Forward Guidance: Opening the Black Box of the New Keynesian Model

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Abstract

The forward-guidance puzzle refers to the implausibly large effects of anticipated future interest rate changes on current output and inflation, as predicted by standard New Keynesian models. We analyze the underlying nonlinear model both theoretically and numerically, tracking the full distribution of prices across firms. We show that large output expansions arise only under extreme and economically implausible circumstances: firms unable to reset prices must sell below marginal cost while meeting unbounded demand, thus accumulating arbitrarily large losses. The model also exhibits asymmetric effects across policy directions, with important implications for the economy at the zero lower bound. When non-reoptimizing firms can at least set prices equal to marginal cost, output and inflation remain bounded, though not all forward-guidance announcements are feasible in equilibrium. Our results identify a neglected microeconomic assumption as the root cause of the puzzle and clarify the limits of New Keynesian models in analyzing large or persistent monetary shocks.

1 Introduction

The New Keynesian (NK) model has been highly successful in capturing the qualitative and quantitative responses of key macroeconomic variables to monetary policy shocks under normal business-cycle conditions (see, among others, Christiano, Eichenbaum, and Evans, 2005, or Smets and Wouters, 2007). Following the Great Financial Crisis, however, nominal interest rates in many advanced economies reached the zero lower bound, prompting central banks to rely on unconventional policy tools. One such tool was forward guidance: explicit announcements about the future path of policy rates, typically committing to keep rates at exceptionally low levels for an extended period.

Several authors have shown that standard New Keynesian models predict implausibly large effects of forward-guidance announcements on current output and inflation. In particular, Del Negro, Giannoni, and Patterson (2013) and Carlstrom, Fuerst, and Paustian (2015), building on the early work of Eggertsson and Woodford (2003), demonstrate that the impact of forward guidance grows exponentially with the length of the announced commitment. For standard parameter values, the model can generate double-digit output gaps following commitments that last only a few quarters.[1]

These predictions stand in sharp contrast to the empirical evidence. During the period in which forward guidance was most actively used (roughly 2008–2015), its observed macroeconomic effects were modest (see e.g. Campbell, Fisher, Justiniano and Melosi, 2017). More broadly, estimates of the U.S. output gap since 1950 indicate that output has rarely deviated from potential by more than a few percentage points (see Figure 1). The extreme booms implied by the New Keynesian model therefore raise serious questions about its suitability for analyzing forward guidance and, more generally, large or persistent monetary interventions.

To understand how such outcomes arise, this paper moves beyond the standard linearized representation of the model and studies the underlying nonlinear economy. While the linearized three-equation system is analytically convenient and almost universally employed, it obscures the microeconomic mechanisms that generate extreme responses. We therefore analyze the behavior of firms in monopolistic competition subject to Calvo-style price rigidity, explicitly keeping track of the entire distribution of prices in the economy.

[1] Carlstrom, Fuerst, and Paustian (2015) demonstrate that the issue arises across a broad range of models featuring nominal rigidities, though it is particularly pronounced in frameworks with inflation indexation and comparatively less severe in sticky information models (e.g., Mankiw and Reis, 2002).

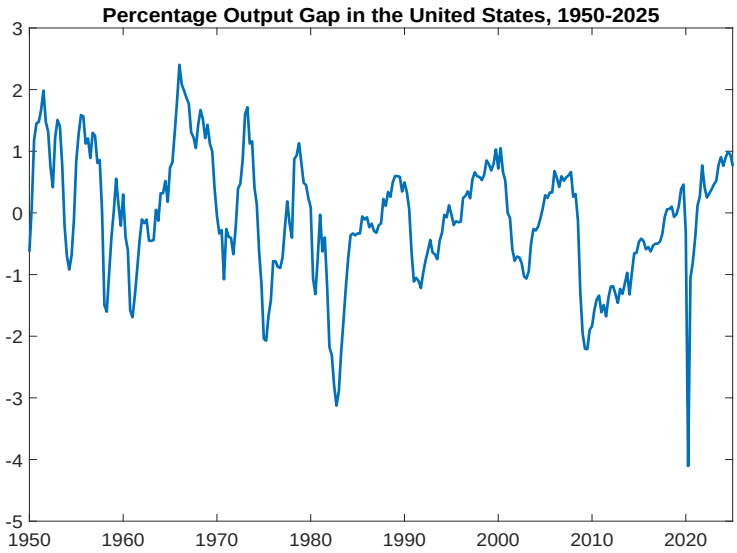


Figure 1: Percentage output gap in the United States, 1950-2025. Output gap is defined as the logarithm of the ratio between realized and potential GDP. Potential GDP is estimated by the Congressional Budget Office and retrieved from FRED database.

Our analysis reveals that the forward-guidance puzzle is driven by a rarely discussed but crucial assumption: firms that are unable to reoptimize prices are nevertheless required to satisfy all demand at their preset price, even when that price falls below marginal cost.

Firms set the price of their product at a positive markup to marginal cost when they can. With inflation, the prices that cannot be reoptimized fall below the optimal level, i.e. the markup shrinks. If inflation is not too large, the markup is compressed but remains positive, and the economy moves closer to the perfect-competition benchmark, generating a modest expansion. When inflation is large or persistent, however, some preset prices may fall below marginal cost. Demand for their goods explodes, and the model sustains large output expansions only by implicitly forcing these firms to incur arbitrarily large losses.

We establish several theoretical results that clarify this mechanism. First, we show that in the nonlinear model inflation is bounded for any strictly positive degree of price rigidity, in stark contrast to the unbounded inflation predicted by the linearized system. Second, we prove that output can exceed the perfect-competition benchmark only if the aggregate price level falls below marginal cost, implying negative aggregate profits. Third, we demon-

strate that, as the fraction of non-reoptimizing firms shrinks, positive output gaps can be sustained only if the prices of a vanishingly small set of firms collapse toward zero relative to marginal cost. These results explain both the magnitude of the forward-guidance effects and the counterintuitive finding, previously documented by Werning (2012) and Cochrane (2017), that the puzzle intensifies as prices become more flexible.[2]

The nonlinear framework also uncovers an asymmetry that is absent from the linearized model, and that, to our knowledge, has not been documented before. While expansionary forward guidance can generate extreme booms, sufficiently aggressive or prolonged contractionary policies may fail to admit an equilibrium altogether, as no price reduction is sufficient to induce a contraction in output of the required magnitude. This non-existence result also applies in environments in which the natural rate is negative and the policy rate is constrained at the zero lower bound, so that simply maintaining the nominal rate at zero can be inconsistent with equilibrium. In this sense, our analysis provides a new perspective on equilibrium feasibility in New Keynesian models at the zero lower bound.[3]

Finally, we consider a minimal modification of the model in which non-reoptimizing firms are allowed to adjust prices up to marginal cost. This modification leaves local dynamics unchanged in normal times but rules out equilibria that rely on firms selling below cost. In the modified model, output and inflation remain bounded, and, as prices become more flexible, output gaps shrink, tending to zero in the flexible-price limit. However, not all forward-guidance announcements are feasible in equilibrium: a too aggressive or too prolonged expansionary policy fails to result in an equilibrium.

Literature Review

Mckay, Nakamura and Steinsson (2016) focus on a slightly different aspect of the forward-guidance puzzle: the effect of an anticipated rate change of a given size and duration is increasing in the horizon of its implementation. They show that such effect is dampened in incomplete markets, in particular when agents are subject to borrowing constraints. Hagedorn, Luo, Manovskii and Mitman (2019) similarly find that the effect of forward guidance is mild

[2] In slightly different contexts, the "paradox of flexibility", i.e. the fact that greater price flexibility increases the real effects of shocks in the New Keynesian model, has been analyzed by Eggertsson and Krugman (2012), Bhattarai, Eggertsson, and Schoenle (2018) and Eggertsson and Garga (2019).

[3] Ascari and Mavroeidis (2022) and Ascari, Mavroeidis and McClung (2023) study different environments in which an equilibrium fails to exist at the ZLB, in the context of a log-linearized New Keynesian model. However, all these environments involve shocks of stochastic duration. In contrast, we find that, in the non-linear model, an equilibrium may fail to exist even in an economy with perfect foresight.

in incomplete markets.

Another strand of the literature focuses on information imperfections: Campbell, Ferroni, Fisher and Melosi (2019) show that the effect of forward guidance is dampened if we take into account that central bank communication is imperfect and agents' expectations do not fully align with central bank's announcements; Angeletos and Lian (2018) find that absence of common knowledge about future policy and fundamentals make forward guidance less powerful. Eggertsson and Schule (2024) contend that the puzzle stems from an implausibly radical policy, in which the central bank would stick to its commitment about future rates regardless of the changes in the economic environment.

The remainder of the paper is organized as follows. Section 2 presents the basic New Keynesian model. Section 3 reviews the forward-guidance experiment in the linearized model. Section 4 analyzes key properties of the nonlinear economy. Section 5 studies the nonlinear response to forward guidance numerically. Section 6 introduces and evaluates a modified model. Section 7 concludes.

2 The Basic New Keynesian Model (BNK)

This section presents the baseline New Keynesian model that serves as the foundation for the analysis. The environment is standard, and the presentation follows closely Gali (2015). Since we focus on the forward guidance experiment, we choose to abstract from productivity or monetary policy shocks. There is perfect foresight after the initial monetary policy announcement.

2.1 Model Setup

2.1.1 Households

Households consume a continuum of goods of measure 1 labeled by i, that are imperfect substitutes. They derive utility from consumption and disutility from labor according to the separable utility function

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} \quad \sigma > 0,\ \phi > 0 \tag{1}$$

where N_t is the work supplied and C_t is a consumption index given by

$$C_t = \left(\int_0^1 C_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} \quad \epsilon > 1 \tag{2}$$

where $C_t(i)$ is the consumption of good i.

The only saving instrument is a 1-period risk-free bond. One unit of the asset B_t can be purchased at $t-1$ at price Q_t and pays one unit of "money" at time t.[4] Households' budget constraint is

$$\int_0^1 P_t(i)C_t(i)di + Q_t B_t = B_{t-1} + W_t N_t \tag{3}$$

where $P_t(i)$ is the price of good i and W_t is the economy-wide wage.

2.1.2 Firms and Price Setting

Each good i is produced by a single firm, similarly labeled by i, with the production function[5]

$$Y_t(i) = N_t(i) \tag{4}$$

where $N_t(i)$ is the amount of work employed by firm i at time t. Production function (4) implies that the marginal cost is equal to the wage. Notice that aggregate labor N_t is simply the sum of the labor for each firm.

$$N_t = \int_0^1 N_t(i)di \tag{5}$$

At any given time t, with probability θ firm i is constrained to keep the previous-period price for its product; with probability $1-\theta$ it is instead a reoptimizer. In the latter case, the firm chooses its price P_t^* to maximize the present value of all the profits it will make while the price remains effective, i.e. until the next time it can reoptimize. For example, with probability θ^k the price P_t^* is still effective at time $t+k$, in which case $t+k$-profits are given by

$$\Pi_{t+k} = P_t^* Y_{t+k;t} - W_{t+k} N_{t+k;t}$$

where $Y_{t+k;t}$ and $N_{t+k;t}$ are the demands for goods and labor at time $t+k$, respectively, for firms whose price has remained fixed at P_t^*. Firms are owned by households, hence profits are discounted at the households' (nominal) stochastic discount factor $D_{t,t+k} = \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+k}}$. The firm's problem is then

$$max_{P_t^*} \sum_{k=0}^{\infty} \theta^k \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+k}} (P_t^* Y_{t+k;t} - W_{t+k} N_{t+k;t}) \tag{6}$$

[4] In a cashless world, "money" is just a unit of account: one unit of money entitles the owner to $1/P$ units of a good with price P.

[5] In a slightly more general version of the basic model the production function could be decreasing-returns-to-scale: $Y_t(i) = zN_t(i)^{1-\alpha}$. We focus on the simplest case $\alpha = 0$. We also normalize the productivity factor z to 1, as the experiments we consider abstract from productivity shocks.

2.1.3 Central Bank

The central bank sets the nominal interest rate $i_t \equiv -log(Q_t)$ at every t. In normal times it uses the Taylor rule:

$$i_t - \rho = \phi_\pi \pi_t + \phi_x x_t \tag{7}$$

where $\pi_t \equiv log\left(\frac{P_t}{P_{t-1}}\right)$ is the inflation rate; $x_t \equiv log\left(\frac{Y_t}{Y_{SS}}\right)$ is the *output gap*, i.e. the output log-deviation from its steady-state level Y_{SS}; $\rho \equiv -log(\beta)$ is the *natural interest rate*.

Exceptionally, at $t = 1$, the central bank announces the (exogenous) rate i_t, $t = 1, ..., N$, it commits to set over these N periods, regardless of the value of the other economic variables (inflation, output, or other).

2.2 Model Equations

The derivation of all model equations is standard, see Gali (2015).

2.2.1 Households' first-order conditions

In addition to the Euler equation and intertemporal labor/consumption trade-off

$$C_t^{-\sigma} = \beta(1 + i_t)\frac{P_t}{P_{t+1}}C_{t+1}^{-\sigma} \tag{8}$$

$$\frac{W_t}{P_t} = C_t^\sigma N_t^\phi \tag{9}$$

where P_t is a price index

$$P_t = \left(\int_0^1 P_t(i)^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}} \tag{10}$$

we have the demand for each good i

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t \tag{11}$$

2.2.2 Firms' first-order conditions

Using the closed-economy resource constraint $C_t = Y_t$, the production function (4) and the demand function (11), it is straightforward to find that the optimal price P_t^*, i.e. the price that maximizes the profit function (6), is

$$P_t^* = \frac{\epsilon}{\epsilon - 1} \sum_{k=0}^{\infty} w_k W_{t+k} \tag{12}$$

$$\text{with} \quad w_k = \frac{\theta^k \beta^k P_{t+k}^{\epsilon-1} C_{t+k}^{1-\sigma}}{\sum_{k=0}^{\infty} \theta^k \beta^k P_{t+k}^{\epsilon-1} C_{t+k}^{1-\sigma}} \tag{13}$$

Given the production function (4) and the demand function (11), we can also rewrite the labor supply (5), the wage (9) and thus the optimal price (12) as

$$N_t = C_t \mathcal{O}_t \tag{14}$$

$$W_t = P_t C_t^{\sigma+\phi} \mathcal{O}_t^{\phi} \tag{15}$$

$$P_t^* = \frac{\epsilon}{\epsilon - 1} \frac{\theta^k \beta^k P_{t+k}^{\epsilon} C_{t+k}^{1+\phi} \mathcal{O}_t^{\phi}}{\sum_{k=0}^{\infty} \theta^k \beta^k P_{t+k}^{\epsilon-1} C_{t+k}^{1-\sigma}} \tag{16}$$

where

$$\mathcal{O}_t \equiv P_t^{\epsilon} \int_0^1 P_t(i)^{-\epsilon} di$$

is the **price-dispersion index**, a key variable in the model, that is equal to 1 if $P(i) = P \ \forall i$. The derivation of (12)-(16) is in Appendix A.

The two fundamental equations of BNK are the Euler equation (8) and the price-setting equation (16).

2.3 Equilibrium

We assume that the system is at the steady state at $t = 0$, with $P_0(i) = P_0 \ \forall i$, zero inflation and

$$C_0 = C_{SS} = \left(\frac{\epsilon - 1}{\epsilon} \right)^{\frac{1}{\sigma+\phi}}$$

Given the sequence for the optimal price P_t^* for $t \geq 1$, we can reconstruct the whole distribution of prices in the economy at all times. The set of all prices at $t \geq 1$ is

$$\{P_0, P_1^*, P_2^*, ...P_t^*\}$$

with shares

$$\{\theta^t, (1-\theta)\theta^{t-1}, (1-\theta)\theta^{t-2}, ..., (1-\theta)\}$$

The price level P_t and the price-dispersion $\mathcal{O}_t$ are then obtained as

$$P_t = \left(\theta^t P_0^{1-\epsilon} + (1-\theta) \Sigma_{k=1}^{t} \theta^{t-k} (P_k^*)^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}} \tag{17}$$

$$\mathcal{O}_t = P_t^{\epsilon} \left(\theta^t P_0^{-\epsilon} + (1-\theta) \Sigma_{k=1}^{t} \theta^{t-k} (P_k^*)^{-\epsilon} \right) \tag{18}$$

An equilibrium is a sequence $\{P_t^*, P_t, \mathcal{O}_t, i_t, C_t\}_{t \geq 1}$ solving:

- The Euler equation (8) and the optimal price setting (16) for $t \geq 1$;
- The Taylor rule (7) for $t > N$;

with P_t and $\mathcal{O}_t$ given by (17)-(18) $\forall t$, and with the exogenous nominal rate i_t announced by the central bank for $t = 1, ..N$.

2.4 The flexible-price case

In the flexible-price case ($\theta = 0$), (12) becomes

$$P_t^* = \frac{\epsilon}{\epsilon - 1} W_t$$

and since all the firms reoptimize and choose this price, we have $P_t(i) = P_t^* \; \forall i$ hence $P_t = P_t^*$ and $\mathcal{O}_t = 1$. Inserting this in (32) we obtain that flexible-price consumption is equal to the steady-state value of consumption:

$$C_{FP} = C_{SS} = \left(\frac{\epsilon - 1}{\epsilon} \right)^{\frac{1}{\sigma + \phi}} \tag{19}$$

Finally, in order for the Euler equation to be satisfied, we must have

$$\frac{P_{t+1}}{P_t} = \frac{1 + i_t}{\beta} \simeq 1 + i_t - \rho \tag{20}$$

The perfect-competition case is found in the limit $\epsilon \to \infty$, in which goods are perfect substitutes. In this case consumption is $C_{PC} = 1$.

2.5 BNKL: the log-linearized version of BNK

The New Keynesian model is almost universally used in its log-linearized form, in our specification reducing to the equations for inflation π_t and the *output gap* $x_t \equiv log\left(\frac{C_t}{C_{SS}}\right)$:

$$\pi_t = \kappa x_t + \beta \pi_{t+1} \tag{21}$$

$$i_t - \rho = \sigma(x_{t+1} - x_t) + \pi_{t+1} \tag{22}$$

with $\kappa = (1 - \theta)\frac{1 - \beta\theta}{\theta}(\sigma + \phi)$, for the period in which the rate i_t is set exogenously, plus the Taylor rule (7) afterwards. (21) is the New Keynesian Phillips curve (NKPC), resulting from the linearization of the price-setting equation. (22) is the linearized Euler equation. The derivation of (21)-(22) is standard (see e.g. Gali (2015)). The equilibrium of the linearized model consists in a sequence $\{\pi_t, x_t, i_t\}_{t \geq 1}$ solving (21)-(22) and the Taylor rule for $t > N$.

3 The forward-guidance experiment in BNKL

This section reviews the forward-guidance experiment in the standard log-linearized New Keynesian model (BNKL). The objective is not to introduce new results, but to establish a benchmark and clarify the source of the forward-guidance puzzle in the canonical framework. The analysis closely follows Del Negro, Giannoni, and Patterson (2013) and Carlstrom, Fuerst, and Paustian (2015).

The baseline policy we focus on is what we call "E-Policy" (expansionary), in which the quarterly nominal rate is set 25 bps below the natural rate for N periods: $\{i_t = \rho - 0.25\%, \}_{t=1,...N}$. This implies that the annualized interest rate is 1% below its steady-state value. At quarter $N+1$ the Taylor rule (7) is restored. We will also consider the specular "C-Policy" (contractionary), where i_t is set 25 bps above the natural rate for the whole duration of the experiment. Since we focus on the workings of the model, we abstract from the reasons behind the decision of the central bank to conduct this experiment.

Solving the linear model

It is well known that in BNKL, the economy returns to the steady state as soon as the Taylor rule is restored, provided that the Taylor rule parameters are such that $\kappa(\phi_\pi - 1) + (1-\beta)\phi_x > 0$ (see Gali, 2015). Assuming that the latter condition holds, the system starts at the steady state at $t = 0$ and returns to the steady state $t = N + 1$. We can then use (21) and (22) and obtain x_t, π_t for $t = 1, ...N$ by backward induction from the boundary condition $x_{N+1} = \pi_{N+1} = 0$. A solution exists for every choice of the exogenous path $\{i_t\}_{t=1,...N}$.

E-Policy equilibrium

It is easy to get an intuition about what the equilibrium is going to look like for the E-Policy $\{i_t = \rho - 0.025\%\}_{t=1,...,N}$. The fact that $\pi_{N+1} = x_{N+1} = 0$ implies that the real rate at $t = N$, $i_N - \pi_{N+1}$, is forced to be 25 bps below the natural rate ρ (or, equivalently, 1% below ρ in annual terms). In order to satisfy the Euler equation (22), we then need a positive output gap, $x_N = \frac{0.0025}{\sigma}$. But (21) tells us that, to have positive output gap at time N, we need positive time-N inflation, $\pi_t = \kappa x_t$. Going backward to time $N-1$, the time-N results for inflation tell us that the time-$(N-1)$ real rate is *even lower* than the time-N real rate. To satisfy the Euler equation, we need x_{N-1}

to be higher than x_N, which forces π_{N-1} to be higher than π_N. Proceeding backward, we obtain the explosive behavior for output gap and inflation that has been noted in previous literature.

To get a quantitative sense of the output gap and inflation path predicted by the model for this experiment, we choose standard values of the parameters, in line with those of Gali (2015). Taking one period to be a quarter, we set $\beta = 0.99$ – so that the annual natural rate is 4% – and we set the utility function parameters $\phi = 1$, $\sigma = 1$. For the production sector, we set $\epsilon = 6$ and $\theta = 2/3$. The latter value for the price-stickiness parameter θ implies that firms reset prices on average every three quarters (see e.g. Galí, Gertler, and López-Salido (2001) and Sbordone (2002)).

The left panel of Figure 2 shows the path of the economy for a forward guidance period of 8 quarters. While output gap and inflation are modest toward the end of the period (for example, in the last two quarters before the end of the forward guidance period, output gap is 59 bps and 29 bps and inflation is 1.13% and 34 bps, respectively), the values in the first period are enormous: 20% for output gap and 61% for inflation (in annual terms).

C-Policy Equilibrium

The equilibrium for the C-Policy experiment is shown in the right panel of Figure 2. It is easy to see that, if $\{[x_t, \pi_t]\}_{t=1,...,N}$ is the solution for a policy $\{i_t\}_{t=1,...,N}$, the solution for the specular policy $\{-i_t\}_{t=1,...,N}$ is just given by $\{[-x_t, -\pi_t]\}_{t=1,...,N}$. Hence, BNKL predicts an equally explosive deflationary and recessive behavior for the solution of the C-Policy experiment.

Sensitivity to Price Rigidity

Another unrealistic feature of the solution has been pointed out by Werning (2012) and Cochrane (2017): for a given length of the forward guidance period, the predicted inflation and output gap is larger the lower is θ, i.e. the less sticky are prices. This means that the closer we get to the flexible-price case, the farther the solution departs from the flexible-price solution. This is shown in Figure 3: the left (right) panel shows the initial-period output and inflation, as a function of θ, in response to a policy lasting 3 periods (4 periods). For $\theta = 0.1$, i.e. in case 90% of the firms are reoptimizers in any period, the initial output gap is 80% if the policy lasts 3 periods and 1500% if it lasts 4 periods! We will find an explanation of this aberrant behavior of the model in the next sections, by analyzing the nonlinear model.

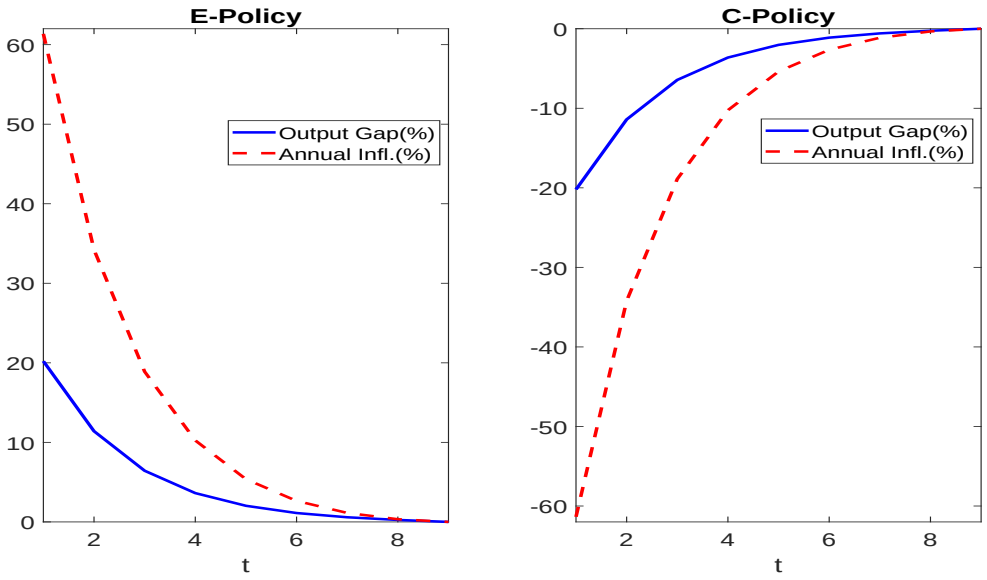


Figure 2: Response of the economy to the E-Policy (left panel) and C-Policy (right panel) experiment lasting 8 periods in the linear model.

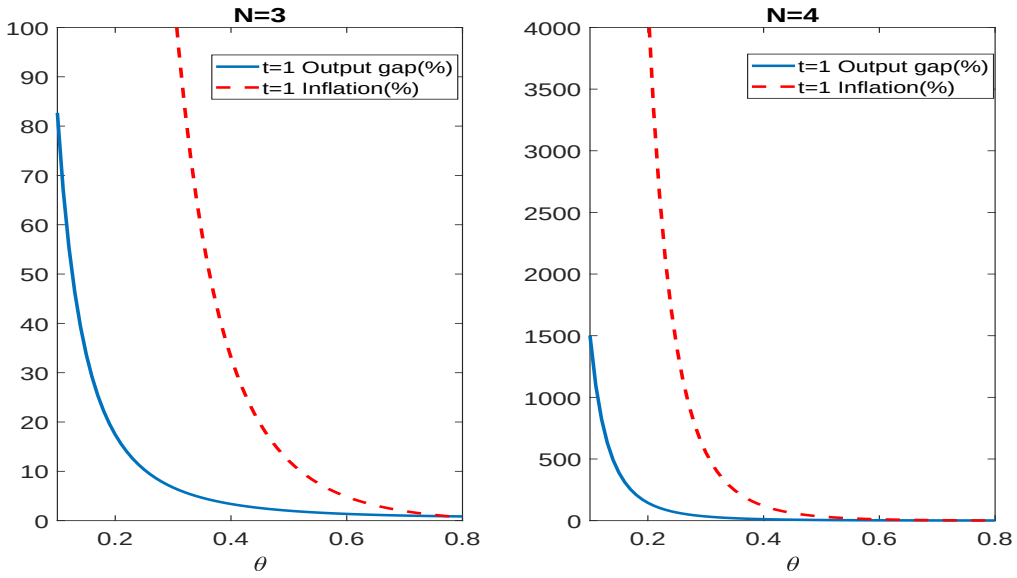


Figure 3: Initial-period output gap and inflation response to an E-Policy experiment lasting 3 periods (left panel) and 4 periods (right panel), as a function of θ.

4 Properties of BNK

This section studies the key properties of the nonlinear model (BNK) and contrasts them with those of its log-linearized counterpart. While previous work has shown that the forward-guidance puzzle persists quantitatively in the nonlinear model (e.g., Carlstrom, Fuerst, and Paustian, 2015), we show that the nonlinear framework delivers qualitatively different insights once the full distribution of prices and the resulting price dispersion are taken into account.

Our analysis highlights three central features. First, inflation is bounded in the nonlinear model for any strictly positive degree of price rigidity, in sharp contrast to the linearized model. Second, output can exceed the perfect-competition benchmark only if the aggregate price level falls below marginal cost, implying negative aggregate profits. Third, as price rigidity vanishes, negative output gaps are not feasible; positive output gaps, on the other hand, can be sustained only through increasingly extreme relative-price distortions, with a vanishingly small set of firms selling at prices arbitrarily close to zero.[6] All detailed proofs and calculations are in Appendix A.

4.1 State Variables and Price Dispersion

Lemma 1: The price-dispersion index $\mathcal{O}_{t-1}$ is the only state variable in the nonlinear New Keynesian model.

Preset prices affect the time-t model equations through their contribution to the variables P_t and $\mathcal{O}_t$. As shown in Appendix A, we can rewrite P_t and $\mathcal{O}_t$ as

$$P_t = (\theta P_{t-1}^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon})^{\frac{1}{1-\epsilon}} \tag{23}$$

$$\mathcal{O}_t = P_t^{\epsilon}(\theta P_{t-1}^{-\epsilon}\mathcal{O}_{t-1} + (1-\theta)(P_t^*)^{-\epsilon}) \tag{24}$$

Since P_{t-1} can be normalized without loss of generality, the entire effect of preset prices $P_{t-1}(i)$ is summarized by the lagged dispersion variable $\mathcal{O}_{t-1}$. A direct implication is that the linearized model, which approximates the economy around a zero-inflation steady state where $\mathcal{O}_t = 1$ to first order, features no state variable.

Lemma 2: For all t, the price-dispersion variable satisfies $\mathcal{O}_t \geq 1$.

[6] The result that inflation is bounded in New Keynesian models has also been obtained by Christiano, Eichenbaum and Johannsen (2024). The other results have not been found before, to our knowledge.

This result follows directly from Jensen's inequality.

The results in Lemma 1 and 2 are well known in the literature on the nonlinear New Keynesian model (see e.g. Christiano, Eichenbaum and Johannsen, 2018); we include them since they serve as building blocks for the analysis to come.

4.2 Inflation and Relative Prices

We begin by characterizing the behavior of inflation. In the nonlinear model, we show that inflation is bounded above whenever there is any degree of price rigidity; we also show that the relative price $\frac{P_t^*}{P_t}$ is bounded below. The latter result will be useful for Proposition 3.

Proposition 1:

(i) For any $\theta > 0$, the gross inflation rate satisfies

$$\frac{P_{t+1}}{P_t} \leq \theta^{\frac{1}{1-\varepsilon}}$$

In contrast, inflation is unbounded in the linearized model.

(ii) The optimal price, relative to the general price level, satisfies

$$\frac{P_t^*}{P_t} \geq (1-\theta)^{\frac{1}{\epsilon-1}} \tag{25}$$

One way to understand these results intuitively is to write the aggregate price level as the demand-weighted average of individual prices (as shown by (11), demand for good i is proportional to $\left(\frac{P_t(i)}{P_t}\right)^{-\epsilon}$)

$$P_t = \int P_t(i) \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} di \tag{26}$$

As the prices chosen by reoptimizing firms rise, demand is increasingly reallocated toward firms with preset prices. This endogenous reweighting makes the aggregate price level progressively less sensitive to further increases in optimal prices, placing a tight upper bound on inflation. Conversely, as the prices chosen by reoptimizing firms decrease, demand is reallocated toward the corresponding goods, making the price level less sensitive to preset prices. Linearization around a zero-inflation steady state eliminates this mechanism, allowing inflation and relative prices to grow (or decrease) without bound.

4.3 Profits, Marginal Cost, and the Output Gap

We next relate output fluctuations to firms' profitability. Aggregate profits can be expressed in closed form as a function of consumption and price dispersion.

Lemma 3: Aggregate profits satisfy

$$\Pi_t = P_t C_t \left(1 - C_t^{\sigma+\varphi} \mathcal{O}_t^{1+\varphi}\right) \tag{27}$$

This expression makes clear that profits depend not only on the level of activity but also on the degree of price dispersion, which raises effective marginal costs.

Proposition 2: If the aggregate price level weakly exceeds the real wage (i.e. marginal cost), output cannot exceed the perfect-competition level. Specifically, if $P_t \geq W_t$ then $C_t \leq 1$, so that the output gap is bounded above: $x_t \leq \frac{1}{\sigma+\varphi} \log\left(\frac{\varepsilon}{\varepsilon-1}\right)$

The economic content of this result is transparent. To induce households to supply labor sufficient to produce output above the perfect-competition benchmark, real wages must rise above productivity. Firms can pay wages in excess of marginal revenue only by operating at a loss. Large output expansions therefore require negative aggregate profits,[7] financed implicitly by households in their role as firm owners.

4.4 The Flexible-Price Limit

We now consider the behavior of the nonlinear model as price rigidity vanishes. This limit is of particular interest given the finding, emphasized by Werning (2012) and Cochrane (2017), that the forward-guidance puzzle becomes more severe as prices become more flexible.

Proposition 3: As $\theta \to 0$:

(i) Strictly negative output gaps are not feasible in equilibrium.

(ii) If there exists $\kappa > 0$ such that $P_t(i)/W_t \geq \kappa$ for all firms, then strictly positive output gaps are not feasible in equilibrium.

The first part establishes that contractionary monetary policies cease to be feasible in the limit in which prices become fully flexible. The proof exploits

[7] This can be seen immediately from (27): since, from Lemma 2, $\mathcal{O}_t \geq 1$, $C_t > 1$ implies necessarily negative aggregate profits.

the result of Proposition 1, part ii): for a given θ, there is a lower bound to the relative price $\frac{P^*}{P}$. As $\theta \to 0$, $\frac{P^*}{P} \geq 1$. As we will see in the next section, with a contractionary monetary policy an equilibrium involves reoptimizing firms lowering their price relative to the general price level, but, as $\theta \to 0$, this becomes unfeasible.

The second part shows that positive output gaps can persist in the limit only if some firms charge prices that are vanishingly small relative to marginal cost. In an equilibrium with strictly positive output gap, reoptimizing firms raise prices by arbitrarily big factors, driving up wages, while an infinitesimal fraction of firms that cannot reoptimize absorb virtually all demand by selling at extremely low prices.

This can be understood by inspecting the price-setting equation (16), which, in the limit $\theta \to 0$, can be written as $\frac{P_t^*}{P_t} = \mathcal{O}_t^{\phi} \left(\frac{C_t}{C_{FP}}\right)^{\sigma+\phi}$. For consumption to be bigger than the flexible-price level C_{FP}, reoptimizing firms have to set their price P^* higher than the general price level. But as the share of non-reoptimizing firms becomes infinitesimal, the general price level (23) tends to coincide with P^*, *unless the ratio between* P^* *(or* W*, since* $P^* = \frac{\epsilon}{\epsilon-1}W$ *in this limit) and the preset prices becomes infinite.*

This result clarifies the paradoxical behavior of the model near the flexible-price limit. The divergence between the New Keynesian and flexible-price allocations relies on increasingly extreme and economically implausible relative-price distortions.

4.5 Implications

Taken together, the results in this section show that the nonlinear New Keynesian model places sharp theoretical restrictions on inflation, output, and equilibrium feasibility that are entirely absent from its linearized version. Extreme output responses to forward guidance are not generic implications of price rigidity, but are instead supported by equilibria in which some firms incur arbitrarily large losses. These properties provide the foundation for the numerical analysis in Section 5, which illustrates how such equilibria arise in practice under forward-guidance policies.

5 The Forward-Guidance Experiment in the Nonlinear Model

This section studies the forward-guidance experiment in the fully nonlinear New Keynesian model (BNK). The objective is twofold. First, we assess

to what extent the extreme predictions of the linearized model persist once nonlinearities and price dispersion are taken into account. Second, we provide a detailed diagnosis of the microeconomic mechanisms that sustain large output responses, thereby clarifying the economic content of the equilibria generated by the model.

5.1 Solving the Nonlinear Model

The nonlinear equilibrium is characterized by the Euler equation and the optimal price-setting condition during the forward-guidance period, and by the Taylor rule thereafter. In contrast to the linearized model, the equilibrium cannot be computed by backward induction. Price dispersion generated during the forward-guidance period persists after the Taylor rule is restored, so that the economy returns to the steady state only asymptotically. Moreover, equilibrium allocations at time t depend on the inherited dispersion $\mathcal{O}_{t-1}$. We therefore solve the model using a recursive algorithm, as described in Appendix B. For robustness, we also consider a "semi-linear" version of the model in the economy is assumed to return immediately to the steady state after the end of the forward-guidance period, thereby shutting down the effects of post-guidance price dispersion. Whenever differences arise between the two approaches, they are explicitly discussed below.

5.2 A Short Expansionary Policy

We begin with a short expansionary forward-guidance policy in which the nominal interest rate is set one percentage point below the natural rate for four quarters. Figure 4 reports the resulting equilibrium paths.

For this experiment, the linear and nonlinear models yield very similar outcomes. Inflation and the output gap remain modest throughout the forward-guidance period, and results from the semi-linear model are virtually indistinguishable from those of the fully nonlinear economy. The bottom-left panel of Figure 4 shows that all prices in the economy remain above the wage at all dates. This includes the smallest price among all firms, called P_{min} in the figure, which, at time t, can be found as $min(P_0, P_1^*, ..., P_t^*)$, as discussed in Section 2.3. As a result, markups are compressed but remain positive, and aggregate profits stay positive as well (bottom-right panel).

In this case, the expansion operates through the standard New Keynesian channel: price rigidity temporarily reduces markups, moving the economy closer to the perfectly competitive benchmark and generating a moderate increase in output. No firm is required to sell below marginal cost, and the equilibrium is economically well behaved.

5.3 A Longer Expansionary Policy

We now consider a longer expansionary policy in which the interest rate is kept one percentage point below the natural rate for eight quarters. The results are reported in Figure 5.

The behavior of inflation differs sharply between the linear and nonlinear models. In the nonlinear economy, inflation is bounded, as established in Proposition 1, and never exceeds the theoretical upper limit implied by price rigidity. In contrast, the linearized model predicts explosive inflation dynamics. Despite this stark difference, the behavior of the output gap is qualitatively similar across models and quantitatively even more extreme in the nonlinear case. In the first period of the forward-guidance announcement, the nonlinear model generates an output gap exceeding 40 percent.

Some differences are noticeable also between the solutions of the semi-linear and nonlinear model. In the non-linear model, inflation stays positive after $t = N$, while the output gap turns *negative* .[8]

The lower panels of Figure 5 reveal how such an expansion is sustained. Throughout most of the forward-guidance period, the minimum price in the economy, and even the aggregate price level, lie below the wage, implying negative aggregate profits. While reoptimizing firms raise their prices aggressively, firms that cannot reoptimize see their relative prices fall. Demand for their goods increases sharply, proportional to the ϵ-power (the 6-th power with our parameters) of their inverse relative price, and the model assumes that these firms satisfy all demand at their preset price. The resulting equilibrium is therefore supported by firms that sell below marginal cost and incur massive losses. In the first period alone, aggregate losses approach 120 percent of output.

These findings confirm Proposition 2: output expansions beyond the flexible-price benchmark require the aggregate price level to fall below marginal cost. The forward-guidance boom is thus not a consequence of intertemporal substitution alone, but of an extreme form of demand reallocation toward loss-making firms.

[8]The intuition for the coexistence of positive inflation and negative output gap when the Taylor rule is restored is the following: since the wage W_t is proportional to the price dispersion variable $\mathcal{O}$, price dispersion generated in the forward guidance period drives real wages up. This drives the optimal price P^* above the price level, creating inflation. In response to inflation, the central bank, following the Taylor rule, increases the nominal interest rate *more than one-for-one* with inflation, inducing an increase of the real rate above the natural rate, hence a contraction of output.

5.4 The Role of Price Rigidity

Figure 6 illustrates how the equilibrium response to an 8-period expansionary policy varies with the degree of price rigidity θ. As θ decreases, both inflation and the output gap in the first period increase sharply, even in the nonlinear model. Although inflation is bounded for any fixed $\theta > 0$, the bound itself diverges as $\theta \to 0$.

The bottom panels of Figure 6 provide a precise microeconomic interpretation. As prices become more flexible, the optimal price chosen by reoptimizing firms increases dramatically relative to the wage, while the minimum price in the economy collapses toward zero. In the limit, a vanishingly small fraction of firms accounts for an arbitrarily large share of demand by selling at prices that are arbitrarily low relative to marginal cost. This mechanism explains why the forward-guidance puzzle becomes more severe as the economy approaches the flexible-price limit, as formalized in Proposition 3.

5.5 Contractionary Policies

We next turn to contractionary forward-guidance policies, in which the nominal interest rate is set above the natural rate. In economies in which the nominal interest rate is constrained by the zero lower bound, this case also captures situations in which, by no active policy of the central bank, the natural rate is negative, while the interest rate is stuck at zero.

Figure 7 compares the linear and nonlinear responses to a six-period contractionary policy. For moderate policy interventions, the two models produce similar dynamics. Prices rise relative to marginal cost, markups widen, and output contracts in a conventional manner.

However, unlike the linearized model, the nonlinear economy does not admit an equilibrium for arbitrarily aggressive or prolonged contractionary policies. The reason is the following: an equilibrium is a sequence $\{P_t^*, P_t, \mathcal{O}_t, i_t, C_t\}_{t\geq 1}$. But by Proposition 1, part of this space is not accessible. In particular, while contractionary policies involve reoptimizing firms lowering their price below the general price level, for any $\theta < 1$ there is a limit to how much lower the optimal price P^* can be relative to the general price level P, as detailed in Proposition 1, part ii). In other words, for a too aggressive or prolonged contractionary policy, there is no price P^* that the reoptimizing firms can choose that is low enough to achieve the desired contraction of the economy. Figure 8 illustrates this result. For each duration of the forward-guidance period, there exists an upper bound on the feasible increase in the nominal interest rate (shown in the left panel), and a lower bound on the feasible contraction of output (middle panel). Beyond this bound, no equilibrium exists.

As price rigidity decreases, the set of feasible contractionary policies shrinks, and in the limit of fully flexible prices (right panel) no contractionary policy is feasible at all, consistently with Proposition 3.

The possible absence of an equilibrium is further illustrated in Figure 9 in the form of an "AD-AS" plot. In this figure we consider the representative case of a C-Policy experiment lasting one period, in an economy close to the flexible-price limit, with $\theta = 0.05$. The Euler equation (the Aggregate Demand curve) and the price-setting equation (the Aggregate Supply curve), taken individually, imply a relationship between output gap and inflation, and the equilibrium (if any) is found at their intersection. The algorithm to compute the two curves is detailed in Appendix B2. As shown in Figure 9, the range of output gaps covered by the AS curve is bounded below, and the AD curve is almost vertical.[9] Increasing the interest rate leaves the AS curve unchanged, while shifting the AD curve to the left, so that, if the interest rate is high enough (right panel), the two curves do not intersect.

This non-existence result represents an asymmetry relative to the expansionary case that is entirely absent from the linearized model.

5.6 Summary

The nonlinear analysis shows that the forward-guidance puzzle survives linearization only in a qualified sense. While inflation dynamics are fundamentally altered once nonlinearities are taken seriously, extreme output responses persist in the case of exansionary policies. These responses, however, are supported by equilibria in which a subset of firms incurs arbitrarily large losses by selling below marginal cost. The puzzle is therefore not a generic implication of intertemporal substitution under price rigidity, but a consequence of equilibrium allocations that rely on economically implausible firm behavior.

In the case of contractionary policies, we find the new result that, if the contraction is aggressive or prolonged enough, or if the economy is sufficiently close to the flexible-price benchmark, an equilibrium may be unfeasible. Importantly, the same mechanism operates when the nominal interest rate is constrained by the zero lower bound, so the result also applies in that environment.

[9] Since the Euler equation relating consumption demand at t and $t+1$ involves $t+1$-inflation, time-t consumption demand depends on time-t inflation only insofar C_{t+1} and π_{t+1} depend on π_t. The latter dependence occurs only through the state variable $\mathcal{O}_t$.

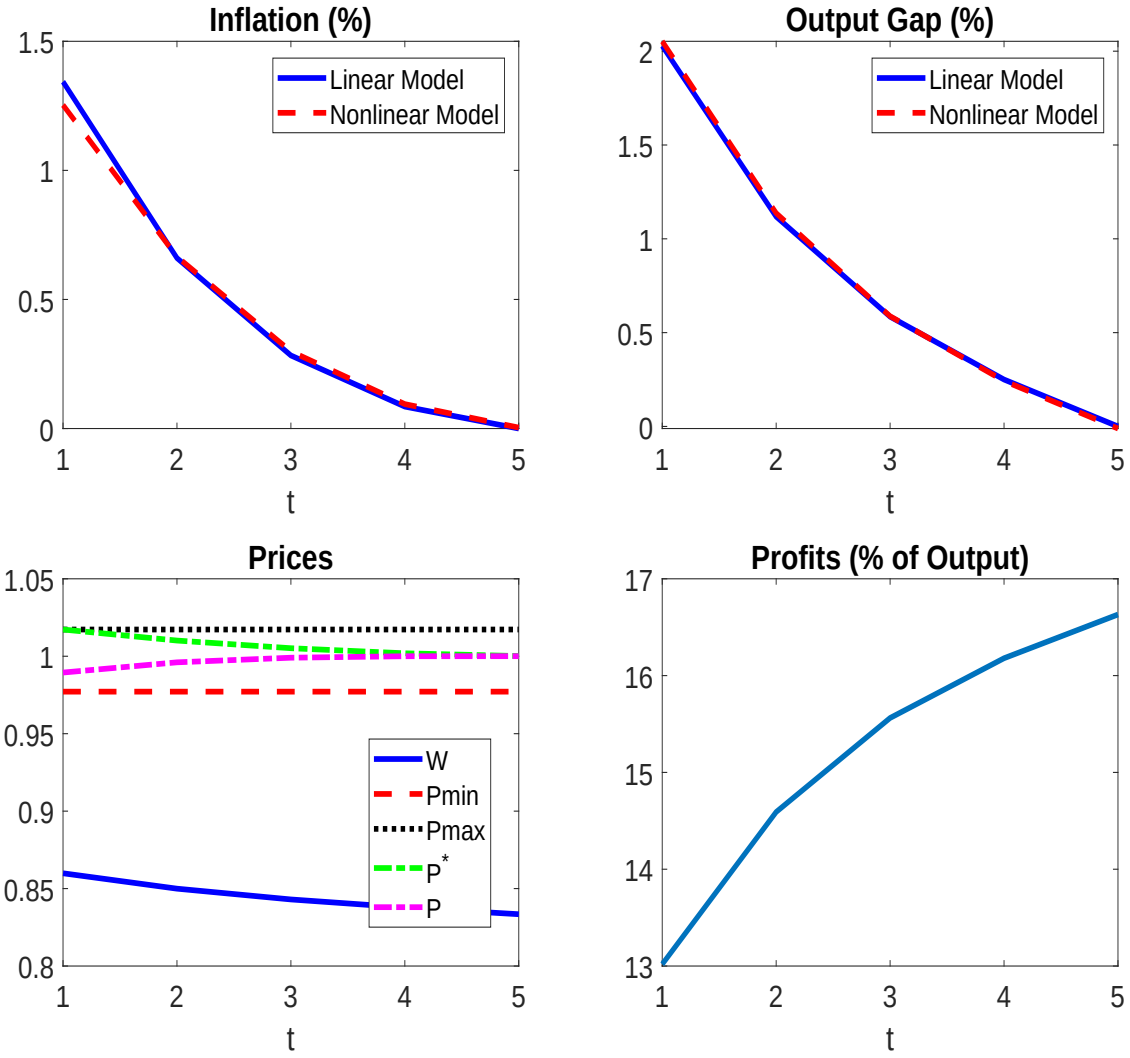


Figure 4: Response of the economy to a 4-period E-Policy experiment, as a function of time. At $t = 5$ the Taylor rule is restored.

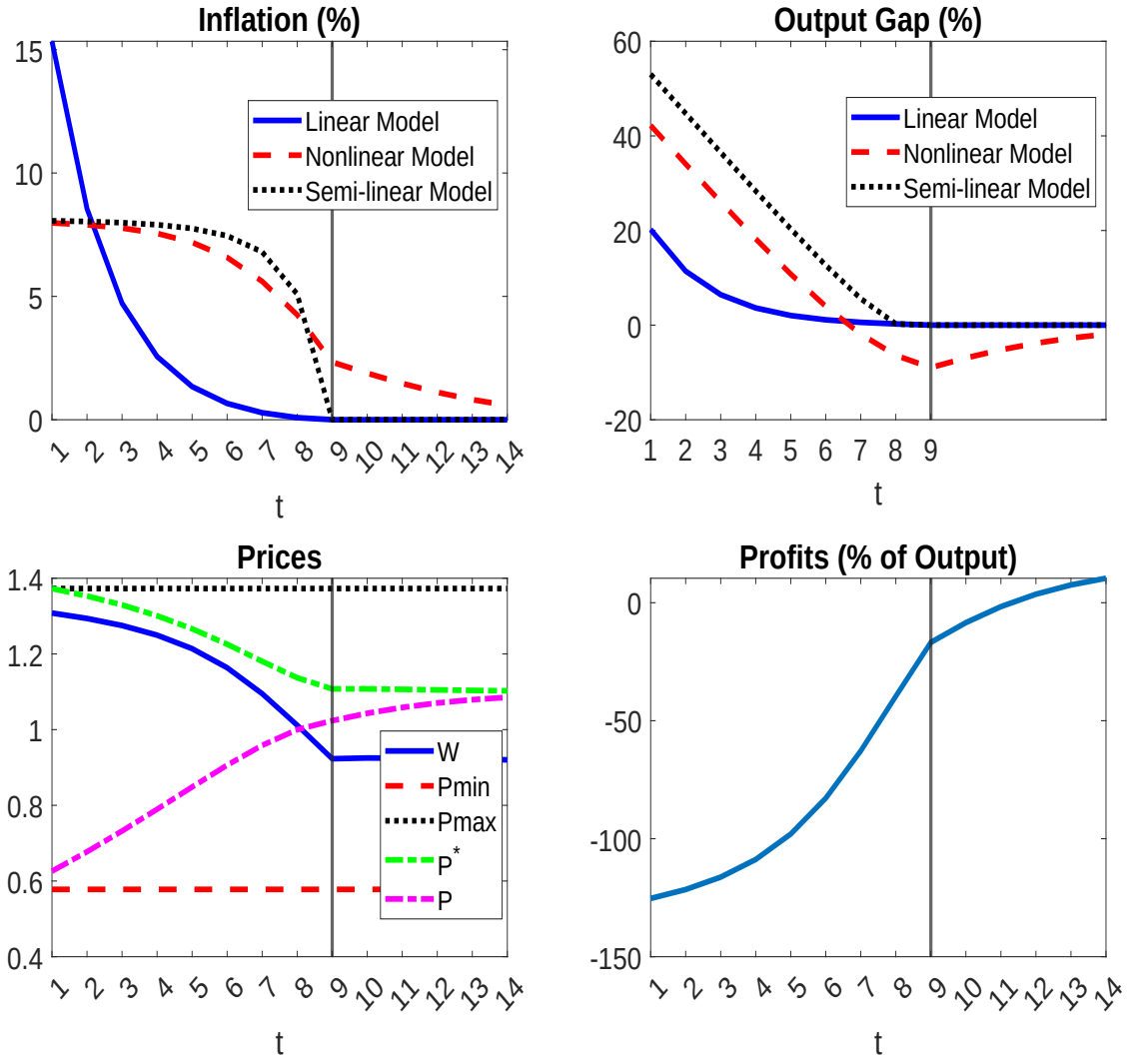


Figure 5: Response of the economy to an 8-period E-Policy experiment, as a function of time. The vertical grey line marks the return to the Taylor rule.

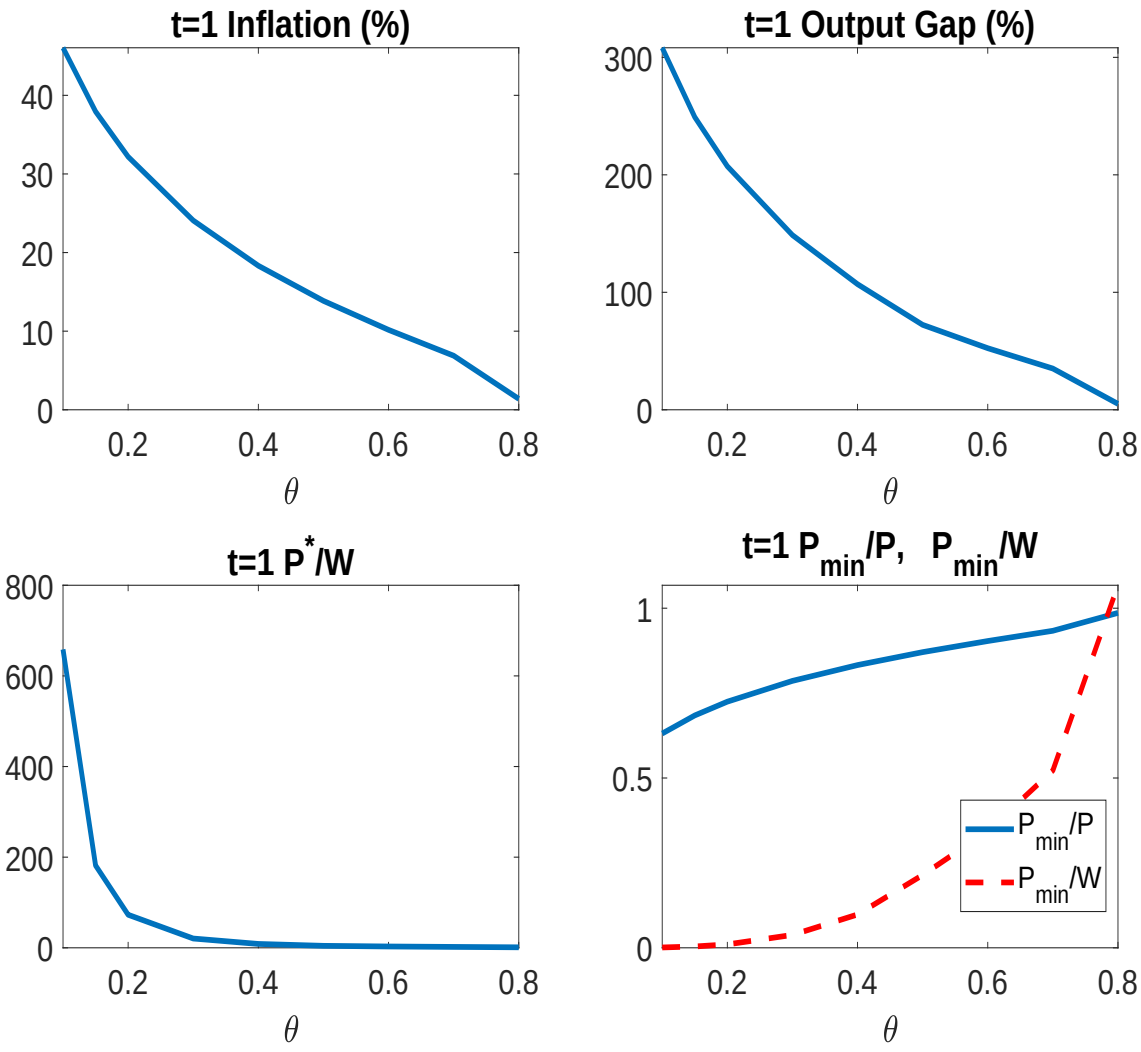


Figure 6: Response of the economy in the first period of an 8-period E-Policy as a function of θ. The top panels show the response of inflation and the output gap. The lower panels show the response of the optimal price as a fraction of the wage (bottom-left panel) and the minimum price as a fraction of the wage and of the general price level (bottom-right panel), all as a function of θ.

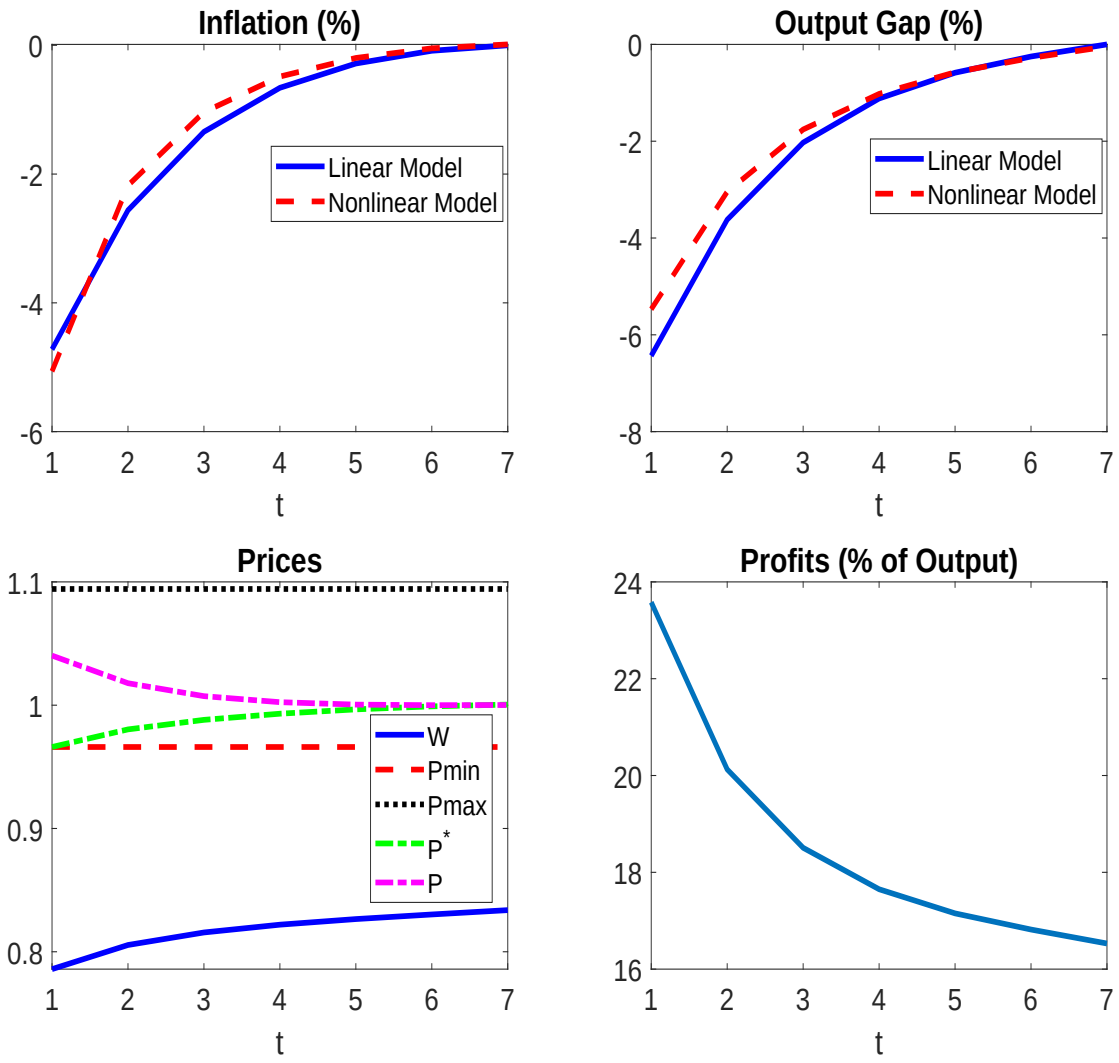


Figure 7: Response of the economy to a 6-period C-Policy experiment, as a function of time.

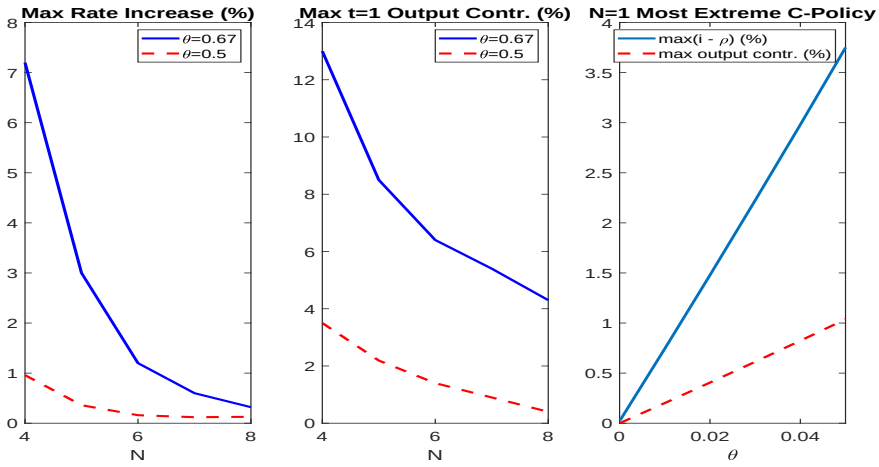


Figure 8: The left panel shows the highest feasible increase in the interest rate above the natural rate (in annnual terms), as a function of the duration of the policy, for two different values of θ. The middle panel shows the maximum $t = 1$ feasible contraction of the economy, as a function of the duration of the policy, for the same two values of θ. The right panel shows the maximum feasible single-period rate increase and ouput contraction, for an economy close to the flexible-price benchmark ($\theta < 0.05$).

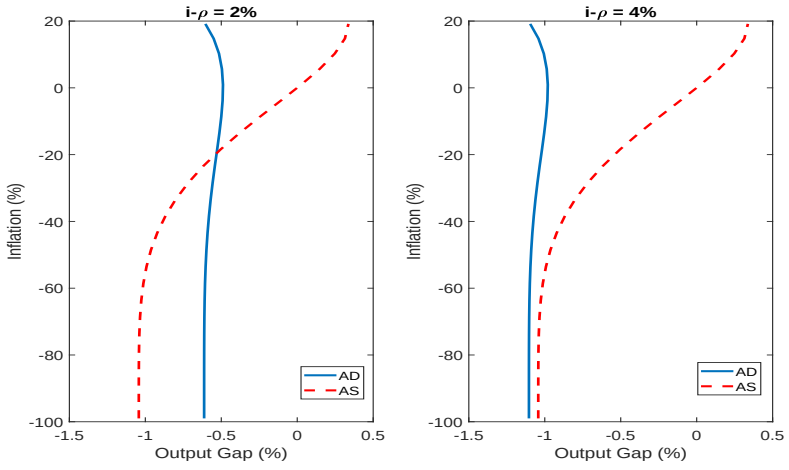


Figure 9: The two panels show the relationship between output gap and inflation implied by the Euler equation (the "AD" curve) and the price-setting equation (the "AS" curve) for a 1-period C-Policy experiment, in an economy with $\theta = 0.05$. In the left (right) panel, the annual rate set by the central bank is 2% (4%) above the natural rate.

6 Toward an Alternative Model

The analysis so far has shown that extreme responses to forward guidance in the New Keynesian model are supported by equilibria in which some firms sell below marginal cost while satisfying unbounded demand. In this section, we discuss a model featuring a minimal modification relative to the baseline BNK that blocks this mechanism. We call this model BNK*. The modification is straightforward. Firms remain subject to Calvo rigidity, but non-reoptimizing firms are allowed to adjust their price upward to marginal cost if their preset price would otherwise imply selling below cost. Formally, the price charged by a non-reoptimizing firm in period t is given by

$$P_t(i) = max(P_{t-1}(i), W_t) \tag{28}$$

All other aspects of the model—preferences, technology, and monetary policy—are unchanged.

This modification preserves the local behavior of the economy around the steady state. In particular, if we consider an experiment in which prices remain above marginal cost all the time, the constraint (28) does not bind and the equilibrium coincides exactly with that of the baseline model.[10] However, in an experiment such as, for example, the long expansionary policy of Section 5.3, the behavior of the baseline and the modified model would diverge.

A full analysis of the modified model is beyond the scope of this paper, if only because this model is significantly less tractable than the baseline BNK. In fact, in BNK* the price-dispersion variable $\mathcal{O}_{t-1}$ is not the only state variable: time-t price level and price dispersion are affected by each individual $P_{t-1}(i)$ such that $P_{t-1}(i) < W_t$. The whole price distribution needs therefore to be carried around in general to find a solution.

We can nevertheless establish three results that set this model apart from the baseline one. First the output gap is upward-bounded. Second, a too aggressive expansionary policy does not result in an equilibrium. Third, in the flexible-price limit no output gap different from 0 is possible and the only feasible interest rate is $i_t = \rho$.

[10] In a model with shocks, even close to steady state firms would take into account the fact that, in case of a large future shock, they can at least reset prices to match marginal cost. The firm's FOC would therefore internalize the optionality embedded in (28). This is not the case in our setup, in which there is perfect foresight after the initial central bank's announcement.

Result 1: The output gap is upward bounded

Proposition 2*: Output cannot exceed the perfect-competition level, i.e. $C_t \leq 1$, so that the output gap is bounded above by $x_t \leq \frac{1}{\sigma+\varphi} \log\left(\frac{\varepsilon}{\varepsilon-1}\right)$

As shown in Appendix A7, the proof of this proposition is the same as its analog in BNK, Proposition 2, together with the fact that, due to the constraint (28), no price can fall below marginal cost in BNK*.

Result 2: Absence of equilibrium for too aggressive expansionary policies

This result is intuitive given that the upper bound on the output gap. As the simplest example, consider a 1-period expansionary policy. As a back-of-the-envelope estimation, let us neglect the effect of $t = 1$ price dispersion on the solution starting at $t = 2$, when the Taylor rule is restored. In other words, let us assume that the economy goes back to the zero-inflation steady state at $t = 2$. Actual calculations confirm that the latter is a very good approximation. We then have $C_2 = C_{SS}$ and $P_2 = P_1$. The Euler equation then implies

$$\left(\frac{C_{SS}}{C_1}\right)^{\sigma} = \beta(1 + i_1)$$

Since C_1 is upward-bounded, it is clear that the space of the *feasible* values of i_1 is downward-bounded. If the central bank sets i_1 below the downward bound, no equilibrium is possible.

As a more precise numerical example, we consider the case of an E-Policy experiment with $N = 1$, with $\theta = 0.1$. This case is comparatively simple since, given that the economy starts at $t = 0$ at the steady state with no price dispersion, at $t = 1$ the price distribution comprises only two different prices: one for the reoptimizing firms and one for the other firms.

We compute the AD and AS curves for this example, using the algorithm detailed in Appendix B3. As shown in the left panel of Figure 10, both curves are vertical for inflation higher than a threshold level. As explained in Appendix B3, this is the level above which constraint (28) binds; beyond this level, all prices move in parallel and real variables are unchanged. The AD curve shifts right (left) when the interest rate is lower (higher). It is visually intuitive that no intersection exists between the two curves if the interest rate is low enough.[11] In our numerical estimation for this example,

[11] It is equally intuitive that no intersection exists if the interest rate is high enough, but that is the case also in BNK.

we find that, when the quarterly interest rate is set lower than 1.03% below the natural rate, no equilibrium is possible.

Result 3: No output gap in the flexible-price limit, and no equilibrium if $i \neq \rho$

Proposition 3*: In the limit $\theta \to 0$, no non-zero output gap is possible.

As shown in Appendix A8, this Proposition can be proven similarly as its analog Proposition 3 in BNK. No additional condition, such as the one in Proposition 3, part ii), is necessary here, since the condition $P(i)/W \geq \kappa > 0 \ \forall i$, is automatically satisfied given the constraint (28).

The right panel of Figure 10 shows that, in the $\theta \to 0$ limit, the AS and AD curves become vertical, overlapping only for one value of the interest rate, $i_t = \rho$. This means that any interest rate different from the natural rate does not result in an equilibrium. The intuition is the following: in the limit $\theta \to 0$, almost all firms are re-optimizers and set $P^* = \frac{\epsilon}{\epsilon-1}W$; the few non-reoptimzers can only create minimal distortion, as all prices must stay at least equal to W. As a result, the price level is entirely determined by P^*, i.e. $P = P^*$, which implies, from the price-setting equation, that it must be $C = C_{SS}$. But from the Euler equation, and given the zero-inflation target in the Taylor rule, $C = C_{SS}$ is only consistent with $i = \rho$.

Discussion

The analysis in this section shows that, in the modified model, no output gap can emerge in the limit $\theta \to 0$, which is consistent with the behavior of the genuine ($\theta = 0$) flexible-price model. However, when $\theta = 0$, any forward guidance announcement is feasible, since inflation, as discussed in Section 2.4, adjusts so as to keep the real rate equal to the natural rate $\rho = -log(\beta)$, and all the real variables at the steady-state level. But for this to happen, a rather peculiar price-setting behavior is required of firms: since the real rate is equal to the nominal rate minus the *next-period* inflation, for the real rate to be constant firms have to set their price reacting to the *previous-period* nominal rate. The latter could never occur for any $\theta > 0$: if even a small fraction of firms are non-reoptimizers, reoptimizing firms are able to affect relative prices, which has real effects. In this case they make an optimal, *forward-looking* decision. We find therefore a discontinuous behavior of the model for $\theta = 0$: while in the limit $\theta \to 0$ only equilibria with the nominal rate equal to the natural rate (and zero inflation) are feasible, for $\theta = 0$ any nominal rate is feasible, with inflation adjusting to ensure the equality of the *real* rate and the natural one.

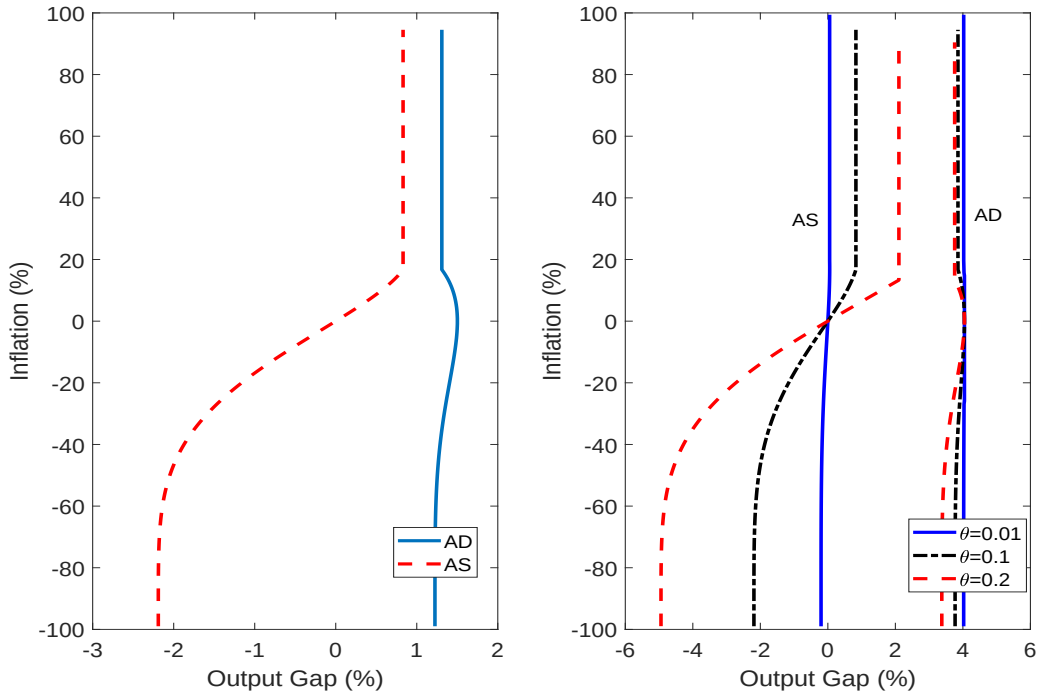


Figure 10: The left panel shows the Aggregate Demand and Aggregate Supply for an experiment with $N = 1$, in an economy with $\theta = 0.1$. To obtain the AD curve, the quarterly rate is set 1.5% below the natural rate. The left panel shows the AD and AS curves for three different values of θ, showing how both curve tend to become vertical in the limit $\theta \to 0$. For this panel, the quarterly interest rate is set 4% below the natural rate.

7 Conclusion

This paper revisits the forward-guidance puzzle through a careful analysis of the nonlinear New Keynesian model that underlies its widely used linearized representation. By explicitly tracking the full distribution of prices and the resulting price dispersion, we show that the extreme responses of output and inflation to forward-guidance announcements do not arise from intertemporal substitution per se, but from an assumption embedded in the Calvo pricing structure: non-reoptimizing firms are required to satisfy all demand at their preset price, even when that price falls far below marginal cost.

In the nonlinear model, large and persistent inflation compresses markups for constrained firms and can eventually drive their prices below marginal cost. When this occurs, the model generates expansions only by implicitly forcing these firms to absorb unbounded losses while meeting arbitrarily large demand. Moreover, as the degree of price rigidity vanishes, positive output gaps can be sustained only if the prices of a vanishingly small fraction of firms collapse toward zero relative to wages. These mechanisms explain both the magnitude of the forward-guidance effects and the counterintuitive result that the puzzle intensifies as prices become more flexible.

The broader insight of the paper extends beyond forward guidance. In environments featuring large or persistent shocks that generate substantial inflation or deflation, the standard New Keynesian model can deliver quantitatively and conceptually misleading predictions unless its microeconomic assumptions are carefully scrutinized. Our results suggest that restoring economically plausible firm behavior is essential for using New Keynesian models to evaluate unconventional monetary policies. Future research should explore alternative forms of nominal rigidity, or possibly demand rationing, that preserve tractability while avoiding the pathological outcomes identified here.

Finally, our analysis has implications for the literature on the zero lower bound. The non-existence of equilibrium under sufficiently strong or prolonged contractionary policies also applies when the natural rate of interest is negative and the policy rate is constrained at the zero lower bound, since just maintaining the nominal rate at zero constitutes a contractionary stance relative to the natural benchmark. From this perspective, our results highlight that equilibrium feasibility can be a binding constraint in New Keynesian models at the zero lower bound.

Appendix A – Proofs and derivations

A1: Derivation of the price-setting equation and of (12)-(16)

Given the production function (4), $N_{t+k;t} = Y_{t+k;t}$ and given the demand equation (11) and the resource constraint $C_t = Y_t \ \forall t$,

$$N_{t+k;t} = Y_{t+k;t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon} C_{t+k} \tag{29}$$

Inserting (29) into (6), we obtrain that the firm's problem at time t is

$$max_{P_t^*} P_t C_t^{\sigma} \Sigma_{k=0}^{\infty} \theta^k \beta^k \left((P_t^*)^{1-\epsilon} P_{t+k}^{\epsilon-1} C_{t+k}^{1-\sigma} - (P_t^*)^{-\epsilon} P_{t+k}^{\epsilon-1} C_{t+k}^{1-\sigma} W_{t+k}\right) \tag{30}$$

The first-order condition gives (12). Next, using $N(i) = Y(i) = C(i)$, where the first equality comes from the production function and the second equality comes from the resource constraint, and using (11), we have

$$N_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t$$

hence aggregate labor (5) is

$$N = C_t P_t^{\epsilon} \int (P_t(i))^{-\epsilon} di = C_t \mathcal{O}_t \tag{31}$$

Inserting (31) into (9) we can express the wage as

$$W_t = P_t C_t^{\sigma+\phi} \mathcal{O}_t^{\phi} \tag{32}$$

and finally, inserting (32) in (12), we have the price setting equation in the form (16).

A2: Lemma 1 – Derivation of (23) and (24)

Define an indicator $\vartheta_t(i)$ that is equal to 1 if firm i *cannot* change its price at t and 1 otherwise. Then we have

$$P_t(i) = \begin{array}{ll} P_t^* & \text{if } \vartheta_t(i) = 0 \\ P_{t-1}(i) & \text{if } \vartheta_t(i) = 1 \end{array}$$

Then for any function f we have

$$f(P_t(i)) = \vartheta(i) f(P_{t-1}(i)) + (1 - \vartheta(i)) f(P_t^*)$$

so in particular

$$P_t(i)^{1-\epsilon} = \vartheta_t(i)P_{t-1}(i)^{1-\epsilon} + (1-\vartheta_t(i))(P_t^*)^{1-\epsilon}$$

In addition we have

$$\theta = \int_0^1 \vartheta_t(i)di$$

Hence the price level can be written as

$$\begin{aligned} P_t &= \left(\int_0^1 P_t(i)^{1-\epsilon}di\right)^{\frac{1}{1-\epsilon}} \\ &= \left(\int_0^1 \vartheta_t(i)P_{t-1}(i)^{1-\epsilon}di + \int_0^1 (1-\vartheta(i))(P_t^*)^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}} \qquad (33) \\ &= \left(\int_0^1 \vartheta_t(i)P_{t-1}(i)^{1-\epsilon}di + (1-\theta)(P_t^*)^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}} \qquad (34) \end{aligned}$$

The first integral on the RHS of (34) can be interpreted as an expected value, and, since the firms that can optimize are chosen randomly, the random variables $\vartheta(i)$ are independent of $P_{t-1}(i)$, so that

$$\int_0^1 P_{t-1}(i)^{1-\epsilon}\vartheta_t(i)di = E[\vartheta_t(i)P_{t-1}(i)^{1-\epsilon}] = E[\vartheta(i)]E[P_{t-1}(i)^{1-\epsilon}] = \theta P_{t-1}^{1-\epsilon} \qquad (35)$$

Inserting (35) in (34) we obtain (23). Similarly, we have

$$\mathcal{O}_t = P_t^\epsilon \left(\int \vartheta(i)P_{t-1}(i)^{-\epsilon}di + (1-\theta)(P_t^*)^{-\epsilon}\right)$$

and

$$\int \vartheta(i)P_{t-1}(i)^{-\epsilon}di = E[\vartheta(i)]E[P_{t-1}(i)^{-\epsilon}] = \theta \int P_{t-1}(i)^{-\epsilon}di$$

(24) follows.

A3: Lemma 2 – $\mathcal{O}_t \geq 1$

$$\begin{aligned} \mathcal{O}_t &= P_t^\epsilon \int_0^1 P_t(i)^{-\epsilon}di = P_t^\epsilon \int_0^1 \left(P_t(i)^{1-\epsilon}\right)^{\frac{\epsilon}{\epsilon-1}}di \\ &\geq P_t^\epsilon \left(\int_0^1 P_t(i)^{1-\epsilon}di\right)^{\frac{\epsilon}{\epsilon-1}} = P_t^\epsilon P_t^{-\epsilon} = 1 \qquad (36) \end{aligned}$$

where the inequality is due to Jensen inequality and the first equality uses (10).

Proposition 1:

i) From (34), and given $\epsilon > 1$,

$$\frac{P_t}{P_{t-1}} = \left(\theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}} \leq \theta^{\frac{1}{1-\epsilon}} \tag{37}$$

ii) Again, from (34) and given $\epsilon > 1$,

$$\frac{P_t}{P_t^*} = \left(\theta\left(\frac{P_{t-1}}{P_t^*}\right)^{1-\epsilon} + (1-\theta)\right)^{\frac{1}{1-\epsilon}} \leq (1-\theta)^{\frac{1}{1-\epsilon}} \tag{38}$$

or equivalently $\frac{P_t^*}{P_t} \geq (1-\theta)^{\frac{1}{\epsilon-1}}$.

A4: Lemma 3

Profits of company i are given by

$$\Pi_t(i) = P_t(i)C_t(i) - W_t C_t(i) \tag{39}$$

where W_t is the economy-wide wage. Using (11)

$$\Pi_t(i) = P_t^{\epsilon} C_t (P_t(i)^{1-\epsilon} - W_t P_t(i)^{-\epsilon}) \tag{40}$$

intergrating across all firms and using (10)

$$\Pi_t = \int_0^1 \Pi_t(i) = P_t C_t (1 - \frac{W_t}{P_t}\mathcal{O}_t) \tag{41}$$

Finally, using (32) we obtain the result

$$\Pi_t = P_t C_t (1 - C_t^{\sigma+\phi}\mathcal{O}^{1+\phi})$$

A5: Proposition 2

The result is immediate considering (32), that we repeat here for convenience.

$$W_t = P_t C_t^{\sigma+\phi}\mathcal{O}_t^{\phi}$$

Since, by Lemma 2, $\mathcal{O}_t \geq 1$, and since $\phi > 0$, it is obvious that C_t can exceed 1 only if $W_t > P_t$. The result that the output gap is smaller than $\frac{1}{\sigma+\phi} log\left(\frac{\epsilon}{\epsilon-1}\right)$ unless $W_t > P_t$ is immediate given that $C_{SS} = \left(\frac{\epsilon-1}{\epsilon}\right)^{\frac{1}{\sigma+\phi}}$.

A6: Proposition 3

i) First, we express this part of the proposition in a more general form:

In the limit $\theta\beta \to 0$

$$C_t \geq \left(\frac{\epsilon - 1}{\epsilon} \frac{(1-\theta)^{\frac{1}{\epsilon-1}}}{\left(\theta^{\frac{1}{1-\epsilon}} \mathcal{O}_{t-1} + (1-\theta)^{\frac{1}{1-\epsilon}}\right)^{\phi}} \right)^{\frac{1}{\sigma+\phi}} \tag{42}$$

This implies that in the limit $\theta \to 0$, with β fixed, $C_t \geq \left(\frac{\epsilon-1}{\epsilon}\right)^{\frac{1}{\sigma+\phi}} = C_{SS}$, i.e. the output gap cannot be negative, which is the formulation of Proposition 3 in Section 4.

To prove (42), consider that

$$\begin{aligned} \mathcal{O}_t &= P_t^{\epsilon}(\theta P_{t-1}^{-\epsilon} \mathcal{O}_{t-1} + (1-\theta)(P_t^*)^{-\epsilon}) \leq \left(\theta \frac{P_t^{\epsilon}}{P_{t-1}^{\epsilon}} \mathcal{O}_{t-1} + (1-\theta)^{\frac{1}{1-\epsilon}}\right) \\ &\leq \left(\theta^{\frac{1}{1-\epsilon}} \mathcal{O}_{t-1} + (1-\theta)^{\frac{1}{1-\epsilon}}\right) \end{aligned} \tag{43}$$

where the first equality uses (24), the first inequality uses the fact that, by Proposition 1, part ii), $(P_t^*)^{-\epsilon} \leq P_t^{-\epsilon}(1-\theta)^{\frac{\epsilon}{1-\epsilon}}$, and the last inequality uses the result of Proposition 1, part i), which implies that $\left(\frac{P_t}{P_{t-1}}\right)^{\epsilon} \leq \theta^{\frac{\epsilon}{1-\epsilon}}$.

Moving now to the limit $\theta\beta \to 0$, we have

$$P_t^* \to \frac{\epsilon}{\epsilon - 1} P_t \mathcal{O}_t^{\phi} C_t^{\sigma+\phi} \tag{44}$$

So, in this limit we have

$$(1-\theta)^{\frac{1}{\epsilon-1}} \leq \frac{P_t^*}{P_t} = \frac{\epsilon}{\epsilon-1} \mathcal{O}_t^{\phi} C_t^{\sigma+\phi} \leq \frac{\epsilon}{\epsilon-1} \left(\theta^{\frac{1}{1-\epsilon}} \mathcal{O}_{t-1} + (1-\theta)^{\frac{1}{1-\epsilon}}\right)^{\phi} C^{\sigma+\phi} \tag{45}$$

where the first inequality uses (38), the second equality uses (44) and the third inequality uses (43) (given $\phi > 0$). This finally establishes

$$C_t \geq \left(\frac{\epsilon - 1}{\epsilon} \frac{(1-\theta)^{\frac{1}{\epsilon-1}}}{\left(\theta^{\frac{1}{1-\epsilon}} \mathcal{O}_{t-1} + (1-\theta)^{\frac{1}{1-\epsilon}}\right)^{\phi}} \right)^{\frac{1}{\sigma+\phi}} \tag{46}$$

ii) For $\theta \to 0$ we have $P_t^* \to \frac{\epsilon}{\epsilon-1} W_t$.

Using (23) and (24), the price level and the dispersion index can be written as

$$\begin{aligned} P_t &= \left(\int_0^1 \vartheta_t(i) P_t(i)^{1-\epsilon} di + (1-\theta) \left(P_t^*\right)^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}} \\ &= P_t^* \left(\int_0^1 \vartheta_t(i) \left(\frac{P_t(i)}{P_t^*} \right)^{1-\epsilon} di + (1-\theta) \right)^{\frac{1}{1-\epsilon}} \qquad (47) \\ \mathcal{O}_t &= P_t^{\epsilon} \left(\int_0^1 \vartheta_t(i) P_t(i)^{-\epsilon} di + (1-\theta) \left(P_t^*\right)^{-\epsilon} \right) \\ &= \left(\frac{P_t}{P_t^*} \right)^{\epsilon} \left(\int_0^1 \vartheta_t(i) \left(\frac{P_t(i)}{P_t^*} \right)^{-\epsilon} di + (1-\theta) \right) \qquad (48) \end{aligned}$$

where $\vartheta_t(i)$ is an indicator that takes value 1 if firm i *cannot* reoptimize at time t. If $P_t(i) > \kappa W_t \ \forall i$, then $P_t(i) > \tilde{\kappa} P_t^* \ \forall i$, with $\tilde{\kappa} = \frac{\epsilon-1}{\epsilon}\kappa$. Hence, given $\epsilon > 1$,

$$\int_0^1 \left(\frac{P_t(i)}{P_t^*} \right)^{1-\epsilon} \vartheta_t(i) di < \tilde{\kappa}^{1-\epsilon} \int_0^1 \vartheta_t(i) di = \tilde{\kappa}^{1-\epsilon} \theta$$

$$\int_0^1 \left(\frac{P_t(i)}{P_t^*} \right)^{-\epsilon} \vartheta_t(i) di < \tilde{\kappa}^{-\epsilon} \int_0^1 \vartheta_t(i) di = \tilde{\kappa}^{-\epsilon} \theta$$

Obviously both the above epressions tend to 0 as $\theta \to 0$. It is then immediate to see from (47), (48) that in this limit $P_t \to P_t^*$ and $\mathcal{O}_t \to 1$. The latter also implies that $W_t = P_t C_t^{\sigma+\phi} \mathcal{O}_t^{\phi} \to P_t^* C_t^{\sigma+\phi}$. Hence

$$lim_{\theta \to 0} P_t^* = \frac{\epsilon}{\epsilon - 1} W_t = \frac{\epsilon}{\epsilon - 1} P_t^* C_t^{\sigma+\phi}$$

which implies that in the limit C_t tends to its steady-state value $\left(\frac{\epsilon}{\epsilon-1}\right)^{\frac{1}{\sigma+\phi}}$.

A7: Proposition 2*

The relationship (9), specifying the equilibrium level of the wage in BNK, is valid also in BNK*, as it is derived from the FOCs of the household, that are identical in the two models. We repeat it here for convenience:

$$W = P \mathcal{O}^{\phi} C^{\sigma+\phi}$$

Lemma 2, stating that $\mathcal{O} \geq 1$, is also still valid. (hence $\mathcal{O}^{\phi} \geq 1$ since $\phi > 0$). Moreover, thanks to the constraint (28), the condition $P \geq W$ in BNK* is automatically satisfied in BNK. It follows immediately that $C \leq 1$.

A8: Proposition 3*

We first show that, for $\theta \to 0$, $P \to P^*$ and $\mathcal{O} \to 1$. In BNK* we have

$$P_t = \left(\theta\left(\int_0^1 max(P_{t-1}(i), W_t)di\right)^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}} \quad (49)$$

$$\mathcal{O}_t = P_t^{\epsilon}\left(\theta\left(\int_0^1 max(P_{t-1}(i), W_t)di\right)^{-\epsilon} + (1-\theta)(P_t^*)^{-\epsilon}\right) \quad (50)$$

Moreover, in the limit $\theta \to 0$ we have

$$P^* \to \frac{\epsilon}{\epsilon - 1}W \quad (51)$$

Hence, in the limit

$$P \geq \left(\theta W^{1-\epsilon} + (1-\theta)(P^*)^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}} = \left(\theta(\frac{\epsilon-1}{\epsilon}P^*)^{1-\epsilon} + (1-\theta)(P^*)^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}} \quad (52)$$

$$P \leq (1-\theta)^{\frac{1}{1-\epsilon}}P^* \quad (53)$$

In (52), the inequality comes from (28) and the equality from (51). (52)-(53) show that $P \to P^*$ in the limit $\theta \to 0$. To show that $\mathcal{O} \to 1$ in this limit, consider that

$$\mathcal{O} \leq P^{\epsilon}\left(\theta W^{-\epsilon} + (1-\theta)(P^*)^{-\epsilon}\right) = (\frac{\epsilon}{\epsilon-1}W)^{\epsilon}\left(\theta W^{-\epsilon} + (1-\theta)(\frac{\epsilon}{\epsilon-1}W)^{-\epsilon}\right) \quad (54)$$

where the inequality is due to (28) and the equality from $P \to P^*$ together with (51).

The expression on the RHS of (54) obviously tends to 1 for $\theta \to 0$, so $\mathcal{O} \leq 1$ in this limit. But since $\mathcal{O} \geq 1$ by Lemma 2, this clearly implies that $\mathcal{O} \to 1$ in this limit. Now, since $W = PC^{\sigma+\phi}\mathcal{O}^{\phi}$, the fact that $P^* \to \frac{\epsilon}{\epsilon-1}W$, together with $P \to P^*$ and $\mathcal{O} \to 1$ shows that, in the limit

$$\frac{\epsilon}{\epsilon-1}P^* = P^*C^{\sigma+\phi}$$

which immediately implies that C tends to its steady state value $C_{SS} = \left(\frac{\epsilon}{\epsilon-1}\right)^{\frac{1}{\sigma+\phi}}$.

Appendix B – Solution Algorithms

B1: General solution for the nonlinear and semilinear model (BNK)

Given that the economy is at the steady state at $t = 0$, with $P_0(i) = P_0 \ \forall i$ and $C_0 = C_{SS}$, the solution of the non-linear BNK model for a forward-guidance (henceforth FG) policy lasting N periods is given by a sequence $\{P_t, P_t^*, C_t, \mathcal{O}_t\}_{t\geq 1}$ satisfying the Euler equation and the optimal price-setting equation:

$$C_t^{-\sigma} = \beta(1+i_t)\frac{P_t}{P_{t+1}}C_{t+1}^{-\sigma} \tag{55}$$

$$P_t^* = \frac{\epsilon}{\epsilon-1}\frac{\sum_{k=0}^{\infty}\theta^k\beta^k P_{t+k}^{\epsilon}C_{t+k}^{1+\phi}\mathcal{O}_{t+k}^{\phi}}{\sum_{k=0}^{\infty}\theta^k\beta^k P_{t+k}^{\epsilon-1}C_{t+k}^{1-\sigma}} \tag{56}$$

with i_t exogenous for $t = 1, ..N$, and additionally satisfying the Taylor rule for $t > N$; subject to the law of motion of the price level

$$P_{t+1} = \left(\theta P_t^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}} \tag{57}$$

and

$$\mathcal{O}_t = P_t^{\epsilon}(\theta P_{t-1}^{-\epsilon}\mathcal{O}_{t-1} + (1-\theta)(P_t^*)^{-\epsilon}) \tag{58}$$

with $\mathcal{O}_0 = 1$.

It is convenient to adopt the normalization in which the price level is 1 in the last period of the FG policy. Our normalization is then $P_N = 1$.

Nonlinear model: Solution for $t \geq N+1$

After the Taylor rule is restored, the optimal price chosen by the unconstrained firms, P_t^*, relative to the previous-time price level P_{t-1}, must be a function of price dispersion in the previous period, $\mathcal{O}_{t-1}$, which is the only state variable. We postulate the following functional form

$$\frac{P_t^*}{P_{t-1}} = f(\mathcal{O}_{t-1}) = 1 + a(\mathcal{O}_{t-1} - 1) + b(\mathcal{O}_{t-1} - 1)^2 \tag{59}$$

where a and b are constant parameters. Obviously when $\mathcal{O}_{t-1} = 1$ (no previous-period price dispersion) $P_t^* = P_{t-1}$, which ensures that no price dispersion is generated and inflation is zero. In this case the economy is at the steady state.

The algorithm to calibrate a and b is the following. Given a value of $\mathcal{O}_N$, and given our price normalization $P_N = 1$, choosing an arbitrary guess value for a and b we can compute a sequence $\{P_t, P_t^*, \mathcal{O}_t\}_{t \geq N+1}$ by

$$\begin{aligned} \frac{P_t^*}{P_{t-1}} &= f(\mathcal{O}_{t-1}) \\ P_t &= \left(\theta P_{t-1}^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}} \\ \mathcal{O}_t &= P_t^{\epsilon}(\theta P_{t-1}^{-\epsilon}\mathcal{O}_{t-1} + (1-\theta)(P_t^*)^{-\epsilon}) \end{aligned}$$

starting from $t = N+1$ and ending at T such that $\mathcal{O}_T$ is sufficiently close to 1. After T we can consider the economy to be at the steady state with constant prices, i.e. $P_{T+s} = P_{T+s}^* = P_T$ and $\mathcal{O}_{T+s} = 1 \ \forall s > 0$.

Given the sequence for the price level P and the Taylor rule, we use the Euler equation to find a sequence $\{C_t\}_{t \geq N+1}$. We then compute a second sequence $\{\tilde{P}_t^*\}_{t \geq N+1}$ through the price-setting equation (56), using the previously computed sequences $\{P_t, C_t, \mathcal{O}_t\}_{t \geq N+1}$ on the RHS. We finally select a and b as the values that minimize the distance between the sequences P_t^* and $\tilde{P}_t^*$.

Nonlinear model: Solution for $1 \leq t \leq N$

We now describe the recursive algorithm to find the solution for $t \leq N$. Each iteration k of the algorithm requires a sequence $\{\mathcal{O}_t^{(k)}\}_{t=1,...,N}$ as an input, and produces a sequence $\{\mathcal{O}_t^{(k+1)}\}_{t=1,...,N}$ as an output, which becomes the input for the next iteration. In a first iteration, we use, as a first guess, $\{\mathcal{O}_t^{(k)} = 1\}_{t=1,...,N}$. The algorithm stops at the iteration when the output sequence is sufficiently close to the input sequence. Each iteration k consists of two steps

- **Step 1:** Given $\mathcal{O}_N^{(k)}$, obtain the "Taylor sequence" (i.e. the sequence starting when the Taylor rule is restored) $\{P_t, P_t^*, C_t, \mathcal{O}_t\}_{t \geq N+1}$ with the algorithm described in the previous subsection (using the calibrated parameters a and b). Notice that, in the first iteration, given the input $\mathcal{O}_N^{(1)} = 1$, the system is at the steady state at $t = N+1$, so the Taylor sequence is simply $P_t = P_N = 1$, $C_t = C_{SS}$, $\mathcal{O}_t = 1 \ \forall t \geq N+1$.

- **Step 2:** Given $P_N = 1$, the exogenous sequence i_t, $t = 1, .., N$, and $\{P_{N+1}, C_{N+1}\}$ from the "Taylor sequence" (found in Step 1), we find C_N through the Euler equation (55), $C_N = C_{N+1}\left(\beta(1+i_N)\frac{P_N}{P_{N+1}}\right)^{-\frac{1}{\sigma}}$. We then use the price-setting equation to solve for P_N^*. Given P_N and P_N^* we then find P_{N-1} through (57). We proceed the same way all the way

backward, first finding C_t through (55) given $\{C_{t+1}, P_{t+1}, P_t\}$ from the previous time step, then P_t^* through (56) and then P_{t-1} through (57). After completing the time step $t = 1$ we also obtain P_0, which represents the price level (common to all good varieties) in the steady state prior to the forward guidance period. We use the resulting sequences $\{P_t\}_{0\leq t\leq N}$, $\{P_t^*\}_{1\leq t\leq N}$ to produce the output sequence $\{\mathcal{O}_t^{(k+1)}\}_{1\leq t\leq N}$, using (58).

Semilinear model

The solution algorithm for the semilinear model is similar to the nonlinear one, except that in every iteration, the Taylor sequence is simply equal to the steady-state one $P_t = P_N = 1$, $C_t = C_{SS}$, $\mathcal{O}_t = 1$ $\forall t \geq N + 1$. Step 2 of each iteration is identical to the corresponding step in the nonlinear model.

B2: AD and AS curves for BNK

In this section we detail the algorithm used to obtain the AD and AS curves in a forward guidance experiment with $N = 1$. The economy starts at the steady state at $t = 0$, with $\mathcal{O}_0 = 1$. At $t = 1$ the central bank sets the interest rate i_1 at its desired level. At $t = 2$ the Taylor rule is restored. We normalize the initial price level to 1: $P_0 = 1$.

AS curve

We consider a large set of possible values of the variable P_1^* (optimal time-1 price). For each of these values, we compute, in sequence:

AS 1) The gross time-1 inflation

$$\frac{P_1}{P_0} = \left(\theta + (1-\theta)\left(\frac{P_1^*}{P_0}\right)^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}}$$

.

AS 2) The time-1 price dispersion

$$\mathcal{O}_1 = P_1^{\epsilon}\left(\theta + (1-\theta)\left(P_1^*\right)^{-\epsilon}\right)$$

AS 3) The Taylor sequence $\{P_t, P_t^*, C_t, \mathcal{O}_t\}_{t\geq 2}$, given the previously computed price dispersion $\mathcal{O}_1$, as described in Appendix B1.

AS 4) The consumption supply C_1^S, by inverting the price-setting equation (i.e. C_1^S is the consumption value that makes P_1^* the optimal time-1 price level, given the previously computed quantities).

AS 5) The "AS output gap" as $log(C_1^S/C_{SS})$.

We are finally able to relate each inflation value to an output gap on the AS curve.

AD curve

For each value of P_1^* we compute the AD curve the following way:

AD 1) We reuse the value of $\mathcal{O}_1$, P_1, inflation and the Taylor sequence already computed to obtain the AS curve.

AD 2) Using C_2, P_2, P_1 from the previous point and the exogenous i_1, we compute consumption demand C_1^D from the Euler equation.

AD 3) We compute the "AD output gap" as $log(C_1^D/C_{SS})$.

We are then able to relate each inflation value to an output gap on the AD curve.

B3: AD and AS curves for BNK*

In this section we detail the algorithm used to obtain the AD and AS curves in a forward guidance experiment with $N = 1$ in BNK*. The economy starts at the steady state at $t = 0$, with $\mathcal{O}_0 = 1$. At $t = 1$ the central bank sets the interest rate i_1 at its desired level. At $t = 2$ the Taylor rule is restored.

The Euler equation (55) and the price-setting equation (56) are the same as in BNK, and of course the Taylor rule for $t \geq N + 1$ is the same, but given the new constraint (28), the law of motion of the price level and the dispersion variable are now the following

$$P_t = \left(\theta \left(\int_0^1 max(P_{t-1}(i), W_t) di \right)^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}} \tag{60}$$

$$\mathcal{O}_t = P_t^{\epsilon} \left(\theta \left(\int_0^1 max(P_{t-1}(i), W_t) di \right)^{-\epsilon} + (1-\theta)(P_t^*)^{-\epsilon} \right) \tag{61}$$

We also reproduce here for convenience the equilibrium expression for the wage

$$W_t = P_t C_t^{\sigma+\phi} \mathcal{O}_t^{\phi} \tag{62}$$

As in the analogous experiment in BNK, detailed in Appendix B2, we consider a large set of values of P_1^*. For each of these values, in order to find one point on the AS curve we need to find the value of C satisfying the price-setting equation, given (60) and (61). Notice that the wage W appearing in (60) and (61) is different for every point: it is constructed using (62), consistently with prices and consumption at each point.

For low levels of P_1^*, the wage remains below P_0, and non-reoptimizing firms maintain their time-0 price; both curves are identical to the corresponding curves in BNK. For P_1^* bigger then a threshold value $\bar{P}$ (possibly different on the AD and AS curve), W_1 exceeds P_0, and all the non-reoptimizing firms reset their price to match W_1; as explained below, each curve becomes vertical above this threshold.

AS curve

Phase 1: We start with the lowest value of P_1^* and define a flag F_S that we set equal to 0, indicating that the constraint (28) is not binding. We will set it equal to 1 when we find the level of P_1^* such that the constraint binds. As long as $F_S = 0$ we follow steps **AS 1)- AS 5)** described in Appendix B2. At the end of these steps, we compute the value of the wage using (62), with the values of prices and consumption obtained in the previous steps, for each point. Once we find the value of P_1^* (that we define $\bar{P}_{AS}$) such that W_1 is within a tolerance level from P_0, we set $F_S = 1$. We also define $\bar{C}^S$ as the consumption value at this point.

Phase 2: For $P_1^* > \bar{P}_{AS}$ (i.e. if $F_S = 1$) it is easy to see that all prices evolve in parallel, i.e. all relative prices (for all periods) keep the values they had for $P_1^* = \bar{P}_{AS}$ and consumption is equal to $\bar{C}^S$, i.e. *the AS curve becomes vertical.*[12]

AD curve

Phase 1: We start with the lowest value of P_1^* and define a new flag $F_D = 0$. As long as $F_D = 0$ we follow steps **AD 1)- AD 3)** described

[12]Given that the price-setting equation, together with (60)-(62), is solved by $P_1^* = \bar{P}_{AS}$, $P_1^n = P_0$ for the non-reoptimizing firms and $C = \bar{C}^S$, it is easy to check that the same system of equations is also solved by any $P_1^* > \bar{P}_{AS}$, with $P_1^n = P_0 \frac{P_1^*}{\bar{P}_{AS}}$ (and all prices for $t \geq 2$ rescaling by the same factor $\frac{P_1^*}{\bar{P}_{AS}}$), $C = \bar{C}^S$ (and all C_t unchanged for $t \geq 2$). In all these cases we have $P_1^n = W_1$ (i.e. constraint (28) is binding), all prices evolve in parallel and real variables stay constant.

in Appendix B2. At the end of these steps, we compute the value of the wage using (62), with the values of prices and consumption obtained in the previous steps, for each point. Once we find the value of P_1^* (that we define $\bar{P}_{AD}$) such that W_1 is within a tolerance level from P_0, we set $F_D = 1$. We define $\bar{C}^D$ as the consumption value at this point.

Phase 2: For $P_1^* > \bar{P}_{AD}$ it is easy to see that all relative prices keep the values they had for $P_1^* = \bar{P}_{AD}$ and consumption is equal to $\bar{C}^D$, i.e. *the AD curve becomes vertical*.[13]

References

[1] Angeletos, George-Marios, and Chen Lian (2018), "Forward guidance without common knowledge", American Economic Review 108, no. 9: 2477-2512.

[2] Ascari, Guido, and Sophocles Mavroeidis (2022), "The unbearable lightness of equilibria in a low interest rate environment", Journal of Monetary Economics 127: 1-17.

[3] Ascari, Guido, Sophocles Mavroeidis, and Nigel McClung (2023), "Coherence without Rationality at the ZLB."

[4] Bhattarai, Saroj, Gauti B. Eggertsson, and Raphael Schoenle (2018), "Is increased price flexibility stabilizing? Redux", Journal of Monetary Economics 100: 66-82.

[5] Calvo, Guillermo A. (1983),"Staggered prices in a utility-maximizing framework." Journal of monetary Economics 12, no. 3 (1983): 383-398.

[6] Campbell, Jeffrey R., Filippo Ferroni, Jonas DM Fisher, and Leonardo Melosi (2019), "The limits of forward guidance." Journal of monetary economics 108: 118-134.

[7] Campbell, Jeffrey R., Jonas DM Fisher, Alejandro Justiniano, and Leonardo Melosi (2017), "Forward guidance and macroeconomic outcomes since the financial crisis." NBER macroeconomics annual 31, no. 1: 283-357.

[13] Similarly to what observed for the AS curve, given that the Euler equation, together with (60)-(62), is solved by $P_1^* = \bar{P}_{AD}$, $P_1^n = P_0$ for the non-reoptimizing firms and $C = \bar{C}^D$, it is easy to check that the same system of equations is also solved by any $P_1^* > \bar{P}_{AD}$, with $P_1^n = P_0 \frac{P_1^*}{\bar{P}_{AD}}$ and $C = \bar{C}^D$.

[8] Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans (2005), "Nominal rigidities and the dynamic effects of a shock to monetary policy." Journal of political Economy 113, no. 1: 1-45.

[9] Christiano, Lawrence, Martin S. Eichenbaum, and Benjamin K. Johannsen (2018), "Does the New Keynesian Model Have a Uniqueness Problem?" No. w24612. National Bureau of Economic Research.

[10] Christiano, Lawrence J., Martin S. Eichenbaum, and Benjamin K. Johannsen (2024) "Measured Inflation and the New-Keynesian Model." Divisions of Research & Statistics and Monetary Affairs, Federal Reserve Board.

[11] Carlstrom, Charles T., Timothy S. Fuerst, and Matthias Paustian (2015), "Inflation and output in New Keynesian models with a transient interest rate peg." Journal of Monetary Economics 76: 230-243.

[12] Cochrane, John H. (2017), "The new-Keynesian liquidity trap." Journal of Monetary Economics 92: 47-63.

[13] Del Negro, Marco, Marc P. Giannoni, and Christina Patterson (2023), "The forward guidance puzzle." Journal of Political Economy Macroeconomics 1, no. 1: 43-79.

[14] Eggertsson, Gauti B., and Vaishali Garga (2019),"Sticky prices versus sticky information: Does it matter for policy paradoxes?", Review of Economic Dynamics 31: 363-392.

[15] Eggertsson, Gauti B., and Paul Krugman (2012), "Debt, deleveraging, and the liquidity trap: A Fisher-Minsky-Koo approach", The Quarterly Journal of Economics 127, no. 3: 1469-1513.

[16] Eggertsson, Gauti B., and Finn D. Schüle (2024), "The Forward Guidance Puzzle is Not a Puzzle", No. w33180. National Bureau of Economic Research.

[17] Eggertsson, Gauti B., and Michael Woodford (2003), "Optimal monetary policy in a liquidity trap".

[18] Galí, Jordi (2015), "Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications." Princeton University Press.

[19] Galı, Jordi, Mark Gertler, and J. David Lopez-Salido (2001), "European inflation dynamics." European economic review 45, no. 7: 1237-1270.

[20] Hagedorn, Marcus, Jinfeng Luo, Iourii Manovskii, and Kurt Mitman (2019), "Forward guidance", Journal of Monetary Economics 102: 1-23.

[21] Mankiw, N. Gregory, and Ricardo Reis (2002), "Sticky information versus sticky prices: a proposal to replace the New Keynesian Phillips curve." The Quarterly Journal of Economics 117, no. 4: 1295-1328.

[22] McKay, Alisdair, Emi Nakamura, and Jón Steinsson (2016), "The power of forward guidance revisited", American Economic Review 106, no. 10: 3133-3158.

[23] Sbordone, Argia M (2002), "Prices and unit labor costs: a new test of price stickiness." Journal of Monetary economics 49, no. 2: 265-292.

[24] Smets, Frank, and Rafael Wouters (2007), "Shocks and frictions in US business cycles: A Bayesian DSGE approach." American economic review 97, no. 3: 586-606.

[25] Werning, Ivan (2012), " Managing a liquidity trap: monetary and fiscal policy", Manuscript.