Welfare Cost of Inflation in Production Networks*

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Preliminary and incomplete

Abstract

We study the welfare cost of inflation in a quantitative multi-sector menu cost production networks model with heterogeneity in nominal rigidities across different sectors. Inflation is costly in the model due to aggregate misallocation—the Domar-weighted within-sector price dispersion—and menu costs paid by firms to adjust prices. We calibrate this model to 65 U.S. sectors by matching several moments of the price change distribution within these sectors and the input shares from the input-output table. At 2% inflation, we find that the welfare cost of inflation is 3 times higher compared to a benchmark one-sector economy. Moreover, going from 2% to 4% inflation, the consumption-equivalent welfare cost of inflation increases by 21 basis points, as opposed to only 6.5 basis points in the one-sector model. Of this 21 basis points, 69.5% of the increase is due to menu costs incurred, and only 30.5% due to an increase in aggregate misallocation that results from higher within-sector price dispersion.

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Key Words: Welfare cost of inflation; production networks; menu costs; misallocation; multi-sector model

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1 Introduction

Following large increases after the Covid pandemic, aggregate inflation rates in many countries have yet to fall down to official central bank target levels.[1] This has rekindled discussions about the welfare cost of inflation and whether there are large costs to society if inflation converges to slightly higher levels than the official central bank targets. But are the welfare costs of inflation substantially, if at all, higher at 4% or 3% compared to 2%? If not, then arguably, it might not be worth it for the central bank to generate an aggregate recession to bring inflation down to the target.

In standard sticky price models, welfare cost of inflation in fact does tend to be low at moderate levels of inflation. As pointed out by Burstein and Hellwig (2008) and Nakamura, Steinsson, Sun, and Villar (2018), this is especially the case in a model where sticky prices are modeled using a state-dependent approach, where firms pay a fixed menu cost to adjust nominal prices and face idiosyncratic firm-specific shocks. These models, however, have a one-sector set-up and thus abstract from important heterogeneity across sectors in the moments of the price change distribution. Moreover, as emphasized by Christiano (2015), it is important to incorporate input-output linkages as this is not only realistic in terms of the production structure to match the data on input shares, but it can also critically amplify the welfare cost of inflation.

Some important research questions arise, which we will address in this paper. Theoretically, what determines the welfare cost of inflation in a multi-sector model with unrestricted input-output linkages? Quantitatively, how can various sectoral moments of the price change distribution and data from the input-output table be used to discipline model parameters and thereby provide a credible answer to the welfare cost of inflation in such a model?

To answer these questions, we study welfare cost of inflation in a quantitative multi-sector menu cost model with production networks and idiosyncratic firm-specific shocks. The model features heterogeneity in the distribution of price changes and in input-output linkages across sectors. Our main results are as follows. We show theoretically that welfare cost of inflation is driven by two forces: aggregate misallocation, which is given by Domar-weighted within-sector price dispersion; and menu costs paid by firms to adjust prices.

To provide a quantitative answer to our main research questions, we calibrate the model to 65

[1] For instance, as of December 2025, the 12-month change in core U.S. PCE (FRED series PCEPILFE) is about 3%. The last time it was within 20 basis point of the 2% target was March 2021 (at 2.2%), about four years and half ago; it then jumped to 3.1% in April 2021. Since January 2024, the 12-month core PCE has ranged between 2.6% and 3.1%. As of December 2025, the 12-month change in U.S. PCE (FRED series PCEPI) is about 2.9%. The last time it was within 20 basis point of the 2% target was February 2021 (at 1.8%), about four years and half ago. Since January 2024, the 12-month PCE has ranged between 2.2% and 2.9%.

U.S. sectors, by matching sectoral moments of the price change distribution (frequency of price changes, kurtosis of price changes, standard deviation of price changes, and the fraction of positive price changes) and the input shares from the input-output table. First, in our production networks economy, at 2% inflation, the welfare cost of inflation is 0.146% (in consumption-equivalent units relative to 0% inflation).[2] Next, at 2% inflation, we find that welfare cost of inflation is 3 times higher in our calibrated production networks economy compared to a counterfactual one-sector economy.

Moreover, going from 2% inflation to 4% inflation increases the welfare cost of inflation by 21 basis points in our production networks economy, but only by 6.5 basis points in the one-sector economy. Finally, when going from 2% to 4% inflation, 69.5% of the increase in welfare cost of inflation in our calibrated economy is due to menu costs and only 30.5% due to an increase in aggregate misallocation. Our production networks model thus provides new insights about both the magnitude and the determinants of the welfare cost of inflation.

We do a series of counterfactual exercises to inspect the key mechanisms that drive our main results. First, as our calibrated multi-sector production networks model features both input-output linkages and sectoral heterogeneity in price-setting moments, in order to disentangle the role of input-output linkages in magnifying the welfare cost of inflation, we shut down production networks and compare a multi-sector economy to its one-sector counterpart, which are calibrated to the same set of sectoral pricing moments. We find that the extent of this amplification is around 1.1 at 2% steady-state inflation, increasing to 1.22 at 8%. Thus, without production networks, the amplification of welfare loss at 2% inflation compared to a one-sector economy falls from 3 times to 1.1 times.

Second, we assess the role played by endogenous vs. exogenous price-setting mechanisms. In our multi-sector calibrated model, firms set prices in a state-dependent way by paying a fixed menu cost to adjust prices. In addition, they also get a free price change opportunity, as in time-dependent pricing models. This latter feature is important in order to match the data on a rich set of sectoral pricing moments, especially the kurtosis of price changes. We calibrate two counterfactual multi-sector production network economies, one with purely time-dependent (Calvo) pricing and the other with purely state-dependent pricing. We calibrate them to a smaller, but common, set of sectoral pricing moments compared to our baseline economy, as we drop the kurtosis of price changes as a targeted moment. We find that the welfare cost of inflation at 2% steady-state inflation

[2] We report the welfare cost of inflation in consumption equivalent terms relative to the zero inflation steady-state, i.e., the percentage change in steady-state consumption under zero inflation that makes the household indifferent between the zero-inflation steady state and the steady state with inflation π. If we were to compute welfare cost relative to the flexible price steady-state, the values would be an order of magnitude higher.

is 4.8 times larger in the purely time-dependent pricing model, while it is 27 times smaller in the purely state-dependent pricing model relative to our baseline model. The key reason is that the time-dependent pricing model generates significantly higher increases in price dispersion while the state-dependent pricing model generates barely any increase in price dispersion.

Our paper is motivated by previous work on one-sector menu cost models that computed the welfare cost of inflation, in particular, Burstein and Hellwig (2008) and Nakamura, Steinsson, Sun, and Villar (2018). These papers pointed out the importance of matching moments from the price change distribution to assess welfare cost of inflation and showed how the combination of state-dependent pricing and firm-specific shocks can lead to differences in welfare cost of inflation compared to standard time-dependent pricing models. In particular, the welfare cost becomes small and does not increase significantly with higher levels of inflation in such a state-dependent pricing model.

In this context, Alvarez, Beraja, Gonzalez-Rozada, and Neumeyer (2018) shows that, in low-inflation environments, the frequency of price changes and the dispersion of relative price changes, presumably key statistics to assess the welfare cost of inflation, do not change with inflation, implying negligible welfare costs of increasing inflation around zero inflation. We develop a rich multi-sector production networks model with state-dependent pricing and firm-specific shocks and show how sectoral misallocation aggregates up to an aggregate welfare cost of inflation. Moreover, we provide a quantitative answer to our main research question by matching several sectoral moments of the price change distribution, as well as the input share from input-output tables.

To connect moments of price changes to the welfare cost of inflation, we draw on the literature linking moments of the distribution of price change and monetary non neutrality. Alvarez, Le Bihan, and Lippi (2016) show that the ratio of kurtosis to frequency of price changes is a sufficient statistic for the real effects of monetary shocks, while Luo and Villar (2021) highlight the importance of the skewness of the price change distribution. More specifically, we build on sufficient-statistics results in Baley and Blanco (2021), Alvarez, Lippi, and Oskolkov (2021), which express the propagation of aggregate shocks in terms of moments of the distribution of actions. Cavallo, Lippi, and Miyahara (2023) apply this framework empirically to quantify welfare cost by recovering price-gap statistics from observed price-change size and timing. Our contribution is to extend this logic to multi-sector economies with input-output linkages and sectoral heterogeneity, deriving the welfare cost of inflation when sectors interact and differ in their moments.

Our paper is also related to the work on multi-sector models with production networks and nominal rigidities, such as the recent work of Ghassibe and Nakov (2025) with state-dependent

pricing; Pasten, Schoenle, and Weber (2020a), Rubbo (2023), Afrouzi and Bhattarai (2023) with time-dependent pricing as well as the earlier and important work of La'O and Tahbaz-Salehi (2022) with information rigidities. While this literature focuses on the propagation of monetary and sectoral shocks and the nature of optimal monetary policy in response to sectoral shocks, we study the cost of inflation and nominal rigidities in a state-dependent pricing model. More broadly, the dispersion induced by nominal rigidities in our model corresponds to price distortions and misallocation in production networks models (see, e.g., Baqaee and Farhi, 2020, Bigio and La'O, 2020). The key distinction is that, in our framework, the degree of these distortions varies with inflation, which is paramount to understanding its welfare cost.

More closely related to our paper are Castro (2019) and Christiano (2015). Castro (2019) considers a multi-sector model with production networks, but in a time-dependent pricing set-up. It finds that the welfare costs of inflation increase by an order of magnitude in the multi-sector model compared to a standard one-sector model. Christiano (2015) shows in a one-sector model with a round-about production structure and time-dependent pricing how input-output linkages amplify the welfare cost of inflation. We build on this insight in a quantitative multi-sector model with unrestricted input-output linkages, state-dependent pricing, and firm-specific shocks. In such a model, we are able to discipline the parameters by matching sectoral moments of the price change distribution (frequency of price changes, kurtosis of price changes, standard deviation of price changes, and the fraction of positive price changes) and the input shares from the input-output table.[3]

To match moments of the sectoral price change distribution, we allow sector-specific firm-level trend productivity. This feature of the model relates our paper to Adam and Weber (2019, 2023), who study the implications of firm-level trend productivity for optimal trend inflation. We differ by abstracting from life-cycle dynamics and turnover. In doing so, we shut down the mechanism by which cohort differences in average productivity, together with sticky prices, keep the relative sizes of newer and older firms away from their flexible-price benchmark.

2 Model

We base our analysis on a multi n-sector New Keynesian model with arbitrary production linkages and heterogeneous price stickiness across sectors. We study the steady state of this model, in a

[3]To build the frequency of price changes for the 65 sectors, we use data from Pasten, Schoenle, and Weber (2020a) while the other sectoral moments of the price change distribution are taken from Hong, Klepacz, Pasten, and Schoenle (2023).

setting with menu costs and firm-specific shocks, with positive trend inflation.

2.1. Environment

Time is continuous and is indexed by $t \in \mathbb{R}_+$. The economy consists of a representative household, n sectors with input-output linkages, and monetary and fiscal authorities. In each sector $i \in [n] \equiv \{1,2,\ldots,n\}$, a unit measure of monopolistically competitive firms use labor and goods from all sectors to produce and supply to a competitive final good producer within the same sector. These final goods are sold to the household and to firms in all sectors.

Household. The representative household demands the final goods produced by each industry, supplies labor in a competitive market, and holds nominal bonds with nominal yield i_t. The household's preferences over consumption C and labor supply L is $U(C) - V(L)$, where U and V are strictly increasing with Inada conditions, and $U''(.) < 0$, $V''(.) > 0$. Household solves:

$$\begin{aligned} & \max_{\{(C_{i,t})_{i\in[n]},L_t,B_t\}_{t\geq 0}} \int_0^\infty e^{-\rho t}[U(C_t) - V(L_t)]\mathrm{d}t \\ s.t. \quad & \textstyle\sum_{i\in[n]} P_{i,t}C_{i,t} + \dot{B}_t \leq W_t L_t + i_t B_t + \text{Profits}_t - P_t T_t, \qquad C_t \equiv \Phi(C_{1,t},\ldots,C_{n,t}) \end{aligned} \tag{1}$$

Here, $\Phi(.)$ defines the consumption index C_t over the household's consumption from sectors $(C_{i,t})_{i\in[n]}$. It is degree one homogeneous, strictly increasing in each $C_{i,t}$, satisfying Inada conditions. L_t is labor supply at wage W_t, $P_{i,t}$ is sector i's final good price, B_t is demand for nominal bonds, Profits_t denote all firms' profits rebated to the household, and T_t is a lump-sum tax, in units of aggregate consumption.

Final Good Producers. A competitive final good producer in each industry i buys from a continuum of intermediate firms in its sector, indexed by $ij : j \in [0,1]$, and produces a final sectoral good using a CES production function. The profit maximization problem of this firm is:

$$\max_{(Y^d_{ij,t})_{j\in[0,1]}} P_{i,t}Y_{i,t} - \textstyle\int_0^1 P_{ij,t}Y^d_{ij,t}\mathrm{d}j \quad s.t. \quad Y_{i,t} = \left[\int_0^1 A_{ij,t}^{\eta_i^{-1}}(Y^d_{ij,t})^{1-\eta_i^{-1}}\mathrm{d}j\right]^{\frac{1}{1-\eta_i^{-1}}} \tag{2}$$

where $Y^d_{ij,t}$ is the producer's demand for variety ij at price $P_{ij,t}$, $Y_{i,t}$ is its production at price $P_{i,t}$, $A_{ij,t}$ is a firm-specific taste shock, and $\eta_i > 1$ is the substitution elasticity across varieties in i.

Thus, demand for variety ij is:

$$Y^d_{ij,t} = \mathscr{D}(P_{ij,t}/P_{i,t}; Y_{i,t}) \equiv A_{ij,t}Y_{i,t}\left(\frac{P_{ij,t}}{P_{i,t}}\right)^{-\eta_i} \quad \text{where} \quad P_{i,t} = \left[\int_0^1 A_{ij,t}P_{ij,t}^{1-\eta_i}\mathrm{d}j\right]^{\frac{1}{1-\eta_i}} \tag{3}$$

Final good producers define a unified good for each industry and have zero value added due to being competitive and having a constant returns to scale (CRS) production.

Intermediate Goods Producers. The intermediate good producer ij uses labor as well as the sectoral goods as inputs and produces with the following CRS production function:

$$Y^s_{ij,t} = Z_{ij,t} F_i(L_{ij,t}, X_{ij,1,t}, \ldots, X_{ij,n,t}) \tag{4}$$

where $Z_{ij,t}$ is ij's idiosyncratic productivity, $L_{ij,t}$ is firm ij's labor demand, and $X_{ij,k,t}$ is its demand for sector k's final good. The function F_i is strictly increasing in all arguments with Inada conditions. We assume that

$$\mathrm{d}\log Z_{ij,t} = \mu_i \mathrm{d}t + \sigma_i \mathrm{d}\mathcal{W}_{ij,t} \tag{5}$$

where $\mathcal{W}_{ij,t}$ is a Wiener process, μ_i and σ_i are, respectively, sector i's productivity drift and standard deviation of idiosyncratic shocks. Importantly, we assume that $A_{ij,t} Z_{ij,t}^{\eta_i - 1} = 1$[4]. The firm's total cost for producing output Y, given $\mathbf{P}_t \equiv (W_t, P_{i,t})_{i\in[n]}$, is:

$$\mathscr{C}_{ij}(Y; \mathbf{P}_t, Z_{ij,t}) \equiv \min_{L_{ij,t}, X_{ij,k,t}} W_t L_{ij,t} + \sum_{k\in[n]} P_{k,t} X_{ij,k,t} \quad s.t. \quad Z_{ij,t} F_i(L_{ij,t}, X_{ij,1,t}, \ldots, X_{ij,n,t}) \geq Y \tag{6}$$

Let sector i's Domar weight be $\lambda_{i,t} \equiv \frac{P_{i,t} Y_{i,t}}{P_t C_t}$. In each sector i, firms can change prices by paying a total menu cost $\lambda_{i,t}\chi_i > 0$ denominated in units of labor. Besides the menu cost, they receive free i.i.d. price change opportunities that arrive at Poisson rate $\theta_i > 0$. Concretely, firm ij's nominal adjustment cost takes the form

$$\mathrm{d}\varphi_{ij,t} = \begin{cases} 0 & \text{, if } \mathrm{d}P_{ij,t} = 0 \text{ or } \mathrm{d}N_{ij,t} = 1 \\ W_t \lambda_{i,t} \chi_i & \text{, if } \mathrm{d}P_{ij,t} > 0 \text{ and } \mathrm{d}N_{ij,t} = 0 \end{cases}$$

where $N_{ij,t}$ is a Poisson counter with arrival rate θ_i. Given its cost and demand functions, firm ij chooses a sequence of prices to maximize its expected discounted flow of real profits. The sequential problem of the firm is given by

$$\max_{(P_{ij,t+s})_{s=0}^{\infty}} \mathbb{E}\left\{ \int_0^\infty e^{-\rho s} \frac{U'(C_{t+s})}{P_{t+s}} \left[\left((1-\tau_{i,t}) P_{ij,t+s} \mathscr{D}\left(\frac{P_{ij,t+s}}{P_{i,t+s}}; Y_{i,t+s}, A_{ij,t+s}\right) - \mathscr{C}_{ij}(Y^s_{ij,t+s}; \mathbf{P}_{t+s}, Z_{ij,t+s}) \right) \mathrm{d}s - \mathrm{d}\varphi_{ij,t+s} \right] \right\}$$

$$s.t. \quad Y^S_{ij,t+s} \geq \mathscr{D}(P_{ij,t+s}/P_{i,t+s}; Y_{i,t+s}, A_{ij,t+s}), \forall s \geq 0$$

where $e^{-\rho s}\frac{U'(C_{t+s})}{P_{t+s}}$ is the stochastic discount rate, $\tau_{i,t}$ is the tax/subsidy rate on sales.

Were prices flexible, maximizing net present value of real profits would be equivalent to choosing *ideal* prices, denoted by $P^*_{ij,t}$, that maximized firms' static profits within every instant. The notion of *ideal* prices plays a key role and is useful to establish here, because as we later show, in our recursive solution method, the firm's sufficient state variable is the price gap, the deviation of its

[4]This assumption has been used in the context of state-dependent pricing models (Woodford, 2009, Alvarez and Lippi, 2014) and it ensures that a firm's price gap is its only state variable to solve its problem.

actual price from its ideal price. Firms ideal prices solve:

$$P^*_{ij,t} = \arg\max_{P_{ij,t}} (1-\tau_{i,t}) P_{ij,t} \mathscr{D}(P_{ij,t}/P_{i,t}; Y_{i,t}, A_{ij,t}) - \mathscr{C}_{ij}(Y^s_{ij,t}; \mathbf{P}_t, Z_{ij,t}) \quad s.t. \quad Y^s_{ij,t} \geq \mathscr{D}(P_{ij,t}/P_{i,t}; Y_{i,t}, A_{ij,t})$$

Monetary and Fiscal Policy. The monetary authority directly controls the path of nominal GDP, $\{M_t \equiv P_t C_t\}_{t\geq 0}$, where P_t is the consumer price index (CPI).[5] The monetary authority sets $\frac{\dot{M}_t}{M_t} = \pi$, where steady-state inflation is given by π. The fiscal authority taxes or subsidizes intermediate firms' sales in each sector i at a possibly time-varying rate $\tau_{i,t}$, lump-sum transferred back to the household.

Market Clearing. Having outlined the environment, we now complete the model description by specifying the market-clearing conditions and the (balanced) government budget constraint. In equilibrium, bonds are in zero net supply, such that $B_t = 0, \forall\, t \geq 0$. Government budget balance therefore implies that total subsidies given to firms across sectors must be equal to lump-sum taxes from consumers, $\sum_{i\in[n]} \int_0^1 \tau_{i,t} P_{ij,t} Y_{ij,t} \mathrm{d}j = -P_t T_t, t \geq 0$.

Total production in sector k is used either for consumption or as an intermediate inputs in production, resulting in $Y_{k,t} = C_{k,t} + \sum_{i\in[n]} \int_0^1 X_{ij,k,t} \mathrm{d}j, k \in [n], \forall\, t \geq 0$. Finally, labor supply is equal to labor demand, which comes either from demand for production or from demand to pay menu costs, $L_t = \sum_{i\in[n]} \int_0^1 L_{ij,t} \mathrm{d}j + \sum_{i\in[n]} \lambda_i \chi_i \int_0^1 \mathbf{1}_{\{\mathrm{d}P_{ij,t}\neq 0 \text{ and } \mathrm{d}N_{ij,t}=0\}} \mathrm{d}j$.

2.2. Recursive Stationary Equilibrium

We solve the recursive formulation of the firm's optimal pricing problem within a recursive stationary equilibrium with steady-state inflation π, as characterized by the following HJB-VI equation

$$\rho v_i(x) = \max\left\{\rho\big(v(x^*_i) - \phi_i\big), f_i(x) + (\mu_i - \pi) v_i'(x) + \frac{\sigma_i^2}{2} v_i''(x) + \theta_i\big(v(x^*_i) - v_i(x)\big)\right\} \tag{7}$$

where x is the firm's price gap, defined as $x \equiv \log\left(P_{ij}/P^*_{ij}\right)$, and it captures how far is firm's current price from its ideal price. $x^*_i \equiv \arg\max_y v_i(y)$ captures the price the firm would set if it gets to reset prices, i.e., the *reset* gap, with $v_i(x^*) = \max_y v_i(y)$ being the value of the firm at the reset gap. $f_i(x) \equiv \left(e^x - \frac{\eta_i - 1}{\eta_i}\right) e^{-\eta_i x}$ is firm's instantaneous return function when the gap is x. ϕ_i is a 'reduced-form' menu cost parameter that is a function of χ_i, taxes, and equilibrium objects. Moreover, note that $v_i(.)$ is sector-specific as different sectors vary in their firm-level productivity processes and probability of free price adjustment.

The optimal policy function that solves Equation (7) is characterized by a reset point and (S, s) bands, $\{x^*_i, \underline{x}_i, \bar{x}_i\}$. Whenever the price gap is in the interior of the bands, i.e. $x \in (\underline{x}_i, \bar{x}_i)$, the firm does

[5] Such policy can be implemented by a cash-in-advance constraint (e.g. La'O and Tahbaz-Salehi, 2022), money in utility (e.g. Golosov and Lucas, 2007) or nominal GDP growth targeting (e.g. Afrouzi and Yang, 2019).

not adjust its price, and the gap evolves according to $\mathrm{d}x = (\mu_i - \pi)\,\mathrm{d}t + \sigma_i\,\mathrm{d}\mathcal{W}$. If, instead, the price gap reaches one of the bounds, $x \notin (\underline{x}_i, \bar{x}_i)$, the firm adjusts by *resetting* its gap to x_i^*. Furthermore, optimality of the policy function requires that its associated value function satisfy value matching, i.e., $v_i(\underline{x}_i) = v_i(x_i^*) - \phi_i$ and $v_i(\bar{x}_i) = v_i(x_i^*) - \phi_i$, and smooth pasting, i.e., $v_i'(\underline{x}_i) = v_i'(\bar{x}_i) = 0$, and the reset gap satisfies $v_i'(x_i^*) = 0$.

Finally, the economy has a steady-state distribution of price gaps G_i, with density $\mathrm{g}_i(x)$, that solves the following Kolmogorov forward equation with its boundary conditions:

$$-(\mu_i - \pi)\frac{\mathrm{d}\mathrm{g}_i(x)}{\mathrm{d}x} + \frac{\sigma_i^2}{2}\frac{\mathrm{d}^2\mathrm{g}_i(x)}{\mathrm{d}x^2} - \theta_i \mathrm{g}_i(x) = 0, \quad \forall x \in (\underline{x}, \bar{x}), \quad x \neq x_i^*$$

where $\int_{\underline{x}_i}^{\bar{x}_i} \mathrm{g}_i(x)\mathrm{d}x = 1$, $\mathrm{g}_i(\underline{x}_i) = \mathrm{g}_i(\bar{x}_i) = 0$. The derivation of the HJB-VI is in Appendix B.4.

A recursive stationary equilibrium with steady-state inflation π is defined below:

Definition 1. *A **recursive stationary equilibrium with steady-state inflation** π is an allocation for the representative household, $\mathscr{A}_h = \{C, L, B, (C_i)_{i\in[n]}\}$, an allocation for the final goods firms, $\mathscr{A}_{final} = \{(Y_i, Y_{ij}^d)_{i\in[n]}\}$, value functions, $(v_i(x))_{i\in[n]}$, and policy functions $(\{x_i^*, \underline{x}_i, \bar{x}_i\})_{i\in[n]}$ for the monopolistic competitive firms, a distribution of price gaps $(\mathsf{G}_i(x))_{i\in[n]}$, a set of monetary policy and fiscal policies $\mathscr{A}_g = \{M_t, T, (\tau_i)_{i\in[n]}\}_{t\geq 0}, \dot{M}_t = \pi M_t$, and a set of aggregate relative prices $\mathscr{P} = \{(P_{i,t}/W_t)_{i\in[n]}, P_t/W_t, i\}_{t\geq 0}$ such that $\mathscr{P}, \mathscr{A}_h$, and $\mathscr{A}_{final}$ are time-invariant and*

1. *Given $\mathscr{P}$ and $\mathscr{A}_g$, $\mathscr{A}_h$ solves the household's problem in Equation (1)*
2. *Given $\mathscr{P}$ and $\mathscr{A}_g$, $\mathscr{A}_{final}$ solves the final good price problem in Equations (2) and (3)*
3. *Given $\mathscr{P}$, $\mathscr{A}_g$, and $\mathscr{A}_{final}$, $(v_i(x))_{i\in[n]}$ and $(\{x_i^*, \underline{x}_i, \bar{x}_i\})_{i\in[n]}$ solve Equation (7)*
4. *$(\mathsf{G}_i(x))_{i\in[n]}$ is stationary*
5. *Labor, bonds, and final sectoral goods markets clear and government budget constraint is satisfied:*

$$Y_k = C_k + \sum_{i\in[n]} \int_0^1 X_{ij,k,t}\mathrm{d}j \;\forall k \in [n]$$

$$L = \sum_{i\in[n]} \int_0^1 L_{ij,t}\mathrm{d}j + \sum_{i\in[n]} \lambda_i \chi_i \int_0^1 \mathbf{1}_{\{\mathrm{d}P_{ij,t}\neq 0 \text{ and } \mathrm{d}N_{ij,t}=0\}}\mathrm{d}j$$

$$B = 0, \quad \sum_{i\in[n]} \int_0^1 \tau_i P_{ij,t} Y_{ij,t} dj = P_t T, \quad \forall t \geq 0$$

3 Welfare Cost of Inflation

We now study, both theoretically and quantitatively, the consumption-equivalent welfare cost of moving from a zero-inflation steady-state to a higher inflation rate steady-state. Our main focus is

on how production networks make inflation more costly.

3.1. Theoretical Decomposition of Welfare Cost of Inflation

We measure the welfare cost of inflation π as the *log* consumption-equivalent loss in zero-inflation steady-state consumption that makes the representative household indifferent to the steady state with inflation π. Let $\Lambda(\pi)$ denote this loss[6], defined by

$$\mathscr{U}(C(\pi), L(\pi)) = \mathscr{U}(e^{-\Lambda(\pi)} C(0), L(0)) \tag{8}$$

where $C(\pi)$ and $L(\pi)$ are steady-state consumption and labor at inflation π, and $C(0)$, $L(0)$ are their zero-inflation counterparts. For our baseline analysis, we assume Golosov and Lucas (2007) preferences, $U(C) = \log(C)$, $V(L) = L$, use Cobb-Douglas consumption aggregator, $\Phi(C_{1,t}, \ldots, C_{n,t}) = \prod_{i\in[n]} \left(C_{i,t}/\beta_i\right)^{\beta_i}$, $\sum_{i\in[n]} \beta_i = 1$, and Cobb-Douglas production function $F_i(L_{ij,t}, X_{ij,1,t}, \ldots, X_{ij,n,t}) = \left(L_{ij,t}/\alpha_i\right)^{\alpha_i} \prod_{k\in[n]} \left(X_{ij,k,t}/a_{ik}\right)^{a_{ik}}$, $\alpha_i + \sum_{k\in[n]} a_{ik} = 1, \forall i \in [n]$.

To disentangle the effects of inflation on welfare, we first derive the notion of aggregate productivity of the economy. Starting from the sectoral production function $F_i(L_{ij,t}, X_{ij,1,t}, \ldots, X_{ij,n,t})$, given a distribution of firms' prices in this sector, we can derive a notion of aggregate sectoral production function with the following form

$$Y_{i,t} = \frac{Z_i^f}{D_{i,t}} \times F_i(L_{it}, X_{i,1,t}, \ldots, X_{i,n,t}) \quad \text{where} \quad D_{i,t} \equiv Z_i^f \times \int_0^1 \frac{A_{ij,t}}{Z_{ij,t}} \left(\frac{P_{ij,t}}{P_{i,t}}\right)^{-\eta_i} \mathrm{d}j \tag{9}$$

where $L_{i,t} = \int_0^1 L_{ij,t}\mathrm{d}j$, $X_{i,k,t} = \int_0^1 X_{ij,k,t}\mathrm{d}j$ are, respectively, the total labor and input k employed in sector i at time t, $Z_i^f \equiv \left[\int_0^1 A_{ij,t} Z_{ij,t}^{\eta_i - 1} \mathrm{d}j\right]^{\frac{1}{\eta_i - 1}}$ is the aggregate sectoral productivity in the flexible-price equilibrium, and $D_{i,t}$ is the measure of inefficient price dispersion introduced by price stickiness. Under our assumption that $A_{ij,t} Z_{ij,t}^{\eta_i - 1} = 1$, $Z_i^f = 1$, and we get $D_{i,t} \equiv \int_0^1 \frac{A_{ij,t}}{Z_{ij,t}} \left(\frac{P_{ij,t}}{P_{i,t}}\right)^{-\eta_i} \mathrm{d}j$.

In the stationary equilibrium, we have that aggregate quantities are constant, and that the distribution of firms is stationary, so we have $L_{i,t} = L_i, X_{i,k,t} = X_{i,k}, k \in [n], D_{i,t} = D_i, \forall i \in [n]$. From the aggregate sectoral production, one can define the aggregate sectoral marginal cost, $\mathsf{MC}_{i,t}$ from the cost minimization problem subject to the production function in Equation (9). That is,

$$\mathsf{MC}_{i,t} \equiv \min_{L_{i,t}, (X_{i,k,t})_{k\in[n]}} W_t L_{i,t} + \sum_{k\in[n]} P_{k,t} X_{i,k,t} \quad s.t. \quad \frac{Z_i^f}{D_i} \times F_i(L_{i,t}, X_{i,1,t}, \ldots, X_{i,n,t}) \geq Y \tag{10}$$

where $D_{i,t} = D_i$ in the stationary equilibrium. Then, we can define the sectoral markup as $\mathscr{M}_i \equiv P_{i,t}/\mathsf{MC}_{i,t}$. In our baseline results, we assume that taxes are set such that $\mathscr{M}_i = 1, \forall i \in [n]$.

Using the household optimality conditions and the labor market clearing, we can also define

[6] For reporting in percent, set $\lambda(\pi) \equiv 1 - e^{-\Lambda(\pi)}$. Up to first order, $\lambda(\pi) \approx \Lambda(\pi)$.

an aggregate production function for the economy $C(\pi) = Z(\pi)L(\pi)$, where $Z(\pi)$ is the aggregate productivity of the economy, defined as the steady-state consumption per unit of steady-state labor. Under the assumption of Cobb-Douglas production functions,

$$Z(\pi) = \frac{1}{\exp\left(\boldsymbol{\beta}'(\mathbf{I}-\mathbf{A})^{-1}\left(\log(\mathscr{M}_i D_i)\right)_{i\in[n]}\right)} \times \frac{1}{\frac{\boldsymbol{\alpha}'(\text{diag}(\mathscr{M}_i))^{-1}\boldsymbol{\lambda}}{\left(1-\sum_{i\in[n]}\frac{\lambda_i\chi_i}{L}\mathsf{N}_i^{menu}\right)}} \tag{11}$$

where $\left(\log(\mathscr{M}_i D_i)\right)_{i\in[n]} \equiv (\log(\mathscr{M}_1 D_1), \log(\mathscr{M}_2 D_2), \ldots, \log(\mathscr{M}_n D_n))'$, $\text{diag}(\mathscr{M}_i)$ is the diagonal matrix with diagonal entries equal to the sectoral markups, $\mathscr{M}_i$, and $\boldsymbol{\lambda} = (\lambda_i)_{i\in[n]} = (P_i Y_i/PC)_{i\in[n]}$ is the vector of sales-based Domar weights. When we further assume $U(C) = \log(C)$ and $V(L) = L$, as in our baseline quantitative case, we get

$$Z(\pi) = \frac{1}{\exp\left(\boldsymbol{\beta}'(\mathbf{I}-\mathbf{A})^{-1}\left(\log(\mathscr{M}_i D_i)\right)_{i\in[n]}\right)} \times \frac{1}{\boldsymbol{\alpha}'\left(\text{diag}(\mathscr{M}_i)\right)^{-1}\boldsymbol{\lambda} + \sum_{i\in[n]}\lambda_i\chi_i\mathsf{N}_i^{menu}} \tag{12}$$

which clarifies the effects of inflation on aggregate productivity.

Equation (11) and Equation (12) thus show that inflation affects aggregate productivity through its impact on sectoral markups, $\mathscr{M}_i$, sectoral price dispersion, D_i, and number of firms paying the menu cost, N_i^{menu}. We can then use the expression for $Z(\pi)$ to substitute for $C(\pi)$ in Equation (8) to express the welfare costs of inflation in terms of aggregate productivity and aggregate labor.

We are now ready to state our main theoretical result on welfare cost of inflation.[7]

Theorem 1. *When preferences are $U(C) = \log(C)$, $V(L) = L$, the consumption aggregator and production functions are Cobb-Douglas, and taxes are such that sectoral markups equal to one, the welfare cost of inflation π relative to zero-inflation steady state is*

$$\Lambda(\pi) = \sum_{i\in[n]}\lambda_i \times \left[\log(D_i(\pi)) - \log(D_i(0))\right] + \sum_{i\in[n]}\lambda_i\chi_i\left[\mathsf{N}_i^{menu}(\pi) - \mathsf{N}_i^{menu}(0)\right] \tag{13}$$

where $\lambda_i = e_i'\boldsymbol{\lambda}$, with $\boldsymbol{\lambda} = (\mathbf{I}-\mathbf{A}')^{-1}\boldsymbol{\beta}$ is the sales-based Domar weights when the sectoral markups are equal to one, and $\mathsf{N}_i^{menu}(\pi)$ is the measure of firms that pay the menu cost in sector i when steady-state inflation is π.

Proof. See Appendix C.

With sticky prices, inflation affects within-sector relative price dispersion, generating misallocation across firms and lowering sector-level productivity. In multi-sector economies, the extent to which each sector affects aggregate productivity is proportional to their cost-based Domar weights,

[7] In Appendices B.5 and B.8, we derive formally and completely the expressions for sectoral price dispersion, aggregate sectoral production functions, and aggregate productivity that we refer to above.

which under sectoral markups equal to one are equal to sales-based Domar weights, λ_i. Since we are calculating the welfare cost of inflation relative to a zero inflation steady state, we also take into account the price dispersion in that steady state that is present due to idiosyncratic firm-level shocks. Taking everything together, we have the first term on the right-hand side of Equation (13).

In the presence of menu costs, inflation also affects the number of firms paying menu costs in each one of the sectors, which is also a source of welfare cost – now, labor is used not only for production, but also, to pay for menu costs. In multi-sector economies, each sector i will have a different number of firms paying menu costs, along with different menu costs. Taking everything together, we have the second term on the right-hand side of Equation (13).

Before heading to the numerical and quantitative results, it is illustrative to analytically consider a particular case in which (1) the profit function is quadratic; (2) there is no drift; (3) $\pi = 0$. In this case, one can show (Baley and Blanco, 2021, Cavallo, Lippi, and Miyahara, 2023) that the price gap dispersion at zero inflation is given by

$$\log(D(0)) \approx \frac{\eta}{2} \times \frac{\text{Var}(\Delta p) \times \text{Kur}(\Delta p)}{6} \tag{14}$$

where $\text{Var}(\Delta p)$ and $\text{Kur}(\Delta p)$ are, respectively, the variance and the kurtosis of *price changes*. Equation (14), along with Equation (13), strongly suggest that both variance and kurtosis of price changes, at a minimum, are key to understanding the welfare costs of inflation. This observation partially motivates the calibration strategy we pursue next.

3.2. Quantitative Analysis of Welfare Costs of Inflation

To quantify the welfare costs of inflation in production networks due to sticky prices, we calibrate the stationary equilibrium of the model to the U.S. economy using sectoral disaggregated data.

3.2.1. Data. To quantify the welfare costs of inflation, we use the Input-Output tables for the U.S. economy from the Bureau of Economic Analysis (BEA) at the Summary level disaggregation in 2012 (65 sectors) to get the sectoral consumption shares ($\boldsymbol{\beta}$) and the input-output matrix ($\mathbf{A}$), and compute the Domar weights ($\boldsymbol{\lambda}$). Figure A.3 shows the distribution of sectoral Domar weights and consumption shares in the data.

We also use sectoral frequency of price adjustments from Pasten, Schoenle, and Weber (2020b) and moments of the distribution of sectoral price changes in the U.S. from Hong, Klepacz, Pasten, and Schoenle (2023). For the moments of the distribution of sectoral price changes in the U.S., the sectoral data from Hong, Klepacz, Pasten, and Schoenle (2023) is sorted into deciles according to their frequency of price adjustment, and then within each decile, the average standard deviation

of price changes, the kurtosis of price changes, and the fraction of positive price changes are calculated . These statistics, along with the sectoral frequency of price adjustment, are used to calibrate $(\phi_i, \theta_i, \sigma_i, \mu_i)_{i\in[n]}$.

3.2.2. Calibration Strategy. We calibrate the model at the monthly frequency, and the calibration consists of both externally and internally calibrated parameters. For externally calibrated parameters, besides the sectoral consumption shares ($\boldsymbol{\beta}$) and input-output matrix ($\mathbf{A}$) for which we use the Input-Output tables from the BEA, we need to calibrate the discount rate (ρ), the sectoral elasticities of substitution $(\eta_i)_{i\in[n]}$, and the steady-state growth rate of nominal GDP (π). For the discount rate, we set $\rho = -\log(0.96)/12$, which is equivalent to a discount rate of 4% annual. For the elasticity of substitution across varieties within-sector, $(\eta_i)_{i\in[n]}$, we assume $\eta_i = 4, \forall\, i \in [n]$. For the steady-state growth rate of nominal GDP, we set $\pi = \log(1.02)/12$, which corresponds to a 2% annual growth rate and is chosen to reflect the 2% U.S. inflation target.

For the internally calibrated parameters $(\phi_i, \theta_i, \sigma_i, \mu_i)$, we target the frequency of price adjustment, standard deviation, kurtosis, and fraction of positive price changes. We use the sectoral frequencies directly from Pasten, Schoenle, and Weber (2020b). As the other moments of the distribution of sectoral price changes from Hong, Klepacz, Pasten, and Schoenle (2023) are only available as averages across deciles sorted by frequency, we use linear interpolation to impute the sector-specific targets. For each sector $i \in [n]$, we compare its frequency against the frequencies of the ten deciles and interpolate the corresponding moments.[8] Finally, we use simulated method of moments (SMM) to internally calibrate $(\phi_i, \theta_i, \sigma_i, \mu_i)$[9].

For the one-sector economy calibration, we target the median of the moments used in the multi-sector calibration. The target values for the one-sector economy are 0.1595 for frequency of price adjustment, 0.0740 for standard deviation, 3.8871 for kurtosis, and 0.6170 for fraction of positive price changes.

We present in Figures A.4 and A.6 in Appendix A examples of model solutions for three specific sectors. There, we show the solutions for the value function; the reset point and (S, s) bands, $\{x_i^*, \underline{x}_i, \bar{x}_i\}$; the density of the stationary distribution of price gaps, $g_i(x)$; the density of the distribution of price changes; and the time until the next price change as a function of the price

[8] For instance, if sector i has a frequency $\text{freq}_i = 0.25$ and the two closest decile average frequencies are $\text{freq}^l = 0.2$, $\text{freq}^h = 0.3$, we set $\text{kurtosis}_i = 0.5 \times \text{kurtosis}^l + 0.5 \times \text{kurtosis}^h$

[9] That is, for each sector $i \in [n]$, we solve the following minimization problem: $\min_{(\phi_i,\theta_i,\sigma_i,\mu_i)} (\mathbf{m}(\phi_i, \theta_i, \sigma_i, \mu_i) - \mathbf{m}_i^{\text{data}})' \mathbf{W} (\mathbf{m}(\phi_i, \theta_i, \sigma_i, \mu_i) - \mathbf{m}_i^{\text{data}})$, where $\mathbf{m}(\phi_i, \theta_i, \sigma_i, \mu_i)$ is the vector of moments generated by the model with parameters $(\phi_i, \theta_i, \sigma_i, \mu_i)$, and $\mathbf{m}_i^{\text{data}}$ is its data counterpart and it is indexed by i because for each sector we target a different set of moments, and $\mathbf{W}$ is a diagonal weight matrix, which we set to identity.

gap.

When solving for the value function and optimal policy in Equation (7), we work with a 'reduced-form' menu cost ϕ_i, which depends, in our baseline results, on the structural menu cost χ_i, the elasticity of substitution η_i, and the stationary distribution of price gaps in sector i. Because there is a one-to-one mapping between χ_i and ϕ_i, conditional on the calibrated parameters $(\phi_i, \theta_i, \sigma_i, \mu_i)$ we can recover χ_i, which is crucial for evaluating welfare under different steady-state inflation rates. That is, for counterfactuals, we recover the structural parameter χ_i[10]. Appendix B.4 and Equation (C.4) show, respectively, how this relationship arises and its final expression, once we impose the assumptions in our baseline results.

3.2.3. Numerical Results. We first show that abstracting from the presence of production networks and sectoral heterogeneity in moments can substantially understate the welfare costs of inflation and the impacts of raising steady-state inflation on welfare. As Table 1 and Figure 1 show, accounting for production networks and sectoral heterogeneity in moments amplify the welfare cost of inflation by ≈ 3 times relative to a one-sector economy at 2% steady-state inflation. Furthermore, going from 2% steady-state inflation to 4% steady-state inflation increases the welfare cost of inflation by around 21 basis points in our baseline economy. In contrast, it increases by around 6.5 basis points in the one-sector economy.

Moving on to what accounts for the welfare costs of inflation, Figure 2 shows that both the price-dispersion and the menu-cost components are quantitatively important. Menu costs, however, clearly play the larger role, as shown in Table 2. The contribution of menu cost paid on welfare cost of inflation is 73.8% at 2% inflation and it decreases slightly to 67.4% at 8% inflation. It is nevertheless the dominant quantitative force at all levels of inflation.

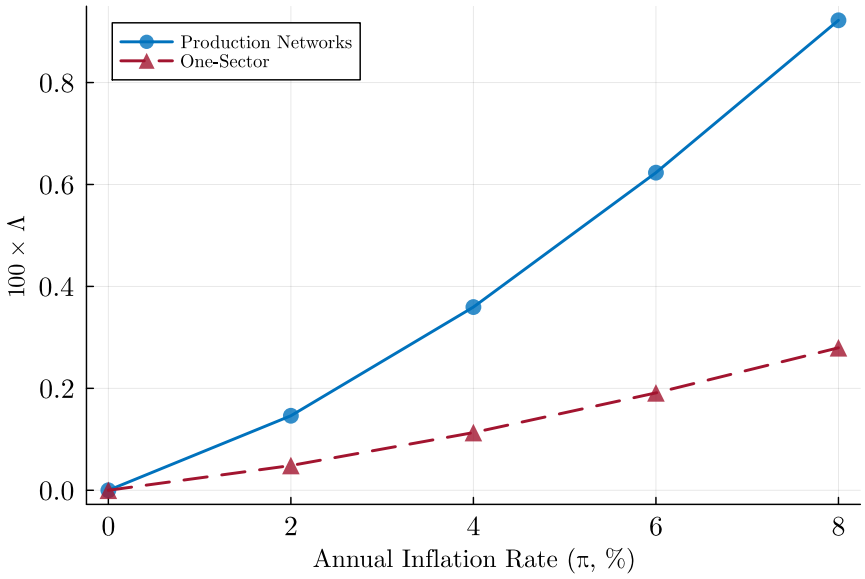
Table 1: Welfare Costs of Inflation: Production Networks × One-Sector

π_{ss}	Production Networks	One-Sector	Ratio
2.0	0.1462	0.0485	3.01
4.0	0.3595	0.1132	3.18
6.0	0.6232	0.1911	3.26
8.0	0.9221	0.2796	3.30

Notes: This table shows the welfare costs of inflation (relative to zero inflation) in the production networks and the one-sector economies. The welfare cost is measured in percent (%). Ratio = $^{\text{Production Networks}}/_{\text{One-Sector}}$.

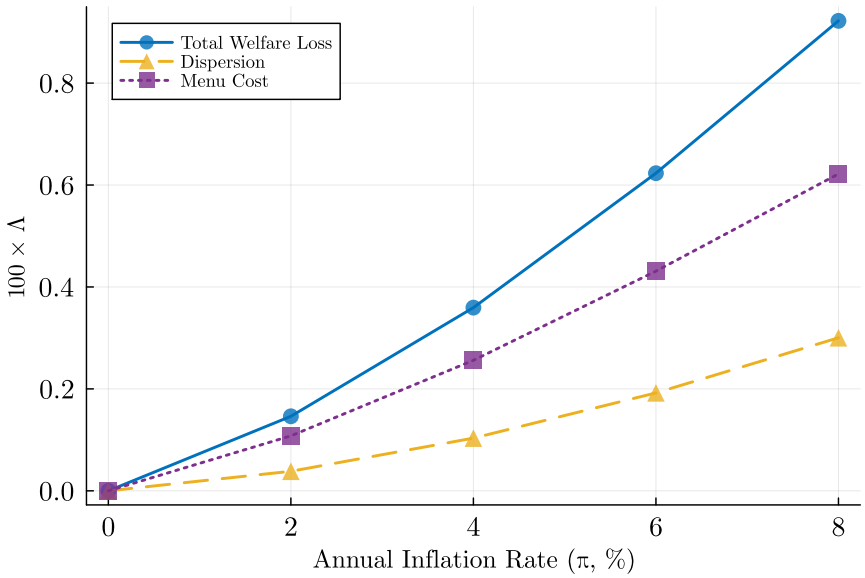
[10]Total menu costs in the economy represent 0.228% of total sales. This magnitude is consistent with existing literature, such as the 0.70% direct estimate from U.S. supermarket chains in Levy, Bergen, Dutta, and Venable (1997) and the 0.34% reported in the benchmark model of Midrigan (2011).

Figure 1: Welfare Cost of Inflation



Notes: This figure plots the welfare costs of inflation as we change the steady-state inflation rate. We measure the welfare cost of inflation in terms of consumption equivalent, i.e., the percentage change in steady-state consumption under zero inflation that makes the representative agent indifferent between the zero-inflation steady state and the steady state with inflation π. This figure compares the production networks economy with a one-sector economy.

Figure 2: Welfare Cost of Inflation Decomposition



Notes: This figure plots the welfare costs of inflation as we change the steady-state inflation rate, along with the contribution coming from dispersion and coming from menu cost. We measure the welfare cost of inflation in terms of consumption equivalent, i.e., the percentage change in steady-state consumption under zero inflation that makes the representative agent indifferent between the zero-inflation steady state and the steady state with inflation π.

Table 2: Welfare Costs of Inflation Decomposition

π_{ss}	Dispersion	Menu Cost	Menu Cost Share
2.0	0.0384	0.1079	73.8
4.0	0.1034	0.2561	71.2
6.0	0.1921	0.4311	69.2
8.0	0.3003	0.6218	67.4

Notes: This table shows the welfare cost of inflation contribution of dispersion and menu-cost components to the welfare costs of inflation (relative to zero inflation) and well as the menu-cost component share in the baseline economy for different steady-state inflation rates. All columns measured in percent (%).

3.3. Inspecting the Mechanisms Behind Welfare Costs of Inflation

We now present additional results and do a series of counterfactuals to inspect the mechanisms that drive our key results presented above.

3.3.1. Change in price change distribution moments as inflation increases. In Equation (13), we show that the welfare cost of inflation is intimately connected to sectoral price dispersions and the number of firms paying the menu costs. Therefore, to further understand what drives the welfare cost of inflation that we find, it is important to see how these moments change as steady-state inflation increases. In Table 3, we show how the 20^{th}, 40^{th}, 60^{th}, and 80^{th} percentiles of the distribution of sectoral log(Dispersion), frequency of price changes, and fraction of non-free price changes with steady-state inflation.

Table 3 corroborates Figure 2 and Table 2 in showing that the menu-cost component is relatively more important for the welfare cost of inflation. Consistent with this, log(Dispersion) rises somewhat modestly across the distribution as steady-state inflation increases, whereas the frequency of price changes and especially, the fraction of non-free price changes, rise more clearly across the distribution as steady-state inflation increases. The fraction of non-free price changes rising with steady-state inflation across the distribution in turn provides the explanation for why the menu cost paid increases with steady-state inflation, as we showed earlier in Figure 2 and Table 2.

For completeness, in Table A.1 we show how the standard deviation of price changes; the kurtosis of price changes; and the fraction of positive price changes, the three other moments we targeted in our calibration, are affected as we vary steady-state inflation. There, as expected, the moment that shows the most clear pattern as we increase steady-state inflation is the fraction of positive price changes, which increases monotonically. Moreover, the standard deviation of price changes increases while the kurtosis of price changes decreases as we increase steady-state inflation. This opposite pattern across the standard deviation and the kurtosis of price changes provides more

insight into why the dispersion does not change much as the inflation rate increases, in line with Equation (14).[11]

Table 3: Moments of the Sectoral Price-change Distribution across Steady-state Inflation

(a) log(Dispersion)

Pct\π	0.0	2.0	4.0	6.0	8.0
20	0.0065	0.0067	0.0069	0.0071	0.0074
40	0.0072	0.0073	0.0074	0.0076	0.0079
60	0.0087	0.0087	0.0089	0.0092	0.0096
80	0.0093	0.0094	0.0097	0.0100	0.0105

(b) Frequency of price change

Pct\π	0.0	2.0	4.0	6.0	8.0
20	0.0755	0.0768	0.0789	0.0816	0.0847
40	0.1163	0.1175	0.1191	0.1209	0.1228
60	0.1908	0.1920	0.1935	0.1951	0.1970
80	0.2297	0.2306	0.2317	0.2330	0.2343

(c) Fraction of non-free price changes

Pct\π	0.0	2.0	4.0	6.0	8.0
20	0.0190	0.0228	0.0274	0.0326	0.0382
40	0.0280	0.0341	0.0414	0.0495	0.0584
60	0.0393	0.0493	0.0617	0.0757	0.0905
80	0.0879	0.1069	0.1381	0.1714	0.2022

Notes: This table shows moments of the sectoral price-change distribution across steady-state inflation rates. Table 3a shows the log(Dispersion); Table 3b shows the frequency of price changes; Table 3c shows the fraction of non-free price changes.

3.3.2. Comparing multi-sector and one-sector economies. Moving from a one-sector to a production networks setting adds two ingredients: input-output linkages and heterogeneity in sectoral price-setting moments. To isolate the latter, we consider a multi-sector economy with no input-output linkages but with heterogeneity across sectors in terms of sectoral price-setting moments. Figure 3 and Table 4 show that such heterogeneity alone makes inflation ≈ 1.1 times more costly than in a one-sector economy at 2%.

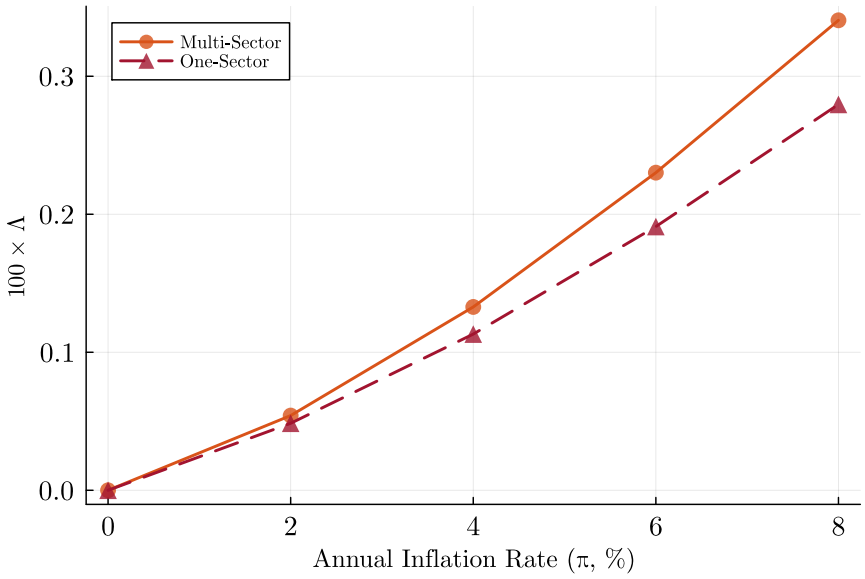
Table 4: Welfare Costs of Inflation: Multi-Sector × One-Sector

π_{ss}	Multi-Sector	One-Sector	Ratio
2.0	0.0541	0.0485	1.12
4.0	0.1328	0.1132	1.17
6.0	0.2301	0.1911	1.20
8.0	0.3405	0.2796	1.22

Notes: This table shows the welfare costs of inflation (relative to zero inflation) in the multi-sector and the one-sector economies. The welfare cost is measured in percent (%). Ratio = $^{\text{Multi-Sector}}/_{\text{One-Sector}}$.

[11] Note, though, that Equation (14) only holds exactly at zero inflation.

Figure 3: Welfare Cost of Inflation



Notes: This figure plots the welfare costs of inflation as we change the steady-state inflation rate. We measure the welfare cost of inflation in terms of consumption equivalent, i.e., the percentage change in steady-state consumption under zero inflation that makes the representative agent indifferent between the zero-inflation steady state and the steady state with inflation π. This figure compares a multi-sector economy with a one-sector economy.

3.3.3. Comparison with a model without sector-specific productivity drift. In our baseline economy, motivated by the heterogeneity in fraction of positive price changes across sectors we see in the data, we consider heterogeneous sector-specific productivity drifts. In this exercise, we set the sector-specific productivity drift to zero, $\mu_i = 0$, for all $i \in [n]$.

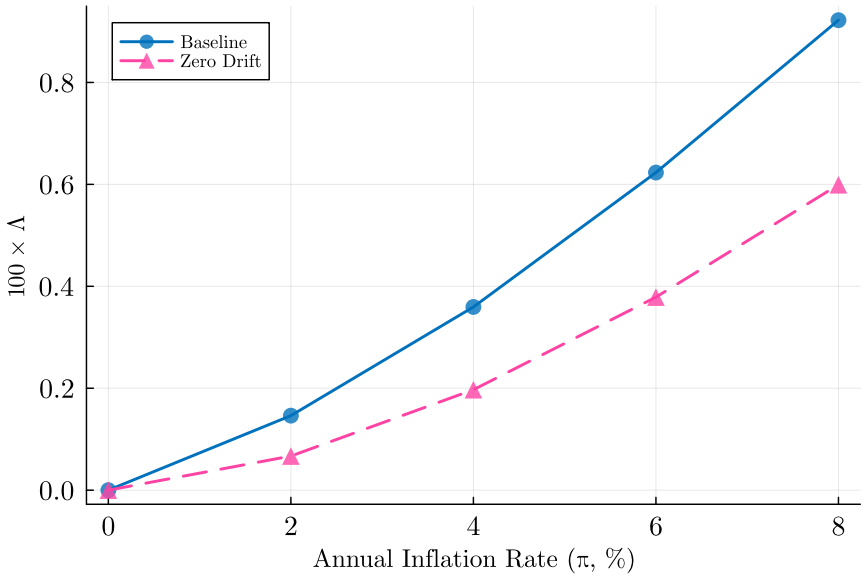
For each sector $i \in [n]$, we then calibrate the menu cost, ϕ_i, the free price change probability θ_i, and the standard deviation of idiosyncratic productivity shocks σ_i to match three sectoral moments: the frequency of price changes, the standard deviation of price changes, and the kurtosis of the distribution of price changes, following Section 3.2.2. With one fewer parameter than in our baseline model with sector-specific productivity drift, we drop the fraction of positive price changes from the set of targeted moments.[12]

Figure 4 and Table 5 show that accounting for non-zero sectoral productivity drift amplifies the welfare cost of inflation relative to the zero sectoral productivity drift economy by 2.2 times at 2% steady-state inflation and 1.5 times at 8%. Moving on to what accounts for the welfare costs of inflation, Figure 5 and Table 6 show that menu costs account for a substantially larger share of the welfare cost relative to dispersion. The contribution of menu cost paid on welfare cost of inflation is 86.6% at 2% inflation and it decreases to 74.3% at 8% inflation. This role of menu cost paid is higher

[12] For the "Calvo" sectors, we target only frequency and standard deviation of price changes.

than in our baseline model as shown in Table 2 earlier.

Figure 4: Welfare Cost of Inflation



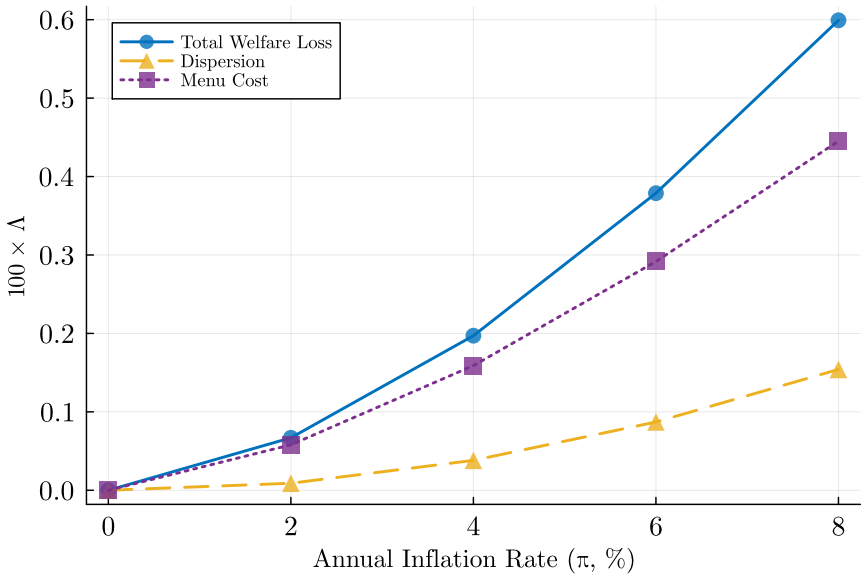
Notes: This figure plots the welfare costs of inflation as we change the steady-state inflation rate. We measure the welfare cost of inflation in terms of consumption equivalent, i.e., the percentage change in steady-state consumption under zero inflation that makes the representative agent indifferent between the zero-inflation steady state and the steady state with inflation π. This figure compares a CalvoPlus economy with non-zero sector-specific productivity drift to a zero sector-specific productivity drift economy.

Table 5: Welfare Costs of Inflation: Baseline × Zero Sectoral Productivity Drift

π_{ss}	Baseline	Zero Drift	Ratio
2.0	0.1462	0.0669	2.18
4.0	0.3595	0.1971	1.82
6.0	0.6232	0.3788	1.65
8.0	0.9221	0.5992	1.54

Notes: This table shows the welfare costs of inflation (relative to zero inflation) in the CalvoPlus economy with non-zero sector-specific productivity drift and a zero sector-specific productivity drift economy. The welfare cost is measured in percent (%).

Figure 5: Welfare Cost of Inflation Decomposition



Notes: This figure plots the welfare costs of inflation as we change the steady-state inflation rate, along with the contribution coming from dispersion and coming from menu cost in the CalvoPlus economy with zero sector-specific productivity drift. We measure the welfare cost of inflation in terms of consumption equivalent, i.e., the percentage change in steady-state consumption under zero inflation that makes the representative agent indifferent between the zero-inflation steady state and the steady state with inflation π.

Table 6: Welfare Costs of Inflation Decomposition

π_{ss}	Dispersion	Menu Cost	Menu Cost Share
2.0	0.0090	0.0580	86.6
4.0	0.0381	0.1590	80.6
6.0	0.0871	0.2917	77.0
8.0	0.1541	0.4451	74.3

Notes: This table shows the welfare cost of inflation contributions of dispersion and menu-cost components to the welfare costs of inflation (relative to zero inflation), as well as the menu-cost component share in the CalvoPlus economy with zero sector-specific productivity drift for different steady-state inflation rates. All columns measured in percent (%).

3.3.4. Comparison with a pure time-dependent pricing model. Under pure time-dependent Calvo pricing, the welfare cost of inflation is given by $\Lambda(\pi) = \sum_{i\in[n]} \lambda_i(\log(D_i(\pi) - \log(D_i(0))$ and is purely driven by price dispersion as there's no menu costs. In the pure Calvo economy, we set menu costs to infinity, $\phi_i \to \infty$ for all $i \in [n]$. For each sector $i \in [n]$, we calibrate the free price change probability θ_i, the standard deviation of idiosyncratic productivity shocks σ_i, and the productivity drift μ_i to match three sectoral moments: the frequency of price changes, the standard deviation of

Table 7: Moments of the Sectoral Price-change Distribution across Steady-state Inflation

(a) log(Dispersion)

Pct\π	0.0	2.0	4.0	6.0	8.0
20	0.0061	0.0061	0.0062	0.0062	0.0064
40	0.0069	0.0069	0.0070	0.0071	0.0073
60	0.0081	0.0081	0.0082	0.0084	0.0087
80	0.0088	0.0088	0.0089	0.0091	0.0094

(b) Frequency of price change

Pct\π	0.0	2.0	4.0	6.0	8.0
20	0.0758	0.0768	0.0787	0.0814	0.0844
40	0.1167	0.1175	0.1189	0.1206	0.1227
60	0.1912	0.1920	0.1932	0.1947	0.1966
80	0.2299	0.2306	0.2316	0.2328	0.2341

(c) Fraction of non-free price changes

Pct\π	0.0	2.0	4.0	6.0	8.0
20	0.0240	0.0270	0.0310	0.0359	0.0415
40	0.0389	0.0429	0.0488	0.0563	0.0650
60	0.0566	0.0628	0.0726	0.0855	0.1007
80	0.1146	0.1269	0.1498	0.1787	0.2089

Notes: This table shows moments of the sectoral price-change distribution across steady-state inflation rates in the CalvoPlus economy with zero sector-specific productivity drift. Table 7a shows the log(Dispersion); Table 7b shows the frequency of price changes; Table 7c shows the fraction of non-free price changes.

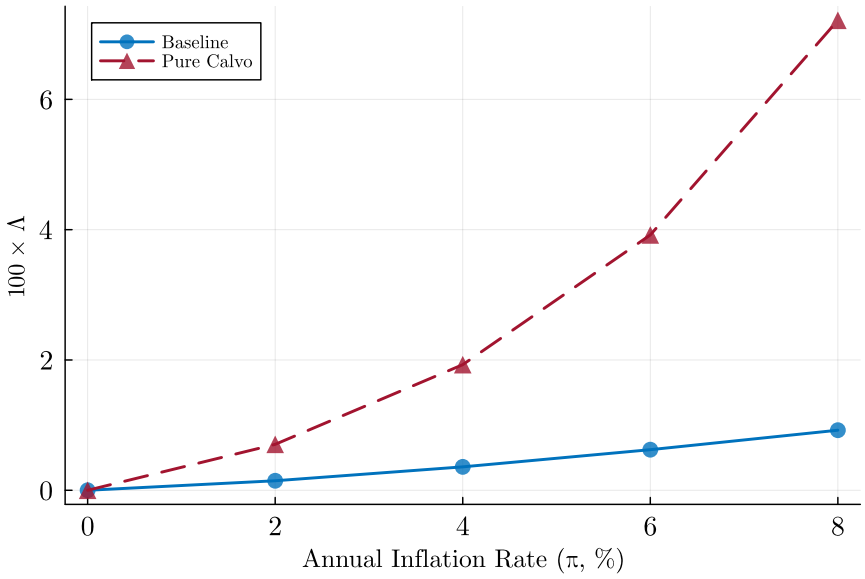
price changes, and the fraction of positive price changes following Section 3.2.2. With one fewer parameter than in our CalvoPlus baseline model, we drop the kurtosis of price changes from the target set.

As expected, Figure 6 shows that the welfare costs of inflation are larger in the pure Calvo economy relative to our baseline at every level of steady-state inflation. The ratio of welfare costs in the Calvo economy relative to the CalvoPlus economy is around 4.8 at 2%, increasing to 7.8 at 8%. Holding fixed the frequency and standard deviation of price changes, and the fraction of positive price changes, but not kurtosis, the comparison makes clear that kurtosis plays a central role in quantifying welfare. The ratio of welfare cost in the Calvo and the CalvoPlus economies is around 4.8 at 2%, 5.4 at 4%, 6.3 at 6%, and 7.8 at 8%.

3.3.5. Comparison with a pure state-dependent pricing model. In the pure menu cost economy, we set the probability of a free price change opportunity to zero, $\theta_i = 0$ for all $i \in [n]$. For each sector $i \in [n]$, we calibrate the menu cost parameter ϕ_i, the standard deviation of idiosyncratic productivity shocks σ_i, and the productivity drift μ_i to match the same three sectoral moments targeted in Section 3.3.4, dropping the kurtosis of price changes as a targeted moment.

As expected, Figure 7 shows that the welfare costs of inflation are smaller in the pure menu cost economy relative to our baseline at every level of steady-state inflation. The ratio of welfare

Figure 6: Welfare Cost of Inflation



Notes: This figure plots the welfare costs of inflation as we change the steady-state inflation rate. We measure the welfare cost of inflation in terms of consumption equivalent, i.e., the percentage change in steady-state consumption under zero inflation that makes the representative agent indifferent between the zero-inflation steady state and the steady state with inflation π. This figure compares a CalvoPlus economy with a pure Calvo economy.

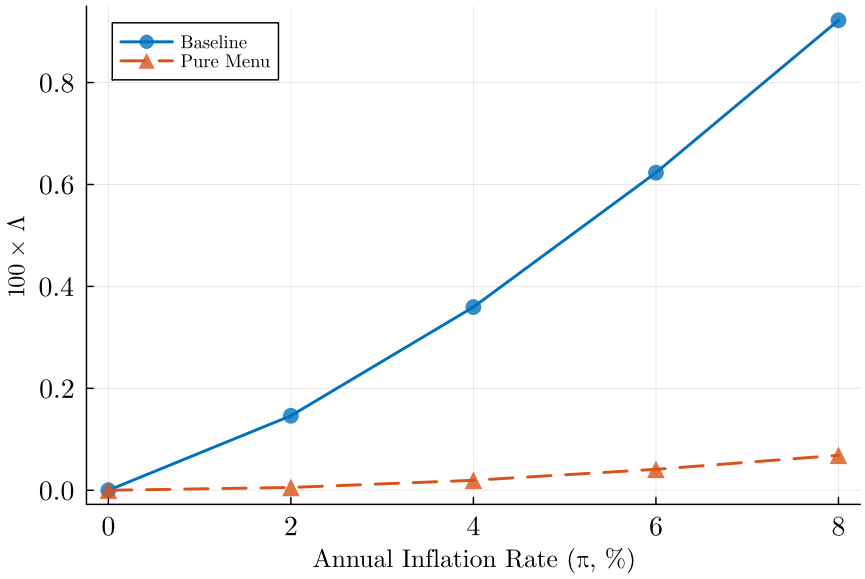
costs in the menu cost economy relative to the CalvoPlus economy is 0.037, 0.054, and 0.074 at 2%, 4%, and 8% inflation. Moreover, in the pure menu cost economy, the welfare cost of inflation does not change in a quantitatively significant way until very high levels of inflation. Similarly to Section 3.3.4, Figure 7 illustrates the importance of targeting kurtosis to quantity the welfare costs of inflation.

3.4. Discussion

In our main results, we analyze the importance of production networks and heterogeneity of sectoral moments considering the welfare cost of inflation relative to *zero-inflation* steady state in a model with idiosyncratic shocks. In this section, we (1) analyze the welfare cost with respect to the *flexible-price* steady state; (2) present results from a one-sector version of our setting to compare directly with the previous literature.

3.4.1. Welfare cost of nominal rigidities. In our baseline results, we compute the welfare cost of inflation with respect to *zero-inflation* steady state. In contrast to the standard New Keynesian model, in our setup the zero inflation and the flexible price steady states do not coincide. As a result, even at zero inflation, there are losses arising from nominal rigidities. We now present the welfare cost of inflation with respect to *flexible-price* steady state. That is, $\Lambda^{\text{nr}}(\pi)$ such that

Figure 7: Welfare Cost of Inflation



Notes: This figure plots the welfare costs of inflation as we change the steady-state inflation rate. We measure the welfare cost of inflation in terms of consumption equivalent, i.e., the percentage change in steady-state consumption under zero inflation that makes the representative agent indifferent between the zero-inflation steady state and the steady state with inflation π. This figure compares a CalvoPlus economy with a pure menu cost economy.

$U(C(\pi), L(\pi)) = U(e^{\Lambda^{\text{nr}}(\pi)} C_{\text{flex}}, L_{\text{flex}})$. In our setup with idiosyncratic shocks, it results in the following theoretical result.

Theorem 2. *When preferences are $U(C) = \log(C)$, $V(L) = L$, the consumption aggregator and production functions are Cobb-Douglas, and taxes are such that sectoral markups equal to one, the welfare cost of inflation π relative to flexible-price steady state is*

$$\Lambda^{\text{nr}}(\pi) = \sum_{i \in [n]} \lambda_i \log(D_i(\pi)) + \sum_{i \in [n]} \lambda_i \chi_i \mathsf{N}_i^{menu}(\pi) \tag{15}$$

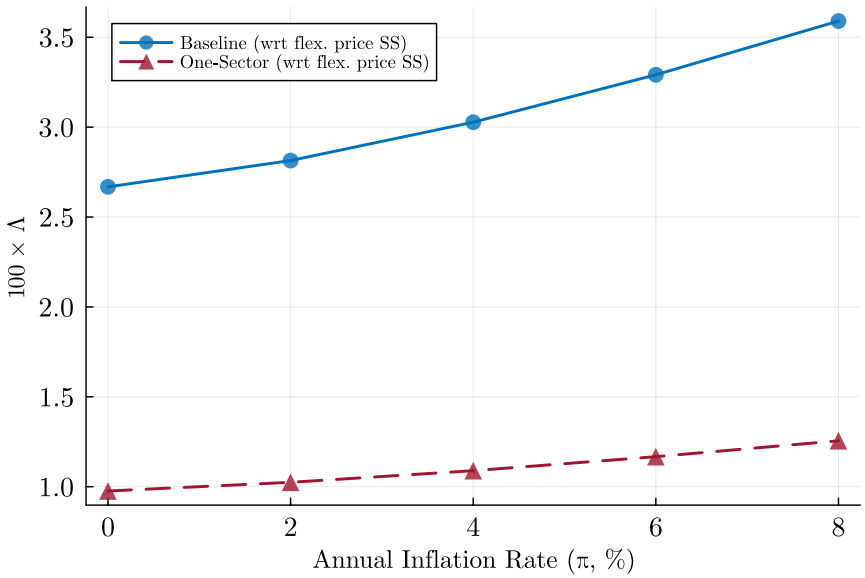
where $\lambda_i = e_i' \boldsymbol{\lambda}$, with $\boldsymbol{\lambda} = (\mathbf{I} - \mathbf{A}')^{-1} \boldsymbol{\beta}$ is the sales-based Domar weights when the sectoral markups are equal to one, and $\mathsf{N}_i^{menu}(\pi)$ is the measure of firms that pay the menu cost in sector i when steady-state inflation is π.

Proof. See Appendix C.1

Figure 8 and Table 8 show how $\Lambda^{\text{nr}}(\pi)$ varies with steady-state inflation for both our baseline and the one-sector economy. In particular, they show that at zero-inflation steady state, there is substantial welfare loss associated with nominal rigidities. The welfare loss at 2% is 2.81%, increasing to 3.02% at 4%. Table 9 shows that dispersion accounts for most of the welfare cost of inflation with respect to the flexible-prices steady state, with menu cost ranging from 18% to 30% from zero to 8 percent steady-state inflation. For the increase in welfare cost as we move to higher

levels of inflation, however, the bulk is accounted for by menu costs as dispersion does not change by much. For instance, when moving from 2% to 4%, menu costs account for 69.5% of the total increase.

Figure 8: Welfare Cost of Inflation With Respect to Flexible-Price Steady State



Notes: This figure plots the welfare costs of nominal rigidities as we change the steady-state inflation rate. We measure the welfare cost of inflation in terms of consumption equivalent, i.e., the percentage change in steady-state consumption under flexible-price steady state that makes the representative agent indifferent between the flexible-price steady state and the steady state with inflation π. This figure compares our baseline economy with a one-sector economy.

Table 8: Welfare Costs of Inflation With Respect to Flexible-Price Steady State

π_{ss}	Baseline (wrt flex. price SS)	One-Sector (wrt flex. price SS)	Ratio
0.0	2.6679	0.9760	2.73
2.0	2.8142	1.0245	2.75
4.0	3.0274	1.0891	2.78
6.0	3.2911	1.1670	2.82
8.0	3.5901	1.2555	2.86

Notes: This table shows the welfare costs of nominal rigidities as we change the steady-state inflation rate. We measure the welfare cost of inflation in terms of consumption equivalent, i.e., the percentage change in steady-state consumption under flexible-price steady state that makes the representative agent indifferent between the flexible-price steady state and the steady state with inflation π. This figure compares our baseline economy with a one-sector economy.

Table 9: Welfare Costs of Inflation With Respect to Flexible-Price Steady State

π_{ss}	Dispersion	Menu Cost	Menu Cost Share
0.0	2.1835	0.4845	18.2
2.0	2.2218	0.5923	21.0
4.0	2.2869	0.7406	24.5
6.0	2.3756	0.9155	27.8
8.0	2.4838	1.1063	30.8

Notes: This table shows the welfare cost of nominal rigidities contribution of dispersion and menu-cost components to the welfare costs of inflation (relative to flexible-price steady state) and well as the menu-cost component share in the baseline economy for different steady-state inflation rates. All columns measured in percent (%).

3.4.2. Relation to previous literature on one-sector models. We now present some results based on a one-sector model to compare directly with the previous literature on welfare cost of inflation in state-dependent pricing models. As this literature uses a pure menu cost economy, we work with a one-sector pure menu cost variant of our setup as well. Thus, we set the probability of free price change opportunity to zero, $\theta = 0$, and recalibrate (ϕ, σ, μ) to match the median frequency, standard deviation, and the median fraction of positive price changes.[13] Note that here we no longer match the data in terms of kurtosis of price changes.

Figure 9 shows that, in this one-sector pure menu cost economy, the welfare cost of steady-state inflation is generally very small. Moreover, it is relatively flat even when we increase steady-state inflation to a large number such as 8%. This is consistent with the results in Burstein and Hellwig (2008) and Nakamura, Steinsson, Sun, and Villar (2018), which feature very similar models and calibration strategies.[14] Moreover, moving from 2% to 4% increases the welfare cost of inflation by a very small amount, 0.3 bps.

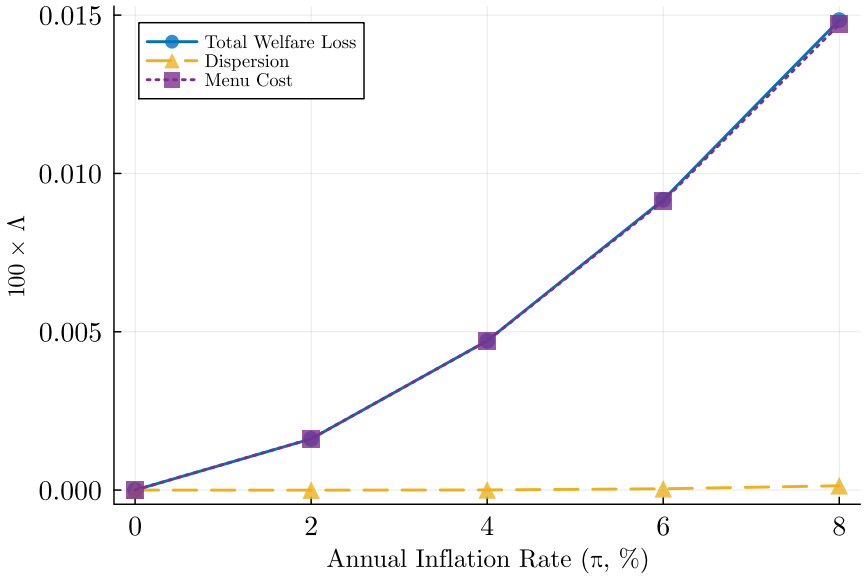
For this one-sector pure menu cost economy, we next decompose the welfare cost of inflation, even though it is quantitatively small, into the two components: misallocation due to price dispersion and menu costs paid to adjust prices. Figure 9 also presents the results of this decomposition

[13] Frequency is calculated from the disaggregated data in Pasten, Schoenle, and Weber (2020a) while other moments are calculated from Hong, Klepacz, Pasten, and Schoenle (2023).

[14] Nakamura, Steinsson, Sun, and Villar (2018) find that the welfare cost of inflation is around 0.4% of flexible-price steady-state equilibrium in a one-sector menu cost model, and that this cost is flat even as steady-state inflation increases to a large number. Instead, we report welfare cost of inflation relative to the zero-inflation steady state. The zero-inflation steady state and the flexible-price steady state are different in these menu cost models with firm-level idiosyncratic shocks, as there is inefficient price dispersion even with zero inflation, and thus, the numbers reported in our paper are different in terms of magnitude. If we compute welfare cost relative to the flexible-price steady state in our calibrated one-sector menu cost model, it is 0.380% at 0% inflation; 0.382% at 2% inflation; 0.385% at 4% inflation; and 0.395% at 8% inflation. Figure A.7 in Appendix A shows in detail the welfare cost when we compute losses relative to the flexible-price steady state.

and shows that all the welfare cost of inflation comes about due to the menu cost paid channel, with price dispersion not increasing even at quite high levels of inflation.

Figure 9: Welfare Cost of Inflation in a One-Sector Pure Menu Cost Economy



Notes: This figure plots the welfare costs of inflation as we change the steady-state inflation rate, along with the contribution coming from dispersion and coming from menu cost in the one-sector economy. We measure the welfare cost of inflation in terms of consumption equivalent, i.e., the percentage change in steady-state consumption under zero inflation that makes the representative agent indifferent between the zero-inflation steady state and the steady state with inflation π.

The result that dispersion does not increase, at all, even at fairly high levels of inflation in the pure one-sector menu cost economy is consistent with the results in Burstein and Hellwig (2008), Nakamura, Steinsson, Sun, and Villar (2018), and Alvarez, Beraja, Gonzalez-Rozada, and Neumeyer (2018). We show this in further detail in Table 10. However, the menu cost paid to adjust prices increases, accounting for *all* of the welfare loss, as inflation increases. This results from a higher frequency of price change, as given in Table 10, and shown to be the case also in the multi-sector production networks economy in Table 3c. Finally, Table 10 shows that as steady-state inflation increases, the standard deviation of price changes decreases while the kurtosis increases. This opposite pattern between the standard deviation and the kurtosis of price changes provides further intuition for why dispersion does not change as the inflation rate increases, in line with Equation (14).[15]

[15]Note again, that Equation (14) only holds exactly at zero inflation.

Table 10: Moments of the Price-change Distribution across Steady-state Inflation in the One-Sector Pure Menu Cost Economy

Statistic\π	0.0	2.0	4.0	6.0	8.0
log (Dispersion)	0.00191	0.00191	0.00191	0.00191	0.00191
Frequency	0.15815	0.15950	0.16208	0.16576	0.17041
Std. Dev. of price changes	0.07541	0.07400	0.07166	0.06859	0.06498
Kurtosis of price changes	1.05159	1.23185	1.54735	2.01149	2.64524
Fraction of positive price changes	0.55642	0.61703	0.67347	0.72463	0.76993

Notes: This table shows moments of the price-change distribution across steady-state inflation rates in the one-sector pure menu cost economy. $\eta = 4$. The menu cost paid as a share of GDP is 0.191%.

4 Conclusion

We study welfare cost of inflation in a quantitative multi-sector menu cost model with production networks and idiosyncratic firms-specific shocks. The model features heterogeneity in both the distribution of price changes and input-output linkages across sectors. We show theoretically that welfare cost of inflation is driven by two forces: aggregate misallocation as given by Domar-weighted within-sector price dispersion; and menu costs paid by firms while adjusting prices.

We calibrate the model to 65 US sectors by matching sectoral moments of the price change distribution and the input shares from the input-output table. At 2% inflation, we find that welfare cost of inflation is 3 times higher in our calibrated multi-sector economy compared to a counterfactual one-sector economy. Moreover, going from 2% inflation to 4% inflation increases the welfare cost of inflation by 21 basis points in our production networks economy, but only by 6.5 basis points in the one-sector economy. Finally, when going from 2% to 4% inflation, 69.5% of the increase in welfare cost of inflation in our calibrated economy is due to menu costs and only 30.5% due to an increase in aggregate misallocation. Our production networks model thus provides new insights about both the magnitude and determinants of welfare cost of inflation.

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APPENDIX
(FOR ONLINE PUBLICATION)

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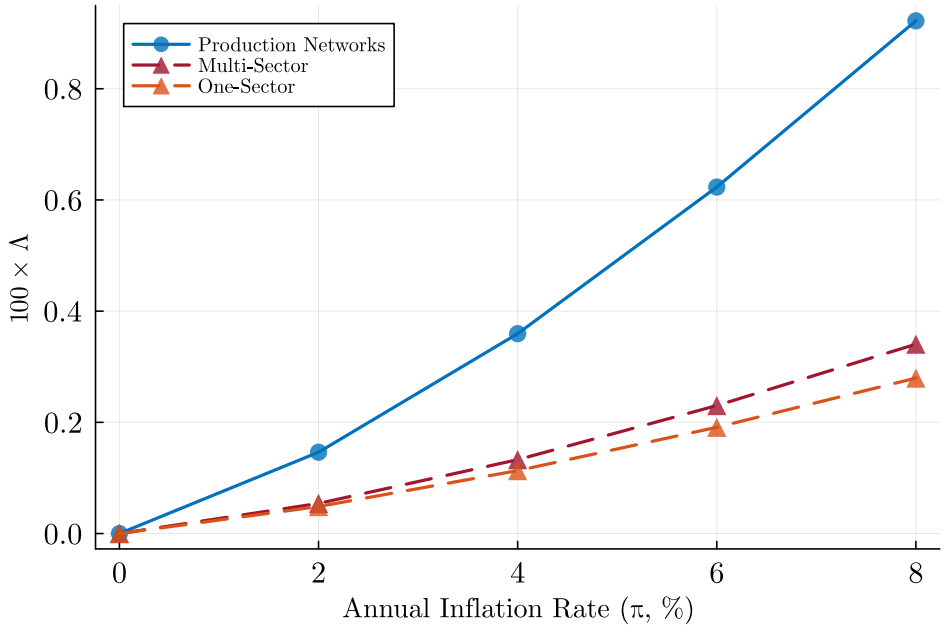
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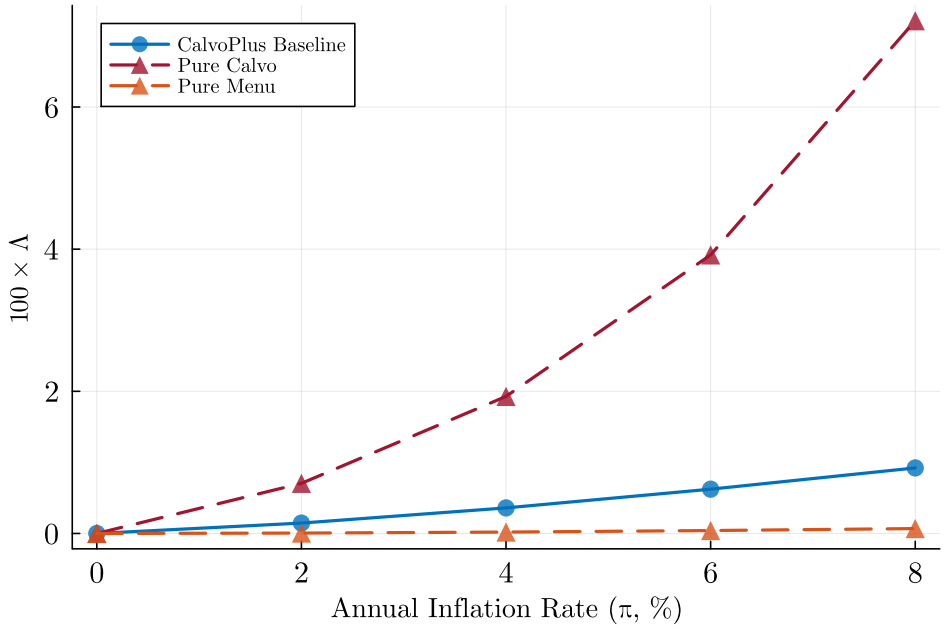
A Figures and Tables

Figure A.1: Welfare Cost of Inflation



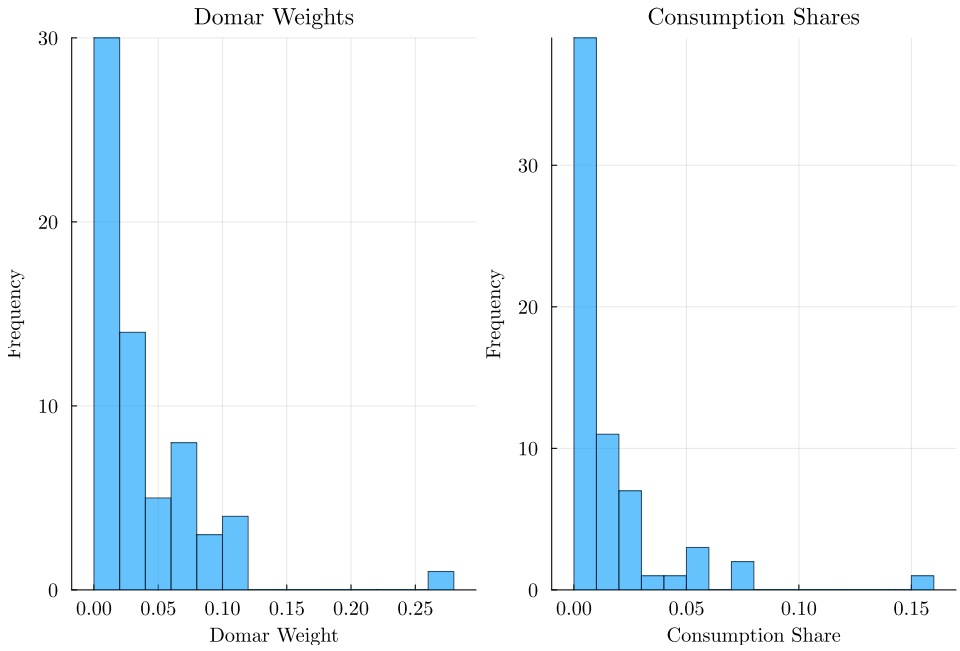
Notes: This figure plots the welfare costs of inflation as we change the steady-state inflation rate. We measure the welfare cost of inflation in terms of consumption equivalent, i.e., the percentage change in steady-state consumption under zero inflation that makes the representative agent indifferent between the zero-inflation steady state and the steady state with inflation π. This figure compares the production networks economy with a multi-sector economy and a one-sector economy.

Figure A.2: Welfare Cost of Inflation



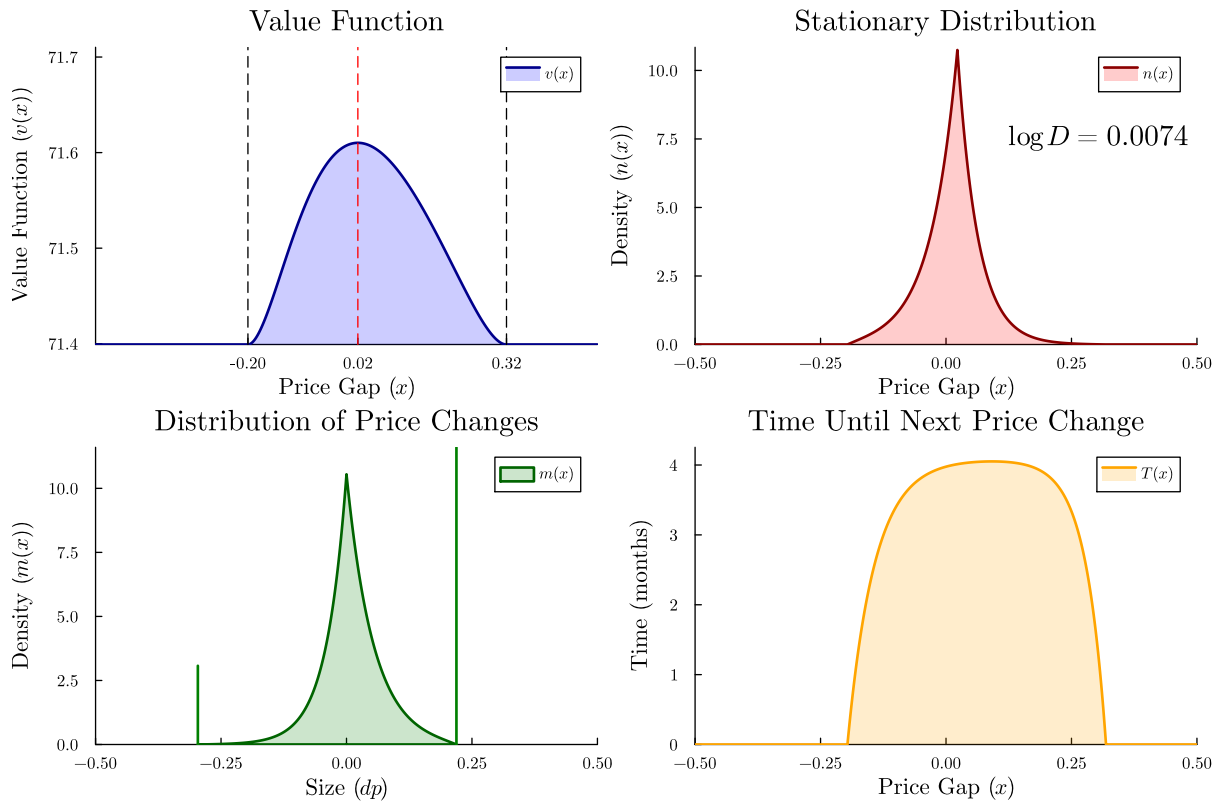
Notes: This figure plots the welfare costs of inflation as we change the steady-state inflation rate. We measure the welfare cost of inflation in terms of consumption equivalent, i.e., the percentage change in steady-state consumption under zero inflation that makes the representative agent indifferent between the zero-inflation steady state and the steady state with inflation π. This figure compares a CalvoPlus baseline economy with the pure Calvo and the pure menu cost economies.

Figure A.3: Distribution of Domar weights and consumption shares



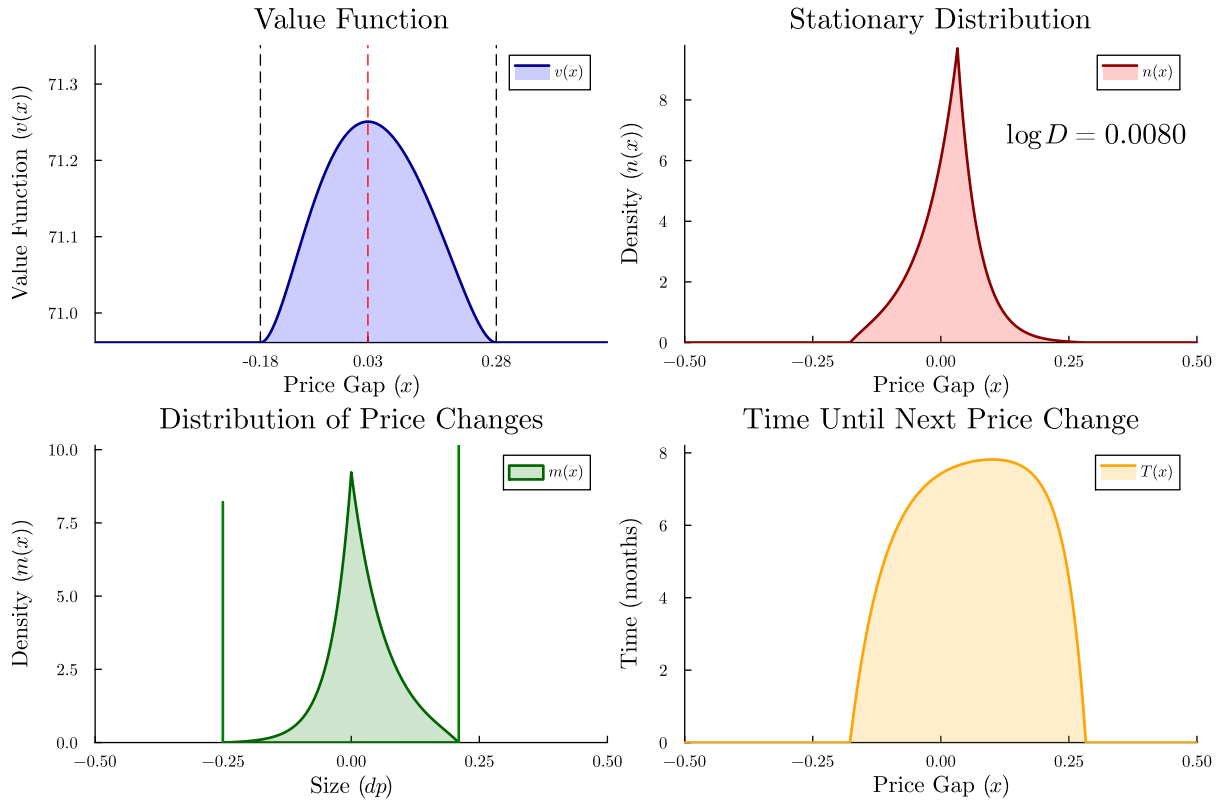
Notes: Left: Histogram of Domar weights. Right: Histogram of consumption shares.

Figure A.4: Wood products



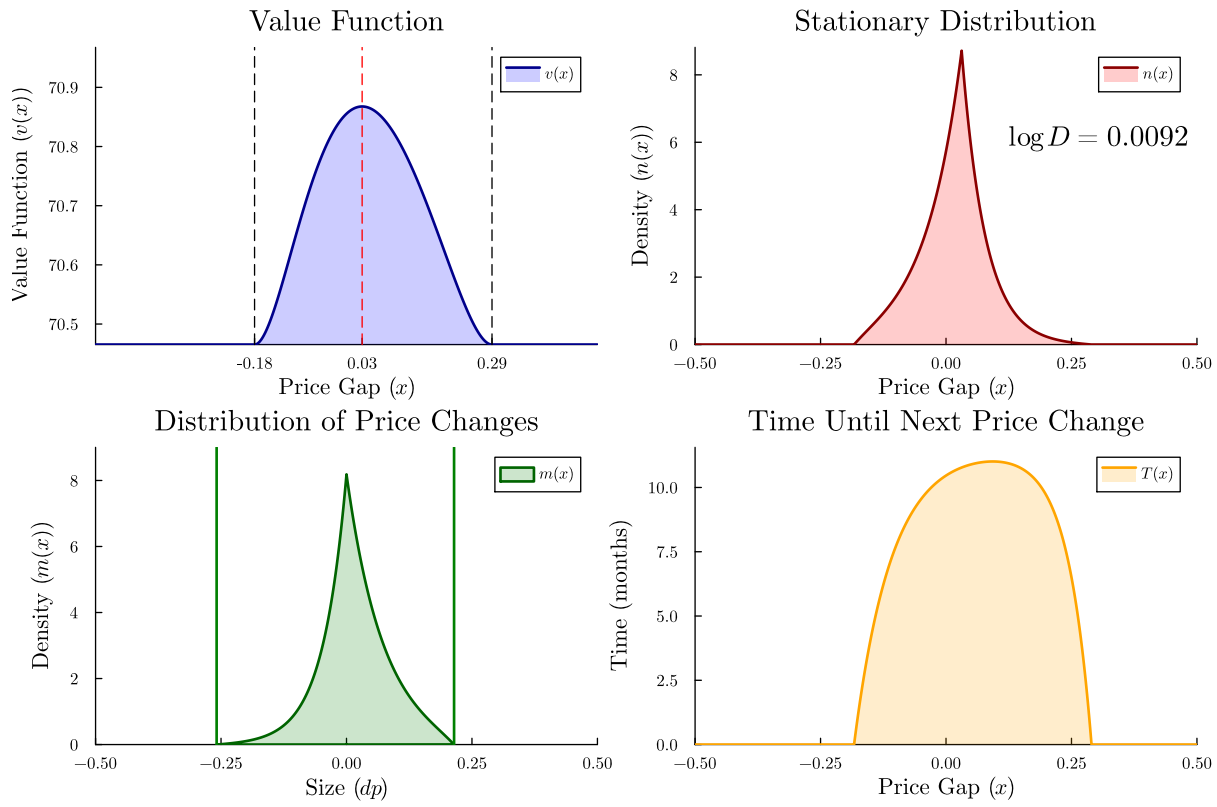
Notes: Sector: Wood products. Top left: Value function. Top right: Stationary distribution of price gaps. Bottom left: Distribution of price changes. Bottom right: Time until next price change. Frequency of price adjustment: 0.2493; Standard deviation price changes: 0.0663; Kurtosis of price changes: 4.8006; Fraction of positive price changes: 0.5719.

Figure A.5: Rental and leasing services and lessors of intangible assets



Notes: Sector: Rental and leasing services and lessors of intangible assets. Top left: Value function. Top right: Stationary distribution of price gaps. Bottom left: Distribution of price changes. Bottom right: Time until next price change. Frequency of price adjustment: 0.1307; Standard deviation price changes: 0.07399; Kurtosis of price changes: 3.8574; Fraction of positive price changes: 0.6337.

Figure A.6: Computer and electronic products



Notes: Sector: Computer and electronic products. Top left: Value function. Top right: Stationary distribution of price gaps. Bottom left: Distribution of price changes. Bottom right: Time until next price change. Frequency of price adjustment: 0.0929; Standard deviation price changes: 0.0823; Kurtosis of price changes: 3.7597; Fraction of positive price changes: 0.6117.

Table A.1: Moments of the Sectoral Price-change Distribution across Steady-state Inflation

(a) Standard deviation of price changes

Pct\π	0.0	2.0	4.0	6.0	8.0
20	0.0648	0.0657	0.0672	0.0690	0.0710
40	0.0679	0.0680	0.0691	0.0696	0.0722
60	0.0782	0.0794	0.0796	0.0809	0.0831
80	0.0836	0.0830	0.0842	0.0869	0.0902

(b) Kurtosis of price changes

Pct\π	0.0	2.0	4.0	6.0	8.0
20	3.5621	3.1322	2.8568	2.4326	2.0979
40	4.2391	3.8547	3.4599	3.0925	2.7849
60	4.5046	4.2155	3.9143	3.6229	3.3541
80	4.8333	4.6105	4.3751	4.2432	4.0233

(c) log(Dispersion)

Pct\π	0.0	2.0	4.0	6.0	8.0
20	0.0065	0.0067	0.0069	0.0071	0.0074
40	0.0072	0.0073	0.0074	0.0076	0.0079
60	0.0087	0.0087	0.0089	0.0092	0.0096
80	0.0093	0.0094	0.0097	0.0100	0.0105

(d) Frequency of price changes

Pct\π	0.0	2.0	4.0	6.0	8.0
20	0.0755	0.0768	0.0789	0.0816	0.0847
40	0.1163	0.1175	0.1191	0.1209	0.1228
60	0.1908	0.1920	0.1935	0.1951	0.1970
80	0.2297	0.2306	0.2317	0.2330	0.2343

(e) Fraction of positive price changes

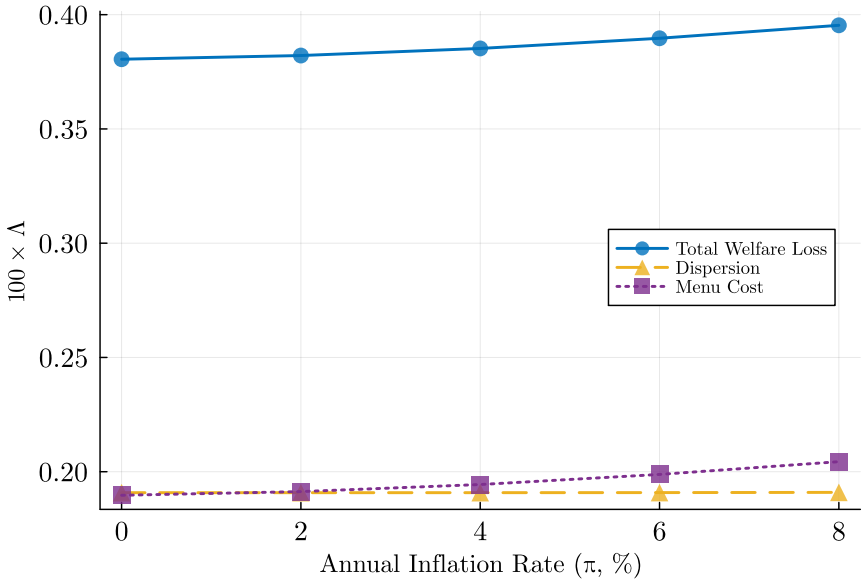
Pct\π	0.0	2.0	4.0	6.0	8.0
20	0.5346	0.5866	0.6226	0.6560	0.6867
40	0.5437	0.6148	0.6591	0.6970	0.7306
60	0.5606	0.6177	0.6799	0.7330	0.7712
80	0.5714	0.6360	0.6992	0.7596	0.8140

(f) Fraction of non-free price changes

Pct\π	0.0	2.0	4.0	6.0	8.0
20	0.0190	0.0228	0.0274	0.0326	0.0382
40	0.0280	0.0341	0.0414	0.0495	0.0584
60	0.0393	0.0493	0.0617	0.0757	0.0905
80	0.0879	0.1069	0.1381	0.1714	0.2022

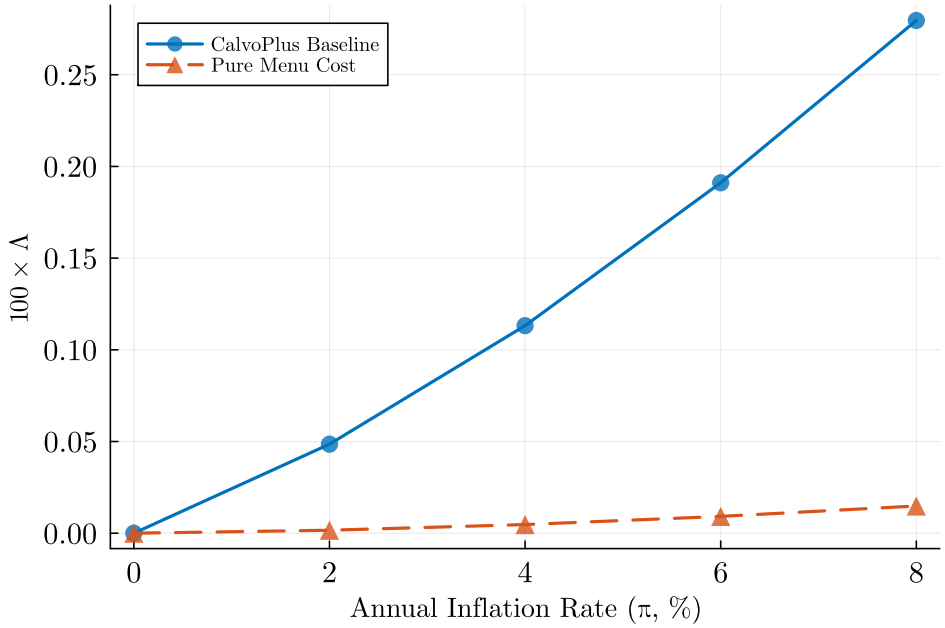
Notes: This table shows moments of the sectoral price-change distribution across steady-state inflation rates. Table A.1a shows the standard deviation of price changes; Table A.1b shows the kurtosis of price changes; Table A.1c shows the log(dispersion); Table A.1d shows the frequency of price changes; Table A.1e shows the fraction of positive price changes; Table A.1f shows the fraction of non-free price changes.

Figure A.7: Welfare Cost of Inflation in a One-Sector Pure Menu Cost Economy



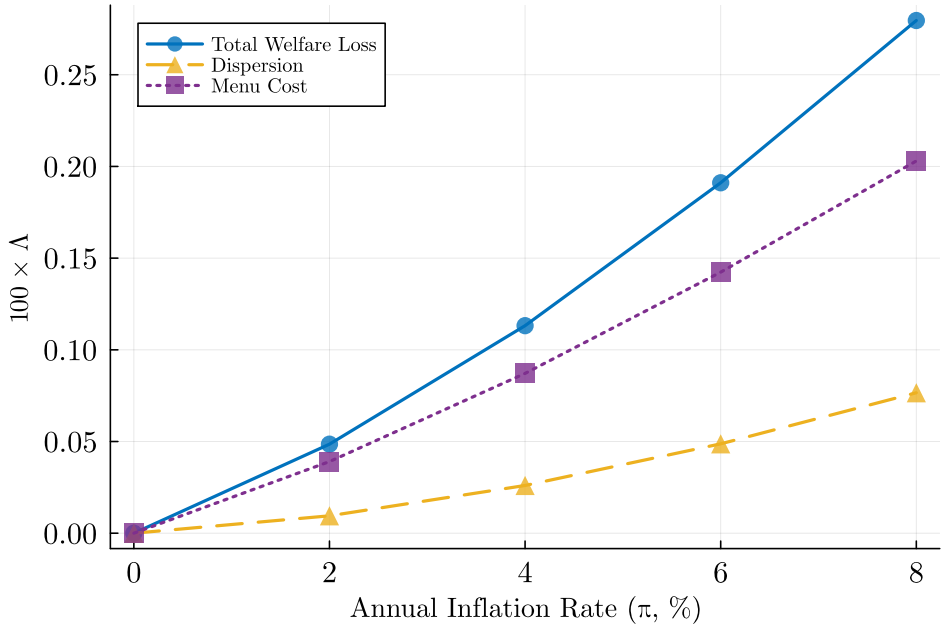
Notes: This figure plots the welfare costs of inflation as we change the steady-state inflation rate, along with the contribution coming from dispersion and coming from menu cost in the one-sector economy. We measure the welfare cost of inflation in terms of consumption equivalent, i.e., the percentage change in steady-state consumption under flexible-price steady state that makes the representative agent indifferent between the flexible-price steady state and the steady state with inflation π.

Figure A.8: Welfare Cost of Inflation in the One-Sector Economy: CalvoPlus × Menu



Notes: This figure plots the welfare costs of inflation as we change the steady-state inflation rate, along with the contribution coming from dispersion and coming from menu cost in the one sector economy. We measure the welfare cost of inflation in terms of consumption equivalent, i.e., the percentage change in steady-state consumption under zero inflation that makes the representative agent indifferent between the zero-inflation steady state and the steady state with inflation π. Calvo Plus × Menu

Figure A.9: Welfare Cost of Inflation Decomposition in One Sector Economy: CalvoPlus



Notes: This figure plots the welfare costs of inflation as we change the steady-state inflation rate, along with the contribution coming from dispersion and coming from menu cost in the one sector economy. We measure the welfare cost of inflation in terms of consumption equivalent, i.e., the percentage change in steady-state consumption under zero inflation that makes the representative agent indifferent between the zero-inflation steady state and the steady state with inflation π. Calvo Plus economy.

B Derivations of Optimality Conditions in the Model

B.1. Household's Optimality Conditions

As in Afrouzi and Bhattarai (2023), we solve the household's consumption problem into two stages: first, given a level of C_t the household minimizes her expenditure on sectoral goods. Then, she decides on the optimal level of C_t as a function of lifetime income. Given the vector of sectoral prices $\mathbf{P}_t = (P_{1,t}, \dots, P_{n,t})$, the household chooses $(C_{i,t})_{i\in[n]}$ to minimize her total expenditure of consuming C_t:

$$\begin{aligned}\mathscr{E}_t(C_t, \mathbf{P}_t) &\equiv \min_{(C_{i,t})_{i\in[n]}} \sum_{i\in[n]} P_{i,t}C_{i,t} \quad s.t. \quad \Phi(C_{1,t}, \dots, C_{n,t}) \geq C_t \\ &= P_t C_t, \quad P_t \equiv \mathscr{E}(1, \mathbf{P}_t)\end{aligned}$$

where the second line follows from the first-degree homogeneity of $\Phi(.)$ and P_t is the cost of a unit of C_t. By Shephard's lemma, the household's expenditure share of sectoral good i is proportional to the elasticity of the expenditure function with respect to the price of i:

$$P_{i,t}C^*_{i,t} = \beta_i(\mathbf{P}_t) \times P_t C_t \quad \text{where} \quad \beta_i(\mathbf{P}_t) \equiv \frac{\partial \ln(\mathscr{E}(C_t, P_t))}{\partial \ln(P_{i,t})}$$

Note that the first-degree homogeneity of the expenditure function implies that $\beta_i(\mathbf{P}_t)$ *only* depends on $\mathbf{P}_t$. Under Cobb-Douglas consumption aggregator, $\Phi(C_{1,t}, \dots, C_{n,t}) = \prod_{i\in[n]} \left(C_{i,t}/\beta_i\right)^{\beta_i}$, these elasticities are *independent* of $\mathbf{P}_t$, and are given by $(\beta_i)_{i\in[n]}$. Concretely,

$$\frac{P_{i,t}C_{i,t}}{P_t C_t} = \beta_i \quad i \in [n] \implies C_{i,t} = \beta_i \frac{P_t C_t}{P_{i,t}} \tag{B.1}$$

The aggregate price, P_t, is given by $P_t = \prod_{i\in[n]} P_{i,t}^{\beta_i}$.

Given the household's expenditure function and the aggregate price index P_t, the optimality conditions for the household are given by the following:

$$\underbrace{\gamma(C_t)\frac{\dot{C}_t}{C_t}}_{\text{marginal loss from saving}} = \underbrace{i_t - \rho - \frac{\dot{P}_t}{P_t}}_{\text{marginal gain from saving}} \quad \text{where} \quad \underbrace{\gamma(C_t) \equiv -\frac{U''(C_t)C_t}{U'(C_t)}}_{\text{inverse elasticity of intertemporal substitution}} \tag{B.2}$$

$$\underbrace{\frac{V'(L_t)}{U'(C_t)}}_{\text{MRS}_{LC}} = \underbrace{\frac{W_t}{P_t}}_{\text{real wage}} \implies \psi(L_t)\frac{\dot{L}_t}{L_t} + \gamma(C_t)\frac{\dot{C}_t}{C_t} = \frac{\dot{W}_t}{W_t} - \frac{\dot{P}_t}{P_t} \quad \text{where} \quad \underbrace{\psi(L_t) \equiv \frac{V''(L_t)L_t}{V'(L_t)}}_{\text{inverse Frisch elasticity of labor supply}} \tag{B.3}$$

Moreover, given a path of $\{M_t\}_{t\geq 0}$ set by monetary policy where $M_t = P_t C_t$ is the nominal GDP, we have:

$$\frac{\dot{P}_t}{P_t} + \frac{\dot{C}_t}{C_t} = \frac{\dot{M}_t}{M_t} \tag{B.4}$$

Note that by combining Equations (B.2) to (B.4) we can write the growth rate of wages as well as the

nominal interest rates as a function of consumption and labor supply growths:

$$\frac{\dot{W}_t}{W_t} = \mu + \psi(L_t)\frac{\dot{L}_t}{L_t} + (\gamma(C_t) - 1)\frac{\dot{C}_t}{C_t}, \qquad i_t = \rho + \frac{\dot{M}_t}{M_t} + (\gamma(C_t) - 1)\frac{\dot{C}_t}{C_t} \tag{B.5}$$

Given Golosov and Lucas (2007) preferences $U(C_t) = \log(C_t)$ and $V(L_t) = L_t$ which imply $\gamma(C_t) = 1$ and $\psi(L_t) = 0$. Plugging these elasticities into the Euler equations above and using we can see how these preference simplify aggregate dynamics by relating interest rates to nominal GDP growth and nominal wages equal to nominal GDP:

$$i_t = \rho + \dot{M}_t / M_t, \qquad W_t = M_t$$

Then, using $\dot{M}_t / M_t = \pi$, we get $i_t = \rho + \pi$

B.2. Firms' Cost Minimization and Input-Output Matrices

The derivations follow closely Afrouzi and Bhattarai (2023). Given wage and sectoral prices, $\mathbf{P}_t = (W_t, P_{i,t})_{i\in[n]}$[16], firm ij chooses inputs to minimize its total cost:

$$\begin{aligned}\mathscr{C}_{ij}(Y^s_{ij,t}; \mathbf{P}_t, Z_{ij,t}) &= \min_{L_{ij,t},(X_{ij,k,t})_{k\in[n]}} W_t L_{ij,t} + \sum_{k\in[n]} P_{k,t} X_{ij,k,t} \quad \text{s.t.} \quad Z_{ij,t} F_i(L_{ij,t}, (X_{ij,k,t})_{k\in[n]}) \geq Y^s_{ij,t} \\ &= \mathsf{MC}_{ij}(\mathbf{P}_t, Z_{ij,t}) \times Y^s_{ij,t}, \quad \mathsf{MC}_{ij}(\mathbf{P}_t, Z_{ij,t}) \equiv \mathscr{C}_{ij}(1; \mathbf{P}_t, 1)/Z_{ij,t}\end{aligned} \tag{B.6}$$

where the second line follows from the first-degree homogeneity of the production function $Z_{ij}F_i(.)$ and $\mathsf{MC}_{ij}(\mathbf{P}_t, Z_{ij,t})$ is ij's marginal cost of production. First-degree homogeneity of the production function implies that marginal costs are independent of the level of production and depend only on the sector's production function, input prices, and ij's idiosyncratic productivity. By Shephard's lemma, firm's expenditure share of any input is the elasticity of the cost function with respect to that input price:

$$W_t L^*_{ij,t} = \alpha_i(\mathbf{P}_t) \times \mathsf{MC}_{ij}(\mathbf{P}_t, Z_{ij,t}) Y^s_{ij,t}, \quad P_{k,t} X^*_{ij,k,t} = a_{ik}(\mathbf{P}_t) \times \mathsf{MC}_{ij}(\mathbf{P}_t, Z_{ij,t}) Y^s_{ij,t}, \quad \forall k \in [n] \tag{B.7}$$

where $\alpha_i(\mathbf{P}_t)$ and $a_{ik}(\mathbf{P}_t)$ are the elasticities of the sector i's cost function with respect to wage and sector k's final good price respectively:

$$\alpha_i(\mathbf{P}_t) \equiv \frac{\partial \ln(\mathscr{C}_{ij}(Y; \mathbf{P}_t, 1)/Z_{ij,t})}{\partial \ln(W_t)}, \qquad a_{ik}(\mathbf{P}_t) \equiv \frac{\partial \ln(\mathscr{C}_{ij}(Y; \mathbf{P}_t, 1)/Z_{ij,t})}{\partial \ln(P_{k,t})} \quad \forall k \in [n] \tag{B.8}$$

with the property that $\alpha_i(\mathbf{P}_t) + \sum_{k\in[n]} a_{ik}(\mathbf{P}_t) = 1$. First-degree homogeneity of the cost function in Equation (6) also implies that these elasticities are only functions of the aggregate wage and sectoral prices, which are directly related to the *cost-based* input-output matrix, denoted by $\mathbf{A}_t \in \mathbb{R}^{n\times n}$, and the labor share vector, denoted by $\boldsymbol{\alpha}_t \in \mathbb{R}^n$:

$$[\mathbf{A}_t]_{i,k} \equiv \frac{\text{total expenditure of sector } i \text{ on sector } k}{\text{total expenditure on inputs in sector } i} = a_{ik}(\mathbf{P}_t), \quad \forall (i,k) \in [n]^2 \tag{B.9}$$

[16] Previously, in household's expenditure minimization problem, $\mathbf{P}_t$ was defined as vector of sectoral prices. Here, without loss of generality and with a slight abuse of notation, we are augmenting this vector with the wage W_t.

$$[\boldsymbol{\alpha}_t]_i \equiv \frac{\text{total expenditure of sector } i \text{ on labor}}{\text{total expenditure on inputs in sector } i} = \alpha_i(\mathbf{P}_t), \quad \forall i \in [n] \tag{B.10}$$

where the second equality holds *only* under firms' optimal expenditure shares and follows from integrating Equation (B.7). Since these elasticities are also homogeneous of degree zero in the price vector $\mathbf{P}_t$, Equations (B.9) and (B.10) imply that in *any equilibrium*, the cost-based input-output matrix and the vector of sectoral labor shares are only a function of the sectoral prices relative to the nominal wage; i.e.,

$$\mathbf{A}_t = \mathbf{A}(\mathbf{P}_t/W_t) = [a_{ik}(\mathbf{P}_t/W_t)], \qquad \boldsymbol{\alpha}_t = \boldsymbol{\alpha}(\mathbf{P}_t/W_t) = [\alpha_i(\mathbf{P}_t/W_t)] \tag{B.11}$$

Under Cobb-Douglas production function, $F_i(L_{ij,t}, X_{ij,1,t}, \ldots, X_{ij,n,t}) = \left(L_{ij,t}/\alpha_i\right)^{\alpha_i} \prod_{k\in[n]} \left(X_{ij,k,t}/a_{ik}\right)^{a_{ik}}$ with $\alpha_i + \sum_{k\in[n]} a_{ik} = 1, \forall i \in [n]$, the elasticities of cost relative to input prices are constant and lead to a *constant* input-output matrix and constant vector of labor shares over time, given by $\mathbf{A} = [a_{ik}]_{i,k\in[n]}$ and $\alpha = (\alpha_1, \ldots, \alpha_n)'$. Given the Cobb-Douglas production function, we have, for a given level of output Y

$$\frac{W_t L^*_{ij}}{\mathsf{MC}_{ij,t}(\mathbf{P}_t, Z_{ijt}) \times Y} = \alpha_i, \quad \frac{P_{k,t} X^*_{ij,k,t}}{\mathsf{MC}_{ij}(\mathbf{P}_t, Z_{ijt}) \times Y} = a_{ik}, \forall k \in [n] \tag{B.12}$$

with the marginal cost of firm ij given by

$$\mathsf{MC}_{ij}(\mathbf{P}_t, Z_{ijt}) = \frac{1}{Z_{ij,t}} W_t^{\alpha_i} \prod_{k\in[n]} P_{k,t}^{a_{ik}}, \quad \forall i \in [n] \tag{B.13}$$

B.3. Firms' Ideal Prices

Having characterized firms' cost functions, we now derive the optimal *ideal* prices, which are the prices that firms would set if prices were flexible. Given marginal cost $\mathsf{MC}_{ij,t}$ in (B.13) and demand functions $Y^d_{ij,t} = A_{ijt}\left(\frac{P_{ij,t}}{P_{i,t}}\right)^{-\eta_i} Y_{i,t}$, firm ij's ideal price solves

$$\max_{P_{ij,t}} (1-\tau_{i,t}) P_{ij,t} A_{ijt} \left(\frac{P_{ij,t}}{P_{i,t}}\right)^{-\eta_i} Y_{i,t} - \mathsf{MC}_{ij,t} \times A_{ijt} \left(\frac{P_{ij,t}}{P_{i,t}}\right)^{-\eta_i} Y_{i,t}$$

which leads to the following optimal prices:

$$P^*_{ij,t} = \frac{1}{(1-\tau_{i,t})} \frac{\eta_i}{\eta_i - 1} \mathsf{MC}_{ij,t} \tag{B.14}$$

B.4. Firms' Recursive Problem

To derive firms' recursive problem, we consider the value of a firm, $V_{ij,t}$, for a sequence of adjustment dates and price changes $\{T_h, \mathrm{d}P_{ij,T_h}\}_{h=1}^{\infty}$, where h indexes the moments at which the firm ij adjust prices[17].

$$V_{ij,t} = \int_0^\infty e^{-\rho s} \frac{U'(C_{t+s})}{P_{t+s}} \Big((1-\tau_{i,t}) P_{ij,t+s} \mathscr{D}(\frac{P_{ij,t+s}}{P_{i,t+s}}; Y_{i,t+s}, A_{ij,t+s}) - \mathscr{C}_{ij}(Y^s_{ij,t+s}; \mathbf{P}_{t+s}, Z_{ij,t+s}) \Big) \mathrm{d}s$$
$$- \sum_{h=1}^{\infty} e^{-\rho T_h} \frac{U'(C_{T_h})}{P_{T_h}} W_{T_h} \lambda_{i,T_h} \chi_i \mathbf{1}_{\{\mathrm{d}P_{ij,T_h} \neq 0 \text{ and } \mathrm{d}N_{ij,T_h} = 0\}}$$

[17] h should is firm-specific, ie, h_{ij}. However, with a slight abuse of notation we omit the index ij

The first term on the right-hand side captures the discounted flow of profits from production, while the second term captures the discounted menu cost expenditures. This expression can be manipulated to get to

$$V_{ij,t} = \int_0^\infty e^{-\rho s}\lambda_{i,t+s}\Big(\frac{P_{ij,t+s}}{\frac{1}{1-\tau_{i,t}}\frac{\eta_i}{\eta_i-1}\mathsf{MC}_{ij,t+s}} - \frac{\eta_i-1}{\eta_i}\Big)\frac{\eta_i}{\eta_i-1}\mathsf{MC}_{ij,t+s}\frac{1}{P_{i,t+s}Y_{i,t+s}}A_{ij,t+s}\Big(\frac{P_{ij,t+s}}{P_{i,t+s}}\Big)^{-\eta_i}Y_{i,t+s}\mathrm{d}s$$
$$- \sum_{h=1}^{\infty} e^{-\rho T_h}\frac{1}{P_{T_h}C_{T_h}}W_{T_h}\lambda_{i,T_h}\chi_i\mathbf{1}_{\{\mathrm{d}P_{ij,T_h}\neq 0 \text{ and } \mathrm{d}N_{ij,T_h}=0\}}$$

where the Domar weights show up after multiplying and dividing by sector i's sales in the first term and using $U(C) = \log C$; and the other expressions show up after substituting the CES demand into $\mathscr{D}(.)$, replacing $\mathscr{C}_{ij}(.)$ with marginal cost times demand, and rearranging terms. In the second term, I use $U(C) = \log C$. Then, we can define the price gap as $X_{ij,t} \equiv \frac{P_{ij,t+s}}{\frac{1}{1-\tau_{i,t}}\frac{\eta_i}{\eta_i-1}\mathsf{MC}_{ij,t+s}} = \frac{P_{ij,t+s}}{P^*_{ij,t+s}}$. With the definition of price gap, we can rewrite the value of the firm in terms of $X_{ij,t}$:

$$V_{ij,t} = \int_0^\infty e^{-\rho s}\lambda_{i,t+s}\Big(X_{ij,t+s} - \frac{\eta_i-1}{\eta_i}\Big)X_{ij,t+s}^{-\eta_i}(1-\tau_{i,t})A_{ij,t+s}\Big(\frac{P^*_{ij,t+s}}{P_{i,t+s}}\Big)^{1-\eta_i}\mathrm{d}s$$
$$- \sum_{h=1}^{\infty} e^{-\rho T_h}\frac{W_{T_h}L_{T_h}}{P_{T_h}C_{T_h}}\frac{\lambda_{i,T_h}\chi_i}{L_{T_h}}\mathbf{1}_{\{\mathrm{d}P_{ij,T_h}\neq 0 \text{ and } \mathrm{d}N_{ij,T_h}=0\}}$$
$$= \int_0^\infty e^{-\rho s}\lambda_{i,t+s}\Big(X_{ij,t+s} - \frac{\eta_i-1}{\eta_i}\Big)X_{ij,t+s}^{-\eta_i}(1-\tau_{i,t})^{\eta_i}\underbrace{A_{ij,t+s}Z_{ij,t+s}^{\eta_i-1}}_{=1}\Big(\frac{\eta_i}{\eta_i-1}\frac{W_t^{\alpha_i}\prod_{k\in[n]}P_{k,t}^{a_{ik}}}{P_{i,t+s}}\Big)^{1-\eta_i}\mathrm{d}s$$
$$- \sum_{h=1}^{\infty} e^{-\rho T_h}\frac{W_{T_h}L_{T_h}}{P_{T_h}C_{T_h}}\frac{\lambda_{i,T_h}\chi_i}{L_{T_h}}\mathbf{1}_{\{\mathrm{d}P_{ij,T_h}\neq 0 \text{ and } \mathrm{d}N_{ij,T_h}=0\}}$$

where in the first line we use the definition of price gaps and ideal prices, in the second line we multiply and divide by L_{T_h} to make it explicit that *labor share* scales the total menu cost paid. In the third line, we replace $P^*_{ij,t+s}$ with its definition and the assumption that $A_{ij,t+s}Z_{ij,t+s}^{\eta_i-1} = 1$. Now, in the stationary equilibrium with steady-state inflation π, aggregate nominal variables grow at a rate π, while real variables are constant. This implies in

$$V_{ij,t} = \int_0^\infty e^{-\rho s}\lambda_i\Big(X_{ij,t+s} - \frac{\eta_i-1}{\eta_i}\Big)X_{ij,t+s}^{-\eta_i}(1-\tau_{i,t})^{\eta_i}\Big(\frac{\eta_i}{\eta_i-1}\frac{W_t^{\alpha_i}\prod_{k\in[n]}P_{k,t}^{a_{ik}}}{P_{i,t+s}}\Big)^{1-\eta_i}\mathrm{d}s$$
$$- \sum_{h=1}^{\infty} e^{-\rho T_h}\frac{W_tL}{P_tC}\frac{\lambda_i\chi_i}{L}\mathbf{1}_{\{\mathrm{d}P_{ij,T_h}\neq 0 \text{ and } \mathrm{d}N_{ij,T_h}=0\}}$$

where we drop t from Domar weights, aggregate consumption, and aggregate labor as they are constant. It is important to note that aggregate wages and prices are still indexed by t as they are growing over time. However, their *ratio* is constant in a stationary equilibrium as they are growing at the same rate. Also, note that, after solving for the optimal policy and the stationary distribution, we need to verify that aggregate prices are indeed growing at the same rate. Now, we factor out

$\lambda_i(1-\tau_{i,t})^{\eta_i}\left(\frac{\eta_i}{\eta_i-1}\frac{W_t^{\alpha_i}\prod_{k\in[n]}P_{k,t}^{a_{ik}}}{P_{i,t+s}}\right)^{1-\eta_i}$ to get

$$V_{ij,t}=\lambda_i(1-\tau_{i,t})^{\eta_i}\left(\frac{\eta_i}{\eta_i-1}\frac{W_t^{\alpha_i}\prod_{k\in[n]}P_{k,t}^{a_{ik}}}{P_{i,t+s}}\right)^{1-\eta_i}\left[\int_0^\infty e^{-\rho s}\left(X_{ij,t+s}-\frac{\eta_i-1}{\eta_i}\right)X_{ij,t+s}^{-\eta_i}\mathrm{d}s\right.$$
$$\left.-\sum_{h=1}^{\infty}e^{-\rho T_h}\frac{W_tL}{P_tC}\frac{1}{L}\frac{\chi_i}{(1-\tau_i)^{\eta_i}\left(\frac{\eta_i}{\eta_i-1}\frac{W_t^{\alpha_i}\prod_{k\in[n]}P_{k,t}^{a_{ik}}}{P_{i,t+s}}\right)^{1-\eta_i}}\mathbf{1}_{\{\mathrm{d}P_{ij,T_h}\neq 0\text{ and }\mathrm{d}N_{ij,T_h}=0\}}\right]$$

Define $\phi_i\equiv\frac{W_tL}{P_tC}\frac{1}{L}\frac{\chi_i}{(1-\tau_{i,t})^{\eta_i}\left(\frac{\eta_i}{\eta_i-1}\frac{W_t^{\alpha_i}\prod_{k\in[n]}P_{k,t}^{a_{ik}}}{P_{i,t}}\right)^{1-\eta_i}}$ and for any variable $Q_{ij,t}$, let $q_{ij,t}\equiv\log Q_{ij,t}$. Then,

$$\bar{V}_{ij,t}=\int_0^\infty e^{-\rho s}\left(e^{x_{ijt+s}}-\frac{\eta_i-1}{\eta_i}\right)e^{-\eta_i x_{ijt+s}}\mathrm{d}s-\sum_{h=1}^{\infty}e^{-\rho T_h}\phi_i\mathbf{1}_{\{\mathrm{d}P_{ij,T_h}\neq 0\text{ and }\mathrm{d}N_{ij,T_h}=0\}}$$

where $\bar{V}_{ij,t}\equiv\frac{V_{ij,t}}{\lambda_i(1-\tau_i)^{\eta_i}\left(\frac{\eta_i}{\eta_i-1}\frac{W_t^{\alpha_i}\prod_{k\in[n]}P_{k,t}^{a_{ik}}}{P_{i,t+s}}\right)^{1-\eta_i}}$. So, given ϕ_i, firm ij can choose adjustment dates and price gaps $\{T_h,x_{ij,T_h}\}_{h=1}^{\infty}$ to maximize $\bar{V}_{ij,t}$, noting that its denominator simply scales $V_{ij,t}$. Now, taking logs of the price gap $X_{ij,t}$, under a stationary equilibrium, if the firm doesn't change prices, $x_{ij,t}$ evolves as $\mathrm{d}x_{ij,t}=(\mu_i-\pi)\mathrm{d}t+\sigma_i\mathrm{d}\mathcal{W}_{ij,t}$. The recursive problem associated with the maximization of the value of the firm above is given by

$$v(x)=\max\left\{\max_y v(y)-\phi_i,f_i(x_i)\mathrm{d}t+(1-\rho dt)\left[(1-\theta_i\mathrm{d}t)\mathbb{E}\left[v(x+\mathrm{d}x)\right]+\theta_i\mathrm{d}t\times\max_y v(y)\right]\right\}$$

where $f_i(x)\equiv\left(e^x-\frac{\eta_i-1}{\eta_i}\right)e^{-\eta_i x}$, which leads to the following Hamilton-Jacobi-Bellman-Variational Inequality equation:

$$\rho v_i(x)=\max\left\{\rho\left(\max_y v(y)-\phi_i\right),f_i(x_i)+(\mu_i-\pi)v_i'(x)+\frac{\sigma_i^2}{2}v_i''(x)+\theta_i\left(\max_y v_i(y)-v_i(x)\right)\right\}$$

Solving the value function results in an optimal policy characterized by $(x_i^*,\underline{x}_i,\bar{x}_i)$

B.5. Aggregate Sectoral Production Function

In this section, we aggregate input demands for firms within a sector to derive a notion of aggregate sectoral production function. For each sector $i\in[n]$, let's consider the total amount of labor and intermediate input k employed for production, $L_{i,t}$ and $X_{i,k,t}$, respectively,

$$L_{i,t}=\int_0^1 L_{ijt}\mathrm{d}j,\quad X_{i,k,t}=\int_0^1 X_{ij,k,t}\mathrm{d}j$$

From the cost minimization problem of an intermediate firm $j\in[0,1]$ in sector $i\in[n]$, we derive the cost share of labor, cost share of intermediate input k and the firm's marginal cost

$$L_{ij,t}=\frac{\alpha_i\mathsf{MC}_{ij,t}}{W_t}\times Y_{ij,t},\quad X_{ij,k,t}=\frac{a_{ik}\mathsf{MC}_{ij,t}}{P_{k,t}}\times Y_{ij,t},\forall k\in[n]$$
$$\mathsf{MC}_{ij,t}=\frac{1}{Z_{ij,t}}W_t^{\alpha_i}\prod_{k\in[n]}P_{k,t}^{a_{ik}},\quad\forall i\in[n]$$

where, under Cobb-Douglas production function, $(\alpha_i)_{i\in[n]}$ and $(a_{i,k})_{i,k\in[n]}$ are parameters. Aggregating the individual labor demands within sector i and using the firm ij's demand function $Y_{ij,t} = A_{ij,t}\left(\frac{P_{ij,t}}{P_{i,t}}\right)^{-\eta_i} Y_{ij,t}$, we can derive the total labor employed in sector i for production as a function of total output produced in i,

$$L_{i,t} = \frac{\alpha_i W_t^{\alpha_i} \prod_{k\in[n]} P_{k,t}^{a_{ik}}}{W_t} Y_{i,t} \int \frac{A_{ij,t}}{Z_{ij,t}} \left(\frac{P_{ij,t}}{P_{i,t}}\right)^{-\eta_i} \mathrm{d}j$$

Similarly, we can aggregate the individual intermediate input demands into sector i's demand for intermediate input k as a function of total output

$$X_{i,k,t} = \frac{a_{ik} W_t^{\alpha_i} \prod_{\ell\in[n]} P_{\ell,t}^{a_{i\ell}}}{P_{k,t}} Y_{i,t} \int \frac{A_{ij,t}}{Z_{ij,t}} \left(\frac{P_{ij,t}}{P_{i,t}}\right)^{-\eta_i} \mathrm{d}j$$

Rearranging the input demand equations, we can express $Y_{i,t}$ as a function of sectoral inputs and prices. Using the Cobb–Douglas structure, we raise each input to the power of its cost share and multiply across all inputs. Finally, applying the condition $\alpha_i + \sum_{k\in[n]} a_{ik} = 1$, we obtain:

$$Y_{i,t} = \frac{1}{\int \frac{A_{ij,t}}{Z_{ij,t}} \left(\frac{P_{ij,t}}{P_{i,t}}\right)^{-\eta_i} \mathrm{d}j} \left(\frac{L_{i,t}}{\alpha_i}\right)^{\alpha_i} \prod_{k\in[n]} \left(\frac{X_{i,k,t}}{a_{ik}}\right)^{a_{ik}}$$

$$Y_{i,t} = \frac{1}{\int \frac{A_{ij,t}}{Z_{ij,t}} \left(\frac{P_{ij,t}}{P_{i,t}}\right)^{-\eta_i} \mathrm{d}j} F_i(L_{i,t}, X_{i,1,t}, \ldots, X_{i,n,t}) \tag{B.15}$$

This expression takes the form of a production function for sector $i \in [n]$, with $Z_{i,t} \equiv \left(\int \frac{A_{ij,t}}{Z_{ij,t}} \left(\frac{P_{ij,t}}{P_{i,t}}\right)^{-\eta_i} \mathrm{d}j\right)^{-1}$ interpreted as the sector's aggregate productivity. When prices are flexible, $Z_i^f = \left[\int A_{ij,t} Z_{ij,t}^{\eta_i - 1} \mathrm{d}j\right]^{\frac{1}{\eta_i - 1}}$. Generically, in the presence of idiosyncratic shocks, there will be price dispersion because some firms will be more productive than others and hence will set lower prices. The magnitude of the flexible-price equilibrium aggregate sectoral productivity will depend on the relationship between idiosyncratic taste and idiosyncratic productivity shocks. When $A_{ij,t} Z_{ij,t}^{\eta_i - 1} = 1, \forall ij$, ie, increases in $Z_{ij,t}^{\eta_i - 1}$ are completely offset by decreases in $A_{ij,t}$, $Z_i^f = 1$. When prices are sticky, price stickiness introduces misallocation of resources which drags aggregate sectoral productivity. Let $D_{i,t} \equiv Z_i^f / Z_{i,t}$ be the measure of inefficient price dispersion introduced by price stickiness. Under the assumption $A_{ij,t} Z_{ij,t}^{\eta_i - 1} = 1, \forall ij$, we have

$$D_i = \int \frac{A_{ij,t}}{Z_{ij,t}} \left(\frac{P_{ij,t}}{P_{i,t}}\right)^{-\eta_i} \mathrm{d}j \tag{B.16}$$

where we drop t because we consider a stationary equilibrium with a stationary distribution of price gaps.

Since we solve for the value function as a function of price gaps $x_{ij,t}$, let's express Equation (B.16) as a function of price gaps. Starting from the definition of $X_{ij,t}$, a firm ij's relative prices can be

written as

$$\frac{P_{ij,t}}{P_{i,t}} = X_{ij,t}\frac{\frac{1}{(1-\tau_i)}\frac{\eta_i}{\eta_i-1}\mathsf{MC}_{ij,t}}{P_{i,t}} \tag{B.17}$$

Substituting Equation (B.17) into the definition of sector i's price index, we get

$$\frac{\frac{1}{(1-\tau_i)}\frac{\eta_i}{\eta_i-1}\mathsf{MC}_{ij,t}}{P_{i,t}} = \frac{1}{Z_{ij,t}\left[\int e^{(1-\eta_i)x_{ij,t}}\mathrm{d}j\right]^{\frac{1}{1-\eta_i}}} \tag{B.18}$$

which, when substituted back into Equation (B.17), yields the following expression for the relative price in terms of sectoral price gaps:

$$\frac{P_{ij,t}}{P_{i,t}} = \frac{1}{Z_{ij,t}}e^{x_{ij,t}}\left[\int e^{(1-\eta_i)x_{ij,t}}\mathrm{d}j\right]^{\frac{1}{\eta_i-1}} \tag{B.19}$$

Now, plugging $P_{ij,t}/P_{i,t}$ into D_i, we get

$$D_i = \int e^{-\eta_i x_{ijt}}\mathrm{d}j \times \left[\int e^{(1-\eta_i)x_{ij,t}}\mathrm{d}j\right]^{\frac{\eta_i}{1-\eta_i}} \tag{B.20}$$

Given the expression for sector i's production function, $Y_{i,t} = \frac{1}{D_i}F_i(L_{i,t}, X_{i,1,t}, \ldots, X_{i,n,t})$ we can calculate the 'as if' aggregate sectoral marginal cost, ie, $\mathsf{MC}_{i,t}$ such that

$$\begin{aligned}\mathsf{MC}_{i,t} &\equiv \min_{L_{i,t},(X_{i,k,t})_{k\in[n]}} W_t L_{i,t} + \sum_{k\in[n]} P_{k,t}X_{i,k,t} \quad s.t. \quad \frac{1}{D_i}\times F_i(L_{i,t}, X_{i,1,t}, \ldots, X_{i,n,t}) \geq Y \\ \implies \mathsf{MC}_{i,t} &= W_t^{\alpha_i}\prod_{k\in[n]} P_{k,t}^{a_{ik}} \times D_i = W_t^{\alpha_i}\prod_{k\in[n]} P_{k,t}^{a_{ik}} \times \int e^{-\eta_i x_{ijt}}\mathrm{d}j \times \left[\int e^{(1-\eta_i)x_{ij,t}}\mathrm{d}j\right]^{\frac{\eta_i}{1-\eta_i}}\end{aligned} \tag{B.21}$$

B.6. Sectoral Markup

Now, define the sector i's markup as $\mathscr{M}_i \equiv {}^{P_{i,t}}/_{\mathsf{MC}_{i,t}}$. Using Equations (B.18) and (B.21), we get

$$\mathscr{M}_i \equiv \frac{P_{i,t}}{\mathsf{MC}_{i,t}} = \frac{1}{(1-\tau_i)}\frac{\eta_i}{\eta_i-1}\frac{\int e^{(1-\eta_i)x_{ijt}}\mathrm{d}j}{\int e^{-\eta_i x_{ijt}}\mathrm{d}j} \tag{B.22}$$

B.7. Market Clearing and Total Value Added

Recall that the sales-based Domar weight of sector $i \in [n]$ is defined as the ratio of the final producer's sales relative to household's total expenditure on consumption, $\lambda_i \equiv {}^{P_{i,t}Y_{i,t}}/_{P_tC_t}$. Substituting the optimal consumption from sector $k \in [n]$ in Equation (B.1) and optimal demand of firms for the final good of sector $k \in [n]$ in Equation (B.7) into the sector k's final good market clearing condition and dividing by the household's total consumption expenditure, we get

$$\lambda_k = \beta_k + \sum_{i\in[n]} a_{ik}\frac{1}{\mathscr{M}_k}\lambda_i \tag{B.23}$$

where $\mathscr{M}_i$ is sector i's markup given in Equation (B.22). Letting $\boldsymbol{\lambda} \equiv (\lambda_i)_{i\in[n]}$ and $\mathscr{M} \equiv \mathrm{diag}(\mathscr{M}_i)$ be, respectively, the vectors of sectoral sales-based Domar weights and the diagonal matrix whose i's

diagonal entry is sector i's markup, we can rewrite Equation (B.23) as

$$\boldsymbol{\lambda} = \left(\mathbf{I} - \mathbf{A}'\mathscr{M}^{-1}\right)^{-1}\boldsymbol{\beta} \tag{B.24}$$

Finally, substituting firms labor demand into labor market condition, we arrive at the following expression for the labor share:

$$\frac{W_t L}{P_t C} = \sum_{i\in[n]} \alpha_i \mathscr{M}_i^{-1}\lambda_i + \frac{W_t}{P_t C}\sum_{i\in[n]} \lambda_i \chi_i \int_0^1 \mathbf{1}_{\{\mathrm{d}P_{ij,t}\neq 0 \text{ and } \mathrm{d}N_{ij,t}=0\}}\mathrm{d}j \tag{B.25}$$

where the second term on the right-hand side captures the number of firms that pay the menu cost – recall that the menu cost is in units of labor. This can also be rewritten as

$$\frac{W_t L}{P_t C} = \frac{\sum_{i\in[n]} \alpha_i \mathscr{M}_i^{-1}\lambda_i}{\left(1 - \sum_{i\in[n]} \frac{\lambda_i \chi_i}{L}\mathsf{N}_i^{menu}\right)} \tag{B.26}$$

where $\mathsf{N}_i^{menu} \equiv \int_0^1 \mathbf{1}_{\{\mathrm{d}P_{ij,t}\neq 0 \text{ and } \mathrm{d}N_{ij,t}=0\}}\mathrm{d}j$

B.8. Aggregate Productivity

To calculate the welfare cost of inflation, it is instructive to define the idea of aggregate productivity of the economy. We start from the labor share expression to express aggregate consumption as a function of labor:

$$C = \underbrace{\frac{1}{(P_t/W_t)} \times \frac{1}{\frac{\sum_{i\in[n]} \alpha_i \mathscr{M}_i^{-1}\lambda_i}{\left(1 - \sum_{i\in[n]} \frac{\lambda_i \chi_i}{L}\mathsf{N}_i^{menu}\right)}}}_{\equiv Z} \times L$$

Note that the expression above resembles a linear production function, with the sole input being labor and the aggregate productivity given by Z above. Under Cobb-Douglas consumption aggregator,

$$\frac{P_t}{W_t} = \prod_{i\in[n]} \left(\frac{P_{i,t}}{W_t}\right)^{\beta_i}$$

Now, using the definition of sectoral markups along with sectoral marginal costs, we get

$$\log\left(\frac{P_{i,t}}{W_t}\right) = \log\left(\mathscr{M}_i D_i\right) + \sum_{k\in[n]} a_{ik}\log\left(\frac{P_{k,t}}{W_t}\right)$$

we can stack the expression above for $i \in [n]$, rearrange it, and multiply by the transpose of the consumption shares vector, $\boldsymbol{\beta}'$, to get

$$\begin{aligned}
\log\left(\frac{P_t}{W_t}\right) &= \boldsymbol{\beta}'(\mathbf{I}-\mathbf{A})^{-1}\Big(\log(\mathscr{M}_i D_i)\Big)_{i\in[n]} \\
\log\left(\frac{P_t}{W_t}\right) &= \boldsymbol{\beta}'(\mathbf{I}-\mathbf{A})^{-1}\left(\log\left(\frac{1}{(1-\tau_i)}\frac{\eta_i}{\eta_i - 1}\left[\int e^{(1-\eta_i)x_{ijt}}\mathrm{d}j\right]^{\frac{1}{1-\eta_i}}\right)\right)_{i\in[n]} \\
\left(\frac{P_t}{W_t}\right) &= \exp\left(\boldsymbol{\beta}'(\mathbf{I}-\mathbf{A})^{-1}\left(\log\left(\frac{1}{(1-\tau_i)}\frac{\eta_i}{\eta_i - 1}\left[\int e^{(1-\eta_i)x_{ijt}}\mathrm{d}j\right]^{\frac{1}{1-\eta_i}}\right)\right)_{i\in[n]}\right)
\end{aligned}$$

This implies in

$$Z(\pi) = \frac{1}{\exp\left(\boldsymbol{\beta}'(\mathbf{I}-\mathbf{A})^{-1}\left(\log\left(\frac{1}{(1-\tau_i)}\frac{\eta_i}{\eta_i-1}\left[\int e^{(1-\eta_i)x_{ijt}}\mathrm{d}j\right]^{\frac{1}{1-\eta_i}}\right)\right)_{i\in[n]}\right)} \times \frac{1}{\frac{\sum_{i\in[n]}\alpha_i\mathscr{M}_i^{-1}\lambda_i}{\left(1-\sum_{i\in[n]}\frac{\lambda_i\chi_i}{L}\mathsf{N}_i^{menu}\right)}} \tag{B.27}$$

C Welfare Cost of Inflation

We measure the welfare cost of inflation in terms of consumption equivalent, i.e., the percentage change in steady-state consumption under zero inflation that makes the representative agent indifferent between the zero-inflation steady state and the steady state with inflation π:

$$\mathscr{U}(C(\pi), L(\pi)) = \mathscr{U}(e^{-\Lambda(\pi)}C_0, L_0)$$

where $\mathscr{U}(.,.)$ is the utility function, $C(\pi)$ and $L(\pi)$ are, respectively, the steady-state consumption and labor supply under inflation π, C_0 and L_0 are, respectively, the steady-state consumption and labor supply under zero inflation, and $\Lambda(\pi)$ is the welfare cost of inflation when inflation is π. Under separable utility function, with $U(C) = \log(C)$ and $V(L) = {}^{L^{1+\psi}}/_{(1+\psi)}$, we have

$$\Lambda(\pi) = \log C_0 - \log C(\pi) - \frac{L_0^{1+\psi}}{1+\psi} + \frac{L(\pi)^{1+\psi}}{1+\psi}$$

Using the relationship between aggregate consumption, productivity and labor, $C(\pi) = Z(\pi) \times L(\pi)$, we get

$$\Lambda(\pi) = \log Z_0 - \log Z(\pi) + \log L_0 - \log L(\pi) - \frac{L_0^{1+\psi}}{1+\psi} + \frac{L(\pi)^{1+\psi}}{1+\psi} \tag{C.1}$$

Equation (C.1) shows that steady-state inflation affects welfare through (1) aggregate productivity; (2) labor supply, with $Z(\pi)$ given by Equation (B.27).

In our baseline results, besides Cobb-Douglas consumption aggregator and production function, we assume that $\psi \to 0$ and that $\mathscr{M}_i = 1, \forall i \in [n]$. That is, we assume linear disutility of labor and that taxes are set such that sectoral markups are equal to one, i.e.,

$$\tau_i = 1 - \frac{\eta_i}{\eta_i - 1}\frac{\int e^{(1-\eta_i)x_{ijt}}\mathrm{d}j}{\int e^{-\eta_i x_{ijt}}\mathrm{d}j}, \quad \forall i \in [n] \tag{C.2}$$

Under these assumptions, the welfare cost of inflation is given by

$$\Lambda(\pi) = \sum_{i\in[n]} \lambda_i \times \left[\log(D_i(\pi)) - \log(D_i(0))\right] + \sum_{i\in[n]} \lambda_i\chi_i\left[\mathsf{N}_i^{menu}(\pi) - \mathsf{N}_i^{menu}(0)\right] \tag{C.3}$$

where $\boldsymbol{\lambda}$ collapses to $(\mathbf{I}-\mathbf{A}')^{-1}\boldsymbol{\beta}$. Equation (C.3) shows that the welfare cost of inflation at steady-state inflation π relative to a zero inflation steady state is given by sum of the Domar-weighted difference between sectoral log dispersions and the difference between the number of firms paying the menu cost across sectors.

Map between ϕ_i and χ_i. In Appendix B.4, we define $\phi_i \equiv \frac{W_t L}{P_t C}\frac{1}{L}\frac{\chi_i}{(1-\tau_{i,t})^{\eta_i}\left(\frac{\eta_i}{\eta_i-1}\frac{W_t^{\alpha_i}\prod_{k\in[n]}P_{k,t}^{a_{ik}}}{P_{i,t}}\right)^{1-\eta_i}}$, which is the menu cost faced by firm ij when it solves its optimal policy function, which is a function of the structural menu cost parameter χ_i. In our baseline results, using Equation (C.2), we get

$$\phi_i = \frac{\eta_i - 1}{\eta_i} \times \chi_i \times \left[\int e^{-\eta_i x_{ijt}} \mathrm{d}j\right] \tag{C.4}$$

C.1. Welfare cost of inflation with respect to flexible-price steady state

We proceed similarly to when we compute the welfare with respect to zero-inflation steady state, except that now we compare to the flexible-price steady state. That is, we are interested in $\Lambda^{\mathrm{nr}}(\pi)$ such that

$$\mathscr{U}(C(\pi), L(\pi)) = \mathscr{U}(e^{-\Lambda^{\mathrm{nr}}(\pi)} C_{\mathrm{flex}}, L_{\mathrm{flex}})$$

where C_{flex}, L_{flex} are the consumption and labor supply under flexible-price steady state. This implies in

$$\Lambda^{\mathrm{nr}}(\pi) = \log Z_{\mathrm{flex}} - \log Z(\pi) + \log L_{\mathrm{flex}} - \log L(\pi) + \frac{L(\pi)^{1+\psi}}{1+\psi} - \frac{L_{\mathrm{flex}}^{1+\psi}}{1+\psi}$$

Now, the new objects we need to compute are Z_{flex} and L_{flex}. We assume Cobb-Douglas consumption aggregator and production function. Under flexible prices, $Z_{i,t}^{agg,f} = \frac{1}{\int \frac{A_{ij,t}}{Z_{ij,t}}\left(\frac{P_{ij,t}^f}{P_{i,t}^f}\right)^{-\eta_i} dj}$, where superscript f stands for flexible-price objects. Using the optimal pricing of a firm ij in period t under flexible prices, one can see that $P_{ij,t}^f / P_{i,t}^f = 1/Z_{ij,t}$. Using this relationship into $Z_{i,t}^{agg,f}$ results in $Z_{i,t}^{agg,f} = 1$, as we assume that $A_{ij,t} Z_{ij,t}^{\eta_i - 1} = 1$. Now, using the aggregate sectoral production, a sector $i \in [n]$ aggregate sectoral marginal cost under flexible prices is given by $\mathsf{MC}_{i,t}^f = (W_t^f)^{\alpha_i}\prod_{i\in[n]}(P_{k,t}^f)^{a_{ik}}$ and the sectoral markup $\mathscr{M}_{i,t}^f \equiv P_{i,t}^f / \mathsf{MC}_{i,t}^f = \frac{1}{1-\tau_i}\frac{\eta_i}{\eta_i - 1}$, where $P_{i,t}^f = \frac{1}{(1-\tau_i)}\frac{\eta_i}{\eta_i-1} W_t^{\alpha_i}\prod_{k\in[n]} P_{k,t}^{a_{ik}}$. Taking log of the sectoral prices, then stacking them in a vector, manipulating the expression, then multiplying by β', we get $\log\left(\frac{P_t}{W_t}\right)^f = \boldsymbol{\beta}'(\mathbf{I}-\mathbf{A})^{-1}(\log \mathscr{M}_{i,t}^f)_{i\in[n]}$, or $\left(\frac{P_t}{W_t}\right)^f = \exp\left(\boldsymbol{\beta}'(\mathbf{I}-\mathbf{A})^{-1}(\log \mathscr{M}_{i,t}^f)_{i\in[n]}\right)$. Assuming that taxes are such that $\mathscr{M}_{i,t}^f = 1$, we get $\left(P_t/W_t\right)^f = 1$. The labor market clearing implies $L_t^f = \sum_{i\in[n]}\int L_t^f \, dj$, recalling that under flexible prices, there's no labor being used to pay menu costs. We can manipulate this expression, to find that

$$C^f = \frac{1}{\boldsymbol{\alpha}'\mathrm{diag}\left(\mathscr{M}_i^f\right)\boldsymbol{\lambda}^f}\frac{1}{(P_t/W_t)^f} L^f$$

where $\boldsymbol{\lambda}^f = (\mathbf{I}-\mathbf{A}'\mathrm{diag}(\mathscr{M}_i^f))^{-1}\boldsymbol{\beta}$. When taxes are such that $\mathscr{M}_i^f = 1$, $\boldsymbol{\alpha}'\mathrm{diag}\left(\mathscr{M}_i^f\right)\boldsymbol{\lambda}^f = \boldsymbol{\alpha}'(\mathbf{I}-\mathbf{A}')^{-1}\boldsymbol{\beta} = \mathbf{1}'(\mathbf{I}-\mathbf{A}')(\mathbf{I}-\mathbf{A}')^{-1}\boldsymbol{\beta} = 1$ and $\left(P_t/W_t\right)^f = 1$. Therefore, $C^f = L^f$. Then using household's intratemporal optimality condition, we get $\left(\frac{W_t}{P_t}\right)^f = C_t^f (L_t^f)^{\psi}$. Since $(W_t/P_t)^f = 1$, and $C^f = L^f = 1$, we get $C^f = L^f = 1$. Now, under Golosov-Lucas preferences, we have

$$\Lambda^{\mathrm{nr}}(\pi) = -\log Z(\pi) - \log L(\pi) + L(\pi) - 1$$

But, as in the derivation of the welfare cost of inflation with respect to zero-inflation steady state, under taxes that sets sectoral markups to one and Golosov-Lucas preferences, $\log Z(\pi) = -\sum_{i\in[n]} \tilde{\lambda}_i \times \log\big(D_i(\pi)\big) - \log\big(1+\sum_{i\in[n]} \lambda_i \chi_i \mathsf{N}_i^{menu}(\pi)\big)$, where $\tilde{\lambda}_i = e_i'(\mathbf{I}-\mathbf{A}')^{-1}\boldsymbol{\beta}$, $L(\pi) = 1+\sum_{i\in[n]} \tilde{\lambda}_i \chi_i \mathsf{N}_i^{menu}(\pi)$, $\log L(\pi) = \log\big(1+\sum_{i\in[n]} \tilde{\lambda}_i \chi_i \mathsf{N}_i^{menu}(\pi)\big)$. Hence,

$$\begin{aligned}
\Lambda^{\text{nr}}(\pi) &= \sum_{i\in[n]} \tilde{\lambda}_i \times \log\big(D_i(\pi)\big) + \log\big(1+\sum_{i\in[n]} \lambda_i \chi_i \mathsf{N}_i^{menu}(\pi)\big) - \log\big(1+\sum_{i\in[n]} \tilde{\lambda}_i \chi_i \mathsf{N}_i^{menu}(\pi)\big) \\
&\quad + 1 + \sum_{i\in[n]} \tilde{\lambda}_i \chi_i \mathsf{N}_i^{menu}(\pi) - 1 \\
\Lambda^{\text{nr}}(\pi) &= \sum_{i\in[n]} \tilde{\lambda}_i \times \log D_i(\pi) + \sum_{i\in[n]} \tilde{\lambda}_i \chi_i \mathsf{N}_i^{menu}(\pi)
\end{aligned}$$

D Solution Method

In this section, we characterize the solution to the value function, $v_i(x), i \in [n]$, and the stationary distribution of price gaps, $\mathsf{g}_i(x), i \in [n]$.

D.1. CalvoPlus model

In the CalvoPlus setting, we have that the value function satisfies the following HJB-VI equation:

$$\rho v(x) = \max\left\{\rho(v(x^*)-\phi), f(x) + (\mu-\pi)v'(x) + \frac{\sigma^2}{2}v''(x) + \theta(v(x^*)-v(x))\right\}$$

with the value matching $v(\underline{x}) = v(x^*)-\phi$, $v(\bar{x}) = v(x^*)-\phi$, and smooth pasting $v'(\underline{x}) = v'(\bar{x}) = 0$ conditions, and optimality of the reset gap $v'(x^*) = 0$. In the interior of the Ss bands, $x \in (\underline{x}, \bar{x})$,

$$\rho v(x) = f(x) + (\mu-\pi)v'(x) + \frac{\sigma^2}{2}v''(x) + \theta(v(x^*)-v(x))$$

or, rearranging,

$$\frac{\sigma^2}{2}v''(x) + (\mu-\pi)v'(x) - (\rho+\theta)v(x) = -f(x) - \theta v(x^*) \tag{D.1}$$

where $f(x) = e^{(1-\eta)x} - \frac{\eta-1}{\eta}e^{-\eta x}$. Equation (D.1) is a second-order, linear non-homogeneous differential equation. First, let's perform a transformation of the problem. Define $u(x) \equiv v(x) - v(x^*)$. Then $u(x^*) = 0$, $u'(x) = v'(x)$, $u''(x) = v''(x)$ and

$$\begin{aligned}
\frac{\sigma^2}{2}u''(x) + (\mu-\pi)u'(x) - (\rho+\theta)u(x) - (\rho+\theta)v(x^*) &= -f(x) - \theta v(x^*) \\
\frac{\sigma^2}{2}u''(x) + (\mu-\pi)u'(x) - (\rho+\theta)u(x) &= -f(x) + \rho v(x^*)
\end{aligned} \tag{D.2}$$

The optimality conditions are then given by $u(\underline{x}) = -\phi$, $u(\bar{x}) = -\phi$, $u'(\underline{x}) = 0$, $u'(\bar{x}) = 0$, $u'(x^*) = 0$, and $u(x^*) = 0$. The solution is given by the sum of the solution to the homogeneous second-order differential equation, $u_h(x)$, and the particular solution to Equation (D.2), $u_p(x)$. We guess that

$u_h(x) = e^{rx}$, which, plugging into the Equation (D.1) homogeneous counterpart, results in

$$\frac{\sigma^2}{2}r^2 + (\mu - \pi)r - (\rho + \theta) = 0$$

The roots of this second-order equation are

$$r_1 = \frac{-(\mu-\pi) + \sqrt{(\mu-\pi)^2 + 2\sigma^2(\rho+\theta)}}{\sigma^2}, \quad r_2 = \frac{-(\mu-\pi) + \sqrt{(\mu-\pi)^2 + 2\sigma^2(\rho+\theta)}}{\sigma^2} \tag{D.3}$$

So that $u_h(x) = c_1 e^{r_1 x} + c_2 e^{r_2 x}$. For the particular solution, we guess $u_p(x) = Ae^{(1-\eta)x} + Be^{-\eta x} + C$, which substituting into Equation (D.2) and matching coefficients results in

$$A = \frac{1}{(\rho+\theta) - (1-\eta)(\mu-\pi) - \frac{\sigma^2}{2}(1-\eta)^2}, \quad B = \frac{-\frac{(\eta-1)}{\eta}}{(\rho+\theta) + \eta(\mu-\pi) - \frac{\sigma^2}{2}\eta^2}, \quad C = -\frac{\rho v(x^*)}{\rho+\theta} \tag{D.4}$$

and we have the following general solution $u(x) = c_1 e^{r_1 x} + c_2 e^{r_2 x} + Ae^{(1-\eta)x} + Be^{-\eta x} + C$. Now, we have six unknowns $(c_1, c_2, C, \underline{x}, \bar{x}, x^*)$ and six conditions given by $u(\underline{x}) = -\phi$, $u(\bar{x}) = -\phi$, $u'(\underline{x}) = 0$, $u'(\bar{x}) = 0$, $u'(x^*) = 0$, and $u(x^*) = 0$. This forms a system of nonlinear equations that can be solved numerically. Once we solve for these unknowns, we recover $v(x^*) = -\frac{\rho+\theta}{\rho}C$, $v(x) = u(x) + v(x^*)$. Then, given the policy function $(\underline{x}, \bar{x}, x^*)$, we can solve for the stationary distribution given by

$$\frac{\sigma^2}{2}g''(x) - (\mu-\pi)g'(x) - \theta g(x) = 0, \quad \forall x \neq x^*, x \in (\underline{x}, \bar{x}) \tag{D.5}$$

where we impose (1) $g(\underline{x}) = g(\bar{x}) = 0$, reflecting the instantaneous adjustment of firms once they reach the boundaries $\underline{x}$ or $\bar{x}$; (2) $g(x^*_-) = g(x^*_+)$, as $g(x)$, reflecting that the density function must be continuous at the reset gap; (3) $\int_{\underline{x}}^{\bar{x}} g(x)dx = 1$, ensuring the $g(x)$ is a probability density function. Whenever the gap reaches the boundaries, it instantaneously moves to the reset gap. The resulting probability density function is not differentiable at that point. Therefore, to characterize the stationary distribution, we need to consider the probability density function in two regions: (a) $x \in [\underline{x}, x^*]$; (b) $x \in (x^*, \bar{x}]$. We guess that $g(x) = e^{rt}$ in each one of the subregions, resulting in the following characteristic equation

$$\frac{\sigma^2}{2}r^2 - (\mu-\pi)r - \theta = 0$$

with roots

$$r_1 = \frac{(\mu-\pi) + \sqrt{(\mu-\pi)^2 + 2\sigma^2\theta}}{\sigma^2}, \quad r_2 = \frac{(\mu-\pi) - \sqrt{(\mu-\pi)^2 + 2\sigma^2\theta}}{\sigma^2} \tag{D.6}$$

and the probability density function is given by

$$g(x) = \begin{cases} A_1 e^{r_1 x} + A_2 e^{r_2 x} & \text{for } \underline{x} \leq x \leq x^* \\ B_1 e^{r_1 x} + B_2 e^{r_2 x} & \text{for } x^* < x \leq \bar{x} \end{cases} \tag{D.7}$$

Now, we have system of four equations, $g(\underline{x}) = g(\bar{x}) = 0$, $g(x^*_-) = g(x^*_+)$, and $\int_{\underline{x}}^{\bar{x}} g(x)dx = 1$ and four unknowns (A_1, A_2, B_1, B_2) that can be solved analytically. Once we have the closed form expression

for $g(x)$, we can calculate the dispersion analytically as

$$D_i = \int e^{\eta_i x} g(x) dx \times \left[\int e^{(1-\eta_i)x} g(x) dx \right]^{\frac{\eta_i}{1-\eta_i}}$$

To calculate any function of the distribution of price changes, and in particular the moments that we are interested in – standard deviation, and kurtosis of price changes, and fraction of positive price changes, we need to calculate the distribution of price changes. To do that, we build on Alvarez, Lippi, and Oskolkov (2021). Given the stationary distribution function in Equation (D.7), the distribution of price changes, $q(\Delta p)$ is given by

$$q(x^* - x) = \frac{g(x)\theta}{N_a}, \quad x \in [\underline{x}, \bar{x}] \tag{D.8}$$

$$\mathrm{d}Q(x^* - \bar{x}) = -\frac{\sigma^2}{2}\frac{g'(\bar{x})}{N_a}, \quad \mathrm{d}Q(x^* - \underline{x}) = \frac{\sigma^2}{2}\frac{g'(\underline{x})}{N_a} \tag{D.9}$$

where the number (frequency) of price changes, N_a satisfies

$$N_a = \theta + \frac{\sigma^2}{2} g'(\underline{x}) - \frac{\sigma^2}{2} g'(\bar{x}) \tag{D.10}$$

Then, with the distribution of price changes $q(\Delta p)$, we can construct any moment of the distribution of price change.

D.2. Pure menu cost model

The pure menu cost model is a particular case of the CalvoPlus model, with $\theta = 0$. Therefore, we can follow Appendix D.1, letting $\theta = 0$. Note that when $\theta = 0$, the roots of the characteristic equation associated with the stationary distribution differential equation are $r_1 = 2\frac{(\mu-\pi)}{\sigma^2}$ and $r_2 = 0$.

D.3. Calvo model

In the Calvo setting, price change opportunities arrive exogenously, and firms don't get to choose whether to change prices or not endogenously. Yet, we can still state the value of a firm that has a given price gap x, as

$$v(x) = f(x)\mathrm{d}t + (1 - \rho \mathrm{d}t)\left[(1 - \theta \mathrm{d}t)\mathbb{E}[v(x + \mathrm{d}x)] + \theta \mathrm{d}t \times v(x^{\#})\right]$$

results in

$$(\rho + \theta)\, v(x) = f(x) + (\mu - \pi)\, v'(x) + \frac{\sigma^2}{2} v''(x) + \theta\, v(x^{\#})$$

$$\frac{\sigma^2}{2} v''(x) + (\mu - \pi)\, v'(x) - (\rho + \theta)\, v(x) = -f(x) - \theta\, v(x^{\#}) \tag{D.11}$$

where $x^{\#} \equiv \arg\max_y v(y)$ is the reset gap. Therefore, $v'(x^{\#}) = 0$. Furthermore, we impose the terminal condition $\lim_{T\to\infty} \mathbb{E}\left[e^{-(\rho+\theta)T} v(x_T)\right] = 0$. Equation (D.11) is a second-order differential equation, with the general solution composed a homogeneous solution and a particular solution.

Recall that $f(x) = e^{(1-\eta)x} - \frac{\eta-1}{\eta}e^{-\eta x}$. The general solution of $v(x)$ will then be given by

$$v(x) = c_1 e^{r_1 x} + c_2 e^{r_2 x} + Ae^{(1-\eta)x} + Be^{-\eta x} + C \tag{D.12}$$

where

$$r_1 = \frac{-(\mu-\pi) + \sqrt{(\mu-\pi)^2 + 2(\rho+\theta)\sigma^2}}{\sigma^2}, \quad r_2 = \frac{-(\mu-\pi) - \sqrt{(\mu-\pi)^2 + 2(\rho+\theta)\sigma^2}}{\sigma^2}$$

$$A = \frac{1}{-\frac{\sigma^2}{2}(1-\eta)^2 - (\mu-\pi)(1-\eta) + (\rho+\theta)}, \quad B = \frac{-\frac{\eta-1}{\eta}}{-\frac{\sigma^2}{2}\eta^2 + (\mu-\pi)\eta + (\rho+\theta)}, \quad C = \frac{\theta v(x^\#)}{(\rho+\theta)}$$

We have four unknowns, $(c_1, c_2, C, x^\#)$, so we need four conditions to pin down these unknowns. First, the optimality of the reset gap, $v'(x^\#) = 0$, provides us with one condition. Then, the terminal condition $\lim_{T\to\infty}\mathbb{E}\left[e^{-(\rho+\theta)T}v(x_T)\right] = 0$ will provide us with the other conditions to pin down the unknowns. It also provides us with constraints on the parameter values. First, the optimal reset gap gives us

$$\left(1 - \frac{\theta}{\rho+\theta}\right)v(x^\#) = c_1 e^{r_1 x^\#} + c_2 e^{r_2 x^\#} + Ae^{(1-\eta)x^\#} + Be^{-\eta x^\#}$$

The terminal condition implies $c_1 = c_2 = 0$. This results in a system of two equations and two unknowns

$$(1-\eta)Ae^{(1-\eta)x^\#} - \eta Be^{-\eta x^{\#*}} = 0$$

$$\left(1 - \frac{\theta}{\rho+\theta}\right)v(x^\#) = Ae^{(1-\eta)x^*} + Be^{-\eta x^\#}$$

Solving for them, we get

$$x^\# = \log\left(\frac{(\rho+\theta) + (\mu-\pi)(\eta-1) - \frac{\sigma^2}{2}(\eta-1)^2}{(\rho+\theta) + (\mu-\pi)\eta - \frac{\sigma^2}{2}\eta^2}\right)$$

and

$$v(x^\#) = \frac{1}{\eta}\frac{\rho+\theta}{\rho}\left((\rho+\theta) + (\mu-\pi)(\eta-1) - \frac{\sigma^2}{2}(1-\eta)^2\right)^{-\eta}\left((\rho+\theta) + (\mu-\pi)\eta - \frac{\sigma^2}{2}\eta^2\right)^{\eta-1}$$

To solve for the stationary distribution, we follow similar steps as in Appendix D.1, except that the Kolmogorov Forward equation is given by

$$\frac{\sigma^2}{2}g''(x) - (\mu-\pi)g'(x) - \theta g(x) = 0, \quad x \in (-\infty, \infty), \quad x \neq x^\#$$

where $g(x^\#)_- = g(x^\#)_+$ and $\int_{-\infty}^{\infty} g(x)dx = 1$. The dispersion in the pure Calvo for sector $i \in [n]$ is given by

$$D_i = \frac{\theta_i}{(\theta_i + \eta(\mu_i - \pi) - \frac{1}{2}\eta^2\sigma_i^2)} \times \left(\frac{\theta_i}{\theta_i + (\eta-1)(\mu_i - \pi) - \frac{1}{2}(\eta-1)^2\sigma_i^2}\right)^{\frac{\eta}{1-\eta}} \tag{D.13}$$

The distribution of price changes will be given by

$$q(x^{\#} - x) = g(x)$$

E Data Appendix

The description follows closely Afrouzi and Bhattarai (2023). We use the input-output (IO) tables from the BEA to construct the input-output linkages across sectors,[18] given by the matrix $\mathbf{A}$; the consumption expenditure shares across sectors, given by the vector $\boldsymbol{\beta}$; and the sectoral labor shares, given by the vector $\boldsymbol{\alpha}$. In particular, to construct $\mathbf{A}$, we use both the "make" and "use" IO tables at the Summary level from the vintage Comprehensive, Release Date: January-23-2014, which gives 65 sectors.[19] The "use" IO table also provides data on the compensation of employees, which is used to construct the sectoral labor shares $\boldsymbol{\alpha}$. Moreover, the "use" IO table also provides data on personal consumption expenditure, which is used to construct the consumption expenditure shares across sectors, $\boldsymbol{\beta}$.

The data on sectoral frequency of price adjustment is taken from Pasten, Schoenle, and Weber (2020b).

E.1. Constructing the Input-Output Matrix

In this subsection, we describe how we use the "Make" and "Use" matrices to get the cost-based industry-by-industry input-output table. Specifically, we use the 2012 "Make" table after redefinitions and the 2012 "Use" table after redefinitions in producers' value.

Recall that the "Make" table is a matrix of Industry-by-Commodity. Given a row, each column shows the values of each commodity produced by the industry in the row. The "Use" table is a matrix of Commodity-by-Industry. Given a column, each row shows the value of each commodity used by the industry (or final use) in the column. In order to create an industry-by-industry IO table, we combine both. We follow the Handbook of Input-Output Table Compilation and Analysis from the UN (United Nations Department of Economic and Social Affairs, 1999) and Concepts and Methods of the United States Input-Output Accounts from the BEA (Horowitz and Planting, 2009). We exclude the government sector, Scraps, Used and secondhand goods, Noncomparable imports, and Rest of the world adjustment.[20]

It is important to note that an industry can produce many commodities. Although each industry

[18] We construct industry-by-industry IO tables. We use industry and sector interchangeably.

[19] The "make" table is a matrix of industries on the rows and commodities on the columns that gives the value of each commodity on the column produced by the industry on the rows. The "use" table is a matrix of commodities on the rows and industries on the columns that gives the value of each commodity on the row that was used by each industry in the column. We combine both matrices to give an industry-by-industry IO matrix.

[20] Baqaee and Farhi (2020) also exclude these sectors. Besides them, we exclude Customs duties, which is an industry with zero commodity use and zero compensation of employees. After excluding these industries and commodities, we end up having 392 commodities and 393 industries. The industry that does not have a corresponding commodity with the same code is 'Secondary smelting and alloying of aluminum', with code 331314

may have its own primary product,[21] an industry can produce more products in addition to its primary ones. These are shown in the "Make" table. Besides that, each industry has its own use of commodities to produce its output. This is shown in the "Use" table. As a result, there is a distinction between industries and commodities, as a given commodity can be produced by different industries while industries can produce different commodities.

In our baseline model, we consider a Cobb-Douglas economy, so we calibrate $\boldsymbol{\beta}$ using consumption shares from the data and $\mathbf{A}$ using the cost-based input-output matrix.

Input-Output Matrix (A) and Labor Shares ($\boldsymbol{\alpha}$). From the "Use" table from the BEA, a given column j gives:

$$\begin{aligned}\text{Total Industry Output}_j = &\text{Total Intermediate}_j \\ &+\text{Compensation of Employees}_j \\ &+\text{Taxes on production and imports, less subsidies}_j \\ &+\text{Gross operating surplus}_j\end{aligned}$$

where $\text{Total Intermediate}_j$ is the sum of the dollar amount of each commodity used by industry j. The total cost is given by

$$\text{Total Industry Cost}_j = \text{Total Intermediate}_j + \text{Compensation of Employees}_j$$

Therefore,

$$\underbrace{P_j Y_j}_{\text{Total Industry Output}} = \underbrace{(1+\omega_j)}_{\text{Wedge}} \underbrace{\left(\sum_i P_i X_{ji} + W L_j\right)}_{\text{Total Industry Cost}}$$

where we implicitly assume that the wedge is attributed to taxes and gross operating surplus. That is

$$(1+\omega_j) \equiv \frac{\text{Total Intermediate}_j + \text{Compensation of Employees}_j + \text{Taxes}_j + \text{Gross Operating Surplus}_j}{\text{Total Intermediate}_j + \text{Compensation of Employees}_j} \tag{E.1}$$

Let $\text{diag}(1+\omega)$ be the diagonal matrix in which each j-th diagonal is the wedge in industry j. We calculate the cost-based IO matrix by first calculating the revenue-based IO matrix and then, using these wedges, recovering the cost-based IO matrix. First, we calculate the revenue-based IO matrix. Let $\mathbf{U}_{(N_C+1)\times N_I}$ be the "Use" matrix (commodity-by-industry) that gives for each cell u_{ij} the dollar value of commodity i used in the production of industry j and in the last row the compensation of employees. Let $\mathbf{M}_{N_I \times N_C}$ be the "Make" matrix (industry-by-commodity) that gives for each cell m_{ij} the dollar value of commodity j produced by i. Let $\mathbf{g}_{N_I \times 1}$ be the vector of industry total output and

[21] According to the BEA, 'each commodity is assigned the code of the industry in which the commodity is the primary product'.

$\mathbf{q}_{N_C \times 1}$ be the vector of commodity output, where N_C is the number of commodities and N_I is the number of industries. Then, define the following matrices

$$\mathbf{B} = \mathbf{U} \times \text{diag}(\mathbf{g})^{-1} \tag{E.2}$$

$$\mathbf{D} = \mathbf{M} \times \text{diag}(\mathbf{q})^{-1} \tag{E.3}$$

where diag($\mathbf{g}$) is the diagonal matrix of vector $\mathbf{g}$ and diag($\mathbf{q}$) is the diagonal matrix of vector $\mathbf{q}$. The matrix $\mathbf{D}$ is a market share matrix. Its entry d_{ij} gives the market share of industry i in the production of commodity j. The matrix $\mathbf{B}$ is a direct input matrix. Its entry b_{ij} gives the dollar amount share of commodity i in the output of industry j. Let

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_I^{N_C \times N_I} \\ \tilde{\boldsymbol{\alpha}}_{1 \times N_I}^{\intercal} \end{bmatrix} \tag{E.4}$$

where $\mathbf{B}_I$ is the part of $\mathbf{B}$ that includes all intermediate inputs and industries and $\tilde{\boldsymbol{\alpha}}'$ includes the labor share in each industry's output. Then, the revenue-based industry-by-industry IO matrix is given by

$$\tilde{\mathbf{A}} = (\mathbf{D}\mathbf{B}_I)^{\intercal} \tag{E.5}$$

To go from the revenue-based IO matrix to the cost-based IO matrix, first recall that

$$[\tilde{\mathbf{A}}]_{ij} = \frac{P_j X_{ij}}{P_i Y_i} \tag{E.6}$$

where $P_j X_{ij}$ is the expenditure of industry i on industry j, $P_i Y_i$ is the revenue of the industry i. The cost-based IO matrix is given by

$$\mathbf{A} = \left[a_{ij}\right]_{i \in [n], j \in [n]}, \; a_{ij} = \frac{P_j X_{ij}}{\mathscr{C}_i} \tag{E.7}$$

where $\mathscr{C}_i \equiv \sum_k P_k X_{ik} + W L_i$ is the total cost of industry i. Since $P_i Y_i = (1+\omega_i)\mathscr{C}_i$, we have that

$$a_{ij} = \frac{P_j X_{ij}}{\mathscr{C}_i} = \frac{P_i Y_i}{\mathscr{C}_i} \frac{\mathscr{C}_i}{P_i Y_i} \frac{P_j X_{ij}}{\mathscr{C}_i} = (1+\omega_i)[\tilde{\mathbf{A}}]_{ij} \tag{E.8}$$

Hence, as in Baqaee and Farhi (2020)

$$\mathbf{A} = \text{diag}(1+\omega_i)\tilde{\mathbf{A}} \tag{E.9}$$

To go from the revenue-based labor share to the cost-based labor share, first recall that

$$\tilde{\alpha}_i = \frac{W L_i}{P_i Y_i} \tag{E.10}$$

where $W L_i$ is the compensation of employees of the industry i. The cost-based labor share is given by

$$\boldsymbol{\alpha} = (\alpha_i)_{i \in [n]}, \; \alpha_i = \frac{W L_i}{\mathscr{C}_i} \tag{E.11}$$

Using similar arguments as above, we have that

$$\alpha_i = \frac{WL_i}{\mathscr{C}_i} = \frac{P_i Y_i}{\mathscr{C}_i}\frac{\mathscr{C}_i}{P_i Y_i}\frac{WL_i}{\mathscr{C}_i} = (1+\omega_i)\tilde{\alpha}_i \tag{E.12}$$

and the cost-based labor shares are given by

$$\boldsymbol{\alpha} = \text{diag}(1+\omega_i)\tilde{\boldsymbol{\alpha}} \tag{E.13}$$

Alternatively, instead of using the vector of industry outputs $\mathbf{g}$, we can calculate the cost-based IO matrix using the vector of industry costs (total intermediate + compensation of employees) $\tilde{\mathbf{g}}$. In this case, define $\tilde{\mathbf{B}} \equiv \mathbf{U}\text{diag}(\tilde{\mathbf{g}})^{-1}$, where, similar to above, $\tilde{\mathbf{B}}$ is composed of $\tilde{\mathbf{B}}_I$ that includes all intermediate inputs and industries and $\boldsymbol{\alpha}^\intercal$ which is the vector with compensation of employees for each industry. Given this decomposition, we can construct the corresponding cost-based IO matrix as $\mathbf{A} = (\mathbf{D}\tilde{\mathbf{B}}_I)^\intercal$. Note that this gives the same cost-based IO matrix as above.

Consumption Share ($\boldsymbol{\beta}$). The "Use" table gives the Personal Consumption Expenditures on each commodity. Since we are working with an industry-by-industry IO matrix, we need to calculate an industry consumption share vector. In order to do that, let C_i be the consumption dollar amount of commodity i, and $\mathbf{c}$ be the vector of the consumption dollar amount of all commodities in the economy. Then, the vector that contains the dollar equivalent consumption amount of each industry is given by $\mathbf{Dc}$:

$$\mathbf{Dc} = \begin{bmatrix} \sum_j d_{1j}c_j \\ \sum_j d_{2j}c_j \\ \vdots \\ \sum_j d_{nj}c_j \end{bmatrix} \tag{E.14}$$

Recall that d_{ij} gives the market share of industry i in the production of commodity j. Therefore, $d_{ij}c_j$ is the amount in dollars spent by households on commodity j produced by i. Then, the sum over j gives the total expenditure in dollars of households on commodities produced by industry i. That is, the total expenditure in dollars of households on industry i. Then,

$$\boldsymbol{\beta} = \frac{\mathbf{Dc}}{\mathbf{1}'\mathbf{Dc}} \tag{E.15}$$

E.2. Constructing the Frequency of Price Adjustment

To get the 2012 summary-level industry frequency of price adjustments from the 2002 detail-level industry frequency of price adjustments, we had to manually match them, mainly using the first three digits of the NAICS code.

The main data consists of frequencies of price adjustment for 341 detailed sectors from Pasten, Schoenle, Weber (2020), originally categorized according to the 2002 Input-Output industry definitions. To align these data with the 2012 Summary IO framework, we map them to the corresponding 2012 summary sectors based on the North American Industry Classification System

(NAICS) prefixes.

For most sectors, we map the 2002 detailed codes to 2012 summary codes using the first three or four digits of their NAICS identifiers. The FPA for a summary industry is calculated as the simple average of the FPAs of all 2002 detailed industries within that NAICS group.

For summary industries where a direct match is unavailable or where industry definitions are revised (e.g., Retail Trade), we assign their FPA values according to three distinct methods:

1. For missing summary-level values, we calculate and assign the average FPA of the broader 2-digit NAICS supersector. The industries for which we use this method are: *Transit and Ground Passenger Transportation* (2012 IO Code 485), *Motion Picture and Sound Recording* (2012 IO Code 512), *Funds, trusts, and other financial vehicles* (IO Code 525), *Computer Systems Design* (2012 IO Code 5415), *Social Assistance* (2012 IO Code 624), and *Performing arts, spectator sports, museums, and related activities* (2012 IO Code 711AS), *Food services and drinking places* (2012 IO Code 722).
2. For summary-level industries that are more disaggregated in 2012 than in 2002, we use their value for 2002. This is the case for Retail Trade. In 2002, there was a single Retail Trade (2002 IO Code 4A0), while in 2012 you have Motor vehicle and part dealers (2012 IO Code 441), Food and beverage stores (2012 IO Code 445), General merchandise stores (2012 IO Code 452), and Other Retail (2012 IO Code 4A0). We use the value of 2002 for all of these summary-level industries.
3. For the summary-level industries that still had missing data, we impute the their FPA using the simple average across all industries with data. These were: *Construction* (2012 IO Code 23), *Management of companies and enterprises* (2012 IO Code 55), *Educational services* (2012 IO Code 61), and *Other Services* (2012 IO Code 81).

Since we use a model in continuous time, after calculating the discrete frequency of price adjustment, we perform the following conversion: $f_i^{cts} = -\ln(1 - f_i)$.