# The Network Origin of Slow Labor Reallocation*

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**Abstract**

How fast do labor markets adjust to technology shocks? This paper introduces a novel network framework to analyze how skill frictions shapes worker reallocation dynamics. Using expert data, I construct an occupation network where links capture feasible transitions based on shared skills. This network is sparse and clustered, with a few critical "bridge occupations" connecting otherwise separate clusters of occupations. Leveraging French administrative data, I show that workers transitioning through these bridges reach higher-wage, lower-unemployment occupations. Next, I develop a job-search model embedded in this occupation network and find that bridge occupations disproportionately influence overall reallocation speed. Quantitatively, the model predicts that robot adoption induces a slow reallocation process—lasting around ten years—and reduces welfare gains by 40%, an order of magnitude higher than previous estimates. Policies targeting bridge occupations significantly accelerate reallocation, outperforming interventions focused on expanding sectors. These findings highlight the crucial role of occupation networks in shaping labor market adjustments and provide new insights for policy design.

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# 1 Introduction

Technological change creates winners and losers in the labor market. As new technologies are introduced, some occupations decline (e.g., cashiers), while others expand (e.g., robotics technicians). In principle, displaced workers should quickly move into growing occupations, mitigating the negative effects of technology shocks. However, empirical evidence shows that labor reallocation is slow, generating prolonged unemployment and welfare losses (Autor et al., 2013; Dix-Carneiro, 2014; Adão et al., 2024; Huckfeldt, 2022). A central reason is the specificity of skills, which are difficult to transfer directly across occupations: cashiers displaced by automation cannot easily move to robotics technicians jobs (Gathmann and Schönberg, 2010; Traiberman, 2019; Lise and Postel-Vinay, 2020).

This paper seeks to understand how skill frictions shape the speed of labor reallocation. This question is critical for designing targeted and effective public policies. But it is also challenging: skill transferability is heterogeneous and asymmetric across pairs of occupations, giving rise to a high-dimensional matrix of frictions – up to 40,000 possible pairs for 200 occupations. Existing approaches typically simplify this complexity by aggregating occupations or considering average frictions. Yet, many technological shocks affect occupations at a very granular level – affecting cashiers but not hand-packers despite similar skills – making aggregation problematic.

The main contribution of this paper is to introduce a new network-based framework to study skill frictions at granular levels and their influence on the speed of reallocation. I construct the *occupation network*, where nodes represent occupations and edges capture feasible transitions based on skill similarity. For example, engineer and computer scientist are connected, but cashier and engineer are not. Recent advances in network theory make it possible to analyze such high-dimensional matrices in a tractable way, allowing me to link reallocation dynamics to the structure of skill frictions. In particular, this approach reveals the importance of a small number of *bridge occupations* that connect otherwise distant clusters of occupations. Intuitively, these bridges can act as bottlenecks or accelerators of mobility between clusters, playing a crucial role for the aggregate reallocation dynamics. The remainder of the paper formalizes this insight.

This paper makes three main contributions. First, I show empirically that the occupation network is divided in clusters, with a few "bridge occupations" connecting distinct clusters of occupations.

Second, I develop a new theory of job search in networked labor markets. I demonstrate that the overall speed of reallocation is tightly linked to job finding rates in bridge occupations, and that slow transition can create important welfare losses. Third, using French administrative data, I estimate a quantitative version of the model and find that robot adoption led to slow reallocation, lasting 40 quarters and reducing welfare gains by 40%. Subsidizing bridge occupations speeds up reallocation three times more effectively than subsidizing growing occupations. I will now outline each part of the paper.

In the first part of the paper, I provide empirical evidence on the structure of the occupation network. I construct the network using a French administrative database on skills (ROME), which lists feasible occupation transitions according to HR experts. I highlight three key features. First, occupations have few connections on average, indicating significant mobility frictions. Second, occupations are grouped into clusters, displaying what Newman (2006) refers to as "community structure." Intuitively, occupations within the same 1-digit group are tightly connected due to their shared skills. Third, some occupations act as 'bridges,' linking different clusters. These occupations, such as logistics technician or sales assistant, typically require diverse skill sets.

Next, I examine how the occupation network's architecture affects worker mobility. Using French employer-employee data, I find that workers are 95% less likely to switch occupations when the transition is not deemed feasible by experts, controlling for time-varying origin and destination occupation fixed effects. I also analyze the impact of transitioning through bridge occupations on long-term outcomes. Workers who pass through bridge occupations are more likely to move into better-paying, lower-risk jobs and to switch 1-digit occupation groups. To address potential self-selection, I control for age, origin occupation, and mobility type. These findings underscore the crucial role of bridge occupations in facilitating transitions to better employment opportunities.

In the second part of the paper, I develop a theory of job search in networked labor markets to build intuition about the mechanisms behind slow reallocation. The model builds on the classical framework of Mortensen and Pissarides (1994), in which search frictions prevent workers from being immediately matched to jobs. I introduce two extensions. First, I allow for heterogeneous labor markets, with multiple occupations. Second, and most importantly. I introduce mobility across occupations via a network structure : workers can search not only within their current occupation, but also in adjacent ones, as defined in the occupation network. In other words, labor markets are

not isolated "islands". This setup unifies two distinct sources of friction – search frictions and skill frictions – and allows me to study how their interaction shapes reallocation speed.

To derive analytical results, I start with a stylized version of the model based on three simplifying assumptions: (i) the network includes three occupations—two peripheral ones connected to a central bridge occupation; (ii) only unemployed workers can search and switch occupations; and (iii) wages and separation rates are exogenous. These assumptions are relaxed later in the quantitative section. The tractability of this setup yields closed-form expressions for the speed of reallocation following permanent shocks, such as those from trade or technological change.

The analysis yields three main insights. First, reallocation can be very slow, with transition time of the order of $1/s$, where $s$ is the separation rate. For a quarterly separation rate of $2.5\%$, this implies a typical reallocation time of roughly 40 quarters. Crucially, this sluggishness does not stem from skill frictions alone, but rather from their interaction with search frictions. Workers in declining occupations still choose to stay in their jobs, as reallocation requires first becoming unemployed – a costly step when job search is frictional. In contrast, without search frictions, marginal workers would be willing to quit their job and reallocate immediately, leading to much faster adjustment.[1]

Second, the location of the shocks in the network is critical for reallocation speed. When a shock hits periphery occupations, reallocation is slower, as these occupations offer fewer transition opportunities. In contrast, when the shock hits a more central occupation, adjustment is faster, as displaced workers can access a wider range of alternative occupations. This can help explain why some technology shocks led to slow reallocation, while others did not.

Third, bridge occupations play a disproportionately important role: reallocation speed is highly sensitive to hiring rates in bridge occupations. By connecting otherwise distant parts of the network, bridge occupations help workers reallocate between different clusters of jobs. When access to bridge occupations is limited – because of low productivity or high hiring costs – bottlenecks arise and the overall reallocation slows down significantly. This result suggests that policy interventions targeting bridge occupations could significantly accelerate reallocation.

Finally, the model provides a framework to quantify the welfare losses caused by slow realloca-

[1] In the quantitative model, workers can also switch occupations while employed, which accelerates reallocation. The qualitative insight from the stylized model still holds: between-occupation transitions occur faster when unemployed.

tion, referred to as *adjustment costs* by Dix-Carneiro (2014); Caliendo et al. (2019). These losses are defined as the difference between the welfare gains under instantaneous reallocation and those achieved under gradual adjustment. The model's tractability allows for a closed-form expression of these losses, highlighting two main forces. First, slower reallocation delays future welfare gains, reducing their present discounted value. Second, workers are misallocated across occupations along the transition, e.g. with labor shortages in expanding occupations, which lowers aggregate output.

In the third part of the paper, I build a quantitative version of the model to test the predictions of the tractable model in a more realistic environment. I relax the assumptions of the tractable model by allowing for arbitrary network structures and more realistic worker behavior, including endogenous search effort, on-the-job search and Nash bargaining on wages. In the quantitative model, workers strategically allocate their search efforts to occupations with higher expected returns, both when employed and unemployed.

Despite its complexity, the quantitative model can be estimated and solved leveraging dynamic hat-algebra techniques from the trade literature. The idea is straightforward: following small shocks, the new equilibrium can be expressed as the product of the pre-shock equilibrium and a first-order approximation of the percentage point change (the so-called "hat changes"). Solving for these hat changes requires only a few key elasticities, significantly reducing the number of parameters to estimate. A key parameter, the search elasticity, can be estimated directly from the data.[2] Importantly, this approach could be applied to a broader set of models that combine discrete choice and search frictions.

Finally, I use the estimated model to simulate labor market adjustments following the introduction of robots. To calibrate the robot shock, I match estimates of the relative wage response and aggregate TFP gains from Webb (2019) and Acemoglu and Restrepo (2022). Occupations involving physical tasks, such as construction or rail operators, decline, while those requiring socio-linguistic skills, like interpreters and journalists, expand. Plotting the distribution of productivity gains and losses in the occupation network, I find that expanding and declining occupations are highly clustered and located at opposite ends of the occupation network, echoing the shock to periphery occupations from the tractable network search model.

There are three main takeaways from this quantitative analysis. First, robot adoption induces

[2]The search elasticity governs by how much search efforts respond to shocks.

slow worker reallocation, of around 40 quarters. This aligns closely with the stylized model's prediction, suggesting that the effects of a larger network and more strategic agents offset each other. Second, this masks important heterogeneity across occupations: a minority of occupations require more than 80 quarters to adjust. Third, and most importantly, the slow dynamics of reallocation generate significant adjustment costs, reducing welfare gains by approximately 40%. This is an order of magnitude higher than previous estimates, such as the 3.5% reduction found by Caliendo et al. (2019) for the US China shock. The key difference lies in the inclusion of both skill and search frictions in my model, which leads to costly, involuntary unemployment as workers reallocate in response to the shock.

Lastly, I explore the potential of targeted policy interventions to accelerate reallocation. I simulate the effects of government employment subsidies aimed at declining occupations (e.g., cashiers), expanding occupations (e.g., robot maintenance technicians), and bridge occupations (e.g., logistics technicians). Targeting bridge occupations leads to the largest reduction in reallocation times, cutting them by three times more than subsidies for expanding occupations. While subsidies for growing occupations increase incentives to switch by raising wages and firm vacancy postings, they do not address skill frictions: workers are willing to switch but lack the necessary skills. In contrast, targeting bridge occupations facilitates reskilling, enabling smoother transitions to expanding jobs. Overall, this suggests that exploiting the network structure of skill frictions can help policymakers design more effective labor market policies.

**Relation to the literature.** My paper connects to three strands of literature. First, it contributes to the literature on labor market dynamics following trade or technology shocks. Motivated by evidence of slow worker reallocation, a series of papers develop structural models of dynamic occupational choice with switching costs to estimate welfare effects (Artuç et al., 2010; Dix-Carneiro, 2014; Caliendo et al., 2019; Traiberman, 2019; Humlum, 2019). More recent work incorporates search frictions to account for the rise in involuntary unemployment triggered by these shocks (Pilossoph, 2012; Dix-Carneiro et al., 2021; Chodorow-Reich and Wieland, 2020; Carrillo-Tudela and Visschers, 2023). I make three contributions. First, I apply network theory to analyze reallocation frictions, highlighting the influence of central occupations on reallocation speed. Second, I extend the hat-algebra method from Caliendo et al. (2019) to include search

frictions. Third, I provide a characterization of the speed of worker reallocation in an environment with rich occupational heterogeneity.[3]

Second, my paper connects to the literature on job search in non-segmented labor markets. A growing body of work emphasizes the substantial overlap in labor market boundaries, whether in geographical space (Schmutz and Sidibé, 2019; Manning and Petrongolo, 2017; Marinescu and Rathelot, 2018) or skill space (Guvenen et al., 2020; Lise and Postel-Vinay, 2020; Böhm et al., 2025). Schubert et al. (2021) and Jarosch et al. (2024) show this matters for the measure of labor market power. I contribute by representing the overlap between labor markets as a network, highlighting the role of central occupations.[4] Unlike this literature, which focuses on mismatch or concentration, my focus is on the speed of worker reallocation.

Third, this paper contributes to the literature on heterogeneity in macroeconomics. An important strand of the literature uses spectral methods to study transition dynamics in economies with heterogeneous agents (Moll, 2014; Gabaix et al., 2016; Alvarez and Lippi, 2022; Baley and Blanco, 2021; Beraja and Wolf, 2021; Kleinman et al., 2023; Liu and Tsyvinski, 2024). I show that these tools can be applied in a labor market context, providing a full analytical characterization in a simplified setting. Additionally, the production network literature highlights that central sectors significantly impact aggregate outcomes (Gabaix et al., 2016; Acemoglu et al., 2012; Carvalho and Tahbaz-Salehi, 2019; Baqaee and Farhi, 2019). I extend this insight to labor markets, showing that central occupations disproportionately affect aggregate worker reallocation speed, paving the way for new research avenues.[5]

[3] In seminal papers, Kambourov (2009), Restrepo (2016) and Adão et al. (2024) study the speed of worker reallocation after trade or technological shocks in stylized two-labor-market settings. Interestingly, they also emphasize the importance of the separation margin (Kambourov (2009)) and stepping-stone occupations (Restrepo (2016)). My paper complements their findings by incorporating rich occupation heterogeneity, allowing for a more direct mapping between theory and data.

[4] An exception is Dabed et al. (2023), who estimate a network-augmented matching function to study to which extent skill overlap facilitates reallocation after technological shocks. I complement their analysis by developing a theory of job search in networked labor markets, to characterize reallocation speed and run counterfactuals.

[5] A literature in complexity economics also emphasizes the network structure of worker flows (see Neffke et al. (2017) or del Rio-Chanona et al. (2021)). I push further their analytical characterization, by incorporating the occupation network in an otherwise standard search and matching models. In addition, I highlight the crucial role of bridge occupations for reallocation speed.

# 2 Empirical Evidence on the Occupation Network

In this section, I construct the occupation network using a French administrative database on skills and analyze its structure, highlighting three stylized facts.

## 2.1 Data

To construct an empirical measure of the skill network, I use the French administrative database 'ROME'.[6] This database provides detailed information on the skills and tasks of occupations and is the French counterpart to the US O*NET database.

The ROME database was developed in 1989 by experts in human resources from the French employment agency Pole Emploi, in collaboration with firms and labor unions. Its original purpose was to improve the matching of job seekers with vacancies based on skill similarity, during a period of substantial structural transformation in labor markets. To find new opportunities for displaced workers, experts relied as little as possible on past worker flows. It has been revised many times since its creation, and here I use the fourth version, which was released in 2023.

## 2.2 Construction of the Main Variables

To begin, I will introduce a few concepts from network theory. A directed weighted network $\mathcal{G}$ consists of nodes $\mathcal{N}$, edges $\mathcal{E}$ and weights $\mathcal{W}$: $\mathcal{G} = \{\mathcal{N}, \mathcal{E}, \mathcal{W}\}$. This network can be represented by an adjacency matrix $\boldsymbol{G} = (g_{ij})_{(i,j)\in\mathcal{N}^2}$, where $g_{ij} \in \mathcal{W}$ is the weight of the edge $i \to j \in \mathcal{E}$. A network is undirected if all nodes are connected in both directions: $i \to j \in \mathcal{E}$ and $j \to i \in \mathcal{E}$. A network is unweighted if all edges weights are equal.

**Nodes.** I construct the nodes of the occupation network using the ROME nomenclature of occupations, defined at the 5-digit level of aggregation, totaling around 500 different occupations. Importantly, occupations are grouped by skill similarity rather than hierarchical rank. For example, the 5-digit category "higher education" includes both "lecturers" and "full professors."

**Edges.** I construct the edges of the skill network using a unique information from ROME: namely,

[6] The acronym ROME stands for *Répertoire Opérationnel des Métiers et Emplois*, which translates to "Operational Nomenclature of Occupations and Jobs".

the list of occupation transitions deemed feasible by experts. The edges of the occupation network are directed, meaning that reverse occupational transitions may not be feasible. For example, a medical doctor can become a nurse, but the reverse may not be true. This directionality partially captures the hierarchical structure of occupations.

**Weights.** To construct the edge weights, I use additional information from the ROME database. Experts distinguish between two types of feasible occupational transitions: "close" transitions can be made immediately, and "distant" transitions require additional training. I assign different numerical weights to each type of transition: $\alpha_{\text{close}}$, $\alpha_{\text{distant}}$, and $\alpha_{\text{self-loops}}$.

The weights measure the degree of skill transferability across occupations, normalized between zero and one. A weight of one indicates perfect accessibility (identical skills), while zero indicates inaccessibility (no skill similarity). Thus, they can be interpreted as the probability that a worker can acquire the skills to move to a new occupation. Formally, $0 \leq \alpha_{\text{distant}} \leq \alpha_{\text{close}} \leq \alpha_{\text{self}} = 1$. The weights are estimated in next section.[7]

## 2.3 Plot of the Occupation Network

Figure 2.1 plots the occupation network, using a standard network layout.[8] Manufacturing occupations are clustered together at the bottom of the network, while high-skilled service occupations, e.g. banking and insurance, are clustered at the top. Occupations with general skills, such as sales, transportation and logistics, or management are found at the center of the plot.[9]

## 2.4 Stylized Facts

This subsection highlights three stylized facts on the occupation network's structure.

**Fact 1.** *The occupation network is very sparse*

A network is sparse if its nodes have few connections relative to the size of the network. This is captured by out-degree centrality, which counts the number of edges originating from a node.

[7] I find $\alpha_{\text{close}} = 0.1$ and $\alpha_{\text{distant}} = 0.05$.

[8] The spring layout treats edges as springs keeping adjacent nodes close, which otherwise want to repel each other.

[9] Many groups of occupations overlap importantly with sectors, e.g. industry. In fact, a lot of occupations are specific to a certain industry but others, like accounting or sales, exist across all sectors exist across all sectors.

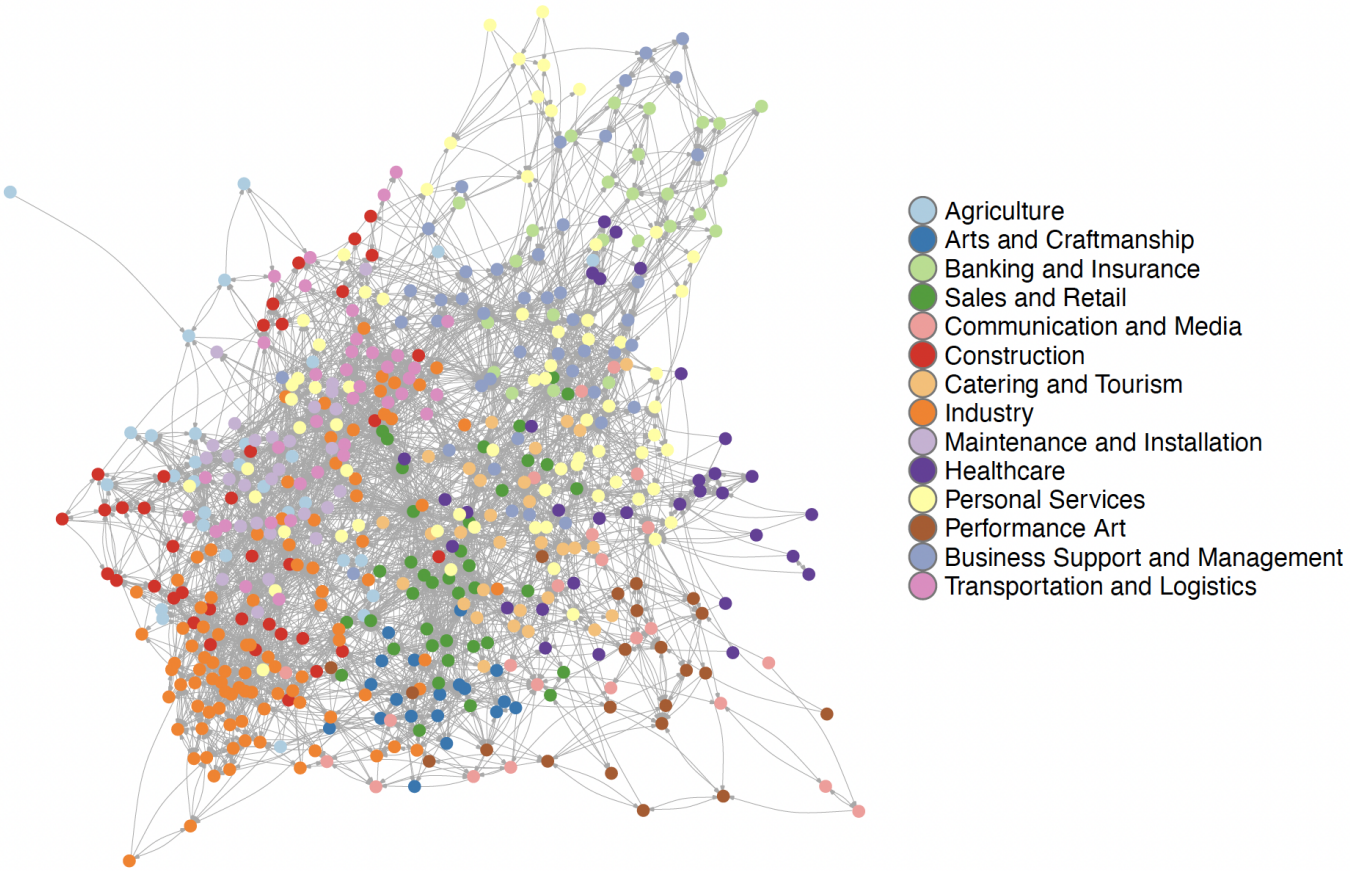



Figure 2.1: **The ROME skill network - with spring layout and close transitions only**

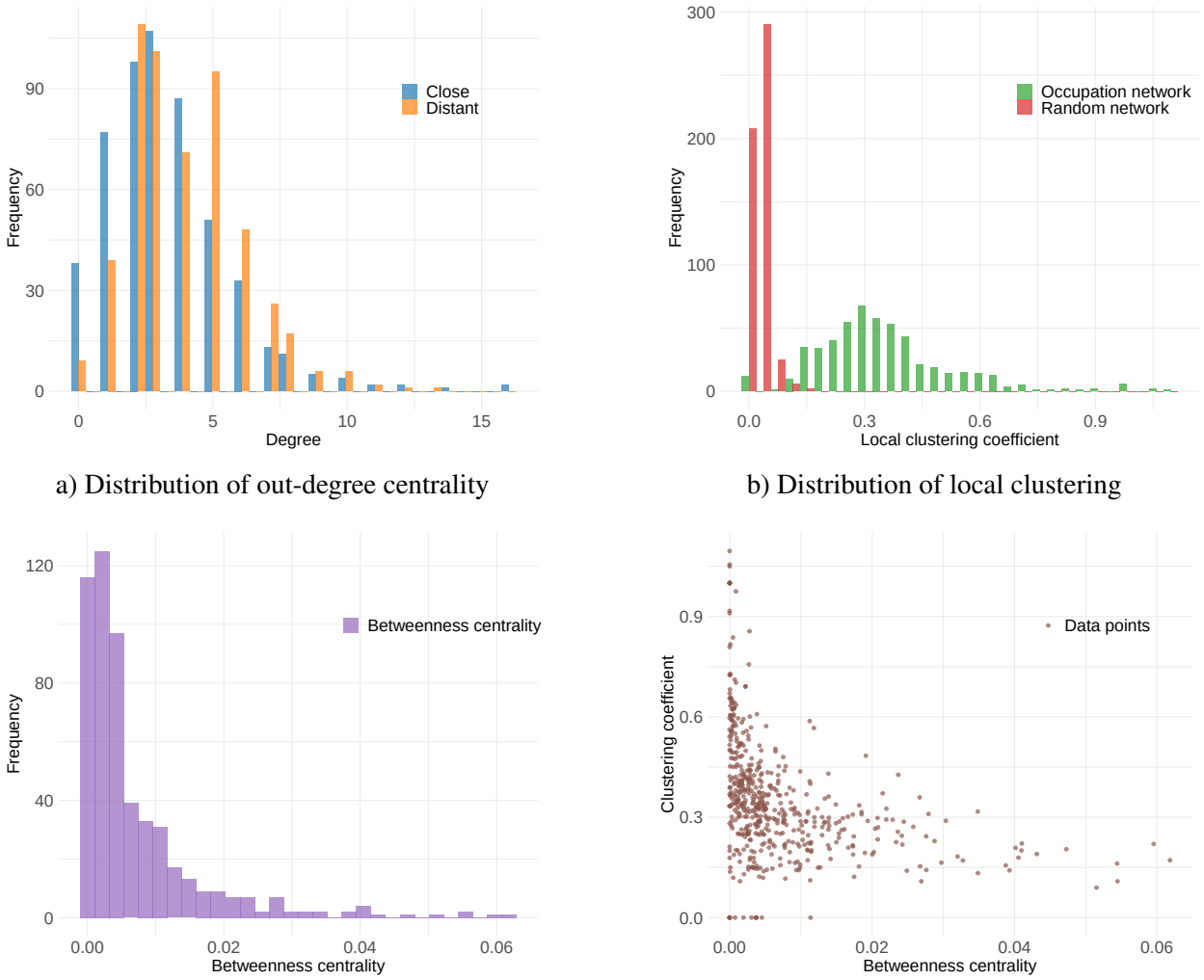
Figure 2.2.a displays the distribution of out-degree centrality indices across occupations. On average, occupations are connected to 7 other occupations in the skill network (out of 500), with a standard deviation of approximately 3. This indicates that occupations have a limited number of reallocation opportunities, suggesting large frictions to occupational mobility.

**Fact 2.** *The occupation network exhibits a community structure*

A network exhibits a community structure when nodes are grouped in tightly-connected clusters that are sparsely connected to each other (Newman (2006)). This is captured by the local clustering coefficient, which measures the share of a node's neighbors that are also connected to each other.

Figure 2.2.b compares the distribution of local clustering coefficients in the occupation network to a random network with similar density. The occupation network exhibits higher clustering than the random network, indicating that occupations tend to be grouped together. Intuitively, occupations

Figure 2.1: **Distribution of network centrality measures in the occupation network**




a) Distribution of out-degree centrality

b) Distribution of local clustering

c) Distribution of betweenness centrality

d) Betweenness vs local clustering

within the same 1-digit family often share similar skills, and hence are tightly-connected. [10]

**Fact 3.** *The occupation network features bridge occupations*

In network analysis, bridges are crucial nodes that act as hubs connecting distinct clusters. Quantitatively, bridges are identified by high betweenness, which measures how often a node lies "in-between" others.[11] I also impose that bridges have low local clustering to ensure they link different clusters rather than just connecting nodes within the same cluster.[12]

[10] In the appendix, I use community detection algorithms to directly identify clusters within the occupation network. The clusters exhibit a high degree of modularity, a standard measure of community structure in network.

[11] Betweenness centrality is computed as the share of shortest paths in the network that pass through a node.

[12] More precisely, bridges are defined as nodes in the top decile of betweenness centrality and the bottom quintile of local clustering coefficient.

Figure 2.2.c plots the distribution of betweenness centrality indices. While most occupations have low betweenness centrality, a small subset has significantly higher values. These bridge occupations act as "hubs" through which a large proportion of reallocation paths between clusters must pass. Examples include logistics technicians, secretaries and business consultants, occupations which typically require a mix of different skill sets. Appendix A provides a list of bridges and map their location in the occupation network.

Intuitively, bridge occupations play a crucial role for reallocation dynamics. If workers find it difficult to transit through bridges, mobility between clusters is severely restricted, impacting reallocation speed. I formalize this intuition in the theoretical model in Section 4.

# 3 Empirical Evidence on Worker Flows

This subsection studies worker trajectories within the occupation network. First, I present the data and the main variables. Then, I study how the occupation network's structure shapes worker mobility. Finally, I show that bridge occupations facilitate worker reallocation.

## 3.1 Data

To measure worker transitions across occupations and employment states, I use the DADS panel, a French employer-employee dataset collected by the Social Security agency. The dataset covers about one-twelfth of the French labor force (all workers born in October) from 2002 to 2010, with data drawn from job contracts. Unique individual identifiers allow tracking individuals across jobs. The dataset includes employment spell dates, occupation codes, wages, hours worked, and demographics such as gender, age, and location.

I restrict the sample to male workers aged 25-55 who held at least two job contracts during the period, representing a group with high labor force attachment. The analysis is further limited to main job contracts in mainland France and the private sector. Unemployment spells are defined as periods of more than 30 days between consecutive employment spells, with occupations based on the last job. I focus on the 2009-2019 period, when firms were required to report 4-digit occupation codes, resulting in a panel of approximately 550,000 workers over ten years. More details are provided in Appendix B.

The main variables of interest are the worker transition rates $\mu_{ijt}^{xy}(t)$, which measure the number of workers in occupation $i$ and employment state $x$ at quarter $t$ who transition to occupation $j$ and employment state $y$ in the next quarter $t+1$.[13]

## 3.2 Gravity Regression

Do workers really transition along the edges of the occupation network built with ROME data? If experts misidentify skill frictions, the previously constructed occupation network becomes irrelevant for understanding the constraints on worker reallocation.

**Specification.** To assess how much worker transitions are constrained by the occupation network, I draw on the analogy with the gravity model in trade. Just as trade economists analyze how geographical distance influences trade flows between countries, I substitute trade flows with worker flows and geographical distance with skill distance.

My baseline specification uses a two-way fixed effects Poisson regression model:[14]

$$\underbrace{\mu_{ijt}^{ue}}_{\substack{\text{transitions} \\ u_i \to e_j}} = \exp\Big\{ \underbrace{\gamma_{it} + \zeta_{jt}}_{\substack{\text{time-varying} \\ \text{origin/destination FE}}} + \underbrace{\beta^{\text{distant}} 1(\text{distant})_{ij} + \beta^{\text{unlinked}} 1(\text{unlinked})_{ij}}_{\text{type of network connection}} \Big\} \eta_{ijt}$$

where $\mu_{ijt}^{x}$ represents the number of transitions to occupation $j$ from unemployed workers in occupation $i \neq i$, $\gamma_{it}$ and $\zeta_{jt}$ are time-varying origin and destination occupation fixed effects, $1(\text{distant})_{ij}$ and $1(\text{unlinked})_{ij}$ are dummy variables for each type of network connection, and $\eta_{ijt}$ is a multiplicative error term.[15] The coefficients of interest, $\beta^{\text{distant}}$ and $\beta^{\text{unlinked}}$, capture the change in the likelihood of switching occupations when a pair is classified as distant or unlinked by experts, relative to close occupations (the omitted control group).

Including time-varying origin and destination fixed effects allows me to control for occupation-specific shocks at both the origin and destination. Without these controls, low transitions might be

[13] For estimation, I also construct occupation-specific wages $w_{it}$ as the average full-time equivalent wage in occupation $i$ at quarter $t$, residualized on age. Unemployment benefits $b_i(t)$ are calculated as $bw_i(t)$, where $b$ is the wage replacement rate, calibrated later. This is a simplification, as actual benefits depend on the last job's wage and duration. The model can incorporate occupation-specific replacement rates, but using a single parameter is more tractable.

[14] The Pseudo-Poisson regression is appropriate for non-negative dependent variables. Excluding the 90% of pairs with no transitions would bias the estimation (see Santos Silva and Tenreyro (2006)).

[15] Transitions not listed as possible by experts are classified as "unlinked."

mistakenly attributed to low skill frictions rather than negative shocks.

**Regression results.** Table 3.1 presents the estimated coefficients. The coefficients are all negative, statistically significant at the 1% level, and economically large. Workers switching occupations are about 95% less likely to transition to an 'unlinked' occupation than to a 'close' one, all else equal.[16]

Table 3.1: **Worker transition probability vs. expert skill frictions**

| | Estimated Coefficients | Marginal Effect |
|---|---|---|
| Distant | -0.87*** | -0.58 |
| | (0.09) | |
| Unconnected | -2.95*** | -0.95 |
| | (0.10) | |
| Observations | 812,912 | |

Notes: Each observation is a quarterly occupation-destination cell. The dependent variable is the number of occupation transitions during a quarter. The variables of interest are dummies capturing whether the transitions are labeled as close, distant, or unconnected by experts. Standard errors are clustered at the origin. * p<0.10, ** p<0.05, *** p<0.01.

This confirms the importance of the occupation network in shaping job transitions. The next subsection examines which network features shape long-term worker reallocation dynamics.

## 3.3 Bridge Regression

I show that worker reallocation is facilitated by transitioning through bridge occupations. Due to the sparsity of the occupation network, workers often cannot move *directly* into occupations outside their skill cluster. However, they can *indirectly* access these occupations by first transitioning through bridge occupations, which connect different clusters. This helps displaced workers acquire new skills, exit declining clusters, and enter expanding ones.

**Specification.** I study how transiting through a bridge occupation affects the long-term outcomes of workers, estimating the following model:

$$\Delta y_{id} = \alpha_{o(id)} + \beta 1(\text{transited through bridge})_{id} + \gamma \boldsymbol{Z}_{id} + u_{id} \tag{1}$$

[16] Marginal effects are calculated using the formula $e^{\beta} - 1$.

where $id$ represents worker identifiers, $o(id)$ is the worker's occupation at the start of the period, $\Delta y_{id}$ is the long-run change in worker outcomes, and $1(\text{transited through bridge})_{id}$ is a dummy variable equal to one if the worker transitioned through a bridge occupation over the period. $\boldsymbol{Z}_{id}$ includes worker-level controls, such as age, and $u_{id}$ is the error term. The coefficient of interest, $\beta$, measures the change in outcomes for workers who transited through bridge occupations relative to those who did not.

Including controls and origin-occupation fixed effects is key for identification. Without these, an endogeneity bias could occur, as transitioning through a bridge may correlate with the other worker characteristics, such as initial occupation or age. As a robustness check, I run an alternative regression limiting the control group to high-mobility workers who made at least two transitions during the period.

**Regression results.** Table 3.2 reports the estimated coefficients for the period 2013-2019.[17] Each column of the regression table corresponds to a different outcome variable: likelihood of changing 1- or 2-digit occupation groups, distance traveled in the occupation network, wage growth, and change in unemployment risk. All coefficients are statistically significant at the 1% percentage threshold and economically large. There are four main take-aways.

First, workers who transit through bridges are about 30 percentage points more likely to switch 1- or 2-digit occupation groups, consistent with the fact that bridge occupations connect clusters of 1- or 2-digit occupation groups.

Second, workers who transit through bridges travel farther in the occupation network, about 0.7 standard deviations of the distance distribution. This suggests that bridge occupations are not just 'rest employment' jobs where displaced workers temporarily stay before returning to their previous occupation. Instead, these workers continue progressing in their career path without reverting to their prior jobs.

Third, workers who transit through bridges experience significantly higher wage growth. Specifically, these workers saw an 8 percentage point cumulative increase in wages, compared to just 3.5% average wage growth over the same period. This indicates that workers who use bridge occupations

[17]The sample is split in two to minimize the number of observations lost due to the balanced panel requirement. Appendix B provides the corresponding estimates for 2008-2012 and for the alternative control group with high-mobility workers.

Table 3.2: **Change in worker outcomes vs. transition through bridge, 2013-2019**

| | Dependent Variables | | | | |
|---|---|---|---|---|---|
| | Change 1-digit Occupation | Change 2-digit Occupation | Distance (Standardized) | Wage Growth | Unemployment Rate Change |
| Through Bridge | 0.296*** | 0.316*** | 0.710*** | 0.083*** | -0.785*** |
| | (0.003) | (0.004) | (0.008) | (0.007) | (0.038) |
| Observations | 196,439 | 196,439 | 196,439 | 196,402 | 196,439 |
| Origin FE | Yes | Yes | Yes | Yes | Yes |

Notes: Each observation is a worker over the period 2013-2019. The dependent variables are the probability of changing 1-digit or 2-digit occupation, distance traveled within the network, wage growth, and change in unemployment risk. The variable of interest is a dummy indicating whether workers transited through a bridge during the period. * p<0.10, ** p<0.05, *** p<0.01.

move to better-paying jobs.

Fourth, workers who transit through bridge occupations end up in jobs with lower unemployment rates, about 0.75 percentage points lower. For comparison, the average unemployment rate in France during this period was around 10%. This suggests that bridge occupations help workers move to jobs with lower unemployment risk.

Taken together, these findings indicate that transiting through bridge occupations significantly benefits workers, enabling access to distant, higher-paying jobs with lower unemployment risks. However, these results are correlational, not causal, and the broader impact on other workers remains unobserved, absorbed in the intercept. To quantify the general equilibrium effects of bridge occupations on workers, a model is required.

# 4 A Tractable Model of Network Search

This section develops a tractable theory of job search with networked occupations. The goal is to provide a clear analytical characterization of worker reallocation dynamics, using new tools from distributional dynamics theory (Moll, 2014; Gabaix et al., 2016; Alvarez et al., 2016).

The model builds on the canonical random search and matching framework of Mortensen and Pissarides (1994). The key new assumption is that workers search for jobs not only within their

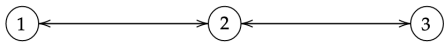
own occupation but also in adjacent occupations within the network, leading to mobility across occupations. Otherwise, the rest of the model remains purposefully very standard.

## 4.1 Setting

**Time.** Time is continuous, starts at zero, runs until infinity, and is discounted at the rate $r$.

**Occupational structure.** There are three occupations, indexed by $i \in \mathcal{N} = \{1, 2, 3\}$, connected by an unweighted and undirected occupation network. Edges indicate that workers can switch occupations.

Figure 4.1 : **A line occupation network with three occupations**




The occupation network is assumed to be a line network, as represented Figure 4.1. Occupation 2 is central in the network, while occupations 1 and 3 are at the periphery.[18] In the quantitative model, I lift this restriction and allow for arbitrary network structures.

**Workers.** There is a unit mass of infinitely-lived workers distributed across occupations. Workers can be either employed or unemployed. Unemployed workers in occupation $i$ are those whose *last job* was in occupation $i$.[19] Employed workers in occupation $i$ earn a wage $w_i(t)$, while unemployed workers receive benefits $b_i$.

The key innovation is that workers search for jobs both within their current occupation and in adjacent ones in the occupation network. For simplicity, only unemployed workers search for jobs, and their search is random. These assumptions are relaxed in the quantitative model in Section 5.

The distribution of workers across occupations and employment states, referred to as the worker distribution, is represented as:

$$\boldsymbol{f}(t) = ((e_i(t))_{i\in\mathcal{N}}, (u_i(t))_{i\in\mathcal{N}})' = (e_1(t), e_2(t), e_3(t), u_1(t), u_2(t), u_3(t))' \tag{2}$$

[18] Hereafter, I will use the terms central and bridge occupations interchangeably.

[19] I abstract from worker birth-death dynamics, as the entry and exit of generations occur on a much slower time scale than the one relevant for this model.

where $e_i(t)$ and $u_i(t)$ are the number of employed and unemployed workers in occupation $i$ at $t$.

**Firms.** There are infinitely many potential firms in each occupation, each incurring a flow cost $c$ to post a job vacancy. When a firm matches with a worker, the vacancy is filled, and the firm-worker pair produces homogeneous output with productivity $y_i$.

This specification abstracts from worker learning dynamics. In the model, reallocated workers instantly acquire the skills of their new occupation and produce at the same level as experienced workers. Relaxing this assumption would reduce tractability.[20] However, this assumption likely overestimates the speed of worker reallocation, leading to more frequent occupational switches than observed in reality.

The firm pays the worker a wage $w_i$ and collects profits of $y_i - w_i$. Wages are set as a fixed fraction of productivity, $w_i = \phi y_i$, where $\phi$ reflects worker bargaining power. Finally, firm-worker matches are subject to exogenous separation shocks at rate $s$.

**Search frictions.** Each occupation is a distinct labor market where search frictions prevent unemployed workers and firms from matching immediately. The number of hires in occupation $i$ at date $t$ is governed by a matching function:

$$h_i(t) = \zeta_i \quad \cdot \left( u_i(t) + \sum_{j \neq i} g_{ij} u_j(t) \right)^{\alpha} \cdot \quad v_i(t)^{1-\alpha_i}$$

where $\zeta_i$ represents the matching efficiency of labor market $i$, $v_i(t)$ is the number of vacancies posted by firms in occupation $i$ at time $t$, $u_i(t) + \sum_{j \neq i} g_{ij} u_j(t)$ is the number of unemployed workers searching *in* occupation $i$ from both occupation $i$ and neighboring occupations $j$, and $\alpha$ is the matching elasticity. The key novelty is that the pool of job seekers includes unemployed workers from neighboring occupations.[21]

The labor market tightness ratio in occupation $i$ at date $t$ writes

$$\theta_i(t) \equiv \frac{v_i(t)}{u_i(t) + \sum_{j \neq i} g_{ij} u_j(t)}$$

[20] If productivity depends on the worker's origin occupation, the model is no longer block-recursive, as the worker distribution would affect the expected value of a match for firms.

[21] This has important implications for the measurement of labor market tightness.

Firms and workers meet randomly, meaning each unemployed worker searching in occupation $i$ has an equal chance of meeting a firm in this occupation. The flow probability of matching with a firm in occupation $i$ at $t$ is given by: $p_i(t) = h_i(t)/\big(u_i(t) + \sum_{j\neq i} g_{ij}u_j(t)\big) = \zeta\theta_i(t)^{1-\alpha}$. Since workers simultaneously search for jobs in all adjacent occupations, their unemployment outflow rate from occupation $i$ is $p_i(t) + \sum_{j\neq i} g_{ij}p_j(t)$.[22] Finally, vacancies in occupation $i$ are filled with flow probability $q_i(t) = h_i(t)/v_i(t) = \zeta\theta_i(t)^{-\alpha}$.

**Productivity shock.** Consider the following scenario. At $t = 0$, the economy is at steady-state with the productivity distribution denoted $\tilde{\boldsymbol{y}}$ and the worker distribution by $\boldsymbol{f}(0) = \tilde{\boldsymbol{f}}_\infty$, where $\tilde{\boldsymbol{f}}_\infty$ represents the pre-shock steady-state worker distribution. Hereafter, $\tilde{x}$ denotes pre-shock variables.

At $t = 0^+$, the economy experiences an unanticipated permanent productivity shock, changing the productivity distribution from $\tilde{\boldsymbol{y}}$ to $\boldsymbol{y}$. The productivity change for each occupation is represented by the vector $\boldsymbol{dy} = (dy_i)_{i\in\mathcal{N}}$, where $dy_i$ is the productivity shock to occupation $i$.[23]

## 4.2 Values

How does firm vacancy posting react to the productivity shock? This is a function of their value functions, detailed below.

**Firm values.** The value of a filled job for firms in occupation $i$ at date $t$ is denoted $J_i(t)$, and the value of an unfilled vacancy in occupation $i$ at date $t$ is denoted $V_i(t)$.

The value of a job filled $J_i(t)$ solves the Bellman equation

$$rJ_i(t) = y_i - w_i + s(V_i(t) - J_i(t)) + \dot{J}_i(t) \tag{3}$$

The firm collects the flow profits $y_i - w_i$. At rate $s$, the firm-worker match gets separated and the firm suffers the loss $V_i(t) - J_i(t)$. The value can vary over time during the transition dynamics.

The value of a vacant job $V_i(t)$ solves the Bellman equation

$$rV_i(t) = -c_i + q(\theta_i)(J_i(t) - V_i(t)) + \dot{V}_i(t) \tag{4}$$

[22] Workers do not match with more than one firm simultaneously due to the continuous time and Poisson assumptions.

[23] I focus on productivity shocks as Cortes et al. (2020) show that most of the decline in routine employment was driven by reduced job finding rates, attributed to productivity shocks in the model, rather than separation shocks.

The firm pays a flow cost $c_i$ to post its vacancy. At rate $q_i$, the vacancy gets filled and the firm earns $J_i(t) - V_i(t)$.

**Free entry.** Potential entrants can freely post vacancies in each occupation, driving the value of posting a vacancy to zero at all times:

$$V_i(t) = 0 \quad \Longrightarrow \quad \frac{c}{q(\theta_i(t))} = J_i(t) \tag{5}$$

This condition determines the equilibrium tightness ratio in each occupation. Equilibrium tightness ratio equalizes the expected benefit of posting a vacancy – the value of a filled vacancy – and the expected cost of posting a vacancy – the flow cost over the expected waiting time.

Importantly, the model assumes the free entry condition holds *at all times*. This assumption is key for tractability but arguably quite restrictive, implying that adjustment in vacancy posting happen infinitely fast relative to other variables, which is counterfactual (see Elsby et al. (2015)).[24] However, allowing for slow vacancy adjustments would only result in slower responses to shocks.

**Transition dynamics.** In theory, the firm values and tightness ratios can vary during the transition dynamics. However, Proposition 1 shows they remain constant during the transition.

**Proposition 1.** (Job posting along the transition)
*Firm values $J_i(t)$, tightness ratios $\theta_i(t)$ and job finding rates $p_i(t)$ are constant along the transition dynamics, and given by*

$$J_i(t) = \frac{(1-\phi)y_i}{r+s} \qquad \theta_i(t) = \left(\frac{\zeta_i}{c_i}\left(\frac{(1-\phi)y_i}{r+s}\right)\right)^{\frac{1}{\alpha}} \qquad p_i(t) = \zeta^{\frac{1}{\alpha}}\left(\frac{(1-\phi)y_i}{c_i(r+s)}\right)^{\frac{1-\alpha}{\alpha}}$$

Therefore, firm values, tightness ratios, and job-finding rates are "jump variables," adjusting instantly to new steady-state levels after productivity shocks. This extends a result from canonical search and matching models to the current framework (see Pissarides (2000)).

This follows from the free entry assumption, which imposes that expected benefits and costs equalize at all times. When a productivity shock occurs, expected benefits from posting vacancies increase instantly. To maintain equilibrium, expected costs must rise simultaneously, achieved by

[24] A nascent literature studies departures from free-entry in search and matching models ( Den Haan et al. (2021)).

an immediate increase in the tightness ratio, which increases the expected filling time.

## 4.3 Laws of Motion

How do workers reallocate across occupations? First, I describe the laws of motion that govern the evolution of the worker distribution across occupations. Then, I introduce a measure for the aggregate speed of reallocation.

**Laws of motion.** The employment in occupation $i$ at date $t$ evolves as

$$\dot{e}_i(t) = p_i \left( u_i(t) + \sum_{j \neq i} g_{ij} u_j(t) \right) - s e_i(t) \tag{6}$$

Workers searching for jobs in occupation $i$ are hired at the rate $p_i$ from a pool of job seekers including unemployed workers from both their own and neighboring occupations $u_i(t) + \sum_{j \neq i} g_{ij} u_j(t)$. Workers in occupation $i$ get separated at rate $s$.

The unemployment in occupation $i$ at date $t$ evolves as

$$\dot{u}_i(t) = s e_i(t) \quad - \left( p_i + \sum_{j \neq i} g_{ij} p_j \right) u_i(t) \tag{7}$$

Newly separated workers from occupation $i$ enter unemployment at rate $s$, while unemployed workers find a job in their own and neighboring occupations at the rate $p_i + \sum_{j \neq i} g_{ij} p_j$.

**Transition matrix.** The laws of motion can be succinctly written as $\dot{\boldsymbol{f}}(t) = \boldsymbol{Q} \boldsymbol{f}(t)$, where $\boldsymbol{f}(t)$ is the worker distribution at date $t$ and $\boldsymbol{Q}(t)$ is the transition matrix.[25]

$$\boldsymbol{Q} \equiv \begin{pmatrix} -s & 0 & 0 & p_1 & p_1 & 0 \\ 0 & -s & 0 & p_2 & p_2 & p_2 \\ 0 & 0 & -s & 0 & p_3 & p_3 \\ s & 0 & 0 & -(p_1 + p_2) & 0 & 0 \\ 0 & s & 0 & 0 & -(p_1 + p_2 + p_3) & 0 \\ 0 & 0 & s & 0 & 0 & -(p_2 + p_3) \end{pmatrix} \tag{8}$$

[25] The matrix $Q(t)$ is the continuous time equivalent of the discrete time transition matrix.

The upper-left and lower-left blocks represent job destructions, while the upper-right and lower-right blocks represent job creations. Worker mobility across occupations is reflected by positive off-diagonal terms in the upper-right block, while the absence of on-the-job search is shown by the null off-diagonal terms in the upper-left block.

**Measure of transition speed.** Measuring the speed of worker reallocation is challenging due to the high dimensionality of distributions. However, the heterogeneous agents literature in macroeconomics has developed valuable metrics for quantifying transition times of distributions (Gabaix et al., 2016; Alvarez and Lippi, 2022; Baley and Blanco, 2021; Beraja and Wolf, 2021).

Building on this body of work, I introduce a new definition of aggregate reallocation time, defined as the normalized cumulative distance to the steady state.

**Definition 1.** (Aggregate worker reallocation time)

*The aggregate worker reallocation time $T$ is defined as*

$$T \equiv \frac{1}{\|\boldsymbol{f}(0) - \boldsymbol{f}_\infty\|_2} \int_0^{+\infty} \|\boldsymbol{f}(t) - \boldsymbol{f}_\infty\|_2 dt \tag{9}$$

The intuition is as follows. The integrand is the distance of the worker distribution to its steady-state. By integrating this distance over time, we obtain the cumulative distance to the steady-state distribution. A larger cumulative distance indicates that the distribution stayed far from the steady-state longer, indicating slower convergence. The normalization constant ensures the measure is not inflated merely by starting farther from the steady-state.[26][27]

As an illustration, consider the dynamics of unemployment in the canonical search and matching model. The unemployment rate evolves as $\dot{u}(t) = s(1 - u(t)) - pu(t)$. The general solution is $u(t) = u_\infty + (u(0) - u_\infty)e^{-(p+s)t}$, implying a transition time of

$$T = \int_0^{+\infty} e^{-(p+s)t}\, dt = \frac{1}{p+s} \sim \frac{1}{p}$$

[26] Unlike the half-life, this metric allows for closed-form solutions in higher dimensions. Additionally, the half-life can be biased, as it focuses only on the first half of the transition dynamics, ignoring slower transitions that occur later.

[27] An alternative measure of transition time is the expected duration, following the approach in Taschereau-Dumouchel and Schaal (2025). It is defined as: $T \equiv \int_0^{+\infty} t \frac{\|\boldsymbol{f}(t) - \boldsymbol{f}_\infty\|_2}{\int_0^{+\infty} \|\boldsymbol{f}(t) - \boldsymbol{f}_\infty\|_2 dt} dt$. The idea is to treat the distance to steady state over time as a probability density function. The expected duration then corresponds to its mean - intuitively, the average time at which the economy is "far" from the new steady state. One can show that this measure is closely related to the cumulative distance to steady state, which is a lower bound for the expected duration.

This is the so-called "characteristics time" in physics, corresponding to the time such that $\frac{u(T)-u_\infty}{u(0)-u_\infty} = e^{-(p+s)T} = e^{-1} \approx 63\%$ of the transition has occurred.[28]

## 4.4 Characterizing Reallocation Speed

What determines the speed of worker reallocation? Obtaining analytical characterization is challenging in the general case, but the simplicity of the tractable model allows me to derive clear closed-form expressions.

**Additionnal simplifying assumptions.** Assume equal productivity $y_P$ in periphery occupations and productivity $y_C$ in the central occupation. Let $p_C$ and $p_P$ denote job finding rate in central and non-central occupations. Also, assume the separation rate $s$ is small compared to job finding rates.

**Reallocation shocks.** I focus on a specific type of productivity shock, which I term *reallocation shocks*: these are productivity shocks where excess employment in some occupations is exactly offset by labor shortages in others. In other words, reallocation shocks do not affect aggregate employment, but only the distribution of employment across occupations. Trade and technology shocks, which unevenly affect different occupations, typically have a strong reallocation component.[29]

Two types of reallocation shocks are examined: periphery and central shocks, defined as

$$\mathbf{dy}_P = (-\Delta, 0, \Delta) \quad \text{and} \quad \boldsymbol{dy}^C = \left(\frac{1}{2}\omega\Delta, -\Delta, \frac{1}{2}\omega\Delta\right)$$

with $\omega \equiv \frac{2p_C+p_P}{2p_C+2p_P}$ accounts for the employment shares and $\Delta > 0$ controls the shock magnitude. The periphery shock affects the periphery occupations in opposite directions: there is excess employment in one periphery occupation and a labor shortage in the other. In contrast, the central shock negatively impacts the central occupation but benefits periphery occupations: excess employment in the central occupation is exactly twice the labor shortage in each periphery occupation.[30]

In Appendix C, I show any reallocation shock can be decomposed as a linear combination of a

[28] In one-dimension, we have $T = \log 2 \cdot t_{1/2}$, with $t_{1/2}$ the half-life, but this relationship is lost in higher dimensions.

[29] Any productivity shock can be decomposed into an aggregate employment shock and a reallocation shock. Trade and technology shocks tend to heavily load on the reallocation component.

[30] These productivity shocks correspond to the eigen-shocks of the economy, as defined by Kleinman et al. (2023). Following eigen-shocks, transition speed is directly determined by the associated eigenvalue.

periphery and central shock:

$$\boldsymbol{dy} = \gamma^C \boldsymbol{dy}^C + \gamma^P \boldsymbol{dy}^P$$

where $\gamma^C$ and $\gamma^P$ are the loadings on periphery and central shock xin the decomposition. Equipped with this, I can now state my main analytical result.

**Proposition 2.** (Reallocation time after any reallocation shock)
*For small shocks, transition time after any reallocation shock is approximately*

$$T \approx \alpha_P \cdot \left(1 + \frac{p_P}{p_C}\right) s^{-1} + (1 - \alpha_P) \cdot s^{-1}$$

*where the weight on transition time after periphery shock $\alpha_P$ is equal to*

$$\alpha_P \equiv \frac{\mid (p_P + p_C)\, \gamma_P \mid}{\mid (p_P + p_C)\, \gamma_P \mid + \mid \frac{(2p_P + p_C)^2}{p_P + p_C} \gamma_C \mid}$$

*Proof:* From eigen-decomposition of the transition matrix $\mathbf{Q}$. See Appendix C for details.

The transition time after reallocation shocks is a weighted average of the transition times after periphery $\left(1 + \frac{p_P}{p_C}\right) s^{-1}$ and central shocks $s^{-1}$. The weights depend on the relative importance of productivity shocks to periphery versus central occupations, as captured by $\gamma^P$ and $\gamma^C$.

There are three main takeaways. First, reallocation is very slow after reallocation shocks. Transition time is on the order of $s^{-1}$, with a typical separation rate of $2.5\%$ per quarter, leading to reallocation times of around 40 quarters. This happens because workers must wait to become unemployed before switching occupations. Allowing for on-the-job search, as in the quantitative model, has little effect, as job-to-job transitions across occupations are rare.

Notably, this transition time is much longer than in the canonical search and matching model, which predicts times of around $p^{-1}$, or 4 quarters in France (see Subsection 4.3). This suggests that labor market heterogeneity helps explain the persistence of shocks.

Second, the centrality of productivity shocks affects reallocation speed. Transition times are longer after shocks to peripheral occupations than to central ones, as $\left(1 + \frac{p_P}{p_C}\right) s^{-1} > s^{-1}$. This is because peripheral occupations offer fewer opportunities for workers to transition, leading to slower reallocation.

Third, bridge occupations matter for worker reallocation speed. The central occupation's job finding rate has a *granular* effect on reallocation time: if $p_C$ approaches zero, the transition time becomes arbitrarily large, while it remains finite if $p_P$, the periphery job finding rate, approaches zero. Intuitively, bridge occupations can act as bottlenecks, especially when workers need to transit between periphery occupations.

## 4.5 Characterizing Welfare Effects

I have shown that reallocation is slow, but it is not immediately obvious why this should matter for workers and firms. This subsection examines the welfare effects of slow reallocation.

**Defining welfare.** Let social welfare over the transition be defined as the present discounted value of aggregate output net of vacancy costs.

$$SW^{\text{slow}} \equiv \int_0^{+\infty} e^{-rt} \sum_{i \in \mathcal{N}} \left[ e_i(t) y_i + (b_i - c_i \theta_i) u_i(t) \right] dt$$

The term $e_i(t)y_i + (b_i - c_i\theta_i)u_i(t)$ is the sum of worker and firm payoffs net of hiring costs in occupation $i$ at date $t$.[31] Summing over occupations yields the *aggregate* flow welfare at date $t$, and multiplying by the discount factor and integrating over time provides the present value of the economy's welfare over the transition. The 'slow' superscript denotes the fact that the worker distribution evolves slowly over time.

By contrast, the social welfare if the economy remains at the old steady-state writes $SW^{\text{old}} \equiv \frac{1}{r} \sum_{i \in \mathcal{N}} \left[ \widetilde{e_{i\infty}} \widetilde{y_i} + (\widetilde{b_i} - c_i \widetilde{\theta_i}) \widetilde{u_{i\infty}} \right]$, where variables denoted by $\widetilde{x}$ represent their old steady-state level. Moreover, the social welfare if the economy were to immediately jump to the next steady-state writes $SW^{\text{new}} \equiv \frac{1}{r} \sum_{i \in \mathcal{N}} \left[ e_{i\infty} y_i + (b_i - c_i \theta_i) u_{i\infty} \right]$.

**Adjustment costs.** To measure the welfare loss from slow reallocation, I follow Caliendo et al. (2019) and define so-called *adjustment costs* $AC$ as follows:

$$AC \equiv \frac{SW^{\text{slow}} - SW^{\text{old}}}{SW^{\text{new}} - SW^{\text{old}}} - 1$$

[31] Note that wages cancel out since they are a transfer, and hiring costs can be written $c_i v_i(t) = c_i \theta_i u_i(t)$.

Adjustment costs compare the welfare change if reallocation is slow (numerator) to a counterfactual scenario with instantaneous reallocation (denominator).[32] For example, if the welfare change under slow reallocation is half that of instantaneous reallocation, $AC = -50\%$. With these definitions, I now present an important result regarding the magnitude of adjustment costs.

**Proposition 3.** (Adjustment cost after any reallocation shock)

*For small shocks, and $p_P \sim p_C$, the adjustment cost after any reallocation shock is approximately*

$$AC \approx \Lambda^P \underbrace{\frac{T^P}{r^{-1}+T^P}}_{\text{speed}} \cdot \underbrace{-1}_{\text{misallocation}} + (1-\Lambda^P) \underbrace{\frac{T^C}{r^{-1}+T^C}}_{\text{speed}} \cdot \underbrace{-\frac{p_C + p_P + p_P^2/(p_C+p_P)}{2p_C + 3p_P + p_P^2/(p_C+p_P)}}_{\text{misallocation}} \tag{10}$$

*where the reallocation times after periphery and central shock are*

$$T^P \equiv \frac{p_P + p_C}{s p_C} \quad \textit{and} \quad T^C \equiv \frac{1}{s}$$

*and where the weight on periphery adjustment costs $\Lambda^P$ writes*

$$\Lambda^P \equiv \frac{\gamma^P 2(2p_C + p_C)}{\gamma^P 2(2p_C + p_C) + \gamma^C (2p_C + 3p_P + {p_P}^2/(p_C + p_P)}$$

Adjustment costs after any reallocation shock is a weighted average of the adjustment costs following central and periphery shock.The weights depend on the relative importance of periphery and central shocks, $\gamma^P$ and $\gamma^C$, in the shock decomposition.

The adjustment costs after periphery and central shocks can be further decomposed into two components: speed and misallocation.The speed term reflects the welfare loss from delaying reallocation, which reduces future gains in present value. The misallocation term captures the loss driven by the misalignement between labor shortages and productivity changes during the transition.

There are three key insights. First, there is a direct relationship between reallocation time and adjustment costs after periphery shocks: the longer the reallocation time, the more delayed the welfare gains, and the lower their present value.

Second, the centrality of shocks matters: adjustment costs are higher after shocks to peripheral

[32] Caliendo et al. (2019) define adjustment costs as $AC = \log \frac{SW^{\text{slow}} - SW^{\text{old}}}{SW^{\text{new}} - SW^{\text{old}}}$. Both definitions are approximately equivalent when $\frac{SW^{\text{slow}} - SW^{\text{old}}}{SW^{\text{new}} - SW^{\text{old}}} \approx 1$.

occupations than central ones, since reallocation is slower after periphery shocks ($T^P > T^C$) and misallocation is more severe ($|\frac{p_C+p_P+p_P^2/(p_C+p_P)}{2p_C+3p_P+p_P^2/(p_C+p_P)}| < 1$).

Finally, the magnitude of adjustment costs can become particularly large after periphery shocks. Misallocation is especially severe when labor shortages and productivity changes are perfectly negatively correlated. Even with moderate reallocation times, such as $T = 40Q$, and a discount rate of $r = 0.03$ as in the quantitative model, adjustment costs remain significant, around $AC^P \approx -50\%$.

# 5 A Quantitative Model of Network Search

While the theoretical model provides clear analytical insights, it abstracts from several key complexities of real-world labor markets. To account for these, I now extend the model to a quantitative framework, incorporating endogenous search effort and a more realistic network structure.

## 5.1 Extensions

**Occupation structure.** There are $N$ distinct occupations, indexed by $i \in \mathcal{N}$, connected by the edges of the occupation network. The network structure is captured by the adjacency matrix $\boldsymbol{G} = (g_{ij})_{(i,j)\in\mathcal{N}^2}$, where $g_{ij} \in (0, 1)$ represents the probability that workers in occupation $i$ can learn the skills of occupation $j$. The matrix $\boldsymbol{G}$ is asymmetric ($g_{ij} \neq g_{ji}$), with each occupation perfectly accessible to itself ($g_{ii} = 1$ for all $i$).[33]

**Workers.** Employed workers can search *on-the-job*, but have less time for search compared to unemployed workers ($\eta_i < 1$). As a result, firm-worker matches can end due to exogenous separation shocks or because the worker found a job in another occupation.

Workers *choose their search effort*, allowing them to partially direct their search toward certain occupations. Unemployed workers are endowed 1 unit of time, while employed workers have $\eta_i$. The search time in occupation $j$ by workers in $i$ and state $x \in \{e, u\}$ is denoted $\tau_{ij}^x$. Search time translates into effective search effort, $\sigma_{ij}^x$, according to:

$$\sigma_{ij}^x = {\tau_{ij}^x}^{\frac{\psi}{\psi+1}}$$

[33]The occupation network is strongly connected, ensuring a positive worker population in all occupations over time.

where $\psi \geq 0$ measures the degree of concavity of the search effort technology. The assumption of decreasing returns to search captures the idea that workers initially contact a few salient firms, but subsequent contacts require more time, resulting in fewer applications per hour invested.

**Wage determination.** Workers and firms negotiate wages using a generalized Nash bargaining rule, where the worker bargaining strength $\phi_i$ determines the surplus split.

Wages are continuously renegotiated during the transition. The model abstracts from wage-setting frictions, such as downward wage rigidity or infrequent negotiations. However, adding these frictions would slow down reallocation dynamics even further.

**Search frictions.** The details of the search frictions are provided in Appendix D. Importantly, the transition rate of workers in occupation $i$ and state $x$ to employment in occupation $j$ date $t$ writes

$$\mu_{ij}^{x}(t) \equiv g_{ij}\sigma_{ij}^{x}(t)p_j(t)$$

where $\sigma_{ij}^{x}(t)$ is the search effort, and $p_j(t)$ the job finding rate per unit of search effort.

The transition rate is the product of three terms: (1) the skill friction $g_{ij}$ reflects whether the transition is *feasible*, (2) the search effort $\sigma_{ij}^{x}$ reflects whether the transition is *desirable* for workers, while (3) the job finding rate per unit of search $p_j(t)$ reflects whether the transition is *desirable* for firms (i.e., their hiring rate).

## 5.2 Values

There are three main margins of adjustment after shocks: workers' allocation of search efforts, bargained wages, and job postings by firms. These decisions are made optimally and depend on agents' value functions, which I now define.

**Workers values.** The value of being employed in occupation $i$ at date $t$ is denoted $E_i(t)$, and the value of being unemployed in occupation $i$ at date $t$ is denoted $U_i(t)$. The value of being

unemployed from occupation $i$ at date $t$ $U_i(t)$ solves

$$rU_i(t) = \max_{\{\tau_{ij}^u(t)\}_j \geq 0} b_i + \sum_{j \in \mathcal{N}} \mu_{ij}^u(t)(E_j(t) - U_i(t)) + \dot{U}_i(t) \tag{11}$$
$$\text{s.t.} \quad \mu_{ij}^u(t) = g_{ij} p_j(t) \tau_{ij}^u(t)^{\frac{\psi}{\psi+1}} \text{ and } \sum_j \tau_{ij}^u(t) = 1$$

Unemployed workers from occupation $i$ receive flow unemployment benefits $b_i$ and allocate their search time $\tau_{ij}^u$ to maximize the expected value of being hired in their own or adjacent occupations. They match with firms in $j$ at the rate $\mu_{ij}^u$, in which case they earn $E_i(t) - U_i(t)$, hereafter the switching gain. The value function can vary over time during the transition dynamics.

The value of being employed in occupation $i$ at date $t$ $E_i(t)$ solves the Bellman equation

$$rE_i(t) = \max_{\{\tau_{ij}^e(t)\}_j \geq 0} w_i(t) + \sum_{j \in \mathcal{N}} \mu_{ij}^e(t)(E_j(t) - E_i(t)) + s_i(U_i(t) - E_i(t)) + \dot{E}_i(t) \tag{12}$$
$$\text{s.t.} \quad \mu_{ij}^e(t) = g_{ij} p_j(t) \tau_{ij}^e(t)^{\frac{\psi}{\psi+1}} \text{ and } \sum_j \tau_{ij}^e(t) = \eta_i$$

Employed workers earn a flow wage $w_i(t)$ and allocate their search time to maximize the expected value of being hired, but have less time to search than unemployed workers ($\eta_i < 1$). They match with firms in $j$ at rate $\mu_{ij}^e(t)$ and gain $E_j(t) - E_i(t)$. At the exogenous rate $s_i$, the firm-worker match is separated, and the worker becomes unemployed. The value function can vary over time during the transition dynamics.

**Optimal search times.** The optimal search times, derived from the first-order conditions of the Bellman equation, are

$$\tau_{ij}^u(t) = \frac{\left(g_{ij} p_j(t) \max\{E_j(t) - U_i(t), 0\}\right)^{\psi+1}}{\sum_k \left(g_{ik} p_k(t) \max\{E_k(t) - U_i(t), 0\}\right)^{\psi+1}} \quad \tau_{ij}^e(t) = \frac{\left(g_{ij} p_j(t) \max\{E_j(t) - E_i(t), 0\}\right)^{\psi+1}}{\sum_k \left(g_{ik} p_k(t) \max\{E_k(t) - E_i(t), 0\}\right)^{\psi+1}} \tag{13}$$

Workers exert higher search time to occupations with (1) high skill similarity $g_{ij}$, (2) high job finding rate $p_j(t)$ or (3) high switching gains, $E_j(t) - U_i(t)$ or $E_j(t) - E_i(t)$. Conversely, they do not exert effort in unconnected occupations, or where switching would result in a loss.

The parameter $\psi$ represents the elasticity of search effort to expected gains from switching

occupations. This formulation encompasses random and directed search as special cases: when $\psi$ approaches zero, workers search randomly across occupations, whereas when $\psi$ approaches infinity, workers fully direct their search effort towards the highest-return occupations. This parameter governs how workers adjust their search in response to economic shocks.

This specification mirrors choice probabilities in discrete choice models. However, there are two main differences. First, effective search efforts $\sigma_{ij}^x$ do *not* sum to one, allowing workers in high-return occupations to search more. Second, conditional on transition probabilities, the Bellman equation is linear in payoffs, significantly simplifying the estimation of values. Further discussion on this comparison can be found in Appendix D.

**Firm values.** The Bellman expressions for firm values are similar to those in the stylized model and are therefore relegated to Appendix D. The main difference is that the value of a filled job now accounts for the possibility of endogenous job separation due to worker on-the-job search.

**Nash bargaining.** When a firm and a worker meet, they bargain over the wage. At any date $t$, the negotiated wage in occupation $i$, denoted $w_i(t)$, maximizes the generalized Nash product

$$\max_{w_i(t)} \left(E_i(t) - U_i(t)\right)^{\phi_i} \left(J_i(t) - V_i(t)\right)^{1-\phi_i}$$

Importantly, the worker's outside option is the value of being unemployed from occupation $i$ onwards, regardless of previous occupations or employment status. This reflects that wages are negotiated only after the worker has transitioned to a new occupation, consistent with the idea that only *ex post* negotiations are credible in a world of incomplete contracts. This assumption ensures analytical tractability by making all workers applying in occupation $i$ have the same outside options and receive the same wage.[34]

The negotiated wage can be expressed as

$$w_i(t) = \phi_i y_i + (1-\phi_i)\bar{w}_i \quad \text{with } \bar{w}_i \equiv \underbrace{b_i + \sum_j \mu_{ij}^u(t)\left(E_j(t) - U_i(t)\right)}_{\text{outside option}} - \underbrace{\sum_j \mu_{ij}^e(t)\left(E_j(t) - U_i(t)\right)}_{\text{option value of on-the-job}} \tag{14}$$

[34] Otherwise, wages would depend on both the current and previous occupations, $w_{ij}$, leading firm and worker values to depend on both—breaking the block-recursive structure of the model.

The proof is detailed in Appendix D.

The wage $w_i(t)$ is a weighted average of the worker-firm productivity $y_i$ and the worker's reservation wage $\bar{w}_i(t)$, with the weight determined by worker's bargaining power $\phi_i$. In other words, Nash bargaining strikes a balance between the maximum wage acceptable to the firm (productivity $y_i$) and the minimum wage acceptable to the worker (reservation wage $\bar{w}_i$).

The worker's reservation wage $\bar{w}_i$ has two components: the value of outside options if negotiation fails, which raises the wage, and the option value of future on-the-job search, which leads workers to accept a lower current wage in anticipation of better future prospects. However, the bargaining channel always dominates, raising wages, since $\mu_{ij}^u(t) \geq \mu_{ij}^e(t)$.

A key new implication is that the reservation wage now depends on job prospects in adjacent occupations within the network, creating spillover effects across occupations. A wage increase in one occupation raises the reservation wages in neighboring occupations, enhancing their negotiated wages. In the appendix, I show that wages are proportional to occupations' Katz-Bonacich centrality in a certain network, implying that centrality increases bargaining power and wages.

**Free entry.** As in the stylized model, potential entrants can freely post vacancies in each occupation, driving the value of posting a vacancy to zero at all times: $V_i(t) = 0$.

An important implication is that tightness ratios are interconnected across occupations via the reservation wage channel, creating negative spillovers in vacancy postings. Increased vacancy postings in neighboring occupations \$j\$ improve workers' search prospects in occupation \$i\$, raising their reservation wages and potentially reducing firms' incentives to post vacancies in that occupation.

## 5.3 Law of Motion

The laws of motion of worker employment and unemployment closely resemble those in the stylized model and are therefore relegated to Appendix D.

Importantly, the worker distribution's law of motion still writes $\dot{\boldsymbol{f}}(t) = \boldsymbol{Q}(t)\boldsymbol{f}(t)$, but where the

quantitative transition matrix $\boldsymbol{Q}(t)$ has a slightly more complex expression:

$$\boldsymbol{Q}(t) = \begin{pmatrix} \underbrace{\text{diag}(\boldsymbol{p})(\boldsymbol{G} \odot \boldsymbol{\sigma}^{\boldsymbol{e}})'\text{diag} - \text{diag}(\boldsymbol{G} \odot \boldsymbol{\sigma}^{\boldsymbol{e}})\boldsymbol{p}) - \text{diag}(\boldsymbol{s})}_{\text{e} \to \text{e}} & \underbrace{\text{diag}(\boldsymbol{p})(\boldsymbol{G} \odot \boldsymbol{\sigma}^{\boldsymbol{u}})'}_{\text{u} \to \text{e}} \\ \underbrace{\text{diag}(\boldsymbol{s})}_{\text{e} \to \text{u}} & \underbrace{-\text{diag}\big((\boldsymbol{G} \odot \boldsymbol{\sigma}^{\boldsymbol{u}})\boldsymbol{p}\big)}_{\text{u} \to \text{u}} \end{pmatrix} \tag{15}$$

where $\odot$ denotes matrix element-wise multiplication.

First, the upper-left block now has non-zero diagonal elements, reflecting on-the-job search by employed workers. Second, the upper-right and lower-right blocks incorporate worker search efforts, accounting for the semi-directed nature of job search. The transition matrix contains all the necessary information to predict the evolution of the worker distribution.

## 5.4 Definition of the Equilibrium

**Definition of the equilibrium.** *The equilibrium is a collection of time-varying value functions* $\{E_i(t)\}_{i\in\mathcal{N}}$, $\{U_i(t)\}_{i\in\mathcal{N}}$, $\{J_i(t)\}_{i\in\mathcal{N}}$, $\{V_i(t)\}_{i\in\mathcal{N}}$, *time-varying search times* $\{\tau^e_{ij}(t)\}_{(i,j)\in\mathcal{N}^2}$, $\{\tau^u_{ij}(t)\}_{(i,j)\in\mathcal{N}^2}$, *time-varying wages* $\{w_i(t)\}_{i\in\mathcal{N}}$, *time-varying labor market tightness ratios* $\{\theta_i(t)\}_{i\in\mathcal{N}}$ *and a time-varying distribution of workers* $\boldsymbol{f}(t)$, *such that:*

*i) The value functions satisfy the worker and firm Bellman equations*

*ii) The search times satisfy the first order conditions of the worker Bellman equations*

*iii) The wages satisfy the Nash bargaining condition at all dates*

*iv) The tightness ratios are such that the free entry condition holds at all dates*

*v) The distribution of workers evolves according to the matrix law of motion*

**Block recursive equilibrium.** The equilibrium in this model is block recursive, as defined by Menzio and Shi (2010) and Menzio and Shi (2011). Block recursivity means that agents' value and policy functions do not depend on the worker distribution. In this model, it is straightforward to verify that the Bellman equations and first-order conditions governing agents' values and optimal actions do not depend on the worker distributions, thus satisfying block recursivity. Block recursivity is crucial for maintaining the tractability of models with heterogeneous agents.

This leads to my solution strategy. First, I solve for agents' optimal actions and values, determining the equilibrium transition rates. Then, I solve for the dynamics of worker reallocation, given the equilibrium matrix of transition rates.

**Stability, existence and unicity of equilibrium.** In theory, the agents' value and policy functions might vary during the transition. However, I prove in Appendix D that the value and policy functions must remain *constant throughout the transition*, as in the canonical search and matching model. Intuitively, this follows from the free entry assumption, as discussed in Subsection 4.2.

I also demonstrate that a steady-state equilibrium always exists. Proving uniqueness is more challenging, but I derive a sufficient condition for uniqueness that can be verified numerically.

## 5.5 Dynamic Hat-Algebra

Solving for counterfactuals in this quantitative model is challenging due to the number of variables and their complex interactions. Additionally, it involves estimating a prohibitively large number of parameters. To address this, I build on the 'hat algebra' method from trade, which significantly reduces the number of parameters to estimate (Jones (1965), Dekle et al. (2008), Caliendo et al. (2019)). This paper extends dynamic hat algebra to include search frictions. Throughout, $\hat{x} \equiv dx/x$ denotes log-changes.

**Proposition 4.** (Dynamic Hat-Algebra)

*Following a small productivity shock $\mathbf{dy}$, the approximate hat-changes in values and actions solve*

$$\hat{E}_i = \frac{1}{rE_i}\left(w_i\hat{w}_i + \sum_j \mu^e_{ij}\Delta^e_{ij}\left(\hat{\mu}^e_{ij} + \hat{\Delta}^e_{ij}\right)\right)$$

$$\hat{U}_i = \frac{1}{rU_i}\left(\sum_j \mu^u_{ij}\Delta_{ij}\left(\hat{\mu}^u_{ij} + \hat{\Delta}^u_{ij}\right)\right)$$

$$\hat{\mu}^u_{ij} = (\psi+1)(1-\alpha)\hat{\theta}_j + \psi\hat{\Delta}^u_{ij} - \psi\sum_j \tau^u_{ij}((1-\alpha)\hat{\theta}_k + \hat{\Delta}^u_{ik})$$

$$\hat{\mu}^e_{ij} = (\psi+1)(1-\alpha)\hat{\theta}_j + \psi\hat{\Delta}^e_{ij} - \psi\sum_j \tau^e_{ij}((1-\alpha)\hat{\theta}_k + \hat{\Delta}^e_{ik})$$

$$\hat{w}_i = \frac{1}{w_i}\left(\phi dy_i + (1-\phi)\sum_j \mu^u_{ij}\Delta^u_{ij}\left(\hat{\mu}^u_{ij} + \hat{\Delta}^u_{ij}\right) - (1-\phi)\sum_j \mu^e_{ij}\Delta^u_{ij}\left(\hat{\mu}^e_{ij} + \hat{\Delta}^u_{ij}\right)\right)$$

$$\hat{\theta}_i = \frac{1}{\alpha}\hat{\Delta}^u_{ii}$$

$$\hat{\Delta}^u_{ij} = \frac{E_j\hat{E}_j - U_i\hat{U}_i}{E_j - U_i}$$

$$\hat{\Delta}^e_{ij} = \frac{E_j\hat{E}_j - E_i\hat{E}_i}{E_j - E_i}$$

*where $\Delta^u_{ij} \equiv (E_j - U_i)$ and $\Delta^e_{ij} \equiv (E_j - E_i)$ denote switching gains.*

The first two equations describe how workers' values respond to changes in agents' actions. Specifically, they capture how values of being employed or unemployed in certain occupations respond to changes in wages and transition rates, following a technological shock. In contrast, the other four equations explain how agents' actions respond to changes in these values, captured by variations in switching gains $\Delta^x_{ij}$. Specifically, they describe how transition rates (proxying for search efforts), wages or tightness ratios adjust to variations in the value of switching occupations and productivity changes.

Several key elasticities govern the magnitude of equilibrium changes. The search elasticity $\psi$ controls how transition rates respond to changes in worker values, while the matching elasticity $\alpha$ determines how tightness ratios adjust to variations in worker values. Worker bargaining power $\phi$ influences how wages respond to both productivity and changes in worker values.

An important implication is that, given baseline transition rates $(\mu^u_{ij})i, j, (\mu^e_{ij})_{i,j}, (s_i)_i$, baseline payoffs $(w_i)_i, (b_i)_i$ (which are observables) and a vector of productivity shocks $(dy_i)$ (an input

in the model), only the discount rate $r$ and the three elasticities $\psi, \alpha, \phi$ are needed to compute counterfactuals. Indeed, there are no additional parameters in the system of equations above.[35]

# 6 Estimation

Few parameters are needed to solve counterfactuals. While most parameters are calibrated, the search elasticity – which determines how workers adjust search efforts in response to shocks – is estimated using a reduced-form regression with structural interpretation. This method leverages the gravity structure of worker transitions across occupations, enabling efficient and transparent estimation.

**Discount rate and replacement rate.** I begin by setting the discount rate $r$ at a quarterly value of $3\%$, corresponds to a time horizon of $8$ years as in Le Barbanchon et al. (2021). The replacement rate $b$ is calibrated at $40\%$.[36]

Given worker payoffs, transition rates and the discount rate, the worker value functions can be computed for each quarter $t$. The vector of worker values at quarter $t$, denoted $\boldsymbol{V}(t) = (E_1(t), \ldots, E_N(t), U_1(t), \ldots U_N(t))'$, is

$$\boldsymbol{V}(t) = (r\boldsymbol{I} - \boldsymbol{Q}'(t))^{-1} \begin{pmatrix} \boldsymbol{w}(t) \\ \boldsymbol{b}(t) \end{pmatrix}$$

The intuition is that worker values are the discounted sum of future income flows, accounting for transitions to other occupations and employment states. Assuming workers do not anticipate further shocks, expected future incomes and expected future transition probabilities are equal to today's, and therefore worker values can be computed.

From this, the switching gains for employed and unemployed workers can be computed as $\boldsymbol{\Delta}^e(t) = (E_j(t) - E_i(t))_{,ij\in\mathcal{N}^2}$ and $\boldsymbol{\Delta}^u(t) = (E_j(t) - U_i(t))_{i,j\in\mathcal{N}^2}$.

[35] Baseline search efforts can be recovered from values and transition rates using the formula: $\tau_{ij}^x = \left(\frac{\mu_{ij}^x \Delta_{ij}^x}{\sum_j \mu_{ij}^x \Delta_{ij}^x}\right)$.

[36] There is no consensus on the calibration of flow unemployment values. This value falls within the mid-range of values reported in the literature, spanning from 0.1 to 0.9 (e.g. Hagedorn and Manovskii (2008), Conlon et al. (2018)). Shimer (2005) sets $b$ at $40\%$ for the US based on monetary benefits (in France, the replacement rate was approximately $70\%$ of net wages at that time). However, this approach ignores non-monetary factors like leisure (utility) and social stigma (disutility). A low replacement rate increases job search incentives, speeding up reallocation, providing a lower bound on reallocation speed.

**Search elasticity.** To estimate search elasticity $\psi$, I estimate how worker transitions between occupations respond to changes in switching gains, controlling for confounding factors.[37]

The model predicts a gravity structure in worker flows:

$$\log \mu^u_{ijt} = (\psi + 1) \log g_{ij} + (\psi + 1) \log p_{jt} - \log N^u_{it} + \psi \log \Delta^u_{ijt}$$

where $\mu^u_{ijt}$ is the transition rate, $g_{ij}$ is the skill friction, $p_j(t)$ is the job finding rate, $N^u_{it}$ is a normalization constant, and $\Delta^x_{ijt}$ is the switching gain.[38]

Ideally, the search elasticity $\psi$ could be estimated directly by regressing transition rates on switching gains. However, skill frictions and job finding rates are not directly observable, leading to a missing variable bias as worker values are correlated with skill frictions and job finding rates. To address this, I control for these unobserved terms with fixed effects (FE): time-invariant origin-destination FE control for skill frictions, while time-varying destination FE control for job finding rates. However, the number of fixed effects quickly becomes too large, leading to non-robust estimates. Following Head and Mayer (2014), I first demean along the time dimension, before running a two-way fixed effect Poisson regression. Further details are provided in Appendix E.

I find a value of $0.09$, much lower than Caliendo et al. (2019)'s quarterly estimate of $0.2$. This means a $1\%$ increase in switching gains results in a $0.09\%$ increase in worker transitions, all else equal. This lower estimate might stem from national differences in willingness to reallocate.

**Matching elasticity and worker bargaining power.** I calibrate the matching elasticity and worker bargaining power $\alpha$ at $\phi = 0.5$, a standard calibration value in macro-labor models.[39]

[37] A key identification assumption is that search elasticity is constant across occupations. **?** relax this assumption in a model without search frictions, estimating heterogeneous elasticities and quantifying their importance for understanding labor market responses to shocks.

[38] In seminal papers, Artuç and McLaren (2015) and Cortes et al. (2020) also exploit the gravity structure of worker flows to estimate occupation switching costs. I extend their approach by incorporating search frictions.

[39] This corresponds to the Hosios condition, which ensures efficiency in search & matching models. Interestingly, the model predicts new sources of inefficiency as firms and workers do not internalize congestion effects on *neighboring* occupations. But the Hosios condition still holds approximately.

# 7 Reallocation Dynamics following the Introduction of Robots

This section uses the estimated quantitative model as a 'laboratory' to explore the effects of the introduction of robots on the French labor market. First, I calibrate the robot shock using off-the-shelf measures from the literature. Then, I study the labor market adjustments after the shock. Finally, I simulate policy interventions and assess their effectiveness in accelerating worker reallocation.

## 7.1 Calibration of the Robot Shock

The robot shock is modeled as a permanent unexpected shock to the vector of occupation productivity. It is decomposed into two components: an aggregate shock, representing economy-wide productivity gains, and a reallocation shock, reflecting the relative effects of technology across occupations.

**Calibration of productivity shocks.** To calibrate the reallocation part of the productivity shocks, I use Webb (2019) measure of occupation exposure to robots. I first apply Webb's estimated effect of robot exposure on wages to predict wage changes.[40] I then infer productivity changes to match the predicted wage response. Details are in Appendix E.

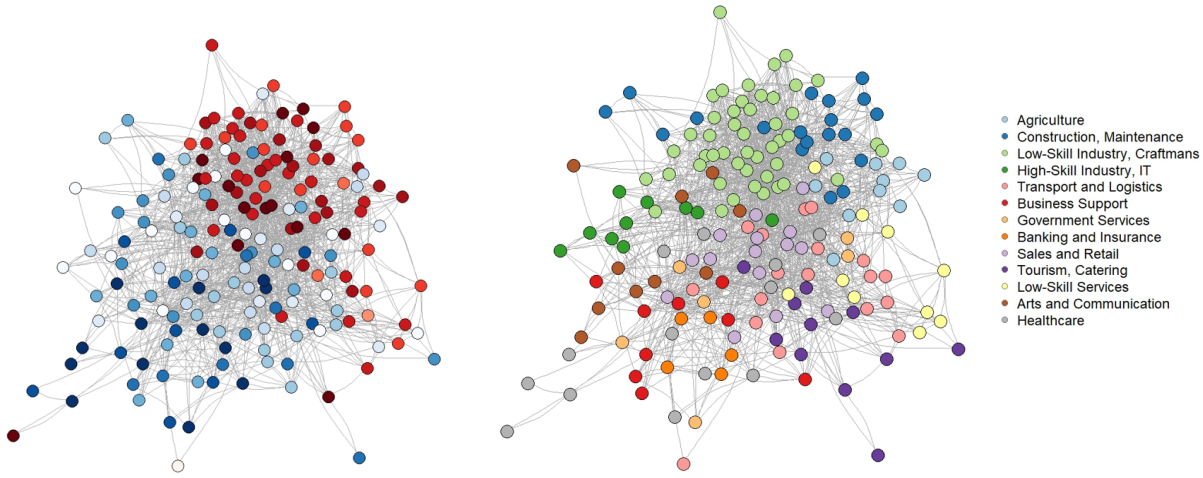
The aggregate part of the productivity shock is calibrated to match the 3.5% aggregate productivity gain reported by Acemoglu and Restrepo (2022) for robots and software in the US from 1980-2016.[41] Since I focus on robots and my data starts in 2009, I calibrate the aggregate productivity gains at approximately 1%.

**Most affected occupations.** Which occupations are most affected by robots? The "declining" occupations – negatively impacted by robots – are those where tasks involve moving objects, such as construction operators, rail operators and industrial unskilled workers. In contrast, the "expanding" occupations – positively affected by robots – involve socio-linguistic skills, including interpreters, telemarketers, journalists and social workers.

Figure 7.1 plots the location of declining and expanding occupations within the network of occupation after the robot shock. It shows that expanding and declining occupations are very clustered, forming two clearly divided groups of occupations with little overlap between them. This

[40] Ideally, I would estimate these effects directly, but the short time span of my data precludes such analysis.
[41] Recovering the GE effects of robot would require a more complex model, beyond the scope of this study.

Figure 7.1: **Expanding (blue) vs. declining (red) occupations in the occupation network**




a) Expanding vs. Declining b) Labels

Notes: The occupation network is aggregated at the level of 3-digit FAP occupations.

suggests that the robot shock strongly resembles the periphery shock from the toy model.

In-between these expanding and declining occupations are bridge occupations, which correspond to maintenance workers, sales representative and sport coach, occupations which typically involve a mix of manual and social skills.

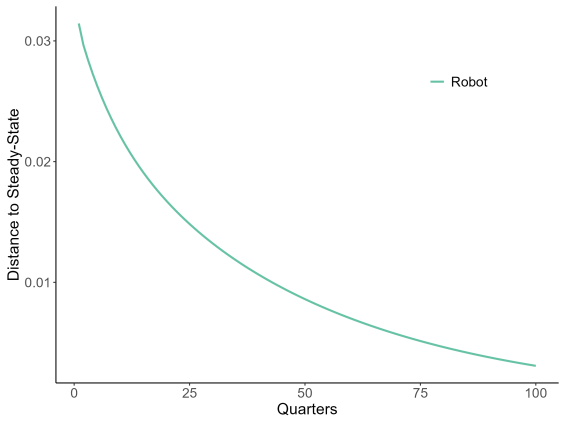
## 7.2 Transition Dynamics

I simulate the economy's transition dynamics after the robot shock. The baseline is calibrated at the time-average of the French economy over 2009-2012.

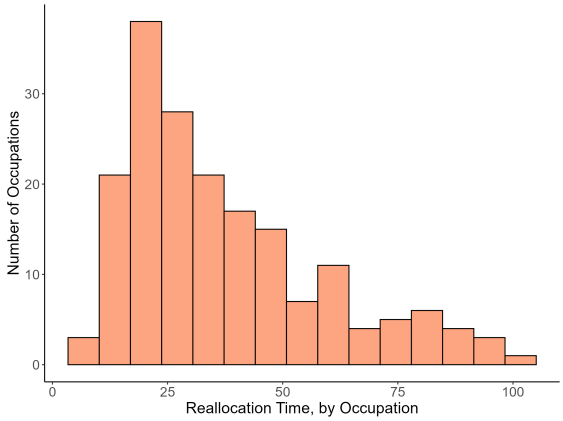
**Aggregate reallocation times.** I find that worker reallocation to the new steady-state distribution is slow, taking approximately 40 quarters.

Figure 7.2.a shows the dynamics of the worker distribution's distance to its steady state after the robot shock. The reallocation time is computed as the area under the curve, normalized by the initial value, yielding a value of around 39 quarters, or about 10 years.

Interestingly, this reallocation time aligns closely with predictions from the simple model, suggesting that the quantitative extensions balance each other. A larger occupation network may slow reallocation by requiring workers to move to more distant occupations, while richer search behaviors—such as on-the-job and semi-directed search—allow workers to switch occupations without waiting for unemployment, thus speeding up reallocation.




(a) Aggregate distance to steady-state




(b) Reallocation time, by occupation

Although the predicted reallocation time might seem short, it should be considered a lower bound. Simplifying assumptions—such as continuously renegotiated wages, free entry in vacancy creation, and immediate skill acquisition—likely overestimate the speed of adjustment.

**Heterogeneous reallocation times.** In addition, I find that the moderate aggregate reallocation time actually masks substantial heterogeneity across occupations.

To capture this, let define reallocation time at the *occupation level* as

$$T_i = \frac{1}{\max_h |m_i(h) - m_i|} \int_0^{+\infty} |m_i(t) - m_i| dt$$

where $m_i(t) \equiv e_i(t) + u_i(t)$ is the mass of workers in occupation $i$ at $t$, and $m_i$ its steady-state level. This metric measures the cumulative distance of an occupation's population from its steady state.

The results reveal significant heterogeneity in transition times across occupations. While most adjust relatively quickly to their new steady-state levels—with a median reallocation time of 25 to 40 quarters—a minority, such as retail assistants, security guards, and skilled warehouse workers, require over 80 quarters. These central network occupations face prolonged adjustment periods due to persistent flows until reallocation is complete.[42]

**Adjustment costs.** Finally, I find that the slow reallocation leads to a large reduction of welfare losses, of around 40%.

[42]Results on unemployment are presented in Appendix F. Interestingly, aggregate unemployment decreases significantly after the shock, despite the small aggregate productivity shock, and the adjustment is relatively fast. However, this masks substantial heterogeneity across occupations, with a minority adjusting very slowly.

Figure 7.3: **Flow social welfare over time**

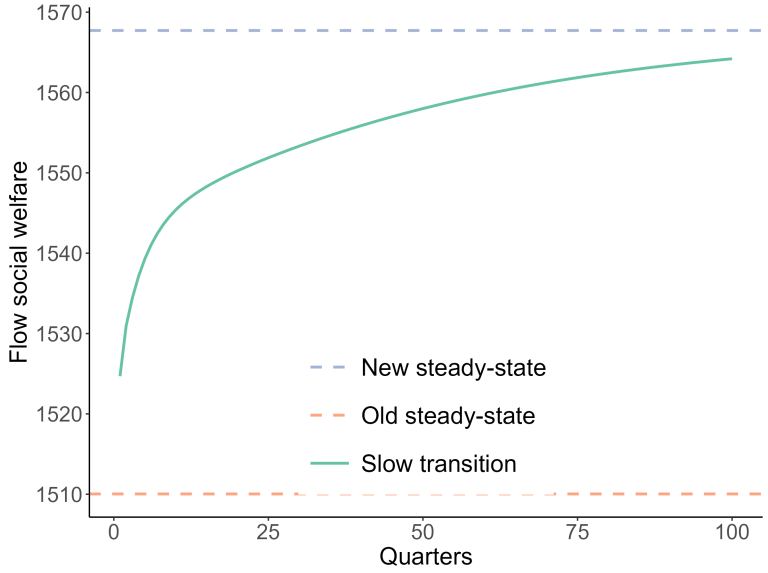



Figure 7.3 shows the evolution of flow social welfare over time under three scenarios. The upward-sloping curve represents social welfare in the slow transition case, where workers reallocate gradually. The top horizontal line represents social welfare in the case where the worker distribution jumps immediately to its new steady state, while the bottom line shows the case where the robot shock does not occur. The figure illustrates that social welfare adjusts slowly, converging toward its new steady-state level.

This slow reallocation results in large adjustment costs, measured graphically as the area between the top line and the upward-sloping curve. I find that adjustment costs amount to approximately -40%, much higher than previous estimates of comparable shocks. For instance, Caliendo et al. (2019) estimate adjustment costs from the China shock at around -3.5%. One reason for this discrepancy is the inclusion of both search and skill frictions. Since unemployed workers are not immediately matched with firms, staying unemployed while waiting to transition to new jobs is costly and leads to welfare losses. This highlights the importance of accounting for search frictions when measuring welfare gains from shocks, as ignoring these frictions could overestimate the benefits of technological advances by nearly a factor of two.

The results demonstrate significant heterogeneity in reallocation times across occupations and sizable adjustment costs. These findings raise important questions for policymakers: how can targeted interventions reduce the welfare losses associated with slow reallocation? In the next

subsection, I explore the potential of targeted subsidies to accelerate reallocation, focusing on occupations that act as bridges within the labor market network.

## 7.3 Policy Experiment

How can policymakers speed up worker reallocation? In this subsection, I simulate the effect of different policy interventions on reallocation speed.

**Hiring subsidies.** I focus on a specific type of policy intervention, namely targeted hiring subsidies. Targeted hiring subsidies, commonly used to support specific groups like young or low-educated workers, involve the government paying part of workers' wages when they find a job. More precisely, I model subsidies as an exogenous permanent productivity increase, shared between workers and firms through Nash bargaining.[43]

Subsidies can be targeted to specific occupations under three schemes: (1) declining occupations, negatively impacted by shocks (e.g., unskilled manufacturing workers); (2) expanding occupations, positively impacted (e.g., telemarketers); and (3) bridge occupations, those linking declining and expanding occupations (e.g., maintenance technicians, sales representatives, or sport coach).

The subsidy is set at 5% of pre-shock productivity for targeted occupations, normalized to ensure comparability across policy schemes.[44] More details can be found in Appendix F.

**Effect on reallocation speed.** I assess the impact of targeted hiring subsidies on reallocation speed. In the simulation, permanent subsidies are introduced post-robot shock, and I compare the resulting reallocation time to the baseline.

Table 7.1: **Change in reallocation time after different targetings, in p.p.**

| | **Declining** | **Expanding** | **Bridges** |
|---|---|---|---|
| $\Delta \log T$ | 0.82 % | -0.73 % | -2.28% |

Table 7.1 shows the percentage-point changes in reallocation times for different targeting

[43] Training is another important policy tool used by the government to accelerate reallocation. Incorporating training would require significant extensions to the quantitative model, which is left for future research.

[44] The total amount of subsidies, unweighted by population, is equal across targeting schemes: $\sum_i s_i^z = S$, where $s_i^x$ is the subsidy to occupation $i$ with targeting $z$ and $S$ is the total amount of subsidy.

strategies. There are two main takeaways. First, targeting declining occupations makes worker reallocation *longer*, as this incentives displaced workers to stay inefficiently long in their own declining occupations. Second, targeting both expanding and bridge occupations speeds up reallocation, with bridge-targeting being three times more effective than targeting expanding occupations.

It is not immediately obvious why targeting bridge occupations is more effective than targeting expanding occupations. Common wisdom suggests that targeting jobs with labor shortages (e.g., web developers) should boost wages and job postings in these occupations, encouraging workers to switch. However, this overlooks the role of skill frictions: workers may lack the skills needed for expanding occupations, limiting their ability to switch, even with higher wages. By targeting bridge occupations, policymakers enable workers to reskill into jobs with skills similar to those in expanding occupations, broadening the pool of workers who can transition effectively.

This shows that leveraging the occupation network structure can help policymakers to design more effective labor market policies. Identifying and targeting the bridge occupations can significantly speed-up the adjustment of labor markets to large structural shocks.

# 8 Conclusion

This paper introduces a novel network-based framework for understanding labor market reallocation in response to technology shocks. By modeling skill frictions as a network of occupations, I provide new insights into the slow dynamics of worker reallocation and its welfare implications. The analysis highlights the critical role of bridge occupations in shaping reallocation speed, and the significant adjustment costs due to slow transitions.

These findings have important implications for labor market dynamics in the context of technological change, trade disruptions, and the green transition. Targeting bridge occupations with policy interventions can significantly accelerate reallocation and reduce adjustment costs. This framework offers a valuable tool for policymakers aiming to mitigate the effects of labor market shocks.

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# 9 Appendix A: The Occupation Network

## 9.1 Alternative Algorithms to Detect Community Structure

Several algorithms have been developed to detect community structures in large networks (e.g. Louvain, Walktrap, Label Propagation, or Edge Betweenness). These methods, either divisive or constructive, aim to maximize metrics like modularity, which compares the actual number of edges within clusters to the expected number if the edges were drawn at random.[45] For example, the Louvain algorithm starts by assigning each node to its own community, then iteratively reassigns nodes to maximize modularity until a local maximum is reached, defining community boundaries. Below is a plot of the occupation network, with nodes colored by their community as identified by the Louvain algorithm.




Figure 4: The occupation network with communities found with Louvain algorithm

[45] Values above 0.3 indicate significant community structure.

## 9.2 Bridge Occupations

An occupation is classified as a bridge if its clustering coefficient is in the lowest decile and its betweenness centrality is in the highest decile of their respective distributions. Below is a list of the top 10 bridge occupations.

| Rank | Betweenness | Clustering | Job Title |
|---|---|---|---|
| 1 | 0.062 | 0.171 | Emergency Management |
| 2 | 0.054 | 0.109 | Recreation Workers |
| 3 | 0.054 | 0.162 | Scientific Research |
| 4 | 0.052 | 0.090 | Technical Sales Representatives |
| 5 | 0.043 | 0.190 | Fine Artists |
| 6 | 0.041 | 0.179 | Logistics Worker |
| 7 | 0.039 | 0.141 | Management Consultant |
| 8 | 0.039 | 0.156 | Sales Secretary |
| 9 | 0.035 | 0.133 | General and Operations Managers |
| 10 | 0.033 | 0.171 | Maintenance Workers |

Figure 1.2 plots the occupation network with node sizes proportional to betweenness centrality, revealing that bridges are concentrated in social services, sales, retail, transport, and logistics.

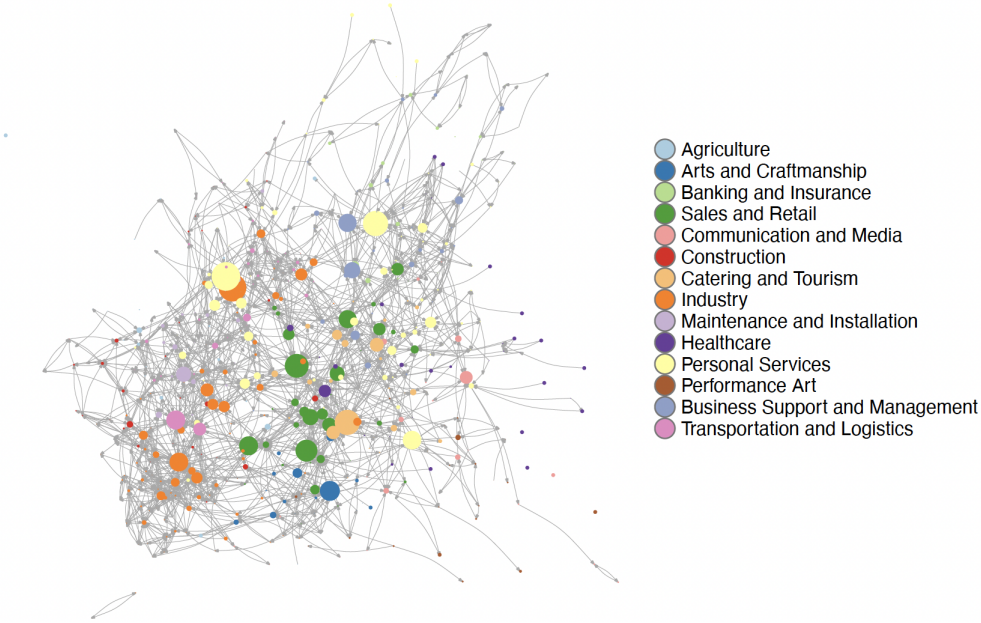



Figure 5: The bridge occupations in the occupation network

# 10 Appendix B: Worker Flows

## 10.1 Data

**Unemployment definition.** I assign occupations to unemployed workers based on their last job, rather than the occupations they are currently searching in. This approach has several advantages: it only requires matched employer-employee data, accounts for workers searching in multiple occupations, and aligns better with theories of skill frictions by reflecting workers' current skills and the difficulty of acquiring new ones.

**Multiple job holding.** Some workers hold multiple jobs despite focusing on main jobs. I interpret this as transitioning between main jobs. To address this, I assign the worker to the job with the most recent start date. If the new job ends before the old one, I drop the new contract, assuming it to be a side job.

**Occupation codes.** The DADS panel provides imperfect 4-digit occupation codes, especially before 2008 when reporting was not mandatory. Missing values are not uniformly distributed, particularly in low-skilled and public sector jobs, so I restrict the analysis to post-2008 data when reporting became mandatory.

Additionally, DADS and ROME use different occupational nomenclatures: DADS uses PCS, which groups by social rank, while ROME groups by skills. For consistency, I map both onto a third nomenclature, FAP, at the 3-digit level, resulting in around 200 distinct occupations.

## 10.2 Construction of the main variables

**Discrete time transition matrix.** Let $\boldsymbol{Q}(t)$ denote the transition matrix between occupations and employment states at quarter $t$. It can be decomposed into four block matrices $\boldsymbol{Q}^{x\to y}(t)$ representing the occupation transition from employment state $x$ to employment state $y$, with $x, y \in \{u, e\}$. For example, $\boldsymbol{Q}^{u\to e}(t)$ represents occupation transitions from unemployment to employment.

The general coefficient of $\boldsymbol{Q}^{x\to y}(t) = (\mu_{ij}^{x\to y}(t))_{i,j\in\mathcal{N}^2}$ is computed as

$$\mu_{ij}^{x\to y}(t) \equiv \frac{\text{\# workers in occupation } i \text{ and state } x \text{ at } t \text{ who are in occupation } j \text{ and state } y \text{ at } t+1}{\text{\# workers in occupation } i \text{ and state } x \text{ at } t}$$

Intuitively, this is akin to conducting a large-scale survey of the entire labor force at a fixed quarterly frequency, recording each worker's occupation and employment state at $t$ and $t+1$. Transitions are as the number of workers who change occupation or state between these dates.

**Time Frequency.** I use a quarterly sampling frequency to balance two risks in measuring transition probabilities. First, *time-aggregation bias* occurs because transitions happen continuously, but are measured at discrete intervals, potentially underestimating actual transitions. While Shimer (2007) offers corrections for two or three states, no general corrections exist for larger state spaces. Shorter intervals (e.g., monthly) would reduce this bias.

Second, *small sample bias* arises when there are too few observations to estimate transition rates accurately. With around 40,000 rates to estimate and limited transitions recorded, small sample bias can significantly affect estimates, especially when no transitions are observed in a given quarter. A shorter sampling interval would worsen this bias.

A quarterly frequency offers a good compromise, reducing time-aggregation bias while limiting small sample bias, ensuring more accurate mobility rate estimates.

**Controlling for Seasonality.** Estimating transition rates is complicated by strong seasonality, with more transitions occurring at certain times, such as January or September.[46] To control for seasonality, I apply a simple strategy: using a moving average of the last four quarterly transition matrices. This assumes that seasonality effects cancel out over time, focusing on relative rather than absolute effects.

**Estimation of the continuous-time transition matrix.** The transition matrix constructed above is a *discrete-time* transition matrix. To convert a discrete-time transition matrix to a continuous-time transition matrix, I use the fact that $\boldsymbol{f}(t+h) = e^{\boldsymbol{Q}h}\boldsymbol{f}(t)$, where $e^{\boldsymbol{Q}h}$ is the discrete-time transition matrix and $\boldsymbol{Q}$ is the underlying continuous-time transition rate matrix. For small time intervals, the matrix exponential can be approximated as $e^{\boldsymbol{Q(t)}h} \approx \boldsymbol{I} + \boldsymbol{Q}(t)h$. Setting $h = 1$, we find $\boldsymbol{Q}(t) \approx e^{\boldsymbol{Q(t)}} - \boldsymbol{I}$.[47]

**Wages.** Let me explain each step in more detail, following the methodology of Le Barbanchon et al.

[46] This is especially true for employment-to-employment transitions, where many within-firm transitions happen on January 1st, as firms often report these changes annually due to the administrative burden of continuous reporting.

[47] This approximation makes sense because the diagonal terms of the discrete-time transition matrix are $1 - \sum_{j \neq i} q_{ij}$, so subtracting one gives $-\sum_{j \neq i} q_{ij}$, ensuring rows sum to zero.

(2021) on the same dataset. First, I measure net wages (in constant euros) rather than gross wages, as the former are more relevant for workers considering job changes. Second, I construct full-time equivalent quarterly net wages. For full-time workers, I calculate daily earnings and multiply by 120 (30 days × 4 months) for quarterly earnings. For part-time workers, I compute hourly earnings and multiply by 35 × 4 × 4. Third, I control for age by regressing wages on a second-order polynomial of age and quarter-specific occupation fixed effects, isolating wage variation by occupation and quarter. Fourth, I adjust for seasonality using a four-quarter moving average.

**Recoding of Skill Frictions.** An important methodological challenge arises from the recoding of expert-based measures of skill frictions into a new occupation nomenclature. Worker occupation transitions use a different code (PCS) than the expert measure of skill frictions (ROME). Both nomenclatures must be converted into a third one, labelled FAP, at the 3-digit level. This is a many-to-one matching: each FAP code is associated with one or many ROME codes.

First, I assign numerical values to ROME skill frictions: 1 for self-connections, 2 for close transitions, 3 for distant transitions, and 4 for unlinked transitions. Second, I compute FAP-level connection values by averaging ROME connection weights across all ROME occupations associated with each FAP occupation:

$$g_{ij}^{FAP} = \frac{1}{|N_i \times N_j|} \sum_{(k,l) \in N_i \times N_j} g_{kl}^{ROME}$$

where $N_i$ denotes the set of ROME occupations associated with FAP $i$, and $N_i \times N_j$ is the set of all ROME connections between two FAP occupations $i$ and $j$. Third, I discretize the continuous FAP connection values into four categories (self, close, distant, unlinked) by partitioning them into intervals: $g_{ij}^{FAP} = 1$ (self), $g_{ij}^{FAP} \in (1, 2.5]$ (close), $g_{ij}^{FAP} \in (2.5, 3.5]$ (distant), and if $g_{ij}^{FAP} \in (3.5, 4]$(unlinked).

## 10.3 Bridge Regression

**Outcome variables.** The long-run difference in worker outcome, $\Delta y_{id}$, is measured as the difference between the outcome at the end of the sub-period (4th quarter of 2012 or 2019) and the outcome at the beginning of the sub-period (1st quarter of 2008 or 2013). I divide the sample into

two periods to avoid dropping too many observations due to the balanced panel assumption (the worker must be present at both the beginning and the end of the period).

I consider several outcome variables: probability of changing 1-digit or 2-digit occupation group, distance traveled in the occupation network (centered and standardized), wage growth, and change in unemployment risk (measured as the difference in the unemployment rate between the origin and destination occupations in percentage points).

The dummy $1(\text{bridge})$ equals one if the worker transited through a bridge occupation, defined as occupations in the top tenth decile of betweenness centrality and bottom tenth decile of clustering coefficient. It excludes workers who start or end in bridge occupations, requiring at least two transitions: from a non-bridge occupation to a bridge occupation, and then to another non-bridge occupation.

**Period 2008-2012.** The Table 2.1 gives the estimated coefficients over the period 2009-2012. All coefficients are statistically significant and have the same sign as in the 2013-2019 period.

Table 1: **Change in worker outcomes vs. transition through bridge, 2009-2012**

| | Dependent Variables | | | | |
|---|---|---|---|---|---|
| | Change 1-digit Occupation | Change 2-digit Occupation | Distance (Standardized) | Wage Growth | Unemployment Rate Change |
| Through Bridge | 0.317*** | 0.342*** | 0.816*** | 0.044*** | -0.345*** |
| | (0.003) | (0.003) | (0.008) | (0.002) | (0.026) |
| Observations | 196,439 | 196,439 | 196,439 | 253,375 | 253,423 |
| Origin FE | Yes | Yes | Yes | Yes | Yes |

Notes: Each observation is a worker over the period 2008-2012. The dependent variables are the probability of changing 1-digit or 2-digit occupation, distance traveled within the network, wage growth, and change in unemployment risk. The variable of interest is a dummy indicating whether workers transited through a bridge during the period. * p<0.10, ** p<0.05, *** p<0.01.

Compared to the subsequent period, workers transiting through bridge occupations have slightly higher chances of changing 1- or 2-digit occupation groups and travel farther in the network. However, their wage growth and reduction in unemployment risk are halved. Despite this, the effect remains economically significant, especially considering that wages declined and unemployment

rates rose over the period.

**Robustness check.** As a robustness check, I restrict the control group to workers who made at least two transitions over the period. This controls for unobserved personality traits, such as high mobility, which could confound the effect of transiting through a bridge occupation.

Table 2.2 presents the estimated coefficients for the 2013-2019 subperiod with the alternative control group. The main take-away is that most results go through. All coefficients are still

Table 2: **Change in worker outcomes vs. transition through bridge, 2013-2019, robust control**

| | Dependent Variables | | | | |
|---|---|---|---|---|---|
| | Change 1-digit Occupation | Change 2-digit Occupation | Distance (Standardized) | Wage Growth | Unemployment Rate Change |
| Through Bridge | 0.103*** | 0.053*** | 0.168*** | 0.049*** | -0.744*** |
| | (0.004) | (0.004) | (0.008) | (0.010) | (0.048) |
| Observations | 110,368 | 110,368 | 110,368 | 110,343 | 110,368 |
| Origin FE | Yes | Yes | Yes | Yes | Yes |

Notes: Each observation is a worker over the period 2008-2012. The dependent variables are the probability of changing 1-digit or 2-digit occupation, distance traveled within the network, wage growth, and change in unemployment risk. The variable of interest is a dummy indicating whether workers transited through a bridge during the period. * p<0.10, ** p<0.05, *** p<0.01.

statistically significant at the $1\%$ threshold. The magnitude of the first three coefficients is reduced but still quite high: workers transiting through bridges still have a higher probability of changing 1-digit and 2-digit occupation groups and travel farther away, but the difference with the control group is reduced. What is more, the magnitude of the fourth and firth coefficients are unchanged, if not slightly increased for unemployment risk.

# 11 Appendix C: Stylized Network Search Model

## 11.1 Firm Values

**Proposition 1.** (Job posting along the transition)

*Firm values $J_i(t)$, tightness ratios $\theta_i(t)$ and job finding rates $p_i(t)$ are constant along the transition dynamics, and given by*

$$J_i(t) = \frac{\phi y_i}{r+s} \qquad \theta_i(t) = \left(\frac{\zeta_i}{c_i}\left(\frac{(1-\phi)y_i}{r+s_i}\right)\right)^{\frac{1}{\alpha}} \qquad p_i(t) = \zeta^{\frac{1}{\alpha}}\left(\frac{(1-\phi)y_i}{c_i(r+s_i)}\right)^{\frac{1-\alpha}{\alpha}}$$

*Proof*: Firm value is unstable, thus the only stable dynamics is the constant-steady-state value. Plugging the free entry condition, the Bellman equation of firm value writes $\dot{J}_i(t) = (r+s_i)J_i(t) - (y_i - w_i)$. Solving for this differential equation gives $J_i(t) = \frac{(1-\phi)y_i}{r+s} + \left(\frac{(1-\phi_i)y_i}{r+s_i} - J_i(0)\right)e^{(r+s_i)t}$. If the initial condition is not equal to the steady-state value of firm value, then firm value diverges over time. Consequently, the only stable trajectory is the constant steady-state value. Using the free entry condition $\frac{c_i}{q(\theta_i)=J_i}$ then gives the equilibrium tightness ratios. Using the functional form the matching function yields the job finding rate.

## 11.2 Approximating worker reallocation dynamics

For small enough shocks, the baseline pre-shock transition matrix is sufficient to compute the dynamics of the worker distribution, as the next lemma shows.

**Lemma** (Approximation of worker reallocation dynamics)
*The deviation from steady-state, $\boldsymbol{\delta f}(t) \equiv \boldsymbol{f}(t) - \boldsymbol{f}_\infty$ evolves approximately as:*

$$\dot{\boldsymbol{\delta f}}(t) \approx \widetilde{\boldsymbol{Q}}\,\boldsymbol{\delta f}(t) \quad \text{with} \quad \boldsymbol{\delta f}_0 \approx \widetilde{\boldsymbol{f}_\infty} - \boldsymbol{f}_\infty$$

*where $\widetilde{\boldsymbol{Q}}$ is the pre-shock transition matrix, and $\widetilde{\boldsymbol{f}_\infty} - \boldsymbol{f}_\infty$ is the deviation in steady-states.*

*Proof:* The proof is in three steps, and extensively leverages properties of first-order approximations. First, for small enough shocks, the post-shock job finding rates can be expressed as $\boldsymbol{p}_{\text{post-shock}} \approx \boldsymbol{p}_{\text{pre-shock}} + \kappa\boldsymbol{\delta y}$ with $\kappa \equiv \frac{1-\alpha}{\alpha}\left(\frac{\zeta(1-\phi)}{c(r+s)}\right)^{\frac{1-\alpha}{\alpha}}\bar{y}^{\frac{1-2\alpha}{\alpha}}$ The new job finding rate is the sum of the old job finding rate and a shock term, which is proportional to productivity shock. Because the shock size can be rescaled by $1/\kappa$, in what follows I simply denote the job finding rate shock by $\Delta$.

Second, for small enough shocks, the post-shock steady-state distribution can be expressed as $\boldsymbol{f}_\infty = \tilde{\boldsymbol{f}}_\infty + \Delta \cdot \boldsymbol{x}_0$, where $\Delta$ controls the magnitude of the shock, and $\boldsymbol{x}_0$ is a vector independent of shock size (which depends on the shock structure, and will specified later). This implies that the

initial deviation from the new steady-state is small, and of order $\Delta$.

Third, a first-order Taylor approximation of the post-shock transition matrix gives $\boldsymbol{Q} \approx \tilde{\boldsymbol{Q}} + \Delta \cdot \boldsymbol{\delta Q}$, where $\Delta$ controls the magnitude of the shock, and $\boldsymbol{\delta Q}$ is a matrix independent of shock size (which depends on the shock structure, and will specified later).

Taking together, these three results imply that

$$\dot{\boldsymbol{\delta f}}(t) \approx (\tilde{\boldsymbol{Q}} + \Delta \cdot \boldsymbol{\delta Q})\boldsymbol{\delta f}(t) \approx \tilde{\boldsymbol{Q}}\boldsymbol{\delta f}(t) + \Delta \cdot \boldsymbol{\delta Q}\boldsymbol{\delta f}(t)$$

Integrating each term yields: $\boldsymbol{\delta f}(t) = \Delta e^{\tilde{\boldsymbol{Q}}t}\boldsymbol{x}_0 + \Delta^2 e^{\boldsymbol{\delta Q}t}\boldsymbol{x}_0$ The second term is of order $\Delta^2$, and hence negligible, yielding the desired result.

To sum up, this means the pre-shock transition matrix is sufficient to simulate worker reallocation dynamics after small shocks.

## 11.3 Spectral decomposition

**Lemma.** (Spectral decomposition)

*The worker distribution at date $t$ can be expressed as a function of the pre-shock transition matrix's eigenvalues and eigenvectors*

$$\boldsymbol{f}(t) \approx \boldsymbol{f}_\infty + \sum_{k=2}^{6} \xi_k e^{\lambda_k t}\boldsymbol{v}_k \tag{16}$$

*where $\lambda_k < 0$ are the eigenvalues of the baseline pre-shock transition matrix $\boldsymbol{Q}$, $\boldsymbol{v}_k$ are the associated right-eigenvectors, and $\xi_k$ are the associated loadings. The loadings depend on the initial distribution: $\xi_k = \boldsymbol{w}_k'\boldsymbol{f_0}$, where $\boldsymbol{w}_k$ are the left-eigenvectors of the transition matrix.*

Intuitively, the eigenvalue-eigenvector decomposition expresses the complex transition dynamics of the worker distribution as a sum of simpler convergence modes. Generally, the worker distribution converges at varying speeds, but along these eigenvectors, each segment converges at the same speed determined by the eigenvalue.

## 11.4 Spectral properties of transition matrix

Here, I turn to characterizing the eigenvalues and left- and right-eigenvectors of the *baseline* pre-shock transition matrix. The details of the computation have limited economic interest, and hence are not shown - but are available upon request.

**Lemma.** *The eigenvalues of the pre-shock transition matrix are approximately*

$$\lambda_1 = 0, \quad \lambda_2 \approx \frac{p_C s}{p_P + p_C + s}, \quad \lambda_3 \approx s + \frac{s p_P p_C}{(p_P + p_C)(2p_P + p_C)}$$
$$\lambda_4 \approx p_P + p_C + s - \frac{2sp_C}{p_P + p_C + s}, \quad \lambda_5 \approx p_P + p_C + s - \frac{sp_C}{p_P + p_C + s}, \quad \lambda_6 \approx 2p_P + p_C + s - \frac{2sp_P}{(2p_P + p_C)}$$

**Lemma.** *The second and third right-eigenvectors of the pre-shock transition rate matrix are approximatively*

$$\boldsymbol{v}_2 \approx \begin{pmatrix} p_C + p_P & 0 & -(p_C + p_P) & s & 0 & -s \end{pmatrix}'$$
$$\boldsymbol{v}_3 \approx \begin{pmatrix} p_C + p_P & - - 2(p_C + p_P) & p_C + p_P & s & -s & s \end{pmatrix}'$$

I do not provide an analytical characterization of other right-eigenvectors here, as it turns out that the loadings on these eigenvectors is negligible for *reallocation* shocks. It is, however, not the case for aggregate shocks - which is not focus of the current focus.

**Lemma.** *The left-eigenvectors of the pre-shock transition rate matrix are*

$$\boldsymbol{w}_2 \approx \mathcal{Z}_2^{-1} \begin{pmatrix} 1 & 0 & -1 & \frac{p_P}{p_P + p_C} & 0 & -\frac{p_P}{p_P + p_C} \end{pmatrix}'$$
$$\boldsymbol{w}_3 \approx \mathcal{Z}_3^{-1} \begin{pmatrix} 1 & -\frac{2p_P}{p_C} & 1 & -\frac{p_P p_C}{(p_P + p_C)(2p_P + p_C)} & \frac{2p_P}{p_C}\frac{p_P p_C}{(p_P + p_C)(2p_P + p_C)} & -\frac{p_P p_P}{(p_P + p_C)(2p_P + p_C)} \end{pmatrix}'$$
$$\boldsymbol{w}_4 \approx \mathcal{Z}_4^{-1} \begin{pmatrix} 1 & -\frac{2s}{p_P + p_C} & 1 & \frac{p_P + p_C}{s} & -2 & \frac{p_P + p_C}{s} \end{pmatrix}'$$
$$\boldsymbol{w}_5 \approx \mathcal{Z}_5^{-1} \begin{pmatrix} s & 0 & -s & -(p_P + p_C) & 0 & -(p_P + p_C) \end{pmatrix}'$$
$$\boldsymbol{w}_6 \approx \mathcal{Z}_6^{-1} \begin{pmatrix} s & \frac{p_P(2p_P + p_C)}{p_C} & s & -(2p_P + p_C) & -\frac{p_P(2p_P + p_C)^2}{sp_C} & -(2p_P + p_C) \end{pmatrix}'$$

where the normalization constants are set such that $\boldsymbol{w}_k^T \boldsymbol{v}_k = 1$.

## 11.5 Reallocation times after periphery and central shocks

In this subsection, I derive an approximation of the aggregate reallocation time after both periphery and central shocks. All proofs are relegated to the online appendix.

**Periphery Shocks.** I begin by analytically characterizing reallocation time following periphery shocks. With the eigenvalues already computed, the next step is to characterize the distribution of loadings in the spectral decomposition, and thus the initial deviation from steady state.

**Lemma.** *After the periphery productivity shock, the initial deviation from steady-state writes*

$$\boldsymbol{\delta f}(0)^{\mathrm{P}} \equiv \tilde{\boldsymbol{f}}_{\infty} - \boldsymbol{f}_{\infty}^{\mathrm{P}} \approx \Delta \mathcal{Z}^{-1}\big( -(2p_P + p_C), 0, (2p_P + p_C), -s, 0, s\big)'$$

*where $\mathcal{Z}$ is the normalization constant of the steady-state worker distribution before the shock.*

Note that the periphery shock is a *reallocation shock*, to the extent that excess employment in one periphery occupation is exactly compensated by a deficit in the other.

Equipped with this result, and the distribution of left eigenvectors, I can now compute the distribution of loadings using the formula: $\xi_k = \boldsymbol{w}_k'(\tilde{\boldsymbol{f}}_{\infty} - \boldsymbol{f}_{\infty,\mathrm{P}})$, where $\boldsymbol{w}_k$ are the left-eigenvectors of the baseline transition matrix.

**Lemma.** *The distribution of loadings after the periphery shock is approximately*

$$\xi_2^P \approx -\Delta \mathcal{Z}^{-1}(p_P + p_C) \quad \xi_3^P \approx 0 \quad \xi_4^P \approx 0 \quad \xi_5^P \approx -s\Delta \mathcal{Z}^{-1}\frac{3p_P + 2p_C}{p_P + p_C} \quad \xi_6^P \approx 0$$

Using the spectral decomposition, one can now solve entirely for the worker distribution at any point in time - and thus compute the aggregate reallocation time.

**Proposition.** *For small shock size $\Delta$, and small separation rates, the transition time after the central productivity shock can be approximated as*

$$T_{\mathrm{C}} \approx \frac{p_C + p_P}{s p_C}$$

**Central shocks.** I begin by analytically characterizing reallocation time following central shocks. The approach is the same as for periphery shocks.

**Lemma.** *After central shock, the initial deviation writes*

$$\boldsymbol{\delta f}(0)^{\mathrm{C}} \equiv \tilde{\boldsymbol{f}}_{\infty} - \boldsymbol{f}_{\infty}^{\mathrm{C}} \approx \Delta \mathcal{Z}^{-1} \begin{pmatrix} -\frac{p_P+p_C}{2} & p_P+p_C & -\frac{p_P+p_C}{2} & -\frac{s}{2} & s & -\frac{s}{2} \end{pmatrix}'$$

*where $\mathcal{Z}$ is the normalization constant of the steady-state worker distribution before the shock.*

Note that the central shock is a *reallocation shock* too, as excess employment in the central occupation is exactly compensated by a deficit in the two periphery occupations.

**Lemma.** *The distribution of loadings after the central productivity shock is approximately*

$$\xi_2^C \approx 0 \quad \xi_3^C \approx \Delta \mathcal{Z}^{-1} \frac{(2p_P+p_C)^2}{p_C} \quad \xi_4^C \approx -s\Delta \mathcal{Z}^{-1} \frac{2p_P+p_C}{2(p_P+p_C)} \quad \xi_5^C \approx 0 \quad \xi_6^C \approx s\Delta \mathcal{Z}^{-1} \frac{2p_P}{p_C}(2p_P+p_C)^2$$

**Proposition.** *For small shock size $\Delta$, and small separation rates, the transition time after the central productivity shock can be approximated as*

$$T_{\mathrm{C}} \approx \frac{1}{s}$$

## 11.6 Reallocation time after any reallocation shock

In this subsection, I show that the reallocation time after *any reallocation* shock can be expressed as a weighted average of reallocation times after periphery and central shocks.

**Lemma.** (Linear decomposition of reallocation shocks)
*All reallocation shocks—defined as productivity shocks where excess and deficit employment offset each other exactly across occupations—can be decomposed as a linear combination of central and periphery productivity shocks.*

$$\boldsymbol{\delta y}_{\text{reallocation}} = \gamma_{\mathrm{P}} \boldsymbol{\delta y}_{\mathrm{P}} + \gamma_{\mathrm{C}} \boldsymbol{\delta y}_{\mathrm{C}}$$

*where $\gamma_P$ is loading on periphery shocks $\gamma_C$ is the loading on central shocks.*

*Proof:* The entire space of productivity shocks is initially three-dimensional. Imposing the condition that excess and deficit employment must sum to zero restricts this space to two dimensions. It is straightforward to verify that central and periphery shocks are linearly independent. I will now show

that any linear combination of these shocks also satisfies the reallocation condition, thus spanning the two-dimensional space of reallocation shocks.

Leveraging the linarity of the first-order approximations, it can be shown that the change in job finding rate, transition matrix and steady-state deviation can be expressed as a linear combination of the associated periphery and central shocks' changes, characterized above.

For small reallocation shocks, the job finding rates can be approximated as follows $\boldsymbol{p}_{\text{after-shock}} \approx \boldsymbol{p}_{\text{before-shock}} + \gamma_{\text{P}}\kappa\Delta\boldsymbol{\delta y}_{\text{P}} + \gamma_{\text{C}}\kappa\Delta\boldsymbol{\delta y}_{\text{C}}$ Similarly, the post-shock transition matrix can be approximated as $\boldsymbol{Q}_{\text{after-shock}} = \boldsymbol{Q}_{\text{pre-shock}} + \gamma_{\text{P}}\Delta\boldsymbol{\delta Q}_{\text{P}} + \gamma_{\text{C}}\Delta\boldsymbol{\delta Q}_{\text{C}}$. Finally, the deviation from the new steady-state is a linear combination of the deviations from steady-state after the periphery and central shocks: $\boldsymbol{\delta f}(0) \equiv \tilde{\boldsymbol{f}}_{\infty} - \boldsymbol{f}_{\infty} = \gamma_{\text{P}}\Delta\boldsymbol{\delta f}_{\text{P}}(0) + \gamma_{\text{C}}\Delta\boldsymbol{\delta f}_{\text{C}}(0)$. Since excess and deficit employment balance out for each of the periphery and central shocks, this balance will also hold for any linear combination of these shocks, thus meeting the condition for a reallocation shock. Therefore, any reallocation shock can indeed be decomposed as a combination of the central and periphery shocks, completing the proof.

**Proposition.** *For small shocks, the transition time after any reallocation shock is*

$$T \approx \alpha_{\text{P}} \cdot \underbrace{\frac{p_C + p_P + s}{p_P} s^{-1}}_{\equiv T_{\text{P}}} + (1 - \alpha_{\text{P}}) \underbrace{\cdot s^{-1}}_{\equiv T_{\text{P}}} \quad \text{with} \quad \alpha_{\text{P}} \equiv \frac{\mid (p_P + p_C)\,\gamma_{\text{P}} \mid}{\mid (p_P + p_C)\,\gamma_{\text{P}} \mid + \mid -\frac{(2p_P + p_C)^2}{p_P + p_C}\gamma_{\text{C}} \mid}$$

*Proof:* The worker distribution at date $t$ can be approximated as

$$\boldsymbol{f}(t) \approx \widetilde{\boldsymbol{f}}_{\infty} - \gamma^P \xi_2^P \, e^{-\lambda_2 t} \boldsymbol{v}_2 - \gamma^C \xi_3^P e^{-\lambda_3 t} \boldsymbol{v}_3$$

Therefore, the transition time is approximately

$$T = \int_0^{+\infty} \frac{\| \gamma^P \xi_2^P \, e^{-\lambda_2 t} \boldsymbol{v}_2 + \gamma^C \xi_3^P e^{-\lambda_3 t} \boldsymbol{v}_3 \|_2}{\| \gamma^P \xi_2^P \boldsymbol{v}_2 + \gamma^C \xi_3^P \boldsymbol{v}_3 \|_2} dt$$

where $\Delta\mathcal{Z}^{-1}$ cancel out from the numerator and denominator.

Now, I use the fact that both right eigenvectors $\boldsymbol{v}_2$ and $\boldsymbol{v}_3$ are orthogonal to each other: that is,

$\boldsymbol{v}_2'\boldsymbol{v}_3 = 0$. Hence, by the Pythagorean theorem, we have

$$\begin{aligned}\|\gamma^P\xi_2^P\, e^{-\lambda_2 t}\boldsymbol{v}_2 + \gamma^C\xi_3^P e^{-\lambda_3 t}\boldsymbol{v}_3\|_2 =& \sqrt{\|\gamma^P\xi_2^P\, e^{-\lambda_2 t}\boldsymbol{v}_2\|_2^{\,2} + \|\gamma^C\xi_3^P e^{-\lambda_3 t}\boldsymbol{v}_3\|_2^{\,2}} \\ =& \sqrt{\left(\|\gamma^P\xi_2^P\boldsymbol{v}_2\|_2 e^{-\lambda_2 t}\right)^2 + \left(\|\gamma^C\xi_2^C\boldsymbol{v}_3\|_2 e^{-\lambda_3 t}\right)^2} \\ \approx& \|\gamma^P\xi_2^P\boldsymbol{v}_2\|_1 e^{-\lambda_2 t} + \|\gamma^C\xi_3^C\boldsymbol{v}_3\|_1 e^{-\lambda_3 t}\end{aligned}$$

where the third line follows from norm approximation: that is, for any vector $\boldsymbol{x}$, $\|\boldsymbol{x}\|_2 \approx \|\boldsymbol{x}\|_1$. Finally, integrating over time yields the desired result.

## 11.7 Adjustment costs

The adjustment costs can be written more compactly in terms of the worker distribution and the vector of agents' payoffs as follows:

$$AC = \frac{SW^{\text{slow}} - SW^{\text{new}}}{SW^{\text{new}} - SW^{\text{old}}} = \frac{\int_0^{+\infty} e^{-rt}(\boldsymbol{f}(t) - \boldsymbol{f}_\infty)^T\boldsymbol{u}dt}{(1/r)\cdot(\boldsymbol{f}_\infty^T\boldsymbol{u} - \widetilde{\boldsymbol{f}_\infty}^T\widetilde{\boldsymbol{u}})}$$

where $\boldsymbol{u} = (y_1, \ldots, b_N - c_N\theta_N)$ is the vector of flow utilites.

By applying spectral decomposition, the adjustment costs can be further decomposed into the transition matrix's eigenvectors and eigenvalues, as clarified in the next lemma.

**Lemma.** (Spectral decomposition of adjustment costs)
*After small productivity shocks, the adjustment costs can be approximated as*

$$AC = \sum_{k=1}^{2N} \underbrace{\frac{r}{r + |\lambda_k|}}_{\text{speed}} \cdot \underbrace{\boldsymbol{w}_k^T(\boldsymbol{f}_0 - \boldsymbol{f}_\infty)}_{\text{loading}} \cdot \underbrace{\frac{\boldsymbol{v}_k^T\boldsymbol{u}}{\boldsymbol{f}_\infty^T\boldsymbol{u} - \widetilde{\boldsymbol{f}_\infty}^T\widetilde{\boldsymbol{u}}}}_{\text{misallocation}} \tag{17}$$

*where $\lambda_k$ is the k-th smallest eigenvalue of the pre-shock transition matrix, while $\boldsymbol{w}_k$ and $\boldsymbol{v}_k$ are the associated left- and right-eigenvectors, respectively.*

The interpretation is as follows: the spectral decomposition breaks down the dynamics of worker transitions into a sum of simpler transition patterns, each characterized by a fixed speed of convergence, given by the eigenvalues of the transition matrix. Along each of these simpler

transition patterns—represented by the eigenvectors of the transition matrix—the welfare effect can be easily quantified as the product of three terms.

The first term $\frac{r}{r+|\lambda_k|}$ captures the effect of *transition speed* on welfare. Smaller eigenvalues $|\lambda_k|$ indicate slower reallocation and higher adjustment costs. This is because slower reallocation delays the realization of long-term benefits from the shock, which reduces the present value of these benefits and increases adjustment costs.

The second term $\boldsymbol{w}_k^T(\boldsymbol{f}_0 - \boldsymbol{f}_\infty)$ reflects the *magnitude* of the simpler transition pattern in the spectral decomposition. It measures the extent to which the initial deviation of the worker distribution from the steady-state $\boldsymbol{f}_0 - \boldsymbol{f}_\infty$ aligns with the right eigenvector $\boldsymbol{v}_k$. For example, if the initial deviation is proportional to $\boldsymbol{v}_k$, then the loading is one for $\boldsymbol{v}_k$ and zero for all other eigenvectors.[48]

The third term $\frac{\boldsymbol{v}_k^T \boldsymbol{u}}{\boldsymbol{f}_\infty^T \boldsymbol{u} - \widetilde{\boldsymbol{f}_\infty}^T \widetilde{\boldsymbol{u}}}$ captures the impact of *transitory misallocation* on aggregate welfare. For example, if labor shortages occur in certain occupations ($\boldsymbol{v}_k(i) < 0$)) where the flow payoff $u_i$ is high, the numerator turns negative, indicating that workers are misallocated over the transition. The denominator scales these misallocations by the total welfare change that would occur if the transition were instantaneous, thus yielding percentage-point variations in welfare due to the slow transition.

By plugging in the analytical characterizations of the eigenvalues and eigenvectors of the baseline transition matrix and simplifying the terms, we obtain the desired results.

# 12 Appendix D: Quantitative Network Search Model

## 12.1 Search frictions

The number of hires in occupation $i$ at date $t$ is now given by

$$h_i(t) = \zeta_i \quad \cdot \left( \sigma_{ii}^u(t) u_i(t) + \sum_{j \neq i} \sigma_{ji}^u(t) u_j(t) + \sum_{j \neq i} \sigma_{ji}^e(t) e_j(t) \right)^{\alpha} \cdot \quad v_i(t)^{1-\alpha_i}$$

[48] Without loss of generality, the left-eigenvectors can be normalized such that $\boldsymbol{W}^T \boldsymbol{V} = \boldsymbol{I}$, where $\boldsymbol{W}$ and $\boldsymbol{V}$ are the matrices of left- and right-eigenvectors, respectively. In other words, $\boldsymbol{w}_l^T \boldsymbol{v}_k = 1$ if $k = l$, and 0 otherwise.

where $v_i(t)$ is the total number of vacancies posted by firms in occupation $i$ at date $t$, and $\sigma^u_{ii}u_i + \sum_{j\neq i}\sigma^u_{ji}g_{ji}u_j + \sum_{j\neq i}\sigma^e_{ji}g_{ji}\eta_j e_j$ represents total search exerted by workers in occupation $i$ at date $t$, from unemployed and employed workers.

The tightness ratio in occupation $i$ at date $t$ is now defined as $\theta_i(t) \equiv v_i(t)/\big(\sigma^u_{ii}(t)u_i(t) + \sum_{j\neq i}\sigma^u_{ji}(t)u_j(t) + \sum_{j\neq i}\sigma^e_{ji}(t)e_j(t)\big)$.

Matching is random conditional on search efforts, so the job finding rate in occupation $i$ at date $t$ per unit of search effort is $p_i(t) = h_i(t)/\big(\sigma^u_{ii}(t)u_i(t) + \sum_{j\neq i}\sigma^u_{ji}(t)u_j(t) + \sum_{j\neq i}\sigma^e_{ji}(t)e_j(t)\big) = \zeta\theta_i(t)^{1-\alpha}$. Similarly, vacancies in occupation $i$ face a flow probability $q_i(t) = h_i(t)/v_i(t) = \zeta\theta_i(t)^{-\alpha}$ of matching with a worker at date $t$.

## 12.2 Firm values

The value of a filled job for firms in occupation $i$ at date $t$ is denoted $J_i(t)$, and the value of an unfilled vacancy in occupation $i$ at date $t$ is denoted $V_i(t)$.

The value of a job filled in occupation $i$ at date $t$ $J_i(t)$ solves a modified Bellman equation

$$rJ_i(t) = \underbrace{y_i - w_i}_{\text{flow profits}} + \underbrace{\left(s_i + \sum_{j\in\mathcal{N}}\sigma^e_{ij}(t)p_j(t)\right)}_{\text{on-the-job adjusted separation rate}}[V_i(t) - J_i(t)] + \dot{J}_i(t) \tag{18}$$

The main differences stems from the job separation rate $s_i + \sum_{j\in\mathcal{N}} g_{ij}\sigma^e_{ij}(t)p_j(t)$, which accounts for the possibility that the employed worker leaves because she has matched with another firm.

The value of a vacant job in occupation $i$ at date $t$ solves the Bellman equation

$$rV_i(t) = -c_i + q(\theta_i)[J_i(t) - V_i(t)] + \dot{V}_i(t) \tag{19}$$

It is the same as the Bellman equation in the tractable model.

## 12.3 Search efforts

In this subsection, I provide an analytical characterization of the equilibrium search time and search effort of both unemployed and employed workers.

**Lemma** (Equilibrium search time of unemployed workers)

*The equilibrium search time of unemployed worker in occupation $i$ searching in occupation $j$ at date $t$ writes*

$$\tau_{ij}^u(t) = \frac{\left(g_{ij}p_j(t)\max\{E_j(t) - U_i(t), 0\}\right)^{\psi+1}}{\sum_k \left(g_{ik}p_k(t)\max\{E_k(t) - U_i(t), 0\}\right)^{\psi+1}}.$$

*Proof:* The first order condition of the unemployed worker problem writes

$$g_{ij}(E_j(t) - U_i(t))\tau_{ij}^u(t)^{\frac{-1}{\psi+1}} = \Lambda_i^u(t),$$

where $\Lambda_i^u(t) \geq 0$ is the Lagrange multiplier associated with the time constraint $\sum_k \tau_{ik}^u = 1$. Intuitively, the optimal search time equalizes the marginal return and the marginal cost of searching.

Rearranging terms, we get

$$\tau_{ij}^u(t) = \left(\frac{g_{ij}(E_j(t) - U_i(t))}{\Lambda_i^u(t)}\right)^{\psi+1}.$$

Summing over $j$ and using the time constraint, we obtain

$$\Lambda_i^u(t)^{\psi+1} = \sum_j \left(g_{ij}(E_j(t) - U_i(t))\right)^{\psi+1}.$$

This yields the desired expression. From this, we can derive optimal search effort:

$$\sigma_{ij}^u(t) = {\tau_{ij}^u}^{\frac{\psi}{\psi+1}} = \frac{\left(g_{ij}p_j(t)\max\{E_j(t) - U_i(t), 0\}\right)^{\psi}}{\left[\sum_k \left(g_{ik}p_k(t)\max\{E_k(t) - U_i(t), 0\}\right)^{\psi+1}\right]^{\frac{\psi}{\psi+1}}}.$$

Let me turn now to the equilibrium search time of employed workers, which has very similar expression from that of unemployed workers.

**Lemma.** (Equilibrium search time of employed workers)

*The equilibrium search time of employed worker in occupation $i$ searching in occupation $j$ at date $t$ writes*

$$\sigma_{ij}^e(t) = {\tau_{ij}^e}^{\frac{\psi}{\psi+1}} = \frac{\eta_i^{\frac{\psi}{\psi+1}}\left(g_{ij}p_j(t)\max\{E_j(t) - E_i(t), 0\}\right)^{\psi}}{\left[\sum_k \left(g_{ik}p_k(t)\max\{E_k(t) - E_i(t), 0\}\right]^{\psi+1}\right]^{\frac{\psi}{\psi+1}}}$$

**Comparison with discrete choice models.** The formulation of optimal search efforts mirrors choice probabilities in *discrete choice models* with Fréchet taste shocks. In such models, the probability that a worker in $i$ chooses occupation $j$ at date $t$ is $\mathbb{P}(j \mid i)(t) = \frac{v_{ij}(t)^{\psi}}{\sum_k v_{ik}(t)^{\psi}}$, where $v_{ij}(t)$ is the expected payoff of occupation $j$ for workers in $i$ at date $t$ and $\psi$ is the shape parameter of the Fréchet distribution.

However, the endogenous search setting differs significantly from discrete choice models in several key aspects. Firstly, normalization constants in discrete choice models must sum to one, which is not required in endogenous search. Secondly, the endogenous search model preserves the linearity of the Bellman equation in terms of transition rates, unlike the non-linear aggregation in discrete choice settings:

$$rU_i(t) = b_i + \left( \sum_k (g_{ij} p_j(t) \max\{E_j(t) - U_i(t)\})^{\psi+1} \right)^{\frac{1}{\psi+1}}.$$

The linearity of the Bellman equation enhances tractability and simplifies estimation of values.

## 12.4 Wages

**Lemma** (Equilibrium wages)

*The equilibrium wage in occupation $i$ at date $t$, denoted $w_i(t)$, is given by*

$$w_i(t) = \phi_i y_i + (1 - \phi_i)\Big(b_i + \sum_j (\mu^u_{ij}(t) - \mu^e_{ij}(t))(E_j(t) - U_i(t))\Big)$$

*Proof:* The equilibrium wage in occupation $i$ at date $t$ maximizes the generalized Nash product $\max_{w_i(t)}(E_i(t) - U_i(t))^{\phi} J_i(t)^{1-\phi}$. The FOC of the maximization program above gives the so-called Nash bargaining condition $(1 - \phi)(E_i(t) - U_i(t)) = \phi J_i(t)$. This is implicitly a function of the wage, through the value functions $E_i(t)$ and $J_i(t)$.

We now use the Bellman equation to make explicit the dependance of the value functions on the wage. The value of a filled job to the firm in $i$ at date $t$, denoted $J_i(t)$, solves $\left(r + s_i + \sum_j \mu^e_{ij}(t)\right) J_i(t) = y_i - w_i(t) + \dot{J}_i(t)$ and the value of being employed in occupation $i$ at date $t$, denoted $E_i(t)$, solves $\left(r + s_i + \sum_j \mu^e_{ij}(t)\right) E_i(t) = w_i(t) + \sum_j \mu^e_{ij}(t) E_j(t) + s_i U_i(t) + \dot{E}_i(t)$ Therefore, the

net value of employment in occupation $i$ at date $t$ is $\left(r + s_i + \sum_j \mu_{ij}^e(t)\right)(E_i(t) - U_i(t)) = w_i(t) - rU_i(t) + \sum_j \mu_{ij}^e(t)(E_j(t) - U_i(t)) + \dot{E}_i(t)$. We can further plug the flow value of unemployment $rU_i(t)$ from the Bellman equation, yielding $\left(r + s_i + \sum_j \mu_{ij}^e(t)\right)(E_i(t) - U_i(t)) = w_i(t) - b_i + \sum_j \mu_{ij}^e(t)(E_j(t) - U_i(t)) - \sum_j \mu_{ij}^u(t)(E_j(t) - U_i(t)) + \dot{E}_i(t) - \dot{U}_i(t)$. Plugging both expressions in the Nash bargaining condition and dividing by the effective discount rate gives $(1-\phi_i)\left(w_i(t) - b_i + \sum_j \mu_{ij}^e(t)(E_j(t) - U_i(t)) - \sum_j \mu_{ij}^u(t)(E_j(t) - U_i(t)) + \dot{E}_i(t) - \dot{U}_i(t)\right) = \phi_i(y_i - w_i(t) + \dot{J}_i(t))$. Note that the time-derivative terms cancel out. Indeed, taking the derivative of the Nash condition with respect to time gives $(1-\phi)(\dot{E}_i(t) - \dot{U}_i(t)) = \phi \dot{J}_i(t)$. Finally, we isolate the wage on the left-hand side and we obtain the desired result.

## 12.5 Vacancy posting

**Lemma** (Equilibrium tightness ratios)

*The equilibrium tightness ratio in occupation $i$ at date $t$, denoted $\theta_i(t)$, is given by*

$$\theta_i(t) = \left(\frac{\zeta_i}{c_i}\frac{1-\phi}{\phi}(E_i - U_i)\right)^{\frac{1}{\alpha}}$$

*Proof:* The equilibrium tightness ratio in occupation $i$ at date $t$ satisfies the free entry condition $V_i(t) = 0$. Plugging this condition in the Bellman equation for the value of vacancy, I get $\frac{c_i}{q(\theta_i(t))} = J_i(t)$. Given $q_i(t) = q(\theta_i(t)) = \zeta_i \theta_i(t)^{-\alpha}$, we get $\theta_i(t) = \left(\frac{\zeta_i}{c_i} J_i(t)\right)^{\frac{1}{\alpha}}$. Making use of the Nash bargaining condition, we have $J_i(t) = \frac{1-\phi}{\phi}(E_i(t) - U_i(t)$. Replacing in the expression above gives the desired result.

**Corollary** (Equilibrium job finding rates)

*The equilibrium job finding rates are given by*

$$p_i(t) = \zeta_i \theta_i(t)^{1-\alpha} = \xi_i \left((E_i(t) - U_i(t))\right)^{\frac{1-\alpha}{\alpha}} \quad \text{with} \quad \xi_i \equiv \zeta_i^{\frac{1}{\alpha}} c_i^{\frac{\alpha-1}{\alpha}} \left(\frac{1-\phi}{\phi}\right)^{\frac{1-\alpha}{\alpha}}$$

## 12.6 Laws of motion

How do workers reallocate across occupations?

**Variations in employment.** The employment in occupation $i$ at date $t$ evolves as follows

$$\dot{e}_i(t) = p_i(t) \underbrace{\left( \sum_{j \in \mathcal{N}} \sigma^u_{ji}(t) u_j(t) + \sigma^e_{ji}(t) e_j(t) \right)}_{\text{search effort by neighboring u and e workers}} - \underbrace{\left( s_i + \sum_{j \in \mathcal{N}} \sigma^e_{ij}(t) p_j(t) \right)}_{\text{OJS-adjusted separation rate}} e_i(t)$$

The variation in employment for occupation $i$ at time $t$ is determined by the balance of new hires and job destructions. The *flow of new hires* is the product of the job finding rate per unit of search effort and the total search effort from neighboring employed and unemployed workers targeting occupation $i$. Conversely, the *flow of job destruction* is the product of the separation rate and the number of employed workers in occupation $i$, adjusted for on-the-job search.

**Variations in unemployment.** The unemployment in occupation $i$ at date $t$ evolves as follows

$$\dot{u}_i(t) = s_i e_i(t) \quad - \quad \underbrace{\left( \sum_{j \in \mathcal{N}} \sigma^u_{ij}(t) p_j(t) \right)}_{\text{unemployment outflow rate adjusted for search effort}} \quad u_i(t)$$

As above, the variation in unemployment for occupation $i$ at date $t$ reflects the difference between job destructions and hires. The *flow of job destruction* is the product of the separation rate—unadjusted for on-the-job search—with the number of employed workers in occupation $i$. Conversely, the *flow of hires* in neighboring occupations is the product of the outflow rate from unemployment—adjusted for search effort—by the mass of unemployed workers originally from occupation $i$.

## 12.7 The Master equation

In this subsection, I show that the dynamics of the whole economy can by described by a single matrix differential equation.

**Lemma 4.** (The Master equation)
*Let $\boldsymbol{V}(t) = (E_1(t), \ldots, E_N(t), U_1(t), \ldots, U_N(t))'$ denote the vector of worker values at date $t$. The vector of worker values $\boldsymbol{V}(t)$ is sufficient to characterize the economy over the transition, and evolves as*

$$\dot{\boldsymbol{V}}(t) = \boldsymbol{G}(\boldsymbol{V}(t))$$

*with $\boldsymbol{G}(\cdot)$ defined as follows*

$$\boldsymbol{G}_i(\boldsymbol{V}) = rE_i(t) - w_i(\boldsymbol{V}(t)) - \sum_j \mu^e_{ij}(\boldsymbol{V}(t))(E_j(t) - E_i(t)) - s_i(U_i(t) - E_i(t)) \tag{20}$$

$$\boldsymbol{G}_{i+N}(\boldsymbol{V}) = rU_i(t) - b_i - \sum_j \mu^u_{ij}(\boldsymbol{V}(t))(E_j(t) - U_i(t)) \tag{21}$$

*and where*

$$w_i(\boldsymbol{V}(t)) = \phi_i y_i + (1-\phi_i)\Big(b_i + \sum_j (\mu^u_{ij}(\boldsymbol{V}(t)) - \mu^e_{ij}(\boldsymbol{V}(t)))(E_j(t) - U_i(t))\Big) \tag{22}$$

$$\mu^u_{ij}(\boldsymbol{V}(t)) = \frac{g_{ij}^{\psi+1}\zeta_j^{\psi+1}\theta_j^{(1-\alpha)(\psi+1)}(\boldsymbol{V}(t))(E_j(t) - U_i(t))_+^{\psi}}{\sum_k g_{ik}^{\psi+1}\zeta_k^{\psi+1}\theta_k^{(1-\alpha)(\psi+1)}(\boldsymbol{V}(t))(E_k(t) - U_i(t))_+\big)^{\psi}} \tag{23}$$

$$\mu^e_{ij}(\boldsymbol{V}(t)) = \frac{\eta_i g_{ij}^{\psi+1}\zeta_j^{\psi+1}\theta_j^{(1-\alpha)(\psi+1)}(\boldsymbol{V}(t))(E_j(t) - E_i(t))_+^{\psi}}{\sum_k g_{ik}^{\psi+1}\zeta_k^{\psi+1}\theta_k^{(1-\alpha)(\psi+1)}(\boldsymbol{V}(t))(E_k(t) - E_i(t))_+\big)^{\psi}} \tag{24}$$

$$\theta_i(\boldsymbol{V}(t)) = \varsigma_j(E_i(t) - U_i(t))^{\frac{1}{\alpha}} \tag{25}$$

*where $\varsigma_j$ is a constant.*

*Proof:* The lemmas above show that all control variables – eg. transition rates, wages and tightness – can be expressed as a function of worker values only. Collecting all equations from these lemmas, I can to construct $\boldsymbol{G}(.)$.

**Proposition** (Linear approximation of the Master equation)

*We compute the total differential of the Master equation with respect to worker values, evaluated at $\boldsymbol{V}_0$. For expositional clarity, we drop the dependence on $\boldsymbol{V}_0$ for all functions.*[49] *Moreover, we define: $\hat{x} = \frac{dx}{x}$. The total differential writes*

$$df_i = rdE_i - dw_i - \sum_j \mu^e_{ij}\Delta^e_{ij}\Big(\hat{\mu}^u_{ij} + \hat{\Delta}^e_{ij}\Big)$$

$$df_{N+i} = rdU_i - \sum_j \mu^u_{ij}\Delta_{ij}\Big(\hat{\mu}^u_{ij} + \hat{\Delta}^u_{ij}\Big)$$

[49] For example, $w_i = w_i(\boldsymbol{V}_0)$, etc.

*with*

$$
\begin{aligned}
d w_i &= (1-\phi)\sum_j \mu^u_{ij}\Delta^u_{ij}\Big(\hat{\mu}^u_{ij}+\hat{\Delta}^u_{ij}\Big) - (1-\phi)\sum_j \mu^e_{ij}\Delta^u_{ij}\Big(\hat{\mu}^e_{ij}+\hat{\Delta}^u_{ij}\Big)\\
\hat{\mu}^u_{ij} &= d\log\mu^u_{ij} = (\psi+1)\hat{p}_j + \psi\hat{\Delta}^u_{ij} - \psi\sum_j \sigma^u_{ij}(\hat{p}_k+\hat{\Delta}^u_{ik})\\
\hat{\mu}^e_{ij} &= d\log\mu^e_{ij} = (\psi+1)\hat{p}_j + \psi\hat{\Delta}^e_{ij} - \psi\sum_j \sigma^e_{ij}(\hat{p}_k+\hat{\Delta}^e_{ik})\\
\hat{\theta}_i &= \frac{1}{\alpha}\hat{\Delta}^u_{ii}\\
\hat{\Delta}^u_{ij} &= \frac{dE_i - dU_i}{\Delta^u_{ij}} \quad \text{and} \quad \hat{\Delta}^e_{ij} = \frac{dE_i - dE_i}{\Delta^e_{ij}}
\end{aligned}
$$

*Proof:* We begin by computing the total differential of $f_{N+i}$. The formula for the total differential of $f_i$ and the total differential of the wage follows the same steps. We have

$$
\begin{aligned}
df_{N+i} &= d\Big(rU_i - \sum_j \mu^u_{ij}\Delta^u_{ij}\Big)\\
&= rdU_i - \sum_j d\left(\mu^u_{ij}\Delta^u_{ij}\right)
\end{aligned}
$$

where the second equality follows from the linearity of the differential operator. Now, we exploit the fact that

$$
\begin{aligned}
d\left(\mu^u_{ij}\Delta^u_{ij}\right) &= d\mu^u_{ij}\cdot\Delta^u_{ij} + \mu^u_{ij}\cdot d\Delta^u_{ij}\\
&= \frac{d\mu^u_{ij}}{\mu^u_{ij}}\mu^u_{ij}\Delta^u_{ij} + \frac{d\Delta^u_{ij}}{\Delta^u_{ij}}\mu^u_{ij}\Delta^u_{ij}\\
&= \mu^u_{ij}\Delta^u_{ij}\left(\hat{\mu}^u_{ij}+\hat{\Delta}^u_{ij}\right)
\end{aligned}
$$

Plugging in the expression above gives the desired expression for the Bellman equation of the value of unemployment.

Now, we compute the hat-change of transition rates from unemployment to employment. The formula for the hat-change of transition rates from employment to employment and the hat-change

of job finding rates follows the same steps. We exploit the fact that

$$\hat{\mu^u_{ij}} = \frac{d\mu^u_{ij}}{\mu^u_{ij}} = d\log\mu^u_{ij}$$

This substantially simplifies computation because log transforms products into sums. Plugging the expression of $\mu^u_{ij}$ we find

$$\hat{\mu^u_{ij}} = d\left((\psi+1)\log g_{ij} + (\psi+1)\log p_j + \psi\log\Delta^u_{ij} - \log N_i\right) = (\psi+1)\hat{p_j} + \psi\hat{\Delta}^u_{ij} - \hat{N_i}$$

where $N_i \equiv \sum_{ik}(g_{ik}p_k\Delta_{ik})^{\psi+1}$ is the normalization constant ensuring that the search probabilities of unemployed workers sum to one: that is, $\sum_{ik}\tau^u_{ik} = 1$. We recover $\hat{N_i}$ by differentiating the constraint, which gives

$$\begin{aligned}
&d\sum_k \tau^u_{ik} = 0\\
\Leftrightarrow &\sum_k \tau^u_{ik}\hat{\tau}^u_{ik} = 0\\
\Leftrightarrow &\sum_k \tau^u_{ik}\left((\psi+1)\hat{p}_k + (\psi+1)\hat{\Delta}^u_{ik} - \hat{N_i}\right) = 0\\
\Leftrightarrow &\hat{N_i} = (\psi+1)\sum_k \tau^u_{ik}\left(\hat{p}_k + \hat{\Delta}^u_{ik}\right)
\end{aligned}$$

where the second equality uses same trick as above, namely: $d\tau^u_{ik} = \tau^u_{ik}\frac{d\tau^u_{ik}}{\tau^u_{ik}} = \tau^u_{ik}\hat{\tau}^u_{ik}$. The third line replaces the hat-change of search efforts $\tau^u_{ik}$ by its expression, which can be found using the same steps as above. The fourth line makes use of the fact that $\sum_k \tau_{ik} = 1$. Plugging $\hat{N_i}$ in the equation above gives the desired expression.

These total differentials can be stored into a single Jacobian matrix denoted $\boldsymbol{D_G}(\boldsymbol{V})$.

## 12.8 Stability, existence and uniqueness

The stability and uniqueness of the equilibrium depend on the properties of this Jacobian matrix.

**Theorem.** (Stability)

*The values are instable over time. Therefore, the only stable solution is that values immediately*

*jump to steady-state and stay constant afterwards.*

*Proof:* The Master equation can be rewritten as: $\frac{d\boldsymbol{V}_t}{dt} = \left[r\boldsymbol{I} - \boldsymbol{Q}(\boldsymbol{V}_t)'\right]\boldsymbol{V}_t$. The transition rate matrix $\boldsymbol{Q}$ depends on worker values through endogenous search efforts and tightness ratios, leading to a *non-linear* matrix differential equation, which is hard to solve in general.

To proceed, we discretize time over a small interval $\Delta$. The Master equation becomes: $\frac{\boldsymbol{V}_{t+\Delta}-\boldsymbol{V}_t}{\Delta} = \left[r\boldsymbol{I} - \boldsymbol{Q}(\boldsymbol{V}_t)'\right]\boldsymbol{V}_t$ which rearranges to: $\boldsymbol{V}_{t+\Delta} = \boldsymbol{A}(\boldsymbol{V}_t)\boldsymbol{V}_t$, where $\boldsymbol{A}(\boldsymbol{V}_t) \equiv \boldsymbol{I} + \Delta\left(r\boldsymbol{I} - \boldsymbol{Q}(\boldsymbol{V}_t)'\right)$.

Assuming that $\boldsymbol{V}_t \neq \boldsymbol{V}_\infty$ (i.e., not at steady state), we have $\boldsymbol{A}(\boldsymbol{V}_t) \neq \boldsymbol{I}$. I show that this implies either values converge to zero or grow without bound: $\|\boldsymbol{V}_{t+\Delta}\| > \|\boldsymbol{V}_t\|$. We have: $\boldsymbol{A}(\boldsymbol{V}_t)^{-1}\boldsymbol{V}_{t+\Delta} = \boldsymbol{V}_t$, and taking the $\infty$-norm: $\|\boldsymbol{A}(\boldsymbol{V}_t)^{-1}\boldsymbol{V}_{t+\Delta}\|_\infty = \|\boldsymbol{V}_t\|_\infty$. By properties of the vector norm, we have: $\|\boldsymbol{A}(\boldsymbol{V}_t)^{-1}\|_\infty \cdot \|\boldsymbol{V}_{t+\Delta}\|_\infty \geq \|\boldsymbol{V}_t\|_\infty$, which gives: $\|\boldsymbol{V}_{t+\Delta}\|_\infty \geq \|\boldsymbol{V}_t\|_\infty \cdot \|\boldsymbol{A}(\boldsymbol{V}_t)^{-1}\|_\infty^{-1}$.

Assuming $\|\boldsymbol{V}_t\|_\infty \to l$ with $l \neq 0$ or $+\infty$, this implies: $l \geq \|\boldsymbol{A}(\boldsymbol{V}_t)\|_\infty l$, which simplifies to: $\|\boldsymbol{A}(\boldsymbol{V}_t)\|_\infty \leq 1$. However, I will show that $\|\boldsymbol{A}(\boldsymbol{V}_t)\|_\infty > 1$, contradicting this assumption and implying that $l$ must be either $0$ or $+\infty$.

By definition of the $\infty$-norm, $\|\boldsymbol{A}(\boldsymbol{V}_t)\|_\infty = \max_i |\lambda_i|$, where $\lambda_i$ is the $i$-th eigenvalue of $\boldsymbol{A}(\boldsymbol{V}_t)$. I show that $\lambda_i > 1$ for all $i$, and thus $\max_i \lambda_i > 1$. The eigenvalues $\nu$ of $\boldsymbol{Q}(\boldsymbol{V}_t)$ satisfy $\nu \leq 0$. Since the eigenvalues of a matrix are the same as its transpose, $\nu$ is also an eigenvalue of $\boldsymbol{Q}(\boldsymbol{V}_t)'$. Thus, $r - \nu > 0$ is an eigenvalue of $r\boldsymbol{I} - \boldsymbol{Q}(\boldsymbol{V}_t)'$, and $1 + \Delta(r - \nu) > 1$ is an eigenvalue of $\boldsymbol{A}(\boldsymbol{V}_t)$, completing the proof.

**Existence and uniqueness.** Let me turn now to the existence and uniqueness of a vector of steady-state worker values. The steady-state vector of worker values solves $\boldsymbol{0} = \boldsymbol{G}(\boldsymbol{V})$ This can be rewritten as a fixed-point problem

$$\boldsymbol{V} = \boldsymbol{H}(\boldsymbol{V})$$

where $\boldsymbol{H}(\boldsymbol{V}) = \boldsymbol{G}(\boldsymbol{V}) + \boldsymbol{V}$. Let $\boldsymbol{D_H}(\boldsymbol{V}) = \boldsymbol{D_H}(\boldsymbol{V}) + \boldsymbol{I}$ denote the associated Jacobian.

**Theorem.** (Existence and uniqueness of steady-state worker values)
*The vector of steady-state worker values always exists. Furthermore, assume that for any* $\boldsymbol{V}$

$$\Re[\lambda(\boldsymbol{D_H})](\boldsymbol{V}) \neq 1$$

*Then the steady-state vector of worker values is unique. Consequently, steady-state worker search efforts, wages and tightness ratios exist and are unique too.*

*Proof:* We now prove the existence of a solution to the fixed-point equation using the Poincaré-Miranda theorem, which is equivalent to the Brouwer fixed-point theorem commonly applied in economics.

The Poincaré-Miranda theorem states: Consider $N$ functions $f_1, \ldots, f_N$ of $N$ variables $x_1, \ldots, x_N$, each bounded by $l_i$ and $L_i$. If $f_i \geq 0$ when $x_i = l_i$ and $f_i \leq 0$ when $x_i = L_i$, then there exists a point where all functions $f_i$ are zero simultaneously.

In our case, we consider $2N$ functions $\{f_i, f_{N+i}\}_i$ of $2N$ variables $\{E_i, U_i\}$, where $E_i$ and $U_i$ are worker values. The lower bound is $l_i = 0$, since worker values cannot be negative, and the upper bound is $L_i = \max_j \frac{y_j}{r}$, the value a worker receives if employed indefinitely at the highest wage a firm is willing to pay.

At the lower bound $U_i = 0$, we have

$$f_{N+i}(0) = -b_i - \sum_j \mu_{ij}^u(\boldsymbol{V}) E_j < 0,$$

since $b_i > 0$, $\mu_{ij}^u > 0$, and $E_j > 0$ for all $i, j$. Similarly, for $E_i = 0$, the same inequality holds.

At the upper bound $U_i = \max_j \frac{y_j}{r}$, we have

$$f_{N+i}\left(\max_j \frac{y_j}{r}\right) \geq r \max_j \frac{y_j}{r} + \sum_j \mu_{ij}^u \left(\max_j \frac{y_j}{r} - E_j\right) > 0,$$

where the second term is positive because $E_j \leq \max_j \frac{y_j}{r}$ by definition of the upper bound.

Thus, the conditions of the Poincaré-Miranda theorem are satisfied, ensuring the existence of a vector where all functions $f_i = 0$ simultaneously. Therefore, a fixed point—and consequently a steady-state vector of worker values—exists.

Finally, uniqueness follows directly from a result in Kellogg (1976).

I am unable to provide an analytical demonstration of this assumption, but numerical computations show that this assumption is always verified.

## 12.9 Reservation wages and network centrality

**Definition.** (Katz-Bonacich centrality)

*Given a graph with adjacency matrix $\boldsymbol{G}$, the index $K_i$ of Katz-Bonacich centrality of the node $i$ is*

$$K_i(\boldsymbol{G}, \boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}) = \alpha_i + \sum_j \gamma_i \beta_j g_{ij} K_j(\boldsymbol{G}, \boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma})$$

The Katz-Bonacich centrality index of node $i$ is the sum of two terms. The first component is a node-specific term $\alpha_i$, which provides an exogenous centrality value to the node. The second component is a linear combination of the Katz-Bonacich centrality indices of node $i$'s neighbors, reflecting how much centrality is conveyed by neighboring nodes. The parameters $\gamma_i$ and $\beta_j$ control how much this effect is dampened as it propagates through the network.

**Proposition.** (Distribution of reservation wages)

*Assume no on-the-job search. The steady-state reservation wages are equal to their occupation's index of Katz-Bonacich centrality in the appropriate network, and they solve the recursion*

$$\bar{w}_i = b_i + \frac{\sum_{k\in\mathcal{N}} \mu^u_{ik}}{r + \sum_{k\in\mathcal{N}} \mu^u_{ik}} \left( \sum_{j\in\mathcal{N}} \frac{\mu^u_{ij}}{\sum_k \mu^u_{ik}} \frac{\phi_j r}{r + s_j} y_j - b_i \right) + \sum_{j\in\mathcal{N}} \frac{1}{r + \sum_k \mu^u_{ik}} \frac{r(1-\phi_j) + s_j}{r + s_j} \mu^u_{ij} \bar{w}_j \quad (26)$$

*Proof:*

The worker reservation wage is closely linked to the Katz-Bonacich centrality index of their occupation, because it follows a similar a recursive relationship. Here, the reservation wage in an occupation is high if it is connected to occupations with high reservation wages too. Indeed, a worker's reservation wage is a function of the wages she could potentially earn in neighboring occupations. Thanks to the Nash sharing rule, the former is a function of the reservation wages in these same neighboring occupations.

A crucial implication is that, everything else being equal, occupations more central within the network have larger outside options. Workers in central occupations enjoy many reallocation possibilities should the bargaining with the firm fail. They can leverage this better outside option in order to secure higher wages. In fact, this prediction is robust to assuming on-the-job search. The wage is still determined by Katz-Bonacich centrality in the general case, though within a more

complex network.

# 13 Appendix E: Estimation

Following Head and Mayer (2014) recommendation, I first implement a demean to reduce the number of fixed effects before running the two-way fixed effects Poisson regression.

First, I drop all observations with no transitions or switching gains over the entire period. Let me briefly explain this sample restriction. Intuitively, occupation pairs with no transitions or gains throughout the time period provide very little information and are likely true "zeros" that can be safely discarded. In contrast, pairs that alternate between zero and positive transitions are informative, and these "zeros" should be retained.

Second, I divide by the average transition over the entire period. This is feasible due to the previous sample restriction. Crucially, I retain the zero transitions for occupation pairs that experienced at least one transition during the time interval. This demeaning procedure allows me to remove time-invariant fixed effects, which are numerous (around 40,000).

Finally, this gives

$$\frac{\mu_{ijt}}{\overline{\mu_{ij}}} = \exp\left\{\log\left(\frac{N_{it}^{-1}}{\overline{N_i^{-1}}}\right) + (\psi+1)\log\left(\frac{p_{jt}}{\overline{p_j}}\right) + \psi\log\left(\frac{\Delta_{ijt}}{\overline{\Delta_{ij}}}\right)\right\} \tag{27}$$

In other words, the gravity equation be rewritten as a Poisson regression with time-varying origin and destination fixed effects. Using standard packages, the search elasticity is then simply estimated as the coefficient associated to switching gains.

# 14 Appendix F: Application to Robots

## 14.1 Occupation exposure to robots

Webb's methodology for measuring occupation exposure involves three steps. First, he groups patents by technology class (e.g., robots or software) and identifies the tasks these patents perform. Second, he examines the tasks carried out by various occupations using US occupation data and

compares them to the tasks performed by the patents. Third, he calculates the overlap between the tasks performed by patents and workers, ranking occupations based on the degree of overlap.

To apply his measure to the French labor market, I map the US ONET occupation categories to the French nomenclature of occupations, specifically FAP. This process involves several crosswalks: first, from ONET to the European nomenclature ISCO, then from ISCO to PCS, and finally from PCS to FAP. After mapping, I rank the occupations based on their exposure and normalize these ranks between zero and one, with zero corresponding to the least exposed occupations. This provides the baseline measure of occupation exposure to various new technologies, and in particular robots.

## 14.2 Calibrating productivity shocks

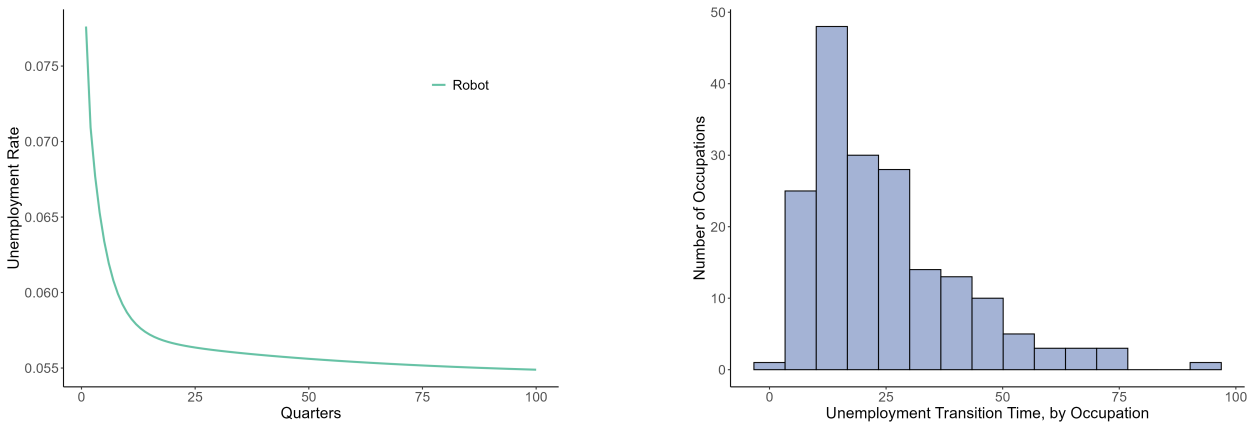
Solving for productivity shocks presents a complex, high-dimensional problem. Indeed, the wage changes induced by productivity shocks cannot be computed in isolation due to their interconnections within the occupational network: wage adjustments in one occupation influence the reservation wages of adjacent occupations, subsequently affecting their wages, and so forth. This interdependence quickly leads to the curse of dimensionality. To circumvent this issue, and because it is not the heart of the paper, I assume the outside option channel is quantitatively small: that is, $dw_i = \phi dy_i + (1-\phi)d\bar{w}_i \approx \phi dy_i$, where $\bar{w}_i$ is the reservation wage in occupation $i$. If worker bargaining power is not too large, this approximation is reasonable.

## 14.3 Unemployment effects

**Aggregate unemployment.** How does aggregate unemployment respond to the robot shock? The aggregate unemployment rate quickly decreases to its new lower steady-state level.

Figure 14.1.a plots the dynamics of the aggregate unemployment rate after the robot shock. There are two main take-aways. First, the figure shows that aggregate unemployment quickly adjusts following the robot shock, with a transition time of only a few quarters. This contrasts with the pattern observed when examining the worker distribution and suggests that the aggregate unemployment rate might be a misleading metric of the underlying movements across disaggregated labor markets.

Second, the robot shock leads to a decrease in the long-term aggregate unemployment rate.




(a) Aggregate unemployment rate

(b) Unemployment transition time, by occupation

Figure 6: Unemployment over the transition

Intuitively, this is because robots reduce employment in high-unemployment occupations, such as manufacturing, while increasing employment in low-unemployment occupations, like services. Perhaps surprisingly, the economy does not experience a temporary increase in unemployment due to the worker reallocation dynamics.

**Heterogeneity in unemployment adjustment speed.** Does unemployment adjust uniformly across occupations? The rapid adjustment of aggregate unemployment actually masks substantial heterogeneity in unemployment adjustment at the occupation level.

I define the unemployment transition time at the occupation level similarly to the occupation-level reallocation times as $T_i^u = \frac{1}{\max_h |u_i(h) - u_i|} \int_0^{+\infty} |u_i(t) - u_i| dt$.

Figure 14.1.b plots the distribution of unemployment transition times for each occupation after the robot shock. There are two main takeaways: First, it shows that, on average, the transition time of unemployment at the occupation level is much longer than at the aggregate level. Intuitively, this is because upward and downward adjustments at the occupation level cancel out in the aggregate, masking the slower adjustments occurring within individual occupations.

Second, there is significant heterogeneity in unemployment transition times across occupations. While the majority of occupations adjust relatively quickly, a small minority adjust very slowly. These occupations typically include journalists, unskilled workers in metallurgy, banking technicians, and architects. These occupations are spread throughout the network, encompassing both expanding and declining occupations, whether central or peripheral.

## 14.4 Targeted subsidies

In practice, I define declining occupations and expanding occupations as the occupation with in the top or bottom deciles of productivity changes. Bridge occupations are defined as the occupation with the top $5\%$ indices of betweenness centrality in the FAP occupation network, where betweenness centrality is computed using declining occupations as sources and expanding occupations as targets. In other words, it represents the occupations *in-between* the declining and expanding occupations.