Price Stability through State-Contingent Maturity Management: The Dilution Rate of Government Debt

Joe Anderson*

Macalester College

March 22, 2026

Abstract

This paper introduces the dilution rate of government debt, the ratio of new short-term debt to unmatured long-term debt, to add state-contingent maturity management to the fiscal theory of the price level. Unlike existing linearized long-term debt approaches requiring constant-duration structures, the dilution rate permits dynamic maturity policy. A dilution rule expands conventional determinacy regions and introduces novel dynamics. Surprise maturity lengthening produces inflationary effects resembling a tax cut. When monetary policy aggressively implements conventional open market operations – quantitative tightening with rising rates, easing with falling rates – credible escalatory maturity lengthening suppresses inflation within the government budget constraint analogously to how the Taylor principle suppresses inflation within the Fisher equation. This theory may explain why 2023–2025 U.S. inflation remained well below its post-COVID peak despite continued deficits.

Keywords: Long-term debt, Fiscal theory of the price level, Federal Reserve, U.S. Treasury, Inflation.

JEL Classification: E31, E52, E58, E63, H63.

*Email: jander11@macalester.edu. I thank Eric Leeper, Anton Korinek, Eric Young, Daniel Murphy, Chris Sims, John Cochrane, Peter Stella, Francisco Roldan, Neil White, and all involved in the UVA macro group for thoughtful input and guidance. I also thank the seminar and conference participants at the Atlanta Fed, Richmond Fed, U.S. Treasury, and Grinnell College who provided useful suggestions. Finally, I thank the University of Virginia's Bankard Fund for financial assistance. This paper was previously circulated with the title "The Dilution Rate of Government Debt: Theory, Measurement, and Policy Implications."

1 Introduction

The U.S. Federal Reserve (Fed) often deploys quantitative easing (QE) and interest rate cuts simultaneously during severe economic downturns, unleashing a powerful two-pronged approach to monetary stimulus. Slashing the federal funds rate reduces short-term borrowing costs for households, firms, and the government, while purchasing assets like U.S. Treasuries replaces long-term obligations with Fed-issued liabilities. Symmetrically, the Fed regularly employs quantitative tightening (QT) in conjunction with rate increases, selling long-term assets to soak up reserves. Since the turn of the century, the U.S. Treasury has accompanied expansionary fiscal policy with reductions in issuance duration relative to non-crisis times, issuing a disproportionately large share of short-term debt during the Great Financial Crisis (GFC) and COVID.

Figure 1 displays the average maturity of Treasury-issued and Fed-held debt and the level of Fed-issued liabilities from 2003–2026. Fed-issued liabilities, which have no duration, increased substantially during the GFC and COVID, with steady growth between the two recessions. Outstanding Fed liabilities have fallen to roughly their 2021 level following the post-COVID inflationary episode. Figure 2 displays the average maturity of outstanding government liabilities (including Fed liabilities) from 2003–2026. During both the GFC and COVID, consolidated government liabilities' average maturity fell by around 12 months. These declines occurred as the Treasury shortened its issuance while the Fed expanded its balance sheet.

Does it matter that the Fed and Treasury manage the U.S. maturity structure in a state-contingent way? Despite these fiscal and monetary policy marriages between U.S. balance sheets and conventional policy tools, a tractable, linearizable theory describing the direct impact of debt duration, including short-term–for–long-term asset swaps, on inflation remains absent. So it is difficult to answer questions like 'does QT suppress inflation independently of interest rate policy?,' 'does QE contribute to low inflation during recessions?,' and 'how does active Treasury issuance affect price dynamics?' This paper outlines new economic theory that addresses these questions.

I investigate a government that freely issues one-period and two-period debt within an otherwise standard fiscal theory of the price level (FTPL) framework, initially developed by Leeper (1991), Sims (1994), and Woodford (1995). The paper's main object of interest is the **dilution rate of government debt**, which measures the amount of short-term government debt outstanding relative to the amount of previously-issued, unmatured long-term debt. The name, inspired by concepts discussed in Cochrane's (2023) book, describes how the government dilutes the real value of its unmatured debt when it issues new debt maturing at

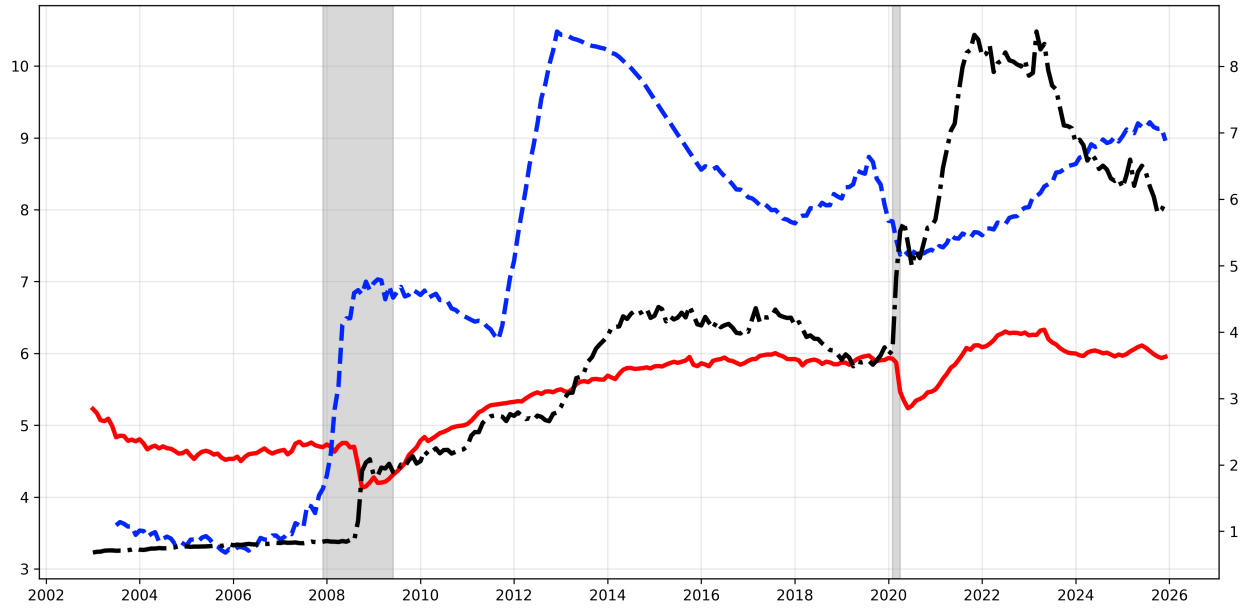


Figure 1: U.S. Debt Average Maturity, Issued and Fed-Held; Fed Liabilities Outstanding

The average maturity of Treasury-issued marketable debt (red solid line) and Fed-held Treasuries (blue dotted line), measured in years (left y-axis). Fed liabilities outstanding (black dash-dotted line), measured in trillions of dollars (right y-axis). Treasury data come from monthly statements of debt. Fed-held debt data come from SOMA. Fed liability data come from the Fed Board's H.4.1 release: Factors Affecting Reserve Balances.

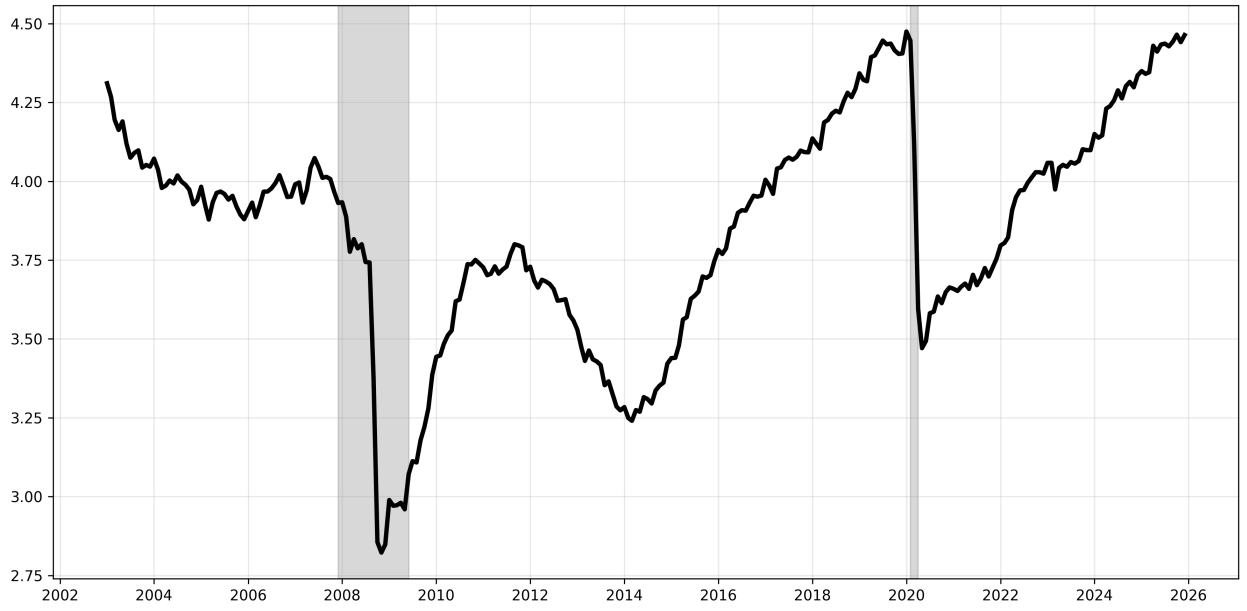


Figure 2: Average Maturity, U.S. Liabilities Outstanding

The average maturity of U.S. outstanding, marketable liabilities. Calculated as the average maturity of [Issued Treasuries]$_t$ + [Fed liabilities outstanding]$_t$ − [Fed-held debt]$_t$ each month from 2003–2026, measured in years. Treasury data come from monthly statements of debt. Fed-held debt data come from SOMA. Fed liability data come from the Fed Board's H.4.1 release: Factors Affecting Reserve Balances.

the same date – similar to a publicly traded company diluting its current market capitalization by issuing additional stock. In addition to a fiscal tax rule and a monetary interest rate rule, the government manages the dilution rate according to an ad-hoc monetary-fiscal composite rule.

The paper's exercise has four major consequences. First, when dilution and taxes are inversely related (i.e., active Treasury issuance), FTPL's 'passive monetary/active fiscal' region extends into the traditional 'passive monetary/passive fiscal' region of the parameter space.[1] When maturing debt rises, fiscal policy raises taxes slightly while lengthening the maturity structure, both of which reduce the following period's maturing debt. Such a policy response neutralizes passive, accommodative fiscal policy and converts it to active fiscal policy as it reduces the variable to which its tax rule reacts. Equilibria in the otherwise passive monetary/passive fiscal region become unique and bounded as a result.

Second, inflationary effects from a surprise maturity lengthening (shortening), regardless of the region of the parameter space, mirror those from a tax cut (hike). When conventional state-contingent open market operations – sales with rate hikes, purchases with rate cuts – are moderate, a surprise tax cut or maturity lengthening increases inflation when fiscal policy is active and has no effect when fiscal policy is passive.[2]

Third, when monetary policy aggressively lengthens (shortens) outstanding debt in conjunction with rate hikes (cuts) – QT, QE – a surprise maturity lengthening (or tax cut) **decreases** inflation under active fiscal policy. Reduced government surpluses would normally result in higher inflation to finance maturing debt, but any increase in inflation triggers an immediate and aggressive maturity lengthening as the central bank vigorously sells long-term debt on open markets. The resulting (contemporaneous and expected future) lengthening is so powerful that it more than compensates for lower primary surpluses, satisfying the government budget constraint without requiring inflation to rise. Expectations about escalatory future maturity lengthening suppress contemporaneous inflationary effects in the GBC in much the same way as the Taylor principle reduces inflation contemporaneously in the Fisher equation when fiscal policy is passive. Under aggressive QT/QE policies, debt maturity replaces inflation as FTPL's primary equilibrating mechanism when fiscal policy is active.

Fourth, a central bank, through the joint setting of interest rates and open market operations, can simultaneously shock its policy rate and the dilution rate to unilaterally coordinate monetary and fiscal policy. For instance, even when fiscal policy is active, a central bank that unexpectedly hikes its policy rate neutralizes resulting inflationary pressures through a contemporaneous negative dilution rate shock (a surprise open-market sale) when

[1] See Leeper's (1991) work for definitions of 'active' and 'passive' monetary and fiscal policy in this context.
[2] Inflationary tax cuts under active fiscal policy is a classic FTPL result.

it commits to aggressive future dilution policy (QT, QE). Regardless of which policy is active, inflation falls upon impact and remains below its steady state level.[3] I construct such a composite shock and discuss its regime-invariant deflationary consequences.

The paper's results point to a politically less expensive alternative to government finance than explicit taxation and inflation: maturity management. Its results also suggest an explanation for why post-COVID U.S. inflation has remained well below its 2022 peak – a year in which the Fed raised rates while implementing QT – despite America's fiscal addiction to deficit financing. Lengthening the government's outstanding maturity structure (and credibly threatening to do so going forward) aided inflation in equating liabilities' market value to the expected present value of current and future U.S. primary surpluses. Finally, using maturity lengthening as a tool to combat inflation raises questions about central bank balance sheets. To maintain policy credibility, the paper's results suggest central banks carry arbitrarily large amounts of long-term debt on their balance sheets and/or directly issue long-term government debt.

Related literature. Wallace (1981) introduces and discusses a Modigliani-Miller-type irrelevance result for outstanding government liabilities: if an equilibrium exists for one path of government portfolios, then that equilibrium also exists for many other paths, leaving the aggregate price level and household consumption unchanged. An unchanged path of taxes net of transfers is necessary for this result. Rocheteau, Wright, and Xiao (2018) break Wallace's (1981) result in a model where money serves an explicit liquidity role, finding that an open market purchase moderately raises the price level. Like Rocheteau, Wright, and Xiao (2018), I define an open market operation as a reserves-for-bonds transaction at market prices. Unlike Rocheteau, Wright, and Xiao (2018), I consider a more traditional framework where bonds and reserves are entirely liquid.[4] I relax Wallace's (1981) fixed-tax assumption by imposing state-contingent fiscal rules as in the FTPL, so that open market operations and maturity issuance decisions affect the economy by shifting how much debt matures in the near term versus further in the future.

A large literature characterizes optimal maturity structures in nonlinear(ized) economies, where leveraging second-order terms in short-term and long-term debt prices is essential. Managing the government's structure carefully completes financial markets or, at least,

[3] When monetary policy is active, this is the standard New Keynesian result. When fiscal policy is active, the on-impact effect comes from Sims's (2011) and Cochrane's (2018) 'stepping on a rake' effect. Dilution policy keeps inflation from offsetting in the medium-to-long run.

[4] As a result, a short-term debt purchase (or sale) is inert.

provides partial insurance against aggregate shocks.[5] Unfortunately, when linearizing around a steady state with short-term and long-term debt, debt objects are perfect substitutes to a first-order approximation absent additional assumptions on debt's no-arbitrage condition (e.g., risk premia).

Woodford (2001) develops an analytically convenient approach by equating a geometrically-declining zero-coupon maturity structure to a perpetuity with geometrically-decaying coupons. Woodford's (2001) method is attractive because it allows the modeler to condense an infinite number of debt objects into a single state variable, allowing for straightforward linearization. Unfortunately, the approach comes with a drawback – the maturity structure's duration must remain constant regardless of the economy's state.[6] Unless the modeler adds additional state variables, the government is prohibited from dynamically responding to economic shocks through changes to the maturity structure's duration. In order to properly (and computationally cheaply) understand the effects of QE, QT, and Treasury issuance behavior in a linearized environment, we need a tractable theory in which the government may freely and state-contingently alter its debt's duration.

Cochrane (2001) and Leeper and Leith (2017) develop a general theory of debt management in a constant endowment setting, allowing for an arbitrarily large number of government debt maturities. They characterize the equilibrium price level as a weighted sum of current and expected future discounted primary surpluses but with a weight structure difficult to penetrate intuitively. Cochrane (2023) writes, "These formulas likely hide additional interesting insights and special cases."

This paper investigates one such case. The two-maturity asset structure dramatically simplifies Cochrane's (2001) and Leeper and Leith's (2017) general formulas while simultaneously bypassing the approximate perfect substitutes issue and overcoming Woodford's (2001) duration restriction. The dilution rate, and the rule governing it, constitutes a state-contingent maturity management policy that survives linearization.

Roadmap. Section 2 introduces the dilution rate of government debt and discusses its role within a government budget constraint. Section 3 implements a joint monetary–fiscal dilution policy rule within an otherwise standard FTPL model and examines equilibrium determinacy and dynamics. Section 4 constructs a composite rate-and-dilution shock and shows its deflationary effects across policy regimes before Section 5 concludes.

[5] Work by Angeletos (2002), Buera and Nicolini (2004) and Faraglia, Marcet, and Scott (2010) use maturities to effectively complete markets. Papers by Lustig, Sleet, and Yeltekin (2008), Debortoli, Nunes, and Yared (2017), and Faraglia et al. (2019) use maturity management to provide insurance in incomplete market environments.

[6] Leeper and Zhou (2021) vary such a portfolio's rate of decay to compare welfare outcomes from policy across debt durations.

2 The Dilution Rate of Government Debt

2.1 The Nominal Dilution Rate

I consider a special case of the government's equilibrium flow government budget constraint (GBC) from papers by Cochrane (2001) and Leeper and Leith (2017) implied by the household budget constraint (HHBC), goods market clearing $C_t = Y_t$, and household optimization. These GBCs put no restrictions on the number of available maturities in which the government may borrow (K) or the maximum available maturity (J). Here I assume the special case where the government may freely borrow in $K = 2$ debt instruments and that its maximum available maturity is $J = 2$ periods. The GBC in real (deflated) terms is

$$\frac{B_{t-1}^{(t)}}{P_t} + \frac{Q_t^{(t+1)} B_{t-1}^{(t+1)}}{P_t} = \frac{Q_t^{(t+1)} B_t^{(t+1)}}{P_t} + \frac{Q_t^{(t+2)} B_t^{(t+2)}}{P_t} + \tau_t \tag{1}$$

where $B_t^{(t+j)}$ is outstanding j-period nominal debt with corresponding price $Q_t^{(t+j)}$. P_t is the economy's aggregate price level and τ_t is a real-valued lump-sum tax net of transfers.

Household optimization over $B_t^{(t+j)}$ determines the price of debt maturing at $t + j$ as

$$Q_t^{(t+j)} = \beta^j \mathbb{E}_t \frac{u'(C_{t+j})}{u'(C_t)} \frac{P_t}{P_{t+j}} = \mathbb{E}_t \frac{1}{r_{t,t+j}} \frac{P_t}{P_{t+j}} \tag{2}$$

where $\beta \in (0, 1)$ and where $r_{t,t+j} = \beta^{-j} u'(C_t) / u'(C_{t+j})$ is defined to be the j-period-ahead product of one-period real returns. Equation (2) reveals a no-arbitrage condition between debt objects $Q_t^{(t+j)} = Q_t^{(t+1)} \mathbb{E}_t Q_{t+1}^{(t+j)} + \text{Cov}_t \left(\frac{1}{r_{t,t+1}} \frac{P_t}{P_{t+1}}, Q_{t+1}^{(t+j)} \right)$. I assume the covariance term is zero, so that the expectations hypothesis holds.[7]

Forward-iterating the flow GBC (1) on the market value of debt reveals the government's present value condition

$$\underbrace{\frac{B_{t-1}^{(t)}}{P_t} + \mathbb{E}_t \frac{B_{t-1}^{(t+1)}}{r_{t,t+1} P_{t+1}}}_{MV(\text{Debt})/P_t} = \underbrace{\mathbb{E}_t \sum_{i=0}^{\infty} \frac{\tau_{t+i}}{r_{t,t+i}}}_{\mathbb{E}[PV(\text{Prim. Spls.})]} + \lim_{i \to \infty} \left(\mathbb{E}_t \frac{B_{t-1+i}^{(t+i)}}{r_{t,t+i} P_{t+i}} + \mathbb{E}_t \frac{B_{t-1+i}^{(t+1+i)}}{r_{t,t+1+i} P_{t+1+i}} \right) \tag{3}$$

where the limit term equals zero via the representative household's transversality condition.

The government's present value condition (3) has a familiar interpretation: the real market value of unmatured and maturing government debt must equal the expected discounted sum of current and future government primary surpluses.

The forward iteration outlined thus far is standard in models that examine fiscal policy's

[7] A zero covariance will follow from more primitive modeling assumptions later in the paper.

role in breaking Ricardian equivalence. Such forward-iteration is sufficient to express P_t alone as a function of current and expected future allocations when the debt takes the form of perpetuities with geometrically declining coupons, introduced in Woodford's (2001) work.[8]

The government's present value condition (3) features an expectation including P_{t+1} within the price of unmatured long-term debt. Current and future debt variables $\left\{\left\{B_t^{(t+j)}, B_{t+1}^{(t+1+j)}, \cdots\right\}_j\right\}$ load into expectations about future price levels recursively; P_{t+1}'s determination includes expectations about P_{t+2} and so on. I express the government's present value condition so that P_t is exclusively a function of current and expected future allocations. When expectations about future debt policy are uncorrelated with those about primary surpluses, the present value condition (3) becomes

$$\underbrace{\frac{B_{t-1}^{(t)}}{P_t}}_{\text{(Mat. Debt)}/P_t} = \underbrace{\tau_t + \mathbb{E}_t \sum_{i=1}^{\infty} \frac{\tau_{t+i}}{r_{t,t+i}} \left(1 + \sum_{h=1}^{i} (-1)^h \Psi_{t,t+h-1}^{-1}\right)}_{\mathbb{E}[PV(\text{Diluted Primary Surpluses})]} \tag{4}$$

where $\Psi_{t,t} = B_t^{(t+1)}/B_{t-1}^{(t+1)}$ is the economy's nominal dilution rate of government debt, following the language from Cochrane's (2023) textbook, and where $\Psi_{t,t+i} = \left(B_t^{(t+1)} \cdots B_{t+i}^{(t+i+1)}\right) / \left(B_{t-1}^{(t+1)} \cdots B_{t+i-1}^{(t+i+1)}\right)$ is the i-period-ahead product of dilution rates (a product of $i+1$ individual dilution rates).[9]

The dilution rate relates the time t stock of outstanding short-term debt to the time $t-1$ stock of outstanding long-term debt. It may be easier to think of the dilution rate as relating short-term to long-term debt outstanding, holding the date of maturity constant.

The rewritten present value condition (4) expresses P_t as a function exclusively of current and expected future allocations. Writing the condition this way also transforms the government's time t debt supply decision from $\left\{B_t^{(t+1)}, B_t^{(t+2)}\right\}$ to $\Psi_{t,t}$. However, the algebraically relevant term in this rewritten condition is not the dilution rate, but the inverse dilution rate $\Psi_{t,t}^{-1}$. The rewritten present value condition (4) collapses to the standard case when a government has no outstanding long-term debt $B_{t-1}^{(t+1)} = 0$. Additionally, it must be the case that the government is expected to issue a non-zero amount of short-term debt each period, or else $\Psi_{t,t+i}^{-1}$ becomes undefined for some i. Interpreting the inverse dilution rate directly is challenging, so I recommend thinking in terms of the (non-inverted) dilution rate and flipping the ratio when examining the algebra.

The government dilutes the real value of its unmatured long-term debt when it issues new short-term debt maturing at the same date, similar to a publicly traded company diluting its

[8] This set of structures includes the 1-period-only case.

[9] Covariance terms make the rewritten present value condition (4) less straightforward while adding little expositional value. I relegate the general formula to its derivation in Appendix B.

current market capitalization by issuing additional stock, reducing price per share. To build intuition for why the maturity structure matters – and to motivate the QT analysis that follows – consider a surprise debt dilution in the spirit of Cochrane's (2023) book. Imagine the government enters time t with previously-issued long-term debt outstanding $B_{t-1}^{(t+1)} > 0$. The outstanding debt's market value is $Q_t^{(t+1)} B_{t-1}^{(t+1)}$, which includes expectations over P_{t+1}. Absent other changes to current or expected policy, a time t budget-neutral increase in short-term debt $B_t^{(t+1)}$ and decrease in long-term debt $B_t^{(t+2)}$ (also known as an open market purchase) increases time $t+1$'s maturing debt, which raises $\mathbb{E}_t P_{t+1}$ according to the rewritten present value condition (4).[10] Household demand for debt maturing at $t+1$ falls with its real (deflated) payout, so that $Q_t^{(t+1)}$ falls. Short-term issuance dilutes long-term debt through $Q_t^{(t+1)}$ which lessens government indebtedness at time t. P_t falls as a result according to the present value condition (3). The mechanism here reflects a trade-off in the timing of inflation: surprise dilution reduces current prices at the cost of higher future prices.[11]

In a steady state with $y_t = y$, $B_t^{(t+j)} = B^{(j)}$ and $\tau_t = \tau$, the rewritten present value condition (4) becomes

$$\frac{B^{(1)}}{P} = \left(\frac{1}{1-\beta}\right)\left(\frac{1}{1+\beta\Psi^{-1}}\right)\tau \tag{5}$$

which is equivalent to a more familiar-looking $\frac{B^{(1)}}{P} + \beta\frac{B^{(2)}}{P} = (1-\beta)^{-1}\tau$ given the definition of Ψ. The duration of the government's steady-state debt portfolio is $D = (\Psi + 2\beta)(\Psi + \beta)$.

The steady state present value condition (5) reveals a long-run restriction on the dilution rate that must hold to ensure equilibrium: $\beta|\Psi^{-1}| < 1$. Or, equivalently, $|B^{(1)}| > \beta|B^{(2)}|$. There must exist a sufficiently large amount of short-term debt (assets) to ensure equilibrium determinacy. In the U.S., most outstanding liabilities (Treasuries + reserves) mature within two years.

2.2 America's Nominal Dilution Rate

Figure 3 displays America's outstanding nominal dilution rate from 1947Q1–2022Q4 where short-term debt outstanding is defined as reserves outstanding plus privately-held Treasuries maturing within one year. The U.S. experienced sharp increases in debt dilution during the early-1950s, early-1970s and during the dot-com, Great Financial Crisis (GFC), and COVID recessions. Dilution fell fairly consistently from the mid-1970s until the late-1980s, where it remained at around 0.85 until the late-1990s. The sample average dilution rate is $\hat{\Psi} = 1.11$ and the sample average of the inverse dilution rate $\hat{\Psi^{-1}} = 0.98$.

[10] Holding other policy fixed constitutes active fiscal policy.
[11] Cochrane (2023) formalizes this thought experiment in his online appendix.

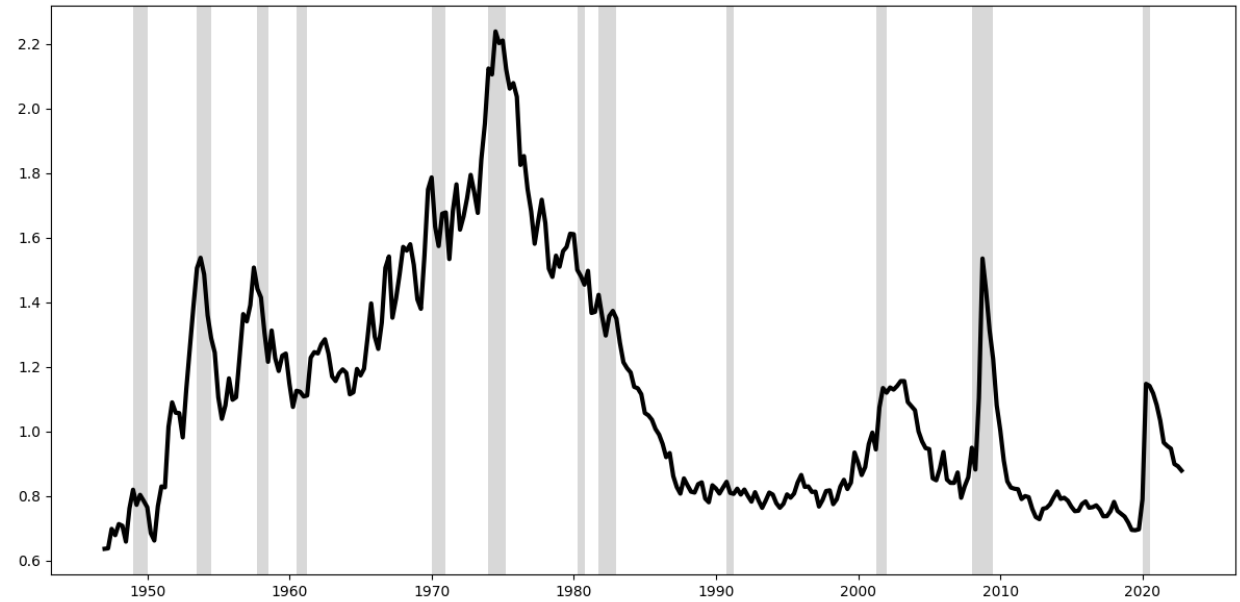


Figure 3: U.S. Nominal Dilution Rate, 1947–2022

The United States' nominal dilution rate from 1947Q1–2022Q4. Short-term debt is the par value sum: [outstanding marketable Treasuries maturing within 1 year]$_t$ + [outstanding Fed liabilities]$_t$ − [Fed-held marketable Treasuries maturing within 1 year]$_t$. Long-term debt is the par value sum: [outstanding marketable Treasuries maturing later than 1 year]$_t$ − [Fed-held marketable Treasuries maturing within 1 year]$_t$. Treasury data come from CRSP, Fed data come from historical Fed balance sheets (1947Q1-2003Q2) and SOMA (2003Q3-2022Q4).

Much of the movement in dilution since the mid-2000s stems from the Fed's increased use of open market operations. Purchasing Treasuries off of open markets for zero-maturity reserves substitutes long-term outstanding debt for ultra-short-term outstanding debt. When the Fed engages in large scale asset purchases, as it did during the GFC and COVID, it increases the numerator of Ψ_t and decreases the denominator of Ψ_{t+1} (absent a response from the Treasury). Conversely, when the U.S. Treasury shortens its issuance, as it did in each of the last three recessions, it increases the numerator of Ψ_t (absent a Fed response).

Ultimately, both fiscal and monetary policy authorities affect movements in the dilution rate. From a modeling perspective, if specifying a dilution rate policy rule, it is reasonable to include reactions to variables that monetary and fiscal policy prioritizes. Additionally, and along the same lines, a shock to such a rule can be interpreted as being fiscally-driven or monetarily-driven. Such an interpretation leads to a re-framing of the nature of monetary policy, adding another theoretical tool to its toolkit. It also re-frames the nature of fiscal policy, adding another theoretical lever it can pull.

2.3 The Real Dilution Rate

The dilution rate may also be written in real terms. I define the real dilution rate of government debt to be $\psi = b_t^{(t+1)}/b_{t-1}^{(t+1)}$ where $b_t^{(t+j)} = B_t^{(t+j)}/P_t$ is the amount of real (deflated) debt outstanding maturing at time $t+j$. The relationship between the real and nominal dilution rates is, therefore, $\Psi_{t,t} = \psi_{t,t}\pi_t$ where $\pi_t = P_t/P_{t-1}$ is the economy's inflation rate.[12][13]

Distinguishing between the real and nominal dilution rates is useful when studying inflation dynamics as opposed to price level determination. For instance, take the government's flow GBC (1) and apply the definitions of real (deflated) debt and the real dilution rate to write

$$\frac{b_{t-1}^{(t)}}{\pi_t} + \frac{Q_t^{(t+1)}\psi_{t,t}^{-1}b_t^{(t+1)}}{\pi_t} = \tau_t + Q_t^{(t+1)}b_t^{(t+1)} + Q_t^{(t+2)}\mathbb{E}_t\psi_{t+1,t+1}^{-1}b_{t+1}^{(t+2)} \tag{6}$$

The real-valued flow GBC (6), written in this way, allows for a richer analysis of monetary and fiscal policy interactions: monetary rate policy sets $Q_t^{(t+1)}$, fiscal tax policy sets τ_t, and a combination of monetary and fiscal maturity management policy sets $\psi_{t,t}^{-1}$ each period.[14]

How does an unexpected shortening of the maturity structure (a dilution shock) affect government debt and inflation paths? How strongly does the maturity structure need to respond to economic variables to achieve a bounded and unique equilibrium? Does dilution policy allow for previously-unidentified policy regimes? Augmenting this version of the flow GBC (6), using the real dilution rate, with assumptions on interest rate, tax, and dilution rate policy unlocks many paths forward to answer such questions.

3 FTPL with Long-Term Debt and a Dilution Rule

In this section, I describe an economy augmented with ad-hoc rules governing monetary (interest rate) policy, fiscal (tax) policy, and jointly-implemented monetary/fiscal maturity (dilution) policy. For notational convenience, I define $a_t = \psi_{t,t}^{-1}$ as the inverse real dilution rate at time t.

[12] This definitional relationship reveals something akin to a Fisher equation in dilution rates, except where inflation is dated at time t.

[13] This transforms equation (5)'s long-run restriction on the dilution rate to $\beta|\psi^{-1}| < \pi$, where ψ^{-1} and π are the steady state inverse real dilution rate and gross inflation, respectively.

[14] I define the price of nominal debt Q_t as the inverse of the gross nominal interest rate: $Q_t = \frac{1}{R_t}$.

3.1 Model Set-Up

I assume constant endowment $Y_t = Y_{t+1} = Y$ so that the model's Fisher relation is

$$Q_t^{(t+1)} = \beta \mathbb{E}_t \frac{1}{\pi_{t+1}} \tag{7}$$

Debt is priced according to

$$Q_t^{(t+1)} = \frac{1}{R_t} \tag{8}$$

where R_t is the economy's nominal interest rate.

Monetary policy follows an interest rate rule as in Taylor's (1993) work

$$R_t = \bar{R} \left(\frac{\pi_t}{\bar{\pi}} \right)^{\alpha} \exp(z_t^R) \tag{9}$$

where $\alpha \geq 0$.

Fiscal policy follows a tax rule as in Leeper's (1991) work

$$\tau_t = \bar{\tau} \left(\frac{b_{t-1}^{(t)}}{\bar{b}} \right)^{\gamma} \exp(z_t^\tau) \tag{10}$$

where $\gamma \geq 0$.

Policy authorities jointly adjust the dilution rate of government debt according to

$$a_t = \bar{a} \left(\frac{\pi_t}{\bar{\pi}} \right)^{\varphi} \left(\frac{b_{t-1}^{(t)}}{\bar{b}} \right)^{\delta} \exp(z_t^a) \tag{11}$$

where $\varphi \geq 0$ regulates the strength to which the central bank engages in open market sales (purchases) when inflation rises (falls), and where $\delta \geq 0$ regulates the strength to which the fiscal authority lengthens (shortens) its issuance when outstanding maturing debt rises (falls). Restricting $\delta \geq 0$ corresponds to observable Treasury issuance patterns: when transfers rise (taxes fall), as in a recession, the Treasury shortens its issuance. Given the tax rule, $\delta \geq 0$ generates this correlation in the model. Restricting $\varphi \geq 0$ corresponds to the Fed's observed balance sheet policy, contingent on inflation realizations.

Government policy is conducted subject to the consolidated flow GBC (6), rewritten here as

$$\frac{b_{t-1}^{(t)}}{\pi_t} + \frac{Q_t^{(t+1)} a_t b_t^{(t+1)}}{\pi_t} = \tau_t + Q_t^{(t+1)} b_t^{(t+1)} + Q_t^{(t+2)} \mathbb{E}_t a_{t+1} b_{t+1}^{(t+2)} \tag{12}$$

Finally, policy disturbances (ε_t^R, ε_t^τ, ε_t^a) are mean-zero, normally distributed, and decay

at constant rates

$$z_t^R = \rho_R z_{t-1}^R + \varepsilon_t^R \tag{13}$$

$$z_t^\tau = \rho_\tau z_{t-1}^\tau + \varepsilon_t^\tau \tag{14}$$

$$z_t^a = \rho_a z_{t-1}^a + \varepsilon_t^a \tag{15}$$

Equations (7)–(10), (12)–(14), and $a_t = a = 0$ yield a traditional, textbook FTPL model. Such a model is discussed in detail in Appendix C.

3.2 Determinacy Regions

I linearize the model described by equations (7)–(15) around a zero-inflation steady state in Appendix D, and I solve the linearized model using the Gensys algorithm developed by Sims (2002). I refer to the model's log-linear approximation as 'the model' for the remainder of the analysis.

3.2.1 Equilibrium Determinacy

The traditional FTPL model described in Appendix C features one backward-looking variable, b_{t-1} (debt maturing at t), and one forward-looking variable, π_t. This paper adds a second forward-looking variable: b_t (debt maturing at $t+1$). A locally unique and bounded equilibrium where π_t, b_{t-1}, and b_t are jointly determined each period must feature two unstable roots – two of the system's three non-ρ eigenvalues must exist outside of the unit circle.

The system's non-ρ eigenvalues satisfy $\lambda_1 = \alpha$ and

$$\{\lambda_2\ ,\ \lambda_3\} = \frac{a\left(1-\beta\delta\right) - 1 \pm \sqrt{\beta a\left(2+2a+\beta a\delta\right)\delta - 4a\left(1-\beta\left(1-a+\beta a\right)\right)\gamma + 1 + 2a + a^2}}{2\beta a} \tag{16}$$

so that real-valued $\lambda_2 > \lambda_3$ for all parameter combinations a, β, α, γ, δ, $\varphi > 0$.

The dilution rate's inflation response parameter φ is notably absent from equation (16)'s eigenvalues. The central bank's dilution policy response parameter φ is not relevant for equilibrium determinacy.

Proposition 1 (*equilibrium determinacy*) *The model is locally unique and bounded if policy parameters α, γ, δ satisfy:*

1. $\alpha > 1$, $|\lambda_2| < 1$, $\lambda_3 < -1$ *(Regime M)*
2. $\alpha < 1$, $\lambda_2 > 1$, $\lambda_3 < -1$ *(Regime F)*
3. $\alpha < 1$, $\lambda_2 < -1$, $\lambda_3 < -1$

4. $\alpha > 1$, $\lambda_2 > 1$, $|\lambda_3| < 1$

5. $\alpha < 1$, $\lambda_2 > 1$, $\lambda_3 > 1$

Proof: The proof can be found in Appendix A.

Determinacy may be achieved in one of five policy regimes. Regime M and Regime F are extensions of Leeper's (1991) 'active monetary/passive fiscal' and 'passive monetary/active fiscal' regimes, respectively. I adopt the 'active/passive' nomenclature going forward, and I distinguish fiscal policy between tax policy (γ) and dilution policy (δ) for determinacy analysis.

Regime M: Active monetary policy ($\alpha > 1$) follows the Taylor principle, adjusting the nominal rate more than 1-for-1 with inflation. Fiscal policy adjusts taxes and shifts debt maturity timing so as to finance otherwise explosive interest expenses, providing the necessary fiscal backing to ensure solvency. The economy's maturing debt stock is determined by a combination of direct redemption through taxes and the acceleration (shortening) or deferral (lengthening) of final debt repayment.

Regime F: Active tax policy (small γ) does not respond strongly to deviations in maturing debt. Passive monetary ($\alpha < 1$) and passive dilution policy (small δ) ensure solvency. Monetary policy holds interest payments down while dilution policy delays debt maturation. When combined, these passive policies ensure the GBC (12) holds.

Other regimes: As will be shown in the next section, only Regimes M and F are relevant for analysis, given reasonable parameter values. It is nonetheless prudent to classify all feasible regimes where unique and bounded equilibria may exist. Three additional regimes arise where determinacy is possible. The third regime features passive monetary and highly-passive tax policy. Dilution policy, on the other hand, is active (large δ), pinning down forward-looking inflation, while tax and interest rate policy combine to ensure the GBC (12) holds. The fourth regime allows for active monetary and active tax policy, where debt dilution alone maintains government solvency. Dilution policy satisfies the GBC (12) by repeatedly switching between issuing short-term debt to buy long-term assets and issuing long-term debt to buy short-term assets. The fifth regime requires a steady state maturity structure skewed extremely long and unrealistically patient households ($\beta < 0.2$), combining to allow for powerful dilutive effects. Small perturbations in this regime result in large deviations in economic variables, but dynamics in this regime are directionally similar to those in Regime F.

3.2.2 Prior Determinacy Analysis

Following Geweke (2010), I conduct a prior determinacy analysis to identify which equilibrium regimes are consistent with realistic parameter values. I sample parameters from

prior distributions and evaluate whether resulting eigenvalues satisfy determinacy conditions, allowing me to rule out theoretically possible but empirically implausible regimes before further analysis.

To begin, I construct a joint distribution of model parameters a, β, α, γ, δ and draw from that distribution $N = 500,000$ times. I am left with a distribution of $n \leq N$ determinate theoretical economies. Treating the model as a transformation from the distribution of model parameters to the distribution of, in this case, eigenvalues allows me to guide the paper's focus. Figure 4 provides a visualization of the five regimes listed in Proposition 1. The first panel labels each regime while the second, third, and fourth panels show various prior determinacy analyses described in Geweke's (2010) book.

I consider three joint distributions of model parameters, each in the form

$$[a, \beta, \alpha, \gamma, \delta] \sim \text{Uniform}\left([\underline{a}, \underline{\beta}, 0, 0, 0], \left[\bar{a}, \bar{\beta}, 2, \bar{\gamma}, \bar{\delta}\right]\right) \tag{17}$$

The prior parameter distributions become progressively more restrictive. The second panel shows determinate equilibria where the prior parameter distribution includes $\underline{a} = 0$, $\underline{\beta} = 0$, $\bar{\beta} = 1$, $\bar{a} = \bar{\gamma} = \bar{\delta} = 10$. The third panel shows determinate equilibria where $\underline{a} = 0$, $\underline{\beta} = 0.7$, $\bar{\beta} = 1$, $\bar{\gamma} = 5$, $\bar{a} = \bar{\delta} = 2$. The final panel shows determinate equilibria where $\underline{a} = \bar{a} = 0.9803$, $\underline{\beta} = \bar{\beta} = 0.9875$, $\bar{\gamma} = 5$, $\bar{\delta} = 2$.[15]

Figure 4 plots a point for each parameter draw resulting in a determinate equilibrium. As non-policy parameters (a, β) and policy parameters (α, γ, δ) become more restricted towards realistic values, the whole distribution of determinate eigenvalue combinations rest in Regime M and Regime F. The prior analysis is clear: equilibria in only Regime M and Regime F are relevant.

3.2.3 Shifting Determinacy Regimes

Calibration: I calibrate the model according to Table 1 for the remainder of the analysis.

FTPL's most compelling result may be Regime F's existence. In Appendix C's traditional FTPL model, Regime F corresponds to the region where $\alpha < 1$ (a relatively weak Taylor rule coefficient) and $\gamma < 1$ (fiscal policy that's relatively unresponsive to maturing debt). When fiscal policy is active, the only way for inflation and maturing debt to be determined is through monetary policy holding rates down to prop debt prices up (keep interest on maturing debt low). Taxes eventually pay down the debt's principal plus interest with the help of monetary policy.

When monetary and tax policy are both passive ($\alpha < 1$, $\gamma > 1$), each works towards

[15] $a = 0.9803$ and $\beta = 0.9875$ are calibrations of non-policy parameters later in the paper.

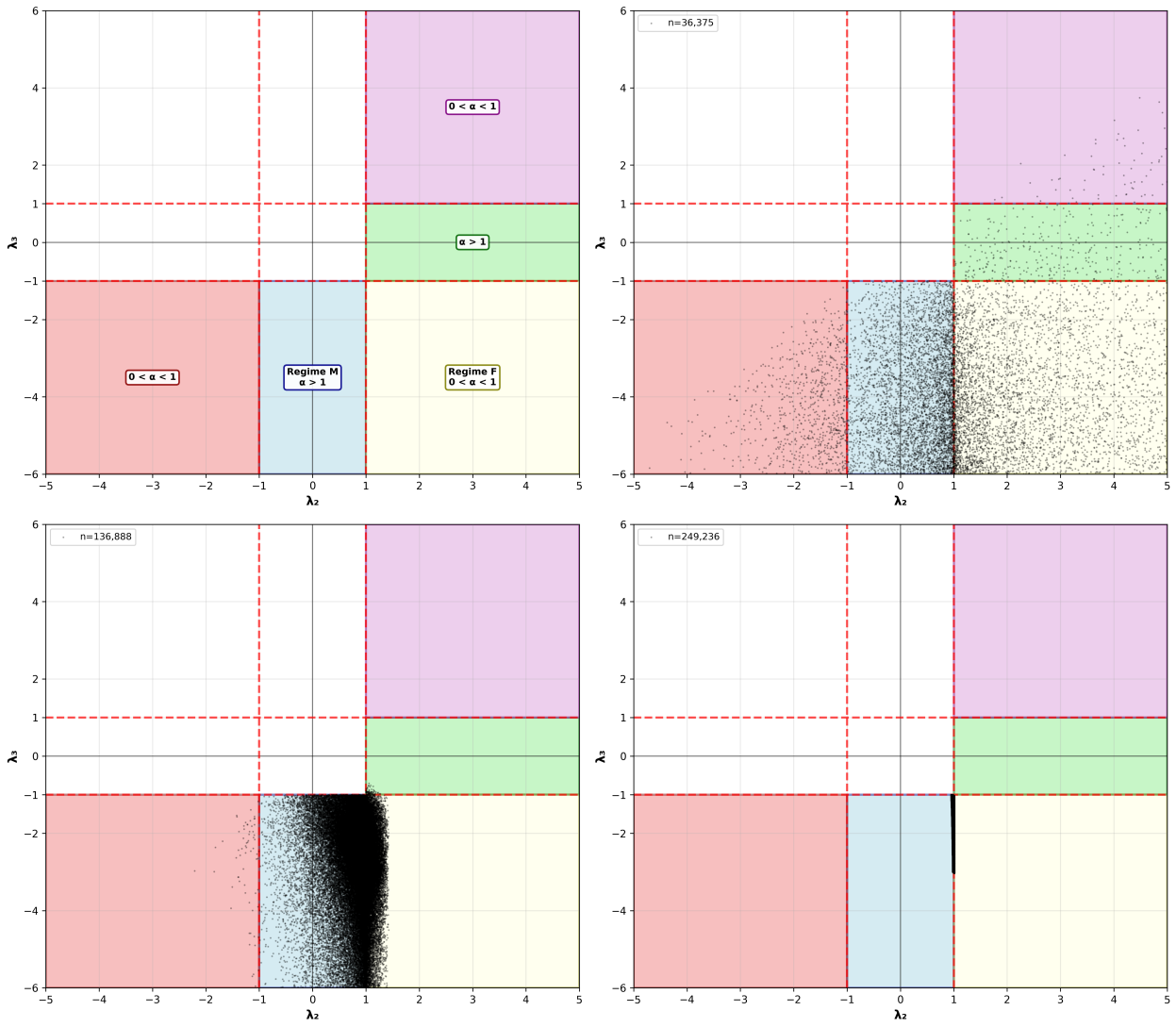


Figure 4: Prior Determinacy Analysis

Determinacy regions in eigenvalue space (λ_2, λ_3) and prior determinacy distributions. Panel (a) shows the five determinacy regions defined by values α, λ_2, and λ_3. Panel (b) displays prior determinacy samples under unrestricted parameter ranges: $a, \delta, \gamma \in [0, 10]$, $\beta \in (0, 1)$. Panel (c) shows samples under restricted parameter ranges: $a, \delta \in [0, 2]$, $\gamma \in [0, 5]$ and $\beta \in [0.7, 1)$. Panel (d) presents samples with calibrated parameters $a = 0.9803$ and $\beta = 0.9875$, while $\delta \in [0, 2]$, $\gamma \in [0, 5]$. The red dashed lines indicate the unit circle boundaries $|\lambda| = 1$. Black dots represent parameter combinations satisfying determinacy.

Parameter	Symbol	Value	Description
Fixed Parameters			
Discount factor	β	0.9875	From Chari, Christiano, and Kehoe (1995)
Steady state inflation	π	1	Zero inflation steady state
Steady state inv. dilution	a	0.9803	Average sample US inverse dilution rate
MP shock persistence	ρ_R	0.16	From Smets and Wouters (2007)
FP shock persistence	ρ_τ	0.94	From Leeper, Plante, and Traum (2010)
Debt shock persistence	ρ_a	0.97	Estimated univariate US inverse dilution ρ
Shocks (Standard Deviations)			
Monetary policy shock	ε_t^R	1.0	One standard deviation (rate hike) shock
Fiscal policy shock	ε_t^τ	1.0	One standard deviation (tax hike) shock
Dilution shock	ε_t^a	-1.0	One standard deviation (shortening) shock

Table 1: Model Calibration

stabilizing the economy's debt position without pinning down forward-looking inflation resulting in multiple equilibria. Adding $\delta > 0$ to the policy mix allows for equilibria in the otherwise 'passive fiscal/passive monetary' regime to be unique and bounded. Figure 5 visualizes how λ_2 and λ_3 vary with δ for certain values of γ. When tax policy would otherwise be considered passive, there exist values of $\delta > 0$ associated with a second explosive root.

Figure 6 displays the way dilution policy affects traditional FTPL determinacy regions. $\delta > 0$ effectively shifts regime F into the region previously-occupied by multiplicity. Higher values of δ validate more combinations of α and γ associated with Regime F, but where these values both traditionally satisfy the criteria for passive policy. For instance, when $\delta = 1$, the parameters $\{\alpha = 0.95, \gamma = 1.25\}$ yield a bounded and unique equilibrium.

Introducing long-term debt allows for a policy tool with which to negate fiscal (tax) passivity.[16] By adjusting the maturity structure, the government may alter which debt obligations are maturing in the following period, shifting the remainder of the debt to be financed later. When the government lengthens its maturity structure in response to rising maturing debt, it systematically pushes a larger portion of that maturing debt burden into the future. Reducing the short-run burden leads to fiscal policy's passive tax rule reacting to a reduced maturing debt obligation, effectively transforming passive fiscal policy into active fiscal policy as the target of its reaction shrinks.

[16]It may also be useful to point out that $\delta < 0$ negates active tax policy, allowing for determinate regimes that would otherwise fall into the 'active/active' region. This may be an idea for future research.

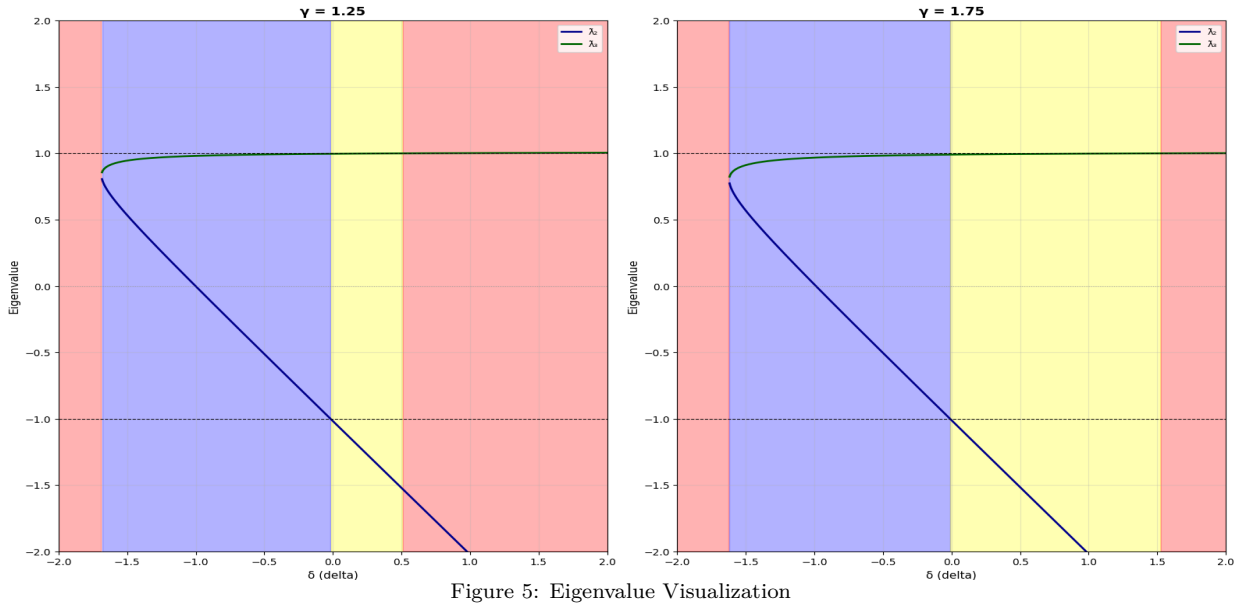


Figure 5: Eigenvalue Visualization

The figure displays λ_2 (green line) and λ_3 (blue line) for various levels of γ, varying δ. When $\delta > 0$, equilibria are determinate even when fiscal and monetary policy would both otherwise be considered passive. Red-shaded regions indicate levels of δ where λ_2 and λ_3 are outside the unit circle. Yellow-shaded regions indicate levels of δ where only λ_3 is outside. Blue shaded regions indicate levels of δ where both λ_2 and λ_3 are inside the unit circle (which cannot be associated with a bounded and unique equilibrium).

3.3 Impulse Response Analysis

How do dilution shocks affect inflation and debt dynamics? How does varying fiscal dilutive responses (active Treasury issuance, δ) and monetary dilutive responses (open market operations, φ) affect the model's dynamics? Why do these effects arise? This section investigates the model's inflation and debt dynamics at various combinations of δ and φ in Regimes M and F.[17]

Until now, this paper has made no distinction between a **determinacy** regime, a combination of policy parameters $(\alpha, \gamma, \delta, \varphi)$ which jointly yield a unique and bounded equilibrium, and a **dynamic** regime, a combination of policy parameters $(\alpha, \gamma, \delta, \varphi)$ which jointly yield qualitatively distinct equilibrium dynamics. While the two coincide in the traditional FTPL model, adding the dilution rule (11) suddenly introduces the need to distinguish between them here.

This section is split into two subsections. First, I explore the dynamic regime within each of Regimes M and F which correspond to FTPL's traditional regimes. Second, I document an additional dynamic regime within each of Regime M and Regime F resulting from aggressive monetary dilution policy, which I denote Regime M^+ and Regime F^+, and explore their properties.

[17]For comparison, I set α and γ in Regimes F's and M's impulse response functions to the same values as in Appendix C's traditional model's impulse response functions.

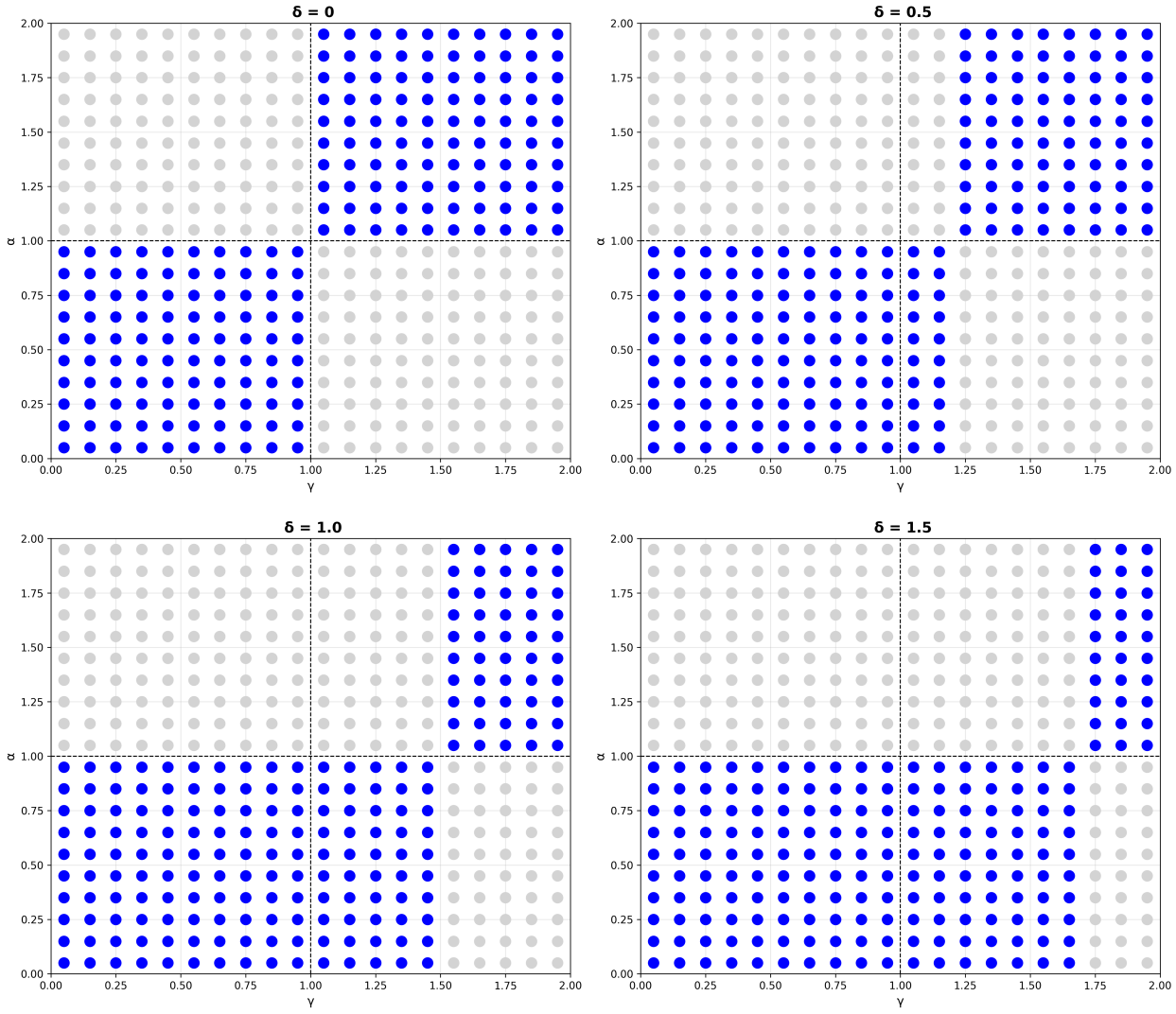


Figure 6: Determinacy Maps Across δ Values

The figure displays determinacy regions in (γ, α) space. The four panels show how increasing dilution policy's sensitivity to outstanding maturing debt affects equilibrium determinacy. The top-left, top-right, bottom-left, and bottom-right panels refer to $\delta \in \{0, 0.5, 1.0, 1.5\}$, respectively. The degree to which the maturity structure shortens when debt falls (taxes fall) extends Regime F and shrinks Regime M. All parameter combinations are searched on a grid with $n = 20$ points per dimension.

The central bank's dilutive response to inflation, while unimportant for determinacy, has major implications for dynamics. This section highlights the paper's main finding: when in Regime F, aggressive, conventional open market operations (e.g., open market sales during inflation, large $\varphi > 0$), moves the economy to Regime F^+ where a surprise maturity lengthening reduces inflation.[18] Inflation and dilution play financing roles in the GBC (12), and, when dilution reacts strongly to inflation, dilution – not inflation – is the main policy lever that ensures government solvency. More specifically, in Regime F^+, expectations about escalatory future maturity lengthening suppress contemporaneous inflationary effects in the GBC in much the same way as the Taylor principle reduces inflation contemporaneously in the Fisher equation in Regime M.

3.3.1 Moderate Maturity Responses and Two Familiar Dynamic Regions

Figures 7 and 8 display inflation and real debt IRFs when responding to positive policy rate shocks ($\varepsilon_t^R \uparrow$), tax shocks ($\varepsilon_t^\tau \uparrow$), and dilution rate shocks ($\varepsilon_t^a \downarrow$) when monetary and fiscal policy adjust the dilution rate proportionally to inflation and maturing debt: $\delta = \varphi = 1$. Figure 7 shows IRFs in Regime M while Figure 8 shows IRFs in Regime F.

This exercise's results are largely consistent with traditional FTPL IRFs: a Regime M surprise policy rate hike reduces inflation and raises short-term debt, a Regime M surprise tax increase has no effect on inflation and reduces short-term debt, and a Regime F tax hike reduces inflation with a small debt decrease. The major difference occurs in Regime F under a monetary policy hike. Like the "stepping on a rake" effect discussed by Sims (2011) and Cochrane (2018), inflation falls on a rate hike's impact. Unmatured debt's market value falls when the nominal interest rate rises, lessening the government's outstanding burden. As in Sims's (2011) and Cochrane's (2018) work, the effect is short-lived as inflation spikes upward and remains high going forward. Monetary policy only shifts inflation from the present to the future, so it is helpless to permanently control inflation without appropriate tax backing.

When the economy experiences a surprise maturity shortening (a decrease in the inverse dilution rate), it largely reacts as if to a tax hike. In Regime M, Ricardian tax policy adjusts to keep inflation constant. One might wonder why a shortening (or dilution) shock would result in less short-term debt on impact (and even further reduced short-term debt afterward). Surprise dilution is overridden by passive fiscal policy. When the government shocks the maturity structure, maturing debt increases which triggers a greater-than-one-for-one increase in taxes. Such a tax response dominates that of the shortening shock, strongly reducing short-term and long-term debt and ensuring inflation remains unchanged.

[18] Reduces inflation on impact without future inflation rising above its steady state level.

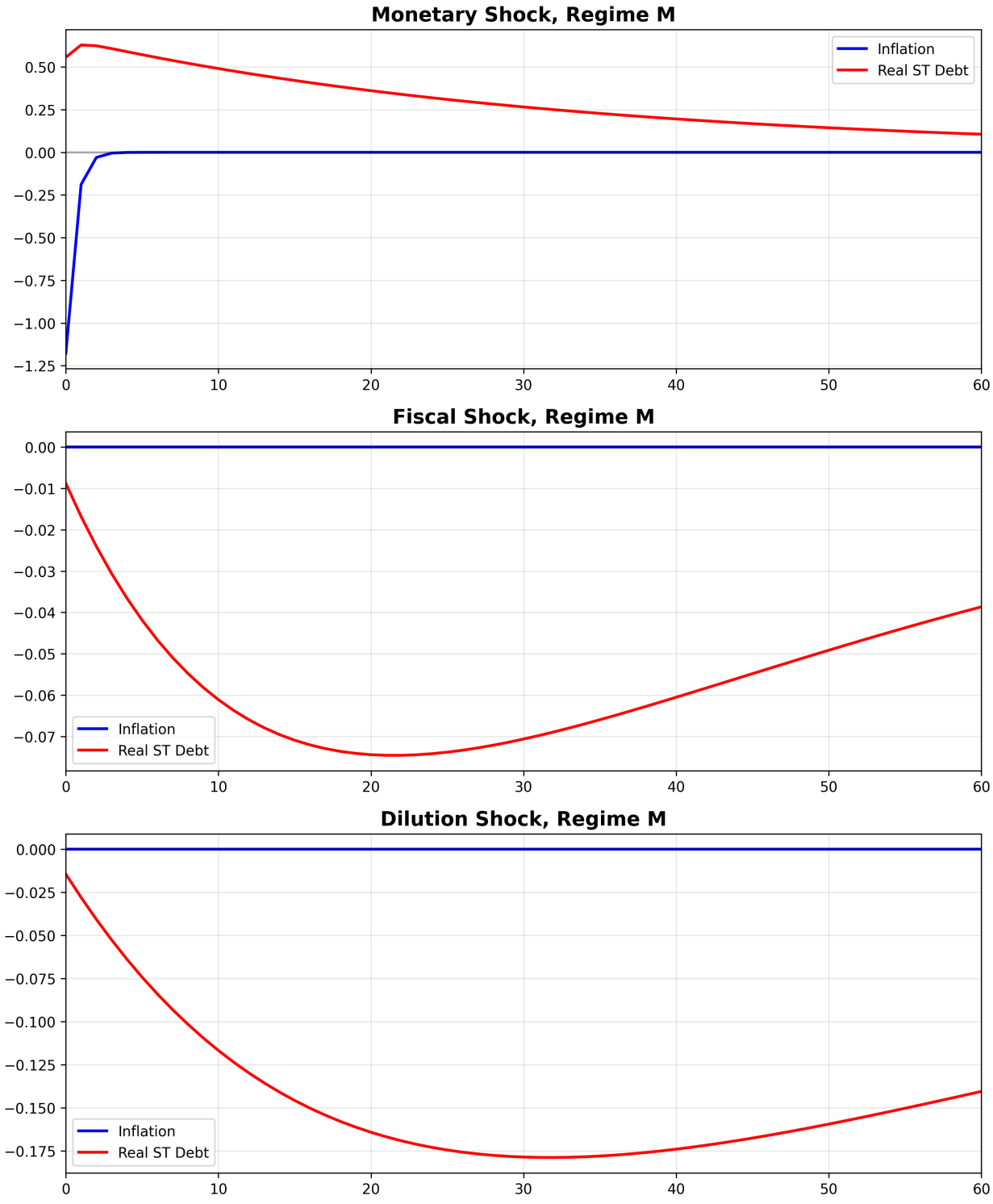


Figure 7: Impulse Response Functions, Regime M

This figure displays IRFs for inflation (blue line) and real short-term debt (red line) following positive one standard deviation shocks to the nominal interest rate ($\varepsilon^R \uparrow$), taxes ($\varepsilon^\tau \uparrow$), and the dilution rate ($\varepsilon^a \downarrow$). The parameter space corresponds to Regime M: $\alpha = 1.01$, $\gamma = 5$. The inverse dilution rate reacts to inflation and real maturing debt proportionally: $\delta = 1.0$, $\varphi = 1.0$. **Top panel:** A surprise rate hike produces a negative inflation response and a positive real short-term debt response. **Middle panel:** A surprise tax hike generates no inflation response and a negative real short-term debt response. **Bottom panel:** A surprise shortening produces no inflation and a negative real short-term debt response.

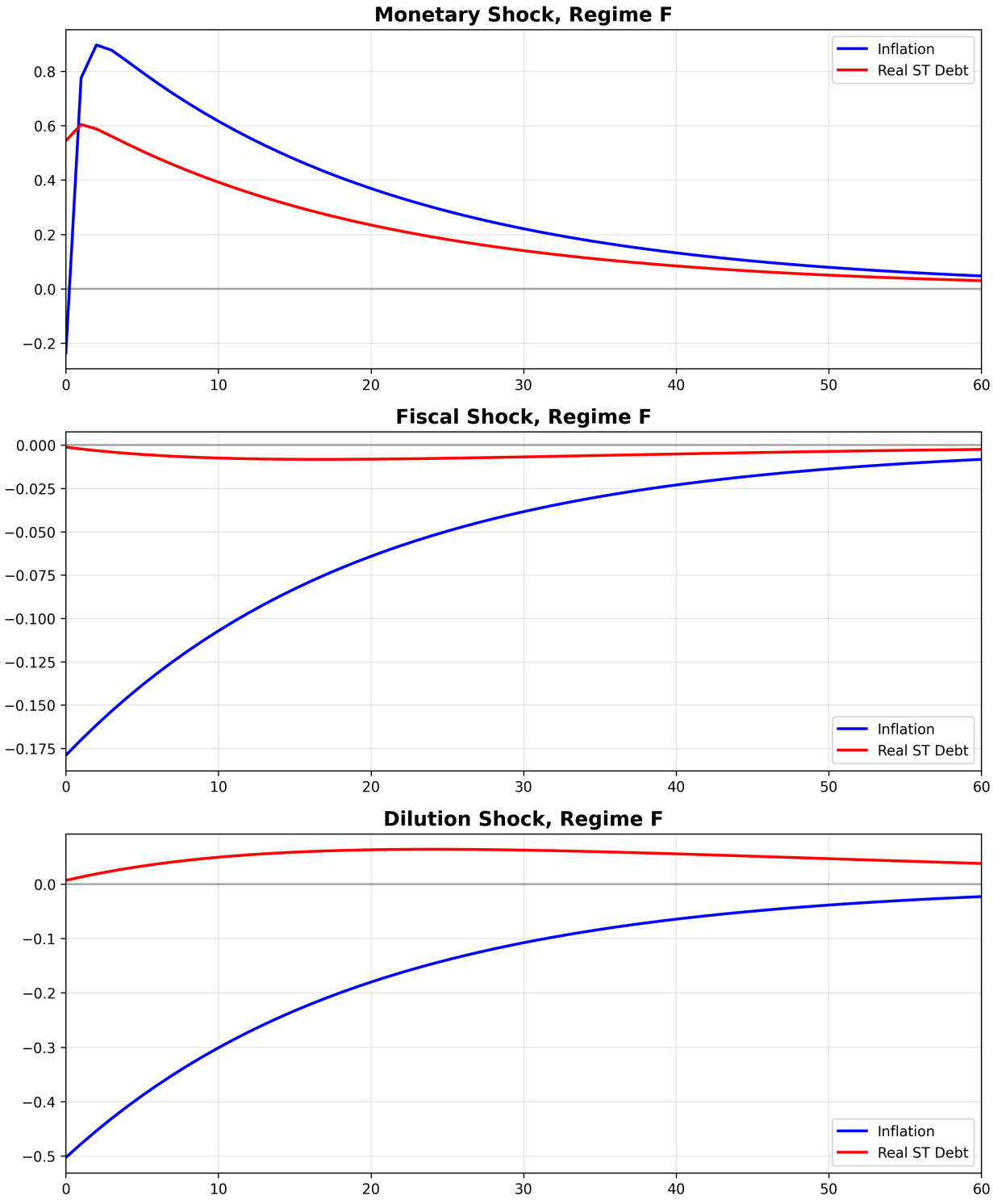


Figure 8: Impulse Response Functions, Regime F

This figure displays IRFs for inflation (blue line) and real short-term debt (red line) following positive one standard deviation shocks to the nominal interest rate ($\varepsilon^R \uparrow$), taxes ($\varepsilon^\tau \uparrow$), and the dilution rate ($\varepsilon^a \downarrow$). The parameter space corresponds to Regime F: $\alpha = 0.95$, $\gamma = 0.05$. The inverse dilution rate reacts to inflation and real maturing debt proportionally: $\delta = 1.0$, $\varphi = 1.0$. **Top panel:** A surprise rate hike shows a positive real short-term debt response and a negative inflation response immediately followed by a positive inflation response. **Middle panel:** A surprise tax hike generates a negative inflation response and a negative real short-term debt response. **Bottom panel:** A surprise shortening produces a negative inflation response a positive real short-term debt response.

In Regime F, a surprise shortening also echoes a surprise tax hike by reducing inflation but through effects distinct from direct fiscal financing. Inflation falls through a more general and persistent version of the deflationary dilution channel outlined in Cochrane's (2023) book and in Section 2. Shortening the time t maturity structure results in more time $t+1$ maturing debt $B_{t+1} \uparrow$ which increases the expected time $t+1$ price level $\mathbb{E}_t P_{t+1} \uparrow$, reducing time t's debt price $Q_t \downarrow$ and the contemporaneous market value of government debt as a result. This shortening is expected to (and does) occur every period until the shock dies off. The effect is similar to stepping on a rake in that the falling market value of long-term debt reduces inflation on impact, but distinct in the sense that the shock's inflationary impact gets continuously pushed into the future. All the while, the Taylor rule ensures interest rates stay low keeping interest expenses depressed. After a length of time, the interest rate and tax responses return the economy back to the steady state maturity structure without inflation cropping up.

3.3.2 Aggressive Maturity Responses and Two New Dynamic Regimes

Adding the dilution rule (11) has, so far, had only a minor effect on the dynamics derived from monetary and fiscal shocks when compared to traditional FTPL. The main finding: a surprise shortening ($\varepsilon_t^a \downarrow$) results in inflation dynamics similar to a tax hike ($\varepsilon_t^\tau \uparrow$) in both Regime M and Regime F.

In this section, I vary δ and φ to examine the degree to which strong and weak maturity management affects inflation and debt dynamics. Figures 9, 10, 11, and 12 explore how inflation and real short-term debt respond to rate, tax, and dilution shocks when $\delta, \varphi \in \{0.1, 1.0, 5.0\}$. Figures 9 and 10 relate to Regime M while Figures 11 and 12 relate to Regime F.

Figures 9 and 11 largely tell the same story: regardless of the size of fiscal policy's dilution response $\delta > 0$, the dynamics in Regimes M and F are qualitatively identical to those in the short-term only model. On-impact and future inflation and short-term debt responses to monetary, fiscal, and dilution shocks move in the same direction as those in Figures 7 and 8. Fiscal maturity management δ is highly important in equilibrium determinacy, but, conditional on the regime, varying δ changes little about the model's dynamics.

Two new dynamic regimes emerge from the model when central bank dilution responses φ is large. One is an extension of regime M and the other is an extension of Regime F. I call these regimes Regime M^+ and Regime F^+, respectively.

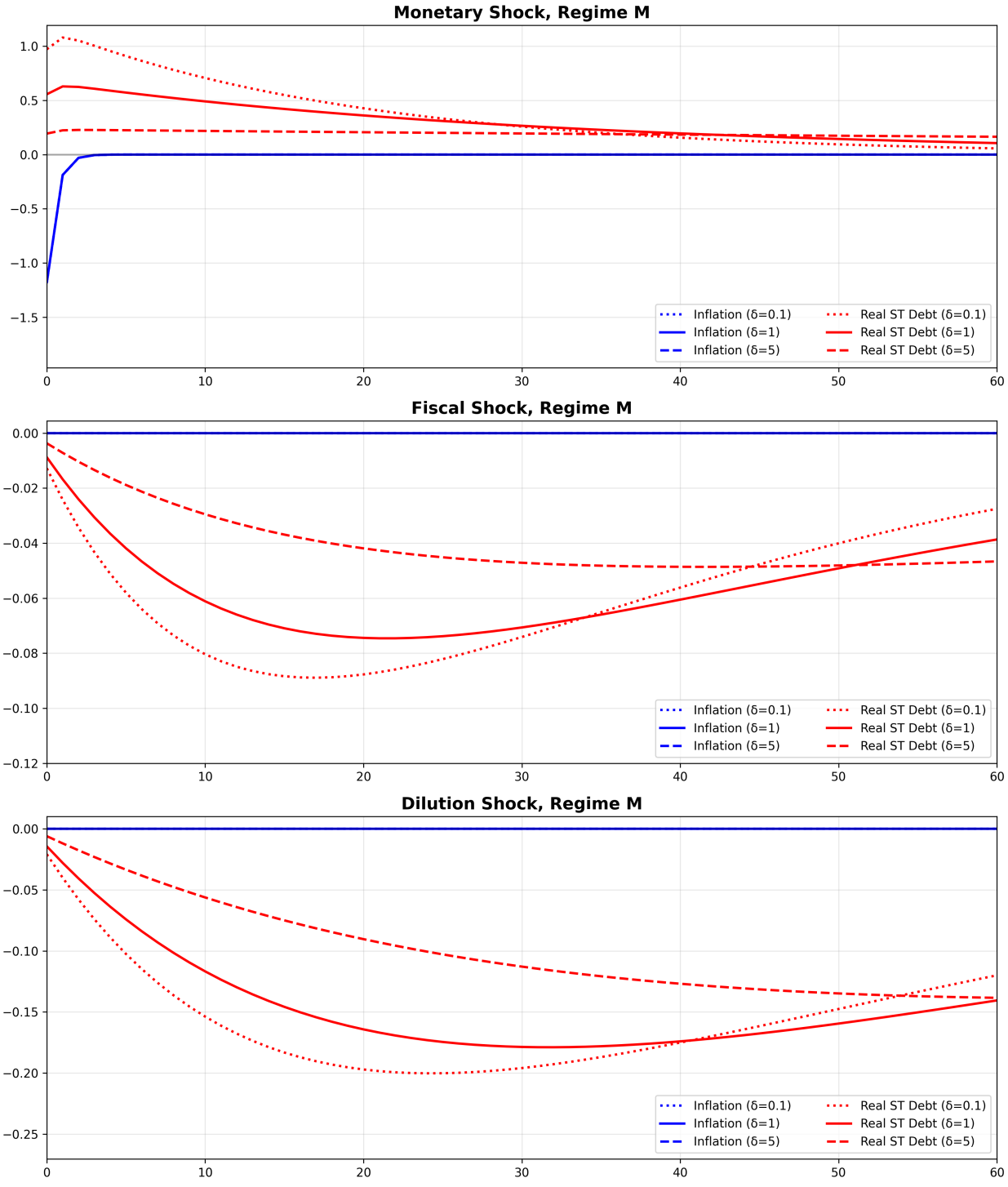


Figure 9: Impulse Response Functions, Regime M; Varying δ

This figure displays IRFs for inflation (blue line) and real short-term debt (red line) following positive one standard deviation shocks to the nominal interest rate ($\varepsilon^R \uparrow$), taxes ($\varepsilon^\tau \uparrow$), and the dilution rate ($\varepsilon^a \downarrow$). The parameter space corresponds to Regime M: $\alpha = 1.01$, $\gamma = 5$. The inverse dilution rate reacts to inflation proportionally: $\varphi = 1.0$. The inverse dilution rate's reaction to maturing debt varies between insensitive, moderately-sensitive, and highly-sensitive: $\delta \in \{0.1, 1.0, 5.0\}$. All IRFs feature identical inflation responses from policy shocks to those from Figure 7. Debt dynamics vary across δ values, but, directionally, no notable differences exist.

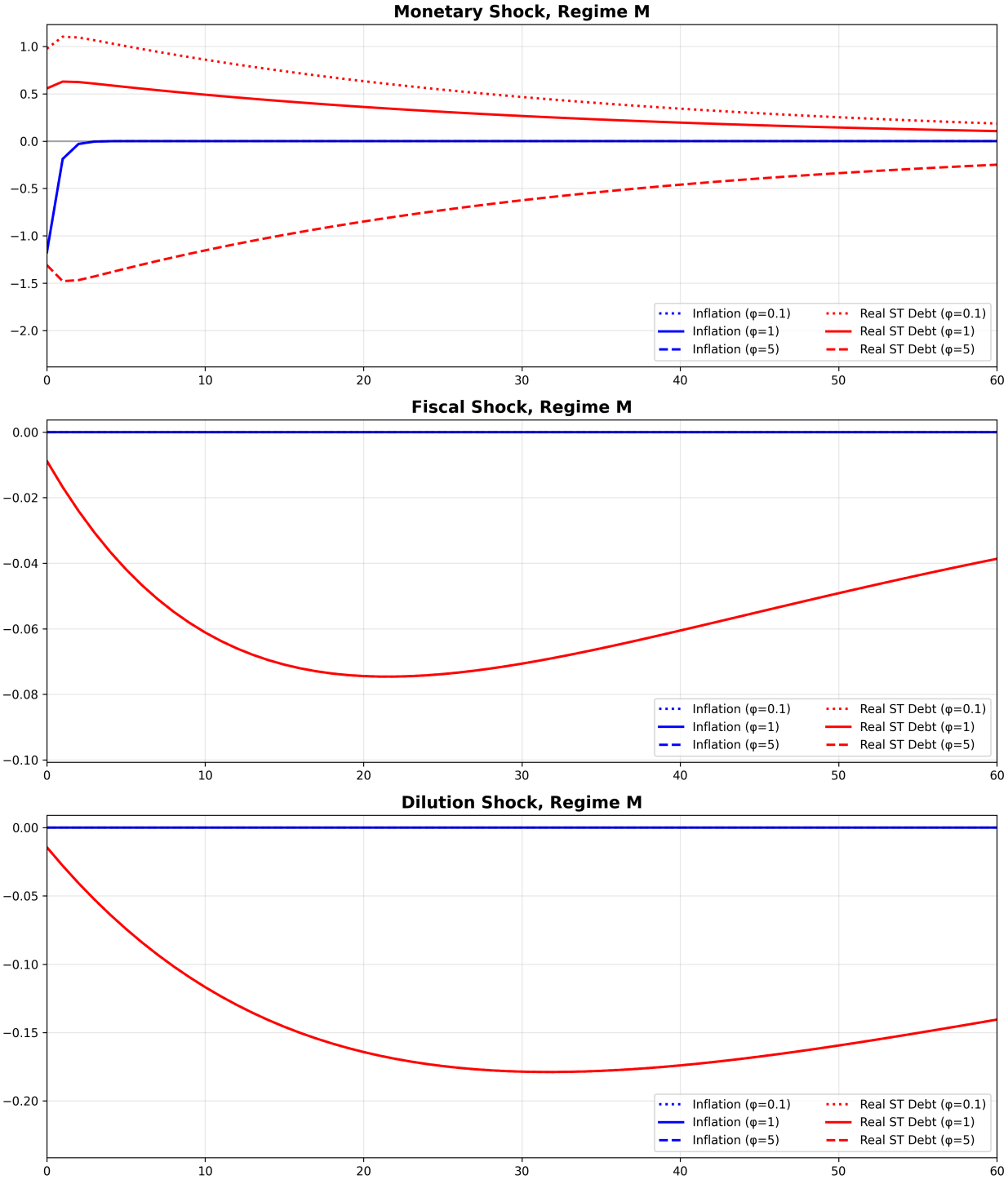


Figure 10: Impulse Response Functions, Regime M; Varying φ

This figure displays IRFs for inflation (blue line) and real short-term debt (red line) following positive one standard deviation shocks to the nominal interest rate ($\varepsilon^R \uparrow$), taxes ($\varepsilon^\tau \uparrow$), and the dilution rate ($\varepsilon^a \downarrow$). The parameter space corresponds to Regime M: $\alpha = 1.01$, $\gamma = 5$. The inverse dilution rate reacts to real maturing debt proportionally: $\delta = 1.0$. The inverse dilution rate's reaction to inflation varies between insensitive, moderately-sensitive, and highly-sensitive: $\varphi \in \{0.1, 1.0, 5.0\}$. All IRFs feature identical inflation responses from policy shocks to those in Figure 7. Debt dynamics vary across φ values for an interest rate shock - for a sufficiently-large φ value, real short-term debt falls as a response to a shock to the policy rate.

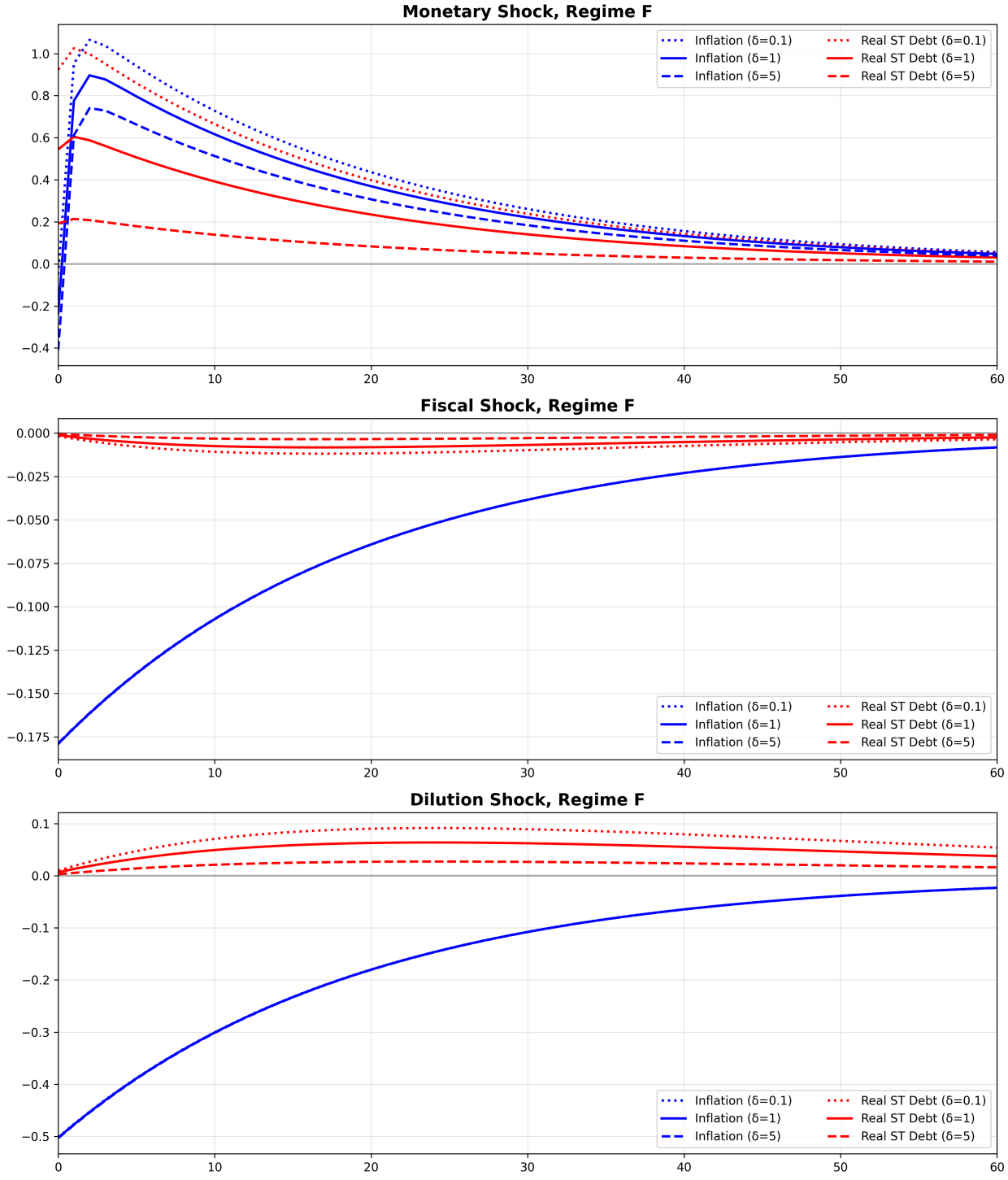


Figure 11: Impulse Response Functions, Regime F; Varying δ

This figure displays IRFs for inflation (blue line) and real short-term debt (red line) following positive one standard deviation shocks to the nominal interest rate ($\varepsilon^R \uparrow$), taxes ($\varepsilon^\tau \uparrow$), and the dilution rate ($\varepsilon^a \downarrow$). The parameter space corresponds to Regime F: $\alpha = 0.95$, $\gamma = 0.05$. The inverse dilution rate reacts to inflation proportionally: $\varphi = 1.0$. The inverse dilution rate's reaction to real maturing debt varies between insensitive, moderately-sensitive, and highly-sensitive: $\delta \in \{0.1, 1.0, 5.0\}$. All IRFs feature relatively unchanged inflation responses from policy shocks from those in Figure 8. Debt dynamics vary across δ values for a rate shock, but, directionally, no notable differences exist across policy shocks.

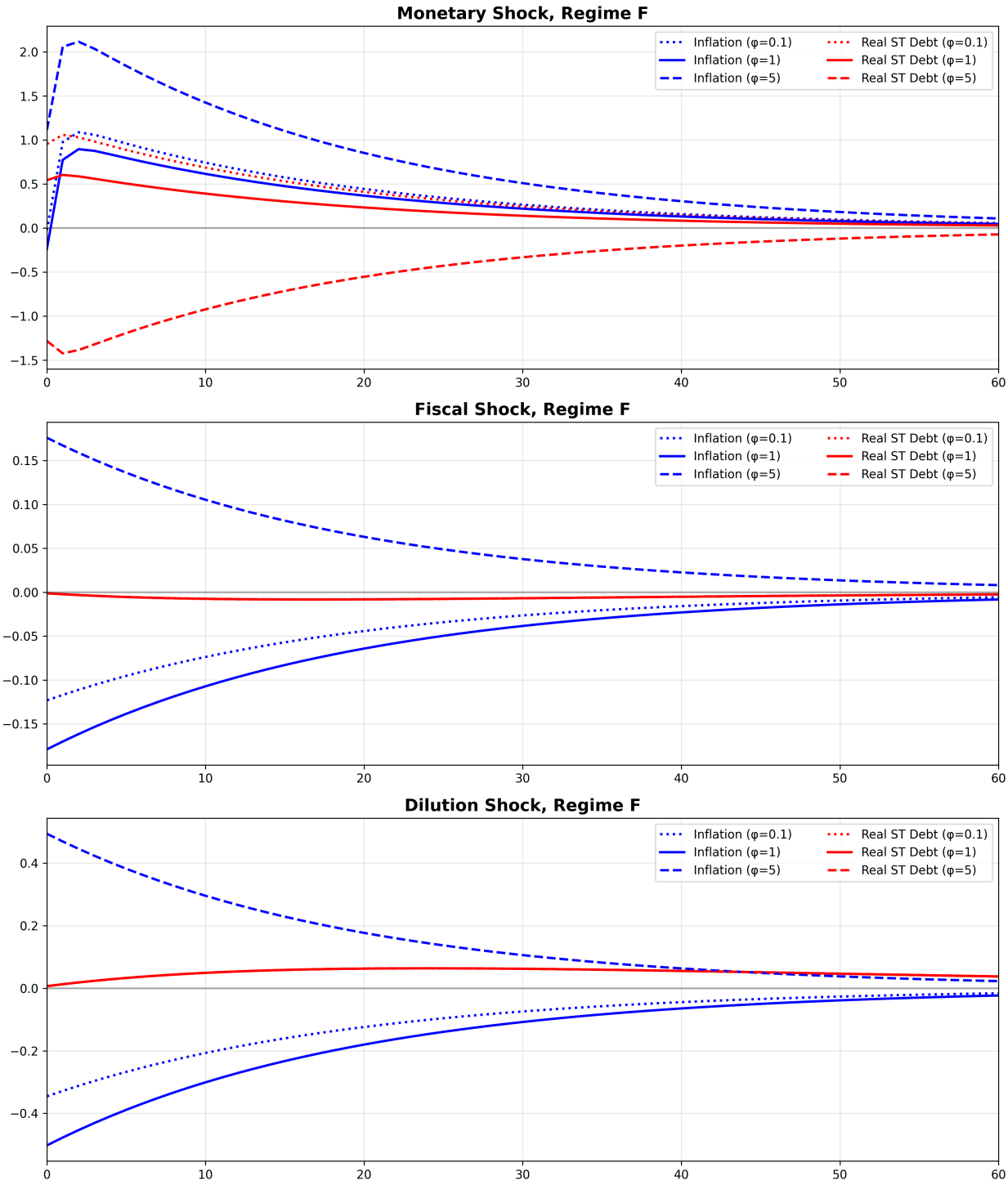


Figure 12: Impulse Response Functions, Regime F; Varying φ

This figure displays IRFs for inflation (blue line) and real short-term debt (red line) following positive one standard deviation shocks to the nominal interest rate ($\varepsilon^R \uparrow$), taxes ($\varepsilon^\tau \uparrow$), and the dilution rate ($\varepsilon^a \downarrow$). The parameter space corresponds to Regime F: $\alpha = 0.95$, $\gamma = 0.05$. The inverse dilution rate reacts to real maturing debt proportionally: $\delta = 1.0$. The inverse dilution rate's reaction to inflation varies between insensitive, moderately-sensitive, and highly-sensitive: $\varphi \in \{0.1, 1.0, 5.0\}$. Debt dynamics vary across φ values for a rate shock - for a sufficiently-large φ value, real debt falls as a response to a shock to the policy rate. Inflation dynamics vary across φ values for fiscal and dilution shocks. For sufficiently high values of φ, inflation rises (falls) in response to a surprise increase in taxes (the inverse dilution rate).

Definition. (***Regime M***$^+$) The economy resides in Regime M$^+$ when Regime M's determinacy conditions hold and when a positive interest rate shock $\varepsilon_t^R > 0$ reduces real short-term debt on impact $b_t^{(t+1)} < 0$.

Definition. (***Regime F***$^+$) The economy resides in Regime F$^+$ when Regime F's determinacy conditions hold and when a positive surplus shock $\varepsilon_t^\tau > 0$ or shortening shock $\varepsilon_t^a < 0$ increases inflation on impact $\pi_t > 0$.

Proposition 2 (*Dynamic Regime Thresholds*) *The model resides in Regime M$^+$ if policy parameters α, γ, δ, φ satisfy:*

1. $\alpha > 1$, $|\lambda_2| < 1$, $\lambda_3 < -1$ *(Regime M)*
2. $\varphi > 1 + \rho_R + \frac{1}{\beta a}$

and the model resides in Regime F$^+$ if policy parameters α, γ, δ, φ satisfy:

1. $\alpha < 1$, $\lambda_2 > 1$, $\lambda_3 < -1$ *(Regime F)*
2. $\varphi > 1 + \alpha + \frac{1}{\beta a}$

Proof: The proof can be found in Appendix A.

Figure 10 displays the qualitative difference: a flip in debt's response to a Regime M monetary shock. When inflation falls from standard New Keynesian mechanics, the central bank begins to engage in strong open market sales, shortening the maturity structure. In Section 3.3.1, this effect was overridden by passive fiscal policy financing the total stock of debt. Now, because φ is large, maturity shortening dominates fiscal debt financing on impact and beyond.

Figure 12 hints at something more interesting: a reversal in the inflationary impact of tax shocks in Regime F$^+$. This second additional dynamic regime reveals dilution policy's usefulness in inflation management. A surprise open market sale (or tax cut) immediately raises the government's financing burden, which would normally require higher inflation to satisfy the GBC. But, when φ is sufficiently high, any rise in inflation triggers large maturity lengthening by the model's central bank. This lengthening response is so aggressive that it more than reverses the original tax cut's inflationary effect. The net result: inflation falls to satisfy the GBC (12).

Maturity management's role with a large φ is a direct analogue to interest rate's role in regime M with a large α. In Regime M, aggressive interest rate responses to inflation

communicate that monetary policy will ever-escalate interest rates should current or future inflation rise. Active monetary policy suppresses inflation through the Fisher equation (7). In Regime F with a large φ, aggressive dilution responses to inflation communicate that monetary policy will ever-escalate maturity lengthening should current or future inflation rise. These large, inflation-contingent open market operations suppress inflation through the flow GBC (12). This lengthening (and expected future lengthening) takes the policy rate's place as the central bank's main inflation-management tool. Handing the central bank the keys to dilution policy gives **monetary policy** a **fiscal tool** which is uses to great effect in Regime F^+.

4 Regime-Invariant Inflation Control

In this section, I construct a surprise, simultaneous rate hike and open market sale that reduces inflation on-impact and beyond in Regime M^+ and F^+ alike. Simultaneously shocking the policy rate and the dilution rate reduces inflation regardless of determinacy regime, effectively allowing the central bank to unilaterally coordinate monetary and fiscal policy. I conjecture that U.S. post-COVID inflation fell due to effects from current and expected maturity management despite continued fiscal expansion.

Figure 13 displays two sets of IRFs resulting from a simultaneously-occurring 1 standard deviation positive shock to the inverse dilution rate ε_t^a and 0.205 standard deviation positive shock to the interest rate ε_t^R when $\delta = 1.0$ and $\varphi = 5.0$.

In Regime M^+, passive (Ricardian) fiscal policy raises taxes to finance increased interest payments, reducing inflation and overall (short-term plus long-term) government debt on impact. As the immediate deflationary effects die off, fiscal policy continues to set taxes to stabilize inflation in response to dilution dynamics.

In Regime F^+, inflation falls on impact from two separate effects: stepping on a rake effects from the rate hike and Section 3.3.2's open market sale effect. After impact, stepping on a rake pushes inflation up while the persistence of the open market sale keeps inflation down. Due to the size of the dilution shock, the open market sale effect dominates and inflation returns back to steady state from below. Short-term debt falls and converges from below due to the substitution from short-term to long-term debt as a result of the open market sales.

A simultaneous rate hike and open market sale reduces inflation regardless of regime. After the COVID pandemic, U.S. inflation surpassed 8% for the first time in 40 years. Despite little response from fiscal policy, the Fed's simultaneous rate hike and QT programming brought inflation down where it remains at roughly 3%. The theory developed in this paper shows

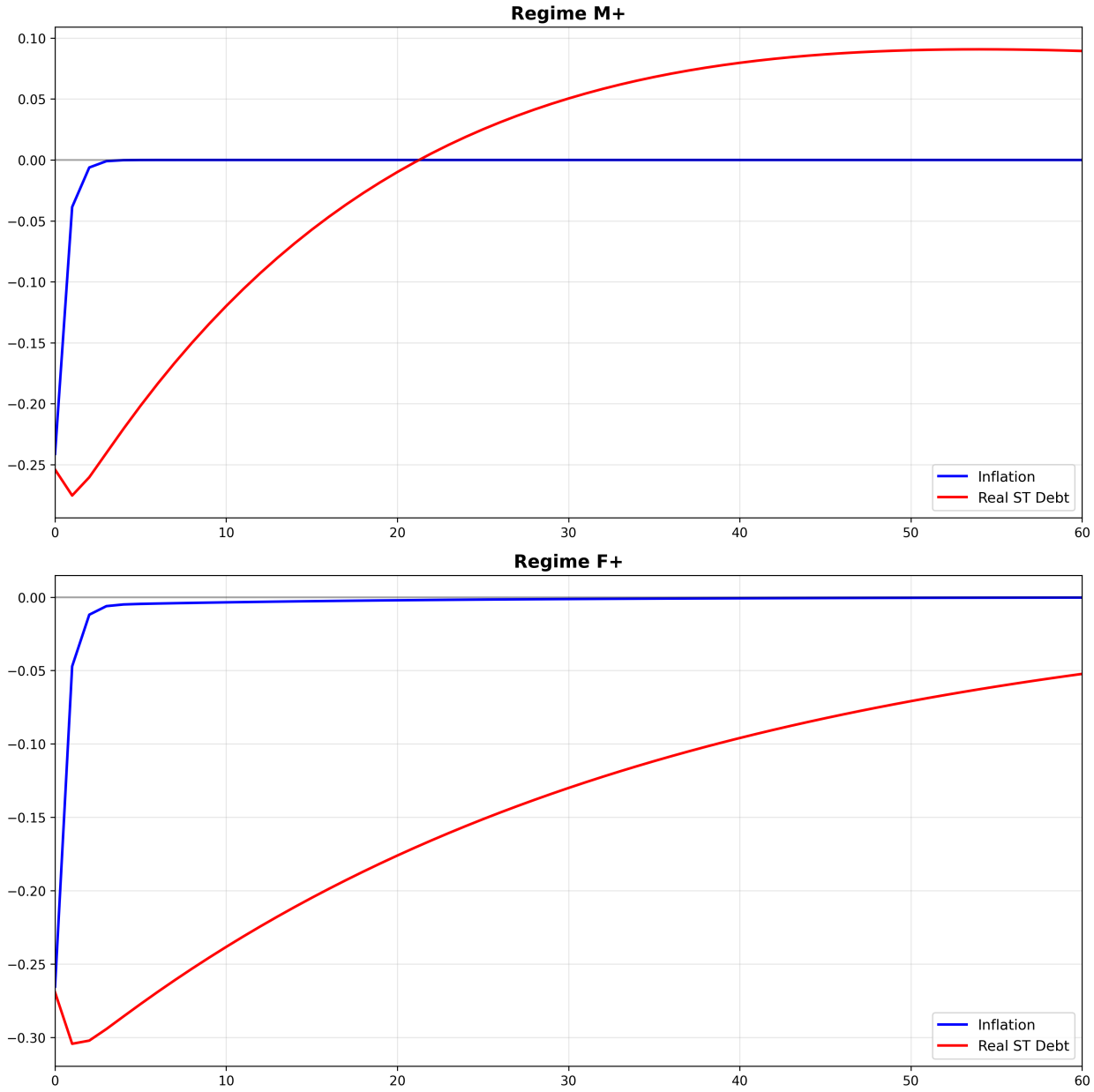


Figure 13: Impulse Response Functions: Regimes M^+ and F^+, Simultaneously Shocking R_t and a_t

This figure displays IRFs for inflation (blue line) and real short-term debt (red line) following a positive 0.205 standard deviation shock to the nominal interest rate ($\varepsilon^R \uparrow$) combined with a negative 1 standard deviation shock to the dilution rate ($\varepsilon^a \uparrow$). The parameter spaces correspond to Regime M^+ (top panel, $\alpha = 1.01$, $\gamma = 5$) and Regime F^+ (bottom panel, $\alpha = 0.95$, $\gamma = 0.05$). The inverse dilution rate reacts to real maturing debt proportionally ($\delta = 1.0$) and is highly-sensitive to inflation ($\varphi = 5.0$). The joint shock to the economy results in deflation without subsequent inflation. The shock in Regime M^+ features an initial decrease, followed by an increase, in short-term debt. The shock in Regime F^+ features an initial and persistent decrease in short-term debt.

how pairing contractionary monetary policy and open market sales can deliver short-run inflation reductions without long-run inflation increases regardless of fiscal policy's behavior. In doing so, it provides a potential explanation for post-COVID inflation reduction absent supply-side shocks.

5 Conclusion

This paper classifies new regimes consistent with the fiscal theory of the price level, it provides a new way to study maturity structure management in linearized economies, and it explores a dilution rule where monetary and fiscal policy separately contribute to the jointly-determined outstanding maturity structure.

In the paper, I introduce the dilution rate of government debt which measures the ratio of contemporaneously-issued short-term debt to previously-issued, unmatured long-term debt. Such a concept is useful for three reasons. First, it can be widely applied to economic models with nominal debt. Second, it survives log-linear approximations common in much of modern macroeconomics. Third, it comes with clean interpretations and tractable algebra. Such an innovation improves on Woodford's (2001) constantly geometrically-declining structure because it allows for analysis of dynamic maturity responses to unexpected economic shocks. I apply this concept to an otherwise-standard FTPL model and investigate theoretical inflation and real debt dynamics from shocks to dilution. These results support the idea that surprise open market operations or changes to issuance duration may aid the government and central bank to achieve policy outcomes using a less politically sensitive tool (relative to surprise inflation and explicit taxation). These effects may also explain why post-2022 U.S. inflation has remained subdued despite persistent deficits. Embedding these policy mixes in alternative frameworks – for instance, models with sticky prices, production economies, or regime-switching – may lead to further understanding about debt dilution and its effects.

I uncover four main findings. First, active Treasury issuance (shortening maturity while cutting taxes) expands FTPL's traditional active fiscal region by neutralizing what would otherwise be passive fiscal policy, creating unique, bounded equilibria. Second, under moderate open market operations, the inflationary effects from a surprise maturity lengthening mirror those from a tax cut, increasing inflation when fiscal policy is active, with no effect when fiscal policy is passive. Third, under aggressive open market operations, this Regime F relationship flips. Tax cuts or maturity lengthening decrease inflation under active fiscal policy because maturity management policy suppresses inflation through the GBC much like the Taylor principle suppresses inflation through the Fisher equation in Regime M. Fourth, the model central bank can unilaterally coordinate monetary and fiscal policy by

simultaneously shocking the interest rate and dilution rate. Such a composite shock produces regime-invariant deflationary effects regardless of which policy regime is active.

The paper's results carry implications for central bank balance sheet design. In Regime F^+, inflation control operates through expectations of future, credible maturity lengthening. Any rise in inflation triggers aggressive open market sales of long-term debt, and the threat of ever-increasing sales disciplines contemporaneous inflation through the government budget constraint. This mechanism requires, however, that the central bank maintain a sufficient stock of long-term government debt available for sale. A central bank that has exhausted its long-term holdings cannot credibly threaten further lengthening, just as a central bank at the zero lower bound cannot credibly threaten further rate cuts. The analogy to the Taylor principle is instructive: in Regime M, the central bank rarely needs to raise rates dramatically because the credible threat of doing so suppresses inflation preemptively. Similarly, in Regime F^+, the central bank may never need to sell its entire arsenal of long-term debt, but the market must believe such a policy is feasible. This logic suggests that central banks seeking to preserve maturity management as a policy tool should maintain a substantial portfolio of long-term government securities. Alternatively, the consolidated government should ensure the central bank has access to a sufficiently large stock of long-term debt that can be sold on open markets when inflationary pressures emerge. The post-COVID experience, in which the Fed entered its tightening cycle holding roughly $5.8 trillion in Treasuries with an average maturity well above that of overall issuance, may represent exactly such a position.

References

Angeletos, George-Marios. 2002. "Fiscal Policy with Non-Contingent Debt and the Optimal Maturity Structure." *Quarterly Journal of Economics* 117 (3):1105–1131.

Blanchard, Olivier J. and Charles M. Kahn. 1980. "The Solution of Linear Difference Models Under Rational Expectations." *Econometrica* 48 (5):1305–1311.

Buera, Francisco and Juan Pablo Nicolini. 2004. "Optimal Maturity Structure of Government Debt without State Contingent Bonds." *Journal of Monetary Economics* 51 (3):531–554.

Chari, V. V., Lawrence J. Christiano, and Patrick J. Kehoe. 1995. "Policy Analysis in Business Cycle Models." In *Frontiers in Business Cycle Research*, edited by Thomas F. Cooley. Princeton, NJ: Princeton University Press, 357–391.

Cochrane, John H. 2001. "Long Term Debt and Optimal Policy in the Fiscal Theory of the Price Level." *Econometrica* 69 (1):69–116.

———. 2018. "Stepping on a Rake: The Fiscal Theory of Monetary Policy." *European Economic Review* 101:354–375.

———. 2023. *The Fiscal Theory of the Price Level*. Princeton University Press.

Debortoli, Davide, Ricardo Cavaco Nunes, and Pierre Yared. 2017. "Optimal Time-Consistent Government Debt Maturity." *The Quarterly Journal of Economics* 132 (1):55–102.

Faraglia, Alisa, Albert Marcet, and Andrew Scott. 2010. "In Search of a Theory of Debt Management." *Journal of Monetary Economics* 57:821–836.

Faraglia, Elisa, Albert Marcet, Rigas Oikonomou, and Andrew Scott. 2019. "Government Debt Management: the Long and the Short of It." *Review of Economic Studies* 86:2554–2604.

Geweke, John. 2010. *Complete and Incomplete Econometric Models*. Princeton, NJ: Princeton University Press.

Leeper, Eric M. 1991. "Equilibria Under 'Active' and 'Passive' Monetary and Fiscal Policies." *Journal of Monetary Economics* 27 (1):129–147.

Leeper, Eric M. and Campbell Leith. 2017. "Understanding Inflation as a Joint Monetary-Fiscal Phenomenon." In *Handbook of Macroeconomics*, vol. 2B, edited by John B. Taylor and Harald Uhlig. Amsterdam: Elsevier Press, 2305–2416.

Leeper, Eric M., Michael Plante, and Nora Traum. 2010. "Dynamics of Fiscal Financing in the United States." *Journal of Econometrics* 156 (2):304–321.

Leeper, Eric M. and Xuan Zhou. 2021. "Inflation's role in optimal monetary-fiscal policy." *Journal of Monetary Economics* 124:1–18.

Lustig, Hanno, Christopher Sleet, and Sevin Yeltekin. 2008. "Fiscal Hedging with Nominal Assets." *Journal of Monetary Economics* 55 (4):710–727.

Rocheteau, Guillaume, Randall Wright, and Sylvia Xiaolin Xiao. 2018. "Open market operations." *Journal of Monetary Economics* 98:114–128.

Sims, Christopher A. 1994. "A Simple Model for Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy." *Economic Theory* 4 (3):381–399.

———. 2002. "Solving Linear Rational Expectations Models." *Computational Economics* 20 (1):1–20.

———. 2011. "Stepping on a Rake: The Role of Fiscal Policy in the Inflation of the 1970's." *European Economic Review* 55 (1):48–56.

Smets, Frank and Raf Wouters. 2007. "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach." *American Economic Review* 97 (3):586–606.

Taylor, John B. 1993. "Discretion versus Policy Rules in Practice." *Carnegie-Rochester Conference Series on Public Policy* 39:195–214.

Wallace, Neil. 1981. "A Modigliani-Miller Theorem for Open-Market Operations." *American Economic Review* 71 (3):267–274.

Woodford, Michael. 1995. "Price-Level Determinacy Without Control of a Monetary Aggregate." *Carnegie-Rochester Conference Series on Public Policy* 43:1–46.

———. 2001. "Fiscal Requirements for Price Stability." *Journal of Money, Credit, and Banking* 33 (3):669–728.

A Appendix: Proofs

Proof for Proposition 1

The linearized system in Appendix D has 2 forward-looking variables (π_t, b_t) and 4 predetermined variables (b_{t-1}, z_t^R, z_t^τ, z_t^a). By the Blanchard and Kahn (1980) conditions, determinacy requires the number of unstable eigenvalues to equal the number of forward-looking variables. Since the persistence parameters ρ_R, ρ_τ, ρ_a are stable by assumption, exactly 2 of the 3 policy-dependent eigenvalues ($\lambda_1 = \alpha$, λ_2, λ_3) must have modulus greater than 1.

This proof is completed in four steps. First, I exhaustively enumerate all eight possible eigenvalue configurations that yield exactly 2 explosive roots. Second, I show that three configurations are infeasible given the condition $\lambda_2 \geq \lambda_3$. Third, I demonstrate that the remaining five configurations are non-empty by constructing parameter examples for each (sufficiency). Fourth, because these five represent all feasible ways to achieve exactly 2 explosive roots, any determinate economy must belong to one of these five regimes (necessity).

Eight possible eigenvalue configurations yield exactly 2 explosive roots in the linearized model:

1a. $\alpha > 1$, $\lambda_2 < -1$, $|\lambda_3| < 1$

1b. $\alpha > 1$, $\lambda_2 > 1$, $|\lambda_3| < 1$

1c. $\alpha > 1$, $|\lambda_2| < 1$, $\lambda_3 < -1$

1d. $\alpha > 1$, $|\lambda_2| < 1$, $\lambda_3 > 1$

2a. $\alpha < 1$, $\lambda_2 < -1$, $\lambda_3 < -1$

2b. $\alpha < 1$, $\lambda_2 > 1$, $\lambda_3 < -1$

2c. $\alpha < 1$, $\lambda_2 > 1$, $\lambda_3 > 1$

2d. $\alpha < 1$, $\lambda_2 < -1$, $\lambda_3 > 1$

where (1c) is Regime M, (2b) is Regime F, and (2a), (1b), and (2c) are the remaining regions in Proposition 1.

These eight configurations are exhaustive: when $\alpha > 1$, exactly one of $\{\lambda_2, \lambda_3\}$ must be explosive, yielding $2 \times 2 = 4$ cases depending on which eigenvalue is explosive and whether it exceeds 1 or is less than -1. When $\alpha < 1$, both $\{\lambda_2, \lambda_3\}$ must be explosive, yielding 4 cases from the combinations of each being greater than 1 or less than -1.

The quadratic formula for λ_2 and λ_3 in equation (16), combined with the parameter restrictions $\beta, a > 0$, imply that all real-valued λ_2, λ_3 must satisfy $\lambda_2 \geq \lambda_3$, since the denominator $2\beta a > 0$ ensures the root with the positive sign is weakly larger. This eliminates three configurations as infeasible: 1a (requires $\lambda_2 < -1 < |\lambda_3| < 1$, violating $\lambda_2 \geq \lambda_3$), 1d (requires $|\lambda_2| < 1 < \lambda_3$, violating $\lambda_2 \geq \lambda_3$), and 2d (requires $\lambda_2 < -1$ and $\lambda_3 > 1$, violating $\lambda_2 \geq \lambda_3$).

The remaining five regimes are nonempty. To show this, I list parameter combinations that satisfy model restrictions and the respective regime's definition:

1b. $a = 4.81$, $\beta = 0.11$, $\alpha = 1.50$, $\gamma = 1.11$, $\delta = 1.25$
$\Rightarrow \lambda_1 = 1.50$, $\lambda_2 = 6.36$, $\lambda_3 = -0.41$

1c. $a = 1.45$, $\beta = 0.37$, $\alpha = 1.50$, $\gamma = 1.46$, $\delta = 1.30$
$\Rightarrow \lambda_1 = 1.50$, $\lambda_2 = 0.99$, $\lambda_3 = -1.45$

2a. $a = 0.35$, $\beta = 0.37$, $\alpha = 0.50$, $\gamma = 2.2$, $\delta = 2.05$
$\Rightarrow \lambda_1 = 0.50$, $\lambda_2 = -1.04$, $\lambda_3 = -6.03$

2b. $a = 0.33$, $\beta = 0.84$, $\alpha = 0.50$, $\gamma = 0.92$, $\delta = 0.55$
$\Rightarrow \lambda_1 = 0.50$, $\lambda_2 = 1.04$, $\lambda_3 = -4.00$

2c. $a = 5.34$, $\beta = 0.15$, $\alpha = 0.50$, $\gamma = 1.29$, $\delta = 0.6$
$\Rightarrow \lambda_1 = 0.50$, $\lambda_2 = 3.71$, $\lambda_3 = 1.11$

where each parameter combination yields exactly 2 eigenvalues with modulus greater than 1 and satisfies its respective regime condition, verified by direct substitution into equation (16).

Finally, since (i) the eight configurations exhaustively enumerate all ways to partition $\{\lambda_1, \lambda_2, \lambda_3\}$ into exactly 2 explosive and 1 stable eigenvalue, (ii) three configurations are infeasible, and (iii) determinacy requires exactly 2 explosive eigenvalues, any determinate economy must belong to one of these five feasible regimes.

Proof for Proposition 2

Define $A \equiv \Gamma_0^{-1}\Gamma_1$ using the canonical form matrices from Appendix D. Given the model structure, A is block-triangular:

$$A = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix}$$

where A_{11} is the upper-left 3×3 block governing the endogenous dynamics of $(\pi_t, \tilde{b}_t, x_t)$ and can be written as

$$A_{11} = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & 0 & 1 \\ A_{31}(\varphi) & A_{32} & A_{33} \end{bmatrix}$$

where A_{32} and A_{33} depend on $(\gamma, \delta, \beta, a)$ but not on φ, and

$$A_{31}(\varphi) = \frac{(\alpha\beta - 1)(1 + a\beta(1 + \alpha - \varphi))}{a\beta^2} \tag{18}$$

Regime F$^+$. In Regime F, $\lambda_2 > 1$ and $\lambda_3 < -1$ are unstable while $\alpha < 1$ is stable. The Blanchard and Kahn (1980) conditions project the system onto the one-dimensional stable eigenspace of A_{11}. The right eigenvector of A_{11} associated with α satisfies $(A_{11} - \alpha I)v_\alpha = 0$, giving

$$v_\alpha \propto \begin{pmatrix} -\left[A_{32} + \alpha(A_{33} - \alpha)\right] / A_{31}(\varphi) \\ 1 \\ \alpha \end{pmatrix}$$

The first component—governing how inflation loads onto the stable eigenspace—is proportional to $1/A_{31}(\varphi)$. The responses of π_0 to each structural shock ε_t^R, ε_t^τ, ε_t^a are proportional to $1/A_{31}(\varphi)$.

From (18), $A_{31}(\varphi) = 0$ requires either $\alpha\beta = 1$ or $1 + a\beta(1 + \alpha - \varphi) = 0$. Since $\alpha < 1$ and $\beta < 1$ in Regime F, the binding condition is

$$1 + a\beta(1 + \alpha - \varphi) = 0 \quad \Longrightarrow \quad \varphi^F = 1 + \alpha + \frac{1}{a\beta}$$

As φ crosses φ^F, A_{31} passes through zero and changes sign. Since every on-impact inflation coefficient is proportional to $1/A_{31}(\varphi)$, all inflation responses reverse sign at $\varphi = \varphi^F$.

Regime M$^+$. In Regime M, $\alpha > 1$ is unstable while $|\lambda_2| < 1$ is stable. The on-impact debt choice $b_t^{(t+1)}$ is determined by the Blanchard and Kahn (1980) condition which projects out the λ_3-unstable component. Using the block-triangular structure of A, this yields

$$b_0^{(1)} = \frac{1}{\lambda_3 - \rho_R}\left[\frac{A_{31}(\varphi)}{\alpha - \rho_R} - A_{12}^{(31)}(\varphi)\right]\varepsilon_t^R$$

where $A_{12}^{(31)}(\varphi) \equiv [A_{12}]_{31}$ is the $(3,1)$ entry of A_{12}, coupling z_t^R into the $b_t^{(t+1)}$ equation. Direct computation gives $A_{12}^{(31)}(\varphi) = (1 - a)/(a\beta) + (1 + \alpha - \varphi) + \rho_R$. Both $A_{31}(\varphi)$ and $A_{12}^{(31)}(\varphi)$ are

affine in φ, so $b_0^{(1)}$ changes sign when

$$\frac{A_{31}(\varphi)}{\alpha - \rho_R} = A_{12}^{(31)}(\varphi)$$

Substituting $\varphi = 1 + \rho_R + 1/(a\beta)$ into (18) gives $A_{31} = (\alpha\beta - 1)(\alpha - \rho_R)/\beta$, and substituting into $A_{12}^{(31)}$ gives $A_{12}^{(31)} = (\alpha\beta - 1)/\beta$. Therefore

$$\frac{A_{31}}{\alpha - \rho_R} = \frac{(\alpha\beta - 1)(\alpha - \rho_R)}{\beta(\alpha - \rho_R)} = \frac{\alpha\beta - 1}{\beta} = A_{12}^{(31)}$$

confirming $b_0^{(1)} = 0$ at $\varphi^M = 1 + \rho_R + 1/(a\beta)$. Since $\lambda_3 < -1$ and $\rho_R < 1$, the prefactor $1/(\lambda_3 - \rho_R) < 0$.

B Appendix: Deriving the Rewritten Present Value Condition

This section derives the rewritten present value condition (4) from the text, as in the paper by Cochrane (2001). I begin with the well-known present value condition (3):

$$\underbrace{\frac{B_{t-1}^{(t)}}{P_t} + \mathbb{E}_t \frac{B_{t-1}^{(t+1)}}{r_{t,t+1} P_{t+1}}}_{MV(\text{Debt})/P_t} = \underbrace{\mathbb{E}_t \sum_{i=0}^{\infty} \frac{\tau_{t+i}}{r_{t,t+i}}}_{\mathbb{E}[PV(\text{Prim. Spls.})]}$$

and subtract both sides by the market value of unmatured debt:

$$\frac{B_{t-1}^{(t)}}{P_t} = \mathbb{E}_t \sum_{i=0}^{\infty} \frac{\tau_{t+i}}{r_{t,t+i}} - \mathbb{E}_t \frac{B_{t-1}^{(t+1)}}{r_{t,t+1} P_{t+1}}$$

then multiply the right-most term by $1 = B_t^{(t+1)} / B_t^{(t+1)}$ to write:

$$\frac{B_{t-1}^{(t)}}{P_t} = \mathbb{E}_t \sum_{i=0}^{\infty} \frac{\tau_{t+i}}{r_{t,t+i}} - \mathbb{E}_t \left(\frac{B_{t-1}^{(t+1)}}{r_{t,t+1} B_t^{(t+1)}} \right) \frac{B_t^{(t+1)}}{P_{t+1}}$$

noticing that $B_t^{(t+1)} / P_{t+1}$ is $B_{t-1}^{(t)} / P_t$ iterated one period into the future.

Apply Section 2's definition of the nominal dilution rate to write:

$$\frac{B_{t-1}^{(t)}}{P_t} = \mathbb{E}_t \sum_{i=0}^{\infty} \frac{\tau_{t+i}}{r_{t,t+i}} - \mathbb{E}_t \left(\frac{\Psi_{t,t}^{-1}}{r_{t,t+1}} \right) \frac{B_t^{(t+1)}}{P_{t+1}}$$

and forward-iterate once on this condition:

$$\frac{B_{t-1}^{(t)}}{P_t} = \mathbb{E}_t \sum_{i=0}^{\infty} \frac{\tau_{t+i}}{r_{t,t+i}} - \mathbb{E}_t \left(\frac{\Psi_{t,t}^{-1}}{r_{t,t+1}} \right) \left[\sum_{i=0}^{\infty} \frac{\tau_{t+1+i}}{r_{t+1,t+1+i}} - \left(\frac{\Psi_{t+1,t+1}^{-1}}{r_{t+1,t+2}} \right) \frac{B_{t+1}^{(t+2)}}{P_{t+2}} \right]$$

which simplifies to:

$$\frac{B_{t-1}^{(t)}}{P_t} = \mathbb{E}_t \sum_{i=0}^{\infty} \frac{\tau_{t+i}}{r_{t,t+i}} - \mathbb{E}_t \Psi_{t,t}^{-1} \sum_{i=0}^{\infty} \frac{\tau_{t+1+i}}{r_{t,t+1+i}} + \mathbb{E}_t \left(\frac{\Psi_{t,t+1}^{-1}}{r_{t,t+2}} \right) \frac{B_{t+1}^{(t+2)}}{P_{t+2}}$$

and continuing this forward-iteration reveals:

$$\frac{B_{t-1}^{(t)}}{P_t} = \mathbb{E}_t \sum_{i=0}^{\infty} \frac{\tau_{t+i}}{r_{t,t+i}} - \mathbb{E}_t \Psi_{t,t}^{-1} \sum_{i=0}^{\infty} \frac{\tau_{t+1+i}}{r_{t,t+1+i}} + \mathbb{E}_t \Psi_{t,t+1}^{-1} \sum_{i=0}^{\infty} \frac{\tau_{t+2+i}}{r_{t,t+2+i}} - \mathbb{E}_t \Psi_{t,t+2}^{-1} \sum_{i=0}^{\infty} \frac{\tau_{t+3+i}}{r_{t,t+3+i}} + \cdots - \cdots$$

so that I can write:

$$\frac{B_{t-1}^{(t)}}{P_t} = \tau_t + \left(\mathbb{E}_t \frac{\tau_{t+1}}{r_{t,t+1}} - \Psi_{t,t}^{-1}\mathbb{E}_t \frac{\tau_{t+1}}{r_{t,t+1}}\right) + \left(\mathbb{E}_t \frac{\tau_{t+2}}{r_{t,t+2}} - \Psi_{t,t}^{-1}\mathbb{E}_t \frac{\tau_{t+2}}{r_{t,t+2}} + \mathbb{E}_t \Psi_{t,t+1}^{-1} \frac{\tau_{t+2}}{r_{t,t+2}}\right) + \cdots$$

where, starting with the term $\mathbb{E}_t \Psi_{t,t+1}^{-1} \frac{\tau_{t+2}}{r_{t,t+2}}$, expectations of future debt policy may be correlated with that of future primary surpluses. When this covariance is zero, $\Psi_{t,t+i}^{-1}$ may be freely factored out of every $\mathbb{E}_t \Psi_{t,t+i-1}^{-1} \frac{\tau_{t+i}}{r_{t,t+i}}$ term. Then, grouping terms by τ_{t+i} allows for equation (4) in the text:

$$\underbrace{\frac{B_{t-1}^{(t)}}{P_t}}_{\text{(Mat. Debt)}/P_t} = \underbrace{\tau_t + \mathbb{E}_t \sum_{i=1}^{\infty} \frac{\tau_{t+i}}{r_{t,t+i}} \left(1 + \sum_{h=1}^{i} (-1)^h \Psi_{t,t+h-1}^{-1}\right)}_{\mathbb{E}[PV(\text{Diluted Primary Surpluses})]} \tag{4}$$

such that the household TVC holds and that the long-run condition $|B_t^{(t+1)}| > \beta |B_t^{(t+2)}|$ as $t \to \infty$ holds.

C Appendix: Traditional FTPL with Only Short-Term Debt

C.1 Model Set-Up

The model is described by equations (7)–(10), (12)–(14) where $a_t = a_{t+1} = a = 0$.

C.2 Determinacy Regions

The traditional FTPL model has one forward-looking variable, π_t, and one backward-looking variable, b_t. Here b_t denotes the government's time t debt choice (maturing at $t+1$) and b_{t-1} is the predetermined stock maturing at t. Depending on the monetary and fiscal policy rule parameters α and γ, the model may lie in one of four distinct parameter regions, two of which are associated with bounded and unique equilibria, one of which returns unbounded equilibria, and the final of which returns multiple equilibria. The two determinate regimes are listed in Table 2.

Regime	**Monetary** (α)	**Fiscal** (γ)
Regime M	$\alpha > 1$ (Active)	$\gamma > 1$ (Passive)
Regime F	$\alpha < 1$ (Passive)	$\gamma < 1$ (Active)

Table 2: Determinate Policy Regimes

The non-ρ eigenvalues of the resulting linearized system are $\lambda_1 = \alpha$ and $\lambda_2 = (1 - \gamma(1-\beta))/\beta$. For a regime to be both bounded and unique, one eigenvalue must be inside the unit circle and the other outside of it.[19] More specifically

$$\alpha > 1 \quad , \quad \frac{1-\gamma(1-\beta)}{\beta} < 1 \quad \text{or} \quad \alpha < 1 \quad , \quad \frac{1-\gamma(1-\beta)}{\beta} > 1$$

Which, as seen in Table 2, can be rewritten as

$$\alpha > 1 \quad , \quad \gamma > 1 \quad \text{or} \quad \alpha < 1 \quad , \quad \gamma < 1$$

The first case ($\alpha > 1, \gamma > 1$) relates to the 'active monetary/passive fiscal' regime (Regime M). In Regime M, the monetary authority responds to fluctuations in inflation by raising

[19] Corresponding to forward-looking π_t and backward-looking b_t.

interest rates more than 1-for-1 while fiscal policy backs monetary policy by following a tax rule aimed at retiring outstanding real debt. The second case ($\alpha < 1$, $\gamma < 1$) relates to the 'active fiscal/passive monetary' regime (Regime F). In Regime F, the fiscal authority is allowed freedom in setting its taxes while monetary policy backs fiscal policy by moving interest rates in an effort to stabilize debt by managing interest payments. No bounded equilibrium exists when both authorities are active ($\alpha > 1$, $\gamma < 1$). No unique equilibrium exists when both authorities are passive ($\alpha < 1$, $\gamma > 1$).

Figure 14 displays a determinacy map outlining the model's four parameter regions visually. Figures 15 and 16 display impulse response functions (IRFs) of inflation and real debt from policy shocks $\left\{\varepsilon_t^R,\ \varepsilon_t^\tau\right\}$. Figure 15 shows IRFs to policy shocks in Regime M ($\alpha = 1.01$, $\gamma = 5$) and Figure 16 shows IRFs to policy shocks in Regime F ($\alpha = 0.95$, $\gamma = 0.05$).

C.3 Calibration and Impulse Responses

I calibrate model variables according to Table 1.[20]

C.4 Displaying the Log-Linearized Traditional Model

This section displays the steps taken to prepare the linearized short-term only model for analysis. The first step is to linearize the equations described above. Variables without time subscripts are steady-state quantities.

$$Q_t = -\mathbb{E}_t \pi_{t+1}$$

$$Q_t = -R_t$$

$$R_t = \alpha \pi_t + z_t^R$$

$$\tau_t = \gamma b_{t-1} + z_t^\tau$$

$$b_{t-1} - \pi_t = \left(\frac{\pi\tau}{b}\right)\tau_t + \beta\left(b_t - R_t\right)$$

$$z_{t+1}^R = \rho_R z_t^R + \varepsilon_{t+1}^R$$

$$z_{t+1}^\tau = \rho_\tau z_t^\tau + \varepsilon_{t+1}^\tau$$

After substituting in policy rules, we have:

$$\mathbb{E}_t \pi_{t+1} = \alpha \pi_t + z_t^R$$

[20] Minus all variables that are present in the paper's model but not in the traditional model.

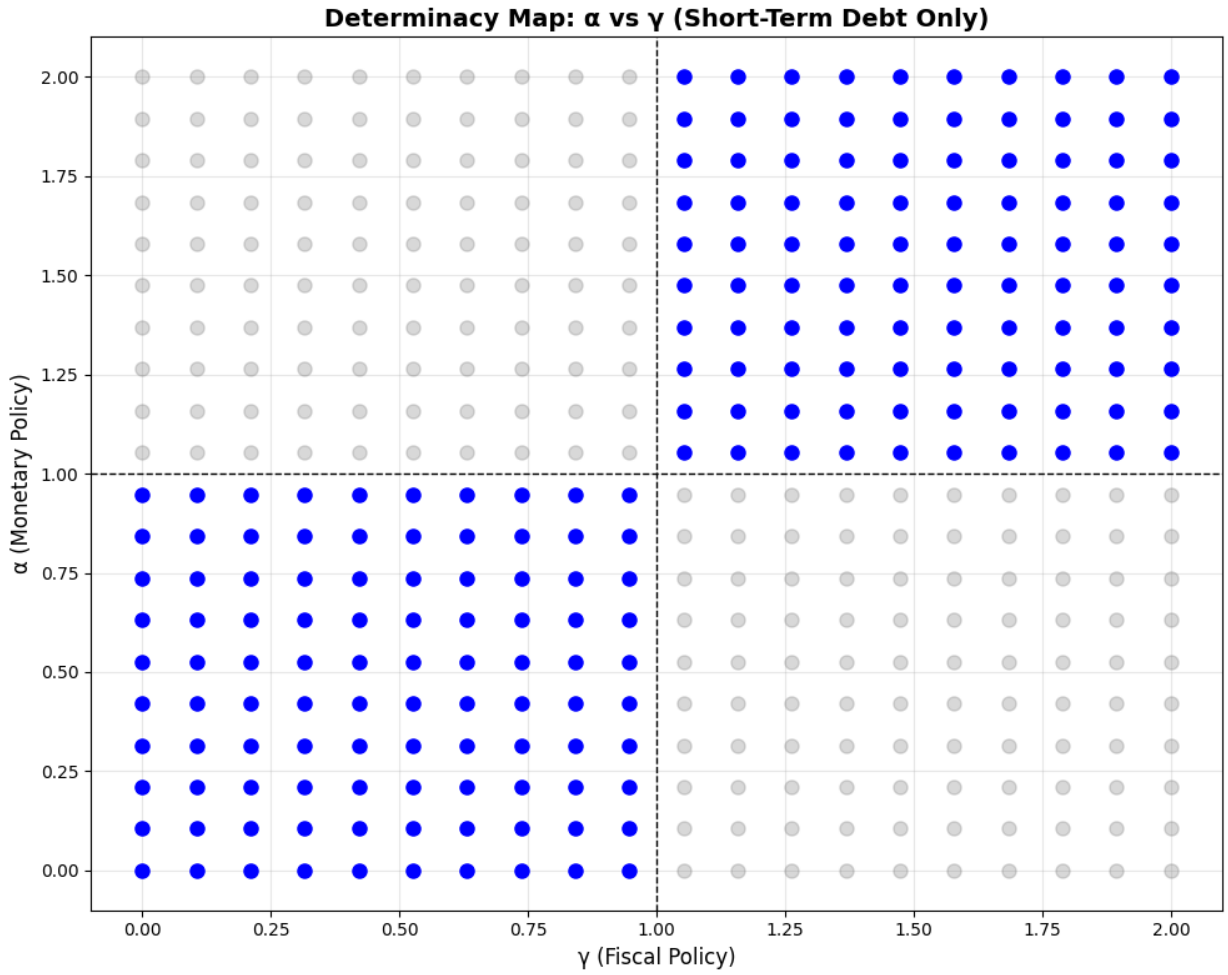


Figure 14: Determinacy Map Across Policy Regimes

The figure displays determinacy regions across different policy parameter combinations. Blue circles indicate determinate equilibria, while gray circles represent indeterminate equilibria. The dashed lines at $\alpha = 1$ and $\gamma = 1$ delineate the boundaries between active and passive policy regimes. The northwest quadrant represents Regime NB, the northeast quadrant represents Regime M, the southwest quadrant represents Regime F, and the southeast quadrant represents Regime NU. All parameter combinations are searched on a grid with $n = 20$ points per dimension.

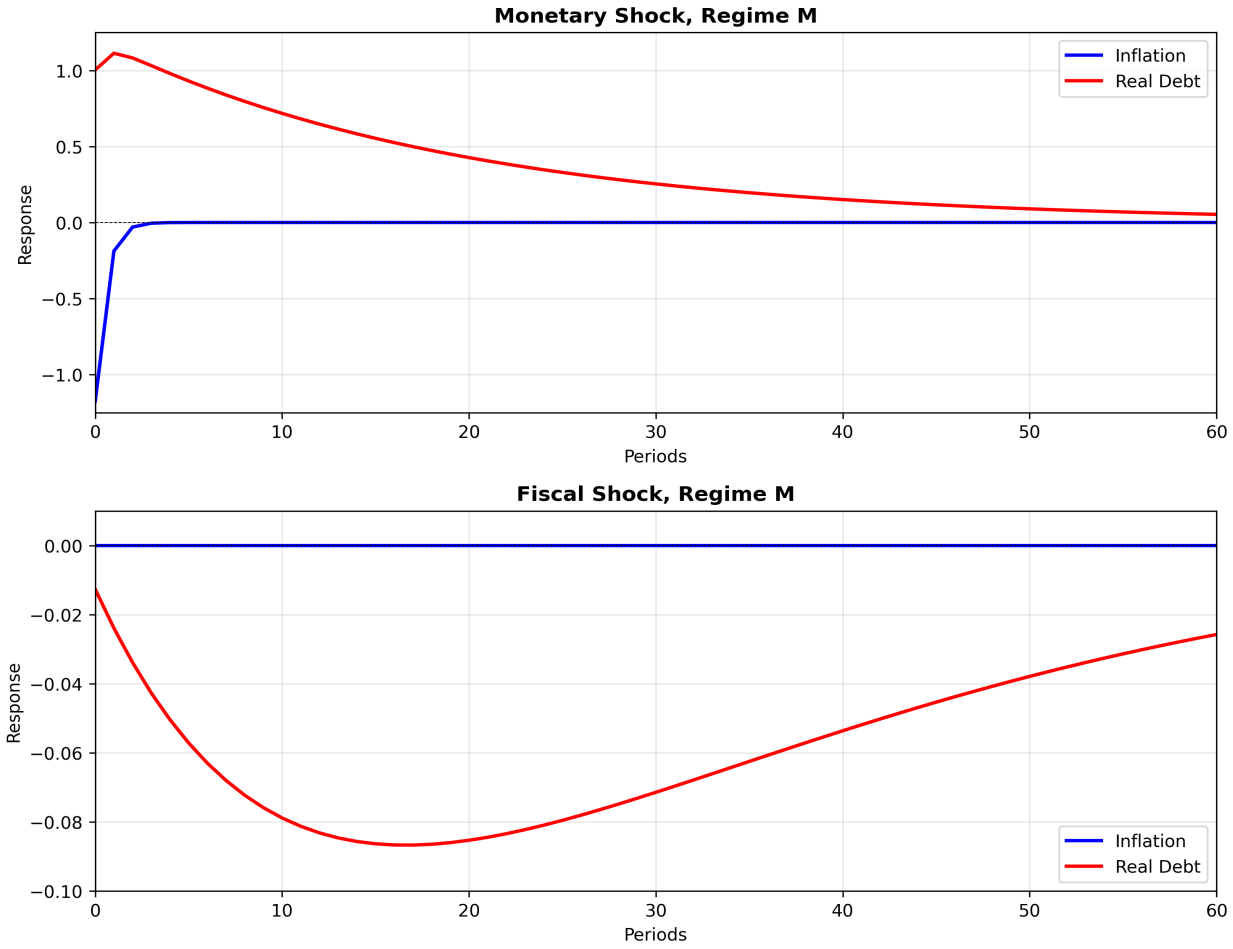


Figure 15: Regime M IRFs

This figure displays IRFs for inflation (blue line) and real maturing debt (red line) following positive, one standard deviation shocks to monetary policy ($\varepsilon^R \uparrow$) and fiscal policy ($\varepsilon^\tau \uparrow$). The parameter space corresponds to Regime M: $\alpha = 1.01$, $\gamma = 5$. **Top panel:** Responses to a monetary policy shock show a negative inflation response and a positive real debt response. **Bottom panel:** A fiscal policy shock generates no inflation response and a negative real debt response.

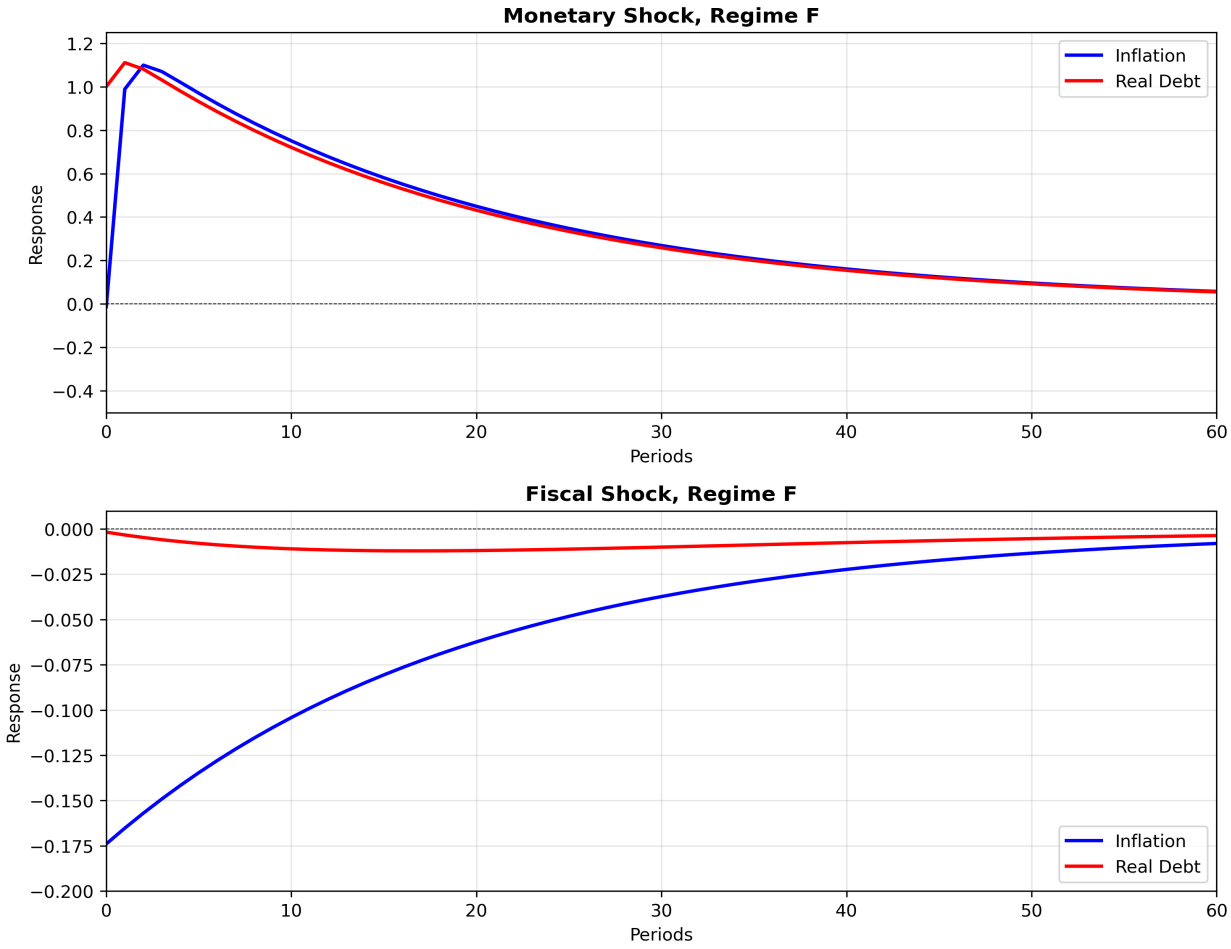


Figure 16: Regime F IRFs

This figure displays IRFs for inflation (blue line) and real maturing debt (red line) following positive, one standard deviation shocks to monetary policy ($\varepsilon^R \uparrow$) and fiscal policy ($\varepsilon^\tau \uparrow$). The parameter space corresponds to Regime F: $\alpha = 0.95$, $\gamma = 0.05$. **Top panel:** Responses to a monetary policy shock show a positive real debt response and a delayed positive inflation response. **Bottom panel:** A fiscal policy shock generates a negative inflation response and a negative real debt response.

$$b_{t-1} - \pi_t = \left(\frac{\pi\tau}{b}\right)(\gamma b_{t-1} + z_t^\tau) + \beta\left[b_t - \left(\alpha\pi_t + z_t^R\right)\right]$$

$$z_{t+1}^R = \rho_R z_t^R + \varepsilon_{t+1}^R$$

$$z_{t+1}^\tau = \rho_\tau z_t^\tau + \varepsilon_{t+1}^\tau$$

Define the forecast error η_{t+1}^π such that:

$$\mathbb{E}_t \pi_{t+1} = \pi_{t+1} - \eta_{t+1}^\pi$$

and substitute it into the system to write:

$$\pi_{t+1} = \alpha\pi_t + z_t^R + \eta_{t+1}^\pi$$

$$b_{t-1} - \pi_t = \left(\frac{\pi\tau}{b}\right)(\gamma b_{t-1} + z_t^\tau) + \beta\left[b_t - \left(\alpha\pi_t + z_t^R\right)\right]$$

$$z_{t+1}^R = \rho_R z_t^R + \varepsilon_{t+1}^R$$

$$z_{t+1}^\tau = \rho_\tau z_t^\tau + \varepsilon_{t+1}^\tau$$

Substituting in the steady state value $\frac{\pi\tau}{b} = 1 - \beta$ and rearranging yields:

$$\pi_{t+1} = \alpha\pi_t + z_t^R + \eta_{t+1}^\pi$$

$$(1 - \beta\alpha)\pi_{t+1} + \beta b_{t+1} - \beta z_{t+1}^R + (1 - \beta) z_{t+1}^\tau = (1 - \gamma(1 - \beta)) b_t$$

$$z_{t+1}^R = \rho_R z_t^R + \varepsilon_{t+1}^R$$

$$z_{t+1}^\tau = \rho_\tau z_t^\tau + \varepsilon_{t+1}^\tau$$

which can be written in canonical form:

$$\Gamma_0 y_{t+1} = \Gamma_1 y_t + \Psi\varepsilon_{t+1} + \Pi\eta_{t+1}$$

Where, given a $\pi = 1$ steady state:

$$\Gamma_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 - \beta\alpha & \beta & -\beta & 1 - \beta \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad , \quad \Gamma_1 = \begin{bmatrix} \alpha & 0 & 1 & 0 \\ 0 & 1 - \gamma(1 - \beta) & 0 & 0 \\ 0 & 0 & \rho_R & 0 \\ 0 & 0 & 0 & \rho_\tau \end{bmatrix}$$

$$\Psi = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \quad , \quad \Pi = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

And:

$$y_t = \begin{bmatrix} \pi_t \\ b_t \\ z_t^R \\ z_t^\tau \end{bmatrix} \quad , \quad \varepsilon_t = \begin{bmatrix} \varepsilon_t^R \\ \varepsilon_t^\tau \end{bmatrix} \quad , \quad \eta_t = \begin{bmatrix} \eta_t^\pi \end{bmatrix}$$

Define Φ:

$$\Phi = [\Psi \ \bar{0} \ \Pi]$$

Where

$$\bar{0} = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}'$$

Then we can rewrite the canonical form:

$$\Gamma_0 x_{t+1} = \Gamma_1 x_t + \Phi \xi_{t+1}$$

Where:

$$\xi_t = \begin{bmatrix} \varepsilon_t^R & \varepsilon_t^\tau & 0 & \eta_t^\pi \end{bmatrix}'$$

D Appendix: Baseline Model in Canonical Form

This section displays the steps taken to prepare the linearized model for analysis. The first step is to linearize the equations described in Section 3 of the text. Variables without time subscripts are steady-state quantities.

$$Q_t = -\mathbb{E}_t\left[\pi_{t+1}\right]$$

$$Q_t = -R_t$$

$$R_t = \alpha\pi_t + z_t^R$$

$$\tau_t = \gamma b_{t-1} + z_t^\tau$$

$$a_t = \varphi\pi_t + \delta b_{t-1} + z_t^a$$

$$b_{t-1} - \pi_t + \frac{\beta a}{\pi}\left(a_t + b_t - \pi_t - \mathbb{E}_t\left[\pi_{t+1}\right]\right)$$

$$= \left(\frac{\pi\tau}{b}\right)\tau_t + \beta\left(b_t - \mathbb{E}_t\left[\pi_{t+1}\right]\right) + \frac{\beta^2 a}{\pi}\left(\mathbb{E}_t\left[a_{t+1}\right] + \mathbb{E}_t\left[b_{t+1}\right] - \mathbb{E}_t\left[\pi_{t+1}\right] - \mathbb{E}_t\left[\pi_{t+2}\right]\right)$$

$$z_{t+1}^R = \rho_R z_t^R + \varepsilon_{t+1}^R$$

$$z_{t+1}^\tau = \rho_\tau z_t^\tau + \varepsilon_{t+1}^\tau$$

$$z_{t+1}^a = \rho_a z_t^a + \varepsilon_{t+1}^a$$

To standardize timing conventions, I define $\tilde{b}_t = b_{t-1}$ and write:

$$Q_t = -\mathbb{E}_t\left[\pi_{t+1}\right]$$

$$Q_t = -R_t$$

$$R_t = \alpha\pi_t + z_t^R$$

$$\tau_t = \gamma\tilde{b}_t + z_t^\tau$$

$$a_t = \varphi\pi_t + \delta\tilde{b}_t + z_t^a$$

$$\tilde{b}_t - \pi_t + \frac{\beta a}{\pi}\left(a_t + \tilde{b}_{t+1} - \pi_t - \mathbb{E}_t\left[\pi_{t+1}\right]\right)$$

$$= \left(\frac{\pi\tau}{b}\right)\tau_t + \beta\left(\tilde{b}_{t+1} - \mathbb{E}_t\left[\pi_{t+1}\right]\right) + \frac{\beta^2 a}{\pi}\left(\mathbb{E}_t\left[a_{t+1}\right] + \mathbb{E}_t\left[\tilde{b}_{t+2}\right] - \mathbb{E}_t\left[\pi_{t+1}\right] - \mathbb{E}_t\left[\pi_{t+2}\right]\right)$$

$$z_{t+1}^R = \rho_R z_t^R + \varepsilon_{t+1}^R$$

$$z_{t+1}^{\tau} = \rho_{\tau} z_t^{\tau} + \varepsilon_{t+1}^{\tau}$$

$$z_{t+1}^{a} = \rho_{a} z_t^{a} + \varepsilon_{t+1}^{a}$$

The GBC is a second order difference equation in $\tilde{b}_t$. To put it into Gensys canonical form, I first define $x_t = \tilde{b}_{t+1}$ to write this system as:

$$\mathbb{E}_t\left[\pi_{t+1}\right] = \alpha \pi_t + z_t^R$$

$$\tau_t = \gamma \tilde{b}_t + z_t^{\tau}$$

$$a_t = \varphi \pi_t + \delta \tilde{b}_t + z_t^a$$

$$\tilde{b}_t - \pi_t + \frac{\beta a}{\pi}\left(a_t + \tilde{b}_{t+1} - \pi_t - \mathbb{E}_t\left[\pi_{t+1}\right]\right)$$

$$= \left(\frac{\pi \tau}{b}\right)\tau_t + \beta\left(\tilde{b}_{t+1} - \mathbb{E}_t\left[\pi_{t+1}\right]\right) + \frac{\beta^2 a}{\pi}\left(\mathbb{E}_t\left[a_{t+1}\right] + \mathbb{E}_t\left[x_{t+1}\right] - \mathbb{E}_t\left[\pi_{t+1}\right] - \mathbb{E}_t\left[\pi_{t+2}\right]\right)$$

$$x_t = \tilde{b}_{t+1}$$

$$z_{t+1}^R = \rho_R z_t^R + \varepsilon_{t+1}^R$$

$$z_{t+1}^{\tau} = \rho_{\tau} z_t^{\tau} + \varepsilon_{t+1}^{\tau}$$

$$z_{t+1}^{a} = \rho_{a} z_t^{a} + \varepsilon_{t+1}^{a}$$

After substituting in policy rules, we have:

$$\mathbb{E}_t\left[\pi_{t+1}\right] = \alpha \pi_t + z_t^R$$

$$\tilde{b}_t - \pi_t + \frac{\beta a}{\pi}\left(\left(\varphi \pi_t + \delta \tilde{b}_t + z_t^a\right) + \tilde{b}_{t+1} - \pi_t - \mathbb{E}_t\left[\pi_{t+1}\right]\right)$$

$$= \left(\frac{\pi \tau}{b}\right)\left(\gamma \tilde{b}_t + z_t^{\tau}\right) + \beta\left(\tilde{b}_{t+1} - \mathbb{E}_t\left[\pi_{t+1}\right]\right) + \frac{\beta^2 a}{\pi}\left(\mathbb{E}_t\left[\varphi \pi_{t+1} + \delta \tilde{b}_{t+1} + z_{t+1}^a\right] + \mathbb{E}_t\left[x_{t+1}\right] - \mathbb{E}_t\left[\pi_{t+1}\right] - \mathbb{E}_t\left[\pi_{t+2}\right]\right)$$

$$x_t = \tilde{b}_{t+1}$$

$$z_{t+1}^R = \rho_R z_t^R + \varepsilon_{t+1}^R$$

$$z_{t+1}^{\tau} = \rho_{\tau} z_t^{\tau} + \varepsilon_{t+1}^{\tau}$$

$$z_{t+1}^{a} = \rho_{a} z_t^{a} + \varepsilon_{t+1}^{a}$$

I simplify the GBC slightly to write:

$$\mathbb{E}_t\left[\pi_{t+1}\right] = \alpha\pi_t + z_t^R$$

$$\tilde{b}_t - \pi_t + \frac{\beta a}{\pi}\left(\left(\varphi\pi_t + \delta\tilde{b}_t + z_t^a\right) + \tilde{b}_{t+1} - \pi_t - \mathbb{E}_t\left[\pi_{t+1}\right]\right)$$

$$= \frac{\beta^2 a}{\pi}\left(\varphi\mathbb{E}_t\left[\pi_{t+1}\right] + \delta\tilde{b}_{t+1} + \rho_a z_t^a + \mathbb{E}_t\left[x_{t+1}\right] - \mathbb{E}_t\left[\pi_{t+1}\right] - \mathbb{E}_t\left[\pi_{t+2}\right]\right)$$

$$+\left(\frac{\pi\tau}{b}\right)\left(\gamma\tilde{b}_t + z_t^\tau\right) + \beta\left(\tilde{b}_{t+1} - \mathbb{E}_t\left[\pi_{t+1}\right]\right)$$

$$x_t = \tilde{b}_{t+1}$$

$$z_{t+1}^R = \rho_R z_t^R + \varepsilon_{t+1}^R$$

$$z_{t+1}^\tau = \rho_\tau z_t^\tau + \varepsilon_{t+1}^\tau$$

$$z_{t+1}^a = \rho_a z_t^a + \varepsilon_{t+1}^a$$

I define forecast errors η_{t+1}^π and η_{t+1}^x such that:

$$\mathbb{E}_t\left[\pi_{t+1}\right] = \pi_{t+1} - \eta_{t+1}^\pi$$

$$\mathbb{E}_t\left[x_{t+1}\right] = x_{t+1} - \eta_{t+1}^x$$

and substitute these into the system to write:

$$\pi_{t+1} - \eta_{t+1}^\pi = \alpha\pi_t + z_t^R$$

$$\tilde{b}_t - \pi_t + \frac{\beta a}{\pi}\left(\left(\varphi\pi_t + \delta\tilde{b}_t + z_t^a\right) + \tilde{b}_{t+1} - \pi_t - \pi_{t+1} + \eta_{t+1}^\pi\right) = \left(\frac{\pi\tau}{b}\right)\left(\gamma\tilde{b}_t + z_t^\tau\right)$$

$$+\beta\left(\tilde{b}_{t+1} - \pi_{t+1} + \eta_{t+1}^\pi\right) + \frac{\beta^2 a}{\pi}\left(\varphi\left(\pi_{t+1} - \eta_{t+1}^\pi\right) + \delta\tilde{b}_{t+1} + \rho_a z_t^a + x_{t+1} - \eta_{t+1}^x - \pi_{t+1} + \eta_{t+1}^\pi - \mathbb{E}_t\left[\pi_{t+2}\right]\right)$$

$$x_t = \tilde{b}_{t+1}$$

$$z_{t+1}^R = \rho_R z_t^R + \varepsilon_{t+1}^R$$

$$z_{t+1}^\tau = \rho_\tau z_t^\tau + \varepsilon_{t+1}^\tau$$

$$z_{t+1}^a = \rho_a z_t^a + \varepsilon_{t+1}^a$$

Next I substitute $\mathbb{E}_t[\pi_{t+2}] = \alpha(\pi_{t+1} - \eta^{\pi}_{t+1}) + \rho_R z^R_t$, obtained from the Fisher equation iterated one period forward, into the GBC to write:

$$\pi_{t+1} = \alpha\pi_t + z^R_t + \eta^{\pi}_{t+1}$$

$$\tilde{b}_t - \pi_t + \frac{\beta a}{\pi}\left(\left(\varphi\pi_t + \delta\tilde{b}_t + z^a_t\right) + \tilde{b}_{t+1} - \pi_t - \pi_{t+1} + \eta^{\pi}_{t+1}\right) = \left(\frac{\pi\tau}{b}\right)\left(\gamma\tilde{b}_t + z^{\tau}_t\right)$$

$$+\beta\left(\tilde{b}_{t+1} - \pi_{t+1} + \eta^{\pi}_{t+1}\right) + \frac{\beta^2 a}{\pi}\left(\varphi\left(\pi_{t+1} - \eta^{\pi}_{t+1}\right) + \delta\tilde{b}_{t+1} + \rho_a z^a_t + x_{t+1} - \eta^x_{t+1} - \pi_{t+1} + \eta^{\pi}_{t+1}\right.$$

$$\left.-\alpha\left(\pi_{t+1} - \eta^{\pi}_{t+1}\right) - \rho_R z^R_t\right)$$

$$x_t = \tilde{b}_{t+1}$$

$$z^R_{t+1} = \rho_R z^R_t + \varepsilon^R_{t+1}$$

$$z^{\tau}_{t+1} = \rho_{\tau} z^{\tau}_t + \varepsilon^{\tau}_{t+1}$$

$$z^a_{t+1} = \rho_a z^a_t + \varepsilon^a_{t+1}$$

$$\mathbb{E}_t\left[\pi_{t+1}\right] = \pi_{t+1} - \eta^{\pi}_{t+1}$$

$$\mathbb{E}_t\left[x_{t+1}\right] = x_{t+1} - \eta^x_{t+1}$$

I can group terms in the GBC to write the system out as

$$\pi_{t+1} = \alpha\pi_t + z^R_t + \eta^{\pi}_{t+1}$$

$$\pi_{t+1}\left(\beta - \frac{\beta a}{\pi} + \frac{\beta^2 a}{\pi} + \frac{\beta^2 a\alpha}{\pi} - \frac{\beta^2 a\varphi}{\pi}\right) + \tilde{b}_{t+1}\left(\frac{\beta a}{\pi} - \beta - \frac{\beta^2 a\delta}{\pi}\right) + x_{t+1}\left(-\frac{\beta^2 a}{\pi}\right)$$

$$= \pi_t\left(1 - \frac{\beta a\varphi}{\pi} + \frac{\beta a}{\pi}\right) + \tilde{b}_t\left(\frac{\pi\tau\gamma}{b} - 1 - \frac{\beta a\delta}{\pi}\right) + x_t\left(0\right) + z^R_t\left(-\frac{\beta^2 a\rho_R}{\pi}\right) + z^{\tau}_t\left(\frac{\pi\tau}{b}\right) + z^a_t\left(\frac{\beta^2 a\rho_a}{\pi} - \frac{\beta a}{\pi}\right)$$

$$+\eta^{\pi}_{t+1}\left(\beta - \frac{\beta a}{\pi} + \frac{\beta^2 a}{\pi} + \frac{\beta^2 a\alpha}{\pi} - \frac{\beta^2 a\varphi}{\pi}\right) + \eta^x_{t+1}\left(-\frac{\beta^2 a}{\pi}\right)$$

$$x_t = \tilde{b}_{t+1}$$

$$z^R_{t+1} = \rho_R z^R_t + \varepsilon^R_{t+1}$$

$$z^{\tau}_{t+1} = \rho_{\tau} z^{\tau}_t + \varepsilon^{\tau}_{t+1}$$

$$z^a_{t+1} = \rho_a z^a_t + \varepsilon^a_{t+1}$$

$$\mathbb{E}_t\left[\pi_{t+1}\right] = \pi_{t+1} - \eta^{\pi}_{t+1}$$

$$\mathbb{E}_t\left[x_{t+1}\right] = x_{t+1} - \eta^x_{t+1}$$

Noticing that $\frac{\pi\tau}{b} = (1-\beta)(1+\frac{\beta a}{\pi})$, I rewrite this system as:

$$\pi_{t+1} = \alpha\pi_t + z^R_t + \eta^\pi_{t+1}$$

$$\pi_{t+1}\left(\beta - \frac{\beta a}{\pi} + \frac{\beta^2 a}{\pi} + \frac{\beta^2 a\alpha}{\pi} - \frac{\beta^2 a\varphi}{\pi}\right) + \tilde{b}_{t+1}\left(\frac{\beta a}{\pi} - \beta - \frac{\beta^2 a\delta}{\pi}\right) + x_{t+1}\left(-\frac{\beta^2 a}{\pi}\right)$$

$$= \pi_t\left(1 - \frac{\beta a\varphi}{\pi} + \frac{\beta a}{\pi}\right) + \tilde{b}_t\left(\gamma\left(1-\beta\right)\left(1+\frac{\beta a}{\pi}\right) - 1 - \frac{\beta a\delta}{\pi}\right) + x_t\left(0\right) + z^R_t\left(-\frac{\beta^2 a\rho_R}{\pi}\right) + z^\tau_t\left(1-\beta\right)\left(1+\frac{\beta a}{\pi}\right)$$

$$+ z^a_t\left(\frac{\beta^2 a\rho_a}{\pi} - \frac{\beta a}{\pi}\right) + \eta^\pi_{t+1}\left(\beta - \frac{\beta a}{\pi} + \frac{\beta^2 a}{\pi} + \frac{\beta^2 a\alpha}{\pi} - \frac{\beta^2 a\varphi}{\pi}\right) + \eta^x_{t+1}\left(-\frac{\beta^2 a}{\pi}\right)$$

$$x_t = \tilde{b}_{t+1}$$

$$z^R_{t+1} = \rho_R z^R_t + \varepsilon^R_{t+1}$$

$$z^\tau_{t+1} = \rho_\tau z^\tau_t + \varepsilon^\tau_{t+1}$$

$$z^a_{t+1} = \rho_a z^a_t + \varepsilon^a_{t+1}$$

$$\mathbb{E}_t\left[\pi_{t+1}\right] = \pi_{t+1} - \eta^\pi_{t+1}$$

$$\mathbb{E}_t\left[x_{t+1}\right] = x_{t+1} - \eta^x_{t+1}$$

Simplifying coefficients in the GBC and multiplying the GBC by π results in:

$$\pi_{t+1} = \alpha\pi_t + z^R_t + \eta^\pi_{t+1}$$

$$\beta\left[\pi - a\left(1-\beta\left(1+\alpha-\varphi\right)\right)\right]\pi_{t+1} + \beta\left[a\left(1-\beta\delta\right) - \pi\right]\tilde{b}_{t+1} - \left(\beta^2 a\right)x_{t+1}$$

$$= \left[\pi + \beta a\left(1-\varphi\right)\right]\pi_t + \left[\gamma\left(1-\beta\right)\left(\pi+\beta a\right) - \pi - \beta a\delta\right]\tilde{b}_t + \left(0\right)x_t - \left[\beta^2 a\rho_R\right]z^R_t + \left[\left(1-\beta\right)\left(\pi+\beta a\right)\right]z^\tau_t$$

$$- \left[\beta a\left(1-\beta\rho_a\right)\right]z^a_t + \beta\left[\pi - a\left(1-\beta\left(1+\alpha-\varphi\right)\right)\right]\eta^\pi_{t+1} - \left(\beta^2 a\right)\eta^x_{t+1}$$

$$\tilde{b}_{t+1} = x_t$$

$$z^R_{t+1} = \rho_R z^R_t + \varepsilon^R_{t+1}$$

$$z^\tau_{t+1} = \rho_\tau z^\tau_t + \varepsilon^\tau_{t+1}$$

$$z^a_{t+1} = \rho_a z^a_t + \varepsilon^a_{t+1}$$

$$\mathbb{E}_t\left[\pi_{t+1}\right] = \pi_{t+1} - \eta^\pi_{t+1}$$

$$\mathbb{E}_t\left[x_{t+1}\right] = x_{t+1} - \eta^x_{t+1}$$

Next I write the system in canonical form:

$$\Gamma_0 y_{t+1} = \Gamma_1 y_t + \Omega\varepsilon_{t+1} + \Pi\eta_{t+1}$$

Where, given a $\pi = 1$ steady state:

$$\Gamma_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \beta\left[1 - a\left(1 - \beta\left(1 + \alpha - \varphi\right)\right)\right] & \beta\left[a\left(1 - \beta\delta\right) - 1\right] & -\beta^2 a & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Gamma_1 = \begin{bmatrix} \alpha & 0 & 0 & 1 & 0 & 0 \\ \left[1 + \beta a\left(1 - \varphi\right)\right] & \left[\gamma\left(1 - \beta\right)\left(1 + \beta a\right) - 1 - \beta a\delta\right] & 0 & -\left[\beta^2 a\rho_R\right] & \left[\left(1 - \beta\right)\left(1 + \beta a\right)\right] & -\left[\beta a\left(1 - \beta\rho_a\right)\right] \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho_R & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho_\tau & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_a \end{bmatrix}$$

$$\Omega = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad , \quad \Pi = \begin{bmatrix} 1 & 0 \\ \beta\left[1 - a\left(1 - \beta\left(1 + \alpha - \varphi\right)\right)\right] & -\beta^2 a \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

And

$$y_t = \begin{bmatrix} \pi_t & \tilde{b}_t & x_t & z^R_t & z^\tau_t & z^a_t \end{bmatrix}' \quad , \quad \varepsilon_t = \begin{bmatrix} \varepsilon^R_t & \varepsilon^\tau_t & \varepsilon^a_t \end{bmatrix}' \quad , \quad \eta_t = \begin{bmatrix} \eta^\pi_t & \eta^x_t \end{bmatrix}'$$

I define Φ:

$$\Phi = \left[\Omega\ \bar{0}\ \Pi\right]$$

where

$$\bar{0} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}'$$

to finally write:

$$\Gamma_0 y_{t+1} = \Gamma_1 y_t + \Phi \xi_{t+1}$$

where

$$\xi_t = \begin{bmatrix} \varepsilon_t^R & \varepsilon_t^\tau & \varepsilon_t^a & 0 & \eta_t^\pi & \eta_t^x \end{bmatrix}'$$

E Appendix: With J-Period Long-Term Debt

A drawback of $J = 2$-period long-term debt is that the duration of the debt stock may not exceed 2 periods. Here I consider the paper's model where long-term debt matures in $J \geq 2$ periods. The flow GBC becomes

$$\frac{B_{t-1}^{(t)}}{P_t} + \frac{Q_t^{(t+J-1)} B_{t-1}^{(t+J-1)}}{P_t} = \frac{Q_t^{(t+1)} B_t^{(t+1)}}{P_t} + \frac{Q_t^{(t+J)} B_t^{(t+J)}}{P_t} + \tau_t$$

so that the forward-iterated GBC is:

$$\underbrace{\frac{B_{t-1}^{(t)}}{P_t} + \mathbb{E}_t \frac{B_{t-1}^{(t+J-1)}}{r_{t,t+J-1}\, P_{t+J-1}}}_{\text{MV(Debt)}/P_t} = \underbrace{\mathbb{E}_t \sum_{i=0}^{\infty} \frac{\tau_{t+i}}{r_{t,t+i}}}_{\mathbb{E}[\text{PV(Surpluses)}]}$$

The nominal dilution rate of government debt generalizes to

$$\Psi_{t,t} = \frac{B_t^{(t+1)}}{B_{t-J+1}^{(t+1)}}$$

where the h-step product of inverse dilution rates, with steps spaced $J-1$ periods apart, is

$$\Psi_{t,\, t+h(J-1)-1}^{-1} = \prod_{k=1}^{h} \Psi_{t+k(J-1)-1,\, t+k(J-1)-1}^{-1}$$

Similar to Appendix B, I subtract the unmatured term from both sides of the forward-iterated GBC to reveal

$$\frac{B_{t-1}^{(t)}}{P_t} = \mathbb{E}_t \sum_{i=0}^{\infty} \frac{\tau_{t+i}}{r_{t,t+i}} - \mathbb{E}_t \frac{B_{t-1}^{(t+J-1)}}{r_{t,t+J-1}\, P_{t+J-1}}$$

and following a similar forward iteration process to Appendix B, this may be written as

$$\frac{B_{t-1}^{(t)}}{P_t} = \tau_t + \mathbb{E}_t \sum_{i=1}^{\infty} \frac{\tau_{t+i}}{r_{t,t+i}} \left(1 + \sum_{h=1}^{\lfloor i/(J-1) \rfloor} (-1)^h \, \Psi_{t,\, t+h(J-1)-1}^{-1} \right)$$

where $\lfloor \cdot \rfloor$ represents the floor function.

In steady state with $Y_t = Y$, $\tau_t = \tau$, $P_t = P$, and constant debt stocks, the present value condition (E) becomes

$$\frac{B^{(1)}}{P} + \frac{\beta^{J-1} B^{(J)}}{P} = \frac{\tau}{1-\beta}$$

where $B^{(1)}$ is steady state total maturing debt and $B^{(J)}$ is steady state J-period issuance per

period, which can be rewritten as

$$\frac{B^{(1)}}{P} = \left(\frac{1}{1-\beta}\right)\left(\frac{1}{1+\beta^{J-1}\Psi^{-1}}\right)\tau$$

which reveals the generalized long-run restriction

$$\beta^{J-1}|\Psi^{-1}|< 1$$

and the duration of the government's debt structure

$$D = \frac{\Psi + J\beta^{J-1}}{\Psi + \beta^{J-1}}$$

Defining the generalized real dilution rate as

$$\psi_{t,t} = \frac{b_t^{(t+1)}}{b_{t-J+1}^{(t+1)}}$$

gives a relationship between the nominal and real dilution rates

$$\Psi_{t,t} = \psi_{t,t}\frac{P_t}{P_{t-J+1}} = \psi_{t,t}\prod_{h=0}^{J-2}\pi_{t-h}$$

so that the the flow GBC may be written as

$$\frac{b_{t-1}^{(t)}}{\pi_t} + \frac{Q_t^{(t+J-1)}}{\pi_t}\,\mathbb{E}_t\Big[a_{t+J-2}\,b_{t+J-2}^{(t+J-1)}\Big] = \tau_t + Q_t^{(t+1)}b_t^{(t+1)} + Q_t^{(t+J)}\,\mathbb{E}_t\Big[a_{t+J-1}\,b_{t+J-1}^{(t+J)}\Big]$$

where each debt variable is short-term.

F Appendix: Data Sources and Construction

This paper uses four main time series from 1949Q1–2022Q4: U.S. tax rate, Fed's policy rate, U.S. real debt dilution rate, and U.S. price levels. Their construction and data sources are below:

- U.S. Tax Rate, constructed as total U.S. receipts over U.S. GDP:
  - Receipts: Section 3, table T30200-Q, line 40 in the National Income and Product Accounts (NIPA) from the BEA and found at `https://apps.bea.gov/iTable/?isuri=1&reqid=19&step=4&categories=flatfiles&nipa_table_list=1`.
  - GDP: Gross domestic product. Section 1, table T10105-Q, line 1 in the National Income and Product Accounts (NIPA) from the BEA. Found at `https://apps.bea.gov/iTable/?isuri=1&reqid=19&step=4&categories=flatfiles&nipa_table_list=1`.
- Fed's Policy Rate, average NY Fed discount rates from 1949Q1–1954Q2 and average Fed Funds rates from 1954Q3–2022Q4:
  - NY Fed discount rates: National Bureau of Economic Research, Release: NBER Macrohistory Database. `https://fred.stlouisfed.org/series/M13009USM156NNBR`
  - Fed Funds effective rate: Source: Board of Governors of the Federal Reserve System (US), Release: H.15 Selected Interest Rates. `https://fred.stlouisfed.org/series/FEDFUNDS`
- U.S. Inflation (1943–2023)
  - Growth rate in annual GDP deflator. Section 1, table T10109-A, line 1 in the NIPA from the BEA. Found at `https://apps.bea.gov/iTable/?isuri=1&reqid=19&step=4&categories=flatfiles&nipa_table_list=1`
- U.S. Real Dilution Rate
  - Constructed as previous-period [(long-term debt outstanding)/(price level)] divided by [(short-term debt outstanding)/(price level)].
  - Long-term debt is Privately-Held Treasurys maturing in more than one quarter.

- Short-term debt calculated as (Reserves Outstanding) + (Privately-Held Treasurys maturing in less than one quarter).
  - Privately-Held Debt (1948Q4–2022Q4). Center for Research in Security Prices (CRSP) U.S. Treasury Database
  - Reserves Outstanding (1949Q1–2002Q4). Calculated as the average reserves outstanding/GDP from 2002–2022 multiplied by GDP from 1942–2001. Member Bank Reserve Account. Source: Center for Financial Stability Release: The Federal Reserve System's Weekly Balance Sheet Since 1914. `https://fred.stlouisfed.org/series/LDMB`
  - Reserves Outstanding (2003Q1–2022Q4). Calculated as (Federal Reserve Notes, Net of F.R. Bank Holdings) + (Deposits with F.R. Banks, Other Than Reserve Balances) + (Other Deposits at the Fed) + (Term Deposits Held by Depository Institutions) - (U.S. Treasury, Supplementary Financing Account) - (Treasury balance in TGA) + (Reverse Repurchase Agreements).
    - Federal Reserve Notes, Net of F.R. Bank Holdings. Table H.4.1.T5 on the Fed's weekly balance sheet and found at `https://fred.stlouisfed.org/series/WLFN`.
    - Deposits with F.R. Banks, Other Than Reserve Balances. Table H.4.1.T5 on the Fed's weekly balance sheet and found at `https://fred.stlouisfed.org/series/WDFOL`.
    - Other Deposits at the Fed. Table H.4.1.T5 on the Fed's weekly balance sheet and found at `https://fred.stlouisfed.org/series/WLODL`.
    - Term Deposits Held by Depository Institutions. Table H.4.1.T5 on the Fed's weekly balance sheet and found at `https://fred.stlouisfed.org/series/TERMT`.
    - U.S. Treasury, Supplementary Financing Account. Table H.4.1.T5 on the Fed's weekly balance sheet and found at `https://fred.stlouisfed.org/series/WLSFAL`.
    - Treasury balance in TGA. Daily Treasury Statements, found at `https://fsapps.fiscal.treasury.gov/dts/issues`.
    - Reverse Repurchase Agreements. Table H.4.1.T5 on the Fed's weekly balance sheet and found at `https://fred.stlouisfed.org/series/WLRRAL`.