Pricing Residential Mortgage Credit Risk in the Post-GFC Era*

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Abstract

Following the Great Financial Crisis (GFC), the Credit Risk Transfer (CRT) bond market emerged as a new asset class in U.S. mortgage market. We develop an asset pricing framework for CRTs consistent with Treasury, corporate, and housing markets. Our analysis reveals that the Government-Sponsored Enterprises compensate investors approximately fairly on average, though they overpay for low-risk tranches and underpay for high-risk ones. Additionally, the post-GFC guarantee fee increases broadly align with underlying credit risk. We find significant reverse cross-subsidization, where high-credit-risk borrowers subsidized low-risk ones. The 2023 reform partially corrected this cross-subsidization.

Keywords: mortgage default risk, prepayment risk, GSE reform, cross-subsidization in household finance

JEL: G12, G21, R31

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1 Introduction

One of the largest financial catastrophes of the last fifty years was the implosion of the private-label U.S. residential mortgage-backed securities market. The government bailed out the banks that had exposure to risky mortgage credit and took the mortgage giants Fannie Mae and Freddie Mac (the GSEs), who had been swept up in the subprime credit bonanza, into conservatorship (Acharya et al., 2011). Since then, the majority of U.S. mortgage credit risk has been borne by the taxpayers.[1] Starting in 2013 and largely out of the public's eye, the GSEs began to lay off the credit risk on the mortgages they had acquired to the private sector. They did so in the form of credit risk transfer (CRT) bonds. What started as a pilot program in 2013 grew into a new asset class, as it came to cover 90% of all newly originated conforming mortgage debt. Since 2013, the GSEs have issued $145 billion of CRT bond principal, passing much of the credit risk on $5.2 trillion of mortgage principal back to the private sector.[2] If we think of the GSEs prior to 2013 as a large insurer of mortgage credit risk, we can think of the GSEs post-2013 as re-insuring a large share of the credit risk with the private sector. The focus of this paper is on the CRT market where most of that re-insurance takes place. Since the demise of the subprime mortgage market during the GFC, it has become the only market where residential mortgage credit risk is traded in significant quantity. Despite its considerable size and importance as a barometer of residential credit risk, the CRT market remains understudied.

We develop a state-of-the-art pricing model that takes into account the institutional features of CRT bonds. Our asset pricing model is consistent with the prices of government and corporate bonds. That is, we assume that the marginal agent in the CRT bond space is an investor who also invests in U.S. government and corporate debt, consistent with anecdotal evidence.[3] Our model also stipulates the dynamics of aggregate labor income risk, layoffs, and aggregate house price risk. We insist on matching the risk premium in the housing market, a

[1] The share of new mortgage originations held on banks' balance sheets grew from below 15% in 2009 to 30% in 2024. Most of this privately-held mortgage debt consists of jumbo mortgages with loan balances above the conforming loan limit and very low default risk. The share of originations of mortgages guaranteed by the Federal Housing Administration and the Veteran's Administration jumped to 20% in the post-GFC period. The share of GSE debt was around 60% between 2008 and 2013, around 45% between 2014 and 2019, 60% in 2020-2021, and around 40% in 2023-24.

[2] What that means is that the U.S. mortgage market has, to a large degree, been *functionally re-privatized*. The GSEs purchase mortgages through the front door, synthetically split the mortgage into two components, one covering the interest rate and prepayment risk and a second component covering the default risk, and sell both pieces off through the back door in the form of default-free agency mortgage-backed securities (MBS) and credit risk transfer bonds, respectively.

[3] Indeed, industry publications from investment banks usually discuss CRT bond spreads relative to corporate credit spreads, since investors in CRT bonds tend to invest in corporate debt as well. In the language of asset pricing, we extract the market prices of interest rate and credit risk from the Treasury and corporate debt markets.

key parameter.

We model mortgage default as triggered by the combination of a liquidity (disposable income) shock and negative home equity. Households face both idiosyncratic and aggregate income and house price risk. The default model produces realistic average mortgage default and loss rates for the various GSE mortgage vintages, and across credit score (FICO) and loan-to-value (LTV) groups. Since mortgage prepayments extinguish the mortgage, they also extinguish the associated credit risk. We build a prepayment model that fits the patterns of empirical prepayment across vintages and coupons well. Prepayments affect the cash-flows to the various CRT bonds in non-trivial ways and are a key component of the valuation.

With the model in hand, our main exercise is to price each of the 329 CRT bonds issued between 2013 and 2025 by Freddie Mac. These bonds differ by vintage, underlying mortgage collateral, and seniority. We use loan-level micro data to infer the initial composition of each CRT mortgage pool.

Our first main result is that, on average across the 329 bonds, our model implies credit spreads that closely match the spreads investors received in the data. The average gap between data and model across the 329 bonds is a mere 30.5 basis points. We obtain this close-to-fair pricing result despite the fact that the asset pricing model did not explicitly target CRT or any other mortgage bond prices. Contrary to received wisdom, the GSEs did not offer excessive compensation to the private sector for taking over credit risk, at least on average.

We arrive at this conclusion because our quantitative model features substantial housing and credit risk premia. These risk premia result in large wedges between mean aggregate income and house price growth rates under the risk neutral and physical measures. Since the pricing of mortgage default risk depends critically on the evolution of income and house prices under the risk-neutral measure, our model delivers high credit spreads for the CRT tranches. While *realized* default rates have been low over the past 12.5 years due to sustained house price appreciation, investors were appropriately compensated for the *ex-ante* risk of high default rates.

Our second main result is that while the average CRT bond was priced fairly, that average hides interesting variation between junior and mezzanine tranches. The junior tranches, which take losses first, have model-implied spreads that are about 320 basis points (bps) higher than in the data, whereas the mezzanine tranches, which take losses only after the junior tranches are wiped out, have model-implied spreads that are about 180 bps lower than in the data. Put differently, CRT investors in the junior bonds received too little risk compensation; these bonds were overpriced. In contrast, mezzanine tranches were underpriced, offering attractive yields to investors given their risk.

The main reason for the high model-implied credit spreads on junior CRT bonds is that the junior bonds carry the most default risk, and that default risk is highly priced in the model. The observed prices suggest that the average CRT investor underestimates the differential credit risk exposure of junior and mezzanine tranches. One potential reasons is that binding leverage constraints generate excess demand for junior tranches, which embed leverage mechanically through their payoff structure.[4] The model's wedge between junior and mezzanine bond spreads is also influenced by prepayment risk. Prepayments return mezzanine tranche principal before junior tranche principal, shortening the length of time that mezzanine CRT investors are exposed to credit risk. As a result, mezzanine tranches have both higher attachment points and a shorter effective duration of credit risk exposure, further lowering their required compensation relative to junior tranches. The complexity of the cash-flow waterfall (and changes therein across vintages) makes it challenging to assess the differential impact of prepayment on the various tranches.

Next, we use our model to price the credit risk on the entire pool of conforming mortgages issued by the GSEs. The resulting credit spread is the fair compensation investors would require to bear all of the credit risk in a given mortgage vintage, not just the risk that the GSEs lay off through the CRT bonds.[5] The fair model-implied credit spread can be compared to the observed guarantee fee (or g-fee) that the GSEs charge mortgage borrowers. The observed g-fee increased from around 20 bps before the GFC to around 40 bps in 2013 and to 65 bps in 2025. Over the past 15 years, the GSEs have tripled the insurance premium they charge mortgage borrowers for taking on credit risk. Did this increase correct a woeful pre-GFC underpricing? Or have the GSEs overdone it, generating excess profits at the expense of homeowners, potentially harming homeownership by unduly increasing the cost of mortgage credit?

Our model shows that the fair compensation for credit risk is 28.8 bps. This compares to an observed average credit risk component of the guarantee fee of 38.7 bps. Our third main result, therefore, is that the post-GFC increase in g-fees was mostly justified. However, in the last two years of the sample, the GSEs are charging about 30 bps more than what the model predicts. We show that about 60% of the model-implied g-fee is a credit risk premium and the remaining

[4]In the data, hedge funds tend to buy the junior tranches while money managers tend to purchase the mezzanine tranches. Hedge funds are targeting higher absolute returns and may be unable to leverage up the mezzanine tranches enough to manufacture the desired return, leading them to hold junior bonds. This demand pressure compresses junior tranche spreads relative to the compensation implied by their underlying credit risk. The overpricing of junior tranches mirrors the "betting-against-beta" phenomenon observed in equities (Frazzini and Pedersen, 2014). Consistent with our model, an index of mezzanine CRT tranches shows lower average realized returns but higher Sharpe ratios than the junior CRT bond index.

[5]CRT bonds do not cover all mortgage debt, even in the later years of the program, and the GSEs do not lay off all of the risk even on the mortgages in the collateral pool underlying a CRT bond. In particular, the GSEs hold on to the tail risk.

40% is compensation for expected loss. We also compute the fair cost of bearing catastrophic losses, which we define as all losses above a 5% loss rate. This tail risk compensation is large, about 58% of the overall g-fee.

Our last set of results concerns the cross-sectional pricing of mortgage credit risk. In the wake of the GFC, the GSEs introduced loan-level price adjustments (LLPAs) that charge high-risk borrowers more than low-risk borrowers. The main differentiators in the LLPAs are the LTV ratio and the FICO score of the borrower. A major change to the LLPA pricing grid in May 2023 reduced LLPAs for riskier (low-FICO) borrowers and increased them for more creditworthy (high-FICO) borrowers. Did the 2023 reform increase the alignment between cross-sectional variation in credit risk and LLPAs or worsen it? Our fourth main result is that the pre-2023 reform LLPA grid overcharged low-FICO borrowers and undercharged high-FICO borrowers. The May 2023 reform significantly reduced the overpricing of low-FICO borrowers, and made the overall pricing closer to actuarially fair. However, the reform did not go far enough to correct this overcharge. The reform also introduced new cross subsidization from middle-FICO, middle LTV borrowers to high-FICO borrowers.

The pricing of mortgage credit risk has taken on renewed urgency in light of the policy momentum to end conservatorship and re-privatize the GSEs.[6] In this emerging regime, where the GSEs may once again operate as private firms—but still benefit from a government guarantee—the pricing of mortgage credit risk becomes a first-order policy concern. If taxpayers are to be compensated for the credit risk that results from the government backstop, it is crucial to determine what a fair compensation (guarantee fee) would be, lest we convert public risk into private profit. This requires a rigorous, market-consistent estimate of the price of mortgage default risk. Our model offers a framework for answering that question—at the level of individual mortgage borrowers, CRT tranches, and entire mortgage vintages.

Related Literature Our study is closely related to the still limited literature on credit risk transfer programs at the GSEs. The descriptive study of Finkelstein et al. (2018) discusses the evolution of the CRT program from 2013–2017 and shows that CRTs reduced the government's exposure to credit loss without disrupting mortgage market liquidity. An important precursor to our work is Golding and Lucas (2020), who examine CRT security pricing using market data. Their exploratory analysis raises important questions but does not construct a structural model

[6]While earlier efforts during the first Trump administration stalled after the 2020 election, the second Trump Administration has revived them with new intensity. Recent equity market activity reflects growing investor expectations of a transition to private ownership: between June 2024 and June 2025, the share prices of Fannie Mae and Freddie Mac rose by 600% and 700%, respectively. News broke of President Trump's intention to privatize the GSEs in August 2025. See `https://fticommunications.com/the-prospects-of-privatization-for-fannie-mae-and-freddie-mac-in-2025/`.

to jointly price interest rate, prepayment, and default risk. O'Neill (2022) develops a valuation model for individual CRT bonds, but employs a reduced-form default model calibrated directly to observed CRT tranche prices.[7] Flanagan (2025) calibrates his model to match subordinate CRT tranche spreads and then uses it to price tail risk and infer the guarantee fee. In contrast, we estimate the stochastic discount factor from Treasury spreads, corporate bond spreads, and housing returns and then use it to jointly price subordinate tranches, tail risk, and g-fee. Our g-fee results are consistent with Flanagan (2025), as we both find that the GSEs have overcharged borrowers post-GFC.

A related strand of the literature studies the design and effectiveness of government risk-transfer programs. Hanson et al. (2019) examine how public risk-sharing mechanisms incentivize investor participation and the pricing of mortgage risk, providing complementary evidence on how government programs allocate mortgage credit risk. While their focus is on program-level incentives rather than structural pricing, their findings reinforce the notion that private investors are willing to absorb substantial mortgage credit exposure under appropriate contractual design.

Kim et al. (2024) examine the distributional impact of g-fees on purchase mortgage originations. As a by-product of the analysis, they construct a model that produces LLPAs across FICO×LTV bins while accounting for borrower default and prepayment. They find cross-subsidization from low- to high-LTV borrowers and from high- to low-FICO score groups. We find the reverse pattern in the FICO dimension. As discussed in Section 7, the discrepancy stems from differences in pricing models, default calibration, and the amount of idiosyncratic default risk within a given FICO×LTV bin. While McGowan and Nguyen (2023) document that the GSEs do not price regional credit risk, we find that they underprice high-FICO and overprice low-FICO borrowers' credit risk.

Gete et al. (2025) uses daily CRT bond spreads to assess investors' views on mortgage default during the Covid-19 period, showing that the forbearance programs in the CARES Act raised expected default rates. They infer a market-implied g-fee from CRT spreads, whereas our paper derives credit spreads in an asset pricing model that jointly prices prepayment and default risk, without using CRT prices as inputs. Capponi et al. (2021) constructs a reduced-form model to assess the impact of mortgage forbearance on refinancing activities, and finds that unemployment risk is a significant driver of default-induced prepayments. Our study directly links unemployment risk to borrowers' liquid wealth in a structural default model.

Our model assumes that the same financial intermediaries who trade corporate credit risk

[7] O'Neill (2022) assumes that tranches are fully wiped out in default, while our approach models default at the individual mortgage level and then aggregates to the bond or pool level.

and interest rate risk are the marginal investors in the CRT bond market, in the spirit of the intermediary-based asset pricing literature (Adrian et al., 2014; He et al., 2017) and related work on mortgage-backed securities (Gabaix et al., 2007; Diep et al., 2021; Fuster et al., 2024).

Our paper is also related to the large literature that studies the origins of the U.S. subprime mortgage crisis (Gerardi et al., 2008; Mian and Sufi, 2009; Davis and Van Nieuwerburgh, 2015; Favilukis et al., 2017). These studies highlight the link between foreclosures and aggregate income and house price risk.

A different stream of literature has explored the link between mortgage risk and the GSEs (Acharya et al., 2010, 2011, 2013; Elenev et al., 2016; Gupta, 2022; Bosshardt et al., 2025). For example, Acharya et al. (2010) caution that government coinsurance programs could devolve into another form of subsidy. We quantify the tail risk—the portion borne by the government. Acharya et al. (2010) also hypothesize that some investors may prefer uninsured, higher-yield mortgage securities and bear the risk themselves. Our results provide empirical evidence for this, showing that the GSEs can issue the riskiest CRT tranches on favorable terms. Frame et al. (2015) discuss hypothetical scenarios where the GSEs are replaced by a private system. Under these circumstances, the market expects the government to intervene under extreme scenarios. We provide a consistent framework for pricing such mortgage tail risk.

We contribute to the literature on prepayment risk (e.g., Duarte, 2008; Chernov et al., 2018; Boyarchenko et al., 2019; Diep et al., 2021; Fuster and Vickery, 2015), which primarily focuses on MBS and mortgage lending, by instead studying the implications of prepayment risk for CRT bond pricing. We also add to the default risk modeling literature (e.g., Deng et al., 2000; Longstaff and Rajan, 2008; Foote et al., 2009; Demyanyk and Van Hemert, 2011; Gyourko and Tracy, 2014; Campbell and Cocco, 2015; Ganong and Noel, 2020a) by calibrating and testing a structural mortgage default model. Defaults are triggered by negative income and negative home equity (e.g., Ganong and Noel, 2020b; Fuster and Willen, 2017). We find that the double trigger model fits the historical default patterns well, adding additional evidence for its validity.

The rest of the paper is organized as follows. Section 2 provides institutional detail. Section 3 sets up and Section 4 estimates the model. The main results for the pricing of CRT tranches are in Section 5, the pricing of the g-fee is in Section 6, and the cross-sectional pricing results are in Section 7. Section 8 concludes. The appendix provides additional details on model derivations, estimation, and institutional aspects of the CRT program.

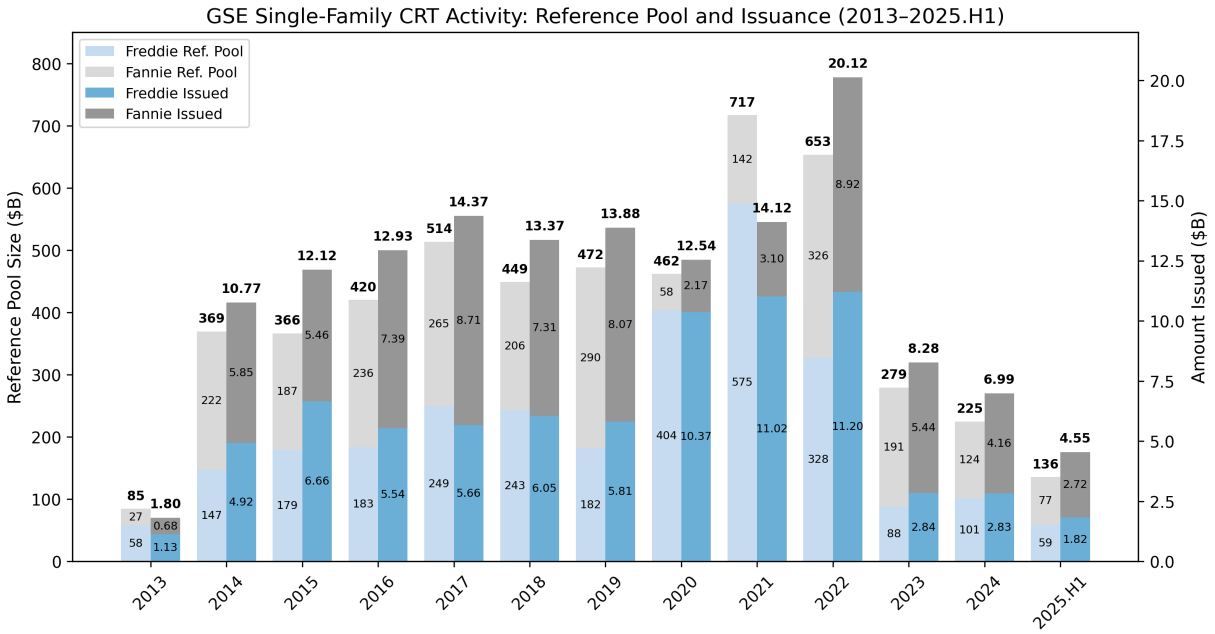
2 Institutional Details

Following the collapse and government bailout of Fannie Mae and Freddie Mac, the GSEs were taken into conservatorship in September of 2008. With taxpayers on the hook for the credit risk in the conforming mortgages purchased by the GSEs, the Federal Housing Financing Agency (FHFA)–the regulator of the GSEs–took three major sets of actions to reduce the taxpayers exposure: increase the guarantee fee, differentiate the pricing of risk through the introduction of the loan-level price adjustments (LLPAs), and launch the credit risk transfer (CRT) program. The former increased the compensation the GSEs received for bearing the credit risk from mortgage borrowers from about 25 basis points per year on each dollar of mortgage principal in 2011 to 65 basis points in 2025. The latter used some of that guarantee fee revenue to pay the private sector to take over some of that credit risk. What started as a pilot program in 2013 grew steadily thereafter, with peak CRT issuance in 2021 and 2022 covering the credit risk on $1 trillion of mortgage principal originated. CRT originations declined in 2023–25 as mortgage originations declined with higher mortgage rates.

There are two types of CRT programs: security issuance (about 2/3 of issuance) and reinsurance transactions (1/3). We focus on the CRT securities. Freddie Mac's securities program is called the Structured Transfer of Agency Credit Risk (STACR) program; Fannie Mae's securities program is called the Connecticut Avenue Securities (CAS) program. Between 2013 and the middle of 2025, $75.9 billion in STACRs were issued covering the credit risk on a reference pool of $2.795 trillion. Over that same period $70.0 billion in CAS were issued covering the credit risk on a reference pool of $2.351 trillion. Together, the GSEs have sold $145 billion in CRT bonds to investors on $5.15 trillion in mortgage principal. The total coverage ratio over the sample period is approximately 2.8% of reference pool principal. Figure 1 shows the evolution of the STACR and CAS programs. We focus on the STACR program, the largest of the two securities' programs. Since the CAS program is structured nearly identically, all our conclusions should apply equally to the CAS program. Our model should also be very useful for pricing the reinsurance deals which are structured similarly to the CRT bonds.[8] Finally, there are similar CRT securities and reinsurance programs on the multi-family side of Fannie Mae and Freddie Mac's business. Again, our analysis should be a useful starting point there as well.

[8]As noted, the GSEs have also laid off credit risk to the private insurance sector in the form of explicit re-insurance through the Agency Credit Insurance Structure program at Freddie Mac and the Credit Insurance Risk Transfer program at Fannie Mae. Between 2013 and the end of 2023, the GSEs issued $60.1 billion in reinsurance deals on a $1.545 trillion mortgage reference pool. Data on the reinsurance deals for 2024 and 2025.H1 are not yet available. Combining CRT securities and reinsurance deals, over $205 billion dollars of CRT instruments have been issued on a reference pool of over $6.7 trillion in single-family mortgage principal.

Figure 1: CRT Issuance



Notes: The left bars for each year, plotted against the left axis, report the size of the reference pool of mortgage collateral underlying the CRT bonds (in $ billions). The right bars for each year, plotted against the right axis, show the amount of CRT bonds issued (in $ billions). Each bar stacks the amounts of Fannie Mae and Freddie Mac activity. The data source is the Urban Institute's *Housing Finance at a Glance June 2025 Chartbook*.

The STACR program works as follows. Every deal delineates a specific reference pool of mortgages, originated during the same short window of time, with principal balances below the conforming loan limit, underwritten following GSE guidelines. Losses on these reference pools are transferred to CRT investors. There are two types of pools. The "DNA" deals refer to reference pools of mortgages with loan-to-value ratios below 80%. The "HQA" deals contain mortgages with LTV ratios between 80% and 97%. In each deal, multiple CRT bonds, known as tranches, are issued. Each tranche has an attachment point, the pool loss rate above which the tranche starts taking losses, and a detachment point, the pool loss rate above which the tranche is fully wiped out. For example, the 2017-DNA1 STACR deal, issued on February 7, 2017, is the first STACR deal of 2017 (hence the 1), refers to a pool of $33.97 billion of 30-year fixed-rate single-family mortgages with LTV ratios below 80%, originated prior to the cutoff date December 15, 2016. Four tranches are sold to CRT investors with attachment and detachment points in parentheses: the B-2 (0.0%–0.5%), B-1 (0.5%–1.0%), M-2 (1.0%–2.55%), and M-1 (2.55%–3.75%) tranches. Losses on the mortgage pool accrue to these tranches in reverse order of seniority: the first 0.5% of losses are transferred to the investors in the B-2 tranche. If losses exceed 0.5%, the B-2 tranche is fully wiped out. Losses above this level get

allocated next to the B-1 tranche investors. If losses exceed 1%, both the B-2 and B-1 tranches are wiped out and the M-2 tranche begins to take losses. Finally, if losses are above 2.55%, the M-1 bondholders begin to take losses. If pool losses exceed 3.75%, all CRT bonds sold to the public are wiped out. The B tranches are referred to as junior bonds, the M tranches as mezzanine bonds. Combined, they are referred to as the subordinate tranches. Losses above 3.75% accrue to the A-H tranche. This tranche, known as the senior tranche, is not sold to the public. In other words, the GSEs (currently, the taxpayers) hold the tail or catastrophic loss risk. In this example, 3.75% is referred to as the credit enhancement (CE) of the senior tranche. The legal maturity of these four CRT tranches is July 29, 2029, or 12.5 years after issuance. Table A.3 lists all tranches included in our study, which is the universe of STACR bonds. The number of tranches sold in a deal ranges from 3 to 5, the attachment and detachment points of tranches vary across deals, as do the CE levels.[9]

Perhaps, one underappreciated aspect of CRT bond pricing is the role of mortgage prepayment. Prior to March 2018, scheduled mortgage payments, i.e., regular principal amortization, repays the principal between senior and CRT tranches pro rata, and in order of seniority within the CRT tranche universe. After March 2018, scheduled mortgage payments only reduce the principal on the CRT bonds if certain conditions are met. Unscheduled principal payments (prepayments and principal recovery after a loss event) are initially just paid to the senior tranche. Once certain conditions are met, they also begin to reduce the principal on the CRT bonds. Within the CRT bond universe, they are paid again in strict order of seniority. Due to prepayments, the amount of tranche principal that is at-risk of credit loss shrinks over the life of the bond, and more rapidly for the mezzanine tranches than for the junior tranches. What may legally be a 12.5-year credit risk exposure may end up only exposing CRT investor capital for a few years years if prepayments are fast. The duration of credit risk exposure is shorter than the legal maturity, and shorter for the M than for the B tranches. This effect is reflected in the compensation CRT investors demand for bearing the credit risk.

In return for taking on mortgage credit risk, CRT bond investors receive a monthly interest payment equal to a short-term reference rate (30-day SOFR since 2021, LIBOR before 2021) plus a fixed tranche spread set at issuance. It is this tranche spread that our asset pricing model solves for, and which we will compare to the observed counterpart. For example, the

[9]There are other differences across deals. In some deals, the first-loss piece is not sold to the CRT investors but retained by the GSEs. In the 2013 and 2014 CRT vintages, a 180-day mortgage delinquency automatically triggered a loss event, and the loss severity was fixed at a pre-determined schedule. Starting in 2015, losses were allocated to CRT investors when the loan got resolved and the severity was the actual realized loss rate. Starting in 2023, some senior (A) tranches were sold to the public and junior tranches were no longer sold. We discuss all of this in detail below. In addition to the senior tranche and sometimes the first-loss tranche, the GSEs retain a 5% vertical slice of the B and M tranches. This vertical risk retention helps align incentives between GSEs and CRT investors.

2017-DNA1's M-1 tranche had a spread of 1.2%, the M-2 tranche a spread of 3.25%, the B-1 a tranche spread of 4.95% and the B-2 bond a spread of 10.0%.[10]

The investors in CRT bonds are a mix of insurance companies, hedge funds, asset managers, broker-dealers, REITs, and pension and sovereign wealth funds. Consistent with the spirit of our model, these investors typically invest in a broad array of fixed income securities. We will assume that the prices of Treasuries and corporate bonds are informative about the sources of risk that are present in the agency credit risk market as well. We will additionally consider house price and labor income risk which directly affect the risk of mortgage delinquency.

According to a 2021 FHFA report (Federal Housing Finance Agency, 2021), the GSEs paid out $12 billion in interest payments to CRT bond investors and $3.0 billion in premium payments to CRT reinsurance counter-parties. In return, the GSEs had received approximately $0.03 billion in loss recuperation via investor bond principal write-downs and $0.02 billion via reinsurance reimbursements. The discrepancy between a $0.05 billion benefit and a $15 billion cost to the GSEs has led some observers to call into question the effectiveness of the CRT program. In the remainder of this paper, we provide a structural model to price the universe of CRT tranches issued since inception. Our model speaks to the question of whether the large payments to hedge funds, asset managers, and private insurers were justified for the risk that was transferred.

3 Model

We set up a flexible dynamic asset pricing model in the spirit of the dynamic affine term structure literature (Singleton, 2002; Ang and Piazzesi, 2003), applied in other contexts by Lustig et al. (2013); Gupta and Van Nieuwerburgh (2021); Koijen et al. (2024). The model is designed and calibrated to match key features of government bond yields and corporate bond returns, alongside key macro-economic variables relevant to the housing market.

3.1 Aggregate Risk and State Vector Dynamics

Time is discrete and indexed by months. We construct a N-dimensional aggregate state vector with $N = 7$ elements:

$$\tilde{z}_t := (r_t, ys_t, cr_t, cs_t, l_t, h_t, k_t)',$$

[10]One appealing design feature of CRT bonds, from the perspective of the GSEs, is that there is no counter-party risk. The investor pays the CRT principal upfront, which is held in escrow, and receives the interest rate on the principal that remains at risk, as well as principal repayments due to scheduled and unscheduled principal payments on the underlying mortgage pool. Losses get subtracted as soon as they are realized from the remaining CRT bond principal.

where $r_t = y_t(12)$ represents the 1-year constant maturity Treasury (CMT) yield; $ys_t = y_t(120) - y_t(12)$ represents the spread between 10- and 1-year CMT yields; cr_t represents the monthly log corporate credit return on BBB-rated bonds; cs_t represents credit spread between BBB-rated corporate and 10-year CMT bonds; l_t represents the monthly layoff rate in the labor market; h_t represents the monthly log change in national house prices; and k_t represents the monthly log change in national disposable per capita income.

We express the state vector as $\tilde{z}_t = \mu + z_t$, where the demeaned state variable z_t follows a first-order VAR process that describes the dynamics of the economy:

$$z_t = \Phi z_{t-1} + \Sigma^{1/2}\varepsilon_t, \tag{1}$$

where the shock vector ε_t is independently and identically distributed standard normal: $\varepsilon_t \sim \mathcal{N}(0, I)$. The matrix $\Sigma^{\frac{1}{2}}$ is the Cholesky decomposition of the residual covariance matrix Σ, a lower-triangular matrix. The ordering of variables in z_t dictates that each orthogonal ε shock affects only itself and the variables ordered after it in the VAR.

Guided by the data, we impose the following zero restrictions on the companion matrix:

$$\Phi = \begin{bmatrix} \phi_{r,r} & \phi_{r,ys} & 0 & 0 & 0 & 0 & 0 \\ \phi_{ys,r} & \phi_{ys,ys} & 0 & 0 & 0 & 0 & 0 \\ \phi_{cr,r} & \phi_{cr,ys} & 0 & \phi_{cr,cs} & 0 & 0 & 0 \\ \phi_{cs,r} & \phi_{cs,ys} & 0 & \phi_{cs,cs} & 0 & 0 & 0 \\ \phi_{l,r} & \phi_{l,ys} & 0 & \phi_{l,cs} & \phi_{l,l} & 0 & 0 \\ \phi_{h,r} & \phi_{h,ys} & 0 & \phi_{h,cs} & 0 & \phi_{h,h} & 0 \\ \phi_{k,r} & \phi_{k,ys} & 0 & \phi_{k,cs} & 0 & 0 & \phi_{k,k} \end{bmatrix},$$

We do not make assumptions on $\Sigma^{\frac{1}{2}}$ and label its entries as follows:

$$\Sigma^{1/2} = \begin{bmatrix} \sigma_{r,r} & 0 & 0 & 0 & 0 & 0 & 0 \\ \sigma_{ys,r} & \sigma_{ys,ys} & 0 & 0 & 0 & 0 & 0 \\ \sigma_{cr,r} & \sigma_{cr,ys} & \sigma_{cr,cr} & 0 & 0 & 0 & 0 \\ \sigma_{cs,r} & \sigma_{cs,ys} & \sigma_{cs,cr} & \sigma_{cs,cs} & 0 & 0 & 0 \\ \sigma_{l,r} & \sigma_{l,ys} & \sigma_{l,cr} & \sigma_{l,cs} & \sigma_{l,l} & 0 & 0 \\ \sigma_{h,r} & \sigma_{h,ys} & \sigma_{h,cr} & \sigma_{h,cs} & \sigma_{h,l} & \sigma_{h,h} & 0 \\ \sigma_{k,r} & \sigma_{k,ys} & \sigma_{k,cr} & \sigma_{k,cs} & \sigma_{k,l} & \sigma_{k,h} & \sigma_{k,k} \end{bmatrix}.$$

3.2 Idiosyncratic Risk

We introduce idiosyncratic income and house price risk for each mortgage borrower i. Denote by H_t^i the house price level and by h_t^i the house price growth rate (log price change) of borrower i. Denote by K_t^i the level of disposable income of borrower i and by k_t^i its growth rate (log change):

$$
\begin{aligned}
H_t^i &= H_0^i \cdot \exp\left(\sum_{s=1}^{t} h_s^i\right), && (2)\\
K_t^i &= K_0^i \cdot \exp\left(\sum_{s=1}^{t} k_s^i\right), && (3)\\
h_t^i &= h_t + \sigma_{h,i}\varepsilon_t^{h,i} = e_6' \cdot \left(\mu + \Phi z_{t-1} + \Sigma^{1/2}\varepsilon_t\right) + \sigma_{h,i}\varepsilon_t^{h,i}, && (4)\\
k_t^i &= k_t + \sigma_{k,i}\varepsilon_t^{k,i} = e_7' \cdot \left(\mu + \Phi z_{t-1} + \Sigma^{1/2}\varepsilon_t\right) + \sigma_{k,i}\varepsilon_t^{k,i}, && (5)
\end{aligned}
$$

where $e_i = (0, \ldots, 0, \underbrace{1}_{i\text{-th position}}, 0, \ldots, 0)^\top$ denotes the i-th canonical vector. Each borrower's house price growth and income growth consists of an aggregate risk and an idiosyncratic risk component, where $(\varepsilon_t^{h,i}, \varepsilon_t^{k,i})$ denote the idiosyncratic shocks and $(\sigma_{h,i}, \sigma_{k,i})$ are the corresponding shock volatilities.

Individual borrowers start out with (H_0^i, K_0^i, A_0^i) at the time of mortgage origination, where A_0^i denotes initial liquid financial wealth. Mortgage borrowers face the risk of job loss. Let $L_{i,t} \sim \text{Bernoulli}(l_t)$ denote the layoff indicator for borrower i in period t, where $l_t \in [0,1]$ is the aggregate layoff rate (an element of the state vector). Conditional on being laid off ($L_{i,t} = 1$), re-employment in the next period occurs with probability *jfr*: $Remp_{i,t} \,|\, L_{i,t} = 1 \;\sim \text{Bernoulli}(jfr)$, with draws independent across borrowers and periods, and where $Remp_{i,t}$ is a binary random variable equal to one if borrower i is rehired in period t and zero otherwise.

3.3 Stochastic Discount Factor

The log of the stochastic discount factor $\log(\mathcal{M}_t) = m_t$ is given by the following process:

$$
m_{t+1} = -\frac{y_t(1)}{12} - \frac{1}{2}\Lambda_t'\Lambda_t - \Lambda_t'\varepsilon_{t+1}, \tag{6}
$$

where the market prices of risk are affine in the state vector:

$$
\Lambda_t = \Lambda_0 + \Lambda_1 z_t, \tag{7}
$$

and the annual yield on a one-month risk-free bond $y_t(1)$ is an affine function of the one-year bond yield r_t and the term-structure slope ys_t:

$$y_t(1) = \delta_0 + \delta_1 e_1' z_t + \delta_2 e_2' z_t. \tag{8}$$

It follows from the pricing equation $\mathbb{E}_t^{\mathbb{P}}\left[\mathcal{M}_{t+1}\exp\left(\frac{y_t(1)}{12}\right)\right] = 1$ that $-\mathbb{E}_t^{\mathbb{P}}[m_{t+1}] - \frac{1}{2}Var_t[m_{t+1}] = \frac{y_t(1)}{12}$. The dynamics of the demeaned VAR under the risk neutral measure $\mathbb{Q}$ follow the same process as in (1) but with risk-neutral companion matrix $\Phi^{\mathbb{Q}} = \Phi - \Sigma^{1/2}\Lambda_1$. The mean of the state vector $\tilde{z}$ under the risk-neutral measure is $\mu^{\mathbb{Q}} = \mu - \Sigma^{1/2}\Lambda_0$.

We explain below how we identify the coefficients in (7) and (8). This leads us to impose the following zero restrictions on the market price of risk coefficients:

$$\Lambda_0' = \begin{pmatrix} \Lambda_0^r & \Lambda_0^{ys} & \Lambda_0^{cr} & 0 & 0 & \Lambda_0^h & 0 \end{pmatrix},$$

$$\Lambda_1 = \begin{pmatrix} 0 & \Lambda_1^{r,ys} & 0 & 0 & 0 & 0 & 0 \\ \Lambda_1^{ys,r} & 0 & 0 & 0 & 0 & 0 & 0 \\ \Lambda_1^{cr,r} & \Lambda_1^{cr,ys} & 0 & \Lambda_1^{cr,cs} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \Lambda_1^{h,r} & \Lambda_1^{h,ys} & 0 & \Lambda_1^{h,cs} & 0 & \Lambda_1^{h,h} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

The model has four sources of priced aggregate risk, associated with the orthogonal shocks to the short rate, the slope of the term structure, credit returns, and house price growth. All four market prices of risk fluctuate over time with the short rate and the slope. The market price of corporate credit risk and house price risk additionally fluctuate with the credit spread, while the market price of house risk additionally depends on lagged house price growth. In our main asset pricing exercise, we estimate both the coefficients in the VAR and the market prices of risk recursively.

3.4 Identifying the Market Prices of Risk

To identify the market prices of risk, we impose that our model prices government bonds, corporate bonds, and house prices accurately. The details are in Appendix A.2. Starting with government bonds, a standard result in this class of models is that the log yield at time t of a

Treasury bond with maturity s is affine in the state vector:

$$\frac{y_t(s)}{12} = -\frac{A_s}{s} - \frac{B_s'}{s} z_t, \tag{9}$$

where the coefficients A_s and B_s satisfy the difference equations (A.1)–(A.2), spelled out in the Appendix. The one-year yield and the difference between the ten- and one-year yields are the first and second elements of the VAR, respectively. That means that they are linear functions of the state variables. But they are also affine in the state by virtue of (9). Matching up the two expressions provides the moment conditions (A.3)–(A.6). We also insist on matching the observed average spread between the 1-year and 3-month Treasury bills. Taken together, this delivers seven equations that uniquely pin down the seven parameters $(\delta_0, \delta_1, \delta_2)$ and $(\Lambda_0^r, \Lambda_0^{ys}, \Lambda_1^{r,ys}, \Lambda_1^{ys,r})$.

Next, we turn to pricing the return on the BBB-corporate bond index. From the first-order condition, we have that:

$$\mathbb{E}_t^{\mathbb{P}}\left[cr_{t+1}\right] - \frac{y_t(1)}{12} + \frac{1}{2} Var_t\left[cr_{t+1}\right] = -Cov_t\left[cr_{t+1}, m_{t+1}\right]. \tag{10}$$

The VAR pins down the left-hand side, while the right-hand side can be computed using the SDF expression (6). Equating the two results yields a system of four equations (A.8)–(A.11) in four unknowns $(\Lambda_0^{cr}, \Lambda_1^{cr,r}, \Lambda_1^{cr,ys}, \Lambda_1^{cr,cs})$.

Finally, we insist on pricing the return on housing. The return on housing consists of a capital gain return plus an income return. The latter is also known as the net rental yield. We assume that the net rental yield is constant and equal to nry per month. Since this is the return on an individual house, part of the return variance is the idiosyncratic component $\sigma_{h,i}^2$:

$$\mathbb{E}_t^{\mathbb{P}}\left[h_{t+1}^i + nry\right] - \frac{y_t(1)}{12} + \frac{1}{2} Var_t\left[h_{t+1}^i\right] = -Cov_t\left[h_{t+1}^i, m_{t+1}\right]. \tag{11}$$

The left-hand side is pinned down by the VAR, while the right-hand side can be computed using the SDF expression in (6). Equating the two results in five equations (A.12)–(A.16) in five unknowns $\left(\Lambda_0^h, \Lambda_1^{h,r}, \Lambda_1^{h,ys}, \Lambda_1^{h,cs}, \Lambda_1^{h,h}\right)$.

3.5 Mortgage and Borrower Liquid Wealth Dynamics

The borrower takes out a 30-year fully-amortizing fixed-rate mortgage (FRM), the standard mortgage product in the United States. The maturity is $T = 360$ months. Let N_0^i be the mortgage principal and $r_0^{m,i}$ the annual rate on the 30-year fixed-rate mortgage of borrower i at

loan origination. Let $N_0 := \sum_{i=1}^{I} N_0^i$ be the aggregate notional of all mortgage loans originated across borrowers in that loan vintage. Absent mortgage default and prepayment, the monthly mortgage payment is:

$$C^i := \frac{\frac{r_0^{m,i}}{12} N_0^i}{1 - \left(1 + \frac{r_0^{m,i}}{12}\right)^{-T}},$$

and unpaid principal balance (UPB) evolves according to:

$$N_t^i = N_0^i \times \frac{\left(1 + \frac{r_0^{m,i}}{12}\right)^{T} - \left(1 + \frac{r_0^{m,i}}{12}\right)^{t}}{\left(1 + \frac{r_0^{m,i}}{12}\right)^{T} - 1}.$$

We model the average 30-year fixed-rate mortgage rate as an affine function of the 10-year CMT bond yield:

$$r_t^m = r_0^m + b \cdot (r_t + ys_t - r_0 - ys_0), \tag{12}$$

where r_0^m is the unconditional mean of the mortgage rate and b its sensitivity to the demeaned 10-year CMT yield.

The borrower's (after-tax) disposable income at mortgage origination is K_0^i. We can back out this income from the observed debt-to-income ratio at the time of mortgage origination d_0^i: $K_0^i = \frac{C^i}{(1+\tau^{inc})d_0^i}$, where τ^{inc} is the income tax rate. Given initial income K_0^i, borrower income dynamics are given by (3) and (5).

We describe the evolution of the borrower's liquid financial wealth, A_t^i, next. This liquid wealth will play an important role in mortgage default. The borrower can either be laid off at time t or not laid off. If she is not laid off, has positive financial wealth at t, and sufficient disposable income to make both her periodic mortgage payment and pay for consumption necessities ($0.8 \cdot K_t^i - C^i > 0$), then she is a saver. Her time-t assets are growing at the risk-free rate of interest and she adds savings from disposable income in the current period. This assumes that 20% of disposable income is required for consumption necessities excluding housing. We assume that such a borrower is able to save 13.5% of her disposable income after the mortgage payment, which is the average savings rate out of post-mortgage disposable income observed in the data.

If she is not laid off, starts the period with positive assets, but has insufficient income to pay for consumption necessities and the mortgage, her assets go down by $0.8 \cdot K_t^i - C^i < 0$.

$$A^i_{t+1} = \begin{cases} \left(\frac{r_t}{12}+1\right) A^i_t + \underbrace{0.135 \cdot (K^i_t - C^i)}_{\text{savings from DPI}} & \text{if not laid off, } 0.8 \cdot K^i_t - C^i \geq 0, A^i_t \geq 0 \\ \left(\frac{r_t}{12}+1\right) A^i_t + (0.8 \cdot K^i_t - C^i) & \text{if not laid off, } 0.8 \cdot K^i_t - C^i < 0, A^i_t \geq 0 \\ \left(\frac{r_t+ccs}{12}+1\right) A^i_t + 0.135 \cdot (K^i_t - C^i) & \text{if not laid off, } 0.8K^i_t - C^i \geq 0, -12K^i_t < A^i_t < 0 \\ \left(\frac{r_t+ccs}{12}+1\right) A^i_t + (0.8 \cdot K^i_t - C^i) & \text{if not laid off, } 0.8K^i_t - C^i < 0, -12K^i_t < A^i_t < 0 \\ \left(\frac{r_t}{12}+1\right) A^i_t - C^i & \text{if laid off, } A^i_t \geq 0 \\ \left(\frac{r_t+ccs}{12}+1\right) A^i_t - C^i & \text{if laid off, } -12K^i_t < A^i_t < 0 \end{cases}$$

If she is not laid off but starts the period in debt ($A^i_t < 0$), she pays an interest rate on her debt that exceeds the risk-free interest rate by a annual consumer credit spread of *ccs*. We assume that consumer debt must not exceed twelve months of income ($-12K^i_t < A^i_t$), a standard unsecured borrowing limit. Whenever a borrower exceeds this limit, we force her to sell the house and repay the mortgage principal. If the borrower has positive home equity, this liquidation does not result in a loss to the lender.

If the borrower is laid off, we assume her income drops to the level that is just sufficient to pay for consumption necessities. Her mortgage must be paid from prior savings.

Finally, if the borrower is laid off at t and was already in debt at t, her debt accumulates further by the amount of the mortgage payment, assuming it stays within the borrowing limit. Otherwise, the mortgage is in default.

3.5.1 Mortgage Default, Loss Resolution, and Loss Severity

Following the consensus in the literature, we adopt the double trigger theory of mortgage default. A borrower i defaults if the equity in the home becomes negative—after considering a fire sale discount fd^i_t and a sales commission *sc*,—and has insufficient liquid assets to pay the mortgage. Define the default event as the first time both conditions are met:

$$t_{i,def} := \inf\{t \leq T : \{A^i_t \leq C^i\} \cap \{(1 - fd^i_t)(1 - sc)H^i_t \leq N^i_t\}\}. \tag{13}$$

We set $\tau^i_t = 1$ if borrower i has defaulted by time t, and $\tau^i_t = 0$ if borrower i has not defaulted by time t and the borrowing constraint on unsecured consumer credit has not been hit. We have $\mathbb{P}(\tau^i_t = 1) = \mathbb{P}(t_{i,def} \leq t)$ and $\mathbb{P}(\tau^i_t = 0) = 1 - \mathbb{P}(\tau^i_t = 1)$.

When a loan defaults, it takes time before the loan gets resolved. Denote the resolution time by $t_{i,res} > t_{i,def}$. The loan resolution results in a loss realization $l^i_{t_{i,res}}$ and a (partial) recovery of

the unpaid principal balance $R^i_{t_{i,resf}}$. The implementation section below discusses how realized loss severity and recovery are computed in the context of CRT transactions.

3.5.2 Mortgage Prepayment

We define the rate incentive of borrower i, $\delta^i_t := r^{m,i}_0 - r^{m,i}_t$, as the difference between the mortgage rate borrower i obtained at loan origination and the mortgage rate she could obtain on a new 30-year fixed-rate mortgage issued in the current period t.

As shown by the prepayment literature, the rate incentive is far and away the most important determinant of mortgage prepayment. We model the monthly prepayment rate, also known as the single month mortality rate, on a mortgage (pool) with interest rate c as a function of the rate incentive:

$$\Gamma^i(\delta^i_t) := \frac{L^c}{1 + e^{-\alpha^c(\delta^i_t - A)}}. \tag{14}$$

The functional form gives rise to the well-known S-shaped prepayment curve, with low but positive prepayment rates at negative prepayment incentives, steeply rising prepayment rates for modestly positive rate incentives, and flattening prepayment rates at high rate incentives.

An individual mortgage borrower prepays each period with probability given by (14). Let ϵ^i_t denote the mortgage prepayment random variable of borrower i: $\epsilon^i_t = 1$ if the borrower has prepaid by time t and $\epsilon^i_t = 0$ if the borrower i has not prepaid by time t.

3.6 Credit Risk Transfer Bond Pricing

Our main exercise is to price the Credit Risk Transfer (CRT) bonds. As explained in Section 2, CRT bonds are structured credit products that pass some of the losses on an underlying mortgage loan pool from the GSEs to private investors. The underlying mortgage pool consists of standard 30-year fixed-rate mortgages originated during a specific time window.

The cumulative loss rate as of time t on a pool of I mortgages originated at time 0, expressed as a share of the initial aggregate loan principal N_0, is:

$$L_t := \sum_{i=1}^{I} \frac{l^i_{t_{i,res}} \mathbf{1}_{t_{i,res} \leq t}}{N_0}, \tag{15}$$

Principal repayments on a mortgage come in two flavors: scheduled principal payments, from amortization, and unscheduled principal payments, which consist of prepayments and recoveries from default resolutions. The cumulative scheduled and unscheduled principal repayments as of time t on a pool of I mortgages originated at time 0, expressed as a share of the initial

aggregate loan principal N_0, is:

$$F_t^S = \sum_{i=1}^{I} \frac{(N_0^i - N_t^i)\mathbf{1}_{\tau_t^i=0,\epsilon_t^i=0}}{N_0}, \tag{16}$$

$$F_t^U = \sum_{i=1}^{I} \frac{R_{t_{def}^i}^i \mathbf{1}_{\tau_t^i=1,\epsilon_t^i=0} + N_0^i \mathbf{1}_{\tau_t^i=0,\epsilon_t^i=1}}{N_0}, \tag{17}$$

where scheduled principal payments come from borrowers who have neither prepaid nor defaulted at of time t ($\mathbf{1}_{\tau_t^i=0,\epsilon_t^i=0} = 1$), and unscheduled principal payments come from borrowers that have prepaid as of time t ($\mathbf{1}_{\tau_t^i=0,\epsilon_t^i=1} = 1$) or have defaulted ($\mathbf{1}_{\tau_t^i=1,\epsilon_t^i=0} = 1$).

In each CRT vintage, multiple bonds (tranches) are issued (sold to investors) with different attachment points b, the pool loss rate above which the tranche begins to take losses, and detachment points b', the pool loss rate at which the tranche is wiped out. We recall that the most senior tranche (called A-H) is always retained by the GSEs. This tranche begins to take losses when the pool loss rate exceeds a credit enhancement level CE, equal to the detachment point of the most senior sold tranche. Tranches sold to the investors are called subordinate tranches.[11] We let M denote the notional of the subordinate tranches relative to the entire pool at deal origination, and M_t the remaining principal outstanding of the subordinate tranches divided by the outstanding notional of the entire pool; $M_0 = M$. It is crucial to understand how the pool's realized losses, scheduled principal payments, and unscheduled principal payments are allocated to the various tranches.

Losses are distributed in reverse order of seniority. The most junior tranche (usually the B-2 tranche) suffers credit losses first. Only after it has been fully wiped out do losses accrue to the next most junior bond (the B-1 tranche), etc. Given the collateral pool's cumulative loss rate L_t as of time t, the cumulative loss rate incurred by a generic $[b, b']$ tranche is:

$$L_t^{[b,b']} := \min\left\{\max\left\{L_t - b, 0\right\}, b' - b\right\} \tag{18}$$

Recoveries from default resolutions are allocated entirely to the senior A-H tranche. Given the size of the A-H tranche (typically well over 90% of the total), CRT bond investors do not receive any payments from recoveries.

Prior to March 2018,[12] scheduled principal payments are allocated to the senior (A-H) tranche and the subordinate tranches pro rata according to the current level of credit en-

[11] The only exception is the A-1 tranche, which is issued only after 2023. A-1 tranche is discussed separately in Appendix A.8.

[12] This refers to all deals from the first CRT deal, 2013 STACR DN1, to the 2018 STACR DNA1 deal issued in March 2018, based on information in the private placement memoranda.

hancement, which is M_t. Among the subordinate tranches, scheduled principal payments are allocated sequentially starting with the most senior. The cumulative scheduled principal cash flow paid to the $[b, b']$ CRT tranche as of time t is:

$$F_t^{S,[b,b']} := \min\left\{(M_t \cdot F_t^S - (M - b'))\mathbf{1}_{M_t \cdot F_t^S > M - b'}, b' - b\right\} \tag{19}$$

The $[b, b']$ tranche only receives scheduled principal cash flows after all tranches senior to it $(M - b')$ have been fully paid off. It gets fully paid off when $M_t \cdot F_t^S \geq M - b$.

After March 2018, scheduled principal payments only flow to the subordinate tranches when (i) cumulative loan loss and (ii) recent loan delinquency levels are sufficiently low, and (iii) the current credit enhancement (CE) is sufficiently high. The indicator variable $\mathbf{1}_{delstop,t}$ is one in periods in which the cumulative loss or recent delinquency threshold is breached. The indicator $\mathbf{1}_{CE,t}$ is one when the CE (of the subordinate tranche) is above the threshold:

$$F_t^{S,[b,b']} := \min\left\{(M_t \cdot F_t^S - (M - b'))\mathbf{1}_{M_t \cdot F_t^S > M - b'}\mathbf{1}_{CE,t}(1 - \mathbf{1}_{delstop,t}), b' - b\right\} \tag{20}$$

Unscheduled principal payments are allocated differently from scheduled principal payments.[13] They are initially directed only to the senior A-H tranche until a pre-specified CE threshold is met, as long as delinquency levels are below pre-specified thresholds. Once these thresholds are reached, prepayments are allocated to the senior A-H tranche and the subordinate tranches pro rata according to the current CE level, which is M_t. Among the subordinate tranches, unscheduled principal payments are allocated sequentially, with the most senior CRT tranche receiving all prepayments until it is fully paid down. The cumulative unscheduled principal cash flow paid to the $[b, b']$ CRT tranche as of time t is:

$$F_t^{U,[b,b']} := \min\left\{(M_t \cdot F_t^U - (M - b'))\mathbf{1}_{M_t \cdot F_t^U > M - b'}\mathbf{1}_{CE,t}(1 - \mathbf{1}_{delstop,t}), b' - b\right\}. \tag{21}$$

Since both scheduled and unscheduled principal payments flow to the senior CRT bonds before they flow to the junior CRT bonds, senior tranches have shorter duration than the junior tranches. Define cumulative principal payments to a tranche as $F_t^{[b,b']} := F_t^{S,[b,b']} + F_t^{U,[b,b']}$.

We define the value-at-risk of tranche $[b, b']$ at time t as:

$$V_t^{[b,b']} := ((b' - b) - L_t^{[b,b']} - F_t^{[b,b']})\mathbf{1}_{L_t^{[b,b']} + F_t^{[b,b']} \leq b' - b} \tag{22}$$

[13] Generally, unscheduled principal payments contain prepayments and default recoveries. Default recoveries exclusively flow to the senior tranche. Hence, for CRT investors, it is prepayments that matter. We discuss this further below in the implementation section.

V represents the remaining credit exposure of a tranche, the difference between the original principal of the tranche ($b'-b$) and the amount of principal repayment and credit losses allocated to that tranche as of time t.

We are now ready to price the CRT bonds. Let T^c be the maturity of the CRT bond expressed in months. The CRT tranche $[b, b']$ pays a tranche spread $ts^{[b,b']}$ over the variable interest rate benchmark sr_t. This interest rate spread is set such that the contract is priced fairly. That is, the present risk-adjusted value of payments on the tranche are equal to the initial investment:

$$\mathbb{E}_0^{\mathbb{P}}\left[\sum_{s=1}^{T^c}\left(\prod_{t=1}^{s}\mathcal{M}_t\right)\left(\underbrace{\frac{ts^{[b,b']}+sr_s}{12}V_{s-1}^{[b,b']}}_{Monthly\ Interest\ Payment}+\underbrace{F_s^{[b,b']}-F_{s-1}^{[b,b']}}_{Monthly\ Cash\ Flow}\right)+\left(\prod_{t=1}^{T^c}\mathcal{M}_\tau\right)\underbrace{V_{T^c}^{[b,b']}}_{Final\ Credit\ Exposure}\right] = \underbrace{b'-b.}_{Initial\ Notional} \quad (23)$$

The CRT investor in this tranche receives principal payments in month s equal to $F_s^{[b,b']}-F_{s-1}^{[b,b']}$, plus an interest rate payment of $ts^{[b,b']}+sr_s$ on the remaining value-at-risk $V_{s-1}^{[b,b']}$ at the start of the month (end of last month) as compensation for bearing credit risk. At the end of the life of the contract, she receives the remaining value-at-risk. Under the risk-neutral pricing measure $\mathbb{Q}$, used in our computation, the CRT tranche pricing formula becomes:

$$\mathbb{E}_0^{\mathbb{Q}}\left[\sum_{s=1}^{T^c}e^{-\sum_{t=1}^{s}\frac{y_t(1)}{12}}\left(\frac{ts^{[b,b']}+sr_s}{12}V_{s-1}^{[b,b']}+(F_s^{[b,b']}-F_{s-1}^{[b,b']})\right)+e^{-\sum_{t=1}^{T^c}\frac{y_t(1)}{12}}V_{T^c}^{[b,b']}\right]=b'-b. \quad (24)$$

3.7 Guarantee Fee

The model-implied guarantee fee represents the fair compensation for bearing the credit risk associated with the entire mortgage pool until maturity, taking as given how market participants price interest rate, corporate credit, and house price risks. Unlike CRT bonds, which have shorter maturity than the underlying mortgages ($T^c < T$), this calculation considers the credit risk exposure over the full life cycle of the loan. As with the CRT tranches, scheduled principal amortization and unscheduled principal prepayments reduce the value that remains at risk. The pricing formula for the g-fee is a special case of the one for a CRT tranche (24), where the tranche now refers to the entire pool, $[b, b'] = [0, 1]$. The model-implied credit spread, g^{fee}, solves:

$$\mathbb{E}_0^{\mathbb{Q}}\left[\sum_{s=1}^{T}e^{-\sum_{t=1}^{s}\frac{y_t(1)}{12}}\left(\frac{g^{\text{fee}}+y_s(1)}{12}V_{s-1}^{[0,1]}+(F_s^{[0,1]}-F_{s-1}^{[0,1]})\right)\right]=1, \quad (25)$$

where we used the fact that $V_T^{[0,1]} = 0$ since the mortgages are fully amortizing. We quantify credit risk for the entire mortgage pool using a g-fee that is expressed relative to the one-month T-bill rate $y_s(1)$.

4 Estimation Results

4.1 VAR

We estimate the VAR in (1) using monthly OLS regressions. Appendix A.1 details the data sources. We use the longest available period for the seven variables in the state vector z, which is from January 2001 until February 2025. For the main asset pricing results, we estimate the VAR using expanding windows.[14] Appendix Table A.1 reports the full-sample VAR estimation results. We note that the volatilities of disposable income growth and the layoff rate increase substantially between the sample that ends in 2019 and the samples that ends in 2020 or later. The other coefficients are more stable over time. A small literature discusses how to best deal with the highly unusual Covid-19 shock in the context of VAR estimation (Lenza and Primiceri, 2022; Carriero et al., 2021; Schorfheide and Song, 2021; Primiceri and Tambalotti, 2020). We believe that a recursive estimation best approximates the information market participants have when pricing successive vintages of CRT bonds. We report robustness to using full-sample VAR coefficients.

4.2 Idiosyncratic Risk

We calibrate the volatility of the monthly idiosyncratic house price growth shock $\epsilon_t^{h,i}$ to $\sigma_{h,i} = 0.0115$, which corresponds to an annual standard deviation of 4%. This number matches the cross-sectional standard deviation of annual MSA-level house price growth during normal times obtained by Sinai and Heathcote (2012, Figure 16).

We set the volatility of the monthly idiosyncratic income shock $\epsilon_t^{k,i}$ to $\sigma_{k,i} = 0.0779$, which corresponds to an annual standard deviation of 27%. This number matches the standard deviation of annual earnings changes for prime-age males in Guvenen et al. (2019).

[14] To price CRT bonds issued in year t, the estimation window depends on the issuance month. If a bond is issued between January and June of year t, we estimate the VAR using data from January 2001 through December of year $t-1$. If a bond is issued between July and December of year t, we estimate the VAR using data from January 2001 through June of year t. For the guarantee fee and LLPA analyses, the VAR is re-estimated once per year to reflect the fact that mortgage originations do not have a single closing date within the year.

We set the monthly job finding rate jfr to 20% to match the observed average unemployment spell duration of 5 months given by the Bureau of Labor Statistics.[15]

4.3 Market Prices of Risk and Asset Prices

Section 3.4 discussed the identification of the coefficients that govern the market price of risk coefficients in (7) and the risk-free rate in (8). For our main asset pricing exercises, we estimate the market price of risk and risk-free rate coefficients recursively. To illustrate the model fit, Appendix A.3 reports the full-sample coefficient estimates. Figure A.1 shows the fit of the model for nominal bond yields of various maturities when market prices of risk are estimated on the full sample. By construction, the model exactly matches the 1- and 10-year yields, while providing a close fit to the 3-month and 5-year yields. These are the most relevant maturities for the pricing of CRT bonds.

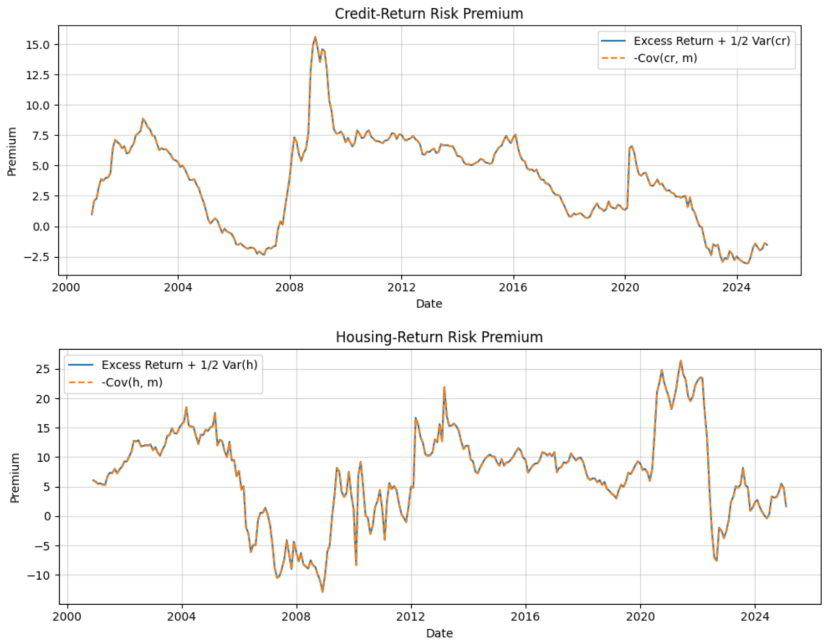
Figure 2 illustrates that our model implies rich time variation in risk premia. The top panel plots the annual risk premium on the BBB-rated corporate bond and the bottom panel the annual housing risk premium. These risk premia are key inputs in the pricing of CRT bonds. Corporate credit and housing risk premia affect mortgage default risk by affecting the drift of aggregate labor income risk and house price risk under the risk neutral measure, respectively. The credit risk premium spikes in 2008 and 2020, increasing the risk of mortgage defaults through the liquidity channel in those periods. The housing risk premium displays a large increase between 2020 and 2021-22, increasing the risk of mortgage defaults through the collateral channel. A high housing risk premium lowers the drift of house price growth under the risk neutral measure, making it more likely that house prices fall below the mortgage balance.[16]

A key parameter, which has a first-order effect on the average housing risk premium, is the net rental yield nry. It subtracts taxes, maintenance and repair costs, and capital expenditures from the gross rental yield. We set $nry = 0.042/12$, which is the national average net rental yield for single-family housing in Eisfeldt and Demers (2015, Table 2). The model's annualized housing risk premium averages to 7.15% over our sample.

[15]Our model captures cyclical fluctuations in unemployment risk through the layoff rate. Allowing the rehiring rate to also vary with the cycle instead or in addition to the layoff rate would likely result in very similar CRT pricing results since it would need to produce the same overall income-driven mortgage default intensities to capture to observed default rates.

[16]The model also implies time-varying long-term bond risk premia. Interest rate risk matters for CRT pricing through prepayments.

Figure 2: Nominal Bond Risk Premium



Notes: The top panel plots the corporate credit risk premium defined in equation (10). The bottom panel plots the housing risk premium in (11). The risk premia are computed either as the expected excess return plus one-half the variance or as minus the conditional covariance between the log SDF and the log return. Market price of risk choices, estimated here on the full sample, ensure these two measures coincide. Risk premium values are expressed as annual percentage points ($\times 1200$).

4.4 Mortgage Dynamics

For the dynamics of the mortgage rate, we estimate a slope of $b = 1.0097$ in (12) using data on the average 30-year FRM rate in the U.S. from 1989 to 2025, retrieved from FRED (series MORTGAGE30US). We calibrate the annual spread of the unsecured consumer credit rate over the one-year Treasury rate to $ccs = 0.0799$ using data from FRED (series TERMCBPER24NS). We use an income tax rate τ^{inc} of 25% to determine the borrower's initial post-tax income from the debt-to-income ratio. We set the sales commission in (13) to $sc = 6\%$, a standard value for the U.S. housing market

4.4.1 Mortgage Default Model Estimation

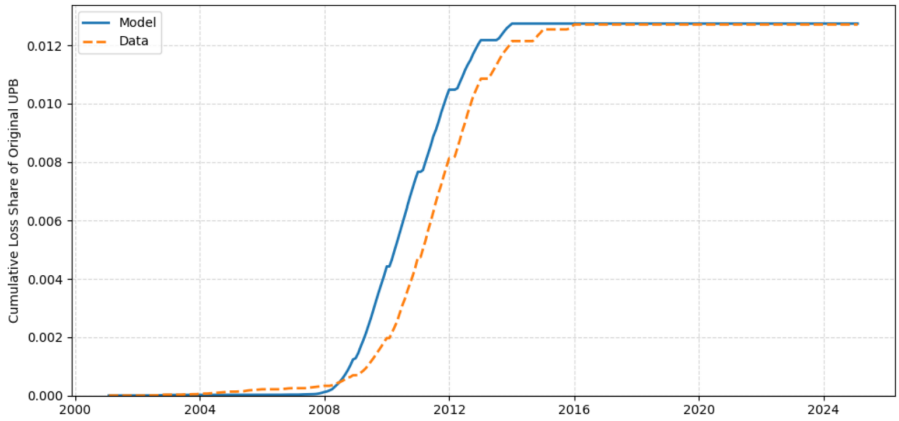
Two objects are crucial for the model's ability to generate realistic mortgage default and loss-given-default estimates. The first one is the fire-sale discount factor fd_t in (13). Gerardi et al. (2015) find that fire-sale discounts in the mortgage market are strongly time-varying; they peaked near 30% during the Great Financial Crisis (GFC), when house prices fell by more than 30% from their peak, and declined to roughly 10–15% in the post-GFC period as housing markets stabilized. Motivated by this evidence, we model a fire-sale discount rate that increases

when past house price growth (HPA) has been persistently negative. The functional form is detailed in Appendix A.4.1.

The second key object is the distribution of liquid assets at the time of mortgage origination $\{A_0^i\}_{i=1}^I$. Gerardi et al. (2018) show that borrowers with adverse liquidity shocks are more likely to default on their mortgage, after controlling for LTV ratios. They also find that liquid financial assets correlate positively with FICO scores. Motivated by this evidence, we model initial liquid financial assets as a multiple of initial monthly income K_0^i with the multiple increasing in the borrower's credit quality (FICO score): $A_0^i = K_0^i \cdot m(\texttt{FICO}^i)$. The functional form for $m(\cdot)$ is detailed in Appendix A.4.2.

The parameters of the fire-sale discount and initial liquid asset-to-income ratio functions are chosen to minimize the distance between the model and the data in terms of (i) the cumulative realized loss rate on all mortgages originated over our sample period, and (ii) the loss rates by FICO × LTV groups. Figure 3 shows that our model exactly matches the aggregate cumulative loss rate of 1.27% at the end of 2024. Figure 4 shows that the model does an excellent job differentiating between high- and low-risk borrowers. The UPB-weighted RMSE between the model-implied and observed loss rates across FICO × LTV bins is only 0.30%. Appendix A.4.3 details the optimization procedure and shows additional implications of the model for mortgage defaults across vintages.

Figure 3: Historical versus Model-Implied Cumulative Loss Rates Across All Vintages



Notes: This figure compares the model-implied cumulative loss rate with the realized cumulative loss rate in the Freddie Mac loan-level dataset, which includes both standard and non-standard mortgages originated between 2001 and 2024.

4.4.2 Prepayment Model Estimation

We estimate the prepayment rate function (14) by coupon. Appendix A.5 provides the details. Figure 5 plots the cumulative prepayment rate over time for select vintages. The model ac-

Figure 4: Mortgage Loss Rates across FICO and LTV Groups



Notes: This figure compares observed (left panel) and model-implied (right panel) conditional loss rates, measured as the loss of UPB shares, across a FICO × LTV grid for loans originated between 2001 and 2024.

curately captures the fact that some vintages, like the 2002.Q1 and 2019.Q1 vintages prepay quickly, with over 80% of mortgage principal extinguished within five years, while others like the 2013.Q1 vintage prepay more slowly due to different time paths of market mortgage rates. Figure 6 shows the UPB-weighted prepayment rate for all vintages, expressed as an annual number, the so-called conditional prepayment rate or CPR.[17] Our prepayment model fits the CPR data quite well.

5 CRT Bond Pricing

Before presenting our main CRT pricing results, we discuss several implementation details.

[17]To go from the single month mortality (SMM), which is a monthly number, to the conditional prepayment rate (CPR), which is an annual number, we use: $CPR = 1 - (1 - SMM)^{12}$.

Figure 5: Cumulative Prepayment Rate for Selected Vintages

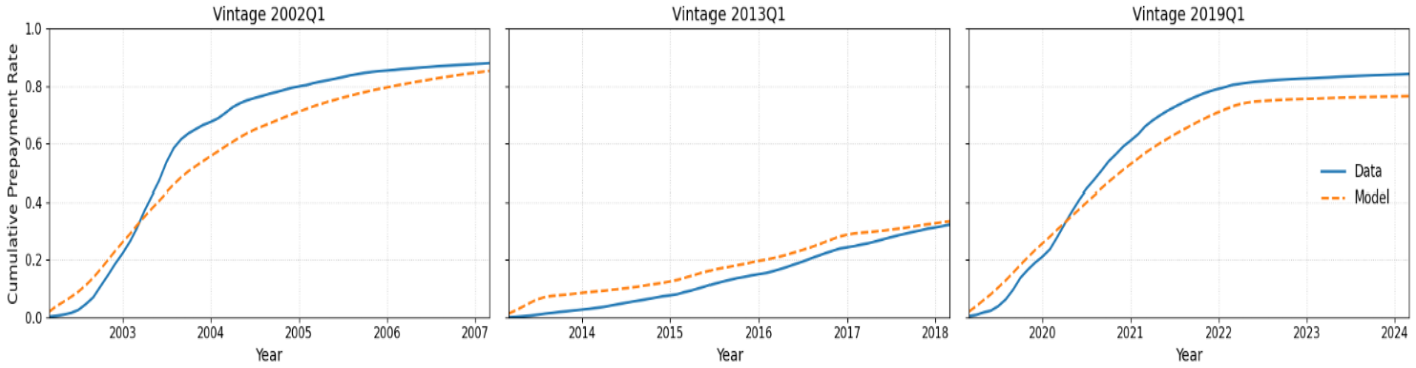
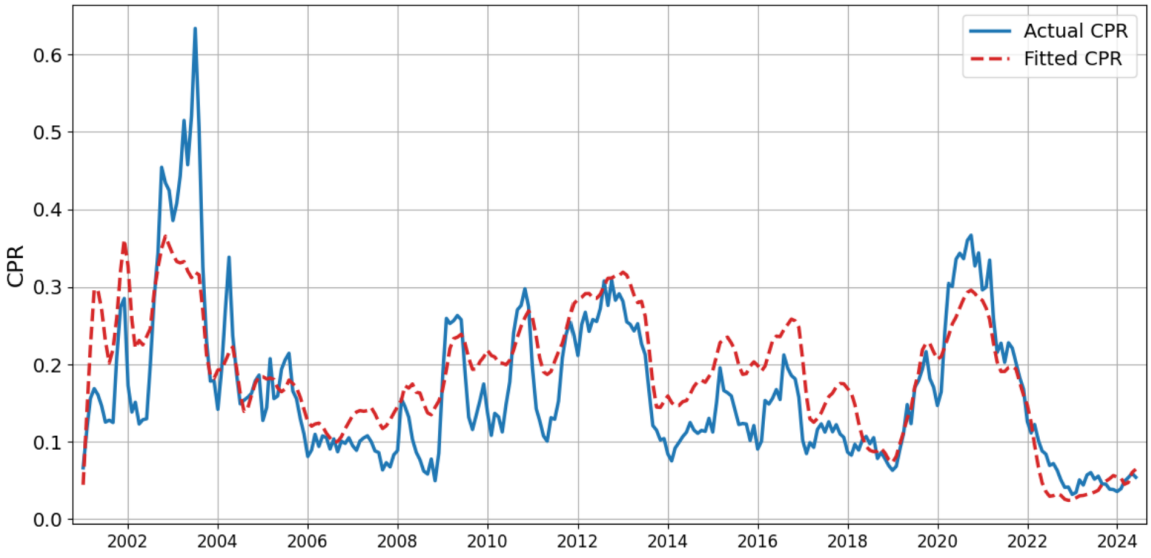


Figure 6: Conditional Prepayment Rate (UPB-weighted average of vintages)



5.1 Implementation Details

Interest Rate Benchmark According to the FHFA Federal Housing Finance Agency (2021), investors receive monthly coupon payments, typically on the 25th day of each month. For the Freddie Mac CRT bonds, the variable-rate payments are tied to the 30-day LIBOR rate until the end of September 2020 and to the 30-day SOFR rate from the start of October 2020 onward. We obtain the 30-day average SOFR and LIBOR rates from FRED. We model these benchmark rates as an affine function of the first two state variables, the one-year Treasury yield r_t and the slope of the yield curve ys_t. To do so, we estimate a regression of one-month LIBOR (SOFR) on a constant and these two state variables for the period 2013.01–2020.12 (2018.01–2025.02). We obtain the coefficient estimates of (0.0052, 0.9466, 0.0259) for the former regression and

(0.0011, 0.9086,-0.2638) for the latter regression.[18]

CRT Maturity and Call Option CRT bonds have a legal maturity that varies across deals, ranging from 10 to 30 years. Table A.3 presents the complete list of tranches we study and has a column indicating the legal maturity. The GSEs also have an option to call the CRT bonds early when the current UPB falls below 10% of the original UPB to avoid the administrative costs of small-balance deals. Based on the observed clean-up call exercise behavior of Freddie Mac, we assume that the clean-up call is always exercised as soon as possible.[19] Starting in October 2021, the legal maturity of CRT tranches became 20 years, but Freddie Mac obtained a 5-year call option. Since none of these CRTs have reached their 5-year mark, we have no data to discipline the exercise behavior of this 5-year call option. Given that much of the credit risk exposure remains after 5 years, we assume that Freddie will not exercise this option but keep its protection alive. Should the clean-up call threshold be met before year 5, the CRT will still be called. Accordingly, we set T^c to be the minimum of the legal maturity and the first month that the deal UPB falls below 10% of the origination UPB.

Calculating Losses Absorbed by CRT Investors Next, we explain in more detail how borrowers' defaults on the underlying collateral pool impact the loss to CRT tranche holders. The realized loss l_t^i of borrower i is the positive difference between the default costs and default credits:

$$\begin{aligned}
&\text{Realized Loss} = \max\{\text{Default Costs} - \text{Default Credits}, 0\} \\
&\text{Default Costs} = \text{Default UPB} + \text{Accrued Interest} + \text{Expenses} \\
&\text{Default Credits} = \text{Net Sales Proceeds} + \text{PMI Recoveries} + \text{Non-PMI Recoveries}
\end{aligned}$$

We define each of these components and their computation in Appendix A.6. This includes a detailed discussion of how the delay between loan delinquency and loss resolution is calibrated, and the way in which private mortgage insurance (PMI) affects the credit risk exposure of CRT investors in the high-LTV deals.

Fixed Severity Vintages For CRT bonds issued in or after 2015 under the realized loss framework, realized losses and principal recoveries are allocated to the tranches at the time

[18]SOFR became actively traded in 2018, and so we prefer to estimate the relationship between SOFR and the two state variables on a slightly longer sample. The results are similar when that relationship is estimated on the post-2020.09 sample instead.

[19]Consistent with this assumption, there are no CRT bonds that remain outstanding that have less than 10% of the original UPB.

of resolution, as described above. The CRT bonds issued during 2013 and 2014 apply a fixed severity schedule instead.[20]

Mortgage Modifications Some delinquent loans receive a modification. The modification may reduce principal or interest or simply extend the maturity, adding the missed payments at the end. When we split cumulative loss rates into those loans that ever received a modification and those that never did, we find that the bulk of losses come from never-modified loans. Figure A.6 shows this break-down for the 2006 vintage, one of the lowest-quality vintages disproportionately subject to post-GFC modification programs. Even this vintage only shows one in six dollars of losses from ever-modified loans. The share is much smaller in the CRT era. This evidence leads us to simply calibrate the model to average losses on all mortgages.

Unscheduled and Scheduled Principal Payments to CRT Investors At each payment date, the GSEs administer tests to determine which types of tranches receive scheduled and unscheduled principal payments. These tests relate to (i) the cumulative net loss rate of the collateral pool, (ii) the delinquency rate over the past six months, (iii) the current credit enhancement. We discuss these cash-flow waterfall mechanics in Appendix A.8.

5.2 Main Results: CRT Tranche Spreads

Our main exercise is to price the various CTR bonds issued by Freddie Mac between the inception of the CRT program in 2013 and the middle of 2025. Table A.3 presents the complete list of tranches we price, indicating the tranche name, legal maturity, attachment and detachment points, minimum credit enhancement, observed tranche spread, and model-implied tranche spread.

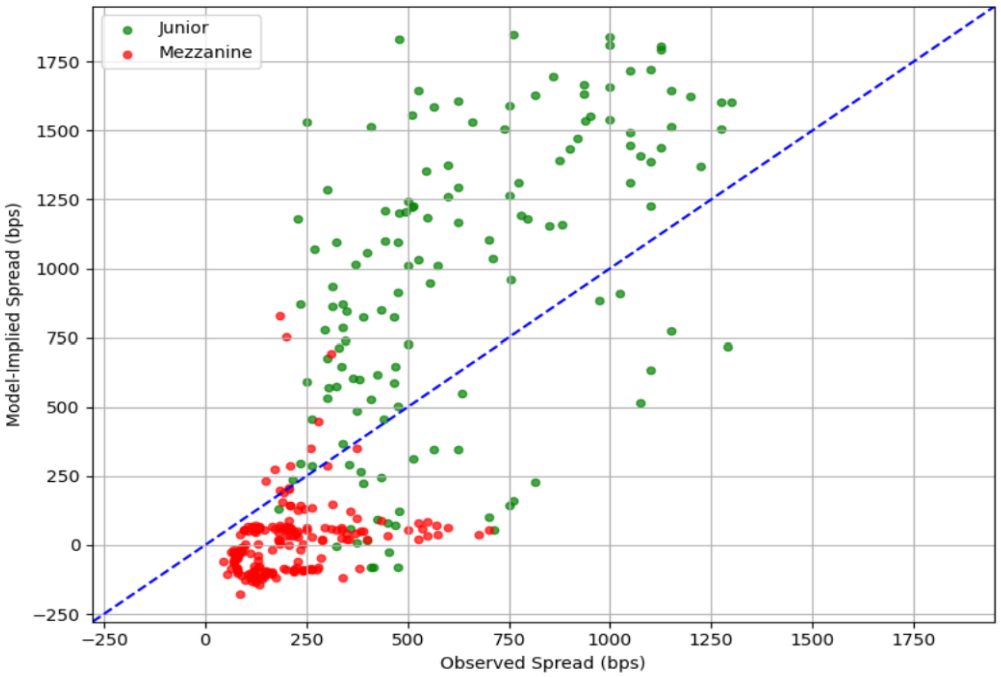
To price each tranche, we construct an initial distribution $(K_0^i, H_0^i, N_0^i, r_0^{m,i})$ by randomly sampling 1,000 households from the underlying loan portfolio that backs the specific deal, reported by Freddie Mac's Clarity Data Intelligence. We construct initial assets A_0^i as in (A.4.2). We draw 5,000 monthly sample paths of the aggregate shocks ϵ_t, and form sample paths for the aggregate state vector z_t from (1) starting from the observed initial state z_0. In each round, we draw i.i.d. idiosyncratic income and house price shocks for the 1,000 households. We also draw i.i.d. shocks for layoffs and loan resolution delay, as described above. The parameters of the VAR in (1), the market price of risk parameters in (7), as well as the prepayment parameters in (14) are estimated recursively: CRT bonds issued in the first (second) half of year t are priced

[20]The STACR 2015-DNA1, priced March 2015, was the first deal priced under the actual loss framework. For the fixed severity loss deals, we apply the stated loss severities when the mortgage becomes 180-days delinquent.

using parameters estimated on a sample ending in December of year $t-1$ (June of year t). We solve for the CRT tranche spreads $ts^{[b,b']}$ from (24), averaging over the 5,000 sample paths to compute expectations. For each CRT bond/vintage we use the corresponding CE thresholds that trigger unscheduled principal paydowns.

Figure 7 summarizes our main results. It plots the observed tranche spreads (on the x-axis) against the model-implied spreads (on the y-axis), using different colors for the different types of tranches. Overall, our model delivers sensible credit spreads across tranches and vintages.

Figure 7: CRT Tranche Spreads: Model vs. Data



Notes: The figure shows the observed CTR tranches spreads at origination in the data for all 329 STACR tranches issued between the start of 2013 and the middle of 2025 on the x-axis and the corresponding model-implied tranche spreads $ts^{[b,b']}$ on the y-axis. Tranche spreads are expressed in basis points in annual yield terms. Junior tranches (in green) are defined as those with attachment points lower or equal to 1%.

Comparing the model-implied and observed spreads across all 329 CRT tranches in our sample, we find that the model-implied CRT spread is on average 30.5 basis points higher than the observed spreads. According to our model, the GSEs have paid CRT investors close to fairly.

We study the relationship between model-implied and observed spreads in Table 1. The dependent variable in each column is the difference between the spread in the model and the spread in the data. Column 1 includes an indicator variable for junior tranches ($Junior = 1$), where we define a junior tranche to be any tranche with an attachment point lower or equal to

1%.[21] The coefficient on *Junior* is 509.2 basis points, indicating that the model implies 321.5 bps (509.2-187.7) higher spreads for the riskiest tranches than what investors actually received for taking on the first-loss risk. In contrast, the model indicates 187.7 basis points lower spread than what was actually paid on the mezzanine CRT tranches. The average fair pricing hides interesting heterogeneity across tranches. According to our model, the GSEs did not compensate CRT investors nearly enough for the riskiest bonds, but over-compensated them for the safer mezzanine tranches. From the perspective of a CRT investor who takes compensation for interest rate, corporate credit, and housing market risk as given, the mezzanine CRT bonds are cheap (the spreads are too high, the prices too low) whereas the junior tranches are expensive (the spreads are too low). Figure 7 confirms the relative pricing of mezzanine (red dots, mostly below the 45-degree line) and junior tranches (green dots, mostly above the 45-degree line).

Column 2 of Table 1 incorporates the role of loan-to-value. The *High-LTV* dummy is one for all tranches that belong to a HA- or HQA-series deal, which refer to collateral pools of loans with LTV ratios above 80 percent. The coefficients on the *High-LTV* dummy and the *High-LTV* $\times$ *Junior* dummy are -17.8 and -79.2, and are not different from zero. The main conclusion is unaffected.

Table 1: Model-Minus-Observed CRT Spreads

	(1)	(2)	(3)	(4)	(5)	(6)
Intercept	-187.7***	-179.9***	-208.9***	-201.0***	-197.4***	-197.8***
Junior	509.2***	541.9***	80.3**	-413.2***	-418.2***	-376.5***
High LTV		-17.8	-8.1	-2.9	-3.2	-2.3
High LTV$\times$*Junior*		-79.2	-119.3**	-42.3	-42.6	-29.0
R^2 (%)	38.0	38.6	3.4	51.1	51.2	46.9
Observations	329	329	329	329	329	329
Average (bps)	30.5	30.5	-199.8	-387.0	-385.6	-365.4

Notes: The table reports regression models with the model-implied credit spread minus the observed credit spread as dependent variable. Column (1) contains an intercept and a *Junior* tranche dummy. Column (2) adds a *High LTV* deal dummy and the interaction of the *High LTV* deal and *Junior* tranche dummies. Column (1) and (2) are for the main model, estimated recursively. Column (3) is for the model estimated on the full sample. Columns (4)-(6) are for the recursive model model without housing premia, without housing and corporate credit risk premia, and without any risk premia, respectively. Statistical significance: $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

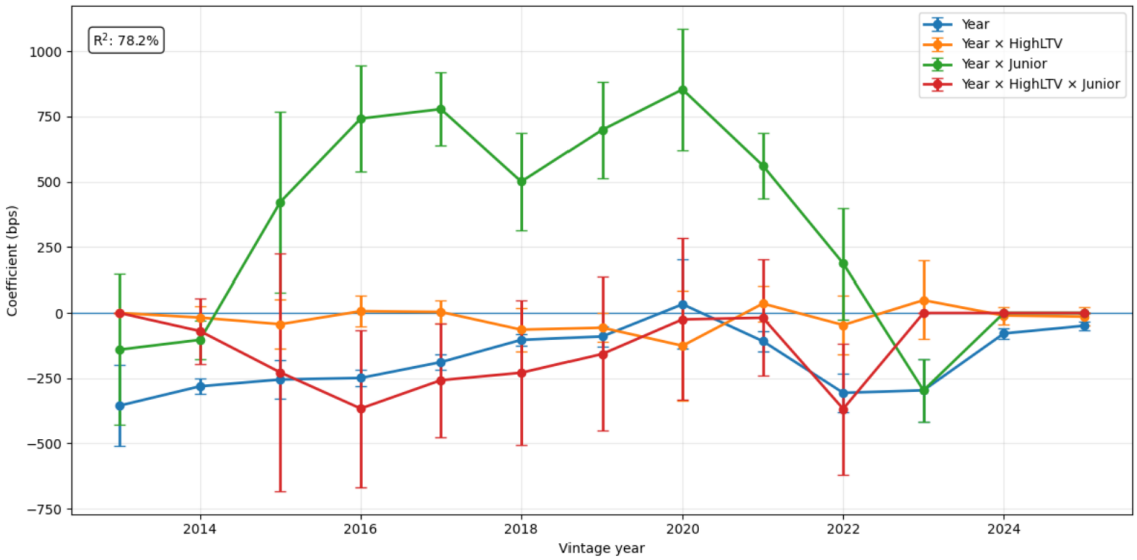
Next, we explore how the pricing gap varies by origination year. We visualize vintage-specific effects from our full specification that includes vintage year fixed effects and their interactions with high LTV and Junior dummies in Figure 8. This specification fits the data well ($R^2 = 78.2\%$), indicating that basic vintage, deal, and seniority characteristics go a long

[21] All B, B-1, and B-2 tranches are junior tranches, but so are some M-3 and M-2 tranches in earlier vintages.

way towards accounting for the cross-sectional variation in CRT mispricing. The blue line shows that mezzanine bonds in low-LTV deals paid spreads that were too high in nearly every year. Table A.4 reports the standard errors on the vintage effects.

The green line shows the overpricing of junior bonds for all vintages from 2015 to 2022. We attribute the initial generosity of the 2013–14 junior tranches to the cost of establishing a new market. Model uncertainty on the part of investors, maybe combined with the recent memories of the GFC, led CRT investors to demand and receive high spreads to take on what turned out to be minimal credit risk. After this initial "learning phase," the excess compensation for taking on junior credit risk reverses in 2015. The red line shows that the junior tranches in high-LTV deals are typically less overpriced than those in low-LTV deals. While high-LTV deals are more exposed to increases in housing risk premia, private mortgage insurance provides an extra layer of protection to CRT investors.[22]

Figure 8: Gap Between Model-Implied and Observed CRT Spreads



Notes: The figure plots the coefficients from a regression that has the difference between the model-implied and observed credit spread on each CRT bond, expressed in basis points, as its dependent variable and a series of fixed effects as its independent variables: vintage year (Year), Year × *High-LTV*, Year × *Junior*, and Year × *Junior* × *High-LTV*. The dots show the point estimates and the whiskers indicate ±1 standard error. There are no junior and/or high-LTV bonds in the last few vintages. The regression model's R^2 is 78.2%.

5.2.1 The Role of Risk Pricing

To better understand the role of risk pricing for our main results, we conduct several exercises. In a first exercise, we replace the recursive parameter estimation of the benchmark model with

[22]The 2013-2014 CRT bonds are issued under a fixed severity framework, when PMI is not relevant. The red line is zero for those two vintages, in those two years, consistent with the relevance of PMI for the post-2015 vintages.

a full-sample estimation of the VAR, market price of risk, and prepayment parameters. Column 3 of Table 1 shows the results. This model generates an average gap of -199.8 basis points across all bonds, compared to 30.5 basis points for the benchmark model with recursive estimation. It produces a similar gap between model and data spreads for mezzanine bonds but a smaller spread for junior bonds.[23] The means of house price and income growth under the $\mathbb{Q}$ measure are substantially lower in 2015–2020 for the benchmark than for the full-sample estimation because credit and housing risk premia are higher in the benchmark model estimation.[24] With lower house price and income growth under $\mathbb{Q}$, mortgage default risk is higher, and so are junior CRT tranche spreads. The substantial difference in pricing underscores the importance of only using information that was actually available to market participants in real time.

In a second exercise, we shut down the housing risk premium. We zero out market prices of risk associated with the aggregate house price growth shock, the sixth row Λ_t in (7). We revert to the benchmark model's assumption of recursively estimating parameters. Column 4 of Table 1 shows the results. This model predicts lower spreads for all tranches, with average spreads that are 387 basis points lower than in the data. The large positive gap between model and data for junior tranche spreads found in the benchmark model disappears and reverses sign. House price growth under $\mathbb{Q}$ is substantially negative, while it is substantially positive under $\mathbb{P}$. When the market price of house price risk is zeroed out, the risk-neutral house price dynamics are close to the physical dynamics. Since one of the conditions for default is for the homeowner to be underwater on the mortgage under the risk-neutral measure, default risk is much lower in the model where house price risk is not priced. In other words, the high model-implied credit spreads of the junior CRT tranches in the benchmark model crucially depend on the estimated market price of aggregate house price risk.

This result leads us to ask a related question: How much large does the housing risk premium need to be for the benchmark model to match the average CRT spreads on the mezzanine (junior) tranches? We find that a monthly housing risk premium of 0.746% (0.454%) results in a zero pricing error for mezzanine (junior) bonds; the housing risk premium in the main results is 0.596%.[25]

Third, we also shut down the pricing of corporate credit risk, zeroing out both the third and sixth rows of Λ_t. Interest rate risk is the only remaining priced risk. Column 5 of Table 1

[23]Column 2 of Table A.4 shows that the difference in the model spreads between the recursive and the full-sample is most pronounced for the junior tranches issued between 2015 and 2020 and nearly vanishes thereafter.

[24]The higher market prices of credit and house price risk result from average credit and housing returns coinciding with below-average quantities of risk in 2015–20.

[25]We engineer a different housing risk premium by changing the parameter nry. Increasing (decreasing) nry from 0.042/12 in the benchmark model to 0.060/12 (0.025/12) results in mezzanine (junior) tranche spreads that match the data. A higher nry results in a lower value for the market price of housing risk Λ_0^h.

shows that this produces similar credit spreads as Column 4 (-387.0 bps versus -385.6 bps).

Fourth, we shut down all risk prices, setting all rows of Λ_t to zero. This is a model of risk-neutral pricing. Column 6 of Table 1 shows the results. This model produces a further 20.2 bps reduction in average credit spreads. Compared to the model with interest rate risk, the model without interest rate risk features slightly lower income growth, which makes the junior tranches slightly less overpriced.

5.2.2 Discussion

How does the mispricing between junior and mezzanine tranches persist? In the data, hedge funds tend to buy the junior tranches while money managers and insurance companies tend to purchase the mezzanine tranches. Targeting high absolute returns, hedge funds may be unable to lever up the mezzanine tranches enough to manufacture the desired return forcing them into the junior tranches. Leverage constraints may prevent the equalization of Sharpe ratios between junior and mezzanine bonds. Frazzini and Pedersen (2014) document a similar puzzle in the equity markets: Investors who would like to lever up low-beta stocks cannot do so cheaply or sufficiently due to margin requirements, borrowing limits, or funding constraints. As a result, they tilt toward high-beta assets instead, pushing high-beta prices up (lowering expected returns) and leaving low-beta assets underpriced (raising expected returns). In corporate bond markets, Frazzini and Pedersen (2014) show that a leveraged portfolio of high-rated bonds outperforms a de-leveraged portfolio of low-rated bonds on a risk-adjusted basis, consistent with leverage-constrained investors overpaying for risky credit. For securitized corporate loans, practitioner studies report that lower-rated CLO tranches offer higher average returns but lower Sharpe ratios and higher market beta than senior tranches, implying that senior tranches deliver superior risk-adjusted performance (e.g. PGIM Fixed Income, 2021; PineBridge Investments, 2024). Evidence from historical average realized returns on an index of mezzanine CRT tranches from AD & Co. indeed show lower returns but higher Sharpe ratios than on the corresponding index of junior CRT tranches; see Table A.5.

Prepayment risk is another potential driver of the relative mispricing of junior and mezzanine tranches. The mezzanine tranches not only take on less credit risk exposure than the junior tranches, they also take it on for less time. Mezzanine bonds have an average duration of 3.9 years compared to 4.9 years for junior bonds, and the duration gap is time-varying (see Figure A.7). The market may not price the cross-sectional and time-series differences in the duration between junior and mezzanine tranches correctly. For example, erroneously ignoring the pro-rata allocation rule for principal payments would extend the duration of the mezzanine bonds to 5.2 years and increase their model-implied spread by 161 basis points.

Finally, we note that the GSEs stopped issuing junior tranches in mid-2023. The last two deals with B tranches paid a high credit spread to CRT investors, indeed a higher spread than what the model suggests. The GSEs may have correctly concluded not to issue junior bonds if CRT investors demanded more spread than warranted. However, if the reason for the policy change were instead that the GSEs incorrectly perceived the high spreads on junior bonds to always have been too large (for example, relative to *realized* losses over the past decade), then the stop-issuance decision would be harder to defend. The model suggests that the GSEs (the taxpayers) have been able to shed junior credit risk to the private sector on attractive terms in most years since 2015.

6 Pricing the Overall Mortgage Guarantee

Having developed a model for pricing mortgage credit risk, we now price the risk in the entire mortgage pool. We compute the guarantee fee g^{fee} produced by the model for each year using (25). The g-fee is a credit spread on an entire vintage year of mortgages, expressed relative to the one-month discount factor. The simulation approach is identical to that used for the CRT tranche pricing, except that we sample 5,000 loans/households from the standard loan origination file. For each mortgage loan, we use the actual first payment date to initialize the state vector. The advantage of our approach, as opposed to approaches that rely on extrapolation from CRT prices, is twofold. First, it does not require an assumption on how the tranche pricing would change were the volume of CRT issuance to increase to cover more (or all) of the mortgage originations. Second, it avoids making arbitrary assumptions on how the price of catastrophic risk (the senior A-H bond retained by the GSEs) relates to the price of CRT bonds.

Prior to the GFC, the g-fee was around 20 basis points (bps) per year. Following the mortgage market meltdown and large loan losses on the pre-GFC vintages, there was consensus that the GSEs had underpriced the risk, with adverse consequences for financial stability (e.g., Elenev et al., 2016). From 2009 to 2012, the average g-fee increased from 22 to 36 bps, and from 2012 to 2013 it jumped from 36 to 51 bps (Federal Housing Finance Agency, 2014). Table 2 shows the evolution of the g-fee in the CRT era. The g-fee increased further from 51 bps in 2013 to 65 bps in 2024. The total g-fee consists of a credit and a non-credit component. The non-credit component is 20 bps (Freddie Mac, 2017), consisting of 8 bps for administering the GSEs, 2 bps for the cost of securitizing loans, and 10 bps of general taxes levied by the Temporary Payroll Tax Cut Continuation Act of 2011. Our model-implied g-fee should be compared to the credit-risk component. In the data, the latter increased from 31 to 45 bps

between 2013 and 2024, and averages 38.7 bps across vintages.

Table 2: Guarantee Fees

Vintage	**Observed**		**Model-implied (bps)**			
	Total	Credit	Credit	EL	CRP	Tail
2013	51	31	24.82	4.97	19.84	11.42
2014	58	38	23.46	5.66	17.80	8.66
2015	59	39	30.27	4.04	26.23	13.84
2016	56	36	36.94	3.96	32.98	20.66
2017	53	33	26.11	5.90	20.21	11.28
2018	55	35	21.93	8.75	13.18	9.28
2019	56	36	31.30	6.68	24.62	16.44
2020	54	34	96.24	2.12	94.13	79.74
2021	56	36	18.80	1.52	17.28	5.31
2022	61	41	15.22	15.04	0.17	5.98
2023	66	46	16.18	27.19	-11.01	8.42
2024	65	45	14.54	21.90	-7.36	7.59
Average	58.71	38.71	28.78	11.05	17.73	16.65

Notes: The first column reports the observed average g-fee by vintage, obtained from the Federal Housing Financing Agency's annual reports to the Congress. It is the sum of the ongoing fee, which is the same for all borrowers, and the average among borrowers of the annual equivalent of upfront loan level price adjustments. The second column reports the credit-risk component (Credit), which subtracts 20 basis points for non-credit related charges from the g-fee. The third column is the model-implied g-fee (Credit). The fourth and fifth column break out the model-implied g-fee into an expected loss (EL) and a credit risk premium (CRP) component. The last column prices a senior tranche which absorbs losses above a 5% loss rate (Tail). The last row weights the costs by the UPB amount of Freddie Mac's entire guarantee portfolio, obtained from its annual 10K reports.

Our model-implied g-fee is reported in the third column of Table 2. Weighted by origination volume, the average g-fee implied by the model is 28.8 bps, or 9.9 bps lower than what the GSEs have been charging since 2013. The 9.9 bps average difference hides some interesting time-series variation. According to our model, the GSEs overcharged for credit risk in all years except for 2016 and especially 2020. In the last four years, the gap between the observed and fairly priced g-fee widened from 17 to 30 bps. In contrast, the GSEs undercharged by over 60 basis points in 2020. The GSEs set guarantee fees that are far less cyclical than what period-by-period fair pricing prescribes. Put differently, the GSEs stabilize mortgage borrowing costs during crises like 2020, making up for it during normal times.

6.1 Decomposing the Guarantee Fee

The fourth and fifth columns of Table 2 decompose the model-implied g-fee into an expected loss (EL) and a credit risk premium (CRP) component. The expected loss is the g-fee that would be set by a risk-neutral insurer, or equivalently the guarantee fee risk neutral investors would accept to take over all the credit risk from the GSEs. The credit risk premium is the extra compensation a risk-averse investor/insurer demands for the fact that mortgage losses go up in bad times. The model's CRP varies both because the quantity of risk fluctuates across vintages and because the market prices of risk vary over time. The weighted average EL across vintages is 11 bps (38% of the g-fee), but ranges from 1.5 bps in 2021 to 27.2 bps in 2023. The CRP is 17.7 bps on average (62% of the g-fee), ranging from 94 basis points in 2020 to -11 bps in 2023. The negative credit risk premium in 2023 and 2024 is a result of negative credit and housing risk premia in 2023 and 2024, shows in Figure 2. The low EL in 2020–21 are noteworthy, given that they occur when the CRP is high. The main reason for the low EL and the high CRP in 2020–21 is fast house price appreciation. On the one hand, high HPA lowers the probability of loss, conditional on default as the underlying collateral has appreciated strongly. This force lowers the EL. On the other hand, the housing risk premium is high in this period of significant macroeconomic risk, which induces a large gap between the expected HPA under the $\mathbb{P}$ and $\mathbb{Q}$ measures and a high CRP.

6.2 What to Charge for Tail Risk?

The last column of Table 2 reports the fair risk premium for bearing catastrophic loss risk. For consistency across vintages, we define catastrophic losses to be the losses in excess of a 5% loss rate. This corresponds approximately to the risk that the GSEs retain in the form of the senior A-H tranche. This tail risk component is hard to infer from CRT prices, i.e., hard to compute without a complete default, prepayment, and interest rate risk model. We find that fair compensation for tail risk exposure is 16.7 bps on average across years, ranging from 5 bps in 2021 to 80 bps in 2020. Hence, catastrophic loss risk exposure that is retained by the GSEs accounts for about half of the overall risk compensation (58% of the average g-fee). This is a surprising result since losses rarely exceed 5%. However, the states of the world in which such large losses do occur are particularly painful, hence the large "disaster risk premium." In a future privatization of the GSEs, where the government remains ultimately on the hook for catastrophic loss risk, and may seek ex-ante compensation for this (explicit or implicit) guarantee, this calculation should be informative. Another implication is that, under current payment rules, CRT bonds lose much of the loss absorption in the few years after issuance.

Since tail loss events take time to materialize, this leaves taxpayers unprotected in severely adverse scenarios.

7 The Cross-Section of Mortgage Risk

Having developed a rich model of mortgage pricing, our last exercise uses the model to study the cross-section of mortgage credit risk.

Loan-level price adjustments (LLPAs) were introduced by the GSEs in April 2008 in response to the financial crisis. LLPAs are an upfront fee charged on conforming mortgages based on a borrower's risk profile. The goal of the LLPAs was to mitigate risk and increase capital for the GSEs. Between April 2008 and April 2023, LLPAs underwent several incremental adjustments.[26] The May 2023 LLPA reform was more substantial. Figure 9 shows borrowers whose LLPAs increased after May 2023 in warmer colors (red), and those that pay lower LLPAs in cooler colors (green). Figure A.8 shows the post-reform LLPA grid. The reform lowered LLPAs for low-FICO (high-credit risk) borrowers, especially those with moderate to high LTV ratios, and increased fees for high-FICO, moderate-LTV borrowers (Federal Housing Finance Agency). It emphasized equity and affordability, drawing political controversy. We use our model to ask the positive question whether the reform brought the pricing closer to or farther from actuarially fair, risk-based pricing, leaving aside the normative question of how much redistribution through the LLPA grid is socially desirable.

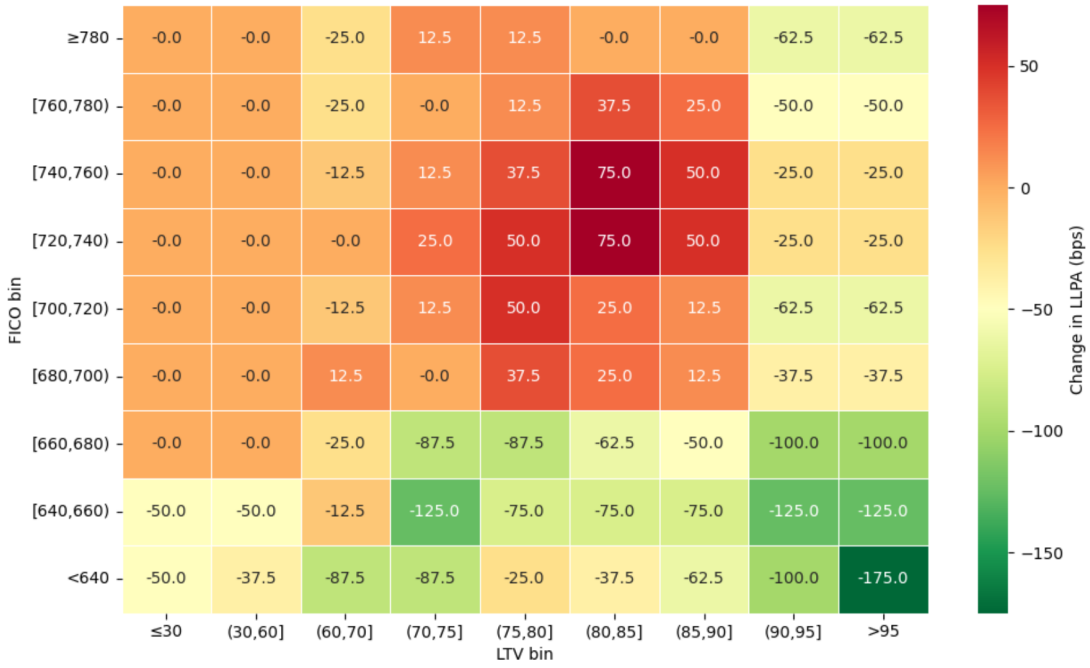
7.1 LLPAs in the Model

To calculate LLPAs in the model, we sample 30,000 loans from the 2024 standard loan file. We group them into the same $(LTV, FICO)$ buckets that are used in the LLPA grid. The sample composition of the 2024 vintage is summarized in Table A.9. Because some buckets are sparsely populated in the data, for pricing accuracy purposes, we ensure that each bucket contains at least 1,000 loans by sampling with replacement when necessary.

We begin by solving for the actuarially fair credit spread for a specific FICO $\times$ LTV group,

[26] An adverse market delivery charge of 0.25% was introduced in late 2008 to reflect a period of heightened credit risk. It was lifted in 2015. Between 2009 and 2011, the GSEs introduced more detailed credit score and LTV buckets, higher pricing for lower-credit and high-LTV borrowers. They also introduced new LLPAs for investment properties, cash-out refinances, condominiums, and multiple-unit properties. LLPAs were increased substantially in 2013–2014, including hikes for middle-credit borrowers (FICO scores between 680 and 740) and for moderate LTV ranges (LTV between 75% and 85%). These increases were reversed in 2015. In January 2022, the FHFA increased LLPAs on high-balance loans and second homes.

Figure 9: Loan Level Price Adjustments: May 2023 Reform



Notes: The graph displays the change in Loan-Level Price Adjustments (LLPA) by credit score (FICO) and loan-to-value (LTV) group. Warm colors (orange/red) indicate higher fees under the new grid while cooler colors (yellow/green) indicate lower fees. Source: Federal Housing Finance Agency

$cs^{[F,L]}$, by applying the asset pricing formula (24):

$$\mathbb{E}_0^{\mathbb{Q}}\left[\sum_{s=1}^{T}\exp\left(-\sum_{t=1}^{s}\frac{y_t(1)}{12}\right)\left(\frac{cs^{(F,L)}+y_s(1)}{12}V_{s-1}^{[F,L]}+(F_s^{[F,L]}-F_{s-1}^{[F,L]})\right)\right]=1, \tag{26}$$

where the terminal value is zero because the mortgages have fully amortized by time T. We simulate 5,000 sample paths of the state variables to compute the expectation.[27]

Next, we subtract the model-implied g-fee, which represents the UPB weighted-average spread among all $(FICO, LTV)$ groups. Denote the relative credit spread by:

$$g^{(F,L)} = cs^{(F,L)} - g^{fee}.$$

Finally, we turn the relative credit spread $g^{(F,L)}$ from an annual credit spread into an upfront amount, expressed as a share of the origination UPB. The LLPA is the present discounted value

[27]We use the same 5,000 draws of the state vector in each vintage and for each (FICO, LTV) group. The draws are the same as those we used in the pricing of CRT tranches and to obtain g-fees.

of the credit spread over the life of the loan:

$$LLPA^{(F,L)} = \frac{\sum_{i\in(F,L)} \sum_{s=1}^{T} \exp\left(-\sum_{t=1}^{s} \frac{y_t(1)}{12}\right) \frac{g^{(F,L)}}{12} N_s^i}{\sum_{i\in(F,L)} N_0^i}, \tag{27}$$

where N_s^i denotes the outstanding balance at time s for loan i. To compute how this balance evolves, we consider both regular principal amortization and a fixed prepayment rate of 1.5% per month.[28] Our approach generates $g^{(F,L)} < 0$ and hence $LLPA^{(F,L)} < 0$ for buckets whose credit risk is below the vintage (UPB-weighted) average, and vice versa for above-average credit risk buckets.

Figure 10 shows model-implied LLPAs for the 2024 vintage. Warm colors (red cells) denote FICO×LTV groups who have higher than average credit risk under actuarially fair pricing, denoted by positive numbers. Cells in green denote low-risk borrowers whose LLPAs are below average, denoted by negative numbers. LLPAs generally increase when reading from the top-left to the bottom-right of the graph.[29] Our model implies that the riskiest borrowers face LLPAs that are 113 bps higher than those charged to the safest borrowers. The observed, post-2023 LLPA grid features more than twice as much dispersion, with a 288 bps spread between the riskiest and safest borrowers. The LLPA spread compression is present along both the FICO and LTV dimensions. Holding the $(75, 80]$ LTV bucket fixed, the spread in LLPAs between the highest and lowest FICO bins is 69 bps in the model but 237 bps in the post-2023 data. Conversely, holding the $[700, 720)$ FICO bucket fixed, the LTV slope is 71 bps in the model but 150 bps in the data. The model suggests excessive differentiation in LLPAs.

7.2 Assessing the 2023 LLPA Reform

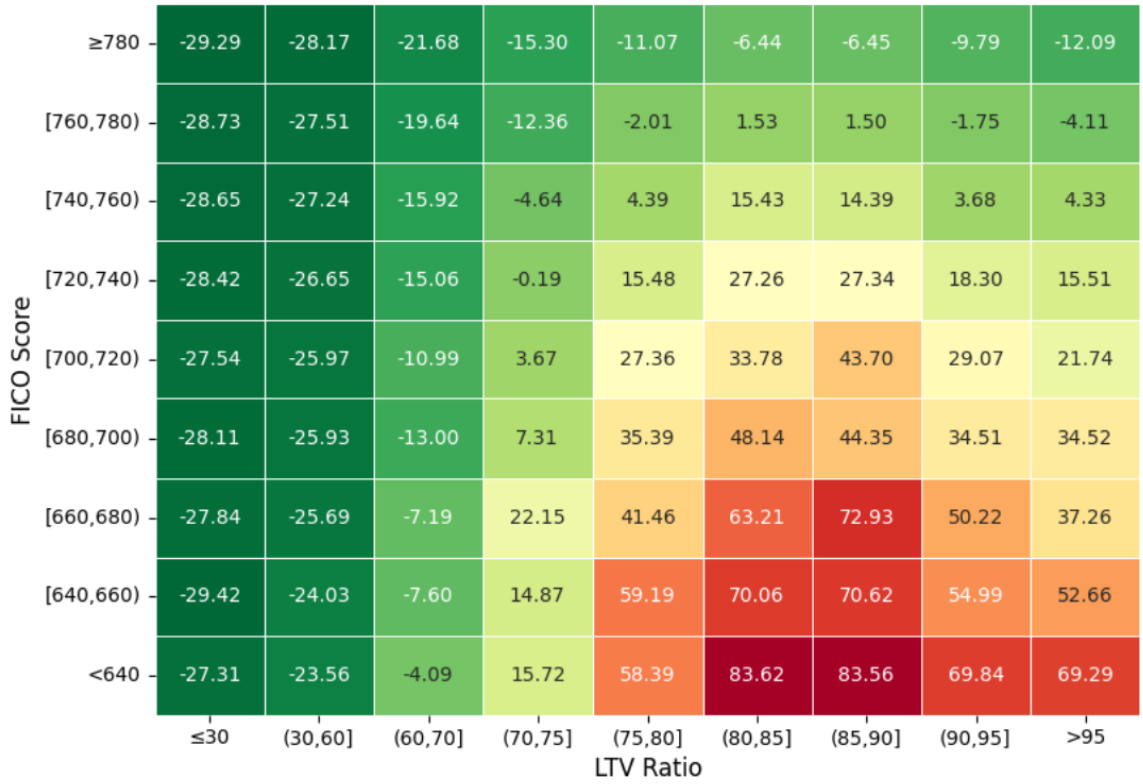
Finally, we ask whether the May 2023 reform brought the LLPA grid closer to the actuarially fair grid. To put model and data on the same footing, we first rescale the LLPAs in the data so that they average to zero on an UPB-weighted average basis.[30] We demean LLPAs in the data

[28]The 1.5% SMM corresponds to a CPR of 16.6%, close to the historical average annualized prepayment rate. We could use our prepayment model to come up with a vintage-specific prepayment rate that differs for each (FICO, LTV) group. However, this may obscure the pricing of credit risk. In reality, the LLPAs do not change when mortgage interest rate levels increase, which in the model and in the data, increases the risk of prepayment.

[29]The careful reader may notice some non-monotonicities in the pricing grid shown in Figure 10. These arise from: (i) the presence of PMI that kicks in and absorbs some of the losses above 80% LTV but not below, (ii) heterogeneous prepayment rates across groups, (iii) heterogeneous debt-to-income ratios across groups, and (iv) heterogeneity in mortgage interest rates.

[30]The LLPAs in the data are weakly positive for each bucket, and hence strictly positive on average. The baseline g-fee pertains to the lowest-risk group. For example, for the 2024 vintage, the baseline g-fee was 28 basis points (48 basis points minus 20 basis points for the non-credit risk related components) and the average

Figure 10: Model-Implied Loan-Level Price Adjustments



Notes: The graph shows model-implied loan-level price adjustments. As explained in the text, these are computed from annual credit spreads in each (FICO,LTV) group relative to the UPB-weighted average credit spread among all loans. These credit spreads are then converted into an upfront fee, expressed as a percent of UPB at origination. These model-implied LLPAs average to zero across the entire matrix, when weighted by the UPB in each cell.

separately for the 2023 (pre-reform) LLPA grid and for the (post-reform) 2024 grid, using the 2023 UPB by group and 2024 UPB by group for the respective weights. We then compute the UPB-weighted root mean squared error (RMSE) between the demeaned LLPAs in model and data across FICO×LTV groups g:

$$RMSE_t = \left(\sum_g w_t^g \left(LLPA_t^{g,data} - LLPA_t^{g,model} \right)^2 \right)^{0.5}$$

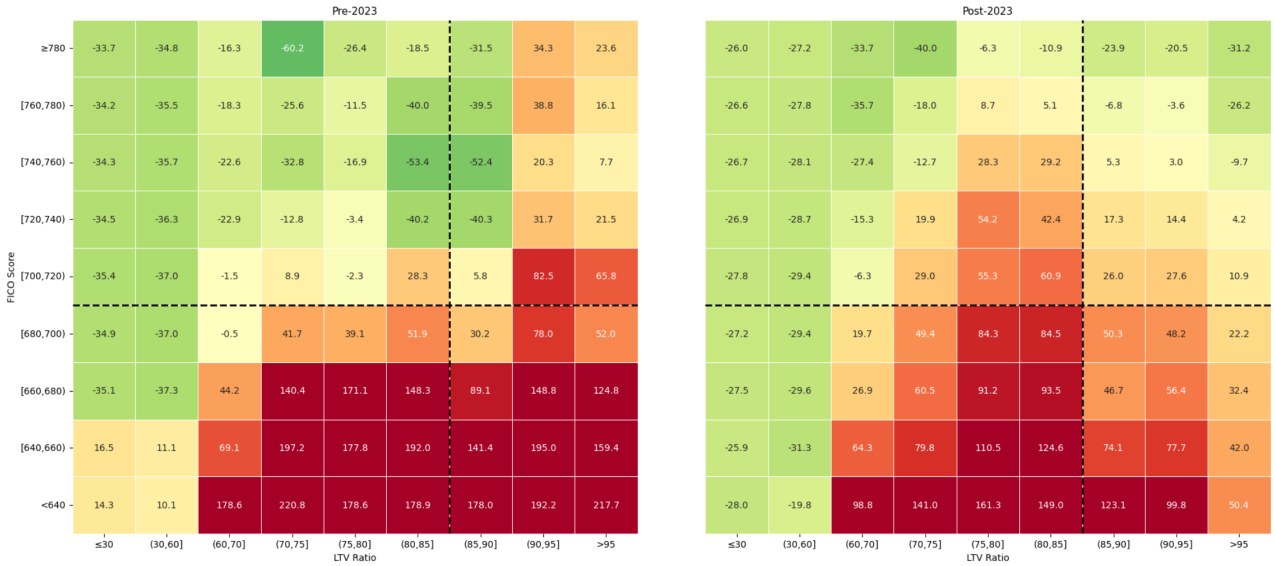
Note how this calculation is agnostic as to whether the average charge for credit risk is too high or too low. It is a purely cross-sectional calculation of the FICO×LTV credit risk surface. We find an overall weighted RMSE of 49.4 bps in the pre-reform schedule and 32.9 bps in the post-reform schedule. The May 2023 reform moved the cross-sectional pricing of mortgage

LLPA added 17 basis points when expressed as an annual spread, for an average g-fee of 45 basis points charged for credit risk in 2024. Table A.6 shows how the g-fees and LLPAs are structured in the data.

credit risk substantially closer to the risk-based pricing implied by our model.

This average improvement masks substantial heterogeneity across the LLPA matrix. To examine this heterogeneity, we compute UPB-weighted RMSEs separately within four quadrants of the LLPA grid, splitting at a FICO score of 720 and an LTV ratio of 85%, and re-normalizing UPB weights to sum to one within each quadrant. The largest improvement occurs in the low-FICO/high-LTV quadrant, where the RMSE declines from 99.1 bps to 45.4 bps. The RMSE also declines in the low-FICO/low-LTV quadrant, from 83.1 bps to 63.7 bps. The gains are more modest in the high-FICO/low-LTV quadrant, where the RMSE declines from 30.6 bps to 26.4 bps, and in the high-FICO/high-LTV quadrant, where the RMSE goes from 33.4 bps to 16.0 bps. Overall, the reform reduces pricing errors across all four regions of the LLPA matrix, with the largest improvements concentrated among riskier borrower segments.

Figure 11: Relative Risk Pricing: Data vs. Model Pre- vs. Post-Reform



Notes: The left panel plots the difference between the LLPAs in the Pre-Reform 2023 data and the LLPAs in the model, while the right panel plots the difference between the LLPAs in the Post-Reform 2024 data and the LLPAs in the (same) model. LLPAs in the data are demeaned by the UPB-weighted average of the LLPAs across all $(FICO, LTV)$ groups where the weights are based on the UPBs in the 2023 vintage in the left panel and the 2024 vintage in the right panel. Warm (red) colors indicate that LLPAs are higher in data than in model, the risk is over-priced, while cool colors (green) indicate that the risk is under-priced.

Figure 11 shows the difference between LLPAs in the data and the model in the pre-reform period in the left panel, and the same difference in the post-reform period in the right panel. Red colors indicate cells in which relative risk is priced higher in the data than in the model, implying that these borrowers are overcharged by the GSEs according to our model. Green colors indicate cells in which relative risk is underpriced in the data. The figure shows that LLPAs for low-FICO and high-LTV borrowers were systematically too high prior to the reform, especially in the low-FICO range (below 680). The May 2023 reform substantially reduced the overpricing, particularly for low-FICO borrowers with high LTV ratios. For example, the riskiest

group of borrowers with FICO scores below 640 and LTV ratios above 95% were overcharged by 217.7 bps in 2023 but only by 50.4 bps in 2024. The reform did not go far enough for most low-FICO borrowers. On the other hand, a group of middle-FICO, middle-LTV borrowers, who were fairly charged pre-reform, became overcharged post-reform. For example, borrowers with FICO scores between 720 and 740 and LTV ratios between 80% and 85% went from being undercharged 40.2 bps to being overcharged 42.4 bps.[31]

In sum, the May 2023 reform substantially reduced reverse cross-subsidization along the FICO dimension, while leaving substantial deviations from actuarially fair pricing and introducing some new overcharging of middle-FICO, middle-LTV borrowers. How this reform affects welfare is beyond the scope of this paper but would depend on the welfare weights the social planner accords to each credit-risk group. Most models of incomplete risk sharing would have the social planner prescribe a subsidy for high marginal-utility (presumably low-FICO, high-LTV) borrowers, paid for by low marginal-utility (high-FICO, low-LTV) borrowers. Richer models would also consider the negative externalities from high leverage and mortgage defaults on he financial system (e.g., Elenev et al., 2016).

7.3 Can the Model Generate Steeper FICO and LTV Slopes?

We noted that the model's LLPA spreads are compressed compared to the data. The benchmark model generates a LLPA spread of 69.5 bps between the lowest- and highest-FICO bins, holding fixed the LTV at $(75, 80]$. Is the model capable of generating the observed, much larger, 237 bps spread for the FICO dimension under different assumptions? Yes. Appendix A.13 shows that giving low-FICO (high-FICO) borrowers a much lower (higher) initial asset-to-income ratio than in the baseline model can increase their relative exposure to aggregate income risk enough to reproduce the observed FICO slope in LLPAs. However, this model generates loss rates that are far too high relative to the historical sample. In sum, the mortgage loss data discipline the model's parameters in ways that make the high FICO slope in LLPAs in the data hard to reconcile with the model. The benchmark model generates a LLPA spread of 71.2 bps between the lowest- and highest-LTV bins, holding fixed the FICO at $[700, 720)$. Is the model capable of generating the observed, much larger, 150.0 bps spread for the LTV dimension under different assumptions? Yes. Appendix A.13 shows that this requires increasing the housing risk premium, which produces more mortgage default risk through the collateral channel, and more so for high-LTV borrowers. However, it requires a counter-factually high net rental yield of

[31] It is important to recognize that borrowers with LTV ratios above 80% are typically required to purchase private mortgage insurance. These premiums represent an additional cost of credit for high-LTV households in addition to the LLPAs and the baseline guarantee fee. Those borrowers with LTVs above 80% who are overcharged in our calculation become even more overcharged once the PMI is taken into account.

6.2%, far above the observed value of 4.2% found in Eisfeldt and Demers (2015).

How come that Kim et al. (2024) find that the FICO slope in observed LLPAs is not steep enough, while we find that it is too steep? The difference in results is driven by three key different modeling assumptions. First, our asset pricing model features a stochastic discount factor driven by aggregate credit, interest rate, and housing risk factors that affect the pricing of LLPAs, which is–as usual–done under the risk-neutral measure. Their model features a constant discount rate and they price LLPAs under the physical measure. Second, their pricing exercise applies the historical default probabilities by FICO×LTV to the current period. Our model generates the same default rates as in the data *along the historical path of state variables*. This path, which included the GFC, was quite unusual. The model implies lower default probabilities along most sample paths, and hence on average, going forward. Third, they assume a single loan in each FICO×LTV bucket, which amounts to an assumption of perfect correlation of default risk across loans within each bucket. Our pricing approach considers 1,000 loans in each bucket and recognizes that there is substantial scope for diversification of idiosyncratic risk across the loans within a bucket. Indeed, if we assume that there is a single loan in each bucket, our model also generates a much larger FICO slope.

8 Conclusion

This paper provides the most comprehensive asset pricing analysis to date of mortgage credit risk after the Great Financial Crisis. The post-GFC era saw the introduction of a large new asset class: Credit Risk Transfer bonds. These instruments have allowed the Government Sponsored Enterprises, Freddie Mac and Fannie Mae, to lay off residential mortgage credit risk to private investors. We analyze all 12.5 years of the CRT market's pricing history. Contrary to received wisdom, which often fails to distinguish between ex-post and ex-ante default risk, we find that the GSEs have paid the CRT investors compensation for taking over the credit risk that was fair on average, consistent with the pricing of risk in treasury and corporate bond as well as housing markets. This fair pricing hides interesting variation across vintages and between junior and mezzanine CRT bonds. Junior tranche spreads were mostly too low, while mezzanine spreads were uniformly too high. CRT investor leverage constraints are one candidate explanation for the relative tranche mispricing. The complex ways in which mortgage prepayment affects CRT tranches with differential duration of credit risk exposure is another possibility. More research would be helpful.

We use the model to compute the fair guarantee fee, the cost of insuring all conforming mortgages in the U.S. The g-fee has increased substantially in the data post-GFC. We find

that the GSEs have charged a g-fee that has been about 10 basis points too high on average. The GSEs subsidize mortgage insurance during crises, like the covid-19 pandemic, making up for it in normal times. About half of the model-implied g-fee is compensation for catastrophic losses, losses the GSEs do not transfer to the private sector. The g-fee calculation is relevant in the context of proposals to re-privatize the GSEs that preserve the government guarantee on agency-backed mortgage-backed securities.

Finally, we use our model to study the relative pricing of credit risk in the cross-section of households. We find evidence for substantial reverse cross-subsidization flowing from low-FICO to high-FICO borrowers before the reform. The loan-level price adjustment reform of May 2023 reduced this cross-subsidization substantially, but did not go far enough. Middle-LTV, middle-FICO borrowers were the losers from this reform.

An important direction for future research is to evaluate whether refinements to the CRT program's design could strengthen the transparency and effectiveness of mortgage credit risk transfer to private investors, in light of institutional constraints including repurchase practices, regulatory capital requirements, and guarantee-fee design.

References

V. Acharya, S. Kon, S. Oncu, M. Richardson, S. Van Nieuwerburgh, and L. White. The government sponsored enterprises. In V. Acharya, T. Cooley, M. Richardson, and I. Walter, editors, *Regulating Wall Street*. John Wiley and Sons, Sept. 2010.

V. Acharya, M. Richardson, S. Van Nieuwerburgh, and L. White. *Guaranteed to Fail: Freddie, Fannie, and the Debacle of U.S. Mortgage Finance*. Princeton University Press, Mar. 2011.

V. Acharya, M. Richardson, S. Van Nieuwerburgh, and L. White. Guaranteed to fail: Fannie mae and freddie mac and what to do about them. *Economist Voice*, 10(1):15–19, 2013.

T. Adrian, E. Etula, and T. Muir. Financial intermediaries and the cross-section of asset returns. *Journal of Finance*, 69(6):2557–2596, 2014.

A. Ang and M. Piazzesi. A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables. *Journal of Monetary Economics*, 50(4):745–787, 2003.

J. Bosshardt, A. Kakhbod, and A. Kermani. Do intermediaries improve GSE lending? evidence from proprietary GSE data. *Journal of Financial Economics*, 170:104082, August 2025. ISSN 0304-405X. doi: 10.1016/j.jfineco.2025.104082.

N. Boyarchenko, A. Fuster, and D. O. Lucca. Understanding mortgage spreads. *Review of Financial Studies*, 32(10):3799–3850, October 2019. doi: 10.1093/rfs/hhz004. URL `https://doi.org/10.1093/rfs/hhz004`.

J. Y. Campbell and J. F. Cocco. A model of mortgage default. *Journal of Finance*, 70(4): 1495–1554, 2015.

A. Capponi, R. Jia, and D. Rios. The effect of mortgage forbearance on refinancing: Evidence from the covid-19 period. Technical Report 68, 2021.

A. Carriero, T. E. Clark, and M. Marcellino. Vars, common factors, and the empirical implications of covid-19. *The Review of Economics and Statistics*, 103(5):1009–1024, 2021. doi: 10.1162/rest_a_00958.

M. Chernov, B. R. Dunn, and F. A. Longstaff. Macroeconomic-driven prepayment risk and the valuation of mortgage-backed securities. *Review of Financial Studies*, 31(3):1132–1183, 2018.

M. Davis and S. Van Nieuwerburgh. Housing, finance, and the macro-economy. In G. Duranton, J. V. Henderson, and W. C. Strange, editors, *Handbook of Regional and Urban Economics*, chapter 12, pages 735–811. Elsevier, 2015.

Y. Demyanyk and O. Van Hemert. Understanding the subprime mortgage crisis. *Review of Financial Studies*, 24(6):1848–1880, 2011.

Y. Deng, J. M. Quigley, and R. Van Order. Mortgage terminations, heterogeneity, and the exercise of mortgage options. *Econometrica*, 68(2):275–307, 2000.

P. Diep, A. L. Eisfeldt, and S. Richardson. The cross section of mbs returns. *Journal of Finance*, 76(5):2093–2151, October 2021.

J. Duarte. The causal effect of mortgage refinancing on interest rate volatility: Empirical evidence and theoretical implications. *The Review of Financial Studies*, 21(4):1689–1731, July 2008. doi: 10.1093/rfs/hhm062.

A. Eisfeldt and A. Demers. Rental yields and hpa: The returns to single family rentals. NBER Working Paper 21804, National Bureau of Economic Research, Cambridge, MA, 12 2015. URL `http://www.nber.org/papers/w21804`.

V. Elenev, T. Landvoigt, and S. Van Nieuwerburgh. Phasing out the gses. *Journal of Monetary Economics*, 81:111–132, Aug. 2016.

J. Favilukis, S. C. Ludvigson, and S. Van Nieuwerburgh. The macroeconomic effects of housing wealth, housing finance, and limited risk sharing in general equilibrium. *Journal of Political Economy*, 125(1):140–223, 2017. doi: 10.1086/689606. URL `https://doi.org/10.1086/689606`.

Federal Housing Finance Agency. FHFA Announces Updates to the Enterprises' Single-Family Pricing Framework. `https://www.fhfa.gov/news/news-release/fhfa-announces-updates-to-the-enterprises-single-family-pricing-framework`. News Release.

Federal Housing Finance Agency. Fannie mae and freddie mac single-family guarantee fees in 2013. Technical report, Federal Housing Finance Agency, 2014. URL `https://www.fhfa.gov/reports/single-family-guarantee-fees/2013`. Accessed: 2025-07-13.

Federal Housing Finance Agency. Performance of fannie mae's and freddie mac's single-family credit risk transfer. Technical report, Federal Housing Finance Agency (FHFA), May 2021. URL https://www.fhfa.gov/document/performance-of-fannie-maes-and-freddie-macs-single-family-credit-risk-transfer. FHFA Research Paper.

D. Finkelstein, A. Strzodka, and J. Vickery. Credit risk transfer and de facto gse reform. *Economic Policy Review*, 24(3):88–116, 2018.

T. Flanagan. Do gses subsidize mortgage lending? Working Paper 2025-03-011, Fisher College of Business, 2025.

C. L. Foote, K. Gerardi, L. Goette, and P. S. Willen. Reducing foreclosures: No easy answers. Technical Report 15063, National Bureau of Economic Research, 2009.

W. Frame, A. Fuster, J. Tracy, and J. Vickery. The rescue of fannie mae and freddie mac. *Journal of Economic Perspectives*, 29(2):25—52, 2015.

A. Frazzini and L. H. Pedersen. Betting against beta. *Journal of Financial Economics*, 111(1): 1–25, 2014.

Freddie Mac. What credit risk transfer tells us about guarantee fees: A single-family white paper. White paper, Freddie Mac, July 2017. URL https://www.freddiemac.com/fmac-resources/perspectives/pdf/CRT-GFee-White-Paper-Final.pdf.

A. Fuster and J. Vickery. Securitization and the fixed-rate mortgage. *Review of Financial Studies*, 28(1):176–211, 2015.

A. Fuster and P. S. Willen. Payment size, negative equity, and mortgage default. *American Economic Journal: Economic Policy*, 9(4):167–191, November 2017. doi: 10.1257/pol.20150237. URL https://www.aeaweb.org/articles?id=10.1257/pol.20150237.

A. Fuster, S. H. Lo, and P. S. Willen. The time-varying price of financial intermediation in the mortgage market. *The Journal of Finance*, 79(4):2553–2602, 2024. doi: 10.1111/jofi.13358. URL https://onlinelibrary.wiley.com/doi/10.1111/jofi.13358.

X. Gabaix, A. Krishnamurthy, and O. Vigneron. Limits of arbitrage: Theory and evidence from the mortgage-backed securities market. *Journal of Finance*, 62(2):557–595, 2007.

P. Ganong and P. Noel. Liquidity versus wealth in household debt obligations: Evidence from housing policy in the great recession. *American Economic Review*, 110(10):3100–3138, 2020a. doi: 10.1257/aer.20181243. URL https://doi.org/10.1257/aer.20181243.

P. Ganong and P. J. Noel. Why do borrowers default on mortgages? NBER Working Paper 27585, National Bureau of Economic Research, July 2020b. URL https://www.nber.org/papers/w27585. Revised August 2022.

K. Gerardi, A. Lehnert, S. M. Sherlund, and P. S. Willen. Making sense of the subprime crisis. *Brookings Papers on Economic Activity*, 2008(2):69–159, 2008.

K. Gerardi, E. Rosenblatt, P. Willen, and V. Yao. Foreclosure externalities: New evidence. Technical Report Working Paper No. 15-24, Federal Reserve Bank of Boston, 2015. URL https://www.bostonfed.org/publications/research-department-working-paper/2015/foreclosure-externalities-new-evidence.aspx.

K. S. Gerardi, K. F. Herkenhoff, L. E. Ohanian, and P. S. Willen. Can't pay or won't pay? unemployment, negative equity, and strategic default. *The Review of Financial Studies*, 31 (3):1098–1139, 2018. doi: 10.1093/rfs/hhx124.

P. Gete, A. Pavlov, A. Tsouderou, and S. Wachter. Forbearance and the cost of credit. Technical report, 2025. URL https://papers.ssrn.com/sol3/papers.cfm?abstract_id=5241320#.

E. Golding and D. Lucas. Credit risk transfer and the pricing of mortgage default risk. Working paper, MIT Golub Center for Finance and Policy, October 2020. URL https://mitsloan.mit.edu/shared/ods/documents?PublicationDocumentID=9586. Preliminary draft analyzing CRT bond pricing using Vista Data Services.

A. Gupta and S. Van Nieuwerburgh. Valuing private equity investments strip by strip. *Journal of Finance*, 76(5):2009–2050, 2021.

D. Gupta. Too much skin-in-the-game?: The effect of mortgage market concentration on credit and house prices. *Review of Financial Studies*, 35(2):814–865, 2022. doi: 10.1093/rfs/hhab027.

F. Guvenen, F. Karahan, S. Ozkan, and J. Song. Lifetime incomes in the united states over six decades. *American Economic Review*, 109(5):1488–1541, 2019. doi: 10.1257/aer.20181586.

J. Gyourko and J. Tracy. Reconciling theory and empirics on the role of unemployment in mortgage default. *Journal of Urban Economics*, 80:87–96, March 2014. doi: 10.1016/j.jue.2014.01.003.

S. Hanson, D. Scharfstein, and A. Sunderam. Social risk, fiscal risk, and the portfolio of government programs. *Review of Financial Studies*, 32(6):2341–2382, 2019. doi: 10.1111/jofi.13358. URL https://onlinelibrary.wiley.com/doi/10.1111/jofi.13358.

Z. He, B. Kelly, and A. Manela. Intermediary asset pricing: New evidence from many asset classes. *Journal of Financial Economics*, 126(1):1–35, 2017.

Y. Kim, F. Liu, and D. Zhang. The distributional effects of gse pricing on home purchases. Technical report, 2024. URL https://papers.ssrn.com/sol3/papers.cfm?abstract_id=5222560.

R. S. J. Koijen, H. K. Lee, and S. Van Nieuwerburgh. Aggregate lapsation risk. *Journal of Financial Economics*, 155:103819, 2024.

M. Lenza and G. E. Primiceri. How to estimate a var after march 2020. *Journal of Monetary Economics*, 135:1–19, 2022. doi: 10.1016/j.jmoneco.2022.03.005.

F. A. Longstaff and A. Rajan. An empirical analysis of the pricing of collateralized debt obligations. *The Journal of Finance*, 63(2):529–563, 2008. doi: 10.1111/j.1540-6261.2008.01330.x. URL https://doi.org/10.1111/j.1540-6261.2008.01330.x.

H. Lustig, S. Van Nieuwerburgh, and A. Verdelhan. The wealth-consumption ratio. *Review of Asset Pricing Studies*, 3(1):38–94, 2013.

D. McGowan and H. Nguyen. To securitize or to price credit risk? *Journal of Financial and Quantitative Analysis*, 58(1):289–323, Feb. 2023.

A. Mian and A. Sufi. The consequences of mortgage credit expansion: Evidence from the u.s. mortgage default crisis. *Quarterly Journal of Economics*, 124(4):1449–1496, 2009. doi: 10.1162/qjec.2009.124.4.1449. URL https://doi.org/10.1162/qjec.2009.124.4.1449.

J. E. O'Neill. The market view of mortgage credit risk. Working paper, UCLA Anderson School of Management, June 2022. URL https://ssrn.com/abstract=4152078. Builds a pricing model using TRACE and hand-collected CRT issuance data; analyzes implied guaranty-fees and portfolio credit risk.

PGIM Fixed Income. The compelling case for global senior clos. White paper, PGIM Fixed Income, 2021. URL https://cdn.pficdn.com/cms/pgim-fixed-income/sites/default/files/PGIM-Fixed-Income-Compelling-Case-for-Global-Senior-CLOs-FInal.pdf.

PineBridge Investments. Clo primer: An introduction to collateralized loan obligations, 2024. URL https://assets.pinebridge.com/m/21d36a807f12b5fb/original/CLO-Primer-Nov-2024.pdf. Investor primer discussing CLO structure, performance, and risk/return.

G. E. Primiceri and A. Tambalotti. Macroeconomic forecasting in the time of covid-19. *Brookings Papers on Economic Activity*, 2020(Summer):1–50, 2020.

F. Schorfheide and D. Song. Real-time forecast evaluation with covid-19 data. *Journal of Econometrics*, 220(1):3–22, 2021. doi: 10.1016/j.jeconom.2020.07.037.

T. M. Sinai and J. Heathcote. House price moments in boom–bust cycles. Technical Report 18059, National Bureau of Economic Research, Cambridge, MA, Jan. 2012. URL https://www.nber.org/papers/w18059.

K. J. Singleton. The structure of risks in equilibrium affine models of bond yields. *Journal of Finance*, 57(3):1307–1341, 2002.

A Appendix

A.1 Data: VAR elements

The variable r_t is the market yield on U.S. Treasury Securities at 1-Year Constant Maturity, Quoted on an Investment Basis (GS1), retrieved from FRED. The variable ys_t is constructed as the difference in ten-year and the one-year treasury yields using Market Yield on U.S. Treasury Securities at 10-Year Constant Maturity, Quoted on an Investment Basis GS10 from FRED. The variable cr_t is constructed as the monthly log change in the ICE BofA BBB US Corporate Total Return Index Value (BAMLCC0A4BBBTRIV), received from FRED. We use Moody's Seasoned Baa Corporate Bond Yield Relative to Yield on 10-Year Treasury Constant Maturity (BAA10Y) retrieved from FRED for s_t. We use the monthly log change in the S&P CoreLogic Case-Shiller U.S. National Home Price Index (CSUSHPINSA), retrieved from FRED, to construct the house price growth rate, h_t. To construct the aggregate disposable income growth rate, k_t, we use seasonally-adjusted disposable personal income per capita in current dollars (A229RC0), retrieved from FRED. For l_t, we use the series seasonally-adjusted total non-farm layoffs and discharges (JTSLDR), retrieved from FRED.

A.2 Identifying Market Prices of Risk

A.2.1 Treasury Bonds

The log yield at time t of a Treasury bond with maturity s is affine in the state vector:

$$\frac{y_t(s)}{12} = -\frac{A_s}{s} - \frac{B_s'}{s} z_t$$

where the coefficients A_s and B_s satisfy the difference equations:

$$A_{s+1} = -\frac{\delta_0}{12} + A_s + \frac{1}{2} B_s' \Sigma \, B_s - B_s' \Sigma^{\frac{1}{2}} \Lambda_0 \tag{A.1}$$

$$\left(B_{s+1}\right)' = \left(B_s\right)' \left(\Phi - \Sigma^{\frac{1}{2}} \Lambda_1\right) - \frac{(\delta_1, \delta_2, 0, \ldots, 0)'}{12}, \tag{A.2}$$

with initial conditions $A_0 = 0$ and $B_0 = (0, 0, \ldots, 0)'$. The proof is standard and omitted for brevity.

Since the one-year yield and the difference between the ten- and one-year yields, both expressed as annual yields, are the first and second elements of the VAR, respectively, we have that:

$$y_t(120) = (e_1 + e_2)' \, \mu + (e_1 + e_2)' \, z_t.$$

By (9), we also have:

$$\frac{y_t(120)}{12} = -\frac{A_{120}}{120} - \frac{B_{120}'}{120} z_t.$$

Matching up these two equations, we obtain the restrictions:

$$A_{120} = -10(e_1 + e_2)'\mu, \tag{A.3}$$

$$B_{120}' = -10(e_1 + e_2)'. \tag{A.4}$$

Similar conditions for the one-year bond imply the restrictions:

$$A_{12} = -e_1'\mu, \tag{A.5}$$
$$B_{12}' = -e_1'. \tag{A.6}$$

Equations (A.3)–(A.6) contain $2 + 2N$ restrictions. Given the zero restrictions imposed on the companion matrix Φ and the Cholesky form of the covariance matrix $\Sigma^{\frac{1}{2}}$, they impose six moment restrictions.

In addition, we insist on matching the average spread between the one-year T-note and the three-month T-bill for an additional, seventh moment:

$$-\frac{A_{12}}{12} + \frac{A_3}{3} = \frac{\text{sample mean of 1Y-3M spread}}{12}. \tag{A.7}$$

We use the seven moment restrictions in (A.3)–(A.7) to solve for seven unknown parameters, namely $(\delta_0, \delta_1, \delta_2)$ and $(\Lambda_0^r, \Lambda_0^{ys}, \Lambda_1^{r,ys}, \Lambda_1^{ys,r})$, obtaining exact identification.

A.2.2 Corporate Bonds

We impose the first-order condition on the BBB-rated corporate bond return cr. Under joint log normality of the SDF and the corporate bond return, we have that:

$$\mathbb{E}_t\left[cr_{t+1}\right] - \frac{y_t(1)}{12} + \frac{1}{2}Var_t\left[cr_{t+1}\right] = -Cov_t\left[cr_{t+1}, m_{t+1}\right]$$

The VAR pins down the left-hand side, while the right-hand side can be computed using the SDF expression (6). The expressions for each side are given by:

$$\begin{aligned}
\mathbb{E}_t\left[cr_{t+1}\right] - \frac{y_t(1)}{12} + \frac{1}{2}Var_t\left[cr_{t+1}\right] &= \mu_{cr} + \phi_{cr,r}r_t + \phi_{cr,ys}ys_t + \phi_{cr,cs}cs_t - \frac{\delta_0}{12} - \frac{\delta_1}{12}r_t - \frac{\delta_2}{12}ys_t \\
&\quad + \frac{1}{2}\left(\sigma_{cr,r}^2 + \sigma_{cr,ys}^2 + \sigma_{cr,cr}^2\right) \\
-Cov_t\left[cr_{t+1}, m_{t+1}\right] &= \Lambda_0^r\sigma_{cr,r} + \Lambda_0^{ys}\sigma_{cr,ys} + \Lambda_0^{cr}\sigma_{cr,cr} + \left(\Lambda_1^{ys,r}\sigma_{cr,ys} + \Lambda_1^{cr,r}\sigma_{cr,cr}\right)r_t \\
&\quad + \left(\Lambda_1^{r,ys}\sigma_{cr,r}\Lambda_1^{cr,ys}\sigma_{cr,cr}\right)ys_t + \Lambda_1^{cr,cr}\sigma_{cr,cr}cs_t
\end{aligned}$$

where r_t, ys_t, cs_t in the above expressions are understood to be demeaned.

Matching up the two sets of expressions results in four restrictions on the market prices of risk:

$$\mu_{cr} - \frac{\delta_0}{12} + \frac{1}{2}\left(\sigma_{cr,r}^2 + \sigma_{cr,ys}^2 + \sigma_{cr,cr}^2\right) = \Lambda_0^r\sigma_{cr,r} + \Lambda_0^{ys}\sigma_{cr,ys} + \Lambda_0^{cr}\sigma_{cr,cr} \tag{A.8}$$
$$\phi_{cr,r} - \frac{\delta_1}{12} = \Lambda_1^{ys,r}\sigma_{cr,ys} + \Lambda_1^{cr,r}\sigma_{cr,cr} \tag{A.9}$$
$$\phi_{cr,ys} - \frac{\delta_2}{12} = \Lambda_1^{r,ys}\sigma_{cr,r} + \Lambda_1^{cr,ys}\sigma_{cr,cr} \tag{A.10}$$
$$\phi_{cr,cs} = \Lambda_1^{cr,cs}\sigma_{cr,cr} \tag{A.11}$$

Given $(\delta_0, \delta_1, \delta_2)$ and $(\Lambda_0^r, \Lambda_0^{ys}, \Lambda_1^{r,ys}, \Lambda_1^{ys,r})$ from the previous step, equations (A.8)–(A.11) ex-

actly identify the four market price of risk parameters $(\Lambda_0^{cr}, \Lambda_1^{cr,r}, \Lambda_1^{cr,ys}, \Lambda_1^{cr,cs})$.

A.2.3 Housing Returns

The first-order condition for the housing return under joint log-normality of the SDF and the housing return can be written as:

$$\mathbb{E}_t\left[h_{t+1}^i + nry\right] - \frac{y_t(1)}{12} + \frac{1}{2}Var_t\left[h_{t+1}^i\right] = -Cov_t\left[h_{t+1}^i, m_{t+1}\right]$$

The left-hand side is pinned down by the VAR, while the right-hand side can be computed using the SDF expression in (6). The expressions for each side are given by:

$$\begin{aligned}
&\mathbb{E}_t\left[h_{t+1}^i + nry\right] - \frac{y_t(1)}{12} + \frac{1}{2}Var_t\left[h_{t+1}^i\right] = \mu_h + nry + \phi_{h,r}r_t + \phi_{h,ys}ys_t + \phi_{h,cs}cs_t + \phi_{h,h}h_t \\
&-\frac{\delta_0}{12} - \frac{\delta_1}{12}r_t - \frac{\delta_2}{12}ys_t + \frac{1}{2}\left(\sigma_{h,r}^2 + \sigma_{h,ys}^2 + \sigma_{h,cr}^2 + +\sigma_{h,cs}^2 + \sigma_{h,l}^2 + \sigma_{h,h}^2 + \sigma_{h,i}^2\right) \\
&-Cov_t\left[h_{t+1}, m_{t+1}\right] = \Lambda_0^r\sigma_{h,r} + \Lambda_0^{ys}\sigma_{h,ys} + \Lambda_0^{cr}\sigma_{h,cr} + \Lambda_0^h\sigma_{h,h} + \left(\Lambda_1^{ys,r}\sigma_{h,ys} + \Lambda_1^{cr,r}\sigma_{h,cr} + \Lambda_1^{h,r}\sigma_{h,h}\right)r_t \\
&+ \left(\Lambda_1^{r,ys}\sigma_{h,r} + \Lambda_1^{cr,ys}\sigma_{h,cr} + \Lambda_1^{h,ys}\sigma_{h,h}\right)ys_t + \left(\Lambda_1^{cr,cs}\sigma_{h,cr} + \Lambda_1^{h,cs}\sigma_{h,h}\right)cs_t + \Lambda_1^{h,h}\sigma_{h,h}h_t
\end{aligned}$$

where r_t, ys_t, cs_t in the above expressions are understood to be demeaned.

Matching up the two sets of expressions results in five restrictions on the market prices of risk:

$$\mu_h + nry - \frac{\delta_0}{12} + \tfrac{1}{2}\sum_{j\in\{r,ys,cr,cs,l,h,i\}}\sigma_{h,j}^2 = \Lambda_0^r\sigma_{h,r} + \Lambda_0^{ys}\sigma_{h,ys} + \Lambda_0^{cr}\sigma_{h,cr} + \Lambda_0^h\sigma_{h,h}, \tag{A.12}$$

$$\phi_{h,r} - \frac{\delta_1}{12} = \Lambda_1^{ys,r}\sigma_{h,ys} + \Lambda_1^{cr,r}\sigma_{h,cr} + \Lambda_1^{h,r}\sigma_{h,h}, \tag{A.13}$$

$$\phi_{h,ys} - \frac{\delta_2}{12} = \Lambda_1^{r,ys}\sigma_{h,r} + \Lambda_1^{cr,ys}\sigma_{h,cr} + \Lambda_1^{h,ys}\sigma_{h,h}, \tag{A.14}$$

$$\phi_{h,cs} = \Lambda_1^{cr,cs}\sigma_{h,cr} + \Lambda_1^{h,cs}\sigma_{h,h}, \tag{A.15}$$

$$\phi_{h,h} = \Lambda_1^{h,h}\sigma_{h,h}. \tag{A.16}$$

Given $(\delta_0, \delta_1, \delta_2)$ and $(\Lambda_0^r, \Lambda_0^{ys}, \Lambda_0^{cr}, \Lambda_1^{r,ys}, \Lambda_1^{ys,r}, \Lambda_1^{cr,r}, \Lambda_1^{cr,ys}, \Lambda_1^{cr,cs})$ from the previous two steps, the five equations (A.12)–(A.16) exactly identify the five market price of risk parameters $\left(\Lambda_0^h, \Lambda_1^{h,r}, \Lambda_1^{h,ys}, \Lambda_1^{h,cs}, \Lambda_1^{h,h}\right)$.

A.3 Full-sample Estimates

Law of Motion for the State Variables Table A.1 presents the full-sample VAR estimation results. Panel A reports the unconditional mean of the state variables, Panel B the VAR companion matrix, and panel C the innovation volatilities (the Cholesky decomposition of the covariance matrix). The entries in Panels A and C have been multiplied by 100 for readability. Note that the interest rate, yield spread, and credit spread observed from FRED are expressed in annual rates, while the other variables are expressed as monthly rates.

Table A.1: VAR Model Parameters

r	ys	cr	cs	l	h	k
Panel A: Monthly Drift Vector $\boldsymbol{\mu}\times 100$						
0.1568· 12	0.1087· 12	0.4386	0.2086 · 12	1.3718	0.3776	0.3074
Panel B: Transition Matrix Φ						
0.9759	0.0601	0	0	0	0	0
0.0043	0.9571	0	0	0	0	0
0.0540	-0.0705	0	0.2475	0	0	0
0.0110	-0.0265	0	0.9693	0	0	0
0.0165	-0.0565	0	0.0552	0.5258	0	0
-0.0232	-0.0246	0	0.0138	0	0.9381	0
-0.0183	-0.0923	0	-0.0671	0	0	-0.5412
Panel C: Covariance Matrix–Cholesky Factor $\boldsymbol{\Sigma}^{1/2}\times 100$						
0.1919	0	0	0	0	0	0
-0.0713	0.1762	0	0	0	0	0
-0.2926	-0.4677	1.8767	0	0	0	0
-0.0899	-0.0484	-0.0896	0.1410	0	0	0
-0.1709	0.0017	-0.0836	0.1046	0.4307	0	0
0.0122	0.0310	0.0313	-0.0163	0.0077	0.2068	0
0.1409	0.0785	-0.2571	-0.1459	0.0581	-0.0987	1.7184

The volatility of disposable income growth is 1.72% per month in the full sample. We note that this estimate is more than twice the estimate of 0.75% obtained for the sample that ends in 2019. The estimate jumps discretely to 1.01% once the year 2020 is added to the sample and to 1.81% once 2021 is added as well, after which it falls back slightly. Similarly, the volatility of the layoff rate over the full sample is 0.43% but only 0.08% in the sample that ends in 2019. It jumps to 0.46% once 2020 is added and remains at 0.46% after 2021 is also added. The other coefficients are more stable over time.

Market Price of Risk Estimates We report below the market prices of risk coefficients estimated on the full sample:

$$\Lambda_0' = \begin{pmatrix} -0.1867 & -0.0286 & 0.1336 & 0 & 0 & 2.5031 & 0 \end{pmatrix},$$

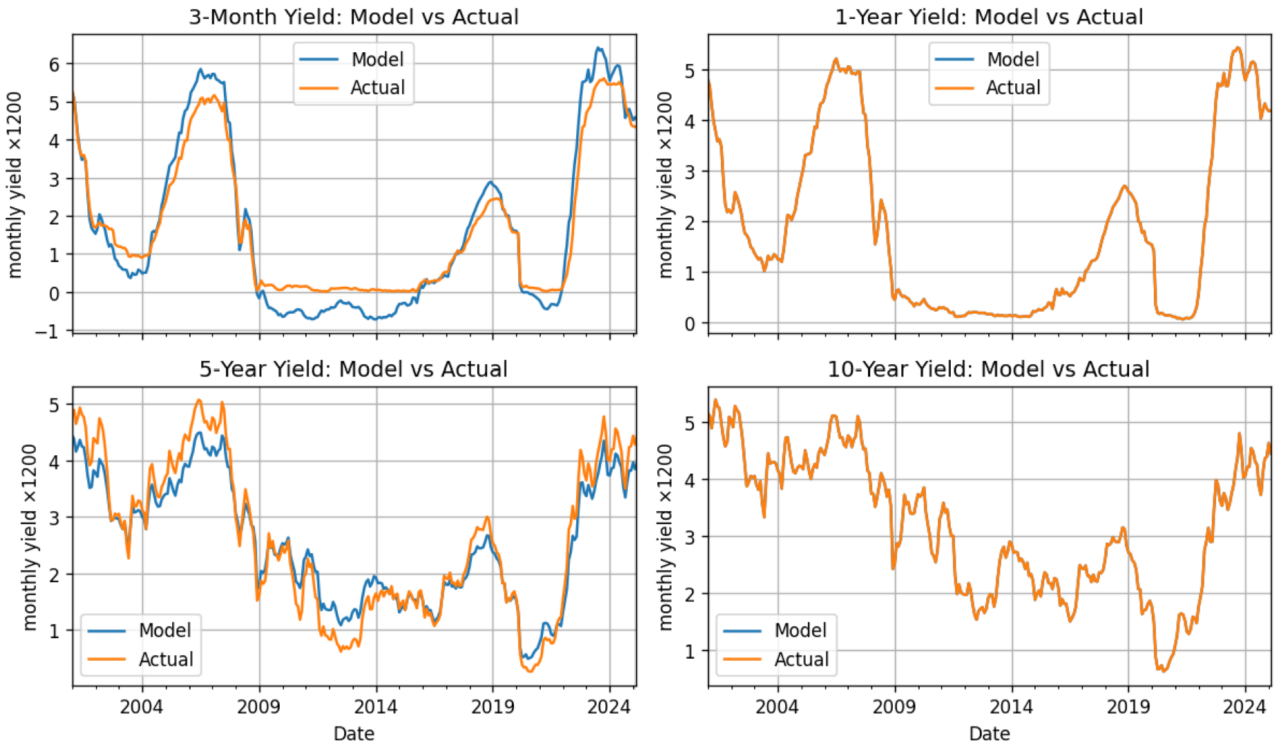
$$\Lambda_1 = \begin{pmatrix} 0 & -45.06 & 0 & 0 & 0 & 0 & 0 \\ -12.75 & 0 & 0 & 0 & 0 & 0 & 0 \\ -5.35 & -2.10 & 0 & 13.19 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -55.20 & 6.39 & 0 & 4.68 & 0 & 453.72 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

The short rate dynamics are given by:

$$(\delta_0, \delta_1, \delta_2) = (0.0167, 1.1586, -0.3798).$$

Bond Yields Figure A.1 shows the fit of the model for nominal bond yields of various maturities when market prices of risk are estimated on the full sample. By construction, the model exactly matches the 1- and 10-year yields, while providing a close fit to the 3-month and 5-year yields. The three-month bond yield is at the zero lower bound for parts of the sample. While an affine term structure model with Gaussian innovations cannot handle the zero lower bound, we observe that the 3-month yield never goes very negative in the model.

Figure A.1: Model-Implied vs. Empirical Treasury Yields



Notes: The figure plots the observed and model-implied constant-maturity yields on Treasuries of maturities of three months (top left panel), one year (top right panel), five years (bottom left panel) and ten years (bottom right panel). The sample begins in January 2001. All yields have been multiplied by 1200, i.e., expressed as annual percentages, for readability.

A.4 Matching Mortgage Loss Rates

A.4.1 Fire-Sale Discount in Mortgage Market

Let n_t the number of months in the past decade that house price appreciation (HPA) was negative:

$$n_t \equiv \sum_{\tau \in \mathcal{W}_t} \mathbf{1}_{h_\tau < 0},$$

where $\mathbf{1}_{h_\tau<0}$ indicates whether seasonally-adjusted monthly HPA is negative in month τ and $\mathcal{W}_t = \{t-119, \ldots, t\}$ denotes the 10-year (120-month) window ending in month t. Define the mean and standard deviation of n_t, computed over expanding windows as:

$$\bar{n}_t \equiv \frac{1}{|\mathcal{T}_t|} \sum_{\tau \in \mathcal{T}_t} n_\tau, \qquad \sigma_{n,t} \equiv \sqrt{\frac{1}{|\mathcal{T}_t| - 1} \sum_{\tau \in \mathcal{T}_t} (n_\tau - \bar{n}_t)^2}.$$

where $\mathcal{T}_t$ is the number of months observed up until month t. Define z_t to be the positive component of the standardized number of negative-HPA months:

$$z_t \equiv \max\left\{0,\ \frac{n_t - \bar{n}_t}{\sigma_{n,t}}\right\}.$$

The variable z_t is high when house price growth in the past decade was more negative than it historically was. We model the fire-sale discount rate as:

$$fd_t \equiv \min\{1,\ \max\{0,\ \kappa_0 \cdot z_t\}\}.$$

The fire-sale discount rate fd_t is a linear function of z_t, truncated to lie in the unit interval.

A.4.2 Initial Liquid Financial Asset Distribution

We model initial liquid financial assets as a multiple of initial monthly income K_0^i with the multiple increasing in borrower credit quality:

$$A_0^i = K_0^i \cdot m(\texttt{FICO}^i),$$

where

$$m(\texttt{FICO}^i) = \begin{cases} \kappa_1 + \max\left\{\dfrac{\texttt{FICO}^i - 580}{S(\texttt{FICO}^i)},\ 0\right\}, & \text{Standard } \texttt{Vintage} < 2009, \texttt{Vintage} \geq 2009 \\ \kappa_2 + \max\left\{\dfrac{\texttt{FICO}^i - 580}{\kappa_4 \cdot S(\texttt{FICO}^i)},\ 0\right\}, & \text{Non-standard } \texttt{Vintage} < 2009, \end{cases}$$

where

$$S(\texttt{FICO}^i) = \begin{cases} 50 - 10\kappa_3, & \texttt{FICO}^i \geq 780, \\ 50 - 9\kappa_3, & 760 \leq \texttt{FICO}^i < 780, \\ 50 - 8\kappa_3, & 740 \leq \texttt{FICO}^i < 760, \\ 50 - 7\kappa_3, & 720 \leq \texttt{FICO}^i < 740, \\ 50 - 6\kappa_3, & 700 \leq \texttt{FICO}^i < 720, \\ 50 - 5\kappa_3, & 680 \leq \texttt{FICO}^i < 700, \\ 50 - 4\kappa_3, & 660 \leq \texttt{FICO}^i < 680, \\ 50 - 3\kappa_3, & 640 \leq \texttt{FICO}^i < 660, \\ 50 - 2\kappa_3, & 620 \leq \texttt{FICO}^i < 640, \\ 50 - \kappa_3, & 600 \leq \texttt{FICO}^i < 620, \\ 50, & 580 \leq \texttt{FICO}^i < 600. \end{cases}$$

We allow the sensitivity of the asset-to-income ratio at origination to the FICO score, governed by $S(\texttt{FICO}^i)$, to depend on the FICO score. We bucket FICO scores into 20-point bins and parameterize the sensitivity by the parameter κ_3. Borrowers with $\texttt{FICO}^i \leq 580$ serve as the baseline group.

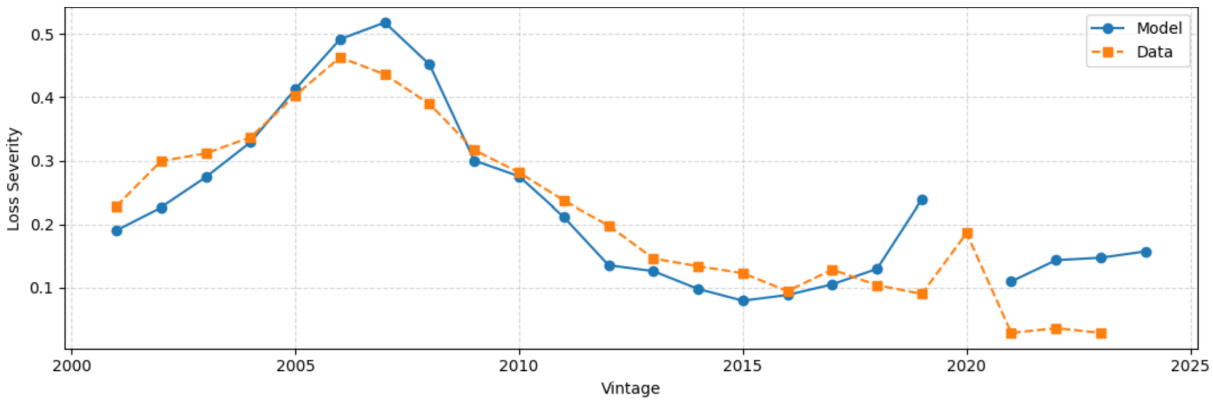
Non-standard mortgages, the GSEs' term that refers to no- or low-documentation mortgages, were widespread prior to the Great Financial Crisis. Freddie Mac reports such loans and their performance in a separate database of non-standard mortgages. Non-standard mortgages represented a substantial and growing share of mortgage originations in the 2003–08 vintages. These non-standard loans disappeared starting in 2009, and post-GFC mortgage underwriting standards tightened substantially and the Dodd-Frank Act's Ability-to-Repay/Qualified Mortgage framework mandated income and assets documentation. Since non-standard mortgages had substantially higher default rates than standard mortgages, we allow for a different income-to-asset mapping for non-standard loans governed by the parameters κ_2 and κ_4.

A.4.3 Estimation

To assess how well the model matches historical mortgage loss rates, we combine loan-level performance data from Freddie Mac's standard Single-Family Loan-Level Dataset with its Non-standard mortgage dataset. Because default rates are substantially higher for non-standard loans, including these loans is essential for accurately capturing aggregate mortgage default and loss behavior. Performance data for non-standard loans are available through 2021.Q2, with no further updates expected. For each mortgage vintage year, we randomly sample 30,000 standard 30-year fixed-rate loans and exclude loans with missing or unknown debt-to-income or loan-to-value ratios. We additionally draw a vintage-specific random sample of non-standard loans, with the sample size chosen to reflect the historical share of non-standard lending in total originations: 10% for vintages from 2001–2005, 60% for the 2006–2007 vintages, 20% for the 2008–2009 vintages, and 5% for all other vintages.

We do a grid search over the parameters $(\kappa_0, \kappa_1, \kappa_2, \kappa_3, \kappa_4)$, defined above, to (i) fit the cumulative aggregate loss rate on all mortgage vintages at the end of 2024, shown in Figure 3, and (ii) minimize the UPB-weighted root mean squared error (RMSE) of model-implied loss rates across FICO–LTV cells, as shown in Figure 4. This optimization delivers the parameter values $(0.17, 3, 1, 2.5, 3)$. Figure A.2 shows that the model also provides a good fit for the loss

Figure A.2: Loss Severity by Vintage



Notes: For each vintage, loss severity is defined as total loss dollars divided by total defaulted UPB (conditional on observing defaults). The model loss rate is undefined and therefore omitted for the 2020 vintage because there are no simulated defaults in 2020. The Data series is undefined and therefore omitted for the 2024 vintage because there are no observed defaults in the data.

severity vintage by vintage.

The parameters imply a fire-sale discount that is 13.34% on average with a standard deviation of 17.0%. It ranges from 0% to 46.7%, with the peak reached in 2009–10. These estimates are comparable to those in the post-GFC literature (e.g., Gerardi et al., 2015).

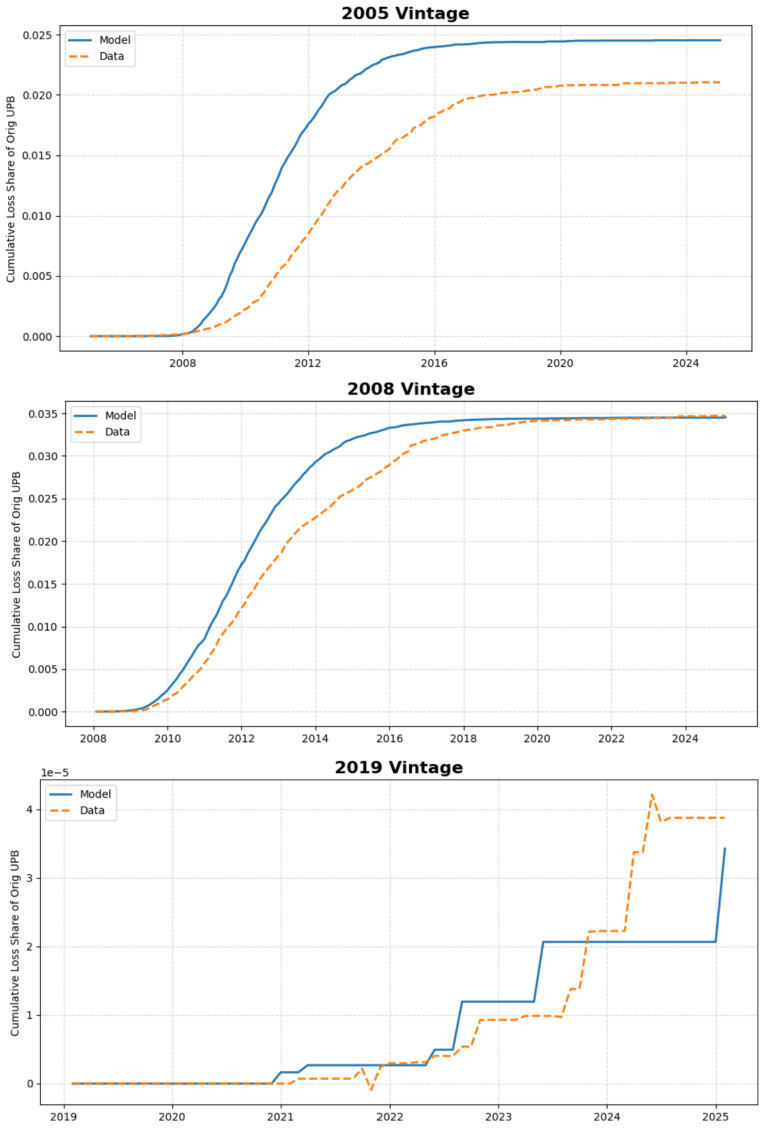
Turning to the initial asset-to-income ratio process, under the estimated parameters for standard loans, an increase in the FICO score from 580 to 590 increases the initial asset-to-income ratio from 3 to 3.2, an increase of 0.2, while an increase in the FICO score from 780 to 790 increases the asset-to-income ratio from 11 to 11.4, an increase of 0.4. For (pre-GFC) non-standard loans, the asset-to-income ratios range from 1 for a FICO score of 580 to 5 for a FICO score of 780.

We also study the time series of mortgage loss rates for a given loan origination year. As an example of a representative pre-GFC vintage, the top panel of Figure A.3 plots the cumulative loss rate over time of the 2005 loan vintage. The model generates a 2.47% cumulative loss rate for this vintage by the year 2024, compared to the observed cumulative loss rate of 2.11%. For the 2008 vintage, one of the all-time riskiest, the middle panel of Figure A.3 shows that our model generates a 3.49% cumulative default rate by the year 2024, nearly identical to the observed value of 3.47%. The bottom panel of Figure A.3 shows a representative post-GFC cohort, the 2019 vintage. It has a cumulative loss rate by the end of 2024 that is 0.0039%. The model predicts a very similar 0.0034% cumulative loss rate. In sum, the model replicates the large differences in default risk across vintages, ranging from 0.0035% to 3.5%.

A.5 Prepayment Model Estimation

We estimate the coefficients of the prepayment function in (14). Specifically, we group all borrowers with an annual mortgage interest rate between c% and $(c+1)$%, and allow (L^c, α^c) to vary by coupon group. We combine coupons 1% and 2% as a single group. Coupons 3%, 4%, 5%, 6%, and 7% are five additional groups. We fix the constant $A = 0.005$. We allow the parameter α^c, which governs the sensitivity of the prepayment rate of a given mortgage pool to the rate incentive, to depend on the age of the mortgage pool (Age, months since origination)

Figure A.3: Cumulative Loss Rates over Time for Selected Vintages



Notes: This figure compares model-implied and observed cumulative loss rates over time for the 2005 mortgage loan vintage (top panel), the 2008 mortgage loan vintage (middle panel), and the 2016 mortgage loan vintage (bottom panel). The observed loss rate combines 30,000 randomly-sampled loans from the Freddie Mac standard loan-level mortgage performance dataset with 3000 (6000, 1500) loans from the Freddie Mac non-standard loan-level performance data set for the 2005 (2008, 2019) vintage.

and the cumulative fraction of the pool that has already been prepaid (*Cum Prepay*):

$$\alpha^c(\text{Age}, \text{Cum Prepay}) = \gamma_0^c + \gamma_1^c \text{Age} + \gamma_2^c \text{Cum Prepay}. \tag{A.17}$$

We also allow L^c to vary by coupon group. L^c is the monthly prepayment rate when the rate incentive is large and positive.

In a given coupon group, there are multiple mortgage pools (vintages) and each pool is observed for multiple months. For each pool-month, we observe the prepayment rate (single month mortality), the age, and the cumulative prepayment fraction. We estimate $(L^c, \gamma_0^c, \gamma_1^c, \gamma_2^c)$ by minimizing the distance between the model-implied and observed prepayment rates. We do this separately for each coupon. The resulting parameter estimates are reported in Table A.2. The best UPB-weighted fit for the annualized prepayment rate achieves a mean absolute error of 4.7% and a root mean squared error of 6.0%.[32]

Table A.2: Prepayment Function Parameter Estimates

Coupon Group	L^c	$\boldsymbol{\gamma_0^c}$	$\boldsymbol{\gamma_1^c}$	$\boldsymbol{\gamma_2^c}$
1–2	0.009529	0.099982	0.049903	-0.000021
3	0.038748	0.105511	1.540497	-0.000127
4	0.047808	306.615386	-0.459396	0.000253
5	0.038475	149.648308	-0.302620	-0.000310
6	0.044321	158.989545	-0.652514	-0.001732
7	0.091098	681.307838	-9.758352	0.000227

Figure A.4 compares the model-implied prepayment rate, where we estimate the prepayment function model coefficients using data until year t in order to form the model-implied prepayment rate in year $t+1$. We start with the sample ending in 2013, then roll the sample forward by one year, and repeat. This out-of-sample prepayment rate still tracks the observed prepayment rate reasonably well. Over the period since 2013, the out-of-sample mean absolute error for the annual prepayment rate is 3.3% and the root mean squared error is 5.1%.

A.6 Realized Loss Calculations

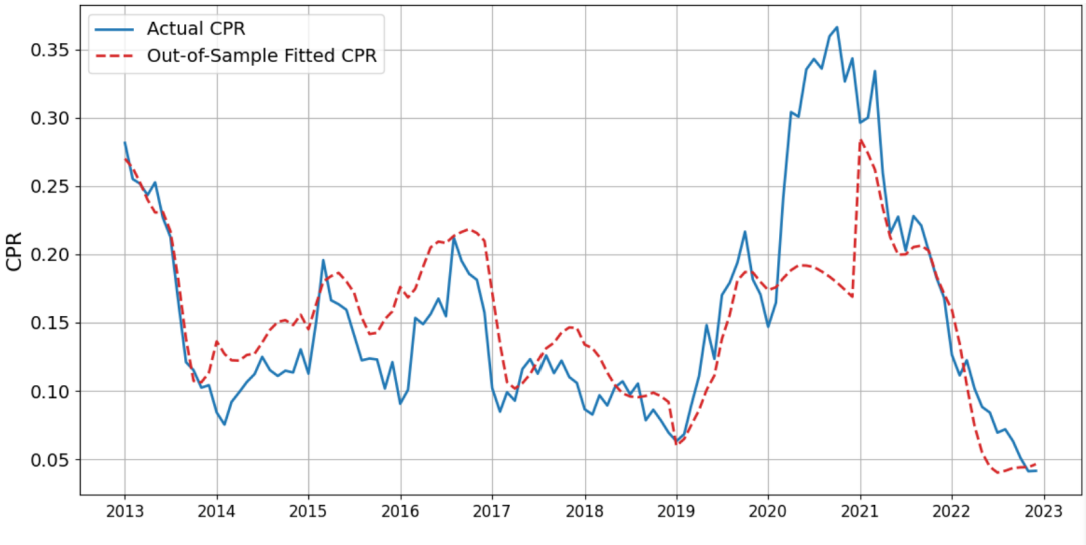
The realized loss l_t^i on mortgage loan i is the positive difference between the default costs and default credits:

$$\begin{aligned}
&\text{Realized Loss} = \max\{\text{Default Costs} - \text{Default Credits}, 0\} \\
&\text{Default Costs} = \text{Default UPB} + \text{Accrued Interest} + \text{Expenses} \\
&\text{Default Credits} = \text{Net Sales Proceeds} + \text{PMI Recoveries} + \text{Non-PMI Recoveries}
\end{aligned}$$

We define each of these components in turn.

[32] For robustness, we have also experimented with a model that imposes the same coefficients $(L^c, \gamma_0^c, \gamma_1^c, \gamma_2^c)$ across all coupons c. That model results in a somewhat worse fit with a mean absolute error of 6.3% and a root mean squared error of 8.0%.

Figure A.4: Out-of-Sample Conditional Prepayment Rate (UPB-weighted).



In the simulation, losses following default are driven by two core components: the unpaid principal balance (UPB) that remains outstanding at the time the default is resolved, and the interest accrued on that defaulted UPB between the default date and the resolution date. In addition to these components, we incorporate default-related expenses—including taxes and insurance, legal fees, maintenance fees, and miscellaneous costs—by applying the median expense ratio observed in Freddie Mac's Single-Family Loan-Level Dataset over 2001–2024, equal to 8.71% of default UPB. These costs are offset by three sources of recovery: net proceeds from the sale of the collateral, payments from private mortgage insurance (PMI), denoted as PMI recoveries, and non-PMI recoveries. The non-PMI recoveries category includes confirmed defect or make-whole proceeds, hazard insurance proceeds, rental receipts, positive escrow balances, and other miscellaneous credits. We set non-PMI recoveries to 0.55% of default UPB, the median value in the same Freddie Mac dataset, and use the median rather than the mean because default UPB is highly right-skewed and heavy-tailed.
If borrower i defaults at time t, the amount of default UPB is

$$\text{Default UPB}_t^i := N_t^i \mathbf{1}_{\tau_t^i=1,\epsilon_t^i=0}$$

The accrued interest is the amount of missed interest rate payments between the first missed payment and final loan resolution on the default UPB. The loan resolution time of a borrower i who defaults at $t_{i,def}$ is: $t_{i,res} = t_{i,def} + \omega^i$. It follows that:

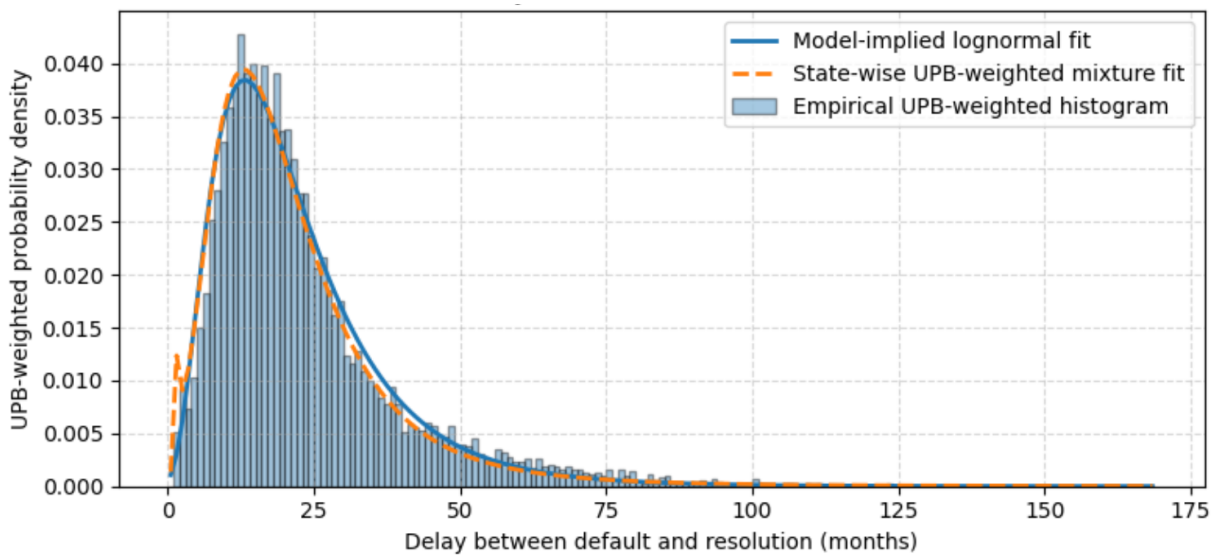
$$\text{Accrued Interest}_t^i = \omega^i \cdot \frac{r_0^{m,i}}{12} \cdot N_t^i.$$

Empirically, the time between default and resolution, ω_i, varies across time and space, and indeed across borrowers. Figure A.5 shows that a log-normal distribution fits the observed delay in the 2000-2024 vintages well. The resulting mean delay is 22 months, the median is 18 months, and the distribution has a long tail extending out to 170 months. Based on this evidence, the resolution delay in the model ω^i is drawn from an i.i.d. lognormal distribution X for which $\log(X - \ell) \sim \mathcal{N}(\ln(e^\mu), \sigma)$. The best fit between model and data results in a

shape parameter $\sigma = 0.544$, location parameter $\ell = -3.331$, and scale parameter $e^{\mu} = 22.121$. The resolution delay plays an important role in the pricing of CRT bonds. In simulations, the mezzanine tranches are typically paid off before significant losses are realized.

As a robustness check, we can fit the resolution-delay model state-by-state with different parameters. This allows s to deal with heterogeneity between judicial and non-judicial foreclosure states, for example. The UPB-weighted mean-squared error of the probability density function for the state-wise model is $1.95 \cdot 10^{-6}$, whereas the MSE for the model-implied fit is $1.86 \cdot 10^{-6}$. Furthermore, this model has very similar implications for the pricing of the 329 CRT tranches in our sample.

Figure A.5: Distribution of Mortgage Default Resolution Delays



Notes: The histogram shows the time lag between mortgage default and mortgage loan resolution in the data for the 2000-2024 vintages in our sample. The solid blue line shows the fitted log-normal curve we assume in our pricing model. The dashed yellow line is the state-wise UPB-weighted mixture fit.

We denote the net proceeds from the sale of the collateral that back loans that defaulted at t as:

$$\text{Net Sales Proceeds}_t^i = R^i_{t_{i,res}} := (1 - fd_{t_{i,res}})(1 - sc)H^i_{t_{i,res}}$$

where fd^i is given by (A.4.1).
GSE mortgages with original LTV above 80% must take out private mortgage insurance (PMI). When loans with PMI coverage default, the mortgage insurer covers (some or all of) the loss. This is relevant for the HAQ series of CRT bonds, which contain loans with origination LTV above 80%. It is also relevant for the LLPA discussion below. PMI coverage depends on the LTV ratio. For each LTV group v, the PMI covers a fixed fraction of the default cost:

$$PMI_t^i = \xi_v \cdot \text{Default Cost}_t^i.$$

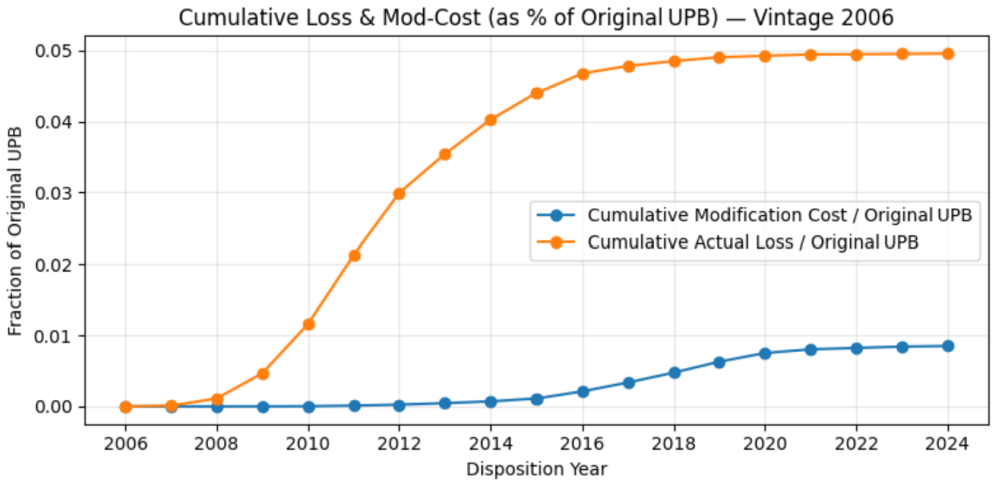
For standard 30-year fixed-rate mortgages, the PMI coverage rate ξ_v is 12% for LTVs at origination in the (80%,85%] range, 25% for LTVs in the (85%,90%] range, 30% for LTVs in the

(90%,95%] range, and 35% for LTVs in (95%, 97%]. PMI coverage automatically terminates when a borrower's LTV falls below 78% of the original home value. At that point, the CRT investor no longer enjoys the additional protection from PMI.[33] We note that the presence of PMI could make it such that the credit risk born by CRT investors in high-LTV pools (the HQA series) is actually lower than that born by CRT investors in low-LTV pools (the DNA series).

A.7 Mortgage Losses: Modified vs Non-modified Loans

We used the 2006 vintage to investigate whether mortgage loan modification significantly affects mortgage loss rates. Figure A.6 breaks out cumulative loss rates for the 2006 vintage by modification status: whether the loan was ever modified during its life. It shows that only one in six dollars of losses on this pool came from modified loans. In the first 10 years of the life of these loans, which is the relevant horizon for CRT investors, a much smaller percentage of losses came from modified loans. In vintages 2013 and later, modification probabilities conditional on mortgage delinquency were very low, typically below 1%.

Figure A.6: Cumulative Loss Rate by Modification Status



Notes: The blue line shows the cost as a percentage of the original UPB due to modifications. The orange line shows the loss rate excluding modification-related losses.

[33]By the Homeowners Protection Act (HOPA) §4902(b), the servicer must terminate the PMI payment with no action by the borrower if the loan falls below 78% of the original value and the loan is current. Article §4902(c) also states that PMI terminates automatically at the midpoint of the loan, i.e., after 15 years for a 30-year mortgage. This is not relevant for us since it is beyond the horizon of CRT bonds. Paragraph §4902(a) of HOPA states that borrowers may also initiate PMI termination when the loan falls below 80% of the original home value, and that the servicer must oblige if the borrower is current, has a good payment history, and the property value has not declined. We do not apply this last rule since few borrowers are aware of this option, and establishing whether the true market value of a property has increased or not is difficult in practice.

A.8 Principal Payments to CRT Investors

This appendix discusses the detailed rules that determine how scheduled and unscheduled principal payments on the reference pool flow through to the cash flows paid to CRT investors.

We begin by defining some terminology. We recall that the senior tranche is the A-H tranche, and is never sold to CRT investors. The subordinate tranches are those sold to CRT investors that are junior to (take credit losses before) the senior tranche. Define the Senior (Subordinate) Reduction Amount as the amount by which the principal of the senior (subordinate) tranche(s) is (are) paid down at each payment date. Define the Senior (Subordinate) Percentage as the ratio of the total outstanding senior (subordinate) tranche notional over the total outstanding notional of the entire pool. The credit enhancement (CE) is one minus the Senior Percentage, which is equal to the Subordinate Percentage. Additionally, there is Offered Reference Tranche Percentage (ORTP) at each payment date, which is defined as the percentage of the notional of the sold tranches at each payment date t divided by the original notional of the pool at time 0. These percentages change as the deal ages.

Scheduled principal payments refer to regular amortization. The part of each month's fixed mortgage payment that constitutes principal repayment grows over time as the mortgage ages. There are two types of *unscheduled* principal payments. One is called the *recovery principal* and the other is called the *unscheduled stated principal*. The recovery principal is the amount of recoveries from realized losses. The unscheduled stated principal consists of prepayments, the most important source of principal payments for both subordinate tranches and the senior tranche.

Recovery Principal All recovery principal payments flow exclusively to the senior tranche. CRT investors never receive principal payments from loss recoveries.

Scheduled Principal Payments For scheduled principal payments, there are two regimes. Prior to March 2018,[34] scheduled principal payments are allocated to the senior (A-H) tranche and the subordinate tranches pro rata according to the current-period Senior and Subordinate Percentages. Among the subordinate tranches, scheduled principal payments are allocated sequentially, starting with the most senior tranche.

After March 2018, the pro-rata allocation of scheduled principal payments to senior and subordinate STACR tranches only occurs when tests (1)–(3) below are satisfied. If any of tests (1), (2), or (3) fails, there is no scheduled principal payment to the subordinate tranches.

Unscheduled Stated Principal Payments STACR subordinate tranches only receive unscheduled stated principal payments if tests (1), (2), and (3) below are satisfied. Unscheduled stated principal payments from the pool are then split pro rata between senior and subordinate tranches according to the current Senior and Subordinate Percentages. Among the subordinate tranches, stated principal payments are sequential, i.e., in strict order of seniority. If any of the tests (1), (2), or (3) fail, then unscheduled stated principal payments, including prepayments, stop flowing to the subordinate tranches and get diverted to the senior tranche.

Freddie Mac currently administers four tests related to principal payments:

[34] This refers to all deals from the first CRT deal, 2013 STACR DN1, to the 2018 STACR DNA1 deal issued in March 2018, according to the information in the private placement memoranda.

1. Cumulative Net Loss Test
2. Minimum Credit Enhancement Test
3. Delinquency Test
4. Supplemental Subordinate Reduction Amount test (SSRA Test)

We describe these tests in detail below, and explain the special role of test (4). We obtain the actual test thresholds for each deal by manually collecting information from the STACR deal documents (private placement memoranda) and Freddie Mac's Data Intelligence.

To illustrate the mechanics, suppose that the current subordinate percentage is 4.6%, all tests are passed, and the M-1 tranche is the most senior subordinate tranche with positive notional balance. Suppose that total (scheduled and stated) principal payments in month t from the reference pool are \$10,000. Then the M-1 tranche holders receive a \$460 cash flow in month t, which reduces the notional outstanding of the M-1 tranche by the same amount. The M-2, B-1 and B-2 tranches receive no principal payments and their notionals remain unchanged.

We now describe the three tests in more detail. The first test verifies whether the current cumulative level of realized loss on the collateral pool (between origination and the current month), as a share of the collateral pool's notional at origination, exceeds a threshold. The loss threshold ranges from 0.1% to 3% percent across deals, and is reported in the deal documents. In addition, within a given deal, the loss threshold varies over the life of the deal. For example, for the 2025-HQA1 deal, the cumulative loss threshold is 0.1% percent at the beginning of the deal and increases gradually to 1.1% after ten years, an increase of 0.1% each year. For Fixed-Severity CRT deals (the 2013 and 2014 vintages), the threshold starts at 0.25% and increases by 0.25% annually, and cumulative net loss is defined as the cumulative 180-day plus delinquency rate.

The second test computes the current credit enhancement (CE), which is the Subordinate Percentage in month t. That threshold ranges from 2% to 7% across deals, as reported in the deal documents.

The CE at origination may or may not be below the threshold. If the CE threshold is not met initially, it usually is met at some point later in the life of the deal.

The third test compares the delinquency rate over the most recent six-month period to a threshold. The definition of the delinquency rate varies across deals. It may refer to the 60-day plus, 90-day plus, or 180-day plus delinquency. The delinquency level is calculated as:

$$\textit{Delinquency level} := \frac{\frac{1}{6}\sum_{j=0}^{5} \text{ Delinquent Principal Balance }_{t-j}}{(\text{ Subordinate Percentage }) \times (\text{UPB}) - \text{Actual Loss Amount}}.$$

The threshold delinquency level is typically 50%. The delinquency test is only administered after month six of the deal. This test is administered after March 2015.

Prior to March 2018, if any of the three test fails, then the investors of the subordinate tranches still receive scheduled principal payments, excluding prepayments. After March 2018, if any of the three test fails, then the investors of the subordinate tranches receive no principal payments at all.

The Supplemental Subordination Reduction Amount After 2018, Freddie Mac began to administer a Supplemental Subordinate Reduction Amount (SSRA) Test to determine whether the subordinate tranches qualify for additional principal amortization. The first CRT bond that receives supplemental subordinate reduction amount is the STACR 2019 DNA4 Deal. It states that, if the CE at time t is sufficiently high, then an additional payment, called the Supplemental Subordinate Reduction Amount, will be allocated to the subordinate tranches. The SSRA test does not interfere with the three tests related to principal payments. Typically, the CE threshold in the SSRA test is either 5.5% or 6.15%, depending on the deal. The first STACR deal that implements a 5.5% SSRA threshold is the 2021-DNA5 Deal. Prior deals all use SSRA equal to 6.15%.

The Supplemental Subordinate Reduction Amount is defined as the outstanding UPB of the reference pool at time t times the difference between the actual credit enhancement in month t, the ORTP, and the threshold credit enhancement in the SSRA test:

$$\text{SSRA}_t = \text{UPB}_t \times \max(\text{ ORTP }_t - \text{ SSRA Threshold, 0}).$$

The GSEs collect the amount $ORTP_0$ times UPB_0 upfront from the CRT investors, and put it in an escrow account. They distribute some of the cash from that account, namely the amount $SSRA_t$ in month t, entirely to the subordinate tranches for those deals that satisfy test 4 in month t. Among the subordinate tranches, the SSRA is paid sequentially. To receive the SSRA, the three aforementioned tests do not need to be passed.

The A-1 Tranche Starting in November 2023, STACRs began to issue a new A-1 tranche, which is *pari passu* with the A-H tranche, which the GSEs retain. In other words, the A-1 bond is not a subordinate tranche but a senior tranche. In the first 36 months of its life, the scheduled and stated principal payments to the A-1 tranche are the same as for the A-H tranche. From month 36, all senior reduction amount, excluding the recovery principal, is allocated to the A-1 Tranche, regardless of whether the three tests are passed or not. In month 40, all remaining notional of the A-1 tranche is paid back as long as the cumulative net loss does not exceed 1% of the pool notional at that date.

A.9 Model-implied CRT tranche Spreads: Tranche by Tranche

We present the full CRT tranche spreads from 2013 to 2025 below. Recall that the interest rate benchmark was LIBOR for STACR deals issued between June 2013 (2013DN1) and Sept 2020 (2020 DNA4), and SOFR thereafter. Fixed severity bonds are labeled as the DN (low-LTV loans) and HQ series (high-LTV loans), while the actual loss deals are called DNA (low-LTV loans) and HQA (high-LTV loans) series.

Table A.3: STACR CRT Tranches: Model Implied vs Observed Spreads (%)

Deal	Tranche	Mat	Attach	Detach	Min CE	Observed Spread	Model Spread
2013-DN1	M-1	10.00	1.95	3.00	3.00	3.40	-1.18
2013-DN1	M-2	10.00	0.30	1.95	3.00	7.15	0.56
2013-DN2	M-1	10.00	1.65	3.00	3.00	1.45	-1.07
2013-DN2	M-2	10.00	0.30	1.65	3.00	4.25	0.94
2014-DN1	M-1	10.00	3.50	4.51	5.00	1.00	-1.25
2014-DN1	M-2	10.00	2.00	3.50	5.00	2.20	-0.92
2014-DN1	M-3	10.00	0.30	2.00	5.00	4.50	0.82
2014-DN2	M-1	10.00	3.50	4.50	3.00	0.85	-1.77
2014-DN2	M-2	10.00	2.00	3.50	3.00	1.65	-0.99
2014-DN2	M-3	10.00	0.30	2.00	3.00	3.60	0.59
2014-DN3	M-1	10.00	3.60	4.60	5.10	1.35	-1.20
2014-DN3	M-2	10.00	2.40	3.60	5.10	2.40	-0.94
2014-DN3	M-3	10.00	0.40	2.40	5.10	4.00	0.15
2014-DN4	M-1	10.00	4.20	5.20	5.70	1.40	-1.21
2014-DN4	M-2	10.00	2.90	4.20	5.70	2.40	-0.94
2014-DN4	M-3	10.00	0.50	2.90	5.70	4.55	-0.27
2014-HQ1	M-1	10.00	4.10	6.50	7.00	1.65	-1.02
2014-HQ1	M-2	10.00	2.55	4.10	7.00	2.50	-0.89
2014-HQ1	M-3	10.00	0.75	2.55	7.00	4.10	-0.81
2014-HQ2	M-1	10.00	4.10	6.10	6.60	1.45	-1.04
2014-HQ2	M-2	10.00	2.25	4.10	6.60	2.20	-0.88
2014-HQ2	M-3	10.00	0.60	2.25	6.60	3.75	0.10
2014-HQ3	M-1	10.00	4.75	6.50	7.00	1.65	-1.07
2014-HQ3	M-2	10.00	3.10	4.75	7.00	2.65	-0.90
2014-HQ3	M-3	10.00	0.85	3.10	7.00	4.75	-0.81
2015-DN1	M-1	10.00	3.50	4.50	5.00	1.25	-1.13
2015-DN1	M-2	10.00	2.50	3.50	5.00	2.40	-0.91
2015-DN1	M-3	10.00	1.00	2.50	5.00	4.15	-0.82
2015-DN1	B	10.00	0.00	1.00	5.00	11.50	7.73
2015-DNA1	M-1	12.50	3.25	4.25	4.75	0.90	-1.09
2015-DNA1	M-2	12.50	2.25	3.25	4.75	1.85	2.00
2015-DNA1	M-3	12.50	1.00	2.25	4.75	3.30	7.14
2015-DNA1	B	12.50	0.00	1.00	4.75	9.20	14.73
2015-DNA2	M-1	12.50	4.50	5.50	6.00	1.15	-1.31
2015-DNA2	M-2	12.50	2.50	4.50	6.00	2.60	-0.85
2015-DNA2	M-3	12.50	1.50	2.50	6.00	3.90	0.52
2015-DNA2	B	12.50	0.00	1.50	6.00	7.55	9.61
2015-DNA3	M-1	12.50	4.85	5.85	6.35	1.35	-1.45
2015-DNA3	M-2	12.50	2.65	4.85	6.35	2.85	-0.46
2015-DNA3	M-3	12.50	1.00	2.65	6.35	4.70	6.45
2015-DNA3	B	12.50	0.00	1.00	6.35	9.35	16.65
2015-HQ1	M-1	10.00	4.75	6.50	7.00	1.05	-1.06
2015-HQ1	M-2	10.00	3.00	4.75	7.00	2.20	-0.87
2015-HQ1	M-3	10.00	1.50	3.00	7.00	3.80	-0.83
2015-HQ1	B	10.00	0.00	1.50	7.00	10.75	5.17
2015-HQ2	M-1	10.00	4.10	5.60	6.10	1.10	-1.13
2015-HQ2	M-2	10.00	2.25	4.10	6.10	1.95	-0.85
2015-HQ2	M-3	10.00	1.00	2.25	6.10	3.25	-0.05
2015-HQ2	B	10.00	0.00	1.00	6.10	7.95	11.79
2015-HQA1	M-1	12.50	4.95	5.95	6.45	1.25	-1.26
2015-HQA1	M-2	12.50	2.70	4.95	6.45	2.65	-0.87
2015-HQA1	M-3	12.50	1.00	2.70	6.45	4.70	0.71
2015-HQA1	B	12.50	0.00	1.00	6.45	8.80	11.57
2015-HQA2	M-1	12.50	5.40	6.40	6.90	1.15	-1.17
2015-HQA2	M-2	12.50	2.95	5.40	6.90	2.80	-0.81
2015-HQA2	M-3	12.50	1.00	2.95	6.90	4.80	1.22

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Deal	Tranche	Mat	Attach	Detach	Min CE	Observed Spread	Model Spread
2015-HQA2	B	12.50	0.00	1.00	6.90	10.50	13.12
2016-DNA1	M-1	12.50	3.95	5.00	5.50	1.45	-1.14
2016-DNA1	M-2	12.50	2.95	3.95	5.50	2.90	0.17
2016-DNA1	M-3	12.50	1.00	2.95	5.50	5.55	9.45
2016-DNA1	B	12.50	0.00	1.00	5.50	10.00	18.12
2016-DNA2	M-1	12.50	4.15	5.00	5.50	1.25	-1.34
2016-DNA2	M-2	12.50	3.25	4.15	5.50	2.20	-0.96
2016-DNA2	M-3	12.50	1.00	3.25	5.50	4.65	5.88
2016-DNA2	B	12.50	0.00	1.00	5.50	10.50	17.18
2016-DNA3	M-1	12.50	4.00	5.00	5.50	1.10	-1.20
2016-DNA3	M-2	12.50	3.05	4.00	5.50	2.00	-0.88
2016-DNA3	M-3	12.50	1.00	3.05	5.50	5.00	7.25
2016-DNA3	B	12.50	0.00	1.00	5.50	11.25	17.95
2016-DNA4	M-1	12.50	4.00	5.00	5.50	0.80	-1.01
2016-DNA4	M-2	12.50	3.00	4.00	5.50	1.30	-0.56
2016-DNA4	M-3	12.50	1.00	3.00	5.50	3.80	6.02
2016-DNA4	B	12.50	0.00	1.00	5.50	8.60	16.96
2016-HQA1	M-1	12.50	4.40	5.50	6.00	1.75	-1.19
2016-HQA1	M-2	12.50	3.20	4.40	6.00	2.75	-0.89
2016-HQA1	M-3	12.50	1.00	3.20	6.00	6.35	5.50
2016-HQA1	B	12.50	0.00	1.00	6.00	12.75	15.05
2016-HQA2	M-1	12.50	4.50	5.50	6.00	1.20	-0.95
2016-HQA2	M-2	12.50	3.00	4.50	6.00	2.25	-0.79
2016-HQA2	M-3	12.50	1.00	3.00	6.00	5.15	3.11
2016-HQA2	B	12.50	0.00	1.00	6.00	11.50	15.14
2016-HQA3	M-1	12.50	4.20	5.50	6.00	0.80	-0.83
2016-HQA3	M-2	12.50	2.85	4.20	6.00	1.35	-0.77
2016-HQA3	M-3	12.50	1.00	2.85	6.00	3.85	2.68
2016-HQA3	B	12.50	0.00	1.00	6.00	9.00	14.33
2016-HQA4	M-1	12.50	4.28	5.50	6.00	0.80	-0.93
2016-HQA4	M-2	12.50	3.05	4.28	6.00	1.30	-0.81
2016-HQA4	M-3	12.50	1.00	3.05	6.00	3.90	2.26
2016-HQA4	B	12.50	0.00	1.00	6.00	8.75	13.90
2017-DNA1	M-1	12.50	2.55	3.75	4.25	1.20	-0.94
2017-DNA1	M-2	12.50	1.00	2.55	4.25	3.25	5.77
2017-DNA1	B-1	12.50	0.50	1.00	4.25	4.95	12.03
2017-DNA1	B-2	12.50	0.00	0.50	4.25	10.00	18.38
2017-DNA2	M-1	12.50	2.30	3.50	4.00	1.20	-0.74
2017-DNA2	M-2	12.50	1.00	2.30	4.00	3.45	7.41
2017-DNA2	B-1	12.50	0.50	1.00	4.00	5.15	12.27
2017-DNA2	B-2	12.50	0.00	0.50	4.00	11.25	18.04
2017-DNA3	M-1	12.50	2.50	3.50	4.00	0.75	-0.81
2017-DNA3	M-2	12.50	1.00	2.50	4.00	2.50	5.91
2017-DNA3	B-1	12.50	0.50	1.00	4.00	4.45	12.10
2017-DNA3	B-2	12.50	0.00	0.50	4.00	11.00	17.21
2017-HQA1	M-1	12.50	3.25	4.25	4.75	1.20	-1.02
2017-HQA1	M-2	12.50	1.00	3.25	4.75	3.55	2.91
2017-HQA1	B-1	12.50	0.50	1.00	4.75	5.00	10.10
2017-HQA1	B-2	12.50	0.00	0.50	4.75	12.75	16.03
2017-HQA2	M-1	12.50	3.00	4.00	4.25	0.80	-0.91
2017-HQA2	M-2	12.50	1.00	3.00	4.25	2.65	4.57
2017-HQA2	B-1	12.50	0.50	1.00	4.25	4.75	10.95
2017-HQA2	B-2	12.50	0.00	0.50	4.25	12.00	16.25
2017-HQA3	M-1	12.50	3.70	4.50	4.75	0.55	-1.05
2017-HQA3	M-2	12.50	1.00	3.70	4.75	2.35	2.98
2017-HQA3	B-1	12.50	0.50	1.00	4.75	4.45	10.99

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Deal	Tranche	Mat	Attach	Detach	Min CE	Observed Spread	Model Spread
2017-HQA3	B-2	12.50	0.00	0.50	4.75	13.00	16.03
2018-DNA1	M-1	12.50	3.10	4.00	4.50	0.45	-0.60
2018-DNA1	M-2	12.50	1.00	3.10	4.50	1.80	1.31
2018-DNA1	B-1	12.50	0.50	1.00	4.50	3.15	8.63
2018-DNA1	B-2	12.50	0.00	0.50	4.50	9.50	15.54
2018-DNA2	M-1	30.00	2.50	3.50	3.75	0.80	-0.47
2018-DNA2	M-2	30.00	1.00	2.50	3.75	2.15	2.35
2018-DNA2	B-1	30.00	0.50	1.00	3.75	3.70	10.14
2018-DNA2	B-2	30.00	0.00	0.50	3.75	10.50	14.95
2018-DNA3	M-1	30.00	3.00	4.00	4.25	0.75	-0.37
2018-DNA3	M-2	30.00	1.10	3.00	4.25	2.10	1.42
2018-DNA3	B-1	30.00	0.60	1.10	4.25	3.90	8.23
2018-DNA3	B-2	30.00	0.10	0.60	4.25	7.75	13.12
2018-HQA1	M-1	30.00	3.20	4.00	4.25	0.70	-0.27
2018-HQA1	M-2	30.00	2.10	3.20	4.25	2.30	-0.59
2018-HQA1	B-1	30.00	0.50	2.10	4.25	4.35	2.44
2018-HQA1	B-2	30.00	0.00	0.50	4.25	11.00	13.88
2018-HQA2	M-1	30.00	3.00	4.00	4.25	0.75	-0.30
2018-HQA2	M-2	30.00	1.15	3.00	4.25	2.30	0.52
2018-HQA2	B-1	30.00	0.65	1.15	4.25	4.25	6.19
2018-HQA2	B-2	30.00	0.10	0.65	4.25	11.00	12.28
2019-DNA1	M-1	30.00	3.00	4.25	4.50	0.90	-0.27
2019-DNA1	M-2	30.00	1.10	3.00	4.50	2.65	1.35
2019-DNA1	B-1	30.00	0.60	1.10	4.50	4.65	8.24
2019-DNA1	B-2	30.00	0.10	0.60	4.50	10.75	14.06
2019-DNA2	M-1	30.00	3.50	4.25	4.50	0.80	-0.23
2019-DNA2	M-2	30.00	1.10	3.50	4.50	2.45	1.29
2019-DNA2	B-1	30.00	0.60	1.10	4.50	4.35	8.52
2019-DNA2	B-2	30.00	0.10	0.60	4.50	10.50	14.45
2019-DNA3	M-1	30.00	3.25	4.25	4.50	0.73	-0.44
2019-DNA3	M-2	30.00	1.10	3.25	4.50	2.05	2.01
2019-DNA3	B-1	30.00	0.60	1.10	4.50	3.25	10.95
2019-DNA3	B-2	30.00	0.10	0.60	4.50	8.15	16.30
2019-DNA4	M-1	30.00	3.00	4.00	4.25	0.70	-0.58
2019-DNA4	M-2	30.00	1.10	3.00	4.25	1.95	1.92
2019-DNA4	B-1	30.00	0.60	1.10	4.25	2.70	10.72
2019-DNA4	B-2	30.00	0.10	0.60	4.25	6.25	16.09
2019-HQA1	M-1	30.00	3.60	4.50	4.75	0.90	-0.09
2019-HQA1	M-2	30.00	1.50	3.60	4.75	2.35	-0.15
2019-HQA1	B-1	30.00	0.60	1.50	4.75	4.40	4.57
2019-HQA1	B-2	30.00	0.10	0.60	4.75	12.25	13.69
2019-HQA2	M-1	30.00	3.50	4.50	4.75	0.70	-0.16
2019-HQA2	M-2	30.00	1.50	3.50	4.75	2.05	-0.01
2019-HQA2	B-1	30.00	0.60	1.50	4.75	4.10	5.28
2019-HQA2	B-2	30.00	0.10	0.60	4.75	11.25	14.37
2019-HQA3	M-1	30.00	3.50	4.50	4.75	0.75	-0.42
2019-HQA3	M-2	30.00	1.50	3.50	4.75	1.85	0.14
2019-HQA3	B-1	30.00	0.60	1.50	4.75	3.00	6.79
2019-HQA3	B-2	30.00	0.10	0.60	4.75	7.50	15.89
2019-HQA4	M-1	30.00	3.25	4.50	4.75	0.77	-0.48
2019-HQA4	M-2	30.00	1.15	3.25	4.75	2.05	0.88
2019-HQA4	B-1	30.00	0.60	1.15	4.75	2.95	7.79
2019-HQA4	B-2	30.00	0.10	0.60	4.75	6.60	15.32
2020-DNA1	M-1	30.00	2.75	3.75	4.00	0.70	-0.64
2020-DNA1	M-2	30.00	1.10	2.75	4.00	1.70	2.74
2020-DNA1	B-1	30.00	0.60	1.10	4.00	2.30	11.78

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Deal	Tranche	Mat	Attach	Detach	Min CE	Observed Spread	Model Spread
2020-DNA1	B-2	30.00	0.10	0.60	4.00	5.25	16.43
2020-DNA2	M-1	30.00	2.50	3.75	4.00	0.75	-0.47
2020-DNA2	M-2	30.00	1.10	2.50	4.00	1.85	8.27
2020-DNA2	B-1	30.00	0.60	1.10	4.00	2.50	15.30
2020-DNA2	B-2	30.00	0.10	0.60	4.00	4.80	18.30
2020-DNA3	M-1	30.00	3.00	4.00	4.50	1.50	-0.94
2020-DNA3	M-2	30.00	1.75	3.00	4.50	3.00	2.88
2020-DNA3	B-1	30.00	0.75	1.75	4.50	5.10	12.24
2020-DNA3	B-2	30.00	0.25	0.75	4.50	9.35	16.34
2020-DNA4	M-1	30.00	3.00	4.00	4.50	1.50	-0.96
2020-DNA4	M-2	30.00	1.75	3.00	4.50	3.75	3.50
2020-DNA4	B-1	30.00	0.75	1.75	4.50	6.00	12.60
2020-DNA4	B-2	30.00	0.25	0.75	4.50	10.00	16.56
2020-DNA5	M-1	30.00	2.50	3.50	3.75	1.30	-0.38
2020-DNA5	M-2	30.00	1.50	2.50	3.75	2.80	4.47
2020-DNA5	B-1	30.00	0.75	1.50	3.75	4.80	12.03
2020-DNA5	B-2	30.00	0.10	0.75	3.75	11.50	16.45
2020-DNA6	M-1	30.00	2.00	3.00	3.25	0.90	-0.27
2020-DNA6	M-2	30.00	1.25	2.00	3.25	2.00	7.55
2020-DNA6	B-1	30.00	0.75	1.25	3.25	3.00	12.88
2020-DNA6	B-2	30.00	0.25	0.75	3.25	5.65	15.84
2020-HQA1	M-1	30.00	3.00	4.25	4.50	0.75	-0.50
2020-HQA1	M-2	30.00	1.10	3.00	4.50	1.90	1.55
2020-HQA1	B-1	30.00	0.60	1.10	4.50	2.35	8.70
2020-HQA1	B-2	30.00	0.10	0.60	4.50	5.10	15.57
2020-HQA2	M-1	30.00	3.00	4.00	4.25	1.10	-1.03
2020-HQA2	M-2	30.00	1.10	3.00	4.25	3.10	6.94
2020-HQA2	B-1	30.00	0.60	1.10	4.25	4.10	15.15
2020-HQA2	B-2	30.00	0.10	0.60	4.25	7.60	18.48
2020-HQA3	M-1	30.00	3.30	4.00	4.50	1.55	-0.98
2020-HQA3	M-2	30.00	1.75	3.30	4.50	3.60	1.21
2020-HQA3	B-1	30.00	0.75	1.75	4.50	5.75	10.13
2020-HQA3	B-2	30.00	0.25	0.75	4.50	10.00	15.39
2020-HQA4	M-1	30.00	3.00	4.00	4.25	1.30	-0.94
2020-HQA4	M-2	30.00	1.75	3.00	4.25	3.15	1.47
2020-HQA4	B-1	30.00	0.75	1.75	4.25	5.25	10.30
2020-HQA4	B-2	30.00	0.25	0.75	4.25	9.40	15.35
2020-HQA5	M-1	30.00	2.75	3.75	4.00	1.10	-0.30
2020-HQA5	M-2	30.00	1.50	2.75	4.00	2.60	3.53
2020-HQA5	B-1	30.00	0.75	1.50	4.00	4.00	10.57
2020-HQA5	B-2	30.00	0.25	0.75	4.00	7.40	15.04
2021-DNA1	M-1	30.00	2.00	2.50	2.75	0.65	-0.25
2021-DNA1	M-2	30.00	1.25	2.00	2.75	1.80	0.18
2021-DNA1	B-1	30.00	0.75	1.25	2.75	2.65	2.87
2021-DNA1	B-2	30.00	0.25	0.75	2.75	4.75	9.14
2021-DNA2	M-1	12.50	2.00	2.50	2.75	0.80	-0.41
2021-DNA2	M-2	12.50	1.25	2.00	2.75	2.30	1.28
2021-DNA2	B-1	12.50	0.75	1.25	2.75	3.40	8.71
2021-DNA2	B-2	12.50	0.25	0.75	2.75	6.00	13.76
2021-DNA3	M-1	12.50	2.00	2.50	2.75	0.75	-0.36
2021-DNA3	M-2	12.50	1.25	2.00	2.75	2.10	1.42
2021-DNA3	B-1	12.50	0.75	1.25	2.75	3.50	8.47
2021-DNA3	B-2	12.50	0.25	0.75	2.75	6.25	12.94
2021-DNA5	M-1	12.50	1.75	2.00	2.00	0.65	-0.86
2021-DNA5	M-2	12.50	1.25	1.75	2.00	1.65	-0.17
2021-DNA5	B-1	12.50	0.75	1.25	2.00	3.05	5.71

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Deal	Tranche	Mat	Attach	Detach	Min CE	Observed Spread	Model Spread
2021-DNA5	B-2	12.50	0.25	0.75	2.00	5.50	11.85
2021-DNA6	M-1	20.00	1.75	2.00	2.25	0.80	-0.41
2021-DNA6	M-2	20.00	1.25	1.75	2.25	1.50	2.30
2021-DNA6	B-1	20.00	0.75	1.25	2.25	3.40	7.87
2021-DNA6	B-2	20.00	0.25	0.75	2.25	7.50	12.66
2021-DNA7	M-1	20.00	1.75	2.25	2.50	0.85	-0.25
2021-DNA7	M-2	20.00	1.30	1.75	2.50	1.80	0.22
2021-DNA7	B-1	20.00	0.75	1.30	2.50	3.65	6.04
2021-DNA7	B-2	20.00	0.25	0.75	2.50	7.80	11.92
2021-HQA1	M-1	12.50	2.50	3.25	3.50	0.70	-0.30
2021-HQA1	M-2	12.50	1.25	2.50	3.50	2.25	0.40
2021-HQA1	B-1	12.50	0.75	1.25	3.50	3.00	5.32
2021-HQA1	B-2	12.50	0.25	0.75	3.50	5.00	12.41
2021-HQA2	M-1	12.50	2.50	3.00	3.25	0.70	-0.54
2021-HQA2	M-2	12.50	1.25	2.50	3.25	2.05	2.06
2021-HQA2	B-1	12.50	0.75	1.25	3.25	3.15	9.35
2021-HQA2	B-2	12.50	0.25	0.75	3.25	5.45	13.53
2021-HQA3	M-1	20.00	2.00	3.25	3.50	0.85	0.38
2021-HQA3	M-2	20.00	1.25	2.00	3.50	2.10	2.87
2021-HQA3	B-1	20.00	0.75	1.25	3.50	3.35	6.48
2021-HQA3	B-2	20.00	0.25	0.75	3.50	6.25	11.68
2021-HQA4	M-1	20.00	2.00	3.50	3.75	0.95	-0.10
2021-HQA4	M-2	20.00	1.25	2.00	3.75	2.35	1.42
2021-HQA4	B-1	20.00	0.75	1.25	3.75	3.75	4.86
2021-HQA4	B-2	20.00	0.25	0.75	3.75	7.00	11.05
2022-DNA1	M-1A	20.00	3.00	4.50	4.75	1.00	0.06
2022-DNA1	M-1B	20.00	2.00	3.00	4.75	1.85	0.05
2022-DNA1	M-2	20.00	1.25	2.00	4.75	2.50	0.54
2022-DNA1	B-1	20.00	0.75	1.25	4.75	3.40	3.68
2022-DNA1	B-2	20.00	0.25	0.75	4.75	7.10	10.37
2022-DNA2	M-1A	20.00	3.40	4.75	4.75	1.30	0.05
2022-DNA2	M-1B	20.00	2.00	3.40	4.75	2.40	0.06
2022-DNA2	M-2	20.00	1.25	2.00	4.75	3.75	0.99
2022-DNA2	B-1	20.00	0.75	1.25	4.75	4.75	5.05
2022-DNA2	B-2	20.00	0.25	0.75	4.75	8.50	11.57
2022-DNA3	M-1A	20.00	3.60	5.25	5.25	2.00	0.33
2022-DNA3	M-1B	20.00	2.00	3.60	5.25	2.90	0.22
2022-DNA3	M-2	20.00	1.25	2.00	5.25	4.35	0.88
2022-DNA3	B-1	20.00	0.75	1.25	5.25	5.65	3.48
2022-DNA3	B-2	20.00	0.25	0.75	5.25	9.75	8.86
2022-DNA4	M-1A	20.00	3.60	5.25	5.25	2.20	0.33
2022-DNA4	M-1B	20.00	2.00	3.60	5.25	3.35	0.23
2022-DNA4	M-2	20.00	1.25	2.00	5.25	5.25	0.81
2022-DNA4	B-1	20.00	0.75	1.25	5.25	6.25	3.45
2022-DNA4	B-2	20.00	0.25	0.75	5.25	10.25	9.11
2022-DNA5	M-1A	20.00	3.60	5.25	5.25	2.95	0.61
2022-DNA5	M-1B	20.00	2.05	3.60	5.25	4.50	0.32
2022-DNA5	M-2	20.00	1.25	2.05	5.25	6.75	0.36
2022-DNA5	B-1	20.00	0.75	1.25	5.25	7.50	1.44
2022-DNA5	B-2	20.00	0.25	0.75	5.25	12.90	7.19
2022-DNA6	M-1A	20.00	3.60	4.75	4.75	2.15	0.53
2022-DNA6	M-1B	20.00	2.20	3.60	4.75	3.70	0.41
2022-DNA6	M-2	20.00	1.30	2.20	4.75	5.75	0.38
2022-DNA7	M-1A	20.00	3.50	4.75	4.75	2.50	0.64
2022-DNA7	M-1B	20.00	2.55	3.50	4.75	5.00	0.55
2022-DNA7	M-2	20.00	1.50	2.55	4.75	7.00	0.54

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Deal	Tranche	Mat	Attach	Detach	Min CE	Observed Spread	Model Spread
2022-HQA1	M-1A	20.00	3.75	5.00	5.00	2.10	0.34
2022-HQA1	M-1B	20.00	2.60	3.75	5.00	3.50	0.22
2022-HQA1	M-2	20.00	1.25	2.60	5.00	5.25	0.23
2022-HQA1	B-1	20.00	0.75	1.25	5.00	7.00	1.01
2022-HQA1	B-2	20.00	0.25	0.75	5.00	11.00	6.35
2022-HQA2	M-1A	20.00	3.00	4.60	4.60	2.65	0.29
2022-HQA2	M-1B	20.00	2.00	3.00	4.60	4.00	0.21
2022-HQA2	M-2	20.00	1.25	2.00	4.60	6.00	0.65
2022-HQA3	M-1A	20.00	3.00	4.95	4.95	2.30	0.31
2022-HQA3	M-1B	20.00	2.00	3.00	4.95	3.55	0.21
2022-HQA3	M-2	20.00	1.25	2.00	4.95	5.35	0.57
2023-DNA1	M-1A	20.00	3.25	5.25	5.25	2.10	0.65
2023-DNA1	M-1B	20.00	2.55	3.25	5.25	3.10	0.60
2023-DNA1	M-2	20.00	1.50	2.55	5.25	5.50	0.84
2023-DNA1	B-1	20.00	0.75	1.50	5.25	8.15	2.29
2023-DNA2	M-1A	20.00	3.25	5.50	5.50	2.10	0.63
2023-DNA2	M-1B	20.00	2.50	3.25	5.50	3.25	0.55
2023-DNA2	M-2	20.00	1.50	2.50	5.50	5.70	0.71
2023-DNA2	B-1	20.00	1.00	1.50	5.50	7.60	1.60
2023-HQA1	M-1A	20.00	3.00	4.00	4.00	2.00	0.55
2023-HQA1	M-1B	20.00	2.00	3.00	4.00	3.50	0.43
2023-HQA1	M-2	20.00	1.50	2.00	4.00	5.50	0.35
2023-HQA2	M-1A	20.00	4.30	5.75	5.75	2.00	0.62
2023-HQA2	M-1B	20.00	3.00	4.30	5.75	3.35	0.55
2023-HQA2	M-2	20.00	2.60	3.00	5.75	3.85	0.51
2023-HQA3	A-1	20.00	4.25	5.25	4.25	1.85	0.63
2023-HQA3	M-1	20.00	3.25	4.25	4.25	1.85	0.71
2023-HQA3	M-2	20.00	2.25	3.25	4.25	3.35	0.65
2024-DNA1	A-1	20.00	4.25	5.50	4.25	1.35	0.57
2024-DNA1	M-1	20.00	2.90	4.25	4.25	1.35	0.63
2024-DNA1	M-2	20.00	2.20	2.90	4.25	1.95	0.60
2024-DNA2	A-1	20.00	4.00	5.50	4.00	1.25	0.60
2024-DNA2	M-1	20.00	2.50	4.00	4.00	1.20	0.64
2024-DNA2	M-2	20.00	2.20	2.50	4.00	1.70	0.60
2024-DNA3	A-1	20.00	4.00	5.00	4.00	1.05	0.50
2024-DNA3	M-1	20.00	2.80	4.00	4.00	1.00	0.54
2024-DNA3	M-2	20.00	2.25	2.80	4.00	1.45	0.56
2024-HQA1	A-1	20.00	4.40	5.50	4.40	1.25	0.63
2024-HQA1	M-1	20.00	3.30	4.40	4.40	1.25	0.70
2024-HQA1	M-2	20.00	2.25	3.30	4.40	2.00	0.66
2024-HQA2	A-1	20.00	4.00	5.00	4.00	1.25	0.45
2024-HQA2	M-1	20.00	3.00	4.00	4.00	1.20	0.52
2024-HQA2	M-2	20.00	2.30	3.00	4.00	1.80	0.48
2025-DNA1	A-1	20.00	4.20	5.75	4.20	0.95	0.55
2025-DNA1	M-1	20.00	2.60	4.20	4.20	1.05	0.66
2025-DNA1	M-2	20.00	1.95	2.60	4.20	1.35	0.67
2025-HQA1	A-1	20.00	4.20	5.75	4.20	0.95	0.55
2025-HQA1	M-1	20.00	2.85	4.20	4.20	1.15	0.66
2025-HQA1	M-2	20.00	2.10	2.85	4.20	1.65	0.67

Table A.4: Model-Minus-Observed CRT Spreads (Baseline and Alternative Models By Year)

	(1)	(2)	(3)	(4)	(5)
2013	-354.8***	-350.0***	-355.0***	-355.1***	-405.6***
2014	-288.4***	-277.6***	-288.7***	-288.7***	-333.5***
2015	-276.4***	-289.0***	-301.0***	-301.2***	-325.0***
2016	-245.6***	-239.1***	-245.4***	-246.3***	-238.1***
2017	-186.2***	-172.5***	-179.5***	-180.1***	-163.2***
2018	-135.0***	-171.4***	-154.4***	-152.8***	-98.8***
2019	-118.1***	-148.7***	-182.3***	-181.9***	-108.7***
2020	-23.6	-193.7***	-245.7***	-245.6***	-215.6***
2021	-93.6***	-104.6***	-157.0***	-156.0***	-141.6***
2022	-319.8***	-315.6***	-296.2***	-289.4***	-286.3***
2023	-266.9***	-265.9***	-174.8***	-162.3***	-151.9***
2024	-82.5***	-79.7***	50.2**	60.6**	-7.5
2025	-55.7***	-56.0***	97.0**	114.4**	-29.2***
2013 × *Junior*	-140.3	-289.6**	-287.7**	-291.2**	-269.5**
2014 × *Junior*	-132.6***	-201.5***	-207.2***	-208.5***	-187.4***
2015 × *Junior*	308.7***	-149.0***	-432.5***	-435.6***	-403.4***
2016 × *Junior*	559.3***	-150.9***	-550.8***	-551.8***	-510.5***
2017 × *Junior*	649.8***	-104.6**	-457.0***	-462.0***	-411.4***
2018 × *Junior*	435.1***	-65.2	-382.1***	-388.5***	-362.6***
2019 × *Junior*	620.5***	49.7	-451.6***	-457.5***	-419.5***
2020 × *Junior*	842.9***	88.0	-402.7***	-402.8***	-382.3***
2021 × *Junior*	554.6***	442.6***	-318.0***	-317.2***	-295.0***
2022 × *Junior*	132.2	194.0*	-413.4***	-414.9***	-424.2***
2023 × *Junior*	-326.0***	-373.6***	-423.2***	-424.8***	-302.4***
R^2 (%)	74.5	53.2	63.2	63.5	60.5
Average	30.5	-199.8	-387.0	-385.6	-365.4

The indicator variable *Junior* equals one for tranches with an attachment point $\leq 1\%$. Years 2024 and 2025 contain no junior tranches. Column (1) is the baseline recursively-estimated model. Column (2) uses the full sample for estimation. Column (3) zeroes out market price of risk (MPR) coefficients associated with house price growth shocks. Column (4) also zeroes MPR coefficients for corporate credit return shocks. Column (5) zeroes all MPR coefficients. Stars indicate robust (HC1) significance: $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$.

A.10 Realized Returns

Table A.5 reports realized returns on CRT junior and mezzanine tranches from AD & Co and compares them to corporate bond returns. Corporate bond returns are reported for our VAR estimation sample and for the CRT sample.

Table A.5: Realized Returns

Asset	Mean Excess	Std. Dev.	Sharpe Ratio
Aug 2001–Feb 2025			
AAA corporate	2.30%	6.50%	0.354
BBB corporate	3.68%	6.80%	0.542
B corporate	5.05%	9.03%	0.560
Jul 2015–Apr 2025			
AAA corporate	0.45%	7.72%	0.058
BBB corporate	1.55%	7.40%	0.209
B corporate	2.85%	7.78%	0.367
CRT junior tranche	10.21%	17.98%	0.568
CRT mezz tranche	3.63%	4.98%	0.731

The table reports average excess returns over a one-month Treasury bill rate, standard deviations of realized returns, and average Sharpe ratios. All numbers are annualized from monthly means and standard deviations. The data are from FRED CRTINDEXTIER0, CRTINDEXTIER1, CRTINDEXTIER2, and CRTINDEXTIER3, corresponding to the AD&Co US Mortgage High Yield Index (Tier 0, Tier 1, Tier 2, Tier 3).

A.11 CRT Tranche Duration

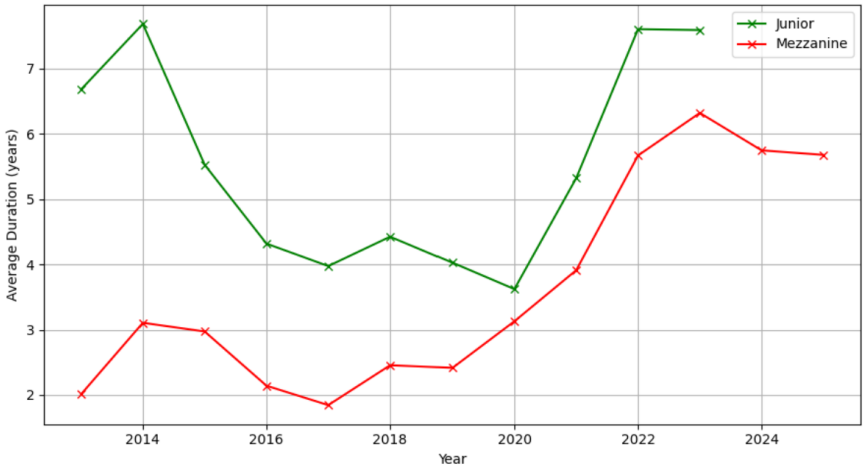
Figure A.7 shows the average duration for junior and mezzanine tranches by vintage in the benchmark model. The higher duration of the mezzanine tranches of the 2023, 2024, and 2025 vintages may be surprising at first sight. Those vintages carry much higher average mortgage rates than the preceding vintages, and higher rates *ceteris paribus* result in higher expected future principal prepayments rates and lower duration. Instead, the higher duration results from lower predicted credit loss than earlier vintages. It is also affected by the introduction of the A-1 bond in these vintages, which we include among the mezzanine bonds. The A-1 bond has a relatively high duration itself, and lengthens the duration of the other mezzanine tranches in the same deal.

A.12 Loan Level Price Adjustments

Figure A.8 shows the current loan level price adjustment grid, as of June 2025. This is the grid that has been in place since the May 2023 LLPA reform.

Figure A.9 shows the empirical distribution of mortgage UPB by (FICO,LTV) cell in the 2024 mortgage vintage.

Figure A.7: Duration of CRT Bonds



Notes: The figure plots the McCauley duration of STACR tranches (expressed in years). At each origination date, we separately average the durations of the Junior and Mezzanine tranches. Junior tranches are those with attachment points lower or equal to 1%.

A.13 Alternative Models of Income and Housing Risk

This appendix considers an alternative model of income risk that generates the observed difference in LLPAs between the lowest-FICO and highest-FICO bins for the LTV $(75, 80]$ group. It also considers an alternative model of house price risk that generates the observed difference in LLPAs between the lowest-LTV and highest-LTV bins for the$[700, 720)$ FICO group.

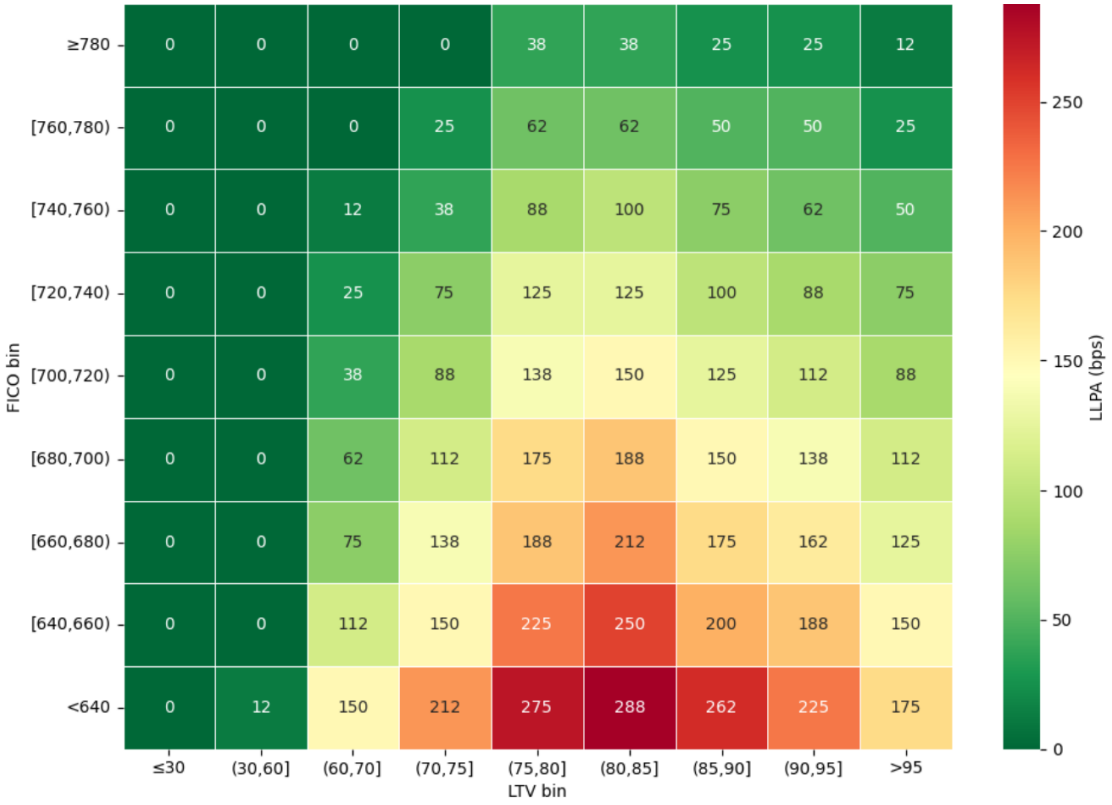
The first alternative model modifies the mapping from FICO scores to initial asset levels, which is given by (A.4.2) in the benchmark calibration, and replaces it by:

$$A_0^i = \left(0.5 + \max\left\{\frac{\text{FICO}^i - 580}{15}, 0\right\}\right) \cdot K_0^i$$

This model gives low-FICO borrowers a much lower initial liquid financial asset-to-income ratio than in the benchmark model and high-FICO borrowers a much higher initial asset-to-income ratio. It thereby increases the exposure of low-FICO borrowers to aggregate income risk, relative to the exposure of high-FICO borrowers, enough to reproduce the observed FICO slope in LLPAs. The resulting FICO slope of the $(75, 80]$ bucket is 237 bps, matching the slope of 237 bps in the data. However, this model generates loss rates that are too high for low-FICO borrowers and too low for high-FICO borrowers relative to the historical sample. Specifically, the UPB-weighted RMSE between observed loss rates in the model and the data is 0.75%, while it was far lower at 0.30% in the main model.

The second alternative model sets the net rental yield to be 6.2% per year. This generates a substantially higher housing risk premium than in the baseline model, and increases the risk of mortgage default through house price declines. This model generates an LTV slope in LLPAs of 150 bps, matching the observed 150 bps for the $[700, 720)$ FICO group. However, this model's net rental yield is far above the net rental yield of 4.2% per year, estimated by Eisfeldt and Demers (2015) in U.S. data.

Figure A.8: Loan Level Price Adjustment Grid Post May 2023 Reform



Notes: Loan Level Price Adjustment grid as of June 2025. The source is the Freddie Mac Single-Family Seller/Servicer Guide, Exhibit 19 (Credit Fees), Bulletin 2025-07, June 4, 2025.

A.14 Structure of G-Fees

Table A.6 describes the components of the guarantee fee charged to borrowers in conforming mortgages, and passed on from loan servicers to the GSEs when the loans are securitized into agency mortgage-backed securities.

Table A.6: Guarantee Fee (G-Fee) Structure Summary

Component	Description
Base G-Fee	- Uniform fee applied to all loans regardless of borrower risk. - Collected monthly as a fixed percentage of unpaid principal balance (UPB). - Typical level: 30–50 basis points (bps) annualized.
Loan-Level Price Adjustments (LLPAs)	- Risk-based add-ons based on loan characteristics: credit score, LTV, loan purpose, product type, etc. - Charged as upfront fees at origination (quoted in % of UPB), but often converted into annualized bps for pricing. - Can range from 0 to 300+ bps upfront, depending on risk.
G-Fee	- Sum of Base G-Fee and annualized LLPAs.
Collection Method	- Ongoing is paid monthly as part of borrower's mortgage payment. Upfront is paid as a lump sum payment at loan origination. - Applied to outstanding UPB; declines with amortization or prepayment.
Use of Proceeds	- Funds GSE credit guarantee operations, capital build-up, and CRT payments. - Supports coverage of expected credit losses, admin costs, capital charge, and uncertainty buffer.

Figure A.9: Empirical Borrower Distribution by FICO and LTV (% of loans) for the 2024 vintage.

