Zombies? Growth Options, Heterogeneous Firms, and the Allocative Effects of Zombie Lending.*

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Abstract

We develop a model with firm dynamics, imperfect competition, and investment options. Firms borrow using credit lines offered by competitive lenders that can optimally provide "zombie" lending, with an interest rate below the risk free rate. We solve and calibrate the model and show that zombie lending is used in equilibrium to subsidise two very different types of firms: "Stagnant zombies" previously borrowed to fund projects that are no longer feasible. They are offered zombie lending so that they can gradually reduce their debt and avoid default. "High-potential zombies" have options to invest in projects that can increase their profitability, and borrow to pursue them. Zombie lending "buys them more time", in the hope they will become more profitable in the future and will thus be able to repay the debt. The main insight of the model is that while both types of zombie firms are consistent with the way such firms are usually identified in the empirical literature, they have opposite implications for misallocation and aggregate productivity and welfare. We use the model to propose novel criteria to identify the different zombie types and new testable predictions of their allocative effects: i) An increase in markup dispersion within industries should increase the share of zombie firms and the share of zombies that in the future recover and reach top quartile performance. ii) Sectors where zombies behave like high-potential ones should exhibit positive TFP responses to zombie prevalence, while those dominated by stagnant zombies should show declining productivity. Using firm-level data from Italian firms between 2007-2016, we provide robust empirical evidence confirming these predictions.

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1. Introduction

"Zombie firms" is a term used to described businesses that are highly indebted and not profitable, but avoid default thanks to the support of their lenders, which provide additional credit (evergreening) and/or lower interest rates (subsidised credit) (among others, see Blattner et al., 2023; Álvarez et al., 2023; Faria-e Castro et al., 2024 and Acharya et al., 2022). These improved lending conditions are usually defined "zombie lending", and this literature shows theoretically and empirically that zombie lending is especially prevalent when banks are weakly capitalised and have additional incentives to delay recognising loan losses.

However, while the literature analyses carefully the incentives of banks to provide zombie lending, less is known about the incentives of different types of firms to use them. Firms that are loss-making but have some prospects of overcoming the current distress might be more likely to take advantage of these loans, while firms with no hope of recovery might still want to default even when loan conditions are softened. In order to understand whether or not such lending is socially inefficient it is necessary to consider the incentives of firm to use it depending on their characteristics, their dynamic behaviour, and the resulting aggregate implications.

In this paper we provide such analysis by developing a model with firm dynamics, imperfect competition, growth options, and profit maximising firms and lenders. Our theoretical analysis shows that the incentives of banks to provide zombie lending are important for its overall magnitude relative to normal lending, but whether or not zombie lending misallocates resources depends primarily on the type of firms that take advantage of it. We show that different equilibrium conditions in different industries affect the mix of zombie types prevailing in them, and determine different aggregate implications, and verify these model predictions with empirical data on Italian firms from Orbis.

In the model, firms maximise the net present value of their dividends, and are heterogenous in terms of growth options. All firms are born with low productivity, and only a fraction of them, called "high-potential", have the option to invest to increase their productivity and become highly profitable "leaders". The firms without this growth option are called "stagnant". 'high-potential firms also face a per-period risk of losing their growth option and becoming stagnant.

Firms cannot issue equity, and after paying a fixed cost to enter they discover their type and apply for credit line contracts offered by lenders, which are ex-ante perfectly competitive, observe the firms' types, and break even in equilibrium for the contracts offered to each type. These contracts include an initial fee, and an interest rate paid every period on the amount borrowed. While lenders can commit to this long term contract,

firms cannot. Initially all firms, whether "high-potential" or "stagnant", have low debt and a positive net present value. However, if their debt increases after a sequence of negative profits shocks, they might find optimal to default.[1] Therefore, in order for the lenders to break even when the credit line is started, the interest rate on borrowing is higher than the risk free interest rate. Finally, lenders have the option to permanently lower the interest rate below the risk free rate, thus providing subsidised zombie lending.

The model features imperfect competition, and the decisions of lenders to provide zombie lending to the different types of firms potentially affects the allocation of resources through various channels: i) By relaxing financial frictions and changing firms value, it affects entry decisions, the number of firms in operation, and aggregate prices and output; ii) By affecting the share of high-potential versus stagnant firms in operation, it determines the share of firms that invest to improve productivity, and thus also aggregate TFP.

We solve the model, and calibrate it so that firms in the simulated industry have realistic levels and volatility of profits, and a realistic distribution of productivity. Consistently with Faria-e Castro et al. (2024) we show that in equilibrium lenders optimally provide zombie lending to highly indebted firms. Intuitively, the value of the credit line for the lender is the net of two components. First, the net present value of interest payments, which is positive. Second, the expected loss in case of default, which is negative. Turning a firm to zombie reduces both the positive component, because the firm pays a lower interest rate, and the negative component, because it is less likely that it will choose to default. When the debt increases, the firm is gradually more likely to default and the second component grows and eventually dominates on the first component. In this case the value of the loan for the lender has turned negative, but it can be made smaller in absolute value by reducing interest payments, to increase the probability the firm will manage to repay the debt and avoid default.

Having shown that in our model rational profit maximising lenders optimally provide zombie lending, the most interesting and novel results of the paper relate to the heterogeneity of zombie firms. We show that firms that are born "stagnant" are very unlikely to become zombies.[2] Conversely, the presence of the growth option of "high potential" firms is key for the existence is equilibrium of two types of zombie firms. the first type is

[1] A defaulting firm cannot continue activity by contracting a new credit line from another lender. Moreover, credit lines also come with a maximum borrowing limit, so in theory the model implies two types of default. Voluntary default, when the firms find optimal to default on their debt because the value of continuing is lower than their outside option, and forced default, when they break the borrowing limit. In the calibrated model only the former type of default happens in equilibrium. Reducing the borrowing limit would allow also the latter to happen, but would not change the model's results.

[2] Intuitively, they are small and generate few earnings, but they are on average profitable, otherwise they would not enter into operation. They borrow in equilibrium to smooth earnings losses, but for a realistically calibrated volatility of earnings, they are unlikely to accumulate so much debt that they enter in the "zombies" region.

firms that were born as high-potential and became stagnant after some periods, because a negative shocks eliminated their growth option. These "stagnant zombies" are firms that previously borrowed to invest into positive NPV projects, and discover that these projects are no longer feasible. It is optimal for lenders to offer zombie lending to these firms because it helps them reduce gradually their debt and avoid default.

The second type are the "high-potential zombies". These are firms that still have growth options that have not been realised yet, and find optimal to borrow to pursue them. When they are highly indebted, these firms are turned to zombie because it is optimal for the lender to "buy them more time", in the hope they will become more profitable in the future and will thus be able to repay the debt.

Interestingly, on the one hand both type of zombies, stagnant and high-potential, are fully consistent with the way zombie firms are identified in the empirical literature: they have high debt relative to the median values in the industry, low profits relative to the interest payments on such debt, are not very young (because in the model it takes time to accumulate debt), and have not been growing in recent years (high-potential zombies have a potential to grow that has not been realised yet).

On the other hand, they have starkly different effects for the misallocation implications of zombie lending. We simulate several industries that differ only in the share of stagnant over high-potential zombies.[3] Then we compare the aggregate outcomes in these industries versus industries identical in everything except that lenders do not have the option to provide zombie lending. We show that zombie lending worsens the allocation of resources, reducing aggregate TFP and welfare, when stagnant zombies are prevalent over high-potential zombies, while it actually improves it when they are less prevalent. How is this novel result affected by banks incentives? We consider alternative simulations in which we model "weakly capitalised" banks as lenders that suffer additional losses proportional to the amount of debt that defaults. We show that the presence of weak banks always increases the amount of zombie lending, but in equilibrium it amplifies more the zombie type that is already more prevalent, and thus its effects on allocative efficiency can be positive or negative depending on whether high-potential or stagnant zombies are the prevalent type.

Finally, we propose two ways to empirically test the novel predictions of the model. For the first test, we simulate several industries that differ only in terms of the productivity of leaders (the high-potential firms that succeeded in improving productivity), while the productivity of new firms is constant and normalised to one. We model imperfect

[3] In practice these industries differ only in the probability high potential firms lose their growth options, because there is a direct positive relation between such probability and the share of Stagnant over High-potential zombies in equilibrium.

competition using the quadratic utility approach of Melitz and Ottaviano (2008), so that more productive firms charge higher markups, and industries with higher productivity of leaders can be empirically mapped into industries with an higher dispersion of markups.

The model thus predicts that the overall share of zombie firms increases in markup dispersion, and that such an increase is entirely driven by the high potential firms. The intuition is that higher markup dispersion increases competitive pressures for low productivity firms, reducing their profits, but also increases the gain from investing in productivity to become leaders. Thus on the one hand high potential laggards invest more heavily in improving productivity, and because their profits are lower (due to stronger competition) accumulate debt faster. On the other hand lenders have more incentives to turn these firms to zombies, to help them become leaders and repay the debt. Conversely the share of stagnant zombies is more or less constant over the different simulations. This result implies the two following testable predictions:

Prediction 1: in an industry, the larger is markups dispersion, the larger is the share of zombie firms.

Prediction 2: An increase in markup dispersion increases the likelihood that firms receiving zombie loans transition in the future to the top of the markup distribution.

Regarding the second test, as mentioned before stagnant and high potential zombies are not distinguishable using the way the literature identifies these firms, but only the stagnant type are "true zombies" in the sense that they misallocate resources. Our model implies additional criteria to separately identify these two types. While both are high-debt firms unable to cover interest payments with their profits, the stagnant type tend to invest less and reduce their debt over time, while the high potential type tend to invest relatively more and increase their debt over time. We test these new identification criteria using the prediction of the model that changes in the amount of these different zombie types have opposite effects on sectorial productivity.

We performs these tests using detailed firm-level data from Italy between 2007-2016. We compute firm-level markups following De Loecker et al. (2020), and we use them to obtain measures of markup dispersion and markup of top firms for every industry and year in the sample. Our model implies that higher markup dispersion across industries correlates with higher markups of top firms, and therefore we can test predictions 1 and 2 using either variations in dispersion or variation in top markups. We use the former as our benchmark measure, but we verify that all results hold when using the latter. Furthermore, our predictions are derived by comparing equilibria across industries and therefore their direct empirical counterpart is cross-industry evidence. Nonetheless, we confirm that all the results hold if we include industry-year fixed effects, thus controlling for all industry invariant characteristics and focusing on within industry variation.

First, we document that increases in markup dispersion correlate with larger zombie shares both across and within sectors. In particular, a one standard deviation increase in sector-level markup dispersion (0.56) is associated with 36 percent higher shares of zombie firms (0.9 percentage points higher). We further show that this markup-zombie relationship is amplified in sectors where markups more directly translate into market dominance. Using firm-specific estimates of how tightly market shares track markup levels, we find that zombie prevalence responds more strongly to markup dispersion in industries where high-markup firms also capture larger market shares. This heterogeneity suggests that zombie lending concentrates where the option value of market leadership is most readily exploitable.

Second, we show that as markup dispersion increases, the likelihood of zombie firms reaching the top quartile of market share and markup distribution over the subsequent four years increases substantially. In other words, the larger the markups for top firms in an industry, the more likely zombies are to ever become top firms in the future, thus confirming the second prediction of the model. Quantitatively, while zombie firms are on average less likely to become leaders than other firms, their chances are significantly better in sectors with higher and more dispersed markups. In particular, increasing the markup dispersion from the bottom 25th to the top 75th percentile distribution increases the likelihood of zombie firms becoming leaders (e.g. being in the top quartile of the market share) by 11.8 percentage points. The effect is even stronger for markups, where the same increase in dispersion relates to a 10.1% increase in reaching the top quartile and offsets 82% of the initial "zombie disadvantage". We further document this positive relation between sector markup dispersion and zombie dynamism also for employment levels and investment rates. Together, these findings strongly support our model's key mechanism that market structure fundamentally shapes zombie firms' potential for future improvements.

These results suggest that not all zombie firms are equally detrimental to resource allocation. If zombies in high markup dispersion sectors are primarily high-potential laggards experiencing financial distress, their survival may actually enhance dynamic efficiency by preserving valuable growth options. Conversely, if zombies are predominantly stagnant firms with no viable recovery path, their persistence would generate the misallocation effects emphasized in prior literature.

Guided by the model's predictions about how these two zombie types behave differently, we devise an empirical-based measure to distinguish between them using their investment and leverage dynamics. High-potential zombies exhibit investment rates and leverage growth comparable to healthy firms, while stagnant zombies invest less and deleverage over time. We thus classify sectors based on the relative distance between zombies and the

rest of the firms in these dimensions. Exploiting this distinction, we show that increases in zombie shares are associated with higher sectoral TFP in industries where zombies behave like high-potential laggards, but with declining TFP where they resemble stagnant firms. This evidence directly confirms the model's prediction that the allocative implications of zombie lending depend critically on which type of zombie firm predominates in a given industry. Standard zombie identification based solely on financial distress metrics cannot make this distinction, potentially obscuring important heterogeneity in the welfare effects of zombie credit across different competitive environments.

2. The model

We consider a firm dynamics model with imperfect competition, financial frictions, and productivity-enhancing investment. Even though the main insights of the model would hold in a monopolistic competition model with constant markups, having a more realistic model in which markups vary with market share and productivity has the advantage to provide clear empirically testable predictions. We introduce variable markups by adopting the quadratic utility approach of Melitz and Ottaviano (2008).

2.1. Preferences

The representative consumer's utility function is given by:

$$U_t = A \int_{i \in N} q_{it} \, di - \frac{\gamma}{2} \int_{i \in N} q_{it}^2 \, di - \frac{\eta}{2} \left(\int_{i \in N} q_{it} \, di \right)^2 + C \tag{1}$$

where q_{it} is the quantity consumed of variety i in period t; N is the set of all varieties; $\gamma > 0$ measures the degree of substitutability between varieties; $\eta > 0$ captures the intensity of competition among firms; C is the consumption of an homogeneous good chosen as numéraire. The consumer's budget constraint is:

$$\int_{i \in N} p_{it} q_{it} \, di + C = I \tag{2}$$

where p_{it} is the price of variety i in period t; I is the consumer's income. Maximizing utility subject to the budget constraint yields the inverse demand function for each variety:

$$p_{it} = A - \gamma q_{it} - \eta Q_t \tag{3}$$

where $Q_t = \int_{i \in N} q_{it} \, di$ is the total consumption of all varieties.

2.2. Technology

Firm i in period t has productivity z_{it} and marginal cost $c_{it} = \frac{w}{z_{it}}$. $w > 0$ is the unitary cost of the variable production input. If the firm chooses to produce, obtains profits equal to:

$$\pi_{it} = (p_{it} - c_{it})q_{it} - F_{it} \tag{4}$$

Where F_{it} is a positive cost of production that is drawn every period from a uniform distribution:

$$F_{it} \in [\underline{F}, \overline{F}]$$

We use (3) to compute $q_{it} = \frac{A - p_{it} - \eta Q_t}{\gamma}$ and we substitute it into profits:

$$\pi_{it} = (p_{it} - c_{it})\frac{A - p_{it} - \eta Q_t}{\gamma} - F_{it} \tag{5}$$

The first order condition for the optimal price yields :

$$p_{it} = \frac{A + c_{it} - \eta Q_t}{2} \tag{6}$$

And the markup is:

$$p_{it} - c_{it} = \frac{A - c_{it} - \eta Q_t}{2} \tag{7}$$

Therefore, firms with higher productivity and lower marginal costs c_{it} in equilibrium have higher markups. We substitute p_{it} and q_{it} to obtain:

$$\pi(z_{it}, F_{it}) = \overline{\pi} - F_{it} \tag{8}$$

Where $\overline{\pi} \equiv \frac{(A - \frac{w}{z_{it}} - \eta Q_t)^2}{4\gamma}$. We denote the expectation of $\pi(z_{it}, F_{it})$ over the value of F_{it} as $\pi^e(z_{it})$:

$$\pi^e(z_{it}) = \overline{\pi} - E(F_{it}) \tag{9}$$

2.3. Firm Types

There are three types of firms. The firms of the first type have high productivity z^{High}. We call them "*leaders*". The second type have a productivity equal to z^{Low}, with $z^{Low} < z^{High}$, but are able to perform productivity-enhancing investments, which with some probability will allow them to reach z^{High}. We call these firms "*high-potential laggards*"(HP). Finally, the third type have also productivity equal to z^{Low}, but are not able to make a productivity-enhancing investment. We call them "*stagnant laggards*"(S). We will index firm types

with the subscript $J \in \{HP, S\}$. Firms draw their types (HP or S) at the time of entry.[4]

2.4. Financial Frictions

We assume firms do not have access to equity finance beyond the financing of the initial entry cost, but only to long term debt financing. Specifically, lenders offer to all new firms a credit line up to a maximum amount $\overline{b}$. The credit line implies an initial fee F^b, and an interest rate on borrowing equal to $r + \tau^G$, where r is the risk free interest rate, and $\tau^G > 0$ is a positive spread. While lenders can commit to the long term debt contract, firms cannot, and will default on the debt if the value of continuing operating is lower than their outside option, which is a fraction of the initial value of the firm. Therefore, after a long enough sequence of negative outcomes, firms accumulate debt and might find it optimal to default rather than to continue.

We introduce zombie lending by assuming the lenders have the option to improve lending conditions to their borrowers by reducing the interest wedge τ^G. We maintain the assumption that lenders can commit, and hence such reduction lasts as long as the borrower remains in operation as laggard firm. Therefore when the lending contract begins there exists a "zombie policy function" $\zeta^J(b_{it}) \in [0, 1]$ that represents probability that a firm with debt $b_{i,t}$ and type J transitions from normal to zombie status in period t. We index zombie status with $G \in \{NZ, Z\}$ where NZ and Z indicate respectively non-zombie firms and zombie firms. Therefore $\tau^{NZ} > \tau^Z > -r$. That is, we allow τ^Z to be negative, but total interest payments $r + \tau^G$ are non-negative. The probability $\zeta^J(b_{it})$ is determined optimally by the lenders in order to maximise the value of the credit line:

$$\zeta^J(b_{it}) \;=\; \mathbb{1}\Big[\Lambda_t^{J,Z}(b_{it}) \;>\; \Lambda_t^{J,NZ}(b_{it})\Big], \tag{10}$$

Where $\Lambda_t^{J,NZ}(b_{it})$ and $\Lambda_t^{J,Z}(b_{it})$ are respectively the value of the credit line for the lender conditional on non-zombie and zombie status, computed at the point in time in which such decision is taken, right before the fixed cost value F_{it} is known (see Section 2.7 for details). Note that condition (10) implies $\zeta^J(b_{it})$ is a binary choice. However, in practice we assume that an arbitrarily small stochastic cost is required to convert the credit line from normal to zombie status, so that $\zeta^J(b_{it})$ changes smoothly from 0 to 1 for the relevant portion of the state space. This assumption is not essential, but helps with the convergence of the iterative procedure to find a numerical solution to the problem.

The debt contract also includes a covenant on the firm, imposing that it can distribute dividends only out of expected profits net of interest payments. Without such covenant,

[4] We assume that all firms enter initially with productivity equal to z^{Low}. Nonetheless, assuming that a fraction of firms are born leaders would not change the qualitative implications of the model.

it would be optimal for firms to borrow up to the limit to increase interest payment and default immediately after. Such moral hazard problem would raise so much ex-ante borrowing costs that no firm would be able to operate in equilibrium.

Summing up, lenders offer to firms credit line contracts that specify maximum borrowing $\bar{b}$, per period interest rate payments of $(r + \tau^{NZ})b_t$ for normal lending, $(r + \tau^{Z})b_t$ for zombie lending, and interest payments of rb_t for firms that are net savers and have negative b_t. Firms are forced to default if they breach the maximum borrowing limit, and can voluntarily default if their continuation value is lower than the exit option. We assume lenders to be perfectly competitive, and able to observe firm types, so credit line contracts are specific to the type of firm, HP or S, and they break even in equilibrium.

2.5. Leaders

A firm becomes leader when it successfully performs a productivity enhancing investment. We assume the current owners do not have the ability to run the firm once it becomes leader, but can sell it to new owners at a price equal to the net present value of future profits, provided they repay the current debt in full first.[5] Then the leader maintains the same productivity level until it dies exogenously with probability $1 - d$. Therefore the value at time t of a firm that became leader with debt b_t, for the current owners, is equal to minus the debt plus the net present value of constant expected profits equal to $\pi^e(z^{High})$ until the firm dies:

$$V_t^{Leader}\left(z^{High}, b_t\right) \;=\; -\,b_t \;+\; \frac{d}{R-d}\,\pi^e\left(z^{High}\right)$$

where $R \equiv 1 + r$ and r is the real interest rate.

2.6. Laggards

In this section we describe the problem and derive the value functions of laggard firms. For brevity, we refer to them simply as "firms". As explained above, their category is defined by the superscripts JG, where $J \in \{HP, S\}$ is their type (High potential or Stagnant) and $G \in \{NZ, Z\}$ is zombie status.

[5] This is equivalent to assuming a covenant in the debt contract that requires such repayment in case of change of ownership of the firm. We make this assumption to simplify the computation of the problem for the leader, but the results would be unaffected by its relaxation.

2.6.1. Timing

At the beginning of each period t, firm i is denoted by its debt b_{it}, by its productivity z_{it}, and by its type J and zombie status G. The timing of each period is organised in the following sequential sub-periods:

i) At the beginning of the period, HP firms lose their capacity to invest to increase productivity with probability $1 > p_s > 0$, and become S firms.

ii) Firms die exogenously with probability $1 - d$ and survive otherwise, with $0 < d < 1$.

iii) Surviving firms are able to continue if their debt is lower than the upper bound limit $\overline{b}$, otherwise are forced to default and liquidate.

iv) Non-zombies with debt b_{it} and type J benefit from a reduction in the interest rate from τ^{NZ} to τ^{Z} with probability $\zeta^{J}(b_{it})$. That is, they acquire zombie status.

v) Firms of type JG and debt b_{it} decide dividend payments $d_{it}^{JG}(b_{it})$. Firms can distribute net savings as dividends. Furthermore, as explained above, firms currently using the credit line are constrained by the value of expected profits net of interest rate payments:

$$0 \leq d_{it}^{JG}(b_{it}) \leq \max[E(\pi(z_{it}, F_{it})) - (r + \tau^{G})\, b_{it}, 0]. \tag{11}$$

After paying dividends, the firms learn the value of the per-period fixed cost F_{it} and decide whether to continue or to voluntary default.

vi) Continuing HP firms learn the cost of investing to improve productivity X_{it}, which is drawn from a uniform distribution with bounds $\underline{X}, \overline{X}$, and decide whether or not to invest to improve productivity.

v) Firms produce, generating profits $\pi(z_{low}, F_{it})$ as defined in (8), pay the interest on debt or receive return from savings, and pay dividends. The law of motion of debt is:

$$b_{i,t+1}^{JG} = (1 + r) b_{it}^{JG} + \tau^{G} b_{it}^{+} - \pi(z^{Low}, F_{it}) + \mathbb{I}_{I}\, X_{it} + d_{it}^{JG} \tag{12}$$

where $(b)^{+} \equiv \max\{x, 0\}$ and $\mathbb{I}_{I}$ is an indicator function equal to 1 if the firm invests to increase productivity, and equal to 0 otherwise.

2.6.2. Liquidation value

Before deriving the value of the firm, it is useful to define its liquidation value in case of exit. Define as $V^{JG}(z^{low}, b_{it})$ the value at the beginning of the period of a firm of type J and zombie status G. Since all firms begin as non-zombies and with zero debt, their initial value is $V^{JNZ}(z^{low}, 0)$. We assume that the liquidation value available for the lenders is a fraction ϕ^{L} of this value. if such value is higher than outstanding debt, then the residual

accrues to the firm. Furthermore, a fraction ϕ^F of the same value can be diverted by the firm in case of exit. Therefore the exit dividend $div^{J,exit}(z^{low}, b_{it})$ for a firm of type J with debt b_{it} is equal to:

$$div^{J,exit}(z^{low}, b_{it}) = \phi^F V^{JNZ}(z^{low}, 0) + \max(\phi^L V^{JNZ}(z^{low}, 0) - b_{it}, 0) \tag{13}$$

The recovery value for the lender in case of exit, $rec^{J,exit}$, is thus equal to:

$$rec^{J,exit} = \min(b_{it} - \phi^L V^{JNZ}(z^{low}, 0), 0) \tag{14}$$

In practice we assume ϕ^F to be small, so that a firm voluntarily defaults only when its debt is very high and its value of continuing is very low, which also means the exit dividend is in equilibrium simply equal to $\phi^F V^{JNZ}(z^{low}, 0)$.

2.6.3. Value function determination

We first consider the value at the end of the period, starting from the decision to invest in improving productivity, and then we move backwards to derive the value at the beginning of the period.

Investment decision

Consider an HP firm with zombie status G, after paying dividends d_{it}^{HPG}, and observing investment cost X_{it}. We define the value conditional on investing and not investing as $V_{it}^{HPG,I}(z^{Low}, b_{it}, F_{it}, X_{it})$ and $V_{it}^{HPG,NI}(z^{Low}, b_{it}, F_{it}, X_{it})$, respectively:

$$V_{it}^{HPG,NI}(z^{Low}, b_{it}, F_{it} \mid d_{it}^{HPG}) = \beta^f V_{it+1}^{HPG}(z^{Low}, b_{i,t+1}^{HPG}). \tag{15}$$

$$V_{it}^{HGP,I}(z^{Low}, b_{it}, F_{it}, X_{it} \mid d_{it}^{HPG}) = \beta^f \Big[\xi\, V_{it+1}^{Leader}(z^{High}, b_{i,t+1}^{HPG}) + (1-\xi)\, V_{it+1}^{HPG}(z^{Low}, b_{i,t+1}^{HPG})\Big]. \tag{16}$$

Here β^f is the firm's discount factor, $\xi \in [0, 1]$ is the probability the investment succeeds in improving productivity and making the firm a leader, $b_{i,t+1}^{HPG}$ is next period debt from (12), and $V_{it+1}^{HPG}(z^{Low}, b_{i,t+1}^{HPG})$ is the value of the firm at the beginning of period $t+1$ if it does not become leader.

Let $X_{it}^{*,G}$ be the investment cost that satisfies $V_{it}^{HPG,NI} = V_{it}^{HPG,I} - X_{it}^{*,G}$. It follows that a firm will invest if $X_{it} < X_{it}^{*,G}$. The ex-ante probability to invest p^{HPG} is:

$$p^{HPG} = \frac{X_{it}^{*,G} - \underline{X}}{\overline{X} - \underline{X}}. \tag{17}$$

when $X_{it}^{*,G} \in [\underline{X}, \overline{X}]$. If $X_{it}^{*,G} < \underline{X}$, then $p^{HPG} = 0$; if $X_{it}^{*,G} > \overline{X}$, then $p^{HPG} = 1$.

Consider now an S firm with zombie status G. This firm has no investment option, and therefore its value is the no-investment one:

$$V_{it}^{SG,NI}(z^{Low}, b_{it}, F_{it} \mid d_{it}^{SG}) = \beta^f V_{it+1}^{SG}(z^{Low}, b_{i,t+1}^{SG}). \tag{18}$$

and $p^{SG} = 0$.

Continuation decision

We now define the continuation decision $V_{it}^{cont,JG}(z^{Low}, b_{it}, F_{it}|d_{it}^{JG})$, where as before G is the zombie status and $J \in \{HP, S\}$, as:

$$\begin{aligned} V_{i,t}^{Cont,JG}(z^{Low}, b_{it}, F_{it}|d_{it}^{JG}) = \max \Big\{ & p^{JG}\, \mathbb{E}_X \left[V_{i,t}^{JG,I}(z^{Low}, b_{it}, F_{it}, X_{i,t} \mid d_{it}^{JG}) \right] \\ & + \left(1 - p^{JG}\right) V_{i,t}^{JG,NI}(z^{Low}, b_{it}, F_{it} \mid d_{it}^{JG}) \Big\}. \end{aligned} \tag{19}$$

Where the expectation term $\mathbb{E}_X \left[V_{i,t}^{JG,I}(z^{Low}, b_{it}, F_{it}, X_{i,t} \mid d_{it}^{SG}) \right]$ relates to the cost of investment $X_{i,t}$, which increases $b_{i,t+1}$ (see Equation 12) and thus reduces future expected dividends. As explained before, for stagnant firms with $J = S$, p^{SG} is equal to zero by assumption.

The firm chooses to continue if $V_{i,t}^{Cont,JG}$ is higher than the exit option $div^{J,exit}(z^{low}, b_{it})$ as defined in (13). $V_{i,t}^{Cont,JG}$ monotonously decreases in F_{it}, because higher realizations of the fixed cost reduce profits, increase debt, and reduce the continuation value. Therefore there exist a value of $F_{it}^{*,G}$ such that $V_{i,t}^{Cont,JG}(z^{Low}, b_{it}, F_{it}^{*,G}|d_{it}^{JG}) = div^{J,exit}(z^{low}, b_{it})$, and the firm will exit for values of $F_{it} \geq F_{it}^{*,G}$. Define the probability of continuation as $p^{cont,JG}$. It follow that the probability of defaulting voluntarily is:

$$1 - p^{cont,JG} \;=\; \frac{F_{it}^{*,G} - \underline{F}}{\overline{F} - \underline{F}}, \tag{20}$$

provided that $F_{it}^{*,G} \in [\underline{F}, \overline{F}]$. If $F_{it}^{*,G} < \underline{F}$, then $p^{cont,JG} = 1$; if $F_{it}^{*,G} > \overline{F}$, then $p^{cont,JG} = 0$.

Dividend decision

At the dividend stage, type JG chooses $d_t^{JG}(b_t)$ before observing F_{it}. The value conditional on optimal dividends $V_{i,t}^{Div,JG}(z^{Low}, b_t)$ is therefore as follows:

$$V_{i,t}^{Div,JG}(z^{Low}, b_{it}) = \max_{d_{it}^{JG}} \Bigg\{ d_{it}^{JG} \;+\; p^{cont,JG}\, \mathbb{E}_F \Big[V_{i,t}^{Cont,JG}(z^{Low}, b_{it}, F_{it} \mid d_{it}^{JG}) \;\Big|\; F_{it} \leq F_{it}^{*,G} \Big] \\ +\; \big(1 - p^{cont,JG}\big)\, div^{exit}(z^{low}, b_{it}) \Bigg\}, \tag{21}$$

subject to the dividend constraint (11) and the law of motion of debt (12).

Initial value

We now define the value of the firm at the beginning of the period. Recall that at the start of period t, an HP firm loses the ability to invest with probability p_s and becomes S. Then exogenous exit occurs with probability $1 - d$. If the firm survives, it is forced to default and liquidate if $b_{it} > \bar{b}$. Firms that are not liquidated and are in NZ status become zombies with probability $\zeta^J(b_{it})$. We define $\mathbb{I}_{it}^b$ as an indicator function equal to one if $b_{it} < \bar{b}$ and equal to 0 otherwise. Then the initial value of a firm not in zombie status is:

$$V_{i,t}^{HP\,NZ}(z^{Low}, b_{it}) = \big(1 - d\,\mathbb{I}_{it}^b\big)\, \mathrm{Div}^{HP,\mathrm{death}}(z^{low}, b_{it}) \\ +\; d\,\mathbb{I}_{it}^b \Big[(1 - p_s)\, \mathcal{B}_{it}^{HP,NZ}(b_{it}) \;+\; p_s\, \mathcal{B}_{it}^{S,NZ}(b_{it}) \Big]. \tag{22}$$

Where

$$\mathcal{B}_{it}^{J,NZ}(b_{it}) \equiv \big(1 - \zeta^J(b_{it})\big)\, V_{i,t}^{Div,J\,NZ}(z^{Low}, b_{it}) + \zeta^J(b_{it})\, V_{i,t}^{Div,J\,Z}(z^{Low}, b_{it}),$$
$$\mathrm{Div}^{HP,\mathrm{death}}(z^{low}, b_{it}) \;\equiv\; (1 - p_s)\, div^{HP,exit}(z^{low}, b_{it}) \;+\; p_s\, div^{S,exit}(z^{low}, b_{it}).$$

The value of a stagnant firm (S) not in zombie status is similar, except that it cannot switch to HP status:

$$V_{i,t}^{S\,NZ}(z^{Low}, b_{it}) = \big(1 - d\,\mathbb{I}_{it}^b\big)\, div^{S,exit}(z^{low}, b_{it}) \;+\; d\,\mathbb{I}_{it}^b \mathcal{B}_{it}^{S,NZ}(b_{it}). \tag{23}$$

Similarly, the values of HP and S firms in zombie status are analogous to those above except that the term $\mathcal{B}_{it}^{J,Z}(b_{it})$ simplifies to $V_{i,t}^{Div,J\,Z}(z^{Low}, b_{it})$:

$$V_{i,t}^{HP\,Z}(z^{Low}, b_{it}) = \big(1 - d\,\mathbb{I}_{it}^b\big)\, \mathrm{Div}^{HP,\mathrm{death}}(z^{low}, b_{it}) \\ +\; d\,\mathbb{I}_{it}^b \Big[(1 - p_s)\, V_{i,t}^{Div,HP\,Z}(z^{Low}, b_{it}) \;+\; p_s\, V_{i,t}^{Div,S\,Z}(z^{Low}, b_{it}) \Big]. \tag{24}$$

$$V_{i,t}^{S\,Z}(z^{Low}, b_{it}) = (1 - d\mathbb{I}_{it}^{b})\, div_{S}^{S,exit}(z^{low}, b_{it}) + d\mathbb{I}_{it}^{b} V_{i,t}^{Div,S\,Z}(z^{Low}, b_{it}). \tag{25}$$

2.7. Lender's problem and loan value

As in the previous section, we refer to type as $J \in \{HP, S\}$ and zombie status as $G \in \{NZ, Z\}$. Furthermore, we define $a_{it}^{JG,+} \equiv (R + \tau^{G} - 1)\, b_{it}^{+}$ as interest payments on the debt when this is positive, indicating that the firm is using the credit line.

Given the dividend choice d_{it}^{JG}, let $\widehat{X}_{it} \geq 0$ denote the realized investment cost (equal to X_{it} if the firm invests, and 0 otherwise). Then the within–period payment to the lender is:

$$\text{pay}_{it}^{JG} \;=\; \min\Big\{\, a_{it}^{JG,+},\; \pi(z^{Low}, F_{it}) - d_{it}^{JG} - \widehat{X}_{it} \,\Big\} \tag{26}$$

Intuitively, if profits net of dividends and the investment cost are smaller than interest payments due, the firm accumulates more debt. Therefore next period usage of the credit line can be written as:

$$\widehat{b}_{i,t+1}^{JG} \;=\; \big[a_{it}^{JG,+} - \big(\pi(z^{Low}, F_{it}) - d_{it}^{JG} - \widehat{X}_{it}\big)\big]_{+}, \tag{27}$$

where $[x]_{+} \equiv \max\{x, 0\}$. Notice the difference between Equation (27) and Equation (12), which defines the law of motion of the firm's financial assets. Equation (27) only keeps track of the amount of the credit line drawn, so $\widehat{b}_{i,t+1}^{JG}$ goes to zero when all the debt is repaid. Conversely b_{t+1} in (12) becomes negative when the firm repays the debt and becomes a net saver.

We define the value of the credit line for the lender as the net present value of the interest payments, and assume lenders discount future payments using the risk free rate. We denote as $L_{i,t}^{Cont,SG}(b_{it}, F_{it})$ the value of the credit line for the lender when the borrower is stagnant with zombie status G and decided to continue operating:

$$L_{i,t}^{Cont,SG}(b_{it}, F_{it}) \;=\; \frac{1}{R}\Big(\text{pay}_{it}^{SG,NI} \;+\; L_{i,t+1}^{SG}\big(\widehat{b}_{i,t+1}^{SG,NI}\big)\Big). \tag{28}$$

The corresponding value $L_{i,t}^{Cont,HPG}(b_{it}, F_{it})$ for the high–potential type (J=HP) is more complicated because with probability p^{HPG} the firm invests in enhancing productivity and with probability ξ becomes a leader, thus repaying entirely all the outstanding debt, as

explained before:

$$L_{i,t}^{Cont,HPG}(b_{it},F_{it}) \;=\; \frac{1}{R}\Bigg\{p^{HPG}\,\mathbb{E}\Big[\mathrm{pay}_{it}^{HPG,I}(X_{it})+\xi\,\widehat{b}_{i,t+1}^{HPG,I}(X_{it}) \\ +(1-\xi)\,L_{i,t+1}^{HPG}\big(\widehat{b}_{i,t+1}^{HPG,I}(X_{it})\big)\;\Big|\;X_{it}\le X_{it}^{*}\Big] \\ +(1-p^{HPG})\Big(\mathrm{pay}_{it}^{HPG,NI}+L_{i,t+1}^{HPG}\big(\widehat{b}_{i,t+1}^{HPG,NI}\big)\Big)\Bigg\}. \tag{29}$$

Furthermore, we define as $\Lambda_t^{J,G}(b_{it})$ the value of the loan before knowing whether the firm will voluntarily continue. Conditional on choosing to default, the payment equals the liquidation value $rec^{J,exit}(b_{it})$ defined in (14):

$$\Lambda_t^{J,G}(b_{it}) \;\equiv\; \big(1-p^{cont,JG}\big)\,rec^{J,exit}(b_{it}) \;+\; p^{cont,JG}\,\mathbb{E}_F\Big[L_{i,t}^{Cont,JG}(b_{it},F_{it})\;\Big|\;F_{it}\le F_{it}^{*,JG}\Big]. \tag{30}$$

Note that $\Lambda_t^{J,G}(b_{it})$ is also the value of the loan used for the decision to zombify the credit line described before in (10). For firms in non-zombie status ($G=NZ$), we further define $\Psi_t^{J,NZ}(b_{it})$ as the expected value of $\Lambda_t^{J,G}(b_{it})$ over the zombification probability:

$$\Psi_t^{J,NZ}(b_{it}) \;\equiv\; \big(1-\zeta^{J}(b_{it})\big)\,\Lambda_t^{J,NZ}(b_{it}) \;+\; \zeta^{J}(b_{it})\,\Lambda_t^{J,Z}(b_{it}). \tag{31}$$

Then the initial values of the loan conditional on types HP and S and zombie status G are:

$$L_{i,t}^{HP,G}(b_{it}) = \big(1-d\,\mathbb{I}_{it}^{b}\big)\,\mathrm{Rec}^{HP,\mathrm{death}}(b_{it}) \;+\; d\,\mathbb{I}_{it}^{b}\,\Psi_t^{HP,G}(b_{it}), \tag{32}$$

$$L_{i,t}^{S,G}(b_{it}) = \big(1-d\,\mathbb{I}_{it}^{b}\big)\,\mathrm{Rec}^{S,\mathrm{death}}(b_{it}) \;+\; d\,\mathbb{I}_{it}^{b}\,\Psi_t^{S,G}(b_{it}), \tag{33}$$

Where the terms

$$\mathrm{Rec}^{HP,\mathrm{death}}(b_{it}) \;\equiv\; (1-p_s)\,rec^{HP,exit}(b_{it})+p_s\,rec^{S,exit}(b_{it}), \qquad \mathrm{Rec}^{S,\mathrm{death}}(b_{it})\equiv rec^{S,exit}(b_{it}) \tag{34}$$

take into account that recovery value depends on whether the firm is HP or S.

2.7.1. Entry

New firms pay an initial cost κ to enter. With probabilities b^{HP} and $1-b^{HP}$ they discover being high potential laggards or stagnant laggards, respectively. Firms are then offered credit lines by competitive lenders. The value of a new credit line to a firm of type J is $L_{i,t}^{J,NZ}(0)+k_L^J$,where k_L^J is the initial fee to open a credit line for a type J firm. Competition

among lenders bring down this cost to zero, so that in equilibrium $k_L^J = -L_{i,t}^{J,NZ}(0)$. Thus the free entry condition is:

$$b^{HP}(V_{i,t}^{HPNZ}\left(z^{Low},0\right) - \kappa_L^{HP}) + (1-b^{HP})(V_{i,t}^{SNZ}\left(z^{Low},0\right) - \kappa_L^{S}) - \kappa = 0 \tag{35}$$

Properties of the loan value

Note that the value of a loan $L^{JG}(b_{it})$ is exactly equal to its face value b_{it} when two conditions are met: first, the lender charges the risk free interest rate, that is, $\tau^{NZ} = \tau^{Z} = 0$. Second, the debt is always repaid in full either because the firm never defaults, or because the repayment is fully guaranteed by the liquidation value of the firm in case of default. Relative to this "neutral" case, the value of the loan increases with the excess interest rate, when $\tau^J > 0$, and decreases with the default probability since in equilibrium when the firm defaults its liquidation value is insufficient to repay the debt. When we calibrate the model, we set τ^{NZ} to match the average observed spread between lending and borrowing rates, and τ^Z to values consistent with the empirical zombies literature. Therefore, the initial type specific fees k_L^J are chose to ensure both types of contracts break even in equilibrim when the credit line is opened.

2.7.2. Aggregate Equilibrium

For given N firms in operation, total quantity produced is $Q = \sum_{i=1}^{N} q_i$. Substituting $q_{it} = \frac{1-p_{it}-\eta Q}{\gamma}$ and rearranging we obtain:

$$Q = \frac{NA - \sum_{i=1}^{N} c_i}{2\gamma + N\eta} \tag{36}$$

After simulating the model for many period, we obtain the steady state distribution of marginal costs. We use it to compute $\sum_{i=1}^{N} c_i$. Then equations 35 and 36 jointly determine the aggregate equilibrium values of Q and N.

3. Calibration

Table 1 illustrate the main parameters in the model, which are calibrated to match a set of relevant empirical moments. Among these parameters, we highlight here those that most closely affect zombie firms. i) The ratio between the markup at the 90th percentile and the median markup. This ratio determines the gains from investing in productivity. It motivates HP laggards to invest more and accumulate debt if they are not successful. In the model this moment is matched by selecting the appropriate difference between z^{low},

which is normalised to 1, and z^{high}. ii) The probability to transit from below median markup to the group of firms with top 10% markups. We match this in the model with the probability that investment in improving productivity succeeds. iii) Average EBIT/sales for the 25th percentile of the distribution. We match this moment with κ, the initial entry cost. This is important because it determines how profitable laggards are when in operation and, jointly with the volatility of profits, their steady state debt distribution. iv) The standard deviation of the EBIT/sales ratio within firms, matched with the dispersion of the per-period cost F. It is important to replicate a realistic volatility of profits because, as mentioned above, it affects the distribution of leverage across firms and is a key factor for the decisions of offering zombie loans. v) The share of defaults. This is jointly determined by all the previous parameters, but also in particular by the probability p_s that HP firms lose their capacity to invest in innovation and become S firms. These firms experience a sudden drop in value relative to their debt and have a high probability of defaulting. vi) The interest rate paid by zombie firms. Following the empirical evidence, we assume they pay 1% less than the AAA interest rate.

The discount factor for firms β^f is set equal to $\frac{1}{R+\epsilon^\beta}$ where ϵ^β is positive and arbitrarily small. In other words, we assume firms have a slight preference for distributing dividends when they are not borrowing from the credit line. This assumption is useful to match the leverage observed in the data, but is not otherwise essential for the results. The other parameters are calibrated to reasonable values, and we check that the main results illustrated next are not affected by varying them inside an empirically relevant interval of values.

4. Model's results

In this section we describe the simulation results of the model.

4.1. What type of firms are converted to zombies?

Table 2 illustrates one of the main findings of this model. The first column shows zombie shares for an economy where there is no risk for high-potential firms to lose the ability to invest and become stagnant ($p_s = 0$). In this economy, there are no zombies among the stagnant types, while high-potential zombies are around 1.5% of the total number of firms.

In order to understand this result, consider first Figure 1. The blue line shows the probability that S firms are turned to zombies as a function of their debt, in the simulation with $p_s = 0$. As debt increases, the value of the firm falls, and at some point it might

Table 1:
Calibration

Moment	Data	Model	Related Parameter	Value
Interest rate	4	4	Real interest rate, r	4%
Hazard rate in long run	4.6%	4.6%	Exogenous exit probability, $1-d$	4.6%
Bankruptcy rate	0.5%	0.52%	transition to S, p_s	2%
Average markup (weighted)	1.43	1.76	Substitutability between varieties, γ	5
Ratio top 90th markups over median	1.87	1.82	Productivity of leaders, z^{High}	3
Profit rate (EBIT to sales) 25th percentile	3.35%	3.03%	Entry cost, κ	2.074
Volatility of Profit rate, within firms	0.04	0.043	Dispersion of F, F^{mult}	1.5
Firms avg. age in years	15	19	Variable input cost, w	6
Average fixed cost over sales	35%	15.6%	Average fixed cost, $\overline{F}$	0.35
Median debt over sales	63%	58%	Intensity of competition, η	0.25
Share transition from below median markup to top 95% markup	0.34%	0.50%	Probability to succeed in enhancing productivity, ξ	2%
Average productivity enhancing investment (Net intangibles investment over sales)	8%	18%	Average investment cost, $\overline{X}$	0.45
Premium over AAA borrowing rate, 75th percentile	2.14%	2.14%	Borrowing premium for laggards, τ	2.14%
Premium over AAA borrowing rate, zombie firms	≤ 0	-0.1%	Borrowing premium for laggard zombies, τ^Z	-1%

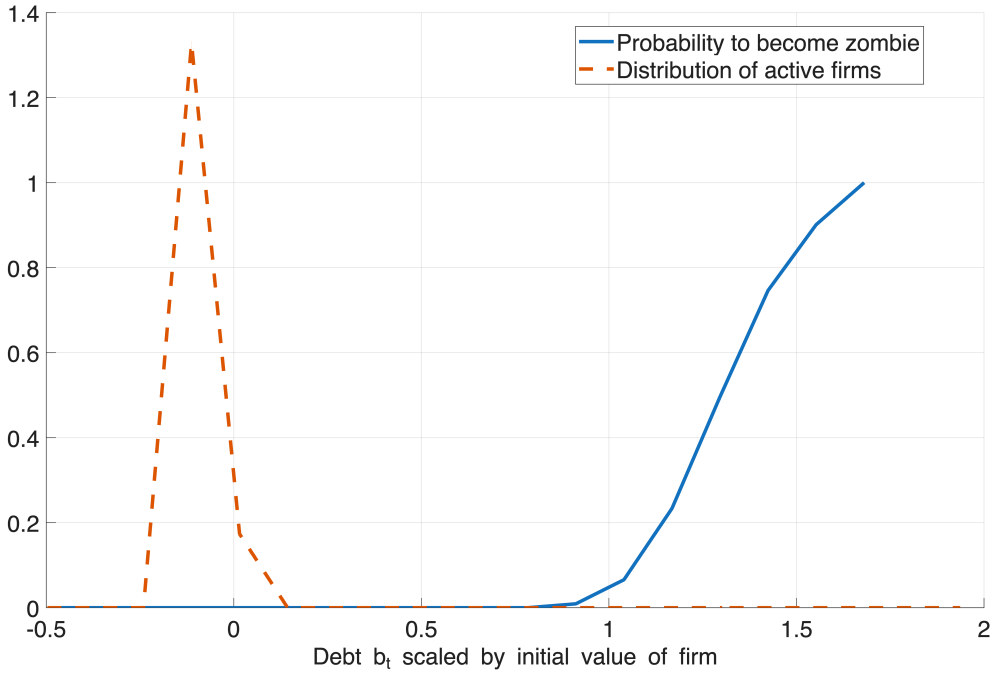
choose to default for high realizations of the per-period cost F_{it}.[6] The blue line turns positive for high levels of debt implying that lenders would find it optimal to turn S firms into zombies if they were close to default. The intuition is that the value of the credit line for the lender is the net of two components. First, the net present value of interest payments, which is positive. Second, the expected loss in case of default, which is negative. Turning a firm to zombie reduces both the positive component, because the firm pays a lower interest rate, and the negative component, because it is less likely that it will choose to default. When the credit line is created and the debt is low, the second component is zero. The firm does not voluntary default, and in any case its liquidation value is higher than the outstanding debt. Therefore the lender will not turn this firm to zombie. When the debt increases, the firm is gradually more likely to default and the second component grows and eventually dominates on the first component. In this case the value of the loan for the lender has turned negative, but it can be made smaller in absolute value by

[6]The blue line is censored to the right because eventually for higher values of debt the firm defaults with 100% probability

Table 2:
Share of zombie firms over total number of firms.

	Model with no transition shock from HP to S	**Benchmark model with 2% transition prob. from HP to S**
Stagnant zombies	0%	0.75%
High potential zombies	1.51%	0.95%

Figure 1:
Probability that stagnant firms become zombies (simulation with no transition from HP to S, $p_s = 0$)



reducing interest payments, to increase the probability the firm will manage to repay the debt and avoid default.

However in Figure 1 the red dashed line represents the distribution of debt holdings among S firms, and shows that these firms never accumulate large levels of debt, and therefore never enter into the high-debt region in which lenders might make them zombies. Stagnant firms are small and generate very little earnings, but they are on average profitable, otherwise they would not stay in operation. They do borrow in equilibrium to smooth losses, but for the realistically calibrated volatility of earnings, they never accumulate so much debt that they enter in the part of the debt distribution in which lenders might offer

zombie lending.[7] They instead have predominantly negative levels of debt, which are very concentrated because savings above a certain level are optimally distributed as dividends.

Note that, on the one hand, this is a quantitative result. If we simulate an economy in which S firms are even less profitable, and their profits are even more volatile, it would be possible that their debt distribution would overlap with values in which firms become zombies. However, on the other hand this would necessarily imply levels of profits volatility much larger that those observed in the data.

Figure 2 is analogous to 1 but shows high-potential firms instead. Because these firms invest to increase productivity, on average their debt is higher than stagnant firms. Moreover, the debt distribution has a thick right tail. When these firms fail repeatedly in improving productivity, their debt becomes very large. Therefore, some of them enter the region in which the blue line is above zero, and become zombies. Interestingly, the probability to become zombies increases above zero much before than the firm reaches debt levels close to default. Intuitively, the lender realises that even though these firms have high debt, they always have the chance to fully repay in the future if they become leaders. By turning them to zombies early relative to the default region, they slow down debt accumulation and "buy these firms more time" to succeed in their investments.

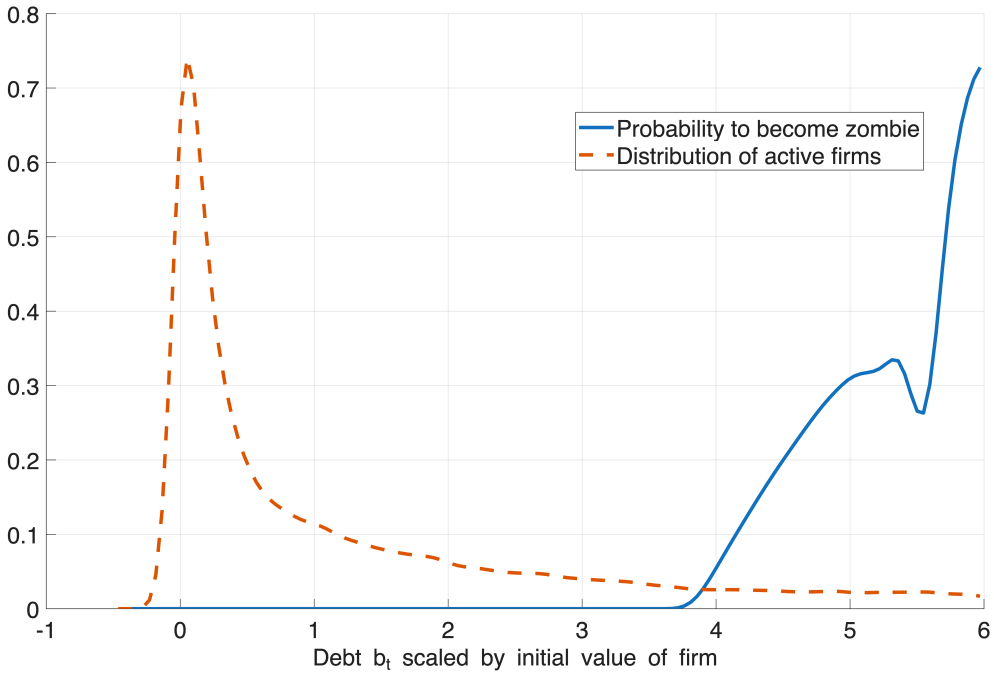
The implication of Figures 1 and 2 is that in the absence of long term shocks to the investment capacity of the firms (that is, when the probability p_s of transitioning from HP to S status is set to zero), stagnant firms are not likely to become zombie firms, while high-potential firms are.

This conclusion is confirmed by column 2 in Table 2. In the benchmark simulation, with a positive probability of transitioning from HP to S status ($p_s = 2\%$), there are both HP and S zombies in equilibrium. For HP zombies the intuition remains the same, with the only difference that there are fewer of them because there are fewer HP firms in equilibrium, given the risk of becoming S type. Conversely, S zombies are caused by the pool of previously HP firms that suffer the transition shock and become S firms. 42% of the firms suffering this shock have too high debt given the reduced value of being S type, and default immediately. A smaller fraction (5%) have instead levels of debt that are high but lower than the exit threshold, and are turned into stagnant zombies. Note that this is a very small fraction of the total number of firms ($2\% * 5\% = 0.1\%$), but making these firms zombies implies increasing their lifespan, so that in equilibrium they account for 0.75% of the total number of firms, while high-potential zombie account for 0.95%.

[7] This does not mean that they never default, as default could happen conditional on a large realization of the fixed cost $F_{i,t}$ even if the firm is not in the zombie region.

Figure 2:

Probability that high potential firms become zombies (simulation with no transition from HP to S, $p_s = 0$)



4.2. Equilibrium implications and empirical identification of zombie types

The previous section highlighted two main results. First, stagnant zombies arise in equilibrium only in the presence of shocks that reduce the long term growth prospects of firms. That is, stagnant zombies are firms that previously borrowed to invest into positive NPV projects, and discover that these projects are no longer feasible. It is optimal for lenders to offer zombie lending to these firms because it helps them reduce gradually their debt and avoid default.

Second, the same long term growth prospects that are a necessary condition for the existence of stagnant zombies, also imply the existence of high-potential zombies. These are firms that still have growth options that have not been realised yet, and find optimal to borrow to pursue them. These firms are turned to zombie because it is optimal for the lender to "buy them more time", in the hope they will become more profitable in the future and will thus be able to repay the debt.

Interestingly, on the one hand both type of zombies, stagnant and high-potential, are

consistent with the way such zombie firms are identified in the empirical literature: they have high debt relative to the median values in the industry, low profits relative to the interest payments on such debt, are not very young (because in the model it takes time to accumulate debt), and have not been growing in recent years (high-potential zombies have a potential to grow that has not been realised yet). On the other hand, they have starkly different effects for the misallocation implications of zombie lending, as it will be shown in the next section.

4.2.1. Testable predictions

Figure 3:
Markups dispersion and zombie shares

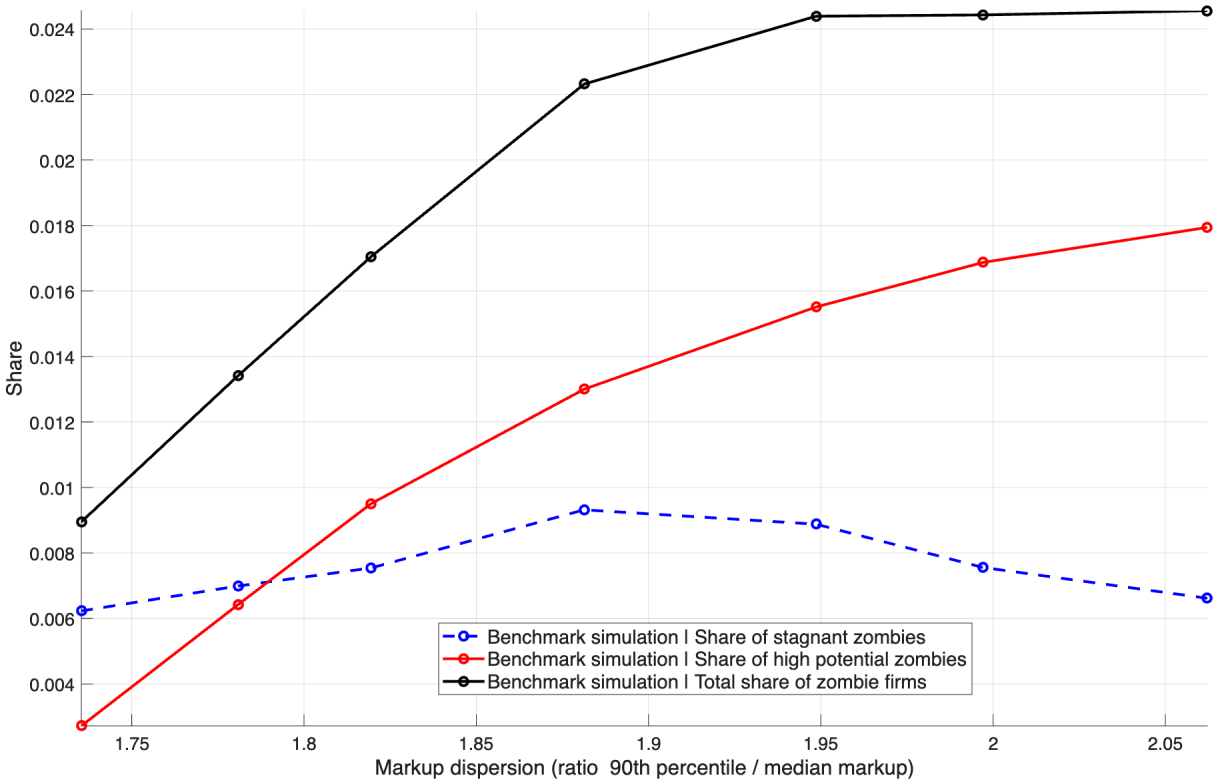


Figure 3 shows the results from simulating several industries that have the same benchmark parameters, except for the leaders' productivity level z^{High}.[8] On the horizontal axis, we report the markup dispersion for each simulation. This exercise is useful for two

[8] All these different industry equilibria satisfy the same free entry condition. In other words, changing z^{High} effectively changes the dispersion in productivity and markups, while equilibrium aggregate prices and quantities adjust so that the expected value at entry is the same across all the simulations.

reasons. First, we can verify how changing technology and market equilibrium affect the share of zombie firms. Second, we can map these different simulations into industries with different markups dispersion, and therefore obtain empirically testable predictions.

The figure shows that the overall share of zombie firms increases in markup dispersion, and that such an increase is entirely driven by the high-potential zombies. The intuition is that higher markup dispersion increases competitive pressures for laggard firms, reducing their profits, but also increases the gain from investing in productivity to become leaders. Thus on the one hand high potential laggards invest more heavily in improving productivity, and because their profits are lower (due to stronger competition) accumulate debt faster and are more likely to reach the region in which they become zombies. On the other hand, lenders have also more incentives to turn these firms to zombies. Because they invest more in improving productivity, high-potential firms are more likely to become leaders and being able to repay the debt. Conversely the share of stagnant zombies is more or less constant over the different simulations.

One implication of Figure 3 is that as markups dispersion increases, zombies are relatively more likely to be high-potential firms, and therefore more likely to become more productive in the future. This is shown in Figure 4, in which the probability that zombie firms become leaders, and therefore among the top firms in terms of productivity and markups, increases in markup dispersion.

It follows that zombie firms are more "dynamic" in sectors with higher top markups, leading to the two following testable predictions of the model:

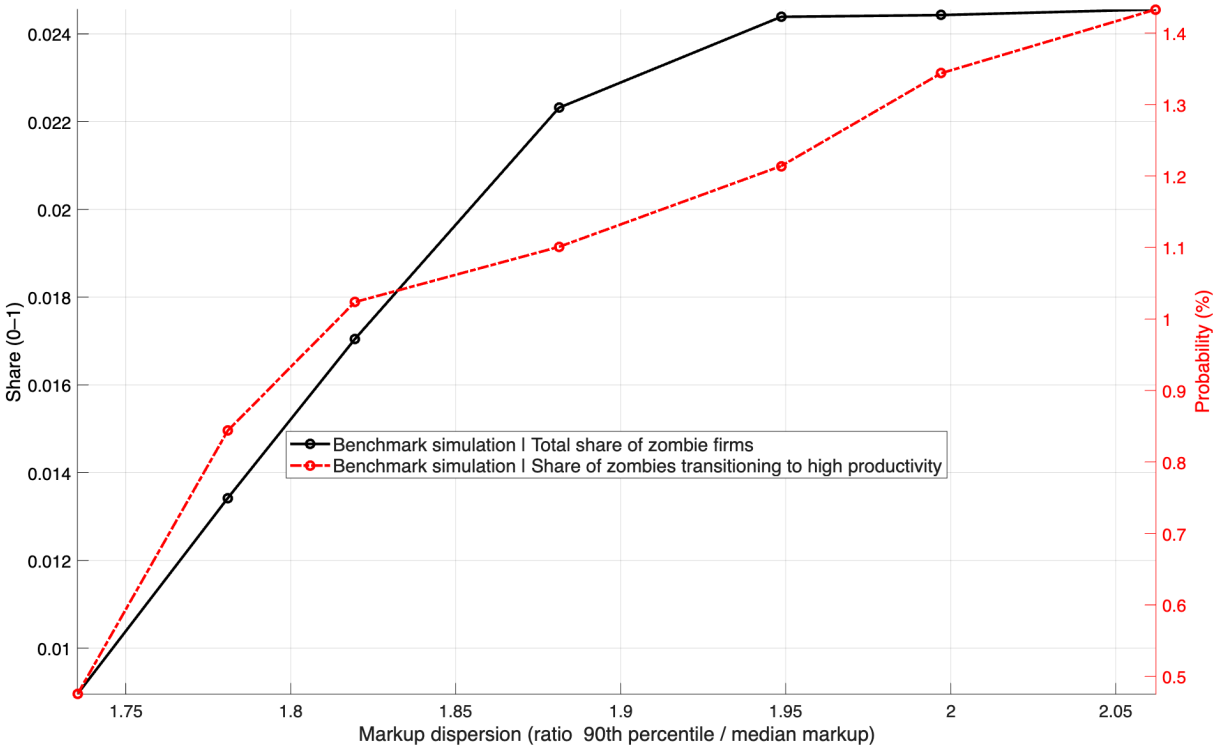
Prediction 1: *An increase in markup dispersion increases the share of firms receiving zombie loans.*

Prediction 2: *An increase in markup dispersion increases the likelihood that firms receiving zombie loans transition in the future to the top of the markup distribution.*

Jointly, these predictions are able to verify whether zombie firms are heterogenous in nature in the data, and whether high-potential zombies become more predominant the more markups are dispersed. Figure 5 shows that these predictions are robust to different parameter values. It compares the benchmark calibration with $p_s = 2\%$, with alternative calibrations with higher value of this probability ($p_s = 3\%$ and $p_s = 4\%$). The higher is p_s, the more high-potential firms face the risk of transitioning to stagnant, the higher is the relative share of stagnant zombies. However regardless of the value of p_s, we still observe an increase in zombie share as markup dispersion increases (top graph) and an increase in the share of zombies investing in productivity improvements (bottom graph). Other changes in the parameters, such as for example an increase in firm-level volatility of profits, also leaves these two predictions unaffected.

Finally, note that these predictions are derived by comparing equilibria across simula-

Figure 4:
Markups dispersion, zombie shares, and transitions to the top



tions and therefore their direct empirical counterpart is cross-industry evidence. Nonetheless, in our empirical analysis below we confirm that all the results hold if we include industry-year fixed effects, thus controlling for all industry invariant characteristics and focusing on within industry variation.

4.2.2. Misallocation implications

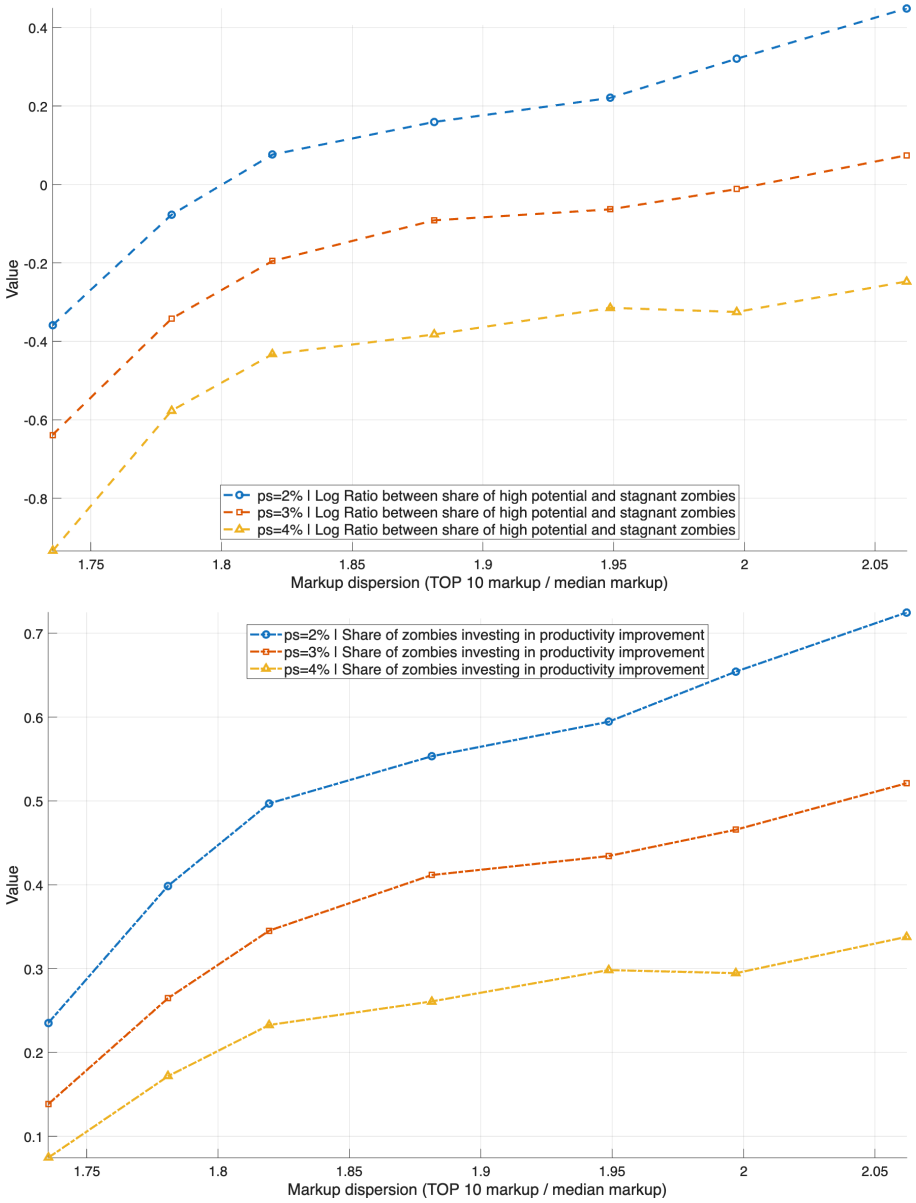
We can now discuss the key question analysed by the literature on zombie firms. How does zombie lending affect the optimal allocation of resources, aggregate productivity, and welfare? In the model, there are two main factors that can potentially affect aggregate outcomes:

i) Financial frictions: firms can only borrow using credit lines with a fixed interest rate. This means that in equilibrium some firms, even though they are on average profitable, are forced to exit because of a "debt hangover" problem, when they accumulate debt to cover losses after a sequence of negative shocks.

ii) Externalities from investing in productivity: Firms (and lenders) do not internalise that firms' investment in productivity has the social benefit of increasing the number of

Figure 5:

Markups dispersion, log ratio between zombie types (top), and share of zombies investing in productivity (bottom)



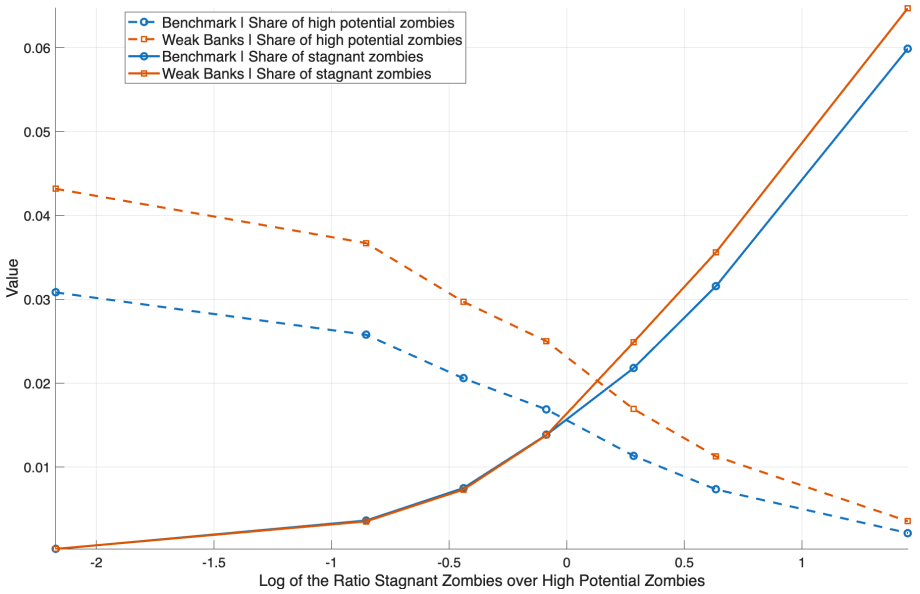
leaders, and thus increasing competition and consumers welfare.

Zombie lending affects both externalities. The option of zombie lending (a reduction in the interest rate on the loans for heavily indebted borrowers) essentially allows some risk sharing between firms and lenders, thus mitigating financial frictions. Moreover, by affecting the relative share of high potential and stagnant zombies, it also affects the magnitude of the second externality.

In addition, in our model zombie lending is an optimal choice of risk-neutral, profit-maximising lenders. However, in the literature it is often highlighted that poorly capitalised banks have special incentives to evergreen loans because recognising losses is especially costly for them. Therefore we include this possibility in a modified version of the model with "weak banks". In this case, when borrowers default, lenders suffer an additional loss, proportional to the amount of defaulting debt. In other words, the cost of default has an higher weight over the interest payments in determining the values of the credit lines, and lenders are more likely to provide zombie lending.

Our objective is to show how the mix of zombie types prevailing in an industry (high-potential vs Stagnant) affects the mapping between these different channels and aggregate outcomes. To this end, we simulate several economies that vary only in the probability p_s to transition from HP to S type.[9] As explained in Figure 5, the ratio of S zombies over HP zombies monotonously increases in p_s. Furthermore, we compare simulation of the benchmark model, and simulations with "weak banks" in which lenders have additional incentives to create zombie firms, because they suffer additional losses if firms default.

Figure 6:
Shares of high potential and Stagnant Zombies.



In Figure 6, these different simulations are ordered according to the log ratio of stagnant zombies over high-potential zombies. The first simulation on the left corresponds to a value of p_s close to zero ($p_s = 0.001\%$), in which almost all zombies are High-Potential.

[9]Since as p_s increases the equilibrium share of HP laggards decreases, we also change the fraction of firms born as HP laggards so that such share is more or less constant across simulations.

Then p_s gradually increases until a maximum value of 10%, in which zombies are almost exclusively stagnant firms. The log scale implies that a value of zero on the horizontal axis corresponds to equal shares between the two types. The figure shows that the presence of weak banks amplifies the share of the predominant zombie type (HP zombies when p_s is low, and Stagnant zombies when p_s is high). It also shows that this amplification effect is stronger for high-potential zombies. As explained before, high potential firms borrow more on average than stagnant firms, and therefore weak banks suffer on average larger damages if they default than if stagnant firms default.

Figure 7 shows aggregate TFP (the weighted average of firm level TFP) relative to a simulation that is identical in everything except that credit line contracts do not include the option to provide zombie lending. Therefore, a value above 1 indicates that such lending improves the allocation of resources, while a value below 1 indicates that it worsens it. The figure shows that the prevailing type of zombie firm is essential in determining its allocative implications. When high potential zombies dominate, in the left part of the figure, zombie lending improves aggregate productivity. Intuitively, the presence of this option helps high-potential firms to stay in operation and increases the share of firms investing to improve productivity. Having more leaders (firms with high productivity) in equilibrium not only increases aggregate TFP, but also it increases competition, increasing quantities and reducing prices, and ultimately improves consumers welfare.

Conversely, when stagnant zombies dominate the opposite effect prevails. an higher share of stagnant firms in the industry reduces transitions to leader status, mitigating aggregate productivity and welfare. The effects are quantitively significant, they range from aggregate TFP increasing by 0.9% when high-potential zombies prevail, to decreasing by 2.5% when stagnant zombies prevail. The calibrated benchmark value of $p_s = 2\%$ (corresponding to -0.44 on the horizontal axis) is a middle case, in which zombie lending has almost no misallocation effects.

Figure 8 proposes the same exercise and adds one additional line for the simulations with weakly capitalised lenders. Not surprisingly, since we already saw in Figure 6 that the presence of these lenders amplifies zombie lending to the prevailing firm type, productivity gains are even higher when the share of stagnant zombies is low, and lower when this is high.

4.3. Discussion

The existing literature on zombie firms argues that evergreening incentives of weak banks create zombie firms, misallocating resources. We explore this issue in a firm dynamics model with imperfect competitions, growth options, and profit maximising firms and lenders.

Figure 7:
Aggregate TFP gain relative to a no-zombies economy.

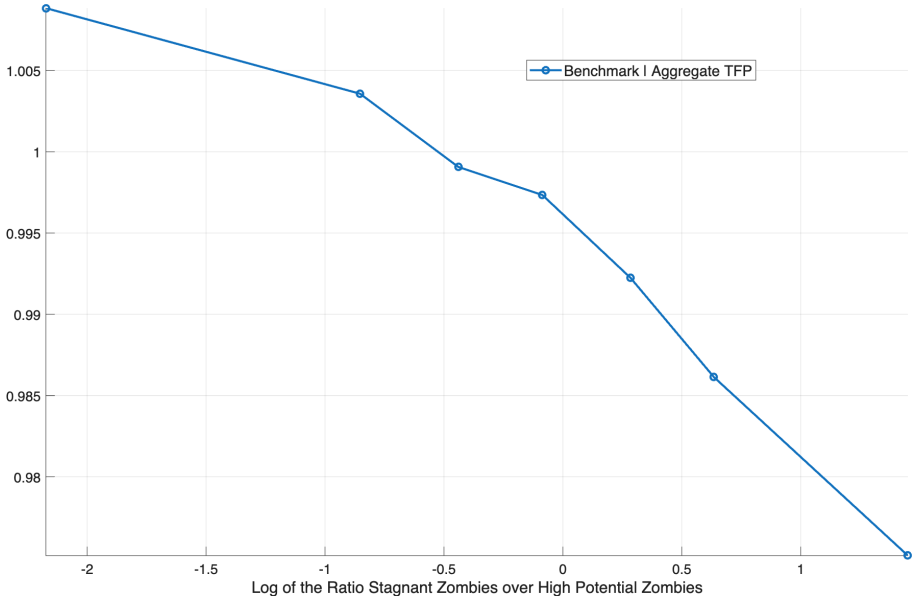
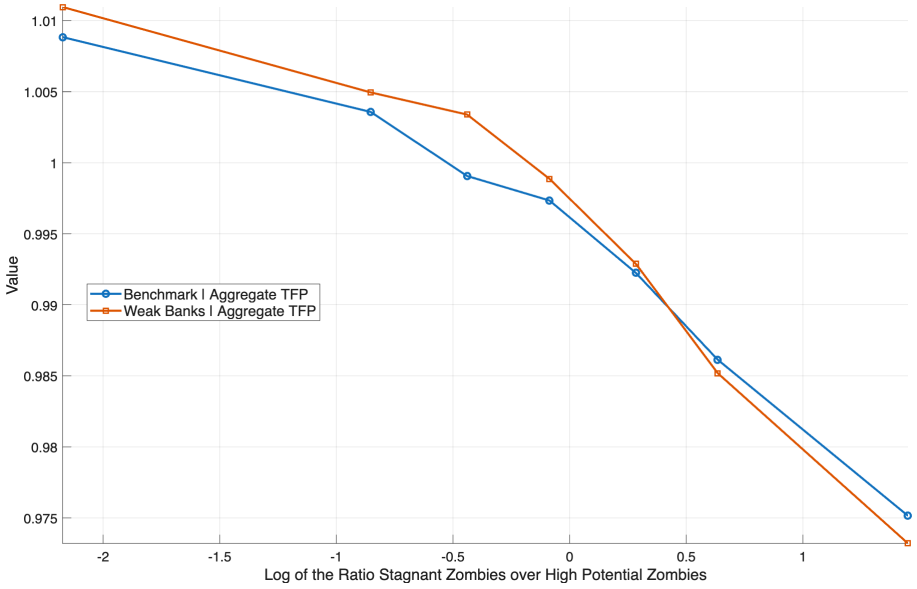


Figure 8:
Aggregate TFP gain relative to a no-zombies economy. The effect of weakly capitalised lenders.



Our model helps to shed new light on a key question to understand the misallocation implications of zombie lending: what type of firms are converted to zombies?

On the one hand, we have shown that our model is consistent with the presence of

"stagnant zombies" that are fully consistent with the previous literature: they are firms that previously borrowed to invest into positive NPV projects, and discover that these projects are no longer feasible. It is optimal for profit maximising banks to offer zombie lending to these firms because it helps them reduce gradually their debt and avoid default. However, because of a negative aggregate externality, providing zombie lending to these firms reduces aggregate TFP and welfare. In addition, the presence of weak banks further increases zombie lending and exacerbates this problem.

On the other hand, our model provides a novel striking results. It shows that the same long term growth prospects that are a necessary condition for the existence of stagnant zombies, also imply the existence of high-potential zombies. These are firms that still have growth options that have not been realised yet, and find optimal to borrow to pursue them. These firms are turned to zombie because it is optimal for the lender to "buy them more time", in the hope they will become more profitable in the future and will thus be able to repay the debt. We also show that both type of zombies, stagnant and high-potential, are consistent with the ways such zombie firms are identified in the empirical literature: they have high debt relative to the median values in the industry, low profits relative to the interest payments on such debt, are not very young (because in the model it takes time to accumulate debt), and have not been growing in recent years (high-potential zombies have a potential to grow that has not been realised yet).

Importantly, the equilibrium share of stagnant over high potential zombies is key for the misallocation implications of zombie lending. For realistic parameter values, we simulate industries in which most zombies are high-potential, and show that zombie lending increases aggregate TFP, and the presence of weak banks amplifies further this positive aggregate effect. In other words, what matters most for the misallocation implications of zombie lending is the types of zombie firms rather than whether or not lenders have additional incentives to evergreen loans.

It is therefore important to verify empirically the presence of high potential zombies. Our model implies clear testable predictions to do so, which will be verified in the empirical section of this paper.

5. Data & Variable Measurements

To test our prediction, we use Orbis data for Italy provided by Bureau van Dick. Orbis contains harmonized information on a representative sample for publicly listed and private Italian firms. Our data covers the years from 2007 to 2016. To clean the raw data, we closely follow Gopinath et al. (2017); Kalemli-Özcan et al. (2015).

The sample construction involves two main steps: First, we obtain sectoral-level data

from Eurostat, including price indices, gross value added, and other industry aggregates. Specifically, we collect data on consumer price indices, and gross value added by industry. These series are processed to construct sector-specific price deflators and value-added measures. We focus particularly on gross value-added series for 10 industry classifications and compile price deflators at the sector level. For manufacturing sectors, we use 2-digit NACE industry deflators when available, while for other sectors we rely on 1-digit industry deflators or aggregate deflators when more granular measures are not available.

Second, we process the Orbis firm-level data to address issues of consolidated accounts, since some firms report both consolidated and unconsolidated statements. Following Gopinath et al. (2017), we retain consolidated accounts when both types are available and, in cases of multiple entries due to changes in accounting years, keep the most recent observation. We then proceed with basic cleaning steps by dropping observations with missing or negative values for key variables including sales, total assets, employment, and operating revenue (Kalemli-Özcan et al., 2015; Díez et al., 2021). Financial sector firms (NACE codes 64-66) are excluded from the analysis.

The data is further refined by implementing a series of internal consistency checks. We verify the coherence of financial ratios, including the shares of fixed assets to total assets, current assets to total assets, and various liability measures. Following De Loecker et al. (2020), we drop firm-year observations with missing information regarding their industry of activity. We also ensure the consistency of debt reporting by comparing different liability measures and removing firms showing unrealistic growth rates in sales or employment (e.g. see Kalemli-Özcan et al., 2015). Finally, we combine the cleaned firm-level variables with the sector-level price deflators. All monetary variables are deflated using the appropriate industry deflators and expressed in 2005 prices. Our final sample provides a comprehensive view of the Italian non-financial business sector.

5.1. Markup Estimation

Following De Loecker et al. (2020), we estimate firm-level markups using the production approach. For any variable input V, the markup is computed as:

$$\mu_{it} = \theta_{it}^{V} \frac{P_{it}Q_{it}}{P_{it}^{V}V_{it}}$$

where θ_{it}^{V} is the output elasticity of variable input V, $P_{it}Q_{it}$ is sales, and $P_{it}^{V}V_{it}$ is total expenditure on the variable input. We estimate output elasticities from a production function:

$$y_{it} = f(v_{it}, k_{it}; \beta) + \omega_{it} + \epsilon_{it}$$

where y_{it} is log output, v_{it} is log variable input, k_{it} is log capital, ω_{it} is productivity, and ϵ_{it} is measurement error. We use the cost of goods sold as our variable input and estimate the production function at the 2-digit industry level using the control function approach of Ackerberg et al. (2015). This allows us to estimate the elasticity of variable inputs in each industry from which we determine the firm-level markups.

5.2. Zombie Identification

We follow Acharya et al. (2024), and combine information on subsidized credit access with measures of firm performance. A firm is classified as a zombie in year t if it simultaneously satisfies a subsidized credit condition and exhibits poor performance. The subsidized credit condition requires that a firm's borrowing cost falls below a benchmark rate: $r_{it} < \tilde{r}_t$, where r_{it} represents firm i's cost of debt in year t, computed as the ratio of interest expenses to total debt averaged over the two years. The benchmark rate $\tilde{r}_t$ represents the minimum interest rate that a top-rated firm would pay in the market, defined as: $\tilde{r}_t = (r_{it}^{p50}|\text{AAA firms})$. The term $\tilde{r}_t$ represents the median credit rate paid by AAA-rated firms in year t.[10]

The poor performance criterion requires that a firm exhibit the following conditions: first, the firm's Interest Coverage Ratio (ICR) falls below the industry median. To avoid the influence of the outliers we consider the ICR over the two years for both the firm level and the industry median. Where the ICR at each point in time is calculated:

$$\text{ICR}_{it} = \frac{\text{EBIT}_{it}}{\text{Interest Expenses}_{it}}$$

EBIT_{it} represents earnings before interest and taxes. This threshold indicates that the firm's operating profits are insufficient to cover its interest expenses. Second, the firm's leverage (debt to asset) is above the sector median. The sector medians are calculated at the two-digit NACE sector level for each year. As argued in Acharya et al. (2024), this identification strategy captures firms that not only receive subsidized credit but also show signs of fundamental economic weakness.

Table 3 presents summary statistics for our key variables. Panel A reports firm-level statistics, while Panel B presents industry-level measures.

The average ROA of 2.4 percent is comparable to other studies of Italian firms (Schivardi et al., 2020; Gopinath et al., 2017). Firms in our sample have an average age of 15 years

[10]While Orbis does not directly provide firm credit ratings, we estimate each firm's rating category based on its Interest Coverage Ratio (ICR) following the methodology established in Acharya et al. (2019) and Acharya et al. (2021). Specifically, we map firms' ICR values to rating categories using the standard mapping between ICR ranges and credit ratings documented in Damodaran's credit risk database.

Table 3:
Summary statistics

Panel A				
Firm level	Mean	SD	p25	p75
$Zombie_{i,t}$	0.023	0.149	0.000	0.000
$Inv.\ Rate_{i,t}$	0.053	0.099	0.003	0.058
$ROA_{i,t}$	0.023	0.065	0.000	0.042
$\mu_{i,t}$	1.426	1.051	0.836	1.549
$\log Assets_{i,t}$	−0.094	1.597	−1.198	0.884
$\log Age_{i,t}$	2.403	0.931	1.792	3.135
$\log Mkt.\ Share_{i,t}$	−10.444	1.844	−11.679	−9.292
Observations	2,996,486			

Panel B				
Sector Level	Mean	SD	p25	p75
$sd(\mu)_{s,t}$	0.887	0.567	0.381	1.301
$\mu_{s,t}^{p^{75}}$	1.974	1.123	1.198	2.545
$\log TFP(Salesweight)_{s,t}$	0.395	0.373	0.115	0.692
$\log sd(\mu)_{s,t}$	−0.362	0.744	−0.966	0.263
$\log Zombies_{s,t}$	−4.372	1.305	−5.187	−3.521
$CPI_{s,t}$	96.994	4.606	94.804	99.615
$ValueAdded_{s,t}$	22.062	380.192	0.257	2.728
Observations	829			

Notes: Data source: Orbis database for Italian firms, 2007-2016. Panel A contains summary statistics for the firm-year panel. Panel B contains summary statistics for the industry-year panel. Financial variables are winsorized at the 1st and 99th percentiles. Profitability measures are scaled by total assets. The sample is restricted to industries with at least 20 firm-year observations and industries that appear for all 10 years in our sample. Panel B reports summary statistics for revenue-weighted standard deviation of markups, top quartile markup level, and log TFP.

with substantial heterogeneity (standard deviation of 13.3 years). Our estimated markups average 1.43, aligning with recent European evidence (Díez et al., 2021). The markup distribution exhibits considerable dispersion, with an interquartile range of 0.71 (from 0.84 to 1.55) and a 95th percentile of 3.53.

The industry sales-weighted standard deviation of markups averages 0.88, consistent with De Loecker et al. (2020)'s findings for U.S. firms. The asset-weighted share of zombie firms within industries averages 1.3 percent, with substantial variation across industries (interquartile range of 2.83 percentage points, from 0.27 to 3.1 percent). At the aggregate level, zombie firms represent 2.5 percent of total firms over our sample period, in line with Acharya et al. (2022).

In terms of sample coverage, firms appear in our sample for an average of 4.6 years,

with the median firm observed for 4 years (interquartile range of 2 to 7 years). This panel participation rate is comparable to other studies using Orbis data (Kalemli-Özcan et al., 2015; Gopinath et al., 2017) and helps mitigate concerns about selection bias from firm survival. Overall, we track 587,098 distinct firms over the 2007-2016 period.

6. Empirical Evidence: Model Validation

In this section, we provide new empirical evidence confirming our model's predictions. We first document a series of novel stylized facts about the relationship between zombie firms and markups. Consistent with our model, we find a strong correlation between markup dispersion and the share of zombie firms. We also show that zombies in highly dispersed markup sectors are more upward mobile, falling in the top categories for market shares, markups, investment, and employment. Finally, we propose a model-driven approach to identify the different types of zombie firms tha have opposite misallocation implications.

6.1. Industry-level Markups Dispersion and Zombie Shares

In this section, we examine the markup-zombie relationship at the sector level. This disaggregated analysis offers two key advantages over the aggregate analysis: first, it enables us to control for sector-specific factors that might simultaneously affect markups and zombie prevalence; second, it allows us to identify the relationship between markups and zombie share within sectors. Exploiting within-sector variation helps mitigate concerns that our results are driven by structural transformation, economy-wide trends, or persistent cross-sector differences.

The aim of this section is to provide a direct test of Prediction 1. According to the model, we can in principle both use the variation in markup dispersion and in the level of top firms markups. However, in practice, a measure of dispersion is more reliable because is likely less affected by outliers, and it does not require taking a stand on what is the right percentile of the distribution to define top firms. Therefore, our main explanatory variable is the log of sectoral sales-weighted markup dispersion. Nevertheless, we provide robustness checks using the top quartile of industry-level markups ($\mu_{s,t}^{75th}$).

Figure 9 presents binscatter plots of log zombie shares against two measures of market structure: log industry sales-weighted markup dispersion ($log(sd(\mu)_{s,t})$) in panels (a) and (b). Panels (a) include year-fixed effects to absorb common time-varying factors, thus exploiting cross-sectional differences across industries. The Panel (b) adds sector fixed effects, thereby focusing on within-sector variation over time.

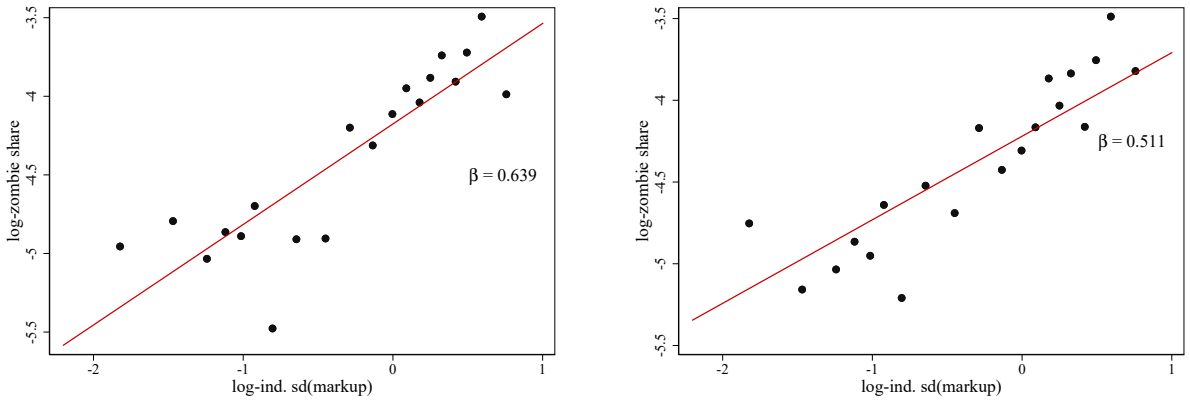
Both specifications reveal a strong positive relationship between markup dispersion and

zombie shares: sectors with higher markup dispersion exhibit higher zombie shares. The stability of the estimated relationship across specifications—with slopes of 0.639 in the cross-section and 0.511 within sectors—suggests this pattern is not driven by unobserved sector-specific characteristics. To ensure our results are not sensitive to the specific measure of market structure, we replicate our analysis using the markup level of industry leaders (defined as the 75th percentile of the sector-year markup distribution) rather than markup dispersion. Figure A1 provides similar results. Finally, Figure A2 shows that the sectoral relationship we document a strong time-series correlation between economy-wide average markups and the aggregate share of zombie firms.

The positive relationship between market power and zombie shares persists: sectors where top firms charge higher markups exhibit systematically higher zombie shares, with magnitudes comparable to our baseline estimates. This robustness to alternative measures reinforces our interpretation that the option value mechanism operates through the rewards available at the top of the market structure.

Figure 9:
Sector Markups dispersion and Zombie Shares

(a) *Cross-Section of Sectors* **(b)** *Within Sector*



Notes: Data source: Orbis database for Italian firms, 2007-2016. Unit of analysis: sector-year. The figure presents binscatter plots relating log zombie shares to log markup dispersion ($log(sd(\mu_{s,t}))$). Left panels estimate: $log(zombie\ share_{s,t}) = \alpha_t + \beta_1 log(sd(\mu)_{s,t}) + \epsilon_{s,t}$, where α_t represents year fixed effects. Right panels add sector fixed effects: $log(zombie\ share_{s,t}) = \alpha_t + \alpha_s + \beta_1 log(sd(\mu)_{s,t}) + \epsilon_{s,t}$. Each dot represents the average value within each bin and estimated β coefficients are reported at the top of each panel.

We formally test this relationship by estimating the following specification at the sector level:

$$log(zombie\ share_{s,t}) = \alpha_t + \alpha_s + \beta_1 log(sd(\mu)_{s,t}) + \Gamma X_{s,t} + \epsilon_{s,t}, \tag{37}$$

where α_t and α_s are time and sector fixed effects, β_1 captures the elasticity of zombie shares with respect to sectoral sales-weighted markup dispersion ($sd(\mu)_{s,t}$), and $X_{s,t}$ is a

vector of sector-time controls including real value-added and sector-specific price indices. Standard errors are clustered at the sector level.

Table 4 presents our main results. Columns (1) and (3) correspond to the binscatter plots in Figure 9, while Columns (2) and (4) add sector-level controls. The patterns strongly support our model's prediction: sectors with higher markup levels—which in our framework corresponds to higher incentives to stay in the market—have higher shares of zombie firms. In the cross-sectional analysis, a 1% higher level in the of industry markup dispersion correlates with a 0.60-0.63% higher share of zombie firms. While less precisely estimated, this relationship persists when exploiting within-sector variation, where a 1% difference in markups relates to a 0.50-0.51% higher difference in zombie shares. The stability of coefficients to the inclusion of controls suggests these correlations are not driven by observable sector characteristics. These results confirm **Prediction 1**, where higher markup dispersion is associated with higher shares of firms receiving zombie loans.

To put these magnitudes in perspective, between 2007 and 2016, the median sector-level markup dispersion ($sd(\mu)_{s,t}$) increased by 2.3% ($\log(0.753_{2016})$ - $\log(0.735_{2007})$), related to a 1.4% (2.3×0.605) higher zombie share based on our estimates. Alternatively, a standard deviation increase in markup dispersion (0.56) relates to a 36% increase in zombie share. The within-sector estimates provide similar magnitudes and signs (1.1%) aligning closely with our model's predictions.

Table 4:
Sector level Zombie Share and Markup Dispersion

Main Dependent: log(zombie share$_{s,t}$)				
	Cross-Section		*Within*	
	(1)	(2)	(3)	(4)
$log(sd(\mu)_{s,t})$	0.639***	0.605***	0.511*	0.504*
	(0.125)	(0.131)	(0.294)	(0.296)
Controls		✓		✓
Year F.E.	✓	✓	✓	✓
Industry F.E.			✓	✓
Clustered Std.Errors	Sector	Sector	Sector	Sector
R^2	0.19	0.20	0.56	0.57
Obs	732	732	732	732

Notes: Data source: Orbis database for Italian firms, 2007-2016. Unit of analysis: sector-year. The dependent variable is the log of zombie shares. The main explanatory variable is the log of sector-level markup dispersion ($log(sd(\mu_{s,t}))$), where markups are weighted by firm sales in an industry. All specifications include year-fixed effects. Columns (1) and (2) report cross-sectional estimates, while columns (3) and (4) add sector-fixed effects. Controls include sector-level real value-added and price indices. Standard errors are clustered at the sector level and reported in parentheses. ***, **, * denote significance at 1%, 5%, and 10% levels, respectively.

Overall, Table 4 and Figure 9 suggest a strong relation between sector-level markups and

zombie share. Table A1, in the appendix, provides further evidence using our alternative measure, the sales-weighted markup level of industry leaders ($\mu_{s,t}^{75th}$). The results are qualitatively the same.

To conclude, this evidence confirms the results presented in Section 4. In the next section, we test **Prediction 2** of our model on the data.

6.1.1. Dynamic Response of Zombie Shares to Markup Dispersion

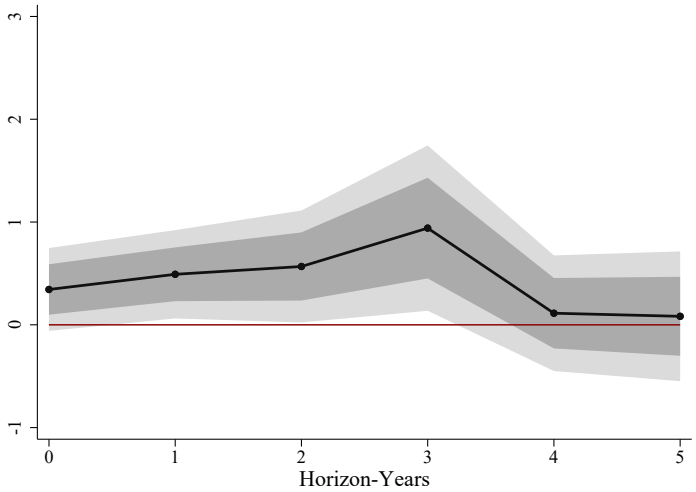
To further examine the relationship between markup dispersion and zombie prevalence, we investigate the evolution of zombie shares following changes in industry markup structure. While our previous cross-sectional and within-sector analyses establish a contemporaneous correlation, understanding the temporal dynamics is crucial for two reasons. First, it allows us to assess whether the relationship between markup dispersion and zombie shares represents a long-run equilibrium or merely a transitory phenomenon. Second, it provides insight into the speed of adjustment in credit allocation following changes in market structure, which has important implications for the persistence of zombie firms in different competitive environments.

We employ a local projection method (Jordà, 2005) to trace out the dynamic response of zombie shares to changes in markup dispersion over a six-year horizon. Specifically, for each horizon $h = 0, 1, ..., 5$, we estimate the following specification at the industry-year level:

$$\Delta^h \log(\text{zombie share}_{s,t+h}) = \alpha_t + \alpha_s + \beta_h \log(\text{sd}(\mu)_{s,t}) + \gamma \Delta^0 \log(\text{zombie share}_{s,t-1}) + \Gamma X_{s,t-1} + \varepsilon_{s,t+h} \quad (38)$$

where $\Delta^h \log(\text{zombie share}_{s,t+h}) = \log(\text{zombie share}_{s,t+h}) - \log(\text{zombie share}_{s,t-1})$ represents the cumulative log change in zombie shares from period $t-1$ to $t+h$. The key coefficient of interest, β_h, captures the cumulative effect of an increase in log markup dispersion on zombie shares at horizon h. For ease of interpretation, we scale the coefficient to a standard deviation increase. We include the lagged change in zombie shares to control for short-run dynamics and potential mean reversion. The vector $X_{s,t-1}$ includes lagged industry-level controls (real value added and sector-specific price indices) to account for business cycle effects and demand conditions. Year fixed effects (α_t) absorb common macroeconomic shocks, while industry fixed effects (α_s) control for time-invariant industry characteristics. Standard errors are clustered at the industry level to account for serial correlation within sectors. Following our empirical specification, we scale the coefficient β_h by the standard deviation of $\log(\text{sd}(\mu)_{s,t})$ to facilitate interpretation in terms of a one-standard-deviation shock.

Figure 10:
Markups dispersion and Zombie Shares Dynamics



Notes: The figure displays impulse responses from local projections estimating the cumulative effect of a one-standard-deviation increase in industry markup dispersion on zombie shares over a six-year horizon. The solid line represents point estimates, the dark shaded area shows 68% confidence intervals, and the light shaded area shows 90% confidence intervals. Standard errors are clustered at the industry level. All specifications include year and industry fixed effects, as well as lagged controls for real value added and sector-specific price indices.

Figure 10 presents the estimated impulse response of zombie shares to a one-standard-deviation increase in markup dispersion. The response is positive and statistically significant, reaching its peak at year 3 with a cumulative increase of 100% percent. While the effect is persistent, it gradually reverts to its mean, which is most likely driven by the limited time span in our sample (2007-2016), not allowing a precise long-run estimates. This persistent positive response is consistent with our model's prediction that higher markup dispersion creates stronger incentives for high-potential firms to remain in operation despite temporary financial distress, thereby increasing the equilibrium share of zombie firms in the industry.

6.1.2. Does Market Structure Amplify the Markup-Zombie Link?

Our model's mechanism relies on the assumption that high-potential zombie firms can transition to industry leaders by capturing the option value of investments. This, in turn, allows the firms to charge a higher markup. In other words, in the model markups, profitability and market share are all perfectly correlated across firms, as is the case in most imperfect competition models with variable markups. However, in the data, the ease of converting high markup into market dominance varies substantially across sectors. In certain industries, higher markups translate into larger market shares as customers

migrate toward higher-quality or lower-cost offerings. In others, factors such as regulatory barriers weaken this link, making high markups not directly translate into large market shares.

To test whether zombie shares are concentrated in sectors where this link to markup and market share is stronger, we construct a sector-level measure of market share responsiveness to markups. Specifically, we estimate $\log(\text{market share}_{i,t}) = \alpha_s + \alpha_t + \gamma_s \log(\text{markup}_{i,t}) + \varepsilon_{i,t}$ separately for each sector, where γ_s captures how tightly market shares track markup levels. We then classify sectors into "high option-value" (γ_s above median) and "low option-value" (γ_s below median) groups, and estimate our baseline specification (Equation 37) separately for each of the two samples.

Table 5:
Markup-Zombie Relationship by Market Structure

	log *Zombie Share*$_{s,t}$			
	LowOption − Value		High Option-Value	
	(1)	(2)	(3)	(4)
$\log(sd(\mu_{s,t}))$	0.662***	0.128	0.725***	0.706*
	(0.163)	(0.326)	(0.188)	(0.348)
Year F.E.	✓	✓	✓	✓
Industry F.E.		✓		✓
Clustered Std.Errors	Sector and Year	Sector and Year	Sector and Year	Sector and Year
R^2	0.24	0.61	0.17	0.54
Obs	364	364	368	368

Notes: Data source: Orbis database for Italian firms, 2007–2016. Unit of analysis: sector-year. Low option-value sectors are those where market share responsiveness to markups (β_s) is below the median; high option-value sectors are above the median. The β_s coefficients are estimated from firm-level regressions of log market share on log markups within each sector. Standard errors clustered at the sector level in parentheses. ***, **, * denote significance at 1%, 5%, and 10% levels, respectively.

Table 5 presents evidence consistent with our model's mechanism, though the differences across sector types are not statistically significant at conventional levels. In the cross-sectional analysis (Columns 1 and 3), the coefficient on markup dispersion is 0.662 in low option-value sectors and 0.725 in high option-value sectors, both economically large and statistically significant. This pattern suggests that a one-standard-deviation increase in markup dispersion (0.56) is associated with approximately 37% higher zombie shares in low option-value sectors and 41% higher zombie shares in high option-value sectors.[11] When exploiting within-sector variation over time (Columns 2 and 4), the point estimates suggest an even stronger differential pattern: the coefficient remains marginally significant in high option-value sectors (0.706, $p < 0.10$) but becomes statistically indistinguishable

[11] Calculated as $\exp(0.662 \times 0.56) - 1 = 0.37$ and $\exp(0.725 \times 0.56) - 1 = 0.41$.

from zero in low option-value sectors (0.128), though we cannot reject equality of these coefficients.

While not definitive, these patterns provide suggestive evidence that the markup-zombie relationship may be amplified in sectors with a stronger relationship between markups and market share. Combined with our findings from **Predictions 1**, this sectoral heterogeneity suggests that zombie lending is not just responding to the magnitude of markup dispersion (potential rewards) but also to ease of converting markup advantages into market share gains (becoming a leader). We would not observe this systematic variation across competitive environments unless an underlying link between markups and market share. Instead, our findings are consistent with zombie lending concentrating in sectors where high-potential firms face more favourable conditions for exploiting their option value.

6.2. Zombies & Firm Dynamism

Having established that sectors with higher markup dispersion exhibit larger shares of zombie firms, we now investigate the behavior of these firms. If zombie firms indeed survive to preserve their option value of future market rents, as our model suggests, we should observe them actively investing to exploit future dominant market positions rather than merely subsisting. Our main contribution is to document that zombie firms in high-markup sectors are more likely to invest and gain market share, with a higher probability of reaching the top quartile of these distributions, thus aligning with our model's predictions.

To formally test these relationships, we examine whether zombie firms in sectors with high markup dispersion are more likely to achieve strong performance across various dimensions over time. Specifically, we analyze whether these firms ever reach the top quartile of the distribution in the subsequent four years for firm-level markups, market share, employment growth, and investment rate. We estimate the following OLS specifications:

$$\begin{aligned}\mathbb{1}\,\textit{Top-Quartile}\; Y_{i,t+1\to t+4} = \alpha_t + \alpha_s + \alpha_{s,t} + \beta_1 \mathbb{1}\textit{Zombie} + \beta_2 sd(\mu)_{s,t}) \\ + \beta_3 \mathbb{1}\textit{Zombie} \times sd(\mu)_{s,t} + \Gamma X_{i,t} + \varepsilon_{i,t} \quad (39)\end{aligned}$$

where $\mathbb{1}\,\textit{Top-Quartile}\; Y_{i,t\to t+4}$ is an indicator variable equal to one if firm i ever reaches the top quartile of the outcome variable Y's distribution during the period from $t+1$ to $t+4$. Our specification includes time-fixed effects (α_t), sector-fixed effects (α_s), and sector-by-time fixed effects ($\alpha_{s,t}$) to account for unobserved heterogeneity across periods and industries. The vector $X_{i,t}$ comprises firm-level control variables that are orthogonal to zombie status to prevent multicollinearity. Specifically, we control for firm size and age.

To account for within-firm correlation and the relative importance of firms in aggregate output, we cluster standard errors at the firm level and weight observations by sales. The coefficient of interest is the interaction between the firm-level zombie indicator ($\mathbb{1}Zombie$) and the sales-weighted sector-level markup dispersion ($sd(\mu)_{s,t}$). This interaction term captures how the relationship between zombie status and the probability of reaching the top performance quartile varies with industry markup dispersion.

While our model simulates different industry equilibria - making year-fixed effects the closest empirical counterpart - our preferred specification includes industry-by-year fixed effects. Although this more stringent specification maps less precisely to the model's setup, it offers the advantage of controlling for unobserved industry-specific factors that might confound our results. This specification exploits both cross-sectional variation across firms within a given year and temporal variation within sectors. In essence, we compare firms with different zombie statuses at varying levels of sector-level markup dispersion, while absorbing any industry-year specific factors.

A concern for the identification of the effect of interest is that the probability of moving across quantiles of market share or investment fluctuates significantly, potentially capturing random fluctuations rather than the effect of interest. In the appendix, we provide transition probabilities for the firms' log market share and investment rate in Table A2 and Table A3. These tables show that the probability of firms remaining in the same bins (diagonal elements) is high, suggesting that firms' performance is stable over time, allowing us to make reliable inferences about zombie firms' behavior. Additionally, this persistence supports our model's prediction that zombie firms in high-markup sectors are likely to invest and gain market share, maintaining their positions once improved.

We formally present the results for Equation 39 in Table 6, documenting that zombie firms in high-markup sectors are more likely to invest and gain market share.

The results reveal substantial heterogeneity in zombie firms' performance across market environments. In our preferred specifications with industry-by-year fixed effects (Columns 3 and 6), zombie firms are 18.9 percent less likely to reach the top quartile of market share and 12.3 percent less likely to achieve top markup performance. Given mean probabilities of 24% and 25% respectively of the outcome variables, these effects represent substantial reductions of approximately 78% (-0.189/0.24) and 49% (-0.123/0.25) relative to the baseline probability. These results are in line with previous findings on the poor performances of zombies (Caballero et al., 2008; Acharya et al., 2019, 2024).

Our key contribution lies in documenting how market structure fundamentally shapes zombie firms' potential for performance improvements. While prior literature has focused on the average negative effects of zombie firms, we show that this performance gap narrows significantly in sectors with higher markup dispersion. Put differently, we empirically

Table 6:

Likelihood of Zombies Reaching Top Quartile of Market Share and Markups

Likelihood: $\mathbb{1}$ *Top quartiles*$:t+1 \to t+4_{f,t}$						
	$\mathbb{1}$ *Top quartile log mkt.share*$_{f,t+1\to t+4}$			$\mathbb{1}$ *Top quartile log*$\mu_{f,t+1\to t+4}$		
	(1)	(2)	(3)	(4)	(5)	(6)
$\mathbb{1}Zombie_{f,t}$	-0.148**	-0.187**	-0.189**	-0.124***	-0.122***	-0.123***
	(0.073)	(0.084)	(0.087)	(0.043)	(0.032)	(0.033)
$sd(\mu)_{s,t}$	0.001	0.017**		0.178***	0.002	
	(0.009)	(0.008)		(0.015)	(0.020)	
$\mathbb{1}Zombie_{f,t} \times sd(\mu)_{s,t}$	0.120**	0.130**	0.132**	0.095*	0.113***	0.112***
	(0.053)	(0.062)	(0.064)	(0.050)	(0.039)	(0.040)
Controls	✓	✓	✓	✓	✓	✓
Year F.E.	✓	✓		✓	✓	
Industry F.E.		✓			✓	
Year by industry F.E.			✓			✓
Clustered Std.Errors	Firm	Firm	Firm	Firm	Firm	Firm
R^2	0.30	0.32	0.32	0.05	0.09	0.09
Obs	2,097,034	2,097,034	2,097,034	2,078,395	2,078,395	2,078,395
Mean Dep.	0.24			0.25		

Notes: Data source: Orbis database for Italian firms, 2007-2016. Unit of analysis: firm-year. The dependent variable is an indicator equal to one if the firm reaches the top quartile of either log market share (Columns 1-3) or log of markups (Columns 4-6) distribution in the subsequent four years: $t+1$ to $t+4$. The table presents estimates of the relationship between zombie status and the probability of reaching top performance, interacting with industry leaders' markup ($sd(\mu_{s,t})$) as of Equation 39. All specifications include controls for firm size (log of assets) and age. Standard errors are clustered at the firm level and reported in parentheses. Observations are weighted by firm sales. ***, **, * denote significance at 1%, 5%, and 10% levels, respectively.

confirm **Prediction 2** in the data: an increase in markup dispersion increases the likelihood that zombie firms transition in the future to the top of the market share and markup distribution. The interaction terms of 0.132 and 0.112 for market share and markups, respectively, are statistically and economically significant, and confirm that the zombie disadvantage systematically diminishes as industry markup dispersion increases, suggesting that market power creates opportunities for zombie firms to transition into industry leaders.

To quantify the strength of this relationship, consider the difference between industries at the 25th percentile of markup dispersion (0.37) versus the 75th percentile (0.9). This interquartile difference in market structure reduces the zombie disadvantage substantially. For market share, zombie firms in high-dispersion industries are only 7 percent less likely to reach the top quartile (-0.189 + 0.132 × 0.9), compared to the 18.9 percent disadvantage in low-dispersion industries.[12] The rise in markup dispersion relates to an increase in the likelihood of zombies being in the top quartile of market share by approximately 11.8

[12]These calculations represent the total effect of being a zombie firm on the likelihood of reaching the top quartile when markup dispersion increases by 0.9 units (the interquartile difference). In other words, they show the net zombie disadvantage conditional on an interquartile markup dispersion increase.

percent (0.132 × 1.08), offsetting 63% of the baseline zombie disadvantage (-0.118/-0.189). The effect is even stronger for markups: the zombie disadvantage nearly disappears in high-dispersion industries, with the penalty shrinking from 12.3 to just 2 percent (-0.123 + 0.112 × 0.9). This substantial reduction is driven by a 10.1 percent higher likelihood of reaching the top quartile of markup distribution (0.112 × 0.9), which offsets 82% of the baseline disadvantage (-0.112/-0.123).

In Table 7, we extend our analysis to examine whether market structure similarly affects zombies' real outcomes, focusing on investment rates and employment growth. In our preferred specifications with industry-by-year fixed effects (Columns (3) and (6)), zombie firms are 2.2 percent less likely to reach the top quartile of investment rates and 1.87 percent less likely to achieve high employment growth. Consistent with the traditional view that zombie firms underinvest and create fewer jobs (Caballero et al., 2008; Blattner et al., 2023; Acharya et al., 2024), we find that the zombie indicator is negatively related to firms being in the top quartile of investment rates and the log of employees.

However, as with market share and markups, this investment and employment gap significantly narrows in sectors with higher markup dispersion. The interaction term of 0.108 for investment rates and 0.119 for employment suggests that zombies' real performance disadvantage diminishes as industry concentration increases. Moving from industries at the 25th percentile of markup dispersion (0.37) to the 75th percentile (1.27) would increase zombies' likelihood of being in the top investment quartile by approximately 9.7 percent (0.108 × 0.9), offsetting about 44% of the baseline investment disadvantage (-0.097/-0.220). Similarly, for employment, this change in market structure offsets 57% of the baseline employment growth disadvantage (0.119 × 0.9/0.187). Consistent with our model predictions, these findings suggest that market power affects zombies' markets, real investment, and employment decisions. In particular, as markup dispersion increases, the likelihood for zombie firms to transition to the top employment quartile grows in line with our **Prediction 2**. Moreover, the table shows that the unconditional effect of markup dispersion (β_2) is not significant. This indicates that, on average, a firm is not more likely to get to the top when dispersion increases. The strong effect of markup dispersions is really only for zombie firms, which confirms our model predictions.

In further robustness checks outlined in Table A4 and Table A5, we provide estimates using sector-level industry leader markup (e.g. μ^{75th}). The results are qualitatively analogous to the main results; however, the point estimates are smaller. An interquartile increase in industry leader markup levels (2.57 - 1.19 = 1.38) would increase zombies' likelihood of being in the top markup distribution, and market share quartile by approximately 8.6% (1.38×0.063), offsetting about 45% and 64% of the baseline investment disadvantage for zombies. We find similar magnitudes for employment and investments. Finally, we also

Table 7:

Likelihood of Zombies Reaching Top Quartile of Investment and Employees

Likelihood: $\mathbb{1}$ *Top quartiles:* $t+1 \to t+4_{f,t}$						
	$\mathbb{1}^{p75}$*investment rate*$_{f,t+1\to t+4}$			$\mathbb{1}^{p75}$*log employees*$_{f,t+1\to t+4}$		
	(1)	(2)	(3)	(4)	(5)	(6)
$\mathbb{1}Zombie_{f,t}$	-0.256***	-0.230***	-0.220***	-0.151***	-0.183***	-0.187***
	(0.061)	(0.044)	(0.046)	(0.055)	(0.067)	(0.070)
$sd(\mu)_{s,t}$	-0.013	-0.030		0.005	0.010	
	(0.010)	(0.038)		(0.009)	(0.009)	
$\mathbb{1}Zombie_{f,t} \times sd(\mu)_{s,t}$	0.139**	0.117***	0.108**	0.110***	0.117**	0.119**
	(0.058)	(0.045)	(0.045)	(0.040)	(0.050)	(0.052)
Controls	✓	✓	✓	✓	✓	✓
Year F.E.	✓	✓		✓	✓	
Industry F.E.		✓			✓	
Year by industry F.E.			✓			✓
Clustered Std.Errors	Firm	Firm	Firm	Firm	Firm	Firm
R^2	0.01	0.03	0.04	0.30	0.34	0.34
Obs	2,074,328	2,074,328	2,074,328	2,097,034	2,097,034	2,097,034
Mean Dep.	0.24			0.23		

Notes: Data source: Amadeus database for Italian firms, 2007-2016. Unit of analysis: firm-year. The dependent variable is an indicator equal to one if the firm reaches the top quartile of either investment rate (Columns 1-3) or log of employees (Columns 4-6) distribution in the subsequent four years: $t+1$ to $t+4$. The table presents estimates of the relationship between zombie status and the probability of reaching top performance, interacting with industry leaders' markup ($sd(\mu)_{s,t}$) as of Equation 39. All specifications include controls for firm size (log of assets) and age. Standard errors are clustered at the firm level and reported in parentheses. Observations are weighted by firm sales. ***, **, * denote significance at 1%, 5%, and 10% levels, respectively.

confirm that profits, measured as Return on Assets (ROA) are more likely to fall in the top quartile of the distribution of firms in the subsequent four years in Table A6 and Table A7.

6.3. Bad vs. Good Zombies: Evidence from TFP Dynamics

Thus far, our analysis has established that zombie firms in high markup-dispersion sectors are more likely to transition to industry leadership, suggesting they may embody high-potential laggards rather than stagnant firms. However, a natural question arises: can we empirically distinguish sectors where zombies predominantly consist of high-potential firms from those where they are primarily stagnant? And if so, do these two types of zombie-intensive sectors exhibit different trajectories for aggregate productivity? Answering these questions is important because it can directly verify in the data the prediction of the model that the allocative implications of zombie lending are positive or negative depending on the type of zombies that prevail in given industries (see Section 4.2.2)

Our model provides guidance on how to make this distinction. High-potential zombies are characterized by two key behaviours: they actively invest in productivity-enhancing activities (captured by capital expenditures) and they leverage up to finance these investments, maintaining the option to transition to leadership. In contrast, stagnant zombies neither invest nor accumulate debt, as they lost their growth opportunities. Rather they slowly reduce their debt holdings and merely subsist until eventual exit. Therefore, we can classify sectors based on how zombie firms' investment and leverage dynamics compare to those of the average firm in their sector.

Specifically, we construct two measures at the sector level. First, we calculate the time invariant difference between the average investment rate of all firms and that of zombie firms within each sector: $\Delta\text{Investment}_s = \text{Investment}_{\text{all firms},s} - \text{Investment}_{\text{zombies},s}$. Second, we compute the analogous difference for leverage growth: $\Delta\text{Leverage}_s = \Delta\text{Leverage}_{\text{all firms},s} - \Delta\text{Leverage}_{\text{zombies},s}$. We then classify sectors into quartiles based on these differences.

In sectors where both differences are large (above the median of $\Delta\text{Investment}_s$ and $\Delta\text{Leverage}_s$), zombie firms invest significantly less and deleverage more than the typical firm, suggesting they consist predominantly of stagnant zombies with limited growth potential. Conversely, in sectors where both differences are small, zombie firms exhibit investment and leverage patterns similar to the average firm, indicating they are more likely to be high-potential zombies actively pursuing productivity improvements. Our model (see Figure 7) predicts that zombie lending should have beneficial effects on aggregate productivity only in the latter type of sector, where zombies represent firms with genuine growth opportunities rather than purely distressed entities.

To test this prediction, we estimate local projection regressions of the form:

$$\text{TFP}_{s,t+h} = \alpha_s + \alpha_t + \beta_h \log(\text{Zombie Share}_{s,t}) + \gamma \text{TFP}_{s,t-1} + \Gamma X_{s,t} + \varepsilon_{s,t+h} \tag{40}$$

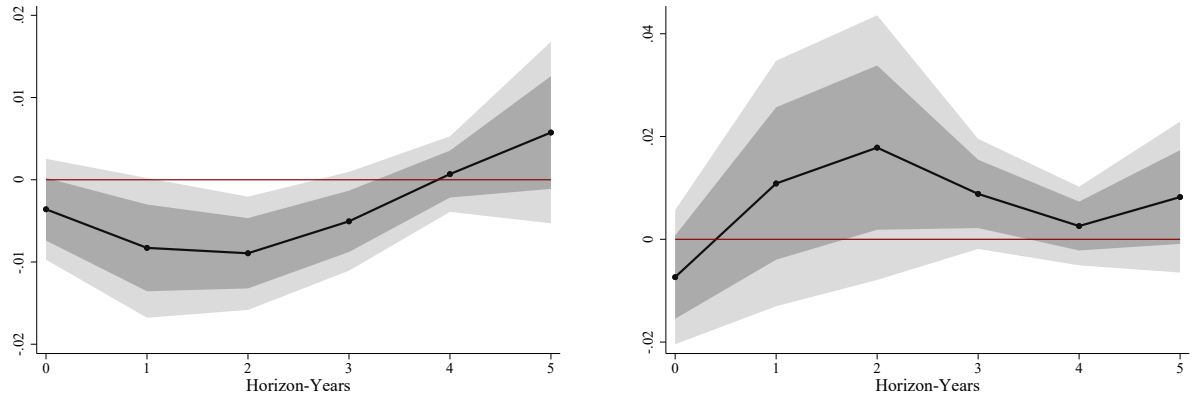
where $h \in \{0, 1, ..., 5\}$ indexes the horizon in years, and we estimate this specification separately for sectors with high versus low investment and leverage differences. The vector $X_{s,t}$ includes controls for sector-level value added and price indices, and we include both sector and year fixed effects. Standard errors are clustered at the sector and year level.

Figure 11 presents the results, plotting the impulse response β_h over horizons 0 to 5 years for both high-difference and low-difference sectors. Panel (a) plots the results for high-difference sectors, where zombies behave like stagnant zombies (low investment, deleveraging), while Panel (b) shows results for low-difference sectors, where zombies resemble high-potential laggards (active investment, leveraging up). The differential response aligns precisely with our model's predictions.

In low-difference sectors (Panel (b)), where zombies resemble high-potential laggards,

Figure 11:
Impulse Responses of log TFP to Zombie Shares by Sector Type

(a) *High Difference Zombies* **(b)** *Low Difference Zombies*



Notes: The figure presents impulse responses from local projection regressions of TFP on log zombie shares, estimated separately for high-difference sectors (Panel a) and low-difference sectors (Panel b). High-difference sectors are those where both the investment gap and leverage growth gap between average firms and zombies exceed their respective medians. Low-difference sectors are those where both gaps are below their medians. The solid line represents the point estimate of β_h at each horizon h, scaled by the standard deviation of log zombie shares. Dark gray shaded areas represent 68% confidence intervals, while light gray areas represent 90% confidence intervals. Standard errors are clustered at the sector level.

an increase in the zombie share is associated with higher TFP over the subsequent years, with the effect peaking around 2 years. The positive trajectory suggests that zombie credit in these sectors supports firms with productivity-enhancing opportunities, consistent with our earlier findings that zombies in favorable environments successfully transition to leadership positions.

In contrast, in high-difference sectors, where zombies exhibit behavior characteristic of stagnant zombies, the same increase in zombie share is associated with declining TFP over the forecast horizon. This negative relationship aligns with the misallocation narrative emphasized in prior literature (Caballero et al., 2008; Blattner et al., 2023; Bonfim et al., 2023; Acharya et al., 2019): when zombie firms are predominantly stagnant entities with limited growth prospects, their survival crowds out resources that could be allocated to more productive firms, depressing aggregate productivity.

6.4. Discussion

Overall, our empirical analysis provides support for both predictions of the model on the relation between rising markups and zombie lending. Interpreted through the lenses of the model, our findings imply that, in high markup sectors, zombie loans can improve

dynamic efficiency and aggregate productivity.

Rather than contrasting previous studies, our findings highlight a novel and complementary mechanism through which zombie lending affects industry dynamics and aggregate efficiency. While the existing literature has primarily focused on the potential misallocation effects of zombie lending (Blattner et al., 2023; Acharya et al., 2022, 2024), our results suggest that in sectors characterized by high markup dispersion, zombie loans may also play a role in promoting dynamic efficiency. As discussed in section 4.3, this complementarity stems from two key insights. First, the mechanisms analyzed in our model could coexist with other factors that generate the misallocation effects highlighted in the literature, particularly in environments where firms are born with persistent productivity differences. Second, our empirical results, by strongly confirming **Prediction 2**, support the conclusion that this novel role of zombie lending is empirically relevant and might partly counteract the negative effects highlighted by the literature, especially in high markup industries. Our contribution lies in identifying a previously unexplored channel through which zombie lending affects industry outcomes while acknowledging that this mechanism operates alongside, rather than instead, the misallocation effects established in the existing literature.

Furthermore, we show theoretically that standard zombie definitions based solely on financial distress metrics (low interest coverage ratios, high leverage) do not distinguish between high-potential and stagnant zombies. Our results suggest that incorporating information on firms' investment and leverage dynamics can improve the identification of welfare-reducing zombie lending. These findings reconcile our earlier results, that zombies in high-markup sectors exhibit strong performance, with the broader literature documenting negative aggregate effects of zombie lending. Both perspectives are valid, but they apply to different types of zombies operating in different competitive environments. Zombie lending concentrated among high-potential laggards in sectors with tight links between markups and market share can have a distinct dynamic efficiency path, while zombie lending to stagnant firms in less dynamic sectors generates the misallocation effects documented elsewhere.

In terms of policy implications, this means it is important to consider the dynamism of a sector, as well as its type of technology and market structure, to correctly evaluate the positive and negative reallocation effects of zombie lending.

7. Conclusion

This paper examines the relationship between rising markups and zombie firms through the lens of a firm dynamic model with heterogeneous markups, financial frictions, and

productivity-enhancing investments. Our theoretical and empirical analysis reveals a novel firm-level mechanism through which markup dispersion shapes the industry composition of zombie firms and, in turn, their potential for future growth. We show that the interaction between market structure and zombie lending has important implications for industry dynamics and aggregate efficiency, as it affects both the selection of which firms receive zombie credit and their subsequent productivity trajectories.

In the model, firms pay a fixed cost to enter and begin operating with low productivity as "laggards". These firms have access to borrowing facilities - e.g. a credit line - and only a fraction of laggards have the potential to invest and become highly productive "leaders". When firms accumulate debt to keep producing after facing negative profits shocks, they may receive a "zombie loan" - subsidized credit that keeps them operating despite their poor performance. As markup dispersion increases, the higher potential profits of industry leaders create stronger incentives for high-potential laggards to remain in operation, even when experiencing net losses, making them more likely to receive a "zombie loan". These firms, upon receiving credit, are more likely to invest in productivity-enhancement technology due to larger rewards for succeeding in it. Our key model predictions thus tell that industries with higher markup dispersion exhibit both a higher share of zombie firms and an increased likelihood for these firms to become industry leaders. The latter prediction stems from the interaction between the option value of becoming a leader and the availability of zombie credit, which allows high-potential firms to continue pursuing productivity improvements despite temporary financial distress.

We test and confirm these predictions using detailed firm-level data from Italy between 2007-2016. First, we document that an increase in sector-level markup dispersion correlates with larger zombie shares both across and within sectors. We further show that this relationship is amplified in industries where markups translate more directly into market dominance, suggesting that zombie lending concentrates where the option value of leadership is most readily exploitable. Second, and more importantly, we show that zombie firms in sectors with higher markup dispersion are substantially more likely to reach the top quartile of the market share and markup distribution, thus being more likely to be industry leaders. Similar patterns emerge for employment levels and investment rates. Critically, we demonstrate that these aggregate patterns mask important heterogeneity across different types of zombie firms. Guided by the model's predictions, we develop an empirical classification method based on zombies' investment and leverage dynamics relative to healthy firms. We show that in sectors where zombies behave like high-potential laggards, exhibiting investment and leverage patterns similar to other firms, increases in zombie shares are associated with higher sectoral TFP. Conversely, in sectors where zombies resemble stagnant zombies, investing less and deleveraging, zombie prevalence

correlates with declining productivity. This heterogeneity directly confirms the model's central prediction that the allocative implications of zombie lending depend fundamentally on which type of zombie firm predominates in a given industry.

Our findings suggest that in high markup sectors, zombie firms might enhance dynamic efficiency by preserving the option value of becoming future market leaders. Rather than contrasting previous studies highlighting the misallocation effects of zombie lending, our results identify a complementary mechanism through which zombie credit affects industry outcomes. This distinction is of particular relevance for sectors with high markup dispersion and significant growth opportunities (e.g. information technology).

In terms of policy implications, our analysis suggests that evaluating the impact of zombie lending requires careful consideration of industry characteristics, particularly the degree of markup dispersion and potential for productivity improvements. While zombie lending creates misallocation in some contexts, our findings indicate that in industries with high markup dispersion, it can help preserve valuable growth options and promote dynamic efficiency. Our results show that incorporating information about firms' investment and leverage behavior can improve the identification of "welfare-reducing" versus "welfare-enhancing" zombie lending. Therefore, policies aimed at addressing zombie lending should be calibrated to account for these industry-specific factors rather than applying a one-size-fits-all approach.

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A. Appendix

B. Aggregate Markups and Zombie Shares

We document the positive relationship between aggregate markups and the prevalence of zombie firms. Figure A2 plots the evolution of sales-weighted markups (solid blue line, left axis) and the asset-weighted share of zombie firms (dotted red line, right axis) from 2007 to 2016. The patterns we uncover are consistent with prior evidence on markup trends and zombie prevalence De Loecker et al. (2020); Díez et al. (2021), Acharya et al. (2024). In our sample, the average sales-weighted markup is 1.24, while the share of zombie firms ranges from 1.5% to 3.5% of total firms, with a mean of 2.5%. These two series show strong co-movements: periods of elevated markups systematically coincide with higher zombie shares. This aggregate evidence provides initial support for our model's prediction that markup dispersion and zombie prevalence are interlinked.

C. Sectoral Level Markups and Zombie Shares

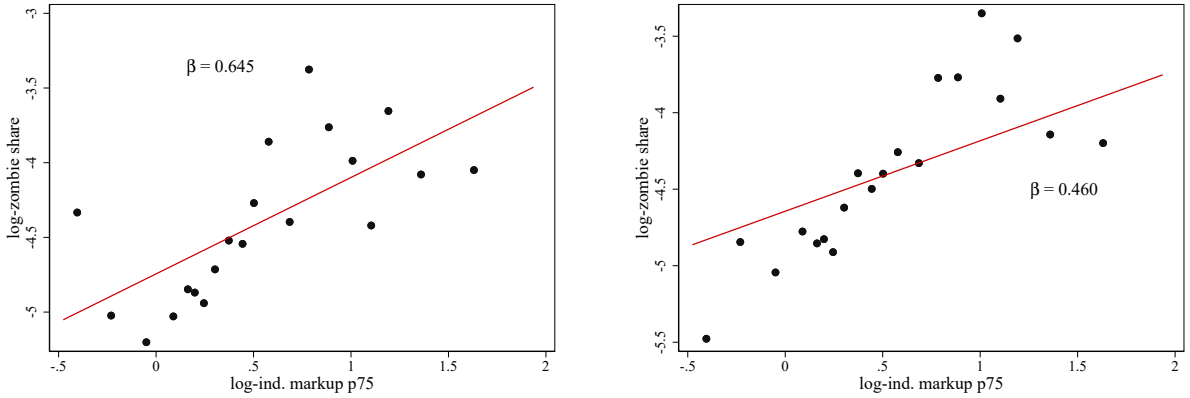
The positive relationship between market power and zombie shares persists using markup levels rather than markup dispersion: sectors where top firms charge higher markups exhibit systematically higher zombie shares, with magnitudes comparable to our baseline estimates. This robustness to alternative measures reinforces our interpretation that the option value mechanism operates through the rewards available at the top of the market structure.

The estimates presented in Table A1 for Equation 37 provide similar patterns to our main results. Between 2007 and 2016, the median markup increase across sectors for industry leaders ($\mu_{s,t}^{75th.}$) increased by 5% ($\log(1.63_{2016})$ - $\log(1.55_{2007})$), implying a 3% higher zombie share based on our estimates. While the within-sector estimates are not significant at conventional levels, their sign and magnitude align with our model's predictions. As mentioned before, these findings align closely with our model's mechanism: when market structure changes lead to higher markup dispersion within a sector, the value of maintaining market presence increases while the one of exiting decreases, leading to a higher prevalence of zombie firms.

While these series show strong co-movement, this aggregate relationship may mask substantial heterogeneity across sectors and could be driven by compositional changes in the economy. In our sectoral regressions, we exploit variation at the sector-year level, which allows us to account for sector-specific characteristics and aggregate time trends.

Figure A1:
Sector level Markups and Zombie Shares

(a) *zombie share and markup level* **(b)** *zombie share and markup level within sector*



Notes: Data source: Orbis database for Italian firms, 2007-2016. Unit of analysis: sector-year. The figure presents binscatter plots relating log zombie shares to log markup level of industry leaders ($log(\mu_{s,t}^{75th.})$). Left panels estimate: $log(zombie\ share_{s,t}) = \alpha_t + \beta_1 log(\mu_{s,t}^{75th.}) + \epsilon_{s,t}$, where α_t represents year fixed effects. Right panels add sector fixed effects: $log(zombie\ share_{s,t}) = \alpha_t + \alpha_s + \beta_1 log(\mu_{s,t}^{75th.}) + \epsilon_{s,t}$. Each dot represents the average value within each bin, and estimated β coefficients are reported at the top of each panel.

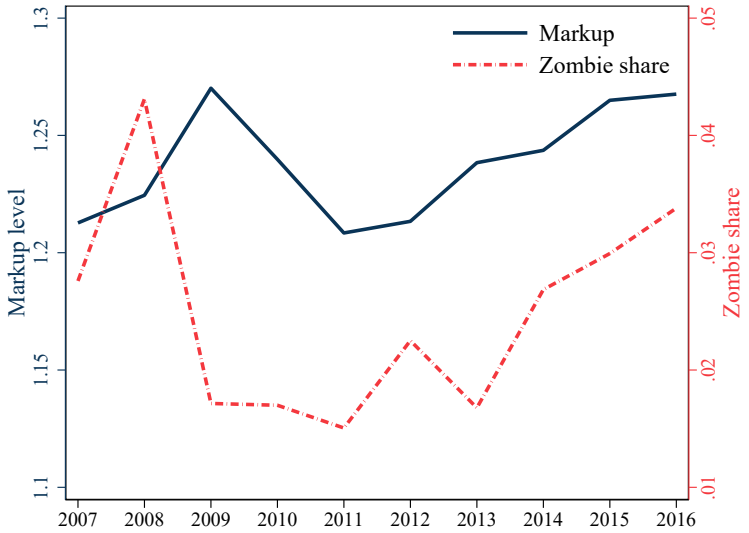
Table A1:
Sector level Zombie Share and Markup Level

Main Dependent log(zombie share$_{s,t}$)				
	Cross-Section		*Within*	
	(1)	(2)	(3)	(4)
$log(\mu_{s,t}^{75th.})$	0.645***	0.602***	0.460	0.476
	(0.197)	(0.188)	(0.438)	(0.440)
Controls		✓		✓
Year F.E.	✓	✓	✓	✓
Industry F.E.	✓	✓	✓	✓
Clustered Std.Errors	Sector	Sector	Sector	Sector
R^2	0.13	0.16	0.56	0.56
Obs	732	732	732	732

Notes: Data source: Orbis database for Italian firms, 2007-2016. Unit of analysis: sector-year. The dependent variable is the log of zombie shares. The main explanatory variable is the log of sector-level markup dispersion ($log(\mu_{s,t}^{75th.})$), where markups are weighted by firm sales in an industry. All specifications include year-fixed effects. Columns (1) and (2) report cross-sectional estimates, while columns (3) and (4) add sector-fixed effects. Controls include sector-level real value-added and price indices. Standard errors are clustered at the sector level and reported in parentheses. ***, **, * denote significance at 1%, 5%, and 10% levels, respectively.

D. Zombie Firms & Innovation: Likelihood of falling in Top-Quartile

Figure A2:
Aggregate Markups and Zombie Share



Note: Orbis database for Italian firms, 2007-2016. The figure shows two series: (1) the aggregate markup (blue solid line, left axis), computed as the sales-weighted average of firm-level markups, and (2) the share of zombie firms (red dashed line, right axis), computed as the asset-weighted fraction of zombie firms in the economy. Both series are reported at an annual frequency.

Table A2:
Transition Probabilities YoY

Quantiles of log market share$_{f,t}$					
To Portfolio Share Quartile	Q_1	Q_2	Q_3	Q_4	Sum
From Portfolio Share Quartile					
Q_1	0.86	0.13	0.01	0.00	1.00
Q_2	0.09	0.76	0.14	0.00	1.00
Q_3	0.01	0.09	0.79	0.11	1.00
Q_4	0.00	0.00	0.06	0.93	1.00

Notes: Data source: Orbis database for Italian firms, 2007-2016. Unit of analysis: firm-year.

Table A3:
Transition Probabilities YoY

Quantiles of investment rate$_{f,t}$					
To Portfolio Share Quartile	Q_1	Q_2	Q_3	Q_4	Sum
From Portfolio Share Quartile					
Q_1	0.45	0.26	0.16	0.12	1.00
Q_2	0.25	0.36	0.24	0.15	1.00
Q_3	0.17	0.25	0.34	0.24	1.00
Q_4	0.14	0.16	0.28	0.42	1.00

Notes: Data source: Orbis database for Italian firms, 2007-2016. Unit of analysis: firm-year.

Table A4:

Likelihood of Zombies Reaching Top Quartile of Market Share and Markups

Likelihood: $\mathbb{1}$*Top quartiles:* $t+1 \to t+4_{f,t}$						
	$\mathbb{1}$*Top quartile log mkt.share*$_{f,t+1\to t+4}$			$\mathbb{1}$*Top quartile log* $\mu_{f,t+1\to t+4}$		
	(1)	(2)	(3)	(4)	(5)	(6)
$\mathbb{1}Zombie_{f,t}$	-0.148*	-0.188**	-0.191**	-0.128**	-0.132***	-0.136***
	(0.081)	(0.092)	(0.095)	(0.050)	(0.038)	(0.040)
$\mathbb{1}Zombie_{f,t} \times \mu_{s,t}^{75th.}$	0.057*	0.062*	0.063*	0.057*	0.062**	0.063**
	(0.031)	(0.036)	(0.037)	(0.032)	(0.026)	(0.027)
Controls	✓	✓	✓	✓	✓	✓
Year F.E.	✓			✓		
Industry F.E.		✓			✓	
Year by industry F.E.			✓			✓
Clustered Std.Errors	Firm	Firm	Firm	Firm	Firm	Firm
R^2	0.30	0.32	0.32	0.07	0.09	0.09
Obs	2,097,034	2,097,034	2,097,034	2,078,395	2,078,395	2,078,395
Mean Dep.	0.24			0.25		

Notes: Data source: Amadeus database for Italian firms, 2007-2016. Unit of analysis: firm-year. The dependent variable is an indicator equal to one if the firm reaches the top quartile of either log market share (Columns 1-3) or log markup (Columns 4-6) distribution in the subsequent four years: $t+1$ to $t+4$. The table presents estimates of the relationship between zombie status and the probability of reaching top performance, interacting with industry leaders' markup ($\mu_{s,t}^{75th.}$) as of Equation 39. All specifications include controls for firm size (log of assets) and age. Standard errors are clustered at the firm level and reported in parentheses. Observations are weighted by firm sales. ***, **, * denote significance at 1%, 5%, and 10% levels, respectively.

Table A5:

Likelihood of Zombies Reaching Top Quartile of Investment and Employees

Likelihood: $\mathbb{1}$*Top quartiles:* $t+1 \to t+4_{f,t}$						
	$\mathbb{1}^{p75}$*investment rate*$_{f,t+1\to t+4}$			$\mathbb{1}^{p75}$*log employees*$_{f,t+1\to t+4}$		
	(1)	(2)	(3)	(4)	(5)	(6)
$\mathbb{1}Zombie_{f,t}$	-0.283***	-0.260***	-0.249***	-0.151**	-0.186**	-0.191**
	(0.064)	(0.047)	(0.049)	(0.060)	(0.073)	(0.077)
$\mathbb{1}Zombie_{f,t} \times \mu_{s,t}^{75th.}$	0.089**	0.080***	0.075***	0.053**	0.057**	0.059*
	(0.036)	(0.028)	(0.028)	(0.023)	(0.029)	(0.030)
Controls	✓	✓	✓	✓	✓	✓
Year F.E.	✓			✓		
Industry F.E.		✓			✓	
Year by industry F.E.			✓			✓
Clustered Std.Errors	Firm	Firm	Firm	Firm	Firm	Firm
R^2	0.01	0.03	0.04	0.30	0.34	0.34
Obs	2,074,328	2,074,328	2,074,328	2,097,034	2,097,034	2,097,034
Mean Dep.	0.24			0.23		

Notes: Data source: Orbis database for Italian firms, 2007-2016. Unit of analysis: firm-year. The dependent variable is an indicator equal to one if the firm reaches the top quartile of either investment rate (Columns 1-3) or log of employees (Columns 4-6) distribution in the subsequent four years: $t+1$ to $t+4$. The table presents estimates of the relationship between zombie status and the probability of reaching top performance, interacting with industry leaders' markup ($\mu_{s,t}^{75th.}$) as of Equation 39. All specifications include controls for firm size (log of assets) and age. Standard errors are clustered at the firm level and reported in parentheses. Observations are weighted by firm sales. ***, **, * denote significance at 1%, 5%, and 10% levels, respectively.

Table A6:
Likelihood of Zombies Reaching Top Quartile of ROA

Likelihood: $\mathbb{1}$ *Top quartile of ROA* : $t+1 \to t+4_{f,t}$			
	$\mathbb{1}$ *Top quartile of ROA* : $t+1 \to t+4_{f,t}$		
	(1)	(2)	(3)
$\mathbb{1}Zombie_{f,t}$	-0.257***	-0.196***	-0.195***
	(0.043)	(0.030)	(0.030)
$\mathbb{1}Zombie_{f,t} \times sd(\mu_{s,t})$	0.064**	0.028	0.027
	(0.029)	(0.021)	(0.021)
Controls	✓	✓	✓
Year F.E.	✓		
Industry F.E.		✓	
Year by industry F.E.			✓
Clustered Std.Errors	Firm	Firm	Firm
R^2	0.01	0.03	0.04
Obs	2,083,803	2,083,803	2,083,803
Mean Dep.	0.23		

Notes: Data source: Amadeus database for Italian firms, 2007-2016. Unit of analysis: firm-year. The dependent variable is an indicator equal to one if the firm reaches the top quartile of either log market share (Columns 1-3) or log markup (Columns 4-6) distribution in the subsequent four years: $t+1$ to $t+4$. The table presents estimates of the relationship between zombie status and the probability of reaching top performance, interacting with industry leaders' markup ($sd(mu_{s,t})$) as of Equation 39. All specifications include controls for firm size (log of assets) and age. Standard errors are clustered at the firm level and reported in parentheses. Observations are weighted by firm sales. ***, **, * denote significance at 1%, 5%, and 10% levels, respectively.

Table A7:
Likelihood of Zombies Reaching Top Quartile of ROA - Markup Level

Likelihood: $\mathbb{1}$ *Top quartile of ROA* : $t+1 \to t+4_{f,t}$			
	$\mathbb{1}$ *Top quartile of ROA* : $t+1 \to t+4_{f,t}$		
	(1)	(2)	(3)
$\mathbb{1}Zombie_{f,t}$	-0.266***	-0.205***	-0.203***
	(0.048)	(0.033)	(0.032)
$\mathbb{1}Zombie_{f,t} \times \mu_{s,t}^{75th.}$	0.037**	0.020*	0.019
	(0.017)	(0.012)	(0.012)
Controls	✓	✓	✓
Year F.E.	✓		
Industry F.E.		✓	
Year by industry F.E.			✓
Clustered Std.Errors	Firm	Firm	Firm
R^2	0.01	0.03	0.04
Obs	2,083,803	2,083,803	2,083,803
Mean Dep.	0.23		

Notes: Data source: Amadeus database for Italian firms, 2007-2016. Unit of analysis: firm-year. The dependent variable is an indicator equal to one if the firm reaches the top quartile of either log market share (Columns 1-3) or log markup (Columns 4-6) distribution in the subsequent four years: $t+1$ to $t+4$. The table presents estimates of the relationship between zombie status and the probability of reaching top performance, interacting with industry leaders' markup ($\mu_{s,t}^{75th.}$) as of Equation 39. All specifications include controls for firm size (log of assets) and age. Standard errors are clustered at the firm level and reported in parentheses. Observations are weighted by firm sales. ***, **, * denote significance at 1%, 5%, and 10% levels, respectively.