# Accounting for Credibility: Fiscal-Monetary Interactions and the Credibility of Central Bank Mandates*

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**Abstract**

We develop a model for fiscal and monetary policy determination in the tradition of Sargent and Wallace (1981). Ex-ante, the government has incentives to delegate monetary policy to a central bank with an inflation targeting mandate. Ex-post, however, the government faces temptations to revoke the mandate to generate seigniorage revenues. The likelihood that the government will adhere to its commitment depends on shocks to fiscal fundamentals and the costs of reneging on the mandate. The economy endogenously transitions between a "monetary-dominant" regime where monetary policy adheres to its commitment and a "fiscal-dominant" regime where the fiscal authority interferes with monetary policy. These two regimes sharply differ in their implications for the comovement of inflation and debt-to-GDP ratios. We use the model as a measurement device to interpret the fiscal and monetary history in Colombia, Chile, and the U.S.

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# 1 Introduction

In the decades that followed the inflation crises of the 1980s and early 1990s, many emerging and advanced economies attempted to stabilize prices by delegating monetary policy to newly independent central banks operating under explicit inflation-targeting mandates. The effectiveness of these measures ultimately rests on whether governments remain willing and able to respect the mandate once macroeconomic conditions change. In many instances, governments have revoked such mandates during crises. For example, in early 2002, Argentina decided to repeal the Convertibility Law and exit from the currency-board regime, which was taken in the face of mounting debt servicing pressures. More recently, there has been mounting political pressure on the Federal Reserve Bank and other central banks around the world, prompting concerns about their independence.[1]

This paper develops a flexible framework for analyzing the credibility of inflation-targeting mandates in the presence of fiscal–monetary interactions, in the tradition of Sargent and Wallace (1981). We study a benevolent government that cannot commit. Ex ante, it may find it optimal to delegate monetary policy to a central bank with a narrow inflation-targeting mandate; ex post, it may choose to override the mandate in order to raise seigniorage and reduce the real value of its liabilities. The incentive to renege depends on fiscal fundamentals, summarized by outstanding public debt and the marginal value of public spending, and on a stochastic institutional cost of deviation that reflects the legal, reputational, and political hurdles associated with overriding the mandate.

The economy endogenously fluctuates between two regimes: a *monetary-dominant* regime in which the target is honored and inflation is insulated from fiscal considerations, and a *fiscal-dominant* regime in which the target is overridden and inflation responds strongly to fiscal conditions. The two regimes generate sharply different dynamics for inflation and government indebtedness. In the monetary-dominant regime, inflation is low and insulated from fiscal considerations, while the debt-to-GDP ratio is high. In the fiscal-dominant regime, inflation is high on average, volatile, and closely related to fiscal considerations, while public debt is low. We show that credible delegation is necessary to support high levels of public debt, yet high debt itself endogenously undermines credibility We use the model to interpret fiscal and monetary histories in Colombia, Chile, and the United States by accounting for the relative role of credibility and fiscal fundamentals in shaping them.

We consider a monetary economy where households work, consume and derive utility from real money balances, like in Calvo (1978). A benevolent government chooses how to finance government expenditures using distortionary labor income taxes, issuing gov-

[1] See, for example, Yellen (2026) and Canepa, Schneider, and Kihara (2026).

ernment debt and money. The fundamental shock in the economy is a preference shock to the marginal utility of government expenditures. This term captures all the reasons a government might find optimal to increase expenditures, such as a recession, an increase in demand for transfers, etc. We show how we can use the equilibrium conditions to write down an indirect utility function for the government over primary surpluses. We can then write the government's problem as choosing policies to maximize its indirect utility function, subject to the government budget constraint and a money demand condition. (A large class of economies admits a representation of the government problem similar to the one we derive, albeit with a different indirect utility function. If this function is concave in surpluses, our conclusions continue to hold.)

If the government could perfectly commit, the Ramsey outcome would follow the Friedman rule and realized inflation would not be sensitive to fiscal shocks.[2] This outcome could be implemented by delegating monetary policy to an independent central bank with a narrow inflation targeting mandate, and have fiscal policy being determined by the treasury. The treasury solves a problem like that in the real economy studied by Aiyagari, Marcet, Sargent, and Seppälä (2002), taking as given a constant flow of seigniorage revenues.

Our objective, however, is that of studying optimal policy when the government lacks such commitment. In those cases, it is well known that the policy game admits a continuum of sustainable equilibria—that is, subgame perfect equilibria (SPE). Most of the previous literature has analyzed two extreme cases, either focusing on the best sustainable equilibrium, which often coincides with the Ramsey outcome and in which policy promises are perfectly credible; or on the Markov perfect equilibrium (often the worst sustainable equilibrium), in which promises about future policies are not at all credible. These approaches, while useful, are not flexible enough to study optimal policy in the empirically plausible case, that is when promises are *imperfectly credible*. For this reason, rather than picking one of these two extremes, we develop a flexible approach to represent the full set of equilibria of the policy game. This nesting model will then be used to study optimal fiscal and monetary policy with imperfect credibility and to select the equilibrium that best describes the data.

Our construction of the nesting model is motivated by the observation that many countries have recently adopted some form of an inflation targeting regime. In our framework, the government seeks to manage expectations by promising to deliver a specific inflation target in the next period. The government in the following period then decides whether to honor this target or to pay a random cost to deviate from it. This cost is exogenous and

[2]This result holds under standard assumptions on preferences for the stand-in household. Even when these assumptions are not satisfied, we can show numerically that the Ramsey outcome has a nominal interest rate close to zero and a roughly constant inflation rate.

is intended to capture various frictions, such as reputational losses, coordination failures that lead to inferior equilibria, institutional constraints like the protection offered by a constitutional court to the independence of the central bank, and the political costs faced by policymakers.[3] We show that this framework captures policy determination and expectation formation in a way that continuously nests both the Ramsey and Markov equilibria, and outcomes in between.

We use this framework to examine how changes in the credibility of promises influence the dynamics of fiscal and monetary outcomes. Specifically, we show that the possibility that the government will not honor the inflation target in the future has important implications for the conduct of fiscal and monetary policy. On the monetary side, the inflation target is raised relative to what the full commitment solution would prescribe, in order to incentivize the government to adhere to it. On the fiscal side, it would be optimal to reduce the amount of debt issued in order to reduce its temptation to abrogate the mandate in the following period. These effects are stronger when the government is more likely to abrogate the mandate, which occurs when the marginal utility of spending is large, public debt is high, and the institutional costs of defaulting are small.

In our application, we calibrate the model to an average of Latin American economies from 1960-2017 and use it to study the drivers of large disinflation episodes. Specifically, in our model there are two ways in which inflationary episodes end. Inflation can decline either due to a reduction in the marginal value of government spending, or due to an increase in the (expected) cost of deviating from the promised inflation. We refer to the former as *fundamental disinflation* and to the latter as *institutional disinflation*. These distinct paths have different implications for the dynamics of public debt: while fundamental disinflations are associated with a declining path for real debt, institutional disinflations are characterized by an increasing path for real debt. The reason for this is that increased institutional costs weaken the incentive effect and allow the government to borrow more. In contrast, reductions in the marginal value of the government spending lower their desire to issue debt and generate inflation. This contrasting comovement is the identification logic we use in the case studies to interpret observed disinflations.[4]

We apply particle filter methods to recover the sequences of fiscal and institutional shocks consistent with observed data on inflation and debt-to-GDP ratios in Chile and Colombia. The experience of these two countries illustrate the two disinflation channels implied by our model. In Colombia, inflation fell sharply while debt rose—the signature

[3] Lohmann (1992) studies the optimal size of such a cost in a Barro-Gordon model, trading off commitment and flexibility.

[4] This approach is similar to the one used by Aguiar and Gopinath (2007) and Bocola and Dovis (2019) in different contexts.

of an institutional disinflation. The particle filter identifies a significant increase in credibility beginning in 1997, and a counterfactual holding institutional costs at zero shows that debt would have declined sharply, contrary to the data. In Chile, both inflation and debt declined, consistent with a fundamental disinflation driven by fiscal consolidation. However, institutional improvements were still necessary: in the late 1990s, inflation continued to fall while debt stabilized, a pattern that fiscal consolidation alone cannot explain. We also extend the analysis to the rise and fall of inflation in the United States during the 1970s-1980s. The Great Inflation of the 1970s reflects a collapse in credibility amid political pressure on the Federal Reserve, while the Volcker disinflation marks a gradual restoration of credibility that enabled rising debt alongside stable low inflation from the mid-1980s onward.

Our case studies provide a model and outcome-based measure of the credibility of the inflation target mandate. Such a measure complements purely legal measures of central bank independence that do not fully capture the credibility of the mandate. For example, for the case of Colombia, we do not identify an increase in the credibility in 1992, the first year after the reform, but only in 1997. In contrast, the credibility index for Colombia in Romelli (2022) increases in 1992 and stays constant at this level afterwards. This may be driven by the fact that it took time for the government to convince private agents that the new institutional arrangement was credible and not just a cosmetic adjustment. Relatedly, Cukierman (2008) finds that legal independence is associated with low inflation only in advanced economies and not in developing ones, indicating that credibility issues can be important in the latter group.

**Related literature** Our paper is related to a large literature that studies optimal monetary and fiscal policy with and without commitment. See, for example, Lucas Jr and Stokey (1983), Aiyagari et al. (2002), Calvo (1978), Chang (1998), Chari and Kehoe (1999), Alvarez, Kehoe, and Neumeyer (2004), Aguiar, Amador, Farhi, and Gopinath (2013), and more recently Espino, Kozlowski, Martin, Sánchez et al. (2023) and Espino, Kozlowski, Martin, and Sánchez (2022). We depart from this literature by considering an outcome where, in addition to fundamental shocks, there are shocks to the cost of deviating from the promised policy path. These costs allow us to span a large class of sustainable equilibrium outcomes and not just focus on the best or Markov equilibrium. Moreover, they allow us to quantify the component of inflation that is driven by institutional changes (the cost of deviation) and fundamentals.

The transition between a monetary and a fiscally dominant regime in the conduct of monetary policy is related to the work of Leeper (1991), Sims (1994), Woodford (1994),

Bianchi (2013), Bianchi and Ilut (2017), Bianchi, Faccini, and Melosi (2023),Cochrane (2023) and Witheridge (2024). Our modeling approach differs from this line of work on two fronts. First, in our model, the switches from one regime to the other are endogenous.[5] Second, the policy chosen in each regime is also endogenous in our model, while papers in this literature consider exogenous monetary and fiscal rules. This allows us to study how changes in the credibility of the monetary policy mandate influence the optimal conduct of fiscal and monetary policy. Moreover, in our model, movements in inflation are the direct consequence of deliberate policy choices as in Sargent and Wallace (1981) and not the result of an equilibrium selection mechanism. Another difference is that in this class of economies, long-run inflation and debt remain constant across regimes, varying only in their response to shocks, which can be quite persistent. In contrast, our economy exhibits different long-run inflation and debt levels in the two regimes.

Closer to our paper is the work on loose commitment in Debortoli and Nunes (2010), Debortoli, Maih, and Nunes (2014), and Debortoli and Lakdawala (2016) that also considers economies where the government can re-optimize its choices with some random probability. In this line of work, the policy is endogenous as in our model, but the regime is exogenous.[6] The endogeneity of the regime is critical for our results because the government chooses a different inflation target and level of debt to influence the decision of the next period's government. This incentive effect is critical for the different debt dynamics in disinflation episodes driven by institutional or fundamental forces.

Our quantitative analysis complements work that studies the joint dynamics of debt and inflation. Among these papers are the seminal narrative analysis in Sargent (1982), and the various chapters in the volume edited by Kehoe and Nicolini (2022).[7] Our contribution is provide a theory of these joint dynamics driven by government incentives. Relatedly, Gao, Kulish, and Nicolini (2025) show that medium-run movements in nominal variables are explained by movements in the inflation target. Our theory is consistent with this finding as the inflation targets and expected inflation fluctuate between regimes and depending on fiscal fundamentals.

Finally, our paper is related to deeper theories of credibility and reputation building

[5] Barthélemy, Mengus, and Plantin (2024) consider endogenous movements between fiscal and monetary dominant regimes as the outcome of a "chicken game." We take a different view: the fiscal side (the government) can always take the central bank independence away – say by changing the central bank mandate – but doing so has a cost. These two approaches have very different implications for debt dynamics: in the chicken game, the fiscal authority issues more debt to induce the central bank to cave. In our model, higher risk of fiscal dominance leads to less debt, and more debt is issued only when monetary dominance is expected. The dynamics predicted by our model is consistent with the narrative around important historical episodes.

[6] Debortoli and Nunes (2010) consider an extension where the probability of re-optimization depends on the state variable but this is an exogenous function.

[7] See also Sargent, Williams, and Zha (2009).

like Atkeson and Kehoe (2001), Atkeson, Chari, and Kehoe (2007), Dovis and Kirpalani (2021), Piguillem and Schneider (2013), King and Lu (2022), Lu, King, and Pasten (2016), Halac and Yared (2021), Halac and Yared (2025), de Aguilar (2024), Kostadinov and Roldán (2024), and Bocola, Dovis, Jørgensen, and Kirpalani (2025). The dynamics of the credibility measure we derive in our application can serve to discipline and discriminate between these models.

# 2 A Sargent-Wallace economy

## 2.1 Environment

Consider an economy that blends elements of Aiyagari et al. (2002) and Calvo (1978). Time is discrete and indexed by $t = 0, 1, ....$ The exogenous state of the economy is $s_t \in S$. We assume that $s_t$ follows a Markov process with transition $\Pr(s_{t+1}|s_t)$. The economy is composed of a stand-in household, competitive firms, and a benevolent government. The stand-in household supplies labor and has utility over private consumption $c(s^t)$, labor $l(s^t)$, real money balances $m(s^t)$, and public consumption $g(s^t)$ given by

$$\sum_t \sum_{s^t} \beta^t \Pr(s^t|s_0)\, \mathcal{U}(c(s^t), l(s^t), m(s^t), g(s^t), s_t) \tag{1}$$

where $\beta$ is the stand-in household discount factor. We assume that

$$\mathcal{U}(c, l, m, g, s) = c - v(l) + v(m) + \theta(s)\, u(g) \tag{2}$$

where $v(l)$ is a strictly increasing and convex function, $\theta(s)$ is a preference shock to the marginal utility of government spending, $v(m)$ and $u(g)$ are strictly increasing and concave functions. We further assume that all functions are twice differentiable and $v$ is such that $v'(l)\, l$ is convex.[8]

The resource constraint for the economy is

$$c(s^t) + g(s^t) \leq l(s^t). \tag{3}$$

The linear production technology is operated by competitive firms.

The government is benevolent but it may have a different discount factor than the stand-in household. Let $\hat{\beta} \leq \beta$ be the government's discount factor. The government finances

[8] For example, this last assumption is satisfied if the labor disutility has a constant Frisch elasticity, $v(l) = \chi l^{1+\psi}/(1+\psi)$, or more generally if $2v''(l) + v'''(l)\, l \geq 0$.

government spending with linear taxes on labor income, $\tau\left(s^t\right)$, by issuing real uncontingent debt, $B\left(s^t\right)$, and by printing money, $M\left(s^t\right)$, injected into the economy via open market operations. We also allow the government to make positive lump-sum transfers, $T\left(s^t\right)$, to the stand-in household. The government budget constraint is

$$\begin{aligned} & P\left(s^t\right) g\left(s^t\right)+P\left(s^t\right) B\left(s^{t-1}\right)+M\left(s^{t-1}\right)+T\left(s^t\right) \\ \leq & \tau\left(s^t\right) W\left(s^t\right) l\left(s^t\right)+Q\left(s^t\right) B\left(s^t\right)+M\left(s^t\right) \end{aligned} \tag{4}$$

where $P\left(s^t\right)$ is the nominal price level, $W\left(s^t\right)$ is the nominal wage, and $Q\left(s^t\right)$ is the price of real debt (in terms of money).

The household problem is to choose $\left\{c\left(s^t\right), l\left(s^t\right), M\left(s^t\right), B\left(s^t\right)\right\}$ to maximize (1) subject to the budget constraint

$$\begin{aligned} & P\left(s^t\right) c\left(s^t\right)+Q\left(s^t\right) B\left(s^t\right)+M\left(s^t\right) \\ \leq & \left(1-\tau\left(s^t\right)\right) W\left(s^t\right) l\left(s^t\right)+M\left(s^{t-1}\right)+P\left(s^t\right) B\left(s^{t-1}\right)+T\left(s^t\right) \end{aligned}$$

where real balances are given by $M\left(s^{t-1}\right) / P\left(s^t\right)$. We adopt the Nicolini (1998)-timing where only money carried over from last period enters the utility function and not newly issued money. This timing convention creates costs to unexpected inflation and guarantees the existence of a Markov perfect equilibrium with positive real balances.

Given initial $\left(M_0, B_0\right)$, an equilibrium is a set of allocations $\left\{c\left(s^t\right), l\left(s^t\right), M\left(s^t\right), B\left(s^t\right)\right\}$, policies $\left\{\tau\left(s^t\right), M\left(s^t\right), B\left(s^t\right), T\left(s^t\right)\right\}$, and prices $\left\{P\left(s^t\right), W\left(s^t\right), Q\left(s^t\right)\right\}$ such that i) the allocation solves household's maximization problem, ii) the government budget constraint holds, iii) $W\left(s^t\right)=P\left(s^t\right)$ and asset markets clear.

## 2.2 Implementable allocations

We next derive implementability conditions that characterize the set of fiscal and monetary outcomes that can be supported as a competitive equilibrium. We focus on fiscal and monetary outcomes –rather than allocations, as is standard in the Ramsey literature– because they are the objects of interest in our empirical analysis.

Optimality for the stand-in household's problem requires that the labor supply satisfies

$$\left(1-\tau\left(s^t\right)\right)=v'\left(l\left(s^t\right)\right), \tag{5}$$

the Euler equation for money holdings holds

$$\frac{1}{P\left(s^t\right)} = \beta E_t \left[ \frac{1}{P\left(s^{t+1}\right)} + v' \left( \frac{M\left(s^t\right)}{P\left(s^{t+1}\right)} \right) \frac{1}{P\left(s^{t+1}\right)} \right], \tag{6}$$

and the price for real bonds is given by

$$\frac{Q\left(s^t\right)}{P\left(s^t\right)} = \beta. \tag{7}$$

Following the insight in Aiyagari (1989) and Aiyagari et al. (2002), we define the static indirect utility function over primary surpluses.[9] Define the real primary surplus as

$$\Delta\left(s^t\right) \equiv \tau\left(s^t\right) l\left(s^t\right) - g\left(s^t\right) - T\left(s^t\right).$$

From the labor supply condition (5), we know that in any competitive equilibrium tax revenues must satisfy the following restriction

$$\tau\left(s^t\right) l\left(s^t\right) = \left(1 - v'\left(l\left(s^t\right)\right)\right) l\left(s^t\right).$$

We can define the static indirect utility function over surpluses as

$$U\left(\Delta, s\right) = \max_{c,l,g} c - v\left(l\right) + \theta\left(s\right) u\left(g\right) \tag{8}$$

subject to the resource constraint (3) and the static implementability constraint

$$\left(1 - v'\left(l\right)\right) l - g \geq \Delta.$$

This function is well-defined for all surplus values below the maximal surplus implied by the static Laffer curve, $\Delta \leq \bar{\Delta} \equiv \max_l \left(1 - v'\left(l\right)\right) l$. Under the assumption that $v\left(l\right) l$ is convex, we have:

**Lemma 1.** *The indirect utility function $U\left(\Delta, s\right)$ is decreasing and concave in $\Delta$ for all $s$. Moreover, $\lim_{\Delta \to \bar{\Delta}} U'\left(\Delta, s\right) = -\infty$ and $U'\left(\Delta, s\right) = 0$ for all $\Delta \leq -g^*\left(s\right)$ implicitly defined by $\theta\left(s\right) u'\left(g^*\left(s\right)\right) = 1$.*

The indirect utility function $U$ captures the costs associated with distortionary labor taxation. In state $s$, the government can implement the efficient level of public spending, $g^*\left(s\right)$, only if it runs a primary deficit of at least $g^*\left(s\right)$, thereby financing public spending

[9] See also Chari, Dovis, and Kehoe (2020).

without relying on distortionary taxes. For any primary surpluses above $-g^*(s)$, $U$ is strictly decreasing in $\Delta$: a larger required primary surplus makes the government to reduce public spending below $g^*(s)$ and to increase distortionary taxation, lowering static welfare. The concavity of $U$ follows from the convex cost of distortionary taxes.

It is convenient to normalize all nominal variables by the amount of money inherited in each period. In particular, let the normalized price level be $p(s^t) = P(s^t)/M(s^{t-1})$, the (gross) money growth rate be $\mu(s^t) \equiv M(s^t)/M(s^{t-1})$, and let

$$\phi(s^t) \equiv 1/p(s^t) = M\left(s^{t-1}\right)/P(s^t)$$

be the inverse of the normalized price level – the price of money in terms of the private consumption good – or the real monetary balances $M(s^{t-1})/P(s^t)$. Using these normalizations and defining

$$H(\phi) \equiv \phi + v'(\phi)\phi,$$

we can rewrite the money demand condition (6) as

$$\mu(s^t)\phi(s^t) = \beta \sum_{s_{t+q}} \Pr(s_{t+1}|s_t) H\left(\phi\left(s^{t+1}\right)\right) \tag{9}$$

and, using (7) to substitute for $Q(s^t)$, the government budget constraint (4) as

$$b\left(s^{t-1}\right) + \phi(s^t) = \Delta(s^t) + \beta b(s^t) + \mu(s^t)\phi(s^t) \tag{10}$$

We can then state the implementability conditions:

**Lemma 2.** *A fiscal and monetary outcome $\{\Delta(s^t), b(s^t), \phi(s^t), \mu(s^t), \pi(s^t)\}$ is implementable as a competitive equilibrium given an initial level of normalized real debt $b_0$ if and only if it satisfies the normalized version of the government budget constraint (10), a no-Ponzi-game condition, the Euler equation for money holdings (9), the feasibility condition for the primary surplus $\Delta(s^t) \leq \bar{\Delta}$, and inflation is given by*

$$1 + \pi\left(s^{t+1}\right) = \frac{\mu(s^t)\phi(s^t)}{\phi(s^{t+1})}. \tag{11}$$

*The associated value for the government is given by*

$$V_0 = \sum_t \sum_{s^t} \hat{\beta}^t \Pr(s^t|s_0) \left[U(\Delta(s^t), s_t) + v(\phi(s^t))\right]. \tag{12}$$

Note that we can combine the government budget constraint (10) with the money demand equation (9), iterating forward and invoking the household's transversality condi-

tions to obtain

$$b\left(s^{t-1}\right)+\phi\left(s^{t}\right)=\sum_{j=0}^{\infty}\sum_{s^{t+j}}\beta^{j}\Pr\left(s^{t+j}|s^{t}\right)\Delta\left(s^{t+j}\right)+\sum_{j=1}^{\infty}\sum_{s^{t+j}}\beta^{j}\Pr\left(s^{t+j}|s^{t}\right)h\left(\phi\left(s^{t+j}\right)\right) \quad (13)$$

where $h\left(\phi\right)\equiv v'\left(\phi\right)\phi$. That is, after any $s^{t}$, the real value of government's liabilities, $b\left(s^{t-1}\right)+\phi\left(s^{t}\right)$, equals the discounted value of current and future primary surpluses and future seigniorage revenues, $h\left(\phi\left(s^{t+j}\right)\right)$.

The conditions in Lemma 2 highlight the main forces shaping equilibrium outcomes. The concavity of the indirect utility function, $U$, gives the government an incentive to smooth the marginal cost of primary surpluses across both time and states as in a standard consumption-smoothing problem. At the same time, a benevolent government trades off the distortion associated with real balances against the cost of distortionary taxation. Distortions in real balances generate seigniorage revenues that relax the government budget constraint (10), allowing the government to run a smaller primary surplus and therefore to reduce taxes and keep government spending closer to its efficient level. Under standard preferences, however, the commitment solution satisfies the Friedman rule: the government issues the optimal quantity of reserves, $v'\left(\phi\right)=0$, and raises no seigniorage revenue. This outcome is not time consistent: ex-post, a benevolent government has an incentive to reduce the value of real balances, $\phi\left(s^{t}\right)$, relative to its promised value, thereby lowering the real value of its liabilities and reducing the costs associated with distortionary taxes.

# 3 Policy determination

We want to study fiscal and monetary outcomes when the government cannot commit. A common approach is to consider a sustainable equilibrium, i.e., a subgame perfect equilibrium (SPE) of the policy game. This equilibrium notion, however, does not sufficiently narrow down the set of outcomes that can occur when the government lacks commitment. Following the logic in Abreu (1988) and Chari and Kehoe (1990), any fiscal and monetary outcome that is consistent with the implementability conditions in Lemma 2 and a *sustainability constraint* that requires that the continuation value for the government after any history is above the value of the worst equilibrium is a valid sustainable equilibrium outcome. To understand why, notice that the incentive for the government to follow the prescribed path of play depends on the consequences of the deviation. For example, one can construct an equilibrium in which the government faces no penalty from such deviations. One can then use this outcome as a threat to support better outcomes on path.

This multiplicity raises a natural question: how should we select among these equilibria, and how do we model the ways the government tries to manage expectations and how private agents coordinate on a continuation equilibrium when there is a deviation?[10] Previous work considers either the best sustainable equilibrium, in which the consequences of a deviation are maximal, or the Markov perfect equilibrium, in which there are no consequences for not respecting the promised or expected policy..

We take a different approach and instead construct a model that is flexible enough to nest these possibilities along with other sustainable equilibria in which policy promises are imperfectly credible. Motivated by the observation that several countries have recently adopted some form of an inflation targeting regime, we assume that the government tries to manage expectations by promising to deliver an inflation target $\pi^*$ next period. The government in the following period can choose whether to maintain the inflation target or to pay a random cost $\xi$ to deviate from the target. This cost is exogenous and captures factors such as reputation losses, coordination of private agents leading to worse equilibria, institutional constraints, and the political costs faced by policymakers. This flexible formulation enables us to confront the data and examine how changes in the credibility of promises — represented by changes in $\xi$— influence the dynamics of fiscal and monetary outcomes.

In Appendix B, we show in a deterministic economy that, given a specific sequence for $\xi_t$, our model can recover any sustainable equilibrium outcome, except for the ones that allows for Pareto improvements: that is, those in which there are deviations in period $t+1$ that increase the government's value in both period $t+1$ and $t$.

**Recursive formulation** We next set up the problem recursively. In addition to the exogenous state and the real value of debt, we also need to include the inflation target that was promised in the previous period, $\pi^*$. It is however more convenient and without loss to keep track of the promised value for real balances $\phi$. In fact, consider a government in period $t$ that promises an inflation target $\pi^*$ for the next period. From (11) we know that

$$1 + \pi^* = \frac{\mu_t \phi_t}{\phi_{t+1}}$$

Thus, from period $t$'s perspective – given the current money growth rate and $\phi_t$ – there is a one-to-one map between the inflation target $\pi^*$ and the promised value for real money balances $\phi_{t+1}$. Thus, we can keep track of promised real money balances and let the state

[10] This problem is beautifully explained in Sargent (2024).

be $x = (b, \phi, s)$.[11]

Note that we do not allow the promised value of real balances to be contingent on the next period's state. In principle, one could imagine that the delegation of monetary policy includes state-dependent escape clauses. However, a justification for our assumption is that it is difficult to detect deviations and impose penalties when promised inflation is state-dependent and information about the state is dispersed, as shown by Piguillem and Schneider (2013).

In any period, the economy can be in either a *monetary dominant* regime (denoted by $md$) where the government satisfies the inflation target and attains value $V_{md}(x)$, or in a *fiscal dominant* regime (denoted by $fd$) where the government deviates from the set target and attains a value net of the cost $\xi$ given by $V_{fd}(b, s)$. The value for the government is then

$$V(b, \phi, s) = \max \left\{ V_{md}(b, \phi, s), V_{fd}(b, s) - \xi(s) \right\} \tag{14}$$

We let $\eta(x)$ be an indicator variable that takes value 1 if it is optimal to satisfy the target and be in the monetary dominant regime and 0 otherwise:

$$\eta(b', \phi', s') = \begin{cases} 1 & \text{if } V_{md}(b, \phi', s') \geq V_{fd}(s') - \xi(s') \\ 0 & \text{if } V_{md}(b, \phi', s') < V_{fd}(s') - \xi(s') \end{cases}. \tag{15}$$

Finally, we let $J(b', \phi', s)$ be the expected marginal value of real balances the following period. As it will be clear, this is a function of newly issued debt $b'$, the next period inflation target summarized by $\phi'$, and the current exogenous state $s$.

**Monetary dominance** The problem for the government when it respects the target is

$$V_{md}(b, \phi, s) = \max_{\Delta, b', \mu, \phi'} U(\Delta, \theta) + v(\phi) + \hat{\beta} \sum_{s'} \Pr(s'|s) V(b', \phi', s') \tag{16}$$

subject to the budget constraint

$$\Delta = b + \phi - \beta b' - \mu\phi$$

and the money-demand condition

$$\mu\phi = J(b', \phi', s)$$

[11] We do not need to track additional state variables—such as the promised marginal utility for the stand-in household—because preferences are quasi-linear in consumption. As a result, there is no time-inconsistency problem arising from interest-rate manipulation.

with $b' \in \left[0, \bar{b}\right]$.

**Fiscal dominance** The problem when the inflation target is not satisfied is similar but now the government can choose the current value for the real money balance $\phi$:

$$V_{fd}(b,s) = \max_{\phi,\Delta,b',\mu,\phi'} U(\Delta,\theta) + v(\phi) + \hat{\beta}\sum_{s'} \Pr\left(s'|s\right) V\left(b',\phi',s'\right) \tag{17}$$

subject to

$$\Delta = b + \phi - \beta b' - \mu\phi$$

$$\mu\phi = J\left(b',\phi',s\right)$$

Note for later that in this case the value of real balances is determined by the static first order condition

$$-U(\Delta,\theta) = v'\left(\phi_{fd}\right). \tag{18}$$

That is, the marginal benefit of real balances is equated to the marginal cost of the primary surplus. Thus, since money supply is pre-determined, the model predicts a higher price level (lower real balances) when the marginal cost of the surplus is high. This happens when either the value of public spending $\theta$ is high or when the amount of inherited debt is high and so the government must run a surplus just to stabilize the level of debt.

**Equilibrium** The expected marginal value for real balances next period is

$$J\left(b',\phi',s\right) = \beta\sum_{s'} \Pr\left(s'|s\right)\left[\eta\left(b',\phi',s'\right) H\left(\phi'\right) + \left(1-\eta\left(b',\phi',s'\right)\right) H\left(\phi_{fd}\left(b',s'\right)\right)\right] \tag{19}$$

We can then define an equilibrium as a set of value functions $V$, $V_{md}$, $V_{fd}$, policy functions $\phi_i, \Delta_i, b_i', \mu_i, \phi_i'$ for $i = md, fd$, a regime choice function $\eta$ and $J$ that satisfy (14), (16), (17), (19), and (15).

Note that our equilibrium notion nests the Ramsey outcome if the support of $\xi$ is large enough so that $\eta = 1$ in every period in which case $J\left(b',\phi',s\right) = \beta H\left(\phi'\right)$ and the government in the current period controls the amount of inflation next period. The model also nests the Markov perfect equilibrium outcome if the cost $\xi$ always equals 0 so that it is always better to be in the fiscal dominant regime, $\eta = 0$, and $J\left(b',\phi',s\right) = \beta\sum_{s'} \Pr\left(s'|s\right) H\left(\phi_{fd}\left(b',s'\right)\right)$.

# 4 Model dynamics and credibility of inflation targets

We now analyze the implied dynamics for debt, inflation targets, and inflation. Before doing so, we consider two benchmarks: the Ramsey outcome and the Markov outcome. In the Ramsey outcome, inflation is approximately constant and insulated from fiscal conditions, and high levels of public debt can be sustained. In the Markov equilibrium, inflation responds strongly to fiscal pressures and debt capacity is sharply limited. The full model interpolates between these extremes depending on the costs $\xi_t$.

A central mechanism emphasized in this section is an incentive effect operating through debt issuance and promised inflation targets. Specifically, current policymakers have incentives to limit borrowing and to choose less ambitious inflation targets when credibility is fragile, as those will reduce the incentives of the government to override the mandate in the future. These incentive motives distort policy relative to the Ramsey benchmark, especially when the probability of switching from monetary to fiscal dominance is high..

## 4.1 Ramsey outcome

The Ramsey outcome is the best allocation consistent with a competitive equilibrium. Given our definition of indirect utility $U(\Delta)$ and the characterization of implementable allocations in Lemma 2, we can write the Ramsey problem as choosing $\{\Delta(s^t), b(s^t), \phi(s^t), \mu(s^t)\}$ to maximize (12) subject to (9), (10) and $b(s^t) \leq \bar{b}$ given $b_0$.

Note that the Ramsey outcome, other than assuming commitment, allows for greater flexibility than the equilibrium considered here because the promised value $\phi(s^{t+1})$ can be contingent on the state of the world in period $t+1$. This can be valuable because it allows the government's total real liabilities to be state-contingent and overcome the assumed market incompleteness for real debt.

To keep the analysis parallel with our recursive formulation of the full model above, we express the Ramsey problem in a quasi-recursive form. The Ramsey outcome from period 1 onward solves the following recursive problem:

$$V_R(b, \phi, s) = \max_{\Delta, b', \phi'(s')} U(\Delta, s) + v(\phi) + \hat{\beta} \sum_{s'} \Pr(s'|s) V_R(b', \phi'(s'), s') \tag{20}$$

subject to

$$\Delta = b + \phi - \beta b' - \beta \sum_{s'} \Pr(s'|s) H(\phi'(s'))$$

and $0 \leq b' \leq \bar{b}$. In period 0, the Ramsey outcome solves (20) with $\phi_0 = \arg\max_\phi V_R(b_0, \phi, s_0)$.

**Proposition 3.** *Suppose* $v(\phi) = \chi \frac{\phi^{1-\eta}}{1-\eta}$ *for* $\eta \in (0,1)$, $\chi > 0$, *and there is a bound on real balances,* $\phi(s^t) \leq \phi^*$. *If* $\sup_{s'} |\theta(s') - \sum_{s'} \Pr(s'|s)\theta(s')|$ *is small enough then the Ramsey outcome has* $\phi'(s^t) = \phi^*$ *for all* $t \geq 1$ *and* $s^t$. *If we impose the additional restriction that* $\phi'(s') = \phi'$ *for all* $s'$ *then* $\phi'(s^{t-1}) = \phi^*$ *for all* $s^{t-1}$ *for all process for* $\theta_t$.

Under our assumptions, there is a trade-off between following the Friedman rule and making real debt state contingent.[12] If the volatility of the marginal value of government expenditures is sufficiently small, the benefits of making the real debt state contingent are small relative to the cost of anticipated inflation and it is optimal to set $\phi(s') = \phi^*$ for all $s'$ next period. Thus, the constraint that the promised value of money must be constant across states in the next period is not binding.

Under the conditions in the proposition, the Ramsey outcome has a fixed inflation level given by[13]

$$1 + \pi_R = \frac{\beta H(\phi^*)}{\phi^*} = \beta\left(1 + \chi(\phi^*)^{-\eta}\right)$$

In particular, the inflation level does not depend on the fiscal fundamentals – the level of debt and $\theta$.

One way to implement the Ramsey outcome is to delegate monetary policy to an independent central bank with a mandate to target an inflation rate of $\pi_R$, while fiscal policy is determined by the treasury, which solves a problem similar to that in the real economy studied by Aiyagari et al. (2002), taking as given a constant flow of seigniorage revenues (which may be negative). The dynamics of surpluses and real debt, when interior, must be consistent with the Euler equation

$$-U'(\Delta, s) = \frac{\hat{\beta}}{\beta} \sum_{s'} \Pr(s'|s) \left[-U(\Delta(s'), s')\right] \tag{21}$$

and the budget constraint. This arrangement, while optimal, relies on the government's ability to commit. In the subsequent sections, we will see how the optimal mandate changes if the government cannot commit.

When the preferences over real money balances take a different form then the iso-elastic one considered in the proposition, we cannot prove that a version of the Friedman rule is optimal. For example, if $v(m)$ is quadratic, the optimal value for $\phi$ in the Ramsey outcome will typically be different from the satiation point and dependent on fiscal fundamentals.

[12] Chari and Kehoe (1999) show how in cash-credit good economy it is possible to simultaneously follow the Friedman rule and use inflation to generate state contingencies.

[13] The case with no bounds on real balances has $\phi^* \to \infty$ and so the inflation rate in the Ramsey outcome equals $\beta - 1$.

However, in numerical simulations, we find that such variations are small and that the delegation of monetary policy to an independent central bank with a fixed inflation target is approximately optimal under commitment. See Figure 2 for a typical outcome where $v(m)$ is quadratic.

## 4.2 Markov outcome

We now turn to the polar opposite case in which the government has no way to commit to inflation and consider a Markov equilibrium outcome. This is a special case of our environment with $\xi(s) = 0$ for all $s$ in which case the fiscal dominant regime is always optimal. Consequently, we can drop $\phi'$ as a choice in (17) since it has no effect on the value. The problem reduces to

$$V_M(b,s) = \max_{\phi,\Delta,b'} U(\Delta,\theta) + v(\phi) + \hat{\beta}\sum_{s'} \Pr(s'|s)\, V_M(b',s') \tag{22}$$

subject to

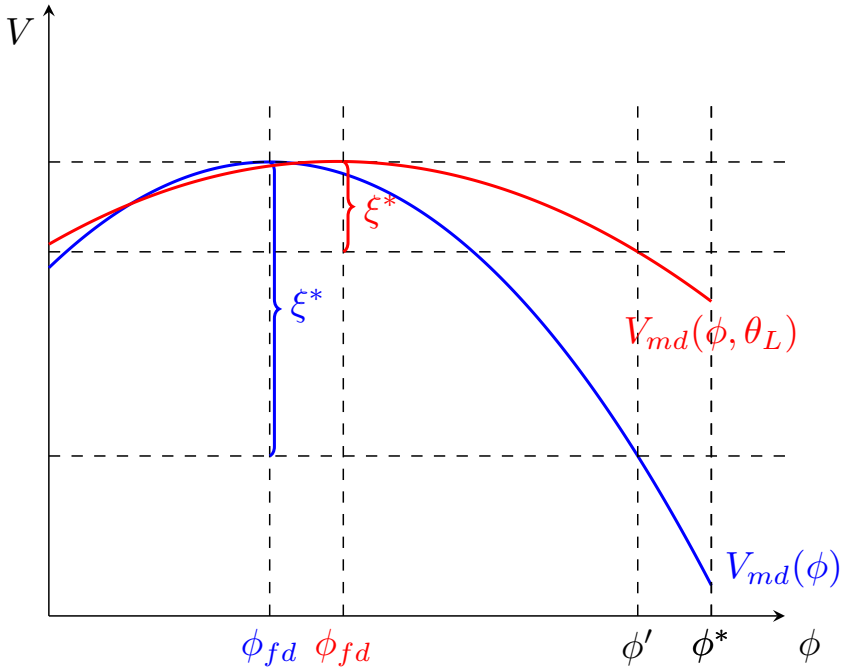
$$\Delta = b + \phi - \beta b' - \beta \sum_{s'} \Pr(s'|s)\, H\left(\phi_M(b',s')\right).$$

The optimum is then characterized by a static optimality condition that relates the surplus to the level of inflation, (18), and an intertemporal optimality condition,

$$\begin{aligned} -U'(\Delta,\theta)\left[1 + \sum_{s'} \Pr(s'|s)\, H'\left(\phi_M(b',s')\right) \frac{\partial \phi_M(b',s')}{\partial b'}\right] &= \frac{\hat{\beta}}{\beta}\sum_{s'} \Pr(s'|s)\, V_M'(b',s') \\ &= \frac{\hat{\beta}}{\beta}\sum_{s'} \Pr(s'|s)\left[-U'(\Delta',s')\right] \end{aligned} \tag{23}$$

The term $\sum_{s'} \Pr(s'|s)\, H'\left(\phi_M(b',s')\right) \frac{\partial \phi_M(b',s')}{\partial b'}$ captures the incentive effect of debt issuance. The current government can influence the choices of next period government only indirectly by affecting the amount of debt inherited by the next period government. Since $\partial \phi_M(b',s')/\partial b' < 0$ and $H' > 0$, this term is negative and acts a tax on debt issuance relative to the Ramsey Euler equation (21). This is because the next period value of real balances is too low from perspective of the current government. The current government can thus boost next period real balances by reducing the amount of debt inherited by its successor, which in turn induces a higher choice of $\phi'$. Therefore, debt issuances are distorted downward relative to the Ramsey outcome.

Figure 1: Credibility of inflation targets




## 4.3 Full model

**Credibility of inflation targets** We first analyze the credibility of an inflation target summarized by a promised $\phi$. The inflation target is satisfied if and only if

$$\xi \geq \xi^* = V_{fd}(b,s) - V_{md}(b,\phi,s) = \max_{\phi_{fd}} V_{md}(b,\phi_{fd},s) - V_{md}(b,\phi,s)$$

As illustrated in Figure 1, deviating from the target allows the government to attain the maximum utility possible net of the cost $\xi$. Thus, a cost $\xi$ greater than the cutoff $\xi^*$ is required for the target to be sustained and for $\eta = 1$. In particular, the cutoff $\xi$ is larger the more ambitious the inflation target. That is, inflation targets closer to the Ramsey value $\phi^*$ are harder to achieve than higher inflation targets.

To make further progress on the characterization, we assume that the value function is concave, as verified in all our numerical solutions. In particular, note that we can write $V_{md}(b,\phi,s)$ as $W(b+\phi,s) + v(\phi)$ because only the total amount of real liabilities, $B = b+\phi$, is relevant for dynamic decisions. We will assume that $W(B)$ is concave. The inflation target is easier to achieve when the marginal utility of government expenditure is lower. This is because the value of reducing the distortions associated with positive surpluses is smaller when $\theta$ is low and government expenditures are less valuable. A similar argument

applies to the level of inherited debt: a lower initial stock of debt $b$ reduces the minimum cost required to satisfy the target, thereby making it more likely that the target can be met.

The next lemma summarizes the above discussion:

**Lemma 4.** *Assume that $W(B,s)$ is concave in $B$. The value of primary surplus is increasing in total government real liabilities. Moreover, if $\phi > \phi_{fd}(b,s)$, then the cutoff $\xi^*(b,\phi,s)$ is increasing in $b$, $\phi$, and $\theta$. That is, the target is less likely to be satisfied if $b$, $\phi$ and $\theta$ are large.*

**Optimal inflation target** We now characterize the optimal inflation target. Recall that inflation target $\pi^*$ for the next period is given by $1+\pi^* = \mu\phi/\phi'$. Thus given current $\mu\phi$, there is an inverse relationship between $\pi^*$ and $\phi'$. We show that it optimal to choose a lower promised $\phi'$ (higher inflation target) than under the Ramsey outcome, in order to better incentivize the next period's government not to deviate from the plan.[14]

To see this, we assume that $\xi$ follows a continuous distribution $f(\xi'|\xi)$. If the optimal inflation target is interior, it must satisfy the following necessary condition whenever the equilibrium objects are differentiable:

$$-U'(\Delta,s)\frac{\partial J(b',\phi',s')}{\partial\phi'} + \hat{\beta}\sum_{s'}\Pr(s'|s)\frac{\partial V(b',\phi',s')}{\partial\phi'} = 0$$

where

$$\begin{aligned}\frac{\partial J(b',\phi',s)}{\partial\phi'}/\beta = &\sum_{s'}\Pr(s'|s)\,\eta(b',\phi',s')\,H'(\phi') \\ &-\sum_{\theta'}\Pr(\theta'|\theta)\frac{\partial\xi^*}{\partial\phi'}\left[H(\phi') - H\left(\phi_{fd}(b',s')\right)\right]f(\xi^*|\xi)\end{aligned}$$

and

$$\sum_{s'}\Pr(s'|s)\frac{\partial V(b',\phi',s')}{\partial\phi'} = \sum_{s'}\Pr(s'|s)\,\eta(b',\phi',s')\left[v'(\phi') + \frac{\partial V(b',\phi',s')}{\partial b'}\right]$$

which can be combined to obtain

$$\begin{aligned}0 = &-U'(\Delta,s)\,\beta H'(\phi') + \hat{\beta}\left[v'(\phi') + \mathbb{E}\left(\frac{\partial V(b',\phi',s')}{\partial b'}|\eta'=1\right)\right] \\ &+\frac{U'(\Delta,s)}{N(b',\phi',s)}\sum_{\theta'}\Pr(\theta'|\theta)\frac{\partial\xi^*}{\partial\phi'}\left[H(\phi') - H\left(\phi_{fd}(b',s')\right)\right]f(\xi^*|\xi)\end{aligned} \tag{24}$$

[14] This is similar to the logic in Dovis and Kirpalani (2021).

where $N(b', \phi', s) \equiv \sum_{s'} \Pr(s'|s)\, \eta(b', \phi', s')$ is the probability that the target is satisfied next period.

The first line in (24) is the optimality condition that must be satisfied in the Ramsey outcome (with $\eta = 1$ always). It equates the marginal benefits of increasing promised $\phi'$ coming from higher resources raised today, $-U'(\Delta, s)\, \beta H'(\phi')$, and a higher value of real balances tomorrow, $\hat{\beta} v'(\phi')$, to the marginal cost of having higher total real liabilities, captured by the term $\hat{\beta}\mathbb{E}\left(\frac{\partial V(b', \phi', s')}{\partial b'} | \eta' = 1\right)$. The term in the second line represents the additional marginal cost of increasing $\phi'$ due to the incentive provision. The term is negative since $\partial \xi^* / \partial \phi' > 0$, $H(\phi') \geq H\left(\phi'_{fd}\right)$ and $U' < 0$. A lower $\phi'$ increases incentives to respect the target ($V_{md} > V_{fd} - \xi'$). This in turn increases the expected marginal value of money as $\phi' > \phi'_{fd}$.

The presence of the last term makes it optimal to choose a lower promised $\phi'$ than under the Ramsey outcome, in order to better incentivize the next period's government not to deviate from the plan. For example, it may not be optimal to promise the Friedman rule under the condition in Proposition 3. Note further that a lower $\phi'$ implies a higher inflation target as the realized inflation if the promised $\phi'$ is delivered is $1 + \pi' = \frac{\mu \phi}{\phi'} = \frac{\beta J(\phi')}{\phi'}$ which is decreasing in $\phi'$.[15]

**Realized inflation and money growth rate** Realized inflation differs from the announced inflation target because of the possibility that the government ignores the target and chooses the statically optimal inflation level. Using the definition of $J$ in (11), we can express the realized inflation level in the monetary-dominant regime as

$$1 + \pi(b, \phi, s_-) = \frac{J(b, \phi, s_-)}{\phi}$$

Note that realized inflation in a given period depends on the inherited debt $b$, the promised value of money holdings, and the state of the economy in the previous period, $s_-$, (and not the current state $s$).

[15] However, if we have CRRA preferences with a maximal amount of real balances, $\phi^*$, the constraint $\phi' \leq \phi^*$ can be binding if

$$\begin{aligned} &-U'(\Delta, s)\, \beta H'(\phi^*) + \hat{\beta}\left[v'(\phi^*) + \mathbb{E}\left(\frac{\partial V(b', \phi^*, s')}{\partial b'} | \eta' = 1\right)\right] \\ &> -\frac{U'(\Delta, s)}{N(b', \phi^*, s)} \sum_{\theta'} \Pr(\theta'|\theta)\, \frac{\partial \xi^*}{\partial \phi'} \left[H(\phi^*) - H\left(\phi_{fd}(b', s')\right)\right] f(\xi^*|\xi) \end{aligned}$$

in which case $\phi' = \phi^*$. In the quantitative analysis we assume quadratic utility for real balances and $\phi' < \phi^R \leq \phi^*$ where $\phi^R$ the promise for real balances in the Ramsey outcome and $\phi^*$ is the satiation point, $v'(\phi^*) = 0$.

If instead we are in the fiscal-dominant regime then inflation depends on the current state as well:

$$1+\pi_{fd}\left(b,\phi,s_{-},s\right)=\frac{J\left(b,\phi,s_{-}\right)}{\phi_{fd}\left(b,s\right)}.$$

This is because $s$ affects the current fiscal surpluses and therefore the chosen $\phi_{fd}\left(b,s\right)$. Since in equilibrium $\phi_{fd}\left(b,s\right)<\phi$, the realized inflation is higher in the fiscal dominant regime than in the monetary dominant regime.

The model also has implications about the growth rate of money supply. In particular, if the perceived commitment to monetary dominance in the future is weak, the government must follow a much tighter monetary policy to implement its target $\phi$. To see this, note that the gross money growth rate is

$$\mu=J\left(b',\phi',s\right)/\phi.$$

If the expected value of money $J\left(b',\phi',s\right)$ falls, then money growth $\mu$ must also fall to implement the target $\phi$.

**Debt dynamics** We next consider the optimal debt issuance. We show that debt issuance is lower than in the Ramsey outcome. This is because of the presence of a wedge in the government's Euler equation that acts as a tax on debt issuance, similarly to the Markov equilibrium. The magnitude of the wedge, and therefore of the amount of underborrowing, depends on $\xi$. If this shock is large then the wedge is small.

The necessary condition for an optimal interior $b'$ is

$$\begin{aligned}-U'\left(\Delta,s\right)\left(1+\frac{\partial J\left(b',\phi',s\right)}{\partial b'}/\beta\right)&=-\frac{\hat{\beta}}{\beta}\sum_{s'}\Pr\left(s'|s\right)\frac{\partial V\left(b',\phi',s'\right)}{\partial b'}\\&=\frac{\hat{\beta}}{\beta}\sum_{s'}\Pr\left(s'|s\right)\left[-U'\left(\Delta\left(s'\right),s'\right)\right]\end{aligned}\tag{25}$$

The term $\frac{\partial J\left(b',\phi',s\right)}{\partial b'}/\beta$ is negative and effectively acts as a tax on debt issuance, leading to lower debt levels relative to the Ramsey Euler equation (21). To see this, note that

$$\begin{aligned}\frac{\partial J\left(b',\phi',s\right)}{\partial b'}/\beta=&\sum_{s'}\Pr\left(s'|s\right)\left(1-\eta\left(b',\phi',s'\right)\right)H'\left(\phi_{fd}\left(b',s'\right)\right)\frac{\partial\phi_{fd}\left(b',s'\right)}{\partial b'}\\&-\sum_{s'}\frac{\partial\xi^{*}}{\partial b'}\left[H\left(\phi'\right)-H\left(\phi_{fd}\left(b',s'\right)\right)\right]\leq0\end{aligned}\tag{26}$$

The first term captures the incentive effect described in the Markov equilibrium, which

operates for next-period states where it is optimal to deviate from the inflation target and enter the fiscal dominant regime ($\eta' = 0$). This term is negative since $H' > 0$ and $\partial \phi_{fd}(b', s') / \partial b' < 0$.[16] The second term is also negative because, as explained above, since the cutoff $\xi^*$ is increasing in the amount of inherited debt, and $H(\phi') > H\left(\phi_{fd}(b', s')\right)$.

It is therefore optimal to reduce debt issuance in order to induce a higher $\phi_{fd}$ in the event of a switch to the fiscal dominant regime in the next period, and to strengthen the next-period government's incentive to adhere to the inflation target. This incentive to limit indebtedness becomes stronger as the probability of switching to the fiscal dominant regime increases.

Heuristically, if there are two distributions for $\xi'$ and one first-order stochastically dominates the other, we should expect that the wedge in the Euler equation (25), $|\frac{\partial J(b', \phi', s)}{\partial b'} / \beta|$, is larger for the dominated distribution. That is, the higher the expected future costs of switching to the fiscal dominant regime, the smaller the wedge becomes. In the limiting case where only the monetary dominant regime is possible next period—as in the Ramsey outcome—the wedge disappears entirely, and there are no downward distortions to debt issuance. This basic intuition is central to understanding the different responses of debt to shocks to $\theta$ or $\xi$, which we study next.

In Appendix C, we prove for a deterministic economy with $\beta = \hat{\beta}$ that a Markov equilibrium outcome is characterized by a higher level of inflation and a lower level of debt relative to the Ramsey outcome that starts with the same level of inherited debt. This result illustrates how greater commitment allows for lower inflation and higher debt..

To better understand these dynamics, we consider a typical equilibrium outcome for a calibrated version of our model.[17] The blue lines in Figure 2 plot a representative path for our model with endogeneous switches. It is instructive to compare these outcomes with those under the Ramsey and Markov equilibria for the same sequence of $\theta$ shocks (red and green lines respectively). We observe that the Markov equilibrium sustains very little debt and exhibits high and volatile inflation.[18] In contrast, the Ramsey equilibrium supports substantially higher debt levels along with lower and more stable inflation.

[16] To see that $\partial \phi_{fd} / \partial b < 0$, note that totally differentiating (18) we have

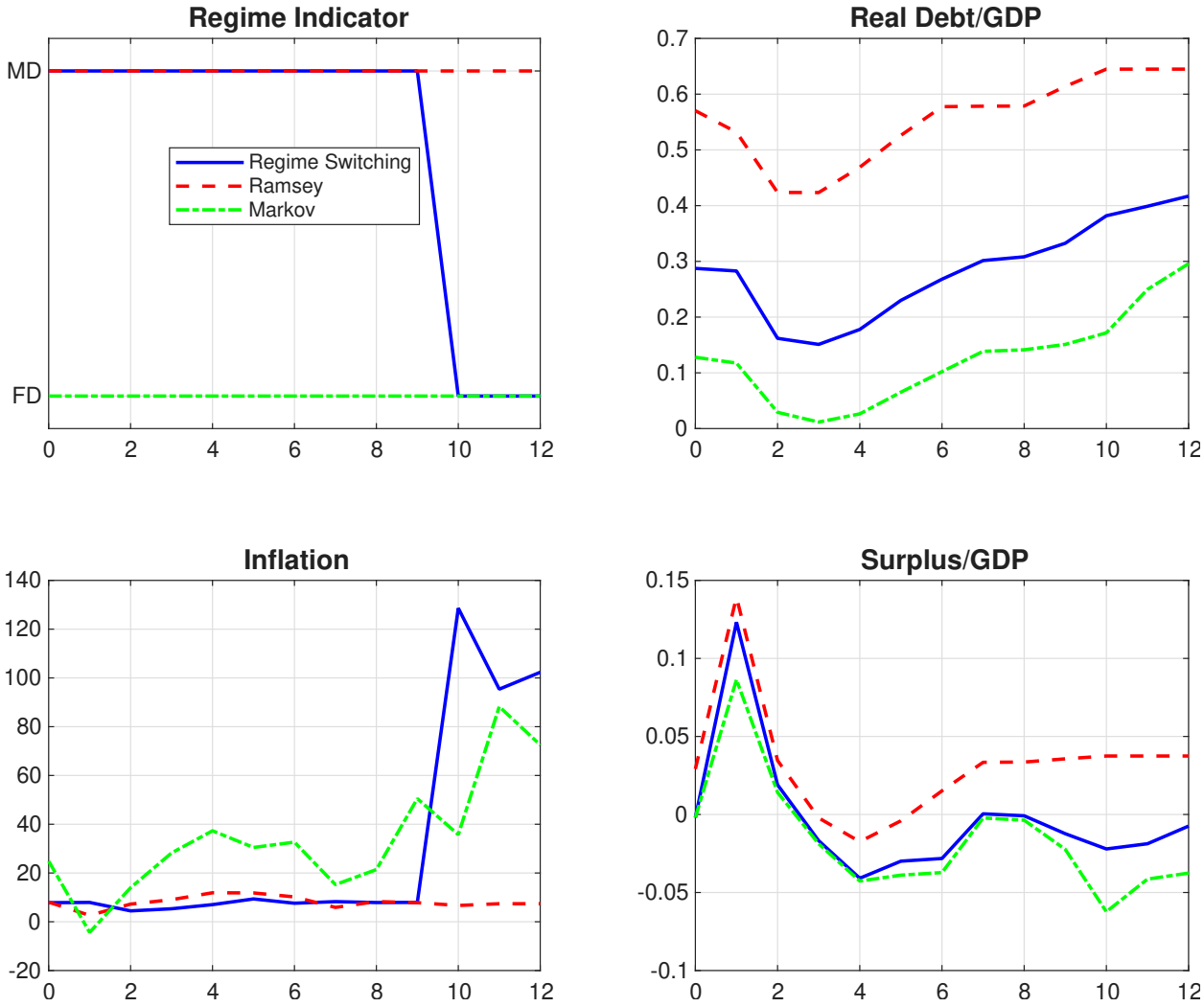
$$\frac{\partial \phi_{fd}}{\partial b} = \frac{-U''(\Delta, \theta)}{v''\left(\phi_{fd}\right)} \frac{\partial \Delta}{\partial b} < 0$$

where the inequality follows because $U$ and $v$ are concave and $\partial \Delta / \partial b > 0$ as showed in Lemma 4.

[17] The calibration is the one for the Latin American (LA) economies presented in Section 6.1.

[18] This finding echoes the result we prove for a deterministic economy with $\beta = \hat{\beta}$ in Appendix B.

Figure 2: Typical outcomes




Notes: The figure plots a representative path from a long simulation of the model calibrated to the Latin American economies (see Section 6.1 for details on the calibration). The solid blue lines report outcomes under endogenous regime switching, the dashed red lines report the Ramsey equilibrium outcome, and the dash-dotted green lines report the Markov equilibrium outcome, all subject to the same sequence of $\theta$ shocks. The top-left panel plots the regime indicator, where MD denotes monetary dominance and FD denotes fiscal dominance. The top-right panel reports the real debt-to-GDP ratio, the bottom-left panel reports inflation in annualized percent, and the bottom-right panel reports the primary surplus-to-GDP ratio. The x-axis represents time in years.

**Endogenous versus exogenous regime switching** A distinctive feature of our model relative to the existing literature on fiscal-monetary regime switching (Leeper (1991), Bianchi (2013)), and (Debortoli and Nunes (2010)) is that the regime is endogenous. To quantify the importance of this feature, we compare the outcome of our model with one where regime switching is exogenous. To make a fair comparison, we simulate data from our calibrated model and then estimate a Markov chain for the regime indicator, ignoring its dependence on state variables. We then solve a version of the model where the regime follows this exogenous process.

Figure 3 displays a long simulation of both models subject to the same sequence of fundamental shocks $\theta$ and the same realized regime path. The top two panels show that both models experience identical regime switches and face the same marginal utility shocks (as illustrated by the top two panels). Despite this, the bottom panels reveal that the two models generate quite different outcomes for both debt and real money balances. In particular, debt in the model with exogenous regime switches is higher than in our (endogenous) model, and real money balances are also higher, reflecting a lower inflation target.

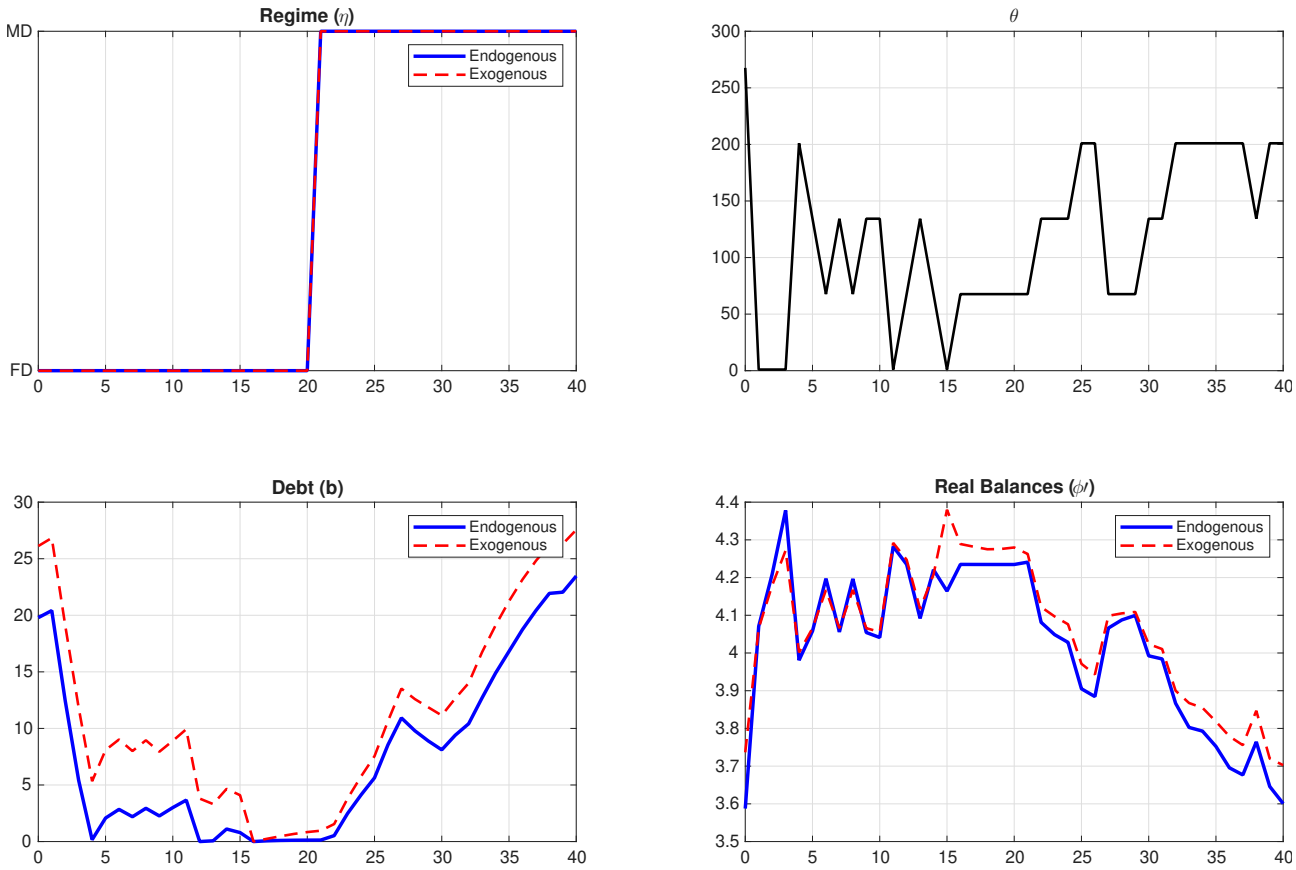
This difference arises because of the incentive effects we discussed earlier. In our model, the government's choices of debt and the inflation target both reflect the need to ensure that future governments honor the mandate. Lower debt and a less ambitious inflation target (lower $\phi'$) make it easier to sustain the monetary-dominant regime. With exogenous regime switches, the government does not internalize how its choices affect future regime outcomes - transition probabilities are fixed. Thus, there is no incentive to strategically limit debt or choose a higher inflation target.

# 5 Two types of disinflation

We now describe how the restrictions implied by the model on equilibrium outcomes can help to disentangle the role played by $\xi_t$ and $\theta_t$. Specifically, in our application we will focus on big disinflation episodes, that is periods during which governments managed to bring inflation under control after years of high inflation. There are two ways in which the model can generate these dynamics. Inflation can decline either due to a reduction in the marginal value of government spending, $\theta$, or due to an increase in the (expected) cost of deviating from the promised inflation. We will refer to the former as *fundamental disinflation* and to the latter as *institutional disinflation*.

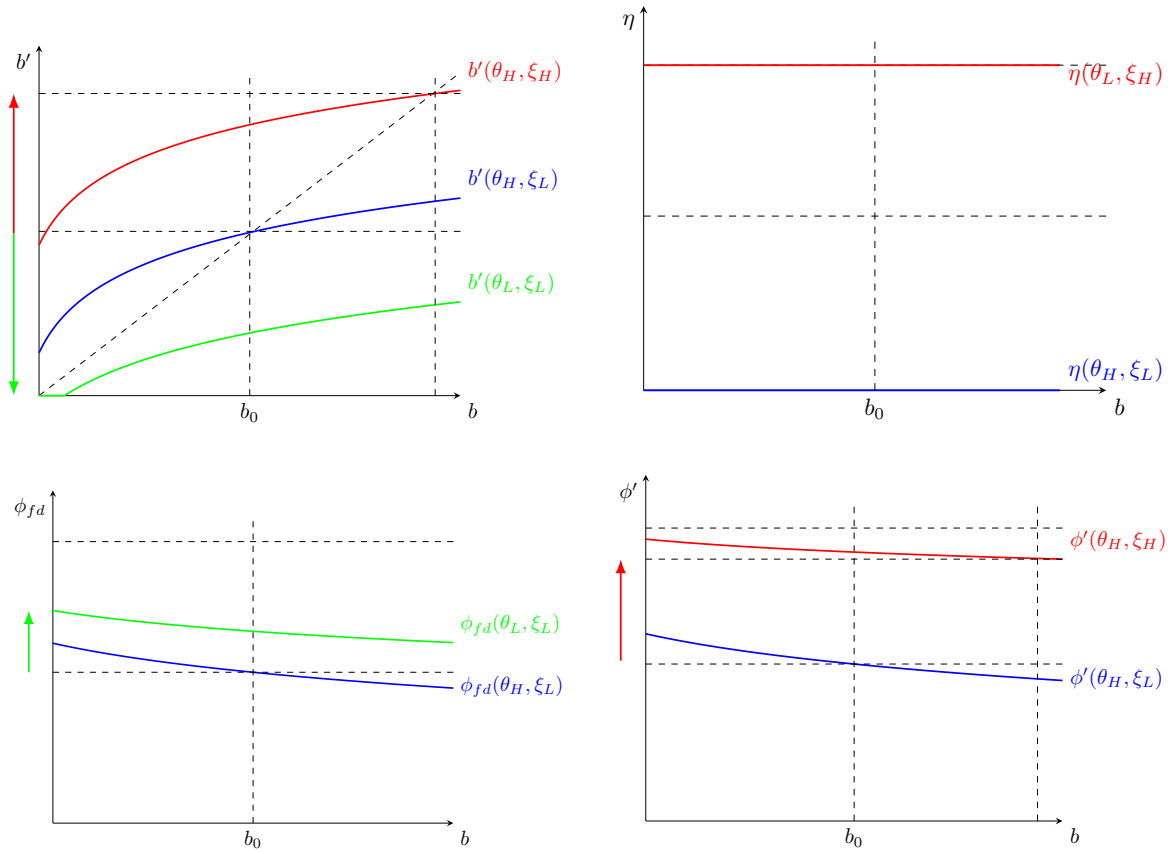
In this section, we will show that these types of disinflation have distinct implications for the dynamics of public debt. In particular, fundamental disinflations are associated

Figure 3: Typical outcomes for endogenous and exogenous models




Notes: The figure compares a simulation of the model with endogenous regime switching (solid blue lines) to a version with exogenous regime switching (dashed red lines), both using the Latin American calibration (see Section 6.1). The exogenous model is constructed by estimating a Markov chain for the regime indicator from simulated data of the endogenous model, ignoring its dependence on state variables. Both models are subject to the same sequence of fundamental shocks $\theta$ and the same realized regime path. The top-left panel plots the regime indicator $\eta$, where MD denotes monetary dominance and FD denotes fiscal dominance. The top-right panel plots the realized marginal utility shock $\theta$. The bottom-left panel reports real debt and the bottom-right panel reports real money balances ($\phi'$). The x-axis represents time in years.

Figure 4: Two types of disinflation




with a declining path for real debt, while institutional disinflations are characterized by an increasing path for real debt. In Section 6 we will use this logic along with data on inflation and debt to disentangle the relative importance of institutional and fundamental considerations.

**Fundamental disinflation** Consider a path in which the realization of $\xi_t$ is low enough so that it is always optimal to be in the fiscal dominant regime. Along this path, the value of real money balances (and inflation) is determined by the static condition (18) and is therefore closely tied to fiscal considerations, as in a Markov equilibrium.

Suppose next that the desirability of government expenditures falls from $\theta_H$ to $\theta_L$ in period $t_0$ but it remains optimal to stay in the fiscal dominant regime. As illustrated in the third panel of Figure 4, the reduction in $\theta$ shifts the policy function $\phi_{fd}(b, \theta)$ upward: for any level of real debt, the government finds it optimal to choose a higher value for real balances, reflecting the lower marginal value of relaxing its budget constraint when government spending is less valuable. This has the effect of reducing inflation.

The optimal policy for debt issuance, by contrast, shifts downward (see the first panel of the figure). This occurs because the government now has stronger precautionary saving motives and therefore chooses to reduce its debt issuance. As a result, a decline in $\theta$ while keeping $\xi$ at a low level (e.g. $\xi = 0$) leads to a decline in inflation and a decrease in real debt.

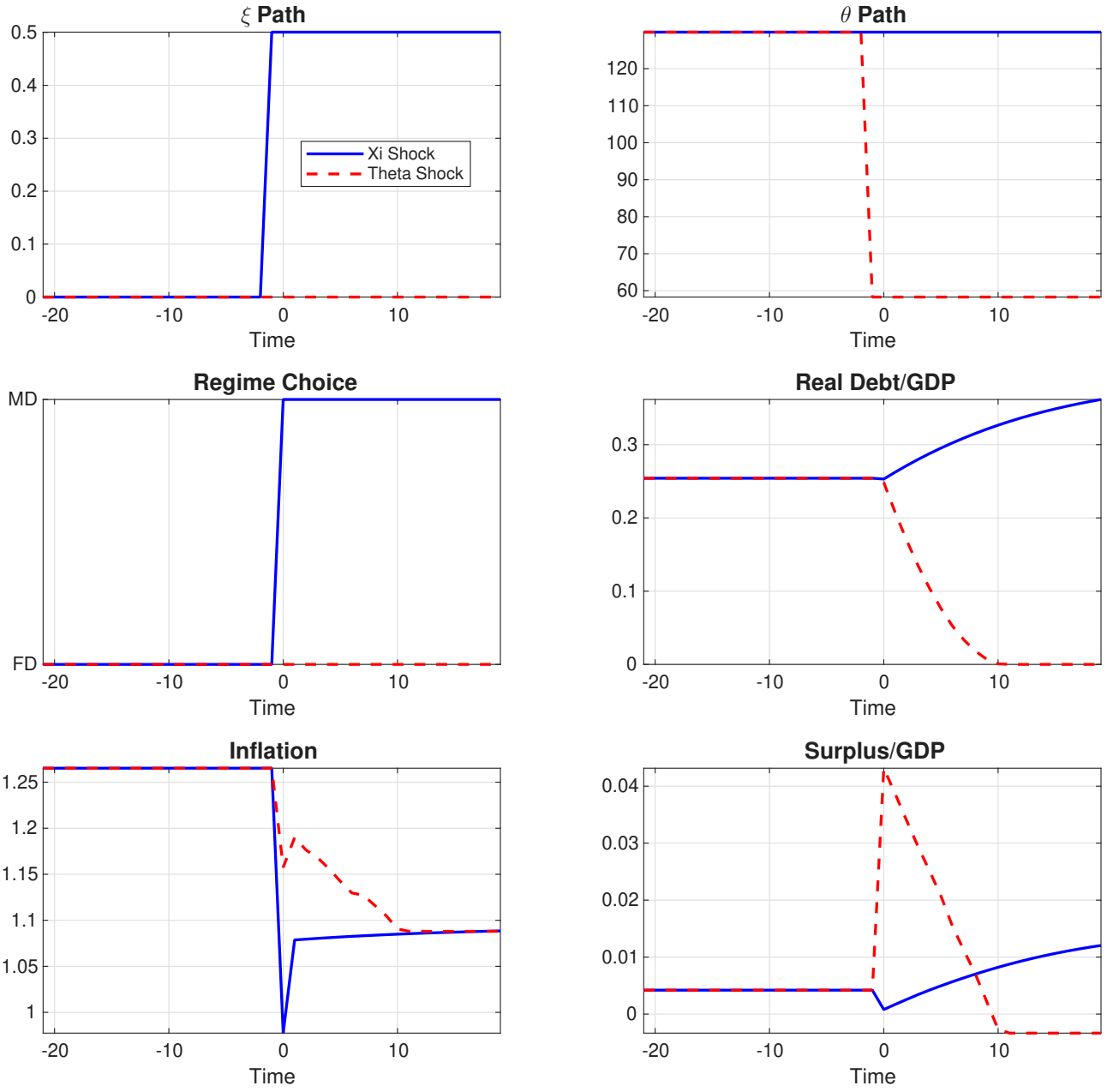
The red dashed lines in Figure 5 depicts the impulse response function (IRF) to a reduction in $\theta$ in our calibrated model (see below for details). The realized shocks are shown in the first two panels. Following the shock, the debt-to-GDP ratio declines, while the inflation rate initially drops sharply in the period when $\theta$ falls, reflecting the lower marginal cost of generating primary surpluses. Inflation then continues to decline gradually, tracking the falling path of debt, which further reduces the marginal cost of surpluses.

**Institutional disinflation** We now consider the effects of an increase in the cost of deviating from the promised inflation target. Specifically, we begin with the same path of low $\xi$ and high $\theta$ considered above, and suppose that at $t_0$, the realized cost of deviation permanently rises from $\xi_L$ to $\xi_H$, with $\xi_H$ high enough to make it optimal to switch to the monetary dominant regime, as shown in the second panel of Figure 4. In this case, the reduction in inflation is driven by the change in the regime. The realized $\phi$ now equals to the promised value of real balances which is higher than the statically optimal level $\phi_{fd}$. Critically, if the process for $\xi$ is persistent, an increase in current $\xi$ implies an increase in the expected value for $\xi'$. Thus, as argued in the previous section, the government now has lower incentives to reduce the amount of debt it issues because the wedge in (25) given by (26) is smaller in absolute value.

There is also another effect at play. As the government shifts to the monetary-dominant regime, the present value of seigniorage revenues, the term $\sum_{j=1}^{\infty} \sum_{s^{t+j}} \beta^j \Pr\left(s^{t+j}|s^t\right) h\left(\phi\left(s^{t+j}\right)\right)$ in the intertemporal government budget constraint (13), falls, and the government must finance the inherited real liabilities with a higher present value of surpluses. Since the government is impatient, these higher surpluses are back-loaded. This also results in an increase in the level of debt issued. Thus, the government increases its debt issuance as shown in the first panel of Figure 4.

The solid blue lines in Figure 5 depict an IRF to an increase in $\xi$ for our calibrated model. The path of real government debt is increasing as argued above. The inflation rate instead initially jumps downward at $t_0$. The level of inflation may overshoot its value a few years out because as the level of debt increases the government may find it optimal to reduce the promised value for real balances (increase the inflation target) to ensure that the target is satisfied.

Figure 5: Impulse responses




Notes: The figure plots impulse response functions for the model calibrated to the Latin American economies (see Section 6.1). The solid blue lines depict the response to a permanent increase in the cost of deviating from the inflation target, $\xi$ (institutional disinflation), while the dashed red lines depict the response to a permanent decrease in the marginal value of government spending, $\theta$ (fundamental disinflation). The top two panels show the paths of the exogenous shocks $\xi$ and $\theta$, respectively. The middle-left panel reports the regime choice, where MD denotes monetary dominance and FD denotes fiscal dominance. The middle-right panel reports the real debt-to-GDP ratio. The bottom-left panel reports gross inflation and the bottom-right panel reports the primary surplus-to-GDP ratio. Time is measured in years, with the shock occurring at time 0.

# 6 Case studies

In this section, we take the model to the data and uses it as a measurement device to interpret actual disinflation episodes through the identification logic developed in the previous section. We fit the model to data from Colombia, Chile, and the United States and we apply the particle filter to recover the realizations of $\{\theta_t, \xi_t\}$ that account for the observed paths of inflation and debt-to-GDP.

## 6.1 Calibration

We assume the following functional form for the preferences of the stand-in household:

$$v(l) = \chi \frac{l^{1+\psi}}{1+\psi}, \quad v(m) = \kappa m - \eta m^2, \quad u(g) = \frac{g^{1-\sigma}}{1-\sigma}.$$

The fundamental shock $\theta_t$ follows an AR(1) process given by:

$$\theta_{t+1} = (1-\rho_\theta)\overline{\theta} + \rho_\theta \theta_t + \sigma_\theta \varepsilon_{t+1}.$$

We assume that $\xi_t$ has a persistent and two transitory components, $\xi_t = \xi_{1t} + \xi_{fd,t} - \xi_{md,t}$. The persistent component $\xi_{1,t}$ is a discretized version of an AR(1) process with parameters $\overline{\xi}, \rho_\xi, \sigma_\xi$ using the Tauchen method where we only allow for weakly positive values. The iid components, $\xi_{fd,t}$ and $\xi_{md,t}$, follow a Gumbel$(\mu, 1/\lambda)$ with mean zero so $\mu + \gamma/\lambda = 0 \rightarrow \mu = -\gamma/\lambda$ for $\lambda > 0$ where $\gamma$ is the Euler-Mascheroni constant.[19] The model solution is described in Appendix C.

The model is parameterized by the preference parameters $(\beta, \hat{\beta}, \chi, \psi, \kappa, \eta, \sigma)$ and the parameters governing the stochastic processes for $\theta_t$ and $\xi_t$, $(\rho_\theta, \sigma_\theta, \bar{\theta})$ and $(\rho_\xi, \sigma_\xi, \lambda)$. We set a subset of parameters to some predetermined values, and we calibrate the remaining to match a set of moments related to fiscal and monetary variables.

We set the inverse of the Frisch elasticity, $\psi$, to 1 and the inverse of the elasticity of substitution, $\sigma$, for government expenditures to the common parameter of 2. We set the stand-in household's discount factor, $\beta$, to 0.95 so that the real interest rate is approximately 5 percent at our chosen annual frequency. Finally, we set $\lambda = 0.2$ and $\lambda = 0.5$ for the Latin American and US calibration, respectively.

We calibrate the other parameters to match key moments for fiscal and monetary variables. We target the average real money balances, the mean, autocorrelation and standard

[19] We introduce the iid shock to ensure convergence of the solution algorithm.

Table 1: Calibration: Parameters

| Parameter | Description | Value | |
|---|---|---|---|
| | | LA Calibration | US Calibration |
| $\beta$ | Households' discount factor | 0.95 | 0.95 |
| $\psi$ | Inverse Frisch elasticity | 1 | 1 |
| $\sigma$ | Inverse EIS gov't expenditures | 2 | 2 |
| $\hat{\beta}$ | Gov'ts discount factor | 0.92 | 0.91 |
| $\chi$ | Labor disutility parameter | 0.015 | 0.021 |
| $\kappa$ | Money-demand parameter | 0.68 | 0.7 |
| $\eta$ | Money-demand parameter | 0.07 | 0.06 |
| $\bar{\theta}$ | Mean $\theta$ | 130 | 2 |
| $\rho_\theta$ | Persistence of $\theta$ | 0.9 | 0.9 |
| $\sigma_\theta^2$ | Variance of $\theta$ | 60 | 20 |
| $\rho_{\tilde{\xi}}$ | Persistence of $\tilde{\xi}$ | 0.998 | 0.99 |
| $\sigma_{\tilde{\xi}}$ | Variance of $\tilde{\xi}$ | 0.112 | 0.3 |
| $\lambda$ | Gumbel parameter | 0.2 | 0.5 |

deviation of primary surpluses, and key moments summarizing the distribution of inflation: average inflation in the first and fourth quartile of the distribution, and the probabilities that inflation remains in either the top or bottom quartile from one period to the next. We consider two calibrations: one for Latin American Economies ("LA calibration") from 1960-2017 and one for the United States ("US calibration").[20] Table 1 reports the parameters value for these two calibrations, and Table 2 reports the target moments in the data and in the model.

Heuristically, the persistence, mean, and variance of primary surpluses are informative for the parameters governing $\theta_t$, $\left(\rho_\theta, \sigma_\theta, \bar{\theta}\right)$. The persistence of inflation in Q1 and Q4 are informative about the persistence of the costs of deviating from the target. The remaining parameters $\chi$, $\eta$, and $\kappa$ are informed by the average inflation in the first quartile (Q1), the average inflation in the fourth quartile (Q4), and the average real money balances.

The model defines a nonlinear state-space system

$$y_t = f\left(S_t\right) + \varepsilon_{yt}$$
$$S_{t+1} = k\left(S_t, \varepsilon_{t+1}\right)$$

where $y_t$ is a vector of observable variables, $S_t = (b_t, \phi_t, \theta_t, \tilde{\xi}_t)$ is the state vector, $\varepsilon_t$ are the innovations to the exogenous states $s_t = (\theta_t, \tilde{\xi}_t)$, and $\varepsilon_{yt}$ is a vector of uncorrelated Gaussian measurement errors. The functions $f$ and $k$ are obtained using the model's nu-

[20]The countries included are Argentina, Bolivia, Brazil, Chile, Colombia, Ecuador, Mexico, Paraguay, Peru, Uruguay, and Venezuela.

Table 2: Calibration: Moments

| Moment | LA Calibration | | US Calibration | |
|---|---|---|---|---|
| | Data | Model | Data | Model |
| Avg. inflation in Q1 | 3.41% | 3.85% | -1% | 1.66% |
| Avg. inflation in Q4 | 57.0% | 58.27% | 9.0% | 7.88% |
| Prob. of staying in Q1 | 0.69 | 0.69 | 0.56 | 0.78 |
| Prob. of staying in Q4 | 0.77 | 0.67 | 0.73 | 0.71 |
| Average debt-to-GDP | 35.38% | 34.87% | 56.0% | 61.86% |
| Avg. real money balances-to-GDP | 9.89% | 6.60% | 8.06% | 12.17% |
| Variance of primary surplus | 10.68 | 12.88 | 29.68 | 4.75 |
| Autocorr. of primary surplus | 0.67 | 0.55 | 0.82 | 0.61 |
| Avg. of primary surplus-to-GDP | 0.42% | 0.95% | 2.42% | 2.58% |

Notes: The table reports targeted moments in the data and their model counterparts for both the Latin American (LA) and US calibrations. The LA calibration uses data from 11 Latin American economies (ARG, BOL, BRA, CHL, COL, ECU, MEX, PAR, PER, URU, VEN) over 1960–2017; the US calibration uses US data. Q1 and Q4 denote the first and fourth quartiles of the inflation distribution, respectively. The probability of staying in Q1 (Q4) is the frequency with which inflation remains in the bottom (top) quartile from one year to the next. Inflation is expressed in annualized percent. The variance and autocorrelation of primary surpluses are computed over the full sample. The model moments are computed from a long simulation.

merical solution. The vector of observables include inflation and the debt-to-GDP ratio, $y_t = (\pi_t, b_t/l_t)$. We use the model to recover the path for $\{\theta_t, \xi_t\}$ given the observed path for these variables by applying a non-linear particle filter. The measurement errors are assumed to be independent and normally distributed, $\varepsilon_{yt} \sim N(0, \Sigma)$ , where $\Sigma = \text{diag}(\sigma_\pi^2, \sigma_b^2)$ with $\sigma_\pi$ and $\sigma_b$ chosen depending on the particular country.[21]

## 6.2 Colombia

We first consider the disinflation episode in Colombia during the 1990s. In 1991, Colombia instituted a new constitution that granted substantial independence to its central bank, Banco de la República, explicitly mandating price stability as its primary objective and significantly insulating monetary policy from political influence. As written in Perez-Reyna and Osorio-Rodríguez (2017)

> The central bank was given technical independence as to the instruments employed to achieve its main task, which was defined solely as the control of inflation. In addition, the monetary board was replaced by a board of governors in which the minister of finance had only one vote (of seven) and no veto power. Finally, the constitution prescribed that any direct loan from the central bank to

[21] We set $\sigma_\pi = 3$pp,$\sigma_b = 2$pp for Colombia, $\sigma_\pi = 1$pp,$\sigma_b = 3$pp for Chile, and $\sigma_\pi = 2$pp,$\sigma_b = 6$pp for the US.

> the central government would require unanimous approval by the members of the board, thus all but forbidding monetary financing under this guise. To date, the independent central bank has never granted any direct loans to the central government.

In 2001, Colombia adopted an explicit inflation targeting regime with a long-term inflation goal of 3%. Prior to the 1991 reform, the central bank lacked autonomy, often making monetary policy susceptible to government pressures. As a result Colombia suffered from persistent high inflation despite the relatively low level of debt. Starting in 1991, inflation started falling.

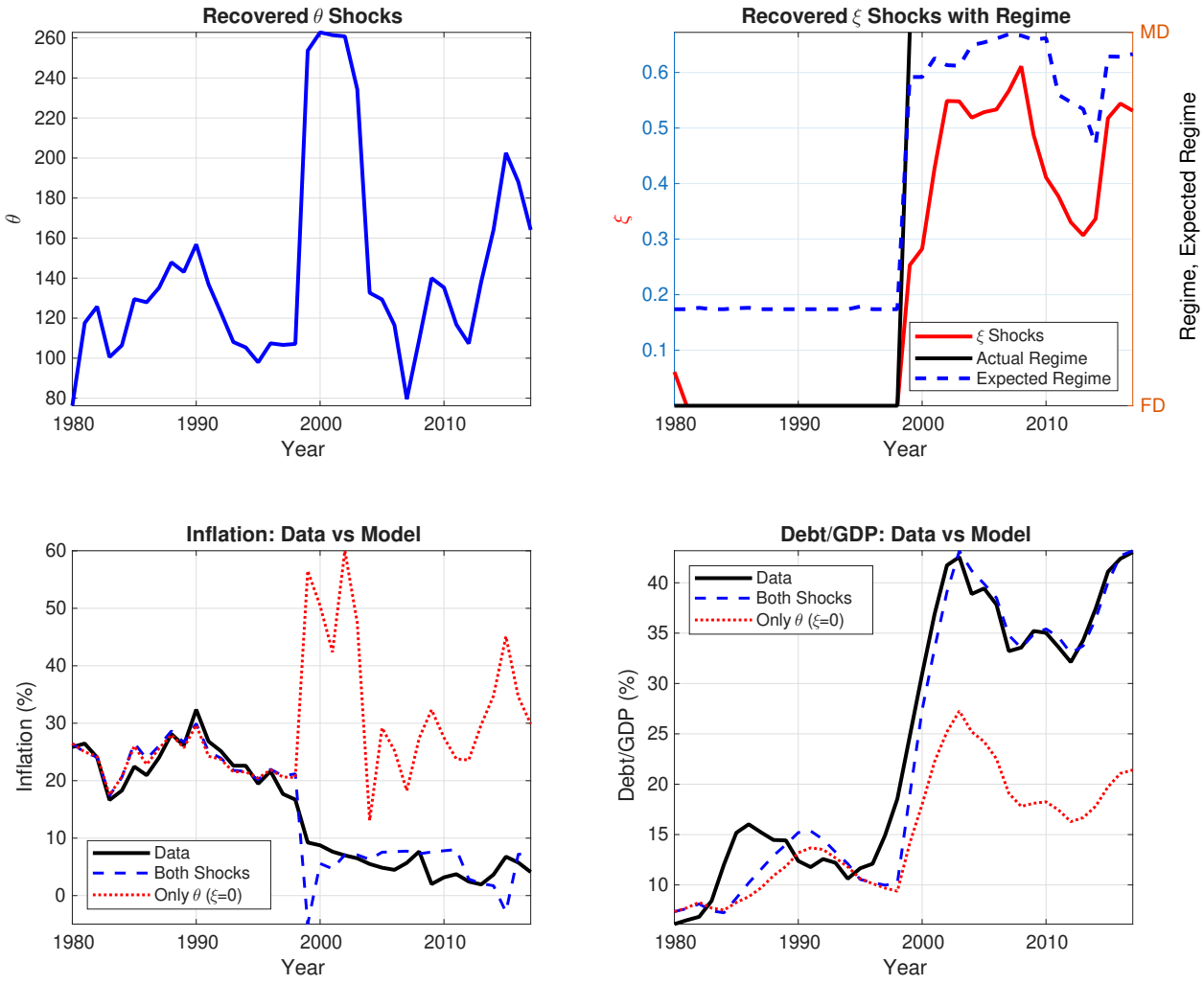
We apply the particle filter to our model and estimate the path of the structural shocks $\{\theta_t, \xi_t\}$ for the periods 1980-2017. The results are presented in Figure 6. The top two panels at the bottom display the resulting series of $\theta$ and $\xi$ shocks. The bottom two panels show that the model implied paths for inflation and debt (the blue dashed lines) closely match the observed trajectories of debt and inflation (solid black lines).

The model accounts for the reduction in inflation in the 1990s with an increase in the cost of deviating from the inflation target in 1997. Such an increase results in a persistent shift to a monetary-dominant regime from 1997 onward. The observed increase in the debt-to-GDP ratio from 1994 to 2002 is driven by the switch to a monetary-dominant regime that allows for greater debt issuances and by higher-than-average realizations of $\theta_t$.[22] The model can account for the stable and low inflation (at least from 1998 onward) and the increasing path for the debt-to-GDP ratio because in the MD regime, fiscal considerations – $\theta_t$ and the inherited debt – have little effect on inflation. Monetary policy is credibly delegated, and there is little spillover to monetary variables from the fiscal side. It is then possible to find a path of $\theta_t$ to match the debt dynamics.

To understand the importance of the detected change in the regime, we also conduct a counterfactual exercise in which we retain the computed sequence of $\theta$ shocks but hold $\xi_t = 0$ throughout the entire period, so it is optimal to always be in the FD regime. This corresponds to the red dotted lines in the second panel. This scenario aims to replicate conditions had Colombia not undertaken a *credible* constitutional reform. As illustrated, without this institutional change, debt would have similarly increased, driven by the high realizations of $\theta_t$, but inflation would have remained constant or even risen during the latter half of the decade. This result underscores the crucial role credible institutional reforms played in simultaneously achieving higher debt levels and declining inflation in Colombia

[22] The increasing path for $\theta_t$ is consistent with the observation that in Colombia the "size of the government almost doubled between 1991 and 1999, as the ratio of central government expenditures to GDP increased from 8.9 percent to 16.9 percent , Perez-Reyna and Osorio-Rodríguez (2017)."

Figure 6: Colombia




Notes: The figure reports results from applying the particle filter to Colombian data over 1980–2017 using the LA calibration (see Section 6.1). The top-left panel plots the recovered path of the fundamental shock $\theta$. The top-right panel plots the recovered credibility shock $\xi$ (solid line, left axis) along with the actual regime (dashed line, right axis) and the expected regime (dash-dotted line, right axis), where MD denotes monetary dominance and FD denotes fiscal dominance. The bottom two panels compare the data (solid black lines) with the model-implied paths under both shocks (dashed blue lines) and under only $\theta$ shocks with $\xi$ set to zero (dotted red lines) for inflation (bottom-left, in annualized percent) and the debt-to-GDP ratio (bottom-right, in percent).

during this period.

Our result of an institutional improvement in Colombia in the mid-90s is consistent with prior narrative analyses and also with the behavior of other macroeconomic variables.[23] Interestingly, the model does not identify the increase in the credibility of the reform in 1992, the first year after the reform, but only in 1998. This is because inflation was still relatively high, and the level of debt only started to increase in that period. This may be driven by the fact that it took time for the government to convince private agents that the new institutional arrangement was credible and not just a cosmetic adjustment. Our measure thus brings a complementary perspective to purely de-jure measures of central bank credibility like Grilli, Masciandaro, and Tabellini (1991); Cukierman (2008); Romelli (2022). For example, the credibility index for Colombia in Romelli (2022) increases in 1992 and stays constant at this level.

The identified switch to a monetary dominant regimes in 1998 coincides with one of the largest contractions and deepest financial crisis in the history of Colombia. Consistently with the recovered large increase in $\theta$, these were times of high fiscal needs for the consolidated government budget constraint as "the central bank also heavily used its lender of last resort facilities, but unlike in the eighties, it was not allowed to finance bailout operations with monetary emission, (Perez-Reyna and Osorio-Rodríguez, 2017)." This pattern is consistent with a deeper theory of reputation based on signaling with hidden types: the institutional setting with an independent central bank became credible when the economy with a large adverse shocks and the government decided not to monetize the debt and increase the monetary base.

The identified switch to a monetary-dominant regimes in 1998 coincides with one of the largest contractions and deepest financial crisis in the history of Colombia. Consistently with the recovered large increase in $\theta$ , these were times of high fiscal needs for the consolidated government budget constraint as "the central bank also heavily used its lender of last resort facilities, but unlike in the eighties, it was not allowed to finance bailout operations with monetary emission, (Perez-Reyna and Osorio-Rodríguez, 2017)." This pattern is consistent with a theory of reputation based on signaling with hidden types: the institutional arrangement centered on an independent central bank became credible when the economy was hit by a large adverse shock and the government nevertheless chose not to monetize the associated debt and expand the monetary base.

[23]For example, starting from the mid 1990s the Colombia government started to issue a progressively larger share of debt to foreigners in local currency (Perez-Reyna and Osorio-Rodríguez, 2017), which would also be consistent with more credible inflation promises, see Du, Pflueger, and Schreger (2020).

## 6.3 Chile

Beginning in the late 1980s, Chile enacted a variety of fiscal and monetary reforms. It tightened public finances and, for roughly three decades, consistently posted budget surpluses (Caputo and Saravia (2018)). Since 2001 these efforts have been formalized by a structural fiscal rule that targets a surplus of 1 percent of GDP. On the monetary front, a 1989 constitutional law granted the Central Bank of Chile full autonomy and the country moved to an explicit inflation regime targeting soon after.

As we did for the case of Colombia, we use the model to filter a sequence of fiscal-needs shocks ($\theta_t$)and credibility shocks ($\xi_t$) to reconcile our model with Chile's post-1990 data shown in Figure 7. In contrast to Colombia, both inflation and the debt-to-GDP ratio declined over this period.

During the first half of the 1990s, the drop in inflation can be replicated either by a fall in fiscal needs or by a rise in the penalty for deviating from the inflation target; each channel on its own is sufficient to match the joint movements in inflation and debt. The distinction between them becomes critical in the second half of the decade: inflation keeps falling while debt-to-GDP merely flattens out. Replicating this pattern requires credibility shocks—an isolated increase in fiscal needs would stabilize the debt ratio but, counterfactually, would drive inflation back up. As in the case of Colombia, we also use a particle filter to compute a path of $\theta$ shocks that only targets the path of inflation when $\xi_t = 0$ for the entire period. Like in Colombia, one can find a path of $\theta_t$ shocks that replicates the declining path of inflation but this would not be able to sustain the level of debt.

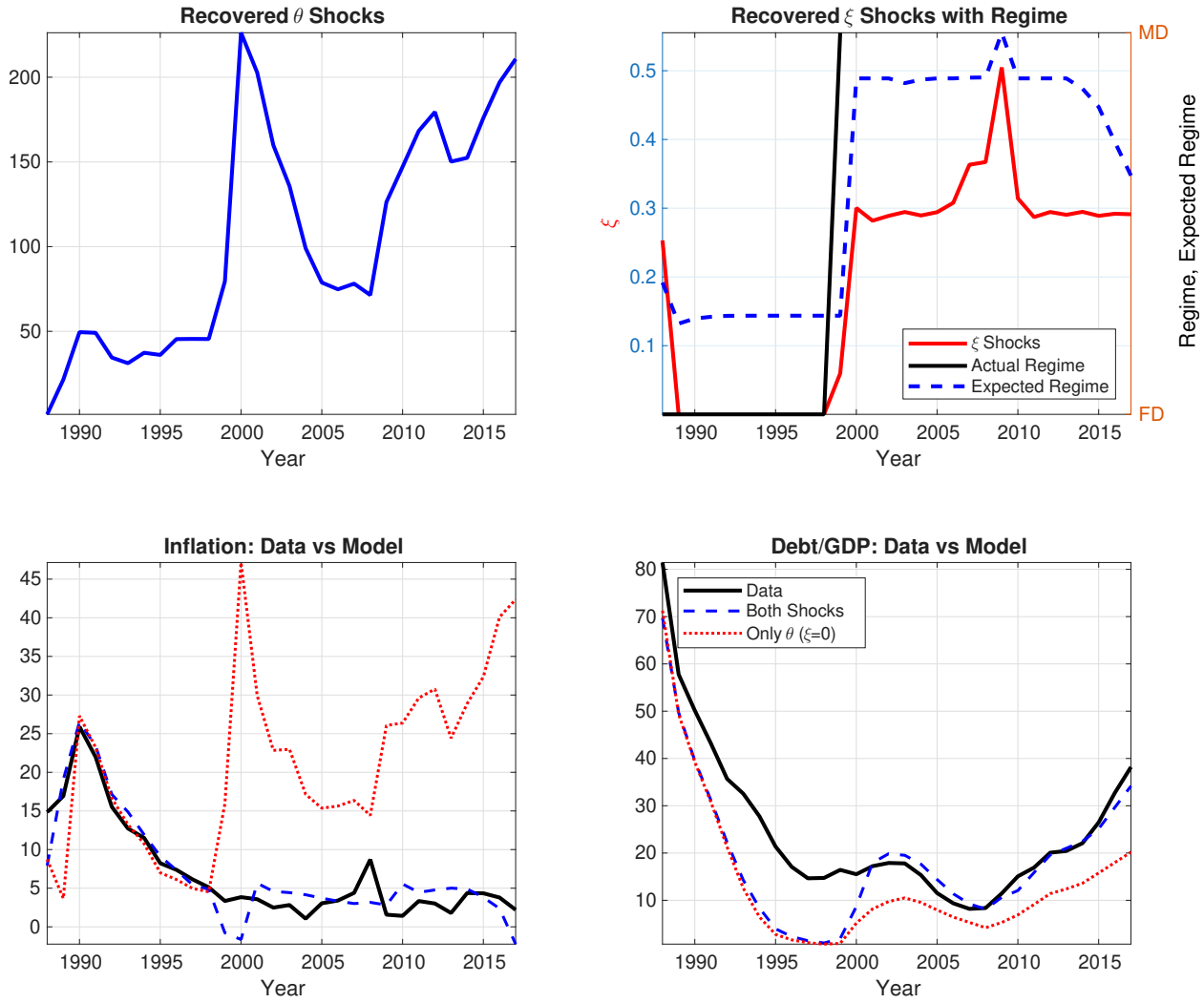
The contrasting experiences of Colombia and Chile illuminate the two disinflation channels implied by our model. In Colombia, the data can only be reconciled with a credibility gain—modeled as positive $\xi_t$ shocks—because a pure drop in fiscal needs would have driven both inflation and the debt ratio lower, which the data do not show. In Chile, the early-1990s disinflation could be matched either by stronger credibility or by lower fiscal needs. Yet our calibrated paths indicate that, once debt-to-GDP leveled off in the mid-1990s, the continued decline in inflation required additional credibility gains.

## 6.4 United States

We apply the same methodology to understand the US inflation and disinflation experience from 1960-2007. To do so, we first recalibrate the model to fit US data from 1914-2017.

The filtering results are presented in Figure 8, which shows the recovered paths of $\theta$ and $\xi$ shocks alongside the model's fit to the data.

Figure 7: Chile



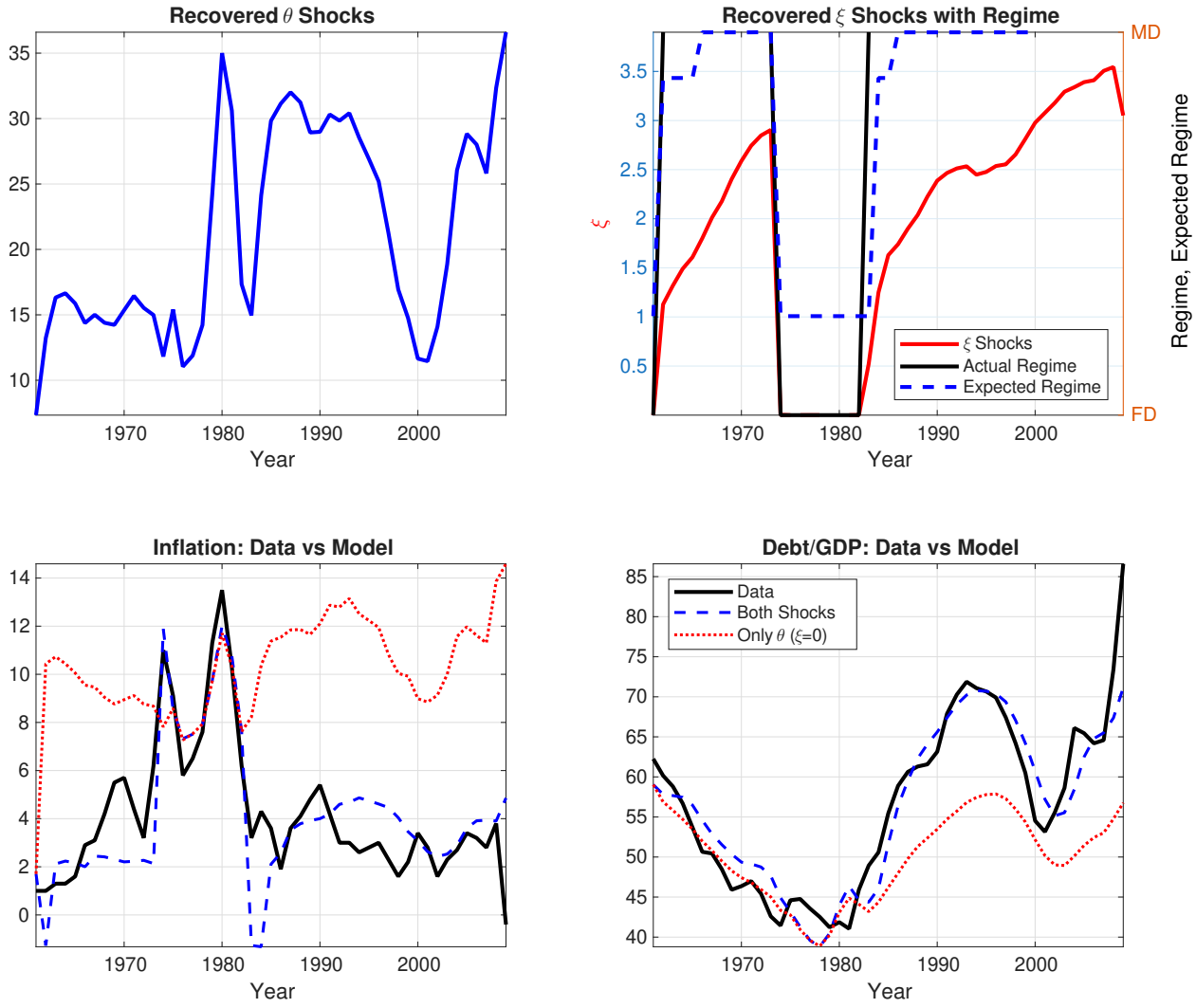

Notes: The figure reports results from applying the particle filter to Chilean data over 1990–2017 using the LA calibration (see Section 6.1). The top-left panel plots the recovered path of the fundamental shock $\theta$. The top-right panel plots the recovered credibility shock $\xi$ (solid line, left axis) along with the actual regime (dashed line, right axis) and the expected regime (dash-dotted line, right axis), where MD denotes monetary dominance and FD denotes fiscal dominance. The bottom two panels compare the data (solid black lines) with the model-implied paths under both shocks (dashed blue lines) and under only $\theta$ shocks with $\xi$ set to zero (dotted red lines) for inflation (bottom-left, in annualized percent) and the debt-to-GDP ratio (bottom-right, in percent).

The model identifies an interesting pattern in the evolution of monetary credibility over this period. The recovered $\xi$ path shows a significant decline in credibility beginning in the late 1960s and persisting through the 1970s. This erosion of credibility coincides with the "Great Inflation" episode—the actual regime indicator shows the economy switching to the fiscal-dominant regime during this period, with inflation rising from around 2% in the mid-1960s to double digits by the late 1970s. The collapse in credibility is consistent with well-documented historical accounts of political pressure on the Federal Reserve during this era. Most notably, President Nixon pressured Fed Chairman Arthur Burns to pursue accommodative monetary policy in the lead-up to the 1972 election, subordinating price stability to short-term political and fiscal objectives (Blinder (2022)). The model attributes this inflationary period to both rising fiscal needs (elevated $\theta_t$)—driven by the costs of the Vietnam War and expanding social programs—and collapsing credibility (declining $\xi_t$), which together pushed the economy into the fiscal dominant regime

The Volcker disinflation beginning in 1979 marks a turning point. The model accounts for the sharp reduction in inflation through a combination of declining fiscal needs and, critically, a restoration of credibility in the early 1980s that triggers a switch back to monetary dominance. The timing is notable: credibility does not recover immediately with Volcker's appointment or the October 1979 policy shift, but rather builds gradually as the Fed demonstrated its willingness to maintain tight policy despite a severe recession. This echoes our finding for Colombia, where credibility lagged formal institutional reform by several years.

Critically, the debt-to-GDP ratio rose substantially from the early 1980s onward, from around 45% to over 70% by 2000, even as inflation declined and stabilized. This rising debt trajectory alongside falling inflation is what identifies the institutional disinflation. To illustrate the importance of credibility gains, we conduct the same counterfactual exercise as for Colombia and Chile: we retain the computed sequence of $\theta$ shocks but hold $\xi_t = 0$ throughout, so the economy remains in the fiscal-dominant regime. The dotted red lines show that without institutional changes, the rising debt levels driven by elevated $\theta_t$ realizations would have generated much higher inflation in the latter half of the sample period. The model would predict debt levels closer to those observed in the data, but inflation would have remained elevated or even increased, contrary to the pattern of stable, low inflation actually observed from the mid-1980s onward. This exercise illustrates that the credibility of the Federal Reserve's commitment to price stability, built through the 1980s, was essential for the United States to simultaneously achieve higher debt levels and low inflation in subsequent decades.

Figure 8: USA




Notes: The figure reports results from applying the particle filter to US data over 1960–2007 using the US calibration (see Section 6.1). The top-left panel plots the recovered path of the fundamental shock $\theta$. The top-right panel plots the recovered credibility shock $\xi$ (solid line, left axis) along with the actual regime (dashed line, right axis) and the expected regime (dash-dotted line, right axis), where MD denotes monetary dominance and FD denotes fiscal dominance. The bottom two panels compare the data (solid black lines) with the model-implied paths under both shocks (dashed blue lines) and under only $\theta$ shocks with $\xi$ set to zero (dotted red lines) for inflation (bottom-left, in annualized percent) and the debt-to-GDP ratio (bottom-right, in percent).

### 6.5 Discussion

So far we have seen how we can use the restrictions of the model to identify the relative importance of institutional and fundamental drivers of disinflation episodes. There are however other factors that drive the dynamics of debt and inflation and that were left out of our model for tractability. Some of these factors may drive inflation and debt in opposite directions, as the institutional cost $\xi_t$ does in our framework, so a relevant question is whether our measurement overstates the importance of institutional forces in driving disinflation episodes. As we argue below, however, while these forces may temporarily generate a negative co-movement between debt and inflation, in the long-run they will eventually lead to a positive co-movement unless the monetary dominant regime becomes credible.

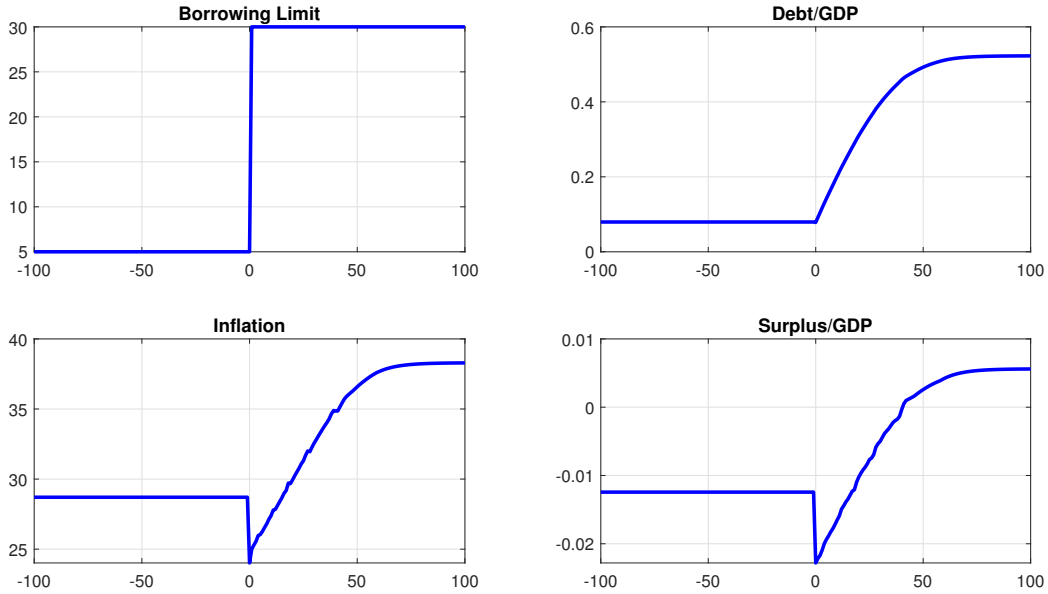
To understand this point, let's consider a fundamental shock that improves the government ability to issue debt, a factor that could lead simultaneously to a decline in inflation and an increase in debt in our framework. Suppose that before period $t_0$ the government is constrained by a tight debt limit, $b' \leq \chi$, but in $t_0$ the debt limit is relaxed. This higher debt capacity allows the (impatient) government to initially run lower surpluses by borrowing more, something that puts less pressure on inflation. Eventually, however, higher surpluses will be required to service the higher level of debt, and this will result in higher inflation if the central bank's credibility does not increase. Thus, higher debt capacity can only be associated with higher debt and lower inflation temporarily, as illustrated in Figure 9, if there is no increase in credibility of the monetary regime. In this sense, the credibility of the monetary-dominant regime is a necessary condition for the ability to support high levels of debt with low levels of inflation.

## 7 Conclusion

This paper develops a framework for analyzing the credibility of inflation-targeting mandates in environments with fiscal-monetary interactions. We model a government that delegates monetary policy to a central bank but retains the option to abrogate the mandate when fiscal pressures mount. The decision to honor or revoke the mandate depends on two forces: fiscal fundamentals, captured by outstanding debt and the marginal utility of public spending, and a stochastic institutional cost of deviation that reflects legal, reputational, and political constraints on government interference with monetary policy.

The model generates endogenous transitions between monetary-dominant and fiscal-dominant regimes. In the fiscal-dominant regime, inflation is high, volatile, and tightly

Figure 9: Relaxation of a tight debt limit




Notes: The figure plots the response of the model with endogenous regime switching to a one-time relaxation of the borrowing limit at time 0, using the Latin American calibration (see Section 6.1). Before time 0, the government faces a tight debt limit $b' \leq \chi$; at time 0, the constraint is permanently relaxed. The credibility shock $\zeta$ is held constant throughout, so there is no change in the monetary regime. The top-left panel reports the borrowing limit, the top-right panel reports the debt-to-GDP ratio, the bottom-left panel reports inflation in annualized percent, and the bottom-right panel reports the primary surplus-to-GDP ratio. Time is measured in years.

linked to fiscal conditions, while debt levels remain low. In the monetary-dominant regime, inflation is low and insulated from fiscal shocks, and the economy can sustain higher levels of public debt. Credible delegation is thus a necessary condition for supporting high debt-to-GDP ratios with low inflation.

We find that the endogeneity of regime switching creates incentive effects that distort government policy choices. Governments strategically limit debt issuance and choose less ambitious inflation targets to reduce future temptations to abrogate the mandate. These incentive effects generate distinct predictions for disinflation episodes: fundamental disinflations driven by reduced fiscal needs produce declining paths for both inflation and debt, while institutional disinflations driven by increased credibility produce declining inflation alongside rising debt. This contrasting behavior allows us to identify the contribution of institutional changes to observed disinflation episodes. We apply the framework to study disinflation experiences in Colombia, Chile, and the United States. In each case, the joint dynamics of inflation and debt allow us to decompose the relative contributions of fiscal consolidation and institutional change to the decline in inflation.

Our recovered credibility measure can help discipline deeper theories of reputation and credibility by mapping their distinct implications into testable restrictions on the path of $\{\xi_t\}$. Mechanisms in the spirit of Atkeson et al. (2007); Halac and Yared (2025, 2021)—building on the logic of Green and Porter (1984)—naturally generate discrete regime changes, with $\xi_t$ switching abruptly between low values (periods in which promises are weakly enforced) and high values (periods in which deviations are prohibitively costly). By contrast, reputation theories based on signaling with hidden types, in which private agents gradually update beliefs about the government's type from observed actions—as in King and Lu (2022); Bocola et al. (2025)—imply a more persistent and smooth evolution of $\{\xi_t\}$. Our formulation is flexible enough to accommodate either pattern, and this flexibility may be empirically important: the end of four big inflations detailed by Sargent (1982) appear closer to abrupt credibility shifts, whereas the end of the Great Inflation in the United States is associated in our estimates with a gradual increase in $\xi_t$ between 1980 and 1984. Developing and formally testing these microfoundations against the inferred dynamics of $\{\xi_t\}$ is an important direction for future research.

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# Appendix

# A Omitted proofs

## A.1 Proof of Lemma 1

Concavity follows from concavity of the objective function and convexity of the constraint set in (8) under the assumption that $v'(l)\,l$ is convex.

To characterize the derivative of $U$ with respect to primary surpluses, it is useful to rewrite the problem in (8) in terms of labor tax revenues and $g$ only. To this end, as noted in the text, in any competitive equilibrium tax revenues, $T$, must satisfy the following restriction

$$T = \left(1 - v'(l)\right) l. \tag{27}$$

We can then define $\ell(T)$ as the largest labor supply that solves the expression above (so we are always on the upward side of the Laffer curve). Defining

$$W(T) = \ell(T) - v(\ell(T)), \tag{28}$$

we can then rewrite problem (8) as

$$U(\Delta, s) = \max_{T,G} W(T) - G + \theta(s)\, u(G)$$

subject to

$$\Delta \leq T - G.$$

The optimality conditions for this problem are

$$\begin{aligned} -W'(T) &= \lambda \\ \theta(s)\, u'(G) &= 1 + \lambda \end{aligned}$$

where $\lambda$ is the multiplier on the constraint. Moreover, by the envelope theorem, we have that $U'(\Delta, s) = -\lambda$ . Thus,

$$U'(\Delta, s) = W'(T). \tag{29}$$

Note that from (28) we have

$$W'(T) = \ell'(T)\left[1 - v'(\ell(T))\right] = \ell'(T)\, T/\ell(T)$$

and from (27), by the implicit function theorem, we have

$$\ell'(T) = \frac{1}{[1 - v'(\ell(T)) - v''(\ell(T))\ell(T)]} = \frac{1}{[T/\ell(T) - v''(\ell(T))\ell(T)]} \leq 0.$$

Hence,

$$W'(T) = \frac{T/\ell(T)}{T/\ell(T) - v''(\ell(T))\ell(T)} = \frac{T}{T - v''(\ell)\ell^2} \leq 0$$

since $\ell'(T) \leq 0$. As $\Delta \to \bar{\Delta}$, then feasibility requies that $T \to T_{max} = \bar{\Delta}$ and $G \to 0$. Since the top of the Laffer curve is characterized by

$$1 - v'(\ell) = \ell v''(\ell) \to T_{max} = v''(\ell(T_{max}))\ell(T_{max})^2$$

then the denominator in the expression for $W'(T_{max})$ converges to 0. Thus,

$$\lim_{\Delta \to \bar{\Delta}} U'(\Delta, s) = \lim_{T \to T_{max}} W'(T) = -\infty.$$

We implicitly define $g^*(s)$ by $\theta(s)u'(g^*(s)) = 1$. To see that $U'(\Delta, s) = 0$ for all $\Delta \leq -g^*(s)$ , note that for such large deficits it is possible to finance the unconstrained optimal amount of government expenditures, $g^*(s)$, without raising any distortionary labor taxes. Thus, $\lambda = 0$ and $U'(\Delta, s) = 0$.

## A.2 Proof of Proposition 3

Consider first the restricted Ramsey problem:

$$V_R(b, \phi, s) = \max_{\Delta, b', \phi'} U(\Delta, s) + v(\phi) + \hat{\beta} \sum_{s'} \Pr(s'|s) V_R(b', \phi', s')$$

subject to

$$\Delta = b + \phi - \beta b' - \beta H(\phi')$$

where we added the restriction $\phi'(s') = \phi'$ for all $s'$.

The first order conditions and the envelope condition imply

$$U'(\Delta, s) = \frac{\hat{\beta}}{\beta} \sum_{s'} \Pr(s'|s) U'(\Delta'(s'), s') \tag{30}$$

and

$$\hat{\beta} v'(\phi') - \beta U'(\Delta, s) h'(\phi') = 0 \tag{31}$$

Under the assumption $v(\phi) = \frac{\phi^{1-\eta}}{1-\eta}$ we have

$$h(\phi) = \phi^{1-\eta}, \quad h'(\phi) = (1-\eta)\phi^{-\eta} = (1-\eta)v'(\phi)$$

Suppose by way of contradiction that $v'(\phi(s')) > 0$. Condition (31) then implies

$$U'(\Delta, s) = \hat{\beta}/\beta \frac{v'(\phi')}{h'(\phi')} = \frac{\hat{\beta}/\beta}{1-\eta} \tag{32}$$

Clearly, if $\eta \in (0,1)$ we have a contradiction since $U'(\Delta) < 0$ so the left side of the equation above is negative while the right side is positive. Thus, it must be that $v'(\phi(s')) = 0$ for all $s'$. *Q.E.D.*

## A.3 Proof of Lemma 4

Letting $B = b + \phi$, define

$$W(B, s) = \max_{\Delta, b', \phi'} U(\Delta, s) + \hat{\beta} \sum_{s'} \Pr(s'|s) V(b', \phi', s') \tag{33}$$

subject to

$$\Delta = b + \phi - \beta b' - J(b', \phi', s)$$

and $b' \leq \bar{b}$. Clearly, $V_{md}(b, \phi, s) = W(b + \phi, s) + v(\phi)$ and

$$V_{fd}(b, s) = \max_{\phi_{fd}} \left\{ W(b + \phi_{fd}, s) + v(\phi_{fd}) \right\}.$$

We first show that the optimal $\Delta$ in (33) is increasing in $B$. From the envelope condition we have $W'(B, s) = U'(\Delta(B), s)$. Thus, because of concavity of $W$, if $B_H > B_L$

$$U'(\Delta_H, s) = W'(B_H, s) < W'(B_L, s) = U'(\Delta_L, s)$$

Hence, concavity of $U$ implies that $\Delta_H > \Delta_L$ as wanted.

Consider next the comparative statics with respect to the cutoff

$$\xi^*(b, \phi, s) = V_{fd}(b, s) - V_{md}(b, \phi, s).$$

We first establish that if $\phi > \phi_{fd}(b, s)$ then $\xi^*(b, \phi, s)$ is increasing in $b$. To see this, note

that

$$\frac{\partial \xi^* (b, \phi, s)}{\partial b} = \frac{\partial V_{fd} (b, s)}{\partial b} - \frac{\partial V_{md} (b, \phi, s)}{\partial b} = U' (\Delta_{fd}, s) - U' (\Delta_{md}, s)$$

where the second equality follows from the envelope condition in problem (33) with $\Delta_{fd} = \Delta \left( b + \phi_{fd} (b, s), s \right)$ and $\Delta_{md} = \Delta (b + \phi, s)$, where $\Delta (B, s)$ is the policy function associated with problem (33). Thus, because of concavity of $U$, $\partial \xi^* / \partial b > 0$ if and only if $\Delta_{fd} < \Delta_{md}$. To see that this is the case, note that, since we are considering $\phi$ such that $\phi > \phi_{fd} (b, s)$, it follows that $B_{md} \equiv b + \phi > B_{fd} \equiv b + \phi_{fd} (b, s)$ and we just proved that $\Delta (B)$ is increasing. Thus, $\Delta_{fd} < \Delta_{md}$ and $\xi^*$ is increasing in $b$.

Consider next $\theta$. Note that

$$\begin{aligned} \frac{\partial \xi^* (b, \phi, s)}{\partial \theta} &= \frac{\partial V_{fd} (b, s)}{\partial \theta} - \frac{\partial V_{md} (b, \phi, s)}{\partial \theta} \\ &= \frac{\partial U \left( \Delta_{fd}, s \right)}{\partial \theta} - \frac{\partial U (\Delta_{md}, s)}{\partial \theta} \\ &= u \left( g \left( \Delta_{fd}, s \right) \right) - u \left( g (\Delta_{md}, s) \right) \end{aligned}$$

where the last equality follows from the envelope condition applied to the indirect utility function (8) where $g (\Delta, s)$ is the optimal policy function associated to the problem. As we show above, $\Delta_{fd} < \Delta_{md}$ and $g$ is dcreasing in $\Delta$. Hence $u \left( g \left( \Delta_{fd}, s \right) \right) > u \left( g (\Delta_{md}, s) \right)$ and $\xi^*$ is increasing in $\theta$.

Finally, note that

$$\frac{\partial \xi^* (b, \phi, s)}{\partial \phi} = - \frac{\partial V_{md} (b, \phi, s)}{\partial \phi} = - \left[ W' (b + \phi, s) + v' (\phi) \right] > 0$$

since we are considering $\phi > \phi^* (b, s)$ and $W$ and $v$ are concave so

$$0 = \left[ W' \left( b + \phi^* (b, s), s \right) + v' \left( \phi^* (b, s) \right) \right] > \left[ W' (b + \phi, s) + v' (\phi) \right].$$

Thus, $\xi^*$ is increasing in $\phi$. *Q.E.D.*

# B Partial equivalence with set of sustainable equilibrium outcomes

Consider an economy without shocks.

**Set of Sustainable Equilibrium Outcomes** Using the logic in Abreu (1988) and Chari and Kehoe (1990), an outcome $x = \{\phi_t, b_t, \Delta_t\}$ is a sustainable equilibrium outcome if and only if it satisfies for all $t$

$$\Delta_t = b_t + \phi_t - \beta b_{t+1} - \beta H(\phi_{t+1})$$

$$W_t + v(\phi_t) \geq \underline{V}(b_t)$$

where $\underline{V}(b_t)$ is the value of the worst sustainable equilibrium given $b_t$, and $\{W_t\}$ is the sequence of associated equilibrium values without the current value of real monetary balance,

$$W_t = U(\Delta_t, \theta_t) + \hat{\beta}\left[v(\phi_{t+1}) + W_{t+1}\right].$$

To characterize the worst equilibrium value, we can follow Abreu, Pearce, and Stacchetti (1990) and Chang (1998). Let $\mathcal{W}(b)$ be the equilibrium value correspondence given the amount of inherited real debt $b$.[24] That is, $\mathcal{W}(b)$ is the set of government values $W$ (ex-monetary balances) and $\phi$ that can be delivered as an sustainable equilibrium given $b$. Given the correspondence $\mathcal{W}$ and $b$, $(W, \phi)$ are supportable as a sustainable equilibrium if there exists $(\Delta, \phi', b', W')$ such that

$$W = U(\Delta, \theta) + \hat{\beta}\left[v(\phi') + W'\right]$$

$$\Delta = b + \phi - \beta H(\phi') - \beta b'$$

$$(W', \phi') \in \mathcal{W}(b)$$

$$W + v(\phi) \geq \underline{V}(b)$$

where

$$\underline{V}(b) = \max_{\mu, b'} \min_{(\phi', W') \in \mathcal{W}(b')} U(\Delta, \theta) + v\left(\frac{\beta H(\phi')}{\mu}\right) + \hat{\beta}\left[v(\phi') + W'\right] \tag{34}$$

subject to

$$\Delta = b + \frac{\beta H(\phi')}{\mu} - \beta H(\phi') - \beta b'$$

since $\phi = \frac{\beta H(\phi')}{\mu}$. Thus, for each $\phi$, there are $\bar{W}(b, \phi)$ and $\underline{W}(b, \phi)$ that describe the max and min value for a given promised $\phi$ and $b$. We can also define the set of expected values $\mathcal{V}(b) = [\underline{V}(b), \bar{V}(b)]$.

One can then find $\mathcal{W}$ as the largest fixed point of the operator implicitly defined by

[24] Recall that governments cannot default on $b$ so it is a state variable for the problem.

the conditions above. Knowing $\mathcal{W}$ we can calculate $\underline{V}(b)$ using (34) that we can use to characterize the entire set of sustainable equilibrium outcomes.

**Our problem** Consider next a version of our problem given a path $\{\theta_t, \xi_t\}$:

$$W_t(b+\phi) = \max_{\Delta, b', \phi'} U(\Delta, \theta_t) + \hat{\beta}\left[W_{t+1}\left(b'+\phi'\right) + v\left(\phi'\right)\right]$$

subject to

$$\Delta = b + \phi - \beta b' - \beta H\left(\phi'\right)$$

$$W_{t+1}\left(b'+\phi'\right) + v\left(\phi'\right) \geq \underline{V}\left(b'\right) + \xi_t$$

Let $s = \{\theta_t, \xi_t\}$ and $y(s) = \{\Delta_t(s), b_t(s), \phi_t(s)\}$ that solves the problem.

The solution is characterized by

$$\begin{aligned} -U'(\Delta_t, \theta_t)\beta H'(\phi_{t+1}) &= -\hat{\beta}(1+\kappa_{t+1})\left[U'(\Delta_{t+1}, \theta_{t+1}) + v'(\phi_{t+1})\right] \\ -U'(\Delta_t, \theta_t)\beta &= -\hat{\beta}(1+\kappa_{t+1})U'(\Delta_{t+1}, \theta_{t+1}) \end{aligned}$$

and the complementary slackness condition

$$\kappa_{t+1}\left[W_{t+1}(\phi_{t+1}) + v(\phi_{t+1}) - \underline{V} - \xi_t\right] = 0 \tag{35}$$

where $\kappa_{t+1}$ is the multiplier on the sustainability constraint. Thus,

$$\begin{aligned} -U'(\Delta_t, \theta_t)\beta h'(\phi_{t+1}) &= -\hat{\beta}(1+\kappa_{t+1})v'(\phi_{t+1}) \\ -U'(\Delta_t, \theta_t)\beta &= -\hat{\beta}(1+\kappa_{t+1})U'(\Delta_{t+1}, \theta_{t+1}) \end{aligned}$$

or

$$\beta H'(\phi_{t+1}) = \frac{\left[U'(\Delta_{t+1}, \theta_{t+1}) + v'(\phi_{t+1})\right]}{U'(\Delta_{t+1}, \theta_{t+1})} \tag{36}$$

$$-U'(\Delta_t, \theta_t)\beta \geq -\hat{\beta}U'(\Delta_{t+1}, \theta_{t+1}) \tag{37}$$

These two are the extra restrictions imposed by our problem.

**Partial equivalence** We then have the following partial equivalence result:

**Proposition.** *Fix* $\{\theta_t\}$*:*

1. *For any* $\{\xi_t\}$ *such that* $\underline{V}(b_{t+1}(s)) \leq \underline{V}(b_{t+1}(s)) + \xi_t \leq \bar{V}(b_{t+1}(s))$ *then* $y(\{\theta_t, \xi_t\})$ *is*

*a sustainable outcome.*

2. *Conversely, if $x$ is a sustainable outcome such that*

$$\beta H'(\phi_{t+1}) = \frac{[U'(\Delta_{t+1}, \theta_{t+1}) + v'(\phi_{t+1})]}{U'(\Delta_{t+1}, \theta_{t+1})} \tag{38}$$

$$-U'(\Delta_t, \theta_t)\beta \geq -\hat{\beta}U'(\Delta_{t+1}, \theta_{t+1}) \tag{39}$$

*then there exists $\{\xi_t\}$ such that $y(\{\theta_t, \xi_t\}) = x$.*

*Proof.* First, consider $s = \{\theta_t, \xi_t\}$ with the associated solution to our problem, $y(s) = \{\Delta_t(s), b_t(s), \phi_t(s)\}$. Suppose that $\{\xi_t\}$ is such that $\underline{V}(b_{t+1}(s)) \leq \underline{V}(b_{t+1}(s)) + \xi_t \leq \bar{V}(b_{t+1}(s))$. A solution to our problem must satisfy the government budget constraint and the sustainability constraint because

$$W_{t+1}(b_{t+1} + \phi_{t+1}) + v(\phi_{t+1}) \geq \underline{V}(b_{t+1}) + \xi_t \geq \underline{V}(b_{t+1})$$

Thus, it is a sustainable equilibrium. (The requirement that $\underline{V}(b_{t+1}) + \xi_t \leq \bar{V}(b_{t+1})$ implies that the constraint set is not empty.)

Second, given $\{\theta_t\}$, consider a sustainable outcome $x = \{\Delta_t, b_t, \phi_t\}$ that satisfies conditions (38)–(39). Let $\{W_t\}$ be the associated continuation values. Construct $\{\xi_t\}$ as

$$\xi_t \equiv W_{t+1}(\phi_{t+1}) + v(\phi_{t+1}) - \underline{V}$$

so that the complementary slackness condition (35) always hold. Next, note that since $x$ is a sustainable outcome then it satisfies the budget constraint and the sustainable outcome $x$ is then feasible in our problem given the constructed $\{\xi_t\}$. Finally, $x = \{\Delta_t, b_t, \phi_t\}$ is optimal because conditions (38) and (39) imply that the sufficient focs (38) and (39) are satisfied. Thus $x$ solves our problem given $s = \{\xi_t, \theta_t\}$. *Q.E.D.*

Conditions (38) and (39) rule out sustainable equilibrium outcomes that allows for "good" deviations. That is, deviations in period $t+1$ that increase the government's value in both period $t+1$ and $t$. In a general sustainable equilibrium, even good deviations may be deterred if private agents coordinate on an unfavorable continuation equilibrium following such a deviation. The solution to our programming problem does not allow the presence of such good deviations. This is because the punishment $\xi_t$ is activated only when the government in period $t+1$ deviates from the policy proposed by the government in period $t$.

# C Debt and inflation in the Markov and Ramsey Equilibria

In this section we provide an analytical result that makes clear the comparison between inflation and level of real debt in the Markov equilibrium and in a Ramsey outcome. To do so, we consider a deterministic economy with $\beta = \hat{\beta}$. We assume that $v(\phi) = \phi - \frac{1}{2}\phi^2$.

**Proposition.** *Consider a Markov equilibrium outcome* $\{\phi_{Mt}, \Delta_{Mt}, \pi_{Mt+1}, b_{Mt+1}\}$ *and a Ramsey outcome* $\{\phi_{Rt}, \Delta_{Rt}, \pi_{Rt+1}, , b_{Rt+1}\}$ *starting from a common inherited level of real debt* $b_0$*. As* $t \to \infty$, $b_{Mt} \to 0$ *while* $b_{Rt} = b_R > 0$ *for all* $t \geq 1$*. Moreover, if* $b_0$ *is small enough then* $\phi_{Rt} > \phi_{Mt}$ *and* $\pi_{Rt+1} < \pi_{Mt+1}$ *for all* $t$*.*

Proof. Consider first the Ramsey equilibrium. Letting $B = b + \phi$ be the total value for real government liabilities, we can write the recursive problem that characterizes the *continuation* Ramsey problem as

$$W_R(B) = \max_{\Delta, B', \phi'} U(\Delta) + \hat{\beta}\left[v\left(\phi'\right) + W_R\left(B'\right)\right]$$

subject to

$$\begin{aligned} \Delta &= B - \beta\left(h\left(\phi'\right) + B'\right) \\ \phi' &\leq B' \leq \bar{b} + \phi' \end{aligned}$$

In period 0 the Ramsey problem is

$$V_{R0}(b_0) = \max_{\phi, \Delta, b'_G, \phi'} U(\Delta) + v(\phi) + \hat{\beta}\left[v\left(\phi'\right) + W_R\left(B'\right)\right]$$

subject to

$$\begin{aligned} \Delta &= b_0 + \phi - \beta\left(h\left(\phi'\right) + B'\right), \\ \phi' &\leq B' \leq \bar{b} + \phi' \end{aligned}$$

where $\bar{b}$ is the upper bound of real debt. When $\beta = \hat{\beta}$, $\phi_t$ is constant from $t \geq 1$ and surpluses are always constant. So the Ramsey outcome is fully characterized by $(\Delta_R, \phi_{R0}, \phi_{R1})$

that solve[25]

$$\begin{aligned} v'(\phi_{R0}) &= -U'(\Delta_R) \\ v'(\phi_{R1}) &= U'(\Delta_R) h'(\phi_{R1}) \\ b_0 + \phi_{R0} &= \frac{\Delta_R}{1-\beta} + \frac{\beta h(\phi_{R1})}{1-\beta} \end{aligned}$$

The Markov equilibrium outcome solves the following problem

$$V_M(b) = \max_{\phi,\Delta,b'} U(\Delta) + v(\phi) + \hat{\beta} V_M(b')$$

subject to

$$\begin{aligned} b + \phi &= \Delta + \beta b' + \beta H(\phi_M(b')) \\ 0 &\leq b' \leq \bar{b} \end{aligned}$$

taking as given the value function $V_M(b')$ and the policy function $\phi_M(b')$ next period. The Markov outcome $\{\Delta_t, \phi_t, b_{t+1}\}$ satisfies

$$\begin{aligned} v'(\phi_t) &= -U'(\Delta_t) \\ -U'(\Delta_t)\left[1 + H'(\phi_M(b_{t+1}))\frac{\partial \phi_M(b_{t+1})}{\partial b'}\right] &= -U'(\Delta_{t+1}) \\ b_t + H(\phi_M(b_{t+1})) &= \frac{1}{\beta}[b_t + \phi_t - \Delta_t] \end{aligned}$$

whenever $b_{t+1} > 0$. Note that

$$H(\phi) = \phi + h(\phi) \rightarrow H'(\phi) = 1 + h'(\phi) = 1 + (1 - 2\phi) = 2(1 - \phi) > 0$$

Thus, since $\frac{\partial \phi_M(b_{t+1})}{\partial b'} < 0$ we have

$$-U'(\Delta_{t+1}) = -U'(\Delta_t)\left[1 - H'(\phi_M(b_{t+1}))\left|\frac{\partial \phi_M(b_{t+1})}{\partial b'}\right|\right] < -U'(\Delta_t)$$

so $\{\Delta_t\}$ is a decreasing sequence. If $\{b_{t+1}\}$ is always interior, $\{\Delta_t\}$ is a decreasing sequence and $\{\phi_t\}$ is an increasing sequence in a bounded set. Thus, they must converge so we have

[25] Note that we need $h'(\phi') < 0$ for an interior solution. With quadratic utility for real money balances we have $h'(\phi) = v'(\phi) + v''(\phi)\phi = 1 - 2\phi$ and $h'(\phi) < 0$ if $\phi > 1/2$.

that as $t \to \infty$

$$b_{M\infty} = \frac{\Delta_{M\infty}}{1-\beta} + \frac{\beta h\left(\phi_{M\infty}\right)}{1-\beta} - \phi_{M\infty} \geq 0.$$

We next argue that the corresponding level of debt must be zero, $b_{M\infty} = 0$.[26] Suppose by way of contradiction that there exists $b > 0$ such that $b'_M(b) \geq b$. We have established that $\{\Delta_t\}$ is strictly decreasing if not constrained by $b' \geq 0$. Then, starting at $b_0 = b$ we have

$$\begin{aligned} b_0 &= -f_M(\Delta_0) + \sum_{t=0}^{\infty} \beta^t \Delta_t + \sum_{t=1}^{\infty} \beta^t h\left(f_M(\Delta_t)\right) \\ b'_M(b_0) &= -f_M(\Delta_1) + \sum_{t=1}^{\infty} \beta^{t-1} \Delta_t + \sum_{t=2}^{\infty} \beta^{t-1} h\left(f_M(\Delta_t)\right) \end{aligned}$$

where we defined

$$f_M(\Delta) \equiv \left(v'\right)^{-1}\left(-U'(\Delta)\right) \to \phi = f_M(\Delta)$$

with $f'_M(\Delta) < 0$. Since $\{\Delta_t\}$ is decreasing and strictly so between 0 and 1, $f'_M < 0$, and $h' < 0$, we have that

$$\begin{aligned} b_0 &= -f_M(\Delta_0) + \sum_{t=0}^{\infty} \beta^t \Delta_t + \sum_{t=1}^{\infty} \beta^t h\left(f_M(\Delta_t)\right) \\ &> -f_M(\Delta_1) + \sum_{t=0}^{\infty} \beta^t \Delta_{t+1} + \sum_{t=1}^{\infty} \beta^t h\left(f_M(\Delta_{t+1})\right) \\ &= b'_M(b_0) \end{aligned}$$

yielding a contradiction because the contradiction hypothesis was that $b'_M(b) \geq b \geq 0$. Thus, $b'_M(b) < b$ for all $b > 0$. By continuity of the policy function $b_M(\cdot)$ it follows that $b_{Mt} \to 0$ as $t \to \infty$.

We next turn to compare the value of real money balances in the two equilibria. Iterating the budget constraint in the Markov equilibrium outcome we have

$$b_0 + \phi_{M0} = \sum_{t=0}^{\infty} \beta^t \Delta_{Mt} + \sum_{t=1}^{\infty} \beta^t h\left(f_M(\Delta_{Mt})\right)$$

[26] Using the envelope condition we can write

$$-V'_M(b_{Mt}) < -\hat{\beta} V'_M(b_{Mt+1})$$

so if $V_M$ is concave then $b_{t+1} < b_t$ and so $\{b_{Mt}\} \to 0 = b_{M\infty}$. The argument in the text does not assume concavity of $V_M$.

Define $f_R(\Delta)$ as the implicit solution to

$$v'(f_R(\Delta)) = U'(\Delta) h'(f_R(\Delta)) \to \phi = f_R(\Delta)$$

with

$$f_R'(\Delta) = \frac{U''h'}{v'' - U'h''}.$$

We know that

$$f_M(\Delta) < f_R(\Delta)$$

since $h'(\phi) = 1 - 2\phi \to -h' \leq 1$ for all $\phi$.

Suppose that $b_0 \approx b_{M\infty}$ then the Markov outcome is already in steady state and $\Delta_{M\infty}$ solves

$$\Phi_M(\Delta_{M\infty}) = 0$$

with

$$\Phi_M(\Delta) \equiv b_{M\infty} + f_M(\Delta) - \frac{\Delta}{1-\beta} - \frac{\beta h(f_M(\Delta))}{1-\beta}$$

The surplus in the Ramsey outcome instead solves

$$\Phi_R(\Delta_R) = 0$$

with

$$\Phi_R(\Delta) \equiv b_{M\infty} + f_M(\Delta) - \frac{\Delta}{1-\beta} - \frac{\beta h(f_R(\Delta))}{1-\beta}$$

Since $h$ is decreasing in the relevant range and $f_M' < 0$, then $\Phi_M(\Delta)$ is strictly decreasing in $\Delta$. Using this observation together with $f_R(\Delta) > f_M(\Delta)$ for all $\Delta$ , we have that

$$\Phi_R(\Delta) > \Phi_M(\Delta) \quad \forall \Delta$$

Thus,

$$\Delta_R > \Delta_{M\infty}$$

which implies that

$$\begin{aligned}
\frac{\beta}{1-\beta} h(\phi_{M\infty}) &= b_{M\infty} + f_M(\Delta_{M\infty}) - \frac{\Delta_{M\infty}}{1-\beta} \\
&> b_{M\infty} + f_M(\Delta_R) - \frac{\Delta_R}{1-\beta} \\
&= \frac{\beta}{1-\beta} h(\phi_{R1})
\end{aligned}$$

Thus, since $h$ is decreasing in the relevant range,

$$\phi_{R1} > \phi_{M\infty}.$$

Since $\{\phi_{Mt}\}$ is an increasing sequence then

$$\phi_{R1} > \phi_{M\infty} \geq \phi_{Mt} \quad \forall t \tag{40}$$

Finally, note that in each regime

$$1 + \pi_t = \frac{\beta H(\phi_t)}{\phi_t} = \frac{\beta\left[\phi_t + (1-\phi_t)\phi_t\right]}{\phi_t} = \beta\left[2 - \phi_t\right]$$

Therefore (40) implies that for all $t \geq 1$

$$1 + \pi_{Rt} = \beta\left[2 - \phi_{R1}\right] < \beta\left[2 - \phi_{Mt}\right] = 1 + \pi_{Mt}$$

*Q.E.D.*

The result is true also if $v(m) = m^{1-\eta}/(1-\eta)$ with $m \leq m^*$. The proof for this result is more straightforward since in the Ramsey outcome is optimal to have $\phi_1 = m^*$ while in the Markov equilibrium it is optimal to have $\phi_{Mt} < m^*$ if $m^*$ is large enough and government expenditures are sufficiently valuable.

The result also remains true if we allow for the government to be more impatient than the stand-in household. If $\beta > \hat{\beta}$, the Ramsey outcome features an increasing path for primary surpluses with $\Delta_{Rt} \to \Delta_{max}$ , the maximal surplus implied by the static Laffer curve. In the Markov equilibrium outcome, if $\hat{\beta}$ is sufficiently close to $\beta$, the incentive effects still dominate the impatience effect and the level of debt still converges to 0 with a decreasing sequence $\{\Delta_{Mt}\}$. Thus, the result in the proposition still holds provided that the gap between $\beta$ and $\hat{\beta}$ is sufficiently small.

# D Model solution

Here we describe how we solved the full model. First, note that we can reduce the number of endogenous state variables to one by considering the problem of the government after $\phi$ is chosen. In this case, the unique state variable is the level of real government liabilities,

$B = b + \phi$. Denote by $W(B, s)$ the value function at this stage. Then

$$V_{md}(b, \phi, s) = W(b + \phi, s) + v(\phi) \tag{41}$$

$$V_{fd}(b, s) = \max_{\phi_{fd}} \left\{ W\left(b + \phi_{fd}\right) + v\left(\phi_{fd}\right) \right\} \tag{42}$$

$$\phi_{fd}(b, s) = \arg\max_{\phi_{fd}} \left\{ W\left(b + \phi_{fd}\right) + v\left(\phi_{fd}\right) \right\} \tag{43}$$

and

$$\eta(b, \phi, s) = \begin{cases} 1 & \text{if } W(b + \phi, s) + v(\phi) \geq \max_{\phi_{fd}} \left\{ W\left(b + \phi_{fd}\right) + v\left(\phi_{fd}\right) \right\} - \xi(s) \\ 0 & \text{otherwise} \end{cases} \tag{44}$$

The value function $W$ solves the Bellman equation

$$W(B, s) = \max_{\Delta, b', \phi'} U(\Delta, \theta) \tag{45}$$
$$+ \hat{\beta} \sum_{s'} \Pr(s'|s) \max \left\{ W\left(b' + \phi', s\right) + v(\phi), \max_{\phi_{fd}} \left\{ W\left(b' + \phi_{fd}\right) + v\left(\phi_{fd}\right) \right\} - \xi(s) \right\}$$

subject to

$$\Delta = B - \beta b' - \beta \sum_{s'} \Pr(s'|s) \left\{ \eta\left(b', \phi', s'\right) H\left(\phi'\right) + \left(1 - \eta\left(b', \phi', s'\right)\right) H\left(\phi_{fd}\left(b', s'\right)\right) \right\}$$

with $\phi_{fd}$ and $\eta$ given by (43) and (44).

Algorithm:

- Let the state space: $B, B' \in [0, \bar{B}]$ , $\phi, \phi' \in [0, \phi^*]$ and $b, b' \in \left[0, \bar{b}\right]$ where $\bar{B} = \bar{b} + \phi^*$
- Start with a guess $W_0$ (starting from the Ramsey value)
- For any $n$, calculate $\phi_{fd}$ and $\eta$ associated with candidate $W_n$ using (43) and (44)
- Compute $W_{n+1}$ using the Bellman operator (45)
- Iterate until convergence of the value function.

Note that if $s = (\theta, \xi)$ and $\theta$ and $\xi$ are independent and $\xi$ is a continuous variable then the

Bellman equation is

$$\begin{aligned} W(B,s) = & \max_{\Delta,b',\phi'} U(\Delta,\theta) \\ & + \hat{\beta} \sum_{\theta'} \Pr(\theta'|\theta) \int_{\xi' \leq \xi^*(b',\phi',\theta')} \left[ \max_{\phi_{fd}} \left\{ W\left(b' + \phi_{fd}, s'\right) + v\left(\phi_{fd}\right) \right\} - \xi' \right] f\left(\xi'|\xi\right) d\xi' \\ & + \hat{\beta} \sum_{\theta'} \Pr(\theta'|\theta) \int_{\xi' \geq \xi^*(b',\phi',\theta')} \left[ W\left(b' + \phi, s'\right) + v\left(\phi\right) \right] f\left(\xi'|\xi\right) d\xi' \end{aligned}$$

subject to

$$\begin{aligned} \Delta = & B - \beta b' - \beta \sum_{\theta'} \Pr(\theta'|\theta) \int_{\xi' \leq \xi^*(b',\phi',\theta')} H\left(\phi_{fd}\left(b', s'\right)\right) f\left(\xi'|\xi\right) d\xi' \\ & - \beta \sum_{\theta'} \Pr(\theta'|\theta) \int_{\xi' \geq \xi^*(b',\phi',\theta')} H\left(\phi'\right) f\left(\xi'|\xi\right) d\xi' \end{aligned}$$

We make the following assumption on $\xi$: we assume that it has a persistent component $\xi_1$ that follows a Markov chain $\Gamma(\xi_1'|\xi_1)$ and two iid components $\xi_{md}$ and $\xi_{fd}$ that follow Gumbel$(\mu, 1/\lambda)$ and

$$\xi = \xi_1 + \xi_{fd} - \xi_{md}$$

We will assume that the expectation of the Gumbel random variables are zero and so $\mu + \gamma/\lambda = 0 \rightarrow \mu = -\gamma/\lambda$ for $\lambda > 0$ where $\gamma$ is the Euler-Mascheroni constant ($\approx$0.5772).

Given a state $s_1 = (\theta, \xi_1)$, the expected value over $\xi_{fd}$ and $\xi_{md}$ is

$$\begin{aligned} \Omega(b,\phi,s_1) \equiv & \int_{\xi_{md}} \int_{\xi_{fd}} \max\left\{ V_{md}(b,\phi,s_1) + \xi_{md}, V_{fd}(b,s_1) - \xi_1 + \xi_{md} \right\} f\left(\xi_{fd}\right) f\left(\xi_{md}\right) d\xi_{fd} d\xi_{md} \\ = & \frac{1}{\lambda} \log\left( \exp\left( \left[ V_{md}(b,\phi,s_1) + \mu \right] \lambda \right) + \exp\left( \left[ V_{fd}(b,s_1) - \xi_1 + \mu \right] \lambda \right) \right) + \frac{\gamma}{\lambda} \\ = & \frac{1}{\lambda} \log\left( \exp\left( V_{md}(b,\phi,s_1) \lambda \right) + \exp\left( \left[ V_{fd}(b,s_1) - \xi_1 \right] \lambda \right) \right) + \mu + \frac{\gamma}{\lambda} \end{aligned}$$

and

$$\begin{aligned} \bar{\eta}(b,\phi,s_1) \equiv & \Pr\left( V_{md}(b,\phi,s_1) + \xi_{md} \geq V_{fd}(b,s_1) - \xi_1 - \xi_{fd} \right) \\ = & \frac{\exp\left( V_{md}(b,\phi,s_1) \lambda \right)}{\exp\left( V_{md}(b,\phi,s_1) \lambda \right) + \exp\left( \left[ V_{fd}(b,s_1) - \xi_1 \right] \lambda \right)} \\ = & \frac{1}{1 + \exp\left( -\lambda \left[ V_{md}(b,\phi,s_1) - V_{fd}(b,s_1) + \xi_1 \right] \right)} \end{aligned}$$

Thus, the Bellman equation is

$$W(B,s_1) = \max_{\Delta,b',\phi'} U(\Delta,\theta) + \hat{\beta}\sum_{s_1'}\Pr\left(s_1'|s_1\right)\Omega\left(b',\phi,s_1'\right)$$

subject to

$$\Delta = B - \beta b' - \beta\sum_{s_1'}\Pr\left(s_1'|s_1\right)\left[\bar{\eta}\left(b',\phi',s_1\right)H\left(\phi'\right) + \left(1-\bar{\eta}\left(b',\phi',s_1'\right)\right)H\left(\phi_{fd}\left(b',s_1'\right)\right)\right]$$

The first order conditions for $\phi'$ and $b'$ are:

$$0 \leq \sum_{s_1'}\Pr\left(s_1'|s_1\right)\left[\frac{-U'(\Delta,\theta)\,\beta H'\left(\phi'\right) + \hat{\beta}\left[W'\left(b'+\phi',s_1'\right)+v'\left(\phi'\right)\right]}{1+\exp\left(-\lambda\Delta V\left(b',\phi',s_1'\right)\right)}\right]$$
$$- U'(\Delta,\theta)\,\beta\sum_{s_1'}\Pr\left(s_1'|s_1\right)\left[\frac{\exp\left(-\lambda\Delta V\left(b',\phi',s_1'\right)\right)\lambda}{\left[1+\exp\left(-\lambda\Delta V\left(b',\phi',s_1'\right)\right)\right]^2}\frac{\partial V_{fd}\left(b',\phi',s_1'\right)}{\partial\phi'}\right]\left[H\left(\phi'\right) - H\left(\phi_{fd}\left(b',s_1'\right)\right)\right]$$

$$0 = -U'(\Delta,\theta)\,\beta + \sum_{s_1'}\Pr\left(s_1'|s_1\right)\hat{\beta}\frac{\exp\left(V_{md}\left(b',\phi',s_1'\right)\lambda\right)\frac{\partial V_{md}}{\partial b'} + \exp\left(\left[V_{fd}\left(b,s_1\right)-\xi_1\right]\lambda\right)\frac{\partial V_{fd}}{\partial b'}}{\exp\left(V_{md}\left(b,\phi,s_1\right)\lambda\right)+\exp\left(\left[V_{fd}\left(b,s_1\right)-\xi_1\right]\lambda\right)}$$
$$- U'(\Delta,\theta)\,\beta\sum_{s_1'}\Pr\left(s_1'|s_1\right)\left[\frac{\exp\left(-\lambda\Delta V\left(b',\phi',s_1'\right)\right)\lambda}{\left[1+\exp\left(-\lambda\Delta V\left(b',\phi',s_1'\right)\right)\right]^2}\frac{\partial\Delta V\left(b',\phi',s_1'\right)}{\partial b'}\right]\left[H\left(\phi'\right) - H\left(\phi_{fd}\left(b',s_1'\right)\right)\right]$$
$$- U'(\Delta,\theta)\,\beta\sum_{s_1'}\Pr\left(s_1'|s_1\right)\left[\frac{\exp\left(-\lambda\Delta V\left(b',\phi',s_1'\right)\right)}{1+\exp\left(-\lambda\Delta V\left(b',\phi',s_1'\right)\right)}H'\left(\phi_{fd}\left(b',s_1'\right)\right)\right]\frac{\partial\phi_{fd}\left(b',s_1'\right)}{\partial b'}$$