Deposit Insurance for Wholesale Creditors: The Role of Liquidity Injection

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Abstract

Deposit insurance reform should be designed jointly with ex-post intervention. We develop a global-games model in which a policymaker sets deposit insurance ex ante, infers bank fundamentals from aggregate withdrawals, and chooses liquidity provision or forced liquidation. While deposit insurance reduces direct run costs, it dampens depositors' responsiveness to private signals, making withdrawals less informative and weakening targeted intervention. To preserve learning, optimal insurance is zero when run costs are low. Conversely, full coverage is optimal when high run costs make run suppression paramount. Ex-post intervention substitutes for deposit insurance at low run costs but complements it at high costs.

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1 Introduction

The 2023 failures of Silicon Valley Bank and First Republic Bank reignited debates over the design of the financial safety net for modern banking systems. One prominent line of discussion concerns deposit insurance reform for wholesale creditors—business and institutional depositors with uninsured balances—whose rapid withdrawals contributed to the failures of these banks. According to the FDIC's report *Options for Deposit Insurance Reform*, the share of uninsured deposits in the banking system has risen, reaching 46.6% in 2021, thereby increasing the system's exposure to run risk. In response, policymakers are evaluating whether broader coverage for wholesale creditors is necessary to restore depositor confidence and strengthen financial stability. This debate has been further shaped by recent analyses of the 2023 banking turmoil in academia, which revisit the design of deposit insurance in light of the fragility created by large uninsured funding bases (Acharya, Richardson, Schoenholtz, and Tuckman 2023).

Deposit insurance reform, however, cannot be evaluated in a vacuum. As highlighted by policymakers and academics alike, the safety net relies on multiple, interacting components. Recent policy work emphasizes that crisis management requires a coherent mix of ex-ante and ex-post measures, rather than isolated reform of any single instrument (Acharya, Carletti, Restoy, and Vives 2024). In the *Wharton Future of Finance* podcast,[1] Loretta Mester, former President of the Federal Reserve Bank of Cleveland, argues that broader deposit protection is not enough without credible access to central bank liquidity facilities, such as the discount window and the Bank Term Funding Program. Similarly, Hyun Song Shin of the Bank for International Settlements emphasizes that deposit insurance must be understood jointly with the lender-of-last-resort function. These views, consistent with Schich (2008), suggest that deposit insurance and emergency liquidity provision should be designed jointly rather than in isolation.

Despite extensive work on deposit insurance and lender-of-last-resort policy, their joint design remains underexplored. The interaction is inherently dynamic. Deposit insurance shapes depositors' incentives to run and therefore the bank's ex-post liquidity need. At the same time, the availability of ex-post intervention changes the value of ex-ante deposit insurance. This paper studies that interaction through a policy-portfolio lens. We ask two questions. How do deposit insurance and ex-post liquidity provision interact? And when a policymaker can

[1] See "The Future of Banking" (Knowledge at Wharton — edited transcript and audio), hosted by Professor Itay Goldstein, Chair of the Wharton Finance Department. Available at https://knowledge.wharton.upenn.edu/article/the-future-of-banking/.

inject liquidity during a crisis, how much deposit insurance should be provided to wholesale creditors?[2]

To address these questions, we develop a bank-run model that builds on Goldstein and Pauzner (2005). A fraction of depositors are retail and fully insured. The remaining depositors are wholesale creditors (hereafter simply "depositors"). Depositors may withdraw early either because they face liquidity needs or because they receive adverse private signals about the bank's fundamentals. To meet withdrawals, the bank must liquidate long-term investment, which can destroy value. The policymaker seeks to maximize economic efficiency by preserving positive-NPV banks while liquidating negative-NPV banks, net of the social cost of bank runs. Policy is chosen in two stages. Ex ante, the policymaker commits to a level of deposit insurance. Ex post, after observing aggregate withdrawals, she chooses whether to inject liquidity or force liquidation.[3]

The key friction is that fundamentals are not directly observed during the crisis. Instead, the policymaker learns from aggregate withdrawals. This signal is informative because it reflects withdrawals by informed depositors who respond to unfavorable private information; yet, it is noisy because it also reflects unknown mechanical liquidity needs, which are an exogenous aggregate shock. Hence, the same withdrawals that create pressure on the bank therefore also help the policymaker infer whether the bank is merely illiquid or fundamentally weak.

This observation generates the central trade-off in the paper. Deposit insurance stabilizes funding by discouraging withdrawals. But precisely because it suppresses withdrawals, it also reduces the extent to which aggregate withdrawals reveal information about fundamentals. A more generous guarantee therefore makes crises quieter, but it also makes the policymaker less informed at the moment when intervention decisions must be made. Ex-post liquidity provision, in turn, is valuable precisely because it can be targeted: it is most effective when withdrawals remain informative. The policymaker must therefore balance two forces. More deposit insurance lowers the direct expected cost of runs,[4] but less deposit insurance preserves a more informative withdrawal signal and improves the efficiency of ex-post intervention.

Our main result is that optimal deposit insurance depends on the marginal cost of runs. When the marginal run cost is small, the informational value of withdrawals dominates. The

[2]We take the view that deposit insurance for retail depositors, who are typically uninformed and already covered under current arrangements, is necessary to prevent inefficient panic runs and protect household welfare.

[3]For notational convenience, we treat forced liquidation as a "negative" liquidity injection and include it in the policy set.

[4]This cost captures, among other things, the premium required to raise emergency funds on short notice, as well as broader externalities associated with runs, including political or reputational costs.

policymaker then chooses the lowest feasible level of deposit insurance, allowing withdrawals to remain highly responsive to fundamentals so that ex-post liquidity support can be targeted efficiently. When the marginal run cost is large, the stabilization motive dominates. The policymaker then chooses full deposit insurance to suppress costly withdrawals. For intermediate run costs, the optimal policy involves partial coverage. Thus, once deposit insurance is embedded in a broader policy portfolio, its role is no longer simply to prevent runs. It also shapes the informativeness of the endogenous signal on which crisis management relies.

We formalize this learning channel using Blackwell's notion of informativeness. Higher deposit insurance dampens depositors' responsiveness to private signals and therefore garbles the signal observed by the policymaker. Because crisis intervention is chosen optimally after withdrawals are observed, Blackwell's theorem implies that a more informative withdrawal signal weakly increases the maximal expected payoff from intervention. This provides a sharp foundation for the learning motive: reducing deposit insurance can raise welfare not because runs are desirable per se, but because the information contained in withdrawals improves the allocation of ex-post liquidity support.

A central implication of the model emerges from comparing our baseline economy, in which the policymaker can inject liquidity ex post, with a benchmark in which deposit insurance is the only policy instrument. This comparison reveals that deposit insurance and liquidity injection can be either substitutes or complements.

They are substitutes when the marginal cost of runs is low. In that case, the policymaker values the information revealed by withdrawals. Because ex-post liquidity support can use that information, the baseline economy calls for less deposit insurance than the benchmark. Without ex-post intervention, by contrast, the informational value of withdrawals is irrelevant: deposit insurance serves only to reduce inefficient liquidation and run costs. The availability of liquidity injection therefore reduces the need for ex-ante insurance.

They become complements when the marginal cost of runs is high. In that region, the policymaker wants to suppress withdrawals, but ex-post liquidity intervention remains useful because it still allows weak banks to be liquidated after stress materializes. In other words, once liquidity support is available as an ex-post sorting device, deposit insurance no longer has to preserve runs as the only mechanism through which weak banks are disciplined. The policymaker can then rely more heavily on deposit insurance ex ante to limit costly withdrawals. The comparison between the benchmark and the baseline is therefore non-monotone: adding ex-post liquidity support lowers optimal deposit insurance for low run costs, but raises it for high run costs.

More broadly, the paper highlights a general mechanism in policy design. An ex-ante policy changes private agents' strategic behavior. That behavior generates an endogenous aggregate signal. The policymaker then learns from that signal and chooses an ex-post action. In our setting, deposit insurance is not only a stabilizing guarantee; it is also a device that coarsens the policymaker's information set at the moment of crisis management. This perspective helps clarify when ex-ante guarantees strengthen ex-post intervention and when they instead undermine the effectiveness of targeted crisis response.

Related literature. Our paper is related to the large literature on bank fragility driven by strategic complementarities in depositor withdrawals and panic-based runs (Diamond and Dybvig 1983). We follow Rochet and Vives (2004) and Goldstein and Pauzner (2005) in using a global-games framework, which is attractive for policy analysis because it delivers a unique equilibrium and links run behavior to fundamentals. Our environment is also related to Chari and Jagannathan (1988), who study withdrawals by uninformed depositors who cannot disentangle liquidity shocks from weak fundamentals. In our model, the corresponding inference problem is faced by the policymaker: after observing aggregate withdrawals, she must infer whether stress reflects fundamental weakness or aggregate liquidity demand.

Our paper also contributes to the literature on policies that mitigate inefficient runs. A central policy in this literature is deposit insurance. Diamond and Dybvig (1983) provide the classic rationale for deposit insurance as a tool to eliminate panic-based runs. Subsequent work emphasizes the associated moral hazard costs (Keeley 1990; Cooper and Ross 2002). More recently, Allen, Carletti, Goldstein, and Leonelloe (2018) compare alternative forms of deposit insurance and show that deposit insurance can be welfare improving despite moral hazard. Other work studies deposit insurance when aggregate liquidity demand is uncertain (Wallace 1988; Chari 1989). We contribute to this literature by studying deposit insurance jointly with ex-post liquidity intervention. This perspective uncovers a new trade-off: deposit insurance not only stabilizes funding, but also weakens the informational content of withdrawals that is valuable for crisis management. Relatedly, Dávila and Goldstein (2023) take a sufficient-statistics approach to the design of deposit insurance. Our analysis complements that approach by showing that optimal coverage depends on the broader policy portfolio within which deposit insurance is embedded.

The paper is also related to the literature on ex-post bailouts and crisis intervention. That literature often treats deposit insurance and bailouts as alternative forms of government guarantees. But the distinction in timing is central: deposit insurance is an ex-ante commitment,

whereas liquidity support is an ex-post response. A large literature studies the distortions created by ex-post bailout policy, especially through its effects on ex-ante incentives (Holmström and Tirole 1998; Farhi and Tirole 2012; Keister 2016; Keister and Narasiman 2016; Chari and Kehoe 2016). Nosal and Ordoñez (2016) show that uncertainty about the source of distress can discipline bailout expectations. Cooper and Kempf (2016) study a policy mix of ex-post deposit insurance and orderly resolution, and Mitkov (2020) analyze optimal ex-post bailout policy under wealth inequality. Our contribution is different. We emphasize the flexibility of ex-post liquidity support as a state-contingent tool and show how its availability changes the optimal design of ex-ante deposit insurance.

Finally, our paper contributes to the literature on government learning from the actions of private agents. Bond and Goldstein (2015) study a setting in which the government learns from financial prices that aggregate investors' private information.[5] Ahnert, Machado, and Pereira (2023) examine government learning from markets when an investor has market power. Goldstein, Ozdenoren, and Yuan (2011) study how a central bank learns from speculative attacks when deciding whether to defend a currency peg, in an environment that also features strategic complementarities. Relative to these papers, our novel contribution is to show how a proactive policy—deposit insurance—changes private agents' incentives, thereby changing the information contained in aggregate behavior and hence the effectiveness of a reactive policy—liquidity injection.

2 A Model of Policy Portfolio for Banks

This section develops a theoretical framework to analyze the interaction between deposit insurance and liquidity intervention. The economy has three periods, indexed by $t = 0, 1, 2$, and is populated by a bank, a policymaker, and a continuum of depositors $i \in [0, 1]$. All agents are risk-neutral, and there is no time discounting.

The Bank. At $t = 0$, the bank raises one unit of capital in total from depositors through demandable deposit contracts and invests it in a divisible long-term project with constant returns to scale. Let θ denote the bank's investment fundamentals. At the beginning of $t = 1$, Nature draws θ from the uniform distribution on $(-\Theta, \Theta)$, where $\Theta > 0$ is arbitrarily large. If held

[5] More broadly, our paper is related to the literature on informational feedback between real decisions and financial markets. See Bond, Edmans, and Goldstein (2012) and Goldstein (2023) for reviews. Much of that literature features strategic substitution in investors' actions.

until $t = 2$, each unit of investment yields a gross return of $R > 1$ with probability $P(\theta)$ and zero with probability $1 - P(\theta)$.

To ensure equilibrium uniqueness, particularly regarding depositors' withdrawal decisions, we follow the standard approach in the global games literature by imposing lower and upper dominance regions. Specifically, there exist θ^L and θ^U with $-\Theta < \theta^L < \theta^U < \Theta$ such that

$$P(\theta) = \begin{cases} 0, & \text{for all } \theta \in \left(-\infty, \theta^L\right], \\ 1, & \text{for all } \theta \in \left[\theta^U, \infty\right). \end{cases} \tag{1}$$

We further assume that $P(\theta)$ is strictly increasing on $\left(\theta^L, \theta^U\right)$ and differentiable on $\mathbb{R}$. Note that although the true support of the bank's investment fundamentals θ is $(-\Theta, \Theta)$, we define the success probability function $P(\theta)$ on $\mathbb{R}$. This is a technical assumption because, as discussed below, depositors hold an improper uniform prior over $\mathbb{R}$. We then refer to $\left(-\infty, \theta^L\right]$ as the lower-dominance region and $\left[\theta^U, +\infty\right)$ as the upper-dominance region.

At $t = 1$, the bank may liquidate part or all of its long-term investment to meet withdrawal demands. Liquidation occurs only when necessary to fulfill withdrawals requested by depositors or required by the policymaker's forced-liquidation action. Each unit of bank investment has liquidation value

$$L(\theta) = \begin{cases} 1, & \text{if } \theta < \theta^U, \\ r_1, & \text{if } \theta \geq \theta^U, \end{cases} \tag{2}$$

where $r_1 > 1$. The latter assumption reflects the intuition that when the bank's project quality is undeniably high, adverse selection costs associated with early liquidation dissipate.[6]

Demand Deposit Contract. The bank signs a demand deposit contract (r_1, r_2) with each depositor at $t = 0$. To focus on the interaction between policy tools during banking crises, the contract is assumed to be exogenously given, as in a competitive banking industry with a standard contract. We assume the parameter ordering $R > r_2 > r_1 > 1$. Under this contract, a depositor requesting a withdrawal at $t = 1$ is promised r_1, while a depositor staying until $t = 2$ is promised up to r_2, subject to the availability of funds. At $t = 2$, the bank's equity holder consumes any residual funds after all depositors are repaid.

[6]This is a standard assumption in Goldstein and Pauzner (2005), used to ensure that agents have no incentive to run in the upper-dominance region.

Depositors. Each depositor i contributes one unit of capital at $t = 0$, and total deposits are normalized to one. A measure $1 - \bar{m}$ of depositors are retail and fully insured; they remain passive and never withdraw at $t = 1$. The remaining $\bar{m}$ fraction are wholesale creditors who are not fully insured and may demand early withdrawal. We assume $\bar{m} \in \left(\frac{R-r_1}{r_1 R - r_1}, \frac{1}{r_1}\right)$. The upper bound prevents mechanical bank failures, so each depositor who requests to withdraw at $t = 1$ obtains r_1.[7] The lower bound ensures that strategic interactions are present. Henceforth, "depositor" refers specifically to wholesale depositors.

At $t = 1$, an aggregate liquidity state $\lambda \sim \mathcal{U}[0, 1]$ is realized. Conditional on λ, each wholesale depositor experiences an individual liquidity shock independently with probability λ. Depositors observe only their own liquidity status. Impatient depositors consume only at $t = 1$ and always withdraw. Patient depositors choose their withdrawal timing based on a private signal $s_i = \theta + \xi_i$, where $\xi_i \sim \mathcal{N}\left(0, \beta^{-1}\right)$ i.i.d. across depositors.

Beliefs. To sharpen the economic trade-off between the policymaker's learning and the social costs of bank runs, we adopt a standard tractability device: a small prior heterogeneity (i.e., agents "agree to disagree"). In particular, the policymaker holds the correct prior

$$f_g(\theta) = \frac{1}{2\Theta} \mathbf{1}\{\theta \in (-\Theta, \Theta)\}, \tag{3}$$

whereas depositors share an improper uniform prior over $\mathbb{R}$. Thus, depositors are misspecified about the support of fundamentals, but they do not know that they are misspecified and update according to their own prior.

We assume that the bank's investment has a positive ex-ante expected NPV from the policymaker's perspective, so that the bank is viable ex ante:

$$\mathbb{E}_g\left[P(\theta)\right] R - 1 > 0. \tag{4}$$

The depositors' improper prior simplifies the analysis of the run stage, while the policymaker's bounded prior ensures that her ex-ante expected welfare is well-defined. Although this assumption is primarily technical, it is also consistent with recent work emphasizing that policymakers and private agents need not share a common prior, especially in environments involving communication and persuasion (Alonso and Câmara 2016; Andrade, Gaballo, Mengus, and Mojon 2019; Herbert 2021).

[7] Goldstein and Pauzner (2005) show that bank failure creates a region of strategic substitution. Incorporating this substitution region introduces technical complications while not necessarily generating new insights. Our analysis remains qualitatively robust when accounting for the possibility of bank failures.

Policymaker and Policy Portfolio. The policymaker uses two tools. At $t = 0$, she commits to a deposit insurance level $K \in [0, r_1]$, guaranteeing a minimum cash flow of K to all depositors regardless of their withdrawal decisions. At $t = 1$, after observing the realized total withdrawal size $m \in [0, \bar{m}]$ and the liquidation-value $L \in \{1, r_1\}$, the policymaker chooses either: (i) a liquidity injection $\ell \in [0, mr_1]$ to meet withdrawals and preserve investment, or (ii) a forced-liquidation action that liquidates the bank's remaining investment at $t = 1$. For notational convenience, we denote the forced-liquidation action by $\ell = mr_1 - 1 < 0$. Liquidity injection cannot be reinvested in the project; that is, it can only be used to meet $t = 1$ withdrawals.

The signal L is observed only by the policymaker after withdrawal decisions are made. Observing $L = r_1$ is equivalent to learning that $\theta \geq \theta^U$, because $L = r_1$ if and only if $\theta \geq \theta^U$. Similarly, observing $L = 1$ is equivalent to learning that $\theta < \theta^U$. Thus, when observing the low liquidation value, the policymaker does not observe the exact realization of θ and still learns from the withdrawal signal m.

We assume that deposit insurance payments and liquidity injections are financed through lump-sum transfers (or deep-pocket taxation) and are welfare-neutral transfers. Thus, welfare effects arise only through their impact on liquidation and run behavior.

Payoffs. The active players are the depositors and the policymaker. If a depositor withdraws early, her payoff is $u_i(d_i = 1) = r_1$.

If a depositor does not withdraw at $t = 1$, her payoff depends on the policymaker's action and on the realized liquidation value. If the policymaker forces full liquidation at $t = 1$, the depositor is paid out of liquidation proceeds at $t = 1$. In this case, the payoff is

$$u_i(d_i = 0) = \begin{cases} r_1, & \text{if } \theta \geq \theta^U, \\ \max\left\{\dfrac{1 - mr_1}{1 - m}, K\right\}, & \text{if } \theta < \theta^U. \end{cases} \tag{5}$$

If the policymaker injects $\ell \geq 0$ liquidity, outcomes differ by region. If $\theta \geq \theta^U$, a depositor who stays receives the promised payoff r_2 independently of m and ℓ. If $\theta < \theta^U$, a depositor who stays receives the following expected payoff:

$$u_i(d_i = 0) = \begin{cases} P(\theta) r_2 + (1 - P(\theta))\, K, & \text{if } \dfrac{1 + \ell - mr_1}{1 - m} R \geq r_2, \\ P(\theta) \dfrac{1 + \ell - mr_1}{1 - m} R + (1 - P(\theta))\, K, & \text{if } r_2 > \dfrac{1 + \ell - mr_1}{1 - m} R > K, \\ K, & \text{if } K \geq \dfrac{1 + \ell - mr_1}{1 - m} R. \end{cases} \tag{6}$$

Conditional on the observed pair (m, L), the policymaker's expected welfare is defined as the total NPV of the bank's long-term investment, net of bank run costs ρm. If $L = 1$, the bank investment has the low liquidation value. In this case, meeting net withdrawals $mr_1 - \ell$ requires liquidating exactly $mr_1 - \ell$ units of investment. The policymaker's expected payoff is therefore

$$u_g(\ell \mid m, L = 1) = [1 - (mr_1 - \ell)] \left(\mathbb{E}_g[P(\theta) \mid m, L = 1] R - 1\right) - \rho m. \tag{7}$$

If $L = r_1$, the bank investment has the high liquidation value, which implies $\theta \geq \theta^U$, and hence $P(\theta) = 1$. In that case, the policymaker's payoff is

$$\begin{aligned} u_g(\ell \mid m, L = r_1) &= \left(1 - \frac{mr_1 - \ell}{r_1}\right)(R - 1) + \frac{mr_1 - \ell}{r_1}(r_1 - 1) - \rho m \\ &= (R - 1) - \frac{mr_1 - \ell}{r_1}(R - r_1) - \rho m. \end{aligned} \tag{8}$$

Here, $\rho \geq 0$ is the marginal bank run cost, capturing, among other things, the premium required to raise emergency liquidity on short notice and other externalities associated with runs, such as the policymaker's reputational cost.

Strategy and Equilibrium. The timeline of our model is shown in Figure 1. At $t = 0$, the policymaker commits to deposit insurance. At $t = 1$, θ and λ are realized. Depositors receive liquidity shocks and private signals and then simultaneously decide whether to withdraw. After observing the realized withdrawal size m and the liquidation value L, the policymaker chooses a $t = 1$ intervention. At $t = 2$, the investment return and all remaining payoffs are realized.

The policymaker's behavior strategy at $t = 0$ is a choice of deposit insurance $K \in [0, r_1]$. A depositor's strategy is a mapping $D_i : [0, r_1] \times \mathbb{R} \to \{0, 1\}$, so that for any K and any private signal $s_i \in \mathbb{R}$, depositor i decides whether to withdraw at $t = 1$. The policymaker's behavior strategy at $t = 1$ is a mapping $\mathcal{L} : [0, r_1] \times [0, \bar{m}] \times \{1, r_1\} \to [0, mr_1] \cup \{mr_1 - 1\}$, so that for any committed deposit insurance K, any withdrawal size $m \in [0, \bar{m}]$, and any liquidation value $L \in \{1, r_1\}$, the policymaker chooses either forced liquidation (i.e., $\ell = mr_1 - 1$) or a liquidity injection $\ell \in [0, mr_1]$.

Definition 1 *The policymaker's deposit insurance K^*, her intervention strategy $\mathcal{L}^*$, and the depositors' strategies $\{D_i^*\}_{i \in [0,1]}$ constitute a monotone perfect Bayesian equilibrium if:*

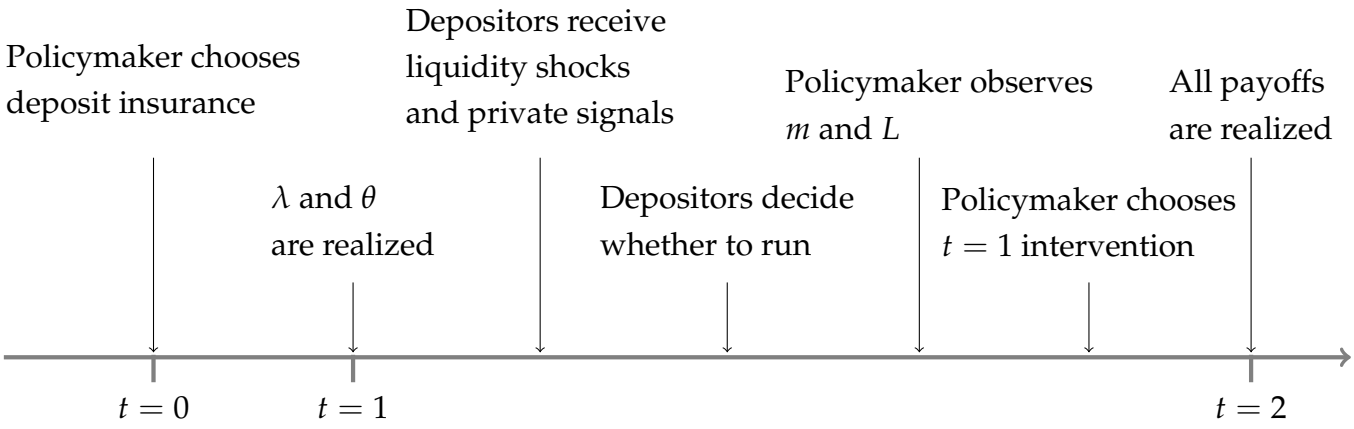


Figure 1: **Timing**

1. *the deposit insurance K^* maximizes the policymaker's expected payoff at $t = 0$, $\mathbb{E}_g[u_g]$, given $\{D_i^*\}_{i\in[0,1]}$ and $\mathcal{L}^*$;*

2. *given any K, other depositors' strategies $\{D_j^*\}_{j\neq i}$, and the policymaker's strategy $\mathcal{L}^*$, depositor i's choice $D_i^*(K, s_i)$ maximizes her expected payoff at $t = 1$ and is monotone in s_i;*

3. *given the depositors' strategies $\{D_i^*\}_{i\in[0,1]}$, for any K, any m, and any $L \in \{1, r_1\}$, the intervention $\ell = \mathcal{L}^*(K, m, L)$ maximizes the policymaker's payoff given her information at $t = 1$, namely the observation (m, L);*

4. *all players update their beliefs according to Bayes' rule under their respective priors whenever possible.*

3 Benchmark: Deposit Insurance Alone

In this section, we examine a benchmark scenario in which the policymaker relies solely on deposit insurance and commits to no ex-post liquidity injection (i.e., $\ell = 0$ at $t = 1$). We solve for the equilibrium by backward induction. Section 3.1 characterizes depositors' equilibrium run behavior given a deposit insurance policy. Section 3.2 characterizes the optimal deposit insurance policy. We compare this benchmark with the policy-portfolio environment in Section 5.

3.1 Panic Bank Run

Given a deposit insurance policy $K \in [0, r_1]$, for fundamentals below the upper-dominance region ($\theta < \theta^U$), the incremental payoff to a patient depositor i from staying with the bank until $t = 2$ (rather than withdrawing at $t = 1$) is

$$\Delta c^{DI}(m, \theta; K) = \begin{cases} P(\theta) r_2 + (1 - P(\theta)) K - r_1, & \text{if } m \leq \dfrac{R - r_2}{r_1 R - r_2}, \\ P(\theta) \dfrac{1 - m r_1}{1 - m} R + (1 - P(\theta)) K - r_1, & \text{if } \dfrac{R - r_2}{r_1 R - r_2} < m < \dfrac{R - K}{r_1 R - K}, \\ K - r_1, & \text{if } m \geq \dfrac{R - K}{r_1 R - K}, \end{cases} \tag{9}$$

where the superscript "DI" stands for "deposit insurance only."

In the upper-dominance region ($\theta \geq \theta^U$), fundamentals are sufficiently strong that $P(\theta) = 1$, and the depositor who stays obtains r_2 at $t = 2$ independently of m. Hence, for $\theta \geq \theta^U$,

$$\Delta c^{DI}(m, \theta; K) = r_2 - r_1.$$

When $\theta < \theta^U$, increasing deposit insurance coverage K increases $\Delta c^{DI}(m, \theta; K)$ for any given (m, θ). Thus, deposit insurance raises the continuation value of staying and discourages runs. Moreover, for a fixed K, the incremental payoff is weakly decreasing in m, reflecting strategic complementarities in withdrawal decisions. In particular, depositors have stronger incentives to run when they expect a larger early withdrawal size, which forces greater premature liquidation. Figure 2 illustrates these forces for $K \in [0, r_1)$.

In a monotone perfect Bayesian equilibrium, the withdrawal decision d_i is weakly decreasing in the private signal s_i. Consequently, there exists a threshold $\hat{s}$ such that depositor i runs if and only if $s_i \leq \hat{s}$ (with ties broken in favor of staying if desired). Given such a threshold, in a state (θ, λ) the early withdrawal size at $t = 1$ is

$$m(\lambda, \theta; \hat{s}) = \bar{m}\left(\lambda + (1 - \lambda) n(\theta; \hat{s})\right), \tag{10}$$

where

$$n(\theta; \hat{s}) = \Phi\Big(\sqrt{\beta}\left(\hat{s} - \theta\right)\Big) \tag{11}$$

is the fraction of patient depositors who run.

Because depositors use an improper uniform prior over $\mathbb{R}$ and observe the private signal $s_i = \theta + \xi_i$ with $\xi_i \sim \mathcal{N}\left(0, \beta^{-1}\right)$, depositor i's posterior over θ is

$$\theta \mid s_i \sim \mathcal{N}\Big(s_i, \beta^{-1}\Big), \tag{12}$$

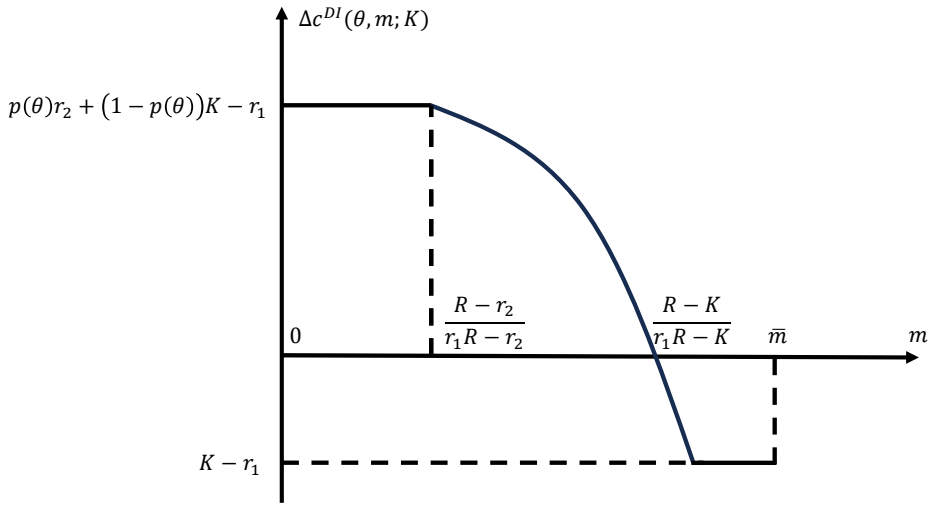


Figure 2: Incremental Payoff from Staying with the Bank (Deposit Insurance Alone)

which has a conditional density

$$f(\theta \mid s_i) = \sqrt{\beta}\, \phi\Big(\sqrt{\beta}\,(\theta - s_i)\Big), \tag{13}$$

where $\phi(\cdot)$ denotes the standard normal PDF.

A depositor who does not experience a liquidity shock updates her belief about the aggregate liquidity state λ. Since $\lambda \sim \mathcal{U}[0,1]$ and the probability of being patient is $1-\lambda$, Bayes' rule implies the posterior density

$$g_p(\lambda) = \frac{(1-\lambda)\cdot 1}{\int_0^1 (1-x)\,\mathrm{d}x} = 2(1-\lambda). \tag{14}$$

Therefore, the expected incremental payoff from staying for a patient depositor observing s_i is

$$V^{DI}(s_i; K, \hat{s}) = \int_{-\infty}^{\infty} \left(\int_0^1 \Delta c^{DI}(m(\lambda, \theta; \hat{s}), \theta; K)\; g_p(\lambda)\; \mathrm{d}\lambda \right) f(\theta \mid s_i)\; \mathrm{d}\theta. \tag{15}$$

In equilibrium, the marginal depositor (with $s_i = \hat{s}$) is indifferent between withdrawing and staying:

$$V^{DI}(\hat{s}; K, \hat{s}) = 0. \tag{16}$$

We the following the traditional method in the literature on global games to prove the uniqueness of the threshold equilibrium for sufficiently precise private signals. In particular, there exists $\hat{\beta}^{DI} > 0$ such that for any $\beta > \hat{\beta}^{DI}$ and any $K \in [0, r_1)$, equation (16) admits a unique solution. This yields a unique equilibrium in the bank run stage for each deposit insurance policy.

Lemma 1 *Suppose that the policymaker commits to zero liquidity injection ($\ell = 0$). There exists $\hat{\beta}^{DI} > 0$ such that for any $\beta > \hat{\beta}^{DI}$ and any deposit insurance coverage $K \in [0, r_1]$, there is a unique threshold-strategy equilibrium. Specifically, depositor i's optimal strategy is*

$$d_i^{DI} = \begin{cases} 1, & \text{if } s_i < \hat{s}^{DI}, \\ 0, & \text{if } s_i \geq \hat{s}^{DI}, \end{cases} \tag{17}$$

where the threshold $\hat{s}^{DI}$ is characterized as follows:

1. *If $K \in [0, r_1)$, then $\hat{s}^{DI}$ is the unique solution to*

$$V^{DI}\left(\hat{s}^{DI}; K, \hat{s}^{DI}\right) = 0.$$

2. *If $K = r_1$, then $\hat{s}^{DI} = -\infty$, i.e., patient depositors never run.*

Moreover, $\hat{s}^{DI}$ is strictly decreasing in K on $[0, r_1)$.

3.2 Optimal Deposit Insurance Policy

Given an equilibrium threshold $\hat{s}$, the realized withdrawal size in state (θ, λ) is $m(\theta, \lambda; \hat{s})$ as in equation (10). The policymaker's realized welfare (equivalently, aggregate welfare net of bank run costs) can be written as

$$u_g^{DI}(\theta, \lambda; \hat{s}) = \begin{cases} (1 - r_1 m(\theta, \lambda; \hat{s}))(P(\theta) R - 1) - \rho\, m(\theta, \lambda; \hat{s}), & \text{if } \theta < \theta^U, \\ (1 - m(\theta, \lambda; \hat{s}))(R - 1) + (r_1 - 1)\, m(\theta, \lambda; \hat{s}) - \rho\, m(\theta, \lambda; \hat{s}), & \text{if } \theta \geq \theta^U. \end{cases} \tag{18}$$

The difference across the two regions reflects the liquidation technology in the model: when $\theta < \theta^U$, each unit of withdrawals requires liquidation of r_1 units of investment (liquidation value 1), whereas when $\theta \geq \theta^U$, each unit of withdrawals requires liquidation of only one unit of investment (liquidation value r_1). Since transfers between the policymaker and private agents are welfare-neutral, deposit insurance affects welfare solely through its impact on the equilibrium run threshold $\hat{s}$ and the induced withdrawal size.

Lemma 1 implies that for any $K \in [0, r_1]$, there exists a unique equilibrium threshold, denoted by $\hat{s}^{DI}(K)$. Moreover, higher deposit insurance raises the continuation value of staying and thus discourages runs, so $\hat{s}^{DI}(K)$ is strictly decreasing in K for $K \in [0, r_1)$. Since a patient depositor is never strictly better off running when $K = r_1$, we have $\hat{s}^{DI}(r_1) = -\infty$.

Let $\hat{s}_0^{DI}$ denote the laissez-faire run threshold under $K = 0$, i.e., the unique solution to

$$V^{DI}\left(\hat{s}_0^{DI}; 0, \hat{s}_0^{DI}\right) = 0.$$

Since $\hat{s}^{DI}(K)$ is strictly decreasing in K and ranges from $\hat{s}^{DI}(0) = \hat{s}_0^{DI}$ to $\hat{s}^{DI}(r_1) = -\infty$, for any target threshold $\hat{s} \in \left[-\infty, \hat{s}_0^{DI}\right]$ there exists a unique deposit insurance level $K \in [0, r_1]$ that implements it.

Corollary 1 *For any $\beta > \hat{\beta}^{DI}$, $\hat{s}^{DI}(K)$ is strictly decreasing in K on $[0, r_1)$ with $\hat{s}^{DI}(r_1) = -\infty$. Consequently, for any target threshold $\hat{s} \in \left[-\infty, \hat{s}_0^{DI}\right]$, there exists a unique $K \in [0, r_1]$ such that $\hat{s}^{DI}(K) = \hat{s}$.*

This implementation result enables us to rewrite the policymaker's problem as choosing the induced run threshold $\hat{s}$. Hence, the policymaker solves

$$\max_{\hat{s} \in \left[-\infty, \hat{s}_0^{DI}\right]} \mathbb{E}_g\left[u_g^{DI}(\theta, \lambda; \hat{s})\right], \tag{19}$$

where the expectation is taken under the policymaker's bounded uniform prior over $[-\Theta, \Theta]$ and $\lambda \sim \mathcal{U}[0, 1]$.

The trade-off is as follows. Lowering $\hat{s}$ (equivalently, increasing K) reduces the withdrawal size m in every state. When $P(\theta) R > 1$, reducing runs preserves productive investment and lowers the run-cost term ρm. When $P(\theta) R < 1$, reducing runs lowers the direct run cost but also delays liquidation of a negative-NPV project. The optimal deposit insurance balances these forces. Proposition 1 characterizes the optimal benchmark policy.

Proposition 1 *Suppose that the policymaker commits to zero liquidity injection (i.e., $\ell = 0$). Let K_{DI}^* denote the optimal deposit insurance policy, and let $\hat{s}_{DI}^*$ denote the equilibrium run threshold induced by K_{DI}^*. Then:*

(i) *For any $\rho \geq r_1$, the optimal policy is full deposit insurance, that is, $K_{DI}^* = r_1$.*

(ii) *Fix any $\rho < r_1$. There exists a $\hat{\beta}^{DI}(\rho) > 0$ such that, for every $\beta > \hat{\beta}^{DI}(\rho)$, the policymaker chooses a unique optimal deposit insurance policy $K_{DI}^* \in (0, r_1)$, which induces a unique equilibrium threshold $\hat{s}_{DI}^* \in \left(-\infty, \hat{s}_0^{DI}\right)$ satisfying*

$$\frac{\mathrm{d}}{\mathrm{d}\hat{s}} \mathbb{E}_g\left[u_g^{DI}(\theta, \lambda; \hat{s})\right] \Big|_{\hat{s} = \hat{s}_{DI}^*} = 0,$$

where $\hat{s}_0^{DI}$ denotes the laissez-faire threshold under $K = 0$. Therefore, defining $\rho^{\mathrm{DI}} = \inf\{\rho : K_{DI}^* = r_1\}$, we conclude $\rho^{\mathrm{DI}} = r_1$.

(iii) *Fix any* $\rho < r_1$. *Then*

$$\lim_{\beta \to \infty} \hat{s}^*_{DI} = P^{-1}\left(\frac{1-\rho/r_1}{R}\right).$$

Proposition 1 shows that, absent ex-post liquidity injection, the policymaker uses deposit insurance to control liquidation risk directly. At the margin, the policymaker trades off the social value of preserving long-term investment against the bank run cost and the benefit of liquidating negative-NPV projects. In particular, the optimal policy tolerates runs only when fundamentals are sufficiently weak. As $\rho \to 0$, the limiting threshold converges to the zero-NPV cutoff $P^{-1}(1/R)$.

4 Policy Portfolio

In this section, we analyze the interaction between ex-ante deposit insurance and ex-post intervention and characterize the equilibrium policy portfolio. We show that when the policymaker can intervene at $t = 1$, the optimal deposit insurance is zero for sufficiently small (but positive) marginal costs of bank runs. The key economic insight is that generous insurance dampens the information content of withdrawals and reduces the effectiveness of targeted intervention. For larger marginal run costs, the optimal deposit insurance is positive; however, relative to the benchmark in which the policymaker commits to no ex-post action, the policymaker may optimally tolerate more panic-driven withdrawals in order to learn from informed depositors and better direct liquidity support.

We conduct the analysis by backward induction. In Section 4.1, we characterize the policymaker's inference about bank fundamentals and her optimal $t = 1$ intervention policy after observing the realized withdrawal size m, taking as given depositors' run threshold $\hat{s}$. In Section 4.2, we solve for depositors' run threshold $\hat{s}$ anticipating the policymaker's $t = 1$ policy. Lastly, Section 4.3 analyzes the optimal ex-ante deposit insurance.

4.1 Policymaker's Learning and $t = 1$ Intervention

Because the policymaker observes both the early withdrawal size m and the liquidation value L at $t = 1$, learning is relevant only when $L = 1$. If instead $L = r_1$, the policymaker learns that $\theta \geq \theta^U$ and hence $P(\theta) = 1$. As we shall argue shortly, in this case, the policymaker's $t = 1$ problem is trivial—she will inject full liquidity ($\ell = mr_1$) for any early withdrawal size.

Hence, we first characterize the policymaker's posterior belief conditional on $(m, L = 1)$ and then study the optimal $t = 1$ intervention policy.

Posterior belief when $L = 1$. Fix a run threshold $\hat{s} \in \mathbb{R}$. Recall from equation (10) that, for given (θ, λ), the total withdrawal size is $m(\lambda, \theta; \hat{s}) = \bar{m}(\lambda + (1-\lambda) n(\theta; \hat{s}))$, where $n(\theta; \hat{s}) = \Phi\left(\sqrt{\beta}(\hat{s} - \theta)\right)$ is the fraction of patient depositors who run.

Suppose that the policymaker observes $L = 1$, which is equivalent to learning that $\theta < \theta^U$. Then the smallest withdrawal size consistent with $(L = 1, \hat{s})$ is attained when aggregate liquidity shock λ is zero and fundamentals approach the upper boundary θ^U. Hence the set of feasible withdrawal realizations given a run threshold $\hat{s}$ is

$$m \in [\underline{m}(\hat{s}), \bar{m}], \text{ where } \underline{m}(\hat{s}) \equiv \bar{m}\,\Phi\left(\sqrt{\beta}\left(\hat{s} - \theta^U\right)\right). \tag{20}$$

Hence, fixed a $\hat{s}$, given a feasible early withdrawal size $m \in [\underline{m}(\hat{s}), \bar{m}]$, the bank fundamentals cannot be too low; otherwise, even when $\lambda = 0$, the withdrawal size would exceed m. Since $\lambda \in [0, 1]$ implies $n(\theta; \hat{s}) \leq m/\bar{m}$, feasibility requires

$$\theta \geq \theta_L(m; \hat{s}), \text{ where } \theta_L(m; \hat{s}) = \hat{s} - \frac{1}{\sqrt{\beta}}\Phi^{-1}\left(\frac{m}{\bar{m}}\right). \tag{21}$$

Because the policymaker's prior support is $(-\Theta, \Theta)$ and the observation $L = 1$ implies $\theta < \theta^U$, the posterior support is

$$\left[\underline{\theta}(m; \hat{s}), \theta^U\right), \qquad \underline{\theta}(m; \hat{s}) \equiv \max\{-\Theta, \theta_L(m; \hat{s})\}. \tag{22}$$

Now fix $\theta < \theta^U$. Conditional on $(\theta, \hat{s})$, $m(\lambda, \theta; \hat{s})$ is affine in λ with slope $\bar{m}(1 - n(\theta; \hat{s}))$. Hence, for $\theta < \theta^U$, the conditional density of m given θ is

$$f(m \mid \theta; \hat{s}, L = 1) = \frac{1}{\bar{m}(1 - n(\theta; \hat{s}))}\mathbf{1}\{m \in [\bar{m}\, n(\theta; \hat{s}), \bar{m}]\}.$$

Conditional on $L = 1$, the policymaker's prior is uniform on $\left(-\Theta, \theta^U\right)$. Therefore, Bayes' rule implies that, for $m \in [\underline{m}(\hat{s}), \bar{m}]$, the policymaker's posterior cdf is

$$G_\theta(\theta \mid m; \hat{s}) = \begin{cases} 0, & \text{if } \theta < \underline{\theta}(m; \hat{s}), \\ \dfrac{\int_{\underline{\theta}(m;\hat{s})}^{\theta} \frac{1}{1-n(x;\hat{s})}\,\mathrm{d}x}{\int_{\underline{\theta}(m;\hat{s})}^{\theta^U} \frac{1}{1-n(x;\hat{s})}\,\mathrm{d}x}, & \text{if } \underline{\theta}(m; \hat{s}) \leq \theta < \theta^U, \\ 1, & \text{if } \theta \geq \theta^U. \end{cases} \tag{23}$$

Note that the policymaker's posterior is relevant only when $L = 1$, we suppress the conditioning on $L = 1$ in the notation for posterior beliefs whenever no confusion arises.

Two features of the posterior are worth emphasizing. First, conditional on $L = 1$, a larger withdrawal size m makes lower fundamentals more likely because it is more consistent with depositors receiving unfavorable private signals. Second, for a given realization of m in the interior region, a higher run threshold $\hat{s}$ makes withdrawals more likely to be panic-driven rather than fundamentals-driven, which shifts the policymaker's inference in a more optimistic direction.

To state these properties formally, we define

$$\tilde{m}(\hat{s}) \equiv \bar{m}\,\Phi\Big(\sqrt{\beta}\,(\hat{s}+\Theta)\Big). \tag{24}$$

At $m = \tilde{m}(\hat{s})$, the feasibility lower bound $\theta_L(m;\hat{s})$ reaches the prior boundary $-\Theta$. Hence, for $m \geq \tilde{m}(\hat{s})$, the posterior support is fixed at $\left[-\Theta, \theta^U\right)$.

Proposition 2 *Suppose that the policymaker observes $L = 1$. Then her posterior belief depends on the early withdrawal size m and on the depositors' cutoff run threshold $\hat{s}$.*

(i) *Fix $\hat{s} \in \mathbb{R}$. For any $m_1, m_2 \in [\underline{m}(\hat{s}), \bar{m}]$ with $m_1 < m_2$:*

- *if $m_1 < \tilde{m}(\hat{s})$, then $G_\theta(\cdot \mid m_1;\hat{s})$ first-order stochastically dominates $G_\theta(\cdot \mid m_2;\hat{s})$;*
- *if $m_1 \geq \tilde{m}(\hat{s})$, then $G_\theta(\cdot \mid m_1;\hat{s}) = G_\theta(\cdot \mid m_2;\hat{s})$.*

Consequently, $\mathbb{E}_g[P(\theta) \mid m;\hat{s}]$ is weakly decreasing in m and strictly decreasing on $(\underline{m}(\hat{s}), \tilde{m}(\hat{s}))$.

(ii) *For any $\hat{s}_1 > \hat{s}_2$ and any withdrawal size $m \in (\underline{m}(\hat{s}_1), \tilde{m}(\hat{s}_2))$, $G_\theta(\cdot \mid m;\hat{s}_1)$ first-order stochastically dominates $G_\theta(\cdot \mid m;\hat{s}_2)$. Consequently,*

$$\mathbb{E}_g[P(\theta) \mid m;\hat{s}_1] > \mathbb{E}_g[P(\theta) \mid m;\hat{s}_2].$$

Proposition 2 implies a saturation property. For a given $\hat{s}$, once withdrawals exceed $\tilde{m}(\hat{s})$, further increases in m are no longer informative because the lower bound of the posterior support is fixed at $-\Theta$. Thus, when the bank investment liquidation value is $L = 1$, the withdrawal signal is informative only on $[\underline{m}(\hat{s}), \tilde{m}(\hat{s}))$. In addition, part (ii) applies to the interior region in which the posterior support boundary remains endogenous. As in the benchmark analysis, the comparative statics with respect to $\hat{s}$ may differ once the support boundary binds, a feature driven by the policymaker's bounded prior.

Optimal $t = 1$ intervention. We next characterize the policymaker's optimal $t = 1$ intervention. Because she observes the bank investment liquidation value L at $t = 1$, the policymaker's optimal $t = 1$ police differs across the two liquidation-value cases.

The case with $L = r_1$ is pretty staightforward. The policymaker learns that $\theta \geq \theta^U$ and hence $P(\theta) = 1$. If she injects $\ell \in [0, mr_1]$, the bank liquidates $(mr_1 - \ell)/r_1$ units of investment, so her payoff, as shown in equation (8), is strictly increasing in ℓ (because $R > r_1$). Hence the policymaker injects the maximum feasible amount, $\ell = mr_1$, for every realization of m.

The case with $L = 1$ is more complicated but more interesting. After observing a feasible withdrawal size $m \in [\underline{m}(\hat{s}), \bar{m}]$, the policymaker chooses $\ell \in [0, mr_1] \cup \{mr_1 - 1\}$. If she injects $\ell \geq 0$, net withdrawals of size $mr_1 - \ell$ must be financed by liquidating exactly $mr_1 - \ell$ units of investment. Her expected payoff is therefore

$$u_g(\ell \mid m, L = 1; \hat{s}) = [1 - (mr_1 - \ell)] \left(\mathbb{E}_g[P(\theta) \mid m, L = 1; \hat{s}] R - 1\right) - \rho m.$$

If instead she chooses forced liquidation, that is, $\ell = mr_1 - 1$, then all bank investment is liquidated at $t = 1$ and the policymaker's payoff is

$$u_g(mr_1 - 1 \mid m, L = 1; \hat{s}) = -\rho m.$$

Because $u_g(\ell \mid m, L = 1; \hat{s})$ is linear in ℓ on $[0, mr_1]$, the optimal intervention in the case with $L = 1$ is bang-bang.

Lemma 2 *Fix depositors' run threshold $\hat{s}$. After observing (m, L), the policymaker's optimal $t = 1$ intervention is*

$$\mathcal{L}^*(m, L, \hat{s}) = \begin{cases} mr_1, & \text{if } L = r_1, \\ mr_1, & \text{if } L = 1 \text{ and } \mathbb{E}_g[P(\theta) \mid m, L = 1; \hat{s}] \geq \dfrac{1}{R}, \\ mr_1 - 1, & \text{if } L = 1 \text{ and } \mathbb{E}_g[P(\theta) \mid m, L = 1; \hat{s}] < \dfrac{1}{R}. \end{cases} \tag{25}$$

Lemma 2 shows that the policymaker always injects full liquidity when the bank investment liquidation value is $L = r_1$. In contrast, in the case of low liquidation value ($L = 1$), her $t = 1$ intervention depends on the expected success probability, which is shaped by both the early withdrawal size m and the run threshold $\hat{s}$.

By Proposition 2, for each fixed $\hat{s}$, $\mathbb{E}_g[P(\theta) \mid m, L = 1; \hat{s}]$ is strictly decreasing in m over the interval $(\underline{m}(\hat{s}), \tilde{m}(\hat{s}))$ and constant over $[\tilde{m}(\hat{s}), \bar{m}]$. Thus, conditional on $L = 1$, the policymaker's best response can be summarized by a cutoff rule in m.

To accommodate both interior and corner cases, we define the policymaker's best-response cutoff for each $\hat{s}$ when the bank investment liquidation value is $L = 1$ as

$$\hat{m}^{BR}(\hat{s}) \equiv \sup\left\{m \in [\underline{m}(\hat{s}), \bar{m}] : \mathbb{E}_g[P(\theta) \mid m; \hat{s}] > \frac{1}{R}\right\}. \tag{26}$$

Corollary 2 *Suppose that $L = 1$. Fix $\hat{s} \in \mathbb{R}$. Then, the policymaker's optimal $t = 1$ policy is characterized by a cutoff rule. Specifically, for every feasible $m \in (\underline{m}(\hat{s}), \bar{m}]$,*

$$\mathcal{L}^*(m, L = 1, \hat{s}) = \begin{cases} mr_1, & \text{if } m \leq \hat{m}^{BR}(\hat{s}), \\ mr_1 - 1, & \text{if } m > \hat{m}^{BR}(\hat{s}), \end{cases} \tag{27}$$

where $\hat{m}^{BR}(\hat{s}) = \bar{m}$ if $\mathbb{E}_g[P(\theta) \mid \tilde{m}(\hat{s}); \hat{s}] \geq \frac{1}{R}$; otherwise, $\hat{m}^{BR}(\hat{s}) \in (\underline{m}(\hat{s}), \bar{m})$ is the unique solution to $\mathbb{E}_g[P(\theta) \mid m; \hat{s}] = \frac{1}{R}$.

Corollary 2 highlights that when $L = 1$, the policymaker still faces uncertainty about fundamentals and learns from the withdrawal signal m. In such a case, small withdrawals are more consistent with relatively strong fundamentals and are therefore more likely to trigger liquidity support, whereas large withdrawals are more consistent with weak fundamentals and are therefore more likely to trigger forced liquidation.

We also argue that in equilibrium, the policymaker will not inject full liquidity for every realized early withdrawal size. In principle, for an arbitrary run threshold $\hat{s}$, Corollary 2 allows the possibility that $\hat{m}^{BR}(\hat{s}) = \bar{m}$, so that conditional on $L = 1$ the policymaker injects full liquidity for any realized early withdrawal size. However, Lemma 3 shows that, in equilibrium, the policymaker's best-response cutoff is strictly interior when Θ is sufficiently large.

Lemma 3 *Suppose that Θ is sufficiently large so that $\Theta > \hat{\Theta} \equiv R(\theta^U - \theta^L) - \theta^L$. Then, for every $\hat{s} \in \mathbb{R}$, $\mathbb{E}_g[P(\theta) \mid \tilde{m}(\hat{s}); \hat{s}] < \frac{1}{R}$. Therefore,*

$$\hat{m}^{BR}(\hat{s}) \in (\underline{m}(\hat{s}), \tilde{m}(\hat{s})) \subset (\underline{m}(\hat{s}), \bar{m}), \tag{28}$$

so the policymaker never injects full liquidity for every realized early withdrawal size.

Intuitively, if $\hat{m}^{BR}(\hat{s}_0) = \bar{m}$, then conditional on $L = 1$ the policymaker would inject full liquidity for any realized early withdrawal size. In that case, a depositor's expected payoff no longer depends on the early withdrawal size, so strategic uncertainty would disappear. Since $\hat{s}_0$ is induced by $K = 0$, the marginal depositor's indifference condition would then be

pinned down by the purely fundamental equation $P(\theta) r_2 = r_1$. For sufficiently large β, the equilibrium threshold would therefore be $P^{-1}(r_1/r_2) < \theta^U$. However, at such a threshold, sufficiently large withdrawals conditional on $L = 1$ still reveal weak fundamentals, so the policymaker would not find it optimal to inject full liquidity for every feasible realization of m.

Iso-m curves. We use iso-m curves to visualize how primitives (λ, θ) map into the observable withdrawal size m and thereby into the optimal $t = 1$ policy. For a fixed run threshold $\hat{s}$, an iso-m curve is the set of pairs (λ, θ) satisfying equation (10). These curves are upward sloping in the (θ, λ) plane: holding $\hat{s}$ fixed, stronger fundamentals reduce withdrawals by patient depositors, so a larger aggregate liquidity need λ is required to generate the same withdrawal size.

Conditional on $L = 1$, since $\hat{m}^{BR}(\hat{s}) \in (\underline{m}(\hat{s}), \bar{m})$, the iso-$\hat{m}^{BR}$ curve separates states in which the policymaker injects full liquidity from those in which she forces liquidation. States below the curve generate relatively small withdrawals and induce full injection, whereas states above the curve generate relatively large withdrawals and induce forced liquidation.

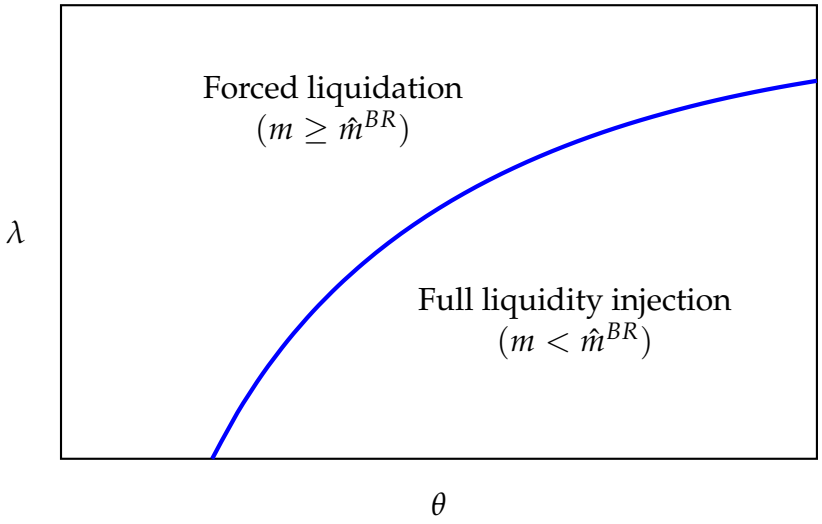


Figure 3: Iso-$\hat{m}^{BR}$ curve (illustrative) and optimal $t = 1$ policy regions.

4.2 Panic Bank Run

We are now in a position to analyze depositors' withdrawal behavior, taking as given the policymaker's optimal $t = 1$ intervention characterized in Section 4.1. By Lemma 2 and Corollary 2, conditional on depositors using a cutoff strategy with threshold $\hat{s}$, the policymaker's best response has a simple structure. If $\theta \geq \theta^U$, she observes $L = r_1$ and injects full liquidity for every

realization of m. If instead $\theta < \theta^U$, she observes $L = 1$ and follows a cutoff rule: injecting full liquidity when the early withdrawal size is below $\hat{m}^{BR}(\hat{s})$ and forces liquidation otherwise.

Incremental payoff from staying. Fix a deposit insurance level K and a run threshold $\hat{s}$. We define a patient depositor's incremental payoff from staying relative to withdrawing early as

$$\Delta c(m, \theta; K, \hat{s}) \equiv u_i(d_i = 0) - r_1.$$

The dependence of $\Delta c(m, \theta; K, \hat{s})$ on m reflects both the strategic complementarity in withdrawal decisions and the effect of the policymaker's $t = 1$ intervention on continuation payoffs. Because the policymaker's intervention rule is itself determined by $\hat{s}$, the incremental payoff also depends on the depositors' run threshold even if the early withdrawal size m is given.

The depositor's continuation payoff differs across bank investment liquidation values. If $\theta \geq \theta^U$, the policymaker observes $L = r_1$ and always injects full liquidity. A patient depositor who stays therefore receives r_2, so

$$\Delta c(m, \theta; K, \hat{s}) = r_2 - r_1 \qquad \text{for all } \theta \geq \theta^U.$$

If instead $\theta < \theta^U$, then the policymaker observes $L = 1$ and follows the cutoff rule characterized in Corollary 2. Hence a pateint depositor who stays has the incremental payoff

$$\Delta c(m, \theta; K, \hat{s}) = \begin{cases} r_2 - r_1, & \text{if } \theta \geq \theta^U, \\ P(\theta) r_2 + (1 - P(\theta)) K - r_1, & \text{if } \theta < \theta^U \text{ and } m \leq \hat{m}^{BR}(\hat{s}), \\ \max\left\{\dfrac{1 - m r_1}{1 - m}, K\right\} - r_1, & \text{if } \theta < \theta^U \text{ and } m > \hat{m}^{BR}(\hat{s}). \end{cases} \tag{29}$$

Figure 4 illustrates the incremental payoff from staying when the bank investment liquidation value $L = 1$. Compared with the benchmark with deposit insurance alone (Figure 2), the main difference is that runs no longer force partial liquidation. Instead, the policymaker either provides full liquidity support when $m < \hat{m}^{BR}(\hat{s})$, thereby avoiding liquidation altogether, or forces the bank to liquidate all bank investment when $m \geq \hat{m}^{BR}(\hat{s})$. Thus, relative to Figure 2, the continuation payoff no longer declines smoothly with withdrawals through partial liquidation; it changes discretely at the policy cutoff $\hat{m}^{BR}(\hat{s})$.

Depositors' cutoff strategy. As in the benchmark with deposit insurance alone, depositors optimally follow a cutoff strategy in their private signal. Let $m(\lambda, \theta; \hat{s})$ be the withdrawal size

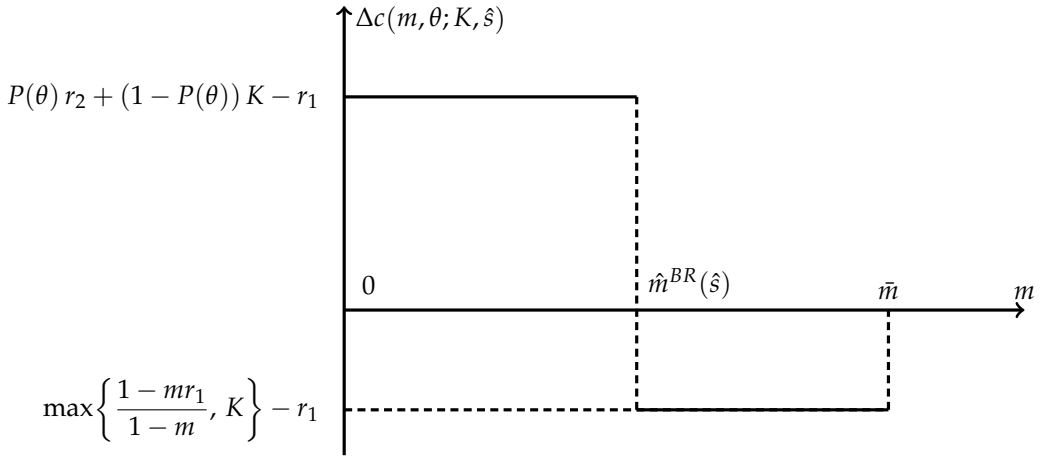


Figure 4: Incremental payoff from staying (policy portfolio).

induced by fundamentals θ, liquidity state λ, and threshold $\hat{s}$ as in equation (10). For a patient depositor with signal s_i, the expected incremental payoff from staying is

$$V\left(s_i; K, \hat{s}\right) = \int_{-\infty}^{\infty} \left(\int_0^1 \Delta c\left(m\left(\lambda, \theta; \hat{s}\right), \theta; K, \hat{s}\right) g_p\left(\lambda\right) \mathrm{d}\lambda \right) f\left(\theta \mid s_i\right) \mathrm{d}\theta, \tag{30}$$

where $g_p\left(\lambda\right)$ is the posterior density of λ conditional on being patient, as characterized in equation (14), and $f\left(\theta \mid s_i\right)$ is depositor i's posterior density of θ given her private signal s_i.

Because the signal L is observed and the policymaker's $t = 1$ intervenes after depositors make their withdrawal decisions, the revised information structure does not alter the global games structure underlying the run stage. Hence, using the same method as in Section 3.1, we obtain the following result.

Lemma 4 *When the policymaker cannot commit not to intervene at $t = 1$, given any $K \in [0, r_1]$, depositors follow a cutoff strategy in equilibrium. Specifically, for any $i \in [0, 1]$,*

$$d_i^* = \begin{cases} 1, & \text{if } s_i \leq \hat{s}, \\ 0, & \text{if } s_i > \hat{s}, \end{cases} \tag{31}$$

where $\hat{s} = -\infty$ if $K = r_1$, and otherwise $\hat{s}$ is the unique solution to

$$V\left(\hat{s}; K, \hat{s}\right) = 0 \qquad \text{for } K \in [0, r_1)\,.$$

Moreover, the equilibrium run threshold $\hat{s}$ is strictly decreasing in K.

Since the equilibrium run threshold $\hat{s}$ strictly decreases in K, we obtain the following equivalence between deposit insurance levels and run thresholds.

Corollary 3 *For any $s^* \in [-\infty, \hat{s}_0]$, there exists a unique deposit insurance policy $K \in [0, r_1]$ such that the equilibrium run threshold satisfies $\hat{s} = s^*$, where $\hat{s}_0$ is defined by*

$$V(\hat{s}_0; 0, \hat{s}_0) = 0. \tag{32}$$

4.3 Optimal Policy Portfolio

We now characterize the optimal deposit insurance policy, taking into account its effect on depositors' equilibrium withdrawal behavior and, through it, on the efficiency of the policymaker's $t = 1$ intervention.

Because deposit insurance is a welfare-neutral transfer in our model, it affects welfare only through its effect on the equilibrium run threshold $\hat{s} = \hat{s}(K)$ and the induced withdrawal size $m(\lambda, \theta; \hat{s}) = \bar{m}\left(\lambda + (1-\lambda)\Phi\left(\sqrt{\beta}(\hat{s} - \theta)\right)\right)$. By Corollary 3, every target threshold $s^* \in [-\infty, \hat{s}_0]$ can be implemented by a unique deposit insurance level $K \in [0, r_1]$. Thus, the policymaker's problem can be reformulated as choosing the induced run threshold $\hat{s} \in [-\infty, \hat{s}_0]$.

Fix an induced run threshold $\hat{s} \in [-\infty, \hat{s}_0]$. Given the optimal $t = 1$ intervention policy characterized in Section 4.1, the policymaker's ex-post welfare in state (θ, λ) can be written as

$$u_g(\theta, \lambda; \hat{s}) = \begin{cases} R - 1 - \rho\, m(\lambda, \theta; \hat{s}), & \text{if } \theta \geq \theta^U, \\ P(\theta) R - 1 - \rho\, m(\lambda, \theta; \hat{s}), & \text{if } \theta < \theta^U \text{ and } m(\lambda, \theta; \hat{s}) \leq \hat{m}^{BR}(\hat{s}), \\ -\rho\, m(\lambda, \theta; \hat{s}), & \text{if } \theta < \theta^U \text{ and } m(\lambda, \theta; \hat{s}) > \hat{m}^{BR}(\hat{s}). \end{cases} \tag{33}$$

The first line corresponds to the case of $L = r_1$, which implies that $\theta \geq \theta^U$. In such a case, the policymaker always injects full liquidity, so the bank's long-term investment is preserved and its realized NPV is $R - 1$. The second and third lines correspond to the two cases where $L = 1$, which implies $\theta < \theta^U$. Following Corollary 2, the policymaker employs a cutoff intervention policy at $t = 1$: injecting full liquidity if the realized withdrawal size is below $\hat{m}^{BR}(\hat{s})$ and forcing liquidation otherwise.

Equivalently, after observing (m, L), the policymaker's optimal continuation value net of the bank run cost is

$$u_g^*(m, L; \hat{s}) = \begin{cases} R - 1 - \rho m, & \text{if } L = r_1, \\ \max\left\{\mathbb{E}_g[P(\theta) \mid m; \hat{s}]\, R - 1,\, 0\right\} - \rho m, & \text{if } L = 1, \end{cases} \tag{34}$$

where the posterior mean $\mathbb{E}_g[P(\theta) \mid m; \hat{s}]$ is computed under the policymaker's posterior when the liquidation value is $L = 1$. The policymaker's ex-ante objective at $t = 0$ is therefore

$$\begin{aligned}\max_{\hat{s}\in[-\infty,\hat{s}_0]} W(\hat{s};\rho) &\equiv \max_{\hat{s}\in[-\infty,\hat{s}_0]} \mathbb{E}_g\left[u_g(\theta,\lambda;\hat{s})\right] \\ &= \max_{\hat{s}\in[-\infty,\hat{s}_0]} \mathbb{E}_g\Bigg[\mathbf{1}\left\{\theta \geq \theta^U\right\}(R-1) \\ &+ \mathbf{1}\left\{\theta < \theta^U\right\}\max\left\{\mathbb{E}_g[P(\theta) \mid m(\lambda,\theta;\hat{s});\hat{s}]\,R-1,\,0\right\} - \rho m(\lambda,\theta;\hat{s})\Bigg].\end{aligned} \tag{35}$$

where the expectation is taken under the policymaker's prior over θ and the common prior over λ.

Equation (34) clarifies the key informational role of aggregate withdrawals. When $L = r_1$, the policymaker already knows that fundamentals are strong and always injects full liquidity, so learning from m is irrelevant. By contrast, when $L = 1$, the continuation value is $\max\left\{\mathbb{E}_g[P(\theta) \mid m; \hat{s}]\,R-1,\,0\right\}$, which is convex in the posterior expected success probability. Hence, holding fixed the distribution of the posterior mean, when the early withdrawal size is more informative about the bank fundamentals, the policymaker's expected continuation value is higher in the sense of Blackwell. In our model, a higher deposit insurance level lowers $\hat{s}$ and makes patient depositors' withdrawal decisions less responsive to fundamentals. As a result, the early withdrawal size becomes less informative about fundamentals precisely.

Equation (35) therefore highlights the central trade-off. Inside the expectation, the first term is constant in $\hat{s}$ since the policymaker will learn perfectly that the bank fundamentals are in the upper dominance region by the high liquidation value. In contrast, the second term and the third term depend on the run threshold $\hat{s}$. In particular, a lower run threshold $\hat{s}$, implemented by a higher deposit insurance coverage, reduces withdrawals and hence lowers the expected run-cost term $\rho\,\mathbb{E}_g[m(\lambda,\theta;\hat{s})]$ (i.e., the expected third term in equation (35)). At the same time, it reduces the informativeness of aggregate withdrawals in the case of low liquidation value and thus weakens the effectiveness of targeted $t = 1$ intervention (i.e., the second term). Thus, deposit insurance suppresses inefficient runs but also destroys information that is useful for ex-post intervention policy.

The trade-off between the informativeness of early withdrawals and the bank run cost then leads to the following characterization of the optimal policy portfolio. Before formally stating

the results, we define two thresholds of the marginal bank run cost as

$$\rho_1^{\text{PP}} = \sup\left\{\rho \geq 0 : \hat{s}_0 \text{ is a maximizer of } W\left(\hat{s};\rho\right) \text{ on } \left[-\infty, \hat{s}_0\right]\right\}, \tag{36}$$

$$\rho_2^{\text{PP}} = \inf\left\{\rho \geq 0 : -\infty \text{ is a maximizer of } W\left(\hat{s};\rho\right) \text{ on } \left[-\infty, \hat{s}_0\right]\right\}, \tag{37}$$

Note that by construction, ρ_1^{PP} and ρ_2^{PP} are defined using the requirement that $\hat{s}_0$ and $-\infty$, respectively, be maximizers of $W\left(\hat{s};\rho\right)$. Equivalently, one could define the same threshold values using the requirement that these corner points be unique maximizers, except possibly at the threshold point itself. Thus, for any $\rho < \rho_1^{\text{PP}}$, $\hat{s}_0$ is the unique maximizer of $W\left(\hat{s};\rho\right)$, and for any $\rho > \rho_2^{\text{PP}}$, $-\infty$ is the unique maximizer.

Proposition 3 presents our main results about optimal policy portfolio.

Proposition 3 *Suppose Θ is sufficiently large so that $\Theta > \hat{\Theta} \equiv R\left(\theta^U - \theta^L\right) - \theta^L$. Then ρ_1^{PP} and ρ_2^{PP} in equations (36) and (37) are well defined and satisfy $0 < \rho_1^{\text{PP}} \leq \rho_2^{\text{PP}} < \infty$. Moreover:*

(i) *If $\rho < \rho_1^{\text{PP}}$, then $K^* = 0$, and the induced equilibrium run threshold is $\hat{s}^* = \hat{s}_0$.*

(ii) *If $\rho > \rho_2^{\text{PP}}$, then $K^* = r_1$, and hence $\hat{s}^* = -\infty$.*

(iii) *If $\rho_1^{\text{PP}} < \rho_2^{\text{PP}}$, then for any $\rho \in \left(\rho_1^{\text{PP}}, \rho_2^{\text{PP}}\right)$, every optimal deposit insurance policy is interior: $K^* \in (0, r_1)$, which induces an equilibrium run threshold $\hat{s}^* \in (-\infty, \hat{s}_0)$.*

In all cases, conditional on the induced run threshold $\hat{s}^*$, the policymaker's optimal $t = 1$ intervention is given by Lemma 2.

The economic intuition of Proposition 3 is as follows. Because deposit insurance is a welfare-neutral transfer, the policymaker chooses K solely by trading off its effect on bank run costs against its effect on the informativeness of withdrawals.

At one extreme, when $K = r_1$, patient depositors never run and $\hat{s} = -\infty$. Conditional on $L = 1$, the realized withdrawal size then reflects only exogenous liquidity needs and is uninformative about fundamentals. When Θ is sufficiently large, the bank's long-term investment has negative expected NPV, so the policymaker forces liquidation whenever $L = 1$. Thus, generous deposit insurance suppresses runs but eliminates the informational role of withdrawals.

At the other extreme, when $K = 0$, the induced run threshold is the laissez-faire threshold $\hat{s}_0$, and aggregate withdrawals are most informative. Conditional on $L = 1$, the policymaker can then use the realized withdrawal size to better distinguish relatively strong banks from relatively weak ones and to target liquidity support more effectively. This informational benefit is valuable because it improves ex-post sorting: banks with stronger fundamentals are more

likely to receive liquidity support, whereas banks with weaker fundamentals are more likely to be liquidated.

The cost of relying more heavily on withdrawals is that it increases the run-cost term ρm. Hence, when ρ is small, the informational benefit of allowing withdrawals to reveal information dominates, and the policymaker prefers minimal deposit insurance. When ρ is large, suppressing withdrawals dominates, and the policymaker prefers full deposit insurance. For intermediate values of ρ, the optimal policy balances these two forces and therefore involves an interior level of deposit insurance.

Taken together, these observations imply that the optimal policy portfolio combines an ex-ante tool, deposit insurance, with an ex-post tool, state-contingent liquidity intervention. Deposit insurance controls the severity of runs, whereas the withdrawal signal generated in equilibrium allows the policymaker to allocate liquidity support more efficiently.

4.4 Decomposing Effects of Deposit Insurance

While the formal proof of Proposition 3 is relegated to the Appendix, we discuss in this subsction how the deposit insurance coverage, through affecting the depositors' withdrawal decisions, shapes the efficiency of the policymaker's $t = 1$ intervention policy, by ignoring the bank run cost. Again, by Corollary 3, each deposit insurance coverage $K \in [0, r_1]$ induces a unique depositors' run threshold $\hat{s} \in [-\infty, \hat{s}_0]$, so we may consider the effects of an increase in $\hat{s}$.

Indeed, an increase in $\hat{s}$ affects the $t = 1$ intervention efficiency operates through two channels: an intensive-margin effect where a change in $\hat{s}$ alters the realized early withdrawal size $m(\lambda, \theta; \hat{s})$ and an extensive-margin effect where a change in $\hat{s}$ shifts the policymaker's posterior belief for a fixed intervention cutoff and changes the optimal intervention cutoff $\hat{m}^{BR}(\hat{s})$. We analyze these channels formally and shows that the key force is the informational effect through the policymaker's posterior inference.

We first discuss the intensive-margin effect of a marginal increase in $\hat{s}$ in the learning-relevant region $(\hat{s}^c, \hat{s}_0]$. Figure 5 illustrates how an increase in the bank run threshold $\hat{s}$ affects the realized withdrawal size. Fix any state (θ, λ). A marginal increase in $\hat{s}$ increases the mass of depositors who withdraw, and therefore increases the realized early withdrawal size m.

If one were to hold the bank's continuation decision fixed, this increase in m would affect welfare through the amount of long-term investment that remains in place. In particular, when the bank's expected NPV is positive, i.e., $\theta > P^{-1}\left(\frac{1}{R}\right)$, as at states A and D, a larger withdrawal size would reduce welfare by causing more premature liquidation. By contrast, when

the bank's expected NPV is negative, as at states B and C, a larger withdrawal size would improve welfare by inducing more liquidation of a negative-NPV investment.

However, this intensive-margin effect is neutralized by the policymaker's $t = 1$ intervention rule. Holding fixed the intervention cutoff $\hat{m}^{BR}$, the policymaker injects full liquidity whenever $m < \hat{m}^{BR}$ and forces liquidation whenever $m \geq \hat{m}^{BR}$. Thus, for states below the blue iso-$\hat{m}^{BR}$ curve (such as A and B), the policymaker chooses $\ell = mr_1$, so the bank's long-term investment is fully preserved. For states above the blue iso-$\hat{m}^{BR}$ curve (such as C and D), the policymaker chooses $\ell = mr_1 - 1$, so the remaining investment is fully liquidated and the continuation investment is zero.

Therefore, for any state strictly away from the iso-$\hat{m}^{BR}$ boundary, a marginal increase in $\hat{s}$ (holding $\hat{m}^{BR}$ fixed) changes the realized withdrawal size m but does not change the continuation investment decision. Hence, the intensive-margin effect of a higher $\hat{s}$ on withdrawals is locally nullified by the policymaker's bang-bang $t = 1$ intervention policy.

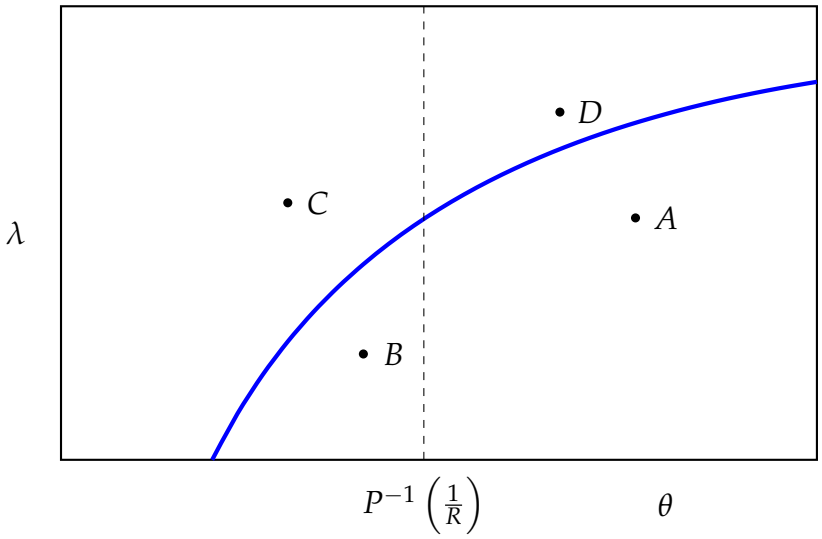
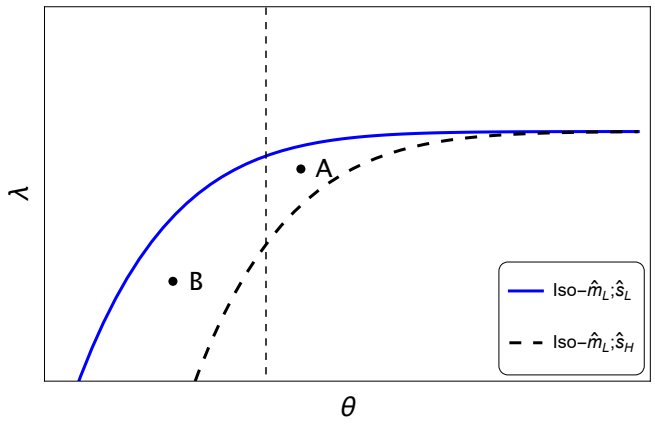


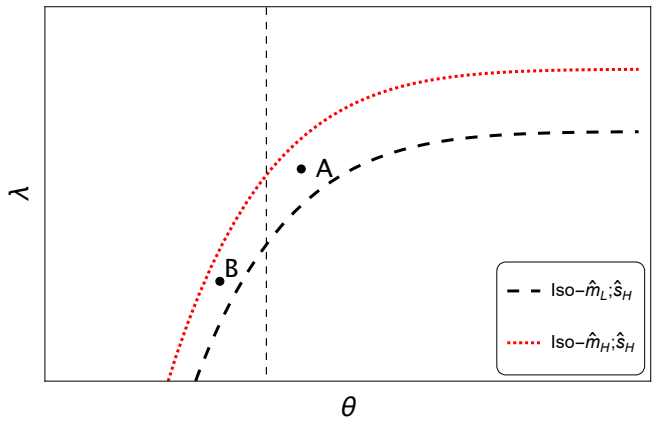
Figure 5: Effect of Increasing $\hat{s}$ on bank investment

We now use Figure 6 to illustrate the extensive margin effect of a higher run threshold through policymaker's learning. Suppose that the run threshold from $\hat{s}_L$ to $\hat{s}_H$ (with $\hat{s}_0 \geq \hat{s}_H > \hat{s}_L > \hat{s}^c$), so that the run thresholds are in the learning-relevant region.

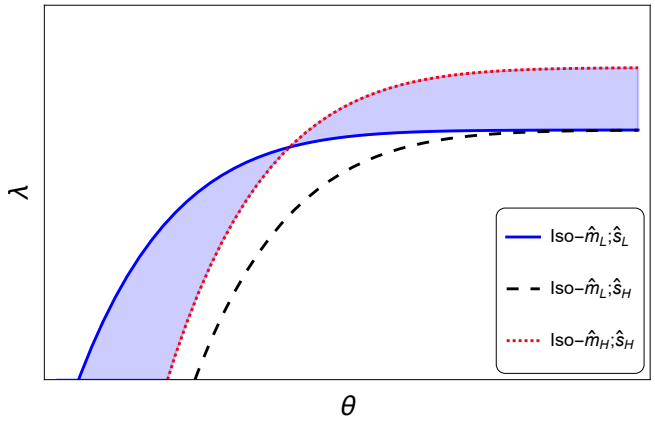
Suppose first that the policymaker keeps the same liquidity-injection cutoff $\hat{m}_L$; that is, she injects full liquidity if $m < \hat{m}_L$ and forces liquidation otherwise. The solid blue curve in panel (a) is the iso-$\hat{m}_L$ curve under $\hat{s}_L$, while the dashed black curve is the iso-$\hat{m}_L$ curve under $\hat{s}_H$. Because a higher run threshold increases withdrawals for every fixed state (θ, λ), the iso-$\hat{m}_L$



(a) Change in $\hat{s}$ holding $\hat{m}$ fixed



(b) Change in $\hat{m}$



(c) Aggregate effect

Figure 6: Effect of increasing $\hat{s}$ on the extensive margin

curve shifts downward when $\hat{s}$ rises from $\hat{s}_L$ to $\hat{s}_H$. Consequently, the states between the blue and dashed curves lose liquidity support: they were below the blue curve (hence $m < \hat{m}_L$) under $\hat{s}_L$, but are now above the dashed curve (hence $m \geq \hat{m}_L$) under $\hat{s}_H$.

The welfare effect of this reclassification depends on fundamentals. If a state has sufficiently weak fundamentals (e.g., point B in panel (a)), losing liquidity support is welfare-improving because it induces liquidation of a negative-NPV investment. If a state has strong fundamentals (e.g., point A), losing liquidity support is welfare-reducing. The key force, however, is improved sorting. Specifically, a higher $\hat{s}$ makes the withdrawal size more sensitive to fundamentals, so for a given cutoff $\hat{m}_L$ the states that are screened out of support are disproportionately tilted toward weaker-fundamental states.

Next, we allow the policymaker to re-optimize her $t = 1$ intervention cutoff. By Corollary 2,

when $\hat{s}$ rises from $\hat{s}_L$ to $\hat{s}_H$, the optimal cutoff increases from $\hat{m}_L$ to $\hat{m}_H$. Panel (b) illustrates this adjustment: holding $\hat{s}_H$ fixed, the iso-curve moves from the dashed black curve (iso-$\hat{m}_L$; $\hat{s}_H$) to the dotted red curve (iso-$\hat{m}_H$; $\hat{s}_H$), which shifts upward and to the left. Therefore, relative to the dashed-black benchmark, the policymaker restores liquidity support to the states between the dashed black and dotted red curves.

Again, the sign of the welfare effect at each state depends on fundamentals: restoring support is welfare-improving in strong-fundamental states (such as A in panel (b)) and welfare-reducing in weak-fundamental states (such as B). However, this cutoff-adjustment effect is second-order at the optimum. Formally, if we write the ex-ante welfare (for a fixed $\hat{s}$) as a function of the intervention cutoff $\hat{m}$, then

$$\left.\frac{\partial \mathbb{E}_g\left[u_g\right]}{\partial \hat{m}}\right|_{\hat{m}=\hat{m}^{BR}(\hat{s})} = 0$$

by the ex-post optimality of the policymaker's intervention rule (envelope theorem). Thus, the marginal welfare effect of the induced change in $\hat{m}^{BR}$ vanishes.

Panel (c) combines the two steps and shows the aggregate extensive-margin reallocation generated by increasing $\hat{s}$ from $\hat{s}_L$ to $\hat{s}_H$. Comparing the initial blue boundary (iso-$\hat{m}_L$; $\hat{s}_L$) with the final red boundary (iso-$\hat{m}_H$; $\hat{s}_H$), liquidity support is reallocated across states in a way that reflects improved learning from withdrawals. Intuitively, lowering deposit insurance (equivalently, increasing $\hat{s}$) makes depositors' run decisions more responsive to their private information about fundamentals, which makes the aggregate withdrawal size m a more informative signal and improves the efficiency of targeted $t = 1$ intervention.

Taken together, the intensive-margin and extensive-margin decompositions imply that the first-order welfare effect of increasing $\hat{s}$ operates through the policymaker's inference (learning) channel and is strictly positive. The intensive-margin effect is locally neutralized by the bang-bang intervention policy, while the induced adjustment in $\hat{m}^{BR}$ has zero first-order effect by the envelope theorem. Therefore, conditional on $\rho = 0$, the policymaker strictly prefers the largest feasible run threshold in that region, namely $\hat{s}_0$, which is implemented by $K = 0$.

Hence, for any given ρ, the policymaker trades off two forces. A higher run threshold $\hat{s}$ makes patient depositors' withdrawal decisions more responsive to their private signals, which improves the policymaker's learning about bank fundamentals and increases the efficiency of the $t = 1$ intervention policy. At the same time, a higher run threshold increases early withdrawals and therefore raises bank run costs.

When ρ is small (in particular, for $\rho < \rho_1^{\text{PP}}$), the marginal bank run cost is small, so the learning benefit of a higher run threshold dominates. As a result, the policymaker optimally chooses the largest feasible run threshold, which is implemented by the minimum deposit insurance level, $K = 0$. At the other extreme, when ρ is large (in particular, for $\rho > \rho_2^{\text{PP}}$), the bank run cost dominates, so the policymaker optimally chooses full deposit insurance, $K = r_1$, which suppresses runs by patient depositors. For intermediate values $\rho \in \left(\rho_2^{\text{PP}}, \rho_2^{\text{PP}}\right)$ (if the set has a non-empty interior), the optimal deposit insurance coverage is interior, balancing these two forces.

5 Role of Liquidity Injection

We now compare the optimal deposit insurance policy in the policy-portfolio environment to that in the benchmark with deposit insurance alone. The comparison clarifies how the availability of ex-post liquidity intervention changes the design of deposit insurance.

In the baseline model, the policymaker observes both the early withdrawal size m and the liquidation-value signal L at $t = 1$. When $L = r_1$, the project is known to be viable and the policymaker injects full liquidity. When $L = 1$, the policymaker uses the withdrawal signal m to decide whether to inject liquidity or force liquidation. Thus, ex-post intervention creates an informational channel: lower deposit insurance induces more informative withdrawals, which can improve the efficiency of the $t = 1$ intervention.

By contrast, in the benchmark model of Section 3, the policymaker cannot intervene at $t = 1$. In that environment, deposit insurance is the only tool available to manage bank runs and liquidation. Accordingly, the optimal benchmark policy balances the benefit of reducing inefficient liquidation of positive-NPV projects, the benefit of allowing liquidation of negative-NPV projects through runs, and the direct bank run cost. As shown in Proposition 1, for any $\rho \in \left(0, \rho^{\text{DI}}\right)$ the optimal benchmark deposit insurance policy is interior; that is, $K_{DI}^*(\rho) \in (0, r_1)$.

In the baseline model, the policymaker instead has two instruments: ex-ante deposit insurance and ex-post intervention. Proposition 3 shows that the optimal policy portfolio depends on the marginal bank run cost. For sufficiently small ρ, the policymaker chooses zero deposit insurance, because preserving informative withdrawals is more valuable than suppressing runs. For sufficiently large ρ, she chooses full deposit insurance, because the run-cost motive dominates and ex-post intervention can still be used to liquidate weak banks conditional on $L = 1$.

These observations imply that the comparison between the baseline model and the bench-

mark is non-monotone in ρ. When the marginal cost of bank run is small, ex-post intervention and deposit insurance are substitutes: the baseline model features lower deposit insurance because ex-post intervention can use the information contained in withdrawals. When the marginal cost of bank run is large, the two tools become complements: because ex-post intervention can still liquidate weak banks at $t = 1$, the policymaker is willing to use more deposit insurance ex ante to suppress costly runs.

Proposition 4 formalizes this comparison.

Proposition 4 *Suppose that the conditions of Propositions 1 and 3 hold, and that Θ is sufficiently large so that*

$$\Theta > \hat{\Theta}^C \equiv \frac{2R(R-1)\left(\theta^U - \theta^L\right)}{r_1 \bar{m}} + (R-1)\left(\theta^U - \theta^L\right) - \theta^L. \tag{38}$$

Then we have:

(i) *For every $\rho \in \left(0, \rho_1^{\mathrm{PP}}\right)$, $K_{PP}^*(\rho) = 0 < K_{DI}^*(\rho)$.*

(ii) *$\rho_2^{\mathrm{PP}} < \rho^{\mathrm{DI}}$, so for every $\rho \in \left(\rho_2^{\mathrm{PP}}, \rho^{\mathrm{DI}}\right)$, $K_{PP}^*(\rho) = r_1 > K_{DI}^*(\rho)$.*

Proposition 4 implies that the effect of allowing $t = 1$ intervention on optimal deposit insurance depends on the bank run cost. For low values of ρ, the baseline model features lower deposit insurance than the benchmark. The reason is that, when runs are not very costly, the policymaker values the information revealed by withdrawals. In the policy-portfolio problem, lower deposit insurance induces a higher run threshold and therefore more informative withdrawals, which improve the efficiency of the $t = 1$ intervention. In the benchmark without ex-post intervention, this learning channel is absent. As a result, the policymaker relies more on deposit insurance to reduce inefficient liquidation, so $K_{PP}^*(\rho) < K_{DI}^*(\rho)$ for all $\rho \in \left(0, \rho_1^{\mathrm{PP}}\right)$.

For higher values of ρ, the ranking is reversed. In particular, part (ii) of Proposition 4 shows that $\rho_2^{\mathrm{PP}} < \rho^{\mathrm{DI}}$. Then, the policy-portfolio problem prescribes full deposit insurance for $\rho \in \left(\rho_2^{\mathrm{PP}}, \rho^{\mathrm{DI}}\right)$, whereas the benchmark continues to use partial coverage. The reason is that, once ex-post intervention is available, deposit insurance no longer needs to preserve runs as the sole mechanism through which weak banks are liquidated. Conditional on $L = 1$, the policymaker can still use the realized withdrawal size to screen banks ex post. Hence, when bank runs are sufficiently costly, deposit insurance and ex-post intervention become complements: the policymaker uses more ex-ante insurance in the baseline model than in the benchmark.

The ranking is generally ambiguous in the intermediate region $\rho \in \left[\rho_1^{\mathrm{PP}}, \rho_2^{\mathrm{PP}}\right]$. In that region, the relative strength of the learning benefit from informative withdrawals and the liquidation-

disciplining role of runs depends on model primitives such as the shape of $P(\theta)$ and the locations of θ^L and θ^U.

6 Conclusion

Banking crises remain a central threat to financial stability and economic efficiency, making the design of the financial safety net a first-order policy question. This paper studies the joint design of two core instruments: ex-ante deposit insurance and ex-post liquidity support. Our central point is that these instruments cannot be evaluated in isolation. Deposit insurance affects not only the incidence and severity of runs, but also the informativeness of aggregate withdrawals for subsequent crisis management. Once ex-post intervention is feasible, this informational channel becomes a key determinant of optimal policy.

In our framework, deposit insurance generates a fundamental trade-off. On the one hand, it discourages withdrawals and thereby reduces the direct costs associated with bank runs. On the other hand, it dampens depositors' responsiveness to private information about bank fundamentals. As a result, aggregate withdrawals become a less informative signal, making it harder for the policymaker to distinguish liquidity-driven withdrawals from fundamentals-driven runs. The optimal level of deposit insurance therefore reflects a balance between the stabilization benefit of suppressing runs and the information cost of weakening the signal that guides ex-post intervention.

Our first main result is that the optimal level of deposit insurance depends on the marginal cost of runs. When run costs are low, the informational value of withdrawals dominates. In that case, the policymaker optimally chooses the lowest feasible level of insurance in order to preserve learning and improve the targeting of ex-post liquidity support. When run costs are high, the run-suppression motive dominates, and full insurance becomes optimal. For intermediate values, the optimal policy involves partial coverage. Thus, in the presence of a lender of last resort, deposit insurance does more than prevent runs: it also shapes the information environment in which crisis management takes place.

We further sharpen this mechanism by comparing the baseline model to a benchmark in which deposit insurance is the only policy instrument. In the benchmark, optimal insurance trades off the reduction in inefficient liquidation of positive-NPV banks against the preservation of run-based discipline for negative-NPV banks, together with the direct cost of runs. Once ex-post liquidity intervention is introduced, this logic changes. The comparison yields a non-monotone ranking of optimal deposit insurance across the two environments. When run

costs are sufficiently small, optimal coverage is lower in the policy-portfolio problem than in the benchmark, because the policymaker places high value on preserving the informational content of withdrawals. By contrast, when run costs are sufficiently large, optimal coverage is higher in the policy-portfolio problem. The reason is that ex-post intervention can substitute for the disciplining role of runs, making it less necessary to rely on limited insurance ex ante to preserve liquidation through withdrawals.

More broadly, our analysis highlights a general lesson for policy evaluation: when multiple instruments jointly shape private behavior and the information available to policymakers, they should be analyzed as a portfolio rather than in isolation. In our setting, ex-ante deposit insurance affects depositors' withdrawal behavior, withdrawals generate an endogenous signal about bank fundamentals, and the policymaker conditions ex-post intervention on that signal. The welfare effect of deposit insurance therefore depends not only on its direct effect on run incentives, but also on how it changes the informativeness of withdrawals for crisis management. This mechanism is likely relevant beyond banking, in settings where policy both affects behavior and alters the information available to future decision-makers.

A Perfectly Informed Policymaker

This appendix considers a model in which the policymaker is perfectly informed about the bank fundamental θ at $t = 1$ when choosing the intervention, while she remains uninformed at $t = 0$ when committing to deposit insurance. As in the baseline model, depositors use an improper uniform prior over $\mathbb{R}$, whereas the policymaker evaluates ex-ante welfare under her bounded uniform prior f_g in equation (3). Because the policymaker observes θ at $t = 1$, she does not need to infer fundamentals from the withdrawal size m. Hence, this model establish a nice "no-learning" benchmark.

We solve the benchmark by backward induction, starting from the policymaker's $t = 1$ intervention.

The $t = 1$ intervention. Recall that, after observing m, the policymaker chooses either a liquidity injection $\ell \in [0, mr_1]$ or forced liquidation, which we denote by $\ell = mr_1 - 1$. Since the policymaker observes θ at $t = 1$, her payoff becomes

$$u_g(\ell \mid m, \theta) = [1 - (mr_1 - \ell)](P(\theta) R - 1) - \rho m.$$

The choice is trivially bang-bang: if the bank investment NPV is positive, the policymaker injects full liquidity to avoid inefficient liquidation; otherwise, she forces liquidation to eliminate a negative-NPV investment.

Lemma 5 *When the policymaker observes θ at $t = 1$, her optimal intervention is*

$$\mathcal{L}^I(m, \theta) = \begin{cases} mr_1, & \text{if } P(\theta) \geq \dfrac{1}{R}, \\ mr_1 - 1, & \text{if } P(\theta) < \dfrac{1}{R}, \end{cases} \tag{39}$$

where $\ell = mr_1 - 1$ denotes forced liquidation. Equivalently, letting $\theta^I \equiv P^{-1}\left(\frac{1}{R}\right)$, the policymaker injects fully if and only if $\theta > \theta^I$.

Lemma 5 implies that, with perfect information, the policymaker intervenes solely based on whether the bank's investment has positive NPV. If $P(\theta) R \geq 1$ (i.e., $\theta \geq \theta^I$), early withdrawals are socially inefficient because they would trigger premature liquidation of a productive investment; the policymaker injects $\ell = mr_1$ and preserves the entire investment. If $P(\theta) R < 1$ (i.e., $\theta < \theta^I$), continuing the investment is inefficient; the policymaker forces liquidation, $\ell = mr_1 - 1$, and the remaining investment is fully liquidated at $t = 1$.

Bank run. Given the intervention rule in Lemma 5, a patient depositor's incremental payoff from staying (rather than withdrawing early and obtaining r_1) is

$$\Delta c^I(m,\theta;K) = \begin{cases} P(\theta)\, r_2 + (1 - P(\theta))\, K - r_1, & \text{if } \theta \geq \theta^I, \\ \max\left\{\dfrac{1-mr_1}{1-m}, K\right\} - r_1, & \text{if } \theta < \theta^I. \end{cases} \tag{40}$$

The first line follows because when $\theta \geq \theta^I$ the policymaker injects full liquidity, so the long-term investment is preserved; since $R > r_2$ and $m < 1$, the bank can repay all remaining depositors r_2 upon success. The second line follows because when $\theta < \theta^I$ the policymaker forces liquidation at $t = 1$; since $\theta^I < \theta^U$, liquidation proceeds are based on value 1 per unit of investment, so a depositor who stays receives $\max\left\{\frac{1-mr_1}{1-m}, K\right\}$ at $t = 1$.

As in the baseline and deposit-insurance-only environments, the global-games structure implies that depositors follow a threshold strategy in equilibrium. Let s^I denote the conjectured run threshold, and define

$$V^I\left(s_i; K, s^I\right) = \int_{-\infty}^{\infty} \left(\int_0^1 \Delta c^I\left(m\left(\lambda, \theta; s^I\right), \theta; K\right) g_p(\lambda)\, \mathrm{d}\lambda \right) f(\theta \mid s_i)\, \mathrm{d}\theta, \tag{41}$$

where $g_p(\lambda)$ is given by (14), $m\left(\lambda, \theta; s^I\right)$ is given by (10), and under depositors' improper uniform prior,

$$f(\theta \mid s_i) = \sqrt{\beta}\, \phi\Big(\sqrt{\beta}\,(\theta - s_i)\Big).$$

Lemma 6 *When the policymaker observes θ at $t = 1$, there exists $\hat{\beta}^I > 0$ such that for any $\beta > \hat{\beta}^I$ and any $K \in [0, r_1]$, depositors follow a unique threshold-strategy equilibrium. Specifically, for any $i \in [0,1]$,*

$$d_i^I = \begin{cases} 1, & \text{if } s_i \leq s^I, \\ 0, & \text{if } s_i > s^I, \end{cases}$$

where $s^I = -\infty$ if $K = r_1$, and otherwise s^I is the unique solution to

$$V^I\left(s^I; K, s^I\right) = 0 \quad \text{for } K \in [0, r_1).$$

Moreover, s^I is strictly decreasing in K on $[0, r_1)$.

Optimal deposit insurance. With perfect information at $t = 1$, the policymaker's intervention rule (39) depends only on θ and not on the informativeness of withdrawals. Consequently, deposit insurance affects welfare only through its effect on the equilibrium withdrawal size m

because of the bank run cost $-\rho m$. Since deposit insurance is a welfare-neutral transfer and $\rho > 0$, the policymaker strictly prefers to minimize inefficient panic withdrawals by maximizing insurance coverage. In particular, setting $K = r_1$ implies $s^I = -\infty$ in Lemma 6, so patient depositors never run, and withdrawals are driven only by liquidity shocks. Therefore, full deposit insurance is optimal.

Proposition 5 *When the policymaker observes θ at $t = 1$ and $\rho > 0$, the optimal policy portfolio is $\left(K^I, \mathcal{L}^I\right)$, where*

$$K^I = r_1$$

and $\mathcal{L}^I$ is characterized in (39).

Proposition 5 highlights that, with perfect information at $t = 1$, the learning motive for tolerating withdrawals disappears. The policymaker can implement the efficient continuation/liquidation decision directly through intervention, and she uses full deposit insurance to eliminate panic-driven runs and reduce run costs.

B Proofs of Lemmas and Propositions

Proof of Lemma 1:

Fix $K \in [0, r_1]$ and a conjectured cutoff $\hat{s} \in \mathbb{R}$. Recall

$$V^{DI}(s_i; K, \hat{s}) = \int_{-\infty}^{\infty} \left(\int_0^1 \Delta c^{DI}(m(\lambda, \theta; \hat{s}), \theta; K)\, g_p(\lambda)\, \mathrm{d}\lambda \right) f(\theta \mid s_i)\, \mathrm{d}\theta,$$

where, under the depositors' improper uniform prior,

$$f(\theta \mid s_i) = \sqrt{\beta}\, \phi\Big(\sqrt{\beta}(\theta - s_i) \Big).$$

Consider first the case where $K = r_1$. If a patient depositor withdraws early, her payoff is r_1. If she stays, her payoff is at least $K = r_1$ if $\theta < \theta^U$ and is r_2 if $\theta \geq \theta^U$. Since the depositor's posterior belief has a full support, her expected payoff from staying is strictly greater than that from running, for any private signal she observes. Hence, when $K = r_1$, $\hat{s}^{DI} = -\infty$, meaning that no patient depositors run.

We now consider the case where $K \in [0, r_1)$. We prove the lemma in two steps. We first show that given other depositors' cutoff strategy with the run threshold $\hat{s}$, depositor i with private signal s_i also employs a cutoff strategy. Then, we show that there is a unique $\hat{s}$ so that the marginal depositor's expected payoff is 0.

To show the first step, for any $\hat{s} \in \mathbb{R}$, we define

$$\Psi^{DI}(\theta; K, \hat{s}) \equiv \int_0^1 \Delta c^{DI}(m(\lambda, \theta; \hat{s}), \theta; K)\, g_p(\lambda)\, \mathrm{d}\lambda.$$

Then

$$V^{DI}(s_i; K, \hat{s}) = \int_{-\infty}^{\infty} \Psi^{DI}(\theta; K, \hat{s})\, f(\theta \mid s_i)\, \mathrm{d}\theta.$$

Fix λ and $\hat{s}$. Since $m(\lambda, \theta; \hat{s}) = \bar{m}\left(\lambda + (1-\lambda)\,\Phi\big(\sqrt{\beta}(\hat{s} - \theta)\big)\right)$, and $\Phi(\cdot)$ is increasing, $m(\lambda, \theta; \hat{s})$ is weakly decreasing in θ. Moreover, for fixed K, the function $\Delta c^{DI}(m, \theta; K)$ is weakly decreasing in m and weakly increasing in θ. Therefore, $\Delta c^{DI}(m(\lambda, \theta; \hat{s}), \theta; K)$ is weakly increasing for each λ, so $\Psi^{DI}(\theta; K, \hat{s})$ is strictly increasing in θ after integrating over λ. Then, because the posterior $\theta \mid s_i'$ first-order stochastically dominates $\theta \mid s_i$ it follows from the fact that $\Psi^{DI}(\cdot; K, \hat{s})$ is weakly increasing that $V^{DI}(s_i'; K, \hat{s}) \geq V^{DI}(s_i; K, \hat{s})$. Hence the expected incremental payoff from staying is increasing in the private signal s_i, so the best response is a cutoff rule.

As $s_i \to -\infty$, the posterior mass concentrates on very low fundamentals, at which $P(\theta) = 0$ and $\Delta c^{DI}(m, \theta; K) = K - r_1$ for all $m \in [0, \bar{m}]$. Therefore, $\lim_{s_i \to -\infty} V^{DI}(s_i; K, \hat{s}) = K -$

$r_1 < 0$. On the other extreme, as $s_i \to +\infty$, the posterior concentrates on very high fundamentals, so eventually $\theta \geq \theta^{UI}$ where $\Delta c^{DI}(m, \theta; K) = r_2 - r_1$ for all $m \in [0, \bar{m}]$. Hence, $\lim_{s_i \to +\infty} V^{DI}(s_i; K, \hat{s}) = r_2 - r_1 > 0$. Since $V^{DI}(s_i; K, \hat{s})$ is continuous and strictly increasing in s_i, there exists a unique best-response cutoff. This completes the proof of the first step.

We now consider the marginal depositor whose private signal realization $\hat{s}$ is just the threshold of other depositors. Fix $K \in [0, r_1)$ and define

$$H(\hat{s}; K) \equiv V^{DI}(\hat{s}; K, \hat{s}).$$

A threshold-strategy equilibrium cutoff is exactly a solution to $H(\hat{s}; K) = 0$.

The function $H(\hat{s}; K)$ is continuous in $\hat{s}$. This function is also strictly increasing for any $\beta > 0$. To see this, take $x = \sqrt{\beta}(\theta - s)$ and $\theta = s + \frac{x}{\sqrt{\beta}}$. Then $f(\theta \mid s)\, \mathrm{d}\theta = \phi(x)\, \mathrm{d}x$ and $n(\theta; s) = \Phi\left(\sqrt{\beta}(s - \theta)\right) = \Phi(-x)$. Hence $m(\lambda, \theta; s) = \bar{m}(\lambda + (1 - \lambda)\Phi(-x))$, which is *independent of* s. Therefore,

$$H(\hat{s}; K) = \int_{-\infty}^{\infty} \left(\int_0^1 \Delta c^{DI} \left(\bar{m}(\lambda + (1 - \lambda)\Phi(-x)), \hat{s} + \frac{x}{\sqrt{\beta}}; K \right) g_p(\lambda)\, \mathrm{d}\lambda \right) \phi(x)\, \mathrm{d}x.$$

For each fixed (x, λ), the only dependence on s is through the second argument $\theta = s + x/\sqrt{\beta}$. Since $\Delta c^{DI}(m, \theta; K)$ is increasing in θ, the integrand is strictly increasing in $\hat{s}$ pointwise. Therefore $H(\hat{s}; K)$ is strictly increasing.

Furthermore, because of the lower- and upper-dominance regions,

$$\lim_{\hat{s} \to -\infty} H(\hat{s}; K) = K - r_1 < 0, \qquad \lim_{\hat{s} \to +\infty} H(\hat{s}; K) = r_2 - r_1 > 0.$$

Since $H(\hat{s}; K)$ is continuous and strictly increasing, there exists a unique $\hat{s}^{DI} \in \mathbb{R}$ such that

$$V^{DI}\left(\hat{s}^{DI}; K, \hat{s}^{DI}\right) = 0.$$

This is the unique threshold-strategy equilibrium cutoff for $K \in [0, r_1)$.

Finally, for $K \in (0, r_1)$, since $V^{DI}\left(\hat{s}^{DI}; K, \hat{s}^{DI}\right)$ is increasing in K. Therefore, $\hat{s}^{DI}(K)$ is strictly decreasing in K.

Q.E.D.

Proof of Corollary 1:

To show that for any target $\hat{s} \in \left[-\infty, \hat{s}_0^{DI}\right]$, there exists a unique $K \in [0, r_1]$ such that $\hat{s}^{DI}(K) = \hat{s}$, where $\hat{s}_0^{DI} = \hat{s}^{DI}(0)$, we first notice that $V^{DI}(\hat{s}; K, \hat{s})$ is continuous in $\hat{s}$ over $(-\infty, \hat{s}_0]$ and is continuous in K over $[0, r_1)$. Therefore, from the equation $V^{DI}\left(\hat{s}^{DI}; K, \hat{s}^{DI}\right) = 0$, $\hat{s}^{DI}$ is a continous function of K.

Fix any finite $s \in \mathbb{R}$. We have $V^{DI}(s; K, s) \to V^{DI}(s; r_1, s)$ as $K \to r_1$. When $K = r_1$, $V^{DI}(s; r_1, s) > 0$ for any finite s. Therefore, for K sufficiently close to r_1, $V^{DI}(s; K, s) > 0$. Since $V^{DI}(s; K, s)$ is strictly increasing and $\hat{s}^{DI}(K)$ is its unique root, this implies $\hat{s}^{DI}(K) < s$. Because s is an arbitrary finite real number, we have

$$\lim_{K \to r_1} \hat{s}^{DI}(K) = -\infty.$$

Since $\hat{s}^{DI}(K)$ is continuous and strictly decreasing on $[0, r_1)$, with $\hat{s}^{DI}(0) = \hat{s}_0^{DI}$ and $\lim_{K \to r_1} \hat{s}^{DI}(K) = -\infty$. Hence its image on $[0, r_1)$ is exactly $\left(-\infty, \hat{s}_0^{DI}\right]$. Therefore, for any target $\hat{s} \in \left(-\infty, \hat{s}_0^{DI}\right]$, there exists a unique $K \in [0, r_1)$ such that $\hat{s}^{DI}(K) = \hat{s}$.

Finally, for the target $\hat{s} = -\infty$, Lemma 1 gives $\hat{s}^{DI}(r_1) = -\infty$. Thus, for every $\hat{s} \in \left[-\infty, \hat{s}_0^{DI}\right]$, there exists a unique $K \in [0, r_1]$ such that $\hat{s}^{DI}(K) = \hat{s}$.

Q.E.D.

Proof of Proposition 1:

By Lemma 1 and Corollary 1, for any $\beta > \hat{\beta}^{DI}$ the policymaker can equivalently choose the induced run threshold $\hat{s} \in \left[-\infty, \hat{s}_0^{DI}\right]$, where $\hat{s}_0^{DI}$ is the laissez-faire threshold under $K = 0$. Let

$$W_{\beta,\rho}(\hat{s}) \equiv \mathbb{E}_g\left[u_g^{DI}(\theta, \lambda; \hat{s})\right].$$

Recall that $m(\theta, \lambda; \hat{s}) = \bar{m}\left(\lambda + (1-\lambda)\Phi\left(\sqrt{\beta}(\hat{s} - \theta)\right)\right)$, so we can calculate that $\frac{\partial m(\theta, \lambda; \hat{s})}{\partial \hat{s}} = \bar{m}(1-\lambda)\sqrt{\beta}\,\phi\left(\sqrt{\beta}(\hat{s} - \theta)\right)$. Also define

$$\Gamma_\rho(\theta) = \begin{cases} r_1(P(\theta)R - 1) + \rho, & \text{if } \theta < \theta^U, \\ R - r_1 + \rho, & \text{if } \theta \geq \theta^U. \end{cases}$$

Then

$$\frac{\partial u_g^{DI}(\theta, \lambda; \hat{s})}{\partial m} = -\Gamma_\rho(\theta),$$

and therefore, differentiating $W_{\beta,\rho}(\hat{s})$ with respect to $\hat{s}$ yields

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}\hat{s}} W_{\beta,\rho}(\hat{s}) &= -\frac{\bar{m}}{2\Theta} \int_{-\Theta}^{\Theta} \int_0^1 \Gamma_\rho(\theta)(1-\lambda)\sqrt{\beta}\,\phi\Big(\sqrt{\beta}(\hat{s} - \theta)\Big)\,\mathrm{d}\lambda\,\mathrm{d}\theta \\ &= -\frac{\bar{m}}{4\Theta} \int_{-\Theta}^{\Theta} \Gamma_\rho(\theta)\sqrt{\beta}\,\phi\Big(\sqrt{\beta}(\hat{s} - \theta)\Big)\,\mathrm{d}\theta. \end{aligned} \tag{42}$$

Let

$$\varphi_\beta(x) \equiv \sqrt{\beta}\,\phi\left(\sqrt{\beta}x\right) \quad \text{and} \quad \tilde{\Gamma}_\rho(\theta) \equiv \Gamma_\rho(\theta)\,\mathbf{1}\{\theta \in [-\Theta, \Theta]\},$$

We first prove part (ii) since the definition of ρ^{DI} in part (i) requires the conclusion in part (ii). we may write

$$\frac{\mathrm{d}}{\mathrm{d}\hat{s}} W_{\beta,\rho}(\hat{s}) = -\frac{\bar{m}}{4\Theta} G_{\beta,\rho}(\hat{s}), \quad \text{where } G_{\beta,\rho}(\hat{s}) \equiv \left(\tilde{\Gamma}_\rho * \varphi_\beta\right)(\hat{s}). \tag{43}$$

To prove part (i), we fix any $\rho \geq r_1$. Then for every $\theta < \theta^U$,

$$\Gamma_\rho(\theta) = r_1\left(P(\theta)R - 1\right) + \rho \geq -r_1 + \rho \geq 0,$$

and for every $\theta \geq \theta^U$,

$$\Gamma_\rho(\theta) = R - r_1 + \rho > 0.$$

Thus equation (43) implies $\frac{\mathrm{d}}{\mathrm{d}\hat{s}} W_{\beta,\rho}(\hat{s}) < 0$ for every finite $\hat{s}$. Hence the policymaker's objective is strictly decreasing in any finite threshold, so the optimum is attained at the lower boundary $\hat{s} = -\infty$, which corresponds to full deposit insurance:

$$K^*_{DI} = r_1.$$

Therefore, for every $\rho \geq r_1$, the optimal policy is full insurance.

Now, to prove part (i), we fix any $\rho < r_1$. Define

$$\theta_\rho \equiv P^{-1}\left(\frac{1 - \rho/r_1}{R}\right).$$

Because $\rho < r_1$, we have $0 < \frac{1-\rho/r_1}{R} < 1$, so $\theta_\rho \in \left(\theta^L, \theta^U\right)$ is well defined. Moreover, $\Gamma_\rho(\theta) < 0$ if and only if $\theta < \theta_\rho$. Hence $\tilde{\Gamma}_\rho$ has exactly one sign change (from negative to positive). Then, by the Variation-Diminishing Property for the Gaussian kernel (Karlin 1968), the convolution of $\tilde{\Gamma}_\rho$ with φ_β has at most one sign change. Therefore, for every $\beta > 0$, $G_{\beta,\rho}$ has at most one sign change, and hence at most one interior solution.

Pick any $\underline{s} \in \left(-\Theta, \theta^L\right)$. Since $\underline{s}$ is a continuity point of $\tilde{\Gamma}_\rho$ and $\tilde{\Gamma}_\rho(\underline{s}) = \Gamma_\rho(\underline{s}) = \rho - r_1 < 0$, so we have $G_{\beta,\rho}(\underline{s}) \to \tilde{\Gamma}_\rho(\underline{s}) = \rho - r_1 < 0$, as $\beta \to +\infty$. Hence there exists $\beta_1(\rho) > 0$ such that for all $\beta > \beta_1(\rho)$, we have $G_{\beta,\rho}(\underline{s}) < 0$, which implies $\frac{\mathrm{d}}{\mathrm{d}\hat{s}} W_{\beta,\rho}(\underline{s}) > 0$.

Now, let

$$\Delta^{DI}_0(m, \theta) \equiv \Delta^{DI}(m, \theta; 0)$$

denote the patient depositor's incremental payoff from staying when $K = 0$. Because $m \leq \bar{m} < 1/r_1$, the third branch in equation (9) never occurs when $K = 0$. For any $m \in [0, \bar{m}]$ and any $\theta \in \mathbb{R}$, we claim that

$$\Gamma_\rho(\theta) > \Delta_0^{DI}(m, \theta). \tag{44}$$

Indeed, when $\theta \geq \theta^U$, $\Gamma_\rho(\theta) - \Delta_0^{DI}(m, \theta) = (R - r_1 + \rho) - (r_2 - r_1) = R - r_2 + \rho > 0$. And when $\theta < \theta^U$, there are two cases. First, if $m \leq \frac{R-r_2}{r_1 R - r_2}$, we have $\Gamma_\rho(\theta) - \Delta_0^{DI}(m, \theta) = [r_1(P(\theta) R - 1) + \rho] - [P(\theta) r_2 - r_1] = P(\theta)(r_1 R - r_2) + \rho > 0$. Second, if $\frac{R-r_2}{r_1 R - r_2} < m < \frac{1}{r_1}$, we have

$$\begin{aligned}
\Gamma_\rho(\theta) - \Delta_0^{DI}(m, \theta) &= [r_1(P(\theta) R - 1) + \rho] - \left[P(\theta) \frac{1 - m r_1}{1 - m} R - r_1\right] \\
&= \rho + P(\theta) R \left(r_1 - \frac{1 - m r_1}{1 - m}\right) \\
&= \rho + P(\theta) R \frac{r_1 - 1}{1 - m} \\
&> 0.
\end{aligned}$$

Therefore, equation (44) holds.

Now we evaluate the derivative at $\hat{s}_0^{DI}$. By definition of $\hat{s}_0^{DI}$,

$$V^{DI}\left(\hat{s}_0^{DI}; 0, \hat{s}_0^{DI}\right) = 0.$$

Using (15), (12), and (14), this is

$$\int_{-\infty}^{\infty} \int_0^1 \Delta_0^{DI}\left(m\left(\lambda, \theta; \hat{s}_0^{DI}\right), \theta\right) 2(1 - \lambda) \varphi_\beta\left(\hat{s}_0^{DI} - \theta\right) \mathrm{d}\lambda \, \mathrm{d}\theta = 0.$$

Hence

$$\int_{-\infty}^{\infty} \int_0^1 \Delta_0^{DI}\left(m\left(\lambda, \theta; \hat{s}_0^{DI}\right), \theta\right) (1 - \lambda) \varphi_\beta\left(\hat{s}_0^{DI} - \theta\right) \mathrm{d}\lambda \, \mathrm{d}\theta = 0.$$

Combining this with the strict pointwise inequality (44), we obtain

$$\int_{-\infty}^{\infty} \int_0^1 \Gamma_\rho(\theta) (1 - \lambda) \varphi_\beta\left(\hat{s}_0^{DI} - \theta\right) \mathrm{d}\lambda \, \mathrm{d}\theta > 0.$$

Then, by equation (42), we have $\frac{\mathrm{d}}{\mathrm{d}\hat{s}} W_{\beta,\rho}\left(\hat{s}_0^{DI}\right) < 0$.

Define a cutoff $\hat{\beta}^{DI}(\rho) \equiv \max\left\{\hat{\beta}^{DI}, \beta_1(\rho)\right\}$, where the first term is the threshold from Lemma 1. Then for every $\beta > \hat{\beta}^{DI}(\rho)$, we have $\frac{\mathrm{d}}{\mathrm{d}\hat{s}} W_{\beta,\rho}(\underline{s}) > 0$ and $\frac{\mathrm{d}}{\mathrm{d}\hat{s}} W_{\beta,\rho}\left(\hat{s}_0^{DI}\right) < 0$. By continuity, $W'_{\beta,\rho}$ has at least one zero in $\left(\underline{s}, \hat{s}_0^{DI}\right) \subset \left(-\infty, \hat{s}_0^{DI}\right)$. Because $G_{\beta,\rho}$ has at most one

zero, equation (43) implies that $W'_{\beta,\rho}$ also has at most one zero. Therefore $W_{\beta,\rho}$ has a unique critical point

$$\hat{s}^*_{DI} \in \left(-\infty, \hat{s}_0^{DI}\right),$$

and this critical point is the unique global maximizer of the threshold problem.

Finally, by Corollary 1, the one-to-one mapping between $K \in [0, r_1]$ and $\hat{s} \in [-\infty, \hat{s}_0^{DI}]$ implies that there exists a unique

$$K^*_{DI} \in (0, r_1)$$

that implements $\hat{s}^*_{DI}$.

Furthermore, combining the result from part (i) that $K^*_{DI} = r_1$ for any $\rho \geq r_1$, we have

$$\rho^{\text{DI}} = \inf\{\rho \geq 0 : \ K^*_{DI} = r_1\} = r_1.$$

To show Part (iii), we fix any $\rho < r_1$. Take any $\varepsilon > 0$ small enough that $\theta_\rho - \varepsilon, \ \theta_\rho + \varepsilon \in (-\Theta, \theta^U)$. Since Γ_ρ is continuous at both points and

$$\Gamma_\rho\left(\theta_\rho - \varepsilon\right) < 0 < \Gamma_\rho\left(\theta_\rho + \varepsilon\right),$$

when β is sufficiently large, we have $G_{\beta,\rho}\left(\theta_\rho - \varepsilon\right) < 0 < G_{\beta,\rho}\left(\theta_\rho + \varepsilon\right)$. Because $G_{\beta,\rho}$ has a unique zero, the unique maximizer $\hat{s}^*_{DI}$ must satisfy

$$\theta_\rho - \varepsilon < \hat{s}^*_{DI} < \theta_\rho + \varepsilon$$

for all sufficiently large β. Since $\varepsilon > 0$ is arbitrary,

$$\lim_{\beta \to \infty} \hat{s}^*_{DI} = \theta_\rho = P^{-1}\left(\frac{1 - \rho/r_1}{R}\right).$$

Q.E.D.

Proof of Proposition 2:

Because the proposition is stated conditional on $L = 1$, we suppress the conditioning on $L = 1$ in the posterior notation throughout the proof. We first derive the posterior belief. Conditional on $L = 1$, the policymaker learns that $\theta < \theta^U$. Since her prior over θ is uniform on $(-\Theta, \Theta)$, her prior conditional on $L = 1$ is uniform on $\left(-\Theta, \theta^U\right)$. For a given $\theta < \theta^U$ and run threshold $\hat{s}$, equation (10) implies that $m\left(\lambda, \theta; \hat{s}\right)$ is affine in λ with slope $\bar{m}\left(1 - n\left(\theta; \hat{s}\right)\right) > 0$. Because $\lambda \sim \mathcal{U}\left[0, 1\right]$, the conditional density of m given θ is

$$f\left(m \mid \theta; \hat{s}\right) = \frac{1}{\bar{m}\left(1 - n\left(\theta; \hat{s}\right)\right)} \mathbf{1}\{m \in \left[\bar{m} n\left(\theta; \hat{s}\right), \bar{m}\right]\}.$$

Hence, for any feasible m, Bayes' rule implies that the posterior density of θ is proportional to $\frac{1}{1-n(\theta;\hat{s})}$ on the set of values of θ that are consistent with both m and $L = 1$. The observation m requires $n(\theta;\hat{s}) \leq m/\bar{m}$, or equivalently $\theta \geq \theta_L(m;\hat{s})$, while the observation $L = 1$ requires $\theta < \theta^U$. Combining these restrictions with the prior support $(-\Theta, \Theta)$, the posterior support is $\left[\underline{\theta}(m;\hat{s}), \theta^U\right)$, where $\underline{\theta}(m;\hat{s}) = \max\{-\Theta, \theta_L(m;\hat{s})\}$. Normalizing the posterior density then yields equation (23).

We now prove the two comparative statics. For part (i), we fix $\hat{s} \in \mathbb{R}$ and take feasible $m_1 < m_2$. If $m_1 < \tilde{m}(\hat{s})$, then $\underline{\theta}(m_1;\hat{s}) > \underline{\theta}(m_2;\hat{s})$. So, for any $\theta < \underline{\theta}(m_1;\hat{s})$, we have $G_\theta(\theta \mid m_1;\hat{s}) = 0 \leq G_\theta(\theta \mid m_2;\hat{s})$. For any $\theta \in \left[\underline{\theta}(m_1;\hat{s}), \theta^U\right)$, equation (23) implies

$$1 - G_\theta(\theta \mid m_j;\hat{s}) = \frac{\int_\theta^{\theta^U} \frac{1}{1-n(x;\hat{s})}\,\mathrm{d}x}{\int_{\underline{\theta}(m_j;\hat{s})}^{\theta^U} \frac{1}{1-n(x;\hat{s})}\,\mathrm{d}x}, \qquad j = 1, 2.$$

The numerator is the same for $j = 1, 2$, whereas the denominator is strictly larger for m_2 because $\underline{\theta}(m_2;\hat{s}) < \underline{\theta}(m_1;\hat{s})$. Hence $G_\theta(\theta \mid m_1;\hat{s}) < G_\theta(\theta \mid m_2;\hat{s})$ for all $\theta \in \left[\underline{\theta}(m_1;\hat{s}), \theta^U\right)$. That is, $G_\theta(\cdot \mid m_1;\hat{s})$ first-order stochastically dominates $G_\theta(\cdot \mid m_2;\hat{s})$.

If instead $m_1 \geq \tilde{m}(\hat{s})$, then $\underline{\theta}(m_1;\hat{s}) = \underline{\theta}(m_2;\hat{s}) = -\Theta$, so equation (23) shows that $G_\theta(\cdot \mid m_1;\hat{s}) = G_\theta(\cdot \mid m_2;\hat{s})$. The claim about $\mathbb{E}_g[P(\theta) \mid m;\hat{s}]$ then follows because $P(\theta)$ is weakly increasing in θ. Moreover, if $m_1 < m_2 < \tilde{m}(\hat{s})$, the first-order stochastic dominance relation is strict, and $P(\theta)$ is not constant on the relevant posterior support because it is strictly increasing on (θ^L, θ^U). Hence $\mathbb{E}_g[P(\theta) \mid m_1;\hat{s}] > \mathbb{E}_g[P(\theta) \mid m_2;\hat{s}]$.

For part (ii), we fix $\hat{s}_1 > \hat{s}_2$ and $m \in (\underline{m}(\hat{s}_1), \tilde{m}(\hat{s}_2))$. Since $\underline{m}(\hat{s})$ and $\tilde{m}(\hat{s})$ are both strictly increasing in $\hat{s}$, the above condition implies $m \in (\underline{m}(\hat{s}_j), \tilde{m}(\hat{s}_j))$ for $j = 1, 2$. Hence, for both $\hat{s}_1$ and $\hat{s}_2$, the posterior lower bound is interior:

$$\underline{\theta}(m;\hat{s}_j) = \theta_L(m;\hat{s}_j) = \hat{s}_j - \frac{1}{\sqrt{\beta}}\Phi^{-1}\left(\frac{m}{\bar{m}}\right).$$

Let $c = \frac{1}{\sqrt{\beta}}\Phi^{-1}\left(\frac{m}{\bar{m}}\right)$. Then $\underline{\theta}(m;\hat{s}_j) = \hat{s}_j - c$, for $j = 1, 2$. Equation (23) the implies that for any $\theta \in \left[\hat{s}_1 - c, \theta^U\right)$,

$$G_\theta(\theta \mid m;\hat{s}_j) = \frac{\int_{\hat{s}_j - c}^{\theta} \frac{1}{\Phi\left(\sqrt{\beta}(x - \hat{s}_j)\right)}\,\mathrm{d}x}{\int_{\hat{s}_j - c}^{\theta^U} \frac{1}{\Phi\left(\sqrt{\beta}(x - \hat{s}_j)\right)}\,\mathrm{d}x}, \qquad j = 1, 2.$$

Changing variables by $u = x - \hat{s}_j$ yields

$$G_\theta\left(\theta \mid m; \hat{s}_j\right) = \frac{\int_{-c}^{\theta - \hat{s}_j} \frac{1}{\Phi\left(\sqrt{\beta}u\right)} \, \mathrm{d}u}{\int_{-c}^{\theta^U - \hat{s}_j} \frac{1}{\Phi\left(\sqrt{\beta}u\right)} \, \mathrm{d}u}.$$

Now fix $\theta < \theta^U$ and let $d = \theta^U - \theta > 0$. Writing $b = \theta^U - \hat{s}$, the posterior cdf becomes

$$H\left(b\right) \equiv \frac{\int_{-c}^{b-d} \frac{1}{\Phi\left(\sqrt{\beta}u\right)} \, \mathrm{d}u}{\int_{-c}^{b} \frac{1}{\Phi\left(\sqrt{\beta}u\right)} \, \mathrm{d}u}.$$

Thus, for fixed m and θ, it suffices to study the monotonicity of $H\left(b\right)$.

Let $q\left(u\right) = \frac{1}{\Phi\left(\sqrt{\beta}u\right)}$. Then,

$$H'\left(b\right) = \frac{q\left(b-d\right)\int_{-c}^{b} q\left(u\right) \, \mathrm{d}u - q\left(b\right)\int_{-c}^{b-d} q\left(u\right) \, \mathrm{d}u}{\left[\int_{-c}^{b} q\left(u\right) \, \mathrm{d}u\right]^2}.$$

Using

$$\int_{-c}^{b} q\left(u\right) \, \mathrm{d}u = \int_{-c}^{b-d} q\left(u\right) \, \mathrm{d}u + \int_{b-d}^{b} q\left(u\right) \, \mathrm{d}u,$$

we obtain

$$H'\left(b\right) = \frac{\left(q\left(b-d\right) - q\left(b\right)\right)\int_{-c}^{b-d} q\left(u\right) \, \mathrm{d}u + q\left(b-d\right)\int_{b-d}^{b} q\left(u\right) \, \mathrm{d}u}{\left[\int_{-c}^{b} q\left(u\right) \, \mathrm{d}u\right]^2}.$$

Since $\Phi\left(\sqrt{\beta}u\right)$ is strictly increasing in u, $q\left(u\right)$ is strictly decreasing, so $q\left(b-d\right) > q\left(b\right)$. Therefore, $H'\left(b\right) > 0$; that is, $H\left(b\right)$ is strictly increasing in b.

Since $\hat{s}_1 > \hat{s}_2$, we have $\theta^U - \hat{s}_1 < \theta^U - \hat{s}_2$, and hence $G_\theta\left(\theta \mid m; \hat{s}_1\right) < G_\theta\left(\theta \mid m; \hat{s}_2\right)$ for all $\theta \in \left[\hat{s}_1 - c, \theta^U\right)$. If $\theta < \hat{s}_1 - c$, then $G_\theta\left(\theta \mid m; \hat{s}_1\right) = 0 \leq G_\theta\left(\theta \mid m; \hat{s}_2\right)$. Therefore $G_\theta\left(\cdot \mid m; \hat{s}_1\right)$ first-order stochastically dominates $G_\theta\left(\cdot \mid m; \hat{s}_2\right)$.

Since $P\left(\theta\right)$ is weakly increasing in θ, this implies $\mathbb{E}_g[P\left(\theta\right) \mid m; \hat{s}_1] \geq \mathbb{E}_g[P\left(\theta\right) \mid m; \hat{s}_2]$. The inequality is strict because the first-order stochastic dominance relation is strict and the posterior support contains an interval of positive length below θ^U, on which $P\left(\theta\right)$ is strictly increasing. Hence $\mathbb{E}_g[P\left(\theta\right) \mid m; \hat{s}_1] > \mathbb{E}_g[P\left(\theta\right) \mid m; \hat{s}_2]$.

Q.E.D.

Proof of Lemma 2:

Fix $\hat{s}$ and an observed pair (m, L). Consider first the case $L = r_1$. By equation (8), for any $\ell \in [0, mr_1] \cup \{mr_1 - 1\}$, the policymaker's payoff is

$$u_g(\ell \mid m, L = r_1) = (R - 1) - \frac{mr_1 - \ell}{r_1}(R - r_1) - \rho m.$$

Because $R > r_1$, this expression is strictly increasing in ℓ. Hence, the unique optimal intervention is the largest feasible choice. That is, $\ell = mr_1$.

Next consider the case $L = 1$. For any liquidity injection $\ell \in [0, mr_1]$, equation (7) implies

$$\begin{aligned} u_g(\ell \mid m, L = 1; \hat{s}) = (1 - mr_1)\left(\mathbb{E}_g[P(\theta) \mid m, L = 1; \hat{s}]\, R - 1\right) \\ + \ell\left(\mathbb{E}_g[P(\theta) \mid m, L = 1; \hat{s}]\, R - 1\right) - \rho m. \end{aligned}$$

Thus, conditional on $L = 1$, the policymaker's payoff is affine in ℓ on $[0, mr_1]$ with slope $\mathbb{E}_g[P(\theta) \mid m, L = 1; \hat{s}]\, R - 1$.

Hence, if $\mathbb{E}_g[P(\theta) \mid m, L = 1; \hat{s}] \geq 1/R$, the payoff is strictly increasing in ℓ, so the best liquidity injection is $\ell = mr_1$. Its payoff is

$$u_g(mr_1 \mid m, L = 1; \hat{s}) = \mathbb{E}_g[P(\theta) \mid m, L = 1; \hat{s}]\, R - 1 - \rho m.$$

By contrast, under forced liquidation, $\ell = mr_1 - 1$, all investment is liquidated and the policymaker's payoff is

$$u_g(mr_1 - 1 \mid m, L = 1; \hat{s}) = -\rho m.$$

Since $\mathbb{E}_g[P(\theta) \mid m, L = 1; \hat{s}]\, R - 1 \geq 0$, it follows that

$$u_g(mr_1 \mid m, L = 1; \hat{s}) \geq u_g(mr_1 - 1 \mid m, L = 1; \hat{s}).$$

Hence $\ell = mr_1$ is optimal.

In contrast, if $\mathbb{E}_g[P(\theta) \mid m, L = 1; \hat{s}] < 1/R$, the payoff is strictly decreasing in ℓ, so the best liquidity injection is $\ell = 0$. Its payoff is

$$u_g(0 \mid m, L = 1; \hat{s}) = (1 - mr_1)\left(\mathbb{E}_g[P(\theta) \mid m, L = 1; \hat{s}]\, R - 1\right) - \rho m.$$

Because $m \leq \bar{m} < 1/r_1$, we have $1 - mr_1 > 0$. Since $\mathbb{E}_g[P(\theta) \mid m, L = 1; \hat{s}]\, R - 1 < 0$, this implies

$$u_g(0 \mid m, L = 1; \hat{s}) < -\rho m = u_g(mr_1 - 1 \mid m, L = 1; \hat{s}).$$

Therefore forced liquidation strictly dominates any liquidity injection, and the optimal intervention is $\ell = mr_1 - 1$.

Q.E.D.

Proof of Corollary 2:

Fix $\hat{s} \in \mathbb{R}$ and suppose that $L = 1$. By Lemma 2, after observing m the policymaker chooses full injection if and only if $\mathbb{E}_g[P(\theta) \mid m; \hat{s}] \geq \frac{1}{R}$, and chooses forced liquidation otherwise.

By part (i) of Proposition 2, for fixed $\hat{s}$, $\mathbb{E}_g[P(\theta) \mid m; \hat{s}]$ is weakly decreasing in m on $[\underline{m}(\hat{s}), \bar{m}]$, strictly decreasing on $(\underline{m}(\hat{s}), \tilde{m}(\hat{s}))$, and constant on $[\tilde{m}(\hat{s}), \bar{m}]$. Therefore, the set of withdrawal realizations for which full injection is optimal is in a cutoff form in which the threshold $\hat{m}^{BR}(\hat{s})$ is defined in equation (26).

We now characterize $\hat{m}^{BR}(\hat{s})$. If $\mathbb{E}_g[P(\theta) \mid \underline{m}(\hat{s}); \hat{s}] \leq \frac{1}{R}$, then the weak monotonicity in m implies $\mathbb{E}_g[P(\theta) \mid m; \hat{s}] \leq \frac{1}{R}$, for all feasible m. Hence, full injection is never optimal for any feasible interior realization of m, so $\hat{m}^{BR}(\hat{s}) = \underline{m}(\hat{s})$.

Now assumethat $\mathbb{E}_g[P(\theta) \mid \tilde{m}(\hat{s}); \hat{s}] \geq \frac{1}{R}$. Since $\mathbb{E}_g[P(\theta) \mid m; \hat{s}]$ is constant on $[\tilde{m}(\hat{s}), \bar{m}]$ and weakly larger for smaller m, we have $\mathbb{E}_g[P(\theta) \mid m; \hat{s}] \geq \frac{1}{R}$ for all feasible m. Hence, full injection is optimal for every feasible realization of m, and $\hat{m}^{BR}(\hat{s}) = \bar{m}$.

Finally, suppose that

$$\mathbb{E}_g[P(\theta) \mid \underline{m}(\hat{s}); \hat{s}] > \frac{1}{R} \quad \text{and} \quad \mathbb{E}_g[P(\theta) \mid \tilde{m}(\hat{s}); \hat{s}] < \frac{1}{R}.$$

By equation (23), the posterior belief varies continuously with m on $(\underline{m}(\hat{s}), \tilde{m}(\hat{s}))$. Hence, $\mathbb{E}_g[P(\theta) \mid m; \hat{s}]$ is continuous on that interval. Since it is also strictly decreasing there by part (i) of Proposition 2, the intermediate value theorem implies that there exists a unique $\hat{m}^{BR}(\hat{s}) \in (\underline{m}(\hat{s}), \tilde{m}(\hat{s}))$ such that $\mathbb{E}_g\left[P(\theta) \mid \hat{m}^{BR}(\hat{s}); \hat{s}\right] = \frac{1}{R}$.

Q.E.D.

Proof of Lemma 3:

Fix $\hat{s} \in \mathbb{R}$ and consider the posterior at $m = \tilde{m}(\hat{s})$. By the definition of $\tilde{m}(\hat{s})$, $\theta_L(\tilde{m}(\hat{s}); \hat{s}) = -\Theta$. Hence, equation (23) implies that the posterior support is $\left[-\Theta, \theta^U\right)$, and the posterior density is proportional to

$$\frac{1}{1 - n(\theta; \hat{s})} = \frac{1}{\Phi\left(\sqrt{\beta}(\theta - \hat{s})\right)}.$$

Let

$$q(\theta; \hat{s}) \equiv \frac{1}{\Phi\left(\sqrt{\beta}(\theta - \hat{s})\right)}.$$

Then

$$\mathbb{E}_g[P(\theta) \mid \tilde{m}(\hat{s}); \hat{s}] = \frac{\int_{-\Theta}^{\theta^U} P(\theta)\, q(\theta; \hat{s})\, \mathrm{d}\theta}{\int_{-\Theta}^{\theta^U} q(\theta; \hat{s})\, \mathrm{d}\theta}.$$

Because $P(\theta) = 0$ for all $\theta \leq \theta^L$ and $P(\theta) \leq 1$ for all θ, we have

$$\int_{-\Theta}^{\theta^U} P(\theta)\ q(\theta;\hat{s})\ \mathrm{d}\theta \leq \int_{\theta^L}^{\theta^U} q(\theta;\hat{s})\ \mathrm{d}\theta.$$

Also,

$$\int_{-\Theta}^{\theta^U} q(\theta;\hat{s})\ \mathrm{d}\theta \geq \int_{-\Theta}^{\theta^L} q(\theta;\hat{s})\ \mathrm{d}\theta.$$

Therefore,

$$\mathbb{E}_g[P(\theta) \mid \tilde{m}(\hat{s});\hat{s}] \leq \frac{\int_{\theta^L}^{\theta^U} q(\theta;\hat{s})\ \mathrm{d}\theta}{\int_{-\Theta}^{\theta^L} q(\theta;\hat{s})\ \mathrm{d}\theta}.$$

Now $q(\theta;\hat{s})$ is strictly decreasing in θ because $\Phi\left(\sqrt{\beta}(\theta - \hat{s})\right)$ is strictly increasing in θ. Hence

$$\int_{\theta^L}^{\theta^U} q(\theta;\hat{s})\ \mathrm{d}\theta \leq \left(\theta^U - \theta^L\right) q\left(\theta^L;\hat{s}\right),$$

and

$$\int_{-\Theta}^{\theta^L} q(\theta;\hat{s})\ \mathrm{d}\theta \geq \left(\Theta + \theta^L\right) q\left(\theta^L;\hat{s}\right).$$

Combining the two inequalities yields

$$\mathbb{E}_g[P(\theta) \mid \tilde{m}(\hat{s});\hat{s}] \leq \frac{\theta^U - \theta^L}{\Theta + \theta^L}.$$

By the assumption

$$\Theta > R\left(\theta^U - \theta^L\right) - \theta^L,$$

we have

$$\frac{\theta^U - \theta^L}{\Theta + \theta^L} < \frac{1}{R}.$$

Therefore,

$$\mathbb{E}_g[P(\theta) \mid \tilde{m}(\hat{s});\hat{s}] < \frac{1}{R}. \tag{45}$$

Now, by part (i) of Proposition 2, we have $\mathbb{E}_g[P(\theta) \mid m;\hat{s}]$ is strictly decreasing in m on $(\underline{m}(\hat{s}), \tilde{m}(\hat{s}))$ and constant on $[\tilde{m}(\hat{s}), \bar{m}]$. Then, equation (45) implies that the policymaker does not inject for any $m \in [\tilde{m}(\hat{s}), \bar{m}]$.

On the other hand, as m approaches $\underline{m}(\hat{s})$ from right, the posterior support shrinks to θ^U, so

$$\mathbb{E}_g[P(\theta) \mid m;\hat{s}] \to P\left(\theta^U\right) = 1 > \frac{1}{R}.$$

Hence, by continuity and strict monotonicity on $(\underline{m}(\hat{s}), \tilde{m}(\hat{s}))$, there exists a unique $\hat{m}^{BR}(\hat{s}) \in (\underline{m}(\hat{s}), \tilde{m}(\hat{s}))$ such that $\mathbb{E}_g\left[P(\theta) \mid \hat{m}^{BR}(\hat{s}); \hat{s}\right] = \frac{1}{R}$.

Q.E.D.

Proof of Lemma 4:

The argument is the same as in the benchmark case with deposit insurance alone. The only difference is that, after withdrawals are realized, the policymaker follows the optimal $t = 1$ intervention rule characterized in Section 4.1. Because the signal L is observed only after depositors choose whether to withdraw, this modification does not change the global games structure of the run stage.

Fix a deposit insurance level K and suppose that other depositors follow a cutoff strategy with threshold $\hat{s}$. Given the policymaker's best responding $t = 1$ intervention rule, a patient depositor's expected incremental payoff from staying is $V(s; K, \hat{s})$ as defined in equation (30). As in the benchmark model, a higher private signal s shifts the posterior distribution of θ upward in the sense of first-order stochastic dominance. Since the payoff from staying is weakly increasing in fundamentals, it follows that $V(s; K, \hat{s})$ is strictly increasing in s. Hence the depositor's best response is characterized by a cutoff rule.

If $K = r_1$, then staying guarantees at least r_1, whereas withdrawing yields exactly r_1. Since we assume a tie-breaking rule in favor of staying, patient depositors never run, so the equilibrium cutoff is $\hat{s} = -\infty$.

Now fix $K \in [0, r_1)$. For sufficiently low signals, the depositor assigns arbitrarily high posterior probability to sufficiently weak fundamentals. In those states, staying yields at most $K < r_1$, so withdrawing is strictly optimal. For sufficiently high signals, the depositor assigns arbitrarily high posterior probability to sufficiently strong fundamentals. In those states, staying yields $r_2 > r_1$, so staying is strictly optimal. Therefore, for any fixed conjectured cutoff $\hat{s}$, the depositor's best response is itself a cutoff strategy.

The remainder of the proof follows from the same standard global-games argument used in the proof of Lemma 1. In particular, because the depositor's payoff satisfies the same monotonicity and single-crossing properties as in the benchmark, given any $K \in [0, r_1)$, the fixed-point equation

$$V(\hat{s}; K, \hat{s}) = 0$$

admits a unique solution. This yields the unique threshold-strategy equilibrium in equation (31).

Finally, a higher deposit insurance level K raises the payoff from staying in every state and strictly raises it in a set of states with positive probability. Hence $V(\hat{s}; K, \hat{s})$ is strictly increasing in K for every fixed $\hat{s}$. Since $V(\hat{s}; K, \hat{s})$ is strictly increasing in $\hat{s}$ at the unique equilibrium, the unique cutoff $\hat{s}$ is strictly decreasing in K.

Q.E.D.

Proof of Corollary 3:

Let $\hat{s}_0 \equiv \hat{s}(0)$ denote the unique cutoff when $K = 0$, equivalently the unique solution to $V(\hat{s}_0; 0, \hat{s}_0) = 0$. Take any target threshold $s^* \in [-\infty, \hat{s}_0]$. If $s^* = -\infty$, choose $K = r_1$ and then $\hat{s}(K) = \hat{s}(r_1) = -\infty$.

Now take $s^* \in (-\infty, \hat{s}_0]$. Since $\hat{s}(K)$ is strictly decreasing in K and $\lim_{K \to r_1} \hat{s}(K) = -\infty$, there exists $K \in [0, r_1)$ such that $\hat{s}(K) = s^*$.

Suppose $K_1, K_2 \in [0, r_1]$ both implement the same cutoff s^*, i.e., $\hat{s}(K_1) = \hat{s}(K_2) = s^*$. If $s^* = -\infty$, then necessarily $K_1 = K_2 = r_1$ because for any $K < r_1$ Lemma 4 yields a finite cutoff. If $s^* > -\infty$, then $\hat{s}(K)$ is strictly decreasing in K.

Therefore, for each $s^* \in [-\infty, \hat{s}_0]$ there exists a unique $K \in [0, r_1]$ such that the equilibrium run threshold satisfies $\hat{s} = s^*$.

Q.E.D.

Proof of Proposition 3:

For $\hat{s} \in [-\infty, \hat{s}_0]$, define

$$W(\hat{s}; \rho) \equiv \mathbb{E}_g\left[u_g(\theta, \lambda; \hat{s})\right] = A(\hat{s}) - \rho B(\hat{s}),$$

where

$$A(\hat{s}) \equiv \mathbb{E}_g\left[u_g(\theta, \lambda; \hat{s})\right]\Big|_{\rho=0}, \qquad B(\hat{s}) \equiv \mathbb{E}_g[m(\lambda, \theta; \hat{s})].$$

By Corollary 3, choosing a deposit insurance policy $K \in [0, r_1]$ is equivalent to choosing an induced run threshold $\hat{s} \in [-\infty, \hat{s}_0]$.

Given Lemma 3, for each finite $\hat{s}$, we can define

$$c(\hat{s}) \equiv \frac{\hat{m}^{BR}(\hat{s})}{\bar{m}} \in (0,1) \quad \text{and} \quad \theta^m(\hat{s}) \equiv \hat{s} - \frac{1}{\sqrt{\beta}}\Phi^{-1}(c(\hat{s})),$$

so that $n\left(\theta^{m}\left(\hat{s}\right);\hat{s}\right)=c\left(\hat{s}\right)$. Since $\hat{m}^{BR}\left(\hat{s}\right)\in\left(\underline{m}\left(\hat{s}\right),\bar{m}\right)$ by Lemma 3, we have $\theta^{m}\left(\hat{s}\right)\in\left(-\Theta,\theta^{U}\right)$ for every finite $\hat{s}$.

Conditional on $\theta<\theta^{U}$, the policymaker injects full liquidity if and only if $m\left(\lambda,\theta;\hat{s}\right)\leq\hat{m}^{BR}\left(\hat{s}\right)$, which is equivalent to

$$\lambda\leq\frac{c\left(\hat{s}\right)-n\left(\theta;\hat{s}\right)}{1-n\left(\theta;\hat{s}\right)}.$$

Hence the probability of support for a bank with fundamentals θ is

$$\Lambda\left(\theta;\hat{s}\right)=\begin{cases}0, & \text{if }\theta\leq\theta^{m}\left(\hat{s}\right),\\ \dfrac{c\left(\hat{s}\right)-n\left(\theta;\hat{s}\right)}{1-n\left(\theta;\hat{s}\right)}, & \text{if }\theta>\theta^{m}\left(\hat{s}\right).\end{cases}\tag{46}$$

When $\hat{s}=-\infty$, patient depositors never run. Since $\Theta>\hat{\Theta}\equiv R\left(\theta^{U}-\theta^{L}\right)-\theta^{L}$, the expected NPV is negative, and hence the policymaker forces liquidation whenever $L=1$. Since the welfare is zero when the policymaker chooses to force liquidation, the ex-ante welfare comes only from the states with $L=r_{1}$. Therefore

$$A\left(-\infty\right)=\frac{\Theta-\theta^{U}}{2\Theta}\left(R-1\right).$$

Then, for any finite $\hat{s}$,

$$A\left(\hat{s}\right)-A\left(-\infty\right)=\frac{1}{2\Theta}\int_{-\Theta}^{\theta^{U}}\left(RP\left(\theta\right)-1\right)\Lambda\left(\theta;\hat{s}\right)\,\mathrm{d}\theta.$$

By the definition of $\hat{m}^{BR}\left(\hat{s}\right)$, the policymaker is indifferent at $m=\hat{m}^{BR}\left(\hat{s}\right)$, so

$$\mathbb{E}_{g}\left[P\left(\theta\right)\mid\hat{m}^{BR}\left(\hat{s}\right);\hat{s},L=1\right]=\frac{1}{R}.$$

Using the posterior density in Proposition 2, this is equivalent to

$$\int_{\theta^{m}\left(\hat{s}\right)}^{\theta^{U}}\frac{RP\left(x\right)-1}{1-n\left(x;\hat{s}\right)}\,\mathrm{d}x=0.\tag{47}$$

Then, equation (46) and equation (47) imply

$$\begin{aligned}A\left(\hat{s}\right)-A\left(-\infty\right)&=\frac{1}{2\Theta}\int_{\theta^{m}\left(\hat{s}\right)}^{\theta^{U}}\left(RP\left(\theta\right)-1\right)\frac{c\left(\hat{s}\right)-n\left(\theta;\hat{s}\right)}{1-n\left(\theta;\hat{s}\right)}\,\mathrm{d}\theta\\&=\frac{1}{2\Theta}\int_{\theta^{m}\left(\hat{s}\right)}^{\theta^{U}}\left(RP\left(\theta\right)-1\right)\left[1-\frac{1-c\left(\hat{s}\right)}{1-n\left(\theta;\hat{s}\right)}\right]\mathrm{d}\theta\\&=\frac{1}{2\Theta}\int_{\theta^{m}\left(\hat{s}\right)}^{\theta^{U}}\left(RP\left(\theta\right)-1\right)\,\mathrm{d}\theta-\frac{1-c\left(\hat{s}\right)}{2\Theta}\int_{\theta^{m}\left(\hat{s}\right)}^{\theta^{U}}\frac{RP\left(\theta\right)-1}{1-n\left(\theta;\hat{s}\right)}\,\mathrm{d}\theta\\&=\frac{1}{2\Theta}\int_{\theta^{m}\left(\hat{s}\right)}^{\theta^{U}}\left(RP\left(\theta\right)-1\right)\,\mathrm{d}\theta.\end{aligned}\tag{48}$$

Therefore,

$$A(\hat{s}) = A(-\infty) + \frac{1}{2\Theta}\int_{\theta^m(\hat{s})}^{\theta^U} (RP(\theta) - 1)\, \mathrm{d}\theta. \tag{49}$$

Now define

$$H(\hat{s}, \theta) \equiv \int_{\theta}^{\theta^U} \frac{RP(x) - 1}{\Phi\left(\sqrt{\beta}(x - \hat{s})\right)}\, \mathrm{d}x.$$

We will apply the Implicit Function Theorem on the equation $H(\hat{s}, \theta^m(\hat{s})) = 0$. In particular,

$$\frac{\mathrm{d}}{\mathrm{d}\theta} H(\hat{s}, \theta) = -\frac{RP(\theta) - 1}{\Phi\left(\sqrt{\beta}(\theta - \hat{s})\right)}.$$

At $\theta = \theta^m(\hat{s})$, we must have $RP(\theta^m(\hat{s})) - 1 < 0$; otherwise, $H(\hat{s}, \theta^m(\hat{s})) > 0$, contradicting $H(\hat{s}, \theta^m(\hat{s})) = 0$. Hence

$$\frac{\mathrm{d}}{\mathrm{d}\theta} H(\hat{s}, \theta) > 0.$$

Therefore, by the implicit function theorem, $\theta^m(\hat{s})$ is continuously differentiable on $(-\infty, \hat{s}_0]$.

Differentiating $H(\hat{s}, \theta)$ with respect to $\hat{s}$,

$$\frac{\mathrm{d}}{\mathrm{d}\hat{s}} H(\hat{s}, \theta) = \int_{\theta}^{\theta^U} (RP(x) - 1) \frac{\sqrt{\beta}\,\phi\left(\sqrt{\beta}(x - \hat{s})\right)}{\Phi\left(\sqrt{\beta}(x - \hat{s})\right)^2}\, \mathrm{d}x.$$

Evaluating at $\theta = \theta^m(\hat{s})$, define

$$G(x; \hat{s}) \equiv \int_{\theta^m(\hat{s})}^{x} \frac{RP(u) - 1}{\Phi\left(\sqrt{\beta}(u - \hat{s})\right)}\, \mathrm{d}u, \qquad x \in \left[\theta^m(\hat{s}), \theta^U\right].$$

Then

$$G(\theta^m(\hat{s}); \hat{s}) = 0 \quad \text{and} \quad G\left(\theta^U; \hat{s}\right) = 0.$$

Moreover, since $RP(u) - 1$ is strictly increasing in u and $RP(\theta^m(\hat{s})) - 1 < 0 < R - 1$, there exists a unique $\theta^0 = P^{-1}(1/R) \in \left(\theta^m(\hat{s}), \theta^U\right)$ such that

$$RP(u) - 1 < 0 \quad \text{for } u < \theta^0, \qquad RP(u) - 1 > 0 \quad \text{for } u > \theta^0.$$

Because the denominator in the definition of G is strictly positive, $G(x; \hat{s})$ first decreases and then increases. Since it is zero at both endpoints, we have

$$G(x; \hat{s}) \leq 0 \qquad \text{for all } x \in \left[\theta^m(\hat{s}), \theta^U\right].$$

Also define

$$g(x; \hat{s}) \equiv \frac{\sqrt{\beta}\,\phi\left(\sqrt{\beta}(x - \hat{s})\right)}{\Phi\left(\sqrt{\beta}(x - \hat{s})\right)}.$$

Then

$$\frac{\mathrm{d}}{\mathrm{d}\hat{s}}H\left(\hat{s},\theta^{m}\left(\hat{s}\right)\right)=\int_{\theta^{m}(\hat{s})}^{\theta^{U}}g\left(x;\hat{s}\right)\left(\frac{\mathrm{d}}{\mathrm{d}x}G\left(x;\hat{s}\right)\right)\mathrm{d}x=-\int_{\theta^{m}(\hat{s})}^{\theta^{U}}g_{x}\left(x;\hat{s}\right)\,G\left(x;\hat{s}\right)\,\mathrm{d}x.$$

by integrating by parts and using $G\left(\theta^{m}\left(\hat{s}\right);\hat{s}\right)=G\left(\theta^{U};\hat{s}\right)=0$. Finally, $\frac{\mathrm{d}}{\mathrm{d}x}g\left(x;\hat{s}\right)<0$ for all x, because of the decreasing inverse Mills ratio. So, the fact that $G\left(x;\hat{s}\right)\leq 0$ for all x and $G\not\equiv 0$ implies $\frac{\mathrm{d}}{\mathrm{d}\hat{s}}H\left(\hat{s},\theta^{m}\left(\hat{s}\right)\right)<0$. Therefore, $\left(\theta^{m}\right)'\left(\hat{s}\right)>0$ by the Implicit Function Theorem.

Therefore, differentiating equation (49) gives

$$A'\left(\hat{s}\right)=-\frac{1}{2\Theta}\left(RP\left(\theta^{m}\left(\hat{s}\right)\right)-1\right)\left(\theta^{m}\right)'\left(\hat{s}\right)>0.$$

That is, $A\left(\hat{s}\right)$ is continuously differentiable and strictly increasing on $[-\infty,\hat{s}_{0}]$.

Moreover, $B\left(\hat{s}\right)$ is also continuously differentiable and strictly increasing on $[-\infty,\hat{s}_{0}]$. Indeed,

$$B\left(\hat{s}\right)=\frac{\bar{m}}{2}+\frac{\bar{m}}{4\Theta}\int_{-\Theta}^{\Theta}\Phi\Big(\sqrt{\beta}\left(\hat{s}-\theta\right)\Big)\,\mathrm{d}\theta,$$

so

$$B'\left(\hat{s}\right)=\frac{\bar{m}\sqrt{\beta}}{4\Theta}\int_{-\Theta}^{\Theta}\phi\Big(\sqrt{\beta}\left(\hat{s}-\theta\right)\Big)\,\mathrm{d}\theta>0.$$

Recall the definitions

$$\mathcal{R}_{0}\equiv\left\{\rho\geq 0:\hat{s}_{0}\text{ is a maximizer of }W\left(\hat{s};\rho\right)\text{ on }\left[-\infty,\hat{s}_{0}\right]\right\},$$

and

$$\mathcal{R}_{\infty}\equiv\left\{\rho\geq 0:-\infty\text{ is a maximizer of }W\left(\hat{s};\rho\right)\text{ on }\left[-\infty,\hat{s}_{0}\right]\right\}.$$

Then $\rho_{1}^{\mathrm{PP}}=\sup\mathcal{R}_{0}$ and $\rho_{2}^{\mathrm{PP}}=\inf\mathcal{R}_{\infty}$.

We first establish the low-ρ region. Define

$$\bar{\rho}_{1}\equiv\inf_{\hat{s}\in[-\infty,\hat{s}_{0})}\frac{A\left(\hat{s}_{0}\right)-A\left(\hat{s}\right)}{B\left(\hat{s}_{0}\right)-B\left(\hat{s}\right)}.$$

Because A and B are both strictly increasing, the ratio is well defined and strictly positive for every $\hat{s}<\hat{s}_{0}$. We next show that $\bar{\rho}_{1}>0$.

Pick any finite $\underline{s}<\hat{s}_{0}$. First consider $\hat{s}\leq\underline{s}$. Since A is strictly increasing,

$$A\left(\hat{s}_{0}\right)-A\left(\hat{s}\right)\geq A\left(\hat{s}_{0}\right)-A\left(\underline{s}\right)>0.$$

Also, since B is increasing and bounded,

$$B\left(\hat{s}_{0}\right)-B\left(\hat{s}\right)\leq B\left(\hat{s}_{0}\right)-B\left(-\infty\right)<\infty.$$

Hence, for every $\hat{s} \leq \underline{s}$,

$$\frac{A(\hat{s}_0) - A(\hat{s})}{B(\hat{s}_0) - B(\hat{s})} \geq \frac{A(\hat{s}_0) - A(\underline{s})}{B(\hat{s}_0) - B(-\infty)} > 0.$$

Next consider $\hat{s} \in [\underline{s}, \hat{s}_0)$. By the Cauchy mean value theorem, for each such $\hat{s}$ there exists $\xi \in (\hat{s}, \hat{s}_0) \subset (\underline{s}, \hat{s}_0)$ such that

$$\frac{A(\hat{s}_0) - A(\hat{s})}{B(\hat{s}_0) - B(\hat{s})} = \frac{A'(\xi)}{B'(\xi)}.$$

Since A' and B' are continuous and strictly positive on the compact interval $[\underline{s}, \hat{s}_0]$, we have

$$\inf_{\xi \in [\underline{s}, \hat{s}_0]} \frac{A'(\xi)}{B'(\xi)} > 0.$$

Therefore,

$$\inf_{\hat{s} \in [\underline{s}, \hat{s}_0)} \frac{A(\hat{s}_0) - A(\hat{s})}{B(\hat{s}_0) - B(\hat{s})} > 0.$$

Combining the two regions shows that $\bar{\rho}_1 > 0$. Moreover, for every $\rho < \bar{\rho}_1$ and every $\hat{s} < \hat{s}_0$,

$$A(\hat{s}_0) - A(\hat{s}) > \rho\Big(B(\hat{s}_0) - B(\hat{s})\Big),$$

or equivalently,

$$W(\hat{s}_0; \rho) > W(\hat{s}; \rho).$$

Hence $\hat{s}_0$ is the unique maximizer of $W(\hat{s}; \rho)$ on $[-\infty, \hat{s}_0]$ whenever $\rho < \bar{\rho}_1$.

We next establish the high-ρ region. Recall the definitions of $c(\hat{s})$, $\theta^m(\hat{s})$, and $\Lambda(\theta; \hat{s})$ above. Because $\Lambda(\theta; \hat{s}) = 0$ for $\theta \leq \theta^m(\hat{s})$, $0 \leq \Lambda(\theta; \hat{s}) \leq c(\hat{s})$ for $\theta > \theta^m(\hat{s})$, and $RP(\theta) - 1 \leq R - 1$, we obtain

$$A(\hat{s}) - A(-\infty) \leq \frac{R-1}{2\Theta} c(\hat{s}) \left(\theta^U - \theta^m(\hat{s})\right). \tag{50}$$

We next show that $\theta^m(\hat{s})$ is uniformly bounded from below. Let

$$f(\theta) \equiv RP(\theta) - 1, \qquad w(\theta) \equiv \frac{1}{1 - n(\theta; \hat{s})}.$$

Then f is strictly increasing in θ, and w is strictly decreasing in θ. Let $\theta^0 \equiv P^{-1}(1/R)$, so that $f(\theta^0) = 0$. Equation (47) can be rewritten as

$$0 = w(\theta^0) \int_{\theta^m(\hat{s})}^{\theta^U} f(\theta)\, d\theta + \int_{\theta^m(\hat{s})}^{\theta^U} f(\theta) \left(w(\theta) - w(\theta^0)\right) d\theta.$$

The second integral is strictly negative: on $[\theta^m(\hat{s}), \theta^0)$, $f(\theta) < 0$ and $w(\theta) - w(\theta^0) > 0$; on $(\theta^0, \theta^U]$, $f(\theta) > 0$ and $w(\theta) - w(\theta^0) < 0$. Hence

$$\int_{\theta^m(\hat{s})}^{\theta^U} f(\theta)\, d\theta > 0.$$

Since $P(\theta) = 0$ for $\theta \leq \theta^L$ and $P(\theta) \leq 1$ for all θ, we have $f(\theta) = -1$ for $\theta \leq \theta^L$, and $f(\theta) \leq R - 1$ for all θ. Therefore,

$$0 < \int_{\theta^m(\hat{s})}^{\theta^U} f(\theta)\, \mathrm{d}\theta \leq -\max\{\theta^L - \theta^m(\hat{s}), 0\} + (R-1)\left(\theta^U - \theta^L\right).$$

It follows that

$$\theta^m(\hat{s}) \geq \theta^L - (R-1)\left(\theta^U - \theta^L\right). \tag{51}$$

Substituting (51) into (50) yields

$$A(\hat{s}) - A(-\infty) \leq \frac{R(R-1)\left(\theta^U - \theta^L\right)}{2\Theta} c(\hat{s}). \tag{52}$$

We next bound $B(\hat{s}) - B(-\infty)$ from below. Since

$$B(\hat{s}) - B(-\infty) = \frac{\bar{m}}{4\Theta} \int_{-\Theta}^{\Theta} n(\theta; \hat{s})\, \mathrm{d}\theta,$$

and since $n(\theta; \hat{s})$ is decreasing in θ with $n(\theta^m(\hat{s}); \hat{s}) = c(\hat{s})$, we have $n(\theta; \hat{s}) \geq c(\hat{s})$ for all $\theta \leq \theta^m(\hat{s})$. Thus

$$B(\hat{s}) - B(-\infty) \geq \frac{\bar{m}}{4\Theta} \int_{-\Theta}^{\theta^m(\hat{s})} n(\theta; \hat{s})\, \mathrm{d}\theta \geq \frac{\bar{m}}{4\Theta} c(\hat{s}) \left(\theta^m(\hat{s}) + \Theta\right).$$

Equation (51) implies

$$B(\hat{s}) - B(-\infty) \geq \frac{\bar{m}}{4\Theta} c(\hat{s}) \left(\Theta + \theta^L - (R-1)\left(\theta^U - \theta^L\right)\right). \tag{53}$$

Because, when $\Theta > \hat{\Theta} \equiv R\left(\theta^U - \theta^L\right) - \theta^L$, the term in parentheses in (53) is strictly positive.

Combining (52) and (53), we conclude that for every finite $\hat{s}$,

$$\frac{A(\hat{s}) - A(-\infty)}{B(\hat{s}) - B(-\infty)} \leq \frac{2R(R-1)\left(\theta^U - \theta^L\right)}{\bar{m}\left(\Theta + \theta^L - (R-1)\left(\theta^U - \theta^L\right)\right)} < \infty.$$

Hence, defining

$$\bar{\rho}_2 \equiv \sup_{\hat{s} \in [-\infty, \hat{s}_0)} \frac{A(\hat{s}) - A(-\infty)}{B(\hat{s}) - B(-\infty)},$$

we have

$$\bar{\rho}_2 < \infty.$$

Moreover, for every $\rho > \bar{\rho}_2$ and every finite $\hat{s} \in (-\infty, \hat{s}_0]$,

$$A(\hat{s}) - A(-\infty) < \rho\left(B(\hat{s}) - B(-\infty)\right),$$

or equivalently,

$$W(-\infty;\rho) > W(\hat{s};\rho).$$

Hence, $-\infty$ is the unique maximizer of $W(\hat{s};\rho)$ on $[-\infty,\hat{s}_0]$ whenever $\rho > \bar{\rho}_2$.

We now show that $\mathcal{R}_0$ is downward closed. Take any $\rho \in \mathcal{R}_0$ and any $\rho' \in [0,\rho)$. For every $\hat{s} < \hat{s}_0$,

$$W(\hat{s}_0;\rho') - W(\hat{s};\rho') = W(\hat{s}_0;\rho) - W(\hat{s};\rho) + (\rho - \rho')\Big(B(\hat{s}_0) - B(\hat{s})\Big).$$

Since $\rho \in \mathcal{R}_0$, the first term on the right-hand side is nonnegative. Since $B(\hat{s})$ is strictly increasing in $\hat{s}$, the second term is strictly positive. Therefore

$$W(\hat{s}_0;\rho') > W(\hat{s};\rho') \qquad \text{for all } \hat{s} < \hat{s}_0,$$

so $\rho' \in \mathcal{R}_0$. Hence $\mathcal{R}_0$ is downward closed.

Similarly, $\mathcal{R}_\infty$ is upward closed. Take any $\rho \in \mathcal{R}_\infty$ and any $\rho' > \rho$. For every finite $\hat{s} \in (-\infty,\hat{s}_0]$,

$$W(-\infty;\rho') - W(\hat{s};\rho') = W(-\infty;\rho) - W(\hat{s};\rho) + (\rho' - \rho)\Big(B(\hat{s}) - B(-\infty)\Big).$$

Since $\rho \in \mathcal{R}_\infty$, the first term is nonnegative. Also, $B(\hat{s})$ is strictly increasing in $\hat{s}$, so the second term is strictly positive. Therefore

$$W(-\infty;\rho') > W(\hat{s};\rho') \qquad \text{for all finite } \hat{s} \in (-\infty,\hat{s}_0],$$

so $\rho' \in \mathcal{R}_\infty$. Hence $\mathcal{R}_\infty$ is upward closed.

Since $[0,\bar{\rho}_1) \subseteq \mathcal{R}_0$, the set $\mathcal{R}_0$ is nonempty, and therefore $\rho_1^{\text{PP}} = \sup \mathcal{R}_0 > 0$. Similarly, since $(\bar{\rho}_2,\infty) \subseteq \mathcal{R}_\infty$, the set $\mathcal{R}_\infty$ is nonempty, and therefore $\rho_2^{\text{PP}} = \inf \mathcal{R}_\infty < \infty$.

We next show that $\rho_1^{\text{PP}} \leq \rho_2^{\text{PP}}$. Suppose not, then for every $\rho \in (\rho_2^{\text{PP}},\rho_1^{\text{PP}})$, the downward-closed property of $\mathcal{R}_0$ implies $\rho \in \mathcal{R}_0$, and the upward-closed property of $\mathcal{R}_\infty$ implies $\rho \in \mathcal{R}_\infty$. Hence, for every such ρ, both $\hat{s}_0$ and $-\infty$ are maximizers of $W(\hat{s};\rho)$. Therefore,

$$W(\hat{s}_0;\rho) - W(-\infty;\rho) = 0 \qquad \text{for all } \rho \in \left(\rho_2^{\text{PP}},\rho_1^{\text{PP}}\right).$$

But

$$W(\hat{s}_0;\rho) - W(-\infty;\rho) = \Big(A(\hat{s}_0) - A(-\infty)\Big) - \rho\Big(B(\hat{s}_0) - B(-\infty)\Big),$$

and since $B(\hat{s}_0) - B(-\infty) > 0$, $W(\hat{s}_0;\rho) - W(-\infty;\rho)$ cannot be constant in ρ on $(\rho_2^{\text{PP}},\rho_1^{\text{PP}})$. This contradiction establishes that $\rho_1^{\text{PP}} \leq \rho_2^{\text{PP}}$.

We now prove parts (i) and (ii). If $\rho < \rho_1^{\text{PP}}$, choose any $\tilde{\rho} \in \mathcal{R}_0$ such that $\rho < \tilde{\rho} \leq \rho_1^{\text{PP}}$. Then for every $\hat{s} < \hat{s}_0$,

$$W(\hat{s}_0; \rho) - W(\hat{s}; \rho) = W(\hat{s}_0; \tilde{\rho}) - W(\hat{s}; \tilde{\rho}) + (\tilde{\rho} - \rho)\Big(B(\hat{s}_0) - B(\hat{s})\Big).$$

Since $\tilde{\rho} \in \mathcal{R}_0$, the first term is nonnegative, and the second term is strictly positive. Hence

$$W(\hat{s}_0; \rho) > W(\hat{s}; \rho) \qquad \text{for all } \hat{s} < \hat{s}_0.$$

Therefore $\hat{s}_0$ is the unique maximizer. By Corollary 3, this implies that the unique optimal deposit insurance policy is $K^* = 0$, and the induced equilibrium run threshold is $\hat{s}^* = \hat{s}_0$. This proves part (i).

If $\rho > \rho_2^{\text{PP}}$, choose any $\tilde{\rho} \in \mathcal{R}_\infty$ such that $\rho_2^{\text{PP}} \leq \tilde{\rho} < \rho$. Then for every finite $\hat{s} \in (-\infty, \hat{s}_0]$,

$$W(-\infty; \rho) - W(\hat{s}; \rho) = W(-\infty; \tilde{\rho}) - W(\hat{s}; \tilde{\rho}) + (\rho - \tilde{\rho})\Big(B(\hat{s}) - B(-\infty)\Big).$$

Since $\tilde{\rho} \in \mathcal{R}_\infty$, the first term is nonnegative, and the second term is strictly positive. Hence

$$W(-\infty; \rho) > W(\hat{s}; \rho) \qquad \text{for all finite } \hat{s} \in (-\infty, \hat{s}_0].$$

Therefore $-\infty$ is the unique maximizer. By Corollary 3, this implies that the unique optimal deposit insurance policy is $K^* = r_1$, and the induced equilibrium run threshold is $\hat{s}^* = -\infty$. This proves part (ii).

Finally, suppose that $\rho_1^{\text{PP}} < \rho_2^{\text{PP}}$ and $\rho \in (\rho_1^{\text{PP}}, \rho_2^{\text{PP}})$. By the definition of ρ_1^{PP}, $\hat{s}_0$ is not a maximizer of $W(\hat{s}; \rho)$ on $[-\infty, \hat{s}_0]$. Similarly, by the definition of ρ_2^{PP}, $-\infty$ is not a maximizer of $W(\hat{s}; \rho)$ on $[-\infty, \hat{s}_0]$. Hence no optimal threshold can lie at either corner. Therefore every maximizer satisfies $\hat{s}^* \in (-\infty, \hat{s}_0)$. By Corollary 3, every optimal deposit insurance policy is interior: $K^* \in (0, r_1)$. This proves part (iii).

In all cases, conditional on the induced run threshold $\hat{s}^*$, the policymaker's optimal $t = 1$ intervention follows from Lemma 2 and Corollary 2.

Q.E.D.

Proof of Proposition 4:

We first prove part (i). Take any $\rho \in (0, \rho_1^{\text{PP}})$. By part (i) of Proposition 3,

$$K_{PP}^*(\rho) = 0.$$

On the other hand, Proposition 1 implies that the benchmark optimal deposit insurance is strictly positive for every $\rho > 0$. Indeed, if $\rho \in \left(0, \rho^{\text{DI}}\right)$, then $K^*_{DI}(\rho) \in (0, r_1)$. Hence, $K^*_{PP}(\rho) = 0 < K^*_{DI}(\rho)$.

We next prove part (ii). From the proof of Proposition 3, we have

$$\rho_2^{\text{PP}} \leq \frac{2R(R-1)\left(\theta^U - \theta^L\right)}{\bar{m}\Big(\Theta + \theta^L - (R-1)\left(\theta^U - \theta^L\right)\Big)}.$$

By condition (38), the right-hand side is strictly smaller than r_1. Therefore $\rho_2^{\text{PP}} < r_1$. By Proposition 1, $\rho^{\text{DI}} = r_1$. Hence, $\rho_2^{\text{PP}} < \rho^{\text{DI}}$.

Now take any $\rho \in \left(\rho_2^{\text{PP}}, \rho^{\text{DI}}\right)$. By part (ii) of Proposition 3, $K^*_{PP}(\rho) = r_1$. At the same time, since $\rho < \rho^{\text{DI}}$, Proposition 1 implies that $K^*_{DI}(\rho) \in (0, r_1)$. Therefore $K^*_{PP}(\rho) = r_1 > K^*_{DI}(\rho)$, which proves part (ii).

Q.E.D.

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