1. $k \leq k_1^O$

In this case the fees are equal $f_{HS} = f_{HS}^O = \Delta - k\lambda_S$ and $f_{LS} = f_{LS}^O = 0$.

Case 2. $k_1^O < k \leq k_1$

In this case the fees of the HS fund are $f_{HS} = \Delta - k\lambda_S$ and $f_{HS}^O = k\lambda_S + \frac{4}{3}k\lambda_N + \frac{1}{3}\Delta$. Since $f_{HS} = \Delta - k\lambda_S \leq k\lambda_S + \frac{4}{3}k\lambda_N + \frac{1}{3}\Delta \leq k\lambda_S + \frac{1}{3}\Delta = f_{HS}^O$, we have that $f_{HS}^O \geq f_{HS}$.

Similarly, the fees of fund LS are $f_{LS} = 0$ and $f_{LS}^O = k\lambda_S + \frac{2}{3}k\lambda_N - \frac{1}{3}\Delta \geq 0$ and therefore $f_{LS}^O \geq f_{LS}$.

Case 3. $k_1 < k \leq k_2^O$

In this case the fees of the HS fund are $f_{HS} = k\lambda_S + \frac{1}{3}\Delta$ and $f_{HS}^O = k\lambda_S + \frac{4}{3}k\lambda_N + \frac{1}{3}\Delta \geq f_{HS}$, respectively, whenever $\lambda_N \geq 0$. Similarly, the fees of fund LS are $f_{LS} = k\lambda_S - \frac{1}{3}\Delta$ and $f_{LS}^O = k\lambda_S + \frac{2}{3}k\lambda_N - \frac{1}{3}\Delta \geq f_{LS}$.

Case 4. $k_2^O < k \leq k_2$

In this case the fees of the HS fund are $f_{HS} = k\lambda_S + \frac{1}{3}\Delta$ and $f_{HS}^O = \frac{1}{2}(R_{HS} + u_0 + k\lambda_N) > f_{HS}$. To see this one can see that $\frac{1}{2}(R_{HS} + u_0 + k\lambda_N) \geq k\lambda_S + \frac{4}{3}k\lambda_N + \frac{1}{3}\Delta$ for all $k \geq \frac{3u_0 + 2R_{LS} + R_{HS}}{5\lambda_N + 6\lambda_S}$. But since $\frac{3u_0 + 2R_{LS} + R_{HS}}{5\lambda_N + 6\lambda_S} < \frac{2u_0 + R_{HS} + R_{LS}}{\lambda_N + 2\lambda_S}$, it implies that this condition is satisfied always in this particular case. Moreover since $k\lambda_S + \frac{4}{3}k\lambda_N + \frac{1}{3}\Delta > k\lambda_S + \frac{1}{3}\Delta = f_{HS}$, it implies that $f_{HS}^O > f_{HS}$.

Similarly, the fees of the LS fund are $f_{LS} = k\lambda_S - \frac{1}{3}\Delta$ and $f_{LS}^O = \frac{1}{2}(R_{LS} + u_0)$. In a similar manner we show that $f_{LS}^O = \frac{1}{2}(R_{LS} + u_0) \geq k\lambda_S + \frac{2}{3}k\lambda_N - \frac{1}{3}\Delta$ for all $k \leq \frac{3u_0 + R_{LS} + 2R_{HS}}{4\lambda_N + 6\lambda_S}$. Since $\frac{3u_0 + R_{LS} + 2R_{HS}}{4\lambda_N + 6\lambda_S} \leq \frac{2u_0 + R_{HS} + R_{LS}}{\lambda_N + 2\lambda_S}$ it implies that the relationship is true for all the parameters in this case and therefore we have that $f_{LS}^O = \frac{1}{2}(R_{LS} + u_0) \geq k\lambda_S + \frac{2}{3}k\lambda_N - \frac{1}{3}\Delta \geq k\lambda_S - \frac{1}{3}\Delta = f_{LS}$.

Case 5. $k_2 < k \leq k_2^O$

In this case the fees of the HS fund are $f_{HS} = \frac{1}{2}(R_{HS} + u_0)$ and $f_{HS}^O = \frac{1}{2}(R_{HS} + u_0 + k\lambda_N) \geq f_{HS}$. The fees of fund LS are $f_{LS} = f_{LS}^O = \frac{1}{2}(R_{LS} + u_0)$.

Consequently, we show that the fee charged to ESG investors in the case there are only ESG funds in the market are always higher or equal than those in the case when both ESG and conventional funds coexist. Notice also that the equality holds if and only if $\lambda_N = 0$, i.e. there are no neutral investors.

In the case of the neutral investors we have shown that $f_{HS}^O \geq f_{HS}$ and that $f_{HS} > f_{HC}$. As a result, the fee charged to the neutral investors when there are only ESG funds are higher or equal than those in the case when both ESG and conventional funds coexist. □

Lemma 4. *In the case when there are in the market both ESG and conventional funds, we have that the total welfare of ESG investors equals to*

$$\begin{aligned} W_{ESG} &= \int_0^x U_{Pi}di + \int_y^{\lambda_S} U_{Pi}di = (R_{HS} - f_S + u_0)\frac{x}{\lambda_S} - k\int_0^x \frac{x_i}{\lambda_S}dx_i + \\ &\quad (R_{LS} - f_{LS} + u_0)\frac{(\lambda_S - y)}{\lambda_S} - k\int_y^{\lambda_S}(\lambda_S - x_i)\frac{1}{\lambda_S}dx_i \\ &= \frac{1}{\lambda_S}\left((R_{HS} - f_{HS} + u_0)x + (R_{LS} - f_{LS} + u_0)(\lambda_S - y)\right) - \frac{1}{2\lambda_S}kx^2 - \frac{k}{2\lambda_S}(\lambda_S - y)^2 \\ &= \begin{cases} W^1_{ESG} & if \qquad k \le k_1, \\ W^2_{ESG} & if \quad k_1 < \ k < k_2, \\ W^3_{ESG} & if \qquad k \ge k_2, \end{cases} \end{aligned} \tag{10}$$

where

$$\begin{aligned} W^1_{ESG} &= R_{LS} + u_0 + \frac{1}{2}k\lambda_S, \\ W^2_{ESG} &= \frac{1}{12}\frac{\Delta^2 + 12k\lambda_S u_0 + 6k\lambda_S(R_{LS} + R_{HS}) - 9k^2\lambda_S^2}{\lambda_S k}, \\ W^3_{ESG} &= \frac{1}{8}\frac{(R_{HS} + u_0)^2}{k\lambda_S} + \frac{1}{8}\frac{(R_{LS} + u_0)^2}{k\lambda_S}. \end{aligned}$$

In the case there are only ESG funds in the market the total welfare of ESG investors is equal

to

$$
\begin{aligned}
W_{ESG}^{O} &= \int_{0}^{x^{O}} U_{Pi}di + \int_{y^{O}}^{\lambda_S} U_{Pi}di = \left(R_{HS} - f_{HS}^{O} + u_0\right)\frac{x}{\lambda_S} - k\int_{0}^{x^{O}} \frac{x_i}{\lambda_S}dx_i + \qquad (11)\\
&\quad \left(R_{LS} - f_{LS}^{O} + u_0\right)\frac{\left(\lambda_S - y^{O}\right)}{\lambda_S} - k\int_{y^{O}}^{\lambda_S} \left(\lambda_S - x_i\right)\frac{1}{\lambda_S}dx_i\\
&= \frac{1}{\lambda_S}\left(\left(R_{HS} - f_{S}^{O} + u_0\right)x^{O} + \left(R_{LS} - f_{LS}^{O} + u_0\right)\left(\lambda_S - y^{O}\right)\right) - \frac{1}{2\lambda_S}k\left(x^{O}\right)^2 + \frac{k}{2\lambda_S}\left(\lambda_S - y^{O}\right)^2\\
&= \begin{cases} W_{ESG}^{O1} & if \qquad\qquad k \le k_1^{O}, \\ W_{ESG}^{O2} & if \quad k_1^{O} < k \le \min\left\{k_2^{O}, \dfrac{\Delta}{2\lambda_N}\right\}, \\ W_{ESG}^{O3} & if \quad k_2^{O} < \ k \le \max\left\{k_2^{O}, \dfrac{\Delta}{\lambda_N}\right\}, \end{cases}
\end{aligned}
$$

where

$$
\begin{aligned}
W_{ESG}^{O1} &= R_{LS} + u_0 + \frac{1}{2}k\lambda_S,\\
W_{ESG}^{O2} &= \frac{1}{36}\frac{k^2\left(4\lambda_N^2 - 45\lambda_S^2 - 36\lambda_N\lambda_S\right) + 18k\lambda_S\left(2u_0 + R_{LS} + R_{HS}\right) - 4k\Delta\lambda_N + \Delta^2}{k\lambda_S},\\
W_{ESG}^{O3} &= \frac{1}{8}\frac{\left(R_{HS} + u_0\right)^2}{k\lambda_S} + \frac{1}{8}\frac{\left(R_{LS} + u_0\right)^2}{k\lambda_S} - \frac{1}{4}\lambda_N\frac{u_0 + R_{HS}}{\lambda_S}.
\end{aligned}
$$

The welfare of investors satisfies the following inequalities: $W_{ESG}^{1} \ge W_{ESG}^{2} \ge W_{ESG}^{3}$, *and* $W_{ESG}^{1} = W_{ESG}^{O1}$, $W_{ESG}^{2} \ge W_{ESG}^{O2}$, $W_{ESG}^{3} \ge W_{ESG}^{O3}$.

Proof. Define $F\left(k\right) = \dfrac{1}{12}\dfrac{\Delta^2 + 12k\lambda_S u_0 + 6k\lambda_S\left(R_{LS} + R_{HS}\right) - 9k^2\lambda_S^2}{\lambda_S k}$. We calculate

$$F'\left(k\right) = -\frac{1}{12}\frac{9k^2\lambda_S^2 + \Delta^2}{k^2\lambda_S} < 0.$$

This implies that $F\left(k\right)$ is increasing and

$$F\left(\frac{\Delta}{3\lambda_S}\right) \ge F\left(k\right) \ge F\left(\frac{1}{2\lambda_S}\left(2u_0 + R_{HS} + R_{LS}\right)\right) \text{ for all } k_1 \le k \le k_2.$$

Notice that $W_{ESG}^{1} = F\left(\dfrac{\Delta}{3\lambda_S}\right)$ and $W_{ESG}^{2} = F\left(k\right)$.

Define also

$$G(k) = \frac{1}{8}\frac{(u_0 + R_{HS})^2}{k\lambda_S} + \frac{1}{8}\frac{(u_0 + R_{LS})^2}{k\lambda_S}.$$

It can be shown that

$$G'(k) = -\frac{1}{8}\frac{2u_0^2 + R_{LS} + R_{HS} + 2u_0 R_{LS} + 2u_0 R_{HS}}{k^2\lambda_S} < 0.$$

so for all $k \geq k_2$, we have that $G(k_2) \geq G(k)$.

Finally since

$$\begin{aligned} F\left(\frac{1}{2\lambda_S}(2u_0 + R_{HS} + R_{LS})\right) &= \frac{1}{24}\frac{4\Delta^2 + 12u_0(u_0 + R_{LS} + R_{HS}) + 3(R_{LS} + R_{HS})^2}{2u_0 + R_{LS} + R_{HS}}, \\ G\left(\frac{1}{2\lambda_S}(2u_0 + R_{HS} + R_{LS})\right) &= \frac{1}{4}\frac{2u_0^2 + R_{LS} + R_{HS} + 2u_0 R_{LS} + 2u_0 R_{HS}}{2u_0 + R_{LS} + R_{HS}}, \end{aligned}$$

we have that

$$F\left(\frac{1}{2\lambda_S}(2u_0 + R_{HS} + R_{LS})\right) - G\left(\frac{1}{2\lambda_S}(2u_0 + R_{HS} + R_{LS})\right) = \frac{1}{24}\frac{\Delta^2}{2u_0 + R_{LS} + R_{HS}} > 0.$$

Therefore it implies that

$W^2_{ESG} = F(k) \geq F\left(\frac{1}{2\lambda_S}(2u_0 + R_{HS} + R_{LS})\right) > G\left(\frac{1}{2\lambda_S}(2u_0 + R_{HS} + R_{LS})\right) \geq W^3_{ESG}$.

Next we show that $W^1_{ESG} = W^{O1}_{ESG}$, $W^2_{ESG} \geq W^{O2}_{ESG}$, $W^3_{ESG} \geq W^{O3}_{ESG}$.

When both markets are covered the fees and quantities are the same, so the welfare is the same $W^1_{ESG} = W^{O1}_{ESG} = R_{LS} + u_0 + \frac{1}{2}k\lambda_S$.

When the funds HS and LS compete for investors we have that

$$\begin{aligned} W^2_{ESG} - W^{O2}_{ESG} = \frac{1}{12}\frac{\Delta^2 + 12k\lambda_S u_0 + 6k\lambda_S(R_{LS} + R_{HS}) - 9k^2\lambda_S^2}{\lambda_S k} \\ - \frac{1}{36}\frac{k^2\left(4\lambda_N^2 - 45\lambda_S^2 - 36\lambda_N\lambda_S\right) + 18k\lambda_S(2u_0 + R_{LS} + R_{HS}) - 4k\Delta\lambda_N + \Delta^2}{k\lambda_S} = \\ \frac{1}{18}\frac{-2k^2\lambda_N^2 + 9k^2\lambda_S^2 + \Delta^2 + 2k\Delta\lambda_N + 18k^2\lambda_N\lambda_S}{k\lambda_S} > 0. \end{aligned}$$

Finally, when the funds HS and LS are local monopolies

$$W^3_{ESG} - W^{O3}_{ESG} = \frac{1}{8}\frac{(R_{HS}+u_0)^2}{k\lambda_S} + \frac{1}{8}\frac{(R_{LS}+u_0)^2}{k\lambda_S} - \left(\frac{1}{8}\frac{(R_{HS}+u_0)^2}{k\lambda_S} + \frac{1}{8}\frac{(R_{LS}+u_0)^2}{k\lambda_S} - \frac{1}{4}\lambda_N\frac{u_0+R_{HS}}{\lambda_S}\right) = \frac{1}{4}\lambda_N\frac{R_{HS}+u_0}{\lambda_S} > 0$$

if $\lambda_N > 0$, and $W^3_{ESG} = W^{O3}_{ESG}$ when $\lambda_N = 0$. □

Proof of Proposition 4. We compare first the welfare of neutral investors. Since in both cases they are investing with a high quality fund and their utility is $R_{HS} - f_\varphi$, $\varphi \in \{HS, HC\}$ and the fee charged by HC, f_{HC} is lower than the fee charged by the high quality ESG fund HS, f_{HS}, we can conclude that the welfare of neutral investors when there are only ESG funds is lower then when there are both ESG and conventional funds. Notice that the welfare of the neutral investors in the case conventional funds coexist with ESG funds is equal to

$$W_N = \int_0^{\lambda_N} U_{Ni}di = \lambda_N\left(R_{HS} - f_{HC}\right),$$

while in the case only ESG funds exists and there is an equilibrium the welfare of neutral investors equals

$$W^O_N = \int_0^{\lambda_N} U^O_{Ni}di = \lambda_N\left(R_{HS} - f_{HS}\right).$$

Let us next study the total welfare of ESG investors. In the case when there are in the market both ESG and conventional funds, we have that the total welfare of ESG investors is given by (10), while in the case when there are in the market only ESG funds the welfare is given by (11).

Notice that in case $k \geq \dfrac{\Delta}{k\lambda_N}$, there is no equilibrium in case only ESG funds exist and therefore the total welfare of ESG investors equals 0. We will therefore study the case $k < \dfrac{\Delta}{k\lambda_N}$. The next cases will be restricted to the case $k < \dfrac{\Delta}{k\lambda_N}$ and therefore, we consider only the intersection of the following cases with the case $k < \dfrac{\Delta}{k\lambda_N}$.

Case A $k^O_2 < k_1$

Case A.1 $k < k^O_1$

Both markets are covered and $W^1_{ESG} = W^{O1}_{ESG}$ it implies that $W_{ESG} = W^O_{ESG}$.

Case A.2 $k^O_1 \leq k < k^O_2$

Market in case the conventional fund exist is covered and the welfare is $W_{ESG} = W^1_{ESG}$.

In case only the ESG funds exists the funds HS and LS compete for investors and the welfare is W^{O2}_{ESG}.

Since $W^1_{ESG} \geq W^2_{ESG}$ and $W^2_{ESG} \geq W^{O2}_{ESG}$ it implies that $W_{ESG} \geq W^O_{ESG}$.

Case A.3 $k^O_2 \leq k < k_1$

The ESG market in case the conventional funds exist is covered and the welfare is $W_{ESG} = W^1_{ESG}$. In the case only the ESG funds exist, the funds HS and LS are local monopolies and the welfare is W^{O3}_{ESG}. Since $W^1_{ESG} \geq W^3_{ESG}$ and $W^3_{ESG} \geq W^{O3}_{ESG}$ it implies that $W_{ESG} \geq W^O_{ESG}$.

Case A.4 $k_1 \leq k < k_2$

In the case in which the conventional funds exist and the funds HS and LS compete for ESG investors investors, the welfare is W^2_{ESG}. In the case only the ESG funds exist, the funds HS and LS are local monopolies and the welfare is W^{O3}_{ESG}. Since $W^2_{ESG} \geq W^3_{ESG}$ and $W^3_{ESG} \geq W^{O3}_{ESG}$ it implies that $W_{ESG} \geq W^O_{ESG}$.

Case A.5 $k_2 \leq k$

In case the conventional funds exist in the market, the funds HS and LS are local monopolies and the welfare is W^3_{ESG}. In the case only the ESG funds exist, the funds HS and LS are also local monopolies and the welfare is W^{O3}_{ESG}. Since $W^3_{ESG} \geq W^{O3}_{ESG}$ it implies that $W_{ESG} \geq W^O_{ESG}$.

Case B $k_1 \leq k^O_2$

Case B.1 $k < k^O_1$

Both markets are covered and $W^1_{ESG} = W^{O1}_{ESG}$ it implies that $W_{ESG} = W^O_{ESG}$

Case B.2 $k^O_1 \leq k < k_1$

The market in case the conventional funds exist is covered and the welfare is $W_{ESG} = W^1_{ESG}$. In case only the ESG funds exist, the funds HS and LS compete for investors and the welfare is W^{O2}_{ESG}. Since $W^1_{ESG} \geq W^2_{ESG}$ and $W^2_{ESG} \geq W^{O2}_{ESG}$ it implies that $W_{ESG} \geq W^O_{ESG}$.

Case B.3 $k_1 \leq k < k^O_2$

In case the conventional funds exist in the market, the funds HS and LS compete for investors and the welfare is W^2_{ESG}. In case only the ESG funds exist, the funds HS and LS

compete for investors and the welfare is W^{O2}_{ESG}. Since $W^{2}_{ESG} \geq W^{O2}_{ESG}$ it implies that W_{ESG} $\geq W^{O}_{ESG}$.

Case B.4 $k^{O}_{2} \leq k < k_2$

In the case the conventional funds exist in the market the funds HS and LS compete for investors and the welfare is W^{2}_{ESG}. In the case only the ESG funds exists the funds HS and HL are local monopolies and the welfare is W^{O3}_{ESG}. Since $W^{2}_{ESG} \geq W^{3}_{ESG}$ and $W^{3}_{ESG} \geq W^{O3}_{ESG}$ it implies that $W_{ESG} \geq W^{O}_{ESG}$.

Case B.5 $k_2 \leq k$

In the case the conventional funds exist, the funds HS and LS are local monopolies and the welfare is W^{3}_{ESG}. In case only the ESG funds exist the funds HS and LS are local monopolies and the welfare is W^{O3}_{ESG}. Since $W^{3}_{ESG} \geq W^{O3}_{ESG}$, it implies that $W_{ESG} \geq W^{O}_{ESG}$. □

Proof of Proposition 6. We consider now that ESG investors may invest in conventional funds but have a disutility $K \geq 0$. As shown in Lemma 1, the low-quality conventional fund LC is inactive in equilibrium. Hence, the only relevant conventional fund is HC, charging an equilibrium fee f_{HC}.

For each ESG fund $\varphi \in \{HS, LS\}$, we define the no-leakage cap as the maximal fee such that at least one ESG investor is weakly indifferent between investing in φ and in HC. These fees are given by 1. Any fee above the corresponding cap induces leakage of ESG investors to HC and yields weakly lower profits.

Let $f^{0}_{\varphi}(k)$ denote the equilibrium fee of ESG fund φ in the baseline segmented economy (i.e., when ESG investors never invest in conventional funds). As shown in Proposition 1, the baseline equilibrium is piecewise in k with three regions A, B, C.

Associated with the baseline equilibrium, we define for each ESG fund φ the baseline residual demand mapping $\Psi_{\varphi}(f_{\varphi}, k)$, which gives the mass of ESG investors investing in fund φ when the ESG market is segmented and the fund charges fee f_{φ}, holding fixed the opponent's equilibrium baseline strategy in the relevant region.

$$\Psi_{HS}(f_{HS}, f_{LS}, k) = \begin{cases} \lambda_S & if \quad k \leq k_1, \\ \dfrac{\lambda_S}{2} + \dfrac{\Delta - (f_{HS} - f_{LS})}{2k}, & if \quad k_1 < k < k_2, \\ \dfrac{R_{HS} - f_{HS} + u_0}{k} & if \quad k \geq k_2, \end{cases}$$

$$\Psi_{LS}(f_{HS}, f_{LS}, k) = \begin{cases} 0 & if \quad k \leq k_1, \\ k\lambda_S - \Psi_{HS}(f_{HS}, f_{LS}, k) & if \quad k_1 < k < k_2, \\ \dfrac{R_{LS} - f_{LS} + u_0}{k} & if \quad k \geq k_2, \end{cases}$$

For each ESG fund $\varphi \in \{HS, LS\}$ we define the constrained candidate equilibrium fee

$$f_{\varphi}^{eq}(k, K) \equiv \min\{f_{\varphi}^{0}(k), \bar{f}_{\varphi}(f_{HC}, K)\}. \tag{12}$$

The equilibrium fees f_{HS} and f_{HC} solve the fixed-point system

$$\begin{aligned} f_{HS} &= \min\{\, f_{HS}^{0}(k),\ \bar{f}_{HS}(f_{HC}, K)\,\}, \\ f_{HC} &= \min\{\, \Delta_C,\ \mathcal{H}(f_{HS})\}, \end{aligned} \tag{13}$$

where $\mathcal{H}(\cdot)$ is the smallest conventional fee that makes HS's best deviation (to capture neutral investors) unprofitable.

Let $q_{S,\varphi}^{eq}(k, K)$ denote ESG demand for fund φ induced by $(f_{HS}^{eq}(k, K), f_{LS}^{eq}(k, K))$, at the constrained equilibrium (after applying the cap). Similarly to the Proposition 1 we determine the optimal fee for the HC fund such that the HS fund does not want to mimic the fee of the HC fund. We find next the fees f_{HC} adn f_{HS} in each of the three cases regions.

The fee f that makes the manager of the HS fund to be indifferent between operating only with the ESG investors and operating with both ESG and neutral investors in the case

the cap for fund HS is binding is such that

$$f_{HS} q^{eq}_{S,HS} = f\left(q^{eq}_{HS}\left(f, f_{LS}\right) + \lambda_N\right). \tag{14}$$

where f is the largest feasible price that still wins neutral investors

$$f = R_{HS} - (R_{HC} - f_{HC}) = R_{HS} - R_{HC} + f_{HC}.$$

The deviation-prevention (indifference) condition becomes defining equality

$$f_{HS} q^{\text{eq}}_{S,HS} \;=\; (R_{HS} - R_{HC} + f_{HC})\left(q^{\text{eq}}_{S,HS} + \lambda_N\right). \tag{15}$$

Solve (15) for f_{HC} to obtain the explicit threshold

$$f_{HC} \;=\; \frac{f_{HS}\, q^{\text{eq}}_{S,HS}}{q^{\text{eq}}_{S,HS} + \lambda_N} \;-\; (R_{HS} - R_{HC}), \tag{16}$$

and we impose nonnegative fee constraint $f_{HC} \geq 0$.

Case A. Exclusionary ESG duopoly First we considered the case when the fund HS covers all the market of ESG investors and fund LS does not participate in the market (sets a fee $f_{LS} = 0$). Two subcases arise depending on whether the HS cap binds.

A.1. *HS Cap does not bind*

In this case $f_{HS} = f^0_{HS}(k) = \Delta - k\lambda_S$, and

$$f^A_K = f^A_e$$

the fee obtained in the baseline case. The fee set by the HC is then

$$f^A_{HC,\,K} \;=\; \min\left\{\Delta_C,\; \max\{0, f^A_K\}\right\}.$$

A.2. *HS Cap binds*

In this case $f_{HS} = \bar{f}_{HS}(f_{HC}, K) = R_{HS} - R_{HC} + f_{HC} + K$ and substituting into (15) we have

$$(R_{HS} - R_{HC} + f_{HC} + K)\,\lambda_S = (R_{HS} - R_{HC} + f_{HC})\,(\lambda_S + \lambda_N)).$$

We solve for f_{HC} and obtain

$$f_K^A = \frac{K\lambda_S}{\lambda_N} - (R_{HS} - R_{HC}). \tag{17}$$

The fee set by the HC fund is then

$$f_{HC,\,K}^A \;=\; \min\left\{\Delta_C,\; \max\{0, f_K^A\}\right\}.$$

Note that this solution is consistent only if the cap binds, i.e. if the value computed for f_{HC} satisfies $f_{HS} = \bar{f}_{HS}(f_{HC}, K) \leq f_{HS}^0(k)$ (see below in consistency check).

Case B. Covered ESG duopoly

In this case the market is covered by the two funds. Three cases arise

B.1. Neither the HS cap nor the LS cap binds

In this case $f_{HS} = f_{HS}^0(k)$ and $q_{S,HS}^{\text{eq}} = q_{S,HS}^0(k)$ and $f_K^B = f_e^B$ the fee obtained in the baseline case. The fee set by the HC is then

$$f_{HC,\,K}^B \;=\; \min\left\{\Delta_C,\; \max\{0, f_K^B\}\right\}.$$

B.2. *HS cap binds, the LS cap does not bind*

In this case only the HS cap binds and LS best-responds. The covered-duopoly demands are

$$q_{HS} = \frac{\lambda_S}{2} + \frac{\Delta - (f_{HS} - f_{LS})}{2k}, \tag{18}$$

$$q_{LS} = \frac{\lambda_S}{2} - \frac{\Delta - (f_{HS} - f_{LS})}{2k}. \tag{19}$$

The no-leakage constraint binds for HS:

$$f_{HS} = \bar{f}_{HS}(K) \equiv (R_{HS} - R_{HC}) + f_{HC} + K. \tag{20}$$

By definition of this case, the LS cap is slack, so LS chooses an interior best response. Given f_{HS}, fund LS solves

$$\max_{f_{LS} \geq 0} \; \Pi_{LS}(f_{LS}, f_{HS}) \equiv f_{LS}\, q_{LS}(f_{HS}, f_{LS}),$$

with q_{LS} given by (19). Substituting (19),

$$\Pi_{LS}(f_{LS}, f_{HS}) = f_{LS}\left(\frac{\lambda_S}{2} - \frac{\Delta - f_{HS} + f_{LS}}{2k}\right).$$

The first-order condition $\partial\Pi_{LS}/\partial f_{LS} = 0$ yields

$$f_{LS}^{BR}(k, f_{HS}) = \frac{k\lambda_S - \Delta + f_{HS}}{2}. \tag{21}$$

Therefore, the candidate equilibrium fees are

$$f_{HS} = \bar{f}_{HS}(K), \qquad f_{LS} = f_{LS}^{BR}\big(k, \bar{f}_{HS}(K)\big) = \frac{k\lambda_S - \Delta + \bar{f}_{HS}(K)}{2}. \tag{22}$$

Substituting (22) into (18)–(19) gives

$$q_{LS} = \frac{\lambda_S}{4} + \frac{\bar{f}_{HS}(K) - \Delta}{4k}, \tag{23}$$

$$q_{HS} = \lambda_S - q_{LS} = \frac{3\lambda_S}{4} + \frac{\Delta - \bar{f}_{HS}(K)}{4k}. \tag{24}$$

Neutral investors choose HC, hence

$$q_{HC} = \lambda_N. \tag{25}$$

This case requires simultaneously: (i) interior covered-duopoly shares for both ESG funds, (ii) HS cap binding, and (iii) LS cap slack.

(i) Interior shares. The conditions $0 < q_{LS} < \lambda_S$ are, using (23),

$$0 < \frac{\lambda_S}{4} + \frac{\bar{f}_{HS}(K) - \Delta}{4k} < \lambda_S,$$

which is equivalent to

$$\Delta - k\lambda_S < \bar{f}_{HS}(K) < \Delta + 3k\lambda_S. \tag{26}$$

(ii) HS cap binding. Let $f_{HS}^{0,B}(k)$ denote the *unconstrained* covered-duopoly equilibrium fee of HS in Region B. In the baseline model,

$$f_{HS}^{0,B}(k) = k\lambda_S + \frac{\Delta}{3}.$$

For the HS cap to bind (so that HS is indeed pinned by (20)), it is sufficient that

$$\bar{f}_{HS}(K) \leq f_{HS}^{0,B}(k) = k\lambda_S + \frac{\Delta}{3}. \tag{27}$$

(iii) LS cap slack. The no-leakage cap for LS is

$$\bar{f}_{LS}(K,k) = (R_{LS} - R_{HC}) + f_{HC} + K + k\lambda_S. \tag{28}$$

Thus, LS cap slack is equivalent to

$$f_{LS}^{BR}\big(k, \bar{f}_{HS}(K)\big) < \bar{f}_{LS}(K,k). \tag{29}$$

Substituting (21) and (20) into (29),

$$\begin{aligned} &\frac{k\lambda_S - \Delta + \bar{f}_{HS}(K)}{2} < (R_{LS} - R_{HC}) + f_{HC} + K + k\lambda_S \\ &\quad \iff \quad \bar{f}_{HS}(K) < 2(R_{LS} - R_{HC}) + 2f_{HC} + 2K + k\lambda_S + \Delta. \end{aligned} \tag{30}$$

Using $\bar{f}_{HS}(K) = (R_{HS} - R_{HC}) + f_{HC} + K$, this becomes

$$R_{HS} - R_{HC} + f_{HC} + K < 2(R_{LS} - R_{HC}) + 2f_{HC} + 2K + k\lambda_S + \Delta,$$

or equivalently,

$$R_{LS} + f_{HC} + K + k\lambda_S > 0,$$

Hence, the LS cap is indeed slack and Case B.2 is feasible whenever (31) and (27) hold.

Case B.2 is feasible if and only if (31) (both ESG funds obtain strictly positive shares) and (27) (HS cap binds) hold. Combining them together this is equivalent to

$$\Delta - k\lambda_S < \bar{f}_{HS}(K) < \; k\lambda_S + \frac{\Delta}{3}. \tag{31}$$

If LS cap does not bind, we solve for f_{HC}

$$(R_{HS} - R_{HC} + f_{HC} + K)q_{S,HS}^{\text{eq}} \;=\; \big(R_{HS} - R_{HC} + f_{HC}\big)(q_{S,HS}^{\text{eq}} + \lambda_N)$$

which gives

$$f_K^B \;=\; \frac{K q_{S,HS}^{\text{eq}}}{\lambda_N} - (R_{HS} - R_{HC}) = \frac{K}{\lambda_N}\left(\frac{3\lambda_S}{4} + \frac{\Delta - \bar{f}_{HS}(K)}{4k}\right) - (R_{HS} - R_{HC}).$$

The fee set by the HC is then

$$f_{HC,\,K}^B \;=\; \min\left\{\Delta_C,\; \max\{0, f_K^B\}\right\}.$$

B.3. *Both HS and LS cap bind*

If $f_{HS} = \bar{f}_{HS}(f_{HC}, K)$, $f_{LS} = \bar{f}_{LS}(f_{HC}, K)$ we substitute the fees in the corresponding residual demand

$$q_{S,HS}^{\text{eq}} = \frac{\lambda_S}{2} + \frac{\Delta - (f_{HS} - f_{LS})}{2k} = \frac{\lambda_S}{2} + \frac{\Delta - (\bar{f}_{HS} - \bar{f}_{LS})}{2k} = \lambda_S$$

and this case is possible only at the inferior margin of the region and the result is the same as in case A.2.

Case C. Uncovered ESG market Finally, we consider the case when the market is not covered, so the fund HS acts as a monopoly.

C.1. *Neither the HS cap nor the LS cap binds*

In this case $f_{HS} = f_{HS}^0(k)$ and $q_{S,HS}^{\text{eq}} = q_{S,HS}^0(k)$ and $f_K^C = f_e^C$, the fee obtained in the baseline case. The fee set by the HC is then

$$f_{HC,\,K}^C \;=\; \min\left\{\Delta_C,\; \max\{0, f_K^C\}\right\}.$$

C.2. *HS cap binds, the LS cap does not bind*

If $f_{HS} = \bar{f}_{HS}(f_{HC}, K)$ we substitute the fee and the corresponding residual demand for fund HS into (16) and solve for f_{HC}. The residual demand for fund HS is

$$q_{S,HS}^{\text{eq}} = \frac{R_{HS} - f_{HS} + u_0}{2k} = \frac{R_{HC} - f_{HC} + u_0 - K}{2k}.$$

The demand for fund LS in this case is the monopoly demand

$$q_{S,HS}^{\text{eq}} = \frac{R_{LS} + u_0}{2k}.$$

We solve for f_{HC}

$$(R_{HS} - R_{HC} + f_{HC} + K)q_{S,HS}^{\text{eq}} \;=\; \big(R_{HS} - R_{HC} + f_{HC}\big)(q_{S,HS}^{\text{eq}} + \lambda_N),$$

$$f_K^C \;=\; \frac{K(R_{HS} + u_0 - K)}{K + 2k\lambda_N} - (R_{HS} - R_{HC}),$$

if $K + 2k\lambda_N > 0$. The fee set by the HC is then

$$f_{HC,\,K}^C \;=\; \min\left\{\Delta_C,\; \max\{0, f_K^C\}\right\}.$$

C.3. *Both HS and LS cap bind*

If $f_{HS} = \bar{f}_{HS}(f_{HC}, K)$ and $f_{LS} = \bar{f}_{LS}(f_{HC}, K)$, we substitute the fee and the corresponding residual demand for fund HS into (16) and solve for f_{HC}. The residual demand for fund HS is

$$q_{S,HS}^{\text{eq}} = \frac{R_{HS} - f_{HS} + u_0}{2k} = \frac{R_{HC} - f_{HC} + u_0 - K}{2k}.$$

The residual demand for fund LS is

$$q_{S,HS}^{\text{eq}} = \frac{R_{LS} - f_{LS} + u_0}{2k} = \frac{R_{HC} - f_{HC} + u_0 - K - k\lambda_S}{2k}.$$

As in case C.2 We solve for f_{HC} and obtain that if $K + 2k\lambda_N > 0$,

$$f_K^C \;=\; \frac{K(R_{HS} + u_0 - K)}{K + 2k\lambda_N} - (R_{HS} - R_{HC}).$$

The fee set by the HC is then

$$f_{HC,\,K}^C \;=\; \min\left\{\Delta_C,\; \max\{0, f_K^C\}\right\}.$$

Next, we identify the three regions by defining $k_1(K)$ and $k_2(K)$. We define

$$k_1(K) \equiv \sup\{k \geq 0 : q_{LS}^{eq}(k, K) = 0\}. \tag{32}$$

Thus, $k_1(K)$ is the largest k for which the low-quality ESG fund cannot secure positive demand under the no-leakage constraint. $k_1(K)$ is the value of k at which LS obtains zero

demand when HS charges its cap, i.e. $f_{HS} = \bar{f}_{HS}(K)$, and LS best-responds. The boundary is $q_{LS} = 0$, which implies

$$k_1(K) = \frac{\Delta - \bar{f}_{HS}(K)}{\lambda_S}.$$

Imposing nonnegativity,

$$k_1(K) = \max\left\{0,\ \frac{\Delta - \bar{f}_{HS}(K)}{\lambda_S}\right\}.$$

Under (1), $\bar{f}_{HS}(K) = (R_{HS} - R_{HC}) + \Delta_C + K$, so

$$k_1(K) = \max\left\{0,\ \frac{\Delta - \left[(R_{HS} - R_{HC}) + \Delta_C + K\right]}{\lambda_S}\right\}. \tag{33}$$

We also define

$$k_2(K) \equiv \inf\{k \geq 0 : q^{eq}_{HS}(k, K) + q^{eq}_{LS}(k, K) < \lambda_S\}. \tag{34}$$

Thus, $k_2(K)$ is the smallest k at which the ESG market becomes uncovered.

In the local-monopoly (uncovered) region, each ESG fund $\varphi \in \{HS, LS\}$ serves investors up to distance d^*_φ pinned down by the participation constraint:

$$R_\varphi - f_\varphi + u_0 - kx^*_\varphi = 0 \quad \Longrightarrow \quad d^*_\varphi = \frac{R_\varphi - f_\varphi + u_0}{k}. \tag{35}$$

With unit density on $[0, \lambda_S]$, monopoly demand is $q_\varphi = d^*_\varphi$ (truncated to $[0, \lambda_S]$). At the coverage boundary, total ESG participation exactly equals λ_S:

$$q_{HS} + q_{LS} = \lambda_S. \tag{36}$$

The unconstrained monopoly fee for fund φ is $(R_\varphi + u_0)/2$. With no-leakage,

$$f^M_{LS} = \frac{R_{LS} + u_0}{2}, \tag{37}$$

$$f^M_{HS} = \min\left\{\frac{R_{HS} + u_0}{2},\ \bar{f}_{HS}(K)\right\}. \tag{38}$$

Thus

$$q_{LS} = \frac{R_{LS} - f^M_{LS} + u_0}{k} = \frac{R_{LS} + u_0}{2k}, \qquad q_{HS} = \frac{R_{HS} - f^M_{HS} + u_0}{k}. \tag{39}$$

Plugging (39) into (36) yields

$$k_2(K) = \frac{R_{HS} - f_{HS}^M + u_0 + \frac{R_{LS}+u_0}{2}}{\lambda_S}. \tag{40}$$

Using (38) gives the piecewise closed form.

Then $f_{HS}^M = (R_{HS} + u_0)/2$, so (40) becomes

$$k_2(K) = \frac{\frac{R_{HS}+u_0}{2} + \frac{R_{LS}+u_0}{2}}{\lambda_S} = \frac{R_{HS} + R_{LS} + 2u_0}{2\lambda_S}. \tag{41}$$

If the cap is binding in monopoly ($\bar{f}_{HS}(K) < (R_{HS} + u_0)/2$), then $f_{HS}^M = \bar{f}_{HS}(K)$, so (40) becomes

$$k_2(K) = \frac{R_{HS} - \bar{f}_{HS}(K) + u_0 + \frac{R_{LS}+u_0}{2}}{\lambda_S}. \tag{42}$$

Under (1), $\bar{f}_{HS}(K) = (R_{HS} - R_{HC}) + \Delta_C + K$, hence

$$k_2(K) = \frac{R_{HC} - \Delta_C - K + \frac{R_{LS}}{2} + \frac{3u_0}{2}}{\lambda_S}. \tag{43}$$

Combining (41)–(43),

$$k_2(K) = \begin{cases} \dfrac{R_{HS} + R_{LS} + 2u_0}{2\lambda_S}, & \text{if } \bar{f}_{HS}(K) \geq \dfrac{R_{HS} + u_0}{2}, \\ \dfrac{R_{HS} - \bar{f}_{HS}(K) + u_0 + \frac{R_{LS}+u_0}{2}}{\lambda_S}, & \text{if } \bar{f}_{HS}(K) < \dfrac{R_{HS} + u_0}{2}, \end{cases} \tag{44}$$

and under (1),

$$k_2(K) = \begin{cases} \dfrac{R_{HS} + R_{LS} + 2u_0}{2\lambda_S}, & \text{if } K \geq \dfrac{R_{HS} + u_0}{2} - ((R_{HS} - R_{HC}) + \Delta_C), \\ \dfrac{R_{HC} - \Delta_C - K + \frac{R_{LS}}{2} + \frac{3u_0}{2}}{\lambda_S}, & \text{if } K < \dfrac{R_{HS} + u_0}{2} - ((R_{HS} - R_{HC}) + \Delta_C). \end{cases} \tag{45}$$

As in baseline case, we have three regions.

Region A($k \leq k_1(K)$)

By definition of $k_1(K)$, $q_{LS}^{eq}(k, K) = 0$. The high-quality ESG fund HS serves all ESG investors who do not leak to HC and charges $f_{HS}^{eq}(k, K)$. Any upward deviation violates

the no-leakage constraint, while any downward deviation lowers profit by concavity of the profit function. Fund LS cannot profitably attract investors. Hence, an exclusionary ESG equilibrium obtains.

Region B ($k_1(K) < k < k_2(K)$)

Both ESG funds obtain strictly positive demand and the ESG market is fully covered. If the cap does not bind for fund φ, then $f_\varphi^{eq} = f_\varphi^0(k)$ is its unique best response in the baseline duopoly. If the cap binds, the constrained maximizer is $f_\varphi^{eq} = \bar{f}_\varphi(K, k)$. Thus, $(f_{HS}^{eq}, f_{LS}^{eq})$ constitutes a Nash equilibrium.

Region C ($k \geq k_2(K)$)

The ESG market is uncovered and each ESG fund acts as a local monopoly. The unconstrained monopoly fee is truncated by the cap whenever binding, yielding $f_\varphi^{eq}(k, K)$. Any deviation either induces leakage or lowers profits. Hence, the uncovered ESG equilibrium is sustained. □

Proof of Proposition 7. When the no-leakage constraint binds for at least one ESG fund, ESG demand is determined by indifference between the ESG fund and the conventional fund HC. We define the *integration residual demand mapping* for fund φ as

$$\varphi_\varphi(f_\varphi, k, K) \equiv \frac{R_\varphi - f_\varphi + u_0 - (R_{HC} - f_{HC} - K)}{k}, \tag{46}$$

truncated to $[0, \lambda_S]$.

When the no-leakage constraint does not bind for any ESG fund, ESG investors never consider HC and demand is given by the baseline residual demand mapping $\Psi_\varphi(f_\varphi, k)$ defined in the proof of Proposition 6.

There exists $K_1 > 0$ such that for $K < K_1$ no ESG fund can set a nonnegative fee satisfying the no-leakage constraint while generating positive demand. Hence,

$$q_{S,HS}^* = q_{S,LS}^* = 0, \qquad q_{N,HC}^* = \lambda_N + \lambda_S.$$

If $K \geq K_1$ and the cap binds for at least one ESG fund, equilibrium fees satisfy $f_\varphi^{eq} =$

$\min\{f_\varphi^0, \bar{f}_\varphi\}$ and ESG demand is given by

$$q_{S,\varphi}^* = \varphi_\varphi(f_\varphi^{eq}, k, K).$$

Market clearing implies

$$q_{N,HC}^* = \lambda_N + \lambda_S - \sum_\varphi q_{S,\varphi}^*.$$

If $K \geq K_1$ and no cap binds, equilibrium fees coincide with the baseline fees $f_\varphi^0(k)$ and ESG demand is

$$q_{S,\varphi}^* = \Psi_\varphi(f_\varphi^0(k), k), \qquad q_{N,HC}^* = \lambda_N.$$

□