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THE ECONOMIC VALUE OF NATIONS (FOR OTHERS)

Hans Gersbach, Paul Maunoir and Kieran Walsh

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Centre for Economic Policy Research
187 boulevard Saint-Germain, 75007 Paris, France
2 Coldbath Square, London EC1R 5HL
Tel: +44 (0)20 7183 8801
www.cepr.org

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THE ECONOMIC VALUE OF NATIONS (FOR OTHERS)

Abstract

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JEL Classification: C67, D57, F17, F41, F51

Keywords: global trade networks, Geoeconomics, value of countries

Hans Gersbach - hgersbach@ethz.ch
KOF ETH Zurich and CEPR

Paul Maunoir - pmaunoir@ethz.ch
KOF ETH Zurich

Kieran Walsh - kwalsh@ethz.ch
KOF ETH Zurich

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The Economic Value of Nations (for Others)*

Hans Gersbach
KOF Swiss Economic Institute
at ETH Zurich and CEPR
Leonhardstrasse 21
8092 Zurich, Switzerland
hgersbach@ethz.ch

Paul Maxence Maunoir
KOF Swiss Economic Institute
at ETH Zurich
Leonhardstrasse 21
8092 Zurich, Switzerland
pmaunoir@ethz.ch

Kieran James Walsh
KOF Swiss Economic Institute
at ETH Zurich
Leonhardstrasse 21
8092 Zurich, Switzerland
kwalsh@ethz.ch

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1 Introduction

What is the economic value of a country to others through trade, relative to the gains it secures for itself? In this paper, we introduce and analyze the *Trade Benefit Ratio* (TBR), a welfare-based general equilibrium index that captures a country's systemic external value in trade relative to its internal value. TBR is defined as the ratio of the welfare loss others would suffer to the country's own loss were it to withdraw from trade. We (i) establish a weak-positivity result for equivalent-variation-based TBRs, (ii) quantify TBRs in an extension of a nonlinear global trade model calibrated to 34 countries/regions and 38 sectors, and (iii) extend the framework to sectors, bilateral decompositions, and coalitions, including a "Cold War 2.0" decoupling scenario. The resulting statistics (levels, sectoral sources, and dispersion) provide a concise lens on latent coercive capacity and exposure in the geopolitics of global value chains.

To provide a basis for interpreting these measures, we begin with a discussion of current patterns of international trade and trade policy. Linked global value chains characterize the economic world today. Current developments display several interrelated phenomena. We refer, e.g., to Aiyar et al. (2023) and Baldwin et al. (2024, 2022, 2023) for a more detailed account of the four patterns that follow. First, the global cross-border flows of goods, services, and capital strongly expanded in the second half of the 20th century but have seemed to stagnate since the global financial crisis, and trade in goods has even declined somewhat. Yet, trade in services has not plateaued and is expected to grow further (Baldwin et al., 2024), and key economic indicators of global openness remain at high levels (Goldberg and Reed, 2023; Franco Bedoya, 2023; Di Sano et al., 2023).

Second, there have been stark recent structural changes in global production, now that China has become the dominant manufacturing location in the world. Remarkably, China has become the world's manufacturing powerhouse in only two decades, producing roughly 30% of the world's manufacturing value-added output.

Third, trade restrictions have increased significantly in recent years, as shown in data from the Global Trade Alert (see, e.g., Aiyar et al. (2023)), and national security is often part of the justification for their imposition. Moreover, in recent years, the EU, the US, and China have launched large-scale industrial subsidy programs to reduce

external dependencies by widening the possibilities for domestic sourcing, with the goal of achieving self-sufficiency in key technologies.

Fourth, "geoeconomics" – in which a country uses its economic strength to exert influence on foreign entities to achieve geopolitical or economic goals – has become more prominent, especially since US President Donald Trump took office. Tariff threats have increasingly entered the geopolitical toolkit, and there are even scenarios such as a "Cold War 2.0," in which a Western bloc decouples from a China-aligned bloc that have started to be contemplated.

A central element of geoeconomics and theories of national power is the comparison between the damage that the government of one country A can impose on another country B and the loss that country A would incur (Hirschman, 1945; Keohane and Nye, 1977). In this paper, we develop the *Trade Benefit Ratio* (TBR) as a metric for measuring the geoeconomic significance of nations through their trade relationships. Building on ideas in the rapidly-growing geoeconomics literature (e.g., Clayton et al. (2025b), Clayton et al. (2025a), and Clayton et al. (2023)), we argue that TBRs offer a simple aggregate measure of how much systemic value a country provides to others via international trade, relative to how much it receives. We use a quantitative general equilibrium model of global trade networks to calculate TBRs for 34 countries and regions.

The trade benefit ratio (TBR) is a summary statistic, specifically the ratio between the total economic loss a country's withdrawal from global trade would cause others, divided by the self-inflicted economic loss. This ratio quantifies the core asymmetry that enables geoeconomic influence: the capacity to inflict disproportionate disruption relative to self-cost. Countries with high TBRs are uniquely positioned to exert pressure or extract concessions via policy.

In principle, the TBR can be measured by changes in utilities of residents in a country if these utilities are assumed to be cardinal, and we provide some intuitive competitive equilibrium examples showing that a TBR can be below 1 or above 1, the latter case indicating that a country's participation in trade generates more external than internal value. With identical homothetic preferences, TBRs can be calculated directly from changes in equilibrium utility. But when countries have different utility functions (as in our quantitative analysis), one must use a money metric, and we opt for equivalent

variation. Can a TBR be negative in the general equilibrium context? In Proposition 2, we prove under fairly general conditions that equivalent-variation TBRs are positive. That is, when one country enters autarky, at least one other country must be hurt, and the welfare losses of the losers must be larger than the total gain of the beneficiaries.[1]

Importantly, a country's withdrawal from global trade entails potentially large general equilibrium effects, that is, changes in the prices and allocations for the remaining countries. A key challenge in implementing the TBR in practice is calculating the new equilibrium that emerges in international trade when one country leaves.

When we move to the actual measurement of TBRs, we thus extend a quantitative general equilibrium trade model with networks of production. Our framework is inspired by the approach of Baqaee and Farhi (2024) which has been employed in Bachmann et al. (2024), Baqaee et al. (2024), and Çakmaklı et al. (2021). The approach of Baqaee and Farhi (2024) allows us to compute, relatively quickly, equilibrium changes with 34 countries/regions, 38 sectors, and a global trade network with production. Due to the nested constant elasticity of substitution (CES) structure, we can compute TBRs using only various sufficient statistics and numerical integration, as described in Baqaee and Farhi (2024) (see our Lemma 1).[2] The sufficient statistics required for the computation of quantitative TBRs come from the 2019 OECD input-output table (OECD, 2023).

To understand the nature of TBR heterogeneity across countries, we calculate three different versions of the ratio: retreat from full trade, retreat from manufacturing only, and retreat from manufacturing and services only. So in the latter two scenarios, we maintain international trade in agriculture and mining.[3] Figure 1 summarizes some of our main findings (see Section 3.3 for full details on TBR calculations).

For our headline "All Sectors" TBR, the top individual (non-aggregated) economies are the US, the UK, China, and Indonesia, and the US is a noticeable outlier, with a TBR close to one. At the bottom end are Japan and Korea, with TBRs around 0.2.

[1] However, in Example 4 we show that negative TBRs are possible using the compensating variation money metric.

[2] In Appendix Propositions 3 and 4, we provide matrix versions of a key theorem in Baqaee and Farhi (2024), characterizing the derivatives of the model with respect to trade barrier shocks. A nonlinear solution for large shocks is then given by numerical integration. In Section 3.3, we describe a simple computational trick we developed that speeds up the otherwise computationally-costly linear algebra. In the end, we can compute a single country's TBR in around 90 minutes.

[3] In each case, we approximate autarky with 300% iceberg shocks, although our results are robust to alternate (but still large) levels.

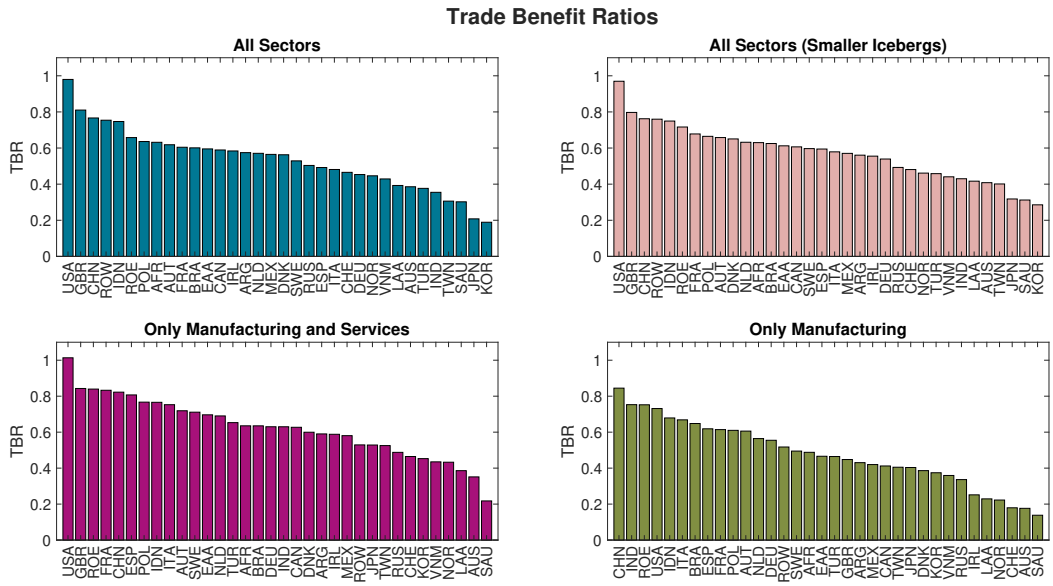


Figure 1: **TBR in Quantitative Model**

Note: "ROE" means Rest-of-EU, "LAA" is a subset of Latin American economies, "EAA" is a subset of east Asian economies, "AFR" is a subset of African economies, and "ROW" means Rest-of-World. The "All Sectors (Smaller Icebergs)" pane assumes 150% icebergs, whereas the other panes use 300%. "Only Manufacturing and Services" excludes ISIC codes A and B (agriculture, forestry, fishing, and mining) from trade barriers, and "Only Manufacturing" includes the C codes only. See Section 3.3 for calculation details.

In the "Only Manufacturing and Services" case, the US TBR actually rises above one. One could interpret this as suggesting that the US has more leverage in manufacturing and services and relatively less in agriculture and mining. The "Only Manufacturing" case, which preserves free trade in agriculture, mining, and services, has a larger effect on the levels and ordering of TBR. India, for example, rises substantially, suggesting its relative value stems more from manufacturing. On the other hand, TBRs fall significantly in several economies, such as the UK and Switzerland, suggesting major contributions to the TBR stemming from trade in services. Comparing across the three cases, some countries appear to have TBRs that are more evenly distributed sectorally, China and Indonesia for example.

In Section 3.3, we also decompose each country's TBR with respect to the foreign countries it affects when entering autarky and uncover several interesting patterns. First, China, Germany, and the US are exposed to many other countries, that is, they benefit from a large number of different trade relationships. In contrast, some countries, especially Australia and Mexico, benefit from trade with relatively few countries.

Second, Australia, Canada, Norway, Russia, and Saudi Arabia often *benefit* from other countries entering autarky, possibly due to their richness in natural resources. Third, some countries (e.g., Mexico) have very concentrated TBRs in the sense that relatively few other countries are affected when they enter autarky. Other countries, like the UK, have dispersed TBR, meaning they benefit many different countries by engaging in trade.[4]

We conclude our analysis with a Cold War 2.0 scenario, in which a US-aligned bloc decouples from a China-aligned one. Extending our TBR notion to coalitions of countries, we calculate similar TBRs for the two blocs, although the scenario entails much larger real income losses in the Chinese coalition.

While our quantitative analysis accounts for nonlinearities, general equilibrium effects, and the richness of the observed input-output tables for dozens of countries and sectors, our framework nevertheless abstracts from many clearly important aspects of trade and power, for example, intertemporal dynamics and financial markets. In Section 6, we highlight some of the key factors excluded from the model and discuss how their omission could affect the measurement of the economic value of countries for others. For example, in our "All Sectors" ranking, the US comes out on top with a high level of TBR. Yet, a substantial fraction of US consumption is effectively financed by foreign private investors and public authorities like central banks, as the US has a persistently large trade deficit. A more comprehensive assessment of how countries benefit other countries would ideally account for cross-border trade in assets, reserve currencies, and financial markets in general. We discuss these issues (and other caveats) in Section 6.

The next section (2) further discusses related literature, and Section 3 introduces the TBR metric, provides some motivating examples, and contains our main theoretical results. Section 4 describes the quantitative model and contains our main quantitative findings, and Section 5 considers a Cold War 2.0 scenario. Some caveats are offered in Section 6, and Section 7 concludes. Full details on using the model to compute the effects of shocks are provided in Appendix A, and Appendix B explains our calibration.

[4]There is a slight inverse relationship between TBR level and concentration, but the pattern is not particularly striking.

2 Relation to the Literature

Of course, there is a large literature on welfare gains from international trade, which we cannot fully summarize here, as this topic has been at the center of academic discussions for many decades.[5] In recent times, several quantitative studies have used models to compute welfare changes from trade relative to autarky or relative to high trade barriers, Eaton and Kortum (2002), Costinot and Rodríguez-Clare (2014), and Fajgelbaum and Khandelwal (2016) for example. Other papers analyze the gains or losses from specific policies, like Caliendo and Parro (2015), who calibrate a multi-sector Eaton–Kortum model to evaluate the welfare impacts of NAFTA.[6] More recently, Eppinger et al. (2021) build on Antràs and Chor (2018) to explore decoupling in intermediate goods trade and the welfare implications for shock propagation. These works focus on *own* country welfare changes but typically do not measure the *external* benefits a country creates for others. Our trade benefit ratio concept complements these welfare analyzes by comparing a country's own trade gains to the total gains it generates for its trading partners.

Our work is also connected to political science: the TBR quantifies a core asymmetry in international relations, described in Keohane and Nye (1977), that enables geoeconomic influence: the capacity to inflict disproportionate disruption relative to self-cost. More recently, Farrell and Newman (2019) introduce the concept of "weaponized interdependence," where control over key nodes (e.g., payment networks or high-tech supply chains) allows states to impose high costs on adversaries while minimizing their own exposure. Countries with high TBRs – such as those controlling critical inputs or intermediate goods – are uniquely positioned to exert pressure or extract concessions via policy. Since the decoupling of countries or coalitions from other countries or coalitions is an increasingly likely possibility in the current climate, insights from our TBR calculations could be relevant for the policy world, with regard, for example, to new Cold War scenarios.

Further, we contribute to the growing geoeconomics literature (see Mohr and Trebesch

[5] See descriptions of this broad literature in Costinot and Rodríguez-Clare (2014) and Arkolakis et al. (2012), for example.

[6] Also in this vein, Fajgelbaum et al. (2019) study the 2018-2019 US-China trade war and show how tariffs disrupted trade flows and caused efficiency losses. Yalcin et al. (2025) demonstrate substantial trade diversion and welfare effects concerning EU sanctions against Russia.

(2025) for a recent survey), especially to the quantification of models. As Mohr and Trebesch (2025) describe, studies of what would now be called geoeconomics – the strategic use of trade and economic tools to achieve geopolitical aims – extend far back into the history of economic thought. The topic retreated somewhat in the economics literature towards the end of the twentieth century, when the contributions seemed to have been more theoretical than empirical or applied. For example, game-theoretic models were widely used to formalize geoeconomic strategies. See, for instance, Eaton and Engers (1992), who model sanctions as a bargaining game where trade restrictions serve as credible threats to influence the target's actions, or McLaren (1997).

More recent geoeconomics work integrates such strategic behavior into gravity-based international trade models with networks, models that lend themselves to structural empirics or quantification based on calibration. Thoenig (2024) and Mayer et al. (2025), for example, combine an international bargaining game with trade in the shadow of war, quantifying the relationships between trade openness and violent conflict. Other network-based analyses, such as those by Clayton et al. (2023) and Clayton et al. (2025b), consider global input-output structures to measure how hegemonic states can exert coercive power by withholding critical intermediate goods or through joint trade and financial threats. These models highlight how asymmetric interdependence translates into strategic leverage in global trade, as detailed in the classic exploration on this issue by Hirschman (1945). Hirschman (1945) observed that when one country becomes disproportionately dependent on another for exports or imports, the stronger partner gains leverage and can translate economic ties into political influence. Conversely, countries may try to shape trade structures deliberately to create dependency and extend national power. Also building on Hirschman (1945), Liu and Yang (2025) show that geoeconomic power arises from asymmetric import dependencies and quantify country-level power using trade theory. They argue that, in a world of increasingly weaponized interdependence, it is not economic size per se that determines strategic power, but rather the positional leverage of a country within trade networks. TBR provides an operational summary measure of this leverage as the ability to impose losses on all others through the withdrawal of economic relationships.[7]

[7]Further contributions to the fast-growing field of geoeconomics include Becko and O'Connor (2025), Broner et al. (2025), Konrad (2023), Kleinman et al. (2024), Kooi (2025), Alekseev and Lin (2025), and Pflueger and Yared (2024).

Our analysis contributes to the geoeconomics literature in three main ways. First, our index is a new equivalent-variation-based welfare object capturing system-wide spillovers, not only bilateral exposure. Second, we offer nonlinear general equilibrium quantification in a global networks model. Third, we decompose TBR by sectors and countries, distinguishing between diffuse vs. concentrated leverage. Closest to us are Liu and Yang (2025) and Mayer et al. (2025). Liu and Yang (2025) develop a theory-based, bilateral measure of international power that rises with a country's asymmetric import dependence on its partner (weighted by sectoral elasticities). Our contribution is systemic and welfare-based: TBR aggregates the general equilibrium welfare losses that a country's withdrawal would impose on all trading partners relative to its own loss, and it extends naturally to coalitions and sectoral slices. Mayer et al. (2025) integrate a quantitative gravity model with a bargaining game of conflict/peace to study how conflict risk alters the optimal degree of decoupling. Their objective is policy evaluation under conflict risk, not a cross-country index of trade-based leverage. In short, these papers illuminate dyadic power and conflict-related trade policy, while our TBR provides a general equilibrium welfare metric of a country's systemic external value (and potential coercive capacity) vs. the whole world.

We also contribute to recent efforts to create new datasets and indexes useful in assessing the origins and impact of economic coercion. The *Global Sanctions Data Base* (GSDB) by Felbermayr et al. (2020), for example, tracks over 1000 sanction episodes since 1950, enabling studies of trade and welfare effects.[8] And Fernández-Villaverde et al. (2024) propose a *Geopolitical Fragmentation Index* that identifies shifts towards trade blocs. We add to this area by quantifying the intuitive TBR metric of geoeconomic power. Clayton et al. (2025a) emphasize the emerging toolkit of geoeconomic instruments: export bans, market access restrictions, financial sanctions, and technological decoupling. The efficacy of these tools depends on the structural relationships amongst economies. TBR serves as a summary statistic that can guide how such tools may be used or impact countries in international trade. For instance:

- A country with a high TBR may possess latent coercive capacity that can be activated through geoeconomic instruments.
- In contrast, a low-TBR country may be more vulnerable to trade-based coercion,

[8] See Yalcin et al. (2025) for a recent update.

suggesting the need for diversification or "friend-shoring."

- Coalitions or alliances can be evaluated by aggregating member TBRs to assess collective geoeconomic power or influence. This is particularly important for the current geopolitical tensions between China and the US and in scenarios like the Cold War 2.0 that we address later.

3 The trade benefit ratio

3.1 The Road Map

This section introduces the *Trade Benefit Ratio* (TBR) as a welfare-based metric for measuring the geoeconomic significance of nations through their trade relationships. Specifically, the trade benefit ratio measures the ratio between the economic value a country provides to other countries through international trade and the economic value it gains itself from engaging in trade. It is constructed via a hypothetical scenario in which the country becomes autarkic and the remaining countries engage in trade.

The trade benefit ratio has a straightforward interpretation: If $\text{TBR}^{(i)} > 1$: country i creates more value for others than it receives – a *net contributor*. If $\text{TBR}^{(i)} < 1$: country i is more dependent on others – a *net beneficiary*. In line with the geoeconomics and political science literatures described in Section 2, a higher TBR should be associated with greater geoeconomic power.

When a country enters into international trade, there is no assurance that more than one other country, let alone a majority, will benefit (see e.g. Gersbach and Haller (2022)). Indeed, as we will see in our quantitative model, many countries can actually *gain* in welfare terms when other countries enter autarky. But, as Proposition 2 in Section 3.3 shows, one stark result holds with considerable generality: all TBRs are weakly positive when measured using the equivalent-variation money metric.

Before turning to the full quantitative model, we fix ideas with the following examples.

3.2 Examples

We start by defining the trade benefit ratio for the case where all countries have the same homothetic utility function, thus making the utility levels comparable. Later on, we will define TBR in a more general setting.

Definition 1

The trade benefit ratio when country i enters autarky is:

$$\mathrm{TBR}^{(i)} = \frac{\sum_{j \neq i} \Delta U_j^{\mathrm{cf}(i)}}{\Delta U_i^{\mathrm{cf}(i)}} \tag{1}$$

*where $\Delta U_j^{\mathrm{cf}(i)}$ is the welfare change of country j for the **counterfactual** ("*cf*") scenario where country i becomes autarkic. We also define the bilateral trade benefit ratio for country j as:*

$$\mathrm{TBR}_j^{(i)} = \frac{\Delta U_j^{\mathrm{cf}(i)}}{\Delta U_i^{\mathrm{cf}(i)}}. \tag{2}$$

We illustrate this simple case with two competitive equilibrium examples. In these examples, the demand and supply of goods of each country are determined by a representative household, international trade (if allowed) is frictionless, and markets are perfectly competitive.

Example 1 (Trade benefit ratios below one)

We consider a private ownership pure exchange economy with three countries (indexed by i and j), each with a representative household, two goods (denoted by x and y), and endowments

$$\omega_1 = (12, 1), \quad \omega_2 = (1, 12), \quad \omega_3 = (6, 3).$$

We assume the representative households have Cobb-Douglas utilities $U_i(x_i, y_i) = \sqrt{x_i y_i}$. Solving for competitive equilibrium, we get the equilibrium utilities

$$U_1 = \frac{211\sqrt{19}}{152} \approx 6.05, \qquad U_2 = \frac{61\sqrt{19}}{38} \approx 7.00, \qquad U_3 = \frac{153\sqrt{19}}{152} \approx 4.39.$$

For the counterfactuals, we define $U_j^{\mathrm{cf}(i)}$ as the utility of country j when country i goes

into autarky and get:

$$U_1^{\mathrm{cf}(1)} = \sqrt{12} \approx 3.46, \qquad U_2^{\mathrm{cf}(1)} = \frac{33\sqrt{105}}{70} \approx 4.83, \qquad U_3^{\mathrm{cf}(1)} = \frac{37\sqrt{105}}{70} \approx 5.42$$
$$U_1^{\mathrm{cf}(2)} = \frac{11\sqrt{2}}{4} \approx 3.89, \qquad U_2^{\mathrm{cf}(2)} = \sqrt{12} \approx 3.46, \qquad U_3^{\mathrm{cf}(2)} = \frac{13\sqrt{2}}{4} \approx 4.59$$
$$U_1^{\mathrm{cf}(3)} = \frac{13}{2}, \qquad U_2^{\mathrm{cf}(3)} = \frac{13}{2}, \qquad U_3^{\mathrm{cf}(3)} = 3\sqrt{2} \approx 4.24.$$

The TBR for country i is defined as

$$\mathrm{TBR}^{(i)} = \frac{\sum_{j \neq i} \left(U_j^{\mathrm{cf}(i)} - U_j \right)}{U_i^{\mathrm{cf}(i)} - U_i},$$

and thus, the computed TBRs are:

$$\mathrm{TBR}^{(1)} \approx 0.44, \quad \mathrm{TBR}^{(2)} \approx 0.55, \quad \mathrm{TBR}^{(3)} \approx 0.33.$$

Hence, trade benefit ratios are less than one: each country, by engaging in trade, creates more value for itself than for the other countries in total.

Example 2 (Trade benefit ratio above one)
As in the previous example, we consider a private ownership pure exchange economy with three countries (indexed by i and j), each with a representative household, two goods (denoted by x and y), and endowments

$$\omega_1 = (6, 1), \quad \omega_2 = (2, 5), \quad \omega_3 = (1, 6).$$

We assume the representative households all have the utility function

$$U_i(x_i, y_i) = \left[\left(\frac{1}{2}\right)^{\frac{1}{\sigma}} x_i^{\frac{\sigma-1}{\sigma}} + \left(\frac{1}{2}\right)^{\frac{1}{\sigma}} y_i^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}.$$

For the base case, we assume $\sigma = 1/5$ and thus that the two goods are complements. Solving for equilibrium, we get the equilibrium utilities

$$U_1 \approx 10.521, \qquad U_2 \approx 5.375, \qquad U_3 \approx 4.088$$

For the counterfactuals, we get:

$$\begin{aligned} U_1^{\mathrm{cf}(1)} &\approx 2.378, & U_2^{\mathrm{cf}(1)} &\approx 4.742, & U_3^{\mathrm{cf}(1)} &\approx 2.383 \\ U_1^{\mathrm{cf}(2)} &\approx 7.000, & U_2^{\mathrm{cf}(2)} &\approx 4.727, & U_3^{\mathrm{cf}(2)} &\approx 7.000 \\ U_1^{\mathrm{cf}(3)} &\approx 4.088, & U_2^{\mathrm{cf}(3)} &\approx 9.234, & U_3^{\mathrm{cf}(3)} &\approx 2.378 \end{aligned}$$

and thus, the TBRs are:

$$\mathrm{TBR}^{(1)} \approx 0.287, \quad \mathrm{TBR}^{(2)} \approx 0.940, \quad \mathrm{TBR}^{(3)} \approx 1.504.$$

Country 3 *has a* ***TBR above 1*** *and is pivotal in the example in the sense that its trade creates more value for others than it gains itself. When country* 3 *enters autarky, the external harm is greater than the domestic harm. In figure 2, we show how the TBRs depend on the value of the elasticity of substitution.*

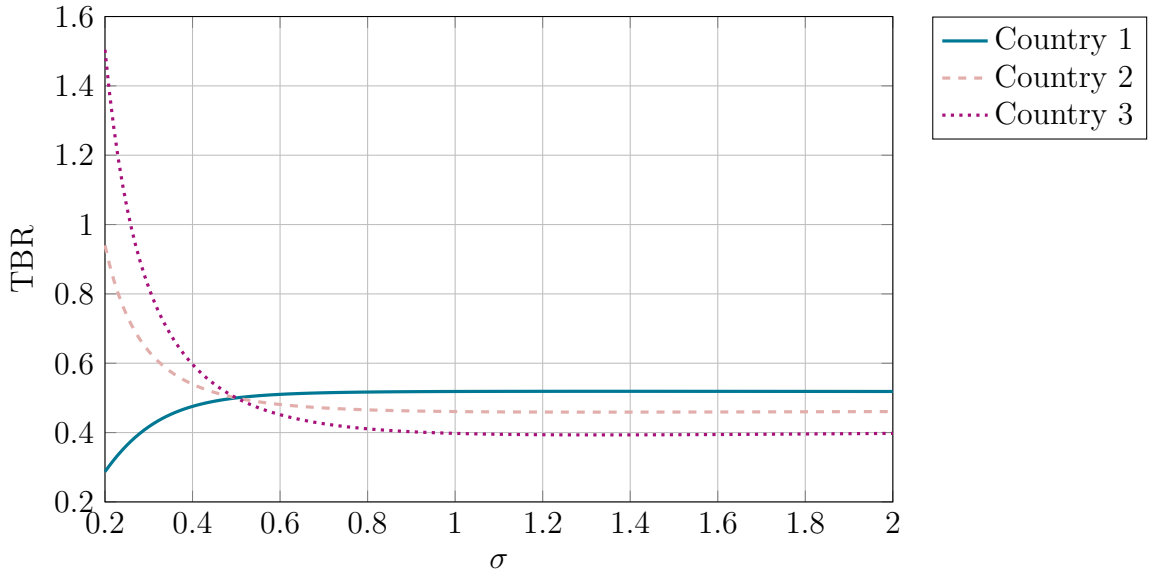


Figure 2: TBRs for a range of substitution elasticities

Example 3 (Eaton-Kortum Model)

In the standard Eaton-Kortum model with perfect competition, zero-gravity in the initial equilibrium, and manufacturing as the only sector, we can compute the trade benefit ratio in closed form.

Consider N *countries indexed by* i, j*, each with a labor endowment* L_i *and a continuum of goods* $\omega \in [0, 1]$*. The efficiency for country* i *of producing* ω *will be drawn from a*

Fréchet distribution:

$$F_i(z) = \exp\big(-T_i z^{\theta}\big),$$

(drawn independently for each i and ω). The agents (firms or consumers) aggregate intermediate goods using the CES function:

$$U = \left[\int_0^1 Q(\omega)^{\frac{\sigma-1}{\sigma}}\,\mathrm{d}\omega\right]^{\frac{\sigma}{\sigma-1}},$$

and during production, labor and intermediates are combined using a Cobb-Douglas aggregator where labor has the share parameter β. In the initial equilibrium we assume that geographical barriers are not present: $d_{ij} = 1$ for all i, j. The welfare of each country is then given by

$$U_i = \gamma^{-\frac{1}{\beta}} T_i^{\frac{1}{1+\theta\beta}} L_i \left[\sum_{j=1}^{N} T_j^{\frac{1}{1+\theta\beta}} \left(\frac{L_j}{L_i}\right)^{\frac{\theta\beta}{1+\theta\beta}}\right]^{\frac{1}{\theta\beta}}$$

where $\gamma^{1-\sigma} = \Gamma\big((\theta+1-\sigma)/\theta\big)$ and $\Gamma(\cdot)$ is the Gamma function. Similarly, the welfare of any country in the counterfactual where country i has gone into autarky is

$$U_j^{\mathrm{cf}(i)} = \gamma^{-\frac{1}{\beta}} T_j^{\frac{1}{1+\theta\beta}} L_j \left[\sum_{j'\neq i}^{N} T_{j'}^{\frac{1}{1+\theta\beta}} \left(\frac{L_{j'}}{L_j}\right)^{\frac{\theta\beta}{1+\theta\beta}}\right]^{\frac{1}{\theta\beta}}$$

if $j \neq i$ and

$$U_i^{\mathrm{cf}(i)} = \gamma^{-\frac{1}{\beta}} T_i^{\frac{1}{\theta\beta}} L_i$$

otherwise. Therefore we get:

$$\mathrm{TBR}^{(i)} = \frac{\sum_{j\neq i} T_j^{\frac{1}{1+\theta\beta}} L_j^{\frac{\theta\beta}{1+\theta\beta}} \left[\left[\sum_{j'\neq i}^{N} T_{j'}^{\frac{1}{1+\theta\beta}} L_{j'}^{\frac{\theta\beta}{1+\theta\beta}}\right]^{\frac{1}{\theta\beta}} - \left[\sum_{j'=1}^{N} T_{j'}^{\frac{1}{1+\theta\beta}} L_{j'}^{\frac{\theta\beta}{1+\theta\beta}}\right]^{\frac{1}{\theta\beta}}\right]}{T_i^{\frac{1}{1+\theta\beta}} L_i^{\frac{\theta\beta}{1+\theta\beta}} \left[\left[T_i^{\frac{1}{1+\theta\beta}} L_i^{\frac{\theta\beta}{1+\theta\beta}}\right]^{\frac{1}{\theta\beta}} - \left[\sum_{j'=1}^{N} T_{j'}^{\frac{1}{1+\theta\beta}} L_{j'}^{\frac{\theta\beta}{1+\theta\beta}}\right]^{\frac{1}{\theta\beta}}\right]}.$$

3.3 Trade Benefit Ratios with Heterogenous Countries

In the examples from Section 3.2, we computed the TBR using a country's change in utility from entering autarky and the sum of utility changes in all other countries. The resulting TBR metric had a clear interpretation because the agents all had the same

utility function.

However, in our quantitative model, the countries have different utility functions due to heterogeneity in the preference parameters. With different utility functions, it is not possible to compare changes in utility levels across countries as easily as before, and a new approach is necessary. Accordingly, for models in which the countries have potentially different utility functions, we adopt the following, more general definition of the trade benefit ratio:

Definition 2

The Trade Benefit Ratio when country i enters autarky is:

$$\mathrm{TBR}^{(i)} = \frac{\sum_{j \neq i} \mathrm{EV}_j^{\mathrm{cf}(i)}}{\mathrm{EV}_i^{\mathrm{cf}(i)}} \tag{3}$$

where $\mathrm{EV}_j^{\mathrm{cf}(i)}$ is the equivalent variation for country j when country i enters autarky.[9] *We also define the bilateral Trade Benefit Ratio for country j when country i enters autarky as:*

$$\mathrm{TBR}_j^{(i)} = \frac{\mathrm{EV}_j^{\mathrm{cf}(i)}}{\mathrm{EV}_i^{\mathrm{cf}(i)}} \tag{4}$$

Observe that, under this definition, the TBR is scale-invariant in the presence of homogeneous utility functions and production sets. In particular, when country i transitions to autarky, its TBR remains unchanged if, within the subset of countries that continue to engage in trade, one country is hypothetically partitioned into two smaller entities, each endowed with the same firms and a proportional share of the original endowments. The following proposition provides conditions under which our two definitions of TBR agree.

Proposition 1

If all the countries have the same continuous, locally nonsatiated, and homogeneous of degree one utility function, then the two definitions of TBR agree.

Proof. Let U_j and $U_j^{\mathrm{cf}(i)}$ be the utilities of country j, respectively, in the original equilibrium and in the counterfactual where country i has entered autarky. Moreover,

[9] One could alternatively express utilities in common units using the prices after the change to autarky, forming a compensating variation TBR. Below, in our quantitative analysis, we do this and form a Fisher TBR (the geometric average of the two versions) and show that it yields very similar results in our full model. We stick with the equivalent variation TBR as our baseline because it is additive with respect to foreign countries and because the status quo is not entering autarky.

let p be the vector of prices in the initial equilibrium, and let $e(\cdot,\cdot)$ be the expenditure function of the representative households.[10] Using the fact that the utility function is homogeneous of degree one, it follows that its expenditure function is linear in utility. Combining with the definition, we get:

$$\begin{aligned}\mathrm{TBR}_j^{(i)} = \frac{\mathrm{EV}_j^{\mathrm{cf}(i)}}{\mathrm{EV}_i^{\mathrm{cf}(i)}} &= \frac{e\left(p, U_j^{\mathrm{cf}(i)}\right) - e\left(p, U_j\right)}{e\left(p, U_i^{\mathrm{cf}(i)}\right) - e\left(p, U_i\right)} \\ &= \frac{\left(U_j^{\mathrm{cf}(i)} - U_j\right) e\left(p, 1\right)}{\left(U_i^{\mathrm{cf}(i)} - U_i\right) e\left(p, 1\right)} = \frac{\Delta U_j^{\mathrm{cf}(i)}}{\Delta U_i^{\mathrm{cf}(i)}}\end{aligned}$$

as claimed. ■

In the following proposition, we find fairly general conditions under which the TBR is always positive.

Proposition 2

We consider a private ownership economy with price-taking behavior, complete markets, perfect information, rational behavior, selfish preferences represented by continuous and locally nonsatiated utility functions, and no externalities. Moreover, we assume that each firm is located in one country and that the profits accrue to the representative consumer of that country only.

Then, if the TBR is defined (i.e., if $\mathrm{EV}_i^{\mathrm{cf}(i)} \neq 0$*), it is weakly positive.*

Proof. We consider the private ownership economy $(\{X_i, u_i(\cdot)\}_{i=1}^I, \{Y_j\}_{j=1}^J, \{\omega_i, \theta_i\}_{i=1}^I)$ with $I \geq 2$ countries, each represented by a representative household, $J \geq 1$ firms, and $L \geq 1$ commodities.[11]

Since each firm is located in one country, we can denote the set of all firms in country i by J_i. By assumption, the countries own the profits generated by the firms located in their borders. This means if $j \in J_i$ then $\theta_{ij} = 1$ and $\theta_{ij} = 0$ otherwise.

We consider the case where country i goes into autarky. Let $\left(p, \{x_{i'}\}_{i'=1}^I, \{y_{j'}\}_{j'=1}^J\right)$ be an equilibrium in the initial economy, and let $\left(\{x_{i'}^*\}_{i'=1}^I, \{y_{j'}^*\}_{j'=1}^J\right)$ be an equilibrium allocation after the change. Now, since $u_j(x_j^*) = U_j^{\mathrm{cf}(i)}$, it must follow for all countries

[10] The expenditure function is the same for all representative households since they have the same utility function.

[11] $X_i \subseteq \mathbb{R}^L$ are the consumption sets, $u_i(\cdot)$ the utility functions, $Y_j \subseteq \mathbb{R}^L$ the production sets, $\omega_i \in \mathbb{R}^L$ the endowments, and $\theta_i \in [0,1]^J$ the ownership shares of the firms.

$e_j\left(p, U_j^{\mathrm{cf}(i)}\right) \leq p \cdot x_j^*$ where $e_j(\cdot,\cdot)$ is the expenditure function of the household in country j. Moreover, by local nonsatiation, $e_j\left(p, U_j\right)$ is equal to the total income of country j. Therefore, we get:

$$\begin{aligned}
\mathrm{EV}_i^{\mathrm{cf}(i)} &\leq p \cdot x_i^* - p \cdot \omega_i - \sum_{j \in J_i} p \cdot y_j \\
&= p \cdot \left(\omega_i + \sum_{j \in J_i} y_j^*\right) - p \cdot \omega_i - \sum_{j \in J_i} p \cdot y_j \\
&= \sum_{j \in J_i} p \cdot \left(y_j^* - y_j\right) \leq 0
\end{aligned}$$

where for the second line we used $x_i^* = \omega_i + \sum_{j \in J_i} y_j^*$ (autarky for country i) and for the last inequality, the fact that firms always maximize profits given prices. Considering now the non-autarky countries, we get

$$\begin{aligned}
\sum_{j \neq i} \mathrm{EV}_j^{\mathrm{cf}(i)} &\leq \sum_{j \neq i} p \cdot x_j^* - \sum_{j \neq i} \left(p \cdot \omega_j + \sum_{k \in J_j} p \cdot y_k\right) \\
&= p \cdot \left(\sum_{j \neq i} \omega_j + \sum_{j \neq i} \sum_{k \in J_j} y_k^*\right) - \sum_{j \neq i} \left(p \cdot \omega_j + \sum_{k \in J_j} p \cdot y_k\right) \\
&= \sum_{j \neq i} \sum_{k \in J_j} p \cdot \left(y_k^* - y_k\right) \leq 0,
\end{aligned}$$

where for the second we used $\sum_{j \neq i} x_j^* = \sum_{j \neq i} \omega_j + \sum_{j \neq i} \sum_{k \in J_j} y_k^*$ (market clearing for the countries not in autarky) and for the last line profit maximization. This concludes the proof.

■

While equivalent-variation TBRs are positive under general conditions (as we have just shown), it is possible to construct simple examples with negative TBRs defined via compensating variation:

Example 4 (Negative trade benefit ratio using compensating variation) *Consider a private ownership pure exchange economy with three countries, each with a representative household, two goods (denoted by x and y), and endowments*

$$\omega_1 = (0.05, 4.45), \quad \omega_2 = (0.05, 3.80), \quad \omega_3 = (3.70, 0.05).$$

We assume the representative households have the utility functions

$$U_i(x_i, y_i) = \left[(a_i)^{\frac{1}{\sigma}} \, x_i^{\frac{\sigma-1}{\sigma}} + (1-a_i)^{\frac{1}{\sigma}} \, y_i^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}},$$

with weights

$$a_1 = 0.80, \quad a_2 = 0.30, \quad a_3 = 0.75$$

and $\sigma = 0.5$*. Normalizing the price of* x *to one, the initial equilibrium price of* y *is* $p_y \approx 0.03$*. Putting country 1 into autarky, the equilibrium price (with trade only amongst countries 2 and 3) is* $p_y \approx 0.5$*. Letting* $P_i = (a_i + (1-a_i)p_y^{1-\sigma})^{\frac{1}{1-\sigma}}$ *denote country* i*'s natural price index, the expenditure function for each country is, due to homothetic utility,* $U_i P_i$*. Therefore, the compensating variation for country* i *(when country 1 enters autarky) is* $\mathrm{CV}_i^{\mathrm{cf}(1)} = \left(U_i^{\mathrm{cf}(1)} - U_i \right) P_i^{\mathrm{cf}(1)}$*. We have* $\mathrm{CV}_2^{\mathrm{cf}(1)} \approx 1.34$ *and* $\mathrm{CV}_3^{\mathrm{cf}(1)} \approx -1.3$*. Since* $U_1^{\mathrm{cf}(1)} - U_1 \approx -0.2$ *and* $\mathrm{CV}_2^{\mathrm{cf}(1)} + \mathrm{CV}_3^{\mathrm{cf}(1)} > 0$*, country 1 has a negative* CV TBR*: countries 2 and 3 gain on net (according to compensating variation) when country 1 enters autarky.*

While both equivalent variation (EV) and compensating variation (CV) are valid money-metric measures of welfare change, we prefer EV for calculating TBRs in our quantitative model. The choice reflects several considerations. In today's globalized trading system, the ex ante perspective is central for geoeconomic analysis. Countries, most prominently the United States in recent years, frequently deploy the threat of trade barriers and sanctions as a strategic instrument. To assess the credibility and potential consequences of such threats, EV is the more appropriate metric: it captures how much other countries would be willing to concede in advance to avert disruption,[12] and conversely, how much the threatening country must itself be prepared to forgo to carry through on its threat. By contrast, CV measures how much compensation at the new price systems would be required to restore countries to their original welfare levels. In the scenario we analyze – one country moving into autarky – ex post compensation values are less relevant, since international transfers are precisely what autarky rules out.

Nevertheless, CV can provide a valuable perspective in the reverse case, when trade barriers are removed. In this setting, CV measures the gain countries experience under

[12] A recent example is the US-Japan trade negotiations, where Japan effectively made monetary concessions to avert higher tariffs, an explicit case of EV logic.

the new, liberalized price system compared to their welfare in the old, protected equilibrium.[13] This makes CV a natural tool for quantifying the benefits of trade opening, since it evaluates how much better off agents are once they face the new prices. It can also help assess adjustment pressures by indicating the size of the welfare improvement that could, in principle, be redistributed to compensate those adversely affected by removing trade barriers.

Comparing our positive EV TBR proposition with the negative CV TBR example, we see that it is at least in principle possible for the two concepts to yield quantitatively different results. On the other hand, using the logic of Proposition 1, EV and CV TBRs coincide when the agents have identical homothetic utility. Moreover, even though the countries have quite different utility functions in our quantitative applications, we show below that EV and CV yield similar country orderings.

4 Quantitative Model

4.1 The Formal Framework

In this section, we introduce our general equilibrium model of global trade networks, then describe the data and calibration, and finally calculate the TBRs.

To perform the quantitative counterfactual exercises for calculating the TBRs and Cold War 2.0 scenario, we follow the approach of Baqaee and Farhi (2024) employed in Bachmann et al. (2024) and Baqaee et al. (2024), as well as the analysis of Çakmaklı et al. (2021). There are C countries and N industries linked together with a nested constant elasticity of substitution (CES) structure. In what follows, for variables related to cross-border trade, superscripts refer to destinations and subscripts refer to sources. Each country $c \in C$ is endowed with a fixed supply of labor L^{kc} in each industry $k \in N$, which it inelastically supplies to the local industry at wage w^{kc}. The representative

[13] Note that putting one country into autarky creates, in principle, parallel price systems: one for the autarky country and one for everyone else. Yet, the parallel price system issue never arises in our particular quantitative applications because we never actually put anyone into autarky; instead, we approximate autarky with extremely large trade barriers.

consumer in country c solves the maximization problem

$$U^c = \max_{x_k^c} \left(\sum_{k \in N} \left(b_k^c\right)^{\frac{1}{\sigma}} \left(x_k^c\right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \quad \text{subject to} \tag{5}$$
$$\sum_{k \in N} P_k^c x_k^c = \underbrace{\sum_{k \in N} w^{kc} L^{kc}}_{I^c},$$

where x_k^c is the country's composite consumption intermediate in industry $k \in N$ and P_k^c is its price. $\sigma > 0$, common across countries, is the consumption elasticity of substitution. The constants $b_k^c \geq 0$, which sum to one, are exogenous taste parameters that correspond to the Cobb-Douglas shares in the $\sigma = 1$ limit. I^c is national income (and, as we discuss below, GDP). By the standard CES argument, maximized utility is equal to real income:

$$U^c = \frac{I^c}{P^c} \tag{6}$$

$$P^c = \left(\sum_{k \in N} b_k^c \left(P_k^c\right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}, \tag{7}$$

where P^c is the natural consumer price index.[14] We assume that world GDP,

$$Y = \sum_{c \in C} \sum_{k \in N} w^{kc} L^{kc},$$

is the numeraire, and it is normalized to one.

The composite consumption intermediates are produced by competitive local importers

[14] Deflating nominal income/GDP by the consumer price index creates the consumer welfare-relevant measure of economic activity in the model, as we saw in Equation 6. Our definition of real income corresponds to real GNE in Baqaee and Farhi (2024) but differs from usual calculations of real GDP by statistical agencies. One could also calculate changes in real GDP in the model, but doing so for large shocks would require a stance on the method of deflation, which varies across countries. And, in any case, real income corresponds to welfare in our theory. See Kehoe and Ruhl (2008) and Oulton (2023) for an in-depth discussion of this topic. Kehoe and Ruhl (2008) note that real income is sometimes referred to as command-basis GDP.

(one for each industry) that minimize the cost of meeting demand x_k^c:

$$\min_{y_{kc'}^c} \sum_{c' \in C} \tau_{kc'}^c p_{kc'} y_{kc'}^c \quad \text{subject to} \tag{8}$$

$$x_k^c = \left(\sum_{c' \in C} \left(\delta_{kc'}^c\right)^{\frac{1}{\theta_k}} \left(y_{kc'}^c\right)^{\frac{\theta_k - 1}{\theta_k}} \right)^{\frac{\theta_k}{\theta_k - 1}},$$

where $p_{kc'}$ is the "raw" price of the industry k good in country c' and $y_{kc'}^c$ is the quantity purchased. The composite consumption producer bundles together industry k goods from around the world according to trade elasticity $\theta_k > 0$ (which depends on the industry but is common across countries) and exogenous weights $\delta_{kc'}^c$ that sum to one.[15] The exogenous "icebergs" $\tau_{kc'}^c \geq 1$, which depend on the source, destination, and industry, represent trade barriers: buying one unit of $y_{kc'}^c$ requires the purchase of $\tau_{kc'}^c$ units (and $\tau_{kc'}^c - 1$ are destroyed).[16] The composite consumption producers price at marginal cost, which, by CES, implies

$$P_k^c = \left(\sum_{c' \in C} \delta_{kc'}^c \left(\tau_{kc'}^c p_{kc'}\right)^{1-\theta_k} \right)^{\frac{1}{1-\theta_k}}. \tag{9}$$

The raw, traded goods are produced in each country by firms (one for each industry) that hire local labor and solve:

$$\min_{l^{kc}, X^{kc}} w^{kc} l^{kc} + P^{kc} X^{kc} \quad \text{subject to} \tag{10}$$

$$y_{kc} = A^{kc} \left(\left(\alpha^{kc}\right)^{\frac{1}{\theta}} \left(l^{kc}\right)^{\frac{\theta-1}{\theta}} + \left(1 - \alpha^{kc}\right)^{\frac{1}{\theta}} \left(X^{kc}\right)^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}},$$

where A^{kc} is (exogenous) productivity, l^{kc} is labor demand, α^{kc} is the labor share, and $\theta > 0$ is the production elasticity. X^{kc} is a bundle of intermediate inputs specific to industry k in country c, and P^{kc} is the corresponding intermediate input cost index. The raw production firms also price at marginal cost, yielding a price p_{kc} with the standard CES form analogous to (9).

There are two other sets of CES bundling firms on the production side of the economy. We describe their notation below but omit the optimization problems, because they are

[15] Following Caliendo and Parro (2015), this formulation can be microfounded in a Ricardian fashion by assuming a continuum of sub-goods in each industry and Fréchet distributed productivities.

[16] As we discuss in Appendix A.2, one could use icebergs of less than one to represent productivity shocks.

analogous to the ones already presented. First, the industry-specific input bundle X^{kc}, analogous to the consumption aggregate (Problem 5), is formed via CES with inputs x_j^{kc} from all industries j and cross-industry elasticity ϵ and industry-specific weights ($\sum_{j \in N} \omega_j^{kc} = 1$). We call these the aggregate intermediate firms. The industry-specific inputs into the aggregate intermediate firms are bundles created by composite intermediate producers, analogous to the composite consumption intermediates in Problem 8. These N^2 production-side importers use the trade elasticity θ_k from before but face distinct weights $\sum_{c' \in C} \delta_{kc'}^{jc} = 1$ and icebergs $\tau_{kc'}^{jc}$ that can depend on the source industry/country kc' and the destination industry/country jc. Their imports are denoted $y_{kc'}^{jc}$ (in contrast with imports destined for final consumption, which were denoted $y_{kc'}^{c}$).

In summary, each of the C countries has N firms that use industry-specific labor and intermediates to produce "raw" goods for export (and domestic use). These are the only "value-added" firms, and their goods can ultimately be used both for final consumption as well as inputs into production. The intermediate inputs of the raw firms are produced by industry-specific aggregate intermediate firms (N in each country). The aggregate intermediate firms use input bundles from all industries, which they buy from composite intermediate firms. Each composite intermediate firm is specialized in a particular industry, so there are N for each industry and a total of N^2 in each country. These are one set of the importers of the economy, and we assume that they have to pay for the icebergs (i.e., icebergs are at the entry port). The other set of importers are the consumption composite intermediate firms, which have weights and icebergs separate from the importers on the production side of the economy. The consumption composite intermediate producers (N per country) sell industry bundles to the representative consumer in each country, which creates and consumes the final consumption good.[17] The representative consumer in each country pays for industry bundles by renting its industry-specific labor to the raw firms.

While the model exhibits a rich global trade network, the model is static and net trade is balanced: in each country, final consumption expenditure equals national income, which is also equal to GDP; since profits are zero and the factors of production are owned locally, national income is the sum of domestic value added (the local wages). In total, there are $\mathcal{N} = C \times (1 + N + N + N^2 + N)$ CES agents in the world economy.

[17] Note that in our quantitative analysis, we interpret final consumption as domestic "absorption," that is, the sum of final consumption and investment by consumers, firms, and governments.

Figure 3 visualizes the model's optimizing agents and the structure of trade.

Given productivities and icebergs, competitive equilibrium consists of wages, production y_{kc}, labor demands, and the various CES demands and price indexes, such that given prices and wages (i) the demands solve the consumer problems and minimize firm costs, (ii) prices equal marginal costs, and (iii) the labor and raw goods markets clear:

$$l^{kc} = L^{kc}$$

$$y_{kc} = \sum_{c' \in C} \tau_{kc}^{c'} y_{kc}^{c'} + \sum_{jc' \in N \times C} \tau_{kc}^{jc'} y_{kc}^{jc'}$$

for all $k, c \in N \times C$.[18]

To calculate the TBRs, we investigate how prices, trade, and welfare change in response to scenarios in which one country (or a coalition of countries) switches into autarky. This is achieved by imposing prohibitively high iceberg costs on all exports and imports the country could make.

Despite the richness of the model, Baqaee and Farhi (2024) show that calibrating and solving the model is easier than it might appear. To compute *log changes* in the model with respect to icebergs, one need not know the initial productivities A^{kc}, labor supplies, taste parameters, or production weights. Baqaee and Farhi (2024) prove that observing the equilibrium expenditure shares and Domar weights (which we define in Appendix A) is sufficient for calculating how the model changes for small shocks.

For our counterfactual exercises, we need the effects of large shocks. The impact of such shocks can then be calculated by dividing into steps and performing numerical integration. Since the derivatives of the model are available in closed form, this procedure

[18]Note that when the labor and raw intermediates markets clear and all agents are optimizing/cost-minimizing, the aggregate and composite intermediate markets also clear. To see this, suppose the raw intermediate firms are optimally supplying y_{kc}. This induces labor demand l^{kc}. So if the labor market is clearing, these firms optimally hire $l^{kc} = L^{kc}$. These production and employment levels induce unique demands for aggregate intermediates X^{kc} and aggregate consumption bundles. Meeting the demand for aggregates induces unique demands (from the aggregate firms) for composite intermediates x_k^c and x_j^{kc}. For the composite intermediate firms to meet these demand, they require the unique amounts of raw intermediates $y_{kc'}^c$ and $y_{jc'}^{kc}$. So as long as these raw intermediate demands (plus additional demand to cover icebergs) sum to the original supplies y_{kc}, all of the other markets are clearing. In other words, the only economically meaningful trade in goods and labor is happening between raw intermediate producers and consumers, and the other markets can be viewed as within-country bundling steps. The advantages of the present formulation are that (i) the CES demands simplify analysis and computation and (ii) it is explicit and transparent about where trade and trade barriers are in the chain.

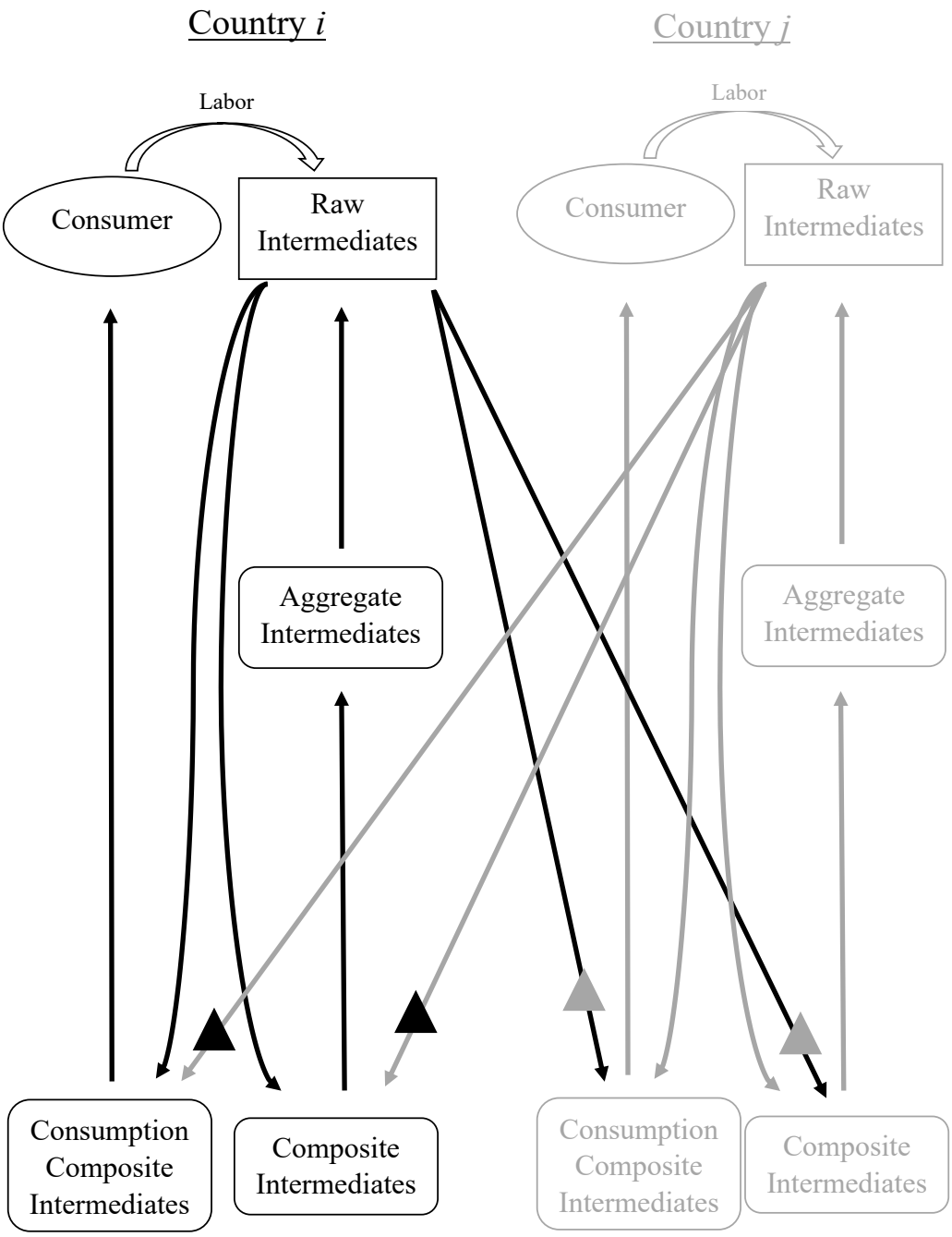


Figure 3: **Model Diagram**

Note: Triangles represent icebergs.

can be performed without numerical optimization and approaches the exact solution as the number of steps in integration is increased. Calibration consists of only two sets of numbers: the elasticities (σ, ϵ, θ, θ_k) and the expenditure shares and Domar weights, which we can calculate from trade input-output tables. Appendix A explains with full rigor how we solve for the effect of shocks in the model. Along the way, we provide a detailed proof of a matrix version of the key theorem in Baqaee and Farhi (2024) that enables the exact calculation of the model's derivatives and global approximation of the effect of shocks.

4.2 Data and Calibration

To use the model to calculate the effects of trade barriers, we require two sets of inputs. First, we need input-output trade data to initialize the model, in particular the expenditure shares of producers and consumers. Second, we need values for the model's elasticities of substitution (σ, ϵ, θ, θ_k). Note that while labor endowments, productivity parameters, and the underlying consumption/production weights affect equilibrium, knowledge of them is not required for performing counterfactuals: as we explain in detail in Appendix A, realized expenditure shares and Domar weights are sufficient for computing the model's derivatives.

We set the initial expenditure shares using the OECD, Inter-Country Input-Output (ICIO) Tables, specifically the 2019 table.[19] For 45 industries and 77 countries (including "rest-of-world" ROW), the OECD ICIO gives the intermediate sales of each industry in each country to all other industry/country pairs, as well as the final-use sales to each country. We perform three adjustments to the raw data before running the model: (1) a standard correction for negative inventories, (2) a trade balance correction, and (3) an aggregation scheme required for computational reasons. The details and logic of our approach are described in Appendix B. After aggregation, we are left with 38 industries (see Appendix Table 2) and 34 countries/regions, which include ROW and several regional aggregates (see Appendix Tables 3 and 4). We take the elasticity parameters (see Appendix Tables 1 and 2) from Baqaee and Farhi (2024) and Eppinger et al. (2021).

We would like to add two cautionary remarks regarding our data. First, while we

[19] See OECD (2023).

use the latest non-COVID year of the OECD ICIO tables (2019), these data may not reflect structural changes in trade patterns that occurred between 2019 and the present. Second, there is always uncertainty surrounding the extent to which IO tables and other statistics, in particular trade statistics on service exports and imports, represent the true underlying situation.

4.3 Calculating TBR

Before showing the main results, we provide some additional details on using the quantitative model to compute TBRs. The following lemma shows that $\text{TBR}^{(i)}$ only depends on the sufficient statistics that determine the new equilibrium as described in Baqaee and Farhi (2024).

Lemma 1

$$\text{TBR}^{(c)} = \frac{\sum_{c' \neq c} \left(\chi^{c'}_{post} \frac{1}{P^{c'}_{post}/P^{c'}_{pre}} - \chi^{c'}_{pre} \right)}{\chi^{c}_{post} \frac{1}{P^{c}_{post}/P^{c}_{pre}} - \chi^{c}_{pre}},$$

where $\chi^c = I^c/Y$ is country c's share of world GDP.

Proof. The lemma follows immediately from the definition of $\text{TBR}^{(c)}$ (3), $U^c = I^c/P^c$ (6), and the normalization of world GDP to one, so $Y_{post}/Y_{pre} = 1$. ■

We show in Appendix A how to calculate χ^c_{post} and $\log(P^c_{post}/P^c_{pre})$ for arbitrary iceberg shocks, and χ^c_{pre} comes from the data (as described in Appendix B). Therefore, Lemma 1 allows us to calculate TBR from our computational solution method and IO tables, *without* knowledge of initial productivities, labor supplies, taste parameters, or production weights.

In what follows, to calculate the quantitative TBR we approximate autarky with two-way 300% icebergs, as our computational procedure cannot handle infinite icebergs, although (as we will see) reducing the icebergs to 150% does not change TBRs much, implying that going beyond 300% would not likely make a big difference. Specifically, one country at a time, we put 300% icebergs on all imports and exports ("All Sectors" below) for the particular country and compute the equilibrium log changes in the model. We also consider two less extreme shocks for each country: (i) "Only Manufacturing and

Services," which excludes agriculture and mining from the icebergs (ISIC sector groups A and B), and (ii) "Only Manufacturing," which only applies icebergs to ISIC C codes and maintains free trade in services, agriculture, and mining. For each shock exercise, we use 30 steps in numerical integration (see Appendix A on how local approximations are integrated to give global solutions).

Shocking the countries one at a time for the four cases (all sectors, smaller icebergs, only manufacturing and services, and only manufacturing) entails running our computational procedure $4 \times C = 136$ times. Since the number of CES agents in the model is $\mathcal{N} = 53{,}006$, the required matrix algebra described in Appendix A might at first seem to render this task impractical with respect to memory and processor time. Fortunately, a simple Matlab trick drastically reduces the required memory and computation time. We describe it here because it may be useful to the related literature.

As an example, suppose Ω is a very large but sparse $Q \times Q$ matrix, and we need $(I-\Omega)^{-1}R$, where R is sparse $Q \times Z$ with $Z \ll Q$. The output is only $Q \times Z$ and thus not memory-intensive, but explicitly forming $(I-\Omega)^{-1}$ would be both infeasible to store and prohibitively expensive to compute. Declaring Ω as sparse is not enough, since the inverse of a sparse matrix is typically dense. Instead, we exploit two features of Matlab. First, the backslash operator "\" solves $(I-\Omega)X = R$ directly, producing the desired product without ever forming the inverse. When the inputs are sparse, Matlab uses sparse LU/Cholesky factorization internally, which preserves memory efficiency. Second, Matlab parallelizes these sparse factorizations across threads. Using 48 cores on ETH's Euler cluster, declaring all large matrices sparse, and consistently writing expressions in the form $(I-\Omega)\backslash R$ rather than $(I-\Omega)^{-1}R$, we can compute the TBR for an individual country in under 90 minutes with about 30 GB of RAM (for 30 numerical-integration steps). In short, although the algebra of our algorithm involves very large matrices and their inverses, in practice the inverses only ever appear premultiplied or postmultiplied by smaller matrices (such as R above).[20] This structure means we never form the dense inverse explicitly. Instead, we solve the corresponding sparse linear systems using Matlab's backslash operator, which yields substantial efficiency gains thanks to sparse factorization and automatic multithreading.

[20] See Appendix Corollary 1.

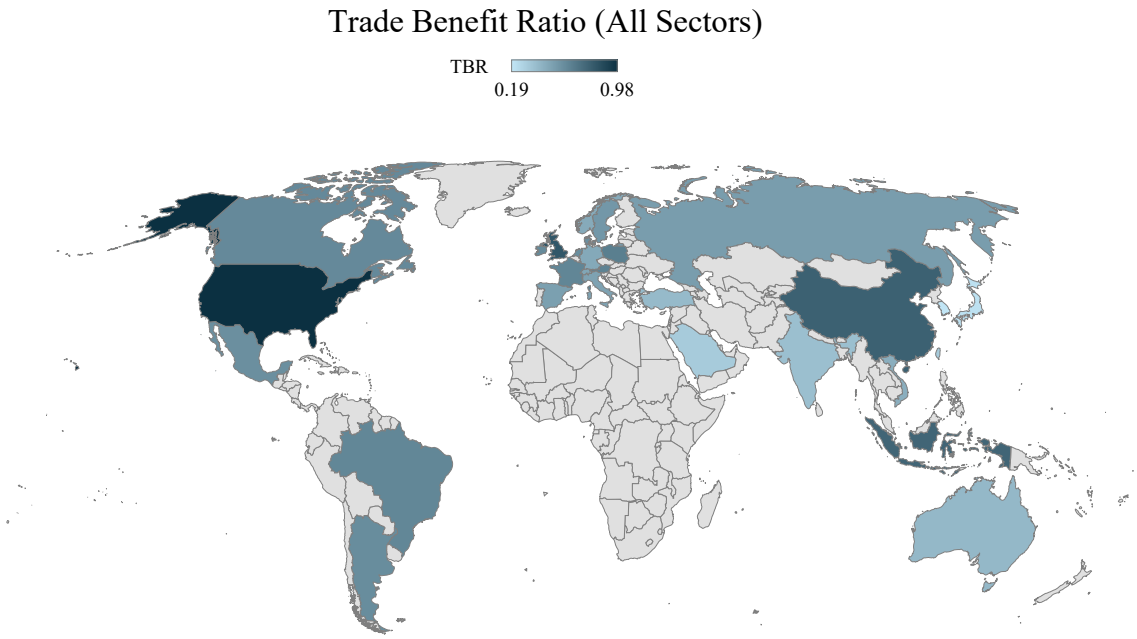


Figure 4: **TBR Map**

Note: Values for regional aggregates are not displayed and the aggregated countries are shaded gray.

4.4 Results

Figure 1 shows TBR in the calibrated model. First consider the top-left panel in which the icebergs are applied to all goods and services. The clear top 5 TBRs are the United States, the United Kingdom, China, Rest-of-World, and Indonesia, and the US is a noticeable outlier, with a TBR close to one. At the bottom end are Japan and Korea, with TBRs around 0.2. The top-right panel uses 150% icebergs (instead of 300%), which slightly compresses the distribution but provides a very similar picture. Figure 4 projects the TBR for all sectors onto a world map. This figure illustrates that there is substantial spatial heterogeneity in TBR, especially in the East, where China and Indonesia are surrounded by relatively low TBR economies.

Recalculating TBR by applying icebergs to fewer industries (the bottom panels of Figure 1) helps reveal the nature of particular economies' relative contributions to global welfare. Excluding agricultural and mining sectors when putting countries into autarky (the bottom-left panel), the US TBR actually rises above one, and the TBR order changes somewhat. The Spain TBR, for example, moves from under 0.6 to around 0.8. The implication is that the US and Spain, for example, have relatively more leverage in manufacturing and services and relatively less in agriculture and

mining.

Restricting the autarky exercises to only manufacturing (and preserving free trade in agriculture, mining, and services) has a larger effect on the levels and ordering of TBR. China, India, and Rest-of-Europe rise to the top of the distribution, while Ireland, Norway, and Switzerland fall substantially to near the bottom of the distribution. Figure 5 shows a scatter plot of the alternate TBRs based on manufacturing and services or only manufacturing against the TBR computed with all sectors. Deviations from the 45° line indicate sensitivities concerning the sectors used to construct TBR. Some of the most dramatic declines in TBR from only blocking manufacturing are in the US, UK, Ireland, and Switzerland, suggesting the economic influence of these countries largely stems from trade in services. In contrast, we can conclude that the power of India and Italy, for example, stems in large part from manufacturing. Calculating TBR with manufacturing and services leads to several substantial increases. Countries including Japan, Korea, Turkey, and Spain appear to derive their influence from manufacturing and services and have much lower TBRs when also restricting trade in agriculture and mining. Other countries, such as China, Indonesia, and Vietnam, appear to have TBRs more evenly distributed with respect to different sectors.

We next ask how concentrated a country's TBR is with respect to the foreign countries it affects when entering autarky? Which countries create value for just one or two other countries, and which ones have a dispersed impact? Which countries actually *gain* when others exit the global trade network? Figure 6 is a heat map decomposition of TBR (all sectors) for each country. Each pixel represents the bilateral TBR $\text{TBR}_j^{\text{cf}(i)}$, so the columns of the heat map sum to the TBR in Equation 3 for each country i. Shades of red correspond to positive TBR components (that is, country i has an adverse effect on country j when i enters autarky). Blue represents the opposite (j benefits when i enters autarky).

The heat map figure reveals at least three interesting patterns. First, looking down the rows, three individual countries – China, Germany, and the US – immediately stand out due to their relatively dark-red columns. The implication is that these countries are exposed to a large number of other countries. Put differently, China, Germany, and the US benefit, in relative terms, from a large number of different trade relationships. Some countries, in contrast, like Australia and Mexico, appear to be exposed to a

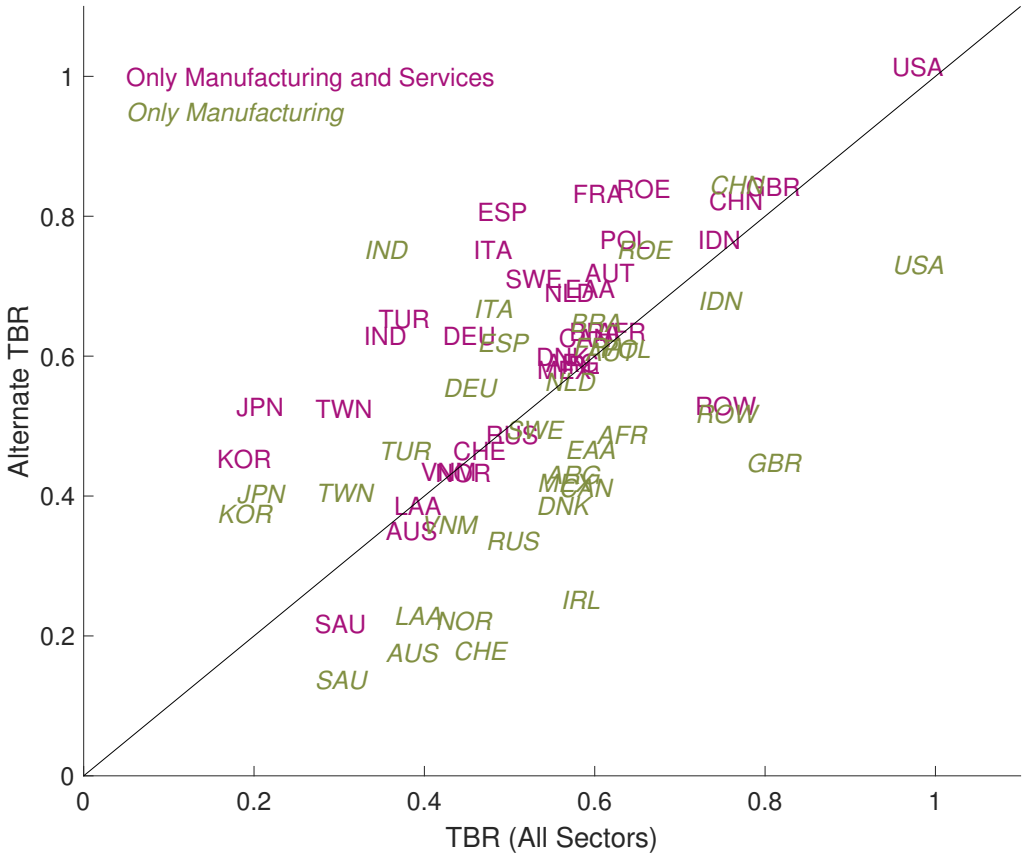


Figure 5: **TBR Version Comparison in Quantitative Model**

Note: "ROE" means Rest-of-EU, "LAA" is a subset of Latin American economies, "EAA" is a subset of east Asian economies, "AFR" is a subset of African economies, and "ROW" means Rest-of-World. "Only Manufacturing and Services" excludes ISIC codes A and B (agriculture, forestry, fishing, and mining) from trade barriers, and "Only Manufacturing" includes the C codes only.

relatively small number of countries: China in the case of Australia and the US in the case of Mexico. The Australia column even has a substantial amount of blue, meaning there are many countries that actually benefit Australia when they enter autarky.

Our second observation is that shades of blue are concentrated amongst only a few countries, especially Australia, Canada, Norway, Russia, and Saudi Arabia. A possible explanation for this is that these countries are rich in relatively homogeneous natural resources and effectively serve as substitutes for each other in the global supply chain.

Third, now looking across the columns, we see there is substantial heterogeneity in the concentration of countries' TBR decompositions. Several countries – Canada and Mexico especially – derive their TBRs predominantly from one or two other countries. With TBRs around 0.6, Canada and Mexico are in the middle of the distribution in terms of bringing relative value to the world, but this foreign value is highly concentrated in the US. On the other end of the spectrum, the UK, Germany, and Denmark have a relatively uniform effect when they enter autarky. Put differently, countries like the UK benefit many different countries from engaging in trade, whereas the gains from Mexico are concentrated in the US.

In Figure 7, we summarize the extent of TBR concentration by forming a TBR HHI for each country:

$$\text{HHI}^c = \frac{\sum_{c' \neq c} \left(\text{TBR}_{c'}^{\text{cf}(c)} \right)^2}{\left(\sum_{c' \neq c} |\text{TBR}_{c'}^{\text{cf}(c)}| \right)^2}. \tag{11}$$

We see in the top panel of Figure 7 that Denmark has the most dispersed TBR decomposition, whereas the external effects of Canada and Mexico are the most concentrated. In the bottom panel, we see that there may be a weak inverse relationship between TBR size and concentration, but the relationship is not striking, and Mexico and Canada are clear exceptions to this pattern.

4.5 TBR and other Macro Variables

One natural concern is that our quantitative TBRs might simply be repackaging standard macro variables related to income levels, economic size, or external balances for instance. Figure 8 suggests that this is not the case, although we leave a full exploration of TBR correlates for future research. We first compare each country's TBR

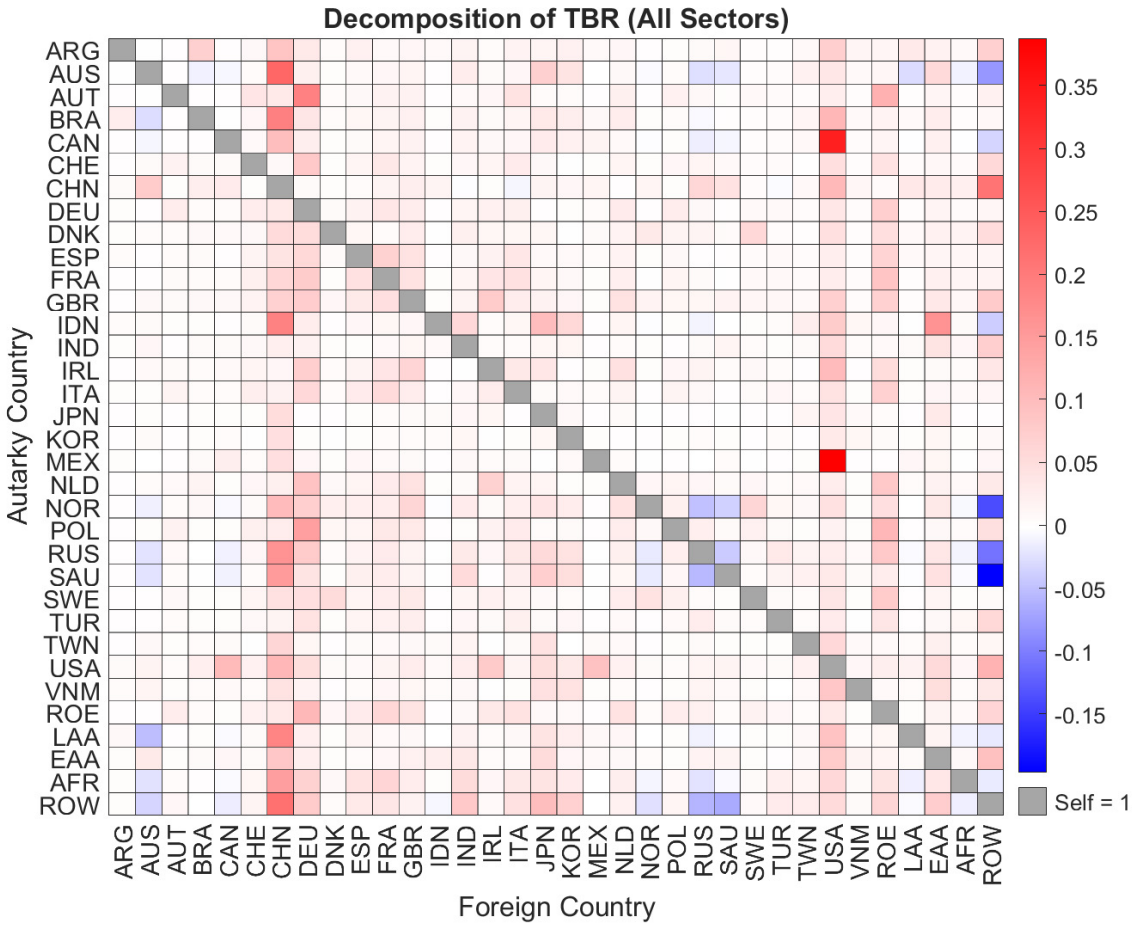


Figure 6: **Country Decomposition of TBR**

Note: The figure decomposes each country's (All Sectors) TBR into the effects on all other countries, so the columns sum (excluding gray) to the bars in the top left panel of Figure 1. Shades of red (blue) indicate an adverse (beneficial) impact on the particular foreign country. "ROE" means Rest-of-EU, "LAA" is a subset of Latin American economies, "EAA" is a subset of east Asian economies, "AFR" is a subset of African economies, and "ROW" means Rest-of-World..

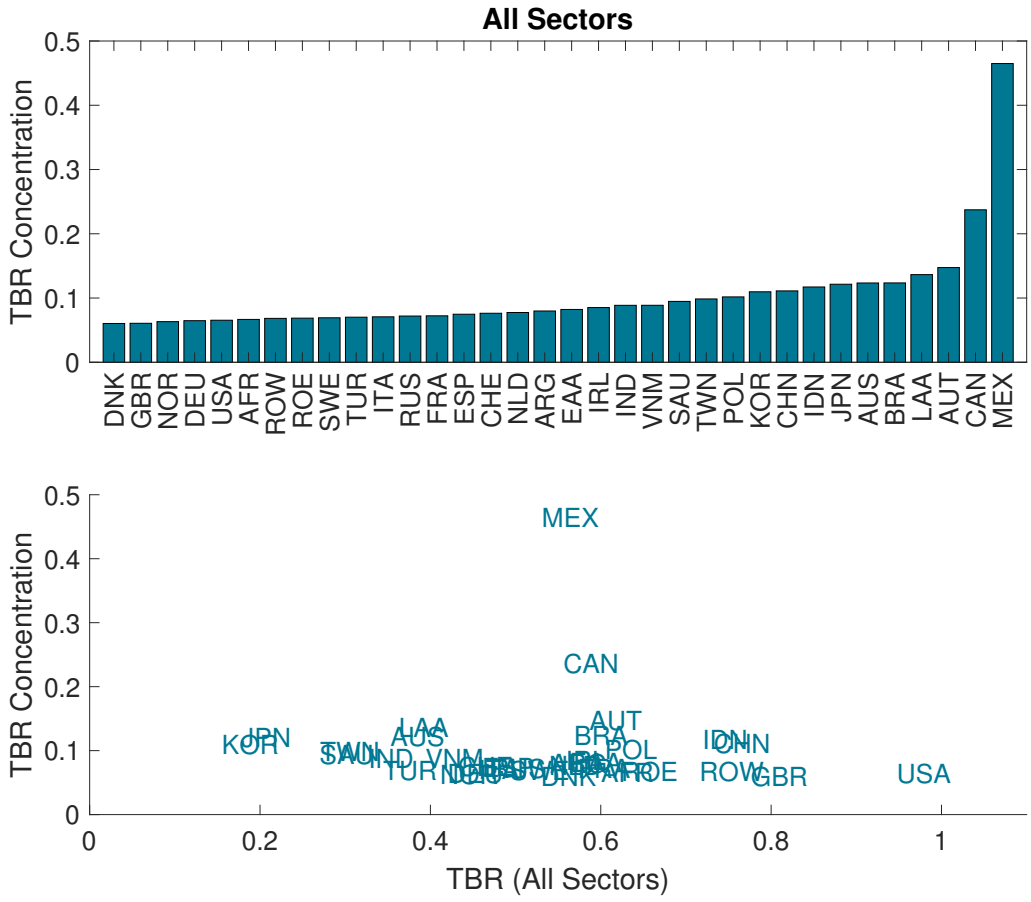


Figure 7: **TBR Concentration**

Note: The top panel shows, for each country, the cross-country TBR HHI described in the main text (11). The bottom panel shows the relationship between TBR and its concentration. "ROE" means Rest-of-EU, "LAA" is a subset of Latin American economies, "EAA" is a subset of east Asian economies, "AFR" is a subset of African economies, and "ROW" means Rest-of-World.

to its GDP per capita (top-left panel), using GDP per capita as a proxy for technological sophistication and proximity to the global productivity frontier. There is no systematic relationship. We then ask whether TBR is closely related to scale. By construction, TBR normalizes a country's external spillover by the harm it would suffer itself, so it is not purely a size measure (recall that, above, we described one sense in which TBR is scale-invariant). The plot of TBR against total GDP (top-right panel) is consistent with this. While the US and China have both high GDP and high TBR, once we exclude those two outliers (as we do in the figure), the association between GDP level and TBR disappears. Finally, we relate TBR to current account balances (bottom panel), since surpluses or deficits could, in principle, reflect cross-border trade or financing dependencies. We see there may be a weak inverse relationship, but the pattern is not particularly striking, and current accounts do not explain most of the variation in TBRs. Below, in Section 6, we discuss potential connections between trade imbalances and TBRs but conclude that the relationship is a priori ambiguous.

4.6 Robustness

Finally, before turning to our Cold War 2.0 hypothetical case study, we check the extent to which our equivalent-variation/Laspeyres formulation of TBR drives our quantitative results thus far. Replacing P^c_{pre} with P^c_{post} in Equation 3, we can form the compensating variation/Paasche version of TBR. Then, we take the geometric average of the two measures to form a Fisher-type money-metric trade benefit ratio (FTBR):

$$\text{FTBR}^{(c)} = \sqrt{\left(\frac{\sum_{c' \neq c}\left(U^{c'}_{post} - U^{c'}_{pre}\right) P^{c'}_{pre}}{\left(U^{c}_{post} - U^{c}_{pre}\right) P^{c}_{pre}}\right)\left(\frac{\sum_{c' \neq c}\left(U^{c'}_{post} - U^{c'}_{pre}\right) P^{c'}_{post}}{\left(U^{c}_{post} - U^{c}_{pre}\right) P^{c}_{post}}\right)} \tag{12}$$

Note that as with the equivalent-variation TBR in Lemma 1, the compensating variation TBR can also be calculated from the log changes generated by our computation, without knowledge of initial productivities, labor supplies, or preference and production weights.

Figure 9 shows scatter plots of the Fisher-type TBRs against the equivalent variation ones. We see that the country orderings and TBR magnitudes are quite similar, although there are a few noticeable differences. For example, the Fisher metric increases

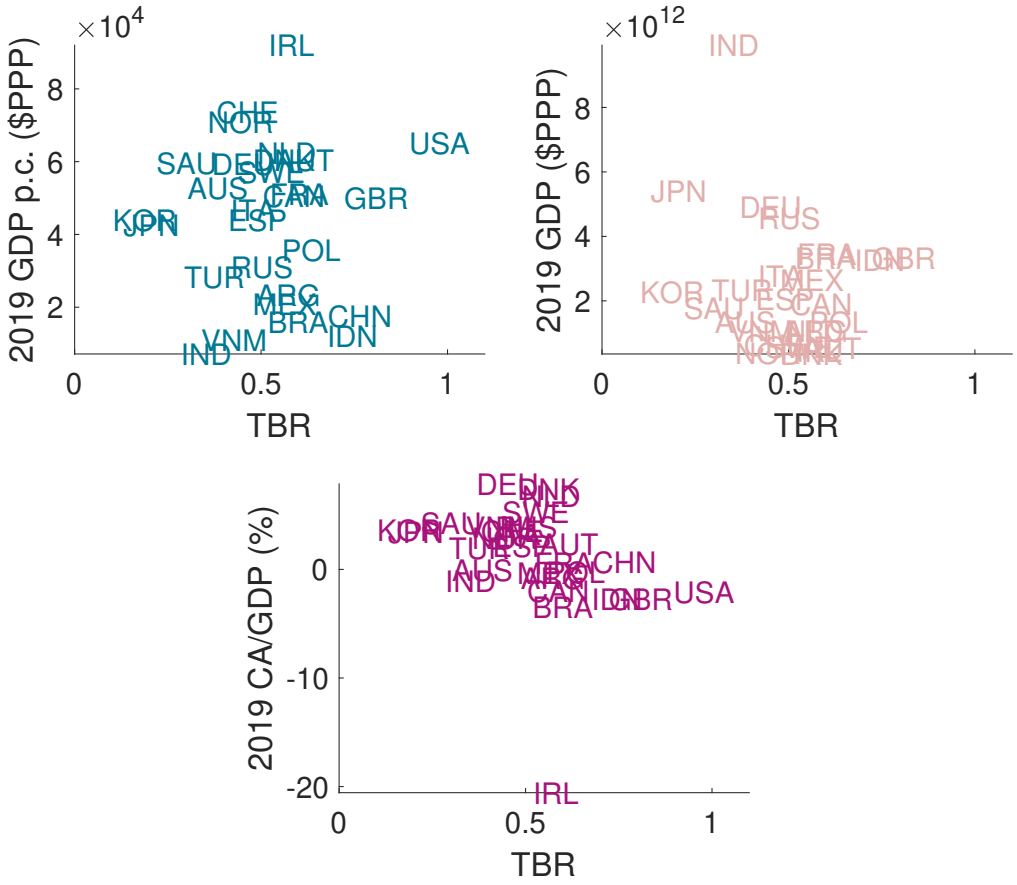


Figure 8: **TBR, Size, and Current Accounts**

Note: The figures plot the All Sectors TBR against (i) 2019 GDP per capita in PPP-adjusted US dollars, (ii) 2019 GDP in PPP-adjusted US dollars (excluding US and China outliers), and (iii) 2019 current acounts (divided by GDP), all from the World Bank's WDI, for our individual countries (except Taiwan). The WDI Chinese statistics do not include Hong Kong.

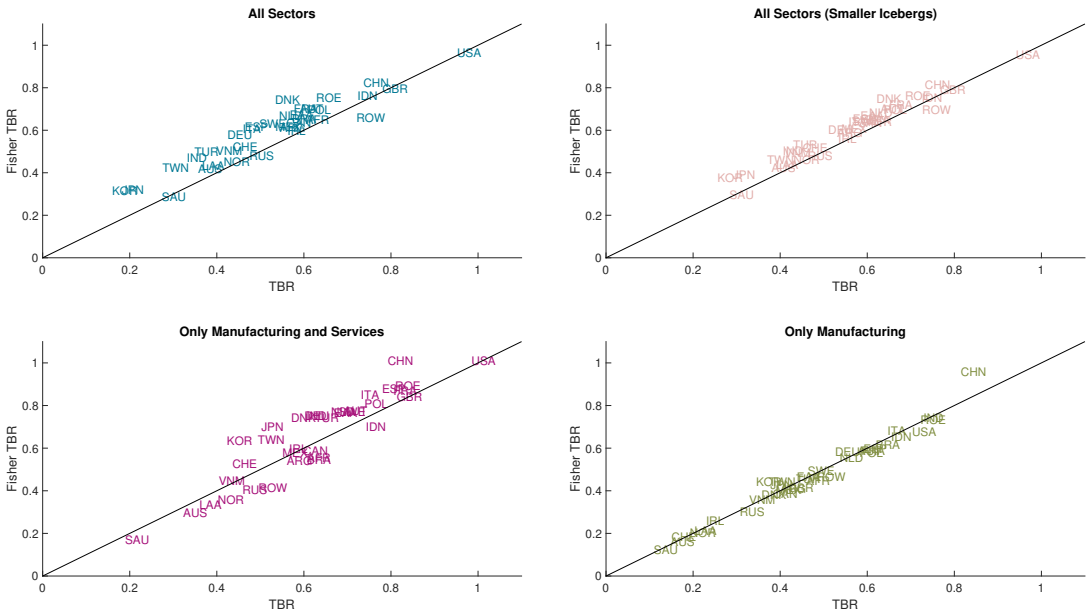


Figure 9: **Fisher TBR**

Note: The figures compare the baseline TBR measures in Figure 1 with the Fisher versions described in the text. "ROE" means Rest-of-EU, "LAA" is a subset of Latin American economies, "EAA" is a subset of east Asian economies, "AFR" is a subset of African economies, and "ROW" means Rest-of-World. "Only Manufacturing and Services" excludes ISIC codes A and B (agriculture, forestry, fishing, and mining) from trade barriers, and "Only Manufacturing" includes the C codes only.

the TBR for China. When icebergs are only applied to manufacturing and services, the increase is particularly substantial, with China's TBR rising above one.

5 Cold War 2.0 and Formation of Two Power Spheres

While we have thus far focused on individual countries, our trade benefit ratio concept can also be applied to investigate the potential power and influence of different coalitions or blocs of countries. Which coalitions would be most effective at generating economic leverage over other countries and blocs? Or, how balanced are hypothetical power spheres that could potentially form and decouple from one another?

While we don't answer these important questions with our present analysis, we provide a geopolitical scenario that illustrates an application of our quantitative TBR framework. We call our scenario Cold War 2.0, borrowing the term from Baqaee et al. (2024)

but using different specifications.[21] This scenario represents an extreme escalation of existing global conflicts. Specifically, we attempt to quantify the economic effects of a decoupling of two power spheres, a decoupling that could stem from conflicts involving Taiwan. A US-aligned power sphere forms, and the US is joined by the EU, Norway, Switzerland, and non-EU G7 (Canada, Japan, and UK) in putting large trade barriers on exports from a second power sphere: China (with Taiwan) and Russia. Specifically, 300% icebergs in all Europe/G7 bloc countries are applied to all exports of goods and services from the China/Taiwan/Russia bloc (which equivalently reciprocates). Countries outside of the two spheres continue to trade with everyone without direct disruption but are affected by spillovers, and trade within the blocs is uninterrupted.

The country-specific economic effects are summarized in the top panel of Figure 10. The real income impacts – the percent changes in I^c/P^c (Equation 6) – are mostly negative and quite large in some cases (more than -8.5% for Taiwan, almost -7% for Russia, -2.65% for China, around -2% for Ireland and Japan, and -2.39% for Poland). Some economies outside of the two power spheres experience non-trivial gains, for example Vietnam ($+2.64\%$), Mexico ($+0.68\%$), and Turkey ($+0.79\%$). The change is -0.94% for the US.

The bottom-left panel of Figure 10 shows the percent real income changes for the blocs as a whole in the Cold War 2.0 scenario. The % loss for the eastern bloc (over 3%) is more than double the loss of the Europe/G7 bloc. But TBR reveals a somewhat different story. We calculate a country bloc TBR by putting the coalition countries into autarky, while maintaining free trade within the bloc, and measuring the equivalent-variation loss outside the bloc relative to the equivalent variation within the bloc.[22] Both the Europe/G7 group and the China/Taiwan/Russia bloc have a TBR of around 0.8. That is, the two blocs have a similar capacity to harm other countries around the world, relative to self-inflicted harm.

[21] In Gersbach et al. (2025), we quantify the economic effects of many other geopolitical scenarios.

[22] For example,

$$\text{TBR}^{(\text{East})} = \frac{\sum_{j \notin \text{East}} \text{EV}_j^{\text{cf(East)}}}{\sum_{j \in \text{East}} \text{EV}_j^{\text{cf(East)}}}.$$

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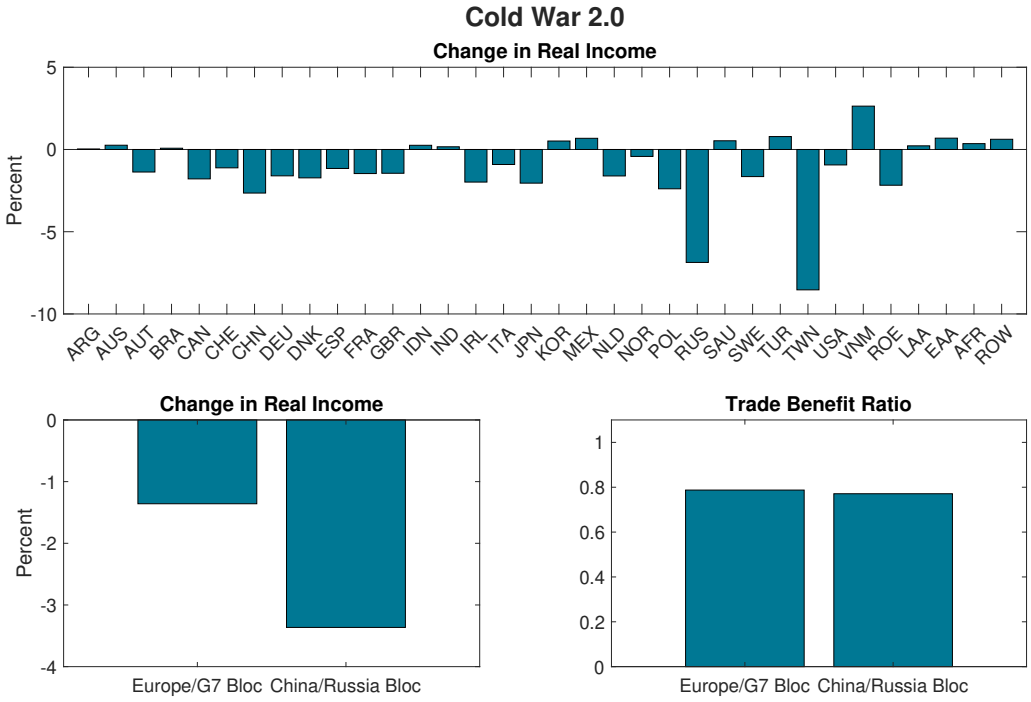


Figure 10: **Cold War 2.0**

Note: In the scenario, a China/Russia bloc and a Europe/G7 bloc put 300% icebergs on each other's exports. "ROE" means Rest-of-EU, "LAA" is a subset of Latin American economies, "EAA" is a subset of east Asian economies, "AFR" is a subset of African economies, and "ROW" means Rest-of-World. Real income % changes in the bottom left panel are averages of the individual member changes, weighted by the average of pre- and post-shock nominal GDP shares. The TBRs in the bottom right panel are calculated by putting the respective blocs into autarky one at a time (via 300% icebergs) with respect to all other countries, while maintaining trade within the bloc.

6 Other Considerations

Our TBR metric is a measure of relative economic value only through the trade network. Equilibrium in our static model represents a long-run and frictionless steady state and accounts neither for short-run effects nor for cross-border financial assets. Here, we highlight other important dimensions of international economic interactions and discuss their potential implications for quantifying the economic value of nations (for others).

The Role of the Capital Account and Financial Assets

Our examples and quantitative TBRs were calculated in static models, assuming balanced trade, but in practice many countries have substantial non-zero current accounts and international investment positions. Accounting for the level and composition of cross-border asset trade would likely modify the domestic and foreign costs of a country entering autarky.

However, at least with complete markets and similar discount rates across countries, we conjecture that TBRs calculated from present value losses in a dynamic model with borrowing and lending need not be so different from TBRs in a static model. Relative to the balanced trade case, a borrower/net importer entering autarky would lose more today, but also less tomorrow (when the debts would have come due). Conversely, the lender/net exporter would lose less today but more tomorrow. So with similar discount rates, the dynamic TBR might not deviate much from the static one, assuming borrowing is not facilitating the gross trade flows through overcoming market frictions.

On the other hand, dynamic TBRs could deviate substantially from the static, balanced trade case if loans, foreign direct investment (FDI), foreign reserves, etc. were necessary for the functioning of the global network, due to market incompleteness and other frictions. For example, the existence of trade linkages between two countries could depend on credit or underwriting from a third country. Or the production network *within* a country could rely on foreign currencies or foreign financial institutions. To some extent, our framework accounts for these factors because we include trade in services. But in such cases, entry into autarky could affect domestic or foreign production on an *extensive* margin, changing the structure of the supply chain itself. Our intuition, at least, is that broader TBR measures explicitly considering financial

markets would increase the relative economic value of financial hub countries.[23]

Relatedly, our TBRs do not capture the effects of international or reserve currencies and the potential "exorbitant privilege" that comes with having such a currency. For instance, one might argue that the large US TBR we calculate is overstated because it does not include the additional harm the US would suffer from losing dollar privilege. But to the extent that there is no easy dollar substitute and the reserve status of the dollar serves a valuable purpose in other countries, one might instead argue that our trade-based US TBR is understated. So while accounting for international usage of the US dollar, euro, yen, Swiss franc, and so on would likely change TBRs, the sign of the effect is not ex ante clear.

Lastly, our framework does not account for FDI, multinational corporations, or other forms of cross-border capital ownership. Assuming foreign-owned capital would remain in its current domicile after an autarky event, accounting for these factors would seem to decrease the TBRs of countries with extensive foreign investments. Again, however, it is difficult to sign the effect ex ante because autarky would decrease the future extension of productive FDI to recipient countries.

Second-Layer Effects

Our quantitative model captures the effects of relative price and demand changes caused by trade shocks, how sectors respond to these changes in their production of goods and services, and the spillover effects on all market participants resulting from these changes. However, our TBR calculations assume a frictionless shift from one trade equilibrium to another and are therefore static in nature, abstracting from the impacts of the transition path. This abstraction leaves out important real-world mechanisms. In practice, trade disruptions often trigger amplifying feedback: higher import barriers raise prices and, when wages or prices are sticky, this can depress output and employment, pushing consumption into a downward spiral. Financial frictions may reinforce the amplification, as firms facing lost trade revenues encounter tighter credit, cut investment, or even default, potentially generating broader financial instability. These macroeconomic channels could substantially magnify the true welfare losses beyond what our model captures.

[23]Economies of scale in traded-goods sectors are another potential mechanism through which trade imbalances could affect TBRs. Cuñat and Zymek (2025) show that long-run imbalances can depress productivity and real income in deficit countries, reducing their costs of autarky.

The framework also abstracts from production stoppages in critical inputs. Shocks to supply chains for rare minerals, semiconductors, or other strategic goods could halt entire industries. For example, a cutoff of Taiwan's semiconductor exports or China's key raw materials would likely cause far greater economic harm than our static estimates suggest. Resulting effects – including disrupted innovation, long-run growth setbacks, and systemic bottlenecks – could significantly amplify both transitional and long-term costs. In sum, while the TBR offers a useful first-order measure of a country's value-for-others through trade, actual geopolitical leverage could be different once these additional effects are taken into account.

In Gersbach et al. (2025), we call these further-reaching effects the "second-layer" effects and provide an extensive discussion and some assessments of how they might amplify or mitigate the costs of geopolitical shocks. Of course, one important avenue for future research is to calculate TBRs when more of these second-layer effects are taken into account. While any autarky or decoupling event would involve costs along these lines, the second-layer effects are particularly relevant for the Cold War 2.0 scenario, and we evaluate them in detail Gersbach et al. (2025).[24]

7 Conclusion

By blocking trade with the outside world, how much harm can a country inflict on others relative to the self-inflicted damage? When a country engages in trade, is it capturing most of the benefits, or are the gains substantially accruing to other countries? As Hirschman (1945), Keohane and Nye (1977), and others have argued, these questions are important for understanding the power of nations. More generally, the rapidly growing field of geoeconomics seeks to use economic theory to understand the source and extent of influence nations can exert on others through threats of trade barriers. We contribute to this area by introducing a new index, the *Trade Benefit Ratio* (TBR), which measures the harm a country can inflict on others by going into autarky, relative to the self-inflicted damage. Put differently, the TBR measures the value a country creates for others through trade, relative to the value it receives. Calibrating and solving a quantitative general equilibrium model of global trade networks for 34

[24] See also Baqaee et al. (2024) for an excellent account of caveats when using a similar trade model to assess the consequences for Germany of a hard decoupling from China.

countries/regions and 38 sectors, we calculate TBRs for each country and apply our framework to a hypothetical Cold War 2.0 scenario.

Our analysis reveals several patterns that could inform strategies regarding future geopolitical conflicts. Some countries, such as the US and UK, disproportionately create external value through services, whereas others, like India, derive potential influence more through manufacturing. TBRs for China and Indonesia, for example, appear more uniform across sectors. Some countries gain from trade with many countries, while others are exposed to a small set of partners. Countries rich in natural resources seem to be substitutes for each other in the sense that the exit of one actually benefits the others. TBRs can be dispersed or concentrated with respect to the other countries affected when one goes into autarky. Mexico and Canada, for example, predominantly impact the US, whereas the UK creates value for many countries.

Beyond offering a new perspective on the distribution of trade benefits, the TBR framework opens avenues for both policy applications and future research. From a policy perspective, the TBR highlights two key dimensions. Geoeconomic leverage arises when high-TBR countries can impose disproportionately large costs on others relative to their own losses, enhancing the credibility of sanctions or trade restrictions as diplomatic tools. Conversely, low-TBR countries are more exposed and may seek diversification or alliance-based insurance. At the same time, alliance and bloc formation can be understood through TBR patterns: countries that generate substantial value for one another are natural candidates for stable coalitions, while asymmetric dependencies may expose fault lines. Policymakers could thus use TBR insights to evaluate the credibility of trade threats and the resilience of international partnerships.

For research, the framework suggests several promising directions. Dynamics and growth matter: incorporating capital accumulation, investment responses, and adjustment costs would show how isolation effects unfold over time. Financial markets and asset flows also deserve attention, since cross-border lending, reserve-currency status, and credit frictions shape countries' external value, especially in cases such as the US, whose trade deficit is financed by foreign capital. Finally, a range of other channels could enrich the analysis, from technology spillovers and richer services trade to labor mobility, sanctions, and climate or security considerations. Understanding how TBR interacts with supply-chain resilience or the transition to low-carbon emissions would

be especially valuable.

In sum, the TBR provides a novel lens on the economic value nations create for others through trade, with clear implications for policy and ample scope for future work.

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A Computing the Effect of Shocks

In this section, we use the method of Baqaee and Farhi (2024) to compute how equilibrium changes in response to iceberg shocks, and we offer a matrix version of their Theorem 3 specialized to our particular economy.[25] First, we show how equilibrium changes in response to small arbitrary changes in icebergs. Next, we explain how to calculate the effect of small productivity shocks by showing they are equivalent to particular icebergs. Finally, we show how to globally approximate the effect of large shocks using Euler's numerical integration method. In short, the strategy is to characterize analytically the derivative of the equilibrium system and then numerically integrate over the first-order changes to get an exact solution.

A.1 Derivatives

At the consumer optimum, it is straightforward to show that the utility of representative consumer $c \in C$ is equal to real income I^c/P^c. Therefore, for a small shock the percent change in utility U^c (5) is

$$d\log U^c = d\log I^c - d\log P^c.$$

Since labor supply is fixed, the first term is

$$d\log I^c = \sum_{k\in N}\left(\frac{w^{kc}L^{kc}}{I^c}\right) d\log w^{kc} \tag{13}$$

[25] Our model has a structure similar to the one used for quantitative exercises in Baqaee and Farhi (2024) and Bachmann et al. (2024). Our notation and exposition are based in part on Çakmaklı et al. (2021).

Let w (without superscripts) be the $CN \times 1$ vector of wages for the world (stacking the wages of country 1, country 2, and so on). Let $\mathcal{F}^c$ be the $1 \times N$ vector of labor income shares $w^{kc}L^{kc}/I^c$ for each country, and define the $C \times CN$ matrix of world labor income shares

$$\mathcal{F} = \begin{pmatrix} \mathcal{F}^1 & 0 & 0 & 0 \\ 0 & \mathcal{F}^2 & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \dots & 0 & \mathcal{F}^C \end{pmatrix}.$$

Then letting I be the vector of country incomes, the matrix version of Equation 13 is

$$d\log I = \mathcal{F} d\log w. \tag{14}$$

Since $\mathcal{F}$ can be constructed from data, we can compute the change in equilibrium nominal income if we know the change in wages.

To get the change in real income (welfare), we need to figure out the change in the consumer price indexes P^c. And it turns out you can solve for all price changes as a linear function of $d\log w$. So if we want to compute the change in welfare from a change in icebergs (the τ's), we just need to compute the change in equilibrium nominal wages. Conveniently, as Baqaee and Farhi (2024) show, the changes in the equilibrium equations reduce to a linear system for the change in wages.

The first step is to show that price changes from small changes in icebergs are a linear function of $d\log w$, but this requires more notation. Let $d\log P$ be the vector of price

changes ordered as follows:

$$d\log P = \begin{pmatrix} \overrightarrow{d\log P^c} \\ \overrightarrow{d\log P^{kc}} \\ \overrightarrow{d\log P_k^c} \\ \overrightarrow{d\log P_j^{kc}} \\ \overrightarrow{d\log p_{kc}} \end{pmatrix},$$

where $\overrightarrow{d\log P^c}$ is the vector of consumer price index changes across countries, $\overrightarrow{d\log P^{kc}}$ is the vector of aggregate intermediate price changes across countries and industries, $\overrightarrow{d\log P_k^c}$ is the vector of price changes for composite intermediates in the consumption sector across countries and industries, $\overrightarrow{d\log P_j^{kc}}$ is the vector of price changes for composite intermediates in the production sector across countries and industries, and $\overrightarrow{d\log p_{kc}}$ is the vector of raw intermediate price changes across countries and industries. $\overrightarrow{d\log P_j^{kc}}$, for example, is $CN^2 \times 1$, and the whole vector has dimension $\mathcal{N} = C(1 + N + N + N^2 + N)$ (the number of firms and consumers). In words, the order is (1) consumer price indexes, (2) aggregate intermediates, (3) consumer composite intermediates, (4) producer composite intermediates, and (5) raw intermediates.

Next, let $G_{cc'}$ be the $N \times N$ matrix of iceberg changes when the composite consumption intermediates in country c import from the raw intermediates from country c'. This is a diagonal matrix with $d\log \tau_{jc'}^c$ on the diagonal and zeros elsewhere. Let $H_{c'}^{kc}$ be the $N \times N$ matrix of iceberg changes when the composite intermediate firms from industry k of country c buy raw intermediates from country c'. This is a diagonal matrix with $d\log \tau_{jc'}^{kc}$ on the diagonal and zeros elsewhere. Using those, we define the matrices of

iceberg changes in the consumption and production sector

$$d\log\tau^{cons} = \begin{pmatrix} G_{11} & G_{12} & \dots & G_{1C} \\ G_{21} & \ddots & \dots & G_{2C} \\ \vdots & \dots & \ddots & \vdots \\ G_{C1} & \dots & \dots & G_{CC} \end{pmatrix}, \qquad d\log\tau^{prod} = \begin{pmatrix} H_1^{11} & H_2^{11} & \dots & H_C^{11} \\ H_1^{21} & H_2^{21} & \dots & H_C^{21} \\ \vdots & \vdots & \ddots & \vdots \\ H_1^{N1} & H_2^{N1} & \dots & H_C^{N1} \\ H_1^{12} & H_2^{12} & \dots & H_C^{12} \\ \vdots & \vdots & \ddots & \vdots \\ H_1^{NC} & H_2^{NC} & \dots & H_C^{NC} \end{pmatrix}$$

which are respectively $CN \times CN$ and $CN^2 \times CN$. Defined the matrix

$$d\log T = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & d\log\tau^{cons} \\ 0 & 0 & 0 & 0 & d\log\tau^{prod} \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

which is $\mathcal{N} \times \mathcal{N}$.

Now we define the expenditure share matrices. Let Ω_c^{cons} be the $1 \times N$ vector of expenditure shares of consumer c on composite consumption intermediates $P_k^c x_k^c / I^c$. Similarly, we define Ω_{kc}^{prod} be the $1 \times N$ vector of expenditure shares of the aggregate intermediate firm k in country c on composite intermediates $P_j^{kc} x_j^{kc} / (P^{kc} X^{kc})$. Using

those we define the expenditure share matrices

$$\Omega^{cons} = \begin{pmatrix} \Omega_1^{cons} & 0 & 0 & 0 \\ 0 & \Omega_2^{cons} & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \dots & 0 & \Omega_C^{cons} \end{pmatrix},$$

$$\Omega^{prod} = \begin{pmatrix} \Omega_{11}^{prod} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \Omega_{21}^{prod} & 0 & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ddots & 0 & \vdots & \vdots & \vdots \\ 0 & \dots & 0 & \Omega_{N1}^{prod} & 0 & \vdots & \vdots \\ 0 & \dots & \dots & 0 & \Omega_{12}^{prod} & 0 & \vdots \\ 0 & \dots & \dots & \dots & 0 & \ddots & 0 \\ 0 & \dots & \dots & \dots & \dots & 0 & \Omega_{NC}^{prod} \end{pmatrix},$$

respectively of size $C \times CN$ and $CN \times CN^2$.

The expenditure share matrices for the composite intermediates have a slightly different shape. For any two countries c, c', let $A_{cc'}$ be the $N \times N$ matrix of the expenditure share of consumption composite intermediates in country c on raw intermediates from country c'. This is a diagonal matrix with $(\tau_{kc'}^c p_{kc'} y_{kc'}^c)/(P_k^c x_k^c)$ on the diagonal and zeros elsewhere.

For any industry k and two countries c, c', we define $B_{c'}^{kc}$ to be the $N \times N$ matrix of the expenditure share of the production composite intermediates of industry k in country c on raw intermediates from country c'. This is a diagonal matrix with $(\tau_{jc'}^{kc} p_{jc'} y_{jc'}^{kc})/(P_j^{kc} x_j^{kc})$ on the diagonal and zeros elsewhere. Using those matrices, we define the expenditure share matrix of consumption composite intermediates on raw

intermediates as well as the expenditure share matrix of production composite intermediates on raw intermediates as

$$\Omega^{comp-cons} = \begin{pmatrix} A_{11} & A_{12} & \dots & A_{1C} \\ A_{21} & \ddots & \dots & A_{2C} \\ \vdots & \dots & \ddots & \vdots \\ A_{C1} & \dots & \dots & A_{CC} \end{pmatrix}, \qquad \Omega^{comp-prod} = \begin{pmatrix} B_1^{11} & B_2^{11} & \dots & B_C^{11} \\ B_1^{21} & B_2^{21} & \dots & B_C^{21} \\ \vdots & \vdots & \ddots & \vdots \\ B_1^{N1} & B_2^{N1} & \dots & B_C^{N1} \\ B_1^{12} & B_2^{12} & \dots & B_C^{12} \\ \vdots & \vdots & \ddots & \vdots \\ B_1^{NC} & B_2^{NC} & \dots & B_C^{NC} \end{pmatrix}.$$

respectively of size $CN \times CN$ and $CN^2 \times CN$.

Finally, for any country c, let Ω_c^{raw} be the $N \times N$ matrix of expenditure shares of the raw production sectors in country c on aggregate intermediates in country c. This is a diagonal matrix with entries $(P^{kc}X^{kc})/(p_{kc}y_{kc})$ on the diagonal and zeros elsewhere. The expenditure shares matrix of raw intermediates on aggregate intermediates is defined as

$$\Omega^{raw} = \begin{pmatrix} \Omega_1^{raw} & 0 & \dots & 0 \\ 0 & \Omega_2^{raw} & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \Omega_C^{raw} \end{pmatrix}$$

which is of size $CN \times CN$. The resulting $\mathcal{N} \times \mathcal{N}$ input/output matrix Ω for the world

is

$$\Omega = \begin{pmatrix} 0 & 0 & \Omega^{cons} & 0 & 0 \\ 0 & 0 & 0 & \Omega^{prod} & 0 \\ 0 & 0 & 0 & 0 & \Omega^{comp-cons} \\ 0 & 0 & 0 & 0 & \Omega^{comp-prod} \\ 0 & \Omega^{raw} & 0 & 0 & 0 \end{pmatrix}.$$

Finally, let $\hat{F}$ be the diagonal $CN \times CN$ labor expenditure share matrix, which has $w^{kc}l^{kc}/p_{kc}y_{kc}$'s on the diagonal, and define the $\mathcal{N} \times CN$ matrix

$$F = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \hat{F} \end{pmatrix}.$$

Let $\mathbf{1}$ be a vector of ones, let $\mathcal{I}$ be the identity matrix, and let $\circ$ be element-by-element multiplication. We can now prove a version of part of Theorem 3 from Baqaee and Farhi (2024).

Proposition 3 (Baqaee and Farhi (2024))

$$d\log P = (\mathcal{I} - \Omega)^{-1} F d\log w + (\mathcal{I} - \Omega)^{-1}(\Omega \circ d\log T)\mathbf{1}$$

Proof. Consider a generic CES producer with production function

$$q = a\left(\sum_k u_k^{\frac{1}{\gamma}} x_k^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}}$$

who faces prices $\tau_k p_k$ and chooses x_k to minimize the cost of producing q. Letting

$\tilde{p} = \left(\sum_k u_k \left(\tau_k p_k\right)^{1-\gamma}\right)^{\frac{1}{1-\gamma}}$ and defining $p = \tilde{p}/a$, at the optimum

$$\frac{\tau_k p_k x_k}{pq} = u_k \left(\frac{\tau_k p_k}{\tilde{p}}\right)^{1-\gamma}$$

and p is the marginal cost of producing q. You get the same solution if the production function is a consumption aggregate, and a consumer with income I maximizes its value. In that case, p is the price index and $pq = I$ at the optimum. And you can think of the inputs as labor, in which case the prices are wages.

Noting that $\log p = \frac{1}{1-\gamma} \log \left(\sum_k u_k \left(e^{\log \tau_k} p_k\right)^{1-\gamma}\right) - \log a$, we have

$$\frac{\partial \log p}{\partial \log \tau_j} = \frac{\sum_k u_k \left(\tau_k p_k\right)^{-\gamma} \tau_k \frac{\partial p_k}{\partial \log \tau_j}}{\tilde{p}^{1-\gamma}} + \frac{u_j \left(\tau_j p_j\right)^{-\gamma} \tau_j p_j}{\tilde{p}^{1-\gamma}} = \sum_k O_k \frac{\partial \log p_k}{\partial \log \tau_j} + O_j,$$

where $O_k = \frac{\tau_k p_k x_k}{pq}$ is the expenditure share, and we used $d \log z/dy = (dz/dy)/z$. It follows that the total derivative is

$$d \log p = \sum_k O_k d \log p_k + \sum_k O_k d \log \tau_k.$$

This derivative holds for each producer of the model, although only the composite intermediate producers have the second direct iceberg term, and the raw intermediate producers only have two inputs: aggregate intermediates and labor. Returning to the notation of the paper and proposition, explicitly distinguishing between labor and intermediate inputs, and appropriately adding zeros in matrices to represent the structure of the model, the price derivative for the entire economy can be written in matrix form as

$$d \log P = \Omega d \log P + F d \log w + (\Omega \circ d \log T)\mathbf{1}.$$

Solving for $d \log P$ completes the proof. ■

The next and most difficult step is to use Equation 17 below – sales equal purchases in equilibrium – to write wage changes as a linear function of price changes. Doing

so requires defining Domar weights. Letting $Y = \sum_{c \in C} I^c$ be world income, a firm's Domar weight (denoted by λ's) is its share of world income. For example, the Domar weight for the firm in country c producing composite intermediate j for industry k is $\lambda_j^{kc} = P_j^{kc} x_j^{kc}/Y$, and the Domar weight for the industry k raw intermediate producer in country c is $\lambda_{kc} = p_{kc} y_{kc}/Y$. Let λ be the vector of Domar weights around the world, in the same order as $d \log P$ but excluding the consumers. This vector has size $\mathcal{M} \times 1$ where $\mathcal{M} := C(N + N + N^2 + N)$. The Domar weight for a consumer is $\chi^c = I^c/Y$, and χ is the vector of representative consumer Domar weights. $\Lambda^{kc} = w^{kc} L^{kc}/Y$ is the Domar weight for labor in country c's industry k. Let Λ be the vector of all Domar weights for labor Λ^{kc} and Λ^c be the country specific vector. Since world income is normalized to one (so $dY = 0$) and labor is fixed, we have $d\chi^c = \sum_{k \in N} \Lambda^{kc} d \log w^{kc}$ and

$$d\chi = \tilde{\Lambda} d \log w, \tag{15}$$

where $\tilde{\Lambda}$ is the $C \times CN$ matrix with Λ^c's on the diagonal. Another consequence of fixed Y is

$$d\Lambda = \text{diag}(\Lambda^{kc}) d \log w, \tag{16}$$

implying the percent change in wages is equal to the percent change in labor Domar weights.

By market clearing, the income of a firm must be equal to expenditure on the firm by consumers and other firms. In other words, letting $\tilde{\Omega}$ be the above input/output matrix but with the first C columns and rows removed (consumption), we must have

$$\lambda' = \chi' \tilde{\Omega}^{cons} + \lambda' \tilde{\Omega} \implies \lambda' = \chi' \tilde{\Omega}^{cons} \Psi, \tag{17}$$

where $\Psi = (\mathcal{I} - \tilde{\Omega})^{-1}$, and $\tilde{\Omega}^{cons}$ is the top C rows of Ω after removing the first C columns. Using $d\Psi = \Psi d\tilde{\Omega} \Psi$ and Equation 17, we have

$$d\lambda' = d\chi' \tilde{\Omega}^{cons} \Psi + \chi' d\tilde{\Omega}^{cons} \Psi + \chi' \tilde{\Omega}^{cons} d\Psi = (d\chi' \tilde{\Omega}^{cons} + \chi' d\tilde{\Omega}^{cons} + \lambda' d\tilde{\Omega}) \Psi. \tag{18}$$

And let $\tilde{F}$ be F with the first C rows removed. Then $\Lambda = \tilde{F}'\lambda$ and

$$d\Lambda' = d\lambda'\tilde{F} + \lambda' d\tilde{F}. \tag{19}$$

Suppose we can write $\lambda' d\tilde{F}$, $\chi' d\tilde{\Omega}^{cons}$, and $\lambda' d\tilde{\Omega}$ as linear functions of $d\log P$ and $d\log w$. Then with Proposition 3 and Equations 15, 16, 18, and 19, we can derive and solve a linear equation for $d\log w$. The next proposition (again corresponding to Theorem 3 of Baqaee and Farhi (2024)) writes $\lambda' d\tilde{F}$, $\chi' d\tilde{\Omega}^{cons}$, and $\lambda' d\tilde{\Omega}$ as linear functions of $d\log P$ and $d\log w$.

Proposition 4

1. $\lambda' d\tilde{F} = d\log w' \operatorname{diag}((1-\theta)\Lambda^{kc}) - d\log P'\Phi_A$, *where*

$$\Phi_A = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \operatorname{diag}((1-\theta)\Lambda^{kc}) \end{bmatrix}$$

   is $\mathcal{N} \times CN$

2. $\chi' d\tilde{\Omega}^{cons} = d\log P'\Phi_B$ *where*

$$\Phi_B = (1-\sigma) \begin{bmatrix} 0 & -\chi \circ \Omega^{cons} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \operatorname{diag}(\lambda_k^c) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

is $\mathcal{N} \times \mathcal{M}$

3. $\lambda' d\tilde{\Omega} = d \log P' \Phi_C + \mathbf{1}' \Phi_D$ *where*

$$\Phi_C = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \text{diag}((1-\theta)\lambda^{kc}) & 0 & -(1-\epsilon)\lambda^{kc} \circ \Omega^{prod} & 0 \\ 0 & 0 & 0 & -\mu \circ \lambda_k^c \circ \Omega^{comp-cons} \\ 0 & 0 & \text{diag}((1-\epsilon)\lambda_j^{kc}) & -\nu \circ \lambda_j^{kc} \circ \Omega^{comp-prod} \\ -\text{diag}((1-\theta)\lambda^{kc}) & 0 & 0 & \text{diag}((1-\theta_k)\lambda_{kc}) \end{bmatrix}$$

$$\Phi_D = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu \circ \lambda_k^c \circ \Omega^{comp-cons} \circ d \log \tau^{cons} \\ 0 & 0 & 0 & \nu \circ \lambda_j^{kc} \circ \Omega^{comp-prod} \circ d \log \tau^{prod} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

where ν *has size* $1 \times CN$ *and is given by* $[1-\theta_k]_{kc}$*. The vector* μ *is of size* $CN \times 1$ *and is given by* ν'*. Moreover, the first matrix has size* $\mathcal{N} \times \mathcal{M}$ *and the second matrix* $\mathcal{N} \times \mathcal{M}$

Remark 1

In the proof that follows, it will sometimes be difficult to distinguish between vectors/matrices and the values inside of them. For example, λ_{kc} *could both refer to the* $CN \times 1$ *vector of Domar weights or to the value* $p_{kc}y_{kc}/Y$*. In general, it should be possible to infer from the context what is meant. However, sometimes, to make sure everything is clear we will use the following notation. If* a_k *are some values indexed over* $k \in I$ *where* I *can be both explicit or implicit, then* $[a_k]_k$ *will refer to the row vector with entries* a_k*.*

Moreover, previously, we defined $\circ$ *as the element-wise multiplication of matrices. We*

extend this operation slightly. If A is an $n \times m$ matrix and v an $n \times 1$ vector, then, $v \circ A$ will be taken to mean element-wise multiplication of v with each column of A. Similarly, if v is an $1 \times m$ vector, then, $v \circ A$ will be taken to mean element-wise multiplication of v with each row of A.

Proof. Recall that

$$d\log P' = [d\log P^{c\prime}, d\log P^{kc\prime}, d\log P_k^{c\prime}, d\log P_j^{kc\prime}, d\log p_{kc}{}'].$$

Concerning the first part, we begin by observing that

$$\begin{aligned}\lambda' d\tilde{F} &= \begin{bmatrix}\lambda^{kc\prime} & \lambda_k^{c\prime} & \lambda_j^{kc\prime} & \lambda_{kc}{}'\end{bmatrix}\begin{bmatrix}0 & 0 & 0 & d\hat{F}\end{bmatrix}^T = \lambda_{kc}{}' d\hat{F} \\ &= \left[\lambda_{kc} d\left(\frac{w^{kc}L^{kc}}{p_{kc}y_{kc}}\right)\right]_{kc}.\end{aligned}$$

The minimization problem of the raw producer (10) implies that labor demand is $\frac{w^{kc}L^{kc}}{p_{kc}y_{kc}} = \alpha^{kc}\left(\frac{w^{kc}}{A^{kc}p_{kc}}\right)^{1-\theta}$. It follows

$$\begin{aligned}d\left(\frac{w^{kc}L^{kc}}{p_{kc}y_{kc}}\right) &= d\left(\alpha^{kc}\left(\frac{w^{kc}}{A^{kc}p_{kc}}\right)^{1-\theta}\right) \\ &= \frac{\alpha^{kc}}{(A^{kc})^{1-\theta}}\frac{(1-\theta)(w^{kc})^{1-\theta}p_{kc}^{1-\theta}d\log w^{kc} - (1-\theta)(w^{kc})^{1-\theta}p_{kc}^{1-\theta}d\log p_{kc}}{p_{kc}^{2(1-\theta)}} \\ &= (1-\theta)\alpha^{kc}\left(\frac{w^{kc}}{A^{kc}p_{kc}}\right)^{1-\theta}\left(d\log w^{kc} - d\log p_{kc}\right) \\ &= (1-\theta)\frac{w^{kc}L^{kc}}{p_{kc}y_{kc}}\left(d\log w^{kc} - d\log p_{kc}\right)\end{aligned}$$

and thus

$$\lambda_{kc} d\left(\frac{w^{kc}L^{kc}}{p_{kc}y_{kc}}\right) = (1-\theta)\frac{w^{kc}L^{kc}}{Y}\left(d\log w^{kc} - d\log p_{kc}\right) = (1-\theta)\Lambda^{kc}\left(d\log w^{kc} - d\log p_{kc}\right)$$

and we are done.

For the second part, we get by definition that

$$\chi' d\tilde{\Omega}^{cons} = \chi' \begin{bmatrix} 0 & d\Omega^{cons} & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & \left[\frac{I^c}{Y} d\left(\frac{P_k^c x_k^c}{I^c}\right)\right]_{kc} & 0 & 0 \end{bmatrix}.$$

Utility maximization of the consumers (5) implies $\frac{P_k^c x_k^c}{I^c} = b_k^c \frac{(P_k^c)^{1-\sigma}}{(P^c)^{1-\sigma}}$, and it follows

$$\begin{aligned} d\left(\frac{P_k^c x_k^c}{I^c}\right) &= d\left(b_k^c \frac{(P_k^c)^{1-\sigma}}{(P^c)^{1-\sigma}}\right) \\ &= b_k^c \frac{(P^c)^{1-\sigma}(1-\sigma)(P_k^c)^{1-\sigma} d\log P_k^c - (P_k^c)^{1-\sigma}(1-\sigma)(P^c)^{1-\sigma} d\log P^c}{(P^c)^{2(1-\sigma)}} \\ &= b_k^c (1-\sigma)\left(\frac{P_k^c}{P^c}\right)^{1-\sigma} \left(d\log P_k^c - d\log P^c\right) \\ &= (1-\sigma)\frac{P_k^c x_k^c}{I^c}\left(d\log P_k^c - d\log P^c\right). \end{aligned}$$

Accordingly, we get

$$\begin{aligned} \chi' d\tilde{\Omega}^{cons} &= \begin{bmatrix} 0 & \left[(1-\sigma)\lambda_k^c (d\log P_k^c - d\log P^c)\right]_{kc} & 0 & 0 \end{bmatrix} \\ &= (1-\sigma) d\log P' \begin{bmatrix} 0 & -\chi \circ \Omega^{cons} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \mathrm{diag}(\lambda_k^c) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \end{aligned}$$

For the third part, it follows by definition that

$$\lambda' d\tilde{\Omega} = \begin{bmatrix} \lambda^{kc\prime} & \lambda_k^{c\prime} & \lambda_j^{kc\prime} & \lambda_{kc}{}' \end{bmatrix} \begin{bmatrix} 0 & 0 & d\Omega^{prod} & 0 \\ 0 & 0 & 0 & d\Omega^{comp-cons} \\ 0 & 0 & 0 & d\Omega^{comp-prod} \\ d\Omega^{raw} & 0 & 0 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} \lambda_{kc}{}' d\Omega^{raw} & 0 & \lambda^{kc\prime} d\Omega^{prod} & \lambda_k^{c\prime} d\Omega^{comp-cons} + \lambda_j^{kc\prime} d\Omega^{comp-prod} \end{bmatrix}.$$

Using the definition, it holds that

$$\lambda_{kc}{}' d\Omega^{raw} = \Big[\frac{p_{kc} y_{kc}}{Y} d\Big(\frac{P^{kc} X^{kc}}{p_{kc} y_{kc}}\Big)\Big]_{kc}, \qquad \lambda^{kc\prime} d\Omega^{prod} = \Big[\frac{P^{kc} X^{kc}}{Y} d\Big(\frac{P_j^{kc} x_j^{kc}}{P^{kc} X^{kc}}\Big)\Big]_{kcj}$$
$$\lambda_k^{c\prime} d\Omega^{comp-cons} + \lambda_j^{kc\prime} d\Omega^{comp-prod} = \Big[\sum_{\tilde{c}} \frac{P_k^{\tilde{c}} x_k^{\tilde{c}}}{Y} d\Big(\frac{\tau_{kc}^{\tilde{c}} p_{kc} y_{kc}^{\tilde{c}}}{P_k^{\tilde{c}} x_k^{\tilde{c}}}\Big) + \sum_{j,\tilde{c}} \lambda_k^{j\tilde{c}} d\Big(\frac{\tau_{kc}^{j\tilde{c}} p_{kc} y_{kc}^{j\tilde{c}}}{P_k^{j\tilde{c}} x_k^{j\tilde{c}}}\Big)\Big]_{kc}.$$

We compute each part separately. Again by CES demand in raw production, it follows

$$\begin{aligned} d\Big(\frac{P^{kc} X^{kc}}{p_{kc} y_{kc}}\Big) &= (1-\alpha^{kc}) d\Big(\frac{(P^{kc})^{1-\theta}}{(A^{kc} p_{kc})^{1-\theta}}\Big) \\ &= (1-\theta)(1-\alpha^{kc}) \Big(\frac{(P^{kc})^{1-\theta}}{(A^{kc} p_{kc})^{1-\theta}}\Big) \big(d\log P^{kc} - d\log p_{kc}\big) \\ &= (1-\theta) \frac{P^{kc} X^{kc}}{p_{kc} y_{kc}} \big(d\log P^{kc} - d\log p_{kc}\big). \end{aligned}$$

Then using CES demand in the production of aggregate intermediates, we obtain

$$\begin{aligned} d\Big(\frac{P_j^{kc} x_j^{kc}}{P^{kc} X^{kc}}\Big) &= \omega_j^{kc} d\Big(\frac{(P_j^{kc})^{1-\epsilon}}{(P^{kc})^{1-\epsilon}}\Big) \\ &= \omega_j^{kc} \Big(\frac{P_j^{kc}}{P^{kc}}\Big)^{1-\epsilon} (1-\epsilon) \big[d\log P_j^{kc} - d\log P^{kc}\big] \\ &= \frac{P_j^{kc} x_j^{kc}}{P^{kc} X^{kc}} (1-\epsilon) \big[d\log P_j^{kc} - d\log P^{kc}\big]. \end{aligned}$$

CES demands for composite consumption importing implies

$$
\begin{aligned}
d\Big(\frac{\tau_{kc}^{\tilde{c}} p_{kc} y_{kc}^{\tilde{c}}}{P_k^{\tilde{c}} x_k^{\tilde{c}}}\Big) &= \delta_{kc}^{\tilde{c}} d\Big(\frac{(\tau_{kc}^{\tilde{c}} p_{kc})^{1-\theta_k}}{(P_k^{\tilde{c}})^{1-\theta_k}}\Big) \\
&= (1-\theta_k)\delta_{kc}^{\tilde{c}}\Big(\frac{\tau_{kc}^{\tilde{c}} p_{kc}}{P_k^{\tilde{c}}}\Big)^{1-\theta_k}\big[d\log p_{kc} + d\log \tau_{kc}^{\tilde{c}} - d\log P_k^{\tilde{c}}\big] \\
&= (1-\theta_k)\frac{\tau_{kc}^{\tilde{c}} p_{kc} y_{kc}^{\tilde{c}}}{P_k^{\tilde{c}} x_k^{\tilde{c}}}\big[d\log p_{kc} + d\log \tau_{kc}^{\tilde{c}} - d\log P_k^{\tilde{c}}\big].
\end{aligned}
$$

And finally, the CES import demands of the composite intermediate firms on the production side give

$$
\begin{aligned}
d\Big(\frac{\tau_{kc}^{j\tilde{c}} p_{kc} y_{kc}^{j\tilde{c}}}{P_k^{j\tilde{c}} x_k^{j\tilde{c}}}\Big) &= \delta_{kc}^{j\tilde{c}} d\Big(\frac{(\tau_{kc}^{j\tilde{c}} p_{kc})^{1-\theta_k}}{(P_k^{j\tilde{c}})^{1-\theta_k}}\Big) \\
&= (1-\theta_k)\delta_{kc}^{j\tilde{c}}\Big(\frac{\tau_{kc}^{j\tilde{c}} p_{kc}}{P_k^{j\tilde{c}}}\Big)^{1-\theta_k}\Big[d\log p_{kc} + d\log \tau_{kc}^{j\tilde{c}} - d\log P_k^{j\tilde{c}}\Big] \\
&= (1-\theta_k)\frac{\tau_{kc}^{j\tilde{c}} p_{kc} y_{kc}^{j\tilde{c}}}{P_k^{j\tilde{c}} x_k^{j\tilde{c}}}\Big[d\log p_{kc} + d\log \tau_{kc}^{j\tilde{c}} - d\log P_k^{j\tilde{c}}\Big].
\end{aligned}
$$

Accordingly, it follows

$$
\begin{aligned}
&(\lambda_k^c)' d\Omega^{comp-cons} + (\lambda_j^{kc})' d\Omega^{comp-prod} \\
&\quad= \Big[(1-\theta_k)\big(\sum_{\tilde{c}} \frac{\tau_{kc}^{\tilde{c}} p_{kc} y_{kc}^{\tilde{c}}}{Y}(d\log p_{kc} + d\log \tau_{kc}^{\tilde{c}} - d\log P_k^{\tilde{c}}) \\
&\qquad\qquad + \sum_{j\tilde{c}} \frac{\tau_{kc}^{j\tilde{c}} p_{kc} y_{kc}^{j\tilde{c}}}{Y}(d\log p_{kc} + d\log \tau_{kc}^{j\tilde{c}} - d\log P_k^{j\tilde{c}})\big)\Big]_{kc} \\
&\quad= \Big[(1-\theta_k)\big(\lambda_{kc} d\log p_{kc} + \\
&\qquad\qquad \sum_{\tilde{c}} \frac{\tau_{kc}^{\tilde{c}} p_{kc} y_{kc}^{\tilde{c}}}{Y}(d\log \tau_{kc}^{\tilde{c}} - d\log P_k^{\tilde{c}}) + \sum_{j\tilde{c}} \frac{\tau_{kc}^{j\tilde{c}} p_{kc} y_{kc}^{j\tilde{c}}}{Y}(d\log \tau_{kc}^{j\tilde{c}} - d\log P_k^{j\tilde{c}})\big)\Big]_{kc}
\end{aligned}
$$

where we used the raw intermediates market clearing condition to get the final equality. To compute the final matrix form of $\lambda' d\tilde{\Omega}$, we will find the matrix form of each of $\begin{bmatrix} \lambda_{kc}{}' d\Omega^{raw} & 0 & 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 & \lambda^{kc\prime} d\Omega^{prod} & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 0 & 0 & \lambda_k^{c\prime} d\Omega^{comp-cons} + \lambda_j^{kc\prime} d\Omega^{comp-prod} \end{bmatrix}$

and then combine them together. Therefore, we get

$$\begin{bmatrix} {\lambda_{kc}}' d\Omega^{raw} & 0 & 0 & 0 \end{bmatrix} = (1-\theta) \begin{bmatrix} \left[\lambda^{kc}(d\log P^{kc} - d\log p_{kc})\right]_{kc} & 0 & 0 & 0 \end{bmatrix}$$

$$= (1-\theta) d\log P' \begin{bmatrix} 0 & 0 & 0 & 0 \\ \mathrm{diag}(\lambda^{kc}) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\mathrm{diag}(\lambda^{kc}) & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & {\lambda^{kc}}' d\Omega^{prod} & 0 \end{bmatrix} = (1-\epsilon) \begin{bmatrix} 0 & 0 & \left[\lambda_j^{kc}[d\log P_j^{kc} - d\log P^{kc}]\right]_{kcj} & 0 \end{bmatrix}$$

$$= (1-\epsilon) d\log P' \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\lambda^{kc} \circ \Omega^{prod} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \mathrm{diag}(\lambda_j^{kc}) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

We get

$$\begin{aligned}
&\begin{bmatrix} 0 & 0 & 0 & \lambda_k^c d\Omega^{comp-cons} + \lambda_j^{kc} d\Omega^{comp-prod} \end{bmatrix} \\
&\quad = \Big[0 \quad 0 \quad 0 \quad \big[(1-\theta_k)\big(\lambda_{kc} d\log p_{kc} + \\
&\qquad\qquad \sum_{\tilde{c}} \frac{\tau_{kc}^{\tilde{c}} p_{kc} y_{kc}^{\tilde{c}}}{Y} (d\log \tau_{kc}^{\tilde{c}} - d\log P_k^{\tilde{c}}) + \sum_{j\tilde{c}} \frac{\tau_{kc}^{j\tilde{c}} p_{kc} y_{kc}^{j\tilde{c}}}{Y} (d\log \tau_{kc}^{j\tilde{c}} - d\log P_k^{j\tilde{c}})\big)\big]_{kc}\Big] \\
&= d\log P' \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\mu \circ \lambda_k^c \circ \Omega^{comp-cons} \\ 0 & 0 & 0 & -\nu \circ \lambda_j^{kc} \circ \Omega^{comp-prod} \\ 0 & 0 & 0 & \mathrm{diag}((1-\theta_k)\lambda_{kc}) \end{bmatrix} + \mathbf{1}' \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu \circ \lambda_k^c \circ \Omega^{comp-cons} \circ d\log \tau^{cons} \\ 0 & 0 & 0 & \nu \circ \lambda_j^{kc} \circ \Omega^{comp-prod} \circ d\log \tau^{prod} \\ 0 & 0 & 0 & 0 \end{bmatrix}
\end{aligned}$$

and we are done. ■

We now combine all of the previous results to get a system of linear equations for $d\log w$ and $d\log P$. Equations (18) and (19) imply that

$$\begin{aligned}
d\lambda' \tilde{F} &= (d\chi' \tilde{\Omega}^{cons} + \chi' d\tilde{\Omega}^{cons} + \lambda' d\tilde{\Omega}) \Psi \tilde{F} \\
d\lambda' \tilde{F} &= d\Lambda' - \lambda' d\tilde{F}.
\end{aligned}$$

For the first of those equations, we can use Proposition 4 as well as Equation (15) to get

$$\begin{aligned}
d\lambda' \tilde{F} &= (d\log w' \tilde{\Lambda}' \tilde{\Omega}^{cons} + d\log P' \Phi_B + d\log P' \Phi_C + \mathbf{1}' \Phi_D) \Psi \tilde{F} \\
&= (d\log w' \tilde{\Lambda}' \tilde{\Omega}^{cons} + d\log P' (\Phi_B + \Phi_C) + \mathbf{1}' \Phi_D) \Psi \tilde{F}.
\end{aligned}$$

Concerning the second equation, we use Proposition 4 as well as Equation (16)

$$\begin{aligned} d\lambda' \tilde{F} &= d\log w' \operatorname{diag}(\Lambda^{kc}) - d\log w' \operatorname{diag}((1-\theta)\Lambda^{kc}) + d\log P' \Phi_A \\ &= d\log w' \operatorname{diag}(\theta\Lambda^{kc}) + d\log P' \Phi_A. \end{aligned}$$

Combining everything together, we get

$$d\log w' \left(\operatorname{diag}(\theta\Lambda^{kc}) - \tilde{\Lambda}'\tilde{\Omega}^{cons}\Psi\tilde{F}\right) = d\log P' \left(\left(\Phi_B + \Phi_C\right)\Psi\tilde{F} - \Phi_A\right) + \mathbf{1}'\Phi_D\Psi\tilde{F}.$$

Finally, we use Proposition 3 and solve for $d\log w$ to arrive at the equation for the changes in wages:

Corollary 1

$$\begin{aligned} &d\log w' \left(\operatorname{diag}(\theta\Lambda^{kc}) - \tilde{\Lambda}'\tilde{\Omega}^{cons}\Psi\tilde{F} - F'(\mathcal{I} - \Omega')^{-1}\left(\left(\Phi_B + \Phi_C\right)\Psi\tilde{F} - \Phi_A\right)\right) \\ &= \mathbf{1}'\left[\Phi_D\Psi\tilde{F} + (\Omega' \circ d\log T')(\mathcal{I} - \Omega')^{-1}\left(\left(\Phi_B + \Phi_C\right)\Psi\tilde{F} - \Phi_A\right)\right]. \end{aligned}$$

However, due to Walras' law, this linear system is not of full rank. Accordingly, to solve it, we need to add to it the equation $\mathbf{1}'d\chi = 0$ (world income is fixed), which is equivalent to

$$\sum_{kc} \Lambda^{kc} d\log w^{kc} = 0.$$

A.1.1 Derivatives of Domar Weights and Expenditure Shares

After having computed $d\log w$ we can use Proposition 3 and 4 as well as Equations (15), (16) and (18) to compute the derivatives of the other variables. We get

$$\begin{aligned}
d\log P =&(\mathcal{I}-\Omega)^{-1}Fd\log w+(\mathcal{I}-\Omega)^{-1}(\Omega\circ d\log T)\mathbf{1}\\
d\chi =&\tilde{\Lambda}d\log w\\
d\Lambda =&\text{diag}(\Lambda^{kc})d\log w\\
d\lambda =&\Psi'(d\chi'\tilde{\Omega}^{cons}+\chi'd\tilde{\Omega}^{cons}+\lambda'd\tilde{\Omega})'\\
=&\Psi'(d\log w'\tilde{\Lambda}'\tilde{\Omega}^{cons}+d\log P'\Phi_B+d\log P'\Phi_C+\mathbf{1}'\Phi_D)'\\
=&\Psi'((\tilde{\Omega}^{cons})'\tilde{\Lambda}d\log w+\Phi_B'd\log P+\Phi_C'd\log P+\Phi_D'\mathbf{1}).
\end{aligned}$$

For the derivatives of the input/output matrix Ω and labor shares $\hat{F}$ we can reuse parts of the proof of Proposition 4. Recall that Ω^{raw} is of size $CN\times CN$ and that

$$\Omega^{raw}=\begin{pmatrix}\Omega_1^{raw}&0&0\\0&\ddots&0\\0&0&\Omega_C^{raw}\end{pmatrix},\qquad \Omega_c^{raw}=\text{diag}\left(\frac{P^{kc}X^{kc}}{p_{kc}y_{kc}}\right)_{kc}$$

$$d\left(\frac{P^{kc}X^{kc}}{p_{kc}y_{kc}}\right)=(1-\theta)\frac{P^{kc}X^{kc}}{p_{kc}y_{kc}}(d\log P^{kc}-d\log p_{kc}).$$

This implies that $d\Omega^{raw}=(1-\theta)\left([d\log P^{kc}]_{kc}\circ\Omega^{raw}-[d\log p_{kc}]_{kc}\circ\Omega^{raw}\right)$.

Recall that Ω^{cons} is of size $C\times CN$ and that

$$\Omega^{cons}=\begin{pmatrix}\Omega_1^{cons}&0&0\\0&\ddots&0\\0&0&\Omega_C^{cons}\end{pmatrix},\qquad \Omega_c^{cons}=\left[\frac{P_k^cx_k^c}{I^c}\right]_k$$

$$d\left(\frac{P_k^cx_k^c}{I^c}\right)=(1-\sigma)\frac{P_k^cx_k^c}{I^c}(d\log P_k^c-d\log P^c).$$

This implies that $d\Omega^{cons} = (1-\sigma)\left([d\log P_k^c]_{kc} \circ \Omega^{cons} - [d\log P^c]'_c \circ \Omega^{cons}\right).$

Recall that Ω^{prod} is of size $CN \times CN^2$ and that

$$\Omega^{prod} = \begin{pmatrix} \Omega_{11}^{prod} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \Omega_{NC}^{prod} \end{pmatrix}, \qquad \Omega_{kc}^{prod} = \left[\frac{P_j^{kc}x_j^{kc}}{P^{kc}X^{kc}}\right]_j$$

$$d\left(\frac{P_j^{kc}x_j^{kc}}{P^{kc}X^{kc}}\right) = (1-\epsilon)\frac{P_j^{kc}x_j^{kc}}{P^{kc}X^{kc}}(d\log P_j^{kc} - d\log P^{kc}).$$

This implies that $d\Omega^{prod} = (1-\epsilon)\left([d\log P_j^{kc}]_{kcj} \circ \Omega^{prod} - [d\log P^{kc}]'_{kc} \circ \Omega^{prod}\right).$

Recall that $\Omega^{comp-cons}$ is of size $CN \times CN$ and that

$$\Omega^{comp-cons} = \begin{pmatrix} A_{11} & \dots & A_{1C} \\ \vdots & \ddots & \vdots \\ A_{C1} & \dots & A_{CC} \end{pmatrix}, \qquad A_{cc'} = \operatorname{diag}\left(\frac{\tau_{kc'}^c p_{kc'} y_{kc'}^c}{P_k^c x_k^c}\right)_k$$

$$d\left(\frac{\tau_{kc'}^c p_{kc'} y_{kc'}^c}{P_k^c x_k^c}\right) = (1-\theta_k)\frac{\tau_{kc'}^c p_{kc'} y_{kc'}^c}{P_k^c x_k^c}(d\log p_{kc'} + d\log \tau_{kc'}^c - d\log P_k^c).$$

This implies that

$$d\Omega^{comp-cons} = [1-\theta_k]'_{kc} \circ \big(d\log \tau^{cons} \circ \Omega^{comp-cons} + [d\log p_{kc}]_{kc} \circ \Omega^{comp-cons} - [d\log P_k^c]'_{kc} \circ \Omega^{comp-cons}\big).$$

Recall that $\Omega^{comp-prod}$ is of size $CN^2 \times CN$ and that

$$\Omega^{comp-prod} = \begin{pmatrix} B_1^{11} & \dots & B_C^{11} \\ \vdots & \ddots & \vdots \\ B_1^{NC} & \dots & B_C^{NC} \end{pmatrix}, \qquad B_{c'}^{kc} = \operatorname{diag}\left(\frac{\tau_{jc'}^{kc} p_{jc'} y_{jc'}^{kc}}{P_j^{kc} x_j^{kc}} \right)_j$$

$$d\left(\frac{\tau_{jc'}^{kc} p_{jc'} y_{jc'}^{kc}}{P_j^{kc} x_j^{kc}} \right) = (1-\theta_j) \frac{\tau_{jc'}^{kc} p_{jc'} y_{jc'}^{kc}}{P_j^{kc} x_j^{kc}} (d\log p_{jc'} + d\log \tau_{jc'}^{kc} - d\log P_j^{kc}).$$

This implies that

$$d\Omega^{comp-prod} = [1-\theta_k]'_{kc} \circ \left(d\log \tau^{prod} \circ \Omega^{comp-prod} + [d\log p_{kc}]_{kc} \circ \Omega^{comp-prod} \right.$$
$$\left. - [d\log P_j^{kc}]'_{kcj} \circ \Omega^{comp-prod} \right).$$

Recall that $\hat{F} = \operatorname{diag}\left(\frac{w^{kc} L^{kc}}{p_{kc} y_{kc}} \right)_{kc}$ is of size $CN \times CN$ and that

$$d\left(\frac{w^{kc} L^{kc}}{p_{kc} y_{kc}} \right) = (1-\theta) \frac{w^{kc} L^{kc}}{p_{kc} y_{kc}} (d\log w^{kc} - d\log p_{kc}).$$

This implies that $d\hat{F} = (1-\theta) \left(\left[d\log w^{kc} \right]_{kc} \circ \hat{F} - [d\log p_{kc}]_{kc} \circ \hat{F} \right)$

A.2 Productivity Shocks

Note that the preceding analysis also applies to productivity shocks, since we can rewrite the productivities A^{kc} as uniform icebergs across all buyers. To see this, write the raw production function (10) as

$$\tilde{y}_{kc} = \frac{y_{kc}}{A^{kc}} = \left((\alpha^{kc})^{\frac{1}{\theta}} \left(l^{kc} \right)^{\frac{\theta-1}{\theta}} + (1-\alpha^{kc})^{\frac{1}{\theta}} \left(X^{kc} \right)^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}$$

the price index obtained by marginal costs pricing as

$$\tilde{p}_{kc} = A^{kc} p_{kc} = \left(\alpha^{kc} (w^{kc})^{1-\theta} + (1-\alpha^{kc}) (P^{kc})^{1-\theta} \right)^{\frac{1}{1-\theta}},$$

and market clearing as

$$\tilde{y}_{kc} = \frac{1}{A^{kc}} \left(\sum_{c' \in C} y_{kc}^{c'} + \sum_{jc' \in N \times C} y_{kc}^{jc'} \right).$$

Then the equilibrium $(\tilde{y}_{kc}, \tilde{p}_{kc})$ with uniform icebergs $\tau_{kc}^{c'} = \tau_{kc}^{jc'} = \frac{1}{A^{kc}}$ and unitary productivity is equivalent to the equilibrium with unitary icebergs and productivity A^{kc}, except $y_{kc} = A^{kc}\tilde{y}_{kc}$ and $p_{kc} = \frac{\tilde{p}_{kc}}{A^{kc}}$ (so the value of production doesn't change). All other variables are identical because in the both cases (1) the buyers pay the same effective prices and (2) the raw producers have the same nominal sales. So to compute the effect of a productivity shock, one must simply calculate the response to the corresponding iceberg shock and then adjust the raw prices.

A.3 Computing the Effect of Large Shocks

In Section A.1, we provided closed-form expressions for how equilibrium changes in response to arbitrary, small iceberg shocks. Section A.2 explained how to use the same equations to calculate the effects of small productivity shocks. Here we explain how, by numerically integrating these derivatives, we can compute equilibrium changes in response to large shocks.

Given elasticities, icebergs $d \log T$, initial expenditure shares F_0 and Ω_0, and Domar weights λ_0, Λ_0, and χ_0, we solve for the change in equilibrium using Euler integration.

Let S be the number of steps. Using Proposition 3 and Corollary 1, calculate $d \log w_1$ and $d \log P_1$ for the shock $d \log T/S$. Then calculate $d\chi_1$, $d\Lambda_1$, $d\lambda_1$, dF_1, and $d\Omega_1$ according to Section A.1.1. This generates the updated variables $\chi_1 = \chi_0 + d\chi_1$, $\Omega_1 = \Omega_0 + d\Omega_1$, and so on. Then repeat the derivatives, starting at the updated matrices and vectors.

After S steps, $\sum_{s=1}^{S} d \log P_s$ and $\sum_{s=1}^{S} d \log w_s$ are the total percent changes in prices and wages, and χ_S, Λ_S, λ_S, F_S, and Ω_S are the resulting Domar weights and expendi-

ture shares. Since world income is normalized to one throughout, the total change in welfare for the countries is the vector

$$\log(\chi_S) - \log(\chi_0) - \sum_{s=1}^{S} d \log P_s^C$$

. As S becomes large, the procedure converges to the true change in equilibrium.

B Calibration Details

While the derivatives in Propositions 3 and 4 and Section A.1.1 depend on the preference, production, and trade elasticity parameters, for which there are estimates from the trade literature, they do not depend directly on the taste and production weights (the δ's, ω's, and so on). Nor do the derivatives depend directly on the initial labor supplies or productivities. Instead, the realized expenditure shares and Domar weights, which can be measured from data, turn out to be sufficient statistics for the underlying unobserved structure of the economy. And the Domar weights and expenditure shares can be easily calculated from the empirical input/output tables, e.g., the OECD ICIO. Therefore, counterfactual exercises require only three inputs: (1) elasticity parameters, (2) iceberg shocks, and (3) an input/output table.

We calibrate the model using the OECD ICIO, specifically the 2019 table, which is the most recent version not during the COVID crisis. The data include 77 countries (including "rest-of-world" ROW) and 45 industries, and we use two parts of the table. The first part is the raw intermediate use matrix we call IO. Each row corresponds to a different country/industry source of goods or services, and each entry of the row gives, in dollars, purchases from that country/industry pair by all country/industry pairs. The second part is the final use matrix. Each row corresponds to a different country/industry source of goods or services, and each entry of the row gives, in dollars, purchases from that country/industry pair for final use in each country. The final

uses are household consumption, non-profits' consumption for households, government consumption, investment, the change in inventories, and direct purchases abroad. For computational reasons, as described in Section B.3, we add some of the countries into ROW, form several regional aggregate economies, and aggregate some of the industries together, leaving us with $C = 34$ and $N = 38$.

By summing across final uses for each country, we create the $CN \times C$ total absorption matrix we call $CONS$, which corresponds to consumption in our model. As discussed in Eppinger et al. (2021) and the appendix of Costinot and Rodríguez-Clare (2014), simply adding columns can generate negative entries for absorption, which are not allowed in the model because there are negative values for inventories and investment. To correct for this, we follow the procedure of Costinot and Rodríguez-Clare (2014) (after performing the aggregation of Section B.3). Let X be the vector of total sales, and let A be IO with each row divided by X, that is, intermediate expenditures as a fraction of (buyer) sales. Letting $CONS$ (corrected) be the sum of the positive parts of absorption, and letting neg be the sum of the negative parts of absorption, we have $X = AX + CONS\mathbf{1} + neg\mathbf{1}$ by construction. Defining $\tilde{X} = (\mathcal{I} - A)^{-1} CONS\mathbf{1}$, the corrected intermediate use table (corrected IO) is $A \circ \tilde{X}'$,[26] and we have $\tilde{X} = A\tilde{X} + CONS\mathbf{1}$ (note that dividing each row of $A \circ \tilde{X}'$ by $\tilde{X}$ simply yields A). Intuitively, this amounts to assuming negative inventories and investments were produced and consumed in the current period. Mechanically, the corrected IO multiplies each row by the vector of $\tilde{X}$ divided element-wise by X, proportionally scaling up current sales to include inventoried prior sales. In what follows, we assume $CONS$ and IO have been adjusted as described.

With adjusted $CONS$ and IO in hand, we can directly compute the expenditure shares corresponding to our model. In keeping with our previous notation, let $IO_{jc'}^{kc}$ be purchases of industry j in country c' by industry k in country c. This corresponds

[26] As in Section A, $\circ$ means element-by-element multiplication when the matrices have the same dimensions and row-by-row or column-by-column element-by-element multiplication when one of the matrices is a vector. So $A \circ \tilde{X}'$ means element-by-element multiplication of each row of A by the row vector $\tilde{X}'$.

to cell $(N \times (c'-1)+j, N \times (c-1)+k)$ of IO. Define $CONS^c_{jc'}$ analogously. Then, assuming the data reflect equilibrium in the model, we have

$$\begin{aligned}
\frac{w^{kc}l^{kc}}{p_{kc}y_{kc}} &= \frac{\sum_{jc'} IO^{jc'}_{kc} + \sum_{c'} CONS^{c'}_{kc} - \sum_{jc'} IO^{kc}_{jc'}}{\sum_{jc'} IO^{jc'}_{kc} + \sum_{c'} CONS^{c'}_{kc}} \\
\frac{P^{kc}X^{kc}}{p_{kc}y_{kc}} &= 1 - \frac{w^{kc}l^{kc}}{p_{kc}y_{kc}} \\
\frac{P^c_k x^c_k}{I^c} &= \frac{\sum_{c'} CONS^c_{kc'}}{\sum_{jc'} CONS^c_{jc'}} \\
\frac{\tau^c_{kc'} p_{kc'} y^c_{kc'}}{P^c_k x^c_k} &= \frac{CONS^c_{kc'}}{\sum_m CONS^c_{km}} \\
\frac{P^{kc}_j x^{kc}_j}{P^{kc}X^{kc}} &= \frac{\sum_{c'} IO^{kc}_{jc'}}{\sum_{ic'} IO^{kc}_{ic'}} \\
\frac{\tau^{kc}_{jc'} p_{jc'} y^{kc}_{jc'}}{P^{kc}_j x^{kc}_j} &= \frac{IO^{kc}_{jc'}}{\sum_m IO^{kc}_{jm}},
\end{aligned}$$

which are all we need to calculate the matrices Ω (goods expenditure shares) and F (labor shares) from Appendix A.

We now turn to the calibration of the other quantities required for computing the model and discuss the issue of the overall trade balance.

B.1 Domar Weights and Trade Imbalances

To implement the solution algorithm in Section A.3, we also need to calculate the Domar weights for labor Λ (value added by industry), the consumer Domar weights χ, and the producer Domar weights λ. One could directly calculate these objects from IO and $CONS$ in the OECD ICIO (after aggregation and the negative inventory correction), but doing so would lead to inconsistencies with the model equations. For example, suppose one were to compute labor Domar weights Λ as industry value added in each country, divided by global value added, and calculated χ by summing across industries within each country. Then Equation 17 (sales equal purchases; $\lambda' = \chi' \tilde{\Omega}^{cons} \Psi)$) immediately gives us the producer Domar weights λ since Ψ only depends on Ω. But the resulting λ would not equal the corresponding Domar weights directly calculated

from the OECD ICIO. And, consequently, the labor shares F in the model would deviate from their empirical counterparts. The reason is that, due to trade imbalances and cross-border factor payments, total value added in each country is not equal to absorption or income in the data. In contrast, while the model exhibits lots of gross trade, trade in the model is balanced in the aggregate, final absorption equals country value added (wages), and GDP equals income.

Alternatively, one could directly calculate λ and χ and then back out Λ using the observed value added shares F. In this case, Λ would not add up to χ (as required in the model), again because of the assumption of balanced trade.

The upshot is that one must either (i) include trade imbalances/cross-border factor payments in the model, which is challenging because the model is static or (ii) perform some additional correction to IO and $CONS$. We adopt the latter approach.

Specifically, we adjust the OECD ICIO to be consistent with the model equations, while respecting the observed expenditure shares Ω and F. Returning to our notation from the negative inventories correction, let A be IO with the rows divided by total sales. Let B be $CONS$ with the rows divided by total spending. So B is a $CN \times C$ matrix of consumption weights. Next define the "ownership matrix" $\mathcal{O}$ that describes how factor payments (value added) are distributed. We assume, following the model, that $\mathcal{O}$ is $C \times CN$ with $1 \times N$ vectors of ones on the diagonal (so GDP equals income). Lastly, let $\mathcal{L}$ be the $CN \times CN$ matrix with the observed labor shares on the diagonal and zeros elsewhere (in the model, we called this $\hat{F}$).

We calculate the $CN \times 1$ vector of sales $\hat{X}$ satisfying

$$\begin{aligned}\hat{X} =& A\hat{X} + B\mathcal{O}\mathcal{L}\hat{X} \\ \mathbf{1}'\mathcal{L}\hat{X} =& 1.\end{aligned}$$

Note that $\mathcal{L}\hat{X}$ is the $CN \times 1$ vector of value added by country/industry, so the second equation normalizes world GDP to one, as the first equation only determines $\hat{X}$ up to

a multiplicative constant (it is the first eigenvector of $A + B\mathcal{O}\mathcal{L}$, which has spectral radius one since the rows sum to one). The solution yields the corrected IO matrix $A \circ \hat{X}'$, the corrected $CONS$ matrix $B \circ (\mathcal{O}\mathcal{L}\hat{X})'$, $\Lambda = \mathcal{L}\hat{X}$, $\lambda = \hat{X}$, and $\chi = \mathcal{O}\mathcal{L}\hat{X}$. And we have

$$\lambda = A\lambda + B\chi,$$

which is the same as Equation 17. Corrected IO and $CONS$ are thus constructed to respect the observed expenditure and labor shares, and implied industry value added sums to total final consumption expenditure in each country:

$$\mathbf{1}'(B \circ (\mathcal{O}\mathcal{L}\hat{X})') = (\mathcal{O}\mathcal{L}\hat{X})' = \chi',$$

which is due to the fact that the rows of B sum to one by construction.

While the corrected matrices match the observed expenditure and labor shares, they produce Domar weights different from those directly calculated from the OECD ICIO. However, the differences are not too large. After implementing our aggregation scheme described in Section B.3 and performing the negative inventory and trade balance corrections described above, the cross-country correlation between the corrected and direct Domar weights is approximately 0.99 for χ, Λ, and λ. For Switzerland, for example, the correlations for Λ and λ are > 0.99, and Switzerland comprises approximately 1% of global value added, with and without the correction.

With Ω, F, Λ, λ, and χ in hand, after choosing elasticity parameters (see Section B.2) and icebergs, we have information sufficient for calculating changes in equilibrium according to Section A. For counterfactual exercises reported in terms of changes, it is not necessary for us to take a stance on the underlying labor supplies, productivities, or preference/production weights. The only assumption we need to proceed is that the corrected OECD ICIO reflects an equilibrium in the model. One can remain agnostic on the extent to which initial patterns in the data are driven by productivity, labor supply, preferences, or technology.

Nevertheless, it is straightforward to calibrate the primitives of the model such that the initial equilibrium matches key features of the observed data. Doing so is not necessary for counterfactuals, but it can be useful for generating synthetic data and comparing solution methods. We describe one such calibration approach here, as it may be of theoretical interest.

Suppose the labor supplies are effective labor (labor force times labor productivity). And suppose we could find a set of labor productivities ensuring that all equilibrium prices and wages are equal to one. Then from the CES demand curves and price indexes, the equilibrium expenditure shares are all equal to the underlying preference and production weights, e.g. $\frac{w^{kc}l^{kc}}{p_{kc}y_{kc}} = \alpha^{kc}$ (as would have been the case if we had originally imposed Cobb-Douglas). The following proposition and remark show how to compute these effective labor supplies.

Proposition 5

Given production weights $\delta^{c}_{kc'}, \omega^{kc}_{j}, \delta^{kc}_{jc'}, \alpha^{kc}$ and taste parameters b^{c}_{k}, we assume that the productivities A^{kc} and icebergs $\tau^{c'}_{kc}, \tau^{jc'}_{kc}$ are all equal to one and that the industry-country labor allocation fulfill

$$\frac{L^{kc}}{\alpha^{kc}} = \sum_{jc'} \left(\delta^{c'}_{kc} b^{c'}_{k} + \delta^{jc'}_{kc} \omega^{jc'}_{k} \frac{1-\alpha^{jc'}}{\alpha^{jc'}} \right) L^{jc'}. \tag{20}$$

Then, there exists an equilibrium in which all the prices and wages are equal to 1 *such that the expenditure shares of firms are exactly equal to the production weights and the expenditure shares of consumers are equal to the taste parameters.*

Remark 2

Writing the system of CN equations above in matrix form as $y = Qy$, where y is the vector formed from L^{kc}/α^{kc}, y is the first eigenvector of Q, which has spectral radius one since the rows sum to one. Assuming Q is irreducible, the solution y is unique up to a multiplicative constant. In that case, imposing $1 = \sum_{kc} L^{kc}$ then uniquely pins down the labor supplies.

Proof. We start by assuming that all the prices and wages are equal to 1. Clearly, it

follows that prices and wages equal marginal costs and satisfy the price indexes. The CES demand curves, the budget constraint of the consumers, and the market clearing condition imply directly that

$$x_k^c = b_k^c \sum_j L^{jc}, \qquad y_{kc'}^c = \delta_{kc'}^c b_k^c \sum_j L^{jc}, \qquad y_{kc} = \frac{L^{kc}}{\alpha^{kc}}, \qquad X^{kc} = \frac{1-\alpha^{kc}}{\alpha^{kc}} L^{kc},$$
$$x_j^{kc} = \omega_j^{kc} \frac{1-\alpha^{kc}}{\alpha^{kc}} L^{kc}, \quad y_{jc'}^{kc} = \delta_{jc'}^{kc} \omega_j^{kc} \frac{1-\alpha^{kc}}{\alpha^{kc}} L^{kc} \quad I^c = \sum_k L^{kc} \quad y_{kc} = \sum_{c'} y_{kc}^{c'} + \sum_{jc'} y_{kc}^{jc'}.$$

If all of those equations hold, then we have a market equilibrium. After plugging in $y_{kc}, y_{kc}^{c'}$ and $y_{kc}^{jc'}$ in the market clearing condition, we get the equation as stated. ■

Assuming the labor supplies satisfy Proposition 5 and total labor is set to one, world income and all prices and wages are one, and the model matches the observed expenditure and labor shares, provided the preference and production weights are set equal to them. It is then straightforward to calculate the Domar weights:

$$\begin{aligned}
\chi^c &= \sum_k L^{kc} \\
\Lambda^{kc} &= L^{kc} \\
\lambda_{kc} &= \frac{L^{kc}}{\alpha^{kc}} \\
\lambda^{kc} &= \frac{1-\alpha^{kc}}{\alpha^{kc}} L^{kc} \\
\lambda_k^c &= b_k^c \chi^c \\
\lambda_j^{kc} &= \omega_j^{kc} \lambda^{kc}
\end{aligned}$$

Because this procedure respects both the model equations and observed expenditure and labor shares, it yields exactly the same Domar weights as those that come from implementing our balanced trade correction described earlier in the section. Therefore, one can think of our correction to the OECD ICIO as redistributing world factors such that assuming net balanced trade implies the observed expenditure and labor shares.

B.2 Elasticities

We take the preference/production elasticities, σ, θ, and ϵ, from Baqaee and Farhi (2024) (Table 1), and we use the trade elasticities $\theta^k - 1$ from Eppinger et al. (2021) (Table 2).

Table 1: Preferences/Technology Elasticity Parameters

Elasticity Parameter	Value	Source
Consumption (σ)	0.9	Baqaee and Farhi (2024)
Labor/Intermediates (θ)	0.5	Baqaee and Farhi (2024)
Cross-Industry (ϵ)	0.2	Baqaee and Farhi (2024)

Note: σ is the cross-industry consumption elasticity of substitution. θ is the raw producer elasticity of substitution between labor and aggregate intermediate inputs. ϵ is the cross-industry elasticity of substitution in creating aggregate intermediate bundles.

B.3 Aggregation

The matrix inverses required for the solution algorithm in Section A do not exist when the underlying input-output table has zero columns or rows. Additionally, zero rows or columns will render the key matrix in the trade balance correction reducible, meaning the correction might not be unique. Furthermore, the computation of the expenditure shares requires that, for any column of the input-output table, the sum of the entries from the same industry but across all countries be strictly positive.[27] Lastly, with $N = 45$ and $C = 77$, the number of agents is $\mathcal{N} = 166{,}397$, which requires the storage and manipulation of extremely large matrices. For all of these reasons, we combine some industries together and put some countries into ROW and regional aggregates.

While aggregation obviously requires many judgment calls, we use the following general approach, which aims to resolve the issues mentioned above while preserving a level of disaggregation useful for global policy questions.[28] First, there are some countries

[27] In other words, this means $\sum_{jc'} CONS^c_{jc'}$ and $\sum_{jc'} IO^{kc}_{jc'}$ both strictly positive for any c, k.

[28] Our aggregation scheme comes, specifically, from Gersbach et al. (2025). This scheme was somewhat tailored to questions of Swiss interest, but it is not so different from the aggregation in Bachmann et al. (2024).

Table 2: Industry Detail and Trade Elasticities

Non-aggregated		
ISIC Rev. 4 code	Description	$\theta_k - 1$
A01_02	Agriculture, hunting, forestry	3.468
A03	Fishing and aquaculture	3.468
B05_06	Mining and quarrying, energy producing products	3.468
C10_C12	Food products, beverages and tobacco	3.468
C13_C15	Textiles, textile products, leather and footwear	3.468
C16	Wood and products of wood and cork	3.468
C17_18	Paper products and printing	3.468
C19	Coke and refined petroleum products	6.027
C20	Chemical and chemical products	3.543
C21	Pharmaceuticals, medicinal chemical and botanical products	11.479
C22	Rubber and plastics products	2.27
C23	Other non-metallic mineral products	1.375
C24	Basic metals	3.206
C25	Fabricated metal products	1.558
C26	Computer, electronic and optical equipment	7.771
C27	Electrical equipment	6.012
C28	Machinery and equipment, n.e.c.	7.868
C29	Motor vehicles, trailers and semi-trailers	4.61
C30	Other transport equipment	2.915
C31_C33	Manufacturing n.e.c.; repair and installation of machinery and equipment	3.714
D	Electricity, gas, steam and air conditioning supply	1.442
E	Water supply; sewerage, waste management and remediation activities	1.442
F	Construction	1.442
G	Wholesale and retail trade; repair of motor vehicles	1.442
H49	Land transport and transport via pipelines	1.442
H50	Water transport	1.442
H51	Air transport	1.442
H52	Warehousing and support activities for transportation	1.442
H53	Postal and courier activities	1.442
I	Accommodation and food service activities	1.442
K	Financial and insurance activities	1.442
L	Real estate activities	1.442
Q	Human health and social work activities	1.442
Aggregated		
Included ISIC codes	Description	$\theta_k - 1$
B07_08, B09	Mining, non-energy and support services	3.468
J58_60, J61, J62_63	Telecomm, IT, publishing and media	1.442
M, P	Science and education	1.442
N, O	Administrative services and defence; compulsory social security	1.442
R, S, T	Other services	1.442

Note: The table lists the 38 industries we consider in our quantitative analysis. The top panel contains the 33 ISIC Rev. 4 industries we use as given in the OECD ICIO. The bottom panel describes how we aggregate the remaining 12 OECD ICIO industries into the 5 used in our analysis. The last column displays the trade elasticity estimates from Eppinger et al. (2021), which correspond to our θ_k parameters (minus one). For C17_18 we use the C17 elasticity in Eppinger et al. (2021).

with either many zero rows/columns or zero rows/columns in industries of general policy importance. These countries are natural to put in aggregates such as Rest-of-

EU (ROE), Subset of Africa (AFR), or ROW. For example, in Belgium and Portugal, the industry "Mining and quarrying, energy producing products" has zero output. The sector considered to be highly relevant for policy questions, those two countries were added to the Rest-of-EU aggregate. Second, there are industries of less interest with either zero rows/columns for many countries or zero rows/columns for countries we would like to consider individually. These industries are natural to combine with related industries. An example of this is the industry "Activities of households as employers" which has zero output for many important countries and was therefore combined with other service-related industries. Third, after removing such natural industries and countries, dealing with the remaining zeros requires more difficult judgment calls, and we proceed by combining some countries and industries to retain countries and sectors of particular interest to us. Finally, to speed up our computations, we put some smaller countries into aggregates.

Our industry aggregation scheme is described in Table 2. The top panel contains the 33 industries we keep at the level of disaggregation in the OECD ICIO. The bottom panel contains the 5 aggregates we create to remove zero rows and columns. Our country aggregation scheme is given in Tables 3 and 4.

Table 3: Countries and Regions included in the Analysis

ARG	Argentina	AUS	Australia
AUT	Austria	BRA	Brazil
CAN	Canada	CHE	Switzerland
CHN	China	DEU	Germany
DNK	Denmark	ESP	Spain
FRA	France	GBR	United Kingdom
IDN	Indonesia	IND	India
IRL	Republic of Ireland	ITA	Italy
JPN	Japan	KOR	Republic of Korea
MEX	Mexico	NLD	Netherlands
NOR	Norway	POL	Poland
RUS	Russia	SAU	Saudi Arabia
SWE	Sweden	TUR	Türkiye
TWN	Taiwan	USA	United States of America
VNM	Vietnam	ROE	Rest-of-EU
LAA	Subset of Latin America	EAA	Subset of East Asia
AFR	Subset of Africa	ROW	Rest-of-World

Table 4: Countries Aggregates

	Rest-of-EU (ROE)		**Subset of East Asia (EAA)**
BEL	Belgium	BGD	Bangladesh
BGR	Bulgaria	BRN	Brunei
CYP	Cyprus	KHM	Cambodia
CZE	Czechia	LAO	Laos
EST	Estonia	MMR	Myanmar
FIN	Finland	MYS	Malaysia
GRC	Greece	PAK	Pakistan
HRV	Croatia	PHL	Philippines
HUN	Hungary	SGP	Singapore
LTU	Lithuania	THA	Thailand
LUX	Luxembourg		
LVA	Latvia		**Subset of Africa (AFR)**
MLT	Malta	CIV	Côte d'Ivoire
PRT	Portugal	CMR	Cameroon
ROU	Romania	EGY	Egypt
SVK	Slovakia	MAR	Morocco
SVN	Slovenia	NGA	Nigeria
		SEN	Senegal
Subset of Latin America (LAA)		TUN	Tunisia
CHL	Chile	ZAF	South Africa
COL	Colombia		
CRI	Costa Rica		**Added to Rest-of-World (ROW)**
PER	Peru	BLR	Belarus
		ISL	Iceland
		ISR	Israel
		JOR	Jordan
		KAZ	Kazakhstan
		NZL	New Zealand
		UKR	Ukraine

Note: The country ROW we use in our analysis is the sum of the original ROW category in the OECD ICIO plus the 7 countries in the bottom panel. CHN refers to China + Hong Kong, which are separated in the raw data but combined for our analysis.