From Labor to Intermediates: Firm Growth, Input Substitution, and Monopsony

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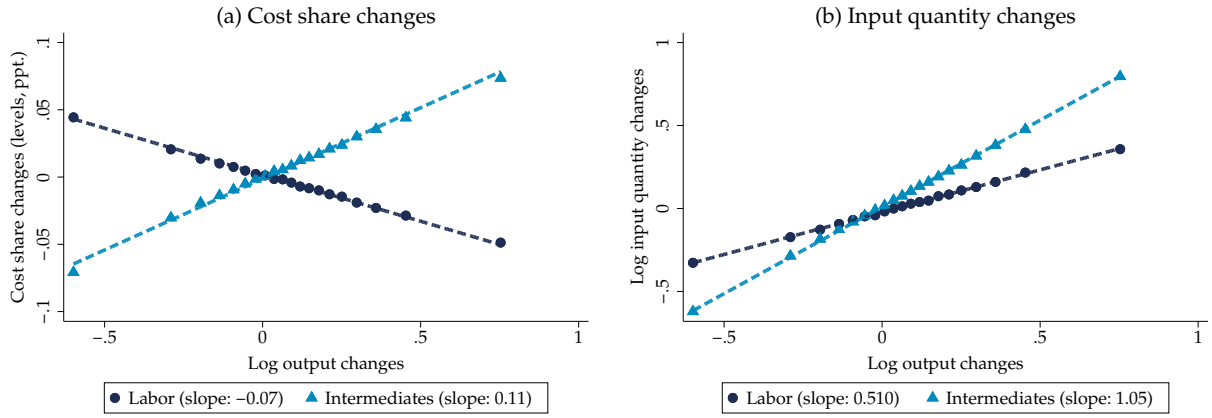
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Abstract

We document and dissect a stylized fact about firm growth: the shift from labor to intermediate inputs. This shift occurs in input quantities, cost and output shares, and output elasticities. We establish this regularity in firm data for Germany and in firm (and industry) data for 11 (20) additional countries, and also in response to exogenous product demand shocks. We explain this regularity through a parsimonious model featuring an elasticity of substitution between intermediates and labor above one, and an increasing shadow price of labor (monopsony or adjustment costs). Our firm growth regressions identify a labor-intermediates substitution elasticity between 1.8 and 4.2. Labor-intermediates substitution also accounts for much of the labor share decline that we document accompanies firm and industry growth.

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Figure 1: The shift from labor to intermediate inputs: within-firm 4-year changes in cost shares and log input quantities against log output growth.



Notes: The figure reports binned scatter plots of the within-firm changes in labor and intermediate cost shares and log input quantities (labor in headcounts, intermediates in deflated expenditures) against log output changes (deflated sales). All panels depict 4-year differences and control for industry-year fixed effects. The data cover German manufacturing firms from 1995 to 2017; further details on the datasets, samples, and empirical specifications are discussed in the main part of the paper.

1 Introduction

How do firms grow? Standard Cobb-Douglas production and competitive input markets predict that firms simply scale up inputs proportionately, with constant cost shares. Our paper dissects and rationalizes systematic departures from this canonical benchmark. Figure 1 illustrates the stylized fact at the core of our analysis: as firms grow, they shift production inputs from labor to intermediate inputs. This shift from labor to intermediates holds across the board: in terms of cost shares and input quantities (as depicted) as well as output shares (i.e., the labor share) and output elasticities.[1] It holds across firm types, industries, and countries. To account for this set of facts, we offer a *parsimonious* model of firm growth.[2] The model leverages—and our analysis provides new estimates for—two features: (i) a labor-intermediates substitution elasticity above one, and (ii) an increasing shadow price of labor relative to intermediates, most likely due to monopsony. This paper is the first dedicated study of how these two forces jointly shape firm growth and how firm growth episodes, in turn, offer new insights into these two core parameters.

We primarily study firm growth in German manufacturing micro data. The data include firm-product-specific sales and real quantities, allowing us to construct firm-

[1]For manufacturing, intermediate inputs primarily consist of product components (e.g., car seats manufactured by other companies) and materials. As we discuss below, temporary agency labor or services play a much smaller role given their small cost shares.

[2]Here, firm growth should be thought of as driven by input-neutral shifters, such as in TFP growth or product demand. Our model formalizes this through cost minimization, taking scale as given, and our empirical analyses include an instrument for growth that relies on product demand shifts.

specific output prices and to address the price biases in output elasticities derived from production function estimation (De Loecker et al., 2016, Bond et al., 2021). We draw on OLS regressions as well as an IV strategy using export demand shocks as a firm growth shifter unrelated to factor-biased price or technological changes. We also establish the shift from labor to intermediates in nonparametric scatter plots of raw firm-level data and, as a complementary analysis, we also infer output elasticities from cost shares.

Additionally, we confirm our main results in administrative firm-level data from 11 additional countries as well as in industry-level data for 20 European countries using harmonized data from the Competitiveness Research Network (CompNet) for manufacturing *and* non-manufacturing industries—and for a subset of outcomes also for US manufacturing industries. We estimate our regressions at various horizons, from 1- to 10-year changes, and across different size and age classes of firms. As our evidence does not point toward a major role of production function non-homotheticities and/or fixed labor inputs in explaining our results, we rationalize our findings with a parsimonious production perspective on input substitutability as described below.

The shift from labor to intermediates in quantities and the reduction in output elasticities of labor accounts for *about one half to one third* of the negative effect of firm growth on the firm-level labor share. (In log changes, the labor share is equal to the output elasticity minus markups and markdowns.) Hence, our framework provides a novel, technological explanation for the negative association between the labor share and firm growth, complementing existing approaches that focus on large firms' product or labor market power. Importantly, unlike the existing literature on cross-sectional firm *size* gradients and concentration (e.g., Autor et al., 2020), we focus on *firm growth* in panel data.[3] Therefore, our additional results on labor shares in growing firms also resonate with the empirical study of Kehrig and Vincent (2021), who study firm growth dynamics in labor shares and highlight the role of demand-side factors (markups).

To rationalize the *full* set of facts, we propose a deliberately *parsimonious* model of firm growth. It leverages the combination of two features typically studied in isolation: substitutability between intermediates and labor, and an increasing relative shadow price of labor. In a nutshell, under an increasing relative shadow price of labor, as firms grow, they lower their demand for labor relative to intermediate inputs. If labor and intermediates are substitutes, this shift toward intermediates translates into a lower output elasticity of labor relative to intermediates. This, in turn, lowers the labor cost share and

[3]We find much smaller (but qualitatively similar) cross-sectional differences in output elasticities by firm *size* than by firm *growth*, perhaps due to firm-specific permanent factors shaping input intensities and output elasticities.

the labor share in output.

Substitution elasticities above one imply that as the intermediate-labor input quantity ratio increases, output elasticities shift from labor to intermediates—consistent with our firm growth facts. In fact, we formally show that this comovement—expressed as the ratio of our reduced-form regression coefficients on firm growth—identifies the substitution elasticity (i.e., elasticity of relative input quantity changes to relative shadow price changes). Our approach to estimating substitution elasticities is new in that it relies on growth-driven within-firm panel comovement in input quantities and output elasticities (including IV-based variation).[4]

Quantitatively, our firm growth regressions identify substitution elasticities well above one, ranging from 1.8 to 2.7 (OLS) and from 3.8 to 4.2 (IV). Our paper situates these values in a systematic meta-analysis of existing estimates. We also apply our (OLS) identification of substitution elasticities to firm micro data for additional 11 countries (and for 20 countries in industry data). For all those countries, we provide new, harmonized estimates of labor-intermediate substitution elasticities, which consistently exceed unity.

To account for *why* growing firms choose to shift their input mix from labor to intermediates, our model features an increasing relative shadow price of labor. A natural source is a finitely elastic firm-specific labor supply curve, i.e., firms holding monopsony power over labor. (Alternatively, adjustment costs may play a role, at least in the short run.) In fact, for a given substitution elasticity and under the assumption of firm cost minimization, our input mix estimates identify the elasticity of the labor (shadow) price to the firm (i.e., the inverse labor supply elasticity). We offer a range of implied elasticities across our specifications and wage measurement approaches, and compare them to existing estimates using data from the review of Sokolova and Sorensen (2021). Our results also imply that supply elasticities appear higher in the long run (i.e., labor markets are more competitive), for our IV estimates, and when we net out markdown shifts or use direct (average) wage estimates rather than implied ones.

[4]Existing studies often combine within-firm/-industry and cross-sectional variation and use arguably exogenous price changes as instruments. We exclusively use within-firm variation, which accounts for firm-specific fixed confounders. Using comovements of input quantities and output elasticities is equivalent to using comovements in input prices and input quantities (due to firms' first order conditions). Existing studies are difficult to compare to our approach, as they typically estimate substitution elasticities between intermediates and a capital-labor bundle rather than labor. Three studies that directly estimate a labor-intermediate substitution elasticity are Doraszelski and Jaumandreu (2018), Huneeus et al. (2022), and Chan (2023), who report estimates ranging from 0.12 to 1.8, 1.05 to 1.62, and 1.6 to 9.6, respectively. Moreover, existing studies also typically assume competitive input markets and do not consider shadow price changes, which are the actual relevant price changes for factor demand adjustments. In this context, Huneeus et al. (2022) also show that accounting for input heterogeneity when constructing intermediate and labor input prices increases estimated substitution elasticities. We provide further discussion in Section 5.

Our paper focuses on *firm*-level growth. Studying aggregate implications would necessitate an input-output network analysis or open economy perspective. However, we confirm that, qualitatively, all our firm-level patterns transfer to the industry level, drawing on CompNet data for 20 countries and the US. Industry-level inputs, cost shares, and output elasticities shift from labor to intermediates, and these inputs are substitutes also at the industry level. Strikingly, at the industry level, the negative association between output growth and labor shares is fully accounted for by labor output elasticities, with no role for markups or wage markdowns. Hence, reductions in the output elasticity of labor may act as a new, production-function-based factor in aggregate labor share declines—but we caveat that potential aggregation is beyond the scope of our paper's focus on micro growth dynamics of individual firms.[5]

Additional related literature. We add to existing studies that document factor substitution in response to firm-specific shocks and trends—though the existing literature has largely focused on capital-labor substitution. Acemoglu and Restrepo (2020) study substitution of labor with robots. Lashkari et al. (2024) analyze how non-homotheticities in the production function cause firms to shift toward higher IT-capital intensities. Hubmer and Restrepo (2021) use Compustat data and show that capital-output elasticities have increased in the largest firms in the most recent years, consistent with automation. Karabarbounis and Neiman (2014) discuss the role of declining capital prices in the global labor share decline. Huneeus et al. (2022) estimate that labor and intermediates are substitutes and show that firms with access to cheaper suppliers have lower labor shares. Dhyne et al. (2022) show that labor adjustments to demand shocks are weaker compared to intermediate inputs and emphasize the role of fixed cost in labor. Our paper also rationalizes these empirical patterns *and* additional ones specific to our mechanism, offering a complementary but distinct mechanism, based on labor-intermediates substitution.[6] Castro-Vincenzi and Kleinman (2024) use aggregate data to study how rising material prices may lower labor shares if labor and materials are complements. We focus on firm-level mechanisms and firm growth, and find empirical evidence showing that intermediates and labor are substitutes and that, as firms and industries grow, shadow prices of labor relative to intermediates increase.

Our results extend evidence rejecting Cobb–Douglas production (e.g., Chirinko et al., 2011, Raval, 2019). We confirm these findings and provide a novel micro-foundation

[5]Similarly, Elsby et al. (2013) argue that China's accession to the WTO and the resulting offshoring of labor-intensive tasks have contributed to aggregate labor share declines in many advanced countries.

[6]While we do not aim to adjudicate between fixed costs and the substitution mechanism, our paper shows that several firm-growth patterns at the core of our analysis cannot be rationalized by at least basic versions of a fixed-cost model.

for endogenous technology (i.e., output elasticity) differences: firm growth changes relative input prices through differential input supply elasticities, inducing shifts in output elasticities under flexible production functions such as CES—beyond non-homotheticities highlighted by Lashkari et al. (2024). This mechanism, for example, offers an alternative micro-foundation for why larger/growing firms outsource a greater share of production (Fort, 2017).

In parallel work, Chan et al. (2024) show that larger firms exhibit higher returns to scale cross-sectionally, due to higher output elasticities of intermediate inputs, and study the resulting efficiency losses under financial frictions. Our two papers complement each other. We focus on within-firm shifts from labor to intermediates as firms grow, analyze implications for firm and industry labor shares, and provide a parsimonious micro-foundation for our findings based on substitution elasticities and imperfect input markets.

Additionally, by focusing on input mix and production function dynamics accompanying firm growth, our study complements existing studies of firm growth using one-input (labor) models and measures of firm size (e.g., Sterk et al., 2021), and we leave firm life cycles for future research because we lack information on firm age in our main dataset. However, using additional data from other countries, we show that our results hold for young and mature growing firms.

Outline. Section 2 provides a formal model and derives predictions. Section 3 describes the firm-level data, sample, and production function estimation. Section 4 uses German firm-level data to empirically establish the shift from labor to intermediates. Section 5 interprets the results quantitatively, identifies parameters of interest, and discusses alternative explanations for our findings. Section 6 draws implications of our findings for firm-level labor shares. Section 7 uses additional firm- and industry-level data to transfer our analysis to other European countries, sectors outside of manufacturing, and US manufacturing industries. Section 8 concludes.

2 Theory

2.1 Firm Optimization

Production function. We consider a constant returns to scale (CRS) constant-elasticity-of-substitution (CES) production function of firm i in period t that transforms labor (L_{it}),

intermediates (M_{it}), and capital (K_{it}) into output (Q_{it}):

$$Q_{it} = \Omega_{it}\Lambda_i^K K_{it}^{1-\kappa}\left(\Lambda_i^{LM}\alpha_i^L L_{it}^{\frac{\sigma-1}{\sigma}} + \Lambda_i^{LM}\alpha_i^M M_{it}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}\kappa}. \quad (1)$$

Ω_{it} represents firm productivity. α_i^L, α_i^M, Λ_i^{LM}, and Λ_i^K are distribution parameters. Capital enters multiplicatively with a Cobb-Douglas exponent $(1-\kappa)$. Labor and intermediates enter as a CES nest with substitution elasticity σ, forming a labor-intermediate bundle with Cobb-Douglas exponent κ. We choose this homothetic CES specification for its analytical simplicity, and as it can fully explain our main empirical finding of a shift from labor to intermediates in production through the substitution elasticity σ (Section 5.3 discusses non-homotheticities).

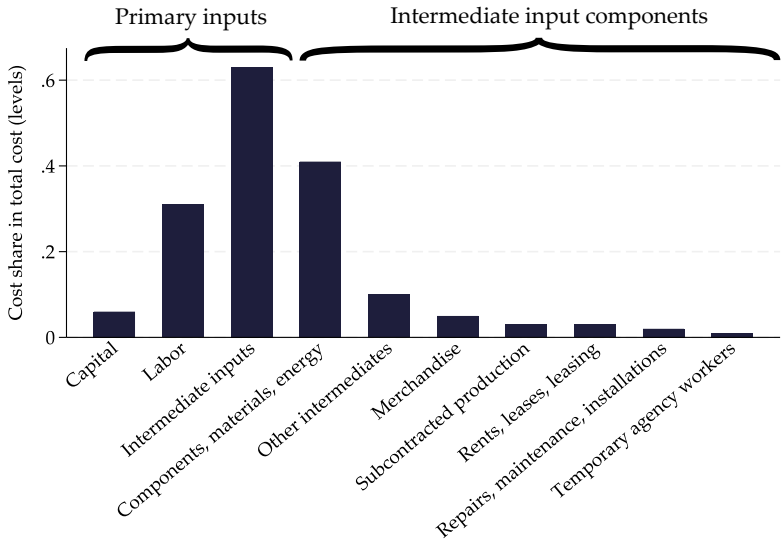
The specification foreshadows our main result, that the substitution from labor to intermediates is the primary pattern accompanying firm growth, with capital intensity shifts being comparatively unimportant. In fact, in the production function in Eq. (1), the capital output elasticity is constant. In our empirical analysis, we will allow output elasticities (and cost shares) of all inputs (including capital) to vary. As the capital output elasticity is small (see below), its level changes in response to output growth are also small, even with sizable percent effects, and one could indeed think of $(1-\kappa)$ as being approximately constant in our context.

Cost shares. Figure 2 details input cost shares for German manufacturing firms (data described in Section 3). The average capital cost share is just 6% (which equals $1-\kappa$ under perfect input markets).[7] The low average capital share, along with our empirical findings (showing relatively small capital cost share and output elasticity changes in levels), further motivates our focus on labor-intermediate substitution. The figure also decomposes the intermediate input cost share. Two thirds of intermediate inputs consist of external product components (e.g., car seats), materials, and energy. Of the remainder, half are classified as "other intermediate inputs," which include services like transport, postage, insurance, and legal services. The other half comprises merchandise, rents, and subcontracted production steps (i.e., a firm contracts another firm to perform specific production tasks). Temporary agency labor represents only about 1% of total costs.

Cost minimization. We rely on cost minimization at a given output level, which introduces firm size and growth as quasi-parameters and permits us to cast our reduced-form empirical regression equations as structural equations. This section focuses on key equations; Appendix C.1 details derivations.

[7]Similarly, under a translog production function, we find a capital output elasticity of 0.12. In our firm-level data, labor's average share of value added is 80%.

Figure 2: Cost shares of production inputs.



Notes: The figure reports average firm-level cost shares for capital, labor, intermediate inputs, and the components of intermediate inputs. Capital costs are approximated as 8% of the capital stock following Dhyne et al. (2024) and the approach used in the CompNet data (CompNet, 2023), which we use in Section 7. Separate information for temporary agency worker cost shares is available from 1999. All other variables are available from 1995. The data cover German manufacturing firms and is described in Section 3.

We allow for imperfect product market competition and input market frictions, such as monopsony power and adjustment costs, creating wedges between the marginal costs of production inputs and their unit costs. We discuss this further in Section 2.2. Cost minimization implies a FOC for each input, labor, capital, and intermediates, $X = \{L, K, M\}$,

$$P_{it}^X \underbrace{\left(1 + \frac{\partial P_{it}^X}{\partial X_{it}} \frac{X_{it}}{P_{it}^X} + \frac{\partial \chi^X}{\partial X_{it}}\right)}_{\gamma_{it}^X} = \lambda_{it} \frac{\partial Q_{it}}{\partial X_{it}}. \tag{2}$$

P^X is the price for input X (e.g., wage for labor L), χ^X is an adjustment cost function, λ_{it} is marginal cost, and γ_{it}^X is the input price wedge, such that $P_{it}^X \gamma_{it}^X$ is the input *shadow price*. We express the adjustment cost function in flexible "quasi-static" terms (as in Bond et al., 2021, see also Appendix C.1) without formulating the dynamic problem (this sidesteps continuation-related terms) to highlight the key take-away from Eq. (2): monopsony power and adjustment costs raise marginal input costs (the overall shadow price) beyond an input's price, P_{it}^X.[8]

Using the production function and cost minimization, we derive two key equations. First, we show how the substitution elasticity is related to and can be identified by the

[8]Under profit maximization, γ_{it}^X equals the wedge between marginal revenue products and input costs.

co-movement of relative output elasticity and input ratio changes:

$$\frac{\frac{\partial Q_{it}}{\partial L_{it}}}{\frac{\partial Q_{it}}{\partial M_{it}}} = \frac{\alpha_i^L}{\alpha_i^M}\left(\frac{L_{it}}{M_{it}}\right)^{\frac{-1}{\sigma}} \Leftrightarrow \frac{\theta_{it}^L}{\theta_{it}^M} = \frac{\alpha_i^L}{\alpha_i^M}\left(\frac{L_{it}}{M_{it}}\right)^{\frac{\sigma-1}{\sigma}} \quad \Rightarrow \frac{\sigma-1}{\sigma} = \frac{\Delta\ln(\theta_{it}^L) - \Delta\ln(\theta_{it}^M)}{\Delta\ln(L_{it}) - \Delta\ln(M_{it})}, \quad (3)$$

where, in the last step, we have taken changes within firms (and used the assumption of constant α_i levels within the firm, departures from which we discuss below). $\theta^X = \frac{\partial Q}{\partial X}\frac{X}{Q}$ is the output elasticity of input $X = \{L, K, M\}$. *Importantly*, Eq. (3) can also be directly derived from cost minimization (without specifying a production function) as an elasticity of relative input quantity changes to relative shadow prices changes, holding capital inputs and prices fixed (see Appendix C.3). If labor and intermediates are substitutes ($\sigma > 1$, consistent with our evidence below), Eq. (3) implies that a decrease in the labor-intermediates ratio will decrease the ratio of the labor output elasticity to the intermediate output elasticity. With Cobb-Douglas ($\sigma = 1$), no such change would occur, and complements ($\sigma < 1$) would imply the opposite.

Second, inserting Eq. (2) into Eq. (3) recovers the implied shadow price ratio change that rationalizes a given shift in input mix at the firm level for a given level of σ:

$$\Delta\ln\left(P_{it}^L\gamma_{it}^L\right) - \Delta\ln\left(P_{it}^M\gamma_{it}^M\right) = \frac{\Delta\ln(L_{it}) - \Delta\ln(M_{it})}{-\sigma}. \quad (4)$$

Hence, for any $\sigma > 0$, an increase in the shadow price of labor relative to intermediates leads to a decrease in the labor-to-intermediate ratio, with σ scaling this relationship (absent input-biased shifts). Analogously, σ guides the size of the implied input price ratio shift that must have rationalized a given shift in the input mix. We will use Eqs. (3) and (4) in our empirical analysis to estimate substitution elasticities and the implied shadow price ratios that accompany the production function dynamics of firm growth.

Cost and output shares. Changing output elasticities have important implications. Appendix C.1 shows that using Eq. (2) for all inputs determines an input X's cost share:

$$CS_{it}^X = \frac{P_{it}^X X_{it}}{P_{it}^X L_{it} + P_{it}^M M_{it} + P_{it}^K K_{it}} = \frac{\frac{\theta_{it}^X}{\gamma_{it}^X}}{\frac{\theta_{it}^L}{\gamma_{it}^L} + \frac{\theta_{it}^M}{\gamma_{it}^M} + \frac{\theta_{it}^K}{\gamma_{it}^K}}. \quad (5)$$

Similarly, reformulating Eq. (2) defines an input's output share (e.g., the labor share in output):

$$OS_{it}^X = \frac{P_{it}^X X_{it}}{P_{it}Q_{it}} = \frac{\theta_{it}^X}{\mu_{it}\gamma_{it}^X}. \quad (6)$$

$\mu_{it} = \frac{P_{it}}{\lambda_{it}}$ is the output price over (total) marginal cost markup. γ_{it}^X is the wage markdown

(the wedge between the input's marginal cost and price). Thus, a decrease in an input's output elasticity leads to a reduction in both the cost share and output share of an input.[9] Eq. (3) highlights that such a decline in the labor output elasticity can result from changes in the labor-intermediate input mix (holding fixed returns to scale and if $\sigma \geq 1$).

2.2 Predictions for Firm Growth

To make predictions about firm growth, we require additional structure and model *firm-specific* labor and intermediate supply as isoelastic, $P_{it}^X = a_{it}^X X_{it}^{\varepsilon_i^X}$ for $X = \{L, M\}$, where a_{it}^X is a baseline input price normalization and ε_i^X is the inverse firm-specific input supply elasticity (i.e., a finite supply elasticity indicates monopsony in input markets).[10] For simplicity, we also assume constant markups, that a firm's capital shadow prices do not depend on its own capital demand, and that γ_{it}^L and γ_{it}^M are determined by the supply functions, which is sufficient to make growth predictions consistent with our empirical results. Importantly, in our empirical analysis, we relax these assumptions and allow for firm- and time-specific markups and wage markdowns (resulting from monopsony/finitely elastic firm-specific labor supply curves, or adjustment costs).

Inserting the input supply functions into Eq. (2) determines labor and intermediate demand as functions of marginal products and parameters (see Appendix C.2):

$$X_{it} = \left(\frac{\lambda_{it}\alpha_i^X}{(1+\varepsilon^X)a_{it}^X}\right)^{\frac{1}{\varepsilon^X}} \left(\frac{\partial Q_{it}}{\partial X_{it}}\right)^{\frac{1}{\varepsilon^X}} \quad \text{for} \quad X = \{L, M\}. \tag{7}$$

Inserting the production function and expressing the resulting equation in terms of the labor-intermediates ratio yields:

$$\frac{L_{it}}{M_{it}} = \varrho_{it}\lambda_{it}^{\frac{\sigma+\kappa-1}{\kappa}\left(\frac{1}{\sigma\varepsilon^L+1}-\frac{1}{\sigma\varepsilon^M+1}\right)} Q_{it}^{\left(\frac{1}{\sigma\varepsilon^L+1}-\frac{1}{\sigma\varepsilon^M+1}\right)}. \tag{8}$$

ϱ_{it} (expression in Appendix C.2) captures effects unrelated to firm growth (i.e., parameters, baseline prices, and TFP). $\lambda_{it}^{\frac{\sigma+\kappa-1}{\kappa}\left(\frac{1}{\sigma\varepsilon^L+1}-\frac{1}{\sigma\varepsilon^M+1}\right)}$ captures the effect of marginal costs, which increase with quantities produced due to increasing supply curves. The key insight from Eq. (8) is that the response of *input ratios* (and thus output elasticities and cost and output

[9] We can express θ_{it}^X in terms of returns to scale ($RTS_{it} = \theta_{it}^L + \theta_{it}^M + \theta_{it}^K$) and the input output elasticity relative to other output elasticities, $\theta_{it}^X = \frac{\theta_{it}^X}{RTS_{it}} RTS_{it}$, which separates returns to scale from the relative technological importance of inputs vis-à-vis other production factors.

[10] Strictly speaking, our analysis does not require firms to exploit upward-sloping firm-specific labor supply; what matters is that shadow input prices increase as output grows. In our empirical analysis we study within-firm changes and include industry-year fixed effects.

Table 1: Growth predictions: different substitution and supply elasticities (ceteris paribus).

	$\sigma < 1$	$\sigma = 1$	$\sigma > 1$	
L **less** elastic than M $(\varepsilon^L)^{-1} < (\varepsilon^M)^{-1}$	$\downarrow$	$\downarrow$	$\downarrow$	
	$\uparrow$	$=$	$\downarrow$	
	$\uparrow$	$=$	$\downarrow$	
	$\uparrow$	$=$	$\downarrow$	
L **as** elastic as M $(\varepsilon^L)^{-1} = (\varepsilon^M)^{-1}$	$=$	$=$	$=$	$\Delta \ln L_{it}/M_{it}$
	$=$	$=$	$=$	$\Delta \ln \theta^L_{it}/\theta^M_{it}$
	$=$	$=$	$=$	$\Delta \ln CS^L_{it}/CS^M_{it}$
	$=$	$=$	$=$	$\Delta \ln OS^L_{it}/OS^M_{it}$
L **more** elastic than M $(\varepsilon^L)^{-1} > (\varepsilon^M)^{-1}$	$\uparrow$	$\uparrow$	$\uparrow$	
	$\downarrow$	$=$	$\uparrow$	
	$\downarrow$	$=$	$\uparrow$	
	$\downarrow$	$=$	$\uparrow$	

Notes: Ceteris paribus refers to constant returns to scale and non-changing market imperfections with output growth ($\Delta \ln Q_{it}$) (but our empirical treatment will relax those expositional assumptions). The shaded area represents the region of the parameter space consistent with our empirical findings. These assumptions are relaxed in the empirical analysis.

shares) to an increase in output (i.e., firm growth) depends on $\frac{1}{\sigma\varepsilon^L+1} - \frac{1}{\sigma\varepsilon^M+1}$.

Growth predictions. Table 1 summarizes our predictions for various assumptions about labor-intermediate substitution elasticities and relative input supply elasticities.[11] This section's discussion remains qualitative. Our quantitative interpretation is presented in Section 5, where we back out the implied values for σ and labor supply elasticities, $\epsilon^L = (\varepsilon^L)^{-1}$, identified by our empirical estimates.

Potential cases. We use a CRS Cobb-Douglas production function with perfect markets as a benchmark to fix ideas (CES with $\sigma = 1$), and consider departures under input complementarity ($\sigma < 1$) and substitutability ($\sigma > 1$). We consider three cases for firm-specific input supply elasticities: labor is more or less elastic than intermediates ($(\varepsilon^L)^{-1} > (\varepsilon^M)^{-1}$ and $(\varepsilon^L)^{-1} < (\varepsilon^M)^{-1}$), and the inputs are equally elastic ($(\varepsilon^L)^{-1} = (\varepsilon^M)^{-1}$).

The shaded, top-right area in Table 1 highlights the combination of parameters implied by our empirical evidence: substitutability and labor being less elastically supplied to the firm than intermediates.

Rejected by evidence: equally elastic supply. We first consider the benchmark of equal supply elasticities, which nests the competitive input prices case (the middle row of Table 1). In this case, input ratios, output elasticities, cost shares, and output shares are constant as firms grow for any value of the substitution elasticity.

[11] We focus on the empirically relevant case of $\sigma + \kappa > 1$, where κ is approximately the sum of labor and intermediate output elasticities or, under constant returns to scale and perfect input markets, the sum of labor and intermediate cost shares. κ is therefore close to unity (see Figure 2 and Appendix Table A.1). Our estimates of σ are well above unity (see Table 3).

Rejected by evidence: Cobb-Douglas and less elastic labor supply. Second, consider the case of an increasing relative shadow price of labor (the top row). With Cobb-Douglas ($\sigma = 1$), firms substitute from labor to intermediates, but output elasticities and thus cost and output shares remain unchanged. That is, even though labor becomes relatively more expensive as firms grow, the quantity substitution from labor exactly offsets the price increase, leaving labor cost and output shares constant.

Rejected by evidence: complements and less elastic labor supply. Now, consider that labor and intermediates are complements ($\sigma < 1$). In this case, a relative reduction in labor quantities increases the output elasticity of labor relative to intermediates. Intuitively, firms reduce their labor-intermediate ratio less than one-to-one with the input price ratio increase, causing cost and output shares of labor to *increase*.

Case supported by evidence: substitutes and monopsony. Input substitutability ($\sigma > 1$) implies the opposite: the reduction in the labor-intermediate input quantity ratio translates into declines in output elasticity ratios and cost and output shares of labor. As labor becomes expensive, firms substitute away from it by more than one-to-one. This scenario aligns with our empirical evidence discussed below (and Figure 1).

Rejected by evidence and implausible: more elastic labor supply. For completeness, we consider the case where intermediate inputs are less elastically supplied than labor. In this scenario, the opposite sign for quantities emerges across the board, irrespective of σ.

Discussion: non-constant input wedges/supply elasticities and markups. As noted, the previous predictions hold ceteris paribus with respect to input wedges and markups. This simplified framework can fully explain our findings. However, in our empirical analysis, we will measure both markups and relative input wedges and explore their dynamics in response to firm growth in Section 6.

Markups tend to rise with firm growth, which dampens input responsiveness to growth relatively more for the more elastically supplied input (this can be seen in Eq. (8) as $\lambda_{it} = \frac{P_{it}}{\mu_{it}}$). However, empirically, this attenuation is small and does not overturn the predicted reduction in the labor-intermediate input ratio, because markups only increase moderately as firms grow.

Regarding changing input wedges, we find that they *amplify* the shift from labor to intermediates, and, thus, increase estimated substitution elasticities, as the relative labor wedge increases with firm growth (intuitively, the difference in supply elasticities in Eq. (8) gets larger as firms grow).

Discussion: factor-augmenting productivity. In Eq. (1), α_i^L and α_i^M are assumed to be constant within the firm. If these parameters changed at different rates within firms, the

estimation of the substitution elasticity would need to account for such changes (i.e., they would enter Eqs. (3) and (4)).[12] Similar to other work, in our empirical section, we address potential biases from unobserved factor-augmenting productivity shocks. We control for industry-year FE to absorb these shocks at the industry level. To account for firm-specific factor-augmenting productivity shocks, we use an IV approach (factor-neutral demand shocks)—where causality goes from factor-neutral output changes to output elasticities (we also discuss non-homotheticities).

3 Firm-level Data

We now describe the German firm-level data and the production function estimation.

Production data. The firm-product-level panel data for Germany's manufacturing sector cover the period of 1995-2017. The data are collected and supplied by the German Statistical Offices. The unit of observation is firms (not establishments or plants, although 90% of firms are single-plant firms).[13] The data include employment, investment (includes tangible and intangible assets), intermediate input costs, wage bills, depreciation, and total gross output. Importantly, as a rare feature, the data also include *real product quantities and sales* at a ten-digit product classification, allowing us to compute firm-product-level prices.[14] The data cover 40% of firms with at least 20 employees and consist of a rotating panel that is redrawn every 4-5 years. Labor is defined as the number of employees on September 30th. All other variables pertain to the full calendar year.[15] We provide details on data access and preparation, capital stock construction, variable definitions, and summary statistics in Appendix D.

Supplementary trade data. To provide *causal* evidence on the relationship between output growth (i.e., growth in response to an input-neutral shifter) and our variables of interest, we merge bilateral trade flows from the United Nations Comtrade Database at the firm-product-year level to the German micro data (1995 to 2017). We map product codes in both datasets into the PRODCOM2002 classification using official concordance tables following the code of Bräuer et al. (2023). As described below, we will use the IV approach by Hummels et al. (2014) to instrument output changes with foreign export demand to

[12] For instance, if α_{it}^L and α_{it}^M entered as time-varying factor-specific technologies, Eq. 3 would change to: $\frac{\sigma-1}{\sigma} = \frac{\Delta \ln(\theta_{it}^L) - \Delta \ln(\theta_{it}^M)}{\Delta \ln(\alpha_{it}^L) - \ln(\alpha_{it}^M) + \Delta \ln(L_{it}) - \Delta \ln(M_{it})}$.

[13] In this dataset, firms are defined as legal units, referring to the smallest legally independent unit that keeps accounts for commercial or tax purposes.

[14] Product examples: "Tin sheets and tapes, thicker than 0.2mm", "Workwear: long trousers, men, cotton."

[15] We will estimate results for 1-, 4-, and 10-year changes—assuaging concerns about timing differences between labor and other variables.

study how inputs respond to exogenous output changes (more precisely, output shifts in response to input-neutral product demand shifters).

Production function estimation. We use two complementary approaches that allow for time-varying and firm-specific output elasticities. Within the CES framework, we can estimate output and thus substitution elasticities from cost shares. Under constant returns to scale and perfect input markets, cost shares equal output elasticities, $\theta_{it}^X = \frac{P^X X}{\sum_{X'} P^{X'} X'}$ (see Eq. (5)). While these are typical assumptions in cross-sectional settings, for our within-firm analyses of changes, it is sufficient to assume *non-changing* returns to scale and input wedges (e.g., from iso-elastic labor supply as in Section 2)—i.e., our changes-based specification (including Eq. (10)) relaxes conventional assumptions when using cost share approaches that assumes away such features.

We produce all results with the cost share specification, but our preferred approach estimates a translog production function, which permits more flexible forms of imperfect labor markets (and non-constant returns to scale) and which can approximate more flexible production functions, but also a CES (see Appendix Figure A.2).[16] We find that input wedges increase with firm growth, causing an upward bias in estimated substitution elasticities in the cost share approach (but cost share and translog results are still *qualitatively* aligned). The translog specification is therefore a more conservative baseline. Importantly, even under a translog, the resulting substitution elasticities remain fully consistent with Eq. (3) as we can derive this equation directly from cost minimization (see Appendix C.3).[17] The translog specification is given by:

$$\begin{aligned}\tilde{q}_{it} = \beta_j^l l_{it} + \beta_j^k k_{it} + \beta_j^m m_{it} + \beta_j^{ll} l_{it}^2 + \beta_j^{kk} k_{it}^2 + \beta_j^{mm} m_{it}^2 + \\ \beta_j^{lk} l_{it} k_{it} + \beta_j^{lm} l_{it} m_{it} + \beta_j^{km} k_{it} m_{it} + \beta_j^{lkm} l_{it} k_{it} m_{it} + \omega_{it} + \nu_{it},\end{aligned} \tag{9}$$

where lower case letters denote logs. ν_{it} is an i.i.d. error term. $\tilde{q}_{it}$ denotes log firm sales deflated by a *firm-specific* price index that we derive from firm-product-level price information. ω_{it} is log total factor productivity. Labor enters Eq. (9) in quantities, intermediates and capital enter as expenditures deflated by 2-digit-industry-year-specific deflators. The production function is estimated for each industry, j, separately (NACE Rev. 1.1 two-digit).

We detail our production function estimation approach in Appendix E, where we also discuss how we address several recently highlighted biases to production function

[16] Using simulated data, Appendix Figure A.2 shows that a translog approach recovers the true output elasticities if there is an underlying true CES production function.

[17] Moreover, the translog function can be viewed as an approximation to any twice-differentiable production function.

estimation. Most importantly, we apply a *correction for price biases (De Loecker et al., 2016) by constructing a firm-specific output price index from our firm-product-level price data* as in Eslava et al. (2004) and by additionally including an input price control function, which proxies for unobserved input price variation using information on firms' output prices and market shares. The latter follows the firm-level adaption of the control function method in De Loecker et al. (2016) by Mertens (2022).[18] As described in more detail in Appendix E, including the input price control function captures, among others, quality differences in inputs that create (unobserved) input price variation across firms. Importantly, we explicitly include labor quantities rather than expenditures in Eq. (9) as labor markets are imperfect (see also Footnote 18).

Using the estimated coefficients from Eq. (9) and the information on input levels, we compute output elasticities for each firm-year observation as $\theta_{it}^{X} = \frac{\partial \tilde{q}_{it}}{\partial x_{it}}$ for input $X = \{L, M, K\}$. As production function coefficients are industry-specific, output elasticities vary across firms within an industry due to heterogeneity in input quantities between firms and across time and vary across industries additionally due to different coefficients (the cost-share approach allows for more flexibility).

While estimating the translog production function allows for imperfect product and input markets and non-constant returns to scale, the identification relies on Hicks-neutrality to formulate a control function for productivity based on flexible production inputs (materials, product components, and energy expenditures; see Appendix E).[19] This is exactly where the main advantage of our alternative cost share approach comes in: it can account for non-Hicks-neutral productivity (at the cost of imposing a constant degree of input market imperfections). We view it as reassuring that the cost share and translog approach both yield qualitatively similar results, despite very different assumptions.

Summary statistics. Appendix Table A.1 provides sample summary statistics.

[18] This approach relies on the positive correlation between output and input prices (e.g., due to high quality outputs requiring high quality inputs). To account for the dependence of input decisions on productivity, we utilize a control function approach similar to Wooldridge (2009) and proxy productivity with information on expenditures on materials, product components, and energy. The control function for productivity also includes (lagged) wages (in addition to the input price control function), which helps address some of the identification concerns raised by Doraszelski and Jaumandreu (2021), particularly relevant in our context, where labor markets are imperfect.

[19] Recent studies (e.g., Demirer, 2025) estimate production functions allowing for labor-augmenting productivity, though at the cost of imposing competitive labor markets. Demirer (2025) shows that biases from ignoring non-Hicks-neutral productivity are significantly smaller under a flexible translog specification compared to Cobb-Douglas—in some cases, differences in estimated output elasticities between Hicks-neutral and non-Hicks-neutral production functions become statistically insignificant when considering the translog specification.

4 Firm-level Evidence: Reduced Form Analysis

4.1 OLS Regressions

Strategy. For each firm i in year t and firm-level outcome $\mathbb{O}_{it}$ (input quantities, cost shares, output shares, output elasticities, and, later, markups and markdowns), we estimate OLS regressions in within-firm log differences across h years (i.e., $\Delta^h x_{it} = x_{it+h} - x_{it}$) of the form:

$$\Delta^h \ln(\mathbb{O}_{it}) = \beta_Q^h \Delta^h \ln Q_{it} + \upsilon_{jt} + \nu_{it}. \tag{10}$$

Q_{it} is deflated sales, and υ_{jt} captures 4-digit NACE rev. 1.1 industry-year fixed effects.[20] The difference specification accounts for unobserved constant firm characteristics. That is, we focus on within-firm and within-industry idiosyncratic firm growth, rather than industry growth or industry responses to shocks. We will study industry-aggregated analogs of this specification in Section 7.[21] ν_{it} is an error term. The regression is unweighted (i.e., each firm has the same weight). We conduct robustness checks by size (sales) quintiles below, finding similar results for all key outcomes. The coefficient of interest is β_Q^h. It captures the percent effect (co-movement) of a one percent change in firms' output on firm-level outcome $\mathbb{O}_{it}$ (compared to the industry-year mean growth). We examine time differences h ranging from one year to ten years.

Results. The first row of each Panel in Table 2 presents OLS regression results (1- to 10-year changes across panels). As a companion exhibit, Figure 3 visualizes firm-level relationships from Table 2 using binned scatter plots and residualizing variables by industry-year fixed effects as in Eq. (10)—revealing linear relationships that support the linear regression specification. As OLS coefficients are precisely estimated, we focus our discussion on point estimates.

Input quantities. Table 2 Columns (2)-(4) and Figure 3 Panels (a)-(c) report effects on input use. We measure labor in head counts. Capital and intermediates are measured as industry-deflated monetary values (Appendix D describes the capital stock calculation).[22]

[20] For this regression, we use the 2-digit industry output price deflator provided by the Statistical Office (but also include industry-year fixed effects). Using our own firm-specific output price index yields, however, similar results. The former may be preferable if there is measurement error in the firm-level output price index or under correlation between input and output prices.

[21] In unreported robustness checks, we also ran specifications with firm effects to take out firm-specific trends. This specification leads to slightly higher coefficient estimates, particularly at longer horizons, likely due to its remaining variation capturing more transitory fluctuations around the firm-specific trends. This consistency of results suggests that our findings are not merely responses to transitory shocks.

[22] Not accounting for firm-specific intermediate input prices may slightly attenuate our results on real intermediate input quantities. This bias is likely small, and we can ballpark it as follows. Percent changes

We discuss hours worked, capacity utilization, and input quality below. (We omit TFP as another factor that accounts for firm growth with inputs scaling (weakly) less than one to one with output, and below, test and find no evidence for increasing returns to scale.)

Intermediate inputs exhibit approximately a unit elasticity with output growth; labor and capital increase by much less. These relative slopes result in a declining ratio of labor (and capital) to intermediate inputs. The small standard errors allow us to reject the hypothesis of proportionate input growth, which would be expected in a Cobb-Douglas model with constant or uniformly shifting (shadow) input prices.

Cost shares. One possible explanation for the shift from labor to intermediates is divergence of input prices, while production remains consistent with a Cobb-Douglas model. In this scenario, cost shares would remain stable, as firms adjust input quantities inversely proportionately to rising input prices. (Changing input wedges are not captured by monetary costs and hence cost shares—we explore their role in Section 6.)

Table 2 Columns (5)-(7) and Figure 3 Panels (d)-(f) report effects on cost shares, i.e., input expenditures divided by total costs.[23] The data reveal a striking shift towards intermediate costs, away from labor (and capital), indicating that firms increasingly outsource production.

Complementing the cost share analysis in logs, we also run the specification in level (ppt.) changes (as in Figure 1), showing that intermediate cost shares absorb the decrease in the labor cost shares and, with a quantitatively much smaller role, the shift in capital cost shares (see Appendix Table A.3).

Output elasticities and returns to scale. With constant input wedges and returns to scale, shifts in cost shares correspond directly to changes in output elasticities (Eq.

can be expressed as $\frac{dM_{it}}{M_{it}} + \frac{d\widetilde{P}^M_{it}}{\widetilde{P}^M_{it}} = \frac{d\widetilde{M}_{it}}{\widetilde{M}_{it}}$, where M_{it} is real quantities (our object of interest), $\widetilde{P}_{it}$ is the *firm-specific* intermediate price deviation from the industry intermediate input price index, and $\widetilde{M}_{it}$ is the measured real intermediate input quantity adjusted for the industry index. Assuming that firm-specific price movements are linked to firm-specific real quantities through firm-specific elasticity ϵ', such that $\frac{d\widetilde{P}^M_{it}}{\widetilde{P}^M_{it}} = \epsilon' \cdot \frac{dM_{it}}{M_{it}}$, we can write: $\frac{dM_{it}}{M_{it}} = \frac{d\widetilde{M}_{it}/\widetilde{M}_{it}}{1+\epsilon'}$. Thus, the estimated regression coefficient on the basis of our industry-deflated intermediate input measure is related to the object of interest by $\rho_{M,Q} = \frac{\rho_{\widetilde{M},Q}}{1+\epsilon'}$. The attenuation is small; for instance, even if we assumed that intermediate inputs were as inelastically supplied as labor, with $\epsilon' \approx 0.1$, then latent real intermediate inputs would only move slightly less elasticically with output than suggested by our coefficient estimate—so that our overall results are unlikely to be significantly affected. Intermediate inputs are likely more elastically supplied than this benchmark suggests, implying negligible bias in our results, although our paper highlights the need for better evidence on firm-specific supply elasticities of intermediates.

[23] We approximate capital costs as 8 percent of the capital stock, following Dhyne et al. (2024) and CompNet (2023). For logged specifications, this homogeneous multiplicative factor is absorbed by industry-year fixed effects. Similarly, an industry-specific capital deflator would drop out. For that reason, we also do not attempt to provide more granular heterogeneous cost of capital measures.

(5)). To account for the possibility of varying input wedges, Table 2 Columns (8)-(10) and Figure 3 Panels (g)-(i) analyze output elasticities based on our translog production function estimates.

We find a strong negative relationship between output growth and labor output elasticities. Table 2 Column (8) reveals that an additional 10 percentage point output growth reduces the (growth of) labor output elasticity by 3 percentage points, with a coefficient of -0.30 (SE 0.004) at a one-year horizon. Figure 3 Panel (g) visualizes the underlying relationship. This shift away from labor output elasticities is accompanied by a significant increase in intermediate output elasticities, as shown in Column (10) and Panel (i). Notably, capital output elasticities decline as well (Column (9) and Panel (h)).

For additional clarity, Appendix Table A.3 reproduces our findings in levels rather than logs for the dependent variables. The level changes in labor and capital output elasticities offset those in intermediate output elasticities, with labor and intermediates driving most of the variation. This result is consistent with the much smaller capital cost shares and output elasticities (Figure 2 and Table A.1).

Notably, we normalize the output elasticities in our regressions by the returns-to-scale parameter to account for any changes in scale. However, the results remain consistent without this adjustment (Table A.3). Column (11) shows that returns to scale are relatively stable in the short term and only change slightly over longer horizons. (When we estimate substitution elasticities, the returns to scale normalization cancels out—see Eq. 3.)

Effects across horizons. Comparing horizons (1, 4, 10 years) across the panels reveals interesting dynamics: the coefficient for labor output elasticities drops from -0.30 to -0.20 and -0.14 for 4- and 10-year horizons, respectively (Column (8)). As we explore in Section 5, the evidence for horizon-dependency could be consistent with short-run labor adjustment costs.

Table 2: Firm-level adjustments in response to firm growth: OLS and IV estimates.

	1st stage	$\Delta \ln(L_{it})$	$\Delta \ln(K_{it})$	$\Delta \ln(M_{it})$	$\Delta \ln\left(\frac{P_{it}^L L_{it}}{C_{it}}\right)$	$\Delta \ln\left(\frac{P_{it}^K K_{it}}{C_{it}}\right)$	$\Delta \ln\left(\frac{P_{it}^M M_{it}}{C_{it}}\right)$	$\Delta \ln\left(\frac{\theta_{it}^L}{RTS_{it}}\right)$	$\Delta \ln\left(\frac{\theta_{it}^K}{RTS_{it}}\right)$	$\Delta \ln\left(\frac{\theta_{it}^M}{RTS_{it}}\right)$	$\Delta \ln(RTS_{it})$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Panel A: 1-year differences											
OLS estimates											
Log output change		0.298***	0.0405***	1.025***	-0.376***	-0.728***	0.238***	-0.304***	-0.315***	0.151***	0.0062***
		(0.0040)	(0.0022)	(0.0033)	(0.0039)	(0.0033)	(0.0022)	(0.0040)	(0.0063)	(0.0015)	(0.0003)
R^2		0.215	0.043	0.741	0.343	0.556	0.340	0.230	0.137	0.325	0.053
IV estimates											
Export demand shock (first stage)	0.0451***										
	(0.0045)										
Log output change		0.324***	0.0612	0.929***	-0.444***	-0.684***	0.171***	-0.327***	-0.229**	0.119***	0.0082
		(0.0543)	(0.0461)	(0.0508)	(0.0521)	(0.0565)	(0.0292)	(0.0623)	(0.0980)	(0.0198)	(0.0053)
R^2	0.205	0.214	0.042	0.736	0.336	0.554	0.321	0.229	0.132	0.314	0.052
First-stage F-stat		102.6	102.6	102.6	102.6	102.6	102.6	102.6	102.6	102.6	102.6
Observations						183,813					
Firms						29,950					
Panel B: 4-year differences											
OLS estimates											
Log output change		0.510***	0.206***	1.045***	-0.286***	-0.649***	0.171***	-0.202***	-0.181***	0.0951***	0.0167***
		(0.0064)	(0.0067)	(0.0045)	(0.0062)	(0.0071)	(0.0034)	(0.0060)	(0.0074)	(0.0025)	(0.0006)
R^2		0.471	0.144	0.860	0.348	0.476	0.355	0.220	0.170	0.268	0.163
IV estimates											
Export demand shock	0.0635***										
	(0.0091)										
Log output change		0.536***	0.262***	0.963***	-0.269***	-0.570***	0.124***	-0.242***	-0.147	0.0823***	0.0222***
		(0.0737)	(0.0973)	(0.0542)	(0.0664)	(0.0969)	(0.0369)	(0.0706)	(0.0938)	(0.0268)	(0.0075)
R^2	0.183	0.470	0.139	0.855	0.347	0.471	0.339	0.217	0.169	0.266	0.156
First-stage F-stat		48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57
Observations						70,936					
Firms						11,492					
Panel C: 10-year differences											
OLS estimates											
Log output change		0.606***	0.395***	1.041***	-0.219***	-0.490***	0.131***	-0.140***	-0.0720***	0.0582***	0.0278***
		(0.0066)	(0.0082)	(0.0043)	(0.0061)	(0.0083)	(0.0033)	(0.0053)	(0.0069)	(0.0024)	(0.0007)
R^2		0.594	0.256	0.901	0.296	0.329	0.303	0.188	0.149	0.200	0.268
Observations						49,915					
Firms						10,595					

Notes: The table reports OLS and IV regressions from estimating the specification in Eq. (10). IV uses foreign demand shocks as instruments (Eq. (12)). Column (1) reports the first-stage regression results. The dependent variables in Columns (2)-(11) are log changes in labor, capital, intermediates, labor, capital, and intermediate cost shares, labor, capital, and intermediate output elasticities over returns to scale, and returns to scale, respectively. All columns report regressions of those dependent variables on changes in log output for 1-, 4-year, and 10-year differences (10-year differences are not available for IV). All regressions control for industry-year fixed effects. Standard errors are clustered at the firm level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

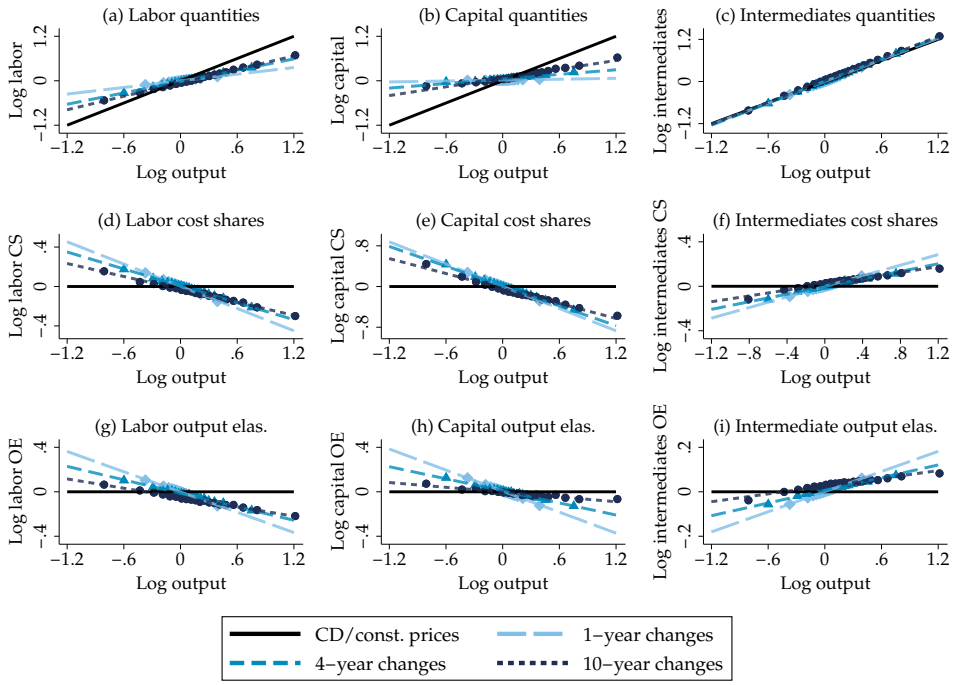
Heterogeneity: types of intermediates. The strong response of intermediates and substitution from labor may partially reflect that intermediate inputs also include intermediate services, unfinished product components, and similar items (see Figure 2). Appendix Table A.4 reports additional regressions for logged cost shares of all available subcategories of intermediates as dependent variables. The effects are as expected. We find strong effects for the main category product components, materials, and energy, possibly capturing offshoring/outsourcing of labor-intensive production steps/product components. Effects are negative for intermediate inputs that are arguably complementary to capital inputs, such as repair, maintenance, and installation services. We find the strongest effects for temporary agency workers, indicating that firms rely on those more flexible labor inputs when growing (see De Leon et al., 2024), particularly in the short run. However, *quantitatively*, this response has a minimal impact on the overall effect, as the average cost share of temporary agency workers is only 1% (Figure 2). Consistent with this analysis, in an unreported robustness check, we have assigned temporary agency workers and subcontracted production to labor rather than intermediate inputs (and divided those costs by the firm's average wage to approximate labor-equivalent quantities) and found similar results across the board.

Heterogeneity: size heterogeneity. Appendix Table A.6 reproduces our regressions splitting firms into five size quintiles. Across all size groups, results align closely with our main results (Table 2). We return to the issue of firm size when discussing non-homotheticities in Section 5.3.

Heterogeneity: industries. Appendix Figure A.3 reports key results by industry. Qualitatively, results are similar across industries—input quantities, cost shares, and output elasticities shift from labor to intermediates. Quantitatively, these shifts differ somewhat across industries, particularly for output elasticities. We have not systematically examined this heterogeneity for lack of clear variation in the underlying parameters. However, all things equal, one would predict a stronger shift from labor toward intermediate quantities in industries with stronger labor monopsony power or higher labor adjustment costs (relative to intermediates). Relatedly, we note that firms generally have substantial wage flexibility within the German collective bargaining system, because coverage status is not universal, chosen by firms (by joining employer associations), and covered firms often pay premia (Jäger et al., 2022). Moreover, this system does not provide clear variation that could systematically mediate monopsony degrees for use in heterogeneity analyses.

Additional analysis: hours worked, capacity utilization, and input quality. From 1999 to 2017, we observe employment in full-time equivalents (FTE), permitting us to investigate

Figure 3: Firm-level adjustments in response to firm growth (OLS, binned scatter plots).



Notes: The figure reports binned scatter plots from estimating the specification in Eq. (10) with OLS for various differences against log output changes for the following dependent variables in log changes: labor, capital, intermediate quantities, output elasticities over returns to scale, and cost shares. It also includes the prediction from a Cobb-Douglas production framework with firms optimizing against constant input price (ratios). All panels report results that are residualized by industry-year fixed effects. German firm-level data.

intensive-margin hours adjustments. Appendix Table A.2 shows that headcount and FTE responses are nearly identical, suggesting that intensive margin hours effects are unlikely to confound our estimates. (Capital adjustments are not our focus, and we do not observe capital utilization.) Regarding input *quality* adjustments, we do not have detailed micro-data on labor quality, but we can gauge such effects by studying compositional effects on the basis of average wages. Average wages slightly increase in response to output growth, with coefficients ranging from 0 to 0.1 (Appendix Table A.9). If interpreted as an increase in labor quality, the efficiency units of labor (quantity plus quality effect) would only slightly attenuate the implied substitution elasticity.[24] The alternative—and our preferred—interpretation of the average wage effect points to upward-sloping labor supply curves (and is quantitatively realistic, see our meta-analysis in Section 5.2). We discuss the limitations of average wages below and instead infer changes in shadow prices

[24]The effect on the wage bill can be imputed as the effect on employment times the average wage, i.e., $\Delta \ln(\bar{w}L) = \Delta \ln(\bar{W}) + \Delta \ln(L)$, such that the labor effect would slightly rise from 0.510 to 0.572 for, e.g., at the 4-year horizon. Foreshadowing our calculation of the substitution elasticity in Section 5.1, and assuming away effects on measured output elasticities, this effect would only slightly change our conclusion about substitution elasticities between intermediates and labor (from 2.25 to 2.29).

as a broader marginal cost measure that is more relevant to our analysis.

Additional analysis: cross-sectional results and firm lifecycle. Our paper focuses on within-firm changes. For completeness, we provide results for the cross-section in Appendix Figure A.1.[25] Finally, while we do not have information on firm age in our main data, our extension to other countries in which age is included shows qualitatively similar effects across firm-age groups (Section 7.2 and Appendix Table A.13).

4.2 Causal Effects: IV Strategy using Export Demand Shocks

We now use an instrumental variable strategy that draws on *foreign* product demand shocks as variation in firm growth that is plausibly unrelated to input-biased shifts in production function parameters, input prices, age effects, or input quality (conditional on industry-year fixed effects). This analysis aims to trace firm growth dynamics corresponding to our structural equation in Section 2, where we focused on shifts in firm output while holding such confounding factors constant.

Across outcomes, IV and OLS yield similar coefficients. This is consistent with our OLS regressions largely reflecting input-unbiased sources of growth such as shifters in product demand, TFP, or input prices across the board.

Strategy. We follow an established literature using trade shocks as exogenous shifters (see Autor et al., 2016 for a review). In particular, we follow Hummels et al. (2014) and instrument changes in firms' output with changes in world export demand (excluding Germany). The method proceeds in two steps. First, we compute the total exports for each product, g, from a country group, n, to the world:

$$EX_{gt} = \sum_{c} ex_{gct}^{n \to world}. \tag{11}$$

n contains Australia, Norway, Sweden, Singapore, New Zealand, Great Britain, Canada, Japan, and the US. c denotes the country. Our country selection follows Dauth et al. (2014), with the exception that we additionally include the US (results are robust to excluding the US). There are two reasons for this selection: We choose other industrialized countries whose economies and export specialization are more plausibly similar to Germany, and excluding Germany's direct neighbors and members of its EUR currency union helps

[25]The cross-sectional analysis reflects qualitatively different forces such as permanent heterogeneity. Our empirical analysis confirms that large firms have lower labor cost shares and labor output elasticities but paints a less clear picture likely due to input price differences and/or other (permanent) heterogeneities (e.g., the α and Λ terms in Eq. (1)). The effect for the labor output elasticity falls to -0.03, again precisely estimated. Scatter plots in Appendix Figure A.1 reveal a slightly concave pattern, consistent with large shorter-run elasticities not extending to cross-sectional variation in firm size, which is right-skewed.

mitigate potential endogeneity concerns arising from unobserved shocks that might be correlated between Germany and other nations.

In the second step, we compute firm-specific export demand shocks by aggregating the product-level shocks from Eq. (11) to the firm level. We calculate weighted averages of product-level trade flows, using the sales shares of products within firms' product portfolios as weights:

$$INS_{it} = \sum_{g} s_{git=0} \ln EX_{gt}. \tag{12}$$

s_{git} denotes the firm-specific sales weight of product g. To limit anticipatory effects (product mix adjustments), we fix the weights for each firm to its first year in the sample. We thus rely on variation from a fixed (internationally tradable) set of products (and including industry-year FEs implies we use between-firm, within-industry variation).[26] Having constructed these firm-specific instruments, we instrument output changes, $\Delta \ln Q_{it}$, in Eq. (10) with export demand shocks, ΔINS_{it}. Importantly, 80% of firms in our German manufacturing firm sample export in a given year. As we do not observe firm-product-specific export shares, our product sales share weights include domestic sales as well. Furthermore, we restrict our IV analysis to 1- and 4-year differences as the first stage is not sufficiently strong (a low F-statistic) for 10-year changes.

IV-results. Table 2, Column (1) shows the first stage coefficient from regressing output growth on our instrument. We find a statistically significant positive association. The F-statistic is 102.6 (48.57) for one-year (four-year) changes. Table 2 Columns (2)-(11) also report the second stage IV results, where we instrument output growth with export demand shocks. Across outcomes, IV coefficients are similar to OLS and confirm the shift from labor to intermediates across the board.

5 Quantitative and Structural Interpretation

We now interpret our reduced form results structurally and quantitatively using Eqs. (3) and (4). Subsequently, we discuss alternative accounts.[27] Throughout, we focus on our *firm growth* estimates.

[26] The aggregation is unconditional on export status at the firm-product level, as export information is reported at the firm level only; in unreported analyses, we confirmed that the instrument power is derived from exporters.

[27] We focus on within-firm changes, which net out fixed, unobserved firm-specific factors. Large firms seem to have higher intermediate-labor ratios, with smaller differences in labor and intermediate input output elasticities (Appendix Figure A.1). As our focus is on firm growth, we omit a detailed discussion of how to rationalize those facts with permanent heterogeneity.

Table 3: Implied substitution elasticities, effects of firm growth on input ratios and input shadow price ratios, and firm-specific labor supply elasticities.

	OLS				IV			
	σ	$\Delta \ln\left(\frac{L_{it}}{M_{it}}\right)$	$\Delta \ln\left(\frac{P^L_{it}\gamma^L_{it}}{P^M_{it}\gamma^M_{it}}\right)$	$\epsilon^L = \frac{1}{\varepsilon^L}$	σ	$\Delta \ln\left(\frac{L_{it}}{M_{it}}\right)$	$\Delta \ln\left(\frac{P^L_{it}\gamma^L_{it}}{P^M_{it}\gamma^M_{it}}\right)$	$\epsilon^L = \frac{1}{\varepsilon^L}$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1-year diff.	2.67	-0.73	0.27	1.10	3.81	-0.61	0.16	2.04
4-year diff.	2.25	-0.54	0.24	2.16	4.16	-0.43	0.10	5.20
10-year diff.	1.84	-0.44	0.24	2.56				

Notes: The table reports substitution elasticities (Columns (1) and (5)) following Eq. (14), changes in input factor ratios (Columns (2) and (6)), implied changes in shadow input price ratios (Columns (3) and (7)) following Eq. (16), and implied labor supply elasticities following Eq. (18), assuming perfectly elastic intermediate input supply (Columns (4) and (8)), based on our OLS (Columns (1)-(4)) and IV (Columns (5)-(8)) regressions from Table 2 that regress log output elasticities over returns to scale and log input quantities on log output in within-firm differences. Consequently, Columns (2) and (6) report coefficient ratios for labor and intermediates from these regressions with respect to firm growth, while all other columns report values implied by our regressions as described in the text.

5.1 Identification of Substitution Elasticity

Identification argument. Our identification of the substitution elasticity rests on Eq. (3), which identifies the labor-intermediates substitution elasticity, σ, from the co-movement of output elasticities and input quantities:

$$\frac{\sigma-1}{\sigma} = \frac{\Delta \ln(\theta^L_{it}) - \Delta \ln(\theta^M_{it})}{\Delta \ln(L_{it}) - \Delta \ln(M_{it})} \quad \Rightarrow \quad \sigma = \frac{1}{1 - \frac{\Delta \ln(\theta^L_{it}) - \Delta \ln(\theta^M_{it})}{\Delta \ln(L_{it}) - \Delta \ln(M_{it})}}. \tag{13}$$

To calculate σ, we use within-firm changes (Δ) *conditional on firm growth*, i.e., we insert the estimated OLS and IV coefficients from our firm growth regressions into Eq. (13).[28] That is, for each right-hand side component of Eq. (13), we plug in the corresponding regression coefficient estimated in Section 4.

$$\widehat{\sigma} = \frac{1}{1 - \frac{\rho_{\Delta \ln(\theta^L),\Delta \ln(Q)} - \rho_{\Delta \ln(\theta^M),\Delta \ln(Q)}}{\rho_{\Delta \ln(L),\Delta \ln(Q)} - \rho_{\Delta \ln(M),\Delta \ln(Q)}}}, \tag{14}$$

where the ρ variables correspond to the regression coefficients from Eq. (10), for various horizons and outcome variables. Hence, relating our reduced-form difference-based estimates of changes in output elasticities and input factors in response to firm growth to a structural equation, identifies the substitution elasticity, σ, *without any assumptions on input prices* (except those related to the estimation of output elasticities). Given the

[28]Compared to the OLS estimates, the IV estimates can be viewed as additionally removing potential confounders, such as input-biased shifters in prices or production function parameters, that may underlie some of the OLS variation in firm growth.

precisely estimated underlying coefficients, β, we focus on point estimates.[29]

Implied parameter values: translog estimates. Table 3 Columns (1) and (4) report the implied values of σ. We find that labor and intermediates are substitutes, with σ exceeding unity in all specifications. The OLS estimates range from 2.67 in the short run to 1.84 in the longer run. IV estimates of σ are around 4 for 1- and 4-year changes (IV estimates for 10-year changes are unavailable).[30] That IV estimates of σ exceed OLS estimates is reassuring. Appendix C.4 shows that this is theoretically consistent with $\sigma > 1$ under the presence of some labor-augmenting productivity shocks that bias OLS substitution elasticities *downward*—working *against* our proposed substitution mechanism—and that is addressed by the IV.

The relatively high substitution elasticities may also reflect that intermediate inputs include services and (labor-intensive) product components (e.g., car seats), that became highly tradable during the period of intense globalization/offshoring that our data covers. It is plausible that substitution elasticities between raw materials (such as steel) or energy and labor would be lower.

Implied parameter values: cost shares. As alternative specification, we report substitution elasticities using cost shares to measure output elasticities in Table A.8 (consistent with the CES production function in Section 2). As expected, estimates of substitution elasticities *increase*. Our cost share estimates allow for a constant degree of input market imperfections, but exclude changes in input wedges (γ_{it}^X). Consequently, variation in γ_{it}^X correlated with output growth loads on the substitution elasticity (see Table 4, showing that $\frac{\gamma_{it}^L}{\gamma_{it}^M}$ increases with output growth). This is intuitive: smaller increases in relative shadow prices imply larger substitution elasticities to explain the observed shift from labor toward intermediates.

Price and quantity comovement. Due to firms' first-order conditions, our identification of σ is equivalent to using comovements in input quantities and (shadow) prices as described in Eq. (4): $\Delta \ln \left(\frac{P_{it}^L \gamma_{it}^L}{P_{it}^M \gamma_{it}^M} \right) = \frac{-1}{\sigma} \Delta \ln \left(\frac{L_{it}}{M_{it}} \right)$. A key challenge in directly using this equation is that *firm-specific* (shadow) prices are unobserved. We report average wage

[29] Bootstrapping the entire estimation is unfortunately not feasible at the statistical office due to computational resource limits.

[30] We do not focus on comparing short- vs. long-run substitution elasticities or attempt to interpret these dynamics in context of the Le Châtelier principle (Milgrom and Roberts, 1996), although we note that both labor and intermediates are presumably quite flexible compared to capital. However, OLS (IV) estimates indicate somewhat smaller (slightly larger) long- than short-run elasticities. These patterns must be interpreted in the context of intermediate-input mix adjustments: in the short run, firms increase inputs *more substitutable with labor*, such as temporary agency workers or subcontracted work, relatively more strongly, while in the longer run, firms increase inputs like rents and leases, repairs and maintenance, or other intermediates relatively more strongly (see Appendix Tables A.4 and A.5).

responses to firm growth in Table A.9, which (under the assumption that intermediate-input prices are absorbed by industry–time fixed effects) is equivalent to studying relative input price changes. Average wage responses are much smaller in absolute value than the corresponding changes in labor-to-intermediate input ratios reported in Table 3, Column (2). This is reassuring and consistent with $\sigma > 1$.[31]

Meta-analysis of substitution elasticity estimates. Figure 4, Panel (a) situates our baseline translog estimates (denoted by "MS") in a meta-analysis of existing estimates. We differentiate between studies that estimate substitution elasticities between intermediates and a capital-labor bundle (diamonds) and between intermediates and labor (triangles). The former approach restricts the substitution elasticity between intermediates and capital to be identical to that between intermediates and labor. It typically identifies the substitution elasticity using the ratio between intermediates and the capital-labor bundle (value added). The latter approach directly relates intermediates to labor, aligns with our methodology, and tends to yield higher substitution elasticities that exceed unity in several studies. Appendix B provides details on the estimates included in Figure 4 Panel (a). We report mean values and estimates often vary significantly within a given study (e.g., values in Chan (2023) range from 1.6 to 9.6). We also note that our estimates are derived from within-firm changes only (many studies use cross-sectional variation, too—e.g., Doraszelski and Jaumandreu, 2018).

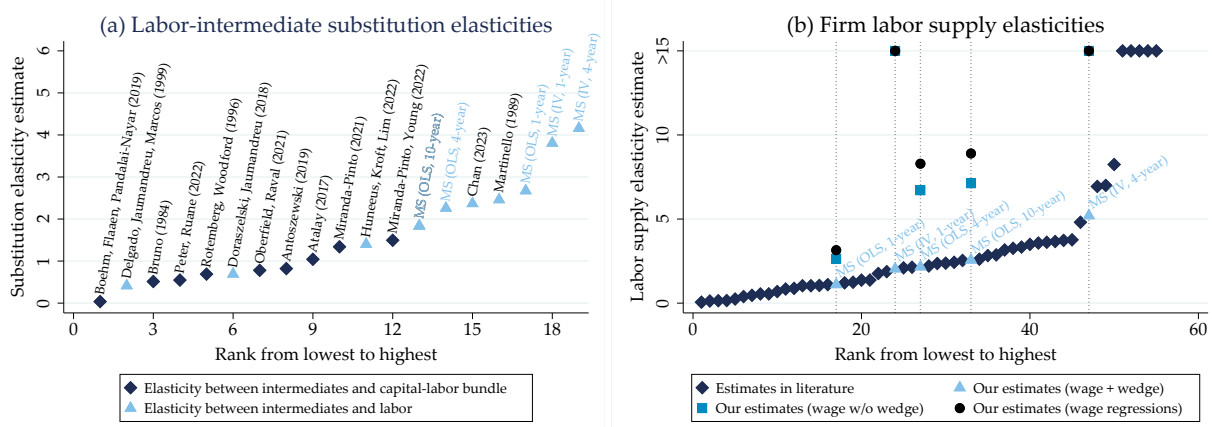
5.2 Identification of Firm-Specific (Relative) Labor Supply Elasticities

Why do firms change their input mix (and output elasticities) as they grow? Firms' cost minimization and changing input prices provide a natural answer, and we now trace out the implied input price ratio, and translate it into an implied firm labor supply elasticity.

Step 1: identifying implied input price ratio changes w.r.t. firm growth. Using our estimates of σ and our firm growth regression coefficients, we can infer the implied (average) change in the shadow price ratio (i.e., the change that is caused by firm growth). We insert our regression coefficients into Eq. (4) to back out the implied effect of firm

[31] In Section 6, we estimate input wedges, $\frac{\gamma^L_{it}}{\gamma^M_{it}}$, and study their changes in response to output growth. Adding changes in input wedges to changes in wages in response to growth will by design yield the reported shadow price changes in Table 3, Column (3) and (7) (this is because of firms' first-order conditions) that are smaller than the input-quantity ratio changes.

Figure 4: Meta-analysis: substitution and labor supply elasticities vs. literature.



Notes: The figure reports estimates of labor-intermediate substitution elasticities (Panel (a)) and labor supply elasticities (Panel (b)) from the literature and from our analysis (our own estimates are indicated by "MS" and in light blue). For substitution elasticity estimates, we focus on samples that encompass the most firms or report mean values (excluding negative estimates when computing the means). Appendix B provides details on the displayed substitution elasticities. Data on labor supply elasticities come from the meta-study by Sokolova and Sorensen (2021). We exclude negative estimates. If available, we report median IV estimates. If no IV approach is used, we report medians of all other estimates. (We prefer medians due to outliers.) We do not implement alternative selection methods (e.g., best practice corrections). Sokolova and Sorensen (2021) note that best practice correction estimates vary by methodology: separations-based approaches yield elasticity estimates between 6.4 (not identified) and 9.9 (identified), while stock-based inverse approaches produce estimates above 20. For a list of studies entering the firm-specific labor supply elasticity estimates, we refer to Sokolova and Sorensen (2021).

growth on within-firm changes in the shadow price ratio:

$$\Delta \ln \left(\frac{P_{it}^L \gamma_{it}^L}{P_{it}^M \gamma_{it}^M} \right) = \frac{\Delta \ln(L_{it}) - \Delta \ln(M_{it})}{-\sigma} \tag{15}$$

$$\Rightarrow \widehat{\rho}_{\Delta \ln(\frac{P^L \gamma^L}{P^M \gamma^M}), \Delta \ln(Q)} = \frac{\rho_{\Delta \ln(L), \Delta \ln(Q)} - \rho_{\Delta \ln(M), \Delta \ln(Q)}}{-\widehat{\sigma}}. \tag{16}$$

We report results in Table 3, Columns (3) and (6). OLS estimates indicate changes in the log price ratio of 0.27 (0.23) in the short (long) run. IV estimates are 0.16 and 0.10 for short- and longer-run changes, respectively. Consequently, as firms grow, the shadow price of labor increases relative to that of intermediate inputs. As a result, firms substitute labor with intermediates and change their modes of production as reflected in their output elasticities.

Potential sources of the relative increase in labor prices. What forces cause relative labor costs to rise as firms grow? One natural explanation is that the firm-specific labor supply elasticity is low (compared to that of intermediates), suggesting the presence of labor monopsony power, which raises wages as firms hire more workers. The persistence of rising relative labor costs even over a 10-year horizon reinforces the monopsony-based explanation. However, the fact that relative labor costs are notably higher in the short run also points to short-run adjustment costs, which are plausibly more significant for labor

than for intermediate inputs. Adjustment costs can thus be a driver of (short-run) changes in input price ratios, and, through that, input quantity and output elasticity ratios.

Step 2: Identifying the firm-specific factor supply elasticities. Although not crucial for our key results (we only require increasing *relative* shadow prices for labor compared to intermediates), we find it useful to discuss implied estimates of the labor supply elasticity. We acknowledge that, compared to the identification of σ, the identification of the labor supply elasticity requires stronger assumptions. As our estimates identify the relative elasticity of labor to that of intermediates, we can infer the *absolute* firm-specific labor supply elasticity under two common assumptions (which are not required for identifying substitution elasticities). First, intermediate inputs are perfectly elastically supplied.[32] Second, the markdown, γ_{it}^L, is constant in firm growth (we still permit a baseline wedge)—an assumption we later relax by measuring γ_{it}^L and studying its firm growth gradient directly. Those two assumptions imply that the input shadow price gradient is solely due to wage increases, allowing us to infer the implied firm-specific labor supply elasticities from our estimated input price changes and the employment effects:

$$\widetilde{\epsilon}^L = \frac{\Delta \ln(L_{it})}{\Delta \ln\left(\frac{P_{it}^L \gamma_{it}^L}{P_{it}^M \gamma_{it}^M}\right)} = \frac{\Delta \ln(L_{it})}{\Delta \ln(P_{it}^L)}, \tag{17}$$

which we identify on the basis of the following ratio:

$$\widehat{\widetilde{\epsilon}}^L = \frac{\rho_{\Delta \ln(L), \Delta \ln(Q)}}{\widehat{\rho}_{\Delta \ln(\frac{P^L \gamma^L}{P^M \gamma^M}), \Delta \ln(Q)}}. \tag{18}$$

That is, we identify the labor supply elasticity by dividing changes in labor quantities from Table 2 by changes in input price ratios from Table 3. Note that this parameter, $\widetilde{\epsilon}^L$, is the *inverse* of the supply elasticity, ε^L, in Section 2. Moreover, the $\widehat{\rho}$ term is *not* a regression coefficient but the inferred input shadow price ratio gradient backed out in Eq. (15) above.

Results: implied firm-specific labor supply elasticities. We report implied labor supply elasticities, ϵ^L, in Table 3. The values range from 1.10 to 2.56 for our OLS and from 2.04 to 5.20 for our IV results. The higher long-run than short-run elasticities either reflect horizon-dependence of labor supply elasticities, or may reflect firm-side adjustment costs (non-constant γ_{it}^L). Nevertheless, even in the long run, supply elasticities remain relatively

[32] We are not aware of comparable estimates of firm-specific supply elasticities for intermediate inputs. Bilal and Lhuillier (2022) make a similar assumption in a labor-only model with labor service purchases (although our intermediates are largely made of goods), while Dobbelaere and Mairesse (2013) and subsequent studies assume perfectly elastic intermediate supply when backing out markdowns and implied labor supply elasticities.

low. These estimates showcase the quantitative basis for the monopsony-driven incentive for firms to shift from labor to intermediate inputs.

Robustness: permitting markdowns and direct wage estimates. By using the shadow price of labor (the product of the wage and wage markdown), our initial labor supply elasticity estimates may be confounded by markdown variation. We now calculate firm-specific labor supply elasticities, under two alternative assumptions. We report and discuss these alternative values in our meta-analysis in Figure 4 Panel (b).

First, we report results that subtract the measured markdown effect on $\gamma_{it}^L/\gamma_{it}^M$ that we estimate in Section 6 when studying the labor share implications. As we find that markdowns *increase* as firms grow, this calculation increases labor supply elasticities. Intuitively, by eliminating the effect of increasing wage markdowns, a given shift in labor now corresponds to a smaller change in labor prices, implying higher supply elasticities.

Second, we also measure wages directly, using the average firm wage (wage bill per head), and run regressions of wage changes on output growth as before. These additional wage regressions are reported in Appendix Table A.9.Using OLS, average wages increase in firm growth (consistent with monopsony). Since those wages move less than the inferred shadow price ratios above, the implied firm-specific labor supply elasticities are again higher than our baseline measure. Using IV, the wage effect point estimates are close to zero; hence, the associated elasticities would be large, but the wider confidence intervals for the wages also accommodate elasticities more consistent with the literature. Overall, we note that average wages are subject to composition bias (e.g., worker quality) and may be confounded by hours responses. We cannot merge matched employer-employee data or data on worker skills to our firm data to use cleaner wage measures.[33]

Meta-analysis of existing parameter estimates. Figure 4 Panel (b) summarizes existing estimates of labor supply elasticities based on the meta-analysis of Sokolova and Sorensen (2021), along with the values our study implies, including the additional estimates from the robustness checks discussed above. While estimates are ranked by size, for our own estimates, we provide robustness check results vertically stacked. Overall, our estimates fall well into the range of existing estimates and highlight a lower short- than long-run labor supply elasticity. We find that supply elasticities are higher for our IV estimates, and when we net out markdown shifts (squares) or use direct (average) wage estimates (dots). We note again that our estimates are based on within-firm growth regressions.

[33]For instance, over a comparable period, Lochner et al. (2020) report a German large-firm wage premium of 10–11 log points, of which 4–6 points are due to person fixed effects, which would suggest higher labor supply elasticities. However, we also note that these regressions capture long-run elasticities. There is an ongoing debate about the level of the elasticity.

5.3 Alternative Mechanisms

Automation. Recent work has highlighted capital-based technological change as a factor reducing labor's importance to firms (Acemoglu and Restrepo, 2020, Hubmer and Restrepo, 2021). Such technological change could explain a decline in labor's output elasticity with firm growth. However, rather than an increase, we document a decline in capital output elasticities with firm growth. Instead, intermediate inputs *unrelated to capital services* gain in importance (as discussed above, maintenance and repair services decline, Appendix Table A.4). While automation is important in shaping changes in firms' production and aggregate labor market trends, it appears not to rationalize our findings in the context of idiosyncratic firm growth.

Unobserved factor-augmenting productivity shocks. One concern could be that output and substitution elasticities could be biased due to unobserved factor-augmenting productivity shocks. We do not believe that such shocks and can explain our findings, for the following reasons:

First, our regressions absorb industry specific factor-augmenting productivity shocks and firm-specific persistent productivity differences. Second, our results for cost shares and output elasticities are similar (Table 2). The cost-share estimates *allow* for factor-biased productivity processes. If anything, using cost share estimates yield even *higher* substitution elasticities (Appendix Table A.8). Third, we employ an instrumental variable approach, using foreign export demand shocks as instruments for firm growth. These shocks are plausibly largely independent of firm-specific factor-augmenting technological shocks. Under this assumption, the IV purges unobserved variation in factor-augmenting productivity, so that Eq. 13 remains valid even in the presence of such shocks.[34]

A specific concern could be the presence of unobserved labor-augmenting productivity (LAP) shocks that can explain our findings only if $\sigma < 1$. In this context, it is important to note that substitution elasticities estimated under IV are *higher*. Appendix C.4 shows that a *downward* bias in OLS is only consistent with LAP shocks if inputs are substitutes (if inputs were complements, OLS estimates would exceed IV estimates). We find that, consistent with theory, correcting for potential bias from unobserved LAP shocks strengthens our evidence of input substitutability.

Finally, to illustrate our identification strategy, we provide a self-contained simulation that is complementary to the paper (accessible on the authors' websites and under this

[34]That is true, despite factor-augmenting implies a modified version of Eq. 13 that includes additional factor-augmenting productivity terms (α_{it}^L and α_{it}^M): $\frac{\sigma-1}{\sigma} = \frac{\Delta \ln(\theta_{it}^L) - \Delta \ln(\theta_{it}^M)}{\Delta \ln(\alpha_{it}^L) - \ln(\alpha_{it}^M) + \Delta \ln(L_{it}) - \Delta \ln(M_{it})}$. Because the IV is orthogonal to these productivity shocks, the original estimating equation remains valid. This approach is similar to studies using instruments to identify σ (e.g., Raval, 2019, Huneeus et al., 2022).

link). The simulation assumes a CES production function with imperfect labor markets and shows that (i) with Hicks-neutral productivity (equivalently, only industry-specific factor-augmenting productivity shocks) and size-dependent monopsony/wedges, our translog-based growth regressions (Eq. 10) recover the true substitution elasticity, and (ii) with firm-specific factor-augmenting productivity shocks and isoelastic labor supply (a fixed wedge), the cost-share approach combined with our IV strategy (a factor-neutral output demand shifter) recover the true substitution elasticity.[35]

Fixed costs. Our data do not allow us to differentiate between fixed and flexible labor inputs. If firms' production involves fixed labor costs (or shared labor inputs) that scale up less than proportionately with flexible labor, labor output elasticities decline as firms grow. Effectively, fixed costs break the homotheticity of a production function.

We expect fixed costs to be particularly relevant for small firms (e.g., fixed overhead costs).[36] However, splitting our firm sample into size (sales) quintiles (by year and industry), yields similar results across size quintiles (Appendix Tables A.6 and A.7). we rely on OLS as IV first stages become weak in this sample split.[37] Whereas we do not find clear evidence for differential labor adjustments, Table A.6 shows that declines in capital output elasticities are moderated in larger firms, consistent with initial capital fixed costs.

To further discuss the role of fixed costs, we formalize a basic fixed-cost model in Appendix C.7. This model permits us to quantify the fixed cost share needed to rationalize our regression results absent the substitution mechanism we explore in this paper. For 1-year changes, our OLS (IV) estimates imply a required fixed labor cost share in total labor costs of 0.51 (0.50) for the average firm, while for 4-year changes, the required fixed labor cost share is 0.28 (0.31).[38] Importantly, the model also reveals that labor fixed costs (or

[35] In this second case, bias in translog-based estimates arises solely from a misspecified production function due to unobserved factor-augmenting productivity. The extent of this bias under a flexible translog is unclear: For markups, Demirer (2025) shows that it can be moderate in some settings—though smaller than under Cobb–Douglas—while negligible in others. To our knowledge no study can jointly address factor-augmenting productivity and size-dependent monopsony/wedges. Our study emphasizes the latter. Jointly addressing both is an important task for future work. Recall: Our goal is not to pin down the precise value of σ, but to show that the data are consistent with $\sigma > 1$, which we document using multiple sources of evidence (OLS and IV, exogenous quantity vs. price variation, industry vs. firm data, and multiple countries).

[36] We emphasize that we refer to true fixed costs, i.e., the part of labor that does not scale with size.

[37] Despite theoretical predictions that large firms exert greater influence over wages, direct empirical evidence on how labor supply elasticities (i.e., firm-specific wage levels) vary with firm size remains limited. In a tentative empirical check, our data indicate a wage-size (output) gradient that is constant in logs, which is inconsistent with higher monopsony power among larger firms (Appendix Figure A.4).

[38] These numbers are huge because they refer to the share of fixed labor in total labor over a given horizon. To put this into perspective: using U.S. census data for manufacturing plants during 1974-2011, Ederhof et al. (2021) define depreciation, rental payments, fringe benefits, and even non-production worker salaries as fixed cost and report a fixed cost share in total cost of 20%. They do not specify the time horizon over which these costs are fixed; we believe that over a four-year horizon many of these cost components—such

broader fixed costs) cannot rationalize the positive effect of firm growth on intermediate output elasticities. While our paper does not aim to adjudicate between fixed costs and our substitution mechanism, we cautiously argue that the data leave substantial room for the mechanism we explore, in addition to a fixed-costs account.

Non-homothetic CES. Our paper rationalizes changes in relative output elasticities through changes in relative input quantities (and prices). Alternatively, intermediate inputs might become more efficient at larger scale, such that firms' relative labor output elasticities decline as a *direct* result of firm growth. Lashkari et al. (2024) discussed such a mechanism in the context of IT inputs. An adaptation of their framework to a CES model and intermediates instead of IT can be described by the following non-homothetic production function:

$$Q_{it} = \Omega_{it}\Lambda_i^K K_{it}^{1-\kappa}\left(\Lambda_i^{LM}\alpha_i^L L_{it}^{\frac{\sigma-1}{\sigma}} + \Lambda_i^{LM}\alpha_i^M\left(\frac{M_{it}}{Q_{it}^{\eta}}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}\kappa}, \tag{19}$$

where $\eta > -1$ is a firm-size scaling parameter that captures the non-homotheticity. If $\eta = 0$, Eq. (19) collapses to Eq. (1). Under this non-homothetic CES, the ratio of output elasticities is (derived in Appendix C.5):

$$\frac{\theta_{it}^L}{\theta_{it}^M} = \frac{\alpha_i^L}{\alpha_i^M}\left(\frac{L_{it}}{M_{it}}\right)^{\frac{\sigma-1}{\sigma}} Q_{it}^{\frac{-\eta}{\sigma}}. \tag{20}$$

That is, the ratio of output elasticities now features an additional *direct* effect of firm size on the output elasticity ratio shaped by η.[39]

Eq. (20) suggests a regression of log changes in the labor-intermediate output elasticity ratio on log changes in input quantity ratios *and* on log changes in output to test between the direct output effect (non-homotheticity) versus the input factor ratio (our substitution channel) in regulating the observed changes in output elasticities. This exercise can be understood as a mediation analysis: If the coefficient on output becomes irrelevant after including the input quantity ratio, the effect of output growth is mediated through the input quantity ratio. We also run a regression using cost shares as dependent variables instead of output elasticities.[40] Appendix Table A.10 reports results from this exercise.

as non-production worker salaries—are in fact adjustable. We note that a definitive benchmarking remains an open question due to lack of a consensus estimate of fixed cost shares.

[39]The translog production function permits general non-homotheticities. Here, we focus on a specific form of non-homotheticity. This can be more broadly interpreted as testing the role of a direct output dependence (non-homotheticity) vs. changing input ratios as mediator.

[40]Translog output elasticities depend directly on input levels, which could limit their variation once we include input quantity ratios as regressors. Our alternative analysis of cost shares as a measure for output

Once we control for the input factor ratio, there is no statistically or economically significant negative effect of output growth on relative output elasticity and cost share ratio changes. This informal test suggests that a non-homotheticity in output is unlikely to be a primary factor behind shifts in labor relative to intermediate output elasticities with firm growth. Instead, the changes in the input ratio and relative input prices appear to explain why growing firms move toward less labor- and more intermediate-intensive production modes, and also offer a natural dynamic explanation for the stronger short-run effects.

6 Implications for Firm-level Labor Shares

We now examine a key implication of the documented growth-induced changes in firms' production modes: the decline in growing firms' labor shares, driven by production function properties (output elasticities) rather than *direct* market power *changes*.

6.1 Labor Share Decomposition and Identification

The labor share. Taking logs of Eq. (6) yields a log decomposition of firms' labor shares in output into three terms: output elasticities, markups, and markdowns:

$$\ln(LS_{it}) = \ln(\theta_{it}^{L}) - \ln(\mu_{it}) - \ln(\gamma_{it}^{L}). \tag{21}$$

Identifying price markups and wage markdowns. To separate the three determinants of the labor share, we construct firm- and time-specific measures of markups and markdowns from our translog production function estimation.

We derive output price markups using the production approach of De Loecker and Warzynski (2012). Assuming that intermediate inputs are a flexible input and that firms take intermediate input prices as given, such that $\gamma_{it}^{M} = 1$, markups, μ_{it}, can be derived from the first-order condition for intermediates within our framework (see Appendix C.6):

$$\mu_{it} = \frac{P_{it}}{\lambda_{it}} = \theta_{it}^{M} \frac{P_{it}Q_{it}}{P_{it}^{M}M_{it}}. \tag{22}$$

Following recent work (e.g., Dobbelaere and Mairesse, 2013), we derive firms' wage markdowns, γ_{it}^{L}, by combining the first-order conditions for intermediates and labor, again

elasticities is not subject to this concern.

assuming that $\gamma_{it}^M = 1$ (see Appendix C.6):

$$\gamma_{it}^L = \frac{\theta_{it}^L}{\theta_{it}^M} \frac{P_{it}^M M_{it}}{P_{it}^L L_{it}}. \tag{23}$$

Importantly, we acknowledge that Eq. (23) also captures factor-augmenting productivity differences between firms that are not captured by output elasticities due to mismeasurement (but they will be captured by cost shares as discussed above).[41]

Total labor wedge. Finally, as a complement, we also study the total labor wedge, which is the combined distortion from markup and labor market imperfections (this is sometimes used as a markup measure, assuming perfect labor markets):

$$\mu_{it}\gamma_{it}^L = \theta_{it}^L \frac{P_{it}Q_{it}}{P_{it}^L L_{it}}. \tag{24}$$

6.2 Results

Strategy. To measure and decompose the effects of firm growth on firm-level labor shares, we estimate the regression model in Eq. (10) for changes in log labor shares, log markups, log markdowns, and log labor wedges as outcome variables. While we focus on the sales labor share because this is the relevant labor share definition that corresponds to the output elasticity of labor and our firm-level production function framework, we also include the value added labor share ($\frac{P_{it}^L L_{it}}{VA_{it}}$) as a robustness check (value added, VA_{it}, is revenue minus intermediate input expenditures).

Results. Table 4 reports the OLS and IV results by time horizon. We add our previous estimates on labor output elasticities to facilitate the analysis. As indicated in Eq. (21), the coefficients on markups, wage markdowns, and output elasticities *sum to* that on the labor share. As before, Figure 5 visualizes the variation underlying the regression estimates.

Declining labor shares. We find a strong negative effect of firm growth on labor shares. OLS results for sales labor shares range from -0.581 to -0.303 for 1- to 10-year differences. Results for value added labor shares, as well as IV estimates, are similar. Again, as for output elasticities, effects somewhat shrink as we widen the time horizons.

[41] While we follow the conventional argument that $\gamma_{it}^M = 1$ is required for measuring μ_{it} and γ_{it}^L as above, we note that γ_{it}^L is effectively a measure of *relative* input cost markdowns for labor vs. intermediate inputs. Assuming exogenous intermediate input prices and flexible intermediate inputs imposes $\gamma_{it}^M = 1$ and hence $\gamma_{it}^L = \frac{\gamma_{it}^L}{\gamma_{it}^M}$. In fact, the relative wedge aligns with our theory as we focus on shifts between intermediates and labor. Furthermore, when we regress the three labor share determinants on output growth below, as in Eq. (10), our specification in *changes* we can relax the assumption that $\gamma_{it}^M = 1$. Instead, it suffices to assume that the intermediate input wedge is constant within a firm ($\gamma_{it}^M = \gamma_i^M$) for these regressions.

Table 4: Firm growth and labor share, markup, markdown, and output elasticity changes.

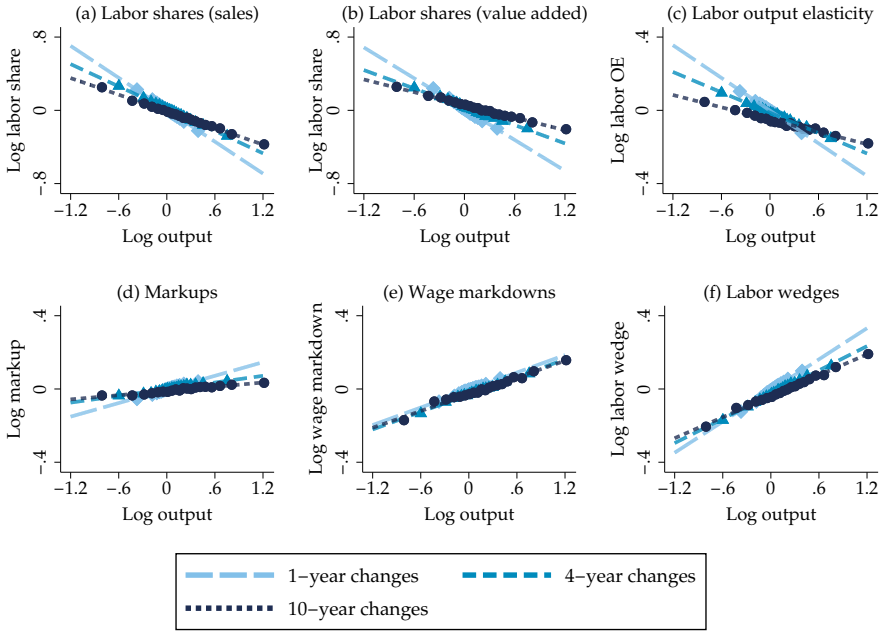
	$\Delta \ln(LS_{it})$ (1)	$\Delta \ln(\mu_{it})$ (2)	$\Delta \ln(\gamma^L_{it})$ (3)	$\Delta \ln(\mu_{it}\gamma^L_{it})$ (4)	$\Delta \ln\left(\frac{\theta^L_{it}}{RTS_{it}}\right)$ (5)	$\Delta \ln(\theta^L_{it})$ (6)	$\Delta \ln\left(\frac{P^L_{it}L_{it}}{VA_{it}}\right)$ (7)
Panel A: 1-year diff.							
OLS estimates							
Log output change	-0.581***	0.124***	0.159***	0.283***	-0.304***	-0.298***	-0.561***
	(0.0040)	(0.0025)	(0.0048)	(0.0044)	(0.0050)	(0.0046)	(0.0076)
R^2	0.534	0.096	0.115	0.191	0.230	0.215	0.190
IV estimates							
Log output change	-0.674***	0.186***	0.169**	0.355***	-0.327***	-0.318***	-0.842***
	(0.0521)	(0.0418)	(0.0759)	(0.0697)	(0.0623)	(0.0645)	(0.120)
R^2	0.524	0.084	0.115	0.185	0.229	0.215	0.160
First-stage F-statistic	102.6	102.6	102.6	102.6	102.6	102.6	102.6
Observations				183,813			
Firms				29,950			
Panel B: 4-year diff.							
OLS estimates							
Log output change	-0.406***	0.0608***	0.160***	0.220***	-0.202***	-0.185***	-0.334***
	(0.0061)	(0.0032)	(0.0065)	(0.0055)	(0.0060)	(0.0063)	(0.0081)
R^2	0.461	0.126	0.193	0.239	0.220	0.201	0.188
IV estimates							
Log output change	-0.443***	0.154***	0.0688	0.223***	-0.242***	-0.220***	-0.603***
	(0.0726)	(0.0444)	(0.0800)	(0.0713)	(0.0706)	(0.0742)	(0.119)
R^2	0.459	0.072	0.180	0.239	0.217	0.199	0.135
First-stage F-statistic	48.57	48.57	48.57	48.57	48.57	48.57	48.57
Observations				70,936			
Firms				11,492			
Panel C: 10-year diff.							
OLS estimates							
Log output change	-0.303***	0.0383***	0.152***	0.191***	-0.140***	-0.112***	-0.231***
	(0.0062)	(0.0030)	(0.0060)	(0.0048)	(0.0053)	(0.0057)	(0.0068)
R^2	0.378	0.125	0.213	0.253	0.188	0.161	0.178
Observations				49,915			
Firms				10,595			

Notes: The table reports OLS and IV regressions estimating the specification in Eq. (10). The dependent variables in Columns (1)–(7) are log changes of the labor share in sales, markups, wage markdowns, labor wedge, labor output elasticity divided by returns to scale, labor output elasticity, and labor share in value added, respectively. Panel A reports 1-year differences and Panel B reports 4-year differences, each showing OLS and IV estimates. Panel C reports 10-year differences (OLS only). All regressions control for industry-year fixed effects. Standard errors are clustered at the firm level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Half to one third of the effect: declining output elasticity of labor. In the short and medium run (1- to 4-year differences), *half* of the negative effect of output growth on labor shares is driven by the declining output elasticity of labor (the output elasticity coefficient is about half the size of the labor share coefficient). For 10-year differences, the declining output elasticity still accounts for one third of the effect (as changing returns to scale become more important over longer horizons).

The remainder: conventional market power effects. The remaining half (short- to medium-run) or two thirds (long-run) of the within-firm effect of firm growth on labor share changes is explained by increasing markups and wage markdowns. As we control for industry fixed effects, output growth reflects an increase in firms' market shares, and a firm's market power is expected to increase with its market share (as in Atkeson and

Figure 5: Firm growth and labor share, markup, markdown, and output elasticity changes (OLS, binned scatter plots).



Notes: The figure reports binned scatter plots estimating the specification in Eq. (10) with OLS for various differences. The graphs report binned scatter plots for changes in the logs of labor shares in sales and value added, labor output elasticities (not divided by returns to scale), markups, wage markdowns, and the labor wedge against log output changes. All panels report results that are residualized by industry-year fixed effects. German firm-level data.

Burstein, 2008 for product markets and Berger et al., 2022 for labor markets).[42]

7 Other Countries and Aggregate Industry Implications

We close our paper by extending the analysis to other countries and sectors using firm and industry data.

7.1 International Industry Panel Data: CompNet Data

CompNet data. We use the 9th vintage of the CompNet data (CompNet, 2023), which is a micro-aggregated database for 22 European countries.[43] The data are provided by the Competitiveness Research Network (henceforth, CompNet). CompNet sources its data from representative administrative firm-level records located within European na-

[42] Also in the cross section, market power effects are more important, consistent with Autor et al. (2020) and De Loecker et al. (2020) (see Figure A.5).

[43] We drop Malta, due to insufficient observations for output elasticities, and the UK, which did not provide industry-level data to CompNet.

tional statistical institutes and central banks (akin to the US Census data). The CompNet team distributes harmonized data collection protocols (i.e., Stata codes) across the data providers and invests significant efforts in harmonizing the input data to maximize comparability across countries. These protocols compute micro-aggregated results. From these results, the CompNet team constructs the CompNet database. The data are aggregated at various levels. We use the country-industry-level data (NACE Rev. 2, two-digit industries), which is the most detailed aggregation level available. The data contain, among other features, country-industry-level information on firms' sales, inputs, expenditures, markups, wage markdowns, and output elasticities (estimated with a translog production function that does not incorporate firm-level price data, see CompNet's User Guide: CompNet, 2023). They cover 1999-2021, and we focus on manufacturing (NACE Rev. 2 two-digit industries 10-33) which has the best coverage. Yet, we also present key results for non-manufacturing industries.[44] Yearly coverage varies across countries as shown in Appendix Table A.11. We focus on the data containing firms with at least 20 employees as this is available for more countries and is consistent with our German micro data (our results hold for the subset of countries for which we have data based on firms of all size classes.) The data are weighted by firm population weights. For further details, we refer to CompNet's User Guide (CompNet, 2023).

While no comparable US data are available, we provide a limited set of results for the US based on the NBER-CES Manufacturing Industry Database (which lacks output elasticities) for 1958-2016.[45]

Commissioning our own firm-level regressions in the CompNet micro data. To supplement the country-industry-level CompNet data, we collaborated with the CompNet team to incorporate additional firm-level regressions into their *10th* vintage data collection. These regressions replicate our firm-level analysis in 11 countries and extend our initial analysis beyond manufacturing. The 10th vintage data include a different sample of countries and covers more recent years than the 9th vintage country-industry data (we document yearly coverage in regression tables). Due to the long running time of codes, we have only received results for the subset of countries reported below in Figure 6.

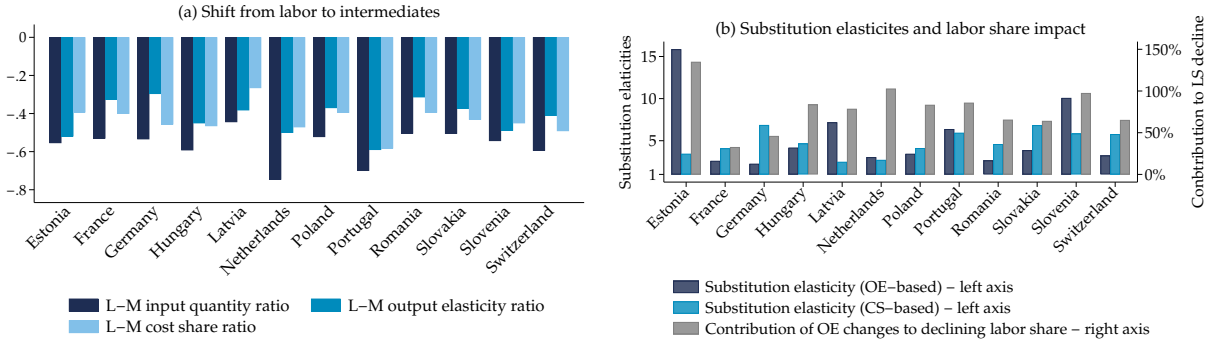
[44]This includes the NACE rev. 2 industries 41-43 (construction), 45-47 (wholesale/retail trade and repair of motor vehicles and motorcycles), 49-53 (transportation/storage), 55-56 (accommodation/food services), 58-63 (information and communication technology), 68 (real estate), 69-75 (professional/scientific/technical activities), and 77-82 (administrative/support service activities).

[45]The NBER CES database currently ends in 2018, but capital, which is required to construct cost shares, is missing after 2016.

7.2 Firm-level Results for 11 other European Countries

Figure 6 summarizes firm-level manufacturing sector results (OLS) for the 11 countries using the CompNet infrastructure. We replicate our previous firm-level analysis across these countries. Panel (a) reports the *ratio* of regression coefficients for labor and intermediate input quantities, cost shares, and output elasticities. Panel (b) reports implied substitution elasticities and the contribution of changes in labor output elasticities to declines in firm-level labor shares with firm growth. Substitution elasticities are reported based on output elasticity (OE) and cost share (CS) coefficients. To facilitate comparison, we include our previous results for German manufacturing firms. Appendix Table A.12 details all underlying regression results and additionally report results including non-manufacturing sectors. All results are highly statistically significant.

Figure 6: Country-specific results from firm-level regressions (manufacturing).



Notes: The figures reports OLS estimates of the specification in Eq. (10) for different countries (manufacturing sector firm micro data). Panel (a) report ratios of coefficients on labor and intermediate input quantities, cost shares, and output elasticities. A negative value indicates a shift from labor to intermediates. Panel (b) reports implied substitution elasticities (left axis) based on the shifts in input quantity ratios and (i) output elasticity ratios (OE-based), and (ii) cost share ratios (CS-based). Additionally, Panel (b) reports the contribution of changes in firms' output elasticities to labor share shares with firm growth (right axis). This is computed by dividing regression coefficients on output elasticities by regression coefficients on labor shares (as in Section 6.2). Appendix Table A.12 reports the underlying regression results in more detail and outcome variable by outcome variable. Results for Germany come from our previous firm-level analysis on the German firm-level data. Other results are based on firm-level data from CompNet data providers.

Results. Our previous firm-level findings for Germany are confirmed across all 11 other countries: input quantities, cost shares, and output elasticities shift from labor to intermediates, with substitution elasticities consistently exceeding unity . The quantitative intensity of the shift from labor to intermediates differs somewhat across countries (we are not aware of consistently estimated cross-country estimates of the labor supply elasticity to the firm). Also, labor output elasticities decline more sharply with firm growth in most countries and, therefore, contribute more strongly to declining labor shares in growing firms. Results for all sectors (Table A.12) are broadly similar to manufacturing.

Young and mature firms. In six of the eleven countries included in the firm-level CompNet data analysis, data on firm age is available. In Appendix Table A.13, we replicate

our analysis for young (no older than five years) and mature (older than five years) manufacturing sector firms. The results are similar between the two groups. If anything, the shift from labor toward intermediates appears somewhat stronger for mature firms. Similar findings by age hold if considering all sectors (not reported).

7.3 Industry-level Dynamics in 20 European Countries and the US

We now study the industry-level analog of our firm-level analysis. We expect divergence in the effect sizes given aggregation biases, reallocation and compositional changes, intra-industry intermediate input patterns now being absorbed, and because market- vs. firm-specific labor supply elasticities may differ.

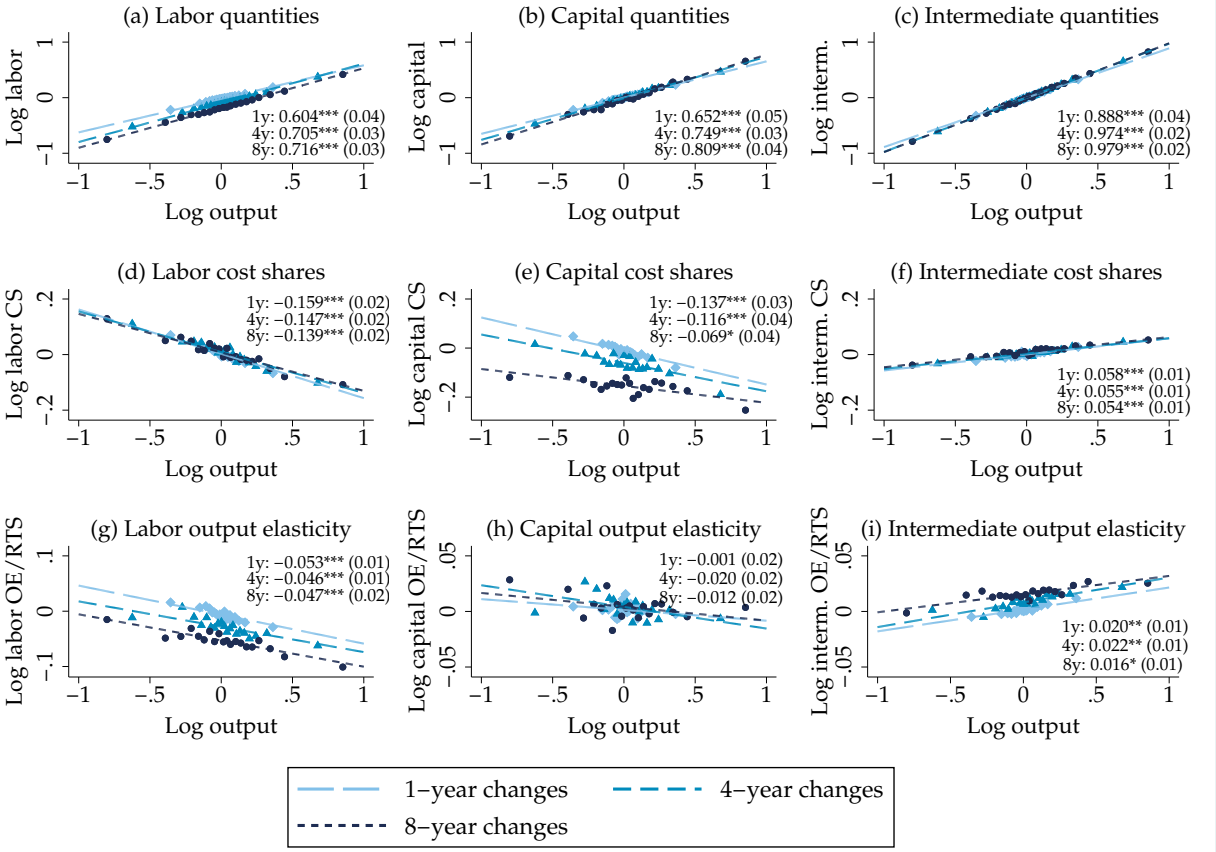
Industry growth regressions. We transfer our firm-level growth analyses to the industry level. We regress changes in log input quantities, log cost shares, and log output elasticities on industry output growth in the aggregated *country-industry* data provided by CompNet. The horizons are 1-, 4-, and 8-year changes (10-year changes are not feasible with CompNet data), controlling for country-year and industry-year fixed effects as we pool country-industry pairs (denoted by c and j, respectively). Compared to the firm-level analysis, this industry analysis could also be read as studying the impact of more aggregate shocks. (Results are similar with country-industry fixed effects.)

Results. Figure 7 present industry-level results (OLS) akin to our previous firm-level results. Our analysis shows that the relationships observed in firm-level data also hold at the European industry level: as industries expand, labor-intermediate ratios, labor cost shares, and labor output elasticities decrease, while intermediate cost shares and output elasticities rise. The point estimates are precisely estimated and indicate that 10% higher industry output growth leads to a 0.5% reduction in the labor output elasticity (adjusted for RTS changes).[46]

Country-specific results. Appendix Table A.14 present results for changes in industry-level labor-intermediate quantity, output elasticity, and cost share ratios in response to industry growth, based on country-specific *regression coefficients* from reproducing Figure 7 by countries (4-year changes). As before, we estimate substitution elasticities using Eq. (14), once based on coefficients on output elasticities from our industry growth regressions, once using cost share coefficients instead. US values are derived from running the same regressions that we run in the European CompNet data with the NBER-CES

[46]Without taking logs, coefficients on industry-level labor, capital, and intermediate output elasticities divided by returns to scale are -0.0112, -0.00003, and 0.0148, respectively (4-year changes). Hence, the increase in intermediate input output elasticities is again largely rationalized by the decline in labor output elasticities.

Figure 7: Industry-level adjustments in response to industry growth (OLS).



Notes: The figure reports binned scatter plots from estimating industry versions of the specification in Eq. (10) with OLS for various differences. Panels (a)-(i) report on regressions of changes in the logs of country-industry-level labor, capital, and intermediate quantities, country-industry-level labor, capital, and intermediate cost shares, and country-industry-level labor, capital, and intermediate output elasticities over returns to scale against country-industry-level log output changes, respectively. All panels control for country-year and industry-year fixed effects and we pool country-industry pairs. Country-industry observations: 5,233 (1-year), 4,207 (4-year) 2,904 (8-year). CompNet manufacturing data.

Manufacturing Industry Database (1958-2016) using cost shares (the data does not include output elasticities).

In almost all countries, industry growth is associated with a shift from industry-level labor to intermediates in terms of quantities, cost shares, and output elasticities (with some quantitative differences). Industry-level labor-intermediate substitution elasticities exceed unity in almost all countries. The US data lack output elasticity estimates, but changes in labor and intermediate quantities and cost shares are similar in Europe and the US. Also, substitution elasticities based on cost shares are similar (approximately 4 in both regions). The substitution elasticity estimate based on directly computed output elasticity movements in Europe is 1.34 for manufacturing industries and 1.48 for non-manufacturing industries.

7.4 Industry Labor Share Implications

As with the firm-level analysis, we now study the industry labor share consequences of the decline of industry labor output elasticities with industry growth. We reiterate that our main focus is on firm-level growth, and drawing aggregate implications would necessitate an input-output network analysis or an open economy perspective.

Labor shares and their drivers. Figure 8 is the European country-industry counterpart to the previous firm-level analyses. We regress country-industry log changes in outcome variables on output growth, controlling for country-year and industry-year fixed effects. Due to aggregation bias, we do not expect that coefficients on output elasticities, markups, and wage markdowns sum to labor share coefficients (as they did in our firm analysis).[47]

Results. The results qualitatively align with our firm-level findings: as industries grow, industry labor shares decline, primarily due to a reduction in labor output elasticities. However, markups also *decline*, and changes in wage markdowns are statistically insignificant, which cannot rationalize the negative relationship between labor shares and industry growth. Instead, the industry analysis points to a substantial role for labor output elasticities. Quantitatively, the slope of the aggregate output elasticity is about a quarter of the slope of the labor share in sales (4-year specification).[48] Notably, as with the firm-level analysis, the value added labor share also declines, highlighting that the rise in intermediates mainly reflects a shift away from labor.

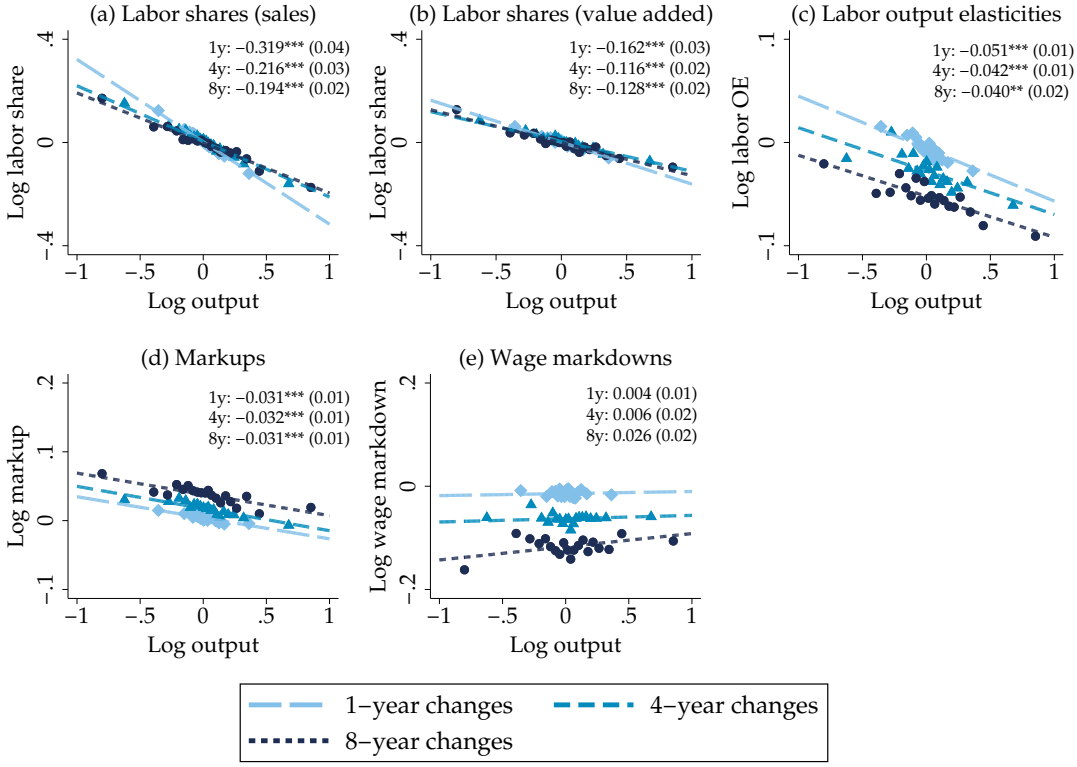
8 Conclusion

We have documented and dissected a stylized fact about firm growth: the declining importance of labor and the increasing importance of intermediate inputs in firms' production. As labor and intermediates function as substitutes, this shift in the input mix reduces (increases) the output elasticity of labor (intermediates). As a result, the labor (intermediates) share falls (increases) in growing firms. This shift from labor to intermediates explains between one-third (10-year horizon) and one-half (1- and 4-year horizon)

[47] Aggregation biases may arise from compositional effects, weighting choices, Jensen's inequality, intra-industry trade, and shifts in labor demand and supply.

[48] In addition to classical aggregation biases, note that under competitive markets, the industry labor share equals the industry output elasticity of labor: $\sum_n s_{it} LS_{it} = \sum_n s_{it} \theta^L_{it}$, where s_{it} is the sales weight. Under imperfect markets, the pass-through from changes in industry output elasticities of labor to changes in industry labor shares is additionally shaped by the distribution of firms' markups and wage markdowns. Nevertheless, qualitatively, it remains true that the partial effect of a change in the aggregate labor-output elasticity on the labor share, holding fixed market imperfections, is positive. Equally, the partial effect (ceteris paribus) of changes in industry markups and wage markdowns on industry labor shares is negative. Importantly, "holding fixed" refers here not only to aggregates, but also to firm distributions.

Figure 8: Labor share, market imperfection, and output elasticity changes in response to industry growth (OLS, binned scatter plots).



Notes: The figure reports binned scatter plots estimating industry versions of Eq. (10) for various differences (OLS). The graphs relate changes in logs of country-industry-level labor shares in sales and value added, labor output elasticities (not divided by returns to scale), markups, and wage markdowns to country-industry-level log output changes. All regressions control for country-year and industry-year fixed effects. Country-industry observations: 5,233 (1-year), 4,207 (4-year) 2,904 (8-year). CompNet data.

of the decline in labor shares in growing firms and accounts for most of the decline in labor shares within growing industries. We establish these patterns using OLS and IV regression in rich German firm-level micro data, administrative firm-level data from 11 additional countries, as well as in micro-aggregated industry data for 20 European countries. Our findings hold for small and large and young and mature growing firms. We rationalize the facts with a parsimonious production function framework characterized by (i) an elasticity of substitution between intermediates and labor that exceeds one, and (ii) an increasing shadow price of labor (e.g., due to monopsony or adjustment costs).

The findings also have broader implications. For instance, many current estimates of factor misallocation and productivity rely on Cobb-Douglas production functions, which assume constant output elasticities. Moreover, our results imply that, generally, any shocks that affect output growth will alter output elasticities, input mixes, and cost shares, with large magnitudes for horizons of up to 10 years. Ignoring this regularity may confound analyses of firm-level effects from productivity shocks, trade, competition, or subsidies

on a wide range of related outcomes in the short and medium term.

Our findings also show how monopsony not only distorts steady state firm sizes but also firm growth. We trace how firms respond to these growth constraints by intensifying their use of intermediate inputs—i.e., by outsourcing production—for which we estimate a high elasticity of substitution with labor. In our case, these intermediate inputs are supplied by other firms and are largely product components and materials (rather than, e.g., temporary agency work). It thus appears that monopsony lengthens the supply chain as firms, or, more broadly, industries and perhaps the aggregate economy, grow.

Finally, the mechanism we highlight in this paper offers avenues for future research, such as firms' life cycles, reallocation, potential implications for aggregate labor shares, global production networks, directed innovation, disaggregated analyses to specific subcategories of intermediate inputs or labor types, and a deeper treatment of capital.

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Online Appendix

A Appendix Tables and Figures

Table A.1: Summary statistics of the German manufacturing sample.

	Mean (1)	**p25** (2)	**Median** (3)	**p75** (4)	**St.Dev.** (5)	**Obs.** (6)
Number of employees	366.63	53	109	272	2559.55	183,813
Real wage (1995 values)	34,162	25,972	33,823	41,485	11,492	183,813
Labor share (sales)	0.30	0.21	0.29	0.38	0.12	183,813
Labor share (value added)	0.80	0.63	0.76	0.88	2.89	183,813
Labor cost share	0.31	0.22	0.30	0.39	0.13	183,813
Capital cost share	0.06	0.03	0.05	0.07	0.04	183,813
Intermediates cost share	0.63	0.54	0.64	0.73	0.14	183,813
Materials, energy, ext. components cost share	0.41	0.29	0.41	0.53	0.17	183,813
Merchandise cost share	0.05	0.00	0.00	0.05	0.10	183,813
Subcontracted production by other companies cost share	0.03	0.00	0.00	0.03	0.06	183,813
Repairs, maintenance, installations cost share	0.02	0.01	0.03	0.04	0.02	183,813
Rents, leases, leasing cost share	0.03	0.01	0.02	0.04	0.03	183,813
Temporary agency worker cost share	0.01	0.00	0.00	0.01	0.03	164,410
Other intermediates cost share	0.10	0.05	0.08	0.12	0.06	183,813
Markup	1.09	0.97	1.05	1.17	0.20	183,813
Wage markdown	1.08	0.71	0.96	1.32	0.55	183,813
Output elasticity of labor	0.31	0.23	0.31	0.38	0.11	183,813
Output elasticity of capital	0.12	0.08	0.11	0.15	0.06	183,813
Output elasticity of intermediates	0.64	0.57	0.64	0.71	0.10	183,813
Returns to scale	1.06	0.98	1.05	1.13	0.12	183,813

Notes: This table presents summary statistics for selected variables from the German manufacturing sector firm-level data. Columns (1)-(5) show the mean, 25th percentile, median, 75th, and standard deviation, respectively. Column (6) reports the number of non-missing observations. German micro-data.

Table A.2: Firm-level adjustments in heads and full time equivalents to firm growth (OLS).

	OLS	OLS	IV	IV
	Log FTE changes	Log head changes	Log FTE changes	Log head changes
Panel A: 1-year changes	(1)	(2)	(3)	(4)
Log output change	0.285***	0.284***	0.259***	0.264***
	(0.00421)	(0.00419)	(0.0531)	(0.0518)
Observations	160,764	160,764	160,764	160,764
N of firms	28,969	28,969	28,969	28,969
First-stage F-Statistic			98.66	98.66
R^2	0.189	0.205	0.188	0.205
Panel B: 4-year changes	(1)	(2)	(3)	(4)
Log output change	0.490***	0.487***	0.500***	0.504***
	(0.00747)	(0.00745)	(0.0811)	(0.0788)
Observations	53,106	53,106	53,106	53,106
N of firms	7,879	7,879	7,879	7,879
First-stage F-Statistic			38.17	38.17
R^2	0.458	0.464	0.458	0.464

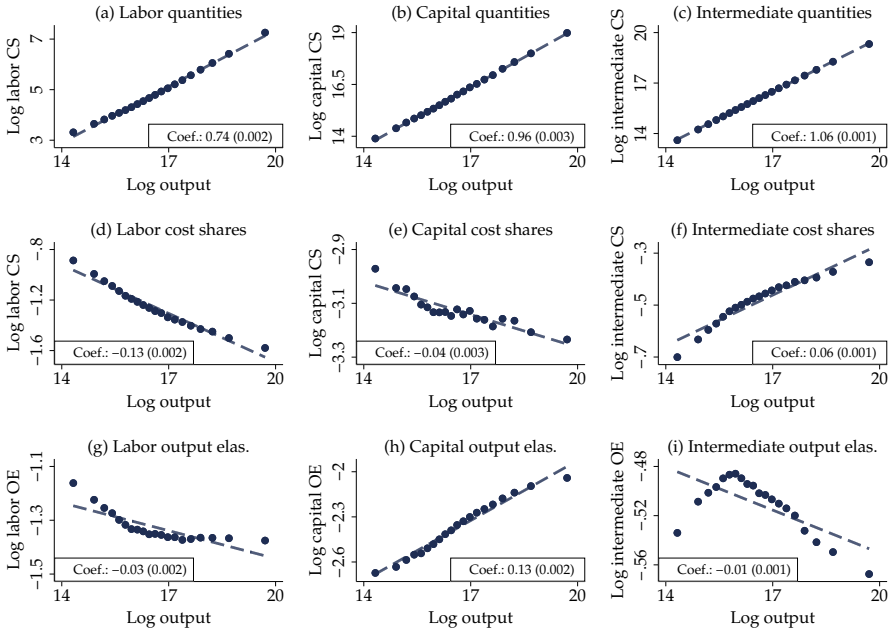
Notes: The table reports OLS and IV regressions from estimating the specification in Eq. (10) for 1- and 4-year differences. The dependent variable is logged employment in changes, once measured in head counts, once measured as full time equivalents. All columns report regressions of those dependent variables on output growth for 1- and 4-year changes. Panels A-B report results for 1- and 4-year differences, respectively. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data. A future version of the paper will be updated to also include the results for 10-year changes (OLS), which remained under disclosure review at the point of circulation.

Table A.3: Firm-level adjustments in cost shares and output elasticities to firm growth. Different specifications: not logged, not dividing by RTS. OLS and IV regressions.

	1st stage	$\Delta \frac{w_{it}L_{it}}{C_{it}}$	$\Delta \frac{r_{it}K_{it}}{C_{it}}$	$\Delta \frac{z_{it}M_{it}}{C_{it}}$	$\Delta \frac{\theta^L_{it}}{RTS_{it}}$	$\Delta \frac{\theta^K_{it}}{RTS_{it}}$	$\Delta \frac{\theta^M_{it}}{RTS_{it}}$	$\Delta \ln(\theta^L_{it})$	$\Delta \ln(\theta^K_{it})$	$\Delta \ln(\theta^M_{it})$	$\Delta\theta^L_{it}$	$\Delta\theta^K_{it}$	$\Delta\theta^M_{it}$	ΔRTS_{it}
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Panel A: 1-year differences														
OLS estimates														
Log output change		-0.103***	-0.0380***	0.141***	-0.0710***	-0.0201***	0.0911***	-0.298***	-0.309***	0.157***	-0.0723***	-0.0203***	0.0991***	0.00647***
		(0.0011)	(0.0003)	(0.0011)	(0.0007)	(0.0003)	(0.0009)	(0.0046)	(0.0064)	(0.00129)	(0.0008)	(0.0004)	(0.0007)	(0.0004)
R^2		0.303	0.419	0.413	0.266	0.191	0.277	0.215	0.132	0.397	0.278	0.183	0.425	0.157
IV estimates														
Export demand shock (first stage)	0.0451***													
	(0.0045)													
Log output change		-0.0933***	-0.0283***	0.122***	-0.0626***	-0.0133***	0.0759***	-0.318***	-0.220**	0.127***	-0.0672***	-0.0135***	0.0914***	0.0107*
		(0.0144)	(0.0042)	(0.0153)	(0.0098)	(0.0039)	(0.0116)	(0.0645)	(0.0995)	(0.0170)	(0.0113)	(0.00455)	(0.0102)	(0.00575)
R^2	0.205	0.294	0.427	0.400	0.309	0.208	0.329	0.215	0.127	0.387	0.277	0.171	0.424	0.049
First-stage F-stat		102.6	102.6	102.6	102.6	102.6	102.6	102.6	102.6	102.6	102.6	102.6	102.6	102.6
Observations							183,813							
Firms							29,950							
Panel B: 4-year differences														
OLS estimates														
Log output change		-0.0692***	-0.0366***	0.106***	-0.0438***	-0.0136***	0.0574***	-0.185***	-0.164***	0.112***	-0.0420***	-0.0124***	0.0729***	0.0185***
		(0.00157)	(0.0006)	(0.00179)	(0.00108)	(0.000525)	(0.0014)	(0.0063)	(0.0077)	(0.0021)	(0.0014)	(0.0007)	(0.0013)	(0.0008)
R^2		0.303	0.419	0.413	0.266	0.191	0.277	0.201	0.160	0.369	0.220	0.154	0.388	0.157
IV estimates														
Export demand shock	0.0635***													
	(0.0091)													
Log output change		-0.0555***	-0.0280***	0.0836***	-0.0465***	-0.0070	0.0536***	-0.220***	-0.125	0.105***	-0.0445***	-0.0005	0.0739***	0.0289***
		(0.0177)	(0.0062)	(0.0201)	(0.0127)	(0.0058)	(0.0157)	(0.0742)	(0.0973)	(0.0227)	(0.0153)	(0.0076)	(0.0143)	(0.00876)
R^2	0.297	0.403	0.402	0.266	0.175	0.276	0.183	0.199	0.158	0.368	0.220	0.121	0.388	0.139
First-stage F-stat		48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57
Observations							70,936							
Firms							11,492							
Panel C: 10-year differences														
OLS estimates														
Log output change		-0.0535***	-0.0275***	0.0810***	-0.0305***	-0.0051***	0.0356***	-0.112***	-0.0441***	0.0860***	-0.0236***	-0.0018***	0.0559***	0.0304***
		(0.0016)	(0.0006)	(0.0018)	(0.0011)	(0.0005)	(0.0014)	(0.00569)	(0.0073)	(0.0021)	(0.00137)	(0.0007)	(0.0013)	(0.0008)
R^2		0.259	0.318	0.347	0.222	0.155	0.201	0.161	0.141	0.322	0.174	0.142	0.334	0.262
Observations							49,915							
Firms							10,595							

Notes: This table reports OLS and IV regressions of the specification in Eq. (10). Dependent variables in Columns (2)–(14) are non-logged changes in labor, capital, and intermediate input cost shares, output elasticities divided by returns to scale, and logged and non-logged elasticities not divided by RTS. Panels A–C correspond to 1-, 4-, and 10-year differences. IV uses foreign demand shocks as instruments (Eq. (12)); Column (1) reports the first stage. Standard errors clustered at the firm level. Significance levels: 0.01***, 0.05**, 0.1*.

Figure A.1: The cross-sectional relationships: output elasticities and cost shares.



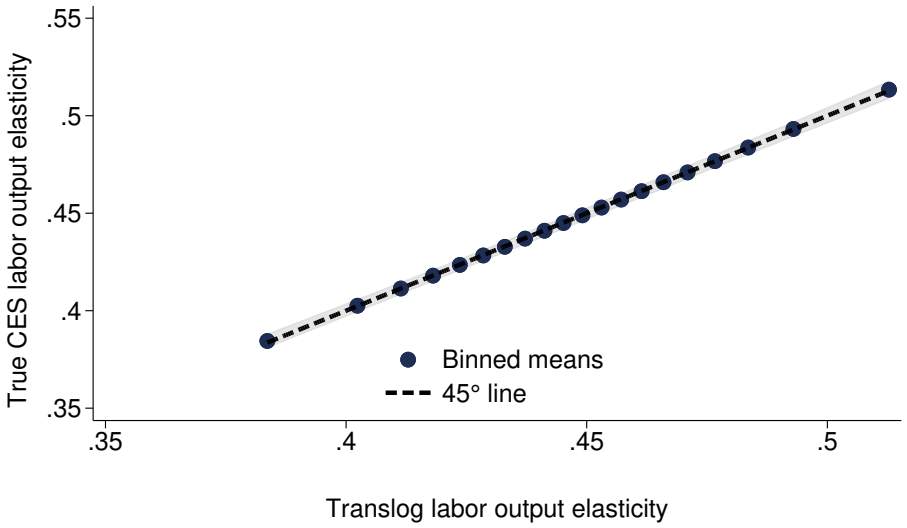
Notes: The figure reports binned scatter plots estimating the specification in Eq. (10) in levels with OLS. Panels (a)-(i) report on regressions of the logs of labor, capital, and intermediate quantities, labor, capital, and intermediate cost shares, and labor, capital, and intermediate output elasticities over returns to scale on log output, respectively. All panels report results that are residualized by industry-year fixed effects. German firm-level data.

Table A.4: Firm-level adjustments in intermediate cost shares by intermediate type (OLS).

	$\Delta \ln(E_{it}^{CS})$	$\Delta \ln(Merch_{it}^{CS})$	$\Delta \ln(Sub_{it}^{CS})$	$\Delta \ln(Rep_{it}^{CS})$	$\Delta \ln(Rent_{it}^{CS})$	$\Delta \ln(Temp_{it}^{CS})$	$\Delta \ln(Other_{it}^{CS})$
Panel A: 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output change	0.297***	0.408***	0.388***	-0.104***	-0.417***	1.165***	-0.107***
	(0.001)	(0.029)	(0.022)	(0.011)	(0.011)	(0.0329)	(0.008)
Observations	183,807	83,975	96,062	176,845	175,391	86,927	183,813
N of firms	29,950	15,277	19,842	29,598	29,340	18,048	29,950
R^2	0.108	0.069	0.061	0.029	0.055	0.133	0.030
Panel B: 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output change	0.187***	0.429***	0.218***	-0.113***	-0.281***	0.578***	-0.054***
	(0.001)	(0.044)	(0.035)	(0.014)	(0.018)	(0.0431)	(0.011)
Observations	70,933	36,495	35,962	68,859	67,827	32,512	70,936
N of firms	11,492	6,096	6,737	11,366	11,149	5,640	11,492
R^2	0.128	0.117	0.115	0.072	0.079	0.150	0.065
Panel C: 10-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output change	0.137***	0.217***	0.148***	-0.072***	-0.218***	0.434***	0.011
	(0.008)	(0.051)	(0.038)	(0.015)	(0.021)	(0.0416)	(0.012)
Observations	48,950	21,102	22,376	46,996	46,143	19,064	48,953
N of firms	10,381	4,542	5,743	10,189	9,996	5,647	10,381
R^2	0.130	0.126	0.120	0.074	0.080	0.124	0.068

Notes: The table reports on OLS regressions from estimating the specification in Eq. (10) for 1-year (Panel A), 4-year (Panel B), and 10-year differences. The dependent variables in Columns (1)-(7) are log changes of raw materials, energy, and external components cost shares, merchandise cost shares, subcontracted production performed by other companies cost shares, repairs, maintenance, and installation cost shares, rents, leases, and leasing cost shares, temporary agency worker cost shares, and other intermediate inputs cost shares, respectively. All columns report regressions of those dependent variables on log output changes for 1-, 4-, and 10-year changes. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Figure A.2: Simulated Data: Translog Estimation Recovers the True CES-based Labor Output Elasticities.



Notes: The figure shows simulation results where we simulated data from a true underlying CES production function (with only labor and intermediates) and estimated output elasticities using a translog production function estimation routine similar to the one described by Eq. 9. In the simulation we directly observe productivity and do not need to implement a control function approach. The simulation is available on the authors' websites and through this link. The figure projects true CES vs. estimated translog-based labor output elasticities. The dashed line reports the 45 degree line. Dots indicated binned mean values. The gray area indicates the 95% Monte Carlo bands from repeatedly running the simulations for 500 times.

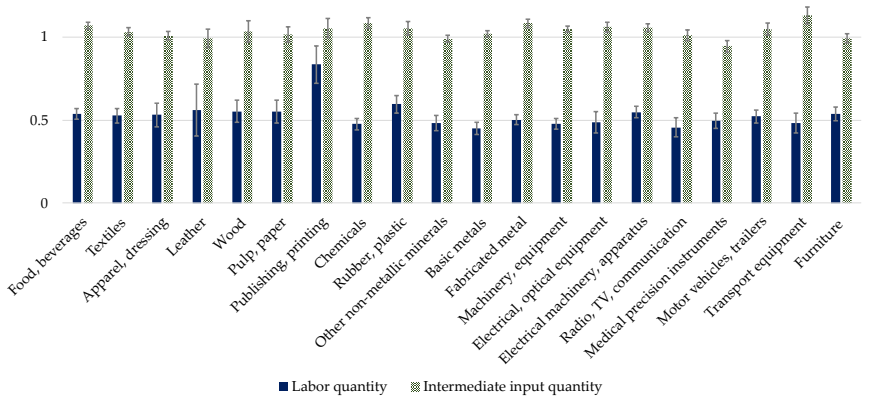
Table A.5: Firm-level adjustments in intermediate type quantities (OLS).

	$\Delta \ln(E_{it})$	$\Delta \ln(Merch_{it})$	$\Delta \ln(Sub_{it})$	$\Delta \ln(Rep_{it})$	$\Delta \ln(Rent_{it})$	$\Delta \ln(Temp_{it})$	$\Delta \ln(Other_{it})$
Panel A: 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output change	1.066***	1.179***	1.151***	0.664***	0.352***	1.932***	0.661***
	(0.007)	(0.003)	(0.0217)	(0.0113)	(0.0107)	(0.0323)	(0.008)
Observations	183,807	83,975	96,062	176,845	175,391	86,927	183,813
N of firms	29,950	15,277	19,842	29,598	29,340	18,048	29,950
R^2	0.467	0.116	0.117	0.077	0.055	0.205	0.109
Panel B: 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output change	1.044***	1.281***	1.062***	0.744***	0.576***	1.427***	0.804***
	(0.011)	(0.044)	(0.0359)	(0.014)	(0.018)	(0.042)	(0.0116)
Observations	70,933	36,495	35,962	68,859	67,827	32,512	70,936
N of firms	11,492	6,096	6,737	11,366	11,149	5,640	11,492
R^2	0.600	0.187	0.182	0.180	0.103	0.232	0.264
Panel C: 10-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output change	1.030***	1.107***	1.026***	0.820***	0.675***	1.322***	0.904***
	(0.009)	(0.051)	(0.039)	(0.015)	(0.022)	(0.041)	(0.012)
Observations	48,950	21,102	22,376	46,996	46,143	19,064	48,953
N of firms	10,381	4,542	5,743	10,189	9,996	5,647	10,381
R^2	0.690	0.207	0.211	0.252	0.134	0.224	0.386

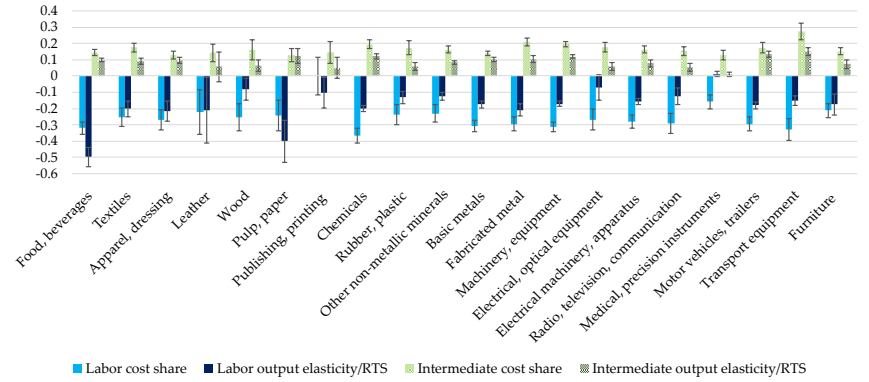
Notes: The table reports on OLS regressions from estimating the specification in Eq. (10) for 1-year (Panel A), 4-year (Panel B), and 10-year differences. The dependent variables in Columns (1)-(7) are log changes of raw materials, energy, and external components, merchandise, subcontracted production performed by other companies, repairs, maintenance, and installation, rents, leases, and leasing, temporary agency worker, and other intermediate inputs, respectively. All columns report regressions of those dependent variables on log output changes for 1-, 4-, and 10-year changes. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Figure A.3: Firm-level adjustments separately by industry, 4-year changes, OLS.

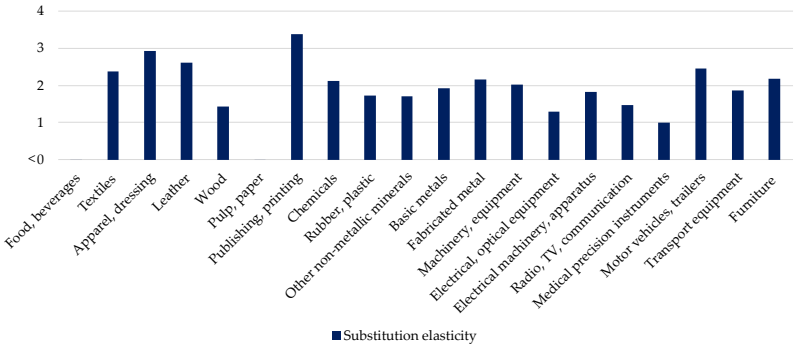
(a) Input quantities



(b) Output elasticities and cost shares



(c) Substitution elasticities



Notes: The figure reports OLS regressions from estimating Eq. (10) in 4-year changes by two-digit NACE manufacturing industries. The dependent variables are logged labor and intermediate input quantities (Panel (a)), cost shares and labor and intermediate output elasticities divided by returns to scale ((Panel (b)), and implied substitution elasticities derived as described in Section 5 (Panel (c)). In Panel (c), values for "Food, beverages" and "Pulp, paper" are truncated at zero as these two estimates are negative (-8.25 and -7.61, respectively) and should not be interpreted, as they result from output elasticity responses exceeding input quantity responses (likely due to measurement errors or small sample sizes). All regressions control for industry-year fixed effects. Standard errors are clustered at the firm level. The error bars indicate 95% confidence intervals. Results for 1- and 10-year changes are similar. German firm-level data.

Table A.6: Firm-level adjustments in output elasticities in response to firm growth: effects by size quintiles (4-year changes).

	$\Delta \ln(L_{it})$	$\Delta \ln(K_{it})$	$\Delta \ln(M_{it})$	$\Delta \ln(CS^L_{it})$	$\Delta \ln(CS^K_{it})$	$\Delta \ln(CS^M_{it})$	$\Delta \ln(\frac{\theta^L_{it}}{RTS_{it}})$	$\Delta \ln(\frac{\theta^K_{it}}{RTS_{it}})$	$\Delta \ln(\frac{\theta^M_{it}}{RTS_{it}})$	$\Delta \ln(RTS_{it})$
Panel A: 1st size quintile	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log output change	0.505***	0.259***	1.051***	-0.248***	-0.594***	0.183***	-0.179***	-0.183***	0.095***	0.024***
	(0.011)	(0.016)	(0.009)	(0.0111)	(0.0172)	(0.0072)	(0.010)	(0.017)	(0.005)	(0.001)
Observations	14,984	14,984	14,984	14,984	14,984	14,984	14,984	14,984	14,984	14,984
N of firms	4,408	4,408	4,408	4,408	4,408	4,408	4,408	4,408	4,408	4,408
R^2	0.558	0.290	0.862	0.426	0.492	0.421	0.337	0.255	0.351	0.357
Panel B: 2nd size quintile	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log output change	0.484***	0.217***	1.034***	-0.280***	-0.625***	0.174***	-0.206***	-0.175***	0.098***	0.018***
	(0.012)	(0.015)	(0.009)	(0.0106)	(0.0153)	(0.0062)	(0.011)	(0.014)	(0.004)	(0.001)
Observations	13,276	13,276	13,276	13,276	13,276	13,276	13,276	13,276	13,276	13,276
N of firms	4,508	4,508	4,508	4,508	4,508	4,508	4,508	4,508	4,508	4,508
R^2	0.540	0.268	0.872	0.456	0.505	0.457	0.383	0.295	0.384	0.325
Panel C: 3rd size quintile	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log output change	0.492***	0.208***	1.032***	-0.299***	-0.630***	0.169***	-0.200***	-0.164***	0.093***	0.017***
	(0.012)	(0.013)	(0.010)	(0.0131)	(0.0134)	(0.0071)	(0.011)	(0.013)	(0.005)	(0.001)
Observations	13,211	13,211	13,211	13,211	13,211	13,211	13,211	13,211	13,211	13,211
N of firms	4,265	4,265	4,265	4,265	4,265	4,265	4,265	4,265	4,265	4,265
R^2	0.546	0.295	0.877	0.461	0.559	0.463	0.353	0.355	0.394	0.312
Panel D: 4th size quintile	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log output change	0.528***	0.178***	1.047***	-0.295***	-0.687***	0.160***	-0.202***	-0.169***	0.094***	0.014***
	(0.015)	(0.012)	(0.009)	(0.0138)	(0.0129)	(0.0063)	(0.013)	(0.010)	(0.005)	(0.001)
Observations	13,276	13,276	13,276	13,276	13,276	13,276	13,276	13,276	13,276	13,276
N of firms	3,532	3,532	3,532	3,532	3,532	3,532	3,532	3,532	3,532	3,532
R^2	0.576	0.287	0.905	0.484	0.620	0.503	0.384	0.417	0.427	0.288
Panel E: 5th size quintile	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log output change	0.529***	0.150***	1.055***	-0.307***	-0.722***	0.162***	-0.196***	-0.206***	0.099***	0.008***
	(0.016)	(0.012)	(0.012)	(0.0165)	(0.0138)	(0.0085)	(0.015)	(0.021)	(0.006)	(0.002)
Observations	11,648	11,648	11,648	11,648	11,648	11,648	11,648	11,648	11,648	11,648
N of firms	2,115	2,115	2,115	2,115	2,115	2,115	2,115	2,115	2,115	2,115
R^2	0.596	0.292	0.910	0.501	0.686	0.537	0.363	0.359	0.456	0.310

Notes: The table reports on OLS regressions from estimating the specification in Eq. (10) for 4-year differences. The dependent variables in Columns (1)-(10) are log changes in labor, capital, and intermediates quantities, labor, capital, and intermediate cost shares, and labor, capital, and intermediate input output elasticities divided by returns to scale, respectively. Panels A-E report results for the 1st, 2nd, 3rd , 4th, and 5th quintile of the firm-level output distribution (computed within year and industry). All columns report regressions of those dependent variables on log output changes for 4-year changes. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm level. Size is measured by sales. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Table A.7: Substitution elasticities, effects of firm growth on input ratios and input shadow price ratios, and firm-specific labor supply elasticities (OLS, 4-year changes, size quintiles).

	σ	$\Delta \ln \left(\frac{L_{it}}{M_{it}} \right)$	$\Delta \ln \left(\frac{P_{it}^L \gamma_{it}^L}{P_{it}^M \gamma_{it}^M} \right)$	
	(1)	(2)	(3)	(4)
Overall	2.259	-0.535	0.236	2.16
1st quintile	2.007	-0.546	0.272	1.86
2nd quintile	2.236	-0.550	0.246	1.97
3rd quintile	2.186	-0.540	0.247	1.99
4th quintile	2.327	-0.519	0.223	2.37
5th quintile	2.277	-0.526	0.231	2.29

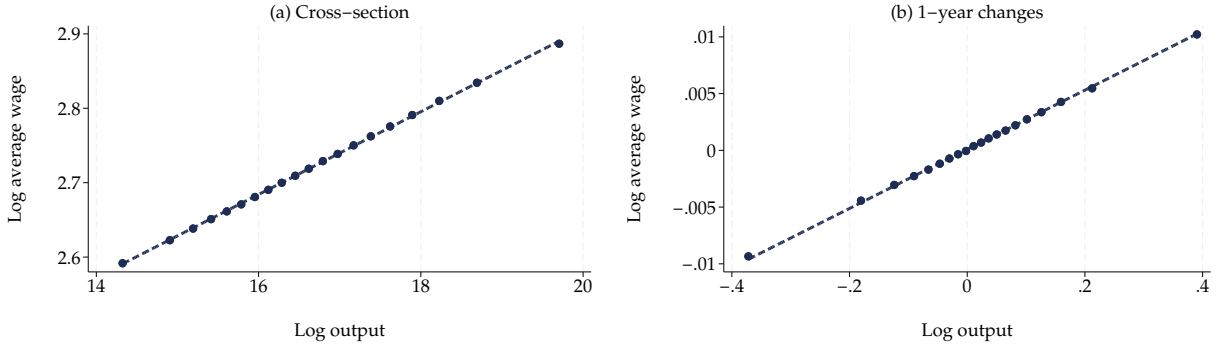
Notes: The table replicates Table 3 by size (i.e., sales) quintiles for OLS. Size quintiles are computed by year and industry.

Table A.8: Substitution elasticities, effects of firm growth on input ratios and input shadow price ratios, and firm-specific labor supply elasticities, based on cost shares.

	OLS			IV		
	σ	$\Delta \ln \left(\frac{w_{it} \gamma_{it}^L}{z_{it} \gamma_{it}^M} \right)$	ϵ_{it}^L	σ	$\Delta \ln \left(\frac{w_{it} \gamma_{it}^L}{z_{it} \gamma_{it}^M} \right)$	ϵ_{it}^L
	(1)	(2)	(3)	(4)	(5)	(6)
1-year diff.	6.434	0.113	2.64	2.28 - inf.	0.00 - 0.35	0.62 - inf.
4-year diff.	6.859	0.078	6.54	12.559	0.034	15.765
10-year diff.	5.118	0.085	7.13			

Notes: The table reports substitution elasticities (Columns (1) and (5)) following Eq. (14), changes in input factor ratios (Columns (2) and (6)), implied changes in shadow input price ratios (Columns (3) and (7)) following Eq. (16), and implied labor supply elasticities following Eq. (18), assuming perfectly elastic intermediate input supply (Columns (4) and (8)), based on our OLS (Columns (1)-(4)) and IV (Columns (5)-(8)) regressions from Table 2 that regress log cost shares and log input quantities on log output in within-firm differences. Consequently, Columns (2) and (6) report coefficient ratios for labor and intermediates from these regressions with respect to firm growth, while all other columns report values implied by our regressions as described in the text. As IV point estimates suggest a substitution elasticity of infinity for 1-year changes, we report intervals using 95% confidence intervals from all point estimates entering the computation for this specification. This yields much larger intervals than directly computing confidence intervals for substitution elasticities and other values.

Figure A.4: Wage-size gradient (OLS).



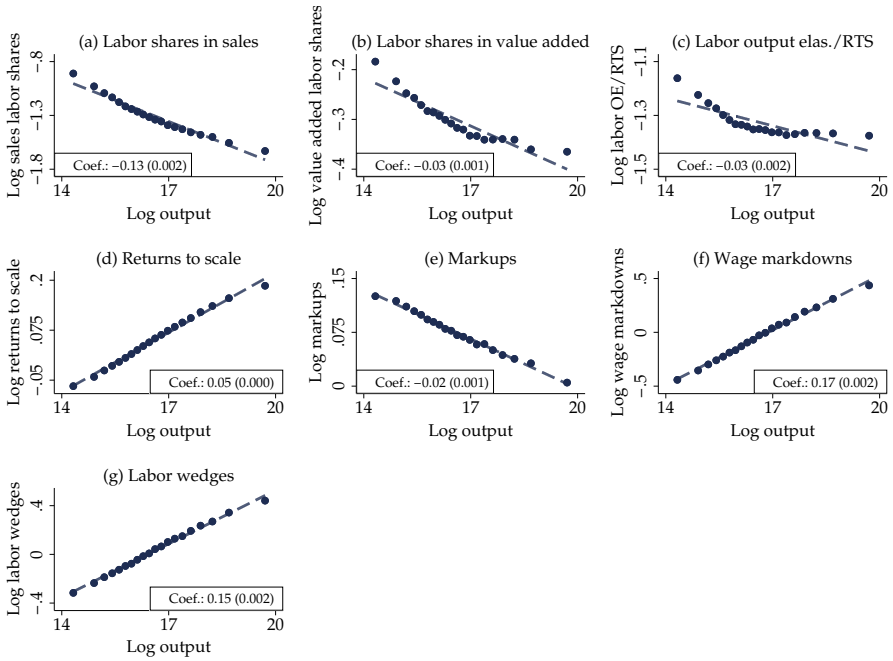
Notes: The figures report binned scatter plots from regressing log wages on log firm size (output). Panel (a) reports cross-sectional results in levels; Panel (b) reports first-difference results (1-year changes). All regressions control for industry-year fixed effects. German firm-level data.

Table A.9: Effect of firm growth on wages. German micro data.

	Dependent variable: Log real wage changes				
	OLS			IV	
	1-year	4-year	10-year	1-year	4-year
Log output change	0.095***	0.062***	0.068***	-0.024	0.031
	(0.003)	(0.003)	(0.003)	(0.056)	(0.046)
Observations	183,813	70,936	48,953	183,813	70,936
N of firms	29,950	11,492	10,381	29,950	11,492
First-stage F-statistic				102.6	48.57
R^2	0.062	0.126	0.156	0.035	0.122

Notes: The table reports OLS and IV regressions of changes in relative shadow prices (Panel A) and average wages (Panel B) on output growth for 1-, 4-, and 10-year differences. Wages are defined as the wage bill divided by the number of employees. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm level. Significance levels: *** 1%, ** 5%, * 10%. German firm-level data.

Figure A.5: The cross-sectional relationships: labor shares, output elasticities, markups, markdowns, labor wedges, and returns to scale.



Notes: The figure reports binned scatter plots estimating the specification in Eq. (10) in levels with OLS. Panels (a)-(g) report on regressions of the logs of labor shares in sales and value added, labor output elasticities divided by returns to scale, returns to scale, markups, wage markdowns, and labor wedges, respectively. All panels report results that are residualized by industry-year fixed effects. German firm-level data.

Table A.10: Testing for the role of non-homotheticities (direct output dependence), OLS results, 4-year changes.

	$\Delta \ln(\frac{\theta^L_{it}}{RTS_{it}})$	$\Delta \ln(\frac{\theta^L_{it}}{RTS_{it}})$	$\Delta \ln(\frac{\theta^L_{it}}{RTS_{it}})$	$\Delta \ln(CS^L_{it})$	$\ln(CS^L_{it})$	$\Delta \ln(CS^L_{it})$	$\Delta \ln(\frac{\theta^L_{it}}{\theta^M_{it}})$	$\Delta \ln(\frac{\theta^L_{it}}{\theta^M_{it}})$	$\Delta \ln(\frac{\theta^L_{it}}{\theta^M_{it}})$	$\Delta \ln(\frac{CS^L_{it}}{CS^M_{it}})$	$\Delta \ln(\frac{CS^L_{it}}{CS^M_{it}})$	$\Delta \ln(\frac{CS^L_{it}}{CS^M_{it}})$
Panel A: 4-year diff. OLS	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Log output change	-0.202***		0.027***	-0.286***		0.003	-0.297***		0.0512***	-0.457***		-0.020***
	(0.006)		(0.005)	(0.006)		(0.003)	(0.00785)		(0.00555)	(0.009)		(0.004)
Log labor-interm. input ratio		0.412***	0.427***		0.538***	0.540***		0.622***	0.651***		0.826***	0.815***
		(0.006)	(0.007)		(0.003)	(0.004)		(0.00656)	(0.00726)		(0.004)	(0.005)
Observations	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936
N of firms	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492
R^2	0.220	0.462	0.463	0.348	0.765	0.765	0.245	0.595	0.597	0.373	0.810	0.810
Panel B: 4-year diff. IV	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)			
Log output change	-0.242***		-0.087	-0.269***		-0.052	-0.324***		-0.0772	-0.393***		-0.053
	(0.071)		(0.075)	(0.066)		(0.048)	(0.0911)		(0.0857)	(0.097)		(0.061)
Log labor-interm. input ratio		0.412***	0.364***		0.538***	0.509***		0.622***	0.579***		0.826***	0.797***
		(0.006)	(0.042)		(0.003)	(0.027)		(0.00656)	(0.0482)		(0.004)	(0.035)
Observations	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936
N of firms	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492
First-stage F-value	48.57		39	48.57		39	48.57		39	48.57		39
R^2	0.217	0.462	0.445	0.347	0.765	0.761	0.244	0.595	0.583	0.369	0.810	0.809

Notes: The table reports OLS (Panel A) and IV (Panel B) regressions from estimating the specification in Eq. (10) for 4-year differences while additionally controlling for log changes of the labor-intermediate input ratio. The IV specification uses foreign demand shocks as instruments (Eq. (12)). The dependent variable in Columns (1)-(3) is the log change in the labor output elasticity divided by returns to scale (we have checked that output elasticities unadjusted for RTS yield very similar results). The dependent variable in Columns (4)-(6) is the log changes in the labor cost share. The dependent variable in Columns (7)-(9) is the log change in the labor output elasticity divided by the intermediate input output elasticity. The dependent variable in Columns (10)-(12) is the log change in the labor output cost share divided by the intermediate input cost share. Standard errors are clustered at the firm level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Table A.11: CompNet summary statistics, averages of industry averages (manufacturing).

Country	Lab. share (sales)	Lab. share (value added)	Lab. cost share	Lab. output elast.	Interm. cost share	Interm. output elast.	Markup	Wage Markdown
Belgium (2000-2020)	0.17	0.42	0.21	0.31	0.74	0.67	1.23	1.26
Croatia (2002-2021)	0.19	0.40	0.25	0.28	0.67	0.64	1.32	1.05
Czech Republic (2005-2020)	0.16	0.59	0.17	0.18	0.78	0.79	1.19	0.75
Denmark (2001-2020)	0.25	0.72	0.26	0.26	0.69	0.74	1.18	0.80
Finland (1999-2020)	0.21	0.69	0.22	0.21	0.75	0.77	1.15	0.84
France (2004-2020)	0.21	0.39	0.31	0.38	0.62	0.57	1.51	1.37
Germany (2001-2018)	0.22	0.74	0.21	0.36	0.69	0.77	1.13	1.53
Hungary (2003-2020)	0.17	0.61	0.17	0.19	0.77	0.77	1.12	0.86
Italy (2006-2020)	0.17	0.63	0.17	0.17	0.78	0.82	1.19	0.77
Latvia (2007-2019)	0.18	0.53	0.21	0.23	0.73	0.75	1.25	0.99
Lithuania (2000-2020)	0.19	0.66	0.20	0.20	0.74	0.76	1.15	1.14
Netherlands (2007-2019)	0.21	0.68	0.22	0.22	0.74	0.75	1.11	0.85
Poland (2002-2020)	0.16	0.54	0.17	0.17	0.78	0.78	1.18	0.79
Portugal (2010-2020)	0.19	0.62	0.20	0.25	0.74	0.74	1.12	1.09
Romania (2005-2020)	0.18	0.61	0.17	0.20	0.77	0.77	1.14	0.90
Slovakia (2000-2020)	0.16	0.50	0.18	0.19	0.76	0.76	1.14	1.09
Slovenia (2002-2021)	0.20	0.64	0.21	0.20	0.74	0.74	1.12	0.83
Spain (2008-2020)	0.17	0.64	0.19	0.21	0.79	0.77	1.17	0.96
Sweden (2003-2020)	0.22	0.38	0.32	0.41	0.62	0.56	1.31	1.52
Switzerland (2009-2020)	0.26	0.70	0.28	0.25	0.68	0.72	1.18	0.79

Notes: The table reports country-level averages of industry-level labor shares (in sales and value added), labor cost shares, labor output elasticities, intermediate input cost shares, intermediate input output elasticities, markups, and wage markdowns, i.e., for each country, we average across country-industry-level values. Country-industry-level aggregates of labor shares are computed from firms' total labor costs, total sales, and total value added. Country-industry cost shares are computed as total input expenditures divided by total costs. Country-industry-level averages of labor output elasticities are sales-weighted aggregates of firm-level values. Country-industry-level average markups are computed as intermediate input cost-weighted averages of firm-level values. Country-industry-level average wage markdowns are derived as labor cost-weighted averages of firm-level values. CompNet manufacturing data. Firms with at least 20 employees.

Table A.12: Firm-level results for other countries: coefficients on log output changes, substitution elasticities, and contribution of labor output elasticity changes to labor share changes (manufacturing separately and all sectors, 4-year changes, OLS).

	$\Delta \ln(L_{it})$ (1)	$\Delta \ln(M_{it})$ (2)	$\Delta \ln(CS^L_{it})$ (3)	$\Delta \ln(CS^M_{it})$ (4)	$\Delta \ln(\frac{\theta^L_{it}}{RTS_{it}})$ (5)	$\Delta \ln(\frac{\theta^M_{it}}{RTS_{it}})$ (6)	$\Delta \ln(\theta^L_{it})$ (7)	$\Delta \ln(LS_{it})$ (8)	$\Delta \ln(\frac{P^L_{it}L_{it}}{VA_{it}})$ (9)	σ^{OE} (10)	σ^{CS} (11)	LS contrib. (12)	Obs. (13)
Panel A: Manufacturing													
France (1995–21)	0.496***	1.027***	-0.088***	0.313***	-0.131***	0.196***	-0.122***	-0.373***	-0.350***	2.60	4.01	33%	282,245
Hungary (2003–22)	0.464***	1.057***	-0.307***	0.159***	-0.330***	0.121***	-0.343***	-0.409***	-0.340***	4.18	4.67	84%	35,902
Poland (2002–22)	0.535***	1.058***	-0.262***	0.134***	-0.281***	0.090***	-0.281***	-0.337***	-0.259***	3.44	4.12	83%	98,385
Portugal (2004–22)	0.414***	1.115***	-0.347***	0.236***	-0.393***	0.198***	-0.403***	-0.469***	-0.352***	6.37	5.94	86%	33,478
Slovakia (2000–23)	0.513***	1.019***	-0.266***	0.166***	-0.273***	0.102***	-0.264***	-0.412***	-0.381***	3.86	6.84	64%	9,605
Slovenia (2002–23)	0.547***	1.091***	-0.285***	0.166***	-0.355***	0.135***	-0.351***	-0.360***	-0.236***	10.07	5.85	98%	7,452
Estonia (2004–23)	0.547***	1.102***	-0.243***	0.151***	-0.393***	0.127***	-0.394***	-0.292***	-0.192***	15.86	3.45	135%	1,627
Latvia (2005–21)	0.495***	0.940***	-0.239***	0.027***	-0.258***	0.125***	-0.257***	-0.327***	-0.423***	7.18	2.49	79%	2,760
Netherlands (2007–22)	0.369***	1.116***	-0.303***	0.169***	-0.358***	0.143***	-0.363***	-0.353***	-0.185***	3.04	2.72	103%	30,210
Romania (2005–23)	0.526***	1.031***	-0.269***	0.126***	-0.232***	0.083***	-0.238***	-0.363***	-0.337***	2.65	4.59	66%	21,642
Switzerland (2009–22)	0.460***	1.055***	-0.303***	0.189***	-0.287***	0.124***	-0.283***	-0.434***	-0.365***	3.23	5.78	65%	11,875
Germany (1995–17)	0.510***	1.045***	-0.286***	0.171***	-0.202***	0.095***	-0.185***	-0.406***	-0.334***	2.26	6.86	46%	70,936
Panel B: All sectors													
France (1995–21)	0.511***	0.975***	-0.049***	0.268***	-0.095***	0.183***	-0.087***	-0.340***	-0.333***	2.50	3.15	26%	589,669
Hungary (2003–22)	0.497***	1.039***	-0.270***	0.125***	-0.276***	0.094***	-0.292***	-0.356***	-0.297***	3.15	3.69	82%	82,062
Poland (2002–22)	0.535***	1.064***	-0.273***	0.128***	-0.276***	0.090***	-0.278***	-0.336***	-0.231***	3.25	4.13	83%	229,205
Portugal (2004–22)	0.501***	1.044***	-0.240***	0.165***	-0.293***	0.149***	-0.301***	-0.361***	-0.305***	5.38	3.93	83%	74,869
Slovakia (2000–23)	0.493***	0.916***	-0.200***	0.130***	-0.202***	0.076***	-0.211***	-0.412***	-0.420***	2.93	4.55	51%	25,395
Slovenia (2002–23)	0.591***	1.080***	-0.259***	0.142***	-0.294***	0.116***	-0.298***	-0.321***	-0.191***	6.19	5.57	93%	17,696
Estonia (2004–23)	0.493***	1.079***	-0.261***	0.142***	-0.348***	0.151***	-0.344***	-0.323***	-0.219***	6.74	3.20	107%	4,020
Latvia (2005–21)	0.529***	0.859***	-0.172***	0.008**	-0.145***	0.074***	-0.132***	-0.320***	-0.361***	2.97	2.20	41%	13,405
Netherlands (2007–22)	0.497***	1.089***	-0.230***	0.130***	-0.257***	0.107***	-0.249***	-0.270***	-0.144***	2.60	2.55	92%	142,659
Romania (2005–23)	0.523***	1.012***	-0.266***	0.096***	-0.153***	0.044***	-0.163***	-0.350***	-0.328***	1.67	3.86	50%	58,441
Switzerland (2009–22)	0.548***	1.058***	-0.232***	0.172***	-0.215***	0.119***	-0.220***	-0.347***	-0.277***	2.90	4.81	63%	28,572

Notes: OLS estimates of Eq. (10) using 4-year differences. Columns (1)–(9) report coefficients on log output growth. Columns (10) and (11) report substitution elasticities based on output elasticities and cost shares, respectively. Column (12) reports the contribution of changes in labor output elasticities (Column 7) to labor share changes (Column 8). Column (13) reports the number of firm-year observations. Panel A reports manufacturing firms only; Panel B reports all sectors. Firm-level data from CompNet data providers and German firm-level data. All regressions include two-digit industry-year fixed effects. Standard errors are clustered at the firm level. We omit standard errors due to space constraints and only report significance levels by stars: Significance: 0.01***, 0.05**, 0.1*. The table with values for all standard errors is available on request.

Table A.13: Firm-level results for other countries: Coefficients on output growth, substitution elasticities, and contribution of labor output elasticity changes to labor share changes (4-year changes, OLS). Young and mature manufacturing firms.

	$\Delta \ln(L_{it})$	$\Delta \ln(M_{it})$	$\Delta \ln(CS^L_{it})$	$\Delta \ln(CS^M_{it})$	$\Delta \ln(\frac{\theta^L_{it}}{RTS_{it}})$	$\Delta \ln(\frac{\theta^M_{it}}{RTS_{it}})$	$\Delta \ln(\theta^L_{it})$	$\Delta \ln(LS_{it})$	$\Delta \ln(\frac{P^L_{it}L_{it}}{VA_{it}})$	σ^{OE}	σ^{CS}	LS contrib. of $\Delta\theta^L_{it}$
Panel A: young firms	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
France (1995-21)	0.477***	1.018***	-0.109***	0.227***	-0.131***	0.195***	-0.122***	-0.318***	-0.305***	2.52	2.64	38%
Obs.: 21,988	(0.008)	(0.010)	(0.007)	(0.008)	(0.004)	(0.006)	(0.005)	(0.007)	(0.008)			
Hungary (2003-22)	0.488***	1.052***	-0.249***	0.134***	-0.287***	0.116***	-0.299***	-0.330***	-0.274***	3.50	3.12	91%
Obs.: 3,966	(0.016)	(0.013)	(0.015)	(0.011)	(0.015)	(0.006)	(0.015)	(0.015)	(0.019)			
Slovakia (2000-23)	0.570***	1.014***	-0.255***	0.134***	-0.248***	0.0830***	-0.242***	-0.375***	-0.370***	3.93	8.07	65%
Obs.: 1,157	(0.031)	(0.031)	(0.031)	(0.021)	(0.028)	(0.013)	(0.029)	(0.031)	(0.051)			
Slovenia (2002-23)	0.590***	1.097***	-0.220***	0.140***	-0.279***	0.140***	-0.273***	-0.266***	-0.163***	5.76	3.45	103%
Obs.: 719	(0.036)	(0.026)	(0.032)	(0.023)	(0.038)	(0.017)	(0.038)	(0.033)	(0.029)			
Latvia (2005-21)	0.563***	0.973***	-0.224**	0.022	-0.205***	0.125***	-0.208***	-0.272***	-0.409***	5.13	2.49	76%
Obs.: 236	(0.053)	(0.053)	(0.090)	(0.0140)	(0.054)	(0.031)	(0.058)	(0.062)	(0.126)			
Netherlands (2007-22)	0.439***	1.107***	-0.251***	0.152***	-0.308***	0.126***	-0.313***	-0.293***	-0.141***	2.85	2.52	107%
Obs.: 1,774	(0.030)	(0.028)	(0.032)	(0.024)	(0.024)	(0.012)	(0.024)	(0.033)	(0.035)			
Panel B: mature firms	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
France (1995-21)	0.498***	1.029***	-0.0824***	0.330***	-0.131***	0.199***	-0.122***	-0.382***	-0.357***	2.64	4.48	32%
Obs.: 253,635	(0.004)	(0.004)	(0.003)	(0.004)	(0.002)	(0.002)	(0.002)	(0.003)	(0.004)			
Hungary (2003-22)	0.454***	1.060***	-0.326***	0.168***	-0.343***	0.124***	-0.356***	-0.434***	-0.354***	4.36	5.41	82%
Obs.: 31,916	(0.008)	(0.007)	(0.009)	(0.006)	(0.007)	(0.003)	(0.008)	(0.009)	(0.011)			
Slovakia (2000-23)	0.491***	1.018***	-0.272***	0.176***	-0.283***	0.108***	-0.274***	-0.428***	-0.391***	3.88	6.67	64%
Obs.: 8,399	(0.018)	(0.022)	(0.015)	(0.012)	(0.015)	(0.008)	(0.016)	(0.018)	(0.028)			
Slovenia (2002-23)	0.533***	1.097***	-0.306***	0.179***	-0.376***	0.137***	-0.371***	-0.387***	-0.244***	11.06	7.14	96%
Obs.: 6,656	(0.017)	(0.014)	(0.016)	(0.012)	(0.019)	(0.008)	(0.019)	(0.016)	(0.016)			
Latvia (2005-21)	0.479***	0.938***	-0.245***	0.0305***	-0.269***	0.127***	-0.267***	-0.338***	-0.424***	7.29	2.50	80%
Obs.: 2,482	(0.026)	(0.024)	(0.029)	(0.006)	(0.024)	(0.010)	(0.025)	(0.024)	(0.042)			
Netherlands (2007-22)	0.363***	1.117***	-0.308***	0.171***	-0.364***	0.145***	-0.369***	-0.359***	-0.188***	3.14	2.74	103%
Obs.: 28,411	(0.010)	(0.008)	(0.012)	(0.007)	(0.008)	(0.004)	(0.009)	(0.012)	(0.008)			

Notes: The table reports OLS regressions from estimating the specification in Eq. (10) for different countries (Columns (1)-(9)) using 4-year differences. The table reports coefficients on log output changes in Columns (1)-(9). The dependent variables in Columns (1)-(9) are log changes in labor and intermediate quantities, cost shares, and output elasticities divided by returns to scale, labor output elasticities, labor shares in sales, and labor shares in value added, respectively. All columns report regressions of those dependent variables on changes in log output for 4-year differences. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data. Columns (10) and (11) report estimates of the substitution elasticity between labor and intermediates based on Eq. (14) using output elasticity coefficients (Column (10)) and cost share coefficients (Column (11)). Column (12) reports the contribution of changes in labor output elasticities (Column (7)) to changes in labor shares (Column (8)). Panel A reports results for firms not older than five years. Panel B reports results for firms older than five years. Firm-level micro data from CompNet data providers for a subset of CompNet countries with information on registration years.

Table A.14: Implied industry-level substitution elasticities and changes in inputs, output elasticities, and cost share ratios based on 4-year differences and by countries.

	$\Delta \ln \left(\frac{L_{cjt}}{M_{cjt}}\right)$	$\Delta \ln \left(\frac{\theta^L_{cjt}}{\theta^M_{cjt}}\right)$	$\Delta \ln \left(\frac{CS^L_{cjt}}{CS^M_{cjt}}\right)$	σ^{OE}	σ^{CS}
	(1)	(2)	(3)	(4)	(5)
Belgium (2000-2020)	-0.23	-0.10	-0.16	1.80	3.36
Croatia (2002-2021)	-0.70	-0.16	-0.64	1.31	12.71
Czech Republic (2005-2020)	-0.40	-0.01	-0.31	1.03	4.67
Denmark (2001-2020)	-0.13	-0.01	-0.09	1.12	3.13
Finland (1999-2020)	-0.09	-0.08	-0.13	5.72	-2.45
France (2004-2020)	-0.40	-0.05	-0.22	1.13	2.25
Germany (2001-2018)	-0.06	0.01	-0.05	0.85	6.59
Hungary (2003-2020)	-0.51	-0.09	-0.38	1.21	4.03
Italy (2006-2020)	-0.18	-0.09	-0.12	2.11	2.89
Latvia (2007-2019)	-0.43	-0.11	-0.25	1.33	2.46
Lithuania (2000-2020)	-0.15	-0.08	-0.11	2.04	3.59
Netherlands (2007-2019)	-0.11	0.03	-0.06	0.81	2.51
Poland (2002-2020)	-0.13	-0.01	-0.09	1.09	2.98
Portugal (2010-2020)	-0.68	0.02	-0.48	0.98	3.42
Romania (2005-2020)	-0.22	-0.03	-0.13	1.17	2.58
Slovakia (2000-2020)	-0.18	-0.09	-0.16	1.98	12.73
Slovenia (2002-2021)	-0.09	0.01	-0.13	0.92	-2.49
Spain (2008-2020)	-0.44	-0.23	-0.16	2.11	1.60
Sweden (2003-2020)	-0.03	-0.04	0.02	-5.77	0.64
Switzerland (2009-2020)	-0.31	-0.19	-0.24	2.63	4.58
Europe (1999-2021, manufac.)	-0.27	-0.07	-0.20	1.34	4.01
USA (1958-2016, manufac.)	-0.31		-0.23		3.88
Europe (1999-2021, non-manufac.)	-0.23	-0.08	-0.15	1.48	2.72

Notes: The table reports implied changes in labor-intermediate input ratios (Column (1)), output elasticity ratios (Column (2)), cost shares (Column (3)), and substitution elasticities based on output elasticity coefficients (Column (4)) and cost share coefficients (Column (5)) as estimated from country-specific versions of the country-industry-level regressions reported in Figure 7 (4-year changes). All results are based on manufacturing industries, except for the last row, which uses non-manufacturing industries. Industries are 2-digit NACE rev. 2 industries for Europe and 6-digit NAICS industries for the US. For US cost shares, capital costs are approximated as 8% of the real capital stock, as nominal values are not reported; labor and material expenditures are reported in nominal terms. CompNet data and NBER-CES Manufacturing Industry Database.

B Details on Meta-Analysis (Figure 4 Panel (a))

We distinguish between studies estimating substitution between intermediates and a capital–labor bundle and those estimating substitution between intermediates and labor. The former impose identical substitution between intermediates and capital and labor and typically do not identify the elasticity from variation in labor-intermediate input (or price) ratios. This restriction could explain why these studies report lower elasticities compared to studies estimating intermediate–labor elasticities directly (as we do). We exclude studies focusing on energy–labor substitution (e.g., Bretschger and Jo, 2024), as energy constitutes only a small share of intermediates. The definition of intermediates also matters: substitution with raw materials or energy is likely weaker than substitution with labor-intensive components (particularly relevant for our case) or intermediate services.

Table B.1: Studies used in the substitution–elasticity meta-analysis.

Substitution elasticities between intermediates and a capital–labor bundle	
Peter and Ruane (2022)	Plant-level data, India, manufacturing. Identification from input-price variation. Mean of OLS (0.479) and IV (0.618) estimates from Table 6: **0.5485**.
Boehm et al. (2019)	Firm-level data, US, manufacturing and non-manufacturing. Identification from short-run variation in output and inputs under constant input prices. Value for all firms in Table 2: **0.037**.
Bruno (1984)	Country-level data (US, UK, Germany, Japan), manufacturing and non-manufacturing. Identification from input-price variation. Mean of non-negative values in Table 3 (0.196, 0.57, 0.337, 0.46, 0.35, 0.132, 0.472, 0.649, 0.812, 0.766, 0.91): **0.514**.
Oberfield and Raval (2021)	Firm-level data, US, manufacturing. Identification from input-price variation. Mean of Table 3 values (1.03, 0.83, 0.69, 0.78, 0.57): **0.78**.
Antoszewski (2019)	Industry-level WIOD data, OECD countries, manufacturing and non-manufacturing. Identification from input-price variation. Mean of all Table 8 values: **0.8211**.
Atalay (2017)	Industry-level data, US, manufacturing and non-manufacturing. Identification from input-price variation. Mean of Table 8 values (1.18, 1.27, 0.84, 0.88): **1.043**.
Rotemberg and Woodford (1996)	Industry-level data, US, manufacturing. Identification from input-price variation. Table 2 value: **0.69**.
Miranda-Pinto (2021)	Industry-level data, multiple countries, manufacturing and non-manufacturing. Identification from input-price variation. Table 4 value for all firms: **1.34**.
Miranda-Pinto and Youngs (2022)	Industry-level data, US, manufacturing and non-manufacturing. Identification from input-price variation. Mean of FE (0.85) and IV (2.14) specifications in Table 2: **1.495**.
Substitution elasticities between intermediates and labor	
Delgado et al. (1999)	Firm-level data, Spain, manufacturing. Identification from input-price variation. Mean of all $1-\beta$ estimates in Table 2: **0.414**.
Doraszelski and Jaumandreu (2018)	Firm-level data, Spain, manufacturing. Identification from input-price variation. Mean of all values in Tables 3–5: **0.691**.
Huneeus et al. (2022)	Firm-level data, Chile, manufacturing and non-manufacturing. Identification from input-price variation. Mean of Table 5 values (1.55, 1.62, 1.05): **1.401**. Authors emphasize that improved input-price measures yield the higher estimates (1.55, 1.62).
Chan (2023)	Firm-task-level data, Denmark, manufacturing and non-manufacturing. Identification from input-price variation. Mean of Table 6 values: **2.368**.
Martinello (1989)	Industry-level data, Canada and US, wood-products industry. Identification from input-price variation and a structural labor-market model. Mean of Table 4 values (2.662, 2.257): **2.46**.

Notes: The table summarizes all studies used in our meta-analysis of substitution elasticities, distinguishing whether elasticities are estimated between intermediates and a capital–labor bundle or directly between intermediates and labor.

C Theoretical Appendix

Throughout this section, the notation follows the main text.

C.1 Derivations for Section 2.1

We define the production function in general terms, $Q_{it}(K_{it}, L_{it}, M_{it}, \Omega_{it}) = Q_{it}(.)$ and write firms' cost minimization as a Lagrangian function:

$$\begin{aligned}\mathcal{L}_{it} = P^L_{it}(L_{it})L_{it} + \chi^L(L_{it})P^L_{it} + P^M_{it}(M_{it})M_{it} + \chi^M(M_{it})P^M_{it} \\ + P^K_{it}(K_{it})K_{it} + \chi^K(K_{it})P^K_{it} - \lambda_{it}(Q_{it} - Q_{it}(.)),\end{aligned} \tag{C.1}$$

where, for simplicity, we model input adjustment costs through convex functions $\chi^X(X_{it})$ with $X = \{L, M, K\}$ and write the optimization in a quasi-static way as in Bond et al. (2021) (this avoids writing out the dynamic problem). One interpretation of this setting is that each input is associated with a baseline quantity, $\bar{X}_{it}$, and that firms incur adjustment costs when choosing an input quantity that differs from $\bar{X}_{it}$. The first-order conditions for each production input lead to Eq. (2) from the main text:

$$\frac{\partial \mathcal{L}_{it}}{\partial X_{it}} = 0 \quad \Rightarrow \quad P_{it}^X \underbrace{\left(1 + \frac{\partial P_{it}^X}{\partial X_{it}} \frac{X_{it}}{P_{it}^X} + \frac{\partial \chi^X}{\partial L_{it}}\right)}_{\gamma_{it}^X} = \lambda_{it} \frac{\partial Q_{it}}{\partial X_{it}}. \tag{C.2}$$

Multiplying Eq. (C.2) by $\frac{X_{it}}{Q_{it}}$ using the definition of the output elasticity, $\theta^X = \frac{\partial Q}{\partial X}\frac{X}{Q}$, and noting that $\lambda_{it} = \frac{P_{it}}{\mu_{it}}$ (price over markup) leads to:

$$X_{it} P_{it}^X = P_{it} Q_{it} \frac{\theta_{it}^X}{\mu_{it} \gamma_{it}^X}. \tag{C.3}$$

Rearranging yields Eq. (6) of the main text:

$$OS_{it}^X = \frac{P_{it}^X X_{it}}{P_{it} Q_{it}} = \frac{\theta_{it}^X}{\mu_{it} \gamma_{it}^X}. \tag{C.4}$$

Similarly, we can define Eq. (C.3) for each input and recover Eq. (5) of the main text:

$$CS_{it}^X = \frac{P_{it}^X X_{it}}{P_{it}^X L_{it} + P_{it}^M M_{it} + P_{it}^K K_{it}} = \frac{\frac{\theta_{it}^X}{\gamma_{it}^X}}{\frac{\theta_{it}^L}{\gamma_{it}^L} + \frac{\theta_{it}^M}{\gamma_{it}^M} + \frac{\theta_{it}^K}{\gamma_{it}^K}}. \tag{C.5}$$

For the next derivations, we rely on the production function (Eq. (1)). The partial derivatives with respect to labor and intermediate yield the first part of Eq. (3):

$$\frac{\frac{\partial Q_{it}}{\partial L_{it}}}{\frac{\partial Q_{it}}{\partial M_{it}}} = \frac{\Omega_{it}^{\frac{\sigma-1}{\sigma\kappa}} K_{it}^{(1-\kappa)\frac{\sigma-1}{\sigma\kappa}} (\Lambda_i^K)^{\frac{\sigma-1}{\sigma\kappa}} \Lambda_i^{LM} \alpha_i^L \kappa L_{it}^{\frac{-1}{\sigma}} Q_{it}^{\frac{\sigma\kappa-\sigma+1}{\sigma\kappa}}}{\Omega_{it}^{\frac{\sigma-1}{\sigma\kappa}} K_{it}^{(1-\kappa)\frac{\sigma-1}{\sigma\kappa}} (\Lambda_i^K)^{\frac{\sigma-1}{\sigma\kappa}} \Lambda_i^{LM} \alpha_i^M \kappa M_{it}^{\frac{-1}{\sigma}} Q_{it}^{\frac{\sigma\kappa-\sigma+1}{\sigma\kappa}}} = \frac{\alpha_i^L}{\alpha_i^M} \left(\frac{L_{it}}{M_{it}}\right)^{\frac{-1}{\sigma_{it}}}. \tag{C.6}$$

Multiplying this expression by $\frac{L_{it}}{Q_{it}} / \frac{M_{it}}{Q_{it}}$ and taking logs and differences (which eliminates the firm fixed effect, $\frac{\alpha_i^L}{\alpha_i^M}$) yields Eq. (3):

$$\frac{\sigma - 1}{\sigma} = \frac{\Delta \ln(\theta_{it}^L) - \Delta \ln(\theta_{it}^M)}{\Delta \ln(L_{it}) - \Delta \ln(M_{it})}. \tag{C.7}$$

C.2 Derivations for Section 2.2

To recover Eq. (7) from the main text, we impose more structure by defining inverse input supply functions as $P_{it}^X = a_{it}^X X_{it}^{\varepsilon^X}$ for $X = \{L, M\}$ and abstract from adjustment costs for labor and intermediates. For simplicity, we also assume that markups and capital shadow costs do not depend on firm scale. Inserting the input supply functions into Eq. (C.1) and dropping adjustment cost terms (except for capital) yields:

$$\mathcal{L}_{it} = a_{it}^L L_{it}^{1+\varepsilon^L} + a_{it}^M M_{it}^{1+\varepsilon^M} + P_{it}^K(K_{it})K_{it} + \chi^K(K_{it})P_{it}^K - \lambda_{it}(Q_{it} - Q_{it}(.)). \tag{C.8}$$

The first-order conditions for $X = \{L, M\}$ yield Eq. (7) from the main text:

$$\frac{\partial \mathcal{L}_{it}}{\partial X_{it}} = 0 \quad \Rightarrow \quad \left(1+\varepsilon^X\right) a_{it}^X X_{it}^{\varepsilon^X} = \lambda_{it}\frac{\partial Q_{it}}{\partial X_{it}} \quad \Rightarrow \quad X_{it} = \left(\frac{\lambda_{it}}{\left(1+\varepsilon^X\right) a_{it}^X}\frac{\partial Q_{it}}{\partial X_{it}}\right)^{\frac{1}{\varepsilon^X}} \tag{C.9}$$

Inserting the production function yields:

$$X_{it} = \left(\frac{\lambda_{it}(\Lambda_i^K)^{\frac{\sigma-1}{\sigma\kappa}}\Lambda_i^{LM}\alpha_i^X\kappa\Omega_{it}^{\frac{\sigma-1}{\sigma\kappa}}}{\left(1+\varepsilon^X\right) a_{it}^X}K_{it}^{(1-\kappa)\frac{\sigma-1}{\sigma\kappa}}Q_{it}^{\frac{\sigma\kappa-\sigma+1}{\sigma\kappa}}\right)^{\frac{\sigma}{(\sigma\varepsilon^X+1)}}. \tag{C.10}$$

Combining this expression for labor and intermediates yields:

$$\frac{L_{it}}{M_{it}} = \frac{\left(\frac{\lambda_{it}(\Lambda_i^K)^{\frac{\sigma-1}{\sigma\kappa}}\Lambda_i^{LM}\alpha_i^L\kappa\Omega_{it}^{\frac{\sigma-1}{\sigma\kappa}}}{(1+\varepsilon^L)a_{it}^L}\right)^{\frac{\sigma}{(\sigma\varepsilon^L+1)}}}{\left(\frac{\lambda_{it}(\Lambda_i^K)^{\frac{\sigma-1}{\sigma\kappa}}\Lambda_i^{LM}\alpha_i^M\kappa\Omega_{it}^{\frac{\sigma-1}{\sigma\kappa}}}{(1+\varepsilon^M)a_{it}^M}\right)^{\frac{\sigma}{(\sigma\varepsilon^M+1)}}}\frac{\left(K_{it}^{(1-\kappa)\frac{\sigma-1}{\sigma\kappa}}Q_{it}^{\frac{\sigma\kappa-\sigma+1}{\sigma\kappa}}\right)^{\frac{\sigma}{(\sigma\varepsilon^L+1)}}}{\left(K_{it}^{(1-\kappa)\frac{\sigma-1}{\sigma\kappa}}Q_{it}^{\frac{\sigma\kappa-\sigma+1}{\sigma\kappa}}\right)^{\frac{\sigma}{(\sigma\varepsilon^M+1)}}}. \tag{C.11}$$

Combining the FOC for capital and using the production function yields capital demand:

$$K_{it} = \left(\frac{\lambda_{it}(1-\kappa)}{P_{it}^K\gamma_{it}^K}\right)Q_{it}. \tag{C.12}$$

Inserting this capital demand equation into Eq. (C.11) yields Eq. (8) from the main text:

$$\frac{L_{it}}{M_{it}} = \varrho_{it}\lambda_{it}^{\left(\frac{\sigma+\kappa-1}{\kappa(\sigma\varepsilon^L+1)}-\frac{\sigma+\kappa-1}{\kappa(\sigma\varepsilon^M+1)}\right)}Q_{it}^{\left(\frac{1}{\sigma\varepsilon^L+1}-\frac{1}{\sigma\varepsilon^M+1}\right)}, \tag{C.13}$$

where $\varrho_{it} = \frac{\left(\frac{(\Lambda_i^K)^{\frac{\sigma-1}{\sigma\kappa}}\Lambda_i^{LM}\alpha_i^L\kappa\Omega_{it}^{\frac{\sigma-1}{\sigma\kappa}}}{(1+\varepsilon^L)a_{it}^L}\left(\frac{(1-\kappa)}{P_{it}^K\gamma_{it}^K}\right)^{\frac{(1-\kappa)(\sigma-1)}{\sigma\kappa}}\right)^{\frac{\sigma}{(\sigma\varepsilon^L+1)}}}{\left(\frac{(\Lambda_i^K)^{\frac{\sigma-1}{\sigma\kappa}}\Lambda_i^{LM}\alpha_i^M\kappa\Omega_{it}^{\frac{\sigma-1}{\sigma\kappa}}}{(1+\varepsilon^M)a_{it}^M}\left(\frac{(1-\kappa)}{P_{it}^K\gamma_{it}^K}\right)^{\frac{(1-\kappa)(\sigma-1)}{\sigma\kappa}}\right)^{\frac{\sigma}{(\sigma\varepsilon^M+1)}}}$ is a function of parameters and markdowns only.

C.3 Deriving the Substitution Elasticity of the Main Text without Imposing Functional Form Assumptions on the Production Function

We focus on how firm growth induces a shift from labor to intermediates. In our setting, growing firms move along labor and intermediate input supply curves, which affects relative input (shadow) prices. The shift from labor to intermediates is governed by the elasticity of relative input quantities with respect to relative shadow price changes (the substitution elasticity). Formally:

$$\sigma_{it}^{REL} = \frac{\Delta \ln(\frac{M_{it}}{L_{it}})}{\Delta \ln(\frac{P_{it}^L\gamma_{it}^L}{P_{it}^M\gamma_{it}^M})}. \tag{C.14}$$

Assuming that firms' minimize cost, it holds that $P_{it}^X = \frac{P_{it}Q_{it}}{X_{it}}\frac{\theta_{it}^X}{\mu_{it}\gamma_{it}^X}$, where γ_{it}^X, μ_{it}, and $P_{it}Q_{it}$ are the input wedge, markup, and sales. Using this, we can rewrite Eq. (C.14) as:

$$\sigma_{it}^{REL} = \frac{\Delta \ln(\frac{M_{it}}{L_{it}})}{\Delta \ln\left(\frac{\theta_{it}^L}{L_{it}}/\frac{\theta_{it}^M}{M_{it}}\right)} \quad \Leftrightarrow \quad \frac{\sigma_{it}^{REL}-1}{\sigma_{it}^{REL}}\frac{\Delta \ln(\theta_{it}^L) - \Delta \ln(\theta_{it}^M)}{\Delta \ln(L_{it}) - \Delta \ln(M_{it})}. \tag{C.15}$$

Eq. (C.15) is identical to the substitution elasticity that we use in the main part of the paper (Eq. (3)) with the exception that a CES production function will yield a constant substitution elasticity that is identical across firms. Eq. (C.15) measures the elasticity of substitution as the responsiveness of input ratios to input shadow price ratios. It provides an intuitive measure of substitution in our context, as firms move along the input supply curves for labor and intermediates (possibly shifting two input prices at the same time), and shadow prices reflect the relevant price ratios for firms' optimization decisions.

C.4 OLS Bias under Labor Augmenting Productivity

Here, we further address the critique that our estimates could be explained through labor-augmenting productivity and input complementarity by theoretically showing that our differences in OLS and IV can provide a coherent picture under input substitution.

We emphasize that our IV estimator already empirically addresses biases from factor-augmenting productivity. The main insight from this section is that unobserved (relatively) labor-augmenting productivity changes tends to push OLS substitution elasticities toward unity, which aligns with our results that substitution elasticities based on IV exceed OLS values (estimated substitution elasticities are above unity in both cases).

Write the omitted variable bias from (relatively) labor augmenting productivity in regression Eq. 10 in terms of ratios:

$$\Delta \ln(\frac{L_{it}}{M_{it}}) = \beta_{L/M}^{OLS} \Delta \ln Q_{it} + \underbrace{\beta_{\Omega} \Delta \ln(\frac{\Omega_{it}^{L}}{\Omega_{it}^{M}})}_{\text{Unobserved factor-augmenting productivity ratio}} + \upsilon_{jt} + \epsilon_{it}. \tag{C.16}$$

Recap: we find $\beta_{L/M}^{OLS} < 0$ (a shift from labor to intermediates). Eq. (3) further becomes $\Delta \ln\left(\frac{\theta_{it}^{L}}{\theta_{it}^{M}}\right) = \frac{\sigma-1}{\sigma}\Delta \ln\left(\frac{L_{it}}{M_{it}}\right) + \frac{\sigma-1}{\sigma}\Delta \ln\left(\frac{\Omega_{it}^{L}}{\Omega_{it}^{M}}\right)$. Insert Eq. (C.16) into this expression:

$$\Delta \ln(\frac{\theta_{it}^{L}}{\theta_{it}^{M}}) = \underbrace{\frac{\sigma-1}{\sigma}\beta_{L/M}^{OLS}}_{\beta_{\theta^L/\theta^M}^{OLS}} \Delta \ln Q_{it} + \underbrace{\frac{\sigma-1}{\sigma}(1+\beta_{\Omega})\Delta \ln(\frac{\Omega_{it}^{L}}{\Omega_{it}^{M}})}_{\text{Unobserved factor-augmenting productivity ratio}} + \upsilon_{jt} + \epsilon_{it}, \tag{C.17}$$

where $\beta_{\theta^L/\theta^M}^{OLS}$ is our estimate of the shift from labor to intermediate output elasticities. If output growth is caused by relatively labor-augmenting productivity shocks (the main identification concern), then we can define: $\psi = \frac{COV\left(\Delta \ln Q_{it}, \Delta \ln\left(\frac{\Omega_{it}^{L}}{\Omega_{it}^{M}}\right)\right)}{Var(\Delta \ln Q_{it})} > 0$, and the omitted variable bias is:

$$\frac{\sigma^{OLS}-1}{\sigma^{OLS}} = \frac{\beta_{\theta^L/\theta^M}^{OLS}}{\beta_{L/M}^{OLS}} = \frac{\sigma-1}{\sigma}\left(1 + \frac{\psi}{\underbrace{\beta_{L/M}^{OLS}}_{<0}}\right). \tag{C.18}$$

Hence, σ^{OLS} is biased toward one if $-1 \leq \frac{\psi}{\beta_{L/M}^{OLS}} \leq 0$. In the extreme case that $\frac{\psi}{\beta_{L/M}^{OLS}} < -1$, the fact that IV exceeds OLS is still consistent with inputs being substitutes.

C.5 Non-homothetic CES Production Function: Derivations

The ratio of marginal products from the non-homothetic production function in Eq. (19) from the main text is written:

$$\frac{\frac{\partial Q_{it}}{\partial L_{it}}}{\frac{\partial Q_{it}}{\partial M_{it}}} = \frac{\Omega_{it}^{\frac{\sigma-1}{\sigma\kappa}} K_{it}^{(1-\kappa)\frac{\sigma-1}{\sigma\kappa}} (\Lambda_i^K)^{\frac{\sigma-1}{\sigma\kappa}} \Lambda_i^{LM} \alpha_i^L \kappa L_{it}^{\frac{-1}{\sigma}} Q_{it}^{\frac{\sigma\kappa-\sigma+1}{\sigma\kappa}}}{\Omega_{it}^{\frac{\sigma-1}{\sigma\kappa}} K_{it}^{(1-\kappa)\frac{\sigma-1}{\sigma\kappa}} (\Lambda_i^K)^{\frac{\sigma-1}{\sigma\kappa}} \Lambda_i^{LM} \alpha_i^M \kappa \left(\frac{M_{it}}{Q_{it}^{\eta}}\right)^{\frac{-1}{\sigma}} Q_{it}^{\frac{\sigma\kappa-\sigma+1}{\sigma\kappa}}} = \frac{\alpha_i^L}{\alpha_i^M} \left(\frac{L_{it}}{M_{it}}\right)^{\frac{-1}{\sigma_{it}}} Q_{it}^{\frac{-\eta}{\sigma}}. \quad \text{(C.19)}$$

Multiplying this expression by $\frac{L_{it}}{Q_{it}} / \frac{M_{it}}{Q_{it}}$ yields Eq. (20) of the main text:

$$\frac{\theta_{it}^L}{\theta_{it}^M} = \frac{\alpha_i^L}{\alpha_i^M} \left(\frac{L_{it}}{M_{it}}\right)^{\frac{\sigma-1}{\sigma}} Q_{it}^{\frac{-\eta}{\sigma}}. \quad \text{(C.20)}$$

C.6 Markups and Wage Markdowns

We now show how we derive the equations for firms' markups and wage markdowns in terms of observables and output elasticities following De Loecker and Warzynski (2012) and Dobbelaere and Mairesse (2013). As discussed in the main text, we assume that intermediates are supplied perfectly elastically and that there are no adjustment costs (as standard in the literature for this derivation). *Strictly speaking, for our firm-level analysis in changes, a weaker assumption is sufficient: we can permit intermediate input market imperfections* γ_{it}^M *but those must be constant over time within a firm, i.e.,* $\gamma_{it}^M = \gamma_i^M$ (indeed, our results provide suggestive evidence for that property). Assuming cost-minimization, the first-order conditions for labor and intermediates are:

$$P_{it}^M = \frac{P_{it}}{\mu_{it}} \frac{\partial Q_{it}}{\partial M_{it}}, \qquad P_{it}^L \gamma_{it}^L = \frac{P_{it}}{\mu_{it}} \frac{\partial Q_{it}}{\partial M_{it}} \quad \text{(C.21)}$$

$$\text{(C.22)}$$

Rearranging the FOC for intermediates yields an expression for the markup:

$$\mu_{it} = \frac{P_{it}}{MC_{it}} = \theta_{it}^M \frac{P_{it} Q_{it}}{P_{it}^M M_{it}}. \quad \text{(C.23)}$$

Combining the FOCs for labor and intermediates recovers the wage markdown:

$$\gamma_{it}^L = \frac{\theta_{it}^L}{\theta_{it}^M} \frac{P_{it}^M M_{it}}{P_{it}^L L_{it}}. \quad \text{(C.24)}$$

Rearranging the FOC for labor yields a measure of combined distortions from markup and labor market imperfections:

$$\mu_{it}\gamma_{it}^{L} = \theta_{it}^{L}\frac{P_{it}Q_{it}}{P_{it}^{L}L_{it}}. \tag{C.25}$$

C.7 Fixed-Cost Model

We now provide a simple assessment of how fixed costs could account for our findings. We do so in a simple model in which a portion of the labor is fixed. To assess the capacity of fixed costs to explain our findings, we compute the latent fixed labor cost share that would be required to explain our results without labor-intermediate substitution and imperfect input markets. To eliminate the role of intermediate-labor input substitutability, we impose a Cobb-Douglas production function.

Firms produce output using labor, intermediates, and capital. Now, total labor L_{it} consists of fixed overhead labor L_{it}^{F} (where the time index in principle permits non-constant amounts, but the key point is that it is not a choice variable) and variable production labor is $L_{it} - L_{it}^{F}$. The production function is:

$$Q_{it} = (L_{it} - L_{it}^{F})^{\Phi^{L}} M_{it}^{\Phi^{M}} K_{it}^{\Phi^{K}}, \tag{C.26}$$

Differentiating Q_{it} with respect to L_{it} yields the output elasticity of *total* labor:

$$\underbrace{\frac{dQ_{it}}{Q_{it}}\frac{L_{it}}{dL_{it}}}_{=\theta_{it}^{L}} = \Phi^{L}\frac{L_{it}}{L_{it} - L_{it}^{F}}. \tag{C.27}$$

Hence, the output elasticity of total labor—the object we study in the main text—can vary due to the presence of fixed labor costs, even though the structural output elasticity of variable production labor is constant at Φ^{L}.

In the fixed-cost model, the fixed portion of labor governs the relationship between firm growth and the output elasticity of total labor:

$$\frac{d\theta_{it}^{L}}{\theta_{it}^{L}} = -\underbrace{\frac{L_{it}^{F}}{L_{it} - L_{it}^{F}}}_{\substack{\text{Fixed labor to} \\ \text{variable labor ratio}}}\frac{dL_{it}}{L_{it}} \Leftrightarrow \underbrace{\frac{L_{it}^{F}}{L_{it}}}_{\substack{\text{Share of fixed} \\ \text{labor in total labor}}} = \frac{-\frac{d\theta_{it}^{L}}{\theta_{it}^{L}}\frac{dL_{it}}{L_{it}}}{1 - \frac{d\theta_{it}^{L}}{\theta_{it}^{L}} \Big/ \frac{dL_{it}}{L_{it}}}. \tag{C.28}$$

We now insert the coefficients from Table 2 for $\frac{d\theta_{it}^{L}}{\theta_{it}^{L}}$ and $\frac{dL_{it}}{L_{it}}$, as they capture the average

change in labor output elasticities (θ_{it}^L) and labor quantities in response to output growth, into Eqs. (C.28). This allows us to infer the fixed labor share required to fully explain our findings without relying on factor substitution and imperfect input markets. For 1-year changes, the OLS (IV) estimates imply a portion of fixed labor in total labor of 0.51 (0.50) for the one-year horizon and 0.28 (0.31) at the four-year horizon.

In addition, the nature of fixed input requirements—strictly speaking, not adjusting in the short run and remaining constant across firm sizes—suggest that the shrinking output elasticity of labor with firm growth should be strongly sized-based.[49] However, we do not find evidence supporting this notion; instead, we document stable effects of firm growth on inputs and output elasticities by firm size classes (Table A.6).

Finally, fixed labor costs cannot explain the shift from the output elasticity of labor to the output elasticity of intermediates. In the above model, the intermediate input output elasticity is fixed at Φ^M:

$$\theta_{it}^M = \Phi^M \quad \Rightarrow \quad \frac{d\theta_{it}^M}{\theta_{it}^M} = 0. \tag{C.29}$$

Our findings that intermediate input output elasticities increase and absorb the labor output elasticity declines contradict this prediction of a fixed-cost model, whereas our proposed substitution mechanism can fully account for our findings.[50] (And if anything, fixed costs in intermediates, too, would predict a decline rather than, as we find, an increase in output elasticities of intermediates.) We caveat that our simple fixed-cost setup may not capture richer versions of fixed costs, including those that may scale with firm size, multiple fixed costs across products or business activities/markets/export status (although we do not find striking heterogeneity across industries in Figure A.3) or fixed costs in other factors, but we believe that our findings above suggest that fixed costs are unlikely to be a parsimonious explanation of the full set of our findings. However, we do not aim to definitively adjudicate between our mechanism and the fixed-cost view.

D Further Details on the German Firm-Product Level data

Data access. The data can be accessed at the Research Data Centres of the Federal Statistical Office of Germany and the Statistical Offices of the German Länder (states). DOIs: 10.21242/42131.2017.00.03.1.1.0, 10.21242/42221.2018.00.01.1.1.0, and 10.21242/42111.2018.00.01.1.1.0.

[49] To see this, fix L^F and let L grow, such that the output elasticity becomes constant and converges to Φ^L.

[50] Of course, *cost shares* would shift towards intermediates, but we directly study output elasticities as well.

Table D.1: Variable definition in the German micro data.

Variable	Definition
L_{it}	Labor in headcounts.
P_{it}^{L}	Firm wage (firm average), defined as gross salary before taxes (including mandatory social costs) + "other social expenses" (including expenditures for company outings, advanced training, and similar costs) divided by the number of employees.
K_{it}	Capital (including tangible and intangible assets) derived by a perpetual inventory method precisely as in Bräuer et al. (2023) (who provided us with their codes). Intangibles include, software, patents, licenses, brand/trademark value, and similar items.
M_{it}	Deflated total intermediate input expenditures, defined as expenditures for raw materials, energy, intermediate services, goods for resale, renting, repairs, and contracted work conducted by other firms.
E_{it}	Deflated expenditures for raw, auxiliary, and operating materials and energy inputs (includes external product components). E_{it} is part of M_{it}.
$Merch_{it}$	Deflated expenditures for merchandise. $Merch_{it}$ is part of M_{it}.
Sub_{it}	Deflated expenditures for subcontracted production performed by other companies. Sub_{it} is part of M_{it}.
Rep_{it}	Deflated expenditures for repairs, maintenance, installation, and assembly. Rep_{it} is part of M_{it}.
$Temp_{it}$	Deflated expenditures for temporary agency workers. $Temp_{it}$ is part of M_{it}.
$Rent_{it}$	Deflated expenditures for rent, leases, leasing. Rep_{it} is part of M_{it}.
$Other_{it}$	Deflated expenditures for Other intermediate costs (insurance, postage, transport, etc.). $Other_{it}$ is part of M_{it}.
$P_{it}^{M}M_{it}$	Nominal values of total intermediate input expenditures.
$P_{it}Q_{it}$	Nominal total revenue, defined as total gross output, including, among others, sales from own products, sales from intermediate goods, revenue from offered services, and revenue from commissions/brokerage.
Q_{it}	Quasi-quantity measure of physical output, i.e., $P_{it}Q_{it}$ deflated by a firm-specific price index (denoted by PI_{it}).
PI_{it}	Firm-specific Törnqvist price index, derived as in Eslava et al., 2004. See Appendix E.1 for its construction.
P_{igt}	Price of a product g.
$share_{igt}$	Revenue share of a product g in total firm revenue.
ms_{it}	Weighted average of firms' product market shares in terms of revenues. The weights are the sales of each product in firms' total product market sales.
G_{it}	Headquarter location of the firm. 90% of firms in our sample are single-plant firms.
D_{it}	A four-digit industry indicator variable. The industry of each firm is defined as the industry in which the firm generates most of its sales.
Exp_{it}	Dummy-variable being one, if firms generate export market sales.
$NumP_{it}$	The number of products a firm produces.

Variable definitions. Table D.1 summarizes variable definitions.

Outlier cleaning. We exclude the top and bottom two percent outliers with respect to value added over revenue and revenue over labor, capital, intermediate input expenditures, and labor costs. We replace quantity and price information with missing values for products displaying a price deviation from the average price in the top and bottom one percent. We drop the sectors 16 (tobacco), 23 (mineral oil and coke), and 37 (recycling) due to insufficient observations counts.

Capital stock estimation. We calculate firm-specific time series of capital stocks following Bräuer et al. (2023) and consider the law of motion:

$$K_{it} = K_{it-1}(1 - depr_{jt-1}) + I_{it-1}. \tag{D.1}$$

K_{it}, $depr_{jt-1}$, and I_{it-1} denote firm i's capital stock, the depreciation rate of capital, and total investment (buildings, equipment, machines, and other investment goods, including, among others, software, patents, licenses). Nominal values are deflated by two-digit industry-level deflators supplied by the German Statistical Office. The industry- and year-

specific depreciation rate is derived from official information on the expected lifetime of capital goods (supplied by the statistical offices, by years, separately for building and equipment capital). The initial capital stock for the perpetual inventory method is derived from reported tax depreciation.[51]

Deriving a time-consistent industry classification. The NACE classification of industries (and thus firms into industries) changed in 2002 and 2008. To recover a time-consistent industry classification (required for the production function estimation), we follow Mertens (2022) and use information on firms' product mix to classify firms into NACE Rev 1.1 industries based on their main production activities. This procedure exploits that the first four digits of the ten-digit GP product classification reported in the German data equal the NACE classification (they indicate the industry of the product). Applying this method requires a consistent reclassification of all products into the GP2002 scheme (which corresponds to the NACE Rev 1.1 scheme). Reclassifying products is, due to the granularity of the ten-digit classification, less ambiguous than reclassifying industries. In the few ambiguous cases, we follow the firms' product mix over the reclassification periods and unambiguously reclassify most products (we observe what firms produce before and after reclassification years). We then attribute every firm to the NACE Rev 1.1 industry in which it generates most of its revenue. When comparing the classification with the one of the statistical offices for the years 2002-2008 (years in which industries are already reported in NACE Rev 1.1), Mertens (2022) finds that this two-digit and four-digit classification of firms into industries matches the classification of the statistical offices in 95% and 86% of all cases, respectively.

E Production Function Estimation in the German Data

We follow Mertens (2022) in estimating the production function. This approach yields firm- and time-specific output elasticities by assuming the following translog production function (throughout, lower case letter denote logs):

$$\tilde{q}_{it} = \boldsymbol{\phi}'_{it}\,\boldsymbol{\beta} + \omega_{it} + \epsilon_{it}\,. \tag{E.1}$$

[51] We do not use the reported tax depreciation when calculating capital stock series as tax depreciation may vary due to state-induced tax incentives and might therefore not reliably reflect the true amount of depreciated capital. As firms likely report too high depreciation levels due to such tax incentives, our first capital values within a capital series are likely overestimated. However, over time, observed investment decisions gradually receive a larger weight in estimated capital stocks, mitigating the impact of the first capital stock. As we estimate very reasonable output elasticities (see Table A.1), we are confident that our capital variables reliably reflect firms' true capital stocks.

$\tilde{q}_{it}$ denotes the log of produced quantities measured as sales deflated by a firm-specific input price index as described below and ϕ'_{it} captures the production inputs capital (k_{it}), labor (l_{it}), and intermediates (m_{it}) and its interactions. The industry-specific production function that we will estimate for each two-digit Nace rev. 1.1 industry is specified in logs as:

$$\begin{aligned}\tilde{q}_{it} =&\beta_l l_{it} + \beta_m m_{it} + \beta_k k_{it} + \beta_{ll} l_{it}^2 + \beta_{mm} m_{it}^2 + \beta_{kk} k_{it}^2 \\ &+ \beta_{lk} l_{it} k_{it} + \beta_{lm} l_{it} m_{it} + \beta_{km} k_{it} m_{it} + \beta_{lkm} l_{it} k_{it} m_{it} + \omega_{it} + \epsilon_{it}.\end{aligned} \tag{E.2}$$

The output elasticity of labor is

$$\frac{\partial \tilde{q}_{it}}{\partial l_{it}} = \beta_l + 2\beta_{ll} l_{it} + \beta_{lm} m_{it} + \beta_{lk} k_{it} + \beta_{lkm} k_{it} m_{it}\,. \tag{E.3}$$

ϵ_{it} is an i.i.d. error term and ω_{it} denotes Hicks-neutral productivity and follows a Markov process. ω_{it} is unobserved to the econometrician, yet firms know ω_{it} before making input decisions for flexible inputs (intermediates in our case). We assume that only firms' input decision for intermediates depends on productivity shocks. Labor and capital do not respond to contemporary productivity shocks. However, our results are similar when allowing labor to respond to productivity innovations. In fact, the CompNet routine for the production function estimation models labor as flexible and we find consistent results in both datasets. In Eq. (E.2), we explicitly include labor quantities rather than labor expenditure, which is preferable in case of imperfect labor markets. Intermediates and capital enter as deflated monetary values, and we account for input price variation through the input price control function as described below.

There are three issues preventing us from estimating the production function in Eq. (E.1) using OLS. *First*, we need to estimate a physical production model to recover the relevant output elasticities. Although we observe product quantities, quantities cannot be aggregated across the various products of multi-product firms. Relying on the standard practice to apply sector-specific output deflators does not solve this issue if output prices vary within industries. While we address this bias below we note that this bias does not affect ratios of output elasticities, as the elasticities themselves are biased to the same extent. *Second*, we do not observe firm-specific input prices for capital and intermediate inputs. If input prices are correlated with input decisions and output levels, an endogeneity issue arises. Unlike the previous identification issue, this bias affects output elasticities differentially. *Third*, the fact that productivity is unobserved and that firms' flexible input decisions depend on productivity shocks, creates another endogeneity problem. We now discuss how we solve these three identification problems.

E.1 Challenge (1): Deriving a Firm-specific Output Price Index

As we cannot aggregate output quantities across different products of a firm (a common problem), we follow Eslava et al. (2004) and construct a firm-specific price index from observed output prices. We use this price index to purge observed firm revenue from price variation by deflating firm revenues with this price index. We construct firm-specific Törnqvist price indices for each firm's composite revenue from its various products in the following way:

$$PI_{it} = \prod_{g=1}^{n} \frac{p_{igt}}{p_{igt-1}}^{1/2(share_{igt}+share_{igt-1})} PI_{it-1} \,. \tag{E.4}$$

PI_{it} is the price index, p_{igt} is the firm-specific price of good, g, that we observe in the data, and $share_{igt}$ is the share of this good in total product market sales of firm i in period t. The growth of the index value is the product of the individual products' price growths, weighted with the average sales share of that product over the current and the last year. The first year available in the data is the base year (i.e., $PI_{it=1995} = 100$). If firms enter after 1995, we follow Eslava et al. (2004) and use an industry average of the computed firm price indices as a starting value. Similarly, we impute missing product price growth information in other cases with an average of product price changes within the same industry.[52] After deflating firm revenue with this price index, we have a quasi-quantity measure of output, which we highlight with a tilde: $\tilde{q}_{it}$.[53]

E.2 Challenge (2): Accounting for Unobserved Input Price Variation

To control for input price variation across firms, we use a firm-level analog of De Loecker et al. (2016) and define a price-control function from firm-product-level output price information that we add to the production function in Eq. (E.1):

$$\tilde{q}_{it} = \boldsymbol{\phi}'_{it}\boldsymbol{\beta} + B_{it}((pi_{it}, ms_{it}, G_{it}, D_{it}) \times \boldsymbol{\phi}^{c}_{it}) + \omega_{it} + \epsilon_{it} \,. \tag{E.5}$$

[52] For roughly 30% of all product observations in the data, firms do not have to report quantities as the statistical office views them as not being meaningful.

[53] Note that, as discussed in Bond et al. (2021), using an output price index does not fully purge firm-specific price variation. There remains a base year difference in prices. Yet, using a firm-specific price index follows the usual practice of using price indices to deflate nominal values, we are thus following the best practice. Moreover, it is the only available approach when pooling multi- and single-product firms. Estimating the production function separately by single-plant firms requires other strong assumptions like perfect input divisibility of all inputs across all products. Finally, our results are also robust to using cost-share approaches to estimate the production function, which requires other assumptions (constant returns to scale, competitive input markets, and the absence of adjustment costs).

$B_{it}(.) = B_{it}((pi_{it}, ms_{it}, G_{it}, D_{it}) \times \boldsymbol{\phi}^c_{it})$ is the price control function consisting of our logged firm-specific output price index (pi_{it}), a logged sales-weighted average of firms' product market sales shares (ms_{it}), a headquarter location dummy (G_{it}) and a four-digit industry dummy (D_{it}). $\boldsymbol{\phi}^c_{it} = [1; \boldsymbol{\phi}_{it}]$, where $\boldsymbol{\phi}_{it}$ includes the production function input terms as specified in Eq. (E.2). These are either in monetary terms and deflated by an industry-level deflator (capital and intermediates) or already reported in quantities (labor). The constant entering $\boldsymbol{\phi}^c_{it}$ highlights that elements of $B(.)$ enter the price control function linearly and interacted with $\boldsymbol{\phi}_{it}$ (a consequence of the translog production function). The idea behind the price-control function $B(.)$ is that output prices, product market shares, and firms' location and industry are informative about firms' input prices. Particularly, we assume that product prices and market shares contain information about product quality and that producing high-quality products requires expensive high-quality inputs. De Loecker et al. (2016) discuss that this motivates adding a control function containing output price and market share information to the right-hand side of the production function to control for unobserved input price variation emerging from input quality differences across firms. We also include year, location, and four-digit industry dummies into $B(.)$ to further absorb the remaining differences in local and four-digit industry-specific input prices.

Conditional on elements in $B(.)$, we assume that there are no remaining input price differences across firms. Although restrictive, this assumption is more general than the ones employed in most other studies estimating production functions without having access to firm-specific price data and which implicitly assume that firms face identical input and output prices within industries.

A notable difference between the original approach of De Loecker et al. (2016) and our version is that they estimate product-level production functions, whereas we transfer their framework to the firm level. For that, we use firm-product-specific sales shares in firms' total product market sales to aggregate firm-product-level information to the firm-level. This implicitly assumes that i) such firm aggregates of product quality increase in firm aggregates of product prices and input quality, ii) firm-level input costs for inputs entering as deflated expenditures increase in firm-level input quality, and iii) product price elasticities are equal across the various products of a firm. These or even stricter assumptions are always implicitly invoked when estimating firm-level production functions.

Finally, note that if some of the above assumptions do not hold, including the price control function is still preferable to omitting it. This is because the price control function can nevertheless absorb some of the unobserved price variations and does not require that input prices vary between firms with respect to all (or any) elements of $B_{it}(.)$. The attractiveness of a price control function lies in its agnostic view about the existence and

degree of input price variation.

E.3 Challenge (3): Controlling for Unobserved Productivity

To address the dependence of firms' intermediate input decisions on unobserved productivity, we employ a control function approach (i.e., Wooldridge, 2009). We base our control function on firms' consumption of energy and materials, which we denote by e_{it} and which are components of total intermediate inputs. Inverting the demand function for e_{it} defines an expression for productivity:

$$\omega_{it} \equiv g_{it}(.) = g_{it}(e_{it}, k_{it}, l_{it}, \mathbf{\Gamma}_{it}). \tag{E.6}$$

$\mathbf{\Gamma}_{it}$ captures state variables of the firm, that in addition to k_{it} and l_{it} affect firms' demand for e_{it}. Ideally, $\mathbf{\Gamma}_{it}$ should include a wide set of variables affecting productivity and demand for e_{it}. We include dummy variables for export (EX_{it}) activities, the log of the number of products a firm produces ($NumP_{it}$), and the average wage a firm pays (P_{it}^{L}) into $\mathbf{\Gamma}_{it}$. The latter absorbs unobserved quality and price differences that shift input demand for e_{it}. Including wages also helps to absorb variation in marginal costs and wage markdowns, which addresses some of the critique raised by Doraszelski and Jaumandreu (2021) because wages are a serially correlated input price that is also correlated with other input prices. Remember that productivity follows a first-order Markov process. Firms can shift this Markov process as described in De Loecker (2013), giving rise to the following law of motion for productivity: $\omega_{it} = h_{it}(\omega_{it-1}, \mathbf{T}_{it-1}) + \xi_{it} = h_{it}(.) + \xi_{it}$, where ξ_{it} denotes the innovation in productivity and $\mathbf{T}_{it} = (EX_{it}, NumP_{it})$ reflects that we allow for learning effects from export market participation and (dis)economies of scope through adding and dropping products to influence firm productivity.[54] Plugging Eq. (E.6) and the law of motion for productivity into Eq. (E.5) yields the equation that we estimate:

$$\tilde{q}_{it} = \boldsymbol{\phi}_{it}'\boldsymbol{\beta} + B_{it}(.) + h_{it}(.) + \epsilon_{it} + \xi_{it}. \tag{E.7}$$

E.4 Identifying Moments

We estimate Eq. (E.7) separately by two-digit NACE Rev. 1.1 industries using a one-step estimator as in Wooldridge (2009).[55] Our estimator uses lagged values of flexible

[54] We would also like to add information on R&D investment to the productivity model but do not observe R&D expenditures for the early years in our data.

[55] We approximate $h_{it}(.)$ by a third-order polynomial in all of its elements, except for the variables in $\mathbf{\Gamma}_{it}$. Those we add linearly. $B_{it}(.)$ is approximated by a flexible polynomial where we interact the output price

inputs (i.e., intermediates) as instruments for their contemporary values to address the dependence of firms' flexible input decisions on realizations of ξ_{it}. Similarly, we use lagged values of terms including firms' market share and output price index as instruments for their contemporary values as we consider these to be flexible variables.[56] We define identifying moments jointly on ϵ_{it} and ξ_{it}:

$$E[(\epsilon_{it} + \xi_{it})\mathbf{Y}_{it}] = 0. \tag{E.8}$$

$\mathbf{Y}_{it}$ includes lagged interactions of intermediate inputs with labor and capital, contemporary interactions of labor and capital, contemporary location and industry dummies, the lagged output price index, lagged market shares, lagged elements of $h_{it}(.)$, and lagged interactions of the output price index with production inputs. Formally this implies:

$$\mathbf{Y}'_{it} = (J_{it}(.), A_{it-1}(.), T_{it-1}(.), \Psi_{it}(.), \boldsymbol{\vartheta}_{it-1}) \ , \tag{E.9}$$

where we defined: $J_{it}(.) = (l_{it},\, k_{it},\, l_{it}^2,\, k_{it}^2,\, l_{it}k_{it},\, G_{it},\, D_{it})$; $A_{it}(.) = (m_{it},\, m_{it}^2,\, l_{it}m_{it},\, k_{it}m_{it},\, l_{it}k_{it}m_{it},\, ms_{it},\, \pi_{it})$; $T_{it}(.) = ((l_{it},\, k_{it},\, l_{it}^2,\, k_{it}^2,\, l_{it}k_{it},\, m_{it},\, m_{it}^2,\, l_{it}m_{it},\, k_{it}m_{it},\, l_{it}k_{it}m_{it}) \times \pi_{it})$; $\Psi_{it}(.) = \sum_{n=0}^{3}\sum_{w=0}^{3-b}\sum_{h=0}^{3-n-b} l_{it-1}^n\, k_{it-1}^b\, e_{it-1}^h$; and $\boldsymbol{\vartheta}_{it-1} = (Exp_{it-1},\, NumP_{it-1},\, P_{it-1}^L)$, with P_{it}^L denoting firms' average wages. Table A.1 reports output elasticities derived from the estimated industry production functions. Average labor, intermediate, and capital output elasticities equal 0.31 (0.11), 0.64 (0.12), and 0.12 (0.10) with standard errors in parentheses.

E.5 Limitations

Our approach addresses biases stemming from input and output prices, which frequently affect existing production function estimates. Nonetheless, we acknowledge the remaining inherent limitations in the general methodology for estimating production functions, which remain subjects of active research. For example, Bond et al. (2021) highlight potential biases in output elasticities if inputs directly influence demand (without observing these inputs), such as intermediates used in marketing. Similarly, aggregating production and non-production labor into a single labor measure can bias output elasticities, though

index with elements in $\boldsymbol{\phi}_{it}$ and add the vector of market shares, the output price index, and the location and industry dummies linearly. Interacting further elements of $B_{it}(.)$ with $\boldsymbol{\phi}_{it}$ creates too many parameters to be estimated. This implementation is similar to De Loecker et al. (2016).

[56] These timing assumptions also address any simultaneity concerns with respect to the price variables entering the right-hand side of our estimation.

this concern might be less critical in manufacturing contexts. Furthermore, incomplete specification of the productivity control function may violate the monotonicity assumption crucial for identification (Doraszelski and Jaumandreu, 2021).[57] Although we extend traditional methods by incorporating wages, export status, and the number of products into our productivity control function, important unobserved factors may still exist that cause the monotonicity assumption to fail. Due to these concerns, we complement our parametric approach with a non-parametric identification of output elasticities based on cost shares (as detailed in the main text).

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