Identifying Non-Gaussian Structural Shocks *

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Abstract

We develop a robust and tractable method to identify SVARs using non-Gaussian features of structural shocks. The approach combines inequality restrictions on higher-order moments with standard set-identifying constraints such as sign restrictions. To benchmark its performances, we apply it to the identification of monetary policy shocks. Combining standard minimal sign restrictions with a constraint that monetary policy shocks be leptokurtic, we recover several key properties documented in earlier work: a correct output response to policy tightening, salient policy narrative episodes, and a plausible central bank reaction function. We then identify sovereign and geopolitical risk shocks, assuming they are skewed and leptokurtic, and uncover sizable macroeconomic effects that remain hidden under conventional identification schemes.

Keywords: Shock identification, skewness, kurtosis, sign restrictions, monetary policy, euro area sovereign risk, geopolitical risk.

JEL classification: C32, E27, E32.

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1 Introduction

Since the seminal contribution of Sims (1980), a wide range of methods have been developed to study the propagation of structural shocks using SVAR models. Most identification schemes rely on exclusion or sign restrictions motivated by economic theory and impose constraints on the shocks' second-order moments, while leaving their higher-order moments unrestricted.[1]

Higher-order moment restrictions are of little use for identification in situations where shocks are nearly Gaussian. Indeed, in the pure Gaussian case limit, the first two moments of shocks fully characterize their distribution, so that higher-order moment restrictions are redundant with restrictions on second-order moments. However, macroeconomic and financial data often exhibit clear non-Gaussian features with normal-time symmetric variations coexisting with relatively frequent large changes that are often asymmetric in nature. Examples are the various types of financial crisis that can lead to sharp recessions, inflation surges induced by large supply disruptions, or abrupt changes in the stance of macroeconomic policies.

This paper makes three contributions. First, it introduces a new method that takes advantage of non-Gaussian features to identify SVARs. More precisely we apply inequality restrictions on some chosen higher-moments, in our applications skewness and kurtosis, of the structural shocks of interest and combine these restrictions with other set identification restrictions, in our applications sign-restrictions.

Second, to gauge the performances of our approach, we apply it to the widely studied monetary policy shocks. We assume monetary policy shocks to be leptokurtic. Indeed, while central banks' decisions are mostly predictable and lead to small policy surprises, unforeseen developments make such decisions sizably deviate from a fixed reaction function, leading to relatively frequent large monetary policy shocks. As we document, that identification assumption is also consistent with the pronounced excess kurtosis observed in a wide range of external proxies for such shocks. Combining the requirement that monetary policy shock kurtosis exceeds a lower bond with standard minimal sign restrictions *à la* Uhlig (2005), we recover key properties emphasized in earlier studies that rely on the use of either an external instrument (e.g. Gertler and Karadi 2015) or narrative restrictions (e.g. Antolin-Diaz and Rubio-Ramírez 2018). So, imposing higher-order moment inequality restrictions can be an alternative to such approaches.

Third, we illustrate the usefulness of our approach with two applications where

[1] See Ramey (2016), Nakamura and Steinsson (2018b) for surveys.

there are no obvious candidates for external instruments or narrative restrictions: the transmission of sovereign risk shocks in the euro area and the effects of geopolitical risk shocks on the U.S. economy. We assume that such shocks are both skewed and leptokurtic, in line with the frequent unexpected large spikes in measures of sovereign and geopolitical risks. Combining that assumption with minimal sign restrictions uncovers effects that standard methods fail to detect.

Existing methods that rely on deviations from Gaussianity to identify structural shocks typically exploit exact parametric restrictions, allowing point identification of the system of structural shocks without the need for additional identification assumptions.[2] These methods can be very powerful, but they are also prone to misspecification and small-sample estimation biases. Such practical challenges can be important for sample sizes typically available when working with macroeconomic variables. And they become more problematic as the dimensionality of the system under study increases.

Our approach seeks to preserve the advantages of using non-Gaussianity for identification while enhancing both robustness and tractability. Specifically, we adopt minimal and non-parametric assumptions to model deviations from normality. We postulate that the economy can be described by a VAR with uncorrelated, non-Gaussian structural shocks. We impose inequality restrictions on higher-order moments—in our applications, skewness and kurtosis—of a limited number of such structural shocks. Using inequality restrictions limits the risk of relying to much on a potentially biased in-sample estimate to achieve identification. In addition, focusing on a limited number of shocks makes the method tractable even in systems with many variables. These more robust and tractable assumptions yield set-identification rather than point-identification. But they can be combined with other constraints, such as sign or narrative restrictions, to narrow the identified set. We discuss how to specify such higher-order inequality restrictions in practice.

We develop a Bayesian methodology to implement our identification approach. We estimate a VAR with non-Gaussian errors and conduct inference, building on Petrova (2022). We then draw orthonormal rotation matrices using the algorithm of Rubio-Ramírez, Waggoner and Zha (2010). We retain the draws that satisfy a set of economic (e.g., sign) restrictions and whose empirical distribution exhibits the degree of asymmetry or fat-tailedness that is chosen. A key feature of our method is also to use inequality restrictions on robust non-parametric estimators of skewness and kurtosis, based on percentile distances in the empirical distribution of shocks. This further helps mitigating small-sample biases often associated with higher-order moment estimation.

[2] See **?** for a survey.

We implement our method to identify the widely studied monetary policy shocks in three different settings, thereby providing three different proofs of concept for our approach. First, we apply it to simulated data from the medium-scale New Keynesian model of Smets and Wouters (2007), estimated on U.S. data. Notably, the recovered monetary policy shock from this model exhibits the most significant excess kurtosis. We identify monetary policy shocks by augmenting the agnostic sign restrictions of Uhlig (2005) with an inequality restriction on excess kurtosis. While a monetary tightening is contractionary in the data-generating model, Uhlig (2005)'s approach fails to uncover this effect. In contrast, our method recovers a clear contractionary response, thereby strongly mitigating the masquerading problem highlighted by Wolf (2020)—linear combinations of positive supply and demand shocks, can "masquerade" as contractionary monetary policy shocks because they also raise interest rates and lower inflation in the short run.

As a second proof of concept, we compare our approach with a method that exploits exact equality constraints on the third and/or fourth moments of the system of structural shocks. While this more restrictive approach allows to point identify the structural shock of interest and performs very well for very large samples, our method is much more accurate when applied to samples of the size typical in macroeconomic studies. The reason is that getting unbiased in-sample estimates of higher-order moments then proves to be quite challenging as these are very sensitive to outliers and minor perturbation of the data.

As a third proof of concept, we revisit the effects of US monetary policy shocks on output and inflation using a VAR model estimated on post-war data. Combining Uhlig (2005)'s minimal sign restrictions with a requirement that the excess kurtosis of monetary policy shocks be greater than a chosen lower bound, we find that a monetary policy tightening has a clear contractionary impact on output. Our approach also overcomes known shortcomings of Uhlig (2005)'s agnostic sign restrictions —such as leading to a mis-classification of the October 1979 Volcker shock as expansionary (Antolin-Diaz and Rubio-Ramírez 2018) or implausible estimates of monetary policy response to output (Arias, Caldara and Rubio-Ramírez 2019). Beyond recovering such key properties, our method is also consistent with the significant excess kurtosis of monetary policy shocks, a robust feature that we observe in a wide range of proxies of monetary policy shocks in the US, the euro area and the UK.

We then apply our identification method to two empirical questions that are relatively underexplored and for which higher-order moment restrictions are particularly natural. First, we estimate a macro-financial VAR for the Euro Area that includes

the Italian-German sovereign bond spread. We focus on large, positive sovereign risk shocks—those that significantly increase spreads and are likely to drive first-order macroeconomic outcomes. To capture this, we impose restrictions on positive skewness and excess kurtosis of the shock to EA sovereign spreads, alongside minimal sign restrictions on the response of the 10-year Italian bond yield, the Italian-German spread, and corporate borrowing costs, that are consistent with structural models such as, for example, Bocola (2016).

We find that sovereign risk shocks lead to an immediate and sharp tightening of credit conditions, with real activity in the euro area responding on impact. These effects largely dissipate within three years, though the rise in unemployment is more persistent. In contrast, when identification relies solely on sign restrictions, the estimated macroeconomic effects are statistically insignificant.

In a second application, we study the impact of shocks to the geopolitical risk index developed by Caldara and Iacoviello (2022). We target large, unexpected tail events, as these are likely to leave the strongest imprint on macroeconomic dynamics. As before, we combine higher-moment restrictions with a minimal set of sign restrictions.

Our findings are consistent with those of Caldara and Iacoviello (2022) and the broader literature on uncertainty shocks (Bloom 2009): geopolitical risk reduces investment and hours worked. However, our estimated effects are substantially larger and more persistent. We also find a lasting tightening of U.S. financial conditions in response to such shocks. Crucially, sign restrictions alone yield highly imprecise and statistically weak estimates of the macroeconomic impact of geopolitical risk.

1.1 Literature review

Our work contributes to the literature that identifies structural VAR models. Recent advances rely on proxy-structural shocks that can be used as external instruments (e.g., Stock and Watson 2012, Mertens and Ravn 2013, Gertler and Karadi 2015, Känzig 2021), or combine sign and narrative restrictions (e.g., Antolin-Diaz and Rubio-Ramírez 2018, Drechsel 2025). We combine higher-order moment inequality restrictions with sign restrictions.

Our method complements these recent contributions. When applied to the identification of monetary policy shocks, it delivers results that are broadly consistent with what is obtained with these alternative approaches. But it can also be applied to studies where narratives or external instruments are not available and assuming that structural shocks are non-gaussian is a natural assumption.

Our paper also contributes to the literature that relies on higher-order moment

restrictions to identify structural VARs. Some methods achieve point identification by postulating that the structural shocks are independent and non-Gaussian and using the implied restrictions on the higher moments of the shocks (see Lanne, Meitz and Saikkonen 2017, Gouriéroux, Monfort and Renne 2017, 2019, Lanne, Liu and Luoto 2023, among others). Drautzburg and Wright (2023) leverage the independence restriction to narrow the set of shocks identified using standard signs restrictions. Other approaches exploit time-variation in the conditional variance of shocks to identify them using parametric assumptions (see Sentana and Fiorentini 2001, Rigobon 2003, Brunnermeier, Palia, Sastry and Sims 2021, among others). Lewis (2025) identifies structural VARs by relying on such conditional heteroskedasticity but without any parametric assumptions on the shock volatility process.

These methods achieve point identification without relying on economic assumptions. However, Montiel Olea, Plagborg-Møller and Qian (2022) underscore two main potential drawbacks. First, they often rely on statistical assumptions that can exclude situations of economic interest. For instance, imposing independence rules out configurations where structural shocks have a variance that decreases or increases at the same time, as it did during the Great Moderation or during Covid. Second, they can suffer from potential weak-identification and lack of robustness issues, especially for samples of limited size typical in macroeconomic applications. Indeed, they involve equality restrictions on point-estimates of higher-order moments for all the shocks in the system, each of these being very sensitive to outliers in small samples.

By contrast, our approach requires much less restrictive assumptions. We merely postulate some deviation from Gaussianity for a set of structural shocks of interest, and we do not impose that the structural shocks of the system be independently distributed. Our method is also more robust and tractable. Indeed, we do not rely on a set of equality constraints applied to a matrix of in-sample estimates of the empirical innovations higher-order moments. We impose inequality restrictions on the higher moments of the structural shocks and use non-parametric robust methods based on the distance between different percentiles of the shock empirical distribution. Because of these less restrictive assumptions, our approach only allows one to set-identify the shocks of interest rather than to point identify the system of shocks. However, this gives us some flexibility and allows us to couple these higher-order moment restrictions with other popular assumptions—motivated by economic analysis—on the signs (or zero) impact of the shock on the endogenous variables, magnitude or elasticity bounds, or narrative restrictions on historical episodes.

2 Identification with higher-order moments

Let us consider that the econometrician observes an n-dimensional vector of *empirical innovations*, ι_t.[3] We postulate that these innovations are linear combinations of a $n-$dimensional vector of *structural shocks*, ν_t, namely,

$$\iota_t = A_o \nu_t,$$

where A_o is the true orthonormal rotation matrix, e.g. $A_o A_o' = A_o' A_o = I$, embedding the structural relationship between shocks and innovations.

To start, consistent with several works achieving identification through restrictions on higher-order moments, we consider that the structural shocks verify the following assumption:

Assumption 1 *The elements of ν_t have unit-variance and are uncorrelated. Some shocks have non-Gaussian third or fourth moments, i.e. $E(\nu_{i,t}^3) = \zeta_i \neq 0$ or $E(\nu_{i,t}^4) = \xi_i \neq 3$ for some i. Moreover, $E(\nu_{i,t}\nu_{j,t}\nu_{k,t}) = 0$ for all $i \neq j, k$ and $E(\nu_{i,t}\nu_{j,t}\nu_{k,t}\nu_{m,t}) = 0$ for all $i \neq j, k, m$ and $E(\nu_{i,t}^2 \nu_{j,t}^2) = 1$ for all $i \neq j$.*

These conditions guarantee that some shocks in the system feature departures from Gaussianity. Moreover, the assumption postulates that all shocks are uncorrelated also for the third and fourth moments so that the co-skewness and co-kurtosis are zero across all structural disturbances. As we will show next, this assumption implies that the empirical innovation third or fourth moments can be expressed as the 'diagonal' matrix containing the third or fourth moments of the structural shocks 'sandwiched' by the matrix A_o, i.e. the true orthonormal rotation matrix, up to a sign switch or column order permutation. In other words, one can retrieve the structural rotation matrix by applying the singular value decomposition to the matrix collecting the sample third or fourth moments of the empirical innovations. We call this approach *equality restrictions* (or statistical) identification.

Our approach rather exploits *inequality restrictions* and requires looser assumptions. We consider two alternative setups. First, we postulate that only the shock of interest has (i) non-Gaussian third or fourth moment and (ii) zero co-skewness and co-kurtosis with the other shocks as follows:

[3] Typically, these innovations will be obtained as the orthogonalized reduced-form residuals of a VAR model estimated on a long time series of data. In this section, we focus on the identification scheme, temporarily setting aside estimation and inference issues, which are discussed in Section 4.

Assumption 2 *The elements of ν_t have unit-variance and are uncorrelated. The n^{th} shock is such that $E(\nu_{n,t}^3) = \zeta_n \neq 0$ or $E(\nu_{n,t}^4) = \xi_n \neq 3$. Moreover, for $j, k, m = 1, ..., n-1$ we have that $E(\nu_{n,t} \cdot \nu_{j,t}\nu_{k,t}) = 0$, $E(\nu_{n,t} \cdot \nu_{j,t}\nu_{k,t}\nu_{m,t}) = 0$ and $E(\nu_{n,t}^2 \cdot \nu_{j,t}^2) = 1$.*

This assumption is used to derive some analytical results that are helpful to understand how our method works. We then consider an even less restrictive version of that assumption where the shock of interest still (i) has non-Gaussian third or fourth moment but (ii) potentially features co-skewness or co-kurtosis with other shocks:

Assumption 3 *The elements of ν_t have unit-variance and are uncorrelated. The n^{th} shock is such that $E(\nu_{n,t}^3) = \zeta_n \neq 0$ or $E(\nu_{n,t}^4) = \xi_n \neq 3$.*

2.1 Equality restrictions

In this section, we show how, under certain conditions, the higher-order moments of the empirical innovations allow to point identify the structural impact matrix, A_o. We leverage a key result from Independent Component Analysis (ICA), which states that if at most one of the components ν_t is Gaussian, then the matrix A_o is point-identified, up to column sign changes and permutations. As a result, all structural shocks can be recovered from the empirical innovations.[4] This result is established in Comon (1994, Theorem 11) and has been further discussed and employed for identification purposes in the literature; see, for instance, Gouriéroux et al. (2019) and Lanne et al. (2017). We focus on the configurations where the shock exhibits either non-zero skewness (third moment) or excess kurtosis (fourth moment).

A spectral decomposition approach. Denote with $\boldsymbol{e}_i$ the $n \times 1$ vector with zeros everywhere except a one in the i^{th} position, and with J_i the $n \times n$ matrix of zeros everywhere except one in the i^{th} position of the main diagonal. We can derive the following proposition:

Proposition 1 *Under Assumption 1, the matrix collecting the squared third moments of the empirical innovations is given by*

$$E(\iota_t\iota_t' \otimes \iota_t')E(\iota_t\iota_t' \otimes \iota_t')' = A_o\Lambda_\zeta A_o',$$

where Λ_ζ is a diagonal matrix collecting the squared third moments of the structural shocks, ζ_i^2, for all i.

[4] ICA exploits higher-order statistical moments to recover statistically independent components from a vector of linearly mixed signals, under the assumption of non-Gaussianity.

The proof is in Appendix A. This spectral decomposition implies that each eigenvalue corresponds to the square of the third moment of a structural shock, and the associated unit-length eigenvector coincides with a column of the original structural (or impact) matrix. Therefore, as long as the shock of interest has a non-zero third moment that differs from those of the other structural shocks, its impact on the empirical innovations can be identified.

Note, however, that this requires exploiting the full set of third moments of the empirical innovations and imposing co-skewness equal to zero across all shocks. Moreover, the identification is only up to a sign change and a permutation of the columns. This means that additional assumptions are needed to point-identify the shock of interest.

Similar results apply to the case of fourth moments.

Proposition 2 *Under Assumption 1, the matrix collecting the fourth moments of the empirical innovations is given by*

$$E(\iota_t \iota_t' \otimes \iota_t' \otimes \iota_t) = P \Lambda_\xi P' + \mathcal{K}^z,$$

where Λ_ξ is a diagonal matrix collecting the excess kurtosis of the structural shocks, the matrix $P = (A_o \otimes A_o)$, and $\mathcal{K}^z$ denote the matrix collecting the fourth moments of the standard Gaussian distribution.

The proof is in Appendix A. If one also assumed that the shock of interest has the largest fourth moment, then the corresponding column of the original rotation matrix could be recovered by taking the first n elements of the leading eigenvector and normalizing them by the square root of the absolute value of its first element, i.e., $P(1\!:\!n, 1)/\sqrt{|P(1,1)|}$. Alternatively, one can follow the procedure described in Kollo (2008), which involves summing the n^2 blocks of squared submatrices of the fourth moments of the empirical innovations, and then performing a spectral decomposition of the resulting matrix. This yields the eigenvalues (i.e., the fourth moments of the structural shocks) and the corresponding eigenvectors (i.e., their impacts on the empirical innovations). Both approaches allow to recover the impact of the non-Gaussian shock.

Limitations While this approach allows to achieve point identification of the system of shocks, it is also prone to three main limitations which are worth emphasizing. These are reminiscent of the ones made by Montiel Olea et al. (2022). First, it requires that structural shocks have zero cross third and fourth moments, so it is not suited for situations where the economy becomes more volatile no matter the source of shocks. Second, the method requires to know the full set of higher-order moments

of the empirical innovations, even when one is interested in identifying the impact of a single shock. The number of such moments grows significantly with the size of the VAR model. This is problematic as estimates of higher-order moments can be very sensitive to outliers or minor perturbation of the data and so these might be imprecise in short samples. By using several potentially imprecise estimates of these higher-order moments, the methodology reinforces the risk of obtaining corrupted estimates of the shock's impact on observables.[5] Third, the eigenvalue decomposition identifies the shock up to a permutation of columns. This implies that one does not know where to locate one shock of interest compared to the others. Related to that, the procedure does not necessarily imply that the shocks have an economic interpretation, as the statistical properties used for identification do not always derive from restrictions implied by economic reasoning.

2.2 Inequality restrictions

Rather than higher-order moment equality restrictions, our approach relies on inequality restrictions. It builds on a key result that maps the higher-order moments of the structural shock of interest to the corresponding column of the impact matrix, which captures the effects of the shock itself on the empirical innovations without specifying how the other structural shocks' higher-order moments are related to each other.

Proposition 3 *Assume Assumption 2 holds. Let $\boldsymbol{\alpha}_n$ denote the n-th (rightmost) column of A_o, the true impact matrix of the structural shock on the empirical innovations. Let $\breve{\nu}_{n,t} = \boldsymbol{a}_n' \iota_t$ be a candidate structural shock with $\boldsymbol{a}_n$ a unit-length vector of weights. When $\boldsymbol{a}_n = \boldsymbol{\alpha}_n$, then the candidate shock ($\breve{\nu}_{n,t}$) has the same higher-order property than the 'true' structural shock.*

The proof is in Appendix A. The proposition offers a necessary condition for identification. Consider a candidate orthonormal rotation matrix A, and an associated candidate structural shock of interest $\breve{\nu}_{n,t} = \boldsymbol{a}_n' \iota_t$ that leads to different higher-order moments than the structural shock, then, according to the proposition, it must be the case that $\boldsymbol{a}_n \neq \boldsymbol{\alpha}_n$.

A related result is that, when an orthonormal rotation matrix leads to a candidate structural shock that does not replicate the true third- and fourth-order moments, then the rotation differs from the true rotation A_o. Importantly, this result holds regardless of shocks co-skewness or co-kurtosis property as stated in the following proposition:

[5] We illustrate this point in section 5 using as laboratory a standard macro New-Keynesian model.

Proposition 4 *Assume Assumption 3 holds. Let $\boldsymbol{\alpha}_n$ denote the n-th (rightmost) column of A_o, the true impact matrix of the structural shock on the empirical innovations. Let $\breve{\nu}_{n,t} = \boldsymbol{a}_n' \iota_t$ be a candidate structural shock with $\boldsymbol{a}_n$ a unit-length vector of weights. When $E\left(\breve{\nu}_{n,t}^3\right) \neq \zeta_n$ or $E\left(\breve{\nu}_{n,t}^4\right) \neq \xi_n$ then $A \neq A_o$.*

Proof. Suppose the proposition is not verified, that is $E\left(\breve{\nu}_{n,t}^3\right) \neq \zeta_n$ and $A = A_o$. Let $S_\nu = E(\nu_t \nu_t' \otimes \nu_t')$ with the (n, n^2)-element of the matrix equal to ζ_n. We have that $\breve{\nu}_t = A\iota_t = A_o\iota_t = \nu_t$, then $E\left(\breve{\nu}_t \breve{\nu}_t' \otimes \breve{\nu}_t'\right) = S_\nu$. This implies that $E\left(\breve{\nu}_{n,t}^3\right) = \zeta_n$, which contraddicts the assumption. Analogous arguments go for the fourth moment. ■

As a consequence, among the set of candidate orthonormal rotations leaving the variance-covariance matrix of the structural shocks unchanged, the ones that do not replicate the higher-order moment properties of a structural shock of interest can be discarded. These necessary conditions can therefore be used as 'filters' to shrink the set of admissible rotations. Were the true higher-order moments of the shock of interest known, one could use a set of equality restrictions $E\left(\breve{\nu}_{n,t}^3\right) = \zeta_n$ and/or $E\left(\breve{\nu}_{n,t}^4\right) = \xi_n$ as filter. However, given that structural shocks are not observed, and that higher-order moments have to be estimated, we consider the more robust approach of imposing inequality restrictions on estimated third- and fourth-order moments of the distribution of the structural shock of interest as stated in the following proposition:

Proposition 5 *Assume that Assumption 3 holds. Let $A(\theta)$ be a generic orthonormal rotation with angle $\theta \in (-\pi, \pi)$, $\boldsymbol{a}_n(\theta)$ denote the n-th (rightmost) column of $A(\theta)$, and $\breve{\nu}_{n,t}(\theta) = \boldsymbol{a}_n(\theta)' \iota_t$ be a candidate structural shock of interest. Let H_n be a set of restrictions on the third and fourth moments of $\breve{\nu}_{n,t}(\theta)$:*

$$H_n = \left\{ |\underline{\zeta}| \leq |E\left(\breve{\nu}_{n,t}(\theta)^3\right)| \leq |\bar{\zeta}| \ \vee \ \underline{\xi} \leq E\left(\breve{\nu}_{n,t}(\theta)^4\right) \leq \bar{\xi} \right\},$$

with $|\underline{\zeta}| \leq \zeta_n \leq |\bar{\zeta}|$ and $|\underline{\xi}| \leq \xi_n \leq |\bar{\xi}|$, then

$$A_o \in \{A(\theta) \mid \theta \in (-\pi, \pi) \ \wedge \ H_n\} \subseteq \{A(\theta) \mid \theta \in (-\pi, \pi)\}.$$

The proof directly follows from Proposition 4.

Restrictions considered Let $\mathcal{S}(x)$ and $K^\star(x)$ denote estimators of respectively the skewness and the excess kurtosis of a variable x.[6] The first type of restrictions is on the shape of the distribution of the structural shock which can be assumed to feature either asymmetry or fat tails or both, as follows:

[6] We discuss robust estimators of these moments in section 4.

Restriction 1 (Asymmetry) *The structural shock x features a longer tail on the right (left) side of the distribution such that its skewness is greater than positive threshold, i.e. $\mathcal{S}(x) > \epsilon_{\mathcal{S}}$ ($\mathcal{S}(x) < \epsilon_{\mathcal{S}}$).*

Restriction 2 (Fat-tailedness) *The structural shock x features tails thicker than the normal distribution such that its excess kurtosis is greater than positive threshold, i.e. $K^{\star}(x) > \epsilon_{\mathcal{K}}$.*

These restrictions can be combined together in situations where the structural shock is assumed to have both an asymmetric and leptokurtic distribution. Alternatively, one can postulate that the structural shock is non-Gaussian without specifying whether the deviation stems from its third or fourth moment as follows:

Restriction 3 (Non-Gaussianity) *The structural shock x is **not** distributed as a Gaussian distribution if the absolute values of skewness or of excess kurtosis is positive, i.e. $|\mathcal{S}(x)| > \epsilon_{\mathcal{S}} > 0$ or $|K^{\star}(x)| > \epsilon_{\mathcal{K}} > 0$.*

Finally, another natural restriction is to impose a ranking on the third or on the fourth moment of the shock of interest as follows:

Restriction 4 (Ranking) *The structural shock x_1 has the largest absolute value of skewness or excess kurtosis (or both) among all the shocks in the system, i.e. x_j with $j = 2, .., n$, so that we have for each $j \neq 1$*

$$|\mathcal{S}(x_1)| > |\mathcal{S}(x_j)|$$
$$|K^{\star}(x_1)| > |K^{\star}(x_j)|.$$

Advantages This approach addresses the three main limitations outlined in Section 2.1. First, it does not require to impose that the shocks are independent. In particular, it does not require to impose zero co-skewness or co-kurtosis across the structural shocks. Second, it only requires estimating the third and/or fourth moment of the structural shock of interest and allows to use robust estimators of these moments based on the percentile of the empirical distribution of the shock, thus avoiding computing a large number of sensitive sample third or fourth moments which can lead to biased estimates. That risk is further mitigated by the fact that we impose an inequality rather than equality restriction, which makes the imprecision in the third or fourth moments estimates having less of an impact on the resulting set of estimates. Third, inequality restrictions achieve set-identification as opposed to point-identification. Rather than a drawback, this allows to combine these restrictions with other assumptions such as

signs, zeros, narrative, magnitude and/or statistical independence. In this sense higher-moments inequality restrictions are *modular*, as they can be flexibly used standalone or in combination with other assumptions.

2.3 An analytical example

We conclude this section offering an analytical example to illustrate that inequality restrictions on higher moments of the structural shock can shrink the identified set of the structural parameter. To this aim, we assume that $n = 2$ and that $\nu_{1,t} \sim (0, 1)$ and $\nu_{2,t} \sim (0, 1)$. Moreover, we assume that the third moments of the structural shocks are zero and one respectively, i.e. $E(\nu_{1,t}^3) = 0$ and $E(\nu_{2,t}^3) = 1$, and cross second and third moments are zero, i.e. $E(\nu_{1,t}\nu_{2,t}) = E(\nu_{1,t}\nu_{2,t}^2) = E(\nu_{1,t}^2\nu_{2,t}) = 0$. The structural equations are defined by

$$\iota_{1,t} = \cos\theta_o \nu_{1,t} - \sin\theta_o \nu_{2,t},$$
$$\iota_{2,t} = \sin\theta_o \nu_{1,t} + \cos\theta_o \nu_{2,t},$$

where θ_o is the 'true' unknown angle of rotation with $\theta_o \in (-\pi/2, \pi/2)$.[7] We assume $\theta_o \neq 0$ else the problem is trivial. The econometrician only observes ι_t. While first and second moments do not depend on θ_o, third moments do. In particular, the matrix of the third moments of the empirical innovation is given by $E(\iota_t\iota_t' \otimes \iota_t') = \underbrace{\begin{pmatrix} \sin^2\theta_o & -\sin\theta_o\cos\theta_o \\ -\sin\theta_o\cos\theta_o & \cos^2\theta_o \end{pmatrix}}_{\Omega} \otimes \begin{pmatrix} -\sin\theta_o & \cos\theta_o \end{pmatrix}$. Since Ω is an idempotent matrix, we have that $E(\iota_t\iota_t' \otimes \iota_t')E(\iota_t\iota_t' \otimes \iota_t')' = \Omega$. The characteristic polynomial of Ω is $(\sin^2\theta_o - \lambda)(\cos^2\theta_o - \lambda) - \sin^2\theta_o\cos^2\theta_o$ and the associated eigenvalues are zero and one respectively. The eigenvector associated with the non-zero eigenvalue is $\begin{pmatrix} -\sin\theta_o & \cos\theta_o \end{pmatrix}'$.

Our preferred approach assumes that the second shock has positive skewness; no assumptions about the third moment of the other shock nor about their co-skewness are imposed. More formally, we only consider the set of rotations such that the following inequality is verified: $E(\breve{\nu}_{2,t}^3) > 0$.

Let A be a generic rotation with angle θ, and let $\breve{\nu}_t = A'\iota_t$. The corresponding population moment is then $E(\breve{\nu}_{2,t}^3) = E(-\sin\theta\iota_{1,t} + \cos\theta\iota_{2,t})^3 = [\text{sign}(\cos\theta_o)\cos(\theta - \theta_o)]^3 > 0$. Since $\text{sign}(\cos\theta_o) = 1$ for all $\theta_o \in (-\pi/2, \pi/2)$, the latter is positive whenever $\cos(\theta - \theta_o) > 0$, which occurs when in the region of points where $\max\{-\pi/2, \theta_o - \pi/2\} <$

[7] We restrict θ_o to $\pm\pi/2$ to have positive entries in the main diagonal of the impact matrix A_o.

$\theta < \min\{\pi/2, \theta_o + \pi/2\}$.[8] This condition allows to shrink the set of admissible rotations which otherwise would be the set $\theta \in (-\pi/2, \pi/2)$; in particular, when θ_o is positive (negative), the lower (upper) bound shrinks.[9]

3 Higher-moment inequality restrictions in practice

Choosing the restriction The higher-order-moment restrictions are structural identification assumptions that can be motivated by theoretical insights, broad narrative analysis, or external instruments.

Considering non-gaussian shocks can be motivated by the presumption of non-linearities in the transmission of shocks. In the spirit of Del Negro and Schorfheide (2009), introducing non-Gaussian structural shocks is a way to model such non-linearities as the impact of non-Gaussian shocks in linear approximation of non-linear models can be observationally equivalent to the impact of gaussian structural shocks in non-linear models. The advantage of our approach is that we do not need to model nor parameterize the non-linearity and we only need to take a stand on the properties of the empirical distribution of the implied structural shock.

Alternatively, assuming non-gaussian shocks can be motivated by broad narrative evidence. The narrative approach to identify structural shocks uses historical records (typically contemporaneous qualitative sources such as newspapers, government reports, and transcripts of policy meetings) to isolate some episodes where policy decisions were made independently of macroeconomic developments (Romer and Romer 1989, Antolin-Diaz and Rubio-Ramírez 2018, Drechsel 2025). Assuming that shocks are non-gaussian can reflect the historical evidence that exogenous policy changes are often large shocks — which can also be asymmetrically distributed — and as for the narrative approach, uses these salient episodes to help achieve identification. The difference is that one does not have to specify ex-ante when such salient shocks occurred.

Finally, non-gaussianity restrictions can be informed by external proxies of the shock of interest. Rather than using an external instrument directly for point identification, it can be used more conservatively, providing information about the *distributional shape* of the target disturbance. Concretely, we discipline the admissible rotations by matching higher-moment features (e.g., tail heaviness) of candidate shocks to those exhibited by an estimated proxy. This avoids leaning on the proxy as a perfectly clean instrument

[8] Details on the solution are provided in section 2.1 of the online appendix.

[9] There is a discontinuity at $\theta_o = 0$. In such a case, the first (second) structural shock coincides with the first (second) empirical innovation, all third moments are zero but the third moment of the second empirical innovation, $E\iota_{2,t}^3$, and there is no point in considering them as a system of mixed signals. More details on the solution can be found in section 2.3 of the online appendix.

for proxy-SVAR identification, which can be fragile in practice. Using the proxy only to calibrate higher-moment inequality restrictions preserves the informational content of the proxy while reducing sensitivity to proxy contamination (see Matthes, Menshikov and Walker 2025), or weak relevance and two-step implementation issues (see Lloyd and Manuel 2024, Lewis and Mertens 2026).

Of course, a number of structural shocks in the system could be fat-tailed or asymmetric. However, while structural shocks might share similar higher order moment properties, they can be separately identified using different sign implications on observables. For example, as we will see in the next sections, both monetary policy shocks and financial shocks can be safely assumed leptokurtic. But, while a monetary policy tightening induces arguably an increase in the policy rate and a tightening of credit conditions, a financial shock is typically associated with tight financial conditions but does not trigger an increase in the policy rate. This example of how economic constraints can be combined with our approach to sharpen the identification of shocks.

Choosing the threshold for the inequality restriction Once the type of restriction is chosen, the thresholds for the inequality restrictions need to be chosen.

This threshold value is a practical modeling choice that determines how much deviation from Gaussianity is postulated, and the modeler can experiment with more or less demanding restrictions. Note that there is a trade-off in this choice: imposing to little deviation from Gaussianity will result in too large identified sets; imposing restrictions that are too far from the data risks returning an empty set.

This choice can be guided by external reference points. For instance, for skewness, a minimal cutoff point is $\epsilon_{\mathcal{S}} = 0.1$. This implies that the median (mode) and the mean are at least ten (thirty) percent apart. Distributions are typically considered symmetric for skewness below that threshold. For kurtosis, the threshold is the dial for *how often do we see an unusually large shock?* As a reference, a random variable distributed –say– as mixture of two normal distributions where the random variable is drawn with a 0.8 (0.2) probability from (three times) the standard normal distribution has a robust excess kurtosis of 1.5. With this distribution, the probability of observing a realization distant two (three) standard deviations away from the mean is 25% (700 %) more likely that in the standard normal case.

The choice can also be informed by an external proxy when it is available. Robust higher-order moment estimates of that proxy can be used to discipline the choice of plausible lower bounds on these higher moments. As emphasized earlier, rather than relying on the external proxy to achieve point identification, it can be used to extract

such robust distributional information.

In general, such higher-moment threshold should be interpreted as a transparent severity parameter that indexes how strongly non-Gaussian the target disturbance must be. Importantly, in the simulations and applications below, we do not treat the threshold as a knife-edge calibrated constant. We report identification paths over a grid of values and emphasize the range over which the qualitative conclusions are stable.

Note that it would be relatively straightforward to extend our approach to choose the higher-order moment thresholds in order to satisfy some probabilistic restrictions. Indeed, one could require that the higher-order moment inequality hold with a targeted probability given the estimation uncertainty. That would amount to computing the moment of interest and its uncertainty via bootstrap (or a Bayesian/resampling step) and then accept the candidate rotation matrix Ω only if $P(\text{moment condition verified}|\Omega) \geq 0.9$.

4 Bayesian Estimation and Identification

Our estimation and identification approach can be described as follows. We postulate that the observed data are generated by a $VAR(p)$ model,

$$y_t = \Phi_1 y_{t-1} + ... + \Phi_p y_{t-p} + \Phi_0 + u_t,$$

where y_t is $n \times 1$ vector of endogenous variables, Φ_0 is a vector of constant and Φ_j are $n \times n$ matrices, and given initial conditions $y_0, \ldots, y_{-p+1}$. We assume that u_t is an $n-$dimensional white noise process with unconditional covariance matrix Σ which is obtained as a linear combination of the unobserved structural shocks, ν_t, i.e. $u_t = \Sigma^{1/2}\iota_t = \Sigma^{1/2}\ \Omega\ \nu_t$, with Ω an orthonormal matrix, $\Omega\Omega' = \Omega'\Omega = I$.

Inference when shocks are non-Gaussian Standard inference on VAR parameters typically postulates a multivariate normal distribution for the reduced form innovations. Such an assumption cannot be considered in our context. We propose to adopt a robust Bayesian approach which allows to construct posterior credible sets which does not request to assume a specific distribution of the reduced form residuals.

Our approach builds on the work of Petrova (2022). She proposes a robust and computationally fast Bayesian procedure to estimate the reduced form parameters of the VAR in the presence of non-Gaussianity. In that configuration, the Quasi Maximum Likelihood (QML) estimator[10] of the VAR parameters is consistent and asymptotically

[10]The QML is the maximum estimator of the quasi-likelihood. The quasi-likelihood in this context coin-

normal. However, inference on the intercept is affected when innovations have a non-symmetric distribution; and inference on Σ is affected when the innovation distribution shows excess kurtosis relative to the multivariate normal density. Petrova (2022) derives closed-form expressions for corrected Bayesian posterior distributions that rely on the asymptotic covariance matrix of the QML estimator. Combined with a prior, one can draw the VAR reduced form parameters from such corrected posterior distribution without specifying the shock distribution and allowing for asymmetries and fat tails (or thin-tails).

Bayesian procedure Assume that we are interested in identifying the last shock, $\nu_{n,t}$. Let $\Sigma^{(j)}$ and $\Phi^{(j)}$ be the j^{th} draw from the reduced form parameters posterior obtained using above. The procedure to identify the structural shock using higher-order restrictions goes as follows:

I. Draw $\breve{\Omega}$ from a uniform distribution with the Rubio-Ramírez et al. (2010) algorithm;

II. Compute the impulse response function and check if the sign (or any other economic) restrictions are verified;

III. Compute the implied structural shocks

$$\breve{\nu}_t^{(j)} = \breve{\Omega}' \left(\Sigma^{(j)}\right)^{-1/2} (y_t - \Phi_1^{(j)} y_{t-1} - ... - \Phi_p^{(j)} y_{t-p} - \Phi_0^{(j)});$$

IV. Compute $\mathcal{S}(\breve{\nu}_{n,t}^{(j)})$ and/or $\mathcal{K}(\breve{\nu}_{n,t}^{(j)})$ and check if the higher-order moment inequality restrictions are satisfied.

If both [II] and [IV] are satisfied, keep the draw $\Omega^{(j)} = \breve{\Omega}$. Then repeat [I] to [IV]. After a suitable number of iterations, the draws are representative of the posterior distribution of the impulse responses of interest. The estimation of the reduced form parameters and the computation of the impulse responses using the higher-order moments is performed using the toolbox described in Ferroni and Canova (2021).[11]

Robust higher-order moment estimators Our procedure requires to have estimates of third and fourth moments. As illustrated above, methods using higher-order moments equality restrictions to identify structural shocks typically require parametric estimates of such moments. A drawback is that these estimates are known to be very

cides with the likelihood of the VAR when *incorrectly* assuming normality of the reduced form residuals.

[11] Estimation is performed using the MATLAB toolbox developed by Ferroni and Canova (2021) which can be found on the Github EMT page.

sensitive to outliers with short samples (see, e.g., Kim and White 2004). In contrast, our approach allows to rely on robust non-parametric estimators, therefore mitigating that issue.

These robust estimators are constructed using the empirical distribution of the structural shocks of interest that are obtained in step III of the above procedure. For third moments or asymmetries, robust measures exploit the standardized distance between median and mean. For fourth moments, or tailedness, robust estimators consider the ratio between the distance of the percentiles in the tails and the distance between the percentiles close to the median. For example, the larger the numerator, the thicker the tails; the smaller the denominator the more clustered is the distribution around the median and the more leptokurtic the shock is.

In what follows, we use the following estimators of skewness ($\mathcal{S}$) and excess kurtosis ($\mathcal{K}^\star$) of a random variable x:

$$\mathcal{S}(x) = \frac{\bar{x} - F^{-1}(0.5)}{\text{std}(x)}, \quad \mathcal{K}^\star(x) = \frac{F^{-1}(0.975) - F^{-1}(0.025)}{F^{-1}(0.75) - F^{-1}(0.25)} - 2.9, \tag{1}$$

where $F^{-1}(\alpha)$ is the α-percentile of the empirical distribution of x and 2.9 is the value of the kurtosis of the Gaussian distribution.

5 Experimental evidence using a medium-scale DSGE model

We illustrate how inequality restrictions on higher moments can improve the identification of the effects of standard economic shocks on macro variables in a laboratory environment. In particular, we focus on the long debated impact of monetary policy on output. Uhlig (2005) finds that monetary policy shocks have no clear effect on output if one uses an agnostic identification procedure. More precisely, he imposes sign restrictions on inflation and interest rate (moving in opposite directions) but remains *agnostic* about the response on output. Wolf (2020) shows that Uhlig (2005)'s result is consistent with a standard New Keynesian (NK) model. The reason is that, when only sign restrictions on inflation and interest rate are imposed, supply and demand shocks tend to *masquerade* as monetary policy shocks. He concludes that pure sign restrictions are quite weak identifying information.

In this section, we show that higher moments inequality restrictions also strongly help resolving the issue of shocks *masquerading* as monetary ones when the latter have a non-Gaussian fourth moment that is sufficiently different from supply and demand shocks. More specifically, we work with the Smets and Wouters (2007) (SW) model, perhaps the most well-known example of an empirically successful structural business

cycle model. In this model the fluctuations in economic activity, labor market variables, prices and interest rate are explained by handful of shocks: technology, risk premium, investment demand, monetary policy, government/exogenous spending, and price and wage markups shocks. We estimate the model on postwar US data on output, consumption, investment, real wages, inflation, interest rate and hours worked as in the original Smets and Wouters (2007)'s work.[12] We set parameters at their posterior mode estimates and consider smoothed estimates of the structural shocks as realizations of these shocks. We then use this model as a laboratory to show how the higher order moments can sharpen identification.

Supply and demand shocks can masquerade as monetary policy shocks A first property of the estimated SW model is that, as for the textbook NK model, demand and supply shocks can masquerade as a monetary policy shock. This means that the combination of supply and demand shocks can generate a sign pattern for inflation and interest rate that is similar to the one resulting from a monetary policy shock. However, while the output reaction to a monetary policy contraction is negative, the masquerading combination of supply and demand shocks have a positive impact on output. As a result, Uhlig (2005)'s agnostic sign restrictions may not be enough to identify the transmission of monetary policy shocks to output.

Estimated monetary policy shocks are leptokurtic A second striking property of the estimated SW model is that structural shocks display some form of fat-tailedness, and even more so for monetary policy shocks.[13] In particular, when we compute a robust measure of kurtosis using equation (1), we find an excess kurtosis for the SW monetary policy shock equal to 2, well above the excess kurtosis for other shocks. In contrast the robust measure of skewness is very close to zero for every structural shock.[14]

Identifying monetary policy shocks in controlled experiments We postulate that the data generating process consists of three shocks, demand supply and

[12] Our implementation of the Smets-Wouters model is based on Dynare (see Adjemian, Bastani, Juillard, Karamé, Maih, Mihoubi, Perendia, Pfeifer, Ratto and Villemot (2011)) replication code kindly provided by Johannes Pfeifer.

[13] This is illustrated in figure 3 in the online appendix which reports the estimated shocks realizations (top panels) and the empirical probability distribution against the normal one (bottom panels). All estimated shocks display some deviations from the gaussian distribution.

[14] These results are consistent with the evidence in Cúrdia, Del Negro and Greenwald (2014). They estimate a version of the SW model which allows for Student$-t$ distributed shocks and stochastic volatility. They show that the estimation on US data clearly favors shocks with leptokurtic distribution.

monetary policy and the econometrician observes output, inflation and interest rate. We are interested in the dynamic effects of monetary policy with a Structural VAR using three identification restrictions: *(i)* sign restrictions only, *(ii)* sign and higher-order moments inequality restrictions as introduced in section 2.2, and *(iii)* higher-order moments equality restrictions using the spectral decomposition approach of section 2.1.

In *(i)*, we postulate that, after a contractionary monetary policy shock, the interest rate increases on impact and for two consecutive quarters and inflation decreases on impact and for two consecutive quarters. In the spirit of Uhlig (2005), no restriction is imposed on output. In *(ii)* we impose the additional restriction that monetary policy shocks are leptokrutic, with different degrees of fat-tails. We report three different sets of results:

1 The identified set of the dynamic response of output to a one percent increase in the nominal interest rate using restrictions *(i)* and *(ii)*.

2 The 90% confidence bands of the dynamic response of output to a one percent increase in the nominal interest rate obtained by estimating a SVAR on a long data sample using restrictions *(i)*-*(iii)*.

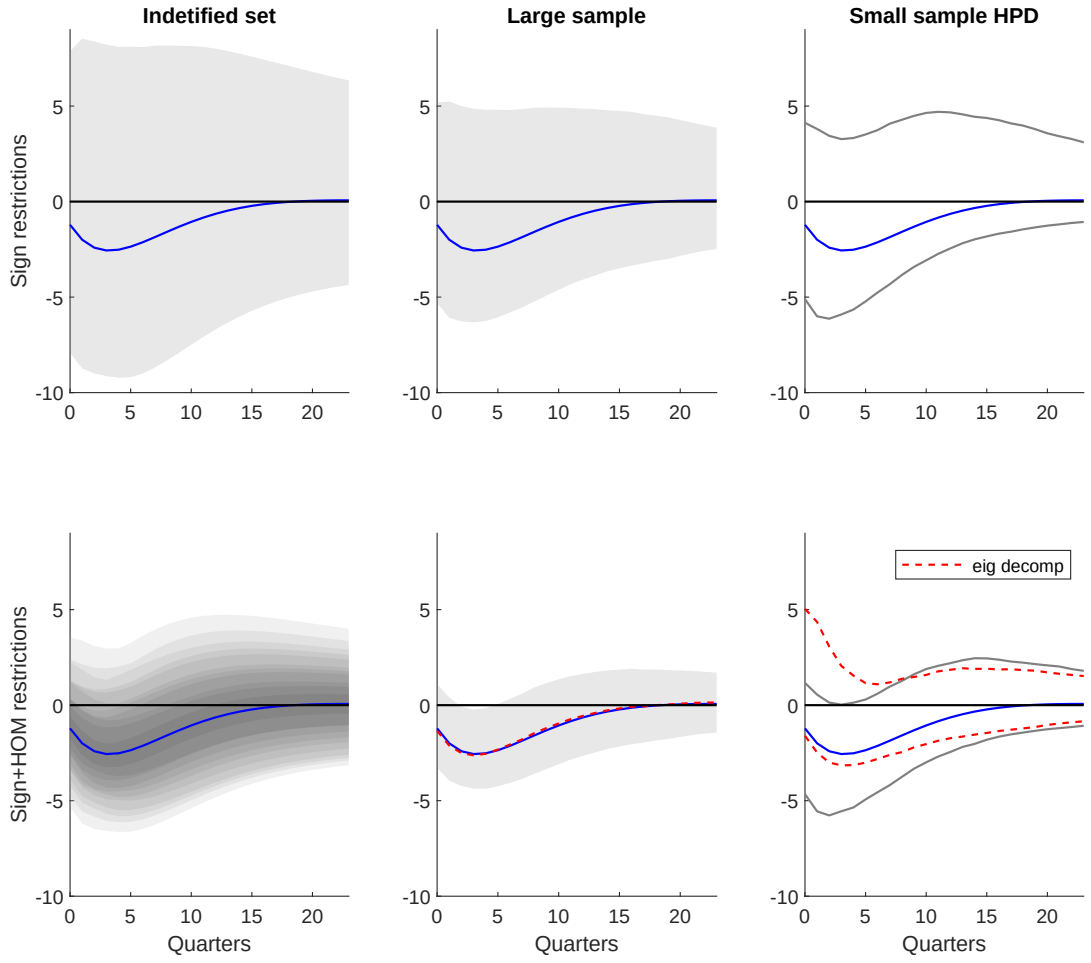
3 The median 68% confidence bounds of the dynamic response of output to a one percent increase in the nominal interest rate obtained by estimating a SVAR on repeated small samples using restrictions *(i)*-*(iii)*.

Figure 1 reports the results of these exercises and in each panel the blue line displays the true monetary policy impulse response derived from the model.

Identified set The first column of figure 1 reports the identified sets of the dynamic response of output to a monetary policy tightening with sign restrictions only (first row), and with sign and higher-order moments inequality restrictions (second row). In particular, we impose that the monetary policy shock excess kurtosis exceeds a threshold value, $\mathcal{K}^{\star}_{mp} >$ `threshold`, and we consider different values of the `threshold`; in particular, we allow it to vary in the following grid `[0:0.2:2]`, where darker gray areas represent larger values of the `threshold`. More details on the construction of the identified set can be found in section 3 of the online appendix. The first panel shows that the sign identified set is extremely large which echoes the finding in Wolf (2020).[15] Interestingly, the identified set shrinks considerably when only minimal mass

[15] Wolf (2020) imposes the sign restrictions for 6 quarters. Results are not sensitive to the choice of the horizon.

Figure 1: Dynamic response of output to a one percent increase in the interest rate.



Notes: First and second row display the results with sign restrictions only and sign and higher-moment inequality restrictions, respectively. Left panels show the identified sets where darker gray areas in the bottom left panel represent larger values of the `threshold`. Middle panels report the 90% confidence sets of responses implied by a SVAR estimated on 50,000 data points (the red dashed line indicates the responses obtained with the spectral decomposition). Right panels report the median value of the upper and lower bounds of the 68% (HPD) sets across artificial short samples of 200 data points obtained with sign restrictions only (top right panel), sign and higher-moment inequality restrictions (bottom right panel in gray) and higher-moment spectral decomposition (bottom right panel in red). In each panel, the blue line is the 'true'.

is added to the tails of the monetary policy distribution, i.e. `threshold` > 0.2. For large enough values of the `threshold`, the identified set mostly consists of negative output trajectories.

Large sample We then study the properties of our identification approach on a large sample. We generate 50,000 observations for output, inflation and interest rate using random draws from the empirical distribution of the technology, risk premium and monetary policy shocks. We assess how different structural VAR restrictions aim-

ing at identifying monetary policy shocks perform on these simulated data. We estimate a VAR with twenty lags[16] and rotate the reduced-form VAR innovations so that the sign restrictions in *(i)* are satisfied. We construct 50,000 accepted rotations using the Haar prior (see Rubio-Ramírez et al. 2010). For each of these rotations, we verify if the monetary policy shock excess kurtosis is larger than 1.6 is also verified and keep the rotation if so (restrictions *(ii)* with `threshold` > 1.6). Finally, in *(iii)*, the rotation matrix is derived from the spectral decomposition of the matrix of fourth moments of the VAR empirical innovations.

The second column of figure 1 reports the impulse responses obtained with each identification schemes. The first row and second column panel reports the 90% identified sets of impulse responses obtained when using sign restrictions only. The second row and second column panel when combining sign and kurtosis inequality restrictions. The red dashed line shows the point-identified impulse responses obtained when using the spectral decomposition of the matrix of fourth moments of the VAR empirical innovations. Also in this simulation experiment, the set of responses identified by sign restrictions is so wide that the impact of monetary policy shock on output is inconclusive. On the contrary, higher order moments help refining the transmission of monetary policy shocks to output. When monetary policy shock excess kurtosis is larger than 1.6, the 90% identified set suggests that most of the output trajectories are negative at least after a few quarters in line with what the theory predicts.[17] As discussed for the identified set the amount of excess kurtosis determines the fraction of positive trajectories of output. For `threshold` values below one, a large fraction of output trajectories are positive after a monetary policy tightening.[18] Interestingly, the identification obtained with the spectral decomposition of the empirical innovation fourth moment matrix is very precise. However, with shorter samples, such precision deteriorates and inequality restrictions perform better as we show in the next section.

Short samples The precision of the identification based on the spectral decomposition deteriorates sharply when working with smaller samples of size typical of macroeconomic studies. Indeed, this approach requires computing all the fourth moments of the VAR empirical innovations and the sample counterparts of these fourth moments can be poorly estimated and very sensitive to outliers in-sample. In contrast,

[16] De Graeve and Westermark (2025) show that long-lag VARs reduce bias and variance, and have better mean-squared error compared to small-lags (e.g. 4) VAR.

[17] Higher moment inequality restrictions also lead to more precise identified sets when one imposes the sign restrictions that a monetary policy shock generates on the observed variables; figure 4 in the appendix report the magnitude of the refinement of the identified set in the context of the SW model.

[18] Not shown here and available in the online appendix (Figure 8).

our identification method only involves estimating the fourth moment of the shock of interest and allows working with robust estimates of that higher-order moment. It performs well also with smaller samples. To illustrate that point, we simulate 500 different set of data each consisting of 200 observations length on output, inflation and interest rate. For each datasets, we estimate a VAR and computed the monetary policy impulse responses using the spectral or eigenvalue decomposition and the sign and higher-moment inequality restrictions.

The third column of figure 1 reports the median value of the upper and lower bounds of the 68% high probability density (HPD) sets across artificial samples obtained with sign restrictions only (top right panel), sign and higher-moment inequality restrictions (bottom right panel in gray) and higher-moment spectral decomposition (bottom right panel in red).[19] With short samples, the spectral decomposition generates very dispersed point estimates which includes positive responses of output after a monetary policy shock. This does not occur with our identification scheme, where point estimate of the responses of output and inflation are robustly estimated to be negative.

6 Empirical Application I: US Monetary Policy

As a proof of concept, we first revisit the effects of monetary policy shocks on output and inflation using U.S. data. This application aims to illustrate how augmenting standard agnostic sign restrictions à la Uhlig (2005) with higher-order moment inequality restrictions can mitigate well-known limitations of this approach. Existing literature has shown that Uhlig (2005)'s method implies an implausible monetary policy reaction function–specifically, a posterior distribution for the policy reaction coefficient to output that is skewed to the left, suggesting that the central bank systematically raises interest rates in response to weak economic conditions (Arias et al. 2019). Moreover, it assigns a high posterior probability to an expansionary shock in October 1979, contradicting the widespread interpretation of that episode as a major contractionary policy shift under Paul Volcker (Antolin-Diaz and Rubio-Ramírez 2018). We document that empirical proxies for U.S. monetary policy shocks, despite their low mutual correlation, consistently exhibit significant excess kurtosis–a feature also present in euro area and UK data. Building on this robust empirical regularity, we impose a lower bound inequality restriction on the excess kurtosis of monetary policy shocks and combine it with the agnostic sign restrictions of Uhlig (2005). Our approach yields markedly sharper inference: it not only recovers a contractionary effect of monetary tightening

[19] Figure 5 in the online appendix reports the dispersion of the (median) point estimate across identification schemes.

on output, but also assigns a large and strictly positive monetary policy shock in October 1979–with posterior mass concentrated entirely above zero–and delivers a policy reaction coefficient to output that is no longer implausibly left-skewed.

6.1 Higher moments of monetary policy shock proxies

We document the third and fourth moment properties of a wide set of estimated monetary policy shocks. In a recent paper, Jarociński (2024) emphasizes that US monetary policy surprises identified from intraday movements of federal funds rate futures observed in a narrow window around FOMC announcements exhibit leptokurticity. In this section, we show that this feature is not specific to daily or intraday variations of interest rates around central bank monetary policy decisions and announcements, nor is it to US monetary policy. This property is observed for a wide range of monetary policy shock proxies obtained with data of different frequency, for different sample-periods, and for different economies.

For the US, we consider the recent monetary policy surprises extracted from high frequency datasets from Gertler and Karadi (2015) (GK), Miranda-Agrippino and Ricco (2021) (MAR) and Jarociński and Karadi (2020) (JK). Some of these proxies control for the information/Delphic effect of monetary policy (as discussed in Campbell, Evans, Fisher and Justiniano (2012) or Nakamura and Steinsson (2018a)). We also look at the raw (unrotated) first three principal components of the intraday variations of the interest rates term structure around FOMC announcements (i.e. USf1-USf3). We also include monetary policy shocks identified with lower frequency data, in particular, the Romer and Romer (2004) (RR)[20] narrative instrument, Sims and Zha (2006) (SZ) monetary policy innovations estimated from a SVAR with regimes shifts, and the monetary policy shock estimated in the Smets and Wouters (2007) (SW) DSGE model. Finally, we also consider the natural language measure constructed by Aruoba and Drechsel (2026) (AD) where they look at the change in the target interest rate which cannot be predicted by the textual information contained in the documents prepared by Federal Reserve staff in advance of policy decisions.

For the Euro Area, we consider the monetary policy surprises constructed in Andrade and Ferroni (2021), which exploits high-frequency variations of future OIS contracts around the ECB monetary policy decisions and press conference and distinguish between conventional (AF(target)) and forward guidance shocks (AF(FWG)) controlling for information/Delphic effects (AF(delphic)). We also look at the unrotated first

[20] We consider here the series constructed in Wieland and Yang (2020) who extend the Romer-Romer (2004) monetary policy shock series.

three principal components of variations of the Euro Area interest rates futures around the ECB decision and communication constructed in Altavilla, Brugnolini, Gürkaynak, Motto and Ragusa (2019) (EAf1-EAf3).

For the UK, we consider monetary policy surprises obtained using the high-frequency data from Gerko and Rey (2017) who looks at change in three-month sterling future rates observed around the release of the minutes of the MPC (GR(minute)) and inflation report (GR(IR)), from Cesa-Bianchi, Thwaites and Vicondoa (2020) (CBTV) who use changes in three months Libor around monetary policy events and the methodology introduced in Gürkaynak, Sack and Swanson (2005), and from Kaminska and Mumtaz (2022) who look at variations of the full yield curve of UK government bonds (KM). We also include the monetary policy shocks obtained by Cloyne and Hurtgen (2016) who employ the Romer-Romer identification approach to construct a monthly measure of UK monetary policy shocks (CH).

Results Table 1 reports the robust measures of skewness and excess kurtosis for each of these proxies along with the 95% confidence intervals obtained by bootstrapping the series for all these various measures. While there is no clear evidence about skewness of the monetary policy shocks, all the proxies have a statistically significant excess kurtosis, although point estimates are very dispersed. It is quite striking that all of these proxies are leptokurtic given that they are obtained using different methods, different datasets, different time spans and countries. As a result, they span relatively different information sets. For example, in the US the HF measures of monetary policy surprises and the narrative instrument constructed by RR have a correlation of about 0.2 only (see figure 1 in the online appendix). So it is not straightforward to assess which proxy better characterizes monetary policy shocks and should be used to instrument the reduced form VAR residuals. Our proposal to use higher-order moment restrictions to identify monetary policy shocks rather leverage on a robust feature observed for various proxies.

6.2 Monetary policy transmission with sign and kurtosis restrictions

VAR specification We consider the dataset studied in Uhlig (2005) which consists of Real GDP (y), GDP Deflator (pi), Commodity Price Index (pcom) and the Federal Funds Rate (FFR) over a 1965m1 to 2003m12 sample.[21] We estimate the VAR

[21] Uhlig (2005) considers also reserves and non borrowing reserves to disentangle monetary policy and money demand shocks. As the point is on monetary policy shocks, we ignore these restrictions. Results hold

Table 1: Proxies of Monetary Policy shocks/surprises: Ex-Kurtosis & Skewness.

	Excess Kurtosis	Skewness	Sample Size	Sample Coverage
SW	2.0 [0.4, 3.2]	-0.0 [-0.1, 0.1]	179	1960-2004
SZ	3.8 [1.8, 6.3]	0.0 [-0.0, 0.1]	518	1960-2003
RR	3.2 [1.8, 5.1]	0.0 [-0.1, 0.1]	468	1969-2007
GK	11.3 [5.9, 18.2]	-0.3 [-0.3, -0.2]	269	1990-2012
MAR	3.3 [1.2, 5.9]	-0.1 [-0.2, 0.0]	228	1991-2009
JK	8.8 [5.4, 15.8]	-0.1 [-0.2, -0.0]	323	1990-2016
AD	3.1[1.4, 7.0]	-0.0[-0.1, 0.1]	313	1982-2008
USf1	1.2 [0.3, 3.5]	0.1 [-0.0, 0.2]	204	1994-2017
USf2	3.0 [1.4, 7.1]	0.1 [-0.0, 0.2]	204	1994-2017
USf3	1.9 [0.5, 4.4]	0.0 [-0.1, 0.1]	204	1994-2017
AF(target)	2.5 [0.6, 5.3]	-0.0 [-0.2, 0.1]	134	2004-2015
AF(delphic)	1.3 [0.2, 3.9]	-0.0 [-0.2, 0.1]	134	2004-2015
AF(FWG)	1.4 [0.2, 3.6]	0.0 [-0.1, 0.1]	134	2004-2015
EAf1	3.4 [1.4, 5.6]	-0.0 [-0.2, 0.1]	197	2002-2019
EAf2	1.5 [0.3, 3.9]	-0.1 [-0.2, 0.0]	197	2002-2019
EAf3	1.1 [0.2, 3.4]	0.0 [-0.1, 0.2]	197	2002-2019
CH	13 [5.9, 38]	0 [-0.1, 0.1]	348	1997-2015
GR(minutes)	2.5 [1.3, 4.9]	-0 [-0.2, 0.1]	211	1997-2015
GR(IR)	3.6 [2.8, 4.6]	-0.1 [-0.2, 0.0]	211	1997-2015
CBTV	3.4 [0.6, 7.5]	-0.1 [-0.2, 0.0]	212	1979-2007
KM	5.3 [2.3, 11.2]	-0.0 [-0.1, 0.1]	235	1997-2016

Note: Bootstrap; median and in parenthesis 95% confidence intervals.

parameters assuming 12 lags. As the QML or least square (LS) estimates of the autoregressive coefficients are asymptotically consistent, so the LS residuals are treated as consistent estimators of the reduced form shocks. We first study the Gaussianity properties of the least square estimates of the whitened VAR reduced form shock. Departures from normality are not fully evident for the output and prices whitened residuals; while there are few observations in the tails that lie far from the normal distribution, the Kolmogorov-Smirnov (K-S) test fails to reject the null that they could have come from a standard normal distribution; their respective $p-$values are 0.12 and 0.42. More evident departures from normality arise for the whitened residuals of the commodity prices and of the federal funds rate where the K-S test $p-$values are 0.07 and 0.00 respectively. The FFR residuals seems to be characterized by a large negative value at the beginning of the 80's. Even after removing the extreme values (min and max) of the FFR residuals, we still reject the null of normality with a $p-$value smaller than 1e-8. As a matter of fact the robust measure of skewness are all very close to zero, whereas the robust measure of kurtosis of the federal funds rate residual is significantly larger than those implied by the Gaussian distribution. Hence, the source of non-Gaussianity seems more evident in the tails thickness rather than in the asymmetry of the distribution.

Identifying assumptions We combine the sign restrictions of Uhlig (2005) with an inequality restriction on the shock fourth moment as follows.

Assumption 4 *An unexpected monetary policy shock induces an increase in the federal funds rate and a decline in the GDP deflator and in the price of commodities on impact and for the following five months.*

Assumption 5 *An unexpected monetary policy shock is leptokurtic and the robust measure of excess kurtosis larger than 1.2, i.e.* $\mathcal{K}^{\star}_{mp} > 1.2$.

The threshold for excess kurtosis is guided by the results of table 1 in section 6.1. We choose a conservative value below the lowest value (1.2) of robust excess kurtosis estimates observed across different proxies of US monetary policy shocks. Still this threshold implies sizeable deviations from Gaussianity.[22]

also when including these variables (see figure 10 in the online appendix).

[22] As a reference, a random variable distributed –say– as mixture of two normal distributions where the random variable is drawn with a 0.8 (0.2) probability from (three times) the standard normal distribution has a robust excess kurtosis of 1.5. With this distribution, the probability of observing a realization distant two (three) standard deviations away from the mean is 25% (700 %) more likely that in the standard normal case.

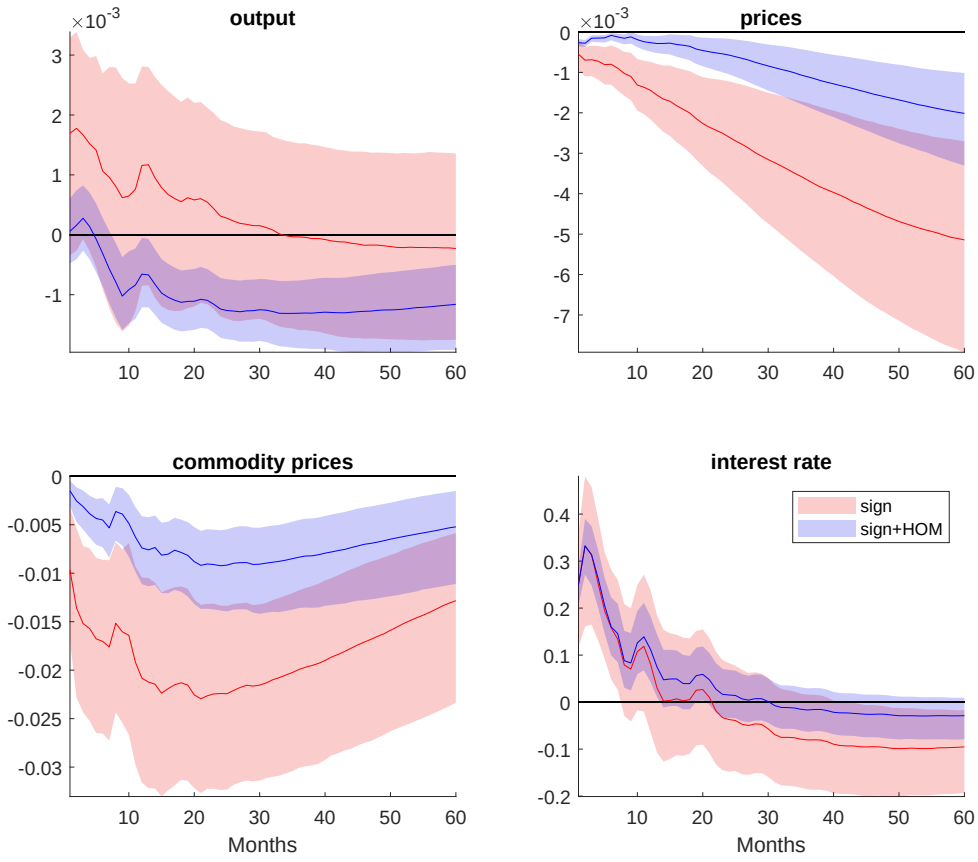


Figure 2: Impulse responses to a monetary policy shock. The colored areas indicate 68% credible sets. Sign restrictions red. Sign and kurtosis ($\mathcal{K}^{\star}_{mp} > 1.2$) restrictions blue.

Impulse responses to a monetary policy shock Figure 2 reports the impulse response function (IRF) to a monetary policy shock. The median IRF obtained with sign restrictions only is in red. The one obtained with both sign and higher moments restrictions is in blue. Shaded areas depict the associated 68% confidence sets (figure 6 in the online appendix illustrates that our findings remain valid when using the 90% confidence bands). We find that monetary policy shocks have no clear effect on output when these are identified only with sign restrictions on interest rate and inflation. When these are complemented with an inequality restriction on the excess kurtosis of monetary policy shocks, almost all the positive responses of output are chopped away and the distribution tilts towards negative outcomes only. Note that the response of the FFR after an initial 25bps increase is almost the same under both identification schemes. Yet, when using inequality restriction on the monetary policy shock kurtosis, prices decline significantly less after a monetary policy shock – the impact is about half

of the size than what is obtained with sign restriction only. So higher-order moments restrictions also affect the estimated slope of the Phillips curve.[23]

Further properties of the identified monetary policy The agnostic identification approach of Uhlig (2005) for studying the real effects of monetary policy is appealing in that it avoids imposing restrictions on the impulse response of output to an exogenous monetary policy shock. However, in addition to its inconclusive implications for output responses, the literature has highlighted two important side effects of this identification strategy that challenge both theoretical consistency and key historical interpretations.

First, the canonical identification fails to align with a widely acknowledged historical episode: the monetary policy actions of October 6, 1979, shortly after Paul Volcker became Chairman of the Federal Reserve. Described by Romer and Romer (2004) as "a major anti-inflationary shock," this episode is commonly regarded as one of the clearest instances of an exogenous contractionary monetary policy shock in the postwar era. Yet, Antolin-Diaz and Rubio-Ramírez (2018) show that the agnostic identification assigns an 11% posterior probability to an expansionary shock at that date–contradicting the prevailing consensus. They further demonstrate that imposing a simple restriction–that the monetary policy shock in October 1979 must be positive–yields a markedly improved identification.

Second, the approach generates a posterior distribution for the policy reaction coefficient (PRC) to output that is heavily left-skewed (see, e.g., Arias et al. (2019)). This implies, counterintuitively, that the central bank tends to raise the federal funds rate in response to weak economic conditions. A more plausible characterization of monetary policy behavior can be achieved by imposing restrictions on the systematic component of policy, as demonstrated in Arias et al. (2019).

Figure 3 presents the October 1979 monetary policy shock (right) and the posterior distributions of the policy reaction coefficient to output (left), comparing the Uhlig (2005) identification (red) with our proposed approach, which employs sign and kurtosis restrictions. Our method proves to be consistent with well-established historical narratives and avoids implausible implications for the monetary policy reaction function.

[23]The confidence bands are obtained using the Haar prior as in Rubio-Ramírez et al. (2010). There is a debate as to whether such a prior is overly informative—and the prior selection does not vanish asymptotically (e.g. Baumeister and Hamilton 2015)—or not (e.g. Arias, Rubio-Ramirez and Waggoner 2026). Giacomini and Kitagawa (2021) propose to compute a 'robust credible region' that relies on a prior class that specifies an 'unrevisable' prior for the structural parameter given the reduced-form estimates. Figure 7 in the online appendix illustrates that our findings remain valid when using such an approach.

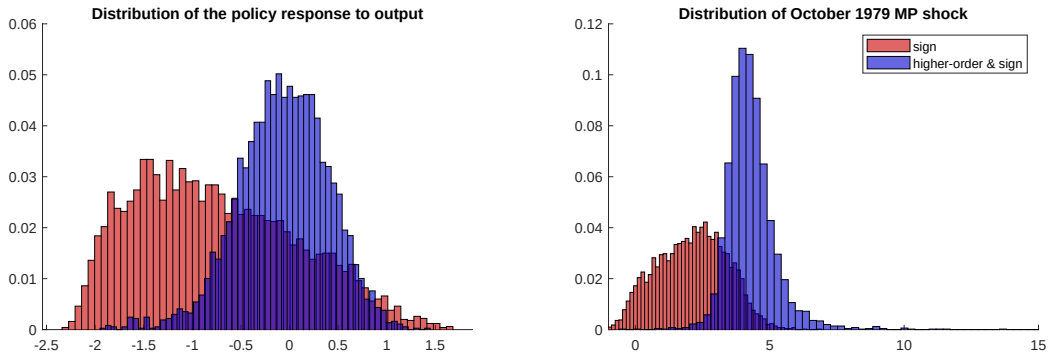


Figure 3: Posterior distribution of the policy reaction coefficient (PRC) to output (left) and of the October 1979 monetary policy contractionary shock (left). Sign restrictions red. Sign and kurtosis ($\mathcal{K}^{\star}_{mp} > 1.2$) restrictions blue.

7 Empirical application II: Sovereign Risk in the Euro Area

In the wake of the Great Recession, the Euro Area has experienced an acute sovereign crisis leading to very large spreads between government bond yields for countries that were exposed to sovereign risk and the other members. These spreads were driven by macroeconomic fundamentals but also by an increase in perceived sovereign risk. As a matter of fact, sovereign spreads dropped very significantly after Mario Draghi made his "Whatever it takes" statement in 2012 which asserted the ECB commitment to intervene on sovereign debt markets to prevent any risk of potential break-up of the euro area. These sharp non-linear increases in sovereign risks are believed to have had large negative macroeconomic consequences in particular through their impact on the balance sheet of financial intermediaries and therefore lending to the private sector and several theoretical works analyze their macroeconomic effect (see Fornaro and Grosse Steffen 2024, for a recent example). However, identifying such a macroeconomic impact of euro area sovereign risk shocks is challenging as they coincided with other shocks — the Great Recession, the ensuing deleveraging shocks, and the contractionary monetary policy shocks implied by the zero lower bond constraint — that also had negative macroeconomic consequences. Some scholars address that issue by using bond market reactions observed in a tight window around key political events. The assumption is that these market reactions capture policy choices that will only affect the debt sustainability but not current and future economic conditions. They then use these market reaction as instruments for the reduced form residuals obtained from country level panel VAR or local projections (e.g. Bahaj 2020, Balduzzi, Brancati, Brianti and Schiantarelli 2023). Other contributions rely on a structural non-linear model that they estimate using country specific data (Bocola 2016) or US regional data and euro-area

country data (Martin and Philippon 2017). We propose to use sign and higher-order moment inequality restrictions in a structural VAR to estimate the impact of sovereign shocks on euro area aggregates.

VAR specification We use monthly data on the EA industrial production (IP, in log), the EA consumer prices measured by core HICP (Core, in log), the unemployment rate (UNR, in percent), a measure of non-financial corporation borrowing costs (NFC spreads, in percent),[24] the one year money market rate (Euribor 1y, in percent), the spread between the 5 year Italian and German bond yield (It-De 5y), and the 10 year government bond yield for Italy (It 10y, in percent), and Germany (De 10y, in percent). The sample period goes from 1999m1 to 2019m12. We estimate a VAR model with six lags and uninformative priors for the parameters of interest. Looking at the reduced form residuals higher moments properties, there are salient departures for normality for some variables: The K-S test rejects the null that the borrowing costs, the one year Euribor and the spread could have come from a standard normal distribution; for the remaining variables the test fails to reject the null hypothesis of normality.

Identifying assumptions We impose the following restrictions on the sign of the impulse responses to a sovereign risk shock.

Assumption 6 *An unexpected spread shock increases the 5 year Italian - German sovereign spread, increases the 10 year Italian government bond yield, and the NFC borrowing costs on impact and for the following month.*

This assumption is consistent with models featuring sovereign default risk and financial intermediaries (e.g. Corsetti, Kuester, Meier and Muller 2013, Bocola 2016). We are agnostic about the reaction of other macroeconomic variables to that shock and therefore leave their reaction unrestricted. Other shocks can potentially satisfy these sign restrictions, so we combine them with higher order moments inequality restrictions capturing what we assume to be a feature specific of sovereign risk shocks: They increase sharply relatively frequently.

Assumption 7 *The spread shock has moderate asymmetry to the right and moderate fat tails, In particular, the skewness larger than 0.2 and excess kurtosis larger than 0.5, i.e.* $\mathcal{S}_{sp} > 0.2$ *and* $\mathcal{K}^{\star}_{sp} > 0.5$.

A skewness value larger than 0.2 only imposes a moderate right-skewness. The latter combined with an excess kurtosis larger than 0.5 implies that the frequency of large

[24] We use the non-financial corporation borrowing costs constructed in Gilchrist and Mojon (2017).

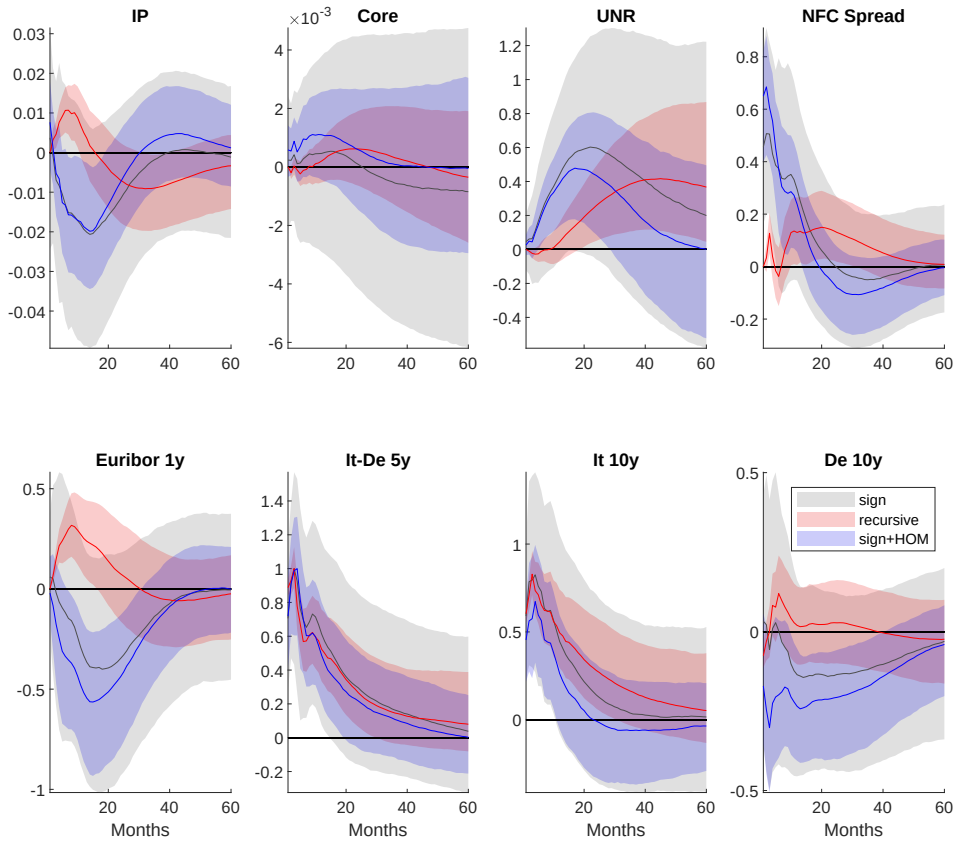


Figure 4: Impulse responses to a sovereign risk shock. Recursive ordering in red. Sign restrictions in gray. Sign and higher-order moment restrictions in blue. Impulse responses are normalized so that the maximum median impact on the spread is 1 percent. The solid line indicates the median response and the colored areas report the 68% confidence bands.

positive events is twice (six times) as large for realizations bigger than two (three) standard deviations relative to the normal case.[25]

Results Figure 4 shows the results obtained with our identification scheme (in blue) and compare them with what a standard recursive ordering identification — where the sovereign risk shock affects the Italian, German sovereign yields and their spread on impact, but the other financial and macroeconomic conditions with a lag — implies (in red). Impulse responses are normalized so that the maximum median effect on the spread over the response horizon is one percent. The identification using signs and higher moments restrictions produces interesting dynamic responses. A sovereign risk

[25] Assuming that the distribution is skewed in addition to leptokurtic makes the shock differ from monetary policy which in the previous section was assumed to be leptokurtic but symmetric.

shock leading to a 100 basis points peak increase in the 5-year italian-german sovereign spread generates a 50 basis points increase in the Italian 10-year bond yield as well as a 25 basis points decline in the German 10-year bond yield, consistent with flight to quality effects. Credit conditions tighten with a NFC spread increasing by 70 basis points, showing a large pass-through from the sovereign risk to the corporate sector. Industrial production contracts by nearly 2 percent 16 months after the shock and the unemployment rate increases by almost 50 basis point 18 months after the shock. The price level for core goods and services increase to reach by a tiny (and non statistically significant) 1 basis points peak one year after the shock. The 1-year Euribor drops as monetary policy eases to stabilize the economy. In less than three years the shock is absorbed. These effects are qualitatively similar to what Bahaj (2020) obtains on average for Ireland, Italy, Portugal, and Spain over a crisis 2007-13 sample period. Our estimates of the macroeconomic effects are larger though. This is even more the case as our estimates are for euro area aggregates and includes no-crisis time. This suggests that country specific sovereign risk shocks have negative spillovers on other monetary union members even if they are not affected by sovereign risk.

The results obtained with a recursive ordering scheme offer a very different picture, with some puzzling patterns. A shock leading to a one percent increase in the sovereign spread triggers an increase in the 10-year Italian bond yield. It also increases the German ones, so there is no flight to quality effect. Credit costs increase but only modestly and with a delay compared to our proposed identification. Industrial production increases in the short run and declines after two years; the unemployment rate increases for several months after the shock reaching its peak in three to four years after the impulse. The price level only modestly increases but it is not significant. Finally the 1-year money market rate increase, signalling monetary policy tightening. Overall, these effects do not seem consistent with what sovereign risk would produce, implying that the identification mixes several structural shocks. Figure 4 also shows with gray areas the results obtained when only the sign restrictions (i.e. assumption 6) are used. The results show that these constraints are not restrictive enough as this scheme does not generate any statistically significant response.[26]

[26]Figures 9 and 10 in the online appendix report also the 90% confidence sets.

8 Empirical Application III: The Macroeconomic Effects of Geopolitical Risk

What are the effects of rising geopolitical tensions on the US economy? Caldara and Iacoviello (2022) develop an index of geopolitical risks (GPR) that can be used to address that question. Such an index is obtained by computing the share of articles mentioning adverse geopolitical events in leading newspapers published in the United States, the United Kingdom, and Canada. Caldara and Iacoviello (2022)'s GPR index captures exogenous events that are not caused, at least within a quarter, by US macroeconomic performances. However, as Caldara and Iacoviello (2022) discuss, the GRP index correlates with other measures of policy shocks. Indeed, major adverse geopolitical events can coincide with exceptional and unpredictable policy decisions—whether from fiscal authorities, such as military spending shocks, or from central banks, such as the Federal Reserve's emergency response to 9/11. Some articles discussing geopolitical tensions may actually also be articles discussing these unforeseen policy decisions. Since these other policy shocks affect macroeconomic aggregates, isolating the macroeconomic impact of geopolitical risk shocks raises an identification challenge. We propose to address this challenge by combining sign and higher order moments restrictions.

VAR specification Caldara and Iacoviello (2022) also provide a structural VAR analysis of the macroeconomic effects of geopolitical risk (GPR) for the US economy. Their VAR consists of eight quarterly variables: (i) the log of the GPR index: (ii) the VIX; (iii) the log of real business fixed investment per capita; (iv) the log of private hours per capita; (v) the log of the S&P 500 index; (vi) the log of the WTI price of oil; (vii) the yield on two-year US Treasuries; (viii) the Chicago Federal Reserve National Financial Conditions Index (NFCI). The estimation sample is 1986:Q1 to 2019:Q4 and the VAR admits two lags.

Identifying assumptions Caldara and Iacoviello (2022) identify a GPR shock using a recursive scheme, with the GPR index ordered first. This assumes that any contemporaneous correlation between the GPR index and the other variables in the VAR comes from the causal effect of the GPR shock on the other variables. As reported in Figure 5 which replicates their approach, a one standard-deviation unforeseen increase in geopolitical risk has a significant contractionary impact on financial conditions and macroeconomic aggregates such as investment and hours. However, as Caldara and Iacoviello (2022) note, the GPR index can also captures other shocks,

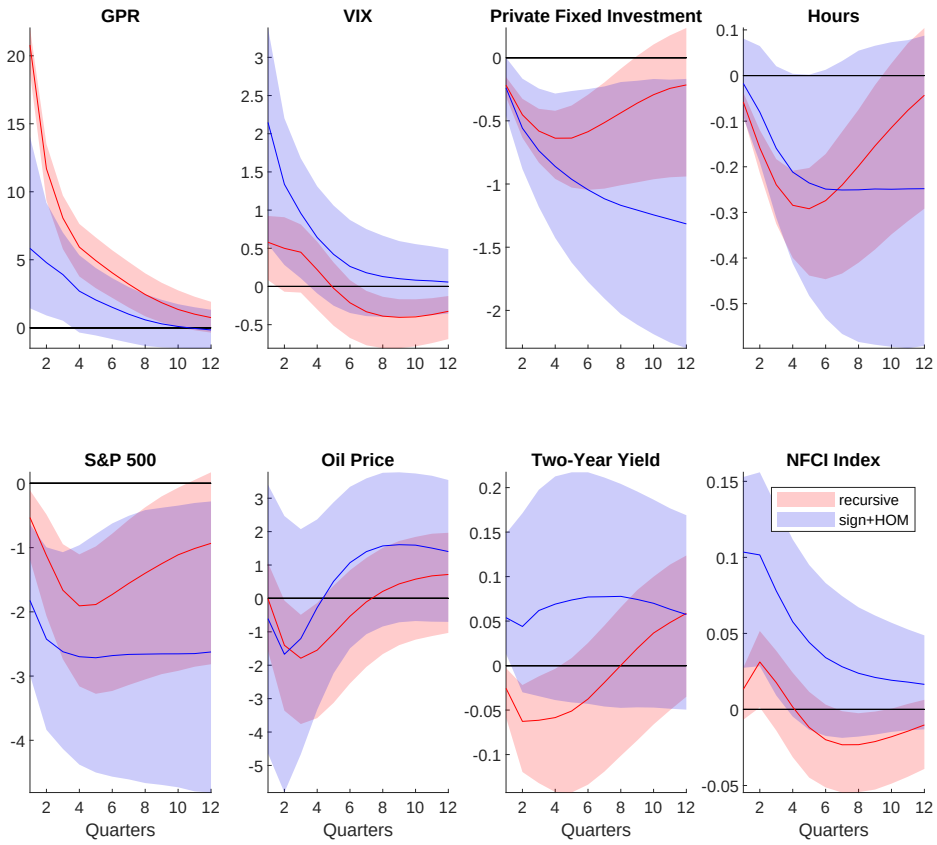


Figure 5: Impulse responses to a geopolitical risk shock. Recursive restrictions in red. Sign and higher-order moment restrictions in blue. For each identification scheme we impulse the shocks with a one standard deviations. Solid line median response and colored areas report the 68% and 90% confidence bands.

such as policy shocks. If these shocks also affect financial and macroeconomic conditions contemporaneously, a recursive ordering may in part capture these as well as the geopolitical risk ones. This could explain why, under a recursive ordering identification scheme, although the geopolitical tensions last well over one year after the initial GPR shock, the VIX starts to decline, the 2-year bond yield declines, and overall financial conditions, as captured by the NFCI index, loosen a few quarters after the shock. One way to address this potential issue is to use sign restrictions requiring that, on impact, the GPR shock increases the GPR index and tightens financial conditions by lowering the S&P500 and increasing the 2-year US Treasury yield. However, unreported results show that these sign-restrictions are too loose and lead to a set of admissible rotations that is too large and median IRFs that are not significantly different from zero. We thus investigate an alternative identification scheme which combines these

sign restrictions with restrictions on higher-order moments of the GPR shock. Indeed, the GPR index is skewed to the right and leptokurtic consistent with the intuition that geopolitical risks are characterized by spikes in international tensions which are large and relatively frequent. We assume that geopolitical risk shocks have a distribution that is also skewed to the right and leptokurtic, as follows.

Assumption 8 *An unexpected GPR shock lowers the SP500 and increase the 2-year US Treasury yield on impact.*

Assumption 9 *The GPR shock has small asymmetry to the right and fat tails, In particular, the skewness larger than 0.1 and excess kurtosis larger than 1.5, i.e.* $\mathcal{S}_{gpr} > 0.1$ *and* $\mathcal{K}^{\star}_{gpr} > 1.5$.

Results As Figure 5 illustrates, consistent with the results in Caldara and Iacoviello (2022), we find evidence of a persistent contractionary effect of GPR shocks on financial conditions and macroeconomic aggregates. The estimated effects obtained with our scheme are less precise compared to the recursive ordering as we get set-identification rather than point identification. But they are larger and more persistent, consistent with the possibility that the recursive identification can capture both the effects of geopolitical surprises and of coincident expansionary fiscal and monetary policy shocks resulting from discretionary decisions taken in the wake of some geopolitical events. Interestingly, the reaction of the 2-year Treasury yield is positive on impact, consistent with an increase in risk premium, but then non-significantly different from zero soon after the shock. This may reflect the offsetting effects that an expansionary fiscal and monetary policy implemented after the geopolitical risk shock would have on that yield. Finally, oil prices do not significantly react to geopolitical shock either. Indeed, depending on the shock, oil prices may in some cases decline due to drop in demand associated with an increased uncertainty, and, in other cases, increase if geopolitical events happen in oil producing countries. Overall, the evidence is in line with models where uncertainty shocks lower investment (Bloom 2009) and employment (Basu and Bundick 2017) and tighten financial conditions (Gilchrist, Sim and Zakrajsek 2014).

9 Conclusion

This paper introduces a novel and tractable identification strategy that leverages non-Gaussian features of structural shocks–specifically skewness and kurtosis–through inequality restrictions. By avoiding strong parametric assumptions and working within

a set-identification framework, our approach offers a robust alternative to conventional moment-based or sign-restriction-only methods and remains tractable even in large systems. It is particularly well suited for environments where shocks may exhibit asymmetries or fat tails, leading to 'relatively' frequent large deviations from the norm.

We show the effectiveness of the method across three applications. First, we show that combining sign restrictions with a kurtosis inequality restriction improves identification of monetary policy shocks and helps resolve known shortcomings in existing agnostic approaches. Second, we uncover significant macroeconomic effects of sovereign risk shocks in the euro area that standard methods fail to detect. Third, we find that geopolitical risk shocks have stronger and more persistent effects on U.S. financial conditions and real activity than estimated with standard methods. In each case, our approach yields sharper and more economically plausible inferences while relying on fewer and weaker identifying assumptions.

A Proofs

Notation Define $\boldsymbol{e}_k$ as the $n\times 1$ vector with zeros everywhere except a one in the k^{th} position, J_k the $n \times n$ matrix of zeros everywhere except one in the k^{th} position of the main diagonal and J_{jk} the $n \times n$ matrix of zeros everywhere except one in the $(j,k)^{th}$ element. Notice that $\boldsymbol{e}_k'\otimes\boldsymbol{e}_k = J_{k,k} = J_k$, $\boldsymbol{e}_j'\otimes\boldsymbol{e}_k = J_{k,j}$, $\boldsymbol{e}_j'\otimes\boldsymbol{e}_k = \boldsymbol{e}_k\boldsymbol{e}_j'$. Denote with I_n the identity matrix of size n and with $\text{vec}(X)$ the column-wise vecotrization of X. The following identities hold $\text{vec}(I_n) = \sum_{i=1}^n \boldsymbol{e}_i\otimes\boldsymbol{e}_i$ and $\text{vec}(I_n)\text{vec}(I_n)' = \sum_{k,j=1}^n J_{k,j}\otimes J_{k,j}$. Define the commutation matrix, $K_{n,n}$, the $(n^2 \times n^2)$ matrix consisting of $n \times n$ blocks where the $(j,i)-$element of the (i,j) block equals one, elsewhere there are all zeros. Notice that $K_{n,n} = \sum_{k,j=1}^n J_{k,j} \otimes J_{j,k}$.

Proof of Proposition 1 The $(n\times n^2)$ matrix collecting the structural shocks third moments can be written as

$$E(\nu_t\nu_t'\otimes\nu_t') = E(\nu_t\nu_t'\otimes[\begin{pmatrix}\nu_{1,t} & \dots & 0\end{pmatrix} + \cdots + \begin{pmatrix}0 & \dots & \nu_{n,t}\end{pmatrix}]) = \sum_{k=1}^n \zeta_k J_k \otimes \boldsymbol{e}_k'.$$

Using the properties of the Kronecker product, i.e. $(A\otimes B)(C\otimes D) = AC\otimes BD$, we have that

$$\left(\sum_{i=1}^n \zeta_i J_i\otimes\boldsymbol{e}_i\right)\left(\sum_{i=1}^n \zeta_i J_i\otimes\boldsymbol{e}_i\right)' = \sum_{i=1}^n \zeta_i^2 J_i' = \begin{pmatrix}\zeta_1^2 & \dots & 0\\ \vdots & \ddots & \vdots\\ 0 & \dots & \zeta_m^2\end{pmatrix} = \Lambda_\zeta.$$

where Λ_ζ is a diagonal matrix collecting the squared third moments of the structural shocks. Notice that the cross product are zero since $(J_i \otimes \boldsymbol{e}_i')(\boldsymbol{e}_k \otimes J_k') = J_k \boldsymbol{e}_i \otimes \boldsymbol{e}_k' J_i' = 0$. Likewise, the matrix collecting the empirical innovation third moments can be expressed as $E(\iota_t \iota_t' \otimes \iota_t') = A_o E(\nu_t \nu_t' \otimes \nu_t')(A_o \otimes A_o)'$. This leads to the following eigenvalue/eigenvector or spectral decomposition of the matrix collecting the squared third moments of the empirical innovations $E(\iota_t \iota_t' \otimes \iota_t')E(\iota_t \iota_t' \otimes \iota_t')' = A_o \Lambda_\zeta A_o'$.

Proof of Proposition 2 We first show that the fourth moments of a multivariate normal distribution are invariant to orthonormal rotation matrix. Denote with $\mathcal{K}^z$ the matrix that collects the fourth moments of the *standard* normal distribution, which are given by

$$\mathcal{K}^z = I_{n^2} + K_{n,n} + \text{vec}(I_n)\text{vec}(I_n)',$$

see Kollo (2008) for more details. First, we show that $\mathcal{K}^z$ is invariant to orthonormal rotations. Using the property of the commutation matrix[27] that $K_{n,n}(A \otimes B) = (B \otimes A)K_{n,n}$, we have that the commutation matrix is invariant to any orthonormal rotation Ω, i.e. $(\Omega \otimes \Omega)' K_{n,n}(\Omega \otimes \Omega) = K_{n,n}(\Omega \otimes \Omega)'(\Omega \otimes \Omega) = K_{n,n}(\Omega' \otimes \Omega')(\Omega \otimes \Omega) = K_{n,n}(\Omega'\Omega \otimes \Omega'\Omega) = K_{n,n}$. Moreover, using the relationship between the vectorization and Kronecker product, i.e. $\text{vec}(ABC) = (C' \otimes A)\text{vec}(B)$, we have that $(\Omega \otimes \Omega)'(\text{vec}(I_n)\text{vec}(I_n)')(\Omega \otimes \Omega) = (\Omega' \otimes \Omega')\text{vec}(I_n)\ ((\Omega' \otimes \Omega')\text{vec}(I_n))' = \text{vec}(\Omega' I_n \Omega)\text{vec}(\Omega' I_n \Omega)' = \text{vec}(I_n)\text{vec}(I_n)'$. Therefore, we have that $(\Omega \otimes \Omega)'\ \mathcal{K}^z\ (\Omega \otimes \Omega) = \mathcal{K}^z$, which proofs that fourth moments of a multivariate normal distribution are invariant to orthonormal rotation matrix.

The $(n^2 \times n^2)$ matrix collecting the full set of structural shocks fourth moments can be written as

$$\begin{aligned}
\mathcal{K} &= E(\nu_t \nu_t' \otimes \nu_t' \otimes \nu_t) \\
&= E(\nu_{1,t}^2 \nu_t \nu_t') \otimes \boldsymbol{e}_1' \otimes \boldsymbol{e}_1 + ... + E(\nu_{n,t}^2 \nu_t \nu_t') \otimes \boldsymbol{e}_n' \otimes \boldsymbol{e}_n + \sum_{j \neq k} E(\nu_{j,t} \nu_{k,t} \nu_t \nu_t') \otimes \boldsymbol{e}_j' \otimes \boldsymbol{e}_k = \\
&= \begin{pmatrix} \xi_1 & \cdots & 0 \\ & \ddots & \\ 0 & \cdots & 1 \end{pmatrix} \otimes \boldsymbol{e}_1' \otimes \boldsymbol{e}_1 + ... + \begin{pmatrix} 1 & \cdots & 0 \\ & \ddots & \\ 0 & \cdots & \xi_n \end{pmatrix} \otimes \boldsymbol{e}_n' \otimes \boldsymbol{e}_n + \sum_{j \neq k} (J_{j,k} + J_{k,j}) \otimes J_{k,j} = \\
&= ((\xi_1 - 1)J_1 + I_n) \otimes J_1 + ... + ((\xi_n - 1)J_n + I_n) \otimes J_n + \sum_{j \neq k} (J_{j,k} + J_{k,j}) \otimes J_{k,j} = \\
&= \sum_{i=1}^{n} (\xi_i - 1) J_i \otimes J_i + I_{n^2} + \sum_{j \neq k}^{n} \sum_{j,k=1} (J_{j,k} + J_{k,j}) \otimes J_{k,j}.
\end{aligned}$$

[27] See e.g. Schott (2016) (Theorem 8.26 at page 342)

Since $\boldsymbol{e}_i' \otimes \boldsymbol{e}_i = J_i$ and $\boldsymbol{e}_j' \otimes \boldsymbol{e}_k = J_{k,j}$, for any i, j, k; and $I_n = J_1 + \cdots + J_n$. It is convenient to express the fourth moments in deviation from the standard normal distribution analogs. To this end define $\xi_i = x_i + 3$; when $x_i = 0$ then the fourth moments coincide with the standard normal distribution ones. We can rewrite the fourth moments of the structural shock as follows

$$\begin{aligned}
\mathcal{K} &= \sum_{i=1}^{n}(x_i+2)J_i \otimes J_i + I_{n^2} + \sum_{j\neq k}^{n}\sum_{j,k=1}(J_{j,k}+J_{k,j}) \otimes J_{k,j} = \\
&= \sum_{i=1}^{n} x_i J_i \otimes J_i + I_{n^2} + \left[\sum_{i=1}^{n} J_i \otimes J_i + \sum_{j\neq k}^{n}\sum_{j,k=1} J_{j,k} \otimes J_{k,j}\right] + \left[\sum_{i=1}^{n} J_i \otimes J_i + \sum_{j\neq k}^{n}\sum_{j,k=1} J_{k,j} \otimes J_{k,j}\right] = \\
&= \sum_{i=1}^{n} x_i J_i \otimes J_i + I_{n^2} + \sum_{j,k=1}^{n} J_{j,k} \otimes J_{k,j} + \sum_{j,k=1}^{n} J_{k,j} \otimes J_{k,j} = \\
&= \sum_{i=1}^{n} x_i J_i \otimes J_i + \underbrace{I_{n^2} + K_{n,n} + \text{vec}(I_n)\text{vec}(I_n)'}_{\mathcal{K}^z} = \sum_{i=1}^{n} x_i J_i \otimes J_i + \mathcal{K}^z = \Lambda_\xi + \mathcal{K}^z.
\end{aligned}$$

It is straightforward now to derive the fourth moments of the empirical innovations in excess of the standard one as $E(\iota_t\iota_t' \otimes \iota_t' \otimes \iota_t) = (A_o \otimes A_o)\Lambda_\xi(A_o \otimes A_o)' + \mathcal{K}^z$.

Proof of Proposition 3 The third moments of the candidate structural shocks are given by

$$\begin{aligned}
E(\breve{\nu}_t\breve{\nu}_t' \otimes \breve{\nu}_t') &= A'E(\iota_t\iota_t'A \otimes \iota_t'A) = A'E(\iota_t\iota_t' \otimes \iota_t')(A \otimes A) = \\
&= \zeta_1 A'A_oJ_1A_o'A \otimes \boldsymbol{e}_1'A_o'A + ... + \zeta_n A'A_oJ_nA_o'A \otimes \boldsymbol{e}_n'A_o'A.
\end{aligned}$$

Notice that for all $k = 1, ..., n$,

$$\begin{aligned}
\boldsymbol{e}_k'A_o'A &= \begin{pmatrix} \boldsymbol{\alpha}_k'\boldsymbol{a}_1 & \boldsymbol{\alpha}_k'\boldsymbol{a}_2 & \dots & \boldsymbol{\alpha}_k'\boldsymbol{a}_n \end{pmatrix}; \\
A'A_oJ_kA_o'A &= \begin{pmatrix} \boldsymbol{a}_1'\boldsymbol{\alpha}_1 & \dots & \boldsymbol{a}_1'\boldsymbol{\alpha}_n \\ & \ddots & \\ \boldsymbol{a}_n'\boldsymbol{\alpha}_1 & \dots & \boldsymbol{a}_n'\boldsymbol{\alpha}_n \end{pmatrix} \begin{pmatrix} 0 & \dots & 0 \\ & 1 & \\ 0 & \dots & 0 \end{pmatrix} \begin{pmatrix} \boldsymbol{\alpha}_1'\boldsymbol{a}_1 & \dots & \boldsymbol{\alpha}_1'\boldsymbol{a}_n \\ & \ddots & \\ \boldsymbol{\alpha}_n'\boldsymbol{a}_1 & \dots & \boldsymbol{\alpha}_n'\boldsymbol{a}_n \end{pmatrix} = \\
&= \begin{pmatrix} (\boldsymbol{a}_1'\boldsymbol{\alpha}_k)(\boldsymbol{\alpha}_k'\boldsymbol{a}_1) & \dots & (\boldsymbol{a}_k'\boldsymbol{\alpha}_k)(\boldsymbol{\alpha}_k'\boldsymbol{a}_1) \\ & \ddots & \\ (\boldsymbol{a}_1'\boldsymbol{\alpha}_k)(\boldsymbol{\alpha}_k'\boldsymbol{a}_n) & \dots & (\boldsymbol{a}_n'\boldsymbol{\alpha}_k)(\boldsymbol{\alpha}_k'\boldsymbol{a}_n) \end{pmatrix} = \\
&= A'\boldsymbol{\alpha}_k\boldsymbol{\alpha}_k'A.
\end{aligned}$$

Therefore we have

$$
\begin{aligned}
E(\breve{\nu}_t\breve{\nu}_t'\otimes\breve{\nu}_t') &= \sum_k \zeta_k A'A_oJ_kA_o'A\otimes \boldsymbol{e}_k'A_o'A = \\
&= \sum_k \zeta_k \left(\boldsymbol{\alpha}_k'\boldsymbol{a}_1\times A'A_oJ_kA_o'A \quad \dots \quad \boldsymbol{\alpha}_k'\boldsymbol{a}_n\times A'A_oJ_kA_o'A\right) = \\
&= \left(\underbrace{\sum_k \zeta_k\boldsymbol{\alpha}_k'\boldsymbol{a}_1\times A'A_oJ_kA_o'A}_{\chi_1} \quad \dots \quad \underbrace{\sum_k \zeta_k\boldsymbol{\alpha}_k'\boldsymbol{a}_n\times A'A_oJ_kA_o'A}_{\chi_n}\right)
\end{aligned}
$$

where χ_j are $n\times n$ matrices. As $E(\breve{\nu}_{n,t}^3)$ occupies the (n,n^2) position in the matrix $E(\breve{\nu}_t\breve{\nu}_t'\otimes\breve{\nu}_t')$, we can focus on the $(n,n)-$element of rightmost $n\times n$ χ_n matrix, i.e.

$$
\chi_n = \sum_k \zeta_k\boldsymbol{\alpha}_k'\boldsymbol{a}_n\times A'A_oJ_kA_o'A = \zeta_1\boldsymbol{\alpha}_1'\boldsymbol{a}_n\times A'A_oJ_1A_o'A+\dots+\zeta_n\boldsymbol{\alpha}_n'\boldsymbol{a}_n\times A'A_oJ_nA_o'A
$$

If $\boldsymbol{a}_n=\boldsymbol{\alpha}_n$, then $\boldsymbol{\alpha}_j'\boldsymbol{a}_n=\boldsymbol{\alpha}_j'\boldsymbol{\alpha}_n=0$ with $j\neq n$ and $\boldsymbol{\alpha}_n'\boldsymbol{a}_n=\boldsymbol{\alpha}_n'\boldsymbol{\alpha}_n=1$. Hence, the latter equation simplifies to

$$
\chi_n = \zeta_nA'A_oJ_nA_o'A
$$

The $(n,n)-$element of χ_n equals $\zeta_n(\boldsymbol{\alpha}_n'\boldsymbol{\alpha}_n)(\boldsymbol{\alpha}_n'\boldsymbol{a}_n)=\zeta_n$. Finally, notice that if $\zeta_n=0$, the impact column vector $\boldsymbol{a}_n$ of the matrix A is not identified.

The fourth moments of the candidate structural shocks

$$
\begin{aligned}
E(\breve{\nu}_t\breve{\nu}_t'\otimes\breve{\nu}_t'\otimes\breve{\nu}_t) &= E(A'\iota_t\iota_t'A\otimes\iota_t'A\otimes A'\iota_t) = E(A'[(\iota_t\iota_t'\otimes\iota_t')(A\otimes A)]\otimes A'\iota_t) = \\
&= (A'\otimes A')E(\iota_t\iota_t'\otimes\iota_t'\otimes\iota_t)(A\otimes A)
\end{aligned}
$$

With similar algebra we get

$$
\begin{aligned}
E(\breve{\nu}_t\breve{\nu}_t'\otimes\breve{\nu}_t'\otimes\breve{\nu}_t) &= (A'\otimes A')(A_o\otimes A_o)E(\nu_t\nu_t'\otimes\nu_t'\otimes\nu_t)(A_o'\otimes A_o')(A\otimes A) = \\
&= (A'\otimes A')(A_o\otimes A_o)\left(\sum_{i=1}^n x_iJ_i\otimes J_i+\mathcal{K}^z\right)(A_o'\otimes A_o')(A\otimes A) = \\
&= \underbrace{(A'\otimes A')(A_o\otimes A_o)\left(\sum_{i=1}^n x_iJ_i\otimes J_i\right)(A_o'\otimes A_o')(A\otimes A)}_{\mathcal{K}^\star}+\mathcal{K}^z.
\end{aligned}
$$

In this case as well, with no departure from the normality (i.e. $x_i=0$ for all i) the rotation matrix cannot be identified. Since we focus on the kurtosis of the shock ordered last, we are interested in the $(n^2,n^2)-$element of the matrix $E(\breve{\nu}_t\breve{\nu}_t'\otimes\breve{\nu}_t'\otimes\breve{\nu}_t)$

we can disregard the matrix $\mathcal{K}^z$ and only consider the $(n^2, n^2)-$ element of the matrix $\mathcal{K}^\star$.

$$\begin{aligned}\mathcal{K}^\star &= (A' \otimes A')(A_o \otimes A_o)\left(\sum_{i=1}^{n} x_i J_i \otimes J_i\right)(A_o' \otimes A_o')(A \otimes A) = \\ &= (A'A_o \otimes A'A_o)\left(\sum_{i=1}^{n} x_i J_i \otimes J_i\right)(A_o'A \otimes A_o'A) = \\ &= \sum_{i=1}^{n} x_i (A'A_o J_i A_o' A) \otimes (A'A_o J_i A_o' A) = \sum_{i=1}^{n} x_i (A'\boldsymbol{\alpha}_i\boldsymbol{\alpha}_i' A) \otimes (A'\boldsymbol{\alpha}_i\boldsymbol{\alpha}_i' A).\end{aligned}$$

The $(n^2, n^2)-$element of $\mathcal{K}^\star$ is the sum of the square of the $(n, n)-$elements of the matrices $A'\boldsymbol{\alpha}_i\boldsymbol{\alpha}_i'A$ times x_i for $i = 1, \ldots, n$, which is given by

$$\begin{aligned}\mathcal{K}^\star_{(n^2,n^2)} &= x_1(\boldsymbol{a}_n'\boldsymbol{\alpha}_1)^2(\boldsymbol{\alpha}_1'\boldsymbol{a}_n)^2 + \cdots + x_m(\boldsymbol{a}_n'\boldsymbol{\alpha}_n)^2(\boldsymbol{\alpha}_n'\boldsymbol{a}_n)^2 = \\ &= x_1(\boldsymbol{a}_n'\boldsymbol{\alpha}_1\boldsymbol{\alpha}_1'\boldsymbol{a}_n)^2 + \cdots + x_m(\boldsymbol{a}_n'\boldsymbol{\alpha}_n\boldsymbol{\alpha}_n'\boldsymbol{a}_n)^2.\end{aligned}$$

Hence we have that $\mathcal{K}^\star_{(n^2,n^2)} + \mathcal{K}^z_{(n^2,n^2)} = x_n + 3 = \xi_n$, if $\boldsymbol{a}_n = \boldsymbol{\alpha}_n$ (since $\boldsymbol{\alpha}_j'\boldsymbol{a}_n = 0$ if $j \neq n$ and $\boldsymbol{\alpha}_n'\boldsymbol{a}_n = 1$).

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