Monetary-Fiscal Nexus and Unintended Equilibria*

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Abstract

Multiple equilibria pose challenges for rule-based central banks with inflation targets. I show that, with non-Ricardian households, multiple steady states prevail. While a targeted steady state always exists, a steady state with higher or lower inflation emerges depending on monetary-fiscal interactions. Specifically, a high-inflation steady state emerges when either monetary or fiscal policy responds weakly to inflation or debt. Its associated local dynamics display determinacy while those around the targeted steady state become explosive. The dynamics of self-fulfilling 'deflationary' equilibria also change. Should they exist, at least one will exhibit determinacy and lower real interest rates.

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1 Introduction

Non-Ricardian households are the defining feature of heterogeneous-agent New Keynesian models (HANK). Households become non-Ricardian if they are subject to uninsurable income risk and borrowing constraints or consist of overlapping generations (OLG). The hallmark of such models is that households' saving in risk-free assets is increasing in the real interest rate in steady state. This contrasts with representative-agent New Keynesian models (RANK) in which households are Ricardian. In the models, households' saving is perfectly elastic at $1/\beta$, the reciprocal of the preference discount factor, which pins down the real interest rate in steady state.

This paper shows that when non-Ricardian households meet rule-based monetary and fiscal authorities, multiple steady states prevail. While there always exists a targeted steady state where inflation and government debt hit the targets set by the government, a steady state with higher or lower inflation emerges depending on the combinations of monetary and fiscal policies. Specifically, a high-inflation steady state emerges when either monetary or fiscal policy responds weakly to inflation or debt.

In addition, determinacy properties change when multiple equilibria prevail. Local dynamics around the targeted steady state exhibit determinacy (a unique equilibrium) when the steady state is unique or co-exist with a low-inflation steady state, while local dynamics around the low-inflation steady state display either indeterminacy or explosiveness. However, when a high-inflation steady state emerges, local dynamics around the targeted steady state lose determinacy and become explosive, and in turn those around the high-inflation steady state display determinacy.

Moreover, the existence and dynamics of self-fulfilling 'deflationary' (or zero-interest) equilibria, studied by Benhabib et al. (2001), also change. While there is a unique zero-interest steady state in RANK, with non-Ricardian households, there may exist two such steady states: one with lower real interest rates and the other with higher real interest rates than that of the targeted steady state. Given the existence, there always exists one with lower interest rates, and its local dynamics display determinacy. This contrasts with RANK, where a zero-interest equilibrium is indeterminate and the steady-state real interest rate is the same as that of the targeted steady state.

These results are shown analytically with minimum possible ingredients. The model used in this paper is a perpetual youth, OLG version of the standard New Keynesian model. To

focus on steady states and equilibrium determinacy, sticky prices and fundamental stochastic shocks are abstracted away. Thereby the model is basically deterministic, except for the possibility of non-fundamental (sunspot) shocks, and prices are flexible. As in Leeper (1991), the central bank sets the nominal interest rate in response to inflation, and the fiscal authority, which issues nominal debt, controls a lump-sum tax (the primary surplus) in response to the amount of debt. The model is reduced to a system of three equations: the aggregate Euler equation, the monetary policy rule subject to the zero lower bound (ZLB) on nominal interest rates, and the fiscal policy rule with the government budget constraint, with three variables: inflation, debt, and the nominal interest rate.

Albeit simple, this model nests various benchmark models. Without the ZLB constraint, the model is reduced to a flexible-price version of Angeletos et al. (2024, 2026) and Rachel and Ravn (2025). In addition, if non-Ricardian households are replaced with Ricardian households, the model is reduced to that of Leeper (1991). Moreover, if fiscal policy responds to debt infinitely strongly as well, the model is reduced to RANK with flexible prices.

The main difference of this model from that of Leeper (1991) appears in the aggregate Euler equation. The aggregate saving in government bonds is increasing in the real interest rate even in steady state, because of the OLG structure of households. The saving is made by those who are alive today. Since the bonds are supported by taxes not only today but also in the future, a part of the principal and interest payments of the bonds, purchased today by incumbent households, will be borne by next generations who will enter the economy in the future. Thus, for the incumbents, government bonds serve as net assets, which are held for consumption in the future. The incumbents are willing to substitute consuming today for holding net assets (consuming tomorrow) more if the real interest rate rises.

Multiple steady states prevail when this aggregate Euler equation is combined with the standard set of equations about monetary and fiscal policies. The supply of debt in real terms depends on inflation, as it devalues nominal debt, and the real interest rate. The real interest rate, in turn, is determined by inflation, as monetary policy sets the nominal rate in response to inflation. Thus, inflation affects the government's capacity to supply debt in two distinct ways, directly and indirectly, which helps generate a non-linear curve between the supply of real debt and inflation. The upward sloping Euler equation can cross the non-linear curve twice, giving rise to a high- or low-inflation steady state in addition to the targeted steady state. In this model, the presence of non-Ricardian households is necessary for the

existence of multiple steady states.

Given that households are non-Ricardian, monetary and fiscal interactions affect the regions of multiple steady states. Specifically, in a positive interest environment, when either monetary or fiscal policy responds strongly to inflation or debt, respectively, a high-inflation steady state emerges if the other policy responds weakly. Outside these regions a low-inflation steady state prevails if monetary policy's response is either strong or weak. In the remaining region, where monetary policy's response is moderate, there exists a unique targeted steady state.

After establishing the existence of multiple steady states, the paper studies the determinacy of local dynamics around each of the steady states. The determinacy properties of local dynamics around the targeted steady state changes relative to those of RANK, as studied by Rachel and Ravn (2025). In the presence of non-Ricardian households, the regions of determinacy and explosiveness expand, while the region of indeterminacy contracts. This paper demonstrates that a high-inflation steady state arises within the expanded explosive region as well as within the contracted indeterminacy region. Furthermore, the local dynamics around high-inflation steady states consistently exhibit determinacy. Consequently, when the targeted equilibrium loses determinacy and becomes explosive, the economy will shift toward a high-inflation equilibrium. Additionally, a low-inflation steady state exists within a subset of the expanded determinate region, where the associated local dynamics exhibit either indeterminacy or explosiveness.

The nature of zero-interest equilibria also changes with non-Ricardian households, as mentioned above. For the same reason as in the case of positive interest rates, there may exist multiple zero-interest steady states when households are non-Ricardian. Specifically, one steady state always features lower real interest rates than in the targeted steady state, and the associated local dynamics display determinacy. In RANK with flexible prices, the Euler equation depends on the expected inflation only, giving rise to indeterminacy. With non-Ricardian households, the Euler equation depends on government bonds as well. Inflation is determined to stabilize the amount of government bonds at the target, which is otherwise explosive. This in turn pins down the expected inflation in the Euler equation. Thus, the zero-interest equilibrium is uniquely determined as inflation is driven by fiscal policy.

The paper's main results of multiple steady states and the determinacy properties may have implications for monetary policy and inflation dynamics in practice. Many central

banks have adopted inflation targeting with a goal of stabilizing inflation at a target rate. Rule-based monetary policy with inflation targets has become a norm of the conduct of monetary policy. However, despite rule-based monetary policy, the paper's results imply that inflation can deviate from the target persistently. The historical episodes include the Global Financial Crisis of 2007-09 and afterward where inflation subdued below targets in many developed countries for several years; the Covid-19 pandemic and afterward where inflation spiked above targets globally. In the model such situations will occur when the economy moves to a high-inflation steady state or falls into a zero-inflation steady state, where the associated local dynamics are both determinate and thereby stable.

Related literature. This paper contributes to the literature on HANK, especially that which uses a perpetual youth OLG framework of Blanchard (1985) and Yaari (1965) with focus on monetary-fiscal interactions. The OLG households can be interpreted as heterogeneous agents subject to borrowing constraints (Farhi and Werning, 2019). Using the perpetual youth OLG framework, Angeletos et al. (2024, 2026) study how debt financing can be self-financed and how HANK generates the predictions of the fiscal theory of the price level (FTPL). Aguiar et al. (2023) characterize the optimal levels of government bonds and the associated monetary responses. Rachel and Ravn (2025) show that the properties of equilibrium determinacy around the targeted steady state change substantially when households are non-Ricardian. Dupraz and Rogantini Picco (2025) derive fiscal requirements as a limit on debt-to-GDP for price stability.[1] This paper shows that multiple steady states prevail in the same framework even in a positive interest rate environment. It then studies monetary-fiscal interactions that lead to multiple steady states and equilibrium determinacy of the associated local dynamics.

The distinguished feature of the model – households' saving in risk-free assets as an increasing function of the real interest rate in steady state – is shared with the Aiyagari-Bewley-Huggett type of HANK, such as McKay et al. (2016), Kaplan et al. (2018), Auclert (2019), and Bayer et al. (2019), and the quantitative OLG variant of HANK, such as Eggertsson et al. (2019), Bielecki et al. (2021), and Braun and Ikeda (2025). Hence, these quantitative HANK models could also have multiple steady states, especially high-inflation

[1] Using a similar framework, Fujiwara and Teranishi (2008) explore asymmetric effects of structural shocks on working and retired households; Gali (2021) studies rational asset bubbles and monetary policy; Del Negro et al. (2023) discuss the forward guidance puzzle.

equilibrium.

This paper builds on the literature on monetary-fiscal interactions and equilibrium determinacy in a positive interest rate environment. How monetary-fiscal policy rules affect the determinacy of inflation has been studied in a flexible-price model with money in the utility function (Leeper, 1991), RANK (Woodford, 2003, Chapter 4), and the perpetual youth, OLG version of HANK (Rachel and Ravn, 2025). These papers focus on the determinacy of an equilibrium around a targeted steady state, while Woodford (1994) and Sims (1994) analyze global dynamics, including sunspot equilibria, in flexible-price models with cash-in-advance constraints and transaction costs, respectively. Loyo (1999) studies an equilibrium with explosive inflation in a flexible-price version of RANK. This paper studies the perpetual youth, OLG version of HANK, as in Rachel and Ravn (2025), but with flexible prices, focusing on the determinacy of inflation around unintended steady states and how the determinacy conditions are related to those pertaining to the targeted equilibrium.

This paper also contributes to the literature on self-fulfilling zero-interest equilibria, initially studied by Benhabib et al. (2001). Self-fulfilling zero-interest equilibria are considered to explain the Japanese-style deflation that began in the mid-1990s and lasted for about a quarter century (Bullard, 2010; Honkapohja, 2015). However, these equilibria are locally indeterminate, leaving questions of how to select among different equilibria for policy analysis. Moreover, the equilibria are not E-stable, meaning that the economy under adaptive learning fails to locally converge to the self-fulfilling zero-interest steady state, as shown by Evans et al. (2008) and Christiano et al. (2018) for RANK with Rotemberg and Calvo pricing frictions, respectively. This paper shows that with non-Ricardian households, self-fulfilling zero-interest equilibria display determinacy, providing a solid foundation for using such equilibria for policy analysis.

The rest of the paper is organized as follows. Section 2 presents the model and defines an equilibrium as well as a steady state. Section 3 studies positive-interest equilibria. Section 4 analyzes zero-interest equilibria. Section 5 concludes the paper.

2 Model

The model I consider is a perpetual youth, OLG version of the standard New Keynesian model. To focus on steady states and local determinacy, nominal rigidities and exogenous

fundamental shocks are abstracted away.

Time is discrete and the horizon is infinite. There is a single numeraire good. The economy consists of households, firms, a central bank, and a fiscal authority. Households face constant mortality risk, $1-\omega \in [0,1)$, between any two consecutive periods. The mass of the total population of households is assumed to be constant and normalized to unity. In every period, the mass $1-\omega$ households exit the economy, and the same number of households enter the economy as newborns. In each period t, the population of households who were born in period $s \leq t$ is given by

$$f_t(s) = (1-\omega)\omega^{t-s} \tag{1}$$

Households work, consume, and save, taking prices as given. Firms are competitive and produce using a linear technology that transforms labor into goods. The central bank sets the nominal interest rate. The fiscal authority imposes lump-sum taxes on households and issues government bonds.

2.1 Households

Households enjoy utility from consumption and leisure. Each households is endowed with $\bar{n} > 0$ units of time in each period. In period t, the life-time utility of a household who was born in period $s \leq t$ is given by

$$\mathbb{E}_t \sum_{j=0}^{\infty} (\beta\omega)^j \left[\log c_{s,t+j} + \psi \log\left(\bar{n} - n_{s,t+j}\right)\right] \tag{2}$$

where $c_{s,t}$ denotes consumption in period t, $n_{s,t}$ denotes hours worked, $0 < \beta < 1$ is a preference discount factor, $\psi > 0$ is a coefficient of the utility of leisure, and $\mathbb{E}_t(\cdot)$ is an expectation operator given information in period t. The life-time utility takes into account the survival probability of $0 < \omega \leq 1$.

Households can save by holding one-period nominal bonds issued by the government. In period t, the nominal price of one unit of bonds is $1/(1+i_t)$, and it returns 1 nominal unit in period $t+1$, where i_t is the nominal interest rate. Households participate in a fair, risk-free insurance regarding their death risk, so that they receive $1/\omega$ nominal units by holding one unit of bonds. Let $B_{s,t}$ denote the amount of nominal government bonds in period t, purchased by the household who was born in period $s \leq t$. Then, the flow budget constraint

of the household in period t can be written as

$$P_t c_{s,t} + \frac{B_{s,t}}{1+i_t} = W_t n_{s,t} + \frac{B_{s,t-1}}{\omega} + P_t z_{s,t} - P_t \tau_t$$

where P_t is the price level, W_t is the nominal wage, $z_{s,t}$ is the real social fund transfer, and τ_t is the real lump-sum tax. In real terms the budget constraint can be written as

$$c_{s,t} + \frac{b_{s,t}}{1+i_t} = w_t n_{s,t} + \frac{b_{s,t-1}}{\omega \pi_t} + z_{s,t} - \tau_t \tag{3}$$

where $b_{s,t} = B_{s,t}/P_t$ is the amount of government bonds in real terms, $w_t = W_t/P_t$ is the real wage, and $\pi_t = P_t/P_{t-1}$ is the gross inflation rate. Let $R_{t+1} \coloneqq (1+i_t)/\pi_{t+1}$ denote the ex-post gross real interest rate. Iterating the flow budget constraint (3) forward yields the life-time budget constraint, given by

$$\mathbb{E}_t \sum_{j=0}^{\infty} \left(\prod_{\ell=1}^{j} \frac{\omega}{R_{t+\ell}} \right) c_{s,t+j} = \mathbb{E}_t \sum_{j=0}^{\infty} \left(\prod_{\ell=1}^{j} \frac{\omega}{R_{t+\ell}} \right) h_{s,t+j} + \frac{b_{s,t-1}}{\omega \pi_t} \tag{4}$$

where $h_{s,t+j} = w_{t+j} n_{s,t+j} + z_{s,t+j} - \tau_{t+j}$ is the wage income net of the transfer and tax. In deriving equation (4) the transversality condition, $\lim_{j\to\infty} \left(\prod_{\ell=1}^{j} \frac{\omega}{R_{t+\ell}} \right) b_{s,t+j} = 0$, is imposed.

Newly-born households enter the economy with no assets: $b_{t,t-1} = 0$. To ensure that there always exists a steady state in which the gross real interest rate $R_{t+1} = (1+i_t)/\pi_{t+1}$ is $1/\beta$ as in RANK, it is assumed that the social fund collects real resources from incumbent households and distributes them to newly-born households as follows:

$$z_{s,t} = \begin{cases} \frac{b^*}{\pi^*} & \text{for } s = t \\ -\frac{1-\omega}{\omega} \frac{b^*}{\pi^*} & \text{otherwise} \end{cases}$$

where b^* is the target amount of per-capita government bonds in real terms and π^* is the target rate of inflation. The social fund is self-financing as $\sum_{s \le t} z_{s,t} f_t(s) = 0$. In this specific steady state with $R_{t+1} = 1/\beta$ and $\pi_t = \pi^*$, the sum of the bonds held from previous period and the social transfer becomes identical for all households: $b_{s,t-1}/(\omega \pi_t) + z_{s,t} = b^*/\pi^*$ for all $s \le t$.

A household who was born in period $s \le t$ maximizes the utility (2) subject to the life-time budget constraint (4). Let $\lambda_{s,t}$ denotes a Lagrange multiplier on the constraint. The first-order conditions with respect to $c_{s,t}$ and $c_{s,t+j}$ are given, respectively, by

$$\frac{1}{c_{s,t}} = \lambda_{s,t}$$

$$\beta^j \frac{1}{c_{s,t+j}} = \lambda_{s,t} \left(\prod_{\ell=1}^{j} \frac{1}{R_{t+\ell}} \right)$$

Combining these equations yields

$$c_{s,t+j} = \beta^j \left(\prod_{\ell=1}^{j} R_{t+\ell} \right) c_{s,t}, \tag{5}$$

Thus, the standard Euler equation holds for individual households.

Regarding the choice of labor supply, to make the aggregation of individual variables simple, it is assumed that all households join a labor union which demands identical hours worked for participants. The labor union chooses hours worked n_t to maximize the average of the sum of wage income and the utility of leisure in terms of a good:

$$\max_{n_t \in [0,1]} \sum_{s \leq t} \left[w_t n_t + \frac{1}{\lambda_{s,t}} \psi \log (\bar{n} - n_t) \right] f_t(s)$$

where $f_t(s)$ is the population distribution, given by equation (1). Note that $\sum_{s \leq t} \lambda_{s,t}^{-1} f_t(s) = \sum_{s \leq t} c_{s,t} f_t(s) = c_t$, where c_t is the aggregate consumption. The solution yields the labor supply curve:

$$\psi c_t = w_t (\bar{n} - n_t) \tag{6}$$

This labor supply curve is identical to that of RANK. Finally, substituting equation (5) into the life-time budget constraint (4) yields

$$c_{s,t} = (1 - \beta\omega) \left[\frac{b_{s,t-1}}{\omega \pi_t} + \mathbb{E}_t \sum_{j=0}^{\infty} \left(\prod_{\ell=1}^{j} \frac{\omega}{R_{t+\ell}} \right) h_{s,t+j} \right] \tag{7}$$

The household consumes a fraction of the real value of nominal government bonds held from the previous period plus the discounted sum of future wage income net of transfers and taxes.

2.2 Firms

The firm sector is simple. Firms have a linear technology that transforms one unit of labor into one unit of the good, given by

$$y_t = n_t \tag{8}$$

Since the firms are competitive, the labor demand is perfectly elastic and the real wage is given by

$$w_t = 1 \tag{9}$$

2.3 Government

Fiscal authority. The fiscal authority issues one-period nominal bonds B_t and finances the redemption by imposing a lump-sum tax on households. For simplicity, government spending is assumed to be zero. Then, the government budget constraint is given by

$$\frac{B_t}{1+i_t} + P_t\tau_t = B_{t-1}$$

In real terms, it can be written as

$$\frac{b_t}{1+i_t} + \tau_t = \frac{b_{t-1}}{\pi_t} \tag{10}$$

The fiscal authority sets the lump-sum tax by following the tax rule, given by

$$\tau_t = \tau^* \left(\frac{b_{t-1}}{b^*}\right)^{\theta} \tag{11}$$

Parameter $\theta \geq 0$ governs feedback from deviations of real government debt from its target level of b^*. Equations (10) and (13) imply that, given the existence of the targeted steady state of $b = b^*$ and $\pi = \pi^*$, the parameter τ^* is given by

$$\tau^* = \frac{b^*}{\pi^*}\left(1 - \frac{\pi^*}{1+i^*}\right) > 0 \tag{12}$$

The inequality holds since the gross real interest rate at the targeted steady state is strictly greater than zero, given by $R^* = (1+i^*)/\pi^* = 1/\beta > 1$. Combining the budget constraint (10) and the tax rule (11) yields the fiscal policy rule for debt as

$$\frac{b_t}{1+i_t} = \frac{b_{t-1}}{\pi_t} - \tau^* \left(\frac{b_{t-1}}{b^*}\right)^{\theta} \tag{13}$$

Central bank. The central bank sets the short-term nominal interest rate by following a monetary policy rule with the zero lower bound (ZLB) of the interest rate:

$$1 + i_t = \max\left\{(1+i^*)\left(\frac{\pi_t}{\pi^*}\right)^{\phi}, 1\right\} \tag{14}$$

where $i^* = (\pi^*/\beta) - 1$ is targeted nominal interest rate. The ZLB constraint limits the gross nominal interest rate to be no less than unity, implying that the net nominal interest rate is bounded below by zero. Parameter $\phi \geq 0$ governs feedback from deviations of inflation from its target rate π^*.

2.4 Aggregation

The Euler equation (5) can be written as

$$1 = \beta \mathbb{E}_t \frac{c_{s,t}}{c_{s,t+1}} \frac{1 + i_t}{\pi_{t+1}} \tag{15}$$

Markets are complete, so that a pricing kernel from period t to $t+1$ is equated among households who were born in period $s \leq t$. Hence, the pricing kernel can be defined as $Q_{t,t+1} = \beta c_{s,t}/c_{s,t+1}$. Aggregating the pricing kernel over $s \leq t$ using equation (7) yields the aggregate version of the Euler equation as

$$1 = \beta \mathbb{E}_t \frac{c_t}{c_{t+1} + \chi \left(\frac{b_t}{\pi_{t+1}} - \frac{b^*}{\pi^*} \right)} \frac{1 + i_t}{\pi_{t+1}} \tag{16}$$

where

$$\chi = \frac{(1-\omega)(1-\beta\omega)}{\omega}$$

The derivation can be found in Appendix A.1.

Combining the labor supply equation (6) and the labor demand equation (9) yields $\psi c_t = \bar{n} - n_t$. The production technology (8) and the good market clearing condition imply $y_t = n_t = c_t$. Since prices are flexible, the output, consumption, and hours worked are constant at $y^* = \bar{n}/(1+\psi)$. Without loss of generality, $\bar{n}$ is set at $1 + \psi$, so that y^* is normalized to unity for notational simplicity. Then, the aggregate Euler equation (16) can be written as

$$1 = \beta \mathbb{E}_t \frac{1}{1 + \chi \left(\frac{b_t}{\pi_{t+1}} - \frac{b^*}{\pi^*} \right)} \frac{1 + i_t}{\pi_{t+1}} \tag{17}$$

2.5 Equilibrium

Competitive equilibrium. A competitive equilibrium for this economy consists of the aggregate Euler equation (17), the fiscal policy rule (13), and the monetary policy rule (14) with the ZLB constraint, with three variables $\{\pi_t, b_t, i_t\}$.

Albeit simple, the model encompasses key models as a special case. If households are Ricardian: $\chi = 0$ (or $\omega = 1$), the economy is essentially the same as that of Leeper (1991) if there were no ZLB constraint, and the discrete-time version of Benhabib et al. (2001). Without the ZLB constraint, the economy is essentially the same as the flexible-price version of Angeletos et al. (2024) and Rachel and Ravn (2025).

Steady states. Let $\{\pi, b, i\}$ denote a set of steady state variables: inflation, debt, and the nominal interest rate. In steady state, the Euler equation (17) is reduced to

$$\frac{b}{\pi} = \frac{1}{\chi}\left[\beta\left(\frac{1+i}{\pi}\right) - 1\right] + \frac{b^*}{\pi^*} \tag{18}$$

Given that households are non-Ricardian ($\chi > 0$), the deb deflated by inflation is increasing in the real interest rate, $(1+i)/\pi$. In steady state, the fiscal policy rule (13) is reduced to

$$\frac{b}{\pi} = \left(1 - \frac{\pi}{1+i}\right)^{\frac{1}{\theta-1}} \left(\frac{\pi}{b^*}\right)^{-\frac{\theta}{\theta-1}} (\tau^*)^{-\frac{1}{\theta-1}} \tag{19}$$

This equation shows that the debt-output ratio, deflated by inflation with output normalized to unity, is increasing (decreasing) in the real interest rate if $\theta > 1$ ($\theta < 1$). For completeness, the monetary policy rule (14) in steady state is written as

$$1 + i = \max\left\{(1+i^*)\left(\frac{\pi}{\pi^*}\right)^{\phi}, 1\right\} \tag{20}$$

The monetary policy rule determines how inflation affects the real interest rate. Given $i > 0$, the real interest rate is increasing (decreasing) in the inflation rate if $\phi > 1$ ($\phi < 1$).

3 Positive-Interest Equilibria

This section studies the model in the case of positive interest rates. Section 3.1 studies steady states of the economy when interest rates are positive. It shows that multiple steady states prevail. Section 3.2 studies the properties of local dynamics around each of the steady states.

3.1 Steady State Analysis

When interest rates are positive, the equations (18), (19), and (20) that characterize steady states can be transformed into the two equations:

$$\frac{b}{\pi} = \mathcal{E}(\pi) := \frac{1}{\chi}\left[\left(\frac{\pi}{\pi^*}\right)^{\phi-1} - 1\right] + \frac{b^*}{\pi^*} \tag{21}$$

$$\frac{b}{\pi} = \mathcal{F}(\pi) := \frac{b^*}{\pi^*}\left[\frac{1-\beta(\pi^*/\pi)^{\phi-1}}{1-\beta}\right]^{\frac{1}{\theta-1}}\left(\frac{\pi^*}{\pi}\right)^{\frac{\theta}{\theta-1}} \tag{22}$$

where τ^* has been substituted out using equation (12). Equation (21) corresponds to the Euler equation. Equation (22) corresponds to the fiscal policy rule. In these equations, the interest rate has been substituted out by using the monetary policy rule.

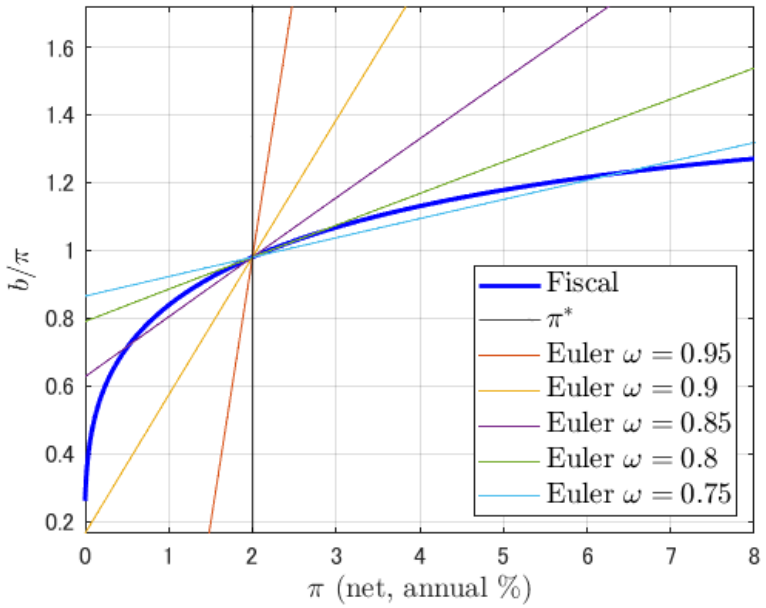
The steady state (π, b) is determined as an intersection of the two curves, $\mathcal{E}(\pi)$ and $\mathcal{F}(\pi)$, which are called as the 'Euler curve' and 'fiscal curve,' respectively. In addition, such π has to satisfy two conditions. First, since the case of positive interest rates is considered, π is high enough to satisfy $i > 0$, or $\pi > \underline{\pi}_i := \beta^{\frac{1}{\phi}}(\pi^*)^{1-\frac{1}{\phi}}$. Second, the real interest rate has to be positive to ensure that debt is positive. This requires $\pi > \underline{\pi}_r := \beta^{\frac{1}{\phi-1}}\pi^*$ if $\phi > 1$, and $\pi < \underline{\pi}_r$ if $\phi < 1$. The inequality flips because the relationship between the real interest rate and inflation flips depending on whether $\phi > 1$. Hence, the domain of π of our interest is $(\underline{\pi}, \infty)$ in the case of $\phi > 1$, where $\underline{\pi} = \max\{\underline{\pi}_i, \underline{\pi}_r\}$, and it is $(\underline{\pi}_i, \underline{\pi}_r)$ in the case of $\phi < 1$.

The set of targeted variables, $(\pi, b) = (\pi^*, b^*)$, constitutes a steady state by construction. There can be other steady states depending on policy parameters ϕ and θ and the parameter ω that governs the degree of which agents are non-Ricardian. The two curves $\mathcal{E}(\pi)$ and $\mathcal{F}(\pi)$ suggest that the sign of the slope of each curve depends on whether $\phi > 1$ and whether $\theta > 1$. Each case is considered in turn.

3.1.1 Case of $\phi > 1$ and $\theta > 1$

I begin by illustrating how the presence of non-Ricardian households affects the existence of additional steady states. Figure 1 plots the fiscal curve $\mathcal{F}(\pi)$ defined by (22) and the Euler curve $\mathcal{E}(\pi)$ defined by (21) for various values of ω. The fiscal curve is independent of ω and increasing in the inflation rate, as shown by the thick line in Figure 1. In RANK ($\omega = 1$), the Euler curve is given by the vertical line at the target inflation rate, which is set at $\pi^* = 2\%$. Since there is only one intersection between the two curves, RANK has a unique steady state at $\pi = \pi^*$.

Figure 1: Positive-interest steady states: $\phi > 1$ and $\theta > 1$



Note: 'Fiscal' plots $\mathcal{F}(\pi)$ defined by (22) and 'Euler' plots $\mathcal{E}(\pi)$ defined by (21) for various values of ω. The unit of time is annual. The parameter values are set as $\phi = 1.5$, $\beta = (1 + 0.01)^{-1}$, $\pi^* = 1 + 0.02$, $b^*/y^* = 1$, and $\theta = 0.05/(1 - \beta)$. The lower end of the x-axis is $\underline{\pi}_i$.

As the degree of which households are non-Ricardian is increased (as ω is lowered from unity), the Euler curve starts shifting clockwise around the targeted steady state of $(\pi, b) = (\pi^*, b^*)$, as shown in Figure 1. When $\omega = 0.95$, the Euler curve exhibits steep positive slopes, close to the vertical line at $\pi = \pi^*$, so the economy continues to have a unique steady state. However, when ω is lowered to 0.85, there emerges an additional intersection at $\pi = \pi^\diamond < \pi^*$. If ω is lowered further to 0.8, the Euler curve becomes flatter, and there emerges a different intersection at $\pi = \pi^\diamond > \pi^*$. When $\omega = 0.75$, the economy has an unintended steady state with a high inflation rate around 6 percent, in addition to the targeted steady state.

Figure 1 illustrates that when households are non-Richardian ($\omega < 1$), the economy features an unintended steady state with low inflation if the Euler curve $\mathcal{E}(\pi)$ is relatively steep and that with high inflation if the Euler curve is relatively flat. The slope of $\mathcal{E}(\pi)$ relative to that of $\mathcal{F}(\pi)$ turns out to be critical not only for the existence and uniqueness of unintended steady states but also for equilibrium determinacy. Thus, it is useful to introduce a function $\Theta(\cdot)$ that will be used repeatedly in this paper:

$$\Theta(\phi, \chi^*, \beta) := 1 + \frac{\chi^*(\beta\phi - 1)}{(\chi^* + \phi - 1)(1 - \beta)} \tag{23}$$

where $\chi^* \coloneqq (\chi b^*)/\pi^*$. The conditions that determine the relative slope are derived in the following lemma.

Lemma 1 *Suppose $\phi > 1$ and $\theta > 1$. Then, $\mathcal{E}'(\pi^*) < \mathcal{F}'(\pi^*)$ if and only if*

$$\theta < \Theta(\phi, \chi^*, \beta) \tag{24}$$

where $\Theta(\phi, \chi^*, \beta)$ is given by (23). Similarly, $\mathcal{E}'(\pi^*) > \mathcal{F}'(\pi^*)$ if and only if

$$\theta > \Theta(\phi, \chi^*, \beta) \tag{25}$$

Proof. See Appendix A.2 ■

With the conditions of relative slopes of $\mathcal{E}(\pi)$ and $\mathcal{F}(\pi)$ at $\pi = \pi^*$ at hand, as stated in Lemma 1, I now relate the existence and uniqueness of unintended steady states to the relative slopes of the Euler curve (21) and fiscal curve (22) in the following proposition.

Proposition 2 (Unintended steady states with $\phi > 1$ and $\theta > 1$) *Suppose $\phi > 1$ and $\theta > 1$, and consider the case of positive interest rates.*

i) There exists a unique high-inflation steady state, $\pi = \pi^\diamond > \pi^*$, if and only if condition (24) is satisfied, i.e., $\theta < \Theta(\phi, \chi^*, \beta)$.

ii) Assume:

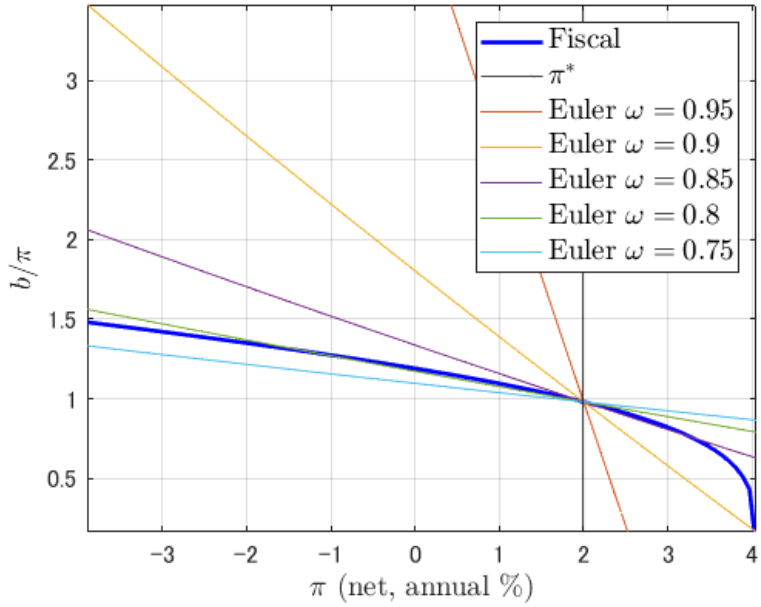
$$\chi^* > \begin{cases} 1 - \beta & \text{if } \beta \geq (1/\pi^*)^{\phi - 1} \\ \Psi(\phi, \theta) & \text{otherwise} \end{cases} \tag{26}$$

where $\Psi(\phi, \theta)$ is given by (A.9) in the appendix. Then, there exists a unique low-inflation steady state, $\pi = \pi^\diamond < \pi^*$, if and only if condition (25) is satisfied, i.e., $\theta > \Theta(\phi, \chi^*, \beta)$.

Proof. See Appendix A.3. ■

Proposition 2(i) shows that a high-inflation steady state is unique and it exists if and only if condition (24) is satisfied, i.e., when the fiscal policy's response to debt, $\theta > 1$, is relatively muted. Proposition 2(ii) shows that, under condition (26), a low-inflation steady state is unique and it exists if and only if condition (25) is satisfied, i.e., when the fiscal policy's response to debt is relatively strong. Condition (26) ensures $\mathcal{E}(\underline{\pi}) > \mathcal{F}(\underline{\pi})$ at the lower end of the domain, $\underline{\pi} = \max\{\underline{\pi}_i, \underline{\pi}_r\}$, as in the cases of $\omega = 0.85$ in Figure 1.

Figure 2: Positive-interest steady states: $\phi < 1$ and $\theta > 1$



Note: 'Fiscal' plots $\mathcal{F}(\pi)$ defined by (22) and 'Euler' plots $\mathcal{E}(\pi)$ defined by (21) for various values of ω. The parameter values are the same as those used in Figure 1 except for ϕ, which is set at 0.5. The domain of the x-axis is set as $(\underline{\pi}_i, \underline{\pi}_r)$.

3.1.2 Case of $\phi < 1$ and $\theta > 1$

Next the case of $\phi < 1$ and $\theta > 1$ is considered. With $\phi < 1$, the real interest rate becomes decreasing in the inflation rate, so that both the Euler curve $\mathcal{E}(\pi)$ and the fiscal curve $\mathcal{F}(\pi)$ become downward sloping. In RANK ($\omega = 1$), the Euler curve is vertical at $\pi = \pi^*$, and thereby a steady state is uniquely determined at the target rate of inflation. As the degree of which households are non-Ricardian is increased (as ω is lowered from unity), the Euler curve shifts counterclockwise, as shown in Figure 2. In the case of $\omega = 0.95$, the Euler curve is so steep that there continues to be a unique steady state at $\pi = \pi^*$. However, as ω is lowered to 0.85, the Euler curve becomes flatter, and there emerges an additional steady state with a higher inflation rate than the targeted rate. Moreover, if ω is lowered further to 0.8, there emerges a different steady state with a lower inflation rate than the targeted rate.

These observations suggest that a high-inflation steady state emerges when the Euler curve is relatively steep and a low-inflation steady state emerges when the Euler curve is relatively flat. These patterns can be formally stated in the following proposition.

Proposition 3 (Unintended steady states with $\phi < 1$ and $\theta > 1$) *Suppose $\phi < 1$ and $\theta > 1$, and consider the case of positive interest rates.*

i) Assume $\chi^* > 1-\beta$. Then, there exists a high-inflation steady state such that $\pi = \pi^\diamond > \pi^*$ if condition (25) is satisfied, i.e., $\theta > \Theta(\phi, \chi^*, \beta)$.

ii) Assume $\chi^* < \Psi(\phi, \theta)$. Then, there exists a low-inflation steady state such that $\pi = \pi^\diamond < \pi^*$ if condition (24) is satisfied, i.e., $\theta < \Theta(\phi, \chi^*, \beta)$.

Proof. See Appendix A.4. ■

The assumption in Proposition 3(i) ensures $\mathcal{E}(\underline{\pi}_r) > \mathcal{F}(\underline{\pi}_r)$, where $\underline{\pi}_r$ is the upper end of the relevant domain of π. The corollary of Lemma 1 is that $|\mathcal{E}'(\pi^*)| > |\mathcal{F}'(\pi^*)| \Leftrightarrow$ condition (25). Hence, there exists a steady state $\pi = \pi^\diamond \in (\pi^*, \underline{\pi}_r)$. Proposition 3(ii) is derived based on a similar idea. Proposition 3 suggests that, given $\phi < 1$, a high-inflation steady state emerges if the fiscal policy's response to debt, $\theta > 1$, is relatively strong; a low-inflation steady state emerges if the fiscal policy's response is relatively muted.

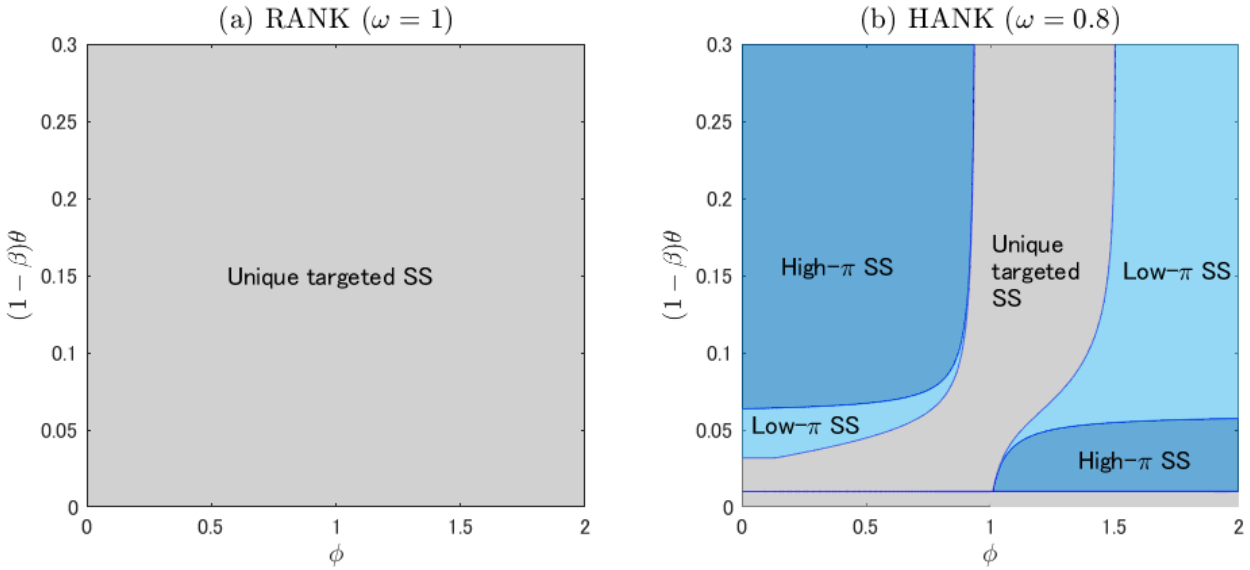
3.1.3 Cases of $\phi \lessgtr 1$ and $\theta < 1$

Finally the cases of $\phi \lessgtr 1$ and $\theta < 1$ are considered. In the case of $\phi < 1$, the fiscal curve $\mathcal{F}(\pi)$ is increasing in π, while the Euler curve $\mathcal{E}(\pi)$ is decreasing. Hence, there is a unique steady state at the target rate of inflation. In the case of $\phi > 1$, the Euler curve $\mathcal{E}(\pi)$ is increasing, while the fiscal curve $\mathcal{F}(\pi)$ is decreasing for $\pi \leq \pi^*$ if $\phi > 1 + (\beta^{-1} - 1)\theta$. Given that β is close to unity, this condition is satisfied for almost all values of $\phi > 1$ since $\theta < 1$. Because of continuity, $\mathcal{F}(\pi)$ is decreasing for $\pi > \pi^*$ as long as π is not extremely high. Hence, within the range of non-extreme values of π, there is a unique steady state at the target rate of inflation. A further discussion with graphical illustrations can be found in Appendix B.1.

3.1.4 Summary of steady states

Figure 3 summarizes the steady state analysis for all cases of monetary and fiscal policy parameters: $\phi \lessgtr 1$ and $\theta \lessgtr 1$. It plots the region of the existence of steady states for each pair of $(\phi, (1-\beta)\theta)$, where θ is scaled by $1-\beta$. In the case of RANK ($\omega = 1$), there is always a unique steady state, which is the targeted steady state $(\pi, b) = (\pi^*, b^*)$ for any pair of monetary and fiscal policies (Figure 3(a)). In HANK ($\omega < 1$), the targeted steady state continues to exist for any pair of monetary and fiscal policies. However, in addition to the targeted steady state, low or high inflation steady states emerge depending on monetary

Figure 3: The summary of positive-interest steady states



Note: 'Unique targeted SS' indicates that the targeted steady state is a unique steady state. 'High-π SS' indicates there is a high-inflation steady state in addition to the targeted steady state. 'Low-π SS' indicates there is a low-inflation steady state in addition to the targeted steady state. The parameter values are same as those in Figure 1.

and fiscal policies (Figure 3(b)). Specifically, a high-inflation steady state emerges when monetary policy's response is relatively strong and fiscal policy's response is relatively muted (south-east region of the figure), or when monetary policy's response is relatively weak and fiscal policy's response is relatively strong (north-west region of the figure). A low-inflation steady state prevails when monetary policy's response is relatively strong and fiscal policy's response is not so muted (east region of the figure), or when monetary policy's response is relatively muted and fiscal policy's response is neither so muted nor so strong (west region of the figure). Hence, with non-Ricardian households ($\omega < 1$), multiple steady states prevail.

3.2 Equilibrium Determinacy

Given the existence of possibly multiple steady states, I now study the determinacy of equilibrium local dynamics around each of the steady states.

3.2.1 Dynamics around the targeted steady state

First, local dynamics around the targeted steady state are considered. Let $\hat{x}_t$ denote a log-deviation of variable x_t from x. Given that interest rates are positive, log-linearizing the

system of equations (13), (14), and (17) around the targeted steady state $(\pi, b, i) = (\pi^*, b^*, i^*)$ yields

$$\hat{b}_t = \frac{1}{\beta}(1-\theta^*)\hat{b}_{t-1} - \left(\frac{1}{\beta} - \phi\right)\hat{\pi}_t$$
$$(1-\chi^*)\,\mathbb{E}_t\hat{\pi}_{t+1} + \chi^*\hat{b}_t = \phi\hat{\pi}_t$$

where $\theta^* \coloneqq (1-\beta)\theta$, $\chi^* \coloneqq (\chi b^*)/\pi^*$, and $\hat{i}_t$ has been substituted out. This constitutes a system of two equations with two variables: one jump variable ($\hat{\pi}_t$) and one pre-determined variable ($\hat{b}_{t-1}$). Deriving the eigenvalues of the system and applying the Blanchard-Kahn conditions (Blanchard and Kahn, 1980) yield the following results.

Proposition 4 (Determinacy regions of the targeted equilibrium) *Suppose $\chi^* < 1$. Then, the determinacy properties of local dynamics around the targeted steady state are classified into four regions.*

I) $\phi > 1-\chi^*$ and $\theta > \Theta(\phi, \chi^*, \beta)$: Determinate.

II) $\phi < 1-\chi^*$ and $\theta < \Theta(\phi, \chi^*, \beta)$: Determinate.

III) $\phi < 1-\chi^*$ and $\theta > \Theta(\phi, \chi^*, \beta)$: Indeterminate.

IV) $\phi > 1-\chi^*$ and $\theta < \Theta(\phi, \chi^*, \beta)$: Explosive.

where $\Theta(\cdot)$ is given by (23).

Proof. See Appendix A.5. ■

Proposition 4 relates equilibrium determinacy conditions to the conditions that determine the relative slopes of the Euler curve and fiscal curve (Lemma 1), which in turn determine the existence of high- or low-inflation steady states (Propositions 2 and 3). Hence, Proposition 4 has implications when multiple steady states arise.

In region I, condition $\theta > \Theta(\phi, \chi^*, \beta)$ holds, which suggests that there may exist a low-inflation steady state (Proposition 2(ii)). In Figure 1 this situation corresponds to 0.85 as the slope of $\mathcal{E}(\pi)$ is steeper than that of $\mathcal{F}(\pi)$ at $\pi = \pi^*$ (Lemma 1). A similar result follows in region II in which Proposition 3 becomes relevant. Hence, Proposition 4 shows that even if a low-inflation steady state emerges, local dynamics around the targeted steady state is determinate.

In region III, condition $\theta > \Theta(\phi, \chi^*, \beta)$ suggests that there may exist a high-inflation steady state (Proposition 3(i)). In Figure 2 this situation corresponds to $\omega = 0.85$. Hence, given $\phi < 1-\chi^*$, if a high-inflation steady state emerges, local dynamics around the targeted steady state become indeterminate.

In region IV, local dynamics around the targeted steady state become explosive, but at the same time, condition $\theta < \Theta(\phi, \chi^*, \beta)$ implies that a high-inflation steady state emerges (Proposition 2(i)). Put differently, given $\phi > 1-\chi^*$, the emergence of a high-inflation steady state implies explosive local dynamics around the targeted steady state. In Figure 1, this situation corresponds to $\omega = 0.8$ or 0.75 as the slope of $\mathcal{E}(\pi)$ is flatter than that of $\mathcal{F}(\pi)$ at $\pi = \pi^*$ (Lemma 1).

Proposition 4 is essentially the same as that in Rachel and Ravn (2025), but here it pertains more about relationships with unintended steady states through Lemma 1 and Propositions 2 and 3. It also encompasses the result of Leeper (1991) as a special case ($\chi^* = 0$ and $\Theta(\phi, 0, \beta) = 1$). The assumption $\chi^* < 1$ of Proposition 4 is not too restrictive, which holds for reasonable values of ω and b^*.[2]

3.2.2 Dynamics around unintended steady states

Now, local dynamics around an unintended steady state, $(\pi, b, i) = (\pi^\diamond, b^\diamond, i^\diamond)$, is considered. The unintended steady state is either a high- or low-inflation steady state in the case of $\phi > 1$ or $\phi < 1$. Log-linearizing the system of equations (13), (14), and (17) around an unintended steady state yields

$$\hat{b}_t = \frac{1}{\beta^\diamond}(1-\theta^\diamond)\hat{b}_{t-1} - \left(\frac{1}{\beta^\diamond} - \phi\right)\hat{\pi}_t$$
$$(1-\chi^\diamond)\,\mathbb{E}_t\hat{\pi}_{t+1} + \chi^\diamond \hat{b}_t = \phi\hat{\pi}_t$$

where

$$\beta^\diamond := \left(\frac{1+i^\diamond}{\pi^\diamond}\right)^{-1},\ \theta^\diamond := (1-\beta^\diamond)\theta,\ \chi^\diamond := \frac{\chi b^\diamond}{\beta(1+i^\diamond)}$$

[2]Condition $\chi^* < 1$ can be rewritten as

$$b^* < \frac{\omega\pi^*}{(1-\omega)(1-\beta\omega)}$$

Since y^* is normalized to unity, b^* can be interpreted as the target level of a debt-output ratio. If $\beta = 0.99$, $\pi^* = 1.02$, and $\omega = 0.8$, as in Figure 3(b), the upper bound is 19.62. Even if ω is lowered to 0.7, the upper bound still exceeds 7.7. Hence, in practice, condition $\chi^* < 1$ is not so restrictive.

The functional form of the system of equations is exactly the same as that pertaining to the targeted equilibrium. What differs is the coefficients only. Hence, the determinacy conditions can be derived exactly in the same way as in Proposition 4. The results are summarized in the following proposition.

Proposition 5 (Determinacy regions of unintended equilibria) *Suppose* $\chi^\diamond < 1$*. Then, the determinacy properties of local dynamics around an unintended steady state are classified into four regions.*

I) $\phi > 1 - \chi^\diamond$ *and* $\theta > \Theta(\phi, \chi^\diamond, \beta^\diamond)$*: Determinate.*

II) $\phi < 1 - \chi^\diamond$ *and* $\theta < \Theta(\phi, \chi^\diamond, \beta^\diamond)$*: Determinate.*

III) $\phi < 1 - \chi^\diamond$ *and* $\theta > \Theta(\phi, \chi^\diamond, \beta^\diamond)$*: Indeterminate.*

IV) $\phi > 1 - \chi^\diamond$ *and* $\theta < \Theta(\phi, \chi^\diamond, \beta^\diamond)$*: Explosive.*

where $\Theta(\cdot)$ *is given by (23).*

The parallel between Proposition 5 about unintended equilibria and Proposition 4 about a targeted equilibrium is useful, but Proposition 5 is not without problems since the determinacy conditions include endogenous variables $\{\pi^\diamond, b^\diamond, i^\diamond\}$. Thus, examining determinacy requires solving for the unintended steady state numerically. However, with an additional parallel to Lemma 1, analytical results can be obtained. As in Lemma 1, the following lemma relates the determinacy conditions in Proposition 5 to the relative slopes of the Euler curve $\mathcal{E}(\pi)$ and the fiscal curve $\mathcal{F}(\pi)$ at $\pi = \pi^\diamond$.

Lemma 6 *In the case of* $\phi > 1 - \chi^\diamond$ *and* $\theta > 1$*:*

$$\mathcal{E}'(\pi^\diamond) > \mathcal{F}'(\pi^\diamond) \quad \Leftrightarrow \quad \theta > \Theta(\phi, \chi^\diamond, \beta^\diamond)$$

In the case of $\phi < 1 - \chi^\diamond$ *and* $\theta > 1$*:*

$$|\mathcal{E}'(\pi^\diamond)| > |\mathcal{F}'(\pi^\diamond)| \quad \Leftrightarrow \quad \theta > \Theta(\phi, \chi^\diamond, \beta^\diamond)$$

Proof. See Appendix A.6 ■

Lemma 6 helps provide sharp predictions about the determinacy of unintended equilibria. As studied in Section 3.1.1, in the case of $\phi > 1$ and $\theta > 1$, if a high-inflation steady state

exists, it is unique and necessarily exhibits $\mathcal{E}'(\pi^\diamond) > \mathcal{F}'(\pi^\diamond)$ as illustrated in the case of $\omega = 0.8$ or 0.75 in Figure 1. Lemma 6 implies that this slope condition is equivalent to $\theta > \Theta(\phi, \chi^\diamond, \beta^\diamond)$, which implies the case I) in Proposition 5. Hence, a high-inflation equilibrium is determinate in the case of $\phi > 1$. Similarly, given the assumptions in Proposition 2(ii), there exists a unique low-inflation steady state with $\mathcal{E}'(\pi^\diamond) < \mathcal{F}'(\pi^\diamond)$ as illustrated in the case of $\omega = 0.85$ in Figure 1. Then, Lemma 6 implies that a low-inflation equilibrium in the case of $\phi > 1$ corresponds to the case IV) in Proposition 5 and thus it is explosive.

In the case of $\phi < 1$ and $\theta > 1$, as studied in Section 3.1.2, a high-inflation steady state exhibits $|\mathcal{E}'(\pi^\diamond)| < |\mathcal{F}'(\pi^\diamond)|$ as illustrated in the case of $\omega = 0.85$ in Figure 2. Then, Lemma 6 implies that this equilibrium falls into the case II) in Proposition 5, so that it is determinate. Similarly, a low-inflation steady state exhibits $|\mathcal{E}'(\pi^\diamond)| > |\mathcal{F}'(\pi^\diamond)|$ as illustrated in the case of $\omega = 0.8$ in Figure 2. Then, Lemma 6 implies that this equilibrium belongs to the case III) in Proposition 5, so that it is indeterminate.

This argument is summarized in the following proposition, where the four cases there correspond to those in Proposition 5.

Proposition 7 (Determinacy of unintended equilibria) *Consider the unintended steady states studied in Propositions 2 and 3. Then:*

I) Given $\phi > 1$, the high-inflation equilibrium is determinate.

II) Given $\phi < 1$, the high-inflation equilibrium is determinate.

III) Given $\phi < 1$, the low-inflation equilibrium is indeterminate.

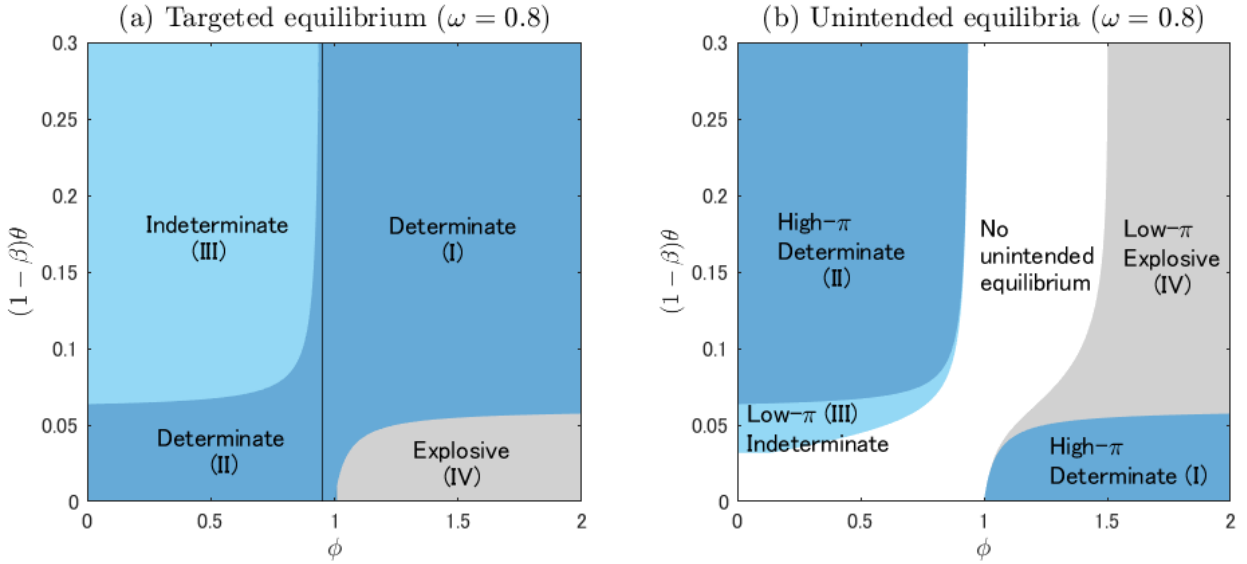
IV) Given $\phi > 1$, local dynamics around the low-inflation steady state is explosive.

3.2.3 Summary of equilibrium determinacy

Figure 4 summarizes the analysis of equilibrium determinacy. The regions, denoted by I, II, III, and IV, corresponds to those in Propositions 4 and 5 in panels (a) and (b), respectively. These regions also correspond to those of Leeper (1991) in the case of $\omega = 1$ in our model, that is, I) *active-passive* monetary-fiscal policies ($\phi > 1$ and $\theta > 1$), II) *passive-active* ($\phi < 1$ and $\theta < 1$), III) *passive-passive* ($\phi < 1$ and $\theta > 1$), and IV) *active-active* ($\phi > 1$ and $\theta < 1$).

In region I), intuitively, inflation is monetary-policy driven. In the targeted equilibrium with non-Ricardian households (Figure 4(a)), relative to the RANK model ($\phi > 1$ and

Figure 4: The summary of equilibrium determinacy



Note: 'High-π' and 'Low-π' indicate local dynamics around a high-inflation steady state and a low-inflation steady state, respectively. The parameter values are same as those in Figure 1.

$(1-\beta)\theta > 0.01$), the region expands leftward as it includes some territory with $\phi < 1$, while it shrinks upward as the explosive region IV) expands from its RANK counterpart ($\phi > 1$ and $(1-\beta)\theta < 0.01$). In region II), inflation is fiscal-policy driven. With non-Ricardian households, the region expands substantially from its RANK counterpart ($\phi < 1$ and $(1-\beta)\theta < 0.01$), while the indeterminacy region III) shrinks substantially from its RANK counterpart ($\phi < 1$ and $(1-\beta)\theta > 0.01$).

The model features determinacy, having a unique equilibrium around the targeted steady state, in vast areas of a set of monetary and fiscal policy parameters $(\phi, (1-\beta)\theta)$ as shown in Figure 4(a). Within such areas, a low-inflation steady state can co-exist, but the model is either explosive or indeterminate around the low-inflation steady states, as shown in Figure 4(b). In particular, under explosive dynamics, the economy diverges from the low-inflation steady state. Even if such a steady state exists, the targeted equilibrium would be more relevant for economic analysis as it exhibits determinacy in the parameter region.

Starting from a set of policies where both monetary and fiscal policies are strongly responsive (north-east area of Figure 4(a)), local dynamics around the targeted steady state loses determinacy and displays explosive dynamics or indeterminacy if either monetary or fiscal policy becomes less responsive, respectively. In both cases, a high-inflation steady state emerges simultaneously and its local dynamics exhibit determinacy. In particular, in

the case of explosive dynamics, the economy would converge to a high-inflation steady state.

The analysis in this section shows that unintended steady states co-exist with the targeted steady state in vast areas of a set of monetary and fiscal policies. Moreover, local dynamics around some of the unintended steady states show either determinacy or indeterminacy, thus constituting equilibria. Hence, the analysis here suggests that these unintended equilibria may be relevant in practice.

4 Zero-Interest Equilibria

This section studies the model in the case of zero interest rates. As studied by Benhabib et al. (2001), a unique self-fulfilling zero-interest (or deflationary) steady state can emerge in RANK when a central bank follows a Taylor rule. Moreover, local dynamics around such a zero-interest steady state display indeterminacy. Section 4.1 studies whether a self-fulfilling zero-interest steady state exists in HANK ($\omega < 1$). Section 4.2 studies whether the determinacy property of zero-interest equilibrium, if any, changes in HANK.

4.1 Steady State Analysis

As stipulated by the monetary policy rule (14), the interest rate hits the zero lower bound if the inflation is low enough to satisfy $\pi \leq \underline{\pi}_i \coloneqq \beta^{\frac{1}{\phi}}(\pi^*)^{1-\frac{1}{\phi}}$. Given $\pi \leq \underline{\pi}_i$, the equations (18), (19), and (20) that characterize steady states can be transformed into the two equations:

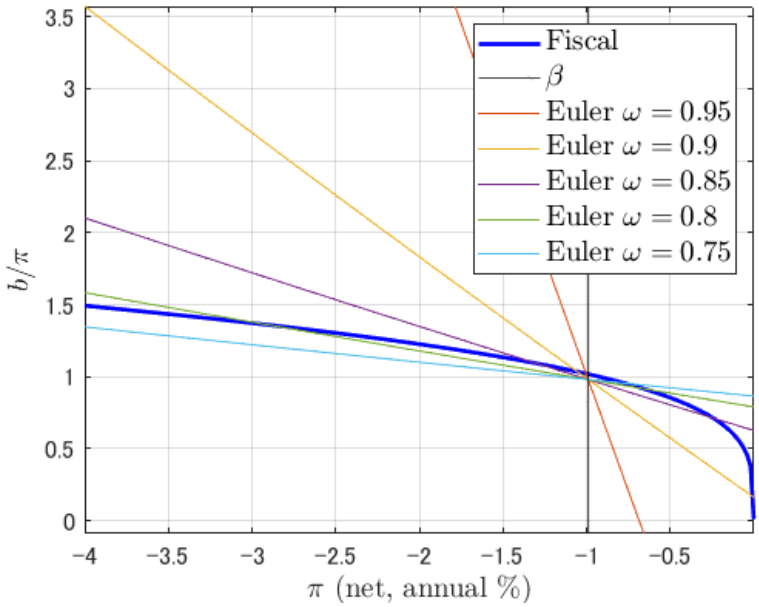
$$\frac{b}{\pi} = \tilde{\mathcal{E}}(\pi) \coloneqq \frac{1}{\chi}\left(\frac{\beta}{\pi} - 1\right) + \frac{b^*}{\pi^*} \tag{27}$$

$$\frac{b}{\pi} = \tilde{\mathcal{F}}(\pi) \coloneqq \frac{b^*}{\pi^*}\left(\frac{1-\pi}{1-\beta}\right)^{\frac{1}{\theta-1}}\left(\frac{\pi^*}{\pi}\right)^{\frac{\theta}{\theta-1}} \tag{28}$$

where τ^* has been substituted out using equation (12). At the zero lower bound the gross real interest rate is given by $1/\pi$. The Euler curve (27) is decreasing in π. The fiscal curve (28) is decreasing in π if $\theta > 1$ and increasing in π if $\theta < 1$. The two cases, $\theta > 1$ and $\theta < 1$, are considered in turn.

I begin by illustrating that there could be multiple zero-interest steady states in the case of $\theta > 1$. Figure 5 plots the fiscal curve (28) and the Euler curve (28) for various values of ω. In the case of RANK ($\omega = 1$), the Euler curve is vertical at $\pi = \beta$, which implies that the real interest rate is identical to that of the targeted steady state. Then, a zero-inflation steady state $\tilde{\pi}$, which is determined as the intersection of the two curves, is unique and given by

Figure 5: Zero-interest steady states: $\theta > 1$



Note: 'Fiscal' plots the fiscal curve defined by (28) and 'Euler' plots the Euler curve defined by (27) for various values of ω. The vertical line 'β' corresponds to the Euler curve $\pi = \beta$ in RANK ($\omega = 1$). The parameter values are the same as those in Figure 1.

$\tilde{\pi} = \beta$. In HANK ($\omega < 1$), as the degree of which households are non-Ricardian is increased (as ω is lowered from unity), the Euler curve shifts counterclockwise. Figure 5 shows that there are two steady states, $\tilde{\pi} > \beta$ and $\tilde{\pi} < \beta$, in the cases of $\omega = 0.9$, 0.85, and 0.8.[3] It is worth noting that in one steady state the real interest rate is lower than that of the targeted steady state, i.e., $1/\tilde{\pi} < 1/\beta = (1 + i^*)/\pi^*$, and in the other steady state, the real interest rate is higher, i.e., $1/\tilde{\pi} > 1/\beta$. These steady state can be called as 'mild-deflation' and 'severe-deflation' steady states, respectively. The presence of severe-deflation steady states implies that a zero-inflation steady state exists even when $\phi < 1$, which is not the case in RANK.[4]

Two steady states emerge because both the Euler and fiscal curves are decreasing in π (increasing in the real interest rate). Since the same parameter values are used in Figures 1 and 5, these figures illustrate that four steady states can co-exist: two positive-inflation steady states and two zero-inflation steady states, for example, in the cases of $\omega = 0.85$ and

[3]For $\omega = 0.75$, there is also a steady state with $\tilde{\pi} < \beta$, although it does not appear in Figure 5.

[4]RANK has a unique zero-inflation steady state, $\pi = \beta$. Since it has to be $\pi \leq \underline{\pi}_i$, it follows that $\pi \leq \beta^{\frac{1}{\phi}}(\pi^*)^{1-\frac{1}{\phi}} < \beta$ when $\phi < 1$. Hence, the monetary policy parameter has to be greater than unity for the existence of the zero-interest steady state in RANK.

$\omega = 0.8$ in Figures 1 and 5.

Next, the case of $\theta < 1$ is considered. In this case, while the Euler curve $\tilde{\mathcal{E}}(\pi)$ continues to be decreasing, the fiscal curve $\tilde{\mathcal{F}}(\pi)$ becomes increasing. Since $\tilde{\mathcal{E}}(\beta) > \tilde{\mathcal{F}}(\beta)$, if a zero-interest steady state exists, it is unique and exhibits $\tilde{\pi} > \beta$.

The analysis above shows that in both cases of $\theta > 1$ and $\theta < 1$, there may exists a zero-interest steady state with a lower real interest than in the targeted steady state. In particular, under mild conditions a steady state with a lower real interest rate than in the targeted steady state, $\tilde{\pi} > \beta$, exists, which is formally stated in the following proposition.

Proposition 8 (Existence of zero-interest steady states with low real interest rates) *Suppose $\underline{\pi}_i \geq 1$. Then, in the case of $\theta > 1$, there exists a zero-interest steady state with $\tilde{\pi} \in (\beta, 1)$ if $\chi^* > 1 - \beta$. In the case of $\theta < 1$, there exists a unique zero-interest steady state with $\tilde{\pi} \in (\beta, 1)$.*

Proof. See Appendix A.7. ■

The condition $\chi^* > 1 - \beta$ in the case of $\theta > 1$ is satisfied for typical parameter values, since β is typically close to unity, and the presence of non-Ricardian households implies that χ^* is positive. Moreover, as discussed in Appendix B.2, the zero-interest steady state $\tilde{\pi} \in (\beta, 1)$ in the case of $\theta > 1$ is unique for almost all values of $\chi^* > 1 - \beta$ if θ is not too large.

4.2 Equilibrium Determinacy

Local dynamics around a zero-interest steady state $(\pi, i, b) = (\tilde{\pi}, 0, \tilde{b})$ are considered. Given that the interest rate is stuck at zero according to the monetary policy rule (14), the log-linearized version of equations (13) and (17) is given by

$$\hat{b}_t = \frac{1}{\tilde{\pi}}(1 - \tilde{\theta})\hat{b}_{t-1} - \frac{1}{\tilde{\pi}}\hat{\pi}_t \tag{29}$$

$$0 = -\tilde{\chi}\hat{b}_t - (1 - \tilde{\chi})\,\mathbb{E}_t\hat{\pi}_{t+1} \tag{30}$$

where $\tilde{\theta} \coloneqq (1 - \tilde{\pi})\theta$ and $\tilde{\chi} \coloneqq \chi\tilde{b}/\beta$. In the case of RANK ($\omega = 1$ so that $\tilde{\chi} = 0$), equation (30) is reduced to $\mathbb{E}_t\hat{\pi}_{t+1} = 0$. It implies that $\hat{\pi}_t = \epsilon_t$ can be a solution if equation (29) is stationary (i.e., $(1 - \tilde{\theta})/\tilde{\pi} < 1$, which is reduced to $\theta > 1$), where ϵ_t is an i.i.d. mean zero non-fundamental (sunspot) shock. Hence, in the case of $\theta > 1$ the zero-inflation equilibrium is indeterminate in RANK.

The determinacy property of HANK with $\tilde{\chi} > 0$ differs from that of RANK. Applying the Blanchard-Kahn conditions to the system of equations (29) and (30) yields the following result.

Proposition 9 (Determinacy regions of zero-interest equilibria) *Assume $\tilde{\chi} < 1$. Then, local dynamics around a zero-inflation steady state is:*

I) Determinate if and only if $\theta < \Theta(0, \tilde{\chi}, \tilde{\pi})$

II) Indeterminate if and only if $\theta > \Theta(0, \tilde{\chi}, \tilde{\pi})$

where $\Theta(\cdot)$ is given by equation (23).

Proof. See Appendix A.8. ■

Proposition 9 implies that the zero-interest equilibrium is determinate in the case of $\theta < 1$. Given $\tilde{\chi} < 1$, the threshold $\Theta(0, \tilde{\chi}, \tilde{\pi})$ is strictly greater than 1, as is clear from its definition (23). Then, the determinacy condition is satisfied for the zero-interest equilibrium with $\theta < 1$. In the case of $\theta > 1$, Proposition 9 alone cannot decide which zero-interest steady state, a mild-deflation steady state or a severe-deflation steady state, displays determinacy or indeterminacy in Figure 5. By connecting the relative slopes of the two curves, $\tilde{\mathcal{E}}(\pi)$ and $\tilde{\mathcal{F}}(\pi)$, and the determinacy condition, as we did in the analysis of positive-interest equilibria, sharp predictions about determinacy can be obtained for mild-deflation and severe-deflation equilibria in the case of $\theta > 1$ as follows.

Proposition 10 (Determinacy of zero-interest equilibria with $\theta > 1$) *Assume $\theta > 1$, $\tilde{\chi} < 1$, and that the zero-interest steady state is unique in the region of $\pi > \beta$ and $\pi < \beta$, respectively, if it exists. Then, the mild-deflation equilibrium is determinate. The severe-deflation equilibrium is indeterminate.*

Proof. See Appendix A.9 ■

To understand why the mild-deflation equilibrium becomes determinate, it is useful to consider a general solution form, $\hat{\pi}_t = \delta_1 \hat{b}_{t-1} + \delta_2 \epsilon_t$, where δ_1 and δ_2 are coefficients and ϵ_t is an i.i.d. mean zero sunspot shock. Determinacy requires that δ_1 be uniquely determined while $\delta_2 = 0$. Substituting the solution form into the Euler curve (30) yields $\delta_1 = -\tilde{\chi}/(1 - \tilde{\chi}) < 0$. Intuitively, when $\hat{b}_t$ increases, the real interest has to rise to induce households to be willing to hold the additional government bonds. At the ZLB, an increase in the real interest rate

is driven by deflation, so that $\hat{\pi}_{t+1}$ has to decrease. This is why the coefficient δ_1 is negative. Substituting the candidate solution into the Fiscal curve (29) yields

$$\hat{b}_t = \underbrace{\frac{1}{\tilde{\pi}}\left(1-\tilde{\theta}+\frac{\tilde{\chi}}{1-\tilde{\chi}}\right)}_{=:\delta_3>0}\hat{b}_{t-1} - \frac{\delta_2}{\tilde{\pi}}\epsilon_t \tag{31}$$

Equation (31) constitutes a solution if $\delta_3 < 1$, i.e., if $\theta > \Theta(0, \tilde{\chi}, \tilde{\pi})$. In this case, δ_2 is indeterminate, corresponding to II) in Proposition 9.

Instead, if $\delta_3 > 1$, i.e., if $\theta < \Theta(0, \tilde{\chi}, \tilde{\pi})$, equation (31) cannot be a solution as $\hat{b}_t$ explodes. In this case, the solution is given by the fiscal curve (29) such that $\hat{b}_t = 0$:

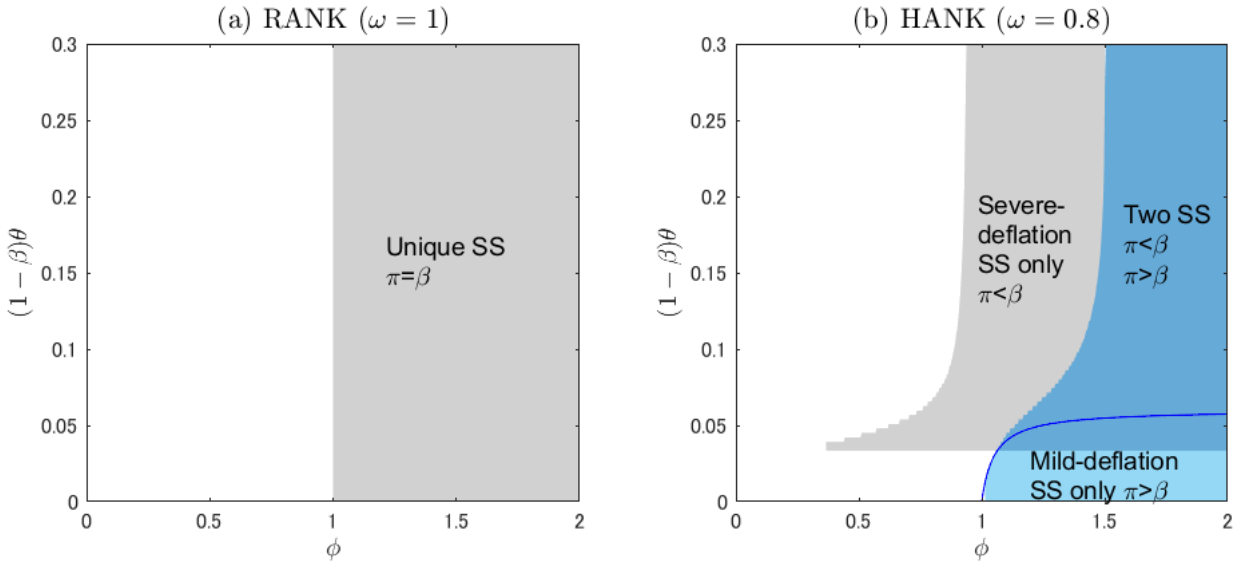
$$\hat{\pi}_t = (1-\tilde{\theta})\hat{b}_{t-1} + \delta_2\epsilon_t \tag{32}$$

with $\delta_2 = 0$. To see why $\delta_2 = 0$ is uniquely determined, suppose contrarily $\delta_2 \neq 0$. Then, substituting equation (32) into the fiscal curve (29) yields $\hat{b}_t = -(\delta_2/\tilde{\pi})\epsilon_t$ while substituting equation (32) into the Euler curve (30) yields $\hat{b}_t = 0$, a contradiction. Hence, in equilibrium, it has to be $\delta_2 = 0$, so that the equilibrium is uniquely determined.

As shown above, the condition $\delta_3 > 1$ in equation (31) is critical for establishing the determinacy of the zero-interest equilibrium. In RANK, $\delta_3 > 1$ is reduced to $\theta < 1$, which means 'active' fiscal policy with 'passive' monetary policy (Leeper, 1991). In HANK, there are two forces that make $\delta_3 > 1$ in spite of $\theta > 1$. First, the presence of non-Ricardian households leads to $\tilde{\chi} > 0$, which increases δ_3, as shown in equation (31). The term involving $\tilde{\chi}$ captures the effect that an increase in $\hat{b}_{t-1}$ is associated with an increase in the real interest rate, which increases $\hat{b}_t$. Second, a low real interest rate in the zero-interest steady state attenuates the effect of $\tilde{\theta}/\tilde{\pi} = (1/\tilde{\pi} - 1)\theta$, where $1/\tilde{\pi} - 1$ corresponds to the net real interest rate. Since the real interest rate in the mild-deflation steady state is lower than that of the targeted steady state, such a low real rate pushes up δ_3. Intuitively, in steady state, a low real rate requires low tax revenue, which attenuates the debt-stabilizing effect of tax that responds to debt, leading to a higher value of δ_3.

Proposition 10, together with Propositions 8 and 9, shows that under mild conditions there exists a self-fulfilling zero-interest steady state around which local dynamics display determinacy. Indeed, as illustrated in Figure 6(b), with non-Ricardian households, there is a large set of monetary-fiscal policy parameter values with which such a steady state exists. Moreover, in the dark blue area above the blue line, there exist two steady states both

Figure 6: The summary of self-fulfilling zero-interest steady states



Note: 'SS' denotes a steady state. The parameter values are same as those in Figure 1. An area below the blue line in panel (b), copied from Figure 4, indicates the existence of a high-inflation steady state where the targeted steady state becomes explosive.

of which exhibit determinacy: a positive-interest targeted steady state and a self-fulfilling zero-interest steady state. This makes contrast with RANK where there is only a unique zero-interest steady state that displays indeterminacy (Figure 6(a)). Furthermore, the real interest rate in such a zero-interest steady state is lower than in the targeted steady state. Low inflation and low real interest rates are consistent with the observations after the global financial crisis of 2007-09 in many countries. In this model, however, low real interest rates are the consequence of a shift in expectations, but not the cause of deflation. Moreover, the determinacy property implies that the economy can fall into the self-fulfilling zero-interest equilibrium for a prolonged period. Hence, the model has potential to explain a pro-longed period of zero interest rates, as the Japanese economy experienced from the late 1990s.

5 Conclusion

This paper has analyzed a perpetual youth, OLG version of HANK, where households are non-Ricardian, and studied its implications for the consequences of monetary-fiscal interactions. With non-Ricardian households, not only local dynamics around the targeted steady state change (Rachel and Ravn, 2025), but multiple steady states prevail depending on monetary-fiscal interactions even in a positive interest rate environment. Specifically, the

targeted steady state can co-exist with a low- or high-inflation steady state. This paper analytically characterizes what combination of monetary-fiscal policies gives rise to multiple steady states and the determinacy conditions of the associated local dynamics. The results imply that a high-inflation equilibrium may be relevant in practice in explaining persistently high-inflation episodes, as it displays determinacy, while at the same time local dynamics around the targeted steady state lose determinacy and exhibit explosiveness. Moreover, with non-Ricardian households, dynamic properties of self-fulling zero-interest equilibria change as well. The paper shows that, given their existence, at least one steady state displays determinacy, as opposed to indeterminacy in RANK. Furthermore, such a determinate steady state features a lower real interest rate than in the targeted steady state. The result suggests that with non-Ricardian households the model has potential to explain a prolonged period of deflation or low inflation, such as that which occurred in Japan from the late-1990s to the early 2020s.

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Appendix

A Derivations and Proofs

A.1 Derivation of the aggregate Euler equation (16)

The aggregate consumption in period t is given by $c_t = \sum_{s\leq t} c_{s,t} f_t(s)$, where the population distribution $f_t(s)$ is given by equation (1). Aggregating individual consumption $c_{s,t}$, given by equation (7), over $s \leq t$ yields

$$c_t = (1-\beta\omega)\left[\frac{b_{t-1}}{\pi_t} + \mathbb{E}_t \sum_{j=0}^{\infty}\left(\prod_{\ell=1}^{j}\frac{\omega}{R_{t+\ell}}\right)(w_{t+j}n_{t+j} + \tilde{z}_{t,t+j} - \tau_{t+j})\right] \tag{A.1}$$

where

$$\tilde{z}_{t,t+j} = \begin{cases} 0 & \text{for } j = 0 \\ -\frac{1-\omega}{\omega}\frac{b^*}{\pi^*} & \text{otherwise} \end{cases}$$

In deriving equation (A.1), the following relationships were used:

$$\sum_{s\leq t} b_{s,t-1} f_t(s) = \sum_{s\leq t-1} b_{s,t-1} f_{t-1}(s)\omega = b_{t-1}\omega$$
$$\sum_{s\leq t} z_{s,t} f_t(s) = \frac{b^*}{\pi^*}(1-\omega) - \frac{1-\omega}{\omega}\frac{b^*}{\pi^*}\sum_{s<t}(1-\omega)\omega^{t-s} = 0$$

The pricing kernel is defined as $Q_{t,t+1} = \beta c_{s,t}/c_{s,t+1}$. Aggregating the pricing kernel over $s \leq t$ yields

$$\begin{aligned} & Q_{t,t+1}\sum_{s\leq t} c_{s,t+1} f_t(s) = \beta c_t \\ \Leftrightarrow \quad & Q_{t,t+1}\frac{1}{\omega}\sum_{s\leq t} c_{s,t+1} f_{t+1}(s) = \beta c_t \end{aligned} \tag{A.2}$$

The aggregate consumption in period $t+1$ is decomposed into two parts as

$$c_{t+1} = \sum_{s\leq t+1} c_{s,t+1} f_{t+1}(s) = \sum_{s\leq t} c_{s,t+1} f_{t+1}(s) + (1-\omega)c_{t+1,t+1} \tag{A.3}$$

From equation (7), the period-t consumption of a household who was born in period t is given by

$$c_{t,t} = (1-\beta\omega)\left[\frac{b^*}{\pi^*} + \mathbb{E}_t \sum_{j=0}^{\infty}\left(\prod_{\ell=1}^{j}\frac{\omega}{R_{t+\ell}}\right)(w_{t+j}n_{t,t+j} + \tilde{z}_{t,t+j} - \tau_{t+j})\right] \tag{A.4}$$

Because of the assumption of a labor union, the labor supply is identical among generations: $n_{s,t+j} = n_{t+j}$ for all $s \leq t$. Then, by using equation (A.1), equation (A.4) for period $t+1$ can be written as

$$c_{t+1,t+1} = c_{t+1} - (1-\beta\omega)\left(\frac{b_t}{\pi_{t+1}} - \frac{b^*}{\pi^*}\right) \tag{A.5}$$

Substituting equations (A.3) and (A.5) into equation (A.2) yields

$$Q_{t,t+1} = \beta \frac{c_t}{c_{t+1} + \chi \left(\frac{b_t}{\pi_{t+1}} - \frac{b^*}{\pi^*} \right)}, \tag{A.6}$$

where $\chi = (1-\omega)(1-\beta\omega)/\omega$. Substituting (A.6) into the individual Euler equation (15) leads to the aggregate Euler equation (16).

A.2 Proof of Lemma 1

Suppose $\phi > 1$ and $\theta > 1$. Since $\mathcal{E}(\pi^*) = \mathcal{F}(\pi^*)$, $\mathcal{E}'(\pi^*) < \mathcal{F}'(\pi^*)$ is equivalent to $d\log\mathcal{E}(\pi)/d\pi < d\log\mathcal{F}(\pi)/d\pi$ evaluated at $\pi = \pi^*$. Taking the log of $\mathcal{E}(\pi)$ defined by (21) and differentiating with respect to π, evaluated at $\pi = \pi^*$, yield

$$\left. \frac{d\log\mathcal{E}(\pi)}{d\pi} \right|_{\pi=\pi^*} = \frac{\phi - 1}{\chi^*} \frac{1}{\pi^*}$$

where $\chi^* \coloneqq (\chi b^*)/\pi^*$. Similarly, taking the log of the fiscal curve $\mathcal{F}(\pi)$ defined by (22) and differentiating with respect to π, evaluated at $\pi = \pi^*$, yield

$$\left. \frac{d\log\mathcal{F}(\pi)}{d\pi} \right|_{\pi=\pi^*} = \left[\frac{1}{\theta - 1} \frac{\beta(\phi - 1)}{1 - \beta} - \frac{\theta}{\theta - 1} \right] \frac{1}{\pi^*}$$

The condition that $\mathcal{E}'(\pi^*) < \mathcal{F}'(\pi^*)$ can be written as

$$\frac{\phi - 1}{\chi^*} < \frac{1}{\theta - 1} \frac{\beta(\phi - 1)}{1 - \beta} - \frac{\theta}{\theta - 1}$$

Since $\theta > 1$, the above inequality can be arranged as

$$\frac{\chi^* + \phi - 1}{\chi^*} \theta < \frac{(\phi - 1)(\beta\chi^* + 1 - \beta)}{(1 - \beta)\chi^*}$$

Since $\phi > 1$, the left-hand side of the inequality is positive, and the inequality can be rearranged as

$$\theta < 1 + \frac{\chi^*(\beta\phi - 1)}{(\chi^* + \phi - 1)(1 - \beta)}$$

which is condition (24). The corollary is that $\mathcal{E}'(\pi^*) > \mathcal{F}'(\pi^*)$ holds if and only if condition (25) holds.

A.3 Proof of Proposition 2

i) High-inflation steady state The steady state is given by the intersection of the two curves: the Euler curve $\mathcal{E}(\pi)$ defined by (21) and the fiscal curve $\mathcal{F}(\pi)$ defined by (22). Since $\phi > 1$, $\mathcal{E}(\pi)$ is monotonically increasing, and $\lim_{\pi\to\infty} \mathcal{E}(\pi) = \infty$. It is either concave or convex. Since $\phi > 1$ and $\theta > 1$,$\mathcal{F}(\pi)$ has the property of $\lim_{\pi\to\infty} \mathcal{F}(\pi) = 0$. To see its properties further, its first derivative is calculated as

$$\mathcal{F}'(\pi) \coloneqq \frac{d\mathcal{F}(\pi)}{d\pi} \propto \left[-\theta + (\theta + \phi - 1)\beta \left(\frac{\pi^*}{\pi} \right)^{\phi - 1} \right] \mathcal{F}(\pi)^{-\theta} \pi^{-\theta - 1}$$

Hence, $\mathcal{F}(\pi)$ is increasing for $\pi < \pi_f^*$ and decreasing for $\pi > \pi_f^*$, where π_f^* is given by $\mathcal{F}'(\pi_f^*) = 0$ as

$$\pi_f^* = \left[\frac{(\theta+\phi-1)\beta}{\theta}\right]^{\frac{1}{\phi-1}} \pi^*$$

In addition, the derivative $\mathcal{F}'(\pi)$ implies that $\mathcal{F}(\pi)$ is concave in the region of $\pi < \pi_f^*$. From the equation above, $\pi_f^* > \pi^*$ holds if and only if

$$\theta < \frac{(\phi-1)\beta}{1-\beta} \tag{A.7}$$

As shown in Lemma 1, $\mathcal{E}'(\pi^*) < \mathcal{F}'(\pi^*)$ if and only if condition (24) holds. In addition, condition (24) can be written as

$$\theta < \frac{(\phi-1)\beta}{1-\beta} \times \frac{\beta^{-1}+\chi^*-1}{\phi+\chi^*-1} \tag{A.8}$$

Since $\theta > 1$, condition (24) implies it is necessary to have $\phi > \beta^{-1}$. Given $\phi > \beta^{-1}$, condition (A.8) implies condition (A.7). Hence, condition (A.7) is satisfied if condition (24) holds.

To summarize the properties of the two curves, they have the following properties under $\phi > 1$ and $\theta > 1$: $\mathcal{F}(\pi^*) = \mathcal{E}(\pi^*)$, $\lim_{\pi\to\infty} \mathcal{F}(\pi) < \lim_{\pi\to\infty} \mathcal{E}(\pi)$, $\mathcal{E}'(\pi) > 0$, $\mathcal{F}(\pi)$ is concave in the region of $\pi < \pi_f^*$, and $\mathcal{E}(\pi)$ is either concave or convex. In addition, condition (24) is equivalent to $\mathcal{F}'(\pi^*) > \mathcal{E}'(\pi^*)$ and $\pi_f^* > \pi^*$, where $\mathcal{F}'(\pi) > 0$ for $\pi < \pi_f^*$ and $\mathcal{F}'(\pi) < 0$ for $\pi > \pi_f^*$. Under the latter conditions, there exists a steady state such that $\pi = \pi^\diamond > \pi^*$. Regarding uniqueness, if the steady state is such that $\pi^\diamond < \pi_f^*$, it is unique since $\mathcal{F}(\pi)$ is increasing and concave and $\mathcal{E}(\pi)$ is increasing and either concave or convex. If the steady state is such that $\pi^\diamond \geq \pi_f^*$, it is also unique since $\mathcal{F}(\pi)$ is decreasing and $\mathcal{E}(\pi)$ is increasing.

ii) Low-inflation steady state Condition (26) ensures $\mathcal{E}(\underline{\pi}) > \mathcal{F}(\underline{\pi})$ at the lower end of the domain, given by $\underline{\pi} = \max\{\underline{\pi}_i, \underline{\pi}_r\}$. The first case of condition (26) corresponds to the case of $\underline{\pi} = \underline{\pi}_r$, and the second case corresponds to the case of $\underline{\pi} = \underline{\pi}_i$, where $\Psi(\phi, \theta)$ is given by

$$\Psi(\phi,\theta) := \frac{1-(\beta/\pi^*)^{\frac{\phi-1}{\phi}}}{1-\left[\frac{1-\beta^{\frac{1}{\phi}}(\pi^*)^{1-\frac{1}{\phi}}}{(1-\beta)(\beta/\pi^*)^{\frac{\theta}{\phi}}}\right]^{\frac{1}{\theta-1}}} \tag{A.9}$$

Given $\mathcal{E}(\underline{\pi}) > \mathcal{F}(\underline{\pi})$ and $\mathcal{E}(\pi^*) = \mathcal{F}(\pi^*)$ for $\underline{\pi} < \pi^*$, since condition (25) is equivalent to $\mathcal{E}'(\pi^*) > \mathcal{F}'(\pi^*)$ as shown in Lemma 1, it implies that there exists a crossing between the two curves, $\mathcal{E}(\pi)$ and $\mathcal{F}(\pi)$, at $\pi = \pi^\diamond \in (\underline{\pi}, \pi^*)$. Regarding uniqueness, note that $\pi^\diamond$ has to be less than π_f^*, where π_f^* is a point at which $\mathcal{F}(\pi)$ is maximized (otherwise, there would be no crossing at $\pi = \pi^* > \pi^\diamond$). Since $\mathcal{F}(\pi)$ is increasing and concave in the region of $\pi < \pi_f^*$, and since $\mathcal{E}(\pi)$ is increasing and either concave or convex, there is at most single crossing of the two curves in the region of $(\underline{\pi}, \pi_f^*)$.

A.4 Proof of Proposition 3

i) High-inflation steady state The targeted inflation rate always constitutes a steady state: $\mathcal{F}(\pi^*) = \mathcal{E}(\pi^*)$. In the case of $\phi < 1$ and $\theta > 1$, both the fiscal curve $\mathcal{F}(\pi)$ defined by (22) and the Euler curve $\mathcal{E}(\pi)$ defined by (21) are decreasing. Since $\phi < 1$, the real interest rate $(1+i)/\pi$ is decreasing, so that the upper end of the domain of π is given by $\underline{\pi}_r := \beta^{\frac{1}{\phi-1}}\pi^*$ at which the net real interest rate becomes zero. The condition, $\chi^* > 1-\beta$, is equivalent to $\mathcal{E}(\underline{\pi}_r) > \mathcal{F}(\underline{\pi}_r) = 0$. Then, there exists a steady state in the interval of $(\pi^*, \underline{\pi}_r)$ if $|\mathcal{E}'(\pi^*)| > |\mathcal{F}'(\pi^*)|$. Since condition (25) holds if and only if $|\mathcal{E}'(\pi^*)| > |\mathcal{F}'(\pi^*)|$, there exists a steady state in the interval of $(\pi^*, \underline{\pi}_r)$ if condition (25) is satisfied.

ii) Low-inflation steady state The targeted inflation rate always constitutes a steady state: $\mathcal{F}(\pi^*) = \mathcal{E}(\pi^*)$. In the case of $\phi < 1$ and $\theta > 1$, both $\mathcal{F}(\pi)$ and $\mathcal{E}(\pi)$ are decreasing. Since the nominal interest rate is bounded below at zero, the lower end of the domain of π is given by $\underline{\pi}_i \coloneqq \beta^{\frac{1}{\phi}}(\pi^*)^{1-\frac{1}{\phi}}$. It can be shown that $\mathcal{F}(\underline{\pi}_i) < \mathcal{E}(\underline{\pi}_i)$ if and only if $\chi^* < \Psi(\phi, \theta)$, where $\Psi(\phi, \theta)$ is given by equation (A.9). To see this, note that $\mathcal{E}(\underline{\pi}_i)$ is given by

$$\mathcal{E}(\underline{\pi}_i) = \frac{1}{\chi}\left[\left(\frac{\beta}{\pi^*}\right)^{\frac{\phi-1}{\phi}} - 1\right] + \frac{b^*}{\pi^*} = \frac{b^*}{\pi^*}\left[1 + \frac{1}{\chi^*}\left(\left(\frac{\pi^*}{\beta}\right)^{\frac{1-\phi}{\phi}} - 1\right)\right]$$

where $\chi^* \coloneqq (\chi b^*)/\pi^*$. Also, $\mathcal{F}(\underline{\pi}_i)$ is given by

$$\mathcal{F}(\underline{\pi}_i) = \left[1 - \beta\left(\frac{\beta}{\pi^*}\right)^{\frac{1-\phi}{\phi}}\right]^{\frac{1}{\theta-1}} \left[\beta^{\frac{1}{\phi}}(\pi^*)^{1-\frac{1}{\phi}}\right]^{-\frac{\theta}{\theta-1}} (b^*)^{\frac{\theta}{\theta-1}} (\tau^*)^{-\frac{1}{\theta-1}}$$

Since $\tau^* = (b^*/\pi^*)(1-\beta)$, $\mathcal{F}(\underline{\pi}_i)$ can be expressed as

$$\begin{aligned}\mathcal{F}(\underline{\pi}_i) &= \left[1 - \beta\left(\frac{\beta}{\pi^*}\right)^{\frac{1-\phi}{\phi}}\right]^{\frac{1}{\theta-1}} \left[\beta^{\frac{1}{\phi}}(\pi^*)^{1-\frac{1}{\phi}}\right]^{-\frac{\theta}{\theta-1}} b^* \left(\frac{\pi^*}{1-\beta}\right)^{\frac{1}{\theta-1}} \\ &= \left[\frac{1 - \beta^{\frac{1}{\phi}}(\pi^*)^{1-\frac{1}{\phi}}}{(1-\beta)(\beta/\pi^*)^{\frac{\theta}{\phi}}}\right]^{\frac{1}{\theta-1}} \frac{b^*}{\pi^*}\end{aligned}$$

Since $\mathcal{F}(\pi)$ is decreasing and $\underline{\pi}_i < \pi^*$, the terms in the square brackets are strictly greater than unity. From these expressions, $\mathcal{F}(\underline{\pi}_i) < \mathcal{E}(\underline{\pi}_i)$ leads to $\chi^* < \Psi(\phi, \theta)$.

Given $\mathcal{F}(\underline{\pi}) < \mathcal{E}(\underline{\pi})$, there exists a steady state in the interval of $(\underline{\pi}_i, \pi^*)$ if $|\mathcal{E}'(\pi^*)| < |\mathcal{F}'(\pi^*)|$. Since condition (24) holds if and only if $|\mathcal{E}'(\pi^*)| < |\mathcal{F}'(\pi^*)|$, this completes the proof of the proposition.

A.5 Proof of Proposition 4

For completeness, firstly the system of log-linear equations is derived. Log-linearizing the system of equations (13), (14), and (17) around the targeted steady state $(\pi, b, i) = (\pi^*, b^*, i^*)$ yields

$$\begin{aligned}\hat{b}_t &= \frac{1}{\beta}(1-\theta^*)\hat{b}_{t-1} - \frac{1}{\beta}\hat{\pi}_t + \hat{i}_t \\ \hat{i}_t &= \phi\hat{\pi}_t \\ 0 &= -\chi^*\hat{b}_t + \hat{i}_t - (1-\chi^*)\,\mathbb{E}_t\hat{\pi}_{t+1}\end{aligned}$$

where $\theta^* \coloneqq (1-\beta)\theta$, $\hat{i}_t = (i_t - i^*)/(1+i^*)$, and $\chi^* \coloneqq (\chi b^*)/\pi^*$. By substituting out the interest rate, the log-linearized system of equations can be rearranged as

$$\begin{aligned}\hat{b}_t &= \frac{1}{\beta}(1-\theta^*)\hat{b}_{t-1} - \left(\frac{1}{\beta} - \phi\right)\hat{\pi}_t \\ (1-\chi^*)\,\mathbb{E}_t\hat{\pi}_{t+1} &+ \chi^*\hat{b}_t = \phi\hat{\pi}_t\end{aligned}$$

In a matrix form the system can be expressed as

$$\begin{bmatrix} 1 & 0 \\ \chi^* & 1-\chi^* \end{bmatrix} \mathbb{E}_t \begin{bmatrix} \hat{b}_t \\ \hat{\pi}_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{1}{\beta}(1-\theta^*) & -\left(\frac{1}{\beta}-\phi\right) \\ 0 & \phi \end{bmatrix} \begin{bmatrix} \hat{b}_{t-1} \\ \hat{\pi}_t \end{bmatrix}$$

or

$$\begin{aligned} \mathbb{E}_t \begin{bmatrix} \hat{b}_t \\ \hat{\pi}_{t+1} \end{bmatrix} &= \begin{bmatrix} 1 & 0 \\ -\frac{\chi^*}{1-\chi^*} & \frac{1}{1-\chi^*} \end{bmatrix} \begin{bmatrix} \frac{1}{\beta}(1-\theta^*) & -\left(\frac{1}{\beta}-\phi\right) \\ 0 & \phi \end{bmatrix} \begin{bmatrix} \hat{b}_{t-1} \\ \hat{\pi}_t \end{bmatrix} \\ &= \underbrace{\begin{bmatrix} \frac{1}{\beta}(1-\theta^*) & -\left(\frac{1}{\beta}-\phi\right) \\ -\frac{(\chi^*/\beta)(1-\theta^*)}{1-\chi^*} & \frac{(\chi^*/\beta)+(1-\chi^*)\phi}{1-\chi^*} \end{bmatrix}}_{:=\mathcal{B}} \begin{bmatrix} \hat{b}_{t-1} \\ \hat{\pi}_t \end{bmatrix} \end{aligned}$$

Since $\hat{b}_{t-1}$ is a predetermined variable and $\hat{\pi}_t$ is a jump variable, the local dynamics is determinate if and only if one of the eigenvalues of $\mathcal{B}$ is inside the unit circle and another is outside the unit circle. The characteristic equation for $\mathcal{B}$ is given by

$$\lambda^2 - \text{tr}(\mathcal{B})\lambda + \det(\mathcal{B}) = 0$$

where λ denotes an eigenvalue. Solving this equation for λ yields

$$\lambda = \frac{\text{tr}(\mathcal{B}) \pm \sqrt{\text{tr}(\mathcal{B})^2 - 4\det(\mathcal{B})}}{2}$$

where

$$\text{tr}(\mathcal{B}) = \frac{1-\theta^*}{\beta} + \frac{(\chi^*/\beta)+(1-\chi^*)\phi}{1-\chi^*} > 0 \tag{A.10}$$

$$\det(\mathcal{B}) = \frac{(1/\beta)(1-\theta^*)}{1-\chi^*}\phi \geq 0 \tag{A.11}$$

Since $\text{tr}(\mathcal{B}) > 0$ and $\det(\mathcal{B}) \geq 0$, there are two distinct eigenvalues that are both non-negative, with one of them is strictly positive. The determinacy condition is $\lambda_1 > 1$ and $0 \leq \lambda_2 < 1$, which is equivalent to: $(\lambda_1 - 1)(\lambda_2 - 1) < 0$. This condition can be written as $\text{tr}(\mathcal{B}) - \det(\mathcal{B}) > 1$, since $\text{tr}(\mathcal{B}) = \lambda_1 + \lambda_2$ and $\det(\mathcal{B}) = \lambda_1\lambda_2$. Substituting equations (A.10) and (A.11) into the condition yields

$$\left(1 - \frac{\phi}{1-\chi^*}\right)\theta^* < 1 - \beta + \frac{\chi^*}{1-\chi^*} - \frac{1-\beta(1-\chi^*)}{1-\chi^*}\phi \tag{A.12}$$

There are two cases for determinacy, depending on the sign of the coefficient on $\theta^* = (1-\beta)\theta$.

I) $\phi > 1-\chi^*$: Condition (A.12) can be written as $\theta > \Theta(\phi, \chi^*, \beta)$, where $\Theta(\cdot)$ is given by (23).

II) $\phi < 1-\chi^*$: Condition (A.12) can be written as $\theta < \Theta(\phi, \chi^*, \beta)$.

The other cases corresponds to either $\lambda_1, \lambda_2 < 1$ (indeterminacy) or $\lambda_1, \lambda_2 > 1$ (explosive). The former case corresponds to $\phi < 1-\chi^*$ and $\theta > \Theta(\phi, \chi^*, \beta)$, as it exhibits lower eigenvalues according to equation (A.11). The latter cases corresponds to $\phi > 1-\chi^*$ and $\theta < \Theta(\phi, \chi^*, \beta)$.

A.6 Proof of Lemma 6

First, consider the case of $\phi > 1 - \chi^\diamond$ and $\theta > 1$. Since $\mathcal{E}(\pi^\diamond) = \mathcal{F}(\pi^\diamond)$, in considering relative slopes, it is equivalent to examine them directly and to examine the relative slopes of log of $\mathcal{E}(\pi^\diamond)$ and $\mathcal{F}(\pi^\diamond)$. Taking the log of the Euler curve (21) and differentiating with respect to π, evaluated at $\pi = \pi^\diamond$, yield

$$\left.\frac{d\log\mathcal{E}(\pi)}{d\pi}\right|_{\pi=\pi^\diamond} = \frac{\phi-1}{\chi^\diamond}\frac{1}{\pi^\diamond}$$

Similarly, taking the log of the fiscal curve (22) and differentiating with respect to π, evaluated at $\pi = \pi^\diamond$, yield

$$\left.\frac{d\log\mathcal{F}(\pi)}{d\pi}\right|_{\pi=\pi^\diamond} = \left[\frac{1}{\theta-1}\frac{\beta^\diamond(\phi-1)}{1-\beta^\diamond} - \frac{\theta}{\theta-1}\right]\frac{1}{\pi^\diamond}$$

Hence:

$$\mathcal{E}'(\pi^\diamond) > \mathcal{F}'(\pi^\diamond) \quad \Leftrightarrow \quad \frac{\phi-1}{\chi^\diamond} > \frac{1}{\theta-1}\frac{\beta^\diamond(\phi-1)}{1-\beta^\diamond} - \frac{\theta}{\theta-1}$$

Arranging this inequality with respect to θ, by paying attention to the inequalities $\phi > 1 - \chi^\diamond$ and $\theta > 1$, yields $\theta > \Theta(\phi, \chi^\diamond, \beta^\diamond)$ if $\phi > 1 - \chi^*$.

Next consider the case of $\phi < 1 - \chi^\diamond$ and $\theta > 1$. In this case, both $\mathcal{E}'(\pi^\diamond)$ and $\mathcal{F}^\diamond(\pi^\diamond)$ are negative. Then, $|\mathcal{E}'(\pi^\diamond)| > |\mathcal{F}'(\pi^\diamond)|$ is equivalent to

$$-\frac{\phi-1}{\chi^\diamond} > -\left[\frac{1}{\theta-1}\frac{\beta^\diamond(\phi-1)}{1-\beta^\diamond} - \frac{\theta}{\theta-1}\right]$$

or

$$\frac{1-\phi}{\chi^\diamond} > \frac{1}{\theta-1}\frac{\beta^\diamond(1-\phi)}{1-\beta^\diamond} + \frac{\theta}{\theta-1}$$

Arranging this inequality with respect to θ, by paying attention to the inequalities $\phi < 1 - \chi^\diamond$ and $\theta > 1$, yields $\theta > \Theta(\phi, \chi^\diamond, \beta^\diamond)$.

A.7 Proof of Proposition 8

Since $\underline{\pi}_i \geq 1$, the upper end of the relevant domain of π is 1. First, consider the case of $\theta > 1$. In this case, both the Euler curve (27) and the fiscal curve (27) are decreasing. In addition, they have the following properties:

$$\tilde{\mathcal{F}}(\beta) = \frac{b^*}{y^*\pi^*}\left(\frac{\pi^*}{\beta}\right)^{\frac{\theta}{\theta-1}} > \frac{b^*}{y^*\pi^*} = \tilde{\mathcal{E}}(\beta)$$
$$\tilde{\mathcal{F}}(1) = 0 < \frac{b^*}{y^*\pi^*} - \frac{1-\beta}{\chi} = \tilde{\mathcal{E}}(1)$$

The inequality in the second line holds because of the assumption $\chi^* > 1 - \beta$. Then, there exists at least one intersection of the two curves (28) and (27) in the interval of $(\beta, 1)$.

In the case of $\theta < 1$, the fiscal curve $\tilde{\mathcal{F}}(\pi)$ becomes increasing in π, while the Euler curve $\tilde{\mathcal{E}}(\pi)$ stays the same. These curves have the following properties:

$$\tilde{\mathcal{F}}(\beta) = \frac{b^*}{y^*\pi^*}\left(\frac{\beta}{\pi^*}\right)^{\frac{\theta}{1-\theta}} < \frac{b^*}{y^*\pi^*} = \tilde{\mathcal{E}}(\beta)$$
$$\lim_{\pi\to 1}\tilde{\mathcal{F}}(\pi) = \infty > \frac{b^*}{y^*\pi^*} - \frac{1-\beta}{\chi} = \tilde{\mathcal{E}}(1)$$

Therefore, there exists a unique zero-interest steady state with $\tilde{\pi} \in (\beta, 1)$.

A.8 Proof of Proposition 9

The system of equations (29) and (30) can be written in a matrix form as

$$\begin{bmatrix} 1 & 0 \\ \tilde{\chi} & 1-\tilde{\chi} \end{bmatrix} \mathbb{E}_t \begin{bmatrix} \hat{b}_t \\ \hat{\pi}_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{1}{\tilde{\pi}}(1-\tilde{\theta}) & -\frac{1}{\tilde{\pi}} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{b}_{t-1} \\ \hat{\pi}_t \end{bmatrix}$$

or

$$\begin{aligned} \mathbb{E}_t \begin{bmatrix} \hat{b}_t \\ \hat{\pi}_{t+1} \end{bmatrix} &= \begin{bmatrix} 1 & 0 \\ -\frac{\tilde{\chi}}{1-\tilde{\chi}} & \frac{1}{1-\tilde{\chi}} \end{bmatrix} \begin{bmatrix} \frac{1}{\tilde{\pi}}(1-\tilde{\theta}) & -\frac{1}{\tilde{\pi}} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{b}_{t-1} \\ \hat{\pi}_t \end{bmatrix} \\ &= \underbrace{\begin{bmatrix} \frac{1}{\tilde{\pi}}(1-\tilde{\theta}) & -\frac{1}{\tilde{\pi}} \\ \frac{1}{\tilde{\pi}}(1-\tilde{\theta}) \times \left(-\frac{\tilde{\chi}}{1-\tilde{\chi}}\right) & -\frac{1}{\tilde{\pi}} \times \left(-\frac{\tilde{\chi}}{1-\tilde{\chi}}\right) \end{bmatrix}}_{:=\tilde{\mathcal{B}}} \begin{bmatrix} \hat{b}_{t-1} \\ \hat{\pi}_t \end{bmatrix} \end{aligned}$$

Since the matrix $\tilde{\mathcal{B}}$ is singular, one eigenvalue is zero, and another eigenvalue is given by

$$\mathrm{tr}(\tilde{\mathcal{B}}) = \frac{1}{\tilde{\pi}} \left(1 - \tilde{\theta} + \frac{\tilde{\chi}}{1-\tilde{\chi}}\right)$$

Given $\tilde{\chi} < 1$, local dynamics around the zero-interest steady state is determinate if and only if $|\mathrm{tr}(\tilde{\mathcal{B}})| > 1$, which can be written as $\tilde{\theta} < 1 - \tilde{\pi} + \tilde{\chi}/(1-\tilde{\chi})$, or $\theta < \Theta(0, \tilde{\chi}, \tilde{\pi})$. Local dynamics around the zero-interest rate is indeterminate if and only if $|\mathrm{tr}(\tilde{\mathcal{B}})| < 1$, which can be written as $\theta > \Theta(0, \tilde{\chi}, \tilde{\pi})$.

A.9 Proof of Proposition 10

Consider the case of $\theta > 1$. The zero-interest steady state is given by the intersections of the two curves (27) and (28). Since the two curves $\tilde{\mathcal{F}}(\pi)$ and $\tilde{\mathcal{E}}(\pi)$ are decreasing and satisfy $\tilde{\mathcal{F}}(\beta) > \tilde{\mathcal{E}}(\beta)$ and $\tilde{\mathcal{F}}(1) < \tilde{\mathcal{E}}(1)$, there exists a zero-interest steady state with $\tilde{\pi} > \beta$ and $|\tilde{\mathcal{F}}'(\tilde{\pi})| > |\tilde{\mathcal{E}}'(\tilde{\pi})|$, which is unique by assumption. In the following, it will be shown that this slope condition is equal to the determinacy condition $\theta < \Theta(0, \tilde{\chi}, \tilde{\pi})$ in Proposition 9. The slope of the Euler curve (27) at $\pi = \tilde{\pi}$ is given by

$$\tilde{\mathcal{E}}'(\tilde{\pi}) = -\frac{\beta}{\chi} \frac{1}{\tilde{\pi}^2}$$

The slope of the fiscal curve (28) at $\pi = \tilde{\pi}$ is given by

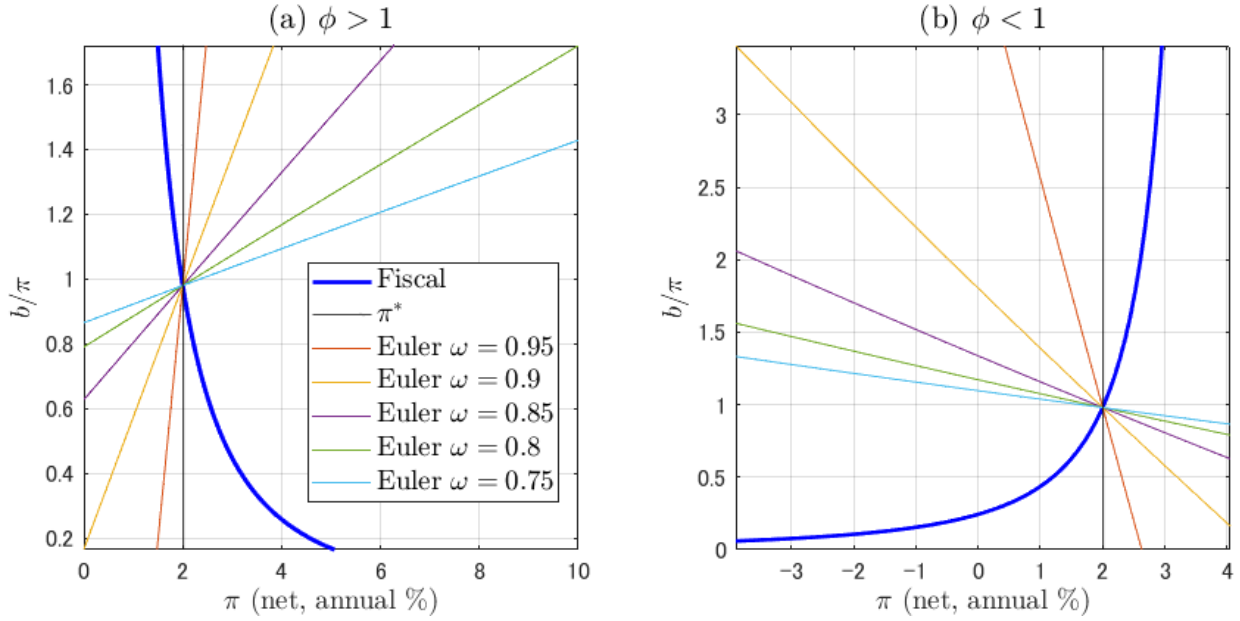
$$\tilde{\mathcal{F}}'(\pi) = (1-\pi)^{\frac{1}{\theta-1}} \pi^{-\frac{\theta}{\theta-1}} (\tau^*)^{-\frac{1}{\theta-1}} (b^*)^{\frac{\theta}{\theta-1}}$$

Since $\tilde{b}/\tilde{\pi} = \tilde{\mathcal{F}}(\tilde{\pi})$ from equation (28), the slope can be expressed as

$$\tilde{\mathcal{F}}'(\tilde{\pi}) = -\frac{\tilde{b}}{\tilde{\pi}} \left(\frac{1}{\theta-1} \frac{1}{1-\tilde{\pi}} + \frac{\theta}{\theta-1} \frac{1}{\tilde{\pi}}\right)$$

Since $\theta > 1$, it is straightforward to show that $|\tilde{\mathcal{F}}'(\tilde{\pi})| > |\tilde{\mathcal{E}}'(\tilde{\pi})|$ is reduced to the determinacy condition $\theta < \Theta(0, \tilde{\chi}, \tilde{\pi})$.

Figure A.1: Positive-interest steady states: $\theta < 1$



Note: 'Fiscal' plots $\mathcal{F}(\pi)$ defined by (22) and 'Euler' plots $\mathcal{E}(\pi)$ defined by (21) for various values of ω. The parameter values are the same as those used in Figure 1 for Panel (a) and Figure 2 for Panel (b), except for θ, which is set at 0.5.

Next, consider a zero-interest steady state with $\tilde{\pi} < \beta$. Since the two curves $\tilde{\mathcal{F}}(\pi)$ and $\tilde{\mathcal{E}}(\pi)$ are decreasing and satisfy $\tilde{\mathcal{F}}(\beta) > \tilde{\mathcal{E}}(\beta)$, the assumption of such a unique steady state implies that the steady state has to display $|\tilde{\mathcal{F}}'(\tilde{\pi})| < |\tilde{\mathcal{E}}'(\tilde{\pi})|$. Similarly to the derivation above, this condition is reduced to $\theta > \Theta(0, \tilde{\chi}, \tilde{\pi})$.

B Additional Analyses

B.1 Positive-interest steady state with $\phi \lessgtr 1$ and $\theta < 1$

The first derivatives of $\mathcal{E}(\pi)$ and $\mathcal{F}(\pi)$ can be derived as

$$\begin{aligned}\mathcal{E}'(\pi) &= -\frac{1}{\chi}(1-\phi)\pi^{\phi-2}(\pi^*)^{1-\phi} \\ \mathcal{F}'(\pi) &= \frac{1}{1-\theta}\left[1-\beta\left(\frac{\pi}{\pi^*}\right)^{1-\phi}\right]^{\frac{1}{\theta-1}}\pi^{-\frac{\theta}{\theta-1}-1}\left[\theta+\frac{(1-\phi)\beta(\pi/\pi^*)^{1-\phi}}{1-\beta(\pi/\pi^*)^{1-\phi}}\right]\end{aligned} \tag{B.1}$$

Given $\theta < 1$, in the case of $\phi < 1$, the signs of the derivatives are $\mathcal{E}' < 0$ and $\mathcal{F}'(\pi) > 0$. Hence, there exists a unique steady state, which is the targeted steady state, as illustrated in Figure A.1(b).

In the case of $\phi > 1$, the Euler curve $\mathcal{E}(\pi)$ becomes increasing. The sign of $\mathcal{F}'(\pi)$ depends on the sign of the terms in the second square brackets in (B.1). It is straightforward to show that $\mathcal{F}'(\pi^*) < 0 \Leftrightarrow \phi > 1 + (\beta^{-1} - 1)\theta$. If $\mathcal{F}'(\pi^*) < 0$, it follows that $\mathcal{F}'(\pi) < 0$ for $\underline{\pi} < \pi < \pi^*$, where $\underline{\pi} = \max\{\underline{\pi}_i, \underline{\pi}_r\}$. Because of continuity, $\mathcal{F}(\pi)$ is decreasing as long as π becomes not too large. Actually, in Figure A.1(a), there appears to be a unique steady state, which is the targeted steady state. However, equation (B.1) implies

that as π becomes extremely large, $(\pi/\pi^*)^{1-\phi}$ becomes smaller, and the sign of $\mathcal{F}'(\pi)$ can be flipped to be positive. Then, they may be another steady state with extremely high inflation. In Figure A.1(a), for $\pi > \pi^*$, $\mathcal{E}(\pi) > \mathcal{F}(\pi)$ as long as π is not extremely large. But, under the parameters of the figure, $\mathcal{F}(\pi)$ becomes strictly greater than $\mathcal{E}(\pi)$ when π reaches around 1e+12. This means that there is an intersection of the two curves between π =1e+11 and 1e+12. The numbers are too large to be analyzed as inflation rates. Hence, in the main text, such an intersection is ignored.

B.2 The uniqueness of the zero-interest steady state with $\tilde{\pi} > \beta$

To analyze uniqueness of the zero-interest steady state with $\tilde{\pi} > \beta$ in the case of $\theta > 1$, note that $\log \tilde{\mathcal{F}}(\pi)$ has the following properties:

$$\frac{d \log \tilde{\mathcal{F}}(\pi)}{d\pi} = -\frac{1}{\theta - 1}\frac{1}{1-\pi} - \frac{\theta}{\theta - 1}\frac{1}{\pi} < 0$$
$$\frac{d^2 \log \tilde{\mathcal{F}}(\pi)}{(d\pi)^2} = -\frac{1}{\theta - 1}\frac{1}{(1-\pi)^2} + \frac{\theta}{\theta - 1}\frac{1}{\pi^2}$$

In addition, the second derivative can be evaluated as follows.

$$\lim_{\pi \to 1} \frac{d^2 \log \tilde{\mathcal{F}}(\pi)}{(d\pi)^2} = -\infty < 0$$
$$\frac{d^2 \log \tilde{\mathcal{F}}(\pi)}{(d\pi)^2}\Big|_{\pi=\beta} \propto -\frac{1}{(1-\beta)^2} + \frac{\theta}{\beta^2} < 0 \quad \Leftrightarrow \quad \theta < \left(\frac{\beta}{1-\beta}\right)^2$$

Since β is close to unity, the condition $\theta < [\beta/(1-\beta)]^2$ is satisfied for reasonable values of θ.[5] Hence, $\log \tilde{\mathcal{F}}(\pi)$ is decreasing and concave if $\theta < [\beta/(1-\beta)]^2$. Next, consider the first and second derivatives of $\log \tilde{\mathcal{E}}(\pi)$.

$$\begin{aligned}
\frac{d \log \tilde{\mathcal{E}}(\pi)}{d\pi} &= -\frac{(\beta/\chi)(1/\pi^2)}{b^*/\pi^* - (1/\chi)(1-\beta/\pi)} < 0 \\
\frac{d^2 \log \tilde{\mathcal{E}}(\pi)}{(d\pi)^2} &= \frac{2(\beta/\chi)(1/\pi^3)}{b^*/\pi^* - (1/\chi)(1-\beta/\pi)} + \frac{(\beta/\chi)^2(1/\pi^4)}{[b^*/\pi^* - (1/\chi)(1-\beta/\pi)]^2} \\
&\propto 2\pi\left[\frac{b^*}{\pi^*} - \frac{1}{\chi}\left(1 - \frac{\beta}{\pi}\right)\right] - \frac{\beta}{\chi} \\
&= \frac{b^*}{\pi^*}\left[\frac{\beta}{\chi^*} - 2\pi\left(\frac{1}{\chi^*} - 1\right)\right]
\end{aligned}$$

In the case of $\chi^* > 1$, the second derivative is always positive, so that the function $\log \tilde{\mathcal{E}}(\pi)$ is convex. In the case of $\chi^* < 1$, $\log \tilde{\mathcal{E}}(\pi)$ is concave over $\pi \in (\beta, 1)$ if

$$\frac{d^2 \log \tilde{\mathcal{E}}(\pi)}{(d\pi)^2}\Big|_{\pi=\beta} < 0 \quad \Leftrightarrow \chi^* < \frac{1}{2}$$

[5] Under the baseline parameter value of $\beta = (1.01)^{-1/4}$, $[\beta/(1-\beta)]^2$ is greater than $160,000$.

The function is convex over $\pi \in (\beta, 1)$ if

$$\frac{d^2 \log \tilde{\mathcal{E}}(\pi)}{(d\pi)^2}\Big|_{\pi=1} > 0 \quad \Leftrightarrow \chi^* > 1 - \frac{\beta}{2}$$

Hence, when β is close to unity,$\log \tilde{\mathcal{E}}(\pi)$ is either concave or convex in the interval of $(\beta, 1)$ for almost all cases, except for the region of $\chi^* \in [1/2, 1 - \beta/2]$.

To summarize, if θ is not too large, satisfying $\theta < [\beta/(1-\beta)]^2$, the function $\log \tilde{\mathcal{F}}(\pi)$ is concave in the interval of $\pi \in (\beta, 1)$. The function $\log \tilde{\mathcal{E}}(\pi)$ is either concave or convex when $\chi \notin (1/2, 1 - \beta/2)$ Under these conditions, the zero-interest steady state with $\tilde{\pi} \in (\beta, 1)$ is unique in the case of $\theta > 1$.