Fiscal Threats to Price Stability When Households are not Ricardian

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March 13, 2026

Abstract

Can monetary policy deliver price stability whatever fiscal policy does? With Ricardian households, no: Public debt must be backed by future fiscal surpluses. But if that fiscal backing holds, monetary policy can deliver price stability by responding sufficiently strongly to inflation. We reconsider the question when households are not Ricardian. First, although public debt need not always be backed by future surpluses, fiscal requirements still exist in a different form: Public debt cannot be so high that its wealth effect on aggregate spending prevents the existence of a natural interest rate. Second, if this condition is satisfied, monetary policy can deliver price stability, but it must respond directly to the level of public debt—not just to the inflation that higher debt creates. We conclude that in practice the main fiscal threat to price stability is institutional: even though debt-contingent rules are feasible, central banks may be reluctant to adopt them.

Keywords: Monetary-Fiscal Interactions, Non-Ricardian Households, Price Stability.

JEL Classification: E31, E62.

*We thank our discussants Sara Casella, Wei Cui and Nigel McClung, as well as Jean Barthélémy, Marco Bassetto, Saroj Bhattarai, Francesco Bianchi, Nicolas Coeurdacier, Fabrice Collard, Bartosz Maćkowiak, Julien Matheron, Leonardo Melosi, Xavier Ragot, Ricardo Reis, Dmitriy Sergeyev and Sebastian Schmidt for very useful discussions, and seminar and conference participants at Banque de France, the ECB, Le Mans University, Sapienza Macro Days, the BSE forum, the SED, the CASFI Annual Research Conference, the EABCN-EUI Joint Conference on Advances in monetary policy and the natural rate of interest, the DNB Annual Research Conference and Sciences Po for very useful comments. The paper previously circulated under the title, *Fiscal Requirements for Price Stability When Households are Not Ricardian*. The views expressed herein are those of the authors and do not necessarily reflect those of the Banque de France, the ECB or the Eurosystem.

Introduction

Give the central bank a clear mandate of price stability. Grant it full independence from the government. And appoint at its head a steadfast governor who will not let anything but its mandate influence its policy. Does the central bank then have all it needs to deliver price stability? Or can fiscal policy hamper its ability to do so? The question is a cornerstone of monetary-fiscal interactions, determining whether monetary policy has the power to insulate inflation from imprudent fiscal decisions, or is ultimately dependent on a well-behaved fiscal authority.

In policy circles, additional constraints on fiscal policy are typically seen as necessary. At the creation of the euro area, the convergence criteria of the Maastricht Treaty and then the Stability and Growth Pact introduced fiscal rules for national governments under the assumption they were necessary to allow the ECB to deliver on its price stability mandate.

Yet the economic literature provides no consensus on whether fiscal policy can threaten the central bank's ability to control inflation. The issue is already contentious when considering Ricardian households. On the one hand, the monetarist view that the control of the price level is ultimately always in the realm of the central bank is embedded in most of the monetary models currently in use. On the other hand, Leeper (1991), Sims (1994), Woodford (1995, 2001) and the subsequent Fiscal Theory of the Price Level (FTPL) literature argue that for the central bank to be able to deliver price stability, current public debt must be backed by future fiscal surpluses.[1] Namely, the real value of nominal public debt $D_{t-1}I_{t-1}$ at stable prices $P_t = P^*$ must be equal to the net present value (NPV) of future real fiscal surpluses T_{t+k},[2]

$$\frac{D_{t-1}I_{t-1}}{P^*} = \sum_{k=0}^{\infty} \frac{1}{\mathcal{R}_{t,t+k}} T_{t+k}, \tag{1}$$

where I_{t-1} is the nominal interest rate from $t-1$ to t and $\mathcal{R}_{t,t+k}$ is the real interest rate from t to $t+k$. If the government does not plan on sufficient future fiscal surpluses to make equation (1) hold, no equilibrium with stable prices exists. The central bank has therefore no chance of delivering it. Instead, it opens the door to fiscal dominance: the possibility for the price level to

[1] Distinct from the Fiscal Theory of the Price Level, the monetarist view was first challenged by Sargent and Wallace (1981)'s unpleasant monetarist arithmetic, which provides a distinct argument based on seigniorage revenues. In the present paper we stick to a cashless economy in which there are no seigniorage revenues from money holdings.

[2] Public debt D_{t-1} denotes the amount the government borrowed at $t-1$, so that the liabilities the government must repay at t are inclusive of interest rates $D_{t-1}I_{t-1}$. It is equal to the face value of zero-coupon bonds issued, which can be denoted $B_{t-1} = D_{t-1}I_{t-1}$.

be determined by fiscal policy, as the value necessary to make the NPV equation (1) hold.[3]

Whether fiscal policy can make the central bank lose control over inflation has been even more debated in recent years as the focus has moved to the case of non-Ricardian households—an assumption with strong appeal given that finite lives, financial frictions, or limited foresight are enough to make households non-Ricardian.[4] Whether non-Ricardian households amplify or diminish the threat that fiscal policy poses to price stability is a priori unclear. On the one hand, non-Ricardian households make it possible for the real interest rate r to be less than the growth rate of the economy g, in which case the NPV equation (1) does not need to hold (Blanchard, 2019). If it does not need to hold, it no longer rules out some fiscal policies as inconsistent with price stability, let alone determines inflation (Bassetto and Cui, 2018).[5] On the other hand, non-Ricardian households can suggest it is even easier for fiscal policy to determine inflation, since public debt then directly affects households' spending, and therefore inflation. Angeletos, Lian, and Wolf (2024b) argue that when households are non-Ricardian, deficits can be as inflationary as under the fiscal dominance of the FTPL, but that contrary to the case of Ricardian households it remains so even when monetary policy is active.

In this paper, we seek to clarify what threats fiscal policy poses to the central bank's ability to deliver price stability when households are not Ricardian.[6] To reconcile contradictory intuitions, we argue it is important to distinguish between two conceptually distinct questions. The first is whether there exist fiscal requirements for price stability: Are some fiscal policies simply inconsistent with price stability, and if so how to characterize them? The second concerns the implementation of price stability: Provided fiscal policy is consistent with price stability, can the central bank make sure price stability obtains, and if so, how? We address both questions in turn, through two main results, and a policy conclusion.

[3]The ability of the NPV equation (1) to determine the price level is what early critiques of the FTPL put into question—e.g. Kocherlakota and Phelan (1999); Buiter (2002); Niepelt (2004).

[4]A note on terminology: In the FTPL, a key distinction is made between Ricardian and non-Ricardian *fiscal policies* (Woodford, 1995). A Ricardian fiscal policy is one that satisfies the intertemporal budget constraint of the government for any price level. Whether a fiscal policy is Ricardian or not is however different from whether households are Ricardian or not—they always are in the FTPL. To avoid confusion, in the paper we use the term *Ricardian* to refer to households only, and avoid using the term to refer to policies.

[5]Bassetto and Cui (2018) also show that the NPV equation remains a valid requirement when r is less than g because of high risk premia. On the role of risk premia in the NPV equation, see also Bohn (1995) and Jiang et al. (2024), who find that the NPV equation fails to hold in the data when the risk of future fiscal surpluses is priced through a realistic asset pricing model.

[6]Our primary interest is in deriving conditions for monetary policy to retain control over inflation, i.e. for monetary dominance. Our focus is not on how fiscal policy determines inflation under fiscal dominance, although some of our results in Section 5 address this question.

To address both questions, we rely primarily on Blanchard (1985)'s model of perpetual youth, which breaks the Ricardian equivalence by assuming that households face a mortality risk,[7] although we also consider alternative models. We focus on the perpetual-youth set-up because it has gained increasing appeal to study economies with non-Ricardian households, as households' mortality risk can be interpreted either literally as biological death—making it an overlapping generations model—or as the risk of hitting borrowing constraints—the financial frictions that are the focus of the HANK literature (e.g. Farhi and Werning, 2019; Wolf, 2025).

To answer the first question we derive the conditions for the existence of an equilibrium with stable prices. We show that when households are not Ricardian, fiscal requirements for price stability still exist but take the form of a limit on the debt-to-GDP ratio. In the canonical specification of the perpetual youth model with log utility, it must be the case that at all points in time,

$$\frac{D_t}{P^* Y_t} < d^*. \tag{2}$$

When the debt-to-GDP ratio is above the threshold d^* or projected to grow above this threshold in the future, no stable price equilibrium exists. The debt-to-GDP limit arises because above it, no interest rate however high can counterbalance the effect of higher debt on aggregate demand. Indeed, when households are not Ricardian, higher public debt increases aggregate demand through a wealth effect, putting upward pressure on inflation. This in itself poses no constraint on the ability of the central bank to maintain price stability. It can counter the inflationary effect of higher debt with higher interest rates, just like it can counter any other inflationary shock, thereby retaining the ultimate control over inflation. In other words, to deliver price stability the central bank needs to deliver the natural rate of interest—the real interest rate consistent with price stability—which increases with the level of public debt when households are not Ricardian.[8] Yet we show that, under log utility, when the debt-to-GDP ratio approaches d^* the natural rate tends to infinity—even infinitely high interest rates are not enough to counter the wealth effect of public debt—resulting in the limit (2).

[7] Our results do not depend on the existence of a mortality risk however. We generalize Blanchard (1985)'s set-up to allow for population growth, which breaks the Ricardian equivalence even when households face no mortality risk.

[8] We use the definition of the natural rate as the real interest rate consistent with price stability. It is the short-term natural rate that moves at business cycle frequency, and not the long-term (steady-state) natural rate. Some authors reserve "natural rate" for the latter and call the short-term natural rate the neutral rate (e.g. Obstfeld, 2023). Here, the short-term natural rate does not necessarily correspond to the real interest rate that would obtain under flexible prices: With non-Ricardian households, the real interest rate that would obtain under flexible prices depends on the monetary and fiscal policies in place.

We further show that the logic extends beyond log utility, focusing this time on steady states. A debt-to-GDP limit d^* still exists as long as the intertemporal elasticity of substitution (IES) is less than one—the case usually considered the more plausible one empirically (Havránek, 2015, e.g.). The overall logic is the same: When debt is too high, no real interest rate can counteract the wealth effect of public debt on aggregate spending. But when the IES is less than one, the limit arises in a slightly different way. When the interest rate is above a certain threshold R^*, the wealth effect of higher interest rates becomes so large that higher interest rates make households save less, not more. This creates an upper bound $d^* = d(R^*)$ on the amount of public debt that households can absorb, this time reached at the finite interest rate R^*. For levels of debt greater than d^*, there is no real interest rate consistent with stable prices.

We show that the debt-to-GDP limit (2) fully characterizes fiscal requirements for price stability. In contrast, households' intertemporal budget constraints impose only very weak restrictions in the non-Ricardian case. In particular, if, from any current level of public debt, the government plans on never raising any fiscal surplus to repay it, this violates no household's intertemporal budget constraint. This does not mean that individual households accumulate explosive amounts of public debt that they intend never to spend. Instead, they sell it to new generations. Intertemporal budget constraints do impose a restriction on the path of future taxes and transfers, but it is a very weak one: in particular, they do not require future fiscal surpluses. The key fiscal requirement comes instead from the need for asset demand to remain compatible with price stability.

The debt-to-GDP limit (2) is specific to non-Ricardian households: When households are Ricardian, the natural rate does not depend on the level of debt so that debt can be arbitrarily high, provided it is backed by future surpluses. Yet the debt limit captures in a different form a popular interpretation of the NPV requirement: fiscal policy cannot be such that the wealth effect of public debt on aggregate spending makes households want to spend more than is compatible with price stability (e.g. Woodford (1995), p.12-13 or Cochrane (2023), Section 1.2). Under the form of the debt limit, this intuition is still alive when households are not Ricardian and still imposes constraints on fiscal policy, even though the NPV equation no longer necessarily holds.

An alternative approach to replace the NPV equation when households are not Ricardian is to say that it must hold up to a bubble (Brunnermeier, Merkel, and Sannikov, 2022; Reis, 2021, 2022). The debt limit (2) characterizes which fiscal policies are consistent with price stability, and does so whether a bubble exists or not. Beyond relating fiscal requirements to the behavior of the

natural rate, it shows that non-Ricardian households make fiscal requirements both looser—when $r < g$—and stricter—by ruling out fiscal policies that bring public debt too high even if it is backed by future surpluses.

The debt-to-GDP limit allows us to recover the common intuition that some very loose fiscal policies are inconsistent with price stability. In particular, it implies that although it is possible for the government never to raise fiscal surpluses without threatening price stability when $r < g$, this is only possible when debt is low enough. Because the real interest rate increases with the level of public debt, there exists a level of public debt above which r is necessarily greater than g, so that in the absence of fiscal surpluses debt-to-GDP necessarily ends up crossing the limit d^*.

Beyond such extreme fiscal policies, the precise value of the debt-to-GDP limit d^* matters to determine which fiscal policies are consistent with price stability and which are not. We show that d^* depends strongly on two statistics: the marginal propensity to consume (MPC) of the households that hold public debt, and their intertemporal elasticity of substitution (IES).

First, d^* decreases strongly with the MPC. In the case of log utility, d^* decreases dramatically as the MPC increases, from about 1700 times annual GDP when the quarterly MPC is 0.01—the value consistent with an interpretation of the model as an overlapping-generations model—to just 4.9 times GDP when it is 0.2—the value consistent with empirical estimates and one that can be rationalized by reinterpreting the model as a HANK model. We emphasize, however, that what matters is the MPC of households *that hold public debt*. To bring the distinction to life, we extend the model to allow for a fraction of hand-to-mouth households with an MPC of one. By varying the share of hand-to-mouth households, an average quarterly MPC of 0.2 can be consistent with any MPC for the households that hold public debt. In our main calibration strategy, we calibrate the extended model to match both the average MPC in the data and the sensitivity of the long-term interest rate to the debt-to-GDP ratio. The latter imposes a rather small MPC for the households that hold public debt, around 0.03 quarterly. In the case of log utility, the debt-to-GDP limit is then 135 times GDP.

But second, d^* also decreases strongly with the IES. Under our main calibration strategy, the debt-to-GDP limit decreases from 135 times GDP in the case of log utility (an IES of 1) to just 5.4 times GDP when the IES is 0.5. While much lower, such a value nevertheless suggests that high levels of public debt can be consistent with price stability provided the central bank finds a way to implement the stable price equilibrium.

We therefore turn to the second question of implementation: Provided fiscal policy is consistent with price stability, how can the central bank make sure price stability obtains? We first assume that the central bank sets nominal interest rates according to a standard Taylor rule, and the government sets taxes according to a similar feedback rule that responds to the level of public debt. In doing so we reconsider Leeper (1991)'s local version of the FTPL in the case of non-Ricardian households. We characterize analytically for which degree of responsiveness of fiscal and monetary policy there exists a unique bounded equilibrium, and when it exists whether inflation is insulated from fiscal shocks.

We show that under a standard Taylor rule where monetary policy responds to inflation only, fiscal shocks always affect inflation, however strong the response of monetary policy to inflation. It is no longer possible to distinguish between a monetary regime that insulates inflation from fiscal shocks and a fiscal regime that does not—the central result of Leeper (1991). This could suggest that price stability is never achievable when households are not Ricardian. Yet, we show that monetary policy can insulate inflation from fiscal shocks and implement the stable price equilibrium if, on top of reacting to inflation, it directly responds to the level of public debt—not just to the inflation that higher debt creates. Doing so allows it to track the natural rate, which depends on the level of public debt when households are not Ricardian. To insulate inflation from fiscal shocks, monetary policy must therefore monitor the level of public debt, in contrast to the idea that monetary dominance obtains when the central bank abstracts from fiscal developments.

So how much of a threat does fiscal policy pose to the central bank's ability to deliver price stability? The debt limit (2) usefully excludes extreme fiscal policies that are intuitively inconsistent with price stability but leaves many fiscal policies consistent with it. And for those policies, implementing price stability is possible, provided the central bank increases interest rates directly with the level of public debt. We conclude that in practice the main threat to price stability from fiscal policy is that central banks may be reluctant to respond directly to the level of public debt. Doing so requires them to express a view on the inflationary effects of fiscal policy and to adjust policy rates accordingly without the easy justification that they are simply reacting to the level of inflation.

Central banks may be reluctant to do so, especially because it could be perceived by the government as interfering with fiscal policy and expose them to the criticism they are stepping outside their roles, ultimately threatening the independence they have been granted. Central banks are at

least reluctant to say publicly that public debt plays a role in their decisions. In an interview with CBS's *60 Minutes* in 2024, Powell (2024) answered the question *"How do you assess the national debt?"* in the following way:[9]

> *"We mostly try very hard not to comment on fiscal policy and instruct Congress on how to do their job when actually they have oversight over us. So, the national debt doesn't play a big role in our thinking—doesn't play any role, actually, in our thinking. When Congress does deficit spending, that can be stimulative, that goes into our models. But we don't—it's not our role at all to be a judge of fiscal policy in any way."*

The main impediment to central banks' ability to insulate inflation from fiscal developments may therefore be institutional. When households are Ricardian, the fact that the central bank can implement price stability without paying attention to the level of debt allows it to rely on a convenient *quid pro quo* interpretation of central bank independence: the fiscal authority refrains from commenting on monetary policy, in exchange for which monetary policy delivers price stability without paying attention to fiscal matters. When households are not Ricardian, the *quid pro quo* interpretation is no longer possible.

By considering what the fiscal requirements for price stability are, this paper connects to the papers that have derived them in the case of Ricardian households, many of them associated with the FTPL literature developed by Leeper (1991), Sims (1994), Woodford (1995, 2001), Bassetto (2002), and Cochrane (2001, 2005).[10] We depart from these by considering non-Ricardian households.

An important stream of the recent FTPL literature considers whether the dynamics of US inflation can be accounted for by models that feature elements of both monetary and fiscal dominance, either because the economy oscillates between regimes of fiscal and monetary dominance (Davig and Leeper, 2007; Bianchi and Ilut, 2017; Bianchi and Melosi, 2017, 2019; Schmidt, 2024), or because some fiscal shocks are funded while others are not (Cochrane, 2022; Bianchi, Faccini, and Melosi, 2023; Smets and Wouters, 2024)—see also Bassetto and Miller (2025) and Corsetti and Maćkowiak (2024) who show how sudden inflation can arise when the economy endogenously shifts to the fiscal regime.[11] We show that when households are not Ricardian the distinction between

[9] The response was edited for length in the broadcast version of the interview, but is available in full in the transcript on CBS's website.

[10] For reviews of the FTPL, see Canzoneri, Cumby, and Diba (2011), Leeper and Leith (2016) and Cochrane (2023), as well as Barthélémy, Mengus, and Plantin (2024). For a review of the FTPL with a special focus on empirical studies, see Bianchi, Melosi, and Rogantini Picco (2024).

[11] On the comparison of the fiscal and monetary regimes and which prevailed when in the US, see also Bhattarai,

monetary and fiscal dominance is already blurred even absent regime switching or different reactions to different fiscal shocks. Elfsbacka-Schmoller and McClung (2024) consider the FTPL in a model with endogenous growth, a dimension from which we abstract. Jarocinski and Maćkowiak (2018), Bianchi, Melosi, and Rogantini Picco (2023) and Maćkowiak and Schmidt (2024) study a monetary union with several national fiscal authorities and study different fiscal-monetary configurations when common euro area debt is issued. We consider an economy with a unique fiscal authority. Barro and Bianchi (2023) analyze the drivers of the recent inflation surge in OECD countries through the lens of the FTPL. Hazell and Hobler (2024) use markets' reaction to the results of Georgia Senate runoff elections to assess how much deficits contributed to the inflation surge in the US.

A fast-growing literature analyzes fiscal-monetary interactions in models where the Ricardian equivalence breaks. A first stream considers models with financial frictions. Cui (2016) studies monetary-fiscal interactions in a model where public debt earns a liquidity premium because private assets face endogenous liquidity frictions. Interestingly, the shape of the determinacy region in his model is very similar to the one we find when monetary policy follows a standard Taylor rule. We show how a monetary rule that responds to public debt can restore the distinct regimes of monetary and fiscal dominance in Leeper (1991). Brunnermeier, Merkel, and Sannikov (2022) show that the government can run permanent negative surpluses and that its intertemporal budget constraint needs to be augmented with a "bubble" term in a model with uninsurable idiosyncratic risk. Kaplan, Nikolakoudis, and Violante (2023) show that r can be less than g and the government can run permanent primary deficits in a canonical HANK model. Mian, Straub, and Sufi (2022) consider a model with bonds-in-the-utility to quantify the maximum level of permanent deficits that the government can run. Mehrotra and Sergeyev (2021) analyze the determinants of the distribution of debt-to-GDP in a model with bonds-in-the-utility and where GDP is subject to risk. Campos et al. (2024) show that, in a HANK model, if the central bank fails to adjust its estimate of the long-term natural rate after a permanent increase in the level of public debt, inflation in the new steady-state is permanently above target. Our implementation result shows that fully insulating inflation from (temporary or permanent) increases in public debt is possible if the central bank's interest-rate rule tracks the short-term natural rate. Hagedorn (2024) contends that the FTPL fails in models with non-Ricardian households because markets are then incomplete.[12]

Lee, and Park (2012, 2014).

[12] Although we deliberately stay away from normative statements and optimal policy, we also connect to papers that study optimal issuance of public debt in models with non-Ricardian households and/or where public debt provides

Another stream considers monetary-fiscal interactions in models with overlapping generations. Precursors are Bénassy (2003, 2005, 2008)—see also Bénassy (2007) for a review—and Leith and Wren-Lewis (2000). Farmer and Zabczyk (2023) study steady-state multiplicity and their determinacy properties in a quantitative OLG model where households live for 62 periods. Aguiar, Amador, and Arellano (2023) analyze when an increase in public debt is Pareto-improving in a perpetual-youth set-up where the government issues real bonds. Consistent with our results, they show that, following an increase in public debt, monetary policy must increase interest rates with debt, otherwise steady-state inflation is permanently higher. Angeletos, Lian, and Wolf (2024a) consider how much deficits can finance themselves in a perpetual-youth model interpreted as a HANK model. Angeletos, Lian, and Wolf (2024b) compare the inflationary effects of deficits in such a model to the ones of the FTPL, under a specific monetary policy rule. Our implementation result derives what monetary policy rule can guarantee that deficits do *not* have an effect on inflation. Rachel and Ravn (2025) extend our determinacy results under a standard Taylor rule to a perpetual-youth economy with sticky prices. They show that under sticky prices as well, a monetary policy that responds only to inflation cannot insulate inflation from fiscal shocks. We show that monetary policy can if it responds to the level of public debt. Both Angeletos, Lian, and Wolf (2024b) and Rachel and Ravn (2025) consider the linearized version of the perpetual-youth model. Our main result—the fiscal requirements for price stability—requires studying the non-linearized version of the model.[13]

Section 1 presents the main model of perpetual-youth with non-Ricardian households. Section 2 derives the fiscal requirements for price stability in this model, as well as in a standard two-generation OLG model. Section 3 considers extensions of the model. Section 4 considers the case of non-log utility with an intertemporal elasticity of substitution different from one. Section 5 considers how the central bank can implement the stable price equilibrium once it exists.

1 An Economy with Non-Ricardian Households

In this section, we lay out the main model of non-Ricardian households we rely on. It is a Blanchard-Yaari perpetual-youth set-up (Blanchard, 1985) in discrete time, under perfect foresight. Relative

liquidity to firms, such as Aiyagari and McGrattan (1998), Bhandari et al. (2017), Azzimonti and Yared (2019), Canzoneri, Cumby, and Diba (2016), Angeletos, Collard, and Dellas (2022) and Bassetto and Cui (2024).

[13] Papers using the perpetual-youth set-up also include Galí (2021), who studies bubble-driven fluctuations in a New-Keynesian perpetual-youth set-up, but without public debt and taxes.

to the baseline version of the Blanchard-Yaari model, we add two features. Neither is necessary for our results, but we allow for them for generality. First, we allow for population growth at rate g, which breaks the Ricardian equivalence even if households are infinitely lived (Weil, 1989). We do so both to meaningfully talk about r and g and in order to stress that our results do not depend on the assumption that households die and buy life-insurance contracts. Second, we allow for individual incomes to shrink over time (Blanchard, 1985, Section III). We do so both for realism to capture the need to save for retirement, and in order to allow for negative interest rates. We assume that the supply-side of the economy is given by an exogenous path for Y_t. Under flexible prices, this exogenous path can be interpreted as the supply-determined level of GDP. Under sticky prices, it can be interpreted as the exogenous path for natural output.

1.1 Of Life and Death

The economy is populated by an infinity of households of different ages. Households face a probability λ of dying each period, independent of how long they have been alive. To insure them against the risk of dying with positive wealth, insurance companies provide them with actuarially fair insurance contracts. The wealth of households that die at t is redistributed to households still alive, in proportion to the financial wealth they held at the end of the previous period, A_{t-1}. The redistributed amount $\lambda A_{t-1} I_{t-1}$ is therefore redistributed to the savings $(1-\lambda)A_{t-1}I_{t-1}$ of the surviving households. Each dollar of saving therefore receives $\lambda/(1-\lambda)$ dollars of annuity.

New households are born every period, with no wealth. We assume that at t a number $(\lambda + g)N_{t-1}$ of households are born, so the population grows at rate g. Since λ of households die every period regardless of their ages, the number of households of age n at t is:

$$N_t(n) = (1-\lambda)^n(\lambda+g)N_{t-n-1} = \frac{\lambda+g}{1+g}\left(\frac{1-\lambda}{1+g}\right)^n N_t. \tag{3}$$

The particular case of the Ricardian representative household arises when there are neither new births nor deaths, $\lambda = 0$ and $g = 0$, in which case all households are identical. Either $\lambda > 0$ or $g > 0$ is enough to break the Ricardian equivalence. From now on, we denote this non-Ricardian case as $\lambda + g > 0$.

1.2 Households

A household i has preferences over its consumption path

$$\sum_{k=0}^{\infty}(\beta(1-\lambda))^k \log(C^i_{t+k}), \tag{4}$$

where β is the preference discount factor. The household effectively discounts the future at the stronger rate $\beta(1-\lambda)$ because it factors in the chance that it won't be there to enjoy consumption tomorrow. Log utility is the canonical specification in the perpetual-youth model. We consider more general CRRA preferences in Section 4.

Household i maximizes its utility (4) subject to its flow budget constraints and a No-Ponzi-scheme constraint. Having saved A^i_{t-1} in period $t-1$, it starts period t with wealth $1/(1-\lambda)A^i_{t-1}I_{t-1}$, where I_t is the nominal interest rate. Its flow budget constraint is

$$\frac{A^i_t}{P_t} + C^i_t = \frac{1}{1-\lambda}\frac{A^i_{t-1}I_{t-1}}{P_t} + (Y^i_t - T^i_t), \tag{5}$$

where C^i_t is its real consumption, Y^i_t its real income, and T^i_t the real taxes it has to pay. Its No-Ponzi-scheme constraint is

$$\lim_{k\to\infty} \frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k}}\frac{A^i_{t+k}}{P_{t+k}} \geq 0, \tag{6}$$

where $R_t = I_t P_t / P_{t+1}$ is the real interest rate and

$$\mathcal{R}_{t,t+k} = \prod_{j=0}^{k-1} R_{t+j}. \tag{7}$$

Like in the flow budget constraint (5) the factor $(1-\lambda)$ enters the No-Ponzi-scheme condition. It takes into account that the wealth the household holds grows at a rate $R/(1-\lambda)$ and not just R since the household also receives the annuity on its wealth.

The optimal behavior of household i can be characterized in the usual way (see Appendix A for a derivation). Household i behaves optimally if and only if its consumption is given by the consumption function

$$C^i_t = \mu\left(\frac{1}{1-\lambda}\frac{A^i_{t-1}I_{t-1}}{P_t} + H^i_t\right) \tag{8}$$

where $\mu = 1-\beta(1-\lambda)$ is the household's marginal propensity to consume and H^i_t is its human

capital

$$H_t^i = \sum_{k=0}^{\infty} \frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k}} (Y_{t+k}^i - T_{t+k}^i), \tag{9}$$

and its No-Ponzi-scheme constraint (6) holds with equality

$$\lim_{k\to\infty} \frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k}} \frac{A_{t+k}^i}{P_{t+k}} = 0. \tag{10}$$

Since condition (10) states that the household leaves no cash on the table, we refer to it as the No-Cash-on-the-Table condition.

1.3 Income Distribution

We assume that a household's income decreases exponentially with its age, as the amount of labor it supplies decreases. This creates a desire to save for one's old age—present in the baseline two-period OLG model—that makes negative real interest rates possible. The household's income is

$$Y_t^i(n) = \kappa(1-\zeta)^n \left(\frac{Y_t}{N_t}\right), \tag{11}$$

where Y_t is aggregate income, $\zeta \in [0,1]$, and κ is a constant determined by the fact that individual incomes must sum to aggregate incomes Y_t. Given the age distribution in the population (3), the constant is

$$\kappa = \frac{1+g-(1-\lambda)(1-\zeta)}{\lambda+g} \text{ if } \lambda+g>0. \tag{12}$$

The assumption that individual incomes decrease with age, $\zeta > 0$, is possible only in the non-Ricardian case $\lambda+g>0$. In the Ricardian case of the representative agent $\lambda+g=0$, all households necessarily get the same income, $Y_t^i = Y_t$. We can then only assume $\zeta = 0$ and $\kappa = 1$.

1.4 Fiscal Policy

We assume no government spending G. We do so to keep focus: We are interested in the effect of the financing of government expenditures through taxes for a fixed path for G which we set to zero to avoid burdening notations. We thus take fiscal policy to consist of an inherited debt level D_{-1}

and a path for the level of aggregate taxes $(T_t)_{t\geq 0}$. The flow budget constraint of the government is

$$\frac{D_t}{P_t} + T_t = \frac{D_{t-1}I_{t-1}}{P_t}, \tag{13}$$

and determines the path for public debt D_t from the government's fiscal policy.

Note that D_t denotes the amount the government borrows at t. It is not the amount it repays at $t+1$ inclusive of interest payments, i.e. the face value of the bonds it issues. Because the latter is the single state variable of the aggregate economy, it will prove useful when analyzing implementation in Section 5. We denote it through[14]

$$B_{t-1} = D_{t-1}I_{t-1}. \tag{14}$$

We assume that individual taxes are imposed proportionally on income, so that they follow the same age profile as income. A household of age n pays taxes

$$T_t^i(n) = \kappa(1-\zeta)^n \left(\frac{T_t}{N_t}\right). \tag{15}$$

1.5 Equilibrium and Stable Price Equilibrium

An equilibrium is defined in the standard way.

Definition 1. *For a given inherited level of public debt $D_{-1}I_{-1}$, distributed as an arbitrary $(D^i_{-1})_i I_{-1}$ in the population, and a given fiscal policy $(T_t)_{t\geq 0}$, an equilibrium consists of paths for the nominal interest rate $(I_t)_{t\geq 0}$, for the price level $(P_t)_{t\geq 0}$, for public debt $(D_t)_{t\geq 0}$, and for individual consumptions and debt holdings $(C^i_t, A^i_t)_{t\geq 0,i}$ s.t.*

1. *All households behave optimally: Given interest rates $(I_t)_{t\geq 0}$, prices $(P_t)_{t\geq 0}$, aggregate incomes $(Y_t)_{t\geq 0}$ and aggregate taxes $(T_t)_{t\geq 0}$—of which individual incomes and individual taxes depend according to (11) and (15)—each household is on its consumption function (8) and satisfies its No-Cash-on-the-Table condition (10).*

[14] Similarly, the single state variable of a household's program is its bond holdings $B^i_{t-1} = A^i_{t-1}I_{t-1}$ and it is possible to work with individual bond holdings instead of individual asset demands. Since introducing individual bond holdings is unnecessary however, we do not introduce notations for them.

2. The goods and debt markets clear:

$$\int_i C_t^i di = Y_t, \tag{16}$$

$$\int_i A_t^i di = D_t. \tag{17}$$

To derive fiscal requirements for price stability, we will be interested in stable price equilibria. A stable price equilibrium is an equilibrium with on-target inflation. For convenience and without loss of generality, we assume that the inflation target is equal to zero, so that keeping inflation on target means keeping the price level constant at a level P^*.

Definition 2. *A stable price equilibrium is an equilibrium where the price level is constant at* $P_t = P^*$ *at all* t.

2 Fiscal Requirements for Price Stability

This section derives the conditions on fiscal policy for a stable price equilibrium to exist: the fiscal requirements for price stability. We first rederive the NPV requirement in the case of Ricardian households then move to the main result of this section: the debt-limit requirement in the case of non-Ricardian households. By restricting our attention to stable price equilibria, we are effectively focusing on the real component of the model. This is by design: we precisely want to disentangle the question of whether a fiscal policy is consistent with price stability from the question of how to implement price stability. We turn to the question of implementation in Section 5, and leverage the full nominal component of the model there.

2.1 The Case of Ricardian Households

For further reference, we first rederive fiscal requirements for price stability in the traditional case where households are Ricardian (Leeper, 1991; Sims, 1994; Woodford, 1995, 2001).

Proposition 1. *Assume households are Ricardian* $\lambda + g = 0$. *Consider an initial level of debt* $D_{-1}I_{-1}$ *and a future tax path* $(T_t)_{t\geq 0}$. *There exists a stable price equilibrium for this fiscal policy*

if and only if

$$\frac{D_{t-1}I_{t-1}}{P^*} = \sum_{k=0}^{\infty} \frac{1}{\mathcal{R}_{t,t+k}} T_{t+k}, \tag{18}$$

where the real interest rate is given by $R_t = \frac{Y_{t+1}}{\beta Y_t}$.

The necessity of condition (18) follows from the intertemporal budget constraint of the representative household

$$\sum_{k=0}^{\infty} \frac{1}{\mathcal{R}_{t,t+k}} C_{t+k} = \frac{D_{t-1}I_{t-1}}{P_t} + \sum_{k=0}^{\infty} \frac{1}{\mathcal{R}_{t,t+k}} (Y_{t+k} - T_{t+k}), \tag{19}$$

which is obtained by combining its flow budget constraint (5) and No-Cash-on-the-Table constraint (10). If the latter did not hold, the household would be holding wealth it plans never to spend, which violates individual optimality. Adding market-clearing (16), it implies equation (18) when prices are stable $P_t = P^*$. The value of the real interest rate follows from the Euler equation of the representative household. Appendix B proves that condition (18) is also sufficient for a stable price equilibrium to exist.

Proposition 1 states that in an equilibrium, the real value of public debt must be equal to the net present value of future fiscal surpluses. Because the intertemporal budget constraint of the representative household and the intertemporal budget constraint of the government are mirror images of each other in the Ricardian case, the requirement (18) can be stated as a requirement on the intertemporal budget constraint of the government. Yet the necessity of condition (18) comes from the optimality condition of the household, not the government's. When the central bank stands ready to unconditionally buy government debt so that the government cannot default, nothing forces the government to satisfy any intertemporal budget constraint. But in an equilibrium, households cannot hold wealth that they plan never to spend.

Equation (18) can therefore be thought of as capturing the following intuition (e.g. Woodford, 1995, p.12-13). If the wealth households own in government bonds is not matched by future taxes to pay, households will have net wealth and spend it. The resulting inflation will dilute their real wealth on the left-hand-side of (18) until it equates the NPV of future surpluses.[15]

[15] Cochrane (2005) provides another interpretation of the NPV equation (18) as a valuation equation, without contradicting the wealth effect intuition.

2.2 The Case of Non-Ricardian Households

The following proposition states our main result on fiscal requirements for price stability.

Proposition 2. *Assume households are non-Ricardian $\lambda + g > 0$. Consider an initial level of debt $D_{-1}I_{-1}$ and a future tax path $(T_t)_{t\geq 0}$. There exists a stable price equilibrium for this fiscal policy if and only if the following two conditions hold.*

1. *For the path of public debt defined by the flow budget constraint of the government (13) and the real rate of interest $R_t^n(D_t)$ defined below in Lemma 5, the real debt-to-GDP ratio always remains below a threshold*

$$\frac{D_t}{P^* Y_t} < d^*, \tag{20}$$

where

$$d^* = \left(\frac{\mu}{1-\mu} \left(1 - \frac{(1-\lambda)(1-\zeta)}{(1+g)} \right) \right)^{-1}. \tag{21}$$

2. *At all times t, future taxes satisfy*

$$\lim_{k\to\infty} \sum_{j=1}^{k} \Omega(k,j) \frac{1}{\mathcal{R}_{t,t+j}^n} T_{t+j} = 0, \tag{22}$$

for coefficient $\Omega(k,j)$ given below in Lemma 3.

Proposition 2 states that when households are not Ricardian, fiscal requirements for price stability take the form of a limit (20) on the debt-to-GDP ratio. Technically there exists a second requirement in the form of condition (22), but it can be disregarded in practice. It only poses very weak constraints on fiscal policy—never raising taxes satisfies it. The next two subsections 2.2.1 and 2.2.2 derive the two fiscal requirements of Proposition 2, explaining where they arise from.

2.2.1 Fiscal Requirement from Households' Intertemporal Budget Constraints

We first follow the logic of the case of Ricardian households and consider what fiscal requirements arise from households' intertemporal budget constraints, or equivalently their No-Cash-on-the-Table

conditions (10). Because there is now an infinity of such constraints, we first need to determine the equilibrium debt holdings of an individual household. Appendix C shows the following lemma.[16]

Lemma 1. *Assume households are not Ricardian $\lambda + g > 0$.*

If all households are on their consumption function (8) and the debt market clears (17), then at time t a household of age $n \leq t$ has a debt holding

$$\frac{A_t^i(n)}{P_t} = \psi_D(n)\frac{D_t}{P_t N_t} - \sum_{k=0}^{n} \phi_T(n,k)\left(\frac{\mathcal{R}_{t-k,t}}{(1+g)^k}\frac{T_{t-k}}{N_{t-k}}\right), \tag{23}$$

where

$$\psi_D(n) = \kappa(1-\zeta)^n \mu \frac{1-x^{n+1}}{1-x}, \tag{24}$$

$$\phi_T(n,k) = \kappa(1-\zeta)^n x^k \left(1 - \mu\frac{1-x^{n-k+1}}{1-x}\right), \tag{25}$$

$$x = \frac{\beta(1+g)}{1-\zeta}. \tag{26}$$

The expression for the coefficient $\psi_D(n)$ on aggregate public debt shows that abstracting from the effect of the past path of taxes, the amount of public debt held by a household either monotonically increases with age to converge to a constant fraction of public debt (when $\zeta = 0$), or is a single-peaked function of age, peaking in middle-age to then gradually shrink to zero (when $\zeta > 0$).

We now determine a necessary and sufficient condition for the No-Cash-on-the-Table conditions of all households to be satisfied. Appendix D shows the following lemma.

Lemma 2. *Assume households are not Ricardian $\lambda + g > 0$.*

Assume all households consume according to the consumption function (8).

All the No-Cash-on-the-Table constraints (10) of all households are satisfied if and only if

$$\lim_{k\to\infty} \frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k}}\frac{\bar{A}_{t+k}}{P_{t+k}} = 0, \tag{27}$$

where $\bar{A}_{t+k}$ is the average debt holding at $t+k$ of households alive at t and still alive at $t+k$.

[16] Since we consider any arbitrary initial distribution of public debt ($A^i_{-1}I_{-1}$), expression (23) holds only for households born after period $t = 0$. For households born before $t = 0$, their debt holdings depend on their initial debt holdings $A^i_{t-1}I_{t-1}$. However, if the initial distribution of public debt has been determined in the same way before $t = 0$ as after $t = 0$, expression (23) is valid for all households of any age.

Intuitively, Lemma 2 states that the No-Cash-on-the-Table conditions of all households are satisfied if and only if the No-Cash-on-the-Table condition of a fictitious average household is satisfied, where the fictitious average household holds debt equal to the average debt holding among households that were already alive at t.

Lemmas 1 and 2 can be combined to determine what requirements households' intertemporal budget constraints impose on fiscal policy. Appendix E shows that they impose condition (22) in Proposition 2.

Lemma 3. *Assume households are not Ricardian* $\lambda + g > 0$.

Consider an inherited debt level $D_{-1}I_{-1}$ *and a future tax path* $(T_t)_{t\geq 0}$. *In an equilibrium, condition* (22) *must hold, where*

$$\Omega(k,j) = (\beta(1-\lambda))^k \theta(j) - \xi(k), \tag{28}$$

$$\xi(k) = \frac{(1-\lambda)^k}{1-x}\left(\mu\left(\frac{1-\zeta}{1+g}\right)^k - \left(\frac{(\lambda+g)\kappa\beta}{1-\zeta}\right)\beta^k\right), \tag{29}$$

$$\theta(0) = 0, \tag{30}$$

$$\forall j \geq 1, \theta(j) = (\beta(1-\lambda))^{-j}\left[\left(1-\frac{\mu}{1-x}\right)\left(1-\left(\frac{(1-\zeta)(1-\lambda)}{1+g}\right)^j\right) + \kappa\frac{\lambda+g}{1+g}\frac{x}{1-x}\left(1-\left(\beta(1-\lambda)\right)^j\right)\right]. \tag{31}$$

As mentioned above, condition (22) only puts a very weak requirement on fiscal policy, contrary to the NPV equation (18) in the case of Ricardian households. In particular, the current debt level D_{t-1} does not appear in condition (22). As a consequence, for any current level of public debt, the fiscal policy of never raising any surplus to repay public debt ($T_t = 0$ at all times) is consistent with all households' intertemporal budget constraints being satisfied. When households are not Ricardian, this does not mean individual households accumulate explosive amounts of debt they intend never to spend. Instead, they sell it to new generations. In addition, condition (22) does not feature inflation. Inflation does nothing to ease this constraint and restore the possibility of an equilibrium.

2.2.2 Fiscal Requirements from the IS Curve

When households are not Ricardian, households' intertemporal budget constraints no longer impose any significant constraint on fiscal policy for price stability. But other fiscal requirements arise,

specific to the case of non-Ricardian households. They arise from the IS curve of the model, derived in Appendix F.

Lemma 4. *In an equilibrium, the following dynamic IS curve holds*

$$Y_t = \frac{1-\zeta}{\beta(1+g)}\frac{1}{R_t}Y_{t+1} + \chi\frac{D_t}{P_t}, \tag{32}$$

where

$$\chi = \frac{\mu}{1-\mu}\left(1 - \frac{(1-\lambda)(1-\zeta)}{(1+g)}\right). \tag{33}$$

In the case of Ricardian households $\lambda + g = 0$ (and so $\zeta = 0$), the model reduces to the standard representative agent model and the dynamic IS curve reduces to the standard Euler equation where public debt does not appear, $\chi = 0$. When households are not Ricardian $\lambda + g > 0$, higher aggregate wealth increases aggregate consumption $\chi > 0$, and therefore puts inflationary pressure on the economy.

Note that this captures the same intuition on the wealth effect of government debt as in the popular interpretation of the NPV requirement (18) (e.g. Woodford, 1995, p.12-13). If anything, this mechanism is more explicit in the case of Non-Ricardian households, since public debt now explicitly increases aggregate demand through the IS equation (32).

Yet, the presence of aggregate wealth in the IS curve (32) does not in itself mean that the central bank loses control over inflation. Many things can affect the level of aggregate demand, for instance exogenous shocks to households' preferences. The claim that monetary policy remains ultimately in control of the price level does not deny that other factors than monetary policy can affect inflation. It only argues that the central bank can use interest rates to counterbalance these shocks and deliver on-target inflation.

The same argument applies in principle to public debt: high public debt increases aggregate demand, but if it creates too much demand, the central bank can increase rates to bring it down. At higher interest rates, households are willing to hold more public debt without spending their extra wealth, making aggregate demand in line with aggregate supply. Specifically, if the central

bank delivers the real interest rate

$$R_t^n(D_t) = \frac{1-\zeta}{\beta(1+g)} \frac{Y_{t+1}}{Y_t} \left(1 - \chi \frac{D_t}{P^* Y_t}\right)^{-1}, \tag{34}$$

in all periods, a stable price equilibrium obtains. The real interest rate (34) is the natural rate of interest—the real interest rate consistent with price stability—and delivering it delivers price stability.

But does there always exist an interest rate level that allows the central bank to bring down demand in line with supply? The following lemma shows there does not when the debt-to-GDP ratio is too high, delivering the fiscal requirement (20).

Lemma 5. *Assume households are not Ricardian* $\lambda + g > 0$.

Consider an inherited debt level $D_{-1}I_{-1}$ *and a future tax path* $(T_t)_{t\geq 0}$. *In a stable price equilibrium, the path of public debt defined by the flow budget constraint of the government* (13) *and the real interest rate* (34) *must be such that the real-debt-to-GDP ratio* $\frac{D_t}{P^* Y_t}$ *always remains below the threshold*

$$d^* = \frac{1}{\chi}, \tag{35}$$

resulting in condition (20).

The debt-to-GDP limit follows from the fact that when the debt-to-GDP ratio is above the limit d^*, no interest rate, however large, can make the IS equation (32) hold. Even an infinitely large real interest rate is not enough to counter the wealth effect of public debt on aggregate consumption and bring aggregate demand down to supply. The natural rate (34) increases with the debt-to-GDP ratio, and becomes infinitely large when the debt-to-GDP ratio approaches d^*.[17]

Lemmas 3 and 5 together show that conditions (20) and (22) are necessary in a stable price equilibrium. Appendix G shows that they are also sufficient, proving the characterization of the stable price equilibrium in Proposition 2.

[17] The limit d^* bears on the amount D_t the government borrows at t. Appendix H shows that the amount $B_t = D_t I_t$ the government repays can be arbitrarily large, but only because the government can face arbitrarily large interest rates—little solace for a government that needs to borrow D_t to close a budget.

2.3 Similar Characterization in a Standard Two-Generation OLG Model

Before discussing the new fiscal requirement of the debt-to-GDP ratio, we show that a similar characterization applies in a standard two-generation OLG model à la Samuelson (1958). Households live for two periods. In period t, households are thus divided between young households born at t and old households born at $t-1$. The population N_t grows at rate g, and is therefore divided between $N_t^{old} = \frac{1}{2+g} N_t$ households born at $t-1$ and $N_t^{young} = \frac{1+g}{2+g} N_t$ households born at t.

A household born at t has preferences over its consumption in periods t and $t+1$

$$\log(C_t^y) + \beta \log(C_{t+1}^o). \tag{36}$$

The household is born with no wealth and maximizes its utility (36) subject to the flow budget constraints

$$C_t^y + \frac{A_t^y}{P_t} = Y_t^y - T_t^y, \tag{37}$$

$$C_{t+1}^o = \frac{A_t^y I_t}{P_t} + Y_{t+1}^o - T_{t+1}^o. \tag{38}$$

We assume that young households receive collectively a share γ of the economy's total income while old households collectively receive the remaining share $1-\gamma$, $Y_t^y = \gamma \frac{Y_t}{N_t^y}$ and $Y_t^o = (1-\gamma)\frac{Y_t}{N_t^o}$. The government still sets aggregate taxes subject to the flow budget constraint (13) and taxes are still imposed proportionally to income, $T_t^y = \gamma \frac{T_t}{N_t^y}$ and $T_t^o = (1-\gamma)\frac{T_t}{N_t^o}$. An equilibrium and a stable-price equilibrium are still defined as in Definitions 1 and 2, where goods-market clearing now takes the form

$$N_t^y C_t^y + N_t^o C_t^o = Y_t. \tag{39}$$

When households live for two periods, their intertemporal budget constraints necessarily hold once they behave optimally. As a consequence, no fiscal requirement arises from intertemporal budget constraints. The following proposition shows however that a real-debt-to-GDP limit similar to (20) characterizes the fiscal policies consistent with price stability (see Appendix I for a proof).

Proposition 3. *For a given initial level of debt $D_{-1}I_{-1}$, consider a future tax path $(T_t)_{t\geq 0}$.*

There exists a stable price equilibrium for this fiscal policy if and only if the path of public debt

defined by the flow budget constraint of the government (13) *and the real interest rate*

$$R_t^n(D_t) = \frac{\frac{1-\gamma}{\gamma\beta}(Y_{t+1} - T_{t+1})}{(Y_t - T_t) - \frac{1+\beta}{\gamma\beta}\frac{D_t}{P^*}} \tag{40}$$

is such that the real-debt-to-GDP ratio always remains below the threshold

$$\frac{D_t}{P^* Y_t} \leq \frac{\gamma\beta}{1+\beta}\left(1 - \frac{T_t}{Y_t}\right). \tag{41}$$

Requirements for price stability still take the form of a limit on the debt-to-GDP ratio. The one difference—beyond the numerical value of the threshold—is that the debt-to-GDP limit now depends on the level of taxes raised in the current period T_t. Lower taxes at t increase the debt-to-GDP limit because in the OLG model the level of debt D_t has a weaker effect on aggregate spending when debt comes from lower taxes $-T_t$ at t than when it comes from legacy debt $D_{t-1}I_{t-1}$. In turn, this is because legacy debt $D_{t-1}I_{t-1}$ is held by old households who have an MPC of one and spend it all while lower taxes $-T_t$ increase aggregate spending only through the average MPC $\gamma \times \frac{\beta}{1+\beta} + (1-\gamma) \times 1$ which is less than one. The distinction is absent in the perpetual youth model where all households have the same MPC μ.

2.4 Are Fiscal Requirements More or Less Stringent with Non-Ricardian Households?

The debt-to-GDP limit (20) rules out some fiscal policies as inconsistent with price stability. Does it rule out more or fewer policies than in the case of Ricardian households? Are fiscal requirements looser or tighter when households are non-Ricardian? The answer is: neither. Some policies that were inconsistent with price stability under Ricardian households are now consistent, but others that were consistent with price stability no longer are.

On the one hand, non-Ricardian households make new fiscal policies consistent with price stability because they make it possible for the real interest rate r to be below the growth rate of the economy g. If a fiscal policy is such that it makes r remain below g, it can fail to satisfy the NPV equation. Yet the debt-to-GDP ratio will steadily decrease over time, so that it will never cross the debt-to-GDP limit. It is therefore consistent with price stability.

An example of particular interest is the policy of never raising any fiscal surplus in the future,

$T_{t+k} = 0$ at all k. Is this consistent with price stability? The answer is: only when the current debt-to-GDP ratio is below a certain threshold $\underline{d}$. The threshold is *not* the threshold d^* defined in equation (20). Indeed, to be compatible with price stability, it must be that not raising fiscal surpluses keeps debt-to-GDP below d^* not just today but in all future periods. The threshold $\underline{d}$ is instead the debt-to-GDP level at which the real interest rate becomes greater than the growth rate of the economy.[18] Indeed, absent fiscal surpluses the debt-to-GDP-ratio $d_t = D_t/(P^*Y_t)$ follows the flow budget constraint of the government

$$d_t = \frac{R_{t-1}}{1+g} d_{t-1}. \tag{42}$$

If the current debt-to-GDP ratio d_{t-1} is already greater than $\underline{d}$ today, subsequent increases in d will only make the interest rate (34) that must prevail under price stability even greater than g, making d grow until it eventually reaches the debt-to-GDP limit d^*. Conversely, if d_{t-1} is below $\underline{d}$ today, d will shrink back by itself, never crossing the debt-to-GDP limit d^*. This is illustrated on Figure 1.

The threshold $\underline{d}$ can easily be obtained analytically. It is given by

$$\underline{d} = \left(1 - \frac{1-\zeta}{\beta(1+g)}\right) d^*. \tag{43}$$

Note that $\underline{d}$ is positive if and only if

$$\frac{1-\zeta}{\beta} < 1+g, \tag{44}$$

that is, if population growth is not too large, and/or if households' income does not shrink too much with their age. When this condition is not satisfied, r is greater than g at all debt levels and not raising fiscal surpluses is never consistent with price stability.

On the other hand, non-Ricardian households also make some policies inconsistent with price stability even though they are when households are Ricardian. When households are Ricardian,

[18] Mian, Straub, and Sufi (2022) argue through a model with bonds-in-the-utility that the condition for a "free lunch" is not $r < g$, but the tighter condition $r < g - \phi$ where ϕ is the semi-elasticity of the real interest rate to debt. For the question of whether not raising fiscal surpluses is consistent with price stability, the standard $r < g$ condition is the relevant one. This is because the question of the consistency with price stability considers a different thought experiment than in Mian, Straub, and Sufi (2022): under what condition the government can run *zero* fiscal surpluses indefinitely, $T_{t+k} = 0$ at all k. Mian, Straub, and Sufi (2022) consider instead under which condition the government can go from a permanent positive deficit to an even higher permanent positive deficit.

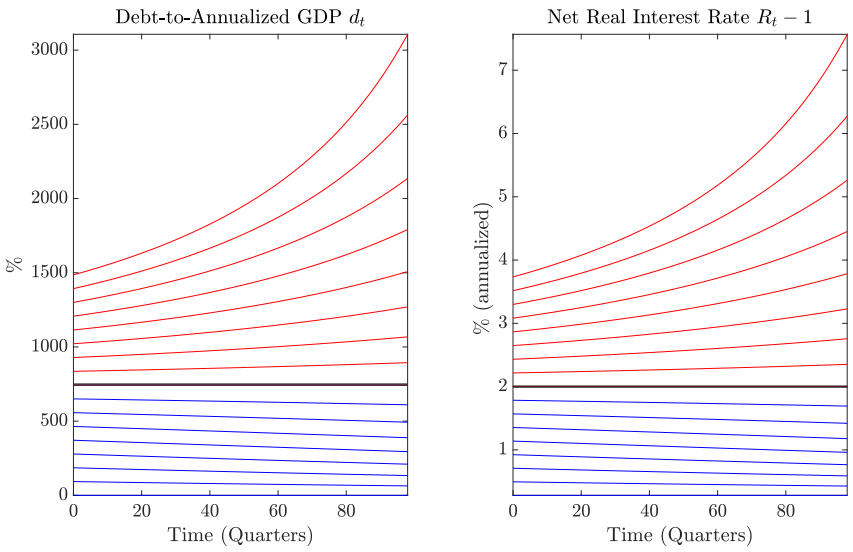


Figure 1: Debt Paths

Note: The left (right) panel plots the dynamics of the debt-to-annual-GDP ratio (the net real interest rate) over time when the government never sets positive fiscal surpluses, for different initial levels of the debt-to-annual-GDP ratio. For an initial debt-to-GDP ratio below $\underline{d}$, the debt-to-GDP ratio gradually shrinks back to zero (blue curves). For an initial debt-to-GDP ratio above $\underline{d}$, the debt-to-GDP ratio gradually increases and eventually diverges to infinity. The calibration is given in the first column of Table 1, OLG.

only high public debt unbacked by future fiscal surpluses threatens price stability. In contrast, the debt-to-GDP limit (20) implies that the absolute level of public debt is a constraint on the existence of a stable price equilibrium. Obviously, future surpluses still matter to assess the existence of a stable price equilibrium, since future surpluses influence the future level of the debt-to-GDP ratio. But a plan to raise taxes in the future is not enough to guarantee the existence of a stable price equilibrium if future taxes are planned to be collected so far into the future that the debt-to-GDP ratio increases above the debt limit before taxes are collected.

As an example of a policy that becomes inconsistent with price stability with non-Ricardian households, consider the following one defined by a surplus rule. Starting from a positive debt position $D_{t-1} > 0$, the government raises in each period a surplus equal to ψ_b of its real legacy debt, $T_{t+k} = \psi_b D_{t+k-1} I_{t+k-1}/P_{t+k}$, with $0 < \psi_b < 1 - \beta$. It implies that the surplus raised at $t+k$ is

$$T_{t+k} = \psi_b (1-\psi_b)^k \mathcal{R}_{t,t+k} \frac{D_{t-1} I_{t-1}}{P_t}. \tag{45}$$

When households are Ricardian, equation (45) implies that the NPV equation (18) is satisfied

so that there exists a stable price equilibrium. This is despite the fact that the real-debt-to-GDP ratio is given by

$$\frac{D_{t+k}}{Y_{t+k}P_{t+k}} = \left(\frac{1}{\beta}(1-\psi_b)\right)^k \frac{D_{t-1}I_{t-1}}{P_tY_t}, \tag{46}$$

so that it diverges to infinity over time. This does not threaten price stability when households are Ricardian: real debt diverges to infinity but so do taxes so households do not sit on net wealth they do not intend to spend.

When households are not Ricardian however, this policy is easily inconsistent with price stability. For instance if $\zeta = g = 0$, we necessarily have

$$\frac{D_{t+k}}{Y_{t+k}P_{t+k}} \geq \left(\frac{1}{\beta}(1-\psi_b)\right)^k \frac{D_{t-1}I_{t-1}}{P_tY_t} \tag{47}$$

so that the debt-to-GDP ratio necessarily diverges to infinity over time, as in the Ricardian case. But since this means it must ultimately cross the threshold d^*, this policy is inconsistent with price stability when households are not Ricardian.[19]

2.5 What Determines the Debt-to-GDP Limit d^*?

An important benefit of the debt-to-GDP limit is to exclude as incompatible with price stability many extreme policies that seem intuitively hard to square with price stability. Never raising positive surpluses when $r > g$ is one example. To exclude such extreme policies, the precise value of d^* is not essential. Yet to determine how much restriction the debt limit puts on fiscal policy, it is important to determine how high d^* is, and what it depends on.

The key determinant of d^* is households' marginal propensity to consume (MPC). The higher households' MPC is, the more of their wealth they want to spend. And the more of their wealth they want to spend, the quicker we reach the point where even infinitely high interest rates cannot prevent them from spending more than the natural level of output.

In turn, the key determinant of households' MPC $\mu = 1 - \beta(1-\lambda)$ is the death probability λ. Under a literal interpretation of the model as an OLG model, λ is the probability of biological

[19] Note that when households are not Ricardian, equation (45) still implies that the NPV equation would hold in equilibrium, were an equilibrium to exist under this policy. The issue is that no stable price equilibrium exists, something that looking at the NPV equation cannot reveal. This highlights the limits of using the NPV equation to characterize fiscal policies consistent with price stability, even when $r > g$.

death and cannot be set very high. As a result, households' MPC must be small. The first column of Table 1 considers such a calibration. We set $\lambda = 0.005$ on a quarterly frequency, i.e. a 2% probability of dying every year. We set $\beta = 0.995$ and $g = 0.005$, i.e. an annual growth rate of 2%. We set the last parameter, the income-shrinking rate ζ, in order to match a steady-state real interest rate of $r_0 = 0.5\%$ at a debt-to-GDP level of 100%. This is the value of the natural rate in the US in 2016 (when debt-to-GDP was 100%) as estimated by Holston, Laubach, and Williams (2017). This gives $\zeta = 0.0043$.

Under this OLG calibration, households' MPC is very small, at 0.01 quarterly. As a result, the debt-to-GDP limit d^* is extremely high, at 1743 times annual GDP. The value of the threshold $\underline{d}$ is also very high, at $\underline{d} = 7.5$, or 750% of GDP.

Yet an MPC of 0.01 is much smaller than the average MPC found in the data. A higher MPC can be obtained through a higher calibration of λ, which can be rationalized if the model is reinterpreted as a proxy for a HANK model (e.g. Del Negro, Giannoni, and Patterson, 2023; Farhi and Werning, 2019). Under this alternative interpretation, the death probability is to be interpreted as the probability for households to hit their borrowing constraints. This allows the model to reproduce any value for the MPC.

As λ—and therefore households' MPC—increases, the debt-to-GDP limit d^* decreases considerably. The plain line of the left panel of Figure 2 shows how d^* decreases as λ increases from 0.005 to 0.196, the value that generates a quarterly MPC of 0.2, consistent with estimates in Fagereng, Holm, and Natvik (2021), keeping all other parameters constant. The debt-to-GDP limit decreases from 1743 to just 4.9 times GDP (Column LARGE-λ of Table 1).

Matching the average MPC in the data overlooks an important distinction however. What matters for the debt-to-GDP limit is the MPC of the households *that hold public debt*. In the perpetual-youth model all households have the same MPC so the distinction is irrelevant. But the importance of the distinction becomes clear once we allow for hand-to-mouth households who have a high MPC but do not hold public debt. We extend the model in this direction in the next section.

3 Extensions

In this section, we consider extensions of the model. In particular, we add hand-to-mouth households and calibrate the model to match the sensitivity of interest rates to the level of public debt.

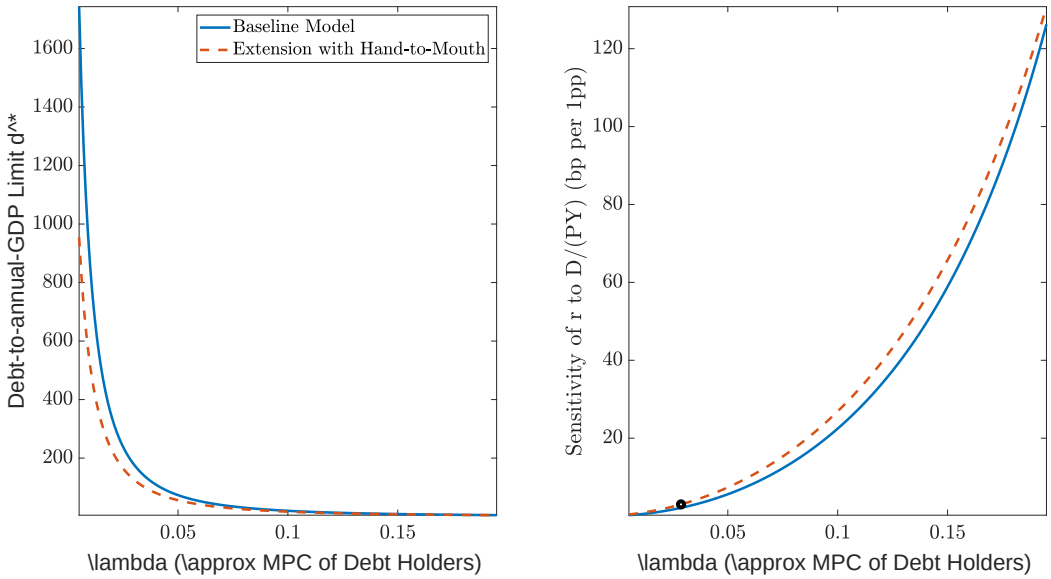


Figure 2: Dependence of the debt-to-GDP Limit on the MPC of Debt Holders

Note: The left panel plots the debt-to-GDP limit d^* (expressed in terms of annual GDP) as a function of the death probability λ, which is about equal to the MPC of non hand-to-mouth households $\mu = 1-\beta(1-\lambda)$. The right panel plots the sensitivity of the long-run real interest rate to the debt-to-GDP ratio as a function of λ. The black circle on the right panel is its empirical value of 3bp for a 1pp increase in debt-to-GDP. In both panels, the relationship is plotted for two versions of the model. The plain lines are for the baseline model of Section 1. It is calibrated with $\beta = 0.995$, $g = 0.005$ and $\zeta = 0.0043$. The dashed lines are for the model's extension with hand-to-mouth households in Section 3.2. It is calibrated with $\beta = 0.995$, $g = 0.005$, $\zeta = 0.011$ and a share of hand-to-mouth households α_Y that is reset as λ varies so that the average MPC in the economy remains constant at 0.2.

3.1 Standard TANK

Before extending the perpetual youth model to allow for hand-to-mouth households, we first highlight the importance of considering hand-to-mouth households by deriving fiscal requirements for price stability in a standard Two-Agent New-Keynesian model (TANK).[20] In such TANK models, the only non-Ricardian households are hand-to-mouth households; others remain Ricardian. As we now show, this makes an important difference to fiscal requirements for price stability.

Assume that a fraction α_Y of aggregate income Y_t goes to hand-to-mouth households that consume their income every period $C_t^i = Y_t^i$. Assume that the remaining households are Ricardian permanent-income households ($\lambda + g = 0$). We assume that hand-to-mouth households collectively pay $\alpha_T \times T_t$ in taxes, with α_T possibly distinct from α_Y. Appendix J shows the following generalization of Proposition 1.

[20] Bhattarai, Lee, and Yang (2023) use a TANK model under fiscal dominance to study fiscal transfers during the Covid crisis.

Table 1: Calibration and Debt Limit

β	0.995			
g	0.005			
	OLG	LARGE-λ	OLG-HTM	MAIN
ζ	0.0043	0.0043	0.0043	0.011
λ	0.005	0.196	0.005	0.029
α_Y	0	0	0.19	0.17
Average MPC	0.01	0.20	0.20	0.20
Sensitivity of r to $B/(PY)$	0.23	128	0.28	3
d^*	1743.1	4.9	1408.5	134.5
$\underline{d}$	7.5	0.02	6.1	1.5

Note: The table gives the average MPC, sensitivity of interest rates to debt, and debt thresholds d^* and $\underline{d}$ in four calibrations of the model: the OLG calibration and LARGE-λ calibration presented in Section 2.5, the OLG+HTM calibration presented in Section 3.2, and the main calibration presented in Section 3.3. The average MPC is the quarterly average MPC. The sensitivity of r to $D/(PY)$ is the derivative of the annualized real interest rate with respect to the debt-to-annual-GDP ratio in a steady-state of the model, expressed in basis points for percentage point increase in the debt-to-annual-GDP ratio. The debt thresholds d^* and $\underline{d}$ are expressed as debt to annual GDP.

Proposition 4. *Consider a standard TANK model where non hand-to-mouth households are Ricardian $\lambda + g = 0$. Consider an initial level of debt B_{-1} and a future tax path $(T_t)_{t\geq 0}$.*

There exists a stable price equilibrium for this fiscal policy if and only if the NPV equation (18) *holds for the interest rate path* $R_t = (Y_{t+1} + \alpha_T/(1-\alpha_Y)T_{t+1})/(\beta(Y_t + \alpha_T/(1-\alpha_Y)T_t))$.

Proposition 4 states that in a standard TANK model the NPV requirement still characterizes the fiscal policies that are consistent with price stability. The only difference is that the equilibrium real interest rate takes a different value than in the absence of hand-to-mouth households.

A TANK model breaks the Ricardian equivalence yet still results in the NPV requirement (18). The result that fiscal requirements for price stability take the form of a limit on the debt-to-GDP ratio when households are not Ricardian therefore needs a more precise statement. What is required for fiscal requirements to take the form of the debt-to-GDP limit is that the households *that hold public debt* are not Ricardian. In a TANK model, the hand-to-mouth households are not Ricardian but they do not hold public debt, so their presence does not change the result that fiscal requirements take the form of the NPV equation.

3.2 Extension with Hand-to-Mouth Households

We now extend the perpetual youth model to allow for a share of hand-to-mouth households, as in Wolf (2025). Assume once again that a fraction α_Y of aggregate income Y_t goes to hand-to-mouth households that consume their income every period $C_t^i = Y_t^i$, and that hand-to-mouth households collectively pay $\alpha_T \times T_t$ in taxes.[21] We continue to assume that among non hand-to-mouth households, taxes are imposed proportional to income, as per (15), where T_t is to be replaced by $(1-\alpha_T)T_t$. Appendix K shows the following generalization of Proposition 2.

Corollary 1. *Consider the model with a share of hand-to-mouth households. Assume the non hand-to-mouth households are not Ricardian $\lambda + g > 0$.*

Consider an initial level of debt $D_{-1}I_{-1}$ and a future tax path $(T_t)_{t\geq 0}$. There exists a stable price equilibrium for this fiscal policy if and only if the two conditions of Proposition 2 are satisfied, but with the amended expression for the real debt-to-GDP threshold

$$\frac{D_t}{P^*Y_t} < d^* \left(1 + \frac{\alpha_T}{1-\alpha_Y}\frac{T_t}{Y_t}\right), \tag{48}$$

$$d^* = \frac{1-\alpha_Y}{\chi}. \tag{49}$$

and the amended expression for the natural rate of interest

$$R_t^n(D_t) = \frac{1-\zeta}{\beta(1+g)} \frac{(1-\alpha_Y)Y_{t+1} + \alpha_T T_{t+1}}{(1-\alpha_Y)Y_t + \alpha_T T_t} \left(1 - \chi \frac{D_t}{P^*Y_t}\right)^{-1}. \tag{50}$$

Corollary 1 states that the requirements of Proposition 2 are robust to the addition of hand-to-mouth households. The debt-to-GDP limit now takes a different value and is slightly affected by the level of taxes at t if hand-to-mouth households pay taxes ($\alpha_T \neq 0$). The higher the taxes hand-to-mouth have to pay, the less they consume and the more output is left for non hand-to-mouth households to consume: non hand-to-mouth households can then hold more public debt before they want to spend more than what is available for them to consume. But fiscal requirements still take the form of a limit on the debt-to-GDP ratio.[22]

[21] As far as the aggregate economy is concerned, how the incomes of hand-to-mouth households are distributed among themselves, e.g. according to age, does not matter.

[22] Note that higher taxes increase the debt limit, while they decrease it in the OLG model of Section 2.3. This is because in the OLG model, households with an MPC of one are old households that hold the legacy debt, while in the perpetual youth model with hand-to-mouth households, households with an MPC of one are hand-to-mouth households that hold no debt.

In addition, for a given calibration, hand-to-mouth households make little change to the quantitative value of the debt-to-GDP limit. Hand-to-mouth households decrease it, but not by much. The third column OLG-HTM of Table 1 reports the debt-to-GDP limit d^* when the model is calibrated as in the OLG calibration of Section 2.5, but with enough hand-to-mouth households (19%) to match an average MPC of 0.20. The debt-to-GDP limit is only slightly lower, to 1409 times annual GDP. The value of $\underline{d}$ is similarly only slightly lower, to 6.1 times annual GDP.

Hand-to-mouth households do not affect d^* much because fiscal requirements for price stability primarily depend on the behavior of the households that hold public debt, and hand-to-mouth households hold none. Hand-to-mouth households do lower the value of the threshold (49) on public debt, by a factor $(1 - \alpha_Y)$. But this is *not* because hand-to-mouth households increase the average MPC. Because hand-to-mouth households do not own wealth, their MPC does not directly matter for fiscal requirements. They affect the debt-to-GDP limit only indirectly. Because they consume a share α_Y of income, the consumption of the non hand-to-mouth households must now be lower. This puts a lower threshold on the maximum level of public debt they can hold in a stable price equilibrium.

Yet, although hand-to-mouth households do not change the debt-to-GDP limit much for a given calibration, they matter because they disentangle the MPC of debt-holders from the average MPC. This makes it possible to match the average MPC in the data for any calibration of λ. This opens the door to alternative approaches to calibrating λ, as we now turn to.

3.3 Matching the Sensitivity of Interest Rates to Public Debt

To calibrate λ in the extended model, we add an additional target beyond the average MPC: the sensitivity of interest rates to the level of public debt. This builds on the calibration strategy Mian, Straub, and Sufi (2022) use to calibrate a bonds-in-the-utility model.[23] For the US, estimates of this sensitivity are found to be around 3 basis points for every percentage point increase in the debt-to-annual-GDP ratio. Laubach (2009) reports a value of 3 to 4bp, Engen and Hubbard (2004) of 3bp, Gamber and Seliski (2019) of 2 to 3bp, Neveu and Schafer (2024) of 2bp, and Plante, Richter, and Zubairy (2025) of 3bp.[24]

[23] Mian, Straub, and Sufi (2022) consider estimates for both the sensitivity of interest rates and the sensitivity of the convenience yield on public debt. There is no convenience yield in our perpetual-youth model so we focus on the effect on interest rates.

[24] Gamber and Seliski (2019) and Neveu and Schafer (2024) use the method of Laubach (2009) on a more recent sample and find a smaller estimate closer to 2bp than 3bp. Plante, Richter, and Zubairy (2025) show that this is

Both the OLG calibration of Section 2.5 and the OLG-HTM calibration of Section 3.2 give much smaller values. Relying on steady-state relationships, the sensitivity of the interest rate to a one percentage point increase in debt-to-GDP is 0.23bp in the former and 0.28bp in the latter, an order of magnitude smaller than in the data.[25] This matters for the value of the debt-to-GDP limit: A calibration that understates how much the natural rate increases with the level of public debt will overstate the maximum amount of public debt consistent with a finite natural rate.

In our main calibration—calibration MAIN in Table 1—we target both the average MPC in the data and the sensitivity of interest rates to the debt-to-GDP ratio. Like Mian, Straub, and Sufi (2022), we consider the steady-state of our model and assume that taxes/transfers to hand-to-mouth are nil, $\alpha_T = 0$. We still set $\beta = 0.995$ and $g = 0.005$ and calibrate the three remaining parameters ζ, α_Y and λ to match three statistics: a real interest rate at 100% debt-to-annual-GDP of $r_0 = 0.5\%$ in steady-state (as in Section 3.2); an average marginal propensity to consume of 0.2 (as in Section 3.2); and a derivative of the real interest rate to the debt-to-annual-GDP ratio in steady-state of 3 bp.

The resulting value for the quarterly death probability is $\lambda = 0.029$, i.e. an MPC of households that hold public debt of $\mu = 0.034$. This is higher than in the OLG and OLG-HTM calibrations of the model, which produce too low a sensitivity of interest rates to debt. Yet this is smaller than the value $\lambda = 0.196$ of the LARGE-λ calibration that matches an average MPC of 0.2 without hand-to-mouth households. Such a high value of λ *overstates* the sensitivity of interest rates to debt, at 128bp for a 1pp increase in debt-to-annual-GDP. This is more than an order of magnitude larger than in the data. That the interest-rate sensitivity is too large when λ is calibrated to a high value is further shown on the right panel of Figure 2. It plots the interest-rate sensitivity to debt as a function of λ keeping all other parameters constant at their value in our main calibration—except on the dashed line corresponding to the extension with hand-to-mouth households where α_Y is reset to still match an average MPC of 0.2 for all values of λ. If the households that hold public debt have a high MPC, the real interest rate becomes extremely sensitive to the level of debt.[26]

due to non-stationarity issues in the recent period and find 3bp when estimating the relationship in first differences instead of levels.

[25] We use the steady-state relationship between r and $B/(PY)$ which is linear, so that the derivative does not depend on the level of debt—see Appendix L for a derivation.

[26] An alternative calibration strategy of the perpetual youth model with hand-to-mouth households is to set λ and α_Y to match the whole profile of intertemporal MPC (iMPC) (Auclert, Rognlie, and Straub, 2018), as in Wolf (2025) and Angeletos, Lian, and Wolf (2024a). Doing so, Wolf (2025) and Angeletos, Lian, and Wolf (2024a) find a value of $\lambda = 0.135$. Such a high value, while less than $\lambda = 0.196$, still produces a sensitivity of the interest rate to debt that is an order of magnitude larger than in the data, as apparent on the right panel of Figure 2.

In our main calibration, the debt-to-GDP limit d^* is 135 times GDP. It is an order of magnitude smaller than in the OLG calibrations of the model, but much larger than when calibrating debt holders to have an MPC of 0.2.[27] Under our main calibration, even though the debt-to-GDP limit excludes many fiscal policies as inconsistent with price stability, very high levels of public debt remain consistent with price stability provided the central bank increases interest rates as needed.

3.4 Blanchard-Yaari Bewley Model

A value of $\lambda = 0.029$ as in our main calibration is better justified through the HANK interpretation of the model. Yet a possible caveat in the HANK interpretation of the model is that it takes the probability of hitting one's borrowing constraint as exogenous, when in a full-fledged HANK model it is endogenous and depends on the level of public debt. Appendix M highlights this point by considering an alternating-endowment model, a particular case of a HANK model which is tractable enough to be solved analytically (Townsend, 1980; Woodford, 1990). As Woodford (1990) shows, the alternating-endowment model is exactly equivalent to the two-generation OLG model of Section 2.3 when the borrowing constraint of low-endowment households binds. This buttresses the case for using the perpetual-youth model in its HANK interpretation. But when public debt is high enough, borrowing constraints never bind even for low-endowment households, as there is enough liquidity in the economy for everyone to self-insure. The model is then equivalent to one with a representative Ricardian household. The real interest rate is constant at $1/\beta$ and no longer increases with public debt. The property that the equilibrium interest rate asymptotes to $1/\beta$ at high levels of public debt is a general property of HANK models (e.g. Chamberlain and Wilson, 2000). If higher and higher levels of public debt do not require higher and higher interest rates, there does not exist a level of public debt beyond which even infinitely high interest rates cannot convince households to hold debt.

Yet in practice households face both borrowing constraints and death. Appendix N considers a model that combines both OLG and HANK departures from the representative agent: a Blanchard-Yaari Bewley model. As in a standard Bewley (1977) model, households face idiosyncratic income risk and a borrowing constraint, and can only borrow and save through non-contingent debt. In

[27] The value of $\underline{d}$ is 150% of GDP, close to the value of 160% that Bayer, Born, and Lütticke (2023) calculate in a HANK model with illiquid capital. It is a bit lower than the 218% of GDP that Mian, Straub, and Sufi (2022) find for their bonds-in-the-utility model. The difference is partly due to the higher r_0 we target: +0.5% based on the estimate of r^* in Holston, Laubach, and Williams (2017) instead of -0.5% in Mian, Straub, and Sufi (2022) based on the value of the real interest rate in 2019. When we target $r_0 = -0.5\%$ as they do, we find a value of 183%.

the Blanchard-Yaari Bewley model, when the debt-to-GDP ratio increases the real interest rate does not asymptote to a finite value. Instead it approaches the behavior of the real interest rate in the Blanchard-Yaari model with overlapping generations alone, since households then have enough liquid wealth to self-insure. The debt-to-GDP limit d^* then applies.

4 Non-Unitary Intertemporal Elasticity of Substitution

The canonical model of perpetual youth assumes logarithmic preferences (4), i.e. an intertemporal elasticity of substitution of one. This makes households' MPC independent of the interest rate and brings many analytical simplifications. In this section, we consider how fiscal requirements for price stability change when the IES is different from one.

4.1 Extension to Non-Log Utility

We consider the more general CRRA preferences

$$\sum_{k=0}^{\infty}(\beta(1-\lambda))^k\frac{(C_{t+k}^i)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}, \tag{51}$$

where σ is the intertemporal elasticity of substitution (IES). The case of log utility (4) is the particular case when the IES σ is equal to one.

As shown in Appendix O, an individual household still consumes according to the consumption function (8), but with an MPC that is no longer constant. Instead, its MPC is a function of the entire path of future interest rates

$$\mu_t=\left(\sum_{k=0}^{\infty}\left(\beta^\sigma(1-\lambda)\right)^k\mathcal{R}_{t,t+k}^{\sigma-1}\right)^{-1}. \tag{52}$$

The IS curve (32) generalizes to

$$Y_t=(\beta R_t)^{-\sigma}\left(\frac{1-\zeta}{1+g}\right)Y_{t+1}+\chi_t\frac{D_t}{P_t}. \tag{53}$$

with a coefficient on public debt

$$\chi_t=\frac{\mu_t}{1-\mu_t}\left(1-\frac{(1-\lambda)(1-\zeta)}{1+g}\right). \tag{54}$$

The IS curve (53) differs from the one under log utility in two ways. First, the IES in the first term in Y_{t+1} is now σ instead of simply one. Second, the coefficient χ_t on public debt is now a function of the future path of interest rates because households' MPC μ_t is.

Because χ_t now depends on the entire future sequence of interest rates, a full dynamic characterization of the fiscal policies consistent with price stability becomes much less tractable. Nevertheless, a simple characterization can still be obtained for time-invariant fiscal policies in steady-state. Assume that GDP per capita is constant and that the government maintains a constant real debt-to-GDP ratio $d = D/(PY)$.[28] Equation (53) can then be rewritten as

$$d(R) = \frac{1}{\chi(R)}\Big(1 - (\beta R)^{-\sigma}(1-\zeta)\Big), \tag{55}$$

where

$$\chi(R) = \frac{\mu(R)}{1-\mu(R)}\left(1 - \frac{(1-\lambda)(1-\zeta)}{1+g}\right), \tag{56}$$

$$\mu(R) = 1 - \beta^{\sigma}(1-\lambda)R^{\sigma-1}. \tag{57}$$

Equation (55) can be read either as giving households' asset demand for a given R, or as giving the natural rate of interest for a given debt-to-GDP ratio d. We now show that whether it gives rise to a limit on debt-to-GDP ratios consistent with price stability crucially depends on whether the IES is smaller or greater than one. We consider both cases in turn.

4.2 Intertemporal Elasticity of Substitution $\sigma < 1$

When the IES is less than one, a positive MPC $\mu(R) > 0$ requires $R > R_0 = \big(\beta^{\sigma}(1-\lambda)\big)^{\frac{1}{1-\sigma}}$ and the MPC is increasing in the real interest rate. In this case, the debt-to-GDP limit still typically exists and is lower than in the log case, as the following proposition shows (see Appendix P for a proof).

Proposition 5. *Assume $\sigma < 1$ and consider a steady-state with constant real debt-to-GDP d.*

1. *If individual income does not shrink too fast with age $1-\zeta > \big(\beta(1-\lambda)\big)^{\frac{\sigma}{1-\sigma}}$, then households' asset demand is increasing in R up to a value R^*, after which it is decreasing. The debt-to-*

[28] In such a steady-state, taxes-to-GDP are equal to $T/Y = (R/(1+g)-1)D/(PY)$ from the flow budget constraint of the government (13).

GDP level $d^* = d(R^*)$ is the maximum amount of public debt that households can absorb.

2. *If individual income shrinks fast with age $1 - \zeta < \left(\beta(1-\lambda)\right)^{\frac{\sigma}{1-\sigma}}$, then asset demand is decreasing in R throughout. There is no limit on the debt-to-GDP ratio.*

Proposition 5 states that there still exists a limit on the debt-to-GDP ratio when the IES is less than one, but with two important differences. First, contrary to the log-utility case, the limit is no longer reached when the natural rate of interest becomes infinite. Instead, it is reached for a finite real interest rate R^*. Overall, the limit on debt-to-GDP still arises for the same reasons: when debt is too high, no real interest rate can counteract the wealth effect of public debt on aggregate spending. But this manifests itself in a slightly different way. When the IES is less than one, the wealth effect of higher interest rates dominates the substitution effect, so that households' MPC increases with the interest rate R. Beyond a certain level R^*, the wealth effect of interest rates is so large that higher interest rates make households save *less*, not more. This makes $d(R^*)$ the highest level of debt that households can absorb. The log-utility case can then be seen as a particular case: When σ tends to 1, the value of R^* tends to infinity, and the debt-to-GDP limit $d(R^*)$ tends to its value under the log-utility case (21).

Second, there is a small qualification. If individual labor income shrinks too fast, then there is no debt limit. Yet, in this case, this is because asset demand is *decreasing* in R throughout. This is because, when households face a strongly declining income profile, their unhedged interest rate exposure (Auclert, 2019) is large, and the wealth effect of an interest rate change always dominates. Since asset demand is increasing in R however (as seen in Section 2.5), this is not the empirically relevant case.[29]

How does an IES smaller than one affect the value of the debt-to-GDP limit d^*? Figure 3 plots households' asset demand curve and the resulting d^* for various values of the IES. The figure is plotted for the OLG calibration of the model in Table 1. As the IES decreases below one, the debt-to-GDP limit d^* decreases considerably, from $d^* = 1743$ times annual GDP for $\sigma = 1$ to just $d^* = 15.1$ for $\sigma = 0.5$.

The much lower value of d^* is robust across different calibration strategies of the model. Table 2 provides the equivalent of Table 1 for an IES of $\sigma = 0.5$ instead of 1 (see Appendix Q for details). In all four calibration strategies, the parameter values required to match the targeted moments are

[29] When individual income does not shrink with age at all $\zeta = 0$, the condition $1 - \zeta > \left(\beta(1-\lambda)\right)^{\frac{\sigma}{1-\sigma}}$ is always satisfied, and there always exists a debt-to-GDP limit.

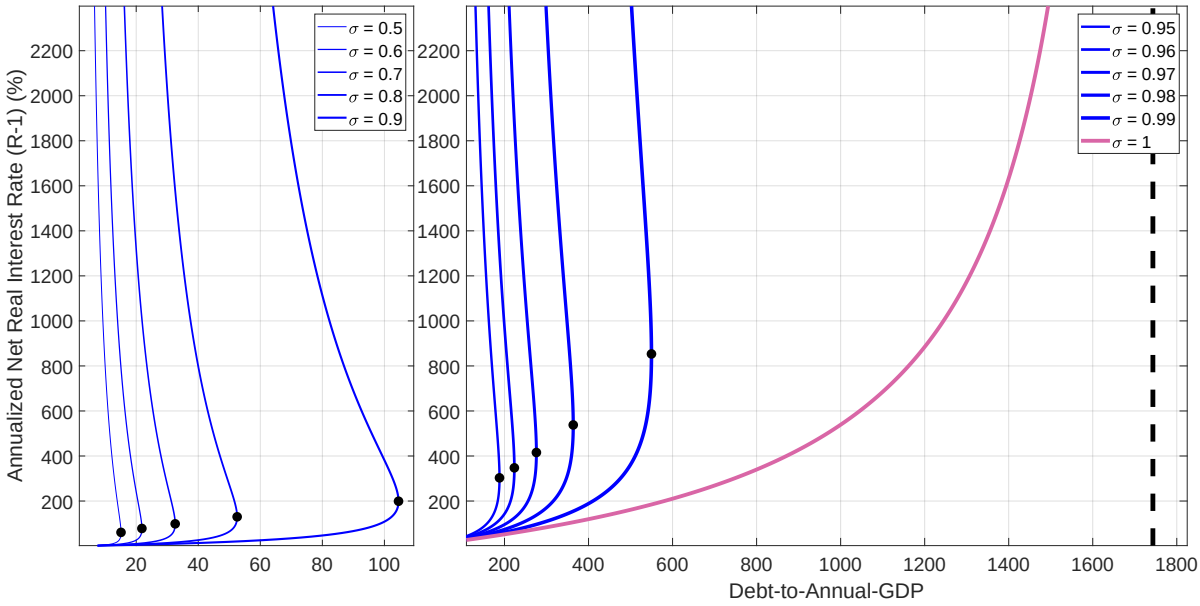


Figure 3: Steady-State Debt-to-GDP Limit d^* for an IES $\sigma < 1$

Note: The figure plots households' steady-state asset-demand schedule (55) for various values of the intertemporal elasticity of substitution $\sigma \leq 1$. On each curve (except the $\sigma = 1$ one), the black circle marks (R^*, d^*), where d^* is the maximum debt-to-GDP ratio households can absorb and R^* is the interest rate at which it is attained. The vertical dashed line corresponds to the value of the debt limit d^* for $\sigma = 1$ (log utility). The figure is split into two panels to make it more readable for low values of σ. All parameters other than σ are set according to the OLG calibration in Table 1.

close to those in the log-utility case of Table 1. Yet in all four calibrations the debt-to-GDP limit d^* is much lower. For our main calibration strategy, the debt-to-GDP limit is reduced from 134.5 times GDP to just 5.4, or 540% of GDP.[30] The LARGE-λ calibration strategy has no solution because the debt-to-GDP limit is then lower than the empirical debt-to-annual-GDP ratio of 100% that the calibration strategy targets at a 0.5% annual interest rate.

Do additional requirements for price stability arise from households' intertemporal budget constraints? In a steady state with a constant real interest rate, households' MPC is constant. As a result, a condition similar to Lemma 3 can be derived following the same logic (which makes use of the fact that the MPC is constant). Household intertemporal budget constraints still add only a weak constraint on fiscal policy.

[30] For the OLG calibration of Table 2, the debt-to-annual-GDP limit d^* is 17.2 and not 15.1 as in Figure 3 because ζ is recalibrated (to 0.0023) in order to match a real interest rate of 0.5% at 100% debt-to-annual-GDP, whereas in Figure 3 it is kept to its value in the OLG calibration for $\sigma = 1$, $\zeta = 0.0043$.

Table 2: Calibration and Debt Limit: Non-Log Utility, IES $\sigma = 0.5$

β		0.995		
g		0.005		
	OLG	LARGE-λ	OLG-HTM	MAIN
ζ	0.0023	0.0023	0.0023	0.0051
λ	0.005	-	0.005	0.019
α_Y	0	0	0.19	0.18
Average MPC	0.01	-	0.20	0.20
Sensitivity of r to $D/(PY)$	0.34	-	0.43	3
d^*	17.2	-	13.8	5.4
$\underline{d}$	4.6	-	3.7	1.5

Note: The table gives the average MPC, sensitivity of interest rates to debt, and debt thresholds d^* and $\underline{d}$ in four calibrations of the model: the OLG calibration and LARGE-λ calibration presented in Section 2.5, the OLG+HTM calibration presented in Section 3.2, and the main calibration presented in Section 3.3. The average MPC is the quarterly average MPC. The sensitivity of r to $D/(PY)$ is the derivative of the annualized real interest rate with respect to the debt-to-annual-GDP ratio in a steady-state of the model, expressed in basis points for percentage point increase in the debt-to-annual-GDP ratio. The debt thresholds d^* and $\underline{d}$ are expressed as debt to annual GDP.

4.3 Intertemporal Elasticity of Substitution $\sigma > 1$

When the IES is greater than one, a positive MPC $\mu(R) > 0$ requires $R < R_0 = \left(\beta^\sigma(1-\lambda)\right)^{\frac{1}{1-\sigma}}$ and the MPC is decreasing in the real interest rate. In this case, there is no longer any limit on the debt-to-GDP ratio (see Appendix P for a proof).

Proposition 6. *Assume $\sigma > 1$ and consider a steady-state with constant real debt-to-GDP d.*

There is no limit on the debt-to-GDP ratio. As the debt-to-GDP ratio increases, the natural interest rate converges to the finite value

$$R_0 = \left(\beta^\sigma(1-\lambda)\right)^{\frac{1}{1-\sigma}}. \tag{58}$$

When the IES is greater than one, increasing the real interest rate becomes even more effective at curbing aggregate spending. It now decreases households' MPC, making them spend less of their wealth. As a result, households can absorb arbitrarily large levels of debt provided a sufficiently high (but finite) interest rate. As debt-to-GDP tends to infinity, the natural rate asymptotes to R_0, for which households' MPC asymptotes to 0. Households are infinitely rich, but they spend an infinitely small share of their wealth, so that aggregate spending can remain in line with aggregate supply.

An IES greater than one is not the most empirically realistic case. Most studies find an IES smaller than one (e.g Havránek, 2015). But the fact that the debt-to-GDP limit disappears in this case highlights how different fiscal requirements for price stability are when households are not Ricardian. They critically hinge on the behavior of the MPC, whereas the MPC plays no role in the NPV requirement (18) that characterizes fiscal requirements when households are Ricardian.

The absence of a debt limit in Proposition 6 makes a sharp contrast with Proposition 5, but it does not mean that price stability is compatible with any fiscal policy. Proposition 6 applies to a steady-state in which the government keeps the debt-to-GDP ratio constant at a level d. As long as $R(d)$ is greater than $1+g$, this requires the government to raise positive fiscal surpluses, $T = (R/(1+g)-1)D/P > 0$. At a very large debt-to-GDP level d, the condition becomes $R_0 > 1+g$. While, interestingly, it is theoretically possible for the limiting rate R_0 to be less than the growth rate of the economy, in practice $R_0 > 1+g$ is satisfied unless the IES σ is unrealistically large.

5 Implementation

Proposition 2 derived fiscal requirements for price stability abstracting from how the central bank can then implement price stability. In this section, we turn to the implementation question. In doing so, we reconsider Leeper (1991)'s analysis of fiscal and monetary dominance in the case of non-Ricardian households. We do so returning to the canonical version of the perpetual youth model with log utility, which allows a direct analytical comparison with the Ricardian case covered by Leeper.

5.1 Local Dynamics

We assume that prices are flexible, as in Leeper (1991)'s original analysis. The dynamics of the aggregate economy reduces to only two equations: the IS curve (32) and the flow budget constraint of the government (13). To study it, it is more convenient to use the repayment value of public debt $B_{t-1} = D_{t-1}I_{t-1}$ instead of the amount borrowed D_{t-1}, since B_{t-1} is the single state variable of the aggregate economy. The IS curve and flow budget constraint can then be written in per

capita terms

$$y_t = \frac{1-\zeta}{\beta}\frac{\Pi_{t+1}}{I_t}y_{t+1} + \chi\left(\frac{b_{t-1}}{\Pi_t} - \tau_t\right), \tag{59}$$

$$b_t\frac{1+g}{I_t} + \tau_t = \frac{b_{t-1}}{\Pi_t}. \tag{60}$$

where $b_{t-1} = B_{t-1}/(P_{t-1}N_t)$, $y_t = Y_t/N_t$ and $\tau_t = T_t/N_t$ are real public debt per capita, GDP-per-capita, and taxes per capita, Π_t is the inflation rate and I_t is the nominal interest rate.

Following Leeper, we take the exogenous level of GDP under flexible prices to be constant, and consider the log-linearized version of these two equations around a steady state with real interest rate R and debt-to-GDP ratio b/y,[31]

$$\hat{i}_t - \hat{\pi}_{t+1} = \eta\left(\hat{b}_{t-1} - \frac{b}{y}\hat{\pi}_t - \hat{\tau}_t\right), \tag{61}$$

$$\hat{b}_t = \frac{R}{1+g}\left(\hat{b}_{t-1} - \hat{\tau}_t\right) + \frac{b}{y}\left(\hat{i}_t - \frac{R}{1+g}\hat{\pi}_t\right), \tag{62}$$

where

$$\eta = \frac{\chi}{\left(\frac{1-\zeta}{\beta R}\right)}. \tag{63}$$

The analysis therefore reduces to the same two-equation system in b_t and π_t as in Leeper (1991). The only difference is that when households are not Ricardian $\eta > 0$, public debt now enters the Fisher equation (61), as higher public debt increases the natural rate of interest.

Following Leeper, we assume monetary and fiscal policies are conducted according to feedback rules. Monetary policy follows a standard Taylor rule and taxes respond to the level of public debt

$$\hat{i}_t = \phi_\pi\hat{\pi}_t + \nu_t^i, \tag{64}$$

$$\hat{\tau}_t = \psi_b\hat{b}_{t-1} - \nu_t^g. \tag{65}$$

Plugging in the policy equations (64) and (65) into the system (61)-(62) gives a 2-by-2 system in $\hat{\pi}_t$ and $\hat{b}_t$.

[31] When log-linearizing we define $\hat{b}_t = db_t/y^*$ to allow for the possibility of a zero level of public debt in steady-state.

5.2 Blurred Lines

Appendix R extends the result of Leeper (1991) on equilibrium determinacy to the case of non-Ricardian households. It shows that when households are not Ricardian, the sharp distinction between a monetary and fiscal regime is no longer applicable.

Proposition 7. *Assume monetary and fiscal policy are given by the feedback rules (64) and (65).*

The economy has a unique bounded equilibrium if and only if

$$\Big(1-\phi_\pi\Big)\left(1-\frac{R}{1+g}(1-\psi_b)\right)+\eta\frac{b}{y}\Big((1-\psi_b)\phi_\pi-1\Big)<0. \tag{66}$$

- *When households are Ricardian $\eta=0$ or when steady-state public debt is zero $b/y=0$, then when a unique bounded equilibrium exists the economy is either in a monetary regime $\phi_\pi>1,\psi_b>1-1/R$ where fiscal shocks ν_t^g have no effect on inflation, or in a fiscal regime $\phi_\pi<1,\psi_b<1-1/R$ where they do.*
- *When households are not Ricardian $\eta>0$ and when steady-state public debt is positive $b/y>0$, then when a unique bounded equilibrium exists fiscal shocks ν_t^g always affect inflation.*

The first item of Proposition 7 is Leeper's classical result. When households are Ricardian $\eta=0$ (and so $g=0$), condition (66) becomes

$$\Big(1-\phi_\pi\Big)\Big(1-R(1-\psi_b)\Big)<0. \tag{67}$$

This determines two regions, represented graphically in the (ϕ_π,ψ_b) plane on the left panel of Figure 4. The first case $\phi_\pi>1,\psi_b>1-(1+g)/R$ is what Leeper calls a regime of monetary dominance. In this case fiscal shocks ν_t^g have no effect on inflation. Indeed, the unique bounded solution can then be obtained by iterating forward equations (61)-(64) to give equilibrium inflation as

$$\hat{\pi}_t=-\sum_{k=0}^{\infty}\left(\frac{1}{\phi_\pi}\right)^{k+1}\nu_{t+k}^i, \tag{68}$$

which is independent of the fiscal shocks ν_t^g. The second case $\phi_\pi<1,\psi_b<1-(1+g)/R$ is what Leeper calls a regime of fiscal dominance. In this case fiscal shocks ν_t^g affect inflation. For instance,

when the interest rate is fully pegged $i_t = 0$, iterating equations (62)-(65) forward gives

$$(1 - \psi_b)\hat{b}_{t-1} - \frac{b}{y}\hat{\pi}_t = -\sum_{k=0}^{\infty} \left(\frac{1}{R(1-\psi_b)} \right)^k \nu^g_{t+k}, \tag{69}$$

which gives the level of inflation necessary to restore the intertemporal budget constraint of the government.

The second item of Proposition 7 considers how the Leeper result changes once we move away from the case of Ricardian households. The determinacy space in this case is represented in the middle panel of Figure 4. It no longer consists of two distinct regions so that there is no straightforward way to distinguish between two regimes.

The absence of a fiscal and monetary regime is clear when looking at the inflationary effect of fiscal shocks. Appendix R shows that when households are not Ricardian, inflation is given by

$$\hat{\pi}_t = \delta \hat{b}_{t-1} + \sum_{k=0}^{\infty} \left(\frac{1}{\upsilon} \right)^{k+1} \left(\left(\delta \frac{b}{y} - 1 \right) \nu^i_{t+k} + \left(\delta \frac{R}{1+g} + \eta \right) \nu^g_{t+k} \right), \tag{70}$$

whenever a unique equilibrium exists, where υ is the root of the economy that is greater than 1 and δ is a positive constant given in Appendix R. Fiscal shocks ν^g_t now always affect inflation.

Figure 5 gives the inflationary effect of a contemporaneous fiscal shock ν^g_t as a function of ϕ_π for fixed values of ψ_b. The upper panel gives the impact effect of the shock on inflation. As monetary policy becomes more reactive, it decreases. But it does so gradually and without ever reaching zero, approaching it only as its asymptotic limit where ϕ_π tends toward infinity. This is in contrast to the Ricardian case, where an active monetary policy $\phi_\pi > 1$ is enough to perfectly insulate inflation from fiscal shocks. In addition, the *cumulative* impact on inflation can be much larger, and even increasing in ϕ_π, as shown on the bottom panel of Figure 5. This is because increasing interest rates more aggressively increases the nominal value of debt, which can increase the real value of debt for a prolonged period, putting sustained inflationary pressure on the economy. Formally, increasing ϕ_π increases the backward-looking root of the economy and therefore the persistence of inflation, as shown in the middle panel of Figure 5. This perverse effect that occurs under fiscal dominance when households are Ricardian can now occur for parameter values typically associated with monetary dominance (high ψ_b).

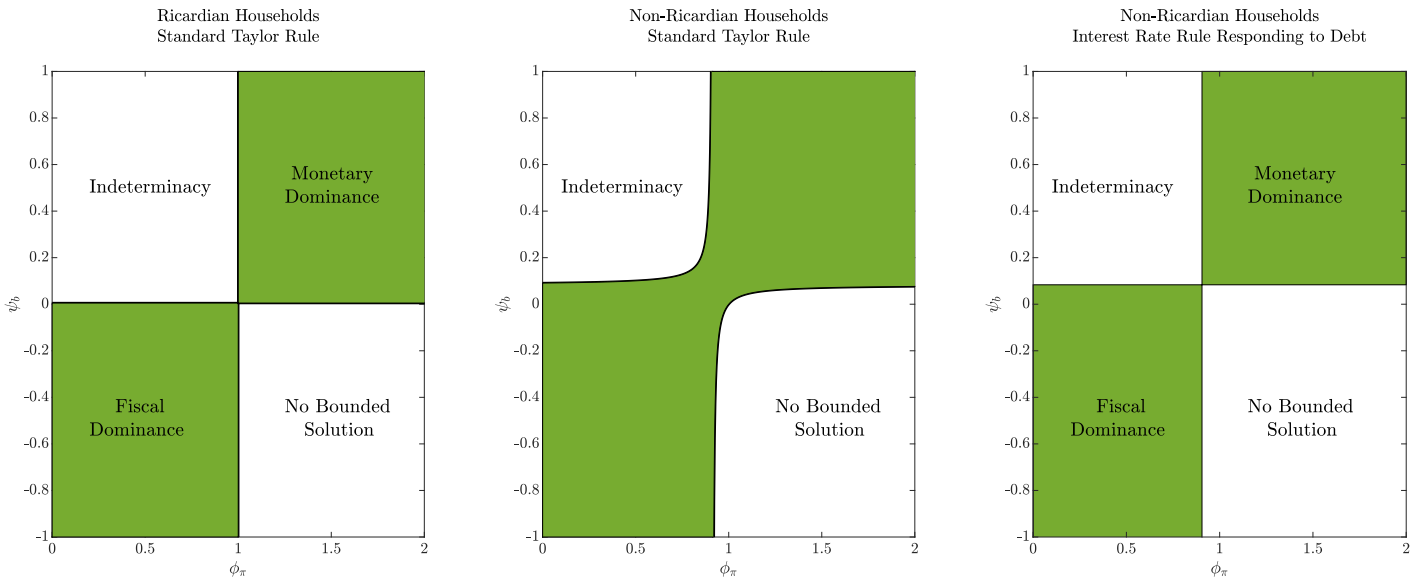


Figure 4: Condition for Equilibrium Uniqueness

Note: The left and middle panels of the figure illustrate Proposition 7; the right panel illustrates Proposition 8. On all panels, in green is the region of parameters (ϕ_π, ψ_b) for which there exists a unique bounded equilibrium: in the economy with Ricardian households and a standard Taylor rule (64)(left panel), in the economy with non-Ricardian households and a standard Taylor rule (64) (middle panel), and in the economy with non-Ricardian households and a monetary policy rule that responds to public debt on top of inflation (71) (right panel). The calibration is for a high $\lambda = 0.10$, $\beta = 0.995$, $g = 0.005$, $b/y = 1 \times 4$ (100% of annual GDP), and sets ζ such that the steady-state real interest rate is 0.5% ($\zeta = 0.09$).

5.3 Implementing Price Stability by Letting Monetary Policy Respond to Debt

The analysis so far would conclude that it is impossible for monetary policy to insulate inflation from fiscal shocks. No degree of responsiveness to inflation ϕ_π in the Taylor rule (64) can fully prevent fiscal shocks from affecting inflation.

Yet this is only the case if one restricts monetary policy to respond to inflation only. We now show that monetary policy can implement the stable price equilibrium if it ventures beyond a simple Taylor rule (64) and follows a policy rule that responds to public debt in addition to inflation. If the reaction function of the central bank is

$$\hat{i}_t = \phi_\pi \hat{\pi}_t + \eta(\hat{b}_{t-1} - \hat{\tau}_t), \tag{71}$$

where η is defined in (63), then monetary policy can again insulate inflation from fiscal shocks. This recovers the notion of a regime of monetary dominance—in the sense defined by the following proposition.

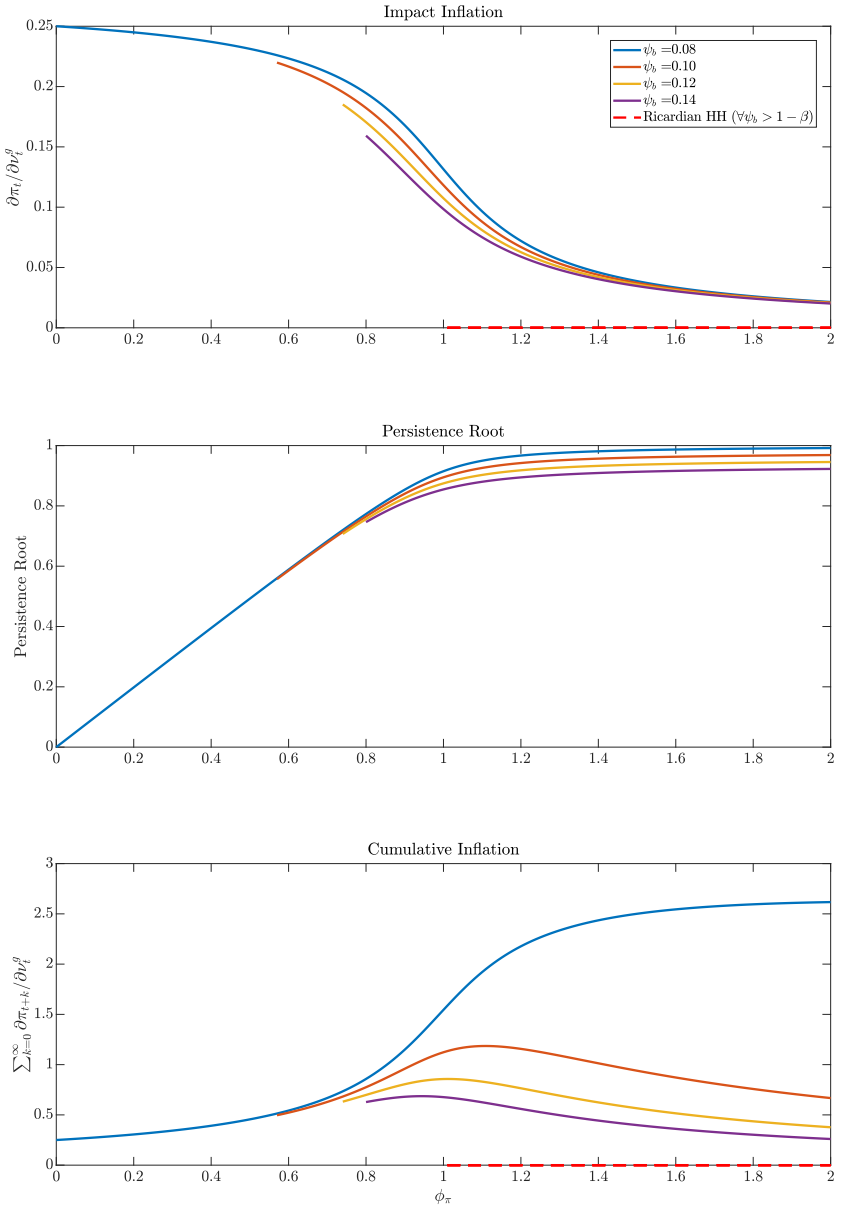


Figure 5: Inflationary Effect of Contemporaneous Fiscal Shock

Note: The figure plots the impact effect of a fiscal shock on inflation, the persistence root, and the cumulative effect on inflation, all three as a function of the reactivity of the Taylor rule ϕ_π, and for a parameter in the fiscal rule $\psi_b = 1 - \frac{1}{\frac{R}{1+g}+\eta\frac{b}{y}}$ or higher. For this first value of ψ_b there exists an equilibrium under all values of ϕ_π in the case of non-Ricardian households. In the case of Ricardian households, there exists an equilibrium only for $\phi_\pi > 0$. The calibration is for a high $\lambda = 0.10$, $\beta = 0.995$, $g = 0.005$, $b/y = 1 \times 4$ (100% of annual GDP), and sets ζ such that the steady-state real interest rate is 0.5% ($\zeta = 0.09$).

Proposition 8. *Assume monetary and fiscal policy are given by the feedback rules* (65) *and* (71).

The economy has a unique bounded equilibrium iff it is in either of the two following regimes

- *Monetary regime* $\phi_\pi > 1 - \eta\frac{b}{y}$ *and* $\psi_b > 1 - \frac{1}{\frac{R}{1+g}+\frac{b}{y}\eta}$. *Fiscal shocks* ν_t^g *have then no effect on inflation.*
- *Fiscal regime* $\phi_\pi < 1 - \eta\frac{b}{y}$ *and* $\psi_b < 1 - \frac{1}{\frac{R}{1+g}+\frac{b}{y}\eta}$. *Fiscal shocks* ν_t^g *then affect inflation.*

See Appendix S for a proof. The two regimes are plotted in the (ϕ_π, ψ_b) plane on the right panel of Figure 4. Just as in the case of Ricardian households (67), in the monetary regime the unique bounded solution can be obtained by solving forward equations (61)-(71), to give

$$\hat{\pi}_t = -\sum_{k=0}^{\infty}\left(\frac{1}{\phi_\pi + \eta\frac{b}{y}}\right)^{k+1}\nu_{t+k}^i. \tag{72}$$

which is independent of the fiscal shocks ν_t^g. Because it insulates inflation from fiscal shocks, monetary policy can be said to be active again. However, this flips on its head the definition of an active monetary policy in Leeper (1991):

> *"I couch active and passive policy in terms of the constraints a policy authority faces. An active authority pays no attention to the state of government debt and is free to set its control variable as it sees fit. A passive authority responds to government debt shocks. Its behavior is constrained by private optimization and the active authority's actions."*

In contrast, the monetary policy (71) delivers monetary dominance *because* it reacts to public debt. Having monetary policy react to public debt is not a standard specification of monetary policy.[32] Yet it follows naturally from the fact that the natural rate of interest—the real interest rate consistent with stable prices—depends on the level of public debt once households are not Ricardian—equation (34). The monetary rule (71) is a Taylor rule augmented with the natural rate as an intercept

$$\hat{i}_t = \hat{r}_t^n + \phi_\pi\hat{\pi}_t, \tag{73}$$

[32]It can be found in some specification of monetary policy however. Bayer, Born, and Lütticke (2023) estimate a Taylor rule that responds to public debt as part of the global Bayesian estimation of a HANK model, and find a positive coefficient.

where in log-linearized form the natural rate is the function of public debt

$$\hat{r}_t^n = \eta(\hat{b}_{t-1} - \hat{\tau}_t). \tag{74}$$

Having the natural rate of interest as an intercept in the Taylor rule is a standard feature in specifications of monetary policy.

5.4 Conditions For Monetary Dominance

Responding directly to public debt restores a notion of monetary dominance, but for it to obtain, the conditions on monetary and fiscal policy are not the same as in the Ricardian case. The threshold on ψ_b is now $\psi_b^* = 1 - \frac{1}{\frac{R}{1+g} + \frac{b}{y}\eta}$. It increases with η—with how non-Ricardian households are—and with the level of public debt. To understand why the threshold ψ_b^* is now higher, consider the dynamics of debt under price stability. Combining the flow budget constraint of the government (62) and the dependence of the natural rate on debt (74) gives the dynamics of debt as

$$\hat{b}_t = \left(\frac{R}{1+g} + \eta\frac{b}{y}\right)(\hat{b}_{t-1} - \hat{\tau}_t). \tag{75}$$

Absent adjustment in taxes, public debt increases at the rate $\frac{R}{1+g} + \eta\frac{b}{y}$, so fiscal surpluses must increase by at least as much—otherwise debt explodes. The term $\eta\frac{b}{y}$ captures that when households are not Ricardian higher public debt increases the natural rate, accelerating how fast debt diverges.

How much does this increase the threshold ψ_b^* for monetary dominance? Here again, the answer depends heavily on the calibration of the death probability λ. When $\lambda = 0.10$ as in Figure 4, the threshold is $\psi_b^* = 0.08$ at a debt-to-annual-GDP ratio of 100%, versus about $\psi_b^* = 0.0025$ when households are Ricardian. When $\lambda = 0.15$, the threshold is higher still, at $\psi_b^* = 0.30$. For such values, monetary dominance requires the government to stabilize debt extremely quickly, arguably unrealistically so. Yet, such high calibrations of λ greatly exaggerate the sensitivity of the natural rate to the level of public debt, as discussed in Section 3.3. When matching a sensitivity of the natural rate to the debt-to-GDP ratio of about 3bp/pp as found by Laubach (2009) and Engen and Hubbard (2004)—which requires a much lower λ of about 0.03—the term $\eta\frac{b}{y}$ only adds about 0.0075 to the threshold ψ_b^*. This is a significant increase, but one that remains consistent with a fiscal policy that only slowly brings debt down following a fiscal expansion. Implementing price stability is therefore feasible for a wide range of fiscal policies. Yet, to do so monetary policy must

increase the policy rate directly with the level of public debt—something central banks may be reluctant to do out of a fear it could be seen as interfering with fiscal policy and threaten central bank independence.

6 Conclusion

Can the central bank deliver price stability whatever fiscal policy the government sets? The NPV requirement that current government debt needs to be backed by future fiscal surpluses has remained controversial, all the more so when considering models with non-Ricardian households where the real interest rate can be less than the growth rate of the economy. In this paper, we have argued that in models with non-Ricardian households, fiscal requirements for price stability can be expressed in a less controversial—and also more intuitive—way. With non-Ricardian households, higher public debt makes households richer. As households spend their higher wealth, aggregate demand increases, putting inflationary pressure on the economy. While monetary policy can increase interest rates to counter the inflationary effects of public debt, it can no longer do so when public debt is too high. For the central bank to keep control over the price level, fiscal policy must be such that it never lets debt reach that level. Provided this fiscal requirement is satisfied, monetary policy can deliver price stability. But to do so, it needs to increase its policy rate with the level of public debt—not just increase it in response to the inflation that higher public debt generates. The main fiscal threat to price stability is institutional: central banks may be reluctant to adopt debt-contingent monetary policy rules, even though they are feasible.

References

Aguiar, M. A., M. Amador, and C. Arellano (2023): "Pareto Improving Fiscal and Monetary Policies: Samuelson in the New Keynesian Model," NBER Working Paper 31297.

Aiyagari, S. R. and E. R. McGrattan (1998): "The Optimum Quantity of Debt," *Journal of Monetary Economics*, 42, 447–469.

Angeletos, G.-M., F. Collard, and H. Dellas (2022): "Public Debt as Private Liquidity: Optimal Policy," Latest circulated version: May 3, 2022. NBER Working Paper 22794 (Nov 2016; rev. June 2021).

Angeletos, G.-M., C. Lian, and C. K. Wolf (2024a): "Can Deficits Finance Themselves?" *Econometrica*, 92, 1351–1390.

——— (2024b): "Deficits and Inflation: HANK meets FTPL," Working paper.

Auclert, A. (2019): "Monetary Policy and the Redistribution Channel," *American Economic Review*, 109, 2333–2367.

Auclert, A., M. Rognlie, and L. Straub (2018): "The Intertemporal Keynesian Cross," NBER Working Paper 25020, National Bureau of Economic Research, Inc.

Azzimonti, M. and P. Yared (2019): "The Optimal Public and Private Provision of Safe Assets," *Journal of Monetary Economics*, 102, 126–144.

Barro, R. and F. Bianchi (2023): "Fiscal Influences on Inflation in OECD Countries, 2020-2022," *NBER Working Paper*.

Barthélémy, J., E. Mengus, and G. Plantin (2024): "Fiscal Dominance: Implications for Bond Markets and Central Banking," Tech. rep., cEPR Discussion Paper No. 19591.

Bassetto, M. (2002): "A Game-Theoretic View of the Fiscal Theory of the Price Level," *Econometrica*, 70, 2167–2195.

Bassetto, M. and W. Cui (2018): "The Fiscal Theory of the Price Level in a World of Low Interest Rates," *Journal of Economic Dynamics and Control*, 89, 5–22.

——— (2024): "A Ramsey Theory of Financial Distortions," *Journal of Political Economy*, 132, 2612–2654.

Bassetto, M. and D. S. Miller (2025): "A Monetary–Fiscal Theory of Sudden Inflations," *Quarterly Journal of Economics*, advance online publication, doi:10.1093/qje/qjaf012.

Bayer, C., B. Born, and R. Lütticke (2023): "The Liquidity Channel of Fiscal Policy," *Journal of Monetary Economics*, 134, 86–117.

Bénassy, J.-P. (2003): "Fiscal Policy and Optimal Monetary Rules in a Non-Ricardian Economy," *Review of Economic Dynamics*, 6, 498–512.

——— (2005): "Interest Rate Rules, Price Determinacy and the Value of Money in a Non-Ricardian World," *Review of Economic Dynamics*, 8, 651–667.

——— (2007): *Money, Interest, and Policy: Dynamic General Equilibrium in a Non-Ricardian World*, Cambridge, MA: MIT Press.

——— (2008): "The Fiscal Theory of the Price Level Puzzle: A Non-Ricardian View," *Macroeconomic Dynamics*, 12, 31–44.

Bewley, T. (1977): "The permanent income hypothesis: A theoretical formulation," *Journal of Economic Theory*, 16, 252–292.

Bhandari, A., D. Evans, M. Golosov, and T. J. Sargent (2017): "Public Debt in Economies with Heterogeneous Agents," *Journal of Monetary Economics*, 91, 39–51.

Bhattarai, S., J. W. Lee, and W. Y. Park (2012): "Monetary–Fiscal Policy Interactions and Indeterminacy in Post-War U.S. Data," *American Economic Review: Papers and Proceedings*, 102, 173–178.

——— (2014): "Inflation Dynamics: The Role of Public Debt and Policy Regimes," *Journal of Monetary Economics*, 67, 93–108.

Bhattarai, S., J. W. Lee, and C. Yang (2023): "Redistribution and the Monetary–Fiscal Policy Mix," *Quantitative Economics*, 14, 817–853.

Bianchi, F., R. Faccini, and L. Melosi (2023): "A Fiscal Theory of Persistent Inflation," *Quarterly Journal of Economics*, 138, 2127–2179.

Bianchi, F. and C. Ilut (2017): "Monetary/Fiscal Policy Mix and Agents' Beliefs," *Review of Economic Dynamics*, 26, 113–139.

Bianchi, F. and L. Melosi (2017): "Escaping the Great Recession," *American Economic Review*, 107, 1030–58.

——— (2019): "The Dire Effects of the Lack of Monetary and Fiscal Coordination," *Journal of Monetary Economics*, 104, 1–22.

Bianchi, F., L. Melosi, and A. Rogantini Picco (2023): "Who Is Afraid of Eurobonds?" CEPR Discussion Paper DP18279, Centre for Economic Policy Research.

——— (2024): "Monetary/Fiscal Policy Mix and Inflation Dynamics," in *Handbook of Inflation*, ed. by G. Ascari and R. Trezzi.

Blanchard, O. (2019): "Public Debt and Low Interest Rates," *American Economic Review*, 109, 1197–1229.

Blanchard, O. J. (1985): "Debt, Deficits, and Finite Horizons," *Journal of Political Economy*, 93, 223–247.

Bohn, H. (1995): "The Sustainability of Budget Deficits in a Stochastic Economy," *Journal of Money, Credit and Banking*, 27, 257–271.

Brunnermeier, M. K., S. Merkel, and Y. Sannikov (2022): "The Fiscal Theory of the Price Level with a Bubble," *NBER Working Paper*.

Buiter, W. H. (2002): "The Fiscal Theory Of The Price Level: A Critique," *Economic Journal*, 112, 459–480.

Campos, R. G., J. Fernández-Villaverde, G. Nuño, and P. Paz (2024): "Navigating by Falling Stars: Monetary Policy with Fiscally-Driven Natural Rates," BIS Working Paper 1172.

Canzoneri, M. B., R. E. Cumby, and B. T. Diba (2011): "The Interaction Between Monetary and Fiscal Policy," in *Handbook of Monetary Economics*, ed. by B. M. Friedman and M. Woodford, Elsevier, vol. 3B, chap. 17, 935–999.

——— (2016): "Optimal Money and Debt Management: Liquidity Provision vs Tax Smoothing," *Journal of Monetary Economics*, 83, 39–53.

Chamberlain, G. and C. A. Wilson (2000): "Optimal Intertemporal Consumption Under Uncertainty," *Review of Economic Dynamics*, 3, 365–395.

Cochrane, J. (2023): *The Fiscal Theory of the Price Level*, Princeton University Press.

Cochrane, J. H. (2001): "Long-Term Debt and Optimal Policy in the Fiscal Theory of the Price Level," *Econometrica*, 69, 69–116.

——— (2005): "Money as stock," *Journal of Monetary Economics*, 52, 501–528.

——— (2022): "A fiscal theory of monetary policy with partially-repaid long-term debt," *Review of Economic Dynamics*, 45, 1–21.

Corsetti, G. and B. Maćkowiak (2024): "Gambling to Preserve Price (and Fiscal) Stability," *IMF Economic Review*, 72, 32–57.

Cui, W. (2016): "Monetary–Fiscal Interactions with Endogenous Liquidity Frictions," *European Economic Review*, 87, 1–25.

Davig, T. and E. M. Leeper (2007): "Fluctuating Macro Policies and the Fiscal Theory," in *NBER Macroeconomics Annual 2006, Volume 21*, National Bureau of Economic Research, Inc, NBER Chapters, 247–316.

Del Negro, M., M. P. Giannoni, and C. Patterson (2023): "The Forward Guidance Puzzle," *Journal of Political Economy Macroeconomics*, 1, 43–79.

Elfsbacka-Schmoller, M. and N. McClung (2024): "Can Growth Stabilize Debt? A Fiscal Theory Perspective," Tech. rep., bank of Finland Research Discussion Paper No. 2/2024.

Engen, E. M. and R. G. Hubbard (2004): "Federal Government Debt and Interest Rates," in *NBER Macroeconomics Annual 2004, Volume 19*, ed. by M. Gertler and K. Rogoff, MIT Press, 83–160.

Fagereng, A., M. B. Holm, and G. J. Natvik (2021): "MPC Heterogeneity and Household Balance Sheets," *American Economic Journal: Macroeconomics*, 13, 1–54.

Farhi, E. and I. Werning (2019): "Monetary Policy, Bounded Rationality, and Incomplete Markets," *American Economic Review*, 109, 3887–3928.

Farmer, R. and P. Zabczyk (2023): "Monetary and Fiscal Policy when People Have Finite Lives," NBER Working Paper 25445, revised November 2023.

Galí, J. (2021): "Monetary Policy and Bubbles in a New Keynesian Model with Overlapping Generations," *American Economic Journal: Macroeconomics*, 13, 121–167.

Gamber, E. and J. Seliski (2019): "The Effect of Government Debt on Interest Rates," CBO Working Paper 2019-01, Congressional Budget Office, cBO Working Paper 2019-01.

Hagedorn, M. (2024): "The Failed Theory of the Price Level," Working paper.

Havránek, T. (2015): "Measuring Intertemporal Substitution: The Importance of Method Choices and Selective Reporting," *Journal of the European Economic Association*, 13, 1180–1204, published 1 December 2015.

Hazell, J. and S. Hobler (2024): "Do Deficits Cause Inflation? A High-Frequency Narrative Approach," CFM Discussion Paper 2024-39.

Holston, K., T. Laubach, and J. Williams (2017): "Measuring the Natural Rate of Interest: International Trends and Determinants," *Journal of International Economics*, 108, S59–S75.

Jarocinski, M. and B. Maćkowiak (2018): "Monetary–Fiscal Interactions and the Euro Area's Malaise," *Journal of International Economics*, 112, 251–266.

Jiang, Z., H. Lustig, S. V. Nieuwerburgh, and M. Z. Xiaolan (2024): "The U.S. Public Debt Valuation Puzzle," *Econometrica*, 92, 1309–1347.

Kaplan, G., G. Nikolakoudis, and G. Violante (2023): "Price Level and Inflation Dynamics in Heterogeneous Agent Economies," *NBER Working Paper*.

Kocherlakota, N. R. and C. Phelan (1999): "Explaining the fiscal theory of the price level," *Quarterly Review*, 23, 14–23.

Laubach, T. (2009): "New Evidence on the Interest Rate Effects of Budget Deficits and Debt," *Journal of the European Economic Association*, 7, 858–885.

Leeper, E. (1991): "Equilibria under 'Active' and 'Passive' Monetary and Fiscal Policies," *Journal of Monetary Economics*, 27, 129–147.

Leeper, E. and C. Leith (2016): "Understanding Inflation as a Joint Monetary-Fiscal Phenomenon," in *Handbook of Macroeconomics*, ed. by J. B. Taylor and H. Uhlig, Elsevier, vol. 2 of *Handbook of Macroeconomics*, chap. 0, 2305–2415.

Leith, C. and S. Wren-Lewis (2000): "Interactions between Monetary and Fiscal Policy Rules," *The Economic Journal*, 110, C93–C108.

Maćkowiak, B. and S. Schmidt (2024): "Fiscal Backing for Price Stability in a Monetary Union," *American Economic Journal: Macroeconomics, Forthcoming*.

Mehrotra, N. and D. Sergeyev (2021): "Debt Sustainability in a Low Interest Rate World," *Journal of Monetary Economics*, 124, 1–18.

Mian, A. R., L. Straub, and A. Sufi (2022): "A Goldilocks Theory of Fiscal Deficits," NBER Working Paper 29707, revised 2024.

Neveu, A. R. and J. Schafer (2024): "Revisiting the Relationship Between Debt and Long-Term Interest Rates," Working Paper 2024-05, Congressional Budget Office.

Niepelt, D. (2004): "The Fiscal Myth of the Price Level," *The Quarterly Journal of Economics*, 119, 277–300.

Obstfeld, M. (2023): "Natural and Neutral Real Interest Rates: Past and Future," NBER Working Paper 31949, National Bureau of Economic Research.

Plante, M. D., A. W. Richter, and S. Zubairy (2025): "Revisiting the Interest Rate Effects of Federal Debt," NBER Working Paper 34018, National Bureau of Economic Research.

Powell, J. (2024): "Interview to CBS's *60 Minutes*," Transcripts available at https://www.cbsnews.com/news/full-transcript-fed-chair-jerome-powell-60-minutes-interview-economy/.

Rachel, L. and M. O. Ravn (2025): "Brothers in Arms: Monetary-Fiscal Interactions Without Ricardian Equivalence," Working paper, University College London, manuscript, version of 29 June 2025.

Reis, R. (2021): "The constraint on public debt when r < g but g < m," Discussion Papers 2111, Centre for Macroeconomics (CFM).

——— (2022): "Debt Revenue and the Sustainability of Public Debt," *Journal of Economic Perspectives*, 36, 103–124.

Samuelson, P. A. (1958): "An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money," *Journal of Political Economy*, 66, 467–482.

Sargent, T. J. and N. Wallace (1981): "Some unpleasant monetarist arithmetic," *Quarterly Review*, 5.

Schmidt, S. (2024): "Monetary-Fiscal Policy Interactions when Price Stability Occasionally Takes a Back Seat," *CEPR Working Paper*.

Sims, C. (1994): "A Simple Model for Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy," *Economic Theory*, 4, 381–99.

Smets, F. and R. Wouters (2024): "Fiscal Backing, Inflation and US Business Cycles," Tech. rep., working Paper.

Townsend, R. M. (1980): "Models of Money with Spatially Separated Agents," in *Models of Monetary Economies*, ed. by J. H. Kareken and N. Wallace, Minneapolis: Federal Reserve Bank of Minneapolis, 265–303.

Weil, P. (1989): "Overlapping Families of Infinitely-Lived Agents," *Journal of Public Economics*, 38, 183–198.

Wolf, C. K. (2025): "Interest Rate Cuts vs. Stimulus Payments: An Equivalence Result," *Journal of Political Economy*, 133, 1235–1275.

Woodford, M. (1990): "Public Debt as Private Liquidity," *American Economic Review*, 80, 382–88.

——— (1995): "Price-level Determinacy Without Control of a Monetary Aggregate," *Carnegie-Rochester Conference Series on Public Policy*, 43, 1–46.

——— (2001): "Fiscal Requirements for Price Stability," *Journal of Money, Credit and Banking*, 33, 669–728.

Fiscal Requirements for Price Stability When Households are not Ricardian

Supplementary Online Appendix

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A Derivation of Individual Optimality

Maximizing (4) subject to the flow budget constraint (5) and the No-Ponzi-scheme constraint (6) gives the Euler equation

$$C_t^i = (\beta R_t)^{-1} C_{t+1}^i \tag{A.1}$$

and the No-Ponzi-scheme condition holding with equality, i.e. the No-Cash-on-the-Table condition (10). Iterating the Euler equation (A.1) forward gives for all $k \geq 0$

$$C_{t+k}^i = (\beta^k \mathcal{R}_{t,t+k}) C_t^i. \tag{A.2}$$

Combining the FBC (5) and the No-Cash-on-the-Table condition (10), the intertemporal budget constraint is

$$\sum_{k=0}^{\infty} \frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k}} C_{t+k}^i = \frac{1}{1-\lambda} \frac{A_{t-1}^i I_{t-1}}{P_t} + H_t^i \tag{A.3}$$

where

$$H_t^i = Y_t^i - T_t^i + \frac{1-\lambda}{R_t} H_{t+1}^i \tag{A.4}$$

is the household's intertemporal wealth, or human capital. Iterated forward, it writes as equation (9).

Injecting (A.2) into the IBC (A.3) gives the consumption function (8). Conversely, equations

(8) and (10) are sufficient for individual optimality.

B Proof of Proposition 1

The text already showed that condition (18) is necessary for a stable price equilibrium. We show that it is sufficient. The proof is constructive. Given an initial level of public debt $D_{-1}I_{-1}$ distributed as $(D^i_{-1}I_{-1})_i$ in the population, consider a path for taxes $(T_t)_{t\geq 0}$. Define the following path for the interest rate

$$R^n_t = \frac{1}{\beta}\frac{Y_{t+1}}{Y_t}. \tag{B.1}$$

For this interest rate path, define individual consumption allocations through the consumption function (8), which in combination with the FBC (5) of household i defines an entire path for its consumption and debt holdings.

We show that this allocation is a stable price equilibrium.

Market-clearing

Start with market-clearing. Since by construction all households are on the consumption function (8), aggregate consumption is given by the aggregate consumption function (O.6), which since $\lambda = g = \zeta = 0$ in the Ricardian case writes

$$C_t = (1-\beta)\left(\frac{A_{t-1}I_{t-1}}{P^*} + \sum_{k=0}^{\infty}\frac{1}{\mathcal{R}^n_{t,t+k}}(Y_{t+k} - T_{t+k})\right). \tag{B.2}$$

For the interest rate path (B.1), we have that

$$\frac{Y_{t+k}}{\mathcal{R}^n_{t,t+k}} = \beta^k Y_t. \tag{B.3}$$

The consumption function (O.6) therefore implies replacing Y_{t+k}

$$C_t = Y_t + (1-\beta)\left(\frac{A_{t-1}I_{t-1}}{P^*} - \sum_{k=0}^{\infty}\frac{T_{t+k}}{\mathcal{R}_{t,t+k}}\right). \tag{B.4}$$

Starting from $t=0$, given that $A^d_{-1} = D_{-1}$ initially, the NPV equation (18) at $t=0$ guarantees market clearing in the goods market at $t=0$. From Walras's law, it implies market clearing in the

debt market as well, $D_0 = A_0$. Continuing by induction, it proves market clearing in all periods.

Individual Optimality

We now check individual optimality, which is characterized by the consumption function (8) and No-Cash-on-the-Table condition (10). The consumption function (8) is satisfied by construction. Combined with the flow budget constraint of the representative household (5), its No-Cash-on-the-Table constraint is equivalent to its intertemporal budget constraint

$$\sum_{k=0}^{\infty} \frac{1}{\mathcal{R}_{t,t+k}} C_{t+k} = \frac{A_{t-1} I_{t-1}}{P^*} + \sum_{k=0}^{\infty} \frac{1}{\mathcal{R}_{t,t+k}} (Y_{t+k} - T_{t+k}). \tag{B.5}$$

Since the goods and assets market clear in all periods, it is equivalent to condition (18), which is satisfied by assumption.

C Proof of Lemma 1

Preliminary: Market-Clearing in the Debt Market

We first simply prove Walras's Law: the assumption of market-clearing in the goods market (16) implies market-clearing in the debt market when combined with flow budget constraints, i.e.

$$\int_i A_t^i di = D_t, \tag{C.1}$$

where the integral is over households alive at t.

By assumption, market-clearing held in period $t = -1$. We show that market-clearing holds in the debt market in all periods by showing that if it holds at $t-1$ then it holds at t. Summing the households' FBC (5) across households alive at t gives

$$C_t + \frac{\int_i A_t^i di}{P_t} = \frac{D_{t-1} I_{t-1}}{P_t} + Y_t - T_t, \tag{C.2}$$

where we used the fact that $\int_{i \text{ alive at } t} D_{t-1}^i di = (1-\lambda) D_{t-1}$, since the remaining λD_{t-1} belonged to households that passed away and has been taken by the insurance fund and redistributed to remaining households as annuities.

Combining (C.2) with goods market-clearing condition (16) gives

$$\frac{\int_i A_t^i di}{P_t} = \frac{D_{t-1} I_{t-1}}{P_t} - T_t, \tag{C.3}$$

which combined with the FBC of the government (13) implies

$$\int_i A_t^i di = D_t, \tag{C.4}$$

which ends the proof.

Preliminary: Defining Aggregate Human Capital

Define aggregate human capital as the sum of individual human capitals

$$H_t = \sum_{k=0}^{\infty} N_t(k) H_t^i(k). \tag{C.5}$$

The human capital (9) of a household of age n can then be written as a function of aggregate human capital per capita

$$H_t^i(n) = \kappa(1-\zeta)^n \left(\frac{H_t}{N_t}\right), \tag{C.6}$$

where aggregate human capital per capita can be written

$$\frac{H_t}{N_t} = \sum_{k=0}^{\infty} \frac{((1-\lambda)(1-\zeta))^k}{\mathcal{R}_{t,t+k}} \left(\frac{Y_{t+k} - T_{t+k}}{N_{t+k}}\right), \tag{C.7}$$

or recursively

$$\frac{H_t}{N_t} = \left(\frac{Y_t - T_t}{N_t}\right) + \frac{(1-\lambda)(1-\zeta)}{R_t} \left(\frac{H_{t+1}}{N_{t+1}}\right). \tag{C.8}$$

Aggregate human capital solves the recursion

$$H_t = (Y_t - T_t) + \frac{(1-\lambda)(1-\zeta)}{(1+g)R_t} H_{t+1}. \tag{C.9}$$

An Intermediary Lemma

We now move to the core of the proof. To prove Lemma 1, we first prove the following lemma:

Lemma C.1. *If all households are on their consumption function* (8) *and the assets market clears*

(16), *then at time* t*, a household of age* $n \leq t$ *has holdings of public debt*

$$\frac{A_t^i(n)}{P_t} = \kappa(1-\zeta)^n \left[\mu \frac{1-x^{n+1}}{1-x} \frac{\mathcal{R}_{t-n,t}}{(1+g)^{n+1}} \frac{D_{t-(n+1)} I_{t-(n+1)}}{N_{t-(n+1)} P_{t-n}} - \sum_{k=0}^{n} \left(x^k + \mu \frac{1-x^k}{1-x}\right) \frac{\mathcal{R}_{t-k,t}}{(1+g)^k} \frac{T_{t-k}}{N_{t-k}}\right], \tag{C.10}$$

where

$$x = \frac{\beta(1+g)}{1-\zeta}. \tag{C.11}$$

Proof. The proof is by induction. For future (frequent) use, first notice that combining the flow budget constraint (5) and the consumption function (8) gives the debt-holding function of a household at t, i.e. how many bonds it holds as a function of inherited bonds and future and present incomes

$$\frac{A_t^i}{P_t} = \beta \frac{A_{t-1}^i I_{t-1}}{P_t} + Y_t^i - T_t^i - \mu H_t^i. \tag{C.12}$$

Base Case(s)

To initialize the recursion, we determine the debt holdings of households of age 0 and of age 1. (The inductive step will show the property for $n+1$ assuming it holds for n and $n-1$, so it requires a double-initialization.) Applying (C.12) to households of age 0, who inherit no financial wealth from the previous period $A_{t-1}^i(0) = 0$, gives

$$\frac{A_t^i(0)}{P_t} = \kappa \left(\frac{Y_t - T_t - \mu H_t}{N_t}\right). \tag{C.13}$$

Aggregating equation (C.12) across all households alive at t and using market-clearing in the debt market gives

$$\frac{D_t}{P_t} = \left(\beta(1-\lambda) \frac{D_{t-1} I_{t-1}}{P_t} + Y_t - T_t - \mu H_t\right). \tag{C.14}$$

Taking the difference between (C.13) and κ/N_t times (C.14) gives

$$\frac{A_t^i(0)}{P_t} - \frac{\kappa}{N_t} \frac{D_t}{P_t} = -\kappa \frac{\beta(1-\lambda)}{(1+g)} \frac{D_{t-1} I_{t-1}}{P_t N_{t-1}}. \tag{C.15}$$

The government's flow budget constraint (13) can be rewritten in per-capita terms as

$$\frac{D_t}{N_t P_t} = \left(\frac{1}{1+g} \frac{D_{t-1} I_{t-1}}{N_{t-1} P_t} - \frac{T_t}{N_t}\right). \tag{C.16}$$

Using (C.16) to replace $D_t/(N_t P_t)$, equation (C.15) can be written

$$\frac{A_t^i(0)}{P_t} = \kappa\left(\mu\frac{1}{(1+g)}\frac{D_{t-1}I_{t-1}}{N_{t-1}P_t} - \frac{T_t}{N_t}\right), \tag{C.17}$$

which proves the base case $n = 0$. The base case $n = 1$ is shown very similarly to the inductive step below, noting that a household of age 0 at t had no debt at $t-1$.

Inductive Step

Assume the property holds for households of age n and $n-1$. We show it then holds for households of age $n+1$. Taking the difference between equation (C.12) applied to a household of age $n+1$ and $(1-\zeta)$ times equation (C.12) applied to a household of age n gives

$$\frac{A_t^i(n+1)}{P_t} - (1-\zeta)\frac{A_t^i(n)}{P_t} = \beta R_{t-1}\left(\frac{A_{t-1}^i(n)}{P_{t-1}} - (1-\zeta)\frac{A_{t-1}^i(n-1)}{P_{t-1}}\right). \tag{C.18}$$

Since the property holds for n and $n-1$, the $t-1$ asset holdings on the right-hand side are

$$\frac{A_{t-1}^i(n)}{P_{t-1}} = \kappa(1-\zeta)^n\left[\mu\frac{1-x^{n+1}}{1-x}\frac{\mathcal{R}_{t-(n+1),t-1}}{(1+g)^{n+1}}\frac{D_{t-(n+2)}I_{t-(n+2)}}{N_{t-(n+2)}P_{t-(n+1)}} - \sum_{k=0}^{n}\left(x^k + \mu\frac{1-x^k}{1-x}\right)\frac{\mathcal{R}_{t-1-k,t-1}}{(1+g)^k}\frac{T_{t-1-k}}{N_{t-1-k}}\right], \tag{C.19}$$

$$\begin{aligned}\frac{A_{t-1}^i(n-1)}{P_{t-1}} = \kappa(1-\zeta)^{n-1}\Bigg[&\mu\frac{1-x^n}{1-x}\frac{\mathcal{R}_{t-(n+1),t-1}}{(1+g)^{n+1}}\frac{D_{t-(n+2)}I_{t-(n+2)}}{N_{t-(n+2)}P_{t-(n+1)}} - \mu\frac{1-x^n}{1-x}\frac{\mathcal{R}_{t-(n+1),t-1}}{(1+g)^n}\frac{T_{t-(n+1)}}{N_{t-(n+1)}} \\ &- \sum_{k=0}^{n-1}\left(x^k + \mu\frac{1-x^k}{1-x}\right)\frac{\mathcal{R}_{t-1-k,t-1}}{(1+g)^k}\frac{T_{t-1-k}}{N_{t-1-k}}\Bigg].\end{aligned} \tag{C.20}$$

where equation (C.20) made use of the government's flow budget constraint (C.16) to replace $D_{t-(n+1)}$. Taking the quasi-difference between the two, the RHS of equation (C.18) can be rewritten as

$$\kappa(1-\zeta)^{n+1}\left[\mu x^{n+1}\frac{\mathcal{R}_{t-(n+1),t}}{(1+g)^{n+2}}\frac{D_{t-(n+2)}I_{t-(n+2)}}{N_{t-(n+2)}P_{t-(n+1)}} - x^{n+1}\frac{\mathcal{R}_{t-(n+1),t}}{(1+g)^{n+1}}\frac{T_{t-(n+1)}}{N_{t-(n+1)}}\right]. \tag{C.21}$$

Meanwhile, the term $(1-\zeta)A_t^i(n)/P_t$ on the left-hand side of equation (C.18) can be rewritten,

using (C.10) and the government flow budget constraint (C.16) to eliminate $D_{t-(n+1)}$, as

$$(1-\zeta)\frac{A_t^i(n)}{P_t} = \kappa(1-\zeta)^{n+1}\left[\mu\frac{1-x^{n+1}}{1-x}\frac{\mathcal{R}_{t-(n+1),t}}{(1+g)^{n+2}}\frac{D_{t-(n+2)}I_{t-(n+2)}}{N_{t-(n+2)}P_{t-(n+1)}} - \mu\frac{1-x^{n+1}}{1-x}\frac{\mathcal{R}_{t-(n+1),t}}{(1+g)^{n+1}}\frac{T_{t-(n+1)}}{N_{t-(n+1)}}\right.$$
$$\left. - \sum_{k=0}^{n}\left(x^k + \mu\frac{1-x^k}{1-x}\right)\frac{\mathcal{R}_{t-k,t}}{(1+g)^k}\frac{T_{t-k}}{N_{t-k}}\right]. \tag{C.22}$$

Injecting equations (C.21) and (C.22) into equation (C.18) gives

$$\frac{A_t^i(n+1)}{P_t} = \kappa(1-\zeta)^{n+1}\left[\mu\left(\frac{1-x^{n+2}}{1-x}\right)\frac{\mathcal{R}_{t-(n+1),t}}{(1+g)^{n+2}}\frac{D_{t-(n+2)}I_{t-(n+2)}}{N_{t-(n+2)}P_{t-(n+1)}} - \sum_{k=0}^{n+1}\left(x^k + \mu\frac{1-x^k}{1-x}\right)\frac{\mathcal{R}_{t-k,t}}{(1+g)^k}\frac{T_{t-k}}{N_{t-k}}\right], \tag{C.23}$$

which ends the proof. □

Finishing the Proof To get from Lemma C.1 to Lemma 1, we just need to express $A_t^i(n)$ as a function of D_t instead of D_{t-n}. Iterate the government's flow budget constraint (C.16) from $t-(n+1)$ to t to obtain

$$\frac{D_t}{N_t P_t} = \frac{\mathcal{R}_{t-n,t}}{(1+g)^{n+1}}\frac{D_{t-(n+1)}I_{t-(n+1)}}{N_{t-(n+1)}P_{t-n}} - \sum_{k=0}^{n}\frac{\mathcal{R}_{t-k,t}}{(1+g)^k}\frac{T_{t-k}}{N_{t-k}}. \tag{C.24}$$

Injecting it in equation (C.10) gives equation (23).

Figure C.1 plots the coefficient $\psi_D(n)$ as a function of age n for different values of ζ.

Sanity Check

Note as a sanity check that we have

$$\sum_{n=0}^{\infty}\psi_D(n)\frac{N_t(n)}{N_t} = 1, \tag{C.25}$$

$$\forall k \in [\![0,n]\!], \sum_{n=k}^{\infty}\phi_T(n,k)N_t(n) = 0, \tag{C.26}$$

so that when enough time has passed so that all the debt holdings of households of all ages is given by (23), the sum of debt holdings by all households sums to D_t.

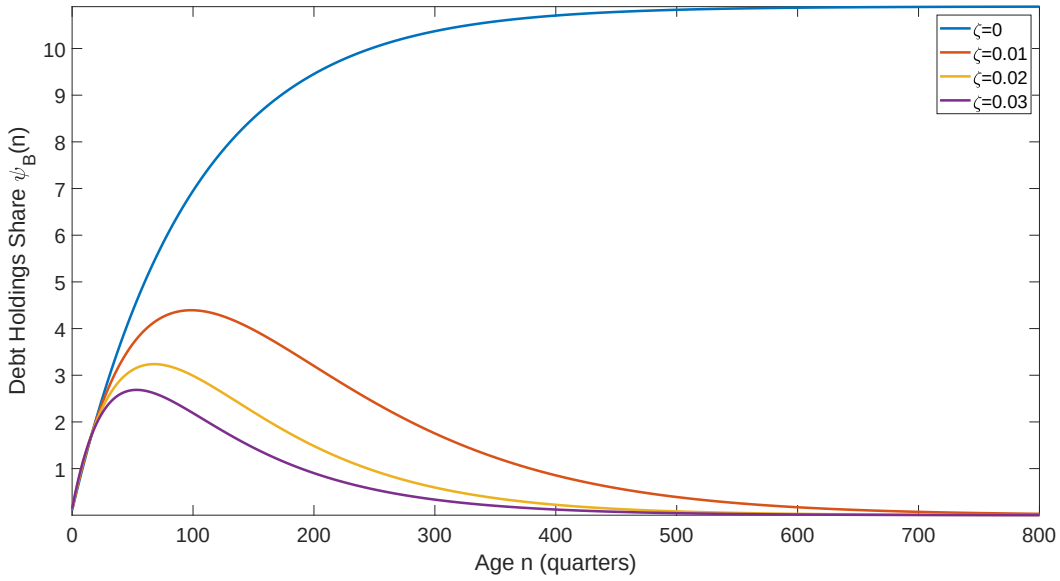


Figure C.1: Individual Debt Holdings as a Function of Age

Note: The figure represents the coefficient $\psi_D(n)$ in the expression of individual debt holdings (23) as a function of the household's age, for different values of the parameter ζ defined in equation (11). The three other parameters are set to $\beta = 0.995$, $\lambda = 0.005$ and $g = 0.005$.

D Proof of Lemma 2

If all households alive at t satisfy their No-Cash-on-the-Table condition (10), then (27) necessarily holds by taking the average over all households alive at t. The lengthier part is to show that (27) is also sufficient.

Let i be a household alive at t, and $n \geq 0$ its age at time t. It is therefore of age $n+k$ at $t+k$. We can decompose its debt holding at $t+k$ into the two following terms

$$\frac{A^i_{t+k}(n+k)}{P_{t+k}} = \kappa(1-\zeta)^n \frac{\bar{A}_{t+k}}{P_{t+k}} + \left(\frac{A^i_{t+k}(n+k)}{P_{t+k}} - \kappa(1-\zeta)^n \frac{\bar{A}_{t+k}}{P_{t+k}} \right). \tag{D.1}$$

It follows that

$$\frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k}} \frac{A^i_{t+k}(n+k)}{P_{t+k}} = \frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k}} \kappa(1-\zeta)^n \frac{\bar{A}_{t+k}}{P_{t+k}} + \frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k}} \left(\frac{A^i_{t+k}(n+k)}{P_{t+k}} - \kappa(1-\zeta)^n \frac{\bar{A}_{t+k}}{P_{t+k}} \right). \tag{D.2}$$

The proof amounts to showing that the second term in (D.2) necessarily tends to zero as $k \to \infty$. Condition (27) then implies that the first term tends to zero since $\kappa(1-\zeta)^n$ is a constant.

We show that the second term in (D.2) necessarily tends to zero. We denote with the superscript *old* the variables aggregated over all households already alive at t. Let $k \geq 0$. Aggregating (C.12) at period $t+k$ across households already alive at t gives

$$\frac{A^{old}_{t+k}}{P_{t+k}} = \left(\beta(1-\lambda)\frac{A^{old}_{t+k-1}I_{t+k-1}}{P_{t+k}} + Y^{old}_{t+k} - T^{old}_{t+k} - \mu H^{old}_{t+k}\right), \tag{D.3}$$

where

$$\begin{aligned} Y^{old}_{t+k} - T^{old}_{t+k} - \mu H^{old}_{t+k} &= \sum_{n=k}^{\infty} N_{t+k}(n)(Y^{i}_{t+k}(n) - T^{i}_{t+k}(n) - \mu H^{i}_{t+k}(n)) \\ &= \left(\frac{(1-\zeta)(1-\lambda)}{(1+g)}\right)^{k}(Y_{t+k} - T_{t+k} - \mu H_{t+k})\,. \end{aligned} \tag{D.4}$$

Injecting (D.4) into (D.3):

$$\frac{A^{old}_{t+k}}{P_{t+k}} = \left(\beta(1-\lambda)\frac{A^{old}_{t+k-1}I_{t+k-1}}{P_{t+k}} + \left(\frac{(1-\zeta)(1-\lambda)}{(1+g)}\right)^{k}(Y_{t+k} - T_{t+k} - \mu H_{t+k})\right). \tag{D.5}$$

The number of households alive at t that are still alive at $t+k$ is $(1-\lambda)^k N_t$. Dividing (D.5) by this to get the average debt holding:

$$\frac{\bar{A}_{t+k}}{P_{t+k}} = \left(\beta\frac{\bar{A}_{t+k-1}I_{t+k-1}}{P_{t+k}} + (1-\zeta)^{k}\left(\frac{Y_{t+k}}{N_{t+k}} - \frac{T_{t+k}}{N_{t+k}} - \mu\frac{H_{t+k}}{N_{t+k}}\right)\right). \tag{D.6}$$

Applying equation (C.12) at $t+k$ to household i gives

$$\frac{A^{i}_{t+k}(n+k)}{P_{t+k}} = \left(\beta\frac{A^{i}_{t+k-1}(n+k-1)I_{t+k-1}}{P_{t+k}} + \kappa(1-\zeta)^{n+k}\left(\frac{Y_{t+k}}{N_{t+k}} - \frac{T_{t+k}}{N_{t+k}} - \mu\frac{H_{t+k}}{N_{t+k}}\right)\right). \tag{D.7}$$

Taking the difference between (D.7) and $\kappa(1-\zeta)^n$ times (D.6) gives

$$\frac{A^{i}_{t+k}(n+k)}{P_{t+k}} - \kappa(1-\zeta)^{n}\frac{\bar{A}_{t+k}}{P_{t+k}} = \beta\left(\frac{A^{i}_{t+k-1}(n+k-1)I_{t+k-1}}{P_{t+k}} - \kappa(1-\zeta)^{n}\frac{\bar{A}_{t+k-1}I_{t+k-1}}{P_{t+k}}\right) \tag{D.8}$$

Iterating backward:

$$\frac{A^{i}_{t+k}(n+k)}{P_{t+k}} - \kappa(1-\zeta)^{n}\frac{\bar{A}_{t+k}}{P_{t+k}} = \beta^{k+1}\mathcal{R}_{t,t+k}\left(\frac{A^{i}_{t-1}(n-1)I_{t-1}}{P_{t}} - \kappa(1-\zeta)^{n}\frac{\bar{A}_{t-1}I_{t-1}}{P_{t}}\right) \tag{D.9}$$

This implies that the second term in (D.2) can be written as

$$\frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k}}\left(\frac{A^i_{t+k}(n+k)}{P_{t+k}}-\kappa(1-\zeta)^n\frac{\bar{A}_{t+k}}{P_{t+k}}\right)=(\beta(1-\lambda))^k\left(\frac{A^i_{t-1}(n-1)I_{t-1}}{P_t}-\kappa(1-\zeta)^n\frac{\bar{A}_{t-1}I_{t-1}}{P_t}\right), \tag{D.10}$$

which tends to zero as $k\to\infty$. This ends the proof.

E Proof of Lemma 3

In an equilibrium, all households must be on their consumption functions (8) so we can apply Lemma 2. In an equilibrium the No-Cash-on-the-Table conditions of all households must be satisfied, so from Lemma 2 condition (27) must hold. We show that it implies the conditions Lemma 3. To derive $\bar{A}_{t+k}$, we calculate A^{old}_{t+k} by calculating its complement: the quantity of public debt that is held at $t+k$ by households born at $t+1$ or later. We denote it A^{young}_{t+k}. It is equal to the sum of the debt held by households of age $0\leq n\leq k-1$ at $t+k$. Since in an equilibrium all households are on their consumption function and the debt market clears, we can use the expression for debt holdings in Lemma 1 to calculate it.

$$\begin{aligned}
\frac{A^{young}_{t+k}}{P_{t+k}}&=\sum_{n=0}^{k-1}\frac{A^i_{t+k}(n)}{P_{t+k}}N_{t+k}(n)\\
&=\left(\frac{\lambda+g}{1+g}\sum_{n=0}^{k-1}\left(\frac{1-\lambda}{1+g}\right)^n\psi_D(n)\right)\frac{D_{t+k}}{P_{t+k}}-\sum_{n=0}^{k-1}\sum_{i=0}^{n}\left(\frac{1-\lambda}{1+g}\right)^n\frac{\lambda+g}{1+g}\phi_T(n,k-j)\mathcal{R}_{t+j,t+k}T_{t+j}\\
&=\left[1-\frac{(1-\lambda)^k}{1-x}\left(\mu\left(\frac{1-\zeta}{1+g}\right)^k-\left(\frac{(\lambda+g)\kappa\beta}{1-\zeta}\right)\beta^k\right)\right]\frac{D_{t+k}}{P_{t+k}}\\
&-\sum_{j=1}^{k}(\beta(1-\lambda))^{k-j}\left[\left(1-\frac{\mu}{1-x}\right)\left(1-\left(\frac{(1-\zeta)(1-\lambda)}{1+g}\right)^j\right)+\kappa\frac{\lambda+g}{1+g}\frac{x}{1-x}\left(1-\left(\beta(1-\lambda)\right)^j\right)\right]\\
&\times\mathcal{R}_{t+j,t+k}T_{t+j}.
\end{aligned} \tag{E.1}$$

We can therefore express $\bar{A}_{t+k}$ as

$$\begin{aligned}\frac{\bar{A}_{t+k}}{P_{t+k}} &= \frac{(D_{t+k} - A^{young}_{t+k})}{P_{t+k}(1-\lambda)^k N_t} \\ &= \frac{1}{1-x}\left(\mu\left(\frac{1-\zeta}{1+g}\right)^k - \left(\frac{(\lambda+g)\kappa\beta}{1-\zeta}\right)\beta^k\right)\frac{D_{t+k}}{N_t P_{t+k}} \\ &+ \frac{1}{(1-\lambda)^k}\sum_{j=1}^{k}(\beta(1-\lambda))^{k-j}\left[\left(1-\frac{\mu}{1-x}\right)\left(1-\left(\frac{(1-\zeta)(1-\lambda)}{1+g}\right)^j\right) + \kappa\frac{\lambda+g}{1+g}\frac{x}{1-x}\left(1-\left(\beta(1-\lambda)\right)^j\right)\right] \\ &\times \mathcal{R}_{t+j,t+k}\frac{T_{t+j}}{N_t}. \end{aligned} \tag{E.2}$$

Condition (27) therefore implies that

$$\frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k}}\frac{\bar{D}_{t+k}}{P_{t+k}} = \xi(k)\frac{D_{t+k}}{N_t\mathcal{R}_{t,t+k}P_{t+k}} + (\beta(1-\lambda))^k\left(\sum_{j=1}^{k}\theta(j)\frac{1}{\mathcal{R}_{t,t+j}}\frac{T_{t+j}}{N_t}\right) \tag{E.3}$$

must tend to zero as $k \to \infty$, where

$$\xi(k) = \frac{(1-\lambda)^k}{1-x}\left(\mu\left(\frac{1-\zeta}{1+g}\right)^k - \left(\frac{(\lambda+g)\kappa\beta}{1-\zeta}\right)\beta^k\right), \tag{E.4}$$

$$\theta(j) = (\beta(1-\lambda))^{-j}\left[\left(1-\frac{\mu}{1-x}\right)\left(1-\left(\frac{(1-\zeta)(1-\lambda)}{1+g}\right)^j\right) + \kappa\frac{\lambda+g}{1+g}\frac{x}{1-x}\left(1-\left(\beta(1-\lambda)\right)^j\right)\right]. \tag{E.5}$$

Using the budget constraint of the government

$$\frac{1}{\mathcal{R}_{t,t+k}}\frac{D_{t+k}}{P_{t+k}} = \frac{D_{t-1}I_{t-1}}{P_t} - \sum_{j=0}^{k}\frac{1}{\mathcal{R}_{t,t+j}}T_{t+j} \tag{E.6}$$

to replace D_{t+k}/P_{t+k}, and multiplying by N_t, this implies equivalently that

$$\xi(k)\frac{D_{t-1}I_{t-1}}{P_t} + \left(\sum_{j=1}^{k}\left((\beta(1-\lambda))^k\theta(j) - \xi(k)\right)\frac{1}{\mathcal{R}_{t,t+j}}T_{t+j}\right) - \xi(k)T_t \tag{E.7}$$

must tend to zero as $k \to \infty$.

Since $\lambda > 0$ or $g > 0$ in the non-Ricardian case, $\xi(k)$ tends to zero as $k \to \infty$, and so the term

in $D_{t-1}I_{t-1}$ tends to zero. The condition is therefore equivalent to

$$\lim_{k\to\infty}\sum_{j=1}^{k}\left((\beta(1-\lambda))^k\theta(j)-\xi(k)\right)\frac{1}{\mathcal{R}_{t,t+j}}T_{t+j}-\xi(k)T_t=0. \tag{E.8}$$

Defining

$$\Omega(k,j)=(\beta(1-\lambda))^k\theta(j)-\xi(k) \text{ for } j\geq 1, \tag{E.9}$$

$$\Omega(k,0)=-\xi(k) \text{ for } j=0, \tag{E.10}$$

this ends the proof.

F Proof of Lemma 4

In an equilibrium, all households are on their consumption function (8). Aggregating the consumption function (8) across households alive at t gives the aggregate consumption function

$$C_t=\mu\left(\frac{A_{t-1}I_{t-1}}{P_t}+H_t\right). \tag{F.1}$$

where we denote by A_{t-1} the aggregate asset holdings inherited from $t-1$ and use the fact that only $(1-\lambda)$ of households who saved assets at $t-1$ are still alive at t: $\int_{i \text{ alive at } t} A^i_{t-1}di=(1-\lambda)\int_i A^i_{t-1}di=(1-\lambda)A_{t-1}$.

Quasi-differentiate the aggregate consumption function (O.6), using $(1-\lambda)(1-\zeta)/((1+g)R_t)$ as the discount factor:

$$C_t-\frac{(1-\lambda)(1-\zeta)}{(1+g)R_t}C_{t+1}=\mu\left[\left(\frac{A_{t-1}I_{t-1}}{P_t}-\frac{(1-\lambda)(1-\zeta)}{(1+g)R_t}\frac{A_tI_t}{P_{t+1}}\right)+Y_t-T_t\right]. \tag{F.2}$$

It simplifies using the aggregate flow budget constraint (C.2) into

$$C_t-\frac{(1-\lambda)(1-\zeta)}{(1+g)R_t}C_{t+1}=\mu\left[C_t+\left(1-\frac{(1-\lambda)(1-\zeta)}{1+g}\right)\frac{A_t}{P_t}\right]. \tag{F.3}$$

Rearranging terms, this can be rewritten as

$$C_t=\frac{1}{\beta R_t}\left(\frac{1-\zeta}{1+g}\right)C_{t+1}+\chi\frac{A_t}{P_t}, \tag{F.4}$$

where χ is defined in equation (33). Note that equation (F.4) is a purely decision-theoretic object. It relies on no assumption of market clearing. In equilibrium, market clearing holds in the debt market $A_t = D_t$. This gives equation (32).

G Proof of Proposition 2

Lemmas 3 and 5 already showed that conditions (22) and (20) are necessary for a stable price equilibrium. We show that they are sufficient. The proof is constructive. Given an initial level of public debt $D_{-1}I_{-1}$ distributed as $(A^i_{-1}I_{-1})_i$ in the population, consider a path for taxes $(T_t)_{t\geq 0}$. Define recursively the following path for the interest rate

$$R_t = \frac{1-\zeta}{\beta(1+g)}\frac{Y_{t+1}}{Y_t}\left(1-\chi\frac{D_t}{P^*Y_t}\right)^{-1}, \tag{G.1}$$

which in combination with the FBC (13) of the government defines an entire path for interest rates and public debt. Because condition (20) is satisfied, equation (G.1) defines a positive finite interest rate. For this interest rate path, define individual consumption allocations through the consumption function (8), which in combination with the FBC (5) of household i defines an entire path for its consumption and debt holdings.

We show that this allocation is a stable price equilibrium.

Market-Clearing

Start with market-clearing. Since by construction all households are on the consumption function (8), aggregate consumption is given by the aggregate consumption function (O.6), which injecting the definition of aggregate human capital (C.7) can be rewritten as

$$C_t = \mu\left(\frac{A_{t-1}I_{t-1}}{P^*} + \sum_{k=0}^{\infty}\frac{a^k}{\mathcal{R}_{t,t+k}}(Y_{t+k}-T_{t+k})\right), \tag{G.2}$$

where

$$a = \frac{(1-\lambda)(1-\zeta)}{1+g}. \tag{G.3}$$

Define

$$Z_{t+k} = a^k \frac{Y_{t+k}}{\mathcal{R}_{t,t+k}}. \tag{G.4}$$

Using the definition of the interest rate path (G.1) which can be written

$$\frac{Y_{t+1}}{R_t} = \frac{\beta(1+g)}{1-\zeta}\left(Y_t - \chi\frac{D_t}{P^*}\right), \tag{G.5}$$

Z_{t+k} satisfies the recursion

$$Z_{t+k} = bZ_{t+k-1} - \chi b\frac{a^{k-1}}{\mathcal{R}_{t,t+k-1}}\frac{D_{t+k-1}}{P^*}, \tag{G.6}$$

where

$$b = \beta(1-\lambda). \tag{G.7}$$

By iteration, it gives

$$Z_{t+k} = b^k Y_t - \chi\left(\sum_{j=0}^{k-1} b^{k-j}a^j\frac{D_{t+j}}{\mathcal{R}_{t,t+j}P^*}\right). \tag{G.8}$$

It follows that (after permuting a double sum)

$$\sum_{k=0}^{\infty} Z_{t+k} = \frac{1}{\mu}\left(Y_t - \chi\beta(1-\lambda)\left(\sum_{j=0}^{\infty} a^j\frac{D_{t+j}}{\mathcal{R}_{t,t+j}P^*}\right)\right). \tag{G.9}$$

Using the budget constraint of the government

$$\frac{D_{t+j}}{\mathcal{R}_{t,t+j}P^*} = \frac{D_{t-1}I_{t-1}}{P^*} - \sum_{i=0}^{j}\frac{1}{\mathcal{R}_{t,t+i}}T_{t+i}, \tag{G.10}$$

it can be rewritten (after permuting a double sum again)

$$\sum_{k=0}^{\infty} Z_{t+k} = \frac{1}{\mu}\left(Y_t - \frac{\chi\beta(1-\lambda)}{1-a}\left(\frac{D_{t-1}I_{t-1}}{P^*} - \sum_{i=0}^{\infty} a^i\frac{T_{t+i}}{\mathcal{R}_{t,t+i}}\right)\right). \tag{G.11}$$

Noticing that $\chi\beta(1-\lambda)/(1-a)=\mu$,

$$\sum_{k=0}^{\infty} Z_{t+k} = \frac{1}{\mu} Y_t - \left(\frac{D_{t-1} I_{t-1}}{P^*} - \sum_{i=0}^{\infty} a^i \frac{T_{t+i}}{\mathcal{R}_{t,t+i}} \right). \tag{G.12}$$

Injecting equation (G.12) into the aggregate consumption function (G.2) implies

$$C_t = Y_t + \mu \left(\frac{A_{t-1} I_{t-1}}{P^*} - \frac{D_{t-1} I_{t-1}}{P^*} \right). \tag{G.13}$$

Because $A_{-1} = D_{-1}$, this implies market clearing in the goods market at $t = 0$, and therefore in the debt market, and so on at all periods by induction.

Individual Optimality

We now check individual optimality, which is characterized by the consumption function (8) and No-Cash-on-the-Table condition (10). Condition (8) is satisfied by construction. We need to show that the No-Cash-on-the-Table constraints of all households are satisfied. Since all households behave according to the consumption function (8), we can apply Lemma 2 and simply show that condition (27) is satisfied. Since all households are on their consumption function and the goods market clears, we can use Lemma 1. That $\frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k+1}} \frac{\bar{B}_{t+k}}{P_{t+k+1}}$ tends to zero as $k \to \infty$ can therefore be rewritten as equation (E.8), which holds by assumption from condition (20).

H Absence of a Limit on the Maturity Value of Public Debt

Using instead the repayment value of debt B_t defined in equation (14), the debt-to-GDP limit (20) can be rewritten as

$$\frac{B_t}{P^* Y_t} \leq d^* R_t. \tag{H.1}$$

Since the real interest rate R_t can potentially be arbitrarily large, this does not necessarily put an upper-bound on the maturity value of debt in a stable price equilibrium. We show by way of an example that the maturity value of debt can indeed take arbitrarily high values in a stable price equilibrium. Assume that the government sets its fiscal policy to maintain a constant level of the maturity value of debt relative to GDP, $b = B_t/(P^* Y_t)$, where b is an arbitrarily large level. That is, it sets taxes each period as $T_t = b(Y_t - Y_{t+1}/R_t)$.

Equation (34) gives the real interest rate consistent with this fiscal policy,

$$R_t = \frac{1-\zeta}{\beta(1+g)}\frac{Y_{t+1}}{Y_t} + \chi b. \tag{H.2}$$

This real rate always exists for any level of the maturity value of debt b. This shows that it is possible for b to be arbitrarily large in a stable price equilibrium. Yet the maturity value of debt is then arbitrarily large only because the real interest rate (H.2) is then arbitrarily large. The government is still unable to borrow a principal larger than d^*, but it can repay an arbitrarily large amount because it repays arbitrarily large interest rates on its debt.

Figure H.1 illustrates the relationship between the amount of debt the government borrows and the amount it repays by plotting the relationship between the two, assuming per-capita GDP is constant and the government maintains debt-to-GDP at a constant level. The left panel plots the amount repaid as a function of the principal borrowed, while the right panel plots the principal borrowed as a function of the amount repaid. Under either representation, the limit d^* on the amount the government can borrow is a constraint on the government's fiscal policy.

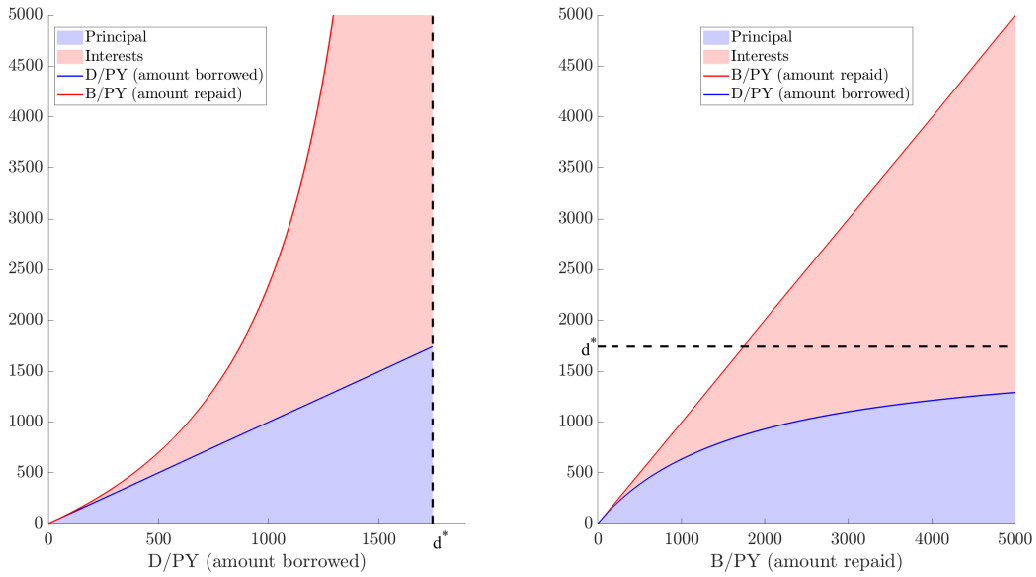


Figure H.1: Amount Borrowed vs. Amount Repaid

Note: The left panel plots the amount $B/(PY)$ the government needs to repay as a function of the principal it borrowed $D/(PY)$. The right panel plots the opposite: the principal $D/(PY)$ the government borrowed as a function of the amount it needs to repay $B/(PY)$. Both panels assume per-capita GDP is constant and the government maintains debt-to-GDP at a constant level. Debt-to-GDP is expressed in terms of annual GDP. The calibration is given in the first column of Table 1, OLG.

I Proof of Proposition 3

The two flow budget constraints (37) and (38) can be combined into the intertemporal budget constraint

$$C_t^y + \frac{1}{R_t} C_{t+1}^o = Y_t^y - T_t^y + \frac{1}{R_t}(Y_{t+1}^o - T_{t+1}^o). \tag{I.1}$$

Combining it with the household's first-order condition, the optimal behavior of households is characterized by the following consumption function,

$$C_t^y = \frac{1}{1+\beta}\left((Y_t^y - T_t^y) + \frac{1}{R_t}(Y_{t+1}^o - T_{t+1}^o)\right), \tag{I.2}$$

$$C_t^o = \frac{A_{t-1}^y I_{t-1}}{P_t} + (Y_t^o - T_t^o). \tag{I.3}$$

Given that households have finite lives and leave no wealth when they die, there is no extra No-Cash-on-the-Table constraint as part of individual optimality. Note that the flow budget constraints of young (37) and old (38) households can be combined to give the aggregate flow budget constraint

$$C_t + \frac{A_t}{P_t} = \frac{A_{t-1} I_{t-1}}{P_t} + Y_t - T_t. \tag{I.4}$$

Necessary Condition: We first show that condition (41) is necessary in equilibrium. Summing up (I.2) and (I.3), and using the fact that the debt market cleared in period $t-1$, $D_{t-1} = N_{t-1}^y A_{t-1}^y$, total aggregate consumption is given by

$$C_t = \frac{1}{1+\beta}\left(\gamma(Y_t - T_t) + \frac{1}{R_t}(1-\gamma)(Y_{t+1} - T_{t+1})\right) + \frac{A_{t-1} I_{t-1}}{P_t} + (1-\gamma)(Y_t - T_t). \tag{I.5}$$

Using the aggregate flow budget constraint (I.4), this can be rewritten

$$Y_t - T_t = \frac{1-\gamma}{\gamma\beta}\frac{1}{R_t}(Y_{t+1} - T_{t+1}) + \frac{1+\beta}{\gamma\beta}\frac{A_t}{P_t}. \tag{I.6}$$

Imposing assets market clearing gives the IS curve and setting the interest rate to infinity gives condition (41).

Sufficient Condition: If condition (41) is satisfied in all periods, define a path for the real interest

rate through

$$R_t^n(D_t) = \frac{\frac{1-\gamma}{\gamma\beta}(Y_{t+1} - T_{t+1})}{(Y_t - T_t) - \frac{1+\beta}{\gamma\beta}\frac{D_t}{P_t}} \tag{I.7}$$

and the government flow budget constraint (13). For this interest rate path, define the consumption of young and old households through the consumption function (I.2) and (I.3), with $A_t^y = D_t/N_t^y$. Individual optimality is satisfied by construction. Rewriting the expression of aggregate consumption (I.5) using the expression (I.7) for R_t shows market clearing $C_t = Y_t$. This ends the proof.

J Proof of Proposition 4

The intertemporal budget constraint of non hand-to-mouth households, denoted with the superscript PI for permanent-income households, is

$$\sum_{k=0}^{\infty} \frac{1}{\mathcal{R}_{t,t+k}} C_{t+k}^{PI} = \frac{A_{t-1} I_{t-1}}{P_t} + \sum_{k=0}^{\infty} \frac{1}{\mathcal{R}_{t,t+k}} (Y_{t+k}^{PI} - T_{t+k}^{PI}). \tag{J.1}$$

Injecting the expression for income $Y_t^{PI} = (1-\alpha_Y)Y_t$ and taxes $T_t^{PI} = (1-\alpha_T)T_t$, as well as the equilibrium requirement that $C_t^{PI} = Y_t - C_t^{HTM} = (1-\alpha_Y)Y_t + \alpha_T T_t$, gives the intertemporal budget constraint of the government (18).

The proof of sufficiency follows the proof of Proposition 1.

K Proof of Corollary 1

Condition (22) can be derived following exactly the same steps as in the case without hand-to-mouth households. Indeed, in the proof, taxes only intervene through the flow budget constraint of the government so they are not affected by the fact that part of the taxes are paid by hand-to-mouth households.

We show that condition (48) is necessary. Following the same steps as without hand-to-mouth households, one can show that equation (F.4) holds for the aggregate consumption of non hand-to-

mouth households,

$$C_t^{PI} = \frac{1}{\beta R_t}\left(\frac{1-\zeta}{1+g}\right) C_{t+1}^{PI} + \chi\left(\frac{A_t}{P_t}\right). \tag{K.1}$$

Since non hand-to-mouth households collectively hold all of public debt, assuming the debt market clears gives

$$C_t^{PI} = \frac{1}{\beta R_t}\left(\frac{1-\zeta}{1+g}\right) C_{t+1}^{PI} + \chi\left(\frac{D_t}{P_t}\right). \tag{K.2}$$

Using the fact that in equilibrium $C_t^{PI} = Y_t - C_t^{HTM} = (1-\alpha_Y)Y_t + \alpha_T T_t$ gives the IS curve

$$(1-\alpha_Y)Y_t + \alpha_T T_t = \frac{1}{\beta R_t}\left(\frac{1-\zeta}{1+g}\right)\Big((1-\alpha_Y)Y_t + \alpha_T T_t\Big) + \chi\left(\frac{D_t}{P_t}\right). \tag{K.3}$$

Condition (48) is obtained when R_t is set to infinity.

The proof of sufficiency follows the proof of Proposition 2.

L Details on the Calibration Strategies of Sections 2.5, 3.2, 3.3

For both the OLG calibration of Section 2.5 and the OLG-HTM calibration of Section 3.2, we set values for β, g and λ, which gives $\mu = 1-\beta(1-\lambda)$. For the OLG calibration, by construction $\alpha_Y = 0$. For the OLG-HTM calibration, we set $\alpha_Y = (AMPC - \mu)/(1-\mu)$, where $AMPC$ is the average MPC target of 0.2. In both calibrations, we then set ζ to match a steady-state real interest rate of 0.5% at a debt-to-GDP ratio of 100%. From equation (K.3) with $\alpha_T = 0$, the steady-state relationship between the annualized net real interest rate r and the debt-to-annual-GDP ratio $D/(4PY)$ is

$$r_0 = 4\left(\frac{1-\zeta}{\beta}\left(\frac{4\chi}{1-\alpha_Y}\frac{D}{4PY}\right)^{-1} - 1\right), \tag{L.1}$$

Equation (L.1) gives an equation in ζ which can be easily solved, knowing that χ depends on ζ through (33).

For the calibration in Section 3.3, we set values for β and g and calibrate the three remaining parameters α_Y, λ and ζ to match three statistics: an average MPC of $AMPC = 0.2$, a derivative of the real interest rate to the debt-to-annual-GDP ratio in steady-state of $\epsilon = 0.03$, and an

annualized net real interest rate at 100% debt-to-annual-GDP in steady-state of $r_0 = 0.5\%$. The three equations relating α_Y, λ and ζ to $AMPC$, ϵ and r_0 are

$$AMPC = \alpha_Y + (1-\alpha_Y)(1-\beta(1-\lambda)), \tag{L.2}$$

$$r_0 = 4\left(\frac{1-\zeta}{\beta}\left(\frac{4\chi}{1-\alpha_Y}\frac{D}{4PY}\right)^{-1} - 1\right), \tag{L.3}$$

$$\epsilon = 16\frac{1-\zeta}{\beta}\frac{\chi}{1-\alpha_Y}\left(1 - \frac{\chi\frac{D}{PY}}{1-\alpha_Y}\right)^{-2}, \tag{L.4}$$

where χ depends on λ and ζ through (33). We solve them for α_Y, λ and ζ.

Equation (L.4) can be used to calculate the sensitivity of the real interest rate to the debt-to-GDP ratio in other calibrations of the model, as reported in Table 1.

M OLG vs. HANK Interpretations and the Alternating Endowment Model

We show the importance of the feedback of the level of public debt on the probability of hitting borrowing constraints through Woodford (1990)'s alternating-endowment model, a particular case of a HANK model which is tractable enough to be solved analytically (see also Townsend (1980)). The economy is populated by two types of infinitely-lived households which cannot borrow. They face idiosyncratic risk because their individual incomes alternate between a low and a high endowment. In odd periods, type-A households receive the high endowment Y_t^h and pay taxes T_t^h while type-B households receive the low endowment Y_t^l and pay taxes T_t^l, and conversely in even periods. Both types of households have the same preferences

$$\sum_{k=0}^{\infty} \beta^k \log(C_{t+k}^i), \tag{M.1}$$

which they maximize subject to their flow budget constraint

$$\frac{A_t^i}{P_t} + C_t^i = \frac{A_{t-1}^i I_{t-1}}{P_t} + Y_t^i - T_t^i, \tag{M.2}$$

and the constraint that they cannot borrow

$$A_t^i \geq 0. \tag{M.3}$$

As Woodford (1990) shows, the model is exactly equivalent to the two-generation OLG model of Section 2.3 when the borrowing constraint of the low-endowment household is always binding. Indeed in this case, the low-endowment household holds no debt so the high-endowment household holds all of public debt. Since the high-endowment household holds positive public debt its borrowing constraint is not binding. At t, it decides how much to save facing the t and $t+1$ budget constraints

$$C_t^h + \frac{A_t}{P_t} = Y_t^h - T_t^h, \tag{M.4}$$

$$C_{t+1}^l = \frac{A_{t-1} I_{t-1}}{P_{t+1}} + Y_t^l - T_t^l, \tag{M.5}$$

where C_t^h and C_t^l are the consumption of high and low-endowment households and A_t is aggregate public debt holdings. This is exactly the same program as in the OLG model of Section 2.3, with high-income households in the stead of young households and low-income households in the stead of old households (with no population growth, $g = 0$).

The exact equivalence between the alternating-endowment model and the two-generation OLG model buttresses the case for using the perpetual-youth model in its HANK interpretation. However, the equivalence only holds when the borrowing constraint of the low-endowment households is binding. Yet whether it is binding depends on the level of public debt. If public debt is high enough there is enough liquidity in the economy for households to self-insure and avoid ever being up against their borrowing constraint in the low-endowment state. The model is then no longer equivalent to the OLG model.

To see it, consider a steady-state where the high and low endowments Y^h and Y^l are constant, and where the government maintains a constant level of public debt D through constant taxes $T = (R-1)D$, which it splits between $T^h = Y^h/Y \times T$ and $T^l = Y^l/Y \times T$. We look for a steady-state equilibrium, i.e. values for R, C^h and C^l.

In both the OLG and the HANK models, we always have

$$R = \frac{1}{\beta}\frac{C^l}{C^h}. \tag{M.6}$$

In the OLG model, we always have in addition that

$$C^h = Y^h - T^h - D, \tag{M.7}$$

$$C^l = RD + Y^l - T^l. \tag{M.8}$$

Together, the three equations (M.6)-(M.7)-(M.8) determine an equation in R that gives the equilibrium real rate.

In the HANK model, the equilibrium is characterized by equations (M.6)-(M.7)-(M.8) only up to the level of public debt such that $C^l = C^h$ and $R = 1/\beta$. Beyond this level, the low-endowment household is no longer up against its borrowing constraint, and is instead on its Euler equation

$$R = \frac{1}{\beta}\frac{C^h}{C^l}, \tag{M.9}$$

so that $C^l = C^h$ and $R = 1/\beta$. There is perfect self-insurance.

Figure M.1 illustrates the difference between the OLG and HANK models by plotting the steady-state relationship between the real interest rate and the real-debt-to-GDP ratio in both the alternating-endowment model and the OLG model. Up to a threshold on the debt-to-GDP ratio, the borrowing constraint of the low-endowment households is binding and the two models are exactly equivalent. But once public debt is above this threshold, borrowing constraints are no longer binding in the alternating-endowment model. The model is then equivalent to one with a representative Ricardian household. As a result, the real rate is constant to $1/\beta$ and no longer increases with public debt. In the OLG model in contrast, the real interest rate keeps increasing with public debt.

N Blanchard-Yaari Bewley Model

We consider a model that combines both OLG and HANK departures from the representative agent: a Blanchard-Yaari Bewley model. As in a standard Bewley (1977) model, households

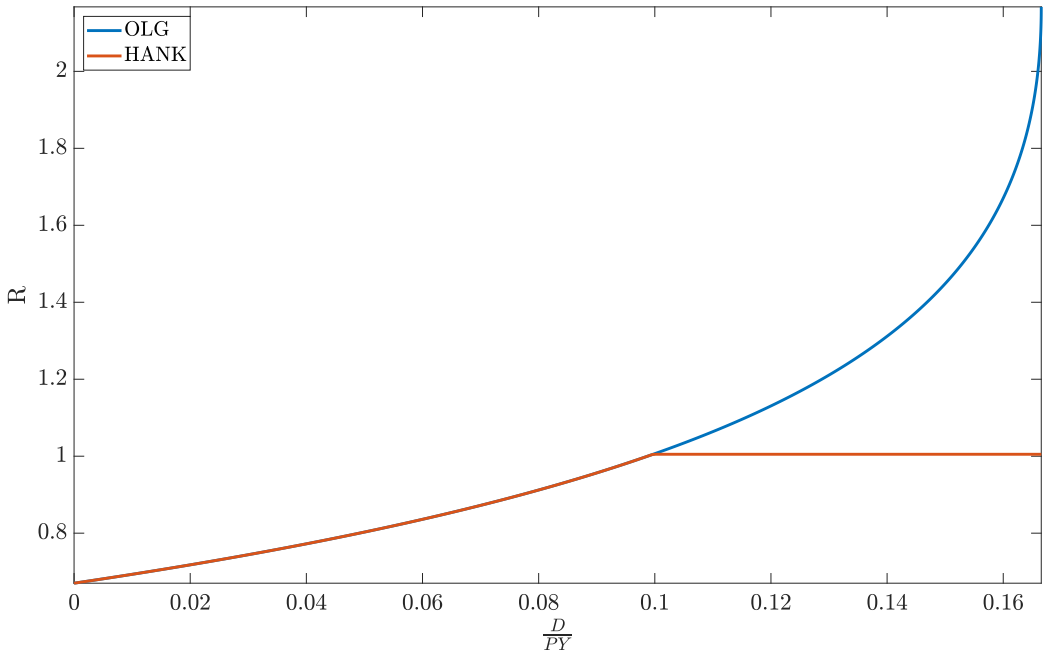


Figure M.1: Steady-State Relationship between Interest Rate and Debt-to-GDP, HANK vs. OLG

Note: The figure plots the steady-state relationship between the real interest rate R and the real debt-to-GDP ratio $D/(PY)$ in the OLG model of Section 2.3 and the alternating-endowment HANK model of Woodford (1990). The models are calibrated with $\beta = 0.995$ and, in the alternating-endowment HANK model, a difference between the high and low endowments equal to 20% of the average endowment. Taxes are distributed proportional to income.

face idiosyncratic income risk and a borrowing constraint, and can only borrow and save through non-contingent debt. As in the Blanchard-Yaari model of Section 1, households are born with no wealth, die with probability λ each period and insure against this mortality risk through life-insurance contracts. For simplicity, we abstract from population growth and from incomes that shrink with age ($g = \zeta = 0$).

A household i maximizes its utility (4) subject to its flow budget constraint (5) and No-Ponzi-scheme constraint (6), but it now faces the additional borrowing constraint

$$A_t^i \geq 0, \tag{N.1}$$

and its income is now given by an AR(1) stochastic process in log

$$\log(Y_t^i) = \rho \log(Y_{t-1}^i) + \varepsilon_t^i, \tag{N.2}$$

where the income processes are independent across households. Taxes are proportional to income. Fiscal policy is still given by a path for aggregate taxes, which still translates into a path for public debt through the flow budget constraint of the government (13). Goods market clearing still imposes equation (16).

We consider the relationship between the real interest rate and debt-to-GDP ratio in a steady-state with stable prices, and compare it to the relationship in a standard Bewley model without Blanchard-Yaari overlapping generations. We calibrate the model quarterly with $\beta = 0.99$, $\rho = 0.98$, a standard deviation of the Gaussian innovations of shocks $\sigma_\varepsilon = 0.12$, and, in the case of the Blanchard-Yaari Bewley model, $\lambda = 0.005$.[1] The relationships between the interest rate and the debt-to-GDP ratio are plotted on Figure N.1.

Two results are apparent. First, in the Blanchard-Yaari Bewley model the real interest rate does not asymptote to a finite value when debt-to-GDP increases, contrary to the Bewley model absent overlapping generations. At high levels of debt, the Blanchard-Yaari Bewley model resembles the Blanchard-Yaari model with overlapping generations alone, since households then have enough liquid wealth to self-insure. The debt-to-GDP limit d^* then applies.

Second however, financial frictions matter for the quantification of the debt limit d^*. At low to medium levels of public debt, financial frictions lower the real interest rate and steepen the slope of the relationship between interest rates and debt. As the debt level increases, the effect of financial frictions fades while the effect of overlapping generations remains. The sensitivity of interest rates to debt therefore decreases. Since what matters for the quantification of the debt limit is the sensitivity at high levels of debt, using the sensitivity at current levels of debt as we did in our main calibration in the third column of Table 1 may understate the debt-to-GDP limit d^*.

This is illustrated on Figure N.1 through the tangent to the curve of the Blanchard-Yaari Bewley model at a debt-to-GDP ratio of 100%. The slope of the tangent is 1.7bp for every 1pp increase in debt-to-annual-GDP, on the same order of magnitude than the value of 3bp in the data that we match in our main calibration. Yet because this slope encompasses the effect of financial frictions it is higher than the sensitivity of the interest rate to debt at the asymptote determined by the Blanchard-Yaari model absent financial frictions. The latter is an order of magnitude lower, at 0.12bp.[2]

[1] We approximate the idiosyncratic income process through Rouwenhorst's method with 11 grid points, and use 500 grid points on the asset dimension.

[2] It is a bit lower than in the OLG and OLG-HTM calibrations of the model in the first two columns of Table 1

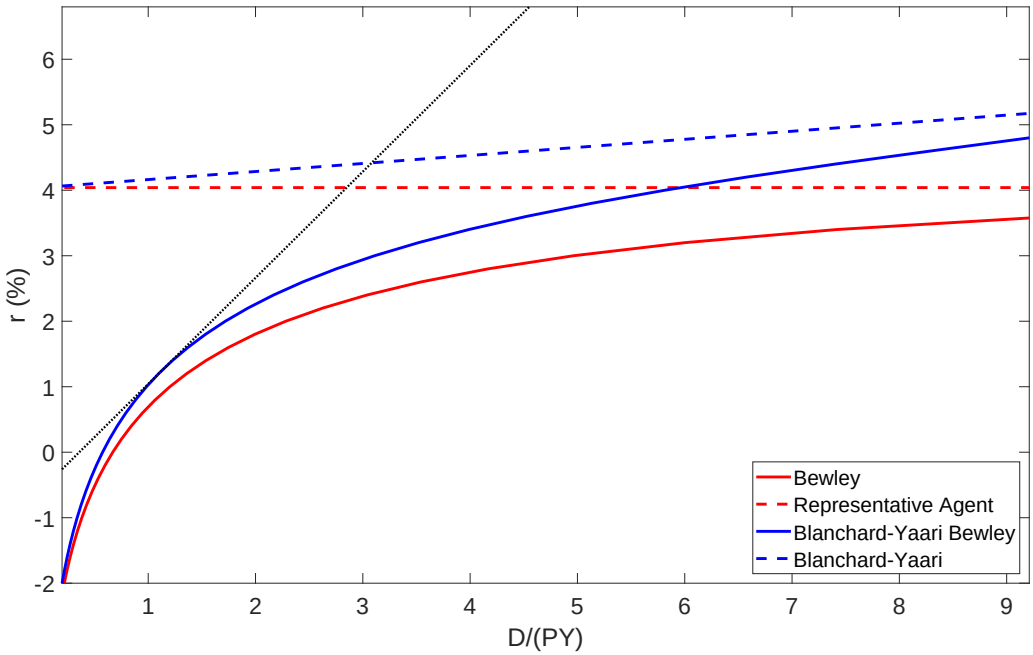


Figure N.1: Steady-State Relationship between Interest Rates and Debt-to-GDP, HANK vs. HANK+OLG

Note: The figure plots the steady-state relationship between the net real interest rate $r = R - 1$ (annualized) and the real debt-to-annual-GDP ratio in four models: the Blanchard-Yaari Bewley model of Section N, the Blanchard-Yaari model of Section 1, a standard Bewley model without overlapping generations, and a Representative Agent model. The models are calibrated with $\beta = 0.99$, $\rho = 0.98$, a standard deviation of innovations to the income process $\sigma_\varepsilon = 0.12$, and in the case of the Blanchard-Yaari HANK-OLG model, $\lambda = 0.005$. The dotted black line is the tangent to the curve of the Blanchard-Yaari Bewley model at a debt-to-annual-GDP ratio of 100%.

O Derivation of the IS Curve for Non-Log Utility

Maximizing (51) for a CRRA utility st. (5) and the No-Ponzi-scheme constraint (6) gives the Euler equation

$$C_t^i = (\beta R_t)^{-\sigma} C_{t+1}^i. \tag{O.1}$$

Iterating it forward gives for all $k \geq 0$

$$C_{t+k}^i = (\beta^k \mathcal{R}_{t,t+k})^{\sigma} C_t^i. \tag{O.2}$$

where it is 0.23bp and 0.28bp because we set $\zeta = g = 0$. But all these are an order of magnitude smaller than in our main calibration in the third column of Table 1.

Combining it with the intertemporal budget constraint (A.3) gives

$$S_t C_t^i = \frac{1}{1-\lambda} \frac{A_{t-1}^i I_{t-1}}{P_t} + H_t^i \tag{O.3}$$

where

$$S_t = \sum_{k=0}^{\infty} \Big(\beta^\sigma (1-\lambda)\Big)^k \mathcal{R}_{t,t+k}^{\sigma-1}, \tag{O.4}$$

which can be written recursively as

$$S_t = 1 + \beta^\sigma (1-\lambda) R_t^{\sigma-1} S_{t+1}. \tag{O.5}$$

Defining $\mu_t = 1/S_t$ the MPC of the household, equation (O.3) can be rewritten as the consumption function (8) with the time-varying MPC μ_t, and equation (O.4) can be rewritten as (52).

Aggregating the consumption function (O.3) across dynasties gives the aggregate consumption function

$$S_t C_t = \frac{A_{t-1} I_{t-1}}{P_t} + H_t. \tag{O.6}$$

where once again we used the fact that $\int_i A_{t-1}^i di = (1-\lambda) A_{t-1}$, since the remaining λA_{t-1} belonging to dynasties that passed away has been taken by insurance fund and redistributed to remaining households as annuities.

Quasi-differentiating the aggregate consumption function (O.6), using $(1-\lambda)(1-\zeta)/((1+g)R_t)$ as the discount factor gives

$$S_t C_t - \frac{(1-\lambda)(1-\zeta)}{(1+g)R_t} S_{t+1} C_{t+1} = \left(\frac{A_{t-1} I_{t-1}}{P_t} - \frac{(1-\lambda)(1-\zeta)}{(1+g)R_t} \frac{A_t I_t}{P_{t+1}}\right) + Y_t - T_t. \tag{O.7}$$

Using the recursive expression (O.5) of S_t and the aggregated flow budget constraint (C.2), this can be rewritten as

$$C_t = (\beta R_t)^{-\sigma} \left(\frac{1-\zeta}{1+g}\right) C_{t+1} + \chi_t \frac{A_t}{P_t}, \tag{O.8}$$

where χ_t is given in (54). Imposing market clearing, this gives the IS curve (53).

P Proof of Propositions 5 and 6

Equation (55) gives households' demand for real asset (relative to GDP) as a function of the interest rate. It can be rewritten

$$d(R) = \Theta \frac{\beta^\sigma R^\sigma - (1-\zeta)}{R - \beta^\sigma (1-\lambda) R^\sigma}, \tag{P.1}$$

where Θ is a positive constant.

P.1 Proof of Proposition 5

We restrict the analysis to values of R such that $\mu(R) > 0$. When $\sigma < 1$, this restricts to the segment $R > R_0$ with

$$R_0 = \left(\beta^\sigma (1-\lambda)\right)^{\frac{1}{1-\sigma}}. \tag{P.2}$$

For $R > R_0$, the sign of $d(R)$ is the sign of the numerator $\beta^\sigma R^\sigma - (1-\zeta)$. It is positive if and only if $R > R_1$, with

$$R_1 = \frac{(1-\zeta)^{1/\sigma}}{\beta}. \tag{P.3}$$

Differentiating (P.1), $d'(R)$ has the same sign as

$$S(R) = (1-\zeta) - (1-\zeta)\beta^\sigma \sigma (1-\lambda) R^{\sigma-1} - (1-\sigma)\beta^\sigma R^\sigma, \tag{P.4}$$

which increases up to a peak at $R_m = (1-\zeta)(1-\lambda)$ then decreases and converges to $-\infty$ as R tends to $+\infty$. From there, we can distinguish two cases, depending on the ordering of R_0 and R_1.

- If $R_1 > R_0$, i.e. if $1 - \zeta > (\beta(1-\lambda))^{\frac{\sigma}{1-\sigma}}$, then $S(R_0) > 0$ so S has exactly one zero R^*, $S(R^*) = 0$. Thus d is increasing up to R^*, then decreasing. The value $d^* = d(R^*)$ is the maximal value of real debt-to-GDP.
- If $R_1 < R_0$, i.e. if $1 - \zeta < (\beta(1-\lambda))^{\frac{\sigma}{1-\sigma}}$, then $S(R_m) < 0$ and therefore S is negative for all $R > 0$, hence d is decreasing throughout $(R_0, +\infty)$, from $+\infty$ to 0.

P.2 Proof of Proposition 6

We restrict the analysis to values of R such that $\mu(R) > 0$. When $\sigma > 1$, this restricts to the segment $R < R_0$. Over $(0, R_0)$, $d(R)$ is negative for $R < R_1$ and positive for $R > R_1$, with R_1 still defined in (P.3). When $\sigma > 1$, we always have $R_1 < R_0$.

Differentiating d, $d'(R)$ still has the sign of S defined in (P.4). When $\sigma > 1$, S is decreasing up to $R_m = (1-\zeta)(1-\lambda)$, then increasing. Since $S(R_m) > 0$, this implies S is positive over $(0, R_0)$, so that d is increasing over $(0, R_0)$, from $-\infty$ to $+\infty$. Debt-to-GDP can therefore be arbitrary large, and R tends to R_0 as d diverges to infinity.

Q Details on the Calibration Strategies in the Non-Log Case

We follow the same four calibration strategies in the non-log-utility case as in the log-utility case. Since some of these calibrations rely on the extension with a share α_Y of hand-to-mouth households of Section 3.2, we first generalize the model to a share of hand-to-mouth households. It is straightforward to show that the steady-state asset-demand function (55) generalizes to

$$d(R) = \frac{1-\alpha_Y}{\chi(R)} \left(1 - (\beta R)^{-\sigma}(1-\zeta)\right), \tag{Q.1}$$

with the expressions (56) for χ and (57) for μ unchanged. Because the asset-demand equation (Q.1) is the same as (55) up to the positive constant $1-\alpha_Y$, the derivation of the debt-to-GDP limit d^* remains the same as in Appendix O. The debt-to-GDP threshold $\underline{d}$ is simply obtained as $\underline{d} = d(1+g)$ using (Q.1).

The expressions for the average MPC and for the sensitivity of the annualized real interest rate to the debt-to-annual-GDP ratio are now

$$AMPC = \alpha_Y + \mu(R)(1-\alpha_Y), \tag{Q.2}$$

$$\epsilon = 16\,\frac{R}{d(R)}\left[\frac{\sigma(1-\zeta)(\beta R)^{-\sigma}}{1-(1-\zeta)(\beta R)^{-\sigma}} - \frac{1-\sigma}{\mu(R)}\right]^{-1}. \tag{Q.3}$$

For the OLG calibration we set values for β, g and λ, and set $\alpha_Y = 0$. We then solve numerically ζ to match a debt-to-annual-GDP ratio of 100% when the real rate is 0.5% using equation (Q.1), knowing that χ depends on ζ through (56).

For the LARGE-λ calibration, we set values for β and g, set ζ at its value in the OLG calibration, and set $\alpha_Y = 0$. We then solve numerically for λ so that the average MPC at a debt-to-annual-GDP ratio of 100% is 0.2 using equation (Q.2), knowing that μ depends on λ through (57).

For the OLG-HTM calibration, we keep β, g, λ and ζ at their values in the OLG calibation and solve for α_Y to match an average MPC of 0.2 at a debt-to-annual-GDP ratio of 100% using equation (Q.2), knowing that μ depends on α_Y because R depends on α_Y.

For the main calibration, we set values for β and g and calibrate the three remaining parameters α_Y, λ and ζ to match three statistics: an average MPC of $AMPC = 0.2$, a derivative of the real interest rate to the debt-to-annual-GDP ratio in steady-state of $\epsilon = 0.03$, and an annualized net real interest rate at 100% debt-to-annual-GDP in steady-state of $r_0 = 0.5\%$. The three equations relating α_Y, λ and ζ to $AMPC$, ϵ and d are equations (Q.2)-(Q.1)-(Q.3). We solve for them numerically.

R Proof of Proposition 7

Plugging in the policy equations (64) and (65) into the system (61)-(62) gives the 2-by-2 system in $\hat{\pi}_t$ and $\hat{b}_t$

$$\left(\phi_\pi + \eta\frac{b}{y}\right)\hat{\pi}_t + \nu_t^i = \hat{\pi}_{t+1} + \eta\left(\left(1-\psi_b\right)\hat{b}_{t-1} + \nu_t^g\right), \tag{R.1}$$

$$\hat{b}_t = \frac{R}{1+g}\left(\left(1-\psi_b\right)\hat{b}_{t-1} + \nu_t^g\right) + \frac{b}{y}\left(\left(\phi_\pi - \frac{R}{1+g}\right)\hat{\pi}_t + \nu_t^i\right). \tag{R.2}$$

The system (R.1)-(R.2) can be written in matrix form

$$A\begin{bmatrix}\hat{\pi}_t \\ \hat{b}_{t-1}\end{bmatrix} = \begin{bmatrix}\hat{\pi}_{t+1} \\ \hat{b}_t\end{bmatrix} + B\begin{bmatrix}\nu_t^i \\ \nu_t^g\end{bmatrix}, \tag{R.3}$$

where

$$A = \begin{bmatrix}\phi_\pi + \eta\frac{b}{y} & -\eta(1-\psi_b) \\ \frac{b}{y}\left(\phi_\pi - \frac{R}{1+g}\right) & \frac{R}{1+g}(1-\psi_b)\end{bmatrix} \tag{R.4}$$

$$B = \begin{bmatrix}-1 & \eta \\ -\frac{b}{y} & -\frac{R}{1+g}\end{bmatrix} \tag{R.5}$$

The economy has a unique bounded solution if and only if the matrix A has one eigenvalue outside the unit circle and one within. Calculating its trace and determinant, its roots are the solution to the quadratic equation

$$P(\upsilon) = \upsilon^2 - \left(\frac{R}{1+g}(1-\psi_b) + \phi_\pi + \eta\frac{b}{y}\right)\upsilon + (1-\psi_b)\phi_\pi\left(\frac{R}{1+g} + \eta\frac{b}{y}\right). \tag{R.6}$$

Necessary Condition: If the economy has a unique equilibrium, then one root is outside the unit circle and one is inside. If so, the roots are necessarily real (complex roots have the same modulus). If roots are real, they are both positive, so having one inside the unit circle and the other outside is equivalent to having one root greater than one and the other smaller than one. In turn, this is equivalent to $P(1) < 0$, which is condition (66).

Sufficient Condition: Conversely, if condition (66) is satisfied, then it means $1 - tr + det < 0$. In this case the determinant of the quadratic equation $\Delta = tr^2 - 4det > (det+1)^2 - 4det = (det-1)^2 > 0$, so both roots are positive. Given that condition (66) is satisfied, it means one root is greater than one and the other outside. Hence there is a unique equilibrium.

When there is a unique equilibrium, let υ be the root of A that is greater than one and let $(1, -\delta)$ be its left eigenvector. The constant δ can be solved to be

$$\delta = \frac{\eta(1-\psi_b)}{\upsilon - \frac{R}{1+g}(1-\psi_b)}. \tag{R.7}$$

Since $\upsilon > \frac{R}{1+g}(1-\psi_b)$, the constant δ is positive. Note that the expression for δ is ill-defined in the case of fiscal dominance of the Ricardian case, $\eta = 0$ and $\upsilon = \frac{R}{1+g}(1-\psi_b)$. In this case, the constant δ can be expressed as

$$\delta = \frac{\phi_\pi - \frac{R}{1+g}(1-\psi_b)}{\frac{b}{y}\left(\phi_\pi - \frac{R}{1+g}\right)}. \tag{R.8}$$

Equation (R.3) implies

$$\upsilon(\hat{\pi}_t - \delta\hat{b}_{t-1}) = (\hat{\pi}_{t+1} - \delta\hat{b}_t) + \left(\left(\delta\frac{b}{y} - 1\right)\nu_t^i + \left(\eta + \delta\frac{R}{1+g}\right)\nu_t^g\right). \tag{R.9}$$

Iterating forward gives equations (70).

Finally, we show that in the particular case $\psi_b = 0$, the impact effect of fiscal shocks on inflation

is the same as in the Ricardian case. When $\psi_b = 0$, we have that the two roots of the polynomial (R.6) are ϕ_π and $\upsilon = \frac{R}{1+g} + \eta\frac{b}{y}$. It follows that $\delta = 1/(b/y)$. Replacing the expression for δ in equation (70) gives

$$b_{t-1} - \frac{b}{y}\hat{\pi}_t = -\sum_{k=0}^{\infty}\left(\frac{1}{\frac{R}{1+g} + \eta\frac{b}{y}}\right)^k \nu^g_{t+k}, \tag{R.10}$$

which implies that in particular $\frac{\partial \pi_t}{\partial \nu^g_t} = \frac{1}{\frac{b}{y}}$.

S Proof of Proposition 8

Equations (R.1)-(R.2) are now

$$\left(\phi_\pi + \eta\frac{b}{y}\right)\hat{\pi}_t + \nu^i_t = \hat{\pi}_{t+1}, \tag{S.1}$$

$$\hat{b}_t = \left(\frac{R}{1+g} + \frac{b}{y}\eta\right)\left(\left(1-\psi_b\right)\hat{b}_{t-1} + \nu^g_t\right) + \frac{b}{y}\left(\left(\phi_\pi - \frac{R}{1+g}\right)\hat{\pi}_t + \nu^i_t\right). \tag{S.2}$$

The system (S.1)-(S.2) can be written as in equation (R.3) but replacing matrix A with the matrix

$$A' = \begin{bmatrix} \phi_\pi + \eta\frac{b}{y} & 0 \\ \frac{b}{y}\left(\phi_\pi - \frac{R}{1+g}\right) & \left(\frac{R}{1+g} + \frac{b}{y}\eta\right)(1-\psi_b) \end{bmatrix} \tag{S.3}$$

The two roots of A' are $\phi_\pi + \eta\frac{b}{y}$ and $\left(\frac{R}{1+g} + \frac{b}{y}\eta\right)(1-\psi_b)$, from which the result follows.