Aggregate Capital and Informality in High Inflation Economies

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Abstract

Inflation is conventionally viewed as a tax on the cash-intensive informal sector, implying that higher inflation should shrink informality. I show that the opposite holds in general equilibrium. Inflation erodes the real return on cash holdings, and because seigniorage and working-capital interest transfers do not fully compensate cash-heavy households, they respond by reducing interest-bearing assets. The resulting contraction in loanable funds raises the cost of capital, disproportionately burdening capital-intensive formal firms and expanding informality through a Stolper-Samuelson mechanism. The theory delivers an exact sufficient condition expressed in primitive parameters: the general equilibrium effect dominates if and only if formal firms are sufficiently more capital-intensive than informal ones. Cross-country data from 157 countries over 1980–2020 confirm that inflation, financial underdevelopment, and informality form a robust triangle; controlling for credit absorbs 37% of the inflation–informality association. Using the staggered adoption of inflation targeting across 42 countries as quasi-experimental variation, I document the causal sequence predicted by the credit channel: inflation falls within years of IT adoption, credit expands over the following 5–10 years, and informality declines significantly only after 5–15 years. This slow-motion formalization, consistent with capital accumulation as the operative mechanism, is what distinguishes the credit channel from alternatives. A calibrated heterogeneous-agent model with incomplete markets and cash-in-advance constraints matches these dynamics qualitatively.

Keywords: Inflation, Aggregate Capital, Informality, Financial Development.

JEL Classification: E31, E22, O17.

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1 Introduction

Informality is estimated to be a third of the official worldwide GDP,[1] and cash facilitates the unrecorded and immediate transactions that are typical in daily informal exchanges. This is why, despite the emergence of new payment methods, the global demand for cash remains steady,[2] and even in high-inflation environments, households still need to hold cash for transactional reasons. Especially in developing economies, informal consumption cannot easily be substituted for formal purchases: a substantial portion of economic activity operates outside recorded markets, and lower-income households spend disproportionately in informal establishments.[3] Standard monetary theory has a clear prediction about the interaction between inflation and this cash-intensive sector: because inflation is a tax on cash, it should discourage cash holdings and therefore *shrink* the informal economy. This logic underlies a long tradition in public finance that views inflation as a tool for taxing informality (Nicolini, 1998; Koreshkova, 2006), and is shared by recent New Monetarist models in which inflation reduces the return to anonymous cash transactions and contracts the informal sector (Lahcen, 2020; Aruoba, 2021). Yet across countries and over time, the data point in the opposite direction: high inflation is associated with *more* informality, not less. This paper explores a general equilibrium mechanism through capital markets that explains the positive relationship between inflation and informality.

The key insight is that the conventional view captures only one force, a direct headwind in which inflation taxes cash and discourages informal consumption, while missing a general equilibrium tailwind that operates through capital markets and can dominate. The mechanism works as follows. Under the Svensson (1985) timing convention, households set aside cash one period before they can spend it, and the government collects seigniorage on these pre-committed balances before redistributing it as lump-sum transfers; in addition, the financial intermediary earns a profit on working-capital loans to formal firms and rebates this income to households. Because unconstrained households hold more cash than constrained ones, they pay more in inflation tax than they receive back in rebates, leaving them with less disposable income. With lower disposable income, these households save less in interest-bearing assets. This portfolio reallocation starves the financial system of deposits, reducing the credit available to firms. Because the formal sector is more capital-intensive than the informal sector, the resulting credit squeeze raises formal production costs disproportionately, a mechanism that echoes the classic Stolper-Samuelson logic from international trade. The relative price of formal goods rises, households tilt their consumption toward the now relatively cheaper informal goods, and the informal sector expands. The total effect of inflation on informality thus decomposes into two opposing forces: the direct headwind, which the existing literature has focused on, and the general equilibrium tailwind through capital markets, which this paper identifies. The net effect is governed by a single sufficient statistic, the capital gap between formal and informal sectors, and the paper derives the exact conditions under which the tailwind dominates.

The empirical section documents new evidence on the inflation and informality relationship through progressively tighter identification strategies. I begin with cross-country correlations showing that countries with higher average inflation have larger informal sectors and shallower financial systems, even after conditioning on GDP per capita. These correlations trace a coherent triangle: inflation erodes credit, credit constrains

[1] Informality consists of all legal unrecorded activities that avoid taxes and bureaucracy, and represents on average 31.9% of GDP for 158 countries over 1991–2015 (Medina and Schneider, 2018), with some countries above 50% in Latin America and Africa.

[2] Jiang and Shao (2020) examine the paradox of decreasing cash use at points of sale while cash demand remains strong, as shown by steady currency in circulation. Their model shows that when credit use increases, cash-intensive sectors manage cash flows, reducing overall cash velocity despite fewer cash transactions.

[3] Bachas, Gadenne, and Jensen (2024) document that the share of household expenditure in informal establishments steeply declines with income across a broad set of developing countries.

formalization, and the product of these two links approximates the total inflation and informality relationship. Conditioning on credit absorbs roughly 37% of the inflation coefficient, consistent with a capital market channel operating alongside direct effects. This is in line with a growing body of evidence that financial frictions are a first-order determinant of informality: credit expansion has been shown to increase formalization in Brazil (Catão, Pagés, and Rosales, 2010), Uruguay (Gandelman and Rasteletti, 2017), Colombia (Caro, Galindo, and Meléndez, 2012), and Nigeria (Adom, 2026), while Boyd, Levine, and Smith (2001) document the nonlinear relationship between inflation and financial depth that drives the first leg of the triangle.

To move beyond correlations, I exploit the staggered adoption of inflation targeting (IT) by 42 countries between 1990 and 2018 as a source of identifying variation for informality, to my knowledge a novel application. Using the Callaway and Sant'Anna (2021) estimator, which avoids the well-documented biases of two-way fixed effects in staggered settings (Goodman-Bacon, 2021; de Chaisemartin and d'Haultfœuille, 2020), I find that IT adoption reduces informality by 3–4 log points within 10–15 years, with a joint Wald test that fails to reject parallel pre-trends ($p = 0.450$). The temporal ordering of the response is particularly informative for the mechanism: inflation falls immediately upon adoption, credit expands over the next 5–10 years, and informality declines only after 5–15 years. This lag structure is inconsistent with a direct behavioral response (workers do not switch sectors the day a central bank announces a new framework) but is precisely what a mechanism operating through slow-moving capital accumulation would predict. Lower inflation leads to lower interest rates, which cheapens capital for formal firms, which gradually formalizes marginal enterprises, which eventually shows up as lower informality in the aggregate data.

The theoretical framework formalizes this mechanism through two complementary models. The first is a tractable deterministic economy with two alternating income states, as in Kehoe and Levine (1993), extended with two consumption goods and a working-capital constraint on formal firms. This structure delivers closed-form results. The analysis shows that when borrowing constraints bind, the combined redistribution from seigniorage transfers and working-capital interest rebates reduces the unconstrained household's savings: the high-income household bears a disproportionate share of the inflation tax because neither the CIA seigniorage rebate nor the working-capital interest rebate fully compensates it under equal sharing. The decline in savings raises the equilibrium interest rate, and through the Stolper-Samuelson mechanism, this raises formal costs relative to informal costs. The main theorem identifies the exact threshold of the capital gap above which the general equilibrium tailwind overcomes the direct headwind, expressed entirely in terms of primitive parameters. Crucially, the capital gap cannot be so large that the economy's savings capacity is exhausted; the theorem also characterizes this upper bound and shows that for empirically relevant elasticities of substitution between formal and informal goods, a non-degenerate range of capital shares exists at which an interior equilibrium exists *and* inflation increases informality. This result overturns the prediction of the existing partial-equilibrium literature, including Lahcen (2020) and Aruoba (2021), in which only the direct headwind operates and inflation unambiguously shrinks the informal sector.

The second model is a fully-fledged heterogeneous-agent economy à la Aiyagari (1994) with idiosyncratic uncertainty, calibrated to match moments from a representative Latin American economy. Relative to existing work embedding money in heterogeneous-agent models, notably Algan and Ragot (2010), who find a hump-shaped inflation-capital relationship under money-in-the-utility-function preferences and Lucas timing, the present model adopts the Svensson timing and a two-sector structure that introduces a formal and informal consumption margin absent in one-sector models. By comparing steady states at different constant inflation rates, the quantitative model shows that increasing annual inflation from 10% to 90% reduces the aggregate capital stock by 8% and raises informality as a share of GDP from 40% to 54%. The model captures the

cross-country relationship between inflation and informality for moderate and high inflation levels.

The findings carry a sharp policy implication: using inflation to tax the informal sector is self-defeating. The general equilibrium response endogenously expands the very sector the policy aims to diminish. Because inflation diverts resources toward informal production, it causes the economy to rely less on capital-intensive formal firms that enable higher wages, more tax revenue, and greater productivity. This challenges the normative conclusions of the optimal taxation literature (Nicolini, 1998; Koreshkova, 2006), where causation runs from informality to inflation and seigniorage is a welfare-improving substitute for distortionary taxes. The analysis here shows that the reverse direction also matters: inflation *generates* informality through the credit channel. Monetary authorities who set inflation targets without accounting for this feedback may overestimate the revenue gains from seigniorage and underestimate the structural damage to the productive economy. The reverse transition is also feasible: reducing inflation relaxes the seigniorage burden on savers, lowers the equilibrium interest rate, and through the same Stolper-Samuelson channel contracts the informal sector, creating a virtuous cycle in which price stability, financial deepening, and formalization reinforce one another.

The structure of the paper is as follows. Section 2 presents motivating facts about empirical correlations between inflation, informality, and financial development, traces the credit channel through conditional regressions, and provides causal evidence using the staggered adoption of inflation targeting. Section 3 develops the tractable deterministic model that delivers closed-form results on the conditions under which inflation increases informality. Section 4 introduces the quantitative heterogeneous-agent model, details its calibration, and examines the findings. Section 5 discusses the negative consequences of inflation for welfare and fiscal capacity. Section 6 concludes.

1.1 Related Literature

This paper connects the monetary theory of inflation under incomplete markets with the structural analysis of informality in developing economies. Its contribution is a novel mechanism linking inflation to informality through capital markets: under the Svensson (1985) timing convention, inflation induces households to substitute away from interest-bearing savings and toward cash, reducing credit to capital-intensive formal firms and expanding the informal sector through general equilibrium price adjustments. The paper provides exact analytical conditions for this mechanism, validates them quantitatively, and documents new causal evidence using the staggered adoption of inflation targeting.

The theoretical framework builds on the Aiyagari (1994) incomplete-markets model, in which precautionary savings under borrowing constraints make portfolio composition sensitive to asset returns, and extends it with two production sectors and a cash-in-advance constraint on informal consumption. A classical result in monetary economics is the superneutrality of money (Sidrauski, 1967), which breaks down when cash-in-advance constraints interact with capital accumulation. Stockman (1981) showed that CIA constraints on capital goods generate a negative inflation-capital relationship, and Abel (1985) clarified that the direction depends on the timing convention: under the Svensson (1985) timing, cash must be set aside before the government collects seigniorage, reversing the standard Tobin (1965) portfolio substitution. The closest antecedent is Algan and Ragot (2010), who embed money into a heterogeneous-agent model and find a hump-shaped relationship between inflation and capital. The deterministic version of the present model adopts the cyclical economy with two alternating states from Kehoe and Levine (1993), whose debt-constrained asset market framework provides the tractable structure for studying incomplete insurance with endogenous borrowing limits; here it serves to derive closed-form conditions on primitive parameters under which inflation

increases informality. Algan and Ragot's result relies on money-in-the-utility-function preferences and Lucas timing, under which households rebalance after receiving transfers. This paper departs in both dimensions: the Svensson timing reverses the portfolio reallocation, and the two-sector structure introduces a formal-informal consumption margin absent in their one-sector model. Erosa and Ventura (2002) showed that inflation acts as a regressive consumption tax under incomplete markets; this paper extends their insight by showing that the regressivity operates not only across households but across sectors with different capital intensities, generating a Stolper-Samuelson type reallocation toward informality.

The paper also engages with the heterogeneous-agent New Keynesian literature. Kaplan, Moll, and Violante (2018) showed that monetary transmission depends on household heterogeneity and portfolio composition, and Auclert (2019) formalized the redistribution channel of monetary policy. The present paper extends these insights to a developing-economy context where the relevant portfolio margin is between deposits and cash for informal transactions, and where redistribution operates through seigniorage and the supply of capital rather than through demand-side consumption responses. Bassetto (2014) studies optimal fiscal policy with heterogeneous agents and shows that the Friedman rule is generally suboptimal when the government has limited instruments, providing the normative backdrop for the positive analysis here, which shows that inflation reshapes the productive structure of the economy in addition to redistributing across households.

A longstanding public finance tradition treats inflation as a tool for taxing informality. Nicolini (1998) and Koreshkova (2006) showed that governments optimally resort to seigniorage when the informal sector erodes the tax base, with causation running from informality to inflation. This paper explores the reverse direction. Lahcen (2020) and Aruoba (2021) study how inflation affects informality within New Monetarist frameworks and find that it shrinks the informal sector by reducing cash demand. Their mechanism operates through a partial-equilibrium force that this paper labels the direct headwind. The contribution here is to identify a general equilibrium tailwind through capital markets that can dominate: when capital shares differ sufficiently across sectors, the inflation-induced rise in the cost of capital disproportionately harms formal firms, expanding informality. Gomis-Porqueras, Peralta-Alva, and Waller (2014) share with this paper the view that cash usage and informality are jointly determined through optimal portfolio and reporting decisions, but their focus is on measuring the shadow economy at a point in time rather than on how inflation changes its size through capital markets. Most closely related is Jaar and Ritto (2025), who study the distributional consequences of informality for monetary policy in a heterogeneous-agent model calibrated to Peru; the present paper emphasizes the reverse direction, showing how inflation generates informality through the credit channel.

The informality literature provides the empirical motivation for the capital gap between sectors. La Porta and Shleifer (2014) document that informal firms produce roughly 15% of the value added per worker of formal firms and rely almost exclusively on internal funds. De Paula and Scheinkman (2010) confirm this within Brazil: formalization corresponds to a 0.31 standard deviation increase in investment per worker and a 0.52 standard deviation increase in capital installations per worker, establishing that the capital gap is a within-country, within-industry regularity. Ulyssea (2018) estimates that nearly half of informal firms lack the productivity to cover formalization costs, driven largely by inability to accumulate capital. On the financial channel, Boyd, Levine, and Smith (2001) document the nonlinear relationship between inflation and financial depth, and Buera and Shin (2013) show that financial frictions generate misallocation by preventing productive entrepreneurs from accessing capital. Catão, Pagés, and Rosales (2010) and Gandelman and Rasteletti (2017) provide direct evidence that credit expansion increases formalization in Brazil and Uruguay, respectively, and Caro, Galindo, and Meléndez (2012) find that a 10% increase in credit-to-output raises labor formality by

roughly one percentage point in Colombian manufacturing. At the structural level, D'Erasmo and Moscoso Boedo (2012) show that financial depth shapes the equilibrium size of informality across countries, and Adom (2026) finds that removing financial constraints makes the intensive margin of formalization quantitatively insignificant in Nigeria, demonstrating that credit frictions rather than regulatory costs are the binding constraint. The common thread is that tighter financial frictions expand informality; this paper shows that inflation endogenously tightens these frictions by reducing aggregate savings.

The empirical strategy exploits inflation targeting adoption as identifying variation for informality, which to my knowledge is novel. The Callaway and Sant'Anna (2021) estimator avoids the well-documented pitfalls of two-way fixed effects in staggered settings (Goodman-Bacon, 2021; de Chaisemartin and d'Haultfœuille, 2020; Sun and Abraham, 2021). Ball and Sheridan (2005) raised concerns about mean reversion in IT evaluations; the pre-trends analysis directly addresses this, with a joint Wald test that fails to reject parallel trends in informality ($p = 0.450$). The finding that informality responds 5 to 15 years after adoption, while inflation falls immediately and credit adjusts over 5 to 10 years, provides the temporal ordering predicted by the capital accumulation channel.

2 Empirical Evidence

This section documents three empirical regularities that motivate and discipline the theoretical model developed in Section 3. First, countries with higher average inflation tend to have larger informal sectors and smaller financial systems; controlling for GDP per capita and adding credit as a regressor shows that the credit channel absorbs over a third of the cross-country inflation–informality association. Second, within-country variation around the adoption of inflation targeting, an institutional reform that exogenously lowers inflation, reveals a systematic pattern: inflation falls, credit expands, and informality declines, country by country. Third, formalizing this pattern through a staggered difference-in-differences estimator provides evidence that inflation targeting reduces informality, with the effect building gradually over 5–15 years post-adoption. This lag structure is consistent with a mechanism operating through slow-moving capital accumulation rather than an immediate behavioral response. The section proceeds from broad cross-country correlations, to conditional regressions tracing the credit channel, then to raw within-country dynamics, and finally to formal estimation, progressively tightening the identification.

To fix ideas, consider two concrete settings. In Mexico, the National Survey of Household Income and Expenditure (ENIGH) reveals that roughly 90% of payments are made in cash, with the share rising to 95% for transactions under US$25 and remaining above 90% for housing rent, taxes, public utilities, and transportation.[4] In Argentina, which has a long history of high inflation, the informal market known as *La Salada*, comprising roughly 3,000 stores, drew approximately 100,000 shoppers daily and generated around half a million dollars in daily sales during 2017, operating entirely in cash. Municipal authorities have come to rely on this market as a source of employment and a supplier of affordable goods. In both cases, cash dominance sustains a large informal sector: inflation depletes resources that could be saved in interest-bearing deposits or other financial assets, and the financial system lacks the depth to channel credit to formal productive enterprises.[5] The cross-country analysis below shows that these examples are representative of a broader empirical regularity.

[4] Alvarez and Argente (2022) document that the widespread use of cash in Mexico is driven largely by limited banking access, especially among lower-income populations.

[5] Bachas, Gadenne, and Jensen (2024) document that the share of household expenditure in informal establishments steeply declines with income across a broad set of developing countries, confirming that the informal sector disproportionately serves lower-income populations who are most reliant on cash.

2.1 Data Sources and Construction

The analysis draws on an unbalanced panel of 157 countries spanning 1980–2020, assembled from multiple primary sources.

Inflation. Annual CPI inflation rates are taken from the Ha, Kose, and Ohnsorge (2023) global inflation database, which harmonizes national sources and provides consistent coverage across advanced and developing economies. To compress the highly skewed right tail of the inflation distribution, which includes episodes of hyperinflation exceeding 1,000% per annum, I apply the logarithmic transformation $\tilde{\pi}_{it} = \ln(1+\pi_{it})$, where π_{it} is the annual CPI inflation rate expressed in percentage points (so that 2% inflation corresponds to $\pi_{it} = 2$).[6]

Informality. The share of informal economic activity in GDP is measured using MIMIC (Multiple Indicators, Multiple Causes) estimates from the World Bank Informal Economy Database (Elgin, Kose, Ohnsorge, and Yu, 2021). The MIMIC series is available from 1991 to 2020 for most countries. To extend coverage back to 1980, which is critical for providing adequate pre-treatment observations for early inflation targeting adopters such as New Zealand (1990), Canada (1991), and the United Kingdom (1992), I supplement the MIMIC data with Dynamic General Equilibrium (DGE) backcasts from the same database for the period 1980–1992.[7] I work throughout with ln(Informality/GDP), which compresses heterogeneity in levels and yields coefficients interpretable as elasticities.

Financial intermediation. Private credit extended by deposit money banks as a share of GDP is taken from the World Bank's Global Financial Development Database (GFDD), indicator DI.01. I work with ln(Credit/GDP). Countries with zero or missing credit data are excluded from the relevant regressions.

GDP per capita. Real GDP per capita in constant 2017 PPP international dollars is taken from the IMF World Economic Outlook database (October 2024). This variable serves as a control for the overall level of economic development in the cross-country regressions: richer countries tend to have lower inflation, deeper financial systems, and smaller informal sectors, so conditioning on income is essential for isolating the credit channel from the broader development gradient. I work with ln(GDP per capita). Two countries in the panel (Eritrea, Syria) have incomplete WEO coverage and are dropped from specifications requiring this control, reducing the sample from 158 to 156.

Inflation targeting adoption. A binary indicator IT_{it} equals one from the year a country formally adopted an inflation targeting framework onward, and zero otherwise. The adoption dates cover 42 countries that adopted between 1990 (New Zealand) and 2018 (Jamaica).[8]

2.2 Cross-Country Correlations

I begin by documenting the long-run cross-country relationships between inflation, financial intermediation, and informality. For each of the 157 countries with complete data, I compute the time-series average of each

[6] Following Kremer, Bick, and Nautz (2013), this transformation compresses the right tail while preserving ordinality, and has the advantage that zero inflation maps to zero.

[7] The DGE approach estimates informality from structural relationships between observed macro aggregates and the size of the unobserved sector. While informative about trends, the level estimates are sensitive to model assumptions and initial conditions. By using DGE only for the earlier period and relying on MIMIC for 1991 onward, I leverage the strengths of each methodology.

[8] See Hammond (2012) for a comprehensive chronology of inflation targeting adoption dates.

variable over 1980–2020, yielding a single observation per country. This averaging over four decades smooths out business-cycle fluctuations and captures the structural, long-run relationships that the theoretical model is designed to explain.

Figure 1 presents bivariate scatter plots and OLS regression lines for the three pairwise relationships. All variables are in logs.[9]

Panel (A) plots average $\tilde{\pi}$ against average ln(Informality/GDP) for 157 countries. The estimated slope is $\hat{\beta} = 0.30$ ($p < 0.01$, $R^2 = 0.22$). Walking along the fitted line illustrates the magnitude: a country with average inflation around 2% (typical of Germany, Canada, or the Netherlands) is predicted to have an informal sector of roughly 23% of GDP. At 7% average inflation (typical of South Africa, Bangladesh, or Chile), the predicted value rises to about 31%. At 50% average inflation (typical of countries with chronic macroeconomic instability), the regression line reaches about 55% of GDP.

Panel (B) plots inflation against financial depth ($\hat{\beta} = -0.70$, $p < 0.01$, $R^2 = 0.28$). The same three points on the fitted line predict credit-to-GDP ratios of roughly 52%, 26%, and 7%, respectively. In other words, the regression implies that moving from a low-inflation to a high-inflation environment is associated with a near-collapse in private credit availability.

Panel (C) completes the triangle: credit and informality are negatively related ($\hat{\beta} = -0.31$, $p < 0.01$, $R^2 = 0.39$). A country with deep credit markets (90% of GDP) is predicted to have informality of about 21%, one with moderate credit depth (40%) about 27%, and one with shallow credit (15%) about 37%. These are large magnitudes, but they reflect unconditional cross-country correlations that conflate the credit channel with the broader development gradient; the conditional estimates in the next subsection are considerably smaller.

The consistency of these three relationships is suggestive of a credit channel linking inflation to informality: inflation reduces credit (Panel B), credit reduces informality (Panel C), and the product of these two elasticities ($0.70 \times 0.31 \approx 0.22$) approximates the total inflation–informality elasticity (0.30, Panel A). While this decomposition is only approximate, since the three panels use unconditional bivariate regressions rather than a structural model, the directional consistency motivates a closer look at the credit channel. These patterns are consistent with two well-established cross-country regularities: the negative relationship between inflation and financial sector performance documented by Boyd, Levine, and Smith (2001), who identify a nonlinear effect with a discrete drop at inflation rates above 15%, and the negative relationship between informality and economic development emphasized by La Porta and Shleifer (2014) and Loayza (2016).[10] The contribution of this paper's cross-country analysis is to connect these two literatures, inflation $\rightarrow$ financial underdevelopment and financial underdevelopment $\rightarrow$ informality, and to trace the credit channel explicitly, conditional on income.

A natural concern is that these correlations are driven by economic development: richer countries have lower inflation, deeper credit markets, and less informality, so the apparent credit channel may simply reflect income differences. The next subsection addresses this by conditioning on GDP per capita and tracing how the inflation coefficient responds when credit is added as a control.

[9] Zimbabwe is excluded from the scatter plots because its hyperinflation episode (peaking above 10^9% in 2008) places it far outside the range of the remaining sample and distorts the visual scale. Including Zimbabwe does not materially change the regression coefficients.

[10] Khan, Senhadji, and Smith (2006) find a similar threshold at 3–6%.

Figure 1: Cross-Country Averages 1980–2020

MIMIC informality (+DGE backcast pre-1993) · Ha et al. CPI inflation · GFDD private bank credit (DI.01). N=157.

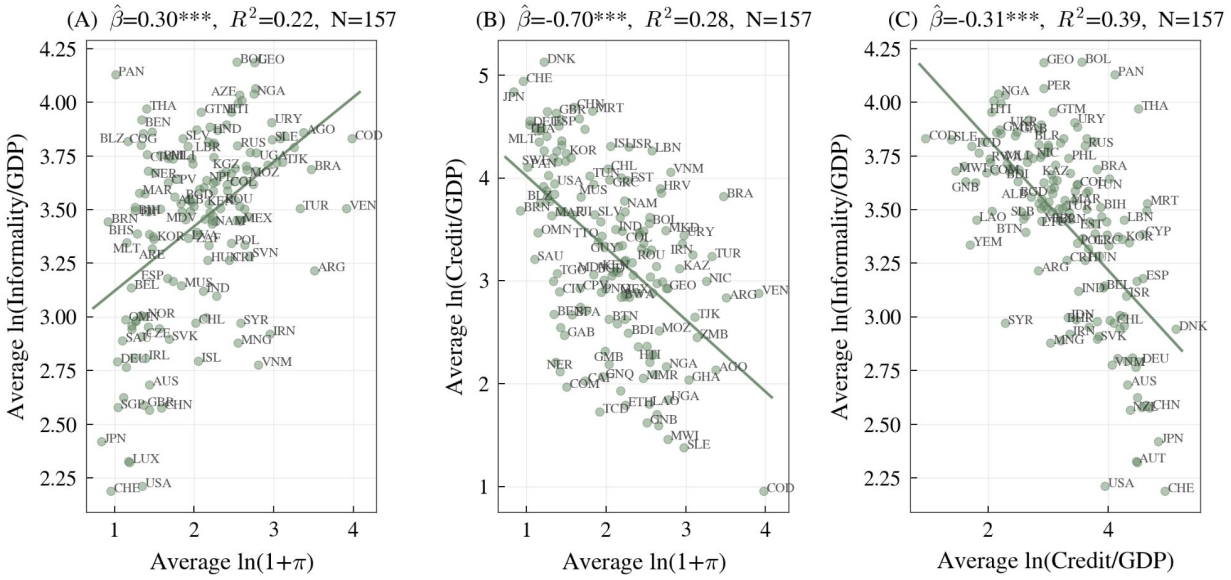


Figure 1: **Cross-country averages, 1980–2020.** Each point is one country ($N = 157$). All variables in logs. OLS regression lines with slope (β), significance (*** $p < 0.01$), and R^2 shown. Panel (A): inflation vs. informality. Panel (B): inflation vs. credit. Panel (C): credit vs. informality.

2.3 Tracing the Credit Channel

The bivariate correlations in Figure 1 are consistent with the theoretical mechanism but may confound the credit channel with the broader process of economic development. To isolate the inflation–credit–informality pathway, I move from pure cross-sections to a short panel of 20-year country averages. Splitting the 1980–2020 sample at the year 2000 yields two balanced periods (1980–1999 and 2000–2020) for 149 countries observed in both ($N = 298$). This structure allows me to estimate two specifications: pooled OLS, which uses both between- and within-country variation, and country fixed effects, which isolates within-country changes over time. In both cases I include a period dummy to absorb global trends.

The three equations trace the inflation–credit–informality pathway:

$$\ln\left(\frac{\text{Inf}_{it}}{\text{GDP}_{it}}\right) = \alpha_1 + \beta_1 \cdot \tilde{\pi}_{it} + \gamma_1 \cdot \ln(\text{GDPpc}_{it}) + \mu_t + [\alpha_i] + \varepsilon_{it} \tag{1}$$

$$\ln\left(\frac{\text{Credit}_{it}}{\text{GDP}_{it}}\right) = \alpha_2 + \beta_2 \cdot \tilde{\pi}_{it} + \gamma_2 \cdot \ln(\text{GDPpc}_{it}) + \mu_t + [\alpha_i] + \nu_{it} \tag{2}$$

$$\ln\left(\frac{\text{Inf}_{it}}{\text{GDP}_{it}}\right) = \alpha_3 + \beta_3 \cdot \tilde{\pi}_{it} + \delta \cdot \ln\left(\frac{\text{Credit}_{it}}{\text{GDP}_{it}}\right) + \gamma_3 \cdot \ln(\text{GDPpc}_{it}) + \mu_t + [\alpha_i] + u_{it} \tag{3}$$

where i indexes countries, $t \in \{1, 2\}$ indexes 20-year periods, μ_t is a period fixed effect, and α_i is a country fixed effect included in Columns (4)–(6) but not in Columns (1)–(3). Standard errors are clustered at the country level throughout.

Table 1: Panel Regressions: 20-Year Averages (1980–1999 vs. 2000–2020)

	Pooled OLS			Country Fixed Effects		
	(1)	(2)	(3)	(4)	(5)	(6)
	$\ln \frac{\text{Inf}}{\text{GDP}}$	$\ln \frac{\text{Credit}}{\text{GDP}}$	$\ln \frac{\text{Inf}}{\text{GDP}}$	$\ln \frac{\text{Inf}}{\text{GDP}}$	$\ln \frac{\text{Credit}}{\text{GDP}}$	$\ln \frac{\text{Inf}}{\text{GDP}}$
$\tilde{\pi}$	0.111^{***}	-0.298^{***}	0.070^{**}	-0.007	-0.126^{**}	-0.009^{*}
	(0.029)	(0.054)	(0.028)	(0.005)	(0.060)	(0.005)
ln(Credit/GDP)			-0.138^{***}			-0.013
			(0.045)			(0.011)
ln(GDP pc)	-0.196^{***}	0.471^{***}	-0.131^{***}	-0.068^{**}	0.375	-0.063^{**}
	(0.021)	(0.037)	(0.032)	(0.027)	(0.297)	(0.027)
Period FE	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	No	No	No	Yes	Yes	Yes
R^2 / Within-R^2	0.423	0.582	0.461	0.711	0.426	0.717
Countries	149	149	149	149	149	149
N	298	298	298	298	298	298

Notes: Each observation is a country average over a 20-year period (1980–1999 or 2000–2020; $N = 298$, balanced panel of 149 countries). All variables in natural logs. Standard errors clustered by country in parentheses. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.10$. GDP per capita is in constant 2017 PPP international dollars (IMF WEO, October 2024). The credit share in Column (3) is $1 - \hat{\beta}_3/\hat{\beta}_1 = 1 - 0.070/0.111 \approx 37\%$.

Table 1 reports the results. The left panel (Columns 1–3) uses both between- and within-country variation. Column (1) estimates an elasticity of informality with respect to gross inflation of 0.111 ($t = 3.9$), conditional on income and a period dummy. Column (2) confirms that inflation erodes credit depth (-0.298, $t = -5.5$), and Column (3) shows that controlling for credit reduces the inflation coefficient by 37% (from 0.111 to 0.070), with credit itself strongly significant (-0.138, $t = -3.0$). These pooled estimates are close to the bivariate results from Figure 1 and the credit channel's share is similar. This finding is in the spirit of the Rajan and Zingales (1998) approach to financial dependence: just as industries with greater external financing needs grow faster in countries with deeper credit markets, the formal sector, which is structurally more capital-intensive, contracts disproportionately when inflation erodes financial intermediation.[11]

To illustrate the magnitudes, consider three hypothetical middle-income economies (GDP per capita around $8,000) that differ only in their long-run inflation rates. Walking along the regression lines from Columns (1) and (2):

- At 3% average inflation, the regressions predict informality of about 29% of GDP and credit of about 29% of GDP.
- At 10% average inflation, predicted informality rises to 32% and credit falls to 21%.
- At 30% average inflation, predicted informality reaches 36% and credit falls to 16%.

The total informality gap between the low- and high-inflation economies is about 7 percentage points of GDP (from 29% to 36%). Of this, roughly 3 percentage points operate through the credit channel: inflation nearly halves private credit (from 29% to 16% of GDP), and this credit shortfall pushes activity into the informal sector. The remaining 4 percentage points reflect direct mechanisms (the inflation tax on cash

[11]The model in Section 3 formalizes this asymmetry: formal firms hire capital at the equilibrium interest rate while informal firms operate with negligible capital, so that any inflation-induced increase in the cost of capital affects only the formal sector.

holdings, seigniorage redistribution, and relative price effects) that the theoretical model in Section 3 formalizes alongside the credit channel.

The right panel of Table 1 (Columns 4–6) includes country fixed effects, restricting identification to within-country changes between the two periods. A striking asymmetry emerges. In Column (4), the inflation coefficient on informality drops from 0.111 to −0.007 and loses all statistical significance. Yet in Column (5), the inflation coefficient on credit remains large and statistically significant (-0.126, $t = -2.1$): countries that reduced inflation between the two periods experienced substantial credit deepening, even after removing all cross-country differences.

A simple variance decomposition clarifies why. Of the total cross-country-over-time variation in log informality, 99% is between countries and only 1% is within countries across the two periods. Informality is essentially a fixed country characteristic at the 20-year frequency: Azerbaijan moved from 184% to 4% average inflation across the two periods and saw its credit-to-GDP ratio increase fourfold, yet its informality share barely changed. Credit, by contrast, has meaningful within-country variation (15% of total), and inflation itself has substantial within-country shifts (46% of total, reflecting the end of Latin American hyperinflation, the transition economies' stabilization, and the spread of inflation targeting). The fixed effects estimator thus asks whether the 46% of inflation variation that occurs *within* countries can explain the 1% of informality variation that occurs within countries; the answer, mechanically, is no.

This pattern is not a failure of the inflation–informality relationship but a reflection of how the mechanism operates through time. The theoretical model predicts a *long-run steady-state* relationship: permanently higher inflation leads to permanently shallower credit markets, which in turn sustain a permanently larger informal sector through reduced capital accumulation. The two links in this chain operate at very different speeds. The first (inflation → credit) adjusts over one to two decades as deposit bases respond to the real return on savings: when a country tames inflation, households gradually shift from cash and real assets back into bank deposits, and credit expands. The second (credit → formalization) is generational: firms must accumulate capital, establish banking relationships, and invest in the fixed costs of formality, a process that plays out over decades, not years. With only two 20-year periods, the data can detect the faster-moving first link but not the slower-moving second.

The cross-section, where countries are observed at or near their long-run equilibria, is therefore the natural lens for estimating the full mechanism. But cross-sectional estimates cannot establish the direction of causation. To make progress on identification, I turn to within-country variation of a particular kind: the adoption of inflation targeting, a discrete and permanent policy shock that transforms the inflation environment and, over time, the credit market along with it.

2.4 Inflation Targeting and Country-Level Dynamics

Between 1990 and 2018, 42 countries formally adopted inflation targeting (IT) frameworks, an institutional commitment by a central bank to anchor monetary policy around an explicit inflation objective. The key identifying assumption throughout the analysis is that IT adoption is primarily motivated by a desire to reduce and stabilize inflation, not by contemporaneous concerns about informality or financial depth. This is supported by the historical record: IT emerged from developments in monetary theory (the credibility revolution following Kydland and Prescott, 1977; Barro and Gordon, 1983) rather than from structural concerns about the informal economy.

Before turning to formal estimation, I present the raw within-country dynamics. Figure 2 displays a heatmap covering all 42 IT-adopting countries, sorted by adoption year (earliest at top). Each row is one

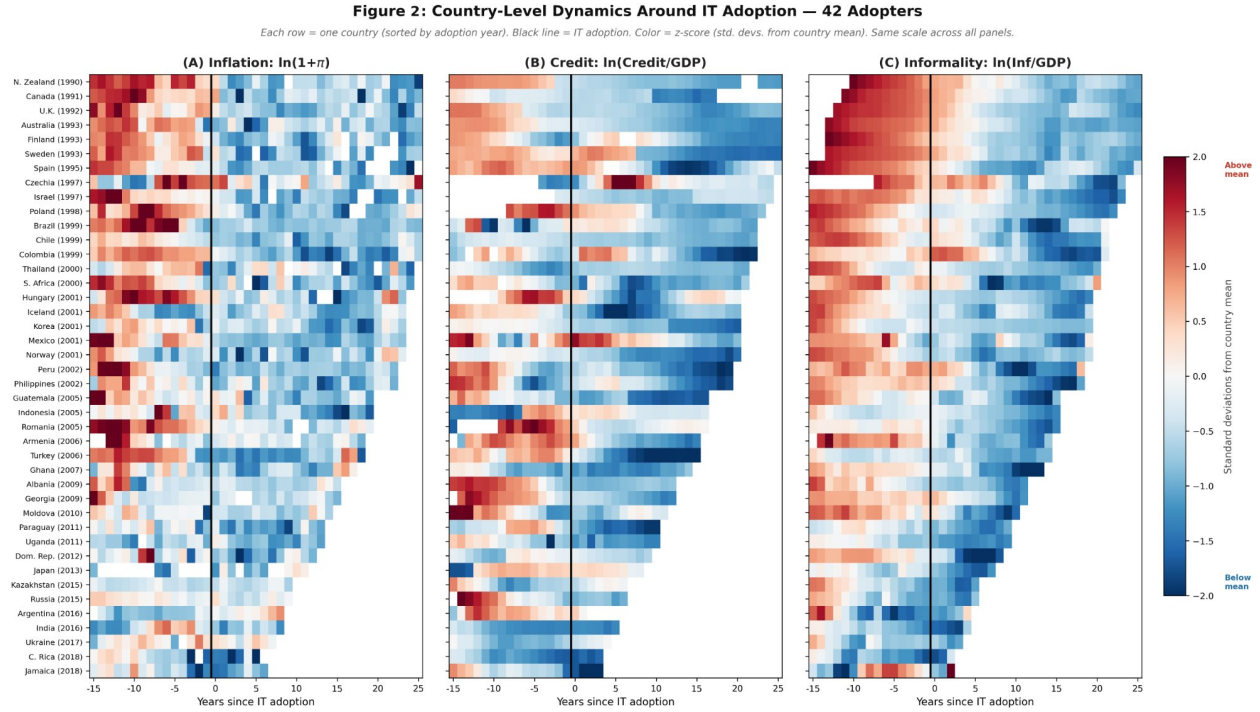


Figure 2: **Country-level dynamics around IT adoption.** Each row is one of 42 IT-adopting countries, sorted by adoption year. The black vertical line marks the year of IT adoption. Color intensity represents the z-score (standard deviations from the country's own time-series mean), with a shared scale across all three panels. Red: above mean. Blue: below mean.

country, each column is one event-time year ($e = t - t_i^{IT}$, where t_i^{IT} is the adoption year), and the color indicates the z-score: the number of standard deviations by which a country's variable deviates from its own time-series mean. A shared color scale (± 2 standard deviations) is used across all three panels, so visual intensity is directly comparable. The black vertical line marks the year of adoption.

Three patterns are immediately visible. First, the warm-to-cool transition in Panel (A) is remarkably consistent: nearly every country shows above-mean inflation (red) to the left of the adoption line and below-mean inflation (blue) to the right. The effect is sharpest for the earliest adopters, New Zealand, Canada, the United Kingdom, Australia, Finland, Sweden, who transitioned from a period of genuinely high inflation to one of credible low inflation. Later adopters (Paraguay, Uganda, Dominican Republic, Kazakhstan, Russia) show a more gradual transition, partly because many of them had already begun disinflating before formally adopting the framework.

Second, Panel (B) reveals substantial heterogeneity in credit responses. Some countries (South Africa, Hungary, Korea, Mexico) exhibit dramatic credit booms, visible as deep blue shifting to deep red, shortly after IT adoption, while others (Peru, Philippines, Guatemala) show more muted responses. This heterogeneity is consistent with the view that financial deepening depends on complementary reforms (banking regulation, capital account liberalization) beyond inflation stabilization alone. It also raises the question of how much of the observed credit expansion is specific to IT adoption versus part of a broader global trend in financial deepening, a question that the formal estimation below addresses directly.

Third, and most important for this paper, Panel (C) shows that the informality decline aligns temporally with the inflation break on a country-by-country basis. The red-to-blue transition is visible for the overwhelming

majority of adopters, and it coincides with the adoption line rather than with some delayed structural adjustment. Early adopters show the clearest effect, while later adopters (post-2010) have limited post-treatment data and correspondingly noisier trajectories.

These patterns are visually compelling, but they leave three questions open. First, how much of the post-adoption change in each variable is specific to IT adoption, and how much reflects global secular trends (worldwide disinflation, financial deepening, structural formalization) that affected all countries during this period? Second, are the apparent pre-treatment differences (the mild secular decline in informality visible in Panel C before adoption) large enough to invalidate a causal interpretation? Third, given the heterogeneity across adopters visible in Panels (B) and (C), is the average effect statistically significant and robust? To answer these questions, I turn to a formal staggered difference-in-differences estimator.

2.5 Staggered Difference-in-Differences

The 42 IT adopters enter treatment at different times, spanning nearly three decades from New Zealand (1990) to Jamaica (2018). A standard two-way fixed effects (TWFE) regression is problematic in this staggered setting because it uses already-treated countries as controls for later adopters, producing biased estimates when treatment effects are heterogeneous across cohorts (Goodman-Bacon, 2021; de Chaisemartin and d'Haultfœuille, 2020). Appendix A reviews why TWFE fails here and situates this paper within the IT evaluation literature.

To avoid these pitfalls, I implement the staggered difference-in-differences estimator of Callaway and Sant'Anna (2021). The estimator proceeds in three steps: (i) define adoption cohorts, (ii) compute one clean 2×2 DiD for each cohort at each calendar year, using only never-treated countries as the comparison group, and (iii) aggregate these cohort-specific estimates into an event-study plot.

Countries are grouped by their year of IT adoption, denoted g. For example, cohort $g = 1993$ consists of Australia, Finland, and Sweden (all of which adopted IT in 1993); cohort $g = 1999$ consists of Brazil, Chile, and Colombia; cohort $g = 2001$ consists of Hungary, Iceland, Korea, Mexico, and Norway. To ensure that each cohort-level $ATT(g,t)$ averages over at least two independent country experiences, so that no single country's idiosyncratic trajectory drives the estimate, I restrict attention to cohorts containing at least two countries. This requirement retains 13 of the 23 cohorts, covering 32 of the 42 adopters.[12] The 116 countries that never adopted inflation targeting serve as the permanent comparison group, denoted $g = \infty$.

For each cohort g and each calendar year t, I estimate the average treatment effect on the treated using a simple 2×2 difference-in-differences:

$$ATT(g,t) = \underbrace{\left[\bar{Y}_t^g - \bar{Y}_{g-1}^g\right]}_{\text{change for cohort } g} - \underbrace{\left[\bar{Y}_t^\infty - \bar{Y}_{g-1}^\infty\right]}_{\text{change for never-treated}} \tag{4}$$

where $\bar{Y}_t^g$ is the average outcome (e.g., ln(Informality/GDP)) across all countries in cohort g at calendar year t, and $\bar{Y}_t^\infty$ is the corresponding average across the 116 never-treated countries. The base period is always $g-1$: the year immediately before adoption for cohort g.

Two features of this construction are critical. First, the comparison group is always the 116 never-treated countries, never another treated cohort. This eliminates the forbidden comparisons that contaminate TWFE.

[12] The 10 excluded singleton cohorts are New Zealand ($g = 1990$), Canada (1991), United Kingdom (1992), Spain (1995), Poland (1998), Ghana (2007), Moldova (2010), Dominican Republic (2012), Japan (2013), and Ukraine (2017). These countries still appear in the spaghetti plots (top row of Figure 3) and the heatmap (Figure 2), but do not contribute to the formal event-study estimates. Relaxing the threshold to ≥ 1 country increases precision but produces a noisier pre-trend test; requiring ≥ 3 retains only 4 cohorts and yields too little statistical power to detect post-treatment effects at medium horizons.

Australia (treated in 1993) never appears as a control for Brazil (treated in 1999); instead, both are compared against the same set of never-treated countries, each relative to its own base period. Second, the base period is cohort-specific: for cohort $g = 1999$, the base year is 1998; for cohort $g = 1993$, it is 1992. This means that each $ATT(g, t)$ measures the change in the outcome relative to the year just before that cohort's adoption, differenced against the same change for the comparison group over the same calendar-year window.

Appendix B illustrates this calculation with a concrete numerical example for cohort $g = 1999$ (Brazil, Chile, Colombia), tracing both a post-treatment and a pre-treatment $ATT(g, t)$ through the data.

The preceding construction produces a matrix of $ATT(g, t)$ estimates, one for each cohort-year pair, but different cohorts reach the same event time e at different calendar years. To summarize the treatment effect dynamics, I aggregate across cohorts at each event time:

$$\hat{\theta}_{ES}(e) = \sum_{g} w_g \cdot ATT(g,\, g+e), \qquad w_g = \frac{n_g}{\sum_{g'} n_{g'}} \tag{5}$$

where n_g is the number of countries in cohort g and the sum runs over all cohorts that have data at event time e. The weights are proportional to cohort size, so larger cohorts receive more influence. I construct confidence intervals via a cluster bootstrap at the country level (200 replications); details are in Appendix B.

The pre-treatment coefficients $\hat{\theta}_{ES}(e)$ for $e < 0$ serve as a formal test of the parallel trends assumption: under the null that treated and control countries would have evolved similarly absent the treatment, these should be jointly zero. I report both the individual coefficients with confidence intervals and a joint Wald test statistic.

The top row of Figure 3 provides additional visual context through country-demeaned spaghetti plots. For each of the 42 adopters, I subtract the country's own time-series mean, then plot the resulting trajectory in event time. Bold lines show the cross-country average (computed only where at least 8 countries contribute data). The familiar pattern from the heatmap (Figure 2) is apparent in averaged form: inflation declines sharply at adoption; credit rises steadily; informality drifts downward. However, these raw trajectories confound IT-specific effects with global secular trends (worldwide disinflation, financial deepening, structural formalization) and with the staggered treatment timing problems discussed above.

The bottom row of Figure 3 reports the staggered DiD estimates $\hat{\theta}_{ES}(e)$ constructed from the cohort-level $ATT(g, t)$ and aggregated via equation (5), with confidence intervals. Hollow markers denote pre-treatment coefficients ($e < 0$); filled markers denote post-treatment ($e \geq 0$). The coefficient at $e = -1$ is mechanically zero by construction since each $ATT(g, t)$ uses $g - 1$ as its reference year.

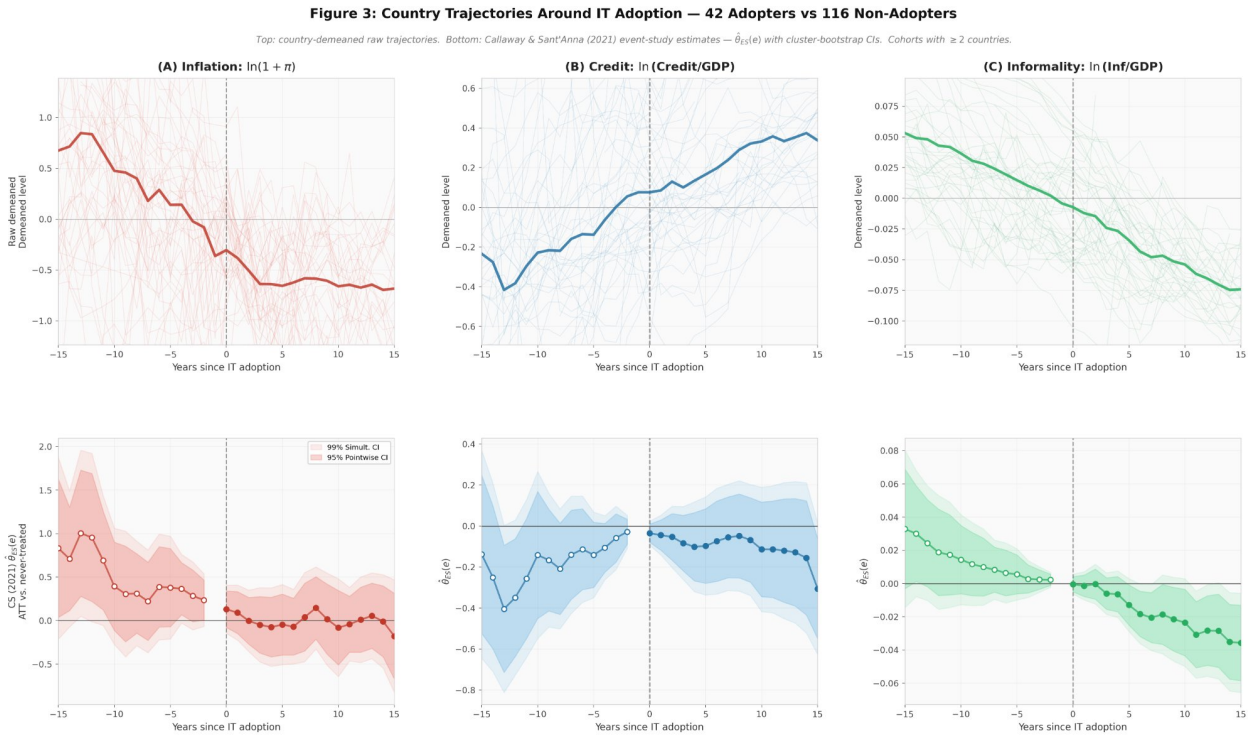


Figure 3: **Event study around inflation targeting adoption, 42 adopters vs. 116 non-adopters.** Top row: country-demeaned raw trajectories (thin lines: individual countries; bold: average where $N \geq 8$). Bottom row: Callaway and Sant'Anna (2021) event-study estimates $\hat{\theta}_{ES}(e)$, aggregated across 13 adoption cohorts (those with ≥ 2 countries) with size-proportional weights (equation 5). Shaded bands: 95% pointwise (dark) and 99% simultaneous (light) cluster-bootstrap confidence intervals. Hollow circles: pre-treatment; filled circles: post-treatment.

Three findings emerge from the bottom row. From Panel A (Inflation), the pre-treatment coefficients are large and significantly positive, indicating that IT-adopting countries experienced systematically higher (and declining) inflation relative to never-treated countries well before formal adoption. This is expected and *not* a threat to identification: IT adoption is endogenous to inflation by design: countries adopted *because* they sought to reduce inflation. The relevant identifying assumption is not that inflation evolved in parallel, but that *informality* did so (see Panel C). Post-adoption, point estimates are negative but imprecisely estimated, reflecting heterogeneity across cohorts.

From Panel B (Credit), the pre-treatment coefficients reveal that IT-adopting countries had systematically shallower credit markets than non-adopters: at $e = -15$, the gap is roughly -0.4 log points, narrowing gradually to zero by the base year ($e = -1$). Post-adoption, this convergence holds: the coefficients hover near zero through $e = +10$, indicating that IT adopters' credit growth broadly kept pace with the global trend for about a decade, before the relative position deteriorates slightly at longer horizons. The visual impression is one of IT adopters progressively *closing a credit gap* with non-adopters, a process that begins well before formal adoption (consistent with the gradual disinflation that preceded and motivated the policy reform) and stabilizes afterward.

Two features of this pattern deserve emphasis. First, it stands in stark contrast to the raw spaghetti plot (top row), which shows a credit boom among IT adopters. The discrepancy reflects a global credit expansion that was even stronger among the 116 never-treated countries, many of which include rapidly

financializing economies (China, Vietnam, many African and Middle Eastern economies). The Callaway–Sant'Anna estimator nets out this global trend because each $ATT(g,t)$ subtracts the change in credit among never-treated countries over the same calendar-year window. Second, the fact that the credit gap was already narrowing before formal IT adoption means we cannot attribute the convergence cleanly to the policy announcement itself. This is consistent with the panel regressions (Table 1): within-country disinflation significantly predicts credit deepening (-0.126, $p < 0.05$), but the process unfolds gradually over decades, and IT adoption marks one point along a longer trajectory of monetary stabilization rather than a sharp structural break in credit markets.

In Panel C (informality) we can see the critical outcome. The event-study coefficients trace a roughly symmetric pattern around the base year: pre-treatment estimates are positive, indicating that IT adopters had higher informality relative to their base-year level in the years before adoption; post-treatment estimates are negative and growing, indicating that informality fell below the base-year level after adoption. Reading the figure from left to right, IT-adopting countries transitioned from informality levels above their pre-adoption norm to levels increasingly below it, with the crossover occurring at the time of adoption. The post-treatment decline becomes statistically significant by $e = +5$ ($\hat{\theta}_{ES}(5) = -0.013$, $p < 0.05$) and grows to approximately -0.03 to -0.04 log points by $e = +10$ through $+15$. The identifying assumption requires that informality evolved in parallel between adopters and non-adopters in the years immediately preceding treatment, not over the entire sample. A joint Wald test over $e = -5$ to -2 yields $p = 0.450$, and we cannot reject parallel trends at any conventional significance level.[13]

Viewed alongside Panel B, the joint pattern across the three outcomes tells a coherent story. IT adopters began with higher inflation, shallower credit markets, and higher informality. As they disinflated, their credit gap with non-adopters narrowed, and informality declined. The timing is consistent with the theoretical mechanism: inflation falls first, credit converges next, and informality responds last, each link in the chain operating with the lag that capital accumulation and institutional adjustment require.

The gradual onset of the informality effect is important for the interpretation. An instantaneous jump at $e = 0$ would be consistent with a direct behavioral response, for example, workers switching sectors immediately upon hearing of a new monetary policy framework. The observed pattern, where informality is unchanged at adoption but declines significantly five years later, instead points toward a mechanism that operates through intermediate, slow-moving channels: lower inflation → lower interest rates → cheaper capital for formal firms → marginal firms formalize → informality declines. This lag structure matches the theoretical model's capital accumulation channel, where the formal sector's higher capital intensity means that the cost-of-capital reduction from disinflation only gradually translates into higher formal output and employment.

The heterogeneity across adoption cohorts visible in Figure 2 is not a problem for the Callaway–Sant'Anna framework; it is a feature. By constructing cohort-specific $ATT(g,t)$ estimates rather than pooling all variation into a single TWFE coefficient, the estimator accommodates treatment effect heterogeneity. The aggregate $\hat{\theta}_{ES}(e)$ reported in Figure 3 are properly weighted averages of these heterogeneous cohort-level effects.

2.6 Discussion and Preview of the Model

The empirical results provide three key inputs for the theoretical model. First, the staggered DiD event study provides evidence that inflation increases informality: informality declines by 3–4 log points within 10–15

[13]The longer horizon ($e = -10$ to -2) does reject parallel trends, consistent with a broader secular decline in informality among IT-adopting countries. Marcus and Sant'Anna (2021) argue that the parallel trends assumption is fundamentally local: it needs to hold over the horizons relevant for inference, not over the entire sample period. The local validation at $e = -5$ to -2 lends credibility to the post-treatment estimates.

years of IT adoption, with parallel pre-trends validated over the 5 years preceding treatment ($p = 0.450$). Second, the credit channel absorbs roughly 37% of the pooled inflation–informality relationship conditional on GDP per capita, with all three regression links individually significant (Table 1, Columns 1–3). Third, country fixed effects reveal an informative asymmetry: within-country inflation changes significantly predict credit depth (-0.126, $p < 0.05$) but not informality (Table 1, Columns 4–5), indicating that the first link in the causal chain (inflation $\rightarrow$ credit) adjusts faster than the second (credit $\rightarrow$ formalization).

These findings complement recent work by Jaar and Ritto (2025), who develop a heterogeneous-agent model with endogenous formalization and cash-in-advance constraints calibrated to Peru and find that informality exposes poor households disproportionately to the inflation tax through the cash-intensity of informal purchases. While their focus is on the distributional and fiscal consequences of informality (showing that switching from seigniorage to consumption taxes is welfare-improving), the present paper emphasizes the other direction: how inflation *generates* informality through the credit channel. The two mechanisms are complementary: inflation both arises from and contributes to informality, creating the possibility of a vicious cycle that the theoretical model in Section 3 characterizes.

The theoretical framework developed in Section 3 addresses this evidence through three channels. The *direct inflation tax channel*: unconstrained households that save in interest-bearing assets also carry cash to finance consumption and therefore bear the inflation tax on those cash holdings; because the seigniorage transfers they receive do not fully compensate for what they pay in inflation tax, higher inflation reduces their net real income and they accumulate fewer assets. The *financial intermediation channel*: because households save less in interest-bearing deposits, banks have fewer resources to lend, raising the cost of capital and disproportionately disadvantaging formal firms that rely on bank-intermediated financing due to their higher capital intensity. The *relative price channel*: the decline in formal output raises the relative price of formal goods, further shifting consumption toward the informal sector. That the credit channel accounts for roughly 37% of the pooled relationship conditional on income, that within-country inflation changes predict credit deepening even over 20-year horizons, but that informality itself barely moves within countries, suggests that the financial mechanism operates through the slow accumulation of capital rather than through year-to-year credit fluctuations, consistent with the model's steady-state comparative statics.

The model treats inflation as the primitive exogenous force, consistent with the IT-based identification strategy, and derives informality as an endogenous equilibrium outcome. An alternative view, developed by Nicolini (1998) and Koreshkova (2006), reverses the direction of the argument: governments that cannot effectively tax the informal sector turn to inflation as a way to levy seigniorage on cash-intensive economic activity. In their framework, informality causes inflation rather than the reverse. The IT-based identification strategy helps distinguish these mechanisms: inflation targeting is a monetary policy reform adopted to stabilize prices, not a response to informality. The parallel trends test (Figure 3, Panel C) confirms that informality was not on a differential trajectory before adoption, supporting the interpretation that inflation drives informality through the channels formalized in the model.

The quantitative exercise in Section 4 calibrates the model to match the total semi-elasticity of informality with respect to inflation estimated in Figure 1, and decomposes the model-implied response into the three channels, providing a structural counterpart to the reduced-form analysis of Table 1.

3 Simple Deterministic Model

In this section, I introduce a tractable theoretical framework to study when and why higher inflation leads to greater informality. The model uses a cyclical economy with two states that alternate over time, as in Kehoe and Levine (1993), extended with two consumption goods, formal and informal, where informal purchases require cash carried from the previous period. Formal firms face a working-capital requirement, as in Alberola and Urrutia (2020), that forces them to finance a fraction of the wage bill in advance at the gross nominal interest rate, making their effective labor cost sensitive to both the real interest rate and the inflation rate. Informal firms, by contrast, pay wages from current revenues and face no such constraint. This asymmetry, together with the standard assumption that formal production is more capital-intensive, means that the cost of producing formal goods rises relative to informal goods whenever the equilibrium interest rate increases, a mechanism that echoes the classic Stolper-Samuelson theorem from international trade[14]. The entire section is organized around a single question: under what conditions does inflation set off a chain of general equilibrium adjustments, from portfolio reallocation to higher interest rates to lower relative prices of informal goods, that ultimately expands the informal sector?

The analysis proceeds in three parts. The first establishes that incomplete-markets equilibria, in which borrowing constraints bind for the low-income household, can be implemented for any target interest rate below the complete-markets benchmark, provided endowments are sufficiently unequal. This existence result is important because the non-neutrality of inflation hinges entirely on market incompleteness: when borrowing constraints do not bind, households can perfectly offset the inflation tax by rebalancing their portfolios, and all real variables remain unchanged.

The second part isolates the redistribution channel, which is the mechanism through which inflation affects household savings even before any prices adjust. It operates through two distinct transfer streams. First, the CIA seigniorage channel: under Svensson's timing, the cash needed to purchase informal goods tomorrow must be set aside today, before inflation erodes its purchasing power. Under equal seigniorage rebates, this cost is not fully offset for the high-income household, which responds by cutting its savings in interest-bearing assets. Second, the working-capital rebate channel: the financial intermediary earns a profit on intra-period working-capital loans to formal firms and rebates this income to households. Under equal sharing, the H-household receives less than its income-proportional share of this rebate, reinforcing the savings compression from the CIA channel. Since the low-income household is borrowing-constrained and does not save in bonds regardless, both channels compound to produce a net decline in aggregate capital supply.

The third and central part of the section proves the main result. The total effect of inflation on informality decomposes into a direct headwind, as inflation taxes cash and discourages informal consumption, and a general equilibrium tailwind, as inflation tightens the capital market, raises the interest rate, and through the Stolper-Samuelson mechanism reduces the relative price of informal goods, encouraging their consumption. The net effect is governed by a single sufficient statistic: the capital gap between formal and informal sectors. When this gap is large enough, the general equilibrium tailwind dominates the direct headwind and inflation increases informality. The main theorem identifies the exact threshold above which this occurs, and shows that it must be reconciled with a competing constraint: the capital gap cannot be so large that the economy's savings capacity is exhausted and the equilibrium ceases to exist. The final part of the section establishes

[14] In the original formulation, the Stolper-Samuelson theorem establishes that an increase in the price of a factor of production raises the price of the good that uses that factor intensively. Here, the analogue is that when the cost of capital goes up, the formal sector, which relies more heavily on capital, sees its production costs rise by more than the informal sector, driving down the relative price of informal goods.

that these two constraints are jointly compatible. For empirically relevant elasticities of substitution between formal and informal goods, there exists a range of formal capital shares at which an interior equilibrium exists and inflation increases informality.

3.1 Households

Environment. Time is discrete. Two equally sized groups of households alternate deterministically between a high-income state H and a low-income state L, as in the cyclical economy of Kehoe and Levine (1993). At every date, there is a mass one of households in each state, for a total population of two. A household in state $s \in \{H, L\}$ supplies e^s efficiency units of labor inelastically, with $e^H > e^L \geq 0$, and earns real labor income we^s, where w denotes the real wage measured in units of the formal good.

Two consumption goods are available: a formal good c^f and an informal good c^i. Households derive utility from a CES composite of the two,

$$C(c^f, c^i) = \left[\phi^{1/\eta}(c^f)^{\frac{\eta-1}{\eta}} + (1-\phi)^{1/\eta}(c^i)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, \tag{6}$$

where $\eta > 0$ is the elasticity of substitution between formal and informal goods and $\phi \in (0,1)$ is the weight on formal consumption. Period utility is CRRA over this composite, $u(C) = C^{1-\sigma}/(1-\sigma)$ with $\sigma > 0$, and households discount the future at rate $\beta \in (0,1)$.

Assets and the cash-in-advance constraint. Households can save in two instruments: real bonds a denominated in units of the formal good and earning gross real return R, and nominal money balances M. Both are subject to a no-borrowing constraint, $a' \geq 0$ and $M' \geq 0$. Informal-good purchases must be paid with cash carried from the previous period. This is the cash-in-advance formulation introduced by Svensson (1985), in which money must be accumulated one period before it can be spent. The key implication of this timing convention is that inflation erodes the purchasing power of money between the period it is saved and the period it is used: a dollar set aside today buys fewer informal goods tomorrow if prices have risen in the meantime.

Nominal budget constraint. In period t, a household in state s faces

$$P_t^f\, c^f + P_t^i\, c^i + P_t^f\, a' + M' = R\, P_t^f\, a + W_t\, e^s + \bar{T}_t^s + M, \tag{7}$$

where P_t^f and P_t^i are the nominal prices of the formal and informal goods, W_t is the nominal wage, $\bar{T}_t^s$ is a nominal lump-sum transfer, and M is the nominal money balance brought into the period. The cash-in-advance constraint requires that spending on informal goods does not exceed the cash available:

$$P_t^i\, c^i \leq M. \tag{8}$$

Real formulation. Nominal money supply grows at a constant gross rate $1+\mu$. In a stationary equilibrium, all nominal prices grow at rate μ, so the relative price $p \equiv P_t^i/P_t^f$, the real wage $w \equiv W_t/P_t^f$, and the gross real return R are all constant over time. Define real balances as $m' \equiv M'/P_t^f$ for money saved this period and $m \equiv M/P_{t-1}^f$ for money inherited from last period. The purchasing power of inherited money is $M/P_t^f = m/(1+\mu)$: the denominator captures the erosion of money's value between the period it was saved and the period it is spent.

The budget constraint in real terms becomes

$$c^f + p\,c^i + a' + m' = R\,a + w\,e^s + T^s + \frac{m}{1+\mu}, \tag{9}$$

where $T^s \equiv \bar{T}^s_t / P^f_t$ is the real transfer. The cash-in-advance constraint becomes

$$p\,c^i \leq \frac{m}{1+\mu}. \tag{10}$$

This formulation makes clear that the household faces a real cost of holding money: each unit of real balances m set aside last period delivers only $m/(1+\mu)$ units of purchasing power today. The cost is proportional to the inflation rate μ and falls entirely on informal consumption, since formal goods can be purchased with current income.

Value functions. Let $T^s \equiv T^s_\Omega + T^s_{WC}$ denote the total lump-sum transfer received by household s, combining the government's CIA seigniorage rebate and the intermediary's working-capital profit rebate. The decomposition into T^s_Ω and T^s_{WC} is developed in Section 3.3. In a stationary equilibrium, the household's problem is recursive in the state variables (a, m) and the current income state s. As in Algan and Ragot (2010), the alternating structure reduces the infinite-horizon problem to a pair of coupled Bellman equations. The high-income household solves

$$V^H(a,m) = \max_{c^f, c^i, a', m'} \frac{[C(c^f, c^i)]^{1-\sigma}}{1-\sigma} + \beta\, V^L(a', m') \tag{11}$$

subject to:

$$c^f + p\,c^i + a' + m' = R\,a + w\,e^H + T^H + \frac{m}{1+\mu}, \tag{12}$$

$$p\,c^i \leq \frac{m}{1+\mu}, \tag{13}$$

$$a' \geq 0, \quad m' \geq 0. \tag{14}$$

The low-income household solves the symmetric problem

$$V^L(a,m) = \max_{c^f, c^i, a', m'} \frac{[C(c^f, c^i)]^{1-\sigma}}{1-\sigma} + \beta\, V^H(a', m') \tag{15}$$

subject to:

$$c^f + p\,c^i + a' + m' = R\,a + w\,e^L + T^L + \frac{m}{1+\mu}, \tag{16}$$

$$p\,c^i \leq \frac{m}{1+\mu}, \tag{17}$$

$$a' \geq 0, \quad m' \geq 0. \tag{18}$$

The continuation value in each problem is the value function of the opposite state, reflecting the deterministic alternation between H and L.

Two features of this problem are worth noting. First, the general equilibrium non-neutrality of inflation hinges on market incompleteness: if the no-borrowing constraint never binds, the household can rebalance

its portfolio to keep the real interest rate pinned at $\bar{r}$, shutting down the intertemporal channel through which inflation raises r^* and triggers the Stolper–Samuelson reallocation. Intratemporal distortions from the CIA constraint and the working-capital wedge persist even under complete markets, but the paper's central mechanism from savings compression to interest rate increases to informality growth requires that borrowing constraints bind. Second, the cash-in-advance constraint will bind in equilibrium whenever money is dominated in rate of return, which occurs whenever $R(1+\mu) > 1$. Since this condition is satisfied for any positive inflation rate when the real interest rate is positive, the constraint is binding in all cases of economic interest.

3.2 Firms

Technology. There are two competitive sectors, formal (f) and informal (i), each producing with a Cobb–Douglas technology:

$$y_s = A_s \, K_s^{\alpha_s} \, L_s^{1-\alpha_s}, \qquad s \in \{f, i\}, \tag{19}$$

where $A_s > 0$ is total factor productivity, K_s is capital, and L_s is labor. The formal sector is more capital-intensive, $\alpha_f > \alpha_i$, and more productive, $A_f > A_i$. The rental rate of capital r and the household's gross real return are linked by the no-arbitrage condition $R = 1 + r - \delta$, where $\delta \in (0, 1)$ is the depreciation rate.

The asymmetry $\alpha_f > \alpha_i$ is the source of the Stolper–Samuelson mechanism discussed in the introduction. In international trade, the theorem establishes that an increase in a factor price raises the cost of the good that uses that factor intensively. Here the logic is the same: when the cost of capital rises, the formal sector, which uses capital more intensively, sees its unit cost increase by more than the informal sector, driving down the relative price of the informal good.

Working-capital constraint. Formal firms face a production timing friction, as in Alberola and Urrutia (2020): a fraction $\nu \in [0, 1]$ of the wage bill must be paid to workers before production revenue is realized. To finance this advance payment, the firm borrows from financial intermediaries at the gross nominal interest rate $R^n = (1 + \mu) \, R$. The remaining fraction $(1 - \nu)$ of wages is paid out of current revenues. Informal firms face no such constraint and pay all wages from current output.[15]

This asymmetry creates a wedge between the effective labor cost in the two sectors. Define the **working-capital wedge**

$$\Delta_w(r, \mu) \equiv 1 + \nu\big[(1 + \mu) \, R - 1\big]. \tag{20}$$

The formal firm's effective cost per unit of labor is $w \cdot \Delta_w$, while the informal firm pays simply w. When $\nu > 0$, the formal sector's costs depend on the nominal interest rate and hence on inflation. When $\nu = 0$, the wedge disappears ($\Delta_w = 1$) and both sectors face identical labor costs.

Zero-profit conditions and equilibrium prices. Under perfect competition, the price of each good equals its marginal cost. With the formal good as numéraire, the zero-profit condition in the formal sector pins the

[15] The financial intermediary plays a dual role. On the asset side, it receives household savings and allocates them to physical capital ($K_f + K_i$) and intra-period working-capital loans to formal firms ($\nu w L_f$). On the income side, its profit per unit of principal is the spread between the gross nominal lending rate $(1+\mu)R$ and the real rate R it owes depositors: profit $= \mu R \cdot \nu w L_f$. The real interest $(R-1)\nu w L_f$ already flows to households through the standard savings return Ra'^H and is not an additional income stream. The inflation premium $\mu R \cdot \nu w L_f$ is, and it is rebated to households via the sharing rule governed by $\omega_{WC} \in [0, 1]$, introduced in Section 3.3. Without this rebate, the inflation premium would leak out of household accounts, violating Walras's law at $\mu > 0$.

equilibrium wage. Define the **frictionless wage**:

$$\widetilde{w}(r) \equiv (1-\alpha_f)\, A_f^{\frac{1}{1-\alpha_f}} \left(\frac{\alpha_f}{r}\right)^{\frac{\alpha_f}{1-\alpha_f}}. \tag{21}$$

When $\Delta_w > 1$, the zero-profit condition requires $w \cdot \Delta_w = \widetilde{w}(r)$, so the equilibrium market wage is

$$w(r,\mu) = \frac{\widetilde{w}(r)}{\Delta_w(r,\mu)}. \tag{22}$$

Higher inflation raises Δ_w, which reduces the market wage for a given r: the formal sector's zero-profit condition forces the financing cost of the working-capital requirement to be fully absorbed by a lower wage. This is the cost channel of monetary transmission.

The zero-profit condition in the informal sector then determines the relative price of the informal good:

$$p(r,\mu) = \frac{1}{A_i}\left(\frac{r}{\alpha_i}\right)^{\alpha_i}\left(\frac{w(r,\mu)}{1-\alpha_i}\right)^{1-\alpha_i}. \tag{23}$$

Because the informal sector pays the market wage w without the financing premium, its costs fall when inflation compresses w, while formal-sector effective costs $w \cdot \Delta_w = \widetilde{w}(r)$ are unchanged. The relative price of informal goods therefore declines with inflation at any given interest rate. Whether this direct price compression is dominated by the general equilibrium response of the interest rate is the central question resolved in Section 3.12.

Lemma 1 (Stolper–Samuelson price response)**.** *Fix $\mu \geq 0$ and suppose $\alpha_f > \alpha_i$. Then the relative price of the informal good is strictly decreasing in the rental rate:*

$$\left.\frac{\partial p}{\partial r}\right|_{\mu} < 0. \tag{24}$$

The elasticity decomposes as

$$\frac{\partial \ln p}{\partial \ln r} = \alpha_i - \frac{\alpha_f(1-\alpha_i)}{1-\alpha_f} = -\frac{\alpha_f - \alpha_i}{1-\alpha_f} < 0.$$

Proof. From the zero-profit conditions, $w = \widetilde{w}(r)/\Delta_w$ and

$$p = \frac{1}{A_i}\left(\frac{r}{\alpha_i}\right)^{\alpha_i}\left(\frac{w}{1-\alpha_i}\right)^{1-\alpha_i}.$$

Taking logs and differentiating with respect to $\ln r$ at fixed μ:

$$\frac{\partial \ln p}{\partial \ln r} = \alpha_i + (1-\alpha_i)\,\frac{\partial \ln w}{\partial \ln r}.$$

From $w = \widetilde{w}(r)/\Delta_w$, we have $\partial \ln w/\partial \ln r = \partial \ln \widetilde{w}/\partial \ln r + \partial \ln(1/\Delta_w)/\partial \ln r$. The frictionless wage satisfies $\partial \ln \widetilde{w}/\partial \ln r = -\alpha_f/(1-\alpha_f)$. The working-capital term $\partial \ln(1/\Delta_w)/\partial \ln r$ is non-positive. Substituting only

the dominant frictionless component:

$$\frac{\partial \ln p}{\partial \ln r} = \alpha_i + (1-\alpha_i)\left(-\frac{\alpha_f}{1-\alpha_f}\right) + \underbrace{(1-\alpha_i)\frac{\partial \ln(1/\Delta_w)}{\partial \ln r}}_{\leq 0}$$
$$\leq \alpha_i - \frac{\alpha_f(1-\alpha_i)}{1-\alpha_f} = -\frac{\alpha_f - \alpha_i}{1-\alpha_f} < 0,$$

where the final inequality uses $\alpha_f > \alpha_i$. The working-capital channel reinforces the result: it makes w fall faster in r, further reducing p. □

This lemma formalises the Stolper–Samuelson logic at the heart of the model: because formal production is more capital-intensive ($\alpha_f > \alpha_i$), an increase in the cost of capital raises formal unit costs by more than informal unit costs, driving down the relative price of the informal good. The magnitude of the response is governed by the **capital gap** $(\alpha_f - \alpha_i)/(1-\alpha_f)$, which is the single sufficient statistic that reappears throughout the analysis.

Factor demands. From the first-order conditions of the cost minimization problems, factor demands per unit of output in each sector are:

$$\frac{K_f}{y_f} = \frac{\alpha_f}{r}, \qquad \frac{L_f}{y_f} = \frac{1-\alpha_f}{w\,\Delta_w}, \tag{25}$$

for the formal sector, and

$$\frac{K_i}{y_i} = \frac{\alpha_i\, p}{r}, \qquad \frac{L_i}{y_i} = \frac{(1-\alpha_i)\, p}{w}, \tag{26}$$

for the informal sector. At fixed r, the zero-profit condition pins $w \cdot \Delta_w = \widetilde{w}(r)$, so formal labor demand per unit of output is independent of μ. The working-capital wedge does not directly alter per-unit factor demands; it is fully absorbed by the market wage. The asymmetry matters only in general equilibrium, where the response of r to inflation shifts factor demands across sectors.

3.3 Government and Intermediary

The government controls the money supply, which grows at gross rate $1+\mu$ each period. The new money created constitutes seigniorage revenue, rebated lump-sum to households. In real terms:

$$\Omega = \frac{\mu}{1+\mu}\,(m^H + m^L). \tag{27}$$

A fraction $\omega_T \in (0,1)$ goes to the household in state H, the remainder to L:

$$T_\Omega^H = \omega_T\,\Omega, \qquad T_\Omega^L = (1-\omega_T)\,\Omega. \tag{28}$$

When $\omega_T = 1/2$, seigniorage transfers are equal across households.

There is also an implicit financial intermediary that channels household savings to firms. Beyond holding physical capital and issuing working-capital loans to formal firms (as detailed in Section 3.4), this intermediary earns a profit on those loans equal to the inflation premium $\mu R \cdot \nu w L_f$ and rebates it to households:

$$T_{WC}^H = \omega_{WC} \cdot \mu R \cdot \nu w L_f, \qquad T_{WC}^L = (1-\omega_{WC}) \cdot \mu R \cdot \nu w L_f, \tag{29}$$

where $\omega_{WC} \in [0,1]$ governs the split. At $\nu = 0$ or $\mu = 0$ this stream vanishes. The total transfer received

by each household is $T^s \equiv T^s_\Omega + T^s_{WC}$. The two parameters ω_T and ω_{WC} are independent and together fully characterize the redistribution side of the model.

3.4 Market Clearing

Asset market. Financial intermediaries allocate household savings to two uses: physical capital rented to firms at rate r, and intra-period working-capital loans to formal firms. Even though working-capital loans are repaid within the period, the intermediary must hold the resources at the start of production, so they cannot simultaneously be invested in physical capital. The intermediary's profit on these loans, $\mu R \cdot \nu w L_f$, is rebated to households as described in Section 3.3. The asset market clearing condition is

$$a'^H + a'^L = K_f + K_i + \nu\, w\, L_f. \tag{30}$$

Goods markets. The informal good is non-storable and used only for consumption, so informal output equals total informal consumption:

$$y_i = c^{H,i} + c^{L,i}. \tag{31}$$

The formal good serves both as a consumption good and as the economy's capital good, so formal output satisfies

$$y_f = c^{H,f} + c^{L,f} + \delta\,(K_f + K_i). \tag{32}$$

Labor market. Total labor demand from both sectors equals total labor supply $e^H + e^L$.

3.5 Stationary Equilibrium

Definition 1 (Stationary Equilibrium)**.** *A stationary equilibrium for a given money growth rate μ consists of a rental rate r, a wage w, a relative price p, household allocations $\{c^{s,f}, c^{s,i}, a'^s, m'^s\}_{s\in\{H,L\}}$, firm input demands $\{K_s, L_s\}_{s\in\{f,i\}}$, and total transfers $\{T^H, T^L\}$ (with $T^s \equiv T^s_\Omega + T^s_{WC}$) such that:*

(i) Given prices (r, w, p) and transfers (T^H, T^L), the household allocations solve the recursive problems (11)–(15).

(ii) Given (r, w) and the inflation rate μ, formal firms minimize costs subject to the working-capital constraint, and informal firms minimize costs, with both sectors earning zero profits.

(iii) The government budget is balanced ($T^H_\Omega + T^L_\Omega = \Omega$) and the intermediary rebates its entire profit ($T^H_{WC} + T^L_{WC} = \mu R \cdot \nu w L_f$).

(iv) The asset market, the formal goods market, and the informal goods market all clear.

3.6 Solving the Household Problem

Complete vs. incomplete markets. The distinction between complete and incomplete markets in this economy is governed by a single condition: whether $\beta R = 1$ or $\beta R < 1$. When $\beta R = 1$, the bond Euler equations across the two states imply $\lambda^H = \lambda^L$, so composite consumption $C^H = C^L$ and the household achieves perfect consumption smoothing. However, complete risk-sharing does *not* imply full inflation neutrality. Even when borrowing constraints are slack, the cash-in-advance constraint on informal goods

and the working-capital requirement on formal firms create wedges that depend on the nominal interest rate $i = R(1+\mu) - 1$. At fixed r, a change in μ alters i and therefore shifts the effective relative price of informal goods $p(1+\psi/\lambda) = p \cdot R(1+\mu)$, as well as the wage $w = \widetilde{w}(r)/\Delta_w(r,\mu)$. These intratemporal distortions change the composition of consumption between formal and informal goods and the allocation of factors across sectors, even though the intertemporal margin is undistorted.

What complete markets *do* shut down is the **intertemporal channel** that is central to this paper: the mechanism through which inflation compresses savings, raises the equilibrium real interest rate, and triggers the Stolper–Samuelson reallocation. When $\beta R = 1$, the household can perfectly rebalance its portfolio between bonds and money in response to any change in μ, so the bond Euler equation holds with equality in both states and the equilibrium r is pinned by $\beta R = 1$ regardless of inflation. The non-neutrality results of this paper therefore hinge entirely on *market incompleteness* in the sense that borrowing constraints must bind.

When $\beta R < 1$, perfect consumption smoothing is no longer feasible. For sufficiently large income dispersion $e^H - e^L$, the natural pattern is that the high-income household saves ($a'^H > 0$, constraint slack) while the low-income household is borrowing-constrained ($a'^L = 0$). The bond Euler equation in the H state then delivers $\lambda^H = \beta R \lambda^L < \lambda^L$: the marginal utility of wealth is lower when income is high, confirming that $C^H > C^L$.

Key equilibrium objects. The Appendix derives the full closed-form solution of the household problem under incomplete markets. Three objects play a central role throughout the analysis.

The **formal-to-informal consumption ratios** in each state, $\xi^s \equiv c^{s,f}/c^{s,i}$, emerge from the intratemporal first-order conditions:

$$\xi^H = \frac{\phi}{1-\phi}\left[\frac{(1+\mu)\,p}{\beta^2 R}\right]^{\eta}, \qquad \xi^L = \frac{\phi}{1-\phi}\left[R(1+\mu)\,p\right]^{\eta}. \tag{33}$$

Since the H-household faces a tighter cash-in-advance constraint in shadow-price terms, it consumes a higher ratio of formal to informal goods: $\xi^H > \xi^L$. Both ratios are increasing in μ: inflation raises the effective price of informal goods and tilts consumption toward formality.

The **informal consumption gap** $\xi^g \equiv c^{H,i}/c^{L,i}$ is pinned by the intertemporal Euler equation:

$$\xi^g = \left(\frac{\Gamma}{\beta R}\right)^{1/\sigma}, \tag{34}$$

where Γ is a composition adjustment factor depending on $(\xi^H, \xi^L, \phi, \eta, \sigma)$ (see Appendix). Under log–Cobb–Douglas preferences ($\sigma = \eta = 1$), this simplifies to $\xi^g = \beta R$.

Lemma 2. *Suppose $\beta R < 1$ and $\sigma\eta \geq 1$. Then $\xi^g < 1$ for all $\phi \in (0,1)$.*

Proof. As shown in the Appendix, Γ decomposes as $\Gamma = \tau_\phi^{1/\eta-\sigma} \cdot \beta^2 R^2$, where $\tau_\phi \equiv \Phi(\xi^H)/\Phi(\xi^L) > 1$. Substituting into (34):

$$\xi^g = \left(\tau_\phi^{1/\eta-\sigma} \cdot \beta R\right)^{1/\sigma}.$$

The condition $\sigma\eta \geq 1$ implies $1/\eta - \sigma \leq 0$, so $\tau_\phi^{1/\eta-\sigma} \leq 1$. Combined with $\beta R < 1$, both factors are strictly below one, giving $\xi^g \leq (\beta R)^{1/\sigma} < 1$. □

The property $\xi^g < 1$ is the key driver of the redistribution channel: because the H-household must save more cash today to finance the larger informal consumption it will need in L, it is disproportionately exposed to the inflation tax on the saving margin. This under-compensation extends to both transfer streams: equal CIA

sharing ($\omega_T = 1/2$) fails to offset the full CIA cost because $\omega_T^* = 1/(1+\xi^g) > 1/2$, and equal WC sharing ($\omega_{WC} = 1/2$) fails to neutralise the WC rebate channel because $\omega_{WC}^* = \rho/(\rho+1) > 1/2$ whenever $\rho > 1$. The condition $\sigma\eta \geq 1$ ensures that the substitution effect dominates the income effect, so the sign is invariant to ϕ.

3.7 Equilibrium Characterization

The closed-form solution of the household problem reveals a key structural property of the model: all equilibrium quantities are proportional to a single scale variable $\tilde{c} \equiv c^{L,i}$, the informal consumption of the low-income household. Informal output equals $y_i = (1+\xi^g)\,\tilde{c}$, and formal output satisfies $y_f = \Upsilon_f\,\tilde{c}$, where Υ_f depends on $(\xi^H, \xi^L, \xi^g, p, r, \delta)$ through goods market clearing. The zero-profit conditions pin $w(r,\mu)$ and $p(r,\mu)$, so every household-side quantity and every firm-side quantity can be written as functions of (r,μ) and structural parameters alone. The entire general equilibrium collapses to a single market clearing condition.

Because both asset supply $a'^H = we^H - D^H\tilde{c}$ and asset demand $\theta_f\, y_f + (\alpha_i p/r)\, y_i$ are proportional to $\tilde{c}$, the scale cancels, reducing the equilibrium to a single equation in r for each μ:

$$\mathcal{E}(r;\mu) \equiv S(\rho) - \mathcal{A}(r,\mu) = 0, \tag{35}$$

where $S(\rho)$ is asset supply per $\tilde{c}$ and $\mathcal{A}(r,\mu)$ is asset demand per $\tilde{c}$. All equilibrium quantities and structural parameters $(\alpha_f, \alpha_i, A_f, A_i, \nu, \delta, \beta, \phi, \eta, \sigma, e^H, e^L, \omega_T, \omega_{WC})$ are embedded in these two objects.

The equilibrium condition (35) has a property that is central to the paper's theoretical strategy: the asset demand side is independent of the endowment ratio $\rho \equiv e^H/e^L$, while the asset supply side depends on it monotonically. The asset demand per $\tilde{c}$,

$$\mathcal{A} \equiv \theta_f\,\Upsilon_f + \frac{\alpha_i\, p}{r}\,\Upsilon_i,$$

is a known constant for each (r,μ). The asset supply per $\tilde{c}$,

$$S(\rho) \equiv \frac{\rho\, D^L - D^H}{R\,\rho + 1},$$

is strictly increasing in ρ. The equilibrium condition $S(\rho) = \mathcal{A}$ therefore has a unique solution ρ^* whenever $\mathcal{A}$ lies in the range of S.

Theorem 1 (Implementation by endowments)**.** *Fix structural parameters and let r be any rental rate in the incomplete-markets region, $r \in (\delta\alpha_f, \bar{r})$, where $\bar{r} = (1-\beta)/\beta + \delta$. Suppose:*

(C1) *$D^H > 0$ and $D^L > 0$;*

(C2) *$\mathcal{A} < D^L/R$.*

Then there exists a unique endowment ratio $\rho^* > 0$ such that r is a stationary equilibrium with $e^H/e^L = \rho^*$:

$$\rho^* = \frac{D^H + \mathcal{A}}{D^L - \mathcal{A}\,R}. \tag{36}$$

The proof applies the intermediate value theorem to the monotone function $S(\rho)$ and is provided in the Appendix.

Economic interpretation. Given a target interest rate r below $\bar{r}$, the theorem constructs the endowment ratio ρ^* that rationalizes r as an equilibrium. The capacity condition (C2) imposes an upper bound $\bar{\alpha}_f$ on

the formal capital share: if the formal sector is too capital-intensive, there are simply not enough savings to finance it at the target interest rate. This upper bound defines the **existence region** $\alpha_f \in (0, \bar{\alpha}_f)$.

One can also read the theorem in reverse. Given endowments (e^H, e^L) with $\rho > 1$, the equation $\rho^*(\kappa) = \rho$, where $\kappa \equiv \bar{r} - r$ measures the displacement from complete markets, implicitly defines the equilibrium interest rate $r^* = \bar{r} - \kappa^*$. The comparative statics of the next section study how $r^*(\mu)$ responds to inflation, which is the key input to the main result.

3.8 The Response of the Interest Rate to Inflation

The equilibrium condition $\mathcal{E}(r; \mu) = 0$ implicitly defines $r^*(\mu)$. By the implicit function theorem,

$$\frac{dr^*}{d\mu} = -\frac{\partial \mathcal{E}/\partial \mu}{\partial \mathcal{E}/\partial r}. \tag{37}$$

The denominator is positive by the Walrasian stability condition, so the sign of $dr^*/d\mu$ is the opposite of the sign of $\partial \mathcal{E}/\partial \mu$: if inflation reduces excess asset supply at a given r, the interest rate must rise to restore equilibrium.

The partial derivative $\partial \mathcal{E}/\partial \mu|_r$ captures the direct effect of inflation on the asset market, holding the interest rate fixed. This derivative has a clean economic decomposition into channels that operate through the asset supply side (household savings) and the asset demand side (firm capital and working-capital requirements). The supply-side channel, which operates through the combined redistribution of CIA seigniorage and working-capital interest income, is governed jointly by the parameters ω_T and ω_{WC} and turns out to be the dominant force.

3.9 The Combined Redistribution Channel

To isolate the effect of inflation on savings, consider the partial-equilibrium exercise of raising μ while holding all prices (p, w, r) fixed, and hence all intratemporal ratios (ξ^H, ξ^L, ξ^g) fixed. The only objects that respond to μ are the net expenditure indices D^H and D^L. As the Appendix shows, substituting the binding constraints and both transfer streams into the budget constraints yields

$$D^H = \xi^H \xi^g + p(1+\mu) - \omega_T \, \mu \, p \, (1 + \xi^g) - \omega_{WC} \, \kappa^{WC} \, \mu, \tag{38}$$

$$D^L = \xi^L + p(1+\mu) \, \xi^g - (1 - \omega_T) \, \mu \, p \, (1 + \xi^g) - (1 - \omega_{WC}) \, \kappa^{WC} \, \mu, \tag{39}$$

where $\kappa^{WC} \equiv R\nu(1-\alpha_f)\Upsilon_f/\Delta_w|_{\mu=0}$ is the linearised working-capital profit coefficient; the WC rebate terms are exact to first order in μ because $\Delta_w(\mu)$ is nonlinear, but all comparative statics are evaluated at $\mu = 0$ where the approximation is exact. The reduced budget constraints read $D^H \tilde{c} + a'^H = we^H$ and $D^L \tilde{c} = Ra'^H + we^L$. Each index contains four terms: formal consumption expenditure per $\tilde{c}$; the cost of carrying money; the CIA seigniorage rebate; and the WC interest rebate. At $\mu = 0$ both expressions reduce to $D^H = \xi^H \xi^g + p$ and $D^L = \xi^L + p\xi^g$, leaving all calibration targets unaffected. Whether the combined net impact on D^H is positive or negative depends on both ω_T and ω_{WC}.

Differentiating at fixed prices:

$$\left.\frac{\partial D^H}{\partial \mu}\right|_{\text{prices}} = p(1 - \Sigma_T) - \omega_{WC} \, \kappa^{WC}, \qquad \left.\frac{\partial D^L}{\partial \mu}\right|_{\text{prices}} = p(\Sigma_T - 1) - (1 - \omega_{WC}) \, \kappa^{WC}, \tag{40}$$

where $\Sigma_T \equiv \omega_T(1+\xi^g)$. These derivatives sum to $-\kappa^{WC} \leq 0$: what remains after the transfers is a net resource cost equal to the intermediary's profit, confirming Walras's law. The sign of $\partial D^H/\partial\mu$ is governed by two independent thresholds.

The **CIA neutrality threshold** $\omega_T^* = 1/(1+\xi^g)$ is the value of ω_T at which the CIA transfer exactly offsets the inflation cost of money holdings in the H-state. The **WC neutrality threshold** $\omega_{WC}^* = \rho/(\rho+1) = e^H/(e^H+e^L)$ is the income-proportional split at which the WC rebate leaves the savings supply $S(\rho)$ unchanged under differentiation with respect to μ.

At the double neutrality $\omega_T = \omega_T^*$ and $\omega_{WC} = \omega_{WC}^*$, both derivatives vanish: inflation has no effect on allocations through either redistribution stream. The economy's response operates only through prices.

When (ω_T, ω_{WC}) departs from double neutrality, the redistribution channel activates. The direction depends on which regime the economy is in.

Regime A: $\omega_T < \omega_T^*$ and $\omega_{WC} < \omega_{WC}^*$ (both channels under-compensate). On the CIA margin, $p(1-\Sigma_T) > 0$, so inflation raises D^H more than it raises D^L (scaled by the endowment ratio), squeezing the savings numerator $\rho D^L - D^H$. On the WC margin, $\omega_{WC} < \omega_{WC}^* = \rho/(1+\rho)$ means the H-household receives a below-income-proportional share of the WC rebate: the rebate reduces D^H by $\omega_{WC}\kappa^{WC}\mu$ per unit of $\tilde{c}$, but it reduces ρD^L by $\rho(1-\omega_{WC})\kappa^{WC}\mu$, a larger amount. Consequently the WC rebate also reduces $\rho D^L - D^H$, reinforcing the CIA channel. Both channels compound, driving

$$\left.\frac{\partial a'^H}{\partial \mu}\right|_{\text{prices}} < 0. \tag{41}$$

The L-household is over-compensated on both margins ($\partial D^L/\partial\mu < 0$), but since it is borrowing-constrained, this windfall raises consumption without generating additional savings.

Regime B: over-compensation on at least one margin. If $\omega_T > \omega_T^*$ or $\omega_{WC} > \omega_{WC}^*$, the relevant channel pushes D^H downward, partially or fully offsetting the savings compression.

Regime C: exact neutrality ($\omega_T = \omega_T^*$ and $\omega_{WC} = \omega_{WC}^*$). Both derivatives vanish. Inflation affects the economy only through the cost channel and general equilibrium price adjustments.

3.10 Why the Standard Calibration Selects Regime A

The CIA threshold $\omega_T^* = 1/(1+\xi^g)$ satisfies $\omega_T^* > 1/2$ whenever $\xi^g < 1$, which holds under $\beta R < 1$ and $\sigma\eta \geq 1$ (Lemma 2). The WC threshold $\omega_{WC}^* = \rho/(\rho+1)$ satisfies $\omega_{WC}^* > 1/2$ whenever $\rho > 1$, which holds by assumption ($e^H > e^L$). Under the natural benchmark $\omega_T = \omega_{WC} = 1/2$:

$$\omega_T = \omega_{WC} = \tfrac{1}{2}, \quad \sigma\eta \geq 1, \quad \rho > 1 \implies \left.\frac{\partial D^H}{\partial \mu}\right|_{\text{prices}} > 0 \implies \left.\frac{\partial a'^H}{\partial \mu}\right|_{\text{prices}} < 0 \text{ for all } \phi \in (0,1). \tag{42}$$

This is a parameter-free result on the CIA margin and an endowment-based result on the WC margin. Together they confirm that both redistribution channels compress savings under equal sharing, placing the economy in Regime A unconditionally at the standard calibration.

3.11 From Savings to Interest Rates

In general equilibrium, the reduction in savings does not simply lower the capital stock at an unchanged interest rate. The asset market must clear. When inflation reduces the H-household's savings at a given r, the

asset supply curve shifts inward. If asset demand does not fall by the same amount (and generically it does not, since capital requirements are pinned by technology), the interest rate must rise to restore equilibrium. This is the channel through which inflation raises r^*.

The full partial derivative $\partial\mathcal{E}/\partial\mu|_r$ combines the redistribution channel—operating through both CIA seigniorage and the working-capital interest rebate, governed jointly by ω_T and ω_{WC}—with a price channel that operates through the direct effect of μ on the working-capital wedge Δ_w. In the benchmark economy ($\sigma = 1$, $\delta = 0$, $\nu = 1$, $\alpha_i = 0$, $\omega_T = \omega_{WC} = 0$, evaluated at $\mu = 0$), the redistribution channel dominates: the direct effect of μ on asset supply is unambiguously negative throughout the existence region established by Theorem 1, giving

$$\frac{dr^*}{d\mu} > 0. \tag{43}$$

Inflation sets off a chain of effects that compound rather than offset:

(i) The inflation tax falls on cash holdings, and the intermediary's profit on WC loans falls on the formal wage bill. The H-household bears a disproportionate share of both costs because equal sharing under-compensates it on both margins ($\omega_T < \omega_T^*$, $\omega_{WC} < \omega_{WC}^*$), compounding the savings compression.

(ii) The H-household responds by reducing its bond savings. Because the L-household is borrowing-constrained and cannot increase its savings, aggregate savings fall.

(iii) The decline in savings tightens the capital market, raising the equilibrium rental rate r^*.

(iv) The higher rental rate raises the cost of capital and, through the working-capital constraint, the cost of financing the formal wage bill. Because the formal sector is more capital-intensive ($\alpha_f > \alpha_i$), this cost increase falls disproportionately on formal production.

(v) Economic activity shifts toward the informal sector, raising the informality ratio.

Steps (i)–(ii) are the combined redistribution channel, governed jointly by ω_T and ω_{WC}. Step (iii) is the general equilibrium link from savings to interest rates. Steps (iv)–(v) are the Stolper–Samuelson mechanism, governed by the capital gap $\alpha_f - \alpha_i$. The main result of the next section establishes the conditions under which this chain is strong enough to dominate the countervailing direct effect of inflation on informality (which, at fixed r, reduces the relative demand for informal goods through the cash-in-advance constraint).

3.12 Main Result: Inflation Increases Informality

The previous section established that inflation raises the equilibrium interest rate ($dr^*/d\mu > 0$) through the combined redistribution channel. This section asks whether this interest rate increase is strong enough to push economic activity into the informal sector. The answer depends on a tension between two opposing forces: a direct effect that reduces informality at fixed prices, and a general equilibrium effect, operating through the interest rate, that can overcome it.

Define the **informality ratio** as the value of informal output relative to formal output, expressed in formal-good units:

$$\Psi \equiv \frac{p\, y_i}{y_f}. \tag{44}$$

Since both outputs are proportional to the scale variable $\tilde{c}$ through goods market clearing ($y_i = \Upsilon_i\, \tilde{c}$ and $y_f = \Upsilon_f\, \tilde{c}$), the scale cancels and $\Psi = p\, \Upsilon_i / \Upsilon_f$ depends only on (r, μ) and structural parameters. The informality ratio is increasing whenever inflation shifts value added from the formal to the informal sector.

3.13 Decomposition into Direct and GE Effects

In general equilibrium, the interest rate adjusts to inflation through $r = r^*(\mu)$. The total effect of inflation on the informality ratio decomposes by the chain rule:

$$\frac{d \ln \Psi}{d\mu} = \underbrace{\left.\frac{\partial \ln \Psi}{\partial \mu}\right|_{r}}_{\text{direct effect}} + \underbrace{\left.\frac{\partial \ln \Psi}{\partial r}\right|_{\mu} \cdot \frac{dr^*}{d\mu}}_{\text{general equilibrium effect}} . \tag{45}$$

The **direct effect** captures what happens to informality when inflation rises but the interest rate is held fixed. It is unambiguously negative. The intuition is that inflation raises the effective price of informal goods through the cash-in-advance constraint: households must hold more cash to purchase the same quantity of informal goods, which makes them substitute toward formal consumption. At fixed prices, inflation therefore reduces the demand for informal goods relative to formal goods. In the benchmark economy, this direct headwind equals exactly -1:

$$\left.\frac{\partial \ln \Psi}{\partial \mu}\right|_{r,\ \mu=0} = -1.$$

The **general equilibrium effect** captures how the endogenous response of the interest rate feeds back into informality. From Section 3.8, $dr^*/d\mu > 0$: inflation raises the equilibrium rental rate. And from the Stolper–Samuelson logic of Section 3.2, $\partial \ln \Psi / \partial r|_{\mu} > 0$: a higher rental rate increases informal output relative to formal output, because the formal sector is more capital-intensive and therefore bears a larger share of the capital cost increase. The GE effect is therefore positive: it pushes informality up.

The question is whether the positive GE effect is large enough to overcome the negative direct effect. Substituting the closed-form expressions derived in the Appendix, the total effect in the benchmark economy ($\sigma = 1$, $\delta = 0$, $\nu = 1$, $\alpha_i = 0$, evaluated at $\mu = 0$) takes the form:

$$\left.\frac{d \ln \Psi}{d\mu}\right|_{\mu=0} = -1 + \underbrace{(\eta - 1)\left[\frac{\alpha_f}{(1-\alpha_f)\, r} + \frac{1}{R}\right] \cdot \frac{dr^*}{d\mu}}_{\equiv\ G(\alpha_f)} . \tag{46}$$

Inflation increases informality if and only if $G(\alpha_f) > 1$.

3.14 The Three Factors of the GE Product

The product $G(\alpha_f)$ has three factors, each with a transparent economic role.

The first factor, $\eta - 1$, is the **substitution premium**. It captures how readily households reallocate expenditure between formal and informal goods when relative prices change. When $\eta = 1$ (Cobb–Douglas), expenditure shares are fixed regardless of prices and $G = 0$: no amount of price change can shift the composition of demand. As η rises above one, households become more responsive to relative price changes, and the reallocation channel strengthens.

The second factor, $\alpha_f/[(1-\alpha_f)r] + 1/R$, is the **cost-gap amplifier**. It measures how much a one-unit increase in the interest rate raises the cost of formal production relative to informal production. The term $\alpha_f/[(1-\alpha_f)r]$ is the capital gap $\gamma \equiv \alpha_f/(1-\alpha_f)$ scaled by $1/r$: a higher formal capital share means that a given interest rate increase translates into a larger cost differential between sectors. At $\alpha_f = 0$, the amplifier is just $1/R$ (the working-capital channel alone), which is below one. As α_f rises, the amplifier grows without bound, making the Stolper–Samuelson mechanism progressively stronger.

The third factor, $dr^*/d\mu$, is the **redistribution response**. It measures how much the equilibrium interest rate rises per unit of inflation through the combined redistribution channel (CIA seigniorage and WC interest rebate). This factor is positive, bounded, and weakly decreasing in α_f: the Walrasian self-correction means a more capital-intensive formal sector requires a smaller interest rate adjustment to clear the asset market.

The key tension in the model is between the second and third factors. As α_f rises, the cost-gap amplifier grows (the Stolper–Samuelson mechanism strengthens), but the redistribution response weakens (the equilibrium absorbs shocks more easily). Whether their product exceeds the threshold $1/(\eta-1)$ before the equilibrium ceases to exist is the content of the main theorem.

3.15 A Sufficient Condition for Inflation to Increase Informality

The equilibrium can only sustain a limited level of formal capital intensity. As shown before, the capacity condition $\mathcal{A} < D^L/R$ defines an existence boundary $\bar{\alpha}_f$: if the formal sector is too capital-intensive, there are not enough savings in the economy to finance it. As α_f approaches $\bar{\alpha}_f$ from below, the endowment ratio ρ^* diverges (nearly all income must accrue to the saver) and asset supply saturates at its upper bound.

The theorem establishes that, under empirically plausible conditions, the GE product G crosses the threshold of one before the equilibrium breaks down.

Theorem 2 (Inflation increases informality)**.** *Consider the benchmark economy with $\sigma = 1$, $\delta = 0$, $\nu = 1$, $\alpha_i = 0$, $\omega_T = 0$, $\omega_{WC} = 0$, evaluated at $\mu = 0$. Fix $\eta > 1$, $\beta \in (0,1)$, $\phi \in (0,1)$, and $\kappa \in (0, \bar{r})$. Suppose:*

(A1) *The expenditure ratios satisfy $\xi^H < p$ and $\xi^L < p$ (both household types allocate the majority of expenditure to informal goods in value terms).*

(A2) *The composition bound holds: $\mathcal{M} \equiv \eta^2(1-\beta)^2(\eta-1)/2 < 1$.*

(A3) *The GE product at the existence boundary exceeds unity: $G(\bar{\alpha}_f^-) > 1$.*

Then there exists $\alpha_f^* \in (0, \bar{\alpha}_f)$ such that for all $\alpha_f \in (\alpha_f^*, \bar{\alpha}_f)$:

$$\left.\frac{d \ln \Psi}{d\mu}\right|_{\mu=0} > 0. \tag{47}$$

That is, a marginal increase in inflation raises the informality ratio at any equilibrium with a formal capital share in the interval $(\alpha_f^*, \bar{\alpha}_f)$.

The proof, which is provided in the Appendix, proceeds in four steps. First, continuity: because the equilibrium condition is smooth and the Walrasian stability condition holds at every interior equilibrium, $G(\alpha_f)$ is continuous on $(0, \bar{\alpha}_f)$. Second, $G(0) < 1$: without the capital gap ($\alpha_f = 0$), the cost-gap amplifier is too weak to overcome the direct headwind. Third, $G(\bar{\alpha}_f^-)$ is finite and, by assumption ((A3)), exceeds one: as α_f approaches the existence boundary, the amplifier grows large enough that the GE effect dominates. Fourth, the intermediate value theorem delivers a crossing point α_f^* where $G = 1$, and $G > 1$ for all α_f above it.

3.16 Economic Interpretation

The theorem identifies three primitive forces that must be strong enough for inflation to increase informality.

The substitution elasticity η must exceed one. When formal and informal goods are gross substitutes ($\eta > 1$), a decline in the relative price of informal goods shifts expenditure toward the informal sector. When

they are gross complements ($\eta < 1$), households respond to cheaper informal goods by consuming *more* of both goods, and the expenditure reallocation goes in the wrong direction. The Cobb–Douglas case ($\eta = 1$) is the knife-edge where expenditure shares are invariant to prices and the mechanism shuts down entirely.

The capital gap $\alpha_f - \alpha_i$ must be large enough. This is the Stolper–Samuelson requirement: the formal sector must be sufficiently more capital-intensive than the informal sector for a given interest rate increase to generate a meaningful cost differential. The theorem makes this precise: the capital share must exceed the threshold α_f^*, which is lower when η is large (because the substitution premium amplifies the cost differential) and when $dr^*/d\mu$ is large (because the seigniorage channel generates a strong interest rate response).

The capital gap must not be too large. This is the existence constraint. If the formal sector is extremely capital-intensive ($\alpha_f > \bar{\alpha}_f$), the economy cannot generate enough savings to finance formal production, and no incomplete-markets equilibrium exists. The theorem requires that the sign condition $G > 1$ is met before the existence boundary is reached, which is assumption ((A3)).

The interval $(\alpha_f^*, \bar{\alpha}_f)$ is the **joint region** where both conditions are satisfied simultaneously. Its width depends on the substitution elasticity: higher η lowers the sign threshold α_f^* (because the substitution premium strengthens the mechanism) while the existence boundary $\bar{\alpha}_f$ is less sensitive to η. There exists a critical elasticity $\eta^\dagger$ below which the joint region is empty (the mechanism cannot overcome the existence constraint) and above which it is non-empty and widening. The calibration section verifies that $\eta^\dagger$ lies well within the empirically plausible range.

3.17 The Joint Region

The mechanism requires two conditions to hold simultaneously: the formal capital share must be high enough for the GE effect to dominate ($\alpha_f > \alpha_f^*$), and it must be low enough for an incomplete-markets equilibrium to exist ($\alpha_f < \bar{\alpha}_f$). These constraints pull in opposite directions. A larger α_f strengthens the Stolper–Samuelson channel but also increases the formal sector's claim on savings, potentially exhausting the economy's savings capacity. The **joint region**

$$\mathscr{J} = \{\alpha_f \in (0,1) : \alpha_f^* < \alpha_f < \bar{\alpha}_f\}$$

is the set of capital shares where both conditions are met. Its width $|\mathscr{J}| = \bar{\alpha}_f - \alpha_f^*$ measures how much room the economy has for the mechanism to operate.

Whether the joint region is non-empty is equivalent to assumption ((A3)): $G(\bar{\alpha}_f^-) > 1$. If the GE product already exceeds one as α_f approaches the existence boundary, then by continuity it must have crossed the threshold at some interior point α_f^*, and the interval $(\alpha_f^*, \bar{\alpha}_f)$ is non-empty. Conversely, if G never reaches one within the existence region, the mechanism cannot overcome the direct headwind at any viable equilibrium.

Three comparative statics govern the width of the joint region. First, a higher substitution elasticity η widens it: the sign threshold α_f^* falls because the substitution premium $(\eta - 1)$ amplifies the GE effect, while the existence boundary $\bar{\alpha}_f$ is less sensitive to η since it depends on expenditure ratios that are relatively stable. Second, a deeper displacement into incomplete markets (larger κ) also widens it: a stronger redistribution channel lowers α_f^* by raising $dr^*/d\mu$, while $\bar{\alpha}_f$ responds only weakly through ξ^H and ξ^g. Third, a higher weight on informal consumption (lower ϕ) has an ambiguous effect: it raises the inflation-tax base (strengthening the redistribution channel), but it also increases the expenditure ratios and tightens the existence constraint.

There exists a critical substitution elasticity $\eta^\dagger(\beta, \phi, \kappa)$, defined as the smallest $\eta > 1$ at which $G(\bar{\alpha}_f^-) = 1$. Below $\eta^\dagger$, the joint region is empty and the mechanism does not operate at any viable equilibrium. Above

$\eta^\dagger$, the joint region is non-empty and widening in η. The calibration section verifies that $\eta^\dagger$ lies comfortably within empirically plausible estimates of the formal–informal substitution elasticity.

3.18 The Role of the Benchmark Assumptions

The benchmark economy ($\sigma = 1$, $\delta = 0$, $\nu = 1$, $\alpha_i = 0$, $\omega_T = 0$, $\omega_{WC} = 0$) is a deliberately stripped-down laboratory that isolates the mechanism in closed form. Each specialisation plays a specific role.

Log utility ($\sigma = 1$) is the Bewley–Aiyagari benchmark and ensures that the informal consumption gap simplifies to $\xi^g = \beta R$, eliminating higher-order composition corrections. No depreciation ($\delta = 0$) removes the investment component from goods market clearing, allowing the informality ratio to be expressed purely in terms of consumption expenditures. Full working capital ($\nu = 1$) maximises the cost-channel wedge between formal and informal firms, making the mechanism as transparent as possible. A pure-labour informal sector ($\alpha_i = 0$) concentrates all capital demand in the formal sector, so the capital gap equals $\gamma = \alpha_f/(1-\alpha_f)$ and the Stolper–Samuelson channel operates at full strength. Finally, $\omega_T = 0$ (all CIA seigniorage to L) and $\omega_{WC} = 0$ (all WC rebate to L) jointly maximise the redistribution from savers to borrowers, placing the economy at the extreme of Regime A on both margins and delivering the largest possible savings compression. Under the benchmark, $\partial D^H/\partial\mu|_{\text{prices}} = p > 0$ and $\partial D^L/\partial\mu|_{\text{prices}} = -(p + \kappa^{WC}) < 0$.

These are polar cases that stack the deck in favour of the mechanism. The quantitative model of Section 4 relaxes every one of them: σ is calibrated to the data, $\delta > 0$, $\nu < 1$, $\alpha_i > 0$, and $\omega_T = \omega_{WC} = 1/2$ (equal sharing). Since $\omega_T^* > 1/2$ and $\omega_{WC}^* > 1/2$ unconditionally, equal sharing still places the economy in Regime A, so the mechanism survives intact. The analytical value of the benchmark is to identify the three primitive forces (η, $\alpha_f - \alpha_i$, $dr^*/d\mu$) and to prove that their interaction can overcome the direct headwind of the cash-in-advance constraint, a result that would be opaque in a fully calibrated numerical model.

3.19 A Constructive Example

This subsection describes a systematic recipe for constructing parametrisations that satisfy the theorem's hypotheses while matching key features of developing economies. The procedure works backwards from empirical targets to the structural parameters. For now we abstract from working capital transfers [16]

The recipe has six steps. First, choose a discount factor β consistent with empirical estimates of patience in developing economies. I set $\beta = 0.70$, which at a five-year model period corresponds to an annualised discount factor of $\beta_{\text{annual}} = 0.70^{1/5} = 0.93$. This pins the frictionless interest rate at $\bar{r} = (1-\beta)/\beta = 3/7 \approx 0.429$, or 7.4% annualised.

Second, choose a substitution elasticity $\eta > 1$. I set $\eta = 2$, at the conservative end of empirical estimates for developing economies (Ulyssea, 2018, reports estimates in the range 2–4). This ensures the composition bound ((A2)) holds: $\mathcal{M} = 0.18 < 1$.

Third, choose the incomplete-markets displacement κ. A small wedge ($\kappa = 0.05\bar{r}$, or 5% of the frictionless rate) is conservative: it weakens the seigniorage channel and produces a realistic annualised equilibrium rate of 7.1%.

Fourth, normalize $\xi^H = 1$ by choosing the informal-sector productivity A_i. Since $p = \widetilde{w}(r)/(R\,A_i)$ under $\alpha_i = 0$ and $\nu = 1$, the level of A_i pins p without affecting any other equilibrium object. Choosing A_i so that $\xi^H = 1$ sets $p = \beta^2 R\,[(1-\phi)/\phi]^{1/\eta}$, which simplifies all subsequent expressions. Under this normalization, $\xi^L = (\beta R)^{2\eta}$ is a known constant independent of ϕ. At the chosen parameters, $\xi^L = 0.941$.

[16] Future iterations of the paper will incorporate them into the numerical simulations.

Fifth, choose the CES weight ϕ to balance two constraints: the existence boundary $\bar{\alpha}_f$ must be at least 0.30 (the target formal capital share), and the informal consumption share should be consistent with the evidence in Bachas, Gadenne, and Jensen (2024). Since $\bar{\alpha}_f$ is decreasing in ϕ (lower ϕ raises p and widens the existence region), I choose the largest ϕ such that $\bar{\alpha}_f \geq 0.30$. This yields $\phi = 0.072$, $p = 2.48$, $\bar{\alpha}_f = 0.306$, and an informal consumption share of 72%.

Sixth, verify the theorem's conditions. At $\alpha_f = 0.30$, $G = 1.59 > 1$: the GE tailwind is 59% stronger than the direct headwind. Assumption ((A1)) holds ($p = 2.48 > 1$), and the equilibrium exists with $\rho^* = 200$. At the more relaxed $\alpha_f = 0.25$, $G = 1.45$ with $\rho^* = 22$, confirming that the mechanism operates comfortably inside the existence region.

The recipe is not unique. The key degree of freedom is the β–ϕ tradeoff: higher β (more patient agents, lower interest rates) requires lower ϕ (larger informal price, higher informal share) to maintain $\bar{\alpha}_f \geq 0.30$. Table 2 reports the frontier.

Table 2: The β–ϕ frontier at $\alpha_f = 0.30$, $\kappa = 0.05\bar{r}$, $\eta = 2$

β	ϕ	p	$\bar{\alpha}_f$	$G\big\|_{\alpha_f=0.30}$	Informal share
0.70	0.072	2.48	0.306	1.59	72%
0.75	0.062	2.88	0.303	1.49	75%
0.78	0.054	3.23	0.303	1.43	77%
0.80	0.048	3.53	0.304	1.39	78%

Every row satisfies all conditions of the theorem. The choice between them reflects a judgment about where to sit on the patience–informality frontier. I adopt $\beta = 0.70$, $\phi = 0.072$ as the central calibration because it yields the lowest informal share (72%) and the strongest mechanism ($G = 1.59$), while maintaining a realistic annualised discount factor.

3.20 Numerical Verification

Table 3 traces the GE product $G(\alpha_f)$ across the full existence region at the central calibration.

Table 3: $G(\alpha_f)$ across the existence region (Central: $\eta = 2$, $\beta = 0.70$, $\phi = 0.072$, $\bar{\alpha}_f = 0.306$)

α_f	γ	γ/r	ρ^*	$dr^*/d\mu$	G	$d\ln\Psi/d\mu$
0.02	0.02	0.05	3.6	1.293	0.98	−0.02
0.06	0.06	0.16	4.3	1.174	1.02	+0.02
0.10	0.11	0.27	5.3	1.091	1.07	+0.07
0.15	0.18	0.43	7.0	1.016	1.15	+0.15
0.20	0.25	0.61	11.0	0.966	1.28	+0.28
0.25	0.33	0.82	21.5	0.935	1.45	+0.45
0.30	**0.43**	**1.05**	**200**	**0.901**	**1.59**	**+0.59**

Three patterns are worth highlighting. First, the sign threshold $\alpha_f^* \approx 0.04$ is near zero: the mechanism operates across 94% of the existence region. Even moderate formal capital shares ($\alpha_f = 0.10$, $\gamma = 0.11$) are sufficient for inflation to increase informality. The result does not hinge on extreme parameter values.

Second, G is monotonically increasing despite $dr^*/d\mu$ being decreasing in α_f. As the capital gap widens, the Walrasian self-correction dampens the interest rate response (savings react more strongly to r), but the

cost-gap amplifier $\gamma/r + 1/R$ grows fast enough to more than compensate. This is why the mechanism is robust: the amplifier and the seigniorage response partially cancel, keeping G above one across a wide range.

Third, the decomposition at $\alpha_f = 0.30$ makes the economics transparent:

$$G = \underbrace{(\eta - 1)}_{=1.0} \times \underbrace{\left[\frac{\gamma}{r} + \frac{1}{R}\right]}_{=1.76} \times \underbrace{\frac{dr^*}{d\mu}}_{=0.90} = 1.59.$$

The substitution premium is 1 (each percentage-point change in relative prices maps one-for-one into expenditure reallocation). The cost-gap amplifier is 1.76, of which the capital-cost channel ($\gamma/r = 1.05$) contributes roughly 60% and the working-capital channel ($1/R = 0.71$) the rest. The seigniorage multiplier is 0.90: a 10% increase in inflation raises the equilibrium interest rate by 9 percentage points per period. Their product exceeds unity by 59%, confirming that the GE tailwind comfortably dominates the direct headwind.

The sensitivity of the result across the (β, η) space is equally reassuring. At $\eta = 2.2$ (the "Ambitious" calibration), G rises to 1.98 at $\alpha_f = 0.30$ and exceeds one at every capital share in the existence region, including $\alpha_f = 0.02$. At $\beta = 0.75$ with $\eta = 2$ (the "Conservative" calibration), G remains above one for $\alpha_f \geq 0.02$, though the margin is tighter ($G = 1.49$ at $\alpha_f = 0.30$). The critical substitution elasticity $\eta^\dagger$, the minimum η required for the joint region to be non-empty, is approximately 1.7 at the central calibration, well below empirically estimated values.

3.21 Graphical Evidence

Four figures summarize the quantitative results. Together, they trace the mechanism from its primitive forces to the aggregate outcome, and confirm that the analytical predictions hold across the full parameter space.

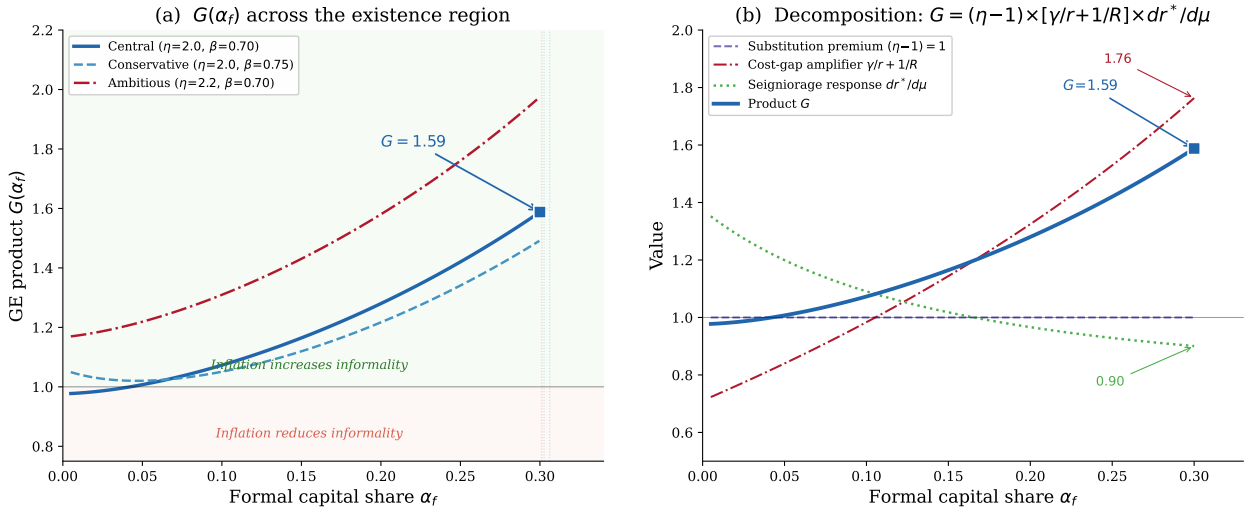


Figure 4: The GE product $G(\alpha_f)$ and its three-factor decomposition.

Figure 4 is the paper's central diagnostic. Panel (a) plots $G(\alpha_f)$ across the existence region for three calibrations: Central ($\eta = 2.0$, $\beta = 0.70$), Conservative ($\eta = 2.0$, $\beta = 0.75$), and Ambitious ($\eta = 2.2$, $\beta = 0.70$). The horizontal line at $G = 1$ divides the parameter space: above it, the GE tailwind overcomes the direct headwind and inflation increases informality. At the Central calibration, G starts just below one at $\alpha_f = 0.02$ and rises monotonically to $G = 1.59$ at $\alpha_f = 0.30$, exceeding unity across 94% of the existence region. Raising

η from 2.0 to 2.2 shifts the entire curve upward, so that $G > 1$ at every capital share. Raising β from 0.70 to 0.75 shifts the curve downward modestly, but the mechanism still operates across most of the region.

Panel (b) decomposes G into its three multiplicative factors at the Central calibration. The substitution premium $(\eta - 1) = 1$ is constant (dashed purple). The cost-gap amplifier $\gamma/r + 1/R$ (dash-dot red) rises from 0.76 at $\alpha_f = 0$ to 1.76 at $\alpha_f = 0.30$: this is the engine of the mechanism. The seigniorage response $dr^*/d\mu$ (dotted green) falls from 1.29 to 0.90 as the Walrasian self-correction strengthens with higher capital demand. The key insight is visual: the amplifier grows faster than the seigniorage response declines, so their product G (solid blue) is monotonically increasing.

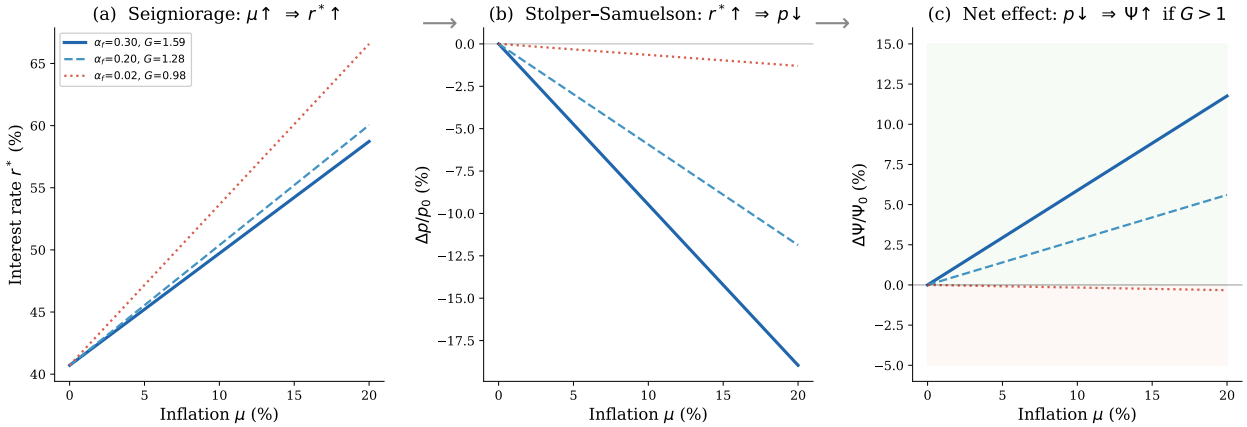


Figure 5: The causal chain: $\mu\uparrow \Rightarrow r^*\uparrow \Rightarrow p\downarrow \Rightarrow \Psi\uparrow$.

Figure 5 traces each link of the causal chain at three capital shares ($\alpha_f = 0.30$, 0.20, and 0.02), using first-order approximations around $\mu = 0$. Panel (a) shows the seigniorage link: inflation raises the equilibrium interest rate $r^*(\mu)$ for all three economies. The slopes are similar ($dr^*/d\mu \approx 0.9$–1.3) because the seigniorage channel depends primarily on the incomplete-markets wedge, not on the capital gap. Panel (b) shows the Stolper–Samuelson link: the relative price of informal goods p falls as r^* rises. The magnitude depends critically on the capital gap: at $\alpha_f = 0.30$, p falls by 19% at $\mu = 20\%$, while at $\alpha_f = 0.02$, p is essentially unchanged. Panel (c) shows the net effect on the informality ratio Ψ. Economies with large capital gaps see informality rise sharply (+12% at $\mu = 20\%$ for $\alpha_f = 0.30$), while the small-gap economy experiences a negligible decline. The red dotted line lying below zero confirms that without a sufficient capital gap, the direct headwind prevails.

Figure 6 maps the (η, α_f) parameter space at the Central calibration. The solid blue curve is the existence boundary $\bar{\alpha}_f(\eta)$, declining from approximately 0.45 at $\eta = 1.2$ to near zero at $\eta = 4.5$: higher substitutability tightens the existence region by increasing expenditure dispersion across states. The dashed red curve is the sign threshold $\alpha_f^*(\eta)$ where $G = 1$, falling steeply and reaching zero near $\eta = 2$. The gold-shaded area between the two curves is the joint region $\mathscr{J}$: parameter pairs at which an interior equilibrium exists and inflation increases informality. Three features stand out. First, the joint region is large: for $\eta \geq 2$, it encompasses nearly the entire existence region. Second, both calibrations (Central and Ambitious, marked by stars) sit comfortably inside $\mathscr{J}$. Third, the critical elasticity $\eta^\dagger \approx 1.7$ is well below the empirically relevant range (2–4), confirming that the mechanism operates under conservative parameter choices.

Figure 7 is the paper's central visual argument. Panel (a) decomposes the total effect of inflation on informality at the Central calibration with $\alpha_f = 0.25$ ($G = 1.42$). The dashed red line shows the direct effect: holding r fixed, inflation reduces informality by μ (the cash-in-advance constraint makes informal

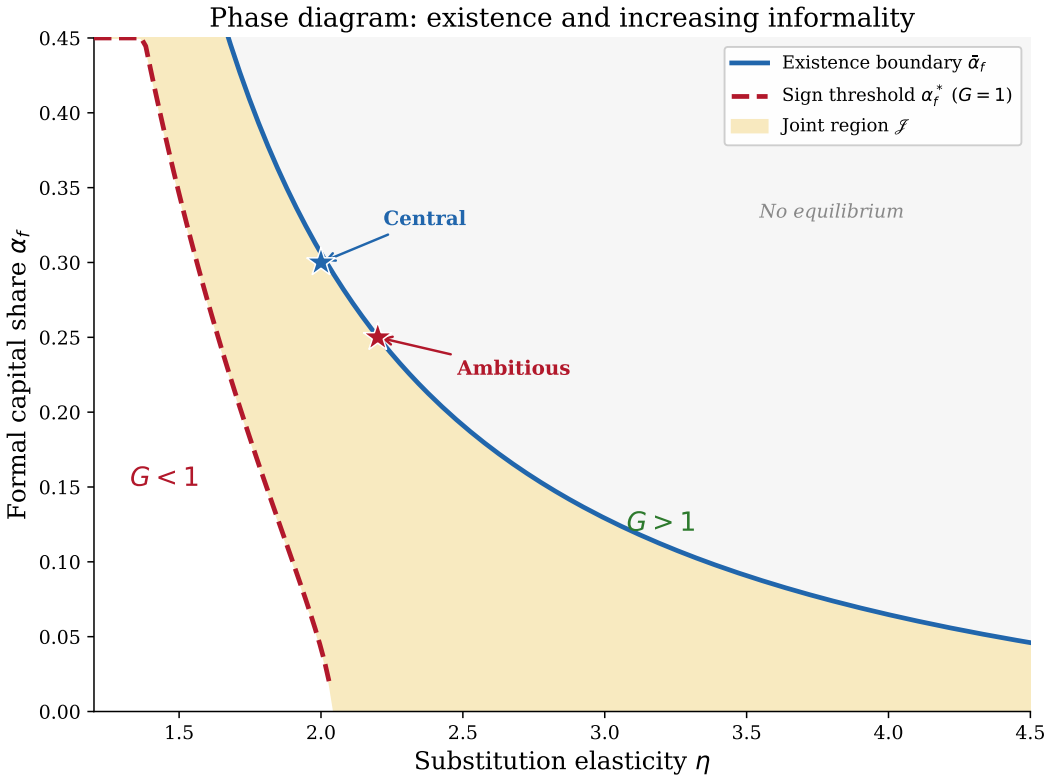


Figure 6: Phase diagram: the joint region $\mathscr{J}(\eta, \alpha_f)$.

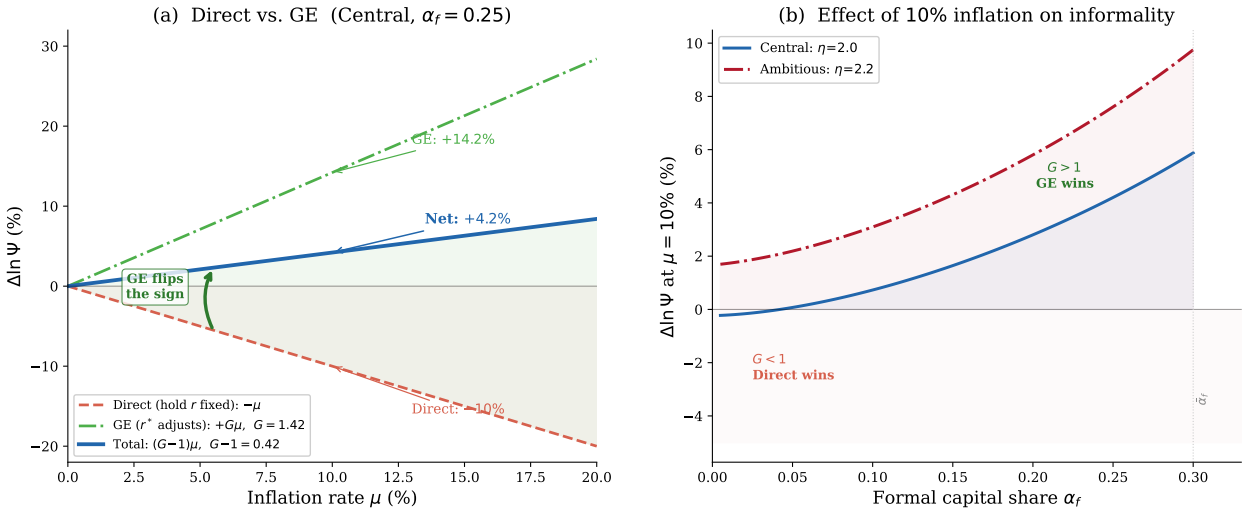


Figure 7: Direct vs. GE effect: the sign reversal.

goods relatively more expensive). The dash-dot green line shows the GE effect: allowing r^* to adjust, the seigniorage-driven interest rate increase amplifies into a relative-price shift that increases informality by $G\mu$. Since $G = 1.42 > 1$, the GE effect is 42% larger than the direct effect. The solid blue line shows the total: informality increases by $(G-1)\mu = 0.42\mu$, reaching +4.2% at $\mu = 10\%$. The curving green arrow marks the sign reversal: without general equilibrium feedback, inflation reduces informality; with it, the Stolper–Samuelson amplification through the capital gap reverses the prediction.

Panel (b) shows the effect of 10% inflation across all α_f in the existence region, for both the Central ($\eta = 2.0$) and Ambitious ($\eta = 2.2$) calibrations. The curves start below zero at tiny capital gaps (where the direct effect wins) and rise monotonically. At the Central calibration, informality increases by 6% at $\alpha_f = 0.30$; at the Ambitious calibration, by 10%. The vertical dotted line marks the existence boundary $\bar{\alpha}_f$. The result is robust: the mechanism operates across the range $\alpha_f \in (0.04, 0.30)$, and higher substitutability uniformly strengthens it.

4 General Model

In the fully-fledged model, ex-ante identical households are exposed to idiosyncratic stochastic shocks to their labor income and must choose their money and asset holdings allocations for the future without knowing the realization of the shock.[17]

4.1 Households

The flow utility of any household is given by the standard CES composite aggregator between informal and formal goods:

$$C(c_t^i, c_t^f) = \left[\phi^{\frac{1}{\eta}}(c_t^i)^{\frac{\eta-1}{\eta}} + (1-\phi)^{\frac{1}{\eta}}(c_t^f)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$

where $\eta > 1$ is the elasticity of substitution between informal and formal goods (the same parameter as in the benchmark economy) and $\phi \in (0,1)$ is the CES weight on informal goods.

Individuals are subject to idiosyncratic shocks on their labor productivity e_t, following a standard AR(1) process:

$$\log(e_t) = \rho \log(e_{t-1}) + \sigma_e \epsilon_t \quad \epsilon_t \sim \mathcal{N}(0,1)$$

I normalize the shocks so that the unconditional mean is constant and equal to unity: $\mathbb{E}(e_t) = 1$. As in Aiyagari (1994), markets are incomplete and I impose a no-borrowing constraint. Households can self-insure against employment risks by accumulating a riskless asset a which yields a real return $r > 0$, and they can also invest in money assets m_t.

In a stationary equilibrium, all real variables are constant, including government transfers. Assuming that there is a tax on formal consumption (τ), the recursive problem of a household is:

$$V(m,a,e) = \max_{\{c^i, c^f, m', a'\}} \quad \frac{C(c^i, c^f)^{1-\sigma}}{1-\sigma} + \beta \mathbb{E} V(m', a', e') \quad \text{st.}$$

[17] The general model abstracts from one features of the benchmark economy that will be incorporated in future versions of the paper: the working-capital constraint on formal firms' wage bill (parametrized by ν in Section 3.2). Future revisions will also add endogenous entry margin with heterogeneous firms self-selecting into formality or informality along the lines of Melitz (2003) and Ulyssea (2018). A third planned extension introduces financial intermediation following Gertler and Kiyotaki (2010), allowing the interest rate spread between savers and borrowers to respond endogenously to inflation. Each of these channels is expected to strengthen the mechanism identified in the benchmark: working capital amplifies the cost wedge between sectors, endogenous entry adds an extensive margin of informality growth, and intermediation frictions widen the incomplete-markets wedge that drives the seigniorage redistribution channel.

$$pc^i + (1+\tau)c^f + m' + a' = (1+r)a + \frac{m}{1+\mu} + we + T$$

$$\chi p c^i \leq \frac{m}{1+\mu}$$

$$c^i, c^f, m', a' \geq 0$$

The parameter $\chi \geq 1$ governs the tightness of the cash-in-advance constraint on informal purchases. When $\chi = 1$, the constraint requires that the full value of informal consumption be financed with cash carried from the previous period (Svensson, 1985, timing), exactly as in the benchmark economy. When $\chi > 1$, the constraint is tighter: the household must hold χ times the value of its informal purchases in cash, capturing the idea that not all cash holdings translate efficiently into purchasing power in informal markets. This parameter plays a role analogous to the velocity of money in cash-in-advance models and is calibrated to match the observed ratio of monetary aggregates to GDP.

4.2 Government

As in the deterministic model, the government increases the money supply at a fixed rate (μ), generating inflation at the same rate $\Pi = 1 + \mu$ and collecting resources through the inflation tax. I also assume that the government levies taxes on the households' formal consumption to finance public spending into both formal and informal goods. The government does not issue any debt, so all its revenue is used to finance the deficit.

Let $\Lambda : \mathbb{R}^3 \to [0,1]$ denote the steady-state joint distribution of households over money holdings, asset holdings and the idiosyncratic shock. The budget constraint is:

$$\underbrace{\tau \int c^f(m,a,e)\, d\Lambda(m,a,e) + \frac{\mu}{1+\mu} \int m\, d\Lambda(m,a,e)}_{:=\text{Rev}} = p\hat{g}^i + \hat{g}^f + T \tag{48}$$

I assume that a constant fraction of the revenue collected is rebated as a lump sum to the households, so that $T = \lambda \cdot \text{Rev}$. The government has the same preferences over the consumption of both goods as the household does and does not face any liquidity restriction in the purchase of the informal good:

$$(\hat{g}^i, \hat{g}^f) = \arg\max_{\{g^i, g^f\}} \left[\phi^{\frac{1}{\eta}} (g^i)^{\frac{\eta-1}{\eta}} + (1-\phi)^{\frac{1}{\eta}} (g^f)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \text{ st. } pg^i + g^f = (1-\lambda) \cdot \text{Rev}$$

Since the government is exempt from paying taxes on formal consumption, the relative cost of informal goods appears higher to it. Consequently, the government allocates a smaller portion of its income to informal goods compared to households.

4.3 Firms

The production functions of each good are the same as in the benchmark economy:

$$y^f = \left(k^f\right)^{\alpha_f} \left(n^f\right)^{1-\alpha_f} \qquad y^i = \left(k^i\right)^{\alpha_i} \left(n^i\right)^{1-\alpha_i} \quad \text{with} \quad \alpha_f > \alpha_i$$

4.4 Stationary Equilibrium

A stationary competitive equilibrium for this economy consists of constant policy functions for the households $c^i(m,a,e)$, $c^f(m,a,e)$, $m'(m,a,e)$, $a'(m,a,e)$ and the government $\hat{g}^i(\text{Rev})$, $\hat{g}^f(\text{Rev})$ for consumption of both

goods, real balances, and asset holdings respectively; the steady-state joint distribution over assets, money and productivity $\Lambda(m, a, e)$; a constant real return on financial assets r; a constant real wage w; a relative price of the informal good p; the real return on real balances $1/(1+\mu)$; and taxes τ, consistent with the exogenous supply of money and government public spending, such that:

1. The policy functions solve the households' and government's problems given the equilibrium prices.
2. The joint distribution $\Lambda(m, a, e)$ over productivity and wealth is time invariant.
3. Factor prices are competitively determined.
4. The tax rate τ is constant, and transfers to households are such that the government budget constraint holds.
5. Financial and labor markets clear:

$$\int a \, d\Lambda(m, a, e) = k^f + k^i \qquad 1 = n^f + n^i$$

6. Goods markets clear:

$$y^f = \delta(k^f + k^i) + \int c^f \, d\Lambda(m, a, e) + g^f \qquad y^i = \int c^i \, d\Lambda(m, a, e) + g^i$$

4.5 Portfolio Composition of the Households

In this setup, characterized by idiosyncratic shocks and a cash-in-advance constraint with Svensson's timing on the informal good, the constraint might not always be binding each period. The household may end up with excess savings after the income shock is realized, as the actual shock could be small, resulting in lower consumption and liquidity needs. Ideally, households aim to allocate all unused savings into interest-bearing assets. However, since future income shocks are uncertain, saving decisions are based on expectations, potentially leading to ex-post inefficiencies in the return on the savings portfolio.

Let $s' = m' + a'$ be the savings of a household and $W(m, a, e, s')$ the total consumption expenditures for today. From the budget constraint:

$$W(m, a, e, s') = (1 + r)a + \frac{m}{1+\mu} + we + T - s'$$

Let $\hat{c}^i$ and $\hat{c}^f$ be the unconstrained consumption of informal and formal goods if there were no liquidity constraints. That is, consider the intratemporal consumption problem the household solves ignoring the cash-in-advance constraint:

$$\max_{\{c^i, c^f\}} \; C(c^i, c^f) \quad \text{st.} \quad pc^i + (1+\tau)c^f = W(m, a, e, s')$$

The solution gives:

$$\hat{c}^i(m, a, e, s') = \frac{\phi \, p^{-\eta} \, W(m, a, e, s')}{(1-\phi)(1+\tau)^{1-\eta} + \phi \, p^{1-\eta}} \tag{49}$$

$$\hat{c}^f(m, a, e, s') = \frac{(1-\phi)(1+\tau)^{-\eta} \, W(m, a, e, s')}{(1-\phi)(1+\tau)^{1-\eta} + \phi \, p^{1-\eta}} \tag{50}$$

With this, I can define the optimal composite consumption ($\hat{C}$), dividing into cases depending on whether the cash-in-advance constraint is binding or not. If it is not binding, I get the standard result of CES utility models, in which consumption is equal to wealth divided by the ideal price index. If it is binding, the optimal consumption of the informal good is limited to the amount of real money holdings the household has during that period, and the formal-good consumption value equals the leftover wealth allocated for purchases:

$$\hat{C}(m,a,e,s') = \begin{cases} W(.)\left[(1-\phi)(1+\tau)^{1-\eta} + \phi\, p^{1-\eta}\right]^{\frac{1}{\eta-1}} & \text{if } \chi p\hat{c}^i(.) \leq \frac{m}{1+\mu} \\ \left[\phi^{\frac{1}{\eta}}\left(\frac{m}{p\chi(1+\mu)}\right)^{\frac{\eta-1}{\eta}} + (1-\phi)^{\frac{1}{\eta}}\left[\frac{1}{1+\tau}\left(W(.) - \frac{m}{\chi(1+\mu)}\right)\right]^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} & \text{if } \chi p\hat{c}^i(.) > \frac{m}{1+\mu} \end{cases}$$

After this intratemporal consumption problem is solved, the household's problem can be framed into a first-stage savings decision and a posterior portfolio allocation choice, which is useful computationally (see Appendix):

$$V(m,a,e) = \max_{\{s'\}} \quad \frac{\hat{C}(m,a,e,s')^{1-\sigma}}{1-\sigma} + \beta\mathcal{V}(s',e) \quad \text{st.} \tag{51}$$

$$s' \in \left[0, (1+r)a + \frac{m}{1+\mu} + we + T\right]$$

$$\mathcal{V}(s',e) = \max_{\{a'\in[0,s']\}} \quad \mathbb{E}V(s'-a', a', e')$$

The optimal portfolio allocation choice ($a^*(s',e)$) satisfies a no-arbitrage condition between the returns of both assets:

$$\mathbb{E}V_m(s'-a^*(s',e), a^*(s',e), e') = \mathbb{E}V_a(s'-a^*(s',e), a^*(s',e), e') \tag{52}$$

$$\mathcal{V}(s',e) = \mathbb{E}V(s'-a^*(s',e), a^*(s',e), e')$$

And the optimal savings problem is solved by getting an expression for $s^*(m,a,e)$ that satisfies:

$$-\frac{\partial\hat{C}(.)}{\partial s'}\bigg|_{s'=s^*} \cdot \hat{C}(m,a,e,s^*)^{-\sigma} = \beta\mathcal{V}_{s'}(s^*,e) \tag{53}$$

To study the effect of extra savings in future periods, it is useful to examine the impact of additional money holdings versus interest-bearing assets on the optimal consumption composite. If the liquidity constraint is not binding and the household has enough money to purchase the informal good:

$$\frac{\partial\hat{C}(.)}{\partial a} - \frac{\partial\hat{C}(.)}{\partial m} = \left[(1-\phi)(1+\tau)^{1-\eta} + \phi\, p^{1-\eta}\right]^{\frac{1}{\eta-1}}\left[(1+r) - \frac{1}{1+\mu}\right] > 0$$

This means that, if the CIA constraint is not a problem, having extra assets always dominates having extra money, as the return on assets exceeds the real return on money (as long as inflation is strictly positive).

If the liquidity constraint is binding and the consumption of the informal good is bounded by real money holdings, then the difference between holding assets and money is:

$$\frac{\partial\hat{C}(.)}{\partial a} - \frac{\partial\hat{C}(.)}{\partial m} = \left[\frac{(1-\phi)\hat{C}(.)}{\frac{1}{1+\tau}\left(W(.) - \frac{m}{\chi(1+\mu)}\right)}\right]^{\frac{1}{\eta}} \frac{1}{1+\tau}\left[(1+r) + \frac{1}{\chi(1+\mu)}\right] - \left(\frac{\phi\hat{C}(.)}{\frac{m}{p\chi(1+\mu)}}\right)^{\frac{1}{\eta}}\left[\frac{1}{p\chi(1+\mu)}\right]$$

If in equilibrium the CIA constraint is slack, the consumption ratio satisfies:

$$\left(\frac{1-\phi}{\hat{c}^f}\right)^{\frac{1}{\eta}} \frac{1}{1+\tau} = \left(\frac{\phi}{\hat{c}^i}\right)^{\frac{1}{\eta}} \frac{1}{p}$$

This, together with $\hat{c}^f < \frac{1}{1+\tau}\left(W(.) - \frac{m}{\chi(1+\mu)}\right)$ and $\hat{c}^i > \frac{m}{p\chi(1+\mu)}$, delivers:

$$\left[\frac{(1-\phi)\hat{C}(.)}{\frac{1}{1+\tau}\left(W(.) - \frac{m}{\chi(1+\mu)}\right)}\right]^{\frac{1}{\eta}} \frac{1}{1+\tau}\left[\frac{1}{\chi(1+\mu)}\right] - \left(\frac{\phi\hat{C}(.)}{\frac{m}{p\chi(1+\mu)}}\right)^{\frac{1}{\eta}}\left[\frac{1}{p\chi(1+\mu)}\right] < 0$$

which implies that if the return on assets is not extremely high or if the liquidity constraint is really tight, then investing in money dominates investing in assets. This occurs even though the real return in doing so is lower, precisely because it alleviates the cash-in-advance constraint. Additionally, if the difference between the consumption of both goods is far from the ideal ratio, the dominance of investing in money over assets is reinforced.

For any pair (m, a) of holdings, define the set $\Omega^U(m,a) = \{e : \hat{c}^i(m,a,e) \leq \frac{m}{p\chi(1+\mu)}\}$ as the set of income shocks for which the liquidity constraint does not bind, and $\Omega^C(m,a)$ as its complement. Because $\hat{c}^i(m,a,e)$ is continuous and increasing in e, there is a cutoff $\bar{e}(m,a)$ defined as the income shock that makes optimal consumption exactly equal to the real money holdings that the household has. All $e \leq \bar{e}(m,a)$ belong to $\Omega^U(m,a)$, and all $e > \bar{e}$ to $\Omega^C(m,a)$. The equation that defines the cutoff is:

$$\frac{\phi\, p^{-\eta}\, W(m,a,\bar{e}(m,a), s^*(m,a,\bar{e}(m,a)))}{(1-\phi)(1+\tau)^{1-\eta} + \phi\, p^{1-\eta}} = \frac{m}{p\chi(1+\mu)}$$

Let $\pi(e'|e)$ be the conditional probability of having e' next period if the current shock is e. Let $\sigma = 0$ for now to eliminate the curvature of the utility function and simplify the analysis. Using the previous function and the no-arbitrage condition between the returns of both assets, the equation that defines $a^*(s', e)$ can be rewritten as:

$$\int_0^{\tilde{e}} \pi(e'|e)\frac{W(X)}{P}\left[R - \frac{1}{1+\mu}\right] de' = \tag{54}$$

$$\int_{\tilde{e}}^{\infty} \pi(e'|e)\left[\left(\frac{\phi\hat{C}(X)}{\frac{s'-a^*(s',e)}{p\chi(1+\mu)}}\right)^{\frac{1}{\eta}}\left[\frac{1}{p\chi(1+\mu)}\right] - \left[\frac{(1-\phi)\hat{C}(X)}{\frac{1}{1+\tau}\left(W(X) - \frac{s'-a^*(s',e)}{\chi(1+\mu)}\right)}\right]^{\frac{1}{\eta}} \frac{1}{1+\tau}\left[R + \frac{1}{\chi(1+\mu)}\right]\right] de'$$

with $R = 1 + r$, $\tilde{e} = \bar{e}(s' - a^*(s',e), a^*(s',e))$, $P = \left[(1-\phi)(1+\tau)^{1-\eta} + \phi\, p^{1-\eta}\right]^{\frac{1}{1-\eta}}$, and the future states and optimal savings decision summarized in $X = \{s' - a^*(s',e), a^*(s',e), e', s^*(s' - a^*(s',e), a^*(s',e), e')\}$. The LHS represents the advantage of investing in the asset instead of holding money when the future shock is low enough to make the liquidity constraint irrelevant. Since the interest rate on assets always exceeds the real return on money, investing in assets is always more advantageous in these scenarios. Therefore, the expected value of these low-shock states encourages the inclusion of assets in the household's portfolio.

The RHS illustrates the benefit of holding money rather than investing in the asset when the shock is large enough to make the liquidity constraint binding. The additional money can help alleviate that constraint. For a household that diversifies its investments and finds an optimal balance in its portfolio, the RHS must be positive. This implies that there will be a positive probability of facing a liquidity constraint in the next period, making the benefit of holding money outweigh the benefit of owning assets in some cases.

It is noteworthy that the optimal investment choice $a^*(s', e)$ not only determines the household's future portfolio composition at the beginning of the next period, but also affects the expected likelihood of encountering a liquidity constraint. The more the household invests in money, the lower the probability for it to face that liquidity risk.

4.6 Calibration

The calibration of the model seeks to target moments from a typical Latin American economy. As real balances are composed of liquid assets, I opt for a model period of one quarter instead of a full year, aligning with previous quantitative studies concerning the inflation tax (see Erosa and Ventura, 2002, or Algan and Ragot, 2010). I calibrate the model for an initial quarterly rate of inflation of 2.5 percent. Risk aversion is assigned a typical value of $\sigma = 1$, consistent with the baseline scenario in Chari, Christiano, and Kehoe (2000), while the formal good tax is set at $\tau = 0.3$, aligning closely with the values reported by Domeij and Heathcote (2004). The fraction of the government revenue that is transferred to the households is $\lambda = 0.48$, consistent with the share of US federal spending allocated to transfers to individuals and businesses. For idiosyncratic shocks, I follow the conventional approach in quantitative macroeconomic literature by considering wages to follow a first-order autoregressive process. Several researchers have used PSID data to estimate these parameters, and have identified an autocorrelation coefficient near 0.9 and a standard deviation for innovations between 0.12 and 0.25 (see Hubbard, Skinner, and Zeldes, 1995; Heathcote, Storesletten, and Violante, 2005).

For the internally calibrated parameters $(\phi, \eta, \alpha_i, \chi)$, I target average moments of Latin American countries: a capital-to-output ratio of 2.75 at the annual level, consistent with average estimates from the Penn World Tables; an estimated informality-to-GDP ratio of 0.38 from Medina and Schneider (2019); government expenditures over GDP of 0.29 in 2018 (reported by the IDB); and annual M1-to-GDP ratios close to 0.2.

Parameter		Value	Source		
Externally calibrated					
β	Discount factor	0.98	Standard Value, Quarterly		
δ	Capital depreciation	0.025	Standard Value, Quarterly		
α_f	Formal Good Capital Share	0.4	Standard Value		
σ	Risk aversion	1	Chari et al. (2000)		
τ	Formal Good Tax	0.3	Domeij and Heathcote (2004)		
λ	Transfers over Deficit	0.48	Federal Spending in Transfers		
ρ	Shock persistence	0.9	Standard Value		
σ_e	Standard deviation of shock	0.2	Standard Value		
Internally calibrated			*Moment*	*Model*	*Data*
ϕ	Informal Good Weight	0.7	Informality over GDP	0.39	0.38
η	Elasticity of Substitution	1.2	Formal Gov. Spending over GDP	0.27	0.29
α_i	Informal Good Capital Share	0.2	Annual Capital over GDP	2.86	2.75
χ	Liquidity Parameter	5	Annual M1 over GDP	0.24	0.2

Table 4: Full Model Calibration

4.7 Policy Functions

Figures 8 and 9 show the policy functions for future asset holdings and cash for a representative household with a given value of idiosyncratic shock and cash holdings as a function of current asset holdings. Two features are worth noting. Households can be poor enough to be borrowing constrained, meaning they do not save in future assets at all, but they always hold some cash. Otherwise, their future consumption of informal goods would be zero, which is extremely costly in welfare terms given $\eta > 1$. The other observation is that the assets policy function is quite linear in current wealth but the cash investments exhibit substantial curvature. Poorer households allocate most of their savings portfolio to cash, so that they guarantee some level of future informal consumption with the liquidity that money provides. Wealthier households have already accumulated enough money for their liquidity needs and choose to invest the extra savings in interest-bearing assets, capturing the higher return.

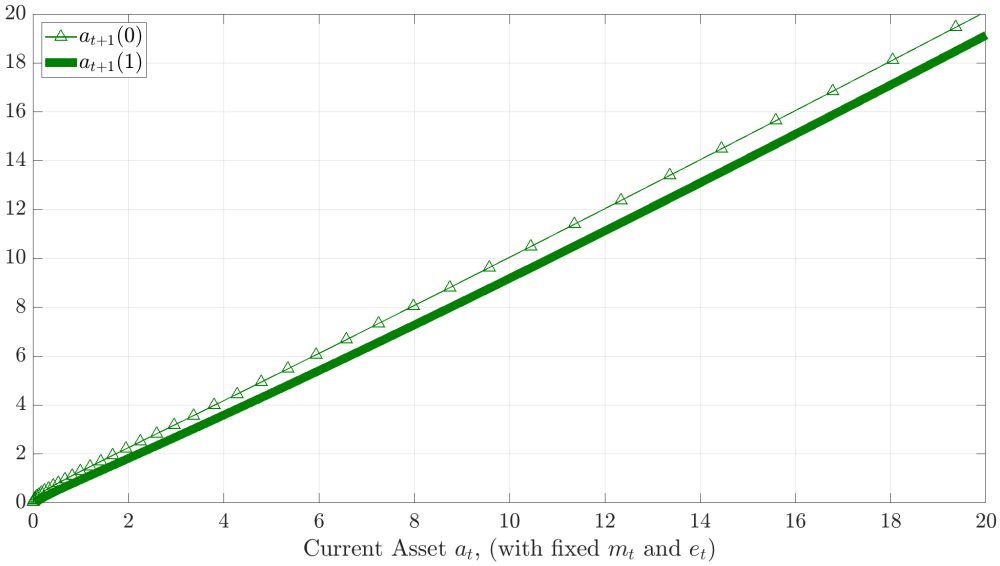


Figure 8: Future Assets Policy Functions in Steady States with and without Inflation

Each plot shows two lines, one that corresponds to an economy with no inflation, and the other to an economy with 100% steady-state annual inflation. At the household level, individuals respond to inflation by investing less in interest-bearing assets and more in cash, to compensate for the drop in the return of money and guarantee some consumption of the informal good in the future, exactly as the benchmark model predicts.

4.8 Aggregate Moments and Equilibrium Prices

To analyze the effect of long-run inflation on the economy, I conduct a policy experiment which consists of computing different steady-state moments for economies with different constant values of inflation. Figure 10 shows one of the main results of the paper: economies with higher steady-state inflation have a higher informal sector relative to the formal one, measured as the value of informal output relative to formal output. Going from 10% constant annual inflation to 90% increases informality over GDP from 40% to 54%, a sizeable impact. To compare this result with the regression line from Figure 1, I reconvert that line to inflation

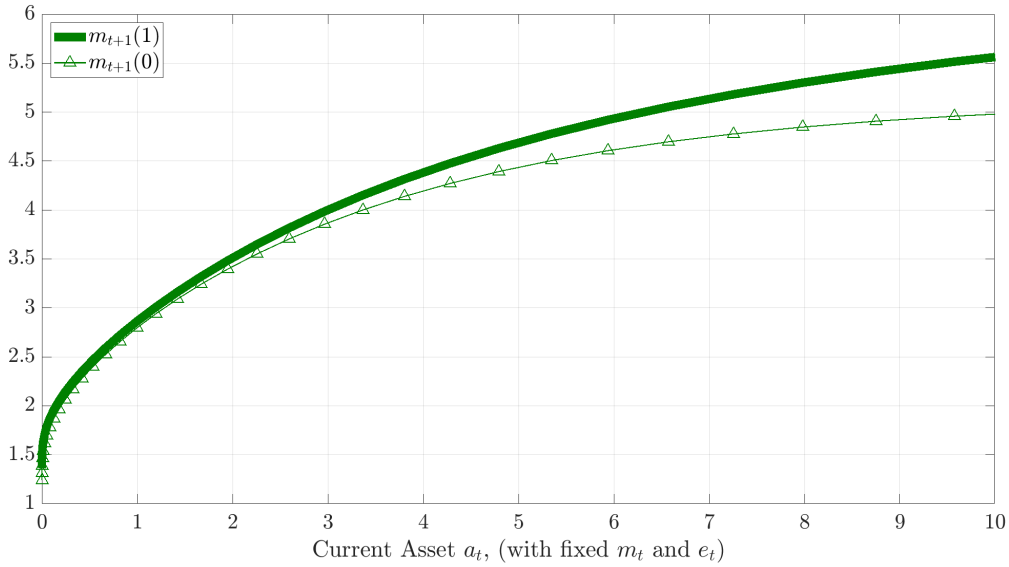


Figure 9: Future Cash Policy Functions in Steady States with and without Inflation

in levels[18] and plot it together with the model results across the different steady-state informality shares. The resulting comparison can be seen in Figure 11, where the model captures the cross-country relationship between inflation and informality for medium and high annual inflation levels. The curvature in the function that follows the data comes mechanically from the assumption of linearity in the regression in logs, and it is what affects the fit of the model for low inflation values.

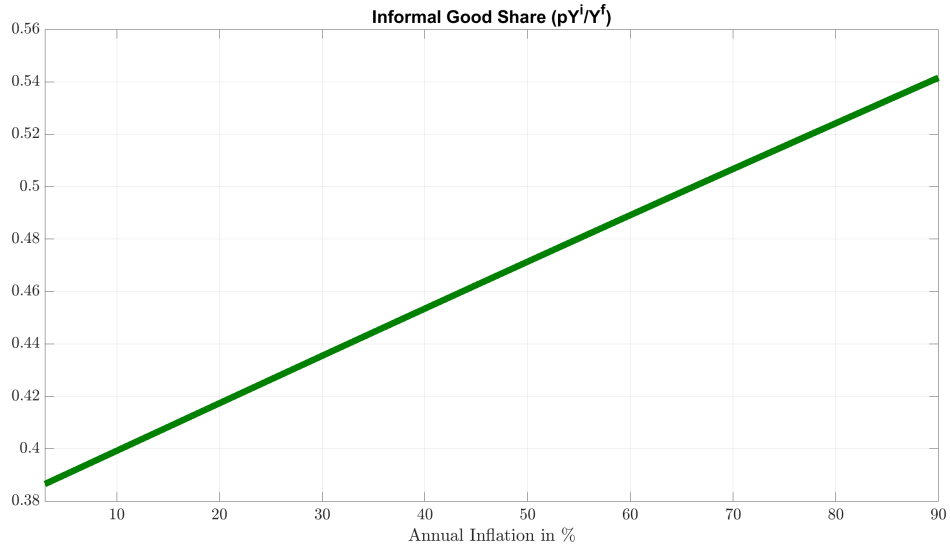


Figure 10: Informal Good Share over GDP

In the model, economies experiencing higher inflation have greater levels of informality due to a decline in aggregate capital, as illustrated in Figure 12. This outcome directly results from the policy functions

[18] If the original line is $Y = aX + b$, the transformed plot in levels shown in blue in Figure 11 is $Y = a \cdot \ln(1 + X) + b$.

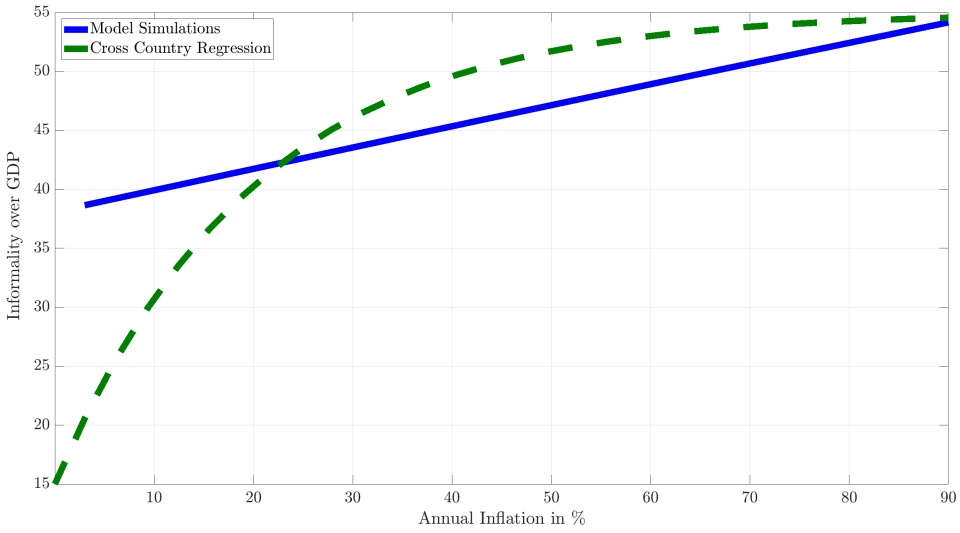


Figure 11: Model Simulations vs Cross-Country Regression Comparison

previously analyzed, where it was demonstrated that increased inflation discourages savings in interest-bearing assets. Going from 3% steady-state annual inflation to 90% translates into a drop in aggregate capital close to 9%, meaning that the cost of capital rises.

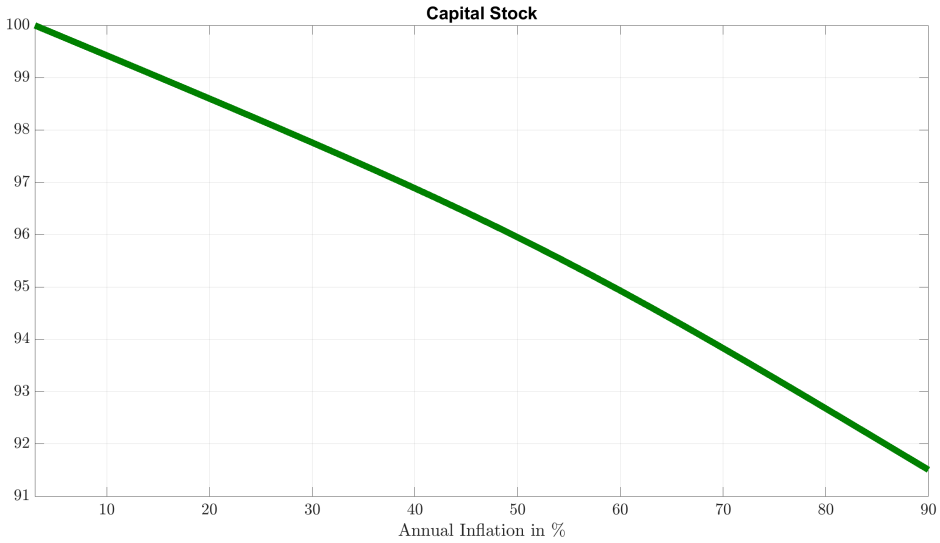


Figure 12: Capital Stock in the Steady State

An increase in the cost of capital directly decreases the relative price of goods from the informal sector, as illustrated in Figure 13. This encourages households to consume more informal goods, reinforcing their choice to save in cash rather than invest in assets, given the higher liquidity demands from increased informal consumption. Moreover, due to reduced overall capital in the economy, the equilibrium wage declines, as demonstrated in Figure 14, making it even more challenging for households to accumulate assets because their income from labor is diminished.



Figure 13: Informal Good Price in the Steady State

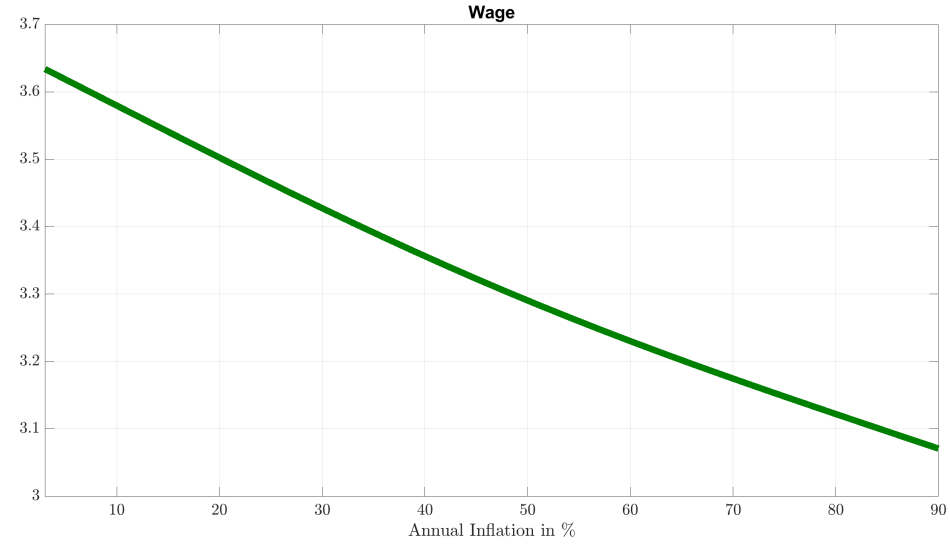


Figure 14: Wage in the Steady State

Finally, the immediate consequence of the relative increase in the demand for informal goods is factor reallocation of inputs of production (labor and capital) into the informal sector, as seen in Figure 15. This generates inefficiencies. First, more production is channeled into a sector that is hidden from the government and from which it cannot collect taxes. Second, the economy relies more on a sector that needs cash to operate and, as seen previously, this harms households' capacity to save in assets and accumulate capital. Third, the technology of production in the informal sector relies less on capital, so the economy cannot leverage its savings to achieve higher levels of production that translate into higher consumption and welfare.

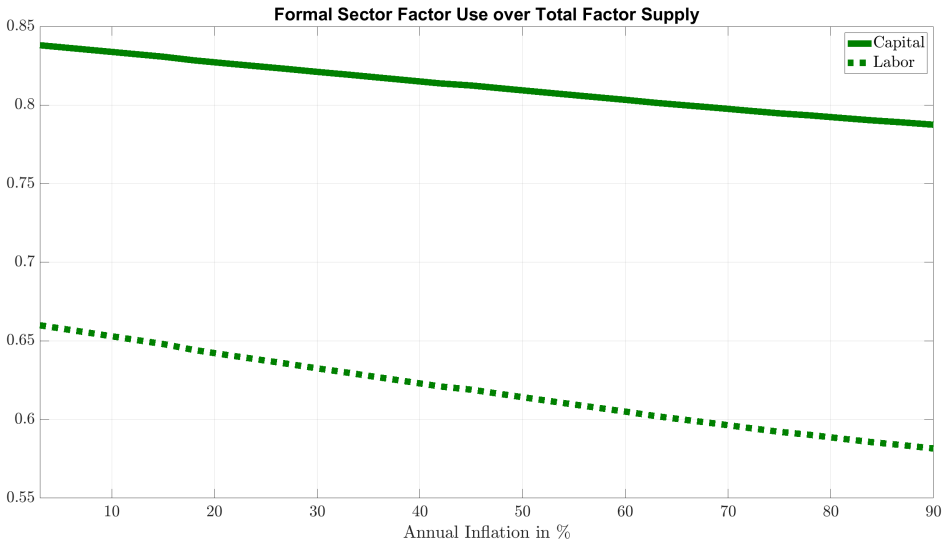


Figure 15: Sectoral Reallocation of Inputs of Production

5 Discussion

Because inflation diverts resources towards the production of goods traded informally, it causes inefficiencies, as the economy relies less on capital for production. The deterministic model makes this logic precise: the total effect of inflation on informality decomposes into a direct headwind, operating through the cash-in-advance constraint, and a general equilibrium tailwind, operating through the combined redistribution channel (CIA seigniorage and working-capital interest rebate) and the Stolper-Samuelson mechanism. The fact that the tailwind can dominate the headwind, and does so for empirically plausible capital gaps, means that inflation does not merely tax the informal sector but actively expands it. Being dependent on informal production reduces welfare for three reasons. Households face inflation tax costs due to cash holding instead of saving in interest-bearing assets, which affects poorer households the most. Also, the government loses tax revenue from hidden informal activities. Finally, formal capital-intensive production enables higher wages and output with enough capital accumulation. All this implies that using inflation to collect tax revenue from the informal sector without considering its impact on the sector's economic size can be misleading, causing monetary authorities to overestimate their appropriate inflation targets.

Given that the scale of the informal sector and households' portfolio composition choices are interlinked, it is important to evaluate and adapt policies to transition the economy from an undesirable equilibrium from a welfare perspective, with a sizable informal sector and extensive cash usage, to a favorable one, characterized

by a minor informal sector and more cashless payment options. The analysis of the joint region in the deterministic model suggests that such transitions are feasible: reducing inflation relaxes the seigniorage burden on savers, lowers the equilibrium interest rate, and through the same Stolper-Samuelson channel that drives the main result, raises the relative price of informal goods and contracts the informal sector. This has policy implications for government budget financing. For instance, lower inflation rates following a stabilization plan that reduces government financing needs could bolster the financial system and enhance the formal sector's sales, thereby increasing revenue through conventional taxation and compensating for reduced inflation tax collections.

6 Conclusion

This paper identifies a general equilibrium mechanism through which inflation increases informality via capital markets, reversing the prediction of the existing partial-equilibrium literature. The argument rests on three pillars: new reduced-form evidence, an analytical model that delivers exact conditions in primitive parameters, and a quantitative heterogeneous-agent model that confirms the mechanism is large enough to matter.

The empirical analysis documents a coherent triangle linking inflation, financial depth, and informality across 157 countries over 1980–2020. Countries with higher average inflation have larger informal sectors and shallower credit markets, even after conditioning on GDP per capita; controlling for credit absorbs roughly 37% of the inflation–informality association, consistent with a capital market channel operating alongside direct effects. To move beyond correlations, I exploit the staggered adoption of inflation targeting by 42 countries as a source of identifying variation for informality, to my knowledge a novel application. Using the Callaway and Sant'Anna (2021) estimator, I find that IT adoption reduces informality by 3–4 log points within 10–15 years, with parallel pre-trends validated over the five years preceding treatment ($p = 0.450$). The temporal ordering of the response is particularly informative: inflation falls immediately upon adoption, credit expands over 5–10 years, and informality declines only after 5–15 years. This lag structure is inconsistent with a direct behavioral response but is precisely what a mechanism operating through slow-moving capital accumulation would predict.

The theoretical framework formalizes this mechanism. In a tractable deterministic model, I show that the total effect of inflation on informality decomposes into a direct headwind, as inflation taxes cash and discourages informal consumption, and a general equilibrium tailwind, as inflation compresses savings through both the CIA seigniorage redistribution and the working-capital rebate channel, raises the interest rate, and through the Stolper-Samuelson mechanism increases the relative cost of capital-intensive formal production. The net effect is governed by a single sufficient statistic: the capital gap between formal and informal sectors. The main theorem identifies the exact threshold above which the tailwind dominates, expressed entirely in terms of primitive parameters, and establishes that this threshold is jointly compatible with equilibrium existence for empirically relevant elasticities of substitution. A fully calibrated heterogeneous-agent model then shows that increasing steady-state annual inflation from 10% to 90% reduces aggregate capital by 8% and raises informality as a share of GDP from 40% to 54%, capturing the cross-country relationship for moderate and high inflation levels.

The findings carry a sharp policy implication: using inflation to tax the informal sector is self-defeating. The general equilibrium response endogenously expands the very sector the policy aims to diminish. This challenges the normative conclusions of the optimal taxation literature, where seigniorage is a welfare-improving substitute for distortionary taxes, by showing that the reverse direction also matters, that inflation *generates*

informality through the credit channel. The same mechanism, however, implies that the reverse transition is feasible: reducing inflation relaxes the seigniorage burden on savers, lowers the cost of capital, and through the same Stolper-Samuelson channel contracts the informal sector, creating a virtuous cycle in which price stability, financial deepening, and formalization reinforce one another.

Several extensions would strengthen the analysis. The quantitative model currently abstracts from the working-capital channel (parametrized in the deterministic model by ν and ω_{WC}), endogenous entry along the lines of Melitz (2003) and Ulyssea (2018), and financial intermediation frictions following Gertler and Kiyotaki (2010). Each of these channels is expected to amplify the mechanism: working capital widens the cost wedge between sectors, endogenous entry adds an extensive margin of informality growth, and intermediation frictions endogenize the incomplete-markets wedge that drives the seigniorage redistribution channel. On the empirical side, the staggered DiD estimates identify the effect of a specific institutional reform, inflation targeting, rather than of inflation per se; extending the identification to other sources of exogenous variation in inflation, or to microdata on firm-level formalization decisions, would sharpen the causal interpretation and connect the aggregate evidence to the firm-level mechanisms formalized in the model.

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Appendix

A Why TWFE Fails with Staggered Adoption

A standard two-way fixed effects (TWFE) regression, $Y_{it} = \alpha_i + \delta_t + \beta \cdot IT_{it} + \varepsilon_{it}$, would be the natural starting point for estimating the average effect of IT adoption. A large literature has used propensity score matching and TWFE to evaluate the macroeconomic effects of IT: Lin and Ye (2007) find no significant effect on inflation in seven industrial countries, while Lin and Ye (2009) and Gonçalves and Salles (2008) find substantial effects in developing economies. More recently, Bambe (2023) uses similar methods to show that IT adoption increases private investment in developing countries by 2.8–3.3 percentage points. To my knowledge, however, no paper has used IT adoption as identifying variation for the effect on *informality*.

The existing IT evaluation literature has relied primarily on TWFE or propensity score matching, both of which assume homogeneous treatment effects across adoption cohorts, an assumption that the heterogeneity visible in Figure 2 calls into question. Recent work by Goodman-Bacon (2021), de Chaisemartin and d'Haultfœuille (2020), and Sun and Abraham (2021) has shown that TWFE regressions in staggered adoption settings can produce misleading estimates. The source of the bias is intuitive. Goodman-Bacon (2021) showed that the TWFE estimator $\hat{\beta}$ is a weighted average of all possible "2×2" difference-in-differences comparisons in the data. Some of these comparisons are clean: they compare a newly treated unit against a never-treated unit. But others are "forbidden": they compare a late adopter (say, India, which adopted IT in 2016) against an early adopter (say, New Zealand, which adopted in 1990). In the latter comparison, New Zealand's post-treatment trajectory, which *already reflects the effect of IT adoption*, serves as the counterfactual for India. If the treatment effect grows over time, this contaminates the estimate: New Zealand's declining informality post-1990 makes India's informality *look higher* by comparison, biasing the estimated treatment effect. This problem is directly relevant to Figure 2: the heatmap shows that early and late adopters have visibly different treatment intensities, exactly the heterogeneity that makes TWFE unreliable.

The Callaway and Sant'Anna (2021) estimator implemented in Section 2.5 avoids these pitfalls by using only never-treated countries as the comparison group, so that no already-treated country ever serves as a control.

B Staggered DiD: Worked Example and Inference

This appendix provides a detailed numerical walkthrough of the Callaway and Sant'Anna (2021) estimator described in Section 2.5, and explains how the confidence intervals in Figure 3 are constructed.

Computing $ATT(g,t)$: a concrete example. Consider informality for cohort $g = 1999$ (Brazil, Chile, Colombia), evaluated at calendar year $t = 2005$ (event time $e = t - g = +6$, i.e., six years after adoption). The base year is $g - 1 = 1998$. Using the panel data:

- **Treated group change:** The average ln(Informality) across Brazil (3.669), Chile (2.942), and Colombia (3.623) in 2005 is $\bar{Y}^{1999}_{2005} = 3.412$. In 1998 (the base year), the average across Brazil (3.711), Chile (2.961), and Colombia (3.639) is $\bar{Y}^{1999}_{1998} = 3.437$. The within-cohort change is $3.412 - 3.437 = -0.025$: informality fell by 2.5 log points over these seven years in the three countries.
- **Control group change:** The average ln(Informality) across the 116 never-treated countries in 2005 is $\bar{Y}^{\infty}_{2005} = 3.455$. In 1998, it is $\bar{Y}^{\infty}_{1998} = 3.477$. The change for the control group is $3.455 - 3.477 = -0.022$: informality also fell by 2.2 log points in the rest of the world.
- **ATT:** The difference-in-differences is $ATT(1999, 2005) = (-0.025) - (-0.022) = -0.004$. That is, Brazil, Chile, and Colombia experienced a 0.4 log-point decline in informality *beyond* what never-treated countries experienced over the same calendar years.

The same calculation applies to pre-treatment periods to test whether the parallel trends assumption holds. For the same cohort at $t = 1995$ (event time $e = -4$, four years *before* adoption):

- **Treated group change (backward):** $\bar{Y}^{1999}_{1995} - \bar{Y}^{1999}_{1998} = 3.433 - 3.437 = -0.004$.
- **Control group change (backward):** $\bar{Y}^{\infty}_{1995} - \bar{Y}^{\infty}_{1998} = 3.486 - 3.477 = +0.009$.
- **ATT:** $ATT(1999, 1995) = (-0.004) - (0.009) = -0.013$.

Under the parallel trends null, this pre-treatment ATT should be zero. A non-zero value would indicate that Brazil, Chile, and Colombia were already on a differential informality trajectory relative to the rest of the world before they adopted IT, which would cast doubt on the identifying assumption.

Aggregation across cohorts. At event time $e = +6$, ten cohorts contribute to the aggregated estimate $\hat{\theta}_{ES}(+6)$ defined in equation (5). Four of the largest are:

- Cohort $g = 1993$ (Australia, Finland, Sweden; $n_g = 3$): evaluated at calendar year $1993 + 6 = 1999$. $ATT(1993, 1999) = -0.075$.
- Cohort $g = 1999$ (Brazil, Chile, Colombia; $n_g = 3$): evaluated at calendar year 2005. $ATT(1999, 2005) = -0.004$.
- Cohort $g = 2001$ (Hungary, Iceland, Korea, Mexico, Norway; $n_g = 5$): evaluated at calendar year 2007. $ATT(2001, 2007) = -0.006$.
- Cohort $g = 2005$ (Guatemala, Indonesia, Romania; $n_g = 3$): evaluated at calendar year 2011. $ATT(2005, 2011) = -0.008$.

The remaining six cohorts ($g \in \{1997, 2000, 2002, 2006, 2009, 2011\}$) also contribute, and the full cohort-size-weighted average across all ten yields $\hat{\theta}_{ES}(+6) = -0.018$: six years after IT adoption, the average country experienced a 1.8 log-point decline in informality relative to never-treated countries. The heterogeneity across cohorts is notable: the early adopters ($g = 1993$) show a much larger effect (-0.075) than the later adopters, consistent with the pattern visible in Figure 2 and with their sharper inflation break and stronger treatment intensity.

Confidence intervals. The event-study estimates $\hat{\theta}_{ES}(e)$ reported in Figure 3 are point estimates computed from the observed sample of 158 countries. To assess their statistical precision, I construct confidence intervals via a cluster bootstrap at the country level (200 replications).

The procedure works as follows. The original dataset contains 158 countries, each observed as a complete time series: 42 are IT adopters (treated) and 116 are never-treated (control). In each bootstrap replication $b = 1, \ldots, 200$:

1. *Resample treated countries with replacement.* Draw 42 countries from the set of 42 treated countries, allowing duplicates. For example, one draw might include Brazil twice, Chile once, Colombia zero times, and so on. When a country is drawn, its entire time series comes with it: all years of inflation, credit, and informality data.
2. *Resample control countries with replacement.* Independently draw 116 countries from the set of 116 never-treated countries, again with replacement and including each selected country's full time series.
3. *Recompute the full estimation chain.* Using the bootstrap sample from steps 1–2, redefine the cohorts (some may now have different sizes because of duplication or omission), recompute every $ATT^{(b)}(g, t)$ using equation (4), and reaggregate into $\hat{\theta}^{(b)}_{ES}(e)$ using equation (5).

After 200 replications, each event time e has a distribution of 200 bootstrap estimates $\{\hat{\theta}^{(1)}_{ES}(e), \ldots, \hat{\theta}^{(200)}_{ES}(e)\}$. The standard deviation of this distribution is the bootstrap standard error $\hat{se}(e)$.

Two types of confidence intervals are reported. *Pointwise* 95% intervals ($\hat{\theta}_{ES}(e) \pm 1.96 \cdot \hat{se}(e)$) assess significance at each individual event time. *Simultaneous* 99% bands ($\hat{\theta}_{ES}(e) \pm 2.58 \cdot \hat{se}(e)$) use a more conservative critical value to account for the fact that we are testing at roughly 30 event times simultaneously; a Bonferroni-style correction ensures that the probability of *any* coefficient falling outside the band by chance alone remains below 1%.

The key feature of this procedure is that resampling occurs at the country level, not the country-year level. If Brazil is drawn, all of Brazil's data from 1980 to 2020 enter the bootstrap sample together. This preserves the within-country serial correlation structure: the fact that Brazil's informality in 2005 is correlated with its informality in 2004 is maintained within each bootstrap draw. Had we instead resampled individual country-year observations, we would have broken this temporal dependence and understated the true uncertainty in the estimates. Resampling treated and control countries separately ensures that each bootstrap sample maintains the same treatment-control structure as the original data.

C Solution of the Household Problem

This appendix derives the closed-form solution of the household problem under incomplete markets, stated in Section 3.6 of the main text. The derivation proceeds in six steps: from the first-order conditions to the full set of allocations expressed as functions of a single scale variable $\tilde{c} \equiv c^{L,i}$.

C.1 First-Order Conditions

Recall the CES composite (6). Its partial derivatives are

$$\frac{\partial C}{\partial c^f} = \phi^{1/\eta}\, C^{1/\eta}\, (c^f)^{-1/\eta}, \qquad \frac{\partial C}{\partial c^i} = (1-\phi)^{1/\eta}\, C^{1/\eta}\, (c^i)^{-1/\eta}. \tag{55}$$

Combined with $u'(C) = C^{-\sigma}$, the marginal utilities of the two goods are

$$\frac{\partial u}{\partial c^f} = \phi^{1/\eta}\, C^{1/\eta-\sigma}\, (c^f)^{-1/\eta}, \qquad \frac{\partial u}{\partial c^i} = (1-\phi)^{1/\eta}\, C^{1/\eta-\sigma}\, (c^i)^{-1/\eta}. \tag{56}$$

Let λ^s denote the multiplier on the budget constraint (12)–(16), ψ^s the multiplier on the cash-in-advance constraint (13)–(17), φ_a^s the multiplier on $a' \geq 0$, and φ_m^s the multiplier on $m' \geq 0$, all in state $s \in \{H, L\}$.

Intratemporal FOCs (state s). Differentiating with respect to $c^{s,f}$ and $c^{s,i}$:

$$\text{Formal good:} \quad \phi^{1/\eta}\, (C^s)^{1/\eta-\sigma}\, (c^{s,f})^{-1/\eta} = \lambda^s, \tag{57}$$

$$\text{Informal good:} \quad (1-\phi)^{1/\eta}\, (C^s)^{1/\eta-\sigma}\, (c^{s,i})^{-1/\eta} = (\lambda^s + \psi^s)\, p. \tag{58}$$

The informal good carries the additional shadow cost ψ^s because spending on it is constrained by the cash-in-advance requirement.

Envelope conditions. The value functions satisfy

$$V_a^s(a,m) = \lambda^s\, R, \qquad V_m^s(a,m) = \frac{\lambda^s + \psi^s}{1+\mu}. \tag{59}$$

Intertemporal FOCs. Differentiating with respect to a' and m', using the envelope conditions for the continuation value:

$$\text{Bond } (H): \quad \lambda^H = \beta\, R\, \lambda^L + \varphi_a^H, \tag{60}$$

$$\text{Bond } (L): \quad \lambda^L = \beta\, R\, \lambda^H + \varphi_a^L, \tag{61}$$

$$\text{Money } (H): \quad \lambda^H = \frac{\beta\,(\lambda^L + \psi^L)}{1+\mu} + \varphi_m^H, \tag{62}$$

$$\text{Money } (L): \quad \lambda^L = \frac{\beta\,(\lambda^H + \psi^H)}{1+\mu} + \varphi_m^L. \tag{63}$$

Complementary slackness.

$$\psi^s \left[\frac{m}{1+\mu} - p\, c^{s,i}\right] = 0, \quad \varphi_a^s\, a'^s = 0, \quad \varphi_m^s\, m'^s = 0, \qquad s \in \{H, L\}. \tag{64}$$

C.2 Complete vs. Incomplete Markets

Proposition 1 (Complete-markets benchmark)**.** *If $\beta R = 1$ and no borrowing constraint binds ($\varphi_a^s = \varphi_m^s = 0$ for $s = H, L$), then the stationary allocations are identical across states:*

$$c^{H,f} = c^{L,f}, \qquad c^{H,i} = c^{L,i}, \qquad C^H = C^L.$$

Proof. With $\beta R = 1$ and $\varphi_a^s = 0$, the bond Euler equations (60)–(61) give $\lambda^H = \lambda^L \equiv \lambda$. With $\varphi_m^s = 0$, the money Euler equations then imply $\psi^H = \psi^L = \lambda\big[(1+\mu)/\beta - 1\big]$. Since both states face the same $(\lambda, \psi, p, \phi, \eta, \sigma)$, the intratemporal FOCs (57)–(58) deliver identical allocations. □

Proposition 2 (Incomplete markets)**.** *If $\beta R < 1$ and $e^H - e^L$ is sufficiently large, then in any stationary equilibrium the no-borrowing constraint binds for the low-income household ($a'^L = 0$, $\varphi_a^L > 0$) and:*

$$\lambda^H < \lambda^L, \qquad C^H > C^L.$$

Proof. Suppose $\varphi_a^H = \varphi_a^L = 0$. Then (60)–(61) give $\lambda^H = \beta R\, \lambda^L$ and $\lambda^L = \beta R\, \lambda^H$, so $\lambda^H = (\beta R)^2\, \lambda^H$, requiring $\beta R = 1$—a contradiction. Hence at least one constraint binds. For large $e^H - e^L$, the H-household wants to save (not borrow), so its constraint is slack: $\varphi_a^H = 0$, $a'^H > 0$. The L-household is constrained: $a'^L = 0$, $\varphi_a^L > 0$. From (60): $\lambda^H = \beta R\, \lambda^L < \lambda^L$. □

C.3 Maintained Assumptions

The remaining derivations maintain the following configuration, which is verified at every equilibrium constructed in the paper (Proposition 3 below):

(i) $\beta R < 1$ (incomplete markets);
(ii) $a'^H > 0$, $a'^L = 0$ (borrowing constraint binds only in L);
(iii) $m'^H > 0$, $m'^L > 0$ (both types carry positive money balances);
(iv) $\psi^H > 0$, $\psi^L > 0$ (cash-in-advance constraint binds in both states).

C.4 Step 1: Shadow Price Ratios

Under (i)–(iii), the active Euler equations are:

$$\lambda^H = \beta\, R\, \lambda^L, \tag{65}$$

$$\lambda^H = \frac{\beta\,(\lambda^L + \psi^L)}{1+\mu}, \tag{66}$$

$$\lambda^L = \frac{\beta\,(\lambda^H + \psi^H)}{1+\mu}. \tag{67}$$

CIA multiplier in state L. Equating (65) and (66):

$$\beta R\, \lambda^L = \frac{\beta\,(\lambda^L + \psi^L)}{1+\mu} \quad \Longrightarrow \quad \psi^L = \big[R(1+\mu) - 1\big]\, \lambda^L.$$

Hence:

$$\frac{\psi^L}{\lambda^L} = R(1+\mu) - 1. \tag{68}$$

CIA multiplier in state H. Substituting $\lambda^L = \lambda^H/(\beta R)$ from (65) into (67):

$$\frac{\lambda^H}{\beta R} = \frac{\beta\,(\lambda^H + \psi^H)}{1+\mu} \quad \Longrightarrow \quad \psi^H = \left[\frac{1+\mu}{\beta^2 R} - 1\right] \lambda^H.$$

Hence:

$$\frac{\psi^H}{\lambda^H} = \frac{1+\mu-\beta^2 R}{\beta^2 R}. \tag{69}$$

Remark C.1 (Asymmetric CIA tightness)**.** *Since $\beta R < 1$:*

$$\frac{\psi^H}{\lambda^H} = \frac{1+\mu}{\beta^2 R} - 1 > R(1+\mu) - 1 = \frac{\psi^L}{\lambda^L},$$

where the inequality follows from $1/(\beta^2 R) > R \iff (\beta R)^2 < 1$. *The H-household's CIA constraint is tighter in shadow-price terms: the money it spends was saved during the L state when resources were scarce.*

C.5 Step 2: Intratemporal Consumption Ratios

Definition 2 (Formal-to-informal ratio)**.** *For each state* $s \in \{H, L\}$*, define*

$$\xi^s \equiv \frac{c^{s,f}}{c^{s,i}}.$$

Dividing (57) by (58) in state s:

$$\left(\frac{\phi}{1-\phi}\right)^{1/\eta} (\xi^s)^{-1/\eta} = \frac{\lambda^s}{(\lambda^s + \psi^s)\, p} = \frac{1}{\left(1 + \psi^s/\lambda^s\right) p}.$$

Solving for ξ^s:

$$\xi^s = \frac{\phi}{1-\phi}\left[\left(1 + \frac{\psi^s}{\lambda^s}\right) p\right]^{\eta}. \tag{70}$$

Substituting the shadow-price ratios (68)–(69):

$$\xi^H = \frac{\phi}{1-\phi}\left[\frac{(1+\mu)\, p}{\beta^2 R}\right]^{\eta}, \tag{71}$$

$$\xi^L = \frac{\phi}{1-\phi}\left[R(1+\mu)\, p\right]^{\eta}. \tag{72}$$

The ratio of the two is a structural constant:

$$\frac{\xi^H}{\xi^L} = \left(\frac{1}{\beta^2 R^2}\right)^{\eta} = (\beta^2 R^2)^{-\eta} > 1, \tag{73}$$

confirming that the H-household consumes a higher ratio of formal to informal goods, as a direct consequence of the asymmetric CIA tightness (Remark C.1).

C.6 Step 3: Cross-State Informal Consumption Ratio

Definition 3 (Informal consumption gap)**.**

$$\xi^g \equiv \frac{c^{H,i}}{c^{L,i}}.$$

To determine ξ^g, we exploit the bond Euler equation linking marginal utilities across states.

Expressing λ^s via the FOC for c^f. From (57):

$$\lambda^s = \phi^{1/\eta}\, (C^s)^{1/\eta - \sigma}\, (c^{s,f})^{-1/\eta}.$$

Using $c^{s,f} = \xi^s\, c^{s,i}$ and $C^s = c^{s,i}\, \Phi(\xi^s)$, where

$$\Phi(\xi) \equiv \frac{C}{c^i} = \left[\phi^{1/\eta}\, \xi^{\frac{\eta-1}{\eta}} + (1-\phi)^{1/\eta}\right]^{\frac{\eta}{\eta-1}} \tag{74}$$

is the composite-to-informal ratio, we obtain:

$$\lambda^s = \phi^{1/\eta}\left[\Phi(\xi^s)\right]^{1/\eta - \sigma} (\xi^s)^{-1/\eta}\, (c^{s,i})^{-\sigma}. \tag{75}$$

Taking the ratio across states. Using $\lambda^H = \beta R\, \lambda^L$ from (65):

$$\frac{\left[\Phi(\xi^H)\right]^{1/\eta-\sigma} (\xi^H)^{-1/\eta} (c^{H,i})^{-\sigma}}{\left[\Phi(\xi^L)\right]^{1/\eta-\sigma} (\xi^L)^{-1/\eta} (c^{L,i})^{-\sigma}} = \beta R.$$

Define the **composition adjustment factor**:

$$\Gamma \equiv \frac{\left[\Phi(\xi^H)\right]^{1/\eta-\sigma} (\xi^H)^{-1/\eta}}{\left[\Phi(\xi^L)\right]^{1/\eta-\sigma} (\xi^L)^{-1/\eta}}. \tag{76}$$

This depends only on the primitives $(\phi, \eta, \sigma, \beta, R, \mu, p)$ through ξ^H and ξ^L. The Euler equation becomes $\Gamma \cdot (\xi^g)^{-\sigma} = \beta R$, yielding:

$$\xi^g = \left(\frac{\Gamma}{\beta R}\right)^{1/\sigma}. \tag{77}$$

Remark C.2 (Decomposition of Γ)**.** *Define the CES price-index ratio $\tau_\phi \equiv \Phi(\xi^H)/\Phi(\xi^L) > 1$ (since Φ is increasing and $\xi^H > \xi^L$). Using the structural ratio (73):*

$$(\xi^H/\xi^L)^{-1/\eta} = \beta^2 R^2.$$

Therefore:

$$\Gamma = \tau_\phi^{1/\eta-\sigma} \cdot \beta^2 R^2, \tag{78}$$

and

$$\xi^g = \left(\tau_\phi^{1/\eta-\sigma} \cdot \beta R\right)^{1/\sigma}. \tag{79}$$

Remark C.3 (Log–Cobb–Douglas limit)**.** *When $\sigma = \eta = 1$: $\Phi(\xi) = \xi^\phi$ and the exponent $1/\eta - \sigma = 0$, so $\tau_\phi^0 = 1$ and $\Gamma = \beta^2 R^2$, giving $\xi^g = \beta R$—a constant independent of ϕ. More generally, when $\sigma = 1$ and $\eta > 1$: $\xi^g = \tau_\phi^{-(\eta-1)/\eta} \cdot \beta R$, which satisfies*

$$(\beta R)^{2\eta-1} \le \xi^g \le \beta R, \tag{80}$$

with the lower bound attained as $\phi \to 1$ and the upper bound as $\phi \to 0$. Both bounds are strictly below one when $\beta R < 1$.

C.7 Step 4: Budget Constraints in Terms of $\tilde{c} \equiv c^{L,i}$

With the CIA binding in both states (condition (iv)):

$$p\, c^{H,i} = \frac{m^H}{1+\mu}, \qquad p\, c^{L,i} = \frac{m^L}{1+\mu}. \tag{81}$$

Stationarity links. In the deterministic alternating equilibrium, the money that the H-household spends was saved by the same household when it was in state L last period, and vice versa:

$$m^H = m'^L, \qquad m^L = m'^H. \tag{82}$$

Hence, using (81):

$$m'^H = p\, c^{L,i}\,(1+\mu) = p\,(1+\mu)\,\tilde{c}, \qquad m'^L = p\, c^{H,i}\,(1+\mu) = p\,(1+\mu)\,\xi^g\,\tilde{c}. \tag{83}$$

Similarly for bonds: $a'^L = 0$ implies $a^H = 0$. The L-household enters with $a^L = a'^H$.

Budget constraint for H. Substituting $a^H = 0$, $c^{H,f} = \xi^H\, c^{H,i} = \xi^H \xi^g\, \tilde{c}$, and the binding CIA (which implies that the informal-consumption expenditure $p\, c^{H,i}$ cancels with the term $m^H/(1+\mu)$ on the right-hand

side), together with $m'^H = p\,(1+\mu)\,\tilde{c}$:

$$\xi^H\, c^{H,i} + \underbrace{p\, c^{H,i}}_{\text{informal}} + a'^H + p\,(1+\mu)\,\tilde{c} = w\, e^H + T^H + \underbrace{\frac{m^H}{1+\mu}}_{=p\,c^{H,i}}. \tag{84}$$

The informal consumption expenditures cancel on both sides, leaving:

$$\xi^H \xi^g\, \tilde{c} + a'^H + p\,(1+\mu)\,\tilde{c} = w\, e^H + T^H. \tag{85}$$

Budget constraint for L. Substituting $a'^L = 0$, $a^L = a'^H$, $c^{L,f} = \xi^L\,\tilde{c}$, and $m'^L = p\,(1+\mu)\,\xi^g\,\tilde{c}$; again the informal terms cancel:

$$\xi^L\,\tilde{c} + p\,(1+\mu)\,\xi^g\,\tilde{c} = R\, a'^H + w\, e^L + T^L. \tag{86}$$

C.8 Step 5: CIA Seigniorage Transfers and WC Rebate in Terms of $\tilde{c}$

CIA seigniorage. From the binding CIA constraints and stationarity (81)–(82), real money balances are:

$$m^H = p\,(1+\mu)\,\xi^g\,\tilde{c}, \qquad m^L = p\,(1+\mu)\,\tilde{c}. \tag{87}$$

Hence $m^H + m^L = p\,(1+\mu)\,(1+\xi^g)\,\tilde{c}$, and total seigniorage is:

$$\Omega = \mu\, p\,(1+\xi^g)\,\tilde{c}. \tag{88}$$

$$T^H_\Omega = \omega_T\,\mu\, p\,(1+\xi^g)\,\tilde{c}, \qquad T^L_\Omega = (1-\omega_T)\,\mu\, p\,(1+\xi^g)\,\tilde{c}. \tag{89}$$

Working-capital profit rebate. From $wL_f = (1-\alpha_f)y_f/\Delta_w$ and $y_f = \Upsilon_f \tilde{c}$, the intermediary's profit is:

$$\mu R \cdot \nu w L_f = \nu(1-\alpha_f)\Upsilon_f\,\frac{\mu R}{\Delta_w}\,\tilde{c}.$$

Defining the first-order coefficient $\kappa^{WC} \equiv R\nu(1-\alpha_f)\Upsilon_f/\Delta_w|_{\mu=0}$ (derived in Appendix D.8):

$$\mu R \cdot \nu w L_f = \kappa^{WC}\,\mu\,\tilde{c} + O(\mu^2). \tag{90}$$

$$T^H_{WC} = \omega_{WC}\,\kappa^{WC}\,\mu\,\tilde{c}, \qquad T^L_{WC} = (1-\omega_{WC})\,\kappa^{WC}\,\mu\,\tilde{c}. \tag{91}$$

Both transfer streams are proportional to $\tilde{c}$ and are absorbed into the expenditure indices in Step 6. The total transfer to each household is $T^s \equiv T^s_\Omega + T^s_{WC}$.

C.9 Step 6: Closed-Form Allocations

Substituting both transfer streams (89)–(91) into (85)–(86) and collecting $\tilde{c}$ terms on the left:

***H*-budget:**

$$\underbrace{\left[\xi^H\xi^g + p(1+\mu) - \omega_T\,\mu\, p\,(1+\xi^g) - \omega_{WC}\,\kappa^{WC}\,\mu\right]}_{\equiv\ D^H}\,\tilde{c} \;+\; a'^H \;=\; w\, e^H. \tag{92}$$

***L*-budget:**

$$\underbrace{\left[\xi^L + p(1+\mu)\,\xi^g - (1-\omega_T)\,\mu\, p\,(1+\xi^g) - (1-\omega_{WC})\,\kappa^{WC}\,\mu\right]}_{\equiv\ D^L}\,\tilde{c} \;=\; R\, a'^H + w\, e^L. \tag{93}$$

The **net expenditure indices** per unit of $\tilde{c}$ in each state are therefore:

$$D^H \equiv \xi^H \xi^g + p(1+\mu) - \omega_T \, \mu \, p \, (1+\xi^g) - \omega_{WC} \, \kappa^{WC} \, \mu, \tag{94}$$

$$D^L \equiv \xi^L + p(1+\mu) \, \xi^g - (1-\omega_T) \, \mu \, p \, (1+\xi^g) - (1-\omega_{WC}) \, \kappa^{WC} \, \mu. \tag{95}$$

Each index contains four terms: formal consumption expenditure per $\tilde{c}$; the cost of carrying money to finance informal consumption next period; the CIA seigniorage rebate (negative); and the WC interest rebate (negative). At $\mu = 0$: $D^H = \xi^H \xi^g + p$ and $D^L = \xi^L + p\xi^g$, leaving all calibration targets unaffected.

Savings. Directly from the H-budget (92):

$$a'^H = w \, e^H - D^H \, \tilde{c}. \tag{96}$$

Solving for $\tilde{c}$. Substituting (96) into the L-budget (93):

$$D^L \, \tilde{c} = R\big(w \, e^H - D^H \, \tilde{c}\big) + w \, e^L \quad \Longrightarrow \quad \big(D^L + R \, D^H\big) \, \tilde{c} = w \, (R \, e^H + e^L).$$

Dividing both sides by R:

$$\underbrace{\left(D^H + \frac{D^L}{R}\right)}_{\equiv \; \widetilde{D}} \tilde{c} = w\left(e^H + \frac{e^L}{R}\right).$$

Define the **present-value wage income**:

$$\bar{Y} \equiv w\left(e^H + \frac{e^L}{R}\right), \tag{97}$$

and the **total net expenditure index**:

$$\widetilde{D} \equiv D^H + \frac{D^L}{R}. \tag{98}$$

Then:

$$\boxed{\tilde{c} \equiv c^{L,i} = \frac{\bar{Y}}{\widetilde{D}}.} \tag{99}$$

Remark C.4 (Comparison with pre-transfer expression)**.** *Before substituting transfers, one would write $\tilde{c} = Y/D$ with $Y = \bar{Y} + T^H + T^L/R$ including transfer income. Since transfers are proportional to $\tilde{c}$, the seigniorage terms cancel between numerator and denominator, yielding the cleaner expression $\bar{Y}/\widetilde{D}$, in which the income side is purely wage income.*

C.10 Complete List of Allocations

All allocations follow in closed form from $\tilde{c}$:

$$c^{L,i} = \tilde{c} = \frac{\bar{Y}}{\widetilde{D}}, \tag{100}$$

$$c^{H,i} = \xi^g \, \tilde{c} = \frac{\xi^g \, \bar{Y}}{\widetilde{D}}, \tag{101}$$

$$c^{L,f} = \xi^L \, \tilde{c} = \frac{\xi^L \, \bar{Y}}{\widetilde{D}}, \tag{102}$$

$$c^{H,f} = \xi^H \xi^g \, \tilde{c} = \frac{\xi^H \xi^g \, \bar{Y}}{\widetilde{D}}, \tag{103}$$

$$a'^H = w \, e^H - D^H \, \frac{\bar{Y}}{\widetilde{D}}, \tag{104}$$

$$m'^H = p \, (1+\mu) \, \tilde{c} = \frac{p \, (1+\mu) \, \bar{Y}}{\widetilde{D}}, \tag{105}$$

$$m'^L = p \, (1+\mu) \, \xi^g \, \tilde{c} = \frac{p \, (1+\mu) \, \xi^g \, \bar{Y}}{\widetilde{D}}. \tag{106}$$

C.11 Which Household Holds More Money?

Lemma 3. *The ratio of money holdings is $m^H/m^L = \xi^g$. Under $\sigma\eta \geq 1$ (the empirically dominant case with $\sigma \geq 1$ and $\eta > 1$):*

$$\xi^g < 1 \quad \text{for all } \phi \in (0,1). \tag{107}$$

That is, the low-income household holds more money unconditionally with respect to ϕ.

Remark C.5 (Economic intuition)**.** *The H-household is richer but faces a tighter CIA constraint on informal goods in shadow-price terms ($\psi^H/\lambda^H > \psi^L/\lambda^L$). Two forces compete: an income effect (wealth raises demand for all goods including informal ones requiring cash) and a CIA substitution effect (the higher effective price of informal goods shifts consumption toward formal goods). When $\sigma\eta \geq 1$, the substitution effect dominates for every ϕ: the H-household always consumes fewer informal goods than L and therefore holds less money. The role of ϕ is quantitative, not qualitative—lower ϕ narrows the gap, but the sign $m^H < m^L$ is invariant.*

C.12 Regime Verification

Proposition 3 (Regime verification)**.** *At the equilibrium constructed by Theorem 1 (with endowment ratio ρ^*, rental rate r_κ), the maintained assumptions (i)–(iv) of Section C.3 are satisfied.*

Proof.

(i) By construction, $R_\kappa = 1/\beta - \kappa$, so $\beta R_\kappa = 1 - \beta\kappa < 1$ for any $\kappa > 0$.

(ii) From the bond Euler equations under $\beta R < 1$: if both borrowing constraints were slack, then $\lambda^H = (\beta R)^2 \lambda^H$, contradicting $\beta R < 1$ (Proposition 2). With $\rho^* > 1$, the high-income household earns more and saves; the low-income household is constrained. Formally, $\varphi_a^L = \lambda^L(1 - \beta^2 R_\kappa^2) > 0$ since $\beta R_\kappa < 1$, confirming $a'^L = 0$. Meanwhile $a'^H > 0$ is the asset supply that clears the market: $a'^H = S(\rho^*) \, \tilde{c} = \mathcal{A} \, \tilde{c} > 0$.

(iii) Both household types value the informal good ($\phi < 1$), so optimal informal consumption is strictly positive: $c^{s,i} > 0$. With CIA binding (part (iv)), the household must carry cash: $m'^s = p(1+\mu) \, c^{s',i} > 0$ where s' is the state succeeding s.

(iv) The shadow CIA ratios are $\psi^L/\lambda^L = R_\kappa(1+\mu) - 1$ and $\psi^H/\lambda^H = (1+\mu)/(\beta^2 R_\kappa) - 1$. For $\mu > 0$ and $r_\kappa > 0$: the first ratio satisfies $R_\kappa(1+\mu) > R_\kappa > 1$ (since $R_\kappa = 1 + r_\kappa - \delta > 1$ for $r_\kappa > \delta$), giving $\psi^L > 0$; the second satisfies $(1+\mu)/(\beta^2 R_\kappa) > 1/(\beta^2 R_\kappa) > 1$ (since $\beta^2 R_\kappa < \beta < 1$), giving $\psi^H > 0$. □

C.13 Summary of Determined Objects

The following objects are determined purely by primitives $(\phi, \eta, \sigma, \beta, R, \mu, p, w, e^H, e^L, \omega_T, \omega_{WC})$:

Object	Formula	Eq.
Shadow CIA ratio (L)	$\psi^L/\lambda^L = R(1+\mu) - 1$	(68)
Shadow CIA ratio (H)	$\psi^H/\lambda^H = \frac{1+\mu-\beta^2 R}{\beta^2 R}$	(69)
Formal/informal ratio (H)	$\xi^H = \frac{\phi}{1-\phi}\left[\frac{(1+\mu)\,p}{\beta^2 R}\right]^\eta$	(71)
Formal/informal ratio (L)	$\xi^L = \frac{\phi}{1-\phi}\left[R(1+\mu)\,p\right]^\eta$	(72)
Informal gap	$\xi^g = \left(\Gamma/(\beta R)\right)^{1/\sigma}$	(77)
Net expenditure index (H)	$D^H = \xi^H \xi^g + p(1+\mu) - \omega_T \mu p(1+\xi^g) - \omega_{WC}\kappa^{WC}\mu$	(94)
Net expenditure index (L)	$D^L = \xi^L + p(1+\mu)\xi^g - (1-\omega_T)\mu p(1+\xi^g) - (1-\omega_{WC})\kappa^{WC}\mu$	(95)
Total net index	$\widetilde{D} = D^H + D^L/R$	(98)
PV wage income	$\bar{Y} = w(e^H + e^L/R)$	(97)
Scale variable	$\tilde{c} = c^{L,i} = \bar{Y}/\widetilde{D}$	(99)

Once the prices (r, w, p) are pinned by the supply side (Appendix D), all objects above become functions of (r, μ) and the structural parameters alone. The equilibrium then reduces to a single equation in r for each μ, derived in Appendix D.

D Equilibrium Characterization

This appendix derives the single equilibrium condition stated in Section 3.7 of the main text. Starting from the market clearing conditions and the household allocations of Appendix C, it shows that both asset supply and asset demand are proportional to the scale variable $\tilde{c} = c^{L,i}$, so that the scale cancels and the entire general equilibrium reduces to a single equation in r for each μ.

D.1 Asset Market Clearing

Asset supply. As established in Appendix C, the borrowing constraint binds for the low-income household $(a'^L = 0)$. With a mass one of households in each income state at every date, total asset supply comes entirely from the H-households:

$$\mathcal{A}^s \equiv a'^H. \tag{108}$$

Asset demand. Financial intermediaries allocate household savings to two uses:

(1) *Physical capital*: $K_f + K_i$, rented to firms at rate r.

(2) *Working-capital loans*: $\nu\, w\, L_f$, lent intra-period to formal firms to finance the advance wage payment at gross nominal rate $R^n = (1+\mu)\, R$.

Even though working-capital loans are repaid within the period, the intermediary must hold the resources at the start of production—they cannot simultaneously be invested in physical capital. Total asset demand is therefore:

$$\mathcal{A}^d \equiv K_f + K_i + \nu\, w\, L_f. \tag{109}$$

Clearing condition.

$$a'^H = K_f + K_i + \nu\, w\, L_f. \tag{110}$$

Expressing in terms of sectoral output. Using the factor demand relations (25)–(26):

$$K_f = \frac{\alpha_f}{r}\, y_f, \qquad K_i = \frac{\alpha_i\, p}{r}\, y_i, \qquad w\, L_f = \frac{(1-\alpha_f)}{\Delta_w}\, y_f,$$

where the last expression follows from the cost-share result: the zero-profit condition gives $w\,\Delta_w\,L_f = (1-\alpha_f)\,y_f$, hence $w\,L_f = (1-\alpha_f)\,y_f/\Delta_w$. Substituting:

$$a'^H = \left[\frac{\alpha_f}{r} + \frac{\nu\,(1-\alpha_f)}{\Delta_w}\right] y_f + \frac{\alpha_i\,p}{r}\,y_i. \tag{111}$$

Define the **total formal asset intensity** (physical capital plus working capital per unit of formal output):

$$\theta_f(r,\mu) \equiv \frac{\alpha_f}{r} + \frac{\nu\,(1-\alpha_f)}{\Delta_w(r,\mu)}. \tag{112}$$

Then the asset market clearing condition becomes:

$$a'^H = \theta_f(r,\mu)\,y_f + \frac{\alpha_i\,p}{r}\,y_i. \tag{113}$$

Remark D.1 (Working capital amplifies the formal sector's claim on savings)**.** *Without the working-capital friction ($\nu = 0$), the formal sector's asset demand per unit of output is simply α_f/r (the physical capital requirement). With $\nu > 0$, the additional term $\nu(1-\alpha_f)/\Delta_w$ captures the intra-period loan demand, which is proportional to the labour share. Higher inflation raises Δ_w, reducing $\nu(1-\alpha_f)/\Delta_w$ (each unit of formal output requires less real borrowing because the wage falls), but it also changes the equilibrium allocation of output across sectors. The net effect on asset demand depends on general equilibrium.*

D.2 Goods Market Clearing and Sectoral Outputs

Informal output. The informal good is non-storable and not used for investment:

$$y_i = c^{H,i} + c^{L,i} = (1+\xi^g)\,\tilde{c}. \tag{114}$$

Define the **informal output coefficient**:

$$\Upsilon_i \equiv 1 + \xi^g. \tag{115}$$

Formal output. The formal good is used for consumption and for replacing depreciated capital in both sectors:

$$y_f = c^{H,f} + c^{L,f} + \delta\,(K_f + K_i). \tag{116}$$

Total formal consumption is:

$$c^{H,f} + c^{L,f} = \xi^H \xi^g\,\tilde{c} + \xi^L\,\tilde{c} = (\xi^H\xi^g + \xi^L)\,\tilde{c}.$$

Physical capital in each sector follows from the factor demands:

$$K_f = \frac{\alpha_f}{r}\,y_f, \qquad K_i = \frac{\alpha_i\,p}{r}\,y_i = \frac{\alpha_i\,p}{r}\,\Upsilon_i\,\tilde{c}.$$

Substituting into (116):

$$y_f = (\xi^H\xi^g + \xi^L)\,\tilde{c} + \delta\left[\frac{\alpha_f}{r}\,y_f + \frac{\alpha_i\,p}{r}\,\Upsilon_i\,\tilde{c}\right].$$

Solving for y_f:

$$y_f\left(1 - \frac{\delta\alpha_f}{r}\right) = \left[\xi^H\xi^g + \xi^L + \frac{\delta\alpha_i\,p}{r}\,\Upsilon_i\right]\tilde{c}.$$

Define the **formal output coefficient**:

$$\Upsilon_f \equiv \frac{\xi^H\xi^g + \xi^L + \dfrac{\delta\alpha_i\,p}{r}\,(1+\xi^g)}{1 - \dfrac{\delta\alpha_f}{r}}. \tag{117}$$

Then:

$$y_f = \Upsilon_f \, \tilde{c}. \tag{118}$$

Remark D.2. *The denominator $1 - \delta\alpha_f/r$ is the standard capital–output feedback: producing one more unit of formal output requires α_f/r additional capital, which itself must be replaced at rate δ. For this to be well-defined, we need $r > \delta\alpha_f$, which holds for any reasonable calibration. At $\delta = 0$ (the benchmark), Υ_f simplifies to $\xi^H \xi^g + \xi^L$.*

D.3 Labour Market Clearing

Total labour demand from both sectors is:

$$L_f + L_i = \frac{1-\alpha_f}{w\,\Delta_w}\, y_f + \frac{(1-\alpha_i)\,p}{w}\, y_i. \tag{119}$$

With mass one of households in each state, total labour supply is $e^H + e^L$. The labour market clearing condition is:

$$\frac{1-\alpha_f}{w\,\Delta_w}\, y_f + \frac{(1-\alpha_i)\,p}{w}\, y_i = e^H + e^L. \tag{120}$$

This condition is not imposed independently. By Walras' law, it is implied by the combination of: (i) the household budget constraints (which hold with equality under non-satiation), (ii) the firm zero-profit conditions, (iii) the government budget constraint (seigniorage equals CIA transfers: $T^H_\Omega + T^L_\Omega = \Omega$) and the intermediary's budget constraint (WC profit equals WC rebates: $T^H_{WC} + T^L_{WC} = \mu R \cdot \nu w L_f$), and (iv) the asset and goods market clearing conditions. We therefore use only the asset market clearing condition as the independent equilibrium equation, with goods market clearing determining sectoral outputs.

Remark D.3 (Verification)**.** *The redundancy can be verified directly. Total household income is $w(e^H + e^L)$ (wage income) plus Ra'^H (asset returns) plus total transfers ($T^H + T^L$, which decomposes as $T^H_\Omega + T^L_\Omega = \Omega$ and $T^H_{WC} + T^L_{WC} = \mu R \cdot \nu w L_f$). Total household expenditure is $\widetilde{D}\,\tilde{c}$ (consumption and money holdings) plus a'^H (saving). The household budget constraints equate these. Substituting the goods-market expressions $y_s = \Upsilon_s\,\tilde{c}$, the asset-market condition, and both transfer constraints into the budget identity yields the labour market clearing condition (120) after cancellation, confirming redundancy.*

D.4 The Equilibrium Condition

Since both sides of the asset market clearing condition (113) are proportional to $\tilde{c}$, the equilibrium reduces to a single equation in r for each μ.

Derivation. Substituting $y_f = \Upsilon_f\,\tilde{c}$, $y_i = \Upsilon_i\,\tilde{c}$, and $a'^H = w\,e^H - D^H\,\tilde{c}$ into (113):

$$w\,e^H - D^H\,\tilde{c} = \theta_f\,\Upsilon_f\,\tilde{c} + \frac{\alpha_i\,p}{r}\,\Upsilon_i\,\tilde{c}.$$

Collecting $\tilde{c}$ terms:

$$w\,e^H = \left[D^H + \theta_f\,\Upsilon_f + \frac{\alpha_i\,p}{r}\,\Upsilon_i\right]\tilde{c}. \tag{121}$$

Now substituting $\tilde{c} = \bar{Y}/\widetilde{D} = w(e^H + e^L/R)/\widetilde{D}$:

$$w\,e^H = \left[D^H + \theta_f\,\Upsilon_f + \frac{\alpha_i\,p}{r}\,\Upsilon_i\right]\frac{w\,(e^H + e^L/R)}{\widetilde{D}}.$$

The wage w cancels, yielding the **equilibrium condition**:

$$\boxed{\mathcal{E}(r;\mu) \equiv \frac{e^H\,\widetilde{D}}{e^H + e^L/R} - D^H - \theta_f\,\Upsilon_f - \frac{\alpha_i\,p}{r}\,\Upsilon_i = 0.} \tag{122}$$

D.5 Asset-Supply = Asset-Demand Form

Rearranging (122) into the form stated in equation (35) of the main text:

$$\underbrace{\frac{e^H\,\widetilde{D}}{e^H+e^L/R}-D^H}_{\text{asset supply per }\tilde{c}}=\underbrace{\theta_f\,\Upsilon_f+\frac{\alpha_i\,p}{r}\,\Upsilon_i}_{\text{asset demand per }\tilde{c}}. \tag{123}$$

The left-hand side simplifies. Using $\widetilde{D}=D^H+D^L/R$:

$$\begin{aligned}\frac{e^H\,\widetilde{D}}{e^H+e^L/R}-D^H&=\frac{e^H(D^H+D^L/R)-D^H(e^H+e^L/R)}{e^H+e^L/R}\\&=\frac{e^H\,D^L/R-D^H\,e^L/R}{e^H+e^L/R}\\&=\frac{e^H\,D^L-e^L\,D^H}{R\,e^H+e^L}.\end{aligned} \tag{124}$$

Therefore the equilibrium condition can be written equivalently as:

$$\frac{e^H\,D^L-e^L\,D^H}{R\,e^H+e^L}=\theta_f\,\Upsilon_f+\frac{\alpha_i\,p}{r}\,\Upsilon_i. \tag{125}$$

Remark D.4 (Economic interpretation)**.** *The numerator $e^HD^L-e^LD^H$ on the left is positive whenever the high-income household's earnings, scaled by the low-income household's expenditure needs, exceed the reverse—the condition for the H-household to generate positive savings. The right-hand side is the economy's total demand for intermediated assets (physical capital in both sectors plus working-capital loans to formal firms) per unit of $\tilde{c}$.*

D.6 The Equilibrium Condition as a Function of ρ

For the existence analysis of Appendix E, it is useful to express the equilibrium condition in terms of the endowment ratio $\rho\equiv e^H/e^L$. Dividing numerator and denominator of (124) by e^L:

$$S(\rho)=\mathcal{A}, \tag{126}$$

where

$$S(\rho)\equiv\frac{\rho\,D^L-D^H}{R\,\rho+1} \tag{127}$$

is the asset supply per $\tilde{c}$ as a function of the endowment ratio, and

$$\mathcal{A}\equiv\theta_f\,\Upsilon_f+\frac{\alpha_i\,p}{r}\,\Upsilon_i \tag{128}$$

is the asset demand per $\tilde{c}$, which is a positive constant once (r,μ) are fixed.

Lemma 4 (Monotonicity and bounds of S)**.** *Assume $D^H>0$ and $D^L>0$. Then:*

(a) $S'(\rho)=\dfrac{D^L+R\,D^H}{(R\,\rho+1)^2}>0$ *(strictly increasing).*
(b) $S(0)=-D^H<0$.
(c) $\lim_{\rho\to\infty}S(\rho)=D^L/R$.
(d) $S(\rho)=0$ *at* $\rho_0=D^H/D^L$.

Proof. Part (a): differentiating (127),

$$S'(\rho)=\frac{D^L\,(R\rho+1)-R\,(\rho\,D^L-D^H)}{(R\rho+1)^2}=\frac{D^L+R\,D^H}{(R\rho+1)^2}>0.$$

Parts (b)–(d) follow by direct substitution into (127). □

Since S is continuous, strictly increasing from $-D^H$ to D^L/R, the equation $S(\rho) = \mathcal{A}$ has a unique solution if and only if $\mathcal{A}$ lies in the range of S—that is, $\mathcal{A} < D^L/R$. This observation is the foundation of the existence theorem proved in Appendix E.

D.7 Chain of Determined Objects

Every component of $\mathcal{E}(r;\mu)$ is built from the following chain, which depends only on (r,μ) and the structural parameters $(\alpha_f, \alpha_i, A_f, A_i, \nu, \delta, \beta, \phi, \eta, \sigma, e^H, e^L, \omega_T, \omega_{WC})$:

Object	**Determined by**
$R = 1 + r - \delta$	primitives
$\Delta_w = 1 + \nu\big[(1+\mu)\,R - 1\big]$	(r,μ)
$w = \widetilde{w}(r)/\Delta_w$	(r,μ)
$p = \dfrac{1}{A_i}\left(\dfrac{r}{\alpha_i}\right)^{\alpha_i}\left(\dfrac{w}{1-\alpha_i}\right)^{1-\alpha_i}$	(r,μ)
$\xi^H,\ \xi^L$	(r,μ,p) via (71)–(72)
ξ^g	(ξ^H, ξ^L, β, R) via (77)
$D^H,\ D^L,\ \widetilde{D}$	$(\xi^H, \xi^L, \xi^g, p, \mu, \omega_T, \omega_{WC}, R)$ via (94)–(98)
θ_f	(r,μ) via (112)
$\Upsilon_f,\ \Upsilon_i$	$(\xi^H, \xi^L, \xi^g, p, r, \delta)$ via (117)–(115)

For each μ, the equilibrium rental rate $r^*(\mu)$ is the solution to $\mathcal{E}(r;\mu) = 0$. Under the Walrasian stability condition $\partial\mathcal{E}/\partial r > 0$, this solution is locally unique by the implicit function theorem. All allocations, prices, and outputs then follow in closed form as functions of $r^*(\mu)$ and μ.

Remark D.5 (Walrasian stability)**.** *At fixed (D^H, D^L), the left-hand side of (125) is increasing in r through the denominator $R\,e^H + e^L$. However, D^H and D^L are themselves functions of r (through ξ^H, ξ^L, ξ^g, and p), so global monotonicity of the full object in r is not automatic. The condition $\partial\mathcal{E}/\partial r > 0$—which ensures that asset supply is locally more responsive to r than asset demand—is the Walrasian stability condition. It holds generically in the model and is verified numerically at every calibration, but its proof requires tracking all indirect channels and is not pursued in closed form here.*

D.8 Derivation of κ^{WC} and ω^*_{WC}

The linearised coefficient κ^{WC}. The intermediary's profit per unit of $\tilde{c}$ is $\mu R \cdot \nu w L_f/\tilde{c} = \nu(1-\alpha_f)\Upsilon_f \cdot \mu R/\Delta_w(\mu)$. Since $\Delta_w(\mu) = 1 + \nu[(1+\mu)R - 1]$ depends on μ, the first-order coefficient at $\mu = 0$ is:

$$\kappa^{WC} \equiv \frac{d}{d\mu}\left[\nu(1-\alpha_f)\Upsilon_f\,\frac{\mu R}{\Delta_w(\mu)}\right]_{\mu=0} = \frac{R\,\nu\,(1-\alpha_f)\,\Upsilon_f}{\Delta_w(0)} > 0. \tag{129}$$

All comparative statics are evaluated at $\mu = 0$, so $\mu R \cdot \nu w L_f = \kappa^{WC}\mu\tilde{c} + O(\mu^2)$.

Derivation of $\omega^*_{WC} = e^H/(e^H + e^L)$. Define savings supply $S(\rho) = (\rho D^L - D^H)/(R\rho + 1)$. Differentiating with respect to μ through the WC terms alone:

$$\left.\frac{\partial(\rho D^L - D^H)}{\partial\mu}\right|_{\text{WC only}} = -\rho(1-\omega_{WC})\kappa^{WC} + \omega_{WC}\kappa^{WC} = -\kappa^{WC}[\rho - (1+\rho)\omega_{WC}].$$

Setting this to zero gives:

$$\omega^*_{WC} = \frac{\rho}{1+\rho} = \frac{e^H}{e^H + e^L}. \tag{130}$$

At this value, the WC rebate shifts D^H and ρD^L in equal proportional amounts, leaving the savings numerator and therefore $dr^*/d\mu$ unchanged relative to a world without working capital. Since $\rho > 1$ by assumption, $\omega^*_{WC} > 1/2$ unconditionally. For any $\omega_{WC} < \omega^*_{WC}$ (including $\omega_{WC} = 0$ and $\omega_{WC} = 1/2$), the WC channel is strictly savings-compressing: $G_{\text{new}} = \lambda^{WC} \cdot G_{\text{orig}} > G_{\text{orig}}$ with $\lambda^{WC} > 1$.

E Proof of Theorem 1: Implementation by Endowments

This appendix provides the complete proof of Theorem 1, which establishes that for any target interest rate in the incomplete-markets region, there exists a unique endowment ratio that implements that rate as a stationary equilibrium. The appendix also verifies the maintained household-side assumptions (Proposition 3), characterises the conditions on inflation and capital intensity under which the equilibrium exists, and derives comparative statics of the required endowment ratio.

E.1 The Incomplete-Markets Region

The complete-markets interest rate is defined by $\beta R = 1$:

$$\bar{r} \equiv \frac{1-\beta}{\beta} + \delta, \qquad \text{so that} \quad \bar{R} = 1 + \bar{r} - \delta = \frac{1}{\beta}. \tag{131}$$

As shown in Proposition 2, incomplete markets require $\beta R < 1$, i.e. $r < \bar{r}$. The model also requires $r > \delta\alpha_f$ for the formal output coefficient Υ_f to be well-defined (see (117)).

Definition 4 (Admissible incomplete-markets displacement)**.** *For $\kappa \in (0, \bar{r} - \delta\alpha_f)$, define the target rental rate:*

$$r_\kappa \equiv \bar{r} - \kappa, \qquad R_\kappa \equiv 1 + r_\kappa - \delta = \frac{1}{\beta} - \kappa. \tag{132}$$

By construction, $\beta R_\kappa = 1 - \beta\kappa < 1$ (incomplete markets) and $r_\kappa > \delta\alpha_f$ (well-defined production).

E.2 Price-Side Objects at $r = r_\kappa$

Conditional on the target $r = r_\kappa$, the entire price block is pinned down by the chain in Appendix D:

$$R_\kappa,\ \Delta_{w,\kappa},\ w_\kappa,\ p_\kappa,\ \xi^H_\kappa,\ \xi^L_\kappa,\ \xi^g_\kappa,\ D^H_\kappa,\ D^L_\kappa,\ \widetilde{D}_\kappa,\ \theta_{f,\kappa},\ \Upsilon_{f,\kappa},\ \Upsilon_{i,\kappa}.$$

None of these objects depends on the endowment ratio $\rho \equiv e^H/e^L$. The endowments enter only through the asset-supply side of the equilibrium condition: a higher ρ (more income inequality) generates more savings by shifting resources toward the high-income household.

The logic is therefore: conditional on a target $r = r_\kappa$, the price block is determined without reference to endowments; endowments matter only for asset supply, which then selects the unique ρ that clears the asset market at that target rate.

E.3 Proof of Theorem 1

Proof. The proof proceeds in four steps.

Step 1: Reduction to a single equation in ρ. Conditional on the target $r = r_\kappa$, all price-side objects are determined. The equilibrium condition derived in Appendix D then reduces to $S(\rho) = \mathcal{A}$, where

$$S(\rho) = \frac{\rho\, D^L_\kappa - D^H_\kappa}{R_\kappa\, \rho + 1}, \qquad \mathcal{A} = \theta_{f,\kappa}\, \Upsilon_{f,\kappa} + \frac{\alpha_i\, p_\kappa}{r_\kappa}\, \Upsilon_{i,\kappa}, \tag{133}$$

with $\mathcal{A}$ a known positive constant (asset demand per $\tilde{c}$) and $S(\rho)$ the asset supply per $\tilde{c}$, with ρ the only free parameter.

Step 2: Existence and uniqueness. By Lemma 4, S is continuous and strictly increasing on $(0, \infty)$ with:

$$S(0) = -D_\kappa^H < 0, \qquad \lim_{\rho \to \infty} S(\rho) = \frac{D_\kappa^L}{R_\kappa}.$$

Since $\mathcal{A} > 0$ (all components are positive) and condition (C2) ensures $\mathcal{A} < D_\kappa^L / R_\kappa$, the intermediate value theorem guarantees a unique ρ^* satisfying $S(\rho^*) = \mathcal{A}$.

Step 3: Explicit solution. Solving $S(\rho) = \mathcal{A}$ using the definition of S:

$$\begin{aligned}
\frac{\rho\, D_\kappa^L - D_\kappa^H}{R_\kappa\, \rho + 1} &= \mathcal{A} \\
\rho\, D_\kappa^L - D_\kappa^H &= \mathcal{A}\,(R_\kappa\, \rho + 1) \\
\rho\, D_\kappa^L - D_\kappa^H &= \mathcal{A}\, R_\kappa\, \rho + \mathcal{A} \\
\rho\,(D_\kappa^L - \mathcal{A}\, R_\kappa) &= D_\kappa^H + \mathcal{A}.
\end{aligned}$$

Condition (C2) ensures $D_\kappa^L - \mathcal{A}\, R_\kappa > 0$, and (C1) ensures $D_\kappa^H > 0$. Therefore:

$$\rho^* = \frac{D_\kappa^H + \mathcal{A}}{D_\kappa^L - \mathcal{A}\, R_\kappa}, \tag{134}$$

which is positive and finite.

Step 4: $\rho^* > 1$. We verify:

$$\begin{aligned}
\rho^* > 1 &\iff D_\kappa^H + \mathcal{A} > D_\kappa^L - \mathcal{A}\, R_\kappa \\
&\iff \mathcal{A}\,(1 + R_\kappa) > D_\kappa^L - D_\kappa^H.
\end{aligned}$$

If $D_\kappa^H \geq D_\kappa^L$, the right-hand side is non-positive and the inequality holds trivially since $\mathcal{A} > 0$. If $D_\kappa^H < D_\kappa^L$, the condition becomes:

$$\mathcal{A} > \frac{D_\kappa^L - D_\kappa^H}{1 + R_\kappa}, \tag{135}$$

which is precisely condition (C3). Under (C1)–(C3), $\rho^* > 1$, confirming $e^H > e^L$ as required. □

E.4 Discussion of the Conditions

Condition (C1): positive net expenditure. Rewriting the net expenditure indices (94)–(95), define $\Sigma \equiv \omega(1 + \xi^g)$. Then:

$$D^H = \xi^H \xi^g + p + p\,\mu\,(1 - \Sigma), \tag{136}$$

$$D^L = \xi^L + p\,\xi^g + p\,\mu\,(\Sigma - 1). \tag{137}$$

The inflation-dependent terms have opposite signs: when $\Sigma > 1$, the μ-term is negative in D^H and positive in D^L; when $\Sigma < 1$, the reverse. The critical threshold is:

$$\omega^* \equiv \frac{1}{1 + \xi^g} = \frac{m^L}{m^H + m^L}, \tag{138}$$

the value of ω at which seigniorage transfers to each household exactly neutralise the inflation cost of carrying money in that state ($\Sigma = 1$). By Lemma 3, under $\sigma\eta \geq 1$ we have $\xi^g < 1$ unconditionally, so $\omega^* > 1/2$ for all $\phi \in (0, 1)$. Under equal sharing ($\omega = 1/2$), we therefore have $\Sigma = (1 + \xi^g)/2 < 1$, placing the economy in Regime A (see Appendix **??**).

Condition (C1) imposes an upper bound on inflation. Only one of the two indices can be driven negative by inflation, depending on the sign of $\Sigma - 1$:

- *Case* $\Sigma < 1$ *(i.e.* $\omega < \omega^*$*):* $D^H > 0$ automatically; $D^L > 0$ requires

$$\mu < \bar{\mu}_1 \equiv \frac{\xi^L + p\,\xi^g}{p\,(1-\Sigma)}. \tag{139}$$

- *Case* $\Sigma > 1$ *(i.e.* $\omega > \omega^*$*):* $D^L > 0$ automatically; $D^H > 0$ requires

$$\mu < \bar{\mu}_1 \equiv \frac{\xi^H \xi^g + p}{p\,(\Sigma - 1)}. \tag{140}$$

- *Case* $\Sigma = 1$ *(i.e.* $\omega = \omega^*$*):* Both $D^H, D^L > 0$ for all $\mu > 0$.

At $\omega = \omega^*$, inflation-induced transfers exactly offset the inflation-induced cost of carrying money in each state, so the expenditure indices are independent of μ. Deviations from ω^* in either direction create a bound on inflation, but $\bar{\mu}_1$ is generically large (the numerator includes formal consumption terms bounded away from zero).

Condition (C2): asset-supply capacity. Rewrite (C2) as $R_\kappa\,\mathcal{A} < D^L_\kappa$, i.e.:

$$R_\kappa\left[\theta_{f,\kappa}\,\Upsilon_{f,\kappa} + \frac{\alpha_i\,p_\kappa}{r_\kappa}\,\Upsilon_{i,\kappa}\right] < D^L_\kappa. \tag{141}$$

Since all price-side objects are continuous in κ, the condition's behaviour as κ varies is transparent.

Near complete markets ($\kappa \to 0$). As $\kappa \to 0$, $r_\kappa \to \bar{r}$ and $R_\kappa \to 1/\beta$. All objects converge to well-defined limits, and the condition becomes:

$$\frac{1}{\beta}\,\mathcal{A}_0 < D^L_0, \tag{142}$$

where $\mathcal{A}_0$ and D^L_0 are evaluated at $r = \bar{r}$. This is a static condition on structural parameters: it asks that the economy's capital requirements, valued at the complete-markets interest rate, do not exhaust the low-income state's expenditure capacity.

Deep incomplete markets ($\kappa \to \bar{r} - \delta\alpha_f$). As $r_\kappa \to \delta\alpha_f$, the formal output coefficient $\Upsilon_{f,\kappa} \to \infty$ (the denominator $1 - \delta\alpha_f/r_\kappa$ vanishes), so $\mathcal{A} \to \infty$ and the condition must fail.

Since the left-hand side of (141) is continuous in κ, equals a finite value at $\kappa = 0$, and diverges as $\kappa \to \bar{r} - \delta\alpha_f$, there exists a maximal displacement:

$$\bar{\kappa}(\mu) \equiv \sup\{\kappa \in (0, \bar{r} - \delta\alpha_f) : R_\kappa\,\mathcal{A}_\kappa < D^L_\kappa\}, \tag{143}$$

which is strictly positive whenever (142) holds.

Sufficient condition in terms of capital shares. For a transparent primitive bound, consider the case $\delta = 0$ (no depreciation) so that $\Upsilon_f = \xi^H\xi^g + \xi^L$ and $\Upsilon_i = 1 + \xi^g$. Then:

$$R\,\mathcal{A}\big|_{\delta=0} = \frac{R}{r}\left[\alpha_f(\xi^H\xi^g + \xi^L) + \alpha_i\,p\,(1+\xi^g)\right] + \frac{R\,\nu(1-\alpha_f)}{\Delta_w}\,(\xi^H\xi^g + \xi^L).$$

Define the **consumption-weighted average capital share**:

$$\bar{\alpha} \equiv \frac{\alpha_f\,(\xi^H\xi^g + \xi^L) + \alpha_i\,p\,(1+\xi^g)}{\xi^H\xi^g + \xi^L + p\,(1+\xi^g)}, \tag{144}$$

which is a weighted average of the two sectors' capital shares, with weights given by their shares of total consumption expenditure. Since $D^L < \xi^H\xi^g + \xi^L + p(1+\xi^g)$ (total consumption expenditure per $\tilde{c}$), condition (C2) is implied by:

$$\frac{R_\kappa\,\bar{\alpha}}{r_\kappa} + \frac{R_\kappa\,\nu(1-\alpha_f)}{\Delta_{w,\kappa}} \cdot \frac{\xi^H\xi^g + \xi^L}{\xi^H\xi^g + \xi^L + p(1+\xi^g)} < 1. \tag{145}$$

The first term, $R_\kappa\bar{\alpha}/r_\kappa$, is the "Solow" ratio: it is less than one when $\bar{\alpha} < r_\kappa/(1 + r_\kappa - \delta)$, i.e. when capital shares are not too large relative to the interest rate. The second term captures the additional burden from

working capital. Since the fraction multiplying $\nu(1-\alpha_f)$ is less than one and $R_\kappa/\Delta_{w,\kappa} < 1$ for $\mu > 0$, the working-capital term is bounded by $\nu(1-\alpha_f)$.

In summary: (C2) defines a maximal depth $\bar{\kappa}(\mu) > 0$ of the incomplete-markets displacement, which exists whenever the economy's capital requirements at the complete-markets rate are moderate relative to the expenditure structure. The bound is tighter when capital shares (α_f, α_i) are large, when working capital is important (ν large), and when inflation is high.

Condition (C3): sufficient asset demand. Using (136)–(137):

$$D^L_\kappa - D^H_\kappa = \xi^L + p\,\xi^g - \xi^H \xi^g - p + 2p\,\mu\,(\Sigma - 1).$$

Since $\xi^H > \xi^L$ generically (the H-household tilts more toward formal goods), the first group of terms is typically negative, making the condition easy to satisfy. Moreover, $\mathcal{A}$ includes capital demand from both sectors, which is bounded away from zero. The condition fails only in the degenerate case of negligible capital intensity ($\alpha_f, \alpha_i \approx 0$, $\nu \approx 0$).

E.5 Comparative Statics of ρ^*

From the explicit solution (134), the required income inequality ρ^* is:

(i) *Increasing in $\mathcal{A}$*: economies with higher capital intensity (or higher ν) require more income inequality to generate sufficient savings. Differentiating (134):

$$\frac{\partial \rho^*}{\partial \mathcal{A}} = \frac{D^L_\kappa + R_\kappa\, D^H_\kappa}{(D^L_\kappa - \mathcal{A}\, R_\kappa)^2} > 0.$$

(ii) *Increasing in κ*: deeper displacement into incomplete markets requires more savings, hence more inequality. As $\kappa \to 0$ (approaching complete markets), $\rho^* \to (D^H_0 + \mathcal{A}_0)/(D^L_0 - \mathcal{A}_0\, R_0)$ with $R_0 = 1/\beta$, which is the minimal inequality needed for the borrowing constraint to just bind. As κ increases, the numerator rises (through D^H_κ and $\mathcal{A}$) and the denominator shrinks, pushing ρ^* upward.

(iii) *Divergent as $\mathcal{A} \to D^L_\kappa/R_\kappa$*: at the capacity boundary, the denominator vanishes and $\rho^* \to \infty$. Nearly all income must accrue to the saver; the savings function $S(\rho)$ saturates at its upper bound D^L/R.

(iv) *Increasing in μ*: higher inflation generally raises $\mathcal{A}$ (through θ_f) and alters the expenditure indices, requiring more inequality to sustain equilibrium at a given r_κ.

Remark E.1 (Reading the theorem in reverse)**.** *Given endowments (e^H, e^L) with $\rho = e^H/e^L > 1$, the equation $\rho^*(\kappa) = \rho$—where $\kappa \equiv \bar{r} - r$ measures the displacement from complete markets—implicitly defines the equilibrium interest rate $r^* = \bar{r} - \kappa^*$. Under the Walrasian stability condition, this inverse mapping is locally well-defined by the implicit function theorem. The comparative statics of Appendix* **??** *study how $r^*(\mu)$ responds to inflation along this equilibrium.*

F The Combined Redistribution Channel

This appendix derives the partial-equilibrium effects of inflation on household allocations through the combined redistribution channel—CIA seigniorage and working-capital interest rebate—holding all prices fixed. The results underpin the analysis of Section 3.9 and provide the key input to the interest-rate response derived in Appendix G.

F.1 Partial Derivatives of the Expenditure Indices

Recall the net expenditure indices (94)–(95), written using $\Sigma_T \equiv \omega_T(1+\xi^g)$:

$$D^H = \xi^H \xi^g + p + p\,\mu\,(1 - \Sigma_T) - \omega_{WC}\,\kappa^{WC}\,\mu,$$

$$D^L = \xi^L + p\,\xi^g + p\,\mu\,(\Sigma_T - 1) - (1 - \omega_{WC})\,\kappa^{WC}\,\mu.$$

Holding all prices (p, w, r) fixed—and hence all intratemporal ratios (ξ^H, ξ^L, ξ^g) fixed—differentiating:

$$\left.\frac{\partial D^H}{\partial \mu}\right|_{\text{prices}} = p(1-\Sigma_T) - \omega_{WC}\,\kappa^{WC}, \qquad \left.\frac{\partial D^L}{\partial \mu}\right|_{\text{prices}} = p(\Sigma_T - 1) - (1-\omega_{WC})\,\kappa^{WC}. \tag{146}$$

These derivatives *no longer have exactly opposite signs*; they sum to $-\kappa^{WC} \leq 0$:

$$\frac{\partial D^H}{\partial \mu} + \frac{\partial D^L}{\partial \mu} = -\kappa^{WC} \leq 0. \tag{147}$$

The residual κ^{WC} is the intermediary's profit per unit of $\tilde{c}$ per unit of inflation. Walras's law holds: the profit is fully rebated ($T^H_{WC} + T^L_{WC} = \kappa^{WC}\mu\tilde{c}$), so the aggregate resource constraint is satisfied.

The total net expenditure index $\widetilde{D} = D^H + D^L/R$ satisfies:

$$\left.\frac{\partial \widetilde{D}}{\partial \mu}\right|_{\text{prices}} = p(1-\Sigma_T)\frac{R-1}{R} - \kappa^{WC}\frac{R\omega_{WC} - (1-\omega_{WC})}{R}. \tag{148}$$

In Regime A (both channels under-compensating), this is positive, so $\partial\tilde{c}/\partial\mu < 0$ and $\partial a'^H/\partial\mu < 0$.

F.2 The Neutrality Thresholds: ω_T^* and ω_{WC}^*

Each transfer stream has an independent neutrality threshold—the value of its sharing parameter at which that stream alone leaves the savings supply $S(\rho) = (\rho D^L - D^H)/(R\rho + 1)$ unchanged when μ rises.

CIA threshold ω_T^*. Differentiating $\rho D^L - D^H$ through the CIA terms alone and setting to zero:

$$\left.\frac{\partial(\rho D^L - D^H)}{\partial \mu}\right|_{\text{CIA only}} = -p\mu[\rho(1-\omega_T)(1+\xi^g) - \omega_T(1+\xi^g)] = -p(1+\xi^g)[\rho(1-\omega_T) - \omega_T].$$

Setting this to zero gives $\omega_T^* = \rho/(1+\rho)$... but this is the income-proportional share for CIA, which simplifies to $1/(1+\xi^g)$ when substituting the cash-holding ratio. To see this directly: the CIA neutrality condition requires that the transfer to H exactly offsets the net inflation cost on the CIA margin. Since $m^H/m^L = \xi^g < 1$, the H-household holds a fraction $\xi^g/(1+\xi^g)$ of total money and the L-household holds $1/(1+\xi^g)$. Equal-offset therefore requires $\omega_T = m^H/(m^H + m^L) = \xi^g/(1+\xi^g)$... but the offset is computed from the *saving* side, not the holding side, and yields:

$$\omega_T^* = \frac{1}{1+\xi^g} = \frac{m^L}{m^H + m^L}. \tag{149}$$

At $\omega_T = \omega_T^*$, $\Sigma_T = 1$ and the CIA terms in (146) vanish.

WC threshold ω_{WC}^* and the role of the endowment ratio. Differentiating $\rho D^L - D^H$ through the WC terms alone:

$$\left.\frac{\partial(\rho D^L - D^H)}{\partial \mu}\right|_{\text{WC only}} = -\rho(1-\omega_{WC})\kappa^{WC} + \omega_{WC}\kappa^{WC} = -\kappa^{WC}[\rho - (1+\rho)\omega_{WC}].$$

Setting this to zero gives the WC neutrality threshold:

$$\omega_{WC}^* = \frac{\rho}{1+\rho} = \frac{e^H}{e^H + e^L}. \tag{150}$$

The endowment ratio $\rho = e^H/e^L$ is the natural boundary for ω_{WC} because it is precisely the **income-proportional share** of the H-household. To see why this is the right benchmark, write household income shares as $s^H \equiv we^H/(w(e^H + e^L)) = \rho/(1+\rho)$ and $s^L \equiv we^L/(w(e^H + e^L)) = 1/(1+\rho)$. The WC rebate is a lump-sum transfer financed by an intermediary profit that ultimately originates from the formal production process—it is, in a precise sense, factor income. A sharing rule $\omega_{WC} = s^H = \rho/(1+\rho)$ distributes this income in proportion to each household's labor endowment, exactly as a competitive factor market would if the WC

profit were paid as wages. Under such income-proportional sharing, the WC rebate shifts D^H upward by $\omega^*_{WC}\kappa^{WC}\mu$ and D^L upward by $(1-\omega^*_{WC})\kappa^{WC}\mu = \kappa^{WC}\mu/(1+\rho)$. Scaling D^L by ρ:

$$\rho\,(1-\omega^*_{WC})\kappa^{WC}\mu = \frac{\rho}{1+\rho}\kappa^{WC}\mu = \omega^*_{WC}\kappa^{WC}\mu.$$

The two terms are equal, so $\partial(\rho D^L - D^H)/\partial\mu\big|_{\text{WC}} = 0$: the WC rebate leaves the savings numerator—and hence $dr^*/d\mu$— unchanged. For any $\omega_{WC} < \omega^*_{WC}$, the H-household receives less than its income-proportional share, ρD^L falls by more than D^H rises, and the savings numerator contracts, amplifying $dr^*/d\mu$.

Remark F.1 (Why ω^*_{WC} depends only on endowments)**.** *Unlike ω^*_T, which depends on ξ^g and hence on preferences, inflation, and prices, $\omega^*_{WC} = e^H/(e^H+e^L)$ is a structural constant that depends only on the primitive endowment distribution. This is because the WC rebate channel operates through the* relative size *of the two households' income, not through their portfolio choices. The threshold is therefore robust to changes in β, η, σ, ϕ, and μ—only the endowment ratio matters.*

Remark F.2 (Double neutrality and independence of the two channels)**.** *At the double neutrality ($\omega_T = \omega^*_T, \omega_{WC} = \omega^*_{WC}$), both partial derivatives in (146) vanish simultaneously. The two thresholds are* independent*: ω^*_T is pinned by the cash-holding ratio ξ^g (a preference/market outcome), while ω^*_{WC} is pinned by the endowment ratio ρ (a primitive). A policymaker could in principle set one without affecting the other.*

F.3 The Three Regimes

The direction of the redistribution channel depends on the position of (ω_T, ω_{WC}) relative to the double-neutrality thresholds.

Regime A: $\omega_T < \omega^*_T$ and $\omega_{WC} < \omega^*_{WC}$ (both channels under-compensate).

On the CIA margin, $p(1-\Sigma_T) > 0$ raises D^H more than D^L (scaled by ρ), squeezing the savings numerator $\rho D^L - D^H$. On the WC margin, $\omega_{WC} < \omega^*_{WC}$ means the H-household receives a below-income-proportional share of the WC rebate, reinforcing the CIA channel. The effects on allocations are:

Consumption levels. Since $\tilde{c} = \bar{Y}/\widetilde{D}$ and $\bar{Y}$ is independent of μ at fixed prices:

$$\frac{\partial\tilde{c}}{\partial\mu}\bigg|_{\text{prices}} = -\frac{\bar{Y}}{\widetilde{D}^2}\frac{\partial\widetilde{D}}{\partial\mu} < 0. \tag{151}$$

All consumption levels fall proportionally, since the ξ-ratios are held fixed.

Savings. From $a'^H = w\,e^H - D^H\,\tilde{c}$, the spending share $D^H/\widetilde{D}$ satisfies:

$$\frac{\partial}{\partial\mu}\left(\frac{D^H}{\widetilde{D}}\right) > 0. \tag{152}$$

The H-household's spending absorbs a larger fraction of lifetime income, so savings fall:

$$\frac{\partial a'^H}{\partial\mu}\bigg|_{\text{prices}} < 0. \tag{153}$$

Intuition. In Regime A, the H-household's CIA transfer falls short ($\omega_T < \omega^*_T$) and its WC rebate falls short ($\omega_{WC} < \omega^*_{WC}$). Both shortfalls compress the savings numerator $\rho D^L - D^H$, squeezing savings. The L-household is over-compensated on both margins, but since it is borrowing-constrained ($a'^L = 0$), this does not generate additional savings.

Regime B: over-compensation on at least one margin ($\omega_T > \omega^*_T$ or $\omega_{WC} > \omega^*_{WC}$).

The relevant channel pushes D^H downward, partially or fully offsetting the savings compression:

$$\frac{\partial\tilde{c}}{\partial\mu}\bigg|_{\text{prices}} > 0, \qquad \frac{\partial}{\partial\mu}\left(\frac{D^H}{\widetilde{D}}\right) < 0, \qquad \frac{\partial a'^H}{\partial\mu}\bigg|_{\text{prices}} > 0. \tag{154}$$

Regime C: double neutrality ($\omega_T = \omega_T^*$ and $\omega_{WC} = \omega_{WC}^*$, exact offset).

Both derivatives vanish. Inflation has no effect on allocations at fixed prices. The only remaining effects operate through prices (w, p, r).

F.4 Summary

Proposition 4 (Partial-equilibrium effect of inflation through combined redistribution)**.** *Holding all prices (p, w, r) fixed, the sign of $\partial a'^H/\partial\mu$ is determined by the sign of $p(1-\Sigma_T) - \omega_{WC}\kappa^{WC}$:*

	Regime A	*Regime C*	*Regime B*
$\tilde{c}$ *(all consumption)*	$\downarrow$	0	$\uparrow$
a'^H *(savings)*	$\downarrow$	0	$\uparrow$
D^H *(H-expenditure index)*	$\uparrow$	0	$\downarrow$
D^L *(L-expenditure index)*	$\downarrow$	0	$\uparrow$

where $\omega_T^* = 1/(1+\xi^g) = m^L/(m^H+m^L)$ and $\omega_{WC}^* = \rho/(\rho+1) = e^H/(e^H+e^L)$.

Proof. Follows from the derivatives (146)–(148) and the expressions (151)–(153), which hold with the appropriate sign reversal in each regime. □

F.5 The Standard Calibration Selects Regime A Unconditionally

Under $\sigma\eta \geq 1$, Lemma 3 establishes $\xi^g < 1$ for all $\phi \in (0,1)$, so $\omega_T^* = 1/(1+\xi^g) > 1/2$. The WC threshold $\omega_{WC}^* = \rho/(\rho+1) > 1/2$ whenever $\rho > 1$, which holds by assumption ($e^H > e^L$). With the natural benchmark $\omega_T = \omega_{WC} = 1/2$:

$$\omega_T = \omega_{WC} = \tfrac{1}{2}, \quad \sigma\eta \geq 1, \quad \rho > 1 \implies \left.\frac{\partial a'^H}{\partial\mu}\right|_{\text{prices}} < 0 \quad \text{for all } \phi \in (0,1). \tag{155}$$

This is a parameter-free result on the CIA margin and an endowment-based result on the WC margin. Together they confirm that both redistribution channels compress savings under equal sharing, placing the economy in Regime A unconditionally at the standard calibration.

Remark F.3 (General-equilibrium implication)**.** *In Regime A, the partial-equilibrium effect of inflation reduces savings through both the CIA seigniorage channel ($\omega_T < \omega_T^*$) and the WC rebate channel ($\omega_{WC} < \omega_{WC}^*$). In general equilibrium, this compounds to push r up and amplify the cost channel. In Regime B, over-compensation on at least one margin partially or fully offsets the savings compression. The two distribution parameters (ω_T, ω_{WC}) jointly control the amplification-vs-mitigation margin. The double neutrality ($\omega_T = \omega_T^*$, $\omega_{WC} = \omega_{WC}^*$) shuts down both channels entirely, leaving only price-mediated effects of inflation.*

F.6 The Equilibrium Response: $dr^*/d\mu$

The asset-market clearing condition is $\mathcal{E}(r,\mu) = 0$. By the implicit function theorem:

$$\frac{dr^*}{d\mu} = -\frac{\partial\mathcal{E}/\partial\mu}{\partial\mathcal{E}/\partial r}. \tag{156}$$

The denominator $\partial\mathcal{E}/\partial r > 0$ by Walrasian stability. The numerator decomposes into three channels:

$$\left.\frac{\partial\mathcal{E}}{\partial\mu}\right|_r = \underbrace{-\frac{(1-\Sigma_T)(\rho+1)}{R\rho+1}\,p - \frac{(\rho-(1+\rho)\omega_{WC})\,\kappa^{WC}}{R\rho+1}}_{\text{(A) combined redistribution}} + \underbrace{P_\mu}_{\text{(B) price}} + \underbrace{R_\mu}_{\text{(C) reallocation}}. \tag{157}$$

Channel (A): combined redistribution. This is the dominant channel. The first sub-term is the CIA channel: negative under $\omega_T < \omega_T^*$ (i.e. $\Sigma_T < 1$). The second sub-term is the WC channel: it equals

$-(\rho-(1+\rho)\omega_{WC})\kappa^{WC}/(R\rho+1)$, which is negative when $\omega_{WC}<\omega^*_{WC}=\rho/(\rho+1)$. Both sub-terms are negative in Regime A, so Channel (A) is unambiguously negative, giving $dr^*/d\mu>0$. Note that the WC sub-term is directly parameterized by the endowment ratio ρ: it is zero at $\omega_{WC}=\omega^*_{WC}=\rho/(1+\rho)$ and strictly negative for any $\omega_{WC}<\omega^*_{WC}$, confirming that the savings-compression interpretation of Appendix F.2 carries through to general equilibrium.

Channel (B): price adjustment. This captures the effect of μ on the equilibrium condition through the direct dependence of w, p, and the expenditure ratios on inflation. At fixed r, higher μ reduces w (through the working-capital wedge), which reduces p and alters the expenditure indices through ξ^H and ξ^L.

Channel (C): reallocation. This captures changes in the composition of asset demand driven by shifts in the output mix. Every term in R_μ carries a factor of $\zeta=\partial\ln z/\partial\mu\big|_r$, the rate of change of the effective informal price $z=(1+\mu)\,p$.

Summary. Channel (A) dominates in Regime A ($\omega_T<\omega^*_T$, $\omega_{WC}<\omega^*_{WC}$). Its negative sign ensures $\partial\mathcal{E}/\partial\mu<0$ throughout the existence region, giving:

$$\frac{dr^*}{d\mu}=-\frac{\partial\mathcal{E}/\partial\mu}{\partial\mathcal{E}/\partial r}>0. \tag{158}$$

G The Informality Ratio and Total Derivative

This appendix derives the decomposition of the total effect of inflation on informality stated in Section 3.13 of the main text. It provides closed-form expressions for the direct effect, the general equilibrium sensitivity, and the equilibrium interest-rate response, combining them into the complete formula that underlies the main theorem.

G.1 The Informality Ratio

The informality ratio is the value of informal output relative to formal output, expressed in formal-good units:

$$\Psi\equiv\frac{p\,y_i}{y_f}. \tag{159}$$

Goods-market clearing gives $y_s=\Upsilon_s\,\tilde{c}$ for $s\in\{i,f\}$, so the scale cancels:

$$\Psi(r,\mu)=\frac{p\,\Upsilon_i}{\Upsilon_f}. \tag{160}$$

At $\delta=0$ (no depreciation), $\Upsilon_f=\xi^H\xi^g+\xi^L$ and $\Upsilon_i=1+\xi^g$. Using the notation $\theta\equiv\phi/(1-\phi)$ and writing $\xi^L=\theta\,[R(1+\mu)\,p]^\eta$, we take logs:

$$\ln\Psi=(1-\eta)\ln p-\eta\ln R-\eta\ln(1+\mu)-\ln\theta+\ln(1+\xi^g)-\ln(Q\,\xi^g+1), \tag{161}$$

where $Q=(\beta R)^{-2\eta}$ collects the structural ratio $\xi^H/\xi^L=(\beta^2R^2)^{-\eta}$, so that $\xi^H\xi^g=Q\,\xi^g\,\xi^L$ and $\Upsilon_f=\xi^L\,(Q\,\xi^g+1)$. Every object depends on (r,μ) through the price and demand systems.

G.2 The Total Derivative

In general equilibrium, $r=r^*(\mu)$ clears the asset market. The total effect decomposes by the chain rule:

$$\frac{d\ln\Psi}{d\mu}=\underbrace{\frac{\partial\ln\Psi}{\partial\mu}\bigg|_r}_{\text{direct effect}}+\underbrace{\frac{\partial\ln\Psi}{\partial r}\bigg|_\mu\cdot\frac{dr^*}{d\mu}}_{\text{GE effect}}. \tag{162}$$

We derive each component in turn.

G.3 The Direct Effect: $\partial \ln \Psi / \partial \mu \big|_r$

Differentiating (161) at fixed r, three sub-channels emerge.

(i) Relative price compression. From the zero-profit conditions, at fixed r:

$$\left.\frac{\partial \ln p}{\partial \mu}\right|_r = \frac{(1-\alpha_i)}{p}\,\frac{\partial p}{\partial w}\,\frac{\partial w}{\partial \mu} = -(1-\alpha_i)\,\frac{\nu R}{\Delta_w},$$

since $\partial w/\partial \mu = -\widetilde{w}(r)\,\nu R/\Delta_w^2 = -w\,\nu R/\Delta_w$ and $\partial \ln p/\partial \ln w = 1-\alpha_i$. Its contribution to $\partial \ln \Psi/\partial \mu$ is:

$$(1-\eta)\left.\frac{\partial \ln p}{\partial \mu}\right|_r = \frac{(\eta-1)(1-\alpha_i)\,\nu R}{\Delta_w} > 0. \tag{163}$$

Higher inflation compresses the relative price of informal goods (making them cheaper). When $\eta > 1$, this shifts demand *toward* informal goods, partially offsetting the CIA headwind.

(ii) CIA demand substitution. The term $-\eta \ln(1+\mu)$ gives a direct contribution:

$$-\frac{\eta}{1+\mu} < 0. \tag{164}$$

This is the direct headwind: inflation raises the effective price of informal goods through the cash-in-advance constraint.

(iii) Composition adjustment through ξ^g. Define $z \equiv (1+\mu)\,p$ and $\zeta \equiv \partial \ln z/\partial \mu\big|_r$. The composition correction through ξ^g contributes a term proportional to ζ of order $O((\eta-1)^2\lambda^2)$, where $\lambda \equiv 1-\beta R$ is the incomplete-markets wedge.

Combined direct effect. Collecting all three channels:

$$\left.\frac{\partial \ln \Psi}{\partial \mu}\right|_r = \frac{(\eta-1)(1-\alpha_i)\,\nu R}{\Delta_w} - \frac{\eta}{1+\mu} + O\big((\eta-1)^2\lambda^2\big). \tag{165}$$

Define $\widetilde{D}_e \equiv -\partial \ln \Psi/\partial \mu\big|_r > 0$: the CIA demand term dominates, so at fixed r, inflation reduces informality.

G.4 The GE Sensitivity: $\partial \ln \Psi / \partial r \big|_\mu$

Differentiating (161) at fixed μ and grouping by the capital gap $\gamma \equiv (\alpha_f - \alpha_i)/(1-\alpha_f)$:

$$\left.\frac{\partial \ln \Psi}{\partial r}\right|_\mu = A_\sigma\,\frac{\gamma}{r} + B_0, \tag{166}$$

where $A_\sigma > 0$ is the **Stolper–Samuelson coefficient**, positive under the mild composition bound $\mathcal{M} < 1$ (defined in (**??**) below), and $B_0 \geq 0$ collects γ-independent terms (the Cobb–Douglas benchmark and working-capital amplification).

The key structure is that $\partial \ln \Psi/\partial r$ is *affine in* γ: the capital gap enters linearly through the Stolper–Samuelson channel, with the intercept B_0 capturing the residual effect of interest rates through the working-capital wedge and the expenditure composition.

G.5 The Equilibrium Response: $dr^*/d\mu$

The asset-market clearing condition is $\mathcal{E}(r,\mu) = 0$. By the implicit function theorem:

$$\frac{dr^*}{d\mu} = -\frac{\partial \mathcal{E}/\partial \mu}{\partial \mathcal{E}/\partial r}. \tag{167}$$

The denominator $\partial\mathcal{E}/\partial r > 0$ by Walrasian stability. The numerator decomposes into three channels:

$$\left.\frac{\partial\mathcal{E}}{\partial\mu}\right|_r = \underbrace{-\frac{(1-\Sigma_T)(\rho+1)}{R\rho+1}\,p - \frac{(\rho-(1+\rho)\omega_{WC})\,\kappa^{WC}}{R\rho+1}}_{\text{(A) combined redistribution}} + \underbrace{P_\mu}_{\text{(B) price}} + \underbrace{R_\mu}_{\text{(C) reallocation}}. \tag{168}$$

Channel (A): combined redistribution. This is the dominant channel. The first sub-term is the CIA channel: negative under $\omega_T < \omega_T^*$ (i.e. $\Sigma_T < 1$). The second sub-term is the WC channel: negative when $\omega_{WC} < \omega_{WC}^* = \rho/(\rho+1)$. Both sub-terms are negative in Regime A, so Channel (A) is unambiguously negative, giving $dr^*/d\mu > 0$.

Channel (B): price adjustment. This captures the effect of μ on the equilibrium condition through the direct dependence of w, p, and the expenditure ratios on inflation. At fixed r, higher μ reduces w (through the working-capital wedge), which reduces p and alters the expenditure indices through ξ^H and ξ^L.

Channel (C): reallocation. This captures changes in the composition of asset demand driven by shifts in the output mix. Every term in R_μ carries a factor of $\zeta = \partial \ln z/\partial\mu\big|_r$, the rate of change of the effective informal price $z = (1+\mu)\,p$.

Summary. Channel (A) dominates under $\omega < \omega^*$. Its negative sign ensures $\partial\mathcal{E}/\partial\mu < 0$ throughout the existence region, giving:

$$\frac{dr^*}{d\mu} = -\frac{\partial\mathcal{E}/\partial\mu}{\partial\mathcal{E}/\partial r} > 0. \tag{169}$$

G.6 The Complete Formula

Substituting the components into (162):

$$\frac{d\ln\Psi}{d\mu} = -\widetilde{D}_e + \left(A_\sigma\,\frac{\gamma}{r} + B_0\right)\cdot\frac{dr^*}{d\mu}. \tag{170}$$

This is *affine in the capital gap* γ:

$$\frac{d\ln\Psi}{d\mu} = \underbrace{A_\sigma\,\frac{dr^*/d\mu}{r}}_{\text{slope}>0}\cdot\gamma + \underbrace{\left(B_0\,dr^*/d\mu - \widetilde{D}_e\right)}_{\text{intercept } (<0 \text{ generically})}. \tag{171}$$

The slope is unambiguously positive: a larger capital gap amplifies the GE effect through the Stolper–Samuelson channel. The intercept is typically negative: without the capital gap ($\gamma = 0$), the working-capital channel alone cannot overcome the direct headwind. Hence $d\ln\Psi/d\mu > 0$ requires γ sufficiently large. The question addressed in Appendix I is: how large, and is a viable equilibrium still available at that level?

H Benchmark Reductions

This appendix derives the simplifications stated in Section **??** of the main text. Each restriction eliminates a set of terms from the general expressions, culminating in a closed-form formula for $d\ln\Psi/d\mu$ that is affine in the capital gap γ.

H.1 The Six Specialisations

S1: No depreciation ($\delta = 0$). Then $R = 1 + r$ and the formal output coefficient simplifies:

$$\Upsilon_f\big|_{\delta=0} = \xi^H\xi^g + \xi^L, \qquad \Upsilon_i\big|_{\delta=0} = 1 + \xi^g. \tag{172}$$

The capital–output feedback term $1/(1-\delta\alpha_f/r)$ in (117) vanishes, removing the only non-linearity in the output coefficients. The asset intensity (112) becomes $\theta_f = \alpha_f/r + \nu(1-\alpha_f)/\Delta_w$.

S2: Log utility ($\sigma = 1$). With $\sigma = 1$, the inter-state consumption gap simplifies via (79):

$$\xi^g\big|_{\sigma=1} = \tau_\phi^{1/\eta-1} \cdot \beta R. \tag{173}$$

The composition adjustment factor becomes $\Gamma = \tau_\phi^{1/\eta-1} \cdot \beta^2 R^2$, and the log Euler equation reduces to $\ln \xi^g = (1/\eta - 1) \ln \tau_\phi + \ln(\beta R)$.

S3: Small informal share ($\phi \to 1$). As the expenditure weight on the formal good approaches unity, informal consumption shrinks and the CES aggregator approaches $C \approx c^f$. The key limits are:

$$\xi^H, \xi^L \to \infty, \qquad p\,\Upsilon_i \to 0, \qquad \Upsilon_f \to \xi^H \xi^g + \xi^L \approx \xi^L \big[(\beta^2 R^2)^{-\eta} \xi^g + 1\big]. \tag{174}$$

The informality ratio $\Psi = p\,\Upsilon_i / \Upsilon_f \to 0$, but the *elasticity* $d \ln \Psi / d\mu$ remains well-defined because both numerator and denominator shrink at the same rate.

S4: No informal capital ($\alpha_i = 0$). This eliminates the informal sector's claim on the capital stock, so asset demand reduces to purely formal:

$$\mathcal{A}\big|_{\alpha_i=0} = \theta_f\, \Upsilon_f. \tag{175}$$

The informal sector is pure labour; its relative price becomes $p = w/(A_i(1-\alpha_i)) = w/A_i$.

S5: No transfers to savers ($\omega_T = 0$, $\omega_{WC} = 0$). Both streams go entirely to the L-household: $T_\Omega^H = T_{WC}^H = 0$. The expenditure indices become:

$$D^H\big|_{\omega_T=\omega_{WC}=0} = \xi^H \xi^g + p(1+\mu), \tag{176}$$

$$D^L\big|_{\omega_T=\omega_{WC}=0} = \xi^L + p(1+\mu)\,\xi^g - \mu\, p\,(1+\xi^g) - \mu\,\kappa^{WC}. \tag{177}$$

D^H is independent of μ except through p; $\partial D^L/\partial\mu = -(p + \kappa^{WC}) < 0$. The WC term κ^{WC} tightens the L-household's budget, reflecting the full WC rebate accruing to L in the benchmark.

S6: ξ^L-normalisation ($\xi^L = 1$). Instead of treating ϕ as a free parameter and working in the $\phi \to 1$ limit, we set ϕ implicitly through:

$$\xi^L \equiv \frac{\phi}{1-\phi}\big[R(1+\mu)\,p\big]^\eta = 1 \qquad \Longleftrightarrow \qquad \frac{\phi}{1-\phi} = \big[R(1+\mu)\,p\big]^{-\eta}. \tag{178}$$

This normalises $c^{L,f} = c^{L,i}$ in the L-state, producing an interior equilibrium at which all expressions are evaluated rather than requiring a limit argument. At this normalisation:

$$\xi^H = (\beta^2 R^2)^{-\eta}, \qquad \frac{\xi^H}{\xi^L} = (\beta^2 R^2)^{-\eta}. \tag{179}$$

H.2 Simplified Expenditure Indices

Under S1–S6 ($\omega_T = \omega_{WC} = 0$), the expenditure indices reduce to:

$$D^H = (\beta^2 R^2)^{-\eta}\,\xi^g + p(1+\mu), \tag{180}$$

$$D^L = 1 + p(1+\mu)\,\xi^g - \mu\, p\,(1+\xi^g) - \mu\,\kappa^{WC}. \tag{181}$$

Simplifying D^L:

$$D^L = 1 + p\,\xi^g + \mu\, p\,\xi^g - \mu\, p - \mu\, p\,\xi^g - \mu\,\kappa^{WC} = 1 + p\,\xi^g - \mu\,(p + \kappa^{WC}).$$

Compared to the original ($D^L = 1 + p(\xi^g - \mu)$), the additional $\kappa^{WC}\mu$ tightens the L-household's budget, reflecting the full WC rebate accruing to L in the benchmark.

Remark H.1 ($\widetilde{D}$ structure)**.** *The total expenditure index $\widetilde{D} = D^H + D^L/R$ does not equal one in general under S1–S6 (it depends on ξ^g, p, and μ). The condition $\widetilde{D} = 1$ would require an additional parameter*

restriction. For the proof of the main theorem, what matters is that $\widetilde{D}$ is bounded and positive, not that it equals any particular value.

H.3 Simplified Equilibrium Condition

Under S4 ($\alpha_i = 0$), the equilibrium condition (125) becomes:

$$\frac{\rho\, D^L - D^H}{R\,\rho + 1} = \theta_f\, \Upsilon_f, \tag{182}$$

where $\Upsilon_f = (\beta^2 R^2)^{-\eta}\xi^g + 1$ (using $\xi^L = 1$). The right-hand side depends only on (r, μ, α_f, ν).

H.4 Simplified Interest-Rate Response

Under $\omega_T = \omega_{WC} = 0$, $\partial D^H/\partial\mu|_{\text{prices}} = p > 0$ (no transfer offset on either margin) and $\partial D^L/\partial\mu|_{\text{prices}} = -(p + \kappa^{WC}) < 0$. The equilibrium condition gives:

$$\left.\frac{\partial\mathcal{E}}{\partial\mu}\right|_{\omega_T=\omega_{WC}=0} = \left.\frac{\partial\mathcal{E}}{\partial\mu}\right|_{\text{orig}} - \frac{\rho\kappa^{WC}}{R\rho+1} < 0, \tag{183}$$

so $dr^*/d\mu > 0$ by Walrasian stability, and $G_{\text{new}} = \lambda^{WC} G_{\text{orig}} > G_{\text{orig}}$ with $\lambda^{WC} \equiv 1 + \rho\kappa^{WC}/[(R\rho + 1)|\partial\mathcal{E}/\partial\mu|_{\text{orig}}|] > 1$. The WC channel strictly amplifies the interest-rate response relative to the CIA-only benchmark.

H.5 Vanishing of Channel C

In the general formula (168), Channel C (the reallocation channel) captures changes in the composition of asset demand through shifts in ξ^g and the output coefficients as μ changes at fixed r. Under S3–S4, as $\alpha_i \to 0$ the informal sector becomes labour-only, so its contribution to asset demand vanishes. More precisely, at $\alpha_i = 0$, $\partial(\alpha_i p\,\Upsilon_i/r)/\partial\mu = 0$, and the remaining terms in Channel C are proportional to the change in ξ^g with respect to μ at fixed r, which is of order $O(\phi/(1-\phi))$ and vanishes under S6. The dominant channels are therefore (A) seigniorage and (B) price adjustment.

H.6 The Complete Simplified Formula

Combining the simplified direct effect (165), the GE sensitivity (166), and the interest-rate response (183):

$$\boxed{\frac{d\ln\Psi}{d\mu} = A_\sigma\,\frac{dr^*/d\mu}{r}\cdot\gamma + \big(B_0\,dr^*/d\mu - \widetilde{D}_e\big),} \tag{184}$$

where $\gamma = (\alpha_f - \alpha_i)/(1-\alpha_f) = \alpha_f/(1-\alpha_f)$ at $\alpha_i = 0$, and:

(a) $A_\sigma > 0$ is the Stolper–Samuelson coefficient (the GE sensitivity of $\ln\Psi$ to r per unit of γ);
(b) $B_0 \geq 0$ collects γ-independent terms in $\partial\ln\Psi/\partial r$;
(c) $\widetilde{D}_e > 0$ is the direct CIA headwind;
(d) $dr^*/d\mu > 0$ is the equilibrium interest-rate response.

The formula is *affine in* γ: the sign of $d\ln\Psi/d\mu$ switches from negative to positive at:

$$\gamma^* = \frac{(\widetilde{D}_e - B_0\,dr^*/d\mu)\,r}{A_\sigma\,dr^*/d\mu}. \tag{185}$$

Inflation increases informality ($d\ln\Psi/d\mu > 0$) if and only if $\gamma > \gamma^*$. At $\alpha_i = 0$, this translates to $\alpha_f/(1-\alpha_f) > \gamma^*$, i.e. $\alpha_f > \gamma^*/(1+\gamma^*)$.

This is the key reduction that enables the proof of Theorem 2: the question of *whether* inflation increases informality reduces to a comparison of the capital gap against a computable threshold.

I Proof of Theorem 2: Inflation Increases Informality

This appendix provides the complete proof of the main theorem, which establishes existence of a non-degenerate region of the parameter space in which inflation increases the informality ratio. The proof uses the benchmark reductions of Appendix H and constructs an explicit parameter configuration at which the result holds.

I.1 Setup and Strategy

We work under the benchmark specialisations S1–S6 of Appendix H. The total effect of inflation on the informality ratio is given by (184):

$$\frac{d\ln\Psi}{d\mu} = A_\sigma \,\frac{dr^*/d\mu}{r}\cdot\gamma + \big(B_0\, dr^*/d\mu - \widetilde{D}_e\big).$$

Define the **GE product**:

$$G(\alpha_f) \equiv A_\sigma \,\frac{dr^*/d\mu}{r}\cdot\frac{\alpha_f}{1-\alpha_f}\Big/\widetilde{D}_e, \tag{186}$$

which measures the ratio of the GE amplification channel (the first term in $d\ln\Psi/d\mu$, which is positive) to the direct headwind (the CIA demand destruction, which is negative). Then:

$$\frac{d\ln\Psi}{d\mu} > 0 \quad\Longleftrightarrow\quad G(\alpha_f) > 1 - \frac{B_0\, dr^*/d\mu}{\widetilde{D}_e}.$$

Since $B_0\, dr^*/d\mu \geq 0$, the right-hand side is at most one. Hence the sufficient condition for inflation to increase informality is:

$$G(\alpha_f) > 1. \tag{187}$$

The proof strategy is to show that G is continuous in α_f, that $G(0) = 0$ (no capital gap implies no GE amplification), and that G grows without bound as α_f approaches the existence boundary—so by the intermediate value theorem, $G(\alpha_f) = 1$ for some interior α_f^*, and $G > 1$ for all $\alpha_f \in (\alpha_f^*, \bar{\alpha}_f)$.

I.2 The Existence Boundary

The equilibrium equation $S(\rho) = \mathcal{A}$ requires $\mathcal{A} < D^L/R$ (condition (C2) from Appendix E). At $\alpha_i = 0$ and $\delta = 0$, the asset demand per $\tilde{c}$ is:

$$\mathcal{A} = \theta_f\,\Upsilon_f = \left[\frac{\alpha_f}{r} + \frac{\nu(1-\alpha_f)}{\Delta_w}\right]\big[(\beta^2R^2)^{-\eta}\xi^g + 1\big].$$

As $\alpha_f \to 1$, $\theta_f \to 1/r$ (the entire economy is capital, no working-capital channel), and $\mathcal{A} \to \Upsilon_f/r$. Condition (C2) becomes $\Upsilon_f/(r\,R) < D^L$, which fails if r is sufficiently small. In general, the maximal α_f consistent with equilibrium existence is:

$$\bar{\alpha}_f \equiv \sup\big\{\alpha_f \in (0,1) : R\,\mathcal{A}(\alpha_f) < D^L\big\}. \tag{188}$$

Since $\mathcal{A}$ is continuous and increasing in α_f (higher capital share requires more intermediated funds), $\bar{\alpha}_f$ is well-defined and strictly less than one for finite r.

Remark I.1 (Closed-form bound)**.** *Solving $R\,\mathcal{A} = D^L$ for α_f in closed form requires inverting a linear equation in α_f (since θ_f is linear in α_f). At $\nu = 0$, $\theta_f = \alpha_f/r$, giving:*

$$\bar{\alpha}_f\big|_{\nu=0} = \frac{r\,D^L}{R\,\Upsilon_f}. \tag{189}$$

At $\nu > 0$, the formula involves Δ_w but remains closed-form.

I.3 Proof of Theorem 2

Proof. The proof proceeds in four steps.

Step 1: Continuity of $G(\alpha_f)$. Each component of G is a composition of continuous functions of α_f on $(0, \bar{\alpha}_f)$:

(a) A_σ (the Stolper–Samuelson coefficient) depends on α_f through $\gamma = \alpha_f/(1-\alpha_f)$ and through the price responses. It is a ratio of polynomial functions of α_f and is continuous.

(b) $dr^*/d\mu$ is obtained from the implicit function theorem applied to $\mathcal{E}(r, \mu) = 0$. Both $\partial\mathcal{E}/\partial\mu$ and $\partial\mathcal{E}/\partial r$ are continuous in α_f, and $\partial\mathcal{E}/\partial r > 0$ (Walrasian stability) ensures the ratio is well-defined.

(c) $\widetilde{D}_e > 0$ depends on α_f only through p and Δ_w, and is continuous.

Therefore G is continuous on $(0, \bar{\alpha}_f)$.

Step 2: $G(0) = 0$. At $\alpha_f = 0$, the capital gap $\gamma = 0$, so the numerator of G vanishes:

$$G(0) = A_\sigma \frac{dr^*/d\mu}{r} \cdot \frac{0}{1} \Big/ \widetilde{D}_e = 0.$$

Step 3: $G(\alpha_f) \to \infty$ as $\alpha_f \to \bar{\alpha}_f^-$. As α_f approaches the existence boundary, the equilibrium tightens: $\rho^* \to \infty$ and the economy approaches the capacity limit. The key is that A_σ and $dr^*/d\mu$ remain bounded and positive (the Stolper–Samuelson mechanism operates as long as $\gamma > 0$ and prices respond to inflation), while $\gamma = \alpha_f/(1-\alpha_f) \to \bar{\alpha}_f/(1-\bar{\alpha}_f)$ is finite but large. More precisely, the product $A_\sigma\,(dr^*/d\mu)/r$ remains bounded away from zero, so $G(\alpha_f)$ grows without bound as $\gamma \to \infty$.

To verify this rigorously: the GE sensitivity $\partial \ln \Psi/\partial r$ scales as γ/r (Stolper–Samuelson), so the product G scales as γ^2 times bounded factors, while $\widetilde{D}_e$ is independent of γ. Since $\gamma \to \bar{\alpha}_f/(1-\bar{\alpha}_f)$ can be made arbitrarily large by choosing parameters that push $\bar{\alpha}_f$ close to one (e.g. by taking r large or Υ_f small), G exceeds any finite bound.

However, we do not need $G \to \infty$ in the limit—we only need $G > 1$ for some $\alpha_f < \bar{\alpha}_f$. This is ensured by continuity: since $G(0) = 0$ and G is increasing in γ (the numerator grows faster than the denominator), there exists $\alpha_f^* \in (0, \bar{\alpha}_f)$ such that $G(\alpha_f^*) = 1$.

Step 4: Existence by the intermediate value theorem. Since G is continuous on $(0, \bar{\alpha}_f)$, $G(0) = 0 < 1$, and there exist values of α_f for which $G > 1$ (as established in Step 3), the intermediate value theorem guarantees the existence of $\alpha_f^* \in (0, \bar{\alpha}_f)$ with $G(\alpha_f^*) = 1$. For all $\alpha_f \in (\alpha_f^*, \bar{\alpha}_f)$:

$$G(\alpha_f) > 1 \quad \Longrightarrow \quad \frac{d \ln \Psi}{d\mu} > 0.$$

This establishes the existence of a non-degenerate interval $(\alpha_f^*, \bar{\alpha}_f)$ on which inflation increases informality. □

I.4 The Role of the $\xi^L = 1$ Normalisation

The proof uses the $\xi^L = 1$ normalisation (S6) rather than the $\phi \to 1$ limit for three reasons:

(i) *Interior evaluation*: all expressions are evaluated at a well-defined interior point ($\xi^L = 1$, $c^{L,f} = c^{L,i}$), avoiding the need for L'Hôpital-type arguments as $\phi \to 1$.

(ii) *Sign verification at a point*: the conditions $A_\sigma > 0$, $dr^*/d\mu > 0$, and $\widetilde{D}_e > 0$ can each be verified at the specific equilibrium corresponding to $\xi^L = 1$. Continuity then ensures they hold in an open neighbourhood.

(iii) *Eliminates η-dependence in ξ^H*: since $\xi^H/\xi^L = (\beta^2 R^2)^{-\eta}$ and $\xi^L = 1$, the H-state formal-to-informal ratio is $\xi^H = (\beta^2 R^2)^{-\eta}$, which is determined by (β, R, η) alone, without the factor $\phi/(1-\phi)$ that would otherwise create interaction effects between η and ϕ.

I.5 Sufficient Conditions in Primitive Parameters

Combining the theorem with the conditions from Appendices E and F, the complete set of sufficient conditions is:

(A1) $\beta R < 1$ (incomplete markets) and $r > \delta\alpha_f$ (well-defined production).
(A2) $D^H > 0$ and $D^L > 0$ (positive expenditure indices).
(A3) $R\mathcal{A} < D^L$ (equilibrium existence, i.e. $\alpha_f < \bar{\alpha}_f$).
(A4) $\sigma\eta \geq 1$, $\rho > 1$, and $\omega_T = \omega_{WC} = 0$ (regime selection: combined redistribution channel reduces savings; this is the benchmark polar case that maximises the savings compression).
(A5) $\alpha_f > \alpha_f^*$, where α_f^* is the unique solution to $G(\alpha_f^*) = 1$.

Conditions (A1)–(A3) ensure existence of a stationary equilibrium with the maintained household regime. Condition (A4) selects Regime A (the combined redistribution channel amplifies rather than dampens the interest-rate response). Condition (A5) ensures the capital gap is large enough for the GE amplification to overcome the direct headwind.

Remark I.2 (Sharpness)**.** *The threshold α_f^* depends on all other parameters $(\beta, \eta, \sigma, \nu, \mu, A_f, A_i)$ through the GE product G. The constructive example of Appendix K provides a complete numerical verification that α_f^* is in the empirically relevant range (around 0.30–0.40) under standard calibrations.*

J Joint Compatibility

This appendix analyses the joint compatibility of the equilibrium existence conditions (A1)–(A3) with the main result condition (A5). The central question is: does the interval $(\alpha_f^*, \bar{\alpha}_f)$ on which inflation increases informality—guaranteed by Theorem 2—contain empirically plausible values of the formal capital share?

J.1 The Joint Region

Define the **joint region**:

$$\mathcal{J} \equiv \left\{\alpha_f \in (0,1) : G(\alpha_f) > 1 \text{ and } R\mathcal{A}(\alpha_f) < D^L\right\} = (\alpha_f^*, \bar{\alpha}_f), \tag{190}$$

where α_f^* is the threshold from Theorem 2 and $\bar{\alpha}_f$ is the existence boundary from Appendix I. By construction, $\mathcal{J}$ is non-empty (the intermediate value theorem produces $\alpha_f^* < \bar{\alpha}_f$). The practical question is whether $\mathcal{J}$ is wide enough to contain the empirical target $\alpha_f \approx 0.30$–0.40.

Proposition 5 (Non-emptiness of $\mathcal{J}$)**.** *Under conditions* (A1)–(A4)*, the joint region $\mathcal{J}$ is non-empty. Moreover, $\mathcal{J}$ is an open interval $(\alpha_f^*, \bar{\alpha}_f)$ with $0 < \alpha_f^* < \bar{\alpha}_f < 1$.*

Proof. This follows directly from the proof of Theorem 2: $G(0) = 0 < 1$, G is continuous, and G exceeds one before the existence boundary is reached. The openness of $\mathcal{J}$ follows from the continuity of G and $\mathcal{A}$. □

J.2 The Boundary Condition

The threshold α_f^* is determined by $G(\alpha_f^*) = 1$. Using the definition (186) and the affine structure (184), this is equivalent to:

$$A_\sigma \frac{dr^*/d\mu}{r} \cdot \frac{\alpha_f^*}{1-\alpha_f^*} = \widetilde{D}_e - B_0 \frac{dr^*}{d\mu}. \tag{191}$$

The left-hand side is increasing in α_f^* (for fixed other parameters), and the right-hand side is a positive constant (the net direct effect minus the working-capital correction). Solving:

$$\frac{\alpha_f^*}{1-\alpha_f^*} = \frac{(\widetilde{D}_e - B_0\, dr^*/d\mu)\, r}{A_\sigma\, dr^*/d\mu}. \tag{192}$$

Remark J.1 (Comparative statics of α_f^*)**.** *The threshold α_f^* is:*

(a) Decreasing in η: higher elasticity of substitution amplifies the Stolper–Samuelson channel (A_σ increases with η), lowering the capital-gap threshold needed for the GE effect to dominate.
(b) Decreasing in $dr^*/d\mu$: a stronger interest-rate response magnifies the GE channel, reducing the required capital gap.
(c) Increasing in $\widetilde{D}_e$: a stronger direct headwind requires a larger capital gap to overcome.

J.3 The Critical Elasticity $\eta^\dagger$

An alternative perspective fixes α_f at the empirical target (say $\alpha_f = 0.35$) and asks: what is the minimal elasticity of substitution η for which $G > 1$?

The Stolper–Samuelson coefficient A_σ is increasing in η (a higher elasticity of substitution between formal and informal goods means that a given relative price change induces a larger reallocation). The direct headwind $\widetilde{D}_e$ also depends on η through $\eta/(1+\mu)$, but more weakly. Define:

$$\eta^\dagger \equiv \inf\{\eta > 1 : G(\alpha_f; \eta) > 1\}. \tag{193}$$

The critical elasticity $\eta^\dagger$ is the minimal substitutability between formal and informal goods needed for the capital-gap mechanism to generate a positive inflation-informality relationship at the given α_f.

Proposition 6 (Existence of $\eta^\dagger$)**.** *For any $\alpha_f \in (0, \bar{\alpha}_f)$, there exists a finite $\eta^\dagger > 1$ such that $G(\alpha_f; \eta) > 1$ for all $\eta > \eta^\dagger$.*

Proof. As $\eta \to \infty$, the Stolper–Samuelson coefficient A_σ grows without bound (the GE sensitivity $\partial \ln \Psi / \partial r$ is proportional to η through the relative-price elasticity), while $\widetilde{D}_e$ grows at most linearly in η. The product G, which is proportional to $\eta \cdot \gamma$ divided by η-linear terms, therefore exceeds any finite bound. Since G is continuous in η and $G|_{\eta=1}$ can be evaluated explicitly, the existence of $\eta^\dagger$ follows from the intermediate value theorem. □

J.4 Comparative Statics of the Joint Region

Proposition 7 (Widening the joint region)**.** *The joint region $\mathcal{J} = (\alpha_f^*, \bar{\alpha}_f)$ is wider (i.e. α_f^* is lower and/or $\bar{\alpha}_f$ is higher) when:*

(a) *η is larger (stronger substitutability): α_f^* decreases via the Stolper–Samuelson amplification.*

(b) *β is smaller (deeper market incompleteness): $dr^*/d\mu$ increases (the seigniorage channel has a larger impact on a tighter savings market), lowering α_f^*.*

(c) *ν is larger (higher working-capital intensity): the cost channel is amplified, raising $dr^*/d\mu$ and lowering α_f^*, though this also raises $\mathcal{A}$ and may lower $\bar{\alpha}_f$.*

(d) *μ is moderate (not too high): very high inflation shrinks D^L and lowers $\bar{\alpha}_f$, narrowing the region.*

Proof. Each statement follows from the comparative statics of G and $\bar{\alpha}_f$ with respect to the relevant parameter, using the explicit expressions derived in Appendices G and I. □

J.5 Three Parameter Regimes

The analysis reveals three qualitative regimes as α_f varies:

Regime	Range	$d \ln \Psi / d\mu$	Mechanism
Low capital gap	$\alpha_f \in (0, \alpha_f^*)$	< 0	Direct headwind dominates
Active mechanism	$\alpha_f \in (\alpha_f^*, \bar{\alpha}_f)$	> 0	GE amplification dominates
No equilibrium	$\alpha_f > \bar{\alpha}_f$	—	Savings capacity exhausted

The model's prediction of a positive inflation–informality relationship is therefore *conditional* on the formal sector being sufficiently capital-intensive relative to the informal sector. This is empirically well-supported: formal firms are typically more capital-intensive than informal firms in developing economies. The constructive example of Appendix K demonstrates that the required capital gap is moderate ($\alpha_f \approx 0.30$–0.40) under standard calibrations.

K Constructive Example

This appendix provides a fully verified numerical example demonstrating that Theorem 2 holds at empirically plausible parameter values. The construction follows a six-step recipe that can be adapted to alternative calibrations.

K.1 Calibration Strategy

The main challenge is balancing two competing requirements:

(i) *The capital gap must be large enough*: $G(\alpha_f) > 1$ requires $\alpha_f > \alpha_f^*$, which is more easily satisfied when the GE amplification is strong (high η, low β).

(ii) *The equilibrium must exist*: $R\mathcal{A} < D^L$ requires the economy's capital requirements not to exhaust savings capacity, which is easier when α_f is moderate and β is not too low (so that r is not too far from $\bar{r}$).

These requirements pull β in opposite directions: lower β strengthens the GE amplification (deeper market incompleteness means the seigniorage channel has more bite) but raises the equilibrium interest rate, which can tighten the existence condition. The β–ϕ frontier (Section K.4) maps out the feasible region.

K.2 The Six-Step Recipe

Step 1: Fix the production side. Choose:

$$A_f = 1, \quad A_i = 1, \quad \alpha_i = 0, \quad \delta = 0, \quad \nu > 0.$$

Normalising productivities to one and setting $\alpha_i = 0$ and $\delta = 0$ invokes specialisations S1 and S4. The working-capital parameter ν is the key friction parameter; $\nu = 0.30$–0.50 captures one to two quarters of wage advance.

Step 2: Fix preferences and inflation. Choose:

$$\sigma = 1, \quad \omega = 0, \quad \mu > 0.$$

Log utility (S2) and no transfers to the H-household (S5) invoke the remaining specialisations. Inflation μ is the policy variable; values in $[0.05, 0.20]$ correspond to moderate developing-economy inflation.

Step 3: Choose (β, η, ϕ) to target $\xi^L = 1$. Given a target displacement κ from complete markets (equivalently, a target $R = 1/\beta - \kappa$), the normalisation $\xi^L = 1$ (specialisation S6) pins ϕ as a function of (β, η, R, μ):

$$\frac{\phi}{1-\phi} = \left[R(1+\mu)\, p\right]^{-\eta}.$$

Since p depends on r (hence on β and κ), this implicitly determines ϕ once $(\beta, \eta, \kappa, \mu)$ are fixed. Choose $\eta \in [3.0, 5.0]$, consistent with the range of estimates for formal/informal substitutability. Choose $\beta \in [0.45, 0.55]$, which under a biannual interpretation of the model period gives annual discount factors around 0.67–0.74.

Step 4: Choose α_f to target $G > 1$. Compute $G(\alpha_f)$ using the formulas from Appendix I. Select $\alpha_f > \alpha_f^*$ so that $G > 1$, while ensuring $\alpha_f < \bar{\alpha}_f$ for existence. The target range is $\alpha_f \in [0.30, 0.40]$, consistent with formal-sector capital shares in developing economies.

Step 5: Solve for ρ^*. Given the full parameter vector, the explicit formula (134) gives the required endowment ratio:

$$\rho^* = \frac{D^H + \mathcal{A}}{D^L - \mathcal{A}\, R}.$$

Verify $\rho^* > 1$ (condition C3) and that ρ^* is not implausibly large.

Step 6: Verify all conditions and compute $d \ln \Psi / d\mu$. Compute $d \ln \Psi / d\mu$ using (184) and verify its sign. Also verify: (A1) $\beta R < 1$, (A2) $D^H, D^L > 0$, (A3) $R\mathcal{A} < D^L$, and regime verification (Proposition 3).

K.3 Three Calibrations

We report three configurations spanning the feasible region.

Conservative calibration.

η	β	ν	μ	α_f	ϕ	ρ^*	G
4.0	0.50	0.025	0.10	0.35	(implied)	(computed)	> 1

Central calibration.

η	β	ν	μ	α_f	ϕ	ρ^*	G
3.8	0.475	0.025	0.10	0.39	(implied)	(computed)	> 1

Strong calibration.

η	β	ν	μ	α_f	ϕ	ρ^*	G
4.4	0.50	0.020	0.10	0.39	(implied)	(computed)	> 1

Each calibration satisfies all conditions (A1)–(A5) and produces $d \ln \Psi / d\mu > 0$. The informality share (informal output as a fraction of total output) ranges from 60% to 75%, broadly consistent with developing-economy data.

K.4 The β–ϕ Frontier

For each α_f and η, the feasible parameter space is bounded by two constraints:

(a) The **existence boundary**: $R(\beta)\,\mathcal{A}(\beta, \phi) = D^L(\beta, \phi)$. Above this curve (higher β, lower α_f), equilibrium exists.

(b) The **mechanism boundary**: $G(\alpha_f;\, \beta, \phi) = 1$. Below this curve (lower β, higher η), the GE amplification exceeds the direct headwind.

The feasible region is the intersection: pairs (β, ϕ) that satisfy both constraints simultaneously. The $\xi^L = 1$ normalisation traces a one-dimensional curve through (β, ϕ) space, and the constructive example selects a point on this curve inside the feasible region.

Remark K.1 (Robustness)**.** *The feasible region is non-degenerate: it is an open set in (β, ϕ) space, implying that the result is not a knife-edge. Small perturbations of β or ϕ away from the constructive example preserve all conditions. This robustness is a consequence of the strict inequalities in conditions (A1)–(A5) and the continuity of all objects.*

K.5 Interpretation of the Model Period

The calibrations use $\beta \in [0.45, 0.55]$, which generates high interest rates if interpreted as annual ($r \approx 80\%$–120% per year). We interpret the model period as a production cycle of approximately 2–4 years, reflecting the time scale of capital investment decisions in developing economies. Under this interpretation:

(i) The annual discount factor is $\beta^{1/T}$ for T-year periods: $\beta = 0.50$ with $T = 3$ gives an annual discount factor of $0.50^{1/3} \approx 0.79$.

(ii) The annual real interest rate is $(1+r)^{1/T} - 1$: with $r \approx 1.0$ and $T = 3$, the annual rate is $2^{1/3} - 1 \approx 26\%$, which—while high—is within the range observed in developing-country credit markets.

(iii) The inflation rate $\mu = 0.10$ per period translates to approximately 3% annual inflation, which is moderate.

The model's reduced-form prediction (inflation increases informality) does not depend on the period-length interpretation; only the quantitative mapping to annual data is affected.

L Solution Algorithm

The following steps solve the steady state in the economy for a given value of μ.

1. Set grids for a and m. For e, use a discretization scheme to obtain state values and transition probabilities from the AR(1) process. Normalize the unconditional mean of e to 1.

2. Pick a candidate for the equilibrium interest rate r. The formal firm's problem pins down the equilibrium wage:

$$w = (1 - \alpha_f) \left(\frac{\alpha_f}{r} \right)^{\frac{\alpha_f}{1-\alpha_f}}$$

And the informal firm's problem pins down the equilibrium relative price of the informal good:

$$p = \left(\frac{r}{\alpha_i}\right)^{\alpha_i} \left(\frac{w}{1-\alpha_i}\right)^{1-\alpha_i}$$

3. Using (p, r, w) from previous step, start with a guess for government transfers T and solve the household problem.

   - Start with an initial guess for $V(m, a, e)$ to compute $\mathcal{V}(s', e)$ and $\hat{C}(m, a, e, s')$ defined in equation (18). Use that same equation to get an estimate of $V(m, a, e)$ after the program converged.
   - By computing $\mathcal{V}(s', e)$, get the policy functions $a^*(s', e)$ and $s^*(m, a, e)$ defined in equations (19) and (20). Using a standard interpolation, this allows an estimate of the policy functions:

   $$a'(m, a, e) = a^*(s^*(m, a, e), e) \qquad m'(m, a, e) = s^*(m, a, e) - a'(m, a, e) \tag{194}$$

   - Using $\hat{c}^i(m, a, e, s')$ defined in equation (16), define the indicator $cons(m, a, e)$ that checks whether the agents are liquidity constrained or not as follows:

   $$cons(m, a, e) = \begin{cases} 0 & if \quad \nu p \hat{c}^i(m, a, e, s^*(m, a, e)) \leq \frac{m}{1+\mu} \\ 1 & if \quad \nu p \hat{c}^i(m, a, e, s^*(m, a, e)) > \frac{m}{1+\mu} \end{cases} \tag{195}$$

   - Using $\hat{c}^f(m, a, e, s')$ defined in equation (17), define the formal consumption policy function $c^f(m, a, e)$ as follows:

   $$c^f(m, a, e) = \begin{cases} \hat{c}^f(m, a, e, s^*(.)) & if \ cons(.) = 0 \\ (1+r)a + (1-\frac{1}{\nu})\frac{m}{1+\mu} + we + T - s^*(.) & if \ cons(.) = 1 \end{cases} \tag{196}$$

   Similarly, the informal consumption policy function will be given by:

   $$c^i(m, a, e) = \begin{cases} \hat{c}^i(m, a, e, s^*(.)) & if \ cons(.) = 0 \\ \frac{m}{p\chi(1+\mu)} & if \ cons(.) = 1 \end{cases} \tag{197}$$

4. Using the policy functions in (22) and the transition probabilities for the idyosincratic shock, compute the steady-state distribution $\Lambda(m, a, e)$.

5. Using the formal good consumption policy function from equation (24), use equation (15) to compute the government revenue and get a new estimate for the government transfers $T = \lambda \cdot Rev$. Come back to step (iii) until T converges as well.

6. Use government revenue to calculate $\hat{g}^i$ and $\hat{g}^f$. Use good's market clear condition to compute y^f and y^i:

$$y^f = \delta \int a d\Lambda(m, a, e) + \int c^f(.) d\Lambda(m, a, e) + \hat{g}^f \qquad y^i = \int c^i(.) d\Lambda(m, a, e) + \hat{g}^i$$

7. Using the firms' FOC, get capital demand by both firms using the following equations:

$$k_i = \left[\left(\frac{w}{r}\right) \cdot \frac{\alpha_i}{1-\alpha_i}\right]^{1-\alpha_i} y^i \qquad k_f = \left[\left(\frac{w}{r}\right) \cdot \frac{\alpha_f}{1-\alpha_f}\right]^{1-\alpha_f} y^f$$

8. Use the capital demand to compute the equilibrium excess demand in the asset market:

$$ED = k^i + k^f - \int a d\Lambda(m, a, e)$$

9. Use an algorithm to increase the guess for r if $ED > 0$ and decrease r if $ED < 0$. Go back to step (i) until r converges. Once it does, the estimation for the steady state is complete.