A Search Model of Financial Bubbles

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Abstract

Financial bubbles are characterized by two stylized facts: explosive asset prices and high, complex trade volumes. This paper endogenously generates both of these features within a rational framework by developing a search-and-matching model that provides a micro-foundation for the "ride-the-tide" behavior of speculators. We show how two main forces—the interaction of heterogeneous agents and the rational expectation of a future collapse risk—drive the explosive price path in our model. A key contribution is the model's empirical tractability, which allows for the structural estimation of unobservable fundamentals and a bubble's potential turning point from market data. While the core mechanisms are established, a formal proof of the dynamic Nash equilibrium is left for future work.

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1 Introduction

Financial bubbles are a persistent feature of market economies, yet a full understanding of their underlying mechanisms remains elusive. A central challenge is to explain their two most prominent empirical regularities: an explosive rise in asset prices followed by a crash, and a sustained period of abnormally high trade volume (Brunnermeier & Oehmke, 2013). While these two features consistently appear together empirically, simultaneously and endogenously generating both within a rational, micro-founded framework theoretically has proven to be a significant hurdle for economic theory.

Existing theoretical frameworks typically fall short of this goal. One strand of literature, often using general equilibrium models, can justify high asset prices as a rational outcome but generally fails to produce the explosive price paths seen in the data, and rarely addresses the issue of trade volume (e.g., Tirole, 1985; Martin & Ventura, 2012; Miao & Wang, 2014). Other models that do capture either explosive prices or high trade volumes often rely on restrictive assumptions, such as exogenous price processes or behavioral biases that are difficult to discipline (e.g., Abreu & Brunnermeier, 2003; Scheinkman & Xiong, 2003). Consequently, a gap persists in the literature for a unified model that can endogenously generate both of the core stylized facts of financial bubbles from the optimizing behavior of rational agents.

This paper fills this gap by building upon the literature that views bubbles as a rational, speculative phenomenon. We develop a search-and-matching model that provides a micro-foundation for the "ride-the-tide" behavior of speculators—the strategy of purchasing an overvalued asset with the expectation of selling it to another speculator at an even higher price in the future. Our key innovation is to model this behavior not as an assumption, but as an endogenous equilibrium outcome that arises from the interactions of heterogeneous agents in a market with trading frictions. This framework allows us to make two primary contributions. First, we endogenously generate an explosive price path that

is consistent with the stylized facts of bubbles. Second, as a direct result of the same underlying mechanism, our model also endogenously produces a sustained period of high and complex trade volume.

The ability of our model to endogenously generate an explosive price path is what makes our framework empirically tractable. Because real-world bubble data is inherently explosive, a model must be able to replicate this dynamic feature to be credibly taken to the data for structural estimation. Our framework is the first to do so based on the rational, micro-founded interactions of agents. This tractability opens up two significant avenues for the applied analysis of bubbles. First, it allows for the estimation of an asset's unobservable fundamental value, D, directly from market data. Second, the expected date of the bubble's turning point, $\bar{t}$, is also an estimable parameter, allowing our model to be used as a theoretically grounded early-warning system.

These results are made possible by the novel mechanism that our search-and-matching framework unlocks. The presence of heterogeneous agents and trading frictions allows us to explicitly model the micro-level transaction patterns that constitute "ride-the-tide" speculation. The explosive price path is then sustained by a backward induction process, where the known risk of future liquidation is priced into current transactions. We show that this explosive equilibrium is not a generic feature, but can only be sustained when the agents' outside investment options are sufficiently unattractive. It is precisely this rich, micro-founded mechanism that allows key unobservables like the fundamental value and the collapse date to be identified from the observable dynamics of prices and trade volumes.

The remainder of the paper is organized as follows. Section 2 establishes the key stylized facts of financial bubbles that motivate our analysis. Section 3 reviews the related literature and identifies the key gap that this paper aims to fill. Section 4 presents and solves our search-and-matching model of bubbles. Section 5 discusses the model's key

results and mechanisms through numerical simulation and counterfactual analysis. Section 6 elaborates on the empirical implications of our framework. Section 7 concludes.

2 Stylized Facts

Before discussing the stylized facts of financial bubbles, we must first acknowledge a foundational challenge: there is no settled definition or identification method for bubbles (Brunnermeier & Oehmke, 2013). A core element of most definitions is a deviation of the asset price from its fundamental value—typically, an overpricing. However, not every overpricing constitutes a bubble (Martin & Ventura, 2018). For an overpricing to be considered a bubble, it must be persistent, yet the required duration of this persistence remains ambiguous.

Historically, economists have used narrative descriptions for bubbles: prices boom, people rush in, prices peak, nerves build, prices crash, and panic spreads (Kindleberger, 1978). However, such narratives provide neither a precise definition nor a testable framework. Luckily, despite this ambiguity, several historical episodes are widely recognized as bubbles, allowing us to extract a set of stylized facts.

As summarized by Brunnermeier and Oehmke (2013), two key features characterize financial bubbles: abnormally high asset prices and abnormally high trade volumes. While these two features match the common intuition of a market "mania", their dynamic patterns are more complex than they first appear.

The abnormally high prices are often described as following an "explosive" path. Unlike in normal periods where asset prices may be volatile or grow steadily, bubble periods are characterized by a dramatic price boom followed by a sharp crash. To determine the shape

of the booming price path, Blanchard and Watson (1982) show that a rational bubble can exist if the asset price grows at an exponential rate, combined with a persistent risk of a sudden collapse. This exponential path, however, is just one theoretical possibility. When we examine price data from actual bubbles, we find that while the price paths are indeed explosive, they rarely follow a simple exponential pattern. The shape of an explosive price path in reality is often more complex.

Regarding trade volume, on average, trade volume is significantly higher during a bubble period than during non-bubble periods. Beyond this, however, the patterns are less clear. For example, the correlation between trade volume and asset price within a bubble period is not stable and remains empirically unclear. Furthermore, there is no consistent evidence on whether trade volume exhibits a clear trend during a bubble. The underlying drivers of these high and complex volume dynamics remain an open question.

The following examples illustrate these two complex features.

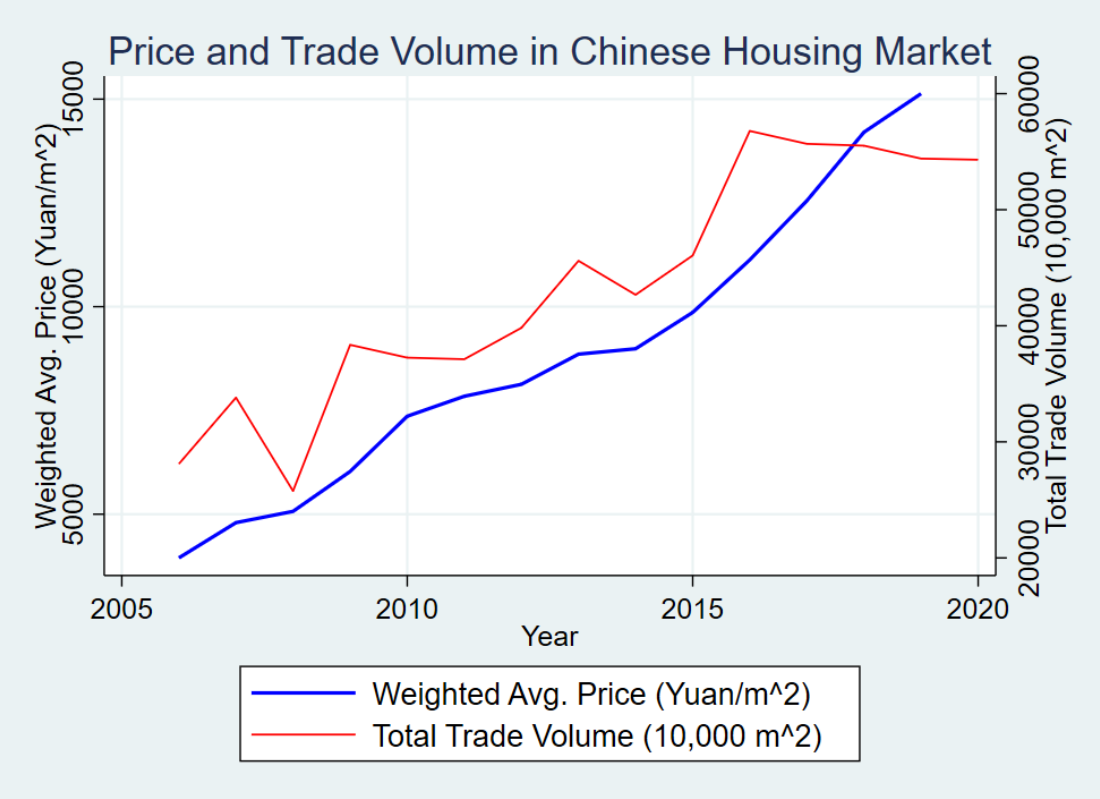


Figure 1: Price and Trade Volume in Chinese Housing Market (2005-2020). The dataset includes 36 major Chinese cities. The price is calculated as a population-weighted average across cities and total trade volume is measured in the unit of 10,000 m^2.

Figure 1 shows the evolution of prices and trade volumes in the housing markets of China's major cities from 2005 to 2020. Both series show a strong upward trend. Since this market is often described as a "bubble", the figure provides a straightforward illustration of the two stylized facts: high prices and high trade volumes.

However, a critical factor makes this interpretation difficult. During this period, the Chinese economy experienced remarkable growth. Therefore, a significant portion of the rising prices and volumes could be driven by fundamental factors, such as rising incomes and newly constructions rather than a simple speculative bubble. Without a structural model that can separate the bubble component from the fundamental component, it is impossible to analyze this bubble or even determine if this is truly a bubble.

Therefore, while the Chinese housing market illustrates the surface features of a bubble, it also highlights the central challenge for empirical work: the need for a structural model to account for unobserved fundamentals.

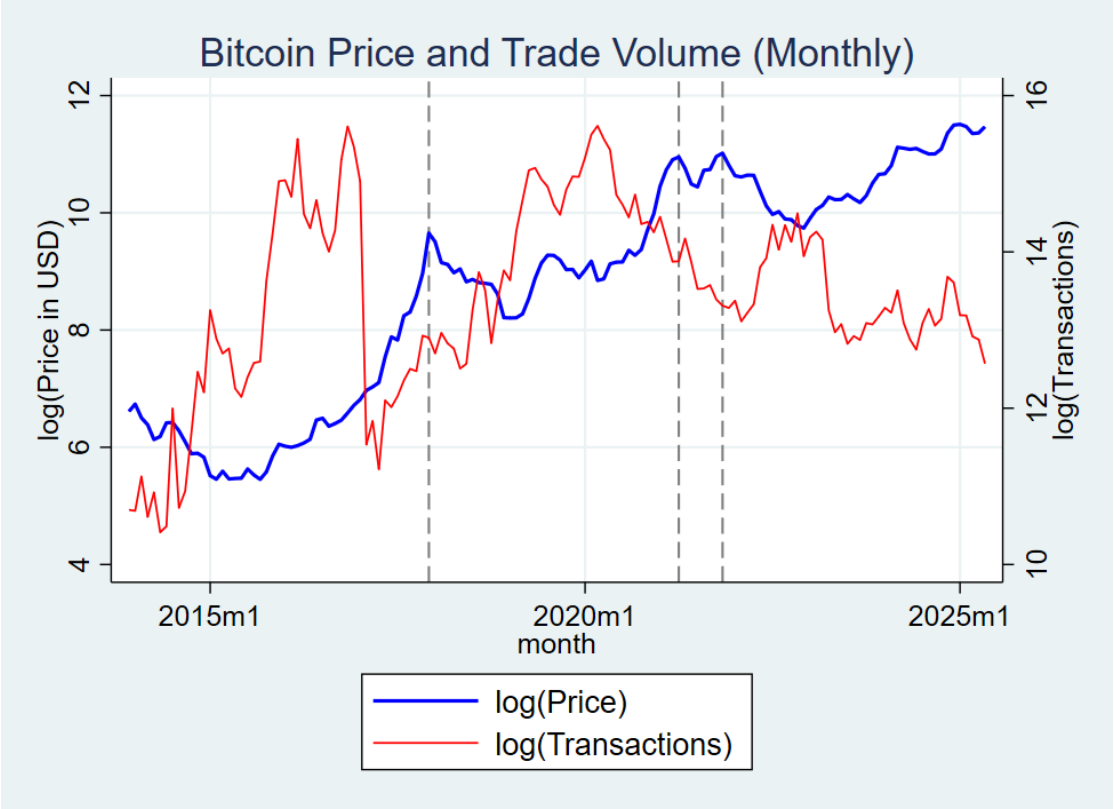


Figure 2: Bitcoin Price and Trade Volume (monthly, in log). source: Yahoo finance.

Figure 2 presents a cleaner example by showing the Bitcoin market. Because Bitcoin has an arguable fundamental value of zero (or close to it), any significant price and trade volume can be considered "abnormally high." This case therefore provides a clearer illustration of a bubble's two key features.

More importantly, this figure highlights the complexity of the trade volume dynamics. While both price and volume are high, they do not exhibit a stable, positive correlation. This pattern of high volume with a weak or unstable correlation to price is also observable in other historical bubbles (e.g., the SZCI index in 2015) and serves as the second key stylized fact.

The patterns observed in the Bitcoin market are not unique to cryptocurrencies. The Dot-com bubble of the late 1990s provides another prominent example of these stylized facts. It featured an explosive rise and fall in technology stock prices that did not follow a simple exponential path. Furthermore, it was accompanied by exceptionally high trading volumes, whose relationship with price movements was also complex and unstable. This reinforces that the stylized facts we identify are general features of major bubble episodes.

In summary, the examples above establish the two stylized facts that motivate this paper. First, financial bubbles are characterized by an explosive price path that is not necessarily exponential. Second, they are accompanied by abnormally high trade volume that follows a complex pattern with no stable correlation to price. Any useful model of financial bubbles should aim to generate both of these features endogenously.

3 Literature Review

Empirical studies on financial bubbles face a common challenge: the lack of a clear definition or identification method. As prominent literature points out (e.g., Brunnermeier & Oehmke, 2013; Martin & Ventura, 2018), identifying a bubble requires knowledge of an asset's fundamental value, which is often unobservable. This inherent difficulty has limited the scope and reliability of empirical work on bubbles.

The theoretical literature can be broadly categorized by its success in explaining one of the two stylized facts, but rarely both.

Blanchard and Watson (1982) were the first to formally define the bubble as an alternative equilibrium where price of a financial asset can be divided into two parts: a fundamental

part that equals the discounted present value of all future cashflow, and a growing bubble part driven by speculative reasons. The model is insightful yet too simple to explain the trade volume.

Following the monetary economists, a large body of literature, often using OLG or DSGE frameworks, focuses on justifying the existence of rational bubbles as efficiency improver (Tirole, 1985; Farhi & Tirole, 2008; Martin & Ventura, 2016; Miao & Wang, 2018; Benhabib, Liu & Wang, 2019). The models usually explain how the existence of asset bubbles can increase the total welfare in a world with distortions, such as credit constraint. While these models are powerful tools for welfare analysis and can generate high asset prices as a rational equilibrium, they fall short in two ways consistent with the stylized facts. First, they typically do not generate an explosive price path. Instead, they often produce a concave or converging path where the economy transitions to a new, higher asset price level from the low non-bubble asset price level. As discussed in Section 2, this does not match the shape of price dynamics in real-world bubbles. Second, these models generally do not account for trade volume. When extensions are made to incorporate trading, they often rely on exogenous shocks or irrational "sunspot" beliefs.

Another strand of literature directly tackles the explosive price path by modeling "ride-the-tide" speculative behavior (e.g., Abreu & Brunnermeier, 2003; Doblas-Madrid, 2012). In these models, rational agents purchase an overpriced asset, believing they can sell it to a "late comer" before the bubble collapses. While these models successfully capture the strategic incentives of speculators, they typically rely on an exogenous price process as a common belief among agents and do not endogenously generate the trade volume dynamics.

In contrast, a separate set of models aims to explain high trade volumes, often by assuming asymmetric information or heterogeneous beliefs, such as over-confidence (e.g., Miller, 1977; Scheinkman & Xiong, 2003). While these models successfully generate high trade

volumes, they typically cannot produce the explosive price paths characteristic of bubbles. To generate such price dynamics, the models would require an ad-hoc assumption of "explosively growing over-confidence," which lacks a clear micro-foundation.

This paper contributes to the literature by developing a model that endogenously generates both an explosive price path and high trade volumes within a unified framework. The model is built on a search-and-matching friction in a market with rational, heterogeneous investors to capture both the efficiency improver idea and the "ride-the-tide" idea. A key feature of this framework is that it does not rely on the observability of important variables, making the model empirically testable.

4 Model and Discussion

4.1 Model Setting

The model is set in discrete time with infinite horizons on both ends:

$$t = \cdots, 0, 1, 2, \ldots, \bar{t} - 1, \bar{t}, \bar{t} + 1, \ldots$$

The analysis focuses on an ongoing bubble, and this paper does not model the initial formation of the bubble.

The financial market consists of a large, fixed number of indivisible assets indexed by $i = 1, 2, 3, \ldots, N$. Each investor can only hold at most one asset at a time. While this indivisibility assumption fits markets such as housing (e.g., one cannot buy half an apartment), it may appear unrealistic in markets like stocks or cryptocurrencies. However, the "assets" in the model need not correspond one-to-one to actual stocks or tokens. One may interpret them as representative assets or units of ownership. However, this may

challenge the other assumption such that each investor can only hold at most 1 asset at a time. Though imperfect, the assumption simplifies the analysis without hurting the validity of the model. This is because the model is not concerned with portfolio choices or risk diversifications across assets.

There are two types of agents in the market: fundamental investors and speculators. We denote the proportion of speculators to be η, and thus the proportion of fundamental investors to be $(1-\eta)$.

The fundamental investors are willing to buy and hold an asset for its fundamental value, which we assume to be a constant D. This value can be interpreted as the standard discounted present value of future cash flows. For example, in the housing market, fundamental investors could be rental agencies and the fundamental value of a house is its expected discounted future rents. In a cryptocurrency market, the fundamental investors could be users who require cryptocurrency for transaction. As noted by Härdle et al. (2020), a significant number of cryptocurrencies are used for anonymous or illegal transactions. For simplicity reason, all assets share the same fundamental value D.

The other group of agents are the "speculators". Each speculator is characterized by a single parameter, ω, which captures their type. Speculators are rational and maximize their intertemporal utility with a discount factor $\beta \in (0,1)$. The instantaneous utility function at the moment of consumption is given by:

$$U(c,\omega) = (\omega_0 - \omega)(c + c_0) + \mathcal{L}$$

For mathematical tractability, I adopt a linear utility function in the current model. In the utility function, the characteristic of the agent ω determines the slope of the function, consumption from the bubble asset is denoted by c and is added up to all other consumptions c_0. The combined consumptions are multiplied by the adjusted agent's

characteristic $(\omega_0 - \omega)$, and are then added with a non-consumption utility $\mathcal{L}$. Linear utility provides the most solvability and corresponds to the standard risk-neutral assumption frequently used in the literature (Blanchard and Watson 1982).

To model heterogeneity in a tractable way, we assume that the speculator type ω is drawn from a binomial distribution. There are only two types of speculators: "high price type" (H) and "low price type" (L)[1]. As we will show, the high-price type is willing to trade at higher prices in equilibrium than the low-price type. A fraction ψ of speculators are high-price type, and the remaining $1 - \psi$ are low-price type:

$$\omega = \begin{cases} \omega_L & \cdots 1 - \psi \\ \omega_H & \cdots \psi \end{cases}$$

We assume $\omega_H < \omega_L$, which, given the utility function, implies that high-price type agents have a higher marginal utility of consumption.

Note that the assumption of a Bernoulli distribution of speculator types is made purely for math simplicity. In principle, we can assume that the types can follow any distributions $(\omega \sim \Omega)$, allowing for a richer spectrum of heterogeneity in trading motives or constraints.

The following section describes the search-and-matching process that governs the interactions between these agents.

[1] In an earlier version of this paper, the "high price type" (H) was referred to as the "risk-tolerant type" (T) and the "low price type" (L) as the "risk-averse type" (A). The current version uses more general terminology to emphasize that agent heterogeneity can stem from various factors—such as liquidity needs, consumption taste, or many other reasons—rather than only differences in risk preference.

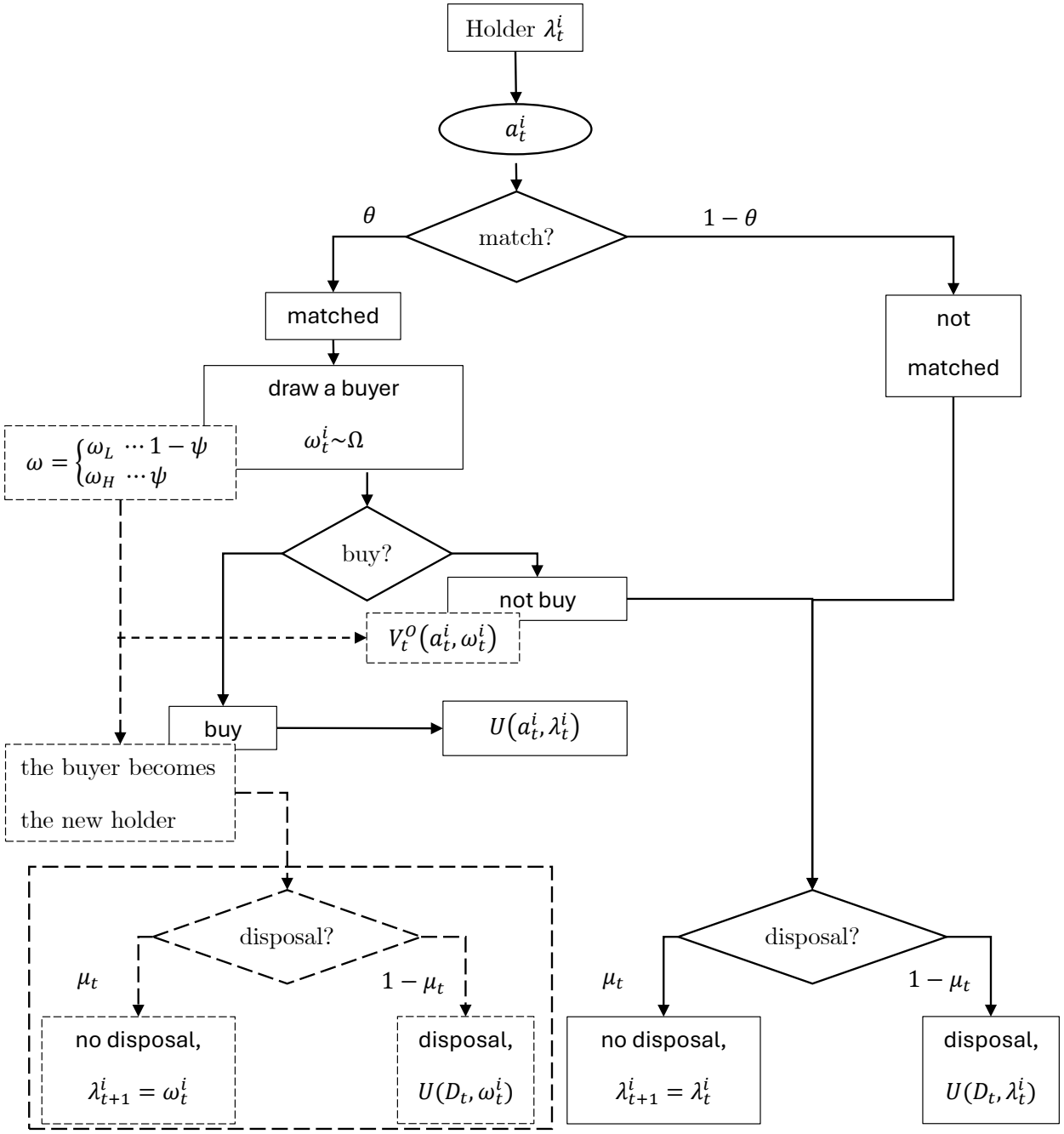


Figure 3: the match and search process in each period. Solid-line nodes represent the decisions and states of the holder of the asset in the beginning of this period; dashed-line nodes represent the matched buyer.

Figure 3 illustrates the search-and-matching process within a single period t.

At the start of each period, each asset is held by a speculator of type $\lambda_t^i \in \{\omega_H, \omega_L\}$. The holder strategically sets an asking price, a_t^i. The optimal asking price in equilibrium, $a_t^{i^*}(\lambda_t^i)$, is determined by the holder's optimization problem, which we solve later. After prices are set, a new generation of buyers, with types $\omega \sim \Omega$, drawn from the population distribution, enters the market. Each holder matches with a buyer with probability θ. The buyer matched is randomly drawn and denoted by her characteristics ω_t^i.

If the holder is not matched with a buyer (with probability $1-\theta$), he enters a "disposal process". This is a random event where the holder survives with the asset to the next period with probability μ_t, or is forced to liquidate the asset to a fundamental investor with probability$1-\mu_t$. A forced liquidation yields a utility of$U(D, \lambda_t^i)$. This event can be interpreted as an exogenous liquidity shock, such as those caused by regulatory changes or personal needs.

If the holder is matched with a buyer, the buyer then strategically chooses whether to purchase the asset at the asking price or not. If the buyer accepts to purchase, the holder receives the payment a_t^i and consumes it immediately with $U(a_t^i, \lambda_t^i)$. The buyer, who is now the new holder, immediately enters the same disposal process. If she survives the potential liquidation shock (with probabilityμ_t), she become the asset holder at the start of period $t+1$.

If the buyer rejects the asking price, the holder also enters the disposal process. The rejecting buyer, however, receives a payoff from an outside option, given by $V^O(a_t^i, \omega_t^i) = U(v_0 + v_1 a_t^i, \omega_t^i)$.

We can provide a clear economic interpretation for the outside payoff, $v_0 + v_1 a_t$. This term represents the net return a buyer would receive by choosing not to invest the amount a_t in the bubble asset, and instead placing it in an alternative investment. Under this

interpretation, the parameter v_1 can be seen as the interest rate or rate of return offered by this alternative market. The parameter v_0 represents a fixed component of this return, which could be interpreted as a base payoff or, if negative, an access cost to the alternative market.

The specification of this outside option is a key element of the model. Intuitively, for a speculative bubble to arise and persist, the alternative to participating in the bubble must be relatively unattractive. This notion aligns with findings in the broader literature, where mechanisms such as credit constraints (Miao & Wang, 2011) or inefficient investments (Martin & Ventura, 2012) can be interpreted as environments with low-value outside options. In our framework, we will show in the Results section that an explosive price path—a core feature of bubbles—can only be sustained under a specific condition on the outside option's functional form. This highlights that explosive rational bubbles are not a generic outcome, but an equilibrium that arises under particular economic conditions. A detailed derivation and discussion of this condition will be presented in Section 4.3.

The survival probability, μ_t, follows a deterministic, time-dependent path. We assume it is governed by a known future date, $\bar{t}$:

$$\mu_t = \begin{cases} \mu \cdots \text{when } t > \bar{t} \\ 1 \cdots \text{when } t \leq \bar{t} \end{cases}$$

that is, before $\bar{t}$, there is no danger of forced liquidation for any speculators; after $\bar{t}$, the probability of forcing liquidation is the same constant $1 - \mu$.

The assumption that forced liquidation begins only after time $\bar{t}$ and is absent before that point is made primarily for mathematical reason. This discrete timing setting helps to produce a tractable equilibrium and does not affect the core intuition of the model. Importantly, the literature offers several extensions that can relax this assumption without changing the main results. For example, Abreu & Brunnermeier (2003) and Doblas-Madrid

(2012) consider the liquidation time itself as a random variable, introducing greater realism while preserving the qualitative features of the bubble collapse.

Beyond these approaches, further refinements are possible. For example, one could generalize the model by allowing fundamental investors to participate in the search process, or by moving the search process from an asking-price decision to a bidding-price decision. Such extensions would strengthen the theoretical foundation of the disposal process without fundamentally altering the model's qualitative dynamics.

These refinements may be implemented in future versions of the model to enhance its realism and generality.

Finally, we aggregate the micro-level interactions to define the macroeconomic variables for the market. We assume a large number of assets, allowing us to describe the evolution of market-level outcomes. The primary aggregate variables are the market price index and trade volume. The price index reflects the weighted average price of all assets, including those held by speculators and fundamental investors. Trade volume is measured in two ways: by the number of assets transacted (shares) and by the total value of these transactions. We specify that forced disposals are counted as transactions in these aggregate measures. The explicit formulas for these macro variables depend on the agents' optimal strategies and will be formally defined in Section 4.2.

4.2 Solving the Model

To solve the model, there are 7 stages: (1) writing the holder's problem; (2) writing the buyer's problem; (3) proposing the strategies after $\bar{t}$; (4) verifying the proposed strategies after $\bar{t}$ as a Nash equilibrium; (5) proposing the strategies before $\bar{t}$; (6) verifying the

proposed strategies before $\bar{t}$ as a Nash equilibrium; (7) deriving the implied the macro-level variables.

Stage 1: The holder's problem

At the beginning of period t, the holder of asset i (λ_t^i) sets an asking price (a_t^i) strategically:

$$a_t^{i^*}(\lambda_t^i) = arg\max_{a_t^i} V_t^h(a_t^i, \lambda_t^i)$$

where $V_t^h(a_t^i, \lambda_t^i)$ is a value function of holder with λ_t^i who sets a_t^i.

As a typical search model would do, we guess that there is a threshold $\bar{\omega}(a_t^i)$, such that if and only if $\omega \leq \bar{\omega}$, the buyer matched would buy the asset. Note that since we only have 2 types of investors, $\bar{\omega}$ can only take 2 values: ω_H and ω_L. I will verify this later.

With the matching and searching mechanism in each period, we can express the value function as:

$$\begin{aligned} V_t^h(a_t^i, \lambda_t^i) = {} & (1-\theta)\left[\mu_t \beta V_{t+1}^h\left(a_{t+1}^{i\,*}(\lambda_t^i), \lambda_t^i\right) + (1-\mu_t)U(D, \lambda_t^i)\right] \\ & + \theta\left(1 - \Omega\left(\bar{\omega}(a_t^i)\right)\right)\left[\mu_t \beta V_{t+1}^h\left(a_{t+1}^{i\,*}(\lambda_t^i), \lambda_t^i\right) + (1-\mu_t)U(D, \lambda_t^i)\right] \\ & + \theta\Omega\left(\bar{\omega}(a_t^i)\right)U(a_t^i, \lambda_t^i) \end{aligned}$$

where θ_t is the probability of being matched with a buyer, μ_t is the probability of NOT getting shocked and therefore staying in the market, β is impatience discount, $U(c, \lambda)$ is an instantaneous utility function.

The 3 parts are: (1) if no buyers were matched, (2) if the buyer matched would not like to buy the asset; (3) if the buyer matched would buy the asset (note that buyer would buy if she has $\omega_t^i \leq \bar{\omega}(a_t^i)$).

A key to solve this model is to notice that after $\bar{t}$, holder's problem stays the same for each t, therefore, when $t > \bar{t}$:

$$a_{t+1}^{i}{}^{*}(\lambda_t^i) = a_t^{i*}(\lambda_t^i)$$

$$V_{t+1}^h(a_{t+1}^{i}{}^{*}(\lambda_t^i), \lambda_t^i) = V_t^h(a_t^{i*}(\lambda_t^i), \lambda_t^i)$$

And we can express the value function as:

$$\begin{aligned} V_t^h(a_t^{i*}, \lambda_t^i) = {} & (1-\theta)\left[\mu\beta V_t^h(a_t^{i*}, \lambda_t^i) + (1-\mu)U(D, \lambda_t^i)\right] \\ & + \theta\left(1 - \Omega\left(\bar{\omega}(a_t^{i*})\right)\right)\left[\mu\beta V_t^h(a_t^{i*}, \lambda_t^i) + (1-\mu)U(D, \lambda_t^i)\right] \\ & + \theta\Omega\left(\bar{\omega}(a_t^{i*})\right)U(a_t^{i*}, \lambda_t^i) \end{aligned}$$

$$(\text{if } t > \bar{t})$$

This can be transformed as: (when $t > \bar{t}$)

$$V_t^h(a_t^{i*}, \lambda_t^i) = \frac{(1-\mu)\left(1 - \theta\Omega\left(\bar{\omega}(a_t^{i*})\right)\right)}{1 - \mu\beta + \mu\beta\theta\Omega\left(\bar{\omega}(a_t^{i*})\right)}U(D, \lambda_t^i) + \frac{\theta\Omega\left(\bar{\omega}(a_t^{i*})\right)}{1 - \mu\beta + \mu\beta\theta\Omega\left(\bar{\omega}(a_t^{i*})\right)}U(a_t^{i*}, \lambda_t^i)$$

This means that the value function can be seen as a discounted weighted sum of the utility provided either when the asset is liquidated or sold.

Stage 2: the buyer's problem

We now turn to the buyer's problem to solve for the expression of $\Omega\left(\bar{\omega}(a_t^i)\right)$.

As a buyer with $\bar{\omega}(a_t^i)$, she is indifferent between buying the asset and become the next holder or not buying the asset and experience the outside option. She has:

$$V^O\left(a_t^i, \bar{\omega}(a_t^i)\right) = \mu_t\beta V_{t+1}^h\left(a_{t+1}^{i}{}^{*}\left(\bar{\omega}(a_t^i)\right), \bar{\omega}(a_t^i)\right) + (1-\mu_t)U\left(D, \bar{\omega}(a_t^i)\right)$$

Stage 3: the equilibrium after $\bar{t}$

We now guess that the best asking price strategy is as described as follows:

Holder \ Buyer	Low-price type	High-price type
Low-price type	Buy	Buy
High-price type	Not Buy	Buy

i.e. High-price type would buy from anyone; low-price type only buys from low-price type. (Note that $\omega_H < \omega_L$) Or we can say that the high-price type set a price so high that only the high-price type would buy, the low-price type set a price that everyone would buy. This implies that the high-price type sets an asking price that is higher than the low-price type's low asking price.

As one could imagine, this further implies that the holders in the non-collapse bubble market will gradually move from low-price types to high-price types.

This suggests that the threshold type of one's best asking price is oneself:

$$\bar{\omega}\left(a_t^{i^*}(\lambda)\right) = \lambda$$

So that we can write: (when $t > \bar{t}$)

$$V^O\left(a_t^i, \bar{\omega}\right) = \frac{(1-\mu)}{1-\mu\beta+\mu\beta\theta\Omega(\bar{\omega})} U(D, \bar{\omega}) + \frac{\mu\beta\theta\Omega(\bar{\omega})}{1-\mu\beta+\mu\beta\theta\Omega(\bar{\omega})} U\left(a_{t+1}^{i\ *}(\bar{\omega}), \bar{\omega}\right)$$

With the assumption that speculators are in a Bernoulli distribution,

(1) for the high-price type: ($\Omega(\bar{\omega}) = \Omega(\omega_H) = \psi$)

$$V^O\left(a_{t+1}^{i\ *}(\omega_H), \omega_H\right) = \frac{(1-\mu)}{1-\mu\beta+\mu\beta\theta\psi} U(D, \omega_H) + \frac{\mu\beta\theta\psi}{1-\mu\beta+\mu\beta\theta\psi} U\left(a_{t+1}^{i\ *}(\omega_H), \omega_H\right)$$

(2) for the low-price type: ($\Omega(\bar{\omega}) = \Omega(\omega_L) = 1$)

$$V^O\left(a_{t+1}^{i\ *}(\omega_L), \omega_L\right) = \frac{(1-\mu)}{1-\mu\beta+\mu\beta\theta} U(D, \omega_L) + \frac{\mu\beta\theta}{1-\mu\beta+\mu\beta\theta} U\left(a_{t+1}^{i\ *}(\omega_L), \omega_L\right)$$

These 2 equations yield to the solution of equilibrium asking prices $a_t^{i^*}(\omega)$ after $\bar{t}$.

Stage 4: Nash equilibrium after $\bar{t}$

A critical step is to verify that the trading strategies proposed in Stage 3 constitute a Nash Equilibrium. Here, we analyze the incentives of agents in the steady state ($t > \bar{t}$). A formal proof requires showing that no agent has a profitable unilateral deviation. However, a rigorous proof is not finished and is expected to be finished in future versions.

Take the high-price type holder as an example, (1) setting any ask price higher than $a_{t+1}^{i}{}^{*}(\omega_H)$ is not favorable, since setting any price higher than this price would lead to nobody buying the asset; (2) setting a price a such that $a_{t+1}^{i}{}^{*}(\omega_H) > a > a_{t+1}^{i}{}^{*}(\omega_L)$, is also obviously not favorable, since still, only high-price type would buy but the holder receives less; (3) only setting a price such that $a \leq a_{t+1}^{i}{}^{*}(\omega_L)$ would induce the low-price type to buy:

In particular, setting any $a = a_{t+1}^{i}{}^{*}(\omega_L)$ leads to

$$V_t^h\left(a_t^{i^*}(\omega_L), \omega_H\right) = \frac{(1-\mu)(1-\theta)}{1-\mu\beta+\mu\beta\theta} U(D, \omega_H) + \frac{\theta}{1-\mu\beta+\mu\beta\theta} U\left(a_t^{i^*}(\omega_L), \omega_H\right)$$

To let the strategy guessed be a Nash equilibrium, we need:

$$V_t^h\left(a_{t+1}^{i}{}^{*}(\omega_L), \omega_H\right) < V_t^h\left(a_{t+1}^{i}{}^{*}(\omega_H), \omega_H\right)$$

That is to say, the alternative to deviate by setting a lower price, specifically the other type's equilibrium price,a_L, is the one worth investigating. This deviation involves a trade-off: the holder receives a lower price upon a sale, but the probability of a sale increases because all buyer types are now willing to purchase. The equilibrium holds if the value

from adhering to the strategy, $V^h(a_H^*, \omega_H)$, is greater than the value from deviating, $V^h(a_L^*, \omega_H)$.

A key advantage of our linear utility framework is that this equilibrium condition is analytically tractable. While we have derived the closed-form expression for the equilibrium conditions, it is too complex to yield immediate economic intuition. A full exposition of this analytical solution is therefore omitted for brevity.

However, we can confirm the existence of this equilibrium through numerical simulations. For a wide range of plausible parameter values, including those used for the simulations in Section 4.3, we verify that $V^h(a_H^*, \omega_H) > V^h(a_L^*, \omega_H)$. This confirms that a parameter space exists in which our proposed separating equilibrium is indeed a Nash Equilibrium. A similar analysis can be applied to the L-type holder's incentives. This provides strong support for the theoretical consistency of our model's core mechanism.

Stage 5: equilibrium before $\bar{t}$

After the derivation of previous stages, for any type of speculator, at $t = \bar{t} + 1$, we have a fixed value function:

$$V_{\bar{t}+1}^h\left(a_t^{i^*}, \lambda_t^i\right) = \frac{V^O\left(a_t^{i^*}, \lambda_t^i\right) - (1-\mu)U\left(D, \lambda_t^i\right)}{\mu\beta}$$

Before $\bar{t}$, however, is a different case. Speculators are not required to liquidate their holdings. (i.e. $\mu_t = 1$) We then have:

$$V_t^h\left(a_t^i, \lambda_t^i\right) = \left(1 - \theta\Omega\left(\bar{\omega}\left(a_t^i\right)\right)\right)\left[\beta V_{t+1}^h\left(a_{t+1}^{i^*}\left(\lambda_t^i\right), \lambda_t^i\right)\right] + \theta\Omega\left(\bar{\omega}\left(a_t^i\right)\right)U\left(a_t^i, \lambda_t^i\right)$$

The following procedure will be similar to stage 1, 2, 3, and 4. For the holder, he solves:

$$\max_{a_t^i} V_t^h(a_t^i, \lambda_t^i)$$

For a buyer matched, the threshold would require:

$$V^o\left(a_t^i, \bar{\omega}(a_t^i)\right) = \beta V_{t+1}^h\left({a_{t+1}^i}^*\left(\bar{\omega}(a_t^i)\right), \bar{\omega}(a_t^i)\right)$$

Notice that as we know the value of the right-hand side for $\bar{t}+1$, and the function form of the left-hand side is also known, we can solve for the asking price in period $\bar{t}$. This process can then be carried out for each period before $\bar{t}$ as a step-back equation.

Recursively using this equation, all asking prices before $\bar{t}$ can be determined by a backward deduction process. The result of this backward deduction should be two series of asking prices, one for the high-price type, the other for the low-price type.

Stage 6: Nash equilibrium before $\bar{t}$

Verifying the Nash Equilibrium for the periods before $\bar{t}$ involves a similar logic, but the analysis is more complex due to the dynamic nature of the problem. Unlike in the steady state, the agent's value function, V_t^h, now depends on the continuation value from the next period, V_{t+1}^h, which is itself changing over time as t approaches$\bar{t}$. This time-varying nature of the value functions makes the analytical derivation of the equilibrium conditions significantly more challenging than in the steady state. A formal proof for this dynamic phase is therefore beyond the scope of the current version of this paper.

Stage 7: the macro-variables

Recall that all assets independently form the market. Now, I call the assets held by speculators the "non-collapse" market. In particular, we introduce 2 more notations: γ_t, denotes the portion of the low-price type holders in the non-collapse market; and η_t,

denotes the portion of all holders in the non-collapse market, in other words, $1-\eta_t$ holders are the fundamental investors.

The price index of the market is (denoted by P_t)

$$P_t = \eta_t[\gamma_t a_t^*(\omega_L) + (1-\gamma_t)a_t^*(\omega_H)] + (1-\eta_t)D$$

which is the weighted sum of all asking prices by all holders.

The trade volume in terms of units of the asset traded is:

$$\eta_t[\gamma_t\theta + (1-\gamma_t)\theta\psi + (1-\mu_t)]$$

which counts both the transaction between speculators and their disposals. Note that the fundamental investors don't trade.

The trade volume in terms of the value traded is:

$$\eta_t[\gamma_t\theta a_t(\omega_L) + (1-\gamma_t)\theta\psi a_t(\omega_H) + (1-\mu_t)D]$$

We can also write a dynamic function for γ_t and η_t:

$$\eta_{t+1} = \eta_t\mu_t$$
$$\gamma_{t+1} = \gamma_t[1-\theta\psi]$$

This could give us a good simulation for each of the 3 macro-variables.

4.3 Result and Discussion

As derived in the previous section, the model is solved using backward induction. In equilibrium, the optimal asking price set by a holder in any period t depends on the holder's type. Our two-type setup therefore yields two distinct asking price paths: one for the high-price type (a_t^H) and one for the low-price type (a_t^L). We now use numerical simulation to illustrate these equilibrium paths and the resulting aggregate market dynamics.

The following simulation is intended to illustrate the qualitative mechanisms of the model, not to match any specific dataset. The parameters are chosen to produce clear graphical patterns that highlight the model's core predictions. A quantitative calibration of these parameters to a specific market, such as the Dot-com or cryptocurrency bubbles, is a promising avenue for future empirical work. Table 1 lists the parameter values used for this simulation.

Table 1

Parameter	Value	Note
θ	0.3	Probability of matching with any buyer
ψ	0.15	Proportion of High-price type
ω_H	0.00	High-price Type Parameter
ω_L	0.04	Low-price Type Parameter
μ	0.9	Disposal process survival probability
$\bar{t}$	45	Disposal process start period
D	0	Disposal price / Fundamental value of assets
β	0.95	Intertemporal discount factor
$U(c,\omega)$	$(\omega_0-\omega)*(c+c_0)+\mathcal{L}$	Instantaneous utility function
c_0	0	Other consumption sources
$\mathcal{L}$	0.01	Other utility sources
ω_0	0.05	Parameter in the instantaneous utility
$V^O(a,\omega)$	$U(v_0+v_1a,\omega)$	Outside option of the buyer with ω
v_0	5	Parameter in the outside option
v_1	-1	Parameter in the outside option

The simulation results generated using these parameters are presented in the figures below.

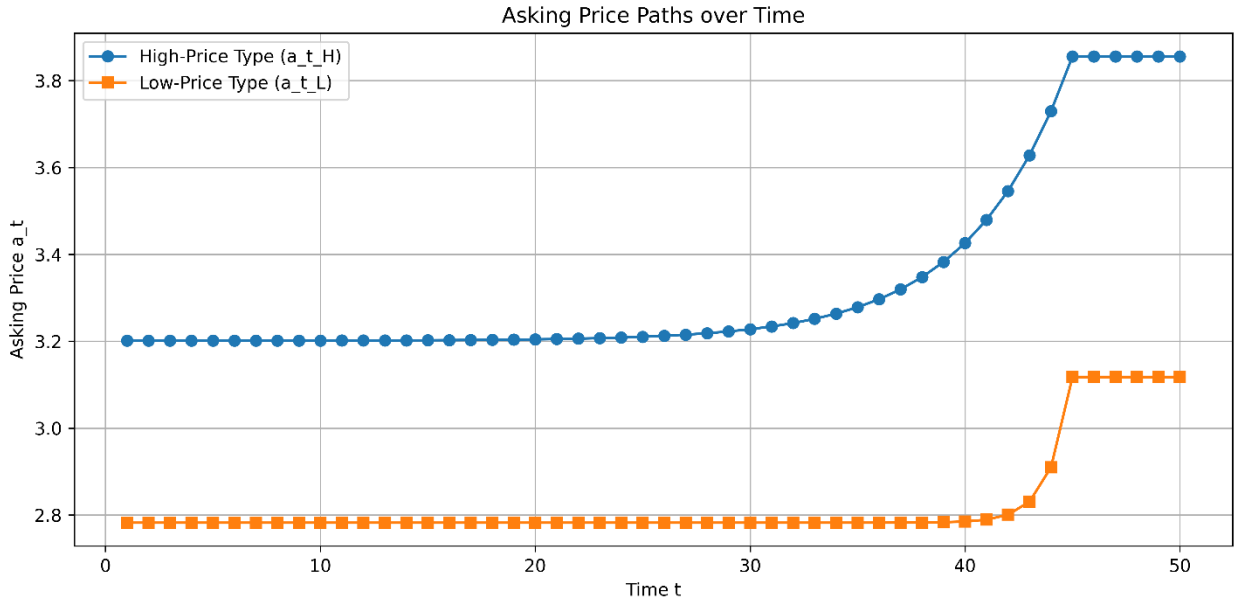


Figure 4: Simulation: Asking Price Paths for the two types of speculators.

Figure 4 displays the equilibrium asking price paths for the two speculator types, a_t^H and a_t^L. The figure reveals three distinct phases in the price dynamics. First, for all periods after the collapse risk begins ($t > \bar{t}$), prices are stable, reflecting the steady-state equilibrium. Second, in the periods leading up to the risk date ($t < \bar{t}$), both price paths are increasing, exhibiting the explosive dynamics characteristic of a bubble. Third, as we look back into the distant past ($t \to -\infty$), the price paths converge to a level that is lower than the post-$\bar{t}$ steady-state price.

The convergence to a lower, stable price in the distant past is intuitive. This state is economically equivalent to a world where the collapse risk is permanently absent (i.e., $\mu = 1$). In such an environment, the holder's problem is identical in every period, leading to a constant asking price.

Specifically, the state in the distant past is described by the following equation:

$$\lim_{t\to-\infty} V_t^h\left(a_t^{i^*},\lambda_t^i\right) = \lim_{\mu\to 1} V_{\bar{t}}^h\left(a_t^{i^*},\lambda_t^i\right) = \lim_{\mu\to 1}\frac{V^O\left(a_t^{i^*},\lambda_t^i\right)-(1-\mu)U\left(D,\lambda_t^i\right)}{\mu\beta}$$

The crucial insight, however, lies in understanding why the price path is explosive. One may notice that the asking price when $\mu \to 1$ is not guaranteed to be lower than when $\mu < 1$. Rather, it depends on the specific setting of the functional form of the outside option V^O. Specifically, a precise analytical condition must be satisfied for the price path to be explosive:

$$v_1 < v_1^{crit} = \frac{\beta\bar{\Omega}\theta(\omega_0-\omega)(D-v_0)}{\left(1-\beta(1-\theta\bar{\Omega})\right)D(\omega_0-\omega)+(1-\beta)U(0,\omega)}$$

Our derivation of v_1^{crit} reveals a trade-off between the two parameters governing the outside option, v_0 andv_1. The value of v_1^{crit} is strongly influenced by the term $(D-v_0)$, the difference between the asset's fundamental value and the outside option's base payoff. In a typical bubble scenario where the alternative investment is initially more attractive $(v_0 > D)$, the critical threshold v_1^{crit} becomes negative. This implies that for an explosive bubble to be sustained, v_1 must be sufficiently low and even negative at some settings. More generally, the condition implies that the overall attractiveness of the outside option must be sufficiently limited.

This finding provides a key economic insight into the origins of rational bubbles. An explosive bubble path cannot be sustained in an environment with highly attractive alternative investments. Instead, it can only arise as a rational equilibrium outcome when outside options are poor. This perspective aligns with and provides a micro-foundation for theories that link bubble formation to broader economic conditions, such as environments with binding credit constraints (Miao & Wang, 2011) or a scarcity of efficient investment opportunities (Martin & Ventura, 2012).

The following figures show the macro-level variables.

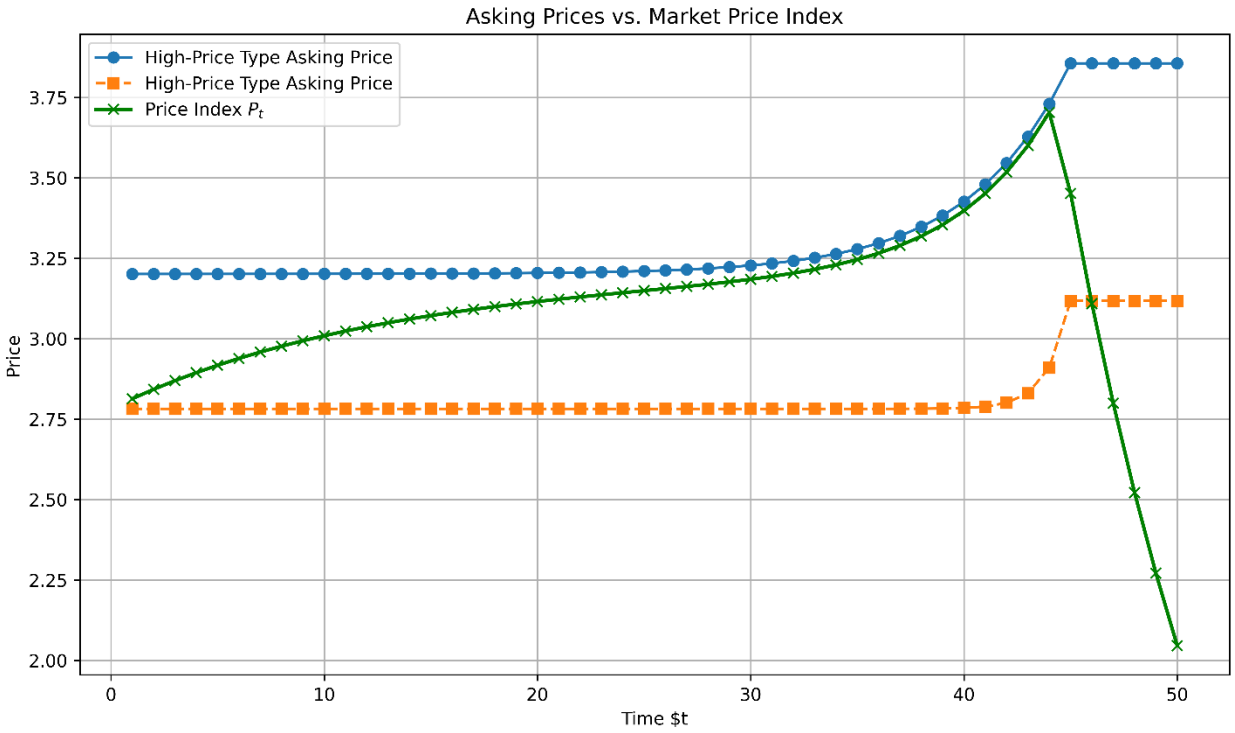


Figure 5: Simulation: the Price Index

Figure 5 illustrates the evolution of the aggregate price index, P_t. The index is a weighted average of the asking prices of high-price and low-price type speculators, and the fundamental price D for assets that have been liquidated. The dynamics of the price index can be understood in three distinct phases.

Phase 1: Early Bubble (Type Shift Driven). In the early periods, the market is populated by both speculator types. Because high-price types are willing to buy from anyone, while low-price types are more restrictive, assets gradually transfer from low-price to high-price holders. This shift in the composition of asset holders drives the aggregate price index upwards, as it converges towards the higher asking price path of the dominant high-price type agents.

Phase 2: Middle Bubble (Expectation Driven). As the market becomes dominated by high-price type holders, the price index is primarily driven by the increase in their asking prices,

a_t^H. This phase highlights a key mechanism of the model. The price accelerates precisely as the known collapse date, $\bar{t}$, approaches. This may seem counter-intuitive, as standard asset pricing models would predict that rising risk should depress prices. However, our result is consistent with the classic insight of Blanchard and Watson (1982): for rational agents to be willing to hold a bubble asset despite increasing risk, the expected capital gain must increase accordingly. Our model provides a micro-foundation for this phenomenon, showing how the interplay of expectations and market frictions leads to price acceleration when risk is highest.

Phase 3: Collapse (Disposal Driven). After period $\bar{t}$, the disposal process begins. In each subsequent period, a fraction $1 - \mu$ of the remaining speculators are forced to liquidate their assets at the fundamental price D. This continuous outflow of assets from the speculative market to fundamental investors leads to a rapid and sustained decline in the aggregate price index.

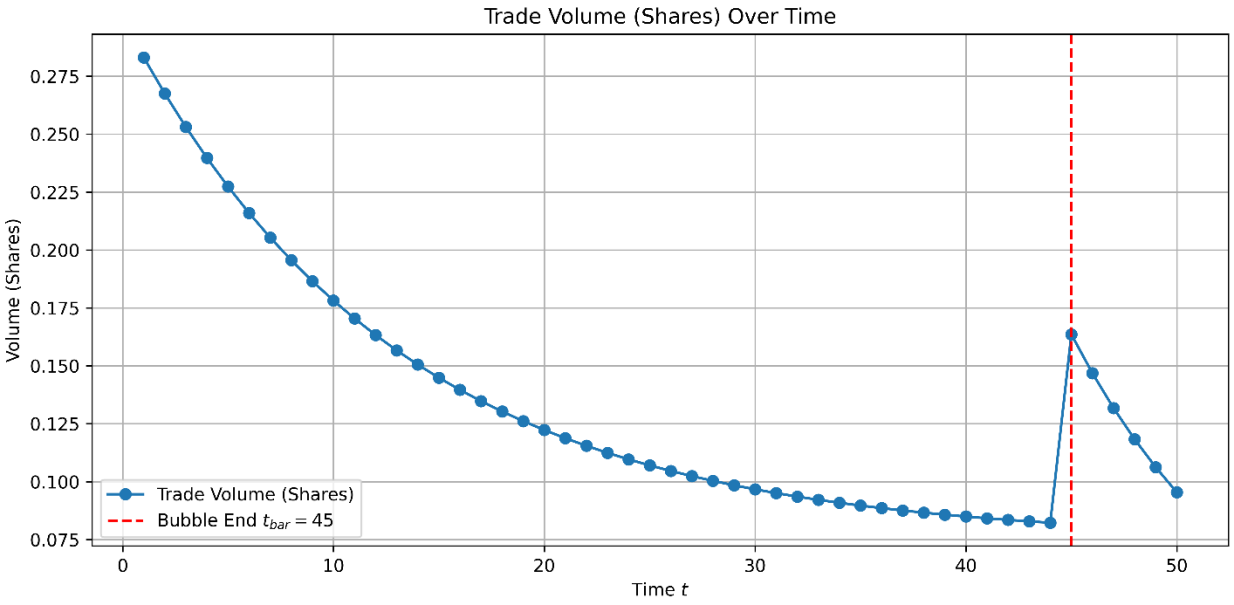


Figure 6: Simulation: Trade Volume measured by Transactions

Figure 6 shows the dynamics of trade volume, measured as the number of transactions per period. Consistent with the stylized facts, the model generates a sustained period of high trade volume. The volume path also exhibits a complex, three-phase pattern that is not trivially correlated with the price index.

Phase 1: Decreasing Volume. In the early phase, volume is high but decreasing. This is a direct result of the shift in holder composition. When the market has many low-price holders, a match is very likely to result in a trade. As high-price holders, who only sell to other high-price types, come to dominate the market, the probability of a successful match for a randomly drawn buyer decreases, leading to a decline in transaction volume. This mechanism offers a potential explanation for the weak price-volume correlation observed in some real-world bubbles like Bitcoin.

Phase 2: Stable Volume. In the middle phase, the market is almost entirely composed of high-price holders. Trade volume stabilizes, with most transactions occurring between high-price type agents.

Phase 3: Spike and Decline. At period $\bar{t}$, the start of the disposal process triggers a sharp spike in transactions, as the first wave of forced liquidations is added to regular trades. Subsequently, as the stock of speculators is depleted through ongoing liquidations, the volume rapidly declines.

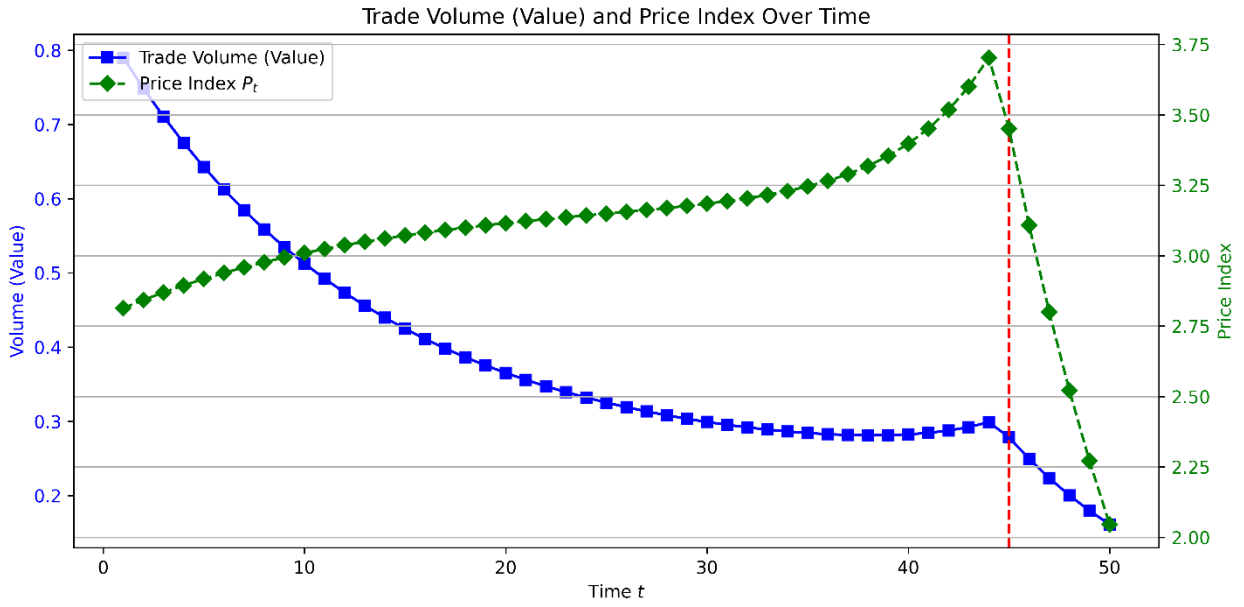


Figure 7: Simulation: Trade Volume measured by Values

Finally, Figure 7 depicts the trade volume measured by transaction value, which is calculated as the product of price and quantity. Its dynamics naturally combine the patterns observed in the previous two figures.

In the early phase, the rising price level initially offsets the decreasing transaction counts, causing the trade value to grow. In the middle phase, as both the transaction count and the rate of price increase stabilize, the trade value becomes relatively flat. Finally, after period $\bar{t}$, the sharp decline in both the price index and the number of speculators leads to a steep drop in the total value of transactions, marking the collapse of the bubble in value terms.

4.4 Summary

This section presented a search model that endogenously generates key features of financial bubbles. By incorporating heterogeneous agents and a terminal disposal risk, the model

produces rich dynamics for both asset prices and trade volumes, solved via backward induction.

Our simulations show that the bubble evolves through three distinct phases. An initial phase is driven by a shift in asset holdings towards more aggressive speculators. A middle phase is characterized by an accelerating price path, driven by the rational expectation of a future collapse. The final phase is a collapse driven by forced liquidations. These dynamics are consistent with the stylized facts of explosive prices and high, complex trade volumes established in Section 2.

While the current specification provides a benchmark, the model's simplicity means it cannot capture every nuance of real-world bubbles. The following section on counterfactual analysis will further explore the model's mechanisms and its potential for extensions.

5 Counterfactual Analysis

This section presents a set of simulations with different parameter settings rather than formal counterfactuals. Since the current version of the model is not calibrated from real-world data, these exercises are meant to illustrate the structural mechanisms of the model rather than to match any specific empirical scenario.

Due to technical issues - most notably, the simplified functional form of the outside option V^O - some simulated outcomes are not empirically plausible. I will illustrate this point by changing the disposal price (i.e., fundamental price) D. Again, the setting of negative sloped outside option V^O is a provisional one for illustrative purpose and will be improved in later versions.

The main focus of this section is on the impact of the matching probability θ. By increasing θ, the model shows how trade volumes can become higher across time. This

may help explain the pattern of the Chinese housing market exhibited in Section 2. In reality, a rising θ could be interpreted as a result of more agents entering the market, thereby increasing the probability of any transactions.



Figure 8: Counterfactuals: Disposal Price D

Figure 8 depicts when the disposal price D is changed. The disposal price is also the fundamental price of the assets. One may notice that after $\bar{t}$, a higher D leads to a slower or softer collapse of the bubble, which is consistent with intuition. However, before the bubble collapses, a higher D leads to lower asking prices, which is not consistent with intuition.

This is because I assumed a negative effect of the asking price on outside options (i.e., $\frac{\partial V^O}{\partial a} < 0$). This issue will be discussed in future versions.

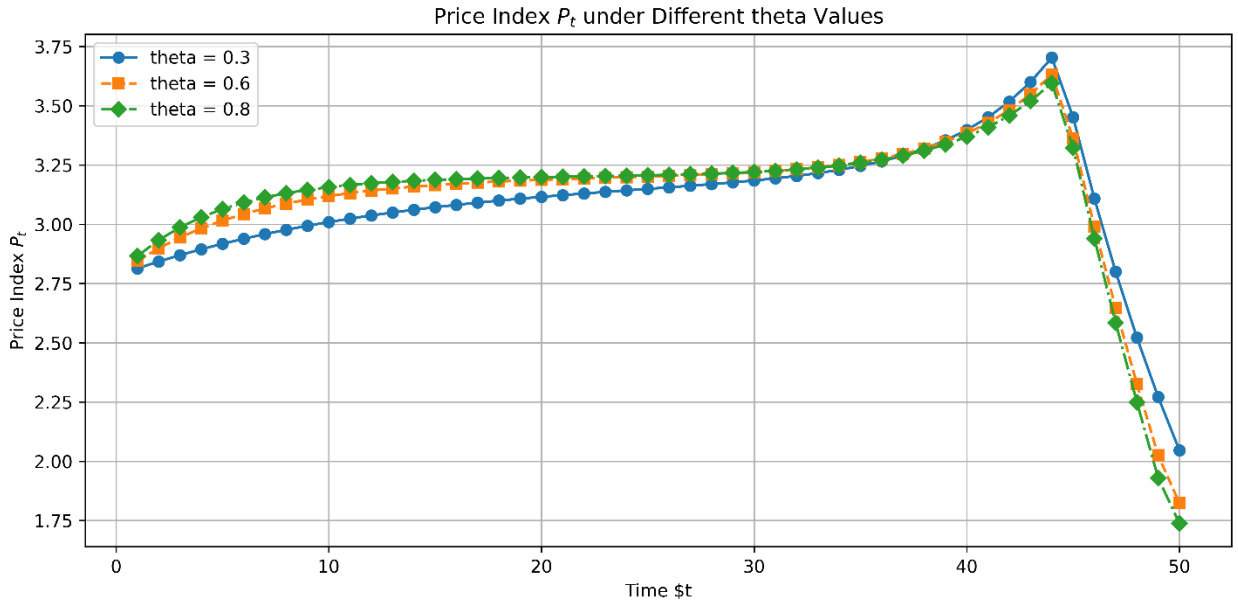


Figure 9: Counterfactuals: Matching Probability θ to the Price Index

Figure 9 above shows when the matching probability θ is changed. The middle phase is still bothered by the same issue as when D is changed. It shows a negative effect of matching probability on asking prices, which is not favorable.

The early phase is consistent with intuition in which a higher matching probability leads to a faster convergence of the price index towards the high-price type's asking prices. It shows a faster shift in holder's types.

The figure below shows the main focus of this section. As the matching probability θ becomes higher, the trade volume significantly rises. This may help explain the Chinese housing market pattern exhibited in Section 2. As one could recall, the Chinese housing market showed a pattern where both prices and trade volumes are growing over time. It may suggest that the matching probability in Chinese housing market has been gradually growing over time. In reality, this may suggest that more people entered the market. The construction data may support this implication.

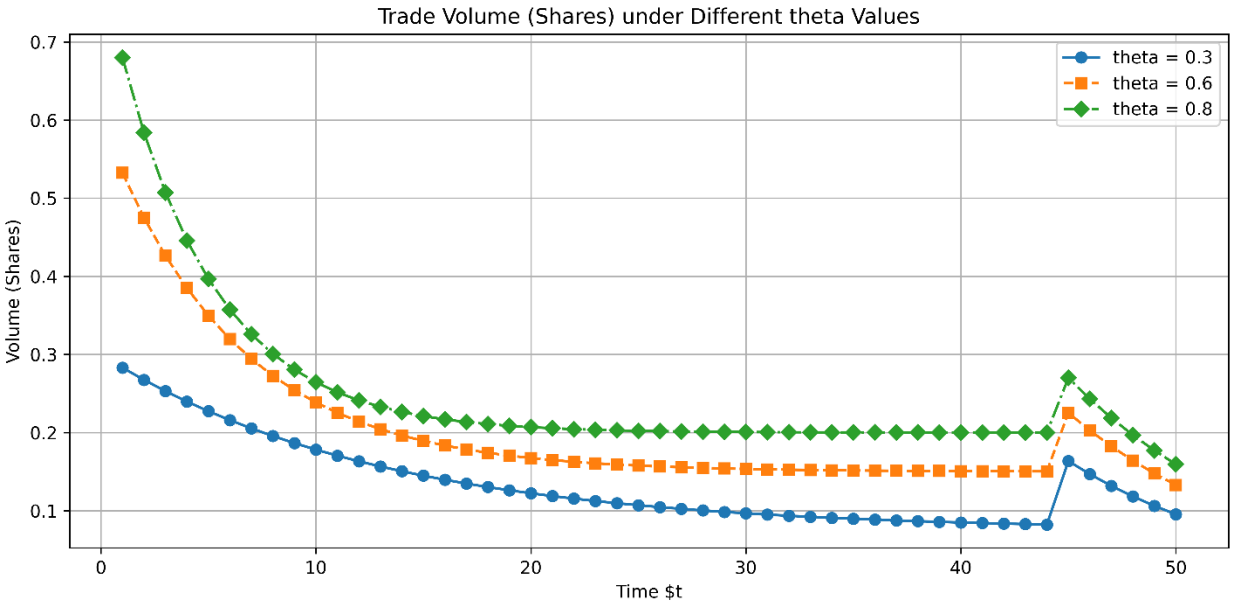


Figure 10: Counterfactuals: Matching Probability θ to Trade Volumes

To sum up, for this model to account for a broader range of real-world bubbles, further extensions are both necessary and feasible.

6 Empirical Implications and Tractability

A key contribution of this paper is the development of a theoretical framework that is empirically tractable. This tractability stems from three core features of the model. First, it endogenously generates explosive price paths and high trade volumes, allowing it to qualitatively match the dynamic patterns of real-world bubble episodes. Second, because these macro-level dynamics are derived from the optimizing behavior of micro-level agents, the model provides a sound basis for structural estimation methods. Third, and most importantly, the model is designed such that key unobservable variables—most notably the asset's fundamental value,D—are identifiable as estimable parameters of the model.

This framework can be taken to data using standard simulation-based estimation techniques, such as the Simulated Maximum Likelihood Estimator (SMLE). The estimation procedure involves finding the set of model parameters, Θ, that maximizes the likelihood of observing the real-world data series for price and trade volume. The model's predictions for price, $P_t^{model}(\Theta)$, and volume, $V_t^{model}(\Theta)$, are linked to the observed data through a set of measurement equations with stochastic errors:

$$P_t^{observed}(\Theta) = P_t^{model}(\Theta) + \epsilon_t^P,$$
$$V_t^{observed}(\Theta) = V_t^{model}(\Theta) + \epsilon_t^V,$$

where $P_t^{observed}$ and $V_t^{observed}$ are the real world observed data, P_t^{model} and V_t^{model} are the model-generated prediction with correct parameters, ϵ_t^P and ϵ_t^V are independent error terms, with $\epsilon_t^P \sim N(0, \sigma_P^2)$ and $\epsilon_t^V \sim N(0, \sigma_V^2)$.

The parameter vector to be estimated, Θ, includes not only the structural parameters governing agent behavior and market structure $(\mu, \theta, \psi, \Omega, \ldots)$, but also the fundamental value D, and, crucially, the expected start date of the collapse risk, $\bar{t}$. The estimator would then be the set of parameters $\widehat{\Theta}$ that maximize the joint log-likelihood function of the observed data. The likelihood functions is:

$$\mathcal{L}(\Theta) = \prod_t \Pr\left(P_t^{observed} \mid \Theta\right) \prod_t \Pr\left(V_t^{observed} \mid \Theta\right)$$

The Maximum Likelihood Estimator is:

$$\widehat{\Theta} = \underset{\text{parameters}}{\operatorname{argmax}}\ \mathcal{L}(\Theta)$$

This estimation framework opens up two significant avenues for empirical research on financial bubbles.

First, by treating the fundamental value D as an estimable parameter, it provides a disciplined method for identifying bubbles and quantifying the magnitude of price misalignment, even in markets where fundamentals are notoriously difficult to measure, such as for cryptocurrencies or during the Dot-com era.

Second, and perhaps most strikingly, the estimability of the collapse date parameter, $\bar{t}$, means this model can be used to forecast the potential turning point of a bubble before it collapses. This provides a theoretically grounded early-warning system, where the

estimated $\hat{\tilde{t}}$ indicates the market's expected horizon for the bubble's final phase. While the benchmark model presented here can be extended to better fit the nuances of specific datasets—for example, by allowing for a time-varying matching probability θ_t to better capture volume dynamics—its core structure provides a flexible and powerful new tool for the empirical analysis of financial bubbles.

7 Concluding Remarks

This paper developed a search-and-matching model of financial bubbles that endogenously generates two of their most prominent stylized facts: explosive asset prices and high trade volumes. We contribute to the literature by providing a micro-founded, rational-agent framework where these dynamics arise as a natural equilibrium outcome, rather than from assumptions of irrationality or exogenous price processes. The model's core mechanism is a search-and-matching friction that captures the strategic "ride-the-tide" behavior of heterogeneous speculators. Our simulations show that this mechanism produces a three-phase bubble dynamic—a type-shift driven initial phase, an expectation-driven explosive middle phase, and a disposal-driven collapse—that is consistent with historical bubble episodes.

We position this model as a tractable benchmark framework for the study of rational bubbles. As such, we acknowledge its current limitations, which also point toward clear directions for future extensions. The functional form of the outside option, while sufficient to generate the core results, could be refined with a more detailed micro-foundation to better reflect real-world investment alternatives. Furthermore, while we have provided analytical and numerical evidence for the existence of the Nash equilibrium, a complete formal proof for the dynamic phase of the bubble remains an important task for future theoretical work.

Looking forward, this framework opens up multiple avenues for both theoretical and empirical research. On the theoretical side, the model can be extended to incorporate policy interventions or to endogenize the timing of the bubble's collapse. On the empirical side, this paper lays the groundwork for a broader research agenda. As outlined in Section 6, the model's structure makes it uniquely suited for structural estimation, allowing researchers to identify bubbles and estimate unobservable fundamentals like an asset's intrinsic value, D, and its potential turning point,$\bar{t}$. The next steps in our research will be to apply this framework to analyze specific historical cases, including the Dot-com bubble of the 1990s and the recent cycles in cryptocurrency markets. Ultimately, this work aims to bridge the gap between the theory of rational bubbles and its empirical application, providing a new set of tools to better understand financial instability.

References

- Abreu, Dilip, and Markus K Brunnermeier. 2003. “Bubbles and Crashes.” *Econometrica* 71 (1): 173–204.
- Benhabib, J., Liu, X., & Wang, P. (2019). Financial markets, the real economy, and self-fulfilling uncertainties. *The Journal of Finance*, *74*(3), 1503-1557.
- Blanchard, Olivier J., and Mark W. Watson. "Bubbles, rational expectations and financial markets." (1982).
- Brunnermeier, Markus K., and Martin Oehmke. "Bubbles, financial crises, and systemic risk." *Handbook of the Economics of Finance* 2 (2013): 1221-1288.
- Chen, Weihua, Shushu Liang, and Donghui Shi. "Who Chases Returns? Evidence from the Chinese Stock Market." *Evidence from the Chinese Stock Market (June 30, 2022)* (2022).
- Doblas-Madrid, Antonio. "A robust model of bubbles with multidimensional uncertainty." *Econometrica* 80, no. 5 (2012): 1845-1893.
- Härdle, W. K., Harvey, C. R., & Reule, R. C. (2020). Understanding cryptocurrencies. *Journal of Financial Econometrics*, *18*(2), 181-208.
- Kindleberger, C. P. (1978). Manias, panics, and rationality. *Eastern Economic Journal*, *4*(2), 103-112.
- Kiyotaki, Nobuhiro, and Randall Wright. "A search-theoretic approach to monetary economics." *The American Economic Review* (1993): 63-77.
- Liao, J., Peng, C., & Zhu, N. (2019). Price and volume dynamics in bubbles.
- Martin, A., & Ventura, J. (2012). Economic growth with bubbles. *American Economic Review*, *102*(6), 3033-3058.
- Martin, A., & Ventura, J. (2016). Managing credit bubbles. *Journal of the European Economic Association*, *14*(3), 753-789.

- Martin, Alberto, and Jaume Ventura. 2018. "The Macroeconomics of Rational Bubbles: A User's Guide." *Annual Review of Economics* 10 (1): 505–39.
- Miao, J., & Wang, P. (2011). Bubbles and credit constraints. *Available at SSRN 1779485*.
- Miao, J., Wang, P., & Zhou, J. (2014). Housing bubbles and policy analysis. *Unpublished Working Paper, Boston University and HKUST*.
- Miao, J., & Wang, P. (2018). Asset bubbles and credit constraints. *American Economic Review*, *108*(9), 2590-2628.
- Miller, E. M. (1977). Risk, uncertainty, and divergence of opinion. *The Journal of finance*, *32*(4), 1151-1168.
- Scheinkman, J. A., & Xiong, W. (2003). Overconfidence and speculative bubbles. *Journal of political Economy*, *111*(6), 1183-1220.
- Tirole, J. (1985). Asset bubbles and overlapping generations. *Econometrica: Journal of the Econometric Society*, 1499-1528.