Our Deficit, Your Problem: Fiscal Sustainability, Exchange Rates, and Inflation*

C. Christopher Hyland† Tobias Kawalec‡

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Abstract

Fiscal expansions can transmit internationally. Empirically, we develop an open-economy present-value framework for the government budget constraint that embeds discount factors, exchange rate expectations, and time-varying foreign-exchange risk premia. Using market value data for U.K. and U.S. public debt covering 1975-2024, we document that real exchange rate and interest rate revisions are materially related to unexpected changes in the value of government debt. Fiscal deficit innovations derived from market-value changes of government debt affect the bilateral real exchange rate and contribute to cross-country inflation spillovers. We then present a continuous-time two-country model in which fiscal shocks in a financial hegemon propagate internationally through exchange rate adjustments that feed back into real interest rates. The model rationalises the empirical magnitudes and predicts "fiscal contagion" across sovereign balance sheets: the associated wealth effects and discount-rate spillovers boost inflation across countries. An estimated version of the model attributes approximately 2 percentage points of the cumulative inflationary pressure in the U.K. since 2021 to spillovers from expansionary U.S. fiscal policy.

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†University of Oxford, Saïd Business School, Exeter College, and Oxford-Man Institute. Email: christopher.hyland@sbs.ox.ac.uk.

‡University of Oxford, Department of Economics and Nuffield College. Email: tobias.kawalec@economics.ox.ac.uk.

1 Introduction

Fiscal deficits and debt sustainability have returned to the forefront of macroeconomic policy debates. Beyond the well-known link between fiscal sustainability and inflation risk, recent developments also highlight the risks that deficits pose to exchange rates. In April 2025, the U.S. dollar depreciated while Treasury yields rose, an unusual combination consistent with a lack of safe-haven inflows into U.S. government debt, hitherto regarded as a safe asset (Jiang et al., 2025). This episode is part of a broader pattern: in the post-Bretton Woods era, large government deficits have repeatedly coincided with abrupt currency movements and puzzling revaluations of foreign public debt. Existing empirical work mostly focuses on book-value debt and is silent on the distinction between expected and realised deficits. Because of this limitation, it struggles to explain why these international spillovers occur or how they can be reconciled with the intertemporal government budget constraint.

This paper revisits deficits and exchange rates from the perspective of an international financial investor, focusing on the notion that cross-border adjustments in exchange rates and discount factors are required for global sovereign debt markets to clear. We leverage this perspective in a rich quantitative model of "fiscal contagion." Households engage in cross-border trade of goods and financial products, with the Home bond featuring a convenience yield. We show that deficit shocks in the Home country directly affect exchange rates, thereby also altering the fiscal position of the Foreign sovereign by changing effective discount rates. In particular, Home-country deficit shocks affect the Foreign country's fiscal position, causing u-shaped movements in bilateral exchange rates as well as fiscally driven inflation in other economies.

Empirically, we start from a present-value identity that embeds the exchange rate, nominal discount rates, and time-varying foreign-exchange risk premia directly in the government budget constraint, building on the present-value logic of Hall and Sargent (2011), Cochrane (2019), and Jiang et al. (2024b). Using market-value data for U.K. (and U.S.) sovereign liabilities spanning 1975–2024, we quantify how much of local debt dynamics is driven by primary surplus news, by global real-rate shocks, and by exchange rate or risk-premium repricing. Our results establish that almost 30% of the *financial* factors driving the year-by-year revaluation of the market value of government debt in the United Kingdom, a representative G10 open economy, can be attributed to movements in the real exchange rate. While the domestic fiscal-surplus process remains the largest determinant of the value of U.K. government debt, accounting for real-exchange-rate-induced movements helps reconcile episodes that appear contradictory when considering only surpluses and interest rate movements in isolation.

To rationalise the empirical observations, we develop a two-country open-economy model in which fiscal shocks in a financial hegemon, identified through the existence of a convenience yield

on its sovereign debt, propagate internationally through the uncovered interest parity condition. We show that the threat of fiscal inflation in the hegemon country can transmit fiscal pressure to other economies by increasing the cost of servicing debt across countries. In the model, we identify a mechanism by which inflationary pressures at the hegemon can spill over to other economies, a mechanism of "international fiscal contagion" that generates fiscally driven inflation spillovers. A deterioration of the Home country's expected surpluses tightens the Foreign intertemporal budget constraint, even when Foreign projected surpluses are unchanged, because currency-denominated discount factors adjust rather than current-account flows. Intuitively, fair valuation of terminal debt streams requires that a domestic devaluation of the real value of debt be matched by a comparable decline in the value of foreign debt. Thus, weaker Home surpluses necessitate either a depreciation of the Home currency or a rise in foreign prices. This mechanism operates through the interest rate charged on foreign government debt in equilibrium, with the nominal exchange rate driving much of the adjustment. In equilibrium, the market-clearing exchange rate between two economies is heavily influenced by changes in relative surpluses; despite the rich goods and asset markets, the model reduces to ten core equations.

Mechanisms linking deficit changes to exchange rate movements and deficit changes in one hegemon country to price-level changes in another country are gaining traction (e.g., Alberola et al. (2021), Ding and Jiang (2024), Mac Mullen (2025)). Recent empirical evidence supports a close nexus between cross-country deficits and exchange rates but assigns only a limited role to government-debt valuation in equilibrium. While valuation may be plausibly secondary in the short run, no-arbitrage arguments holding in the long run provide a useful guide to the exchange rate movements required to sustain a fair market value of government debt over time. Put differently, the literature often overlooks the consistency between short-run exchange rate fluctuations and their long-run implications, which are best observed in a relatively low-frequency object such as the aggregate market value of government debt.

Our contribution addresses these gaps. We focus explicitly on exchange rate determination from the perspective of the government debt valuation equation, recovering the link between fiscal deficits, exchange rate movements, and the observed market value of government debt. Our main empirical exercise provides a detailed narrative of how fiscal deficit changes and exchange rate movements interact in jointly determining the equilibrium value of government debt. The long-run perspective spanning the last 50 years of macroeconomic history in the U.K. furthermore allows us to rationalise seemingly inconsistent episodes of debt valuation movements through corresponding exchange rate adjustments. This analysis is consistent with mechanisms in which changes in expected, rather than realised, deficits drive fiscal equilibrium.

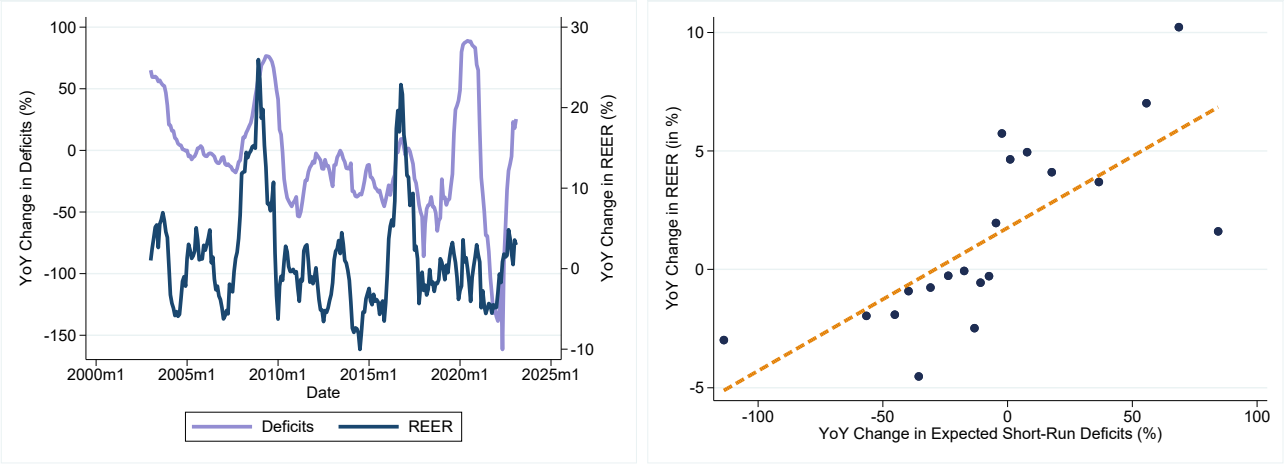
The accompanying theoretical framework shows how expected deficits in one country affect both domestic and foreign inflation by inducing exchange rate and interest rate movements that

jointly determine the real value of government debt in equilibrium. The deviation from a standard complete-markets benchmark that we propose is minimal: the existence of convenience yields on government bonds, which are non-pecuniary benefits associated with holding quasi-safe assets and which have been documented in relation to government debt (Bianchi and Bigio, 2022; Jiang et al., 2024a; Valchev, 2020). Yet this small deviation is sufficient to generate the fiscal contagion mechanism that we observe in the data, with one caveat: both the magnitude and the direction of the effects depend intricately on the fiscal-monetary policy mix present in both economies. Refining the insights of Jiang (2022), we identify two distinct conditions, each of which is on its own sufficient to generate fiscal contagion.

The first is that the Home fiscal authority engages in expansionary government spending without a commitment to repay the additional expenditure in equivalent real terms in the future. This condition alone is sufficient to generate fiscal contagion *irrespective of the behaviour of the Foreign fiscal and monetary authorities*. The second is sharply differing convenience yield functions, whereby temporary increases in non-pecuniary utility flows are increasing in bond holdings in one country but decreasing in another. In this latter case, *no* policy combination is able to neutralise the fiscal contagion effect, including commitments to balanced budgets and restrictive monetary authorities in both countries. Either condition is sufficient on its own, though both mechanisms can also operate simultaneously.Economically, the fiscal expansion at Home can generate a positive wealth effect as follows: if the Home fiscal authority does not commit to repaying the additional expenses through equivalent taxation in the future, but the Home monetary authority attempts to fight the real expansion through restrictive monetary policy, then Foreign bonds must be repriced as well through offering an equivalent return after having taken convenience premia into account. This increases the relative value of Foreign sovereign bonds due to their superior tax-backing, which are predominantly held in the Foreign country, inducing a positive wealth effect for Foreign households.

To further support the claim that expected deficits plausibly matter for exchange rate movements, Figure 1 provides descriptive evidence of a possible link between deficit expectations and exchange rates. The left panel plots the evolution of monthly changes to the REER (on a year-on-year basis) alongside monthly changes to expected fiscal deficits over the subsequent two years (again depicted on a year-on-year basis) in the United Kingdom. The right panel presents a binscatter of the two quantities against each other over the same sample period. Deficit expectations are cross-sectional averages of forecasts by private-sector institutions (banks and independent research firms), generated from a newly collected set of private-sector forecasters (HM Treasury, 2025).

Taken together, Figure 1 suggests a plausible correlation between deficit *expectations* and real exchange rate movements: as panel (b) shows, higher expected deficits correlate with depreciations of the domestic currency. The figure is purely descriptive; it abstracts from realised deficits and does not identify causality. As panel (a) indicates, domestic deficit expectations are far from the



(a) Changes to the REER and expected deficits over time.

(b) Binscatter of Δ expected fiscal deficits against Δ REER.

Figure 1: Dynamics of expected deficits and the REER in the U.K.

only driver of realised REER fluctuations, consistent with the broader literature.

Literature Review

The title of our contribution echoes the book of Rogoff (2025). In this paper, however, our central focus is the role of sovereign *deficit* dynamics for exchange rates, where we consider both domestic and foreign deficits in the way they shape exchange rate dynamics of a given country.

We primarily contribute to the literature on sovereign deficits and their ties to inflation and exchange rate dynamics. The paper is also adjacent to work on fiscally driven price-level determination, which depends on the broader fiscal-monetary policy mix. That literature was inaugurated by Sargent and Wallace (1981), with the subsequent work of Leeper (1991) and Woodford (1995) laying much of the modern groundwork for models that assign at least a share of inflationary dynamics to fiscal aggregate demand management. Literature reviews in this field are provided by Leeper and Leith (2016) and Cochrane (2023), among others. Support for fiscally driven inflation has been developed both theoretically (Ascari et al., 2023; Bassetto et al., 2024; Bassetto and Cui, 2018; Bianchi et al., 2023; Bigio et al., 2024; Campos et al., 2024; Caramp and Silva, 2023; Corsetti and Maćkowiak, 2024; Kaplan et al., 2024; Miao and Su, 2024) and empirically (Ascari et al., 2024; Banerjee et al., 2022; Barro and Bianchi, 2024; Chen et al., 2022a; Cochrane, 2022a; Reichlin et al., 2023).

While most of this work is set in closed-economy environments, a small but growing literature brings these insights to international models. Early explorations linking fiscal price-level determination and exchange rate dynamics include Bergin (2000), Dupor (2000), Sims (1997), Sims (1999), and Woodford (1998). Initial applications of such ideas include Benigno and Missale (2004) and Corsetti and Maćkowiak (2006), who focus on the ties between public debt and currency stabil-

ity, especially during episodes of severe currency turmoil. International aspects of government debt valuation and associated effects on inflation and exchange rates were further explored in the run-up to the Global Financial Crisis by Giannitsarou et al. (2006) and Gourinchas and Rey (2007).

Alongside the renewed interest in deficit-inflation links, research has revisited the role of deficits in exchange rate determination. Schmitt-Grohé and Uribe (2003) provided an early analysis of the role of exchange rate adjustments relative to inflation in financing fiscal deficits within emerging markets. Valchev (2020) establishes that excess returns on currency holdings can be traced to endogenous movements in bond convenience yields, which in turn depend on the prevailing interactions between monetary and fiscal policy. Particularly related to our contribution are Alberola et al. (2021) and Jiang (2022). Alberola et al. (2021) distinguish between Ricardian and non-Ricardian regimes, tying unexpected exchange rate movements (relative to a complete-market frictionless benchmark) to non-Ricardian fiscal regimes in which government debt lacks sufficient fiscal backing. Jiang (2022) makes explicit some of the implications of applying fiscal price-level determination to exchange rate dynamics, pointing out that real exchange rate adjustments are linked to government surplus shocks through the cost of issuing government debt, among other factors.

The paper also relates to the large macro-finance literature on nominal and real exchange rate determination, which we cannot survey fully here. A partial review is provided by Itskhoki (2021), including an overview of the many puzzles related to exchange rate dynamics that standard complete-market models fail to resolve.[1] Of particular relevance are recent contributions on the exchange rate disconnect (Bacchetta and Van Wincoop, 2004; Chari et al., 2002; Corsetti et al., 2008; Duarte, 2003; Engel and West, 2005; Itskhoki and Mukhin, 2021), as our main argument links the exchange rate back to one specific macroeconomic fundamental, namely aggregate demand managed by the fiscal authority. In that sense, our paper also connects to recent work pushing back against the narrative that exchange rates are disconnected from fundamentals. Bodenstein et al. (2024) link exchange rate dynamics to trade balance shocks, which are reduced-form demand disturbances. Kekre and Lenel (2024) attribute the majority of cross-country exchange rate variation to demand movements, viewed through the lens of resulting interest rate differentials. Mac Mullen (2025) shows that these interest rate differentials can be partially attributed to fiscal deficit dynamics, in line with the evidence and models presented here.

By analysing interactions between fiscal hegemons and countries that lack the sovereign-debt privilege conferred by financial dominance, we also speak to this literature. Ellison and Scott (2020) provide a long-run perspective on the valuation of U.K. national debt that served as a cornerstone

[1] These include the possibility of a disconnect from macroeconomic fundamentals (Meese and Rogoff, 1983), the forward premium puzzle (Fama, 1984), the slightly negative correlation of real exchange rates and relative consumption (Backus and Smith, 1993; Corsetti et al., 2008), the Purchasing Power Parity puzzle (Rogoff, 1996), and the lower volatility of the terms of trade relative to the real exchange rate (Atkeson and Burstein, 2008), among others.

for our analysis; our contribution is the addition of the international dimension, the extension of their data sample, and an increased focus on inflationary factors. Jiang et al. (2024b) focus extensively on the U.S. experience in recent decades, finding evidence of a persistent mispricing of U.S. sovereign debt. Considering the evidence presented in Chen et al. (2022b), this dominant position of U.S. sovereign debt was inherited from the United Kingdom in the early 20th century. Ding and Jiang (2024) make explicit a number of channels through which monetary and fiscal policy are intertwined in the presence of a dominant fiscal and financial hegemon.[2] Jiang et al. (2021) link the valuation of the U.S. Dollar to convenience yields enjoyed by foreign investors holding safe U.S. assets, and Jiang (2021) argue that the U.S. fiscal cycle links currency risk premia of the Dollar to those of other currencies. Jiang et al. (2023) and Jiang et al. (2024a) are closest to our argument, showing how convenience yields on Dollar-denominated debt enter as a wedge in the Euler equations that determine exchange rates.

The paper is organised as follows: section 2 lays out the present value accounting framework and derives its open economy linearisation, from which we derive a market-implied measure of surplus/deficit innovations. We then use that measure to link deficits, exchange rates, and inflation in an empirical way. Section 3 develops a continuous-time model to show how fiscal innovations transmit across countries through exchange rate movements, bond flows, and interest rates, culminating in nominal and real effects of fiscal expansions across borders in ways consistent with our empirical findings.[3] Section 4 concludes with policy implications for debt management and for the coordination of fiscal and exchange rate policies. Further details on the data, additional model derivations, and all proofs can be found in the appendices.

2 Empirical Framework

The correct measurement of surplus and deficit innovations is non-trivial due to the fundamental endogeneity of fiscal policy. No single method is considered a clear-cut standard for measuring fiscal innovations (Ramey, 2016, 2019). The consideration of international aspects of cross-border bond trade likely amplifies existing concerns while making these questions even more important for understanding exchange rate determination and the international transmission of inflation.

Anticipating our modelling effort in Section 3, our empirical framework places the pricing of government debt on financial markets at its centre. Informed by the market-value identities of Cochrane (2019), we develop an empirical open-economy framework in which we price U.K. and U.S. sovereign debt in line with a government debt valuation equation, taking into account exchange rate adjustments. We embed exchange rates and the UIP condition in the pricing of government

[2] This literature, in turn, can be linked to insights from the literature on dominant currency paradigms, surveyed in Gopinath and Itskhoki (2022).

[3] A simple two-period version of the model can be found in appendix A.

debt to show that foreign discount-rate news and currency movements enter fiscal constraints on equal footing with domestic surpluses and inflation. We then use the resulting identity, which holds as an accounting relation in every model, to decompose debt dynamics into domestic channels (surpluses, deficits, growth) and international channels (UIP, foreign surpluses and deficits). While our use of the *market value* of government debt (as opposed to its *face value*) is not novel (see Cochrane (2019); Hall and Sargent (2011)), it is a necessary ingredient for recovering the only feasible surplus surprises consistent with the value of government debt itself.

Data. We collect U.K. public debt market values from Ellison and Scott (2020) and update them through 2024 with the Gilt database of Cairns and Wilkie (2024). U.S. debt market values are taken from Cochrane (2019), again suitably updated. Macroeconomic aggregates for both countries are from Müller et al. (2025). The real exchange rate is recovered from the BIS Real Effective Exchange Rate index. We use market values throughout rather than face (book) values. Book-value measures omit capital gains within the portfolio, so the period-by-period budget identity requires ad-hoc valuation adjustments and any present-value decomposition is biased (Hall and Sargent, 2011). Market values embed time-varying real yields and risk premia by construction, ensuring that the decompositions below close exactly without a residual discrepancy, which allows us to recover the model-consistent surplus/deficit surprise.[4]

2.1 The Open-Economy Debt Valuation Identity

We begin by describing the procedure that allows us to recover realised surplus and deficit innovations. We first present the baseline result of Cochrane (2019) before extending it to our open-economy setting.

Single-country baseline. We define the nominal end-of-period market value of government liabilities as

$$V_t \equiv M_t + \sum_{j=0}^{\infty} Q_t^{(t+1+j)} B_t^{(t+1+j)}, \tag{1}$$

where M_t is non-interest-bearing money, $B_t^{(t+j)}$ is the face value of a zero-coupon bond due at $t+j$, and $Q_t^{(t+j)}$ is its time-t price. The log debt-to-GDP ratio is $v_t \equiv \log(V_t/P_tY_t)$ and the log nominal holding-period return on the portfolio is $r_{t+1}^n \equiv \log R_{t+1}^n$, where

$$R_{t+1}^n \equiv \frac{M_t + \sum_{j=1}^{\infty} Q_{t+1}^{(t+j)} B_t^{(t+j)}}{M_t + \sum_{j=1}^{\infty} Q_t^{(t+j)} B_t^{(t+j)}}.$$

[4]We recover the primary surplus residually from $s_{t+1} = v_t + r_{t+1}^n - \pi_{t+1} - g_{t+1} - \rho v_{t+1}$, so the constructed surplus satisfies the flow identity exactly. Official surplus measures, e.g., from NIPA in the U.S., omit mark-to-market valuation changes, understating fiscal effort when bond prices fall and overstating it when prices rise (Hall and Sargent, 2011).

Log-linearising the nonlinear flow identity following Campbell and Shiller (1988) and Cochrane (2019) (derivation in Appendix C.1) yields the following result.

Lemma 1 (Linearised Flow Identity; Cochrane, 2019, 2022b)**.** *The linearised government debt valuation equation is given by*

$$\rho\, v_{t+1} = v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} - s_{t+1}, \tag{2}$$

where π_{t+1} is CPI inflation, g_{t+1} is log real GDP growth, s_{t+1} is the real primary surplus scaled by GDP, and ρ is the discount factor.

Equation (2) is a pure accounting identity that must hold exactly period by period in market values. It states that the debt ratio next period is determined by today's debt ratio, augmented by the nominal return on the government bond portfolio, and eroded by inflation, growth, and the primary surplus. Crucially, the identity is silent on causation: in a Ricardian world, surpluses adjust to satisfy (2) for any given path of prices and interest rates; in a fiscal-theory world, prices and interest rates adjust to satisfy it for a given path of surpluses. Our empirical strategy remains agnostic between these interpretations, since the identity holds under both.

Open-economy extension. Consider a Home country, labelled H, and a Foreign country, labelled F. Applying (2) to each country separately yields

$$\rho_H v^H_{t+1} = v^H_t + r^{n,H}_{t+1} - \pi^H_{t+1} - g^H_{t+1} - s^H_{t+1}, \tag{3}$$
$$\rho_F v^F_{t+1} = v^F_t + r^{n,F}_{t+1} - \pi^F_{t+1} - g^F_{t+1} - s^F_{t+1}. \tag{4}$$

Let $e_t \equiv \log \mathcal{E}_t$ denote the log nominal exchange rate. For concreteness, we let the Home country be the U.K. and the Foreign country be the U.S., so that an increase in e_t represents a depreciation from the U.K. perspective. The UIP condition with a mean-zero risk-premium shock is

$$r^{n,H}_{t+1} - r^{n,F}_{t+1} = \Delta e_{t+1} + \varphi_{t+1}, \qquad \mathbb{E}_t[\varphi_{t+1}] = 0. \tag{5}$$

The key step in our open-economy extension is to substitute (5) into (3), replacing the Home nominal return with its UIP-implied counterpart. This yields the central empirical equation by which we recover surpluses:

$$\rho_H v^H_{t+1} = v^H_t + \underbrace{\left(r^{n,F}_{t+1} + \Delta e_{t+1} + \varphi_{t+1}\right)}_{\text{external discount factor}} - \pi^H_{t+1} - g^H_{t+1} - s^H_{t+1}. \tag{6}$$

The foreign discount rate $r^{n,F}_{t+1}$, the exchange rate depreciation Δe_{t+1}, and the UIP premium φ_{t+1} enter the U.K. budget constraint on exactly the same footing as domestic inflation and growth. In effect, we treat U.S. discount rates as an anchor for international financial flows, with U.K.

discount rates following as a direct function thereof after accounting for exchange rate-related revaluations.

The economic content of equation (6) is best understood through a concrete example. Suppose that U.S. Treasury yields rise by 100 basis points because markets revise down expected U.S. surpluses, raising $r^{n,F}_{t+1}$ in (4). Under the UIP link, this increase in the Foreign discount rate feeds one-for-one into the U.K. budget constraint through the external discount factor in (6). The U.K. debt ratio must therefore be higher next period than it would have been absent the U.S. shock, unless at least one of the remaining terms adjusts. This observation is the core of the international transmission mechanism: the U.K. intertemporal budget constraint tightens in response to a purely Foreign fiscal deterioration.

The adjustment can come through at least one of four channels. *Fiscal inflation* erodes the real value of outstanding Sterling debt, effectively taxing existing bondholders. Stronger *growth* raises the tax base and partially self-finances the higher debt burden (Angeletos et al., 2024). A *Dollar depreciation* occurs as investors reallocate away from Dollar assets, mechanically reducing the Sterling value of the external discount factor. Finally, *UIP compression* arises if the convenience yields on U.S. treasuries falls, narrowing the wedge between Home and Foreign returns. Any combination of these four adjustments that satisfies (6) is consistent with U.K. fiscal solvency without requiring any change in U.K. surpluses.

Two features of this result merit emphasis. The first is that the transmission is symmetric in magnitude but asymmetric in direction: a U.S. deficit shock tightens the U.K. constraint, but the reverse need not hold with the same force, because the UIP substitution replaces the Home return with the Foreign return, not vice versa. This asymmetry reflects a modelling choice that assigns the anchor role to the larger economy's discount rate, consistent with the empirical dominance of U.S. Treasury yields in global fixed-income markets. The second is that the identity (6) places no restriction on which channel does the adjusting. In the data, all four channels operate simultaneously, and the backward and forward decompositions in Section 2.2 are designed to measure the relative contribution of each. This is the empirical counterpart of the fiscal contagion mechanism documented in our theoretical model. Appendix C provides a full microfoundation.

2.2 Backward and Forward Decompositions

Before examining the direct effects of surpluses on exchange rates and inflation, we first analyse the properties of the market-implied surpluses we recover. Following Cochrane (2019), we decompose the drivers of the value of U.K. government debt using both backward and forward perspectives.

2.2.1 Backward Decomposition

Iterating (6) forward from a base year (setting $\rho_H \approx 1$ throughout)[5] yields the backward accounting identity:

$$v_t^H = v_0^H + \sum_{j=1}^{t} \left[r_j^{n,F} + \Delta e_j + \varphi_j \right] - \sum_{j=1}^{t} \pi_j^H - \sum_{j=1}^{t} g_j^H - \sum_{j=1}^{t} s_j^H. \tag{7}$$

This is a pure arithmetic attribution: it measures, rather than tests, the present-value relation (Cochrane, 2019). Each term cumulates one channel's contribution, and their sum equals the realised debt path at every date by construction.

Figure 2 applies (7) to U.K. data from 1975, with each series starting at zero in that year. Table 1 defines each series formally. Three results stand out. First, primary surpluses account for approximately half of the cumulative change in the U.K. debt ratio since 1975; the remainder is accounted for by valuation effects, split broadly between the global discount factor ($r^{n,F} - \pi^H - g^H$) and the exchange rate channels (REER and UIP). Second, every major regime break identified by Bordo et al. (2022) appears as a kink in a valuation curve rather than in the surplus path, confirming that discount-rate repricing is closely related to observed debt dynamics. Third, the open-economy channels are quantitatively material: a permanent 10 % Sterling depreciation raises the domestic-currency value of Dollar coupons by the same 10 %, while a 200 bp UIP shock adds roughly 10% of GDP to the market value of outstanding debt without any change in cash surpluses.[6]

Series	Formula (base 1975)	Interpretation
v	$v_0^H + \sum_{j\le t}(r_j^{n,F} + \Delta e_j + \varphi_j - \pi_j^H - g_j^H - s_j^H)$	Realised market-value debt/GDP ratio.
$-s$	$v_0^H - \sum_{j\le t} s_j^H$	Counterfactual driven solely by realised surpluses.
$r^n - \pi - g$	$v_0^H + \sum_{j\le t}(r_j^{n,F} - \pi_j^H - g_j^H)$	Cumulated real discount-rate contribution.
r^n	$v_0^H + \sum_{j\le t} r_j^{n,F}$	Cumulated foreign nominal return.
$-\pi$	$v_0^H - \sum_{j\le t} \pi_j^H$	Inflation erosion of the real debt value.
$-g$	$v_0^H - \sum_{j\le t} g_j^H$	Growth contribution.
REER	$v_0^H + \sum_{j\le t} \Delta e_j$	Cumulated Sterling depreciation.
UIP	$v_0^H + \sum_{j\le t} \varphi_j$	Cumulated FX risk-premium shocks.

Table 1: Components of the backward decomposition, equation (7), and their mapping to Figure 2.

[5] To be precise, we set $\rho_H \simeq 0.97$ at annual frequency. Suppressing it keeps notation light and has negligible effect on the decomposition over the sample.

[6] A detailed exposition of the historical episodes can be found in Appendix C.2.1.

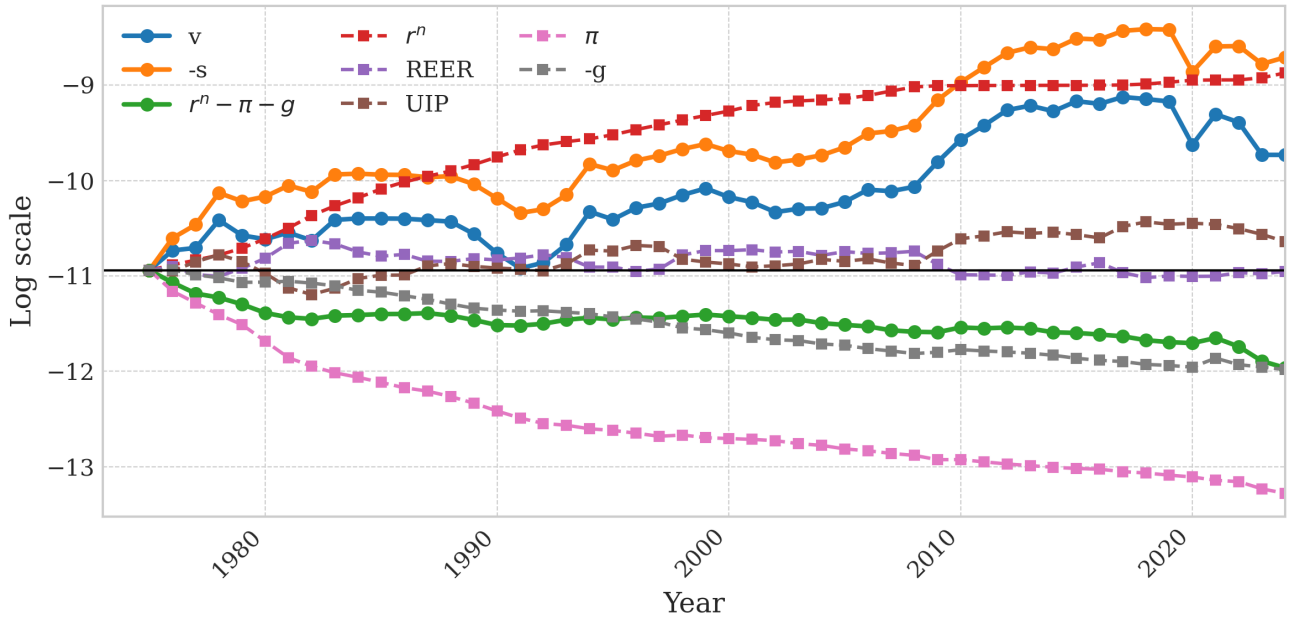


Figure 2: Backward decomposition of U.K. government debt, 1975–2024. Each path cumulates one term of identity (7) from the 1975 base year. The vertical sum of all series equals the realised market-value debt-to-GDP ratio (blue line, v) at every date.

2.2.2 Forward Decomposition

The forward decomposition replaces realised values with conditional expectations: the present-value contribution of each channel at date t is the amount that channel alone would need to provide to balance the intertemporal budget constraint given information at t. Taking expectations of the iterated identity yields the forward balance condition:[7]

$$v_t^H = PV_t^{(s)} + PV_t^{(g)} - PV_t^{(d)}, \qquad PV_t^{(d)} \equiv PV_t^{(r)} - PV_t^{(\pi)}, \tag{8}$$

where each $PV_t^{(k)}$ is the expected discounted sum of component k from date t forward.

VAR implementation. We bring (8) to the data via a first-order annual VAR following Cochrane (2019, 2022b):

$$x_{t+1} = Ax_t + \varepsilon_{t+1}, \qquad \varepsilon_{t+1} \sim \mathcal{N}(0, \Sigma), \tag{9}$$

where the vector x_t contains the foreign nominal return $r_t^{n,F}$, U.K. surplus/deficit innovations s_t^H, inflation π_t^H, growth g_t^H, and the log debt ratio v_t^H, as well as the real exchange rate change Δe_t, the ex-ante UIP premium φ_t, and short and long U.S. Treasury yields as predictors of future returns. All eigenvalues of A lie strictly inside the unit circle, ensuring finite long-run multipliers.

[7] The derivation of the relevant steps is provided in Appendix C.2.

With row-selection vectors a_k picking out component k, the present-value contributions are

$$PV_t^{(k)} \equiv a_k'(I - A)^{-1}Ax_t, \qquad k \in \{s,\, g,\, r,\, \pi,\, \Delta e,\, \varphi\}, \tag{10}$$

yielding an infinite-horizon present value in a single matrix multiplication in which all projections are internally consistent with the accounting identity.

Figure 3 and Table 2 present the results of the forward decomposition exercise. Deficit/surplus news and real discount-rate news are the dominant contributors to debt-ratio fluctuations across the full sample. The exchange rate and UIP channels are nevertheless economically significant: they reconcile the relative undervaluation of U.K. debt implied by the surplus process alone in the pre-2000 period and the relative overvaluation thereafter.[8]

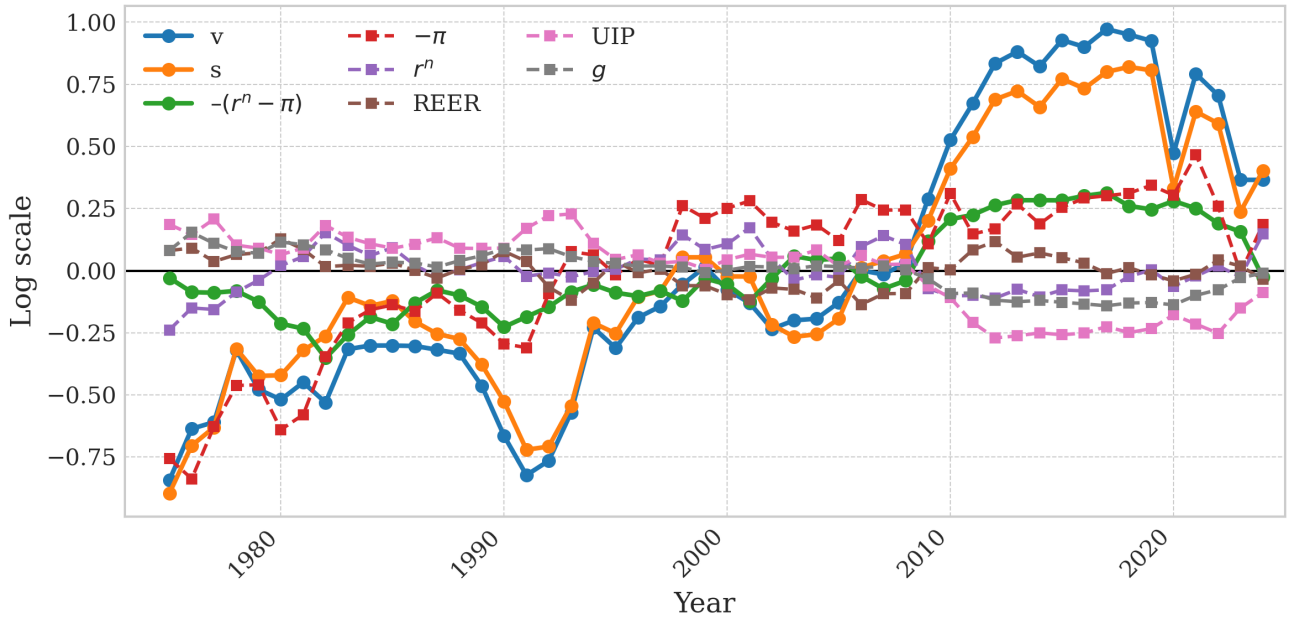


Figure 3: Forward decomposition of the value of U.K. government debt. The blue line is the realised debt ratio v_t^H. Every other series is the corresponding present-value component $PV_t^{(k)}$ from (10), formed with information available at date t. By construction, $v_t^H = PV_t^{(s)} + PV_t^{(g)} - PV_t^{(d)}$.

2.3 Exploiting Deficit Shocks to Analyse Exchange Rate Movements

In our derivation of market value-consistent surpluses, we have already shown how UIP deviations and cross-border interest rate spillovers matter significantly for the equilibrium valuation of government debt. We now leverage the recovered surplus and *deficit* processes for the U.K. and the

[8] A historical variance decomposition assigning shares of total variance to each channel is provided in Appendix C.2.

Series	Object	Economic meaning
v	v_t^H	Market-value debt/GDP ratio.
s	$PV_t^{(s)}$	PV of expected future primary surpluses.
$-(r^n - \pi)$	$-PV_t^{(d)}$	PV of expected future real discount rates.
r^n	$PV_t^{(r)}$	PV of expected future nominal returns.
$-\pi$	$-PV_t^{(\pi)}$	PV of expected future U.K. inflation.
REER	$PV_t^{(\Delta e)}$	PV of expected future exchange rate changes.
UIP	$PV_t^{(\varphi)}$	PV of expected future UIP-premium shocks.
g	$PV_t^{(g)}$	PV of expected future real growth.

Table 2: Components of the forward decomposition in Figure 3. By construction, $v_t^H = PV_t^{(s)} + PV_t^{(g)} - PV_t^{(d)}$.

U.S. to determine how much each contributes to medium-run movements in U.K. real exchange rates. We view this exercise as informative about the plausible secular determinants of U.K. exchange rates, quantifying how much of the observed variance can be attributed to movements in domestic versus foreign deficits, where foreign deficits matter through foreign interest rates and the associated exchange rate and UIP deviation channels described above.

Table 3 summarises the results of this exercise, linking the deficit innovations implied by the forward decomposition of the VAR in Section 2.2 to observed year-on-year movements in U.K. real exchange rates, that is, after accounting for differences in inflation.[9]

The results are striking. Deficit expectation revisions relate to observed movements in U.K. real exchange rates significantly and in the direction implied by the government debt valuation equation: a deficit innovation in the U.K. is associated with a *depreciation* of the U.K. real exchange rate, with a 1% deficit-to-GDP innovation corresponding to a 37 basis point relative *depreciation*. U.S. deficit dynamics matter an order of magnitude less, as expected and as prescribed by the framework. Quantitatively, a 1% deficit-to-GDP innovation in the U.S. is associated with a 1.8 basis point relative *appreciation* of U.K. real exchange rates. The second column confirms that the *differential* between U.K. and U.S. surpluses relates most materially to annual exchange rate movements. The third column introduces an interaction with domestic recessions, revealing that in such periods deficit innovations may plausibly matter through unconventional channels not otherwise captured.[10]

The observed links between deficits and the exchange rate are fairly robust over time, as we demonstrate using the following local projection on our exogenous deficit measure:

[9] We weight the U.S. deficit innovations by relative U.S./U.K. GDP to reflect the relative size of both countries. Not weighting the deficit does not affect the qualitative results.

[10] In Appendix D we provide additional results based on the Engel and West (2005) reverse-regression approach, showing that exchange rates predict materialised deficits in the U.K. The link between exchange rates and deficits follows an "overshooting" pattern.

	Dependent variable: Δ U.K. REER		
	(1)	(2)	(3)
$\Delta\mathbb{E}(\text{Deficit})_t^{U.K.}$	0.370***	0.352***	0.358***
	(0.077)	(0.075)	(0.107)
$\Delta\mathbb{E}(\text{Deficit})_t^{U.K.} \times \mathbf{1}^{g_Y<0}$			−2.370***
			(0.615)
$\Delta\mathbb{E}(\text{Deficit})_t^{U.S.}$	−0.018***		−0.010**
	(0.005)		(0.004)
$\Delta\mathbb{E}(\text{Deficit})_t^{U.S.} \times \mathbf{1}^{g_Y<0}$			0.210***
			(0.061)
$\Delta\mathbb{E}(\text{Deficit})_t^{U.K.-U.S.}$		0.016***	
		(0.006)	
Constant	-0.006	-0.022	-0.001
	(0.010)	(0.014)	(0.009)
Observations	25	25	25
Adjusted R^2	0.469	0.320	0.523

Note: HAC-robust standard errors. *p<0.1; **p<0.05; ***p<0.01

Table 3: Long-run correlates of U.K. exchange rates.

$$\Delta e_{t+h} = \alpha_h + \beta_h^{UK}\,\text{Def}_t^{UK} + \beta_h^{US}\,\text{Def}_t^{US} + \gamma_h\, e_{t-1} + u_{h,t}, \qquad h = 0, 1, \ldots, H,$$

where Δe_{t+h} is the change in the log real exchange rate index and the right-hand side variables are the U.K. and U.S. deficit measures, respectively. Figure 4 plots the local projection coefficients, showing that the exchange rate response to expected deficit innovations is relatively persistent across horizons.

Does the same relationship hold true for materialized inflation rates, in that both domestic and foreign deficits can influence inflation? We re-estimate the previous local projection, but on observed CPI inflation rates:

$$\Delta P_{t+h}^{CPIUK} = \alpha_h + \beta_h^{UK}\,\text{Def}_t^{UK} + \beta_h^{US}\,\text{Def}_t^{US} + \gamma_h\, P_{t-1}^{CPIUK} + u_{h,t}, \qquad h = 0, 1, \ldots, H,$$

where ΔP_{t+h}^{CPIUK} is the change in the log consumer price index (i.e., cumulative inflation) and the right-hand side variables are the U.K. and U.S. deficit measures, respectively. Figure 5 plots the local projection coefficients. While U.K. deficits matter quantitatively more (even though the effect is econometrically not significant), we can also clearly observe that the U.S. deficit surprises matter both quantitatively and economically for movements of the consumer price index in the U.K.

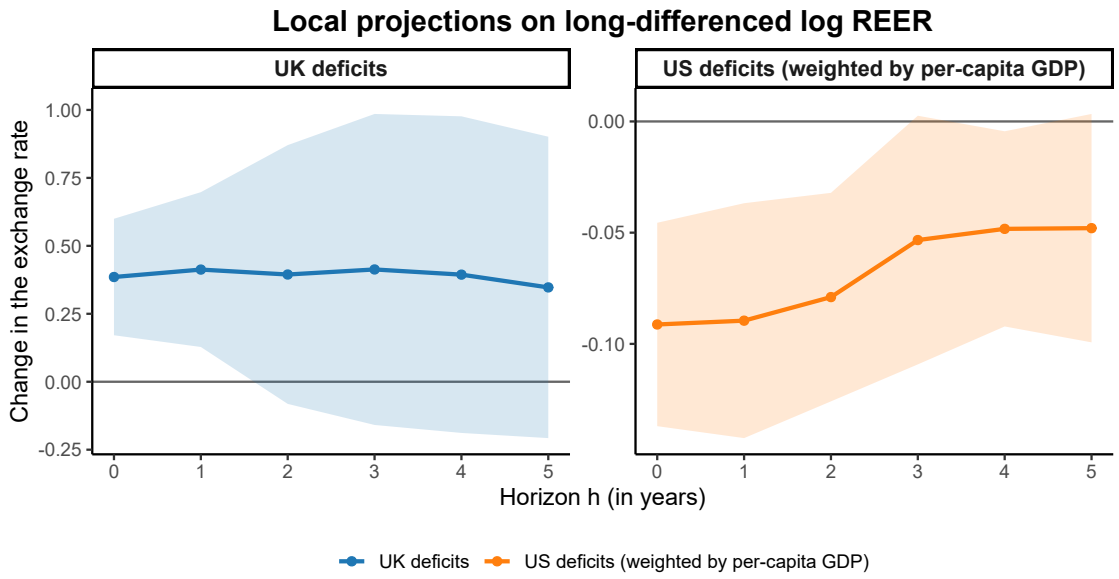


Figure 4: The effect of deficit surprises in the U.K. and the U.S. on the U.K. REER.

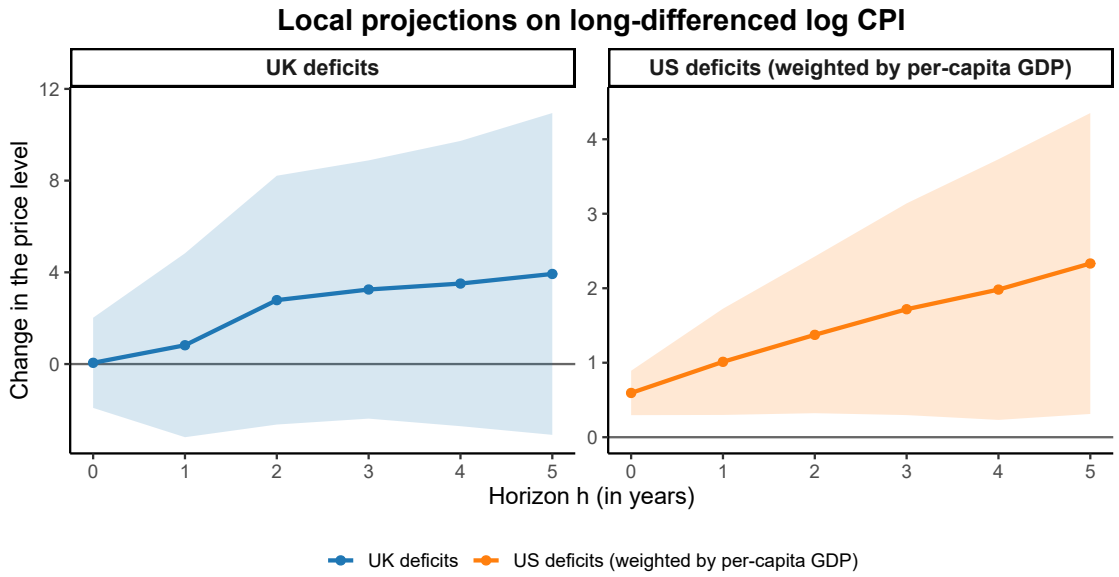


Figure 5: The effect of deficit surprises in the U.K. and the U.S. on the price level in the U.K.

In sum, our long-run historical analysis, centred on the dynamics underpinning the valuation of government debt, underscores that the domestic debt valuation equation is necessarily linked to the UIP condition and to fluctuations in the real exchange rate. We have further shown how measured surplus/deficit innovations contribute to observed movements in U.K. real exchange and inflation rates, which holds true not only for U.K. surplus/deficit innovations, but also for those of the U.S.

The link between expected surplus and deficit dynamics and exchange rates constitutes the focal novelty of the paper. In this section, we have established this link from a longer-run perspective, focusing on the mechanics of the government debt valuation equation. We complement these results with a *shorter-run* perspective in Appendix E, where we utilise domestic short-run expectations of fiscal deficits to analyse their contribution to observed exchange rate revaluations.

Because we have focused on real exchange rates, the adjustment mechanism can plausibly operate through either changes in the nominal (spot) USD-GBP exchange rate or through differences in price level movements between the U.S. and the U.K. We now turn to a quantitative model in which we rationalise this international contagion mechanism of fiscal policy, explaining the observed adjustment through movements in the nominal exchange rate and differences in inflation.

3 Continuous-Time Model

We now develop a continuous-time model of internationally transmitted fiscal spillovers. The core mechanism highlights how fiscal shocks propagate internationally through exchange rate and discount rate adjustments. Both countries produce and trade differentiated goods, so that fiscal policy affects the foreign economy not only through asset-pricing channels but also through shifts in aggregate demand. Nominal rigidities in both countries generate a supply-side response to these demand disturbances, giving rise to fiscally driven inflation spillovers. Households and firms in each country transact freely in goods and asset markets, each economy features its own intermediate and final good producers operating under Rotemberg price adjustment costs, and both monetary and fiscal authorities operate under potentially distinct policy regimes. This combination of wealth effects and expenditure switching, together with the fiscal-monetary policy mix, determines the international transmission of fiscal shocks.

Despite this richness, the competitive equilibrium reduces to a concise system of ten equations: two IS curves, two Phillips curves, two debt-valuation equations, a bond market clearing condition, a terms-of-trade equation, the evolution of the exchange rate, and a no-arbitrage condition. The present section states the model's ingredients and the key equilibrium objects; all proofs are collected in Appendix F.[11]

[11] In Appendix A, we provide a complementary two-period model that outlines our main mechanism more simply.

3.1 Domestic Household

The representative Home household maximises

$$\int_0^\infty e^{-\rho t}\left\{\log C_t - \frac{N_t^{1+\phi}}{1+\phi} + \nu(b_{Ht}, C_t;\, \theta_t)\right\} dt \tag{11}$$

subject to the flow budget constraint

$$P_t C_t + \dot{B}_{Ht} + E_t \dot{B}_{Ft} = i_t B_{Ht} + E_t i_t^* B_{Ft} + W_t N_t + T_t, \tag{12}$$

where C_t is a CES aggregate of Home and Foreign goods (with expenditure weight γ on Foreign goods and intratemporal elasticity η), N_t is labour supply, $b_{Ht} \equiv B_{Ht}/P_t$ denotes real Home bond holdings, and $\nu(\cdot)$ is a convenience flow earned on those bonds with θ_t to be discussed shortly. Cost minimisation over the CES bundle yields the standard demand system and the CPI definition

$$P_t = \left[(1-\gamma)P_{Ht}^{1-\eta} + \gamma P_{Ft}^{1-\eta}\right]^{1/(1-\eta)}, \tag{13}$$

with CPI inflation given by $\pi_t \equiv \dot{P}_t/P_t$, the real exchange rate by $S_t \equiv E_t P_t^*/P_t$, and the terms of trade by $\text{tot}_t \equiv P_{Ft}/P_{Ht}$. The full intratemporal problem is standard and collected in Appendix F.1.

Convenience yield. Following Jiang et al. (2023, 2024a), we introduce a convenience yield on Home bonds to break perfect risk-sharing and resolve the Backus and Smith (1993) puzzle.[12] We adopt an isoelastic specification in real bond holdings relative to steady state:

$$\nu(b_{Ht}, C_t;\, \theta_t) = \frac{\bar{\Psi}\,\theta_t\,\bar{b}_H}{(1-\zeta_H)\,C_t}\left[\left(\frac{b_{Ht}}{\bar{b}_H}\right)^{1-\zeta_H} - 1\right], \qquad \zeta_H > 0. \tag{14}$$

Scaling by $1/C_t$ makes the convenience flow complementary to consumption rather than separable, so that $\nu_C = -\nu_t/C_t$. The process θ_t captures time-varying perceived liquidity of Home debt. The marginal convenience value of real Home bond holdings is

$$\Psi_t \equiv \nu_B\, C_t = \bar{\Psi}\,\theta_t\left(\frac{b_{Ht}}{\bar{b}_H}\right)^{-\zeta_H}. \tag{15}$$

The convenience yields are the sole non-standard component of the Home household utility function. Three composites summarise how the convenience yield enters the pricing kernel:

[12] In fact, the Backus-Smith puzzle is broken in our model whenever Home bonds bear some convenience flows that are not equal in consumption-equivalent terms across countries, which can either be induced through deficit-finance or different slopes of each country's demand function for convenience-bearing debt. In the later section, both mechanisms are implicitly explored, and they both induce a correlation between cross-country consumption growth differences and changes to the exchange rate of $\rho(\Delta c_t - \Delta c_t^*, e_t) \approx 0.4$, with the correlation being -1 in the case without convenience yields and without fiscal deficit-finance.

$$\mu_t \equiv \tfrac{1-\nu_t}{C_t}, \tag{16}$$

$$\Phi_t \equiv \tfrac{\Psi_t}{1-\nu_t}, \tag{17}$$

$$\chi_t \equiv \tfrac{\dot{\nu}_t}{1-\nu_t}. \tag{18}$$

Here μ_t is the marginal utility of consumption adjusted for the service flow from bonds, Φ_t is the *effective convenience yield* (the non-pecuniary return that lowers the required nominal yield on Home debt), and χ_t is the *preference drift* induced by time variation in ν_t, which enters the Euler equation as a time-varying discount rate.

Optimality conditions. Household optimisation yields the optimal labour supply condition, two asset Euler equations, and a modified UIP condition (see Appendix F.1 for the derivation):

$$C_t N_t^{\phi} = \frac{1-\nu_t}{P_t} W_t, \tag{19}$$

$$\frac{\dot{C}_t}{C_t} = i_t - \pi_t - \rho + \Phi_t - \chi_t, \tag{20}$$

$$\frac{\dot{C}_t}{C_t} = i_t^* - \pi_t - \rho + \tfrac{\dot{E}_t}{E_t} - \chi_t. \tag{21}$$

The effective convenience yield Φ_t enters only the Home-bond Euler equation (20): Home bonds yield a non-pecuniary return that lowers the nominal rate required for the Euler equation on Home bonds to hold. Differencing (20) and (21) yields the modified uncovered interest parity condition

$$i_t - i_t^* = \frac{\dot{E}_t}{E_t} - \Phi_t. \tag{22}$$

The convenience yield introduces a wedge into UIP: relative to the standard condition, the Home rate can fall short of the Foreign rate by Φ_t for the same expected exchange rate appreciation. This is the channel through which Home-bond demand feeds into exchange rate determination and, ultimately, into the cross-country transmission of fiscal shocks.

3.2 Foreign Household

The Foreign household problem is symmetric to the Home household's, with one important asymmetry: Foreign households derive a convenience yield from holding *Home-currency* bonds, not from their own government's debt. The full derivation is in Appendix F.2; here we state the key objects and results. In line with convention, starred variables describe Foreign-country quantities.

The Foreign consumption aggregate is a CES bundle over Foreign and Home goods with the

same home-bias parameter γ and elasticity η as the Home household, yielding the Foreign CPI $P_t^* = [(1-\gamma)P_{Ft}^{*\,1-\eta} + \gamma P_{Ht}^{*\,1-\eta}]^{1/(1-\eta)}$ and Foreign terms of trade $\text{tot}_t^* \equiv P_{Ht}^*/P_{Ft}^*$.

The Foreign household maximises

$$\int_0^\infty e^{-\rho^* t}\left\{\ln C_t^* - \frac{N_t^{*\,1+\phi^*}}{1+\phi^*} + \nu\left(\frac{B_{Ht}^*}{E_t};\, \theta_t^*\right)\right\} dt \tag{23}$$

subject to $P_t^* C_t^* + \dot{B}_{Ft}^* + E_t^{-1}\dot{B}_{Ht}^* = i_t^* B_{Ft}^* + E_t^{-1} i_t B_{Ht}^* + W_t^* N_t^* + T_t^*$. The convenience yield follows the same isoelastic form as (14), but is scaled by the exchange rate to express Home bond holdings in Foreign currency units:

$$\nu\left(\frac{b_{Ht}^*}{E_t}, C_t^*;\, \theta_t^*\right) = \frac{\bar{\Psi}^*\,\theta_t^*\,\bar{b}_H^*}{(1-\zeta_F)\,C_t^*\,\bar{E}}\left\{\left[\left(\frac{b_{Ht}^*}{\bar{b}_H^*}\right)\left(\frac{\bar{E}}{E_t}\right)\right]^{1-\zeta_F} - 1\right\}, \quad \zeta_F > 0, \tag{24}$$

with marginal convenience value

$$\Psi_t^* \equiv \bar{\Psi}^*\,\theta_t^*\left[\left(\frac{b_{Ht}^*}{\bar{b}_H^*}\right)\left(\frac{\bar{E}}{E_t}\right)\right]^{-\zeta_F}, \tag{25}$$

and effective composites $\Phi_t^* \equiv \Psi_t^*/(1-\nu_t^*)$ and $\chi_t^* \equiv \dot{\nu}_t^*/(1-\nu_t^*)$, defined analogously to (17)–(18). Note that Φ_t^* depends on both b_{Ht}^* and E_t: a depreciation of the Home currency reduces the Foreign-currency value of Home bond holdings, compressing the convenience wedge and feeding directly into the Foreign Euler equation.

Optimisation yields the Foreign counterparts to (19)–(21):

$$C_t^* N_t^{*\phi^*} = \frac{1-\nu_t^*}{P_t^*} W_t^*, \tag{26}$$

$$\frac{\dot{C}_t^*}{C_t^*} = i_t^* - \pi_t^* - \rho^* - \chi_t^*, \tag{27}$$

$$\frac{\dot{C}_t^*}{C_t^*} = \left(i_t - \frac{\dot{E}_t}{E_t}\right) - \pi_t^* - \rho^* + \Phi_t^* - \chi_t^*. \tag{28}$$

The asymmetry relative to the Home block is visible in (27): Foreign households derive *no* convenience from their own bonds, so Φ_t^* appears only in the Home-bond Euler equation (28). Differencing the two Euler equations yields the Foreign UIP condition

$$i_t - i_t^* = \frac{\dot{E}_t}{E_t} - \Phi_t^*, \tag{29}$$

which is consistent with (22): both households impose the same no-arbitrage wedge $\Phi_t = \Phi_t^*$ in equilibrium.

3.3 Risk-Sharing

Differencing the Home and Foreign Euler equations on their respective own-currency bonds, (20) and (27), and substituting the UIP condition (22), yields the international risk-sharing condition (Itskhoki and Mukhin, 2021):

$$\frac{\dot{C}_t}{C_t} - \frac{\dot{C}^*_t}{C^*_t} - \frac{\dot{Q}_t}{Q_t} = (\rho^* - \rho) + \underbrace{(\chi^*_t - \chi_t)}_{\text{convenience wedge}}, \qquad Q_t \equiv \frac{E_t P^*_t}{P_t}. \tag{30}$$

Equation (30) generalises the complete-markets benchmark in two directions. First, possibly asymmetric time preferences ($\rho \neq \rho^*$) drive a wedge between relative consumption growth and real exchange rate movements even absent any financial friction. Second, the convenience term $\chi^*_t - \chi_t$ captures the imperfect risk-sharing induced by time-varying liquidity valuations: when the perceived convenience of Home bonds rises relative to Foreign bonds, Home households smooth consumption less aggressively, generating excess co-movement between Home consumption and the exchange rate. When $\rho = \rho^*$ and $\chi_t = \chi^*_t = 0$, condition (30) collapses to the standard perfect risk-sharing benchmark $\dot{C}_t/C_t - \dot{C}^*_t/C^*_t = \dot{Q}_t/Q_t$.

3.4 Production

The supply side follows a standard two-tier New Keynesian specification in both countries. A competitive final good sector aggregates a unit continuum of differentiated intermediates via Dixit–Stiglitz technology, while monopolistically competitive intermediate producers set prices subject to Rotemberg adjustment costs.

Final good producers. Cost minimisation by the competitive Home final good producer yields the familiar isoelastic demand schedule for each intermediate variety j:

$$Y_{Ht}(j) = \left[\frac{p_{Ht}(j)}{P_{Ht}}\right]^{-\varepsilon} Y_{Ht}, \tag{31}$$

with aggregate price index $P_{Ht} \equiv \left[\int_0^1 p_{Ht}(j)^{1-\varepsilon} dj\right]^{1/(1-\varepsilon)}$. The Foreign sector is symmetric, yielding $Y^*_{Ft}(j) = [p^*_{Ft}(j)/P^*_{Ft}]^{-\varepsilon} Y^*_{Ft}$ with P^*_{Ft} defined analogously. The full Dixit–Stiglitz aggregation problem is standard and relegated to Appendix F.3.

Intermediate good producers. Each Home intermediate producer j operates the linear technology $Y_{Ht}(j) = A_t n_t(j)$, where A_t is aggregate productivity and $n_t(j)$ is labour input. Nominal

rigidities take the form of Rotemberg quadratic price adjustment costs (Rotemberg, 1982):

$$\Theta_t(j) = \frac{\theta}{2}\Big(\frac{\dot{p}_{Ht}(j)}{p_{Ht}(j)}\Big)^2 P_{Ht} Y_{Ht}.$$

In the presence of convenience yields, the household's nominal stochastic discount factor satisfies $-\dot{M}_t^n/M_t^n = i_t + \Phi_t$, so intermediate firms discount nominal profits at $i_t + \Phi_t$ rather than i_t alone. Under symmetric equilibrium pricing $p_{Ht}(j) = P_{Ht}$ (Galí, 2008), the firm's price-setting problem delivers the following result.

Lemma 2 (Rotemberg inflation dynamics)**.** *Home producer price inflation $\pi_{Ht} \equiv \dot{P}_{Ht}/P_{Ht}$ satisfies*

$$\Big[i_t + \Phi_t - \pi_{Ht} - \tfrac{\dot{Y}_{Ht}}{Y_{Ht}}\Big]\pi_{Ht} = \frac{\varepsilon - 1}{\theta}(mc_t - 1) + \dot{\pi}_{Ht}, \tag{32}$$

where $mc_t \equiv \frac{\varepsilon}{\varepsilon-1}\frac{W_t}{P_{Ht}A_t}$ is real marginal cost. Foreign producer price inflation π^*_{Ft} satisfies the symmetric condition with starred variables and $\Phi_t = 0$, since Foreign bonds carry no convenience yield.

Proof. See Appendix F.4. □

Equation (32) is the continuous-time analogue of the forward-looking pricing equation familiar from discrete-time Rotemberg models. The left-hand side bracket is the firm's effective discount rate, augmented by Φ_t relative to the standard case: convenience yields tighten the discount rate applied to future profits, compressing current inflation for a given path of marginal costs. Inflation rises whenever $mc_t > 1$, that is, whenever real marginal cost exceeds its flexible-price target, with sensitivity governed by $(\varepsilon - 1)/\theta$. As $\theta \to 0$, prices become fully flexible and (32) collapses to $mc_t = 1$.

3.5 Aggregate Dynamics

We now characterise the aggregate demand and supply blocks of the two-country economy. Goods-market clearing requires that world demand absorbs each country's output:

$$Y_{Ht} = C_{Ht} + C^*_{Ht}, \tag{33}$$

$$Y^*_{Ft} = C_{Ft} + C^*_{Ft}, \tag{34}$$

and we impose the law of one price for each traded good, $P_{Ht} = E_t P^*_{Ht}$ and $P_{Ft} = E_t P^*_{Ft}$.

Output. Combining goods-market clearing, the demand systems of both households, and the risk-sharing condition (30) yields the following characterisation of equilibrium output.

Lemma 3 (Output)**.** *Equilibrium Home and Foreign output are*

$$Y_{Ht} = A_t\big(\tfrac{\varepsilon-1}{\varepsilon}\big)^{\frac{1}{1+\phi}}(1-\nu_t)^{\frac{1}{1+\phi}}\big[(1-\gamma)+\gamma\,\mathrm{tot}_t^{1-\eta}\big]^{-\frac{1}{1+\phi}}\big[(1-\gamma)+\gamma Q_t^{\eta-1}e^{-\Xi_t}\big]^{\frac{1}{1+\phi}}, \tag{35}$$

$$Y^*_{Ft} = A^*_t\big(\tfrac{\varepsilon-1}{\varepsilon}\big)^{\frac{1}{1+\phi^*}}(1-\nu^*_t)^{\frac{1}{1+\phi^*}}\big[(1-\gamma)+\gamma(\mathrm{tot}^*_t)^{1-\eta}\big]^{-\frac{1}{1+\phi^*}}\big[(1-\gamma)+\gamma Q_t^{1-\eta}e^{\Xi_t}\big]^{\frac{1}{1+\phi^*}}, \tag{36}$$

where the cumulative risk-sharing wedge is $\Xi_t \equiv \int_0^t(\rho^*-\rho+\chi^*_u-\chi_u)\,du$.

Proof. See Appendix F.5. □

Each expression admits an intuitive four-way factorisation. The first factor is productivity A_t, scaled by the markup. The second is the convenience-yield discount $(1-\nu_t)^{1/(1+\phi)}$, which suppresses output by raising the effective cost of labour through the marginal utility of wealth. The third is a terms-of-trade competitiveness term. The fourth is a risk-sharing term governed by the wedge Ξ_t, which cumulates the cross-country gap in impatience and in the rate of change of convenience valuations: when Home bonds become relatively more attractive over time ($\chi_t < \chi^*_t$), Foreign households accumulate more Home assets, appreciating the real exchange rate and shifting demand toward Home goods.

Natural rate and IS curves. Taking the time derivative of (35) under flexible prices gives the natural real interest rate.

Proposition 4 (Natural Real Interest Rate)**.** *Under flexible prices, the Home natural real rate is*

$$r^n_t = \underbrace{(\rho+g_t)}_{\text{closed-economy}} - \underbrace{\Phi_t}_{\text{convenience}} + \underbrace{\frac{\phi}{1+\phi}\chi_t}_{\text{pref. drift}} + \underbrace{\mathcal{T}_t}_{\text{ToT}} + \underbrace{\mathcal{R}_t}_{\text{risk sharing}}, \tag{37}$$

where $g_t \equiv \dot{A}_t/A_t$*, and the open-economy channels are*

$$\mathcal{T}_t \equiv \Big(\tfrac{\phi}{1+\phi}-\tfrac{1}{1-\eta}\Big)\frac{\gamma(1-\eta)\,\mathrm{tot}_t^{1-\eta}}{(1-\gamma)+\gamma\,\mathrm{tot}_t^{1-\eta}}\frac{\dot{\mathrm{tot}}_t}{\mathrm{tot}_t}, \tag{38}$$

$$\mathcal{R}_t \equiv \frac{\phi}{1+\phi}\frac{\gamma Q_t^{\eta-1}e^{-\Xi_t}}{(1-\gamma)+\gamma Q_t^{\eta-1}e^{-\Xi_t}}\Big[(1-\eta)\tfrac{\dot{Q}_t}{Q_t}+\dot{\Xi}_t\Big], \tag{39}$$

with $\dot{\Xi}_t = (\rho^*-\rho)+\chi^*_t-\chi_t$*. The Foreign natural rate* r^{n*}_t *is symmetric with starred variables and without the convenience term* Φ_t*, since Foreign bonds carry no convenience yield in our baseline.*

Proof. See Appendix F.6. □

Proposition 4 generalises the standard Ramsey natural rate $\rho+g_t$ along three dimensions. The convenience yield lowers r^n_t by Φ_t, because Home bonds are partial substitutes for saving

and the market-clearing real rate is correspondingly depressed relative to a model without safe-asset demand. The term χ_t captures the effect of time-varying convenience on intertemporal substitution. Finally, the open-economy terms $\mathcal{T}_t$ and $\mathcal{R}_t$ transmit terms-of-trade movements and imperfect risk-sharing into the natural rate; both vanish as $\gamma \to 0$.

Log-differentiating Lemma 3 yields the following decomposition of output growth into an IS block, a terms-of-trade channel, and a risk-sharing channel.

Proposition 5 (Output Growth)**.** *Home output growth decomposes as*

$$\frac{\dot{Y}_{Ht}}{Y_{Ht}} = \underbrace{\left(i_t - \pi_t - \rho + \Phi_t - \chi_t\right)}_{\text{IS (with convenience + MRS drift)}} - \underbrace{\frac{\eta(1+\phi)}{1+\phi\eta}\,\mathcal{T}_t}_{\text{Terms of Trade}} - \underbrace{\frac{1+\phi}{\phi}\,\mathcal{R}_t}_{\text{Risk sharing}}, \tag{40}$$

where $\Phi_t \equiv \Psi_t/(1-\nu_t)$ *and* $\chi_t \equiv \dot{\nu}_t/(1-\nu_t)$*. Foreign output growth is*

$$\frac{\dot{Y}^*_{Ft}}{Y^*_{Ft}} = \underbrace{\left(i^*_t - \pi^*_t - \rho^* - \chi^*_t\right)}_{\text{Foreign IS (no } \Phi^*_t\text{)}} - \underbrace{\frac{\eta(1+\phi^*)}{1+\phi^*\eta}\,\mathcal{T}^*_t}_{\text{Terms of Trade}} - \underbrace{\frac{1+\phi^*}{\phi^*}\,\mathcal{R}^*_t}_{\text{Risk sharing}}, \tag{41}$$

with $\chi^*_t \equiv \dot{\nu}^*_t/(1-\nu^*_t)$*. The open-economy channels* $\mathcal{T}_t$ *and* $\mathcal{R}_t$ *are defined in* (38)–(39)*, and the Foreign channels* $\mathcal{T}^*_t$ *and* $\mathcal{R}^*_t$ *are defined symmetrically.*

Proof. See Appendix F.7. □

Subtracting the natural rate from the output growth expressions in Lemma 3 yields the IS curves.

Proposition 6 (IS Curves)**.** *The Home and Foreign output-gap dynamics are*

$$\dot{x}_t = (i_t - \pi_t - \rho) - g_t + \left(\Phi_t - \tfrac{\phi}{1+\phi}\chi_t\right) - \mathcal{T}_t - \mathcal{R}_t, \tag{42}$$

$$\dot{x}^*_t = (i^*_t - \pi^*_t - \rho^*) - g^*_t - \tfrac{\phi^*}{1+\phi^*}\chi^*_t - \mathcal{T}^*_t - \mathcal{R}^*_t, \tag{43}$$

where $x_t \equiv \ln Y_{Ht} - \ln Y^n_{Ht}$*,* $x^*_t \equiv \ln Y^*_{Ft} - \ln Y^{n*}_{Ft}$*, and starred open-economy channels are defined symmetrically to* (38)–(39)*.*

Proof. See Appendix F.8. □

The IS curves (42)–(43) contain a standard real-rate-gap term plus three open-economy corrections. The convenience-yield term Φ_t enters only the Home IS curve, directly paralleling its appearance in the Home Euler equation (20) and the natural rate (37). The terms-of-trade channel $\mathcal{T}_t$ links demand conditions to relative price competitiveness, while the risk-sharing channel $\mathcal{R}_t$ transmits imperfect international consumption smoothing into output gaps. Crucially, these channels imply that a Home fiscal shock, by altering Φ_t, the real exchange rate Q_t, and thus Ξ_t, affects

Foreign output gaps directly through (43), even absent any change in Foreign monetary or fiscal policy.

Phillips curves. Linearising the Rotemberg ODE (32) around the non-stochastic steady state yields the New Keynesian Phillips curves.

Proposition 7 (Linearised NKPC)**.** *The Home and Foreign linearised New Keynesian Phillips curves are*

$$\bar{\delta}_H \, \hat{\pi}_{Ht} = \kappa \big[\tilde{c}_t + \phi \, x_t + \gamma \, \tau_t \big] + \dot{\hat{\pi}}_{Ht}, \tag{44}$$

$$\bar{\delta}_F \, \hat{\pi}^*_{Ft} = \kappa \big[\tilde{c}^*_t + \phi^* x^*_t + \gamma \, \tau^*_t \big] + \dot{\hat{\pi}}^*_{Ft}, \tag{45}$$

where $\kappa \equiv (\varepsilon - 1)/\theta$ *is the slope,* $\bar{\delta}_H \equiv \bar{\imath} + \bar{\Phi} - \overline{\dot{Y}_H / Y_H} - \bar{\pi}_H$ *and* $\bar{\delta}_F \equiv \bar{\imath}^* - \overline{\dot{Y}^*_F / Y^*_F} - \bar{\pi}^*_F$ *are steady-state discount rates,* $\tilde{c}_t \equiv \hat{C}_t - \hat{C}^n_t$ *is the consumption gap, and* $\tau_t \equiv \ln \mathrm{tot}_t - \ln \mathrm{tot}^n_t$ *is the terms-of-trade gap.*

Proof. See Appendix F.10. □

Three features of (44)–(45) distinguish this open-economy Phillips curve from its closed-economy counterpart. The steady-state discount rate $\bar{\delta}_H$ embeds the convenience yield $\bar{\Phi}$, so that a higher convenience yield compresses current inflation for a given marginal cost path. Both Phillips curves include a consumption gap $\tilde{c}_t$, arising from the non-separability of the convenience yield in household preferences. The terms-of-trade gap τ_t enters directly, reflecting the fact that import prices feed into real wages and hence into marginal cost. Together, these amendments imply that a Home fiscal expansion, by raising $\tilde{c}_t$ and shifting τ_t, generates inflationary pressure in the Foreign economy through (45). This is the supply-side counterpart to the demand channel documented in the IS curves above.

3.6 Policy

Fiscal rules. Home and Foreign fiscal authorities each face a consolidated nominal flow budget constraint. For the Home government,

$$\dot{B}_t = i_t B_t + P_t G_t - T_t, \tag{46}$$

where $B_t = B_{Ht} + B^*_{Ht}$ is the total face value of Home nominal liabilities, held by domestic and foreign households respectively. Applying the no-Ponzi transversality condition to the real flow identity implied by (46) yields the Home intertemporal debt-valuation equation (derivation in Appendix F.11):

$$b_t = \int_t^\infty \exp\Big(- \int_t^u (i_s - \pi_s) \, ds \Big) s_u \, du, \tag{47}$$

where $b_t \equiv B_t/P_t$ and $s_t \equiv (T_t - P_t G_t)/P_t$ is the real primary surplus. Note that convenience yields enter (47) only indirectly, through their effect on the equilibrium nominal rate i_t; they do not appear in the flow identity itself.

Each fiscal authority closes its policy with a feedback rule for the primary surplus:

$$\hat{s}_t = \psi_b \hat{b}_t + \upsilon_t^s, \qquad \dot{\upsilon}_t^s = -\varphi_s \upsilon_t^s + \sigma_s \varepsilon_t^s, \tag{48}$$

$$\hat{s}_t^* = \psi_b^* \hat{b}_t^* + \upsilon_t^{s*}, \qquad \dot{\upsilon}_t^{s*} = -\varphi_s^* \upsilon_t^{s*} + \sigma_s^* \varepsilon_t^{s*}, \tag{49}$$

where hats denote log-deviations from steady state and ε_t^s, ε_t^{s*} are i.i.d. innovations. The parameter ψ_b governs fiscal *activeness*: when $\psi_b \geq \bar{r}$ the surplus adjusts sufficiently to stabilise debt and the fiscal authority is *passive* (Ricardian); when $\psi_b < \bar{r}$ the government commits to deficit finance and the fiscal authority is *active*. The Foreign block is symmetric, with parameters $(\psi_b^*, \varphi_s^*, \sigma_s^*)$ interpreted analogously.

Monetary rules. Both central banks follow continuous-time Taylor rules:

$$i_t = r_t^n + \bar{\pi} + \phi_\pi(\pi_t - \bar{\pi}) + \phi_x x_t, \tag{50}$$

$$i_t^* = r_t^{n*} + \bar{\pi}^* + \phi_\pi^*(\pi_t^* - \bar{\pi}^*) + \phi_x^* x_t^*, \tag{51}$$

where $\bar{\pi}$ and $\bar{\pi}^*$ are the respective inflation targets and r_t^n and r_t^{n*} are the open-economy natural rates from Proposition 4. The monetary authority is considered *active* when $\phi_\pi > 1$ under $\phi_y = 0$ (the Taylor principle holds) and *passive* when $\phi_\pi < 1$ under $\phi_y = 0$.[13]

Debt valuation with Taylor rules. Substituting the implied ex-post real rates $i_t - \pi_t = r_t^n + (\phi_\pi - 1)\hat{\pi}_t + \phi_x x_t$ and $i_t^* - \pi_t^* = r_t^{n*} + (\phi_\pi^* - 1)\hat{\pi}_t^* + \phi_x^* x_t^*$ into (47) and its Foreign analogue gives the Taylor-rule augmented valuation equations:

$$b_t = \int_t^\infty \exp\Big(-\int_t^u \big[r_\tau^n + (\phi_\pi - 1)\hat{\pi}_\tau + \phi_x x_\tau\big]\, d\tau\Big) s_u \, du, \tag{52}$$

$$b_t^* = \int_t^\infty \exp\Big(-\int_t^u \big[r_\tau^{n*} + (\phi_\pi^* - 1)\hat{\pi}_\tau^* + \phi_x^* x_\tau^*\big]\, d\tau\Big) s_u^* \, du. \tag{53}$$

Equations (52)–(53) are the two central equations governing price-level determination in the model. When B_{t-} (or B_{t-}^*) is predetermined, each equation pins down the contemporaneous CPI from fiscal solvency alone. This is the Fiscal Theory channel: under a passive monetary/active fiscal regime ($\phi_\pi < 1$, $\psi_b < \bar{r}$), the price level adjusts on impact to satisfy (52), with convenience yields entering through their effect on r_t^n.

[13] For $\phi_y \neq 0$, the boundaries under which the eigenvalue associated with monetary policy on the aggregate demand-side is explosive will be altered.

The pair (52)–(53) also encodes the international transmission mechanism of the paper. A deterioration in Home surpluses $\{s_u\}$ or a rise in the Home natural rate r_t^n (itself partly a function of Φ_t and hence of θ_t) lowers b_t for given B_{t-}, requiring a jump in P_t. Through the real exchange rate and the risk-sharing condition (30), this Home price adjustment feeds into the Foreign discount kernel in (53), tightening the Foreign intertemporal budget constraint even when $\{s_u^*\}$ is unchanged.

3.7 Equilibrium

As shown in Appendix H, the model reduces to a system of ten equations in ten endogenous variables. A log-linearised competitive equilibrium is a sequence $\{x_t,\ x_t^*,\ \hat{\pi}_{Ht},\ \hat{\pi}_{Ft}^*,\ \tau_t,\ \hat{b}_t,\ \hat{b}_t^*,\ \hat{b}_{Ht},\ \hat{b}_{Ht}^*,\ \hat{e}_t\}$ driven by exogenous processes $\{\hat{g}_t,\ \hat{g}_t^*,\ \hat{\theta}_t,\ \hat{\theta}_t^*,\ \upsilon_t^s,\ \upsilon_t^{s*}\}$ that satisfies the following system.

IS curves.

$$\dot{x}_t = (\phi_\pi - 1)\big(\hat{\pi}_{Ht} + \gamma\dot{\tau}_t\big) + \phi_x x_t, \tag{54}$$

$$\dot{x}_t^* = (\phi_\pi^* - 1)\big(\hat{\pi}_{Ft}^* - \gamma\dot{\tau}_t\big) + \phi_x^* x_t^*. \tag{55}$$

Phillips curves.

$$\dot{\hat{\pi}}_{Ht} = \bar{\delta}_H\,\hat{\pi}_{Ht} - \kappa\big[(1+\phi)x_t + \gamma\tau_t\big], \tag{56}$$

$$\dot{\hat{\pi}}_{Ft}^* = \bar{\delta}_F\,\hat{\pi}_{Ft}^* - \kappa\big[(1+\phi^*)x_t^* - \gamma\tau_t\big], \tag{57}$$

where $\kappa \equiv (\varepsilon-1)/\theta$, $\bar{\delta}_H \equiv \bar{\imath} + \bar{\Phi} - \overline{\dot{Y}_H/Y_H} - \bar{\pi}_H$, and $\bar{\delta}_F \equiv \bar{\imath}^* - \overline{\dot{Y}_F^*/Y_F^*} - \bar{\pi}_F^*$.

Terms-of-trade dynamics.

$$\begin{aligned}\Lambda\dot{\tau}_t + (1-\phi_\pi)\hat{\pi}_{Ht} - (1-\phi_\pi^*)\hat{\pi}_{Ft}^* = {}& g_t^{\Delta} + (\beta_H+\beta_F)(\rho^*-\rho) \\ &+ \Big(\tfrac{\phi}{1+\phi} - (\beta_H+\beta_F)\Big)\tfrac{\bar{\psi}\bar{\theta}\bar{b}_H}{\bar{C}}\,\hat{\dot{b}}_{Ht} \\ &+ \Big((\beta_H+\beta_F) - \tfrac{\phi^*}{1+\phi^*}\Big)\tfrac{\bar{\psi}^*\bar{\theta}^*\bar{b}_H^*}{\bar{C}^*\bar{E}}\big(\hat{\dot{b}}_{Ht}^* - \dot{\tau}_t + \hat{\pi}_{Ft}^* - \hat{\pi}_{Ht}\big) \\ &+ \phi_x x_t - \phi_x^* x_t^*.\end{aligned} \tag{58}$$

Debt dynamics.

$$\begin{aligned}\hat{\dot{b}}_t = {}& (\bar{r} - \psi_b)\hat{b}_t + \Big[\Big((\phi_\pi - 1) - \beta_H\tfrac{\bar{\psi}^*\bar{\theta}^*\bar{b}_H^*}{\bar{C}^*\bar{E}}\Big)\hat{\pi}_{Ht} + \beta_H\tfrac{\bar{\psi}^*\bar{\theta}^*\bar{b}_H^*}{\bar{C}^*\bar{E}}\hat{\pi}_{Ft}^* + \phi_x x_t + (g_t - \bar{g}) - \bar{\Psi}\hat{\theta}_t + \bar{\Psi}\zeta_H\hat{b}_{Ht} \\ &+ \beta_H(\rho^*-\rho) + \Big(\tfrac{\phi}{1+\phi} - \beta_H\Big)\tfrac{\bar{\psi}\bar{\theta}\bar{b}_H}{\bar{C}}\hat{\dot{b}}_{Ht} + \beta_H\tfrac{\bar{\psi}^*\bar{\theta}^*\bar{b}_H^*}{\bar{C}^*\bar{E}}\hat{\dot{b}}_{Ht}^* + \Big(\xi_H - \beta_H\tfrac{\bar{\psi}^*\bar{\theta}^*\bar{b}_H^*}{\bar{C}^*\bar{E}}\Big)\hat{\dot{\tau}}_t\Big] - \upsilon_t^s,\end{aligned} \tag{59}$$

$$\begin{aligned}\hat{\dot{b}}_t^* = {}& (\bar{r}^* - \psi_b^*)\hat{b}_t^* + \Big[\Big((\phi_\pi^* - 1) + \Big(\tfrac{\phi^*}{1+\phi^*} - \beta_F\Big)\tfrac{\bar{\psi}^*\bar{\theta}^*\bar{b}_H^*}{\bar{C}^*\bar{E}}\Big)\hat{\pi}_{Ft}^* - \Big(\tfrac{\phi^*}{1+\phi^*} - \beta_F\Big)\tfrac{\bar{\psi}^*\bar{\theta}^*\bar{b}_H^*}{\bar{C}^*\bar{E}}\hat{\pi}_{Ht} + \phi_x^* x_t^* \\ &+ (g_t^* - \bar{g}^*) - \beta_F(\rho^*-\rho) + \beta_F\tfrac{\bar{\psi}\bar{\theta}\bar{b}_H}{\bar{C}}\hat{\dot{b}}_{Ht} + \Big(\tfrac{\phi^*}{1+\phi^*} - \beta_F\Big)\tfrac{\bar{\psi}^*\bar{\theta}^*\bar{b}_H^*}{\bar{C}^*\bar{E}}\hat{\dot{b}}_{Ht}^* + \Big(\xi_F^* - \Big(\tfrac{\phi^*}{1+\phi^*} - \beta_F\Big)\tfrac{\bar{\psi}^*\bar{\theta}^*\bar{b}_H^*}{\bar{C}^*\bar{E}}\Big)\hat{\dot{\tau}}_t\Big] - \upsilon_t^{s*}.\end{aligned} \tag{60}$$

Exchange rate.

$$\dot{\hat{e}}_t = \dot{\tau}_t - \dot{\hat{\pi}}^*_{Ft} + \dot{\hat{\pi}}_{Ht}. \tag{61}$$

Bond-market clearing.

$$\hat{b}_t = \omega_H \hat{b}_{Ht} + \omega_F \big(\hat{b}^*_{Ht} + (1-2\gamma)\hat{\tau}_t - \hat{e}_t\big). \tag{62}$$

No-arbitrage condition.

$$\hat{\theta}_t - \left(\zeta_H - \frac{\bar{\Psi}\bar{\theta}\bar{b}_H}{\bar{C}}\right)\hat{b}_{Ht} = \hat{\theta}^*_t - \left(\zeta_F - \frac{\bar{\Psi}^*\bar{\theta}^*\bar{b}^*_H}{\bar{C}^*\bar{E}^*}\right)\big(\hat{b}^*_{Ht} - \hat{e}_t\big) \tag{63}$$

The system (54)–(63) has a transparent block structure. The IS curves (54)–(55) and Phillips curves (56)–(57) form the New Keynesian core of each economy, linked through the terms-of-trade gap τ_t. Equation (58) governs τ_t jointly and is the primary channel through which a fiscal shock in one country reaches the other: a Home deficit innovation υ^s_t shifts $\dot{\hat{b}}_{Ht}$, altering $\dot{\tau}_t$, which feeds simultaneously into both IS curves and both Phillips curves. The debt equations (59)–(60) reveal the full cross-country structure of fiscal contagion: the Home debt path depends on $\hat{\pi}^*_{Ft}$ and $\dot{\hat{b}}^*_{Ht}$, and symmetrically the Foreign debt path inherits $\hat{\pi}_{Ht}$ and $\dot{\hat{b}}_{Ht}$, so that neither country's fiscal position is insulated from the other's. Equation (61) ties the nominal exchange rate to the inflation differential and the terms of trade, while (62)–(63) close the asset side by allocating Home bond holdings between domestic and foreign investors through the convenience-yield equalisation condition. Parameter definitions for Λ, β_H, β_F, ξ_H, and ξ^*_F are collected in Appendix H.2.

3.8 Calibration and Simulation

We now parametrise the quantitative model and provide simulation results, focusing on the role of convenience yields and the fiscal contagion mechanism. Table 4 summarises the baseline parametrisation, with the left panel specifying fiscal and monetary policy and the right panel covering the remaining model parameters.

The left panel of Table 4 summarises four policy configurations. The first column describes the *monetary-led policy mix* (AM/PF), in which both monetary authorities satisfy the Taylor principle so that inflationary pressure is met with a real interest rate tightening. This behaviour induces an explosive root in the evolution of inflation, requiring each fiscal authority to cover deviations of debt from steady state through corresponding adjustments to the marginal tax rate; accordingly, we set $\psi_b \geq \rho$ and $\psi^\star_b \geq \rho^*$. The second column describes the *fiscally-led policy mix* (AF/PM), in which fiscal policy is expansionary with respect to deviations of the debt stock from steady state and taxes are not raised to cover deficits. To ensure non-explosive behaviour of nominal variables, monetary policy accommodates fiscal policy by forgoing an increase in the real interest rate in response to inflationary pressure, indicated by $\phi_\pi < 1$ and $\phi^\star_\pi < 1$. The third column

Param.	AM/PF	AF/PM	HD	H-AF/ F-AM
ϕ_π	1.5	0.5	1.5	0.5
ϕ_π^*	1.5	0.5	0.5	1.5
ϕ_x	0.0	0.0	0.0	0.0
ϕ_x^*	0.0	0.0	0.0	0.0
ψ_b	1.0	0.0	0.0	0.0
ψ_b^*	1.0	0.0	1.0	1.0

(a) Policy scenarios

Param.	Value	Targets/Sources
κ	0.019	Hazell et al. (2022)
$\phi = \phi^*$	2.00	Standard value
$\delta = \delta^*$	0.005	Standard quarterly discount factor
$\rho = \rho^*$	0.005	Standard quarterly discount factor
γ	0.25	(Inverse of) home bias
η	2	Bajzik et al. (2019)
$\bar{\Psi}$	0.03	Conv. yield steady-state at Home
$\bar{\Psi}^*$	0.017	Conv. yield steady-state at Foreign
ζ_H	0.25	Convenience yield sensitivity Home
ζ_F	0.9	Convenience yield sensitivity Foreign
ω_H	0.68	U.S. Treasury (2025)
B/Y	1.2	Debt-to-GDP ratio in the U.S. in 2025
B^*/Y^*	0.95	Debt-to-GDP ratio in the U.K. in 2025
$\bar{E}$	1.25	Nominal exchange rate, assumed

(b) Baseline calibration

Table 4: Calibration: policy scenarios and baseline parameters.

describes *Home Dominance* (HD), in which the Home monetary *and* fiscal authorities are both active while both Foreign authorities are passive. The fourth column considers the mixed case (H-AF/F-AM), in which the Home fiscal authority and the Foreign monetary authority are active while the Home monetary authority and the Foreign fiscal authority are passive; this configuration reflects a scenario in which the Home fiscal authority can commit to deficit finance while the Foreign monetary authority responds aggressively to inflationary threats.

The right panel summarises the non-policy parameters. Most deep parameters, such as the slope of the Phillips curve and the Frisch elasticity of labour supply, are standard in the literature. Empirically disciplined parameters, such as the debt-to-GDP ratio and the share of bonds held abroad, are set to their means since 2010. The convenience yield level and shape parameters are chosen to satisfy two considerations: the convenience yield is relatively small as a share of overall utility but higher at Home, and Foreign households' convenience yield function is more sensitive to changes in bond supply.[14]

With this calibration in hand, we examine the model-implied responses to changes in Home convenience yields and Home fiscal policy, conditioning on the prevailing fiscal-monetary policy mix. This conditioning is relevant because the policy mix determines the strength of relative wealth effects in response to demand-side shocks.

3.8.1 Convenience Yield Shocks

We begin with shocks to the convenience yield on domestic bonds, captured by an exogenous change to θ. Figure 6 shows the response of domestic and foreign output gaps, inflation, terms

[14] In due course, we plan to estimate these parameters using simulated method of moments, disciplined by empirical movements in convenience yields.

of trade, and the nominal exchange rate to a temporary but persistent 100 basis point *decrease* in the convenience yield on Home bonds enjoyed by Home households.

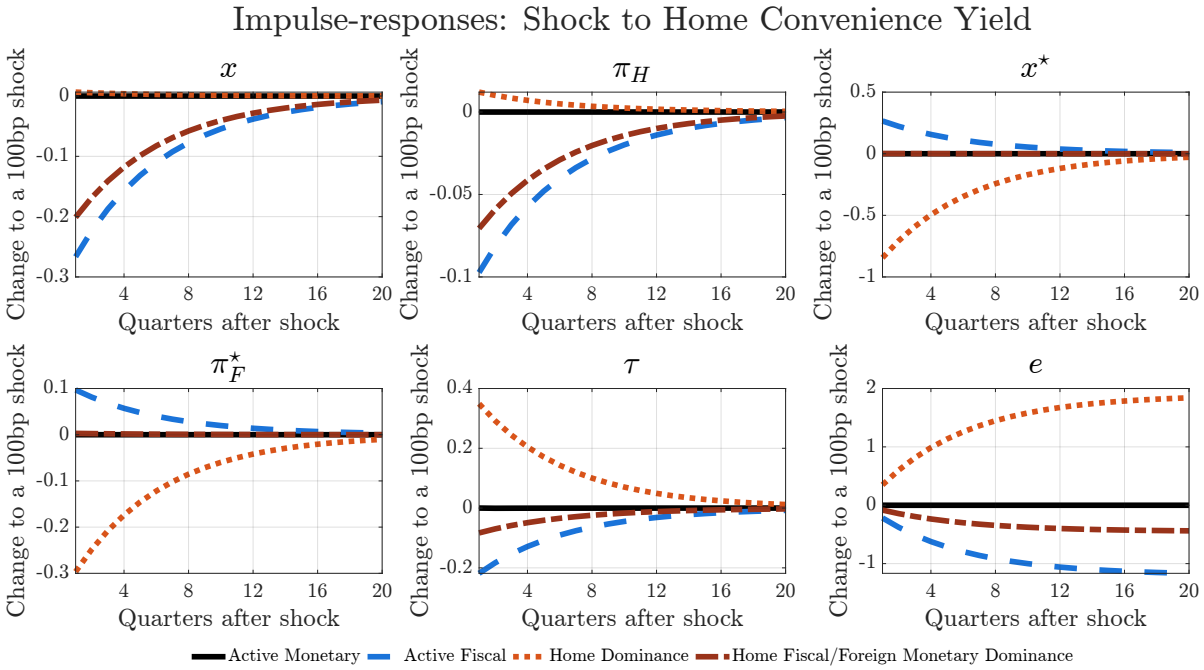


Figure 6: Impulse response functions following a Home convenience yield shock.

Under the monetary-led policy mix, the shock has no effect on the nominal or real allocation. This neutrality follows directly from monetary policy: any disinflationary pressure from the negative demand shock is fully absorbed through an adjustment of the nominal interest rate that prevents a deflationary spiral in equilibrium. Equivalently, this neutrality can be understood through the intertemporal government budget constraint, since households expect any budget imbalances to be fully offset by corresponding adjustments to taxation.

Under the fiscally-led policy mix, the decrease in convenience yields operates as a negative demand shock at Home: output and inflation fall domestically but rise in Foreign, consistent with a relative shift of wealth as Home households are the majority holders of Home government debt, which has declined in value. The terms of trade and the exchange rate appreciate, because the high elasticity of substitution leads Home households to substitute toward Home goods, preventing a favourable terms-of-trade shift for the Foreign country.

Under Home Dominance, the dominant country can effectively export the entire shock onto a depreciation of its own currency, nullifying most of the effects on output and inflation. This occurs through a fiscal expansion that fosters the depreciation, accommodated by a central bank that retains the attractiveness of Home government bonds by adjusting its monetary instrument accordingly. Under the mixed-policy configuration (H-AF/F-AM), the Foreign monetary authority's restrictive behaviour succeeds in insulating Foreign from the real effects of the shock, so that outcomes approximate those of the fiscally-led case but with the allocative effects for Foreign

neutralised.

3.8.2 Fiscal Spending Shocks

Figure 7 presents the consequences of an expansionary Home government spending shock. As with the convenience yield shock, the monetary-led policy mix generates no real effects, by the same logic. Under the fiscally-led mix, the fiscal expansion is not expected to be met by a future increase in Home taxes, effectively boosting Home household wealth since the government's obligations are not expected to be covered by future surpluses. Home households feel richer. Foreign households, by contrast, face additional pressure on their investment in Home government bonds, since those bonds are not backed by a commensurate stream of future surpluses. The Home currency depreciates, reducing the Foreign-currency value of these investments. In total, the fiscal expansion at Home translates into a positive demand shock domestically and a negative demand shock abroad. This is the mechanism by which "Our" deficit at Home becomes "Your" problem in Foreign.

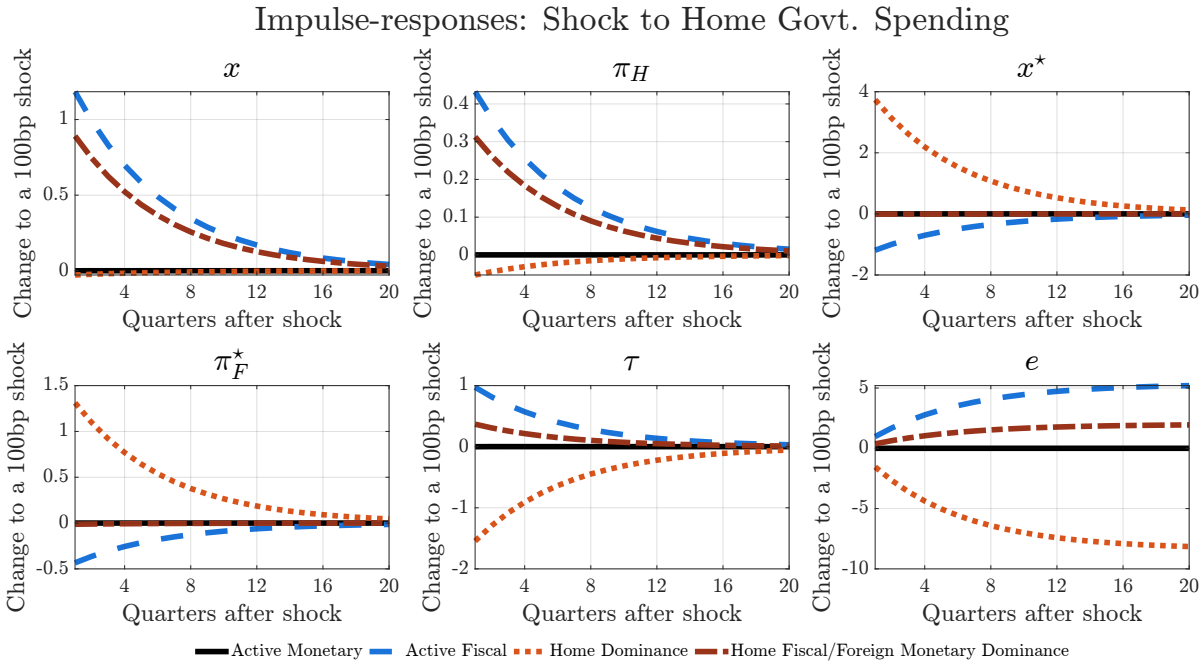


Figure 7: Impulse response functions following an expansionary Home government spending shock.

Under Home Dominance, the aggressive monetary stance at Home ensures that the fiscal expansion does not generate a positive output gap or producer-side price pressure domestically. As interest rates rise significantly, the Home currency appreciates, but Home households do not translate this into a real or nominal expansion because the interest rate tightening offsets the fiscal stimulus. Foreign households, however, benefit from the appreciation of their Home bond holdings in terms of their own currency, contributing to a wealth-driven expansion abroad. Under the mixed-policy configuration, the Home fiscal expansion continues to generate a positive domestic demand

shock alongside a depreciation, but at a smaller magnitude.

3.8.3 The Role of Convenience Yield Levels

Both the level and the shape of the convenience yield function matter significantly for model dynamics, operating through changes to the non-pecuniary value of government bonds as well as through the budget path itself. In the baseline parametrisation, an increase in bond supply decreases the convenience yield earned by households in both countries symmetrically. Inspecting equation (63), which captures the full effect of the convenience function up to first order, makes clear that the shape parameters (ζ_H, ζ_F) and level parameters $(\bar{\Psi}, \bar{\Psi}^*)$ of the convenience yield specification are both consequential.

We now investigate what happens when this symmetry is broken. We set $\bar{\Psi} = 6.03$ while leaving $\bar{\Psi}^* = 0.017$ unchanged. Combined with the smaller elasticity of the convenience yield function at Foreign $(1/\zeta_F > 1/\zeta_H)$, this places us in a regime in which Home households derive substantial utility from bonds while being elastic in their demand, whereas Foreign households' utility from Home bonds continues to decrease in their holdings. We view this asymmetry as a plausible description of the pre-Covid period, in which fiscal expansions could boost the non-pecuniary utility of bond holdings at Home while decreasing them at Foreign.

While the model dynamics in the baseline parametrisation are fairly standard, high levels of convenience yields overturn some of the previous results. We begin again with changes to convenience yields. Figure 8 presents the impulse responses.

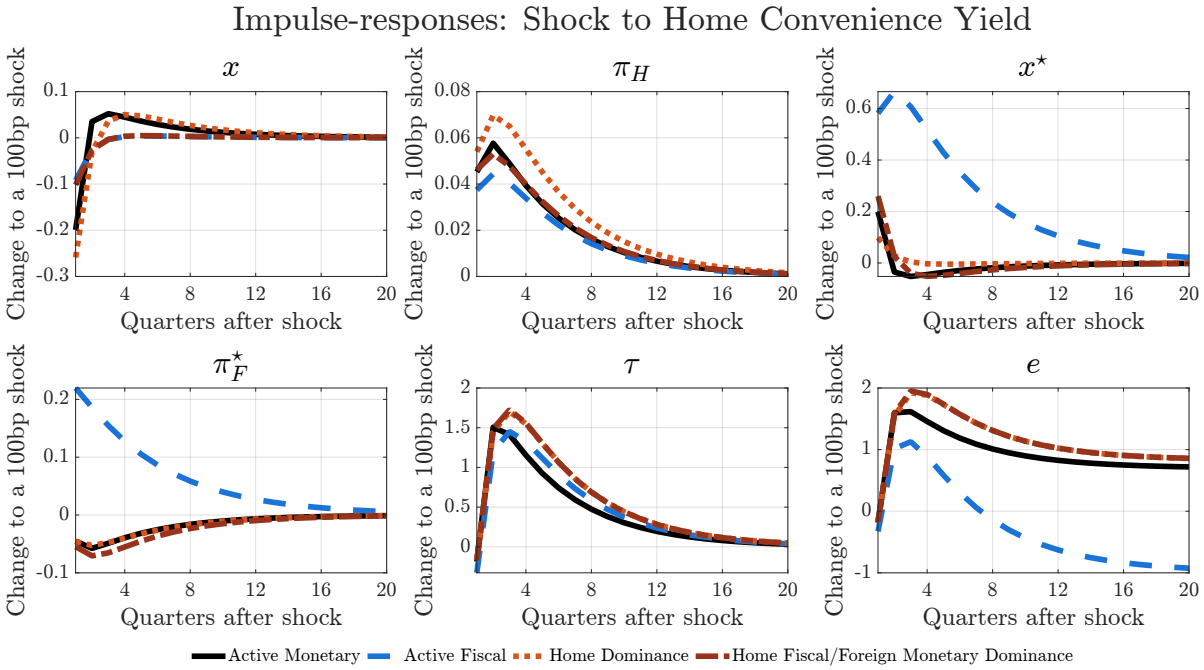


Figure 8: Impulse response functions following a Home convenience yield shock under a higher level of convenience yields.

The asymmetry in convenience flows clearly matters when convenience yields themselves change. Home households lose on impact because they valued the convenience flows from bond holdings more highly. The decrease in convenience yields amounts to a tightening of the Home fiscal position, since budget balance requires more stringent fiscal policy when households in both countries are temporarily less willing to pay a premium to hold Home government debt. Consequently, Home households expect more stringent taxation, generating a negative output gap on impact irrespective of the policy scenario. No policy authority can fully neutralise this impact, because the convenience yield change has asymmetric effects across the two countries and necessarily entails a reshuffling of the present discounted value of household wealth.

Under the monetary-led mix, the central bank accommodates by easing, reducing the future fiscal burden and generating an expansion at Home after the initial period, so that the output gap turns positive and inflation rises. The Foreign economy initially enjoys a positive output gap as the deterioration of the Home fiscal position makes Foreign bonds relatively more valuable, reducing the expected tax burden abroad. Thereafter, a restrictive monetary stance pushes the Foreign output gap into slightly negative territory, accompanied by modest disinflationary pressure. Consistent with the initial loss of confidence in the Home fiscal position and the subsequent monetary reaction, the terms of trade and the exchange rate initially depreciate from the Home perspective before slowly appreciating.

Under the fiscally-led mix, the quantitative effects on output and inflation at Foreign are significantly larger, because the absence of a monetary-induced fiscal tightening abroad allows the full positive wealth effect to materialise as a demand shock through the increased attractiveness of Foreign government bonds. At Home, active fiscal policy mutes the output and inflation responses somewhat, since fiscal policy can accommodate the decrease in non-pecuniary utility flows without monetary constraints. Relative to steady state, the Home currency ends up *appreciating* in the medium run, because the absence of fiscal restraint at Foreign eventually reduces the attractiveness of Foreign sovereign bonds.

The Home Dominance and mixed-policy configurations yield broadly similar outcomes, coinciding exactly in terms of the terms of trade and the exchange rate. The only material difference is that under the mixed-policy case, Foreign government bonds remain even more attractive due to the higher interest paid on them, so that Home households must absorb additional fiscal revaluation pressure from the relative unattractiveness of their own government's bonds.

We next turn to the effects of an expansionary Home government spending shock under the high-convenience-yield parametrisation. Figure 9 presents the results.

Consider first the monetary-led case. The monetary authorities cannot neutralise the effect of the Home fiscal expansion: Home households feel richer because the additional bond supply from the fiscal expansion generates positive utility *even after accounting for the future taxes required to*

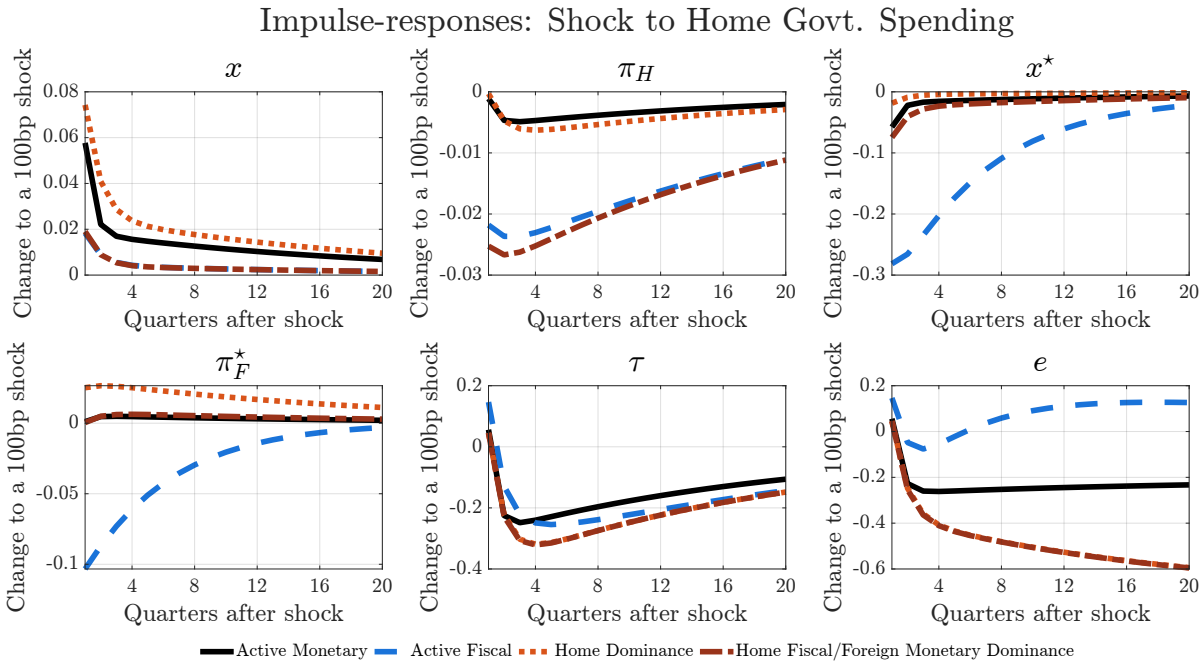


Figure 9: Impulse response functions following an expansionary Home government spending shock under a higher level of convenience yields.

close the deficit, while Foreign households suffer losses in their convenience yields. The monetary authorities cannot overcome this relative wealth shift even under restrictive policies. Consequently, the Home economy enjoys a positive output gap with slight disinflationary pressure, with the opposite obtaining in Foreign, and the Home currency appreciates.

Under the fiscally-led mix, the fiscal spending measure is less expansionary at Home but generates more disinflationary pressure. This result may appear surprising at first. To understand it, note that x_t measures the Home output *gap*, so the expansion relative to the natural output increase from government spending is simply smaller, reflecting a modestly reduced crowding-in of private consumption. The rationale is that the absence of fiscal backing makes the increase in bond supply following the fiscal expansion more pronounced, and with Home households *enjoying* the increased debt supply, they substitute some goods consumption toward bond holdings, contributing to the observed disinflationary pressure. For the Foreign economy, a compounding negative shock materialises: Home households demand fewer Foreign goods while Foreign households' savings in Home bonds depreciate both in terms of asset backing and utility flow, exacerbating the negative demand shock abroad. The exchange rate and terms of trade, which appreciate in the medium run, provide only a partial offset.

In sum, the government spending increase exhibits the type of *fiscal contagion* that the model is designed to isolate, irrespective of the precise fiscal-monetary mix: a Home government spending increase translates into a real expansion at Home, Home *dis*inflation, a temporary Home appreciation, *and* a negative Foreign demand shock operating through a relative positive wealth effect for

the Home household and increased borrowing costs for the Foreign government.

Under Home Dominance, the Home monetary authority's tightening ensures that Home sovereign bonds remain more attractive across the board, amplifying the domestic real expansion and making Home households even wealthier. This provides a partial remedy for the negative demand shock at Foreign, which now sees barely any movement in its output gap but suffers some inflationary contagion due to increased goods demand from Home. The steepness of the Home wealth gain manifests in a sharp appreciation of the Home currency.

Finally, under the mixed-policy configuration, the Foreign monetary authority is able to almost fully neutralise the output and inflation effects of the Home fiscal expansion for the Foreign economy. The aggressive monetary stance at Foreign acts primarily as a credible threat against an inflationary fiscal stance at Foreign itself, effectively neutralising the wealth effects of Foreign bonds, which constitute the majority of Foreign household savings. As a corollary, this restrictive stance also increases the relative attractiveness of Foreign sovereign bonds for Home households, even though the non-pecuniary utility flow from Home government bonds is increasing in supply. In total, the domestic real expansion is smaller, the disinflationary pressure at Home is unaffected, and the effects on the Foreign economy are minimised. This exercise highlights the importance of Foreign authorities coordinating to keep their own bonds attractive in response to a Home fiscal expansion, though the optimal nature of such coordination will depend crucially on the shape and slope of the convenience flows associated with sovereign debt in both countries.

3.9 Estimation

We now conduct a Bayesian estimation of the model at quarterly frequency, covering the period from 1997-Q1 (following the Bank of England's operational independence) through 2024-Q4.

Data. We use data obtained from FRED. The observables are conventional output gap estimates for the U.S. and the U.K., demeaned quarterly inflation, and terms of trade derived from the USD-GBP spot exchange rate adjusted through relative import/export price levels computed from nominal and real trade quantities. Total and local bond issuances are defined as demeaned quarter-on-quarter growth rates in observed market values of total bond stocks after removing their first-order trend.

In total, we match 8 data series: Home and Foreign output gaps, Home and Foreign local inflation, terms of trade, Home and Foreign total bond issuance, and the quantity of Home bonds held domestically. The estimation varies the policy parameters ϕ_π, ϕ_x, $\phi_\pi^\star$, $\phi_x^\star$, ψ_b, and ψ_b^*, as well as the convenience yield shape parameters ζ_H and ζ_F and the steady-state convenience yield levels $\bar{\Psi}$ and $\bar{\Psi}^\star$. We additionally vary the persistence and standard deviations of all six shock processes

and allow for measurement error in the convenience yield and the Foreign output gap, as those data measures are relatively imprecisely estimated.[15]

Estimation method. We employ Bayesian methods following Herbst and Schorfheide (2016). To obtain the unconditional posterior distribution, we use a Markov Chain Monte Carlo mechanism with 25,000 draws per chain, employing the tailored randomised block Metropolis-Hastings algorithm of Chib and Ramamurthy (2010) across two chains to achieve an acceptance rate of approximately 25%.

Prior distributions are specified as follows. Autoregressive coefficients follow relatively narrow Beta distributions, while standard deviations follow wide inverse-gamma distributions to allow the data to determine the relative importance of each shock process. The shape and level parameters of the convenience yield function are uniformly distributed with wide bounds to remain maximally agnostic. Likewise, the monetary and fiscal policy coefficients are uniform on wide intervals so that the data inform the policy schedule.[16]

Policy and convenience yield parameters			**Shock persistence & s.d.**		
Parameter	Estimate	Std. Error	Parameter	Estimate	Std. Error
ϕ_π	1.0063	0.0017	$\rho_\theta = \rho^*_\theta$	0.0933	0.0384
ϕ^*_π	0.7561	0.0433	$\rho_{\hat{g}}$	0.9977	0.0010
ϕ_x	-0.1123	0.0046	$\rho^*_{\hat{g}}$	0.8715	0.0179
ϕ^*_x	0.0286	0.0238	ρ_ν	0.8917	0.0212
ψ_b	0.0317	0.0361	ρ^*_ν	0.9643	0.0532
ψ^*_b	-0.0107	0.0028	$\sigma_{e,\theta} = \sigma^*_{e,\theta}$	0.0131	0.0009
ζ_H	0.0113	0.0012	$\sigma_{e,\hat{g}}$	0.0156	0.0016
ζ_F	0.6627	0.0751	$\sigma^*_{e,\hat{g}}$	0.0197	0.0030
$\bar{\Psi}$	6.03	0.8755	$\sigma_{e,\nu}$	0.9781	0.1245
$\bar{\Psi}^*$	0.017	0.1215	$\sigma^*_{e,\nu}$	0.3724	0.0205

Log-data density: −709.457

Table 5: Maximum likelihood estimates using U.S. and U.K. data since 1997.

Results. Table 5 presents the estimation results. In both the U.S. and the U.K., the data favour a *fiscally-led policy mix*, with marginal tax rates relatively unreactive to deviations of domestic government debt from steady state and with the Taylor principle not satisfied after accounting for the reaction to output. Estimated steady-state convenience yields are high, particularly at Home, validating their role as the principal mechanism breaking perfect cross-country risk sharing. The estimated persistence and standard deviations of the shock processes are broadly standard.

[15] We impose common persistence and standard deviation for the convenience yield shock at Home and at Foreign, which ensures that the model matrices are of full rank in the estimation.

[16] Table G.1 in the appendix summarises the prior distributions in detail.

Historical decomposition. We now decompose the historical variation in the key nominal variables of interest: Home (U.S.) inflation, Foreign (U.K.) inflation, and the exchange rate, captured here through the terms of trade. Figure 10 presents the contribution of individual shocks to movements in nominal U.S. and U.K. variables since 2015.

The grey bars represent estimated initial conditions; the coloured bars represent the contributions of each shock component as indicated by the legend. The post-Covid fiscal expansion in the U.S. (red bars) emerges as the predominant driver of U.S. inflation, accounting for approximately 50% of total excess cyclical inflationary pressure over 2021–2024. Over the same period, the post-Covid fiscal expansion in the U.K. (light blue bars) accounted for roughly 40% of total inflationary pressure in the U.K., with its weight rising in later quarters. The U.S. fiscal expansion contributed primarily to a depreciation of the U.S. terms of trade, while the U.K. fiscal expansion contributed to a modest appreciation.

The novel mechanism highlighted in this paper concerns the spillover of U.S. deficits (red bars) to U.K. inflation (centre panel). Through the channels discussed in the impulse response analysis, a positive relative wealth effect driven by higher demand for Foreign goods, combined with a relative deterioration of the U.K. fiscal position, transmitted U.S. fiscal expansion into U.K. price dynamics, contributing a cumulative total of approximately 2 percentage points to the U.K. price level since 2021.

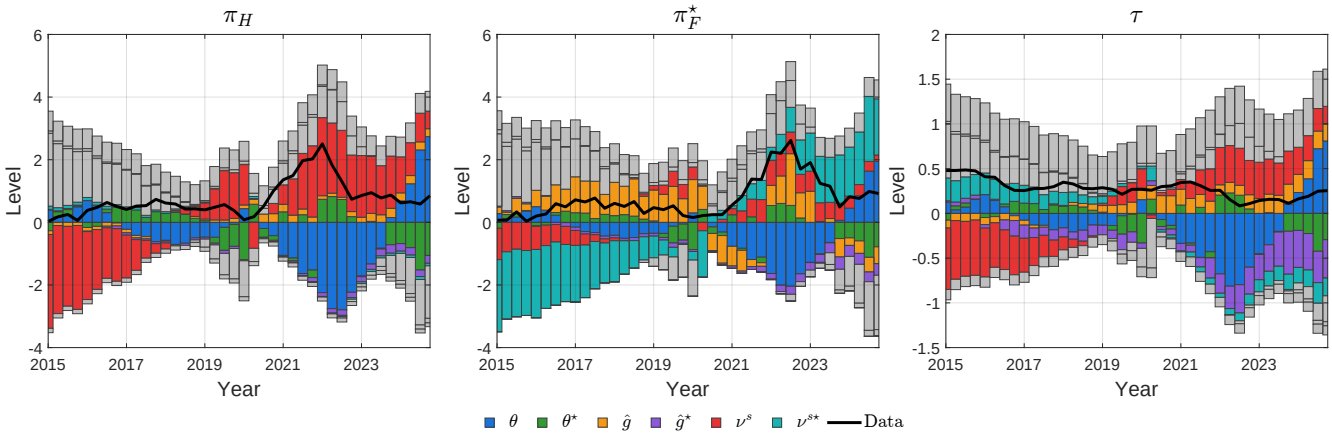


Figure 10: Historical shock decomposition implied by the model.

Complementing this decomposition, Figure 11 depicts the hypothetical path of output gaps, inflation rates, Home bond issuance, and the nominal exchange rate had U.S. deficits been halved since 2021. This exercise confirms the intuition from the historical decomposition and further illustrates how a more restrained fiscal expansion would have dampened the overshooting of output above its natural level, as well as reduced the contribution of U.S. fiscal policy to Dollar movements over the same period.

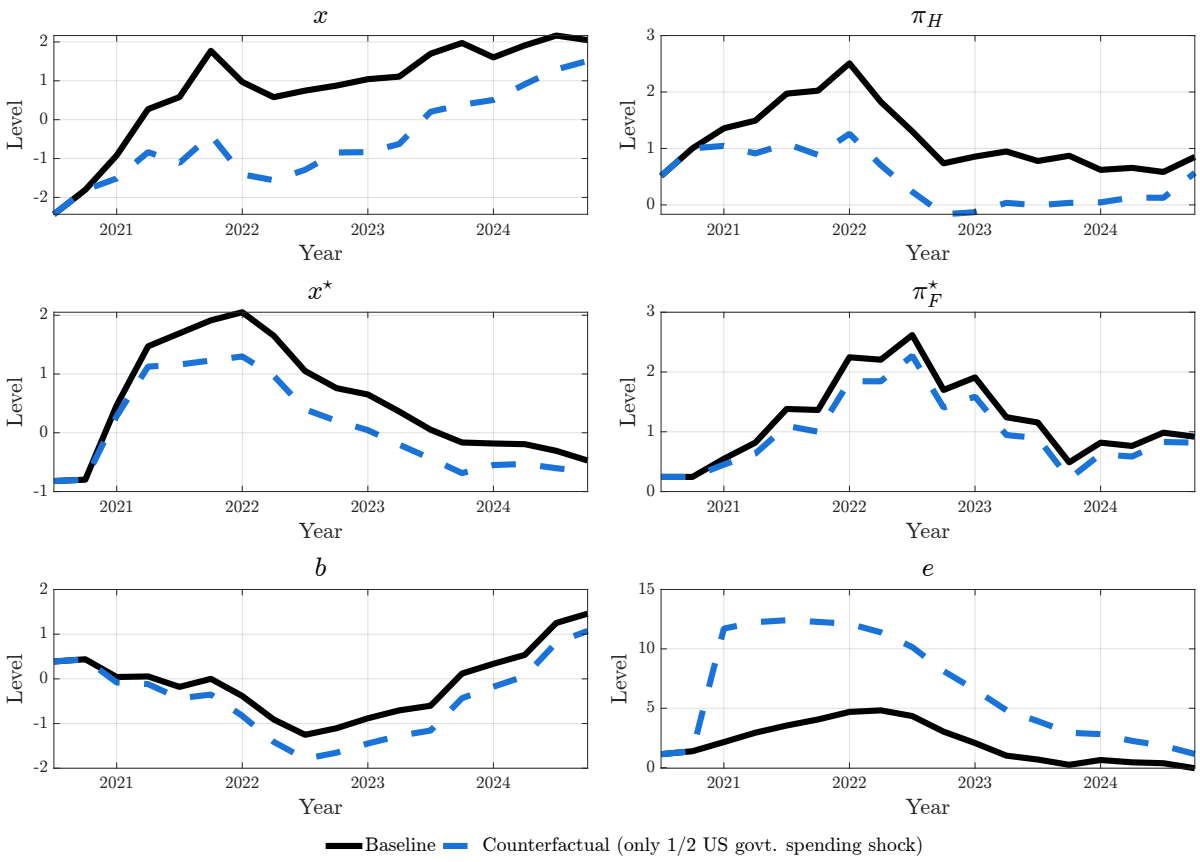


Figure 11: Counterfactual: halving the U.S. fiscal deficit since 2021.

4 Conclusion

This paper shows that fiscal sustainability in open economies is jointly disciplined by expected primary surpluses and by the currency-denominated discount factor that global investors apply to those surpluses. This linkage connects fiscal innovations to exchange rates and allows fiscal policy to spill over internationally. Using market-value debt data for the United Kingdom and the United States since 1975, we apportion roughly 50 percent of unexpected year-to-year changes in the U.K. debt ratio to revisions in surplus and deficit expectations, and the remaining 50 percent to discount-rate news, with the latter split across global yields, real exchange rate revisions, and time-varying UIP premia. The revisions in surplus expectations, in turn, are highly correlated with corresponding revisions in observed exchange rates. Our continuous-time two-country model with convenience yields rationalises these findings: expected fiscal shocks in a financial hegemon propagate internationally through exchange rate adjustments that feed directly into the real discount rates faced by other sovereigns, generating what we term "fiscal contagion." In the model, deficits in the financial hegemon spill over to the cost of borrowing for the government of the other economy, yielding an expected future appreciation of the hegemon's currency that equates cross-country financial flows over a finite horizon. Through this mechanism, the current market value of bonds in the non-hegemon economy decreases, leading to an international transmission of fiscal inflation conditional on either a lack of sufficient fiscal backing or a transmission of real wealth arising from differences in the slope of the demand function for convenience-bearing bonds.

These results carry several policy implications. Debt management strategies that focus narrowly on flow deficits risk misdiagnosing solvency when large portions of debt dynamics stem from the repricing of existing liabilities. Credible policies that stabilise the currency-denominated discount factor, whether through inflation control via conventional and unconventional monetary policy, the anchoring of FX risk premia, or issuance in a diversified mix of currencies, can materially ease the fiscal burden even when deficits are large. More broadly, prudent coordination of fiscal, monetary, and exchange rate policies can mitigate cross-border spillovers; ignoring them risks pro-cyclical tightening precisely when domestic stabilisation is most needed.

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A A Simple Model of International Fiscal Spillovers

This section presents a minimal model emphasizing a fiscally-led inflation mechanism in a standard international macro context, with plausible relevance of fiscal deficits for nominal exchange rate movements and international spillovers through interest rate adjustments. What we therefore show here is a minimal mechanism of how fiscal sustainability concerns in one country can spill over to the government debt valuation of another country. The interesting messages of the model are kept fully nominal, with no relevance for international goods flows, which we nullify here for expositional simplicity. One particular result will be the possibility to distinguish between *announcements* of spending at Home and their factual implementation, with particular relevance for the announcements through expectations-driven revisions of the Uncovered Interest Rate Parity (UIP) condition.

A.1 Environment

The primitives of the model consist of a simplified iteration of the framework of Bassetto and Miller (2025), but with a diminished focus on the information structure. Instead, we add an international dimension facilitating the transmission of fiscal shocks between countries. Denote the Home country by H, the Foreign country by F, and let stars (*) denote variables in the Foreign country, in line with standard notation. Heuristically, we will think of the Home country as an international hegemon determining international spillovers, and of the Foreign country as a small open economy subject to the financial influence of the hegemon.There are two periods:

1. t=1: there exists an initial steady-state inherited from t=0. In period one, the price levels P_1, P_1^*, as well as the nominal exchange rate $\mathcal{E}_1$, are all determined through the respective government budget constraints and cross-border debt holdings inherited from the initial period. There exists an information set about a "government spending shock". This is a shock to Home government spending in period 2, G_2, whose plausible size is revealed. Additionally, there is a prior about the policy regime in place at Home. Two policy regimes are possible: a 'fiscally-led' regime, in which taxes are a fixed quantity and the spending need will be financed through inflation; and a 'monetary-led' regime in which taxes adjust to ensure that a target price level at Home is achieved. The Foreign fiscal authority levies fixed taxes $\bar{T}^*$ in all periods, while Foreign interest rates on debt issuance i_1^* are determined in equilibrium through the UIP condition.

2. t=2: The tax and monetary regimes from Home unfold, yielding the terminal Home price level P_2. Note that the Foreign price level P_2^* will already be perfectly revealed in period 1, being a function of equilibrium interest rates on Foreign debt from period 1 to period 2.

Home households Relative to standard macroeconomic models, the sole key difference is that we introduce a reduced-form preference term on behalf of the Foreign households for the bonds issued by Home.[17]

Home households maximize the utility borne from consumption of the domestic good, adjusted for the disutility from supplying labour:[18]

$$max\ U = max \sum_{t=1}^{2} \beta^t \mathbb{E}_t \left[u(c_t) - L_t \right],$$

where c_t is consumption of the representative household, L_t is labour supplied, and the production function of the household is given by $y_t = L_t$. Disutility from labour is kept linear on purpose to simplify determination of the real equilibrium in the presence of fiscal shocks.

Each household takes as given government policy and the price level. Define B_{Ht-1} as household holdings of Home bonds at the beginning of period t, having taken them over from period $t-1$. The same applies to holdings of Foreign bonds by the Home household, defined as B_{Ft-1}. Then, the domestic household budget constraint is given by:

$$B_{Ht-1}(1+\bar{i}) + B_{Ft-1}\mathcal{E}_t(1+i^*_{t-1}) + P_t L_t = B_{Ht} + B_{Ft}\mathcal{E}_t + P_t(c_t + T_t). \tag{A.1}$$

Note that no debt is left outstanding in the final period for either household in equilibrium; that is, $B_{H2} = B_{F2} = B^*_{H2} = B^*_{F2} = 0$.

Home Fiscal and Monetary Authority Next, consider the Home government. The government faces an already accrued (inherited) stock of debt B_0, held by both Home and Foreign households in some proportion. It can raise lump-sum taxes T_t in each period. We follow Bassetto and Miller (2025) in assuming that $G_1 = 0$ without loss of generality. In period 1, lump-sum taxes will be set to cover interest payments, such that $T_t = \frac{B_{t-1} - \frac{B_t}{1+\bar{i}}}{P_t}$ for $t = 1$. In period $t = 2$, Home government spending and taxes are uncertain and plausibly disjoint, allowing for altered government debt dynamics.

In line with Bassetto and Miller (2025), two monetary-fiscal policy regimes are possible:

- ML (Monetary-led): in this regime, $T_2 = G_2 + \frac{B_1}{P_2^T}$, where P_2^T is an externally supplied targeted price level in period 2. Then, any spending requirement G_2 is fully soaked up by corresponding taxes, while the primary surplus $\frac{B_1}{P_2^T}$ soaks up the cost of outstanding bond holdings.
- FL (Fiscally-led): in regime FL, taxes do not respond to the spending requirement: $T_2 = \bar{T}$,

[17] A similar taste term for the Home household can be included, but it would merely obstruct the core message through more involved algebra. Also, we could express the model through a local preference for domestic bonds, but this would not materially change the outcomes at the cost of more involved algebra again.

[18] Note that there is no goods trade in equilibrium, but only trade in financial products at this stage.

where $\bar{T}$ is an exogenously supplied known constant.

Fundamentally, the assumption of introducing tax changes in period 2, but their announcement taking place in period 1, is meant to distinguish the effects of announced deficits from the effects once the deficits actually materialize. It therefore allows us to capture the effects of fiscal spending announcements on exchange rates through their forward-looking nature.

Monetary policy in the Home economy will be captured by a strict nominal interest rate peg as already in use in the Home budget constraint, $i_t = \bar{i} \, \forall t$.

Foreign block The *Foreign* household is described by a similar problem as the domestic household, but with all variables being starred and the exchange rate adjustment in the budget constraint reflecting that the Foreign household considers the Foreign bonds as domestic, and, crucially, an additional idiosyncratic preference term capturing the utility from holding Home bonds. Formally, the representative Foreign household maximizes:

$$max \; U^* = max \sum_{t=1}^{2} \beta^t \mathbb{E}_t \left[u(c_t^*) - L_t^* + \phi_F \frac{B_{Ht}^*}{P_t^*} \right].$$

The additional term at the end, $+\phi_F \frac{B_{Ht}^*}{P_t^*}$, captures the utility from holding Home bonds. ϕ_F is an exogenous parameter capturing the weight of the disutility term, and $\frac{B_{Ht}^*}{P_t^*}$ is the real quantity of Home bonds held by Foreign households. We use a linear disutility term for simpler calculations here, but a quadratic disutility term is possible.

This maximization is performed subject to the per-period budget constraint of the Foreign household:

$$B_{Ft-1}^*(1 + i_{t-1}^*) + \frac{B_{Ht-1}^*}{\mathcal{E}_t}(1 + \bar{i}) + P_t^* L_t^* = B_{Ft}^* + \frac{B_{Ht}^*}{\mathcal{E}_t} + P_t^*(c_t^* + T_t^*). \tag{A.2}$$

Now, we consider the government of Foreign. The Foreign government differs from the Home government in a number of important aspects:

- First, there is no uncertainty with respect to Foreign government spending; that is, we assume $G_t^* = 0 \; \forall t$ for simplicity.
- In terms of the policy in use, we postulate that taxes levied in the terminal period by the Foreign government are fixed; that is, $T_2^* = \bar{T}^*$. The terminal period price level in the Foreign economy will be determined by the terminal government budget constraint, with the price level adjusting to ensure that the fiscal budget constraint holds.
- Foreign interest rates are allowed to be time-varying: i_t^* is not constant. It adjusts, however, not based on a policy rule, but instead in a way that allows the UIP, adjusted for the wedge term ensuring a preference for the Foreign bonds, to hold.

Information structure: We introduce an explicit information structure that allows us to track the effect of each policy regime in conjecture with government spending on the Home and Foreign price level.

In $t = 1$, both Home and Foreign households have a (shared) prior about the policy regime and spending G_2 of Home. To keep it simple, we denote by ψ_1 the prior that the Home government will be in regime FL, and by $1 - \psi_1$ the prior that the Home government will be in regime ML. These priors will be the same in both Home and Foreign. The government spending variable can follow an arbitrary distribution $\mathcal{G}$.

Optimality Conditions: We provide detailled derivations in appendix B. The domestic household's equilibrium conditions are characterised by:

$$u'(c_t) = 1, \qquad t = 1, 2; \tag{A.3}$$

$$u'(c_t) = \beta(1 + \bar{i})\,\mathbb{E}_t\left[u'(c_{t+1})\frac{P_t}{P_{t+1}}\right], \qquad t = 1; \tag{A.4}$$

$$u'(c_t) = \beta(1 + i_t^*)\,\mathbb{E}_t\left[u'(c_{t+1})\frac{P_t}{P_{t+1}}\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right], \qquad t = 1. \tag{A.5}$$

For the foreign household, their equilibrium optimality conditions are characterised by:

$$u'(c_t^*) = 1, \qquad t = 1, 2; \tag{A.6}$$

$$u'(c_t^*) = \beta(1 + \bar{i})\,\mathbb{E}_t\left[u'(c_{t+1}^*)\frac{P_t^*}{P_{t+1}^*}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right] + \phi_F\mathcal{E}_t, \qquad t = 1; \tag{A.7}$$

$$u'(c_t^*) = \beta(1 + i_t^*)\,\mathbb{E}_t\left[u'(c_{t+1}^*)\frac{P_t^*}{P_{t+1}^*}\right], \qquad t = 1. \tag{A.8}$$

Note that, in general, no Euler equations apply in period 2 as all bond holdings in that period are set to zero in that period by household optimality.

Definition 8 (Competitive equilibrium)**.** *An allocation* $\{c_t, c_t^*, L_t, L_t^*, B_{Ht}, B_{Ft}, B_{Ht}^*, B_{Ft}^*, B_t, B_t^*\}_{t=1,2}$*, nominal prices* $\{P_t, P_t^*, \mathcal{E}_t\}_{t=1,2}$*, fiscal policy* $\{T_t, T_t^*\}_{t=1,2}$*, monetary policy* $\{\bar{i}, i_t^*\}_{t=1,2}$*, and government spending* $\{G_2\}$ *are called a competitive equilibrium in this economy, when:*

- *all time-t variables are t-measurable;*
- *given* $\{P_t, \bar{i}, T_t, P_t^*, i_t^*, T_t^*\}$ *and* $\{G_2\}$*, the variables* $\{c_t, L_t, B_{Ht}, B_{Ft}\}_{t=1,2}$ *solve equations (A.3), (A.4), (A.5), and (A.1); and the variables* $\{c_t^*, L_t^*, B_{Ht}^*, B_{Ft}^*\}_{t=1,2}$ *solve equations (A.6), (A.7), (A.8), and (A.2), with* $B_{H2} = B_{F2} = B_{H2}^* = B_{F2}^* = 0$*.*
- *The domestic government satisfies its budget constraint and policy rules:*
  - $B_t = B_{t-1}(1 + \bar{i}) + P_t(G_t - T_t),\ t = 1, 2$*, where* $B_2 = 0$*.*

- $T_2 = G_2 + \frac{B_1(1+\bar{i})}{P_2^T}$ *in regime ML;* $T_2 = \bar{T}$ *in regime FL. Taxes in periods 1 and 2 adjust to ensure a constant real debt burden*
- *Interest rates are pegged to the value* $\bar{i}$*.*

• *The Foreign government satisfies its budget constraint and policy rules:*

- $B_t^* = B_{t-1}^*(1 + i_{t-1}^*) + P_t^*(G_t^* - T_t^*)$ $t = 1, 2, 3$ *and* $B_2^* = 0$*.*
- *Taxes in all periods are fixed. In* $t = 1$*, taxes offset the cost of serving maturing debt, while in* $t = 2$*, taxes are simply given by some exogenous level* $\bar{T}^*$*.*
- *Interest rates* i_t^* *adjust to make sure that household optimality is achieved as the sole free parameter ensuring that the UIP conditions on both households hold.*

• *Markets clear:*

- $C_t + G_t = L_t$ *for* $t = 1, 2$*,*
- $C_t^* + G_t^* = L_t^*$ *for* $t = 1, 2$*,*
- $B_t = B_{Ht} + B_{Ft}$ *for* $t = 1, 2$*,*
- $B_t^* = B_{Ht}^* + B_{Ft}^*$ *for* $t = 1, 2$-

In total, we have 17 endogenous variables in 17 conditions.

This model gives rise to the following two UIP conditions:

$$\frac{1}{\mathcal{E}_t} = \beta(1 + \bar{i})\, \mathbb{E}_t\left[\frac{P_t^*}{P_{t+1}^*}\frac{1}{\mathcal{E}_{t+1}}\right] + \phi_F, \tag{A.9}$$

$$\mathcal{E}_t = \beta(1 + i_t^*)\, \mathbb{E}_t\left[\frac{P_t}{P_{t+1}}\mathcal{E}_{t+1}\right]. \tag{A.10}$$

A.2 Solving for equilibrium

We now solve for the equilibrium of this model. We start from the terminal period ($t = 2$) and move backwards in solving the model.

To pin down the terminal exchange rate $\mathcal{E}_2$ without invoking the UIP (as there are no bonds outstanding in period 3), we start by recognizing that the model set-up requires all cross-border bond holdings to unravel at the terminal period. In an accounting sense, cross-border flows of financial goods in the terminal period must allow the Home country to repatriate the Foreign holdings of Home bonds, and vice versa. Therefore, the terminal exchange rate reflects the price level-adjusted flows that allow cross-country bond holdings to unravel. The logic is that, in equilibrium,

$$P_2(1+\bar{i})B^*_{H1} = \mathcal{E}_2 P^*_2(1+i^*_1)B_{F1} \qquad \Leftrightarrow \qquad \mathcal{E}_2 = \frac{P_2}{P^*_2}\frac{(1+\bar{i})}{(1+i^*_1)}\frac{B^*_{H1}}{B_{F1}}, \tag{A.11}$$

such that the non-domestic bond holdings in period 1 in combination with their respective interest rates are fully informative about the nominal exchange rate in period 2. This assumption allows us to close the outstanding stock of cross-border financial accounts in the terminal period.

In economic terms, this assumption equates the value of Foreign debt held by Home and Home debt by Foreign in the last period. Therefore, it facilitates a *positive* correlation between the value of Home and Foreign debt in the terminal period. Assume a fiscal expansion in Home, reducing the real value of Home debt in the terminal period. By this assumption, the real value of Foreign debt denoted in the Home currency must also decrease, allowing for a simple reduced-form international transmission of fiscal shocks.

This is, in a sense, an assumption about absolute hegemony of Home: since Foreign fiscal policy is assumed to be fixed, a fiscal deterioration at Home must translate to a fiscal deterioration in Foreign by depressing the market value of Foreign debt. To some extent, this rationalizes the taste for Home bonds in Foreign, manifest in its utility function.

As alluded to previously, terminal bond holdings are zero by household optimality: $B_2 = B^*_2 = B_{H2} = B_{F2} = B^*_{H2} = B^*_{F2} = 0$. Additionally, for our further analysis, we denote the measure of fiscal spending in the Home country conditional on begin in the fiscally-led regime as $\mu_1 \equiv \mathbb{E}_1(G_2|FL)$.

Conditional on the fiscal policy regime in place, we are able to pin down the price level in the terminal period at Home through the specification of the fiscal policy regime in place:

$$P_2 = \begin{cases} P^T_2 & \text{if ML,} \\ \frac{B_1(1+\bar{i})}{\bar{T}-G_2} & \text{if FL.} \end{cases} \tag{A.12}$$

Therefore, the level of taxes levied at Home in the terminal period is given by:

$$T_2 = \begin{cases} \frac{B_1(1+\bar{i})}{P^T_2} + G_2 & \text{if ML,} \\ \bar{T} & \text{if FL.} \end{cases} \tag{A.13}$$

Since $G_1 = 0$ and $u'(c_t) = 1\ \forall t$, we can simplify the Euler equation applying in period 1 in the Home household, which is:

$$u'(c_1) = \beta(1+\bar{i})\mathbb{E}_t\left[u'(c_2)\frac{P_1}{P_2}\right],$$

by recognizing that household optimality prescribes constant consumption across time, such that the Euler equation becomes:

$$\frac{1}{P_1} = \beta(1+\bar{i})\left[\psi_1\frac{\bar{T}-\mu_1}{B_1(1+\bar{i})} + (1-\psi_1)\frac{1}{P^T_2}\right]. \tag{A.14}$$

Alternatively, we can express also the price level explicitly as:

$$P_1 = \frac{1}{\beta(1+\bar{i})} \frac{1}{\left[\psi_1 \frac{\bar{T}-\mu_1}{B_1(1+\bar{i})} + (1-\psi_1)\frac{1}{P_2^T}\right]}, \tag{A.15}$$

which means that we have now pinned down the domestic price level in both period 1 and period 2. It generally does not depend on the exchange rate and Foreign fiscal or monetary policy. This is because our assumptions implicitly allowed us to characterize the Home policy authorities as the 'dominant' ones, imposing their tax and interest policy on the other country through the exchange rate, with little effect vice versa. We are therefore taking here the stark assumption of an 'absolute hegemon' in terms of the monetary and fiscal policy spillovers.

Now, we focus on the Foreign country as well as the international dimension. Our main guiding point will be the two UIP conditions (A.9) and (A.10). Both conditions have to hold. However, since the Home and Foreign price levels are fully informed by the fiscal budget constraint, and the Home interest rate is fixed, equation (A.9) will be cleared by $\mathcal{E}_t$ only. Consequently, equation (A.10) will be cleared by i_t^*, the Foreign bond interest rate, as it is the only free variable ensuring that the UIP on Foreign bonds held by the domestic household holds.

To aid our discussion, we first solve for the Foreign price level in period 1. By the Foreign Euler condition on Foreign bonds,

$$\frac{1}{P_t^*} = \beta(1+i_t^*)\mathbb{E}_t\left[\frac{1}{P_{t+1}^*}\right].$$

We evaluate this at $t=1$. Since

$$P_2^* = \frac{(1+i_1^*)B_1^*}{\bar{T}^*} \tag{A.16}$$

by the Euler equation of Foreign, we find that

$$\frac{1}{P_1^*} = \beta\frac{\bar{T}^*}{B_1^*} \tag{A.17}$$

fully determines the price level. In particular, as P_2^* is fully known conditional on i_1^*, we can pin down the price level in Foreign at $t=1$ as:

$$P_1^* = \frac{1}{\beta}\frac{B_1^*}{\bar{T}^*},$$

where i_0^* was inherited from the initial steady-state.

What is left is to determine are $\mathcal{E}_1$, $\mathcal{E}_2$, and i_1^*, which will be jointly given by our two UIP conditions and the terminal exchange rate condition.

We solve here for $(1+i_1^*)$, which will be the central object of interest capturing spillovers from expansionary fiscal policy at Home.

Proposition 9. *The equilibrium gross interest rate of Foreign in period 1 is given by:*

$$(1+i_1^*) = \frac{\phi_F}{\left[\psi_1 \frac{\bar{T}-\mu_1}{B_1(1+\bar{i})} + (1-\psi_1)\frac{1}{P_2^T}\right] \frac{B_1^*}{\bar{T}^*} \left\{\frac{B_{H1}^*}{B_{F1}} - \frac{B_{F1}}{B_{H1}^*}\right\}}. \tag{A.18}$$

This solution for the equilibrium interest rate delivers a clear message: if expectations on fiscally-led policy mixes at Home grow in period 1, the equilibrium interest rate on Foreign debt issued in period 1 *increases*. Formally:

Corollary 10 (Co-movement between Foreign interest rates and Home fiscal policy perceptions)**.** *If the terminal price level at Home is higher under the fiscally-led policy mix ($P_2|_{FL} > P_2|_{ML}$) and either of the following two sets of conditions hold true:*

1. *Foreign has a positive convenience yield from holding Home bonds and $B_{H1}^* > B_{F1}$; that is, Foreign holds more Home bonds than vice versa; or*
2. *Foreign has a negative convenience yield from holding Home bonds and $B_{H1}^* < B_{F1}$; that is, Home holds more Foreign bonds than vice versa,*

the co-movement between Foreign interest rates in period 1 and the probability of a fiscally-led policy mix at Home are positive; that is:

$$\frac{\partial(1+i_1^*)}{\partial\psi_1} > 0. \tag{A.19}$$

This result is, at first, counter-intuitive. Even though Foreign households do not enjoy Home debt in any case, fiscal policy that makes Home bonds relatively less attractive is accompanied by an increase in the cost of borrowing in Foreign. A deterioration of Home fiscal conditions correlates with a deterioration in Foreign fiscal conditions, even if Foreign fiscal policy is unchanged. This result is squarely linked to the specification of the terminal exchange rate above, which induced the linkage in fiscal sustainability between Home and Foreign. Thus, an alternative fiscal-monetary policy specification limiting the factual hegemony of Home in the determination of the exchange rate would alter our results. Our intuitions are therefore related to Ding and Jiang (2024).

At constant taxes and no changes to the aggregate bond issuance in Foreign, this higher cost of borrowing in period 1 directly translates to a higher price level in period 2 (and, thus, also higher inflation from period 1 to period 2).

Corollary 11. *The terminal price level in Foreign co-moves positively with the probability of a fiscally-led policy mix at Home:*

$$\frac{\partial P_2^*}{\partial\psi_1} > 0. \tag{A.20}$$

Proof. Follows immediately from equations (A.19) and (A.16). □

A higher probability of a Fiscally-led regime in Home ($\psi_1 \uparrow$) increases the domestic price level generally by equation (A.15). Additionally, with our specification of international financial markets, this increase in the Home price level must yield an increase in the interest rates charged on Foreign bonds for the UIP to hold. This, in turn yields an *increase* in the *Foreign terminal price level* P_2^* through the government budget constraint of Foreign, given by $P_2^* = \frac{(1+i_1^*)B_1^*}{\bar{T}^*}$. Therefore, in our simple model, we identify a minimum viable mechanism by which a deterioration in Fiscal conditions at Home spills over to higher inflation from $t = 1$ to $t = 2$ in Foreign, manifesting through a deterioration of the fiscal balance of Foreign.

Figure A.1 presents these comparative statics graphically. Both the expected price level at Home, here denoted by P_2^e, as well as the materialized price level in Foreign P_2^* (which is fully determined in period one through the Foreign government budget constraint) increase in the Foreign interest rate i_1^*, which in turn increases in the probability of being in the fiscally-led regime. The sensitivity of the Foreign price level to the Home fiscal shock is of a similar order of magnitude in level terms to the Home price level sensitivity, which is a consequence of the fiscal shock being comparatively 'large' from the perspective of Foreign due to their lower bond issuance. Thus, the *relative* change of prices at Foreign is larger than at Home. This resonates the mechanism of Barro and Bianchi (2024), by which a given fiscal deterioration has larger effects on the price level when the stock of sovereign debt is *smaller*, as a relatively larger change in the price level is needed to ensure that the real deterioration of the fiscal balance is sufficiently large.

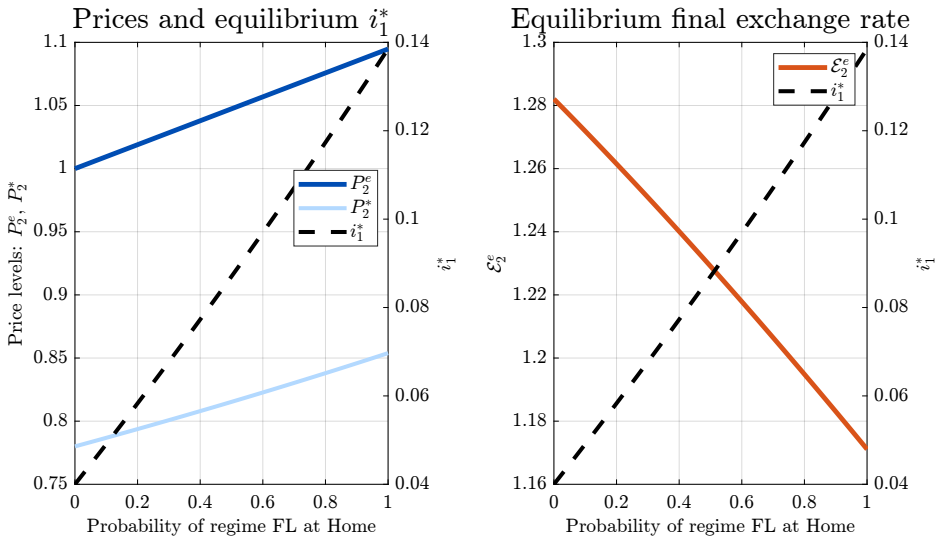


Figure A.1: Comparative statics with respect to ψ_1, the probability of Home being in regime FL, for prices and equilibrium exchange rates. Calibration: $B_1 = 1$, $B_1^* = 0.15$, $B_{H1}^* = B_{F1} = 0.1$, $\bar{i} = 0.04$, $\beta = 0.9$, $\bar{T} = 1$, $\bar{T}^* = 0.2$, $\mu_1 = 0.05$, $P_2^T = 1$, $\phi_F = 0.03$.

As for the exchange rate, note that there are two plausibly opposing effects, in line with equation

(A.11): there is a possible direct effect on the terminal exchange rate if the factual policy regime is actually FL through the deterioration of Home fiscal conditions, and an opposing effect from the spillovers on the fiscal budget of Foreign. In our calibration, the latter effect dominates, which is evidenced by the relatively larger deterioration of the real value of debt at Foreign relative to Home. Here, therefore, a higher probability of the fiscally-led regime in Home leads to an *appreciation* of the Home currency in the terminal period. Effectively, this is the international extension of the mechanism presented in Barro and Bianchi (2024): because the relative adjustment of the Foreign fiscal position is larger in response to the deterioration of the Home fiscal condition, the bilateral exchange rate can appreciate from the perspective of Home.

B Omitted Proofs of Simple Model

Proof of Proposition 9

Proof. We take equation (A.10) and rewrite it to:

$$(1+i_1^*) = \frac{\mathcal{E}_1}{\beta \mathbb{E}_1\left[\frac{P_1}{P_2}\mathcal{E}_2\right]}.$$

We now insert here equation (A.11) after rewriting it to $\frac{\mathcal{E}_2}{P_2} = \frac{1}{P_2^*}\frac{(1+\bar{i})}{(1+i_1^*)}\frac{B_{H1}^*}{B_{F1}}$, yielding:

$$\mathcal{E}_1 = \beta \frac{P_1}{P_2^*}(1+\bar{i})\frac{B_{H1}^*}{B_{F1}}.$$

Use now equation (A.9) to replace $\mathcal{E}_1$ to obtain:[19]

$$\frac{1}{\beta(1+\bar{i})\mathbb{E}_1\left[\frac{P_1^*}{P_2^*}\frac{1}{\mathcal{E}_2}\right] + \phi_F} = \beta \frac{P_1}{P_2^*}(1+\bar{i})\frac{B_{H1}^*}{B_{F1}}.$$

Replacing $\mathcal{E}_2$ again:

$$\frac{1}{\beta \mathbb{E}_1\left[\frac{P_1^*}{P_2}(1+i_1^*)\frac{B_{F1}}{B_{H1}^*}\right] + \phi_F} = \beta \frac{P_1}{P_2^*}(1+\bar{i})\frac{B_{H1}^*}{B_{F1}}.$$

We now replace the price levels (both materialized and expected) using equations (A.12), (A.14), (A.15), (A.16), and (A.17) to obtain:

[19] Note that P_2^* is known in period 1 as it only depends on variables set in $t=1$, so we can drop the expectations operator.

$$\frac{1}{\beta\frac{1}{\beta}\frac{B_1^*}{\bar{T}^*}\left[\psi_1\frac{\bar{T}-\mu_1}{B_1(1+\bar{i})}+(1-\psi_1)\frac{1}{P_2^T}\right](1+i_1^*)\frac{B_{F1}}{B_{H1}^*}+\phi_F}=\frac{1}{\left[\psi_1\frac{\bar{T}-\mu_1}{B_1(1+\bar{i})}+(1-\psi_1)\frac{1}{P_2^T}\right]}\frac{\bar{T}^*}{(1+i_1^*)B_1^*}\frac{B_{H1}^*}{B_{F1}}. \tag{B.1}$$

Note here that for $\phi_F = 0$, $(1+i_1^*)$ would be undertermined by this equation. As in standard models, we would require $i_1^* = \bar{i}$ for bond markets to clear. Rewriting this expression, we obtain a closed-form solution for the foreign interest rate in equilibrium that applies for bonds issued in period 1:

$$(1+i_1^*) = \frac{\phi_F}{\left[\psi_1\frac{\bar{T}-\mu_1}{B_1(1+\bar{i})}+(1-\psi_1)\frac{1}{P_2^T}\right]\left\{\frac{B_1^*}{\bar{T}^*}\frac{B_{H1}^*}{B_{F1}}-\frac{B_1^*}{\bar{T}^*}\frac{B_{F1}}{B_{H1}^*}\right\}}. \tag{B.2}$$

□

Proof of Corollary 10

Proof. Simply taking the first derivative of equation (A.18) yields:

$$\frac{\partial(1+i_1^*)}{\partial\psi_1} = -\phi_F\left(\frac{1}{\left[\psi_1\frac{\bar{T}-\mu_1}{B_1(1+\bar{i})}+(1-\psi_1)\frac{1}{P_2^T}\right]}\right)^2\left\{\frac{B_1^*}{\bar{T}}\frac{B_{H1}^*}{B_{F1}}-fracB_1^*\bar{T}^*\frac{B_{F1}}{B_{H1}^*}\right\}\left(\frac{\bar{T}-\mu_1}{B_1(1+\bar{i})}-\frac{1}{P_2^T}\right).$$

Now, the assumption that $P_2|_{FL} > P_2|_{ML}$ means that $\frac{B_1(1+\bar{i})}{\bar{T}-\mu_1} > P_2^T$, which in turn implies $\frac{\bar{T}-\mu_1}{B_1(1+\bar{i})} < \frac{1}{P_2^T}$ if $B_{H1}^* > B_{F1}$. Therefore, the last bracket is negative, negating the initial minus sign. All other elements are positive, hence this partial derivative is indeed larger than zero. Case 2 follows analogously. □

C Additional details on the empirical framework

C.1 Deriving the linearised government debt value equation

We break this derivation up into multiple steps. Note that this derivation follows Cochrane (2019) closely and is reproduced here for completeness.

Step 1. Describing the value of government debt. Recall that the nonlinear nominal government budget constraint is given by

$$M_t + \sum_{j=1}^{\infty} Q_{t+1}^{(t+j)} B_t^{(t+j)} \;=\; P_{t+1} sp_{t+1} + M_{t+1} + \sum_{j=1}^{\infty} Q_{t+1}^{(t+1+j)} B_{t+1}^{(t+1+j)}.$$

Here, sp_{t+1} is the *real primary surplus* (i.e. does *not* include interest payments). At the beginning of period $t+1$, money M_t and bonds $B_t^{(t+1+j)}$ are outstanding.

Recall by definition that the nominal return on the portfolio of government debt (holding period returns) is

$$R_{t+1}^n \equiv \frac{M_t + \sum_{j=1}^{\infty} Q_{t+1}^{(t+j)} B_t^{(t+j)}}{M_t + \sum_{j=1}^{\infty} Q_t^{(t+j)} B_t^{(t+j)}}. \tag{C.1}$$

We can re-arrange this expression to

$$R_{t+1}^n \left(M_t + \sum_{j=1}^{\infty} Q_t^{(t+j)} B_t^{(t+j)} \right) = M_t + \sum_{j=1}^{\infty} Q_{t+1}^{(t+j)} B_t^{(t+j)}.$$

Re-write the index on the left-hand side

$$R_{t+1}^n \left(M_t + \sum_{j=0}^{\infty} Q_t^{(t+1+j)} B_t^{(t+1+j)} \right) = M_t + \sum_{j=0}^{\infty} Q_{t+1}^{(t+j)} B_t^{(t+j)}$$

Now, use the following definition of the nominal end-of-period market value of debt

$$V_t \equiv M_t + \sum_{j=0}^{\infty} Q_t^{(t+1+j)} B_t^{(t+1+j)},$$

to get

$$M_t + \sum_{j=1}^{\infty} Q_{t+1}^{(t+j)} B_t^{(t+j)} = R_{t+1}^n V_t.$$

Plug into equation (C.1) to get

$$R_{t+1}^n V_t = P_{t+1} sp_{t+1} + M_{t+1} + \sum_{j=1}^{\infty} Q_{t+1}^{(t+1+j)} B_{t+1}^{(t+1+j)}.$$

For the right-hand side, note that we can iterate the nominal end-of-period market value of debt one period forward to get

$$V_{t+1} \equiv M_{t+1} + \sum_{j=0}^{\infty} Q_{t+1}^{(t+2+j)} B_{t+1}^{(t+2+j)}.$$

We can rewrite the index

$$V_{t+1} = M_{t+1} + \sum_{j=1}^{\infty} Q_{t+1}^{(t+1+j)} B_{t+1}^{(t+1+j)}.$$

Plugging this in allows us to conclude

$$V_{t+1} = R^n_{t+1} V_t - P_{t+1} sp_{t+1}.$$

The nominal value of government debt is increased by the nominal rate of return, and decreased by primary surpluses.

Step 2. Introduce stationarity. First, divide through by P_{t+1}

$$\begin{aligned} \leftrightarrow \quad & V_{t+1} = V_t R^n_{t+1} - P_{t+1} sp_{t+1} \\ \Rightarrow \quad & \frac{V_t}{P_t} R^n_{t+1} \frac{P_t}{P_{t+1}} = \frac{V_{t+1}}{P_{t+1}} + sp_{t+1}. \end{aligned}$$

Next, divide by output at time $t+1$

$$\begin{aligned} \leftrightarrow \quad & \frac{V_t}{P_t} R^n_{t+1} \frac{P_t}{P_{t+1}} \frac{1}{Y_{t+1}} = \frac{V_{t+1}}{Y_{t+1} P_{t+1}} + \frac{sp_{t+1}}{Y_{t+1}} \\ \Rightarrow \quad & \frac{V_t}{P_t Y_t} \frac{R^n_{t+1}}{G_{t+1}} \frac{1}{\Pi_{t+1}} = \frac{V_{t+1}}{P_{t+1} Y_{t+1}} + \frac{sp_{t+1}}{Y_{t+1}}, \end{aligned}$$

where we defined GDP growth $G_{t+1} \equiv \frac{Y_{t+1}}{Y_t}$ and inflation rate $\Pi_{t+1} \equiv \frac{P_{t+1}}{P_t}$.

Step 3. Linearisation. Take logs of both sides to get

$$\log\left(\frac{V_t}{P_t Y_t}\right) + \log\left(R^n_{t+1}\right) - \log\left(G_{t+1}\right) - \log\left(\Pi_{t+1}\right) = \log\left(\frac{V_{t+1}}{P_{t+1} Y_{t+1}} + \frac{sp_{t+1}}{Y_{t+1}}\right).$$

We define $v_t \equiv \log\left(\frac{V_t}{P_t Y_t}\right)$, $r^n_{t+1} \equiv \log\left(R^n_{t+1}\right)$, $\pi_{t+1} \equiv \log\left(\Pi_{t+1}\right)$,and $g_{t+1} \equiv \log\left(G_{t+1}\right)$. Thus,

$$v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} = \log\left(\frac{V_{t+1}}{P_{t+1} Y_{t+1}} + \frac{sp_{t+1}}{Y_{t+1}}\right).$$

Finally, define the new variable $sy_{t+1} \equiv \frac{sp_{t+1}}{Y_{t+1}}$.

$$v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} = \log\left(sy_{t+1} + \frac{V_{t+1}}{P_{t+1} Y_{t+1}}\right).$$

We linearise this equation in the *level* of the surplus, not its log since the surplus may be negative. We Taylor expand the right-hand side of the above expression around the steady-state values

(sy, v):

$$\begin{aligned}\log\left\{sy_{t+1} + \frac{V_{t+1}}{P_{t+1}}\right\} &= \log\left\{sy_{t+1} + \exp\left(\log\left\{\frac{V_{t+1}}{P_{t+1}}\right\}\right)\right\} = \log\left\{sy_{t+1} + e^{v_{t+1}}\right\} \\ &\approx \log\left\{sy + e^{v}\right\} + \frac{1}{sy + e^{v}}\left(sy_{t+1} - sy\right) + \frac{e^{v}}{sy + e^{v}}\left(v_{t+1} - v\right) \\ &= \left[\log\left\{sy + e^{v}\right\} - \frac{e^{v}}{sy + e^{v}}\left[v + sye^{-v}\right]\right] + \frac{1}{sy + e^{v}}sy_{t+1} + \frac{e^{v}}{sy + e^{v}}v_{t+1}.\end{aligned}$$

Therefore, the linearisation is

$$\log\left\{sy_{t+1} + \frac{V_{t+1}}{P_{t+1}}\right\} = \left[\log\left\{sy + e^{v}\right\} - \frac{e^{v}}{sy + e^{v}}\left[v + sye^{-v}\right]\right] + \frac{1}{sy + e^{v}}sy_{t+1} + \frac{e^{v}}{sy + e^{v}}v_{t+1}.$$

We first compute the steady–state relationships. First, do the following manipulation

$$v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} = \log\Big\{sy_{t+1} + \exp\big(\log\{\tfrac{V_{t+1}}{P_{t+1}Y_{t+1}}\}\big)\Big\}.$$

Now evaluate at steady state

$$\begin{aligned}&\leftrightarrow \quad v + r^n - \pi - g = \log\big\{sy + e^{v}\big\} \\ &\Rightarrow \quad \tfrac{e^{v}}{sy+e^{v}} = e^{\pi+g-r^n}.\end{aligned}$$

Now we substitute steady-state values into the Taylor expansion. Using

$$\log\{sy + e^{v}\} = v + r^n - \pi - g, \qquad \frac{1}{sy + e^{v}} = \frac{e^{\pi+g-r^n}}{e^{v}}, \qquad \frac{e^{v}}{sy + e^{v}} = e^{\pi+g-r^n},$$

we can plug this into the right-hand side of the linearisation

$$\log\left\{sy_{t+1} + \frac{V_{t+1}}{P_{t+1}}\right\} = \left[\log\left\{sy + e^{v}\right\} - \frac{e^{v}}{sy + e^{v}}\left[v + sye^{-v}\right]\right] + \frac{1}{sy + e^{v}}sy_{t+1} + \frac{e^{v}}{sy + e^{v}}v_{t+1}.$$

to get the right-hand side to be

$$\left[(v + r^n - \pi - g) - e^{\pi+g-r^n}\big(v + sye^{-v}\big)\right] + \frac{e^{\pi+g-r^n}}{e^{v}}\,sy_{t+1} + e^{\pi+g-r^n}\,v_{t+1}.$$

After manipulating the right-hand side, it becomes:

$$\leftrightarrow \quad \left[(v + r^n - \pi - g) - e^{\pi+g-r^n}v - 1 + e^{\pi+g-r^n}\right] + \frac{e^{\pi+g-r^n}}{e^{v}}\,sy_{t+1} + e^{\pi+g-r^n}\,v_{t+1}.$$

Now define

$$\rho \;\equiv\; e^{\pi+g-r^n}.$$

Plug this in the right-hand side expression, which becomes

$$\begin{aligned} \leftrightarrow \quad & \Big[(v + r^n - \pi - g) - \rho v + \rho - 1\Big] + \frac{\rho}{e^v}\, s y_{t+1} + \rho\, v_{t+1} \\ \Rightarrow \; & (r^n - \pi - g) + (1-\rho)(v-1) + \frac{\rho}{e^v}\, s y_{t+1} + \rho\, v_{t+1}. \end{aligned}$$

So now go back to

$$v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} = \log\left(s y_{t+1} + \frac{V_{t+1}}{P_{t+1} Y_{t+1}}\right).$$

Plug in our log-linearised expression for the right-hand side

$$v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} = (r^n - \pi - g) + (1-\rho)(v-1) + \frac{\rho}{e^v}\, s y_{t+1} + \rho\, v_{t+1}\ .$$

We suppress the small constant terms (since we treat all variables as mean-zero).

$$v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} = \underbrace{\Big[(r^n - \pi - g) + (1-\rho)(v-1)\Big]}_{\text{tiny constant}} + \frac{\rho}{e^v}\, s y_{t+1} + \rho\, v_{t+1}.$$

Therefore,

$$v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} = \frac{\rho}{e^v}\, s y_{t+1} + \rho\, v_{t+1}.$$

Define the scaled surplus innovation

$$s_{t+1} \;\equiv\; \frac{\rho}{e^v}\, s y_{t+1},$$

so that the equation becomes

$$v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} = s_{t+1} + \rho\, v_{t+1}.$$

Finally, re-arrange to get the linearised flow identity $\rho\, v_{t+1} = v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} - s_{t+1}$.

C.2 Further Details on the VAR Framework

This appendix presents the derivations underpinning the VAR model estimated in Section 2. The derivations follow Cochrane (2019), which we extend to an open-economy framework. We begin with a narrative of the major U.K. fiscal episodes visible in Figure C.1, broadly in line with Ellison and Scott (2020), before turning to the backward and forward decompositions and the variance decomposition.

The last half-century covered by Figure C.1 divides naturally into two broad regimes, each punctuated by acute crises. The first regime, spanning roughly 1970 to 2000, is characterised by the transition from fiscal dominance to a credible nominal anchor. The March 1972 "dash for growth"

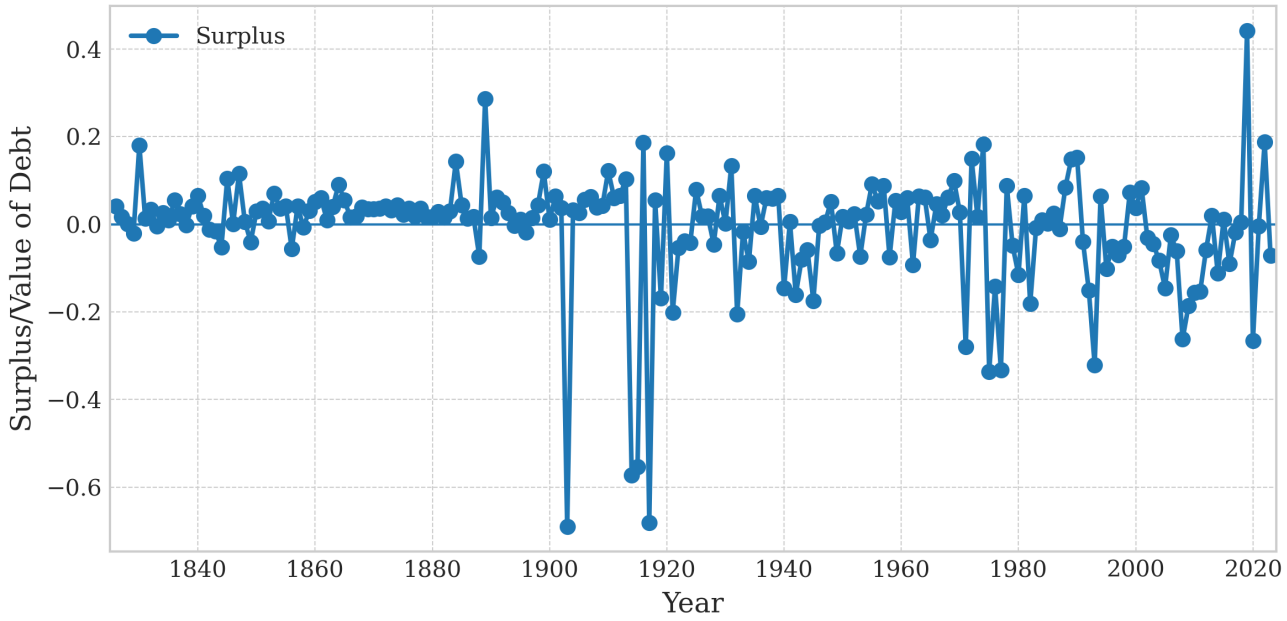


Figure C.1: Implied U.K. surpluses to ensure the government flow identity holds.

budget combined large tax cuts with relaxed credit controls, pushing borrowing requirements to £3.4 billion, then 5% of GDP. The resulting demand boom collided with the 1973 oil shock, sending CPI inflation into double digits and gilt yields to post-war highs. For the first time in three centuries, Britain ran substantial primary deficits outside of wartime. Market participants began to price a fiscal dominance regime: Sterling sold off, gilt investors demanded an inflation premium, and the surplus-over-debt ratio remained below zero until the 1976 IMF programme. The 1980 Medium-Term Financial Strategy then imposed cash limits on departmental spending, turning a structural deficit into a modest surplus within four years, and subsequent positive surpluses persisted through the Thatcher administration. This consolidation was interrupted by the twin blows of a deep domestic recession and "Black Wednesday" in 1992–93: when speculative pressure forced Sterling out of the Exchange Rate Mechanism, the Treasury raised base rates to 15% in a failed defence of the parity, output contracted, and tax receipts collapsed. On the valuation side, the real exchange rate dropped 15% within weeks, instantly re-inflating the domestic-currency value of foreign-currency gilts. The subsequent adoption of inflation targeting stabilised expectations, and by 1995 the surplus ratio had drifted back towards zero. Between 1996 and 2000, robust productivity growth, a high tax base, and restrained departmental spending kept the cash deficit near balance, while falling global real rates reduced the discount factor applied to future surpluses, so that only small primary surpluses were required to keep the ratio flat.

The second regime, from the early 2000s onward, is dominated by two large crisis-driven fiscal expansions. The Global Financial Crisis produced the deepest post-war trough in the surplus ratio during 2009–12: GDP fell 4.5% in 2009, policy aimed at stabilising the banking system pushed the

headline deficit above 10% of GDP, and automatic stabilisers added a further two percentage points. Simultaneously, Bank of England gilt purchases suppressed real yields to multi-decade lows, which the accounting identity treats as requiring even larger negative surpluses to match the observed market value of debt; public sector net debt jumped from 41% to 81% of GDP. From 2013, the coalition's fiscal consolidation and a stabilisation of real rates nudged the ratio back toward zero, though the path was flat rather than rising: low discount factors meant a balanced-budget stance was sufficient for debt stabilisation without the large primary surpluses of the 1980s. Modest surpluses in 2017–19 briefly lifted the ratio into positive territory, but the Covid-19 pandemic reversed these gains. Emergency spending, furlough schemes, and revenue shortfalls generated a deficit of 14.6% of GDP in 2020, yet in Figure C.1 the 2020 observation *rises* to +0.42. The explanation lies in the valuation channel: gilt real yields turned sharply negative and inflation picked up, generating a large *implied* surplus in present-value terms even as the cash balance deteriorated. This is a Lucas and Stokey (1983) state-contingent financing of debt through surprise inflation. The episode encapsulates the fiscal-theory message: debt dynamics hinge on the path of discount factors and prices as much as on the arithmetic of tax minus spending.

Taken together, Figure C.1 shows that shifts in the surplus-over-debt ratio align closely with the regime breaks and policy narratives documented by Bordo et al. (2022). Episodes that placed acute pressure on public finances, whether war financing, the 1970s stagflation, or the Global Financial Crisis, drive the surplus ratio deep into negative territory, whereas phases of deliberate consolidation or windfall receipts keep it persistently positive. Crucially, the accounting framework highlights that shifts in valuation terms, through unexpected inflation and movements in discount rates, can do as much of the adjustment work as the cash-flow surpluses themselves.

C.2.1 Additional Details on the Backward Decomposition

The present-value identity in Section 2 converts a sequence of forward flows into one equilibrium condition that we can take to the data and decompose into economically meaningful components. We now derive this identity. First, we iterate both flow identities forward to arrive at present-value identities for home and foreign debt:

$$v_t^H = \sum_{j=1}^{\infty} \rho_H^{j-1} s_{t+j}^H + \sum_{j=1}^{\infty} \rho_H^{j-1} g_{t+j}^H - \sum_{j=1}^{\infty} \rho_H^{j-1} \left[r_{t+j}^{n,F} + \Delta e_{t+j} + \varphi_{t+j} - \pi_{t+j}^H \right], \tag{C.2}$$

$$v_t^F = \sum_{j=1}^{\infty} \rho_F^{j-1} s_{t+j}^F + \sum_{j=1}^{\infty} \rho_F^{j-1} g_{t+j}^F - \sum_{j=1}^{\infty} \rho_F^{j-1} \left[r_{t+j}^{n,F} - \pi_{t+j}^F \right]. \tag{C.3}$$

Investors price U.S. and U.K. bonds jointly. Persistent Dollar appreciation or a sustained fall in the UIP premium lowers the discount factor for U.S. surpluses while raising it for U.K. surpluses,

binding the two budget constraints in expectation. Rewriting the forward identity for the home country yields

$$v_t^H = \sum_{j\geq 1} \rho_H^{j-1} s_{t+j}^H + \sum_{j\geq 1} \rho_H^{j-1} g_{t+j}^H - \sum_{j\geq 1} \rho_H^{j-1} r_{t+j}^{n,F} - \sum_{j\geq 1} \rho_H^{j-1} \Delta e_{t+j} - \sum_{j\geq 1} \rho_H^{j-1} \varphi_{t+j} + \sum_{j\geq 1} \rho_H^{j-1} \pi_{t+j}^H. \tag{C.4}$$

Equation (C.4) holds ex ante as well as ex post. Taking expectations conditional on information at time t gives

$$\begin{aligned} v_t^H = {} & \underbrace{\mathbb{E}_t\left[\sum_{j\geq 1} \rho_H^{j-1} s_{t+j}^H\right]}_{PV_t^{(s)}} + \underbrace{\mathbb{E}_t\left[\sum_{j\geq 1} \rho_H^{j-1} g_{t+j}^H\right]}_{PV_t^{(g)}} - \underbrace{\mathbb{E}_t\left[\sum_{j\geq 1} \rho_H^{j-1} r_{t+j}^{n,F}\right]}_{PV_t^{(r)}} \\ & - \underbrace{\mathbb{E}_t\left[\sum_{j\geq 1} \rho_H^{j-1} \Delta e_{t+j}\right]}_{PV_t^{(\Delta e)}} - \underbrace{\mathbb{E}_t\left[\sum_{j\geq 1} \rho_H^{j-1} \varphi_{t+j}\right]}_{PV_t^{(\varphi)}} + \underbrace{\mathbb{E}_t\left[\sum_{j\geq 1} \rho_H^{j-1} \pi_{t+j}^H\right]}_{PV_t^{(\pi)}}, \end{aligned} \tag{C.5}$$

where $\rho_H \simeq 0.97$ is the annual Campbell–Shiller discount factor. For convenience we gather the real discount-rate components as

$$PV_t^{(d)} \equiv PV_t^{(r)} - PV_t^{(\pi)}.$$

Equation (C.5) can therefore be written as

$$v_t^H = PV_t^{(s)} + PV_t^{(g)} + PV_t^{(\pi)} - \left[PV_t^{(r)} + PV_t^{(\Delta e)} + PV_t^{(\varphi)}\right]. \tag{C.6}$$

Our goal is to turn the forward-looking identity in (C.6) into measurable objects that can be tracked year by year.

Turning to the trends in the evolution of U.K. debt visible in Figure 2, five distinct episodes emerge, though they are best understood in terms of the interaction between flow and valuation channels rather than in isolation.

The Sterling crisis of 1975–81 illustrates the dominance of valuation adjustment over fiscal retrenchment. The $-s$ line rises rapidly, reflecting large primary deficits, yet the debt ratio v increases only modestly because a cumulative 25% depreciation (the REER series) and double-digit inflation (the $-\pi$ series) erode the real value of gilts. Bordo et al. (2022) date this as the first peacetime episode of fiscal dominance since the 1690s, and the backward decomposition confirms that the adjustment came predominantly through the price level and the exchange rate.

During the subsequent North Sea revenue and disinflation period of 1982–2001, a very different pattern obtains. The $-s$ line flattens as small primary surpluses emerge, while the r^n series climbs as global real rates rise with the Volcker disinflation, so the $r^n - \pi - g$ bundle pulls the debt ratio

upward even in the face of favourable flows. Roughly half of the cumulative drift in debt over these two decades is valuation-driven rather than cashflow-driven. Embedded within this period, Sterling's 1992 departure from the ERM produces a sharp kink in the REER line, offset by a drop in r^n as gilts rally; these opposing valuation effects leave the debt path broadly unchanged, masking the cyclical widening and subsequent narrowing of primary balances.

The Global Financial Crisis of 2007–13 creates the largest wedge in the sample. Primary deficits widen sharply, and simultaneously the r^n line falls as world safe real rates plunge; the two forces reinforce each other, pushing the debt series sharply higher. Cochrane (2019) notes that in the U.S. decomposition approximately 52% of the post-2008 rise in debt is discount-rate news; the U.K. share is similar *only* once exchange rate channels are included, hinting at different debt dynamics for financial hegemons relative to smaller open economies.

Finally, the pandemic shock of 2020–22 generates record cash deficits alongside a large valuation gain: gilt real yields turn deeply negative, inflation re-accelerates, and the UIP curve spikes as investors demand a wider risk premium on Sterling assets. Roughly one half of the pandemic-era debt increase is therefore absorbed by prices rather than by flows.

C.2.2 Key Macroeconomic Episodes through the Lens of the Forward Decomposition

We now revisit the key U.K. macroeconomic episodes through the lens of the forward decomposition, following Figure 3.

The 1992 ERM crisis produces a one-off 15% Sterling depreciation that pushes the REER path upward while the UIP premium widens, making Sterling assets riskier. Expected surpluses dip with the recession but rebound rapidly after 1997, leaving the overall debt valuation little changed. A sharper break occurs with the Global Financial Crisis of 2008–12, when markets price a decade of large primary deficits (the s path plummets) and a near-zero real discount rate (the green curve surges). The two forces reinforce each other, driving v_t sharply higher. From 2013 onward, the headline fiscal plans envisage medium-term surpluses, steepening the s path. The mid-2016 Brexit referendum then causes the REER line to drop and the UIP component to turn negative as investors demand a higher Sterling risk premium, mechanically reducing the contemporaneous debt value. At the start of the pandemic, emergency spending in 2020 raises both s and v_t above 0.8 log points, yet the discount-rate term stays positive because gilt real yields plunge below -2%.

C.2.3 Variance Decomposition

Equation (C.4) writes the scaled market value of debt v_t^H as the sum of six discounted forward series. We now ask how much of the unconditional variance of v_t^H is attributable to each factor.

Taking the variance of (C.4) and abusing notation slightly gives

$$\text{var}(v) = \text{cov}(v, \Sigma s) - \text{cov}\big[v, \Sigma(r^f + \Delta rer + uip - \pi - g)\big].$$

Breaking the second covariance into its five sub-covariances and dividing by var(v) converts each component into a variance share.

Following Cochrane (2019), we construct two alternative transformations of the data before performing the variance calculations. The first is an innovation series: for each variable x_t, the innovation is $\check{x}_t = x_t - \mathbb{E}_{t-1}[x_t]$, obtained from the same VAR that underlies the forward decomposition; cumulating the innovations isolates the portion of debt dynamics that was unanticipated at the time, providing an accounting of "news" rather than levels. The second is a filtered series, obtained by applying the high-pass operator $\theta(L) = 1 - \frac{1}{3}(L + L^2 + L^3)$ to each raw series, so that for debt, for example, $\tilde{v}_t = v_t - \frac{1}{3}(v_{t-1} + v_{t-2} + v_{t-3})$, and analogously for $s, \pi, g, r^{n,F}, \Delta e, \varphi$. The filter removes low-frequency swings, such as wartime run-ups and long post-war declines, which dominate the unconditional variance but carry little business-cycle information; what remains is a medium-frequency measure that moves with recessions and policy cycles. Because identity (C.4) is algebraic, we may filter after forming the identity, and all terms still sum exactly.

Throughout the paper we report results for all three datasets: the plain raw sums, the innovation sums that isolate true surprises, and the filtered sums that highlight cyclical movements. The results are shown in Table C.1. A positive covariance means that the variable raises the variance of debt; a negative number means it moves opposite to debt, dampening variance. For compactness we report the five valuation channels with the same sign as they enter equation (C.4). Row 2 plus Row 3 equals unity by construction, and Rows 4–8 sum to Row 3, ensuring internal consistency across all three perspectives.

	Plain	Innovation	Filtered
var(v)	1.000	1.000	1.000
cov($v, \Sigma s$)	0.832	0.940	0.890
$-\text{cov}[v, \Sigma(r^f + \Delta rer + uip - \pi - g)]$	0.168	0.060	0.110
cov($v, \Sigma r^f$)	-0.452	-0.421	-0.328
cov($v, \Sigma \Delta rer$)	0.012	-0.000	0.061
cov($v, \Sigma uip$)	-0.275	-0.137	-0.230
cov($v, \Sigma \pi$)	-0.406	-0.415	-0.262
cov($v, \Sigma g$)	-0.142	-0.082	-0.125

Table C.1: Variance decomposition.

The dominant finding across all three perspectives is the overwhelming importance of surplus news, which account for 83% of the variance in the plain series, 94% in the innovation series, and 89% in the filtered series. This aligns with the central prediction of the fiscal theory: revisions in

expected primary surpluses are the primary driver of debt valuation.

The combined contribution of the valuation channels offsets the surplus share but never overturns it, reducing it by roughly 17 percentage points in the plain series and 6 percentage points in the innovation series. Among the individual valuation channels, the discount rate is the single largest offset, with a covariance of -0.45 in the plain series and -0.42 in the innovation series: periods in which nominal discount rates are expected to be high coincide with lower debt valuations, consistent with standard asset-pricing intuition. The UIP and inflation channels are also materially important, though their magnitudes are approximately half that of the discount-rate channel; unexpected declines in the UIP premium (-0.28 in the plain series) or in future inflation (-0.41) both push v_t upward. By contrast, real exchange rate news are small on average, with the innovation-series covariance near zero, implying that shocks to the expected path of the real exchange rate rarely move debt valuation contemporaneously; the filtered view allocates a modest 6% share, suggesting that slower-moving terms-of-trade trends play some role. Growth news are modest and negative: better expected growth raises the denominator (future consumption) faster than the numerator (real surpluses), so v_t co-moves negatively with Σg at roughly -0.14 in the plain series.

Overall, the decomposition confirms that, even in an open-economy setting with currency and risk-premium channels, fiscal news remain the dominant force. Financial variables primarily modulate its impact through discounting rather than overturning it.

D Linking deficits to exchange rates

Complementing section 2.3, we provide additional results akin to the 'reverse regression' approach from Engel and West (2005) and Jiang (2021).

$$\frac{s^i_{t+k}}{B^i_{t+k}} = \alpha^{(k)} + \beta^{(k)} \Delta \log e_t^{(-i)/i} + \delta_1^{(k)} \frac{s^i_t}{B^i_t} + \delta_2^{(k)} \Delta \frac{s^i_t}{B^i_t} + \gamma_1^{(k)} \frac{s^{-i}_t}{B^{-i}_t} + \gamma_2^{(k)} \Delta \frac{s^{-i}_t}{B^{-i}_t} + \varepsilon^i_{t+k}, \qquad \text{(D.1)}$$

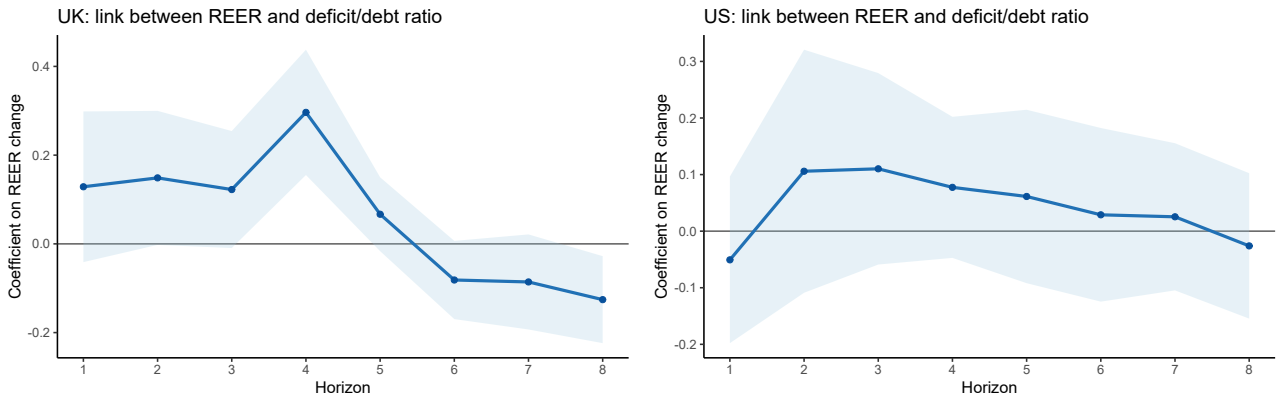
that is; we test the link between the deficit/debt-ratio at time $t+k$ and the change in the real exchange rate as well as the level and the first-difference of *both* domestic and foreign deficit/debt-ratio levels. The results of this exercise are presented in table D.1 and figure D.1a for the U.K. and in table D.2 and figure D.1b for the U.S. In a nutshell, current appreciations are related to smaller local government deficits, but in the case of the U.K., the phenomenon is reversing at very long horizons.

	Dependent variable: deficit-to-debt ratio at t+k							
	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8
Δ REER	0.129	0.149	0.122	0.296***	0.067	−0.081	−0.086	−0.126**
	(0.104)	(0.092)	(0.080)	(0.086)	(0.051)	(0.054)	(0.065)	(0.060)
$\frac{s}{B}$-UK	0.187	−0.491**	−0.858***	−1.109***	−0.905**	−0.287	−0.142	−0.219
	(0.256)	(0.201)	(0.224)	(0.200)	(0.410)	(0.260)	(0.289)	(0.478)
Δ $\frac{s}{B}$-UK	0.184	0.090	0.729**	0.729*	0.689	1.389**	0.604	0.706
	(0.202)	(0.276)	(0.348)	(0.405)	(0.584)	(0.626)	(0.461)	(0.608)
$\frac{s}{B}$-US	0.481**	0.814***	0.827***	0.897***	0.684	−0.005	−0.125	−0.192
	(0.194)	(0.229)	(0.256)	(0.188)	(0.421)	(0.281)	(0.291)	(0.409)
Δ $\frac{s}{B}$-US	−0.073	−0.018	−0.647*	−0.960	−0.752	−0.824	−0.038	−0.149
	(0.246)	(0.220)	(0.386)	(0.588)	(0.673)	(0.667)	(0.458)	(0.560)
Constant	−0.010	−0.024	−0.048***	−0.058***	−0.067***	−0.095***	−0.097***	−0.109***
	(0.010)	(0.015)	(0.016)	(0.010)	(0.016)	(0.012)	(0.013)	(0.017)
Obs.	29	28	27	26	25	24	23	22
R^2	0.659	0.436	0.365	0.548	0.287	0.354	0.251	0.387
Adj. R^2	0.585	0.308	0.214	0.435	0.099	0.175	0.030	0.195

Table D.1: UK: Engel and West (2005)-style regressions for the deficit-to-debt ratio. Newey-West standard errors in parentheses. *p<0.1; **p<0.05; ***p<0.01.

	Dependent variable: deficit-to-debt ratio at t+k							
	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8
Δ REER	−0.051	0.106	0.110	0.077	0.061	0.029	0.025	−0.026
	(0.090)	(0.131)	(0.103)	(0.076)	(0.093)	(0.094)	(0.079)	(0.078)
$\frac{s}{B}$-UK	−0.183***	−0.407***	−0.581***	−0.544***	−0.500***	−0.482**	−0.455**	−0.412*
	(0.070)	(0.109)	(0.135)	(0.138)	(0.175)	(0.190)	(0.181)	(0.225)
Δ $\frac{s}{B}$-UK	0.284***	0.405*	0.413**	0.283	0.126	0.131	−0.020	−0.169
	(0.096)	(0.215)	(0.191)	(0.220)	(0.217)	(0.250)	(0.361)	(0.432)
$\frac{s}{B}$-US	0.726***	0.453***	0.345**	0.263**	0.306**	0.350**	0.354*	0.282
	(0.108)	(0.171)	(0.170)	(0.128)	(0.121)	(0.155)	(0.193)	(0.197)
Δ $\frac{s}{B}$-US	0.139	0.057	−0.130	−0.322*	−0.267*	−0.301	0.001	0.140
	(0.123)	(0.183)	(0.197)	(0.169)	(0.160)	(0.219)	(0.270)	(0.261)
Constant	−0.038***	−0.079***	−0.100***	−0.103***	−0.095***	−0.088***	−0.087***	−0.093***
	(0.012)	(0.018)	(0.016)	(0.015)	(0.016)	(0.016)	(0.019)	(0.026)
Obs.	52	51	50	49	48	47	46	45
R^2	0.624	0.374	0.329	0.293	0.254	0.244	0.230	0.204
Adj. R^2	0.583	0.305	0.253	0.211	0.165	0.152	0.133	0.102

Table D.2: US: Engel and West (2005)-style regressions for the deficit-to-debt ratio. Newey-West standard errors in parentheses. *p<0.1; **p<0.05; ***p<0.01.



(a) United Kingdom

(b) United States

Figure D.1: Engel and West (2005)-style reverse regression

E Private sector deficit forecast revisions

In this section, we present more in-depth evidence in favour of a direct link between deficits, exchange rates, and inflation, in line with the predictions implied by the government debt valuation equation, and thereby supporting the results of section 2 by using private-sector data on deficit expectations in place of the deficit innovations recovered in the VAR framework. To that goal, we now turn to a novel dataset of financial forecasters in the United Kingdom, evaluating to which degree forecasters link deficits, exchange rates, and inflation, and whether a stronger link corresponds to an improved forecast performance. A part of our contribution is the assembly of a near-30-year-long dataset of macroeconomic forecasts made by private forecasters in the United Kingdom, which has henceforth not been created to the best of our knowledge.

This exercise is broadly in the line of work of Cochrane (2018) and Barro and Bianchi (2024): we use an *innovation representation* of the government debt valuation equation in the context of international holdings of government debt to inform forecast revisions pinning down a plausible link between deficits and exchange rates.

Data and estimation

The data that we use in this section is the universe of private sector forecasts on various macroeconomic variables, collected by the U.K. Treasury (HM Treasury, 2025). While aggregate data on average forecasts is available in machine-readable format, the individual forecasts were initially not available as such. We hand-collected all forecaster-specific data since September 1997 on a monthly basis.

In the process of collecting the data, the U.K. Treasury asks a select number of banks and financial institutions (labelled 'City Forecasters') as well as a large number of independent forecasting

bodies without direct vested interest (labelled 'Non-City forecasters') about their same-year and one-year-ahead predictions for a battery of macroeconomic and financial variables. Not all forecasters update their forecasts each month, but the median forecaster who is in the sample for at least three years provides updates multiple times over the year. Of particular interest to us are the forecasts on sovereign deficits (officially "Public Sector Net Borrowing", or PSNB), exchange rates, real interest rates (recovered through the corresponding forecasts made on the Bank of England lending rate and CPI/RPI inflation), and of output growth.

We begin by providing descriptive statistics related to the forecast dataset. Overall, we have 88 unique forecasters in our data-sample, spanning almost 28 complete years (Sep-1997 until Jun-2025) yielding a total of 15,089 unique forecaster-month observations that contain at least some novel forecast being made.[20]

Figure E.1 provides information on the number of unique forecasters appearing in the data and providing novel forecasts at least once a year. We observe a steady downwards trend in the number of forecasters who provide sufficient information to be considered in out analysis. With the exception of the two most recent years, however, there are continually at least 30 individual forecasters available in each year that provide at least one sufficient set of forecasts to be considered in the analysis.

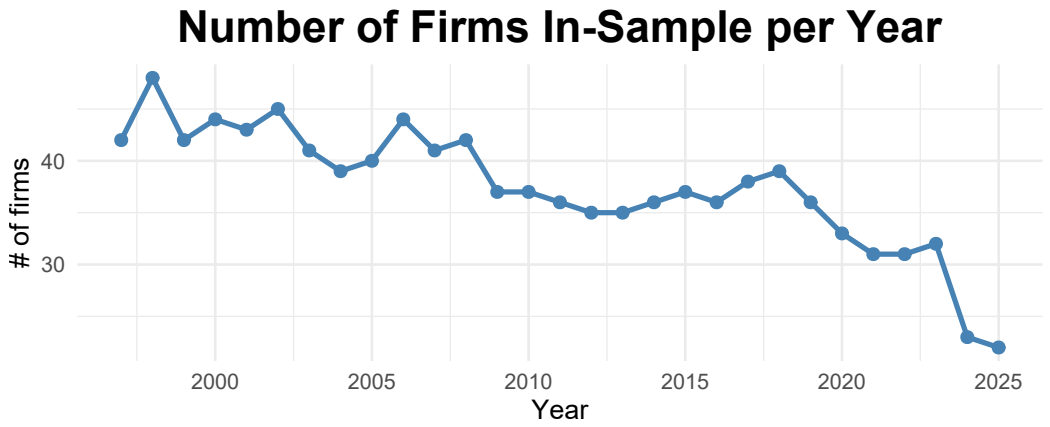


Figure E.1: Number of unique forecasters appearing in the data in each year.

Of particular interest, of course, is the frequency of forecast revisions. Forecasts are made in rolling two-year windows, and so the number of forecast changes *within* a given year is informative for the number of valid observations that we can obtain. Figure E.2 provides this information, showing how often firms in the sample on average revise their forecasts for a given forecasting window. Conditioning on the forecasters who provide sufficiently complete observations of forecasts,

[20] In the data, we generally drop the forecasts made by the European Commission, the OECD, and the IMF, since their forecasts reflect general model estimates that are relatively non-reflective of novel information coming in at monthly frequencies.

we observe a relatively high number of forecast revisions, averaging 6-8 per year per forecaster. These within-year forecast revisions are at the centre of our analysis.

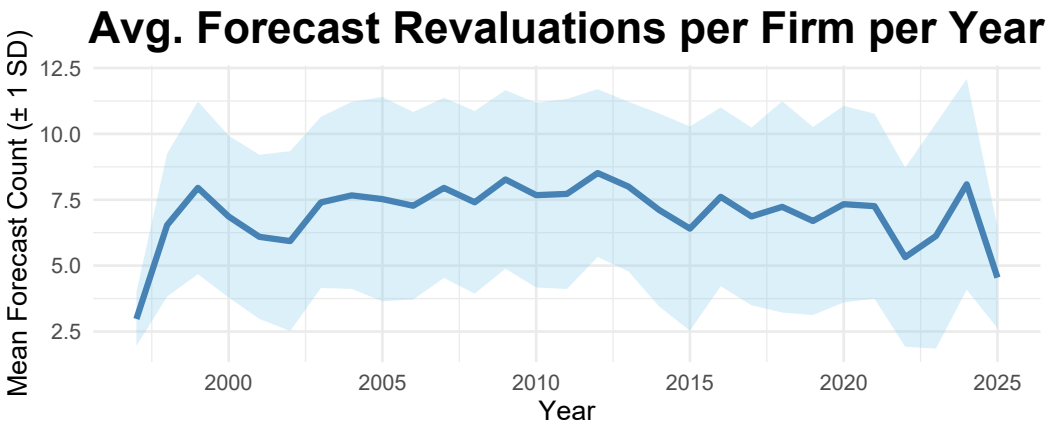


Figure E.2: Number of forecast revisions per firm per year, on average. Shaded bands indicate one standard deviation around the mean of per-firm forecast revisions

In table E.1 we provide information on the average forecast revisions in the sample of forecasters. Generally, the median revision of each individual variable is centred around zero. Since the mean of the PSNB (deficit) forecast revisions is larger than the median, this indicates a possible slight right-skew in the distribution of forecast revisions. Overall, forecasters appear to stick to their original forecasts (made at the beginning of a given forecast year), unless they are receiving novel information that forces them to amend their forecasts in accordance with their needs.

Variable	Mean	Median	SD	Q1	Q3
$\Delta\mathbb{E}_t$(PSNB) (in GBP Billion)	1.111	0.000	13.188	-0.995	1.848
$\Delta\mathbb{E}_t(g_Y)$ (in %)	-0.052	0.000	0.616	-0.100	0.100
$\Delta\mathbb{E}_t(\pi_{RPI})$ (in p.p.)	0.027	0.000	0.580	-0.100	0.200
$\Delta\mathbb{E}_t(\mathcal{E})$ (in %)	0.000	0.000	0.057	-0.003	0.008

Table E.1: Summary Statistics for forecast revisions across the entire sample.

Given our particular interest in the forecasts of deficits (PSNB) and of exchange rates, figure E.3 depicts the distribution of forecast revisions in both variables across time. As is to be expected, the average forecast revision is generally not significantly different from zero, but there is substantial disagreement in terms of the actual revisions. In terms of the forecasts for the exchange rate (right-hand panel), the Brexit episode stands out as one of consensus expectations of depreciation. As for deficit (PSNB) forecasts, the Covid-episode and the subsequent stimulus packages were generally underestimated when forecasts were made, followed by a gradual upwards revision of deficit expectations alongside each individual year.

To complement the depiction of the forecasts, we also consider the average of the *actual* forecasts made by the forecasters. Table E.2 summarizes the average forecasts of our four main

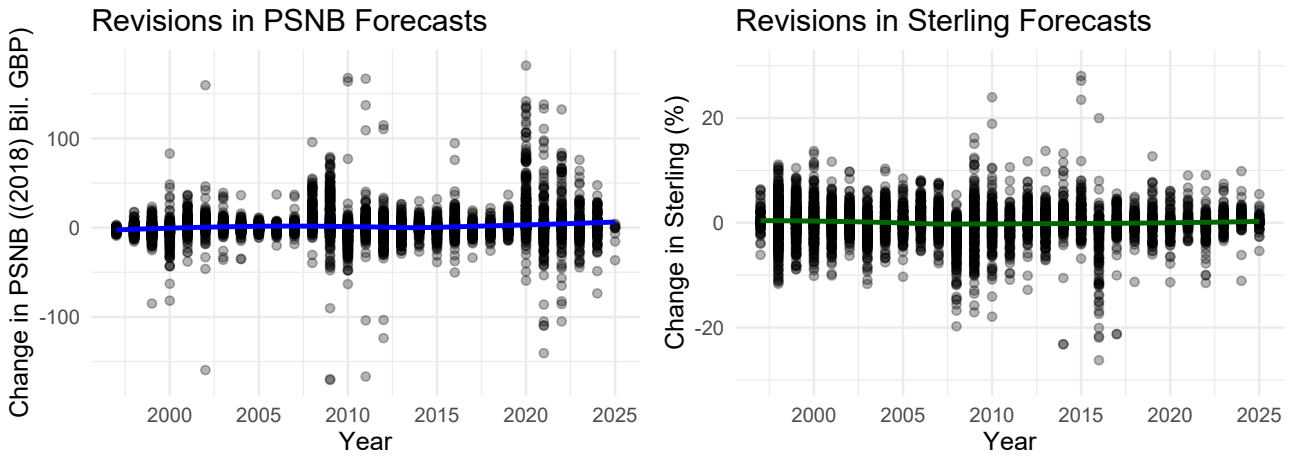


Figure E.3: Distribution of forecast revisions in exchange rates and deficits over time.

variables of interest. Since 1997, average annual deficits were predicted to hover around 60 billion GBP in real (2018) terms, while real output growth has been predicted to remain on average slightly below 2%. In contrast, retail price inflation expectations were closer to 3 percentage points, while the average exchange rate index reflects broad expectations of depreciations relative to the benchmark period, which here is January 2005.

Variable	Mean	Median	SD	Q1	Q3
PSNB (in GBP Billion)	60.241	43.575	54.801	26.318	90.074
g_Y (in %)	1.749	1.900	2.003	1.200	2.600
π_{RPI} (in p.p.)	2.881	2.800	1.446	2.300	3.400
$\mathcal{E}_{\text{Sterling}\|\text{Basket}}$ (in %)	90.889	92.600	9.446	82.100	98.931

Table E.2: Summary statistics of actual forecasts made by forecasters

Econometric specification: let T be the forecast horizon, $t-1$ be the date of the previous forecast, and t be the current date. Then, we estimate generally the following equation:

$$\Delta\mathbb{E}_{it}\mathcal{E}_{(T-j)\to T} = \beta_0 + \beta_1\Delta\mathbb{E}_{it}(-\hat{s}_T) + \beta_2\Delta\mathbb{E}_{it}\hat{y}_T + \beta_3\Delta\mathbb{E}_{it}\hat{r}_T + \Gamma Z_{it} + \varepsilon_{it}. \tag{E.1}$$

Our main specification, equation (E.1), is made up of the following elements: $\Delta\mathbb{E}_{it}\hat{e}_{(T-12)\to T}$ on the left-hand side captures the forecast revision from $t-1$ to t ($\Delta\mathbb{E}_{it}$) of the expected change in the exchange rate from the end-of-last-year (which was j months ago) until the end of the year, which is the time period T. On the right-hand side, we have a forecaster-specific fixed-effect α_i.[21] Next, β_1 describes the main coefficient of interest, showing the effect of revisions in expected deficits (negative surpluses) for the year ending at T for the ability to forecast exchange rates. The next coefficients β_2 and β_3 are symmetrically the coefficients on revisions of output growth

[21] We also employ alternative specifications with a joint forecaster-year fixed-effect, as per the tables below.

and the expected real interest rate earned in the year ending at period T. Finally, ΓZ_{it} captures further control variables, and ε_{it} is the estimation error. The fact that we generally control for other forecast revisions in the estimation process of forecast revisions is thus both informed by economic theory *and* in line with evidence presented by Crump et al. (2025) that forecast revisions are occurring jointly, with important effects being missed when considering only mutual pairs of such revisions.

Results

We now present our results. First, we establish that forecasters on average link revisions indicating larger deficits to corresponding depreciations. Additionally, we provide evidence that those forecasters whose forecast revisions are closer aligned with the deficit-exchange rate-mechanism are, on average, also better in terms of their own exchange rate forecasting performance, rationalizing the link between deficits and exchange rates from the perspective of mutual innovations.

	Outcome: $\Delta\mathbb{E}(\mathcal{E}_{\text{Sterling}\|\text{Basket}})$				
	(1)	(2)	(3)	(4)	(5)
$\Delta\mathbb{E}$(PSNB)	-0.0005**	-0.0005**	-0.0005**	-0.0005**	-0.0002***
	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(5.6×10^{-5})
$\Delta\mathbb{E}(g_Y)$	0.0043**	0.0053***	0.0024	0.0033	0.0047***
	(0.0018)	(0.0018)	(0.0022)	(0.0021)	(0.0011)
$\Delta\mathbb{E}(r_{t+1})$	0.0037*	0.0040**			0.0001
	(0.0018)	(0.0017)			(0.0010)
$\Delta\mathbb{E}(i_{t+1})$			0.0133***	0.0141***	
			(0.0038)	(0.0034)	
$\Delta\mathbb{E}(\pi_{RPI,t+1})$			-0.0017	-0.0020	
			(0.0015)	(0.0014)	
Observations	6,433	6,433	6,433	6,433	5,041
R^2	0.03265	0.01379	0.03565	0.01745	0.16020
Forecaster-year joint FE	✓		✓		✓
Forecaster FE		✓		✓	

Table E.3: Forecast revision regressions across the universe of forecast data. *p<0.1; **p<0.05; ***p<0.01; firm-clustered standard errors in parentheses.

Table E.3 relates changes in the end-of-year exchange rate forecasts to concurrent changes in other forecasts, with a set of fixed effects as specified below. The link between deficit expectation revisions and exchange rate expectation revisions is broadly robust across our specifications. In numbers, a 1 billion GBP (in 2018 GBP terms) increase in deficit expectations corresponds to a

0.05 percentage point *decrease* in end-of-year expectations of weighted exchange rates. Given that forecast revisions can amount to 100 billion GBP (as evidenced in figure E.3), we can think of this mechanism as being also economically significant in addition to statistical significance.

In figure E.4, we visualize the main point made by table E.3: conditional on firm-specific effects and other forecast updates, we generally observe a negative relation between exchange rate forecast revisions and deficit forecast revisions. Thus, a higher deficit forecast corresponds ceteris paribus to a downwards revision in expected year-end exchange rates.

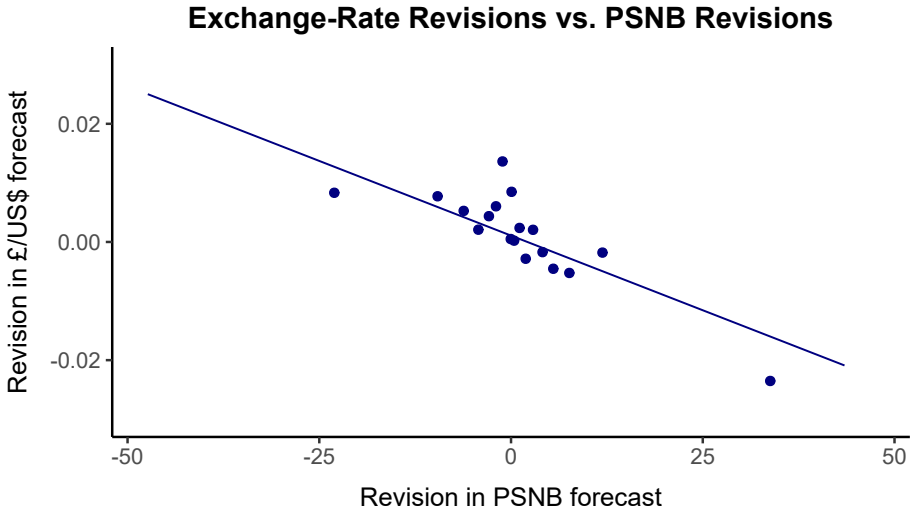


Figure E.4: Binned scatter plot showing the partial correlation between deficit forecast revisions and exchange rate forecast revisions, partialling out firm fixed-effects and forecast revisions for output and real interest rates.

The results of table E.3 and figure E.4 partial out forecaster-specific effects. However, there is substantial heterogeneity across forecasters in how they link deficits to exchange rate revisions. This informs a further interesting question: if a forecaster presumes a tighter link between deficits and depreciations, does that correspond also to a better ability of forecasting exchange rates? We now address this question, borrowing the definition of Mincer and Zarnowitz (1969) to evaluate what makes a *good* forecast.

In detail, we proceed as follows: First, we estimate *within each firm* the following specification:

$$\mathcal{E}_T = \alpha_{i0} + \alpha_{i1}\mathbb{E}_{it}(\mathcal{E}_T) + e_i,$$

to recover the estimate α_{i1}, which captures the degree to which the forecasts made by forecaster i match the actual outcome of the exchange rate index. A perfect forecaster would obtain $\alpha_{i0} = 0$ and $\alpha_{i1} = 1$. We then rank the coefficients α_{i1} in terms of their absolute distance from 1, and check to which degree this "rank" (which is tantamount to forecasting performance) relates to the coefficient) β_{i1} recovered from a firm-specific estimation of equation (E.1), which therefore comes

without firm-specific fixed effects. Having recovered both these elements, we estimate:

$$\hat{\beta}_{i1} = \gamma_1 \, \text{rank}(|\hat{\alpha}_{i1} - 1|) + u_i, \tag{E.2}$$

with the estimated coefficient $\hat{\gamma}_1$ being indicative whether better forecasters are actually characterized as those who link deficits broadly 'more' to exchange rate revaluations. We present the results of this estimation in table E.4.

	Dependent variable: $\hat{\beta}_{PSNB,ER}$	
	Filtered ($n \geq 10$ obs.)	Full sample
	(1)	(2)
rank($\lvert\hat{\alpha}_{i1} - 1\rvert$)	0.838***	0.809***
	(0.157)	(0.220)
Observations	52	61
R^2	0.337	0.100
F Statistic	25.936***	6.648**
Note:		*p<0.1; **p<0.05; ***p<0.01
	Heteroscedasticity-robust standard errors in parentheses.	

Table E.4: Rank regressions depicting the link between forecast accuracy and the firm-specific regression coefficient linking forecast revisions in deficits to forecast revisions in exchange rate

The coefficients for the full sample as well as the coefficients for the restricted sample (where we estimate the firm-specific deficit-exchange-rate link only for firms appearing in at least 10 years) support the notion that forecasters whose models predicate a closer link between deficits and depreciations are also the forecasters whose exchange rate forecasting performance is, on average, better. Thus, it appears that leaning into models of exchange rate determination informed by the government debt valuation equation is a rational way of forecasting exchange rates, conditional on any given information set available at time t. While this is clearly *not* a test of the government debt valuation equation as such, it lends support to the idea of focussing on deficits and deficit expectations when trying to determine expected movements in exchange rates.

Our empirical evidence informed by forecast revisions in the U.K. across almost 30 years helped us establish two major facts: one, exchange rates contribute significantly to the revision of the equilibrium value of government debt in open economies; and second, the link between fiscal deficits and exchange rates implied by the government debt valuation equation appears to be not only present in the forecasts made by firms with a vested interest in the valuation of government debt, but those firms that are more heavily leaning into determining exchange rates with the help of the mechanisms implied by the debt valuation equation are also, on average, better forecasters. Equipped

with those two pieces of evidence, we now move on to rationalize our findings theoretically: first, with a tractable model isolating our main mechanism linking between fiscal policy and exchange rates; and second, with a full-scale general equilibrium model allowing for rich international trade patterns.

F Omitted derivations from the text

F.1 Domestic Household: Full Problem

This appendix collects the full setup of the domestic household problem, including the intratemporal allocation block and the derivation of the optimality conditions stated in Section 3.1.

Intratemporal allocation. The consumption aggregate is the CES bundle

$$C_t = \left[(1-\gamma)^{1/\eta} C_{Ht}^{(\eta-1)/\eta} + \gamma^{1/\eta} C_{Ft}^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)}, \tag{F.1}$$

where $1-\gamma$ is the Home expenditure share (home bias) and $\eta > 0$ is the intratemporal elasticity of substitution. Given total nominal expenditure Z_t, the household solves

$$\max_{C_{Ht},\, C_{Ft}} \; C_t \quad \text{s.t.} \quad P_{Ht}C_{Ht} + P_{Ft}C_{Ft} = Z_t,$$

yielding demand functions

$$C_{Ht} = (1-\gamma)\left(\tfrac{P_{Ht}}{P_t}\right)^{-\eta} C_t, \tag{F.2}$$

$$C_{Ft} = \gamma\left(\tfrac{P_{Ft}}{P_t}\right)^{-\eta} C_t, \tag{F.3}$$

and the CPI dual consistent with (F.1):

$$P_t = \left[(1-\gamma) P_{Ht}^{1-\eta} + \gamma P_{Ft}^{1-\eta}\right]^{1/(1-\eta)}. \tag{F.4}$$

CPI inflation is $\pi_t \equiv \dot{P}_t/P_t$, the real exchange rate is $S_t \equiv E_t P_t^*/P_t$, and the terms of trade are $\text{tot}_t \equiv P_{Ft}/P_{Ht}$.

Intertemporal problem. The household maximises (11) subject to (12). The convenience yield $\nu(\cdot)$ takes the isoelastic form (14), with $b_{Ht} \equiv B_{Ht}/P_t$ and $\nu_C = -\nu_t/C_t$. The current-value Hamiltonian is

$$\mathcal{H} = \left(\log C_t - \tfrac{N_t^{1+\phi}}{1+\phi} + \nu(B_{Ht};\theta_t)\right) + \lambda_t\Big(i_t B_{Ht} + E_t i_t^* B_{Ft} + W_t N_t + T_t - P_t C_t\Big).$$

The necessary conditions with respect to C_t, N_t, B_{Ht}, and B_{Ft} are, respectively:

$$\frac{1-\nu_t}{C_t} = \lambda_t P_t, \tag{F.5}$$

$$N_t^{\phi} = \lambda_t W_t \frac{1}{1-\nu_t}, \tag{F.6}$$

$$\dot{\lambda}_t = \rho\lambda_t - \lambda_t i_t - \nu_B(B_{Ht};\theta_t), \tag{F.7}$$

$$\dot{\lambda}_t = \rho\lambda_t - \lambda_t i_t^* - \lambda_t \tfrac{\dot{E}_t}{E_t}. \tag{F.8}$$

Combining (F.5) and (F.6) gives the labour supply condition (19) in the main text. Differentiating (F.5) with respect to t and substituting into (F.7) yields the Home Euler equation (20), where the effective convenience yield Φ_t and preference drift χ_t are as defined in (17)–(18). The same substitution into (F.8) yields the Foreign Euler equation (21). Differencing the two Euler equations gives the UIP condition (22). Optimality also requires the transversality conditions

$$\lim_{T\to\infty} e^{-\rho T}\lambda_T B_{HT} = 0, \qquad \lim_{T\to\infty} e^{-\rho T}\lambda_T E_T B_{FT} = 0, \tag{F.9}$$

ruling out explosive bond positions.

F.2 Foreign Household: Full Problem

Intratemporal allocation. The Foreign consumption aggregate is

$$C_t^* = \left[(1-\gamma)^{1/\eta} C_{Ft}^{*(\eta-1)/\eta} + \gamma^{1/\eta} C_{Ht}^{*(\eta-1)/\eta}\right]^{\eta/(\eta-1)},$$

where $1-\gamma$ is the Foreign home bias and η is the intratemporal elasticity. Cost minimisation yields demand functions

$$C_{Ft}^* = (1-\gamma)\left(\tfrac{P_{Ft}^*}{P_t^*}\right)^{-\eta} C_t^*,$$
$$C_{Ht}^* = \gamma\left(\tfrac{P_{Ht}^*}{P_t^*}\right)^{-\eta} C_t^*,$$

with Foreign CPI $P_t^* = [(1-\gamma)P_{Ft}^{*\,1-\eta} + \gamma P_{Ht}^{*\,1-\eta}]^{1/(1-\eta)}$ and Foreign terms of trade $\text{tot}_t^* \equiv P_{Ht}^*/P_{Ft}^*$.

Intertemporal problem and Hamiltonian. Let $x_t \equiv B_{Ht}^*/E_t$ denote Home bond holdings in Foreign currency units, with $b_{Ht}^* \equiv B_{Ht}^*/P_t^*$. The current-value Hamiltonian is

$$\mathcal{H}^* = \left[\ln C_t^* - \tfrac{N_t^{*1+\phi^*}}{1+\phi^*} + \nu(x_t, C_t^*; \theta_t^*)\right] + \lambda_t^*\left[i_t^* B_{Ft}^* + \left(i_t - \tfrac{\dot{E}_t}{E_t}\right)x_t + W_t^* N_t^* + T_t^* - P_t^* C_t^*\right].$$

The first-order conditions with respect to C_t^*, N_t^*, B_{Ft}^*, and x_t are:

$$\begin{aligned}\frac{1-\nu_t^*}{C_t^*} &= \lambda_t^* P_t^*,\\ N_t^{*\phi^*} &= \lambda_t^* W_t^* \tfrac{1}{1-\nu_t^*},\\ \dot{\lambda}_t^* &= \rho^* \lambda_t^* - \lambda_t^* i_t^*,\\ \dot{\lambda}_t^* &= \rho^* \lambda_t^* - \lambda_t^* \big(i_t - \tfrac{\dot{E}_t}{E_t}\big) - \nu_x(x_t;\theta_t^*).\end{aligned}$$

Combining the first two conditions gives labour supply (26). Differentiating the first condition and substituting into the third gives the Foreign-bond Euler equation (27); substituting into the fourth gives the Home-bond Euler equation (28). Differencing the two Euler equations yields the Foreign UIP condition (29). The transversality conditions are

$$\lim_{T\to\infty} e^{-\rho^* T}\lambda_T^* B_{FT}^* = 0, \qquad \lim_{T\to\infty} e^{-\rho^* T}\lambda_T^* \tfrac{B_{HT}^*}{E_T} = 0.$$

F.3 Production: Full Setup

Final good producers. In each country, a unit continuum of differentiated intermediate goods is aggregated by competitive final good producers via a Dixit–Stiglitz technology. For the Home economy:

$$Y_{Ht} = \Big[\int_0^1 Y_{Ht}(j)^{(\varepsilon-1)/\varepsilon} dj\Big]^{\varepsilon/(\varepsilon-1)}, \qquad \varepsilon > 1.$$

Taking input prices $p_{Ht}(j)$ as given, the retailer minimises total cost $\int_0^1 p_{Ht}(j)Y_{Ht}(j)dj$ subject to the aggregator, yielding demand schedule (31) and aggregate price index $P_{Ht} = [\int_0^1 p_{Ht}(j)^{1-\varepsilon} dj]^{1/(1-\varepsilon)}$. The Foreign sector is entirely symmetric:

$$Y_{Ft}^* = \Big[\int_0^1 Y_{Ft}^*(j)^{(\varepsilon-1)/\varepsilon} dj\Big]^{\varepsilon/(\varepsilon-1)},$$

with demand schedule $Y_{Ft}^*(j) = [p_{Ft}^*(j)/P_{Ft}^*]^{-\varepsilon} Y_{Ft}^*$ and price index $P_{Ft}^* = [\int_0^1 p_{Ft}^*(j)^{1-\varepsilon} dj]^{1/(1-\varepsilon)}$.

Intermediate good producers: flexible prices. Each Home intermediate producer j has production function $Y_{Ht}(j) = A_t n_t(j)$. Under flexible prices, cost minimisation and the isoelastic demand (31) give the standard constant-markup pricing rule:

$$p_{Ht}(j) = \frac{\varepsilon}{\varepsilon - 1}\frac{W_t}{A_t}.$$

This is the flexible-price benchmark for real marginal cost, $mc_t^n = 1$, used when characterising the natural (flex-price) allocation and defining the output gap $x_t \equiv \ln Y_{Ht} - \ln Y_{Ht}^n$. The symmetric Foreign flexible-price condition is $p_{Ft}^*(j) = \frac{\varepsilon}{\varepsilon-1}\frac{W_t^*}{A_t^*}$.

Intermediate good producers: sticky prices. Per-period profit for Home firm j is

$$\Pi_t(p_{Ht}(j)) = p_{Ht}(j)Y_{Ht}(j) - \frac{W_t}{A_t}Y_{Ht}(j).$$

Rotemberg quadratic adjustment costs are $\Theta_t(j) = \frac{\theta}{2}(\dot{p}_{Ht}(j)/p_{Ht}(j))^2 P_{Ht}Y_{Ht}$. With convenience yields, the household's nominal SDF satisfies $-\dot{M}_t^n/M_t^n = i_t + \Phi_t$, so the firm discounts at $i_t + \Phi_t$. The optimal control problem is

$$V_0(p_{H0}) = \max_{\{p_{Ht}(j)\}_{t\geq 0}} \int_0^\infty \exp\Big\{-\int_0^t [i_s + \Phi_s]\, ds\Big\}\big[\Pi_t(p_{Ht}) - \Theta_t(j)\big]\, dt.$$

Imposing symmetry $p_{Ht}(j) = P_{Ht}$ and applying the maximum principle yields the Rotemberg ODE (32), as shown in the following proof.

F.4 Proof of Lemma 2

Proof. The state variable is $p_{Ht}(j)$, the control variable is $\dot{p}_{Ht}(j)$, and the co-state variable is η_t. In setting up the Hamiltonian, we note that the law of motion for $p_{Ht}(j)$ is trivially given by

$$\dot{p}_{Ht}(j) = \dot{p}_{Ht}(j).$$

We can form the current value Hamiltonian as

$$\mathcal{H}(p_{Ht}(j); \dot{p}_{Ht}(j); \eta_t) = \left[p_{Ht}(j)\left(\frac{p_{Ht}(j)}{P_{Ht}}\right)^{-\epsilon} Y_{Ht} - \frac{W_t}{A_t}\left(\frac{p_{Ht}(j)}{P_{Ht}}\right)^{-\epsilon} Y_{Ht}\right] - \left[\frac{\theta}{2}\left(\frac{\dot{p}_{Ht}(j)}{p_{Ht}(j)}\right)^2 P_{Ht}Y_{Ht}\right] + \eta_t \dot{p}_{Ht}(j). \tag{F.10}$$

The presence of convenience yields only affects the *discount rate* in the costate equation; the instantaneous profit and adjustment cost terms are unchanged.

Necessary and sufficient conditions for firm optimality conditions are

$$\theta \frac{\dot{p}_{Ht}(j)}{p_{Ht}(j)} \frac{P_{Ht}}{p_{Ht}(j)} Y_{Ht} = \eta_t, \tag{F.11}$$

$$\dot{\eta}_t = (i_t + \Phi_t)\eta_t - \left[(1-\epsilon)\left(\frac{p_{Ht}(j)}{P_{Ht}}\right)^{-\epsilon} Y_{Ht} + \epsilon \frac{W_t}{p_{Ht}(j)} \frac{1}{A_t}\left(\frac{p_{Ht}(j)}{P_{Ht}}\right)^{-\epsilon} Y_{Ht} + \theta\left(\frac{\dot{p}_{Ht}(j)}{p_{Ht}(j)}\right)^2 \frac{P_{Ht}}{p_{Ht}(j)} Y_{Ht}\right]. \tag{F.12}$$

The first order condition with respect to the control variable $\dot{p}_{Ht}(j)$ is:

$$[\dot{p}_{Ht}(j)]: \quad -\theta \frac{\dot{p}_{Ht}(j)}{p_{Ht}(j)}\left(\frac{1}{p_{Ht}(j)}\right) P_{Ht}Y_{Ht} + \eta_t = 0.$$

The envelope condition with respect to the state variable $p_{Ht}(j)$ is given by

$$\dot{\eta}_t = \big(i_t + \Phi_t\big)\eta_t - \mathcal{H}_{p_H},$$

where, as usual for a current-value Hamiltonian with discount rate $i_t + \Phi_t$, the costate evolves at rate $i_t + \Phi_t$ minus the partial derivative of $\mathcal{H}$ with respect to the state $p_{Ht}(j)$. Here $\mathcal{H}_{p_H}$ is the current value Hamiltonian differentiated with respect to the state variable $p_{Ht}(j)$:

$$\begin{aligned}
\mathcal{H}_{p_H} &= \frac{\partial}{\partial p_H}\left(\left[p_{Ht}(j)\left(\frac{p_{Ht}(j)}{P_{Ht}}\right)^{-\epsilon} Y_{Ht} - \frac{W_t}{A_t}\left(\frac{p_{Ht}(j)}{P_{Ht}}\right)^{-\epsilon} Y_{Ht}\right] - \left[\frac{\theta}{2}\left(\frac{\dot{p}_{Ht}(j)}{p_{Ht}(j)}\right)^2 P_{Ht} Y_{Ht}\right] + \eta_t \dot{p}_{Ht}(j)\right) \\
&\leftrightarrow \frac{\partial}{\partial p_H}\left(\left[(p_{Ht}(j))^{1-\epsilon}\left(\frac{1}{P_{Ht}}\right)^{-\epsilon} Y_{Ht} - \frac{W_t}{A_t}\left(\frac{p_{Ht}(j)}{P_{Ht}}\right)^{-\epsilon} Y_{Ht}\right] - \left[\frac{\theta}{2}\left(\frac{\dot{p}_{Ht}(j)}{p_{Ht}(j)}\right)^2 P_{Ht} Y_{Ht}\right] + \eta_t \dot{p}_{Ht}(j)\right) \\
&\leftrightarrow (1-\epsilon)\frac{p_{Ht}(j)^{-\epsilon}}{P_{Ht}^{-\epsilon}} Y_{Ht} - (-\epsilon)\frac{W_t}{A_t}\frac{p_{Ht}(j)^{-\epsilon-1}}{P_{Ht}^{-\epsilon}} Y_{Ht} - \frac{\theta}{2}(2)\left(\frac{\dot{p}_{Ht}(j)}{p_{Ht}(j)}\right)\left(-\frac{\dot{p}_{Ht}(j)}{p_{Ht}(j)^2}\right) P_{Ht} Y_{Ht} \\
&\leftrightarrow (1-\epsilon)\left(\frac{p_{Ht}(j)}{P_{Ht}}\right)^{-\epsilon} Y_{Ht} + \epsilon\frac{W_t}{A_t} p_{Ht}(j)^{-1}\left(\frac{p_{Ht}(j)}{P_{Ht}}\right)^{-\epsilon} Y_{Ht} + \theta\left(\frac{\dot{p}_{Ht}(j)}{p_{Ht}(j)}\right)^2\left(\frac{1}{p_{Ht}(j)}\right) P_{Ht} Y_{Ht} \\
&\Rightarrow (1-\epsilon)\left(\frac{p_{Ht}(j)}{P_{Ht}}\right)^{-\epsilon} Y_{Ht} + \epsilon\frac{W_t}{p_{Ht}(j)}\frac{1}{A_t}\left(\frac{p_{Ht}(j)}{P_{Ht}}\right)^{-\epsilon} Y_{Ht} + \theta\left(\frac{\dot{p}_{Ht}(j)}{p_{Ht}(j)}\right)^2 \frac{P_{Ht}}{p_{Ht}(j)} Y_{Ht}.
\end{aligned}$$

So plugging this back into the first order condition for the state variable, we get

$$\dot{\eta}_t = \big(i_t + \Phi_t\big)\eta_t - \left[(1-\epsilon)\left(\frac{p_{Ht}(j)}{P_{Ht}}\right)^{-\epsilon} Y_{Ht} + \epsilon\frac{W_t}{p_{Ht}(j)}\frac{1}{A_t}\left(\frac{p_{Ht}(j)}{P_{Ht}}\right)^{-\epsilon} Y_{Ht} + \theta\left(\frac{\dot{p}_{Ht}(j)}{p_{Ht}(j)}\right)^2 \frac{P_{Ht}}{p_{Ht}(j)} Y_{Ht}\right].$$

We restrict our attention to symmetric domestic firm pricing.

Assumption 12 (Symmetric Equilibrium)**.** *We restrict attention to the symmetric equilibrium*

$$p_{Ht}(j) = P_{Ht}. \tag{F.13}$$

The symmetric equilibrium for the firm pricing decision is then

$$\theta \pi_{Ht} Y_{Ht} = \eta_t, \tag{F.14}$$

$$\dot{\eta}_t = \big(i_t + \Phi_t\big)\eta_t - \left[(1-\epsilon) Y_{Ht} + \epsilon\frac{W_t}{P_{Ht}}\frac{1}{A_t} Y_{Ht} + \theta \pi_{Ht}^2 Y_{Ht}\right], \tag{F.15}$$

where we define the producer inflation rate as $\pi_{Ht} \equiv \frac{\dot{P}_{Ht}}{P_{Ht}}$.

Take the firm optimality condition for the control variable and impose symmetry $p_{Ht}(j) = P_{Ht}$:

$$\begin{aligned}
\theta\frac{\dot{p}_{Ht}(j)}{p_{Ht}(j)}\frac{P_{Ht}}{p_{Ht}(j)} Y_{Ht} &= \eta_t \\
\Rightarrow \quad \theta \pi_{Ht} Y_{Ht} &= \eta_t.
\end{aligned}$$

Take the firm optimality condition for the state variable and impose symmetry:

$$\begin{aligned}\dot{\eta}_t &= \big(i_t + \Phi_t\big)\eta_t - \left[(1-\epsilon)\left(\frac{p_{Ht}(j)}{P_{Ht}}\right)^{-\epsilon} Y_{Ht} + \epsilon \frac{W_t}{p_{Ht}(j)}\frac{1}{A_t}\left(\frac{p_{Ht}(j)}{P_{Ht}}\right)^{-\epsilon} Y_{Ht} + \theta\left(\frac{\dot{p}_{Ht}(j)}{p_{Ht}(j)}\right)^2 \frac{P_{Ht}}{p_{Ht}(j)} Y_{Ht}\right] \\ &= \big(i_t + \Phi_t\big)\eta_t - \left[(1-\epsilon) Y_{Ht} + \epsilon \frac{W_t}{P_{Ht}}\frac{1}{A_t} Y_{Ht} + \theta \pi_{Ht}^2 Y_{Ht}\right].\end{aligned}$$

We now evaluate the firm's symmetric equilibrium condition:

$$\dot{\eta}_t = \big(i_t + \Phi_t\big)\eta_t - \left[(1-\epsilon) Y_{Ht} + \epsilon \frac{W_t}{P_{Ht}}\frac{1}{A_t} Y_{Ht} + \theta \pi_{Ht}^2 Y_{Ht}\right].$$

We can use the firm's symmetric equilibrium condition for the control variable and differentiate with respect to time:

$$\begin{aligned} & \eta_t = \theta \pi_{Ht} Y_{Ht} \\ \leftrightarrow \quad & \frac{d}{dt}\eta_t = \frac{d}{dt}\Big(\theta \pi_{Ht} Y_{Ht}\Big) \\ \Rightarrow \quad & \dot{\eta}_t = \theta \dot{\pi}_{Ht} Y_{Ht} + \theta \pi_{Ht} \dot{Y}_{Ht}. \end{aligned}$$

Now plug this back into the firm's symmetric equilibrium condition:

$$\begin{aligned} & \dot{\eta}_t = \big(i_t + \Phi_t\big)\eta_t - \left[(1-\epsilon) Y_{Ht} + \epsilon \frac{W_t}{P_{Ht}}\frac{1}{A_t} Y_{Ht} + \theta \pi_{Ht}^2 Y_{Ht}\right] \\ \leftrightarrow \quad & \theta \dot{\pi}_{Ht} Y_{Ht} + \theta \pi_{Ht} \dot{Y}_{Ht} = \big(i_t + \Phi_t\big)\big[\theta \pi_{Ht} Y_{Ht}\big] - \left[(1-\epsilon) Y_{Ht} + \epsilon \frac{W_t}{P_{Ht}}\frac{1}{A_t} Y_{Ht} + \theta \pi_{Ht}^2 Y_{Ht}\right] \\ \leftrightarrow \quad & \dot{\pi}_{Ht} + \pi_{Ht}\frac{\dot{Y}_{Ht}}{Y_{Ht}} = \big(i_t + \Phi_t\big)\pi_{Ht} - \frac{(1-\epsilon)}{\theta} - \frac{\epsilon}{\theta}\frac{W_t}{P_{Ht}}\frac{1}{A_t} - \pi_{Ht}^2 \\ \Rightarrow \quad & \dot{\pi}_{Ht} + \frac{\epsilon - 1}{\theta}\left[\left(\frac{\epsilon}{\epsilon - 1}\frac{W_t}{P_{Ht}}\frac{1}{A_t}\right) - 1\right] = \left[i_t + \Phi_t - \frac{\dot{Y}_{Ht}}{Y_{Ht}} - \pi_{Ht}\right]\pi_{Ht}. \end{aligned}$$

Rearranging the final expression gives the claimed result in the statement of the lemma for Home producers. The result for Foreign producers follows by a symmetric argument.

□

F.5 Proof of Lemma 3

Proof. We here prove the decomposition of output in lemma 3.

Step 1. Labor supply and consumption as a function of Y_{Ht}. From Home price setting (producer prices),

$$P_{Ht} = \frac{\varepsilon}{\varepsilon - 1}\frac{W_t}{A_t}, \tag{F.16}$$

so

$$\frac{W_t}{P_t} = \frac{\varepsilon - 1}{\varepsilon} \frac{P_{Ht}}{P_t} A_t.$$

With consumption-scaled convenience yields, the intratemporal optimality condition is

$$C_t N_t^{\phi} = (1 - \nu_t) \frac{W_t}{P_t}.$$

Combine the two:

$$C_t N_t^{\phi} = (1 - \nu_t) \frac{\varepsilon - 1}{\varepsilon} \frac{P_{Ht}}{P_t} A_t.$$

Using $Y_{Ht} = A_t N_t$ (so $N_t = Y_{Ht}/A_t$) gives

$$C_t \left(\frac{Y_{Ht}}{A_t} \right)^{\phi} = (1 - \nu_t) \frac{\varepsilon - 1}{\varepsilon} \frac{P_{Ht}}{P_t} A_t,$$

hence

$$C_t = (1 - \nu_t) \frac{\varepsilon - 1}{\varepsilon} \frac{P_{Ht}}{P_t} A_t^{1+\phi} Y_{Ht}^{-\phi}. \tag{F.17}$$

Step 2. CPI and the relative price of Home goods. The Home CPI is

$$P_t = \left[(1 - \gamma) P_{Ht}^{1-\eta} + \gamma P_{Ft}^{1-\eta} \right]^{1/(1-\eta)}.$$

Raise both sides to $1 - \eta$ and divide through by $P_{Ht}^{1-\eta}$:

$$\left(\frac{P_t}{P_{Ht}} \right)^{1-\eta} = (1 - \gamma) + \gamma \left(\frac{P_{Ft}}{P_{Ht}} \right)^{1-\eta}.$$

Let $\text{tot}_t \equiv P_{Ft}/P_{Ht}$ and define

$$G_t \equiv (1 - \gamma) + \gamma \, \text{tot}_t^{1-\eta}.$$

Then

$$\left(\frac{P_t}{P_{Ht}} \right)^{1-\eta} = G_t \quad \Longrightarrow \quad \frac{P_{Ht}}{P_t} = G_t^{-1/(1-\eta)}. \tag{F.18}$$

Step 3. Home goods market clearing and the role of Q_t. Home and Foreign demand for the Home good are

$$C_{Ht} = (1 - \gamma) \left(\frac{P_{Ht}}{P_t} \right)^{-\eta} C_t, \qquad C_{Ht}^* = \gamma \left(\frac{P_{Ht}^*}{P_t^*} \right)^{-\eta} C_t^*.$$

Goods market clearing,

$$Y_{Ht} = C_{Ht} + C_{Ht}^*,$$

can be written

$$Y_{Ht} = (1 - \gamma) \left(\frac{P_{Ht}}{P_t} \right)^{-\eta} C_t + \gamma \left(\frac{P_{Ht}^*}{P_t^*} \right)^{-\eta} C_t^*.$$

Use the law of one price for the Home good, $P_{Ht} = E_t P^*_{Ht}$, and the real exchange rate

$$Q_t \equiv \frac{E_t P^*_t}{P_t} \quad \Longrightarrow \quad P^*_t = \frac{P_t Q_t}{E_t}.$$

Then

$$\frac{P^*_{Ht}}{P^*_t} = \frac{P_{Ht}/E_t}{P_t Q_t/E_t} = \frac{P_{Ht}}{P_t} \cdot \frac{1}{Q_t},$$

so

$$\left(\frac{P^*_{Ht}}{P^*_t}\right)^{-\eta} = \left(\frac{P_{Ht}}{P_t}\right)^{-\eta} Q_t^{\eta}. \tag{F.19}$$

Plugging (F.19) into market clearing yields

$$\begin{aligned} Y_{Ht} &= (1-\gamma)\left(\frac{P_{Ht}}{P_t}\right)^{-\eta} C_t + \gamma \left(\frac{P_{Ht}}{P_t}\right)^{-\eta} Q_t^{\eta} C^*_t \\ &= \left(\frac{P_{Ht}}{P_t}\right)^{-\eta} \Big[(1-\gamma) C_t + \gamma Q_t^{\eta} C^*_t\Big]. \end{aligned} \tag{F.20}$$

Using (F.18),

$$Y_{Ht} = G_t^{\eta/(1-\eta)} \Big[(1-\gamma) C_t + \gamma Q_t^{\eta} C^*_t\Big]. \tag{F.21}$$

Step 4. Risk sharing with consumption-scaled convenience yields. With consumption-scaled convenience yields, the risk-sharing condition (from the two Eulers plus UIP) is

$$\left[\frac{\dot C_t}{C_t} - \frac{\dot C^*_t}{C^*_t}\right] - \frac{\dot Q_t}{Q_t} = \rho^* - \rho + (\chi^*_t - \chi_t), \qquad \chi_t \equiv \frac{\dot\nu_t}{1-\nu_t},\ \chi^*_t \equiv \frac{\dot\nu^*_t}{1-\nu^*_t}. \tag{F.22}$$

Note that $\frac{d}{dt}(\ln C_t) = \dot C_t / C_t$ and similarly for C^*_t and Q_t, so we can rewrite (F.22) as

$$\frac{d}{dt}\left(\ln C_t - \ln C^*_t - \ln Q_t\right) = \rho^* - \rho + (\chi^*_t - \chi_t).$$

Integrating from 0 to t and absorbing the integration constant into the initial value of Ξ_t gives

$$\ln C_t - \ln C^*_t - \ln Q_t = \Xi_t, \qquad \Xi_t \equiv \int_0^t (\rho^* - \rho)\, du + \int_0^t (\chi^*_u - \chi_u)\, du.$$

Equivalently,

$$\ln C^*_t = \ln C_t - \ln Q_t - \Xi_t \quad \Longrightarrow \quad C^*_t = C_t\, Q_t^{-1}\, e^{-\Xi_t}. \tag{F.23}$$

Step 5. Home output in closed form. Substitute (F.23) into (F.21):

$$\begin{aligned} Y_{Ht} = G_t^{\eta/(1-\eta)} \Big[&(1-\gamma) C_t + \gamma Q_t^{\eta} C^*_t \\ &+ \gamma Q_t^{\eta} \big(C_t Q_t^{-1} e^{-\Xi_t}\big)\Big] \\ &= C_t\, G_t^{\eta/(1-\eta)} \Big[(1-\gamma) + \gamma Q_t^{\eta-1} e^{-\Xi_t}\Big]. \end{aligned}$$

Now substitute C_t from (F.17):

$$Y_{Ht} = \left[(1-\nu_t)\frac{\varepsilon-1}{\varepsilon}\frac{P_{Ht}}{P_t}A_t^{1+\phi}Y_{Ht}^{-\phi}\right]G_t^{\eta/(1-\eta)}\left[(1-\gamma)+\gamma Q_t^{\eta-1}e^{-\Xi_t}\right].$$

Rearranging,

$$Y_{Ht}^{1+\phi} = (1-\nu_t)\frac{\varepsilon-1}{\varepsilon}\frac{P_{Ht}}{P_t}A_t^{1+\phi}G_t^{\eta/(1-\eta)}\left[(1-\gamma)+\gamma Q_t^{\eta-1}e^{-\Xi_t}\right].$$

Use (F.18), i.e. $\frac{P_{Ht}}{P_t} = G_t^{-1/(1-\eta)}$:

$$\begin{aligned} Y_{Ht}^{1+\phi} &= (1-\nu_t)\frac{\varepsilon-1}{\varepsilon}A_t^{1+\phi}G_t^{-1/(1-\eta)}\,G_t^{\eta/(1-\eta)}\left[(1-\gamma)+\gamma Q_t^{\eta-1}e^{-\Xi_t}\right] \\ &= (1-\nu_t)\frac{\varepsilon-1}{\varepsilon}A_t^{1+\phi}G_t^{\frac{-1+\eta}{1-\eta}}\left[(1-\gamma)+\gamma Q_t^{\eta-1}e^{-\Xi_t}\right]. \end{aligned}$$

Since $\frac{-1+\eta}{1-\eta} = -1$, we have

$$G_t^{\frac{-1+\eta}{1-\eta}} = G_t^{-1} = \left[(1-\gamma)+\gamma \mathrm{tot}_t^{1-\eta}\right]^{-1}.$$

Thus

$$Y_{Ht}^{1+\phi} = (1-\nu_t)\frac{\varepsilon-1}{\varepsilon}A_t^{1+\phi}\left[(1-\gamma)+\gamma \mathrm{tot}_t^{1-\eta}\right]^{-1}\left[(1-\gamma)+\gamma Q_t^{\eta-1}e^{-\Xi_t}\right],$$

and taking the $(1+\phi)$-th root yields

$$Y_{Ht} = A_t\left(\frac{\varepsilon-1}{\varepsilon}\right)^{\frac{1}{1+\phi}}(1-\nu_t)^{\frac{1}{1+\phi}}\left[(1-\gamma)+\gamma \mathrm{tot}_t^{1-\eta}\right]^{-\frac{1}{1+\phi}}\left[(1-\gamma)+\gamma Q_t^{\eta-1}e^{-\Xi_t}\right]^{\frac{1}{1+\phi}}.$$

This is the desired expression for Y_{Ht}.

Step 6. Foreign output. The Foreign block is symmetric. Demand for the Foreign good (in Foreign and Home) is

$$C_{Ft} = \gamma\left(\frac{P_{Ft}}{P_t}\right)^{-\eta}C_t, \qquad C_{Ft}^* = (1-\gamma)\left(\frac{P_{Ft}^*}{P_t^*}\right)^{-\eta}C_t^*.$$

Foreign goods market clearing gives

$$Y_{Ft}^* = C_{Ft} + C_{Ft}^* = \gamma\left(\frac{P_{Ft}}{P_t}\right)^{-\eta}C_t + (1-\gamma)\left(\frac{P_{Ft}^*}{P_t^*}\right)^{-\eta}C_t^*.$$

Using $P_{Ft} = E_t P_{Ft}^*$ and $Q_t = E_t P_t^*/P_t$,

$$\left(\frac{P_{Ft}^*}{P_t^*}\right)^{-\eta} = \left(\frac{P_{Ft}}{P_t}\right)^{-\eta}Q_t^{\eta},$$

so

$$Y_{Ft}^{*}=\left(\frac{P_{Ft}}{P_t}\right)^{-\eta}\Big[\gamma C_t+(1-\gamma)Q_t^{\eta}C_t^{*}\Big].$$

Inverting (F.23) gives $C_t = C_t^{*}Q_t e^{\Xi_t}$, hence

$$\gamma C_t+(1-\gamma)Q_t^{\eta}C_t^{*}=C_t^{*}\Big[\gamma Q_t e^{\Xi_t}+(1-\gamma)Q_t^{\eta}\Big]=C_t^{*}Q_t^{\eta}\Big[(1-\gamma)+\gamma Q_t^{1-\eta}e^{\Xi_t}\Big],$$

and therefore

$$Y_{Ft}^{*}=\left(\frac{P_{Ft}}{P_t}\right)^{-\eta}C_t^{*}Q_t^{\eta}\Big[(1-\gamma)+\gamma Q_t^{1-\eta}e^{\Xi_t}\Big].$$

Using the Foreign intratemporal condition with convenience,

$$C_t^{*}(N_t^{*})^{\phi^{*}}=(1-\nu_t^{*})\frac{W_t^{*}}{P_t^{*}},\qquad Y_{Ft}^{*}=A_t^{*}N_t^{*},$$

and Foreign price setting

$$P_{Ft}^{*}=\frac{\varepsilon}{\varepsilon-1}\frac{W_t^{*}}{A_t^{*}}\quad\Longrightarrow\quad\frac{W_t^{*}}{P_t^{*}}=\frac{\varepsilon-1}{\varepsilon}\frac{P_{Ft}^{*}}{P_t^{*}}A_t^{*},$$

one obtains, in exact analogy to (F.17),

$$C_t^{*}=(1-\nu_t^{*})\frac{\varepsilon-1}{\varepsilon}\frac{P_{Ft}^{*}}{P_t^{*}}{A_t^{*}}^{1+\phi^{*}}(Y_{Ft}^{*})^{-\phi^{*}}.$$

Defining

$$G_t^{*}\equiv(1-\gamma)+\gamma(\mathrm{tot}_t^{*})^{1-\eta},\qquad \mathrm{tot}_t^{*}\equiv\frac{P_{Ht}^{*}}{P_{Ft}^{*}},$$

one can show (symmetrically to (F.18)) that

$$\frac{P_{Ft}^{*}}{P_t^{*}}=(G_t^{*})^{-1/(1-\eta)}.$$

Substituting C_t^{*} and P_{Ft}^{*}/P_t^{*} into the expression for Y_{Ft}^{*} and repeating the algebra as in the Home case yields

$$(Y_{Ft}^{*})^{1+\phi^{*}}=(1-\nu_t^{*})\frac{\varepsilon-1}{\varepsilon}{A_t^{*}}^{1+\phi^{*}}(G_t^{*})^{-1}\Big[(1-\gamma)+\gamma Q_t^{1-\eta}e^{\Xi_t}\Big],$$

and taking the $(1+\phi^{*})$-th root gives

$$Y_{Ft}^{*}=A_t^{*}\left(\frac{\varepsilon-1}{\varepsilon}\right)^{\frac{1}{1+\phi^{*}}}(1-\nu_t^{*})^{\frac{1}{1+\phi^{*}}}(G_t^{*})^{-\frac{1}{1+\phi^{*}}}\Big[(1-\gamma)+\gamma Q_t^{1-\eta}e^{\Xi_t}\Big]^{\frac{1}{1+\phi^{*}}},$$

which matches the stated expression once we substitute $G_t^{*}=(1-\gamma)+\gamma(\mathrm{tot}_t^{*})^{1-\eta}$. □

F.6 Proof of Proposition 4

Proof. We highlight how consumption-scaled convenience yields modify the natural rate in three ways.

Step 1: Real rate from the Euler equation with consumption-scaled convenience yields.

With convenience flows scaled by consumption, the Home household's intertemporal condition can be written as

$$\frac{\dot{C}_t}{C_t} = i_t - \pi_t - \rho + \underbrace{\frac{\Psi_t}{1-\nu_t}}_{\Phi_t} - \underbrace{\frac{\dot{\nu}_t}{1-\nu_t}}_{\chi_t}.$$

Defining the ex-post real interest rate in the usual way,

$$r_t \equiv i_t - \pi_t,$$

we obtain the Euler-based real rate expression

$$r_t = \rho + \frac{\dot{C}_t}{C_t} - \Phi_t + \chi_t. \tag{F.24}$$

This makes clear how convenience enters: the real rate equals the Ramsey term ρ plus consumption growth, *minus* the effective convenience wedge Φ_t and *plus* an adjustment χ_t coming from the time-variation of the consumption weight $(1-\nu_t)$.

Step 2: Consumption as a function of technology, prices, and convenience.

The Home intratemporal first-order condition with consumption-scaled convenience yields is

$$C_t N_t^{\phi} = (1-\nu_t)\frac{W_t}{P_t}. \tag{F.25}$$

Technology is linear in labor:

$$Y_{Ht} = A_t N_t \quad \Rightarrow \quad N_t = \frac{Y_{Ht}}{A_t}.$$

Under flexible prices, the Home PPI satisfies

$$P_{Ht} = \frac{\varepsilon}{\varepsilon-1}\frac{W_t}{A_t} \quad \Rightarrow \quad \frac{W_t}{P_t} = \frac{\varepsilon-1}{\varepsilon}\frac{P_{Ht}}{P_t}A_t.$$

Substituting this into (F.25) yields

$$C_t N_t^{\phi} = (1-\nu_t)\frac{\varepsilon-1}{\varepsilon}\frac{P_{Ht}}{P_t}A_t.$$

Using $N_t = Y_{Ht}/A_t$, we have

$$N_t^{\phi} = \left(\frac{Y_{Ht}}{A_t}\right)^{\phi} = \frac{Y_{Ht}^{\phi}}{A_t^{\phi}},$$

so that

$$C_t \frac{Y_{Ht}^{\phi}}{A_t^{\phi}} = (1-\nu_t)\frac{\varepsilon-1}{\varepsilon}\frac{P_{Ht}}{P_t}A_t,$$

or equivalently

$$C_t = (1-\nu_t)\frac{\varepsilon-1}{\varepsilon}\frac{P_{Ht}}{P_t}A_t^{1+\phi}Y_{Ht}^{-\phi}. \tag{F.26}$$

Relative to the standard case, convenience simply multiplies the MRS by the factor $(1-\nu_t)$.

Demand side and open economy structure.

On the demand side, the Home CPI is

$$P_t = \left[(1-\gamma)P_{Ht}^{1-\eta} + \gamma P_{Ft}^{1-\eta}\right]^{\frac{1}{1-\eta}}.$$

Let the terms of trade be $\text{tot}_t \equiv P_{Ft}/P_{Ht}$, and define

$$G_t \equiv (1-\gamma) + \gamma\,\text{tot}_t^{1-\eta}.$$

Then standard CES algebra implies

$$\frac{P_{Ht}}{P_t} = G_t^{-1/(1-\eta)}.$$

Using Home and Foreign demand for the Home good, together with risk sharing, we showed that under flexible prices output can be written as

$$Y_{Ht} = A_t\left(\frac{\varepsilon-1}{\varepsilon}\right)^{\frac{1}{1+\phi}}(1-\nu_t)^{\frac{1}{1+\phi}}G_t^{-\frac{1}{1+\phi}}Z_t^{\frac{1}{1+\phi}},$$

where

$$Z_t \equiv (1-\gamma) + \gamma Q_t^{\eta-1}e^{-\Xi_t}, \qquad Q_t \equiv \frac{E_tP_t^*}{P_t}.$$

Compared to the no-convenience case, the only change is the additional output factor $(1-\nu_t)^{1/(1+\phi)}$, which reflects the fact that convenience tilts the MRS in favor of leisure and hence alters the labor input needed to support a given level of output.

Step 3: Expressing consumption growth in terms of g_t, $\mathcal{T}_t$, and $\mathcal{R}_t$.

Insert the expression for Y_{Ht} into (F.26). First compute

$$Y_{Ht}^{-\phi} = A_t^{-\phi}\left(\frac{\varepsilon-1}{\varepsilon}\right)^{-\frac{\phi}{1+\phi}}(1-\nu_t)^{-\frac{\phi}{1+\phi}}G_t^{\frac{\phi}{1+\phi}}Z_t^{-\frac{\phi}{1+\phi}}.$$

Substituting into (F.26), we obtain

$$C_t = (1-\nu_t)\frac{\varepsilon-1}{\varepsilon}\frac{P_{Ht}}{P_t}A_t\left(\frac{\varepsilon-1}{\varepsilon}\right)^{-\frac{\phi}{1+\phi}}(1-\nu_t)^{-\frac{\phi}{1+\phi}}G_t^{\frac{\phi}{1+\phi}}Z_t^{-\frac{\phi}{1+\phi}}.$$

Collecting terms,

$$\frac{\varepsilon-1}{\varepsilon}\left(\frac{\varepsilon-1}{\varepsilon}\right)^{-\frac{\phi}{1+\phi}}=\left(\frac{\varepsilon-1}{\varepsilon}\right)^{\frac{1}{1+\phi}},\qquad (1-\nu_t)(1-\nu_t)^{-\frac{\phi}{1+\phi}}=(1-\nu_t)^{\frac{1}{1+\phi}}.$$

Hence we obtain

$$C_t=\left[\tfrac{\varepsilon-1}{\varepsilon}\right]^{\frac{1}{1+\phi}}(1-\nu_t)^{\frac{1}{1+\phi}}G_t^{\frac{\phi}{1+\phi}-\frac{1}{1-\eta}}A_tZ_t^{-\frac{\phi}{1+\phi}}. \tag{F.27}$$

Step 4: Log-differentiating C_t and isolating $\mathcal{T}_t$ and $\mathcal{R}_t$.

Taking logs of (F.27),

$$\ln C_t=\frac{1}{1+\phi}\ln\left[\tfrac{\varepsilon-1}{\varepsilon}\right]+\frac{1}{1+\phi}\ln(1-\nu_t)+\left(\frac{\phi}{1+\phi}-\frac{1}{1-\eta}\right)\ln G_t+\ln A_t-\frac{\phi}{1+\phi}\ln Z_t.$$

The first term is a constant and disappears when differentiating. Using $\frac{d}{dt}\ln X_t=\dot{X}_t/X_t$ gives

$$\frac{\dot{C}_t}{C_t}=\frac{1}{1+\phi}\frac{d}{dt}\ln(1-\nu_t)+\left(\frac{\phi}{1+\phi}-\frac{1}{1-\eta}\right)\frac{\dot{G}_t}{G_t}+\frac{\dot{A}_t}{A_t}-\frac{\phi}{1+\phi}\frac{\dot{Z}_t}{Z_t}. \tag{F.28}$$

For the $(1-\nu_t)$ term,

$$\frac{d}{dt}\ln(1-\nu_t)=\frac{-\dot{\nu}_t}{1-\nu_t}=-\chi_t,$$

so we can rewrite (F.28) as

$$\frac{\dot{C}_t}{C_t}=g_t+\left(\frac{\phi}{1+\phi}-\frac{1}{1-\eta}\right)\frac{\dot{G}_t}{G_t}-\frac{\phi}{1+\phi}\frac{\dot{Z}_t}{Z_t}-\frac{1}{1+\phi}\chi_t, \tag{F.29}$$

with $g_t\equiv\dot{A}_t/A_t$.

Terms-of-trade channel. Recall

$$G_t=(1-\gamma)+\gamma\,\mathrm{tot}_t^{1-\eta},\qquad \mathrm{tot}_t\equiv\frac{P_{Ft}}{P_{Ht}}.$$

Differentiating,

$$\dot{G}_t=\gamma(1-\eta)\,\mathrm{tot}_t^{-\eta}\,\dot{\mathrm{tot}}_t,$$

so that

$$\frac{\dot{G}_t}{G_t}=\frac{\gamma(1-\eta)\,\mathrm{tot}_t^{-\eta}\dot{\mathrm{tot}}_t}{(1-\gamma)+\gamma\,\mathrm{tot}_t^{1-\eta}}=\frac{\gamma(1-\eta)\,\mathrm{tot}_t^{1-\eta}}{(1-\gamma)+\gamma\,\mathrm{tot}_t^{1-\eta}}\left(\frac{\dot{\mathrm{tot}}_t}{\mathrm{tot}_t}\right).$$

Therefore the contribution of G_t to (F.29) is

$$\left(\frac{\phi}{1+\phi}-\frac{1}{1-\eta}\right)\frac{\dot{G}_t}{G_t}=\left(\frac{\phi}{1+\phi}-\frac{1}{1-\eta}\right)\frac{\gamma(1-\eta)\,\mathrm{tot}_t^{1-\eta}}{(1-\gamma)+\gamma\,\mathrm{tot}_t^{1-\eta}}\left(\frac{\dot{\mathrm{tot}}_t}{\mathrm{tot}_t}\right),$$

which we identify as

$$\mathcal{T}_t = \left(\frac{\phi}{1+\phi} - \frac{1}{1-\eta}\right) \frac{\gamma(1-\eta)\,\mathrm{tot}_t^{1-\eta}}{(1-\gamma)+\gamma\,\mathrm{tot}_t^{1-\eta}} \left(\frac{\dot{\mathrm{tot}}_t}{\mathrm{tot}_t}\right). \tag{F.30}$$

Risk-sharing channel and Ξ_t. By construction,

$$Z_t = (1-\gamma) + \gamma Q_t^{\eta-1} e^{-\Xi_t}.$$

Differentiating $Q_t^{\eta-1}e^{-\Xi_t}$ and simplifying,

$$\frac{\dot{Z}_t}{Z_t} = -\frac{\gamma Q_t^{\eta-1}e^{-\Xi_t}}{(1-\gamma)+\gamma Q_t^{\eta-1}e^{-\Xi_t}}\Big((1-\eta)\frac{\dot{Q}_t}{Q_t} + \dot{\Xi}_t\Big). \tag{F.31}$$

Under consumption-scaled convenience yields, the risk-sharing condition becomes

$$\Big(\frac{\dot{C}_t}{C_t} - \frac{\dot{C}_t^*}{C_t^*}\Big) - \frac{\dot{Q}_t}{Q_t} = (\rho^* - \rho) + (\chi_t^* - \chi_t),$$

and integrating in time, with $\ln C_t - \ln C_t^* - \ln Q_t = \Xi_t$, implies

$$\dot{\Xi}_t = (\rho^* - \rho) + \chi_t^* - \chi_t,$$

as stated in the proposition.

Substituting (F.31) into (F.29), the contribution of the Z_t term is

$$-\frac{\phi}{1+\phi}\frac{\dot{Z}_t}{Z_t} = \frac{\phi}{1+\phi}\frac{\gamma Q_t^{\eta-1}e^{-\Xi_t}}{(1-\gamma)+\gamma Q_t^{\eta-1}e^{-\Xi_t}}\Big((1-\eta)\frac{\dot{Q}_t}{Q_t} + \dot{\Xi}_t\Big),$$

which we label as the risk-sharing channel:

$$\mathcal{R}_t = \frac{\phi}{1+\phi}\frac{\gamma Q_t^{\eta-1}e^{-\Xi_t}}{(1-\gamma)+\gamma Q_t^{\eta-1}e^{-\Xi_t}}\Big((1-\eta)\frac{\dot{Q}_t}{Q_t} + \dot{\Xi}_t\Big). \tag{F.32}$$

Step 5: Assembling the natural real rate.

Collecting (F.29), (F.30), and (F.32), we can summarise consumption growth as

$$\frac{\dot{C}_t}{C_t} = g_t + \mathcal{T}_t + \mathcal{R}_t - \frac{1}{1+\phi}\chi_t. \tag{F.33}$$

Substitute (F.33) into the Euler-based expression (F.24), replacing r_t with r_t^n as we previously invoked the assumption of flexible prices:

$$r_t^n = (\rho + g_t) - \Phi_t + \frac{\phi}{1+\phi}\chi_t + \mathcal{T}_t + \mathcal{R}_t.$$

□

F.7 Proof of Proposition 5

Proof. **Home.** Start from Home goods market clearing (F.21):

$$Y_{Ht} = \Big[(1-\gamma) + \gamma\, \mathrm{tot}_t^{1-\eta}\Big]^{\eta/(1-\eta)} \Big[(1-\gamma)C_t + \gamma Q_t^{\eta} C_t^*\Big].$$

Define

$$G_t \equiv (1-\gamma) + \gamma\, \mathrm{tot}_t^{1-\eta},$$

so that

$$Y_{Ht} = G_t^{\eta/(1-\eta)} \Big[(1-\gamma)C_t + \gamma Q_t^{\eta} C_t^*\Big].$$

With consumption-scaled convenience yields, risk sharing implies (see the earlier lemma)

$$C_t^* = C_t\, Q_t^{-1} e^{-\Xi_t}, \qquad \dot{\Xi}_t = (\rho^* - \rho) + \chi_t^* - \chi_t,$$

where $\chi_t \equiv \dot{\nu}_t/(1-\nu_t)$ and $\chi_t^* \equiv \dot{\nu}_t^*/(1-\nu_t^*)$ are the MRS-drift terms associated with the time variation in the convenience weights. Substituting C_t^*:

$$Y_{Ht} = G_t^{\eta/(1-\eta)} C_t \Big[(1-\gamma) + \gamma Q_t^{\eta-1} e^{-\Xi_t}\Big].$$

Define

$$Z_t \equiv (1-\gamma) + \gamma Q_t^{\eta-1} e^{-\Xi_t},$$

so that

$$Y_{Ht} = C_t\, G_t^{\eta/(1-\eta)} Z_t.$$

Log-differentiating:

$$\frac{\dot{Y}_{Ht}}{Y_{Ht}} = \frac{\dot{C}_t}{C_t} + \frac{\eta}{1-\eta}\frac{\dot{G}_t}{G_t} + \frac{\dot{Z}_t}{Z_t}. \tag{F.34}$$

(i) IS term. With consumption-scaled convenience yields, the Home Euler equation is

$$\frac{\dot{C}_t}{C_t} = i_t - \pi_t - \rho + \Phi_t - \chi_t, \qquad \Phi_t \equiv \frac{\Psi_t}{1-\nu_t}, \quad \chi_t \equiv \frac{\dot{\nu}_t}{1-\nu_t}.$$

This gives the IS block in the decomposition.

(ii) Terms-of-trade channel. We have

$$G_t = (1-\gamma) + \gamma\, \mathrm{tot}_t^{1-\eta}, \qquad \mathrm{tot}_t \equiv \frac{P_{Ft}}{P_{Ht}}.$$

Differentiate:

$$\dot{G}_t = \gamma(1-\eta)\, \mathrm{tot}_t^{-\eta} \dot{\mathrm{tot}}_t,$$

and divide by G_t:

$$\frac{\dot{G}_t}{G_t} = \frac{\gamma(1-\eta)\,\mathrm{tot}_t^{1-\eta}}{(1-\gamma)+\gamma\,\mathrm{tot}_t^{1-\eta}}\left(\frac{\dot{\mathrm{tot}}_t}{\mathrm{tot}_t}\right).$$

By definition,

$$\mathcal{T}_t \equiv \left(\frac{\phi}{1+\phi}-\frac{1}{1-\eta}\right)\frac{\gamma(1-\eta)\,\mathrm{tot}_t^{1-\eta}}{(1-\gamma)+\gamma\,\mathrm{tot}_t^{1-\eta}}\left(\frac{\dot{\mathrm{tot}}_t}{\mathrm{tot}_t}\right).$$

Therefore

$$\frac{\eta}{1-\eta}\frac{\dot{G}_t}{G_t} = -\frac{\eta(1+\phi)}{1+\phi\eta}\,\mathcal{T}_t.$$

(iii) Risk-sharing channel. Recall

$$Z_t = (1-\gamma)+\gamma Q_t^{\eta-1}e^{-\Xi_t}.$$

Differentiate using the product rule on $Q_t^{\eta-1}e^{-\Xi_t}$:

$$\dot{Z}_t = -\gamma Q_t^{\eta-1}e^{-\Xi_t}\Big((1-\eta)\tfrac{\dot{Q}_t}{Q_t}+\dot{\Xi}_t\Big).$$

Divide by Z_t:

$$\frac{\dot{Z}_t}{Z_t} = -\frac{\gamma Q_t^{\eta-1}e^{-\Xi_t}}{(1-\gamma)+\gamma Q_t^{\eta-1}e^{-\Xi_t}}\Big((1-\eta)\tfrac{\dot{Q}_t}{Q_t}+\dot{\Xi}_t\Big).$$

Define the risk-sharing channel

$$\mathcal{R}_t \equiv \frac{\phi}{1+\phi}\,\frac{\gamma Q_t^{\eta-1}e^{-\Xi_t}}{(1-\gamma)+\gamma Q_t^{\eta-1}e^{-\Xi_t}}\Big((1-\eta)\tfrac{\dot{Q}_t}{Q_t}+\dot{\Xi}_t\Big),$$

so that

$$\frac{\dot{Z}_t}{Z_t} = -\frac{1+\phi}{\phi}\,\mathcal{R}_t.$$

Combining (i)–(iii) in (F.34):

$$\begin{aligned}\frac{\dot{Y}_{Ht}}{Y_{Ht}} &= \underbrace{\big(i_t-\pi_t-\rho+\Phi_t-\chi_t\big)}_{\text{IS}} + \underbrace{\frac{\eta}{1-\eta}\frac{\dot{G}_t}{G_t}}_{\text{ToT}} + \underbrace{\frac{\dot{Z}_t}{Z_t}}_{\text{Risk sharing}} \\ &= \big(i_t-\pi_t-\rho+\Phi_t-\chi_t\big) - \frac{\eta(1+\phi)}{1+\phi\eta}\,\mathcal{T}_t - \frac{1+\phi}{\phi}\,\mathcal{R}_t,\end{aligned}$$

which is exactly (40).

Foreign. The Foreign side is symmetric. Foreign demand and goods market clearing can be written as

$$Y_{Ft}^* = \Big[(1-\gamma)+\gamma(\mathrm{tot}_t^*)^{1-\eta}\Big]^{\eta/(1-\eta)}\Big[(1-\gamma)C_t^*+\gamma Q_t^{1-\eta}e^{\Xi_t}C_t\Big],$$

where $\mathrm{tot}_t^* \equiv P_{Ht}^*/P_{Ft}^* = (\mathrm{tot}_t)^{-1}$.

Using risk sharing $C_t = C_t^* Q_t e^{\Xi_t}$, define

$$G_t^* \equiv (1-\gamma) + \gamma(\text{tot}_t^*)^{1-\eta}, \qquad Z_t^* \equiv (1-\gamma) + \gamma Q_t^{1-\eta} e^{\Xi_t},$$

and obtain

$$Y_{Ft}^* = C_t^* (G_t^*)^{\eta/(1-\eta)} Z_t^*.$$

Log-differentiating:

$$\frac{\dot{Y}_{Ft}^*}{Y_{Ft}^*} = \frac{\dot{C}_t^*}{C_t^*} + \frac{\eta}{1-\eta}\frac{\dot{G}_t^*}{G_t^*} + \frac{\dot{Z}_t^*}{Z_t^*}.$$

With no convenience on the foreign risk-free bond, the Foreign Euler equation is

$$\frac{\dot{C}_t^*}{C_t^*} = i_t^* - \pi_t^* - \rho^* - \chi_t^*, \qquad \chi_t^* \equiv \frac{\dot{\nu}_t^*}{1-\nu_t^*}.$$

The terms-of-trade part is

$$\frac{\eta}{1-\eta}\frac{\dot{G}_t^*}{G_t^*} = -\frac{\eta(1+\phi^*)}{1+\phi^*\eta}\mathcal{T}_t^*,$$

where

$$\mathcal{T}_t^* \equiv \left(\frac{\phi^*}{1+\phi^*} - \frac{1}{1-\eta}\right)\frac{\gamma(1-\eta)\,(\text{tot}_t^*)^{1-\eta}}{(1-\gamma)+\gamma\,(\text{tot}_t^*)^{1-\eta}}\left(\frac{\dot{\text{tot}}_t^*}{\text{tot}_t^*}\right).$$

For the risk-sharing term, note

$$Z_t^* = (1-\gamma) + \gamma Q_t^{1-\eta} e^{\Xi_t},$$

so

$$\dot{Z}_t^* = \gamma Q_t^{1-\eta} e^{\Xi_t}\Big((1-\eta)\tfrac{\dot{Q}_t}{Q_t} + \dot{\Xi}_t\Big),$$
$$\frac{\dot{Z}_t^*}{Z_t^*} = \frac{\gamma Q_t^{1-\eta} e^{\Xi_t}}{(1-\gamma)+\gamma Q_t^{1-\eta} e^{\Xi_t}}\Big((1-\eta)\tfrac{\dot{Q}_t}{Q_t} + \dot{\Xi}_t\Big).$$

Define

$$\mathcal{R}_t^* \equiv \frac{\phi^*}{1+\phi^*}\,\frac{\gamma Q_t^{1-\eta} e^{\Xi_t}}{(1-\gamma)+\gamma Q_t^{1-\eta} e^{\Xi_t}}\Big(-(1-\eta)\tfrac{\dot{Q}_t}{Q_t} - \dot{\Xi}_t\Big),$$

so that the combination of Q_t and Ξ_t terms in $\dot{Y}_{Ft}^*/Y_{Ft}^*$ can be written in the compact "$-(1+\phi^*)/\phi^*\,\mathcal{R}_t^*$" form used in (41) once risk-sharing is imposed via $\dot{\Xi}_t = (\rho^* - \rho) + \chi_t^* - \chi_t$.

Collecting the IS, terms-of-trade, and risk-sharing pieces yields (41). This completes the proof. □

F.8 Proof of Proposition 6

Proof. We proceed in three steps: (i) relate the output gap to actual and natural output growth, (ii) substitute the output growth decompositions with convenience and MRS drift, and (iii) use the

natural rate decomposition to obtain the IS curves.

Domestic IS curve.

By definition, the Home output gap is the log deviation of actual output from its natural (flexible price) level:

$$x_t \equiv \ln Y_{Ht} - \ln Y_{Ht}^n.$$

Differentiating and using $\frac{d}{dt} \ln X_t = \dot{X}_t / X_t$ for any positive process X_t,

$$\dot{x}_t = \frac{\dot{Y}_{Ht}}{Y_{Ht}} - \frac{\dot{Y}_{Ht}^n}{Y_{Ht}^n}.$$

Thus, the dynamics of the output gap depend on the difference between actual and natural output growth.

From the Output-Growth proposition with consumption-scaled convenience yields, Home *actual* output growth can be written as

$$\frac{\dot{Y}_{Ht}}{Y_{Ht}} = \Big(i_t - \pi_t - \rho + \Phi_t - \chi_t \Big) \; - \; \frac{\eta(1+\phi)}{1+\phi\eta} \mathcal{T}_t \; - \; \frac{1+\phi}{\phi} \mathcal{R}_t. \tag{F.35}$$

The first term is a generalized IS component: the standard ex-ante real rate $i_t - \pi_t - \rho$ plus two preference-driven wedges:

- $\Phi_t = \Psi_t/(1-\nu_t)$ is the *effective* convenience yield (in units of the marginal utility of consumption),
- $\chi_t = \dot{\nu}_t/(1-\nu_t)$ captures the drift in the convenience weight in utility (intertemporal MRS scaling).

The second and third terms collect the open economy contributions:

- $\mathcal{T}_t$ is the terms-of-trade channel (relative price of Home vs. Foreign goods),
- $\mathcal{R}_t$ is the risk-sharing/expenditure-switching channel, driven by the real exchange rate and the risk-sharing wedge.

Under flexible prices (the natural allocation), the same decomposition applies except that the real rate $i_t - \pi_t$ is replaced by the natural real rate r_t^n. The convenience block $(\Phi_t - \chi_t)$ and the open economy terms are unchanged, since they are driven by preferences and international goods market structure rather than price stickiness. Hence

$$\frac{\dot{Y}_{Ht}^n}{Y_{Ht}^n} = \Big(r_t^n - \rho + \Phi_t - \chi_t \Big) \; - \; \frac{\eta(1+\phi)}{1+\phi\eta} \mathcal{T}_t \; - \; \frac{1+\phi}{\phi} \mathcal{R}_t. \tag{F.36}$$

Subtracting the natural output-growth equation (F.36) from the actual one (F.35) gives

$$\begin{aligned}\dot{x}_t &= \frac{\dot{Y}_{Ht}}{Y_{Ht}} - \frac{\dot{Y}^n_{Ht}}{Y^n_{Ht}} \\ &= \left[(i_t - \pi_t - \rho + \Phi_t - \chi_t) - \tfrac{\eta(1+\phi)}{1+\phi\eta}\,\mathcal{T}_t - \tfrac{1+\phi}{\phi}\,\mathcal{R}_t\right] \\ &\quad - \left[(r^n_t - \rho + \Phi_t - \chi_t) - \tfrac{\eta(1+\phi)}{1+\phi\eta}\,\mathcal{T}_t - \tfrac{1+\phi}{\phi}\,\mathcal{R}_t\right].\end{aligned}$$

Inside the brackets:

- the terms-of-trade and risk-sharing pieces, $-\frac{\eta(1+\phi)}{1+\phi\eta}\mathcal{T}_t$ and $-\frac{1+\phi}{\phi}\mathcal{R}_t$, are identical under actual and natural allocations and therefore cancel;
- the convenience block $(\Phi_t - \chi_t)$ also cancels.

We are left with the standard gap relation between the actual and natural real rates:

$$\begin{aligned}\dot{x}_t &= (i_t - \pi_t - \rho + \Phi_t - \chi_t) - (r^n_t - \rho + \Phi_t - \chi_t) \\ &= (i_t - \pi_t) - r^n_t.\end{aligned}$$

Next, apply the open economy natural-rate decomposition (proved earlier):

$$r^n_t = (\rho + g_t) \;-\; \Phi_t \;+\; \frac{\phi}{1+\phi}\chi_t \;+\; \mathcal{T}_t \;+\; \mathcal{R}_t, \tag{F.37}$$

with $g_t \equiv \dot{A}_t/A_t$. Substituting (F.37) into $\dot{x}_t = (i_t - \pi_t) - r^n_t$ yields

$$\begin{aligned}\dot{x}_t &= (i_t - \pi_t) - \left[(\rho + g_t) - \Phi_t + \tfrac{\phi}{1+\phi}\chi_t + \mathcal{T}_t + \mathcal{R}_t\right] \\ &= (i_t - \pi_t - \rho) - g_t + \Phi_t - \frac{\phi}{1+\phi}\chi_t - \mathcal{T}_t - \mathcal{R}_t,\end{aligned}$$

which is precisely the claimed Home IS curve.

Foreign IS curve.

The foreign output gap is defined analogously:

$$x^*_t \equiv \ln Y^*_{Ft} - \ln Y^{n*}_{Ft}, \qquad \dot{x}^*_t = \frac{\dot{Y}^*_{Ft}}{Y^*_{Ft}} - \frac{\dot{Y}^{n*}_{Ft}}{Y^{n*}_{Ft}}.$$

From the foreign Output-Growth proposition with convenience on Home currency bonds, foreign *actual* output growth is

$$\frac{\dot{Y}^*_{Ft}}{Y^*_{Ft}} = \left(i^*_t - \pi^*_t - \rho^* - \chi^*_t\right) \;-\; \frac{\eta(1+\phi^*)}{1+\phi^*\eta}\,\mathcal{T}^*_t \;-\; \frac{1+\phi^*}{\phi^*}\,\mathcal{R}^*_t,$$

where there is no foreign convenience term Φ^*_t on the foreign risk-free bond: in our setup, foreign households derive convenience from Home currency bonds, not from their own government debt.

Under flexible prices, foreign natural output growth satisfies the same decomposition with $i_t^* - \pi_t^*$ replaced by the foreign natural rate r_t^{n*}:

$$\frac{\dot{Y}_{Ft}^{n*}}{Y_{Ft}^{n*}} = \left(r_t^{n*} - \rho^* - \chi_t^*\right) \; - \; \frac{\eta(1+\phi^*)}{1+\phi^*\eta}\,\mathcal{T}_t^* \; - \; \frac{1+\phi^*}{\phi^*}\,\mathcal{R}_t^*.$$

Subtracting again delivers the real interest rate gap relationship

$$\dot{x}_t^* = (i_t^* - \pi_t^*) - r_t^{n*}.$$

The foreign natural real rate can be decomposed as

$$r_t^{n*} = (\rho^* + g_t^*) \; + \; \frac{\phi^*}{1+\phi^*}\,\chi_t^* \; + \; \mathcal{T}_t^* \; + \; \mathcal{R}_t^*,$$

with $g_t^* \equiv \dot{A}_t^*/A_t^*$ and again no foreign convenience term on the foreign risk-free asset. Substituting into $\dot{x}_t^* = (i_t^* - \pi_t^*) - r_t^{n*}$ gives

$$\begin{aligned}\dot{x}_t^* &= (i_t^* - \pi_t^*) - \left[(\rho^* + g_t^*) + \tfrac{\phi^*}{1+\phi^*}\chi_t^* + \mathcal{T}_t^* + \mathcal{R}_t^*\right] \\ &= (i_t^* - \pi_t^* - \rho^*) - g_t^* - \frac{\phi^*}{1+\phi^*}\chi_t^* - \mathcal{T}_t^* - \mathcal{R}_t^*,\end{aligned}$$

which is exactly the stated foreign IS curve.

This completes the proof. □

F.9 Deriving the non-linear Phillips Curve

As an intermediate step to the linearized Phillips Curve, we characterize the non-linear Phillips Curve in this model as the integral representation of the Rotemberg pricing ODE (32).

Proposition 13 (Nonlinear Phillips Curve)**.** *Home producer price inflation equals the present discounted value of future real marginal cost deviations:*

$$\pi_{Ht} = \int_t^\infty \exp\Big\{-\int_t^u \Big[(i_s + \Phi_s) - \tfrac{\dot{Y}_{Hs}}{Y_{Hs}} - \pi_{Hs}\Big]\,ds\Big\}\frac{\varepsilon - 1}{\theta}(mc_u - 1)\,du, \tag{F.38}$$

with $mc_t \equiv \frac{\varepsilon}{\varepsilon-1}\frac{W_t}{P_{Ht}A_t}$*. The Foreign analogue holds with starred variables and* $\Phi_s = 0$*.*

Proof. Start from the continuous-time sticky price (Rotemberg-type) price-setting ODE for Home firms. With the household nominal SDF discounting nominal profits at rate

$$-\frac{\dot{M}_t^n}{M_t^n} \;=\; i_t + \Phi_t, \qquad \Phi_t \equiv \frac{\Psi_t}{1-\nu_t},$$

the law of motion for producer price inflation is

$$\dot{\pi}_{Ht} + \frac{\varepsilon - 1}{\theta}\big(mc_t - 1\big) = \Big[\big(\, i_t + \Phi_t \,\big) - \frac{\dot{Y}_{Ht}}{Y_{Ht}} - \pi_{Ht}\Big]\, \pi_{Ht}. \tag{F.39}$$

(When there is no convenience yield, $\nu_t = 0$ so $\Phi_t = 0$ and we recover the baseline ODE with i_t only.)

Define the effective discount rate

$$\delta_t \;\equiv\; \big(\, i_t + \Phi_t \,\big) - \frac{\dot{Y}_{Ht}}{Y_{Ht}} - \pi_{Ht}.$$

Rearrange (F.39) as the Ricatti ODE (due to being quadratic in inflation)

$$\dot{\pi}_{Ht} - \delta_t\, \pi_{Ht} \;=\; -\frac{\varepsilon - 1}{\theta}\,(mc_t - 1).$$

Consider the integrating factor

$$I_t \equiv \exp\Big\{ -\int_0^t \delta_s ds \Big\} = \exp\Big\{ -\int_0^t \Big[\big(\, i_s + \Phi_s \,\big) - \frac{\dot{Y}_{Hs}}{Y_{Hs}} - \pi_{Hs}\Big] ds \Big\}.$$

Multiplying the ODE by I_t gives

$$\frac{d}{dt}\big(I_t\, \pi_{Ht}\big) = -\, I_t\, \frac{\varepsilon - 1}{\theta}\,(mc_t - 1).$$

Integrate forward from t to ∞:

$$\int_t^\infty \frac{d}{du}(I_u\, \pi_{Hu})\, du = -\int_t^\infty I_u \frac{\varepsilon - 1}{\theta}(mc_u - 1)\, du.$$

Impose the no-bubble condition

$$\lim_{u\to\infty} I_u\, \pi_{Hu} = 0,$$

so that

$$-I_t\, \pi_{Ht} = -\int_t^\infty I_u \frac{\varepsilon - 1}{\theta}(mc_u - 1)\, du,$$

or equivalently

$$\pi_{Ht} = I_t^{-1} \int_t^\infty I_u \frac{\varepsilon - 1}{\theta}(mc_u - 1)\, du.$$

Since

$$I_t^{-1} I_u = \exp\Big\{ \int_0^t \delta_s ds \Big\}\, \exp\Big\{ -\int_0^u \delta_s ds \Big\} = \exp\Big\{ -\int_t^u \delta_s ds \Big\},$$

we obtain the stated integral representation:

$$\pi_{Ht} = \int_t^\infty \exp\Big\{ -\int_t^u \Big[\big(\, i_s + \Phi_s \,\big) - \frac{\dot{Y}_{Hs}}{Y_{Hs}} - \pi_{Hs}\Big] ds \Big\}\, \frac{\varepsilon - 1}{\theta}(mc_u - 1)\, du.$$

For Foreign firms, convenience attaches to *home currency* safe assets, not to the foreign

currency risk-free bond. Thus the foreign nominal SDF discounts at i_t^* with no wedge, and the foreign price-setting ODE is

$$\dot{\pi}_{Ft}^* + \frac{\varepsilon - 1}{\theta}(mc_t^* - 1) = \Big[i_t^* - \frac{\dot{Y}_{Ft}^*}{Y_{Ft}^*} - \pi_{Ft}^*\Big]\pi_{Ft}^*.$$

Repeating exactly the same integrating factor steps (with $\delta_t^* \equiv i_t^* - \dot{Y}_{Ft}^*/Y_{Ft}^* - \pi_{Ft}^*$ and no Φ_t term) yields

$$\pi_{Ft}^* = \int_t^\infty \exp\Big\{-\int_t^u \Big[i_s^* - \frac{\dot{Y}_{Fs}^*}{Y_{Fs}^*} - \pi_{Fs}^*\Big] ds\Big\} \frac{\varepsilon - 1}{\theta}(mc_u^* - 1)\, du,$$

which is the foreign integral representation in the proposition. □

F.10 Proof of Proposition 7

Proof. We proceed in five steps for the Home economy and then sketch the foreign case.

Step 1. Rotemberg price-setting ODE with consumption-scaled convenience yields.

Under Rotemberg pricing, Home producer price inflation satisfies the (nonlinear) Phillips curve

$$\dot{\pi}_{Ht} + \frac{\varepsilon - 1}{\theta}(mc_t - 1) = \underbrace{\Big[(i_t + \Phi_t) - \tfrac{\dot{Y}_{Ht}}{Y_{Ht}} - \pi_{Ht}\Big]}_{\delta_t} \pi_{Ht}, \tag{F.40}$$

where the real marginal cost is

$$mc_t \equiv \frac{\varepsilon}{\varepsilon - 1}\frac{W_t}{P_{Ht}A_t},$$

and the *effective* convenience wedge is

$$\Phi_t \equiv \frac{\Psi_t}{1 - \nu_t}.$$

Here ν_t is the convenience flow in utility, Ψ_t the marginal convenience benefit in units of consumption, and Φ_t rescales convenience in units of the nominal rate. The term

$$\delta_t \equiv (i_t + \Phi_t) - \frac{\dot{Y}_{Ht}}{Y_{Ht}} - \pi_{Ht}$$

is the effective nominal discount rate relevant for firms' price-setting problem, which includes the convenience adjustment Φ_t.

Equation (F.40) can be written compactly as

$$\dot{\pi}_{Ht} = \delta_t\, \pi_{Ht} - \kappa\,(mc_t - 1), \qquad \kappa \equiv \frac{\varepsilon - 1}{\theta}. \tag{F.41}$$

Step 2. Linearization around a zero-inflation steady state.

Linearize (F.41) around a steady state with nominal rate $\bar{i}$, effective convenience wedge $\bar{\Phi}$, output growth $\overline{\dot{Y}_{Ht}/Y_{Ht}}$, and steady-state producer price inflation $\bar{\pi}_H$. Write

$$\pi_{Ht} = \bar{\pi}_H + \hat{\pi}_{Ht}, \qquad \delta_t = \bar{\delta}_H + \hat{\delta}_t,$$

and define the log-deviation of marginal cost

$$\hat{mc}_t \equiv \ln(mc_t) - \ln(\overline{mc}),$$

where $\overline{mc} = 1$ (steady-state marginal cost equals one under monopolistic competition with constant markup).

By definition,

$$\delta_t = (i_t + \Phi_t) - \frac{\dot{Y}_{Ht}}{Y_{Ht}} - \pi_{Ht},$$

so at steady state

$$\bar{\delta}_H = \bar{i} + \bar{\Phi} - \overline{\dot{Y}_{Ht}/Y_{Ht}} - \bar{\pi}_H.$$

Because $\bar{\pi}_H$ is constant, $\dot{\pi}_{Ht} = \dot{\hat{\pi}}_{Ht}$. Around $mc_t = 1$, a first-order approximation gives

$$mc_t - 1 \approx \hat{mc}_t.$$

Substituting $\pi_{Ht} = \bar{\pi}_H + \hat{\pi}_{Ht}$, $\delta_t = \bar{\delta}_H + \hat{\delta}_t$, and $mc_t - 1 \simeq \hat{mc}_t$ into (F.41) yields

$$\dot{\hat{\pi}}_{Ht} = (\bar{\delta}_H + \hat{\delta}_t)(\bar{\pi}_H + \hat{\pi}_{Ht}) - \kappa\, \hat{mc}_t.$$

Expanding,

$$\dot{\hat{\pi}}_{Ht} = \bar{\delta}_H \bar{\pi}_H + \bar{\delta}_H \hat{\pi}_{Ht} + \bar{\pi}_H \hat{\delta}_t + \hat{\delta}_t \hat{\pi}_{Ht} - \kappa\, \hat{mc}_t.$$

The product $\hat{\delta}_t \hat{\pi}_{Ht}$ is second order and is dropped at first order. We then impose a zero steady-state inflation target, $\bar{\pi}_H = 0$, so all terms involving $\bar{\pi}_H$ vanish, and obtain the linear pricing equation

$$\dot{\hat{\pi}}_{Ht} = \bar{\delta}_H\, \hat{\pi}_{Ht} - \kappa\, \hat{mc}_t. \tag{F.42}$$

Step 2'. From deviations to gaps.

Under flexible prices, the natural (flex-price) real marginal cost equals one:

$$mc_t^n = 1 \quad \Rightarrow \quad \hat{mc}_t^n = \ln(mc_t^n) - \ln(\overline{mc}) = \ln(1) - \ln(1) = 0.$$

Define the marginal cost *gap* as the difference between log marginal costs in the actual and natural allocations:

$$\tilde{mc}_t \equiv \ln(mc_t) - \ln(mc_t^n).$$

Since $\hat{mc}_t^n = 0$, we have

$$\tilde{mc}_t = \hat{mc}_t - \hat{mc}_t^n = \hat{mc}_t.$$

Thus (F.42) may be written directly in terms of the marginal cost gap:

$$\dot{\hat{\pi}}_{Ht} = \bar{\delta}_H \, \hat{\pi}_{Ht} - \kappa \, \tilde{mc}_t. \tag{F.43}$$

This is the linearized NKPC up to expressing $\tilde{mc}_t$ in terms of observables.

Step 3. Real marginal cost with convenience in the intratemporal condition.

With consumption-scaled convenience yields, the household intratemporal conditions are

$$U_C = \frac{1-\nu_t}{C_t}, \qquad U_N = -N_t^{\phi}.$$

The labor supply FOC equates the MRS to the real wage:

$$\frac{U_N}{U_C} = -\frac{W_t}{P_t} \implies C_t N_t^{\phi} = (1-\nu_t)\frac{W_t}{P_t}.$$

Rearranging,

$$\frac{W_t}{P_t} = \frac{C_t N_t^{\phi}}{1-\nu_t}.$$

We want $\frac{W_t}{P_{Ht}}$, not $\frac{W_t}{P_t}$. Using the CPI–PPI wedge,

$$\frac{W_t}{P_{Ht}} = \frac{W_t}{P_t} \cdot \frac{P_t}{P_{Ht}} = \frac{C_t N_t^{\phi}}{1-\nu_t} \frac{P_t}{P_{Ht}}.$$

Take logs and linearize. Let hats denote log-deviations from steady state. Then

$$\hat{W}_t - \hat{P}_{Ht} = \left(\hat{C}_t + \phi \, \hat{N}_t\right) - \widehat{(1-\nu_t)} + \widehat{\left(\frac{P_t}{P_{Ht}}\right)}.$$

Real marginal cost is

$$mc_t = \frac{\varepsilon}{\varepsilon - 1}\frac{W_t}{P_{Ht} A_t} \quad \Rightarrow \quad \ln mc_t = \ln \frac{\varepsilon}{\varepsilon - 1} + \ln W_t - \ln P_{Ht} - \ln A_t,$$

so in log-deviation form,

$$\hat{mc}_t = (\hat{W}_t - \hat{P}_{Ht}) - \hat{A}_t.$$

Using the production function $Y_{Ht} = A_t N_t$, we have $\hat{N}_t = \hat{Y}_{Ht} - \hat{A}_t$. Substituting gives

$$\begin{aligned}
\hat{mc}_t &= \big(\hat{C}_t + \phi\, \hat{N}_t\big) - \widehat{(1-\nu_t)} + \widehat{\Big(\frac{P_t}{P_{Ht}}\Big)} - \hat{A}_t \\
&= \hat{C}_t + \phi(\hat{Y}_{Ht} - \hat{A}_t) - \widehat{(1-\nu_t)} + \widehat{\Big(\frac{P_t}{P_{Ht}}\Big)} - \hat{A}_t \\
&= \hat{C}_t + \phi\, \hat{Y}_{Ht} + \widehat{\Big(\frac{P_t}{P_{Ht}}\Big)} - (1+\phi)\hat{A}_t - \widehat{(1-\nu_t)}.
\end{aligned}$$

In the flexible price (natural) allocation,

$$\hat{mc}_t^{\,n} = \hat{C}_t^{\,n} + \phi\, \hat{Y}_{Ht}^{\,n} + \widehat{\Big(\frac{P_t^n}{P_{Ht}^n}\Big)} - (1+\phi)\hat{A}_t^n - \widehat{(1-\nu_t^n)}.$$

Technology is common across allocations, so $\hat{A}_t^n = \hat{A}_t$. We impose that the real Home-bond position is identical under the flexible price equilibrium so that

$$b_{Ht}^n = b_{Ht}.$$

Under the isoelastic convenience-flow specification and normalization, $\bar{\nu} = 0$ and the first-order approximation is

$$\nu_t \;\approx\; \kappa_H\, \hat{b}_{Ht}, \qquad \kappa_H \;\equiv\; \frac{\bar{\Psi}\,\bar{\theta}\,\bar{b}_H}{\bar{C}}, \qquad \hat{b}_{Ht} \equiv \ln\Big(\frac{b_{Ht}}{\bar{b}_H}\Big).$$

Hence, this implies

$$\nu_t^n \;\approx\; \kappa_H\, \hat{b}_{Ht}^n \;=\; \kappa_H\, \hat{b}_{Ht} \;\approx\; \nu_t,$$

so that $(1-\nu_t^n)$ and $(1-\nu_t)$ have the same first-order log deviations:

$$\widehat{(1-\nu_t^n)} \;=\; \widehat{(1-\nu_t)}.$$

Therefore these wedge terms cancel when we form the *gap*:

$$\begin{aligned}
\tilde{mc}_t &\equiv \hat{mc}_t - \hat{mc}_t^{\,n} \\
&= (\hat{C}_t - \hat{C}_t^{\,n}) + \phi(\hat{Y}_{Ht} - \hat{Y}_{Ht}^{\,n}) + \left[\widehat{\Big(\frac{P_t}{P_{Ht}}\Big)} - \widehat{\Big(\frac{P_t^n}{P_{Ht}^n}\Big)}\right].
\end{aligned}$$

Define the consumption and output gaps as

$$\tilde{c}_t \equiv \hat{C}_t - \hat{C}_t^{\,n}, \qquad x_t \equiv \hat{Y}_{Ht} - \hat{Y}_{Ht}^{\,n},$$

so

$$\tilde{mc}_t = \tilde{c}_t + \phi\, x_t + \left[\widehat{\Big(\frac{P_t}{P_{Ht}}\Big)} - \widehat{\Big(\frac{P_t^n}{P_{Ht}^n}\Big)}\right]. \tag{F.44}$$

Thus real marginal cost gaps are driven by (i) the consumption gap, (ii) the output gap, and (iii) the CPI–PPI wedge gap. Crucially, because the natural allocation is defined conditional on the same inherited real bond position b_{Ht} (and the same θ_t), the MRS wedge $(1-\nu_t)$ affects both allocations symmetrically to first order and therefore drops out of $\tilde{mc}_t$; convenience enters the NKPC only through $\bar{\delta}_H$.

Step 4. Mapping the CPI–PPI wedge gap into the terms-of-trade gap.

The Home CPI and the terms of trade are

$$P_t = \Big[(1-\gamma)P_{Ht}^{1-\eta} + \gamma P_{Ft}^{1-\eta}\Big]^{\frac{1}{1-\eta}}, \qquad \text{tot}_t \equiv \frac{P_{Ft}}{P_{Ht}}.$$

Then

$$\ln\frac{P_t}{P_{Ht}} = \frac{1}{1-\eta}\ln\Big[(1-\gamma) + \gamma\,\text{tot}_t^{1-\eta}\Big].$$

Differentiate w.r.t. $\ln \text{tot}_t$:

$$\begin{aligned}\frac{\partial \ln(P_t/P_{Ht})}{\partial \ln \text{tot}_t} &= \text{tot}_t\,\frac{\partial \ln(P_t/P_{Ht})}{\partial \text{tot}_t} \\ &= \text{tot}_t \cdot \frac{\gamma(1-\eta)\,\text{tot}_t^{-\eta}}{(1-\gamma)+\gamma\,\text{tot}_t^{1-\eta}} \\ &= \frac{\gamma\,\text{tot}_t^{1-\eta}}{(1-\gamma)+\gamma\,\text{tot}_t^{1-\eta}}.\end{aligned}$$

In a symmetric steady state, $\overline{P}_{Ht} = \overline{P}_{Ft} \Rightarrow \overline{\text{tot}} = 1$, so

$$\left.\frac{\partial \ln(P_t/P_{Ht})}{\partial \ln \text{tot}_t}\right|_{\overline{\text{tot}}=1} = \frac{\gamma \cdot 1^{\,1-\eta}}{(1-\gamma)+\gamma\cdot 1^{\,1-\eta}} = \gamma.$$

A first-order Taylor expansion of $\ln(P_t/P_{Ht})$ around $\overline{\text{tot}} = 1$ yields

$$\ln\frac{P_t}{P_{Ht}} \approx \ln\frac{\overline{P}}{\overline{P}_H} + \gamma\big(\ln \text{tot}_t - \ln\overline{\text{tot}}\big).$$

Analogously, in the natural (flex-price) allocation,

$$\ln\frac{P_t^n}{P_{Ht}^n} \approx \ln\frac{\overline{P^n}}{\overline{P_H^n}} + \gamma\big(\ln \text{tot}_t^n - \ln\overline{\text{tot}^n}\big).$$

Subtracting gives the CPI–PPI wedge *gap*:

$$\begin{aligned}\widehat{\left(\frac{P_t}{P_{Ht}}\right)} - \widehat{\left(\frac{P_t^n}{P_{Ht}^n}\right)} &= \gamma\Big[\big(\ln \text{tot}_t - \ln\overline{\text{tot}}\big) - \big(\ln \text{tot}_t^n - \ln\overline{\text{tot}^n}\big)\Big] \\ &= \gamma\big(\ln \text{tot}_t - \ln \text{tot}_t^n\big).\end{aligned}$$

Define the terms-of-trade gap

$$\tau_t \equiv \ln \mathrm{tot}_t - \ln \mathrm{tot}_t^n.$$

Then the wedge gap is

$$\widehat{\left(\frac{P_t}{P_{Ht}}\right)} - \widehat{\left(\frac{P_t^n}{P_{Ht}^n}\right)} = \gamma\, \tau_t.$$

Substituting in (F.44) gives

$$\tilde{mc}_t = \tilde{c}_t + \phi\, x_t + \gamma\, \tau_t. \tag{F.45}$$

Step 5. Substitute the marginal cost gap into the pricing ODE.

Recall the linearized pricing ODE in gap form (F.43):

$$\dot{\hat{\pi}}_{Ht} = \bar{\delta}_H\, \hat{\pi}_{Ht} - \kappa\, \tilde{mc}_t.$$

Using the decomposition (F.45), we obtain

$$\dot{\hat{\pi}}_{Ht} = \bar{\delta}_H\, \hat{\pi}_{Ht} - \kappa\big[\tilde{c}_t + \phi\, x_t + \gamma\, \tau_t\big].$$

Rearranging yields the stated Home NKPC:

$$\bar{\delta}_H\, \hat{\pi}_{Ht} = \kappa\big[\tilde{c}_t + \phi\, x_t + \gamma\, \tau_t\big] + \dot{\hat{\pi}}_{Ht}.$$

Note that convenience affects the NKPC only via the effective discount factor $\bar{\delta}_H$, through $\bar{\Phi}$; it does not generate additional "gap" terms, because the MRS wedge $(1-\nu_t)$ cancels in $\tilde{mc}_t$.

Foreign NKPC.

The foreign derivation is analogous, with two key differences.

(i) Discounting of profits. Foreign convenience attaches to *home currency* bonds, not to the foreign currency risk-free asset. Thus foreign firms' profits (in foreign currency) are discounted at rate i_t^*, not $i_t^* + \Phi_t^*$. The effective foreign pricing rate is

$$\delta_t^* \equiv i_t^* - \frac{\dot{Y}_{Ft}^*}{Y_{Ft}^*} - \pi_{Ft}^*,$$

with steady-state value

$$\bar{\delta}_F \equiv \bar{i}^* - \overline{\dot{Y}_{Ft}^*/Y_{Ft}^*} - \bar{\pi}_F^*.$$

(ii) Intratemporal condition and marginal cost gaps. The foreign intratemporal condition has its own MRS wedge $(1-\nu_t^*)$, but—as in the Home case—this wedge appears symmetrically in the actual and natural allocations and cancels out of the marginal cost *gap*. The foreign CPI–PPI

wedge is mapped into the foreign terms of trade $\text{tot}_t^* \equiv P_{Ht}^*/P_{Ft}^*$ in exactly the same way, producing

$$\tilde{mc}_t^* = \tilde{c}_t^* + \phi^* x_t^* + \gamma\, \tau_t^*,$$

with $\tilde{c}_t^* \equiv \hat{C}_t^* - \hat{C}_t^{*n}$, $x_t^* \equiv \hat{Y}_{Ft}^* - \hat{Y}_{Ft}^{*n}$, and $\tau_t^* \equiv \ln \text{tot}_t^* - \ln \text{tot}_t^{*n}$.

Linearizing the foreign Rotemberg ODE

$$\dot{\pi}_{Ft}^* + \frac{\varepsilon - 1}{\theta}(mc_t^* - 1) = \left[i_t^* - \tfrac{\dot{Y}_{Ft}^*}{Y_{Ft}^*} - \pi_{Ft}^*\right]\pi_{Ft}^*$$

around a zero-inflation steady state and repeating the same steps as for Home yields

$$\dot{\hat{\pi}}_{Ft}^* = \bar{\delta}_F\, \hat{\pi}_{Ft}^* - \kappa\, \tilde{mc}_t^* = \bar{\delta}_F\, \hat{\pi}_{Ft}^* - \kappa\big[\tilde{c}_t^* + \phi^* x_t^* + \gamma\, \tau_t^*\big],$$

which rearranges to

$$\bar{\delta}_F\, \hat{\pi}_{Ft}^* = \kappa\big[\tilde{c}_t^* + \phi^* x_t^* + \gamma\, \tau_t^*\big] + \dot{\hat{\pi}}_{Ft}^*,$$

completing the proof. □

F.11 Fiscal Policy Blocks: Full Derivations

Home government. Let $b_t \equiv B_t/P_t$ and $\pi_t \equiv \dot{P}_t/P_t$. Dividing the nominal flow constraint (46) by P_t and using $\dot{b}_t = \dot{B}_t/P_t - \pi_t b_t$ gives the real debt law of motion,

$$\dot{b}_t = (i_t - \pi_t)\, b_t - s_t, \tag{F.46}$$

where $s_t \equiv (T_t - P_t G_t)/P_t$ is the real primary surplus. Equation (F.46) is a linear ODE in b_t; its integrating factor $\mathcal{I}_t \equiv \exp(-\int_0^t (i_\tau - \pi_\tau) d\tau)$ satisfies $\frac{d}{dt}(\mathcal{I}_t b_t) = -\mathcal{I}_t s_t$. Integrating from t to T and imposing the no-Ponzi condition $\lim_{T\to\infty} \mathcal{I}_T b_T = 0$ yields (47). Note that convenience yields affect i_t through the household Euler equation but do not enter the flow identity (46) directly.

Foreign government. The Foreign nominal flow constraint is

$$\dot{B}_t^* = i_t^* B_t^* + P_t^* G_t^* - T_t^*, \tag{F.47}$$

where $B_t^* = B_{Ft}^* + B_{Ft}$ by Foreign bond-market clearing. An identical derivation to the Home case gives the Foreign real law of motion

$$\dot{b}_t^* = (i_t^* - \pi_t^*)\, b_t^* - s_t^*, \tag{F.48}$$

and, under the no-Ponzi condition, the Foreign valuation equation

$$b_t^* = \int_t^\infty \exp\Big(-\int_t^u (i_s^* - \pi_s^*)\, ds\Big) s_u^*\, du. \tag{F.49}$$

Foreign bonds carry no convenience yield in our baseline specification, so i_t^* in (F.48) is unaffected by Φ_t^* directly; convenience enters only indirectly through the equilibrium exchange rate and the risk-sharing condition.

Taylor-rule substitution. Combining (50) with (47) replaces $i_t - \pi_t$ with $r_t^n + (\phi_\pi - 1)\hat{\pi}_t + \phi_x x_t$, yielding (52). The analogous substitution of (51) into (F.49) yields (53). Evaluated just before issuance (B_{t-} predetermined), each equation pins down the contemporaneous price level. Under passive monetary–active fiscal regimes, P_t (respectively P_t^*) jumps to satisfy fiscal solvency, with the size of the jump governed by the present value of expected surpluses discounted at the natural rate.

G Further details on the quantitative model

Parameter	Distribution	Mean	Std. Dev.
Policy and Behavioral Parameters			
ϕ_π	Uniform	0.50	0.7217
ϕ_π^*	Uniform	0.50	0.7217
ϕ_x	Uniform	0.00	0.0866
ϕ_x^*	Uniform	0.00	0.0866
Debt Adjustment Parameters			
ψ_b	Uniform	0.00	0.0500
ψ_b^*	Uniform	0.00	0.0500
Convenience Yield Parameters			
ζ_H	Uniform	50.00	28.8646
ζ_F	Uniform	50.00	28.8646
$\bar{\Psi}$	Uniform	50.00	28.8646
$\bar{\Psi}^*$	Uniform	50.00	28.8646
Shock Persistence Parameters			
ρ_θ	Beta	0.50	0.18
$\rho_{\hat{g}}$	Beta	0.85	0.08
$\rho_{\hat{g}}^*$	Beta	0.85	0.08
ρ_{u_s}	Beta	0.50	0.18
$\rho_{u_s}^*$	Beta	0.50	0.18
Shock Standard Deviations			

σ_θ	Inverse Gamma	0.10	2.00
$\sigma_{\hat{g}}$	Inverse Gamma	0.10	2.00
$\sigma^*_{\hat{g}}$	Inverse Gamma	0.10	2.00
σ_{u_s}	Inverse Gamma	0.10	2.00
$\sigma^*_{u_s}$	Inverse Gamma	0.10	2.00

Table G.1: Prior Distributions for the estimation in section 3.9.

H Reducing the dimensionality of the two-county continuous-time model

We now characterise the log-linear dynamics around the symmetric steady state with $\overline{P}_H = \overline{P}_F$, $\overline{\text{tot}} = 1$ and $\bar{Q} = 1$. Hats denote log-deviations from steady state (or gaps in the case of x_t, x^*_t), and bars denote steady-state values. All linearised relations in this subsection are obtained by log-linearising the equilibrium conditions of the model.

Home block. The home IS curve and Taylor rule are

$$\dot{x}_t = (i_t - \pi_t) - r^n_t, \qquad i_t = r^n_t + \bar{\pi} + \phi_\pi(\pi_t - \bar{\pi}) + \phi_x x_t.$$

Substituting the Taylor rule into the IS curve eliminates r^n_t and yields

$$\dot{x}_t = (\phi_\pi - 1)\,\hat{\pi}_t + \phi_x x_t, \qquad \hat{\pi}_t \equiv \pi_t - \bar{\pi}.$$

The home linearised PPI New Keynesian Phillips curve is

$$\bar{\delta}_H\,\hat{\pi}_{Ht} = \kappa\big[\tilde{c}_t + \phi x_t + \gamma\tau_t\big] + \dot{\hat{\pi}}_{Ht}, \qquad \bar{\delta}_H \equiv \bar{i} + \bar{\Phi} - \overline{\dot{Y}_{Ht}/Y_{Ht}} - \bar{\pi}_H,$$

where $\tilde{c}_t \equiv \hat{C}_t - \hat{C}^n_t$ is the consumption gap and $\tau_t \equiv \ln \text{tot}_t - \ln \text{tot}^n_t$ the terms-of-trade gap. Here

$$\kappa \equiv \frac{\varepsilon - 1}{\theta}, \qquad \Phi_t \equiv \frac{\Psi_t}{1 - \nu_t}$$

is the effective (consumption-scaled) convenience yield, and

$$\chi_t \equiv \frac{\dot{\nu}_t}{1 - \nu_t}$$

is the drift in the convenience weight.

Consumption gap equals output gap. With consumption-scaled convenience yields, the home Euler

equations are

$$\frac{\dot{C}_t}{C_t} = i_t - \pi_t - \rho + \Phi_t - \chi_t, \qquad \frac{\dot{C}^n_t}{C^n_t} = r^n_t - \rho + \Phi_t - \chi_t,$$

so that

$$\dot{\tilde{c}}_t \equiv \frac{\dot{C}_t}{C_t} - \frac{\dot{C}^n_t}{C^n_t} = (i_t - \pi_t) - r^n_t = \dot{x}_t.$$

With $\tilde{c}_0 = x_0 = 0$, it follows that $\tilde{c}_t = x_t$ for all t. Hence the home NKPC can be written in solved form as

$$\dot{\hat{\pi}}_{Ht} = \bar{\delta}_H \hat{\pi}_{Ht} - \kappa\big[(1+\phi)x_t + \gamma\tau_t\big].$$

Open economy identities. At a symmetric steady state with $\overline{P}_H = \overline{P}_F$ (hence $\overline{\text{tot}} = 1$), the CPI–PPI wedge satisfies

$$\frac{\partial \ln(P_t/P_{Ht})}{\partial \ln \text{tot}_t}\Big|_{\overline{\text{tot}}=1} = \frac{\gamma}{(1-\gamma)+\gamma} = \gamma.$$

A first-order expansion therefore gives

$$p_t - p_{Ht} \approx \gamma\tau_t \quad \Rightarrow \quad \pi_t - \pi_{Ht} \approx \gamma\dot{\tau}_t.$$

Using $\bar{\pi} = \bar{\pi}_H = 0$,

$$\hat{\pi}_t = \hat{\pi}_{Ht} + \gamma\dot{\tau}_t.$$

By symmetry,

$$\hat{\pi}^*_t = \hat{\pi}^*_{Ft} + \gamma\dot{\tau}^*_t.$$

By the law of one price, $P_{Ht} = E_t P^*_{Ht}$ and $P_{Ft} = E_t P^*_{Ft}$, so

$$\text{tot}_t = \frac{P_{Ft}}{P_{Ht}} = \frac{P^*_{Ft}}{P^*_{Ht}} = (\text{tot}^*_t)^{-1} \quad \Rightarrow \quad \tau_t = -\tau^*_t, \quad \dot{\tau}_t = -\dot{\tau}^*_t.$$

Hence

$$\hat{\pi}^*_t = \hat{\pi}^*_{Ft} - \gamma\dot{\tau}_t.$$

Finally, since $\tau_t = \ln P_{Ft} - \ln P_{Ht}$,

$$\dot{\tau}_t = \pi_{Ft} - \pi_{Ht} = \hat{\pi}_{Ft} - \hat{\pi}_{Ht}$$

(steady-state PPI inflations equal zero). Using $P_{Ft} = E_t P^*_{Ft}$ and zero trend in $\bar{E}$,

$$\hat{\pi}_{Ft} = \dot{\hat{e}}_t + \hat{\pi}^*_{Ft}, \qquad \hat{e}_t \equiv \ln E_t - \ln \bar{E}.$$

Combining these,

$$\dot{\tau}_t = \hat{\pi}_{Ft} - \hat{\pi}_{Ht} = \big(\dot{\hat{e}}_t + \hat{\pi}^*_{Ft}\big) - \hat{\pi}_{Ht} \quad \Rightarrow \quad \dot{\hat{e}}_t = \dot{\tau}_t - \hat{\pi}^*_{Ft} + \hat{\pi}_{Ht}.$$

The real exchange rate $q_t \equiv \hat{e}_t + \hat{p}^*_t - \hat{p}_t$ satisfies

$$\dot{q}_t = \dot{\hat{e}}_t + \hat{\pi}^*_t - \hat{\pi}_t = \big(\dot{\tau}_t - \hat{\pi}^*_{Ft} + \hat{\pi}_{Ht}\big) + \big(\hat{\pi}^*_{Ft} - \gamma\dot{\tau}_t\big) - \big(\hat{\pi}_{Ht} + \gamma\dot{\tau}_t\big) = (1-2\gamma)\dot{\tau}_t.$$

Thus $q_t = (1-2\gamma)\tau_t$ up to a constant (set to zero at the symmetric steady state).

Foreign block. By symmetry, the foreign IS curve is

$$\dot{x}^*_t = (\phi^*_\pi - 1)\,\hat{\pi}^*_t + \phi^*_x x^*_t,$$

and the foreign NKPC is

$$\bar{\delta}_F\,\hat{\pi}^*_{Ft} = \kappa\big[(1+\phi^*)x^*_t + \gamma\tau^*_t\big] + \dot{\hat{\pi}}^*_{Ft}, \qquad \bar{\delta}_F \equiv \bar{i}^* - \overline{\dot{Y}^*_{Ft}/Y^*_{Ft}} - \bar{\pi}^*_F.$$

Using $\tau^*_t = -\tau_t$,

$$\dot{\hat{\pi}}^*_{Ft} = \bar{\delta}_F\,\hat{\pi}^*_{Ft} - \kappa\big[(1+\phi^*)x^*_t - \gamma\tau_t\big].$$

IS Curves Using the CPI–PPI relations,

$$\hat{\pi}_t = \hat{\pi}_{Ht} + \gamma\dot{\tau}_t, \qquad \hat{\pi}^*_t = \hat{\pi}^*_{Ft} - \gamma\dot{\tau}_t,$$

the IS curves become

$$\dot{x}_t = (\phi_\pi - 1)\big(\hat{\pi}_{Ht} + \gamma\dot{\tau}_t\big) + \phi_x x_t, \qquad \dot{x}^*_t = (\phi^*_\pi - 1)\big(\hat{\pi}^*_{Ft} - \gamma\dot{\tau}_t\big) + \phi^*_x x^*_t.$$

Terms of trade dynamics With consumption-scaled convenience yields, UIP linearises to

$$i_t - i^*_t = \frac{\dot{E}_t}{E_t} - \Phi_t \quad \Rightarrow \quad \hat{i}_t - \hat{i}^*_t = \dot{\hat{e}}_t - \hat{\Phi}_t.$$

The linearised Taylor rules are

$$\hat{i}_t = \hat{r}^n_t + \phi_\pi\hat{\pi}_t + \phi_x x_t, \qquad \hat{i}^*_t = \hat{r}^{n*}_t + \phi^*_\pi\hat{\pi}^*_t + \phi^*_x x^*_t.$$

Hence

$$\dot{\hat{e}}_t = (\hat{r}^n_t - \hat{r}^{n*}_t) + \big[\phi_\pi\hat{\pi}_t - \phi^*_\pi\hat{\pi}^*_t\big] + \big[\phi_x x_t - \phi^*_x x^*_t\big] + \hat{\Phi}_t.$$

We now calculate each component of the above expression.
Using the open economy natural rate decompositions,

$$r^n_t = (\rho + g_t) - \Phi_t \;+\; \frac{\phi}{1+\phi}\chi_t \;+\; \mathcal{T}_t \;+\; \mathcal{R}_t, \qquad g_t \equiv \dot{A}_t/A_t,$$

$$r^{n*}_t = (\rho^* + g^*_t) + \frac{\phi^*}{1+\phi^*}\chi^*_t \;+\; \mathcal{T}^*_t \;+\; \mathcal{R}^*_t, \qquad g^*_t \equiv \dot{A}^*_t/A^*_t,$$

we obtain, in deviations from steady state,

$$\hat{r}_t^n = (g_t - \bar{g}) - \hat{\Phi}_t + \frac{\phi}{1+\phi}\chi_t + \mathcal{T}_t + \mathcal{R}_t,$$

$$\hat{r}_t^{n*} = (g_t^* - \bar{g}^*) + \frac{\phi^*}{1+\phi^*}\chi_t^* + \mathcal{T}_t^* + \mathcal{R}_t^*.$$

The $-\hat{\Phi}_t$ term in $\hat{r}_t^n$ cancels the $+\hat{\Phi}_t$ above, so there is *no level convenience term* in $\hat{e}_t$.

At $\overline{\text{tot}} = 1$, the ToT components are

$$\mathcal{T}_t = \lambda_T \dot{\tau}_t, \qquad \lambda_T \equiv \gamma(1-\eta)\left(\frac{\phi}{1+\phi} - \frac{1}{1-\eta}\right),$$

$$\mathcal{T}_t^* = -\lambda_T^* \dot{\tau}_t, \qquad \lambda_T^* \equiv \gamma(1-\eta)\left(\frac{\phi^*}{1+\phi^*} - \frac{1}{1-\eta}\right),$$

so

$$\mathcal{T}_t - \mathcal{T}_t^* = (\lambda_T + \lambda_T^*)\dot{\tau}_t.$$

For risk sharing, linearising around $Q_t = \bar{Q} = 1$ and $\Xi_t = \bar{\Xi} = 0$ gives

$$\mathcal{R}_t = \beta_H\big((1-\eta)\dot{q}_t + \dot{\Xi}_t\big), \qquad \mathcal{R}_t^* = \beta_F\big(-(1-\eta)\dot{q}_t - \dot{\Xi}_t\big),$$

$$\beta_H \equiv \gamma\frac{\phi}{1+\phi}, \qquad \beta_F \equiv \gamma\frac{\phi^*}{1+\phi^*},$$

But note that

$$\mathcal{R}_t^* = -\beta_F\big((1-\eta)\dot{q}_t + \dot{\Xi}_t\big)$$

Hence

$$\mathcal{R}_t - \mathcal{R}_t^* = (\beta_H + \beta_F)\left[(1-\eta)\dot{q}_t + \dot{\Xi}_t\right].$$

Substituting in $q_t = (1-2\gamma)\tau_t$ gives us

$$\mathcal{R}_t - \mathcal{R}_t^* = (\beta_H + \beta_F)\left[(1-\eta)(1-2\gamma)\dot{\tau}_t + \dot{\Xi}_t\right].$$

From the risk-sharing condition,

$$\dot{\Xi}_t = (\rho^* - \rho) + (\chi_t^* - \chi_t),$$

we therefore get

$$\mathcal{R}_t - \mathcal{R}_t^* = (\beta_H + \beta_F)\left[(1-\eta)(1-2\gamma)\dot{\tau}_t + (\rho^* - \rho) + (\chi_t^* - \chi_t)\right].$$

We therefore get that the difference in natural rates is

$$\hat{r}_t^n - \hat{r}_t^{n*} = (g_t - \bar{g}) - (g_t^* - \bar{g}^*) - \hat{\Phi}_t + \frac{\phi}{1+\phi}\chi_t - \frac{\phi^*}{1+\phi^*}\chi_t^* + (\lambda_T + \lambda_T^*)\,\dot{\tau}_t$$
$$+ (\beta_H + \beta_F)\left[(1-\eta)(1-2\gamma)\dot{\tau}_t + (\rho^* - \rho) + (\chi_t^* - \chi_t)\right].$$

Collecting ToT and risk-sharing terms, we obtain the exchange rate dynamics

$$\begin{aligned}\dot{\hat{e}}_t &= (\hat{r}_t^n - \hat{r}_t^{n*}) + \left[\phi_\pi \hat{\pi}_t - \phi_\pi^* \hat{\pi}_t^*\right] + \left[\phi_x x_t - \phi_x^* x_t^*\right] + \hat{\Phi}_t \\ &= \left[(g_t - \bar{g}) - (g_t^* - \bar{g}^*) + \frac{\phi}{1+\phi}\chi_t - \frac{\phi^*}{1+\phi^*}\chi_t^* + (\lambda_T + \lambda_T^*)\,\dot{\tau}_t\right] \\ &\quad + \left[(\beta_H + \beta_F)\left[(1-\eta)(1-2\gamma)\dot{\tau}_t + (\rho^* - \rho) + (\chi_t^* - \chi_t)\right]\right] + \left[\phi_\pi \hat{\pi}_t - \phi_\pi^* \hat{\pi}_t^*\right] + \left[\phi_x x_t - \phi_x^* x_t^*\right].\end{aligned}$$

Now we simplify. Combine the $\dot{\tau}_t$ terms to get:

$$\left[(\lambda_T + \lambda_T^*) + (\beta_H + \beta_F)(1-\eta)(1-2\gamma)\right]\dot{\tau}_t$$

Now combine the χ terms. We have

$$\frac{\phi}{1+\phi}\chi_t - \frac{\phi^*}{1+\phi^*}\chi_t^* + (\beta_H + \beta_F)(\chi_t^* - \chi_t).$$

Grouping coefficients, we get

$$\left(\frac{\phi}{1+\phi} - (\beta_H + \beta_F)\right)\chi_t + \left((\beta_H + \beta_F) - \frac{\phi^*}{1+\phi^*}\right)\chi_t^*$$

In summary, we have that the exchange rate dynamics follow

$$\begin{aligned}\dot{\hat{e}}_t &= (g_t - \bar{g}) - (g_t^* - \bar{g}^*) + (\beta_H + \beta_F)(\rho^* - \rho) \\ &\quad + \left[(\lambda_T + \lambda_T^*) + (\beta_H + \beta_F)(1-\eta)(1-2\gamma)\right]\dot{\tau}_t \\ &\quad + \left(\frac{\phi}{1+\phi} - (\beta_H + \beta_F)\right)\chi_t + \left((\beta_H + \beta_F) - \frac{\phi^*}{1+\phi^*}\right)\chi_t^* \\ &\quad + \left[\phi_\pi \hat{\pi}_t - \phi_\pi^* \hat{\pi}_t^*\right] + \left[\phi_x x_t - \phi_x^* x_t^*\right].\end{aligned} \tag{H.1}$$

Nominal depreciation is driven by (i) relative growth, (ii) relative impatience, (iii) ToT movements through CPI/PPI and real-exchange-rate channels, (iv) liquidity-valuation shocks, and (v) differences in how the two central banks react to inflation and output gaps. Substitute in $\dot{\hat{e}}_t =$

$\dot{\tau}_t - \hat{\pi}^*_{Ft} + \hat{\pi}_{Ht}$:

$$\begin{aligned}\dot{\tau}_t - \hat{\pi}^*_{Ft} + \hat{\pi}_{Ht} = (g_t - \bar{g}) - (g^*_t - \bar{g}^*) + (\beta_H + \beta_F)(\rho^* - \rho) \\ + \Big[(\lambda_T + \lambda^*_T) + (\beta_H + \beta_F)(1-\eta)(1-2\gamma)\Big]\dot{\tau}_t \\ + \left(\frac{\phi}{1+\phi} - (\beta_H + \beta_F)\right)\chi_t + \left((\beta_H + \beta_F) - \frac{\phi^*}{1+\phi^*}\right)\chi^*_t \\ + \left[\phi_\pi \hat{\pi}_t - \phi^*_\pi \hat{\pi}^*_t\right] + \left[\phi_x x_t - \phi^*_x x^*_t\right].\end{aligned}$$

Recall the CPI-PPI identities:

$$\begin{aligned}\hat{\pi}_t &= \hat{\pi}_{Ht} + \gamma\dot{\tau}_t \\ \hat{\pi}^*_t &= \hat{\pi}^*_{Ft} - \gamma\dot{\tau}_t.\end{aligned}$$

So then rewrite the Taylor rule reaction to inflation:

$$\begin{aligned}\phi_\pi \hat{\pi}_t - \phi^*_\pi \hat{\pi}^*_t &= \phi_\pi\left[\hat{\pi}_{Ht} + \gamma\dot{\tau}_t\right] - \phi^*_\pi\left[\hat{\pi}^*_{Ft} - \gamma\dot{\tau}_t\right] \\ &= \phi_\pi \hat{\pi}_{Ht} - \phi^*_\pi \hat{\pi}^*_{Ft} + \gamma(\phi_\pi + \phi^*_\pi)\dot{\tau}_t.\end{aligned}$$

Substituting the rule into the terms-of-trade equation gives

$$\begin{aligned}\dot{\tau}_t - \hat{\pi}^*_{Ft} + \hat{\pi}_{Ht} = (g_t - \bar{g}) - (g^*_t - \bar{g}^*) + (\beta_H + \beta_F)(\rho^* - \rho) \\ + \Big[(\lambda_T + \lambda^*_T) + (\beta_H + \beta_F)(1-\eta)(1-2\gamma)\Big]\dot{\tau}_t \\ + \left(\frac{\phi}{1+\phi} - (\beta_H + \beta_F)\right)\chi_t + \left((\beta_H + \beta_F) - \frac{\phi^*}{1+\phi^*}\right)\chi^*_t \\ + \left[\phi_\pi \hat{\pi}_{Ht} - \phi^*_\pi \hat{\pi}^*_{Ft} + \gamma(\phi_\pi + \phi^*_\pi)\dot{\tau}_t\right] + \left[\phi_x x_t - \phi^*_x x^*_t\right].\end{aligned}$$

Then re-arrange

$$\begin{aligned}\dot{\tau}_t + (1-\phi_\pi)\hat{\pi}_{Ht} - (1-\phi^*_\pi)\hat{\pi}^*_{Ft} = (g_t - \bar{g}) - (g^*_t - \bar{g}^*) + (\beta_H + \beta_F)(\rho^* - \rho) \\ + \Big[(\lambda_T + \lambda^*_T) + (\beta_H + \beta_F)(1-\eta)(1-2\gamma)\Big]\dot{\tau}_t + \gamma(\phi_\pi + \phi^*_\pi)\dot{\tau}_t \\ + \left(\frac{\phi}{1+\phi} - (\beta_H + \beta_F)\right)\chi_t + \left((\beta_H + \beta_F) - \frac{\phi^*}{1+\phi^*}\right)\chi^*_t \\ + \left[\phi_x x_t - \phi^*_x x^*_t\right].\end{aligned}$$

Define the new constants

$$\begin{aligned}\Lambda &\equiv 1 - \Big[(\lambda_T + \lambda^*_T) + (\beta_H + \beta_F)(1-\eta)(1-2\gamma)\Big] - \gamma(\phi_\pi + \phi^*_\pi), \\ g^\Delta_t &\equiv (g_t - \bar{g}) - (g^*_t - \bar{g}^*).\end{aligned}$$

We then have

$$\begin{aligned}\Lambda\dot{\tau}_t + (1-\phi_\pi)\hat{\pi}_{Ht} - (1-\phi^*_\pi)\hat{\pi}^*_{Ft} = g^{\Delta}_t + (\beta_H+\beta_F)(\rho^*-\rho) \\ + \left(\frac{\phi}{1+\phi} - (\beta_H+\beta_F)\right)\chi_t + \left((\beta_H+\beta_F) - \frac{\phi^*}{1+\phi^*}\right)\chi^*_t \\ + \left[\phi_x x_t - \phi^*_x x^*_t\right].\end{aligned}$$

Recall

$$\chi_t \equiv \frac{\dot{\nu}_t}{1-\nu_t}, \qquad \chi^*_t \equiv \frac{\dot{\nu}^*_t}{1-\nu^*_t}.$$

Consider the Home convenience flow:

$$\nu_t = \frac{\bar{\psi}\,\theta_t\,\bar{b}_H}{(1-\zeta_H)\,C_t}\left[\left(\frac{b_{Ht}}{\bar{b}_H}\right)^{1-\zeta_H} - 1\right].$$

At the steady state we set $b_{Ht} = \bar{b}_H$, $\theta_t = \bar{\theta}$, and $C_t = \bar{C}$. Then

$$\left(\frac{b_{Ht}}{\bar{b}_H}\right)^{1-\zeta_H} - 1 = 1^{1-\zeta_H} - 1 = 0.$$

Hence, mechanically,

$$\boxed{\bar{\nu} = 0.}$$

So then around the symmetric steady state, we have $\bar{\nu} = \bar{\nu}^* = 0$. Using

$$\frac{1}{1-\nu_t} = 1 + \nu_t + O(\nu_t^2),$$

we have

$$\chi_t = \dot{\nu}_t\Big(1 + \nu_t + O(\nu_t^2)\Big) = \dot{\nu}_t + O(\nu_t\dot{\nu}_t), \qquad \chi^*_t = \dot{\nu}^*_t\Big(1 + \nu^*_t + O((\nu^*_t)^2)\Big) = \dot{\nu}^*_t + O(\nu^*_t\dot{\nu}^*_t).$$

Since ν_t and $\dot{\nu}_t$ (and likewise $\nu^*_t, \dot{\nu}^*_t$) are first-order objects, the products $\nu_t\dot{\nu}_t$ and $\nu^*_t\dot{\nu}^*_t$ are second order and are dropped. Hence, to first order,

$$\chi_t \approx \dot{\nu}_t, \qquad \chi^*_t \approx \dot{\nu}^*_t.$$

With home consumption-scaled convenience,

$$\nu_t = \frac{\bar{\psi}\,\theta_t\,\bar{b}_H}{(1-\zeta_H)\,C_t}\left[\left(\frac{b_{Ht}}{\bar{b}_H}\right)^{1-\zeta_H} - 1\right].$$

Let $\hat{b}_{Ht} \equiv \ln(b_{Ht}/\bar{b}_H)$. Then by Taylor approximation

$$\left(\frac{b_{Ht}}{\bar{b}_H}\right)^{1-\zeta_H} = \exp\left((1-\zeta_H)\hat{b}_{Ht}\right) \approx 1 + (1-\zeta_H)\hat{b}_{Ht},$$

so then

$$\left[\left(\frac{b_{Ht}}{\bar{b}_H}\right)^{1-\zeta_H} - 1\right] \approx (1-\zeta_H)\hat{b}_{Ht}.$$

So now combining, we have

$$\nu_t \approx \frac{\bar{\psi}\,\bar{\theta}\,\bar{b}_H}{(1-\zeta_H)\,\bar{C}}(1-\zeta_H)\hat{b}_{Ht}.$$

Hence

$$\nu_t \approx \frac{\bar{\psi}\,\bar{\theta}\,\bar{b}_H}{\bar{C}}\,\hat{b}_{Ht}.$$

Differentiating,

$$\dot{\nu}_t \approx \frac{\bar{\psi}\,\bar{\theta}\,\bar{b}_H}{\bar{C}}\,\dot{\hat{b}}_{Ht} \qquad \Longrightarrow \qquad \boxed{\chi_t \approx \frac{\bar{\psi}\,\bar{\theta}\,\bar{b}_H}{\bar{C}}\,\dot{\hat{b}}_{Ht}.}$$

Foreign convenience flow. For Foreign investors holding Home-currency bonds,

$$\nu_t^* = \frac{\bar{\psi}^*\,\theta_t^*\,\bar{b}_H^*}{(1-\zeta_F)\,C_t^*\,\bar{E}}\left\{\left[\left(\frac{b_{Ht}^*}{\bar{b}_H^*}\right)\left(\frac{\bar{E}}{E_t}\right)\right]^{1-\zeta_F} - 1\right\}.$$

Define $\hat{b}_{Ht}^* \equiv \ln(b_{Ht}^*/\bar{b}_H^*)$ and $\hat{e}_t \equiv \ln(E_t/\bar{E})$. Then

$$\left(\frac{b_{Ht}^*}{\bar{b}_H^*}\right)\left(\frac{\bar{E}}{E_t}\right) = \exp(\hat{b}_{Ht}^* - \hat{e}_t),$$

and

$$\left[\left(\frac{b_{Ht}^*}{\bar{b}_H^*}\right)\left(\frac{\bar{E}}{E_t}\right)\right]^{1-\zeta_F} = \exp\left((1-\zeta_F)(\hat{b}_{Ht}^* - \hat{e}_t)\right) \approx 1 + (1-\zeta_F)(\hat{b}_{Ht}^* - \hat{e}_t).$$

Therefore

$$\nu_t^* \approx \frac{\bar{\psi}^*\,\bar{\theta}^*\,\bar{b}_H^*}{\bar{C}^*\,\bar{E}}\,(\hat{b}_{Ht}^* - \hat{e}_t),$$

so

$$\dot{\nu}_t^* \approx \frac{\bar{\psi}^*\,\bar{\theta}^*\,\bar{b}_H^*}{\bar{C}^*\,\bar{E}}\,(\dot{\hat{b}}_{Ht}^* - \dot{\hat{e}}_t) \qquad \Longrightarrow \qquad \chi_t^* \approx \frac{\bar{\psi}^*\,\bar{\theta}^*\,\bar{b}_H^*}{\bar{C}^*\,\bar{E}}\,(\dot{\hat{b}}_{Ht}^* - \dot{\hat{e}}_t).$$

Using the previously derived identity

$$\dot{\hat{e}}_t = \dot{\tau}_t - \hat{\pi}_{Ft}^* + \hat{\pi}_{Ht},$$

we can rewrite

$$\boxed{\chi_t^* \approx \frac{\bar{\psi}^*\,\bar{\theta}^*\,\bar{b}_H^*}{\bar{C}^*\,\bar{E}}\left(\dot{\hat{b}}_{Ht}^* - \dot{\tau}_t + \hat{\pi}_{Ft}^* - \hat{\pi}_{Ht}\right).}$$

Now starting from

$$\begin{aligned}\Lambda\dot{\tau}_t + (1-\phi_\pi)\hat{\pi}_{Ht} - (1-\phi^*_\pi)\hat{\pi}^*_{Ft} = g^\Delta_t + (\beta_H+\beta_F)(\rho^*-\rho) \\ + \left(\frac{\phi}{1+\phi} - (\beta_H+\beta_F)\right)\chi_t + \left((\beta_H+\beta_F) - \frac{\phi^*}{1+\phi^*}\right)\chi^*_t \\ + \left[\phi_x x_t - \phi^*_x x^*_t\right].\end{aligned}$$

and using the first-order approximations above yields

$$\begin{aligned}\Lambda\dot{\tau}_t + (1-\phi_\pi)\hat{\pi}_{Ht} - (1-\phi^*_\pi)\hat{\pi}^*_{Ft} = g^\Delta_t + (\beta_H+\beta_F)(\rho^*-\rho) \\ + \left(\frac{\phi}{1+\phi} - (\beta_H+\beta_F)\right)\frac{\bar{\psi}\,\bar{\theta}\,\bar{b}_H}{\bar{C}}\,\dot{\hat{b}}_{Ht} \\ + \left((\beta_H+\beta_F) - \frac{\phi^*}{1+\phi^*}\right)\frac{\bar{\psi}^*\,\bar{\theta}^*\,\bar{b}^*_H}{\bar{C}^*\,\bar{E}}\left(\dot{\hat{b}}^*_{Ht} - \dot{\tau}_t + \hat{\pi}^*_{Ft} - \hat{\pi}_{Ht}\right) \\ + \left[\phi_x x_t - \phi^*_x x^*_t\right].\end{aligned} \tag{H.2}$$

Debt dynamics Real budget constraints and surplus rates. Real government budget constraints are

$$\dot{b}_t = (i_t - \pi_t)\,b_t - s_t, \qquad \dot{b}^*_t = (i^*_t - \pi^*_t)\,b^*_t - s^*_t,$$

with real debt stocks $b_t \equiv B_t/P_t$ and $b^*_t \equiv B^*_t/P^*_t$. Since primary surpluses s_t, s^*_t need not remain strictly positive off steady state, define *surplus rates* (in interest-rate units) by scaling with steady-state debt:

$$\hat{b}_t \equiv \ln\!\Big(\frac{b_t}{\bar{b}}\Big), \qquad \hat{s}_t \equiv \frac{s_t - \bar{s}}{\bar{b}}, \qquad \hat{s}^*_t \equiv \frac{s^*_t - \bar{s}^*}{\bar{b}^*}, \qquad s_t = \bar{s} + \bar{b}\,\hat{s}_t,\;\; s^*_t = \bar{s}^* + \bar{b}^*\,\hat{s}^*_t,$$

and $\bar{s} = \bar{r}\,\bar{b}$, $\bar{s}^* = \bar{r}^*\,\bar{b}^*$. Fiscal feedback rules are

$$\hat{s}_t = \psi_b\,\hat{b}_t + \upsilon^s_t, \qquad \hat{s}^*_t = \psi^*_b\,\hat{b}^*_t + \upsilon^{s*}_t,$$

where υ^s_t and υ^{s*}_t follow OU processes. Now define the ex-post real rates

$$r^b_t \equiv i_t - \pi_t, \qquad r^{b*}_t \equiv i^*_t - \pi^*_t.$$

Let $(\bar{b}, \bar{r}, \bar{s})$ denote the Home steady state, with

$$\bar{r} \equiv \bar{i} - \bar{\pi}, \qquad \bar{s} = \bar{r}\,\bar{b}.$$

Equivalently

$$b_t = \bar{b}\,e^{\hat{b}_t}, \qquad s_t = \bar{s} + \bar{b}\,\hat{s}_t = \bar{b}(\bar{r} + \hat{s}_t).$$

Analogously for Foreign,

$$b_t^* = \bar{b}^* e^{\hat{b}_t^*}, \qquad s_t^* = \bar{s}^* + \bar{b}^* \hat{s}_t^* = \bar{b}^* (\bar{r}^* + \hat{s}_t^*), \qquad \bar{s}^* = \bar{r}^* \, \bar{b}^*.$$

Differentiate $b_t = \bar{b} e^{\hat{b}_t}$:

$$\dot{b}_t = \frac{d}{dt}\big(\bar{b} e^{\hat{b}_t}\big) = \bar{b} e^{\hat{b}_t} \dot{\hat{b}}_t = b_t \, \dot{\hat{b}}_t.$$

Substitute into the budget constraint and divide by $b_t = \bar{b} e^{\hat{b}_t}$:

$$\begin{aligned} b_t \dot{\hat{b}}_t = r_t^b \, b_t - s_t \quad &\Rightarrow \quad \dot{\hat{b}}_t = r_t^b - \frac{s_t}{b_t} \\ &= r_t^b - \frac{\bar{b}(\bar{r} + \hat{s}_t)}{\bar{b} e^{\hat{b}_t}} = r_t^b - (\bar{r} + \hat{s}_t) e^{-\hat{b}_t}. \end{aligned}$$

Write $r_t^b = \bar{r} + \big(r_t^b - \bar{r}\big)$ and use the first-order expansion $e^{-\hat{b}_t} \approx 1 - \hat{b}_t$. Then

$$\begin{aligned} \dot{\hat{b}}_t &\approx \bar{r} + (r_t^b - \bar{r}) - (\bar{r} + \hat{s}_t)(1 - \hat{b}_t) \\ &= \bar{r} + (r_t^b - \bar{r}) - (\bar{r} + \hat{s}_t) + (\bar{r} + \hat{s}_t)\hat{b}_t \\ &\approx \bar{r} \, \hat{b}_t + (r_t^b - \bar{r}) - \hat{s}_t, \end{aligned}$$

where the product term $\hat{s}_t \, \hat{b}_t$ is second order and dropped. Thus the log-linearised Home debt law is

$$\dot{\hat{b}}_t = \bar{r} \, \hat{b}_t + \big[(i_t - \pi_t) - \bar{r}\big] - \hat{s}_t.$$

By the same steps,

$$\dot{\hat{b}}_t^* = \bar{r}^* \, \hat{b}_t^* + \big[(i_t^* - \pi_t^*) - \bar{r}^*\big] - \hat{s}_t^*.$$

With

$$\hat{s}_t = \psi_b \, \hat{b}_t + \upsilon_t^s, \qquad \hat{s}_t^* = \psi_b^* \, \hat{b}_t^* + \upsilon_t^{s*},$$

substitution yields

$$\dot{\hat{b}}_t = (\bar{r} - \psi_b) \, \hat{b}_t + \big[(i_t - \pi_t) - \bar{r}\big] - \upsilon_t^s, \qquad \dot{\hat{b}}_t^* = (\bar{r}^* - \psi_b^*) \, \hat{b}_t^* + \big[(i_t^* - \pi_t^*) - \bar{r}^*\big] - \upsilon_t^{s*}.$$

With Taylor rules written in real-rate form,

$$(i_t - \pi_t) - \bar{r} = \hat{r}_t^n + (\phi_\pi - 1)\hat{\pi}_t + \phi_x x_t, \qquad (i_t^* - \pi_t^*) - \bar{r}^* = \hat{r}_t^{n*} + (\phi_\pi^* - 1)\hat{\pi}_t^* + \phi_x^* x_t^*,$$

and CPI–PPI relations (around $\overline{\text{tot}} = 1$)

$$\hat{\pi}_t = \hat{\pi}_{Ht} + \gamma \, \dot{\hat{\tau}}_t, \qquad \hat{\pi}_t^* = \hat{\pi}_{Ft}^* - \gamma \, \dot{\hat{\tau}}_t,$$

we obtain

$$\dot{\hat{b}}_t = (\bar{r} - \psi_b)\,\hat{b}_t + \Big[(\phi_\pi - 1)\big(\hat{\pi}_{Ht} + \gamma\dot{\hat{\tau}}_t\big) + \phi_x x_t + \hat{r}_t^{\,n}\Big] - \upsilon_t^{s},$$

$$\dot{\hat{b}}^*_t = (\bar{r}^* - \psi^*_b)\,\hat{b}^*_t + \Big[(\phi^*_\pi - 1)\big(\hat{\pi}^*_{Ft} - \gamma\dot{\hat{\tau}}_t\big) + \phi^*_x x^*_t + \hat{r}_t^{\,n*}\Big] - \upsilon_t^{s*}.$$

Use the natural-rate decompositions

$$\hat{r}_t^{\,n} = (g_t - \bar{g}) - (\Psi_t - \bar{\Psi}) + \frac{\phi}{1+\phi}\chi_t + \mathcal{T}_t + \mathcal{R}_t, \qquad \hat{r}_t^{\,n*} = (g^*_t - \bar{g}^*) + \frac{\phi^*}{1+\phi^*}\chi^*_t + \mathcal{T}^*_t + \mathcal{R}^*_t.$$

Since $\Psi_t > 0$, with $\hat{\Psi}_t \equiv \ln(\Psi_t/\bar{\Psi})$ we have (to first order)

$$\Psi_t - \bar{\Psi} \;\approx\; \bar{\Psi}\,\hat{\Psi}_t,$$

so the Home natural-rate term can be written as $-(\Psi_t - \bar{\Psi}) \approx -\bar{\Psi}\,\hat{\Psi}_t$. Linearised ToT and risk-sharing terms (around $\bar{q} = \overline{\text{tot}} = 1$) are

$$\mathcal{T}_t = \lambda_T\,\dot{\hat{\tau}}_t, \qquad \mathcal{T}^*_t = -\lambda^*_T\,\dot{\hat{\tau}}_t, \qquad \dot{\hat{q}}_t = (1 - 2\gamma)\dot{\hat{\tau}}_t,$$

and using the sign convention from earlier sections,

$$\mathcal{R}_t = \beta_H\big((1-\eta)\dot{\hat{q}}_t + \dot{\Xi}_t\big), \qquad \mathcal{R}^*_t = -\beta_F\big((1-\eta)\dot{\hat{q}}_t + \dot{\Xi}_t\big), \qquad \dot{\Xi}_t = (\rho^* - \rho) + \chi^*_t - \chi_t.$$

Hence

$$\mathcal{T}_t + \mathcal{R}_t = \big[\lambda_T + \beta_H(1-\eta)(1-2\gamma)\big]\dot{\hat{\tau}}_t + \beta_H\dot{\Xi}_t,$$

$$\mathcal{T}^*_t + \mathcal{R}^*_t = \big[-\lambda^*_T - \beta_F(1-\eta)(1-2\gamma)\big]\dot{\hat{\tau}}_t - \beta_F\dot{\Xi}_t.$$

Substituting into the debt evaluation equation and collecting $\dot{\hat{\tau}}_t$ terms gives

$$\dot{\hat{b}}_t = (\bar{r} - \psi_b)\,\hat{b}_t + \Big[(\phi_\pi - 1)\hat{\pi}_{Ht} + \phi_x x_t + (g_t - \bar{g}) - \bar{\Psi}\,\hat{\Psi}_t + \tfrac{\phi}{1+\phi}\chi_t + \beta_H\dot{\Xi}_t + \xi_H\,\dot{\hat{\tau}}_t\Big] - \upsilon_t^{s},$$

$$\dot{\hat{b}}^*_t = (\bar{r}^* - \psi^*_b)\,\hat{b}^*_t + \Big[(\phi^*_\pi - 1)\hat{\pi}^*_{Ft} + \phi^*_x x^*_t + (g^*_t - \bar{g}^*) + \tfrac{\phi^*}{1+\phi^*}\chi^*_t - \beta_F\dot{\Xi}_t + \xi^*_F\,\dot{\hat{\tau}}_t\Big] - \upsilon_t^{s*}.$$

Recall the convenience wedge was

$$\Psi_t = \bar{\Psi}\,\theta_t\Big(\tfrac{b_{Ht}}{\bar{b}_H}\Big)^{-\zeta_H}.$$

Taking log deviations around $\bar{\theta} = 1$ gives the percent-deviation form

$$\hat{\Psi}_t = \hat{\theta}_t - \zeta_H\,\hat{b}_{Ht}, \qquad \hat{\theta}_t \equiv \ln(\theta_t/\bar{\theta}), \;\; \hat{b}_{Ht} \equiv \ln(b_{Ht}/\bar{b}_H).$$

Then use the definition of (log)-linearisation

$$-(\Psi_t - \bar{\Psi}) \approx -\bar{\Psi}\hat{\Psi}_t = -\bar{\Psi}\hat{\theta}_t + \bar{\Psi}\zeta_H\hat{b}_{Ht}.$$

So then we have

$$\dot{\hat{b}}_t = (\bar{r} - \psi_b)\,\hat{b}_t + \Big[(\phi_\pi - 1)\hat{\pi}_{Ht} + \phi_x x_t + \hat{g}_t - \bar{\Psi}\hat{\theta}_t + \bar{\Psi}\zeta_H \hat{b}_{Ht} + \tfrac{\phi}{1+\phi}\chi_t + \beta_H \dot{\Xi}_t + \xi_H\,\dot{\hat{\tau}}_t\Big] - \upsilon_t^s,$$

$$\dot{\hat{b}}^*_t = (\bar{r}^* - \psi^*_b)\,\hat{b}^*_t + \Big[(\phi^*_\pi - 1)\hat{\pi}^*_{Ft} + \phi^*_x x^*_t + \hat{g}^*_t + \tfrac{\phi^*}{1+\phi^*}\chi^*_t - \beta_F \dot{\Xi}_t + \xi^*_F\,\dot{\hat{\tau}}_t\Big] - \upsilon_t^{s*}.$$

Now recall

$$\dot{\Xi}_t = (\rho^* - \rho) + (\chi^*_t - \chi_t).$$

Substituting this in yields

$$\begin{aligned}\dot{\hat{b}}_t = (\bar{r} - \psi_b)\,\hat{b}_t + \Big[&(\phi_\pi - 1)\hat{\pi}_{Ht} + \phi_x x_t + \hat{g}_t - \bar{\Psi}\hat{\theta}_t + \bar{\Psi}\zeta_H \hat{b}_{Ht} + \tfrac{\phi}{1+\phi}\chi_t + \beta_H(\rho^* - \rho) + \beta_H(\chi^*_t - \chi_t)\\ &+ \xi_H\,\dot{\hat{\tau}}_t\Big] - \upsilon_t^s,\end{aligned}$$

$$\begin{aligned}\dot{\hat{b}}^*_t = (\bar{r}^* - \psi^*_b)\,\hat{b}^*_t + \Big[&(\phi^*_\pi - 1)\hat{\pi}^*_{Ft} + \phi^*_x x^*_t + \hat{g}^*_t + \tfrac{\phi^*}{1+\phi^*}\chi^*_t - \beta_F(\rho^* - \rho) - \beta_F(\chi^*_t - \chi_t)\\ &+ \xi^*_F\,\dot{\hat{\tau}}_t\Big] - \upsilon_t^{s*}.\end{aligned}$$

Collecting χ_t, χ^*_t terms gives

$$\begin{aligned}\dot{\hat{b}}_t = (\bar{r} - \psi_b)\,\hat{b}_t + \Big[&(\phi_\pi - 1)\hat{\pi}_{Ht} + \phi_x x_t + (g_t - \bar{g}) - \bar{\Psi}\hat{\theta}_t + \bar{\Psi}\zeta_H \hat{b}_{Ht} + \beta_H(\rho^* - \rho)\\ &+ \Big(\tfrac{\phi}{1+\phi} - \beta_H\Big)\chi_t + \beta_H\chi^*_t + \xi_H\,\dot{\hat{\tau}}_t\Big] - \upsilon_t^s,\end{aligned}$$

$$\begin{aligned}\dot{\hat{b}}^*_t = (\bar{r}^* - \psi^*_b)\,\hat{b}^*_t + \Big[&(\phi^*_\pi - 1)\hat{\pi}^*_{Ft} + \phi^*_x x^*_t + (g^*_t - \bar{g}^*) - \beta_F(\rho^* - \rho)\\ &+ \beta_F\chi_t + \Big(\tfrac{\phi^*}{1+\phi^*} - \beta_F\Big)\chi^*_t + \xi^*_F\,\dot{\hat{\tau}}_t\Big] - \upsilon_t^{s*}.\end{aligned}$$

Now recall we derived

$$\begin{aligned}\chi_t &= \frac{\bar{\psi}\,\bar{\theta}\,\bar{b}_H}{\bar{C}}\,\dot{\hat{b}}_{Ht},\\ \chi^*_t &= \frac{\bar{\psi}^*\,\bar{\theta}^*\,\bar{b}^*_H}{\bar{C}^*\,\bar{E}}\,\big(\dot{\hat{b}}^*_{Ht} - \dot{\tau}_t + \hat{\pi}^*_{Ft} - \hat{\pi}_{Ht}\big).\end{aligned}$$

Substitute this in to get the final expression:

$$\begin{aligned}\dot{\hat{b}}_t = (\bar{r} - \psi_b)\,\hat{b}_t + \Big[&\Big((\phi_\pi - 1) - \beta_H\frac{\bar{\psi}^*\,\bar{\theta}^*\,\bar{b}^*_H}{\bar{C}^*\,\bar{E}}\Big)\hat{\pi}_{Ht} + \beta_H\frac{\bar{\psi}^*\,\bar{\theta}^*\,\bar{b}^*_H}{\bar{C}^*\,\bar{E}}\,\hat{\pi}^*_{Ft} + \phi_x x_t + (g_t - \bar{g}) - \bar{\Psi}\hat{\theta}_t + \bar{\Psi}\zeta_H\hat{b}_{Ht}\\ &+ \beta_H(\rho^* - \rho) + \Big(\tfrac{\phi}{1+\phi} - \beta_H\Big)\frac{\bar{\psi}\,\bar{\theta}\,\bar{b}_H}{\bar{C}}\,\dot{\hat{b}}_{Ht} + \beta_H\frac{\bar{\psi}^*\,\bar{\theta}^*\,\bar{b}^*_H}{\bar{C}^*\,\bar{E}}\,\dot{\hat{b}}^*_{Ht} + \Big(\xi_H - \beta_H\frac{\bar{\psi}^*\,\bar{\theta}^*\,\bar{b}^*_H}{\bar{C}^*\,\bar{E}}\Big)\dot{\hat{\tau}}_t\Big] - \upsilon_t^s,\end{aligned} \tag{H.3}$$

$$\begin{aligned}\dot{\hat{b}}^*_t = (\bar{r}^* - \psi^*_b)\,\hat{b}^*_t + \Big[&\Big((\phi^*_\pi - 1) + \Big(\tfrac{\phi^*}{1+\phi^*} - \beta_F\Big)\frac{\bar{\psi}^*\,\bar{\theta}^*\,\bar{b}^*_H}{\bar{C}^*\,\bar{E}}\Big)\hat{\pi}^*_{Ft} - \Big(\tfrac{\phi^*}{1+\phi^*} - \beta_F\Big)\frac{\bar{\psi}^*\,\bar{\theta}^*\,\bar{b}^*_H}{\bar{C}^*\,\bar{E}}\,\hat{\pi}_{Ht} + \phi^*_x x^*_t + (g^*_t - \bar{g}^*)\\ &- \beta_F(\rho^* - \rho) + \beta_F\frac{\bar{\psi}\,\bar{\theta}\,\bar{b}_H}{\bar{C}}\,\dot{\hat{b}}_{Ht} + \Big(\tfrac{\phi^*}{1+\phi^*} - \beta_F\Big)\frac{\bar{\psi}^*\,\bar{\theta}^*\,\bar{b}^*_H}{\bar{C}^*\,\bar{E}}\,\dot{\hat{b}}^*_{Ht} + \Big(\xi^*_F - \Big(\tfrac{\phi^*}{1+\phi^*} - \beta_F\Big)\frac{\bar{\psi}^*\,\bar{\theta}^*\,\bar{b}^*_H}{\bar{C}^*\,\bar{E}}\Big)\dot{\hat{\tau}}_t\Big] - \upsilon_t^{s*}.\end{aligned} \tag{H.4}$$

Isoelastic convenience yields The convenience wedge in the Euler equation is

$$\Psi_t \equiv \nu_B(B_{Ht};\theta_t)\, P_t C_t, \qquad b_{Ht} \equiv \frac{B_{Ht}}{P_t}.$$

Assume

$$\nu(B_{Ht};\theta_t) = \frac{\bar{\Psi}\,\theta_t\,\bar{b}_H}{(1-\zeta_H)\,C_t}\left[\left(\tfrac{b_{Ht}}{\bar{b}_H}\right)^{1-\zeta_H} - 1\right], \qquad (\zeta_H \neq 1),$$

which implies

$$\Psi_t = \bar{\Psi}\,\theta_t\left(\tfrac{b_{Ht}}{\bar{b}_H}\right)^{-\zeta_H}.$$

Taking log deviations around $\bar{\theta} = 1$ gives the percent-deviation form

$$\hat{\Psi}_t = \hat{\theta}_t - \zeta_H\,\hat{b}_{Ht}, \qquad \hat{\theta}_t \equiv \ln(\theta_t/\bar{\theta}), \;\; \hat{b}_{Ht} \equiv \ln(b_{Ht}/\bar{b}_H).$$

Foreign agents derive convenience from *Home-currency* bonds B^*_{Ht}. Let $b^*_{Ht} \equiv B^*_{Ht}/P^*_t$, $\hat{b}^*_{Ht} \equiv \ln(b^*_{Ht}/\bar{b}^*_H)$ and $\hat{e}_t \equiv \ln(E_t/\bar{E})$, and assume

$$\Psi^*_t = \bar{\Psi}^*\,\theta^*_t\left[\left(\tfrac{b^*_{Ht}}{\bar{b}^*_H}\right)\left(\tfrac{\bar{E}}{E_t}\right)\right]^{-\zeta_F}.$$

Hence, in percent (log) deviations,

$$\hat{\Psi}^*_t = \hat{\theta}^*_t - \zeta_F\big(\hat{b}^*_{Ht} - \hat{e}_t\big).$$

Bond–market clearing Market clearing for home-currency denominated debt is

$$B_t = B_{Ht} + B^*_{Ht} \quad \Rightarrow \quad b_t \equiv \frac{B_t}{P_t} = b_{Ht} + \left(\tfrac{P^*_t}{P_t}\right) b^*_{Ht} = b_{Ht} + \left(\tfrac{Q_t}{E_t}\right) b^*_{Ht},$$

so, in log deviations around $\bar{Q} = 1$,

$$\hat{b}_t = \omega_H\,\hat{b}_{Ht} \;+\; \omega_F\,\left(\hat{b}^*_{Ht} + \hat{q}_t - \hat{e}_t\right), \qquad \omega_H \equiv \frac{\bar{b}_{Ht}}{\bar{b}_t}, \quad \omega_F \equiv 1 - \omega_H,$$

where $\hat{q}_t \equiv \ln(Q_t/\bar{Q})$. Then plug in

$$q_t = (1-2\gamma)\tau_t,$$

to get

$$\hat{b}_t = \omega_H\,\hat{b}_{Ht} \;+\; \omega_F\,\left(\hat{b}^*_{Ht} + (1-2\gamma)\tau_t - \hat{e}_t\right). \tag{H.5}$$

No-arbitrage with effective wedges. The frictionless arbitrage condition requires that *effective* wedges are equalised across borders:

$$\frac{\Psi_t}{1-\nu_t} = \frac{\Psi^*_t}{1-\nu^*_t}.$$

Define the effective wedge (Home and Foreign)

$$\widetilde{\Psi}_t \;\equiv\; \frac{\Psi_t}{1-\nu_t}, \qquad \widetilde{\Psi}^*_t \;\equiv\; \frac{\Psi^*_t}{1-\nu^*_t}.$$

Then no-arbitrage is simply $\widetilde{\Psi}_t = \widetilde{\Psi}^*_t$, equivalently

$$\ln \widetilde{\Psi}_t = \ln \widetilde{\Psi}^*_t, \qquad \ln \widetilde{\Psi}_t = \ln \Psi_t - \ln(1-\nu_t), \quad \ln \widetilde{\Psi}^*_t = \ln \Psi^*_t - \ln(1-\nu^*_t).$$

Log-linearisation. Using $\ln \widetilde{\Psi}_t = \ln \Psi_t - \ln(1-\nu_t)$, we can write

$$\begin{aligned}\hat{\widetilde{\Psi}}_t \equiv \ln\Big(\frac{\widetilde{\Psi}_t}{\bar{\widetilde{\Psi}}}\Big) &= \ln\Big(\frac{\Psi_t}{\bar{\Psi}}\Big) - \Big[\ln(1-\nu_t) - \ln(1-\bar{\nu})\Big] \\ &= \hat{\Psi}_t \;-\; \Big[\ln(1-\nu_t) - \ln(1-\bar{\nu})\Big],\end{aligned}$$

and analogously for Foreign.

Next, take a first-order Taylor expansion of $\ln(1-\nu_t)$ around $\bar{\nu}$:

$$\ln(1-\nu_t) \;\approx\; \ln(1-\bar{\nu}) \;+\; \frac{\partial \ln(1-\nu)}{\partial \nu}\Big|_{\bar{\nu}} (\nu_t - \bar{\nu}) \;=\; \ln(1-\bar{\nu}) \;-\; \frac{\nu_t - \bar{\nu}}{1-\bar{\nu}}.$$

Substituting into the expression for $\hat{\widetilde{\Psi}}_t$ gives

$$\hat{\widetilde{\Psi}}_t \;\approx\; \hat{\Psi}_t \;+\; \frac{\nu_t - \bar{\nu}}{1-\bar{\nu}}, \qquad \hat{\widetilde{\Psi}}^*_t \;\approx\; \hat{\Psi}^*_t \;+\; \frac{\nu^*_t - \bar{\nu}^*}{1-\bar{\nu}^*}.$$

Imposing $\hat{\widetilde{\Psi}}_t = \hat{\widetilde{\Psi}}^*_t$ yields the linearised no-arbitrage condition

$$\hat{\Psi}_t + \frac{\nu_t - \bar{\nu}}{1-\bar{\nu}} \;=\; \hat{\Psi}^*_t + \frac{\nu^*_t - \bar{\nu}^*}{1-\bar{\nu}^*}. \tag{$*$}$$

Convenience-flow linearisation and portfolio-weight coefficients. To avoid notational confusion, we keep ν_t and ν^*_t for the *convenience flows* (the objects entering $1-\nu_t$ and $1-\nu^*_t$), and we introduce separate notation for the local *portfolio-weight* coefficients that govern how these flows move with bond positions.

Start from the isoelastic convenience-flow specification (Home):

$$\nu_t \;=\; \frac{\bar{\Psi}\,\theta_t\,\bar{b}_H}{(1-\zeta_H)\,C_t}\left[\left(\frac{b_{Ht}}{\bar{b}_H}\right)^{1-\zeta_H} - 1\right], \qquad \hat{b}_{Ht} \equiv \ln\Big(\frac{b_{Ht}}{\bar{b}_H}\Big).$$

Rewrite the power term using exponentials:

$$\left(\frac{b_{Ht}}{\bar{b}_H}\right)^{1-\zeta_H} = \exp\big((1-\zeta_H)\hat{b}_{Ht}\big).$$

Then

$$\nu_t = \frac{\bar{\Psi}\,\theta_t\,\bar{b}_H}{(1-\zeta_H)\,C_t}\Big[\exp\big((1-\zeta_H)\hat{b}_{Ht}\big)-1\Big].$$

Using the first-order approximation $\exp(x)-1 \approx x$, we obtain

$$\nu_t \;\approx\; \frac{\bar{\Psi}\,\theta_t\,\bar{b}_H}{(1-\zeta_H)\,C_t}\,(1-\zeta_H)\hat{b}_{Ht} \;=\; \frac{\bar{\Psi}\,\theta_t\,\bar{b}_H}{C_t}\,\hat{b}_{Ht}.$$

Because $\hat{b}_{Ht}$ is already first-order, any first-order variation in the prefactor $\bar{\Psi}\,\theta_t/C_t$ would enter only through products like $\hat{\theta}_t\,\hat{b}_{Ht}$ or $\hat{C}_t\,\hat{b}_{Ht}$, which are second-order and therefore dropped. Hence we evaluate the prefactor at steady state and define the *Home portfolio-weight coefficient*

$$\kappa_H \;\equiv\; \frac{\bar{\Psi}\,\bar{\theta}\,\bar{b}_H}{\bar{C}}.$$

Under the normalisation in the isoelastic form, $\bar{\nu}=0$, so the first-order mapping is

$$\nu_t - \bar{\nu} \;=\; \nu_t \;\approx\; \kappa_H\,\hat{b}_{Ht}. \tag{**}$$

For Foreign investors holding Home bonds, we first define the *Foreign portfolio-weight coefficient*

$$\kappa_F \;\equiv\; \frac{\bar{\Psi}^*\,\bar{\theta}^*\,\bar{b}^*_H}{\bar{C}^*\,\bar{E}^*},$$

and since $\bar{\nu}^*=0$ under the same normalisation,

$$\nu^*_t - \bar{\nu}^* \;=\; \nu^*_t \;\approx\; \kappa_F\,(\hat{b}^*_{Ht} - \hat{e}_t). \tag{***}$$

Substituting into no-arbitrage. Using (**)–(***) in (*), and using $\bar{\nu}=\bar{\nu}^*=0$ (which makes $1-\bar{\nu}=1-\bar{\nu}^*=1$ exactly), the linearised no-arbitrage condition becomes

$$\hat{\Psi}_t + \kappa_H\,\hat{b}_{Ht} \;=\; \hat{\Psi}^*_t + \kappa_F\,(\hat{b}^*_{Ht} - \hat{e}_t). \tag{****}$$

Eliminating $\hat{\Psi}_t$ and $\hat{\Psi}^*_t$ using bond demands. The isoelastic bond-demand schedules imply

$$\Psi_t = \bar{\Psi}\,\theta_t\left(\frac{b_{Ht}}{\bar{b}_H}\right)^{-\zeta_H}, \qquad \Psi^*_t = \bar{\Psi}^*\,\theta^*_t\left[\left(\frac{b^*_{Ht}}{\bar{b}^*_H}\right)\left(\frac{\bar{E}^*}{E_t}\right)\right]^{-\zeta_F}.$$

Taking logs and subtracting steady-state logs yields

$$\hat{\Psi}_t = \hat{\theta}_t - \zeta_H\,\hat{b}_{Ht}, \qquad \hat{\Psi}^*_t = \hat{\theta}^*_t - \zeta_F\,(\hat{b}^*_{Ht} - \hat{e}_t).$$

Substituting these into (****) and collecting terms gives

$$\hat{\theta}_t - (\zeta_H - \kappa_H)\hat{b}_{Ht} \;=\; \hat{\theta}^*_t - (\zeta_F - \kappa_F)\,(\hat{b}^*_{Ht} - \hat{e}_t).$$

Finally, substituting the definitions of the portfolio-weight coefficients $\kappa_H = \bar{\Psi}\,\bar{\theta}\,\bar{b}_H/\bar{C}$ and $\kappa_F = \bar{\Psi}^*\,\bar{\theta}^*\,\bar{b}^*_H/(\bar{C}^*\,\bar{E}^*)$ delivers the stated reduced-form no-arbitrage condition:

$$\hat{\theta}_t - \left(\zeta_H - \frac{\bar{\Psi}\,\bar{\theta}\,\bar{b}_H}{\bar{C}}\right)\hat{b}_{Ht} \;=\; \hat{\theta}^*_t - \left(\zeta_F - \frac{\bar{\Psi}^*\,\bar{\theta}^*\,\bar{b}^*_H}{\bar{C}^*\,\bar{E}^*}\right)(\hat{b}^*_{Ht} - \hat{e}_t). \tag{H.6}$$

H.1 Equilibrium System

The log-linearised competitive equilibrium consists of the ten endogenous variables $\{x_t,\, x^*_t,\, \hat{\pi}_{Ht},\, \hat{\pi}^*_{Ft},\, \tau_t,\, \hat{b}_t,\, \hat{b}^*_t,\, \hat{b}_{Ht},\, \hat{b}^*_{Ht},\, \hat{e}_t\}$ satisfying the following system, driven by the exogenous processes $\{\hat{g}_t,\, \hat{g}^*_t,\, \hat{\theta}_t,\, \hat{\theta}^*_t,\, \upsilon^s_t,\, \upsilon^{s*}_t\}$. Composite parameter definitions are in Section H.2 below.

IS curves.

$$\dot{x}_t = (\phi_\pi - 1)\big(\hat{\pi}_{Ht} + \gamma\dot{\tau}_t\big) + \phi_x x_t, \tag{H.7}$$

$$\dot{x}^*_t = (\phi^*_\pi - 1)\big(\hat{\pi}^*_{Ft} - \gamma\dot{\tau}_t\big) + \phi^*_x x^*_t. \tag{H.8}$$

New Keynesian Phillips curves.

$$\dot{\hat{\pi}}_{Ht} = \bar{\delta}_H\,\hat{\pi}_{Ht} - \kappa\big[(1+\phi)x_t + \gamma\tau_t\big], \tag{H.9}$$

$$\dot{\hat{\pi}}^*_{Ft} = \bar{\delta}_F\,\hat{\pi}^*_{Ft} - \kappa\big[(1+\phi^*)x^*_t - \gamma\tau_t\big], \tag{H.10}$$

where

$$\kappa \equiv \frac{\varepsilon - 1}{\theta}, \qquad \bar{\delta}_H \equiv \bar{\imath} + \bar{\Phi} - \overline{\dot{Y}_{Ht}/Y_{Ht}} - \bar{\pi}_H, \qquad \bar{\delta}_F \equiv \bar{\imath}^* - \overline{\dot{Y}^*_{Ft}/Y^*_{Ft}} - \bar{\pi}^*_F.$$

Terms-of-trade dynamics.

$$\begin{aligned}\Lambda\dot{\tau}_t + (1-\phi_\pi)\hat{\pi}_{Ht} - (1-\phi^*_\pi)\hat{\pi}^*_{Ft} = {}& g^\Delta_t + (\beta_H + \beta_F)(\rho^* - \rho) \\ &+ \left(\tfrac{\phi}{1+\phi} - (\beta_H + \beta_F)\right)\tfrac{\bar{\Psi}\bar{\theta}\bar{b}_H}{\bar{C}}\,\dot{\hat{b}}_{Ht} \\ &+ \left((\beta_H + \beta_F) - \tfrac{\phi^*}{1+\phi^*}\right)\tfrac{\bar{\Psi}^*\bar{\theta}^*\bar{b}^*_H}{\bar{C}^*\bar{E}}\big(\dot{\hat{b}}^*_{Ht} - \dot{\tau}_t + \dot{\hat{\pi}}^*_{Ft} - \dot{\hat{\pi}}_{Ht}\big) \\ &+ \phi_x x_t - \phi^*_x x^*_t.\end{aligned} \tag{H.11}$$

Debt dynamics.

$$\begin{aligned}\dot{\hat{b}}_t &= (\bar{r}-\psi_b)\hat{b}_t \\ &\quad + \Big[\Big((\phi_\pi-1)-\beta_H\tfrac{\bar{\psi}^*\bar{\theta}^*\bar{b}^*_H}{\bar{C}^*\bar{E}}\Big)\hat{\pi}_{Ht}+\beta_H\tfrac{\bar{\psi}^*\bar{\theta}^*\bar{b}^*_H}{\bar{C}^*\bar{E}}\hat{\pi}^*_{Ft}+\phi_x x_t+(g_t-\bar{g})-\bar{\Psi}\hat{\theta}_t+\bar{\Psi}\zeta_H\hat{b}_{Ht} \\ &\quad\quad +\beta_H(\rho^*-\rho)+\Big(\tfrac{\phi}{1+\phi}-\beta_H\Big)\tfrac{\bar{\psi}\bar{\theta}\bar{b}_H}{\bar{C}}\dot{\hat{b}}_{Ht}+\beta_H\tfrac{\bar{\psi}^*\bar{\theta}^*\bar{b}^*_H}{\bar{C}^*\bar{E}}\dot{\hat{b}}^*_{Ht} \\ &\quad\quad +\Big(\xi_H-\beta_H\tfrac{\bar{\psi}^*\bar{\theta}^*\bar{b}^*_H}{\bar{C}^*\bar{E}}\Big)\dot{\hat{\tau}}_t\Big]-\upsilon^s_t, \end{aligned} \tag{H.12}$$

$$\begin{aligned}\dot{\hat{b}}^*_t &= (\bar{r}^*-\psi^*_b)\hat{b}^*_t \\ &\quad + \Big[\Big((\phi^*_\pi-1)+\Big(\tfrac{\phi^*}{1+\phi^*}-\beta_F\Big)\tfrac{\bar{\psi}^*\bar{\theta}^*\bar{b}^*_H}{\bar{C}^*\bar{E}}\Big)\hat{\pi}^*_{Ft}-\Big(\tfrac{\phi^*}{1+\phi^*}-\beta_F\Big)\tfrac{\bar{\psi}^*\bar{\theta}^*\bar{b}^*_H}{\bar{C}^*\bar{E}}\hat{\pi}_{Ht}+\phi^*_x x^*_t \\ &\quad\quad +(g^*_t-\bar{g}^*)-\beta_F(\rho^*-\rho)+\beta_F\tfrac{\bar{\psi}\bar{\theta}\bar{b}_H}{\bar{C}}\dot{\hat{b}}_{Ht}+\Big(\tfrac{\phi^*}{1+\phi^*}-\beta_F\Big)\tfrac{\bar{\psi}^*\bar{\theta}^*\bar{b}^*_H}{\bar{C}^*\bar{E}}\dot{\hat{b}}^*_{Ht} \\ &\quad\quad +\Big(\xi^*_F-\Big(\tfrac{\phi^*}{1+\phi^*}-\beta_F\Big)\tfrac{\bar{\psi}^*\bar{\theta}^*\bar{b}^*_H}{\bar{C}^*\bar{E}}\Big)\dot{\hat{\tau}}_t\Big]-\upsilon^{s*}_t. \end{aligned} \tag{H.13}$$

Nominal exchange rate.

$$\dot{\hat{e}}_t = \dot{\tau}_t-\hat{\pi}^*_{Ft}+\hat{\pi}_{Ht}. \tag{H.14}$$

Bond-market clearing.

$$\hat{b}_t = \omega_H\hat{b}_{Ht}+\omega_F\big(\hat{b}^*_{Ht}+(1-2\gamma)\hat{\tau}_t-\hat{e}_t\big). \tag{H.15}$$

No-arbitrage condition.

$$\hat{\theta}_t-\left(\zeta_H-\frac{\bar{\Psi}\bar{\theta}\bar{b}_H}{\bar{C}}\right)\hat{b}_{Ht}=\hat{\theta}^*_t-\left(\zeta_F-\frac{\bar{\Psi}^*\bar{\theta}^*\bar{b}^*_H}{\bar{C}^*\bar{E}^*}\right)\big(\hat{b}^*_{Ht}-\hat{e}_t\big). \tag{H.16}$$

H.2 Parameter Definitions

Terms-of-trade and risk-sharing coefficients.

$$\lambda_T\equiv\gamma(1-\eta)\Big(\tfrac{\phi}{1+\phi}-\tfrac{1}{1-\eta}\Big), \qquad \lambda^*_T\equiv\gamma(1-\eta)\Big(\tfrac{\phi^*}{1+\phi^*}-\tfrac{1}{1-\eta}\Big),$$

$$\beta_H\equiv\gamma\,\tfrac{\phi}{1+\phi}, \qquad \beta_F\equiv\gamma\,\tfrac{\phi^*}{1+\phi^*}.$$

λ_T and λ^*_T govern the response of relative prices to expenditure switching, while β_H and β_F capture the labour-supply-weighted share of international expenditure in each country's natural rate.

Debt-equation slope coefficients.

$$\xi_H \equiv \gamma(\phi_\pi - 1) + \lambda_T + \beta_H(1-\eta)(1-2\gamma),$$
$$\xi_F^* \equiv -\gamma(\phi_\pi^* - 1) - \lambda_T^* - \beta_F(1-\eta)(1-2\gamma).$$

These collect the net effect of terms-of-trade movements on each country's real debt via its impact on the real discount rate.

Terms-of-trade inertia coefficient.

$$\Lambda \equiv 1 - \left[(\lambda_T + \lambda_T^*) + (\beta_H + \beta_F)(1-\eta)(1-2\gamma)\right] - \gamma(\phi_\pi + \phi_\pi^*).$$

Λ scales the speed of terms-of-trade adjustment; the condition $\Lambda > 0$ is sufficient for the ToT equation (H.11) to be well-posed.

TFP growth differential.

$$g_t^\Delta \equiv (g_t - \bar{g}) - (g_t^* - \bar{g}^*).$$

Convenience-yield portfolio composites.

$$\Omega_H \equiv \frac{\omega_H}{\zeta_H}, \qquad \Omega_F \equiv \frac{\omega_F}{\zeta_F}, \qquad \varpi \equiv \left(\frac{\omega_H}{\zeta_H} + \frac{\omega_F}{\zeta_F}\right)^{-1}.$$

Ω_H and Ω_F weight the two convenience-yield shocks in the no-arbitrage condition (H.16), and ϖ is the harmonic mean that normalises their combined effect on equilibrium bond positions.