Dynamic Carbon Emission Management*

M. Cecilia Bustamante† Francesca Zucchi‡

January 29, 2026

Abstract

Carbon regulation poses the corporate challenge of developing optimal carbon management policies. We provide a unified model characterizing how firms manage emissions through production, heterogeneous green investment, and the trading of carbon credits. We show that carbon pricing incentivizes firms to reduce emissions through immediate yet transient abatement projects, but has an ambiguous impact on green innovation. In economies where carbon pricing discourages innovation, subsidies to green innovation complement—rather than substitute—carbon pricing, jointly lowering current emissions through abatement projects and accelerating the transition to greener technologies through innovation.

Keywords: Carbon pricing, Carbon Emissions, Carbon Abatement, Green Innovation, Sustainability.

JEL Classification Numbers: G30; G31; G12; D62; O33

*We thank Ufuk Akcigit, James Archsmith, Adelina Barbalau, Giorgia Barboni, Andreas Barth, Patrick Bolton, Alexander David, Zhenyu Gao, Peter Henry, Jeong Ho (John) Kim, Stefano Lovo, Adair Morse, Erwan Morellec, Lakshmi Naaraayanan, Conny Olovsson, Markus Parlasca, Loriana Pelizzon, Alessio Piccolo, Louis Preonas, Adriano Rampini, Stefano Sacchetto, Jan Schneemeier, Lin Shen, Javier Suarez, Rick Van der Ploeg, Ole Wilms, Federica Zeni, and participants at the CEPR/EUI/Bank of Italy/EABCN conference, the CEPR Endless Summer conference, CFEA Meetings, CICF, CREST-CMAP Quantitative Sustainable Economics and Finance Seminar, ECB, EDHEC, EFA, Esade spring workshop, the European Winter Finance Summit, FIRS, Goethe workshop "Financial Regulation – Going Green," HEC Paris, ITAM Finance Conference, LBS Summer Symposium, Northeastern University, 2nd Sustainable Finance Conference at the University of Luxembourg, the UBC Winter Finance Conference, the University of Maryland Climate Finance Symposium, the CEPR WE ARE IN conference (Riksbank), WFA 2024, and WU Vienna for useful feedback. Any remaining errors are our own. The views expressed in the paper are those of the authors and should not be interpreted as representing those of the European Central Bank or the Eurosystem.

†University of Maryland. E-mail: mcbustam@umd.edu

‡European Central Bank and CEPR. E-mail: francesca.zucchi@ecb.europa.eu

1 Introduction

Over the past decade, a pronounced divergence has emerged across countries in their approaches to climate policy. Whereas some jurisdictions have aimed to ambitious reductions in carbon emissions by adopting explicit regulatory instruments such as carbon taxes or emissions trading systems, other have forcefully stayed away or have even retreated from direct carbon regulation, leaving carbon management largely to voluntary initiatives rather than mandatory action. Despite this policy divide, firms in both types of jurisdictions have shown engagement in green investment of various profiles, calling for a deeper understanding of what drives firms to manage their emissions and the role of carbon regulation thereof.

We thus propose a novel theoretical framework to study a firm's rich set of corporate decisions which jointly determine the optimal *carbon management policy*, and then analyze how such corporate decisions feed into the design of environmental regulation. Starting from the canonical premise in finance that corporate decisions should maximize shareholder value, this paper characterizes the trade-offs faced by firms under the most common carbon regulatory systems—carbon tax and carbon trading systems—as well as under laissez-faire. A key yet overlooked aspect is that green investment is *heterogeneous*: Firms are increasingly trying to abate their current emissions *and* to move production to low-carbon technologies, two goals requiring investments of various horizons and cost profiles.[1] Accordingly, we propose a continuous-time dynamic model in which firms best handle carbon emissions by balancing the scale of production, the engagement in immediate abatement measures to offset emissions, and the investment in innovative solutions to permanently shift towards more sustainable technologies.

By acknowledging the existence of green investments of various types, the model allows both a firm's carbon emissions and its transition to more sustainable technologies to be endogenous, evolving over time as management maximizes shareholder value. On the one end, abatement projects make an immediate dent on a firm's carbon emissions by offsetting them, but their effect is short-lived as they do not affect the technological sustainability of

[1] See, for instance, the reports by McKinsey & Company (2022) and The World Bank (2024).

the firm's production processes. On the other end, green innovation permanently reduces emissions per unit of production by advancing a firm's technological sustainability *if* a breakthrough materializes, although breakthroughs take time and are uncertain. Crucially, we acknowledge that green breakthroughs bring along *productivity gains* that go above and beyond the savings on the costs directly associated with carbon pricing. Empirically, breakthroughs making the firm more sustainable have been documented to lower operating costs (Popp, 2001; Acemoglu, Aghion, Bursztyn, and Hemous, 2012)—for instance, associated with energy savings—and to spur greater customer demand and larger sales (Derrien, Krueger, Landier, and Yao, 2025; Meier, Servaes, Wei, and Xiao, 2025; Chiu, Hsu, Li, and Tong, 2025).

We solve the model in steps and start by considering a benchmark *laissez-faire* environment, featuring no carbon regulation. In this case, firms pay no cost associated with their carbon emissions and, thus, have no incentives to offset them through abatement projects. Yet, if technological sustainability brings along productivity gains (on top of reducing emissions per unit of production), then profit-maximizing firms have incentives to engage in green innovation even absent carbon pricing so to reap such gains and expand their production and revenues, consistent with the Jevons paradox (or "rebound effect") and the recent evidence in Bolton, Kacperczyk, and Wiedemann (2023).[2] The idea that firms invest in green innovation even absent carbon pricing is evocative of the current US landscape where, despite the lack of carbon pricing at the federal level, polluting firms engage in green innovation (see e.g., Cohen, Gurun, and Nguyen, 2025).[3]

Next, we study optimal carbon management under a carbon tax system. By pricing emissions, a carbon tax compels firms to internalize their environmental impact. As a result, firms not only produce less than under laissez-faire—thereby naturally reducing its gross

[2]The Jevons paradox (also known as rebound effect) occurs when technology increases the efficiency with which a resource is used, but the falling cost of use induces increases in demand enough that resource use is increased rather than reduced. The Khazzoom–Brookes postulate provides a consistent assessment. Recently, Bolton, Kacperczyk, and Wiedemann (2023) provide empirical support for the Jevons paradox in the context of green innovation.

[3]In a similar vein but focusing instead on banks, Morse and Sastry (2025) discuss conditions under which net zero lending commitments emerge as an endogenous choice of the bank even absent a regulatory push.

emissions—but also undertake abatement projects that immediately lower net emissions and, in turn, firms' carbon tax burden. In turn, the effect of the carbon tax on green innovation is more nuanced, as it reflects the interaction of two opposing forces. On the one hand, green breakthroughs reduce emissions for a given scale of production, lowering the firm's carbon tax liability and strengthening incentives to invest in green innovation. On the other hand, when such breakthroughs also bring along productivity gains, they enable the firm to expand output which, however, is less attractive under a carbon tax than under laissez-faire because the associated additional emissions must be paid for, then weakening innovation incentives. The model shows that either force can dominate, yielding an ambiguous cross-sectional relationship between carbon pricing and green innovation. Specifically, the carbon tax has a negative effect on green innovation in jurisdictions where it is low and, for a given carbon tax rate, for firms whose green breakthroughs reduce emissions only modestly. When abstracting from productivity gains, however, only the first strength is at work and, thus, a carbon tax unambiguously spurs green innovation.

We next consider the carbon trading system, in which costly carbon credits give firms the right to deploy a fixed volume of emissions into the atmosphere and, thus, produce.[4] Because carbon credits are tradable, firms in need of credits can buy them in the carbon markets, whereas firms with excess of credits may choose to sell them. We show that the key insights regarding optimal corporate policies under a carbon tax relative to laissez-faire continue to hold under a carbon trading system; namely, we show that the carbon trading system reduces production, spurs abatement, and has an ambiguous impact on green innovation due to the interaction of the two offsetting strengths described in the carbon tax case. In addition, we study the firm's optimal policies as a function of its stock of credits. We find that the firm manages production and green investment in a precautionary fashion, to avoid resorting to the costly carbon credit market.[5] When the firm has a low credit balance, it

[4]In different jurisdictions, this scheme has been labeled as "emissions trading scheme" (ETS) or "cap-and-trade." The most established ETS is the European Union's one (EU ETS). Other major ETS are those in China, the UK, Switzerland, and South Korea. In North America, there are subregional schemes in California, Quebec, and in Northeastern United States (so called "Regional Greenhouse Gas Initiative")

[5]This result aligns with previous findings in dynamic inventory models, such as Bolton, Chen, and Wang

cuts on production and increases its green investment to reduce its consumption of credits. The model reveals that a large balance of carbon credits *reduces* firm commitments to curb emissions, consistent with the evidence in De Jonghe, Mulier, and Schepens (2020).

Considering jointly our findings on the carbon trading and carbon tax systems, a number of important testable implications follow. First, compared to laissez-faire, carbon pricing systems are effective at reducing carbon emissions, consistent with the evidence in Fowlie, Holland, and Mansur (2012), Martin, Muuls, and Wagner (2016), Metcalf and Stock (2023), or Martinsson, Sajtos, Stromberg, and Thomann (2024). Our model describes the mechanism through which this is the case, stemming from both a reduction in production and a pickup in investment in abatement. More importantly, our analysis highlights a novel and previously unexplored effect of carbon pricing on green innovation. On top of incorporating the standard strength whereby carbon pricing encourages green innovation by lowering the cost of carbon regulation per unit of production, our framework identifies an additional strength which only arises when acknowledging the existence of productivity gains associated with green breakthroughs that go beyond the mere saving on carbon regulation—either because of (energy) cost savings or increased demand for greener products. As both strengths can dominate, our model can rationalize the mixed evidence on the impact of carbon pricing on green innovation, see, e.g., Brown, Martinsson, and Thomann (2022), Brown, Martinsson, Stromberg, and Thomann (2025), Shi, Zhou, and Meinerding (2023), and Kaenzig (2025). Our model suggests that overlooking productivity gains from sustainability may lead to a flawed assessment of the effect of carbon pricing on a firm engagement in green innovation, as well as the green investment mix.

Given that several jurisdictions around the world contemplate green subsidies as part of their policy toolkit, we then consider their impact on corporate policies through the lens of our model. We show that the provision of a subsidy to green innovation bundled with carbon pricing can support the twofold goal of reducing emissions in the present as well as support the transition to greener technologies. Irrespective of the carbon pricing

(2011) and Décamps, Mariotti, Rochet, and Villeneuve (2011).

system, such subsidy boosts a firm's investment in green innovation while preserving the firm's abatement efforts, overall increasing a firm's engagement in green investment. Thus, the introduction of such a subsidy shifts the green investment mix towards green innovation. Our model thus suggests a "stick and carrot" approach to environmental regulation, contemplating both carbon pricing (the stick) and green innovation subsidies (the carrot).

Our main predictions continue to hold once we endogenize carbon regulation and thus allow a benevolent regulator to optimize over both carbon pricing and an innovation subsidy, then taking into account the general equilibrium effect of firm-level policies. We confirm that carbon pricing can shift the green investment mix of polluting firms towards abatement when acknowledging productivity gains from green breakthroughs, though a subsidy to green innovation can mitigate this shift. Furthermore, the general equilibrium setting shows that carbon pricing and green innovation subsidies target distinct goals: Optimal carbon pricing guarantees that net emissions are reduced in the present already, whereas the optimal innovation subsidy ensures a faster, innovation-driven transition to sustainable technologies. While we are not the first work to suggest to pair up carbon pricing with subsidies, the rationale in our model is new.[6] In previous contributions, there is typically only one type of green investment to address environmental externalities, so carbon taxes and innovation subsidies act as *substitutes*. By considering different types of green investment, we identify a distinct role for carbon taxes and innovation subsidies, whereby the former regulates firm's immediate incentives to reduce net emissions through abatement, whereas the latter induces sustainable technological progress. Hence, carbon pricing and the green innovation subsidy effectively *complement* each other in equilibrium.

Lastly, consistent with the evidence showing the heterogeneous effects of climate regulation on firms (e.g.,, Martin, de Preux, and Wagner, 2014)—and in spite of the perceived conflict between the interests of environmental regulators and those of businesses—carbon regulation need not reduce firm value in our model. For firms with positive net emissions

[6]See, e.g., Acemoglu et al. (2012) or Acemoglu, Akcigit, Hanley, and Kerr (2016). From a different perspective, Oehmke and Opp (2025) suggest to pair up carbon taxes with subsidies in the presence of financing frictions. While in the current paper we abstract from this aspect, we investigate the role of financing frictions in the companion paper (Bustamante and Zucchi, 2025a).

(i.e., net polluters), carbon regulation is largely a cost, which lowers firm value and reduces investment in green innovation compared to laissez-faire. In turn, for firms with negative net emissions (i.e., net cleansers), carbon regulation can yield gains from selling credits or rewards on negative emissions, then raising firm value. Our analysis then corroborates the discussion among regulators and academics alike (see, e.g., Blanchard, Gollier, and Tirole, 2023) that carbon pricing should recognize and reward net cleansers, in line with the idea that negative emissions are necessary to achieve net zero pledges.

Related literature Our model lies at the intersection of models in corporate finance studying firm incentives to engage in green investment (Heinkel, Kraus, and Zechner, 2001; Dangl, Halling, Yu, and Zechner, 2024; Allen, Barbalau, and Zeni, 2024; Barbalau and Zeni, 2025; Hong, Wang, and Yang, 2023; Oehmke and Opp, 2025; Heider and Inderst, 2023, among many others) and macro models studying carbon regulation and directed technical change (Acemoglu et al., 2012, 2016; Gans, 2012; Aghion, Bergeaud, De Ridder, and Van Reenen, 2025). Our model contributes to the first strand by endogenizing both a firm's carbon emissions *and* its transition to greener technologies by allowing for green investments of various profiles, then uncovering novel implications on how carbon regulation jointly affects green innovation and the green investment mix. We also contribute to the second strand by showing that these novel implications continue to hold when closing the model in general equilibrium, uncovering the complementary nature of carbon pricing and subsidies to green innovation.

Our work provides theoretical grounds that rationalize the growing evidence on how carbon pricing affects green investment. Brown et al. (2025) find that carbon pricing unambiguously leads to greater investment in abatement projects, consistent with our model. Looking at green innovation, instead, the available evidence is mixed, which is also consistent with the model. On the one hand, Brown, Martinsson, and Thomann (2022) show carbon regulation leads high polluting firms adopt existing technologies rather than developing new clean ones.[7] Similarly, Shi, Zhou, and Meinerding (2023) find that greater stringency in an

[7]Specifically, they show that firms do not increase their patenting despite a greater engagement in R&D

ETS system does not spur the supply of green technology, and instead firms reduce emissions through other channels. On the other hand, Kaenzig (2025) finds that carbon policy shocks within the EU ETS have a positive effect on patenting, and Brown et al. (2025) report carbon pricing leads to higher green R&D investment in a sample of Swedish firms subject to high carbon prices. Our model can guide future work on this area by isolating the competing forces explaining the impact of carbon prices on green R&D.

Our focus on acknowledging productivity gains stemming from sustainability—above and beyond the savings on carbon regulation—relies on previous contributions in climate economics and are backed up by a growing evidence. Consistent with Popp (2001) and Acemoglu et al. (2012), we acknowledge that green technological breakthroughs reduce firms' operating costs, which can be due to a reduction in expenditures associated with energy, raw materials, and services (Porter and Van der Linde, 1995). Other works suggest instead that firms face higher demand for their products when consumers care about the firm's sustainability. In this strand, Derrien et al. (2025) show that negative environmental and social news trigger downgrades in earnings forecasts at all horizons, whereas Meier et al. (2025) show that firms with higher environmental and social ratings attain greater local sales.

Methodologically, our analysis in the carbon credit system relates to the dynamic inventory models in corporate finance. Previous contributions in this strand have focused on dynamic cash management, with notable contributions including Bolton, Chen, and Wang (2011, 2013), Décamps et al. (2011), Hugonnier, Malamud, and Morellec (2015), or Malamud and Zucchi (2019).[8] Our paper characterizes instead a novel type of corporate inventory management: The management of carbon credits, which are accumulated, bought, and sold.

The paper is organized as follows. Section 2 describes the model. Section 3 analyzes optimal corporate policies in a laissez-faire benchmark. Section 4 studies optimal corporate

activities, which they interpret as polluting firms not developing new clean products or technologies but rather expanding their capacity to adopt and assimilate external knowledge and technical knowhow.

[8]Other contributions in this strand are Décamps, Gryglewicz, Morellec, and Villeneuve (2017), Della Seta, Morellec, and Zucchi (2020), Zucchi (2024), Dai, Giroud, Jiang, and Wang (2024), or Bruegem, Marfe, and Zucchi (2025).

policies under the carbon tax system, whereas Section 5 focuses on the carbon trading scheme. Section 6 develops further cross sectional implications. Section 7 investigates a subsidy to green innovation. In Section 8, we endogenize carbon pricing and the innovation subsidy in general equilibrium. Section 9 concludes. Technical developments and proofs are gathered in the Appendix.

2 The model

We design a dynamic model to study firms' optimal management of carbon emissions. The model is flexible enough to acknowledge that firms may operate under different regimes: laissez-faire, a carbon tax system, or a carbon trading system. We define the firm's "greenness" g_t as the degree of technological sustainability of the firm. The firm can increase its greenness by investing in green innovation.

Time is continuous, and the economy admits a constant risk free rate denoted by r.

2.1 The firm

Production and carbon emissions Through its production process, the firm generates carbon emissions. Denoting by Y_t the firm's endogenous scale of production, the ensuing flow of carbon emissions is described by the following dynamics:

$$dE_t = (\nu Y_t - \xi g_t - s_t g_t)dt + \sigma Y_t dW_t, \tag{1}$$

where the first term represents the firm's expected net emissions, whereas the second term the volatility of emissions. All else equal, emissions are greater and more volatile if the firm's scale of production is larger, with W_t representing a standard Brownian motion and σ being a positive constant.

The first term in equation (1) implies that net emissions result from gross emissions and the firm's efforts to reduce them through green investment. Namely, the parameter ν

represents the industry-specific emission intensity per unit of production, whereas the term ξg_t is firm-specific and captures the reduction in emissions thanks to the accumulation of green breakthroughs, with the parameter ξ indicating the extent to which greenness reduces the firm's emissions. Hence, the higher the firm's greenness—increasing over time thanks to the firm's green innovation breakthroughs—the lower the firm's net emissions for a given scale of production Y_t. Last, the term s_t represents the firm's engagement in emission abatement, on which we elaborate further below.[9]

Importantly, our modeling in equation (1) is flexible and does not impose restrictions on the sign of expected net emissions ($\nu Y_t - \xi g_t - s_t g_t$), which are endogenous and depend on firm choices. If this term is positive, the firm does not fully offset its gross emissions and exhibits positive net emissions (also referred to as net polluter). If this term is negative, the firm more than offsets its emissions and then exhibits negative net emissions (also referred to as net cleanser).

Cash flows and productivity gains from sustainability. We assume that the firm's cash flows are given by

$$\Pi(Y_t, g_t) = Y_t \left(a - \frac{b}{g_t} Y_t \right). \tag{2}$$

Y_t represents the endogenous production choice, and the term in parentheses lends itself to a twofold interpretation following previous works in environmental economics.[10]

Under the first interpretation, it represents the per-unit revenues net of production costs, where $\frac{bY_t^2}{g_t}$ is an operating expense capturing the idea that more (less) sustainable production processes entail lower (greater) costs—in line with previous work suggesting that sustainability improves resource productivity, and can generate a reduction of expendi-

[9] We assume the reduction in the firm's emissions due to abatement and green innovation to be subtractive (rather than being a share) to allow for negative emissions (as explained below) and to preserve stationarity in the infinite-horizon model. In Appendix A.7, we assess the robustness of our results when allowing green innovation to reduce emission intensity from ν to ν/g (with $g > 1$) by assuming that innovation is not a continuous process but just one-off (then resulting in a two-stage model).

[10] Inspired by Porter and Van der Linde (1995), Ambec and Lanoie (2008) or Kruse, Mohnen, Pope, and Sato (2020) highlight the presence of a cost channel and a revenues channel through which environmental innovation can also affect economic performance, which map well into our interpretations below.

tures associated with energy, raw materials, or services (Porter and Van der Linde, 1995).[11] Consistently, the evidence and previous models suggest that green breakthroughs lead to savings on operating costs (Popp, 2001; Acemoglu et al., 2012).

Under the second interpretation, the term in parentheses represents the firm's inverse demand function, i.e., the price at which output is sold. Within this interpretation, the price sensitivity to Y_t is scaled by g_t to capture that consumers are willing to pay a greater price for products of greener firms, consistent with the accumulating evidence (see, e.g., Derrien et al. (2025); Meier et al. (2025); or Chiu et al. (2025)).[12]

Irrespective of the interpretation, our assumption on the firm's cash flows flexibly accommodates several narratives pointing to the existence of productivity gains associated with sustainability, which go beyond the simple savings on carbon regulation. When this is the case, even absent carbon regulation, firms may have incentives to become more sustainable. In the extension reported in Appendix A.6, we allow for the per-unit earnings in equation (2) to be positively affected by emission reductions too. This extension is relevant under the second interpretation, whereby consumers' demand reflects a preference for lower net emissions. The model's main insights continue to hold under this alternative formulation.

Abatement and green innovation We assume that the firm can undertake two types of green investment: abatement and green innovation.

The specification in equation (1) implies that the firm can limit its emissions by investing in abatement. Examples of abatement are afforestation and reforestation projects or purchasing electricity from renewable sources. That is, abatement does not affect the firm's production technology—as it does not affect the firm's greenness g_t—but simply offsets the firm's gross emissions.[13] Denoting the firm's engagement in carbon abatement by s_t, we

[11] Note that the term aY_t can nest revenues net of any linear cost of production: Namely, $a \equiv a_r - a_c$ where a_r is the marginal revenue and a_c is any linear cost of production.

[12] Derrien et al. (2025) show that analysts significantly downgrade earnings forecasts on a firm following negative ESG news on such firm. They show that the negative revision of earnings forecasts reflects expectation of lower sales. Meier et al. (2025) show that environmental and social ratings positively relate to local sales using granular barcode-level sales data from retail stores. Additionally, Chiu et al. (2025) show that greener products are associated with increased sales.

[13] Specifically, following the taxonomy in Requate (2005), abatement is an "end of the pipe" way of

assume that the cost associated with such projects is given by the quadratic specification $\frac{s_t^2}{2}\theta g_t$, where θ is a positive parameter.[14]

On top of abatement, the firm also invests in green innovation. Differently from abatement, green innovation permanently improves the firm's degree of sustainability g_t. Our modeling of innovation builds on the endogenous growth literature (see Aghion, Akcigit, and Howitt (2014) for a survey). Namely, if the firm spends the quadratic cost $\frac{z_t^2}{2}\zeta g_t$, then a green breakthrough arrives at Poisson rate ϕz_t. The innovation rate z_t is an endogenous choice of the firm—a greater z_t entails greater cost, but also increases the likelihood of attaining a green breakthrough. When a breakthrough happens, the firm's greenness g_t increases by a factor $\lambda > 1$. A higher level of greenness g_t brings along long-term benefits to the firm. First, for a given scale of production, the firm exhibits lower net emissions—and, thus, it is less polluting—as captured by the term ξg in equation (1).[15] Second, it leads to an increase in the firm's earnings per unit of production by unlocking productivity gains, as captured by equation (2) and irrespective of the underlying interpretation.

Whereas both abatement and green innovation aim at making the firm more sustainable, they differ in the horizon and scope of their impact. Abatement aims at reducing the expected flow of emissions in the present—i.e., it cleans up (some of) the firm's gross emissions. Yet, it represents a short-lived effort as it does not change the firm's technology. Conversely, green innovation has long-lasting effects as it leads to a permanent increase in the sustainability of the firm's technology, which in turn makes the firm permanently less polluting for a given scale of production. It also brings permanent productivity gains, as discussed above. However, green breakthroughs take time to materialize and have an uncertain outcome.

reducing emissions—i.e., gross emissions are unchanged but subsequently decreased.

[14]Because both the cost and the gain from abatement (i.e., the ensuing reduction in the firm's net emissions as described in equation (1)) scale with g_t, we can think of the two effects canceling out and, thus, the optimal abatement as being independent of the firm's degree of sustainability.

[15]As already mentioned, we assume that the reduction in emission for a given volume of production is subtractive for tractability. However, in Appendix A.7, we show that the main insights of the model continue to hold when allowing innovation to reduce the emission intensity from ν to ν/g.

2.2 Carbon pricing

Carbon tax system A number of countries around the world have implemented carbon taxes—i.e., taxes levied on a firm's carbon footprint. Under carbon tax systems, a central authority sets a predetermined price that firms have to pay for a given volume of emissions. We denote such price—the carbon tax rate—as κ, which is levied on the firm's net emissions. If the firm's expected net emission flow is positive, then the firm is effectively charged the carbon tax. The central authority then typically uses the proceeds of carbon taxes to support green projects (see, for instance, The World Bank (2024)), then offering rebates to firms that exhibit negative net emissions or a direct reward to these firms, on the grounds that negative emissions are necessary to achieve net zero pledges (Blanchard, Gollier, and Tirole, 2023). To capture this aspect, we assume that firms with negative net emissions—or "net cleansers," as they contribute negatively to pollution—enjoy a rebate at rate κ on their negative net emissions.[16]

Carbon trading system Under the carbon trading system (or emission trading systems), carbon credits are akin to permission slips that enable the firm to release carbon emissions into the atmosphere. Once the firm emits a fixed quantity of carbon emissions (typically, a tonne of carbon dioxide), one credit is retired. Accumulating credits entails a benefit, as carbon credits guarantee the firm the ability to carry on producing. Carbon credits also entail a maintenance/storage cost χ, which is proportional to the firm's stock of credits.[17]

The credit system is nested into a trading scheme. Namely, if the firm exhausts its credits, it either has to stop emitting or to buy credits. In turn, if the firm finds itself with an excess of credits, it can sell them. Consistent with the trend observed in recent years, we assume that trading is centralized and the firm buys its credits on a carbon credit platform at the price $\gamma > 0$. In turn, the firms can sell credits at the cost $\gamma(1-\psi)$, with $\psi > 0$ representing the compensation of the platform.[18] If the firm has no carbon credits to cover

[16] For simplicity, we assume that the tax on positive net emissions and the rebate on negative net emissions have the same magnitude.

[17] In several jurisdictions, carbon credits require maintenance because they need to be certified.

[18] Effectively, platforms ease the needs of firms with an excess of credits (sellers) and firms with a shortfall

emissions, it incurs a proportional penalty Γ. Throughout the analysis, we consider the relevant case in which $\Gamma > \gamma$, meaning that the firm has incentives to buy carbon credits (as opposed to incurring the penalty) when running out of them.

The dynamics of the firm's carbon credits then satisfy:

$$dC_t = -dE_t + dP_t - dO_t + dI_t. \tag{3}$$

The above equation shows that the firm's emission flow (defined in equation (1)) depletes the firm's stock of carbon credits. The process P_t represents the purchased credits, which replenish the firm's credit stock. Conversely, O_t captures the credits that the firm sells, which deplete the firm's credit stock. Finally, the process I_t is the inflow of credits when a firm attains a green breakthrough. Namely, the firm is awarded a lumpy amount of credits at no charge, which replenishes the firm's credit balance.[19]

2.3 Optimality

The firm maximizes its value by managing production Y_t, abatement s_t, and its green innovation rate z_t. In addition, under the carbon trading system, the firm manages its stock of carbon credits C_t by choosing both the upper bound at which the firm starts selling credits (which we denote by C^*) as well as the optimal buy/sell strategy (P_t and O_t).

To better single out the effect of carbon pricing on firm's choices, we start by investigating a laissez-faire benchmark (which no carbon regulation) in the next section. We next solve for optimal policies and value under carbon pricing, first looking at the carbon tax case (section 4) and then at the carbon trading system (section 5). We also consider the impact

of credits (buyers). The assumption that the fee is charged to sellers is consistent with the functioning of major carbon credit platforms, see, e.g., https://ctxglobal.com/the-exchange/.

[19]This assumption is consistent with the idea that firms which are innovative in the green spectrum are rewarded some credits for free. For instance, in the EU ETS, manufacturing industries receive a share of their emission allowances for free, based on benchmarks that reward most efficient installations in each sector (so called "free allocation to industrial installations"). For tractability, we assume that upon a breakthrough the firm can replenish its balance all the way to an endogenous upper bound, at which the firm starts selling credits.

of pairing up carbon pricing with subsidies to green innovation in Section 7.

Finally, we show that the main takeaways of our analysis continue to hold when we endogenize carbon regulation (section 8), including both carbon pricing and a subsidy to green innovation. In so doing, we investigate how our rich firm-level analysis maps into implications for optimal carbon regulation.

3 Optimal policies under laissez-faire

We start by studying a laissez-faire environment with no carbon regulation, i.e., in which the firm does not incur any cost for its carbon emissions.

No productivity gains from sustainability As a special case within this laissez-faire environment, we first consider optimal policies when abstracting from productivity gains associated with sustainability, so that equation (2) boils down to

$$\Pi(Y_n^B) = Y_n^B \left(a - bY_n^B\right).$$

In this setting, green breakthroughs solely reduce emissions for a given scale of production Y_n^B (i.e., as gross emissions νY_n are lowered by ξg). Because carbon emissions are not taxed under laissez faire, green breakthroughs bring no financial reward nor impact on firm value. Following standard arguments, firm value satisfies:

$$rV_n^B = \max_{Y_n^B, z_n^B, s_n^B} \left[Y_n^B \left(a - bY_n^B\right) - \frac{(z_n^B)^2}{2} g\zeta - \frac{(s_n^B)^2}{2} g\theta\right]. \tag{4}$$

The left-hand side of this equation is the return required by the investors, whereas the right-hand side is the net cash flows when sustainability has no productivity gains and, thus, no impact on cash flows. As a result, green breakthroughs and abatement are expenses with

no associated benefit, which yields

$$s_n^B = 0 \quad \text{and} \quad z_n^B = 0. \tag{5}$$

That is, the firm does not engage in any type of green investment when emissions are not costly (there is no carbon pricing) nor there are productivity gains associated with sustainability (i.e., cash flows are independent of g_t). Deriving the optimal production rate and solving for firm value yields $V_n^B = \frac{a^2}{4br}$, which confirms that firm value is independent of carbon emissions and of sustainability g_t.

Acknowledging productivity gains from sustainability Next, we acknowledge the existence of productivity gains associated with sustainability, as discussed in the assumptions and captured by equation (2).[20] In this case, even absent carbon pricing, green breakthroughs lead to greater cash flows and, thus, increase firm value. Thus, firm value is a function of greenness $V^B(g_t)$ and satisfies the following equation:

$$rV^B(g) = \max_{Y^B, z^B, s^B} \; \phi z^B \left[V^B(\lambda g) - V^B(g)\right] + \left[Y^B\left(a - \frac{b}{g}Y^B\right) - \frac{(z^B)^2}{2}g\zeta - \frac{(s^B)^2}{2}g\theta\right]. \tag{6}$$

This equation differs from equation (4) in that sustainability affects cash flows and, thus, green breakthroughs affect firm value—as captured by the first term on the right-hand side. The second term is the expected net cash flow to shareholders on each time interval. To obtain the firm's optimal policies and value, we conjecture that firm value scales with greenness, i.e., $V^B(g_t) = g_t v^B$, where v^B is the scaled firm value. We also define $y^B = Y_t^B/g_t$ as the scaled production quantity. Substituting into equation (6) and maximizing with

[20] As explained in the assumptions, this modeling is amenable to different interpretations familiar with the environmental economics literature, such as decreased operating costs as firms move away from non-renewable inputs of production, or increased demand for sustainable products. In the companion paper Bustamante and Zucchi (2025a), we further study benefits from greenness associated with improved financing costs (consistent with investors' tilting strategies). In that environment, we show that the main implications of carbon pricing on firm policies continue to hold.

respect to y gives the optimal production choice $y^B = \frac{a}{2b}$ which is independent of the firm's emissions: As the firm does not need to pay for its emissions, they are not accounted for when setting the optimal production rate.

Consider next the optimal green investment choices stemming from the maximization of equation (6). The firm continues having no incentives to invest in abatement as in the case with no productivity gains, because abatement entails a cost but does not bring any upside. Thus, optimal abatement continues to be

$$s^B = 0$$

in this case too. Differently, the firm invests in green innovation to reap the associated productivity gains, and the optimal innovation rate is:

$$z^B = \frac{\phi}{\zeta} v^B (\lambda - 1). \tag{7}$$

That is, if sustainability brings along productivity gains that boost the firm's cash flows and value, then a value-maximizing firm has incentives to invest in green innovation even in the absence of carbon pricing. In unscaled terms, the firm's production increase upon a green breakthrough consistent with the Jevons paradox, and the firm's cash flows increase too. Plugging optimal policies back into the (scaled) HJB equation gives a closed-form expression for firm value:

$$v^B = \zeta \frac{r - \sqrt{r^2 - \frac{a^2}{b}\frac{\phi^2}{2\zeta}(\lambda - 1)^2}}{\phi^2 (\lambda - 1)^2}. \tag{8}$$

This expression shows that firm value does not depend on its carbon emissions under laissez-faire. Unscaled firm value, however, is positively affected by the firm's greenness.

Takeaways We summarize our results under laissez-faire in the proposition below.

Proposition 1 *Under laissez-fare:*

(1) Abatement: The firm does not invest in abatement unconditionally ($s^B = s_n^B = 0$);
(2) Green innovation: The firm exhibits a positive engagement in green innovation only if breakthroughs bring along productivity gains ($z^B > 0$). If there are no productivity gains, the firm does not invest in green innovation ($z_n^B = 0$).

Proposition (1) highlights the importance of accounting for productivity gains associated with sustainability. Absent productivity gains, firms take no steps under laissez-faire to abate their emissions or to make their production technology more sustainable. When acknowledging such productivity gains, conversely, firms invest in green innovation even under laissez-faire, so as to reap such benefits and increase their cash flows, consistent with firm maximization objectives. This is consistent with the US experience whereby, in spite of the absence of carbon pricing at the federal level, firms still exhibit engagement in green innovation (see, e.g. Cohen, Gurun, and Nguyen, 2025).

4 Optimal policies in a carbon tax system

We next turn to investigate optimal policies under the carbon tax system. Under this system, we denote firm value by $V^\tau(g)$, which continues to be a function of greenness. Using standard arguments, firm value satisfies:

$$rV^\tau(g) = \max_{Y^\tau, z^\tau, s^\tau} \; \phi z^\tau \left[V^\tau(\lambda g) - V^\tau(g)\right] + \left[Y^\tau p(Y^\tau) - \frac{(z^\tau)^2}{2} g\zeta - \frac{(s^\tau)^2}{2} g\theta - \kappa(Y^\tau \nu - \xi g - s^\tau g)\right]. \tag{9}$$

This equation differs from equation (6) in the last term on the right-hand side, as the firm is taxed on its net emissions. To solve for the firm's optimal policies, we again use the scaling property (see Appendix A.2 for details). The optimal production quantity then satisfies

$$y^\tau = \frac{a - \kappa\nu}{2b}, \tag{10}$$

which implies that the larger the carbon tax rate, the lower the firm's production rate and more so if the emission intensity ν is larger. Because the firm's scale of production directly impacts its gross emissions—and, thus, affects how much the firm is levied—y^τ is consistently below the laissez-faire (unconstrained) level y^B and more so if κ is greater. That is, by making emissions costly, the carbon tax makes the firm internalize the externality generated by its production process. Moreover, the firm's optimal abatement satisfies

$$s^\tau = \frac{\kappa}{\theta}, \tag{11}$$

which implies that the firm invests more in abatement if the carbon tax rate is larger. By imposing a direct price on emissions, the carbon tax provides an incentive to firms to take immediate actions to curtail them. Lastly, the firms' optimal innovation rate satisfies:

$$z^\tau = \frac{\phi}{\zeta}(\lambda - 1)v^\tau(\kappa). \tag{12}$$

The next proposition follows.

Proposition 2 *Under the carbon tax system:*
(1) Abatement: Firms engage in abatement $s^\tau > s_n^B = s^B = 0$ compared to no engagement under laissez-faire, and more so if κ is higher, unconditionally.
(2) Green innovation: The impact of the carbon tax on green innovation is ambiguous: Whereas $z^\tau > z_n^B = 0$ (absent productivity gains), z^τ can be higher or lower than z^B (acknowledging productivity gains). Namely, $z^\tau < z^B$ if κ is sufficiently low.

Proposition 2 summarizes how the carbon tax impacts a firm's green investment. First, it illustrates that the firm invests more in abatement the greater the carbon tax rate κ is: Indeed, a higher carbon tax rate increases the benefit from abatement (i.e., the saving associated with offsetting emissions is higher) while leaving the cost unchanged, then leading to an unambiguous increase in the firm's optimal abatement rate.

By contrast, the impact of the carbon tax on green innovation is nuanced, as it reflects

the interaction of two distinct forces. First, by imposing a price on emissions, the carbon tax encourages polluting firms to reduce emissions for a given scale of production. In the absence of productivity gains from green breakthroughs, this is the only force at work, implying that the carbon tax unambiguously increases green innovation. Consistently, the optimal innovation rate is zero under laissez-faire when ignoring productivity gains from innovation (as $z_n^B = 0$, see Section 3), whereas it is positive in the presence of the carbon tax.[21]

A second, opposing force emerges once we account for the productivity gains generated by green breakthroughs, which go beyond the savings on the carbon tax. When green innovation brings along such productivity gains, the firm has incentives to expand its scale of production after a breakthrough consistent with the Jevons paradox, and thereby enjoy higher revenues. Under a carbon tax, however, such expansion is less attractive than under laissez-faire because the additional associated emissions must be paid for. Put differently, by making production more costly for polluting firms, the carbon tax erodes part of the value created by technological breakthroughs. As a result, the value-to-sustainability ratio v^τ may be lower under a carbon tax than under laissez-faire, v^B.[22]

When acknowledging such productivity gains, the overall effect of the carbon tax on innovation is therefore ambiguous. Namely, the negative effect is more likely to dominate in jurisdictions where the carbon tax rate κ is sufficiently low, which does not motivate the firm to significantly reduce emissions and then leaves firms largely net polluters. By contrast, the positive effect prevails in jurisdictions where the carbon tax rate κ is sufficiently high, which motivates the firm to significantly curb emissions—even offsetting them fully, then becoming net cleansers—as gains from emission reduction are substantial. For a given level of the carbon tax, the negative (positive) effect is also more likely to hold for those firms for which green breakthroughs lead to only a modest (large) reduction in carbon emissions

[21] The simplified framework in Appendix A.7 further isolates this strength. Additionally, such Appendix A.7 shows the robustness of our main result in a two-state version of the model in which the reduction in emissions (per unit of production) associated with a green breakthrough is not subtractive.

[22] The idea that returns to innovation rise with scale is consistent with models of innovation-driven growth; see, for example, Akcigit and Kerr (2018).

for a given scale of production, meaning that the parameter ξ is sufficiently low (high).

The following corollary stems directly from Proposition 2 and comparing the optimal policies with the laissez-faire environment described in Section 3.

Corollary 3 *Expected scaled net emissions are lower under the carbon tax system compared to laissez-faire, $\nu y^{\tau} - s^{\tau} - \xi < \nu y^{B} - s^{B} - \xi$, and more so as κ is greater. Moreover, when acknowledging productivity gains from sustainability, the introduction of the carbon tax leads firms to tilt their green investment mix towards abatement.*

Because firms cut production and engage in green investment when subject to the carbon tax, they reduce their scaled net emissions compared to laissez-faire, and more so if κ is higher. The inverse relation between the price of carbon and emissions is consistent with Martinsson et al. (2024) among others. Furthermore, when acknowledging the presence of productivity gains associated with sustainability, the introduction of a carbon tax leads firms to shift their green investment mix from being solely focused on green innovation (as described in Section (3)) to also including abatement.

To summarize, our analysis emphasizes the importance of acknowledging the presence of productivity gains associated with sustainability—providing firms with incentives to engage in green innovation even absent carbon regulation—to properly understand how the introduction of a carbon tax will affect a firm's engagement to becoming greener. As shown in Appendix A.6, our main results continue to hold when allowing gains from lower carbon emissions (rather than higher levels of sustainability)—this is the case, for instance, if firm's customers value not only greenness but also the firm's net emissions.

5 Optimal policies under the carbon trading system

We next analyze the optimal policies under the carbon trading system. Given that carbon credits are costly, the firm does not buy them until it runs out of them. In turn, the firm does not sell credits as long as the marginal value of credits inside the firm is greater than

the marginal gain from selling them. As a result, we conjecture that there is a region $[0, C^*]$ in which the firm accumulates credits and does not trade them. At $C = 0$, the firm buys credits. Conversely, at the upper bound C^*, the firm sells credits.

In the region $[0, C^*]$, the dynamics of firm value satisfy the following Hamilton-Jacobi-Bellman (HJB) equation:

$$\begin{aligned} rV(C,g) = \max_{z,Y,s} \quad & -(Y\nu - \xi g - sg)V_C + \frac{\sigma^2}{2}Y^2 V_{CC} + \phi z\left(V(C^*, \lambda g) - V(C,g)\right) \\ & + \left(Yp(Y) - \frac{z^2}{2}g\zeta - \frac{s^2}{2}g\theta - \chi C\right). \end{aligned} \tag{13}$$

The right-hand side is the expected change in firm value on each time interval. The first term is the effect of a change in the credit balance on firm value. If $Y\nu - \xi g - sg > 0$, the firm depletes its credit balance in expectation. The second term on the right hand side is the effect of emission volatility. The third term is the effect of a green innovation breakthrough, in which case the firm's greenness as well as its credit balance increase. The last term is the expected net cash flow to shareholders on each time interval.

As in the carbon tax case, we define quantities scaled by greenness. Namely, we define

$$V(C,g) = g\, v\left(\frac{C}{g}\right) \equiv g v(c) \tag{14}$$

where $v(c)$ denotes firm value scaled by greenness and c denotes scaled credits so that $C = gc$. The scaled upper bound for the carbon credit balance is denoted by $c^* = C^*/g$ and is endogenously determined. We also define scaled production by $y = Y/g$. Substituting into equation (13) gives the scaled HJB (see equation (30) in Appendix A.3), which we differentiate with respect to y to get the optimal production quantity:

$$y(c) = \frac{a - \nu v'(c)}{2b - \sigma^2 v''(c)}. \tag{15}$$

Similar to the carbon tax case, the optimal scale of production $y(c)$ internalizes the pollution externality (i.e., production becomes a function of the firm's emission intensity ν). Equation

(15) implies that the greater the marginal value of a carbon credit (v') or the greater the firm's emission intensity (ν), the lower the optimal scale of production.

Consider next the optimal abatement policy, obtained by differentiating the scaled HJB equation with respect to s:

$$s(c) = \frac{v'(c)}{\theta}. \tag{16}$$

The greater the marginal value of carbon credits, the more the firm engages in abatement to reduce its net emissions, then slowing its consumption of carbon credits.

Finally, maximizing the scaled HJB equation with respect to z gives the optimal green innovation rate:

$$z(c) = \frac{\phi}{\zeta}(\lambda v(c^*) - v(c)). \tag{17}$$

Substituting equations (15), (16), (17) into the HJB equation gives an ordinary differential equation (ODE), which we solve subject to following boundary conditions. First, when the firm runs out of credits at $c = 0$, it resorts to the credit market. Recall that buying credits entails a proportional cost γ. Thus, the following boundary condition holds:

$$v'(0) = \gamma, \tag{18}$$

which implies that, when the firm buys credits, the marginal value of credits inside the firm equals their marginal cost.[23] Next, the firm sells credits exceeding the endogenous upper bound c^*. Hence, the following boundary condition holds at c^*:

$$v'(c^*) = \gamma(1 - \psi). \tag{19}$$

That is, at c^*, the marginal value of credits inside the firm is equal to the gain associated

[23] Realistically, we assume that buying credits does not put at stake the viability of the firm—i.e., it does not push the firm below its liquidation value. I.e., the following inequality: $v(0) > \ell$ holds, where we denote by ℓ the firm's liquidation value.

with selling them.[24] Last, the threshold c^* is pinned down by the super-contact condition:

$$v''(c^*) = 0, \tag{20}$$

which guarantees that the threshold is optimally chosen (Dumas, 1991). The next proposition describes firm value as a function of the stock of carbon credits c.

Proposition 4 *Under the carbon trading scheme, firm value is increasing and concave in its stock of carbon credits c.*

Proposition 4 shows that firm value is concave in the stock of carbon credits. Because carbon credits are costly, the firm optimally accumulates them in a precautionary perspective, to avoid having to resort to the costly carbon credit market too often. Indeed, keeping a non-negative carbon credit balance acts as an operating constraint. This result is similar to the finding of previous inventory models (see Bolton, Chen, and Wang, 2011; Décamps et al., 2011), in which financially constrained firms find it optimal to accumulate cash to buffer cash flow shocks and mitigate exposure to costly or uncertain external funding.

Carbon pricing versus laissez-faire We compare policies in the carbon trading system versus laissez-faire, while drawing parallels with the carbon tax system. Consider first the firm's production decision. As in the carbon tax case, the firm in the carbon trading system produces less than under laissez-faire, i.e., $y(c) < y^B$ for any $c \leq c^*$. Recall from Proposition 4 that, under the carbon trading system, the firm is effectively risk averse for any $c < c^*$ (i.e., $v'' < 0$). Thus, for any $c < c^*$, the marginal value of credits inside the firm is greater than the marginal gain of selling them (i.e., $v'(c) > \gamma(1-\psi)$). Equation (15) then implies that scale of production always falls below that associated with laissez-faire, consistent with the evidence in Martin, Muuls, and Wagner (2016) for the EU ETS. To avoid having to resort to buying costly credits, the firm reduces its need for credits by lowering production

[24] For any $c > c^*$, firm value is linear, as the firm sells all the credits exceeding such threshold.

and, thus, emissions.[25]

We proceed to analyze the firm's optimal green investment under the carbon trading system. The next proposition summarizes our results.

Proposition 5 *Under the carbon trading system:*
(1) Abatement: The firm has incentives to invest in abatement, i.e., $s(c) > s^B = s_n^B = 0$ *compared to no engagement under laissez-faire, unconditionally;*
(2) Green innovation: The impact of carbon trading on green innovation is ambiguous: Whereas $z(c) > z_n^B = 0$ *(absent productivity gains),* $z(c)$ *can be higher or lower than* z^B *(acknowledging productivity gains).*

Similar to the carbon tax case, Proposition 5 shows that the firm exhibits positive investment in abatement under the carbon trading system, and more so as the cost of carbon credits increases. By abating more, the firm seeks to reduce its net emissions and, hence, its need to resort to the costly carbon credit market.

As for the carbon tax, Proposition 5 also shows that the carbon trading system has a more nuanced impact on green innovation. In the absence of productivity gains from sustainability, the introduction of a carbon trading system would unambiguously spur investment in green innovation, which would then go from zero ($z_n^B = 0$, as described in Section 3) to positive. When instead acknowledging the presence of productivity gains above and beyond the saving on carbon regulation, another strength arises. In the presence of such gains, green breakthroughs lead the firm to increase production consistent with the Jevons paradox which, however, is less desirable when emissions need to be paid for. This strength then curbs incentives to engage in green innovation. As for the carbon tax case, we find that this second strength is more likely to prevail on the first if the price of carbon credits γ is relatively low. In turn, if the price of carbon credits γ is sufficiently large, the innovation rate $z(c)$ is greater under the carbon trading system than under laissez-faire.

[25]While the proponents of the carbon trading system typically emphasize that the ability to buy credits allows the firm to carry on producing even when it reaches its maximum emission allowance—being the distinctive feature of carbon pricing compared to the command-and-control (CAC) systems used in early pollution regulation—our model shows that this system gives rise to a precautionary behavior.

The next result stems from Proposition 5.

Corollary 6 *The firm exhibits lower scaled net emissions under the carbon trading system than under the laissez-faire benchmark in expectation, i.e.,* $y\nu - \xi - s < y^B\nu - \xi - s^B$. *Moreover, when acknowledging productivity gains from green breakthroughs, the carbon trading system shifts the green investment mix of net polluters towards abatement.*

Corollary 6 resonates with Corollary 3 in the carbon tax case. As shown above, the firm exhibits a higher scale of production in the laissez-faire environment and, at the same time, does not invest in abatement. Therefore, the firm pollutes more in expectation in that environment than under the carbon trading system, a result similar to our conclusion in the carbon tax system. The reduction in emissions triggered by carbon trading systems is empirically documented, e.g., by Fowlie, Holland, and Mansur (2012), Martin, Muuls, and Wagner (2016), or Kumar and Purnanandam (2024). In addition, when acknowledging the presence of productivity gains associated with sustainability, the carbon trading system leads firms to invest in both abatement and green innovation, then shifting the firm's green investment mix towards abatement. Hence, our analysis shows that carbon pricing leads to a similar response in firms' policies, irrespective of whether it is implemented by means of a tax or credit trading system.

In the carbon trading system, firms with negative net emissions can profit from the sale of carbon credits, which then has implications for firm value.

Proposition 7 *If the firm's expected net emissions are sufficiently negative under the carbon trading system, firm value in the carbon trading system can exceed its counterpart in the laissez-faire benchmark at least at* c^*.

In contrast with the long-standing perception of a fundamental conflict of interest between the interest of environmental regulators and businesses, our analysis shows that carbon pricing is not necessarily value-decreasing for corporations.[26] That is, the credit system

[26] A similar result holds for the carbon tax case when it contemplates a reward to firms with negative emissions—i.e., net cleansers enjoy a greater value compared to laissez-faire.

can represent both a gain and a cost for firms, consistent with evidence showing heterogeneous effects of climate considerations and regulations on firms, e.g. Martin, de Preux, and Wagner (2014), or Sautner, van Lent, Vilkov, and Zhang (2023). In particular, the carbon trading system represents an opportunity for profits for a firm whose net emissions are sufficiently negative (or 'net cleanser'). In expectation, a net cleanser accumulates (as opposed to consumes) credits, which are sold when the credit balance reaches its upper bound c^*. Such gain thus increases firm value beyond its laissez-faire benchmark.

Optimal policies with carbon credit management We now investigate optimal policies as a function of carbon credits.

Proposition 8 *Under the carbon trading scheme:*
(1) Production $y(c)$ increases with the stock of credits c and so do revenues $y(c)\,(a - by(c))$;
(2) Abatement $s(c)$ and green innovation $z(c)$ decrease with the stock of credits c;
(3) The firm's expected net emissions $(\nu y(c) - \xi - s(c))$ increase with the stock of credits c.

Proposition 8 shows that the larger the firm's stock of carbon credits, the greater its production rate. That is, when the stock of credits is low, the firm decreases production so to curtail its emissions and reduces its consumption of carbon credits, to avoid having to resort to the costly carbon credit market. By contrast, the firm increases production as its stock of carbon credits is larger, so revenues also increase with c.

Proposition 8 also illustrates that the firm's abatement and innovation policies are complementary to its production choices in keeping emissions in check. Namely, when c is low, the firm actively reduces its emissions not only by scaling down production but also by investing more in abatement and green innovation, to avoid having to resort to the costly carbon credit market. As c increases, the firm's engagement in abatement or in green innovation decreases. Overall, the proposition suggests that the firm manages production, abatement, and innovation policies to steer its carbon credit balance away from the $c = 0$ boundary, at which point it has to buy costly credits in order to sustain production.

As a direct implication of the monotonicity of s and y in c, Proposition 8 also highlights that the firm's expected net emissions increase as c increases. Because a larger stock of credits gives the right to the firm to emit more pollutants into the atmosphere, the firm takes advantage of it by increasing production and, hence, polluting more. Hence, having a large carbon credit balance relaxes the firm's commitment to curb emissions. Consistently, De Jonghe, Mulier, and Schepens (2020) document that, within the EU ETS, polluting firms have weaker incentives to become greener if they have more emission allowances.

6 Green investment in the cross section

Our analysis so far has focused primarily on analytical predictions associated to corporate policies under either the carbon tax or the carbon trading system. In this section, we illustrate these visually and provide additional implications for the cross section of brown and green firms, which we relate to existing empirical findings.[27]

Sensitivity of green investment to carbon pricing Our analytical results suggest a similar response of firm policies to carbon pricing, irrespective of it being a carbon tax or a carbon trading system.

As illustrated by Figure 1, and consistent with the results in Proposition 2 and 5, we observe that the impact of carbon pricing is positive on abatement whereas it is ambiguous on green innovation. In the second panel, green innovation decreases with the carbon price (κ or γ) if it is relatively low. In this case, the gain from emission reduction through green innovation is not significant enough to motivate the firm to increase its investment in green technologies, and the firm remains a net polluter. By contrast, green innovation increases with the carbon price if it is sufficiently large. In this case, the gains from emission reductions are substantial to motivate the firm to significantly curb emissions and even offset them fully, then endogenously becoming a net cleanser. Furthermore, the right panel in 1 shows that the innovation share—i.e., the weight of green innovation in the overall green

[27] Appendix A.4 reports the main parameterization underlying our analysis.

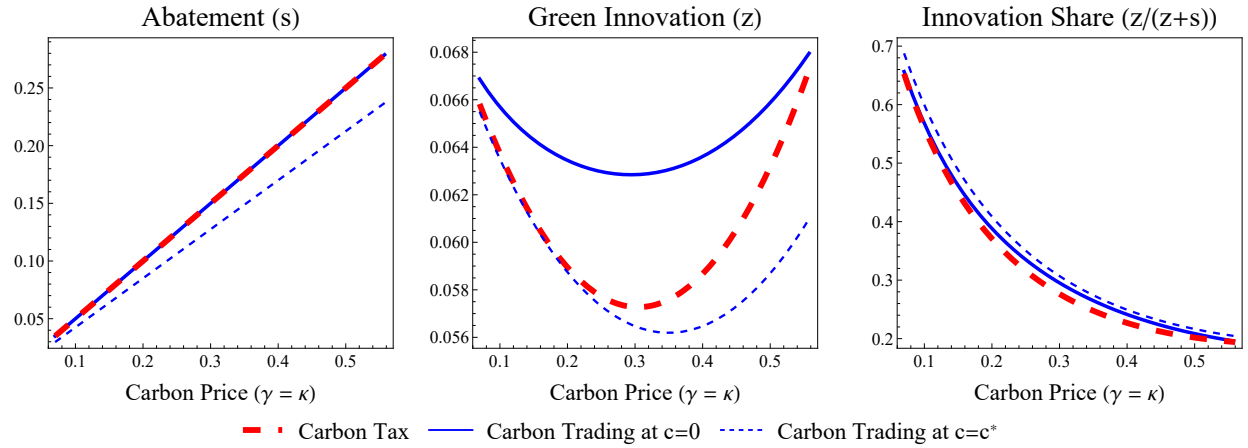


Figure 1: GREEN INVESTMENT AND CARBON PRICING. This figure shows optimal abatement, green innovation, and the innovation share (i.e., the weight of innovation in the firm's green investment mix) as a function of the carbon price, both in the carbon tax system (κ, red line) and carbon credit system (γ, blue lines, plotting the values of these quantities at $c = 0$ and at $c = c^*$).

investment mix—always decreases with carbon pricing. This last result is straightforward for those firms that exhibit a negative sensitivity of green innovation to carbon pricing. For those firms that exhibit a positive sensitivity of green innovation to carbon pricing, it is driven by the fact that abatement is more sensitive to changes in the carbon price than green innovation.[28]

Additional implications on brown and green firms Recent years have seen a rapid expansion of empirical work devoted to studying brown versus green firms, in particular how they differ in their policies. To this end, empiricists have relied on different strategies to identify green versus brown firms. Some works in the literature rely on an industry-level classification, as some industries are structurally more or less polluting than others (see, e.g., Brown, Martinsson, and Thomann, 2022). Other papers sort firms instead as green or brown based on their firm-level carbon emissions, controlling for both industry and firm fixed-effects (see, e.g., Bolton, Kacperczyk, and Wiedemann, 2023).

[28]Recall that abatement increases with the marginal value of carbon credits ($v'(c)$), as per equation (16). Because the marginal value of carbon credits increases with γ, so does the firm's optimal engagement in abatement. Similarly, under the carbon tax system, abatement increases with κ.

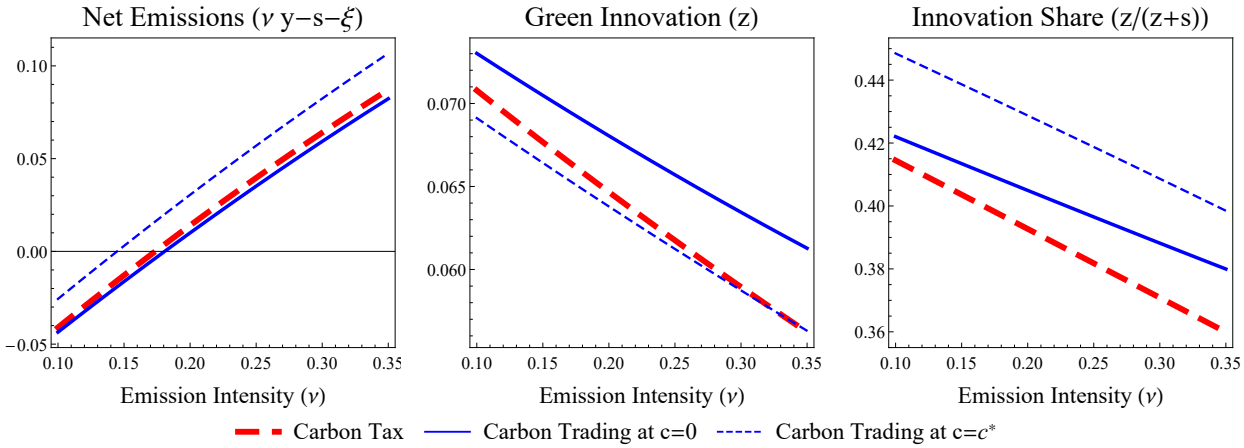


Figure 2: EMISSION INTENSITY AND INNOVATION. The figure shows the net carbon emissions, the optimal innovation rate, and the innovation share (i.e., the weight of innovation in the firm's green investment mix) as a function the degree of industry-specific emission intensity ν and both in the carbon tax system (κ, red line) and carbon credit system (γ, blue lines, plotting the values of these quantities at $c = 0$ and at $c = c^*$).

Our theoretical approach is rich enough to derive implications that can inspire empirical testing adopting either of the two approaches highlighted above. In particular, the industry-specific component of firm's carbon footprint is captured by sorting on the parameter ν.[29] When firms are subject to carbon pricing—being carbon tax or carbon trading systems—they downsize their production, and more so if they operate in a dirtier industry with a higher ν. The middle and right panels of Figure 2 illustrate that, all else equal, firms with a higher emission intensity reduce their engagement in green innovation.

Even given their industry characteristics, firms can pursue emission-reduction efforts. That is, whereas ν describes the emission intensity of the industry in which the firm operates (e.g., an oil company will structurally have a higher ν than a service company), ξ represents how much a firm can improve its own specific degree of sustainability through green innovation. Figure 3 shows that the greater ξ, the more the firm invests in innovation, both in absolute terms and relative to total green investment. Thus, Figure 2 and

[29] As explained, for a given level of production and for a given emission intensity (i.e., ν), a firm can become less polluting by engaging in green investment. Hence, our model nests an industry-specific component as well as a firm-specific one, which stems from the firm's efforts in becoming green.

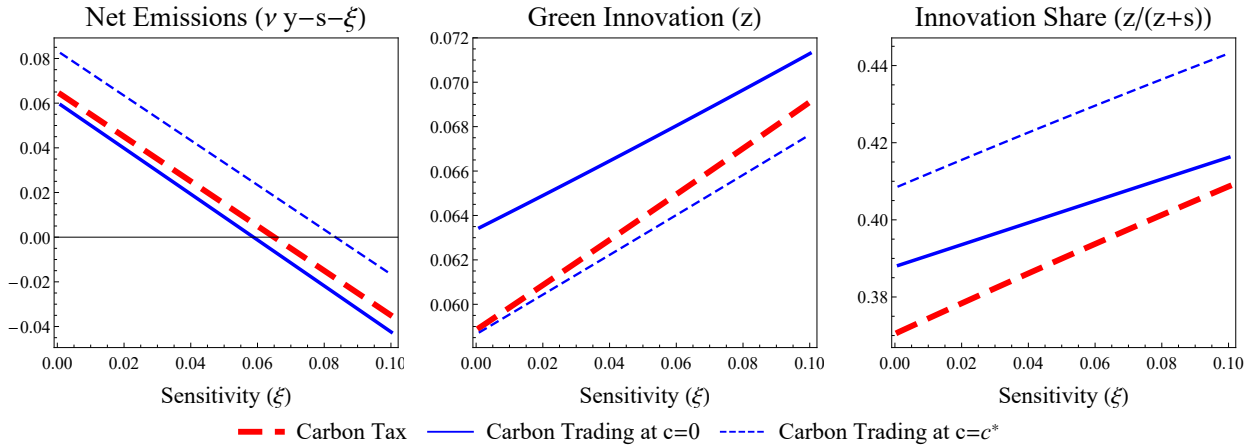


Figure 3: SUSTAINABILITY, INNOVATION, AND THE INNOVATION SHARE. The figure shows the net carbon emissions, the optimal innovation rate, and the innovation share as a function of parameter ξ, both in the carbon tax system (κ, red line) and carbon credit system (γ, blue lines, plotting the values of these quantities at $c = 0$ and at $c = c^*$).

Figure 3 jointly imply that ξ and ν have an opposite effect on green innovation—which effect dominates is an empirical question. This analysis implies that relying solely on an industry perspective may be problematic because it abstracts from the heterogeneous, firm-level efforts that drive actual outcomes. Aligned with this argument, the literature reports an ambiguous cross-sectional relation: On the one hand, Brown et al. (2025) find that Swedish firms in high polluting industries are relatively more engaged in green R&D, also consistent with Cohen, Gurun, and Nguyen (2025); on the other hand, using global data, Brown, Martinsson, and Thomann (2022) find that high polluting firms engage in adopting existing technologies rather than developing new ones.

Consider next the empirical approach which sorts firms according to their carbon emissions. Our model provides guidance to empirical works adopting this approach too. Importantly, carbon emissions are endogenous in the model: Corollary 3 and 6 indeed indicate that a greater carbon pricing leads to lower scaled net emissions. Notably, our specification in equation (1) allows net emissions to go negative if the carbon price is sufficiently large. In addition, in the carbon trading system, a firm can dynamically be a net polluter or cleanser depending on its credit balance, consistent with our result that firms undertake precaution-

ary policies when they expect to run out of credits.[30] Results (2) and (3) of Proposition 8 jointly imply that firms exhibiting the higher emissions are less engaged in green innovation, controlling for any other source of technological differences across firms.

Our model is then capable of reproducing the negative relation between emissions and green innovation documented by Bolton, Kacperczyk, and Wiedemann (2023). Visually supporting this relation, the left and middle panels in Figures 2 and 3 jointly imply that, regardless of carbon regulatory regime, the relation between emissions and green innovation is negative.

7 Subsidy to green innovation

A key result of our analysis is that if there are gains associated with becoming more sustainable on top of the mere saving on carbon regulation, then carbon pricing—being a carbon tax or trading system—may partly erode these gains. A natural question arises as to whether there are complementary tools that regulators can harness to encourage the investment in green innovation.

In this section, we focus on one such tool: A subsidy to green innovation. To this end, we assume that the subsidy covers a portion ι of the firm's cost of innovation, which then decreases to $(1-\iota)\frac{\zeta}{2}z^2 g_t$. Therefore, ι represents the magnitude of the subsidy, decreasing the effective cost of innovation. While the expression of the optimal abatement and production policies remain the same in this case, the optimal green innovation policy is

$$z^\tau = \frac{\phi}{\zeta(1-\iota)} v^\tau(\iota)\,(\lambda - 1) \tag{21}$$

in the carbon tax system, whereas it is

$$z(c) = \frac{\phi}{\zeta(1-\iota)}(\lambda v(c^*, \iota) - v(c, \iota))$$

[30] In particular, a firm might be a net cleanser if its credit balance is low, in the attempt to avert having to resort to the costly carbon credit market, while being a net polluter if the credit balance is sufficiently high.

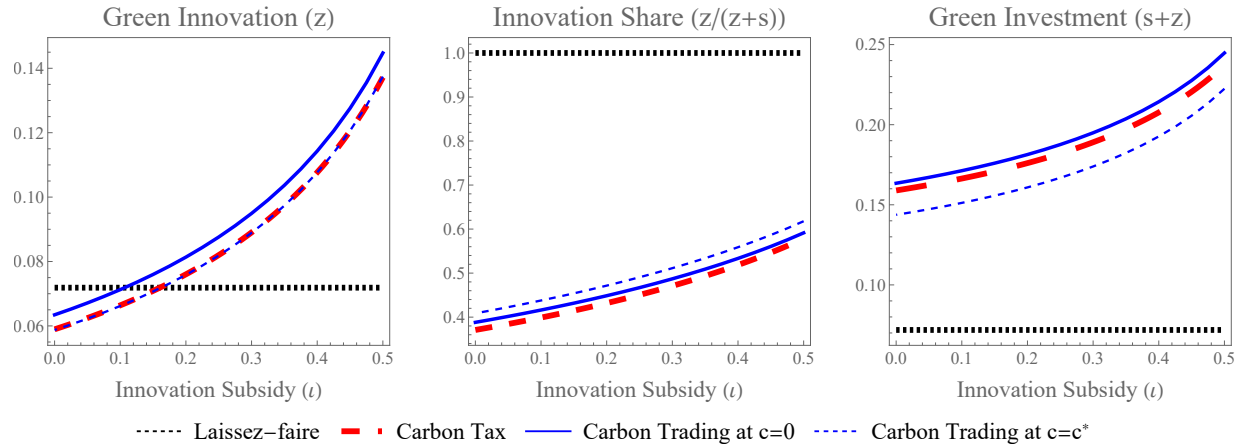


Figure 4: THE IMPACT OF THE INNOVATION SUBSIDY. The figure shows the optimal innovation rate, the innovation share, and total green investment (considering both abatement and innovation) as a function of the innovation subsidy ι in the carbon tax system (κ, red line) and carbon credit system (γ, blue lines, plotting the values of these quantities at $c = 0$ and at $c = c^*$). For comparison, the black dotted line represents the laissez-faire case (no carbon pricing nor subsidies).

in the carbon trading system. These equations suggest that because the innovation subsidy decreases the effective cost of green innovation, the firm's optimal innovation rate should increase. Moreover, the subsidy impacts firm value in both systems, so it should affect innovation expenditures through this channel too.

Figure 4 investigates quantitatively the impact of the innovation subsidy under both the carbon tax and the carbon trading systems. On the horizontal axis, we vary the magnitude of the subsidy ι. The figure shows that the subsidy effectively increases the firm's investment in innovation as well as the innovation share—and more so as the subsidy increases. That is, the subsidy effectively tilts the green investment mix to green innovation. Moreover, the subsidy spurs the total amount that firms spend on becoming green (abatement plus green innovation), as shown in the right panel.[31] In the next section, we investigate whether this

[31]Under the carbon trading system, abatement slightly decreases in the presence of the subsidy for interior levels of the stock of carbon credits (whereas it does not at $c = 0$ or at $c = c^*$, where the value of credits is pinned down by γ and ψ). The reason is that the subsidy reduces the marginal value of carbon credits which, in turn, reduces the marginal benefit from investing in abatement (as illustrated by the numerator of equation (16)). Yet, such reduction in abatement is rather small and, overall, the increase in green innovation is the most notable firm outcome associated with the subsidy.

continues to be the case in general equilibrium, when we endogenize carbon regulation.

8 Endogenous carbon regulation

The main goal of our paper is to understand how firms respond to carbon pricing when allowing for a rich set of corporate policies, encompassing green investment projects of various horizon and risk profiles. To this end, so far we have analyzed corporate policies taking carbon regulation as exogenous. In this section, we investigate whether our main results continue to hold if we allow a benevolent regulator to optimally choose both carbon pricing and the innovation subsidy.[32]

The economy There is a representative agent with logarithmic preferences

$$E\left[\int_0^\infty e^{-\delta t}\ln \mathcal{C}_t\right], \tag{22}$$

where $\mathcal{C}_t$ denotes consumption and $\delta > 0$. Consumption is drained by the social cost of carbon, which we define below. The representative household is endowed with labor L, which is supplied inelastically and yields a competitive wage $\mathcal{W}_t$.

There is a final good sector that serves as the numeraire of the economy, whose output is denoted by $\mathcal{Y}_t$. This final good is produced competitively with a production function using labor and a continuum of inputs:

$$\mathcal{Y}_t = \int_0^1 \left(aY_{jt} + fg_{jt}L\right) dj, \tag{23}$$

where the second term captures that if the intermediate inputs are greener, labor becomes more productive.[33] To nest our analysis so far in general equilibrium, we consider that all

[32] In this section, we focus on the carbon tax case. Under the carbon trading system, the central authority sets the quantity of credits that are exchanged, whereas the market price of carbon credits is instead pinned down by demand and supply forces. Our partial equilibrium analysis in the carbon trading system (Section 5) collapses to the carbon tax case (Section 4) when there is no fee ψ.

[33] This is consistent with the idea that labor becomes less productive as pollution leads to global warm-

intermediate goods are produced by polluting firms subject to carbon pricing—in particular, we assume that the intermediate firms are those analyzed so far in the paper, which not only produce but also invest in abatement and green innovation. In general equilibrium, we focus on one interpretation of equation (2) so to work out the accounting for the different sectors of the economy (details are reported in Appendix A.5). Namely, we assume the first interpretation—i.e., greener production processes are associated with lower operating costs.[34] When solving the maximization problem of the final good sector, the resulting cash flows of the intermediate firm producers coincide with that assumed in equation (2), indicating that our general equilibrium setting nests the partial equilibrium analysis from previous sections.[35]

We focus on a "balanced greening path" equilibrium, in which aggregate quantities become more sustainable at the constant, endogenous rate G, which aggregates the greening of all sectors of the economy. An equilibrium is an allocation such that: (i) intermediate firms set production, abatement, and green innovation to maximize value (as investigated so far); (ii) the final good sector maximizes profits; (iii) the representative household maximizes utility from consumption; (iv) a central authority sets carbon pricing and the innovation subsidy to maximize the utility of the representative agent; (v) all markets clear.

Endogenous aggregate quantities The representative agent's budget constraint implies:

$$\mathcal{C}_t = \mathcal{W}_t L + \mathcal{D}_t + \mathcal{T}_t - \Omega \int_0^1 (\nu Y_{jt} - \xi g_{jt} - s_{jt} g_{jt})\, dj, \tag{24}$$

which spells out the components of consumption. The first term represents labor income. The second term $\mathcal{D}_t$ represents dividends from the intermediate good sector, which are given

ing, see Hsiang, Kopp, Jina, Rising, Delgado, Mohan, Rasmussen, Muir-Wood, Wilson, and et al. (2017). Consistently, Acharya, Johnson, Sundaresan, and Tomunen (2024) show that heat stress exposure decreases labor productivity, thus impacting firm cash flows.

[34] In a previous version of the paper, we adopted the second interpretation, whereby greener production processes are associated with greater demand. Under this interpretation, it is the final good sector that bears a convex cost $\frac{b}{2g_{jt}} Y_{jt}^2$ due to its sourcing of inputs from polluting firms, which is greater the lower the degree of greenness (and, thus sustainability) of the input. The qualitative takeaways continue to hold under this alternative interpretation.

[35] See Appendix A.5 for the analytical details and calculations.

by the flow of revenues of these firms net of abatement and innovation costs, and net of the cost imposed by carbon pricing on their emissions (see Appendix A.5 for details). The third term $\mathcal{T}_t$ represents the flow coming from the regulator, which is given by the proceeds from carbon pricing net of the subsidy to innovation.[36] The last term of the equation is the social cost of carbon, which aggregates net emissions in the different sectors j. The parameter Ω captures the social cost per emission, eroding consumption. Along the balanced greening path, all quantities green at the endogenous rate G supported by green innovation in the intermediate sectors j with $e^{Gt} = \int_0^1 g_{jt} dj$.

Endogenous carbon regulation The regulator optimally sets carbon pricing and the innovation subsidy to maximize the inter-temporal utility of the representative agent. The corresponding first order conditions with respect to carbon pricing and the innovation subsidy are then given, respectively, by:

$$\left(\frac{\partial y}{\partial \kappa}\nu - \frac{\partial s}{\partial \kappa}\right)(\kappa - \Omega) + \left(\frac{\phi}{\delta}(\lambda - 1)\mathcal{C}_0 - \zeta z\right)\frac{\partial z}{\partial \kappa} = 0, \tag{25}$$

$$\left(\frac{\phi}{\delta}(\lambda - 1)\mathcal{C}_0 - \zeta z\right)\frac{\partial z}{\partial \iota} = 0, \tag{26}$$

which characterize how optimal carbon regulation targets the twofold goal of reducing emissions and of ensuring sustainable technological change. The first equation illustrates that carbon pricing reduces consumption due to lower production and higher abatement but, at the same time, it reduces aggregate carbon emissions, then overall alleviating the imprint of the social cost of carbon on consumption.[37] In turn, equation (26) reveals that the optimal innovation subsidy balances the short-run reduction in consumption due to increased

[36] The proceeds from carbon pricing do not affect the representative agents' consumption directly: Carbon pricing reduces the dividend coming from the intermediate sector, and yet the consumer eventually receives the proceeds from the regulator. Similar arguments hold for the innovation subsidy, but flows run in the opposite direction.

[37] In fact, the last term in the first-order condition with respect to κ (equation (25)) is zero, as equation (26) needs to be zero too. If the central authority used carbon pricing only (i.e., no innovation subsidy), such term need not to be zero, meaning that the regulator would need to address both short- and long-term goals with one instrument only. We analyze this case numerically in the next paragraph.

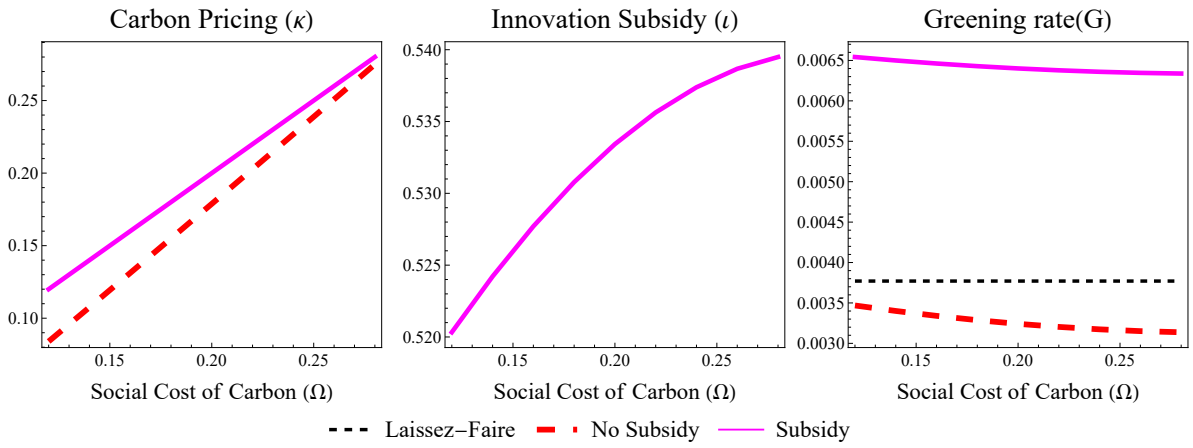


Figure 5: ENDOGENOUS CARBON REGULATION. The figure shows the optimal carbon price, the optimal innovation subsidy, and the greening rate of the economy. The short-dashed black line represents the laissez-faire benchmark, the red line represents the environment in which the regulator sets carbon pricing only, whereas the solid magenta line represents the environment in which the regulator pairs carbon pricing with the innovation subsidy.

investment in innovation with the long-term benefit of greening the economy at a faster pace.

Implications We provide further implications by solving the equilibrium numerically.[38] Figure 5 considers optimal carbon regulation in two cases: In the first, the regulator sets both carbon pricing and the innovation subsidy (solid magenta line); in the second, the regulator uses carbon pricing only (dashed red line). The figure illustrates how optimal carbon regulation and the greening rate of the economy vary with the social cost Ω. Carbon pricing increases with the social cost of carbon, not surprisingly. Moreover, the greening rate of the economy G is the lowest absent the subsidy, whereas it is the highest when carbon regulation contemplates an innovation subsidy on top of carbon pricing. As shown in Figure 6, this result is explained by the fact that, absent the innovation subsidy, carbon pricing can lead polluting firms to invest less in innovation, as illustrated in our analysis in

[38]In addition to our baseline calibration parameters (reported in Appendix A.4), we consider $\delta = 0.02$, $f = 0.15$, and normalize L to 1. Our choice for f ensures the ratio of wages over value added in the final goods sector (i.e., the labor share) is in the range of 40% in our numerical analyses. We run comparative statics for the social cost of carbon.

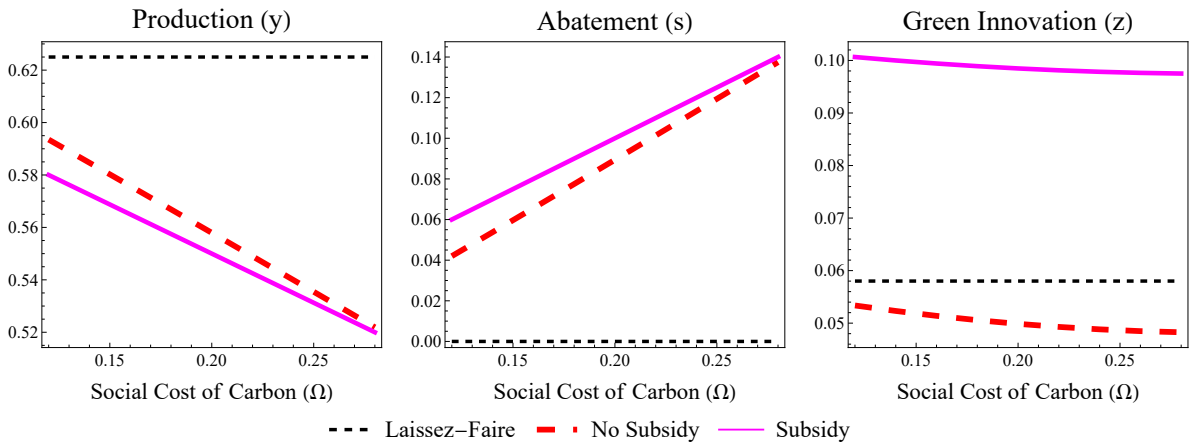


Figure 6: Optimal carbon regulation and corporate policies. The figure shows the production, abatement, and innovation policies of the intermediate-good sector when carbon regulation is endogenous. The short-dashed black line represents the laissez-faire benchmark, the red line represents the environment in which the regulator sets carbon pricing only, whereas the solid magenta line represents the environment in which the regulators pairs carbon pricing with the innovation subsidy.

partial equilibrium.

Figure 6 illustrates the optimal corporate policies with endogenous carbon regulation as a function of the social cost of carbon Ω. Consistent with our analyses in partial equilibrium, we confirm that carbon pricing shifts resources from green innovation towards abatement when acknowledging that producing with browner inputs is costlier—equivalently, when green breakthroughs yield productivity gains, as in our partial equilibrium analysis. In turn, the innovation subsidy helps support green technological change. Crucially, the increase in green innovation is not done at the expense of lower abatement: As the innovation subsidy complements carbon pricing, firms engage in both higher abatement and higher innovation (magenta line) compared to the case with carbon pricing only (i.e., without the innovation subsidy, red dotted line). Both types of investments target environmentally-friendly goals: Abatement starts reducing emissions in the present, green innovation ensures that the transition is technology-based (thus, carbon-efficiency gains are permanent). Moreover, the figure confirms our result in partial equilibrium that carbon pricing reduces production below the laissez-faire benchmark.

Several implications follow from our analysis. First, when endogenizing carbon regulation, our analysis continues to indicate that carbon pricing can lead firms to shift towards abatement when acknowledging that browner products are increasingly costly—and, thus, there are productivity gains from becoming greener. Second, complementing carbon pricing with a subsidy to green innovation stimulates higher green investment in both abatement and innovation, propelling a reduction in net scaled emissions and a faster greening path (captured by a larger G). Carbon pricing and subsidies to green innovation effectively complement each other, because they target distinct goals: Carbon pricing guarantees that emissions are reduced in the short-term, whereas the innovation subsidy fosters a technology-based transition. Whereas other works suggest to pair carbon pricing with subsidies to green innovation (Acemoglu et al., 2012, 2016), the underlying rationale in this model is different and novel. In previous studies, green innovation is the only type of green investment to address the environmental externality, so carbon taxes and innovation subsidies act as substitutes. By contrast, we allow firms to invest in abatement too. Allowing for heterogeneous green investment then enables us to identify novel roles for carbon pricing and subsidies to green innovation. Namely, carbon pricing increases the benefit from abatement, whereas subsidies to green innovation help support green innovation by reducing its effective cost and, furthermore, enable the regulator to set a higher carbon price. We then highlight the complementary nature of carbon pricing and innovation subsidies.

9 Concluding remarks

The control of firms' carbon emissions by regulators poses a new challenge in the corporate world, which involves maximizing shareholder wealth by developing an optimal carbon management policy. This paper provides a unified model to study, precisely, how firms should optimally manage carbon emissions through production, green investment of various types, and the management of carbon credits. The framework is broad enough to allow a comparison of optimal corporate policies under the main carbon pricing schemes observed

internationally: carbon tax and carbon credit systems.

The main takeaways of our model are as follows. First, carbon pricing has a negative effect on a firm production rate and a positive effect on its engagement in abatement projects, overall reducing firm emissions. Second, carbon pricing has an ambiguous effect on green innovation when acknowledging the presence of productivity gains from sustainability that are independent of carbon regulation: Relative to laissez-faire, green innovation can decrease in jurisdictions where carbon prices are sufficiently low and, for a given level of the carbon tax, for firms whose green breakthroughs reduce net emissions only modestly. Third, when optimally set in general equilibrium, we show that carbon pricing and the green innovation subsidy can effectively complement each other: Both abatement and green innovation are higher when carbon pricing is combined with the subsidy.

Overall, our paper delivers new insights that speak to the long- and short-term effects of carbon regulation—a macroeconomic concern—but more importantly to the microeconomic and corporate aspects of carbon management. Our analysis can be extended in many ways—which we intend to examine in future research—to further uncover how the firms' new need to manage emissions can affect existing theories on corporate decision making.

A Appendix

A.1 Proof of the results in Section 3

Consider first the case with no productivity gains from sustainability. Maximizing equation (4) gives $s_n^B = z_n^B = 0$, as abatement and green innovation are costs with no associated benefit. Also, maximizing the same equation with respect to the production rate gives $Y_n^B = a/(2b)$. Substituting this back into the HJB equation then gives the expression for firm value reported in the main text.

Consider now the case in which there are productivity gains associated with sustainability, i.e., the firm's cash flows are as reported in the assumption. Using the scaling property gives the scaled HJB equation:

$$rv^B = \max_{y^B, z^B, s^B} \quad \phi z^B v^B (\lambda - 1) + \left(y^B \left(a - b y^B \right) - \frac{(z^B)^2}{2} \zeta - \frac{(s^B)^2}{2} \theta \right).$$

Substituting the optimal policies y^B, z^B, s^B reported in the main text into this equation gives:

$$\frac{\phi^2}{2\zeta} (\lambda - 1)^2 (v^B)^2 - r v^B + \frac{a^2}{4b} = 0$$

which we solve with respect to v^B and obtain equation (8).

Proof of Proposition 1 The result simply stems from the optimal policies above and reported in the main text. ■

A.2 Proof of the results in Section 4

To solve for the firm's optimal policies in the carbon tax system, we again use the scaling property and get:

$$rv^\tau = \max_{y^\tau, z^\tau, s^\tau} \quad \phi z^\tau (\lambda - 1) v^\tau + \left(y^\tau (a - b y^\tau) - \frac{(z^\tau)^2}{2} \zeta - \frac{(s^\tau)^2}{2} \theta - \kappa (y^\tau \nu - s^\tau - \xi) \right). \quad (27)$$

Plugging the optimal policies into the above equation we have

$$\frac{\phi^2}{2\zeta}\left(\lambda-1\right)^2\left(v^\tau\right)^2 - rv^\tau + \frac{\left(a-\kappa\nu\right)^2}{4b} + \frac{\kappa^2}{2\theta} + \kappa\xi = 0.$$

so we solve for firm value

$$v^\tau = \frac{r - \sqrt{r^2 - \frac{\phi^2}{\zeta}\left(\lambda-1\right)^2\left[\frac{\left(a-\kappa\nu\right)^2}{2b} + \frac{\kappa^2}{\theta} + 2\kappa\xi\right]}}{\frac{\phi^2}{\zeta}\left(\lambda-1\right)^2} \tag{28}$$

Proof of Proposition 2 Part (1) of the proposition is straightforward given that $s^\tau > 0$ under the carbon tax system, whereas $s^B = 0$ under laissez.faire. Given the expression for s^τ, it is obvious that investment in abatement increases with κ.

Part (2) of the proposition claims that the impact of the carbon tax on green innovation has a more nuanced impact. First, recall from Section 3 that the firms does not invest at all in green innovation absent productivity gains from green breakthroughs, i.e., $z_n^B = 0$. When instead acknowledging productivity gains from green breakthroughs, $z^B > 0$ under laissez-faire too. Under the carbon tax case, the expression for the optimal investment in innovation continues having the same functional form as in laissez-faire but for the fact that the value-to-sustainability ratio differs in the two environments. Thus, z^τ can be higher or smaller than z^B if v^τ versus v^B is. By calculations, we have $v^B > v^\tau$ if the following inequality holds:

$$\nu\frac{2a-\kappa\nu}{2b} - \frac{\kappa}{\theta} - 2\xi > 0. \tag{29}$$

Straightforward calculations then imply that if the carbon tax κ is sufficiently low (namely, if $\kappa < \frac{\frac{a\nu}{b}-2\xi}{\frac{\nu^2}{2b}+\frac{1}{\theta}}$), condition (29) holds and, thus, $v^B > v^\tau$. In this case, $z^B > z^\tau$, meaning that green innovation is lower under the carbon tax. If, instead $\kappa > \frac{\frac{a\nu}{b}-2\xi}{\frac{\nu^2}{2b}+\frac{1}{\theta}}$, then (29) does not hold—in this case, $v^B < v^\tau$ and $z^B < z^\tau$ too, meaning that the green innovation is larger under the carbon tax.

For a given κ, we can also show that condition (29) holds if the parameter ξ (capturing how much greenness relates to emission reductions) is sufficiently low, namely if $\xi < \frac{1}{2}\left(\frac{\nu a}{b} - \frac{\kappa\nu^2}{2b} - \frac{\kappa}{\theta}\right)$. In this case, $z^B > z^\tau$. If, instead, $\xi > \frac{1}{2}\left(\frac{\nu a}{b} - \frac{\kappa\nu^2}{2b} - \frac{\kappa}{\theta}\right)$, then $z^B < z^\tau$. ■

A.3 Proof of the results in Section 5

To solve the firm's problem when it accumulates credits, we use the homogeneity property explained in the main text. That is, we have:

$$V_C(C,g) = v'(c) \quad V_{CC}(C,g) = \frac{v''(c)}{g},$$

which we substitute into equation (13) and get the scaled HJB:

$$rv(c) = \max_{z,y,s} \; (s+\xi-y\nu)v' + \frac{\sigma^2}{2}y^2v'' + \phi z\left(\lambda v(c^*) - v(c)\right) + \left(y(a-by) - \frac{z^2}{2}\zeta - \frac{s^2}{2}\theta - \chi c\right). \tag{30}$$

Differentiating equation (30) gives the optimal policies reported in equations (15), (16), and (17). Plugging these policies into (30) gives the following equation:

$$rv(c) = \frac{\phi^2}{2\zeta}(\lambda v(c^*) - v(c))^2 + \frac{(v'(c))^2}{2\theta} + \frac{(a-\nu v')^2}{2(2b-\sigma^2 v'')} - \chi c + \xi v'. \tag{31}$$

At $c = c^*$, this equation boils down to

$$\frac{\phi^2}{2\zeta}(\lambda-1)^2 v^2(c^*) - rv(c^*) + \frac{\gamma^2(1-\psi)^2}{2\theta} + \frac{(a-\nu\gamma(1-\psi))^2}{4b} - \chi c^* + \xi\gamma(1-\psi) = 0,$$

which we solve with respect to $v(c^*)$:

$$v(c^*) = \frac{\zeta}{\phi^2(\lambda-1)^2}\left[r - \sqrt{r^2 - 2\frac{\phi^2}{\zeta}(\lambda-1)^2\left[\frac{\gamma^2(1-\psi)^2}{2\theta} + \frac{(a-\nu\gamma(1-\psi))^2}{4b} - \chi c^* + \xi\gamma(1-\psi)\right]}\right]$$

Next we show Proposition 4.

Proof of Proposition 4 $v''(c) \leq 0$ means that v' is monotonically decreasing for any c. Towards a contradiction, suppose that there are two credit levels $0 < c_1 < c_2 < c^*$ such that $v'(c_1) = v'(c_2) = \Gamma > \gamma(1-\psi)$. This means that there is a $\bar{c} \in [c_1, c_2]$ at which v' attains a local maximum—i.e., $v''(\bar{c}) = 0$, $v''(c_1) > 0$, and $v''(c_2) < 0$. At c_1, firm value satisfies:

$$rv(c_1) = \frac{\phi^2}{2\zeta}\left(\lambda v(c^*) - v(c_1)\right)^2 + \frac{(v'(c_1))^2}{2\theta} + \frac{(a-\nu v'(c_1))^2}{2(2b-\sigma^2 v''(c_1))} - \chi c_1 + \xi v'(c_1) \tag{32}$$

whereas at c_2:

$$rv(c_2) = \frac{\phi^2}{2\zeta}\left(\lambda v(c^*) - v(c_2)\right)^2 + \frac{(v'(c_2))^2}{2\theta} + \frac{(a-\nu v'(c_2))^2}{2(2b-\sigma^2 v''(c_2))} - \chi c_2 + \xi v'(c_2). \tag{33}$$

Subtracting (33) from (32) gives:

$$\begin{aligned} r\left[v(c_1) - v(c_2)\right] =& \frac{\phi^2}{2\zeta}\left[\left(\lambda v(c^*) - v(c_1)\right)^2 - \left(\lambda v(c^*) - v(c_2)\right)^2\right] \\ &+ \frac{(a-\nu v'(c_1))^2}{2}\left[\frac{1}{(2b-\sigma^2 v''(c_1))} - \frac{1}{(2b-\sigma^2 v''(c_2))}\right] - \chi(c_1 - c2). \end{aligned} \tag{34}$$

Because v' is positive over the interval $[c_1, c_2]$, then $v(c_1) < v(c_2)$. Therefore, the left-hand side of this equation is negative. In turn, the first term on the right-hand side is positive, because $c_1 < c_2$ and $v(c)$ is increasing in c. Because $v''(c_1) > 0$ and $v''(c_2) < 0$, the second term is positive too. Finally $-\chi(c_1 - c_2) > 0$ too. Thus, equation (34) does not hold. It then means such two levels c_1 and c_2 do not exist. v' then monotonically decreases in c, and the claim holds. ■

As an implication of the above proposition we have that the firm always produces below its laissez-faire counterpart. I.e., the numerator of $y(c)$ (see equation (15)) is smaller than a, as $v' \geq \gamma(1-\psi)$. In turn, because $v'' < 0$, the denominator of $y(c)$ is equal or greater than $2b$ as $v'' \leq 0$. As a result, because the quantity $y(a-by)$ is non decreasing in y up to $y = a/(2b)$, then revenues are also uniformly smaller in the trading system vis-à-vis the

laissez-faire environment.

We now prove Proposition 8—it is indeed useful to show this first, as the results on monotonicity helps prove Proposition 7, in turn useful to prove Proposition (5).

Proof of Proposition 8 We start by showing part (1) of the claim. Differentiating the optimal policy gives:

$$y'(c) = \frac{-v''(c)\nu + \sigma^2 y(c) v'''(c)}{2b - \sigma^2 v''} \tag{35}$$

As $v'' \leq 0$ (by Proposition 4) and $y \geq 0$, then to prove the claim we simply need to show that $v''' > 0$ for any c. Differentiating equation (31) gives:

$$rv'(c) = \quad -\frac{\phi^2}{\zeta}(\lambda v(c^*) - v(c))v'(c) + \frac{v'(c)v''(c)}{\theta} + \frac{a - \nu v'}{2b - \sigma^2 v''}\left(-\nu v''(c) + \frac{\sigma^2(a - \nu v')v'''}{2(2b - \sigma^2 v'')}\right) - \chi + \xi v''.$$

At $c = c^*$, the equation becomes:

$$rv'(c^*) = \quad -\frac{\phi^2}{\zeta}(\lambda v(c^*) - v(c^*))v'(c^*) + \frac{a - \nu v'(c^*)}{2b}\left(\frac{\sigma^2(a - \nu v'(c^*))v'''(c^*)}{2(2b)}\right) - \chi.$$

Because the left-hand side is positive (recall that $v'(c^*) = \gamma(1 - \psi)$) whereas the first and last terms on the right-hand side are negative, $v'''(c^*)$ must be positive for the equation to hold. Toward a contradiction, suppose this is not the case and there is a point c^{**} at which $v'''(c^{**}) = 0$. Hence, there are two levels $c_1 < c^{**} < c_2$ such that $v''(c_1) = v''(c_2)$ (and negative by Lemma 4), and $v'''(c_1) < 0$ and $v'''(c_2) > 0$. Thus, we have at c_1

$$\begin{aligned} rv'(c_1) = &-\frac{\phi^2(\lambda v(c^*) - v(c_1))v'(c_1)}{\zeta} + \frac{v'(c_1)v''(c_1)}{\theta} \\ &+ \frac{a - \nu v'(c_1)}{2b - \sigma^2 v''(c_1)}\left(-\nu v''(c_1) + \frac{\sigma^2(a - \nu v'(c_1))v'''(c_1)}{2(2b - \sigma^2 v''(c_1))}\right) - \chi + \xi v''(c_1) \end{aligned} \tag{36}$$

and at c_2

$$rv'(c_2) = -\frac{\phi^2(\lambda v(c^*) - v(c_2))v'(c_2)}{\zeta} + \frac{v'(c_2)v''(c_1)}{\theta} + \frac{a - \nu v'(c_2)}{2b - \sigma^2 v''(c_1)}\left(-\nu v''(c_1) + \frac{\sigma^2(a - \nu v'(c_2))v'''(c_2)}{2(2b - \sigma^2 v''(c_1))}\right) - \chi + \xi v''(c_2). \tag{37}$$

Subtracting (36) from (37) gives

$$r(v'(c_2) - v'(c_1)) = \frac{\phi^2}{\zeta}\left[(\lambda v(c^*) - v(c_1))v'(c_1) - (\lambda v(c^*) - v(c_2))v'(c_2)\right] + \frac{(v'(c_2) - v'(c_1))v''(c_1)}{\theta} + \frac{\nu^2 v''(c_1)(v'(c_2) - v'(c_1))}{2b - \sigma^2 v''(c_1)} + \sigma^2 \frac{(a - \nu v'(c_2))^2 v'''(c_2) - (a - \nu v'(c_1))^2 v'''(c_1)}{2(2b - \sigma^2 v''(c_1))^2}.$$

By Proposition 4, the left-hand side is negative, whereas the first, second, and third terms on the right-hand side are positive. Moreover, under the conjecture, $v'''(c_2) > 0$ and $v'''(c_1) < 0$ hold, and the last term is positive too. Hence, the ODE would not hold so it must be that v''' does not switch sign, so $y(c)$ is monotonically increasing. The result about the monotonicity of revenues stems from the monotonicity of $y(c)$. In fact, revenues are $y(c)(a - by(c))$, whose derivative is $(a - 2by(c))$, which is indeed positive.

Part (2) of the claim stems from Proposition (4), as $s'(c) = v''(c)/\theta$. In turn, $z'(c) = -\frac{\phi}{\zeta}v'(c)$–as $v'(c) \geq \gamma(1 - \psi) > 0$ for any c.

Part (3) of the proof stems from the monotonicity of $y(c)$ and $s(c)$. The claim follows. ■

Proof of Proposition 7 The inequality $v(c^*) > v^B$ boils down to:

$$a - \gamma(1 - \psi)\left[\frac{\nu}{2} + \frac{b}{\theta\nu} + \frac{2\xi b}{\gamma(1 - \psi)\nu} - 2\chi c^* \frac{b}{\gamma^2(1 - \psi)^2\nu}\right] < 0. \tag{38}$$

Consider now the firm's net emissions. Suppose that these are positive for at least some values of c. Because they are monotonic increasing in c, it implies that they are at least

positive at c^*.

$$\nu y(c^*) - s(c^*) - \xi = \frac{a\nu - \nu^2\gamma(1-\psi)}{2b} - \frac{\gamma(1-\psi)}{\theta} - \xi > 0$$

By calculations, this boils down to

$$a - \gamma(1-\psi)\left[\nu + \frac{2b}{\theta\nu} + \frac{2b\xi}{\nu\gamma(1-\psi)}\right] > 0$$

However, this inequality cannot coexist with equation (38). The claim follows. ■

Proof of Proposition 5 Part (1) of the claim simply stems from the fact that the firm does not invest in abatement in the laissez-faire benchmark, whereas $s(c) \geq \frac{\gamma(1-\psi)}{\theta} > 0$ for any c.

Consider now part (2), positing that the impact of the carbon trading system on innovation is ambiguous. The case with no productivity gains is analogous to the carbon tax case. Consider now the case with productivity gains, in which $z(c)$ can be both higher or lower than z^B. Namely, $z(c^*) > z^B$ if $v(c^*) > v^B$, which in turn boils down to condition (38), which we can rewrite as follows

$$a - \gamma(1-\psi)\left[\frac{\nu}{2} + \frac{b}{\theta\nu}\right] - \frac{2\xi b}{\nu} + 2\chi c^* \frac{b}{\gamma(1-\psi)\nu} < 0. \tag{39}$$

As a direct effect, increasing γ makes the left hand side smaller—thus, the condition is more likely to hold.[39] As a result, $v(c^*) > v^B$ is more likely to hold if γ is sufficiently large—thus, $z(c^*) > z^B$ and, because $z(c)$ decreases with c (as shown in Proposition 8), then $z(c) > z^B$ for any c. In turn, if γ is sufficiently small, condition (39) does not hold. In this case, $v(c^*) < v^B$, meaning that $z(c^*) < z^B$. ■

[39] In fact, the impact of c^* is second order.

A.4 Details on the analysis in Section 6

We hereby describe our baseline parameterization. We assume that the risk free rate is equal to 0.02. We normalize ϕ to 1 as in other innovation models—hence, z effectively represents the arrival rate of a green breakthrough.[40] We assume that λ is equal to 1.065, which is consistent with the innovation step size estimated by Acemoglu et al. (2016). We assume that the cost ζ is four times larger than α, meaning that innovation is much costlier than abatement. We assume that the emission intensity ν is equal to 0.3, so to match the average emissions around the world per purchasing power parity of GDP as reported by the World Bank. Moreover, we assume that the volatility coefficient σ is 0.3, which implies that the volatility of emissions is about 15% in line with the findings in Ramadorai and Zeni (2024). Whereas carbon price is the main dimension with respect to which the comparative statics are derived, in other comparative statics we assume that both the carbon tax rate κ as well as the price of carbon credits γ are equal to 0.2, so that the expected net emissions' flow of our baseline firm is positive under both carbon pricing systems.[41] Moreover, we assume that the maintenance cost χ is 0.0125, and the transaction fee ψ is 15%, consistent with the information on trading fees reported by carbon exchanges.[42] We set ξ to 0.001 and explore our results for higher values in our comparative statics. We set the parameters characterizing the firm cash flows to $a = 0.5$ and $b = 0.4$, which give a markup of around 30% at c^* (consistent with Hall, 2018) and a return on assets consistent with that of dividend-paying firms in Farre-Mensa and Ljungqvist (2016).

A.5 Proof of the results in Section 8

As clarified in the main text of Section 8, in our general equilibrium setting we stick to the first interpretation of equation (2) (i.e., that green breakthroughs warrant lower operating

[40] See, for instance, Akcigit, Hanley, and Serrano-Velarde (2021), Akcigit and Kerr (2018), or Bustamante and Zucchi (2025b).

[41] The price of carbon varies widely within and across jurisdictions. As a result, we assess the robustness of our results extensively through comparative statics.

[42] The fees reported by exchanges live in a broad range, up to 20%. In general, smaller exchanges charge larger fees. Moreover, tax treatment of the proceeds from selling credits effectively increases the value of ψ.

costs borne by intermediate firms), so to work out the accounting. Namely, the final good sector maximizes:

$$\mathcal{Y}_t - \int_0^1 p_{jt} Y_{jt} dj - \mathcal{W}_t L$$

gives the following demand function for the intermediate products $p_{jt} = a$, which is the gross revenue to the intermediate firms used in our partial equilibrium analysis. In the following, for simplicity and without loss of generality, we normalize L to one.

The expected increase in sustainability in the input j is given, per time interval dt, by

$$E_{t-}[dg_{jt}] = g_{jt-}(\lambda - 1)\phi z_{jt} dt.$$

Breakthroughs occur at independent Poisson times in different industries j. By the law of large numbers, the growth rate of sustainability gives

$$\int_0^1 g_{jt} dj = e^{Gt} \tag{40}$$

as we assume a balanced greening path.[43]

Under the assumption of the balanced greening path, the output of the consumption good sector is given by

$$\mathcal{Y}_t^* = \int_0^1 \left(a \frac{a - \nu\kappa}{2b} g_{jt} + f g_{jt} \right) dj = \left(a \frac{a - \nu\kappa}{2b} + f \right) \int_0^1 g_{jt} dj$$

Moreover, because the final good sector is competitive, the ensuing equilibrium wage is

$$\mathcal{W}_t = f \int_0^1 g_{jt} dj$$

In turn, the aggregate dividends to the representative agent from the final good sector

[43] Initial greenness is normalized to one for any j.

amounts to:

$$\mathcal{D}_t = \int_0^1 \left[Y_{jt} p_{jt} - \frac{b}{g_t} Y_{jt}^2 - \frac{(z_{jt})^2}{2} \zeta (1-\iota) g_{jt} - \frac{(s_{jt})^2}{2} g_{jt} \theta - \kappa (Y_{jt} \nu - s_{jt} g_{jt} - \xi g_{jt}) \right] dj.$$

Namely, the flow of dividends to the representative agent aggregates revenues net of abatement and innovation expenditures and net of carbon pricing in the different product lines. Finally, the net flow to the representative agent coming from the central authority (collecting carbon taxes and distributing subsidies) is given by the proceeds from the carbon tax net of the subsidy to the intermediate good sector, which are given by

$$\mathcal{T}_t = \int_0^1 \kappa (Y_{jt} \nu - s_{jt} g_{jt} - \xi g_{jt}) dj - \iota \int_0^1 \frac{(z_{jt})^2}{2} \zeta g_{jt} dj.$$

Using equation (24) and under the assumption of a balanced greening path, the consumption of the representative agent satisfies $\mathcal{C}_t = \mathcal{C}_0 e^{Gt}$, with

$$\mathcal{C}_0 = y(a - by) - \frac{z^2}{2}\zeta - \frac{s^2}{2}\theta - \Omega\left(\nu y - s - \xi\right) + f.$$

The agent maximizes utility from consumption so that the equilibrium interest rate satisfies $r = \delta + G$. Moreover, the social planner maximizes the inter-temporal utility of the representative agent, which is given by:

$$E\left[\int_0^\infty e^{-\delta t} \ln \mathcal{C}_t\right] = \frac{1}{\delta}\left[\ln(\mathcal{C}_0) + \frac{G}{\delta}\right].$$

Taking the first derivative of the above with respect to the carbon price κ and the innovation subsidy ι gives the first-order conditions reported in the main text (equations (25) and (26), respectively).

A.6 Extension: Net emissions affecting consumer demand

In this Appendix, we extend the model to allow not only sustainability but also carbon emissions to affect the firm's cash flows. This extension is particularly relevant under the second interpretation of the firm's cash flows, whereby consumers have a preference not only for more sustainable firms but also for firms exhibiting lower net emissions.[44] Namely, we assume that the firm's cash flows now satisfy:

$$Y_t \left[a - \frac{1}{g_t} \left(bY_t + d \left(\nu Y_t - s g_t - \xi g_t \right) \right) \right] \quad a > 0 \, , b > 0 \, , d > 0. \tag{41}$$

Plugging this in the firm's optimization problem and maximizing gives:

$$\begin{aligned} y =& \frac{a + d(s + \xi) - \kappa\nu}{2(b + d\nu)} \\ s =& \frac{\kappa + dy}{\theta} \end{aligned}$$

whereas z satisfies the same expression as in the baseline studied in the main text. Note that s and y are now a function of each other. Further calculations give an expression for s and y as a function of the primitive parameters,

$$\begin{aligned} y =& \frac{a\theta - \kappa(\theta\nu - d) + d\theta\xi}{2b\theta + d(2\theta\nu - d)} \\ s =& \frac{\kappa(2b + \nu d) + d(a + d\xi)}{2b\theta + d(2\theta\nu - d)}. \end{aligned}$$

First of all, similar to the formulation analyzed in the main text, the firm under-produces compared to laissez-faire and even more so if κ is larger, unless its net emissions are sufficiently negative. To see this,

$$\frac{dy}{d\kappa} = \frac{-(\theta\nu - d)}{2b\theta + d(2\theta\nu - d)}. \tag{42}$$

which is negative as long as $(\theta\nu - d) > 0$.[45] Indeed, if the opposite inequality held, the firm

[44] For simplicity and brevity, we focus on the carbon tax case.

[45] Note that the denominator is positive otherwise the production rate of the firm would be negative too.

would exhibit negative net emissions, as net emissions satisfy the following expression

$$\nu y - s - \xi = \frac{a(\theta\nu - d) - \theta\nu(\kappa\nu + d\xi) - 2b(\kappa + \theta\xi)}{2b\theta + d(2\theta\nu - d)}. \tag{43}$$

Because the second and the third terms at the numerator are negative, then net emissions are for sure negative if $(\theta\nu - d) < 0$.

Let us now turn to considering green investment. We can prove the following proposition, which is the equivalent of Proposition 2 of our baseline framework.

Proposition 9 *In the presence of a carbon tax:*
(1) The firm increases its investment in abatement compared to laissez-faire, and more so if κ is higher;
(2) The impact on green innovation is nuanced compared to laissez-faire.

Proof. Claim (1) is straightforward, as $s > 0$ unambiguously increases with κ. Consider now claim (2). Plugging the expressions of the optimal y, s, and z into the HJB and solving with respect to v gives

$$v = \frac{\zeta}{\phi^2(\lambda-1)^2}\left[r - \sqrt{r^2 - \frac{(\lambda-1)^2\phi^2\left[\theta a\left(a - 2\kappa\nu\right) + \theta\left(\kappa\nu + d\xi\right)^2 + 2ad(\kappa + \theta\xi) + 2b\kappa(\kappa + 2\theta\xi)\right]}{\zeta\left[2b\theta + d(2\theta\nu - d)\right]}}\right] \tag{44}$$

Taking the first derivative of the above gives

$$\frac{dv}{d\kappa} = -\frac{a(\theta\nu - d) - \theta\nu(\kappa\nu + d\xi) - 2b(\kappa + \theta\xi)}{2b\theta + d(2\theta\nu - d)} \frac{1}{\sqrt{r^2 - \frac{(\lambda-1)^2\phi^2\left[\theta a(a-2\kappa\nu) + \theta(\kappa\nu+d\xi)^2 + 2ad(\kappa+\theta\xi) + 2b\kappa(\kappa+2\theta\xi)\right]}{\zeta[2b\theta + d(2\theta\nu - d)]}}} \tag{45}$$

It is possible to show that this first derivative is negative if κ is sufficiently small, whereas it turns positive if κ is sufficiently large. The claim follows. ■

It is then worth noting that one difference of this setting compared to the one considered in the baseline model is that s is positive in the laissez-faire case with no carbon tax. At the same time, however, the main result continues to hold—that is, the carbon tax leads the

firm to invest more in abatement (and the more so the larger κ is) and has an ambiguous effect on green innovation.

We therefore confirm that the main result of our model are confirmed when allowing consumers to exhibit a higher demand for firms with lower emissions.

A.7 Extension: Two-stages model

While the model considered in the main text is more appropriate to describe and study green innovation—leading, by its very nature, to continuous improvements that shape the transition to greener technologies at the firm (captured by ϕz) and at the aggregate level (captured by G in our Section 8)—this appendix reports a simplified two-stage version of it, which allows us to show robustness of the main results. In the first stage, the firm invests in green innovation. If the firm attains a successful breakthrough as a consequence of its engagement in green innovation (whose rate is endogenously chosen), the second stage starts and the firm stops investing in green innovation thereafter. (If, instead, the innovation process continued forever—i.e., in the second stage too—then we would fall back to the infinite horizon model in the main text with continuous innovation improvements). Throughout, we focus on the case of firms with positive net emissions.

We denote firm value in the first and second stage by V_1 and V_2, respectively. To isolate the strengths at play, we consider two variants of this simplified model. In the first (reported in the first subsection below), we assume that a green breakthrough only reduces the firm's emission intensity from ν to ν/g (with $g > 1$), so the per-unit cash flow is simply $(a - bY_2)$ similar to the case analyzed at the beginning of Section 3. In the second version (reported in the second subsection below), on top of the reduction in the emission intensity, a breakthrough brings along productivity gains as in the main text—i.e., the per-unit cash flows are given by $\left(a - b\frac{Y_2}{g}\right)$ subject to the same interpretation as for equation (2) in the main text (see the discussion therein).[46]

[46] As in the main text, these additional benefits can be motivated as a reduction in the operating cost or an increased demand for the firm's product.

Hence, not only this extension allows us to confirm that the assumption (made for the sake of tractability) of the innovation-driven reduction in emission intensity being subtractive is harmless in the main model (i.e., breakthroughs increasing g lead to a ξg drop in carbon emissions), but also to single out the incentives associated to green innovation driven by (1) saving on the carbon tax thanks to the associated reduction in emission intensity, and (2) additionally reaping productivity gains from green breakthroughs.

Green innovation only reduces emission intensity If a green breakthrough merely reduces emission intensity (i.e., we shut down the presence of related productivity gains), then the only associated benefit is a reduction in the carbon tax liability. We solve the model backwards, starting from firm policies and value in the second stage, i.e., when the firm has already attained a breakthrough. In this case, V_2 satisfies

$$rV_2 = \max_{Y_2, s_2} \quad Y_2(a - bY_2) - \frac{\theta s_2^2}{2} - \kappa \left(Y_2 \frac{\nu}{g} - s_2 \right) \tag{46}$$

with production and abatement rates satisfying, respectively:

$$Y_2 = \frac{ag - \kappa\nu}{2bg}$$

and

$$s_2 = \frac{\kappa}{\theta}.$$

It is worth noting that an increase in g leads to an increase in the firm's production rate, thanks to the reduction in costs associated with the carbon tax liability. The result is consistent with the Jevons paradox discussed in the main text and aligns with the results in the baseline version of the model. That is, even *absent* productivity gains from a greater g, the improved carbon efficiency induces the firm to increase production.

Now, consider firm value before the breakthrough:

$$rV_1 = \max_{Y_1, s_1, z} \quad Y_1 \left(a - bY_1\right) - \frac{\theta s_1^2}{2} - \frac{\zeta z^2}{2} - \kappa \left(Y_1 \nu - s_1\right) + \phi z \left(V_2 - V_1\right) \tag{47}$$

where the last term on the right-hand side captures the impact of a technological breakthrough, which makes firm value switch to V_2. Before the breakthrough, the firm maximizes not only over abatement and production, but also over the innovation rate z. Namely, the optimal production rate before the breakthrough satisfies:

$$Y_1 = \frac{a - \kappa\nu}{2b},$$

the optimal abatement rate continues to be $s_1 = \frac{\kappa}{\theta}$, i.e., it does not change following a breakthrough. Lastly, the optimal innovation rate satisfies:

$$z = \frac{\phi(V_2 - V_1)}{\zeta}.$$

As in the baseline model, the key driver of the firm's engagement in green investment is driven by how much firm value increases upon a breakthrough, which is $V_2 - V_1$ in this extension (in turn, it is $(\lambda - 1)v$ in our baseline model).

Substituting optimal choices into the HJB and solving for firm value gives

$$V_2 - V_1 = -\frac{r\zeta}{\phi^2} + \frac{\sqrt{\zeta}}{\phi^2\sqrt{2bg}}\sqrt{2bg^2r^2\zeta + (g-1)\phi^2\kappa\nu(2ag - (1+g)\kappa\nu)}$$

Taking the first derivative with respect to κ gives

$$\frac{d(V_2 - V_1)}{d\kappa} = \frac{(g-1)\sqrt{\zeta}\nu(ag - (1+g)\kappa\nu)}{g\sqrt{2b}\sqrt{2bg^2r^2\zeta + (g-1)\kappa\nu\phi^2(2ag - (1+g)\kappa\nu)}}$$

By calculations, the first derivative of $V_2 - V_1$ with respect to κ is positive if $ag - (1+g)\kappa\nu$ is also positive. This quantity can be shown to be positive if the firm's carbon emissions decrease after the breakthrough—which, indeed, is the only reason why the firm has incentives to invest in innovation in this variant of the model, i.e., to reduce its carbon tax liability. Hence, green innovation is greater with the carbon tax when it only reduces emissions (but does not induce productivity gains), as it reduces the cost of carbon regulation borne by

the firm.

Acknowledging productivity gains associated with green innovation Now, on top of reducing the firm's emissions—and then inducing a saving on the carbon tax—we assume that green innovation brings along productivity gains (such as cost reductions or increased demand for the firm's product, as per the two interpretations in the main text). In this case, after the breakthrough, firm value V_2 satisfies

$$rV_2 = \max_{Y_2, s_2} Y_2 \left(a - b\frac{Y_2}{g} \right) - \frac{\theta s_2^2}{2} - \kappa \left(Y_2 \frac{\nu}{g} - s_2 \right)$$

so the optimal choices post breakthrough are given by

$$Y_2 = \frac{ag - \kappa\nu}{2b}$$

whereas the abatement rate continues to satisfy $s_2 = \frac{\kappa}{\theta}$. As in the baseline model, the greater the carbon tax, the lower the optimal production rate and the bigger the abatement rate. Also, the production rate continues to be increasing with g (similar to the main text and to the version of the model considered in the previous section).

Before the breakthrough, firm value continues to satisfy equation (47), and so are the related corporate choices. In this case too, the key driver of the firm's engagement in green investment is driven by how much firm value increases upon a breakthrough, which is captured by the difference $V_2 - V_1$. Substituting optimal choices into the HJB gives

$$V_2 - V_1 = -\frac{r\zeta}{\phi^2} + \frac{\sqrt{\zeta}}{\phi^2\sqrt{2bg}} \sqrt{2bgr^2\zeta + (g-1)\phi^2(a^2 g - \kappa^2\nu^2)}$$

Taking the first derivative with respect to κ gives

$$\frac{d(V_2 - V_1)}{d\kappa} = -\frac{(g-1)\sqrt{\zeta}\kappa\nu^2}{\sqrt{2bg}\sqrt{2bgr^2\zeta + (g-1)\phi^2(a^2 g - \kappa^2\nu^2)}}$$

Because $g > 1$ (as innovation reduces the firm's emission intensity ν), then this derivative

is unambiguously negative. That is, the carbon tax erodes the firm's incentives to invest in innovation, consistent with our baseline model.

Takeaways The analysis in this tweaked version of the model then confirms our results in the infinite horizon model in the main text, as summarized below. First, the firm engages in abatement under the carbon tax compared to no engagement absent the carbon tax, and more so if κ is higher. Second, the impact on green innovation is nuanced:

- If a green breakthrough only brings along a reduction in emission intensity but does *not* bring any productivity gain: The firm's engagement in green innovation increases with the carbon tax (and is therefore greater than laissez-faire).
- If a green breakthrough does bring along productivity gains on top of a reduction of emission intensity: The carbon tax erodes the productivity gains associated with green innovation, then eroding the associated benefits compared to laissez-faire.

We conclude that, in gauging the impact of carbon regulation on green investment, it is important to take into account the existence of productivity gains from green breakthroughs, on top and beyond the mere savings from the carbon tax.

References

Acemoglu, D., P. Aghion, L. Bursztyn, and D. Hemous. 2012. "The Environment and Directed Technical Change." *American Economic Review* 102:131–166.

Acemoglu, D., U. Akcigit, D. Hanley, and W. Kerr. 2016. "Transition to Clean Technology." *Journal of Political Economy* 124:52–104.

Acharya, V., T. Johnson, S. Sundaresan, and T. Tomunen. 2024. "Is Physical Climate Risk Priced? Evidence from Regional Variation in Exposure to Heat Stress." Working Paper.

Aghion, P., U. Akcigit, and P. Howitt. 2014. "What Do We Learn from Schumpeterian Growth Theory?" *In: Aghion, P., Durlauf, S. (Eds.), Handbook of Economic Growth, Vol. 2, Elsevier, Amsterdam*, pp. 515–563.

Aghion, P., A. Bergeaud, M. De Ridder, and J. Van Reenen. 2025. "Lost in Transition: Financial Barriers to Green Growth." Working Paper.

Akcigit, U., D. Hanley, and N. Serrano-Velarde. 2021. "Back to Basics: Basic Research Spillovers, Innovation Policy and Growth." *Review of Economic Studies* 88:1–43.

Akcigit, U., and W. Kerr. 2018. "Growth Through Heterogenous Innovation." *Journal of Political Economy* 126:1374–1443.

Allen, F., A. Barbalau, and F. Zeni. 2024. "Reducing Carbon using Regulatory and Financial Market Tools." Working Paper.

Ambec, S., and P. Lanoie. 2008. "Does It Pay to Be Green? A Systematic Overview." *Academy of Management Perspective* 22:45–62.

Barbalau, A., and F. Zeni. 2025. "The Optimal Design of Green Securities." Working Paper.

Blanchard, O., C. Gollier, and J. Tirole. 2023. "The Portfolio of Economic Policies Needed to Fight Climate Change." *Annual Review of Economics* 15:689–722.

Bolton, P., H. Chen, and N. Wang. 2013. "Market Timing, Investment, and Risk Management." *Journal of Financial Economics* 109:40–62.

—. 2011. "A Unified Theory of Tobin's q, Corporate Investment, and Financing." *Journal of Finance* 66:1545–1578.

Bolton, P., M. Kacperczyk, and M. Wiedemann. 2023. "The CO2 Question: Technical Progress and the Climate Crisis." Working Paper.

Brown, J., G. Martinsson, P. Stromberg, and C. Thomann. 2025. "Carbon Pricing and Investment." Working Paper.

Brown, J., G. Martinsson, and C. Thomann. 2022. "Can Environmental Policy Encourage Technical Change? Emission Taxes and R&D Investment in Polluting Firms." *Review of Financial Studies* 35:4518–4560.

Bruegem, M., R. Marfe, and F. Zucchi. 2025. "Corporate Polices and the Term Structure of Risk." Working Paper.

Bushnell, J., H. Chong, and E. Mansur. 2013. "Profiting from Regulation: Evidence from the European Carbon Market." *American Economic Journal: Economic Policy* 5:78–106.

Bustamante, M., and F. Zucchi. 2025a. "Financially Constrained Carbon Management." Working Paper.

—. 2025b. "Innovation, Industry Equilibrium, and Discount Rates." Working Paper.

Ceglar, A., I. Jaccard, M. Parker, A. Popov, and F. Zucchi. 2025. "The Economics of Natural Capital." ECB Working Paper.

Chiu, W., P.H. Hsu, K. Li, and T. Tong. 2025. "Green Products." Working Paper.

Cohen, L., U. Gurun, and Q. Nguyen. 2025. "The ESG-Innovation Disconnect: Evidence from Green Patenting." *Management Science*, Forthcoming.

Dai, M., X. Giroud, W. Jiang, and N. Wang. 2024. "A q theory of internal capital markets." *Journal of Finance* 79:1147–1197.

Dangl, T., M. Halling, J. Yu, and J. Zechner. 2024. "Stochastic Social Preferences and Corporate Investment Decisions." Working Paper.

De Haas, R., and A. Popov. 2023. "Finance and Green Growth." *Economic Journal* 133:637–668.

De Jonghe, O., K. Mulier, and G. Schepens. 2020. "Going Green by Putting a Price on Pollution: Firm-level Evidence from the EU." Working Paper.

Décamps, J.P., S. Gryglewicz, E. Morellec, and S. Villeneuve. 2017. "Corporate Policies with Permanent and Transitory Shocks." *Review of Financial Studies* 30:162–210.

Décamps, J.P., T. Mariotti, J.C. Rochet, and S. Villeneuve. 2011. "Free cash flows, issuance costs, and stock prices." *Journal of Finance* 66:1501–1544.

Della Seta, M., E. Morellec, and F. Zucchi. 2020. "Short-term Debt and Incentives for Risk-taking." *Journal of Financial Economics* 137:179–203.

Derrien, F., P. Krueger, A. Landier, and T. Yao. 2025. "ESG News, Future Cash Flows, and Firm Value." *Journal of Finance* 80:3499–3554.

Dumas, B. 1991. "Super-Contact and Related Optimality Conditions." *Journal of Economic Dynamics and Control* 15:675–685.

Farre-Mensa, J., and A. Ljungqvist. 2016. "Do Measures of Financial Constraints Measure Financial Constraints?" *Review of Financial Studies* 29:271–308.

Fowlie, M., S. Holland, and E. Mansur. 2012. "What do Emission Market Deliver and to Whom? Evidence from Southern California's NOX Trading Program." *American Economic Review* 102:965–993.

Gans, J. 2012. "Innovation and Climate Change Policy." *American Economic Journal: Economic Policy* 4:125–145.

Hall, R. 2018. "New Evidence on the Markup of Prices over Marginal Costs and the Role of Mega-Firm in the US Economy." NBER Working Paper.

Heider, F., and R. Inderst. 2023. "Environmental Policy with Financing Frictions." Working Paper.

Heinkel, R., A. Kraus, and J. Zechner. 2001. "The effect of Green Investment on Corporate Behavior." *Journal of Financial and Quantitative Analysis* 36:431–449.

Hong, H., N. Wang, and J. Yang. 2023. "Mitigating Disaster Risks in the Age of Climate Change." *Econometrica* 91:1763–1802.

Hsiang, S., R. Kopp, A. Jina, J. Rising, M. Delgado, S. Mohan, D. Rasmussen, R. Muir-Wood, P. Wilson, and M.O. et al. 2017. "Estimating Economic Damage From Climate Change in the United States." *Science* 356:1362–1369.

Hugonnier, J., S. Malamud, and E. Morellec. 2015. "Capital Supply Uncertainty, Cash Holdings, and Investment." *Review of Financial Studies* 28:391–445.

Kaenzig, D. 2025. "The Unequal Economic Consequences of Carbon Pricing." *American Economic Review*, Forthcoming.

Kruse, T., M. Mohnen, P. Pope, and M. Sato. 2020. "Green Revenues, Profitability and market valuation: Evidence from global firm level dataset." Working Paper.

Kumar, M., and A. Purnanandam. 2024. "Carbon Emissions and Shareholder Value: Causal Evidence from the US Power Utilities." Working Paper.

Malamud, S., and F. Zucchi. 2019. "Liquidity, Innovation, and Endogenous Growth." *Journal of Financial Economics* 132:519–541.

Martin, R., L. de Preux, and U. Wagner. 2014. "The Impact of a Carbon Tax on Manufacturing: Evidence from Microdata." *Journal of Public Economics* 117:1–14.

Martin, R., M. Muuls, and U. Wagner. 2016. "The Impact of European Union Emission Trading Scheme on Regulated Firms: What is the Evidence after Ten Years?" *Review of Environmental Economics and Policy* 10:129–148.

Martinsson, G., L. Sajtos, P. Stromberg, and C. Thomann. 2024. "The effect of carbon pricing on firm emissions: Evidence from the Swedish CO2 Tax." *Review of Financial Studies* 37:1848–1886.

McKinsey & Company. 2022. "2021 ESG Report: Accelerating Sustainable and Inclusive Growth." Mckinsey and Company Report.

—. 2021. "The state of internal carbon pricing." McKinsey and Company Article.

Meier, J., H. Servaes, J. Wei, and S. Xiao. 2025. "Do Consumers Care About ESG? Evidence from Barcode-Level Sales Data." Working paper.

Metcalf, G., and J. Stock. 2023. "The Macroeconomic Impact of Europe's Carbon Taxes." *American Economic Journal: Macroeconomics* 15:265–286.

Morse, A., and P. Sastry. 2025. "The Economics of Net Zero Banking." *Annual Review of Financial Economics* 17:343–361.

Oehmke, M., and M. Opp. 2025. "A Theory of Socially Responsible Investment." *Review of Economic Studies* 92:1193–1225.

Popp, D. 2001. "The Effect of New Technology on Energy Consumption." *Resources and Energy Economics* 23:215–239.

Porter, M., and C. Van der Linde. 1995. "Toward a New Conception of the Environment-Competitiveness Relationship." *Journal of Economic Perspectives* 9:97–118.

Ramadorai, T., and F. Zeni. 2024. "Climate Regulation and Emissions Abatement: Theory and Evidence from Firms' Disclosures." *Management Science* 70:8217–9119.

Requate, T. 2005. "Dynamic Incentives by Environmental Policy Instruments—A Survey." *Ecological Economics* 54:175–195.

Sautner, Z., L. van Lent, G. Vilkov, and R. Zhang. 2023. "Firm-level Climate Change Exposure." *Journal of Finance* 78:1449–1498.

Shi, M., S. Zhou, and C. Meinerding. 2023. "Do Past Emissions Affect Firms' Supply of Green Technologies: Evidence from Technology-Related Job Postings." Working Paper.

The World Bank. 2024. "State and Trends of Carbon Pricing." The World Bank Report.

Zucchi, F. 2024. "The Real Effects of Financing and Trading Frictions." *Journal of Financial and Quantitative Analysis* 59:3835–3870.