Supply Chain Risk, Trade and Economic Fragility*

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We study how equilibrium patterns of production, trade, and input sourcing in complex supply chains are shaped by aggregate risk. We develop a quantitative model of sequential production under risk, with inputs for each production stage sourced from multiple origins. Input production is subject to aggregate shocks that can vary by origin-destination and production stage. We analytically characterize country-stage input sourcing as a function of the variances of global shocks and production elasticities within and across *all* stages, and derive results showing the welfare impact of increases in risk, and the impact of risk on comparative advantage. We quantify the model using 3 stages of production and 50 countries. Counterfactuals show that comparative advantage flattens in a risky world, as countries diversify away from the relatively most productive regions to mitigate risk. Increasing supply chain complexity disproportionately lowers the volatility of welfare for riskier countries, as these countries benefit from the increased opportunities to diversify as the number of production stages increases. We also quantify the welfare impacts of an increase in Chinese-origin and US-origin risk, and an increase in U.S. trade costs in a world with risky global supply chains.

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1 Introduction

Recent years have underscored the importance of *risk* in shaping the global economy. Amid political conflicts, pandemics, natural disasters, and trade wars, policymakers and firms have become increasingly concerned with the fragility of complex global value chains. Traditional models of trade and supply chains, emphasizing deterministic comparative advantage and non-stochastic iceberg costs, offer limited guidance for how production and sourcing might respond in an environment with risk. When firms face aggregate uncertainty, supply chains may be reconfigured in ways that shape global patterns of production, trade flows, prices, and welfare. Understanding the role of risk in shaping the geography of production is now a first-order question in both academic and policy circles.

Empirical evidence shows firms are adapting. Firm-level analyses suggest that risk exposure and supply-chain fragility affect input sourcing, diversification, and trade volumes (Blaum et al., 2025; Castro-Vincenzi, 2025; Ma and Ramondo, 2026).[1] However, we lack a quantitative framework to understand how these micro adaptations to aggregate risk translate into changes in global production patterns, specialization, and welfare. Our paper asks: in a world with complex supply chains, how does the presence of aggregate risk—whether geographic, specific to a supply chain stage, or both—shape global production, trade patterns, and global welfare?

We address this question by developing a quantitative general equilibrium multi-country model of sequential production under risk.[2] The model features risk averse households, and retailer firms that produce non-tradeable final goods using labor and a bundle of tradable intermediate inputs. Intermediate production is sequential: the intermediate input bundle consists of inputs from S stages, where each stage combines the output of the previous stage with a bundle of new inputs in a CES aggregator. The stage s specific input bundle features varieties that can be sourced from all locations globally. This production function allows for flexible patterns of substitution, both within a stage across origins, and across stages.[3] Facing this production

[1] Abundant empirical evidence also exists in surveys run by think-tanks and policy institutes. For instance, findings from McKinsey Global Institute (2020) and Deloitte (2023) document that a majority of multinational producers are diversifying suppliers and rethinking supply-chain concentration in response to rising uncertainty.

[2] Throughout, our emphasis is on risk, or a change in the variance of shocks, holding the mean constant. However, our quantification permits studying changes in the mean as well.

[3] Antràs and de Gortari (2020) develops a model with sequential production without risk, where

function, firms place ex-ante input orders from a set of sourcing locations globally, before aggregate shocks are realized. Shocks can be specific to an origin-destination pair i, j and stage-of-production s, and can have an arbitrary correlation structure. These shocks capture a wide range of risks: geopolitical tensions such as US–China trade restrictions, stage-specific sectoral disruptions such as rare earth embargoes, or climate-induced production volatility. After shocks are realized, inputs are delivered, all stages of production are assembled, final output is produced with the intermediate bundle and labor, and all markets clear.[4]

In this environment, production decisions are no longer governed by costs and technology alone. When choosing their optimal input mix, firms face a trade-off between cost-minimization and diversification of risk across locations. The sequential nature of the supply chain provides additional opportunities for diversification at every stage, but also implies that upstream sourcing decisions impact downstream output. We show that in this setting, the optimal input sourcing shares equal the expected output elasticity of the inputs across states of the world. In other words, expenditure shares are higher on inputs with greater expected average marginal benefit to production.[5]

Under certain distributional assumptions on the shock structure, we analytically characterize this expected output elasticity, and therefore firms' optimal sourcing shares in a sequential production function. We show how this elasticity responds to changes in the variances of the i, j, s shocks, in partial equilibrium. This structure yields closed-form propagation formulas that map small changes in risk into changes in sourcing shares that are reminiscent of a Leontief inverse. This allows us to track how changes in risk at one node propagate through the supply chain to production patterns globally, holding global relative wages fixed. We then turn to the general equilibrium of this model, and characterize how risk impacts welfare. An increase in a country's own risk unambiguously lowers its welfare, while increases in risk in

stages Cobb-Douglas. Costinot et al. (2013) use a sequential supply chain with risk, where stages are Leontief, and there is no substitution across origins within a stage. In contrast, our production function can feature any substitution elasticity across stages and within stages across origins.

[4]In an environment without risk, the assumption that assembly takes place after all inputs are delivered is equivalent to assuming firms make optimal choices of inputs for each stage, and assemble each stage, before choosing the bundle for the next stage. In an environment with risk, the assumption there is no reoptimization of input orders between the realization of shocks for each stage makes the framework tractable, as it eliminates the need to keep track of the history of an i, j, s vector of shocks in each stage.

[5]The model is efficient, and this decentralization implements the choices of the planner.

other locations or stages have ambiguous effects. Intuitively, higher risk in a foreign location reshapes global production patterns, shifting demand for a country's goods and its terms of trade. These shifts can have distributional impacts on welfare: while global welfare declines as risk increases at any one node, these general equilibrium adjustments can generate heterogeneous welfare effects across countries, with some experiencing gains.

We then analytically contrast optimal sourcing in a deterministic benchmark with a stochastic environment where sourcing decisions reflect both cost and risk. We construct a measure of relative comparative advantage across origins and stages following Costinot et al. (2012). In our framework, this measure reflects risk-adjusted comparative advantage, and is a function of fundamental productivities in each origin and stage adjusted by their bilateral, stage-specific risk. We decompose this measure into a Ricardian comparative advantage (RCA) component, containing destination-specific fundamental origin-stage productivities, and a risk wedge. We characterize the covariance of this risk wedge with the RCA term, and show that under certain parameters, the covariance is negative. Risk can therefore attenuate fundamental comparative advantage: to hedge risk, countries reallocate sourcing away from dominant exporters within a stage towards smaller exporters, and this effect is stronger for the largest exporters.

Finally, we provide a welfare change decomposition that can be implemented across counterfactuals to reveal three distinct channels: a recipe effect (reallocation toward safer input bundles), a risk–recipe effect (hedging within the chain), and a risk-aversion effect (certainty-equivalent utility losses). These results highlight the trade-off between efficiency and resilience absent from deterministic frameworks.

The framework stresses the role of two central forces: diversification (firms multi-source within stages to hedge risk), and the sequential propagation of shocks across stages. Prior to quantification, we present transparent comparative statics in a two-country three-stage environment to illustrate how increases in the variance of shocks affect sourcing, production location, welfare, and comparative advantage. In equilibrium, bilateral stage-specific risk reconfigures sourcing through two forces: direct effects, as firms shift toward more reliable suppliers, and propagation effects, as these shifts alter input demand and prices at all other stages.

The model features a gravity-like equation in expectation, but under risk, the independence of irrelevant alternatives (IIA) condition is violated, complicating esti-

mation. The key analytical object—the expected output elasticity of the final goods aggregator with respect to each input order—depends on the variance of bilateral shocks, the elasticity of substitution within and across stages, and the sequential stage weights. We develop a PPML estimation approach to estimate substitution elasticities both within stages (across sourcing locations) and across stages (along the production chain), along with other model parameters such as productivities and trade costs, using the model-consistent moment condition for bilateral trade flows by stage. Intuitively, productivity is identified by an exporter-stage effect: more productive countries export more, on average, to all destinations. Trade costs are identified using standard time-invariant gravity measures. Substitution elasticities within stage are identified by shifts in relative sourcing across locations in response to changes in relative trade costs, while substitution across stages is identified by relative shifts in stage weights in response to stage-aggregated trade cost shifts.

We estimate the model with 50 countries and 3 stages of production, using bilateral HS-6 trade flows from BACI and tariffs from Teti (2025) for 1995-2023, augmented with tariff data from TRAINS for more recent years. We map sectors into production stages using groupings based on upstreamness measures from Antràs et al. (2012).[6] Our estimates imply that varieties within stages are substitutes, while stages are complements, with complementarity close to Leontief. Given the parameter estimates, bilateral-stage risk is identified as the variance of the fitted model's structural residuals. While our analytical results require stronger assumptions on the independence of shocks across locations, in our quantification, risk is correlated across space, consistent with the comovement of the structural residuals across bilateral-stage pairs.

We contrast the calibrated model under risk to a deterministic benchmark. Welfare declines for nearly all countries in the risky environment, with only the least risky countries on average experience small-to-neutral welfare effects, as demand shifts towards them with risk. We decompose welfare changes into three components: demand shifts, changes in terms of trade, and risk-return tradeoffs. For the riskiest countries, negative demand shifts are a large component of welfare losses under risk. These countries also experience positive changes in input prices (terms-of-trade), however, as their domestic inputs are now relatively cheaper. We then examine how observed comparative advantage changes under risk. A key finding is that risk flattens com-

[6]For pairs with no bilateral trade flows, bilateral tariffs are imputed using MFN or PTA rates that would apply to such trade if it existed to ensure maximum coverage.

parative advantage: the ranking of exports by risk-adjusted comparative advantage is flatter than under RCA alone. As highlighted in the theory, firms tilt sourcing away from the relatively most productive regions, and towards relatively less productive regions, implying the risk wedge is negatively correlated with Ricardian comparative advantage. Our results that aggregate risk flattens revealed comparative advantage are conceptually related to Ma and Ramondo (2026), who show that uncertainty weakens specialization by offsetting economies of scale. In contrast, our mechanism operates through endogenous multisourcing and sequential propagation of risk along global value chains, even in the absence of scale economies.

Our analytical results suggest changes in risk reconfigure production and trade patterns. To illustrate this in the quantification, we consider counterfactuals where the variance of shocks from China increases (in all stages), or the variance of shocks in the US increases (in all stages), in both cases such that aggregate risk globally increases by 5%. An increase in risk from China leads all countries to shift sourcing away from China across all stages. However, the shifts are smaller upstream relative to downstream, as the cross-stage complementarity implies that maintaining the supply of the upstream inputs is relatively more important. This endogenous "decoupling" arises even in the absence of tariffs, and captures the geopolitical fragmentation observed in recent trade patterns (Grossman et al., 2023a; Kleinman et al., 2025). Chinese welfare and input prices decline sharply. Increases in US-specific risk have similar effects on sourcing patterns and US welfare.

Trade policy also interacts nontrivially with risk. In a counterfactual where the United States imposes higher trade costs, the decline in welfare in a risky environment is, on average, larger than in the deterministic world. This is because the US is a relatively low risk and productive country. In a deterministic environment, therefore, the US has very little foreign sourcing of inputs, and so the increase in trade costs results in smaller changes in sourcing shares and declines in demand globally. In the baseline risky equilibrium, the US sources relatively more globally purely for diversification. Raising trade costs in this setting results in relatively lower demand for most foreign countries in the risky equilibrium compared to a deterministic benchmark, lowering welfare. Note that the US remains more diversified after the trade cost increase in a risky environment relative to a deterministic environment.

Finally, in a counterfactual we consider changes in welfare and its volatility when moving from a deterministic to a risky environment with more complex supply chains,

featuring 9 stages of production rather than 3 as in the baseline. While more stages of production provide more opportunities for the amplification of risk through the supply chain, they also provide increased opportunities for diversification. For most countries, the volatility of welfare remains similar to the environment, with 3 stages of production. However, for the riskiest countries, there is a marked decline in volatility in the more complex world. These countries are inherently risky, and the additional diversification opportunities provided by more stages of production outweigh any amplification of risk through greater supply chain complexity for this group.

Related Literature. Our new theoretical framework bridges existing models in the literature on trade under uncertainty, sequential production, and production networks. In contrast to sequential and input -network models emphasizing deterministic comparative advantage (Antràs and Chor, 2013; Antràs and de Gortari, 2020; Yi, 2003, 2010), we introduce uncertainty at each stage of production and derive expressions for how sourcing shares and welfare adjust to changes in the variances of shocks. Compared with the Leontief stage structure with risk analyzed in Costinot et al. (2013), our model features multiple sourcing locations for each production stage, flexibly allows for any elasticity of substitution both across and within stages, and is embedded in a quantitative multi-country general equilibrium framework.

Our quantitative framework links sourcing under uncertainty to the geography of production in global value chains. While prior work studies the organization of production in deterministic environments (Yi, 2003, 2010; Antràs and de Gortari, 2020; Baldwin and Venables, 2013; Johnson and Moxnes, 2023; Ma and Ramondo, 2026), we show that aggregate risk fundamentally reshapes specialization patterns. In equilibrium, sourcing decisions reflect not only relative costs and technologies, but also reliability and diversification. As a result, longer and more fragmented supply chains can either amplify or mitigate aggregate volatility, depending on substitution elasticities and the concentration of risk across stages. Like in Ma and Ramondo (2026), increases in risk flatten the ranking of trade based on Ricardian comparative advantage by shifting production away from dominant exporters to hedge risk.

Additionally, we connect theories of shock propagation in production networks with trade theory under uncertainty. Existing network models emphasize shock propagation through fixed technological linkages (Barrot and Sauvagnat, 2016; Boehm et al., 2019; Carvalho and Tahbaz-Salehi, 2019; Acemoglu and Tahbaz-Salehi, 2024), while trade models under uncertainty abstract from multi-stage production. Our framework

integrates these perspectives by showing how bilateral, stage-level variance shocks propagate globally when firms choose inputs along the production chain in an environment with uncertainty. This structure allows us to analytically characterize how the position of risk within the supply chain (upstream versus downstream) shapes sourcing adjustments, production reallocation, and welfare across countries.

More broadly, our model connects to the literature on trade and volatility (Allen and Atkin, 2022; Caselli et al., 2020), firm resilience (Grossman et al., 2023b; Castro-Vincenzi et al., 2024), and input specificity (Barrot and Sauvagnat, 2016; Boehm et al., 2019), while integrating the multi-stage structure of global value chains that is central to modern trade (Yi, 2010; Johnson and Noguera, 2012; Johnson and Moxnes, 2023). In sum, our paper lies at the intersection of international trade, macroeconomic risk, and production network theory. The model provides a foundation for analyzing current debates on supply-chain resilience, deglobalization, and industrial policy in a world of rising uncertainty.

The rest of our paper is structured as follows. Section 2 sets up the model, derives analytical results, and performs comparative statics. Section 3 calibrates the model, and outlines our solution approach. Section 4 contains counterfactuals. Section 5 concludes.

2 Theory

Setting. The economy consists of J regions indexed by $i, j \in J$, with S production stages indexed by $s \in S$. Each region i is endowed with L_i immobile workers, perfectly competitive retailers producing a non-traded final good, and perfectly competitive intermediate-goods producers.

Timing. The model is static. Retailers place intermediate-input orders $\{M_{ji,s}\}$—where $M_{ji,s}$ denotes the quantity of stage-s inputs ordered by destination i from origin j—before bilateral, stage-specific shocks $\boldsymbol{\chi} = \{\chi_{ji,s}\}$ realize; once shocks realize, orders are delivered, final goods are assembled, and all markets clear.

2.1 Households

The representative household in region i supplies labor L_i inelastically and maximizes $\log q_i(\boldsymbol{\chi})$ subject to

$$p_i(\boldsymbol{\chi})\, q_i(\boldsymbol{\chi}) = Y_i(\boldsymbol{\chi}) \equiv w_i(\boldsymbol{\chi}) L_i,$$

where $q_i(\boldsymbol{\chi})$ is consumption of the regional final good, $p_i(\boldsymbol{\chi})$ is its price, $w_i(\boldsymbol{\chi})$ is the wage in region i, and $Y_i(\boldsymbol{\chi})$ is household income. The resulting stochastic discount factor is $\Lambda_i(\boldsymbol{\chi}) = 1/Y_i(\boldsymbol{\chi})$.

2.2 Intermediate Goods Producers

Competitive suppliers in region i produce tradable intermediates with technology $M_{ji,s} = a_{i,s}\ell^M_{ji,s}$, where $a_{i,s}$ is productivity and $\ell^M_{ji,s}$ is labor used in producing stage-s inputs destined for j. Intermediates are priced at marginal cost $p_{i,s}(\boldsymbol{\chi}) = w_i(\boldsymbol{\chi})/a_{i,s}$. Iceberg trade costs $\tau_{ji,s}$ imply that the price paid by destination i for inputs from origin j satisfies $p_{ji,s}(\boldsymbol{\chi}) = \tau_{ji,s} p_{i,s}(\boldsymbol{\chi})$.

2.3 Retailer Problem

Perfectly competitive retailers in i produce the final good with

$$q_i(\boldsymbol{\chi}) = \varphi_i\, \ell_i(\boldsymbol{\chi})^{\beta}\, M_i(\boldsymbol{\chi})^{1-\beta}, \tag{1}$$

where φ_i is final-good productivity, $\ell_i(\boldsymbol{\chi})$ is labor input, $\beta \in (0,1)$ is the labor share, and $M_i(\boldsymbol{\chi})$ is a CRS aggregator of intermediate inputs across stages and origins.

Ex-post problem. When retailers assemble the final good, input orders $M_{ji,s}$ have been placed and shocks $\boldsymbol{\chi}$ are realized. The retailer chooses labor and how to allocate delivered inputs to maximize profits:

$$\begin{aligned} \max_{q_i, \{x_{ji,s}\}, \ell_i} \quad & p_i(\boldsymbol{\chi})\, q_i(\boldsymbol{\chi}) - w_i(\boldsymbol{\chi})\ell_i(\boldsymbol{\chi}) \\ \text{s.t.} \quad & M_i(\boldsymbol{\chi}) = f\big(x_{ji,s}(\boldsymbol{\chi})\big) \\ & x_{ji,s}(\boldsymbol{\chi}) \le \chi_{ji,s} M_{ji,s}, \quad \forall\, j \in \Omega_{i,s},\ s \in S, \end{aligned} \tag{2}$$

where $x_{ji,s}(\boldsymbol{\chi})$ is the quantity of stage-s inputs from j actually used in production, $f(\cdot)$ is the CRS input aggregator, and $\Omega_{i,s} \subseteq J$ is the endogenous set of stage-s suppliers

for i. Shocks $\chi_{ji,s}$ scale the quantity of inputs available for production—capturing disruptions that may be partner- or stage-specific (e.g., natural disasters, export bans, or destination-stage productivity shocks). As profits monotonically increase in $M_i(\boldsymbol{\chi})$, the input constraint always binds: the firm uses all delivered inputs.

Ex-ante problem. The first-order condition with respect to $M_{ji,s}$ suggests:

$$\mathbb{E}_{\boldsymbol{\chi}}\left[p_i(\boldsymbol{\chi})^{\frac{1}{1-\beta}} w_i(\boldsymbol{\chi})^{-\frac{\beta}{1-\beta}} \beta^{\frac{1}{1-\beta}} \left(\frac{1-\beta}{\beta}\right) \frac{\partial M_i(\boldsymbol{\chi})}{\partial M_{ji,s}}\right] = \mathbb{E}[p_{ji,s}(\boldsymbol{\chi})]. \tag{3}$$

In Appendix A.1.1 we show that, given a numeraire w_1, relative wages w_i and input prices $p_{ji,s}$ are deterministic. With competitive pricing $p_i(\boldsymbol{\chi}) = w_i(\beta L_i/M_i(\boldsymbol{\chi}))^{1-\beta}(\beta\phi_i)^{-1}$, equation (3) simplifies to

$$(1-\beta) w_i L_i \, \mathbb{E}_{\boldsymbol{\chi}}[\mathcal{E}_{ji,s}(\boldsymbol{\chi})] = p_{ji,s} M_{ji,s}, \tag{4}$$

where $\mathcal{E}_{ji,s}(\boldsymbol{\chi}) := \frac{\partial M_i(\boldsymbol{\chi})}{\partial M_{ji,s}} \frac{M_{ji,s}}{M_i(\boldsymbol{\chi})}$ is the point-wise output elasticity of the input aggregator M_i with respect to the pre-shock order $M_{ji,s}$. Total input expenditure is $(1-\beta)w_i L_i$, so the expenditure share on inputs from (j,s) equals the *expected output elasticity*:

$$s_{ji,s} = \frac{p_{ji,s} M_{ji,s}}{(1-\beta) w_i L_i} = \mathbb{E}[\mathcal{E}_{ji,s}(\boldsymbol{\chi})]\,. \tag{5}$$

Thus, the share allocated to a given origin-stage input reflects how sensitive M_i is to that input on average across all states. We next specialize $f(\cdot)$ to a sequential production function and characterize this elasticity in this setting.

2.4 Sequential Production

Let the stage-s input bundle for destination i be $B_{i,s} := \left[\sum_{j\in\Omega_{i,s}} (\chi_{ji,s} M_{ji,s})^{\rho_s}\right]^{1/\rho_s}$, where $\rho_s \in (-\infty, 1)$ governs within-stage substitutability. Define the sequential aggregator:

$$y_{i,1} = B_{i,1}, \tag{6}$$

$$y_{i,s} = \left(\alpha_{i,s}\, y_{i,s-1}^{\eta} + B_{i,s}^{\eta}\right)^{1/\eta}, \quad s = 2, \ldots, S, \tag{7}$$

$$M_i = y_{i,S}, \tag{8}$$

where $\alpha_{i,s} > 0$ is a weight on the cumulative output from all prior stages, $y_{i,s-1}$, and $\eta \in (-\infty, 1]$ governs substitutability across stages. The final intermediate aggregate M_i is the output of the last stage S. Define within-stage CES quantity shares

$$\omega_{ji,s}(\boldsymbol{\chi}) := \frac{(\chi_{ji,s} M_{ji,s})^{\rho_s}}{\sum_{j' \in \Omega_{i,s}} (\chi_{j'i,s} M_{j'i,s})^{\rho_s}},$$

which capture the share of origin j in the stage-s input bundle after shocks realize, and stage weights

$$\lambda_{i,s}(\boldsymbol{\chi}) := \frac{B_{i,s}^{\eta}}{\alpha_{i,s} y_{i,s-1}^{\eta} + B_{i,s}^{\eta}},$$

which measures the contribution of the new stage-s bundle relative to cumulative prior-stage output.

Lemma 1 *The point-wise elasticity of $M_i(\boldsymbol{\chi})$ with respect to $M_{ji,s}$ is*

$$\mathcal{E}_{ji,s}(\boldsymbol{\chi}) = \omega_{ji,s}(\boldsymbol{\chi})\, \lambda_{i,s}(\boldsymbol{\chi}) \prod_{t=s+1}^{S} \big[1 - \lambda_{i,t}(\boldsymbol{\chi})\big], \quad s = 1, \ldots, S. \tag{9}$$

Proof. See Appendix A.3.1. ■

The elasticity decomposes into three multiplicative terms: the within-stage share $\omega_{ji,s}$ of origin j; the stage weight $\lambda_{i,s}$, which measures how much the stage-s bundle matters for M_i; and the product $\prod_{t>s}(1 - \lambda_{i,t})$, which captures how much of the contribution of stage s survives through all downstream stages.

Special cases. If $\eta = 0$, stages are Cobb-Douglas and $\mathcal{E}_{ji,s} = a_s \theta_{ji,s}$ is deterministic—where a_s is the constant cost share of stage s in M_i and $\theta_{ji,s}$ is origin j's constant share within stage s—so expenditure shares are invariant to risk. If $\eta = 1$, stages are perfect substitutes and $\mathcal{E}_{ji,s}(\boldsymbol{\chi}) = \omega_{ji,s}(\boldsymbol{\chi})\, B_{i,s}/M_i(\boldsymbol{\chi})$: each stage contributes in proportion to its quantity share in M_i.

As $\eta \to -\infty$, the aggregator is Leontief across stages, with $\lambda_{i,s}(\boldsymbol{\chi}) = \mathbf{1}\{B_{i,s} < \alpha_{i,s}^{1/\eta} y_{i,s-1}\}$, so $\lambda_{i,s} = 1$ only if stage s is the bottleneck and $1 - \lambda_{i,t} = 1$ only if stage s continues to be the binding constraint at each downstream stage $t > s$. An input contributes to final output only if its stage is the tightest constraint all the way through the hierarchy; else its elasticity is zero. Cost share $s_{ji,s}$ equals the probability that stage s is the binding constraint times the CES within-stage share.

Gravity equation. Combining (9) and (5) and substituting $p_{ji,s} = \tau_{ji,s} w_j / a_{j,s}$, optimal expenditure on inputs $X_{ji,s} = p_{ji,s} M_{ji,s}$ is

$$X_{ji,s} = \left(\tau_{ji,s} \frac{w_j}{a_{j,s}}\right)^{-\frac{\rho_s}{1-\rho_s}} ((1-\beta) w_i L_i)^{\frac{1}{1-\rho_s}} \left[\underbrace{\mathbb{E}_{\boldsymbol{\chi}} \left[\frac{\chi_{ji,s}^{\rho_s}}{\sum_{j' \in \Omega_{i,s}} (\chi_{j'i,s} M_{ji,s})^{\rho_s}} \lambda_{i,s}(\boldsymbol{\chi}) \prod_{t>s} (1 - \lambda_{i,t}(\boldsymbol{\chi})) \right]}_{= \Psi_{ji,s}} \right]^{\frac{1}{1-\rho_s}}. \tag{10}$$

Equation (10) takes a gravity form in expectation. $\Psi_{ji,s}$ encodes how the distribution of shocks $\mathbb{G}(\boldsymbol{\chi})$ shapes origin j's attractiveness as a stage-s supplier, and is unobserved in the data. Because $\Psi_{ji,s}$ is correlated with trade costs $\tau_{ji,s}$ and bilateral quantities, IIA fails: a shock or policy change affecting one stage alters choices within *and* across all other stages in ways that cannot be summarized by standard source- and destination-country price indices, confounding standard gravity estimation.

2.4.1 Changes in Sourcing Shares with Risk

We characterize how sourcing shares respond to changes in the variance of shocks. Assuming $\chi_{ji,s}$ are independently log-normal with mean one, a version of Stein's lemma (see Appendix A.2.1) yields the following partial-equilibrium result.

Proposition 1 *Let $\chi_{ji,s} = \exp\left(-\frac{1}{2}\sigma_{ji,s}^2 + \sigma_{ji,s}\varepsilon_{ji,s}\right)$ with $\varepsilon_{ji,s} \sim \mathcal{N}(0,1)$, where $\sigma_{ji,s}^2$ is the variance of the log shock from origin j in stage s to destination i. Let*

$$\vartheta_{\ell|ji,s'|s}(\boldsymbol{\chi}) := \frac{\partial \ln \mathcal{E}_{\ell i,s'}(\boldsymbol{\chi})}{\partial \ln \chi_{ji,s}}, \qquad \zeta_{\ell|ji,s'|s}(\boldsymbol{\chi}) := \frac{\partial^2 \ln \mathcal{E}_{\ell i,s'}(\boldsymbol{\chi})}{\partial (\ln \chi_{ji,s})^2}$$

denote the slope and curvature of the output elasticity for (ℓ, s') with respect to shocks from (j,s). Then the effect of a change in $\sigma_{ji,s}^2$ on the expenditure share of origin ℓ in stage s' is

$$\frac{\partial s_{\ell i,s'}}{\partial \sigma_{ji,s}^2} = \frac{1}{2} \mathbb{E}_{\boldsymbol{\chi}} \left[\mathcal{E}_{\ell i,s'} \left(\vartheta_{\ell|j,s\downarrow s'}^2 + \zeta_{\ell|j,s\downarrow s'} - \vartheta_{\ell|j,s\downarrow s'} \right) \right]. \tag{11}$$

Proof. See Appendix A.3.2. ■

Proposition 1 gives the direct effect of increased risk from (j,s) on all sourcing shares of i—within and across stages—holding input orders fixed. In this sequential production function setting, explicit expressions for the slopes ϑ and curvatures ζ

in terms of only model parameters and CES quantity shares can be derived and are given in Appendix A.1.2.

Total variance effects in partial equilibrium. We next allow all bilateral orders $\mathbf{M}$ to adjust endogenously. Stacking the $J \times S$ first-order conditions (10) as $\mathbf{F}(\mathbf{M}, \boldsymbol{\sigma}^2) = \mathbf{0}$ and applying the implicit function theorem gives $d\mathbf{M}/d\sigma^2_{ji,s} = -[\partial_{\mathbf{M}}\mathbf{F}]^{-1}\partial_{\sigma^2_{ji,s}}\mathbf{F}$. Defining the diagonal cost matrix $\mathbf{D} = \text{diag}\{p_{ji,s}\}/((1-\beta)w_i L_i)$, which converts input orders into cost shares, and the unit-free substitution matrix $\mathbf{A} = \mathbf{D}^{-1}\Gamma$ (with spectral radius $\rho(\mathbf{A}) < 1$), where

$$\Gamma_{(ji,s),(\ell i',s')} = \frac{\mathbb{E}_{\boldsymbol{\chi}}\left[\mathcal{E}_{ji,s}\,\vartheta_{(ji,s)\leftarrow(\ell i',s')}\right]}{M_{\ell i',s'}}$$

collects the cross-elasticities of expected output elasticities with respect to all input orders, the total variance effect on all expenditure shares is

$$\frac{d\mathbf{s}}{d\sigma^2_{ji,s}} = \mathbf{D}(\mathbf{I}-\mathbf{A})^{-1}\mathbf{D}^{-1}\mathbf{h}, \qquad \mathbf{h} := \partial_{\sigma^2_{ji,s}}\mathbb{E}_{\boldsymbol{\chi}}[\boldsymbol{\mathcal{E}}], \tag{12}$$

where $\mathbf{h}$ collects the direct variance effects from Proposition 1. Analogous to the Leontief inverse in input-output models, $(\mathbf{I}-\mathbf{A})^{-1}$ propagates the shock through technological substitution and complementarity within and across all stages. In contrast to roundabout production structures where similar Leontief inverse propagation results are derived, equation (12) is obtained here with sequential production. Since wages w_i are held fixed, equation (12) is a partial equilibrium characterization.

2.5 General Equilibrium

The equilibrium is characterized by the input-order conditions (10) and labor market clearing in each region j:

$$\sum_i \sum_s \tau_{ji,s}\frac{M_{ji,s}}{a_{j,s}} = (1-\beta)L_j. \tag{13}$$

The left-hand side is the total labor demanded in j for intermediate production across all destinations and stages; the right-hand side is the labor endowment net of the fraction βL_j employed directly by retailers. Equation (13) implicitly pins down wages w_j. As shown in Appendix A.1.1, exactly one vector of relative wages clears all

markets, so wages are non-stochastic. The competitive equilibrium coincides with the social planner's solution (see Appendix A.1.3).

2.6 Welfare

Welfare in region i is

$$\mathcal{W}_i = \beta \log \beta + \beta \log L_i + (1-\beta)\, \mathbb{E}_{\boldsymbol{\chi}}[\log M_i(\{M_{ji,s}\}, \boldsymbol{\chi})]\,. \tag{14}$$

In Appendix A.4 we show that, to first order, the equilibrium welfare change from a baseline economy decomposes as:

$$\begin{aligned}\frac{\Delta \mathcal{W}_i}{1-\beta} = \underbrace{\Delta \log w_i}_{\text{(i) wage}} &- \underbrace{\sum_{\ell} \bar{s}_\ell^{\text{EQ}}\, \Delta \log p_\ell}_{\text{(ii) terms of trade}} + \underbrace{\sum_{\ell} \bar{s}_\ell^{\text{EQ}}\, \Delta \log s_\ell}_{\text{(iii) input mix}} \\ &- \underbrace{\sum_{\ell} (\bar{s}_\ell^{\text{EQ}} - \bar{s}_\ell^{\text{RN}})\, \Delta \log r_\ell}_{\text{(iv) insurance cost}} + \underbrace{\Delta R_i}_{\text{(v) risk}} + \mathcal{O}(\|\Delta r\|^2),\end{aligned} \tag{15}$$

where the summation is over all origin-stage links $\ell = (j, s)$, $\bar{s}_\ell^{\text{EQ}}$ is the equilibrium expenditure share on link ℓ, $\bar{s}_\ell^{\text{RN}}$ is the corresponding risk-neutral share (i.e., the share that would prevail if the firm ignored covariances when choosing inputs), p_ℓ is the input price on link ℓ, and r_ℓ is a risk measure associated with link ℓ.

Term (i) captures GE wage adjustments driven by demand reallocation across countries. Term (ii) measures the terms-of-trade effect in intermediate input prices at the original production technique; losses are larger when shocks hit heavily used origin-stage links. Terms (iii) and (iv) are absent under Cobb-Douglas ($\eta = 0$), where expenditure shares do not respond to risk. Term (iii) is the welfare gain from shifting the input mix toward links with higher average marginal value $\mathbb{E}[\mathcal{E}_\ell]$. Term (iv) is the *cost of insurance*: if the risk-neutral share $\bar{s}_\ell^{\text{RN}}$ exceeds the equilibrium share $\bar{s}_\ell^{\text{EQ}}$, the firm underweights inputs with high expected elasticity in good states in order to hedge against bad states, reducing expected output. Term (v), $R_i = \mathbb{E}[\log \tilde{M}_i] - \log \mathbb{E}[\tilde{M}_i]$, is the certainty-equivalent welfare change from risk—the gap between the log of expected output and the expected log of output. It is large and positive when sourcing is diversified across negatively correlated origins and stages, and large and negative when upstream stages are volatile or positively correlated and

sourcing is concentrated.

Proposition 2 *In the competitive equilibrium, the effects of fundamentals on welfare are:*

$$\frac{\partial \mathcal{W}_i}{\partial \sigma^2_{ji,s}} = \frac{1-\beta}{2}\,\mathbb{E}_{\boldsymbol{\chi}}\left[\mathcal{E}_{ji,s}(\boldsymbol{\chi})\left(\vartheta_{ji,s}(\boldsymbol{\chi})-1\right)\right], \tag{16}$$

$$\frac{\partial \mathcal{W}_i}{\partial a_{js}} = (1-\beta)\,\frac{s_{ji,s}}{a_{js}}, \qquad \frac{\partial \mathcal{W}_i}{\partial \tau_{ji,s}} = -(1-\beta)\,\frac{s_{ji,s}}{\tau_{ji,s}}, \qquad \frac{\partial \mathcal{W}_i}{\partial \mu_{ji,s}} = (1-\beta)\,s_{ji,s}, \tag{17}$$

where $\mu_{ji,s} := \mathbb{E}[\chi_{ji,s}]$ *is the mean of the shock from origin* j *in stage* s*.*

Proof. See Appendix A.3.3. ■

Proposition 2 shows that small changes in productivity and mean-shock (which are similar to destination-specific productivity increases) unambiguously raise welfare. Trade cost increases lower it. However, the sign of the variance effect – the impact of an small increase in *risk* from j, s on i's welfare– in (16) is ambiguous and depends on whether the stage is a bottleneck and the scope for reallocation across stages. Notice that the proof of this proposition relies on the envelope condition, and so the results apply to small deviations from the initial equilibrium. Corollary 1 states the restrictions on the parameter space in which small increases in risk unambiguously lowers welfare.

Corollary 1 *Welfare* $\mathcal{W}_i$ *is decreasing in the variance of shocks* $\sigma^2_{ji,s}$ *(i.e.,* $\frac{\partial \mathcal{W}_i}{\partial \sigma^2_{ji,s}} \leq 0$*) if (1)* $\eta \leq 0$*, or (2)* $\rho_s \leq \frac{1-\eta(\omega_{ji,s}(\boldsymbol{\chi})-\mathcal{E}_{ji,s}(\boldsymbol{\chi}))}{1-\omega_{ji,s}(\boldsymbol{\chi})}, \forall \boldsymbol{\chi} \in \mathbf{G}(\boldsymbol{\chi})$*.*

Proof. See Appendix A.3.4. ■

As a good s from country j becomes more binding—either because stages are complements with $\eta < 0$ or with a low ρ_s—the scope for adjustment diminishes, increasing the likelihood that higher variance lowers welfare. The restriction on ρ_s in the second condition requires ρ_s to be sufficiently small so as to prevent country i from mitigating the welfare loss through reallocation across stages.

2.7 Comparative Advantage and Risk

We now examine how risk shapes comparative advantage relative to a deterministic setting. Assuming a uniform ρ across stages, the gravity equation (10) implies that relative bilateral exports from $j \neq j'$ across stages $s \neq s'$ satisfy:

$$\Delta^{s,s'}_{jj',i} := \log\left(\frac{X_{j'i,s}}{X_{ji,s}}\right) - \log\left(\frac{X_{j'i,s'}}{X_{ji,s'}}\right) = \frac{\rho}{1-\rho}\left[\log\frac{a_{j',s}\tau_{ji,s}}{a_{j,s}\tau_{j'i,s}} - \log\frac{a_{j',s'}\tau_{ji,s'}}{a_{j,s'}\tau_{j'i,s'}}\right] + \left[\log\frac{\Psi_{j'i,s}}{\Psi_{ji,s}} - \log\frac{\Psi_{j'i,s'}}{\Psi_{ji,s'}}\right]. \tag{18}$$

Here $\Delta^{s,s'}_{jj',i}$ is the double-difference in log exports: how much more j' exports relative to j in stage s compared to stage s', for importer i. The first term on the right is classical Ricardian comparative advantage, captured by the relative origin-stage productivities adjusted for bilateral trade costs. The second is a *risk wedge* that modifies specialization patterns beyond what productivity fundamentals alone predict. Defining risk-adjusted productivity $\widetilde{a}_{ji,s} := (a_{j,s}/\tau_{ji,s})\Psi_{ji,s}^{(1-\rho)/\rho}$, equation (18) becomes

$$\Delta^{s,s'}_{jj',i} = \frac{\rho}{1-\rho}\left[\log\frac{\widetilde{a}_{j'i,s}}{\widetilde{a}_{ji,s}} - \log\frac{\widetilde{a}_{j'i,s'}}{\widetilde{a}_{ji,s'}}\right]. \tag{19}$$

Bilateral exports are therefore ranked by risk-adjusted productivities rather than raw fundamentals. In the deterministic benchmark, $\Psi_{j'i,s}/\Psi_{ji,s} = 1$ and (19) reduces to classical Ricardian specialization.[7]

To characterize how the risk wedge moves with uncertainty, we need one additional object: the sensitivity of the stage-s cascade term $\Theta_{i,s}$ to changes in the stage-s price index. Recall from Lemma 1 that the elasticity $\mathcal{E}_{ji,s}$ contains the product $\Theta_{i,s} = \lambda_{i,s}\prod_{t>s}(1-\lambda_{i,t})$. This product, which we call the *cascade term* equals the unconditional cost share of stage s in the full intermediate aggregate M_i (see Appendix A.2.3). Let $\Theta^0_{i,s} := \Theta_{i,s}(\mathbf{1})$ denote its value at the deterministic benchmark $\boldsymbol{\chi} = \mathbf{1}$, and let $P_{i,s}(\boldsymbol{\chi}) := \left[\sum_{j\in\Omega_{i,s}}(p_{ji,s}/\chi_{ji,s})^{\frac{\rho_s}{1-\rho_s}}\right]^{(1-\rho_s)/\rho_s}$ be the state-by-state CES price index for stage s. The *cascade sensitivity* is defined as

$$\kappa_{i,s} := \left.\frac{\partial \log \Theta_{i,s}}{\partial \log P_{i,s}}\right|_{\boldsymbol{\chi}=\mathbf{1}} = \frac{\eta}{\eta-1}(1-\Theta^0_{i,s}), \tag{20}$$

where the steps to derive equation (20) are shown in Appendix A.2.3. Intuitively, $\kappa_{i,s}$ measures how much the overall importance of stage s in M_i responds when inputs at that stage become more expensive. When stages are complements ($\eta < 0$) and stage s does not already dominate ($\Theta^0_{i,s} < 1$), we have $\kappa_{i,s} > 0$: a deterioration

[7] See Appendix A.5 for the general-ρ version of (18).

of stage-s inputs causing an increase in their price raises stage s's equilibrium cost share, amplifying the effect of within-stage shocks on the overall aggregate. Under Cobb-Douglas across stages ($\eta = 0$), cost shares are constant, $\kappa_{i,s} = 0$ and there is no cascade amplification.

2.7.1 Local Attenuation of Comparative Advantage

We now characterize precisely how the risk wedge moves with uncertainty under certain restrictions. We parameterize the degree of uncertainty by a common variance scale $v \geq 0$ for all shocks (with $v = 0$ the deterministic benchmark) and ask how $\Delta^{s,s'}_{jj',i}$ changes as v increases from zero. Let $s^0_{ji,s}$ denote the standard CES deterministic within-stage import share at $v = 0$:

$$s^0_{ji,s} := \frac{\left(\tau_{ji,s} w_j / a_{j,s}\right)^{\rho/(\rho-1)}}{\sum_{r\in\Omega_{i,s}} \left(\tau_{ri} w_r / a_{r,s}\right)^{\rho/(\rho-1)}}. \tag{21}$$

The deterministic share gap $\Delta s^0_{i,s}(j, j') := s^0_{j'i,s} - s^0_{ji,s}$ measures how much more dominant exporter j' is relative to j in stage s in the absence of uncertainty.

Proposition 3 *Assume uniform $\rho \in (0, 1)$ across stages and mean-one lognormal shocks with a common variance scale $v \geq 0$:*

$$\log \chi_{ji,s} = -\tfrac{1}{2} v + \sqrt{v}\, \varepsilon_{ji,s}, \qquad \varepsilon_{ji,s} \overset{\text{iid}}{\sim} \mathcal{N}(0, 1).$$

Then,

(A) Stage-level wedge response. *For any stage s, the effect of a marginal increase in uncertainty on the exporter difference in the risk wedge is, locally at $v = 0$:*

$$\frac{\partial}{\partial v}\Big(\log \Psi_{j'i,s} - \log \Psi_{ji,s}\Big)\Big|_{v=0} = -\frac{\rho(\rho + \kappa_{i,s})}{1-\rho}\left(s^0_{j'i,s} - s^0_{ji,s}\right). \tag{22}$$

(B) Specialization response. *Differencing across stages, the effect on $\Delta^{s,s'}_{jj',i}$ is:*

$$\frac{\partial \Delta^{s,s'}_{jj',i}}{\partial v}\bigg|_{v=0} = -\frac{\rho}{1-\rho}\Big[(\rho + \kappa_{i,s})\big(s^0_{j'i,s} - s^0_{ji,s}\big) - (\rho + \kappa_{i,s'})\big(s^0_{j'i,s'} - s^0_{ji,s'}\big)\Big]. \tag{23}$$

(C) Attenuation of comparative advantage. *Assume additionally that $\Omega_s =$*

$\Omega_{s'} = \{j, j'\}$ *(two exporters) and* $\kappa_{i,s} = \kappa_{i,s'}$ *(equal cascade sensitivity across stages). Then*

$$\operatorname{sign}\left(\left.\frac{\partial \Delta_{jj',i}^{s,s'}}{\partial v}\right|_{v=0}\right) = -\operatorname{sign}\left(\mathrm{CA}_{jj'}^{s,s'}\right),$$

where $\mathrm{CA}_{jj'}^{s,s'} := \log\frac{a_{j',s}\tau_{ji,s}}{a_{j,s}\tau_{j'i,s}} - \log\frac{a_{j',s'}\tau_{ji,s'}}{a_{j,s'}\tau_{j'i,s'}}$ *is the Ricardian comparative-advantage index. For small* $v > 0$*, introducing uncertainty therefore shrinks* $|\Delta_{jj',i}^{s,s'}|$ *toward zero.*

(D) Cascade amplification. *If* $\eta < 0$ *(stages are complements) and* $\Theta_{i,s}^0 \in (0,1)$*, then* $\kappa_{i,s} > 0$ *and the attenuation coefficient* $\rho(\rho + \kappa_{i,s})/(1-\rho)$ *exceeds the single-stage benchmark* $\rho^2/(1-\rho)$*: cross-stage complementarity with sequential production amplifies the compression of comparative advantage.*

Proof. See Appendix A.3.5. ■

The mechanism behind Proposition 3 operates through two channels. First, the *within-stage diversification channel*: injecting mean-one variance into sourcing decisions creates a Jensen effect that reallocates expected sourcing weight away from dominant suppliers toward smaller ones within each stage. The key curvature statistic is $s_{j'i,s}^0 - s_{ji,s}^0$—the dominant exporter in a stage, who has a larger deterministic share, loses relatively more from the introduction of uncertainty. Second, the *cascade channel*: within-stage shocks also move the stage price index $P_{i,s}$, which in turn shifts the ultimate cost share $\Theta_{i,s}$ of stage s in M_i. When stages are complements and the stage's cascade term is less than one ($\kappa_{i,s} > 0$), this cascade effect amplifies the within-stage compression, as the stage where a dominant exporter is most important also sees a larger increase in its cost share when shock realizations are bad. Parts (C) and (D) of the proposition narrows focus to a special case. Under two exporters and symmetric cascade sensitivity, share gaps are monotone in Ricardian comparative advantage indices, delivering a clean sign result: uncertainty always attenuates comparative advantage. Further, the compression of Ricardian comparative advantage in this setting is amplified with multiple stages relative to the single stage case, when stages are complements. Intuitively, cross-stage complementarity amplifies the risk-diversification motive, resulting in larger equilibrium shifts away from dominant exporters towards smaller exporters to mitigate risk.

Proposition 3 characterizes the local response of $\Delta_{jj',i}^{s,s'}$ to uncertainty for a fixed importer-exporter-stage combination. We now aggregate across all pairs to characterize the sign of the cross-sectional covariance between the Ricardian comparative-

advantage index and the risk wedge, which is the object most directly relevant for empirical estimation of the mapping from fundamentals to specialization.

Proposition 4 *Maintain the assumptions of Proposition 3. Let*

$$W^{s,s'}_{jj',i}(v) := \log\frac{\Psi_{j'i,s}}{\Psi_{ji,s}} - \log\frac{\Psi_{j'i,s'}}{\Psi_{ji,s'}}$$

be the uncertainty wedge, so that $\Delta^{s,s'}_{jj',i} = \frac{\rho}{1-\rho}\mathrm{CA}^{s,s'}_{jj'} + W^{s,s'}_{jj',i}(v)$. *Define the* risk-compression index $D_{i,s}(j,j') := (\rho + \kappa_{i,s})\,\Delta s^0_{i,s}(j,j')$ *and its cross-stage difference* $B^{s,s'}_i(j,j') := D_{i,s}(j,j') - D_{i,s'}(j,j')$. *Then, as* $v \downarrow 0$,

$$\mathrm{Cov}(\mathrm{CA}, W(v)) = -\frac{\rho}{1-\rho}\, v\, \mathbb{E}[\mathrm{CA}\cdot B] + o(v). \tag{24}$$

In particular, $\mathrm{Cov}(\mathrm{CA}, W(v)) < 0$ *for all sufficiently small* $v > 0$ *whenever* $\mathbb{E}[\mathrm{CA}\cdot B] > 0$ *and* $\mathrm{Var}(\mathrm{CA}) > 0$. *A sufficient condition is* $\mathrm{CA}^{s,s'}_{jj'} \cdot B^{s,s'}_i(j,j') > 0$ *for all* (i,j,j',s,s') *with* $\mathrm{CA}^{s,s'}_{jj'} \neq 0$.

Proof. See Appendix A.3.6. ■

Proposition 4 establishes that the pair-level attenuation in Proposition 3 can aggregate into a negative cross-sectional covariance between the CA index and the risk wedge, systematically weakening the empirical mapping from Ricardian fundamentals to specialization. The key sufficient statistic is the deterministic share gap $\Delta s^0_{i,s}(j,j')$: mean-one risk reallocates expected sourcing weight away from dominant suppliers toward smaller ones, and the magnitude of this compression is scaled by $(\rho+\kappa_{i,s})$, where within-stage curvature ρ captures direct compression and $\kappa_{i,s}$ captures cascade amplification through the stage's ultimate cost share. The sufficient condition $\mathrm{CA}\cdot B > 0$ then requires that this compression force be strongest precisely where Ricardian fundamentals predict the greatest specialization, ensuring the wedge moves opposite to CA. In our quantitative analysis (Section 3) we evaluate the empirical strength of this attenuation force.

2.8 Comparative Statics

In general, the terms in equation 12 could amplify or offset each other depending on the full set of parameters and endogenous elasticities in the model. To illuminate this, and to illustrate differential impacts of upstream and downstream risk, we next turn to a simple two-location setting for comparative statics.

We assume that both countries i, j are symmetric in productivities and labor endowments, as well as in the shock variances. We consider scenarios with two stages of production to illustrate the role of elasticities for sourcing shares and output, and a scenario with three stages to illustrate the role of upstream vs. downstream risk. In the initial free trade equilibrium, both countries choose identical sourcing shares of every stage from each other, and have identical output and real wages.

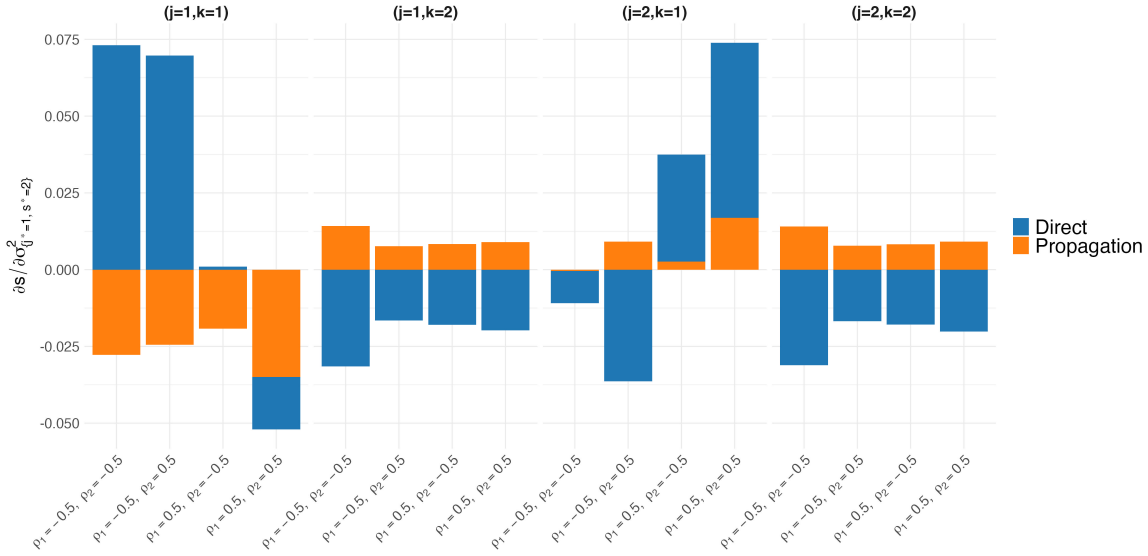
Small changes in $\sigma^2_{1,1}$. We first consider a small shock to $\sigma^2_{1,1}$, starting from the symmetric equilibrium in a two stage economy. This helps clarify the role of the terms in equation (12) (the direct effect $\mathbf{h}$ and the propagation/indirect effect $(\mathbf{I} - \mathbf{A})^{-1}$) in the changes in sourcing patterns across countries and stages. Recall (12) is an exact decomposition holding prices and wages fixed. With an infinitesimal increase in the risk of production in country 1, stage 1, the decomposition in equation (12) completely summarizes the changes in sourcing, and by implication the changes in production, in the economy, as in equilibrium wages and prices do not respond.

Panel A of Figure 1 contains the results for varied combinations of within-stage elasticities, where $\eta = -1$. As stages of production are complements, the direct shock effect typically leads to opposing effects on stage 1 sourcing across locations. The direction of the effect depends on ρ_1: when varieties in stage 1 are also complements, the direct effect of increased stage-1 risk in location 1 is to increase the stage 1 sourcing share from location 1. This effect is slightly moderated when varieties in stage 2 are substitutes. Here, ρ_2 plays a role in the stage-weight λ_2. When stage 1 varieties are substitutes, the direct effect is to increase the sourcing share of stage 1 from location 2, and decrease the sourcing share from location 1. Stages are complements in this scenario, and the upstream stage is now riskier. Across all combinations of within-stage elasticities, therefore, the direct effect is that retailers substitute away from sourcing stage 2, and increase the share of stage 1 in overall production.

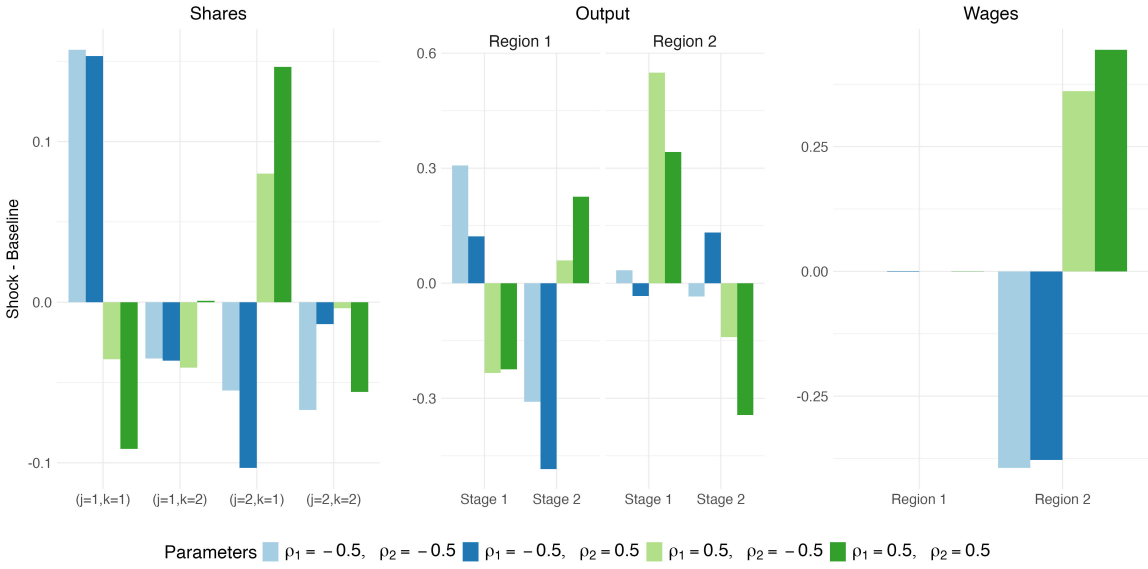
The propagation terms in orange are often substantially larger than the direct effect. In all cases, they reduce ex-ante orders from location 1 in stage 1. The propagation term includes the reallocation of input orders in $\mathbf{M_i}$, and the output elasticity of $\mathbf{M_i}$ depends on ρ_1, ρ_2 and η, determining the magnitude of this effect.

General equilibrium. We next consider the general equilibrium effects of a larger shock, where we increase $\sigma^2_{1,1}$ by 50%. The bottom panel of Figure 1 illustrates the changes in sourcing shares, output and relative wages for $\eta = -1$. Following the large shock, the general equilibrium pattern of changes in sourcing shares is qualita-

Figure 1: Direct and Indirect Effects on Sourcing Patterns



(a) Panel A: Small change in $\sigma^2_{1,1}$



(b) Panel B: Large change in $\sigma^2_{1,1}$

Note. The figures illustrate the changes in sourcing in both countries $j = 1, 2$ and both stages of production $k = 1, 2$ when $\sigma^2_{1,1}$ increases infinitesimally (Panel A) or by 50% (Panel B). The bars in the top panel decompose the changes in sourcing shares following equation (12). The blue bars show the direct effect of the shock on sourcing shares, and the orange bars show the propagation/indirect effects. The bars in the bottom panel show the general equilibrium responses in sourcing shares, output and wages under different values of ρ_s.

tively similar to those following the small variance shock above, but the magnitudes are amplified. That is, general equilibrium wage and price effects do not undo the pattern of changes in sourcing predicted by equation (12). The middle panels of the figures contain the changes in equilibrium output of each stage in each region, reflecting the reorganization of production in each country following the shock. With complementarity across stages, global output of stage 1 increases with an increase in $\sigma^2_{1,1}$. Finally, the right panels illustrate changes in relative wages. Relative wages in country 2 decrease when $\rho_1 < 0$, as the riskier input from country 1 is now more critical in production and stages are complements.

Figure 2 illustrates the change in welfare of a country relative to a deterministic environment, as the risk in the economy increases. We assume a common variance scale v in both countries to parameterize σ^2, and plot the change in welfare of country 1 as v increases. Welfare decreases in risk, with the risk aversion component of the welfare decomposition equation (15) a relatively large component of the welfare decline for high v.

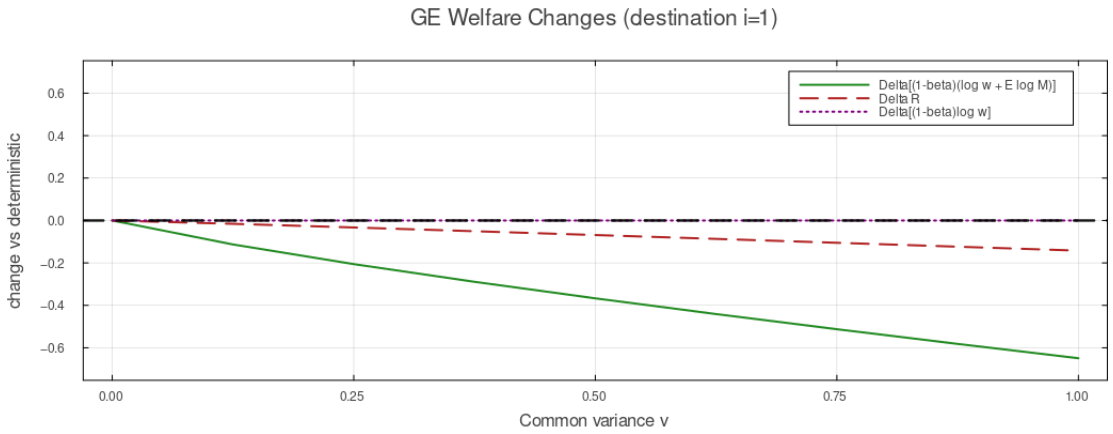
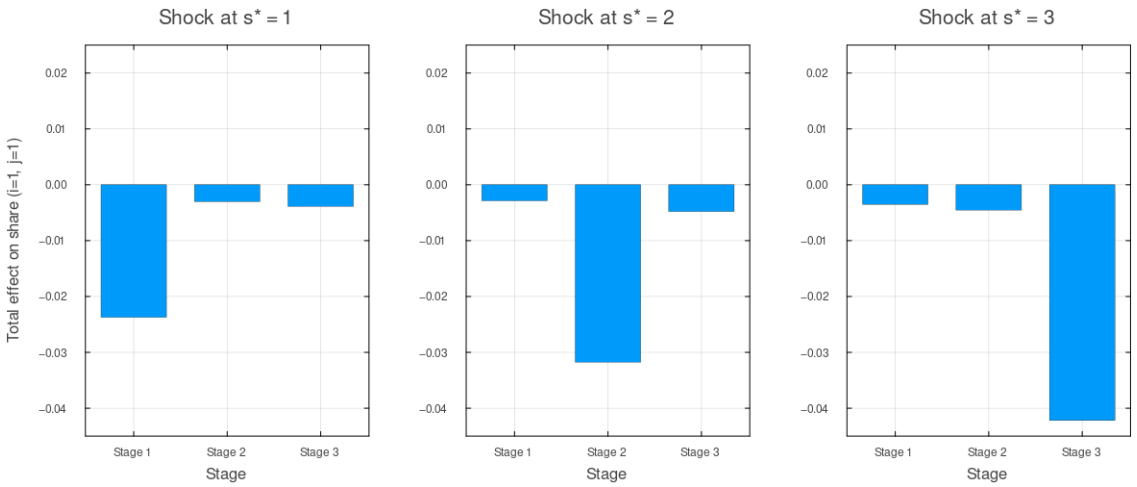


Figure 2: Welfare changes due to risk

Note. The figure plots the decomposition of welfare change from a deterministic economy to a risky economy parameterized by common variance scale v, in the asymmetric two-country three-stage model. We use $\rho_s = 0.70$, $\eta = -5.0$, $\alpha_{1,2} = \alpha_{1,3} = 2/3$, $\alpha_{2,2} = \alpha_{2,3} = 1$, $\tau_{ij,s} = 1 \, \forall \, i, j, s$, $a_1 = [1.4, 1.0, 0.8]$, and $a_2 = [0.8, 1.0, 1.4]$. The solid green line reports total welfare change, $\Delta[(1-\beta)(\log w + \mathbb{E} \log M)]$. The dashed red line reports the risk-aversion component $\Delta R_i = \mathbb{E}[\log \tilde{M}_i] - \log \mathbb{E}[\tilde{M}_i]$ (term (v), equation 15). The dotted purple line isolates the wage component $\Delta[(1-\beta) \log w]$ (term (i), equation 15).

Position effects: upstream vs downstream risk. We increase the stages of production to 3 to illustrate the role of changes in upstream vs. downstream risk.

Figure 3: Position effects: upstream vs downstream risk



Note. The figures illustrate the changes in sourcing in country 1 across all stages of production $s = 1, 2, 3$ for increases in risk of a stage. The left panel illustrates changes in sourcing when $\sigma^2_{1,1}$ increases, the middle panel when $\sigma^2_{1,2}$ increases and the right panel when $\sigma^2_{1,3}$ increases. Throughout, stages are complements ($\eta = -0.5$) and locations within a stage are substitutes $\rho_s = 1$.

We assume $\alpha_2 = 0.66$, $\alpha_3 = 1$, $\eta = -0.5$ and $\rho = 1$. Figure 3 illustrates the results for changes in risk of each stage. Increased risk in any stage in country 1 results in decreased sourcing shares of that stage. Position effects play a visible role in the magnitude of changes in sourcing. For an increase in upstream risk, the decrease in the sourcing share of the riskier stage is smallest, while an increase in downstream risk of stage 3 results in a larger decrease in stage 3 sourcing.

Attenuation of comparative advantage with risk In a last comparative static, we illustrate the attenuation of comparative advantage with risk. Figure 4 illustrates how the log-ratio difference term $\Delta^{s,s'}_{jj',i}$ (equation 18) changes as the risk in the economy (parameterized by a common variance scale v) increases. While there is no change in the Ricardian CA term, the risk wedge changes as v increases, driving the $\Delta^{s,s'}_{jj',i}$ terms towards zero, between all stage-pairs.

3 Quantification

3.1 Data

Our quantitative analysis requires data on bilateral trade flows by production stage and measures of trade costs (e.g., tariffs) to estimate ρ_s and η. We further require

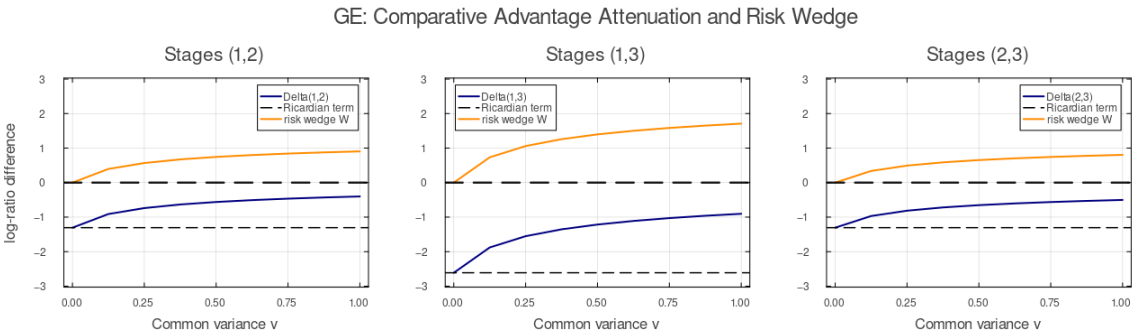


Figure 4: Risk-adjusted Comparative Advantage

Note. The figure plots the change in comparative advantage from a deterministic economy to a risky economy parameterized by common variance scale v iin the asymmetric two-country three-stage model. We use $\rho_s = 0.70$, $\eta = -5.0$, $\alpha_{1,2} = \alpha_{1,3} = 2/3$, $\alpha_{2,2} = \alpha_{2,3} = 1$, $\tau_{ij,s} = 1 \forall i, j, s$, $a_1 = [1.4, 1.0, 0.8]$, and $a_2 = [0.8, 1.0, 1.4]$. The solid blue line plots the double-difference $\Delta^{s,s'}_{jj',i}$ (equation 18). The dashed gray line plots the Ricardian comparative advantage component $\frac{\rho}{1-\rho}\left[\log\frac{a_{j',s}\tau_{ji,s}}{a_{j,s}\tau_{j'i,s}} - \log\frac{a_{j',s'}\tau_{ji,s'}}{a_{j,s'}\tau_{j'i,s'}}\right]$. The solid yellow line plots the uncertainty wedge, $W^{s,s'}_{jj',i}(v) = \log\frac{\Psi_{j'i,s}}{\Psi_{ji,s}} - \log\frac{\Psi_{j'i,s'}}{\Psi_{ji,s'}}$. The three panels correspond to stage pairs $(1,2)$, $(1,3)$, and $(2,3)$, respectively.

data on country size and other region-specific variables for model calibration.

For computational reasons, we focus on the top 50 countries by size. Size is proxied by 2016 GDP, obtained from the CEPII gravity database (Conte et al., 2022). We restrict the list to countries also present in the OECD (2023) ICIO tables (which are used for robustness checks), and drop Iraq and Taiwan due to minimal tariff coverage. Our final 50 country sample is listed in Appendix Table C1.

For the number of production stages S, we use the measure of upstreamness from Antràs et al. (2012). This measure ranks sectors by their upstreamness, based on their position in input-output tables, and is the most commonly used upstreamness measure in the literature. There are discrete values for 19 manufacturing sectors, which we group into $S = 3$ stages of production by partitioning them into three equally-sized groups according to their upstreamness measure.[8] We concord these stages, which are ISIC-based, to NAICS codes using the concordance provided by the U.S. Census Bureau. We then concord the NAICS codes to HS codes using the concordance by Pierce and Schott (2012). Appendix table C2 lists the stages of production.

We obtain bilateral trade flows from the BACI version of UN-COMTRADE (Gaulier

[8]While the baseline measure has discrete values for 41 sectors, the remaining sectors are services sectors and do not map to any Harmonized System (HS) codes in our bilateral trade data.

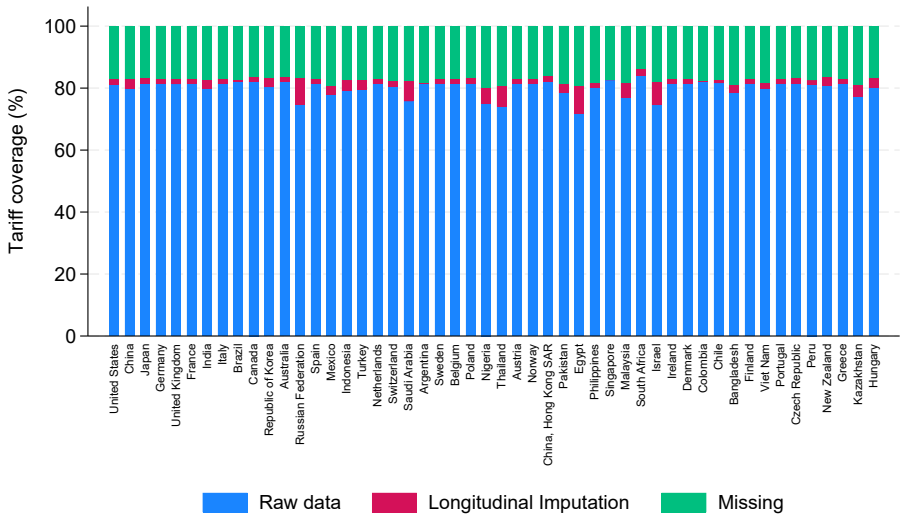
and Zignago, 2010), produced by the CEPII, for the years 1995-2023. The data include exporter, importer, HS-6 product code, value and quantity traded annually. Over this period, the HS-6 product code definitions are subject to periodic revisions. We concord all HS-6 vintages in the data to HS-2017 whenever the mapping across pairs of vintages is unique (1:1), or when multiple HS codes of a vintage map uniquely to a 2017 HS-code, following Boehm et al. (2023). We construct the universe of origin-destination-product trade flows by imputing zero flows for any country-pair-HS-2017-year where there are no reported trade flows in BACI. We map the final HS-codes to the 3 stages using the process described above. These data are then used for estimation, and aggregated at the exporter-stage and importer-stage level to construct export and import shares by stage.

For tariff data, we use the database from Teti (2025), supplemented by information from UNCTAD-TRAINS. As with the trade flows, we map all tariffs to the HS-2017 vintage, using only unique mappings or cases where n HS-codes map to a single 2017 HS code. In the latter case, the implied vintage-2017 tariff is the weighted average tariff of the n codes being replaced, where the weights are the trade flows in the codes in the last year prior to the vintage change.

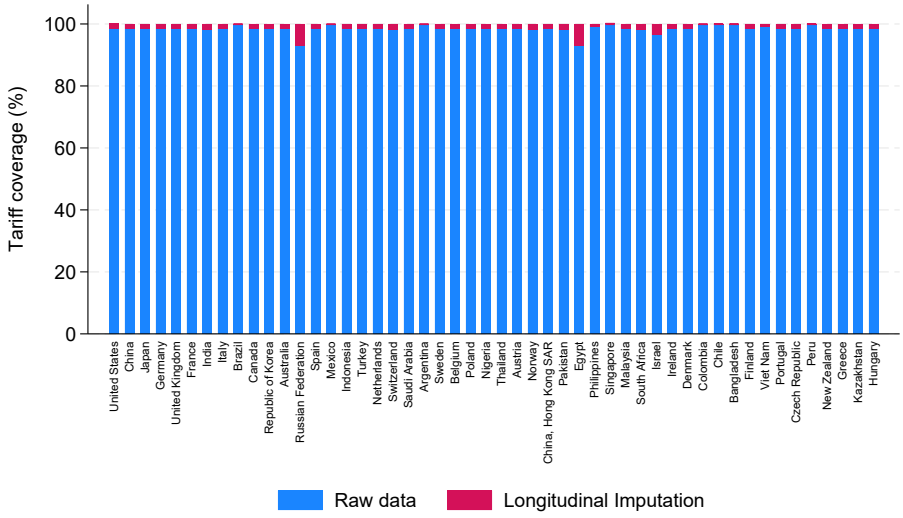
An important step in constructing our tariff data is obtaining bilateral tariffs for zero observed trade flows. While in some cases tariff coverage is more comprehensive than trade flows, in most cases the absence of observed trade between i, j corresponds to a missing bilateral tariff rate. In these cases, as a baseline, we assume the importer's prevailing MFN rate for that HS-6 product would apply, should there be bilateral trade in that product. If the importer's HS-6 MFN rate is not available, we leverage the panel dimension of the data and use a simple set of rules to fill in the missing tariff rate. Appendix C.3 details the steps involved. Figure 5 illustrates our tariff coverage. When bilateral trade flows are observed, they have an associated tariff rate 99.4% of the time. When bilateral trade flows are zero, the coverage is 77.9%. For our parameter estimation, we restrict the sample to HS-codes with tariff coverage over 95%. For stage-specific bilateral tariffs, we compute both a simple average and a trade-weighted average of the HS6-level bilateral tariffs in that stage.

Additionally, to estimate trade barriers, we use standard gravity variables– common language, distance and colonial ties – which we obtain from the CEPII gravity database (Conte et al., 2022). Finally, we obtain regional population from the Penn World Tables Version 10.01 (Feenstra et al., 2015).

Figure 5: Tariff Coverage



(a) All HS



(b) HS in sample

Note. The figure plots the share of bilateral trade-flow observations for which a tariff rate is available. Tariff data are drawn from Teti (2025) supplemented by UNCTAD-TRAINS; the imputation procedure is described in Section 3.1 and Appendix C.3. Blue bars indicate raw tariff data; red bars indicate longitudinal imputation; green bars indicate missing observations. Panel (a) covers all HS codes in the trade flow data. Panel (b) restricts to the HS codes used in gravity estimation, which require tariff coverage exceeding 95%.

3.2 Calibration

The key parameters for our calibration are ρ_s and η, which help determine the two elasticities of substitution in our framework: across products within a stage, and across stages of production. Typically these parameters are estimated using gravity equations. As equation 10 makes clear, in this model IIA is violated, and trade flows have a gravity-like formulation in expectation. Standard estimation of gravity using exporter-stage and importer-stage fixed effects will be subject to bias, arising from the non-separable i, j, s term inside the expectation in equation 10. We therefore turn to maximum likelihood estimation (MLE) to estimate these parameters, together with other essential parameters such as origin-stage productivities and bilateral trade costs. Note that a standard gravity estimation with controls for the i, j, s term is equivalent to a second-order approximation of the model MLE. As robustness, we also estimate the standard gravity equation with controls and compare parameter estimates with those obtained from the MLE in Appendix B.

3.2.1 Estimation Strategy

The model delivers a gravity-like equation for bilateral trade flows by stage, in expectation. Equation (10) implies

$$\ln X_{ji,st}^{Data} = -\phi_s \ln \tau_{ji,st} - \phi_s \ln w_{j,t} + \phi_s \ln a_{j,s} + \frac{1}{1-\rho_s} \ln\big((1-\beta) w_{i,t} L_{i,t}\big) + \frac{1}{1-\rho_s} \ln \Psi_{ji,st} + \varepsilon_{ji,st}, \tag{25}$$

where $\phi_s \equiv \rho_s/(1-\rho_s)$ and

$$\Psi_{ji,st} \equiv \mathbb{E}_{\boldsymbol{\chi}_t}\left[\frac{(\chi_{ji,s,t})^{\rho_s}}{\sum_{j'} \big(\chi_{j'i,s,t}\, M_{j'i,st}\big)^{\rho_s}} \lambda_{i,s,t}(\boldsymbol{\chi}_t) \prod_{u=s+1}^{S} \big(1 - \lambda_{i,u,t}(\boldsymbol{\chi}_t)\big) \right]. \tag{26}$$

Given data on bilateral-stage trade flows $X_{ji,st}^{Data}$ and bilateral trade costs parameterized as $\ln \tau_{ji,st} = \ln(1 + tar_{ji,st}) + \beta^{Dist} \ln Dist_{ji} + \beta^{Lang}(1 - \text{comlang}_{ji})$, we estimate (25) by PPML, iterating until convergence. The parameters to be estimated are β^{Dist}, β^{Lang}, β^{Border},$\{a_{j,s}\}$, $\{\alpha_{i,s}\}$, η, and $\{\rho_s\}_{s=1}^{S}$. For given parameter values, $\Psi_{ji,st}$ is computed from the general equilibrium of the full model, with the expectation simulated by Monte Carlo. A key challenge is that standard gravity estimators assume the

independence-of-irrelevant-alternatives property, while our model does not: a trade-policy shock affecting pair (i, j, s) shifts $\Psi_{j'i,s,t}$ for all other pairs through the GE system. We address this by constructing $\Psi_{ji,st}$ endogenously at each iteration, yielding a model-consistent estimator. The full algorithm is detailed in Appendix B.

Parameter identification. We now discuss the economic intuition underlying the identification of each structural parameter.

Within-stage substitution elasticities ρ_s. The parameter ρ_s governs the elasticity of substitution across source countries within production stage s. The transformed elasticity $\phi_s = \rho_s/(1-\rho_s)$ enters equation (25) as the coefficient on the log bilateral cost $-\ln \tau_{ji,st}$: a one-percent reduction in bilateral trade costs raises the trade share of origin j in stage s by ϕ_s percent. We allow ρ_s to vary freely across stages, accommodating the possibility that early-stage inputs are more substitutable across origins than specialized late-stage components.

Identification of ϕ_s comes from cross-sectional variation in bilateral tariffs within a given stage, conditional on exporter×year and importer×year fixed effects that absorb all monadic determinants of trade such as origin and destination time-varying supply and demand shocks. Intuitively, if buyers can easily substitute across source countries within a stage, a tariff increase on one bilateral route sharply diverts flows toward alternatives—implying a large ϕ_s. When inputs from different origins are poor substitutes—because they embody relationship-specific investments or specialized capabilities—the same tariff change induces only modest reallocation, yielding a small ϕ_s. The stage-specific nature of our estimates allows the data to reveal whether substitutability is heterogeneous along the value chain, a feature that can be quantitatively important for the propagation of supply-chain risk.

Exporter-stage productivities $a_{j,s}$. Conditional on ϕ_s, we recover exporter-stage productivities from the exporter×stage fixed effects in the PPML regression: $\widehat{a}_{j,s} = \exp(\hat{\gamma}_{j,s}/\phi_s)$, normalized so that $\widehat{a}_{US,s} = 1\ \forall s$. Origins that ship systematically large volumes in a given stage to *all* destinations must possess higher stage-specific productivity. The fixed-effect structure ensures inference is not confounded by bilateral pair-specific factors (absorbed by pair fixed effects) or destination-specific demand conditions (absorbed by importer×year fixed effects). What remains in the exporter×stage margin is the country's absolute advantage at that production stage.

Trade-cost coefficients β^{Dist}, β^{Lang}, β^{Border}. The gravity coefficients are standard,

and capture the ad-valorem equivalent costs of distance, linguistic dissimilarity, and crossing an international border. They are recovered from the bilateral-pair residuals of the PPML regression, projected on log distance, a common-language indicator, and a foreign-country indicator, conditional on all monadic fixed effects. Identification follows canonical structural gravity logic: conditional on exporter-stage and importer-stage characteristics, pairs that are geographically closer, linguistically similar, or within the same national border trade systematically more, and the magnitude of these associations identifies the respective cost elasticities.

Stage weights $\alpha_{i,s}$. The parameters $\alpha_{i,s}$ govern how each importer allocates expenditure across production stages. Given the across-stage elasticity η, they are updated to match observed stage expenditure shares by setting $\alpha_s^{\text{new}} \propto \big(\mathbb{E}[\lambda_{i,s,t} \prod_{u>s}(1 - \lambda_{i,u,t})]\big)/S_{s,t}^{\text{Data}}$, with the normalization $\alpha_1 = 1$. A country that devotes a larger fraction of its intermediate input bill to stage s is assigned a higher technological weight on that stage. Country-specific $\alpha_{i,s}$ accommodate heterogeneity in production structures—for instance, assembly-intensive versus resource-processing economies—within stage dimension which contains the vertical structure of the value chain.

Cross-stage substitution parameter η. The parameter η governs the complementarity or substitutability of production stages. With $\eta < 0$ stages are complements: a disruption at one stage propagates to others and cannot easily be offset by expanding production elsewhere. If $\eta \to 0$ (Cobb-Douglas) this amplification is muted.

We estimate η by evaluating the full nested estimation procedure over a grid of candidate values and selecting the one that maximizes the log-likelihood of the model. The identifying variation is the time-series movement in relative stage expenditure shares induced by tariff-driven cost shifters: when tariffs on inputs used intensively at stage s decline, expenditure should reallocate toward that stage, with the magnitude governed by η. Differencing relative to a base stage eliminates importer×time level effects—such as aggregate demand growth—that would otherwise confound identification. Under strong complementarity ($\eta \ll 0$) a tariff reduction at one stage induces only modest reallocation, because firms need all stages in roughly fixed proportions; under stronger substitutability, the same shock produces a pronounced shift in the stage composition of expenditure. Because sourcing shares are endogenous, we instrument for them using a shift-share constructed from baseline import weights interacted

with changes in bilateral tariffs (see Appendix B).[9]

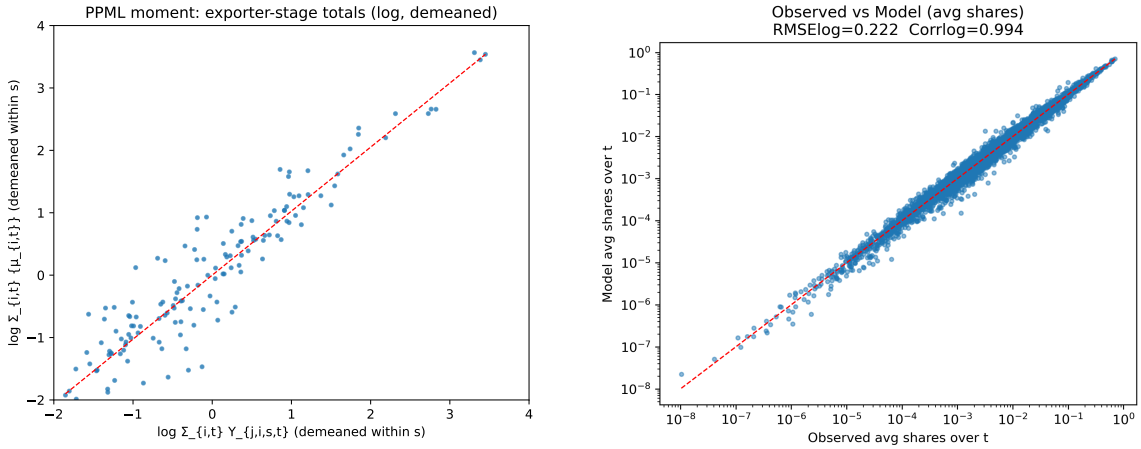
Shock volatilities $\sigma_{j,s}$ and correlations $R^J \otimes R^S$. While the analytical results in Section 2 assumed that the shocks were iid and log-Normal, in the quantification we take a more flexible approach. In our baseline, the stochastic structure of risk is characterized by exporter×stage volatilities $\sigma_{j,s}$ and a separable covariance structure $\Sigma = \text{diag}(\sigma) \cdot (R^J \otimes R^S) \cdot \text{diag}(\sigma)$, where R^J captures cross-country correlations and R^S captures cross-stage correlations. Both objects are estimated post-convergence of the main estimation from structural residuals. Volatilities $\hat{\sigma}_{j,s}$ are identified from the time-series dispersion of each exporter-stage residual; the correlation matrices R^J and R^S are recovered from the co-movement of standardized residuals across countries and stages via an iterative flip-flop algorithm. The Kronecker structure reduces free correlation parameters from $O((JS)^2)$ to $O(J^2 + S^2)$, and is economically motivated by the observation that common macroeconomic shocks generate cross-country correlations that operate similarly at all stages, while technology linkages generate cross-stage correlations that are similar across countries. The full procedure is detailed in Appendix B.5.

Goodness of fit. In Figure 6, we show the model fit against targeted moments (total exports by origin-stage, in deviations from average, left panel) and untargeted moments (the export shares of an origin-stage to a destination in each period). The model fits very well with a correlation of 0.994 between the model untargeted moments and data.

3.2.2 Parameter estimates

Figure 7 illustrates the productivities of countries by stage (relative to the US) and the average country risk by stage (averaging across all covariances). Countries are sorted by the simple average across stage of the plotted variable. Countries such as Germany and Japan are relatively more productive than the US, while Australia is substantially more productive in the most upstream stage. Countries such as Bangladesh, Pakistan, and Nigeria are the least productive, overall. In terms of average risk, the right panel shows that most European countries are less risky than countries such as Kazakhstan,

[9] Note that in principle it would be possible to estimate a cross-stage specific η (in our application, two separate η values. However, once the data are aggregated to 50 countries and 3 stages, there is insufficient variation in the panel over time to estimate multiple η values. Therefore, we continue to maintain the assumption of a single η across stages.



(a) Total Exports by Origin-Stage

(b) Export Shares

Figure 6: Goodness of Fit: Model vs Data

Note. The figure assesses model fit by comparing model-implied moments to their empirical counterparts from BACI. Panel (a) plots log total exports by origin-stage, $\log \sum_i X^{\text{Data}}_{ji,s,t}$, demeaned within stage s, against the model-implied equivalent. Each dot is one origin–stage observation. Panel (b) plots the model-implied average bilateral sourcing share of origin j in stage s for destination i (averaged over time) against its empirical counterpart, on a log-log scale. Each dot is origin–destination–stage observation.

Table 1: Parameter Estimates

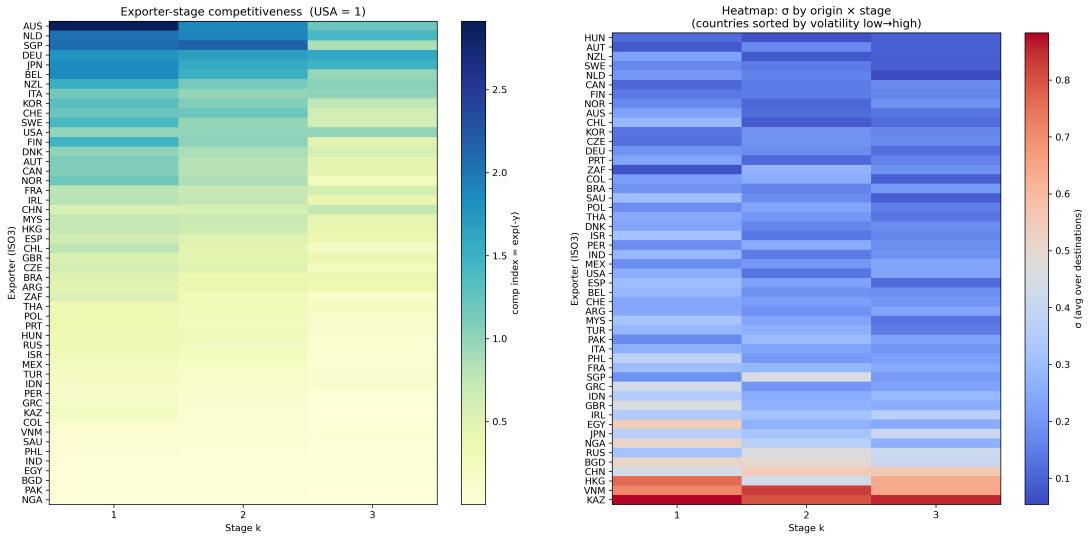
		Stage		
Parameter		$s=1$	$s=2$	$s=3$
Within-stage substitution	$\hat{\rho}_s$	0.86	0.83	0.69
Stage weight (range across i)	$\hat{\alpha}_{i,s}$	1	[0.01,2.23]	[0.44,9.17]
Cross-stage substitution	$\hat{\eta}$		-5.9	

Note. The table reports structural parameter estimates from the nested estimation procedure described in Section 3.2.1. $\hat{\rho}_s$ is the within-stage elasticity of substitution across source countries at stage s. $\hat{\alpha}_{i,s}$ is the destination-specific weight on stage s in the sequential CES aggregator; the reported range reflects heterogeneity across destination countries i, with $\alpha_1 = 1$ normalized for the final stage. $\hat{\eta}$ is the cross-stage substitution elasticity.

Vietnam, China or Russia. In both panels, there is often substantial heterogeneity across stages.

While we are not aware of empirically estimated measures of our precise definition η, there is a related theoretical literature, which often assumes Cobb-Douglas

Figure 7: Parameter Estimates



(a) Estimated competitiveness$_{j,k}$ (b) Estimated $\sigma_{j,k}$

Note. Panel (a) displays estimated exporter-stage competitiveness, normalized so that US = 1, recovered from the exporter-stage fixed effects of the PPML gravity regression (See Appendix B.4 for more details). Values above 1 (darker blue) indicate greater productivity relative to the US. Countries are ordered from highest to lowest average competitiveness. Panel (b) displays estimated origin-stage shock volatilities $\hat{\sigma}_{j,s}$, computed from the time-series dispersion of structural residuals (See Appendix B.5 for more details). Higher values (darker red) indicate greater supply-chain risk. Countries are ordered from lowest to highest simple average volatility across stage $\bar{\sigma}_j = \frac{1}{3}\sum_s \hat{\sigma}_{j,s}$. In both panels, the three columns correspond to stages $s = 1, 2, 3$ from most upstream (left) to most downstream (right).

aggregators across stages.[10]

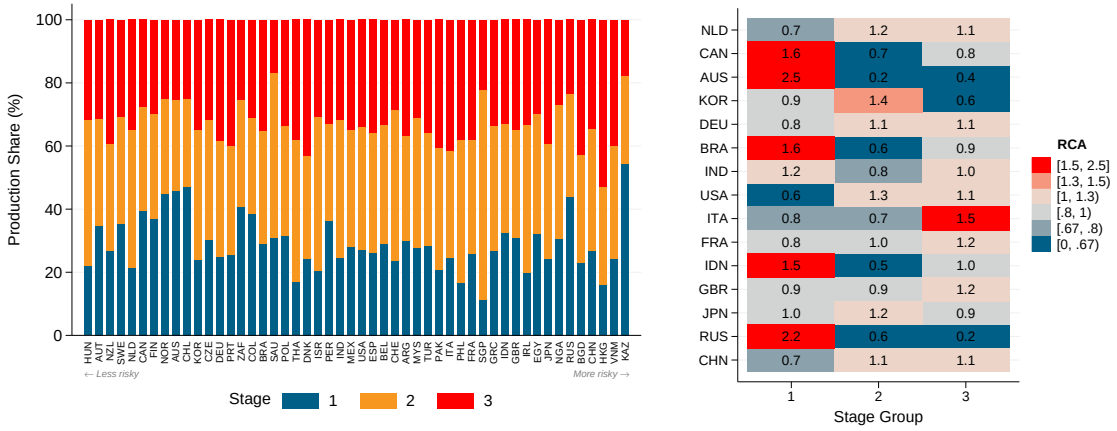
4 Quantitative Results

We first illustrate heterogeneity in production patterns in our baseline economy in the calibrated model. We then turn to counterfactuals. Our first counterfactual contrasts the baseline risky economy to a deterministic setting, focusing on how comparative advantage changes. We then consider counterfactuals contrasting the impact of upstream risk increases to downstream risk increases, to highlight the role of position effects in the model. The model can be used to study how production patterns and

[10]For some papers in this area, see Caliendo and Parro (2015), Antràs and de Gortari (2020), Johnson and Moxnes (2023), Lee and Yi (2018), Wei and Xie (2018), Sposi et al. (2020), Yi (2003), and also Yi (2010).

welfare changes in response to geopolitical events. For instance, we show counterfactuals where the origin risk of China (or US) increases, and where US trade costs increase. Finally, we consider a counterfactual economy with 9 stages of production, to highlight the interaction of sequential production in amplifying or mitigating risk.

4.1 Baseline Economy



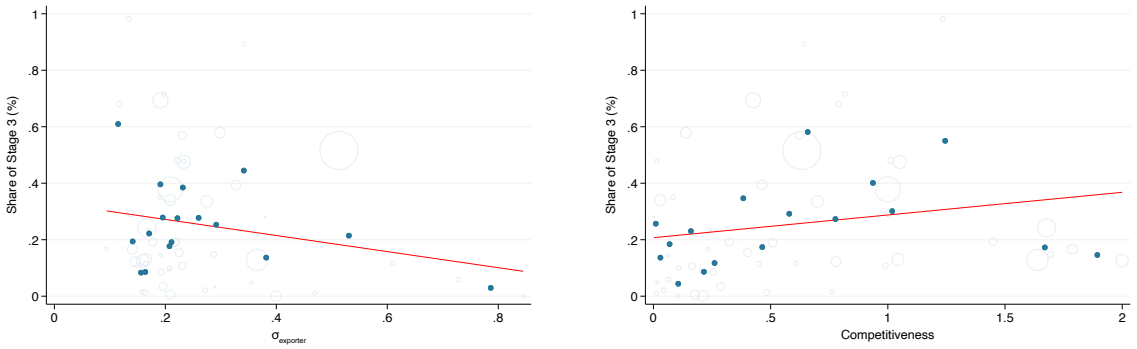
(a) Within country production share

(b) RCA, top 15 countries

Figure 8: Allocation of Production

Note. The left panel figure plots the share of production value by stage and by country of importer. The right panel figure plots the revealed comparative advantage (RCA), or commonly refers as Balassa Index, for the top 15 countries based on total income (wage times labor). For both panels, countries are ordered from lowest to highest simple average volatility across stage.

Figure 8 characterizes the baseline equilibrium. The left panel plots each country's within-country production share across the three stages, with countries ordered along the horizontal axis by average exporter risk $\bar{\sigma}_j$. There is substantial heterogeneity in production structure across countries: some economies concentrate overwhelmingly in upstream stages while others are primarily downstream producers, and this heterogeneity is not fully explained by risk alone. As discussed above, risk is itself a complex, multi-dimensional object here—it encompasses not only marginal volatilities $\sigma_{j,s}$ but also cross-country and cross-stage covariances—so the ordering by average $\bar{\sigma}_j$ provides only a partial characterization of a country's risk. Consistent with this, substantial dispersion in production shares remains conditional on risk, so that a country's position in the value chain reflects both its stage-specific volatilities and the full covariance



(a) Against country riskiness

(b) Against country competitiveness

Figure 9: Average production stage

Note. The left panel figure plots each country's average production stage against its risk level. Average production stage is the average country stage, weighted by production share. Both variables are residualized by productivity. The right panel figure plots each country's average production stage against its average productivity level. Both variables are residualized by country's risk level.

structure of shocks. The right panel of Figure 8 reports revealed comparative advantage (RCA) indices for the fifteen largest economies across the three stages. There is pronounced heterogeneity in comparative advantage: exporters of minerals, metals, and primary commodities such as Canada, Australia, and Russia exhibit strong RCA in upstream (stage 1) production, while more industrialized economies tend toward downstream stages.

Figure 9 illustrates some patterns in the baseline economy: riskier countries tend to specialize relatively more in upstream stages (left panel). With complementarity across stages ($\eta < 0$), an increase in upstream risk induces buyers to source more from risky upstream origins—since the binding constraint on production is more likely to fall on the upstream stage. This raises the expected marginal value of upstream inputs, and in equilibrium, riskier origins have relatively larger shares of upstream production. Consistent with Costinot et al. (2013), more productive countries are, on average, more downstream, though the relationship is quantitatively weak in the cross-section, as productivity differences interact with risk and trade costs in shaping equilibrium specialization. Overall, these patterns indicate that both productivity and risk are important for determining where countries sit in global value chains in the calibrated model.

4.2 Counterfactuals

4.2.1 Deterministic Economy

In the first counterfactual, we contrast the baseline economy to a world without supply-chain risk. Propositions 3 and 4 established that, in the risky world, bilateral trade flows are ranked by risk-adjusted comparative advantage $\tilde{a}_{ji,s} \equiv (a_{j,s}/\tau_{ji,s})\Psi_{ji,s}^{(1-\rho)/\rho}$ rather than Ricardian fundamentals alone, with the risk wedge $\mathcal{W}_{jj',i}^{s,s'}$ compressing the double-difference $\Delta_{jj',i}^{s,s'}$ relative to its deterministic counterpart. That said, the assumptions underlying these propositions (a constant ρ, and iid shocks) are violated in the fully calibrated model, so the counterfactual helps to evaluate this mechanism quantitatively.

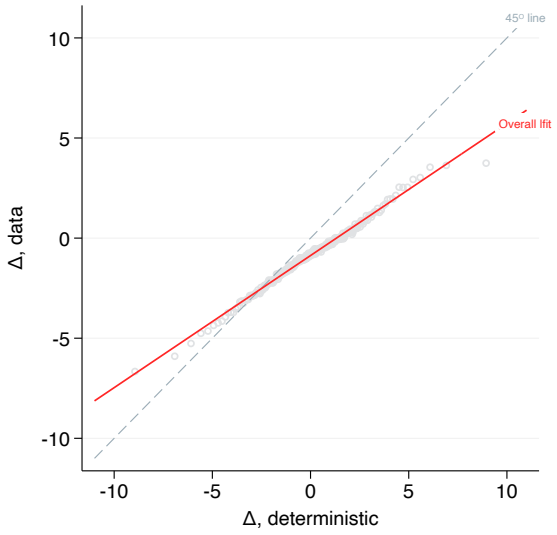


Figure 10: Double difference, data against deterministic

Note. The figure plots the double-difference $\Delta_{jj',i}^{s,s'}$, defined in (18), computed from observed data against the corresponding value from the deterministic counterfactual equilibrium. Each dot represents one country × stage pair, binned into 300 groups.

Figure 10 and Figure 11 confirms the insights of these propositions hold in the quantification. Figure 10 plots the double difference in the data against the deterministic counterfactual for all country×stage pairs: the scatter line is flatter than the 45-degree line, indicating that there is more diversification in reality than what Ricardian fundamentals alone would predict. Figure 11 decomposes the welfare difference between the baseline (risky) and deterministic economies. Largely consistent with

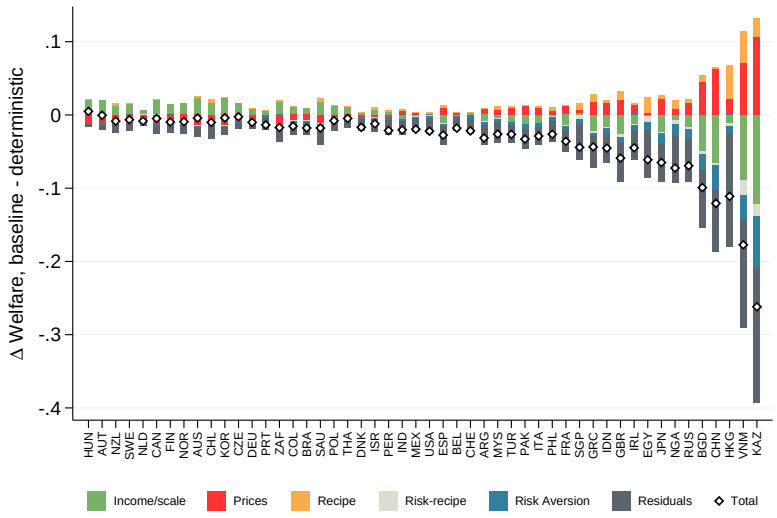


Figure 11: Change in welfare, baseline (risky) - deterministic

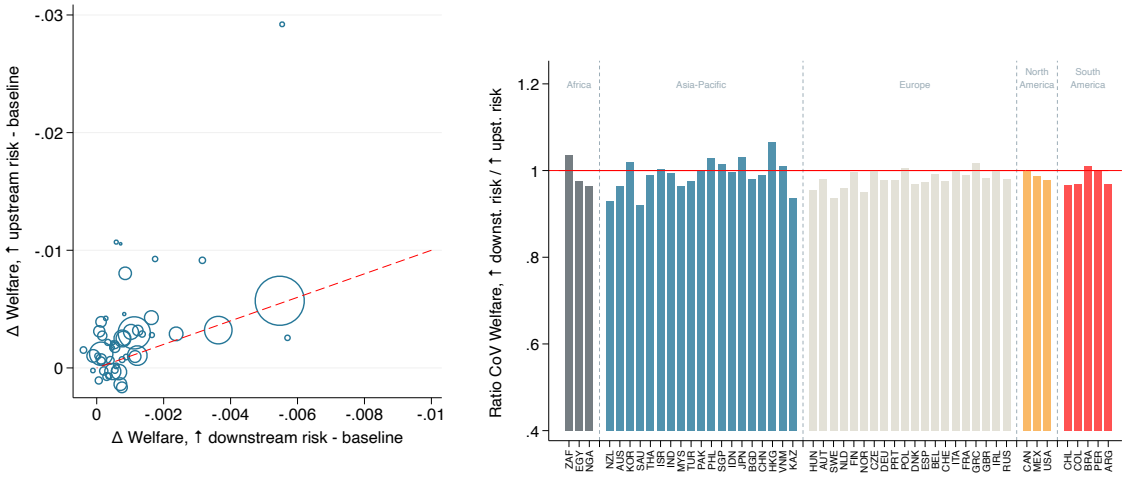
Note. The figure plots the welfare decomposition, defined in (15), when moving from the deterministic economy to the baseline risky equilibrium. Income/scale bar represents the wage adjustment $\Delta \log w_i$. Prices bar represents the terms-of-trade effect $-\sum_a \bar{s}_a^{EQ} \Delta \log p_a$. Recipe bar represents input-mix reallocation $\sum_a \bar{s}_a^{EQ} \Delta \log s_a$. Risk-recipe bar represents the insurance cost $-\sum_a (\bar{s}_a^{EQ} - \bar{s}_a^{RN}) \Delta \log r_a$. Risk Aversion bar represents ΔR_i. The diamond denotes the total welfare change, $\Delta \mathcal{W}_i$. Countries are ordered from lowest to highest simple average volatility across stage.

the analytical result that $\partial \mathcal{W}_i / \partial \sigma_{ji,s}^2 \leq 0$ when $\eta \leq 0$, welfare is lower in the risky world for nearly all countries. For less risky countries, the income effects contribute positively to welfare in the risky world, as demand shifts towards them raising wages. The converse is true for riskier locations, which see negative income/wage terms. The income and price components partially offset each other, as in risky countries, the negative GE impact of a decline in demand for their output results in an improved relative price term in the decomposition (with trade costs, countries purchase a significant share of their own output, and so a decrease in own wages reduces the prices of inputs into production). Overall, risk depresses *real* wages and lowers real output through the distortion of sourcing decisions, with the reallocation of production toward more diversified—if less efficient—sourcing recipes reducing expected output. For the riskiest countries, the risk-aversion channel accounts for a substantial share of the welfare loss.[11] The magnitude of welfare losses is heterogeneous across countries.

[11] The least risky country on average, Hungary, sees a small welfare gain in the risky world relative to the deterministic world, with increased demand for its inputs driving up the GE income term.

We note that the welfare decomposition in Equation (15) is first order, and with as large a change in risk as moving from a risky to a riskless world, the higher order residual components in the decomposition are large as well.

4.2.2 Upstream vs. Downstream Risk



(a) Change in welfare: upstream vs. downstream scenario

(b) Welfare volatility ratio

Figure 12: Increase Upstream Risk vs. Increase Downstream Risk

Note. The left panel plots the change in welfare in the increased upstream risk counterfactual against the increased downstream risk counterfactual. The dotted red line is the 45-degree line. The right panel plots the ratio of the welfare coefficient of variation under increased downstream risk to that under increased upstream risk. In both counterfactuals, we scale the relevant σ_{jk} values such that aggregate risk increases by 5%.

We next ask whether the position of where risk increases in the value chain matters, comparing an increase in upstream (stage 1) risk against an equal-sized increase in downstream (stage 3) risk, where both shocks are calibrated to raise aggregate risk by 5%.[12] Corollary 2 in the Appendix establishes that, under the condition $\eta \leq \rho_{\text{up}} \leq \rho_{\text{dn}}$ and when the upstream sector is large enough, upstream risk generates larger welfare losses than an equal-sized downstream shock. While our estimated parameters do not lie precisely in the interior of the corollary's sufficient condition (while $\eta < \rho_1, \rho_3$,

[12] Aggregate risk is defined as $\sum_{js}\sum_{j's'} \varsigma_{js}\varsigma_{j's'}\sigma_{js}\sigma_{j's'}R_{jj'}R_{ss'}$, where ς_{js} are trade-weighted exposure shares. The required scale factors for σ_{js} differ between the upstream and downstream scenarios because the two sets of origins have different initial volatilities and trade weights; see Appendix C.4 for details.

estimated $\rho_3 < \rho_1$), the quantitative results are broadly consistent with the analytical prediction.

The left panel of Figure 12 shows that, for the vast majority of countries, an increase in upstream risk reduces welfare by more than an equal-sized increase in downstream risk, with most observations lying above the 45-degree line. The right panel considers just the impact on the volatility of welfare: it plots the ratio of the coefficient of variation of welfare under downstream risk to that under upstream risk. For 31 of 50 countries, this ratio is below one, meaning that upstream risk increases generate greater output volatility than downstream risk increases of the same aggregate magnitude.

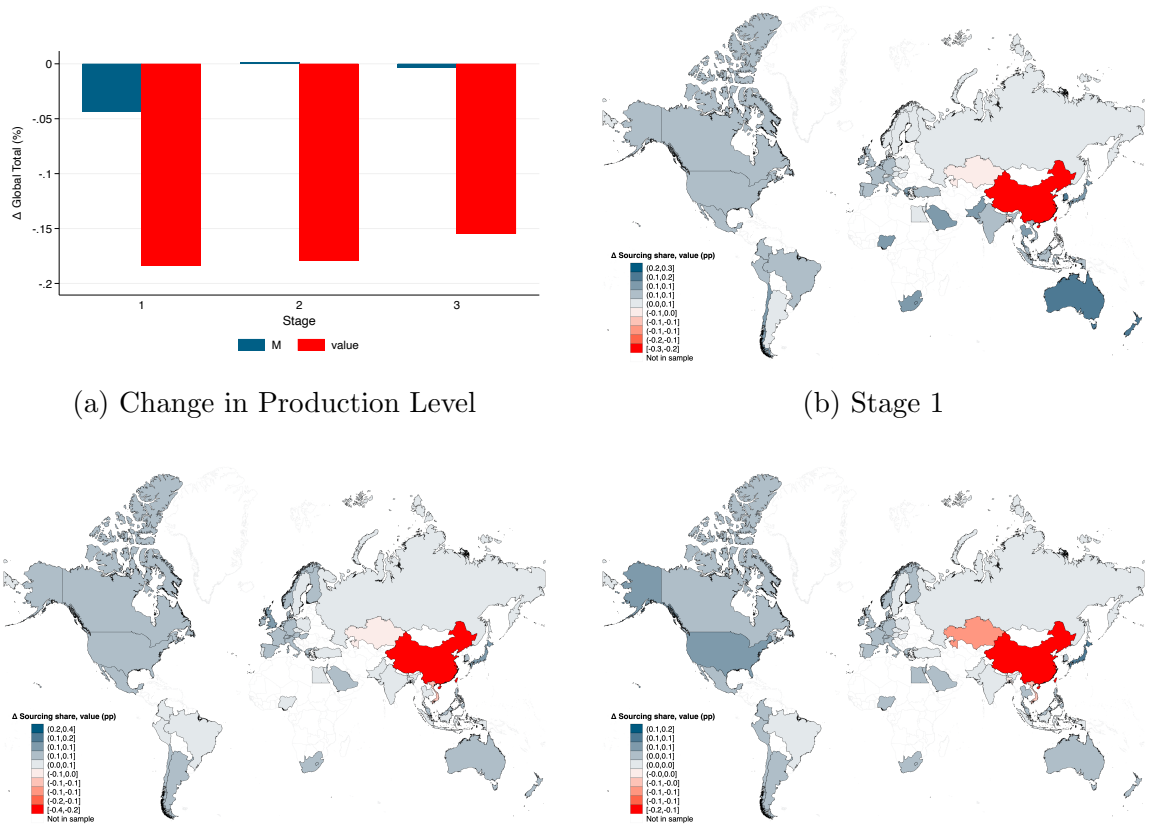
4.2.3 Origin risk: Increased risk from China

We next consider an increase in origin-specific risk, asking how the global economy responds to a 5% increase in aggregate risk driven entirely by an increase in China's supply-chain volatility. China's size and integration into all three stages of global manufacturing make this a particularly policy-relevant scenario, and our model's separation of risk by origin and stage allows us to trace the propagation of such a shock through the value chain.

At the aggregate level, increased China risk reduces total production in all three stages, as buyers collectively reduce their reliance on Chinese inputs (Figure 13, Panel A). The production decline is largest in the more upstream stages, reflecting the difficulty of substituting away from a dominant supplier, particularly in upstream stages where complementarity with downstream production amplifies the effect.

The remaining panels of the figure illustrate changes in each country's sourcing shares from China by production stage. They reveal an asymmetry across stages: upstream sourcing shares from China fall only modestly, while downstream sourcing shares decline more substantially and are reallocated toward the rest of the world. Countries want to diversify away from China in response to higher risk, but the estimated $\eta < 0$ implies that cross-stage complementarity mutes reallocation in upstream stages relative to downstream ones. This asymmetry reflects a fundamental tension in the model: while higher upstream risk raises the option value of diversifying away from China at that stage, complementarity with downstream stages mitigates the shift towards alternative suppliers.

The welfare decomposition for countries outside China (Figure 14) shows that



(a) Change in Production Level

(b) Stage 1

(c) Stage 2

(d) Stage 3

Figure 13: Effects of Increased China Risk on Production and Sourcing Patterns

Note. Panel (a) plots the change in total input production by stage from the baseline to the increased China risk counterfactual. China risk is increased such that aggregate risk rises by 5%. Panels (b)–(d) plot changes in each country's sourcing share from China by stage between the baseline and the increased China risk counterfactual.

the effects are heterogeneous and shaped by the competing forces of the price and income channels. As buyers reduce their sourcing from China, Chinese input prices fall—Chinese input prices decline by 0.26%. At the same time, the shift towards other sourcing locations puts upward pressure on wages in most countries, resulting in a positive contribution from the income term but a negative overall terms-of-trade effect on intermediate input prices for most countries. On average, the two channels roughly offset, yielding modest net welfare effects outside China. China itself experiences a welfare loss of 0.46%, driven by the combined effect of reduced demand for its exports, lower wages, and increased domestic uncertainty.

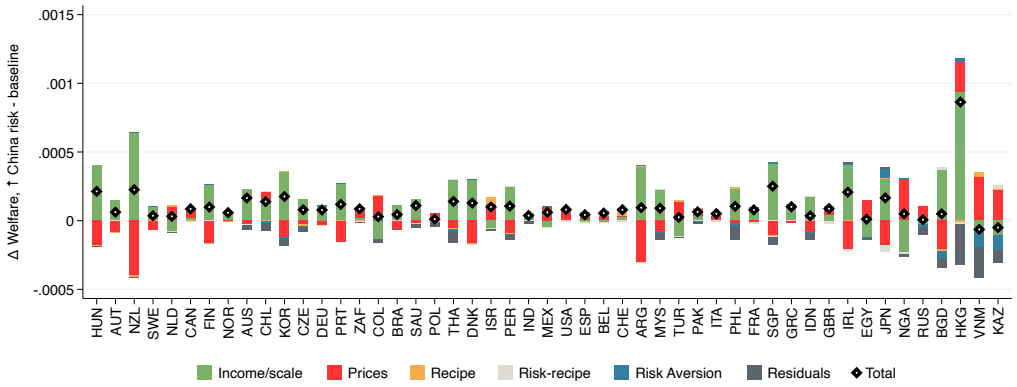


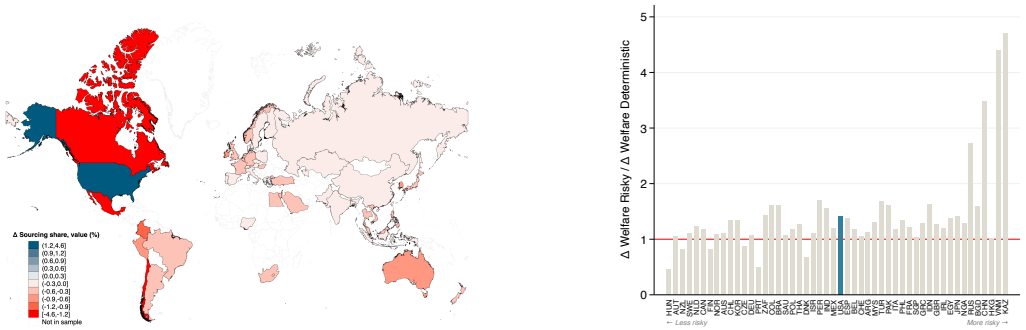
Figure 14: Decomposition of Welfare Change, Increased China Risk Counterfactual

Note. The figure plots the welfare change decomposition, defined in (15), when moving from baseline to the increased China risk counterfactual. China risk is increased such that aggregate risk rises by 5%. Income/scale bar represents the wage adjustment $\Delta \log w_i$ (term (i)). Prices bar represents the terms-of-trade effect $-\sum_a \bar{s}_a^{EQ} \Delta \log p_a$ (term (ii)). Recipe bar represents input-mix reallocation $\sum_a \bar{s}_a^{EQ} \Delta \log s_a$ (term (iii)). Risk-recipe bar represents the insurance cost $-\sum_a (\bar{s}_a^{EQ} - \bar{s}_a^{RN}) \Delta \log r_a$ (term (iv)). Risk Aversion bar represents ΔR_i (term (v)). Lastly, Residuals bar collect higher-order terms from the first-order approximation. The diamond denotes the total welfare change, $\Delta \mathcal{W}_i$. Countries are ordered from lowest to highest simple average volatility across stage.

4.2.4 Increased US trade cost

Our fourth counterfactual introduces a trade cost shock, increasing US bilateral trade costs by 17.5 percentage points—calibrated to match the average of recently proposed US tariff increases—and compares outcomes in the risky and deterministic economies. In contrast to the other counterfactuals, here we keep global risk patterns the same. An increase in US trade costs with all destinations is analogous to a global decrease in US-destined inputs delivered. In other words, in this counterfactual, the mean 'shock' to inputs delivered from all other countries to the US changes. We then contrast the changes in production patterns and welfare with this mean shock in a risky environment, with the same shock in a deterministic environment (a diff-in-diff). This experiment speaks directly to the interaction between trade policy and supply-chain risk: does the presence of risk amplify or attenuate the welfare costs of a trade cost increase?

Panel A of Figure 15 shows the changes in sourcing shares of all countries from the US, averaged across stages. The shock raises the US domestic sourcing share, as higher trade costs prompt US assembling firms to source more domestically. Other countries reduce their sourcing shares from the US across all stages, reallocating



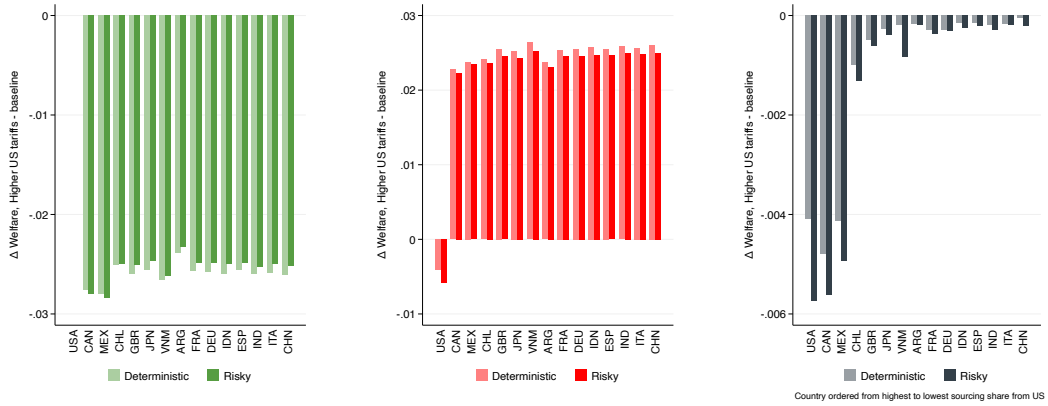
(a) Change in country sourcing share from the US (b) Welfare loss ratio

Figure 15: Increased US trade cost

Note. The figure reports the effects of a 17.5 percentage point increase in US bilateral trade costs, calibrated to the average of recently proposed US tariff increases. Panel (a) maps the change (in percentage points) in each country's average sourcing share from the US, averaged across stages. Blue indicates an increase; darker red indicates a larger decline. Panel (b) plots the ratio of the welfare change in the risky economy to that in the deterministic economy ($\Delta W^{\text{risky}}/\Delta W^{\text{det}}$) for each country. Values above the red line at 1 indicate that risk amplifies the welfare cost of the shock. Countries are ordered from lowest to highest simple average volatility across stage.

toward alternative suppliers, as the increased US demand for its own goods deteriorates foreign terms-of-trade. The reallocation is more pronounced for countries that were initially more dependent on US inputs—primarily close trading partners such as Canada and Mexico. Panel B of the figure plots the ratio of welfare losses (the change in welfare before and after the trade cost increase) in the risky economy relative to the deterministic economy (the diff-in-diff). A ratio above one indicates that risk amplifies the welfare cost of the trade shock. For the majority of countries—and especially for riskier economies—the ratio exceeds one: welfare losses are larger in a world with supply-chain uncertainty.

To understand the mechanism for this finding, Figure 16 decomposes the welfare changes following the shock into the income, price and overall terms in the risky economy and the deterministic economy for a subset of countries. While for large input suppliers to the US – Canada and Mexico – the income decline is larger in the risky environment than in the deterministic one, most countries see slightly smaller declines in the demand/income reallocation term in the risky environment. This is because, in the risky environment, *before and after* the shock, the US – a relatively safe country– diversifies its sourcing more than in the deterministic benchmark. However, in the deterministic environment, the larger decline in wages in most countries translates into improved prices for the input bundle used by most countries, more



(a) Income/scale (b) Price (c) Total

Figure 16: Welfare Change Decomposition, selected countries

Note. This figure plots the change in welfare due to the increase in US trade costs by decomposition for the (darker bar) and deterministic (lighter bar) environments. The decomposition follows equation (15). Panel (a) reports the the wage adjustment $\Delta \log w_i$ (term (i)). Panel (b) reports the terms-of-trade effect $-\sum_a \bar{s}_a^{EQ} \Delta \log p_a$ (term (ii)). Panel (c) reports the total welfare change $\Delta \mathcal{W}_i$. Countries are ordered from highest to lowest baseline sourcing share from the US.

than offsetting the direct wage effects of shifting US demand. On net, as a result, most countries see larger welfare declines in the risky environment.

4.2.5 Supply chain complexity

Our final counterfactual examines the role of supply chain complexity, which we define as the number of stages in the sequential production function. We compare welfare volatility in an economy with 3 stages of production against one with 9 stages, relative to the deterministic benchmark for each case. This experiment asks whether more complex supply chains—with more sequential stages and hence more opportunities for diversification but also more opportunities for shock propagation—are more or less volatile.

Figure 17 reports the change in the coefficient of variation of welfare from the deterministic benchmark for both the 3-stage and 9-stage economies, along with the difference-in-differences. The central finding is that output volatility modestly declines with supply chain complexity for most countries. The 9-stage economy exhibits smaller welfare volatility relative to its deterministic benchmark than the 3-stage economy does, yielding a negative difference-in-differences for the majority of countries. The mechanism is that a more complex supply chain provides more op-

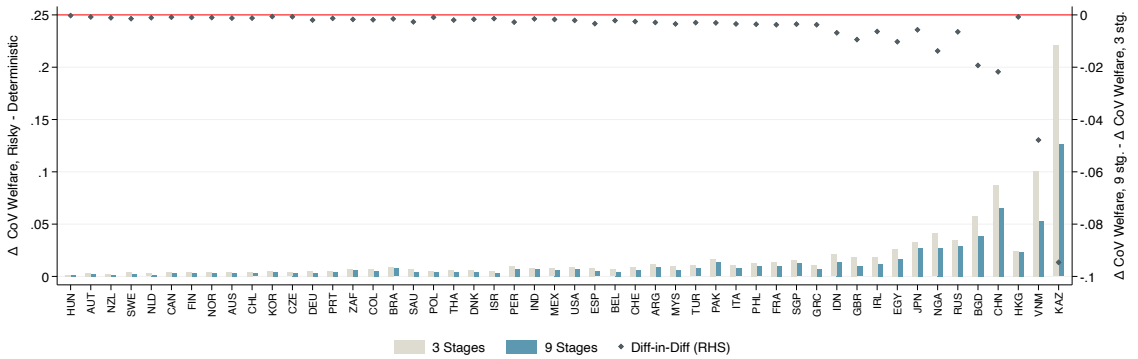


Figure 17: Welfare volatility change from deterministic case

Note. This figure plots the change in welfare volatility from the deterministic to the risky environment under 3 and 9 stages of production. Light gray bars (left axis) report the change in coefficient of welfare (CoV) under 3 stages of production; blue bars (left axis) report the change in CoV under 9 stages of production. Diamond markers (right axis) report the difference-in-differences $\Delta\text{CoV}^{9\text{-stg}} - \Delta\text{CoV}^{3\text{-stg}}$; a negative value indicates that welfare volatility increases more under 3 stages than under 9 stages, i.e., greater complexity reduces volatility. Countries are ordered from lowest to highest simple average volatility across stage.

portunities to diversify risk across a larger number of stages. There is substantial heterogeneity across countries. Riskier regions exhibit a larger reduction in volatility when moving from 3 to 9 stages, because they have the most to gain from the additional diversification opportunities afforded by a more complex value chain. Greater complexity allows them to insure against their higher baseline risk through more diversified sourcing across stages.

5 Conclusion

This paper develops a quantitative general equilibrium model of sequential production under supply-chain risk and uses it to study how risk shapes the global distribution of production, comparative advantage, and welfare. Input orders are placed ex ante across multiple countries and stages, and delivered quantities are subject to bilateral, stage-specific shocks with heterogeneous variances and a rich covariance structure. We derive analytical results on how risk attenuates comparative advantage, how the position of a disruption in the value chain governs its aggregate impact, and how welfare responds to changes in bilateral and origin-specific risk. We then estimate the model on bilateral trade data for 50 countries and 3 production stages over 1995–2023.

The quantitative analysis delivers several findings. Risk meaningfully attenuates revealed comparative advantage: observed trade flows are flatter than Ricardian fundamentals alone would predict, and a deterministic economy with the same productivities and trade costs would exhibit substantially sharper specialization. The position of a disruption in the value chain matters: upstream risk increases welfare volatility by more than an equal increase in downstream risk for the large majority of countries, because upstream risk cascades through the full sequential production process via cross-stage complementarity. Origin-specific shocks to China's volatility reduce global production with reallocation that is muted upstream but more pronounced downstream, for the same reason. A large increase in US trade costs generates welfare losses that are amplified in the risky economy relative to the deterministic benchmark. Finally, greater supply chain complexity reduces welfare volatility for the riskiest countries by expanding diversification opportunities relative to simpler value chains. Our results highlight that the global distribution of production reflects not only comparative advantage and trade costs but also the equilibrium response to aggregate risks affecting supply chains.

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Supplemental Online Appendix

A Theory Appendix

A.1 Additional Results

A.1.1 Deterministic wage and price

Lemma 2 *Given a numeraire w_1, relative wages w_i and input prices $p_{ji,s}$ are deterministic.*

Proof. Ex-post, after input orders $M_{ji,s}$ have been placed, retailer in each region i face an inelastic residual labor supply equal to $\bar{L}_i = L_i - \sum_s \sum_j \frac{M_{ji,s}}{a_{i,s}}$. Retailer's labor demand from the ex-post problem FOC is:

$$\ell_i(\boldsymbol{\chi}) = \frac{\beta\, p_i(\boldsymbol{\chi})\, q_i(\boldsymbol{\chi})}{w_i(\boldsymbol{\chi})} = \frac{\beta\, w_i(\boldsymbol{\chi})\, L_i}{w_i(\boldsymbol{\chi})} = \beta L_i \tag{27}$$

with second equality coming from household budget constraint (and its utility is local nonsatiated). Notice that retailer's labor demand does not depend on the realization of shocks $\boldsymbol{\chi}$. Since retailers are competitive with CRS, profits are zero in every state. Plug in (27) to the retailers profit function yields:

$$(1-\alpha)\, w_j(\boldsymbol{\chi})\, L_j \;=\; \sum_i \sum_s \frac{\tau_{ij,s}\, w_i(\boldsymbol{\chi})}{a_{i,s}}\, M_{ij,s} \qquad \forall\, j,\ \forall\, \boldsymbol{\chi}. \tag{28}$$

This is a linear system of $\mathbf{w}(\boldsymbol{\chi}) = A\mathbf{w}(\boldsymbol{\chi})$, where A is non-stochastic (it depends only on ex-ante order $\{M_{ij,s}\}$, number of labor L_j, iceberg trade cost τ, and productivity $\mathbf{a}$). So $\mathbf{w}(\boldsymbol{\chi})$ must be eigenvector of A with eigenvalue 1 in *every* state. Note that A is a non-negative matrix. If the economy is connected (every country has positive trade with at least one other country, so that intermediate orders link all regions directly or indirectly), then A is irreducible. By Perron-Frobenius Theorem, the eigenvector of A is unique up to a scaling. Since A is deterministic, given a numeraire w_1, relative wage w_i will be unique and doesn't depend on shock realization, i.e. deterministic. Since w_j is deterministic, input prices, $p_{ji,s} = \frac{\tau_{ji,s}\, w_j}{a_{j,s}}$, are immediate constants. ■

A.1.2 Explicit expressions for the slopes and curvatures

Recall that

$$\mathcal{E}_{ji,s}(\boldsymbol{\chi}) = \omega_{ji,s}(\boldsymbol{\chi})\,\lambda_s(\boldsymbol{\chi}) \prod_{t=s+1}^{S} \big[1 - \lambda_t(\boldsymbol{\chi})\big] \tag{29}$$

where,

$$\omega_{ji,s}(\boldsymbol{\chi}) = \frac{(\chi_{ji,s} M_{ji,s})^{\rho_s}}{\sum_{j'} (\chi_{j'i,s} M_{j'i,s})^{\rho_s}}, \quad \lambda_s(\boldsymbol{\chi}) = \frac{B_s^{\eta}}{\alpha_s y_{s-1}^{\eta} + B_s^{\eta}}, \quad B_s = \Big[\sum_{j} (\chi_{ji,s} M_{ji,s})^{\rho_s}\Big]^{1/\rho_s}.$$

Then, the derivative of $\mathcal{E}_{ji,s}(\boldsymbol{\chi})$ with respect to $\chi_{ji,s}$:

$$\begin{aligned}\frac{\partial \mathcal{E}_{ji,s}}{\partial \chi_{ji,s}} = \Big[\frac{\partial \omega_{ji,s}}{\partial \chi_{ji,s}}\Big] \lambda_s \prod_{t>s} (1 - \lambda_t) + \; \omega_{ji,s} \Big[\frac{\partial \lambda_s}{\partial \chi_{ji,s}}\Big] \prod_{t>s} (1 - \lambda_s) \\ + \; \omega_{ji,s} \lambda_s \sum_{k=s+1}^{S} \Big[\frac{\partial (1 - \lambda_k)}{\partial \chi_{ji,s}}\Big] \prod_{\substack{t>s \\ t \neq k}} (1 - \lambda_t).\end{aligned} \tag{30}$$

The derivative of $\omega_{ji,s}(\boldsymbol{\chi})$:

$$\frac{\partial \omega_{ji,s}}{\partial \chi_{ji,s}} = \rho_k \, \frac{\omega_{ji,s}\big(1 - \omega_{ji,s}\big)}{\chi_{ji,s}}$$

Derivative of the new block B_s:

$$\frac{\partial B_s}{\partial \chi_{ji,s}} = \omega_{ji,s} \, \frac{B_s}{\chi_{ji,s}}.$$

Logit-form derivative:

$$\frac{\partial \lambda_s}{\partial B_s} = \eta \, \lambda_s \, (1 - \lambda_s) \, \frac{1}{B_s}.$$

Chain–rule combination:

$$\frac{\partial \lambda_s}{\partial \chi_{ji,s}} = \eta \, \lambda_s \, (1 - \lambda_k) \, \omega_{ji,s} \, \frac{1}{\chi_{ji,s}}$$

For stage $t > s$:

$$1 - \lambda_t = \frac{y_{t-1}^{\eta}}{y_{t-1}^{\eta} + B_t^{\eta}}, \qquad \frac{\partial(1-\lambda_t)}{\partial y_{t-1}} = \eta\,\lambda_t\,(1-\lambda_t)\,\frac{1}{y_{t-1}}.$$

Recursive chain:

$$\frac{\partial y_{t-1}}{\partial \chi_{ji,s}} = (1-\lambda_{t-1})\,\frac{\partial y_{t-2}}{\partial \chi_{ji,s}} = \cdots = \Big[\prod_{u=s+1}^{t-1}(1-\lambda_u)\Big]\,\lambda_s\,\omega_{ji,s}\,\frac{1}{\chi_{ji,s}}.$$

Thus

$$\frac{\partial(1-\lambda_t)}{\partial \chi_{ji,s}} = \eta\,(1-\lambda_t)\,\lambda_t\,\Big[\prod_{u=s+1}^{t-1}(1-\lambda_u)\Big]\,\lambda_s\,\omega_{ji,s}\,\frac{1}{\chi_{ji,s}}$$

Combining all these and after simplification, the derivative becomes

$$\frac{\partial \mathcal{E}_{ji,s}}{\partial \chi_{ji,s}} = \frac{\mathcal{E}_{ji,s}(\boldsymbol{\chi})}{\chi_{ji,s}}\left[\rho_s\big(1-\omega_{ji,s}\big) + \eta\,(\omega_{ji,s} - \mathcal{E}_{ji,s})\right]. \tag{31}$$

By definition of $\vartheta_{ji,s}(\boldsymbol{\chi})$:

$$\vartheta_{ji,s}(\boldsymbol{\chi}) := \frac{\partial \ln \mathcal{E}_{ji,s}(\boldsymbol{\chi})}{\partial \ln \chi_{ji,s}} = \frac{\chi_{ji,s}}{\mathcal{E}_{ji,s}}\,\frac{\partial(1-\lambda_t)}{\partial \chi_{ji,s}} = \rho_s\big(1-\omega_{ji,s}\big) + \eta\,(\omega_{ji,s} - \mathcal{E}_{ji,s}). \tag{32}$$

By definition:

$$\zeta_{ji,s}(\boldsymbol{\chi}) = \rho_s\omega_{ji,s}(1-\omega_{ji,s})(\eta-\rho_s) - \eta\mathcal{E}_{ji,s}\vartheta_{ji,s} \tag{33}$$

A.1.3 Planner's Problem

Consider the social planner's problem that maximizes the weighted sum of countries' expected utility:

$$\max_{\{\ell_i\},\,\{M_{ji,s}\}} \quad \sum_{i\in J}\Upsilon_i\Big[\beta\log\ell_i \;+\; (1-\beta)\,\mathbb{E}_\chi\big[\log M_i(\chi)\big]\Big] \tag{34}$$

subject to resource constraint for labor,

$$\ell_j + \sum_{i\in J}\sum_{s\in S}\frac{\tau_{ji,s}\,M_{ji,s}}{a_{j,s}} \;\le\; L_j, \qquad \forall\, j\in J \tag{35}$$

where $\Upsilon_i > 0$ are pareto weights, and $M_i(\chi)$ is the sequential aggregator of intermediate inputs across stages and origins as defined in Section 2. The planner's problem is a standard concave maximization problem over a convex constraint set. The Karush-Kuhn-Tucker (KKT) conditions are therefore necessary and sufficient for a global optimum. Let Ξ_j denote the multiplier on region j's resource constraint. The Lagrangian is:

$$\mathcal{L} = \sum_{i \in J} \Upsilon_i \Big[\beta \log \ell_i + (1-\beta)\, \mathbb{E}_\chi[\log M_i(\chi)] \Big] + \sum_{j \in J} \Xi_j \Bigg(L_j - \ell_j - \sum_i \sum_s \frac{\tau_{ji,s}\, M_{ji,s}}{a_{j,s}} \Bigg). \tag{36}$$

First order conditions The FOC with respect to ℓ_j is:

$$\frac{\partial \mathcal{L}}{\partial \ell_j} = \frac{\Upsilon_j\, \beta}{\ell_j^*} - \Xi_j = 0 \qquad \Longrightarrow \qquad \Xi_j = \frac{\Upsilon_j\, \beta}{\ell_j^*}. \tag{FOC-ℓ}$$

The FOC with respect to $M_{ji,s}$:

$$\frac{\partial \mathcal{L}}{\partial M_{ji,s}} = \Upsilon_i (1-\beta) \frac{\mathbb{E}_\chi[\mathcal{E}_{ji,s}(\chi)]}{M^*_{ji,s}} - \Xi_j \frac{\tau_{ji,s}}{a_{j,s}} = 0\,, \tag{FOC-M}$$

where $\mathcal{E}_{ji,s}(\chi) = \frac{\partial M_i(\chi)}{\partial M_{ji,s}} \frac{M_{ji,s}}{M_i(\chi)}$ is the point-wise output elasticity.

Equivalence with Competitive Equilibrium Allocation Set Υ_i and Ξ_i to Negishi weights and multipliers, i.e.:

$$\Upsilon_i = w_i L_i\,, \tag{37}$$

$$\Xi_i = w_i\,, \tag{38}$$

where $\{w_i\}_{i \in J}$ are the competitive equilibrium wages. Plug in (37) & (38) to (FOC-ℓ), we get:

$$w_i = \frac{w_i L_i \beta}{\ell_i^*} \qquad \Longrightarrow \qquad \ell_i^* = \beta L_i\,, \tag{39}$$

which is the Competitive Equilibrium solution for labor.

Plug in (37) & (38) to (FOC-M), we get:

$$w_i L_i(1-\beta)\frac{\mathbb{E}_\chi[\mathcal{E}_{ji,s}(\chi)]}{M^*_{ji,s}} = \underbrace{w_j \frac{\tau_{ji,s}}{a_{j,s}}}_{=p_{ji,s}}$$

$$\mathbb{E}_\chi[\mathcal{E}_{ji,s}(\chi)] = \frac{p_{ji,s}M^*_{ji,s}}{(1-\beta)w_i L_i} \tag{40}$$

which is the Competitive Equilibrium solution for expenditure share. Therefore, the competitive equilibrium coincides with the social planner's solution.

A.1.4 Upstream Shock vs Downstream shock

Corollary 2 *If* $\eta < 0$, *and* $\frac{\omega_{ji,1}}{\omega_{ji,3}} \geq \frac{\lambda_3}{(1-\lambda_2)(1-\lambda_3)} \geq \frac{\omega_{ji,1}(1-\omega_{ji,1})}{\omega_{ji,3}(1-\omega_{ji,3})}$ $\forall \boldsymbol{\chi} \in \mathbf{G}(\boldsymbol{\chi})$, *then*

$$\eta \leq \rho_1 \leq \rho_3 \implies \left|\frac{\partial W_i}{\partial \sigma^2_{ji,1}}\right| \geq \left|\frac{\partial W_i}{\partial \sigma^2_{ji,3}}\right|$$

Intuition: If country j upstream good becoming less substitutable than the downstream good, the welfare impact of higher risk in upstream is bigger than downstream. However, need to ensure that country j importance as source within upstream industry (relative to within downstream) is bigger than importance of downstream relative to upstream in final goods production.

Proof. We suppress $\boldsymbol{\chi}$ for brevity. For this part, it is useful to note that:

$$\vartheta_{ji,k} = \rho_k\big(1-\omega_{ji,k}\big) + \eta\left(\omega_{ji,k} - \mathcal{E}_{ji,k}\right) \tag{41}$$

$$= (\rho_k - \eta)(1-\omega_{ji,k}) - \eta(\mathcal{E}_{ji,k} - 1) \tag{42}$$

If $\eta \leq 0$, $\vartheta_{ji,k} \leq 1$. Hence, $\left|\frac{\partial W_i}{\partial \sigma^2_{ji,1}}\right| \geq \left|\frac{\partial W_i}{\partial \sigma^2_{ji,3}}\right|$ iff $\forall \boldsymbol{\chi} \in \mathbf{G}(\boldsymbol{\chi})$:

$$\mathcal{E}_{ji,1}(\rho_1-\eta)(1-\omega_{ji,1}) - \mathcal{E}_{ji,3}(\rho_3-\eta)(1-\omega_{ji,3}) - \eta(\mathcal{E}_{ji,1}-\mathcal{E}_{ji,3})(\mathcal{E}_{ji,1}+\mathcal{E}_{ji,3}-1) \leq 0 \tag{43}$$

One can show that:

- $\frac{\omega_{ji,1}}{\omega_{ji,3}} \geq \frac{\lambda_3}{(1-\lambda_2)(1-\lambda_3)} \iff \mathcal{E}_{ji,1} \geq \mathcal{E}_{ji,3}$
- $\eta < 0 \ \wedge \ \frac{\omega_{ji,1}}{\omega_{ji,3}} \geq \frac{\lambda_3}{(1-\lambda_2)(1-\lambda_3)} \implies -\eta(\mathcal{E}_{ji,1}-\mathcal{E}_{ji,3})(\mathcal{E}_{ji,1}+\mathcal{E}_{ji,3}-1) \leq 0$

Hence,

$$\mathcal{E}_{ji,1}(\rho_1 - \eta)(1 - \omega_{ji,1}) - \mathcal{E}_{ji,3}(\rho_3 - \eta)(1 - \omega_{ji,3}) \leq 0 \implies (43) \tag{44}$$

$$\frac{\rho_1 - \eta}{\rho_3 - \eta} \leq \frac{\mathcal{E}_{ji,3}}{\mathcal{E}_{ji,1}} \frac{(1 - \omega_{ji,3})}{(1 - \omega_{ji,1})} \implies (43) \tag{45}$$

Since $\frac{\lambda_3}{(1-\lambda_2)(1-\lambda_3)} \geq \frac{\omega_{ji,1}(1-\omega_{ji,1})}{\omega_{ji,3}(1-\omega_{ji,1})}$, $\frac{\mathcal{E}_{ji,3}}{\mathcal{E}_{ji,1}} \frac{(1-\omega_{ji,3})}{(1-\omega_{ji,1})} \geq 1 \; \forall \boldsymbol{\chi} \in \mathbf{G}(\boldsymbol{\chi})$. Therefore,

$$\rho_1 \leq \rho_3 \implies \left| \frac{\partial W_i}{\partial \sigma^2_{ji,1}} \right| \geq \left| \frac{\partial W_i}{\partial \sigma^2_{ji,3}} \right|$$

■

A.2 Preliminary Results for Proofs

To help us proves our propositions, it is useful to know the following results

A.2.1 Stein's Lemma

Lemma 3 *Let*

$$\chi_{ji,s} = \exp\!\big(-\tfrac{1}{2}\sigma^2_{ji,s} + \sigma_{ji,s}\,\varepsilon_{ji,s}\big), \qquad \varepsilon_{ji,s} \sim \mathcal{N}(0,1), \qquad v_{ji,s} \equiv \ln \chi_{ji,s},$$

and let $f : \mathbb{R} \to \mathbb{R}$ be twice continuously differentiable with bounded first and second derivatives. Then

$$\boxed{\frac{\partial}{\partial \sigma^2_{ji,s}} \mathbb{E}\big[f(v_{ji,s})\big] \;=\; \frac{1}{2}\,\mathbb{E}\big[f''(v_{ji,s}) - f'(v_{ji,s})\big].} \tag{46}$$

Proof. Step 1 (set–up). Define the mapping

$$v(\sigma, \varepsilon) = \mu(\sigma) + \sigma\varepsilon, \qquad \mu(\sigma) = -\tfrac{1}{2}\sigma^2,$$

so that $v = \ln \chi$ and $S(\sigma) \equiv \mathbb{E}[f(v)]$.

Step 2 (differentiate w.r.t. σ). Because $\partial v / \partial \sigma = \varepsilon - \sigma$,

$$\frac{\partial S}{\partial \sigma} = \mathbb{E}\big[f'(v)\,(\varepsilon - \sigma)\big].$$

Step 3 (apply Stein's lemma). For a standard normal variable $X \sim \mathcal{N}(0,1)$ and any differentiable g with integrable derivative, $\mathbb{E}[X\, g(X)] = \mathbb{E}[g'(X)]$ ("classical Stein identity"). Choose $g(X) = f'\big(\mu + \sigma X\big)$. Then $g'(X) = f''(v)\,\sigma$; hence

$$\mathbb{E}[\varepsilon\, f'(v)] = \mathbb{E}[X\, g(X)] = \mathbb{E}[g'(X)] = \sigma\, \mathbb{E}[f''(v)].$$

Step 4 (substitute back). Insert the Stein result into Step 2:

$$\frac{\partial S}{\partial \sigma} = \sigma\, \mathbb{E}\big[f''(v) - f'(v)\big].$$

Step 5 (convert to a σ^2 derivative). For $\sigma > 0$, $\dfrac{\partial}{\partial \sigma^2} = \dfrac{1}{2\sigma}\, \dfrac{\partial}{\partial \sigma}$. Therefore

$$\frac{\partial S}{\partial \sigma^2} = \frac{1}{2\sigma}\, \frac{\partial S}{\partial \sigma} = \frac{1}{2}\, \mathbb{E}\big[f''(v) - f'(v)\big],$$

which is exactly (46). ■

Remark: If the *location* term $-\frac{1}{2}\sigma^2$ were held fixed while varying σ (i.e. dropping the mean-one normalisation), the $-f'(v)$ component would disappear and one would recover the textbook version of Stein's lemma:

$$\frac{\partial}{\partial \sigma^2} \mathbb{E}\big[f(v)\big] = \tfrac{1}{2}\, \mathbb{E}[f''(v)].$$

A.2.2 Local variance derivative under mean-one lognormals and scale invariance

Lemma 4 *Let $\varepsilon \sim \mathcal{N}(0, I_N)$ and define*

$$x(v) := -\tfrac{1}{2} v\, \mathbf{1} + \sqrt{v}\, \varepsilon, \qquad v \geq 0,$$

where $\mathbf{1}$ is the N-vector of ones. Let $f : \mathbb{R}^N \to \mathbb{R}$ be C^2 and satisfy

$$f(x + c\mathbf{1}) = f(x) \qquad \forall\, c \in \mathbb{R}. \tag{47}$$

(A sufficient condition is that $f(x) = F(e^x)$ for some C^2 function $F : \mathbb{R}^N_+ \to \mathbb{R}$ homogeneous of degree zero: $F(t\chi) = F(\chi)$ for all $t > 0$.) Then

$$\frac{d}{dv}\,\mathbb{E}[f(x(v))]\Big|_{v=0} = \frac{1}{2}\sum_{a=1}^{N} f_{aa}(0),$$

where f_{aa} denotes the second partial derivative with respect to x_a.

Proof. Under the mean-one lognormal parameterization,

$$\frac{d}{dv}\,\mathbb{E}[f(x(v))] = \frac{1}{2}\,\mathbb{E}\left[\sum_{a=1}^{N}\big(f_{aa}(x(v)) - f_a(x(v))\big)\right],$$

where f_a is the first partial derivative with respect to x_a. Translation invariance (47) implies $\sum_a f_a(x) = 0$ for all x (differentiate $c \mapsto f(x + c\mathbf{1})$ at $c = 0$). Evaluating at $v = 0$ (so $x(0) = 0$) yields the claim. ■

Within-stage price indices. Fix an importer i and a stage s with within-stage elasticity $\sigma_s := 1/(1-\rho_s) > 1$.[13] Let the (deterministic) delivered unit cost of origin r's stage-s variety in destination i be

$$p_{ri,s} := \tau_{ri,s}\frac{w_r}{a_{r,s}}.$$

The *deterministic* CES unit-expenditure price index for the stage-s composite is

$$P_{i,s} \;:=\; \left(\sum_{r\in\Omega_s} p_{ri,s}^{1-\sigma_s}\right)^{\frac{1}{1-\sigma_s}}. \tag{48}$$

Under multiplicative productivity shocks $\chi_{ri,s}$, the effective (shock-adjusted) delivered price is $p_{ri,s}(\boldsymbol{\chi}) := p_{ri,s}/\chi_{ri,s}$ and the corresponding *state-by-state* CES price index is

$$P_{i,s}(\boldsymbol{\chi}) \;:=\; \left(\sum_{r\in\Omega_s}\left(\frac{p_{ri,s}}{\chi_{ri,s}}\right)^{1-\sigma_s}\right)^{\frac{1}{1-\sigma_s}}. \tag{49}$$

In what follows, whenever we take local derivatives at the deterministic benchmark ($v = 0$), we evaluate $P_{i,s}(\boldsymbol{\chi})$ at $\boldsymbol{\chi} = \mathbf{1}$ so that $P_{i,s}(\mathbf{1}) = P_{i,s}$.

[13] The price-index definitions in this paragraph and in Lemma 5 allow stage-varying ρ_s. The uniform-ρ restriction is imposed only in Proposition 3.

A.2.3 Cascade term equals the top-level CES cost share

Lemma 5 *Fix an importer i. Let the across-stage aggregator be CES with power parameter η and associated elasticity $\sigma_\eta := 1/(1-\eta)$.*[14] *Let $P_{i,s}(\boldsymbol{\chi})$ be the within-stage price index (49) and let $\alpha_s > 0$ be stage weights.*

Define the unit-expenditure index of the composite of stages $1,\dots,t$ recursively by

$$\Pi_{i,1}(\boldsymbol{\chi})^{1-\sigma_\eta} := \alpha_1 P_{i,1}(\boldsymbol{\chi})^{1-\sigma_\eta}, \qquad \Pi_{i,t}(\boldsymbol{\chi})^{1-\sigma_\eta} := \Pi_{i,t-1}(\boldsymbol{\chi})^{1-\sigma_\eta} + \alpha_t P_{i,t}(\boldsymbol{\chi})^{1-\sigma_\eta}, \;\; t = 2,\dots,S. \tag{50}$$

For each t, define the conditional *cost share of stage t in the composite of stages $1,\dots,t$ by*

$$\lambda_{i,t}(\boldsymbol{\chi}) := \frac{\alpha_t P_{i,t}(\boldsymbol{\chi})^{1-\sigma_\eta}}{\Pi_{i,t}(\boldsymbol{\chi})^{1-\sigma_\eta}} \in (0,1). \tag{51}$$

Define the cascade term *by*

$$\Theta_{i,s}(\boldsymbol{\chi}) := \lambda_{i,s}(\boldsymbol{\chi}) \prod_{t=s+1}^{S} \big(1 - \lambda_{i,t}(\boldsymbol{\chi})\big). \tag{52}$$

Then $\Theta_{i,s}(\boldsymbol{\chi})$ equals the unconditional top-level CES cost share of stage s:

$$\Theta_{i,s}(\boldsymbol{\chi}) = \frac{\alpha_s P_{i,s}(\boldsymbol{\chi})^{1-\sigma_\eta}}{\Pi_{i,S}(\boldsymbol{\chi})^{1-\sigma_\eta}} = \frac{\alpha_s P_{i,s}(\boldsymbol{\chi})^{1-\sigma_\eta}}{\sum_{t=1}^{S} \alpha_t P_{i,t}(\boldsymbol{\chi})^{1-\sigma_\eta}}. \tag{53}$$

Moreover, holding $\{P_{i,t}\}_{t\neq s}$ fixed,

$$\frac{\partial \log \Theta_{i,s}}{\partial \log P_{i,s}} = (1-\sigma_\eta)\big(1 - \Theta_{i,s}\big). \tag{54}$$

Proof. From (51) and (50),

$$1 - \lambda_{i,t}(\boldsymbol{\chi}) = \frac{\Pi_{i,t-1}(\boldsymbol{\chi})^{1-\sigma_\eta}}{\Pi_{i,t}(\boldsymbol{\chi})^{1-\sigma_\eta}}.$$

Hence the product in (52) telescopes:

$$\prod_{t=s+1}^{S} \big(1 - \lambda_{i,t}(\boldsymbol{\chi})\big) = \frac{\Pi_{i,s}(\boldsymbol{\chi})^{1-\sigma_\eta}}{\Pi_{i,S}(\boldsymbol{\chi})^{1-\sigma_\eta}}.$$

[14] The Cobb–Douglas case corresponds to $\sigma_\eta = 1$ and is obtained as a limit; the sensitivity formula (54) continues to hold by continuity.

Multiplying by $\lambda_{i,s}(\boldsymbol{\chi}) = \alpha_s P_{i,s}(\boldsymbol{\chi})^{1-\sigma_\eta} / \Pi_{i,s}(\boldsymbol{\chi})^{1-\sigma_\eta}$ yields the first equality in (53). Iterating (50) gives $\Pi_{i,S}(\boldsymbol{\chi})^{1-\sigma_\eta} = \sum_{t=1}^{S} \alpha_t P_{i,t}(\boldsymbol{\chi})^{1-\sigma_\eta}$, which gives the second equality. Differentiating $\log \Theta_{i,s}$ in (53) w.r.t. $\log P_{i,s}$ yields (54). ■

A.3 Proofs

A.3.1 Proof of Lemma 1

Lemma 1 *The point-wise elasticity of $M_i(\boldsymbol{\chi})$ with respect to $M_{ji,s}$ is*

$$\mathcal{E}_{ji,s}(\boldsymbol{\chi}) = \omega_{ji,s}(\boldsymbol{\chi})\, \lambda_{i,s}(\boldsymbol{\chi}) \prod_{t=s+1}^{S} \big[1 - \lambda_{i,t}(\boldsymbol{\chi})\big], \quad s = 1, \ldots, S. \tag{9}$$

Proof.

1. Derivative inside a CES nest. From the standard CES formula,

$$\frac{\partial B_s}{\partial M_{ji,s}} \;=\; \omega_{ji,s}\, \frac{B_s}{M_{ji,s}}. \tag{55}$$

2. Derivatives between stages. For $s \geq 2$ differentiate $y_s = (y_{s-1}^{\eta} + B_s^{\eta})^{1/\eta}$:

$$\frac{\partial y_s}{\partial y_{s-1}} = (1 - \lambda_s)\, \frac{y_s}{y_{s-1}}, \tag{56}$$

$$\frac{\partial y_s}{\partial B_s} = \lambda_s\, \frac{y_s}{B_s}. \tag{57}$$

When $s = 1$ we have $\partial y_1 / \partial B_1 = 1 = \lambda_1$.

3. Chain-rule from the input to the top. Let $k \in \{1, \ldots, S\}$ be the stage containing the input $M_{ji,k}$. Moving upward through the unique path,

$$\begin{aligned} \frac{\partial M_i}{\partial M_{ji,k}} &= \Big[\prod_{t=k+1}^{S} (1 - \lambda_t) \frac{y_t}{y_{t-1}} \Big] \left(\lambda_k \frac{y_k}{B_k} \right) \left(\omega_{ji,k} \frac{B_k}{M_{ji,k}} \right) \\ &= \lambda_k\, \omega_{ji,k}\, M_i \Big[\prod_{t=k+1}^{S} (1 - \lambda_t) \Big] \frac{1}{M_{ji,k}}, \end{aligned} \tag{58}$$

where telescoping cancels all factors y_t / y_{t-1} down to $M_i / M_{ji,k}$.

Multiplying by $M_{ji,k} / M_i$ gives equation 9. ■

A.3.2 Proof of Proposition 1

Proposition 1 *Let* $\chi_{ji,s} = \exp\!\left(-\frac{1}{2}\sigma^2_{ji,s} + \sigma_{ji,s}\varepsilon_{ji,s}\right)$ *with* $\varepsilon_{ji,s} \sim \mathcal{N}(0,1)$*, where* $\sigma^2_{ji,s}$ *is the variance of the log shock from origin* j *in stage* s *to destination* i*. Let*

$$\vartheta_{\ell|ji,s'|s}(\boldsymbol{\chi}) := \frac{\partial \ln \mathcal{E}_{\ell i,s'}(\boldsymbol{\chi})}{\partial \ln \chi_{ji,s}}, \qquad \zeta_{\ell|ji,s'|s}(\boldsymbol{\chi}) := \frac{\partial^2 \ln \mathcal{E}_{\ell i,s'}(\boldsymbol{\chi})}{\partial (\ln \chi_{ji,s})^2}$$

denote the slope and curvature of the output elasticity for (ℓ, s') *with respect to shocks from* (j,s)*. Then the effect of a change in* $\sigma^2_{ji,s}$ *on the expenditure share of origin* ℓ *in stage* s' *is*

$$\frac{\partial s_{\ell i,s'}}{\partial \sigma^2_{ji,s}} = \frac{1}{2}\,\mathbb{E}_{\boldsymbol{\chi}}\Big[\mathcal{E}_{\ell i,s'}\Big(\vartheta^2_{\ell|j,s\downarrow s'} + \zeta_{\ell|j,s\downarrow s'} - \vartheta_{\ell|j,s\downarrow s'}\Big)\Big]. \tag{11}$$

Proof. Take $f\big(v_{ji,s}\big) = \mathcal{E}_{\ell i,s'}$. Let $s < s'$. Along the unique path $s \to s'$ only the branch weights move, giving

$$f' = \mathcal{E}_{\ell i,s'}\,\vartheta_{\ell|j,s\downarrow s'}, \quad f'' = \mathcal{E}_{\ell i,s'}\left(\vartheta^2_{\ell|j,s\downarrow s'} + \zeta_{\ell|j,s\downarrow s'}\right).$$

Insert f' and f'' into Lemma 3; this yields (11).

Now, Let $s' < s$. Only branch weights on the upward path $\lambda_s, \lambda_{s-1}, \ldots, \lambda_{s'+1}$ move, giving

$$f' = \mathcal{E}_{\ell i,s'}\,\vartheta_{\ell|j,s\downarrow s'}, \qquad f'' = \mathcal{E}_{\ell i,s'}\left(\vartheta^2_{\ell|j,s\downarrow s'} + \zeta_{\ell|j,s\downarrow s'}\right).$$

Insert f' and f'' into Lemma 3; this yields (11). ■

A.3.3 Proof of Proposition 2

Proposition 2 *In the competitive equilibrium, the effects of fundamentals on welfare are:*

$$\frac{\partial \mathcal{W}_i}{\partial \sigma^2_{ji,s}} = \frac{1-\beta}{2}\,\mathbb{E}_{\boldsymbol{\chi}}\big[\mathcal{E}_{ji,s}(\boldsymbol{\chi})\big(\vartheta_{ji,s}(\boldsymbol{\chi}) - 1\big)\big], \tag{16}$$

$$\frac{\partial \mathcal{W}_i}{\partial a_{js}} = (1-\beta)\,\frac{s_{ji,s}}{a_{js}}, \qquad \frac{\partial \mathcal{W}_i}{\partial \tau_{ji,s}} = -(1-\beta)\,\frac{s_{ji,s}}{\tau_{ji,s}}, \qquad \frac{\partial \mathcal{W}_i}{\partial \mu_{ji,s}} = (1-\beta)\,s_{ji,s}, \tag{17}$$

where $\mu_{ji,s} := \mathbb{E}[\chi_{ji,s}]$ *is the mean of the shock from origin* j *in stage* s*.*

Proof. We proceed in several steps.

For country i,

$$Y_i(\boldsymbol{\chi}) = (\beta L_i)^{\beta} M_i(\boldsymbol{\chi})^{1-\beta}, \qquad \mathcal{W}_i := \mathbb{E}_{\boldsymbol{\chi}}\big[\ln Y_i(\boldsymbol{\chi})\big] = \beta \ln(\beta L_i) + (1-\beta)\,\mathbb{E}_{\boldsymbol{\chi}}[\ln M_i(\boldsymbol{\chi})].$$

Let $q_{ji,s} := \chi_{ji,s} M_{ji,s}$ denote the effective input, and let

$$\mathcal{E}_{ji,s}(\boldsymbol{\chi}) := \frac{\partial M_i}{\partial q_{ji,s}} \frac{q_{ji,s}}{M_i}$$

Under the CRS assumption, the equilibrium expenditure share satisfies

$$s_{ji,s} := \frac{p_{ji,s} M_{ji,s}}{(1-\beta) w_i L_i} = \mathbb{E}_{\boldsymbol{\chi}}\big[\mathcal{E}_{ji,s}(\boldsymbol{\chi})\big]. \tag{59}$$

We also use the log–elasticity

$$\vartheta_{ji,s}(\boldsymbol{\chi}) := \frac{\partial \ln \mathcal{E}_{ji,s}(\boldsymbol{\chi})}{\partial \ln \chi_{ji,s}} = \frac{1}{\mathcal{E}_{ji,s}(\boldsymbol{\chi})} \frac{\partial \mathcal{E}_{ji,s}(\boldsymbol{\chi})}{\partial v_{ji,s}}, \qquad v_{ji,s} := \ln \chi_{ji,s}.$$

The lognormal shock is parameterized as

$$v_{ji,s} = \mu_{ji,s} - \tfrac{1}{2}\sigma_{ji,s}^2 + \sigma_{ji,s}\varepsilon_{ji,s}, \qquad \varepsilon_{ji,s} \sim \mathcal{N}(0,1).$$

Given equilibrium wages w_i and prices $p_{ji,s}$, the country–i problem can be written as

$$\max_{\{M_{ji,s}\}} \mathcal{W}_i(\theta) = \mathbb{E}_{\boldsymbol{\chi}}[\ln Y_i(\boldsymbol{\chi})] \quad \text{s.t.} \quad \sum_{j,k} p_{ji,s}(\theta)\, M_{ji,s} \le (1-\beta) w_i L_i,$$

where θ denotes a generic parameter.

Remark: In the full planner's problem, a social planner chooses $\{M_{ji,s}\}_{i,j,k}$ and labor allocations $\{\ell_i^F, \ell_i^M\}_i$ to maximize a weighted sum of expected utilities, subject to all global resource and trade–balance constraints. Under log utility and CRS production, one can show that there exist welfare weights $\{\alpha_i\}_i$ such that the planner's FOCs coincide with the competitive equilibrium, and that in any such efficient allocation labor is split as $\ell_i^F = \beta L_i$, $\ell_i^M = (1-\beta)L_i$ in every country. Conditional on this optimal labor split and on the equilibrium wage w_i and input prices $p_{ji,s}$, the global planner's problem *separates by country* into exactly the country–i problem above, with an intermediate–goods budget $(1-\beta)w_i L_i$. Thus, analyzing this "local" problem is equivalent to applying the envelope theorem to the global planner and then focusing

on country i.

The associated Lagrangian for country i is

$$\mathcal{L}_i(M, \lambda_i; \theta) = \mathbb{E}_{\boldsymbol{\chi}}\big[\ln Y_i(\boldsymbol{\chi}; M, \theta)\big] + \lambda_i\Big((1-\beta)w_i L_i - \sum_{j,k} p_{ji,s}(\theta)\, M_{ji,s}\Big). \tag{60}$$

Let $(M^*(\theta), \lambda_i^*(\theta))$ solve this problem. Then

$$\mathcal{W}_i(\theta) = \mathcal{L}_i\big(M^*(\theta), \lambda_i^*(\theta); \theta\big).$$

By the first–order conditions we have, at the optimum,

$$\frac{\partial \mathcal{L}_i}{\partial M_{ji,s}} = 0 \quad \text{and} \quad \frac{\partial \mathcal{L}_i}{\partial \lambda_i} = 0.$$

Differentiating $\mathcal{W}_i(\theta)$ with respect to θ and using these equalities, the envelope theorem gives

$$\frac{\partial \mathcal{W}_i}{\partial \theta} = \left.\frac{\partial \mathcal{L}_i}{\partial \theta}\right|_{M^*(\theta),\, \lambda_i^*(\theta)}. \tag{61}$$

We now apply (61) parameter by parameter.

1. Derivative with respect to the log–location $\mu_{ji,s}$.

The parameter $\mu_{ji,s}$ affects only the distribution of $\chi_{ji,s}$, and thus enters only through the objective; the budget constraint does not depend on $\mu_{ji,s}$. Hence, from (61),

$$\frac{\partial \mathcal{W}_i}{\partial \mu_{ji,s}} = \mathbb{E}_{\boldsymbol{\chi}}\left[\frac{\partial \ln Y_i(\boldsymbol{\chi})}{\partial \mu_{ji,s}}\right]. \tag{62}$$

Using the chain rule,

$$\frac{\partial \ln Y_i}{\partial \mu_{ji,s}} = \frac{\partial \ln Y_i}{\partial v_{ji,s}}\,\frac{\partial v_{ji,s}}{\partial \mu_{ji,s}} = \frac{\partial \ln Y_i}{\partial v_{ji,s}},$$

because $v_{ji,s}$ is affine in $\mu_{ji,s}$ with unit slope.

From the definition of Y_i,

$$\ln Y_i(\boldsymbol{\chi}) = \beta \ln(\beta L_i) + (1-\beta)\ln M_i(\boldsymbol{\chi}),$$

so (holding $M_{ji,s}$ fixed by the envelope argument)

$$\frac{\partial \ln Y_i}{\partial v_{ji,s}} = (1-\beta)\,\frac{\partial \ln M_i}{\partial v_{ji,s}} = (1-\beta)\,\frac{\partial \ln M_i}{\partial \ln \chi_{ji,s}}.$$

By the definition of the pointwise elasticity and the fact that $q_{ji,s} = \chi_{ji,s} M_{ji,s}$,

$$\frac{\partial \ln M_i}{\partial \ln \chi_{ji,s}} = \mathcal{E}_{ji,s}(\boldsymbol{\chi}).$$

Therefore,

$$\frac{\partial \ln Y_i}{\partial \mu_{ji,s}} = (1-\beta)\,\mathcal{E}_{ji,s}(\boldsymbol{\chi}).$$

Plugging into (62) and using (59),

$$\frac{\partial \mathcal{W}_i}{\partial \mu_{ji,s}} = (1-\beta)\,\mathbb{E}_{\boldsymbol{\chi}}[\mathcal{E}_{ji,s}(\boldsymbol{\chi})] = (1-\beta)\,s_{ji,s}.$$

2. Derivative with respect to the variance $\sigma^2_{ji,s}$.

Again, $\sigma^2_{ji,s}$ enters only via the distribution of $v_{ji,s} = \ln \chi_{ji,s}$, not in the constraint. Thus,

$$\frac{\partial \mathcal{W}_i}{\partial \sigma^2_{ji,s}} = \frac{\partial}{\partial \sigma^2_{ji,s}} \mathbb{E}_{\boldsymbol{\chi}}\big[\ln Y_i(\boldsymbol{\chi})\big] = \frac{\partial}{\partial \sigma^2_{ji,s}} \mathbb{E}\big[f(v_{ji,s})\big],$$

with $f(v_{ji,s}) := \ln Y_i(\boldsymbol{\chi})$.

By Lemma 3,

$$\frac{\partial}{\partial \sigma^2_{ji,s}} \mathbb{E}\big[f(v_{ji,s})\big] = \frac{1}{2}\,\mathbb{E}\big[f''(v_{ji,s}) - f'(v_{ji,s})\big], \tag{63}$$

where derivatives are with respect to $v_{ji,s}$.

We already computed

$$f'(v_{ji,s}) = \frac{\partial \ln Y_i}{\partial v_{ji,s}} = (1-\beta)\,\mathcal{E}_{ji,s}(\boldsymbol{\chi}).$$

Differentiating again,

$$f''(v_{ji,s}) = (1-\beta)\,\frac{\partial \mathcal{E}_{ji,s}}{\partial v_{ji,s}} = (1-\beta)\,\mathcal{E}_{ji,s}(\boldsymbol{\chi})\,\vartheta_{ji,s}(\boldsymbol{\chi}),$$

by the definition of $\vartheta_{ji,s}$.

Substituting into (63),

$$\begin{aligned}\frac{\partial \mathcal{W}_i}{\partial \sigma^2_{ji,s}} &= \frac{1}{2}\,\mathbb{E}\Big[(1-\beta)\mathcal{E}_{ji,s}\vartheta_{ji,s} - (1-\beta)\mathcal{E}_{ji,s}\Big] \\ &= \frac{1-\beta}{2}\,\mathbb{E}_{\boldsymbol{\chi}}\Big[\mathcal{E}_{ji,s}(\boldsymbol{\chi})\left(\vartheta_{ji,s}(\boldsymbol{\chi})-1\right)\Big].\end{aligned}$$

3. Derivative with respect to productivity a_{js}.

The parameter a_{js} only enters the budget constraint via the price

$$p_{ji,s} = \tau_{ji,s}\frac{w_j}{a_{js}}.$$

It does not enter the objective. Hence, by (61), the derivative of $\mathcal{W}_i$ with respect to a_{js} comes purely from the constraint term in (60):

$$\begin{aligned}\frac{\partial \mathcal{W}_i}{\partial a_{js}} &= \left.\frac{\partial \mathcal{L}_i}{\partial a_{js}}\right|_{M^*,\lambda_i^*} \\ &= \lambda_i^*\left(-\sum_{j',s'} M^*_{j'i,s'}\frac{\partial p_{j'i,s'}(a)}{\partial a_{js}}\right). \qquad (64)\end{aligned}$$

Only $p_{ji,s}$ depends on a_{js}, so

$$\frac{\partial \mathcal{W}_i}{\partial a_{js}} = -\lambda_i^*\, M^*_{ji,s}\frac{\partial p_{ji,s}}{\partial a_{js}}.$$

Using $p_{ji,s} = \tau_{ji,s}w_j/a_{js}$,

$$\frac{\partial p_{ji,s}}{\partial a_{js}} = -\tau_{ji,s}\frac{w_j}{a^2_{js}} = -\frac{p_{ji,s}}{a_{js}}.$$

Therefore,

$$\frac{\partial \mathcal{W}_i}{\partial a_{js}} = \lambda_i^*\,\frac{p_{ji,s}M^*_{ji,s}}{a_{js}}.$$

From the FOCs of the Lagrangian and CRS, one can show (and we assume) that the Lagrange multiplier on the intermediate–goods budget is

$$\lambda_i^* = \frac{1}{w_i L_i}.$$

Combining this with (59), which implies $p_{ji,s}M^*_{ji,s} = (1-\beta)w_i L_i\, s_{ji,s}$, we obtain

$$\frac{\partial \mathcal{W}_i}{\partial a_{js}} = \frac{1}{w_i L_i}\,\frac{(1-\beta)w_i L_i\, s_{ji,s}}{a_{js}} = (1-\beta)\,\frac{s_{ji,s}}{a_{js}}.$$

4. Derivative with respect to trade costs $\tau_{ji,s}$.

The parameter $\tau_{ji,s}$ enters only via $p_{ji,s}$:

$$p_{ji,s} = \tau_{ji,s}\frac{w_j}{a_{jk}}.$$

Again applying (61) to the constraint term in (60),

$$\frac{\partial \mathcal{W}_i}{\partial \tau_{ji,s}} = -\lambda_i^*\, M^*_{ji,s}\frac{\partial p_{ji,s}}{\partial \tau_{ji,s}}.$$

Differentiating the price,

$$\frac{\partial p_{ji,s}}{\partial \tau_{ji,s}} = \frac{w_j}{a_{jk}} = \frac{p_{ji,s}}{\tau_{ji,s}},$$

we get

$$\frac{\partial \mathcal{W}_i}{\partial \tau_{ji,s}} = -\lambda_i^*\,\frac{p_{ji,s}M^*_{ji,s}}{\tau_{ji,s}}.$$

Using $\lambda_i^* = 1/(w_i L_i)$ and $p_{ji,s}M^*_{ji,s} = (1-\beta)w_i L_i\, s_{ji,s}$, we obtain

$$\frac{\partial \mathcal{W}_i}{\partial \tau_{ji,s}} = -\frac{1}{w_i L_i}\,\frac{(1-\beta)w_i L_i\, s_{ji,s}}{\tau_{ji,s}} = -(1-\beta)\,\frac{s_{ji,s}}{\tau_{ji,s}}.$$

■

A.3.4 Proof of Corollary 1

Corollary 1 *Welfare $\mathcal{W}_i$ is decreasing in the variance of shocks $\sigma^2_{ji,s}$ (i.e., $\frac{\partial \mathcal{W}_i}{\partial \sigma^2_{ji,s}} \leq 0$) if (1) $\eta \leq 0$, or (2) $\rho_s \leq \frac{1-\eta(\omega_{ji,s}(\boldsymbol{\chi})-\mathcal{E}_{ji,s}(\boldsymbol{\chi}))}{1-\omega_{ji,s}(\boldsymbol{\chi})}, \forall \boldsymbol{\chi} \in \mathbf{G}(\boldsymbol{\chi})$.*

Proof. Given equation (16), $\frac{\partial \mathcal{W}_i}{\partial \sigma^2_{ji,s}} \leq 0$ if $\vartheta_{ji,s} \leq 1$. Recall that:

$$\vartheta_{ji,s} = \rho_k\big(1-\omega_{ji,s}\big) + \eta\,(\omega_{ji,s} - \mathcal{E}_{ji,s}) \tag{65}$$

Since $\rho_k \leq 1$ and $0 \leq 1-\omega_{ji,s} \leq 1$, the first term on the right hand side will always

be smaller or equal to 1. Notice as well:

$$\omega_{ji,s} - \mathcal{E}_{ji,s}(\boldsymbol{\chi}) = \omega_{ji,s} - \omega_{ji,s}(\boldsymbol{\chi})\,\lambda_s(\boldsymbol{\chi}) \prod_{t=s+1}^{S} \big[1 - \lambda_t(\boldsymbol{\chi})\big] \tag{66}$$

$$= \omega_{ji,s}(1 - \lambda_s(\boldsymbol{\chi}) \prod_{t=s+1}^{S} \big[1 - \lambda_t(\boldsymbol{\chi})\big]) \tag{67}$$

$$\geq 0 \tag{68}$$

The last inequality is because $\lambda_k(\boldsymbol{\chi}) \leq 1$. Therefore, if $\eta \leq 0$, the second term on the right hand side will always be smaller or equal to 0. If $\eta > 0$, $\frac{\partial \mathcal{W}_i}{\partial \sigma^2_{ji,s}} \leq 0$ if $\forall \boldsymbol{\chi} \in \mathbf{G}(\boldsymbol{\chi})$:

$$\rho_k \leq \frac{1 - \eta\,(\omega_{ji,s} - \mathcal{E}_{ji,s})}{1 - \omega_{ji,s}}$$

■

A.3.5 Proof of Proposition 3

Proposition 3 *Assume uniform $\rho \in (0,1)$ across stages and mean-one lognormal shocks with a common variance scale $v \geq 0$:*

$$\log \chi_{ji,s} = -\tfrac{1}{2}v + \sqrt{v}\,\varepsilon_{ji,s}, \qquad \varepsilon_{ji,s} \overset{\text{iid}}{\sim} \mathcal{N}(0,1).$$

Then,

(A) Stage-level wedge response. *For any stage s, the effect of a marginal increase in uncertainty on the exporter difference in the risk wedge is, locally at $v = 0$:*

$$\frac{\partial}{\partial v}\Big(\log \Psi_{j'i,s} - \log \Psi_{ji,s}\Big)\Big|_{v=0} = -\frac{\rho(\rho + \kappa_{i,s})}{1-\rho}\left(s^0_{j'i,s} - s^0_{ji,s}\right). \tag{22}$$

(B) Specialization response. *Differencing across stages, the effect on $\Delta^{s,s'}_{jj',i}$ is:*

$$\frac{\partial \Delta^{s,s'}_{jj',i}}{\partial v}\bigg|_{v=0} = -\frac{\rho}{1-\rho}\Big[(\rho + \kappa_{i,s})\big(s^0_{j'i,s} - s^0_{ji,s}\big) - (\rho + \kappa_{i,s'})\big(s^0_{j'i,s'} - s^0_{ji,s'}\big)\Big]. \tag{23}$$

(C) Attenuation of comparative advantage. *Assume additionally that $\Omega_s = \Omega_{s'} = \{j, j'\}$ (two exporters) and $\kappa_{i,s} = \kappa_{i,s'}$ (equal cascade sensitivity across stages).*

Then

$$\operatorname{sign}\left(\left.\frac{\partial \Delta_{jj',i}^{s,s'}}{\partial v}\right|_{v=0}\right) = -\operatorname{sign}\left(\mathrm{CA}_{jj'}^{s,s'}\right),$$

where $\mathrm{CA}_{jj'}^{s,s'} := \log\frac{a_{j',s}\tau_{ji,s}}{a_{j,s}\tau_{j'i,s}} - \log\frac{a_{j',s'}\tau_{ji,s'}}{a_{j,s'}\tau_{j'i,s'}}$ *is the Ricardian comparative-advantage index. For small* $v > 0$*, introducing uncertainty therefore shrinks* $|\Delta_{jj',i}^{s,s'}|$ *toward zero.*

(D) Cascade amplification. *If* $\eta < 0$ *(stages are complements) and* $\Theta_{i,s}^0 \in (0,1)$*, then* $\kappa_{i,s} > 0$ *and the attenuation coefficient* $\rho(\rho+\kappa_{i,s})/(1-\rho)$ *exceeds the single-stage benchmark* $\rho^2/(1-\rho)$*: cross-stage complementarity with sequential production amplifies the compression of comparative advantage.*

Proof. Define the within-stage Herfindahl index $H_{i,s} := \sum_{r\in\Omega_s}(s_{ri,s}^0)^2$. Fix a stage s and write $x_{mi,s} := \log\chi_{mi,s}$. At the deterministic benchmark, CES demand implies $M_{ri,s} \propto p_{ri,s}^{-\sigma}$ (with $\sigma = 1/(1-\rho)$), hence $M_{ri,s}^\rho / \sum_m M_{mi,s}^\rho = s_{ri,s}^0$. Using $(\chi_{ri,s}M_{ri,s})^\rho = \chi_{ri,s}^\rho M_{ri,s}^\rho$, the integrand inside $\Psi_{ji,s}$ in (10) can be written as

$$\frac{\chi_{ji,s}^\rho}{\sum_{r\in\Omega_s}(\chi_{ri,s}M_{ri,s})^\rho}\,\Theta_{i,s}(\boldsymbol{\chi}) = \underbrace{\frac{1}{\sum_{m\in\Omega_s} M_{mi,s}^\rho}}_{\text{independent of } j}\ \underbrace{\frac{\chi_{ji,s}^\rho}{\sum_{r\in\Omega_s} s_{ri,s}^0\chi_{ri,s}^\rho}}_{=:R_{ji,s}(\boldsymbol{\chi}_s)}\ \Theta_{i,s}(\boldsymbol{\chi}),$$

where $\Theta_{i,s}(\boldsymbol{\chi}) = \lambda_{i,s}(\boldsymbol{\chi})\prod_{t>s}(1-\lambda_{i,t}(\boldsymbol{\chi}))$ is the cascade term. The prefactor $(\sum_m M_{mi,s}^\rho)^{-1}$ is independent of exporter j and cancels from exporter ratios and exporter differences in $\log\Psi_{ji,s}$. Writing $R_{ji,s}(x_s) := \exp(\rho x_{ji,s})/\sum_{r\in\Omega_s} s_{ri,s}^0\exp(\rho x_{ri,s})$, define

$$f_{ji,s}(x) := R_{ji,s}(x_s)\,\Theta_{i,s}(x), \qquad x = \log\boldsymbol{\chi}.$$

Up to exporter-invariant terms,

$$\log\Psi_{ji,s} = \frac{1}{1-\rho}\log\mathbb{E}\big[f_{ji,s}(x(v))\big] + (\text{terms independent of } j).$$

Step 1 (local variance derivative). Scaling all shocks by a common factor adds a constant to every component of x. The ratio $R_{ji,s}$ is homogeneous of degree zero in $\boldsymbol{\chi}_s$. By Lemma 5, $\Theta_{i,s}$ is a CES share in $\{P_{i,t}(\boldsymbol{\chi})\}_t$ and therefore homogeneous of degree zero in $\boldsymbol{\chi}$. Hence $f_{ji,s}(x+c\mathbf{1}) = f_{ji,s}(x)$ and Lemma 4 yields

$$\left.\frac{\partial}{\partial v}\log\Psi_{ji,s}\right|_{v=0} = \frac{1}{2(1-\rho)}\cdot\frac{\sum_a f_{ji,s,aa}^0}{f_{ji,s}^0}, \qquad f_{ji,s}^0 := f_{ji,s}(0).$$

If a indexes a shock outside stage s, then $R_{ji,s,a} \equiv R_{ji,s,aa} \equiv 0$ and $f_{aa}/f = \Theta_{aa}/\Theta$, which is independent of exporter j; such terms cancel in $\log \Psi_{j'i,s} - \log \Psi_{ji,s}$. Thus it suffices to sum over $a \in \Omega_s$.

Step 2 (Hessian decomposition within stage s). For $a \in \Omega_s$,

$$\frac{f_{aa}}{f} = \frac{R_{aa}}{R} + \frac{\Theta_{aa}}{\Theta} + 2\frac{R_a}{R}\frac{\Theta_a}{\Theta}.$$

The term Θ_{aa}/Θ is exporter-invariant and drops out of exporter differences. Define within-stage shares under x_s (suppressing the importer subscript i and stage subscript s from the shorthand):

$$s_a(x_s) := \frac{s^0_{ai,s}\, e^{\rho x_{ai,s}}}{\sum_{r\in\Omega_s} s^0_{ri,s}\, e^{\rho x_{ri,s}}}, \qquad H_{i,s}(x_s) := \sum_{r\in\Omega_s} s_r(x_s)^2.$$

At $x = 0$, $s_a(0) = s^0_{ai,s}$ and $H_{i,s}(0) = H_{i,s}$. A direct calculation gives

$$\frac{R_a}{R} = \rho\big(\mathbf{1}\{a = j\} - s_a\big), \qquad \sum_{a\in\Omega_s} \frac{R_{aa}}{R} = 2\rho^2\big(H_{i,s} - s^0_{ji,s}\big).$$

Next, from state-by-state CES price index, eq. (49), one obtains at $x = 0$:

$$\left.\frac{\partial \log P_{i,s}}{\partial x_{ai,s}}\right|_0 = -s^0_{ai,s}.$$

By Lemma 5 evaluated at $\boldsymbol{\chi} = \mathbf{1}$,

$$\kappa_{i,s} := \left.\frac{\partial \log \Theta_{i,s}}{\partial \log P_{i,s}}\right|_{\boldsymbol{\chi}=\mathbf{1}} = (1-\sigma_\eta)(1-\Theta^0_{i,s}),$$

and therefore, by the chain rule,

$$\frac{\Theta^0_a}{\Theta^0} = \left.\frac{\partial \log \Theta_{i,s}}{\partial x_{ai,s}}\right|_0 = \left.\frac{\partial \log \Theta_{i,s}}{\partial \log P_{i,s}}\right|_0 \cdot \left.\frac{\partial \log P_{i,s}}{\partial x_{ai,s}}\right|_0 = -\kappa_{i,s}\, s^0_{ai,s}.$$

Hence

$$\sum_{a\in\Omega_s} 2\frac{R^0_a}{R^0}\frac{\Theta^0_a}{\Theta^0} = 2\rho\kappa_{i,s}\big(H_{i,s} - s^0_{ji,s}\big).$$

Step 3 (assemble and difference). Combining within-stage and cross terms gives

$$\left.\frac{\partial \log \Psi_{ji,s}}{\partial v}\right|_0 = \frac{\rho(\rho+\kappa_{i,s})}{1-\rho}\left(H_{i,s} - s^0_{ji,s}\right) + (\text{terms independent of } j).$$

Taking exporter differences cancels $H_{i,s}$ and j-invariant terms, yielding (22) (part (A)). Differencing across stages yields (23) (part (B)).

Step 4 (exact sign under two exporters and equal κ). Under $\Omega_s = \{j, j'\}$, define

$$d_s := \log\left(\frac{a_{j',s}}{a_{j,s}}\right) - \log\left(\frac{\tau_{j'i}w_{j'}}{\tau_{ji,s}w_j}\right).$$

Then $s^0_{j'i,s} = \exp((\sigma-1)d_s)/(1+\exp((\sigma-1)d_s))$ and

$$s^0_{j'i,s} - s^0_{ji,s} = 2s^0_{j'i,s} - 1 = \tanh\left(\tfrac{\sigma-1}{2}\, d_s\right).$$

Since $\tanh(\cdot)$ is strictly increasing,

$$\operatorname{sign}\Big[\left(s^0_{j'i,k} - s^0_{ji,k}\right) - \left(s^0_{j'i,k'} - s^0_{ji,k'}\right)\Big] = \operatorname{sign}(d_k - d_{k'}).$$

Sector-invariant trade costs and stage-invariant wages imply $d_k - d_{k'} = \mathrm{CA}^{k,k'}_{jj'}$. If also $\kappa_{i,k} = \kappa_{i,k'}$, then (23) reduces to a negative constant times $\left(s^0_{j'i,k} - s^0_{ji,k}\right) - \left(s^0_{j'i,k'} - s^0_{ji,k'}\right)$, giving the sign result in part (C). Part (D) follows from $\eta < 0 \Rightarrow \sigma_\eta < 1 \Rightarrow \kappa_{i,s} > 0$ when $\Theta^0_{i,s} \in (0,1)$. ■

A.3.6 Proof of Proposition 4

Proposition 4 *Maintain the assumptions of Proposition 3. Let*

$$W^{s,s'}_{jj',i}(v) := \log\frac{\Psi_{j'i,s}}{\Psi_{ji,s}} - \log\frac{\Psi_{j'i,s'}}{\Psi_{ji,s'}}$$

be the uncertainty wedge, so that $\Delta^{s,s'}_{jj',i} = \frac{\rho}{1-\rho}\mathrm{CA}^{s,s'}_{jj'} + W^{s,s'}_{jj',i}(v)$. *Define the* risk-compression index $D_{i,s}(j,j') := (\rho+\kappa_{i,s})\,\Delta s^0_{i,s}(j,j')$ *and its cross-stage difference* $B^{s,s'}_i(j,j') := D_{i,s}(j,j') - D_{i,s'}(j,j')$. *Then, as* $v \downarrow 0$,

$$\mathrm{Cov}(\mathrm{CA}, W(v)) = -\frac{\rho}{1-\rho}\, v\, \mathbb{E}[\mathrm{CA}\cdot B] + o(v). \tag{24}$$

In particular, $\mathrm{Cov}(\mathrm{CA}, W(v)) < 0$ *for all sufficiently small* $v > 0$ *whenever* $\mathbb{E}[\mathrm{CA} \cdot B] > 0$ *and* $\mathrm{Var}(\mathrm{CA}) > 0$. *A sufficient condition is* $\mathrm{CA}_{jj'}^{s,s'} \cdot B_i^{s,s'}(j,j') > 0$ *for all* (i,j,j',s,s') *with* $\mathrm{CA}_{jj'}^{s,s'} \neq 0$.

Proof. *Step 1 (linearize* W *in the variance parameter).* Under mean-one normalization, $W(0) = 0$ at the deterministic benchmark and $W(v) = v\,W'(0) + o(v)$ as $v \downarrow 0$.

Step 2 (identify $W'(0)$ *with deterministic shares).* By Proposition 3 (B),

$$W'(0) = \frac{\partial W}{\partial v}\bigg|_{v=0} = -\frac{\rho}{1-\rho}\,B.$$

Combining with Step 1 gives $W(v) = -(\rho/(1-\rho))\,v\,B + o(v)$.

Step 3 (take covariances). Using the symmetry in exporter ordering, $\mathbb{E}[\mathrm{CA}] = \mathbb{E}[W(v)] = 0$, so $\mathrm{Cov}(\mathrm{CA}, W(v)) = \mathbb{E}[\mathrm{CA}\,W(v)]$. Substituting the expansion from Step 2 yields (24).

Step 4 (sufficient conditions). If $\mathrm{CA}_{jj'}^{s,s'} \cdot B_i^{s,s'}(j,j') > 0$ for all (i,j,j',s,s') with $\mathrm{CA}_{jj'}^{s,s'} \neq 0$, then $\mathrm{CA}\,B > 0$ pointwise, hence $\mathbb{E}[\mathrm{CA}\,B] > 0$ and (24) implies $\mathrm{Cov}(\mathrm{CA}, W(v)) < 0$ for small v. ■

A.4 Welfare decomposition

Comparing two scenarios (baseline 0 and counterfactual 1),

$$\Delta \mathcal{W}_i = (1-\beta)\,\Delta \mathbb{E}_\chi[\log M_i(\{M_{ji,s}\}, \boldsymbol{\chi})]\,. \tag{69}$$

By CRS of $M_i(\cdot,\chi)$ and letting total intermediate spending be $E_i = (1-\beta)w_i L_i$, write $M_{ji,s} = E_i\,r_{ji,s}$ where

$$r_{ji,s} \;\equiv\; \frac{M_{ji,s}}{E_i} \;=\; \frac{s_{ji,s}}{p_{ji,s}}, \qquad s_{ji,s} = \frac{p_{ji,s}M_{ji,s}}{E_i}.$$

Define the unit-expenditure aggregator $\tilde{M}_i(r,\chi) := M_i(\{r_{ji,s}\},\chi)$ so that

$$\log M_i(\{M\},\chi) = \log E_i + \log \tilde{M}_i(r,\chi), \quad \log E_i = \log(1-\beta) + \log w_i + \log L_i.$$

Hence

$$\mathbb{E}_\chi[\log M_i(\{M\},\chi)] = \log E_i + \mathbb{E}_\chi[\log \tilde{M}_i(r,\chi)], \tag{70}$$

$$\Delta W_i = (1-\beta)\,\Delta \log w_i + (1-\beta)\,\Delta \mathbb{E}_\chi[\log \tilde{M}_i(r,\chi)]. \tag{71}$$

Let

$$F(r) := \mathbb{E}_\chi[\log \tilde{M}_i(r,\chi)], \qquad G(r) := \log \mathbb{E}_\chi[\tilde{M}_i(r,\chi)], \qquad R_i(r) := F(r) - G(r).$$

Then the Jensen identity gives

$$\Delta F \;=\; \underbrace{\Delta G}_{\text{Risk Neutral mean}} \;+\; \underbrace{\Delta R_i}_{\text{Risk Aversion}}\;. \tag{72}$$

For each link $a \equiv (j,s)$ define the pointwise elasticity

$$\varepsilon_a(\chi) := \frac{\partial \log \tilde{M}_i}{\partial \log r_a}(r,\chi).$$

Two natural shares (gradients) follow:

$$s_a^{\mathrm{EQ}}(r) := \mathbb{E}_\chi[\varepsilon_a(\chi)], \qquad s_a^{\mathrm{RN}}(r) := \mathbb{E}_w[\varepsilon_a(\chi)], \qquad w(\chi) = \frac{\tilde{M}_i(r,\chi)}{\mathbb{E}[\tilde{M}_i(r,\chi)]}.$$

Notice that $s_a^{\mathrm{RN}}(r)$ is changing the measure.

By envelope/chain rule,

$$\frac{\partial F}{\partial \log r_a} = s_a^{\mathrm{EQ}}(r), \qquad \frac{\partial G}{\partial \log r_a} = s_a^{\mathrm{RN}}(r). \tag{73}$$

Take a smooth path from r_0 to r_1 and apply a first-order expansion (using midpoint shares $\bar{s}$):

$$\Delta F = \sum_a \bar{s}_a^{\mathrm{EQ}}\,\Delta \log r_a + \mathcal{O}(\|\Delta r\|^2), \quad \Delta G = \sum_a \bar{s}_a^{\mathrm{RN}}\,\Delta \log r_a + \mathcal{O}(\|\Delta r\|^2). \tag{74}$$

Since $\Delta \log r_a = \Delta \log s_a - \Delta \log p_a$, we can add and subtract $\sum_a \bar{s}_a^{\mathrm{EQ}} \Delta \log r_a$ inside

(72) to obtain

$$\Delta F = \underbrace{\sum_a \bar{s}_a^{\mathrm{EQ}} \Delta \log s_a}_{\textbf{(B) Recipe (equal-weight)}} - \underbrace{\sum_a \bar{s}_a^{\mathrm{EQ}} \Delta \log p_a}_{\textbf{(C) Prices}} - \underbrace{\sum_a (\bar{s}_a^{\mathrm{EQ}} - \bar{s}_a^{\mathrm{RN}}) \Delta \log r_a}_{\textbf{(B') Risk–recipe interaction}} \tag{75}$$

$$+ \underbrace{\Delta R_i}_{\textbf{(D) Risk Aversion}} + \mathcal{O}(\|\Delta r\|^2). \tag{76}$$

Multiplying by $(1-\beta)$ and reinstating the income term yields the decomposition in Equation (15).

A.5 Comparative Advantage and Risk

Consider the following expression for (value) trade flows at stage s:

$$X_{ji,s} = \left(\tau_{ji,s}\frac{w_j}{a_{j,s}}\right)^{-\frac{\rho_s}{1-\rho_s}} ((1-\beta)w_i L_i)^{\frac{1}{1-\rho_s}} \underbrace{\left[\mathbb{E}_\chi\left[\frac{(\chi_{ji,s})^{\rho_s}}{\sum_{m\in\Omega_s}(\chi_{mi,s}M_{mi,s})^{\rho_s}}\lambda_s(\boldsymbol{\chi})\prod_{t=s+1}^{S}[1-\lambda_t(\boldsymbol{\chi})]\right]\right]^{\frac{1}{1-\rho_s}}}_{:=\Psi_{ji,s}}. \tag{77}$$

Throughout, define the within-stage elasticity $\sigma_s := 1/(1-\rho_s)$.

Following Costinot, Donaldson, and Komunjer (2012), fix an importer i, two exporters $j \neq j'$, and two stages $s \neq s'$, and define the difference-in-differences in bilateral exports:

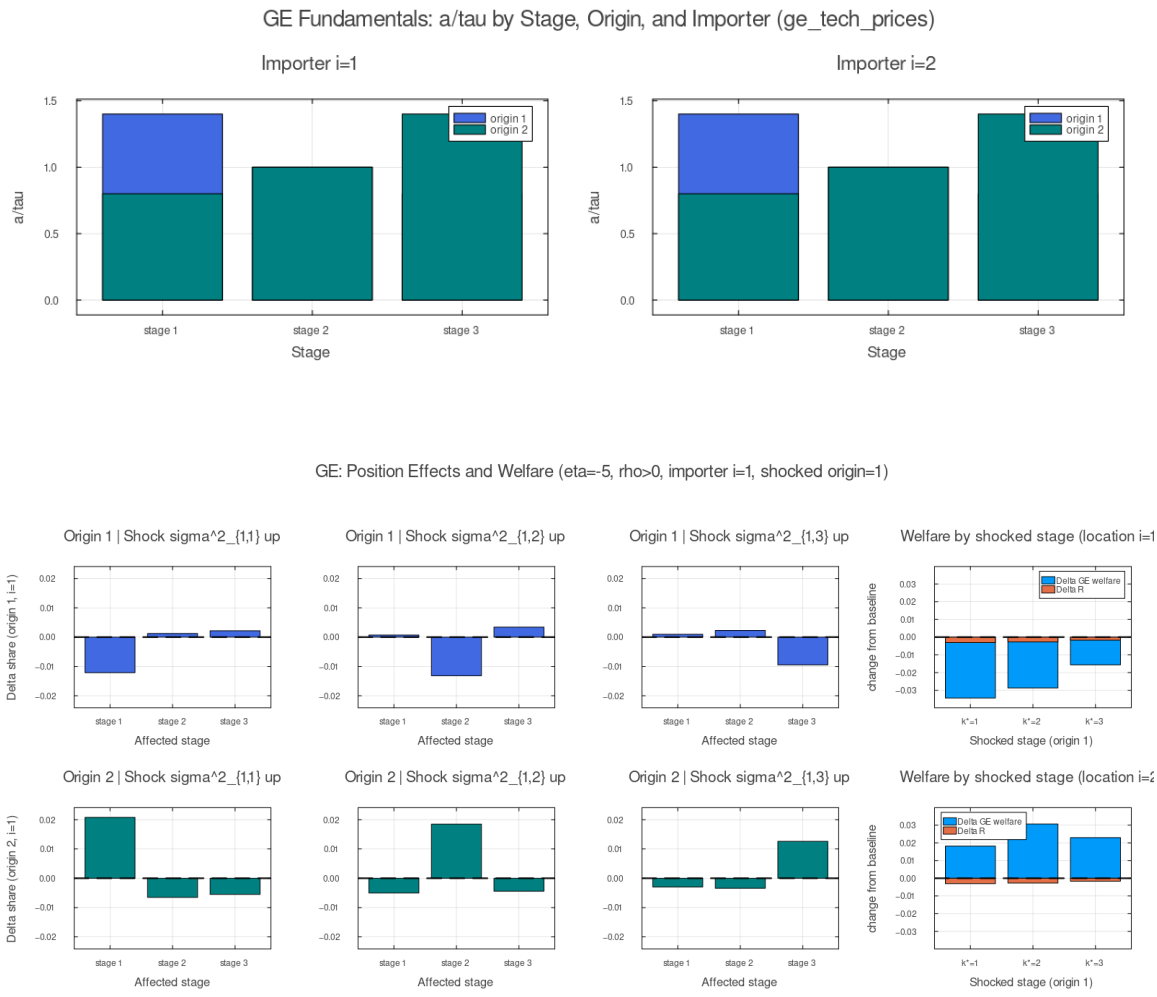
$$\Delta_{jj',i}^{s,s'} := \log\left(\frac{X_{j'i,s}}{X_{ji,s}}\right) - \log\left(\frac{X_{j'i,s'}}{X_{ji,s'}}\right). \tag{78}$$

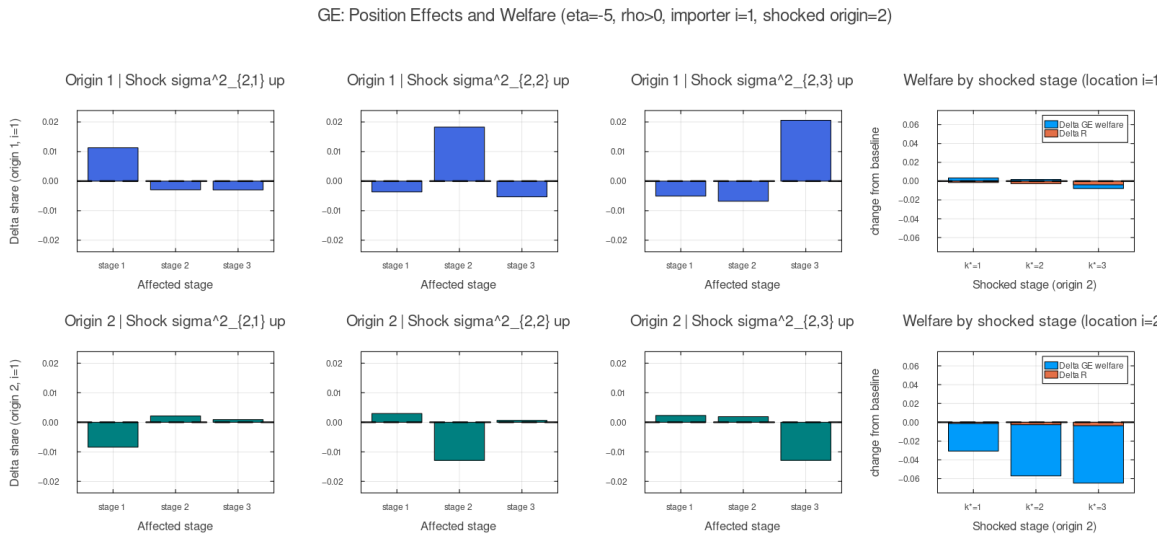
General expression (no restrictions on ρ or τ). Using (77) stage-by-stage and noting that the importer term $((1-\beta)w_iL_i)^{1/(1-\rho_s)}$ cancels in exporter ratios, we obtain:

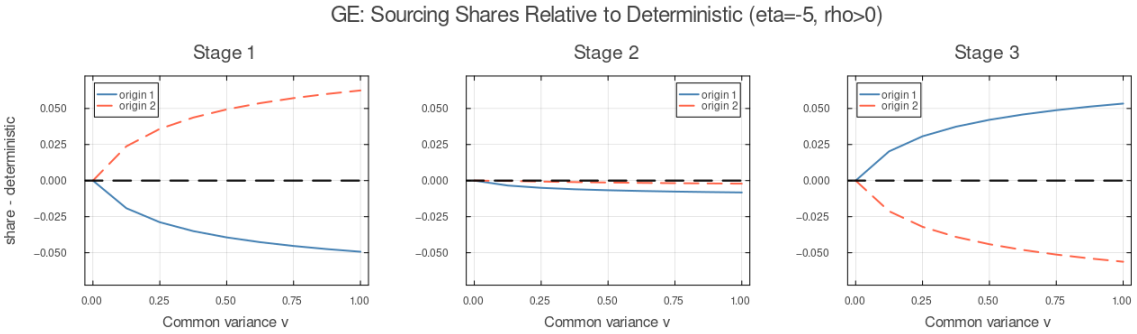
$$\Delta_{jj',i}^{s,s'} = -\frac{\rho_s}{1-\rho_s}\log\left(\frac{\tau_{j'i,s}\,w_{j'}/a_{j',k}}{\tau_{ji,s}\,w_j/a_{j,s}}\right) + \frac{\rho_{s'}}{1-\rho_{s'}}\log\left(\frac{\tau_{j'i,s'}\,w_{j'}/a_{j',s'}}{\tau_{ji,s'}\,w_j/a_{j,s'}}\right)$$
$$+ \log\left(\frac{\Psi_{j'i,s}}{\Psi_{ji,s}}\right) - \log\left(\frac{\Psi_{j'i,s'}}{\Psi_{ji,s'}}\right). \tag{79}$$

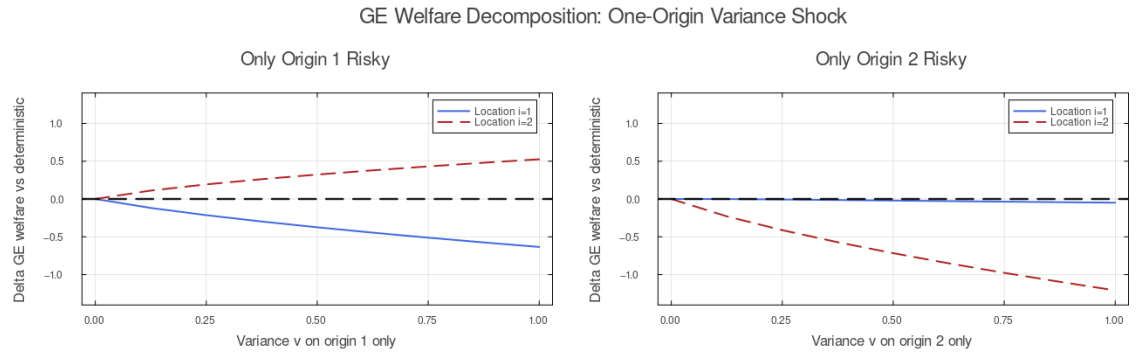
A.6 Comparative Statics

For this comparative statics, we use $\rho_s = 0.70$, $\eta = -5.0$, $\alpha_{1,2} = \alpha_{1,3} = 2/3$, $\alpha_{2,2} = \alpha_{2,3} = 1$, $\tau_{ij,s} = 1 \, \forall \, i, j, s$, $a_1 = [1.4, 1.0, 0.8]$, and $a_2 = [0.8, 1.0, 1.4]$.




GE: Position Effects and Welfare (eta=-5, rho>0, importer i=1, shocked origin=2)
Origin 1 | Shock sigma^2_{2,1} up
Origin 1 | Shock sigma^2_{2,2} up
Origin 1 | Shock sigma^2_{2,3} up
Welfare by shocked stage (location i=1
Delta share (origin 1, i=1)
change from baseline
Delta GE welfare
Delta R
stage 1
stage 2
stage 3
Affected stage
k*=1
k*=2
k*=3
Shocked stage (origin 2)
Origin 2 | Shock sigma^2_{2,1} up
Origin 2 | Shock sigma^2_{2,2} up
Origin 2 | Shock sigma^2_{2,3} up
Welfare by shocked stage (location i=2
Delta share (origin 2, i=1)


GE: Sourcing Shares Relative to Deterministic (eta=-5, rho>0)
Stage 1
Stage 2
Stage 3
share - deterministic
origin 1
origin 2
Common variance v


GE Welfare Decomposition: One-Origin Variance Shock
Only Origin 1 Risky
Only Origin 2 Risky
Delta GE welfare vs deterministic
Location i=1
Location i=2
Variance v on origin 1 only
Variance v on origin 2 only

B Estimation Appendix

B.1 Algorithm Overview

The structural parameters $\Theta \equiv \{\eta,\, \{\rho_s\}_{s=1}^{S},\, \{a_{j,s}\},\, \{g_{ji,s}\},\, \{\alpha_{i,s}\},\, \beta^{Dist},\, \beta^{Lang},\, \beta^{Border}\}$ are estimated by a nested algorithm combining PPML estimation of the structural gravity equation (25) with general-equilibrium model inversion. The algorithm has three nested layers:

- *Layer 1 (outer loop)* searches over a grid $\mathcal{G}$ of candidate values for η, selecting the value that maximizes the log-likelihood of the model.
- *Layer 2 (middle loop)* iterates, for a given η, between running the PPML/GE inner loop and updating the destination-stage weights $\alpha_{i,s}$ to match observed stage expenditure shares.
- *Layer 3 (inner loop)* iterates between solving for GE wages and input shares, constructing model-implied offsets $\ln \Psi_{ji,st}$, and running PPML to update $\{\rho_s\}$, productivities $\{a_{j,s}\}$, the landed-cost wedge $\{g_{ji,s}\}$, and gravity coefficients.

The full procedure is also implemented in a deterministic warm-start block ($\sigma_{js} = 0$) to obtain good initial values for all parameters before activating the full stochastic simulation.

B.2 Layer 1: Grid Search over η

Let $\mathcal{G} = \{\eta^{(1)}, \ldots, \eta^{(G)}\}$ be the grid, sorted by $|\eta|$ in ascending order to facilitate warm-starting. For each $\eta^{(g)} \in \mathcal{G}$:

(i) Run the full nested estimation (Layers 2–3) to convergence, yielding $\hat{\theta}(\eta^{(g)}) = \{\hat{\rho}_s, \hat{a}_{j,s}, \hat{\alpha}_{i,s}, \hat{\beta}^{Dist}, \hat{\beta}^{Lang}, \hat{\beta}^{Border}\}$.

(ii) Record the maximized log-likelihood $\mathcal{L}(\eta^{(g)})$ of the PPML gravity system.

Select $\hat{\eta} = \arg\max_{\eta \in \mathcal{G}} \mathcal{L}(\eta)$.

GMM identification of η. An equivalent characterization of the estimator is GMM: for each importer i, stage s, and year t, define the relative log stage shares differenced against stage 1,

$$\tilde{r}^{D}_{i,s,t} = \ln S^{D}_{i,s,t} - \ln S^{D}_{i,1,t}, \qquad \tilde{r}^{M}_{i,s,t} = \ln \Theta_{i,s,t} - \ln \Theta_{i,1,t}, \tag{80}$$

where $S^D_{i,s,t}$ is the observed stage-s expenditure share and $\Theta_{i,s,t} \equiv \mathbb{E}[\lambda_{i,s,t} \prod_{u>s}(1 - \lambda_{i,u,t})]$ is the model-implied expected stage mass, both demeaned over t within each (i, s) cell. The shift-share instrument is

$$Z_{i,s,t} = \sum_j \omega_{ji,s} \big(\ln \tau_{ji,st} - \ln \bar{\tau}_{ji,s}\big), \tag{81}$$

with $\omega_{ji,s}$ the baseline sourcing shares and $\bar{\tau}_{ji,s}$ the baseline tariff. Defining $\tilde{Z}_{i,s,t} = (Z_{i,s,t} - Z_{i,1,t})_{\text{demeaned}}$, the GMM criterion is

$$Q(\eta) = \big\|\bar{g}(\eta)\big\|^2, \qquad \bar{g}_s(\eta) = \frac{1}{IT} \sum_{i,t} \tilde{Z}_{i,s,t} \big(\tilde{r}^D_{i,s,t} - \tilde{r}^M_{i,s,t}\big), \quad s \neq 1. \tag{82}$$

Maximizing the log-likelihood over the grid is equivalent to minimizing $Q(\eta)$, exploiting the variation in how relative stage shares respond to tariff-induced cost shifters.

B.3 Layer 2: Productivity and Stage-Weight Updates

For a given η, Layer 2 alternates between the PPML/GE inner loop (Layer 3) and a Newton update of $\alpha_{i,s}$.

Step 2a: Inner loop. Run up to N_{inner} iterations of Layer 3 (described below) to update $(\rho_s, a_{j,s}, g_{ji,s}, \beta^{Dist}, \beta^{Lang})$.

Step 2b: Rebuild GE. Solve the GE model for each year t with the updated parameters to obtain equilibrium wages w_t and input orders $M_{ji,s,t}$, and compute model-implied stage masses $\Theta_{i,s}$ via Monte Carlo:

$$\Theta_{i,s} = \frac{1}{D} \sum_{d=1}^{D} \lambda^{(d)}_{i,s} \prod_{u>s} \big(1 - \lambda^{(d)}_{i,u}\big), \tag{83}$$

where $\lambda^{(d)}_{i,s}$ is the stage-s expenditure share for draw d, computed from the sequential CES aggregator.

Step 2c: Update $\alpha_{i,s}$. Define $S^{\text{Data}}_{i,s} = T^{-1} \sum_t S^D_{i,s,t}$ and $S^M_{i,s} = \Theta_{i,s} / \sum_{s'} \Theta_{i,s'}$. The approximate elasticity of $S^M_{i,s}$ with respect to $\alpha_{i,s}$ in the sequential CES is $(1 - S^M_{i,s})$.

The Newton correction is

$$\Delta \ln \alpha_{i,s} = -\frac{\ln S_{i,s}^{\text{Data}} - \ln S_{i,s}^{M}}{\max\{1 - S_{i,s}^{M},\, 0.05\}}, \tag{84}$$

applied with relaxation factor $\lambda_\alpha = 0.8$ and capped at $|\Delta \ln \alpha_{i,s}| \leq \bar{\Delta}_\alpha$, with the normalization $\alpha_S = 1$.

Convergence. Layer 2 terminates when $\max_s |\Delta\rho_s|$, $\max_{j,i,s} |\Delta g_{ji,s}|$, $\max_{i,s} |\Delta \ln \alpha_{i,s}|$, and $\max(|\Delta\beta^{Dist}|, |\Delta\beta^{Lang}|)$ are all below their respective tolerances.

B.4 Layer 3: PPML/GE Inner Loop

Each iteration of the inner loop consists of the following steps.

Step 3a: Build trade costs. Combine ad-valorem tariffs with the landed-cost wedge and productivities to form the trade cost used in the GE solver:

$$\tau_{ji,s,t}^{GE} = \begin{cases} \tau_{ji,s,t}^{tar} \cdot \exp\big(g_{ji,s} + \ln a_{j,s} - \ln a_{i,s}\big) & j \neq i, \\ \tau_{ii,s,t}^{tar} & j = i, \end{cases} \tag{85}$$

where $g_{ji,s} \equiv \ln \tau_{ji,s} - \ln a_{j,s} + \ln a_{i,s}$ is the landed-cost wedge: the total cost disadvantage of sourcing stage-s inputs from country j relative to domestic production in i. On initial iterations or for pairs where g is unavailable, a gravity imputation is used.

Step 3b: Solve GE. Solve the GE system for each year t using dampened fixed-point iteration to obtain equilibrium wages w_t and input orders $M_{ji,s,t}$.

Step 3c: Construct PPML offset. For each observation (j, i, s, t), construct

$$\mathsf{offset}_{ji,st} = \frac{1}{1-\rho_s} \ln \Psi_{ji,st} - \phi_s \ln w_{j,t} + \frac{1}{1-\rho_s} \ln\big((1-\beta) w_{i,t} L_{i,t}\big), \tag{86}$$

where $\ln \Psi_{ji,st}$ is computed via Monte Carlo over D draws as defined in (26).

Step 3d: Run PPML. Estimate equation (25) via `ppmlhdfe` with the offset from Step 3c, stage-specific tariff regressors $z_{s,ji,t} = -\ln \tau_{ji,st}^{tar} \cdot \mathbf{1}\{s = m\}$, and three sets of high-dimensional fixed effects: exporter×stage×year ($\delta_{j,s,t}$), importer×stage×year ($\delta_{i,s,t}$), and bilateral×stage ($\delta_{ji,s}$). The PPML coefficients $\hat{\phi}_s$ map to $\hat{\rho}_s = \hat{\phi}_s/(1+\hat{\phi}_s)$. Standard errors are clustered at the exporter-importer-stage level.

Step 3e: Recover structural objects. Three structural objects are extracted from the estimated fixed effects.

Productivities. The exporter×stage×year fixed effect $\hat{\delta}_{j,s,t}$, purged of its wage component, is averaged over t and normalized against the US reference country:

$$\ln \hat{a}_{j,s} = \frac{1}{T}\sum_{t} \hat{\delta}^{adj}_{j,s,t} - \frac{1}{T}\sum_{t} \hat{\delta}^{adj}_{USA,s,t}, \qquad \hat{a}_{USA,s} = 1 \ \ \forall s. \tag{87}$$

To preserve cross-stage relative prices, only the country-level component of the update $\Delta \ln a_j = S^{-1}\sum_s \Delta \ln a_{j,s}$ is applied.

Landed-cost wedge. The within-(i,s,t) relative-to-domestic differencing of fixed effects yields

$$\delta^{cost}_{j|i,s,t} = \left(\delta^{nw}_{j,s,t} + \delta_{ji,s}\right) - \left(\delta^{nw}_{i,s,t} + \delta_{ii,s}\right), \tag{88}$$

where $\delta^{nw}_{j,s,t}$ denotes the exporter-stage-time fixed effect purged of the wage piece. By construction, (88) annihilates the importer×stage×time fixed effect. The landed-cost wedge is recovered as

$$\hat{g}_{ji,s} = -\,\delta^{cost}_{j|i,s,t}/\hat{\phi}_s, \qquad \hat{g}_{ii,s} = 0 \ \ \forall\,(i,s). \tag{89}$$

Gravity coefficients. The bilateral-stage fixed effect $\hat{\delta}_{ji,s}$ is decomposed into exporter-stage and importer-stage components; the residual $\varepsilon^{pair}_{ji,s}$ is projected on $\ln Dist_{ji}$, comlang_{ji}, and a foreign dummy to recover $\hat{\beta}^{Dist}$ and $\hat{\beta}^{Lang}$.

Step 3f: Update state. Update $(\rho_s, a_{j,s}, g_{ji,s}, \beta^{Dist}, \beta^{Lang})$; converge when all changes are below their respective tolerances.

Justification for PPML. Although shocks are lognormally distributed, PPML (quasi-maximum likelihood under a Poisson working model) is consistent for the true parameters provided the conditional mean is correctly specified, regardless of the form of the conditional variance (Silva and Tenreyro, 2006). Each PPML sub-problem, given offsets and fixed effects, is globally concave in the linear predictor with a unique maximizer. With a continuous GE map, the updates constitute block coordinate ascent on the Poisson pseudo-log-likelihood, so the objective increases monotonically and any limit point is stationary.

B.5 Variance–Covariance Estimation

After convergence of the structural estimation, we estimate the stochastic structure of shocks $\chi_{ji,s,t}$ from model residuals. The procedure yields exporter×stage volatilities $\hat{\sigma}_{j,s}$ and a Kronecker-structured correlation matrix $\hat{R}^J \otimes \hat{R}^K$.

Structural residuals. Using converged parameters, compute model-implied bilateral levels $\mu_{ji,s,t} = \tau^{GE}_{ji,s,t}(w_{j,t}/a_{j,s})M_{ji,s,t}$. Aggregating over importers to reduce noise,

$$u_{j,s,t} = \ln\Big(\textstyle\sum_i X^{Data}_{ji,s,t}\Big) - \ln\Big(\textstyle\sum_i \mu_{ji,s,t}\Big). \tag{90}$$

Residuals are winsorized at the 1st and 99th percentiles and then stage×year demeaned: $\tilde{u}_{j,s,t} = u_{j,s,t} - \bar{u}_{s,t}$, where $\bar{u}_{s,t} = I^{-1}\sum_j u_{j,s,t}$.

Volatility estimation. Marginal volatilities are estimated from the sample standard deviation of the demeaned residuals (requiring at least 5 periods per (j,s) cell):

$$\hat{\sigma}_{j,s} = \sqrt{\frac{\sum_t \big(\tilde{u}_{j,s,t} - T^{-1}\textstyle\sum_t \tilde{u}_{j,s,t}\big)^2}{T-1}}. \tag{91}$$

The mean-one property of lognormal shocks is validated via the diagnostic $\hat{m}_{j,s} + \hat{\sigma}^2_{j,s}/2 \approx 0$.

Kronecker correlation structure. We impose the separable covariance structure $\Sigma = \mathrm{diag}(\sigma)\cdot(R^J \otimes R^K)\cdot\mathrm{diag}(\sigma)$, reducing free correlation parameters from $O((JK)^2)$ to $O(J^2+K^2)$. This parsimony is economically motivated: common macroeconomic shocks generate cross-country correlations that operate similarly at all stages, while technology linkages generate cross-stage correlations that are similar across countries. Let $Z_t \in \mathbb{R}^{J\times K}$ be the matrix of standardized residuals $\tilde{u}_{j,s,t}/\hat{\sigma}_{j,s}$. We model $Z_t \sim \mathrm{MN}(0, R^J, R^K)$ and estimate R^J and R^K by the Dutilleul flip-flop algorithm, which alternates conditional MLEs:

$$R^{J,(m+1)} \propto \frac{1}{(T-1)K}\sum_t Z_t \left(R^{K,(m)}\right)^{-1} Z_t', \tag{92}$$

$$R^{K,(m+1)} \propto \frac{1}{(T-1)J}\sum_t Z_t' \left(R^{J,(m+1)}\right)^{-1} Z_t. \tag{93}$$

After each update the matrix is projected onto the positive-semidefinite cone (eigenvalues clamped at 10^{-8}), shrunk toward the identity with intensities $\xi^J = 0.30$ and $\xi^K = 0.02$ (the higher shrinkage for R^J reflects its smaller effective sample size $T \cdot K$ relative to J), and rescaled to unit diagonal. Convergence is declared when $\max(\|R^{J,(m+1)} - R^{J,(m)}\|_\infty, \|R^{K,(m+1)} - R^{K,(m)}\|_\infty) < 10^{-6}$.

Generating correlated χ-draws. Given $\hat{\sigma}_{j,s}$, $\hat{R}^J$, $\hat{R}^K$, let L^J and L^K be their Cholesky factors. For each Monte Carlo draw $d = 1, \ldots, D$: (i) draw $Z^{(d)} \sim N(0, I_{J\times K})$ i.i.d.; (ii) correlate: $W^{(d)} = L^J Z^{(d)} (L^K)'$; (iii) set $\chi_{j,s}^{(d)} = \exp\!\big(-\frac{1}{2}\hat{\sigma}_{j,s}^2 + \hat{\sigma}_{j,s} W_{j,s}^{(d)}\big)$. By construction $\mathbb{E}[\chi_{j,s}] = 1$ for all (j, s), so the shocks are mean-preserving.

B.6 Gravity estimation

B.6.1 Estimating ρ_s

We alternatively estimate ρ_s using an approximation of the gravity equation. In Appendix B.6.1, we derive a simplified form of the estimating equation. Here, we use the fact that our model generates the following relationship:

$$\begin{aligned} Log\, Exp_{ji,s,t} = &- \frac{\rho_s}{1-\rho_s} Log\, \tau_{ji,s,t} - \frac{\rho_s}{1-\rho_s} Log\, p_{j,s,t} + \frac{1}{1-\rho_s} Log\, (1-\beta) w_{i,t} L_{i,t} \\ &+ \frac{1}{1-\rho_s} Log\, \mathbb{E}_\chi \left[\frac{(\chi_{ji,s,t})^{\rho_s}}{B_{s,t}^{\rho_s}} \frac{B_{s,t}^{\eta}}{y_{i,S,t}^{\eta}} \right] \end{aligned} \tag{94}$$

This gravity equation has many familiar terms. The outcome represents the value of trade flows from j to i for stage-k products, in year t. As the coefficients vary by stage k, we estimate $k = 1, ...S$ different specifications, one for each stage.

However, the last term (inside the expectations operator) is new to the gravity framework. Intuitively, given the structure of trade, trade between any two pairs of countries will be affected by changes in trade flows from the exporter to other countries (say, due to other bilateral tariff changes). As a result, we include an additional control for weighted tariffs between each exporter and other destinations, interacted with exporter fixed effects. We do not use actual trade flows between other parties, as those are determined endogenously. Further, we do not observe domestic absorption by stage for each country in the dataset. The advantage of policy-driven tariff changes is that they are an exogenous control. Under the plausible assumption

that they are correlated with changes in trade flows of the exporter to other countries, they will absorb the variation due to the last term of the estimating equation.

Our products are the HS-6 level, spanning the 19 production stages delineated in Antràs et al. (2012). We first estimate a product-level specification using PPML:

$$Exp_{ijt,h} = exp\left(\alpha_h + \beta_h \cdot \log\left(1 + \text{tariff}_{ijt,h}\right) + \sum_{j' \in J} \Delta_{j'} 1_{j'=j} \tau_{it,h}^{other} + \gamma_{jt,h} + \gamma_{it,h} + \gamma_{ij} + \varepsilon_{ijt,h}\right) \tag{95}$$

This specification is estimated for each HS-6 product h separately. Our coefficient of interest is β_h, as it corresponds to $\log\left(1 + \text{tariff}_{ijt,h}\right)$, which is our empirical driver of $Log\,\tau_{ji,s,t}$. We include a set of high-dimensional fixed effects that control for the various other terms in Equation 94. For instance, origin-by-year $\gamma_{jt,h}$ fixed effects account for changes in origin specific shocks for each product (e.g., $Log\,p_{j,s,t}$), while destination-by-year fixed effects $\gamma_{it,h}$ capture destination-specific changes by product or across all products (e.g., $Log\,(1-\beta)w_{i,t}L_{i,t}$). Origin-by-destination $\gamma_{ij,h}$ fixed effects, similarly control for origin-product time-invariant relationships that may affect various products. As described before, we also control for the exporter fixed effect interacted with a measure of tariff exposure for exporters:

$$\tau_{it,h}^{other} \equiv \sum_{l \in J \setminus \{i\}} \frac{\sum_{j \in J \setminus \{i\}} X_{ljt,h} \log\left(1 + \text{tariff}_{ljt,h}\right)}{\sum_{l' \in J \setminus \{i\}} \sum_{j \in J \setminus \{i\}} X_{l'jt,h}} \quad . \tag{96}$$

This (along with the help of fixed effects, like $\gamma_{ij,h}$) provides a reasonable control for the expectations term in Equation 94 solely derived from exogenous factors.

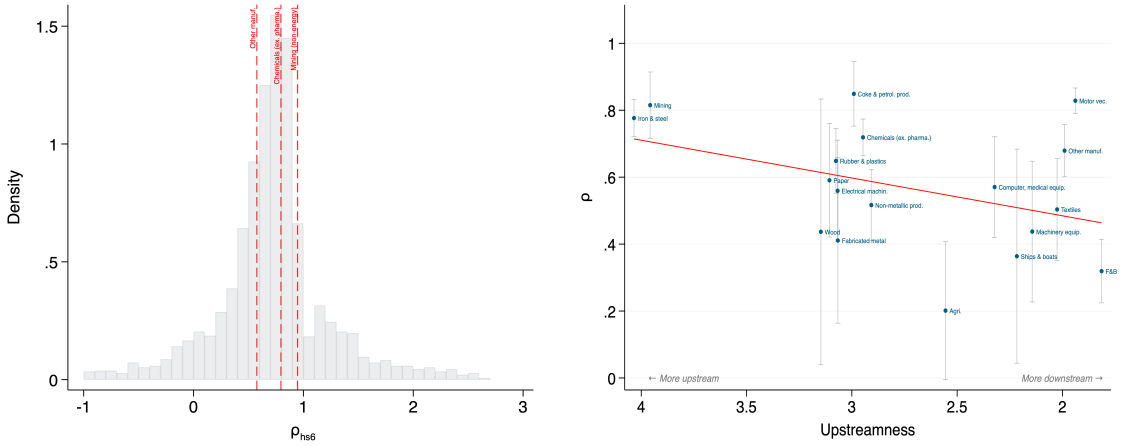
We plot a histogram of ρ_s^{hs6} in Figure B1a. The histogram shows that at the product-level, 75.4% values of ρ_s^{hs6} lie between 0 and 1.

There is, however, meaningful heterogeneity in the parameter estimates, much of which reflects differences across stages. The vertical dotted lines plot the median value of ρ_s^{hs6} for a few of the stages. 'Mining' (one of the most upstream stages) has one of the highest median value of ρ, while, 'Other manufacturing' (one of the most downstream stages) has one of the lowest median values of ρ. Intermediate stages, like 'Chemicals' have values in between.

Next, we estimate the same gravity regressions for each stage s, where each obser-

vation is at the importer i, exporter j, HS-6 product h, year t level.[15]

Figure B1: Estimated ρ



(a) Product level (b) Stage level

Note. The left panel figure plots the distribution of elasticity at product level $\hat{\rho}_h$. The right panel figure plots the estimated output elasticity for each stage $\hat{\rho}_s$ against the stage's upstreamness. The gray line is 95% confidence interval, calculated using delta method.

Since we estimate this regression for each s, we estimate $\hat{\beta}_s$. This allows us to recover $\hat{\rho}_s$:

$$\hat{\rho}_s = \frac{\hat{\beta}_s}{\hat{\beta}_s - 1}$$

Figure B1b describes our results. We plot the distribution of ρ_s across the upstreamness measure of that stage, derived from (Antràs et al., 2012). We see a meaningful relationship between upstreamness and ρ_k. More upstream stages (like "iron and steel" or "mining") have higher ρ_s than stages that are of intermediate upstreamness ("rubber and plastics", "electrical machinery"), which have still higher

15

$$Exp_{ijt,s(h)} = exp\Big(\alpha_{s(h)} + \beta_{s(h)} \cdot \log\left(1 + \text{tariff}_{ijt,s(h)}\right) + \sum_{j' \in J} B_{j'} 1_{j'=j} \tau^{other}_{it,s(h)} + \gamma_{jt,s(h)} + \gamma_{it,s(h)} + \gamma_{ij,s(h)} + \varepsilon_{ijt,h}\Big) \quad (97)$$

where $\gamma_{jt,h}$ are exporter-by-year-by-HS6 product fixed effects, $\gamma_{it,h}$ are importer-by-year-by-HS6 product fixed effects, and $\gamma_{ij,h'}$ are importer-exporter-HS4 product fixed effects. We again control for the exported fixed effect interacted with a measure of tariff exposure for exporters.

ρ_k than downstream industries ("ships and boats", "machinery equipment"). The flexibility of our estimation approach allows us to capture this heterogeneity.

$$p_{ji,s}M_{ji,s} = (\tau_{ji,s}p_{j,s})^{-\frac{\rho_s}{1-\rho_s}} \left[\mathbb{E}_\chi \left[\frac{(\chi_{ji,s})^{\rho_s}}{\sum_{j' \in \Omega_s} (\chi_{j'i,s}M_{j'i,s})^{\rho_s}} \lambda_s(\boldsymbol{\chi}) \prod_{t=s+1}^{S} \left[1 - \lambda_t(\boldsymbol{\chi})\right] \right] (1-\beta) w_i L_i \right]^{\frac{1}{1-\rho_s}}. \quad (10)$$

We now introduce a time t subscript for estimation purposes, as we have yearly data. We further define expenditures $Exp_{ji,s,t} \equiv p_{ji,s,t}M_{ji,s,t}$, to be i's total expenditure on stage s products purchased from j in year t. Taking logs, generates a modified-gravity equation:

$$\begin{aligned} Log\, Exp_{ji,s,t} = &- \frac{\rho_s}{1-\rho_s} Log\, \tau_{ji,s,t} - \frac{\rho_s}{1-\rho_s} Log\, p_{j,s,t} + \frac{1}{1-\rho_s} Log\, (1-\beta) w_{i,t} L_{i,t} \\ &+ \frac{1}{1-\rho_s} Log\, \mathbb{E}_\chi \left[\frac{(\chi_{ji,s})^{\rho_s}}{\sum_{j' \in \Omega_s} (\chi_{j'i,s}M_{j'i,s})^{\rho_s}} \lambda_s(\boldsymbol{\chi}) \prod_{t=s+1}^{S} \left[1 - \lambda_t(\boldsymbol{\chi})\right] \right] \end{aligned} \quad (98)$$

This gravity equation has many familiar terms. The outcome represents the value of trade flows from j to i for stage-k products, in year t. As the coefficients vary by stage k, we estimate $k = 1, ...S$ different specifications, one for each stage. We include origin by time fixed effects $\gamma_{j,k,t}$ to control for variables that vary at the origin-by-stage-by-time level, such as the terms associated with $p_{j,k,t}$. Similarly, destination by time fixed effects $\gamma_{i,k,t}$ control for variables that vary at the destination-by-stage-by-time level, such as $w_{i,t}L_{i,t}$.

However, the last term (inside the expectations operator) is new to the gravity framework. Intuitively, given the structure of trade, trade between any two pairs of countries will be affected by changes in trade flows from the exporter to other countries (say, due to other bilateral tariff changes). As a result, we include an additional control for weighted tariffs between each exporter and other destinations, interacted with exporter fixed effects. We do not use actual trade flows between other parties, as those are determined endogenously. Further, we do not observe domestic absorption by stage for each country in the dataset. The advantage of policy-driven

tariff changes is that they are an exogenous control. Under the plausible assumption that they are correlated with changes in trade flows of the exporter to other countries, they will absorb the variation due to the last term of the estimating equation.

Next, we derive a simplified form of the estimating equation. First, we must simplify the term:

$$+\frac{1}{1-\rho_k} Log\, \mathbb{E}_\chi \left[\frac{(\chi_{ji,s})^{\rho_s}}{\sum_{j' \in \Omega_s} (\chi_{j'i,s} M_{j'i,s})^{\rho_s}} \lambda_k(\boldsymbol{\chi}) \prod_{t=k+1}^{S} \left[1 - \lambda_t(\boldsymbol{\chi})\right] \right] \tag{99}$$

The product of a recurrence can be challenging to deal with, so we first want to simplify:

$$P_{k:S} \equiv \Big[\prod_{t=k+1}^{S} (1-\lambda_t) \Big]$$

Notice,

$$(1-\lambda_t) = \frac{y_{t-1}^{\eta}}{y_t^{\eta}}$$

We want

$$P_{k:S} \;=\; \prod_{t=k+1}^{S} \left(\frac{y_{t-1}}{y_t} \right)^{\eta},$$

Assume $y_t, B_t > 0$ so powers are well-defined.

Substituting:

$$\prod_{t=k+1}^{S} \frac{(y_{t-1})^{\eta}}{(y_t)^{\eta}} = \prod_{t=k+1}^{S} \left(\frac{y_{t-1}}{y_t} \right)^{\eta} = \left[\prod_{t=k+1}^{S} \frac{y_{t-1}}{y_t} \right]^{\eta}.$$

The product telescopes:

$$\prod_{t=k+1}^{S} \frac{y_{t-1}}{y_t} = \frac{y_k}{y_{k+1}} \cdot \frac{y_{k+1}}{y_{k+2}} \cdots \frac{y_{S-1}}{y_S} = \frac{y_k}{y_S}.$$

Therefore:

$$\prod_{t=k+1}^{S}(1-\lambda_t)=\left(\frac{y_k}{y_S}\right)^{\eta}.$$

As a result, we can write:

$$\left[\frac{(\chi_{ji,s})^{\rho_k}}{\sum_{j'\in\Omega_s}(\chi_{j'i,s}M_{j'i,s})^{\rho_k}}\,\lambda_k(\boldsymbol{\chi})\prod_{t=k+1}^{S}\left[1-\lambda_t(\boldsymbol{\chi})\right]\right]=\frac{(\chi_{ji,k})^{\rho_k}}{B_k^{\rho_k}}\,\frac{B_k^{\eta}}{y_S^{\eta}}$$

This is allows us to simplify the term in equation to be

$$+\frac{1}{1-\rho_k}Log\,\mathbb{E}_{\chi}\left[\frac{(\chi_{ji,s})^{\rho_s}}{\sum_{j'\in\Omega_s}(\chi_{j'i,s}M_{j'i,s})^{\rho_s}}\,\lambda_k(\boldsymbol{\chi})\prod_{t=k+1}^{S}\left[1-\lambda_t(\boldsymbol{\chi})\right]\right]=+\frac{1}{1-\rho_k}Log\,\mathbb{E}_{\chi}\left[\frac{(\chi_{ji,k})^{\rho_k}}{B_k^{\rho_k}}\,\frac{B_k^{\eta}}{y_S^{\eta}}\right] \tag{100}$$

Moving forward, we use the fact that the pointwise elasticity can be written as:[16]

$$\mathcal{E}_{ji,k}(\boldsymbol{\chi})=\frac{(\chi_{ji,k}M_{ji,k})^{\rho_k}}{B_k^{\rho_k}}\,\frac{B_k^{\eta}}{y_S^{\eta}} \tag{101}$$

[16]Essentially, the term $\lambda_k(\boldsymbol{\chi})\prod_{t=k+1}^{S}[1-\lambda_t(\boldsymbol{\chi})]$ is a recursive product that telescopes to $\frac{B_k^{\eta}}{y_k^{\eta}}\frac{y_k^{\eta}}{y_S^{\eta}}$. So we can write:

$$\begin{aligned}\mathcal{E}_{ji,k}(\boldsymbol{\chi})&=\omega_{ji,k}(\boldsymbol{\chi})\,\lambda_k(\boldsymbol{\chi})\prod_{t=k+1}^{S}\left[1-\lambda_t(\boldsymbol{\chi})\right]\\&=\omega_{ji,k}(\boldsymbol{\chi})\,\frac{B_k^{\eta}}{y_k^{\eta}}\frac{y_k^{\eta}}{y_S^{\eta}}\\&=\frac{(\chi_{ji,k}M_{ji,k})^{\rho_k}}{B_k^{\rho_k}}\,\frac{B_k^{\eta}}{y_S^{\eta}}\end{aligned}$$

$$Log\, Exp_{ji,k,t} = -\frac{\rho_k}{1-\rho_k} Log\, \tau_{ji,k,t} - \frac{\rho_k}{1-\rho_k} Log\, p_{j,k,t} + \frac{1}{1-\rho_k} Log\, (1-\beta) w_{i,t} L_{i,t} + \frac{1}{1-\rho_k} Log\, \mathbb{E}_\chi \left[\frac{(\chi_{ji,k,t})^{\rho_k}}{B_{k,t}^{\rho_k}} \frac{B_{k,t}^{\eta}}{y_{i,S,t}^{\eta}} \right] \tag{102}$$

Indeed, if we take a first-order approximation around the mean (set $\chi_{ji,k,t} = 1$), we can derive a simpler expression as well.[17]

B.6.2 Estimating η

To estimate η, which governs the elasticity of substitution across stages, we leverage changes in relative prices between stages. In Appendix B.6.2, we derive the familiar relationship:

$$Log\, \left(\frac{Exp_{i,k,t}}{Exp_{i,k',t}} \right) = -\frac{\eta}{1-\eta} Log\, \left(\frac{P_{i,k,t}}{P_{i,k',t}} \right) , \tag{104}$$

where The price index for $i-k$ is defined as:

$$P_{i,k,t} \equiv \left[\sum_{j'} p_{ij',k,t}^{\frac{-\rho_k}{1-\rho_k}} \right]^{\frac{-(1-\rho_k)}{\rho_k}} , \tag{105}$$

and total expenditure on stage k by location i is $Exp_{i,k,t} \equiv \sum_{j'} p_{ij',k,t} M_{ij',k,t}$.

We construct price indices based on the fact that the γ_{jkt} fixed effects from the estimation of Equation 97 captures $p_{j,k,t}$. As we show in Appendix B.6.2, we can derive an empirical equivalent of the $i-k$ price index to be:

$$P_{i,k,t} = \left[\sum_{j} \tau_{ijkt}^{\frac{-\rho_k}{(1-\rho_k)}} exp(\gamma_{jkt}) \right]^{\frac{-(1-\rho_k)}{\rho_k}} \tag{106}$$

[17]

$$Log\, Exp_{ij,k,t} = -\frac{\rho_s}{1-\rho_s} Log\, \tau_{ji,k,t} - \underbrace{\frac{\rho_s}{1-\rho_s} Log\, p_{j,k,t}}_{\gamma_{jkt} \text{ fixed effect}} + \underbrace{\frac{\eta-\rho_k}{1-\rho_k} Log\, B_{ik} + \frac{1}{1-\rho_k} Log\, \frac{(1-\beta) w_i L_i}{y_{iS}^{\eta}}}_{\gamma_{ikt} \text{ fixed effect}} \tag{103}$$

Next, we create an analogous instrumental variable that leverages changes only in tariffs, rather than overall prices:

$$IV_{ik,t} \equiv \left[\sum_{j'} \tau_{ij',k,t}^{\frac{-\rho_k}{1-\rho_k}} \right]^{\frac{-(1-\rho_k)}{\rho_k}} \tag{107}$$

We estimate the following specification:

$$Log\ Exp_{i,k,t} = \beta\ Log\ P_{i,k,t} + \gamma_{ik} + \gamma_{it} + \gamma_{kt} + \delta \tau_{i,k,t}^{other} + \epsilon_{i,k,t}\ , \tag{108}$$

where, γ_{ik} are destination-stage fixed effects, γ_{it} are destination-year fixed effects, and γ_{kt} are stage-by-year fixed effects. Just like in the previous section, we control for price changes (as proxied for by tariffs) of other trading partners, $\tau_{i,k,t}^{other}$, as defined in Equation 96.

Table B1: Estimated elasticity across stage, $\hat{\eta}$

	OLS	IV
η	-0.26*	-0.52*
	(0.150)	(0.278)
ik FE	Y	Y
it FE	Y	Y
kt FE	Y	Y
Tariff control	Y	Y
First stage F-stat		1182.27

Note. Number in parantheses is standard error (clustered at importer), calculated using the delta method. * Statistically significant at 10%. We use CEPII-BACI trade dataset and grouped the 19 stages into 3 big stages.

Our estimate of η is derived from the fact that $\hat{\beta} = \frac{-\hat{\eta}}{1-\hat{\eta}}$. We compute standard errors using the delta method. In practice, we group the nineteen stages into three, as most production processes go through only a handful of stages. We use the upstreamness measure as shown in Figure B1b, and group all stages into three equally sized groups. So the most upstream group consists of industries like mining, steel and wood; the middle group of chemicals, non metallic products, and the most downstream has products like motor vehicles.

Table B1 shows our results. Our OLS specification suggests a value of η that is about -0.26, whereas our 2SLS estimate is of -0.52, with a strong first stage. We use -0.52 as our estimate of η in our exercises.

To derive the estimating equation, we begin with Equation 5:

$$\frac{p_{ji,k}M_{ji,k}}{(1-\beta)w_iL_i} \equiv s_{ji,k} = \mathbb{E}_\chi\left[\mathcal{E}_{ji,k}(\boldsymbol{\chi})\right] = \mathbb{E}_\chi\left[\omega_{ji,k}(\boldsymbol{\chi})\,\lambda_k(\boldsymbol{\chi})\prod_{t=k+1}^{S}\left[1-\lambda_t(\boldsymbol{\chi})\right]\right] \quad (5)$$

We substitute in Equation 101, and derive:

$$p_{ij,k}M_{ij,k} = \mathbb{E}_\chi\left[\frac{(\chi_{ji,k}M_{ji,k})^{\rho_k}}{B_k^{\rho_k}}\frac{B_k^\eta}{y_S^\eta}\right](1-\beta)w_iL_i \quad (109)$$

We now take a first order approximation around mean, and so set $\chi_{ji,k}=1$, and no longer require the expectations operator.

Next, we take the ratio with respect to origin j':

$$\frac{p_{ij,k}M_{ij,k}}{p_{ij',k}M_{ij',k}} = \frac{(M_{ji,k})^{\rho_k}}{(M_{j'i,k})^{\rho_k}} \quad (110)$$

$$\frac{p_{ij,k}M_{ij,k}}{p_{ij',k}M_{ij',k}} = \frac{\left(\frac{p_{ji,k}M_{ji,k}}{p_{ij,k}}\right)^{\rho_k}}{\left(\frac{p_{j'i,k}M_{j'i,k}}{p_{ij',k}}\right)^{\rho_k}} \quad (111)$$

$$\left(\frac{p_{ij,k}M_{ij,k}}{p_{ij',k}M_{ij',k}}\right)^{1-\rho_k} = \left(\frac{p_{ij,k}}{p_{ij',k}}\right)^{-\rho_k} \quad (112)$$

$$\left(\frac{p_{ij,k}M_{ij,k}}{p_{ij',k}M_{ij',k}}\right) = \left(\frac{p_{ij,k}}{p_{ij',k}}\right)^{\frac{-\rho_k}{1-\rho_k}} \quad (113)$$

Cross multiply, and take sum over j':

$$\underbrace{p_{ij,k}M_{ij,k}}_{\equiv Exp_{ij,k}}\underbrace{\sum_{j'}p_{ij',k}^{\frac{-\rho_k}{1-\rho_k}}}_{\equiv P_{ik}^{\frac{-\rho_k}{1-\rho_k}}} = p_{ij,k}^{\frac{-\rho_k}{1-\rho_k}}\underbrace{\sum_{j'}p_{ij',k}M_{ij',k}}_{\equiv Exp_{ik}} \quad (114)$$

where $Exp_{ij,k}$ is the expenditure on k-stage products, sources from j by i, and Exp_{ik} is the total expenditure on k from i (summed over origins j). The price index for $i-k$ is defined as:

$$P_{ik} \equiv \left[\sum_{j'} p_{ij',k}^{\frac{-\rho_k}{1-\rho_k}} \right]^{\frac{-(1-\rho_k)}{\rho_k}} \tag{115}$$

So we are left with the familiar expression of the lower nest expenditure allocation:

$$\frac{Exp_{ij,k}}{Exp_{i,k}} = \left(\frac{p_{ij,k}}{P_{i,k}} \right)^{\frac{-\rho_k}{1-\rho_k}} \tag{116}$$

Cross multiply:

$$Exp_{ij,k} = \left(\frac{p_{ij,k}}{P_{i,k}} \right)^{\frac{-\rho_k}{1-\rho_k}} Exp_{i,k} \tag{117}$$

$$p_{ij,k} M_{ij,k} = p_{ij,k}^{\frac{-\rho_k}{1-\rho_k}} \left[Exp_{i,k} P_{i,k}^{\frac{\rho_k}{1-\rho_k}} \right] \tag{118}$$

$$M_{ij,k} = p_{ij,k}^{\frac{-1}{1-\rho_k}} \left[Exp_{i,k} P_{i,k}^{\frac{\rho_k}{1-\rho_k}} \right] \tag{119}$$

$$M_{ij,k}^{\rho_k} = p_{ij,k}^{\frac{-\rho_k}{1-\rho_k}} \left[Exp_{i,k} P_{i,k}^{\frac{\rho_k}{1-\rho_k}} \right]^{\rho_k} \tag{120}$$

Take sum over j:

$$\left(\sum_j M_{ij,k}^{\rho_k} \right)^{\frac{1}{\rho_k}} = \left[Exp_{i,k} P_{i,k}^{\frac{\rho_k}{1-\rho_k}} \right] \left(\sum_j p_{ij,k}^{\frac{-\rho_k}{1-\rho_k}} \right)^{\frac{1}{\rho_k}} \tag{121}$$

$$\left(\sum_j M_{ij,k}^{\rho_k} \right)^{\frac{1}{\rho_k}} = \left[Exp_{i,k} P_{i,k}^{\frac{\rho_k}{1-\rho_k}} \right] (P_{i,k})^{\frac{-\rho_k}{1-\rho_k} \frac{1}{\rho_k}} \tag{122}$$

$$\left(\sum_j M_{ij,k}^{\rho_k} \right)^{\frac{1}{\rho_k}} = Exp_{i,k} P_{i,k}^{-1} \tag{123}$$

$$B_{ik} = Exp_{i,k} P_{i,k}^{-1} \tag{124}$$

Now, going back to our optimal shares Equation 109:

$$p_{ij,k} M_{ij,k} = \mathcal{E}_{ji,s}(\boldsymbol{\chi} = 1)\,(1-\beta) w_i L_i \tag{125}$$

$$= \frac{(M_{ji,k})^{\rho_k}}{B_k^{\rho_k}} \frac{B_k^{\eta}}{y_S^{\eta}} (1-\beta) w_i L_i \tag{126}$$

$$= \frac{p_{ij,k} M_{ij,k}}{\sum_{j'} p_{ij',k} M_{ij',k}} \frac{B_k^{\eta}}{y_S^{\eta}} (1-\beta) w_i L_i \tag{127}$$

$$\sum_{j'} p_{ij',k} M_{ij',k} = \frac{B_k^{\eta}}{y_S^{\eta}} (1-\beta) w_i L_i \tag{128}$$

Define total expenditure $Exp_{ik} \equiv \sum_{j'} p_{ij',k} M_{ij',k}$. Taking ratio with another stage k':

$$\frac{Exp_{ik}}{Exp_{ik'}} = \left(\frac{B_{ik}}{B_{ik'}}\right)^{\eta} \tag{129}$$

Replacing from Equation 124, we know:

$$\frac{Exp_{ik}}{Exp_{ik'}} = \left(\frac{B_{ik}}{B_{ik'}}\right)^{\eta} \tag{130}$$

$$\frac{Exp_{ik}}{Exp_{ik'}} = \frac{\left(Exp_{i,k} P_{i,k}^{-1}\right)^{\eta}}{\left(Exp_{i,k'} P_{i,k'}^{-1}\right)^{\eta}} \tag{131}$$

$$\left(\frac{Exp_{ik}}{Exp_{ik'}}\right)^{1-\eta} = \left(\frac{P_{i,k}}{P_{i,k'}}\right)^{-\eta} \tag{132}$$

Take logs:

$$\boxed{Log\left(\frac{Exp_{ik}}{Exp_{ik'}}\right) = -\frac{\eta}{1-\eta} Log\left(\frac{P_{i,k}}{P_{i,k'}}\right)} \tag{104}$$

This is our primary estimating equation.

Deriving the $i-k$ price index. From the gravity equation 97, we first estimate the fixed effect γ_{jkt}. We then derive:

$$P_{i,k,t} = \left[\sum_j \tau_{ijkt}^{\frac{-\rho_k}{(1-\rho_k)}} exp(\gamma_{jkt})\right]^{\frac{-(1-\rho_k)}{\rho_k}} \tag{133}$$

To see this, let us start with the fact that in the empirical estimation of the gravity equation, the $j-k-t$ fixed effect captures information on prices.

We know:

$$\begin{aligned}
\gamma_{jkt} &= Log\, p_{jkt}^{\frac{-\rho_k}{(1-\rho_k)}} \\
exp(\gamma_{jkt}) &= p_{jkt}^{\frac{-\rho_k}{(1-\rho_k)}} \\
\tau_{ijkt}^{\frac{-\rho_k}{(1-\rho_k)}} exp(\gamma_{jkt}) &= (\tau_{ijkt} p_{jkt})^{\frac{-\rho_k}{(1-\rho_k)}} \\
\tau_{ijkt}^{\frac{-\rho_k}{(1-\rho_k)}} exp(\gamma_{jkt}) &= (p_{ijkt})^{\frac{-\rho_k}{(1-\rho_k)}} \\
\sum_j \tau_{ijkt}^{\frac{-\rho_k}{(1-\rho_k)}} exp(\gamma_{jkt}) &= \sum_j (p_{ijkt})^{\frac{-\rho_k}{(1-\rho_k)}} \\
\left[\sum_j \tau_{ijkt}^{\frac{-\rho_k}{(1-\rho_k)}} exp(\gamma_{jkt})\right]^{\frac{-(1-\rho_k)}{\rho_k}} &= P_{ikt}
\end{aligned} \tag{134}$$

We use this to create an empirical construct of our price index.

C Quantitative Appendix

C.1 Country in sample

Table C1: Top 50 country list

Countries	
United States	China
Japan	Germany
United Kingdom	France
India	Italy
Brazil	Canada
Republic of Korea	Russian Federation
Spain	Australia
Mexico	Indonesia
Turkey	Netherlands
Switzerland	Saudi Arabia
Argentina	Sweden
Poland	Belgium
Thailand	Nigeria
Austria	Norway
Egypt	China, Hong Kong SAR
Israel	Denmark
Philippines	Singapore
Malaysia	South Africa
Ireland	Pakistan
Colombia	Chile
Finland	Bangladesh
Portugal	Viet Nam
Greece	Czech Republic
Peru	New Zealand
Kazakhstan	Hungary

C.2 Stages and Industry Descriptions

Table C2: Stages and Industry Descriptions

Stage	Industry Description
1	Iron & steel
	Mining and quarrying (energy): coal, lignite, crude petroleum and natural gas
	Mining and quarrying (non-energy): metal ores and other mining and quarrying
	Wood and products of wood and cork
	Pulp, paper, paper products, printing and publishing
	Rubber & plastics products
2	Electrical machinery & apparatus, nec
	Fabricated metal products, except machinery & equipment
	Coke, refined petroleum products and nuclear fuel
	Other non-metallic mineral products
	Agriculture, hunting, forestry and fishing
	Computer, electronic and optical products
	Chemicals excluding pharmaceuticals
3	Building & repairing of ships & boats
	Machinery & equipment, nec
	Textiles, textile products, leather and footwear
	Other manufacturing
	Motor vehicles, trailers & semi-trailers
	Food products, beverages and tobacco

C.3 Tariff Algorithm

1. Use the bilateral simple AHS for year t
2. Use the simple MFN for year t
3. Check at $t + 1$:

(a) If $t+1$ use AHS, use AHS for year $t-1$. If AHS $t-1$ missing, continue checking AHS until $t-5$

(b) If $t+1$ use MFN, use MFN $t-1$. If MFN $t-1$ missing, continue checking MFN until $t-5$

4. If still missing, check the other measure (so if in 2 you check for AHS, now check with MFN from $t-1$ to $t-5$, vice versa

C.4 Aggregate Risk Measure

Our counterfactual exercises require increasing risk in specific parts of the supply chain—for instance, raising upstream risk or downstream risk by a comparable amount, or increasing risk originating from a particular country. Individual origin-stage variances σ_{js}^2 do not map one-to-one into aggregate risk, because the contribution of any single σ_{js} to overall risk depends on how large that origin-stage pair is in global trade and on how correlated its shocks are with those of other origin-stage pairs. A country-stage pair that accounts for a large share of global exports and is positively correlated with other major exporters contributes far more to aggregate risk than an equally volatile but small and uncorrelated pair. An aggregate risk measure is therefore essential for designing counterfactuals that are comparable across different parts of the supply chain.

We define aggregate risk as the variance of a trade-weighted portfolio of origin-stage shocks:

$$aggregate_risk = \sum_{js} \sum_{j's'} \varsigma_{jk}\, \varsigma_{j's'}\, \sigma_{js}\, \sigma_{j's'}\, R_{jj'}\, R_{ss'}, \tag{135}$$

where $\varsigma_{js} = \frac{\sum_i X_{jis}}{\sum_j \sum_i \sum_k X_{jis}}$ is the export share of origin j in stage s in global trade, σ_{js} is the standard deviation of origin-stage (j,s) shocks, $R_{jj'}$ is the correlation between country j and j', and $R_{ss'}$ is the correlation between stage s and s'.

The measure in equation (135) has a natural portfolio interpretation. Just as the variance of a financial portfolio depends on asset weights, individual volatilities, and pairwise correlations, aggregate supply-chain risk depends on each origin-stage pair's share in global trade (ς_{js}), its shock volatility (σ_{js}), and the correlation structure across countries ($R_{jj'}$) and stages ($R_{ss'}$).