A dynamic push-pull model of equilibrium unauthorized migration*

Armando R. Lopez-Velasco†

March 21, 2026

Abstract

This paper constructs a dynamic micro-founded model for the study of equilibrium unauthorized immigration. Agent's decision to migrate considers income in their home country and expected income abroad given the realization of a migration sunk cost and taking into account the probabilities of border and inland enforcement actions that might lead to deportation in an infinite horizon context. The model produces a migration decision-rule and equilibrium unauthorized immigration flows consistent with push and pull factors of migration. Under sunk costs that are uniformly distributed the model yields an analytic solution. Empirical implications of this theory are studied with calibrated versions of the model for Mexico, Honduras, Guatemala and El Salvador for the decade of the 1990s. The quantitative exercises find support for the theory. For Mexico, stagnant unskilled wages after the peso crisis (as opposed to GDP percapita which recovered relatively quickly), combined with rising unskilled wages in the US and strong Mexican population growth explain almost 80% of the change in unauthorized immigrants during the decade.

Keywords: *Immigration Policy, Unauthorized Immigration, Illegal Immigration, Push-Pull*

JEL*: F22, J61*

---

*Declaration of interest: none. Any errors are solely the responsibility of the author of the current paper.

†Department of Economics, The University of Texas Rio Grande Valley, Edinburg, TX, 78539. Email: armando.lopezvelasco@utrgv.edu. Phone (956) 665 - 3693, Fax (956) 665-5020. https://sites.google.com/site/armandolopezvelascowebpage/research

1 Introduction

This paper develops a dynamic model to study the determinants of illegal, or unauthorized, immigration flows—an issue of central policy importance in many countries given the economic, social, and fiscal challenges associated with large migration inflows (see, among others, Alesina et al., 2023). Migration flows are widely understood to respond to a variety of forces, commonly referred to as "push" and "pull" factors (Lee, 1966), including socioeconomic conditions in origin countries, economic opportunities in destination countries, and the type and intensity of immigration enforcement policies implemented by host economies. Because destination countries typically exert only indirect and imperfect control over unauthorized migration, migration flows and stocks should be viewed as equilibrium outcomes. Changes in economic conditions or policy parameters affect individual migration incentives and behavior, which in turn jointly determine the equilibrium level of unauthorized migration observed in a given country.

Despite the vast size of the immigration literature, relatively few contributions develop fully dynamic models with explicit microfoundations for migration decisions. Recent work by Chassamboulli and Peri (2015, 2020) highlights that much of the existing analysis evaluates immigration policy through reduced-form or static neoclassical frameworks, where changes in immigration are modeled primarily as shifts in labor supply. While these approaches have been highly informative for understanding wage and employment effects, they are less suited to capturing the intertemporal trade-offs faced by potential migrants or the incentive mechanisms through which policies affect migration behavior over time. Relatedly, Ortega and Peri (2013) emphasize that progress in the field requires a deeper understanding of the microfoundations of aggregate migration flows, particularly the nature of migration costs and the channels through which immigration policies shape those costs. The present paper contributes to this agenda by developing a dynamic model of unauthorized migration that explicitly links individual decision-making, policy-driven migration costs, and equilibrium migration flows and stocks.

In the model, the agents' migration decisions are optimal ("optimal" from a dynamic programming perspective), and these decisions are endogenously consistent with push-pull factors of immigration. These factors are arguably more important for understanding unauthorized migration than legal flows since in the latter case the host countries can determine the amount of legal migration by selecting the number of permits awarded to potential migrants, while controlling unauthorized

immigration imperfectly through enforcement tools and which also interacts with the supply of unauthorized immigrants.[1]

In the model, agents are risk-neutral and live forever. In each period, foreign agents (potential migrants) observe their current migration cost, which is the realization of a random variable, and decide whether to migrate. Migration is an attempt rather than a guaranteed outcome, since agents may be stopped when attempting to cross the border unlawfully. An agent chooses to migrate if the expected present value of income as a migrant (the value of migration) exceeds the expected present value of income when remaining in the origin country (the value of staying). The migration decision is endogenous and forward-looking, incorporating rational expectations, the risk of being stopped at the border, and the probability of deportation in each period while living as an unauthorized immigrant.

The resulting dynamic problem is stationary and takes the form of an optimal-stopping problem with a recursive formulation. As such, standard dynamic programming techniques apply.

Agents located in the foreign economy—potential unauthorized migrants—choose each period whether to attempt to migrate. Depending on this decision and its outcome, they end the period in one of the following states. First, an agent may optimally choose to remain in the origin country because the expected benefits of migration do not outweigh the migration costs in the current period. Second, an agent may attempt to migrate but end the period back in the origin country because the migration attempt is unsuccessful: the agent is stopped at the border and returned, loses the migration cost, and is unable to work during that period. Third, an agent may successfully migrate, avoid being stopped at the border, losing only the migration cost but working in the host economy during the period.

Foreign agents who are living as unauthorized immigrants in the host economy (following a previously successful migration attempt) face a constant stochastic probability of returning to the sending country. This captures both the risk of deportation and the possibility of being subject to an exogenous return shock. Consequently, when starting a period as an unauthorized migrant, the

[1] In the US, for the period 2000-2021, about 67% of permanent residence permits were family-based . About 15% were given for employment-based reasons and for refugees and asylees about 13%. The remaining were given for other categories. Within the family-based category, there is no cap on permits for immediate relatives of American citizens (spouses, children younger than 21, and Parents of American citizens). Because of this, it is likely that push-pull factors would also explain the immediate relative categories as there are no institutional constraints on the number of permits. But for other categories (employment based, non immediate relatives) the number of permits available represent a real constraint and thus rationing is required (H1B visa in the form of a lottery) and other visas subjected to annual caps. See Lopez-Velasco (2023) for more information on these magnitudes.

agent faces two potential outcomes: either remaining in the host country—if not hit by the shock—and working during that period, or being deported, in which case the agent is forced to return to the origin country, does not work in the host economy during that period, and incurs deportation or relocation costs.

The ex-ante random cost of migrating captures reality in a simple way as people face different situations related to jobs, distance and other factors (e.g. health, networks, family arrangements, etc.) that makes it easier for some and harder for others to migrate at a given point in time. Agents which optimally choose to stay can be seen as waiting optimally for better conditions to migrate.

Since agents differ in each period only by their migration cost—drawn from a stationary distribution of migration costs—heterogeneity in behavior arises endogenously. For some agents it is optimal to migrate, while others optimally choose to remain in the origin country or to return due to external factors, such as enforcement actions or exogenous shocks that induce return migration. Through this mechanism, the model links the determinants of individual migration decisions to both the flows and the stock of unauthorized immigrants in the host country. For a given parameterization, the model admits a unique steady state for the share of unauthorized immigrants residing in the host economy.

In its more general formulation, without specifying a particular distribution of migration sunk costs, the model produces an equilibrium migrating decision that is homogeneous of degree zero in the income (income at the home and host countries) and cost variables (relocation costs, migration sunk cost) of the model. That is, if the incomes in the host and sending economies grow at the same rate and the migration costs are affected proportionally, then the equilibrium probability of migration stays unchanged.

The paper derives the evolution of the share of foreign population living in the host economy. At steady state, this share is a function of the threshold migration cost that defines the per-period probability of migrating. With this threshold held constant, the share is decreasing in the enforcement probabilities (the probabilities of being stopped at the border and the probability of deportation once inside the country), as well as on the probability of having to return due to some other exogenous reason; and increasing in the equilibrium migrating probability. But the threshold cost itself (as well as the implied migrating probability) is an endogenous variable and thus a function of the enforcement probabilities and of the rest of the parameters of the model.

The model assumes a uniform distribution of migration sunk costs in the sender country, as

in Chassamboulli and Peri (2015). While standard numerical techniques, like value function iteration algorithms, can solve the problem, this paper demonstrates that an analytic solution for the equilibrium migration probability exists. This is a welcome result since few infinite-horizon, optimal-stopping problems have known analytic solutions. The solution shows that the equilibrium migration probability is linear in relative income, defined as the ratio of income in the host economy to income in the sending economy. Moreover, the analytic solution reveals the interaction of different push and pull factors explaining unauthorized migration, as opposed to just a list of factors without an organizing framework for explaining migration forces nor their interactions as some social scientists have mentioned about push-pull models of migration (Skeldon, 1990).

The equilibrium probability of migration is increasing in income of the host country and decreasing in (i) income in the sending country, (ii) in enforcement probabilities, and (iii) in the probability of exogenous return. More specifically, it is increasing in relative income in the host country. This probability is also decreasing in the interest rate (under a mild and easily satisfied parametric condition), reflecting the fact that a lower discount rate raises the present value of the expected benefits from migration. In addition, the migration probability increases with transfers available in the host country and decreases with transfers available in the sending country. The same expression further shows that the sensitivity of migration to changes in relative income depends on the enforcement regime, while uniform income growth in both countries leaves the equilibrium migration probability unchanged.

The analytic expression for the equilibrium migration probability can be used to show that the rate used to discount the future and uncertain income flows implicit in the decision to migrate is the sum of the interest rate and the overall probability of return to the sender country. Put into other words, the model endogenously corrects the interest rate used to discount future uncertain income flows which define whether an agent migrates or not. This also suggests that the present value calculations are particularly sensitive to this rate.

If the model incorporates population growth, the equation describing the evolution of the share of foreigners residing in the host economy remains unchanged. What is new, however, is that—holding relative incomes constant—in the steady state, the number of unauthorized immigrants grows at the same rate as the population of the sending country. This distinction is useful when using the model to disentangle the contribution of population growth from that of other factors, such as changes in relative income, in explaining observed levels of unauthorized migration.

The paper then performs some quantitative exercises. It calibrates the model to the 1990–2000 period for Mexico, Guatemala, Honduras, and El Salvador, the main sending countries of unauthorized immigrants to the United States during this decade. Together, these countries accounted for approximately 71% of the total stock of unauthorized immigrants in the U.S. in 1990 and about 75% in 2000. Mexico alone was by far the largest source country, representing roughly 58% of the unauthorized immigrant stock in 1990. This period is particularly suitable for calibration–relative to other periods, because U.S. immigration policy remained relatively stable and the U.S. economy experienced steady growth. In contrast, the following decade was marked by significant policy and economic disruptions, including the tightening of immigration enforcement after the September 11, 2001 attacks and the macroeconomic effects of the 2008–2009 housing and financial crisis.

The paper then asks whether the model, calibrated for year 1990 is able to predict the observed stock of unauthorized immigrants in year 2000 for each of these countries by feeding changes in relative income (where relative income is initially defined as the average GDP percapita in the US in dollars divided by the same concept in the foreign country), in the presence of a given enforcement regime. The cases of Guatemala, Honduras and El Salvador all produce qualitative predictions consistent with the data, while the Mexican case appears (initially) not to be well-explained by the model.

The case of Mexico receives more attention, not only because of the sheer size of the Mexican migration flows and the importance of the Mexican border, but also because of the better quality of the data which allows for deeper analysis and additional insights. During this period, Mexico suffered from the peso crisis, which resulted in a temporary but painful recession and a corresponding (temporary) decrease in Mexican GDP percapita as well as relative to the US GDP percapita.

In the case of Mexico, as with the other countries, the model is initially fed with the decade-average relative GDP per capita, holding all other parameters unchanged. Because Mexico's GDP per capita improved on average relative to that of the United States over the decade, the model predicts a decline in the share of Mexicans aged 15–64 living in the U.S. as unauthorized immigrants—from about 9.1% to roughly 6.6%—rather than the slight increase observed in the data, to approximately 10.2%. In contrast, for the other three countries, this static, decade-average exercise yields predictions that are qualitatively consistent with the observed evolution of unauthorized migration. Nevertheless, this approach abstracts from substantial year-to-year variation in relative income and from the speed with which migration flows adjust to temporary shocks—limitations that are partic-

ularly important in the Mexican case.

The paper shows that convergence to the steady-state share of foreigners residing in the host country is relatively slow. For Mexico, the estimated speed of convergence is approximately 4.5% per year, which implies a half-life of about 15 years. As a result, assuming that the economy reaches its steady state within a single decade is likely unrealistic. Moreover, changes in relative income naturally shift the steady state itself. When the model allows for year-by-year variation in relative GDP per capita, its predictions for Mexico improve. This exercise provides a test of both the model's quantitative predictions and its dynamic implications.

In the early 1990s, prior to the 1994–1995 peso crisis, average hourly wages in Mexico grew broadly in line with GDP per capita. Following the crisis, however, average wages declined sharply—both in real terms and when measured in U.S. dollars—and failed to recover over the remainder of the decade. This stands in contrast to Mexican GDP per capita, which rebounded quickly after the crisis and resumed a sustained growth path.

As a result, relative GDP per capita between Mexico and the United States improved substantially over the period. The ratio of U.S. to Mexican GDP per capita (in dollars) fell from 7.57 in 1990 to 4.83 in 2000, implying that U.S. GDP per capita declined by approximately 36 percent relative to Mexico's. By contrast, the opposite pattern emerges for wages: the relative U.S. hourly wage (measured against Mexico's) *increased* by roughly 36 percent over the same period, rather than *decreasing*.

When the model is fed with the time series for relative average hourly wages—rather than relative GDP per capita—the model gives a better prediction. Average hourly wages for the United States are taken from Heathcote et al. (2010), while comparable data for Mexico are drawn from Binelli and Attanasio (2010). Under this specification, the model predicts that approximately 9.3% of Mexicans were living in the United States in unauthorized status by the year 2000. Although this remains slightly below the observed value of 10.2%, it represents a substantial improvement over the steady-state prediction of 6.6% and over the model that incorporates year-by-year variation in relative GDP per capita, which predicts 8.3%.

Because income measured by GDP per capita and unskilled wages diverged substantially in Mexico during the 1990s, GDP per capita is likely a poor proxy for the economic opportunities faced by potential unauthorized migrants. Since most unauthorized immigrants are likely to earn wages close to those of unskilled workers in the United States, a more relevant measure is the relative

hourly wage of unskilled labor. When using this metric, the relative unskilled wage (U.S. to Mexico) rises by approximately 73 percent over the decade—nearly double the 36 percent increase observed for average wages. In levels, this ratio was slightly above 10 in 1990, indicating that roughly ten hours of unskilled work in Mexico were required to earn the equivalent of one hour of unskilled work in the United States. By the end of the decade, the ratio had increased to about 18, substantially strengthening the economic incentives for migration from Mexico to the U.S.[2]

Feeding the model with the time series for relative unskilled hourly wages—constructed using the same data sources described above—yields the best quantitative performance. Under this specification, the model predicts that approximately 9.6% of Mexicans were living in the United States in unauthorized status by the year 2000. This prediction is very close to the observed value of 10.2% and represents a substantial improvement over the initial predictions based on steady-state assumptions or alternative income measures.

The change in Mexican unauthorized migration for the period is then decomposed into 3 different forces: population growth, change in relative GDP percapita, and the change in relative unskilled wages as compared to the evolution of GDP percapita. During the period, the total change was a net increase of about 1.8 millions of Mexicans living in the US as unauthorized immigrants, of which the model predicts an increase of about 1.45 millions during the period, almost 80% of the observed change.

The decomposition takes the following form. Due to the strong population growth of Mexico, about 1.16 million more Mexicans migrated to the US during the decade. Due to the evolution of GDP percapita (had wages kept up with GDP in Mexico), Mexicans living in the US would actually have decreased by about 556,000 nationals (due to people deciding not to migrate, as well as people who return or are deported or hit with the return shock), and the divergence between Mexican unskilled wages when compared to the evolution of GDP percapita adds additional unauthorized immigration of about 844,000 Mexicans during the decade. Hence both population growth and the unequal evolution of wages in Mexico relative to GDP percapita, together with the continued increases in US wages end up explaining the change in the stock of unauthorized immigrants from Mexico during that decade.

This paper is related to several strands in the immigration literature. It is related to the literature

[2]This income ratio does not correct for the fact that unauthorized immigrants receive a lower amount (about 14 to 24% less according to Kossoudji and Cobb-Clark (2002)), than if they had work permits (legal migrants).

on the "push" and "pull" factors of immigration, terms coined in the classic paper of Lee (1966) when he describes how immigrants are attracted to other countries due to better economic opportunities, as well as pushed from their countries due to adverse conditions (economic or otherwise). While the particular "push" and "pull" words were not used until the work of Lee, some of the ideas were implicit in the work of others. For example, Adam Smith's work (1776) discusses internal migration in the form of labor mobility as a response to wage/opportunities differences between regions, as well as a response to poverty, and criticized legal barriers to migration. Similarly, Ravenstein (1889) discussed extensively about forces that today are known as push and pull factors, including lack of jobs, low wages, political instability, natural disasters, economic opportunities, among other forces. The renowned Harris-Todaro (1970) model also considers migration from rural to urban cities as a phenomenon that depends mainly on expected income differences, which is of course one of the main push-pull factors of migration.

While push-pull factors of migration help motivate and rationalize the findings in many empirical papers (see for example Ortega and Peri (2009, 2013) for general migration or Orrenius (2001) for unauthorized migration), some social scientists like Skeldon (1990) and more recently de Haas (2021) have argued that albeit these factors should matter in some measure to migration flows, they criticize that they are not part of a coherent "explanatory system". Ortega and Peri (2013) have also mentioned how the literature lacks microfoundations in the empirical specifications that end up identifying push and pull factors of migration (e.g. in the "gravity" specifications of migration flows, see references in Ortega and Peri (2009)). Thus the first contribution of the current paper is to provide a model which can formalize the different push and pull factors of migration with explicit microfoundations. In doing so, the paper offers new insights and quantitative implications.

An additional contribution of the paper is to decompose the increase in the number of Mexicans living in the USA during the 1990's. Broadly speaking, the model shows that strong Mexican growth of the labor force during the period and the evolution of the unskilled wages end up explaining about 1.45 millions of additional Mexicans in the US (compared to about 1.8 millions observed during the period), while population growth and the evolution of GDP percapita would explain only about 600,000 additional Mexicans living in the US.

The closest dynamic models related to the proposed model are probably the small recent literature using the search-matching framework for immigration questions, which includes the work of Ortega (2001), Liu (2010), Chassamboulli and Palivos (2014) and Chassamboulli and Peri (2015,

2020). This literature focuses mostly on understanding labor markets issues of immigration. Even though the current paper is closely related in some areas (a dynamic decision that requires the use of dynamic programming methods), the current paper does not use the matching framework for several reasons.

First, within search-matching models, general equilibrium effects of immigration, along with the famous "immigration surplus" (see Borjas, 1995), tend to dissipate relatively quickly while capital adjusts. As shown in dynamic assessments of the immigration surplus (Ben-Gad, 2004, 2008), these gains are largely transitory, a conclusion reinforced by broader surveys of the literature (Blau and Mackie, 2017). Moreover, as emphasized in Lopez-Velasco (2025), U.S. enforcement expenditures substantially exceed the magnitudes typically attributed to the immigration surplus, further limiting its relevance as a central organizing mechanism for unauthorized migration dynamics.

Second, search-matching models of immigration typically emphasize steady state comparisons, whereas the present model implies a relatively slow speed of convergence (between 4.1% and 4.5% for Mexico, Guatemala, and Honduras) and hence it is important to analyze the model out of steady state, which the current framework is designed to capture and analyze.

Third, the present paper focuses on additional determinants of the migration decision that extend beyond labor-market considerations. These include other push and pull factors—such as transfers associated with the welfare state—that are typically absent from search-matching models of immigration. A notable exception is Battisti et al. (2017), who introduce a welfare state into a matching framework; however, in such setting migration flows are taken as exogenous. More broadly, when sending countries are small relative to the host economy, country-level unauthorized migration flows are unlikely to generate meaningful effects on host-country labor-market outcomes, even in the short run, further reducing the relevance of a matching-based general equilibrium approach for the questions studied here.

The paper is also related to the literature on immigration enforcement including Ethier (1986), Bond and Chen (1987), Guzman et al. (2008), Bandyopadhyaya and Pinto (2017), Basu et al. (2022) and Lopez-Velasco (2025). The enforcement aspects on the majority of these papers is different, as models in this literature are static. Notable exceptions include Guzman et al. (2008) which uses dynamic analysis in the form of studying an overlapping generations model with capital accumulation and a smuggling technology of unauthorized immigrants, and the paper by Basu et al. (2022) which models enforcement as a probability of being stopped and returned at the border in

the context of circular migration in an infinite-horizon environment. Similarly, the search-matching model of Chassamboulli and Peri (2020) incorporates a probability of deportation, an element that is conceptually related to the enforcement mechanism in the present framework.

The paper is organized as follows. Section 2 introduces the model and presents several propositions from the theory. Section 3 presents the parameterization of the model and performs some quantitative experiments. Section 4 contrasts the predictions of the model and compares them to observed data. Section 5 performs a decomposition of the forces explaining the evolution of the Mexican population living as unauthorized immigrants. Section 6 presents sensitivity analysis. Section 7 concludes. Most proofs are relegated to the Appendices.

2 The Model

There are two locations, a host/developed economy and a foreign/developing economy. Agents living in the host economy earn a constant wage w each period, are taxed at some exogenous rate $0 < \tau < 1$ and receive some transfer $b \geq 0$.[3] Net income of natives in the host economy is defined as $y \equiv w(1-\tau) + b$ and foreigners who stay in their home (sending) country earn $\widetilde{y} = \widetilde{w}(1-\widetilde{\tau}) + \widetilde{b}$, where a tilde (e.g. $\widetilde{y}$) is used to represent variables for agents of foreign origin in their home country.

Let k be defined as the sunk cost of migration. This cost is realized at the beginning of the period before a migration decision is made and depending on its realization agents optimally decide whether to attempt to migrate or to stay. The heterogeneity in costs captures the idea that agents have different situations related to distinct networks, family commitments, psychological costs of migrating, etc. This cost is the realization of an identically and independently distributed (*i.i.d.*) non-negative random variable with a non-degenerate cumulative distribution function (CDF) given by $F(k)$.

If an agent realizes a low-enough cost which induces the agent to attempt to migrate, there is a probability q that the agent will be stopped at the border and deported immediately, resulting in no wages earned during the period and the loss of the migration sunk cost.[4] If the agent is able

[3] The current model assumes no wage effects from immigration, which is arguably appropriate when analyzing migration from small economies to a large developed one. In addition, the "immigration surplus" (Borjas, 1995) is temporary and empirically small compared to the cost of border enforcement (see Lopez-Velasco, 2025).

[4] According to Orrenius (2001), about 70% or more of unauthorized migrants from Mexico were using coyote services in the 1990's. One of the implications of this is that the majority of those people coming to the US did not have some visa (e.g. tourist) that they could use to overstay. Thus the importance of including enforcement at the border, in addition to deportation once in the inside.

to migrate– thus not stopped at the border, the agent earns a fraction $0 < \lambda < 1$ of the typical native's income during each period the agent is able to stay as an unauthorized immigrant. This reflects that unauthorized immigrants do not fully qualify for transfers in the host country and that they might participate in the underground economy among other things.[5] The paper uses the term "unauthorized immigration" throughout the paper and is taken to mean the same thing as "illegal immigration".

Each period, unauthorized immigrants face a constant probability p of deportation and a probability e of returning home due to an exogenous shock. In each of these cases, having to return exogenously implies a cost for the agents, given by $k_D > 0$, in addition to the opportunity cost of not being able to work in the host country during that period. The total probability of exogenous-return to home is defined as $d \equiv (p + e)$.

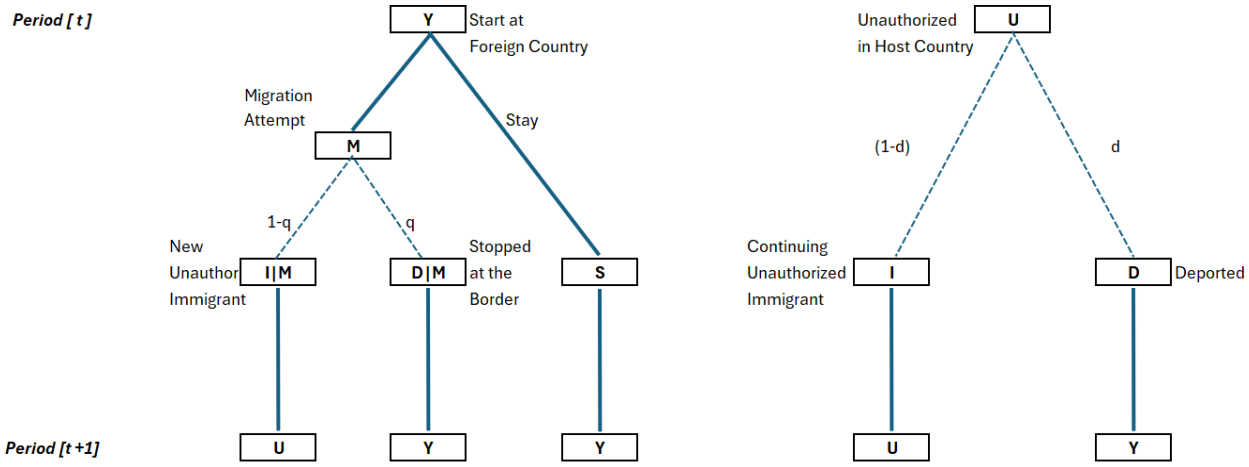
Agents are risk-neutral and live forever, with utility discount factor $\beta \equiv \frac{1}{1+r} < 1$ for some exogenous interest rate $r > 0$, and utility given by $E\sum_{0}^{\infty}\beta^t c_t$ where t is time period, E is the mathematical expectation and c_t is consumption at time t.

At the individual level, the decision to migrate is of the optimal-stopping-time type, and depends on the realization of the migration sunk cost k. The recursive structure of the problem allows for dynamic programming tools as developed in the context of search theory (see excellent expositions of these models in McCall and McCall (2007) or Ljunqvist and Sargent (2018)) to be used.

Figure 1 illustrates the possible states in which an agent from the foreign country can be and its timing. If an agent starts the period at the sending country (node Y), the agent decides whether to stay (S) or whether to attempt to migrate (M). If the agent attempts to migrate, either the agent is succesful in crossing the border undetected, denoted by state $I|M$, or the agent is stopped in its attempt and is thus precluded from migrating in state $D|M$. An agent that either starts as unauthorized immigrant (starts at node U), or arrived the previous period (coming from state $I|M$) can remain in the country undetected and thus working as unauthorized immigrant in state I, or is deported/returned with some probability in state D. Agents who stay (from state S in the previous period), agents who are stopped at the border and returned (from state $D|M$), or agents who are deported (from state D) all start at node Y at the beginning of the next period.

Figure 1. Potential states for foreigners living in sending or in host countries

[5] See Kolker (2022) or Lopez-Velasco (2025) for a discussion on the specific taxes and benefits that unauthorized immigrants participate/qualify in the US.



The value for an agent who optimally stays in the current period is given by

$$V(S) = \widetilde{y} + \beta E\left\{\max\left[V(S), V\left(M, k'\right)\right]\right\}, \tag{1}$$

where $\widetilde{y}$ is the income from working in their home country, $V(S)$ is the value of staying, $V(M, k')$ is the future value of migrating and the term in key brackets is the expected discounted value of next period's migrating decision if the agent stays in the current period.

For an agent that attempts to migrate in the current period, the value is

$$V(M, k) = qV(D|M, k) + (1 - q)V(I|M, k), \tag{2}$$

where q is the probability of being arrested and returned immediately and $V(D|M, k)$ is the corresponding value, where the agent foregoes the migration sunk cost k. In turn, $V(I|M, k)$ is the value when the agent successfully migrates in the current period and receives income λy. These values are given by

$$V(A|M, k) = -k + \beta E\left\{\max\left[V(S), V\left(M, k'\right)\right]\right\}, \tag{3}$$

$$V(I|M, k) = -k + \lambda y + \beta\left\{dV(D) + (1 - d)V(I)\right\}. \tag{4}$$

The term $\{dV(D) + (1 - d)V(I)\}$ in (4) is the expected utility for a current unauthorized immigrant to continue working in an unauthorized fashion or return home/being deported in the

next period.

The value of an unauthorized immigrant who remains in the host country—thus not deported, in the current period is

$$V(I) = \lambda y + \beta \{d * V(D) + (1-d) * V(I)\}. \tag{5}$$

Finally, the value of exogenously having to return to their country of origin (whether for deportation or being hit with the exogenous return shock) is

$$V(D) = -k_D + \beta E \left\{ \max \left[V(S), V\left(M, k'\right) \right] \right\}, \tag{6}$$

where the agent loses the deportation/relocation cost k_D that captures the significant costs (economic, psychological, etc.) of having to suddenly come back and the discounted value of the future migration choice once in their home country.

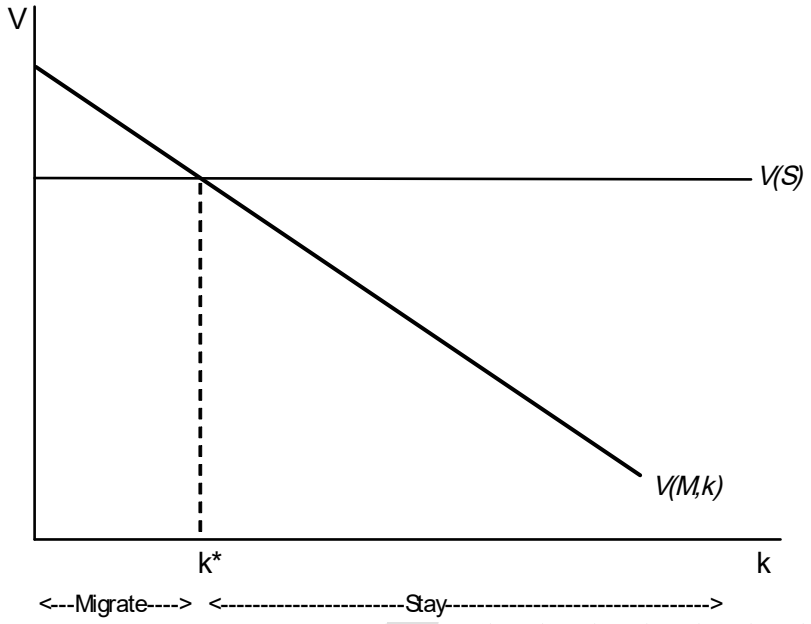
2.1 The Decision to Migrate

Agents in the developing country migrate if $V(M,k) \geq V(S)$, where the value of migrating $V(M,k)$ is decreasing in k (see Appendix). The decision depends on some threshold migration cost k^*, defined as the cost which which makes an agent indifferent between migrating and staying, $V(M,k^*) = V(S)$. This decision is summarized as

$$\begin{aligned} Migrate\ if\ k \leq k^*, \\ Stay\ if\ k > k^*. \end{aligned} \tag{7}$$

The value $V(M,k)$ is a straight line with negative slope in the space of (k,V), while $V(S)$ is some constant. Figure 2 presents a sketch of these values and implied optimal decision rule. Since $V(M,k)$ is decreasing in k (see Appendix A.1) , while $V(S)$ does not depend on the sunk cost k, $V(M,k)$ and $V(S)$ can cross at most only once, depending on the particular parameterization, thus guaranteeing that if a solution exists, the threshold k^* is unique.

Figure 2. The Migration Decision Rule



The threshold k^* is implicitly defined by the following expression (see Appendix A.1 for derivation)

$$k^* = \left\{ \frac{(1-q)\,\lambda y - (1-q)\,\beta d k_D}{[1-\beta\,(1-d)]} \right\} - \left[1 + \frac{\beta\,(1-q)}{[1-\beta\,(1-d)]} \right] \widetilde{y} + \left[\frac{\beta\,(1-q)}{[1-\beta\,(1-d)]} \right] F\left[k^*\right] \left\{ E\left[k | k \leq k^*\right] - k^* \right\}, \tag{8}$$

which is easier to understand if β is replaced by $\frac{1}{1+r}$, in which case the above expression can be written as

$$k^* = \frac{(1-q)\left[\lambda y\,(1+r) - k_D d\right]}{r+d} - \widetilde{y}\left[1 + \frac{1-q}{r+d}\right] - \left(\frac{1-q}{r+d}\right) F\left[k^*\right] \left\{ k^* - E\left[k' | k' \leq k^*\right] \right\}. \tag{9}$$

The terms can be understood in the following way. The term $\left(\frac{1-q}{r+d}\right) \lambda y\,(1+r)$ represents the expected present value of current and future incomes ($EPV\left[IncomeUnautho\right]$) of an unauthorized immigrant, taking into account the uncertainty about crossing the border as they can be stopped and returned in the attempt, as well as the possibility of deportation/exogenous return each future period the agent remains in the host economy. Even though d is not part of the preference discount factor (as utility flows are discounted by the factor $\beta = \frac{1}{(1+r)}$), the recursive structure of the problem makes the optimal decision discount the expected future income flows with the discount rate $(r+d)$.

Hence the higher r+d are, the lower the value placed on the future (and uncertain) income flows.

The term $\frac{(1-q)dk_D}{r+d}$ represents the present value of the expected period-loss in income due to the possibility of deportation/exogenous return (denoted by $EPV\left[DeportLosses\right]$). The term $\widetilde{y}\left[1+\frac{1-q}{r+d}\right]$ represents the present value of domestic income when the relevant discount rate is $(r+d)$, denoted by $EPV[IncomeHome]$. Finally, the term $\left(\frac{1-q}{r+d}\right)F\left[k^*\right]\left\{k^*-E\left[k'|k'\leq k^*\right]\right\}$ represents the expected present value of the benefits of waiting to migrate (a benefit that is lost if the person migrates in the current period).

The optimal policy to this dynamic program is such that the total expected benefits of migration are higher than the total opportunity costs of staying in their home country, where these calculations take into account both the time-value of money and the uncertainty of these flows. The decision to migrate (7) can alternatively be stated as the following: Agent with current cost k migrates if

$$EPV\left(IncomeUnautho\right)\geq EPV\left(DeportLosses\right)+EPV\left(IncomeHome\right)+EPV\left(Waiting\right)+k$$

Since agent types are indexed by their realization of the migration sunk cost k^*, and because in any period all agents with $k<k^*$ choose to migrate, the probability that a randomly sampled agent decides to migrate is $F\left[k^*\right]$. By the law of large numbers, $F\left[k^*\right]$ also represents the share of agents in the sender country who attempt to migrate in that period. Taking into account the probability of being stopped at the border, the fraction of agents in the population who successfully migrate is thus given by $(1-q)F\left[k^*\right]$.

An important implication of equation (9) is now stated.

Proposition 1 *Given parameters* $\left\{w,\widetilde{w},\tau,\widetilde{\tau},b,\widetilde{b},\lambda,\beta,k_D,d\right\}$, *and given a non-degenerate cumulative distribution function* F *such that a threshold* $k_0^*>0$ *exists and which implies the existence of an equilibrium migrating probability* $F\left[k_0^*\right]>0$, *then scaling the income* $\{y,\widetilde{y}\}$ *and cost parameters of the model* $\{k,k_D\}$ *by some factor* $\gamma>0$ *yields a scaled equilibrium threshold cost* γk_0^* *and an unchanged equilibrium probability of migration.*

Proof. See Appendix A2.

The above proposition states that when income $\{y,\widetilde{y}\}$ in each country grows by the same factor, together with the assumption that the costs of migrating k and the deportation cost k_D increase by

the same growth factor, then the equilibrium migrating probability is unchanged. This is the first quantitative statement that one can obtain from the current model.

Push-pull principles say that certain factors tend to "push" people out of their countries and "pull" factors tend to attract people into a host country. But there is no quantitative statement attached to these ideas, only an expected sign of a relationship along the lines of "higher expected income at the host destination tends to attract migrants". Proposition 1 offers a quantitative relationship between income growth in foreign and sender countries (ceteris paribus) and the equilibrium migration probability, albeit one that is very restrictive in the sense that there is no quantitative statement outside of assuming this particular case of equal growth. Additional structure into the model in the following sections provide additional insights.

2.2 The evolution of the stock of unauthorized immigrants

The model has implications for the evolution of the stock of immigrants in the host economy. Let $\widetilde{N}_t$ be defined as the number of foreigners staying in their own country and U_t be defined as the number of unauthorized immigrants residing in the host economy. Then the total population of foreign origin is $\widetilde{N}_t + U_t$.

The number of agents who optimally choose to stay is $(1 - F[k^*])\,\widetilde{N}_{t-1}$. The total number of agents returning home (previously classified as unauthorized immigrants) due to enforcement actions or exogenous reasons is given by $d * U_{t-1} \equiv (p + e)\,U_{t-1}$. The number of potential migrants who are stopped at the border and returned to their home country is $qF(k^*)\,\widetilde{N}_{t-1}$. The number of newly arrived unauthorized immigrants (successfully crossing the border) in the current period is $(1 - q)\,F(k^*)\,\widetilde{N}_{t-1}$, while the number of unauthorized immigrants that continue residing in the host country is given by $(1 - d)\,U_{t-1}$. Hence, evolution of $\widetilde{N}_t$ and U_t is given by

$$\widetilde{N}_t = (1 - (1 - q)\,F(k^*))\,\widetilde{N}_{t-1} + dU_{t-1},$$

$$U_t = (1 - d)\,U_{t-1} + (1 - q)\,F(k^*)\,\widetilde{N}_{t-1}, \tag{10}$$

where for simplicity of the exposition the number of foreigners and natives is assumed constant (the case with population growth is relegated to the Appendix), with

$$\widetilde{N}_t + U_t = \widetilde{N}_{t-1} + U_{t-1}. \tag{11}$$

Define $\widetilde{\mu}_t \equiv \frac{U_t}{U_t + \widetilde{N}_t}$, which is the percentage of the foreign country population living as unauthorized immigrants in the host country. Then equations (10) and (11) imply that $\widetilde{\mu}_t$ evolves according to

$$\begin{aligned} \widetilde{\mu}_t &= (1-d)\,\widetilde{\mu}_{t-1} + (1-q)\,F\left[k^*\right](1-\widetilde{\mu}_{t-1}) \\ &= (1-q)\,F\left(k^*\right) + \left(1 - \left[d + (1-q)\,F\left(k^*\right)\right]\right)\widetilde{\mu}_{t-1}, \end{aligned} \tag{12}$$

which is linear and globally stable since $0 < \left|\frac{d\widetilde{\mu}_t}{d\widetilde{\mu}_{t-1}}\right| = \left|1 - \left[d + (1-q)\,F\left(k^*\right)\right]\right| < 1$. Convergence to the unique steady state is monotonic if $0 < \left[d + (1-q)\,F\left(k^*\right)\right] < 1$ and occurs in cycles otherwise. The steady state is therefore given by

$$\widetilde{\mu}_{ss} = \frac{(1-q)\,F\left(k^*\right)}{d + (1-q)\,F\left(k^*\right)} \equiv \frac{(1-q)\,F\left(k^*\right)}{(p+e) + (1-q)\,F\left(k^*\right)}. \tag{13}$$

The steady state share of unauthorized immigrants ($\widetilde{\mu}_{ss}$) is increasing in k^*; that is, the higher the migration-cost threshold is, the higher the probability to migrate (a higher supply of migrants) and therefore the bigger the share of foreigners living as unauthorized immigrants in the steady state. Conditional on a particular k^*, $\widetilde{\mu}_{ss}$ is decreasing in the enforcement probabilities q, p, and on the exogenous return shock e (and therefore on d). But since k^* is an implicit function of all parameters, changing these probabilities (or other parameters) also affect the threshold k^*. This dependence is addressed later in proposition 4.

Using the results of proposition 1 and equation (13), when both the sender and host countries grow at the same rate, then the share of unauthorized immigrants living in the host economy should stay constant in the long run. The "long run" qualifier is important since the initial condition of the migration share (12) might be out of steady state, where in this case growing at the same rate implies that the transition towards the steady state share (13) is unaffected by growth.

The model under population growth is analyzed in Appendix A.3. Broadly speaking, whether population growth is present or not, equation (13) continues to apply. What changes under population growth is that an alternative measure—the share of unauthorized immigrants expressed as a

percentage of the host country's population—is no longer stationary.

2.3 The probability to migrate under uniformly distributed migration costs

The expressions above in equations (9) and (13), as well as proposition 1 are very general, as they hold for any non-degenerate probability density function and parameterization that yields a positive probability. But additional insights can be obtained if the migration sunk-cost is assumed to be distributed uniformly as in Chassamboulli and Peri (2015), with $k \sim U[0, k_{\max}]$ and where the lowest cost to migrate is normalized to 0 and the maximum cost is given by some constant $k_{max} > 0$. These assumptions imply that $F[k^*]$ is given by $F[k^*] = \int_0^{k^*} \frac{dk}{k_{\max}} = \frac{k^*}{k \max}$. Then the term $F[k^*] E[k'|k' \leq k^*]$ is given by $\int_0^{k^*} k' dF(k') = \int_0^{k^*} \frac{k'}{k_{\max}} dk' = \frac{1}{k_{\max}} \left[\frac{(k^*)^2}{2}\right]$. Hence the term $F[k^*] \{k^* - E[k'|k' \leq k^*]\}$ under these assumptions is given by

$$F[k^*] \left\{k^* - E\left[k'|k' \leq k^*\right]\right\} = \frac{1}{2} \frac{(k^*)^2}{k_{\max}}. \tag{14}$$

Replacing equation (14) into (9) yields a quadratic equation in k^*, which can therefore be solved analytically. The result of this is formalized in the following proposition.

Proposition 2 *If the sunk-cost of migration k is i.i.d. uniform in $[0, k_{\max}]$, the threshold level k^* and the equilibrium probability of attempting to migrate $F[k^*]$ are analytic expressions, given by*

$$k^* = k_{\max} \left(\frac{r+d}{1-q}\right) \left\{ \left[1 + \frac{2(1-q)}{r+d} \left(\frac{(1-q)[\lambda y(1+r) - dk_D] - \widetilde{y}[r+d+(1-q)]}{k_{\max}(r+d)}\right)\right]^{\frac{1}{2}} - 1 \right\}, \tag{15}$$

$$F[k^*] = \left(\frac{r+d}{1-q}\right) \left\{ \left[1 + \frac{2(1-q)}{r+d} \left(\frac{(1-q)[\lambda y(1+r) - dk_D] - \widetilde{y}[r+d+(1-q)]}{k_{\max}(r+d)}\right)\right]^{\frac{1}{2}} - 1 \right\}. \tag{16}$$

Proof. See Appendix A.4.

Although numerical methods—such as value-function iteration—are commonly employed to solve the recursive problem given by equations (1) to (7), the availability of analytic solutions for the threshold k^* and the corresponding equilibrium probability $F[k^*]$ is notable. This result is especially striking given the scarcity of closed-form solutions in infinite-horizon dynamic programming

problems. Analytic solutions transparency for understanding the economic mechanisms and trade-offs at play.

For an easier expression of the threshold value and probability, it is possible to simplify equation (16), by using the Taylor approximation to the function $f(x) = x^{\frac{1}{2}}$ around $x = 1$, which is given by $f(x) \approx \frac{1}{2}(1 + x)$. Appendix A.5 shows that the equation is approximately given by

$$F[k^*] \approx \frac{(1-q)[\lambda y(1+r) - dk_D] - \widetilde{y}[r + d + (1-q)]}{(r+d)k_{\max}}. \tag{17}$$

In the above expression, the cost concepts $k_{\max}$ and k_D warrant further discussion. First, the maximum migration cost k_{max} parameterizes the whole distribution of migration costs and can be thought of parameterizing the average migration cost in the country. In the context of a sending economy with a changing income, this cost is unlikely to remain constant. For simplicity and following one of the implications of proposition 1, this cost defined as proportional to local income in the sending economy, with $k_{max} = \kappa\widetilde{y}$ for some constant $\kappa > 0$. In words, this assumption intends to capture the value of amenities and other location-specific goods, which would naturally be valued at a higher level when local income is higher, among other things. Similarly, the cost that agents incur in case of deportation is intended to capture, for example, the loss of assets (not explicitly modeled) when an agent is unexpectedly deported. This cost is therefore assumed to be proportional to the income in the host country where $\sigma *$ (income unauthorized agent), given by $k_D = \sigma * \lambda y$, for some constant $\sigma > 0$.

In the absence of these assumptions, that is, if k_D and $k_{\max}$ were not assumed proportional to those income levels, then identical growth in the sending and host countries would imply an ever increasing equilibrium share of unauthorized migration. Thus balanced growth in both countries yields a constant probability of migration (and constant steady state share of unauthorized immigrants) only if costs are assumed proportional to income variables. These assumptions ($k_{max} = \kappa\widetilde{y}$ and $k_D = \sigma\lambda y$) produce a slightly simpler expression of (17), formalized in the following proposition.

Proposition 3 *Under the assumption that the deportation cost k_D is proportional to unauthorized immigrant's income λy (with $k_D = \sigma\lambda y$) and parameter that defines the moments of the uniform distribution given by k_{max} is proportional to $\widetilde{y}$ ($k_{max} = \kappa\widetilde{y}$), then the equilibrium probability of*

migration is approximately linear in relative income $\left(\frac{y}{\widetilde{y}}\right)$, *given by*

$$F[k^*] \approx \frac{-(1+r+d-q)}{(r+d)\kappa} + \left[\left(\frac{1-q}{r+d}\right)\left(\frac{\lambda}{\kappa}\right)(1+r-d\sigma)\right]\left(\frac{y}{\widetilde{y}}\right). \tag{18}$$

Proof. See Appendix A.5.

Propositions 2 and 3 are much more specific than proposition 1, where proportional increases in income and cost variables of both countries have no effect on the equilibrium migrating probability. Equation (18) allows for easy evaluation of the different factors that determine the equilibrium probability of migration. Comparative statics with respect to all parameters are formalized in proposition 4.

Proposition 4 *The equilibrium probability of migrating* $F[k^*]$ *is increasing in (1) relative income* $\left(\frac{y}{\widetilde{y}}\right)$ *and (2) in the parameter* λ *which captures percentage of native's income that is available for unauthorized immigrants. The equilibrium probability of migrating is decreasing in (i) the parameter* κ *which governs the average sunk cost of migrating, (ii) the probability of being stopped and repatriated at the border* q*, (iii) the parameter controlling the cost of deportation* (σ)*, (iv) the total probability of return/deportation* $d \equiv p + e$ *and (v) in the interest rate* r *if the parameters are such that* $d(\sigma+1) + \frac{\widetilde{y}}{\lambda y} < 1$ *(thus increasing in* β *when* $d(\sigma+1) + \frac{\widetilde{y}}{\lambda y} < 1$*).*

Proof. See Appendix.

The only parameter which requires some parametric condition in order to sign the effect is the interest rate r. In the empirical applications that follow, the condition was always satisfied easily so that the probability to migrate was decreasing in r.

3 Parameterization/Calibration

The paper parameterizes the model for four countries that historically have represented the biggest number of unauthorized immigrants in the US: Mexico, Guatemala, Honduras and El Salvador where some parameters are common to all countries. The assumptions underlying the model are more likely to be satisfied for relatively small source countries and for a large host economy such as the United States, where general equilibrium effects can be safely ignored for Guatemala, Honduras, and El Salvador. Mexico, which is substantially larger than the other sending countries and represents the most important source of unauthorized immigration to the U.S., is nevertheless included. While

general equilibrium effects may be more relevant in Mexico's case, existing work on the dynamic "immigration surplus" suggests that such effects are likely to be small and transitory (see, for example, Ben-Gad, 2004, 2008), particularly given that each period in the model corresponds to one year rather than a month. Moreover, key data are considerably harder to obtain for the smaller countries, and the measurement error in unauthorized migration flows is likely to be substantially higher than in the case of Mexico.

There are several reasons why the chosen period is appropriate for analysis. First, official data on unauthorized immigrants is scant before the 1990's. The decade of the 1990's saw a relatively calm period of steady growth for the US, and it had (arguably) a fairly stable immigration enforcement regime. There is also relatively good quality data on unauthorized immigration flows from Warren and Warren (2013) that allows for the computation of probabilities q, p and e for the relevant period. By contrast, the period following the terrorist attacks of September 11, 2001, complicates the analysis considerably, as immigration enforcement policies and practices likely underwent substantial and possibly time-varying changes in response to heightened security concerns

Relatively accurate data on unauthorized immigration flows at the country level is hard to obtain. However, data for the relevant countries is available for both 1990 and 2000 from the Department of Homeland Security (DHS), which is discussed later.

For the parameterization of the model, the paper first discusses the relevant probabilities that can be directly measured: 1) the probability of being stopped at the border and returned (q), 2) the probability of inland enforcement actions and deportation (p), and 3) the probability of having to return to the home country for an exogenous reason (e), which are now described.

For q, Espenshade (1990) provides earlier estimates (previous to the 1990's) for Mexican immigrants of a probability of being stopped at the border of 50%, but fluctuating between 40% and 60%. Espenshade mentions that this probability increased after IRCA was passed in 1986. Using data of Warren and Warren (2013) yields a probability fluctuating between 60% and 70% for the period 1990 -2000, and an average level of 65%. In the experiments that follow, this parameter is set at a level of 65%, while sensitivity analysis is later performed for an alternative probability level of 50%.

For the probability of inland enforcement actions p, this number fluctuates between 0.8% and 1.37% per year for the decade, and an average of 1.03%. In the case of e (the exogenous emigration rate), this rate fluctuates between 2.51% and 2.94%, with an average of 2.65%. Since the model

requires the sum of p and e (given by d), this sum yields 3.68%. Chassamboulli and Peri (2015) estimate the sum of these 2 probabilities ($p + e$) at a level of 4.5% using different sources and assumptions for a different period (2001-2009). The paper sets $p + e \equiv d = 4\%$ and considers alternative values in the sensitivity section.

Since from equation (18) what matters is relative income for the calibration of the model, the level of income in the host economy is normalized to a level of $y = 100$. In 1990 GDP percapita in current US dollars was \$3,154 in Mexico and \$23,889 in the US, thus 7.57 times the GDP percapita in Mexico and which implies a model parameter for the foreign economy (Mexico) of $\widetilde{y} = 13.20$. Following Chassamboulli and Peri (2015), the annual interest rate is set at 5% per year, implying a parameter β given by $\frac{1}{1.05}$.

According to Kossoudji and Cobb-Clark (2002), unauthorized immigrants had a wage penalty between 14% and 24% relative to workers with legal status in their sample, implying the parameter λ to be between .76 and 0.86. The paper sets $\lambda = 0.8$ but other values are considered in the sensitivity section.[6]

The deportation cost faced by unauthorized immigrants is represented by the parameter σ (where σ is the ratio of the deportation/relocation cost to income as unauthorized worker, $\sigma = \frac{k_D}{\lambda y}$). Empirical information on the magnitude of this cost is extremely limited. Deportation often entails substantial direct and indirect burdens: many households in the United States include members without legal status, and the removal process frequently involves extended periods in detention (see, for example, Donato et al. 1992), as well as the loss of assets or the need to liquidate them quickly. Reliable numerical estimates of these costs—whether related to forgone income, legal expenditures, or the psychological toll associated with detention and family separation—are largely unavailable. For the purposes of the quantitative analysis, this cost is set to one period of forgone earnings in the host economy ($\sigma = 1$) which, with annual data is perhaps a large number. Sensitivity analysis later shows that this assumption is practically irrelevant (whether this cost is 0 or 200% of the annual earnings in the host country) on the model's predictions once the model is calibrated to target shares of unauthorized immigrants (see sensitivity section).

Conditional on the rest of the parameters, choosing a higher upper bound (as governed by κ)

[6] Unauthorized immigrants do not qualify for many public programs and some of them do work in jobs that do not report income taxes, though they do pay other type of taxes. Following the discussion in Lopez-Velasco (2025), taxes paid by unauthorized immigrants are assumed to be about the same size as the benefits they can qualify for. This implies that λ can be identified with just the wage penalty that unauthorized immigrants pay for their legal status. For details on transfers that unauthorized immigrants do not qualify, see Kolker (2022).

for the migration sunk cost leads to a higher average cost of migration and from expression (13) this implies a lower probability of migrating and a smaller fraction of the population living as unauthorized immigrants in the host country (lower $\widetilde{\mu}_{ss}$). Hence, it is possible to select κ to target the steady state share $\widetilde{\mu}_{ss}$.

For the computation of the share of unauthorized Mexicans (relative to the Mexican population) living in the US, start with the reported number of unauthorized Mexicans in 1990 at almost 2 million (see appendix for details on these flows). This number ignores the very significant number of Mexican nationals which obtained legal status through IRCA legislation in 1986, about 2.3 million (See Zong and Batalova (2014)) who had been living in the US for at least 3 years (among other requirements in the law) in order to gain legal status.[7] Assuming that those individuals who were already in the US would remain there in the absence of such legalization program, the total number of foreigners in 1990 would have been about 4.3 million Mexicans.[8] Dividing this number by the population aged 15-64 in Mexico of 47,094,701 yields a share of about 9.13% of the Mexican population living in the US in 1990. [9]

Table 1. Calibration of Model for Mexico

Parameter	*Value*	*Target/Source*
q	65%	Average, data Warren and Warren (2013) and DHS
$d \equiv p + e$	4%	Average, data Warren and Warren (2013) and DHS
λ	0.8	Kossoudji & Cobb-Clark (2002): wage penalty between 14 and 24%
β	.952381	$r = 5\%$ annual interest rate as in Chassamboulli and Peri (2015)
k_D	80	Annual income at host $(\lambda * y)$
κ	1645.6	Calibration targeting $\mu_{ss} = .0913$
y, w	100	Normalization
$\widetilde{y}, \widetilde{w}$	13.21	Ratio US to Mexico GDP percapita \$23,889/\$3,154
τ	28.3	Avg consumption+labor tax US (Carey and Rabesona, 2002)
$\widetilde{\tau}$	22	Avg consumption+labor taxes for Mexico (Anton, 2005)
b	28.3	Endogenous from y and τ
$\widetilde{b}$	2.88	Endogenous from $\widetilde{y}$ and $\widetilde{\tau}$

While the model can be used for analysis with the parameters already discussed, in order to quantify the effects of individual components of income, the parameterization of these magnitudes is discussed first.

[7] IRCA (signed in November of 1986) legalized people who arrived to the US before Jan 1st of 1984, plus some additional conditions (good moral character, proving continuous residence, among others). Thus those legalized already were in the US for at least 3 years.

[8] The model is to calibrate these magnitudes in 1990, hence if we were to use only the Mexican citizens in 1990, that would severely underestimate the share of foreigners living in the US due to the significant one-time legalization through IRCA.

[9] Using the male population aged 15-64 produces very similar comparative statics and predictions for year 2000. Hence total population aged 15-64 was used throughout the paper.

According to Carey and Rabesona (2002), the average effective tax rates on labor and consumption in the US for the 1990-2000 decade (improving on the seminal work of Mendoza et al. (1994) on average tax rates) was 28.3%. The tax rate in the model in the host economy is set at $\tau = 28.3\%$.

Similar work on taxes for Mexico is available from Anton (2005), who uses Carey and Rabesona's (2002) methodology to produce tax rates for the required period in the case of consumption taxes. However, data for labor income under such methodology is only available since 1993. Adding both tax rates for Mexico results in a total tax rate that is between 20% and 24%, depending on specific assumptions required for the calculations of tax rates in Anton's calculations. The tax rate in Mexico is set at the average of this range, given by $\widetilde{\tau} = 22\%$.

The transfers are set at the level of the taxes collected so that transfers equal taxes, with $b = \tau w$. This implies that net income + transfers in the model can be identified with gross income (so that $y = w$). Given the numbers above, this means setting in the host economy $w = 100$, $b = .283 * 100 = 28.3$ (implying $y = 100$) and in the sending economy (Mexico) setting $w = 13.21$, $\widetilde{b} = .22 * 13.21 = 2.9062$ (implying $\widetilde{y} = 13.21$).[10]

Given the target of $\widetilde{\mu}_{ss} = 9.13$ and the rest of the parameters, the model requires $\kappa = 1645.8$, with an implied threshold level of k^* of 249.64, with an equilibrium annual probability of migration at steady state of $F[k^*] = .01148$ (almost 1.15%/year). These numbers are practically identical whether the model uses the analytic solution or a value function iteration algorithm is used (Matlab).

3.1 Changes in parameters in calibrated model for Mexico: Push-pull factors of unauthorized migration

This section documents the effects on the endogenous variables from changing the exogenous variables and parameters by 5%. The quantitative exercises only consider the case of Mexico, partly because the direction of the effects are the same across countries and including all of them does not yield additional insights; and also because tax information is only available for Mexico. Table 2 summarizes the effects of these experiments.

[10] Assuming that governments return tax revenue as transfers is a convenient assumption which helps to understand the quantitative effects of changes in income components (taxes, wages, and transfers) on the model's endogenous variables. More realistically, systematic differences can exist in the percentage of transfers returned to the public in developed and developing economies. These differences arise partly due to varying transfer compositions, the revenue and tools used by the government to obtain that revenue, and the quality of institutions overseeing those transfers, among other important factors. However, for simplicity, lack of data—particularly since such data would be difficult to obtain for the developing countries in this study—and because these issues are likely of secondary importance compared to the main issues in the paper, the main exercises do not address whether transfers in the host economy are relatively larger than in the sending economies.

When income in the host country y increases by 5% while everything else remains constant, both $V(M,k)$ and $V(S)$ increase in diagram 2. The value $V(M,k)$ rises due to a higher expected return on migrating, and $V(S)$ increases slightly because some foreigners, who currently face high migration costs, see improved future migration returns if they have a lower cost in the future. The threshold level k^* increases, which implies an increase in the probability of migrating from 1.15% to 1.22% per year. This results in a new steady state, with a higher share of the Mexican population living as unauthorized immigrants in the US, reaching now 9.65% of the foreign population. Notice that the increase of 5% in income leads to a probability that is almost 6% higher (1.15%*1.06≈1.22%), but translates in this case into a new steady state share of the population that is 5.7% higher ($\frac{9.65}{9.13} - 1 = 5.7\%$).

When the foreign income (Mexico's percapita income) increases by 5%, the probability to migrate decreases slightly to 1.08%. The steady state share of unauthorized immigrants decreases slightly to 8.63% (a decrease of 5.5% in the steady state share of immigrants).

Table 2. Results of increasing parameters by 5%

$Case \longrightarrow$	$Base$	Δy	$\Delta\widetilde{y}$	Δd	Δq	$\Delta\beta = -5\%$	$\Delta\kappa$	$\Delta\sigma$
$F[k^*]$ in %	1.148	1.22	1.08	1.12	1.04	0.74	1.09	1.15
k^*	249.6	252.7	246.4	243.3	225.2	160.3	249.6	249.0
μ_{ss} in %	9.13	9.65	8.63	8.53	7.6	6.06	8.73	9.11

$Case \longrightarrow$	$Base$	$\Delta\lambda$	Δw	$\Delta\widetilde{w}$	$\Delta\tau$	$\Delta\widetilde{\tau}$	Δb	$\Delta\widetilde{b}$
$F[k^*]$ in %	1.148%	1.22	1.2	1.09	1.13	1.16	1.17	1.13
k^*	249.6	265.4	260.9	247.1	245.2	250.4	254.1	248.9
μ_{ss} in %	9.13	9.65	9.50	8.74	8.98	9.25	9.28	9.02

When the probability of being stopped and returned at the border rises from 65% to 68.25%, corresponding to a 5% increase rather than a 5 percentage-point increase, the equilibrium probability of migration falls to 1.04%. Consequently, a smaller fraction of foreigners finds it optimal to attempt migration. This change leads to a sizable reduction in the steady-state share of unauthorized immigrants, which declines to 7.6%.

When the probability of being deported or exogenously return to home increases by 5% (from 4% to 4.2%), the annual probability to migrate decreases to 1.12% and the new steady state share of unauthorized immigrants decreases to 8.53%.

When the parameter β decreases 5% to a new level of 0.9044 (implying a very big change in the interest rate from 5% to approximately 10%), leads to a significantly lower probability of migrating, of only 0.74%, with an associated steady state total share of the population of 6.06%.

When the cost of returning/deportation σ (determining the deportation cost k_D) increases by 5%, it really does not affect the probability of migrating, as it reduces from 1.148%/year to 1.145% per year and an essentially unchanged share of unauthorized immigrants as percentage of the foreign population is now 9.11% (as opposed to 9.13%).

When the parameter scaling migration sunk costs κ (controlling the the upper bound on the uniform distribution $k_{\max}$) increases by 5%, interpreted as increasing the average cost of migrating for all agents uniformly by 5%, the effect on the probability of migrating is small as it decrease from 1.15% to 1.09% annually, while the steady state share of the population living as unauthorized immigrants decreases to 8.73%.

When the parameter that governs the share of income that the unauthorized get relative to natives (λ) increases 5% from 0.8 to 0.84, the effect on the probability of migration and equilibrium share of unauthorized immigrants is numerically identical to an increase in the host income of 5% since what matters for the calculations are the product of this parameter and income in the host economy (λy).

Table 2 also presents changes in the individual components of income, namely changes in wages, tax rates and transfers at both the host and sender economies, while leaving constant the other components of income. Broadly speaking these changes also reflect additional push and pull effects of the model in that higher wages, transfers or lower taxes in the host economy pull migrants (both the probability and share of migrants increase), while lower wages, transfers or higher taxes in the sender economy pushes agents to become unauthorized migrants.

4 Using the Model: Steady State Analysis

This section uses the model calibrated for Mexico for the year 1990 to study whether changes in relative income during the 1990-2000 period can explain the share of the Mexican population living in the US in 2000. The next section also considers Guatemala, Honduras, and El Salvador, each requiring its own calibration of country-specific parameters.

The stock of unauthorized immigrants in 2000 also requires an adjustment corresponding to the one made for 1990. About 2.3 million Mexican migrants were granted legal status through IRCA (see Zong and Batalova, 2014), and this number was added to represent the steady-state share of Mexicans living in the US in 1990. Similarly, since many of these individuals would likely remain in

the US 10 years later, even without a reform offering a path to legalization, the observed numbers in 2000 also need adjustment for how many would remain after 10 years without the reform.

From Warren and Warren (2013), it is possible to compute aggregate rates of emigration, legalization and deportation (for all nationalities of unauthorized immigrants). Average legalization rates are 0.85% (.0085) for the decade, and an emigration rate of 2.65%, while the probability of deportation actions in the interior was about 1.03% during this period. Thus after 10 years, and using these rates for the correction of the 2.3 millions of unauthorized Mexicans legalized by IRCA, approximately $2.3m * (1 - .0265 - .0085 - .0103)^{10} = 2.3m * 0.62903 = 1.4468m$ would remain in the US in absence of IRCA in year 2000.

According to data from the Department of Homeland Security (DHS), there were 4.68 millions of unauthorized Mexicans living permanently in the US in year 2000 (see Appendix). Adding the 1.4468m computed above, yields $4.68 + 1.4468 = 6.127$ millions of Mexican citizens that would be living in unauthorized fashion in the US in 2000. Divided by the 15-64 years old Mexican population in year 2000 of $59,831,998$ yields an (adjusted) unauthorized share of $\frac{6.127}{59.832} = 10.2\%$ for year 2000.

During the 1990-2000 period, GDP percapita in current dollars in the US grew at an average of 4.3% per year, while the Mexican counterpart grew at an average growth rate of 9.1%. As the relative GDP percapita (US to Mexican) changed during the decade, the average ratio was 5.79 for this period, lower than the initial ratio of 7.57 in 1990. From proposition 4, a lower relative income is expected to lower the steady state share of unauthorized Mexicans living in the US. Feeding the average ratio of 5.79 yields a predicted share of only 6.6% at the new steady state for year 2000. But this concept actually increased from 9.13% in 1990 to 10.2% in 2000 (including IRCA adjustments). Explaining this (apparent) discrepancy will be the task of the rest of the paper, but the next section first performs a similar exercise for the other countries as in the current section.

4.1 The cases of Guatemala, Honduras and El Salvador

The cases of Guatemala, Honduras and El Salvador are also affected by the IRCA reforms, but much less than in Mexico's case. The appendix shows the particular sources and numbers for those countries and similar adjustments as those performed for Mexico. The reference numbers and adjustments are summarized in table 3.

The idea is to parameterize the model so that each of the considered country's steady state shares of unauthorized immigrants is consistent with the (IRCA-adjusted) shares of the population

living in the US in 1990. This implies using common parameters $\{d, q, r, \lambda, \sigma, y\}$, and then calibrate those that are country-specific, given by $\{\widetilde{y}, \kappa\}$. As explained before, relative income ratios matter for the equilibrium probability of migration, which are fed to the model just like the case of Mexico above.

The shares of the population living as unauthorized immigrants in the US (IRCA-adjusted) in 1990 are 6.2% for Guatemala, 2.5% for Honduras, and a staggering 27.5% in the case of El Salvador which at the time was still suffering from civil war that started in 1979. GDP per capita in US dollars was $847 for Guatemala, $988 for Honduras and $892 for El Salvador. In year 2000, these numbers are respectively, $1,648 in Guatemala, $1,092 in Honduras, and $1,982 in El Salvador. The average relative GDP percapita in 1990 were 28.2 for Guatemala, 24.2 for Honduras and 26.8 for El Salvador, while the average for the 1990-2000 period was 21.95 for Guatemala, 30.76 for Honduras, and 20.76 for El Salvador (details shown in the Appendix B.1).

Table 3. Unauthorized stocks and adjustments to data

	Year	Mexico	Guatemala	Honduras	El Salvador
Population 15-64	1990	47,094,701	4,623,196	2,525,144	2,994,266
	2000	59,831,998	6,132,001	3,548,843	3,461,307
Unauthorized in US	1990	1,985,691	237,639	48,696	677,989
	2000	4,680,000	290,000	160,000	430,000
IRCA Adjustments	1990	2,300,000	50,000	15,078	146,000
	2000	1,446,769	31,452	9,485	91,838
Unauthorized IRCA-adjusted	1990	4,285,691	287,639	63,774	823,989
	2000	6,126,769	321,452	169,485	521,838
Shares (Unadjusted)	1990	4.2%	5.1%	1.9%	22.6%
	2000	7.8%	4.7%	4.5%	12.4%
Shares IRCA-adjusted	1990	9.1%	6.2%	2.5%	27.5%
	2000	10.2%	5.2%	4.8%	15.1%

Table 4 reports the country-specific parameterizations together with the corresponding steady-state predictions for the year 2000, using average relative income ratios over the period as inputs. For the Central American countries, the results all move in the predicted direction. Guatemala and El Salvador experienced faster income growth than the United States, implying a decline in US relative income levels (measured as U.S. GDP per capita relative to home-country GDP per capita). Consistent with the model's predictions, the observed shares of unauthorized immigrants from these countries also declined. Quantitatively, the model predicts a steady-state share of 4.8% for Guatemala, compared with an observed value of 5.2%, and 22.4% for El Salvador, compared with an observed share of 15.1%.

In the case of Honduras, U.S. relative income increased over the period, and the model therefore predicts an increase in the share of the Honduran population living as unauthorized immigrants. This prediction is consistent with the data. Quantitatively, the model implies a steady-state share of 3.2%, compared with an observed value of 4.8%. Overall, while the model does not fully match the levels in all cases, it performs well in capturing the direction of changes in the share of the population living in the host economy.

With respect to magnitudes, the largest gaps between model predictions and observed outcomes arise in Honduras and El Salvador. Both cases, however, are characterized by additional push–pull factors during the decade that are not explicitly accounted for in the model. In Honduras, a major additional push factor was Hurricane Mitch, which struck in 1998, devastating large segments of the economy and displacing a substantial share of the population. In El Salvador, by contrast, there was the removal of an important push factor: the civil war that began in 1979 ended in 1992, enabling a significant number of Salvadorans to return to their home country. While these factors are not incorporated into the quantitative exercise, they operate in a direction consistent with explaining the remaining discrepancy between the model's predictions and the observed data.

Table 4. Parameterization for each country and Prediction in year 2000

	Mexico	Guatemala	Honduras	El Salvador
GDP/Pop (1990)	\$3,154	\$847	\$988	\$892
$y/\widetilde{y}$ (1990)	7.57	28.2	24.2	26.8
k^*	249.7	297	294	296
$F[k^*]$	1.15	0.76	0.29	4.33
$\mu_{ss}-$targeted (1990)	9.13	6.2	2.5	27.5
$y/\widetilde{y}$ Avg (1990-2000)	5.79	21.95	30.76	20.76
$\widetilde{\mu}_{ss}$ -Observed (2000)	10.2	5.2	4.8	15.1
$\widetilde{\mu}_{ss}$ -Predicted (2000)	6.6	4.8	3.2	22.4
Qualitative Prediction	X	✓	✓	✓

4.2 Prediction using year by year changes in relative GDP percapita

Table 4 above considers predictions at steady state. But the speed at which the model converges to a new steady state matters since there are only 10 years between 1990 and 2000, in addition to the fact that the calculations use a constant relative income ratio at a new steady state while in reality there are fluctuations in this ratio. In this section, instead of feeding the model with an average ratio of the relative incomes, the model is now fed the year-by-year sequence of annual relative incomes

(using relative GDP percapita), to see what the end result is after 10 years. This type of prediction is (arguably) harder to do since it also involves the speed at which the adjustments take place.

Given equation (12), it is straightforward to show that the speed of convergence of the share of the population living as unauthorized immigrants is given by

$$Speed = -\ln\left(1 - \left[d + (1-q)\,F\left(k^*\right)\right]\right).$$

Using the calibrated parameters, the implied speed of convergence is approximately 4.5% per year for Mexico, and very similar values are obtained for Guatemala and Honduras. For El Salvador, the speed of convergence is somewhat higher, at about 5.7% per year. The corresponding half-life—the approximate time required for the model to close half of the gap between the current level and the steady state—is $\frac{\ln 2}{.045} \approx 15.4$ years for Mexico, while for El Salvador it is $\frac{\ln 2}{.057} \approx 12.2$ years.

Table 5 reports the predicted shares of the population living in the United States. These calculations improve the results for Mexico, as the model now predicts a final share of 8.13%, compared with 6.6% in the steady-state analysis. Although this value remains below the observed share of 10.2%, it nevertheless represents a substantial improvement relative to the steady-state prediction. For Guatemala, the model continues to perform well, though the predicted share now lies slightly below the observed level, in contrast to the previous section where the prediction exceeded the observed value. In the cases of Honduras and El Salvador, the model still captures the correct qualitative direction of change, but the predicted levels remain further from the observed shares in 2000. As discussed earlier, additional push and pull factors—absent from the quantitative exercise—are likely to account for the remaining discrepancies in these two cases.

Table 5. Observed and Predicted Shares for each country, dynamic prediction

	Mexico	Guatemala	Honduras	El Salvador
Share $\widetilde{\mu}_{ss}$ Targeted (1990), %	9.13	6.2	2.5	27.5
Share observed (2000), %	10.2	5.2	4.8	15.1
Predicted Share $\widehat{\widetilde{\mu}}_t$ (2000), %	8.13	5.7	2.8	25.1
Speed of Convergence, %	4.5	4.4	4.2	5.7

In addition, estimates of the stock of unauthorized immigrants rely on multiple assumptions to construct the underlying data (see Warren and Warren, 2013). Because migration flows from the other three countries are relatively small compared to those from Mexico, and because the

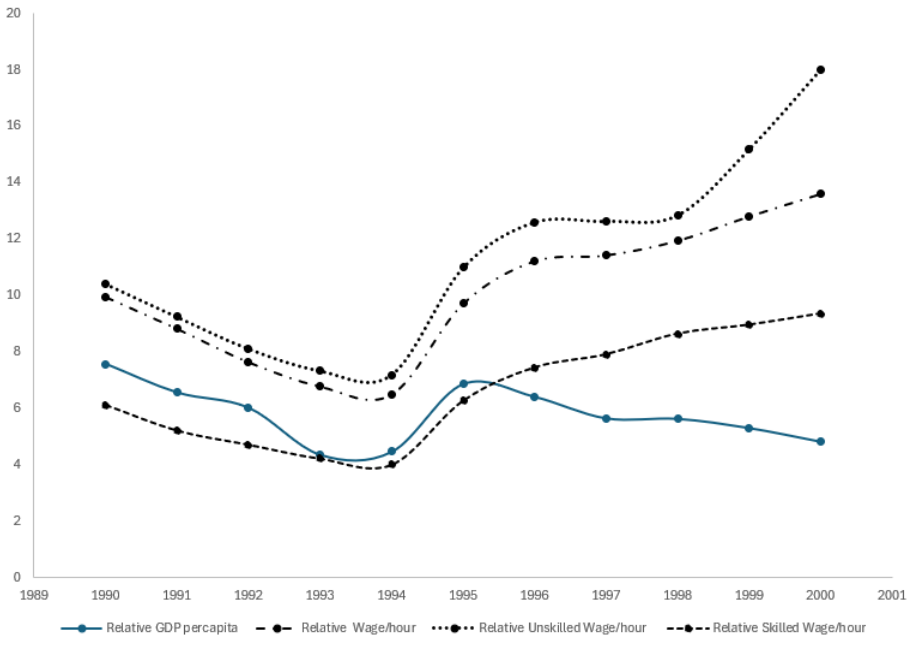
construction of these flows itself depends on several identifying assumptions, measurement error is likely to be more severe for these countries than for Mexico.

4.3 Using relative wages instead of relative GDP percapita

The previous sections used relative GDP per capita for the analysis, but relative wages are likely more suitable for studying this model, as it aims to capture relevant opportunities for potential unauthorized migrants. Wage data for Mexico during the specific period is available from Binelli and Attanasio (2010), who generously shared their dataset for the current paper. Similarly, Heathcote et al. (2010) generously shared their dataset for the American case.

Binelli and Attanasio (2010) used the official Mexican wage data from INEGI (the Mexican National Statistical Institute), using two different surveys: the Mexican Employment Survey (ENEU) and the Mexican Survey on Income and Expenditure (ENIGH), covering 1987 to 2002. Both ENIGH and ENEU provide similar information, but the ENIGH offers a more detailed breakdown of income types. This study uses the ENIGH dataset, except for the growth rate from 1990 to 1992 when it is not available and in which case it is obtained from the ENEU. They compute the average wage per hour for all wage earners between 25 and 60 years old at the time of the interview, including salaried and self-employed workers, regardless of whether they are formal or informal workers.

Figure 3. Evolution of several measures of relative income (US to Mexico)



More specifically, the paper uses the average wage per hour of men, which (from their original dataset) is converted into current dollars in order to compute the average relative wage per hour—that is, the U.S. wage per hour of U.S. males divided by the Mexican wage per hour of Mexican males. The dataset also allows for distinguishing between workers with a college degree and those without one, which is useful for capturing average unskilled wages per hour.

For the U.S., very similar wage-per-hour data are available from Heathcote et al. (2010), who construct several measures of income, consumption, and wealth for the United States between 1967 and 2006. In their case, they provide several samples; the one relevant for this study is based on the Current Population Survey (CPS), selecting individuals aged 25 to 60 who work at least 260 hours per year. Their dataset is reported in real terms, so the series are first converted into current dollars to construct comparable measures of relative wages per hour. Since they deflate the series with the help of Bureau of Labor Statistics CPI-U series for all items, the same deflator is used to express the magnitudes back into current dollars.

For the U.S. skill premium, the dataset does not indicate whether individuals completed college, so—following standard practice in the literature—wage premia are constructed by identifying workers with 16 or more years of schooling.

Figure 3 displays several time series that capture relative income. It shows the evolution of relative GDP percapita (US magnitude relative to Mexico's) displayed by the continuous line which displays a different evolution than the rest of the series after the peso crisis, the evolution of relative average-wage/hour for men, the evolution of relative skilled-wage/hour for men (which is the series that intersects relative GDP percapita), and the evolution of relative-unskilled wage/hour for men (the top series) for the relevant period.

Several things are apparent from the figure. First, all series capture a significant effect due to the peso crisis. Second, from 1990 to 1994 (before the peso crisis hit), all measures of relative income were decreasing, meaning that Mexican income was gaining on its American counterpart. Third, after the peso crisis GDP percapita quickly rebounded and by the end of the period relative GDP percapita was very close to the levels in 1993 and 1994. Fourth, the recovery in relative GDP percapita did not translate into a recovery in relative wages: all measures of relative wages show a worsening of opportunities in Mexico compared to the US. While in the US the wages were tracking the evolution of GDP percapita, this did not happen in Mexico where wages were stagnant while GDP percapita was growing. Finally, while relative skilled wages and relative average wages move

very similarly, relative unskilled wages shows additional divergence in the last years of the decade as unskilled wages in Mexico in dollars remained relatively flat after the peso crisis while American unskilled wages were growing continuously during the decade.

Table 6. Comparison of Predictions for Mexico* for year 2000

Observed		Predictions		
	Static	Dynamic		
	Relative GDP percapita	Relative GDP percapita	Relative Wages	Relative Unskilled Wages
10.24%	6.6%	8.13%	9.26%	9.61%

* Static: Steady state prediction by using average ratio of relevant period.
Dynamic: End of period result after year by year simulation given annual observations.

Although income (or wages per hour) measures are put in current dollars, their ratio is a real magnitude as it measures how many hours a worker in Mexico has to work in order to be able to obtain the same amount of dollars as working one hour in the US.[11] Relative average wages started at a level of 10, so that (in average) spending 10 hours of work in Mexico yield the same dollar amount as working 1 hour in the US. By the end of the period, relative average wages implied a ratio of 14 to 1. But in the case of relative unskilled wages, arguably more relevant for potential unauthorized migrants, this number starts just above 10 but at the end of the period this number was 18 to 1. These numbers are very different when compared to the evolution of relative GDP percapita, which decreases by the end of the decade and most likely fails to reflect the trade-offs of potential migrants.

In order to see the effects of the different time series of relative income, table 6 presents the different predictions implied by the different series. The table also displays the results obtained previously when using relative GDP percapita (both static and dynamic).

Using relative average wage/hour improves the prediction of the model, with a predicted share in year 2000 of 9.26% (compared to 8.13% when relative GDP percapita is used, and much better than the 6.6% obtained when the model was analyzed at steady state). Then using average relative unskilled wages produces the best results for the model, predicting a share of 9.61% of the population, much closer to the final 10.2% observed.

[11] The fact that many immigrants send remittances back home also means that relative income when each component is measured in dollars is arguably a better measure than putting magnitudes in purchasing power parity as exchange rate effects (reflected more easily in nominal magnitudes) matter for the purchasing power of those remittances.

5 A Decomposition of the Migration Flows for Mexico

The analysis from the previous section allows some counterfactuals. There are some forces that can be investigated, but in order to do this exercise, some auxiliary assumptions are needed. First, assuming that wages are proportional to GDP percapita (a relatively good assumption for the US as the labor share is approximately constant, but not a good assumption for Mexico where wages relative to GDP percapita decrease after the peso crisis), it is possible to obtain a counterfactual of "what would be the share of the Mexican population living in the US, if Mexican wages had followed the evolution of Mexican GDP". This results in a share of the population living in the US that starts at 9.13% in 1990 and ends in 8.20% in year 2000.

Then if unskilled Mexican wages evolved proportionally to average wages, the share in the model is predicted to be 9.26% in year 2000. Put it in other words, wages not recovering as much as GDP percapita add slightly more than 1 percentage point (out of a base of 8.20%) to the share of unauthorized migrants living in the US. Finally, the distance from 9.26 to 9.61 (0.35 p.p.) represents the extra change due to the additional divergence between wages and relative unskilled wage per hour (non-college relative wages). All in all, the model explains a final level of 9.6% in the relevant variable, which compares favorably to the value of 10.2%.

The calculations can be put in levels. The 15-64 population in Mexico in year 2000 was 59,831,998 (WDI). Applying the 9.61% from the model yields a total of about 5.75m in year 2000 predicted to be living in the US (while observed would be about $10.24\%*59,831,998 = 6.13m$). In terms of changes in population, relative to the initial 1990 stock of unauthorized immigrants in the US (IRCA-adjusted) which is about 4.3m, yields a change predicted by the model of 5.75m-4.3m=1.45m. This is to be compared with the change observed for the period, which is $10.2\%*59,831,998 - 9.1\%*47,094,701 = 1.827m$. Thus the model predicts $1.45/1.827 = 79.4\%$ of the change in the number of unauthorized immigrants during the period.

Table 7 shows that the evolution of GDP percapita (year-by-year) does not help explaining an increase in the number of unauthorized population, as it would imply a decrease of about 556,000 during the decade (a difference of 8.20%-9.13%=-0.93%, multiplied by the change in population between 1990 and 2000 of 12,737,297 which yields about 556,438).

The divergence of relative unskilled wages with relative GDP percapita in Mexico explains an increase of about 844,000 unauthorized immigrants during the decade. This is computed as the

difference of shares under relative unskilled wages and a share that follows the evolution of relative GDP percapita (9.61% - 8.20%). That number multiplied by current population in year 2000 (59,831,998) yields approximately 844,000 which are attributed to the distinct evolution of unskilled wages relative to relative GDP percapita.

Table 7. Decomposition of Change in Unauthorized Population

	Change Levels (1000's)	% of Predicted	% of Total Change
Relative GDP percapita	-556	-38.4	-30.5
Relative Unskilled Wages	844	58.2	46.2
Mexican Population Growth	1,163	80.2	63.7
Total	1,450	100	79.4

The strong population growth of the relevant age group (15-64) in Mexico importantly contributes for the more than 1.16m of Mexican nationals arriving in the US during the period, where the change in the population was 12,737,297, which multiplied by the initial share of 9.13% yields said number. Broadly speaking, strong population growth and the unequal evolution of wages when compared to GDP percapita in Mexico explain the majority of the change in the stock of unauthorized Mexican immigrants in the US. Table 7 summarizes those numbers. [12]

The model predicts that during the first four years of the decade—before the peso crisis, when Mexican wages in dollar terms were rising faster than U.S. wages—unauthorized immigration would be predicted to increase by roughly 71,500 individuals per year, for a total of about 286,000 for the period. In contrast, during the final six years of the decade, after the crisis reduced real Mexican wages relative to both Mexican GDP per capita and continuing U.S. wage growth, the model projects a much sharper rise: an average of nearly 194,000 unauthorized immigrants per year, amounting to about 1,163,000 over the period.

6 Sensitivity Analysis

Table 8 presents the effects on the steady state share of the sender population living in the host economy. It repeats the exercises in table 2 but for different baseline parameter values. It considers

[12] Alternatively, one can understand the exercise in the following manner. If the model takes into account population growth and the evolution of relative GDP into consideration, the total effect would be +1.163 (Pop growth) - 556,000 (relative GDP evolution) for a total effect of an increase of only about 606,000 unauthorized immigrants during the decade. Population growth + evolution of relative unskilled wages yields a total effect of about 1.45m. The difference between these numbers 1.45m (Pop growth + wage evolution) and 606,000 (Pop growth + evolution of GDP) is 844,000 that can be attributed to the effect of relative unskilled wages when compared to using relative GDP percapita.

alternative values of the total return probability (deportation plus exogenous hit) of 3% and 5% (baseline of 4%). It considers an alternative probability of detention at the border of $q' = 50\%$ (baseline of 65%), consistent with earlier evidence of Espenshade (1990) for a previous period. The value of the share of wage and benefits available for unauthorized immigration (λ) uses alternative values of 60% and 100% (baseline of 80%).

The main parameterization of the model uses an interest of $r = 5\%$, so two additional levels of $r' = 3\%$ and $r' = 8\%$ are also used. Finally, the paper considers alternative values for the parameter σ that captures the proportionality factor for the deportation/return cost as percentage of income available at the host economy (where the cost is given by $k_D = \sigma\lambda y$) and where the baseline was set at a level of $\sigma = 1$. The values considered are $\sigma' = 0$ and $\sigma' = 2$. When σ is set at 0, there is no direct cost of deportation but there is only the opportunity cost of not being able to work in the host economy. When σ is set at 2, the deportation cost is set at 200% of the annual income for an unauthorized immigrant living in the host economy and more generally could also capture other costs not captured in the model (e.g. psychological).

Because the model is calibrated to replicate the observed steady-state share of the population living as unauthorized migrants, the results prove to be highly robust. The table shows not only that the direction of the effects remains unchanged across specifications, but also that the quantitative implications are virtually identical. Under alternative assumptions, most entries display the same magnitudes, with column values remaining nearly constant across the cases considered.

Table 8. Sensitivity analysis: Steady state $\widetilde{\mu}_{ss}$ when parameters change of 5% for alternative parameterizations of the model

Case	Δy	$\Delta\widetilde{y}$	Δd	Δq	$\Delta\beta = -5\%$	$\Delta\kappa$	$\Delta\sigma$	$\Delta\lambda$
Baseline	9.65	8.63	8.53	7.60	6.06	8.73	9.11	9.65
$d' = .03$	9.64	8.63	8.57	7.60	5.80	8.73	9.11	9.64
$d' = .05$	9.65	8.63	8.50	7.59	6.29	8.73	9.10	9.65
$q\prime = 0.5$	9.64	8.64	8.53	8.30	6.11	8.73	9.11	9.64
$\sigma' = 0$	9.64	8.63	8.55	7.60	6.05	8.73	9.13	9.64
$\sigma' = 2$	9.65	8.62	8.51	7.60	6.07	8.73	9.09	9.65
$r' = .03$	9.65	8.63	8.48	7.61	5.54	8.73	9.11	9.65
$r' = .08$	9.65	8.63	8.57	7.59	6.60	8.73	9.11	9.65
$\lambda' = 0.6$	9.70	8.58	8.52	7.58	6.01	8.73	9.11	9.70
$\lambda' = 1$	9.65	8.63	8.53	7.61	6.08	8.73	9.11	9.65

As a consequence of the robustness of the model, predicting the share of Mexicans living as unauthorized immigrants in the US in year 2000 as in the previous sections under alternative parameterizations presented in the current section yields practically identical results, with very small

differences not worth reporting (e.g. 9.62% instead of 9.61%). All in all, uncertainty about some of the parameters appears to be irrelevant for the predictions of the model.

7 Conclusions

This paper develops a dynamic model to study equilibrium unauthorized migration by explicitly modeling individual migration decisions within an optimal-stopping framework. The resulting model is consistent with the logic of push-pull forces that shape migration incentives. The framework yields a closed-form expression for the probability of migrating, which in turn delivers a closed-form characterization of the law of motion for the share of the population in a sending country residing as unauthorized immigrants in the host economy. In particular, the probability of migration is linear in relative income between the host and sending countries, decreasing in enforcement probabilities, and its sensitivity to changes in income depends critically on the enforcement regime in place.

The paper aims to fill some gaps in the immigration literature. First, as noted by Ortega and Peri (2013), relatively few studies provide a theoretical account of the microfoundations underlying aggregate migration flows. Relatedly, while empirical work based on push–pull factors is widespread—including gravity-type specifications enriched with economic and policy variables (see, for example, the references in Ortega and Peri, 2009)—these factors are typically introduced in a reduced-form manner. To the best of the authors' knowledge, there is no existing static or dynamic model that integrates as many push-and-pull determinants of migration within a unified structural framework. The present paper seeks to address these shortcomings by developing a dynamic model that formally links individual migration decisions to equilibrium unauthorized migration flows.

The paper parameterizes the model for the study of the cases of Mexico, Guatemala, Honduras and El Salvador for the decade of the 1990's and finds support for the theory. Mexico, having the biggest economy and the most unauthorized immigrants in the US, as well as the country with the best data among the group, receives most of the attention in the analysis.

The calibrated model shows how the probability of migrating depends on multiple factors, but it is particularly sensitive to the deportation probability, as well as to the interest rate at which agents discount future uncertain income flows.

For the study of the 1990-2000 period, the model keeps most parameters constant (consistent with an interpretation of a relatively constant enforcement regime) and then studies how the evolution of

relative income and of the population predict the future stock of unauthorized Mexican migrants. When relative GDP percapita is used for Mexico in a steady state prediction (assuming that year 2000 represents another steady state), the model suggests a significant decrease in the share of unauthorized immigrants (contrary to measured magnitudes) from 9.1% in 1990 to a predicted level of about 6.6% in year 2000, while the observed value was about 10.2%.

The paper shows that the speed of convergence to the steadsy state is relatively slow, about 4.5% or so for Mexico, with an implied half-life of about 15 years. Because of this, the paper considers mainly predictions out of steady state, by feeding the model the year by year observations of relative GDP percapita for the decade. This improves the prediction to a level of 8.3% by year 2000 (compared to the 6.6% predicted at steady state but still somewhat short of the 10.2% observed). Taking into account strong population growth in Mexico (an increase of 26% of the population in the 15-64 age bracket), the model predicts just a small net increase of only about 600,000 Mexicans living as unauthorized immigrants during the decade, compared to a net increase of about 1.8 million. This divergence is explained away by using other time series that capture better the relative wages of unauthorized immigrants and which in turn improve the predictions relative to using relative GDP percapita.

Mexican wages did not keep pace with GDP per capita growth following the peso crisis. Although Mexico's GDP per capita gained roughly 36% relative to that of the United States over the decade (and in equilibrium attracting a smaller flow of Mexican workers), the opposite occurred with respect to average hourly wages: U.S. wages increased (coincidentally) by about 36% relative to Mexican wages during the same period (attracting more a bigger flow of Mexican workers). The divergence is even more pronounced for unskilled labor, where relative unskilled wages (US relative to Mexico) rose by approximately 73%.

In levels, relative unskilled wages were approximately 10 in 1990, rising to about 18 by 2000. Put differently, in 1990 it took roughly 10 hours of work in an unskilled job in Mexico to earn the same amount in dollars that an unauthorized Mexican worker could earn in the United States; by 2000, this number had increased to about 18 hours, providing ample incentive for more mexican migrants to arrive to the US in unauthorized fashion. When the model uses this time series rather than relative GDP per capita, it predicts a net increase of about 1.45 million unauthorized Mexican immigrants—nearly 80% of the observed increase of roughly 1.8 million over the decade.

The current framework introduces a very flexible model that can be generalized to the study of

multiple issues where the migration decision is central to the analysis. For example, generalizations of the model could study the more recent phenomenon of overstaying of temporary non-working visa permits (simply setting $q = 0$), the impact of a guest worker program on potential migrants' incentives, political turmoil and higher crime in developing economies by assuming different location-specific death probabilities, the effects of legalization on migrants incentives, as well as the cost-effectiveness of altering the probability of inland deportation versus border enforcement as compared to other policy options, among other possibilities. Similarly, the current model could be used as a subset of equations in dynamic stochastic general equilibrium models where the migration decision requires explicit modelling. These are just some examples of extensions that can be considered within the current framework, but their consideration requires their own space and discussion and are thus left for future research.

Acknowledgements

The author thanks Chiara Binelli for kindly sharing the dataset from the paper "Mexico in the 1990s: The Main Cross Sectional Facts" (Binelli and Attanasio, 2010). The author also thanks Jonathan Heathcote for kindly sharing the dataset from the paper "Unequal We Stand: An Empirical Analysis of Economic Inequality in the United States, 1967–2006" (Heathcote, Perri, and Violante, 2010).

Declaration of generative AI and AI-assisted technologies in the writing process.

During the preparation of this work the author used RYTR for grammar purposes in order to improve readability of the manuscript, as well as ChatGPT and MICROSOFT COPILOT for readilbility and some quick simple calculations. After using these tools, the author reviewed and edited the content as needed and takes full responsibility for the content of the published article.

References

[1] Attanasio, O & Binelli, C. (2009). Code and data files for "Mexico in the 1990s: the Main Cross-Sectional Facts," Computer Codes 09-201, Review of Economic Dynamics. Available at https://ideas.repec.org/c/red/ccodes/09-201.html

[2] Binelli, C., & Attanasio, O. (2010). Mexico in the 1990s: The main cross-sectional facts. Review of Economic Dynamics, 13(1), 238–264. https://doi.org/10.1016/j.red.2009.10.003

[3] Acemoglu, D., & Autor, D. (2011). Skills, tasks, and technologies: Implications for employment and earnings. In D. Card & O. Ashenfelter (Eds.), *Handbook of labor economics* (Vol. 4, Part B, pp. 1043–1171). Elsevier. https://doi.org/10.1016/S0169-7218(11)02410-5

[4] Alesina, A., Miano, A. & Stantcheva, S. (2023), Immigration and redistribution. The Review of Economic Studies, 90(1), 1–39. Available from: https://doi.org/10.1093/restud/rdac011

[5] Antón, Arturo (2005), Average effective tax rates in Mexico. Economía Mexicana Nueva Época 14(2), 185-215. http://hdl.handle.net/11651/1769

[6] Baker, B. (2018, December). Illegal alien population residing in the United States: January 2015 (Population Estimates). U.S. Department of Homeland Security, Office of Immigration Statistics. https://www.dhs.gov/sites/default/files/publications/15_0708_plcy_population-estimates.pdf

[7] Bandyopadhyaya, S. & Pinto, S. M. (2017), Unauthorized immigration and fiscal competition, European Economic Review 92, 283 - 305.

[8] Basu, A. K., Chau, N. H., Park, B. (2022), Rethinking border enforcement, permanent and circular migration, Economic Modelling 108, article 105733. Available from https://doi.org/10.1016/j.econmod.2021.105733.

[9] Battisti, M., Felbermayr, G., Peri, G., & Poutvaara, P. (2018). Immigration, search and redistribution. Journal of the European Economic Association, 16(4), 1137–1188. https://doi.org/10.1093/jeea/jvy010

[10] Blau, F.D. & Mackie, C. (Eds.) (2017) The economic and fiscal consequences of immigration. Washington: The National Academies Press. Available from: https://doi.org/10.17226/23550

[11] Bond, E. & Chen, T.-J. (1987) The welfare effects of illegal immigration. Journal of International Economics, 23(3-4), 315–328. Available from: https://doi.org/10.1016/0022-1996(87)90058-4

[12] Borjas, G.J. (1995) The economic benefits from immigration. The Journal of Economic Perspectives, 9(2), 3–22. Available from: https://doi.org/10.1257/jep.9.2.3

[13] Carey, D. & Rabesona, J. (2002), Tax Ratios on Labour and Capital Income and on Consumption, OECD Economic Studies 35: 129-74.

[14] Chassamboulli, A. & Palivos, T. (2014) A search-equilibrium approach to the effects of immigration on labor market outcomes, International Economic Review 55 (1), 111-129

[15] Chassamboulli, A. & Peri, G. (2015) The labor market effects of reducing the number of illegal immigrants. Review of Economic Dynamics, 18(4), 792–821. Available from: https://doi.org/10.1016/j.red.2015.07.005

[16] Chassamboulli, A. & Peri, G. (2020) The economic effect of immigration policies: analyzing and simulating the U.S. case. Journal of Economic Dynamics and Control, 114, 103898. Available from: https://doi.org/10.1016/j.jedc.2020.103898

[17] Cramer, E. E. (2024). Honduran immigrants. EBSCO Research Starters. Retrieved from https://www.ebsco.com/research-starters/social-sciences-and-humanities/honduran-immigrants

[18] Donato, K. M., Durand, J., & Massey, D. S. (1992). Stemming the tide? Assessing the deterrent effects of the Immigration Reform and Control Act. Demography, 29(2), 139–157. https://doi.org/10.2307/2061724.

[19] Espenshade, T. J., Bean, F. D, Goodis, T. A. & White, M. J. (I990). Immigration policy in the United States: Future prospects tor the immigration reform and control act of I986, pp. 59-84 in: G. Roberts (ed.), Population policy: Contemporary issues. NewYork: Praeger.

[20] Ethier, W.J. (1986) Illegal immigration: the host country problem. The American Economy Review, 76, 56–71.

[21] Gammage, S. (2007, July 26). El Salvador: Despite end to civil war, emigration continues. Migration Policy Institute. Retrieved from https://www.migrationpolicy.org/article/el-salvador-despite-end-civil-war-emigration-continues

[22] Guerreiro, J., Rebelo, S., Teles, P. (2020), What is the optimal immigration policy? Migration, jobs, and welfare, Journal of Monetary Economics 113: 61-87. Available at https://doi.org/10.1016/j.jmoneco.2020.04.009.

[23] Guzman, M., Haslag, J. & Orrenius, P. (2008) On the determinants of optimal border enforcement. Economic Theory, 34(2), 261–296. Available from: https://doi.org/10.1007/s00199-006-0166-7

[24] Hanson, G. H. (2006). Illegal migration from Mexico to the United States. Journal of Economic Literature, 44(4), 869–924.

[25] Harris, J. R., & Todaro, M. P. (1970). Migration, unemployment and development: A two-sector analysis. The American Economic Review, 60(1), 126–142.

[26] Heathcote, J., Perri, F., & Violante, G. L. (2010). Unequal we stand: An empirical analysis of economic inequality in the United States, 1967–2006. Review of Economic Dynamics, 13(1), 15–51. https://doi.org/10.1016/j.red.2009.10.010

[27] Jonathan Heathcote & Fabrizio Perri & Giovanni L. Violante, 2010. "Code and data files for "Unequal We Stand: An Empirical Analysis of Economic Inequality in the United States: 1967-2006"," Computer Codes 09-214, Review of Economic Dynamics.

[28] Jonas, S. (2013, March 27). Guatemalan migration in times of civil war and post-war challenges. Migration Policy Institute. Retrieved from https://www.migrationpolicy.org/article/guatemalan-migration-times-civil-war-and-post-war-challenges

[29] Kolker, A.F. (2022) Unauthorized immigrants' eligibility for federal and state benefits: overview and resources. Technical Report R47318, US Congressional Research Service. Available from: https://crsreports.congress.gov/product/pdf/R/R47318

[30] Kossoudji, S. A., & Cobb-Clark, D. A. (2002). Coming out of the shadows: Learning about legal status and wages from the legalized population. Journal of Labor Economics, 20(3), 598–628. https://doi.org/10.1086/339611

[31] Lopez-Velasco, A.R. (2025) On the design of an optimal immigration policy. Economic Inquiry, 63(1), 47–97. Available from: https://doi.org/10.1111/ecin.13260

[32] Lopez-Velasco, A. R. (2023). A snapshot of immigration flows into the US. Center for Economic Border Studies. Border Business Briefs, 19(1), Summer. https://www.utrgv.edu/rcvcobe/_files/documents/pdf/fall-2023.pdf

[33] Lee, E. (1966). A theory of migration. Demography, 3(1), 47–57. https://doi.org/10.2307/2060063

[34] Liu. X. (2010). On the macroeconomic and welfare effects of illegal immigration, Journal of Economic Dynamics and Control 34(12): 2547-2567,

[35] McCall, B., & McCall, J. (2007). The economics of search. Routledge. https://doi.org/10.4324/9780203496039

[36] Mendoza, E. G., Razin, A., & Tesar, L. L. (1994). Effective tax rates in macroeconomics: Cross-country estimates of tax rates on factor incomes and consumption. *Journal of Monetary Economics, 34*(3), 297–323. https://doi.org/10.1016/0304-3932(94)90021-3

[37] Ljungqvist, L., & Sargent, T. J. (2018). Recursive macroeconomic theory (4th ed.). MIT Press. https://doi.org/10.7551/mitpress/12345.001.0001

[38] Orrenius, P. M. (2001). Illegal immigration and enforcement along the U.S.-Mexico border: An overview. Economic and Financial Review, 2001(Q3), 2–11. Federal Reserve Bank of Dallas. https://www.dallasfed.org

[39] Ortega, F., & Peri, G. (2009). The causes and effects of international migrations: Evidence from OECD countries 1980–2005 (NBER Working Paper No. 14833). National Bureau of Economic Research. https://doi.org/10.3386/w14833

[40] Ortega, F., & Peri, G. (2013). The effect of income and immigration policies on international migration. Migration Studies, 1(1), 47–74. https://doi.org/10.1093/migration/mns004

[41] Ravenstein, E. G. (1889). The laws of migration. Journal of the Royal Statistical Society, 52(2), 241–305. https://doi.org/10.1111/j.2397-2335.1889.tb00043.x

[42] Sjaastad, L.A. (1962) The Costs and Returns of Human Migration. Journal of Political Economy, 70, 80-93.

[43] Smith, A. (1776). An inquiry into the nature and causes of the wealth of nations. W. Strahan and T. Cadell.

[44] U.S. Immigration and Naturalization Service, Office of Policy and Planning. (2003). Estimates of the unauthorized immigrant population residing in the United States: 1990 to 2000. U.S. Department of Homeland Security. https://www.dhs.gov/xlibrary/assets/statistics/publications/Ill_Report_1211.pdf

[45] Warren, R., & Warren, J. R. (2013). Unauthorized immigration to the United States: Annual estimates and components of change, by state, 1990 to 2010. International Migration Review, 47(2), 296-329. https://doi.org/10.1111/imre.12022

[46] Zong, J., & Batalova, J. (2014, October 9). Mexican immigrants in the United States. Spotlight. Migration Policy Institute. Available at https://www.migrationpolicy.org/article/mexican-immigrants-united-states-2013

Appendix

(Intended to be offered as an online annex)

A Proofs

A.1 Derivation of threshold level k^*

Start by taking $V(I)$ in equation (5), which can be solved for $V(I)$ to yield

$$V(I) = \frac{y\lambda}{[1-\beta(1-d)]} + \frac{\beta d V(D)}{[1-\beta(1-d)]}, \tag{A.1}$$

which is a term used later. The next term to consider is $V(M,k)$ in equation (2), which for simplicity is repeated here as

$$V(M,k) = qV(A|M,k) + (1-q)V(I|M,k).$$

Replace (3) into (2) to obtain

$$V(M,k) = -k + \beta q E\left\{\max\left[V(S), V\left(M,k'\right)\right]\right\} + (1-q)\left\{V(I)\right\}.$$

Replace $V(I)$ into (A.1) to obtain an intermediate expression for $V(M,k)$. Notice that it depends on the expectational term $E\left\{\max\left[V(S), V(M,k')\right]\right\}$ as well as on $V(D)$.

$$\begin{aligned} V(M,k) = -k &+ \frac{(1-q)y\lambda}{[1-\beta(1-d)]} \\ &+ \beta q E\left\{\max\left[V(S), V\left(M,k'\right)\right]\right\} + \frac{(1-q)\beta d}{[1-\beta(1-d)]}V(D). \end{aligned} \tag{A.2}$$

The next step is to replace $V(D)$ from equation (6) into the above equation, in order to obtain

$$\begin{aligned} V(M,k) = -k &+ \frac{(1-q)y\lambda}{[1-\beta(1-d)]} - k_D\frac{(1-q)\beta d}{[1-\beta(1-d)]} \\ &+ \left(\frac{q(1-\beta)+d\beta}{[1-\beta(1-d)]}\right)\beta E\left\{\max\left[V(S), V\left(M,k'\right)\right]\right\} \end{aligned}$$

The above term requires the evaluation of the expectation term, $E\left\{\max\left[V(S), V(M,k')\right]\right\}$ which is derived below. First consider the maximization inside the expectation. The migrating decision rule implies that

$\max\left[V(S), V(M,k')\right] = V(S)$ for all $k' > k^*$ (Stays)
$\max\left[V(S), V(M,k')\right] = V(M,k')$ for all $k' \leq k^*$ (Migrates)

Since k is an i.i.d. random variable, with a CDF given by $F(k)$, then the expectation term can be written as

$$E\left\{\max\left[V(S), V(M,k')\right]\right\} = P\left[k' \leq k^*\right] * E\left[V(M,k') | k' \leq k^*\right] + P\left[k' > k^*\right] * V(S).$$

Since $P\left[k' > k^*\right] = \left[1 - F(k^*)\right]$ and $P\left[k' \leq k^*\right] * E\left[V(M,k') | k' \leq k^*\right] = \int_0^{k^*} V(M,k')\, dF(k')$, then

$$E\left\{\max\left[V(S), V\left(M,k'\right)\right]\right\} = \int_0^{k^*} V\left(M,k'\right) dF\left(k'\right) + V(S)\left[1 - F(k^*)\right]. \tag{A.3}$$

Now consider $V(S)$ in equation (1), given by

$$V(S) = \widetilde{y} + \beta E\left\{\max\left[V(S), V\left(M, k'\right)\right]\right\}.$$

Replace the expectation in (A.3) into the above expression in order to get

$$V(S) = \widetilde{y} + \beta \int_0^{k^*} V\left(M, k'\right) dF\left(k'\right) + \beta V(S)\left[1 - F\left(k^*\right)\right]$$

which can be solved for $V(S)$ as

$$V(S) = \frac{\widetilde{y} + \beta \int_0^{k^*} V\left(M, k'\right) dF\left(k'\right)}{\left\{1 - \beta\left[1 - F\left(k^*\right)\right]\right\}} \tag{A.4}$$

Now replace the above term into the expectation term (A.3) to obtain

$$E\left\{\max\left[V(S), V\left(M, k'\right)\right]\right\} = \int_0^{k^*} V\left(M, k'\right) dF\left(k'\right) + \frac{\left[\widetilde{y} + \beta \int_0^{k^*} V\left(M, k'\right) dF\left(k'\right)\right]\left[1 - F\left(k^*\right)\right]}{\left\{1 - \beta\left[1 - F\left(k^*\right)\right]\right\}}$$

which can be rewritten by combining similar terms as

$$E\left\{\max\left[V(S), V\left(M, k'\right)\right]\right\} = \frac{\left[1 - F\left(k^*\right)\right]\widetilde{y}}{\left\{1 - \beta\left[1 - F\left(k^*\right)\right]\right\}} + \frac{1}{\left\{1 - \beta\left[1 - F\left(k^*\right)\right]\right\}} \int_0^{k^*} V\left(M, k'\right) dF\left(k'\right). \tag{A.5}$$

Replace the above expectation term (A.5) into $V(M,k)$ in equation (A.2) in order to obtain

$$V(M,k) = -k + \frac{(1-q)y\lambda}{[1-\beta(1-d)]} - k_D \frac{(1-q)\beta d}{[1-\beta(1-d)]} + \left(\frac{q(1-\beta)+d\beta}{[1-\beta(1-d)]}\right)\beta\left\{\frac{[1-F(k^*)]\widetilde{y}}{\{1-\beta[1-F(k^*)]\}} + \frac{1}{\{1-\beta[1-F(k^*)]\}}\int_0^{k^*} V(M,k')\,dF(k')\right\}$$

Simplifying yields $V(M,k)$

$$V(M,k) = -k + \Omega\left(F\left(k^*\right)\right) + \alpha \int_0^{k^*} V\left(M, k'\right) dF\left(k'\right) \tag{A.6}$$

for terms α and $\Omega\left(F\left[k^*\right]\right)$ given by

$$\alpha\left(F\left(k^*\right)\right) \equiv \beta\left(\frac{q(1-\beta)+d\beta}{[1-\beta(1-d)]}\right)\frac{1}{\left\{1-\beta\left[1-F\left(k^*\right)\right]\right\}} \tag{A.7}$$

$$\Omega\left(F\left(k^*\right)\right) \equiv \frac{(1-q)\lambda y - (1-q)\beta d k_D}{[1-\beta(1-d)]} + \alpha\left[1 - F\left(k^*\right)\right]\widetilde{y}. \tag{A.8}$$

Notice in equation (A.6) that, if the parameterization is such that k^* exists, then the second and third terms are just constants and $V(M,k)$ is of the form $V(M,k) = -k$+constant.

The next step is to evaluate $\int_0^{k^*} V(M,k')\,dF(k')$. Evaluate this integral by substituting (A.6) into the integral. Thus obtain

$$\int_0^{k^*} V(M,k')\,dF(k') = -\int_0^{k^*} k'dF(k') + \left[\Omega(F(k^*)) + \alpha(F(k^*)) \int_0^{k^*} V(M,k')\,dF(k')\right] \int_0^{k^*} dF(k')$$

Therefore

$$\int_0^{k^*} V(M,k')\,dF(k') = -\int_0^{k^*} k'dF(k') + \left[\Omega(F(k^*)) + \alpha(F(k^*)) \int_0^{k^*} V(M,k')\,dF(k')\right] F(k^*)$$

$$\int_0^{k^*} V(M,k')\,dF(k') - \alpha(F(k^*))F(k^*) \int_0^{k^*} V(M,k')\,dF(k') = -\int_0^{k^*} k'dF(k') + F(k^*)\Omega(F(k^*))$$

which can be solved for $\int_0^{k^*} V(M,k')\,dF(k')$ as

$$\int_0^{k^*} V\left(M,k'\right)dF\left(k'\right) = \frac{F(k^*)\Omega(F(k^*)) - \left(\int_0^{k^*} k'dF(k')\right)}{1-\alpha((F(k^*)))F(k^*)} \tag{A.9}$$

Therefore $V(M,k)$ can be written by substituting the above term for the integral in (A.6)

$$V(M,k) = -k + \Omega(F(k^*)) + \alpha(F(k^*)) \left[\frac{\left(-\int_0^{k^*} k'dF(k')\right) + F(k^*)\Omega(F(k^*))}{1-\alpha(F(k^*))F(k^*)}\right]$$

which simplifies to

$$V(M,k) = -k + \frac{\Omega(F(k^*))}{1-\alpha F(k^*)} - \frac{\alpha\left(\int_0^{k^*} k'dF(k')\right)}{1-\alpha F(k^*)}. \tag{A.10}$$

One more step is required before deriving k^*, which is to substitute the integral (A.9) into $V(S)$ in equation (A.4). This yields some value (a constant in this case) given by

$$V(S) = \frac{\widetilde{y}}{\{1-\beta[1-F(k^*)]\}} + \frac{\beta F(k^*)\Omega - \beta\int_0^{k^*} k'dF(k')}{[1-\alpha F(k^*)]\{1-\beta[1-F(k^*)]\}} \tag{A.11}$$

Expressions (A.10) and (A.11) can now be used to define the threshold k^*. Since k^* satisfies $V(M,k^*) = V(S)$, then

$$V(S) = \frac{\widetilde{y}}{\{1-\beta[1-F(k^*)]\}} + \frac{\beta F(k^*)\Omega - \beta\int_0^{k^*} k'dF(k')}{[1-\alpha F(k^*)]\{1-\beta[1-F(k^*)]\}} = -k^* + \frac{\Omega}{1-\alpha F(k^*)} - \frac{\alpha\left(\int_0^{k^*} k'dF(k')\right)}{1-\alpha F(k^*)} = V(M,k^*)$$

Solving for k^* and combining similar terms yields

$$k^* = \frac{\Omega}{1-\alpha F(k^*)}\left[\frac{(1-\beta)}{\{1-\beta[1-F(k^*)]\}}\right] - \frac{\widetilde{y}}{\{1-\beta[1-F(k^*)]\}} \tag{A.12}$$
$$+ \left[\frac{\beta}{\{1-\beta[1-F(k^*)]\}} - \alpha\right] \frac{\left(\int_0^{k^*} k'dF(k')\right)}{[1-\alpha F(k^*)]}.$$

While the above expression can be used for some purposes (e.g. see the proof in next subsection), the threshold value can be simplified further. Start by replacing (A.7) and (A.8) into the above expression (except $\alpha \equiv \alpha\left(F\left(k^*\right)\right)$ appearing in the denominator, which will be later replaced), this yields

$$k^* = \frac{\frac{(1-q)\lambda y-(1-q)\beta dk_D}{[1-\beta(1-d)]}+\alpha[1-F(k^*)]\widetilde{y}}{1-\alpha F(k^*)}\left[\frac{(1-\beta)}{\{1-\beta(1-F(k^*))\}}\right]-\frac{\widetilde{y}}{\{1-\beta(1-F(k^*))\}}$$
$$+\left[\frac{\beta}{\{1-\beta(1-F(k^*))\}}-\beta\left(\frac{q(1-\beta)+d\beta}{[1-\beta(1-d)]}\right)\frac{1}{\{1-\beta[1-F(k^*)]\}}\right]\frac{\left(\int_0^{k^*}k'dF(k')\right)}{[1-\alpha F(k^*)]}$$

Which can be written as

$$k^* = \left[\frac{1}{\{1-\beta(1-F(k^*))\}}\right]\left\{\frac{\frac{(1-q)\lambda y-(1-q)\beta dk_D}{[1-\beta(1-d)]}+\alpha[1-F(k^*)]\widetilde{y}}{1-\alpha F(k^*)}\left[(1-\beta)\right]-\widetilde{y}+\beta\left[1-\left(\frac{q(1-\beta)+d\beta}{[1-\beta(1-d)]}\right)\right]\frac{\left(\int_0^{k^*}k'dF(k')\right)}{[1-\alpha F(k^*)]}\right\}$$

Then

$$k^*\left\{1-\beta\left(1-F\left(k^*\right)\right)\right\} = \frac{\frac{(1-q)\lambda y-(1-q)\beta dk_D}{[1-\beta(1-d)]}+\alpha[1-F(k^*)]\widetilde{y}}{1-\alpha F(k^*)}\left[(1-\beta)\right]-\widetilde{y}+\beta\left[1-\left(\frac{q(1-\beta)+d\beta}{[1-\beta(1-d)]}\right)\right]\frac{\left(\int_0^{k^*}k'dF(k')\right)}{[1-\alpha F(k^*)]}$$

Add $\widetilde{y}$ to both sides to get

$$k^*\left\{1-\beta\left(1-F\left(k^*\right)\right)\right\}+\widetilde{y} = \frac{\frac{(1-q)\lambda y-(1-q)\beta dk_D}{[1-\beta(1-d)]}+\alpha[1-F(k^*)]\widetilde{y}}{1-\alpha F(k^*)}\left[(1-\beta)\right]+\beta\left[1-\left(\frac{q(1-\beta)+d\beta}{[1-\beta(1-d)]}\right)\right]\frac{\left(\int_0^{k^*}k'dF(k')\right)}{[1-\alpha F(k^*)]}$$

Multiply both sides by $\left[1-\alpha F\left(k^*\right)\right]$, this yields

$$\left[k^*\left\{1-\beta\left(1-F\left(k^*\right)\right)\right\}+\widetilde{y}\right]\left[1-\alpha F\left(k^*\right)\right] = \left\{\frac{(1-q)\lambda y-(1-q)\beta dk_D}{[1-\beta(1-d)]}+\alpha\left[1-F\left(k^*\right)\right]\widetilde{y}\right\}\left[(1-\beta)\right]$$
$$+\beta\left[1-\left(\frac{q(1-\beta)+d\beta}{[1-\beta(1-d)]}\right)\right]*\int_0^{k^*}k'dF\left(k'\right)$$

Simplify

$$k^*\left\{1-\beta\left(1-F\left(k^*\right)\right)\right\}\left[1-\alpha F\left(k^*\right)\right]+\widetilde{y}\left[1-\alpha F\left(k^*\right)\right] =$$
$$\frac{(1-q)\lambda y-(1-q)\beta dk_D}{[1-\beta(1-d)]}\left[(1-\beta)\right]+\alpha\left[1-F\left(k^*\right)\right]\widetilde{y}\left[(1-\beta)\right]+\beta\left[1-\left(\frac{q(1-\beta)+d\beta}{[1-\beta(1-d)]}\right)\right]\left(\int_0^{k^*}k'dF\left(k'\right)\right)$$

Send $\widetilde{y}\left[1-\alpha F\left(k^*\right)\right]$ to the right

$$k^*\left\{1-\beta\left(1-F\left(k^*\right)\right)\right\}\left[1-\alpha F\left(k^*\right)\right] = \left\{\frac{(1-q)\lambda y-(1-q)\beta dk_D}{[1-\beta(1-d)]}\left[(1-\beta)\right]\right\}+(1-\beta)\alpha\widetilde{y}$$
$$-(1-\beta)\alpha F\left(k^*\right)\widetilde{y}-\widetilde{y}+\widetilde{y}\alpha F\left(k^*\right)+\beta\left[1-\left(\frac{q(1-\beta)+d\beta}{[1-\beta(1-d)]}\right)\right]\left(\int_0^{k^*}k'dF\left(k'\right)\right)$$

Cancel terms on the right hand side to get

$$k^*\left\{1-\beta\left(1-F\left(k^*\right)\right)\right\}\left[1-\alpha F\left(k^*\right)\right] = \left\{\frac{(1-q)\lambda y-(1-q)\beta dk_D}{[1-\beta(1-d)]}\left[(1-\beta)\right]\right\}+\widetilde{y}\left[\alpha\left\{1-\beta\left[1-F\left(k^*\right)\right]\right\}-1\right]$$
$$+\beta\left[1-\left(\frac{q(1-\beta)+d\beta}{[1-\beta(1-d)]}\right)\right]\left(\int_0^{k^*}k'dF\left(k'\right)\right) \tag{A.13}$$

The term $\left\{1-\beta\left(1-F\left(k^*\right)\right)\right\}\left[1-\alpha F\left(k^*\right)\right]$ simplifies as follows, once (A.7) is replaced:

$$\left\{1-\beta\left(1-F\left(k^*\right)\right)\right\}\left[1-\alpha F\left(k^*\right)\right] = \left\{1-\beta\left(1-F\left(k^*\right)\right)\right\}\left[1-\beta\left(\frac{q(1-\beta)+d\beta}{[1-\beta(1-d)]}\right)\frac{F(k^*)}{\{1-\beta[1-F(k^*)]\}}\right]$$

$$\left\{1-\beta\left(1-F\left(k^*\right)\right)\right\}\left[1-\alpha F\left(k^*\right)\right] = \left\{1-\beta\left(1-F\left[k^*\right]\right)\right\}-\beta F\left(k^*\right)\left(\frac{q(1-\beta)+d\beta}{[1-\beta(1-d)]}\right)$$

$\{1-\beta(1-F(k^*))\}[1-\alpha F(k^*)] = (1-\beta)+\beta F[k^*]\left[1-\left(\frac{q(1-\beta)+d\beta}{[1-\beta(1-d)]}\right)\right]$

Then can use this result into (A.13), where the left hand side can be written as

$\text{LHS}=k^*\{1-\beta(1-F(k^*))\}[1-\alpha F(k^*)] = k^*\left\{(1-\beta)+F[k^*]\beta\left[1-\left(\frac{q(1-\beta)+d\beta}{[1-\beta(1-d)]}\right)\right]\right\}$

$\text{LHS}=k^*(1-\beta)+k^*F[k^*]\beta\left[1-\left(\frac{q(1-\beta)+d\beta}{[1-\beta(1-d)]}\right)\right]$

Thus obtain

$$k^*(1-\beta)+k^*F[k^*]\beta\left[1-\left(\frac{q(1-\beta)+d\beta}{[1-\beta(1-d)]}\right)\right] = \left\{\frac{(1-q)\lambda y-(1-q)\beta dk_D}{[1-\beta(1-d)]}[(1-\beta)]\right\}$$
$$+\widetilde{y}[\alpha\{1-\beta[1-F(k^*)]\}-1]+\beta\left[1-\left(\frac{q(1-\beta)+d\beta}{[1-\beta(1-d)]}\right)\right]\left(\int_0^{k^*}k'dF(k')\right)$$

Send $k^*F[k^*]\beta\left[1-\left(\frac{q(1-\beta)+d\beta}{[1-\beta(1-d)]}\right)\right]$ to the other side to obtain

$$k^*(1-\beta) = \left\{\frac{(1-q)\lambda y-(1-q)\beta dk_D}{[1-\beta(1-d)]}[(1-\beta)]\right\}+\widetilde{y}[\alpha\{1-\beta[1-F(k^*)]\}-1]$$
$$+\beta\left[1-\left(\frac{q(1-\beta)+d\beta}{[1-\beta(1-d)]}\right)\right]\left(\int_0^{k^*}k'dF(k')\right)-k^*F[k^*]\beta\left[1-\left(\frac{q(1-\beta)+d\beta}{[1-\beta(1-d)]}\right)\right]$$

on the RHS we have that the following term

$$\beta\left[1-\left(\frac{q(1-\beta)+d\beta}{[1-\beta(1-d)]}\right)\right]\left(\int_0^{k^*}k'dF(k')\right)-k^*F[k^*]\beta\left[1-\left(\frac{q(1-\beta)+d\beta}{[1-\beta(1-d)]}\right)\right]$$

can be written as

$$\beta\left[1-\left(\frac{q(1-\beta)+d\beta}{[1-\beta(1-d)]}\right)\right]\left\{\left(\int_0^{k^*}k'dF(k')\right)-k^*F[k^*]\right\}$$

Then we can write

$$k^*(1-\beta) = \left\{\frac{(1-q)\lambda y-(1-q)\beta dk_D}{[1-\beta(1-d)]}[(1-\beta)]\right\}+\widetilde{y}[\alpha\{1-\beta[1-F(k^*)]\}-1]$$
$$+\beta\left[1-\left(\frac{q(1-\beta)+d\beta}{[1-\beta(1-d)]}\right)\right]\left\{\left(\int_0^{k^*}k'dF(k')\right)-k^*F[k^*]\right\}$$

The term $\alpha\{1-\beta[1-F(k^*)]\}$, after inserting (A.7) can now be written as

$$\alpha\{1-\beta[1-F(k^*)]\} = \left[\beta\left(\frac{q(1-\beta)+d\beta}{[1-\beta(1-d)]}\right)\frac{1}{\{1-\beta[1-F(k^*)]\}}\right]\{1-\beta[1-F(k^*)]\} = \beta\left(\frac{q(1-\beta)+d\beta}{[1-\beta(1-d)]}\right)$$

Therefore one can write in the previous equation that

$$k^*(1-\beta) = \left\{\frac{(1-q)\lambda y-(1-q)\beta dk_D}{[1-\beta(1-d)]}[(1-\beta)]\right\}-\widetilde{y}\left[1-\beta\left(\frac{q(1-\beta)+d\beta}{[1-\beta(1-d)]}\right)\right]$$
$$+\beta\left[1-\left(\frac{q(1-\beta)+d\beta}{[1-\beta(1-d)]}\right)\right]\left\{\left(\int_0^{k^*}k'dF(k')\right)-k^*F[k^*]\right\}$$

The term $\left[1-\beta\left(\frac{q(1-\beta)+d\beta}{[1-\beta(1-d)]}\right)\right]$ can be written as $\left[(1-\beta)+\beta\left\{1-\left(\frac{q(1-\beta)+d\beta}{[1-\beta(1-d)]}\right)\right\}\right]$ to obtain

$$k^*(1-\beta) = \left\{\frac{(1-q)\lambda y-(1-q)\beta dk_D}{[1-\beta(1-d)]}[(1-\beta)]\right\}-\widetilde{y}\left[(1-\beta)+\beta\left\{1-\left(\frac{q(1-\beta)+d\beta}{[1-\beta(1-d)]}\right)\right\}\right]$$
$$+\beta\left[1-\left(\frac{q(1-\beta)+d\beta}{[1-\beta(1-d)]}\right)\right]\left\{\left(\int_0^{k^*}k'dF(k')\right)-k^*F[k^*]\right\}$$

Divide by $(1-\beta)$, obtain

$$k^* = \left\{\frac{(1-q)\lambda y-(1-q)\beta dk_D}{[1-\beta(1-d)]}\right\}-\widetilde{y}\left[1+\frac{\beta}{(1-\beta)}\left\{1-\left(\frac{q(1-\beta)+d\beta}{[1-\beta(1-d)]}\right)\right\}\right]$$
$$+\frac{\beta}{(1-\beta)}\left[1-\left(\frac{q(1-\beta)+d\beta}{[1-\beta(1-d)]}\right)\right]\left\{\left(\int_0^{k^*}k'dF(k')\right)-k^*F[k^*]\right\}$$

Obtain

$$k^* = \left\{\frac{(1-q)\lambda y-(1-q)\beta dk_D}{[1-\beta(1-d)]}\right\}-\widetilde{y}\left[1+\frac{\beta}{(1-\beta)}\left\{1-\left(\frac{q(1-\beta)+d\beta}{[1-\beta(1-d)]}\right)\right\}\right]$$
$$+\frac{\beta}{(1-\beta)}\left[1-\left(\frac{q(1-\beta)+d\beta}{[1-\beta(1-d)]}\right)\right]\left\{\left(\int_0^{k^*}k'dF(k')\right)-k^*F[k^*]\right\}$$

The term $\left[1-\left(\frac{q(1-\beta)+d\beta}{[1-\beta(1-d)]}\right)\right]$ can be written as $\frac{(1-\beta)(1-q)}{[1-\beta(1-d)]}$. Thus replacing into the above expression yields

$$k^*=\left\{\frac{(1-q)\lambda y-(1-q)\beta dk_D}{[1-\beta(1-d)]}\right\}-\widetilde{y}\left[1+\frac{\beta(1-q)}{[1-\beta(1-d)]}\right]+\left[\frac{\beta(1-q)}{[1-\beta(1-d)]}\right]\left\{\left(\int_0^{k^*}k'dF\left(k'\right)\right)-k^*F\left[k^*\right]\right\}$$

Now the term $\int_0^{k^*}k'dF\left(k'\right)$ is by definition given by $F\left[k^*\right]E\left[k|k\leq k^*\right]$. Therefore

$$\left(\int_0^{k^*}k'dF\left(k'\right)\right)-k^*F\left[k^*\right]=\left\{F\left[k^*\right]E\left[k|k\leq k^*\right]-k^*F\left[k^*\right]\right\}=F\left[k^*\right]\left\{E\left[k|k\leq k^*\right]-k^*\right\}$$

Which yields

$$k^*=\left\{\frac{(1-q)\lambda y-(1-q)\beta dk_D}{[1-\beta(1-d)]}\right\}-\widetilde{y}\left[1+\frac{\beta(1-q)}{[1-\beta(1-d)]}\right]+\left[\frac{\beta(1-q)}{[1-\beta(1-d)]}\right]F\left[k^*\right]\left\{E\left[k|k\leq k^*\right]-k^*\right\}$$

which is expression (8) in the text.

The above expression can be further simplified. Start by rewriting it as

$$k^*=\left\{\frac{(1-q)\lambda y}{[1-\beta(1-d)]}-\frac{(1-q)\beta dk_D}{[1-\beta(1-d)]}\right\}-\widetilde{y}\left[1+\frac{\beta(1-q)}{[1-\beta(1-d)]}\right]+\left[\frac{\beta(1-q)}{[1-\beta(1-d)]}\right]F\left[k^*\right]\left\{E\left[k|k\leq k^*\right]-k^*\right\}$$

$$k^*=\left\{\frac{\beta(1-q)}{[1-\beta(1-d)]}\frac{\lambda y}{\beta}-\frac{(1-q)\beta dk_D}{[1-\beta(1-d)]}\right\}-\widetilde{y}\left[1+\frac{\beta(1-q)}{[1-\beta(1-d)]}\right]+\left[\frac{\beta(1-q)}{[1-\beta(1-d)]}\right]F\left[k^*\right]\left\{E\left[k|k\leq k^*\right]-k^*\right\}$$

The term $\frac{\beta(1-q)}{[1-\beta(1-d)]}$ can be simplified. Since $\beta=\frac{1}{1+r}$, then

$$\frac{\beta(1-q)}{[1-\beta(1-d)]}=\frac{\frac{1}{1+r}(1-q)}{\left[1-\frac{(1-d)}{1+r}\right]}=\frac{\frac{1}{1+r}(1-q)}{\left[\frac{(1+r)-(1-d)}{1+r}\right]}=\frac{\frac{1}{1+r}(1-q)}{\left[\frac{(1+r-1+d}{1+r}\right]}=\frac{(1-q)}{r+d}$$

Using this result yields

$$k^*=\left(\frac{(1-q)\lambda y(1+r)-(1-q)dk_D}{r+d}\right)-\widetilde{y}\left[1+\frac{1-q}{r+d}\right]-\left(\frac{1-q}{r+d}\right)F\left[k^*\right]\left\{k^*-E\left[k|k\leq k^*\right]\right\}$$

which is equation (9) in the text.

Q.E.D.

A.2 Proof of Proposition 1: $F\left[k^*\right]$ is constant if income and cost variables are scaled

Let $\widehat{k}=\gamma k$, where γ is a scaling factor. The idea is to prove that $F\left[\widehat{k}^*\right]$ is independent of the scaling factor γ whenever income in both countries y and $\widetilde{y}$, as well as cost k_D are also scale by the same factor γ.

First, rewrite equation (9) in a simpler way as

$$k^*=A\left(y,\widetilde{y},k_D\right)-\left(\frac{1-q}{r+d}\right)F\left[k^*\right]\left\{k^*-E\left[k|k\leq k^*\right]\right\} \tag{A.14}$$

where

$$A\left(y,\widetilde{y},k_D\right)\equiv\frac{(1-q)\lambda y(1+r)-(1-q)dk_D-\widetilde{y}\left[r+d+(1-q)\right]}{r+d} \tag{A.15}$$

thus $A\left(\cdot\right)$ is linear in its arguments, so that $A\left(\gamma y,\gamma\widetilde{y},\gamma k_D\right)=\gamma A\left(y,\widetilde{y},k_D\right)$.

Second. Since $F\left[k\right]$ is given, and since γ is just scaling k, then by definition the CDF of the scaled variable G is given by $G\left[\widehat{k}\right]=F\left[\frac{\widehat{k}}{\gamma}\right]$.

Third. The expectation $E\left[\widehat{k}|\widehat{k}\leq\widehat{k}^{*}\right]$ can be rewritten as $\gamma E\left[k|k\leq k^{*}\right]$. The proof of this is trivial since $E\left[\widehat{k}|\widehat{k}\leq\widehat{k}^{*}\right]=E\left[\gamma k|\gamma k\leq\gamma k^{*}\right]=\gamma E\left[k|k\leq k^{*}\right]$.

Fourth. From equation (A.14), we can rewrite by scaling all relevant variables, and using the distribution of the scaled variable $G\left[\widehat{k}\right]$, then we can write that $\widehat{k}^{*}$ is, by definition, given by

$\widehat{k}^{*}=A\left(\gamma y,\gamma\widetilde{y},\gamma k_{D}\right)-\left(\frac{1-q}{r+d}\right)G\left[\widehat{k}^{*}\right]\left\{\widehat{k}^{*}-E\left[\widehat{k}|\widehat{k}\leq\widehat{k}^{*}\right]\right\}$.

Using linearity of the term $A\left(y,\widetilde{y},k_{D}\right)$, linearity of the expectational term $E\left[\widehat{k}|\widehat{k}\leq\widehat{k}^{*}\right]$, as well as using the fact that $G\left[\widehat{k}\right]=F\left[\frac{\widehat{k}}{\gamma}\right]$ yields a new expression

$\widehat{k}^{*}=\gamma A\left(y,\widetilde{y},k_{D}\right)-\left(\frac{1-q}{r+d}\right)F\left[\frac{\widehat{k}}{\gamma}\right]\left\{\widehat{k}^{*}-\gamma E\left[k|k\leq k^{*}\right]\right\}$

Fifth. Now conjecture that the relationship between thresholds is given $\widehat{k}^{*}=\gamma k^{*}$. Substituting the conjecture yields

$\gamma k^{*}=\gamma A\left(y,\widetilde{y},k_{D}\right)-\left(\frac{1-q}{r+d}\right)F\left[\frac{\gamma k^{*}}{\gamma}\right]\left\{\gamma k^{*}-\gamma E\left[k|k\leq k^{*}\right]\right\}$

Canceling γ on both sides, and rewriting $F\left[\frac{\gamma k^{*}}{\gamma}\right]$ as $F\left[k^{*}\right]$ yields again (A.14), thus confirming the conjecture. Since the threshold k^{*} is unique due to the fact that $V\left(M,k\right)$ and $V\left(S\right)$ can cross only once (in which case $k=k^{*}$), it follows that $\widehat{k}^{*}=\gamma k^{*}$ and $F\left[k^{*}\right]=G\left[\widehat{k}^{*}\right]$.

Q.E.D.

A.3 The Evolution of the Share of Foreign Population Living Abroad under Population Growth

To accommodate growth of the population, assume that the population in the host country grows at the gross rate η, so that $N_{t}=\eta N_{t-1}$ (for $\eta>0$), while the sending country grows at the rate (in this case their population is divided between unauthorized immigrants and stayers at home) $\widetilde{\eta}$, hence $\left(\widetilde{N}_{t}+U_{t}\right)=\widetilde{\eta}\left(\widetilde{N}_{t-1}+U_{t-1}\right)$.[13] Then in this case the evolution of the different magnitudes is given by

$$\widetilde{N}_{t}=\left(1-\left(1-q\right)F\left[k^{*}\right]\right)\widetilde{\eta}\widetilde{N}_{t-1}+d\widetilde{\eta}U_{t-1} \tag{A.16}$$

$$U_{t}=\left(1-q\right)F\left[k^{*}\right]\widetilde{\eta}\widetilde{N}_{t-1}+\left(1-d\right)\widetilde{\eta}U_{t-1} \tag{A.17}$$

Then the above expression can be rewritten as (12). However, what is different at a steady state is both the level of unauthorized immigration as well as the rate of unauthorized immigrants as percentage of natives. Hence, it is of interest to also study the share of unauthorized immigrants as percentage of the natives in the host country, defined as $\mu_{t}\equiv U_{t}/N_{t}$. It can be shown that (see next section for proof) the evolution of this share is given by

$$\mu_{t}=\widetilde{n}_{t}\cdot\widetilde{\mu}_{t} \tag{A.18}$$

which is the product of the relative population (foreigners to the host economy) given by $\widetilde{n}_{t}\equiv\frac{\widetilde{N}_{t}+U_{t}}{N_{t}}$ and the share of foreigners who live as unauthorized immigrants $\widetilde{\mu}_{t}$.

In equation (A.18), remember that $\widetilde{\mu}_{t}$ evolves autonomously, while the term $\widetilde{n}_{t}\equiv\frac{\widetilde{N}_{t}+U_{t}}{N_{t}}$ is discussed below.

[13] To keep the presentation as simple as possible, it is assumed that the children of unauthorized immigrants remain citizens of the sender country.

Since $\widetilde{n}_t \equiv \frac{\widetilde{N}_t+U_t}{N_t}$, and using the fact that $\left(\widetilde{N}_t + U_t\right) = \widetilde{\eta}\left(\widetilde{N}_{t-1} + U_{t-1}\right)$ and $N_t = \eta N_{t-1}$, then it is trivial to show that $\widetilde{n}_t$ also evolves autonomously, with

$$\widetilde{n}_t = \frac{\widetilde{\eta}}{\eta}\widetilde{n}_{t-1}.$$

Hence μ_t from equation (12) evolves as the product of two autonomous variables. While $\widetilde{\mu}_t$ evolves towards a constant share $\widetilde{\mu}_{ss}$, the population ratio $\widetilde{n}_t$ grows or shrinks over time since $\widetilde{n}_t = \left(\frac{\widetilde{\eta}}{\eta}\right)^s \widetilde{n}_{t-s}$. Thus under regimes with identical population growth rates we have that $\frac{\widetilde{\eta}}{\eta} = 1$ for all time periods, and the population ratio $\widetilde{n}_t$ is just some constant. This implies, however, that in levels, at steady state the stock of unauthorized immigrants grows at the common growth rate of the population in both countries.

Another interesting case to consider is when the population growth rate of the sender country is higher than that of natives. In this case the steady state, which exists for $\widetilde{\mu}_t$, leads to an ever growing share of unauthorized immigrants in the host country (an increasing μ_t).

A.3.1 Evolution of Unauthorized Stock as Percentage of Host Population

Start from defining μ_t as $\mu_t \equiv \frac{U_t}{N_t}$, which is the ratio of unauthorized immigrants to the host population. From the definition, multiply the right hand side by $\frac{\widetilde{N}_t+U_t}{\widetilde{N}_t+U_t}$ and reorder the terms in order to obtain

$\mu_t \equiv \frac{U_t}{N_t}\frac{\widetilde{N}_t+U_t}{\widetilde{N}_t+U_t} = \left(\frac{U_t}{\widetilde{N}_t+U_t}\right)\left(\frac{\widetilde{N}_t+U_t}{N_t}\right)$

Since by definition we have that $\frac{U_t}{\widetilde{N}_t+U_t} \equiv \widetilde{\mu}_t$ and $\frac{\widetilde{N}_t+U_t}{N_t} \equiv \widetilde{n}_t$, then we can write

$\mu_t = \widetilde{\mu}_t \cdot \widetilde{n}_t$

Q.E.D.

A.4 Proof of Proposition 2

Under a uniform distribution with $k \sim U[0, k_{\max}]$ it is the case that

$E[k|k \leq k^*] = \frac{k^*}{2}$ and $F[k^*] = \frac{k^*}{k_{\max}}$

Therefore equation (A.14) can be written under this distributional assumption as

$k^* = A - \left(\frac{1-q}{r+d}\right)\frac{(k^*)^2}{2k_{\max}}$

which is a quadratic equation in k^* given by

$\left(\frac{1-q}{r+d}\right)\frac{(k^*)^2}{2k_{\max}} + k^* - A = 0.$

The general solution to the quadratic equation $aX^2 + bX + c = 0$ yields a positive solution (the only one that makes sense in the current application) given by $x = -\frac{b}{2a} + \left(\frac{b^2}{4a^2} - \frac{c}{a}\right)^{\frac{1}{2}}$ where we have that $b = 1$, $a = \left(\frac{1-q}{r+d}\right)\frac{1}{2k_{\max}}$ and $c = -A$. Therefore the solution is given by

$$k^* = -\frac{1}{2\left(\frac{1-q}{r+d}\right)\frac{1}{2k_{\max}}} + \left(\frac{1}{4\left[\left(\frac{1-q}{r+d}\right)\frac{1}{2k_{\max}}\right]^2} + \frac{A}{\left(\frac{1-q}{r+d}\right)\frac{1}{2k_{\max}}}\right)^{\frac{1}{2}}$$

which can be rewritten as

$$k^* = -\left(\frac{r+d}{1-q}\right)k_{\max} + \left(\frac{1}{4\left[\left(\frac{1-q}{r+d}\right)\frac{1}{2k_{\max}}\right]^2} + \frac{A\left(\frac{1-q}{r+d}\right)\frac{1}{2k_{\max}}}{\left[\left(\frac{1-q}{r+d}\right)\frac{1}{2k_{\max}}\right]^2}\right)^{\frac{1}{2}}$$

And further simplified to

$$k^* = \left(\frac{r+d}{1-q}\right)k_{\max}\left\{\left[1 + \left(\frac{1-q}{r+d}\right)\frac{2A}{k_{\max}}\right]^{\frac{1}{2}} - 1\right\}.$$

Finally, replacing (A.15) into k^* yields

$$k^* = \left(\frac{r+d}{1-q}\right) k_{\max} \left\{ \left[1 + 2 \left(\frac{1-q}{r+d}\right) \left(\frac{(1-q)\,\lambda y\,(1+r) - (1-q)\,dk_D - \widetilde{y}\,[r+d+(1-q)]}{k_{\max}\,(r+d)} \right) \right]^{\frac{1}{2}} - 1 \right\},$$

which is precisely equation (15).

Similarly, the equilibrium probability $F[k^*]$ is given by definition as $F[k^*] \equiv \frac{k^*}{k_{\max}}$, which is equation (16).

Q.E.D.

A.5 Proof of Proposition 3

The function $x^{\frac{1}{2}}$, can be approximated with a first-order Taylor expansion around $x = 1$ as $x^{\frac{1}{2}} \approx 1 + \frac{1}{2}(x-1) = \frac{1}{2}(1+x)$.

Let $x \equiv 1 + 2\left(\frac{1-q}{r+d}\right)\left(\frac{(1-q)\lambda y(1+r)-(1-q)dk_D-\widetilde{y}[r+d+(1-q)]}{k_{\max}(r+d)}\right)$, which is the term in key brackets in equation (15). Then

$$x^{\frac{1}{2}} \approx \frac{1}{2}\left(1 + \left[1 + 2\left(\frac{1-q}{r+d}\right)\left(\frac{(1-q)\lambda y(1+r)-(1-q)dk_D-\widetilde{y}[r+d+(1-q)]}{k_{\max}(r+d)}\right)\right]\right)$$
$$\approx \frac{1}{2}\left(2 + 2\left(\frac{1-q}{r+d}\right)\left(\frac{(1-q)\lambda y(1+r)-(1-q)dk_D-\widetilde{y}[r+d+(1-q)]}{k_{\max}(r+d)}\right)\right)$$
$$\approx 1 + \left(\frac{1-q}{r+d}\right)\left(\frac{(1-q)\lambda y(1+r)-(1-q)dk_D-\widetilde{y}[r+d+(1-q)]}{k_{\max}(r+d)}\right)$$

Therefore we can write for (15), replacing the term with the square root can be written as

$$k^* \approx \left(\frac{r+d}{1-q}\right) k_{\max}\left\{1 + \left(\frac{1-q}{r+d}\right)\left(\frac{(1-q)\lambda y(1+r)-(1-q)dk_D-\widetilde{y}[r+d+(1-q)]}{k_{\max}(r+d)}\right) - 1\right\}$$
$$k^* \approx \left(\frac{r+d}{1-q}\right) k_{\max}\left\{\left(\frac{1-q}{r+d}\right)\left(\frac{(1-q)\lambda y(1+r)-(1-q)dk_D-\widetilde{y}[r+d+(1-q)]}{k_{\max}(r+d)}\right)\right\}$$

which simplifies to

$$k^* \approx \frac{(1-q)\,\lambda y\,(1+r) - (1-q)\,dk_D - \widetilde{y}\,[r+d+(1-q)]}{(r+d)}$$

and thus the probability of migration attempt is approximately given by

$$F[k^*] \approx \frac{(1-q)\,\lambda y\,(1+r) - (1-q)\,dk_D - \widetilde{y}\,[r+d+(1-q)]}{(r+d)\,k_{\max}}$$

Under the assumption that $k_{max} = \kappa\widetilde{y}$ for $\kappa > 0$ and a cost of deportation that is proportional to the host country income (opportunity costs), with $k_D = \sigma\lambda y$, then we can write

$$F[k^*] \approx \frac{(1-q)\lambda y(1+r)-(1-q)d\sigma\lambda y-\widetilde{y}[r+d+(1-q)]}{(r+d)\kappa\widetilde{y}}$$

which can be written as

$$F[k^*] \approx \frac{-[r+d+(1-q)]}{(r+d)\,\kappa} + \frac{\lambda\,(1-q)\,[(1+r)-d\sigma]}{(r+d)\,\kappa}\left(\frac{y}{\widetilde{y}}\right)$$

Q.E.D.

A.6 Proof of proposition 4 [Comparative Statics]

$$\frac{dF[k^*]}{d\lambda} = \left[\left(\frac{1-q}{r+d}\right)\left(\frac{1}{\kappa}\right)(1+r-d\sigma)\right]\left(\frac{y}{\widetilde{y}}\right) > 0$$

$$\frac{dF[k^*]}{dy} = \left[\left(\frac{1-q}{r+d}\right)\left(\frac{\lambda}{\kappa}\right)(1+r-d\sigma)\right]\frac{1}{\widetilde{y}} > 0$$

$\frac{dF[k^*]}{d\left(\frac{y}{\widetilde{y}}\right)} = \left[\left(\frac{1-q}{r+d}\right)\left(\frac{\lambda}{\kappa}\right)(1+r-d\sigma)\right] > 0$

$\frac{dF[k^*]}{d\widetilde{y}} = -\left[\left(\frac{1-q}{r+d}\right)\left(\frac{\lambda}{\kappa}\right)(1+r-d\sigma)\right]\left(\frac{y}{\widetilde{y}^2}\right) < 0$

$\frac{dF[k^*]}{d\kappa} = -\frac{1}{\kappa}\left[\frac{-(1+r+d-q)}{(r+d)\kappa} + \left[\left(\frac{1-q}{r+d}\right)\left(\frac{\lambda}{\kappa}\right)(1+r-d\sigma)\right]\left(\frac{y}{\widetilde{y}}\right)\right] = -\frac{1}{\kappa}F[k^*] < 0.$
Notice that this expression implies that $\frac{dF[k^*]}{d\kappa}\frac{\kappa}{F[k^*]} = -1$.

$\frac{dF[k^*]}{d\sigma} = -\left(\frac{1-q}{r+d}\right)\left(\frac{\lambda}{\kappa}\right)(d\sigma)\left(\frac{y}{\widetilde{y}}\right) < 0$

$$\begin{aligned}\frac{dF[k^*]}{d[d]} &= \frac{(1-q)}{(r+d)^2\kappa} + \frac{\lambda y(1-q)}{\widetilde{y}\kappa}\left\{\frac{(r+d)(-\sigma)-(1+r-d\sigma)}{(r+d)^2}\right\} \\ &= \frac{(1-q)}{(r+d)^2\kappa} + \frac{\lambda y(1-q)}{\widetilde{y}\kappa}\left\{\frac{-\sigma r-d\sigma-(1+r)+d\sigma}{(r+d)^2}\right\} \\ &= \frac{(1-q)}{(r+d)^2\kappa} + \frac{\lambda y(1-q)}{\widetilde{y}\kappa}\left\{\frac{-\sigma r-(1+r)}{(r+d)^2}\right\} \\ &= \frac{(1-q)}{(r+d)^2\kappa} - \frac{(1-q)}{(r+d)^2\kappa}\frac{\lambda y}{\widetilde{y}}\{\sigma r+(1+r)\} \\ &= \frac{(1-q)}{(r+d)^2\kappa}\left[1-\frac{\lambda y}{\widetilde{y}}\{\sigma r+(1+r)\}\right]\end{aligned}$$
whose sign depends on $\left[1-\frac{\lambda y}{\widetilde{y}}\{\sigma r+(1+r)\}\right]$. This number is negative since $\lambda y > \widetilde{y}$. That is $1-\frac{\lambda y}{\widetilde{y}}\{\sigma r+(1+r)\} < 0$ implies $\widetilde{y} < \lambda y\{\sigma r+(1+r)\}$, which is true and therefore $\frac{dF[k^*]}{d[d]} = \frac{(1-q)}{(r+d)^2\kappa}\left[1-\frac{\lambda y}{\widetilde{y}}\{\sigma r+(1+r)\}\right] < 0.$

$$\begin{aligned}\frac{dF[k^*]}{dq} &= \frac{1}{(r+d)\kappa} - \left[\left(\frac{1}{r+d}\right)\left(\frac{\lambda}{\kappa}\right)(1+r-d\sigma)\right]\left(\frac{y}{\widetilde{y}}\right) \\ &= \frac{1}{(r+d)\kappa} - \frac{1}{(r+d)\kappa}\lambda(1+r-d\sigma)\left(\frac{y}{\widetilde{y}}\right) = \frac{1}{(r+d)\kappa}\left[1-\lambda(1+r-d\sigma)\left(\frac{y}{\widetilde{y}}\right)\right]\end{aligned}$$
the sign depends on whether $\lambda(1+r-d\sigma)y > \widetilde{y}$.
Since from the definition of the equilibrium probability in (18)we have
$F[k^*] > 0$ iff $\frac{-(1+r+d-q)}{(r+d)\kappa} + \left[\left(\frac{1-q}{r+d}\right)\left(\frac{\lambda}{\kappa}\right)(1+r-d\sigma)\right]\left(\frac{y}{\widetilde{y}}\right) > 0$,
this implies that
$\left[\left(\frac{1-q}{r+d}\right)\left(\frac{\lambda}{\kappa}\right)(1+r-d\sigma)\right]\left(\frac{y}{\widetilde{y}}\right) > \frac{(1-q+r+d)}{(r+d)\kappa} > 0$
which after canceling the denominator term $(r+d)\kappa$ implies the condition
$F[k^*] > 0$ iff $(1-q)\lambda(1+r-d\sigma)y > (1-q+r+d)\widetilde{y} > 0$.
Therefore $\lambda(1+r-d\sigma)y$ is greater than $(1-q+r+d)\widetilde{y}$ since
$\lambda(1+r-d\sigma)y > (1-q)\lambda(1+r-d\sigma)y > (1-q+r+d)\widetilde{y}$
Therefore $\frac{dF[k^*]}{dq} < 0$.
Q.E.D.

$$\begin{aligned}\frac{dF[k^*]}{dr} &= d\frac{\frac{-(1+r+d-q)}{(r+d)\kappa}}{dr} + \left(\frac{y}{\widetilde{y}}\right)\left(\frac{\lambda}{\kappa}\right)\left[\frac{d\left\{\left(\frac{1-q}{r+d}\right)(1+r-d\sigma)\right\}}{dr}\right] \\ &= -\left[\frac{(r+d)\kappa-(1+r+d-q)\kappa}{(r+d)^2\kappa^2}\right] + \left(\frac{y}{\widetilde{y}}\right)\left(\frac{\lambda}{\kappa}\right)\left\{\left(\frac{1-q}{r+d}\right) + (1+r-d\sigma)\left[-\frac{1-q}{(r+d)^2}\right]\right\} \\ &= \left[\frac{(1-q)\kappa}{(r+d)^2\kappa^2}\right] + \left(\frac{y}{\widetilde{y}}\right)\left(\frac{\lambda}{\kappa}\right)\left\{\frac{1-q}{r+d} - (1+r-d\sigma)\left[\frac{1-q}{(r+d)^2}\right]\right\}\end{aligned}$$

$$= (1-q) + \left\{ \left(\frac{\lambda y}{\widetilde{y}} \right) (1-q)(r+d) - \left(\frac{\lambda y}{\widetilde{y}} \right) (1+r-d\sigma)[1-q] \right\}$$

Under what conditions is $\frac{dF[k^*]}{dr} < 0$?

$\frac{dF[k^*]}{dr} < 0$ iff $(1-q) + \left\{ \left(\frac{\lambda y}{\widetilde{y}} \right) (1-q)(r+d) - \left(\frac{\lambda y}{\widetilde{y}} \right) (1+r-d\sigma)[1-q] \right\} < 0$

iff (dividing everything by $(1-q)$)

$$1 + \left\{ \left(\frac{\lambda y}{\widetilde{y}} \right) (r+d) - \left(\frac{\lambda y}{\widetilde{y}} \right) (1+r-d\sigma) \right\} < 0 \text{iff}$$

$$1 + \left(\frac{\lambda y}{\widetilde{y}} \right) (r+d) < \left(\frac{\lambda y}{\widetilde{y}} \right) (1+r-d\sigma)$$

iff

$$\widetilde{y} + (\lambda y)(r+d) < (\lambda y)(1+r-d\sigma)$$

iff

$$\widetilde{y} + (\lambda y) r + (\lambda y) d < (\lambda y)(1-d\sigma) + (\lambda y) r$$

iff

$$\widetilde{y} + (\lambda y) d < (\lambda y)(1-d\sigma)$$

iff

$$\widetilde{y} < (\lambda y)(1-d(\sigma+1))$$

iff

$$\frac{\widetilde{y}}{\lambda y} + d(\sigma+1) < 1$$

Thus $\frac{dF[k^*]}{dr} < 0$ iff $d(\sigma+1) + \frac{\widetilde{y}}{\lambda y} < 1$.

Q.E.D.

B Unauthorized Immigration Stocks and IRCA adjustments

The following calculations detail how the IRCA-adjusted shares were constructed. The sources are the same as those for Mexico (e.g. population is from World Bank's World Development Indicators, while the construction of the unauthorized stock follows the instructions for Mexico).

Estimates of the stock of unauthorized immigrant population for the 1990 and 2000 years are available from the Department of Homeland Security, specifically at

https://ohss.dhs.gov/topics/immigration/unauthorized/population-estimates. The earlier estimates are available in the report "Estimates of the Unauthorized Immigrant Population residing in the United States: 1990 to 2000 " from the U.S. Immigration and Naturalization Service, Office of Policy and Planning (2003). These estimates are shown in table A1. The table also includes information available for year 2000 which uses a more recent/modern methodology to compute the stocks of unauthorized immigrants which is available from the "Illegal alien population residing in the United States: January 2015 (Population Estimates)" from Baker (2018) at the Department of Homeland Security, available at https://ohss.dhs.gov/sites/default/files/2023-12/18_1214_PLCY_pops-est-report.pdf.

Table A1. Stocks of Unauthorized Immigrants in 1000's

	Sources			Inferred
	2003 Report		2018 Report	
Year→	2000	1990	2000	1990
Mexico	4808	2040	4680	1985.691
El Salvador	189	298	430	677.989
Guatemala	144	118	290	237.639
Honduras	138	42	160	48.696

The information for 1990 is inferred by applying the observed growth rates from 1990 to 2000 under the old methodology and applied to the more recent estimates in year 2000. For example,

under the old methodology the stock of unauthorized immigrants relative to the same concept in 1990 is 4808/2040 = 2.356863 for Mexico. Thus with the newer estimate of 4,680 for Mexico in year 2000, it is inferred a level of 4680/2.356863 = 1985.691 (in 1000's). The same calculations are performed for each country next.

Guatemala

For Guatemala, start with a stock of unauthorized in 1990 of about 237,639, compared to a 15-64 population of Guatemala of 4,623,196. Thus without the IRCA adjustment, the share of Guatemalan population living unauthorized in the US would be $237,639/4,623,196 = 5.1\%$. In year 2000, the stock of unauthorized immigrants from Guatemala were 290,000, while the 15-64 population was 6,132,001. Thus the share (unadjusted with respect to IRCA reforms) in year 2000 was $290,000/6,132,001 = 4.7293 \times 10^{-2}$. About 4.7%

IRCA adjustments are the following. According to the Migration Policy Institute (Jonas, S. 2013) about 50,000 Guatemalans legalized their status due to IRCA. This changes the stock of unauthorized immigrants in 1990 to about 237,639+50,000=287,639, and then applying the same adjustments as for the Mexican case $50,000(1 - .0265 - .0085 - .0103)^{10} = 31,451$, results in increasing the unauthorized stock in year 2000 by about 31,451. Thus the IRCA adjusted shares are $287,639/4,623,196 = 6.2\%$ in 1990 and $(290,000 + 31,451)/6,132,001 = 5.2\%$ in year 2000.

Honduras

For Honduras, the number of unauthorized immigrants following the same calculations as for Mexico. Start with a stock of about 48,696 unauthorized immigrants in 1990, while the population was 2,525,144. The (not IRCA-adjusted) share is $48,696/2,525,144$ which is approximately 1.9%. The number of unauthorized immigrants in year 2000 is 160,000 and the 15-64 years old population from El Salvador was 3,548,843. Thus the unadjusted share in year 2000 is $160,000/3,548,843 = 4.5\%$.

According to Cramer (2024), IRCA legalized 15,078 hondureans. Thus doing the same IRCA adjustments as in Mexico's case yield an IRCA-adjusted share of $(48,696 + 15,078)/2,525,144 = 2.5\%$ in 1990 and then $\left(160,000 + 15,000(1 - .0265 - .0085 - .0103)^{10}\right)/3,548,843$ which is an IRCA-adjusted share of about 4.8%.

El Salvador

For El Salvador, following the same calculations as in the above cases yields a stock of about 677,989 unauthorized immigrants in 1990, with a 15-64 population of 2,994,266. Thus the unadjusted share is $677,989/2,994,266$, which is about 22.6%. The stock of unauthorized immigrants in year 2000 is 430,000 and the 15-64 population is 3,461,307 for an unadjusted share in year 2000 of $430,000/3,461,307. = 0.124\,23 = 12.4\%$.

According to Gammage (2007), IRCA legalized about 146,000 Hondurans. Thus doing the same IRCA adjustments as for the other countries, the shares are $(677,989 + 146,000)/2,994,266 = 27.5\%$ in 1990 and then for year 2000 the adjustment implies and adjusted share of

$\left(430,000 + 146,000(1 - .0265 - .0085 - .0103)^{10}\right)/3,461,307 = 15.1\%$.