The Price of Exchange Data*

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Abstract

Exchanges increasingly generate revenues from selling transaction data, in addition to their traditional business of matching traders. Given exchanges' market power, this raises questions about the quality and the pricing of these data, and exchanges' contribution to market transparency. We study the optimal design of an exchange that sells both access to its trading platform and transaction data generated by traders' activity. The market for transaction data reduces trading fees, but exchanges sometimes choose to remain fully opaque. Policies aimed at facilitating access to exchange data can backfire if they target separately the price and the quality of transaction data.

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1 Introduction

Data have become a fundamental input into economic activity, leading to the emergence of markets for information, in which data are sold by brokers or digital platforms such as Google, Meta, and Amazon (see Bergemann and Bonatti (2019)). In line with this evolution, exchanges, who provide platforms to match traders, increasingly derive revenues from the sale of data generated by trading activity. Similarly to digital platforms' data, the value of exchange data increases with activity on their platform.

However, unlike in other digital markets, exchanges primarily sell this data to their own users, who use these data to inform their trading decisions. Consequently, for exchanges, the market for data and the market for matching services—historically exchanges' core business—are intrinsically interdependent.[1]

Our paper develops a theoretical framework to analyze this two-way interaction. This framework allows us to examine how data sales influence liquidity, price discovery, and welfare under various policy regimes (e.g., "laissez-faire", a cap on data fees etc.), thereby providing guidance for policies governing the sale of data in securities markets.

We consider a model in which two risky assets, denoted 1 and 2, trade on platforms operated by a single exchange.[2] The markets for the two assets open sequentially (asset 1 trades first). In each asset market, a monopolist dealer sets bid and ask quotes. Each dealer has a specific clientele that consists of liquidity traders, with a private motive for buying or selling the asset, and speculators, who can learn the asset's payoff at a cost.

[1] For instance, Hendershott et al. (2025) shows that the introduction of a new data product by the NYSE in 2015 triggered an increase in trading activity by both subscribers and non-subscribers.

[2] Asset 1 might be an index ETF and asset 2 a futures contract on the same index. For instance, the iShares MSCI World ETF trades on NYSE Arca, while the ICE MSCI World Futures trades on the Intercontinental Exchange, which owns the NYSE that operates NYSE Arca. Glosten (2020) notes that exchanges are best viewed as "monopolistic competitors" because their products (in particular, their data) are not perfect substitutes. Thus, for simplicity, we consider a single exchange.

The exchange charges a trading fee per share traded in each asset. In addition, the exchange can sell data (hereafter, "transaction data") about the outcome of the first round of transactions. Because the payoffs of the two assets are correlated, the order flow for asset 1 is informative about the payoff of asset 2, as it reflects speculators' demand. Importantly, the exchange can add noise to the realized order flow for asset 1 to generate the transaction data that it sells. By varying the precision of the signal (the "quality" of data), the exchange chooses its degree of opaqueness from being "dark" (the exchange does not sell transaction data) to being fully transparent (transaction data reveals the order flow without noise).

Before trading starts, the exchange sets the trading fees for assets 1 and 2, the fee it charges for access to transaction data (data fees), and the quality of its data. Investors in asset 2 then decides whether to purchase the data, and speculators decide whether to produce information about the payoff of the asset in which they are active. All agents are risk-neutral and maximize their expected profits (there are no noise traders in the model). Liquidity traders' aggregate demand and supply decreases with the total cost of trading (dealers' spread plus the exchange trading fee) while the mass of speculators adjusts such that they break even after accounting for trading costs and the cost of information acquisition, c.

In this framework, the dealer in asset 2 is willing to pay for transaction data because these data contain information about the payoff of asset 2, which she can then use to set her quotes and mitigate her exposure to adverse selection.[3] Thus, our framework highlights an important determinant of the demand for transaction data: they allow intermediaries to price assets more accurately and to reduce adverse selection costs generated by informed

[3]In the equilibrium of our model, neither speculators nor liquidity traders have incentives to purchase the transaction data, because the dealer's quote reveal the information contained in the data. In practice, not all trading firms buy market data. For instance, Hendershott et al. (2025) find that 20% of trading firms (brokers, investment banks, prop trading firms or market making firms) in their sample do not buy any data from the NYSE.

speculation. Since the exchange is a monopolist producer of its data, its data fee equals the increase in the dealer's expected profit from observing market data.[4]

As a baseline, we first consider the case in which the exchange cannot sell data. Then, the exchange's choice of trading fees for asset 1 and 2 are independent of each other. In each asset market, the exchange sets the fee to maximize its profit, given the decreasing demand for matching services from liquidity traders and speculators. We call "baseline trading fees" the optimal trading fees when the exchange cannot sell data.

Data sales change the trade-offs the exchange faces when setting its trading fees in several ways. To build intuition, consider first the case in which the exchange can freely choose the informativeness of its data—that is, it is not constrained by the requirement that this informativeness cannot exceed that of the order flow of asset 1. The value of data for dealer 2 increases with the informativeness of the data and with her exposure to adverse selection resulting from speculators' trading activity in asset 2. To increase this activity, the exchange reduces its trading fee for asset 2 relative to the baseline. In addition, speculators' information advantage and therefore trading activity decline with the informativeness of the data. To mitigate this issue, the exchange further reduces its trading fee for asset 2 and limits the informativeness of its data. The exchange's incentives to limit the informativeness of its data are stronger when the speculators' cost of information c is high.

This argument highlights a conflict between maximizing data sale revenue and maximizing trading revenues for asset 2, as the former objective pushes the exchange to reduces its fee relative to the baseline. In other words, the sale of data cannibalizes the traditional sources of revenues of exchanges. When the cost of information acquisition by speculators is large, this cannibalization effect trumps potential profits from data sale. In this case, the

[4]This requires dealers to earn strictly positive expected profits, at least when purchasing the data. To ensure that dealers get rents, we assume that dealers are monopolists for simplicity.

exchange prefers to be dark, that is, it does not sell any data.

So far, this discussion assumes that the exchange can fully control the informativeness of its data. However, these data reflect transaction activity for asset 1, and thus they cannot be more informative than the order flow for asset 1 itself. While the exchange can downgrade the quality of its data at no cost by adding noise to the order flow, increasing the informativeness of the order flow requires lowering the trading fee for asset 1, relative to the baseline. Whether the exchange seeks to increase the informativeness of the order flow crucially depends on the cost of acquiring information for speculators, c.

When c is large, the informativeness of the order flow is not a limiting factor, since the exchange optimally chooses to sell a signal of lower precision than the signal conveyed by the order flow in asset 1. Therefore, the exchange sets the same fee for asset 1 as in the baseline case, and it downgrades the quality of its data to achieve the desirable level of informativeness. In particular, as mentioned above, when c is large enough, the exchange is dark.

However, when c is low enough, the negative impact from the sale of information on speculators' trading activity is muted, so the exchange wants to sell high-quality data. In this case, the informativeness of the order flow in asset 1 becomes a limiting factor when the ratio of the elasticity of liquidity traders' demand to asset volatility, r_{ev} is high. Intuitively, this ratio measures the relative importance of liquidity traders in the order flow, which is negatively related to its informativeness. Thus, when r_{ev} is high, the exchange lowers its trading fee for asset 1 to incentivize speculators to acquire information about this asset, thereby enhancing order-flow informativeness, and it chooses to be fully transparent. Instead, when r_{ev} is low, the same conclusions as in the high-c case apply, because the order flow is highly informative: the exchange leaves the fee for asset 1 unchanged and chooses an

intermediate level of transparency.

A standard result in the literature on information sales in financial markets (e.g., Admati and Pfleiderer (1986), Cespa (2008), García and Sangiorgi (2011)) is that it can be optimal for an information seller to add noise to its signal about an asset's payoff. Our finding that it can be optimal for the exchange to sell opaque transaction data is similar in spirit. However, the problem of an exchange selling data differs from that of a pure information seller in two important ways. First, the informativeness of the signal available to the exchange is endogenous, and it depends on its trading fees. In markets dominated by liquidity traders, the exchange reduces it trading fee and chooses to be fully transparent, as the order flow is not very informative. Second, the sale of data cannibalizes revenues from trading fees. When this effect is strong, the exchange chooses to be completely dark, that is, it does not sell any transaction data. Such an outcome can never optimal for a pure information seller.

Cespa and Foucault (2014) and Easley et al. (2016) also study data sales by exchanges. However, liquidity traders in these models are noise traders, whose demand is inelastic to trading fees. This precludes an analysis of the effect of trading fees on their decision and welfare. In contrast, in our paper, the exchange jointly prices its matching and data services, accounting for the effect of its fees on all participants' trading decisions and how they affect both its trading and data sell revenues.

Our paper also relates to the broader literature on the industrial organization of exchanges in securities markets (e.g., Gehrig (1993), Degryse et al. (2009), Colliard and Foucault (2012), Chao et al. (2018)). Some papers in this literature consider the sale of fast trading technologies by exchanges (e.g., Biais et al. (2015), Pagnotta and Philippon (2018), Cespa and Vives (2021), or Budish et al. (2024)). There are two important differences with this literature. First, in our setup, trading speed plays no role. In particular, dealers buy data because it

reflects speculators' private information and thereby helps them reduce their exposure to adverse selection, not because it enables them to react faster to the arrival of public news. The former motivation for buying data exists even when dealers are not exposed to arbitrageurs reacting quickly to news (e.g., in slow markets). Second, we analyze in detail how the sale of data affects exchanges' choices of trading fees and show that data sales generally result in lower trading fees. This can explain why, over time, trading fees have declined while revenues from data sales have increased for trading platforms.[5]

Last, our paper is related to the vast theoretical literature on transparency in securities markets (e.g., Madhavan 1995, Pagano and Roell 1996, Bolton et al. 2016, Asriyan et al. 2017, Vairo and Dworczak 2023). This literature distinguishes pre-trade transparency—information on posted quotes before trading—from post-trade transparency—information on past trades. In our model, the exchange is always pre-trade transparent (quotes can be observed at zero cost), but it controls its post-trade transparency through the precision of its signal about past transactions and through its data fee. This is a key difference from the existing literature, which typically takes as given the information that exchanges provide to investors. In contrast, our analysis views transparency as one of the instruments available to exchanges to maximize their total profits.[6] It shows that the optimal pricing policy and transparency of the exchange depends on the characteristics of the asset. This offers a possible explanation for why the level of transparency varies across asset classes (e.g., corporate bonds trade in less transparent markets than stocks), even though there is no constraint anymore on using similar matching technologies across all assets.

[5]See Chapter 4 in Duffie et al. (2022).

[6]See Hendershott and Jones (2005) for an empirical analysis of the effects of one major trading platform in U.S. equity markets choosing to go dark.

2 The Model

The economy has four dates $t \in \{0, 1, 2, 3\}$. There is an asset with payoff $v \in \{-\sigma, +\sigma\}$ at $t = 3$. The probability of a high payoff $v = \sigma$ is $p_0 = 1/2$. The economy is populated by liquidity traders, speculators, market makers, and an exchange (operator) who runs the platform where the asset is traded at dates $t \in \{1, 2\}$. All agents are risk-neutral.

Liquidity Traders

At each trading date $t \in \{1, 2\}$, a continuum of short-lived liquidity traders either wish to buy or sell the asset. Let $m_t^{L+} \sim \mathcal{U}[0, \bar{m}]$ be the mass of liquidity buyers, and $m_t^{L-} \sim \mathcal{U}[0, \bar{m}]$ be the random mass of liquidity sellers. Variables m_t^{L+} and m_t^{L-} are uncorrelated at any date $t = 1, 2$ and also uncorrelated across time.

A date-t liquidity buyer derives a private benefit $\varepsilon_t^+ \sim \mathcal{U}[0, \bar{\varepsilon}]$ from holding the asset, while a date-t liquidity seller derives a private benefit $\varepsilon_t^- \sim \mathcal{U}[0, \bar{\varepsilon}]$ from shorting the asset. This private benefit reflects idiosyncratic hedging needs. The variables ε_t^+ and ε_t^- are uncorrelated for each $t = 1, 2$ and uncorrelated across time. This specification implies that liquidity traders are price-sensitive; the private benefit measures their willingness to trade.

Each liquidity trader can submit a unit order to buy or to sell the asset on the exchange. Let $\tilde{D}_t^L$ ($\tilde{S}_t^L$) denote the aggregate demand (supply) from liquidity traders at date t.

Speculators

There are two generations of short-lived speculators, who trade either at date 1 or at date 2. Speculators have no private benefit for the asset, but they can acquire information about its payoff. At date 0, each speculator of generation $t \in \{1, 2\}$ can pay a cost c to learn the asset payoff privately. The endogenous mass of (informed) speculators is denoted μ_t. Each speculator can submit a unit order to buy or to sell the asset to the exchange. Let

$\tilde{D}_t^I$ ($\tilde{S}_t^I$) denote the aggregate demand (supply) from Informed speculators at date t.

Market Makers

At each trading date $t \in \{1, 2\}$, one short-lived Market Maker (MM) sets a bid price, b_t to execute sell orders and an ask price, a_t, to execute buy orders. The MM sets these quotes prior to observing the total demand flow $\tilde{D}_t = \tilde{D}_t^L + \tilde{D}_t^I$ and the total supply flow $\tilde{S}_t = \tilde{S}_t^L + \tilde{S}_t^I$ from traders. While market making is unlikely to be monopolistic in practice, it helps to obtain simple closed-form solutions. In addition, we will show that several important results do not depend on this market structure.

The Exchange

The exchange is open at both trading ddates,$t \in \{1, 2\}$ and the exchange operator is the only long-lived agent in our model. The exchange charges a trading fee f_t at date t, which is paid by traders (rather than the MM) without loss of generality. In addition, the exchange sells information about the date-1 order flow to the date-2 MM.[7] This order flow contains information about the asset value to the extent that speculators trade at date 1. To predict the asset value, the order flow is informationally equivalent to the posterior probability that $v = \sigma$, denoted $\bar{p}$. Thus, we assume that the exchange sells a signal about $\bar{p}$ (rather than about the order flow). The posterior $\bar{p} \in [0, 1]$ has an endogenous probability distribution, denoted $\bar{\pi}(.)$, that we will characterize as an outcome of the date-1 trading game.

The exchange can add noise to the true posterior $\bar{p}$. We model noise as a (random) variable with unit mass on the uninformed posterior $p_0 = \frac{1}{2}$. Formally, the signal p sold by

[7]We will show that in equilibrium, the (informed) MM quotes reveal her signal. Hence, no date-2 trader would pay for a signal that they can infer from these quotes at no cost.

the exchange obtains from the true posterior $\bar{p}$ as follows:

$$p \equiv \begin{cases} \bar{p} & (\text{prob } \tau) \\ \frac{1}{2} & (\text{prob } 1-\tau) \end{cases}, \tag{1}$$

where $\tau \in [0,1]$ is the signal precision chosen by the exchange. With probability τ $(1-\tau)$, the signal is equal to the true posterior $\bar{p}$ (pure noise). With definition (1), upon observing noisy signal p, an agent posterior is equal to the signal itself, that is,[8]

$$\forall\, \tau \in [0,1], \quad \mathbb{P}[v = \sigma \,|\, p] = p \tag{2}$$

In the first part of our analysis, we will analyze the trading game at date 2 for given MM priors p_2 that the asset value is high. Then, we will endogenize these date-2 priors as realizations of the equilibrium distribution of the exchange signal.

Assumptions

To avoid issues with equilibrium existence under adverse selection, we impose the following condition on parameters.

Assumption 1. $\bar{\varepsilon} \geq \sigma$

Assumption 1 ensures that liquidity traders' gains from trade are large, relative to the information advantage of speculators. This implies that speculators are priced out before the last liquidity trader as trading costs increase. As we will show, this guarantees that an equilibrium with trading always exists. Next, a market for information exists only if some speculators enter the market. The next assumption ensures that this is the case.

[8] Our specification of noise implies that this property holds for any distribution $\bar{\pi}$ of the true posterior. Indeed, a signal $p \neq \frac{1}{2}$ can only be generated by the true posterior, while signal $p = \frac{1}{2}$ does not move the prior, whether it is a realization of the true posterior $\bar{p}$ or noise.

Assumption 2. $\bar{\varepsilon} \leq \sqrt{2}(\sigma - c)$

Intuitively, Assumption 2 holds when the information advantage of a speculator, σ is large relative to the information acquisition cost, c. Observe that Assumption 1 and 2 hold jointly only if $\bar{\varepsilon} \in [\sigma, \sqrt{2}\sigma]$ and $c < \sigma - \bar{\varepsilon}/\sqrt{2}$.

Timing At date 0, there are two stages. First, the exchange chooses its design, that is, it sets its trading fees $\boldsymbol{f} = (f_1, f_2)$ and the signal precisions $\boldsymbol{\tau}$. Then, speculators of each generation $t \in \{1, 2\}$ decide whether to acquire private information, while the date-2 MM decides whether to purchase the exchange signal. Next, the two trading periods take place. At each date $t \in \{1, 2\}$, the date-t MM post quotes and date-t investors (liquidity traders or speculators) decide whether to hit these quotes. At the end of date 1, if the date-2 MM has purchased the signal, it receives it from the exchange. We proceed backwards to solve the model. In Section 3, we solve for the trading equilibrium at date 1 and 2. Next, in Section 4, we characterize the speculators' and date-2 MM decisions to acquire information. Then, we solve for the optimal exchange design in Section 5.

3 Trading Stages

There are two trading stages. In each stage $t \in \{1, 2\}$, the mass of speculators μ_t, and the exchange fee f_t are given. The other determinant of the trading game is the information set of the MM, given by its prior p_t that $v = \sigma$. Given these variables, the trading game is otherwise time-invariant. The date-1 MM is uninformed and has the same prior $p_1 = \frac{1}{2}$ as liquidity traders. If the date-2 MM bought the exchange signal, she possesses super information relative to the liquidity trader. In this case, her prior p_2 that the asset value is

high is endogenous. The following conjecture will simplify our analysis.

Conjecture 1. *The equilibrium support of the prior p_2 of the date-2 MM is* $\{0, 1/2, 1\}$.

Conjecture 1 trivially holds if the date-2 MM does not buy the exchange signal, since then $p_2 = p_1 = \frac{1}{2}$. The crux of the statement is that a date-2 MM that buys the signal either knows the asset value perfectly ($p_2 \in \{0, 1\}$), or stays uninformed ($p_2 = \frac{1}{2}$). We will prove this conjecture when we derive the equilibrium of the date-1 trading stage.

At date 2, the trading game is a signaling game, because the (potentially) informed MM moves first by setting quotes. The MM type is her prior $p_2 \in \{0, 1/2, 1\}$ that the asset value is high. It is well-known that such games can have multiple equilibria. A notable feature of our environment, however, is that the symmetric-information equilibrium, in which the MM information is public, is also an equilibrium of the signaling game. We select this equilibrium to simplify the analysis. Thus, in the main text, we solve for the date-2 equilibrium as if the MM info $p_2 \in \{0, 1/1, 1\}$ were public. In the interest of space, we relegate the proof that the symmetric-information outcome is an equilibrium of the signaling game to Appendix B.[9]

An equilibrium of the trading games consists in bid and ask prices $(b^*(p), a^*(p))$ for each possible value of the *commonly-known* prior about the asset value $p \in \{0, 1/2, 1\}$. Let

$$v(p) \equiv \mathbb{E}[v|p] = 2\sigma p - \sigma, \tag{3}$$

be the conditional expected value of the asset given the prior p. We proceed in three steps to characterize the equilibrium. First, given a commonly-known prior $p \in \{0, 1/2, 1\}$, we

[9]The intuition for this surprising result is that the requirement to post two-sided quotes prevents deviations. A MM with bad news about the asset is tempted to post the same ask price as a MM with good news, because this would mislead liquidity traders into thinking that the asset has a high value. However, to mimic that better type, it must also mimic its bid, which it knows is overpriced. Since the MM expects more sell orders than buy orders following bad news, she stands to lose more from buying at an inflated bid than she gains from selling at an inflated ask.

derive investors' strategies. Second, we solve the maximization problem of the MM. Finally, we characterize the equilibrium distribution of the exchange signal to verify Conjecture 1.

3.1 Investors' Strategies

We derive traders' demand and supply functions for the asset, for any common prior $p \in [0,1]$. Given the MM quotes (a,b), and the exchange fee, f, a liquidity buyer with private benefit $\varepsilon^+ \in [0,\bar{\varepsilon}]$ buys if and only if the sum of the asset value, $v(p)$ and her private benefit, ε^+ exceeds the total trading cost, that is, if and only if

$$v(p) + \varepsilon^+ \geq a + f. \tag{4}$$

Symmetrically, a liquidity seller with private benefit $\varepsilon^- \in [0,\bar{\varepsilon}]$ sells if and only if

$$\varepsilon^- - v(p) \geq f - b, \tag{5}$$

that is she sells the asset if her private benefit plus the payoff of a short position exceeds the trading cost, which is equal to the exchange fee less the MM bid.

With a random mass $\tilde{m}^+$ ($\tilde{m}^-$) of potential liquidity buyers (seller), the random aggregate liquidity demand and supply functions are

$$\tilde{D}^L(a,p) = \tilde{m}^+ \Pr[\varepsilon^+ \geq a + f - v(p)] = \tilde{m}^+ \left(1 - \frac{a - v(p) + f}{\bar{\varepsilon}}\right), \tag{6}$$

$$\tilde{S}^L(b,p) = \tilde{m}^- \left(1 - \frac{v(p) - b + f}{\bar{\varepsilon}}\right). \tag{7}$$

Now, consider speculators' strategies. Speculators, who know the asset value v, buy (sell) if $v = \sigma$ ($v = -\sigma$), provided that the buying cost $f + a$ (selling cost $f - b$) does not exceed

the gross profit σ. The demand and supply function from speculators are thus given by

$$\tilde{D}^I(a, v) = \mathbf{1}_{\{v=\sigma\}}\mathbf{1}_{\{a+f\leq\sigma\}}\mu, \tag{8}$$

$$\tilde{S}^I(b, v) = \mathbf{1}_{\{v=-\sigma\}}\mathbf{1}_{\{f-b\leq\sigma\}}\mu, \tag{9}$$

where μ is the mass of speculators and $\mathbf{1}$ the indicator function.[10]

3.2 Equilibrium of the Trading Game

Now, we may state the MM problem and derive the equilibrium quotes. Given a common prior $p \in \{0, 1/2, 1\}$, the MM sets its bid b and ask a to solve the following problem:

$$\max_{(a,b)} \Pi^{MM}(a, b; p) \equiv \mathbb{E}\Big[\tilde{D}(a - \tilde{v}) + \tilde{S}(\tilde{v} - b) \,|\, p\Big], \tag{10}$$

where total demand $\tilde{D}$ (supply $\tilde{S}$) obtains by summing (6) and (8) ((7) and (9)). When $p \in \{0, 1\}$, speculators have no information advantage. The MM then faces a standard monopolist problem with price-elastic liquidity traders. When the MM is less informed than speculators, that is, when $p = \frac{1}{2}$, she faces adverse selection. In that case, the MM also sets quotes to minimize losses with speculators.

Proposition 1. *Given a mass μ of speculators and an exchange fee f such that*

$$\sigma > \frac{1}{2}(\bar{\varepsilon} + f) + \frac{1}{2}\frac{\bar{\varepsilon}}{\bar{m}}\mu \tag{11}$$

[10]Speculators may randomize between trading and abstaining when $a + f = \sigma$ or $f - b = \sigma$, but this case will not arise in equilibrium. See the proof of the next result, Proposition 1, for details.

the equilibrium bid and ask prices satisfy

$$v(p) - b^*(p) = a^*(p) - v(p) = \begin{cases} \frac{1}{2}(\bar{\varepsilon} - f) & \text{if} \quad p \in \{0, 1\} \\ \frac{1}{2}(\bar{\varepsilon} - f) + \frac{1}{2}\frac{\bar{\varepsilon}}{\bar{m}}\mu & \text{if} \quad p = \frac{1}{2} \end{cases} \tag{12}$$

Proposition 1 characterizes the (mid)spread $a^*(p) - v(p) = v(p) - b^*(p)$ charged by the MM for all values of $p \in \{0, 1/2, 1\}$. This spread measures the component of the total trading cost charged by the MM. When the asset value is publicly known, that is ($p \in \{0, 1\}$), speculators do not trade because they have no information advantage. Then, the spread reflects only the pricing power of the MM over liquidity traders. This spread increases with $\bar{\varepsilon}$, which is the inverse elasticity of the liquidity traders' demand. The MM rebates half of the exchange fee to traders to sustain demand. When the MM is uninformed ($p = \frac{1}{2}$), the spread has an adverse selection component, $\frac{1}{2}\frac{\bar{\varepsilon}}{\bar{m}}\mu$, due to speculative trading. This adverse selection spread increases with the ratio of speculators μ to the expected mass of liquidity traders, $\bar{m}/2$.

This equilibrium characterization given by equation (12) is valid only if equation (11) holds. This condition states that speculators enjoy a strictly positive trading gain, when $p = 1/2$, given by $\sigma - a^*(1/2) - f$, As our analysis will show, this condition holds in any equilibrium with speculator entry. Intuitively, if speculators cannot profit from their information advantage, they would not pay the information cost c ex-ante.[11]

Proposition 1 characterizes the trading equilibrium at both dates. At date 1, the MM has no signal, so $p_1 = \frac{1}{2}$. It also characterizes the equilibrium prices at date 2 for any possible MM signal $p_2 \in \left\{0, \frac{1}{2}, 1\right\}$ under Conjecture 1. Before verifying this Conjecture, we provide the next result to complete the equilibrium description.

[11]For completeness, the proof of Proposition 1 provides a complete characterization of the equilibrium quotes, also when condition (11) does not hold.

Proposition 2. *Let μ be the mass of speculators, f the exchange fee, and $p \in \{0, 1/2, 1\}$, the MM signal. The following results hold:*

1. *The expected trading volume is*

$$V(p, f, \mu) = \frac{\bar{m}}{2}\left(1 - \frac{f}{\bar{\varepsilon}}\right) + \frac{1}{2}\mu \mathbb{1}_{p=\frac{1}{2}}, \tag{13}$$

2. *the MM profit is*

$$\Pi^{MM}(p, f, \mu) = \begin{cases} \frac{\bar{m}}{4\bar{\varepsilon}}\left(\bar{\varepsilon} - f + \mu\frac{\bar{\varepsilon}}{\bar{m}}\right)^2 - \mu\sigma & \text{if} \quad p = \frac{1}{2} \\ \frac{\bar{m}}{4\bar{\varepsilon}}\left(\bar{\varepsilon} - f\right)^2 & \text{if} \quad p \in \{0, 1\} \end{cases}, \tag{14}$$

3. *and informed speculator profit is*

$$\Pi^{I}(p, f, \mu) = \begin{cases} \max\left\{0, \sigma - \frac{1}{2}(\bar{\varepsilon} + f) - \frac{1}{2}\frac{\mu\bar{\varepsilon}}{\bar{m}}\right\} & \text{if} \quad p = \frac{1}{2} \\ 0 & \text{if} \quad p \in \{0, 1\}. \end{cases} \tag{15}$$

First, observe from equation (15) that speculators benefit from their information advantage only when the MM is uninformed ($p = \frac{1}{2}$). This explains why the expected trading volume increases by the mass of speculators, μ, when $p = \frac{1}{2}$, relative to the case when $p \in \{0, 1\}$. Finally, the MM enjoys its largest profit when $p \in \{0, 1\}$, as it faces no adverse selection in this case. Next, we verify Conjecture 1 about the date-2 MM signal.

3.3 Equilibrium Exchange Signal

To complete the characterization of the trading stages, we solve for the distribution of the MM signal p_2. By definition (1), this signal is a noisy version of the posterior $\bar{p}$ formed by observing the date-1 order flow realized on the exchange. The case in which the MM does not purchase the signal corresponds to precision $\tau = 0$.

To simplify the analysis, we assume that the exchange sells information about the realization of date-1 demand only, rather than the joint realization of demand and supply.[12] That is, the signal without noise is defined as

$$\bar{p}(D_1^*) \equiv \Pr[v = \sigma \,|\, D_1^*], \tag{16}$$

where D_1^* is the total equilibrium demand at date 1. Once we characterize $\bar{p}$ and its distribution $\bar{\pi}$, we obtain that of the exchange signal p_2 thanks to equation (1).

First, we characterize the equilibrium demand at date 1, using results from the previous section. From equation (8), the total equilibrium demand by speculators is $\tilde{D}_1^{I,*} = \mu \mathbb{1}_{v=\sigma}$, that is, speculators buy if and only if the asset value is high. The equilibrium demand from liquidity traders obtains by substituting the equilibrium ask $a^*(p_1)$, given by (12) in demand function (8). We obtain

$$\tilde{D}_1^{L,*} \sim \mathcal{U}\left[0, \frac{\bar{m}}{2}\left\{1 - \frac{f}{\bar{\varepsilon}}\right\} - \frac{\mu}{2}\right] \tag{17}$$

Demand from liquidity traders is random, because the potential mass of liquidity traders $\tilde{m}^+$ is itself random. However, unlike speculative demand, it is uncorrelated with the asset value. Total equilibrium demand obtains as the sum of liquidity and speculative demand,

[12] This assumption is made for tractability, but it does not affect our results significantly. By construction, the signal based on joint supply and demand, given by $\Pr[v = \sigma \,|\, (D_1^*, S_1^*)]$ is more precise than the signal we consider, but its qualitative properties are similar.

$\tilde{D}_1^* = \tilde{D}_1^{L,*} + \tilde{D}_1^{I,*}$. For a given asset value $v \in \{-\sigma, \sigma\}$, total equilibrium demand is uniformly distributed. Speculative demand shifts the support of that distribution by μ when $v = \sigma$. Hence, low (high) realizations of equilibrium demand indicate a low (high) asset value. The next result characterizes this inference problem.

Lemma 1. *Given exchange fee f and a mass μ of date-1 speculators, the conditional equilibrium distribution of posterior $\bar{p}$ given the order flow, defined in (16), is*

$$\bar{\pi}^*(\bar{p} \,|\, v) = \begin{cases} 1 - q(\mu, f) & \text{if} \quad \bar{p} = \frac{1}{2} \\ q(u, f) & \text{if} \quad \bar{p} = \frac{1}{2}(1 + \frac{v}{2\sigma}) \end{cases} \tag{18}$$

where $v \in \{-\sigma, \sigma\}$ and

$$q(\mu, f) \equiv \min \left\{ \frac{2\mu}{\bar{m}\left(1 - \frac{f}{\bar{\varepsilon}}\right) - \mu}, 1 \right\} \tag{19}$$

is the probability that the signal reveals (perfectly) the asset value.

Lemma 1 shows that equilibrium demand either reveals the true asset value, with probability $q(\mu, f)$ or it reveals nothing with probability $1 - q(\mu, f)$. In particular, Lemma 1 together with equation (1) verifies Conjecture 1, which states that the date-2 posterior can only take values in $\{0, 1/2, 1\}$. The simplicity of the inference problem follows from the assumption that liquidity trading is uniformly distributed. As we observed, speculative demand shifts the support of the distribution of total demand by μ when the asset value is high. An extremely high (low) demand can only arise with (without) speculative trading. Hence, in this case, the signal perfectly reveals the asset value. Instead, an intermediate level of demand can as likely be generated with speculative trading as without. This implies that the date-2 MM does not update its prior upon observing such a signal.

Lemma 1 also shows that a sufficient statistics for the true posterior $\bar{p}$ is the probability

$q(\mu, f)$ that it reveals the asset value. This probability is equal to the mass of speculators divided by the mass of liquidity traders, an intuitive measure of the informativeness of the order flow. The more speculators trade (higher μ), the more equilibrium demand commoves with the underlying asset value, which increases the informativeness of the signal. Equation (19) shows that the informativeness of the order flow, $q(\mu, f)$ increases with the exchange fee f, because a larger fee discourages liquidity traders from participating. As we will show later, however, this partial-equilibrium comparative statics ultimately reverses, because the mass of speculators is endogenous to the exchange fee.

Finally, we derive the properties of the exchange signal, p_2, which is a noisy version of the true posterior $\bar{p}$. The next result follows from Lemma 1 and definition (1).

Proposition 3. *Given precision $\tau \in [0, 1]$, a mass of informed speculators μ and a fee f, the equilibrium distribution of the signal sold by the exchange is given by*

$$\pi^*(p) = \tau \bar{\pi}^*(p) + (1 - \tau)\mathbb{1}_{p=\frac{1}{2}} \tag{20}$$

with $\bar{\pi}^*(.)$ is the equilibrium distribution of the true posterior, given by (18).

Proposition 3 shows that adding noise to the original signal shifts mass from states in which the asset value is revealed to the state in which the signal is uninformative ($p = \frac{1}{2}$). In the limit when $\tau = 0$, the signal is pure noise, which nests the case without information sale. In that case, the date-2 MM is as uninformed as the date-1 MM. Proposition 3 implies that the informativeness of the exchange signal is the product of the true posterior informativeness $q(\mu, f)$ and of the precision τ.

Definition 1. *Given a mass μ of date-1 speculators, an exchange fee f and a precision τ, the exchange signal informativeness is $x \equiv \tau q(\mu, f)$.*

4 Information Acquisition

In this section, we characterize the speculator's incentives to acquire private information and the value of the exchange information for the date-2 MM. In particular, this step endogenizes the mass of speculators at each date as a function of the exchange policies

4.1 Equilibrium Speculation

First, consider the speculators' incentives to acquire information. An atomistic speculator's expected profit can be written as a function of the exchange trading fee f, the precision x of the signal received by the MM, and the mass μ of (other) speculators of that generation. From Proposition 2, a speculator's expected profit is given by

$$E\Pi^I(f,x,\mu) = (1-x)\max\left\{0, \sigma - \frac{1}{2}(\bar{\varepsilon}+f) - \frac{1}{2}\frac{\mu\bar{\varepsilon}}{\bar{m}}\right\}. \tag{21}$$

Observe that equation (21) applies to date 1 by setting the informativeness x of the date-1 MM's signal to 0. The speculator pays the information acquisition cost c if it is lower than the expected profit from speculation $E\Pi^I(f,x,\mu)$. Free entry drives the speculator's profit to zero, which determines the equilibrium mass of speculators.

Lemma 2. *Given exchange fee f, and informativeness x of the MM signal, the equilibrium mass of speculators is given by*

$$\mu^*(f,x) = \frac{2\bar{m}}{\bar{\varepsilon}}\max\left\{0, \sigma - \frac{1}{2}(\bar{\varepsilon}+f) - \frac{c}{1-x}\right\} \tag{22}$$

Intuitively, the equilibrium mass of speculators is larger when the potential information advantage σ is large, the cost of information c is low, when there is more liquidity trading

(higher $\bar{m}$), and liquidity traders are more price-elastic (lower $\bar{\varepsilon}$). Equation (22) shows that information acquisition by the MM amounts to an increase in the cost of private information from c to $c(1-x)^{-1}$ from the point of view of speculators. Intuitively, as the MM signal becomes more informative (x increases), speculators' profit and entry decreases. It is immediate to verify that, if speculators enter, that is, if $\mu^*(f,x) > 0$, equilibrium conjecture (11) that underpins Proposition 1 holds. Hence, all our results above now hinge on the result that speculators enter the market in equilibrium, which we will verify in the next section.[13]

Having solved for the equilibrium mass of speculators, we can express the informativeness of the exchange signal as a function of its policies. The next result obtains by substituting $\mu^*(f_1, 0)$ from equation (22) into expression (19).

Corollary 1. *Let f_1 be the date-1 exchange fee, the informativeness of the signal is*

$$x(f_1,\tau) \equiv \tau q(f_1) = \begin{cases} \tau \min\left\{\frac{2(\sigma-c)-\bar{\varepsilon}-f_1}{\bar{\varepsilon}-(\sigma-c)}, 1\right\} & \text{if} \quad \sigma - c > \frac{1}{2}(\bar{\varepsilon}+f) \\ 0 & \text{otherwise} \end{cases} \tag{23}$$

With a slight abuse of notation, we now write $q(f_1) = q(\mu^*(f_1,0), f_1)$ for the informativeness of the order flow, evaluated at the equilibrium mass of speculators for date 1. The informativeness of the order flow increases with $\sigma - c$, the information advantage of speculators net of cost, and it decreases with $\bar{\varepsilon}$, which measures the willingness to trade of liquidity traders. Interestingly, Corollary 1 shows that the signal informativeness decreases with the exchange fee, f_1. This result is not obvious, because the fee lowers the willingness to trade both for speculators and liquidity traders. However, in equilibrium, free entry of speculators implies that the marginal liquidity trader does not change with the fee. To see this, observe

[13] To see why positive entry implies condition (11), observe that the latter is equivalent to $\sigma > a^*(1/2) + f$ while positive speculator entry requires $\sigma - c > a^*(1/2) + f$

that the marginal date-1 liquidity trader has a private benefit equal to the total trading cost,

$$\hat{\varepsilon}(f_1) = a^*(1/2) + f_1 = \sigma - c \tag{24}$$

such that it is indifferent between trading and abstaining. The second inequality is the free condition for speculators that pins down the mass of date-1 speculators. Overall, equation (24) reveals that in any equilibrium with speculator entry, liquidity trading at date 1 is exogenous to the exchange policies. In equilibrium, a reduction in the exchange fee stimulates only speculative trading and thus increases the informativeness of the true posterior $\bar{p}$.[14]

4.2 Value of Exchange Information

Next, we determine the date-2 MM's willingness to pay for an exchange signal. The value of information for the date-2 MM is the extra expected profit it earns when informed. Using the equilibrium distribution of p_2 characterized in Proposition 3, together with Proposition 2, we can express the value of information as a function of the mass of speculators μ, the exchange fee f, and the informativeness of the exchange signal x.

$$VI^{MM}(\mu, f, x) \equiv \mathbb{E}_{p_2}[\Pi^{MM}(p_2, f)] - \Pi^{MM}\left(\frac{1}{2}, f\right) = \mu x \left[\sigma - \frac{1}{2}(\bar{\varepsilon} - f) - \frac{\mu}{4}\frac{\bar{\varepsilon}}{\bar{m}}\right]. \tag{25}$$

Given a mass of speculators μ, the value of information to the MM increases with the informativeness of the signal x. In equilibrium, however, speculation $\mu^*(f, x)$ decreases with the informativeness of the MM signal, because it lowers the speculators' information advantage vis-à-vis the MM. When taking into account the endogeneity of speculative volume

[14] Interestingly, the condition that speculators enter remains necessary and sufficient for this result, for any distribution of liquidity traders' private benefit, or any competitive structure of the market-making and exchange businesses.

μ, the value of the exchange signal to the MM is not monotonic in informativeness.

Lemma 3. *For any trading fee $f \leq 2(\sigma - c) - \bar{\varepsilon}$, the equilibrium value of the signal, $VI^{MM}(\mu^*(f,x), f, x)$ is maximized for an intermediate level of informativeness $\hat{x}(f) \in (0,1)$.*

Lemma 3 shows that the equilibrium value of the exchange signal is hump-shaped in its informativeness. The MM's willingness to pay for the exchange signal increases with its informativeness x, for a given mass of speculator μ. However, as the MM acquires better information (higher x), speculators' profit decreases, which implies that the equilibrium mass of speculators, $\mu^*(f,x)$, decreases as well, as shown by equation (22). This effect lowers the MM's willingness to pay for the exchange signal, as reducing the information gap with a decreasing mass of speculators becomes less valuable. Overall, these two effects combined explain why the value of information is nonmonotonic in informativeness.

Lemma 3 characterizes the optimal informativeness for a pure information seller. Due to the nonmonotonicity of the signal value with precision, the seller optimally adds noise to the trading signal. This echoes the classic result by Admati and Pfleiderer (1986) in a slightly different environment. As an exchange, however, the information seller in our model also operates the platform that generates the trading signal and on which the signal is used. As we show next, these features generate additional predictions regarding optimal trading fees and can even reinforce the classic opacity result by Admati and Pfleiderer (1986).

5 Exchange Design

Finally, we turn to the analysis of the exchange design stage. The exchange sets trading fees (f_1, f_2) for both dates and the precision of its signal, τ, to maximize revenues from trading fees and from selling information to the date-2 MM. As a monopolist, the exchange sets a

price for the signal, equal to the willingness to pay of the date-2 MM, given by equation (25). Revenues from trading fees depend on the expected trading volume, given by

$$EV(\mu, f, x) = \frac{\bar{m}}{2}\left(1 - \frac{f}{\bar{\varepsilon}}\right) + (1 - x)\frac{\mu}{2}, \tag{26}$$

which obtains by computing the expected value over the posterior p of $V(p, f, \mu)$, given by equation (13). We write the expected volume as a function of the signal informativeness, x, the trading fee f, and the mass of speculators μ for clarity. However, the expected volume depends ultimately only on the former two variables, because the mass of speculators is determined endogenously by equation (22). Finally, observe that expression (26) applies to date 1 by setting $(\mu, f, x) = (\mu_1, f_1, 0)$. Indeed, the date-1 market maker, who receives no information about the asset value, effectively possesses a signal with zero precision ($x = 0$).

Problem 1 (Exchange Design Problem)**.** *An optimal design for the exchange consists of trading fees* (f_1^*, f_2^*) *and a signal precision* τ^* *that solve the following problem*

$$\max_{(f_1, f_2, \tau)} \Pi^E(f_1, f_2, x) \equiv EV(\mu_1, f_1, 0)f_1 + EV(\mu_2, f_2, x)f_2 + VI^{MM}(\mu_2, f_2, x) \tag{27}$$

where the masses of speculators are $\mu_1 = \mu^*(f_1, 0)$ *and* $\mu_2 = \mu^*(f_2, x)$*, given by* (22) *and the signal informativeness is* $x = \tau q(f_1)$*, with* $q(f_1)$ *given by* (23)*.*

In words, the exchange maximizes the sum of the revenues from trading fees at date 1 and 2 and the revenue from the signal sold to the date-2 MM. As a monopolist, the exchange internalizes the impact of its policies on speculative trading, μ_1 and μ_2. The constraint $x = \tau q(f_1)$ states that the informativeness of the signal is the product of its precision $\tau \in [0, 1]$ chosen by the exchange and the informativeness of the order flow, $q(f_1)$, which

depends on the date-1 exchange fee f_1 (see Corollary 1). The latter is an upper bound on the signal informativeness, because the exchange can only lower the precision of the signal, via its choice of $\tau \in [0, 1]$, not increase it. To increase the informativeness of the signal, the exchange must reduce the fee f_1 that it charges at date 1 to attract more speculators.

To characterize the solution to Problem 1, the following concept will prove useful

Definition 2. *The desirable signal informativeness x^*_{unc} is the value of x that solves Problem 1 without the constraint $x \leq q(f_1)$*

Ignoring constraint $x \leq q(f_1)$ breaks the link between two subproblems faced by the exchange when maximizing overall profit. The first problem is to maximize trading revenues $EV_1 f_1$ at date 1—the first term of Π^E in equation (27). The second problem consists in maximizing the sum of the date-2 revenues from trading fees, $EV_2 f_2$ and the value of information, V^{MM}_{info}, respectively the second and third term of equation (27). This second objective depends on the date-1 fee only via the constraint $x \leq q(f_1)$ on signal informativeness. Ignoring this constraint thus decouples this two problems.By definition, the desirable optimal informativeness x^*_{unc} solves the second problem. This concept provides a recipe to study Problem 1 in full. Indeed, if constraint $x \leq q(f_1)$ does not bind at the solution of the decoupled problems, that solution coincides with the constrained optimum.

5.1 Benchmark: No Information Sale

First, we consider the case in which the exchange does not sell information. This analysis sets a benchmark for the level of fees, against which to assess the impact of the information market. Formally, we solve Problem 1 under the constraint $\tau = 0$.

When the exchange earns revenues only from trading fees, it faces a repeated static problem. The information market is the only connection between the two periods of trading,

because all market participants other than the exchange are short-lived. The expected trading volume EV, defined in (26), decreases with the fee. Thus, the exchange solves the classic problem of a monopolist facing a downward sloping demand curve.

Proposition 4. *When there is no information market ($\tau = 0$), there is positive speculator entry in equilibrium and the optimal exchange fee in both periods is the "baseline" level*

$$f_1^* = f_2^* = \bar{f} \equiv \frac{1}{2}(\sigma - c) \tag{28}$$

The level of fees in equation (28) will serve as a reference point for the analysis of the impact of the information market on trading costs. The exchange charges half of the speculator profit net of information acquisition cost. Remember that the total trading cost for investors is the sum of the exchange fee and the MM midspread. In our model, the zero-profit condition for speculators implies that this cost is equal to $\sigma - c$, as shown by equation (24). The exchange and the MM thus capture and split the rents of the speculators, with the exchange acting as a Stackelberg leader.[15]

Before moving to the analysis of the information market, we provide a useful benchmark for the informativeness of the order flow. Suppose the exchange sets the date-1 exchange fee as in Proposition 4, that is, when it earns revenues only from trading fees. The next result derives the informativeness of the date-1 order flow in this case

Corollary 2. *When the information market is shut down ($\tau = 0$), the informativeness of*

[15] We assume that the exchange and the market making business are monopolistic for simplicity However, the speculators' break-even condition pins down the equilibrium *trading cost* $a^*(1/2) + f^* = \sigma - c$ *for any market structure*. For instance, if exchanges do not charge fees, because e.g., of competition, equilibrium speculation would adjust until the MM spread $a^*(1/2)$ is equal to $\sigma - c$.

the (unobserved) order flow, called baseline informativeness, is

$$\bar{q} \equiv q(\mu^*(\bar{f}, 0), \bar{f}) = \min\left\{\frac{\frac{3}{2}(\sigma - c) - \bar{\varepsilon}}{\bar{\varepsilon} - (\sigma - c)}, 1\right\}$$

Corollary 2 gives the informativeness of the order flow when the exchange cannot sell a signal. Hence, it will generically not coincide with the equilibrium informativeness when the market for information is open ($\tau > 0$). Yet, the baseline informativeness level $\bar{q}$ is a useful reference for the analysis of the information market. It characterizes the most precise signal that the exchange can sell, *while* maximizing trading revenues at date 1 ($f_1 = \bar{f}$). Hence, any signal with precision $x \leq \bar{q}$ can be designed without changing the baseline fee, because the exchange can instead set any precision $\tau < 1$. It is only when the exchange wants to sell a signal with precision $x \geq \bar{q}$ that it needs to lower the date-1 fee below $\bar{f}$ to increase the informativeness of the order flow.

5.2 The Market for Information

Now, we analyze the full model in which the exchange optimally chooses fees at both dates, (f_1, f_2) and the precision of the signal it sells, τ. In particular, we characterize the optimal signal informativeness for the exchange and the impact of the market for information on the trading costs, relative to the benchmark fees of Proposition 4.

Problem 1 shows that the exchange fees and its choice of signal interact in several ways. First, the informativeness of the date-1 order flow, which is an upper bound for the signal informativeness x, decreases with the trading fee, f_1. Second, selling information to the date-2 MM reduces speculative trading μ_2 at date 2, which lowers the revenues from trading fees. Third, the date-2 trading fee itself affects the value of the signal to the date-2 MM, as

shown by equation (25). Despite the subtle intricacies of the fee and information policies, the following result emerges from our analysis.

Proposition 5. *If the exchange sells information ($\tau^* = 0$), the trading fees satisfy $f_2^* < \bar{f}$ and $f_1^* \leq \bar{f}$, that is, the market for information lowers the exchange trading fees.*

The ability to sell trading signals induces the exchange to lower trading fees, relative to the baseline fee $\bar{f}$ derived in Proposition 4. The market for information affects the date-1 fee f_1 and the date-2 fee f_2, via two distinct channels. First, the date-2 fee decreases to balance the negative effect of selling information on speculative trading. As shown by equation (22), speculative trading declines as the informativeness of the exchange signal increases, because speculators loses their information advantage vis-à-vis the date-2 MM. The exchange lowers its date-2 fee to mitigate this effect, because it benefits from speculative trading via the value of information itself and the revenues from trading fees, The date-2 fee and the precision of the signal are costs for speculators, so the exchange optimally lowers the former when it increases the latter.

Proposition 5 further states that the date-1 fee may only decline, relative to the benchmark $\bar{f}$ from Proposition 4, when there is a market for information. Importantly, the exchange changes its date-1 fee, relative to the baseline level $\bar{f}$ only to increase the informativeness of the order flow above its baseline level $\bar{q}$. When it seeks instead to decrease informativeness, the exchange uses the precision tool $\tau \in [0, 1]$, which is free relative to the date-1 fee that affects date-1 revenues. This asymmetry combined with the Lemma 1 imply the result, since Lemma 1 shows that the informativeness of the order flow decreases with the date-1 fee. Hence, the equilibrium fee may only be lower than its baseline level $\bar{f}$.

In Proposition 5, the result on date-2 fee holds if the exchange wishes to sell information, while the date-1 fee is strictly lower only if the exchange wants to increase the informative-

ness of the signal, relative to the baseline level. This leaves us with two questions. Does the exchange wish to sell a trading signal (extensive margin)?What is the desirable informativeness x^*_{unc} of this signal? To shed some light on these questions, we analyze first a simple case, and then provide a numerical analysis for the general case.

5.3 Special case $c = 0$

In this section, we analyze the special case in which private information is free for speculators ($c = 0$). This assumption simplifies the analysis, because it removes the link between the precision of the exchange signal x, and equilibrium speculation, μ_2 at date 2. This can be seen by setting $c = 0$ in equation (22), which becomes

$$\mu_2(f) = \frac{2m}{\bar{\varepsilon}} \left(\sigma - \frac{1}{2}(\bar{\varepsilon} + f) \right) \tag{29}$$

When $c = 0$, we can show that the objective function of the exchange is

$$\Pi^E = \frac{\bar{m}}{\bar{\varepsilon}}(\sigma - f_1)f_1 + \frac{\bar{m}}{\bar{\varepsilon}}(\sigma - f_2)f_2 + \tau q(f_1) \left[\left(\sigma - \frac{1}{2}\bar{\varepsilon} \right)^2 - \frac{1}{4}f_2^2 \right] \frac{\bar{m}}{\bar{\varepsilon}} \tag{30}$$

where $q(f_1)$ is the informativeness of the order flow, given by equation (23).

Without the negative feedback from the exchange signal to informed speculation, the exchange wishes to sell the most precise signal possible.

Lemma 4. *When speculators' private information is costless ($c = 0$), the exchange chooses the maximum precision $\tau^* = 1$, and the date-2 fee is given by*

$$f_2^* = \frac{2\sigma}{4 + q(f_1)} \tag{31}$$

Lemma 4 implies that the exchange never adds noise to the order flow when selling a signal to the date-2 MM. As mentioned above, when information acquisition by speculators is costless, equilibrium speculation does not depend on the precision of the signal. This implies that, as a seller of information, the exchange no longer wishes to garble the signal generated at date 1, unlike in Admati and Pfleiderer (1986). Furthermore, Lemma 4 characterizes the optimal date-2 fee as a function of the date-1 fee. Given that the signal informativeness satisfies $q(f_1) \in (0, 1]$, equation (31) confirms the general finding of Proposition 5 that the exchange strictly lowers its date-2 when it sells information.

Lemma 4 leaves us to characterize the optimal date-1 trading fee f_1^* and the implied informativeness $q(f_1^*)$ of the exchange signal.

Proposition 6. *When speculators' private information is costless ($c = 0$), there exists $\hat{\varepsilon} \in (\frac{5}{4}(\sigma - c), \sqrt{2}(\sigma - c))$ such that*

1. *If $\bar{\varepsilon} \leq \frac{5}{4}(\sigma - c)$, $f_1^* = \bar{f}$, and $x^* = q(f_1^*) = 1$*
2. *If $\bar{\varepsilon} \in \left(\frac{5}{4}(\sigma - c), \hat{\varepsilon}\right)$, $f_1^* = 3\sigma - 2\bar{\varepsilon} < \bar{f}$ and $x^* = q(f_1^*) = 1$,*
3. *If $\bar{\varepsilon} \in \left(\hat{\varepsilon}, \sqrt{2}(\sigma - c)\right)$, $f_1^* \in [3\sigma - 2\bar{\varepsilon}, \bar{f}]$, and it is increasing with $\bar{\varepsilon}$, which implies while $x^* = q(f_1^*) < 1$, and it is decreasing with $\bar{\varepsilon}$*

Proposition 6 characterizes the incentives of the exchange to lower its date-1 fee to increase the informativeness of its trading signal. It shows that the exchange needs to lower its fees only when $\bar{\varepsilon}/\sigma$ is high. This ratio measures the willingness to trade of liquidity traders, relative to speculators. When it is low, the date-1 order flow is very informative. The exchange can then kill two birds with one stone. By setting the date-1 fee equal to its baseline level, $f_1 = \bar{f} = \sigma/2$ it also generates the desired trading signal with maximum

informativeness, $q(f_1^*) = 1$. However, when $\bar{\varepsilon}/\sigma$ is large, the order flow does not fully reveal the asset value, that is, the baseline informativeness $\bar{q}$ is strictly lower than one. This induces the exchange to lower its fee in order to increase the informativeness of the order flow. Proposition 6 shows that for intermediate values of $\bar{\varepsilon}/\sigma$, the exchange sets the fee to provide a fully informative trading signal to the date-2 MM, at the expense of the fee revenues. However, when $\bar{\varepsilon}/\sigma$ is too large, providing a fully informative signal is no longer optimal, due to the large drop in fee revenues. Then, the exchange chooses the date-1 fee f_1^* to trade off fee revenues with the informativeness of the signal.[16]

Assuming that speculators' information is costless allows us to characterize precisely the equilibrium signal informativeness and the equilibrium fees in this case. However, some key effects are shut down when $c = 0$. First, from a normative point of view, the direct welfare costs from speculation disappear, because no resources are spent on private information. Second, as we explained above, it shuts down any negative effect of the market for information on speculative activity. Next, we consider the general case $c > 0$ to study how the optimal exchange design changes when this effect is present.

5.4 General case $c > 0$

When private information is costly for speculators ($c > 0$), Proposition 5 already shows that the exchange sets lower trading fees when it can sell information. Furthermore, we can derive

[16] In cases 1 and 2 of Proposition 6, date-2 speculators acquire information even if they enjoy no trading gain, as the date-2 MM is perfectly informed when $x = 1$. This outcome obtains as the limit of the equilibrium as c converges to 0. Strictly speaking, when $x = 1$, there exists another equilibrium of the information acquisition game with zero speculation. However, this cannot be part of an equilibrium of the exchange design game, because the exchange would then choose $x = 1 - \Delta x$, with Δ infinitesimally small, to stimulate speculation. Hence, the only equilibrium of the full model is as stated in Proposition 6.

the optimal date 2 fee as a function of the trading signal informativeness. We have

$$f_2^*(x) = \frac{\sigma - c - 2\frac{cx}{1-x}}{2 + \frac{x}{2}} \tag{32}$$

As claimed in Proposition 5, $f_2^*(x) < \bar{f} = \frac{\sigma - c}{2}$ when $x > 0$, that is, when the market for information is active. However, the dependence of the date-2 fee on signal informativeness, $x = \tau q(f_1)$ and the fact informativeness depends on the date-1 fee imply that there is no closed-form solution to Problem 1. To show how results change when speculation is costly, we provide a numerical analysis in Figure 1. The top (bottom) plots correspond to a low (high) value of the speculator's information acquisition cost c.

Consider first the low value of c. On the left panel, the blue line represents the equilibrium signal informativeness x^*. First, consider the dashed line that represents the desirable signal informativeness x^*_{unc} from Definition 2. The first key difference with the case $c = 0$ is that the exchange does not wish to maximize the informativeness of its trading signal, that is, $x^*_{unc} < 1$ when $c > 0$. When speculation is costly, selling information to the date-2 MM reduces speculative trading, which reduces both the value of the signal itself and the revenues from trading fees. Hence, the exchange no longer wishes to reveal the asset value to the date-2 MM, that is, $x^*_{unc} < 1$. Figure 1 reveals that for small values of $\bar{\varepsilon}$, the equilibrium signal informativeness (blue line) coincides with the desirable informativeness (dashed line).This configuration mirrors Case 1 of Proposition 6 when $c = 0$. Then, the exchange sets the date-1 fee equal to the baseline fee ($f_1 = \bar{f}$) that maximizes the date-1 revenues from trading fees *only*. In this region, the baseline informativeness $q(\bar{f})$, given by the red line, exceeds the desirable informativeness $\hat{x}$. Hence, the exchange simply sets precision τ^* such that the equilibrium signal informativeness satisfies $x^* = \tau^* q(\bar{f}) = \hat{x}$. When ε is high instead, a

trade-off arises between date-1 trading revenues and optimal informativeness. In this case, the desirable informativeness (dashed line) cannot be attained with the baseline fee. The exchange can increase the signal informativeness only by decreasing its date-1 trading fee below $\bar{f}$, as shown on the top right panel of Figure 1. In this case, the exchange trades off two objectives: maximizing revenues from trading fees and designing an optimal trading signal. This explains why $f_1 < \bar{f}$ and the equilibrium signal informativeness satisfies $x^* < x^*_{unc}$ (blue line below the dashed line) in this region.

Now consider the case when the cost c of speculators' private information is high (bottom panels of Figure 1). The comparison between desirable informativeness x^*_{unc} and equilibrium informativeness x^* still applies. However, when c is high, the desirable signal informativeness decreases, relative to the low-c case. Intuitively, as c increases, the negative effect of selling information to the date-2 MM on equilibrium speculation becomes much stronger. This limits the exchange appetite for selling information. With the value of c considered here, this argument has two noteworthy implications. First, the conflicts between trading fee revenues and signal informativeness never arises, as the benchmark signal informativeness (red line) exceeds the desirable signal informativeness (dashed line) for all values of $\bar{\varepsilon}$. Furthermore, when $\bar{\varepsilon}$, the exchange prefers not to sell any information, $\hat{x} = 0$. In this case, while the exchange has the ability to sell a trading signal, the equilibrium outcome is the same as if the market for information was exogenously shut down (Section **??**).

This last result hints at an important distinction between an exchange who sells information and a pure information seller. In the latter case, Admati and Pfleiderer (1986) showed that the seller optimally adds noise to the signal. However, a pure information seller always provides some information, because it has only one source of revenue. The exchange, however, also operates a trading platform where the signal sold is used by market participants.

Selling a signal undermines speculation, which reduces revenues from trading fees. When the cost of private information is large, this effect is so strong that the exchange prefers to remain completely opaque.

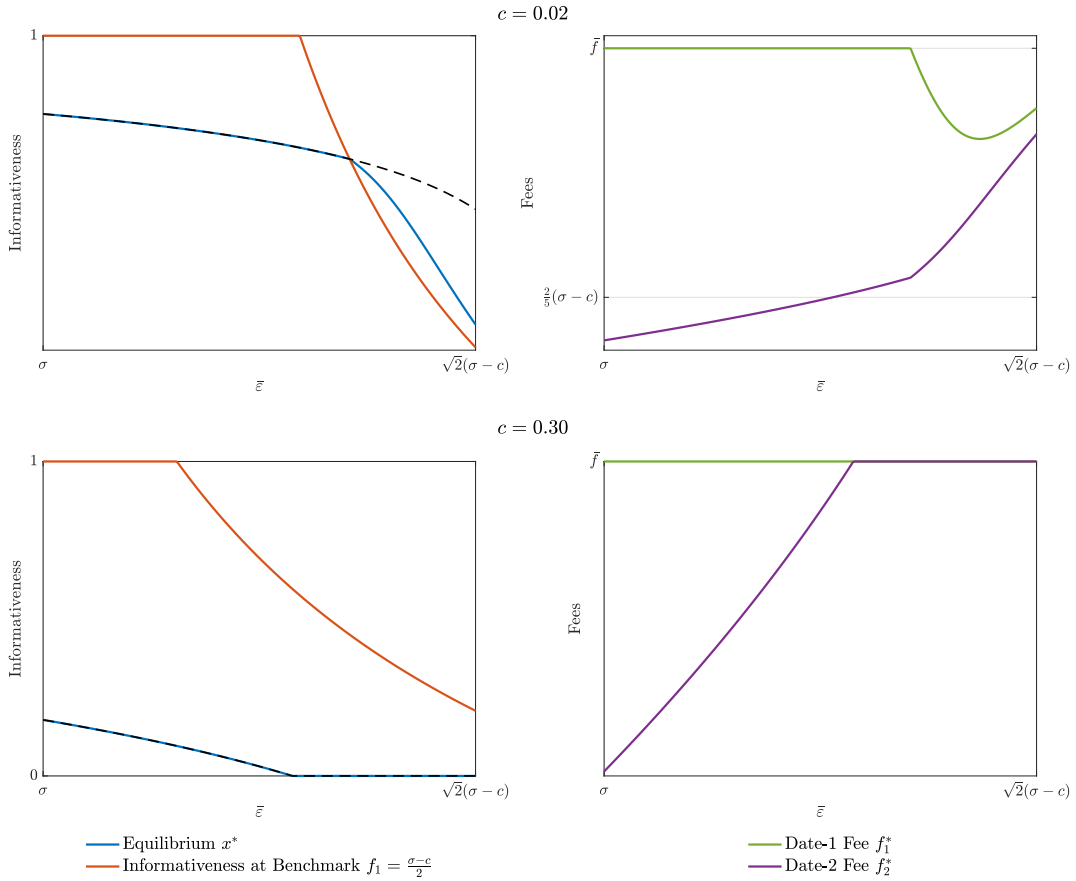


Figure 1. Numerical solution for exchange design Problem 1. The top (bottom) graphs correspond to a low (high) value of c. Left-column graphs show the equilibrium informativeness of the exchange signal. Right-column show the equilibrium trading fees.

6 Policy Analysis and Extensions

First, we determine the welfare effects of the market for information when the exchange chooses its optimal policies.

Definition 3. *Social welfare is given by*

$$W = \frac{\bar{m}}{\bar{\varepsilon}} \int_{a_1+f_1}^{\bar{\varepsilon}} \varepsilon \mathrm{d}\varepsilon + \frac{\bar{m}}{\bar{\varepsilon}} \mathbb{E}\left[\int_{a_2(p)-v(p)+f_2}^{\bar{\varepsilon}} \varepsilon \mathrm{d}\varepsilon \right] - (\mu_1 + \mu_2)c \tag{33}$$

Given that all agents have linear preferences, monetary transfers are neutral for social welfare. Hence, social welfare is equal to the gains from trade realized by liquidity traders, net of the private information acquisition costs paid by speculators.

[TO BE CONTINUED]

7 Conclusion

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Appendix

A Proofs

A.1 Proof of Proposition 1

First, observe that demand $\tilde{D}$ (supply $\tilde{S}$) depends only on the ask (bid) price. This implies that the MM problem is separable: each quote can be determined independently. Below, we solve for the optimal ask price. Then, we argue that the bid price solves a symmetric problem.

Optimal Ask Price The ask component of the MM profit is

$$\Pi^{MM}_{ask}(a;p) \equiv \mathbb{E}\big[(\tilde{D}^I + \tilde{D}^L)(a-\tilde{v}) \,|\, p\,\big] \tag{34}$$

$$= \mathbb{E}[\tilde{D}^L(a-\tilde{v}) \,|\, p] + \mathbb{E}[\tilde{D}^I(a-\tilde{v}) \,|\, p] \tag{35}$$

To characterize the informed speculators' demand, $\tilde{D}^I$, let $y^I(a)$ be the probability that an informed speculator with signal σ hits quote a. We have:

$$y^I(a) = \begin{cases} 0 & \text{if} \quad a+f > \sigma \\ y \in [0,1] & \text{if} \quad a+f = \sigma \\ 1 & \text{if} \quad a+f < \sigma \end{cases} \tag{36}$$

Rewriting equation (35) and using the definition of $y^I(a)$, we obtain that the MM solves the

following problem to determine the optimal ask $a^*(p)$

$$a^*(p) = \arg\max_a \frac{\bar{m}}{2} \max\left\{1 - \frac{a - v(p) + f}{\bar{\varepsilon}}, 0\right\} (a - v(p)) + p\mu y^I(a)(a - \sigma) \tag{37}$$

Consider first the simplest cases, $p \in \{0, 1\}$, in which the speculator enjoys no information advantage. When $p = 0$, the second term of (37) disappears. Assuming that liquidity demand is positive, we find

$$a^*(0) = \frac{1}{2}(\bar{\varepsilon} - f) + v(0), \tag{38}$$

by taking the first-order derivative of $\Pi^{MM}(a; 0)$ with respect to a. Liquidity demand is indeed positive at this quote if $a^*(0) + f - v(0) < \bar{\varepsilon}$, which is equivalent to $f < \bar{\varepsilon}$. Under Assumption 1, this inequality is implied by condition (11).

Next, consider the case $p = 1$. We guess and verify that $y^I(a) = 0$. Similar steps as above imply that

$$a^*(1) = \frac{1}{2}(\bar{\varepsilon} - f) + v(1). \tag{39}$$

It is straightforward that $a^*(1) + f > \sigma$ and thus that $y^I(a^*(1)) = 0$. The MM cannot benefit by charging $a \leq \sigma - f$, because she would reduce her profit on liquidity traders and induce trading by speculators. Again, liquidity demand is positive when condition (11) holds.

Finally, consider the case $p = \frac{1}{2}$, for which we prove an auxiliary result.

Lemma 5. *Let $\mu \geq 0$ and $f \geq 0$ the exchange fee. Let $\underline{f}(\mu) \equiv 2\sigma - \bar{\varepsilon} - \frac{\bar{\varepsilon}}{\bar{m}}\mu$ and $\bar{f} \equiv 2\sigma - \bar{\varepsilon}$.*

The equilibrium ask price for $p = \frac{1}{2}$ *is given by*

$$a^*(1/2) = \begin{cases} \frac{1}{2}(\bar{\varepsilon} - f) + \frac{1}{2}\frac{\bar{\varepsilon}}{\bar{m}}\mu & \text{if} \quad f \leq \underline{f}(\mu) \\ \sigma - f & \text{if} \quad f \in (\underline{f}(\mu), \bar{f}] \\ \frac{1}{2}(\bar{\varepsilon} - f) & \text{if} \quad f \geq \bar{f} \end{cases} \tag{40}$$

Before we prove this result, observe that the results for the ask price in Proposition 1 follows directly from this Lemma, because condition (11) is equivalent to $f \leq \bar{f}(\mu)$.

To prove Lemma 5, we consider in turn the cases without and with speculation. Suppose first that speculators do not trade, that is, $y^I = 0$. The optimal ask price is then $a^* = \frac{1}{2}(\bar{\varepsilon} - f)$. Hence, the conjecture $y^I(a^*) = 0$ holds if and only if $a^* + f > \sigma$, which is equivalent to $f \geq \bar{f}$. Next, conjecture that $y^I = 1$. Taking the first-order derivative of (37) with respect to a for $p = \frac{1}{2}$, we obtain

$$a^* - v(1/2) = \frac{1}{2}(\bar{\varepsilon} - f) + \frac{1}{2}\frac{\bar{\varepsilon}}{\bar{m}}\mu$$

The conjecture $y^I(a^*) = 1$ is valid if and only if $a^* + f \leq \sigma$, which is equivalent to $f \leq \underline{f}(\mu)$. Finally, conside the interval $f \in [\underline{f}(\mu), \bar{f}]$, which is nonempty when $\mu > 0$. In this interval, by construction, neither conjecture $y^I = 1$, nor conjecture $y^I = 0$ is compatible with equilibrium. And it must be that $y^I(a) \in (0, 1)$. Speculators find it optimal to mix if and only if $a^* + f = \sigma$. This concludes the proof of Lemma 5.

Optimal Bid We show that, to determine the optimal bid price, the MM solves the

same problem as when it chooses the ask price. The bid component of the MM profit is

$$\Pi^{MM}_{bid}(b;p) = \mathbb{E}[\tilde{S}^L(b-\tilde{v})\,|\,p] + \mathbb{E}[\tilde{D}^I(b-\tilde{v})\,|\,p] \tag{41}$$

$$= \mathbb{E}[\tilde{m}^-]\max\left\{1 - \frac{v(p)-b+f}{\bar{\varepsilon}}, 0\right\}(v(p)-b) + (1-p)\mu y^I(-b)(-b-\sigma) \tag{42}$$

$$= \Pi^{MM}_{ask}(-b; 1-p). \tag{43}$$

For the second line, we used the definition of y^I in (36). The last equality obtains because $v(1-p) = -v(p)$. This proves that $b^*(p) = -a^*(1-p)$ as shown in (12). This step concludes the proof.

A.2 Proof of Proposition 2

Consider first the expected trading volume given prior p. The expected volume from liquidity traders is

$$\begin{aligned} V^L(p,f) &= \frac{\bar{m}}{2}\left(1 - \frac{a^*(p) - v(p) + f}{\bar{\varepsilon}}\right) + \frac{\bar{m}}{2}\left(1 - \frac{v(p) - b^*(p) + f}{\bar{\varepsilon}}\right) \\ &= \frac{\bar{m}}{4}\left(1 - \frac{f}{\bar{\varepsilon}}\right) - \frac{1}{4}\mu\mathbb{1}_{p=\frac{1}{2}} + \frac{\bar{m}}{4}\left(1 - \frac{f}{\bar{\varepsilon}}\right) - \frac{1}{4}\mu\mathbb{1}_{p=\frac{1}{2}} \end{aligned} \tag{44}$$

For informed speculators, we have

$$V^I(p, f_e) = \frac{1}{2}\mu\mathbb{1}_{p=\frac{1}{2}} + \frac{1}{2}\mu\mathbb{1}_{p=\frac{1}{2}}$$

Summing $V^L(p,f)$ and $V^I(p,f)$, we obtain the expression for $V(p,f)$ in equation (13).

The equilibrium profit of the MM given prior $p \in \{0, 1/2, 1\}$ is

$$\Pi^{MM}(p, f) \equiv \Pi^{MM}\big(a^*(p), b^*(p); p\big) = 2\Pi^{MM}_{ask}(a^*(p), p), \tag{45}$$

where $\Pi^{MM}(a, b; p)$ is defined in equation (10) and Π^{MM}_{ask} is the ask component of the MM profit, defined in equation (37). Starting from equation (**??**), we get

$$\Pi^{MM}_{ask}(a^*(p), p) = \frac{\bar{\varepsilon}}{2\bar{m}}\left(\frac{\bar{m}}{2}\left[1 - \frac{f}{\bar{\varepsilon}}\right] - \frac{1}{2}\mu \mathbb{1}_{p=\frac{1}{2}}\right)^2 - \frac{1}{2}\mu\sigma \tag{46}$$

$$= \frac{\bar{m}}{8\bar{\varepsilon}}\left(\bar{\varepsilon} - f - \mu \mathbb{1}_{p=\frac{1}{2}}\right)^2 - \frac{1}{2}\mu\sigma \tag{47}$$

Combining (45) and (47), we obtain equation (14).

Finally, consider the profit for speculators. We have

$$\Pi^{I}(p, f, \mu) = \frac{1}{2}\max\{0, \sigma - a^*(p) - f\} + \frac{1}{2}\max\{0, \sigma - f + b^*(f)\}$$

Susbtituting for $a^*(f)$ and $b^*(f)$ using equation (12), we obtain equation (15).

A.3 Proof of Lemma 1

From equation (17), the conditional density of total equilibrium demand when the asset value is low is given by

$$f(D_1^* \,|\, v = -\sigma) = \begin{cases} 0 & \text{if} \quad D_1^* < 0 \\ \left(\frac{\bar{m}}{2}\left\{1 - \frac{f}{\bar{\varepsilon}}\right\} - \frac{\mu}{2}\right)^{-1} & \text{if} \quad D_1^* \in \left[0, \frac{\bar{m}}{2}\left\{1 - \frac{f}{\bar{\varepsilon}}\right\} - \frac{\mu}{2}\right], \\ 0 & \text{if} \quad D_1^* > \frac{\bar{m}}{2}\left\{1 - \frac{f}{\bar{\varepsilon}}\right\} - \frac{\mu}{2} \end{cases} \tag{48}$$

while the conditional density when the asset value is high is

$$f(D_1^* \,|\, v = \sigma) = f(D_1^* - \mu \,|\, v = -\sigma) \tag{49}$$

By Bayes law, the posterior that the asset value is high satisfies

$$\bar{p}(D_1^*) = \Pr[v = \sigma \,|\, D_1^*] = \frac{f(D_1^* \,|\, v = \sigma) \Pr[v = \sigma]}{f(D_1^* \,|\, v = \sigma) \Pr[v = \sigma] + f(D_1^* \,|\, v = -\sigma) \Pr[v = -\sigma]} \tag{50}$$

$$= \frac{f(D_1^* \,|\, v = \sigma)}{f(D_1^* \,|\, v = \sigma) + f(D_1^* \,|\, v = -\sigma)} \tag{51}$$

From equations (48) and (49), we obtain

$$\bar{p}(D_1^*) = \begin{cases} 0 & \text{if} \quad D_1^* \in [0, \mu] \\ \frac{1}{2} & \text{if} \quad D_1^* \in \left[\mu, \frac{\bar{m}}{2}\left\{1 - \frac{f}{\bar{\varepsilon}}\right\} - \frac{\mu}{2}\right] \\ 1 & \text{if} \quad D_1^* \in \left[\frac{\bar{m}}{2}\left\{1 - \frac{f}{\bar{\varepsilon}}\right\} - \frac{\mu}{2}, \frac{\bar{m}}{2}\left\{1 - \frac{f}{\bar{\varepsilon}}\right\} + \frac{\mu}{2}\right] \end{cases} \tag{52}$$

To complete the proof, we characterize the conditional distribution of $\bar{p}$ in equilibrium. First, consider the case $v = \sigma$. We have

$$\Pr[\bar{p} = 0 \,|\, v = \sigma] = \Pr[D_1^* \leq \mu \,|\, v = \sigma] = 0 \tag{53}$$

$$\Pr\left[\bar{p} = \frac{1}{2} \middle| \, | \, v = \sigma\right] = \Pr\left[D_1^* \in \left[\mu, \frac{\bar{m}}{2}\left\{1 - \frac{f}{\bar{\varepsilon}}\right\} - \frac{\mu}{2}\right] \middle| \, v = \sigma\right] = \frac{\frac{\bar{m}}{2}\left\{1 - \frac{f}{\bar{\varepsilon}}\right\} - 3\frac{\mu}{2}}{\frac{\bar{m}}{2}\left\{1 - \frac{f}{\bar{\varepsilon}}\right\} - \frac{\mu}{2}} \tag{54}$$

$$\Pr\left[\bar{p} = 1 | \, | \, v = \sigma\right] = \Pr\left[D_1^* \geq \frac{\bar{m}}{2}\left\{1 - \frac{f}{\bar{\varepsilon}}\right\} - \frac{\mu}{2} \middle| \, v = \sigma\right] = \frac{\mu}{\frac{\bar{m}}{2}\left\{1 - \frac{f}{\bar{\varepsilon}}\right\} - \frac{\mu}{2}} \tag{55}$$

By symmetry, the conditional probability when $v = -\sigma$ is given by

$$\forall p \in \{0, 1/2, 1\}, \quad \Pr[\bar{p} = p \,|\, v = -\sigma] = \Pr[\bar{p} = 1 - p \,|\, v = \sigma]$$

Define $q(\mu, f)$ as in equation (19), the distribution of $\bar{p}$ is indeed given by (18). This concludes the proof.

A.4 Proof of Lemma 3

From the expression of $\mu^*(f, x)$ in equation (22), there exists $\bar{x}(f) \in (0, 1)$ such that $\mu^*(f, x) = 0$ if $f \geq \bar{x}(f)$. The assumption $f \leq 2(\sigma - c) - \bar{\varepsilon}$ implies that $\mu^*(f, 0) > 0$. Hence, $\mu^*(f, x) > 0$ in a neighborhood of $x = 0$ by continuity.

When $\mu^* > 0$, we have (25), to obtain

$$V_I^{MM}(\mu^*(f,x), f, x) = \frac{2\bar{m}}{\bar{\varepsilon}} x \left[\sigma - \frac{1}{2}(\bar{\varepsilon} + f) - \frac{c}{1-x}\right] \left[\sigma - \frac{1}{2}(\bar{\varepsilon} - f) - \frac{1}{2}\left(\sigma - \frac{1}{2}(\bar{\varepsilon} + f) - \frac{c}{1-x}\right)\right] \tag{56}$$

$$= \frac{\bar{m}}{\bar{\varepsilon}} x \left[\sigma - \frac{1}{2}(\bar{\varepsilon} + f)\right] \left[\sigma - \frac{1}{2}\bar{\varepsilon} + \frac{3}{2}f\right] - \frac{\bar{m}}{\bar{\varepsilon}} \frac{c^2 x}{(1-x)^2}$$
$$- \frac{\bar{m}}{\bar{\varepsilon}} \frac{cx}{1-x} \left[\sigma - \frac{1}{2}\bar{\varepsilon} + \frac{3}{2}f - \sigma + \frac{1}{2}\bar{\varepsilon} + \frac{1}{2}f\right] \tag{57}$$

$$= \frac{\bar{m}}{\bar{\varepsilon}} x \left\{ \left[\sigma - \frac{1}{2}(\bar{\varepsilon} + f)\right] \left[\sigma - \frac{1}{2}\bar{\varepsilon} + \frac{3}{2}f\right] - 2\frac{fc}{1-x} - \frac{c^2}{(1-x)^2} \right\} \tag{58}$$

Let $f_1(x) = -x/(1-x)$ and $f_2(x) = -x/(1-x)^2$. We have

$$f_1'(x) = -(1-x)^{-2}, \qquad f_1''(x) = -2(1-x)^{-3} \tag{59}$$

$$f_2'(x) = -(1-x)^{-2} - 2(1-x)^{-3}, \quad f_2''(x) = -2(1-x)^{-3} - 6(1-x)^{-4} \tag{60}$$

This proves that both f_1 and f_2 are strictly concave for $x \in [0,1)$. Hence, their sum is strictly concave and $V_I^{MM}(\mu^*(f,x),f,x)$ is also strictly concave, because its second order derivative has the same sign as that of f_1+f_2. Given that $V_I^{MM}(\mu^*(f,x),f,x)$ is equal to 0 for $x > \bar{x}(f)$, it must remain positive over the interval $[0,\bar{x}(f)]$. Hence, there exists a unique maximum $\hat{x}(f) \in (0,\bar{x}(f))$.

A.5 Proof of Proposition 4

As observed in the main text, the exchange solves the same static problem at both dates. This problem is

$$\max_f EV(\mu^*(f,0),f,0) \times f = \left[\frac{\bar{m}}{2}\left(1-\frac{f}{\bar{\varepsilon}}\right)+\frac{1}{2}\max\{\mu^*(f,0),0\}\right] f \tag{61}$$

$$= \left[\frac{\bar{m}}{2}\left(1-\frac{f}{\bar{\varepsilon}}\right)+\frac{m}{\bar{\varepsilon}}\max\left\{\sigma-c-\frac{1}{2}(\bar{\varepsilon}+f),0\right\}\right] f \tag{62}$$

Let $\bar{f} = 2(\sigma-c)-\bar{\varepsilon}$ be the fee such that $\mu^*(f,0)=0$ if and only if $f \geq \bar{f}$. First, let us solve for f_I^*, the fee that maximizes (62) under the constraint $f < \bar{f}$, Guessing that $f_I^* < \bar{f}$, f_I^* solves

$$\max_f \frac{\bar{m}}{\bar{\varepsilon}}\big(\sigma-c-f\big)f$$

which yields $f_I^* = \frac{1}{2}(\sigma-c)$. Observe that $f_I^* < \bar{f}$ under Assumption 2. Next, consider the optimization problem under the constraint $f > \bar{f}$ such that $\mu^*(f,0)=0$. From equation (62), the solution is then $f_{NI}^* = \max\left\{\bar{f},\frac{\bar{\varepsilon}}{2}\right\}$. Observe that $\frac{\bar{\varepsilon}}{2} > \bar{f}$ if and only if $\bar{\varepsilon} > \frac{4}{3}(\sigma-c)$. Hence, by continuity of the exchange profit function at $f=\bar{f}$, the optimum is f_I^* if $\bar{\varepsilon} < \frac{4}{3}(\sigma-c)$. Thus, we are left to compare the exchange profit at f_I^* and $f_{NI}^* = \frac{\bar{\varepsilon}}{2}$ when $\bar{\varepsilon} \in \left[\frac{4}{3}(\sigma-c),\sqrt{2}(\sigma-c)\right]$, with the upper bound imposed by Assumption 2. The exchange

enjoys a larger profit at f_I^* than at f_{NI}^* if and only if

$$\begin{aligned} & 0 \leq \frac{\bar{m}}{\bar{\varepsilon}}(\sigma - c - f_I^*) f_I^* - \frac{\bar{m}}{2}\left(1 - \frac{f_{NI}^*}{\bar{\varepsilon}}\right) f_{NI}^* \\ \Leftrightarrow \quad & 0 \leq \frac{1}{4}(\sigma - c)^2 - \frac{1}{8}\bar{\varepsilon}^2 \end{aligned}$$

The last condition is equivalent to $\bar{\varepsilon} \leq \sqrt{2}(\sigma - c)$, which is Assumption 2.

A.6 Proof of Proposition 5

To prove the result, we first rewrite the exchange profit as follows:

$$\Pi^E = EV(\mu^*(f_1, 0), f_1, 0) f_1 + EV(\mu^*(f_2, x), f_2, x) f_2 + VI^{MM}(\mu_2^*(f_2, x), f_2, x) \tag{63}$$

$$\begin{aligned} &= EV(\mu^*(f_1, 0), f_1, 0) f_1 + \left[\frac{\bar{m}}{2}\left(1 - \frac{f_2}{\bar{\varepsilon}}\right) + \frac{1}{2}\big[1 - x\big]\mu^*(f_2, x)\right] f_2 \\ &\quad + x\mu^*(f_2, x)\left[\sigma - \frac{1}{2}(\bar{\varepsilon} - f_2) - \frac{1}{4}\frac{\bar{\varepsilon}}{\bar{m}}\mu^*(f_2, x)\right] \end{aligned} \tag{64}$$

$$\begin{aligned} &= EV(\mu^*(f_1, 0), f_1, 0) + \left[\frac{\bar{m}}{2}\left(1 - \frac{f_2}{\bar{\varepsilon}}\right) + \frac{1}{2}\mu^*(f_2, x)\right] f_2 \\ &\quad + x\mu^*(f_2, x)\left[\sigma - \frac{1}{2}\bar{\varepsilon} - \frac{1}{4}\frac{\bar{\varepsilon}}{\bar{m}}\mu^*(f_2, x)\right] \end{aligned} \tag{65}$$

From Lemma 2, we have

$$\mu^*(f, x) = \mu^*(f, 0) - \frac{2\bar{m}}{\bar{\varepsilon}}\frac{x}{1 - x}c \tag{66}$$

Substituting for the first occurrence of $\mu^*(f_2, x)$ in (65), thanks to equation (66), and substituting $\mu^*(f_2, x)$ in the second line of (65) thanks to equation (22), we obtain

$$\begin{aligned}\Pi^E = & EV(\mu^*(f_1, 0), f_1, 0)f_1 + EV(\mu^*(f_2, 0), f_2, 0)f_2 \\ & - \frac{\bar{m}}{\bar{\varepsilon}} \frac{x}{1-x} c f_2 + \frac{\bar{m}}{\bar{\varepsilon}} \left[\left(\sigma - \frac{1}{2}\bar{\varepsilon} \right)^2 - \left(\frac{1}{2} f_2 + \frac{c}{1-x} \right)^2 \right] x \end{aligned} \tag{67}$$

The terms on the first line of (67) correspond to the exchange profit function without information sale, as can be seen by setting $x = 0$. The second line of (67) thus collects the new profit components when the exchange sells information.

Now, we prove the statements of Proposition 5 thanks to the optimality conditions of the exchange problem. The exchange chooses (f_1, f_2, τ) to maximize Π^E, given in equation (67), subject to the constraint $x = \tau q(f_1)$, where $q(f_1)$ is given by (23). Here, we guess and verify that speculative volume is positive in equilibrium. In this case, we showed in the proof of Proposition 4 that

$$EV(\mu^*(f, 0), f, 0) = \frac{\bar{m}}{\bar{\varepsilon}}(\sigma - c - f) \tag{68}$$

The first-order condition with respect to the date-2 free thus writes

$$0 = \sigma - c - 2f_2 - \frac{x}{1-x} c - x \left(\frac{1}{2} f_2 + \frac{c}{1-x} \right) \tag{69}$$

Solving for f_2 as a function of x, we find

$$f_2 = 2 \frac{\sigma - c - \frac{2cx}{1-x}}{4 + x}, \tag{70}$$

which is strictly lower than $\bar{f} = (\sigma - c)/2$ when $x > 0$.

Next, consider the optimality conditions with respect to f_1 and τ. Observe that these variables affect the profit terms in the second line of (67) only via the informativeness $x = \tau q(f_1)$. The first-order conditions with respect to τ and f_1 are given respectively by

$$0 = q(f_1)\frac{\partial \Pi^E}{\partial x} + \lambda_{\tau 0} - \lambda_{\tau 1}, \tag{71}$$

$$0 = \sigma - c - 2f_1 + \tau q'(f_1)\frac{\partial \Pi^E}{\partial x}, \tag{72}$$

where $\lambda_{\tau 0}$ ($\lambda_{\tau 0}$) is the Lagrange multiplier on the constraint $\tau \geq 0$ ($\tau \geq 1$). Three cases are possible. First, $\tau^* \in (0,1)$, which implies that $\lambda_{\tau 0} = \lambda_{\tau 1} = 0$ and thus $\frac{\partial \Pi^E}{\partial x} = 0$. Then $f_1 = \bar{f}\frac{\sigma - c}{2}$. The same conclusion applies if $\tau = 0$. Finally, suppose that the constraint $\tau = 1$ binds. Then we have $\frac{\partial \Pi^E}{\partial x} > 0$, which implies that $f_1 < \bar{f}$ since $q'(f_1) < 0$, since, in this case, $q(f_1) < 1$. This concludes the proof.

A.7 Proof of Lemma 4

Setting $c = 0$ in equation (67) from the proof of Proposition 5, the

First, let us derive the optimal precision τ^*. Provided that $f_2^* < \sigma - \frac{1}{2}\bar{\varepsilon}$, which we verify in the next step of the proof, the exchange objective strictly increases with τ, which implies that $\tau^* = 1$ is optimal.

Next, the optimality condition with respect to f_2 is given by

$$0 = \sigma - 2f_2 - \frac{1}{2}q(f_1)f_2 \tag{73}$$

Equation (31) in the statement of the proposition follows directly from (73). Observe that $f_2 \leq \sigma/2$ because $q(f_1) > 0$, which verifies our conjecture about f_2.

A.8 Proof of Proposition 6

Using the results from Lemma 4, the exchange objective is to maximize $\Pi^E\big|_{\tau=1}$ over (f_1, f_2), given by equation (30), subject to

$$q(f_1) = \min\left\{\underbrace{\frac{2\sigma - \bar{\varepsilon} - f_1}{\bar{\varepsilon} - \sigma}}_{\equiv \bar{q}(f_1)}, 1\right\} \tag{74}$$

First, we characterize the candidate optima for f_1. Given the kink in function $q(f_1)$, there are three possible candidates for the optimal date-1 fee. First, $f_1 = \bar{f}$, that is, the fee that maximizes the date-1 revenues from trading fees, given by $\frac{\bar{m}}{\bar{\varepsilon}}(\sigma - c - f_1)f_1$. The second possibility is $f_1 = \hat{f}_1$, where $\hat{f}_1 \equiv 3\sigma - 2\bar{\varepsilon}$ is such that $\bar{q}(\hat{f}_1) = 1$. The last possibility is that f_1 solves the following optimality condition:

$$\begin{aligned} 0 &= \sigma - 2f_1 + \bar{q}'(f_1)\left[\left(\sigma - \frac{1}{2}\bar{\varepsilon}\right)^2 - \frac{1}{4}\left(f_2(f_1)\right)^2\right] \\ &= \sigma - 2f_1 - \frac{1}{\bar{\varepsilon} - \sigma}\left[\left(\sigma - \frac{1}{2}\bar{\varepsilon}\right)^2 - \frac{1}{4}\left(f_2(f_1)\right)^2\right] \end{aligned} \tag{75}$$

We label f_1^c this candidate value for the level of the date-1 fee.

Case 1: $f_1 = \bar{f}$

This fee is optimal when $\bar{q}(\bar{f}) \geq 1$, as fee $f_1 = \bar{f}$ then achieves both objective of maximizing the date-1 trading revenues and the informativeness of the signal. Substituting for $\bar{f} = \sigma/2$ in $\bar{q}(\bar{f})$, we obtain that condition $\bar{q}(\bar{f}) \geq 1$ is equivalent to $\bar{\varepsilon} \leq \frac{5}{4}\sigma$. In this case, we thus have $q(f_1^*) = 1$ and, using equation (31), $f_2^* = \frac{2}{5}\sigma$. This implies that either fees $\hat{f}$ or f_1^c are optimal when $\bar{\varepsilon} > \frac{5}{4}\sigma$.

Case 2: $\bar{\varepsilon} > \frac{5}{4}\sigma$

In this parameter region, we are left to determine whether the exchange chooses $f_1 = \hat{f}$ or $f_1 = f_1^c$. By construction, in this region $\bar{f} > \max\{\hat{f}_1, f_1^c\}$.

First, we argue that $f = \hat{f}_1$ if and only if $\hat{f}_1 > f_1^c$, which is equivalent to $\bar{q}(f_1^c) > 1$ by definition of $\hat{f}$. Indeed, for any $f_1 \in [f_1^c, \hat{f}]$, the first-order derivative of the objective function with respect to f_1 is equal to $\frac{\bar{m}}{\bar{\varepsilon}}(\sigma - 2f_1)$, which is positive because $\hat{f} \leq \bar{f}$. To conclude that $f_1 = \hat{f}_1$ is optimal when $\hat{f}_1 > f_1^c$, we need to show that the derivative of the objective function is negative for $f_1 \in [\hat{f}_1, \bar{f}]$. Let $G(f_1)$ be the right-hand side of optimality condition (75). We have

$$G'(f_1) = -2 + \frac{1}{2(\bar{\varepsilon} - \sigma)} f_2(f_1) f_2'(f_1), \tag{76}$$

$$= -2 + 2\frac{\sigma^2}{(\bar{\varepsilon} - \sigma)^2} \left[\frac{1}{4 + q(f_1)}\right]^3, \tag{77}$$

where for the second line, we used the fact that $q(f_1) = \bar{q}(f_1)$ for any $f_1 \geq \hat{f}$. Observe that $G'(f_1) < 0$ if

$$\sigma^2 < 64(\bar{\varepsilon} - \sigma)^2, \tag{78}$$

which holds under Assumption 1. This implies that $G'(f_1) < 0$ for all $f_1 \in [\hat{f}_1, \bar{f}]$ and thus that $\hat{f}_1$ dominates any $f_1 \in [\hat{f}_1, \bar{f}]$. This proves that $f_1^* = \hat{f}_1$ when $\hat{f}_1 \geq f_1^c$, as claimed above.

Hence, we are left to show that there exists $\hat{\varepsilon}$ such that $\hat{f}_1 > f_1^c$ if and only $\bar{\varepsilon} \leq \hat{\varepsilon}$. For this, we will show that $\hat{f}_1$ (f_1^c) is strictly decreasing (increasing) with $\bar{\varepsilon}$ for $\bar{\varepsilon} \in \left[\frac{5}{4}\sigma, \sqrt{2}\sigma\right]$. The fact that $\hat{f}_1 = 3\sigma - 2\bar{\varepsilon}$ is strictly decreasing is trivial, Furthermore, we have $\lim_{\bar{\varepsilon} \to 5/4\sigma} = \bar{f}$ and $\lim_{\bar{\varepsilon} \to \sqrt{2}\sigma} = (3 - 2\sqrt{2})\sigma$.

Next, consider f_1^c. To study the monotonicity of f_1^c with respect to $\bar{\varepsilon}$, we apply the implicit function theorem to equation (75). We showed in the previous step that $G_{f_1} < 0$.

Next, compute

$$G_{\bar{\varepsilon}} = \frac{1}{(\bar{\varepsilon}-\sigma)^2} \underbrace{\left[\left(\sigma - \frac{1}{2}\bar{\varepsilon}\right)^2 - \frac{1}{4}\left(f_2(f_1)\right)^2\right]}_{\geq 0} - \frac{1}{\bar{\varepsilon}-\sigma}\left[-\left(\sigma - \frac{1}{2}\bar{\varepsilon}\right) - \frac{1}{2}f_2\frac{\partial f_2}{\partial q}\frac{\partial q}{\partial \bar{\varepsilon}}\right] \tag{79}$$

From equation (**??**), we have $\frac{\partial f_2}{\partial q} < 0$, and, from equation (74), we have $\frac{\partial q}{\partial \bar{\varepsilon}} < 0$. These observations, together with the fact that $\bar{\varepsilon} < 2\sigma$ from Assumption 1 imply that $G_{\bar{\varepsilon}} > 0$. Hence, by the implicit function theorem, we obtain

$$\frac{\partial f_1^c}{\partial \bar{\varepsilon}} = -\frac{G_{\bar{\varepsilon}}}{G_{f_1}} > 0,$$

That is, f_1^c is strictly increasing with $\bar{\varepsilon}$. From equation (75), we obtain that $\lim_{\bar{\varepsilon}\to 5/4\sigma} f_1^c < \bar{f}$, because the terms between brackets is strictly positive since $f_2 \leq \bar{f}$. Hence, to prove the existence of a threshold $\hat{\varepsilon}$, we are left to show that

$$\lim_{\bar{\varepsilon}\to\sqrt{2}\sigma} f_1^c > \lim_{\bar{\varepsilon}\to\sqrt{2}\sigma} \hat{f}_1 = (3-2\sqrt{2})\sigma \tag{80}$$

For this, observe first that since $q(f_1) \leq 1$, we have $f_2(f_1) \geq \frac{2\sigma}{5}$ from equation (**??**). This implies that

$$\forall\, \bar{\varepsilon} \in \left[\frac{5}{4}\sigma, \sqrt{2}\sigma\right], \quad f_1^c \geq h(\bar{\varepsilon}) \equiv \frac{\sigma}{2} - \frac{1}{2}\frac{\left(\sigma - \frac{1}{2}\bar{\varepsilon}\right)^2 - \frac{1}{4}\left(\frac{2\sigma}{5}\right)^2}{\bar{\varepsilon}-\sigma}, \tag{81}$$

where $h(.)$ is also strictly increasing with $\bar{\varepsilon}$. Hence, to prove the desired result, it is enough

to show that $h(\sqrt{2}\sigma) > \lim_{\bar{\varepsilon}\to\sqrt{2}\sigma} \hat{f}_1 = (3-2\sqrt{2})\sigma$. We have

$$h(\sqrt{2}\sigma) - (3-2\sqrt{2})\sigma = \sigma\left[\frac{1}{2} - \frac{1}{2}\frac{\left(1-\frac{\sqrt{2}}{2}\right)^2 - \frac{1}{25}}{\sqrt{2}-1} - (3-2\sqrt{2})\right] > 0 \tag{82}$$

This concludes the proof.

A.9 Steps for Definition 3

To compute social welfare, we sum the expected utilities of all agents. We have

$$\begin{aligned} W &= \Pi^E + \sum_{t\in\{1,2\}}\left[\Pi_t^{MM} + \Pi_t^I + \Pi_t^L\right] \qquad (83)\\ &= \mu_1 f_1 + \bar{m}\left(1 - \frac{a_1+f_1}{\bar{\varepsilon}}\right) f_1 + \mathbb{E}_p\left[\bar{m}\left(1 - \frac{a_2(p)-v(p)+f_2}{\bar{\varepsilon}}\right) f_2 + \mu_2 f_2 \mathbf{1}_{p=\frac{1}{2}}\right]\\ &\quad + \bar{m}\left(1 - \frac{a_1+f_1}{\bar{\varepsilon}}\right) a_1 + \mu_1(a_1-\sigma)\\ &\quad + \mathbb{E}_p\left[\bar{m}\left(1 - \frac{a_2(p)-v(p)+f_2}{\bar{\varepsilon}}\right)(a_2(p)-v(p)) + \mu_2 a_2(1/2)\mathbf{1}_{p=\frac{1}{2}}\right]\\ &\quad + \mu_1(\sigma - c - a_1 - f_1) + \mu_2\left(\mathbb{E}_p\left[\mathbf{1}_{p=\frac{1}{2}}(\sigma - a_2(p) - f_2)\right] - c\right)\\ &\quad + \frac{\bar{m}}{\bar{\varepsilon}}\int_{a_1+f_1}^{\bar{\varepsilon}} (\varepsilon - a_1 - f_1)\mathrm{d}\varepsilon + \frac{\bar{m}}{\bar{\varepsilon}}\mathbb{E}_p\left[\int_{a_2(p)-v(p)+f_2}^{\bar{\varepsilon}} (\varepsilon - a_2(p) + v(p) - f_2)\mathrm{d}\varepsilon\right] \qquad (84) \end{aligned}$$

We did not include the exchange revenues from the sale of information VI^{MM}, because this is a cost to the date-2 MM. Summing the first three lines of (84), we obtain

$$\begin{aligned}
W = &\mu_1(f_1 + a_1 - \sigma) + \mathbb{E}_p\left[\mu_2(f_2 + a_2(1/2) - \sigma)\mathbf{1}_{p=\frac{1}{2}}\right] \\
&+ \bar{m}\left(1 - \frac{a_1 + f_1}{\bar{\varepsilon}}\right)(a_1 + f_1) + \mathbb{E}_p\left[\bar{m}\left(1 - \frac{a_2(p) - v(p) + f_2}{\bar{\varepsilon}}\right)(a_2(p) - v(p) + f_2)\right] \\
&+ \mu_1(\sigma - c - a_1 - f_1) + \mu_2\left(\mathbb{E}_p\left[\mathbf{1}_{p=\frac{1}{2}}(\sigma - a_2(p) - f_2)\right] - c\right) \\
&+ \frac{\bar{m}}{\bar{\varepsilon}}\int_{a_1+f_1}^{\bar{\varepsilon}} \varepsilon \mathrm{d}\varepsilon - \bar{m}\left(1 - \frac{a_1 + f_1}{\bar{\varepsilon}}\right)(a_1 + f_1) \\
&+ \frac{\bar{m}}{\bar{\varepsilon}}\mathbb{E}_p\left[\int_{a_2(p)-v(p)+f_2}^{\bar{\varepsilon}} \varepsilon \mathrm{d}\varepsilon\right] - \mathbb{E}_p\left[\bar{m}\left(1 - \frac{a_2(p) - v(p) + f_2}{\bar{\varepsilon}}\right)(a_2(p) - v(p) + f_2)\right]
\end{aligned} \tag{85}$$

Regrouping terms of equation (85), we obtain equation (33). This concludes the proof.

B Implementation as a Separating Equilibrium

We prove the claim that the date-2 (symmetric-information) equilibrium described in Section 3 is an equililibrium of the original signaling game. The symmetric-information equilibrium is characterized by the set of equilibrium quotes

$$\mathcal{Q} \equiv \{(a^*(p), b^*(p))\}_{p\in\{0,0.5,1\}}$$

given by equations (**??**) and (**??**). The mapping from a posterior $p \in \{0, 0.5, 1\}$ to these quotes is invertible. Hence, we can define an equilibrium belief mapping $\beta^* : \mathcal{Q} \mapsto \{0, 0.5, 1\}$ such that for all $(a, b) \in \mathcal{Q}$, $\beta^*(a, b)$ is the (unique) posterior value p such that $(a, b) = (a^*(p), b^*(p))$.

In the signaling game, liquidity traders do not know the posterior probability of a high asset value, $p \in \{0, 0.5, 1\}$. Instead, they from beliefs about p, based on the quotes posted by the MM. Our objective is to show that the symmetric-information equilibrium is implementable as a separating equilibrium of the signaling game. For this, we need to show that given the conjectured equilibrium beliefs β^*, defined above, a MM with type $p \in \{0, 0.5, 1\}$ maximizes its profit by setting the symmetric-information equilibrium quotes $(a^*(p), b^*(p))$ instead of any other quotes in $\mathcal{Q}$.

For this analysis, we defined $\Pi^{MM}(p'; p)$ as the payoff of a type-p MM who posts quotes $(a^*(p'), b^*(p'))$ for $(p, p') \in \{0, 0.5, 1\}^2$. This payoff is given by

$$\begin{aligned}\Pi^{MM}(p'; p) = \mathbb{E}\left[\tilde{D}^L(a^*(p'), p')\right]\big(a^*(p') - v(p)\big) + \mathbb{E}\left[\tilde{S}^L(b^*(p'), p')\right]\big(v(p) - b^*(p')\big) \\ + \mathbb{E}\big[\tilde{D}^I(a^*(p'), v)(a^*(p') - v)\,|\,p\,\big] + \mathbb{E}\big[\tilde{S}^I(b^*(p'), v)(v - b^*(p'))\,|\,p\,\big] \qquad (86)\end{aligned}$$

Observe that upon seeing quotes $(a^*(p'), b^*(p'))$, liquidity traders update their expectation of the asset value to $v(p')$, while the true expected value is $v(p)$. This is why the second argument of demand $\tilde{D}^L$ and supply $\tilde{S}^L$ is the posterior p' corresponding to the posted prices, rather than the true posterior p. Informed speculators, on the other hand, know the true value v, so the MM quotes have no informational content for them. For this analysis, it is useful to rewrite the MM profit as follows:

$$\begin{aligned}\Pi^{MM}(p';p) = &\mathbb{E}\left[\tilde{D}^L(a^*(p'),p)\right]\big(a^*(p') - v(p)\big) + \mathbb{E}\left[\tilde{S}^L(b^*(p'),p)\right]\big(v(p) - b^*(p')\big)\\ &+ \left\{\mathbb{E}\left[\tilde{D}^L(a^*(p'),p')\right] - \mathbb{E}\left[\tilde{D}^L(a^*(p'),p)\right]\right\}\big(a^*(p') - v(p)\big)\\ &+ \left\{\mathbb{E}\left[\tilde{S}^L(b^*(p'),p')\right] - \mathbb{E}\left[\tilde{S}^L(b^*(p'),p)\right]\right\}\big(v(p) - b^*(p') - \big)\\ &+ \mu p x_B^I(a^*(p'))(a^*(p') - \sigma) + \mu(1-p)x_S^I(b^*(p'))(-\sigma - b^*(p')) \qquad (87)\end{aligned}$$

where in the last line of (87), we used equations (8) and (9) to substitute for $\tilde{D}^I$ and $\tilde{S}^I$ respectively. Next, using the linearity of liquidity traders' aggregate demand and supply, and rearranging terms, we obtain

$$\begin{aligned}\Pi^{MM}(p';p) = &\mathbb{E}\left[\tilde{D}^L(a^*(p'),p)\right]\big(a^*(p') - v(p)\big) + \mathbb{E}\left[\tilde{S}^L(b^*(p'),p)\right]\big(v(p) - b^*(p')\big)\\ &+ \mu p x_B^I(a^*(p'))(a^*(p') - \sigma) + \mu(1-p)x_S^I(b^*(p'))(-\sigma - b^*(p'))\\ &+ \frac{\bar{m}}{2\bar{\varepsilon}}(v(p') - v(p))\big[a^*(p') + b^*(p') - 2v(p)\big] \qquad (88)\end{aligned}$$

The terms in the first two lines are equal to the symmetric-information profit function $\Pi_p(.,.)$ evaluated at quotes $(a^*(p'), b^*(p'))$. In addition, we showed that the mapping

$$\hat{\Pi}_p(a,b) = \Pi_p(a,b) - \mu p\big[x_B^I(a) - x_B^I(a^*(p))\big](a-\sigma) + \mu(1-p)\big[x_S^I(b) - x_S^I(b^*(p))\big](-\sigma - b)$$

is quadratic and separable in its arguments (a, b) with quadratic coefficient $-\frac{\bar{m}}{2\bar{\varepsilon}}$. By definition of the symmetric-information equilibrium quotes, $\hat{\Pi}_p(a, b)$ is maximized at $(a^*(p), b^*(p))$. Since it is a quadratic function, we can thus write

$$\hat{\Pi}_p(a,b) = \hat{\Pi}_p(a^*(p), b^*(p)) - \frac{\bar{m}}{2\bar{\varepsilon}}\left[\big(a^*(p)-a\big)^2 + \big(b^*(p)-b\big)^2\right] \tag{89}$$

$$= \Pi^{MM}(p;p) - \frac{\bar{m}}{2\bar{\varepsilon}}\left[\big(a^*(p)-a\big)^2 + \big(b^*(p)-b\big)^2\right] \tag{90}$$

where the second equality follows from the fact that $\hat{\Pi}_p(a^*(p), b^*(p)) = \Pi_p(a^*(p), b^*(p)) = \Pi^{MM}(p;p)$. Hence, we obtain

$$\begin{aligned}\Pi^{MM}(p';p) = \Pi^{MM}(p;p) &- \frac{\bar{m}}{2\bar{\varepsilon}}\left[\big(a^*(p)-a^*(p')\big)^2 + \big(b^*(p)-b^*(p')\big)^2\right] \\ &+ \frac{\bar{m}}{2\bar{\varepsilon}}(v(p')-v(p))\big[a^*(p')+b^*(p')-2v(p)\big] \\ &+ \mu p\big[x_B^I(a^*(p')) - x_B^I(a^*(p))\big](a^*(p')-\sigma) \\ &+ \mu(1-p)\big[x_S^I(b^*(p')) - x_S^I(b^*(p))\big](-\sigma - b^*(p')) \qquad (91) \\ = \Pi^{MM}(p;p) &+ \frac{\bar{m}}{2\bar{\varepsilon}}K(p';p) + L(p';p) \qquad (92)\end{aligned}$$

where

$$K(p';p) \equiv -\big(a^*(p)-a^*(p')\big)^2 - \big(b^*(p)-b^*(p')\big)^2 + (v(p')-v(p))\big[a^*(p')+b^*(p')-2v(p)\big] \tag{93}$$

$$L(p';p) \equiv \mu p\big[x_B^I(a^*(p')) - x_B^I(a^*(p))\big](a^*(p')-\sigma) + \mu(1-p)\big[x_S^I(b^*(p')) - x_S^I(b^*(p))\big](-\sigma - b^*(p')) \tag{94}$$

A sufficient condition for the result is that $L(p';p) \leq 0$ and $K(p';p) \leq 0$ for all values of

$(p, p') \in \{0, 0.5, 1\}^2$.

Proof that $K \leq 0$

Our results in the main text show that for all $p' \in \{0, 0.5, 1\}$,

$$a^*(p') + b^*(p') = 2v(p')$$

Next, for any $(p, p') \in \{0, 0.5, 1\}^2$

$$a^*(p') - a^*(p) = v(p') - v(p) + \frac{1}{2}\frac{\mu\bar{\varepsilon}}{\bar{m}}\left(\mathbf{1}_{p'=0.5} - \mathbf{1}_{p=0.5}\right) \tag{95}$$

$$b^*(p') - b^*(p) = v(p') - v(p) - \frac{1}{2}\frac{\mu\bar{\varepsilon}}{\bar{m}}\left(\mathbf{1}_{p'=0.5} - \mathbf{1}_{p=0.5}\right) \tag{96}$$

This implies that

$$K(p'; p) \leq -2(v(p') - v(p))^2 + 2(v(p') - v(p))^2 = 0$$

Proof that $L \leq 0$

Consider first $p = 0$. We have

$$L(p'; 0) = \mu x_S^I(b^*(p'))(-\sigma - b^*(p'))$$

For any $p' \neq 0$, $b^*(p') > -\sigma$ which implies that $L(p'; 0) \leq 0$ with a strict inequality for $p' \neq 0$.

Next, consider $p = 1$. We have

$$L(p'; 1) = \mu x_B^I(a^*(p'))(-\sigma + a^*(p'))$$

For any $p' \neq 0$, $a(p') < \sigma$ which implies that $L(p'; 1) \leq 0$ with a strict inequality for $p' \neq 1$.

Finally, consider the case $p = \frac{1}{2}$. We have

$$L(p';0.5) = \frac{1}{2}\mu\Big\{\big[x_B^I(a^*(p')) - x_B^I(a^*(p))\big](a^*(p') - \sigma) + \big[x_S^I(b^*(p')) - x_S^I(b^*(p))\big](-b^*(p') - \sigma)\Big\}$$

For $p' = 0$, we have $x_S^I(b^*(p')) = 0$ and $x_B^I(a^*(p')) = x_B^I(a^*(p)) = 1$. This implies that

$$L(0;0.5) = \frac{1}{2}\mu\big[b^*(p') + \sigma\big] = \frac{1}{2}\mu\big[-\sigma - \frac{1}{2}(\bar{\varepsilon} - f) + \sigma\big] < 0$$

For $p' = 1$, we have $x_S^I(b^*(p')) = x_S^I(b^*(p)) = 1$ and $x_B^I(a^*(p')) = 0$. This implies that

$$L(1;0.5) = -\frac{1}{2}\mu\big[a^*(p') - \sigma\big] = -\frac{1}{2}\mu\big[\sigma + \frac{1}{2}(\bar{\varepsilon} - f) - \sigma\big] < 0$$

This proves that $L(p';p) \leq 0$ for all $(p, p') \in \{0, 0.5, 1\}$, which concludes the proof.