How Do Rapid Sectoral Shifts Affect Workers? Theory and An Application to the Climate Transition*

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Abstract

Sectoral shifts require costly labor reallocation for workers, fueling concerns about how quickly they occur. We study how the pace of such sectoral shifts affects workers at risk of displacement. We develop a life-cycle model with skill heterogeneity and job ladders where labor demand gradually rises in one sector and declines in another. The model reveals three novel insights. First, workers' lifetime earnings are strongly non-linear and even non-monotonic in the horizon over which the transition unfolds. Second, more and more workers benefit on the extensive margin as the transition accelerates, but the tail of losses becomes thicker on the intensive margin. Third, labor market frictions are important to quantify these non-linearities. We apply our model to the climate transition and find substantial earnings losses from a transition ending in 2060. Completing the transition ten years earlier generates small average losses, but raises losses in the tail by a fifth.

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1 Introduction

Secular changes in economic activity require substantial labor reallocation from declining sectors to growing ones. These sectoral shifts happen constantly, and include the contraction of manufacturing and the rise of services, sectoral reallocation due to technological change, or, looking ahead, the transition from polluting to clean industries. The labor reallocation that accompanies these sectoral shifts is often costly for workers attached to the declining sector, who forgo specific human capital and experience periods of unemployment over the transition, with lasting effects on their earnings (Jacobson et al., 1993; Davis and von Wachter, 2011). These reallocation costs fuel concern about how quickly sectoral shifts take place. For instance, survey evidence and voting behavior suggest that workers are concerned about the labor market impact of the climate transition, which poses a significant obstacle for the support to ambitious climate goals (Dechezleprêtre et al., 2025; Bombardini et al., 2025). Despite these crucial political economy implications, we know surprisingly little about how different transition timelines affect workers' outcomes.

This paper studies how the *pace* at which a sectoral shift occurs affects the earnings distribution among workers initially employed in the declining sector. For instance, in the climate context, we study how completing the energy transition by 2060, 2050, 2045, etc. affects the distribution of earnings losses among workers initially attached to dirty industries, who are the most at risk of displacement and the most likely to oppose this transition. It is essential to determine how these workers fare under different scenarios and how expensive it might be to compensate them to raise support for a faster transition. While much of the literature focuses on particular historical transitions,[1] we conduct a *counterfactual* analysis asking how workers' earnings would change if the government were to intervene to accelerate or slow down a transition. Our goal is to develop general conceptual insights that are portable to a variety of sectoral shifts, and apply these insights to a context which is relevant for today's policymakers: the climate transition.

We develop a model that captures the displacement and reemployment experience of workers initially attached to a declining industry. The model features finitely-lived workers with heterogeneous skills. Changes in preferences and tech-

[1] See, e.g., Burstein et al. (2019), Porzio et al. (2022), Braxton and Taska (2023).

nology induce a transition of economic activity away from a declining sector and towards a growing sector. As the first sector declines, workers may keep looking for employment in this sector where jobs are destroyed, transition to the growing sector where jobs are created, or claim an outside option which captures the rest of the economy and early retirement.

The government can accelerate or slow down this transition by implementing sector-specific subsidies, thereby shifting the path of marginal revenue products of labor (MRPL) and firms' incentives to create jobs across sectors. We study how government interventions that change these paths of MRPL — corresponding to more or less ambitious timelines — affect the distribution of workers' present discounted value of future earnings or "lifetime earnings."

Three novel conceptual insights emerge from our analysis. First, workers' lifetime earnings are highly non-linear or even non-monotonic in the horizon over which the transition unfolds. The non-linearity implies that shortening a transition by 1, 5 or 10 years has dramatically different implications for workers. Indeed, workers will experience a given shift in labor demands by the time they retire. A faster transition not only means that workers will be exposed to this shift for longer; it also exposes them to a larger shift which they otherwise would not have experienced before retirement. These two effects reinforce one another and imply that a faster transition has a non-linear effect on lifetime earnings. Extrapolating from small changes around a status quo transition can thus be very misleading.

Workers' earnings changes can even be *non-monotonic* in the pace of the transition: some workers suffer at first when the transition accelerates, but benefit from an even faster one. This non-monotonicity results from the tension between two opposing forces: a faster transition displaces workers earlier from the declining sector, but it also creates more high-paying jobs in the growing sector before they retire. We show analytically that the relative importance of the second effect grows as the transition accelerates. In particular, workers generally suffer the most for *medium-speed* transitions which are fast enough that workers experience all of the earnings drop in the declining sector, but slow enough that workers retire before benefiting from good job opportunities in the growing sector.

The second insight is that the government faces a trade-off as it accelerates the transition: on the one hand, more and more workers benefit on the *extensive margin* as the transition accelerates further; on the other hand, the tail of losses becomes

thicker on the *intensive margin*. These results have important political economy implications: the median worker might prefer a faster transition, but a minority of workers might oppose it more and more vocally. We also find that older workers — who participate more in the political process — are *less* likely to benefit from a faster transition, as they have a shorter horizon over which to recoup the short-run losses experienced after being displaced from the declining sector.

After establishing these results analytically in a frictionless model with perfectly competitive labor markets, we develop a quantitative version of our model that nests this frictionless benchmark and incorporates features that are important empirically to account for earnings losses. First, unemployment spells and job ladders are crucial to explain workers' earnings losses after separation (Jacobson et al., 1993; Huckfeldt, 2022). We therefore allow for costly vacancy creation and directed search on-the-job (Menzio and Shi, 2011), which generate unemployment scarring as workers fall from the ladder. Second, human capital depreciation has long been recognized as another important source of unemployment scarring (Ljungqvist and Sargent, 1998; Schmieder et al., 2016). We therefore allow workers to accumulate multi-dimensional skills on-the-job and dis-accumulate them off-the-job (Lise and Postel-Vinay, 2020). Finally, there are large gross flows between industries (Pilossoph, 2014) and job-to-job switches often entail wage cuts (Sorkin, 2018). Accordingly, we assume that workers face a two-tier noisy discrete choice across sectors and across jobs so that their search is partially directed.

Our third main finding is that accounting for labor market frictions is important to quantify the non-linearities stemming from a faster transition. We repeat our counterfactual analysis in the quantitative model and find that workers' earnings remain highly non-linear in the transition horizon. We decompose the contribution of each quantitative feature by adding them one at a time, starting from the frictionless benchmark, until we reach the full model. The two key additions that most affect the non-linearity are the presence of hiring costs and the existence of a job ladder. Creating and filling a vacancy is costly in the model, as in the data (Hagedorn and Manovskii, 2008; Muehlemann and Strupler Leiser, 2018), and firms only create vacancies if they expect to recoup these costs. As the transition accelerates, workers' earnings change for two reasons: firms' marginal revenues go up in the growing, which raises the wage offered to workers for a given piece rate; in *addition*, the presence of a fixed hiring cost ν means that firms are now able able to post

vacancies at higher piece rates in that sector, leading to a sharper increase workers' earnings as they cycle through jobs. The opposite happens in the declining sector. Overall, we conclude that labor market frictions are key to quantify not just the level of earnings losses from displacement, echoing the findings of Kambourov (2009), but also how they *change* as a transition speeds up.

While these insights are applicable to any shifts in economic activity where one sector expands at the expense of another one, we apply the model empirically to one particular context: the climate transition in the United States. This application is especially relevant for today's policymakers: it concerns 10-15 million workers who are currently employed in polluting industries, roughly the size of manufacturing employment in the early 2000s; its pace is partly under the control of the government; and most of the transition has yet to take place. Whether the government decides to speed up or slow down the climate transition, it needs to know how workers' outcomes vary with the horizon over which it takes place. We start by using wage data to estimate the paths of MRPLs realized so far in clean and polluting industries. The prospective nature of our analysis means that we have to extrapolate these realized MRPLs several decades into the future. We thus consider different scenarios where the MRPLs either change at the same pace as they have so far, or take off more aggressively after 2025.

We then estimate the distribution of workers' skill endowments in this particular context, using the realized and projected paths of MRPLs. To do so, we leverage the block recursive property of competitive search equilibria, meaning that workers' policy functions can be solved independently from the distribution of skill endowments. The extent and timing at which workers reallocate between polluting and clean industries in the phase of the transition observed so far allows us to back out the distribution of skill endowments, accounting for all the model's frictions and the presence of substantial gross flows.

In our main scenario, we find that most workers initially attached to polluting industries have already experienced significant earnings losses from the climate transition. The average worker has lost 5.4% of cumulative earnings over the period 2005–2025, with some workers losing more than 10%. These numbers are comparable to existing estimates in the context of trade liberalization and import competition (Autor et al., 2016; Traiberman, 2020) and equivalent to a third of polluting-attached workers having experienced one layoff over the period 2005–

2025 in a difficult labor market (Davis and von Wachter, 2011). These average cumulative earnings loss for incumbent workers over this period is roughly $67,000 or $670 billion in aggregate, which is in line with the aggregate earnings decline in polluting industries that we estimate in the Quarterly Census of Employment and Wages.

We then use our model to perform counterfactuals and ask how changing the pace of the remaining transition, from 2025 onwards, would affect workers' earnings. While the *average* earnings losses from completing the transition in 2050 or 2040 instead of 2060 are relatively modest, the losses *conditional* on losing are substantial: workers lose an additional $9,388 on average when the transition is completed in 2050, and $22,239 when it is completed in 2040. These losses are partly offset by the fact that the number of workers who benefit increases as the transition accelerates. Consistent with our conceptual insights, we thus find that policymakers face a tradeoff when accelerating the transition: doing so creates more winners on the extensive margin, but thickens the tail of lifetime earnings losses on the intensive margin.

Our work connects to a literature studying historical transitions to particular labor demand shocks. Lagakos and Waugh (2013) and Porzio et al. (2022) focus on past structural transformation and its effect on workers' wages. Burstein et al. (2019) and Braxton and Taska (2023) find that workers incur substantial earnings losses when technological changes make their skills obsolete. Dix-Carneiro (2014), Traiberman (2020) and Galle et al. (2023) quantify the impact of various trade liberalizations episodes. Adão et al. (2024) show that the adoption of new technologies takes more time when workers do not possess the right skills to operate these technologies. In contrast to this literature, we do not study one particular historical episode. Instead, we ask a counterfactual question: how large would earnings losses be if the government intervened to speed up or slow down a given transition? We uncover strong non-linearities and non-monotonicities in earnings losses as transitions get faster, and show that accounting for skill heterogeneity, costly hiring, and unemployment scarring is key to quantify these non-linearities. Our focus on non-linearities also sets our paper apart from recent work considering first order deviations from status quo transitions (Lehr and Restrepo, 2023; Seo and Oh, 2024).

We contribute to a literature asking whether policymakers should slow the

adoption of new technologies. Guerreiro et al. (2021), Costinot and Werning (2023) and Lehr and Restrepo (2023) argue that taxing automation can be optimal to mitigate their distributional consequences. Beraja and Zorzi (2024) show that the pace of technological transitions is inefficiently fast when workers cannot smooth consumption after being displaced. Braxton and Taska (2024) investigate the role of retraining subsidies to insure workers against technological change. Studying sectoral shifts where one sector grows at the expense of another, we find that some workers who suffer from a faster transition *locally* might, in fact, prefer an even faster one *globally* if it accelerates sufficiently to create enough high-paying jobs in the growing sector.

Our climate application adds to a growing body of work on the costs and benefits of the energy transition. A large literature documents substantial societal gains from a faster climate transition (Barrage and Nordhaus, 2023; Bilal and Känzig, 2025; Nath et al., 2024). At the same time, Bilal and Stock (2025) note that there has been comparatively little work on worker reallocation and earnings losses due to this transition, which motivates our application. Empirical work documents the ongoing reallocation of workers towards green industries (Curtis and Marinescu, 2023; Curtis et al., 2023). Consoli et al. (2016) and OECD (2024) argue that green jobs typically require more education and training, and are more intensive in cognitive skills. Walker (2011, 2013) and Känzig (2023) document substantial and long-lasting earnings losses due to environmental regulation, confirming that workers' skills are imperfectly transferable to clean industries.[2] Hafstead and Williams (2018) study the effect of carbon taxes on unemployment in models where workers have uniform skill across sectors.[3] Relative to the existing literature, our analysis estimates the distribution of workers' skills that matches the observed labor reallocation to clean industries, conducts counterfactuals to explore how the set of winners and losers changes as the transition accelerates from 2025 onwards, and emphasizes the importance of non-linear effects.

[2] A complementary set of papers studies how policy can influence the speed of this transition (Acemoglu et al., 2012, Acemoglu et al., 2023).

[3] Hafstead and Williams (2026) review this literature and emphasize labor market frictions and skill mismatch as avenues for future research, both of which play a central role in our paper.

2 Frictionless Benchmark

We introduce a frictionless model to illustrate the core economic mechanisms as transparently as possible. We will extend this model in Section 5 to account for realistic labor market frictions. Time is continuous and indexed by $t \geq 0$. There are two sectors: $k = C$ where labor demand grows over time, and $k = D$ where labor demand declines. With an eye toward our application, sector $k = C$ can be thought of as the "clean'" sector where jobs are "created," and sector $k = D$ can be thought of as the "dirty" sector where jobs are "destroyed." We also allow for an outside option $k = O$, which represents sectors unaffected by the transition or the value of retiring early.

2.1 Workers

Demographics and preferences. Workers live for J years and are replaced by new ones upon retirement. We let τ denote a worker's cohort or birth date. Workers supply labor inelastically and consume in ages $j \in [0, J)$. Their preferences are given by

$$\int_0^J \exp\left(-\rho j\right) u\left(c_j\right) dj \tag{2.1}$$

where c_j is consumption of the final good at age j, $u(c) \equiv c^{1-\sigma}/\left(1-\sigma\right)$ is the flow utility function, and $0 < \rho < \infty$ is the discount rate.

Skills. Workers have heterogeneous skills across sectors. Let ψ be their skill in sector D, and $\hat{\psi}$ be their relative skill in sector C. Workers transfer a share $b^O \geq 0$ of their skill ψ to the outside option. Therefore, a worker's skills are

$$h\left(\psi, \hat{\psi}, k\right) = \begin{cases} \psi & \text{if } k = D \\ \psi \times \hat{\psi} & \text{if } k = C \\ \psi \times b^O & \text{if } k = O \end{cases} \tag{2.2}$$

All workers are endowed with a reference skill $\psi = 1$ and draw an endowment of relative skill $\hat{\psi}$ from a distribution $G\left(\hat{\psi}\right)$. We suppose for now that these skills are fixed over a worker's life, and relax this assumption in our quantitative model.

2.2 Firms

Final good. A representative firm produces a final good by combining the output of the declining and growing sectors according to

$$Y = Z_t F_t \left(X_t^D, X_t^C \right) \equiv Z_t \left[\sum_{k=D,C} \left(\varsigma_t^k \right)^{\frac{1}{\phi}} \left(X_t^k \right)^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}}, \tag{2.3}$$

where Z_t is aggregate TFP, X_t^k are intermediate goods produced in the two sectors, and $\phi > 0$ is the elasticity of substitution between these intermediates. As usual, the aggregate production function can be alternatively interpreted as households' preferences over these two goods. Accordingly, the time-varying ς_t^k coefficients capture both intermediate-specific productivity improvements and changes in preferences for these goods. In our application to the climate transition, we will think of the two goods as being imperfect substitutes so $\phi \in (1, \infty)$, e.g., electric vehicles (EVs) and internal combustion engines (ICEVs) are substitutes.[4] We allow for long-term growth so $Z_t \equiv Z_0 \exp(gt)$ for some growth rate $g \geq 0$.

Intermediates. The two intermediate goods are produced using capital and labor

$$X_t^k = \left(K_t^k \right)^{\alpha} \left(n_t^k \right)^{1-\alpha}, \tag{2.4}$$

where $\alpha \in (0,1)$ is the capital share. In equilibrium, labor used in a sector equals its supply

$$n_t^k = \int h\left(\psi, \hat{\psi}, k\right) d\mu_t^k \left(\psi, \hat{\psi}\right),$$

where $\mu_t^k \left(\psi, \hat{\psi}\right)$ denotes the measure of workers employed in sector k and period t with skills $\left(\psi, \hat{\psi}\right)$. A continuum of entrepreneurs invests in sector-specific capitals which evolve according to

$$dK_t^k = \left(I_t^k - \delta^K K_t^k \right) dt,$$

where $\delta^K > 0$ is the depreciation rate. Gross investment $I_t^k = \left(z_t^k\right)^{\gamma} \left(e_t^k\right)^{1-\gamma}$ is installed by combining units of the final good z_t^k and entrepreneurs' time e_t^k, given

[4] Of course, the two intermediate goods cannot be perfect complements. Otherwise, they would perfectly co-move and there would be no sectoral shift in the first place.

some elasticity $\gamma \in (0,1)$. Entrepreneurs' time is in fixed supply so $e_t^D + e_t^C = 1$ in equilibrium. This capacity constraints can impede the investment boom in the growing sector.

2.3 Markets and Government

Labor and goods markets. Labor markets are perfectly competitive in our frictionless benchmark. Workers may supply labor to growing sector $k = C$ or the declining one $k = D$ and earn $\exp(gt)\, h\left(\psi, \hat{\psi}, k\right) w_t^k$ where w_t^k is the de-trended wage per efficiency unit in a sector. They can alternatively claim the outside option and receive a flow endowment of $h\left(\psi, \hat{\psi}, O\right)$ of final goods which grows exogenously at the rate g that prevails along a BGP. Workers freely choose between $k = C, D, O$ at every instant in this frictionless model. Product markets are also competitive, and firms sell their output to the final goods producer at a de-trended price P_t^k.

Government. The government can speed up or slow down the transition using subsidizes or taxes τ_t^k on investment in each sector, e.g., an investment tax credit, accelerated depreciation for certain types of equipment, subsidized loans for manufacturing investments, etc.[5] We suppose that the government balances its budget in every period using lump sum taxes on entrepreneurs or workers.

Financial Markets. Our analysis focuses on the case with risk-neutral workers, i.e., $\sigma = 0$, which is standard in the type of search models that we use in our quantitative analysis (Menzio and Shi, 2011; Bilal et al., 2022). It is equivalent to assume that $\sigma > 0$ but that workers can borrow or save freely. In both cases, workers' objective and what we characterize is the present discounted value of earnings.

2.4 Equilibrium

An equilibrium consists of labor supplies n_t^k and capital stocks K_t^k, and wages w_t^k and goods prices P_t^k such that workers supply labor optimally given wages, entrepreneurs demand labor and invest optimally given wages and prices, and labor and goods markets clear. We characterize the equilibrium in Appendix A.1 and explain how we solve the model numerically in Appendix A.2. Next, we describe

[5] As we discuss in Section 3, our results carry through when the government subsidizes households' purchases of the goods produced in each sector.

how the transition unfolds.

3 Status Quo and Counterfactual Transitions

We first discuss the status quo transition that would prevail without further intervention. We then explain our counterfactuals.

3.1 Status Quo Transition

The *status quo* transition is the one that prevails when the government sticks to the path of policy interventions that is currently expected. The economy rests on its initial balanced growth path before the transition begins in period $t = 0$. Sector C is not active initially: there is no demand for the good it produces, e.g., EVs have not been invented or are not valued yet. The transition begins in $t = 0$ due to an exogenous change in technology or preferences as captured by the coefficients ς_t^k in expression (2.3), e.g., increased awareness of climate change or a change in government mandates for greener goods.

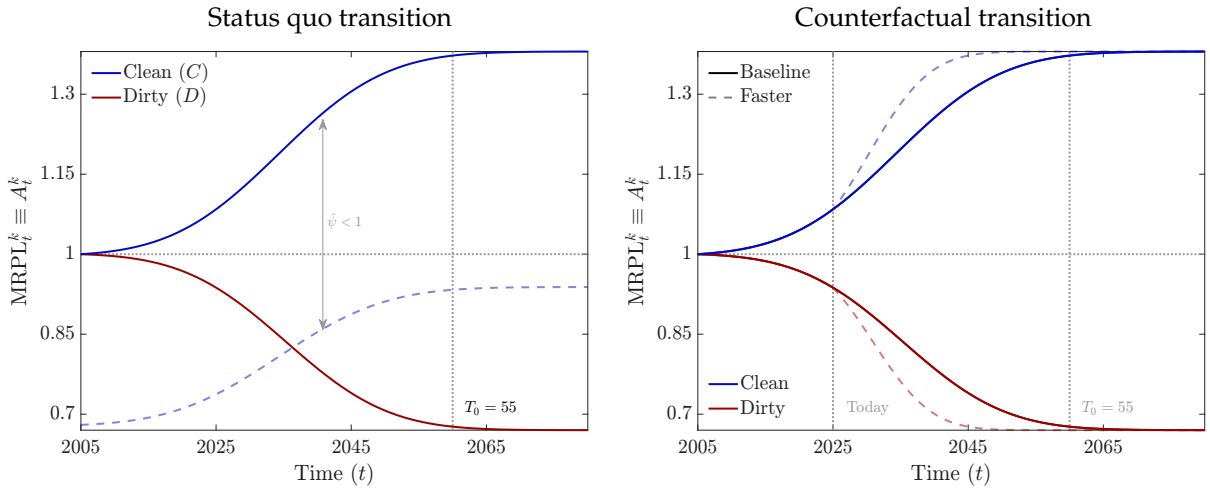
To illustrate how the sectoral shift affects workers, it is useful to define the following de-trended marginal revenue products of labor (MRPLs) in the two sectors

$$A_t^k \equiv \text{MRPL}_t^k = \underbrace{F_{k,t}\left(X_t^D, X_t^C\right)}_{\equiv P_t^k} \underbrace{(1-\alpha)\left(\frac{K_t^k}{n_t^k}\right)^{\alpha}}_{=\partial X_t^k(\cdot)/\partial n_t^k}, \tag{3.1}$$

where X_t^k still denotes sectoral outputs (2.4). These MRPLs reflect changes in technology or preferences F_t or ς_t^k, but also the endogenous response of sectoral investments and capital stocks K_t^k, labor supplies n_t^k and the de-trended equilibrium prices $P_t^k \equiv F_{k,t}$. As we show in Appendix A.1, these marginal revenue products of labor are *sufficient statistics* to characterize workers' outcomes over the transition: other features of the economy matter only to the extent that they affect A_t^k.[6] Sectoral MRPLs therefore provide a convenient way to express our counterfactuals. We will measure these MRPLs empirically using wage data in our application to the climate transition in Section 6.

[6] This observation carries through in our full model of Section 5 which nests our frictionless benchmark (see Appendix C.2).

Figure 3.1: Marginal revenues product of labor A_t^k



Notes: The left panel illustrates the paths of MRPLs A_t^k in the two sectors under the status quo transition. The solid blue corresponds to the growing sector C and the solid red curve corresponds to the sector D. The dashed blue curve indicates the earnings of a worker with $\hat{\psi} < 1$ who supplies labor to sector C. The right panel illustrates a faster transition parametrized by expression (3.2) with $\eta > 1$.

By definition, a sectoral shift is a transition where labor demand, i.e., the MRPL, rises in one sector and falls in the other, leading to a labor reallocation between the two. For example, the service sector expands, while manufacturing declines relative to trend; a new and more productive technology takes over an aging one (Violante, 2002); or labor demand rises in EV production and falls in ICEV production. The left panel of Figure 3.1 illustrates a particular sectoral shift.[7] The red curve corresponds to the equilibrium path of A_t^D in the declining sector, while the blue curve corresponds to A_t^C in the growing sector sector. In this benchmark model, workers reallocate to the growing sector when $\hat{\psi} A_t^C$ — the dashed curve on the left panel — exceeds A_t^D, which dampens the rise in MRPLs in the growing sector.

We choose the scale of the economy so the MRPL in the declining sector starts at $A_0^D = 1$ and normalize the sequence ς_t^D accordingly, without loss of generality. In turn, the initial MRPL in the growing sector A_0^C is not separately identified from the mean of the distribution of relative skills G: workers may decide to supply labor to the clean sector either because sectoral productivity is high, or because they have high idiosyncratic skills there. We thus set $A_0^C = 1$ by normalizing the

[7] The numerical parameters used for this example are listed at the end of Appendix A.3.

sequence ς_t^C accordingly.[8]

With our application in mind, Figure 3.1 assumes that the sectoral shift begins in 2005 and is mostly completed by 2060. In the following, we will use $T_0 \geq 0$ to denote the period when a large and fixed share of sectoral shift in MRPLs is completed (say 98%). In our example, the de-trended MRPLs follow S-curves in the two sectors. These S-curves are characteristic of technology adoption or diffusion (Griliches, 1958) and reflect changes in F_t or ς_t^k. For instance, information diffuses slowly and car adjustments are infrequent, so EV adoption is slow. While the S-shapes are not crucial to our analytical results, we will maintain them throughout for illustration.

3.2 Counterfactuals

We study how workers initially attached to the declining sector D — those who are most likely to oppose the transition — would fare if the government intervened to speed up the sectoral shift. We suppose that the government intervenes starting from today's period $t_0 \in [0, T_0)$. The government may want to intervene for various reasons that depend on the context and that are not modeled explicitly. For instance, in our application, the government might intervene to meet climate commitments.[9] We then ask: how do workers' earnings change if the climate transition were to be completed 1, 5, or 10 years earlier?

To capture this idea, we suppose that the government uses its short-term subsidies and taxes to accelerate the sectoral shift so that

$$A_t^k(\eta) = A_0^k + \Phi_t^k\left(t_0 + \eta\,(t - t_0)\right) \times \left(A_\infty^k - A_0^k\right), \quad \text{for } t \geq t_0, \tag{3.2}$$

where the parameter $\eta > 0$ governs the speed of transition and Φ_t^k corresponds to the share of the sectoral shift completed in a given sector at any given point in time under the status quo. By construction, the paths of A_t^k coincide with the status

[8] The coefficients ς_t^k change smoothly over time, starting at zero in the growing sector. The marginal revenue product of labor (3.1) therefore starts at 1 after normalization, even in the limit $t \downarrow 0$ where n_t^C is small.

[9] Other well-known reasons to speed up a given transition include: shifting resources from previously protected industries (Mussa, 1984), reducing the uncertainty associated with a reform (Rodrik, 1989; Calvo and Drazen, 1998), overcoming the resistance from lobbies and a tendency to favor the status quo (Fernandez and Rodrik, 1991; Rodriguez-Clare et al., 2025), or addressing coordination failures (Murphy et al., 1989; Matsuyama, 1992).

quo transition when $\eta = 1$. Increasing η compresses the timeline of the transition, ensuring that the same transition is completed sooner at some horizon $T < T_0$, as depicted in the gray curves in the right panel of Figure 3.1. While specification (3.2) provides a convenient and intuitive way to parameterize the counterfactual pace of transition, we establish in Appendix B that our conceptual results hold under more general perturbations of the paths of A_t^k.

The government can use its short-term subsidies and taxes to implement such changes in MRPLs, e.g., investment tax credits or direct subsidies for clean investment or tighter emission standards for polluting technologies. We show in Appendix A.3 how to solve for these subsidies, accounting for the endogenous response of sectoral investments, labor supplies and goods prices.[10] Note that the production side of the economy matters for the exact mapping between subsidies and A_t^k and hence the fiscal cost of an intervention. However, the production side does *not* affect how these MRPLs affect workers's labor market outcomes — the focus on this paper. The fiscal cost of these subsidies is mostly incidental to our paper and deserves a separate treatment. For completeness, we will still present some related results in our quantitative application in Section 6.

In the following, we will conduct a *positive* analysis: we study how a faster sectoral shifts in MRPLs affects workers' labor market outcomes, how many of them benefit or lose from the transition, and how expensive it would be to compensate them as a function of how ambitious the transition is. Our goal is *not* to conduct an optimal policy analysis, which would require us to take a stance of who should be compensated, what Pareto weights we assign to them, which workers or cohorts bear the fiscal cost of the subsidies, etc. Instead, our analysis provides a crucial first step in this larger agenda by exploring how a faster sectoral shift affects the earnings distribution.

Climate application. While our conceptual insights apply to any sectoral shifts, it is useful to specialize equation (3.2) to our climate application to fix ideas. The Kyoto Protocol was the first major climate agreement. It established binding targets for

[10] Of course, the government must adopt larger subsidies than it would in partial equilibrium to offset to general equilibrium dampening coming from worker reallocation and decreasing returns at the sector-level. While the intervention that implements the counterfactual of Figure 3.1 requires a mix of subsidies and taxes, unilateral subsidies to the growing sector can achieve the same goal qualitatively when the goods are sufficiently substitutable. They generate an investment boom in the growing sector, which comes at the expense of the declining sector.

greenhouse gas (GHG) emissions for the 192 countries that ratified it, including the US. The Protocol entered into force in 2005. We therefore interpret period $t = 0$ as January 2005. In turn, the Paris Climate Agreement of 2015 set a goal of net-zero emissions for developed economies by the year 2050. The US, however, pulled out of the Paris Agreement and is not currently on track to meet this target by 2050. The median response in a recent survey of US energy CEOs suggest that 2060 is a more realistic timeline (Bain & Company, 2025). We therefore assume that the status quo transition in our model runs from 2005 to roughly 2060, corresponding to $T_0 = 55$ in Figure 3.1. For concreteness, we refer to these dates throughout our analysis. Our counterfactuals consist of speeding up the transition starting in $t_0 = 2025$ — the exercise that is most relevant to policymakers today.[11]

4 Economic Mechanisms

We use our frictionless benchmark to build new conceptual insights about how different transition timelines affect workers' outcomes during a sectoral shift.

4.1 The Non-Linear Effects of Rapid Transitions

Consider a worker initially attached to the declining sector. The left panel of Figure 4.1 plots their path of earnings over time under the status quo transition, in the case where the worker has a relative productivity $\hat{\psi} < 1$ in the growing sector. To keep the exposition simple, we abstract momentarily from the outside option, e.g., workers can only switch between manufacturing and services, or within car-manufacturing from ICEVs to EVs.[12] For illustration, we assume that workers are active between the ages of 25 to 65, i.e., $J = 40$. Workers discount the future at rate $\rho = 5\%$ and long-run growth is 2%.[13]

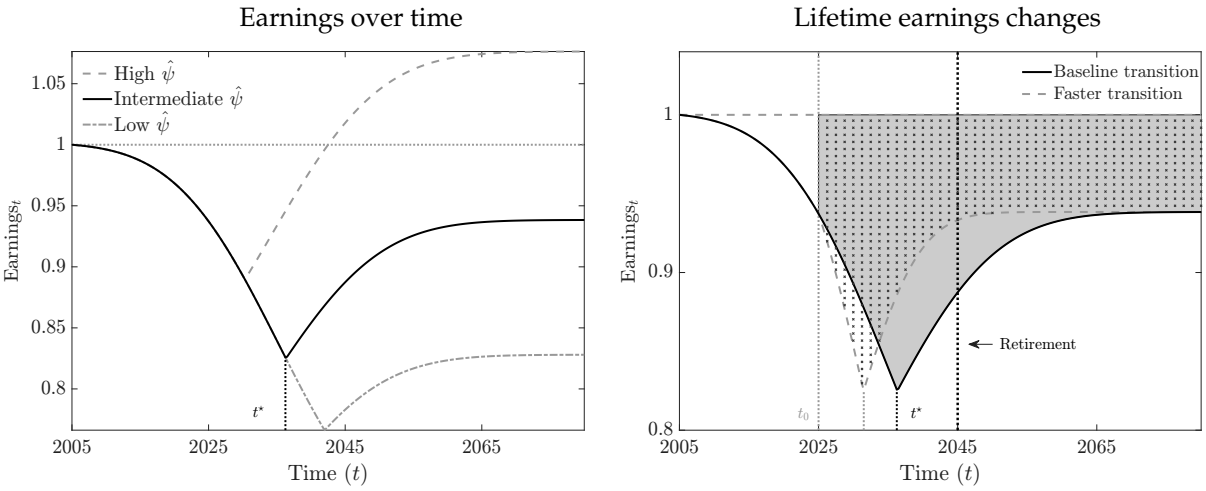
Workers supply labor to the growing sector if and only if $\hat{\psi} \geq A_t^D / A_t^C$. The right-hand side of this inequality shrinks over time. Therefore, a worker remains in

[11] Our analysis focuses on short-run subsidies and taxes that speed up (or slow down) the transition, so we treat the long-run levels as fixed. Of course, coordinated interventions across the globe can reduce climate damages, thereby raising aggregate output in the long-run. Despite being a large economy, the US accounts for only 13% of global CO2 emissions, so the unilateral interventions we focus here have minimal effect on climate damages.

[12] That is, we set $b^O < A_\infty^D$ for now. We will discuss the role of this outside option shortly.

[13] The effective discount rate is thus 3%, as we show in Appendix C.3 for our full model.

Figure 4.1: Lifetime earnings changes for various transition speeds



Notes: The left panel plots a worker's earnings over time in our frictionless benchmark, for various productivities $\hat{\psi}$ in the clean sector. The right panel indicates lifetime earnings changes under the status quo transition and when transition ends 15 years earlier.

the declining sector D until an idiosyncratic switching time $t^\star \geq 0$, which depends on their relative skill $\hat{\psi}$ and the particular transition that they face.[14] The worker supplies labor to the growing sector C after that. A worker actually reallocates across sectors if their switching time occurs during their life, i.e., $t^\star < J + \tau$.

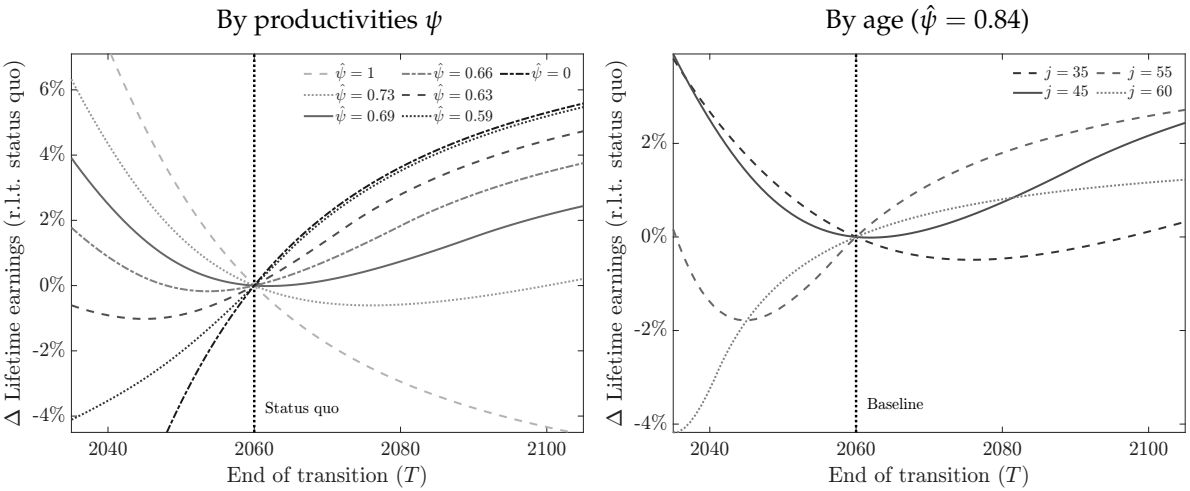
Workers' earnings fall while they remain in the declining sector, reflecting the path of A_t^D. Their earnings rise after they switch to the growing sector, reflecting the path of $\hat{\psi} A_t^C$, but may never fully recover. Workers with a higher relative skill $\hat{\psi}$ switch earlier and benefit more in the long-run, as depicted by the dashed line. The opposite happens when the worker has a lower relative skill.

The effect of the status quo transition on workers' lifetime earnings is given by the area between the realized earnings path and the one that they would have experienced without the transition, discounted at rate ρ until retirement. We shade this area in gray on the right panel of Figure 4.1. This area is negative when a worker's relative skill $\hat{\psi}$ is relatively low, since they lose from the early phase of the transition and do not make up for that before retirement.

Consider now a faster transition, as depicted by the checked region on the right panel of Figure 4.1. It has two opposing effects on a worker's lifetime earnings. On the one hand, it steepens the fall in their earnings while they remain in the declin-

[14] Formally, the idiosyncratic t^* is the period when $\hat{\psi} = A_t^D / A_t^C$, with $t^\star \equiv 0$ or $t^\star \equiv \infty$ if no solution exists and the worker either switches immediately or never switches.

Figure 4.2: Average lifetime earnings changes



Notes: The left panel plots workers' average lifetime earnings changes in our frictionless benchmark as a function of the transition horizon T, for various values of the relative skill $\hat{\psi}$. Earnings changes are relative to the status quo transition ending in 2060, and transitions are parameterized as in equation (3.2). All workers have 20 years remaining in their careers (median age). The right panel repeats the same exercise for various ages as of 2025, fixing relative productivity to $\hat{\psi} = 0.84$.

ing sector. On the other hand, it means that the worker switches to the growing sector earlier, that their earnings grow faster once they supply labor there, and that a larger share of the transition is completed before they retire.

Our goal is to understand the interplay between these two forces as the transition accelerates. Figure 4.2 plots changes in lifetime earnings as a function of the year T when the transition is mostly complete.[15] Faster transitions are parametrized as in equation (3.2) and we set $\eta \equiv (T_0 - t_0) / (T - t_0)$ so the transition is mostly completed in period T instead of T_0. For each of these transitions, we compute the change in lifetime earnings from 2025 onwards as

$$\mathcal{W}\left(j, \hat{\psi}; \eta\right) \equiv \frac{W_{t_0}\left(j, \hat{\psi}; \eta\right) - W_{t_0}\left(j, \hat{\psi}; \eta_0\right)\big|_{\eta_0 = 1}}{W_{t_0}\left(j, \hat{\psi}; \eta_0\right)\big|_{\eta_0 = 1}}, \tag{4.1}$$

so all lifetime earnings changes are zero for a transition ending in 2060 by construction.[16] The left panel repeats this experiment for various workers with different

[15] As before, T denotes the period when 98% of the the transition is complete.

[16] In this section, we present results for various cross-section of workers' ages. All workers of a given age share the same denominator in expression (4.1) in our frictionless model. When pooling earnings changes across cohorts in our quantitative model (Section 6), we take into account the fact that various cohorts have different remaining horizons and hence different denomina-

relative skills $\hat{\psi}$ and who have 20 years of work remaining. In turn, the right panel does the same for workers of different cohorts with $\hat{\psi} = 0.84$.

This figure conveys our first main insight: changes in lifetime earnings are highly non-linear or even non-monotonic as a function of the horizon over which the sectoral shift unfolds. We now explain why this is the case.

Non-linearities. A faster transition creates winners and losers among workers, depending on their relative skill $\hat{\psi}$. Figure 4.2 reveals that, at the margin, a worker can gain or lose much more as the transition accelerates further. For instance, a worker with $\hat{\psi} = 0.86$ is mostly indifferent at first when the transition accelerates starting from 2060, but they benefit more and more as the pace of the transition picks up. In other words, accelerating the transition by 1, 5, or 10 years has a vastly different effect on workers' lifetime earnings. Policymakers should be cautious about extrapolating from small changes around the status quo ending in 2060.

To understand the intuition behind these non-linearities, consider a worker with $\hat{\psi} = 0$ who never switches to the growing sector C, regardless of the transition speed. This worker's change in lifetime earnings is given by the negative "triangle" between their earnings with no transition (1) and $A_t^D < 1$ which declines over time. In our example, $A_t^D \simeq 0.88$ under the status quo by the time the worker retires (Figure 3.1). As the transition accelerates, this negative area expands for two reasons: the economy reaches $A_t^D \simeq 0.88$ sooner, so the worker experiences a lower income for longer before retirement; and the economy experiences an *even larger* decline in A_t^D by the time the worker retires. These effects compound one another and translate into non-linear changes in lifetime earnings, as illustrated by Figure B.1 in Appendix.[17] The opposite happens for a workers with $\hat{\psi} = 1$ who switches immediately to the growing sector. These strong non-linearities arise even when the transition evolves linearly over time (as opposed to being S-shaped), as we establish in Appendix B.[18]

Non-monotonicities. A faster transition can even have a *non-monotonic* effect on workers' lifetime earnings: some workers lose from a faster transition at first, but eventually benefit from an even faster one. In particular, consider the worker with

tors.

[17] If the transition finishes before the worker's retirement, the earnings loss becomes the area of a trapezoid, generating an additional non-linearity.

[18] The result supposes *some* form of discounting, coming from the discount rate $\rho > 0$ or finite lives.

$\hat{\psi} = 0.84$ on the left panel of Figure 4.2. Their earnings changes are U-shaped: they would benefit from either a slower or a faster transition relative to the status quo. For this worker, a *medium-speed* transition that ends in 2060 represents the worst of all worlds. It is sufficiently fast that the worker experiences maximal losses in the declining sector before they switch sectors, but sufficiently slow that they retire before benefiting meaningfully from earnings gains in the growing sector.

To understand the intuition behind these U-shapes, consider a worker who retires at the exact moment when they would have switched to the growing sector, i.e., $t^* = J + \tau$. This worker spends their whole career in the declining sector and thus clearly prefers a slower transition. Now consider a marginally faster transition that leads the worker to switch to sector C before retirement. As the right panel of Figure 4.1 illustrates, a faster transition has two effects: workers experience earnings losses earlier when they discount them less; but they also benefit from stronger gains in the growing sector C before they retire. One can show that that the first effect dominates if the transition accelerates sufficiently relative to the status quo. As a result, changes in lifetime earnings $\mathcal{W}\left(j, \hat{\psi}; \eta\right)$ defined in (4.1) are U-shaped as a function of the speed of transition.

A worker who suffers from faster transition *locally* might, therefore, prefer an even faster one *globally* if it accelerates sufficiently to create enough high-paying jobs in the growing sector. It is worth noting that all workers will experience these non-monotonicities for *some* transition horizon, except those with very large productivities $\hat{\psi} \geq \hat{A}_{t_0}^{-1}$ who have already switched to sector C in period t_0 and those with very low productivities $\hat{\psi} \leq \hat{A}_{\infty}^{-1}$ who will never switch for any horizon. In our quantitative application, roughly half of workers would experience non-monotonicities for some transition speed.

Proposition 1 (Non-monotonicity)**.** *Lifetime earnings $\mathcal{W}\left(j, \hat{\psi}; \eta\right)$ are non-monotonic, i.e. U-shaped, in the speed of sectoral shifts η for all workers with $\hat{\psi} \in (A_{\infty}^{D}/A_{\infty}^{C}, A_{t_0}^{D}/A_{t_0}^{C})$. In contrast, they increase monotonically in η for workers with $\hat{\psi} \geq A_{t_0}^{D}/A_{t_0}^{C}$ who have already switched in period t_0 and decrease monotonically for those with $\hat{\psi} \leq A_{\infty}^{D}/A_{\infty}^{C}$ who will never switch.*

Proof. See Appendix B. □

The right panel of Figure 4.2 shows that a worker's age also matters for these non-monotonicities. We repeat the same exercise as above, for various ages in 2025,

fixing relative productivity $\hat{\psi} = 0.84$. Older workers have a shorter remaining horizon over which they can benefit from earnings gains in the growing sector. They are therefore more likely to experience earnings losses as the transition accelerates and it takes a faster transition for them to reach the trough of their U-shape.

The introduction of an outside option does not change these qualitative insights — only the *quantitative* predictions of the model, as we discuss in Section 5. This outside option puts a lower bound on workers' earnings; it is akin to truncating the MRPL in the declining sector D from below. Workers who never or always switch to the growing sector C still experience non-linear earnings changes as the transition accelerates further, and workers who switch to the growing sector at some point still experience non-monotonic earnings changes.

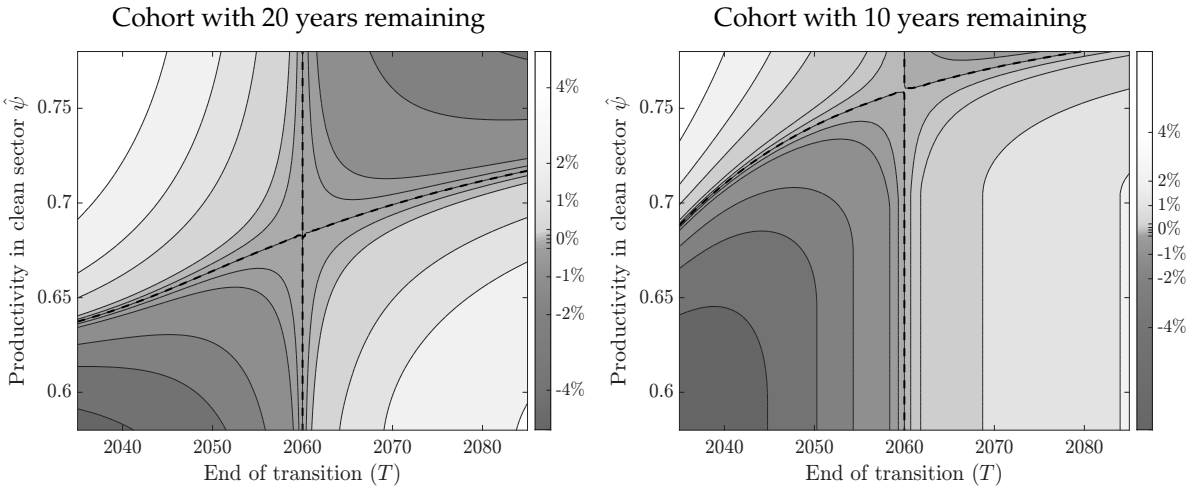
4.2 Political Economy Implications

An important corollary to the non-monotonicity result is that the set of workers who prefer a faster transition is endogenous and increases as the transition accelerates further. To understand this corollary, we plot in Figure 4.2 isoquants of lifetime earnings changes relative to the status quo transition in terms of the end date of the transition T (x-axis) and workers' relative productivities $\hat{\psi}$ (y-axis). The left panel corresponds to workers with 20 years of remaining work life, while the right panel corresponds to those with only 10 years left. The region on the left of 2060 corresponds to a faster transition, and the region on the right corresponds to a slower one.[19] Darker colors indicate larger losses, while lighter ones indicate smaller losses or even gains relative to the status quo. Intuitively, workers with a high relative skill $\hat{\psi}$ in sector C benefit more from, or are hurt less by, a faster transition and move to lighter and lighter regions. Those with a low $\hat{\psi}$ lose more and gradually move to darker and darker regions. For intermediate values of $\hat{\psi}$, workers experience non-monotonic earnings changes: they move to a darker region at first, cross the trough of their U-shape, and then move to ever lighter regions.

This figure reveals our second main insight: as the transition accelerates further, more workers prefer an even faster transition at the margin. Graphically, more workers cross from a darker region to a lighter one as we move to the left starting from 2060; either because they always benefit from a faster transition (high $\hat{\psi}$), or

[19] The central symmetry reflects that workers who prefer a faster transition locally also dislike a slower one, for instance.

Figure 4.3: Lifetime earnings change isoquants



Notes: This figure plots isoquants of lifetime earnings changes in terms of the end date of the transition T and workers' relative productivities $\hat{\psi}$. All earnings changes are relative to a baseline transition ending in 2060, and faster transitions are parameterized as in equation (3.2). The left panel plots isoquants for workers with 20 years remaining in their careers, while the right panel considers workers with 10 years remaining.

because more workers cross the trough of their U-shape (intermediate $\hat{\psi}$) and then prefer a faster transition. In other words, if the government already puts measures in place to accelerate the transition, it might be easier to garner more votes on the *extensive margin* to accelerate it further.

Similarly, the share of workers who benefit *on net* relative to 2060, i.e., globally and not just locally, increases as the transition accelerates. Graphically, more and more workers cross the dashed curves that separate positive and negative isoquants as we move to the left starting in 2060. These dashed curves are upward-sloping in Figure 4.3, reflecting again the U-shapes discussed in Section 4.1.

At the same time, a faster transition worsens losses on the *intensive margin* among workers with low skill in sector C who do not reallocate. Therefore, the government faces a trade-off as it considers accelerating the transition: doing so increases the mass of workers who benefit on the extensive margin; but it also generates a thicker tail of losses on the intensive margin among workers who stay in place in the declining sector. This trade-off has important political economy considerations: for instance, the median worker might prefer a faster transition, but a smaller and vocal share of workers might oppose it increasingly.

Comparing the left and right panel reveals that accelerating the transition tends

to benefit younger cohorts at the expense of older ones, since older workers are less likely to reach the trough of their U-shapes for a given transition horizon. In practice, older workers are more likely to participate in the political process (Census Bureau, 2025), potentially hindering public support for rapid transitions, which is particularly relevant for the climate transition (Dechezleprêtre et al., 2025).

4.3 Summary and Discussion

Our analysis reveals two novel conceptual insights. First, workers' lifetime earnings are highly non-linear and even non-monotonic as a function of the horizon over which a sectoral shift unfolds. In particular, medium-speed transitions can maximize losses for workers by displacing them early from the declining sector, without improving their prospects sufficiently in the growing sector before they retire. Second, a faster transition always increases the mass of workers who benefit on the extensive margin, but generates a thicker tail of losses on the intensive margin.

A few additional remarks are in order. First, the qualitative insights developed here do *not* depend on workers' expectations about the future. Indeed, workers base their labor supply decisions on current MRPLs (or wages) in our frictionless benchmark. Second, we consider counterfactuals where the government intervenes to change the speed of the transition. A different but related exercise would be to change the final levels of the transition and compare steady states directly, i.e., abstract from transitions. This second comparison also generates a non-monotonicity (or size-dependent effect of shocks) which has not been documented in the literature: incumbent workers might lose from small shocks (akin to a slow sectoral shift in our analysis) but can gain from larger ones (akin to faster shift). However, this alternative exercise differs from ours in that it does *not* generate non-linearities for workers who do not switch sectors, whereas earnings are strongly non-linear for these workers as the transition speeds up, for the reasons laid out in Section 4.1 and as illustrated by Figure B.1 in Appendix. Third, this paper studies sectoral shifts, i.e., scenarios where one sector rises at the expense of another. This scenario is the natural case to think about job displacement and captures numerous transitions in practice. Still, one could imagine *another* form of transitions where one sector takes off without affecting the other, so that no worker is ever displaced. Our non-linearity result still carries through in this case

(Appendix B). But, by construction, there cannot be any non-monotonicity, since no worker ever loses from that alternative transition.

Finally, note that the boundary of firms is not defined in this model. Therefore, $k = C, D$ really represent *technologies* — not necessarily sectors. In particular, the two technologies might co-exist in the same industry or firm, e.g., the same car manufacturer might produce ICEVs and EVs. The paths of A_t^k reflect firms' incentives to operate one technology or the other. In turn, the relative skill $\hat{\psi}$ reflects workers' comparative advantage at operating one or the other. Workers with a high $\hat{\psi}$ are more likely to operate the C technology, either by switching between employers or by being re-assigned to the new technology by their current employer.[20] In both cases, technology adoption is disruptive for workers. Empirically, Braxton and Taska (2023) find that technology adoption leads to displacement and substantial earnings losses for incumbent workers who do not possess the right skills. Kogan et al. (2020) document a higher separation risk in industries that innovate more, as workers' skills become obsolete more rapidly. This dynamic also plays out in the climate context: environmental regulations lead to layoffs (Greenstone, 2002; Walker, 2011) and green jobs tend to require more cognitive skills and experience (Consoli et al., 2016; Marin and Vona, 2019).[21]

5 The Role of Labor Market Frictions

To derive clear analytical insights, our frictionless benchmark abstracted from important empirical features. In particular, there was no proper job destruction or creation over the transition; and workers did not experience unemployment or the scarring that comes with it, both of which are crucial to explain earnings losses after displacement (Jacobson et al., 1993; Huckfeldt, 2022). Accordingly, we now aug-

[20] Our full model accounts for the fact that these switches are costly for the firm and worker. For instance, the firm must train the worker on a specific production process. Training costs are a large component of hiring costs in practice (Muehlemann and Strupler Leiser, 2018) and are incurred both when hiring outside the firm or re-assigning a worker within the firm. Similarly, workers must typically apply for jobs and wait to be matched, both for different firms and for openings in a different department or plant.

[21] For instance, ICEVs use many components that are not present in EVs and vice versa, such as catalytic converters, fuel tanks, engines, and batteries, which all rely on specific skills. The European Association of Automotive Suppliers projects half a million jobs will be lost from an EV transition, while the Economic Policy Institute similarly projects large layoffs in the US. Layoffs and retraining are already occurring at Ford and Volkswagen (accessed July 2, 2025).

ment the labor market block of our model by introducing costly vacancy creation, job ladders, and skill accumulation. We show that accounting for labor market frictions is crucial to quantify not only the level of earnings losses from displacement, but also how they *change* non-linearly with the transition speed.

5.1 A Realistic Model of the Labor Market

Skill accumulation. Workers have multi-dimensional skills: a general skill $\left(\psi^G\right)$ and skills that are specific to each sector $\left(\psi^D, \psi^C\right)$. Workers' idiosyncratic productivity in the declining and growing sectors are thus $\psi^G\psi^D$ and $\psi^G\psi^C$, respectively. At birth, workers are endowed with $\psi^G = \psi^D = 1$, and draw an initial $\psi^C \sim G$. The general skill accumulates at rate $\delta_j^+ \geq 0$ when the worker is employed in sectors D or C, at rate $\delta_j^O \geq 0$ when the worker claims the outside option, and $\delta^U \leq 0$ when unemployed. Specific skills grow at rate $\hat{\delta} \geq 0$ when employed in the corresponding sector. As before, we define a workers' reference skill as their skill in the declining sector $\psi \equiv \psi^G\psi^D$ and their relative skill as $\hat{\psi} \equiv \psi^C/\psi^D$. Workers' productivity $h\left(\psi, \hat{\psi}, k\right)$ is still defined as in equation (2.2).

Labor services. There is now a continuum of atomistic firms in each sector, which can hire one worker each to produce labor services. Each atomistic firm produces an amount of labor services equal to the productivity $h\left(\psi, \hat{\psi}, k\right)$ of the worker they hire. They sell these labor services at a de-trended price $\mathcal{P}_t^k$ to firms in their sector, which use them to produce varieties of the corresponding intermediate good.[22]

Directed search and matching. Workers can become unemployed and experience earnings scarring, as they slide off the job ladder and dis-accumulate skills during unemployment. We formalize this idea by assuming that workers and firms producing labor services search for each other in frictional labor markets. Search is directed, as in Menzio and Shi (2011), both within and across sectors. Workers can search on-the-job, giving rise to a job ladder. Specifically, labor markets are indexed by a worker's age j, their skills $\left(\psi, \hat{\psi}\right)$, the sector k of the job, and the piece rate π that workers receive. Employed workers therefore earn $\pi h\left(\psi, \hat{\psi}, k\right)\mathcal{P}_t^k$ and firms claim the rest of revenues.

A constant returns to scale matching function yields a job-finding probability for workers $f(\theta)$ within each market, where θ is the market tightness defined as

[22] In our frictionless model of Section 2, the prices of labor services coincide with the wages w_t^k.

the ratio of vacancies to job-seekers, and f is increasing and concave. The corresponding job-filling probability for firms is $q(\theta) \equiv f(\theta)/\theta$ which decreases with θ. Workers search for jobs with intensity $\lambda^U > 0$ off-the-job and $\lambda^E > 0$ on-the-job, giving rise to a job-ladder in the declining and growing sectors.

We allow for imperfectly directed search across markets to rationalize two facts: there are large gross flows between sectors (Pilossoph, 2014); and job switches often entail wage cuts (Sorkin, 2018). Accordingly, we allow for a two-tier discrete choice problem. In the lower tier, workers searches *within* a given sector $k = D, C, O$. Specifically, a worker of type $\mathbf{x} \equiv (j, \psi, \hat{\psi})$ in market $s \equiv (k, w)$ searches in market s' with probability

$$\underline{p}_t(s'; \mathbf{x}, s) \equiv \frac{\exp\left(f\left(\theta_t(\mathbf{x}, s')\right)\left[W_t(\mathbf{x}, s') - W_t(\mathbf{x}, s)\right]/\zeta\right)}{\sum_{\hat{s}} \exp\left(f\left(\theta_t(\mathbf{x}, \hat{s})\right)\left[W_t(\mathbf{x}, \hat{s}) - W_t(\mathbf{x}, s)\right]/\zeta\right)}, \tag{5.1}$$

where the sum over the markets $\hat{s}$ within the same sector associated with different piece rates $\pi \in [0, 1]$.[23] Workers are more likely to search in a given market or option s' if the expected improvement in their value is larger. This search behavior can be microfounded by introducing idiosyncratic amenity shocks across jobs that are distributed i.i.d. according to a Type-1 Extreme Value distribution with scale parameter ζ (Artuç et al., 2010) or by assuming that workers have to acquire costly information about the jobs that they apply to (Matêjka and McKay, 2015).[24] The parameter ζ captures the degree to which search is directed: search is purely directed as $\zeta \to 0$ as in canonical directed search, and purely random when $\zeta \to \infty$.[25] In the upper tier, workers search *across* sectors $k = D, C, O$. A worker in sector k searches in sector k' with probability

$$\bar{p}_t(k'; \mathbf{x}, k) \equiv \frac{\exp\left(\left[\overline{W}_t(\mathbf{x}, k') - \overline{W}_t(\mathbf{x}, k)\right]/z\right)}{\sum_{\hat{k}} \exp\left(\left[\overline{W}_t\left(\mathbf{x}, \hat{k}\right) - \overline{W}_t(\mathbf{x}, k)\right]/z\right)}, \tag{5.2}$$

where $\overline{W}_t(\mathbf{x}, k)$ is the expected value conditional on searching in the lower tier

[23] There is no job ladder for the outside option O, i.e., the only piece rate available is 1.

[24] The first formulation would increase workers' values by allowing them to enjoy job-specific amenities. On the contrary, the second would decrease them by assuming that workers have to acquire costly information. We thus follow Traiberman (2020) and Lentz et al. (2026) and do not include the amenity value or information cost in workers' values. These values can therefore be directly interpreted as workers' lifetime earnings.

[25] Firms know the characteristics of the worker they are matching with, which preserves the block-recursive nature of the equilibrium (as in Menzio and Shi, 2011).

associated with sector k, and z is the scale parameter associated with the upper tier. Matches are destroyed exogenously at rate $\chi > 0$.

Firm entry. All firms are owned by a continuum of infinitely-lived, risk-neutral and deep-pocket entrepreneurs who discount future flows at rate ρ. We let $V_t\left(j, \psi, \hat{\psi}, s\right)$ denote the value of a firm employing a worker of age j with skills $\left(\psi, \hat{\psi}\right)$ in market $s = (k, w)$. Firms can post vacancies for a worker with reference skill ψ at a cost $\nu\psi \exp(gt) > 0$, where ν captures both the vacancy and hiring cost.[26] This cost grows at rate g to ensure that the economy admits a balanced-growth path. There is free entry in each market s so that

$$q\left(\theta_t\left(j, \psi, \hat{\psi}, s\right)\right) V_t\left(j, \psi, \hat{\psi}, s\right) \leq \nu\psi \exp(gt) \tag{5.3}$$

in every period t, with equality if firms post a positive measure of vacancies.

Outside sector and unemployment. The rest of the economy is unaffected by the transition. Workers receive an income of $\psi b^O \exp(gt)$ of final goods when claiming the outside option, and $\psi b^U \exp(gt)$ when they are unemployed, with $b^O, b^U \geq 0$. We think of the outside option as capturing self-employment and jobs with limited scope for professional progression that workers initially attached to declining sectors can easily transition to. Accordingly, we abstract from the job-ladder in the outside option. We set the job finding rate f^O so it coincides with the average job finding rate in available jobs in the initial steady state. New cohorts ($\tau > 0$) are born unemployed.

Equilibrium. We assume that firms and workers have rational expectations. We state workers' and firms' values recursively and define an equilibrium in Appendix C. We also establish that firms' and workers' values are linear in $\psi \exp(gt)$, which allows us to de-trend the model using the growth rate g as in Section 3. This property also implies that it is without loss of generality to assume that $\psi = 1$ at birth, and allows us *not* to track ψ as an explicit state variable in our numerical implementation, making the simulation and estimation of the model computationally manageable. Given paths of MRPLs across sectors, the labor market part of the model satisfies the usual block recursive property of directed search models (Menzio and Shi, 2011), which also allows us to solve it efficiency. Workers now

[26] Assuming that the vacancy posting cost scales with ψ captures the fact that searching for a junior employee is cheaper compared to a more experienced, senior manager.

Table 5.1: External calibration

Parameter	**Description**	**Value**	**Source**
J	Lifespan	40	Career length
ρ	Discount rate	0.03	3% interest rate
α	Elast. of matching function	1.6	Schaal (2017)
δ_j^+	General skill accum. (D, C)	Age-specific	CPS ORG
δ^U	General skill accum. (U)	-0.096	Schmieder et al. (2016)
δ^O	General skill accum. (O)	0	Earnings grow at rate g

face idiosyncratic risk and maximize expected utility. We continue to assume that workers are risk neutral. Workers' search decisions, and hence the distributional impact of the transition on earnings, would be the same if workers were risk-averse ($\sigma > 0$) but faced complete markets for idiosyncratic risk.

Finally, note that the frictionless benchmark of Section 2 obtains as a limit of our full model when vacancy posting is free ($\nu \to 0$), search is perfectly directed ($\zeta \to 0, z \to 0$), unemployment pays less than employment (b^U is small) and skills are fixed over workers' lives ($\delta_j^+ = \delta_j^O = \delta^U = \hat{\delta} = 0$). We show in Appendix A.3 that the government can still implement the sort of interventions described in Section 3.2 in this full model.

5.2 Calibration

External Calibration. Table 5.1 summarizes the external calibration. Workers live for $J = 40$ years and discount the future at rate $\rho = 3\%$.[27] Following Den Haan et al. (2000), the matching function is $f(\theta) = (1 + \theta^{-\alpha})^{-1/\alpha}$ with $\alpha = 1.6$ as in Schaal (2017). We directly measure the skill accumulation rate $\delta_j^+ + \hat{\delta}$ in the Current Population Survey (CPS) using on-the-job wage growth.[28] We let $\delta_j^U = -0.096$, corresponding to a 0.8% loss in human capital per month of unemployment (Schmieder et al., 2016). Finally, we set $\delta_j^O = 0$ so workers' income increases at the same rate at which the economy grows when they claim the outside option.

Internal Calibration. We calibrate the remaining parameters to match various fea-

[27] This is the effective discount rate net of the growth rate g along a BGP (see Appendix C.3).

[28] With an eye towards our application in Section 6, we measure this wage growth for workers attached to polluting sectors before the start of the climate transition.

Table 5.2: Internal calibration

Parameter	Description	Value	Moment	Source	Target	Model
Search						
λ^U	Search intensity (U)	5.910	UE rate (M)	Shimer (2005)	25%	25%
λ^E	Search intensity (E)	0.657	EE rate (M)	Fujita et al. (2024)	2.5%	2.5%
z	Noise (upper nest)	0.245	LS elast. (sectors)	Galle et al. (2023)	2	2
ζ	Noise (lower nest)	0.168	Share of wage cuts	Grigsby et al. (2021)	30%	30%
Job creation & destruction						
ν	Vacancy post. cost	0.299	Average markdown	Berger et al. (2022)	20%	20%
χ	Job destruction rate	14.3%	Unemployment rate	-	5%	5%
Benefits and Skill						
b^O	Benefit in O	0.587	Wage change (EUO)	CPS ORG	-18%	-18%
b^U	Benefit in U	0.322	Average UI replac.	US UI rules	45%	45%
$\hat{\delta}$	Specific skill accum.	0.4%	Net realloc. by age	CPS	–	–

tures of US labor markets. We think of our calibration period as the year 1995, before the growing sector emerges. Table 5.2 summarizes the internal calibration. We calibrate the search intensity for unemployed workers λ^U by targeting a monthly UE rate of 25% (Shimer, 2005), and the one for employed workers λ^E by matching a monthly EE rate of 2.5% (Fujita et al., 2024). We pick the degree of noise in search in the upper nest z to match an elasticity of labor supply of 2 across sectors (Lagakos and Waugh, 2013; Hsieh et al., 2019; Galle et al., 2023) following a change in b^O in the outside sector. In turn, we choose the degree of noise of noise in search in the lower nest ζ so that 30% of job-switchers experience a wage cut (Grigsby et al., 2021; Sorkin, 2018).

Workers' average piece rate, or equivalently firms' average markdown, determines equilibrium profits and thus firms' incentives to create vacancies during the transition. The vacancy-posting cost ν determines the barriers to entry in the labor markets and hence the average markdown: a higher ν will imply high markdowns in equilibrium by the free entry condition (5.3). We target an average markdown of 20% based on the evidence of Berger et al. (2022). We choose the separation rate χ to match a steady state unemployment rate of 5%. We set the unemployment benefit b^U to obtain an average replacement rate of 45% over workers' wage before separation, reflecting a mix of home production and unemployment insurance.

The two remaining parameters are the flow payoff of the outside option b^O and the rate at which workers accumulate specific skills $\hat{\delta}$. These two parameters are

application-specific and depend on which occupations and sectors are affected by the transition. To illustrate the mechanics of the model, we pick the same values as in our climate application. There, we use $b^O = 0.587$ to match the empirical replacement rate for workers initially attached to polluting industries who lose their job and transition to the rest of the economy (O) — we compute and match the exact same moment in our model. Finally, we structurally estimate $\hat{\delta} = 0.4\%$ by matching the propensity of different cohorts to reallocate across sectors, as we explain in Section 6.

Other parameters. For the reasons discussed in Section 3.2, the production side of the economy matters for the exact map between subsidies and A_t^k and hence the fiscal cost of implementing these MRPLs, but *not* how these MRPLs affect workers's labor market outcomes — the focus on this paper. We therefore relegate the calibration of the production side to our quantitative application in Section 6.

5.3 The Role of Labor Market Frictions

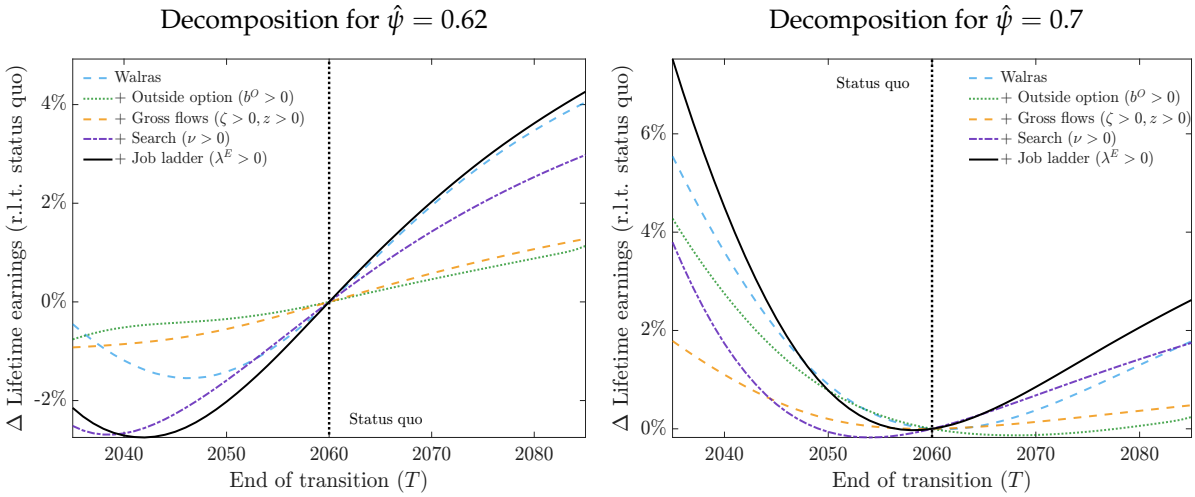
Figure F.1 in Appendix F reproduces the analysis of Figure 4.2, using our full model. We plot lifetime earnings changes in 2025 as a function of the transition horizon, relative to the status quo transition ending in 2060, for workers with various relative skills $\hat{\psi}$ in 2025 and 20 years of remaining work life.[29] We reach the same conclusion as in Section 4: workers' outcomes are strongly non-linear and even non-monotonic as a function of the transition horizon.

While the qualitative predictions are similar, the *quantitative* results are quite different between our full model and its frictionless counterpart. To understand which features of the labor market explain these differences, Figure 5.1 introduces one feature after another — namely, an outside option, gross flows, costly vacancy creation, and job-ladders — starting with the frictionless model of Section 2 with skill accumulation (in dashed light blue), until we reach our full model. As we add each additional feature, we recalibrate all other parameters in order to match the moments described in Section 5.2.[30] The left panel focuses on a worker with

[29] Since workers can be employed in different markets s, we must condition on some market to construct this plot. We look here at workers who are unemployed at the beginning of 2025 — keeping in mind that the UE rate is high.

[30] Table D.1 in Appendix D.4 describes these successive calibrations.

Figure 5.1: The role of labor market frictions



Notes: This figure plots workers' average lifetime earnings changes in 2025 in our full model as a function of the transition horizon T. Earnings changes are relative to the status quo transition ending in 2060, and transitions are parameterized as in equation (3.2). All workers have 20 years remaining in their careers. We consider workers who are unemployed as of January 2025. The two panels correspond to different values of relative productivity $\hat{\psi}$. The dashed light blue curve corresponds to our frictionless benchmark from Section 2 augmented with skill accumulation. The other lines add one feature after another until we obtain our full model corresponding to the solid black curve.

$\hat{\psi} = 0.62$ and the right one on another worker with $\hat{\psi} = 0.7$. Both have 20 years of remaining work life. We next describe how these features affect workers' lifetime earnings changes and the exact mechanisms.

First, we introduce an outside option ($b^O > 0$), which corresponds to the green dotted curve. As expected, the outside option is mostly irrelevant for high-$\hat{\psi}$ workers who reallocate earlier to the growing sector. On the contrary, the presence of an outside option flattens earnings changes substantially for low-$\hat{\psi}$ as the transition accelerates, since these workers are more likely to claim this outside option (e.g., become self-employed or retire early).[31] Then, we introduce gross flows across sectors ($z > 0$), which corresponds to the dashed orange curve.[32] Unsurprisingly, earnings changes are flatter since workers are more likely to jump back-and-forth between sectors.

[31] The fact that the green curve can lie below the blue one does *not* imply that the presence of an outside option worsens losses for workers. Indeed, the earnings changes (4.1) are defined relative to lifetime earnings under the status quo, which is different across models. Workers suffer more in the first place under the status quo if they do not have access to an outside option.

[32] We also introduce gross flows within sectors ($\zeta > 0$) as soon as we introduce search frictions.

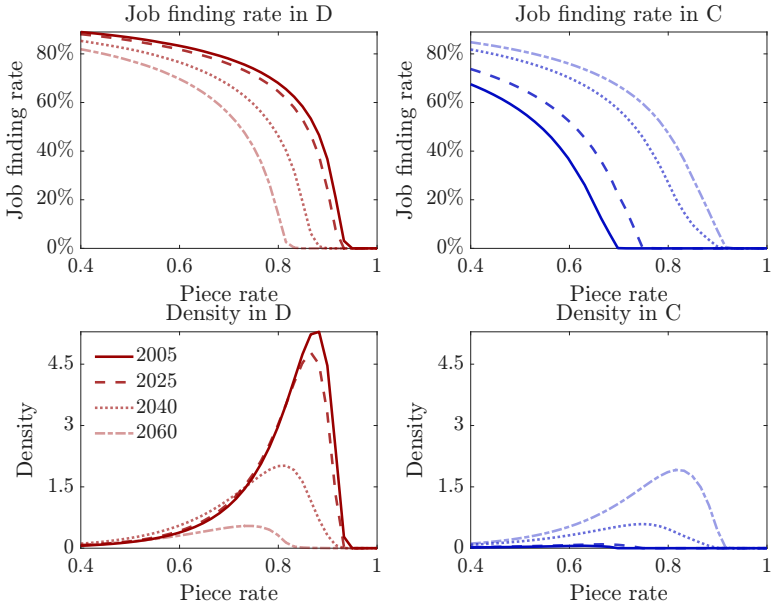
We then introduce costly vacancy creation ($\nu > 0$), which corresponds to the dot-dashed purple curve. Subsequently, we add on-the-job search ($\lambda^E > 0$) which gives rise to a job-ladder, to obtain our full model corresponding to the solid black curve. The non-linearities are much more pronounced in the full model relative to its counterpart with no labor market imperfections (dashed orange curve). Overall, the worker with low relative skill $\hat{\psi}$ loses much more in the full model as the transition accelerates, whereas the one with a high $\hat{\psi}$ gains considerably more. In other words, the additional labor market frictions are important to quantify the implications of a faster transition and the associated non-linearities.

To understand why costly vacancy creation and job-ladders generate such strong non-linearities, Figure 5.2 plots the evolution of the job finding rate (first row) and the distribution of piece rates (second row) in our full model at various points of the status quo transition for a worker endowed with $\hat{\psi} = 0.7$ at birth. The first column corresponds to the declining sector D and the second corresponds to the growing sector C. As the MRPL falls in the declining sector D, firms create fewer vacancies there and the job-finding rate plummets. The opposite happens in the growing sector C. Workers leave sector D gradually to join sector C. Over time, the distribution of piece rates shifts to the left in the declining sector, and to the right in the growing sector. Accelerating the transition causes these shifts to occur sooner. For comparison, Figure F.2 in Appendix F plots the same information in 2040 for workers endowed with various relative skills $\hat{\psi}$ at birth.

Figure 5.2 illustrates the role that costly vacancy creation play in shaping workers' earnings changes. Creating a vacancy is costly ($\nu > 0$), both in the model and in the data, so firms only create vacancies if they expect to recoup these costs. As the transition unfolds or accelerates, the earnings of a worker employed in the growing sector C increase for two reasons: firms' marginal revenues A_t^C go up and workers claim a given piece rate π of those; in *addition*, the presence of a fixed hiring cost ν means that firms are able to post vacancies at higher and higher piece rates π, further increasing workers' earnings as they cycle through jobs.[33] As a result, high-$\hat{\psi}$ workers (who are more likely to reallocate to sector C) experience larger earnings gains in the model with costly vacancy posting (dash-dotted pur-

[33] While our results are particularly easy to understand in terms of piece rates, we obtained very similar ones when firms and workers contract on fixed wages as opposed to piece rates (see the earlier version Grigsby and Zorzi, 2026). The same principle applies: the average wage changes more when fixed hiring affect which jobs are posted.

Figure 5.2: Job finding rates and the distribution of piece rates



Notes: The first row plots the job finding rate as a function of the piece rate posted by firms, in various periods under the status quo transition, for workers born with relative productivity $\hat{\psi} = 0.7$. The second row plots the equilibrium distribution of piece rates, for the same periods and the same productivity. The first column corresponds to the declining sector, and the second to the growing one.

ple curve) compared to its frictionless counterpart (dashed orange curve). The non-linearities are stronger as workers' earnings depends on the *product* of A_t^C and their average piece rate π in the growing sector. The opposite happens in the declining sector D: firms' marginal revenues A_t^D goes down *and* firms post fewer (or no longer post) jobs with high piece rates π. As a result, low-$\hat{\psi}$ workers (who are more likely to remain in sector D) experience larger losses in the model with costly vacancy posting. Eventually, these workers switch to sector D, so their earnings resemble those of higher-$\hat{\psi}$ workers.

The presence of a job ladder ($\lambda^E > 0$) reinforces these non-linearities, as it allows workers to claim jobs with higher and higher piece rates π in the growing sector C. On the contrary, the job ladder collapses in the declining sector D, which generates larger losses for workers who remain in that sector.

In summary, our full model offers a third conceptual insight: accounting for labor market frictions is key to quantify how workers' earnings *change* with the transition horizon and the associated non-linearities and non-monotonicities. These frictions generate stronger earnings increases in the growing sector, as firms post

jobs with larger and larger piece rates; and the opposite happens in the declining sector where the job-ladder collapses. Our finding that labor market frictions matter even for medium-run phenomena echoes the finding of Kambourov (2009), who reaches a similar conclusion in a model with one-time permanent sectoral shocks.

6 Application to the Climate Transition

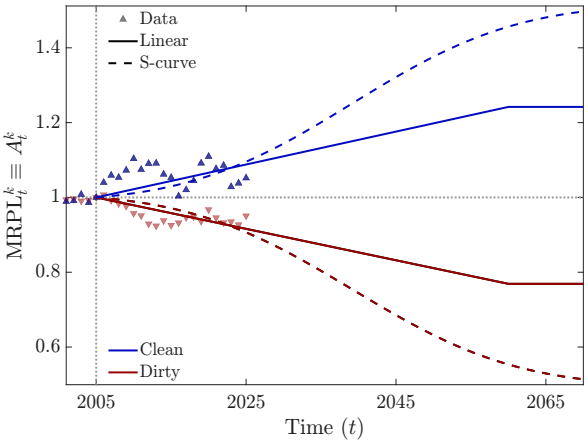
Having developed general insights that apply to various sectoral shifts, we now present a quantitative application to the climate transition. This application is particularly relevant for today's policymakers: the climate transition carries direct environmental benefits, providing a motive to accelerate it; at the same time, there are 10-15 million workers currently employed in polluting industries who are at risk of displacement; furthermore, most of the transition has yet to happen, and the government can affect its pace using various taxes and subsidies. Appendix E provides more details on the empirical component of our analysis.

6.1 Sector Definitions and Paths of MRPLs

Sector definitions. We interpret the growing sector C as "clean" industries, and the declining sector D as polluting or "dirty" industries. Following OECD (2024), polluting industries are those in the top decile by emissions intensity. In turn, "clean" industries either produce green goods (Bontadini and Vona, 2023) or involve green tasks (Granholm, 2024).

Paths of A_t^k. Workers are paid a constant piece rate in our model, so workers' detrended wage growth conditional on remaining on the job is precisely equal to the growth in the MRPLs A_t^k. Accordingly, we discipline the paths of A_t^k by matching realized data on sectoral wage growth from 2005–2025. Specifically, we compute 12-month wage growth using data from the Current Population Survey (CPS) Outgoing Rotation Group supplement (ORG), which asks workers about their hourly wages. We residualize job-stayers' wage growth against age, sex and education fixed effects to account for life cycle wage growth and differential trend growth

Figure 6.1: Estimated and projected paths of A_t^k



Notes: This figure plots the implied path of de-trended MRPLs A_t^k in clean sectors (blue curves) and polluting ones (red curves). The squares correspond to the estimated paths of A_t^k constructed by compounding wage growth of job-stayers in the CPS ORG residualized against sex, age and education fixed effects. The solid lines correspond to linear projections until 2060. The dashed curves correspond to projections based on an S-curve (see text for details).

rates by education.[34] We then compute average residualized wage growth by sector $g_{k,t}^w$, and chain these growth rates to obtain an *in-sample* estimate of MRPLs $\tilde{A}_t^k = \tilde{A}_{t-1}^k \left(1 + g_{k,t}^w\right)$ over the period 2005–2025, with $\tilde{A}_0^k \equiv 1$.[35] The triangles in Figure 6.1 correspond to these estimates: the MRPL goes down in the polluting sector (red curve), and up in the clean sector (blue curve).

Our climate application is a *prospective* analysis: it relies on assumptions about the trajectory of the US economy several decades in the future, about which there is of course substantial uncertainty. We therefore project the future path of A_t^k using parametric forms chosen to best match the in-sample estimates $\tilde{A}_t^k$, and explore the sensitivity of our results to these parameterizations. As before, we suppose that the transition begins in 2005, that the government can intervene starting in 2025, and that the status quo transition is mostly completed by 2060.

We must start by choosing a *shape* for the path of MRPLs, i.e., calibrate a function $\Phi(t)$ in expression (3.2) which describes the share of the transition that is completed as a function of calendar time.[36] In our first scenario, the transition is lin-

[34] Appendix E presents robustness exercises where we change the set of variables that we residualize against, as well as various implementation details.

[35] As discussed in Section 3.1, normalizing the MRPLs to 1 in $t = 0$ is without loss of generality.

[36] Due to the lack of available data, we assume that this function is common across sectors, i.e., the

ear over the period 2005–2060 and we choose the slope to minimize the quadratic distance between $\Phi(t)$ and the in-sample MRPLs. The corresponding A_t^k are indicated by the solid curves in Figure 6.1. In this case, the MRPLs reach $A_\infty^C = 1.24$ and $A_\infty^D = 0.77$ in 2060. In our second scenario, the transition is S-shaped to allow for the possibility that it might accelerate in the coming years. In this case, we suppose that $\Phi(t)$ is given by $\Phi^\star(t;\mu,\sigma)$ where $\Phi^\star(t;\mu,\sigma)$ is the CDF of a normal distribution with mean μ and standard deviation σ truncated below so that $\Phi^\star(0) = 0$. The parameters μ and σ are chosen to match data and forecasts from the U.S. Energy Information Administration (EIA) on the share of power generated by fossil fuels and renewable sources between 2005 and 2050 — a proxy for overall clean technology adoption. Figure E.1 in Appendix E shows that the truncated normal CDF describes these power generation data well. Given this shape, we choose the long-run levels A_∞^k in expression (3.2) to minimize the quadratic distance between A_t^k and the in-sample MRPLs. The corresponding A_t^k are indicated by the dashed curves in Figure 6.1 and reach $A_\infty^C = 1.50$ and $A_\infty^D = 0.49$ in the long-run.

6.2 Estimating the Skill Distribution

Applying our framework to a particular episode requires one additional piece of information relative to the steady state calibration of Section 5.2: the distribution of relative skills $\hat{\psi}$. In our model, this distribution is determined by two objects: the distribution of skill endowments at birth $G(\hat{\psi})$, and the subsequent rate at which workers accumulate specific skills $\hat{\delta}$. We now estimate these objects by matching workers' reallocation in the phase of the transition for which we have data (2005–2025).

Identification. We identify the distribution of skill endowments by matching the observed labor reallocation over 2005–2025 between the polluting and clean sectors, given the estimated paths of A_t^k. The idea is best understood by inspecting the left panel of Figure 4.1. In our frictionless benchmark, workers switch sectors if and only if $\hat{\psi} \geq A_t^D / A_t^C$, so the share of workers who have switched by period t is $1 - G\left(A_t^D / A_t^C\right)$. We can therefore use changes in the share of workers who have reallocated over time to back out the distribution $G(\hat{\psi})$, in the spirit of Lagakos and Waugh (2013), Burstein et al. (2019) or Porzio et al. (2022). Of course, labor re-

declining and growing sectors converge to their new respective BGPs with the same half-life.

allocation is much richer in our full model compared to its frictionless counterpart: workers have different employment statuses, they accumulate skills, there are job ladders, large gross flows, etc. However, the same idea applies: workers endowed with a larger $\hat{\psi}$ are more likely to switch earlier to clean industries, allowing us to estimate the distribution G based on reallocation flows.[37]

To disentangle the distribution of endowments G from the skill accumulation rate $\hat{\delta}$, we compare two cohorts born in different periods. Older workers have spent more time accumulating specific skills in polluting industries by 2005. As a result, they are less likely to reallocate to the clean sector, as in the data.

Estimation. The estimation problem consists of choosing a distribution G and a rate of specific skill accumulation $\hat{\delta}$ to solve

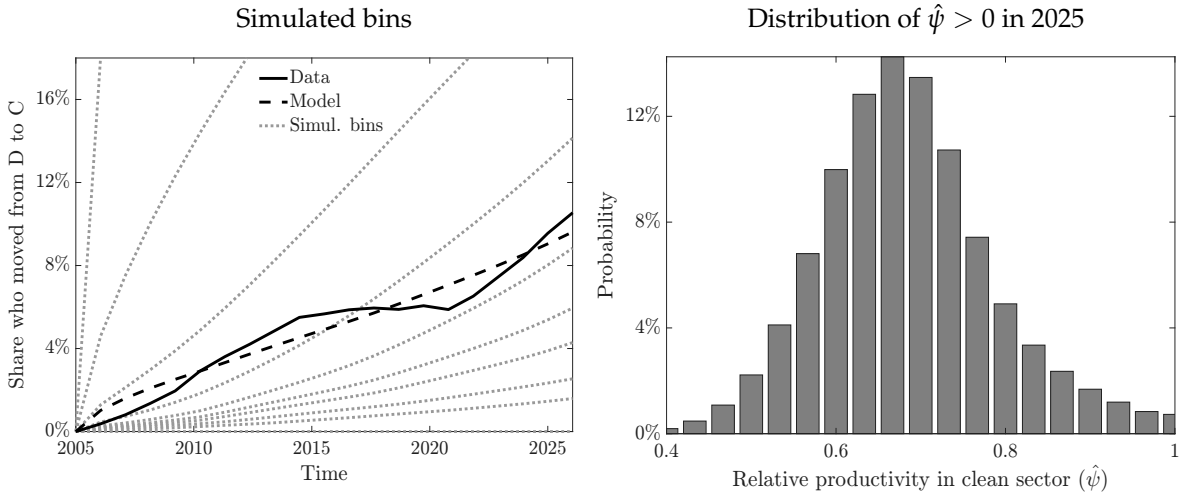
$$\min_{\left\{G,\hat{\delta}\right\}} \sum_{\tau} \int_{2005}^{2025} \left(z_{\tau,t}^{\text{data}} - \int_{0}^{\infty} z_{\tau,t}^{\text{model}} \left(\hat{\psi}^{\text{endow}}; \hat{\delta} \right) G \left(\mathrm{d}\hat{\psi}^{\text{endow}} \right) \right)^2 \mu_\tau dt, \tag{6.1}$$

where $z_{\tau,t}^{\text{data}}$ is the empirical share of workers in cohort τ who have reallocated from polluting to clean industries between period 0 and t, while $z_{\tau,t}^{\text{model}} \left(\hat{\psi}^{\text{endow}}; \hat{\delta} \right)$ is the same share computed in the model among workers in that cohort who were endowed with relative skill $\hat{\psi}^{\text{endow}}$ at birth, and μ_τ is the mass of workers in each cohort. The model-induced series depend on the skill accumulation rate $\hat{\delta}$. Empirically, we measure $z_{\tau,t}^{\text{data}}$ in the CPS as the share of workers initially attached to polluting industries who reallocated to a clean industry. By definition, a worker is initially "attached" to polluting industries if they are employed there for four consecutive months during their first CPS rotation.[38] As discussed throughout this paper, we focus on these workers because they are the most at risk of displacement by the climate transition and the most likely to oppose it, so predicting their outcomes and understanding how expensive it would be to compensate them is of

[37] In principle, workers could reallocate because they have a non-pecuniary value of working in clean sectors. This distinction is unimportant for the exercise that we conduct in this paper. To understand why, consider a version of our model where the relative flow payoff in sector C is $\hat{\psi} \times \xi \times \Lambda \times A_t^C / A_t^D$, where ξ is the idiosyncratic amenity and Λ is the sectoral one, both of which are time-invariant as in Dix-Carneiro (2014). Given paths of A_t^k, our estimation would back out the distribution of $\hat{\psi}\xi\Lambda$, which is precisely the relevant ones for our counterfactuals. In this case, our "earnings changes" should be interpreted as *welfare* changes inclusive of the amenity values.

[38] The CPS does not allow us to track workers over long periods. We therefore weight individuals in later waves by the probability that they would have been attached to the polluting sector in the pre-2005 period according to their observables. See Appendix E for details.

Figure 6.2: Estimation of the skill distribution



Notes: The left panel plots the share of workers initially attached to polluting industries who moved to clean industries. The solid curve is the data from the CPS (see Appendix E). Workers are weighted by the probability that they are attached to the polluting sector based on observables in the data. We consider workers aged 25-40 in 2005, so they do not retire before 2025. The dotted curves correspond to simulated bins associated to various values of $\hat{\psi}^{\text{endow}}$ under the estimated $\hat{\delta}$. The dashed curve corresponds to the model-generated reallocation when using the estimated distribution of $\hat{\psi}^{\text{endow}}$. The right panel plots the estimated cross-sectional distribution of relative productivity $\hat{\psi}$ in the population in 2025.

the utmost importance.[39]

Solving the estimation problem (6.1) is challenging given our large state space and the presence of job ladders. We leverage the block recursivity of our directed search model (Menzio and Shi, 2011) to tackle this problem efficiently. This property allows us to solve for workers' optimal reallocation $z^{\text{model}}_{\tau,t}\left(\hat{\psi}^{\text{endow}};\hat{\delta}\right)$ separately for each value of $\hat{\psi}^{\text{endow}}$ and *independently* from the distribution G. We then choose the distribution G that assigns weights to these optimal policies in order to fit workers' reallocation over time. We focus on workers aged 25 to 40 when the transition began in 2005, since these cohorts have not retired yet by 2025. We repeat this experiment for various values of $\hat{\delta}$ and keep the one that best predicts the reallocation across different cohorts.

We illustrate our approach on the left panel of Figure 6.2. The solid black line

[39] While our estimation matches the reallocation of these workers specifically, workers from the rest of the economy can also reallocate to clean sectors. Conditional on the paths of MRPLs, the presence of these workers does not affect our estimation for the reasons discussed in Section 3.1. We account for these other workers when quantifying the fiscal cost of the subsidies τ^k_t that implement our counterfactuals [in progress].

corresponds to the empirical reallocation of the workers initially attached to polluting industries.[40] Their reallocation is relatively slow at first and picks up towards the end of the period. Roughly 10% of these workers had reallocated to clean industries by 2025. The overall timing of the reallocation and the relatively small annual reallocation rates in this early phase of the transition are consistent with existing findings in the literature (IMF, 2022; Curtis and Marinescu, 2023).

The dotted curves correspond to the model-implied net reallocations associated with different bins of relative skill endowment $\hat{\psi}$ at birth under the linear transition of Figure 6.1. The levels and shapes of these curves are different from one another and do not overlap, allowing us to identify the mass assigned to these bins.

The estimation problem consists of choosing the weights on these dotted curves to best match the empirical series. Following a long tradition (e.g. Lagakos and Waugh, 2013; Hsieh et al., 2019; Galle et al., 2023), our baseline approach supposes that ψ^C and ψ^D are drawn independently from Fréchet distributions at birth, so that their ratio $\hat{\psi}$ follows a log-logistic distribution. Having already computed labor reallocation for each bin of relative skill endowment $\hat{\psi}$ at birth, optimizing over the location and scale of this log-logistic distribution is computationally costless — making the estimation manageable. We also consider a variant where $\hat{\psi}$ is drawn from a log-normal distribution at birth, as is often assumed in Roy models of selection (Erosa et al., 2022).

We repeat this exercise for various values of the skill accumulation rate $\hat{\delta}$ to estimate the one that best matches the reallocation for cohorts aged 25-30 years and 35-40 years in 2005. Figure F.4 in Appendix shows how different values of $\hat{\delta}$ affect workers' labor supply to the clean sector for different cohorts.

Our model matches labor reallocation well, as depicted by the dashed line on the left panel of Figure 6.2. We estimate that $\hat{\psi}$ is distributed with scale 0.719 and shape 14.199 under the log-logistic assumption. We obtain a very similar distribution of relative skills when assuming log-normality instead (Figure F.3 in Appendix). The model also fits reallocation by cohort well, as shown in Figure F.4. We estimate a value of relative skill accumulation $\hat{\delta} = 0.4\%$, which lies in the range

[40] The CPS has less granular industry codes than 6-digit NAICS, which prevents us from observing workers who switch from ICEV and EV production for instance (Bachmann et al., 2026). We therefore attribute a share of dirty manufacturing employment to clean manufacturing over time in proportion to the share of cars sold which use alternative fuels, i.e., a proxy for clean technology adoption in the economy.

of Kambourov and Manovskii (2009).

It is worth noting that our parametric assumption helps overcome a well-known identification challenge: in the frictionless model, workers whose relative skill $\hat{\psi}$ lies below A^C_{2025}/A^D_{2025} do not reallocate in-sample, so the distribution cannot be identified below this cutoff. The presence of noisy search and large gross flows substantially expands the region of $\hat{\psi}$ that is identified, as workers have a positive probability of reallocating even below this cutoff. In addition, our parametric distributional assumptions helps us extrapolate beyond the observed reallocation over the period 2005–2025.

The right panel of Figure 6.2 plots the cross-sectional distribution of relative skills $\hat{\psi}$ induced by our estimates of G and $\hat{\delta}$ in 2025. We find that most workers initially attached to polluting industries exhibit substantial skill specificity (i.e., $\hat{\psi} < 1$), reflecting the relatively slow reallocation observed in the early phase of the transition.[41] This finding echoes existing evidence showing that skill requirements are different in clean and polluting industries and that workers lose specific human capital when leaving polluting industries (Walker, 2013; Consoli et al., 2016; OECD, 2024).[42] Overall, about 80% of workers have a $\hat{\psi}$ between $A^D_\infty/A^C_\infty = 0.77/1.24$ and $A^D_{2005}/A^C_{2005} = 0.92/1.09$; these workers are the ones who, if the transition accelerates sufficiently, will reallocate in some $t > 0$ and experience non-monotonic earnings changes.[43]

In addition to the steady state moments discussed in Section 5.2, our model replicates many labor market features specific to the climate application. Our estimation ensures that the model matches the reallocation to clean industries from workers initially attached to polluting ones, both overall and by cohort. In addition, net flows to the C sector are much smaller (less than 1% per year) than gross flows between sectors (around 2.5% per *month*), as in the data. While our calibration matches the replacement rate associated with the outside option, we do not

[41] Monetary migration costs are absorbed in our estimates of the $\hat{\psi}$ distribution. The costs of switching from D to C that we estimate — on the order of 30% of lifetime earnings or $360,000 — are much larger than the typical monetary costs involved in moving.

[42] Note that $\hat{\psi}$ captures any reason why a worker's productivity may differ in clean and polluting industries, such as the relative productivity of a worker's region for clean industries, e.g., a coal-rich region or a sunny desert. While the evidence suggests that migration of incumbents is mostly unresponsive to local labor demand shocks (Autor et al. 2025; Greenland et al. 2019), future work can extend our analysis by introducing a richer geography.

[43] This back-of-the-envelope does not account for skill accumulation. Below, we use our model to quantify the share of workers who experience non-monotonicites for each transition horizon.

target gross flows to O in steady nor net reallocation flows over the transition. In the model, the probability of an E-E switch from polluting industries to the rest of the economy, as captured by the outside option, is 0.35% per month in steady state. In the data, this figure was 0.4% before the transition (1996–2005). These probabilities increased to 0.5% by 2025, both in the model and in the data, as more workers reallocated to O. In summary, our model matches well the flows between C, D and O . Going forward, the model generates a small increase in unemployment over the transition — peaking at less than 0.1 p.p. (or 2%) above its initial steady state level by 2045. This prediction is consistent with the view that the transition should not raise unemployment meaningfully (Causa et al., 2024; Hafstead and Williams, 2018).

6.3 Earnings Losses from the Climate Transition

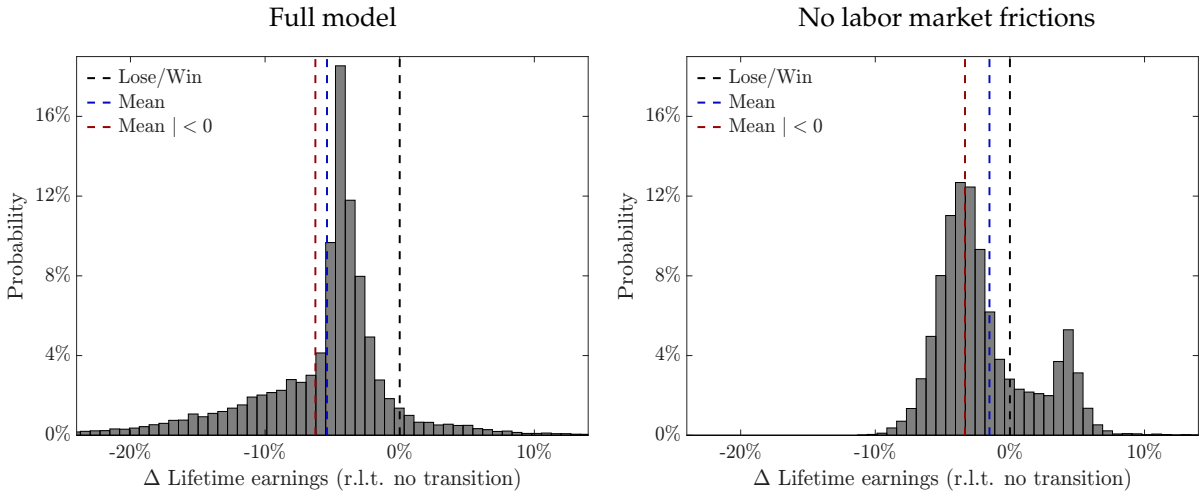
Realized losses over 2005–2025. We start by using our model to quantify earnings changes that workers have already experienced due to the climate transition. Figure 6.3 plots workers' cumulative earnings changes over the period 2005–2025, expressed as a share of their cumulative earnings had there been no transition.[44] We first focus on the left panel, which corresponds to our full model.

Most workers initially attached to polluting industries lose from the status quo transition, reflecting that $\hat{\psi} < 1$ for these workers. The average worker has lost 5.4% of cumulative earnings so far, and the average loss is 6.3% conditional on losing. Some workers experience substantially larger losses, on the order of 10–20%. Figure F.5 in Appendix puts a dollar value on these numbers by expressing each workers' earnings change relative to yearly average earnings had there been no transition. Using a common denominator allows us to take into account that some workers have supplied labor for longer than others. The average worker initially attached to polluting industries has lost about one year of income over 2005–2025, or $67,250 in the 2005 Quarterly Census of Employment and Wages (QCEW). Multiplying this figure by the 10 million workers initially attached to these industries, we obtain an aggregate earnings loss of roughly 670 billion dollars.[45] These

[44] We restrict attention to workers who are present over the entire period. We simulate the same workers from 2005–2025 with and without transition, subject to the same sequences of idiosyncratic shocks in these two scenarios.

[45] There were 11.3 million workers employed in polluting industries in 2005, but only 9.9 million are categorized as *attached* to these industries (Appendix E).

Figure 6.3: Cumulative earnings changes over 2005–2025



Notes: This figure plots the cross-sectional distribution of cumulative earnings losses for workers present over the period 2005–2025, expressed as a share of cumulative earnings had there be no transition. The left panel corresponds to our full model, while the right panel corresponds to the frictionless model with skill accumulation, an outside option and gross flows, i.e., the blue curves in Figure 5.1.

large losses can explain why many workers oppose the climate transition (Dechezleprêtre et al., 2025).

For context, the losses that we obtain are comparable to those estimated by Autor et al. (2016) due to exposure to Chinese imports or Traiberman (2020) following trade liberalizations in Denmark. Colmer et al. (2026) document losses of 1.8 times average annual earnings over a shorter period (2012–2019) for workers initially attached to coal — an industry that has been more affected by the transition than the broader set that we consider here. Davis and von Wachter (2011) find that workers experience an earnings loss of 2.8 times yearly earnings if they are displaced during a recession. The average loss that we obtain is thus equivalent to a third of polluting-attached workers having experienced one such layoff over a 20-year period. Finally, the aggregate earnings loss of $670 billion that we find is in line with a decline of roughly $1 trillion estimated using the QCEW.[46] Of course, we expect the model to produce a somewhat smaller figure since workers have the ability to leave polluting industries. Overall, compensating workers who lose from the tran-

[46] To estimate this number, we start by deflating all earnings by core CPI. We then compute a counterfactual sequence of total earnings in polluting sectors if they had grown at the same rate as the U.S. economy's total earnings. We take the difference between actual earnings in polluting industries and this counterfactual sequence. Finally, we sum up these differences over 20 years.

sition would be relatively expensive. For comparison, the U.S. Congress budgets roughly half a billion per year for the Trade Adjustment Assistance for Workers (TAA), which assists workers who lost their job to foreign competition. While sizable, the numbers that we obtain are small relative to the social benefits of reducing carbon emissions estimated in the literature (Bilal and Känzig, 2025).

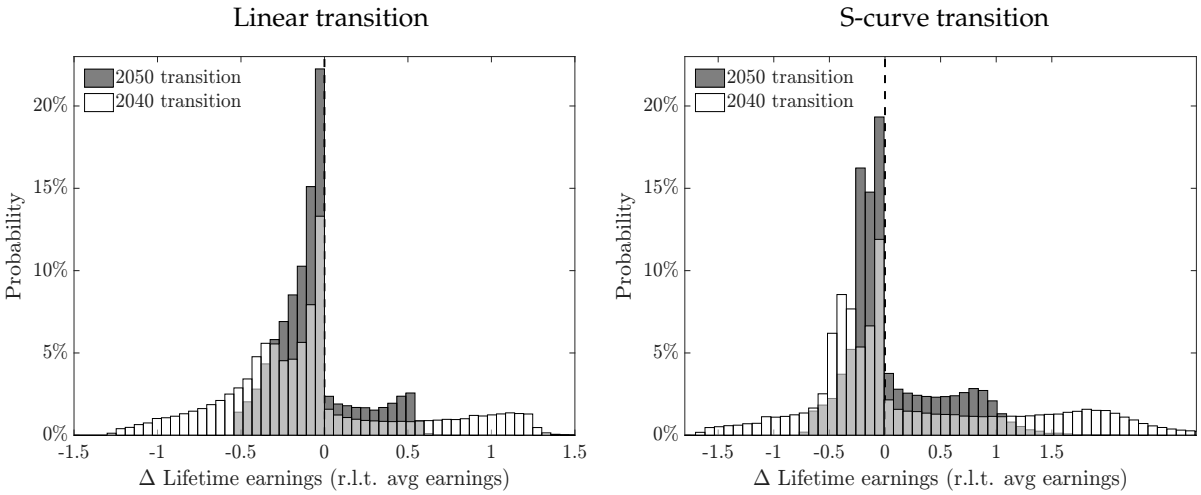
Finally, the right panel of Figure 6.3 reports the same earnings changes, but in a version of the model with no labor market imperfections (corresponding to the dashed orange curve in Figure 5.1). The average loss is 3 times smaller in this frictionless model, and the left tail of losses is much thinner. Indeed, the frictionless model does not account for the scarring that comes with repeated unemployment spells and the collapse of the job ladder in polluting industries. These findings echoes our results from Section 5.3: labor market frictions are essential to quantify workers' earnings losses over the transition and, ultimately, how they change if the government intervenes to speed up the transition.

Counterfactuals from 2025 onwards. We now conduct our prospective analysis. Figure F.6 in Appendix plots the distribution of incumbents' remaining lifetime earnings changes from 2025 onwards, which corresponds to the grey area in Figure 4.1. The left panel expresses these changes as a share of remaining lifetime earnings had there been no transition, while the right panel expresses them relative to average yearly income in 2025 (this common denominator accounts for differences in remaining lifespans across workers). We find that workers lose the equivalent of an *extra* year of average earnings (or \$67,250) from 2025 onwards, compared to what they already lost over the period 2005–2025.

Turning to our counterfactuals, Figure 6.4 plots the distributions of earnings changes in 2025 from speeding up the transition such that it ends in 2050 (in gray) or 2040 (in white), relative to maintaining the status quo transition ending in 2060. All earnings changes are expressed relative to yearly average earnings in 2025, for the same reasons discussed above. We focus first on the left panel, which corresponds to the linear transition that we use as a baseline.

Two main features stand out. First, earnings changes are indeed non-linear in the speed of the transition: the distribution associated with the 2040 transition is not simply a linearly "stretched" version of the one for the 2050 transition. In particular, the left tail gets thicker as the transition accelerates. A transition ending in 2040 leads some workers to lose an additional year of lifetime earnings relative to

Figure 6.4: Counterfactual earnings changes



Notes: This figure plots the cross-sectional distribution of lifetime earnings changes associated with faster transitions ending in 2050 or 2040. The changes are expressed relative to yearly average earnings had there been no transition. The left panel corresponds to a linear transition, and the right panel to an S-curved transition as shown in Figure 6.1.

the status quo transition, i.e., the losses in the tail increase by roughly one fifth relative to Figure F.6. At the same time, the number of workers who benefit increases as the transition accelerates.

The average worker would lose an additional $2,616 in remaining lifetime earnings (3.9% times yearly average earnings) from a transition ending in 2050 rather than 2060. This figure translates to an aggregate cost of $26 billion when accounting for 10 million workers initially attached to polluting industries. At the same time, 23.8% of workers would benefit from such a faster transition. Of course, workers who gain might not directly compensate those who lose. Conditional on losing, workers lose $9,388 (14% times yearly average earnings) with an aggregate cost of $72 billion. When it comes to the transition ending in 2040, the average workers would lose $4,128. More workers benefit from this faster transition (27.6%). And the loss conditional on losing is now $22,239 or $161 billion in aggregate. These results echo our insight of Section 4.2: policymakers who consider speeding up the climate transition face a tradeoff between increasing the share of winners on the extensive margin, and generating a thicker tail of losers on the intensive margin.

Appendix F presents a number of additional results. Figure F.7 reports average changes in lifetime earnings by age, which are non-linear and non-monotonic as

the transition accelerates. Figure F.8 plots the share of workers who experience U-shaped changes in lifetime earnings as the transition accelerates starting from 2060. In particular, about 10% of workers who initially prefer a slower transition starting from 2060 would, in fact, prefer to speed up the transition further relative to 2050. That figure rises to 17% for the 2040 transition.

S-curved transition. Finally, we explore the sensitivity of our results by considering an S-shaped transition that picks up after 2025 (see Figure 6.1). The realized earnings losses over 2005-2025 are very similar to those obtained with a linear transition, as shown in Figure F.10 in Appendix, since we fit the same empirical series in-sample. In turn, the S-shaped transition produces similar counterfactual losses from 2025 onwards, despite larger changes in the paths of A_t^k (Figure F.11 in Appendix). While wages decrease more in the declining sector, workers' losses are bounded below by their ability to claim the outside option and workers are more likely to switch to the growing sector earlier and benefit from wage gains there.

7 Conclusion

This paper studies how the pace of sectoral shifts affects workers at risk of displacement. We develop a life-cycle model with skill heterogeneity, costly hiring and job ladders in which labor demand gradually rises in one sector and declines in another. Our analysis reveals three novel insights. First, workers' lifetime earnings are strongly non-linear and even non-monotonic in the horizon over which the transition unfolds. Second, more and more workers benefit on the extensive margin as the transition accelerates, but the tail of losses becomes thicker on the intensive margin. Third, accounting for labor market frictions is essential to quantify how earnings losses change non-linearly as the transition accelerates.

We apply our model in the context of the climate transition. We estimate the skill distribution by leveraging the block recursivity of directed search models. We find that workers initially attached to polluting industries have lost about 5% of cumulative earnings so far, with substantial heterogeneity. They are predicted to lose another 5% on average in lifetime earnings over the next decades. Compensating these workers would cost in the hundreds of billions of dollars. Accelerating the transition so it ends around 2050 would produce relatively modest *average* earnings losses, but losses *conditional* on losing are substantial for workers with the

least versatile skills.

The goal of our paper is to explain transparently the source and determinants of the non-linear and non-monotonic relationship between the pace of sectoral shifts and workers' outcomes. Future work can build on our analysis and enrich it quantitatively by introducing many sectors, aggregate uncertainty about the transition path, a rich geography with local demand spillovers, or by carrying out a broader optimal policy analysis accounting for the fiscal cost of policy interventions.

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Online Appendix

A Equilibrium in the Frictionless Benchmark

This appendix . Section A.1 characterizes the equilibrium. Section A.2 explains how we solve it numerically. Section A.3 shows formally that the government can use short-run subsidies to perturb the MRPLs as described in Section 3.2 in the main text and we present a numerical example.

A.1 Equilibrium Characterization

Workers. Labors supplies take a particularly simple form in this benchmark model. Workers supply labor to the growing sector if and only if

$$\psi\hat{\psi}w_t^C > \psi w_t^D \Leftrightarrow \hat{\psi}w_t^C > w_t^D, \tag{A.1}$$

where w_t^k is the de-trended wage that prevails in a given sector. Therefore, workers' labor supplies are

$$n_t^D \equiv \int_0^{A_t^D/A_t^C} \psi^D\left(\hat{\psi}\right) G\left(d\hat{\psi}\right) \quad \text{and} \quad n_t^C \equiv \int_{A_t^D/A_t^C}^{\infty} \psi^C\left(\hat{\psi}\right) G\left(d\hat{\psi}\right) \tag{A.2}$$

where $\psi^D\left(\hat{\psi}\right) \equiv 1$ and $\psi^C\left(\hat{\psi}\right) \equiv \hat{\psi}$. Note that labor supplies are entirely determines by the sequence of MRPLs A_t^k across sectors.

Final good producer. In equilibrium, the de-trended price of intermediate goods is

$$P_t^k = \left(\varsigma_t^k\right)^{\frac{1}{\phi}} \left(X_t^k\right)^{-\frac{1}{\phi}} \left[\sum_m \left(\varsigma_t^m\right)^{\frac{1}{\phi}} \left(X_t^m\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{1}{\phi-1}} \tag{A.3}$$

for each sector $k = D, C$. The supply of intermediates is

$$X_t^k = \left(K_t^k\right)^{\alpha} \left(n_t^k\right)^{1-\alpha} \tag{A.4}$$

where

$$n_t^{k'} = \int \mathbf{1}_{\{k(s)=k'\}} h\left(\psi, \hat{\psi}, k\left(s\right)\right) d\mu_t\left(j, \psi, \hat{\psi}, s\right) \tag{A.5}$$

is the labor supply in a sector $k' = D, C$, with $k(s)$ denoting the sector that a given worker in market s is employed in.

Entrepreneurs' problem. Before characterizing the entrepreneurs' optimal demand for labor and investment problem, it is useful to solve for their optimal time allocation problem. The cost of installing I_t^D and I_t^C units of investment is

$$\begin{aligned} \mathcal{C}\left(I_t^D, I_t^C\right) \equiv \min_{\left\{z_t^k, e_t^k\right\}} \; & z_t^D + z_t^C \\ \text{s.t. } & \left(z_t^h\right)^{\gamma} \left(e_t^h\right)^{1-\gamma} \geq I_t^h \;,\; h = D, C \\ & e_t^D + e_t^C \leq 1 \end{aligned} \tag{A.6}$$

The resulting cost function is

$$\mathcal{C}_t\left(I_t^D, I_t^C\right) = \left[I_t^D + I_t^C\right]^{\frac{1}{\gamma}}, \tag{A.7}$$

so the marginal cost or price of the two equipment goods are equalized

$$\varphi_t^k = \varphi_t \equiv \frac{d}{dI^k}\mathcal{C}_t\left(I_t^D, I_t^C\right) = \frac{1}{\gamma}\left[I_t^D + I_t^C\right]^{\frac{1}{\gamma}-1} \tag{A.8}$$

Note that this marginal cost increases with the amount of investment produced: if entrepreneurs invest more in the clean sector, i.e. I_t^D goes up, they dedicate more installation effort e_t^D to that sector, thereby crowding out their installation effort in the polluting sector. Thus, an investment boom in the clean sector, i.e. I_t^D goes up, leads to an investment decline in the polluting sector when the goods are sufficiently substitutable.[47]

In turn, the firm's optimal demand for labor and capital solves

$$\begin{aligned} \max_{\left\{I_t^k, K_t^k, n_t^k\right\}} \; & \int_0^{\infty} \exp\left(-r^\star t\right) \left[\sum_k P_t^k \left(K_t^k\right)^{\alpha} \left(n_t^k\right)^{1-\alpha} - \left(1-\tau_t^k\right)\varphi_t I_t^k - w_t^k n_t^k\right] dt \\ & dK_t^k = \left(I_t^k - \delta K_t^k\right) dt \;,\; k = D, C, \end{aligned} \tag{A.9}$$

[47] Of course, production and investment would co-move perfectly in the two sector if the two goods were perfect complements.

where P_t^k is the price of each intermediate good and A_t^k is the price of labor services in each sector (as discussed above). Both prices are de-trended using the BGP growth rate $g > 0$. The interest rate $r^\star \equiv r - g = \rho - g$ is the effective discount rate after taking into account that all prices grow at rate $g > 0$ in equilibrium. To economize on notation, we will directly use r to denote this effective discount rate in the rest of this appendix.

The solution to this investment problem satisfies

$$(1-\alpha)\, P_t^k \left(\frac{K_t^k}{n_t^k}\right)^{\alpha} = w_t^k \tag{A.10}$$

and

$$\alpha P_t^k \left(\frac{K_t^k}{n_t^k}\right)^{\alpha-1} = (r+\delta)\, \vartheta_t^k - \dot{\vartheta}_t^k, \tag{A.11}$$

where the multipliers on the laws of motion of capital are

$$\vartheta_t^k = \left(1-\tau_t^k\right) \varphi_t, \tag{A.12}$$

in each sector $k = D, C$. The solution to the firms' problem is characterized by the optimality conditions (A.10)–(A.12), together the laws of motion for capital and the usual transversality condition. Comparing the definition (3.1) and equation (A.10) reveals that the wage coincides with the marginal revenue product of labor $A_t^k = w_t^k$ in each sector.

Government. The government chooses a path of subsidies $\{\varsigma_t^k\}$ and sets lump sum taxes to balance its flow budget

$$T_t = \sum_k \left(\tau_t^k P_t^k X_t^k + \varsigma_t^k I_t^k\right) \tag{A.13}$$

A.2 Solving for the Transition Equilibrium

Investment. Combining (A.3)–(A.4) and (A.11),

$$\alpha\left(\varsigma_t^k\right)^{\frac{1}{\phi}}\left[\sum_k\left(\varsigma_t^m\right)^{\frac{1}{\phi}}\left(\frac{\left(K_t^m\right)^{\alpha}\left(n_t^m\right)^{1-\alpha}}{\left(K_t^k\right)^{\alpha}\left(n_t^k\right)^{1-\alpha}}\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{1}{\phi-1}}\left(\frac{K_t^k}{n_t^k}\right)^{\alpha-1}$$
$$=\left(r+\delta\right)\vartheta_t^k-\dot{\vartheta}_t^k \tag{A.14}$$

where the multipliers are given by

$$\vartheta_t^k=\left(1-\tau_t^k\right)\varphi_t, \tag{A.15}$$

and capital evolves as

$$dK_t^k=\left(I_t^k-\delta K_t^k\right)dt \tag{A.16}$$

in each sector $k=D,C$.

Labor demands and supplies. Combining expressions (A.3)–(A.4) and the labor demands (A.10) gives the MRPLs

$$\left(1-\alpha\right)\left(\varsigma_t^k\right)^{\frac{1}{\phi}}\left[\sum_m\left(\varsigma_t^m\right)^{\frac{1}{\phi}}\left(\frac{\left(K_t^m\right)^{\alpha}\left(n_t^m\right)^{1-\alpha}}{\left(K_t^k\right)^{\alpha}\left(n_t^k\right)^{1-\alpha}}\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{1}{\phi-1}}\left(\frac{K_t^k}{n_t^k}\right)^{\alpha}=A_t^k \tag{A.17}$$

in each sector $k=D,C$. As we have established in Appendix A.1, the labor supplies n_t^k depend exclusively on the paths of de-trended MRPLs $\mathbf{A}\equiv\left\{A_t^k\right\}$ so

$$n_t^k\equiv n_t^k\left(\mathbf{A}\right) \tag{A.18}$$

Price of investment. Finally, the marginal cost of investment is

$$\varphi_t=\frac{1}{\gamma}\left[I_t^D+I_t^C\right]^{\frac{1}{\gamma}-1} \tag{A.19}$$

Steady states. Before solving for the transition, we must solve for the initial and final steady states, which determine the initial stocks of capital K_0^k in each sector and the prices A_∞^k and φ_∞ that prevail in the long-run. Solving for these two steady states

is very similar so we focus on the initial one and drop time indices. Fix prices A^k and c. These MRPLs imply labor supplies n^k_∞ using expression (A.18). Using the optimality condition (A.14),

$$\alpha \left(\varsigma^k\right)^{\frac{1}{\phi}} \left[\sum_k \left(\varsigma^m\right)^{\frac{1}{\phi}} \left(\frac{\left(K^m\right)^\alpha \left(n^m\right)^{1-\alpha}}{\left(K^k\right)^\alpha \left(n^k\right)^{1-\alpha}}\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{1}{\phi-1}} \left(\frac{K^k}{n^k}\right)^{\alpha-1} = \left(r+\delta\right)\vartheta^k \tag{A.20}$$

in each sector $k = D, C$. Therefore,

$$\left(\frac{K^D_t}{K^C_t}\right)^{\alpha\frac{\phi-1}{\phi}-1} = \left(\frac{\varsigma^C_t}{\varsigma^D_t}\right)^{\frac{1}{\phi}} \times \left(\frac{n^D_t}{n^C_t}\right)^{-(1-\alpha)\frac{\phi-1}{\phi}} \times \frac{\left(r+\delta\right)\vartheta^D_t}{\left(r+\delta\right)\vartheta^C_t} \tag{A.21}$$

and

$$K^D_t = \left[\frac{\left(r+\delta\right)\vartheta^D_t}{\alpha\left(\varsigma^D_t\right)^{\frac{1}{\phi}}\left[\left(\varsigma^D_t\right)^{\frac{1}{\phi}} + \left(\varsigma^C_t\right)^{\frac{1}{\phi}}\left(\frac{K^D_t}{K^C_t}\right)^{-\alpha\frac{\phi-1}{\phi}}\left(\frac{n^C_t}{n^D_t}\right)^{(1-\alpha)\frac{\phi-1}{\phi}}\right]^{\frac{1}{\phi-1}}}\right]^{\frac{1}{\alpha-1}} n^D_t, \tag{A.22}$$

which pin down the capital stocks K^D and K^C. In turn, the labor demands (A.17) pin down new MRPLs A^k. Furthermore, expressions (A.19) and (A.16) imply a new price of investment

$$\varphi = \frac{1}{\gamma}\left(\delta\sum_k K^k\right)^{\frac{1}{\gamma}-1} \tag{A.23}$$

We then update the MRPLs A^k and the price of investment φ iteratively until convergence.

Transitions. We proceed similarly to solve for the transition dynamics. We guess sequences for the MRPLs A^k_t and the price of investment φ_t that converge to their long-run levels A^k_∞ and φ_∞. Given those, we use expression (A.18) to solve for workers' labor supplies n^k_t and expressions (A.10)–(A.12) to solve for the entrepreneurs' optimal sequences of investment I^k_t and capital K^k_t. The labor demands (A.17) im-

ply new sequences for the MRPLs and expression (A.19) implies new prices of investment. We then update these sequences iteratively until convergence.

A.3 Subsidies that Implement Changes in MRPLs

We now show that the government can use short-run subsidies to perturb the MRPLs as described in Section 3.2 in the main text.

Solving for the subsidies. Combining expressions (A.3)–(A.4) and (A.10),

$$(1-\alpha)\left(\varsigma_t^k\right)^{\frac{1}{\phi}}\left[\sum_m\left(\varsigma_t^m\right)^{\frac{1}{\phi}}\left(\frac{X_t^m}{X_t^k}\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{1}{\phi-1}}X_t^k=A_t^k n_t^k \tag{A.24}$$

for each sector $k=D,C$. We established in Appendix A.1 that labor supplies n_t^k depend exclusively on the paths of de-trended MRPLs $\mathbf{A}\equiv\left\{A_t^k\right\}$. Therefore, the sectoral outputs can also expressed exclusively as a function of $\mathbf{A}$ and parameters. Making this dependence explicit,

$$X_t^k(\mathbf{A})=\frac{A_t^k n_t^k(\mathbf{A})}{(1-\alpha)\left(\varsigma_t^k\right)^{\frac{1}{\phi}}}\left[\left(\varsigma_t^k\right)^{\frac{1}{\phi}}+\left(\varsigma_t^{-k}\right)^{\frac{1}{\phi}}\left(\frac{X_t^{-k}(\mathbf{A})}{X_t^k(\mathbf{A})}\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{1}{1-\phi}}, \tag{A.25}$$

where $-k$ denotes the other sector. Therefore,

$$\frac{X_t^D(\mathbf{A})}{X_t^C(\mathbf{A})}=\Omega_t(\mathbf{A})\left[\frac{\left(\varsigma_t^D\right)^{\frac{1}{\phi}}+\left(\varsigma_t^C\right)^{\frac{1}{\phi}}\left(\frac{X_t^C(\mathbf{A})}{X_t^D(\mathbf{A})}\right)^{\frac{\phi-1}{\phi}}}{\left(\varsigma_t^D\right)^{\frac{1}{\phi}}\left(\frac{X_t^D(\mathbf{A})}{X_t^C(\mathbf{A})}\right)^{\frac{\phi-1}{\phi}}+\left(\varsigma_t^C\right)^{\frac{1}{\phi}}}\right]^{\frac{1}{1-\phi}}, \tag{A.26}$$

where

$$\Omega_t(\mathbf{A})\equiv\frac{A_t^D n_t^D(\mathbf{A})\left(\varsigma_t^D\right)^{-\frac{1}{\phi}}}{A_t^C n_t^C(\mathbf{A})\left(\varsigma_t^C\right)^{-\frac{1}{\phi}}}>0 \tag{A.27}$$

Simplifying expression (A.26),

$$\frac{X_t^D(\mathbf{A})}{X_t^C(\mathbf{A})}=\Omega_t(\mathbf{A})^{\frac{\phi}{\phi-1}} \tag{A.28}$$

Therefore, $X^C(\mathbf{A})$ and $X^D(\mathbf{A})$ are uniquely pinned down by expressions (A.25) and (A.28). In turn, using the definition of sectoral outputs (A.4),

$$K_t^k(\mathbf{A}) = X_t^k(\mathbf{A})^{\frac{1}{\alpha}} n_t^k(\mathbf{A})^{-\frac{1-\alpha}{\alpha}} \tag{A.29}$$

for each sector $k = D, C$. Given these sequences of capital stocks, the gross investments $I_t^k(\mathbf{A})$ and the marginal cost of investment $\varphi_t(\mathbf{A})$ are given by expressions (A.8) and the laws of motion for capital.

The subsidies τ_t^h that implement the path of MRPLs $\mathbf{A}$ are the ones that ensure that firms indeed want to hold capital stocks $K_t^k(\mathbf{A})$ given the marginal cost of investment $\varphi_t(\mathbf{A})$. Specifically, these subsidies can be solved from the optimality conditions (A.11)–(A.12). Specifically,

$$\begin{aligned} &\alpha\left(\varsigma_t^k\right)^{\frac{1}{\phi}}\left[\sum_m\left(\varsigma_t^m\right)^{\frac{1}{\phi}}\left(\frac{X_t^m(\mathbf{A})}{X_t^k(\mathbf{A})}\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{1}{\phi-1}}\left(\frac{K_t^k(\mathbf{A})}{n_t^k(\mathbf{A})}\right)^{\alpha-1} \\ &\quad = (r+\delta)\,\vartheta_t^k(\mathbf{A}) - \dot{\vartheta}_t^k(\mathbf{A}), \end{aligned} \tag{A.30}$$

for each sector $k = D, C$, which defines a system of two independent ordinary differential equations with terminal conditions $\lim_{t\to\infty}\vartheta_t^k(\mathbf{A}) = \bar{\vartheta}^k(\mathbf{A})$ with $\bar{\vartheta}^k(\mathbf{A})$ denoting the long-run level of the multipliers. Finally, the subsidies $\tau_t^k(\mathbf{A})$ can be backed out residually using the definition of the marginal cost (A.8) and the optimality condition (A.12), so

$$\tau_t^k = 1 - \gamma\vartheta_t^k(\mathbf{A})\left[I_t^D(\mathbf{A}) + I_t^C(\mathbf{A})\right]^{\frac{\gamma-1}{\gamma}} \tag{A.31}$$

for each sector $k = D, C$. It is worth noting that the derivation above carries through in our full model with skill accumulation, costly vacancy-posting and job ladders introduced in Section 5. Of course, the exact labor supplies n_t^k are different in this case, but they still depend exclusively on the paths of $\mathbf{A} \equiv \{A_t^k\}$.

A numerical example. We now present a numerical example showing the types of subsidies needed to implement our counterfactuals. This example is meant as an illustration rather than a proper quantification since the paths of MRPLs A_t^k and the distribution of relative skills G are not known at this stage. We will estimate those

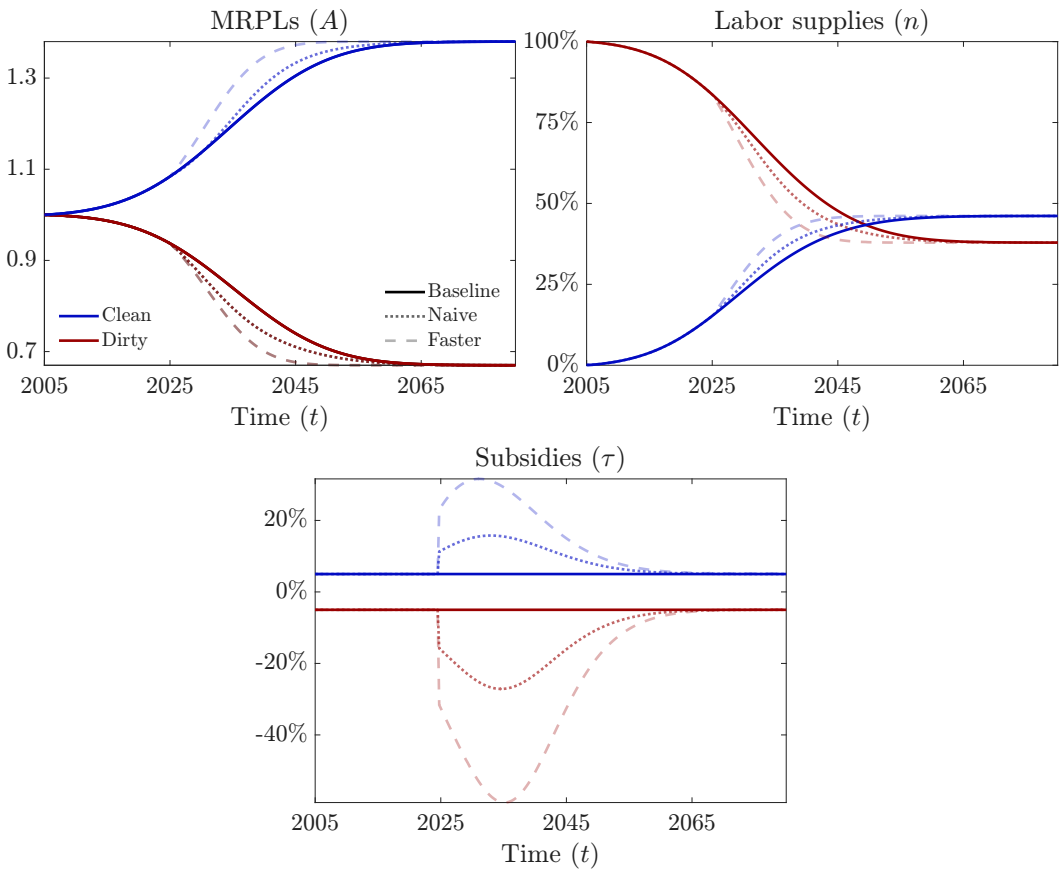
in Section 6 as part of our climate application. For now, we consider the particular MRPLs plotted in Figure 3.1 in the main text. We assume that the distribution G is uniform over a finite interval. The upper bound is 1 so no discrete mass of workers reallocates in $t = 0$. In turn we choose the lower bound to ensure that a small majority of workers employed in the growing (clean) sector in the long-run.[48] We set the discount rate to $\rho = 5\%$. The capital share takes the standard value $\alpha = 1/3$ and we choose a depreciation rate of $\delta = 0.1$. We set the elasticity of substitution to $\phi = 5$, which lies in the range considered by Acemoglu et al. (2012). In our quantitative application, we will consider a range going from $\phi = 2$ to $\phi = 10$ to acknowledge the uncertainty surrounding this parameter. Finally, note that the parameter γ governs the inverse price elasticity of the supply for investment goods. We pick $\gamma = 0.95$, which lies between the values used by McKay and Wieland (2021) and Orchard et al. (2025). The government implements constant subsidies and taxes of $\tau_t^C = 5\%$ and $\tau_t^D = -5\%$ under the status quo, as we assume in our quantitative application of Section 6. For the reasons discussed in Section 3.1, we consider a sequence of coefficients ϑ_t^k that are S-shaped over time, resulting in the status quo displayed in the left panel of Figure 3.1.

Figure A.1 illustrates how the government can use its subsidies to implement a faster sectoral shift in MRPLs. The top left panel corresponds to A_t^k in the declining (red) and growing (blue) sectors. The solid curves correspond to the status quo, while the dashed curves correspond to the counterfactual of interest where the transition is completed sooner. We will discuss the dotted curves momentarily. The top right panel plots the labor supplies associated with the status quo and counterfactual transitions: workers reallocate faster when the sectoral shift is completed earlier. Finally, the bottom panel plots the subsidies that implement these two transitions.

Of course, workers' reallocation and the presence of decreasing returns at the sector-level dampen the change in MRPLs in general equilibrium. As a result, the government must adopt larger subsidies than it would in partial equilibrium to implement the same change in MRPLs. To illustrate that, we first compute the subsidies that would have implemented the new MRPLs in partial equilibrium. These subsidies correspond to the dotted curve in the lower panel. Workers reallocate to

[48] In our climate application, this outcomes lies which lies in the range spanned by McKinsey & Company (2022), IRENA (2023) and Mayfield et al. (2023).

Figure A.1: Subsidies that implement MRPLs



Notes: The left panel illustrates the paths of MRPLs A_t^k in the two sectors under the status quo transition. The solid and dashed curves indicate the growing sector C and the declining sector D, respectively. The gray curve indicates the earnings of a worker with $\hat{\psi} < 1$ who supplies labor to sector C. The right panel illustrates a faster transition parametrized by expression (3.2) with $\eta > 1$.

the growing sector, as these subsidies raise the relative MRPLs and hence wages. This labor reallocation and endogenous changes in the price of the two goods lean against the initial change in MRPLs, resulting in the dotted curves in the top right panel. The government must therefore "scale up" its subsidies or taxes to offset this effect.

B Earnings Changes in the Frictionless Benchmark

In this appendix, we derive formally the results that we presented in Sections 3–4 using our frictionless benchmark, and under weaker restrictions on the transitions

than the ones we considered in the main text. Throughout, we index workers by their birth date τ and their relative skill $\hat{\psi}$ in the growing C sector. We define t_0 to be the date at which the government first intervenes to affect the speed of transition. We let $A^k(t,\eta)$ be the de-trended marginal product of labor in sector k, where we have made explicit the dependence on both calendar time t and η, which parametrizes how quickly sectoral MRPLs change after t_0. We define $\hat{A}(t,\eta) \equiv A^C(t,\eta)/A^D(t,\eta)$ as the relative MRPL between the growing and declining sectors. Finally, we maintain minimal regularity assumptions on $A^k(t,\eta)$, summarized below

Assumption 1. *De-trended marginal revenue products of labor $A^k(t,\eta)$ satisfy:*

1. *$A^C(t,\eta)$ increases with t and η and $A^D(t,\eta)$ decreases in t and η, for all $t \geq t_0$, and strictly so over some interval in the neighborhood of t_0;*
2. *$A^k(t,\eta)$ is continuous in t and η for all k; and*
3. *$\lim_{\eta\to\infty} A^k(t,\eta) = A^k_\infty$ and $A^k(t,0) = A^k_{t_0}$ for all $t \geq t_0$ and k.*

The first assumption defines the kinds of transitions that we consider: productivity falls in the declining D sector (relative to trend) while it rises in the growing C sector. A larger η speeds up the transition in that it implies a faster MRPL growth in the growing sector and a faster decline in the declining sector starting from t_0. The second assumption is standard. The third assumption states that, in the limit as η goes to infinity, the MRPLs instantly jump to their long run level A^k_∞, and permanently stay at their initial level $A^k_{t_0}$ if $\eta = 0$. We define $\hat{A}_\infty$ and $\hat{A}_{t_0}$ similarly. Whenever is convenient, we also assume that $A^k(t,\eta)$ is differentiable almost everywhere in its arguments. This set of assumptions is strictly weaker than those imposed in Section 3.

B.1 Always, Never and Occasional Switchers

It is useful to first partition the workers into three groups based on their reallocation patterns. Let $t^*(\hat{\psi};\eta)$ denote the period in which a worker of skill $\hat{\psi}$ would switch from sector D to C under transition speed η if they lived forever, and let $\bar{t}(\hat{\psi};\eta)$ denote the final period after t_0 when the worker is employed in sector D.

Lemma 1. *The set of workers $(\tau,\hat{\psi})$ can be partitioned into:*

1. ***Always switchers*** *with* $\hat{\psi} \geq \hat{A}_{t_0}^{-1}$ *who move immediately to the C sector so* $\bar{t}\left(\tau, \hat{\psi}; \eta\right) = t^{\star}\left(\hat{\psi}; \eta\right) = t_0$ *irrespective of* τ *and* η*.*

2. ***Never switchers*** *with* $\hat{\psi} \leq \hat{A}_{\infty}^{-1}$ *who never move to the C sector so* $\bar{t}\left(\tau, \hat{\psi}; \eta\right) = t^{\star}\left(\hat{\psi}; \eta\right) = \infty$ *irrespective of* τ *and* η*.*

3. ***Occasional switchers*** *with* $\hat{\psi} \in \left(\hat{A}_{\infty}^{-1}, \hat{A}_{t_0}^{-1}\right)$ *who potentially move to sector C if they are sufficiently young and the transition is sufficiently short. For these workers,*

$$\bar{t}\left(\tau, \hat{\psi}; \eta\right) = \min\left\{\max\left\{t^{\star}\left(\hat{\psi}; \eta\right), t_0\right\}, J + \tau\right\} \tag{B.1}$$

where the cutoff $t^{\star}\left(\hat{\psi}; \eta\right)$ *satisfies*

$$\hat{\psi} = \hat{A}\left(t^{\star}\left(\hat{\psi}; \eta\right), \eta\right)^{-1} \tag{B.2}$$

and is therefore continuous, strictly decreasing in $\hat{\psi}$*, strictly decreasing in* η*, and satisfies* $t^{\star}\left(\hat{\psi}; \eta\right) \to t_0$ *as* $\eta \to \infty$*.*

Proof. A worker supplies labor to the growing sector if and only if

$$\hat{\psi} A^{C}(t, \eta) \geq A^{D}(t, \eta) \quad \text{or} \quad \hat{\psi} \geq \hat{A}(t, \eta)^{-1} \tag{B.3}$$

during their lifetime. Note that the relative MRPL $\hat{A}_t(\eta)^{-1}$ decreases over time. Condition (B.3) always holds for workers with $\hat{\psi} \geq \hat{A}_{t_0}^{-1}$ so they always supply labor to sector C for any $t \geq t_0$. On the contrary, this condition never holds for workers with $\hat{\psi} \leq \hat{A}_{\infty}^{-1}$ so they never supply labor to sector C. For any intermediate $\hat{\psi} \in \left(\hat{A}_{\infty}^{-1}, \hat{A}_{t_0}^{-1}\right)$, there exists a cutoff $t^{\star}\left(\hat{\psi}; \eta\right)$ that satisfies (B.2) since $\hat{A}(t, \eta)$ is a continuous function of t. Workers supply labor to the declining sector until $t^{\star}\left(\hat{\psi}; \eta\right)$ and then supply labor to the growing sector. It is clear from condition (B.3) that $t^{\star}\left(\hat{\psi}, \eta\right)$is continuous because $\hat{A}(t, \eta)$ is itself continuous. Furthermore, it decreases with the relative productivity $\hat{\psi}$ since $\hat{A}(t, \eta)^{-1}$ decreases over time. In turn, a faster transition implies a smaller cutoff since $\hat{A}(t, \eta)^{-1}$ decreases with the pace of the transition η. Furthermore, the cutoff $t^{\star}\left(\hat{\psi}; \eta\right) \to t_0$ as $\eta \to \infty$ by Assumption 1 since $\hat{\psi} > \hat{A}_{\infty}^{-1}$. Finally, a worker only switches in their lifetime if the cutoff $t^{\star}\left(\hat{\psi}; \eta\right)$ occurs before they retire in period $J + \tau$. □

B.2 Non-Linearity of Lifetime Earnings

We now establish that earnings changes are non-linear for always and never switchers as a function of the horizon over which the transition unfolds. In other words, workers are affected non-linearly as the transition ends 5, 10 or 15 years earlier, and extrapolating from small changes around the status quo transition can be misleading. We will turn our attention to occasional switchers later in this appendix.

A worker's lifetime earnings starting from period t_0 are given by

$$W_{t_0}\left(\tau, \hat{\psi}; \eta\right) = \int_{t^+(\tau)}^{\bar{t}\left(\tau,\hat{\psi};\eta\right)} e^{-\rho\left(t-t^+(\tau)\right)} A^D\left(t,\eta\right) dt + \hat{\psi} \int_{\bar{t}\left(\tau,\hat{\psi};\eta\right)}^{J+\tau} e^{-\rho\left(t-t^+(\tau)\right)} A^C\left(t,\eta\right) dt, \tag{B.4}$$

where $t^+(\tau) \equiv \max\{\tau, t_0\}$ is the first period where worker is present after t_0.[49] We focus on incumbent workers so $\tau \in (t_0 - J, t_0]$ and $t^+(\tau) = t_0$.

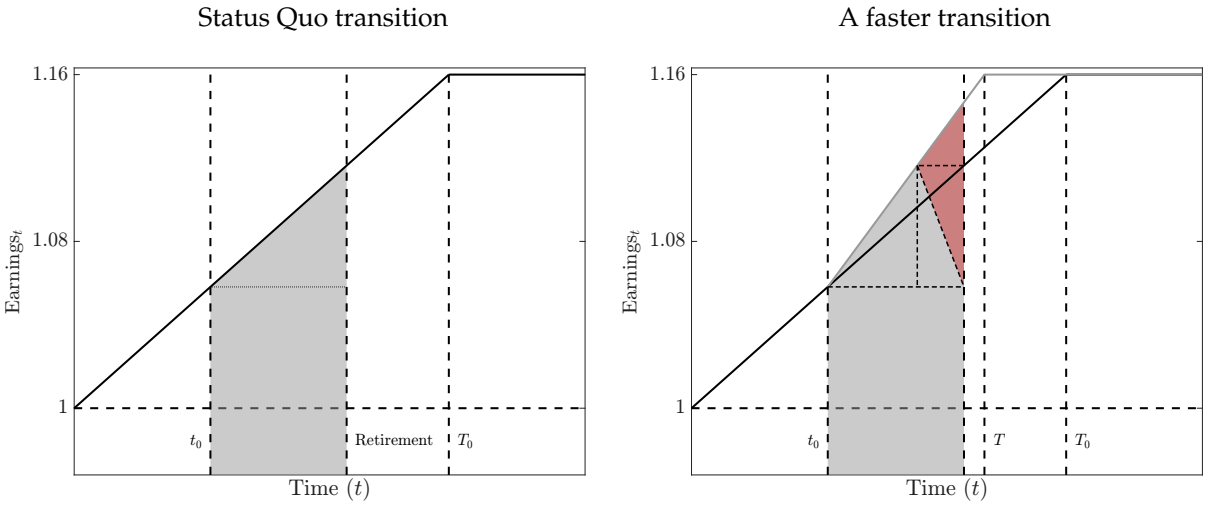
We start by inspecting always switchers whose outcomes are unrelated to sector D. Their lifetime earnings become $W_{t_0}\left(\tau, \hat{\psi}; \eta\right) = \hat{\psi} \int_{t_0}^{J+\tau} e^{-\rho(t-t_0)} A^C\left(t,\eta\right) dt$. Starting from a status quo horizon that ends in period T_0, we are interested in characterizing changes in lifetime earnings as a function of the horizon T over which the transition is completed. Figure B.1 illustrates the path of earnings for an always switcher with $\hat{\psi} = 1$. The left panel corresponds to the status quo transition. As a convenient benchmark, suppose that the transition is linear over time. The shaded gray area corresponds to workers' lifetime earnings (B.4) in the case where there is no discounting, i.e., $\rho = 0$.[50] The right panel of Figure B.1 considers a faster transition ending in period $T < T_0$. In the following, we let $\Delta T \equiv T_0 - T$. The sum of the shaded gray and red areas correspond to lifetime earnings in this case.

It is straightforward to show that lifetime earnings are convex in the horizon ΔT. This result can be illustrated geometrically. By construction, the shaded gray areas are the same in the two panels of Figure B.1 since they have the same base and height. Thus, lifetime earnings increase by the two shaded red triangles on the right panel of Figure B.1 as the transition accelerates. The area of the bottom triangle scales linearly with ΔT. On the contrary, the area of the top triangle grows non-linearly with ΔT since its base is linear in ΔT but its height increases too.

[49] Note we index workers' values by cohort τ rather than age j for the purposes of Appendix B. One could equivalently index workers by their age as of time 0 given by $j_0 \equiv \tau + t_0$.

[50] The gray area extends down all the way to zero. We truncated the plot to improve readability.

Figure B.1: Non-linear earnings changes



Notes: This figure plots the path of earnings over time for an always switcher worker with $\hat{\psi} = 1$. The left panel illustrates a status quo transition ending in periods T_0. The shaded area corresponds to lifetime earnings when there is no discounting, i.e., $\rho = 0$. The right panel illustrates a faster transition ending in $T < T_0$. The two shaded areas correspond to lifetime earnings in this case.

As the transition ends earlier and earlier, every additional $\Delta T = 1$ year steepens the line in Figure B.1 more and more, leading to non-linear changes in lifetime earnings.[51] Note that this result holds regardless of how A_t^D changes over time. The case where the worker is a never switcher is symmetric.

Finally, it is worth noting that the transition above made two assumptions to convey the insights transparently. First, we abstracted from discounting. Obviously, exponential discounting can reinforce the non-linearities documented above. Second, we considered a linear transition. Naturally, non-linearities in the functions $\Phi^k(t)$ in expression (3.2) can strengthen the non-linearities in earnings.

B.3 Proof of Proposition 1

We now establish that earnings changes are non-monotonic for occasional switchers as a function of the transition horizon. In the following, we use the terminology, assumptions and notation previously introduced in this Appendix.

Before we establish this non-monotonicity result, let us briefly discuss the case of always switchers with $\hat{\psi} \geq \hat{A}_{t_0}^{-1}$, who switch immediately to the growing sector

[51] Of course, workers' earnings are bounded by the flat region in Figure B.1, which might itself introduce additional non-linearities.

(Lemma 1). Their earnings increase over time, so they are trivially made better off when the transition is completed faster. On the contrary, never switchers with $\hat{\psi} \leq \hat{A}_{\infty}^{-1}$ stay in the declining sector. Their earnings decrease over time, so they are made worse off by a faster transition.

Consider now an occasional switcher with $\hat{\psi} \in \left(\hat{A}_{\infty}^{-1}, \hat{A}_{t_0}^{-1}\right)$. The simple idea behind the non-monotonicity is that occasional switchers spend their whole careers in sector D when η is small enough and thus behave like never switchers over that range, but essentially behave like always-switchers when η is high enough that they work most of their career in sector C.

Formally, consider an incumbent worker born in $\tau \in (t_0 - J, t_0]$. By Lemma 1 and the intermediate value theorem, there exists some $\eta_l \equiv \eta_l\left(\tau, \hat{\psi}\right)$ for which $t^\star\left(\hat{\psi}; \eta_l\right) = J + \tau$ so the worker switches to the growing sector exactly at retirement. The worker never switches in their lifetime if $\eta < \eta_l$. Over that region, lifetime earnings (B.4) are $W_{t_0}\left(\tau, \hat{\psi}; \eta\right) = \int_{t_0}^{J+\tau} e^{-\rho(t-t_0)} A^D\left(t, \eta\right) dt$ and decrease with η by Assumption 1, so the worker is worse off as the transition accelerates.

Now consider a pace η with $0 \leq t^\star\left(\hat{\psi}; \eta\right) < J + \tau$, i.e., the worker switches at some point before retirement. There exists such a pace by Lemma 1 and Assumption 1. In this case, the change in lifetime earnings (B.4) is

$$\frac{dW_{t_0}\left(\tau, \hat{\psi}; \eta\right)}{d\eta} = \int_{t_0}^{t^\star\left(\hat{\psi};\eta\right)} e^{-\rho(t-t_0)} \frac{\partial A^D\left(t, \eta\right)}{\partial \eta} dt + \hat{\psi} \int_{t^\star\left(\hat{\psi};\eta\right)}^{J+\tau} e^{-\rho(t-t_0)} \frac{\partial A^C\left(t, \eta\right)}{\partial \eta} dt, \tag{B.5}$$

where we have used the fact that the switching time $t^\star\left(\hat{\psi}; \eta\right)$ is chosen optimally. We will show that there exists some $\eta_h \equiv \eta_h\left(\tau, \hat{\psi}\right)$ with $\eta_h > \eta_l$ such that the expression above is positive for all $\eta > \eta_h$.

First, note that the second term of the right-hand side of (B.5) is positive since $A^C\left(t, \eta\right)$ increases with η. In particular, it converges to $\hat{\psi}\left(A_{\infty}^C - A_0^C\right)$ as $\eta \to \infty$ by Assumption 1. Therefore, it suffices to show that

$$\lim_{\eta \to \infty} \int_{t_0}^{t^\star\left(\hat{\psi};\eta\right)} e^{-\rho(t-t_0)} \frac{\partial A^D\left(t, \eta\right)}{\partial \eta} dt = 0 \tag{B.6}$$

Note that, for any finite η, Assumption 1 implies

$$0 > \int_{t_0}^{t^\star(\hat{\psi};\eta)} e^{-\rho(t-t_0)} \frac{\partial A^D(t,\eta)}{\partial \eta} dt > \int_{t_0}^{t^\star(\hat{\psi};\eta)} \frac{\partial A^D(t,\eta)}{\partial \eta} dt \tag{B.7}$$

since $\partial A^D(t,\eta)/\partial\eta \leq 0$ and strictly so in a neighborhood of t_0. Moreover,

$$\int_{t_0}^{t^\star(\hat{\psi};\eta)} \frac{\partial A^D(t,\eta)}{\partial \eta} dt > \frac{\partial}{\partial \eta} \int_{t_0}^{t^\star(\hat{\psi};\eta)} A^D(t,\eta)\, dt \tag{B.8}$$

since $t^\star\left(\hat{\psi};\eta\right)$ decreases with η. We will now establish that

$$\lim_{\eta\to\infty} \frac{\partial}{\partial \eta} \int_{t_0}^{t^\star(\hat{\psi};\eta)} A^D(t,\eta)\, dt = 0 \tag{B.9}$$

Note that

$$\frac{\partial}{\partial \eta} \int_{t_0}^{t^\star(\hat{\psi};\eta)} A^D(t,\eta)\, dt < 0 \tag{B.10}$$

both because $t^\star\left(\hat{\psi},\eta\right)$ and $A^D(t,\eta)$ decrease with η. Suppose by contradiction that expression (B.9) does not hold. Then, for each $\bar{\eta}$, there exists an $\epsilon < 0$ and an infinite number of subsets with positive measure in $(\bar{\eta},\infty)$ such that

$$\frac{\partial}{\partial \eta} \int_{t_0}^{t^\star(\hat{\psi};\eta)} A^D(t,\eta)\, dt < \epsilon \tag{B.11}$$

for each η in these subsets. Then,

$$\begin{aligned} 0 &= \int_{t_0}^{t^\star(\hat{\psi};\eta)} A^D(t,\eta)\, dt \Big|_{\eta=\infty} \\ &= \int_{t_0}^{t^\star(\hat{\psi};\eta)} A^D(t,\eta)\, dt \Big|_{\eta=\bar{\eta}} + \int_{\bar{\eta}}^{\infty} \left\{ \frac{\partial}{\partial \tilde{\eta}} \int_{t_0}^{t^\star(\hat{\psi};\tilde{\eta})} A^D(t,\tilde{\eta})\, dt \right\} d\tilde{\eta} \\ &= -\infty \end{aligned} \tag{B.12}$$

which yields the desired contradiction. Therefore, expression (B.9) holds. Combining (B.7)–(B.8) and using the squeeze theorem, it must be that equation (B.6) holds

as well. Thus,

$$\lim_{\eta \to \infty} \frac{dW_{t_0}\left(\tau, \hat{\psi}; \eta\right)}{d\eta} > 0, \tag{B.13}$$

which shows that there exists indeed a pace $\eta_h > \eta_l$ above which workers benefit from a transition that ends sooner. Workers' welfare is therefore non-monotone in the transition horizon for incumbent workers.

Having established the desired result for incumbent cohorts, we now consider a future cohort born in $\tau \in (t_0, \infty)$. We derive η_l as above, such that the worker would switch only at retirement and thus prefers a slower transition for $\eta < \eta_l$. We may now define η_h as the speed such that the worker would switch to the growing sector at birth; at this point, the worker spends their whole life in the growing sector and prefers a faster transition for $\eta > \eta_h$. Thus, lifetime earnings are also non-monotonic in the transition horizon for these new cohorts. □

C Full Model

This appendix provides additional derivations and proofs for our full model introduced in Section 5. Section C.1 states workers' and firms' values recursively. Section C.2 defines the equilibrium formally. Section C.3 shows that workers' and firms' values scale linearly with $\psi \exp(gt)$, where for g is the long-run growth rate of the economy and ψ is a worker's reference skill.

C.1 Workers' and Firms' Values

Skill accumulation. Before stating workers' and firms' values, it is useful to perform a change of variable and define the "reference" skill $\psi \equiv \psi^G \psi^D$ and the "relative" skill $\hat{\psi} \equiv \psi^C / \psi^D$. Using the laws of motions that we described in Section 5, the reference skill ψ grows at rate

$$\delta_j^\star(k) \equiv \begin{cases} \delta_j^+ + \hat{\delta}_j & \text{if } k = D \\ \delta_j^+ & \text{if } k = C \\ \delta_j^k & \text{if } k = O, U \end{cases} \tag{C.1}$$

depending on the worker's employment status. The relative skill grows at rate

$$\hat{\delta}_j^{\star}(k) \equiv \begin{cases} -\hat{\delta}_j & \text{if } k = D \\ \hat{\delta}_j & \text{if } k = C \\ 0 & \text{if } k = O, U \end{cases} \tag{C.2}$$

In the following, we let

$$h\left(\psi, \hat{\psi}, k\right) = \begin{cases} \psi & \text{if } k = D \\ \psi \times \hat{\psi} & \text{if } k = C \\ \psi \times b^k & \text{if } k = O, U \end{cases} \tag{C.3}$$

denote a workers' productivity depending on their employment status. Workers are now indexed by five states: their age j; their reference skill ψ; their relative skill in the clean sector $\hat{\psi}$; their employment status $k = D, C, O, U$; and, if employed, their wage w. The last two states can be combined into a single state s which corresponds to a market (k, w) when the worker is employed, or to O or U when the worker claims the outside option or is unemployed. We omit the dependence of values on η for notational convenience.

Workers' values. The value of an employed worker of age j with skills ψ and $\hat{\psi}$ who is currently in submarket s satisfies

$$\begin{aligned} \rho W_t\left(j, \psi, \hat{\psi}, s\right) = {} & \pi(s)\, h\left(\psi, \hat{\psi}, k(s)\right) \mathcal{P}_t^{k(s)} \exp(gt) \\ & + \lambda^E \sum_{s'} \mathcal{S}_t\left(s'; j, \psi, \hat{\psi}, s\right) \left[W_t\left(j, \psi, \hat{\psi}, s'\right) - W_t\left(j, \psi, \hat{\psi}, s\right)\right] \\ & + \chi\left[W_t\left(j, \psi, \hat{\psi}, U\right) - W_t\left(j, \psi, \hat{\psi}, s\right)\right] \\ & + \frac{d}{d\epsilon} W_{t+\epsilon}\left(j + \epsilon, \psi \exp\left(\delta_j^{\star}(k(s))\,\epsilon\right), \hat{\psi} \exp\left(\hat{\delta}_j^{\star}(k(s))\,\epsilon\right), s\right)\Big|_{\epsilon=0} \end{aligned} \tag{C.4}$$

for each age $j \in [0, J)$, where $W_t\left(j, \psi, \hat{\psi}, U\right)$ is the value of being unemployed, which is defined below. Workers discount flows at rate ρ. When employed, they earn $\pi(s)\, h\left(\psi, \hat{\psi}, k(s)\right) \mathcal{P}_t^{k(s)} \exp(gt)$ for each productive unit of labor $h\left(\psi, \hat{\psi}, k(s)\right)$ that they provide, where $\pi(s)$ is the piece rate in their current market, $h(\cdot)$ is given by expression (C.3), and $\mathcal{P}_t^k$ is the price of labor services in sector k de-trended using the long-run growth rate g. Workers have the opportunity to search for on-the-job at rate λ^E. They search in submarket $s' \equiv (k', \pi')$ and the probability of a

successful search is

$$\mathcal{S}_t\left(s';j,\psi,\hat{\psi},s\right)\equiv\bar{p}_t\left(k\left(s'\right);\mathbf{x},k\left(s\right)\right)\underline{p}_t\left(s';j,\psi,\hat{\psi},s\right)f\left(\theta_t\left(j,\psi,\hat{\psi},s'\right)\right), \tag{C.5}$$

where $\bar{p}_t$ and $\underline{p}_t$ are the search probabilities in the upper tier (5.2) and lower tier (5.1), respectively, and the f term is the job finding rate in a given market. Matches are dissolved exogenously at rate χ. Finally, there is drift in age, over time, and in the skills ψ and $\hat{\psi}$ at rates (C.1)–(C.2).[52] The boundary condition is $W_t\left(J,\psi,\hat{\psi},s\right)=0$ upon retirement at age $j=J$. The terminal conditions are similar for the values below so we omit them. Lump sum transfers from the government do not distort workers' labor supply decisions when workers are risk-neutral or have access to perfect borrowing and saving, as we assume in this paper (Sections 2.3 and 5.1). Therefore, we exclude these transfers from workers' values, allowing us to interpret them as the present discounted value of future earnings — the object of interest in this paper.

The value of claiming the outside option satisfies

$$\begin{aligned}\rho W_t\left(j,\psi,\hat{\psi},O\right)=&\,h\left(\psi,\hat{\psi},O\right)\exp\left(gt\right)\\&+\lambda^E\sum_s\mathcal{S}_t\left(s;j,\psi,\hat{\psi},O\right)\left[W_t\left(j,\psi,\hat{\psi},s\right)-W_t\left(j,\psi,\hat{\psi},O\right)\right]\\&+\chi\left[W_t\left(j,\psi,\hat{\psi},U\right)-W_t\left(j,\psi,\hat{\psi},O\right)\right]\\&+\frac{d}{d\epsilon}W_{t+\epsilon}\left(j+\epsilon,\psi\exp\left(\delta_j^\star\left(O\right)\epsilon\right),\hat{\psi},O\right)\Big|_{\epsilon=0}\end{aligned} \tag{C.6}$$

for each age $j\in[0,J)$, since they receive benefits $h\left(\psi,\hat{\psi},O\right)\exp\left(gt\right)$, search for jobs with intensity λ^E, and separate from the outside option exogenously at rate χ. Similarly, the value of an unemployed worker satisfies

$$\begin{aligned}\rho W_t\left(j,\psi,\hat{\psi},U\right)=&\,h\left(\psi,\hat{\psi},U\right)\exp\left(gt\right)\\&+\lambda^U\sum_s\mathcal{S}_t\left(s;j,\psi,\hat{\psi},U\right)\left[W_t\left(j,\psi,\hat{\psi},s\right)-W_t\left(j,\psi,\hat{\psi},U\right)\right]\\&+\frac{d}{d\epsilon}W_{t+\epsilon}\left(j+\epsilon,\psi\exp\left(\delta_j^\star\left(U\right)\epsilon\right),\hat{\psi},s\right)\Big|_{\epsilon=0}\end{aligned} \tag{C.7}$$

for each age $j\in[0,J)$, since they receive benefits $h\left(\psi,\hat{\psi},U\right)\exp\left(gt\right)$ and search

[52] These drifts are characterized by the single index ϵ.

for jobs with intensity λ^U. Workers' search probabilities are then given by in the upper tier are

$$\bar{p}_t\left(k';\mathbf{x},k\right)\equiv\frac{\exp\left(\left[\overline{W}_t\left(\mathbf{x},k'\right)-\overline{W}_t\left(\mathbf{x},k\right)\right]/z\right)}{\sum_{\hat{k}}\exp\left(\left[\overline{W}_t\left(\mathbf{x},\hat{k}\right)-\overline{W}_t\left(\mathbf{x},k\right)\right]/z\right)},\tag{C.8}$$

where $\mathbf{x}\equiv\left(j,\psi,\hat{\psi}\right)$ denotes a worker's state excluding its current market s and $\overline{W}_t\left(\mathbf{x},k\right)$ is the expected value conditional on searching in the lower tier associated with sector k. In turn, the search probabilities in the lower tier are

$$\underline{p}_t\left(s';\mathbf{x},s\right)\equiv\frac{\exp\left(f\left(\theta_t\left(\mathbf{x},s'\right)\right)\left[W_t\left(\mathbf{x},s'\right)-W_t\left(\mathbf{x},s\right)\right]/\zeta\right)}{\sum_{\hat{s}}\exp\left(f\left(\theta_t\left(\mathbf{x},\hat{s}\right)\right)\left[W_t\left(\mathbf{x},\hat{s}\right)-W_t\left(\mathbf{x},s\right)\right]/\zeta\right)},\tag{C.9}$$

Labor supplies. Sectoral labor supplies are given by

$$n_t^{k'}=\int\mathbf{1}_{\{k(s)=k'\}}h\left(\psi,\hat{\psi},k\left(s\right)\right)d\mu_t\left(j,\psi,\hat{\psi},s\right)\tag{C.10}$$

in a sector $k'=D,C$, with $k\left(s\right)$ denoting the sector that a given worker in market s is employed in, and $\mu_t\left(j,\psi,\hat{\psi},s\right)$ is the distribution of workers' idiosyncratic states.
Firms' values. The value of a filled job in submarket s is given by

$$\begin{aligned}rV_t\left(j,\psi,\hat{\psi},s\right)=&\left[1-\pi\left(s\right)\right]h\left(\psi,\hat{\psi},k\left(s\right)\right)\mathcal{P}_t^{k(s)}\exp\left(gt\right)-\chi V_t\left(j,\psi,\hat{\psi},s\right)\\&-\lambda^E\sum_{s'}\mathcal{S}_t\left(s';j,\psi,\hat{\psi},s\right)V_t\left(j,\psi,\hat{\psi},s\right)\\&+\frac{d}{d\epsilon}V_{t+\epsilon}\left(j+\epsilon,\psi\exp\left(\delta_j^\star\left(k\left(s\right)\right)\epsilon\right),\hat{\psi}\exp\left(\hat{\delta}_j^\star\left(k\left(s\right)\right)\epsilon\right),s\right)\Big|_{\epsilon=0}\end{aligned}\tag{C.11}$$

for each age $j\in[0,J)$, where $\mathcal{P}_t^k$ denotes the de-trended prices of labor services produced in the declining or growing sectors. Firms discount flows using the equilibrium interest rate $r\equiv\rho$.[53] They claim a share $1-\pi\left(s\right)$ of their revenues as profits. The match is dissolved either exogenously at rate χ, or because workers voluntarily quit due to on-the-job search. The opportunity for on-the-job search arrives at rate λ^E and the probability that the individual searches there successfully in a given market is $\mathcal{S}_t\left(s';j,\psi,\hat{\psi},s\right)$. The firm's value also features a drift in workers' skills, in their age, and over time. The boundary condition is $V_t\left(J,\psi,\hat{\psi},s\right)=0$

[53] The equilibrium interest rate is $r=\rho$ since entrepreneurs have linear utility.

when the worker retires at age $j = J$.

Tightnesses. Free entry in each market implies that equilibrium tightnesses $\theta_t\left(j, \psi, \hat{\psi}, s\right)$ satisfy

$$q\left(\theta_t\left(j, \psi, \hat{\psi}, s\right)\right) V_t\left(j, \psi, \hat{\psi}, s\right) \leq \nu \psi \exp\left(gt\right) \tag{C.12}$$

in every period t and market s, with equality if a positive measure of vacancies are posted. Posting a vacancy for a worker with reference skill ψ comes at a cost $\nu\psi \exp\left(gt\right) > 0$ which grows at rate g to ensure that the economy admits a balanced-growth path where vacancy posting costs do not vanish. Firms post vacancies until the expected value of a vacancy equals the vacancy-posting cost. This expected value is the product of the value of the match $V_t\left(j, \psi, \hat{\psi}, s\right)$ and the probability of finding match $q\left(\theta_t\left(j, \psi, \hat{\psi}, s\right)\right)$. Firms do not post any vacancies if the expected value of the vacancy is lower than its cost.

C.2 Equilibrium Definition

The production side of the economy is the same as the one described in Appendix A.1. The only difference comes from the labor supply block and the fact that we now call w_t^k the price of labor services $\mathcal{P}_t^k$. By definition, the prices of labor services still coincide with the MRPLs, i.e., $\mathcal{P}_t^k = w_t^k = A_t^k$, as in Appendix A.1.

Definition 1. Given a path of government subsidies $\left\{\tau_t^k, \varsigma_t^k\right\}$, an equilibrium consists of sequences of workers' and firms' values $W_t\left(j, \psi, \hat{\psi}, s\right)$ and $V_t\left(j, \psi, \hat{\psi}, s\right)$, search probabilities $\mathcal{S}_t\left(s'; j, \psi, \hat{\psi}, s\right)$, tightnesses $\theta_t\left(j, \psi, \hat{\psi}, s\right)$, a distribution of workers' states $\boldsymbol{\mu}_t\left(j, \psi, \hat{\psi}, s\right)$, sectoral outputs X_t^k, sectoral labor supplies n_t^k, sectoral TFPs a_t^k, prices of intermediates P_t^k , prices of labor services $\mathcal{P}_t^k$, and lump sum taxes T_t such that:

1. Workers' values satisfy (C.4)–(C.7)
2. Firms' values satisfy (C.11)
3. The search probabilities satisfy (C.5)
4. The tightnesses satisfy (C.12)

5. The distribution is consistent with the flows implied by workers' and firms' values (see Appendix D.3)
6. Sectoral outputs are given by (A.4)
7. Sectoral labor supplies are given by (C.10)
8. The prices of intermediates satisfy (A.3)
9. The prices of labor services satisfy (A.10) with $\mathcal{P}_t^k \equiv w_t^k$
10. Lump sum taxes balance the government's budget (A.13)

Before characterizing the equilibrium further, it is worth noting that firms' vacancy posting and workers' labor supplies are pinned down by the coupled system of partial differential equations that define workers' and firms' values (C.4)–(C.7) and (C.11), together with the search probabilities given by expressions (C.5) and (C.8)–(C.9) and the free entry condition (C.12). This is a closed system that is parametrized by the prices of labor services $\mathcal{P}_t^k = A_t^k$, i.e., the MRPLs fully characterize workers' outcomes and the rest of the economy matters for workers' labor market outcomes only to the extent that it affects A_t^k. Finally, note that lump sum taxes T_t are non-distortionary since workers are risk-neutral or have access to perfect borrowing and saving (Sections 2.3 and 5.1). Therefore, these taxes do not affect workers' labor supplies or any equilibrium variables characterized in this appendix.

C.3 Normalized and De-Trended Values

This appendix establishes that workers' and firms' are linearly homogenous in $\psi \exp(gt)$, where g still denotes the rate at which output grows along a balanced-growth path. This homogeneity property makes our analysis computationally feasible because it allows us to 1) characterize de-trended values that are constant over a balanced-growth path and 2) not keep track of the reference skill ψ in our state space. Formally, we will show that workers' and firms' values satisfy

$$W_t\left(j, \psi, \hat{\psi}, s\right) = \psi \exp(gt)\, W_t^\star\left(j, \hat{\psi}, s\right) \tag{C.13}$$

$$V_t\left(j, \psi, \hat{\psi}, s\right) = \psi \exp(gt)\, V_t^\star\left(j, \hat{\psi}, s\right), \tag{C.14}$$

for some $W_t^\star$ and $V_t^\star$ that do not depend on workers' reference skill ψ, and depend on time only through the de-trended prices of labor services $\mathcal{P}_t^k$ which coincide with the MRPLs A_t^k (Appendix C.2). While establishing this property, we will also show that

$$\theta_t\left(j,\psi,\hat{\psi},s\right)=\theta_t^\star\left(j,\hat{\psi},s\right)\quad,\quad\mathcal{S}_t\left(s';j,\psi,\hat{\psi},s\right)=\mathcal{S}_t^\star\left(s';j,\hat{\psi},s\right) \tag{C.15}$$

for some $\theta_t^\star$ and $\mathcal{S}_t^\star$ that do not depend on workers' reference skill ψ and depend on time only through A_t^k. In the following, we use the fact that $h\left(\psi,\hat{\psi},k\right)\equiv\psi h^\star\left(\hat{\psi},k\right)$ where $h^\star\left(\hat{\psi},k\right)\equiv h\left(1,\hat{\psi},k\right)$ using the definition (C.3).

We proceed following a guess and verify approach. First, consider the value of a worker employed in market s. Using the definition (C.4), the properties above and expanding the time derivative with respect to ϵ, we obtain

$$\begin{aligned}\rho\frac{W_t\left(j,\psi,\hat{\psi},s\right)}{\psi\exp\left(gt\right)}&=\pi\left(s\right)h^\star\left(\hat{\psi},k\left(s\right)\right)A_t^{k(s)}\\&+\lambda^E\sum_{s'}\mathcal{S}_t^\star\left(s';j,\hat{\psi},s\right)\left[W_t^\star\left(j,\hat{\psi},s'\right)-W_t^\star\left(j,\hat{\psi},s\right)\right]\\&+\chi\left[W_t^\star\left(j,\hat{\psi},U\right)-W_t^\star\left(j,\hat{\psi},s\right)\right]\\&+\left(g+\delta_j^\star\left(k\left(s\right)\right)\right)W_t^\star\left(j,\hat{\psi},s\right)+\hat{\psi}\hat{\delta}_j^\star\left(k\left(s\right)\right)\frac{\partial}{\partial\hat{\psi}}W_t^\star\left(j,\hat{\psi},s\right)\\&+\frac{\partial}{\partial j}W_t^\star\left(j,\hat{\psi},s\right)+\frac{\partial}{\partial t}W_t^\star\left(j,\hat{\psi},s\right),\end{aligned} \tag{C.16}$$

which confirms that $W_t\left(j,\psi,\hat{\psi},s\right)$ can indeed be expressed as (C.13), where the normalized and de-trended value evolves as

$$\begin{aligned}\rho^\star W_t^\star\left(j,\hat{\psi},s\right)&=\pi\left(s\right)h^\star\left(\hat{\psi},k\left(s\right)\right)A_t^{k(s)}\\&+\lambda^E\sum_{s'}\mathcal{S}_t^\star\left(s';j,\hat{\psi},s\right)\left[W_t^\star\left(j,\hat{\psi},s'\right)-W_t^\star\left(j,\hat{\psi},s\right)\right]\\&+\chi\left[W_t^\star\left(j,\hat{\psi},U\right)-W_t^\star\left(j,\hat{\psi},s\right)\right]\\&+\delta_j^\star\left(k\left(s\right)\right)W_t^\star\left(j,\hat{\psi},s\right)+\hat{\psi}\hat{\delta}_j^\star\left(k\left(s\right)\right)\frac{\partial}{\partial\hat{\psi}}W_t^\star\left(j,\hat{\psi},s\right)\\&+\frac{\partial}{\partial j}W_t^\star\left(j,\hat{\psi},s\right)+\frac{\partial}{\partial t}W_t^\star\left(j,\hat{\psi},s\right)\end{aligned} \tag{C.17}$$

with $\rho^\star\equiv\rho-g$. These normalized and de-trended values depend on time only

through the de-trended MRPLs A_t^k. Proceeding similarly for the values (C.6)–(C.7) also confirms that $W_t\left(j,\psi,\hat{\psi},O\right)$ and $W_t\left(j,\psi,\hat{\psi},U\right)$ scale with $\psi\exp\left(gt\right)$ and yields the laws of motion for the normalized values, which we spell out explicitly in Appendix D.1.

Turning to firm's values (C.11), the same steps confirm that $V_t\left(j,\psi,\hat{\psi},s\right)$ can indeed be expressed as (C.14), where the normalized value evolves as

$$\begin{aligned} r^\star V_t^\star\left(j,\hat{\psi},s\right) = & \left[1-\pi\left(s\right)\right]h^\star\left(\hat{\psi},k\left(s\right)\right)A_t^{k(s)} \\ & -\lambda^E\sum_{s'}\mathcal{S}_t^\star\left(s';j,\hat{\psi},s\right)V_t^\star\left(j,\hat{\psi},s\right) \\ & -\chi V_t^\star\left(j,\hat{\psi},s\right) \\ & +\delta_j^\star\left(k\left(s\right)\right)V_t^\star\left(j,\hat{\psi},s\right)+\hat{\psi}\hat{\delta}_j^\star\left(k\left(s\right)\right)\frac{\partial}{\partial\hat{\psi}}V_t^\star\left(j,\hat{\psi},s\right) \\ & +\frac{\partial}{\partial j}V_t^\star\left(j,\hat{\psi},s\right)+\frac{\partial}{\partial t}V_t^\star\left(j,\hat{\psi},s\right), \end{aligned} \tag{C.18}$$

where $r^\star\equiv r-g$ with $r=\rho$ in equilibrium. Again, these normalized and de-trended values depend on time only through A_t^k.

Having verified the properties (C.13)–(C.14), we now show that the properties (C.15) also hold. Using the free entry condition (C.12) and the property (C.14), we obtain $\theta_t\left(j,\psi,\hat{\psi},s\right)=\theta_t^\star\left(j,\hat{\psi},s\right)$ for all ψ, where

$$q\left(\theta_t^\star\left(j,\hat{\psi},s\right)\right)V_t^\star\left(j,\hat{\psi},s\right)\leq\nu \tag{C.19}$$

with the usual complementary slackness condition, which confirms that $\theta_t^\star\left(j,\hat{\psi},s\right)$ does not depend on the reference skill ψ and depends on time only through changes in A_t^k. Finally, using the search probabilities (C.5) and (C.8)–(C.9) and the property (C.13), we obtain $\mathcal{S}_t\left(s';j,\psi,\hat{\psi},s\right)=\mathcal{S}_t^\star\left(s';j,\hat{\psi},s\right)$ for all ψ, where

$$\mathcal{S}_t^\star\left(s';j,\hat{\psi},s\right)\equiv\bar{p}_t\left(k\left(s'\right);j,\hat{\psi},k\left(s\right)\right)\underline{p}_t\left(s';j,\hat{\psi},s\right)f\left(\theta_t\left(j,\hat{\psi},s'\right)\right), \tag{C.20}$$

with

$$\bar{p}_t\left(k';\tilde{\mathbf{x}},k\right)\equiv\frac{\exp\left(\left[\overline{W}_t^\star\left(\tilde{\mathbf{x}},k'\right)-\overline{W}_t^\star\left(\tilde{\mathbf{x}},k\right)\right]/z\right)}{\sum_{\hat{k}}\exp\left(\left[\overline{W}_t^\star\left(\tilde{\mathbf{x}},\hat{k}\right)-\overline{W}_t^\star\left(\tilde{\mathbf{x}},k\right)\right]/z\right)}, \tag{C.21}$$

and

$$\underline{p}_t\left(s';\tilde{\mathbf{x}},s\right)\equiv\frac{\exp\left(f\left(\theta_t^\star\left(\tilde{\mathbf{x}},s'\right)\right)\left[W_t^\star\left(\tilde{\mathbf{x}},s'\right)-W_t^\star\left(\tilde{\mathbf{x}},s\right)\right]/\zeta\right)}{\sum_{\hat{s}}\exp\left(f\left(\theta_t^\star\left(\tilde{\mathbf{x}},\hat{s}\right)\right)\left[W_t^\star\left(\tilde{\mathbf{x}},\hat{s}\right)-W_t^\star\left(\tilde{\mathbf{x}},s\right)\right]/\zeta\right)},\tag{C.22}$$

with $\tilde{\mathbf{x}}\equiv\left(j,\hat{\psi}\right)$, which confirms that $\mathcal{S}_t^\star\left(s';j,\hat{\psi},s\right)$ does not depend on the reference skill ψ and depends on time only through A_t^k.

The homogeneity property (C.13) implies that we can economize one state variable when computing workers' and firms' values or iterating on the distribution of workers' states (Appendix D). In particular, none of the moments that we target in Section 5.2 depend on the level of the reference skill ψ. In Section 6.3, we compute workers' cumulative earnings losses over the period 2005–2025 (Figure 6.3). In this case, the level of ψ and its evolution are relevant and we use stochastic simulations to track the evolution of workers' states over time.

D Computational Appendix

We state workers' and firm's values recursively in Appendix D.1 and introduce some useful notation. We discretize the resulting system of differential equations in Appendix D.2 and describe how we solve it numerically. We then explain how we iterate on the distribution of workers' idiosyncratic states in Appendix D.3. Finally, we provide details about our calibration strategy in Appendix D.4.

D.1 Workers' and Firm's Values

We now formulate workers' and firms' values recursively. As shown in Appendix C.3, we do not need to include workers' reference skill ψ in our state space. Relative to that appendix, we state workers' values in all employment states, i.e., whether they are employed, unemployed, or claim the outside option. To save on notation, we let $W_t\left(j,\hat{\psi},s\right)\equiv W_t^\star\left(j,\hat{\psi},s\right)$ denote workers' normalized values defined in Appendix C.3. We define the functions V_t, θ_t, p_t, δ_j, $\hat{\delta}_j$ and h and the constant ρ similarly, dropping the $\star$ superscript relative to Appendices C.1 and C.3.

Workers. Workers are indexed by their age j; their relative skill $\hat{\psi}$ in the clean sector;

and their employment state s. The value of an employed worker satisfies

$$\begin{aligned}\rho W_t\left(j,\hat{\psi},s\right) = {} & \pi\left(s\right) h\left(\hat{\psi},k\left(s\right)\right) A_t^{k(s)} \\ & + \lambda^E \sum_{s'} \mathcal{S}_t\left(s';j,\hat{\psi},s\right)\left[W_t\left(j,\hat{\psi},s'\right) - W_t\left(j,\hat{\psi},s\right)\right] \\ & + \chi\left[W_t\left(j,\hat{\psi},U\right) - W_t\left(j,\hat{\psi},s\right)\right] \\ & + \delta_j\left(k\left(s\right)\right) W_t\left(j,\hat{\psi},s\right) + \hat{\psi}\hat{\delta}_j\left(k\left(s\right)\right)\frac{\partial}{\partial\hat{\psi}} W_t\left(j,\hat{\psi},s\right) \\ & + \frac{\partial}{\partial j} W_t\left(j,\hat{\psi},s\right) + \frac{\partial}{\partial t} W_t\left(j,\hat{\psi},s\right) \end{aligned} \tag{D.1}$$

for all ages before retirement $j \in [0, J)$, relative productivities $\hat{\psi}$ in the clean sector, and markets $s \equiv (k, w)$. The expression above uses the fact that $\mathcal{P}_t^k = A_t^k$, as discussed in Appendix C.2. As explained above, we have removed $\star$'s for notational convenience so that, for instance, $\delta_j^\star(k)$ from equation (C.1) becomes $\delta_j(k)$. Similarly, the probability that a worker searches successfully in market s' when they receive a search opportunity and are employed in sector s is given by expressions (C.20)–(C.22) after removing the $\star$'s.

In turn, the value of a worker who claims the outside option is

$$\begin{aligned}\rho W_t\left(j,\hat{\psi},O\right) = {} & h\left(\hat{\psi},O\right) \\ & + \lambda^E \sum_{s} \mathcal{S}_t\left(s;j,\hat{\psi},O\right)\left[W_t\left(j,\hat{\psi},s\right) - W_t\left(j,\hat{\psi},O\right)\right] \\ & + \chi\left[W_t\left(j,\hat{\psi},U\right) - W_t\left(j,\hat{\psi},O\right)\right] \\ & + \delta_j\left(O\right) W_t\left(j,\hat{\psi},O\right) + \frac{\partial}{\partial j} W_t\left(j,\hat{\psi},O\right) + \frac{\partial}{\partial t} W_t\left(j,\hat{\psi},O\right), \end{aligned} \tag{D.2}$$

Finally, the value of an unemployed worker satisfies

$$\begin{aligned}\rho W_t\left(j,\hat{\psi},U\right) = {} & h\left(\hat{\psi},U\right) \\ & + \lambda^U \sum_{s} \mathcal{S}_t\left(s;j,\hat{\psi},U\right)\left[W_t\left(j,\hat{\psi},s\right) - W_t\left(j,\hat{\psi},U\right)\right] \\ & + \delta_j\left(U\right) W_t\left(j,\hat{\psi},U\right) + \frac{\partial}{\partial j} W_t\left(j,\hat{\psi},U\right) + \frac{\partial}{\partial t} W_t\left(j,\hat{\psi},U\right) \end{aligned} \tag{D.3}$$

The boundary conditions are

$$W_t\left(J,\hat{\psi},s\right)=W_t\left(J,\hat{\psi},O\right)=W_t\left(J,\hat{\psi},U\right)=0 \tag{D.4}$$

when they retire at age $j=J$, for all relative productivities $\hat{\psi}$ and markets s.

Firms. The value of a firm satisfies

$$\begin{aligned}rV_t\left(j,\hat{\psi},s\right)=&\left[1-\pi\left(s\right)\right]h\left(\hat{\psi},k\left(s\right)\right)A_t^{k(s)}\\&-\lambda^E\sum_{s'}p_t\left(s';j,\hat{\psi},s\right)f\left(\theta_t\left(j,\hat{\psi},s'\right)\right)V_t\left(j,\hat{\psi},s\right)\\&-\chi V_t\left(j,\hat{\psi},s\right)\\&+\delta_j\left(k\left(s\right)\right)V_t\left(j,\hat{\psi},s\right)+\hat{\psi}\hat{\delta}_j\left(k\left(s\right)\right)\frac{\partial}{\partial\hat{\psi}}V_t\left(j,\hat{\psi},s\right)\\&+\frac{\partial}{\partial j}V_t\left(j,\hat{\psi},s\right)+\frac{\partial}{\partial t}V_t\left(j,\hat{\psi},s\right),\end{aligned} \tag{D.5}$$

for all ages before retirement $j\in[0,J)$, relative productivities $\hat{\psi}$ in the clean sector, and markets $s\equiv(k,w)$, where $r\equiv\rho$ is the interest rate. The boundary condition is

$$V_t\left(J,\hat{\psi},s\right)=0 \tag{D.6}$$

when workers retire at age $j=J$, for all relative productivities $\hat{\psi}$ and markets s.

Free entry. The equilibrium tightnesses $\theta\left(j,\hat{\psi},s\right)$ are pinned down by the firms' free entry condition

$$q\left(\theta_t\left(j,\hat{\psi},s\right)\right)V_t\left(j,\hat{\psi},s\right)=\nu \tag{D.7}$$

if this equation has a solution given $q\left(\cdot\right)\leq 1$, and $\theta_t\left(j,\hat{\psi},s\right)=0$ otherwise.

D.2 Solving for Workers' and Firms' Values

Discretized system. We start by discretizing the differential equations of Appendix D.1. We let $\Delta>0$ denote the time step between consecutive periods t or ages j, and $\hat{\Delta}>0$ denote the distance between grid points for the relative skill $\hat{\psi}$. We let $\Delta\equiv 1/4$ so each period corresponds to a quarter. Our finite difference scheme is implicit in markets s, i.e., we evaluate the continuation values associated to changes in s in period t. As usual, the coefficients in front of these continuation

values are evaluated in $t+\Delta$ to obtain a system that is linear in the current values. On the contrary, our scheme is explicit in the relative skills $\hat{\psi}$, which allows us to parallelize computations over $\hat{\psi}$ to speed up our code. As usual, the scheme is explicit in age j and time t, i.e., we evaluate the continuation values associated to aging or the passing of time in period $t+\Delta$.

After discretization, the value of being employed becomes

$$\begin{aligned}
\rho W_t\left(j,\hat{\psi},s\right) = & \pi\left(s\right) h\left(\hat{\psi},k\left(s\right)\right) A_t^{k(s)} \\
& + \lambda^E \sum_{s'} \mathcal{S}_{t+\Delta}\left(s';j+\Delta,\hat{\psi},s\right)\left[W_t\left(j,\hat{\psi},s'\right) - W_t\left(j,\hat{\psi},s\right)\right] \\
& + \chi\left[W_t\left(j,\hat{\psi},U\right) - W_t\left(j,\hat{\psi},s\right)\right] \\
& + \delta_j\left(k\left(s\right)\right) W_t\left(j,\hat{\psi},s\right) \\
& + \hat{\psi}\hat{\delta}_j\left(k\left(s\right)\right) \frac{W_{t+\Delta}\left(j+\Delta,\hat{\psi}+\tilde{\Delta}\left(k\left(s\right)\right),s\right) - W_{t+\Delta}\left(j+\Delta,\hat{\psi},s\right)}{\tilde{\Delta}\left(k\left(s\right)\right)} \\
& + \frac{W_{t+\Delta}\left(j+\Delta,\hat{\psi},s\right) - W_t\left(j,\hat{\psi},s\right)}{\Delta}
\end{aligned} \tag{D.8}$$

for all markets $s \equiv (k,w)$, where $\Delta > 0$ denotes the time step. In turn, the step $\hat{\Delta}$ for the relative skill $\hat{\psi}$ depends on which sector the worker is employed in. We let $\tilde{\Delta}\left(k\left(s\right)\right) \equiv \hat{\Delta} > 0$ when $k\left(s\right) = C$ so $\hat{\psi}$ increases and $\tilde{\Delta}\left(k\left(s\right)\right) \equiv -\hat{\Delta} < 0$ when $k\left(s\right) = P$ so $\hat{\psi}$ decreases.

In turn, the value of claiming the outside option satisfies

$$\begin{aligned}
\rho W_t\left(j,\hat{\psi},O\right) = & h\left(\hat{\psi},O\right) \\
& + \lambda^E \sum_{s} \mathcal{S}_{t+\Delta}\left(s;j+\Delta,\hat{\psi},O\right)\left[W_t\left(j,\hat{\psi},s\right) - W_t\left(j,\hat{\psi},O\right)\right] \\
& + \chi\left[W_t\left(j,\hat{\psi},U\right) - W_t\left(j,\hat{\psi},O\right)\right] \\
& + \delta_j\left(O\right) W_t\left(j,\hat{\psi},O\right) + \frac{W_{t+\Delta}\left(j+\Delta,\hat{\psi},O\right) - W_t\left(j,\hat{\psi},O\right)}{\Delta}
\end{aligned} \tag{D.9}$$

and the value of being unemployed satisfies

$$\begin{aligned}\rho W_t\left(j,\hat{\psi},U\right) = & \, h\left(\hat{\psi},U\right) \\ & + \lambda^U \sum_s \mathcal{S}_{t+\Delta}\left(s;j+\Delta,\hat{\psi},U\right)\left[W_t\left(j,\hat{\psi},s\right) - W_t\left(j,\hat{\psi},U\right)\right] \\ & + \delta_j\left(U\right) W_t\left(j,\hat{\psi},U\right) + \frac{W_{t+\Delta}\left(j+\Delta,\hat{\psi},U\right) - W_t\left(j,\hat{\psi},U\right)}{\Delta} \end{aligned} \tag{D.10}$$

Finally, the value of a firm becomes

$$\begin{aligned} rV_t\left(j,\hat{\psi},s\right) = & \left[1-\pi\left(s\right)\right] h\left(\hat{\psi},k\left(s\right)\right) A_t^{k(s)} \\ & - \lambda^E \sum_{s'} \mathcal{S}_{t+\Delta}\left(s';j+\Delta,\hat{\psi},s\right) V_t\left(j,\hat{\psi},s\right) \\ & - \chi V_t\left(j,\hat{\psi},s\right) \\ & + \delta_j\left(k\left(s\right)\right) V_t\left(j,\hat{\psi},s\right) \\ & + \hat{\psi}\hat{\delta}_j\left(k\left(s\right)\right) \frac{V_{t+\Delta}\left(j+\Delta,\hat{\psi}+\tilde{\Delta}\left(k\left(s\right)\right),s\right) - V_{t+\Delta}\left(j+\Delta,\hat{\psi},s\right)}{\tilde{\Delta}\left(k\left(s\right)\right)} \\ & + \frac{V_{t+\Delta}\left(j+\Delta,\hat{\psi},s\right) - V_t\left(j,\hat{\psi},s\right)}{\Delta}, \end{aligned} \tag{D.11}$$

for all markets $s \equiv (k,w)$, where $\tilde{\Delta}(k)$ was defined above.

Workers' and firms' values are characterized by the system (D.8)–(D.11) together with the boundary conditions (D.4) and (D.6), the search probabilities (C.20)–(C.22), and the free entry condition (D.7). Note that this system is independent of the distribution of workers' states given paths of A_t^k — this is the block-recursive property of directed search models (Menzio and Shi, 2011).

Numerical solution. We explain below how we solve for workers' and firms' values.

Algorithm. Solve for workers' and firms' values.

Step 0. Let the terminal values for workers $W_T\left(J,\hat{\psi},s\right)$, $W_T\left(J,\hat{\psi},O\right)$ and $W_T\left(J,\hat{\psi},U\right)$ and firms $V_T\left(J,\hat{\psi},s\right)$ satisfy the boundary conditions (D.4) and (D.6) at retirement at age J. Initiate the recursion by setting the time index to $t = T - \Delta$ and the age index to $j = J - \Delta$.

Step 1. Compute the search probabilities $\mathcal{S}_{t+\Delta}\left(s';j+\Delta,\hat{\psi},s\right)$ using (C.20)–(C.22), and the tightnesses $\theta_{t+\Delta}\left(j+\Delta,\hat{\psi},s\right)$ using (D.7).

Step 2. Solve for workers' values:

Solve for the values

$$\mathbf{W}_t\left(j,\hat{\psi}\right)=\left[\mathbf{\Lambda}\left(j\right)-\mathbf{M}_{t+\Delta}\left(j+\Delta,\hat{\psi}\right)\right]^{-1}\mathbf{B}_t\left(j,\hat{\psi}\right), \tag{D.12}$$

where the vector $\mathbf{W}_t\left(j,\hat{\psi}\right)$ contains workers' values, the matrix $\mathbf{M}_{t+\Delta}\left(j+\Delta,\hat{\psi}\right)$ captures workers' transitions across markets and employment statuses which are part of the implicit scheme, the vector $\mathbf{B}_t\left(j,\hat{\psi}\right)$ captures workers' flow earnings and the evolution of the relative skill $\hat{\psi}$, time t and age j, and the matrix $\mathbf{\Lambda}\left(j\right)$ captures discounting and the evolution of the reference skill ψ. Formally, workers' values are

$$\mathbf{W}_t\left(j,\hat{\psi}\right)_{[s,1]}\equiv W_t\left(j,\hat{\psi},s\right), \tag{D.13}$$

the transition matrix is

$$\mathbf{M}_{t+\Delta}\left(j,\hat{\psi}\right)\equiv\begin{bmatrix}\mathbf{M}_{1,1,t+\Delta}\left(j,\hat{\psi}\right) & \mathbf{M}_{1,2,t+\Delta}\left(j,\hat{\psi}\right) & \mathbf{M}_{1,3,t+\Delta}\left(j,\hat{\psi}\right)\\ \mathbf{M}_{2,1,t+\Delta}\left(j,\hat{\psi}\right) & M_{2,2,t+\Delta}\left(j,\hat{\psi}\right) & M_{2,3,t+\Delta}\left(j,\hat{\psi}\right)\\ \mathbf{M}_{3,1,t+\Delta}\left(j,\hat{\psi}\right) & M_{3,2,t+\Delta}\left(j,\hat{\psi}\right) & M_{3,3,t+\Delta}\left(j,\hat{\psi}\right)\end{bmatrix}, \tag{D.14}$$

and the flows and discounting are

$$\mathbf{B}_t\left(j,\hat{\psi}\right)\equiv\begin{bmatrix}\mathbf{B}_{1,t}\left(j,\hat{\psi}\right)\\ B_{2,t}\left(j,\hat{\psi}\right)\\ B_{3,t}\left(j,\hat{\psi}\right)\end{bmatrix}\quad,\quad\mathbf{\Lambda}\left(j\right)\equiv\begin{bmatrix}\mathbf{\Lambda}_1\left(j\right) & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \Lambda_2\left(j\right) & 0\\ \mathbf{0} & 0 & \Lambda_3\left(j\right)\end{bmatrix} \tag{D.15}$$

We start by describing the terms involving employed workers. We define the square matrix

$$\mathbf{M}_{1,1,t+\Delta}\left(j,\hat{\psi}\right)_{[s,s']}\equiv\lambda^E\mathcal{S}_{t+\Delta}\left(s';j+\Delta,\hat{\psi},s\right)+\mathbf{N}_{t+\Delta}\left(j,\hat{\psi}\right)_{[s,s']} \tag{D.16}$$

for each pair of markets $s=(k,w)$ and $s'=(k',w')$, with

$$\mathbf{N}_{t+\Delta}\left(j,\hat{\psi}\right)_{[s,s']}\equiv\begin{cases}-\lambda^E\sum_{\hat{s}}\mathcal{S}_{t+\Delta}\left(\hat{s};j+\Delta,\hat{\psi},s\right)-\chi & \text{if } s'=s\\ 0 & \text{otherwise}\end{cases}, \tag{D.17}$$

and the column vectors

$$\mathbf{M}_{1,2,t+\Delta}\left(j,\hat{\psi}\right)_{[s,1]} = \lambda^{E}\mathcal{S}_{t+\Delta}\left(O;j+\Delta,\hat{\psi},s\right) \tag{D.18}$$

$$\mathbf{M}_{1,3,t+\Delta}\left(j,\hat{\psi}\right)_{[s,1]} = \chi \tag{D.19}$$

We also define the column vectors

$$\begin{aligned}\mathbf{B}_{1,t}\left(j,\hat{\psi}\right)_{[s,1]} \equiv{}& \pi\left(s\right)h\left(\hat{\psi},k\left(s\right)\right)A_{t}^{k(s)} \\ &+ \hat{\psi}\hat{\delta}_{j}\left(k\left(s\right)\right)\frac{W_{t+\Delta}\left(j+\Delta,\hat{\psi}+\tilde{\Delta}\left(k\left(s\right)\right),s\right)-W_{t+\Delta}\left(j+\Delta,\hat{\psi},s\right)}{\tilde{\Delta}\left(k\left(s\right)\right)} \\ &+ \frac{W_{t+\Delta}\left(j+\Delta,\hat{\psi},s\right)}{\Delta},\end{aligned} \tag{D.20}$$

and the diagonal matrix

$$\mathbf{\Lambda}_{1}\left(j\right)_{[s,s']} = \begin{cases}\rho+\frac{1}{\Delta}-\delta_{j}\left(k\left(s\right)\right) & \text{if } s'=s \\ 0 & \text{otherwise}\end{cases} \tag{D.21}$$

We now turn to workers claiming the outside option. We define the row vector

$$\mathbf{M}_{2,1,t+\Delta}\left(j,\hat{\psi}\right)_{[1,s]} \equiv \lambda^{E}\mathcal{S}_{t+\Delta}\left(s;j+\Delta,\hat{\psi},O\right), \tag{D.22}$$

for each market $s=(k,w)$, the scalar

$$\begin{aligned}M_{2,2,t+\Delta}\left(j,\hat{\psi}\right) = {}& \lambda^{E}\mathcal{S}_{t+\Delta}\left(O;j+\Delta,\hat{\psi},O\right) \\ &- \lambda^{E}\sum_{s}\mathcal{S}_{t+\Delta}\left(s;j+\Delta,\hat{\psi},O\right)-\chi\end{aligned} \tag{D.23}$$

and the scalar

$$M_{2,3,t+\Delta}\left(j,\hat{\psi}\right) \equiv \lambda^{E}\mathcal{S}_{t+\Delta}\left(U;j+\Delta,\hat{\psi},O\right)+\chi. \tag{D.24}$$

We also define the scalars

$$B_{2,t}\left(j,\hat{\psi}\right) \equiv h\left(\hat{\psi},O\right)+\frac{W_{t+\Delta}\left(j+\Delta,\hat{\psi},O\right)}{\Delta} \tag{D.25}$$

and

$$\Lambda_2(j) \equiv \rho + \frac{1}{\Delta} - \delta_j(O) \tag{D.26}$$

Finally, we consider unemployed workers. We define the row vector

$$\mathbf{M}_{3,1,t+\Delta}\left(j,\hat{\psi}\right)_{[1,s]} \equiv \lambda^U \mathcal{S}_{t+\Delta}\left(s; j+\Delta, \hat{\psi}, U\right), \tag{D.27}$$

for each market $s = (k, w)$, the scalar

$$M_{3,2,t+\Delta}\left(j,\hat{\psi}\right) = \lambda^U \mathcal{S}_{t+\Delta}\left(O; j+\Delta, \hat{\psi}, U\right), \tag{D.28}$$

and the scalar

$$M_{3,3,t+\Delta}\left(j,\hat{\psi}\right) \equiv -\lambda^U \sum_s p_{t+\Delta}\left(s; j+\Delta, \hat{\psi}, U\right) f\left(\theta_{t+\Delta}\left(j+\Delta, \hat{\psi}, s\right)\right) \tag{D.29}$$

We also define the scalars

$$B_{3,t}\left(j,\hat{\psi}\right) \equiv h\left(\hat{\psi}, U\right) + \frac{W_{t+\Delta}\left(j+\Delta, \hat{\psi}, U\right)}{\Delta} \tag{D.30}$$

and

$$\Lambda_3(j) \equiv \rho + \frac{1}{\Delta} - \delta_j(U). \tag{D.31}$$

Step 3. Solve for the firms' values:

Solve for the values

$$\mathbf{V}_t\left(j,\hat{\psi}\right) = \left[\mathbf{C}(j) - \mathbf{P}_{t+\Delta}\left(j,\hat{\psi}\right)\right]^{-1} \mathbf{D}_t\left(j,\hat{\psi}\right), \tag{D.32}$$

where the vector $\mathbf{V}_t\left(j,\hat{\psi}\right)$ contains firms' values, the matrix $\mathbf{P}_{t+\Delta}\left(j+\Delta,\hat{\psi}\right)$ captures workers' transitions across markets and employment statuses which are part of the implicit scheme, the vector $\mathbf{D}_t\left(j,\hat{\psi}\right)$ contains firms' flow profits and and the evolution of the relative skill $\hat{\psi}$, time t and age j, the matrix $\mathbf{C}(j)$ captures discounting and the evolution of the reference skill ψ. Formally, the firms' values are

$$\mathbf{V}_t\left(j,\hat{\psi}\right)_{[s,1]} \equiv V_t\left(j,\hat{\psi},s\right), \tag{D.33}$$

with the square matrix

$$\mathbf{P}_{t+\Delta}\left(j,\hat{\psi}\right)_{[s,s']} \equiv \begin{cases} -\lambda^E \sum_{\hat{s}} \mathcal{S}_{t+\Delta}\left(\hat{s}; j+\Delta, \hat{\psi}, s\right) - \chi & \text{if } s' = s \\ 0 & \text{otherwise} \end{cases}, \tag{D.34}$$

the column vector

$$\begin{aligned} \mathbf{D}_t\left(j,\hat{\psi}\right)_{[s,1]} = & \left[1-\pi\left(s\right)\right] h\left(\hat{\psi}, k\left(s\right)\right) A_t^{k(s)} \\ & + \hat{\psi}\hat{\delta}_j\left(k\left(s\right)\right) \frac{V_{t+\Delta}\left(j+\Delta, \hat{\psi}+\tilde{\Delta}\left(k\left(s\right)\right), s\right) - V_{t+\Delta}\left(j+\Delta,\hat{\psi},s\right)}{\tilde{\Delta}\left(k\left(s\right)\right)} \\ & + \frac{V_{t+\Delta}\left(j+\Delta,\hat{\psi},s\right)}{\Delta}, \end{aligned} \tag{D.35}$$

and the square matrix

$$C\left(j\right)_{[s,s']} = \begin{cases} r + \frac{1}{\Delta} - \delta_j\left(k\left(s\right)\right) & \text{if } s' = s \\ 0 & \text{otherwise} \end{cases}. \tag{D.36}$$

for all markets $s = (k, w)$ and employment statuses.

Step 4. Set the age $j' = j - \Delta$, repeat Steps 1–3, and continue until $j = 0$.

Step 5. When solving for transitions, set $t' = t - \Delta$, repeat Steps 1–4 in that period, and continue until $t = 0$. This last step is redundant when solving for stationary values along a BGP.

D.3 Distribution of Workers' States

We denote the density of workers' states by $\mu_t\left(j,\hat{\psi},s\right)$. As in Appendix D.1, we do not keep track of a worker's reference skill ψ when iterating over the distribution.

As usual, the law of motion for the distribution is closely related to the recursive formulation of workers' values (Appendix D.2). In fact, the matrix $\mathbf{M}_t\left(j,\omega\right)$ encodes the information required to iterate on the distribution. This is intuitive: this matrix captures the outflows from the current state and the inflows into subsequent ones. The only flows that are not captured by $\mathbf{M}_t\left(j,\omega\right)$ are the changes in age j and the relative productivity in the clean sector $\hat{\psi}$ that are included in $\mathbf{B}_t\left(j,\hat{\psi}\right)$ as part of the explicit scheme.

Abstracting momentarily from changes in $\hat{\psi}$, the law of motion for the distribution is

$$\frac{d}{d\epsilon}\boldsymbol{\mu}_{t+\epsilon}\left(j+\epsilon,\hat{\psi}\right)=\mathbf{M}_t\left(j,\omega\right)'\boldsymbol{\mu}_t\left(j,\hat{\psi}\right) \tag{D.37}$$

for all ages $j\in[0,J-\Delta)$, where $\mathbf{M}_t\left(j,\omega\right)'$ is the matrix transpose of $\mathbf{M}_t\left(j,\omega\right)$ and

$$\begin{gathered}\boldsymbol{\mu}_t\left(j,\hat{\psi}\right)_{[s,1]}\equiv\mu_t\left(j,\hat{\psi},s\right)\quad\boldsymbol{\mu}_t\left(j,\hat{\psi}\right)_{[\text{end-1},1]}\equiv\mu_t\left(j,\hat{\psi},O\right)\\ \boldsymbol{\mu}_t\left(j,\hat{\psi}\right)_{[\text{end},1]}\equiv\mu_t\left(j,\hat{\psi},U\right)\end{gathered} \tag{D.38}$$

Integrating (D.37) over a short time interval Δ,

$$\boldsymbol{\mu}_{t+\Delta}\left(j+\Delta,\hat{\psi}\right)=\boldsymbol{\mu}_t\left(j,\hat{\psi}\right)\exp\left(\mathbf{M}_t\left(j,\hat{\psi}\right)'\Delta\right), \tag{D.39}$$

where $\exp\left(X\right)\equiv\sum_{k=0}^{\infty}1/k!X^k$ is the matrix exponential. We iterate over this law of motion over intervals over duration Δ, starting in period $t=0$. The non-linear formulation (D.39) is computationally more intensive than a simple linear approximation of expression (D.37). However, it allows us to compute workers' values over relative long time intervals, e.g., a quarter $\Delta\equiv1/4$, while still accounting for multiple, high-frequency changes in employment status that occur over shorter periods, e.g., a month. Doing so is important to ensure that our model-simulated moments (such as monthly job flows) are consistent with their empirical counterparts measured in the CPS (Appendix D.4).

As mentioned, the law of motion above does not account for changes in the relative productivity $\hat{\psi}$ captured by our explicit scheme. We account for these transitions by computing

$$\boldsymbol{\mu}_{t+\Delta}\left(j+\Delta,\hat{\psi}\right)=\sum_{\tilde{\psi}}\mathbf{Q}\left(\hat{\psi},\tilde{\psi}\right)\boldsymbol{\mu}_t\left(j,\tilde{\psi}\right)\exp\left(\mathbf{M}_t\left(j,\tilde{\psi}\right)'\Delta\right), \tag{D.40}$$

where the sum is over the initial relative productivities $\tilde{\psi}$. The matrices $\mathbf{Q}\left(\hat{\psi},\tilde{\psi}\right)_{[s,s']}$ are diagonal and their elements $[s,s]$ indicate the probability that $\tilde{\psi}$ switches to $\hat{\psi}$ depending on workers' current employment status s. The relative productivity evolves deterministically by $\hat{\delta}\left(k\left(s\right)\right)\Delta$ after a time step Δ if the worker is employed i.e., $k\left(s\right)=P,C$. When this new value does not fall on a grid point for $\hat{\psi}$, we allocate the corresponding mass to the neighboring points following the non-stochastic simulation method of Young (2010). Accordingly, we set the diagonal

elements to

$$\mathbf{Q}\left(\hat{\psi},\tilde{\psi}\right)_{[s,s]} \equiv \begin{cases} 1-q\left(\hat{\psi}_0\left(\tilde{\psi},s\right),\tilde{\psi},s\right) & \text{if } \hat{\psi}=\hat{\psi}_0\left(\tilde{\psi},s\right) \\ q\left(\hat{\psi}_0\left(\tilde{\psi},s\right),\tilde{\psi},s\right) & \text{if } \hat{\psi}=\hat{\psi}_1\left(\tilde{\psi},s\right), \\ 0 & \text{otherwise} \end{cases} \tag{D.41}$$

where $\hat{\psi}_0\left(\tilde{\psi},s\right)$ is the largest point on the grid for relative productivities that is smaller or equal to $\tilde{\psi}+\hat{\delta}\left(k\left(s\right)\right)\Delta$ and $\hat{\psi}_1\left(\tilde{\psi},s\right)\equiv\hat{\psi}_0\left(\tilde{\psi},s\right)+\hat{\Delta}$ is the smallest point on the grid that is strictly larger than $\tilde{\psi}+\hat{\delta}\left(k\left(s\right)\right)\Delta$, and

$$q\left(\hat{\psi}_0,\tilde{\psi},s\right)\equiv\left(\tilde{\psi}+\hat{\delta}\left(k\left(s\right)\right)\Delta-\hat{\psi}_0\right)/\hat{\Delta} \tag{D.42}$$

allows us to assign masses in inverse proportion to the distance between $\tilde{\psi}+\hat{\delta}\left(k\left(s\right)\right)\Delta$ and the closest grid points. All the off-diagonal elements are zero.

An alternative approach would be to extend the law of motion (D.39) by stacking the vectors $\boldsymbol{\mu}_t\left(j,\hat{\psi}\right)$ for all values of $\hat{\psi}$, and define a larger transition matrix that also accounts for flows between values of $\hat{\psi}$. This approach turns out to be computationally demanding since this new transition matrix is very large and prevents numerical parallelization. The only difference between our approach (D.40) and this alternative one is that we suppose that the state $\hat{\psi}$ evolves only once at the end of the time interval Δ, whereas the alternative approach accounts for potentially multiple changes over this time interval.

Finally, the law of motion (D.40) has a boundary at age $j=J$ when workers retire. The flow of retirees, and hence newborns, is

$$\mu_t^R=\int\int\mu_{t-\Delta}\left(J,d\hat{\psi},ds\right) \tag{D.43}$$

Newborn workers are endowed with a reference skill $\psi=1$ (which we do not need to keep track of) and a relative skill $\hat{\psi}\sim G$. Accordingly, the distribution of workers' states among newborn workers is

$$\mu_t\left(0,d\hat{\psi},s\right)=\begin{cases}\mu_t^R\times G\left(d\hat{\psi}\right) & \text{if } s=U \\ 0 & \text{otherwise}\end{cases} \tag{D.44}$$

since newborns start unemployed.

Stationary distribution and transition. As explained in Section 3, the de-trended economy starts from a stationary distribution where sector C is absent, i.e., $A^C \equiv 0$. We normalize the stationary distribution so it has a unit mass.

D.4 Calibration

We discretize our continuous time model into short periods corresponding to a quarter, i.e., $\Delta \equiv 1/4$. We explain below how we compute the moments that we use in our steady state calibration (Section 5.2).

State space reduction. We think of our calibration period as the year 1995, before the growing sector emerges. The fact that the good produced in sector C is not valued means that $P^c = 0$ so there are no vacancies posted in that sector and no workers supply labor there (Sections 3.1 and 5.1). Therefore, our initial BGP does not depend on the distribution of relative skill $\hat{\psi}$ and, in particular, on its endowment distribution G. Formally, one can guess and verify that the de-trended values satisfy $W_t\left(j, \hat{\psi}, s\right) = W_t\left(j, s\right)$ and $V_t\left(j, \hat{\psi}, s\right) = V_t\left(j, s\right)$ for any relative productivities $\hat{\psi}$ along the initial BGP.[54] Furthermore, this initial BGP does not depend on $\hat{\delta}_j$ itself, only on $\delta_j^+ + \hat{\delta}_j$ which we directly measure in the data (Section 5.2). Without loss of generality, we can therefore normalize $\hat{\psi} = 1$ and set $\hat{\delta} = 0$ for our steady state calibration. This property allows us to discipline internally the seven parameters in Table 5.2 to match steady state moments *before* structurally estimating the distribution G and the rate $\hat{\delta}$.

Of course, different distributions of skill endowments G and skill accumulation rates $\hat{\delta}$ imply different stationary distributions of relative skill $\hat{\psi}$, which will matter for the economy's behavior during the transition. The structural estimation thus involves simulating the initial stationary distribution of $\hat{\psi}$ given G and $\hat{\delta}$ — accounting for the endogenous evolution of $\hat{\psi}$ — and then simulating workers forward over the transition.

Employment flows. We compute the EE, EU and UE probabilities over one month, i.e., $\Delta^m \equiv 1/12$. When doing so, we account for the possibility that workers might

[54] When doing so, we use the fact that: $h\left(\hat{\psi}, s\right) = h\left(s\right)$ does not depend on $\hat{\psi}$ whenever the worker is employed in sector D, claims the outside option, or is unemployed; $\frac{\partial}{\partial \hat{\psi}} W_t\left(j, \hat{\psi}, s\right) = 0$ for all employment states since $W_t\left(j, \hat{\psi}, s\right) = W_t\left(j, s\right)$ under our guess; and $\theta_t\left(j, s\right) = 0$ in all markets s in sector C since $V_t\left(j, s\right) \leq 0$ so that $f\left(\theta_t\left(j, s\right)\right) = 0$.

Table D.1: Internal calibration (all models)

Parameter	Description	Frictionless	+ Outside	+ Gross	+ Search	+ Ladders
Search						
λ^U	Search intens. (U)	—	—	—	9.367	5.910
λ^E	Search intens. (E)	—	—	—	0	0.657
z	Noise (upper tier)	—	—	0.068	0.431	0.245
ζ	Noise (lower tier)	—	—	—	0.174	0.168
Job creation & destruction						
ν	Vacancy post. cost	—	—	—	0.835	0.299
χ	Job destruction	—	—	—	0.144	0.143
Benefits						
b^O	Benefit in O	—	0.82	0.82	0.617	0.587
b^U	Benefit in U	—	—	—	0.333	0.322

experience multiple job transitions within the same month. For instance, a worker who first experiences an EU transition followed by an UE one during the same month will be counted in the EE probability, which ensures that the moment we compute in the model is comparable to the EE probability measured using the CPS.

To compute the EU probability, we use the law of motion (D.39) since we do not account for changes in $\hat{\psi}$ within a given quarter (Appendix D.3). We make two changes compared to this law of motion. First, we initialize it using the distribution of states conditional on employment $\boldsymbol{\mu}_t^E$ instead of the unconditional distribution $\boldsymbol{\mu}_t$. Second, we set the time step to $\Delta^m = 1/12$ instead of $\Delta = 1/4$. We iterate on this new law of motion for all ages j and relative productivities $\hat{\psi}$. The EU probability corresponds to the mass of workers with state $s = U$ after a month. We proceed analogously to compute the UE probability.

In turn, we combine two moments to compute the EE probability over a month: the probability that an employed worker is again employed in a month in *any* market; and the probability that the worker stays in their current job over that period.[55] The EE probability is the difference between the these two quantities. The first moment is computed in a way similar to the one described above. To compute the second moment, we use the same law of motion as before, except with an alternative matrix $\widetilde{\mathbf{M}}_t\left(j, \hat{\psi}\right)$ in which all the *inflows*, i.e., the positive terms, are all zero. This matrix lets workers "flow out" of the system as they move out of their current

[55] A worker who leaves their current job for unemployment and subsequently moves back to the same market (i.e., to a different job) is counted in the EE probability, consistently with the data.

job. The remaining mass of workers after a month corresponds to the probability that a worker stays in their current job.

Labor supply elasticity. We compute the labor supply elasticity across sectors by increasing the value of b^O by 1% permanently and measuring how the numbers of workers who claim the outside option changes in the long-run.

Calibrated parameters. Table D.1 describes the calibrated parameters across the various models that we consider in our decomposition in Section 5.3. In particular, "Costly," "Gross," "Ladders," and "Outside" correspond to the models where we introduce costly vacancy creation, gross flows, job ladders, and the outside option, respectively. The rightmost column corresponds to our full model.

E Details on the Climate Application

E.1 Empirics

Sector definitions and S-curves. To define polluting industries (D), we use data on emissions intensity — measured as the kilograms of CO2-equivalent gas emissions per dollar of output — by 6-digit NAICS industry codes from the Supply Chain Greenhouse Gas Emission Factors published by the Environmental Protection Agency (EPA). An industry is deemed polluting if it is in the top decile of emissions intensities, weighted by employment.[56] These emissions data are missing for some key sectors, such as the energy sector. Some industries are not polluting themselves but produce or sell a polluting output, e.g., gas stations, or are related to fossil fuel extraction and power generation, e.g., fossil fuel electric power generation or pipelines for oil and gas. We handcode these industries as polluting.

To define clean industries (C), we combine two approaches. First, we follow the output-based approach of Bontadini and Vona (2023), who find that 95% of the production of "green goods" is concentrated in just 13 out of 119 4-digit European NACE industries and these industries do not produce polluting goods.[57] We clas-

[56] Employment information comes from the Quarterly Census of Employment and Wages (QCEW).

[57] The list of green goods employed by Bontadini and Vona (2023) is based on the Combined List of Environmental Goods (CLEG) of the OECD and considers goods which are useful for the green transition. For instance, solar panels and wind turbines are classified as "green goods" even if their production is emission-intensive, since the goods themselves are green by nature.

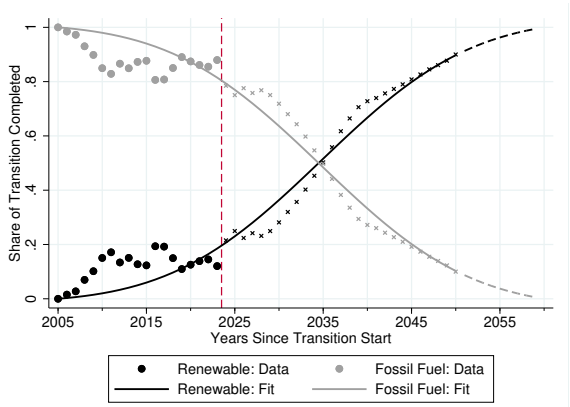
sify these 13 sectors as clean. We supplement this output-based approach with the task-based approach of the US Energy Employment Report (Granholm, 2024). We use data from the 2012 Green Goods and Services (GGS) survey produced by US Bureau of Labor Statistics (BLS) and define a sector as clean if at least 50% of their employees work on green tasks. Finally, we handcode battery manufacturing as green, since this is a crucial input to electrification of transport, and is not included by any of the two classifications above. Industries that are neither clean nor dirty correspond to the outside option (O).

The largest polluting sectors are gasoline stations (840,000), various sectors associated with freight or trucking (1.3 million combined), air travel (447,500) and plastic product manufacturing (307,000). The largest clean sectors are semiconductor manufacturing (a key input to solar panels), other motor vehicle parts manufacturing (e.g. car batteries), and recyclable material wholesalers. Around 11 million jobs remained in the polluting sectors as of 2025, suggesting a substantial remaining scope for job displacement.

To calibrate the shape $\Phi(t)$ of the S-curve for the MRPLs, we use data from the US Energy Information Authority (US EIA), which provides information and projections on energy production from 2005 to 2050. We calculate the share of energy produced by renewable sources in every year. We then normalize the share of the transition completed as of 2005 to be 0 (corresponding to the start of the transition for workers initially attached to polluting sectors), and assume that the projected share in 2050 reaches 90% of its final level since the transition might not be fully complete until 2060 (Section 3.2). This yields the data points in Figure E.1. A normal CDF, truncated between 2005 and 2060, fits the data well and suggests that the transition should accelerate after 2025 and half of it should be completed by 2035. The parameters of the normal CDF that best fit these data have mean corresponding to roughly 2039 and standard deviation 17.2.

To understand how we estimate the paths of A_t^k under the status quo transition, note that job-stayers' wages would exactly track A_t^k absent human capital accumulation and trend growth. Using data from the CPS ORG, we thus residualize 12-month wage growth against age, sex, and education fixed effects. In our model, human capital grows grows at the same rate in both sectors, but differentially over the life cycle, which is captured by age fixed effects. In turn, the sex and education fixed effects capture differential trend growth rate by education and sex which we

Figure E.1: Estimated S-curve of energy generation transmission

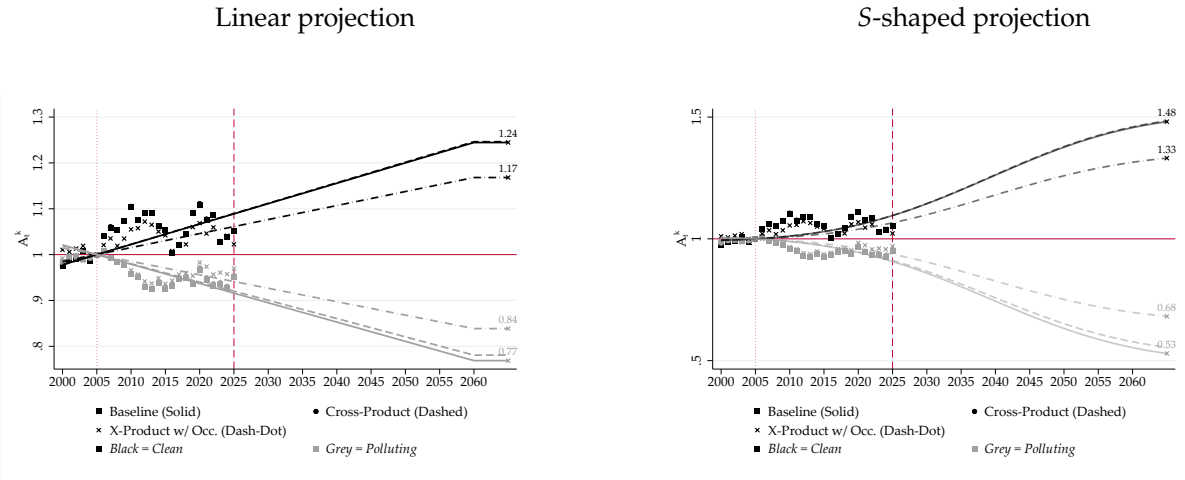


Notes: This figure plots the estimated shape $\Phi(t)$ of the S-curve for the sectoral MRPLs, which is assumed to take the form of a truncated normal CDF. The data comes from the US Energy Information Administration (EIA). Dashed lines indicate projections through 2050.

do not explicitly model. The residualization therefore purges wage growth from both the background trend and life cycle human capital accumulation. We then calculate the average residualized wage growth by sector and year. We construct empirical paths for the MRPLs by cumulating these residualize wage growths. Finally, we we use these empirical MRPLs to obtain our implied path of A_t^k by fitting either a linear (for our baseline) or S-shaped $\Phi(t)$ function.

Figure E.2 plots the empirical series and the fitted ones. The markers correspond to the estimated MRPLs and the curves correspond to our fitted A_t^k. The square markers and the solid curves correspond to our preferred specification, where we residualize wage growth against age, sex and education fixed effects. The circles and dashed curves residualize against the cross-product of age, sex and education fixed effects, rather than including them separately. This alternative specification allows for different education and sex groups to have different life cycle wage growth profiles, and implies very similar paths for A_t^k. Finally, the X markers and dash-dot lines residualize against the cross product of age, sex, education *and* occupation fixed effects. It is not obvious that wage growth should be residualized against occupation effects: the D or C sectors might differ in their occupation mixes, so residualizing against these occupations removes some of the variation that we intend to capture. Nevertheless, this residualization purges the data from potential pre-existing labor demand trends that are unresponsive to the

Figure E.2: Estimated and projected paths of A_t^k (robustness)



Notes: This plot reproduces Figure 6.1 in the main text, residualizing wage growth against different variables. The black lines report the path of A_t^k for the clean sector, while the gray lines represent the dirty sector. The scatter plot points represent the empirical series of MRPLs constructed by cumulating residualized 12-month wage growth among job-stayers. Squares and solid curves represent our baseline set of controls — sex, age and education fixed effects. Circles and dashed curves residualize against the cross product of these controls. X's and dash-dot lines residualize against the cross product of our controls and occupation fixed effects. The left panel projects using a linear projection through 2060, while the right panel uses the S-shaped transition whose shape is chosen to match projections of the share of clean energy sales based on US EIA data.

climate transition. Our preferred specification without occupation fixed effects predicts a slightly milder climate transition. The linear transition implies that A_∞^C falls from 1.24 in the baseline to 1.17 after these controls, and A_∞^D rises from 0.77 to 0.84 with these controls. If the transition is *S*-shaped instead, A_∞^C goes from 1.42 to 1.29 and A_∞^D rises from 0.59 to 0.72. We present sensitivity to these transitions in Appendix F [in progress].

Current Population Survey – Cleaning and targets. The source of many of the moments used in our calibration is the Current Population Survey (CPS), which is a large, nationally-representative survey of U.S. households. The CPS has a rotating panel structure whereby households enter the sample for a four-month rotation, before leaving for 8 months and returning for a subsequent four months. Respondents are asked additional questions about their usual weekly earnings and hourly wage in the final month of each rotation (month 4 and 8 in the sample). All wages are deflated by the CPI-U. We use the IPUMS sample of Flood et al. (2024), restricting attention to those aged 25 to 65. We use the survey to measure employment in

clean and polluting sectors, and to discipline the value of the outside option. We use the provided sample weights throughout out analysis.

To construct our empirical moments, we must merge our sector definitions with the industries in the CPS. The CPS uses Census industry codings, which are generally at a coarser level of aggregation than the 6-digit NAICS codes that we classify as polluting, clean, or other. We thus adopt an iterative approach to assign CPS industry codes to polluting versus clean sectors. We first merge our CPS data with Census industry to NAICS crosswalks provided by the Census, available here. We then merge these coarser CPS-NAICS codings with our finer clean versus polluting 6-digit NAICS classifications at progressively coarser levels beginning with 6-digit NAICS. At each level, we assign clean vs polluting vs other sectors according to the classification of the majority employment in lower-level NAICS industries.

We label a worker as "polluting-attached" in the pre-transition period (1995–2004) if they are employed in the polluting sector for four consecutive months during the first rotation. Among workers employed in the polluting sector in the final month of their first CPS rotation, 87.9% were polluting-attached before 2005.

The CPS does not allow us to track these polluting-attached workers over long periods. Instead, we weight individuals in subsequent years (2005–2025) by the likelihood that they were polluting-attached in 2005 based on their current observables. In effect, we study how the transition affects workers who, based on observable characteristics, would likely have been working in the polluting sector if the transition had not occurred. We use information on workers' education, race and marital status, as well as geographic information which the CPS partially provides down to the county level. The Census Bureau has changed confidentiality rules regarding individuals living in small geographic areas over time. As a result, some counties and metro areas (MSAs) appear in some waves of the CPS but not others. In order to have a consistent probability of being polluting-attached, we require consistent geographic codings over time; we therefore focus on county and MSA codes that are present in every year from 2004 to 2025. This yields 194 counties and 209 MSAs. State information is included for all years.

We start by computing the probability that an individual i observed in year t was polluting-attached in 2005. To do so, we use the pre-transition period (1995–2004) to regress a dummy variable $P_{i,t}^{\text{attached}}$ indicating whether a worker is polluting-attached on a vector of observables $X_{i,t}$ (education $\times$ race $\times$ marital status $\times$ sex

$\times$ geography). For later years (2005-2025), we use the estimated fixed effects in this regression to obtain a predicted probability $\hat{P}_{i,t}^{\text{attached}}$ that each worker was polluting-attached in 2005 based on their current observables $X_{i,t}$. Since some combinations of observables do not exist in all years, we run the regressions $P_{i,t}^{\text{attached}}$ at various geographic levels and use the fixed effects corresponding to the finest geographic level available for each individual i in period t (county $\rightarrow$ MSA $\rightarrow$ state $\rightarrow$ national) to compute $\hat{P}_{i,t}^{\text{attached}}$. The individuals most likely to be polluting-attached given their observables are less-educated men in traditional mining communities (e.g. parts of Texas and West Virginia). About 25% of individuals are assigned a zero likelihood of being polluting-attached and are thus dropped.

We use this information to calculate the share of workers attached to polluting industries in 2005 who subsequently supply labor in clean or polluting industries in every period. First, we compute the number of workers initially attached to polluting industries who now supply labor to clean industries $C_t = \sum_{i \in C} \hat{P}_{i,t}^{\text{attached}}$ for each period $t \geq 2005$. We define $D_t = \sum_{i \in D} \hat{P}_{i,t}^{\text{attached}}$ similarly for polluting industries. We then compute $z_t^{\text{data}} = (C_t - C_{2005}) / D_{2005}$, and we compute the exact same moment in the model for every distribution G that we consider.

To correct for misspecification from overly coarse industry classifications and within-industry shifts in manufacturing, we ascribe a fraction of polluting manufacturing employment D_t in the data to clean industries C_t. For instance, the industry classifications associated with motor vehicle manufacturing do not distinguish between ICEV and EV vehicles. We therefore allocate employment in motor vehicle manufacturing to clean and polluting sectors based on the share of cars sold that run on alternative fuel (hybrid, plug-in hybrid, electric, or biofuel) — which proxies for clean technology adoption.[58] We proceed in the same way for other industries within manufacturing, using the same shares as we use for car manufacturing.[59] We then extract the trend component of C_t, computed as a moving average over 12 quarters before and after a target date, applying a correction to the end of the sample which accounts for the fact that there are fewer observations

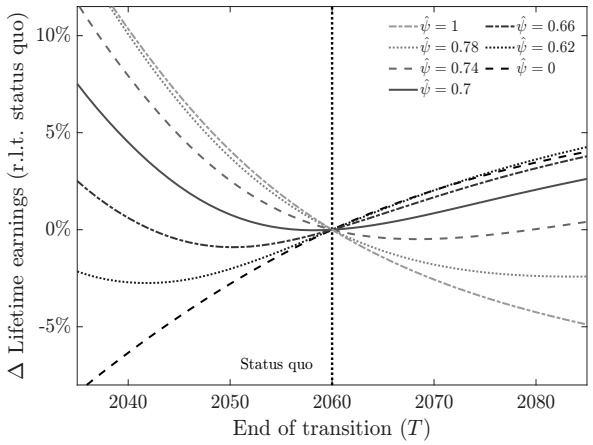
[58] To calculate these shares, we use data from the US Bureau of Transportation on total car sales by year, available from here, and on alternative energy vehicle sales, available here (both accessed June 3, 2025). Total car sales data go through 2021 while alternative-fuel sales go to 2024. We therefore extrapolate total car sales assuming a constant log-linear trend.

[59] We maintain the following four manufacturing sectors as polluting over the full transition: Petroleum products, Petrochemical manufacturing, Oil and gas field machinery and equipment, and Motor vehicle gasoline engine and parts manufacturing.

available. The resulting series for C_t is the one that we use to compute z_t^{data} using the formula above. Figure 6.2 plots z_t^{data} over 2005–2025. Finally, we construct the same series for different cohorts to obtain the $z_{\tau,t}^{\text{data}}$ series that we use in our estimation (6.1).

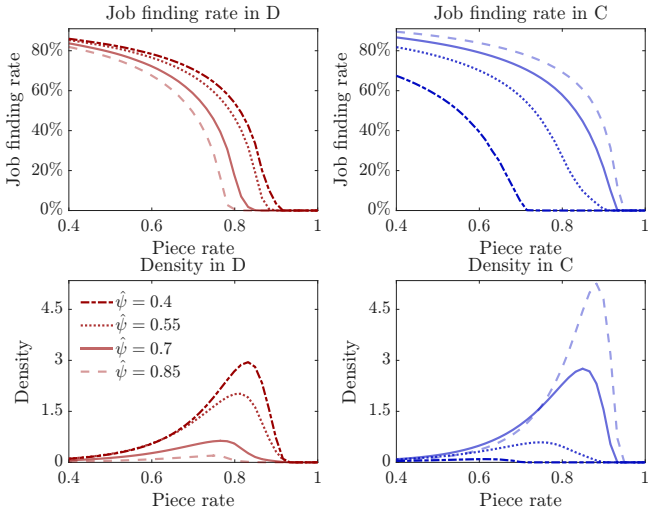
F Additional Quantitative Results

Figure F.1: Counterfactual lifetime earnings changes



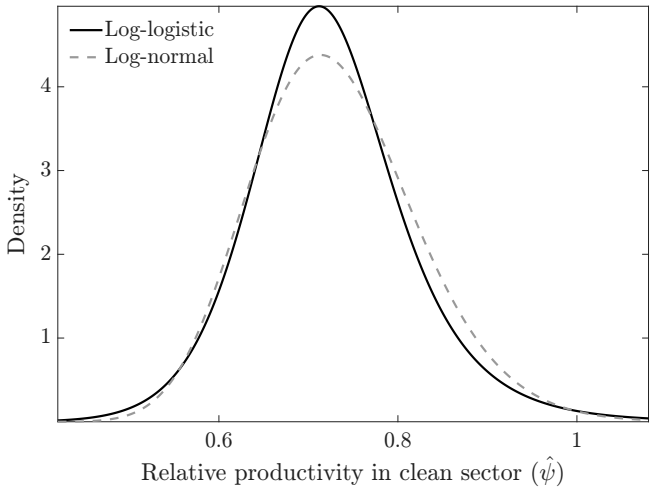
Notes: This figure plots workers' average lifetime earnings changes in 2025 in our full model as a function of the transition horizon T, for various values of relative productivity $\hat{\psi}$. Earnings changes are relative to the status quo transition ending in 2060, and transitions are parameterized as in equation (3.2). All workers have 20 years remaining in their careers and start with the same employment status ($s = U$).

Figure F.2: Job finding rates and the distribution of piece rates



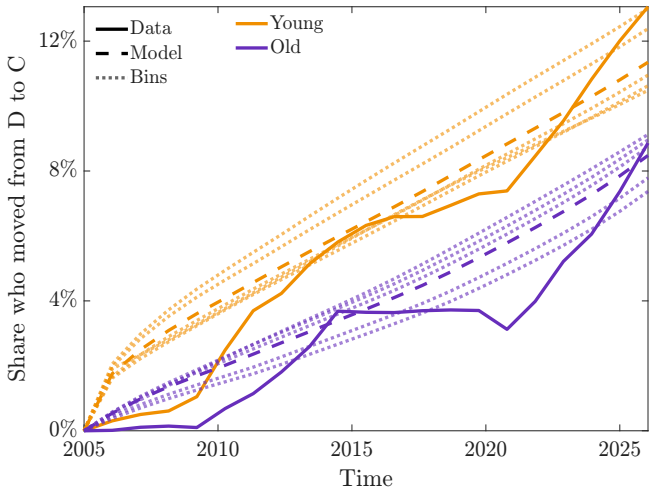
Notes: The first row plots the job finding rate as a function of the piece rate posted by firms, in the year 2040 under the status quo transition, for workers with various relative productivities $\hat{\psi}$. The second row plots the equilibrium distribution of piece rates, for the same period and the same productivities. The first column corresponds to the declining sector, and the second to the growing one.

Figure F.3: Estimated distribution of $\hat{\psi}$ at birth



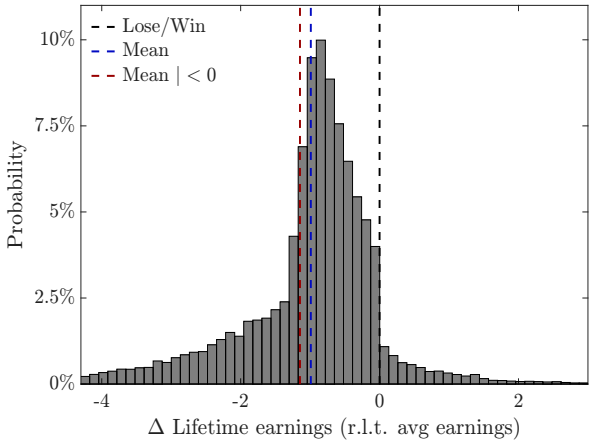
Notes: This figure plots the estimated distribution of $\hat{\psi}$ at birth. The solid curve corresponds to our preferred specification where G is a log-logistic distribution. The dashed curve corresponds to a variant where G is a log-normal distribution.

Figure F.4: Labor reallocation by cohort



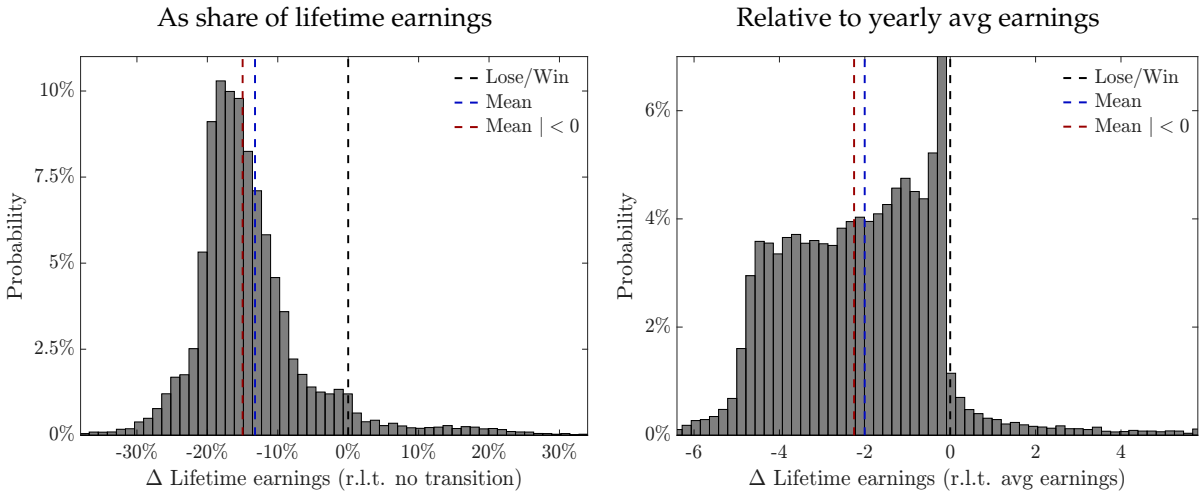
Notes: This figure plots the share of workers initially attached to polluting industries who moved to clean industries, for the cohorts used in our estimation. The orange curves correspond to workers aged 25-30 in 2005, whereas the purple curves correspond to those aged 35-40 in 2005. The solid curve is the data from the CPS (see Appendix E). Workers are weighted by the probability that they are attached to the polluting sector based on observables in the data. The dotted curves correspond to the model-generated reallocations for different values of the skill accumulation rate $\hat{\delta}$ and the associated distributions G of relative skills. The dashed curves corresponds to the estimated value of $\hat{\delta}$.

Figure F.5: Cumulative earnings losses (2005–2025) relative to yearly avg earnings



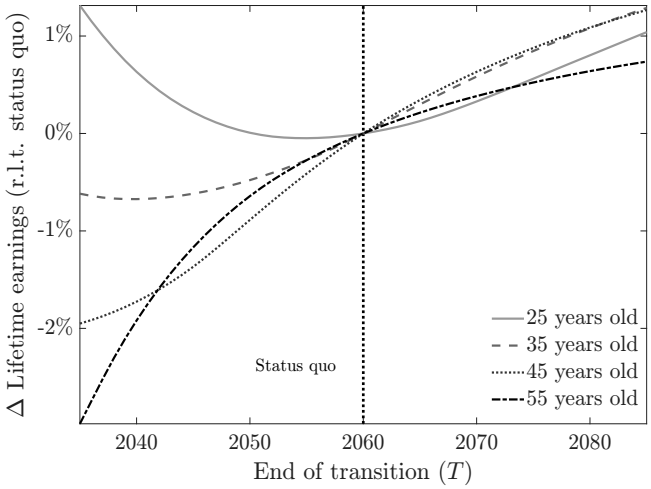
Notes: This figure plots the cross-sectional distribution of cumulative earnings losses for workers present over the period 2005–2025, expressed as a share of average yearly 2005 income.

Figure F.6: Lifetime earnings changes from 2025 onwards



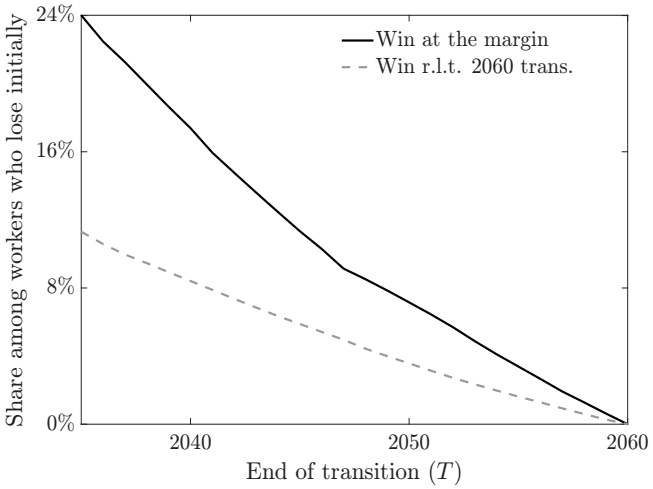
Notes: This figure plots the cross-sectional distribution of lifetime earnings changes in 2025 associated with the status quo transition ending in 2060. The left panel expresses these changes as a share of lifetime earnings in 2025 had there been no transition. The right panel expresses the same changes as a share of share of 2025 average yearly income.

Figure F.7: Counterfactual lifetime earnings changes by age



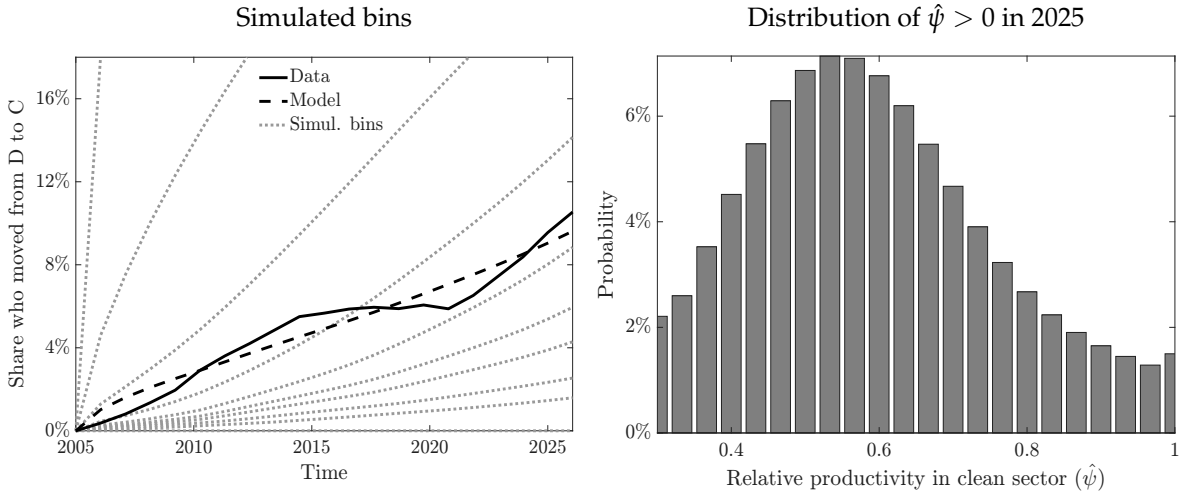
Notes: This figure plots average lifetime earnings changes in 2025 in our full model as a function of the transition horizon, for various ages. All changes are expressed as a share of lifetime earnings under the status quo transition ending in 2060, and the counterfactuals are parameterized as in equation (3.2). We average workers within each age using the distribution that prevails in 2025 in our climate application.

Figure F.8: Share of workers who experience non-monotonicities



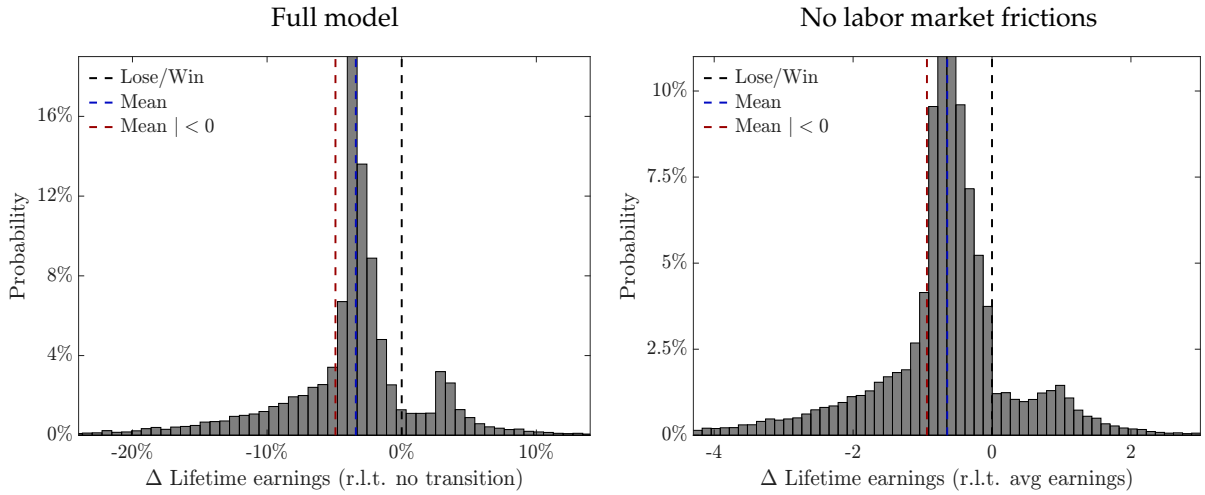
Notes: This figure plots the share of workers who experience non-monotonic changes in lifetime earnings as a function of the transition horizon. This share is expressed relative to workers who lose at first when accelerating the transition, starting from the status quo one ending in 2060. The solid black curve indicates the share of workers who cross the trough of their U-shape at some point as the transition accelerates, i.e., who gain *at the margin*, and the dashed gray curve indicates the share of workers who gain relative to the status quo transition, i.e., who gain *in level*.

Figure F.9: Estimation of the skill distribution (S-shaped transition)



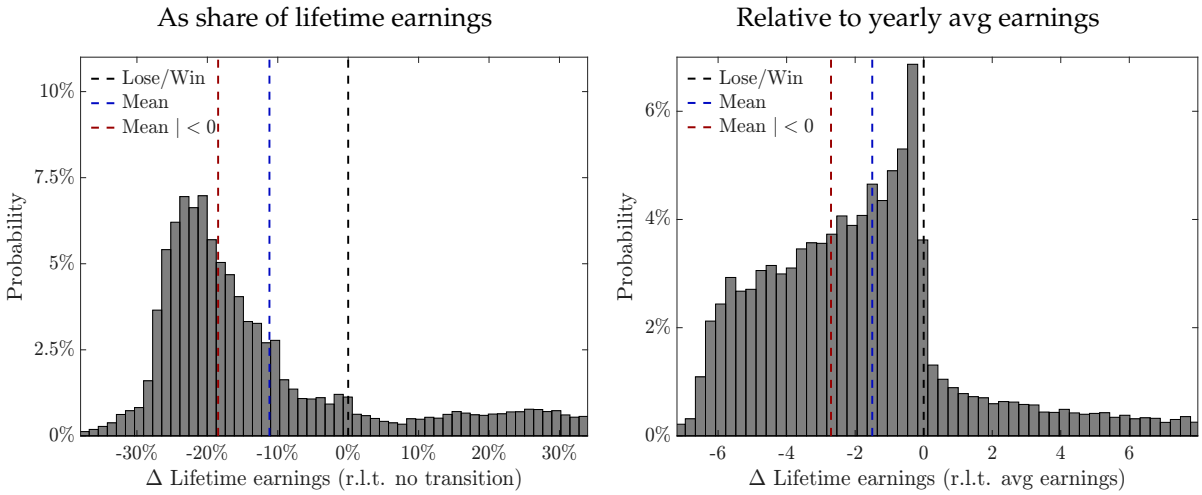
Notes: This plot reproduces Figure 6.2, this time assuming an S-shaped transition instead of a linear one (see Figure 6.1). The left panel plots the share of workers initially attached to polluting industries who moved to clean industries. The solid curve is the data from the CPS (see Appendix E). The dotted curves correspond to simulated bins associated to various values of $\hat{\psi}^{\text{endow}}$ under the estimated $\hat{\delta}$. The dashed curve corresponds to the model-generated reallocation when using the estimated distribution of $\hat{\psi}^{\text{endow}}$. The right panel plots the estimated cross-sectional distribution of relative productivity $\hat{\psi}$ in the population in 2025.

Figure F.10: Cumulative earnings losses over 2005–2025 (S-shaped transition)



Notes: This plot reproduces the left panel of Figure 6.3 and Figure F.5, this time assuming an S-shaped transition instead of a linear one (see Figure 6.1). The left panel plots the cross-sectional distribution of cumulative earnings losses for workers present over the period 2005–2025, relative to cumulative earnings had there been no transition. The right panel expresses the same changes relative to 2005 yearly average income.

Figure F.11: Lifetime earnings changes from 2025 onwards (S-shaped transition)



Notes: This plot reproduces Figure F.6, this time assuming an S-shaped transition instead of a linear one (see Figure 6.1). This figure plots the cross-sectional distribution of lifetime earnings changes in 2025 associated with the status quo transition ending in 2060. The left panel expresses these changes as a share of lifetime earnings in 2025 had there been no transition. The right panel expresses the same changes as a share of share of 2005 average yearly income.

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