Volatility in Solar Renewable Energy Certificates: Jumps and Fat Tails

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Abstract

Economies are increasingly adopting renewable energy certificates as a policy mechanism to support the transition away from reliance on fossil fuels. We investigate the price of solar renewable energy certificates (SRECs) in New Jersey, allowing for the potential presence of jumps and time-varying volatility. We find that both features play an important role in the stochastic process describing SREC price returns. We also simulate the implied probability that at least one jump would occur in any given week. These implied probabilities indicate that jumps played a consistently important role in both SRECs and electricity prices. Jumps in SRECs appear to have been particularly noteworthy between late 2011 and early 2013, a period when electricity prices in New Jersey were relatively high. Using an investment under uncertainty framework, we describe analytically the impact of jump upon investment.

JEL Codes: L14, L71, O33, O34, Q40
Keywords: Investment under uncertainty, Solar energy prices, Geometric Brownian Motion, Jumps, Fat tails

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1 Introduction

"Change is inevitable, but transformation is by conscious choice." Amara (2014)

Renewable energy sources form the centerpiece of a strategy aimed at reducing green house gas (GHG) emissions. Indeed, a fundamental transformation of the energy system is likely to be required if mankind is to limit potential future damages from climate change (Nemet, 2019). An important part of this energy transition is likely to entail widespread adoption of renewables in general, and most likely solar power in particular.[1]

Solar power generation produces less GHG emissions than electricity deriving from the combustion of fossil fuels. The Energy Information Administration (*EIA*) forecast that solar power, which only accounted for 3% of US generation in 2020, will grow to 20% by 2050 (U.S. Energy Information Administration, 2021). Indeed, between 2014 and 2020, solar generation from photovoltaic (PV) has increase by 389%, in the US. Such growth has been driven by increases in both the Utility scale generation (475%) and small scale (rooftop) generation has also experienced significant growth (271%).[2]

Numerous policy and regulatory frameworks have been proposed to incentivize the instillation of renewable energy, so as to achieve long term reductions in carbon emissions. Many US states have introduced a renewable portfolio standard (RPS) to stimulate the production of energy from renewable sources. One policy, that can be used to comply with the

[1] (Nemet, 2019, p. 14) writes: "[m]ajor changes are needed to adapt energy systems in order to ensure that PV [photovoltaic] is a central, rather than peripheral, component of energy systems."

[2] The EIA defines utility-scale solar to be generation of at least 1 MW of capacity, while small-scale solar generation would be less than 1 MW of capacity. For a thorough discussion of the various factors that have contributed to the rise of solar energy see Nemet (2019).

RPS mandate, is the use of 'renewable energy certificates' (RECs). These RECs require a tradable certificate be issued to the generator of each MWh of renewable energy, who is then allowed to sell the REC in a market. New Jersey (NJ) was an earlier adopter of this system. Having adopted ambitious future targets for solar generation, NJ instituted a market for solar renewable energy certificates (SRECs), production credits awarded to owners of grid-connected solar PV systems (Coulon et al., 2015).

Like many sources of renewable energy, the lion's share of the cost associated with a solar project is born up front. This raises the potential for slower adoption of solar technology. With large up-front costs, the economic benefits of an investment is substantially driven by the flow of earnings after installation, be they in the form of reduced energy bills (for homeowners) or credits from adopting the new technology such as SRECs (Davis and Owens, 2003). This benefit flow must be compared against the (sunk) cost of investment. When either benefits or costs are uncertain, the problem becomes on of "investment under uncertainty" (Dixit and Pindyck, 1994).

For this type of problem, the decision-maker may choose at any point in time to invest immediately or to delay investment. If the payoff to the investment can either increase or decrease, as will naturally occur in the presence of uncertainty, delaying allows the decision-maker to reduce the potential for *ex post* regrets after making an investment that falls in value. In this way, delaying the investment can generate an increase in expected payoff; this increase in value is akin to the value associated with a financial option.[3] A key point is

[3] Many applications in this genre refer to the investment under uncertainty problem with the moniker "real options".

that factors that increase the option value would strengthen the incentive to delay (Dixit and Pindyck, 1994; Kellogg, 2014). These factors include changes in the underlying variance of the source of uncertainty, but they also can include features that contribute to fat tails in the "stochastic process". Such fat tails can arise because of time-varying volatility or the presence of unanticipated dramatic changes, sometimes called "jumps".

Our goal in this paper is to formally assess the empirical importance of jumps and time-varying stochasticity in SREC prices. Using maximum likelihood estimation, we evaluate the percentage change in three key energy prices: SRECs, electricity spot prices and natural gas spot prices. We find that allowing for jumps and time-varying volatility provides statistically important improvements over models that do not account for such features, for each of these energy prices. The preferred econometric specification, which allows for both time-varying volatility and jumps, can then be used to estimate the implied probability that at least one jump would occur over a particular time interval (in our application, one day). Upon constructing these jump probabilities, we find that the estimated jump probability for SREC prices is large at a number of points in time, in particular from late 2011 to early 2013 – a period of relatively high electricity prices in New Jersey. Because the possibility of jumps raises the option value of waiting to invest, one conjecture is that tighter markets could be associated with a slower pace of investment – which in this application suggests an additional factor that could retard the transition to solar power.[4] We also discuss the im-

[4] At the same time, the implied probability of jumps in the natural gas prices were much smaller. One important distinction between the markets for SRECs on the one hand and natural gas on the other is the thickness of the market: the volume of trades in SRECs was much smaller, and the market much less mature, than the market for natural gas. This contrast raises the possibility that a thinner market, which one might expect to be less intensely competitive, might be tied to a greater tendency for jumps. An alternative explanation is

plications of allowing for fat tails in SREC price returns, for example by including jumps. Here we analytically examine a decision-making under uncertainty problem in the manner of Dixit and Pindyck (1994), both when commodity prices do not include the potential for jumps and where they do. We show that investment is delayed by increases in the probability of a jump in SREC prices occurring, the expected value of the jump size, or the variance associated with jump size. By contrast, investment is accelerated by increases in the probability of a jump in electricity prices occurring, the expected value of the jump size, or the variance associated with jump size.

2 Literature Review

Increasing interest from state and local policymakers in the development of renewable energy markets has led to the rapid adoption of renewable portfolio standards (RPS). Many of the state RPS laws include a “solar carve-out", which mandates a percentage of power supplied derives from solar generation sources. Hence, RPS creates demand for renewable energy and stimulates investment in renewable energy generation. Krasko and Doris (2013) report that, between 1996 and 2007, more than half of U.S. states had formalized RPS. The authors examine the use of state policy as a tool to support the development of distributed generation photovoltaic (PV) markets. Using cross sectional data, the authors find evidence suggesting that market supporting policies can be effective at increasing overall PV capacity.

SRECs are financial instruments, created by a state, that are granted to producers of solar

that gas can be stored while electricity can not, suggesting that both electricity prices and solar credits could be more prone to larger and more frequent jumps in price.

power when they generate electricity from solar sources in certain states and under certain conditions (Cohen et al., 2022). SRECs have become an important tool for increasing the proportion of solar energy collected for power generation. However, the nascent nature of this market means that the SREC market experiences significant price volatility. Lee et al. (2017) report eight states in the U.S. have SREC markets, with Massachusetts and New Jersey showing the highest residential average retail prices (2013). The higher expected price makes residential installation more attractive, allowing residents to obtain a future revenue stream based on SREC prices. On the other hand, the high volatility causes uncertainty of profits and discourages the installation of solar PV systems.[5]

Using county level data for 13 Northeastern states, Crago and Chernyakhovskiy (2017) examine the impact of policy incentives for residential PV capacity. These authors report that, while rebates are a highly significant predictor of solar capacity additions, SREC prices were not significant. They further argue that the lack of significance *may be due to the volatility in the prices* during the period studied (emphasis added). Under North Carolina's RPS compliance market, installation of solar power has only enabled a few large solar power producers to compete with utility companies to finance, install, and operate solar generating systems (Gaul and Carley, 2012).

The potential that heightened price volatility can reduce investment is well-documented

[5]There is the additional concern that prolific adoption of solar panels could induce a measurable increase in electricity supply, thereby lowering price – and hence the value of the investment in solar panels. Indeed, Mwampashi et al. (2021) argue that widespread adoption of rooftop solar in Australia's National Electricity Market depressed the level of spot prices while increasing price volatility; both of these effects would exert an adverse impact on the value of investment in rooftop solar in the first instance. Jha and Gordon (2021) document a similar price effect, arguing that rooftop solar adoption in Western Australia is sufficiently impactful as to induce utilities to occasionally curtail power deliveries.

in the energy literature. Pindyck (2004) examines the behavior of volatility in both the natural gas and oil markets, arguing that volatility alters the incentives to invest in production facility, storage, and transport infrastructure. Indeed, volatility is a key element for the pricing of derivatives, hedging strategies and the decision to invest in physical capital (real options). Relatedly, Efimova and Serletis (2014) show that GARCH models explain price volatility for crude oil, natural gas and electricity. Mason and Wilmot (2016a) show that delaying investment in physical assets could occur because of discontinuities – or "jumps" – in the relevant market price. Similarly, in an analysis of the market for renewable identification numbers (RINs), Mason and Wilmot (2016b) show that a stochastic model which includes jumps provides a compelling explanation of the data. Their results indicate the statistical importance of incorporating jumps in a model of price returns.

The empirical importance of jumps has been documented for a variety of energy markets, including oil (Askari and Krichene, 2008; Gronwald, 2012; Postali and Picchetti, 2006; Wilmot and Mason, 2013), natural gas (Benth et al., 2008; Mason and Wilmot, 2016a), carbon permits in the European Union trading market (Alberola et al., 2008; Chevallier and Sévi, 2014; Daskalakis et al., 2009; Hammoudeh et al., 2014), and coal (Wilmot, 2016; Xiaoming et al., 2012).[6] A natural consequence of jumps in energy commodity prices is that electricity spot prices can also exhibit jumps (Benth et al., 2008; Huisman and Mahieu,

[6] Indeed, Chevallier and Ielpo (2014) argue that many energy prices are subject to these effects. These energy markets can sometimes be correlated Gonzalez-Pedraz et al. (2014); Asche et al. (2006) and Panagiotidis and Rutledge (2007) demonstrate a link between UK natural gas prices and European crude prices. On the other hand, (Brown and Yücel, 2008, p. 48) argue that there has been a fundamental change in the relationship between oil and gas prices after 2000. And while the data from 2000 – 2006 is reflective of emerging technological trends, it predates the major burst in gas production that followed the widespread application of fracking. With prices subject to jumps, it is not unexpected that related derivatives would also exhibit jumps – for example, the convenience yield associated with holding crude oil stockpiles (Mason and Wilmot, 2020).

2003). The jumps in these energy prices can be large: Huisman and Mahieu (2003) show that daily jumps in electricity prices can be on the order of 30%. Similarly, Thompson et al. (2009, p. 227) observe "high [natural gas] price spikes far outside normal seasonal equilibrium levels;" and Chen and Forsyth (2010, p. 359) argue it "is not uncommon to see spot gas price jumps ... as much as 20% in a single day."

Similar to this earlier literature, there is evidence that the SREC market in NJ has exhibited volatile price dynamics (Coulon et al., 2015); this has the potential to reduce investments in solar power generation due to the increased risk from large price swings. These authors propose a stochastic model to examine the behavior of SREC market prices, which includes a feedback mechanism for power generation decisions. Utilizing monthly data, over the period July 2008 through March 2014, they model the historical dynamics of SREC prices. The model requires numerical solution algorithm, which the authors argue must be carefully chosen, as the historical SREC price behavior is extremely challenging to capture or understand in a classical econometric price model.

Understanding the nature of these price movements has important implications for large-scale investment decisions, particularly when these decisions entail up-front costs that are costly (or impossible) to reverse. Here too there is a long tradition in the literature of analyzing investment decisions when key ingredients, such as underlying prices, are stochastic. Much of this literature follows the seminal work of Dixit and Pindyck (1994). A key insight here is that there is a value to waiting – reflecting the potential for the gain from investing to increase over time; this value is referred to as the "option value" of delaying invest-

ment. In general, the larger is the option value from delaying, the later will the investment be undertaken. Of direct relevance to our analysis, Torani et al. (2016) evaluate a model of investment in solar projects under uncertainty. Their model assumes solar installation prices follow geometric Brownian motion; they allow for neither GARCH-style volatility nor jumps in prices.[7] By contrast, we do allow for both GARCH and jumps in our model of SREC prices, and find that both aspects are statistically important.

These results have clear implications for a broad range of decisions involving renewable energy, from larger decisions (such as a utility's decision to install a substantial solar farm) to smaller decisions (such as a homeowner's decision to install rooftop solar). There is a robust literature that has studied problems of investment under uncertainty; many of these papers study applications to energy and climate economics. A number of authors have studied applications to non-renewable energy resources, such as oil or gas, often with a focus on production from a single well. A representative sample of these papers includes Almansour and Insley (2017); Conrad and Kotani (2005); Cortazar and Casassus (1998); Davis and Lund (2018); Insley (2017); Kellogg (2014); Mason (2001); Muehlenbachs (2015) and Paddock et al. (1988). The broad consensus from this literature is that the development of energy resources can often be characterized in a fashion that is consistent with real option theory. In particular, the notion of the option value of waiting, which is central to our discussion, is empirically relevant in these applications.

[7] They also argue that electricity price uncertainty plays an important role in impacting the volatility of solar markets.

3 Empirical Framework

3.1 Econometric Methods

To present the stochastic process under investigation, let P_t denote the price of *SREC* at time t. We start by describing a geometric Brownian motion *(GBM)* process with trend μ_0 and dispersion σ:

$$dP_t = \mu_0 P_t d_t + \sigma P_t dz_t. \tag{1}$$

In Eq. 1, dz represents an increment of a Wiener process, $dz = \xi\sqrt{dt}$, where ξ is an identically and independently distributed (i.i.d.) random variable with mean zero and variance one. (Dixit and Pindyck (1994)). Let x_t denote the percentage change in price, $x_t = dP_t/P_t$. When P_t follows a *GBM* process then x_t is normally distributed with variance σ^2 and mean $\mu = \mu_0 - \sigma^2/2$. This gives the *pure diffusion (PD)* model[8]

$$dx = \mu + \sigma dz. \tag{2}$$

As we observed in Section 2 above, many authors have argued that continuous stochastic processes, such as *GBM*, are insufficient for explaining discontinuous movements, or "jumps", in prices. To introduce jumps into the above model, we adapt the methodology in Merton (1976) – wherein returns are composed of two types of changes. The first, 'nor-

[8] For expositional compactness we henceforth drop the time subscript where no confusion will result.

mal' fluctuations, are represented through the geometric Brownian motion process, while the second – 'abnormal' shocks due to the arrival of new information – are modeled through a discontinuous process. The discontinuities are described by a Poisson distribution jump intensity, λ, which describes the number of events N_t that occur in a time interval $(t, t+dt)$:

$$P(N_t = j) = \frac{\exp(-\lambda dt)(\lambda dt)^j}{j!}. \tag{3}$$

With this structure, the change in the number of events in an interval of width dt is

$$dN_t = \begin{cases} 0 & \text{with probability } 1 - \lambda dt \\ 1 & \text{with probability } \lambda dt \end{cases} \tag{4}$$

so that the expected number of events is λdt, and the parameter λ may be interpreted as the arrival rate of jumps.

If a jump occurs it results in the price YP (*i.e.*, it renders an increase that is proportional to the original price). The size of the jump is $YP - P = (Y-1)P$. We assume $\ln(Y)$ is a normally distributed random variable with mean θ and variance δ^2 – so that Y is lognormally distributed – which is independent of the distribution for the arrival of a jump. Altogether, the contribution from abnormal fluctuations can be written as:

$$J = (Y-1)\lambda dt.$$

The *jump-diffusion* (*JD*) process for the log-price returns can then be described by

$$dx = \mu dt + \sigma dz + J, \tag{5}$$

where dz has the same properties assumed in equation (1). It is natural to assume that dz and J are independent, since the first component reflects ordinary movements in price while the second component reflects unusual changes in price.

An alternative explanation for the "fat tails" that are often observed in commodity price data is that P_t is subject to time-varying volatility. An example of such a phenomenon is the "generalized autoregressive conditional heteroskedastic" (GARCH) framework. Adapting the pure diffusion model to allow for this form of time-varying volatility gives the *GARCH–diffusion* (*GPD*) process:

$$dx_t = \mu + \sqrt{h_t} dz_t, \tag{6}$$

where the conditional variance, h_t is described by the (discrete time) process

$$h_t = \kappa + \alpha_1 \left(x_{t-1} - \mu\right)^2 + \beta_1 h_{t-1}. \tag{7}$$

Note that when $h_t = \sigma^2$ the GARCH–diffusion model *(GPD)* reduces to pure diffusion model *(PD)*. On the other hand, when $\kappa > 0$ and $\alpha_1 + \beta_1 < 1$, the unconditional variance of the volatility of the process exists and equals $\frac{\kappa}{1-\alpha_1-\beta_1}$. The process described in equations (6) – (7) is characterized by four parameters, μ, κ, α_1 and β_1. There is a general consensus in the literature is that a GARCH model with a limited number of terms performs reasonably well

(Akgiray, 1989; Hansen and Lunde, 2005; Sadorsky, 2006), and so we focus our analysis on this more parsimonious representation.[9] Allowing for jump discontinuities then results in the *GARCH jump-diffusion* (*GJD*) process:

$$dx_t = \mu + \sqrt{h_t} dz_t + J_t, \tag{8}$$

where h_t is described by equation (7).

We evaluate the four models using maximum likelihood estimation methods, which are known to provide consistent and invariant estimates, with asymptotically normal distributions of the parameters. To this end, we note that the parameters of our four candidate models – *PD, JD, GPD, GJD* – may be nested into the general log-likelihood function

$$L\left(\phi, x_t\right) = -T\lambda + \sum_{t=1}^{T} \ln\left[\sum_{n=0}^{\infty} \frac{\lambda^n}{n!} \frac{1}{\sqrt{h_t + n\delta^2}} \exp\left(\frac{-\left(x_t - \mu - n\theta\right)}{2\left(h_t + n\delta^2\right)}\right)\right] + K, \tag{9}$$

where K is the constant $\frac{T\ln(2\pi)}{2}$, n indexes the number of jumps, the parameter vector is $\phi = (\mu, \sigma, \lambda, \alpha_1, \beta_1, \theta, \delta)$ and the description of h_t given in equation (7).[10] In this framework, the GPD model corresponds to the parameter restriction $\lambda = \theta = \delta = 0$; the JD model corresponds to the restriction $\alpha_1 = \beta_1 = 0$; and the PD model corresponds to the restriction $\alpha_1 = \beta_1 = \lambda = \theta = \delta = 0$.

[9] Duan (1997) shows that the diffusion limit of a large class of GARCH(1,1) models contain many diffusion processes allowing the approximation of stochastic volatility models by the GARCH process.

[10] Under the PD and JD models, the variance is a constant, which corresponds to $h_t = \kappa$ in eq. (7). Consistent with eqs. (2) and (5) we denote variance by σ^2 in these two models. In the GPD and GJD models variance is h_t. To facilitate estimation based on eq. (9), the number of jumps was truncated at 10 (Ball and Torous, 1985). Allowing more jumps did not significantly change the point estimates.

Comparing any pair of potential models can thus be framed as a test of an appropriate parameter restriction. For example, the comparison of the PD and GPD models is conducted by testing the parameter restriction $\alpha_1 = \beta_1 = 0$; the comparison of the PD and JD models is conducted by testing the parameter restriction $\lambda = \theta = \delta = 0$. The empirical validity of the parameter restriction of interest can be evaluated by use of the likelihood ratio test. This approach compares the likelihood function under a particular restriction, $L(\phi^R;x)$, to that of the unrestricted or less restricted likelihood function, $L\left(\hat{\phi};x\right)$. Under the null hypothesis that the restriction is empirically valid, the decrease in the likelihood function associated with the restriction will be small. Such an approach can be used to make pairwise-comparisons between a more general model and a more restricted model. The test statistic is the log-likelihood ratio

$$LR = 2[L(\hat{\phi};x) - L(\phi^R;x)];$$

under the null hypothesis this statistic will be distributed as a Chi-square random variable with m degrees of freedom, where m is the number of parameter restrictions.

3.2 Data

Our empirical analysis is based on daily price observations. We obtained SREC prices from the New Jersey Clean Energy program website.[11] Natural gas prices are based on the Henry

[11] Accessed at https://njcleanenergy.com/renewable-energy/project-activity-reports/srec-pricing/srec-pricing-archive.

Hub spot price,[12] as the natural gas market is widely thought to be national. For electricity, we used the PJM Western hub price,[13] as this range includes New Jersey.

Table 1 provides summary statistics for the log-returns, calculated as $x_t = ln(P_t) - ln(P_{t-1})$, for each of the prices returns series.[14] The statistics indicate a significant amount of variation among the three variables, though SERC returns exhibit the lowest relative level of dispersion. Evidence of asymmetry in the distribution of returns is also highest for SERC returns, while electricity prices exhibit negative skewness. The "fat tails" are evident from the large values of kurtosis. The normality hypothesis of the Kolmogorov-Smirnov test is rejected, for each time-series. Lastly, Table 1 contains the results of the modified ADF test of nonstationarity. In each of the log return series the null hypothesis of a unit root is rejected, in favor of stationarity.[15]

These results are corroborated by Figures 1 and 2. Figure 1 displays the log returns for the three energy series. Notably, large sudden changes in the returns are evident, and appear with less frequency for natural gas returns, though on a larger scale for SREC solar price returns. Also, the large changes in SREC solar price returns are evident, and the positive shocks appear much larger in amplitude than the negative changes. Figure 2 displays "quantile-quantile" plots, for the three return series. This plot compares the quantiles of the empirical distribution (measured on the y-axis) against the quantiles of a theoretical

[12] Accessed at https://www.eia.gov/dnav/ng/ng_pri_fut_s1_d.htm.

[13] Accessed at https://www.eia.gov/electricity/wholesale/.

[14] This is the discrete time approximation of $dP/P = dln) = (P)$.

[15] While not reported in the table, the SREC solar credit, natural gas, and electricity prices are nonstationary (in levels), and thus first differencing (e.g. log returns) renders the series stationary. The findings are supported by traditional unit root tests, the results of which are available upon request.

normal distribution (measured on the x-axis). If the empirical distribution is close to a normal distribution, the relation will be well described by a straight line. As demonstrated in the figure, there are significant departures from a linear relation, indicating that the date is not well-described by a normal distribution. Indeed, the significant departures observed in along the ends of the plots are indicative of significant leptokurtosis and cast doubt on the assumptions of an underlying GBM process.

4 Empirical Results

4.1 Maximum Likelihood Estimation

Using maximum likelihood methods, we estimate the parameters for the four stochastic processes (*PD, JD, GPD, GJD*), using the daily returns series. The results of the estimation are presented in Table 2, while the likelihood ratio test statistics, for the pairwise comparison of the four models, are presented in Table 3.

Examining the pure diffusion results, the estimated drift parameter, μ, is positively and statistically significant for SREC returns, though not statistically significant for either natural gas or electricity prices.[16] For each series, the estimated instantaneous rate of variance, σ, is statistically significant. This same pattern holds when examining the drift and variance of the JD models, though allowing for jumps markedly reduces the estimated instantaneous rate of variance. The estimated jump intensity, λ, is statistically significant for each of the

[16] To improve convergence of the estimation procedure, we rescaled the price return series for natural gas and electricity by multiplying by 100; the SREC returns were not rescaled.

three series. The estimated value of λ is largest for the SREC series; this estimate suggests jumps occur quite frequently for SREC returns – roughly two out of every three days. Finally, the estimated mean jump size, θ, is positive when the estimate is significant.

Comparing the results of the PD and GPD models, we see that allowing for GARCH also improved the models predictive power: the log-likelihood function shows improvement, in each of the three series. The estimated coefficients associated with the GARCH model, κ, α_1 and β_1, are statistically significant across each of the three series. Additionally, all three series demonstrate persistent volatility, with $\alpha_1 + \beta_1$ close to 1. Notably, the magnitude of the estimated value of κ is large for electricity price returns – pointing to large overall variation in that series. Importantly, our results indicate that combining GARCH with the jump model (GJD) yield the best results, demonstrating the importance of including jumps. We note that the probability of a jump occurring has fallen across all three series, while the GARCH terms remain statistically significant. Evidently, the inclusion of GARCH captures some of the estimated variance originally interpreted as jumps in the JD model, it does not render the jumps irrelevant. Indeed, based on the decreased values of λ, and notable increases in θ, the average jump size, one conceivable theory is that the small perturbations are captured by the GARCH model while the larger discontinuous moves are represented as jumps.

4.2 Likelihood Ratio Test Results

Results of the pairwise likelihood ratio tests are presented in Table 3. Each entry in the table is a test of the form $LR_{X,Y}$, where the null hypothesis is that the appropriate stochastic process describing the data is X and the alternative hypothesis is that the appropriate stochastic process describing the data is Y. The probability that the null hypothesis is preferred to the alternative, based upon the Chi-squared distribution, is presented in the parentheses below each test statistics. The results indicate that the null hypothesis is rejected at the 1% level of significance in every case. Such a result is interpreted as demonstrating the statistically important gain in predictive power associated with adopting the added complexity associated with moving to specification Y, from X. Thus, for example, including GARCH improves upon the JD model, while folding in jumps improves upon the GARCH model. Importantly, the preferred specification allows from both jumps and time-varying volatility (*GJD*), for each of the three series.[17]

[17] One might be concerned that jumps in SREC prices are asymmetric, *i.e.*, that unanticipated positive shocks exert different effects than unanticipated negative shocks. To assess the potential for such effects on our results, we considered a variation on the GARCH(1,1) model that includes an asymmetric response parameter (known as "EGARCH"). We then collected the residuals from this approach and compared them against the residuals from the GARCH(1,1) process discussed in the main body of the paper, and found the two series were highly correlated. Then, we estimated a jump model using the residuals and compared the likelihood statistic from this estimation to the likelihood function from the EGARCH analysis, so as to evaluate the hypothesis that jumps are no longer statistically important when asymmetries are allowed. The test results strongly reject that hypothesis, corroborating the results outlined in the body of the paper. These results are available upon request.

4.3 Implied Probabilities

Figure 3 gives a visual representation of the implied probabilities of at least one jump in the log price returns, based on the JD model. The first panel shows the probabilities for the SREC solar credit returns. Given the frequency and persistence of implied probabilities that are well above 0.5, it is apparent that jumps play an important role in both SREC and electricity markets. We note too that the implied jumps close to 1 in the SREC markets are particularly common in 2012; this is intriguing as electricity prices in New Jersey were particularly large during this period. In contrast to the other two markets, the implied jump probabilities in the natural gas are smaller, and less frequently close to 1.

One possible explanation for this distinction is that the natural gas market is thicker than the other markets. In addition, the SREC market is noticeably less mature, having only come into fruition within the last twenty years; by contrast, natural gas markets have been functioning for many years. Perhaps market thickness as well as maturity might create a form of insulation from large transitory events – thereby rendering jumps more important in younger or thinner markets, such as the SREC market. An alternative explanation is that gas can be stored while electricity cannot, suggesting that both electricity prices and solar credits could be more prone to larger and more frequent jumps in price. In addition, natural gas supply tends to respond to gas demand; by contrast, in permit markets (such as the market for solar credits) the supply is commonly fixed at an exogenous level. The inability of supply to adjust to changes in demand in credit markets would then seem likely to induce greater variation in prices – along with a greater tendency for abrupt changes.

5 Solar Investment Under Uncertainty with Jumps

In light of the results above, which highlight the empirical importance of fat tails in solar credit price returns – and in particular the potential for jumps – a key issue here is the implication for investment in rooftop solar: do jumps influence investment decisions, and if so how? Addressing this issue requires an analysis of the investment decision with and without jumps; to that end, one needs to determine the value associated with forestalling the investment together with a determination of the value that is realized by investing. The former includes an "option value of waiting" to invest, an important element in the analysis. The latter is the expected present discounted value of the flow of rewards. Central to this investigation is a comparison of the value of forestalling the investment against the expected discounted flow just described. In this section, we provide such an analysis.

In this section, we provide such an analysis. To illustrate the basic ideas, we adapt the analysis in Torani et al. (2016), who considers a problem where a decision-maker such as a homeowner is contemplating an investment in a solar project. The investment incurs an immediate cost that is irreversible, but that provides the decision-maker with an asset that generates a stream of benefits. In the solar installation case, there are two forms of benefits: the individual obtains a stream of solar credits, based on the volume of electricity the installed unit produces – these credits can be monetized in the solar credit market (with per unit value equal to the "solar renewable energy credit" we discussed above); the individual may also sell electricity back to the grid, at a price that is related to the spot price of electricity. The key features of this problem are that there is an economically significant

up-front cost of investing, K, that is either irreversible or so expensive to reverse as to be qualitatively irreversible; and the stream of future rewards depends on two per-unit values or prices, both of which follow some sort of stochastic process. For now, we assume each of these prices follows a geometric Brownian motion (GBM) process; these processes may be correlated.

A key question here regards timing: when should the individual undertake the investment? Answering this question requires a determination of the value associated with forestalling the investment together with a determination of the value that is realized by investing – what Torani et al. (2016) call the value of a "live investment". The former includes an "option value of waiting" to invest, an important element in the analysis. The latter is the expected present discounted value of the flow of rewards; in light of the GBM assumption, this expected value is straightforward to calculate. A central point of the investigation is determining the value of forestalling the investment, which is then compared against the expected discounted flow just described.

5.1 Model formulation without jumps

We start by fleshing out the problem under geometric Brownian motion (GBM) process. One can express the stochastic evolution of any geometric Brownian motion process X as

$$dX/X = \mu dt + \sigma dz, \tag{10}$$

where dz is an increment of a Wiener process, μ is a deterministic drift term and σ characterizes the magnitude of stochastic variability in X. This form implies that X is lognormally distributed (*i.e.*, $\ln(X)$ is Normally distributed). We denote the two key prices in this framework as P_{sc} – the price of a solar credit – and P_E – the price of a unit of electricity. Following Torani et al. (2016), we assume the amount of electricity produced from the investment is a constant, which we denote as q_S, and that the amount of electricity used is a constant $q_E < q_S$. We start by working through the problem when each of P_{sc} and P_E follow a GBM process; we denote the drift term associated with the process governing the solar credit price as $\mu = \mu_{sc}$, the drift term associated with the process governing the price of electricity as $\mu = \mu_E$, the term describing stochastic variability in P_{sc} as σ_{sc}, and the term describing stochastic variability in P_E as σ_E. Later, we analyze the problem when these processes are also subject to the potential for jumps.[18] The value of a live investment is (Torani et al., 2016)

$$F(P_{sc}, P_E) = \mathcal{E}\left\{ \int \left(P_{sc} q_S + \gamma P_E (q_S - q_E) \right) e^{-rt} dt \right\},$$

where r is the discount rate, γ is a parameter that captures the degree to which electricity sold back to the grid is discounted, and $\mathcal{E}$ is the expectations operator. This value is readily computed as

$$V_0 = \frac{P_{sc} q_S}{r - \mu_{sc}} + \frac{\gamma P_E (q_S - q_E)}{r - \mu_E}.$$

[18] One aspect of the GBM process is that changes tend to exert an effect for a considerable length of time. An alternative approach would be to use a model in which the effect of changes in prices tend to dissipate relatively more rapidly – for example, a mean-reverting process. Analysis such a process is more complicated, though the broad principles we describe in this section still apply Dixit and Pindyck (1994).

At any moment where the decision to undertake the investment has yet to be made there are two possible decisions: either invest now or wait. The decision to invest now yields the payoff $V_0 - K$ (where K is the sunk cost of investing). The decision to wait earns a flow payoff of 0 (since no action has been taken there are no flow benefits), while the option value, $F(P_{sc}, P_E)$, is retained. Delay will deliver anticipated change in $F(P_{sc}, P_E)$ (which can be thought of as the anticipated capital gains) less the foregone capitalized option value (which can be thought of as the interest earned on the net returns). If delaying is optimal, these two effects balance out Dixit and Pindyck (1994). The optimal value function must satisfy therefore satisfy the Bellman equation:

$$rF(P_{sc}, P_E) = \frac{1}{\mathrm{dt}} E\big[\mathrm{d}(F)\big], \tag{11}$$

where the expression on the right-hand side is the so-called Itô operator. The left-hand side of eq. (11) measures the capitalized option value, while the right-hand side is the sum of flow payoffs plus anticipated capital gains.

One proceeds by expanding the expression on the right-hand side using Itô's lemma,

which gives rise to[19]

$$\frac{1}{\mathrm{dt}}E\left[\mathrm{d}(F)\right] = \mu_{sc}P_{sc}\frac{\partial F}{\partial P_{sc}} + \mu_E P_E \frac{\partial F}{\partial P_E} + (\rho\sigma_{sc}P_{sc}\sigma_E P_E)\frac{\partial^2 F}{\partial P_E \partial P_{sc}} + \frac{1}{2}\left[\sigma_{sc}^2 P_{sc}^2 \frac{\partial^2 F}{\partial P_{sc}^2} + \sigma_E^2 P_E^2 \frac{\partial^2 F}{\partial P_E^2}\right], \tag{12}$$

where ρ is the correlation between the prices of solar credits and electricity. It will be convenient to rewrite the problem, recognizing that F is homogeneous of degree one (Torani et al., 2016), so that

$$F(P_{sc}, P_E) = P_E f(x), \tag{13}$$

where $x = P_{sc}/P_E$. Because both P_{sc} and P_E both follow a GBM process (and so are lognormally distributed), so is their ratio $x = P_{sc}/P_E$:

$$dx = \mu_x dt + \sigma_x dz, \tag{14}$$

where dz_x is an increment of a standard Wiener process. The drift and dispersion parameters

[19] One way to think about this step is that a Taylor's series expansion has been executed, with higher order terms dropped. Thus, we only retain the terms $\mu_{sc}P_{sc}$ for dP_{sc}, $\mu_E P_E$ for dP_E, and $(\rho\sigma_{sc}P_{sc}\sigma_E P_E)$ for $d[P_{sc}P_E]$.

for x can be obtained as:[20]

$$\mu_x = \mu_{sc} - \mu_E - \rho\sigma_{sc}\sigma_E + \sigma_E^2; \tag{15}$$

$$\sigma_x^2 = \sigma_{sc}^2 + \sigma_E^2 - 2\rho\sigma_{sc}\sigma_E. \tag{16}$$

It will be convenient to rewrite the Bellman equation in terms of f; to this end we interpret the various partial derivates of F in terms of derivatives of f:

$$\frac{\partial F}{\partial P_{sc}} = f'(x); \quad \frac{\partial F}{\partial P_E} = f(x) - xf'(x); \quad \frac{\partial^2 F}{\partial P_E \partial P_{sc}} = -x^2 f''(x)/P_E;$$
$$\frac{\partial^2 F}{\partial P_{sc}^2} = f''(x)/P_E; \quad \frac{\partial^2 F}{\partial P_E^2} = x^2 f''(x)/P_E.$$

Inserting these expressions into eq. (12) using eq. (13), recalling eq. 16, and simplifying, we obtain:

$$rf(x) = \mu_{sc} x f'(x) + \mu_E\big(f(x) - xf'(x)\big) + \frac{\sigma_x^2}{2} x^2 f''(x). \tag{17}$$

Eq. 17 is a (log) linear second-order differential equation, the solution to which takes the form

$$f(x) = ax^b.$$

[20] Aplying Itô's lemma to the function $x = G(P_{sc}, P_E) = P_{sc}/P_E$ one has

$$dx = \frac{1}{P_E} dP_{sc} - \frac{P_{sc}}{P_E^2} dP_E - \frac{1}{P_E^2} dP_{sc} P_E + \frac{1}{2}\frac{2P_{sc}}{P_E^3}(dP_E)^2 + h.o.t.$$

where "h.o.t." refers to terms above dt in magnitude (which can be omitted, as they go to zero faster than dt). Applying eq. (10) one can substitute for dP_{sc} and dP_E; collecting terms in dt and dz yields the expressions in eqs. (15) and (16).

The parameter b is determined by the quadratic

$$\mathcal{Q}(b) = \frac{\sigma_x^2}{2} b(b-1) + \mu_{sc} b + \mu_E (1-b) - r = 0. \tag{18}$$

Since $\mathcal{Q}$ is convex and both $\mathcal{Q}(0)$ and $\mathcal{Q}(1)$ are negative, there is one negative and one positive root (which exceeds one) to this quadratic; of these, the negative root is economically irrelevant and so the economically meaningful root exceeds one (Dixit and Pindyck, 1994; Torani et al., 2016). For later reference we denote this value as $\hat{b}$. It is straightforward to show that $\hat{b}$ depends positively on σ_x^2 and negatively on μ_{sc}.

The value function $f(x)$ can be interpreted as the value of an option to invest in the future Dixit and Pindyck (1994). This option value is compared against the net benefits from investing now, v_0. It is optimal to invest when the net benefit from acting now just equals the option value of waiting; this implies a cutoff value x^*, which is implicitly defined by the "value-matching" and "smooth-pasting" conditions; it is straightforward to derive[21]

$$x^* = \frac{bK}{b-1}, \tag{19}$$

which a linear relation between P_{sc}^* and P_E^* on the transition boundary. Because investment is delayed until x rises to this cutoff value, and as the transition boundary shifts up with an

[21] The value-matching condition is $f(x^*) = v_0(x^*) - K/P_E$ and the smooth-pasting condition is $f'(x^*) = v_0'(x^*)$. Using the functional form for f given above, it is easy to see that the smooth-pasting condition implies

$$b f(x^*) = \frac{x^* q_S}{r - \mu_{sc}}.$$

Combining this expression with the value-matching condition leads directly to eq. 19.

increase in b, it follows that investment will tend to be undertaken sooner the larger is μ_{sc} or the smaller is σ_x.[22]

5.2 Model formulation with jumps

Now suppose the value x evolves according to the mixed jump-diffusion process. We assume for now that jumps occur only in P_{sc}; later we consider a version where both solar credits and electricity prices are subject to jumps. We assume changes in x are composed of two types of changes: 'typical' fluctuations, represented through the GBM process, and 'abnormal' fluctuations, due to the arrival of new information or some unusual event. We model the arrival of these abnormal fluctuations as following a Poisson process.[23] The probability a jump will occur during a brief time interval of length dt is then λdt, where $\lambda > 0$ is a parameter measuring the arrival frequency of jumps.

Should a jump occur, we assume it induces the price $Y_{sc}P_{sc}$ where $\ln(Y_{sc})$ is a normally distributed random variable, with mean θ_{sc} and variance δ_{sc}^2, that is independent of the distribution for the arrival of a jump. The magnitude of the jump is thus $(Y_{sc} - 1)P_{sc}$.

Recalling the characterization of a GBM process from eq. (10), one may describe the

[22] These features can also be characterized in terms of the option value. Because a larger option value raises the benefits from delay, it will tend to push back in time the moment at which the decision to invest is taken. Intuitively, an increase in the variance of the stochastic process raises the option value because of the potential for a more dramatic future increase in the underlying value x; delaying investment allows the decision maker to strategically take advantage of such future movements. This effect is more important the larger is the initial investment K.

[23] Some authors model price jumps using a Lévy process, an approach that requires an *ex ante* definition of a jump. For example, Benth et al. (2008) define a jump as an observation that falls outside of 2 standard deviations from the mean. Other authors assume jumps follow a Poisson process; one advantage of this approach is that there is no need to arbitrarily define a jump *ex ante*.

combined jump - GBM process by

$$dP_{sc}/P_{sc} = \mu_{sc}dt + \sigma_{sc}dz_{sc} + (Y_{sc} - 1)\lambda_{sc}dt. \tag{20}$$

If a jump occurs it may move the price directly into that region where investment is undertaken, or it may leave price in the continuation region. In the latter event the function characterizing the continuation value is still given by eq. (11), but now the term on the right-hand side is impacted by the jump component; this changes the expression in eq. (12) into:

$$\begin{aligned}\frac{1}{\mathrm{dt}}E\left[\mathrm{d}(F)\right] = \mu_{sc}\frac{\partial F}{\partial P_{sc}}P_{sc} + \mathcal{E}_Y\left[\lambda_{sc}F\left(Y_{sc}P_{sc}, P_E\right) - F\left(P_{sc}, P_E\right)\right] + \mu_E P_E\frac{\partial F}{\partial P_E} \\ +(\rho\sigma_{sc}P_{sc}\sigma_E P_E)\frac{\partial^2 F}{\partial P_E \partial P_{sc}} + \frac{1}{2}\left[\sigma_{sc}^2 P_{sc}^2\frac{\partial^2 F}{\partial P_{sc}^2} + \sigma_E^2 P_E^2\frac{\partial^2 F}{\partial P_E^2}\right],\end{aligned} \tag{21}$$

where the expectation in the final term on the right-hand side is take with respect to the random variable Y_{sc}. The only difference between eq. (21) and eq. (12) is the term

$$\begin{aligned}\Omega &= \mathcal{E}_Y\left[\lambda_{sc}\Big(F\left(Y_{sc}P_{sc}, P_E\right) - F\left(P_{sc}, P_E\right)\Big)\right] \\ &= \mathcal{E}_Y\left[\lambda_{sc}P_E\Big(f\left(Y_{sc}P_{sc}/P_E\right) - f\left(P_{sc}/P_E\right)\Big)\right] \\ &= \lambda_{sc}P_E\Big(\mathcal{E}_Y\left[f\left(Y_{sc}P_{sc}/P_E\right)\right] - f\left(P_{sc}/P_E\right)\Big).\end{aligned} \tag{22}$$

The solution to the differential equation describing the continuation value remains a

power function with exponent b, which implies

$$\mathcal{E}_Y\left[f\left(Y_{sc}P_{sc}/P_E\right)\right] = f\left(P_{sc}/P_E\right)\mathcal{E}_Y\left[Y_{sc}^b\right].$$

Because Y_{sc} is lognormal so is Y_{sc}^b; its expected value is $\mathcal{E}_Y\left[Y_{sc}^b\right] = e^{b\left(\theta_{sc}+\frac{\delta_{sc}^2}{2}\right)}$. This then leads to the revised equation governing the implied value of b:

$$\begin{aligned} 0 = \widetilde{\mathcal{Q}}(b) &= \frac{\sigma_{sc}^2}{2}b(b-1) + \mu_{sc}b + (1-b)\mu_E - r + \lambda_{sc}\left(\mathcal{E}_Y\left[Y_{sc}^b\right] - 1\right) \\ &= \frac{\sigma_{sc}^2}{2}b(b-1) + \mu_{sc}b + \mu_E(1-b) - r + \lambda_{sc}\left(e^{b\left(\theta_{sc}+\frac{\delta_{sc}^2}{2}\right)} - 1\right). \end{aligned} \tag{23}$$

While the solution for b must be obtained numerically, we can compare this solution to $\hat{b}$, the solution to eq. (18). Inserting $\hat{b}$ into the right hand side of eq. (23) and using eq. (18), we infer that

$$\widetilde{\mathcal{Q}}(\hat{b}) = \lambda_{sc}\left(e^{b\left(\theta_{sc}+\frac{\delta_{sc}^2}{2}\right)} - 1\right).$$

since $\lambda_{sc} > 0$ and $\hat{b} > 1$ it is clear that $\widetilde{\mathcal{Q}}(\hat{b}) > 0$ so long as $\theta_{sc} + \frac{\delta_{sc}^2}{2} > 0$. As the function $\widetilde{\mathcal{Q}}$ is increasing for values of $b > 0$ it then follows that the solution to eq. (23), call it $\hat{b}_{sc}$, must be smaller than $\hat{b}$ under those conditions. In turn, this implies that the problem that includes jumps renders a more concave continuation value function, and therefore increases the cutoff value of x at which investment will occur, so long as θ_{sc} exceeds some threshold negative value.

While determination of specific magnitudes requires numerical simulation it is possi-

ble to anticipate various comparative statics. First, the potential for delayed investment is increasing in the jump probability for SREC. Second, the potential for delayed investment is increasing in the expected magnitude of jumps in SREC and the variance of this jump magnitude.[24]

6 Discussion

In this study we investigated the price of solar renewable energy certificates (SRECs) in New Jersey – production credits awarded to owners of grid-connected solar PV systems – allowing for the potential presence of jumps and time-varying volatility. Our results point to the importance of both of these features in modeling SREC price returns. In addition to providing a statistically important improvement in explanatory power of the empirical model, our results allow us to simulate the implied probability that at least one jump would occur in any given month. Following this approach, we developed implied jump probabilities for price returns for SRECs, as well as natural gas and electricity price returns. Our results point to the consistently important role of jumps in both SRECs and electricity prices, but less so for natural gas prices. Jumps in SRECs appear to have been of particular significance between late 2011 and early 2013, a period when electricity prices in New Jersey were particularly high – presumably providing heightened incentives for homeowners to invest in

[24] While it is analytically more complex to analyze a model allowing for jumps in both prices, we can intuit the impact of allowing for jumps in the ratio $x = Psc/P_E$. Such a model would be similar to the one we analyzed above, and so we expect comparative static statements about the price of solar credits could be extended to the ratio. Specifically, since both prices are log-normally distributed so too would be their ratio; as such the drift term of the ratio would be the difference between drift terms of underlying prices ($\mu_{sc} - \mu_E$), so that increases in μ_E would encourage investment as would an increases in the probability of a jump in electricity prices.

rooftop solar. Two important factors that distinguish SREC and natural gas markets are the relative thickness of the market and its maturity. Natural gas markets are commonly thought to be quite thick and mature, whereas SREC markets have been in place for a much shorter period, and there are far fewer suppliers of solar projects. This points to the potentially important role of market structure in driving fat tails in price returns.[25]

The rapid growth in solar PV capacity has been driven by the falling price of installing PV and the implementation of public policies (Nemet, 2019). Even so, rooftop solar panel installations are a non-trivial expense to the homeowner; Dastrup et al. (2012) estimate the cost to be on the order of 3-4 % of the home value. Perhaps because of this feature, Mamkhezri et al. (2020, p. 178) have argued that "there is a diminishing return in support for [r]ooftop solar." As we noted above, the presence of uncertainty can retard investment, particularly in projects with largely irreversible up-front investments such as rooftop solar installations. The obvious alternative would be a large-scale solar farm; but the private incentive to undertake such a large investment also depends importantly on the option value of waiting to invest. This delaying effect is larger the larger is the variation in the stochastic element of concern (here, SREC prices). Importantly, the presence of jumps mimics an increase in variability. As such, our finding that both jumps and time-varying price volatility provide a statistically important explanation of the pattern in SREC prices is of some concern, as there is reason to fear these effects will increase the option value of waiting –

[25] Of course, market structure can exert an important influence on price levels – either by allowing sellers to charge higher prices or by allowing sellers to capitalize on cost reductions, for example through scale economies (Gillingham et al., 2016). Our point is that more concentrated markets may be more susceptible to the influence of unexpected events that could cause dramatic changes in prices.

and thereby retard the rate of investment in solar energy. The upshot is that development of solar farms and similar investments are likely to occur at a somewhat slower pace than one might prefer – slowing the energy transition.

This effect is reminiscent of the potential delay in investment that arises when learning-by-doing effects are important (van Benthem et al., 2008). When learning-by-doing effects are present, private decision-makers undervalue the potential future benefits associated with present actions, *i.e.* they overweight current effects. In contrast, when investments the option value of waiting induces delayed investment, decision-makers see increased present gains in part because of the desire to capture potential future gains that might arise if jumps occur.

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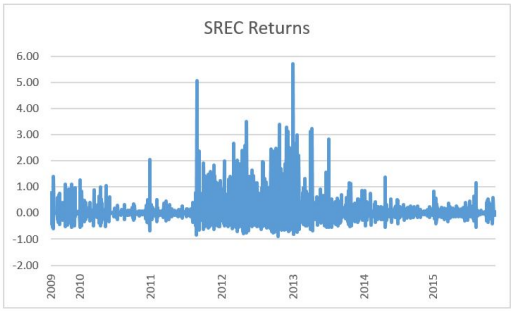
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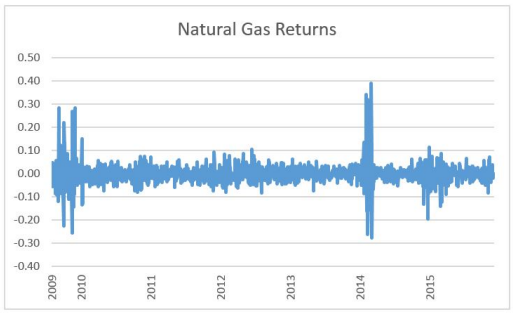
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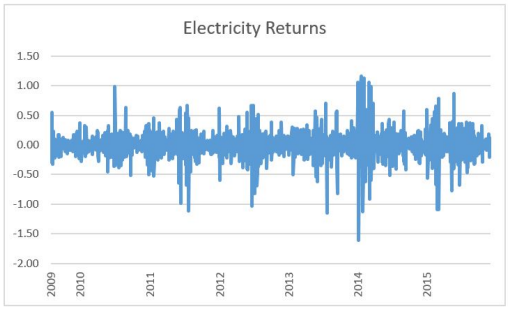
Figure 1: Time series of price returns



(a) SREC



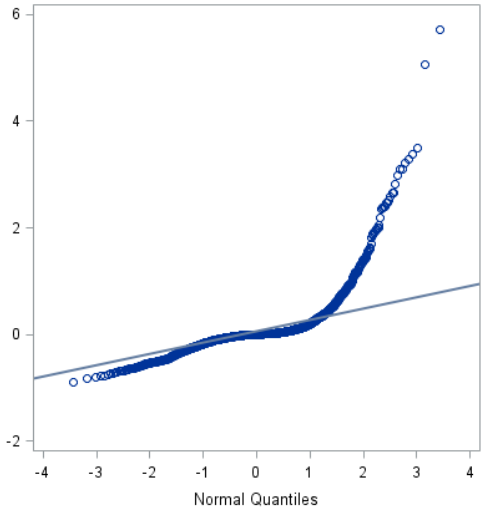
(b) Natural gas



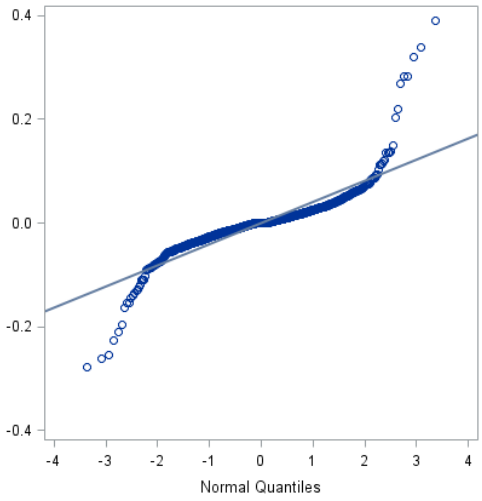
(c) Electricity

Source: Authors' calculations, based on data from
(a) https://njcleanenergy.com/renewable-energy/project-activity-reports/srec-pricing/srec-pricing-archive;
(b) https://www.eia.gov/dnav/ng/ng_pri_fut_s1_d.htm;
(c) https://www.eia.gov/electricity/wholesale/.

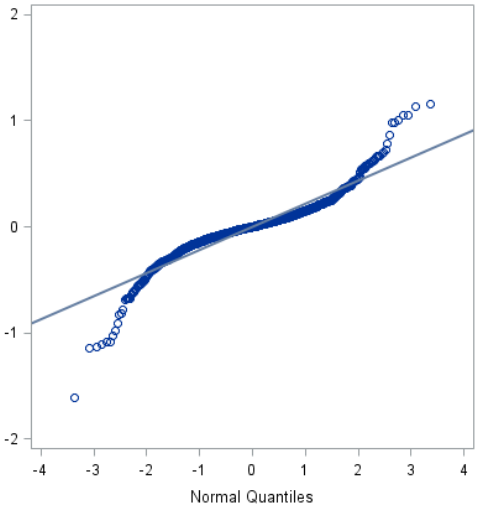
Figure 2: Quantile–quantile plots



(a) SREC



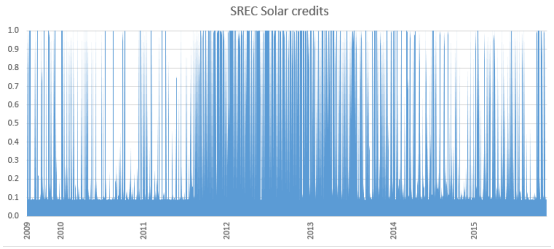
(b) Natural gas



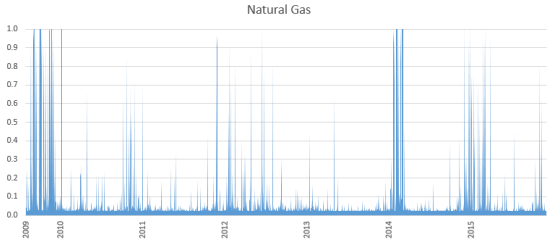
(c) Electricity

Source: Authors' calculations, based on data from
(a) https://njcleanenergy.com/renewable-energy/project-activity-reports/srec-pricing/srec-pricing-archive;
(b) https://www.eia.gov/dnav/ng/ng_pri_fut_s1_d.htm;
(c) https://www.eia.gov/electricity/wholesale/.

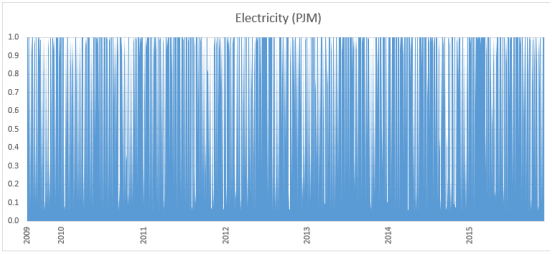
Figure 3: Fitted probabilities based on the Jump Diffusion model results



(a) SREC



(b) Natural gas



(c) Electricity

Source: Authors' calculations, based on results from Table 2.

Table 1: Summary Statistics: Solar Price Returns

Variable	Solar Returns	Natural Gas Returns	Electricity Returns
Sample Range			
Start	Aug 01, 2009	Aug 01, 2009	Aug 01, 2009
End	Nov 30, 2015	Nov 30, 2015	Nov 30, 2015
Summary Statistics			
Mean	0.0648	-0.00029	-0.0003
Median	-0.0014	0.00000	0.00000
Minimum	-0.8867	-0.27844	-1.6136
Maximum	5.7102	0.39007	1.1624
Variance	0.2137	0.00166	0.0471
Std. Dev.	0.4623	0.04071	0.2171
Coeff. of Variation	713.6	-14,172.5	-84,591.0
Skewness	4.1873	1.11170	-0.3023
Kurtosis	29.4956	20.22889	7.6107
n	2099	1599	1606
Test of Normality			
Kolmogoro Smirnov	0.2508	0.1133	0.1017
p-value	<0.01	<0.01	<0.01
Unit Root Test			
Modified Dickey Fuller	-4.806	-4.909	-4.446
1% critical-value†	-3.48	-3.48	-3.48
Lags	20	23	24

Note: Statistics are based on a sample of daily observations. Solar returns are based on SREC prices; natural gas returns based on Henry Hub spot prices; and electricity returns are based on PJM prices.
†: Critical values are based on Elliot et al. (1996).

Table 2: Estimation of the Parameters for Solar Price Returns

Variable	μ	σ	κ	α_1	β_1	λ	θ	δ
				Solar Credits				
PD	0.0648***	0.4622***	.	.	.	.	.	.
	(0.010)	(0.007)						
JD	-0.0069***	0.0536***	.	.	.	0.6673***	0.1074***	0.4540***
	(0.002)	(0.032)				(0.037)	(0.014)	(0.17)
GPD	0.0236***	.	0.0011***	0.1084***	0.8816***	.	.	.
	(0.004)		(0.000)	(0.009)	(0.009)			
GJD	0.0017	.	0.0007***	0.3899***	0.3216***	0.2065***	0.3818***	0.6744***
	(0.001)		(0.0001)	(0.058)	(0.050)	(0.019)	(0.056)	(0.045)
				Natural Gas				
PD	-0.02872	4.0695***	.	.	.	.	.	.
	(0.043)	(0.072)						
JD	-0.0529	2.3915***	.	.	.	0.0988***	0.2385***	10.3349***
	(0.046)	(0.076)				(0.019)	(0.416)	(9.63)
GPD	-0.0829	.	0.2716***	0.1404***	0.8431***	.	.	.
	(0.063)		(0.068)	(0.017)	(0.017)			
GJD	-0.0704	.	0.5013***	0.1160***	0.8010***	0.0195***	0.9577 ***	10.3349***
	(0.062)		(0.105)	(0.016)	(0.024)	(0.007)	(2.77)	(3.029)
				Electricity Prices				
PD	-0.0449	21.7033***	.	.	.	.	.	.
	(0.090)	(0.383)						
JD	0.2161	10.8724***	.	.	.	0.3169***	-0.7611	32.7599***
	(0.725)	(0.591)				(0.055)	(1.358)	(2.822)
GPD	0.3107	.	19.6518***	0.2463***	0.7350***	.	.	.
	(0.332)		(3.879)	(0.029)	(0.026)			
GJD	-0.8772	.	7.9596**	0.2219***	0.6945***	0.1654***	10.3549***	21.7218***
	(0.490)		(5.520)	(0.027)	(0.032)	(0.062)	(2.836)	(4.758)

The standard errors are presented in the parentheses.
***, **, and * indicate statistical significance at the 1%, 5% and 10% levels, respectively.

Table 3: Likelihood Ratio Test Results

Variable	$LR_{PD,JD}$	$LR_{PD,GPD}$	$LR_{JD,GJD}$	$LR_{GPD,GJD}$
Solar Credits (*SREC*)	2539.02	1714.60	570.62	1395.04
	(0.000)	(0.000)	(0.000)	(0.000)
Natural Gas (*HH*)	812.45	999.34	254.05	67.16
	(0.000)	(0.000)	(0.000)	(0.000)
Electricity (*PJM*)	540.64	634.21	246.59	153.02
	(0.000)	(0.000)	(0.000)	(0.000)

Note: Based on a sample of 2009 observations. *p-values* are given in the parentheses.