Privilege Lost?

The Rise and Fall of a Dominant Global Currency

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Abstract

Economic size matters for three key aspects of a dominant currency: safety, liquidity, and insurance. First, larger countries tend to be more diversified, increasing their debt capacity and safety. Secondly, the larger the share of a country in the supply of safe assets, the more liquid and attractive its bonds are. Finally, the larger a safe country's share in the world economy, the more its bonds appreciate in downturns, providing insurance for investors. We argue that these three aspects reinforce each other and show how they erode if the dominant country grows less than the rest of the world.

Keywords: Dominant Currency, Safe Assets, US Dollar, Default

JEL Classification: E42, F02, F33, N10

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1 Introduction

"The US Treasury market is [...] a really important feature to promoting the dollar's continued leadership around the globe. [..] Having that reliable, safe and readily accessible and tradeable asset is critical. It was critical to the British in their time as the leaders in currency. It was critical to the Dutch before that."

- Gary Gensler, chair of the SEC, FT March 2024.

The United States and the US Dollar play a crucial role in the international financial system. Dollar-denominated US government bonds are used extensively as safe assets by investors around the world. This special demand can give rise to yield differentials, lowering the required interest rates the US needs to pay when compared to other countries. This became known as the US' *exorbitant privilege*, a term coined in the 1960s by Valéry Giscard d'Estaing, at the time the French Minister of Finance, when discussing the position of the USD in the Bretton Woods system.

More recently, this expression has often been applied to return differentials between the US investment position compared to the rest of the world. Gourinchas and Rey (2007), Maggiori (2017) and Gourinchas and Rey (2022) describe how the US ends up holding riskier foreign assets than the rest of the world, who in turn holds a larger proportion of safe assets from the US. This implies that the US earns a higher return in normal times ('exorbitant privilege') while suffering losses in crisis (exorbitant duty'). Some recent papers such as Jiang et al. (2022) and Atkeson et al. (2025) argue that the privilege has waned or even disappeared since the Great Financial Crisis. The debate is still open whether this was due to a shift in expected return differentials or a higher than expected frequency of crisis. This paper goes back to the classic meaning of the exorbitant privilege and focuses on the cost of debt and demand for liquid safe assets.

Motivated by that, we ask how a country obtains the position of a global safe haven in the first place, how it might eventually lose that position and how safety is linked to convenience yields. We argue that both safety and size are necessary conditions for becoming a dominant currency, but also that these two properties are inherently connected. The larger the economy, the more diversified it will tend to be and so, *ceteris paribus*, the degree of safety will rise with size. This safety leads to lower interest rates and also a larger debt capacity, reflected in a higher debt-to-GDP ratio. At the same time, safety and size alone are not sufficient for a country to earn the exorbitant privilege.

We identify two key channels, both linked to country size, through which convenience

yields emerge. First, larger countries are able to issue a larger amount of bonds, implying deeper financial markets. This increases the probability that investors match with lenders to finance their investment opportunities. Relative size differences then generate a *liquidity* premium. Secondly, countries with a larger share in the world economy appreciate during downturns, generating an *insurance* premium. We demonstrate how both of these channels require the safety status of the dominant country to generate convenience yields. Finally, we show how rate differentials between two countries decline as relative sizes converge. This eventually leads to a loss of the convenience yield privilege when both countries are equally large and safe, as the liquidity and insurance status erode.

Interest rate differentials in our model are composed of risk, liquidity and insurance premia, with these three aspects reinforcing each other. While an abundant supply of bonds is good from the perspective of liquidity provision, there is a trade-off as the country's risk premium increases at the same time. As in the Triffin dilemma (Triffin, 1961), a country that acts as the global liquidity provider needs to be prudent not to overextend itself. A country's capacity to act as a global liquidity provider is therefore directly linked to its economic size, as larger countries can more sustainably fulfill this role. Furthermore, a loss of the exorbitant privilege also implies an erosion in debt capacity. Safety and insurance are linked to each other, as the large safe heaven status relies on the dominant country remaining safe during crises. Finally, the insurance and liquidity channels also reinforce each other, as payoffs of investment opportunities in large and liquid countries are more valuable in bad states than those in a small country. Increased demand due to the insurance channel then increases debt capacity, ultimately leading to a larger bond supply and enhanced liquidity.

We develop a model in which a government strategically defaults on its debt obligations when repayment is too costly relative to default. Investors anticipate this possibility and demand higher or lower interest rates depending on the distribution of the underlying shocks. In larger economies, idiosyncratic shocks originate from more and more sources and therefore become less granular and less important, effectively diversifying fundamental risk.

We discuss which features in an economy determine whether there is growth with or without diversification. The first feature is the correlation of shocks. A large economy with a low shock correlation across units enjoys a lower aggregate shock variation. This decreases the likelihood of an aggregate tail-event that could lead to a default. Unlike optimal currency areas, an optimal debt area requires a low correlation of shocks.

Furthermore, we also formalize how these idiosyncratic shocks are aggregated. This aggregation reflects how shocks propagate through the economy, but also the government's institutional ability to perform transfers across units[1]. Growth, together with low shock correlation and such institutional features, decrease the risk premia.

We embed this framework in a two-country open economy model in which investors can save in bonds of two countries and governments behave strategically. We show that the relative size of both countries matters for interest rate spreads. As the small country grows and diversifies, default becomes less likely, its interest rate falls and debt-to-GDP rises, while the larger country's interest rate increases slightly. We also provide stylized facts showing that economic size is correlated with safety, liquid bond markets and an insurance effect.

To generate liquidity premia, we also include investment opportunities that are easier to finance when bonds are safe and abundant. The larger the country's supply of assets relative to the rest of the world, the more liquid they will be in line with the empirical evidence presented. Liquidity is modeled similar to Chahrour and Valchev (2022). However, if a country's share of the world economy falls, it will see its liquidity premium erode. In this environment, we discuss the role of financial development for liquidity yields. If one country is financially more developed, interest rate spreads can persist even with equally large and safe economies.

Insurance yields arise in our setup because investors want to hedge against shocks to their consumption. When a country is in a recession, its good will be scarce and its price high. But the larger the country, the larger the impact in the marginal utility of global investors, due to the good's share in their consumption basket. The larger this positive covariance between the price of the underlying good and marginal utility, the stronger is the insurance value of the bond. Bonds of (safe) larger countries will therefore be more expensive than bonds of a smaller country whose shocks affect the stochastic discount factor of global investors less, as in Hassan (2013). We highlight an important connection between insurance and safety: only if there is negligible default risk can this channel capitalize on the size of the country and generate *insurance* convenience yields. We also demonstrate how risk aversion of investors can drive up the insurance premium.

Our model highlights that acquiring the exorbitant privilege does not depend on only one characteristic, but on the interaction between safety, liquidity, and the ability to provide insurance for global investors. These characteristics are always in comparison

[1]This can also be interpreted as a proxy for institutional quality and its ability to provide inter-regional or sectoral smoothing.

relative to other countries, and are crucially a function of size: While Switzerland is indeed very safe, its smaller size implies that its bond market is less liquid and the country as a whole struggles to provide sufficient insurance for the rest of the world. The Euro area in contrast is large, but its bond market is too fragmented to obtain a comparable status to the US.

Literature Our paper emphasizes how size influences the three key characteristics - safety, liquidity and insurance- for a dominant currency issuer and how those interact with each other. We contribute to the literature that emphasizes the role of economic size in explaining currency returns. Hassan (2013) shows that bonds of larger countries feature lower interest rates, as they insure against shocks that affect a larger part of the world economy. The more important is the country for global GDP volatility, the stronger is this effect. Therefore, larger countries would also be 'privileged' in the sense of low cost of debt. Our model includes this *insurance* channel explicitly and highlights the safety status of the country as a necessary condition for insurance. By including fundamental risk we also ensure that higher volatility of the larger country does not necessarily increase its advantage as a global insurer. If higher volatility worsens debt sustainability, then the asset's insurance value is reduced. We also discuss how investors' risk aversion can drive up the insurance premium.

Farhi and Maggiori (2018) develop a model of the international monetary system, in which a hegemon issues international reserves to the rest of the world. The hegemon has –as in our model– a lack of commitment and can default on its reserves. The stability of the monetary system is determined by how many reserves are issued by the hegemon, which ultimately determines the probability of a default. They also discuss how the system changes if competition arises from other reserve issuers. Although they consider strategic risk, in their model the reserve issuer status of a country is determined ex-ante. In this paper, we want to understand the determinants of currency dominance, so all countries issue bonds but some are endogenously more valuable than others due to being safer, more liquid and/or having a better insurance value. Our approach also explicitly considers the size of the country as an important determinant for its debt capacity, the insurance value of its bonds and their liquidity. We also emphasize that the issuance of debt involves a fundamental trade-off between liquidity on the one hand, and safety and insurance on the other hand.

What makes government debt safe? He et al. (2019) highlight strategic interactions between investors and the size of a country's debt market. Our paper focuses on the

determination of a country's fundamentals as its size grows, and on strategic competition between governments' bond supply. We also allow for an endogenously changing aggregate amount of bonds, with governments taking into account both the impact on safety and liquidity.

Jiang et al. (2021) develop a model of the global financial cycle in which the US dollar plays a special role as a provider of safe assets. They assume that US bonds are safe and that convenience yields arise via money-in-utility. Our paper asks how such safety is generated in the first place and how this safety links to the emergence of convenience yields when compared to other foreign bonds. We therefore put the focus on the question why the US became the safe haven and overtook other countries. On China, Clayton et al. (2025) discuss how its slow entry into the international financial market can be rationalized via reputation building to gain credibility as a safe currency issuer. Our paper focuses instead on the underlying macroeconomic fundamentals that are required for a currency to become dominant. Brunnermeier et al. (2024) discuss what determines the fundamental price of such safe assets. They emphasize that, beyond the pure cash value, the price also includes service flows via insurance. Our model emphasizes service values such as liquidity and insurance as well, but highlights how a reduction in safety can compromise the liquidity and insurance status.

The literature that discusses the US' exorbitant privilege has mostly focused on return differentials between the international investment position of the US, when compared to the rest of the world vis-a-vis the US. Gourinchas and Rey (2007) highlighted that the net external position to the US was akin to being the "world banker", borrowing short and lending long. In Gourinchas and Rey (2022), the same authors describe how the US position also has an element of an insurer for the rest of the world, as it provides safe assets while holding risky assets. The return differentials then arise because of risk premia and the US provides wealth transfers to the rest of the world during crises (the 'exorbitant duty'). The exorbitant privilege is therefore akin to an insurance premium during normal times. A similar view is taken by Maggiori (2017) who shows that the more financially developed country will take the riskier investment positions and should therefore be short in debt and long in equity. We demonstrate that such countries have lower borrowing costs due to their liquidity, as their safe assets can be easily sold or used as collateral when investment opportunities arise. Furthermore, we argue that the larger country is also safer because it is more diversified, increasing its debt capacity as it will have a lower probability of default for a given debt-to-GDP ratio. We emphasize that the dominant country also needs this safety status to act as a global insurer.

Some recent papers argue that the privilege has waned or even disappeared in the years since the Great Financial Crisis. Jiang et al. (2022) show that the ex-post return differential was no longer positive for the US from 2010 to 2019, and similarly Atkeson et al. (2025) demonstrate that the US' NFA position has substantially deteriorated. Our paper contributes to this strand of literature by explicitly modeling why US (government) assets are considered to be more safe and how the US obtains (and might lose) this position. Vicquéry (2022) provides systematic accounts for the historical structure of the international financial system, describing how the British Empire was at the center of the international monetary system, but was then overtaken by the US. Eichengreen et al. (2017) document in a similar way the evolution of the international financial system, noting how several currencies can be at the same time relevant for the global economy during a transition phase.

Choi et al. (2022) note that the US is the most important provider of safe assets and could exert market power over its asset supply. This results in under provision of US bonds and a markup with lower interest rates for the United States reflecting the exorbitant privilege. In our paper, we analyze changes in relative country size and the impact of supply on market depth and liquidity. We also consider governments as price makers for the provision of safe assets. Bianchi et al. (2023) provide a theory why convenience yields arise. They argue that financial institutions have precautionary demand for (safe) US assets as they are subject to volatile financial flows. This demand gives rise to convenience yields. We model the additional liquidity element of safe bonds as the ability to liquidate assets to finance investment opportunities, which in turn drives up demand for the safest bond and consequently lowers its interest rates. We also highlight the role of a country's financial development in explaining the magnitude of such liquidity yields.

Coppola et al. (2023) establish a liquidity-based theory in which economic agents decide to denominate their debt in the most liquid market. The government of the most liquid market has then the strongest incentive to invest to make its market even more liquid, reinforcing its status as a dominant currency. While the authors argue that it is not economic size that matters, we emphasize that there is a crucial connection between size, safety and liquidity. Economic size may not be a sufficient condition, but it can be a key distinguishing characteristic, especially among safe advanced economies. Large countries can provide the world with a sufficient amount of safe and liquid assets without overextending, mitigating the Triffin dilemma. Our findings on the importance of economic size are also in line with Richmond (2019) who provides evidence that countries that constitute a larger share of world GDP feature lower interest rates. Our way to

model the emergence of liquidity yields is very close to Chahrour and Valchev (2022) who emphasize the role of the overall size of an asset market. The larger the share of the bond in world markets, the easier it is to facilitate transactions, which is in line with the empirical evidence we present on bond bid-ask spreads and country size. Doepke and Schneider (2017) also show how agents coordinate on the usage of one dominant currency which increases matching efficiency.

Lastly, there is a literature that measures the role of convenience or liquidity yields for US government bonds. Krishnamurthy and Vissing-Jorgensen (2012) analyze the aggregate demand for treasury debt and find that the special status of US treasury in terms of liquidity and safety drive down its interest rate by on average 73 basis points between 1926 and 2008 with 46bp attributed to liquidity and 27bp to safety. This special status of US debt saves the government interest rate costs of about 0.25 % of its GDP, a fact that is also present in our analysis as higher diversification and liquidity allow for a larger debt capacity of the dominant global currency country. More recently, Du et al. (2018) document a clear trend in the decline of US convenience yields, in particular for medium and long-run maturities. Our model also yields a decomposition of interest rate spreads into a credit risk, liquidity and insurance components. We show that convenience yields are a function of relative country size and that a decline in the hegemon's share in global activity reduces convenience yields.

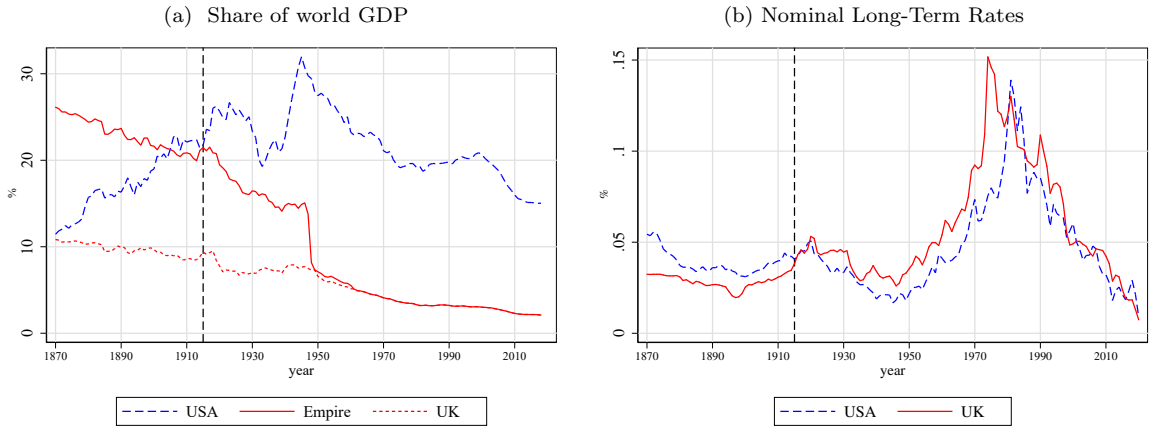
2 Historical and stylized facts

There are not many instances where a country lost its position as the global dominant currency issuer, but one well-known example is the USD taking the spot of the British Sterling, as the British Empire waned. In Figure 1 (a), we highlight the historical evolution of the share of world GDP of the United States and the British Empire, consisting out of the United Kingdom and all of its colonies. Towards the end of the 19th century and the beginning of the 20th century, the US share of world GDP grew substantially, eventually overtaking the British Empire and becoming the world's largest economy at around 1915 (vertical dashed line).

Figure 1 (b) documents that this change in the position as the world's largest economy also coincided with a systematic change in interest rates that both central governments in Washington and London had to pay. As the right panel shows, the long-term interest rates of 10 year government bonds were higher for the United States throughout the

period before 1915 [2]. After World War 1, the role reversed and the US was the country that paid lower interest rates. This might not only reflect the loss of the status as the entity with the greatest GDP, but also the loss of the safety status.

Figure 1: World GDP and interest rates



Note: World GDP share of the United States, the UK and the British Empire. The British Empire includes all countries and colonies for which the Empire has established control in a given year. GDP is in real 2011 USD, adjusted for relative price differences. *Source:* Bolt et al. (2018) and own calculations. Nominal long-term interest rates (10 year) of the US and the UK. *Source*: Jordà et al. (2019) and own calculations.

We furthermore examine different measures of interest rate spreads in Table A2 and Figure A1, which take into account inflation and exchange rate movements. The evidence for an important shift after 1915 is present in all these measures, as is the waning of the privilege after the Great Financial Crisis. This is indicative of the privileged position the United States occupied for almost a century, as the global provider of safe assets and the USD as the dominant global currency.

More recently, the difference between both countries eroded substantially. Since 2009 spreads are very close to zero across the board and no longer even positive when looking at implied yields rather than bond returns, reflecting the possibility that the US could be losing its exorbitant privilege, as argued by Atkeson et al. (2025) and Jiang et al. (2022). As we will argue in this paper, this is also related to the accelerated fall in the share of the United States in global GDP since the 2000s.

Before Sterling became the dominant currency, the Dutch florin served as the leading international currency in Europe, even though Spain was larger and received vast inflows of silver and gold from the Americas. However, by the time the Bank of Amsterdam created the Dutch florin in 1609, the Spanish crown had a recent history of serial defaults, the most recent of which just 2 years prior.[3] Spain was also experiencing large inflation,

[2] Chen et al. (2022) and Pflueger and Yared (2024) make a similar point about historical interest rate differentials between the dominant currency providers and other countries.

[3] See Reinhart et al. (2003).

in part exacerbated by the fiscal issues that prompted its government to issue a debased copper coinage ('vellón') that further eroded trust in the currency. In our model, we are able to incorporate explicitly this link between size, safety and liquidity. Spain was large, but not safe and therefore not liquid either. Moreover, as Coppola et al. (2023) argue, the Dutch were also more financially developed, which we also show provides it another advantage over Spain.

2.1 The role of size: safety, liquidity, and insurance

At the heart of our argument is that size combined with a sufficient degree of diversification allows a government to create safe, liquid bonds that can be used to insure global investors. Such governments have a higher debt capacity and can issue large amounts of bonds without increasing default risk too much.[4] A sufficiently large debt market then allows for the emergence of additional returns (convenience yields), as larger markets are more liquid and it becomes easier to facilitate transactions with that bond. In this section, we provide some simple stylized facts highlighting the correlations between size, safety, insurance and liquidity.

Size and Volatility Larger countries tend to be less volatile than smaller ones, as can be seen in Figure 2 (a). This chart plots the standard deviation of GDP growth rates against the country's average log share of world GDP (PPP, 1980 to 2023). Moreover, countries with exceptionally large volatility are small, or have a low degree of diversification in the economy, such as the large commodity producers highlighted in red in Figure 2 (a). This correlation is in line with Di Giovanni and Levchenko (2012) who document a similar correlation for the period 1970-2006. Depending on the correlation of shocks and institutional features, our model is able to generate both large countries with low aggregate volatility and large countries with high volatility.

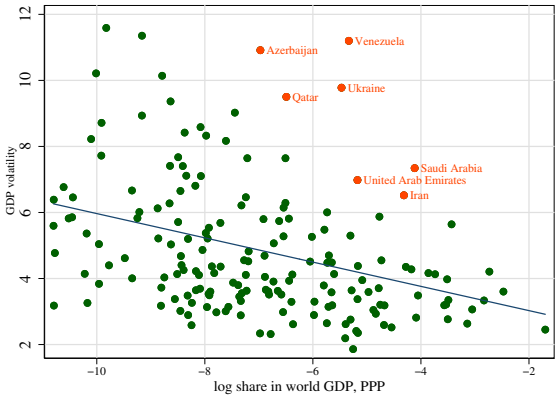
Size and Safety Using estimates of a government's default probability provided by Bloomberg[5] we show in Figure 2 (c) that there is a significant and negative correlation between size and safety. Larger countries like the US, Germany or Japan tend to have lower probabilities of default. Using the dataset of Asonuma and Trebesch (2016) for actual default events, we furthermore document in Figure 2 (d) that large countries have

[4] Imbs and Wacziarg (2003) noted that there's a U-shape in diversification as a country develops. That result is expressed relative to per capita income, as it concerns itself with development stages rather total size of the economy.

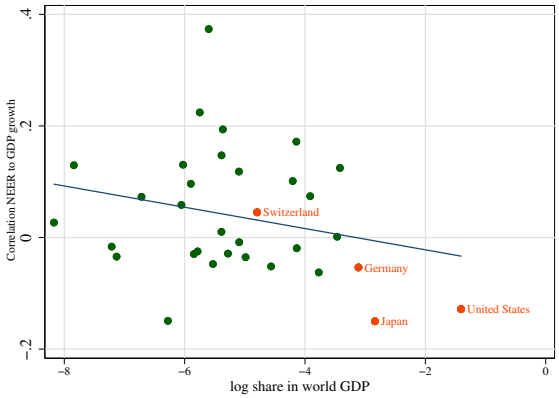
[5] Bloomberg uses its own SRSK model to estimate these probabilities of default based on several factors such as real GDP growth, foreign reserve to debt ratio, bank non-performing loan ratio, among others.

Figure 2: Size, Safety, Liquidity and Insurance

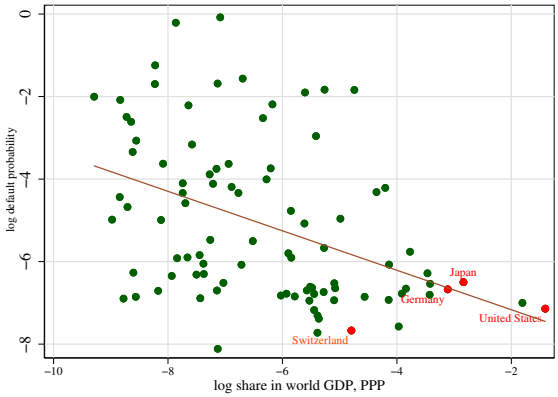
(a) Size and GDP volatility



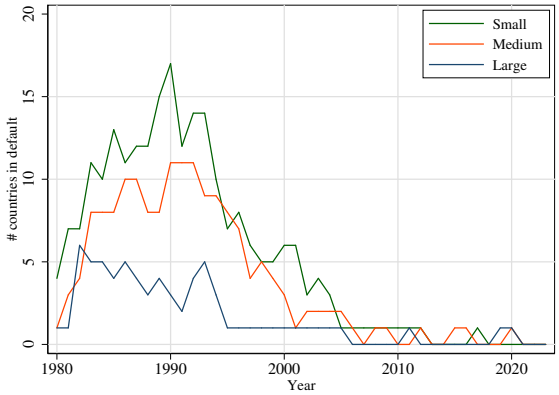
(b) Size and NEER-GDP correlation



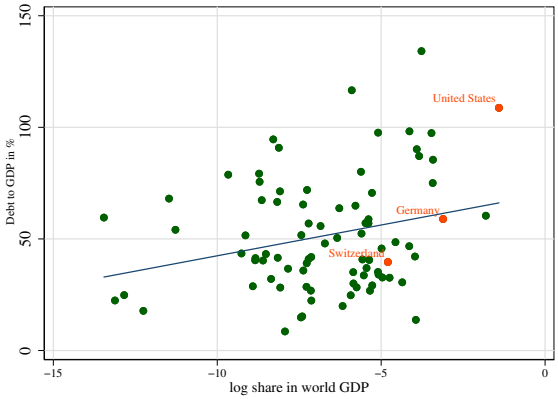
(c) Size and default probability



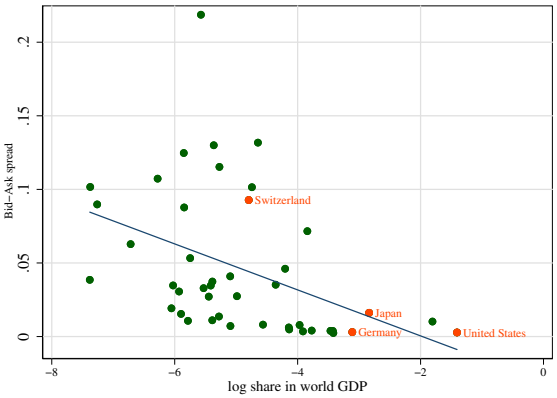
(d) Size and actual default



(e) Size and Debt-to-GDP



(f) Size and Government Bid-Ask spreads



Note: Panel (a) shows the log GDP share and GDP volatility of 176 countries over time. Mean between 1980-2023, GDP in PPP with IMF data. Panel (b) shows the log GDP share and the correlation between the Nominal Effective Exchange Rate and global cyclical GDP growth of 33 OECD countries Panel (c) shows the probability of default of 99 countries for one year government bonds according to Bloomberg CDS data correlated with the corresponding share in world GDP, all data in log. Panel (d) shows the number of countries in a default state, sorted into three groups of small, medium and large countries. Each group has the same number of countries. Panel (e) shows the log GDP share and debt-to-GDP ratio of 84 countries in 2019 with IMF data. Extreme outliers in terms of debt-to-GDP are excluded (Greece, Japan and Venezuela). Panel (f) shows the log GDP share and the average Bid-Ask spread in 2024 of 10-year government bonds of 47 countries.*Source: IMF, Bloomberg, Asonuma and Trebesch (2016)*

historically defaulted less than medium sized countries, while medium-sized countries have defaulted less than small countries. This empirical pattern supports a key feature of the model in which large countries tend to be safer. This correlation is also important for the global insurance status, given that such bonds need to remain safe during downturns.

Size and Insurance A key prediction of the insurance channel is that larger countries should be the ones that appreciate during global downturns as their goods and bonds become more valuable. Using the correlation of the nominal effective exchange rate and the cyclical component of global GDP growth, we indeed find evidence that larger countries such as the US or Japan tend to appreciate during downturns, whereas smaller countries' NEER moves together with the global cycle.[6]

Size and Liquidity An important implication of our model is that countries with better fundamentals have higher debt capacity and more liquid markets. We should then observe higher debt-to-GDP ratios for larger countries with developed financial markets. This can also be observed in the data. In Figure 2 (e), a positive correlation between size and debt-to-GDP ratios around the world is apparent. The correlation of size and debt capacity is an important requirement in our model, since the ability to supply safe bonds abundantly is a key factor in determining liquidity premia. In panel (f), we also find that larger countries tend to have lower bid-ask spreads for their 10 year government bonds, with US and Germany having some of the most liquid markets. While Switzerland is indeed very safe and with highly developed financial markets, its spreads are substantially higher as its bond market lacks depth relative to larger countries. Bid-ask spreads are an appropriate measure of liquidity in bond markets because they capture the cost of trading. A narrow spread means buyers and sellers are close in their valuations, so trades can happen quickly and at low cost.

3 Baseline model

We consider an economy with 2 countries, Home (H) and Foreign (F). In each country, governments decide on the supply of debt to be emitted and whether to repay their legacy debt. As standard in the endogenous default literature, we assume the governments are impatient relative to global investors, providing a motive to borrow. Due to limited commitment, the government has the option to (costly) default in period 2.

[6]This relationship is robust to using real effective exchange rates instead of nominal ones.

The two countries can be asymmetric in terms of their size, N and N^*. Size is defined as the number of economic units n that the country consists of. For now, units are ex-ante identical within and across countries and later we will explore heterogeneity in unit size across countries. We use the concept of unit as an umbrella term, which encompasses very different forms of growth. It could be interpreted as colonies or regions during the days of the British Empire, or as sectors, of an economy growing in complexity. The fundamental aspect that defines a unit in our framework is that each is a source of production, subject to idiosyncratic shocks. If economic growth occurs via an increase in the number of units, then it leads to diversification, as we will detail later. However, if it occurs by a proportional increase in the size of each unit, this will be a form of growth without diversification. The model is therefore able to distinguish the different impact of growing with and without diversification. For simplicity, we assume that units are ex-ante symmetric and the total mass of investors (and therefore their wealth) is proportional to world size $N + N^*$.

Section 3.1 details the government problem and output properties as a function of size, shock correlation and institutional characteristics. Section 3.2 details the investor's behavior in which the insurance motive becomes clear, while section 3.3 explains the setup for the investment opportunities to give rise to liquidity premia.

3.1 Government

We assume that the central government receives utility from a nation-wide aggregate of government consumption G_t:

$$\mathbb{E}\left[\log(G_1) + \delta \log(G_2)\right]. \tag{1}$$

δ is the time discount factor and log preferences imply some degree of (constant relative) risk aversion and willingness for intertemporal consumption smoothing. Government collects tax receipts by taxing the domestic good Y_t, which is an aggregator of N unit-specific endowments $y_{n,t}$:

$$Y_t = \left(\sum_{n=1}^{N} \omega_n^{\frac{1}{\theta}} y_{n,t}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}.$$

ω_n is a measure of unit size, and $y_{n,t}$ indicates unit n endowment in period t. We assume that the endowment is known and identical in the first period, while it is unit-specific

and stochastic in the second. The shock process is the same for each unit with a cross-unit correlation ρ[7], and θ is the elasticity of substitution of endowments across units. When $\theta \to \infty$, output is simply the (weighted) sum of endowments across all units, as if endowment in one unit is perfectly substitutable with endowment from other units. For lower values, endowments become more complementary, implying that endowment dispersion across units in the second period will reduce the overall aggregator. $\theta = 1$ corresponds to a Cobb Douglas aggregator while the extreme case of $\theta \to 0$ gives the Leontief case, in which the amount of domestic goods Y_t is equal to the lowest endowment realization across all units. This specification in general can limit the diversification effect of size. We discuss the importance of shock correlation and the elasticity of substitution in Section 4.1.2 and 4.1.1.

We also consider a country-specific TFP component Z that is stochastic and scales output across all regions, providing an additional limit to diversification. Output in each region in priod 2 is given by

$$y_{n,2} = Z \cdot \epsilon_{n,2} \tag{2}$$

where $\epsilon_{n,2}$ is the unit-specific shock at time 2. The nationwide component is independent of the number of units, it does not feature a diversification effect. Aggregate output is a combination of the country-specific shock Z and the un-diversified component of the unit-specific shocks.

In period 1, the government issues bonds B to Home and Foreign investors at price Q. The government budget constraint is then

$$G_1 = QB_1 + \tau Y_1 \tag{3}$$

where τ denotes the tax rate which is assumed to be fixed for simplicity.

As in Hassan (2013), we assume that each unit of bonds pays out $\hat{P}_2$ units of Home goods, where $\hat{P}_2$ is the price, expressed in home goods, of the investor's consumption basket in period one.[8] More details on the payouts are provided in the investor's section 3.2. The government either pays back all the debt B to investors, or it partly defaults. In case of default, it only pays back a share $\kappa < 1$ of the original debt repayment to

[7]Later, we will calibrate the marginal probability distribution to be uniformly distributed between $\underline{u}$ and an upper limit $\bar{u}$, with the cross-unit correlation given by a copula, but for now the results are more general. We also consider equally sized units. For more details, see Appendix B.1.

[8]Having the bonds payout $\hat{P}_2$ is not strictly necessary in our model, but we keep the setup as close to Hassan (2013) as possible.

investors and it suffers a dead-weight loss of ψ of output Y_2 that cannot be recovered by the government or investors. Government consumption is then given by

$$G_2 = \begin{cases} \tau Y_2 - B\hat{P}_2 & \text{if no default} \\ (1-\psi)\tau Y_2 - \kappa B\hat{P}_2 & \text{if default} \end{cases}$$

The government defaults if country wide consumption G_2 is bigger under default than under repayment, that is if default costs are smaller than the debt haircut:

$$\psi\tau Y_2 < (1-\kappa)B\hat{P}_2 \tag{4}$$

Default therefore happens in recessions, when aggregate output is very low. Given the stochastic process for Y_2, this inequality is satisfied with probability p. The (ex-ante) default probability is therefore a function of debt, bond prices, model parameters and country size . The impact of having more units on the default probability crucially depends on the shock correlation and the institutional features as we detail in Section 4.1.2 and 4.1.1.

3.1.1 The bond supply schedule

The government maximizes (1) subject to the aggregator and the payouts in each period and state. In our baseline setup, governments act as price-makers, behaving as monopolistic suppliers of their own bonds. The government chooses the quantity of bonds B to maximize utility subject to the budget constraint and default options:

$$\begin{aligned} \max_{B} \quad & \log(G_1) + \delta\big[(1-\mathcal{P}(B))\mathbb{E}(\log G_2|repay) + \mathcal{P}(B)\mathbb{E}\left[\log G_2|default\right]\big] \\ \text{s.t.} \quad & G_1 = \tau Y_1 + \mathcal{Q}(B, B^*)B \\ & G_2 = \begin{cases} \tau Y_2 - B\hat{P}_2 & \text{if repay} \\ (1-\psi)\tau Y_2 - \kappa B\hat{P}_2 & \text{if default} \end{cases} \end{aligned}$$

Governments are price-makers engaging in Cournot competition. They internalize how their choice of debt B affects probabilities of default and prices, represented above by the functions $\mathcal{P}$ and $\mathcal{Q}$, respectively. The price of bonds $Q = \mathcal{Q}(B, B^*)$ is also dependent on the foreign supply B^*, as long as investors are not risk-neutral. In the section below, we will show why this is the case. Since we have Cournot competition, each country takes the

other's choice of debt as given and the Nash equilibrium holds when there is no incentive to deviate by either country. The problem of the Foreign government is analogous, with all variables being denoted by a star * and with the distinction that endowments are in country-F specific goods. We solve the two period-model via an iterative value function algorithm which takes into account how prices and default probabilities depend on each government's quantity of supplied bonds. More details on the algorithm can be found in Appendix B.6.

3.2 Investors

There is a mass of global investors proportional to the number of units in both countries $N + N^*$. Investors are ex-ante homogeneous and have CRRA preferences

$$U = C_1^{1-\sigma} + \beta E\left(C_2^{1-\sigma}\right) \tag{5}$$

where $\beta < 1$ is the time discount factor and σ the relative risk aversion coefficient. The aggregate consumption good is also a CES composite of home and foreign goods:

$$C_t = \left(\alpha^{1/\eta}\, {C_{H,t}}^{\frac{\eta-1}{\eta}} + (1-\alpha)^{1/\eta}\, {C_{F,t}}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \tag{6}$$

$C_{j,t}$ indicates consumption of country j goods. The weights are allowed to vary according to the relative size of countries.

We normalize the price of the home good to 1 and assume that there are no trade frictions, so the law of one price always holds for both goods. The CES aggregator implies that the price index of the consumption basket (in terms of home goods) is $P_t = \left(\alpha + (1-\alpha)\, {P_{F,t}}^{1-\eta}\right)^{\frac{1}{1-\eta}}$. We also assume weights are proportional to the share of each country in the world economy such that $\alpha = \frac{E(Y)}{E(Y)+E(Y^*)}$. Since units are equal sized in the baseline model, this becomes (for now) equivalent to $N/(N+N^*)$.

Following Hassan (2013), the H bond pays out $\hat{P}_2 = P_1$ at $t = 2$, reflecting that bonds pay the exact number of H goods required to buy one unit of H's consumption bundle fixed at first-period prices. The F bond is similar, just that the value of the payment is fixed in terms of F goods. When expressed in terms of the numeraire (home goods) we have $\hat{P}_2^* = P_1/P_{F,1}$.

If H goods are scarce in period 2, P_2 is low and the H bond provides a larger real return (in terms of the consumption basket). The reverse effect is true if F is scarce,

$P_{F,2}$ is high relative to $P_{F,1}$, so the F bond pays higher real returns than expected. The main difference is that the larger the country, the larger the comovement of prices with marginal utility due to its size in the consumption basket. For example, an atomistic country's endowment being low still generates increases in its price, but this will not correlate with movements in the marginal utility of global investors. Therefore, the larger the share of a country's good in the consumption basket, the higher the insurance value of its bond. This is akin to Hassan (2013), although the mechanism there relied on tradable vs non-tradable goods.

Aggregate expenditure by investors in period 1 is given by

$$C_{H,1} + P_{F,1}C_{F,1} \equiv P_1 C_1 = (1-\tau)(Y_1 + Y_1^* P_{F,1}) - QB - Q^* B^* P_{F,1} \tag{7}$$

Investors can freely trade goods with each other, before buying bonds from both governments using the corresponding good.

In period 2 governments pay back their debt or choose to default. The investors budget constraint is:

$$C_{H,2} + P_{F,2}C_{F,2} = (1-\tau)(Y_2 + Y_2^* P_{F,2}) + BR + B^* R^* P_{F,2} \tag{8}$$

where

$$R = \begin{cases} \hat{P}_2 + R^I & \text{if no H default} \\ \kappa \hat{P}_2 & \text{if H defaults} \end{cases}, \quad R^* = \begin{cases} \hat{P}_2^* + R^{I*} & \text{if no F default} \\ \kappa \hat{P}_2^* & \text{if F defaults} \end{cases}$$

where R^I and R^{I*} indicate additional returns from investment opportunities that are further detailed in section 3.3.

Investors maximize lifetime utility (5) subject to the intratemporal aggregator (6), the period 1 budget constraint (7), and the state-dependent budget constraint in period 2 (8). Investors are price takers and do not internalize the aggregate impact of their choices on prices and default probabilities. The full investor optimization problem and the derivation of the first-order conditions are provided in Appendix B.3.

The optimal intratemporal decision at time t is characterized by

$$P_{F,t} = \left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{\eta}} \left(\frac{C_{F,t}}{C_{H,t}}\right)^{-\frac{1}{\eta}},$$

which determines the relative price of good F as a function of relative consumption and the trade elasticity,

The Euler equations give rise to an expression for the safe asset discount, which illustrates how the insurance aspect lowers the price for assets of a big country:

$$0 = \mathbb{E}\left[C_{H,2}^{-\frac{1}{\eta}} C_2^{\frac{1}{\eta}-\sigma}\left(\frac{R}{Q} - \frac{P_{F,2}R^*}{P_{F,1}Q^*}\right)\right]$$

If there is a shortage of home goods, then P_{F2} decreases and conversely a shortage of foreign goods leads to higher $P_{F,2}$. In that sense, it is generally desirable to hold the bonds of the good for which there is a shortage of. Moreover, the derivative of marginal utility is directly related to the good's share within the consumption basket. This means that holding bonds of the larger country is particularly valuable since the covariance between marginal utility and its return will be larger than for the smaller country. Whenever goods are perfectly substitutable there are no price movements to allow for a distinction in bond returns. If agents are risk neutral, there is no covariance of marginal utilities with prices either. Insurance therefore requires both imperfect substitution and investor risk-aversion. For realistic calibrations of η under log utility, this effect generates an insurance discount for the larger country as in Hassan (2013).[9]

3.3 Investment opportunities

We also assume that in the beginning of the second period, with probability χ, an investor i gains access to an investment opportunity in country j. This investment can be financed with country-j specific bonds *if and only if* country j is not in a default state. In that case, those with investment opportunities have the possibility to receive an additional return in terms of country j goods of R^+ on their (scalable) investment. The other mass of investors $(1-\chi)$ do not have direct access to these country-specific investment opportunities. However, they can lend their net worth to those that do. This lending procedure is subject to search and matching frictions. Financial intermediaries specialized in H country bonds collect $\mathcal{B} = (1-\chi)B$ bonds and match it to the mass of investors with investment opportunities $X = \chi(N + N^*)$. Following Chahrour and Valchev (2022),

[9] If goods are very complementary, this intuition breaks down and can even reverse. In the case $\eta = \sigma = 1$, the two effects exactly cancel out as in Cole and Obstfeld (1991), so for $\sigma = 1 > \eta$ it would actually be the smaller country who would earn that premium.

we assume the number of matches is given by the following matching function:

$$\mathcal{M} = m\frac{\mathcal{B}X}{\mathcal{B}+X}$$

where the parameter m represents velocity of asset holdings, reflecting the number of transactions a certain amount of bond holdings can sustain. It therefore also reflects the degree of financial development. The probability of a successful matching is then given by

$$\mu = \frac{\mathcal{M}}{X} = m\frac{\mathcal{B}}{\mathcal{B}+X}. \tag{9}$$

With the foreign matching being analogous, but a function of foreign debt B^* and foreign matching technology m^*. For a given matching technology, countries with a larger debt market $\mathcal{B}$ relative to the global economy will have a higher probability of generating a match. This is in line with the empirical evidence for higher liquidity in large economies presented in Section 2.1.

The match surplus R^+ is distributed via Nash bargaining, so a fraction ν goes to the borrower with investment opportunities while $1-\nu$ is allocated to lenders. The intermediaries are perfectly competitive and receive zero profits. Finally, investors have the possibility of absconding with a fraction of the borrowed funds. As in Kiyotaki and Moore (1997), the participation constraint introduces a borrowing limit. Given the standard nature of the friction, we do not model the full agent-principal problem and summarize it through a reduced form borrowing limit, which is a fraction ζ of the borrower's own funds b. Conditional on no default, the expected additional return R^I due to the existence of investment opportunities is:

$$E\left(R^I \,|\text{no default}\right) = \chi R^+ + \chi\mu\zeta\nu R^+ + (1-\chi)R^D \tag{10}$$

where R^D is the return earned from depositing funds with intermediaries.[10]. In this expression, the first term is the expected gain from getting an investment opportunity and investing his own funds. The second term reflects the gain from an investor with an investment opportunity if it matched with a lender. The last is the return that those without investment opportunities get from depositing their funds with financial intermediaries.

[10]Given intermediaries are perfectly competitive, this is simply the ratio of total fund earnings divided by the total deposited funds: $R^D = \frac{\mu\chi(1-\nu)\zeta R^+\mathcal{B}}{(1-\chi)\mathcal{B}}$.

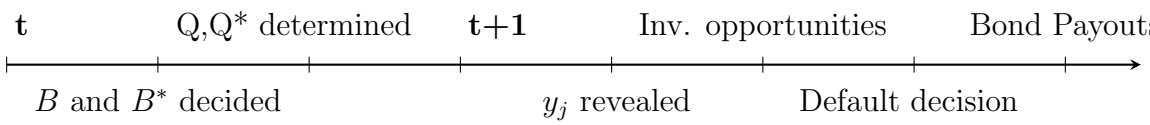
The unconditional expected return for each country is then:

$$E(R^I) = (1-p)\left(\chi R^+ + \chi\mu\zeta\nu R^+ + (1-\chi)R^D\right) \tag{11}$$

The main difference between additional returns of home and foreign bonds (per unit) lies in the default probability p, the matching probability μ and the good delivered by them (H vs F). The matching probability depends on the financial depth of the bonds market $\mathcal{B}$, which is a function of the current market value of the respective bond. As in Chahrour and Valchev (2022), the number of matches increases more than the increase in bond volume, making it therefore more likely that a borrower or a lender will find a match. However, safety also plays an important role. Only bonds that are not in a default state can be used to finance investment opportunities. Large and safer bond markets will therefore be privileged from the perspective that matching between borrowers and lenders is easier. This in turn generates larger expected returns for the corresponding asset. Lowering credit risk not only lowers risk premia, but also increases convenience yields.

3.4 Timing and equilibrium

Figure 3: Timeline of baseline model



The timing of the events is the following: The government chooses the supply of debt B, B^*. Investors then decide whether to invest in bonds or consume, with prices Q, Q^* clearing the bond market. Once the next period begins, unit-specific shocks y_j are revealed and it becomes public knowledge which government will default on their debt. After that each investors can have an investment opportunity with probability $1-\chi$ while the intermediation between those with and without investment opportunities takes place. The bonds are paid out (or defaulted upon) and finally endowments are received and agents consume their remaining wealth. An illustration of the timeline is shown in Figure 3.

The investors optimize their portfolio under this timeline, giving rise to first order conditions that shape their demand curve for bonds. The full set of optimization problems,

together with the budget constraint and the first order conditions, are described in more detail in Appendix B.3.

The equilibrium requires that the first order conditions of all investors and all governments are satisfied for a set of variables $\{C_H, C_F, P_F, B, B^*, Q, Q^*\}$. Bond markets clear, as well as goods markets according to the following conditions:

$$\mathcal{Y}_{Ht} = C_{Ht} + G \quad \text{and} \quad \mathcal{Y}_{Ft} = C_{Ft} + G^*$$

where $\mathcal{Y}_i$ is the stochastic output that includes potential gains from investment opportunities and losses from default. Furthermore, the budget constraints hold in all states.

3.5 Calibration

This section provides a calibration for key parameters of the model. Table 1 shows the parameters, the corresponding calibrated values and their description.

The upper part of the table calibrates the properties of the economy. We calibrate the bounds of TFP and the unit endowment process such that for $N = 50$ and $\rho = 0$, the largest country has an output volatility of 0.03 (target USA), while the smallest country N=2 has an output volatility of 0.12, approximately the largest volatility observed in Fig 2, panel a). Furthermore, we calibrate the output loss in the case of default to 8.5% of GDP. This number is motivated by the literature, that finds an average output loss of 1.099% per year (Trebesch and Zabel, 2017) and an average duration of 8 years (Uribe and Schmitt-Grohé, 2017). Together, these two numbers imply a net present value of the output losses (discounted at β) equal to around 8.5%. The average haircut for investors in the event of default is set to $\kappa = 0.46$, in line with the estimates of Graf von Luckner et al. (2025).

Table 1: Calibration

	Value	Description
Economy		
Y_1	1	Endowment in period 1
Y_2	$U[1-\frac{1}{3}, 1+\frac{1}{3}]$	Endowment process in period 2
std(Z)	0.052	Calibrated such that std($\mathcal{Y}$)=0.03
ρ	$\in [0,1]$	Unit shock correlation, baseline =0
N	50	Size of Home (dominant) country
N^*	$\in [1,...,50]$	Size of Foreign country
κ	0.46	Average haircut of 46%
ψ	0.085	Output loss of 8.5% if default
Investors		
σ	$\in \{0,1,1.5\}$	Risk aversion, baseline risk neutral
η	$\in \{3,\infty\}$	Trade elasticity, baseline =3
α	$\frac{E(Y)}{E(Y)+E(Y^*)}$	Consumption weights reflect relative size
β	0.96	Time discount factor
χ	0.33	Share investors with inv. opportunity
ν	0.8	Share investors in Nash bargaining
m	$\in \{2,8\}$	Matching efficiency, baseline =8 as in Chahrour and Valchev (2022)
ζ	0.5	Borrowing limit
R^+	0.05	Investment opportunity, additional 5% return
Government		
δ	0.5	Time discount factor, impatient government
τ	0.4	endowment taxation rate
θ	$\in [0,\infty)$	Unit smoothing parameter, baseline = ∞

Next, we calibrate parameters that govern the investors' behavior. For the baseline, we calibrate investors to be risk neutral. In Section 4.3, we relax the assumption of risk neutrality and consider investors with $\sigma > 0$. The trade elasticity in the baseline is set to 3, corresponding to some degree of substitutability between Home and Foreign goods. This is in line with empirical estimates by Feenstra et al. (2018) who find estimates between 2 and 4. We also consider the case with perfect substitution ($\eta = \infty$) that nests the one good setup. We make sure that the share parameter α in the consumption aggregator is in line with the share of the country in the global economy at the deterministic steady. The time discount factor for investors is set to $\beta = 0.96$, implying yearly real rates of 4% for safe assets without convenience yields. The government discount factor is set to $\delta = 0.5$, meaning it is very impatient and has a strong incentive to issue debt in the first period. We set $\tau = 0.4$, implying governments retain around 40% of the economy's endowment. We focus on the case with $\theta \to \infty$ and $\rho = 0$ which implies relatively large diversification effects of growing economies.

Regarding investment opportunities, we choose a value of χ=0.33, indicating that about one third of investors will have a direct investment opportunity. R^+ is calibrated such that investment opportunities have a return of 5%, being in line with a 1pp credit spread between safe government bonds and relatively safe corporate bonds, as documented by Valenzuela (2016). At the same time, we calibrate the Nash bargaining parameter ν to 80% indicating that investors with opportunities retain the majority of additional surplus, while intermediaries receive only a small share. The parameter m which proxies financial market development or the average credit duration is set to 8, as in Chahrour and Valchev (2022). Together with χ at 33%, a zero default probability of bonds, and the extra lending via financial markets investors expect to have close to 2% more returns on their bonds. With these values, interest rate spreads are at 35bp between countries with similar safety but a 1:10 size differential, as in the risk averse case in Figure 11, panel b . This is in line with Diamond and Van Tassel (2021) who find an average convenience yield of 35 bp [11].

4 Results

In this section, we highlight how key economic variables evolve as a function of economic size. We first discuss what is necessary for size to generate a diversification effect that makes a country's bonds safe. Then, we demonstrate how one country can lose its exorbitant privilege of low interest rate, when another country grows in terms of relative size. Finally we disentangle the importance of the liquidity and insurance channels, and their connection to each other.

4.1 Growth and diversification

As shown in the empirical evidence in Section 2.1, larger countries generally exhibit lower GDP volatility and lower default probabilities. However, this pattern does not hold universally as the episode of the Spanish empire in 1600 indicates or the high output volatility of large commodity exporters nowadays. This subsection spells out conditions under which size also implies safety. The model can therefore account for cases in which large countries are relatively unsafe.

[11] Other literature use different ways to measure convenience yields, resulting in different estimates. For example Krishnamurthy and Vissing-Jorgensen (2012) find slightly higher average US convenience yields for liquidity of 46 basis points. Our calibration is therefore on the conservative side, but we note our model can create a large set of yield differentials depending on safety difference, risk aversion, trade elasticities and liquidity yields.

4.1.1 Elasticity of substitution θ

θ controls how the shock in one unit transmits to aggregate output. We consider a polar case of perfect substitutability and discuss features in intermediate cases that include Cobb Douglas and Leontief to understand how economic shocks influence the default decision.

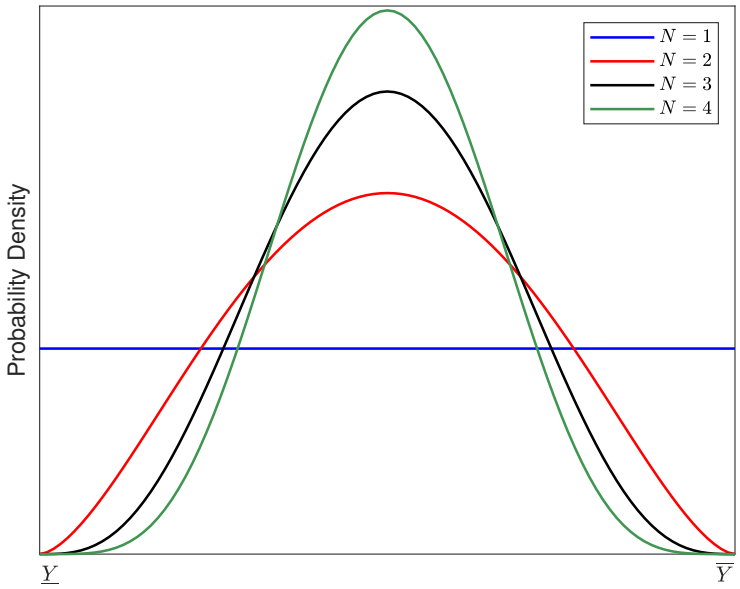
$\theta \to \infty$ (perfect substitutability) For $\theta \to \infty$, the CES aggregator sums up each unit. The government then defaults if the (weighted) average endowment $\bar{y}_2$ is lower than the haircut $1-\kappa$ on average repayment costs per unit, that is if

$$\psi\tau\bar{y}_2 < (1-\kappa)\frac{B\hat{P}_2}{N}.$$

where $\bar{y}_2 = \sum_{n=1}^{N} \omega_n y_{2n}$ is the weighted average nationwide output. $\bar{y}_2$ has a probability distribution that is a function of N, and the inequality is satisfied with a certain probability (the default probability p). Under equal weights, if y_{2n} is uniform and IID, the relevant distribution of the sum is an Irwin-Hall distribution.

As long as the unit processes are not perfectly correlated ($\rho \neq 1$), considering more units makes the overall variability of the left hand side smaller for $\theta \to \infty$. Therefore, the inequality that determines default is less likely to be satisfied with larger N.[12]

Figure 4: PDF of aggregate output for $\theta \to \infty$ and $\rho = 0$



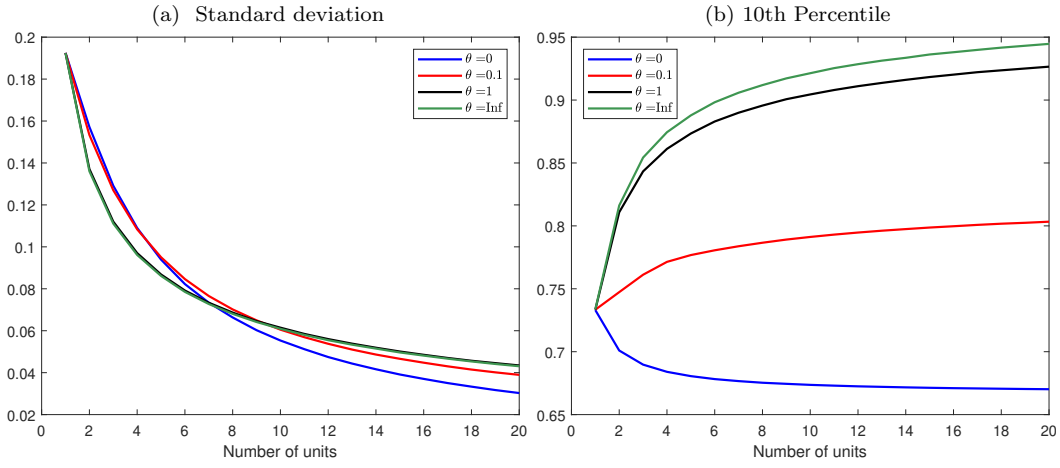
Note: Figure shows the PDF of $\bar{y}_2$ ($\theta \to \infty$). It is plotted for $N = \{1, 2, 3, 4\}$ where $y_{2n} \sim U(\underline{Y}, \overline{Y})$ and $\rho = 0$.

[12] As long as the probability of default is lower than 50%.

Figure 4 displays the probability distribution function of average national output for the polar case and varies the number of units between $N = 1$ (blue line) and $N = 4$ (green line). In the case of perfect substitutability ($\theta \to \infty$), the probability mass of the sum of uniforms increasingly centers around the mean while the tail mass shrinks substantially. The more units you have, the more likely it is that the average output is close to the mean. If mean output is large enough, this makes default less likely the more units there are.

$0 \leq \theta < \infty$ **(Cobb Douglas and intermediate)** For intermediate values, we can compute the probability mass of the tails to assess the extent countries are able to diversify. Lower values of θ indicate some degree of complementarity between units. This implies that the mean output decreases with smaller θ and the standard deviation of output decreases with a rising number of units. Overall, there is a diversification effect of having more units, even for small θ as there is less and less mass at the tail, as indicated in Figure 5. This is true for the case of Cobb Douglas ($\theta = 1$), as well as for small values such as $\theta = 0.1$. Only if production is very complementary, such as the Leontief case with $\theta = 0$, the mass concentrates increasingly on the left which indicates rising tail risks.

Figure 5: Average output properties as a function of size



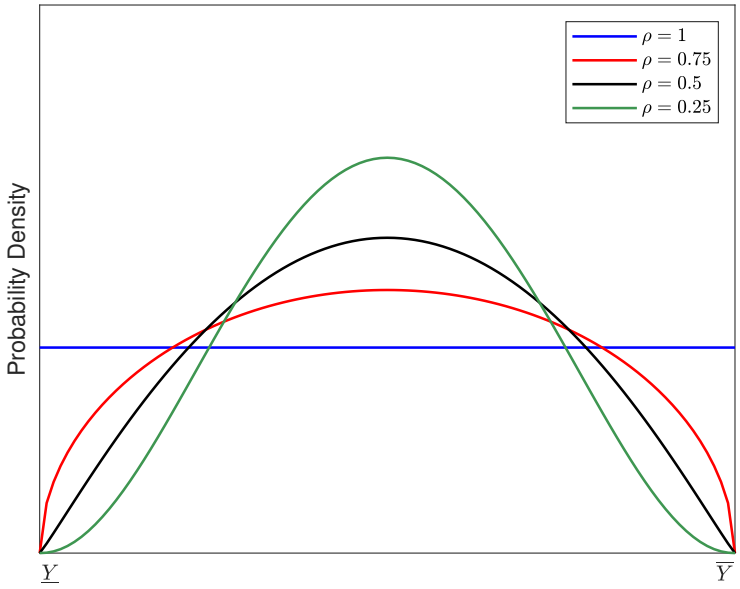
Note: Graph (a) shows the standard deviation, while (b) computes the 10th percentile of average output as a function of the number of units, for different values of θ. The shock process is $U(\underline{Y}.\overline{Y})$ and $\rho = 0$.

4.1.2 Cross-sectional output correlation ρ

Shock correlation affects the impact of size on diversification but also introduces limits to the degree of diversification that size can achieve. With positive correlation, economies with infinite units will still feature aggregate risk. This correlation could be interpreted

as a measure of production complementarities, common components or simply reflect a measure for the diversity across economic units in a particular country.

Figure 6: PDF of average aggregate output, N=4



Note: Figure shows the PDF of $\bar{Y}_2$ ($\theta \to \infty$). It is plotted for $N = 4$ where $y_{2j} \sim U(\underline{Y}, \overline{Y})$ with correlation ρ across units.

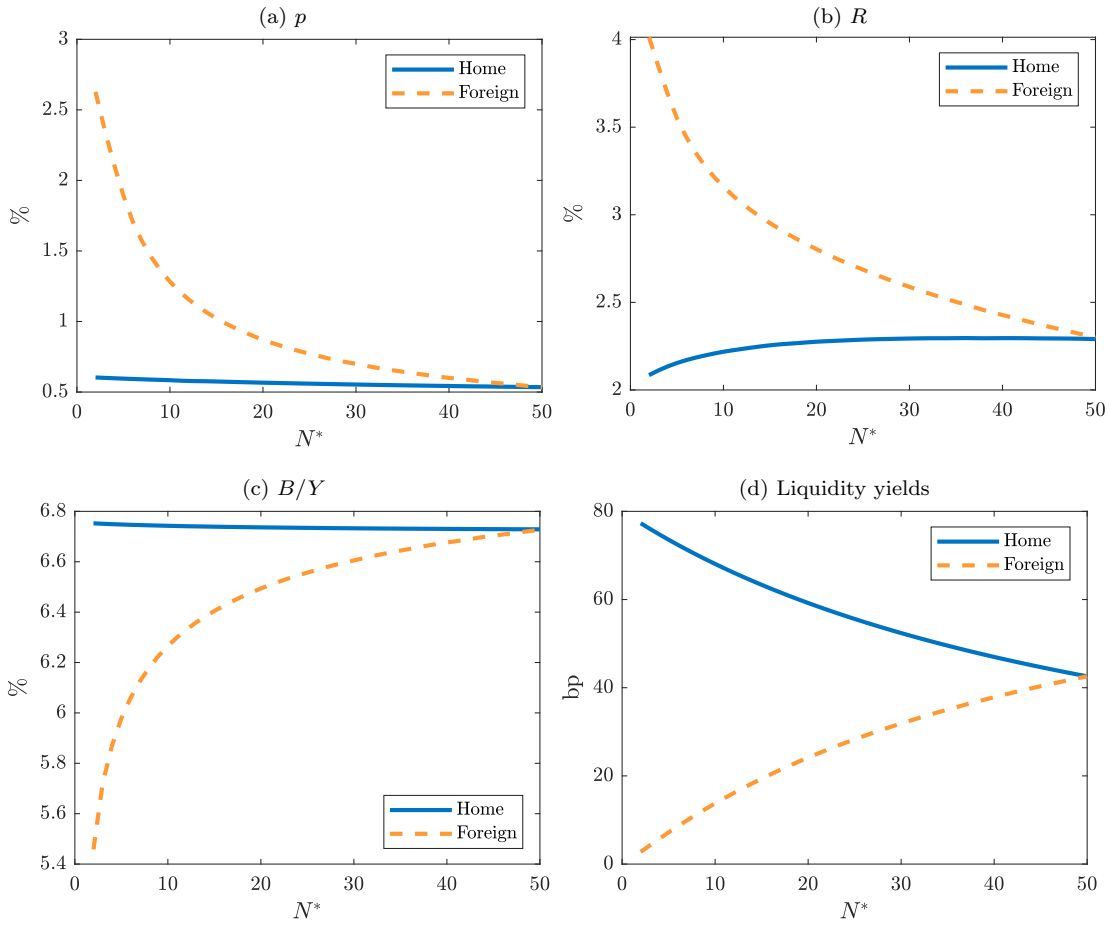
The higher the correlation across units, the closer the distribution gets to the original uniform distribution. In the extreme case of perfect correlation (the horizontal blue line), adding more units does not alter the aggregate distribution, as all units draw the same shock. As a consequence, the lower the correlation across units, the more diversified an economy gets and the less likely will be a default of the government. In a sense, an *optimal debt area* requires low (or even negative) correlation of shocks across units, as large nationwide recessions become less likely the more diversified the country is. Indeed, the literature has emphasized the importance of tail risks, by observing that interest rate spikes and sovereign default usually coincide with heavy recessions, see for example Arellano (2008), Neumeyer and Perri (2005) and Uribe and Yue (2006). In our setup more units with low shock correlation create safer and less variable fundamentals for the nationwide government, increasing the debt capacity of the country.

4.2 Privilege Lost: Challenger size and vanishing bond spreads

Next, we consider an initial hegemon country H that enjoys significantly lower interest rates relative to another growing country F. We set initially $N^* = 2$, and $N = 50$, so that there is an important size difference between the two countries, and then explore what would happen if F was closer or equal in size to the H country. We first highlight the case

of risk neutral investors and later relax that assumption. Figure 7, displays the values for default probability, interest rates[13], Debt-to-GDP ratio and liquidity yields as a function of F's size N^* under perfect substitution across economic units ($\theta = \infty$).

Figure 7: Main variables under perfect substitution



Note: Values of key variables for Home (solid blue) and Foreign (dashed orange) as a function of N^*, with risk neutral investors, $\eta = 3$, $R^+ = 5\%$, $\rho = 0$, $\theta = \infty$.

In panel (a), we see that probability of default falls for F as N^* grows, while the debt-to-GDP ratio rises substantially in (c). As explained before, when country F grows, its debt capacity increases as well and governments use this opportunity to raise additional funds. F's probability of default falls as it is optimal for the government to benefit both in terms of quantities and prices, from the higher debt capacity. Eventually, when countries are exactly identical, the privilege is no longer there, and all variables equalize.

For the original dominant country, it is important to understand what happens to liquidity yields. We depict liquidity yields as a function of N^* in Figure 7 (d). These yields are defined as the expected gain from the ability to fund investment opportunities

[13]For exposition purposes, we present the results in terms of yields/interest rates expressed in terms of the Home good $R = 1/Q - 1$ and $R^* = 1/(P_{F1}Q^*/\mathbb{E}P_{F2}) - 1$.

via the search and matching market:

$$R^{liq} = (1-p)\left(\chi\mu\zeta\nu R^{+} + (1-\chi)R^{D}\right) \tag{12}$$

and analogously for country F, just with p^*, μ^* and $\mathcal{B}^*$. The expression includes elements from equation (11) without the direct gains from investment opportunities. As discussed in the previous section, this expression is a function of liquidity (the matching probabilities μ) and the probability of default p. As F becomes safer, the fall in the probability of default p^* improves convenience yields, but also liquidity μ^* improves as the country (and its debt market depth $\mathcal{B}$) grows. At the same time, the dominant country share of the world economy is falling, not only is it eroding its *relative* advantage with respect to F but also reducing the *absolute* value of liquidity yields.

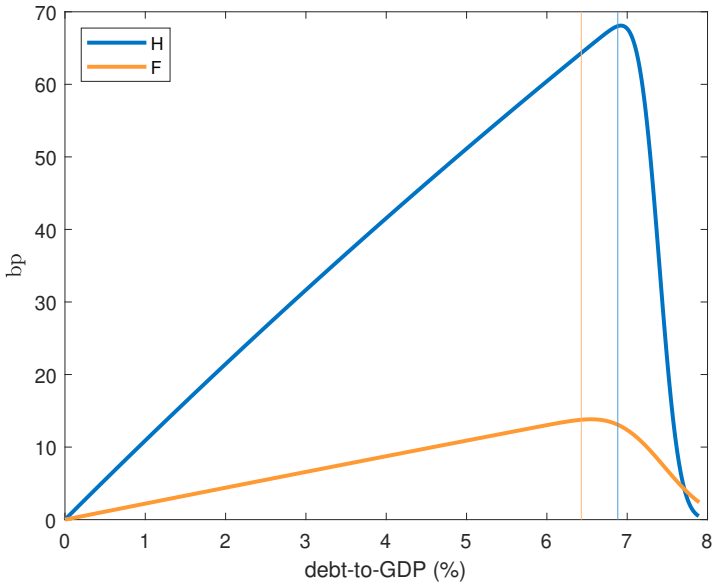
The dominant currency country therefore faces a dilemma. On the one hand, the more of its bonds are held and traded across the world, the better its liquidity properties (absent default), but on the other hand it needs to be prudent in bond emissions so that its debt remains safe and therefore useful as a financing currency[14]. For the smaller country, relative growth improves both dimensions. It increases debt capacity, and with it better safety and liquidity services. Figure 8 illustrates the liquidity *Laffer*-style curve that governments face for the case with $N = 50$ and $N^* = 10$. We plot expected liquidity returns from equation (12), as a function of each country's possible choices of debt-to-GDP, assuming the other country remains at their optimal choice. Liquidity returns increase for both countries initially, but much more so for H. This reflects not only the larger debt capacity but also the relative differences in total debt supply as a proportion of the world economy. Liquidity gains are also quickly eroded once probabilities of default begin to lift-off, leading to a sharp drop after the peak. Both countries are on the left side of the graph, while the bigger country is closer to the peak, reflecting the higher risk-aversion of governments relative to investors.

For the original anchor, convenience yields reduce when the other country grows, which forces a reduction in the debt-to-GDP ratio. It is optimal for H to make its debt safer as its dominant status erodes, while interest rates increase nevertheless due to lower convenience yields.

As a next step, we disentangle the spreads between the Home and Foreign country. Given the model outlined above, there are three sources of spreads: First, credit risk in the form of default probability. If one country is more likely to default than the other,

[14]This reflects elements of the traditional Triffin dilemma, Triffin (1961).

Figure 8: Debt and liquidity



Note: Liquidity yields from equation (12) as a function of debt to GDP. H has size $N = 50$, F has $N^* = 10$. Vertical lines indicate optimal debt-to-GDP in equilibrium. $R^+ = 5\%, \eta = 3$.

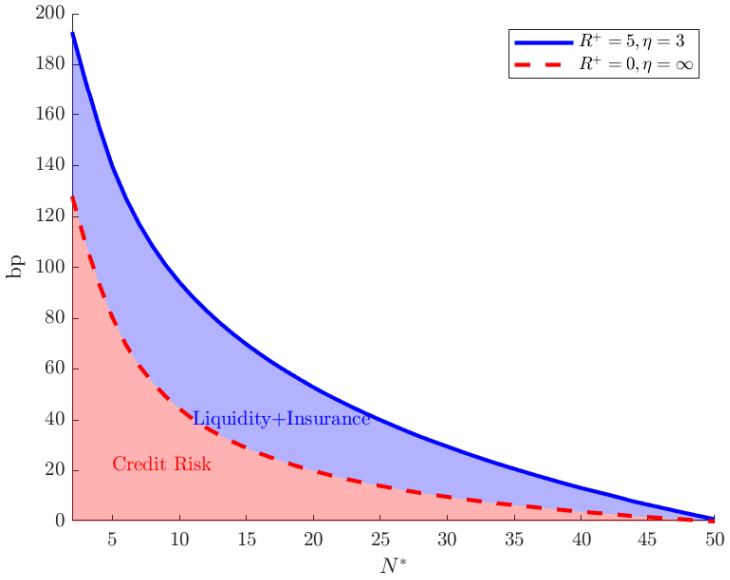
investors want higher interest rates. Second, there is the liquidity component arising from the ability to finance investment opportunities, as explained above. The last source of spreads is the insurance aspect. As bonds pay out in country-specific goods, and the price of these goods fluctuates with the global state of the economy, the value of each bond depends on the state of the world. Negative shocks in a country make its good more valuable, ceteris paribus. Provided there is no default, these payouts are more valuable as well, since it is also a state of low aggregate consumption C_2. The larger the country, the larger the drop in aggregate consumption due to its weight in the consumption basket. Larger countries will therefore have a larger insurance value, since (on average) their real return is higher in states of low aggregate consumption.[15]

There are two parameters that control the liquidity and insurance channels. R^+ determines the additional returns on investment opportunities. If set to zero, the liquidity channel is completely eliminated. For insurance, $\eta = \infty$ implies that both goods are perfectly substitutable, this means that the price of both goods is the same and the unit value of bond payouts in full repayment states is the same. Figure 9 shows the spreads between both countries in our baseline and in a model without the liquidity and insurance channels.

The initial spreads in the baseline are at around 190 basis points, with credit risk being the largest component. As the small country grows, it diversifies its economy and

[15]It is more intuitive to think about an infinitesimally small country. A strong recession in that country will raise the price of its own good, but it will not move the aggregate consumption basket of investors and therefore has little insurance value for them.

Figure 9: Spreads



Note: Spreads ($R^* - R$) between Foreign and Home country. The solid blue line indicates the case for, $\eta = 3$, $R^+ = 5\%$, while the dashed red line shows the case without liquidity and insurance, $\eta = \infty$, $R^+ = 0$.

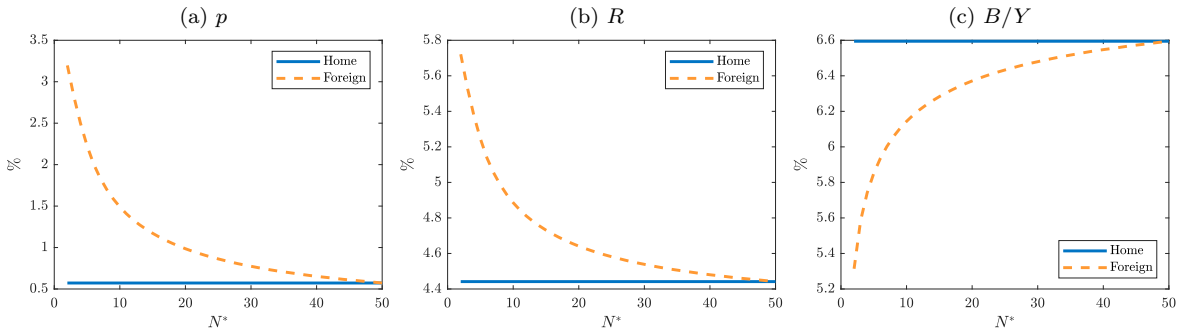
the increase in debt capacity reduces credit risk rapidly, as can be seen by the convex relationship between the probability of default and N^* (Fig 9). This is despite the rise in debt-to-GDP (Fig 7, panel c), as F takes advantage of its higher debt capacity. Liquidity yields reduce more progressively, with the convergence being still convex but much closer to linear in N^* (Fig 7, panel d). As N^* approaches N, the spread becomes mostly driven by the liquidity and insurance components.

We zoom in on the case where only credit risk matters, with $R^+ = 0$ and $\eta = 3$ in Figure 10. H is relatively insulated with respect to N^*. With risk-neutral investors, the Home country's debt, default probability and interest rates do not depend on N^*, since neither the insurance or the liquidity channel are present. At the same time, the larger size of F helps it diversify, reducing credit risk and interest rates, with spreads eventually becoming zero due to the countries being identical when $N^* = N$.

How important is liquidity relative to insurance? To answer this, we need a setup in which the insurance channel is properly isolated. There is however a crucial connection between safety and insurance, as only bonds that do not default in a bad state of the economy can fully reap the benefits from the insurance channel. At the same time it is true that default is especially likely in bad states. Moreover, changing the number of units alters the volatility of F. This implies that the share of F on global volatility does not necessarily increase as it grows.

Because safety and insurance are interconnected in an important way, we show another

Figure 10: Main variables without liquidity and insurance specification



Note: Steady state values of key variables for Home (solid blue) and Foreign (dashed orange) as a function of N^*, with risk neutral investors, $\eta = \infty$ and $R^+ = 0\%$.

setup in the next subsection in which we abstract from the credit risk channel to better isolate the insurance channel.

4.3 Insurance and liquidity in small and safe countries

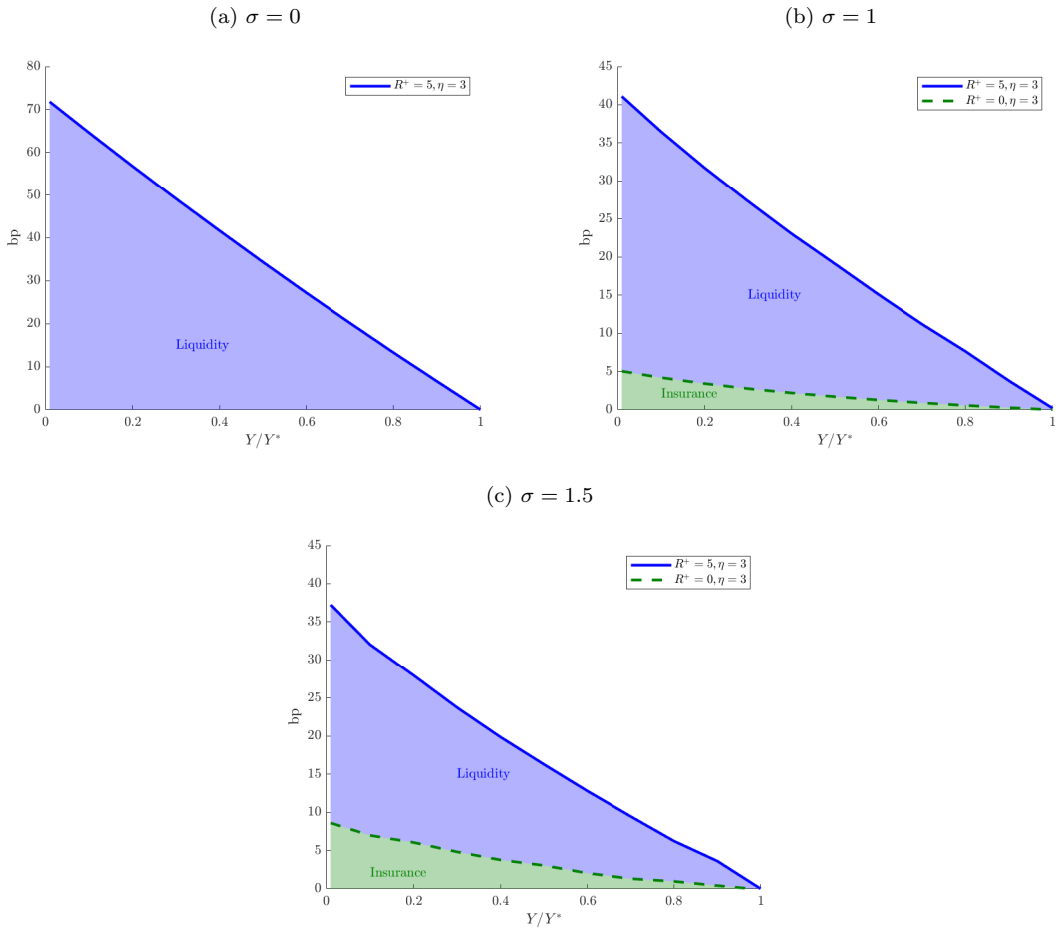
In this subsection, we turn off the credit risk channel by enabling countries to commit to repayment and consider countries that are small and diversified. We do so by setting the number of economic units in both countries to be equal, but allowing variation in the size of each unit. We also ensure that the consumption basket share α remains consistent with the country's global GDP share.

This implies that both countries have the same level of diversification and would choose the same debt-to-GDP if size was irrelevant. However, this will not be the case due to liquidity differences in their bond supply and also due to the insurance effect described in the previous section.

We focus on the interest rate spread as we increase the risk aversion of investors. In all cases, we set the trade elasticity to $\eta = 3$ again. The first graph plots the spread with risk neutral ($\sigma = 0$) investors. We then move to more risk-averse investors with $\sigma = 1$ and $\sigma = 1.5$.

The entire spread differential can be explained by liquidity yields when investors are risk neutral, as there is no demand for insurance. With risk neutrality, globally bad states that originate from recessions in the large country are not weighted more in the Euler equation. The absence of this hedging motive with risk neutral investors implies no insurance discount when $\sigma = 0$. This changes as soon as there is some risk aversion, as high marginal utility states and price movements weigh payoffs from the larger country more. With $\sigma = 1$ insurance yields are 5 basis points and rise to almost 10 bp when $\sigma = 1.5$. Note

Figure 11: Spreads with risk averse investors



Note: Spreads ($R^* - R$) between Foreign and Home country. The solid blue line indicates the case for, $\eta = 3$, $R^+ = 5\%$, while the dashed green line shows the case without liquidity and with insurance only, $\eta = 3$, $R^+ = 0$. Investors are risk neutral in panel (a) risk averse with $\sigma = 1$ in panel (b), and even more risk averse with $\sigma = 1.5$ in panel (c). There are two output states Y_{low} and Y_{high} for both countries.

that $\sigma > 0$ also leads to non infinite IES, which makes investors less willing to tilt their consumption towards *t+1* too much. This reduces total debt supply, and explains the lower liquidity yields as σ grows. Additionally the uncertainty in idiosyncratic liquidity payoffs reduces the value of liquidity returns. The more risk averse investors are, the lower debt capacity and the more important is the insurance motive. So the share of convenience yields explained by the insurance channel grows with investor risk-aversion. Nevertheless, for a given investor risk-aversion, the insurance channel actually reinforces the liquidity advantage of the larger country. Since liquidity payouts of H are also in H goods, they are more valuable in the bad state, as they increase the availability of scarce goods for bond holders. The insurance status therefore reinforces the liquidity channel.

Gourinchas and Rey (2022) characterize the position of the US as an insurer for global investors: While the US enjoys in normal times an "exorbitant privilege" in form of higher returns on its asset position, this privilege turns into a duty in times of a global crisis, in

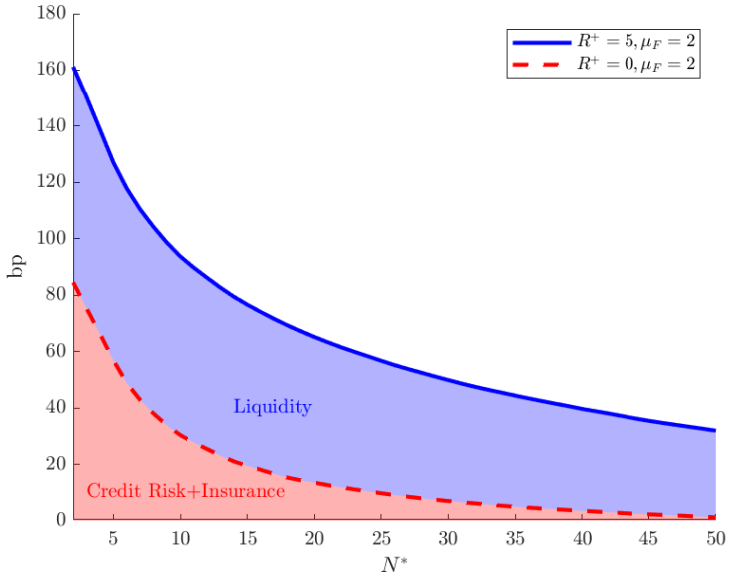
which the US effectively transfers wealth towards the rest of the world. In our model, the dominant country also plays that role, through the insurance channel of Hassan (2013).

Although Gourinchas and Rey (2022) formulate the notion of privilege and duty through the heterogeneous portfolio composition of the US and the rest of the world, even without those elements the hegemon assumes the role of global insurer, since on average the real return on its bonds will be higher in negative global states due to currency appreciation. This leads ex-ante to a lower price of its bonds, but it is accompanied by a transfer of wealth to foreign bond holders during crises.

4.4 Size and financial development

Next we consider an asymmetric setup in which one country is less financially developed than another. We implement this by setting a lower value to the asset velocity parameter m^* of the foreign version of equation (9). We set $m^* = 2 < m = 8$. This implies that the matching process in F is less efficient. For example, the matching probability for investors is 4 times higher in the large financially developed country, if both had the same size and debt-to-GDP. We go back to the setup of Section 4.2 with credit risk, risk-neutral investors, and varying numbers of units. Similarly to before, we look at the spread as F grows, turning the liquidity channel on in the baseline with $R^+ = 5\%$ and off with $R^+ = 0\%$.

Figure 12: Spreads with heterogeneous financial development



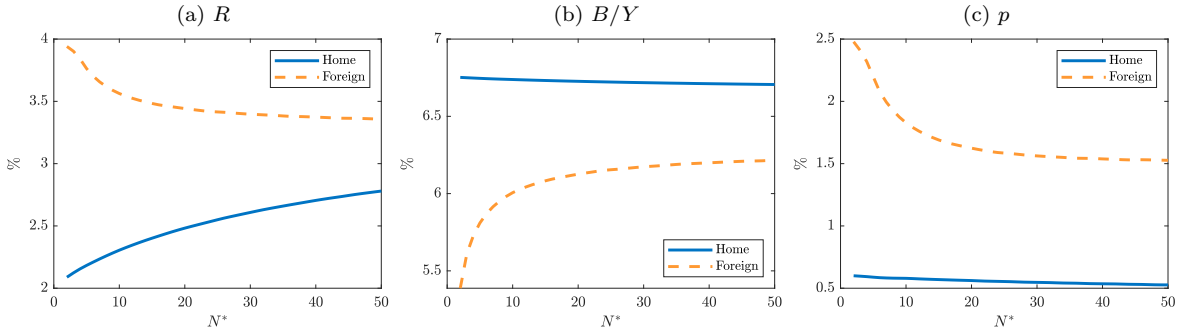
Note: Spreads $(R^* - R)$ between Foreign and Home country. The solid blue line indicates the case for, $\eta = 3$, $R^+ = 5\%$, while the dashed red line shows the case without liquidity but with insurance, $\eta = 3$, $R^+ = 0$. The credit velocity parameter m is set to 2 for F and 8 for H.

In such a setup, spreads of 40 basis points persist even if one country overtakes the other in terms of GDP size. While the financially developed country enjoys additional liquidity yields, its debt to GDP ratio is slightly higher. This means that the dominant country is more leveraged and slightly more risky than the financially less developed country. Spreads are almost entirely explained by liquidity yields, while the risk component plays a less meaningful role here. Additional figures for this case are in the Appendix Figure A2.

4.5 Lack of diversification

If one country has a higher correlation of shocks between units than another, it diversifies its output less when it grows. This is what we consider in the setup here, where H has a correlation of 0 as before, while F has a correlation of 0.1. A slightly higher shock correlation leads to a substantially less diversified tax base. This results in a default probability for the Foreign country of 1.5%, if it has the same size as the Home country which only has a 0.5% default probability. As a result, the debt capacity of the Home country is larger, while having lower interest rates at the same time.

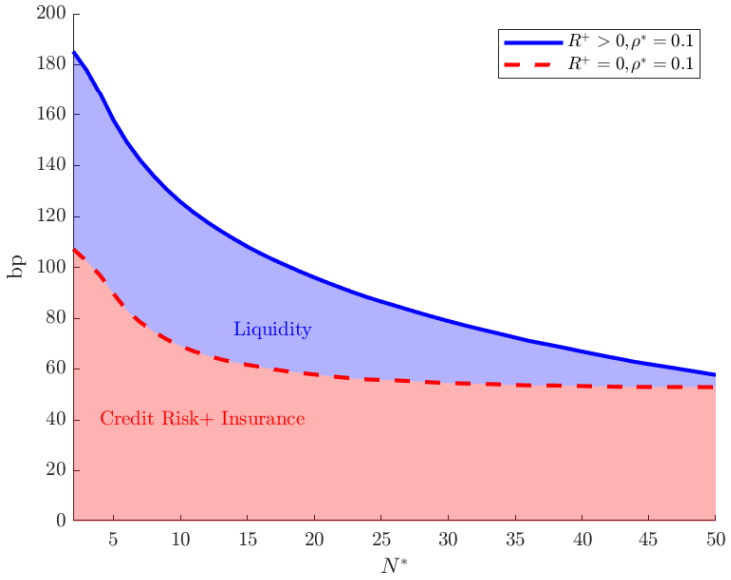
Figure 13: Variables with different diversification



Note: Steady state values of key variables for Home (solid blue) and Foreign (dashed orange) as a function of size with risk neutral investors, $\sigma = 0$, $\eta = 3$, $R^+ = 5\%$ and $\rho_F = 0.1$.

When considering the spread decomposition as before, the credit (and insurance) component are more prominent, determining the spreads even when they are equally large. The liquidity component is still responsible for a small amount of spreads, as H's debt market is larger.

Figure 14: Spreads with heterogeneous shock correlation



Note: Spreads $(R^* - R)$ between Foreign and Home country. The solid blue line indicates the case for, $\eta = 3$, $R^+ = 5\%$, $\rho_F = 0.1, \rho_H = 0$ while the dashed red line shows the case without liquidity but with insurance and risk, $\eta = 3$, $R^+ = 0$.

5 Conclusion

We develop a model of the international financial system in which governments can default on their outstanding debt. In this paper, we focus on the role of economic size, and how it interacts with three key aspects of a dominant currency: safety, liquidity and insurance. We highlight that acquiring the exorbitant privilege does not depend on only one of these aspects, but on the interaction between all three characteristics. Safety, liquidity and insurance are always in comparison relative to other countries, and are crucially a function of size. While Switzerland is indeed very safe, its smaller size implies that its bond market is less liquid and the country as a whole struggles to provide sufficient insurance for the rest of the world. The Euro area in contrast is large, but its bond market is too fragmented to obtain a comparable status to the US. China is large too, but not yet sufficiently financially developed for global investors.

The position of the United States can nevertheless erode if the global economy grows more than the US. This leads to a gradual decrease in the liquidity and insurance properties of US bonds, and therefore lowers interest rate spreads. If there is another country becoming larger (and potentially safer) its bonds gradually receive convenience yields as well at the expense of the incumbent. Our model can explain how global interest rates on public debt declined while debt-to-GDP ratios were rising and US convenience yields were falling.

Finally, we show that the more efficient the country's financial markets are, the larger the convenience yields for a given bond supply. Large and safe countries might nevertheless find it hard to challenge the dominant currency if they do not match the financial development of the hegemon.

References

Arellano, C. (2008). Default risk and income fluctuations in emerging economies. *American economic review*, 98(3):690–712.

Asonuma, T. and Trebesch, C. (2016). Sovereign debt restructurings: Preemptive or post-default. *Journal of the European Economic Association*, 15(1):175–214.

Atkeson, A., Heathcote, J., and Perri, F. (2025). The end of privilege: A reexamination of the net foreign asset position of the united states. *American Economic Review*, 115(7):2151–2206.

Bianchi, J., Bigio, S., and Engel, C. (2023). Scrambling for dollars: International liquidity, banks and exchange rates. Technical report, NBER Working Paper.

Bolt, J., Inklaar, R., De Jong, H., and Van Zanden, J. L. (2018). Rebasing 'maddison': new income comparisons and the shape of long-run economic development. *GGDC Research Memorandum*, 174:1–67.

Brunnermeier, M. K., Merkel, S., and Sannikov, Y. (2024). Safe assets. *Journal of Political Economy*, 132(11):3603–3657.

Chahrour, R. and Valchev, R. (2022). Trade finance and the durability of the dollar. *The Review of Economic Studies*, 89(4):1873–1910.

Chen, Z., Jiang, Z., Lustig, H., Van Nieuwerburgh, S., and Xiaolan, M. Z. (2022). Exorbitant privilege gained and lost: Fiscal implications. Technical report, National Bureau of Economic Research.

Choi, J., Kirpalani, R., and Perez, D. J. (2022). The macroeconomic implications of us market power in safe assets. Technical report, National Bureau of Economic Research.

Clayton, C., Dos Santos, A., Maggiori, M., and Schreger, J. (2025). Internationalizing like china. *American Economic Review*, 115(3):864–902.

Cole, H. L. and Obstfeld, M. (1991). Commodity trade and international risk sharing: How much do financial markets matter? *Journal of Monetary Economics*, 28(1):3–24.

Coppola, A., Krishnamurthy, A., and Xu, C. (2023). Liquidity, debt denomination, and currency dominance. Nber working paper 30984.

Di Giovanni, J. and Levchenko, A. A. (2012). Country size, international trade, and aggregate fluctuations in granular economies. *Journal of Political Economy*, 120(6):1083–1132.

Diamond, W. and Van Tassel, P. (2021). Risk-free rates and convenience yields around the world. *Federal Reserve Bank of New York Staff Reports, no. 1032*.

Doepke, M. and Schneider, M. (2017). Money as a unit of account. *Econometrica*, 85(5):1537–1574.

Du, W., Im, J., and Schreger, J. (2018). The u.s. treasury premium. *Journal of International Economics*, 112:167–181.

Eichengreen, B., Mehl, A., and Chitu, L. (2017). How global currencies work. In *How Global Currencies Work*. Princeton University Press.

Farhi, E. and Maggiori, M. (2018). A model of the international monetary system. *The Quarterly Journal of Economics*, 133(1):295–355.

Feenstra, R. C., Luck, P., Obstfeld, M., and Russ, K. N. (2018). In search of the armington elasticity. *Review of Economics and Statistics*, 100(1):135–150.

Gourinchas, P.-O. and Rey, H. (2007). From World Banker to World Venture Capitalist: US External Adjustment and the Exorbitant Privilege. In *G7 Current Account Imbalances: Sustainability and Adjustment*, NBER Chapters, pages 11–66. National Bureau of Economic Research, Inc.

Gourinchas, P.-O. and Rey, H. (2022). Exorbitant Privilege and Exorbitant Duty. CEPR Discussion Papers 16944, C.E.P.R. Discussion Papers.

Graf von Luckner, C. M., Meyer, J., Reinhart, C. M., and Trebesch, C. (2025). Sovereign haircuts: 200 years of creditor losses. *IMF Economic Review*, 73(1):150–195.

Hassan, T. A. (2013). Country size, currency unions, and international asset returns. *The Journal of Finance*, 68(6):2269–2308.

He, Z., Krishnamurthy, A., and Milbradt, K. (2019). A model of safe asset determination. *American Economic Review*, 109(4):1230–1262.

Imbs, J. and Wacziarg, R. (2003). Stages of Diversification. *American Economic Review*, 93(1):63–86.

Jiang, Z., Krishnamurthy, A., and Lustig, H. (2021). Dollar safety and the global financial cycle. Technical report, Working Paper.

Jiang, Z., Richmond, R. J., and Zhang, T. (2022). A portfolio approach to global imbalances. Technical report, NBER Working Paper.

Jordà, Ò., Knoll, K., Kuvshinov, D., Schularick, M., and Taylor, A. M. (2019). The rate of return on everything, 1870–2015. *The Quarterly Journal of Economics*, 134(3):1225–1298.

Kiyotaki, N. and Moore, J. (1997). Credit cycles. *Journal of political economy*, 105(2):211–248.

Krishnamurthy, A. and Vissing-Jorgensen, A. (2012). The aggregate demand for treasury debt. *Journal of Political Economy*, 120(2):233–267.

Maggiori, M. (2017). Financial intermediation, international risk sharing, and reserve currencies. *American Economic Review*, 107(10):3038–71.

Neumeyer, P. A. and Perri, F. (2005). Business cycles in emerging economies: the role of interest rates. *Journal of monetary Economics*, 52(2):345–380.

Pflueger, C. E. and Yared, P. (2024). Global hegemony and exorbitant privilege. *University of Chicago, Becker Friedman Institute for Economics Working Paper*, (2024-95).

Reinhart, C. M., Rogoff, K. S., and Savastano, M. A. (2003). Debt intolerance. *Brookings Papers on Economic Activity*, 34(1):1–74.

Richmond, R. J. (2019). Trade network centrality and currency risk premia. *The Journal of Finance*, 74(3):1315–1361.

Trebesch, C. and Zabel, M. (2017). The output costs of hard and soft sovereign default. *European Economic Review*, 92:416–432.

Triffin, R. (1961). Gold and the dollar crisis: the future of convertibility. *Yale University Press.*

Uribe, M. and Schmitt-Grohé, S. (2017). *Open economy macroeconomics*. Princeton University Press.

Uribe, M. and Yue, V. Z. (2006). Country spreads and emerging countries: Who drives whom? *Journal of international Economics*, 69(1):6–36.

Valenzuela, P. (2016). Rollover risk and credit spreads: Evidence from international corporate bonds. *Review of Finance*, 20(2):631–661.

Vicquéry, R. (2022). The rise and fall of global currencies over two centuries. *Banque de France Working Paper Series*, 882.

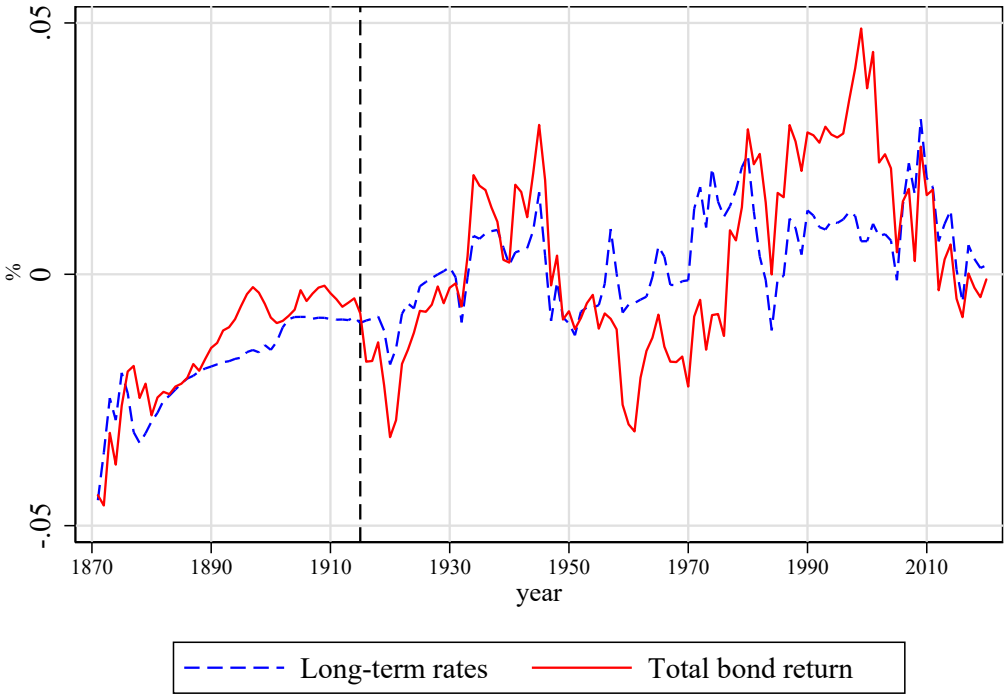
A Additional Tables and Figures

Table 2: UK-US long term interest spreads over time

Spread rate	**1870-1914**	**1915-2008**	**2009-2020**
Real long rate	-1.81%	0.13%	-0.24%
Real total bond return	-1.59%	0.17%	0.16%
USD long rate	-1.34%	1.29%	-0.11%
USD total bond returns	-1.13%	1.33%	0.07%
Real USD long rate	-1.36%	1.27%	-0.10%
Real USD total bond returns	-1.13%	1.32%	0.11%

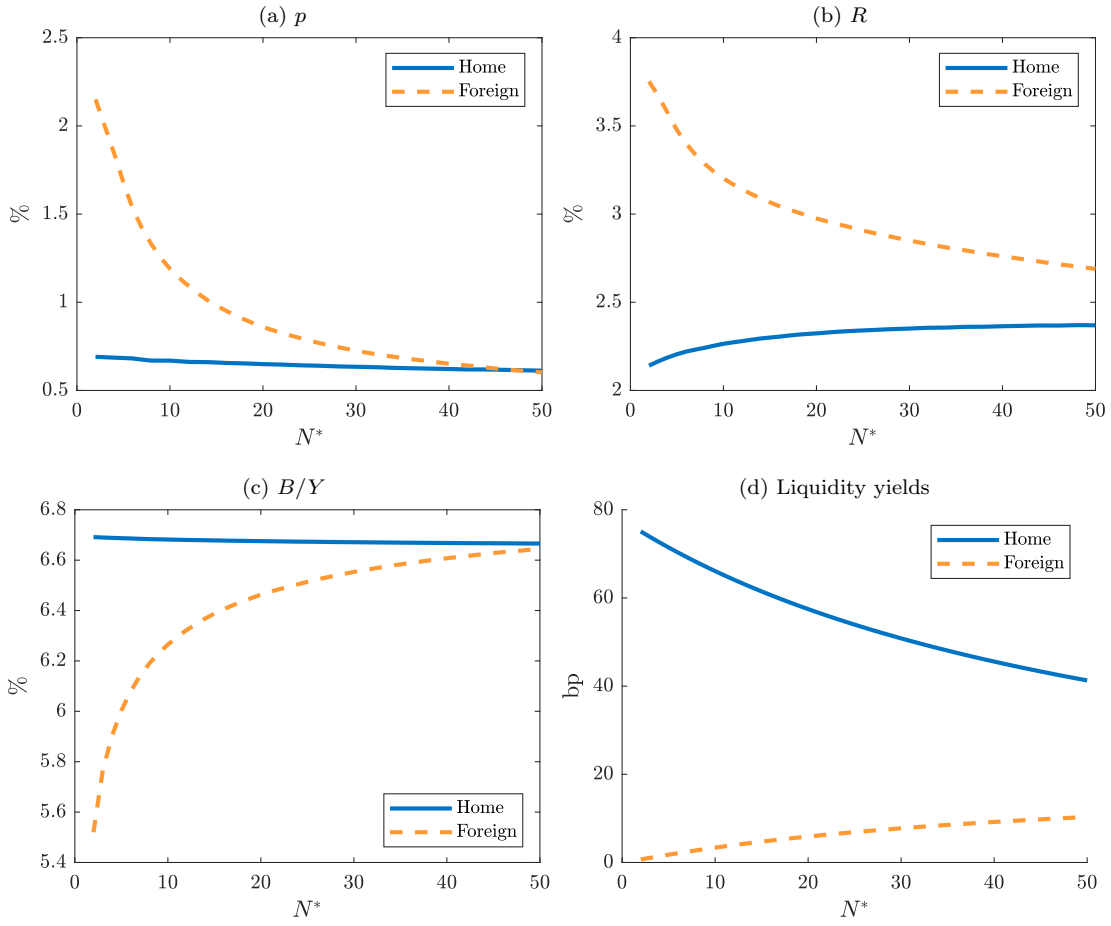
Note: Yearly average spreads of long term interest rates between the United Kingdom and the United States, excluding the World War periods of 1914-1919 and 1939-1947. *Source*: Jordà et al. (2019) and own calculations.

Figure A1: Interest Rate Spreads



Interest rates spreads in USD are calculated the following way: You invest one USD in UK bonds in Pounds to exchange rates in period t and receive the interest rate in period $t+1$ in Pound, then exchanging those back to USD the current exchange rate. This is compared to US long-term bond interest rate payments in US Dollar. Spreads are very volatile, we apply a quarter-centennial moving average to smooth out short-run fluctuations. *Source*: Jordà et al. (2019) and own calculations.

Figure A2: Main variables with heterogeneous financial development



Note: Values of key variables for Home (solid blue) and Foreign (dashed orange) as a function of N^*, with risk neutral investors, $\eta = 3$, $R^+ = 5\%$, $m = 8, m^* = 2$.

B Online Appendix: Model and derivations

B.1 Correlation across units

We introduce correlation across units by making use of Gaussian copulas. A Gaussian copula is a joint probability distribution function over the hypercube $[0,1]^N$ which can be mapped to the joint distribution of y_{2n} for $n = 1, .., N$. One particular feature of Gaussian copulas is that marginal probability distributions remain uniform and it can be fully specified by a simple correlation matrix Ω. For simplicity, we set the non-diagonal elements of Ω to be all equal to ρ, so we can characterize the copula with a single parameter.[16] Since marginal distributions are uniform, when $\rho = 0$ we have the same distribution as in the previous section, with each y_{2n} being drawn from a iid uniform.

B.2 Government: Intertemporal problem and default

The government problem is characterized by

$$\begin{aligned}
\max_{B} \quad & \log(G_1) + \delta\left[(1-\mathcal{P}(B))\mathbb{E}(\log G_2|repay) + \mathcal{P}(B)\mathbb{E}\left[\log G_2|default\right]\right] \\
\text{s.t.} \quad & G_1 = \tau Y_1 + \mathcal{Q}(B, B^*)B \\
& G_2 = \begin{cases} \tau Y_2 - B\hat{P}_2 & \text{if repay} \\ (1-\psi)\tau Y_2 - \kappa B\hat{P}_2 & \text{if default} \end{cases} \\
& Y_t = Z_t \left(\sum_{j=1}^{J} \omega_j^{\frac{1}{\theta}} y_{t,j}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}.
\end{aligned}$$

where Z is aggregate TFP. The default probability p is determined by the shock process for aggregate output:

for $\theta \to \infty$ a government defaults if aggregate output is sufficiently small:

$$\bar{Y}_2 \equiv \sum_{j=1}^{J} Y_{2j} < \frac{(1-\kappa)B\hat{P}_2}{\psi Z} \equiv \tilde{Y}_2^{\infty}.$$

[16] Since Ω is a correlation matrix, diagonal elements are all equal to 1, and we set all other elements to ρ. Note that ρ will generally be related, but not exactly equal, to the cross-sectional correlation, except in the cases of perfect ($\rho = 1$) or no ($\rho = 0$) correlation where the two coincide.

for a given density function f, the probability of default is then a function of B, Q and other parameters, $p = f(\bar{Y}_2 < \tilde{Y}_2^{\infty})$. If f in uniformly distributed as in the example of the main text, the sum of uniformly distributed variables follows an Irwin-Hall distribution.

for $\theta \to 1$

$$\tau Z \prod_{j=1}^{J} Y_{2j}^{\omega_j} - B\hat{P}_2 < \tau Z \prod_{j=1}^{J} (1-\psi) Y_{2j}^{\omega_j} - \kappa \hat{P}_2 B$$

implicit function which gives $\tilde{Y}_2^1$

for $\theta \to 0 \quad G_t \to \min g_{it}$ default if

$$\min_j \{Y_{2j}\} < \frac{(1-\kappa)B\hat{P}_2}{\psi Z} \equiv \tilde{Y}_2^0.$$

for iid uniforms between $[a, b]$ the density function of the minimum value is

$$F(y) = \begin{cases} 1 - \left(1 - \frac{y}{b-a}\right)^n & : y \in (a,b) \\ 0 & : y < a \\ 1 & : y > b \end{cases} \qquad f(y) = \begin{cases} \frac{n}{b-a}\left(1 - \frac{y}{b-a}\right)^{n-1} & : y \in (a,b) \\ 0 & : \text{ otherwise} \end{cases}$$

B.3 Investor problem

Investors in H maximize a CRRA utility function, as follows:

$$\max_{B,B^*,C_H,C_F,C} U = C_1^{1-\sigma} + \beta E_1\left(C_2^{1-\sigma}\right) \tag{13}$$

$$\text{s.t.} \quad [\mu]C = \left(\alpha^{\frac{1}{\eta}} C_H^{\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}} C_F^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \tag{14}$$

$$[\lambda_1] \qquad P_1 C_1 = (1-\tau)(Y_1 + P_{F,1} Y_1^*) - QB - Q^* P_{F1} B^* \tag{15}$$

$$[\lambda_2] \qquad P_2 C_2 = (1-\tau)(Y + P_{F,2} Y_2^*) + RB + R^* P_{F,2} B^* \tag{16}$$

$$R = \begin{cases} \hat{P}_2 + R^I & \text{if no default} \\ \kappa \hat{P}_2 & \text{if default} \end{cases} \tag{17}$$

$$R^* = \begin{cases} \hat{P}_2^* + R^{I*} & \text{if no default} \\ \kappa \hat{P}_2^* & \text{if default} \end{cases} \tag{18}$$

$$R^I = f(B, \Theta) \tag{19}$$

$$R^{I*} = f^*(B^*, \Theta) \tag{20}$$

where R are the state contingent payouts that include direct payments $\hat{P}_2$ with or without default and investment opportunities R^I. Θ is a vector including all parameters and states (idiosyncratic and aggregate). Idiosyncratic states include having an investment opportunity (or not), as well as matching (or not) in that case.

We take the derivative with respect to C_{H1}, C_{H2}, B to retrieve the Euler equation and the intratemporal conditions at each time t:

$$C_{H1}^{-\frac{1}{\eta}} C_1^{\frac{1}{\eta}-\sigma} = \beta \mathbb{E}\left[C_{H2,s}^{-\frac{1}{\eta}} C_{2,s}^{\frac{1}{\eta}-\sigma} \frac{R_s}{Q}\right]$$

$$\frac{P_{Ht}}{P_{Ft}} = \left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{\eta}} \left(\frac{C_{Ht}}{C_{Ft}}\right)^{-\frac{1}{\eta}}$$

Normalizing $P_{Ht} = 1$ as the numeraire we have:

$$P_{Ft} = \left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{\eta}} \left(\frac{C_{Ht}}{C_{Ft}}\right)^{\frac{1}{\eta}}$$

$$(P_{Ft})^{\eta} = \frac{1-\alpha}{\alpha} \frac{C_{Ht}}{C_{Ft}}$$

we then use the budget constraint in period 2 to get an expression for C_{H2} as a function of endowments and state dependent payouts and plug this into the previous equation to

get a price for traded goods.

Subtract the two Euler equations to get the safe asset discount:

$$0 = \mathbb{E}\left[(C_{H2}(s))^{-\frac{1}{\eta}} C_2(s)^{\frac{1}{\eta}-\sigma}\left(\frac{R_s}{Q} - \frac{P_{F2}R_s^*}{P_{F,1}Q^*}\right)\right]$$

B.4 Price index

Consumption is CES

$$C = \left(\alpha^{\frac{1}{\eta}} C_H^{\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}} C_F^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}$$

$$\max PC - P_H C_H - P_F C_F$$

The first order condition is given by:

$$C_H = \alpha\left(\frac{P_H}{P}\right)^{-\eta} C, \quad C_F = (1-\alpha)\left(\frac{P_F}{P}\right)^{-\eta} C$$

Plug into budget constraint:

$$PC = P_H^{1-\eta} P^\eta C\alpha + (1-\alpha) C P_F^{1-\eta} P^\eta$$

Solve for P:

$$P = \left(\alpha P_H^{1-\eta} + (1-\alpha) P_F^{1-\eta}\right)^{\frac{1}{1-\eta}}$$

B.5 Period 1 and 2 prices

We can also compute the price of period 1 as a function of endowments and asset trading. Removing the time indices momentarily for brevity, we depart from the following equations

in period 1:

$$
\begin{aligned}
C_H + G = Y, \quad C_F + G^* = Y^* \\
C_H = \alpha \left(\frac{P_H}{P}\right)^{-\eta} C, \quad C_F = (1-\alpha)\left(\frac{P_H}{P}\right)^{-\eta} C \\
G = \tau Y + QB, \quad G^* = \tau Y^* + Q^* B^* \\
PC = (1-\tau)P_H Y + (1-\tau)P_F Y^* - QP_H B - Q^* P_F B^*
\end{aligned}
$$

We begin by plugging in the demand function and the government budget constraint into the resource constraint

$$\alpha \left(\frac{P_H}{P}\right)^{-\eta} C + \tau Y + QB = Y,$$

Plug this expression in the investors budget constraint

$$\alpha \left(\frac{P_H}{P}\right)^{-\eta} \frac{(1-\tau)P_H Y + (1-\tau)P_F Y^* - QP_H B - Q^* P_F B^*}{P} + \tau P_H Y + QP_H B = P_H Y$$

Normalizing $P_H = 1$ and using the price index we have

$$\alpha \left(\frac{1}{\left(\alpha + (1-\alpha)P_F^{1-\eta}\right)^{\frac{1}{1-\eta}}}\right)^{-\eta} \frac{(1-\tau)Y + (1-\tau)P_F Y^* - QB - Q^* P_F B^*}{P} + \tau Y + QB = Y$$

We can now solve for P_F to obtain

$$P_{F1} = \left(\frac{(1-\alpha)[(1-\tau)Y_1 - QB]}{\alpha\left[(1-\tau)Y_1^* - Q^* B^*\right]}\right)^{\frac{1}{\eta}}$$

where we reinsert the time indices. We can solve analogously for P_{F2}, obtaining in case of no default

$$P_{F2} = \left(\frac{(1-\alpha)[(1-\psi)^{\mathbb{I}^d}(1-\tau)Y_2 + BR]}{\alpha\left[(1-\psi)^{\mathbb{I}^{d^*}}(1-\tau)Y_2^* + B^* R^*\right]}\right)^{\frac{1}{\eta}}$$

where $\mathbb{I}^d$ and $\mathbb{I}^{d^*}$ are indicator variables taking the value of 1 when H or F default, respectively, and 0 otherwise. Note that R and R^* also include possible haircuts and output generated from investment opportunities.

B.6 Pseudo-algorithm

1. Discretize the governments choice space $\mathbf{B}$ and $\mathbf{B}^*$
2. Guess a solution for each government $\tilde{B}$ and $\tilde{B}^*$
3. For every point in the choice space $\mathbf{B}$, recover $(Q, Q^*, P_{F,1})$ from the investors optimality conditions and the resource constraint, assuming $B^* = \tilde{B}^*$.

   $\rightarrow$ Compute $V(\mathbf{B})$ and select the optimal choice $B \in \mathbf{B}$ that maximizes it
4. Repeat the same process analogously for every point in $\mathbf{B}^*$, recovering optimal B^*
5. Compare optimal B and B^* with guess and update $\tilde{B}$ and $\tilde{B}^*$ accordingly
6. Repeat steps 3 to 5 until convergence