Intergenerational Mobility and Long-Term Care

Alexis Smith (University of Wisconsin - Madison) *

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Abstract

This project investigates how long-term care risk facing grandparents impacts the economic mobility of grandchildren in the United States. In order to understand the mechanisms that underpin this relationship, I estimate an overlapping generations model in which parents and grandparents interact with limited commitment to make both long-term care and childcare choices, as well as investments in children. I then use the model to evaluate the impact of changes to Medicaid's asset limit for long-term care eligibility and subsidies for informal elder-caregivers on intergenerational mobility.

*amsmith67@wisc.edu.

1 Introduction

Intergenerational mobility in the United States is low. Wealth and earnings tend to persist across generations, and the factors that contribute to this are diverse but are underpinned by parents' ability to invest in their children. Parents are constrained by their ability to access credit and borrow, (Braxton et al., 2023; Caucutt and Lochner, 2020), and by where they live (Anstreicher, 2024; Chetty et al., 2016). However, long-term care concerns facing grandparents may also impact parents' ability to invest in their children. Further, poorer families may be disproportionately impacted, which may contribute to the persistence of earnings and wealth across generations in the US.

Long-term care risk is one of the largest risks facing aging adults and their families. Informal care is very common in the United States, particularly among low-income families. This is, in part, driven by the fact that formal long-term care is often cost prohibitive. Families facing both childcare and eldercare needs simultaneously may have a harder time investing in their children, both in terms of time spent with children and financial investments, including higher education. However, because long-term care is risky and hard to insure against, even grandparents who never develop care needs may dis-save more slowly than would be optimal in the absence of these long-term care concerns to insure against the cost of potential care needs. This may result in under-investment in children if families would otherwise use these funds to help pay for care and schooling of children.

In this project, I ask whether and to what extent long-term care risk facing grandparents dampens the economic mobility of children and how this varies across the income distribution. Further, without accounting for the impact of long-term care risk on children, we likely understate the benefit of insuring families against this risk.

To address this, I first summarize a number of empirical facts that suggest long-term care concerns facing grandparents are related to the ability of parents to invest in their children. I then develop an overlapping generations model in which parents and grandparents interact with limited commitment to make investment, care, and savings decisions. I estimate the model using data from the Health and Retirement Study and the Panel Study of Income Dynamics. I then use the model to quantify the impact of long-term care risk on the economic mobility of children and decompose the effects into those coming from families who face these dual care needs directly and from the savings choices of all families, even those who never develop long-term care needs.

Further, in order to inform how long-term care insurance policies may lessen the persistence of poverty across generations, I evaluate the impact of two policy changes on

the investments in and outcomes of children. First, relaxing Medicaid's asset testing may ease the burden of caregiving for parents and allow grandparents who face long-term care needs to pass on more of their savings to parents rather than spending on formal long-term care. Additionally, this may allow them to make transfers to parents at earlier ages, thus boosting investments in children. Second, I evaluate subsidies for informal caregivers. This is effectively subsidizing informal, rather than formal, insurance, and I use the model to evaluate which is more effective at improving the economic mobility of children.

The existing literature on long-term care in economics has largely focused on the impact of informal care on the economic outcomes of aging parents and their adult children. The ability of family members to provide informal care reduces Medicaid utilization (Barczyk and Kredler, 2018) and slows nursing home entry (Barczyk et al., 2022). Further, informal care also suppresses demand for long-term care insurance (Mommaerts, 2024) and exacerbates adverse selection issues in the long-term care insurance market (Ko, 2022). This project, in contrast, investigates the effects of informal care on not only grandparents and parents, but children as well. Accounting for the impacts on children is important for quantifying the benefits of insuring families against long-term care risk, which may be understated in past studies.

Beyond the direct impacts of eldercare needs, there are also several studies that document the impact of informal insurance from family members on economic decisions. Parents save to insure their children against income risk, with the amount varying by the riskiness of the child's occupation (Boar, 2021). For young men, the option for coresidence with one's parents provides labor market insurance and helps explain why that group has a low elasticity of labor supply (Kaplan, 2012). Thus family-provided insurance plays an important role in determining outcomes across generations. In this project, I build on this literature by investigating how long-term care risk facing grandparents may impact not only parents but their children as well and whether changes in long-term care insurance policies could help reduce the persistence of poverty across generations.

A largely separate literature documents several factors that contribute to the persistence of economic outcomes across generations, including the role of human capital investments made by parents (Becker and Tomes, 1986; Lee and Seshadri, 2019), borrowing constraints facing parents (Caucutt and Lochner, 2020), parents' ability to access credit (Braxton et al., 2023), and neighborhood and migration decisions (Anstreicher, 2024; Chetty et al., 2016). Further, the structure of the immediate family, namely the presence of two parents, positively impacts a child's future income (Chang et al., 2024). I extend this literature by accounting for the impacts of a third generation, the grandpar-

ent generation, and eldercare risk on the intergenerational mobility of children in the US. Further, in order to do so, I develop a model of overlapping generations in which parents and grandparents interact with limited commitment to make choices over the entire life-cycle–a novel contribution to this literature.

The results of this study will shed light on the impact of long-term care risk on the persistence of poverty and inequality across generations. Decomposing the effects into direct and indirect mechanisms will help inform which policies are most effective in accomplishing this goal.

2 Empirical Facts

Long-term care (LTC) covers difficulty with (Instrumental) Activities of Daily Living. These include activities such as bathing, dressing, managing medications, and housekeeping and are not acute medical events but rather long-term difficulties. These care needs are risky, while 70% of individuals aged 65 will need LTC at some point, there is substantial heterogeneity in both the timing and amount of care.

Long-term care risk facing grandparents may impact the intergenerational mobility of children in the United States directly, for families facing these care needs as well as indirectly for families concerned about developing these care needs in the future. Here, I present suggestive evidence that both of these channels are important.

First, one in five elder-caregivers also care for at least one child under 12 (Chari and Valli, 2021). Among families facing these simultaneous care needs, parents may be unable to invest as much in their children as they would absent eldercare needs. Both time and monetary investments in children are important for development, and investments in earlier stages make investments later on more productive (Heckman and Mosso, 2014; Caucutt and Lochner, 2020; Cunha et al., 2010). Parents in families that require both eldercare and childcare may have a harder time making both time and monetary investments.

Further, low-income parents may be more severely affected. Poorer grandparents receive more informal care from parents (Figure 1), which may impact a parent's ability to make time investments in her child. Additionally, many parents, especially single parents, rely on grandparent-provided childcare (Anstreicher and Venator, 2022). However, grandparents who have care needs of their own are often unable to provide such care, which means parents must fill these hours themselves or with formal childcare. This may be more difficult for lower-income parents and single parents, thus the impact of long-term care needs may be more concentrated at the bottom of the income distribution,

further perpetuating intergenerational inequality.

Monetary investments may also be impacted; parents who provide intensive eldercare are 20% less likely to have an educational savings account than otherwise similar parents (Bogan, 2015). This suggests that eldercare responsibilities impact the ability of parents to finance their child's education which is an important driver of economic mobility.

In addition to the impact that eldercare needs have on time and monetary investments made by parents, these care needs may make it harder for families to build generational wealth. Long-term care expenses and Medicaid asset testing could dampen grandparents' ability to pass on their wealth. Medicaid, rather than Medicare, covers long-term care for aging adults, and thus recipients are subject to strict income and asset requirements. Thus, long-term care expenses and Medicaid asset testing may wipe out assets of low-and-middle income families that would have otherwise been passed down. Given strict eligibility rules for Medicaid, the US spends less on formal care per capita than many other developed countries, which may help explain why poverty is more persistent across generations in the United States than in many of these countries (Figure 2).

While the above mechanisms are most salient for families facing dual care needs, even children in families that do not face child and eldercare needs simultaneously may be impacted by long-term care concerns. Given that long-term care needs are risky, there may be indirect effects on children if families' make saving choices to self-insure against this risk, even if these care needs never materialize. Using data from the Health and Retirement Study, I show that even before care needs are realized, grandparent health expectations are correlated with parent education, controlling for grandparent age, wealth, education and parent age (Figure 3). While there are likely several underlying causes for this, it suggests that expectations about long-term care risk do have an impact on educational attainment–one driver of economic mobility[1].

3 Model

In order to quantify the importance of direct and indirect effects of long-term care risk on intergenerational mobility, I develop a model in which families make childcare and eldercare choices, save, and invest in children. While it may be possible to capture some of the effect of long-term care risk on children through exogenous variation in things like grandparent mortality or changes in long-term care policy, these strategies cannot address the relative impacts of direct versus indirect effects. Using a structural model also allows

[1] In the future, I plan to explore how these expectations impact savings behavior as one potential mechanism that may help explain this link.

me to evaluate a range of counterfactual policies. Further, data with suitable sample sizes necessary for these empirical strategies in the US context is sparse. Using a structural model allows me to utilize data from multiple data sets.

Thus, I develop a model to understand how families facing long-term care risk make care and investment decisions in children. Dynasties in the model are infinitely lived and made up of overlapping generations. There are three generations within a family alive at any given time–a child, a parent, and a grandparent. The child is cared for by their parent, and parents and grandparents interact with limited commitment to make consumption, savings, childcare, and eldercare choices as well as monetary and time investments in children. Limited commitment is important to capture the degree of coordination around these choices while maintaining the separation of each agent's assets.

3.1 Timeline

Time is discrete and runs forever. A period in the model is 10 years. I model infinitely lived dynasties and within each dynasty there are three overlapping generations: a child (k), a parent (p) and a grandparent (g). Each agent in the model spends three periods in each generation: three as a child, then three as a parent, and then three as a grandparent. Thus, a family within a dynasty may be in one of three stages: young ($t = y$), middle-age ($t = m$), or old ($t = o$).

In the model, children receive time and monetary investments from parents and require childcare in the first two periods of their lives. In the third period, they may or may not attend college. These children then become parents, and enter the labor market where they face earnings risk and make childcare, eldercare, and labor supply choices. Parents also make a savings decision. After three periods as a parent, the parent becomes a grandparent and the child becomes a parent. Grandparents face eldercare risk, may help with childcare, and make a savings decision. At the end of three periods as a grandparent, the grandparent passes away, and may leave a bequest to the parent, who becomes a grandparent. Thus, while individuals in the model live for nine periods, a new generation is born every three periods and thus the dynasty is infinitely lived. This structure is outlined in Figure 4. Below, I outline these life stages within each generation in more detail.

3.2 Children

When children are born into the model, their initial skill level depends on the skill of the parent and a randomly drawn component. The persistence of ability across generations

is governed by ρ_s. A child who enters the model at time t has an initial skill level is given by

$$s_y^k = \rho_s s_y^p + (1 - \rho_s)d \tag{1}$$

where the random component of a child's initial skill, d, is drawn from a normal distribution with mean μ_{sk} and variance σ_{sk}^2.

In the first two stages of life, the child's skill development depends on monetary (x_t) and time (T_t) investments.

$$s_{t+1}^k = s_t^k(1 + T_t^{\omega_t} x_t^{(1-\omega_t)}) \tag{2}$$

This formulation incorporates dynamic complementarity as skills in the next period depend on the skills the child has today, which is an important feature of child skill development (Heckman and Mosso, 2014; Caucutt and Lochner, 2020; Cunha et al., 2010). It also captures the complementarity of monetary and time investments, which are particularly important in this context as eldercare may crowd out the parent's ability to invest time and money in the child. The relative productivity of time and monetary investments varies over the two stages and is captured by ω_t.

The time input (T_t) in the child's skill development function is a composite of time from parents (hp_t^k), grandparents (hg_t^k), or market-based childcare (hm_t^k). I assume that young and middle-age children require Hk_t hours of care, and that these childcare hours can be met by the parent, grandparent, or market (or any combination therein). However, each of these inputs has a different productivity in the child's skill development, such that the time investment component is given by:

$$T_t = \left[\frac{\omega_p s_p}{Z} (\frac{hp_t^k}{H_k})^{\rho_\omega} + \frac{\omega_g}{Z} (\frac{hg_t^k}{H_k})^{\rho_\omega} + \frac{\omega_m}{Z} (\frac{hm_t^k}{H_k})^{\rho_\omega} \right]^{\frac{1}{\rho_\omega}} \tag{3}$$

where $Z = \omega_p s_p + \omega_g + \omega_m$. In this formulation, the productivity of parent time scales with parent skill.

Old children do not require childcare, but may or may not attend college. Thus their skill development is a function of the skills they have accumulated thus far and a linear return to college education.

$$s_y^p = s_o^k + \omega_X X_o \tag{4}$$

I model college-going as a binary choice, such that $X_o = \{0, 1\}$. Thus, the skills that a young parent has in the model are a product of the skills accumulated over her childhood,

culminating in a college choice.

3.3 Parents

Parents receive a wage offer that correspond to their skill level:

$$log(w_t) = s_t^p \tag{5}$$

Parents then make a discrete labor supply choice, where they may choose not to work, to work part-time or to work full-time such that $L_t^p = \{0, \text{part-time}, \text{full-time}\}$. Their labor income is then given by $w_t L_t^p$. In addition to labor income, the total earnings of parents includes an exogenous component that is a function of the parents skill ($I(s_t^p)$) and an earnings shock ϵ_t. Exogenous earnings here are meant to represent those of a spouse or partner, and are correlated with the skill level of the parent given that spouses tend to have similar levels of education in the data. Earnings risk follows a normal distribution with mean of zero and variance σ_ϵ^2.

$$I_t^p = w_t L_t^p + I(s_t^p) + \epsilon_t \tag{6}$$

Parents' skills also develop over their three stages as a parent:

$$s_{t+1}^p = \alpha_{s_p} s_t^p + \alpha_L L_t^p \tag{7}$$

Where the return to last period's skills are given by α_{s_p} and return to experience is given by α_L.

3.4 Grandparents

Grandparents do not make labor market choices, and spend all their time on leisure or childcare. They receive income I_t^g, meant to represent pension and social security income. Middle-age and old grandparents face risk in the form of shocks to the number of long-term care hours needed (H_t^g). In order to help cover these shocks, I model a Medicaid benefit given by:

$$M_t = \max\{0, c_f - p^g hm_t^g - a_t^g - I_t^g\} \tag{8}$$

Grandparents are eligible for Medicaid if their care costs ($p^g hm_t^g$), asset holdings, (a_t^g) and income (I_t^g) exceed a consumption floor (c_f). Additionally, only healthy grandparents

are able to provide childcare.

3.5 Constraints

Families face both time and budget constraints. Children in young and middle-age families require Hk_t hours of childcare. These hours may be met by the parent (hp_t^k), the grandparent (hg_t^k), or market care (hm_t^k) for price p^k.

$$H_t^k = hp_t^k + hg_t^k + hm_t^k \tag{9}$$

Similarly, middle-age and old families may face eldercare needs. The total number hours of eldercare a grandparent requires (H_t^g) are drawn randomly. These hours may be met by the parent hp_t^g or by market-based formal eldercare (hm_t^g), at price p^g such that:

$$H_t^g = hm_t^g + hp_t^g \tag{10}$$

Given these care needs, the parent's time constraint is given by:

$$T = hp_t^k + hp_t^g + L_t^p + \ell_t^p \tag{11}$$

where she spends her time on labor, leisure, childcare, and eldercare depending on her choices and the childcare and eldercare needs a family faces in a given period.

The grandparent's time constraint is given by:

$$T = hg_t^k + \ell_t^g \tag{12}$$

where she spends all her time on leisure and childcare if she provides it. I model childcare provision as subtracting from the leisure of the grandparent. While many grandparents likely enjoy spending time with their grandchildren, the model captures a level of care that is more intensive than occasional visits or time supervised by a parent.

Turning to the family budget constraint, in all stages, families make consumption (c_t^p, c_t^k, c_t^g) and savings (a_t^p, a_t^g) choices. Parent's income is made up of their labor income ($w_t L_t^p$), spousal income ($I_t^p(s_t^p)$), and an earnings shock (ϵ_t). Grandparents receive income I_t^g each period, meant to approximate pension income. Finally, care and educational costs vary over each stage, and are summarized by B_t. Thus, the family budget constraint is given by:

$$c_t^k + c_t^p + c_t^g = w_t L_t^p + I_t^p(s_t^p) + \epsilon_t + I_t^g + R(a_{t-1}^p + a_{t-1}^g) - a_t^p - a_t^g - B_t \tag{13}$$

Young families make monetary investments in their child (x_t) and may pay for formal childcare if they choose to use it (hm_t^k) Thus, the care and investment cost for young families is given by:

$$B_y = x_y + p^k hm_y^k \tag{14}$$

Middle-age families also make monetary investments in children and decide whether to pay for formal childcare. Middle-age grandparents are also subject to eldercare shocks, and the family may choose to use for formal eldercare (hm_t^g). Additionally, grandparents may qualify for Medicaid (M_t) to help cover care costs. The Medicaid benefit is described in Equation 8. Thus, care and investment costs for middle-age families is given by:

$$B_m = x_m + p^k hm_m^k + p^g hm_m^g - M_m \tag{15}$$

Finally, old families may also require eldercare, and may receive a Medicaid benefit if eligible. Children in old families no longer require childcare, but may go to college, and thus families incur the cost of college (p^X) if the child goes to college. So, the care and educational costs old families face is given by:

$$B_o = p^X X_o + p^g hm_o^g - M_o \tag{16}$$

I describe the budget constraint of the family in each stage, though it could equivalently have been written as one for the parent and one for the grandparent with transfers between the parent and grandparent.

3.6 Preferences

In each stage, parents have preferences over their own consumption, c^p, and the consumption over their child, c^k, weighted by λ. Because parents make all choices for the child, the child's consumption is a component of the parent's utility function. Parents also have preferences over leisure, weighted by α_ℓ.

$$u_t^p(c_t^p, c_t^k, \ell_t^p) = \frac{c^{p(1-\gamma_c)}}{1-\gamma_c} + \lambda \frac{c^{k(1-\gamma_c)}}{1-\gamma_c} + \alpha_\ell \frac{\ell^{p(1-\gamma_\ell)}}{1-\gamma_\ell} \tag{17}$$

Grandparents have preferences over their own consumption, c^g, and leisure, ℓ^g, as well as the type of eldercare that they receive. Currently grandparents may rely on informal care from parents or formal care, which may be covered by Medicaid. The parameter z_F gives the (dis)-utility from receiving formal care ($F_t = 1$). The parameter z_M gives the

additional (dis)-utility from receiving Medicaid-funded formal care. This reflects the fact that Medicaid-funded care is often of lower quality than other formal care options. The Medicaid benefit is described in Equation 8. Grandparent preferences are given by:

$$u_t^g(c_t^g, \ell_t^g) = \frac{c^{g(1-\gamma_c)}}{1-\gamma_c} + \alpha_\ell \frac{\ell^{g(1-\gamma_\ell)}}{1-\gamma_\ell} + z_M \mathbb{1}_{M_t>0} + z_F \mathbb{1}_{F_t=1} \tag{18}$$

The γ_c and γ_ℓ parameters govern the curvature of consumption and leisure and α_ℓ controls the how much weight individuals place on leisure relative to consumption.

3.7 Family Problem

Each dynasty maximizes the Pareto weighted sum of the parent and grandparent utilities in every period. Agents discount the future at rate β. Grandparents are altruistic and place a weight of η on the parent's utility in each period. Thus, the problem facing a dynasty is given by:

$$V_0 = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \{\theta_t u_t^p + (1-\theta_t)[u_t^g + \eta u_t^p]\} \right\} \tag{19}$$

I model the interactions between the parent and grandparent with limited commitment. A unitary model would imply full insurance between the parent and grandparent, and the literature generally rejects full insurance within extended family networks (Hayashi et al., 1996; Attanasio et al., 2019). A cooperative model would also imply that the family has only a single asset pool shared by the parent and grandparent. However, maintaining separation between the assets of the parent and the grandparent is particularly important in this context given that Medicaid covers long-term care, and thus beneficiaries are subject to Medicaid's asset test[2].

Given that the data rejects a fully cooperative model, both non-cooperative and limited commitment models are common when modeling complex family interactions (Kaplan, 2012; Voena, 2015; Mommaerts, 2024). I have chosen limited commitment because the level of coordination between families suggests a level of cooperation beyond that in a non-cooperative model (Mommaerts, 2024).

The individual value functions for the parent and the grandparent respectively are give by

$$V_t^p(z_t) = \max_{j_t} u_t^p + \beta E V_{t+1}^p(z_{t+1}) \tag{20}$$

[2]Individuals may not have more than $2000 in liquid assets in most states.

$$V_t^g(z_t) = \max_{j_t} u_t^g + \beta E v_{t+1}^g(z_{t+1}) + \eta V_t^p(z_t) \tag{21}$$

where $v(z_t) = u_t^g + \beta E v_{t+1}^g(z_{t+1})$. Then, since parents become grandparents at the end of the old stage, the parent's continuation value when $t = o$ is given by $V_{o+1}^p(z_{o+1}) = V_y^g(z_y)$. Since grandparents pass away at the end of the old family stage, their continuation value at $t = o$ is given by $v_{o+1}^g(z_{o+1}) = 0$.

Thus, the recursive family problem is given by:

$$V_t(z_t) = \max_{j_t} \theta_t V_t^p + (1 - \theta_t) V_t^g \tag{22}$$

and is subject to the time constraints (equations 9, 10, 11, and 12) and the budget constraint (equation 13). Because parents and grandparents interact with limited commitment, each is also subject to their own voluntary participation constraint. The state variables are given by z_t and the choice variables are given by j_t. These are summarized in Table 1.

The voluntary participation constraints for each stage are given by:

$$\mu_t^p : V_t^p(z_t) \geq W_t^p(z_t) \tag{23}$$

$$\mu_t^g : V_t^g(z_t) \geq W_t^g(z_t) \tag{24}$$

Where the outside options W_t^p and W_t^g reflect an inability of parents and grandparents to coordinate on transfers or care within a dynasty. However, grandparent are still altruistic and may leave a bequest. The Lagrange multipliers on these constraints, μ_t^p and μ_t^g dictate how the Pareto weights evolve over time in order to maintain cooperation while feasible. Appendix A provides more details on the outside options and Appendix B provides more details on how I solve the model.

3.8 Model Discussion

This model allows me to capture both the direct and indirect effects of long-term care risk on intergenerational mobility. However, the model does make a number of abstractions from the real world, including marriage, divorce, and fertility choices. While these mechanisms may be important, I do not consider them here in order to focus on intergenerational interactions while maintaining tractability. The model does incorporate heterogeneity in spousal income as well as earnings risk as well as single parent households, which help to capture the effects of marriage and divorce without introducing another choice. Further, while large expansions in LTC policies may have impacts on fertility de-

cisions, many European countries have extensive long-term care insurance programs and persistently low birth rates, so these effects may be secondary.

The model also abstracts from the existence of siblings and a second set of grandparents. This is again to maintain tractability and focus on intergenerational interactions, but it is also informed by what is available in the data. Figure 6 outlines a family tree across three generations. In this family, there are four grandparents, each parent has a sibling, and there are two children. However, in the data, I am at most able to observe half of this family tree (shaded in blue). Given this, along with a focus on intergenerational interactions, the structure of them model collapses this family tree into a grandparent generation, made up of two grandparents, a parent generation, made up of two parents, and a child generation, currently made up of one child. This is described in Figure 5.

4 Estimation

To evaluate the impact of long-term care on intergenerational mobility, I will estimate the model using data from the Panel Study of Income Dynamics and the Health and Retirement Study. I first set a number of parameters outside the model and then use the simulated method of moments to structurally estimate the remaining parameters.

4.1 Data

The Panel Study of Income Dynamics (PSID) is a longitudinal survey that has followed a representative sample of descendants from approximately 5,000 families since 1968. The main survey of the PSID contains detailed data on the labor market choices and outcomes of parents, and the Child Development Supplement (PSID-CDS) has information on the investments and educational outcomes of grandchildren. Given the PSID follows families, I can link individuals across generations which is particularly useful in this context.

For more detailed information on the grandparent generation, I use the Health and Retirement Study (HRS). The HRS is a longitudinal panel of approximately 20,000 people. Surveys have been conducted biennially since 1992, with extensive information about respondents' health and care needs, long-term care arrangements, insurance, and assets. The survey also contains some information about the children of the respondents.

For statistics related to savings, earnings, and labor supply choices of parents and investments in and educational outcomes of children, I use the PSID. My sample consists of children in the 1997, 2002 and 2007 waves of the CDS and their parents. While the CDS has also sampled children more recently, I restrict to these years to observe children

sampled at later ages. A 12-year-old child sampled in 1997 would be 34 in 2019, which allows me to capture some earnings data for these children. Additionally, the CDS includes investments made in children, so I restrict to parents of CDS children to capture their investment decisions in tandem with their labor supply choices.

Summary statistics for the children are presented in Table 2, and information on their children is reported in Table 3. In total, the sample is made up of 6,185 children in 4,444 households. About 35% of these households are headed by single parents. One-third of the children in my sample eventually attend college.

For more detailed information on the grandparent generation, including detailed information on long-term care needs, I turn to the HRS. I use data from 1992 to 2022 and keep individuals who respond in at least three waves. I also drop respondents who do not have children or grandchildren throughout the sample period. My sample consists of 31,789 individuals in 18,575 households. Summary statistics for this sample are presented in Table 4. In my sample, 33% of grandparents provide childcare at some point in their lives, and 53% need eldercare beyond what is provided by their spouse.

4.2 First-Stage Parameters

I first set a number of values outside the model. These are largely based on values in the literature and scaled to take account for the length of a period in my model and are summarized in Table 5. Those not taken from the literature are described below.

First, the hours of childcare that children need in the model are calculated as follows. Young children in the model are between the ages of 0 and 10. From the ages between 0 and 4, I assume children need full-time childcare: 8 hours per day 7 days a week. While a day is 16 hours, I limit this to active care time, rather than time that would be spent with the child anyways (i.e. secondary care time). After age 4, they may enter kindergarten, and thus some care time is taken up by school hours. Between the ages of 5 and 10, I assume children need 4 hours of active care time 5 days a week and 8 hours two days a week (on the weekends). Averaging these, we get 2,400 hours annually.

Middle children in the model are middle and high school aged, and thus even outside of school time need fewer active care hours. I assume they need four hours per day five days a week. This is time that may be filled by parents, grandparents or "childcare", which may include things like after-school extracurriculars or tutoring. Thus, these children need 1040 hours of active care.

Turning to the cost of childcare, a full day of childcare can range from $6,500 to $15,600. Assuming 8 hours a day, this is 3 to 7.5 per hour. Half-day care ranges from $5,900 to

$9,200, which works out to 5 to 9 per hour. Hourly caregivers range from $15 to $35 per hour (Poyatzis and Livingston, 2024). Given these, I set the cost of childcare to be $10 per hour.

The cost of college is also taken as the sum of an average over four years. Two years of average tuition ($9,750 each) and two years of tuition plus room and board ($35,248 each) which gives an annual cost of $22,500, or $90,000 for four years (Hanson, 2026).

The weights on time in the child skill production function comes from (Lee and Seshadri, 2019). There, a model-period is 6 years, thus, I rescale their parameters to fit a ten-year period. Time is more important for young children than middle-age children.

The literature finds mixed results on the effectiveness of grandparent and market care relative to parent care (Milovanska-Farrington, 2021; Bernal and Keane, 2010; Caucutt et al., 2026). Given this, coupled with the fact that most studies find small effects, I set the returns to parent time, grandparent time and market time to be equal. Additionally, parent time scales with parent skill, which may help explain why these studies find mixed results and is included in the model. The substitutability of skill inputs is set to be 0.63, implying that market care, parental care and grandparent care are imperfect substitutes in the child skill production function.

Turning to eldercare, part-time care costs $20,000 annually which is $20/hour and full-time care is $61,700 annually or $30/hour (Brown and Finkelstein, 2007). Further, a semi-private room in a nursing home ranges from $112,420-186,000 ($25/hour assuming 12 hours of care a day). Home care is around $51,000 annually or $33 per hour and assisted living is $66,132 annually ($15/hour assuming 12 hours of care a day) (OPM, 2026). Averaging these, I set the hourly cost of care at $20 per hour in the model.

In addition to the parameters set based on values in the literature, I estimate a number of parameters based on my PSID and HRS samples that do not require the full structure of the model. These are summarized in Table 6.

The mean and standard deviation of child skills are based on the mean and variance of the letter-word tests in the PSID-CDS. The return to college is based on the earnings difference between children in the CDS who attend college and those who do not, accounting for initial skill differences.

The returns to skill and labor market experience for parents come from a regression of hourly earnings on lagged earnings, experience, and age for working mothers. The standard deviation of income shocks comes from the variance of the residual in a regression of family income on age, year, and education for working-age parents of CDS children.

Finally, for information on grandparents' income over time and long-term care risk, I use my HRS sample. Grandparent income is calculated as the mean household income

of grandparents in each stage. For long-term care risk, I compute the total hours of care needed by a household (so that of both grandparents if two are present). I then remove the hours of care from spouses and use the number of hours met formally and by children when calculating the risk of developing care needs. This implicitly assumes that spouses cover care before any other source, when they do not have care needs of their own. The number of hours of care across each care source is summarized in Figure 7.

4.3 Estimated Parameters

Taking the first stage parameters as given, I will then estimate the remaining parameters using the simulated method of moments. I will estimate the curvature on consumption and leisure (γ_c and γ_ℓ), the weight on leisure (α_ℓ), the formal care and Medicaid utility parameters (z_M and z_F), the consumption floor for Medicaid eligibility (c_f), and the degree to which grandparents are altruistic towards parents (η). These parameters are summarized in Table 7.

In order to match the data moments, I will use the Nelder-Mead algorithm to minimize equation 25 with respect to the parameter vector Θ,

$$M(\Theta) = (m_d - m_M(\Theta))' W (m_d - m_M(\Theta)) \tag{25}$$

where W is a diagonal weighting matrix which puts equal weight on each of the moments, m_d is a vector of moments from the HRS and PSID data, and $m_m(\Theta)$ is a vector of moments calculated from simulated data. Standard errors will be calculated using the standard sandwich formula.

4.4 Model Fit

Here, I present preliminary results from a calibrated model. To estimate the parameters, I will target twelve moments including the Medicaid rate, the fraction of mothers working full-time, informal care rates over the wealth distribution, and the savings of families over the life course. These moments are presented in Table 8 and Figure 8.

In addition to these targeted moments, I also show how the model matches Medicaid, and formal and informal eldercare and childcare over the wealth distribution in Figure 9. Figure 10 shows moments related to investment and labor supply over the wealth distribution. Given the model is not yet estimated, I would not expect these moments to match up very well, but they are generally trending the right direction and I show them here to give an idea of what I will show once the model is estimated.

5 Counterfactuals

In this section I describe the counterfactuals I plan to investigate once the full model is estimated. I will first quantify the impact of long-term care on intergenerational mobility. I will then conduct two policy-related counterfactuals.

5.1 Quantifying the Impact of LTC on Integenerational Mobility

To quantify the impact of eldercare needs and eldercare risk on intergenerational mobility, I will regress average child earnings rank on average parent earnings rank:

$$y_c = \alpha_1 + \alpha_2 y_p + \epsilon \tag{26}$$

where the coefficient of interest is α_2. To compute the effect of eldercare risk on intergenerational mobility, I will re-solve the model, but with some families requiring a known number of hours of long-term care in advance. This maintains the impact of long-term care and the overall amount of long-term care needed but removes the uncertainty within a family. To compute the effect of eldercare needs, I eliminate eldercare needs entirely in the model.

5.2 Policy Counterfactuals

Beyond quantifying the impact of long-term care on intergenerational mobility, I will also use the model to evaluate the effects of changes in long-term care policy. I will explore two policy changes that help insure families against long-term care risk:

1. Increases in Medicaid's asset limit.
2. Subsidies for informal caregivers.

Given the structure of my model, I will also account for the cost of implementing each of these counterfactuals–either with taxes on labor supply or estate taxes. Further, I can compare the efficacy of each of these taxation schemes.

I currently model long-term care insurance as a benefit given by $M_t = \max\{0, c_f - p^g h m_t^g - a_t^g - I_t^g\}$. This formulation captures the fact that, in the United States, Medicaid covers long-term care, and thus beneficiaries must cover the cost of care until they meet the asset requirements. The consumption floor, c_f, represents the basket of other social

services that would be available to grandparents if they are poor enough to qualify for Medicaid[3].

I can use this formulation to evaluate changes to the eligibility requirements. The current asset limit is $2,000 in most states, though in 2024 California eliminated their asset test. To test different asset limits, I will add an asset limit, a_{lim}, which changes the amount that grandparents must spend out of their savings on formal eldercare before they are eligible for Medicaid:

$$M_t = \max\Big\{0, c_f - p^g hm_t^g - I_t^g - \max\{a_t^g - a_{lim}, 0\}\Big\}$$

Expansions in Medicaid long-term care coverage would mostly impact families in the middle of the wealth distribution, as low wealth families are already eligible and high wealth families are unlikely to take up Medicaid care. Broadly, means-tested social insurance programs provide strong incentives for low-income individuals not to save (Hubbard et al., 1995; Scholz et al., 2006), and expansions in eligibility would likely reduce savings among newly eligible families. However, this may enable families to invest more in children as they know that long-term care insurance is available if the grandparent requires care, and that they are themselves insured against future long-term care needs as well.

I will also evaluate the impact of expanding subsidies for informal care. Historically Medicaid has only provided benefits for formal eldercare, in recent years most states have allowed informal caregivers to receive some payments through the program. While informal care subsidies have been shown to reduce Medicaid utilization (Barczyk and Kredler, 2018) and decrease labor supply of caregivers (Skira, 2015) in two generation models, I will evaluate how these subsidies impact children as well. These subsidies may allow sandwiched parents to substitute work for caregiving. They may also adjust their time and monetary investments in their children as a result, and the model will allow me to quantify the overall impact of these subsidies as well as the margins along which families adjust.

6 Conclusion

In this project, I aim to investigate the extent to which long-term care risk impacts the economic mobility of children. To do so, I first provide evidence of the link between long-

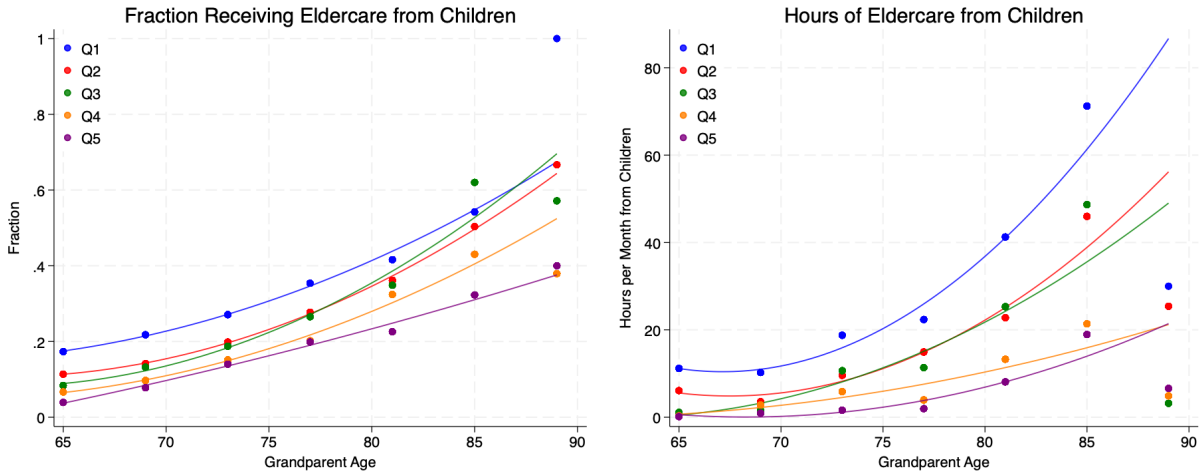
[3]For example, many elderly people who use Medicaid also qualify for SNAP, the Supplemental Nutrition Assistance Program.

term care and intergenerational mobility. I then construct a structural model in which parents and grandparents interact with limited commitment to make informal care choices and parents make investments in children.

I will use the model to quantify the impact of both eldercare needs and eldercare risk on intergenerational mobility. I will also evaluate the impact of changes to Medicaid's asset limits and subsidies for informal caregivers.

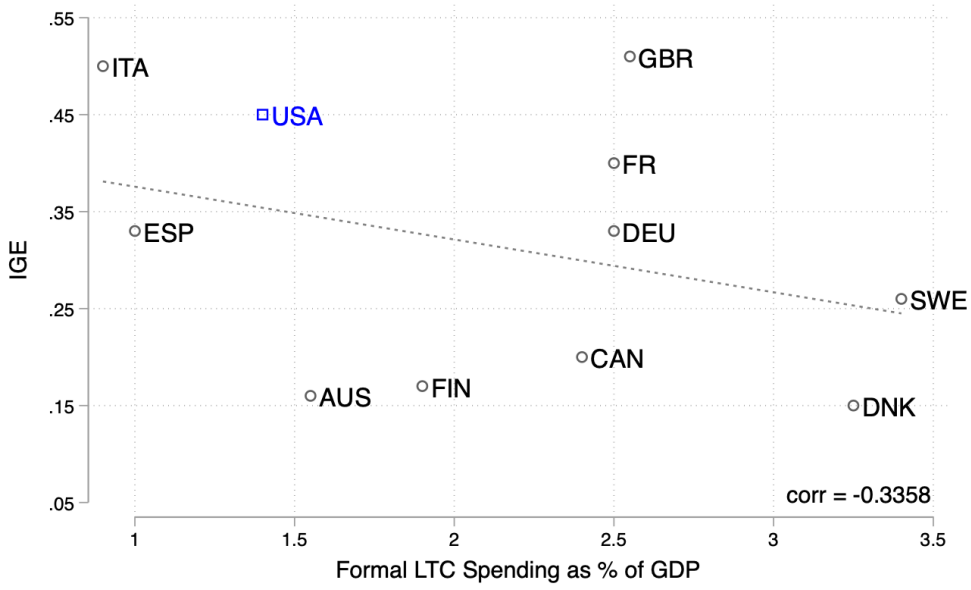
7 Figures

Figure 1: Informal Eldercare from Adult Children by Wealth Quintile



Note: The left panel includes all includes all grandparents over the age of 64 in the Health and Retirement Study, 1992-2018. The right panel includes all grandparents over the age of 64 who receive care from a child. Asset quintiles are computed using all assets (housing and non-housing) at age 65.

Figure 2: Intergenerational Earnings Elasticity and Formal Long-Term Care Spending



Sources: IGE: Corak (2006), LTC: Gruber, McGarry, and Hanzel (2023)

Figure 3: Pr(work limiting health problem within 10 years) and parent education (HRS)

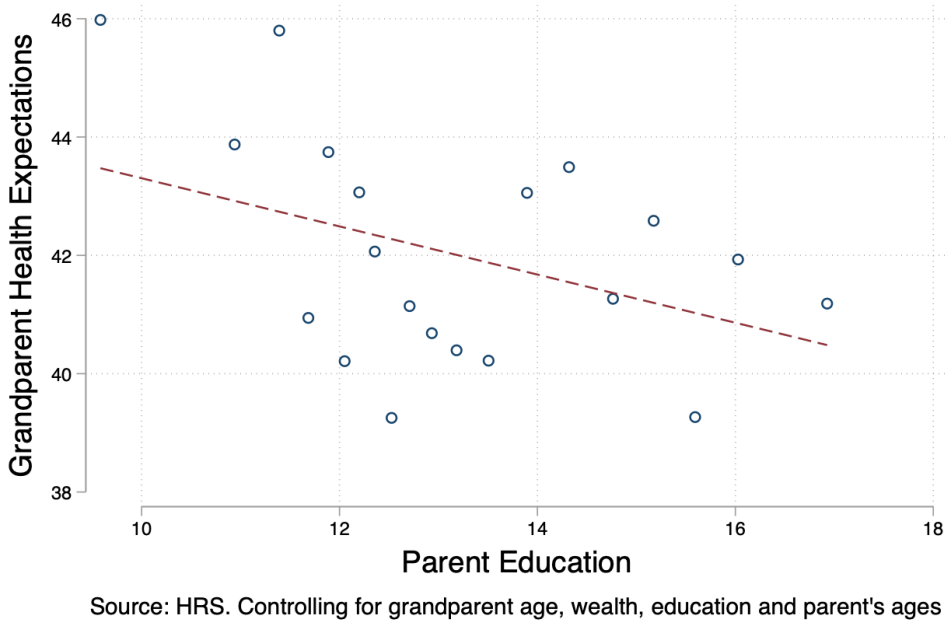


Figure 4: Model Timeline

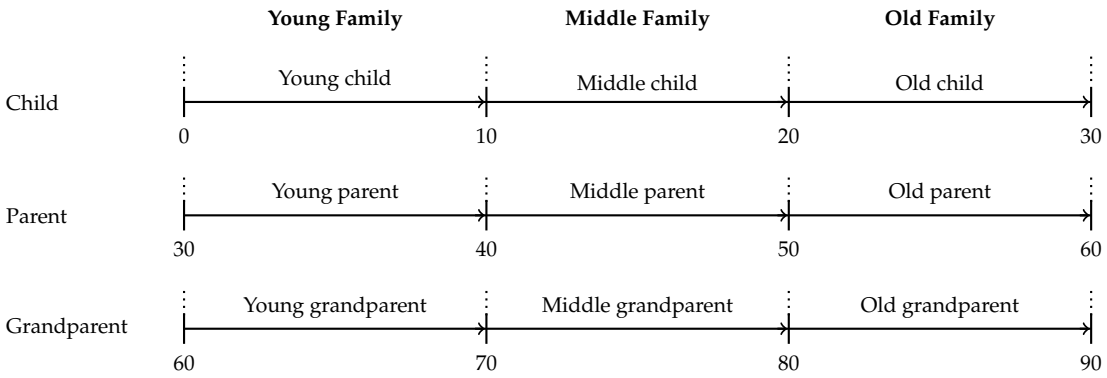
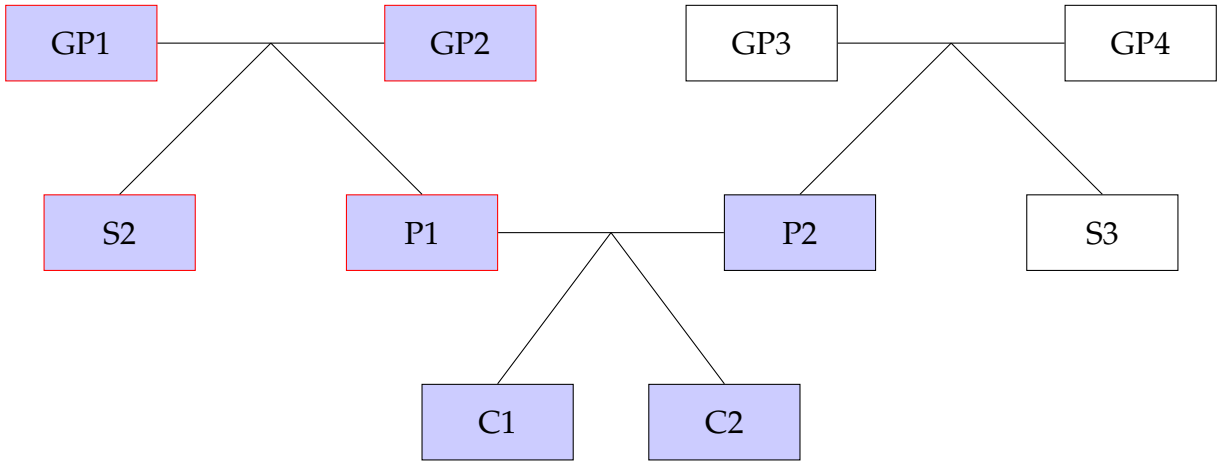


Table 1: State and Choice Variables

Type	Variable	Elements
State	z_y	$\{\theta_y, a_y^p, a_y^g, s_y^p, s_y^k, \epsilon_y\}$
State	z_m	$\{\theta_m, a_m^p, a_m^g, s_m^p, s_m^k, \epsilon_m, H_m^g\}$
State	z_o	$\{\theta_o, a_o^p, a_o^g, s_o^p, s_o^k, \epsilon_o, H_o^g\}$
Choice	j_y	$\{c_y^k, c_y^p, c_y^g, a_m^p, a_m^g, L_y^p, \ell_y^p, x_y, hm_y^k, hp_y^k, hg_y^k, \ell_y^g, \mu_y^p, \mu_y^g\}$
Choice	j_m	$\{c_m^k, c_m^p, c_m^g, a_o^p, a_o^g, L_m^p, \ell_m^p, x_m, hm_m^k, hp_m^k, hg_m^k, hm_m^g, hp_m^g, \ell_m^g, \mu_m^p, \mu_m^g\}$
Choice	j_o	$\{c_o^k, c_o^p, c_o^g, a_y^p, a_y^g, L_o^p, \ell_o^p, X_o, hm_o^g, hp_o^g, \ell_o^g, \mu_o^p, \mu_o^g\}$

Figure 5: Family Tree: Data



Note: Red boxes indicate data available in the HRS. Blue shading indicates data available in the PSID.

Figure 6: Family Tree: Model

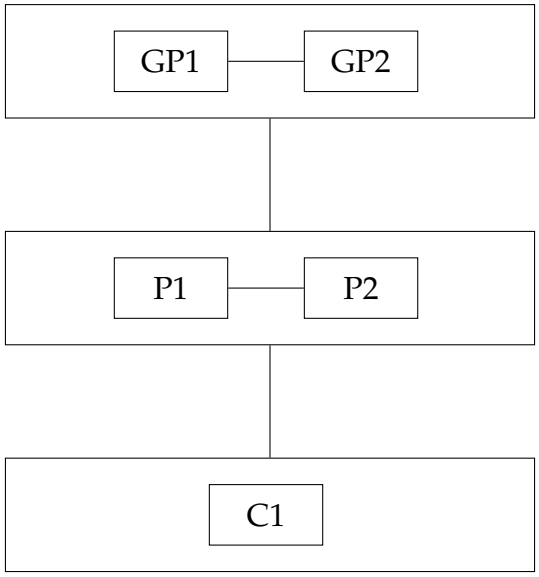


Table 2: PSID Children Summary Statistics

	Mean	Std. Dev.
Age	7.20	3.50
Mean investment	1,166.15	2,287.27
Mean letter-word test score	33.64	14.79
Fraction attend college	0.33	0.47
Observations	6,185	

Table 3: PSID Parents Summary Statistics

	Mothers		**Fathers**	
	Mean	Std. Dev.	Mean	Std. Dev.
Age	35.99	8.36	38.45	8.75
Years of education	13.63	2.39	13.46	2.60
Fraction married	0.69	0.46	0.96	0.20
Fraction working full-time	0.38	0.49	0.82	0.38
Mean family income	42,412	44,534	51,530	48,555
Mean family Assets	312,859	1,116,205	418,322	1,313,276
Observations	4,401		3,147	

Table 4: HRS Grandparent Summary Statistics

	Mean	Std. Dev.
Age	68.45	10.84
Fraction married	0.54	0.50
Grandparents ever provide childcare	0.33	0.47
Grandparents ever need LTC	0.53	0.50
Family income	53,115	186,927
Family assets	363,445	1,026,418
Observations	18,575	

Table 5: Calibrated Parameters

Parameter	Description	Source	Value
β	Discount (annual)	Lockwood (2018)	0.95
R	Interest Rate (annual)	Mommaerts (2024)	1.03
T	Total hours (annual)	16 hrs/day *365 days/year	5824
λ	Weight on child consumption	Lee and Seshadri (2019)	0.3
ρ_s	Persistence of ability	Lee and Seshadri (2019)	0.23
p^k	Price of childcare (hourly)	Poyatzis and Livingston (2024)	10.0
p^g	Price of eldercare (hourly)	OPM (2026)	20.0
P_X	College cost (four years)	Hanson (2026)	90000.0
ω_1	Weight on time (young)	Lee and Seshadri (2019)	0.82
ω_2	Weight on time (middle)	Lee and Seshadri (2019)	0.68
ρ_ω	Substitutability of skill inputs	See text	0.63
ω_p	Return to parental time	See text	0.33
ω_g	Return to grandparent time	See text	0.33
ω_m	Return to market time	See text	0.33
H_y^k	Hours childcare - young (annual)	See text	2400
H_m^k	Hours childcare - middle (annual)	See text	1040

Table 6: Externally Estimated Parameters

Parameter	Description	Value
Child Skills		
μ_d	Mean initial child skill	0.6
σd	Std. dev initial child skill	0.17
ω_X	Return to college	0.48
Labor Market Returns		
α_{sp}	Return to parental skill	0.57
α_L	Return to parental experience	0.16
σ_ϵ	Std. deviation of wage shocks	25,749.93
Grandparents		
I_t^g	Grandparent income (annual)	[59000, 46000, 33000]
P_m^{Hg}	Middle-age LTC risk	[0.91, 0.03, 0.06]
P_o^{Hg}	Old-age LTC risk	[0.79, 0.03, 0.16]

Figure 7: Care Hours by Care Source

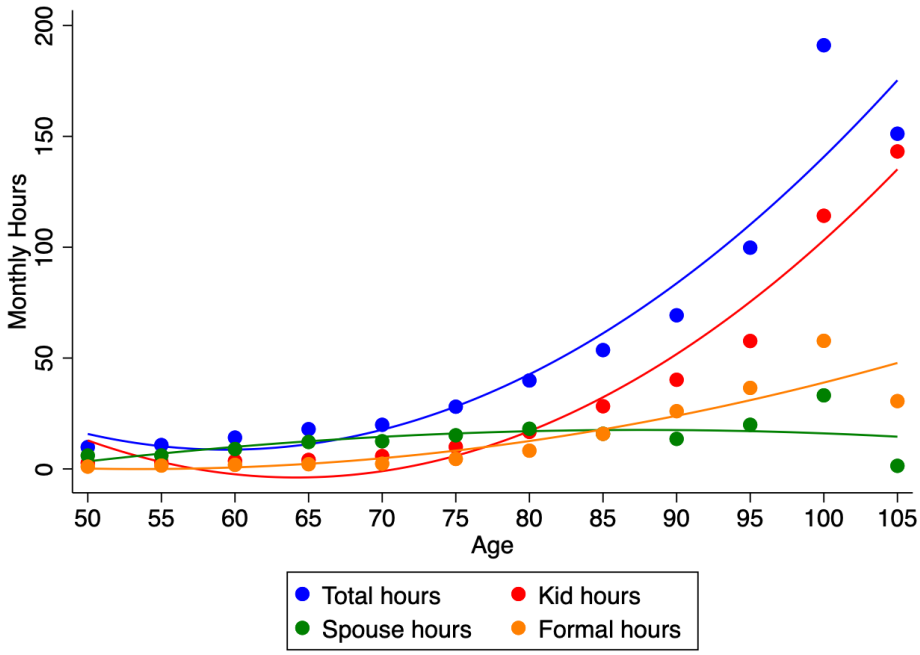


Table 7: Calibration of Parameters to Estimated

Parameter	**Description**	**Value**
γ_c	Consumption curvature	2.2
γ_l	Leisure curvature	2.2
α_ℓ	Leisure weight	0.6
z_M	Medicaid dis-utility (10^{-7})	-4.5
z_F	Formal care dis-utility (10^{-7})	-2.0
c_f	Consumption floor	0.0
η	Altruism	0.16

Table 8: Targeted Moments

Moment	Data	Model
Medicaid rate	0.14	0.02
Full-time work	0.69	0.59
Informal eldercare rates		
Q1	0.12	0.46
Q2	0.12	0.34
Q3	0.09	0.24
Q4	0.05	0.15

Figure 8: Savings over Time

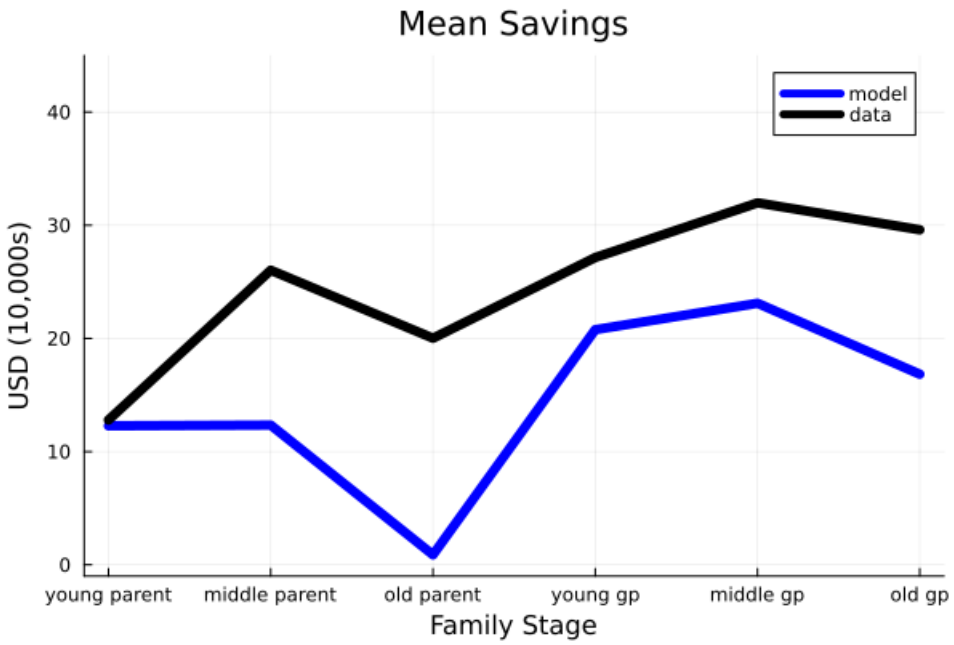


Figure 9: Care by Wealth

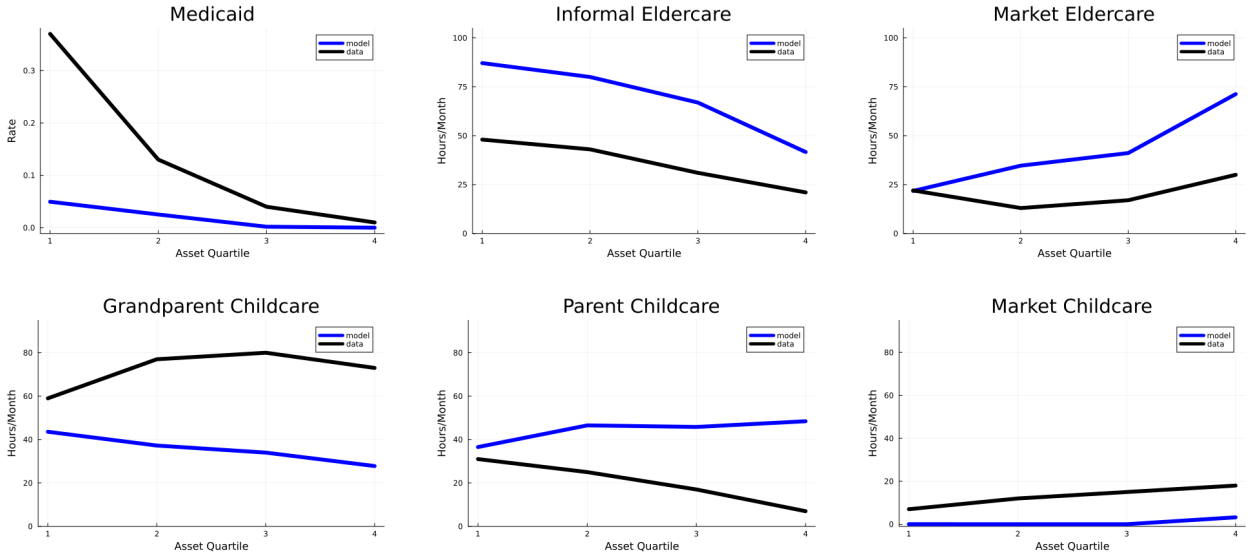
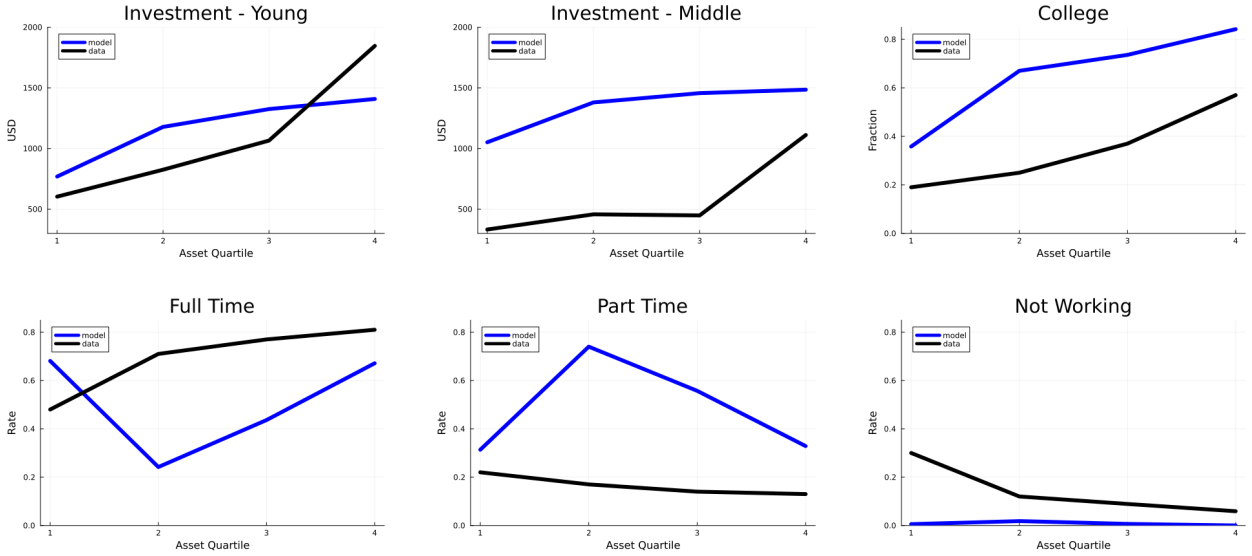


Figure 10: Investment and Labor Supply by Wealth



References

Anstreicher, Garrett, "Spatial Influences in Upward Mobility," *Journal of Political Economy*, 2024.

_ and Joanna Venator, "To Grandmother's House We Go: Childcare Time Transfers and Female Labor Mobility," *Working Paper*, 2022.

Attanasio, Orazio P., Costas Meghir, and Corina Mommaerts, "Insurance in Extended Family Networks," *National Bureau of Economic Research*, 2019, (21059).

Barczyk, Daniel and Matthias Kredler, "Evaluating Long-Term-Care Policy Options, Taking the Family Seriously," *The Review of Economic Studies*, 2018.

_ , Sean Fahle, and Matthias Kredler, "Save, Spend or Give? A Model of Housing, Family Insurance, and Savings in Old Age," *The Review of Economic Studies*, 2022.

Becker, Gary and Nigel Tomes, "Human Capital and the Rise and Fall of Families," *Journal of Labor Economics*, 1986.

Bernal, Raquel and Michael P. Keane, "Quasi-structural estimation of a model of childcare choices and child cognitive ability production," *Journal of Econometrics*, 2010, *156* (1), 164–189.

Boar, C., "Dynastic precautionary savings," *The Review of Economic Studies*, 2021, p. 2735–2765.

Bogan, Vicki L, "Household Asset Allocation, Offspring Education, and the Sandwich Generation.," *American Economic Review: Papers and Proceedings*, 2015, *105* (5), 611—-615.

Braxton, J. Carter, Nisha Chikhale, Kyle Herkenhoff, and Gordon Phillips, "Intergenerational Mobility and Credit," 2023.

Brown, J R and A Finkelstein, "Why is the market for long-term care insurance so small?," *Journal of Public Economics*, 2007, p. 1967–1991.

Caucutt, Elizabeth M. and Lance Lochner, "Early and Late Human Capital Investments, Borrowing Constraints, and the Family," *The Journal of Political Economy*, 2020.

_ , _ , Joseph Mullins, and Youngmin Park, "Child Skill Production: Accounting for Parental and Market-Based Time and Goods Investments," *Journal of Political Economy*, jan 2026, *134* (1), 150–209.

Chang, Yoosoon, Steven N. Durlauf, Bo Hu, and Joon Y. Park, "The Effects of Parental Income and Family Structure on Intergenerational Mobility: A Trajectories-Based Approach," *National Bureau of Economic Research*, 2024, (34179).

Chari, A. and E. Valli, "The effect of subsidized childcare on the supply of informal care: Evidence from public kindergarten provision in the US," *Journal of Health Economics*, 2021.

Chetty, Raj, Nathaniel Hendren, and Lawrence F Katz, "The effects of exposure to better neighborhoods on children: New evidence from the moving to opportunity experiment," *American Economic Review*, 2016, *106* (4), 855–902.

Corak, Miles, "Do Poor Children Become Poor Adults? Lessons from a Cross-Country Comparison of Generational Earnings Mobility," *IZA Discussion Paper*, 2006.

Cunha, Flavio, James J. Heckman, and Susanne M. Schennach, "Estimating the Technology of Cognitive and Noncognitive Skill Formation," *Econometrica*, 2010, *78* (3), 883–931.

Gruber, Jonathan, Kathleen M. McGarry, and Charles Hanzel, "Long-Term Care around the World," *NBER Working Paper*, 2023.

Hanson, Melanie, "Average Cost of College & Tuition," https://educationdata.org/average-cost-of-college 2026. Last updated February 14, 2026.

Hayashi, Fumio, Joseph G. Altonji, and Laurence J. Kotlikoff, "Risk-Sharing between and within Families," *Econometrica*, 1996, *64* (2), 261–294.

Heckman, James J. and Stefano Mosso, "The Economics of Human Development and Social Mobility," *Annual Review of Economics*, 2014, *6*, 689–733.

Hubbard, R. Glenn, Jonathan Skinner, and Stephen P. Zeldes, "Precautionary Saving and Social Insurance," *Journal of Political Economy*, 1995, *103* (2), 360–399.

Kaplan, Greg, "Moving Back Home: Insurance Against Labor Market Risk.," *Journal of Political Economy*, 2012, pp. 446–512.

Ko, Ami, "An Equilibrium Analysis of the Long-Term Care Insurance Market," *Review of Economic Studies*, 2022, p. 1993–2025.

Lee, Tim and Ananth Seshadri, "On the Intergenerational Transmission of Economic Status," *Journal of Political Economy*, 2019.

Lockwood, Lee M., "Incidental Bequests and the Choice to Self-Insure Late-Life Risks," *American Economic Review*, 2018, *108* (5), 2513–2550.

Milovanska-Farrington, Stefani, "The Effect of Parental and Grandparental Supervision Time Investment on Children's Early-Age Development," *Research in Economics*, sep 2021, *75* (3), 286–304.

Mommaerts, Corina, "Long-Term Care Insurance and the Family," *Journal of Political Economy*, 2024.

OPM, Federal Long Term Care Insurance Program, "Long Term Care Costs," https://www.ltcfeds.gov/long-term-care/costs 2026.

Poyatzis, Georgia and Gretchen Livingston, "NEW DATA: Childcare Costs Remain an Almost Prohibitive Expense," https://blog.dol.gov/2024/11/19/new-data-childcare-costs-remain-an-almost-prohibitive-expense November 2024. U.S. Department of Labor Blog.

Scholz, John Karl, Ananth Seshadri, and Surachai Khitatrakun, "Are Americans Saving 'Optimally' for Retirement?," *Journal of Political Economy*, 2006, *114* (4), 607–643.

Skira, Meghan, "Dynamic Wage and Employment Effects of Elder Patient Care," *International Economic Review*, 2015, pp. 63–93.

Voena, Alessandra, "Yours, Mine, and Ours: Do Divorce Laws Affect the Intertemporal Behavior of Married Couples?," *American Economic Review*, 2015.

Appendix

A Outside Options

I model the outside options of parents and grandparents as a non-cooperative game. I assume that cross-generation time and monetary transfers are not possible, with the exception of a bequest at the end of the grandparent's life. Because it is a non-cooperative game, I assume the following timing structure. The grandparent makes her choices first, anticipating parent's choices, and the parent follows.

A.1 Parents

Parents maximize

$$W_t^p(z_t) = \max_{j_t^p} \frac{c^{p(1-\gamma_c)}}{1-\gamma_c} + \lambda \frac{c^{k(1-\gamma_c)}}{1-\gamma_c} + \alpha_\ell \frac{\ell^{p(1-\gamma_\ell)}}{1-\gamma_\ell} + \beta E W_{t+1}^p(z_{t+1}) \tag{A.1}$$

subject to their own budget constraint

$$c_t^p + c_t^k = w_t L_t + R a_{t-1}^p - a^p - p^k h m_t^k - x_t - p^X X_t \tag{A.2}$$

and time constraint

$$T = h p_t^k + L_t^p + \ell_t^p \tag{A.3}$$

Additionally, childcare hours must be met by the market or parent such that

$$H_t^k = h p_t^k + h m_t^k \tag{A.4}$$

The parent's continuation value when $t = o$ is given by $W_{o+1}^p(z_{o+1}) = W_y^g(z_y)$. The parent's choice variables are $j_t^p = \{c_t^p, c_t^k, h k_t^p, h k_t^m, X_t, a_t^p\}$.

A.2 Grandparents

Grandparents maximize

$$W_t^g(z_t) \max_{j_t^g} \frac{c^{g(1-\gamma_c)}}{1-\gamma_c} + \alpha_\ell \frac{\ell^{g(1-\gamma_\ell)}}{1-\gamma_\ell} + z_m \mathbb{1}_{m_t=1} + \beta E W_{t+1}^g(z_{t+1}) + \eta W_t^p(z_t) \tag{A.5}$$

subject to her own budget constraint, given by:

$$c_t^g = I^g + R a_{t-1}^g - a_t^g - M_t - p^g h m_t^g \tag{A.6}$$

and time constraint, given by:

$$T = \ell_t^g \tag{A.7}$$

Any eldercare hours must be met formall, such that

$$H_t^g = hm_t^g \tag{A.8}$$

The grandparent's continuation value when $t = o$ is given by $W_{o+1}^g(z_{o+1}) = 0$. The grandparent's choice variables are given by $j_t^g = \{c_t^g, a_t^g, hm_t^g\}$.

B Model Solution

To solve the model, I use value function iteration. The young grandparent's value function appears in the old parent's value function and the grandparent is altruistic, thus the parent's value function appears in the grandparent's value function, which makes it an infinite horizon model. Further, because families make decisions jointly, I solve the family problem in each family stage by maximizing the Pareto-weighted value functions of the parent and grandparent. The solution algorithm is given by the following:

1. Solve the non-cooperative problem.
2. Solve the fully cooperative model, with θ as a state variable.
   (a) Guess the value functions.
   (b) Solve the family problem at each stage using the guesses.
   (c) Update the value functions and repeat until the difference between the updated value functions and the value functions from the previous iteration are sufficiently close.
3. Use the solution to the fully cooperative model as the starting guess in the limited commitment model. Update the Pareto weights and asset distribution to ensure both parties remain in the arrangement. The solution proceeds as follows:
   (a) Solve the problem with all of the state variables of the fully cooperative problem and a state variable for the asset split between the parent and the grandparent. For each combination of state variables, solve the problem at each θ. The choice variables include all of the choice variables from the cooperative problem and the asset split tomorrow.
   (b) For a given set of state variables, check the commitment constraints at each θ. There are three possible scenarios (because there are always gains from cooperation, both constraints will never bind at the same time):
      i. Neither constraint binds.
      ii. The parent's binds. In this case, find μ_t^p such that $V_t^p(z_t, \mu_t^p) - V_t^{po}(z_t) = 0$. This gives the solution to the problem:

$$\max_{j_t} (\theta_t + \mu_t^p) V_t^p(z_t) + (1 - \theta_t) V_t^g(z_t)$$

I then update and normalize the Pareto weight to reflect the commitment constraints, such that $\hat{\theta}_t = \frac{\theta_t + \mu_t^p}{1+\mu_t^p}$.

iii. The grandparent's binds. In this case, find μ_t^g such that $V_t^g(z_t, \mu_t^g) - V_t^{go}(z_t) = 0$. This gives the solution to the problem:

$$\max_{j_t} (\theta_t) V_t^p(z_t) + (1 - \theta_t + \mu_t^g) V_t^g(z_t)$$

I then update and normalize the Pareto weight to reflect the commitment constraint, such that $\hat{\theta}_t = \frac{\theta_t}{1+\mu_t^g}$.

iv. Neither binds. In this case, I assign the parent and grandparent the value of their outside option, which assumes no cooperation going forward.

(c) Iterate until convergence of the value functions.

Why does this method of updating the Pareto weight work? Consider the Lagrangian that incorporates the commitment constraints:

$$\mathcal{L} = \max_{j_t} \theta_t V_t^p(z_t) + (1-\theta_t) V_t^g(z_t) + \mu_t^p (V_t^p(z_t) - V_t^{po}(z_t)) + \mu_t^g (V_t^g(z_t) - V_t^{go}(z_t)) \quad \text{(B.1)}$$

By complementary slackness:

$$\mu_t^p = 0$$

or

$$V_t^p(z_t) - V_t^{po}(z_t) \leq 0$$

So, if $V_t^p(z_t) < V_t^{po}(z_t)$ then we must find μ_t^p such that $(\theta_t + \mu_t^p) V_t^p(z_t) = V_t^{po}(z_t)$. Then, we evaluate the objective function at the new optimum j_t^* implied by the updated pareto weight $\hat{\theta}_t$. However since $\mu_t^p (V_t^p(z_t) - V_t^{po}(z_t)) = 0$ after the adjustment, we evaluate the objective at θ_t rather than $\hat{\theta}_t$.