When Do Persistent Supply Shocks Call for Hawkishness?

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Abstract

Central banks typically rely on the following heuristic: Look through transitory supply shocks to stabilize the output gap, but pivot to stabilizing inflation if shocks are persistent. Yet baseline models only support this heuristic for markup shocks, a particular type of supply shock. For the most common supply shocks—productivity, labor supply, energy prices—stabilizing the output gap remains close to optimal regardless of the shock's persistence. What, then, rationalizes a hawkish response to persistent supply shocks? The paper shows that risks of expectations de-anchoring are insufficient: even with backward-looking expectations, output gap stabilization remains close to optimal. Instead, real wage rigidity provides a rationale. When real wages are sufficiently rigid, optimal policy shifts toward inflation stabilization when shocks are persistent, whether or not expectations can de-anchor. This suggests that signs of labor market rigidities are more essential to assess the need for a hawkish pivot than signs of poorly anchored expectations.

Keywords: Supply Shocks, De-anchoring, Real Wage Rigidity.

JEL Classification: E52, E31, E58.

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Introduction

Powell (2024) calls it *"standard thinking"* and Lagarde (2021) *"what monetary policy should normally [do]"*: In the face of supply shocks, central banks typically rely on the following heuristic.[1] If the supply shock is transitory, react moderately and prioritize stabilizing the output gap, even if it implies letting inflation rise temporarily above target. But if the supply shock is persistent, shift the focus to stabilizing inflation, as the risk of de-anchoring inflation expectations looms. This heuristic structured much of the debate on the monetary policy response to the inflation surge of 2021-2023. Dovish defenders of central banks' response insisted on the transitory nature of the shock, while proponents of a more hawkish response argued it was persistent.

Given the importance of this supply shock heuristic for policymaking, it would be expected to align with the policy recommendations of baseline monetary models. Yet, workhorse New-Keynesian models provide little support for the idea that monetary policy should tilt toward stabilizing inflation when supply shocks are more persistent. In those models, monetary policy should tilt toward stabilizing inflation only for persistent *markup* shocks—a particular type of supply shock. For the main types of supply shocks that central banks face in practice—productivity shocks such as supply chain disruptions, labor supply shocks such as changes in participation and sectoral reallocation, oil/gas price shocks, and other relative-price shocks—optimal monetary policy is very close to stabilizing the output gap, however persistent the shock may be. This is obviously the case when the divine coincidence holds and stabilizing the output gap also stabilizes inflation. But it continues to be the case when stabilizing the output gap means letting price inflation depart from target, either because both prices and nominal wages are sticky (Erceg, Henderson, and Levin 2000) or because prices are sticky in two or more sectors (La'O and Tahbaz-Salehi 2022, Rubbo 2023). This is so even though the standard New-Keynesian model puts a very large weight on inflation

[1] Powell (2024): *"Standard thinking has long been that, as long as inflation expectations remain well anchored, it can be appropriate for central banks to look through a temporary rise in inflation. [...] Beginning in October, the data turned hard against the transitory hypothesis. [...] We recognized that and pivoted beginning in November."* Lagarde (2021): *"Monetary policy should normally "look through" temporary supply-driven inflation, so long as inflation expectations remain anchored."* See also e.g. Lagarde (2025) for a statement after the inflation surge: *"Classically, monetary policy [...] "looks through" or reacts less to supply shocks that push output and inflation in opposite directions if they are sufficiently small and transitory."*

volatility—possibly an unrealistically large one (Nakamura et al. 2018).

Does it mean that central banks' supply shock heuristic is wrong, or that standard New-Keynesian models miss an important aspect of reality? If so, what assumptions and departures from the standard New-Keynesian set-up are necessary to recover the heuristic? This is the question this paper addresses. Its message is that central banks' supply shocks heuristic can be rationalized, but that, contrary to conventional wisdom, what can justify a hawkish pivot for persistent supply shocks is not de-anchoring risks. Instead, it is the existence of sufficiently strong real wage rigidity.

To study this question, the paper takes as its backbone a baseline New-Keynesian model with both price and nominal wage rigidities (Erceg, Henderson, and Levin 2000).[2] It considers four types of supply shocks: price markup shocks, wage markup shocks, productivity shocks, and labor supply shocks. As mentioned above, in the baseline version of the model the supply shock heuristic only applies to markup shocks. Against productivity and labor supply shocks, including persistent ones, optimal policy is very close to stabilizing the output gap. Stabilizing the output gap is even exactly optimal for a particular (but not extreme) set of calibrations.

The conclusion of the standard model may not be overly surprising however. The main argument to justify a hawkish pivot when shocks are persistent is the risk of de-anchoring inflation expectations. Yet, the baseline New-Keynesian model abstracts from de-anchoring risks by assuming rational expectations. The paper, therefore, first reconsiders the model assuming that expectations are adaptive (following Preston (2005)'s long-horizon approach) instead of rational, so that they can de-anchor.

The paper's first main result is that de-anchoring risks do not significantly alter the optimal policy recommendations of the model. The optimal monetary response to productivity and labor supply shocks is still very close to stabilizing the output gap, regardless of how persistent the shocks are. Once again, stabilizing the output gap is even exactly optimal for a particular set of calibrations, although a somewhat narrower set than in the case of rational expectations. Optimal policy does recommend a shift to stabilizing inflation in response to

[2]With a single sector and price or nominal wage rigidity alone, the divine coincidence obtains, so the question of stabilizing inflation or the output gap is moot. It takes two nominal rigidities to make a real one and to create a monetary policy trade-off.

persistent markup shocks, but this is already the case under rational expectations.

The reason why stabilizing the output gap still works well is ultimately the same as under rational expectations. When there are both price and wage rigidities, monetary policy does not have much impact on the real wage. What it does affect is whether a decrease in the real wage occurs through price increases or wage decreases, or any combination of the two. But stabilizing the output gap turns out to deliver a very good divide between price increases and wage decreases, which minimizes the distortions arising from price and wage dispersion, all while avoiding fluctuations in the output gap. De-anchoring risks do not challenge this logic. Under the optimal policy, long-term inflation expectations depart from target for some time, but then converge back to target.

The paper's second main result is that while de-anchoring risks per se do not rationalize a hawkish pivot when supply shocks are persistent, something else does: a sufficient degree of real wage rigidity. I introduce real wage rigidity in a way that is both a natural counterpart to nominal wage rigidity and allows for analytical tractability. Unions set *real* wages under staggered Calvo wage-setting. During the spell of a real wage, nominal wages are automatically indexed to price inflation so that the real wage remains constant. Like under Calvo nominal wage rigidity, real wages are not perfectly rigid and respond to shocks, in a forward looking manner (under rational expectations at least). I show that if real wage rigidity is sufficiently high, the optimal response to productivity and labor supply shocks shifts from stabilizing the output gap when the shock is transitory to stabilizing more price inflation when the shock is persistent. This rationalizes central banks' supply shock heuristic, but applies even if inflation expectations are rational and cannot de-anchor.

What real wage rigidity changes is that stabilizing the output gap no longer splits a decrease in the real wage between price increases and wage decreases. Instead, when real wages are sufficiently rigid, both prices and wages move in the same direction. This creates much price and wage dispersions. To minimize price and wage dispersions, monetary policy is willing to create a negative output gap. All the more so that the productivity or labor supply shock is persistent, since the risk of a sustained inflation-wage co-movement is then all the larger.

Together, the paper's two main results imply that de-anchoring risks are neither sufficient

nor necessary to rationalize a hawkish pivot for persistent supply shocks. The paper's bottom line is therefore that the risks of expectations de-anchoring are less central to assessing the need for a hawkish pivot than is commonly assumed. Conversely, signs of real wage rigidity are at least equally important to monitor.

It is also important to emphasize what the paper's results do *not* say. First, they do not say that expectations de-anchoring never matters for optimal policy. The paper's first result does imply that de-anchoring risks per se do not create a rationale for creating a negative output gap to ward off a persistent inflationary shock. Yet this result considers the situation when monetary policy is optimal at all times and has always been in the past. Central banks can make mistakes though. Given that they take decisions in real-time in the face of considerable uncertainty, they are even bound to make decisions that will appear sub-optimal in hindsight. Following such mistakes, whether expectations can de-anchor does make an important difference. Assume, for instance, that monetary policy has been too loose, for instance because the central bank underestimated the output gap in real time (Orphanides 2001). As a result, the output gap has been positive and inflation has run higher than it should have under optimal (full-information) policy. In this case, targeting a zero output gap is still close to optimal under rational expectations. But if expectations are backward-looking, monetary policy should run a negative output gap to bring inflation down to target faster. Yet, even in this case, the recommendation to run a negative output gap does not follow from the fact that long-run inflation expectations are running above target. Long-term inflation expectations move above target under the fully optimal policy as well. What calls for a negative output gap is if long-term inflation expectations have been moving more above target than under the optimal policy.

Second, the paper's results do not say that, absent real wage rigidity, the central bank should not react to persistent productivity and labor supply shocks. Optimal policy is dovish in the absence of sufficient real wage rigidity, but it is dovish in the usual sense of stabilizing the output gap at the expense of stabilizing price inflation. Stabilizing the output gap typically requires increasing interest rates. A recommendation that monetary policy should not react to shocks at all is difficult to obtain in monetary models, and the models of this paper are no exception. There is a common argument for not reacting at all however: the

notion that monetary policy works with "long and variable lags" (Friedman 1961). If a shock is short-lived, any benefit of a rate hike may then come too late.[3] The paper's results do not rely on the "long and variable lags" argument.[4] The paper assumes that monetary policy can control aggregate demand without delays and asks whether it should use this ability to stabilize the output gap or to stabilize price inflation.

The paper connects to several literatures. It first builds on the papers that consider optimal monetary policy in economies with nominal rigidities on several goods. This includes the combination of price and wage rigidity in Erceg, Henderson, and Levin (2000), but also the combination of price rigidities in several sectors (Aoki 2001, Benigno 2004, Eusepi, Hobijn, and Tambalotti 2011, La'O and Tahbaz-Salehi 2022, Rubbo 2023, Afrouzi, Bhattarai, and Wu 2024). A robust conclusion of this literature, which restricts to rational expectations and abstracts from real wage rigidity, is that stabilizing the output gap is a very good proxy to optimal monetary policy.

The paper connects to the adaptive learning literature that reconsiders optimal policy recommendations under backward-looking expectations. Gaspar, Smets, and Vestin (2010), Molnár and Santoro (2014), Eusepi and Preston (2018) consider models with price rigidity and markup shocks as supply shocks and show that monetary policy should stabilize price inflation more than under rational expectations when expectations can de-anchor. I depart by focusing on the optimal policy response to productivity and labor supply shocks and by assuming both price and wage rigidities.

Blanchard and Galí (2007) and Blanchard and Galí (2010) study the implications of real wage rigidity in New-Keynesian models, highlighting how it breaks the divine coincidence.[5]

[3]For a recent empirical investigation of the argument, see Buda et al. (2023), who actually find that the impact of monetary policy shocks on output can be seen as early as a few days after the shock.

[4]For this reason, the paper abstains from referring to central banks' supply shock heuristic as the "looking through" heuristic: *"Look through temporary shocks but not persistent ones"*. The term "looking through" is often used to mean what is meant in this paper: stabilizing the output gap instead of inflation. For instance Carney (2017), *"Monetary policy makers may be more inclined to "look through" the direct, temporary effects of commodity price shocks on inflation, which means they do not intend to engineer output gaps to offset the inflationary effects in response."* But it is also used to mean not reacting in the sense of keeping interest rates unchanged. To avoid any ambiguity, the paper mostly avoids the term. On the history of the concept of "looking through", see Nelson (2025).

[5]On real wage rigidity in mid-scale New-Keynesian models, see e.g. Christiano, Eichenbaum, and Trabandt (2016), who model real wage rigidity through Hall and Milgrom (2008) 's alternating offer bargaining within a search and matching framework.

I show how it can rationalize central banks' supply side heuristic, while de-anchoring risks do not. Rudd (2022) expresses skepticism about the empirical importance of inflation expectations for inflation dynamics. I similarly argue that expectations de-anchoring is less central than commonly assumed, but from a normative perspective.

The paper finally connects the literature on wage-price spirals, as revived by Blanchard (1986) and more recently Lorenzoni and Werning (2023b).[6] It emphasizes the different normative implications of expectations de-anchoring and sticky aspirations (in the form of real wage rigidity), two elements that are typically considered central to a wage-price spiral. While Lorenzoni and Werning (2023b) build their analysis on the case of rational expectations and flexible aspirations, its final section considers a model with both adaptive expectations and real rigidities. In contrast to them, I consider optimal policy under these two assumptions, and highlight the difference between the normative implications of adaptive expectations and real wage rigidity.

More broadly, the paper connects to studies that seek to account for the 2021-2023 inflation surge and its monetary policy response, such as Erceg, Lindé, and Trabandt (2024). They consider the effect of cost-push supply shocks, while I contrast the different implications of cost-push and other supply shocks.

The paper is organized as follows. Section 1 lays out the backbone of the models used throughout the paper. Section 2 recalls optimal policy results in the standard model under rational expectations. Section 3 considers the model under adaptive expectations and de-anchoring risks. Section 4 considers the model under real wage rigidity.

1 Price and Wage Setting

This section presents the backbone of the models used throughout the paper: nominal price and wage rigidities modeled through Calvo staggered price/wage-setting. It presents it without making any assumptions on expectations formation. Under rational expectations it is essentially the model of Erceg, Henderson, and Levin (2000), written with four distinct types of supply shocks: productivity shocks, labor-supply shocks, price markup shocks, and wage

[6] See also Lorenzoni and Werning (2023a).

markup shocks.

1.1 Firms' Price-Setting

Firm i produces good i from a composite labor input N_t through the production function

$$Y_t^i = A_t(N_t^i)^\alpha, \tag{1}$$

where A_t is aggregate productivity. It takes as given the nominal wage W_t and the demand curve it faces, derived from consumers' standard CES preferences,

$$Y_t^i = \left(\frac{P_t^i}{P_t}\right)^{-\theta_t^p} Y_t, \tag{2}$$

where Y_t is aggregate demand, P_t the aggregate price level, P_t^i the price firm i sets for its good, and θ_t^p the elasticity of demand at t (equal to consumers' elasticity of substitution across goods).

Denote log-deviations with lowercase variables. Firm i faces a probability μ^p of not being able to reset its price in any given period. Appendix A shows that when firm i does reset its price, it resets it to

$$p_t^* = (1-\beta\mu^p)\sum_{k=0}^{\infty}(\beta\mu^p)^k\tilde{\mathbb{E}}_t\left(\zeta_p\left(w_{t+k} - mrt_{t+k} + m_{t+k}^p\right) + \frac{\zeta_p(1-\alpha)\theta^p}{\alpha}p_{t+k}\right), \tag{3}$$

where

$$mrt_t = \frac{1}{\alpha}a_t - \frac{1-\alpha}{\alpha}y_t \tag{4}$$

is the economy-wide marginal rate of transformation, m_t^p is an exogenous price markup shock which is a direct function of fluctuations in the elasticity of demand θ_t^p, θ^p is the steady-state value of θ_t^p, and

$$\zeta^p = \frac{1}{1+\frac{(1-\alpha)\theta^p}{\alpha}}, \tag{5}$$

The notation $\tilde{\mathbb{E}}$ stands for firm i's subjective expectations, on which no further assumption is made for the moment. They may or may not be rational.

1.2 Unions' Wage-Setting

The composite labor input N_t is a CES aggregator of a continuum of labor types N_t^j with elasticity of substitution θ_t^w. The nominal wage W_t^j of labor type N_t^j is set by labor union j, which faces the labor demand

$$N_t^j = \left(\frac{W_t^j}{W_t}\right)^{-\theta_t^w} N_t. \tag{6}$$

Union j represents the interests of workers who value consumption and dislike labor through the preferences

$$\tilde{\mathbb{E}}_0 \sum_{t=0}^{\infty} \beta_t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \mathcal{E}_t \int_j \frac{(N_t^j)^{1+\psi}}{1+\psi} dj \right), \tag{7}$$

where C_t is aggregate consumption, $\mathcal{E}_t$ is a labor-supply shock.

Union j faces a probability μ^w of not being able to reset its wage every period. Appendix A shows that when it does reset its wage it resets it to

$$w_t^* = (1-\beta\mu^w) \sum_{k=0}^{\infty} (\beta\mu^w)^k \tilde{\mathbb{E}}_t \left(\zeta_w \left(p_{t+k} + mrs_{t+k} + m_{t+k}^w \right) + (1-\zeta^w) w_{t+k} \right), \tag{8}$$

where

$$mrs_t = \psi n_t + \sigma c_t + e_t, \tag{9}$$

is the economy-wide marginal rate of substitution, m_t^w is an exogenous wage markup shock which is a direct function of fluctuations in the elasticity of labor demand θ_t^w, θ^w is the steady-state value of θ_t^w, and

$$\zeta^w = \frac{1}{1+\psi\theta^w}. \tag{10}$$

From the Calvo staggered pricing assumptions, price and wage inflation relate to the resetting price (3) and resetting wage (8) through

$$\pi_t^p = \frac{1-\mu^p}{\mu^p}(p_t^* - p_t), \tag{11}$$

$$\pi_t^w = \frac{1-\mu^w}{\mu^w}(w_t^* - w_t). \tag{12}$$

The real wage $\omega_t = w_t - p_t$ follows the dynamics

$$\omega_t = \omega_{t-1} + \pi_t^w - \pi_t^p. \tag{13}$$

1.3 Types of Supply Shocks

The model captures four types of supply shocks: productivity shocks a_t, labor supply shocks e_t, price markup shocks m_t^p and wage markup shocks m_t^w. All of them are assumed to follow AR(1) processes in log, with autoregressive coefficients ρ^a, ρ^e, ρ^{mp}, ρ^{mw}, and standard deviations of the innovations σ_ε^a, σ_ε^e, σ_ε^{mp}, σ_ε^{mw}.

1.4 Welfare

To evaluate different monetary policies, I use the model's microfounded loss function, corresponding to a second-order approximation of the representative household's utility function (7) (see Rotemberg and Woodford 1999, Woodford 2003, or Appendix B for a re-derivation). Regardless of the assumptions made on the private-sector's expectations, I always assume that the central bank has rational expectations. Assuming that the steady-state is efficient, the loss function of the central bank can be written in permanent-consumption equivalent terms and in distance to the first best,

$$\mathcal{L} = \frac{1}{2}(1-\beta)\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\lambda_x (x_t)^2 + \lambda_p (\pi_t^p)^2 + \lambda_w (\pi_t^w)^2\right], \tag{14}$$

where $x_t = y_t - y_t^e$ is the output gap defined in distance to the efficient level of output, and where the weights are given by

$$\lambda_x = \sigma + \frac{\psi + 1 - \alpha}{\alpha}, \tag{15}$$

$$\lambda_p = \frac{\theta^p}{\kappa_c^p}, \tag{16}$$

$$\lambda_w = \frac{\alpha\theta^w}{\kappa_c^w}. \tag{17}$$

where

$$\kappa_c^p = \frac{(1-\beta\mu^p)(1-\mu^p)}{\mu^p}\zeta^p, \tag{18}$$

$$\kappa_c^w = \frac{(1-\beta\mu^w)(1-\mu^w)}{\mu^w}\zeta^w. \tag{19}$$

Under Calvo staggered price and wage-setting, the microfounded weights λ_p and λ_w on price and wage inflation arise from relative price and wage dispersion. Under standard calibrations, they are typically much larger than the weight on the output gap λ_x: the Calvo model assigns very large costs to inflation. The Calvo costs of inflation may be unrealistically high (Nakamura et al. 2018). Throughout the paper, however, I stick to the microfounded Calvo weights. All results are therefore derived under conservatively hawkish preferences. The paper will show that even with those, it is actually difficult to get the model to depart from the dovish recommendation of stabilizing the output gap.

1.5 Monetary Policy

The paper's interest lies in how dovish or hawkish a model's optimal policy recommendations are—distinguishing between different shocks and different degrees of shock persistence. To help capture how hawkish or dovish a model's recommendations are, it is useful to consider a special class of monetary policies, defined through the objective that the central bank pursues (Svensson 2003). These are the rules stabilizing a weighted average of price inflation

and the output gap

$$(1-\phi)\pi_t^p + \phi x_t = 0, \tag{20}$$

where ϕ is between zero and one. The higher ϕ, the more dovish the policy. Within this class of policies, the best policy corresponds to the value of ϕ that minimizes the central bank's loss (14). Focusing on the policies (20) is helpful for capturing a model's recommendations, but it is not necessary. The paper will also consider fully optimal monetary policy.

Note that I use the terms hawkish and dovish to describe policies, not preferences. A hawkish policy is one that is close to stabilizing price inflation. A dovish policy is one that is close to stabilizing the output gap. I will say a model's recommendation is dovish if its optimal policy recommendation is close to a policy rule (20) with ϕ close to one. I do not use hawkish and dovish to describe varying preferences.[7] I will always use the microfounded preferences (14) to evaluate policies.

1.6 Calibration

Unless otherwise stated, all figures are plotted for the default calibration in the first column of Table 1. The calibration is intended to be as standard as possible. The discount factor β corresponds to 2% per year. The labor share is $\alpha = 2/3$. The inverse Frisch elasticity of labor supply is $\psi = 2$. The income effect on labor supply is $\sigma = 1$. The steady-state elasticities of both goods and labor demand are $\theta^p = \theta^w = 7$. Firms change their prices every three quarters ($\mu^p = 0.66$) and unions change their wages every five quarters ($\mu^w = 0.8$).

2 Standard New-Keynesian Recommendations

This section considers whether the standard New-Keynesian model under rational expectations rationalizes central banks' supply shock heuristics. It shows that it does for markup

[7] If a model has dovish recommendations, it can be said to be a dovish model. However, a model's policy recommendations do not arise from the model's "preferences" alone—as captured by its microfounded loss. They arise from the combination of its loss function and its perspective on how different policies affect the economy.

Table 1: Default Calibration

	Default Calibration	Exact-Result Calibration
β	0.995	-
α	2/3	-
ψ	2	1/3
σ	1	0
θ^p	7	-
θ^w	7	10.5
μ^p	0.66	0.8
μ^w	0.8	-

Note: The table gives the default calibration used in the paper, as well as one example of an exact-result calibration satisfying the conditions of Proposition 1. The calibration is quarterly.

shocks, but not for productivity and labor supply shocks.

2.1 Phillips Curves

Under rational expectations, the price-setting and wage-setting equations can be rewritten to give the price and wage New-Keynesian Phillips curves

$$\pi_t^p = \kappa_c^p(\omega_t - \omega_t^e) + \kappa_y^p x_t + \beta \mathbb{E}_t \pi_{t+1}^p + \kappa_c^p m_t^p, \tag{21}$$

$$\pi_t^w = -\kappa_c^w(\omega_t - \omega_t^e) + \kappa_y^w x_t + \beta \mathbb{E}_t \pi_{t+1}^w + \kappa_c^w m_t^w, \tag{22}$$

where $\omega_t - \omega_t^e$ is the distance of the real wage to the level of real wages ω_t^e that would obtain in an economy under perfect competition and absent nominal rigidities, where the slopes of the Phillips curves wrt. the real wage κ_c^p and κ_c^w are defined in (18)-(19), and where the slopes of the Phillips curves wrt. the output gap are

$$\kappa_y^p = \frac{1-\alpha}{\alpha}\kappa_c^p, \tag{23}$$

$$\kappa_y^w = \left(\frac{\psi}{\alpha} + \sigma\right)\kappa_c^w \tag{24}$$

See Appendix C for a derivation.

2.2 Should the Central Bank Be Dovish or Hawkish?

Should the central bank respond to supply shocks in a dovish way, stabilizing the output gap? Or should its response be hawkish, stabilizing price inflation instead? Does it depend on the type of supply shock? Does it depend on the persistence of the shock? The responses of the standard New-Keynesian model to these questions are summed up in Figure 1. For each type of supply shock, the figure plots the loss function of the central bank as a function of the weight ϕ on the output gap in the objective (20) of the central bank, under the assumption that the economy is hit by only this one type of shock. It does so for different levels of persistence of the shock, captured by different values of the auto-regressive coefficient of the AR(1) process. For a more concrete interpretation, the value of the auto-regressive coefficient is expressed through the corresponding shock half-life.[8]

For price markup shocks—commonly referred to as "cost-push shocks"—the model makes prescriptions that are very much in line with central banks' supply shock heuristic. When the shock is transitory (low half-life), the central bank minimizes its loss by being dovish, i.e. by putting a high weight on stabilizing the output gap (large ϕ), at the expense of stabilizing price inflation. But as the shock becomes more persistent, the central bank should become more hawkish, placing increasing weight on stabilizing price inflation at the expense of the output gap.

Yet, the recommendation to adopt a more hawkish stance for more persistent shocks is specific to price markup shocks. For productivity and labor supply shocks, the model concludes instead that the central bank should stabilize the output gap regardless of how persistent the shock is. This contradicts central banks' supply shock heuristic. For wage markup shocks, the recommendation is yet different: the central bank should always stabilize price inflation at the expense of the output gap, even when the shock is transitory.

2.3 Why Stabilizing the Output Gap Works Well

Why is stabilizing the output gap a good policy in response to productivity and labor-supply shocks? Two observations are key. First, when there is both price and wage rigidity,

[8] A half-life of N quarters corresponds to a value of $\rho = (1/2)^{\frac{1}{N}}$.

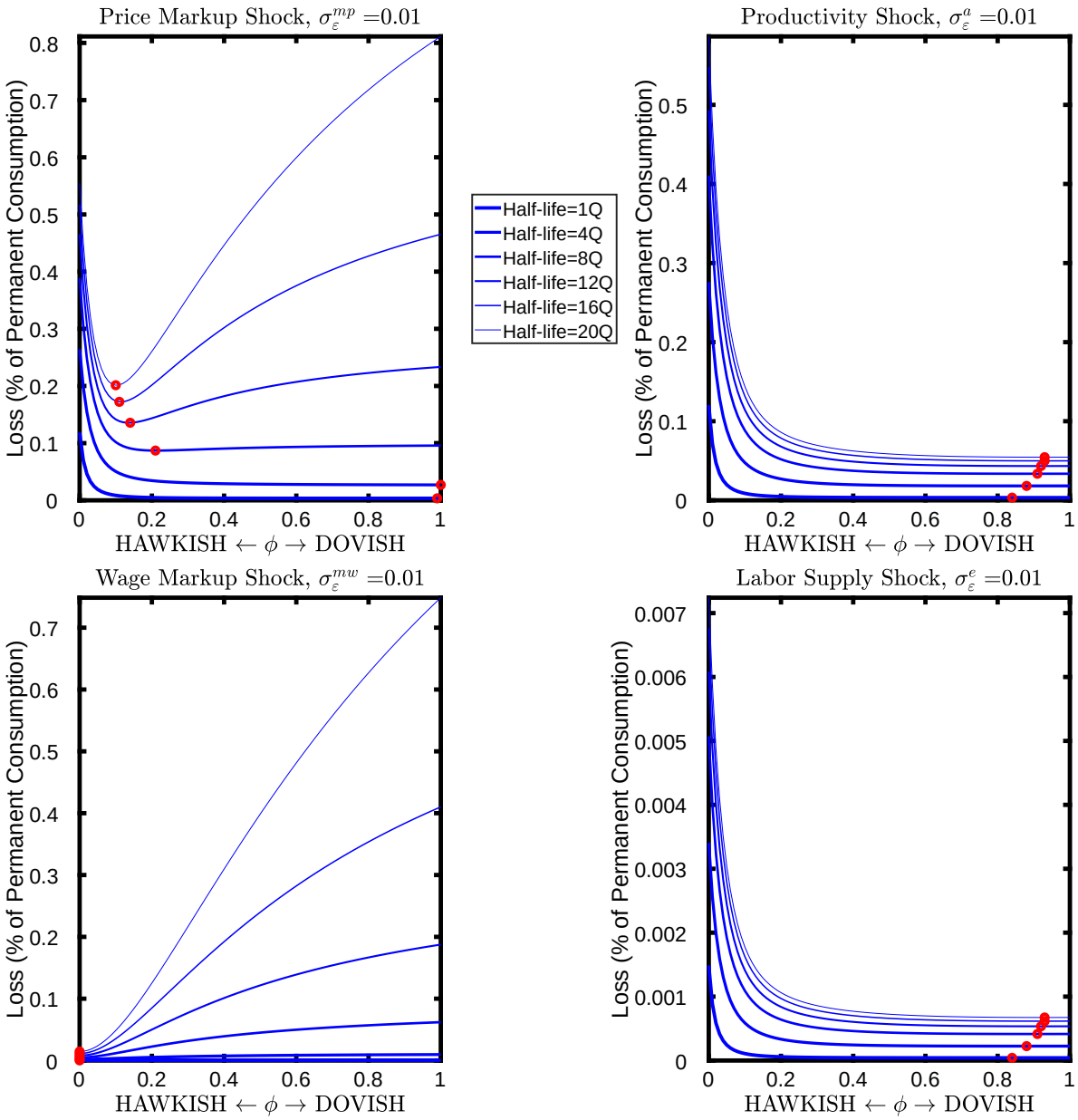


Figure 1: Loss Function under Rational Expectations

Note: For each type of supply shock, the figure plots the loss function of the central bank (14) as a function of the weights ϕ on the output gap in the objective (20) of the central bank, under the assumption that the economy is hit by this one type of shock only. It does so for different levels of persistence of the shock, captured by the half-life of the AR(1) process. On each curve, the red dot corresponds to the minimum of the curve, that is the optimal policy within the class of policies (20). The calibration is given in the "Default Calibration" column of Table 1.

monetary policy ultimately has little effect on the dynamics of the real wage. To see this, take the difference between the wage (22) and price (21) Phillips curves to get the dynamics

of the real wage as

$$\Delta_t = -\left(\kappa_c^w + \kappa_c^p\right)(\omega_t - \omega_t^e) + \left(\kappa_y^w - \kappa_y^p\right)x_t + \beta \mathbb{E}_t \Delta_{t+1} + \kappa_c^w m_t^w - \kappa_c^p m_t^p. \tag{25}$$

where Δ_t is the growth rate of real wages

$$\Delta_t = \omega_t - \omega_{t-1}. \tag{26}$$

Monetary policy can affect the path of the real wage by allowing for a larger or smaller output gap x_t, but the output gap affects the growth rate of wages through the difference between the two slopes $\kappa_y^w - \kappa_y^p$. If the price and wage Phillips curves have about the same slope, then monetary policy has little effect on the dynamics of the real wage. Intuitively, higher aggregate demand pushes both firms to increase their prices (because of decreasing returns to scale in production) and unions to increase their wages (because of the decreasing marginal utility of labor and a possible income effect on labor supply). When $\kappa_y^w = \kappa_y^p$, higher aggregate demand pushes prices and wages by the same amount, and the real wage is unaffected. Even beyond this particular case, the effect on the real wage is small—at least unless there is a very strong asymmetry between the degrees of price and wage rigidity. Ultimately, what monetary policy can affect is not so much the real wage but whether the change in the real wage occurs through a fall in nominal wages, an increase in prices, or some combination of the two.

Second, stabilizing the output gap turns out to deliver a very good split of a real wage increase between price increases and wage decreases. To see this, define the weighted average of price and wage inflation

$$\pi_t^{DC} = \left(\frac{\kappa_c^w}{\kappa_c^p + \kappa_c^w}\right)\pi_t^p + \left(\frac{\kappa_c^p}{\kappa_c^p + \kappa_c^w}\right)\pi_t^w, \tag{27}$$

which satisfies

$$\pi_t^{DC} = \left(\frac{\kappa_c^w \kappa_y^p + \kappa_c^p \kappa_y^w}{\kappa_c^w + \kappa_c^p}\right)x_t + \beta \mathbb{E}_t \pi_{t+1}^{DC} + \left(\frac{\kappa_c^p \kappa_c^w}{\kappa_c^w + \kappa_c^p}\right)(m_t^p + m_t^w), \tag{28}$$

In response to productivity and labor-supply shocks, stabilizing the output gap stabilizes the weighted average π_t^{DC}. Following Rubbo (2023), I call the index the Divine Coincidence index.[9] In itself, there is nothing particularly divine about the fact that stabilizing the output gap stabilizes a particular weighted average of price and wage inflation. What *is* divine about π_t^{DC} is that stabilizing it turns out to be very close to minimizing the distortions arising from price and wage inflation.[10]

To start with, if π_t^{DC} remains constant, it means price inflation and wage inflation have different signs: an increase in the real wage is split between price increases and wage decreases. This is already a good idea, better than relying only on price increases or on wage decreases alone, since welfare losses are quadratic in price and wage inflation. But better still, the exact weights in π_t^{DC} have very good properties. Stabilizing π^{DC} is closer to stabilizing price inflation the more rigid prices are relative to wages, making the adjustment occur through nominal wages instead. Since price dispersion is relatively more costly the more rigid prices are, this is good. A particular set of calibrations buttresses this point, as shown in Woodford (2003) and Galí (2008).

Proposition 1. *Consider the model under rational expectations. If $\kappa_y^w = \kappa_y^p$ and $\theta^p = \alpha\theta^w$, then the optimal monetary policy in response to productivity and labor supply shocks is to stabilize the output gap at all times $x_t = 0$, under both commitment or discretion.*

To see why the result holds, make the change of variables from (π_t^p, π_t^w) to $(\bar{\pi}_t, \Delta_t)$, where $\bar{\pi}_t$ is the following weighted average of price and wage inflation

$$\bar{\pi}_t = \frac{\lambda_p}{\lambda_p + \lambda_w}\pi_t^p + \frac{\lambda_w}{\lambda_p + \lambda_w}\pi_t^w. \tag{29}$$

The central bank's loss can then be rewritten as

$$\mathbb{W} = -\frac{1}{2}(1-\beta)\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\lambda_x (x_t)^2 + (\lambda_p + \lambda_w)\bar{\pi}_t^2 + \frac{1}{\lambda_p^{-1} + \lambda_w^{-1}}\Delta_t^2\right]. \tag{30}$$

[9] Rubbo uses the term to define a weighted average of price inflation rates across sectors in a multisector model with input/output linkages and wage flexibility. The idea is the same however.

[10] Divine as this is, even in the case of Proposition 1 the optimal policy does not achieve the first best, in contrast to the case of the divine coincidence under price rigidity alone.

As explained before, if $\kappa_y^w = \kappa_y^p$ then the dynamics of the real wage is independent of monetary policy and the central bank trades off stabilizing $\bar{\pi}_t$ and stabilizing x_t. Now, if in addition $\lambda_p \kappa_c^p = \lambda_w \kappa_c^w$, which is equivalent to $\theta^p = \alpha \theta^w$, then $\bar{\pi}_t$ corresponds to the Divine Coincidence index π_t^{DC}, so there is no trade-off between stabilizing $\bar{\pi}_t$ and stabilizing the output gap x_t for any shock that enters the Phillips curves through ω_t^e only: productivity and labor supply shocks. For these shocks, it is possible to set both $\bar{\pi}_t = 0$ and $x_t = 0$ at all times, which minimizes the loss function (30).

While the exact optimality of stabilizing the output gap only obtains in the particular cases of Proposition 1, stabilizing the output gap remains very close to the optimal monetary policy under standard calibrations. Crucially, since none of the reasons why stabilizing the output gap is a good policy hinge on the shock being transitory, it applies similarly to persistent shocks.

2.4 Abstracting from Wage Dispersion Losses

The logic behind Proposition 1 can suggest that the dovish recommendation to stabilize the output gap rather than price inflation depends heavily on taking the welfare costs of wage dispersion into account. Stabilizing price inflation only shifts the real-wage adjustment to nominal wages, which only shifts welfare costs to wage dispersion. While these welfare losses are real and the natural counterpart of the welfare costs of price inflation, this can raise an objection. Stabilizing nominal wages is not part of any major central bank's mandate, and central banks rarely articulate it as a policy objective. If the recommendation to stabilize the output gap depends on taking into account the losses from wage dispersion, it can be seen as conflicting with central banks' mandates. The result may then primarily be interpreted as a recommendation to revise these mandates to better reflect the welfare costs of each policy.

The good properties of stabilizing the output gap do not depend on including the welfare costs of wage inflation in the loss function however. Figure 2 plots the loss function for different degrees of dovishness and hawkishness, but this time both with and without including the last wage dispersion term $\lambda_w (\pi_t^w)^2$ in (14). To maintain readability, the figure is plotted for a single degree of shock persistence: a half-life of 16 quarters. The figure shows that, abstracting from the costs of wage inflation, stabilizing the output gap remains close

to optimal. The loss from pursuing a hawkish policy is smaller when abstracting from the costs of larger wage dispersion. But hawkish policies with a small ϕ are still the costliest ones, and more costly than the high-ϕ dovish policies. The exact optimality of stabilizing the output gap no longer obtains even under the set of calibrations in Proposition 1. But stabilizing the output gap is still close to optimal.

Why is stabilizing the output gap still a good idea? The microfounded Calvo weights are still conservatively hawkish: they put a large weight on price inflation and a small weight on the output gap. Yet, stabilizing price inflation requires very large variations in the output gap. As a result, for hawkish policies the loss from output gap variations $\lambda_x x_t^2$ is very large.

This can be seen on Figure 2, which plots the response of the economy to a persistent negative productivity shock under four policies: stabilizing price inflation, stabilizing the output gap, the fully optimal policy, and the optimal policy abstracting from the costs of wage dispersion. The latter two are optimal policies under commitment, characterized in Appendix D. Stabilizing price inflation requires a large drop in the output gap. While it avoids an increase in price inflation, it only shifts the adjustment in the real wage to a decrease in nominal wages. Under the microfounded loss function, stabilizing price inflation only trades inefficient price dispersion for inefficient wage dispersion. This has little benefit and brings output gap costs. The optimal monetary policy is therefore very close to stabilizing the output gap. Under the loss function that abstracts from the costs of wage dispersion, there is some benefit in replacing inefficient price dispersion with wage dispersion, but this benefit still comes at a large output gap cost. Optimal policy allows for a slightly negative output gap, but it is still much closer to stabilizing the output gap than to stabilizing price inflation.

The model's dovish recommendations only require the presence of wage rigidity in addition to price rigidity, whether or not the welfare costs of wage dispersion are taken into account. Absent wage rigidity, the real wage can adjust without a negative output gap. Stabilizing the output gap is still optimal, but so is stabilizing price inflation since the divine coincidence holds, so that both are equivalent.

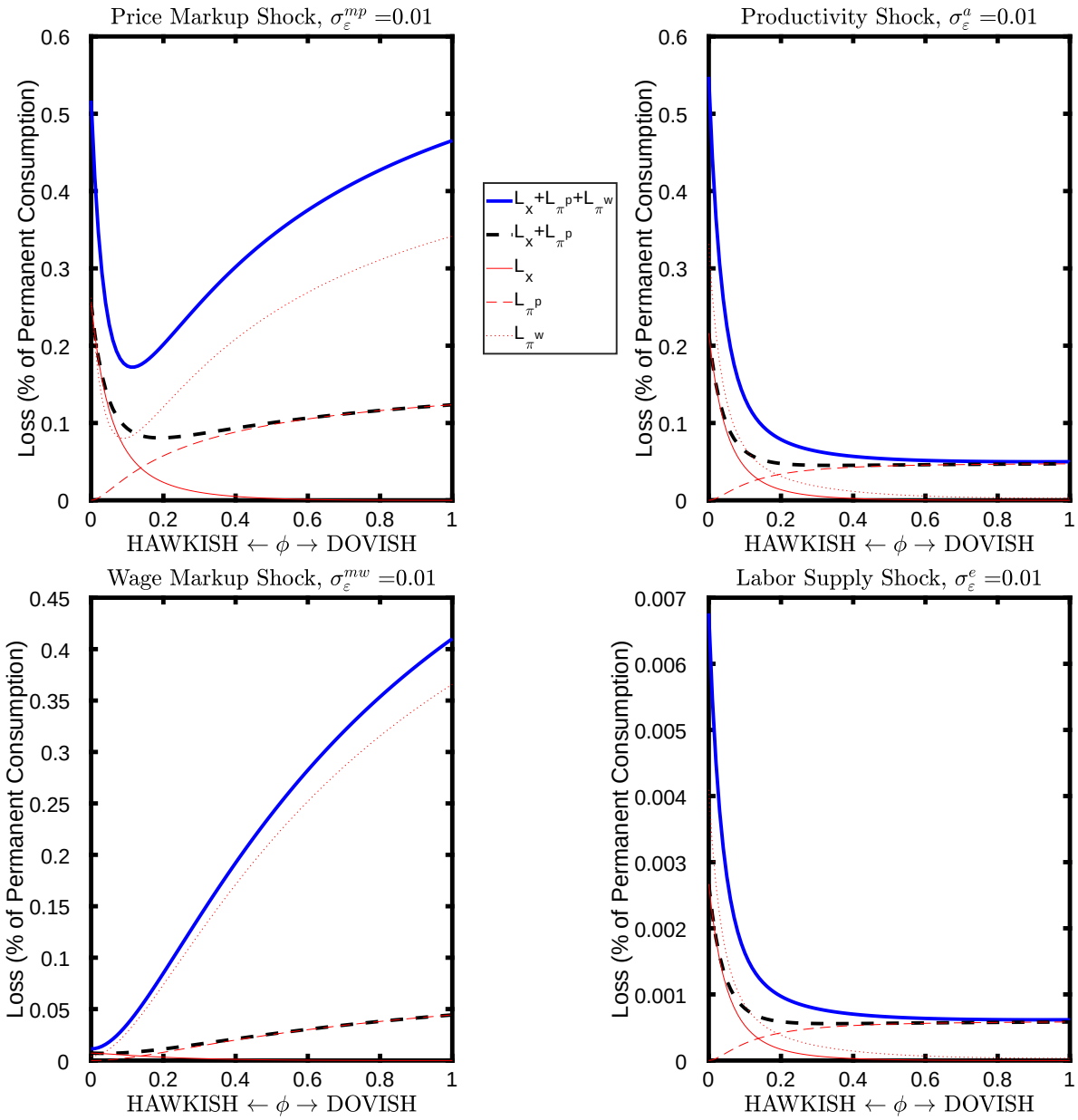


Figure 2: Decomposition of the Loss Function under Rational Expectations

Note: For each type of supply shock, the thick plain line plots the loss function of the central bank (14) as a function of the weights ϕ on the output gap in the objective (20) of the central bank, under the assumption that the economy is hit by this one type of shock only. The thick dashed line plots the loss function abstracting from the costs arising from wage inflation. The thin lines, plain, dashed and dotted, show the losses arising from the output gap, from price inflation, and from wage inflation. The calibration is given in the "Default Calibration" column of Table 1. The persistence of the shock corresponds to a half-life of 16 quarters.

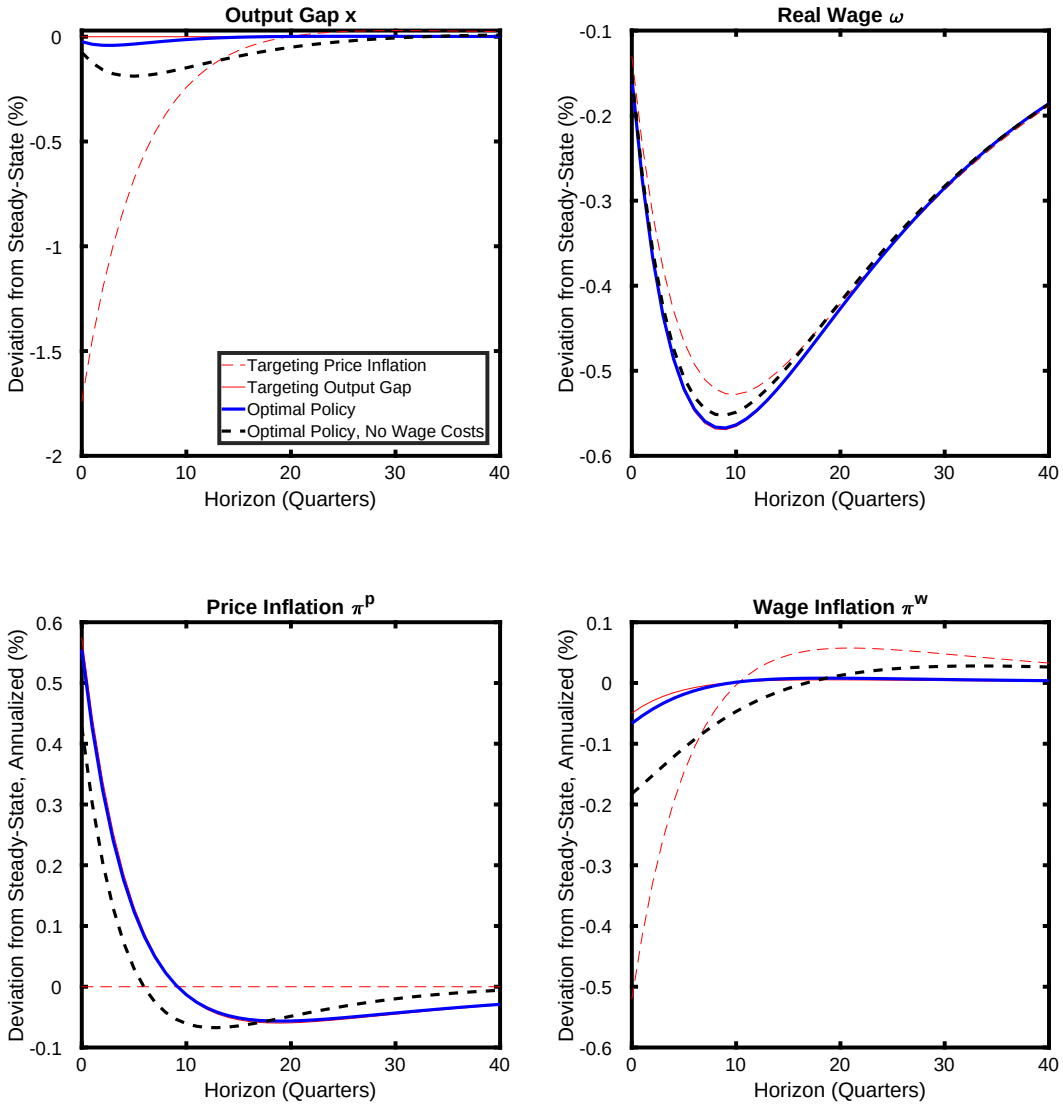


Figure 3: Response to a Negative Productivity Shock under Rational Expectations

Note: The figure plots the response of the output gap, price inflation, wage inflation and the real wage to a persistent negative productivity shock with a half-life of 16 quarters. For each variable, the panel plots the response under 4 policies: stabilizing price inflation, stabilizing the output gap, the optimal policy and the optimal policy abstracting from wage dispersion costs. The calibration is given in the "Default Calibration" column of Table 1.

3 Adaptive Expectations and De-anchoring Risks

That the standard New-Keynesian model does not capture central banks' supply shock heuristic may not be surprising: Because it assumes rational expectations, it does not capture

de-anchoring risks. Yet the risk of expectations de-anchoring is the main rationale put forward to justify a shift to a more hawkish policy when shocks are persistent. This section drops the assumption of rational expectations and assumes instead that inflation expectations are adaptive, so that they can de-anchor. It shows that de-anchoring risks do not change the model's dovish recommendations: stabilizing the output gap remains very close to optimal for productivity and labor supply shocks, however persistent the shock.

3.1 Expectation Formation

I assume that firms and unions form expectations according to adaptive expectations. Adaptive expectations are a particular form of adaptive learning when households and firms learn only about the average inflation rate, without learning the relationship between inflation and its potential predictors. I model adaptive expectations following the "long-horizon approach" of Preston (2005) (in a model with price rigidity only).[11] Firms and unions form expectations about all variables that matter to their optimal resetting price and wage in equations (3)-(8). They perfectly observe all time-t variables. They form expectations on future prices and future nominal wages from the current price and current nominal wage, together with their expectations of future price and wage inflation

$$\tilde{E}_t(p_{t+k}) = p_t + \sum_{i=1}^{k} \tilde{E}_t(\pi^p_{t+i}), \tag{31}$$

$$\tilde{E}_t(w_{t+k}) = w_t + \sum_{i=1}^{k} \tilde{E}_t(\pi^w_{t+i}). \tag{32}$$

Firms and unions need to form expectations on all exogenous variables that enter their program: price inflation π^p, wage inflation π^w, aggregate demand y for firms and aggregate employment n for unions, the shocks a, m^p for firms and the shocks e and m^w for unions. To focus on the de-anchoring of inflation expectations, I assume that they expect output, employment and the shocks to remain constant to their steady-state values. I assume that their expectations of price and wage inflation is the same at all future horizons. They are

[11] The alternative is to follow the "Euler equation approach" that replaces the forward-looking expectations in the Phillips curves (21) and (22) by adaptive versions. Since the derivation of the Phillips curves (21) and (22) makes use of the assumption of rational expectations however, this can be internally inconsistent.

formed under constant-gain learning

$$\pi_t^{p*} = \gamma\pi_t^p + (1-\gamma)\pi_{t-1}^{p*}, \tag{33}$$

$$\pi_t^{w*} = \gamma\pi_t^w + (1-\gamma)\pi_{t-1}^{w*}, \tag{34}$$

where γ is the constant learning gain that I assume is the same for both price and wage inflation.

3.2 Phillips Curves

Appendix E derives the price and wage Phillips curves under adaptive expectations,

$$\pi_t^p = \kappa_c^p\left(\omega_t - \omega_t^e\right) + \kappa_y^p x_t + \zeta^p\beta(1-\mu^p)\omega_t + \frac{\beta(1-\mu^p)}{1-\beta\mu^p}\left(\pi_{t-1}^{p*} + \zeta^p\Delta_{t-1}^*\right) + \kappa_c^p m_t^p, \tag{35}$$

$$\pi_t^w = -\kappa_c^w\left(\omega_t - \omega_t^e\right) + \kappa_y^w x_t - \zeta^w\beta(1-\mu^w)\omega_t + \frac{\beta(1-\mu^w)}{1-\beta\mu^w}\left(\pi_{t-1}^{w*} - \zeta^w\Delta_{t-1}^*\right) + \kappa_c^w m_t^w. \tag{36}$$

where $\Delta_{t-1}^* = \pi_{t-1}^{w*} - \pi_{t-1}^{p*} = \gamma\Delta_t + (1-\gamma)\Delta_{t-1}^*$, defining $\Delta_t^* = \gamma\Delta_t + (1-\gamma)\Delta_{t-1}^*$.

Expectations of both price and wage inflation enters both Phillips curves. Firms need to form expectations on price inflation to assess what the price of their competitors will be, and they need to form expectations on wage inflation to assess what their nominal marginal cost will be. Under rational expectations, expectations of wage inflation can be derived from expectations of price inflation and expectations of future real wages, which themselves depend on expectations of future output and shocks. But doing so requires knowledge of general equilibrium relationships that firms are here assumed not to have. Some variants of adaptive learning can seemingly bypass the need for firms to form expectations on wage inflation. I consider such an alternative later on.

3.3 Should the Central Bank Be Dovish or Hawkish?

Does the possibility for expectations to de-anchor recover central banks' supply shock heuristic? Figure 4 represents the ranking of the different policies (20) in the same way as in Figure

1, but this time under adaptive expectations. The results are broadly unchanged. Against persistent markup shocks, dovish policies become very costly. But against productivity and labor-supply shocks, even very dovish policies continue to have a small cost, and are considerably less costly than very hawkish policies. The loss function is now slightly increasing with ϕ for values of ϕ getting closer to 1. But the increase is small and is actually calibration-specific. As shown below, stabilizing the output gap is still exactly optimal in a particular set of calibrations.

3.4 Why Stabilizing the Output Gap Still Works Well

The reasons why stabilizing the output gap still works well are essentially the same as under rational expectations. First, it is still the case that monetary policy has ultimately little effect on the real wage. To see this, consider the case where the frequency of price and wage changes is the same, $\mu^w = \mu^p$—denote it μ. Taking the difference between the wage Phillips curve (36) and price Phillips curve (35) gives the dynamics of the real wage as

$$\begin{aligned}\Delta_t = &-\left(\kappa_c^w + \kappa_c^p\right)(\omega_t - \omega_t^e) + \left(\kappa_y^w - \kappa_y^p\right)x_t - \beta(1-\mu)(\zeta^w + \zeta^p)\omega_t \\ &+ \frac{\beta(1-\mu)}{1-\beta\mu}(1-\zeta^w-\zeta^p)\Delta_{t-1}^* + \kappa_c^w m_t^w - \kappa_c^p m_t^p, \end{aligned} \tag{37}$$

If $\kappa_y^w = \kappa_y^p$, then the real wage is independent of monetary policy. Once again, what monetary policy can affect is not the real wage, but whether real wage decreases occur through decreases in nominal wages or increases in prices.

Second, it is still the case that stabilizing the output gap delivers a good split of the real wage between price increases and wage decreases. Still restricting to the case $\mu^w = \mu^p = \mu$, and noticing that $\kappa_c^w \zeta^p = \kappa_c^p \zeta^w$ in this case, the divine coincidence index (27) satisfies

$$\pi_t^{DC} = \left(\frac{\kappa_c^w \kappa_y^p + \kappa_c^p \kappa_y^w}{\kappa_c^w + \kappa_c^p}\right) x_t + \frac{\beta(1-\mu)}{1-\beta\mu}\pi_{t-1}^{DC*} + \left(\frac{\kappa_c^p \kappa_c^w}{\kappa_c^w + \kappa_c^p}\right)(m_t^p + m_t^w), \tag{38}$$

where $\pi_t^{DC*} = \left(\frac{\kappa_c^w}{\kappa_c^w+\kappa_c^p}\right)\pi_t^{p*} + \left(\frac{\kappa_c^p}{\kappa_c^w+\kappa_c^p}\right)\pi_t^{w*}$ is the long-term expected Divine Coincidence index, which satisfies $\pi_{DC^*} = \gamma\pi_t^{DC} + (1-\gamma)\pi_{t-1}^{DC^*}$. Stabilizing the output gap in response to productivity and labor supply shocks still stabilizes the Divine Coincidence index, although

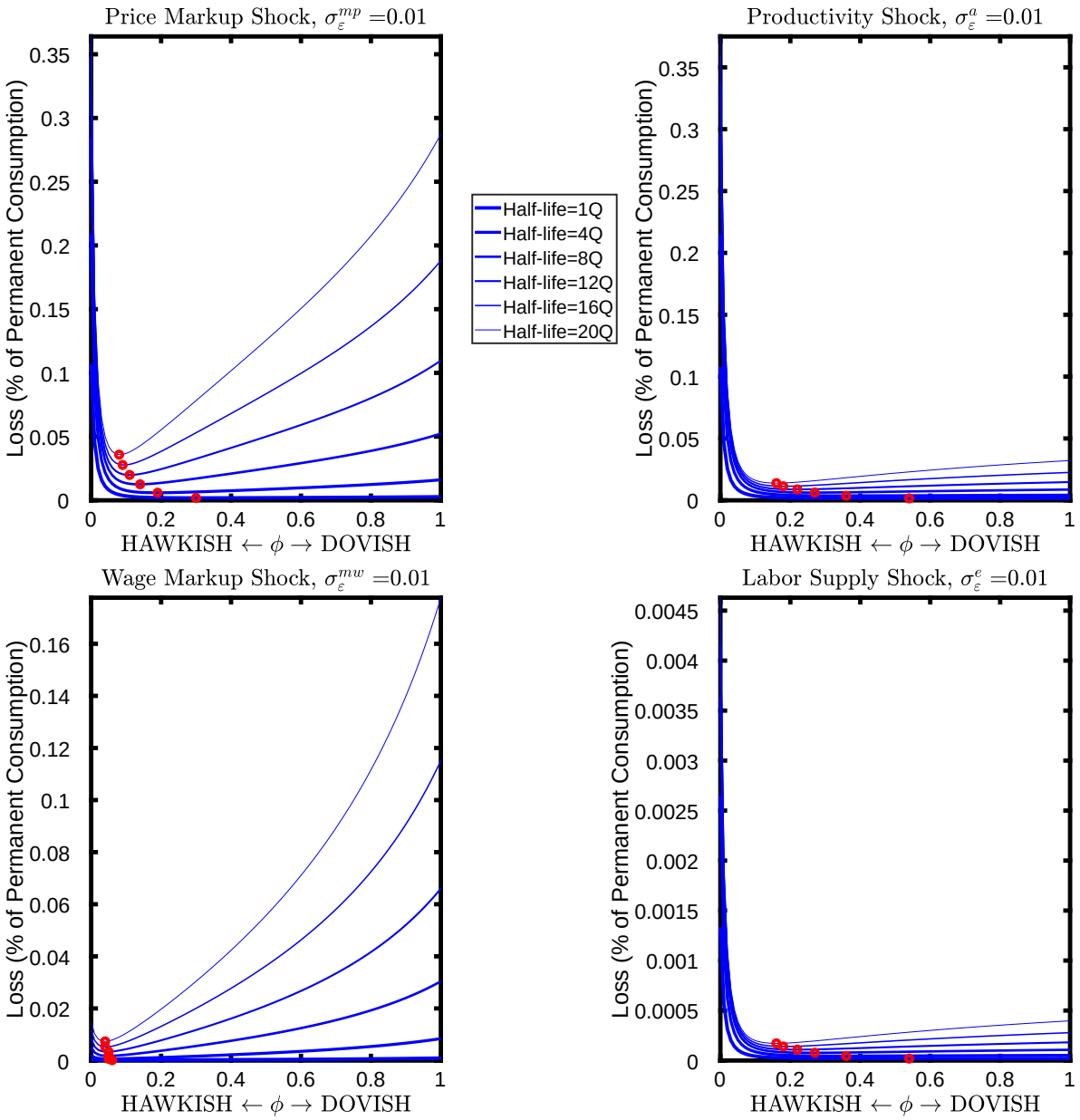


Figure 4: Loss Function under Adaptive Expectations

Note: For each type of supply shock, the figure plots the loss function of the central bank (14) as a function of the weights ϕ on the output gap in the objective (20) of the central bank, under the assumption that the economy is hit by this one type of shock only. It does so for different levels of persistence of the shock, captured by the half-life of the AR(1) process. On each curve, the red dot corresponds to the minimum of the curve, that is the optimal policy within the class of policies (20). The calibration is given in the "Default Calibration" column of Table 1.

this now holds exactly only when restricting to the case $\mu^w = \mu^p$.

In turn, and just like under rational expectations, if in addition $\theta^p = \alpha\theta^w$, then the Divine Coincidence index corresponds to the weighted average (29) of price and wage inflation, and

stabilizing it is the fully optimal policy that minimizes the loss function (30).[12] The three conditions $\mu^w = \mu^p$, $\theta^p = \alpha\theta^w$, and $\kappa_y^w = \kappa_y^p$ are equivalent to the conditions stated in the proposition below.

Proposition 2. *Consider the model under adaptive expectations. If $\mu^w = \mu^p$, $\theta^p = \alpha\theta^w$, and $\psi = 1 - \alpha$ and there is no income effect on labor supply $\sigma = 0$, then the optimal monetary policy in response to productivity and labor supply shocks is to stabilize the output gap at all times $x_t = 0$.*

The conditions of Proposition 2 are the same as the conditions of Proposition 1, plus the extra condition that the frequencies of price and wage changes are the same, $\mu^w = \mu^p$. They are therefore more restrictive, imposing further symmetry between price and wage rigidity for the exact optimality of stabilizing the output gap to hold.[13]

But the underlying intuition is the same as in the case of rational expectations. Under adaptive expectations, letting price inflation overshoot its target increases long-term inflation expectations, which creates persistently high inflation. Yet, it does not follow that a hawkish policy of stabilizing the price inflation is preferable. Stabilizing price inflation still only replaces inefficient price dispersion by inefficient wage dispersion, and still comes at a large cost to the output gap. Figure 5 plots the response of the economy to a persistence negative productivity shock under the same four policies as in Figure 3 for rational expectations: stabilizing the output gap, stabilizing price inflation, the fully optimal policy and the optimal policy abstracting from the welfare costs of wage dispersion. The latter too are characterized in Appendix F. In the first 40 quarters, the optimal policy is if anything even closer to stabilizing the output gap than under rational expectations. (The more increasing loss for very dovish policies on Figure 4 comes from deviations at very high horizons of the impulse response function.) In addition, the good properties of stabilizing the output gap are once again robust to abstracting from the costs of inefficient wage dispersion.

[12] Stabilizing the output gap is once again the optimal policy under both discretion and commitment, but under adaptive expectations there is never any difference between discretion and commitment anyway.

[13] No income effect on labor supply σ does not need to be interpreted as an infinite intertemporal elasticity of substitution. See e.g. Gali (2011) for an approach to calibrating the two independently of each other.

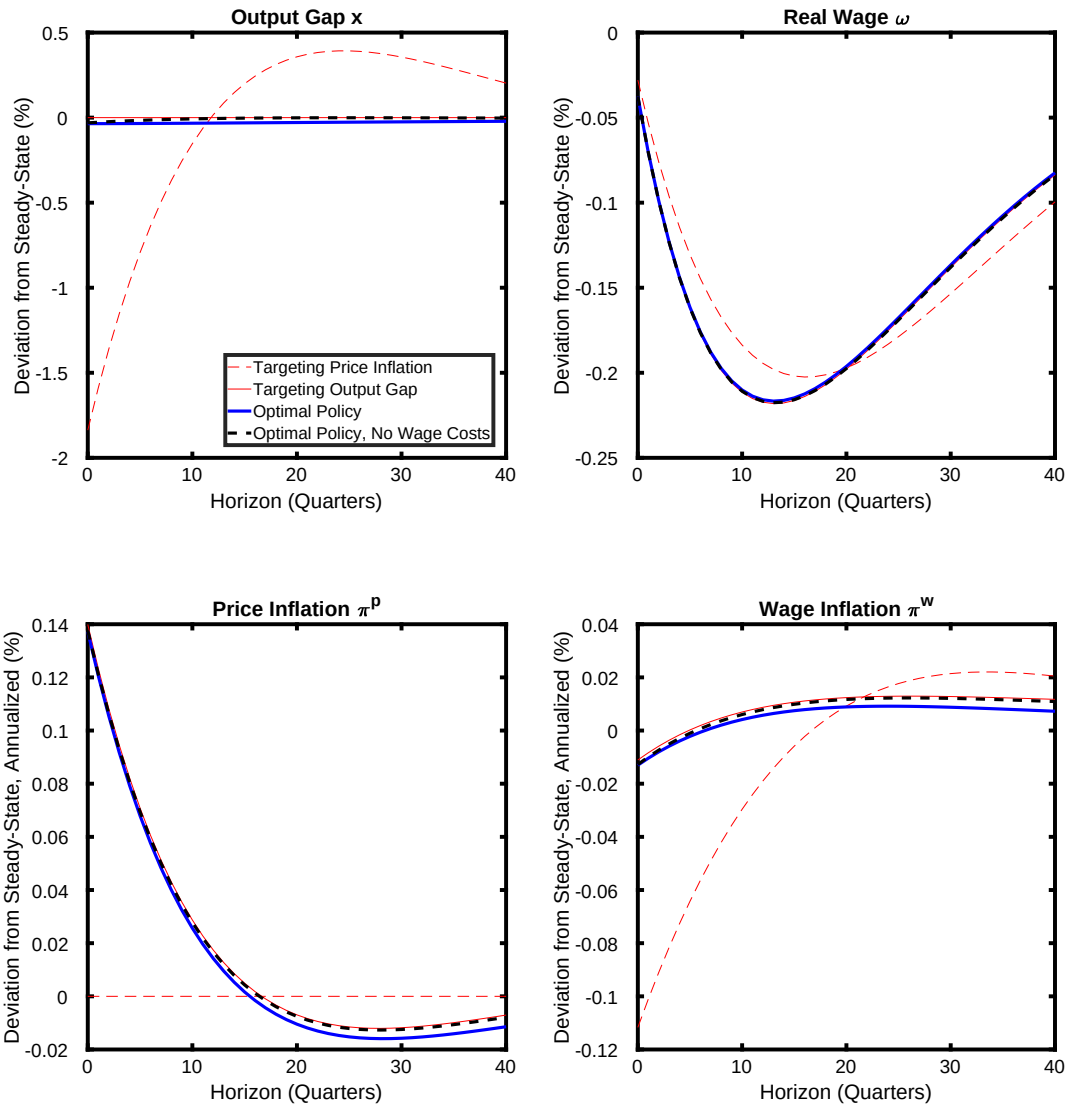


Figure 5: Response to a Negative Productivity Shock under Adaptive Expectations

Note: The figure plots the response of the output gap, price inflation, wage inflation and the real wage to a persistent negative productivity shock with a half-life of 16 quarters. For each variable, the panel plots the response under 4 policies: stabilizing price inflation, stabilizing the output gap, the optimal policy and the optimal policy abstracting from wage dispersion costs. The calibration is given in the "Default Calibration" column of Table 1.

3.5 Past Mistakes and Course Correction

The previous result shows that stabilizing the output gap remains optimal when expectations are backward-looking and can de-anchor, but it is important to highlight what the result

does *not* say. The result concerns the case when monetary policy is optimal at all times and has always been optimal in the past. Then, de-anchoring risks make little change to optimal policy, which should seek to stabilize the output gap at all times. Yet backward-looking expectations do matter for how the central bank should react if it realizes that it has made mistakes in the past, departing from stabilizing the output gap. Such mistakes—or at least what appears to be mistakes in hindsight—are bound to happen given that central banks make decisions in real-time in the face of considerable uncertainty.

Figure 6 plots the response of the economy to the same persistent negative productivity shock, but assumes that in the first ten quarters following the shock, monetary policy stabilizes *output* instead of the output gap, creating a positive output gap. Monetary policy departs from optimal policy, it is not necessarily because it actively wants to overheat the economy. It may be because the central bank wanted to close the output gap but underestimated the drop in natural output caused by the shock. (Orphanides 2001) argues this is what happened in the US in the 1970s.

When the central bank realizes it has run too accommodative a policy and shifts to optimal policy, creating a negative output gap is then justified. The cost of a negative output gap is outweighed by the benefit of speeding up the return of inflation to the target. Optimal policy does not attempt to bring price deflation all the way to what it would have been if the policy had been optimal from the beginning, however. In particular, wage inflation remains positive. But the policy shift is sharp, allowing to revert the dynamics of inflation expectations.

The recommendation to create a negative output gap is driven by backward-looking expectations and does not arise under rational expectations. There, optimal monetary policy from quarter 10 onward is very little affected by past mistakes. When switching to optimal policy, the central bank should cut the output gap to zero, but not to create a negative output gap, as shown in Appendix G. This is because rational expectations make the dynamics very forward-looking and little dependent on past state variables.[14] Under adaptive expectations

[14]The economy under rational expectations is not entirely forward-looking though. The real wage is a state variable and since optimal policy is taken under commitment, bygones are not treated as bygones. But the backward-looking component of the system does not make past mistakes have much influence on future policy.

in contrast, backward-looking inflation expectations make the past matter.

The reversal of inflation expectations—which include long-term expectations here since expectations are the same at all horizons—fits the narrative that a negative output gap is needed to re-anchor expectations. Yet the fact that long-term inflation expectations increase above target is no sign that policy has been non-optimal: they increase above target under optimal policy as well, just less so. The need for a negative output gap therefore does not follow from long-term inflation expectations increasing above target. It comes if long-term inflation expectations have been increasing more than they should have under optimal policy.

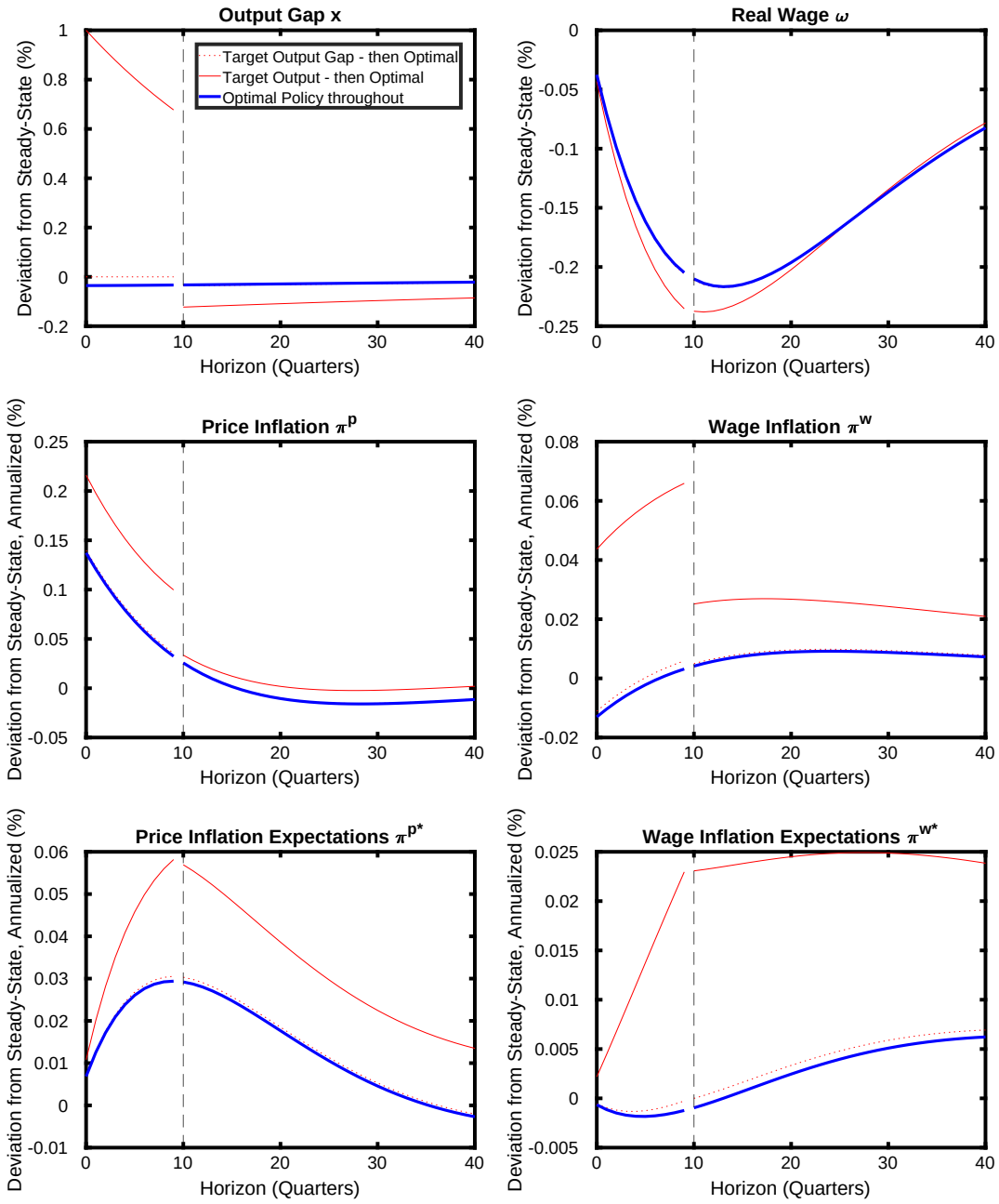


Figure 6: Optimal Policy After Overly Accommodative Policy, Adaptive Expectations

Note: The figure plots the response of the output gap, the real wage, price inflation, wage inflation, price inflation expectations and wage inflation expectations to a persistent negative productivity shock with a half-life of 16 quarters, under three scenarios. First, monetary policy keeps the output gap constant to zero from quarters 0 to 10, then switches to fully optimal policy. Second, monetary policy keeps output constant to its steady-state value from quarters 0 to 10, then switches to optimal policy. Third, monetary policy follows the optimal policy throughout. The calibration is given in the "Default Calibration" column of Table 1.

3.6 Robustness to Alternative Assumption on Learning

Equation (32) assumes that firms and unions expect that future nominal wages will be current nominal wages plus their expectations of future wage inflation. Alternatively, one can assume they form their expectations of future nominal wages by thinking first in terms of future *real* wages. They learn the level of future real wages through

$$\omega_t^* = \gamma\omega_t + (1-\gamma)\omega_{t-1}^*. \tag{39}$$

Their expectations of future nominal wages is then their expectations of future real wages plus their expectations of the future price level,

$$\tilde{E}_t(w_{t+k}) = \tilde{E}_t(\omega_{t+k}) + \sum_{i=1}^{k} \tilde{E}_t(\pi_{t+i}^p) + p_t. \tag{40}$$

Away from rational expectations, this is not equivalent to forming expectations directly on nominal wages. Both assumptions have been used in the literature. Assumption (32) is used for instance in Lorenzoni and Werning (2023b). Assumption (40) is typical in studies that assume nominal wage flexibility, for instance Preston (2005). Indeed, there it is standard to assume that firms understand the equilibrium dependence of the future real wage on future output, and that they learn future output (e.g. Preston 2005).

While both assumptions produce slightly different dynamics, they yield the same normative recommendation. Figure 7 represents the ranking of the different policies (20) under the adaptive expectations variant. Once again, for productivity and labor supply shocks, optimal policy is very close to stabilizing the output gap. Details on the model under this adaptive expectations variant are given in Appendix H.

4 Real Wage Rigidity and Real Wage Catch Up

This section introduces a feature distinct from adaptive expectations and de-anchoring risks: real wage rigidity, and shows that it rationalizes central banks' supply shock heuristic. It does so even when assuming rational expectations so that expectations never de-anchor.

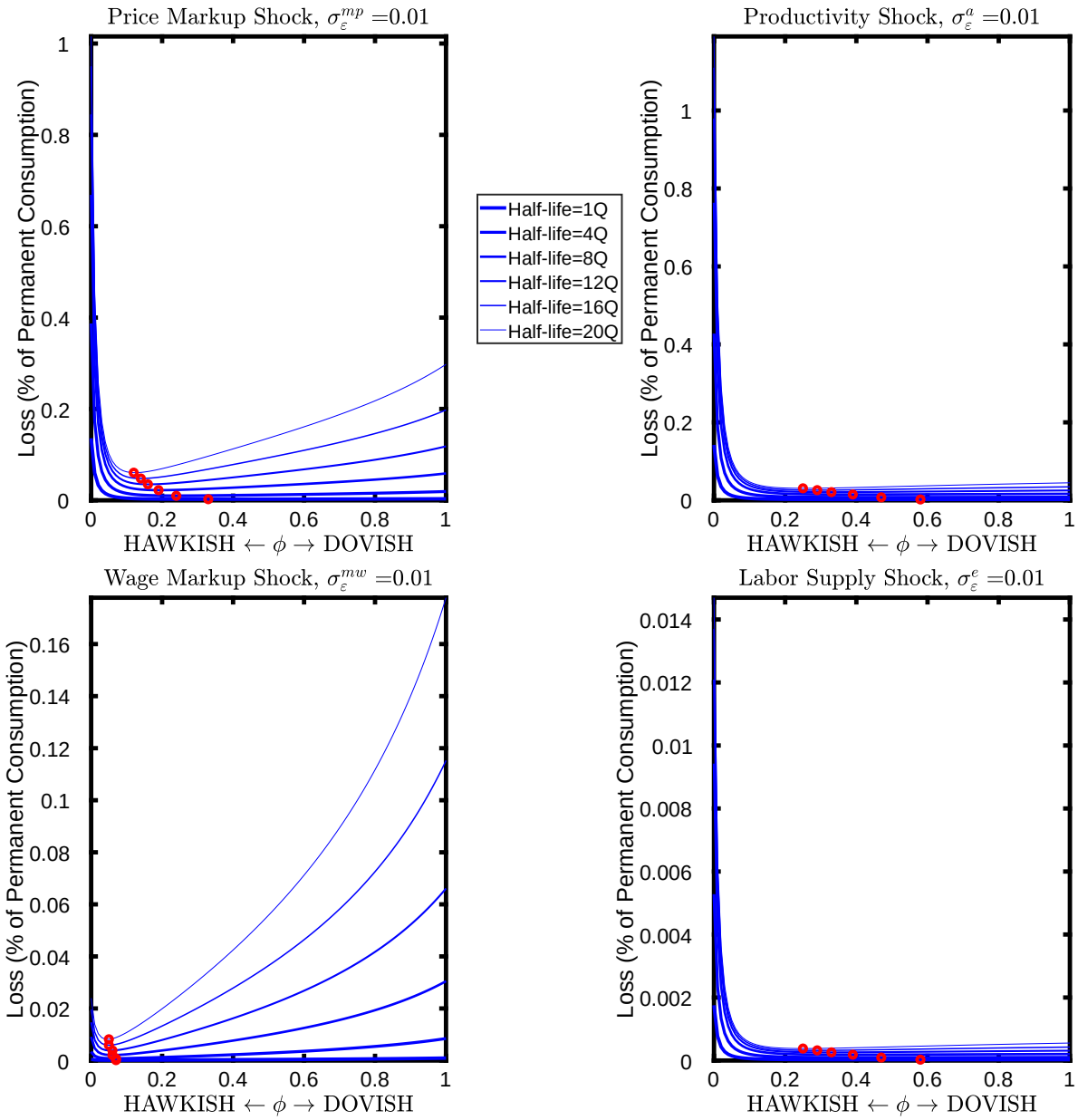


Figure 7: Loss Function under Adaptive Expectations Variant

Note: For each type of supply shock, the figure plots the loss function of the central bank (14) as a function of the weights ϕ on the output gap in the objective (20) of the central bank, under the assumption that the economy is hit by this one type of shock only. It does so for different levels of persistence of the shock, captured by the half-life of the AR(1) process. On each curve, the red dot corresponds to the minimum of the curve, that is the optimal policy within the class of policies (20). The calibration is given in the "Default Calibration" column of Table 1.

4.1 Calvo Staggered Real Wage Setting

I model real wage rigidity in a way that is a direct parallel to nominal wage rigidity in the standard model. This will allow to pinpoint how shifting from nominal to real wage rigidity

changes monetary policy trade-offs. The assumption can be called Calvo staggered *real* wage setting. Unions reset *real* wage with a probability μ^w every period. During the spell of a real wage, nominal wages are automatically indexed on inflation every period so that the real wage remains constant. Optimization under these constraints implies that, when the union resets its *real* wage, it resets it to the weighted average of the future real wage it would reset under flexible wages. The wage-setting equation (8) is replaced by

$$\omega_t^* = (1-\beta\mu^w)\sum_{k=0}^{\infty}(\beta\mu^w)^k\tilde{\mathbb{E}}_t\left(\zeta_w\left(mrs_{t+k}+m_{t+k}^w\right)+(1-\zeta^w)\omega_{t+k}\right). \tag{41}$$

while the equation (12) is now replaced by the following equation linking the resetting wage to the rate of change in the real wage

$$\omega_t - \omega_{t-1} = \frac{1-\mu^w}{\mu^w}(w_t^* - w_t), \tag{42}$$

Under rational expectations, equations (41) and (42) imply the new (real) wage Phillips curve

$$\omega_t - \omega_{t-1} = \kappa_c^w(\omega_t^e - \omega_t + m_t^w) + \kappa_y^w x_t + \beta\mathbb{E}_t(\omega_{t+1} - \omega_t). \tag{43}$$

The price Phillips curve is still (21).

Shifting from nominal to real wage rigidity makes changes to the central bank's loss function, since the indexation of nominal wages on inflation implies that wage inflation does not necessarily create wage dispersion. Appendix B shows that the microfounded loss function (14) is now

$$\mathbb{W} = -\frac{1}{2}(1-\beta)\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left[\lambda_x(x_t)^2+\lambda_p(\pi_t^p)^2+\lambda_w(\pi_t^w-\pi_t^p)^2\right]. \tag{44}$$

Wage dispersion is now related to changes in the *real wage*, $\Delta_t = \pi_t^w - \pi_t^p$.

4.2 Should the Central Bank Be Dovish or Hawkish?

How does real wage rigidity change the desirability of stabilizing price inflation versus the output gap? Figure 8 represents the ranking of the different policies (20) as in Figure 1, but now with real wage rigidity (and still rational expectations). This time, the model can rationalize central banks' supply shock heuristic for productivity and labor supply shocks. Optimal monetary policy is very close to stabilizing the output gap when the shock is transitory, but stabilizing the output gap becomes more and more costly as the shock becomes more persistent. As a result, optimal policy becomes hawkish for persistent shocks.

Figure 8 is plotted under the calibration of the frequency μ_w of wage changes (and everything else) in the default calibration of Table 1: 0.8, or about every 5 quarters. This is a typical calibration for the frequency of changes in *nominal* wages. The frequency of *real* wage changes has no reason to be the same however. Workers and unions may well take more than five quarters to accept the idea of a real wage change, in which case μ_w should be set higher. Appendix I replicates Figure I.1 under an alternative calibration of $\mu^w = 0.9$, keeping all other parameters constant. The cost of stabilizing the output gap becomes all the stronger, and the case for shifting to a more hawkish stance when shocks are persistent all the stronger as well. Conversely, Figure I.2 replicates the figure under an alternative calibration of $\mu^w = 0.5$. Fully stabilizing the output gap no longer produces as large a loss and is no longer worse than fully stabilizing price inflation.

4.3 The Worldly Coincidence Index

To understand why real wage rigidity undermines the benefits of stabilizing the output gap, consider how stabilizing the output gap now affects price and wage inflation. Stabilizing the output gap no longer coincide with stabilizing the Divine Coincidence index (27). Yet a new linear combination of price and wage inflation does. Define

$$\pi_t^{WC} = \left(\frac{\kappa_c^w - \kappa_c^p}{\kappa_c^p + \kappa_c^w}\right)\pi_t^p + \left(\frac{\kappa_c^p}{\kappa_c^p + \kappa_c^w}\right)\pi_t^w. \tag{45}$$

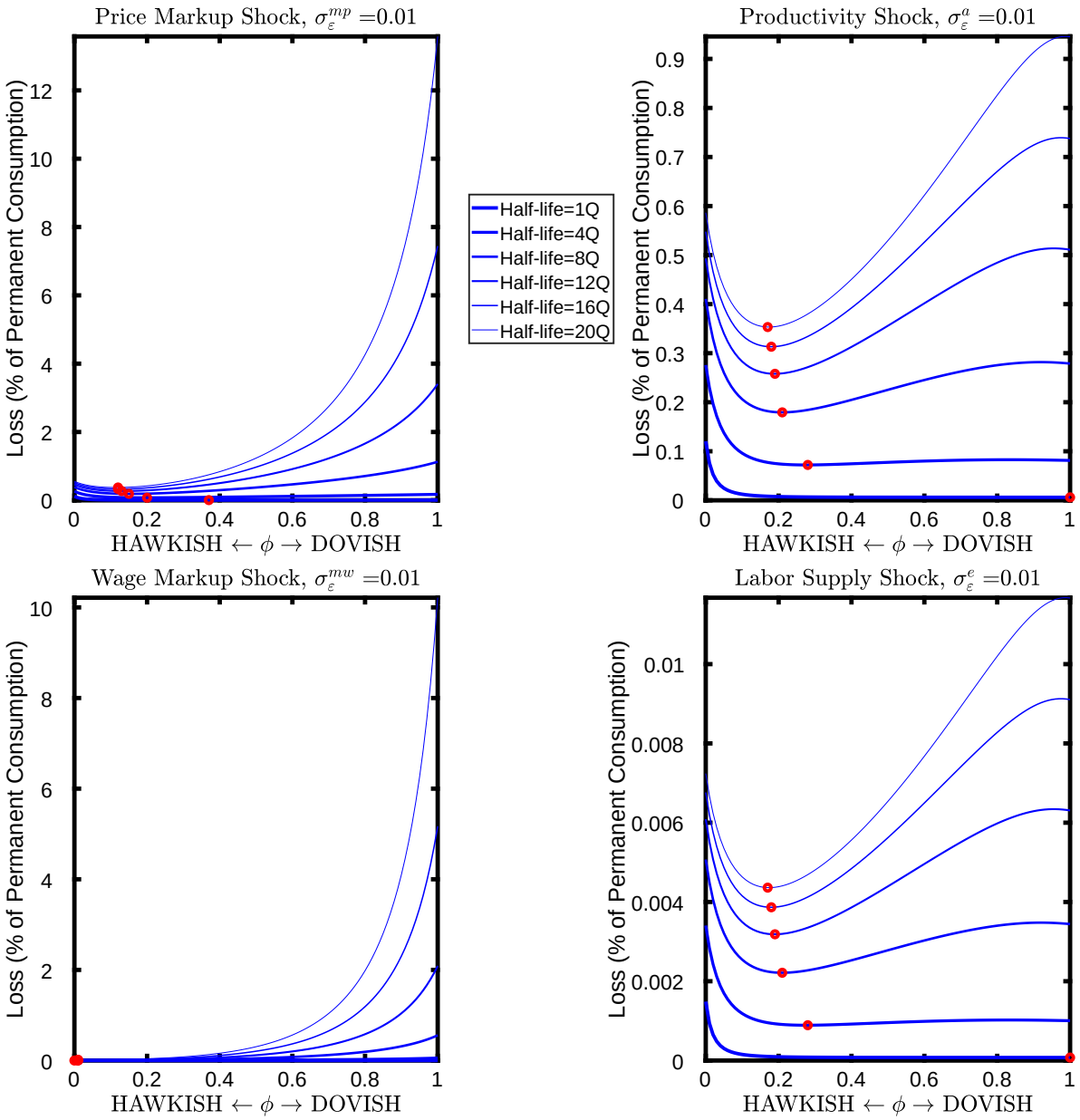


Figure 8: Loss Function under Real Wage Rigidity and Rational Expectations

Note: For each type of supply shock, the figure plots the loss function of the central bank (44) as a function of the weights ϕ on the output gap in the objective (20) of the central bank, under the assumption that the economy is hit by this one type of shock only. It does so for different levels of persistence of the shock, captured by the half-life of the AR(1) process. On each curve, the red dot corresponds to the minimum of the curve, that is the optimal policy within the class of policies (20). The calibration is given in the "Default Calibration" column of Table 1.

It satisfies

$$\pi_t^{WC} = \left(\frac{\kappa_c^w \kappa_y^p + \kappa_c^p \kappa_y^w}{\kappa_c^w + \kappa_c^p}\right) x_t + \beta \mathbb{E}_t \pi_{t+1}^{WC} + \left(\frac{\kappa_c^p \kappa_c^w}{\kappa_c^w + \kappa_c^p}\right)(m_t^p + m_t^w), \tag{46}$$

so that in the face of productivity or labor supply shocks, stabilizing the output gap stabilizes it. While it does coincide with stabilizing the output gap, stabilizing it is far from minimizing the loss function of the central bank. There is nothing divine about the coincidence. I call the weighted average (45) the Worldly Coincidence (WC) index.

Why is stabilizing the Worldly Coincidence index π_t^{DC} and the output gap x_t far from optimal? Contrary to the Divine Coincidence index under nominal wage rigidity, stabilizing π_t^{WC} does not typically split the change in real wages between price increases and wage decreases. Instead, as soon as $\kappa_c^w < \kappa_c^p$—i.e. when real wages are stickier than nominal prices—stabilizing it causes both price and wages to move in the same direction—to increase following an inflationary shock. This is therefore far from minimizing the amount of price and wage dispersion. To spread the real-wage adjustment between price increases and wage decreases, a negative output gap is needed. This creates a trade-off between stabilizing the output gap and minimizing the costs of price and wage inflation.

The cost of stabilizing the output gap is all the larger the stickier real wages are. Figure I.1 in Appendix I plots the same graphs as Figure 8 but with $\mu_w = 0.9$ so that $\kappa_c^w > \kappa_c^p$. In this case, stabilizing the output gap does make prices and wages move in opposite directions, and is therefore not too costly a policy. In contrast, Figure I.2 plots the figure with $\mu_w = 0.9$ so that κ_c^w is even larger than κ_c^p still. Stabilizing the output gap is then even more costly.

4.4 Combining Real Wage Rigidity and De-anchoring Risks

Importantly, real wage rigidity recovers central banks' supply shock heuristic even under rational expectations, which rules out de-anchoring risks. But what if, on top of real wage rigidity, expectations can de-anchor? As derived in Appendix L, the real wage Phillips curve is now

$$\omega_t - \omega_{t-1} = \kappa_c^w(\omega_t^e - \omega_t + m_t^w) + \kappa_y^w x_t - \beta(1-\mu^w)\zeta^w\omega_t + \frac{\beta(1-\mu^w)}{1-\beta\mu^w}(1-\zeta^w)(\pi_{t-1}^{w*} - \pi_{t-1}^{p*}). \tag{47}$$

The price Phillips curve is still (35).

Figure 9 represents the ranking of the different policies (20) as in Figure 1 but under

the joint assumptions of real wage rigidity and adaptive expectations.[15] The model once again recovers central banks' supply shocks heuristic. But the figure is overall similar to the case of real wage rigidity and rational expectations. What is driving the need for a hawkish pivot in the face of persistent productivity and labor supply shocks is real wage rigidity, not de-anchoring risks.

Conclusion

Although it may not have this reputation, the standard New-Keynesian set-up makes rather dovish recommendations on how to best respond to supply shocks. Except for mark-up shocks—which are not the main supply shocks central banks face in practice—optimal policy is very close to stabilizing the output gap, at the cost of letting price inflation deviate from target. Yet by assuming rational expectations, the New-Keynesian set-up abstracts from one main argument for a hawkish response to persistent supply shocks: not stabilizing price inflation runs the risk of de-anchoring long-term inflation expectations. Relying on the standard New-Keynesian set-up with both price and wage rigidity, but replacing rational expectations with backward-looking expectations that can de-anchor, this paper has argued that introducing de-anchoring risks does not challenge the soundness of stabilizing the output gap instead of price inflation. What can challenge it is instead real wage rigidity.

[15] Appendix L considers the joint assumption of real wage rigidity and the adaptive expectations variant. Results are very similar.

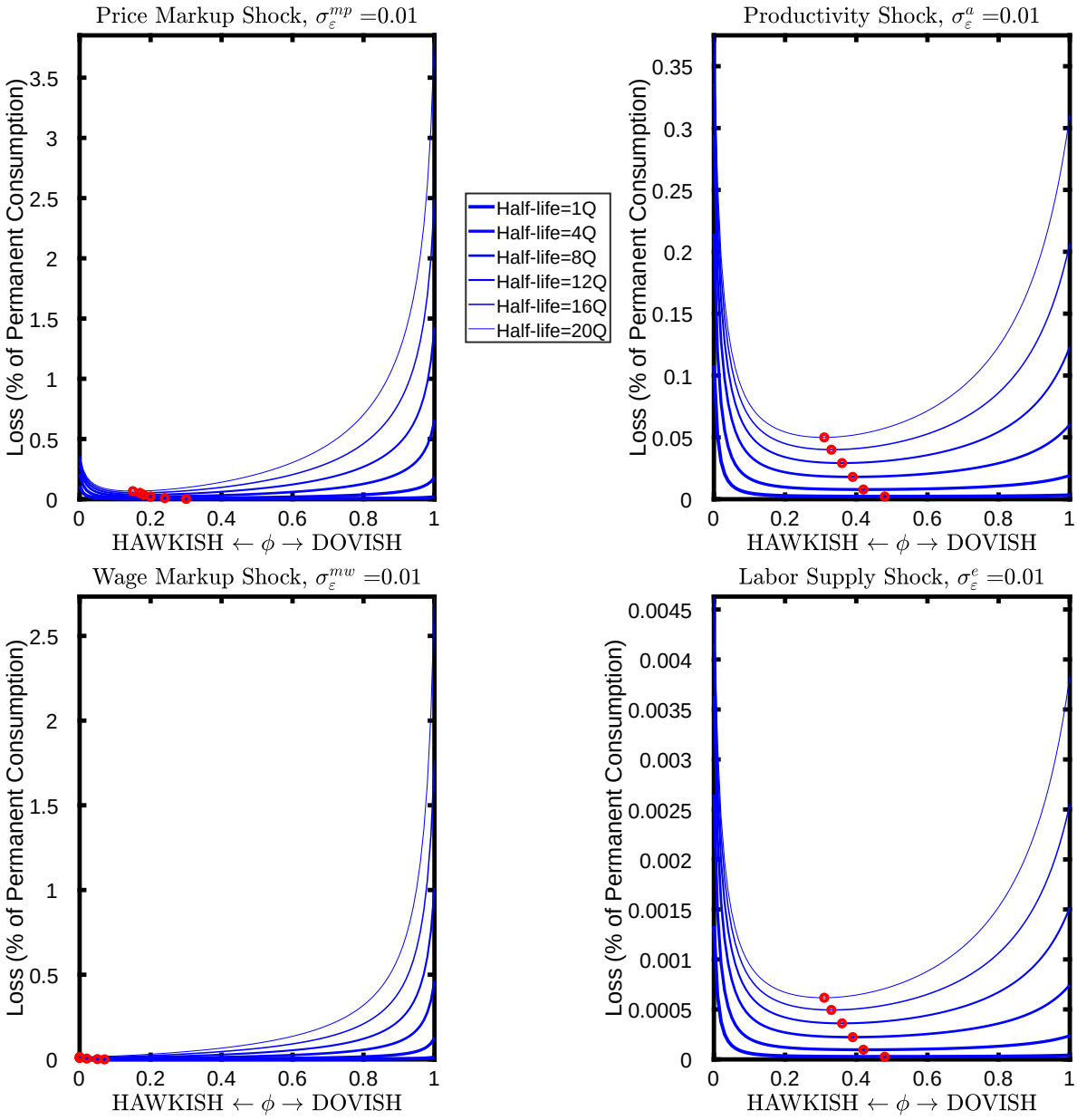


Figure 9: Loss Function under Real Wage Rigidity and Adaptive Expectations

Note: For each type of supply shock, the figure plots the loss function of the central bank (44) as a function of the weights ϕ on the output gap in the objective (20) of the central bank, under the assumption that the economy is hit by this one type of shock only. It does so for different levels of persistence of the shock, captured by the half-life of the AR(1) process. On each curve, the red dot corresponds to the minimum of the curve, that is the optimal policy within the class of policies (20). The calibration is given in the "Default Calibration" column of Table 1.

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A Derivation of Price and Wage Setting

A.1 Firm i's price-setting

Absent price rigidity, firm i would set the price

$$P_t^{i,flex} = \mathcal{M}_t^p \times C'_N(Y_t^i), \tag{A.1}$$

where

$$\mathcal{M}_t^p = \theta_t^p/(\theta_t^p - 1) \tag{A.2}$$

is the price markup and

$$C'_N(Y_t^i) = \frac{W_t}{A_t F'(N_t^i)} \tag{A.3}$$

is firm i's marginal cost.

Denoting log-deviations with lowercase variables, firm i's price absent price rigidity is

$$p_t^{i,flex} = w_t - mrt_t - \frac{(1-\alpha)\theta^p}{\alpha}(p_t^i - p_t) + m_t^p \tag{A.4}$$

where

$$mrt_t = \frac{1}{\alpha}a_t - \frac{1-\alpha}{\alpha}y_t \tag{A.5}$$

is the economy-wide marginal rate of transformation and θ^p is the steady-state value of θ_t^p.

Firm i sets prices under Calvo staggered price-setting. Each period, it faces a probability $1-\mu^p$ of being able to reset its price. When it does, profits-maximization makes it reset it to

$$p_t^* = (1-\beta\mu^p)\sum_{k=0}^{\infty}(\beta\mu^p)^k\tilde{\mathbb{E}}_t\left(p_{t+k}^{i,flex}\right), \tag{A.6}$$

where $\tilde{\mathbb{E}}$ denotes firm i's subjective expectations, on which no further assumption is made for

the moment. They may or may not be rational. Plugging in the expression for the flexible price (A.4) gives equation (3) for the resetting price.

A.2 Union j's wage-setting

Absent wage rigidity, the union would set the wage

$$W_t^{j,flex} = \mathcal{M}_t^w \frac{P_t \mathcal{E}_t (N_t^j)^\psi}{C_t^{-\sigma}}. \tag{A.7}$$

where

$$\mathcal{M}_t^w = \theta_t^w / (\theta_t^w - 1) \tag{A.8}$$

is the wage markup.

Denoting log-deviations with lowercase variables, union j's wage absent wage rigidity is

$$w_t^{j,flex} = p_t + mrs_t - \psi\theta^w (w_t^i - w_t) + m_t^w \tag{A.9}$$

where

$$mrs_t = \psi n_t + \sigma c_t + e_t, \tag{A.10}$$

is the economy-wide marginal rate of substitution and θ^w is the steady-state value of θ_t^w. Note that in equilibrium, given goods and labor market-clearing the marginal rate of substitution can be rewritten

$$mrs_t = \left(\frac{\psi}{\alpha} + \sigma\right) y_t - \frac{\psi}{\alpha} a_t + e_t. \tag{A.11}$$

but we stick to the expression (A.10) here since equation (A.11) would implicitly impose that unions necessarily take into account goods and labor market-clearing when forming expectations, which would be premature at this stage. Instead, for the moment I only assume that unions and firms understand the equations that explicitly enter their program,

such as the demand curve they face.

Union j sets wages under Calvo staggered wage-setting. Each period, it faces a probability $1-\mu^w$ of being able to reset its wage. When it does profits-maximization makes it reset it to

$$w_t^* = (1-\beta\mu^w)\sum_{k=0}^{\infty}(\beta\mu^w)^k\tilde{\mathbb{E}}_t\left(w_{t+k}^{j,flex}\right), \tag{A.12}$$

where $\tilde{\mathbb{E}}$ denotes union j's subjective expectations, on which we make no further assumption for the moment. They may or may not be rational. Plugging in the expression for the flexible wage (A.9) gives equation (8) for the resetting wage.

B Derivation of the Loss Function

Step 1: Second-Order Approximation to the Period Utility

The preferences of the representative household are

$$\mathcal{U} = \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t U_t, \tag{B.1}$$

where

$$U_t = \frac{C_t^{1-\sigma}}{1-\sigma} - \mathcal{E}_t\int_j \frac{(N_t^j)^{1+\psi}}{1+\psi}dj. \tag{B.2}$$

A second-order approximation to U gives

$$dU = \left((u'_C.C)\left(\frac{dC}{C}\right) + \frac{u''_{CC}.C^2}{2}\left(\frac{dC}{C}\right)^2\right) - \int_j\left((v'_N.N)\left(\frac{dN^j}{N}\right) + (v'_N.N)\left(\frac{dN^j}{N}\right)e + \frac{v''_{NN}.N^2}{2}\left(\frac{dN^j}{N}\right)^2\right) \tag{B.3}$$

Using the fact that at second order

$$\frac{dX}{X} = x + \frac{1}{2}x^2, \tag{B.4}$$

and that $u'_{CC}C^2 = -\sigma u'_C C$, $v'_{NN}N^2 = \psi v'_N N$, this can be rewritten as

$$dU = (u'_C.C)\left(c + \frac{1-\sigma}{2}c^2\right) - (v'_N.N)\int_j \left(n^j + n^j \times e + \frac{1+\psi}{2}(n^j)^2\right) dj. \tag{B.5}$$

Since at second order

$$\int_j n^j + \frac{1+\psi}{2}\int_j (n^j)^2 = n + \frac{1+\psi}{2}n^2 + \frac{\theta^w}{2\zeta^w}\mathbb{V}ar^j(w^j), \tag{B.6}$$

$$\int_j n_t^j \times e = \frac{1}{\alpha}(y-a) \times e, \tag{B.7}$$

$$n = \frac{1}{\alpha}\left((y-a) + \frac{1}{2}\frac{\theta^p}{\zeta^p}\mathbb{V}ar^i(p^i)\right), \tag{B.8}$$

$$n^2 = \left(\frac{1}{\alpha}(y-a)\right)^2, \tag{B.9}$$

it can be rewritten as

$$\begin{aligned} dU = {} & (u'_C.C)\left(c + \frac{1-\sigma}{2}c^2\right) \\ & - \left(\frac{v'_N.N}{\alpha}\right)\left((y-a) + (y-a)\times e + \frac{1+\psi}{2\alpha}(y-a)^2 + \frac{1}{2}\frac{\theta^p}{\zeta^p}\mathbb{V}ar^i(p^i) + \frac{\alpha\theta^w}{2\zeta^w}\mathbb{V}ar^j(w^j)\right). \end{aligned} \tag{B.10}$$

Imposing market clearing and the fact that the steady-state is efficient $u'_C.C = (v'_N.N)/\alpha$,

$$dU = -(u'_C.C)\left(\frac{\sigma-1}{2}y^2 + a + (y-a)\times e + \frac{1+\psi}{2\alpha}(y-a)^2 + \frac{1}{2}\frac{\theta^p}{\zeta^p}\mathbb{V}ar^i(p^i) + \frac{\alpha\theta^w}{2\zeta^w}\mathbb{V}ar^j(w^j)\right).$$

Up to terms independent of policy and dividing by $u'_C.C$ to get the utility in consumption equivalent

$$dU = -\frac{1}{2}\left(\left(\sigma + \frac{1-\alpha+\psi}{\alpha}\right)x^2 + \frac{\theta^p}{\zeta^p}\mathbb{V}ar^i(p^i) + \frac{\alpha\theta^w}{\zeta^w}\mathbb{V}ar^j(w^j)\right). \tag{B.11}$$

Hence the central bank's loss function is equal, up to a second-order approximation, to

$$\mathcal{L} = -\mathcal{U} = \frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left(\left(\sigma + \frac{1-\alpha+\psi}{\alpha}\right)x_t^2 + \frac{\theta^p}{\zeta^p}\mathbb{V}ar^i(p_t^i) + \frac{\alpha\theta^w}{\zeta^w}\mathbb{V}ar^j(w_t^j)\right). \tag{B.12}$$

This first step makes no assumptions beyond those regarding preferences (except at the

end for goods market clearing and the assumption that the steady-state is efficient).

Step 2: Expression of Price Dispersion

Under the assumption of Calvo staggered price setting, $\sum \beta^t \mathbb{V}ar^i(p^i)$ can be rewritten as a function of the inflation path. The distribution of prices p_t^i at t is a mixture of a dirac at p_t^* with probability $1-\mu$, and the past distribution of prices at $t-1$ with probability μ^p. Hence,

$$\mathbb{V}ar^i(p_t^i) = E_t^i(p_t^i - p_t) = (1-\mu^p)(p_t^* - p_t)^2 + \mu^p \mathbb{E}_t^i(p_{t-1} - p_t)^2. \tag{B.13}$$

It follows that

$$\mathbb{V}ar^i(p_t^i) = \frac{\mu^p}{1-\mu^p}\pi_t^2 + \mu^p \mathbb{V}ar_t^i(p_{t-1}). \tag{B.14}$$

Iterating backward,

$$\mathbb{V}ar^i(p_t^i) = \frac{\mu^p}{1-\mu^p}\sum_{i=0}^{t}(\mu^p)^{t-i}\pi_{t+i}^2 + (\mu^p)^{t+1}\mathbb{V}ar_{-1}^i(p_{-1}), \tag{B.15}$$

so that

$$\sum_{t=0}^{\infty}\beta^t \mathbb{V}ar^i(p_t^i) = \frac{\mu^p}{(1-\mu^p)(1-\beta\mu^p)}\sum_{t=0}^{\infty}(\pi_t^p)^2, \tag{B.16}$$

up to an initial condition independent of policy.

This second step relies on the assumption of Calvo staggered price, but makes no assumption on private sector's expectations. (It does assume the central bank has rational expectations, which is a maintained assumption of the paper.)

Step 3: Expression of Wage Dispersion

Under Calvo nominal wage-setting, following the same steps as for price dispersion gives for

wage dispersion

$$\sum_{t=0}^{\infty}\beta^t\mathbb{V}ar^j(w_t^j)=\frac{\mu^w}{(1-\mu^w)(1-\beta\mu^w)}\sum_{t=0}^{\infty}(\pi_t^w)^2. \tag{B.17}$$

Injecting (B.16) and (B.17) into (B.11) gives (14).

Under real wage rigidity instead of nominal wage rigidity, equations (B.11) and (B.16) are still valid. Equation (B.17) is replaced by

$$\sum_{t=0}^{\infty}\beta^t\mathbb{V}ar^j(w_t^j)=\frac{\mu^w}{(1-\mu^w)(1-\beta\mu^w)}\sum_{t=0}^{\infty}(\pi_t^w-\pi_t^p)^2. \tag{B.18}$$

Injecting (B.16) and (B.18) into (B.11) gives (44).

C Derivation of the Phillips Curves under Rational Expectations

Under rational expectations, the price and wage-setting equations (3)-(8) can be written recursively as

$$p_t^*=(1-\beta\mu^p)\left(\zeta^p\left(\omega_t-mrt_t+m_t^p\right)+p_t\right)+(\beta\mu^p)\mathbb{E}_t p_{t+1}^*, \tag{C.1}$$

$$w_t^*=(1-\beta\mu^w)\left(\zeta^w\left(mrs_t-\omega_t+m_t^w\right)+w_t\right)+(\beta\mu^w)\mathbb{E}_t w_{t+1}^*. \tag{C.2}$$

Using the fact that

$$\pi_t^p=\frac{1-\mu^p}{\mu^p}(p_t^*-p_t), \tag{C.3}$$

$$\pi_t^w=\frac{1-\mu^w}{\mu^w}(w_t^*-w_t), \tag{C.4}$$

it can be rewritten as the price and wage NKPC

$$\pi_t^p = \kappa_c^p\Big(\omega_t - mrt_t + m_t^p\Big) + \beta\mathbb{E}_t\pi_{t+1}^p, \tag{C.5}$$

$$\pi_t^w = \kappa_c^w\Big(mrs_t - \omega_t + m_t^w\Big) + \beta\mathbb{E}_t\pi_{t+1}^w, \tag{C.6}$$

where

$$\omega_t = w_t - p_t \tag{C.7}$$

denotes the real wage. The NKPC (C.5) and (C.6) can be rewritten as (21) and (22) where the efficient level of output corresponds to $mrs_t = mrt_t$ and is given by

$$y_t^e = \frac{\frac{1+\psi}{\alpha}a - \varepsilon}{\frac{1-\alpha+\psi}{\alpha} + \sigma}, \tag{C.8}$$

$$\tag{C.9}$$

and the level of real wages ω_t^e that would obtain in an economy under perfect competition and absent nominal rigidities is given

$$\omega_t^e = \frac{\frac{\sigma+\psi}{\alpha}a + \frac{1-\alpha}{\alpha}e}{\frac{1+\psi-\alpha}{\alpha} + \sigma}. \tag{C.10}$$

D Optimal Policy Under Rational Expectations

Let $\eta_t^1, \eta_t^2, \eta_t^3$ be the Lagrange multipliers on the constraints (21), (22) and (13). The first-order conditions of the central bank's program wrt. x_t, π_t^p, π_t^w and ω are

$$\lambda_x x_t + \kappa_y^p\eta_t^1 + \kappa_y^w\eta_t^2 = 0, \tag{D.1}$$

$$\lambda_p\pi_t^p - \eta_t^1 + \eta_{t-1}^1 + \eta_t^3 = 0, \tag{D.2}$$

$$\lambda_w\pi_t^w - \eta_t^2 + \eta_{t-1}^2 - \eta_t^3 = 0, \tag{D.3}$$

$$\kappa_c^p\eta_t^1 - \kappa_c^w\eta_t^2 + \eta_t^3 - \beta\mathbb{E}_t\eta_{t+1}^3 = 0. \tag{D.4}$$

The optimal policy that abstracts from the costs of wage dispersion is characterized in the same way, except that $\lambda_w = 0$ in equation (D.3).

E Derivation of the Phillips Curves under Adaptive Expectations

We derive the price Phillips curve under adaptive expectations (35). The wage Phillips curve then follows the same steps. Subtracting p_t (which is perfectly observed by firms) from the expression of the resetting price (3) and using the fact that time-t variables are perfectly observed by the firm gives

$$\begin{aligned} p_t^* - p_t = (1-\beta\mu^p)\zeta^p(\omega_t - mrt_t + m_t^p) \\ + (1-\beta\mu^p)\sum_{k=1}^{\infty}(\beta\mu^p)^k \tilde{E}_t\Big(\zeta^p\Big(w_{t+k} - p_t - mrt_{t+k} + m_{t+k}^p\Big) + (1-\zeta^p)(p_{t+k} - p_t)\Big), \end{aligned} \tag{E.1}$$

Using the assumptions on expectations formation of future prices and nominal wages (33) and (34) it rewrites

$$\begin{aligned} p_t^* - p_t = (1-\beta\mu^p)\zeta^p(\omega_t - mrt_t + m_t^p) \\ + (1-\beta\mu^p)\zeta^p\sum_{k=1}^{\infty}(\beta\mu^p)^k \tilde{E}_t\Big(\omega_t - mrt_{t+k} + m_{t+k}^p\Big) \\ + \sum_{k=1}^{\infty}(\beta\mu^p)^k \tilde{E}_t\Big(\zeta^p\pi_{t+k}^w + (1-\zeta^p)\pi_{t+k}^p\Big). \end{aligned} \tag{E.2}$$

Assuming expectations at all horizons are the same,

$$\begin{aligned} p_t^* - p_t = (1-\beta\mu^p)\zeta^p(\omega_t - mrt_t + m_t^p) \\ + \zeta^p(\beta\mu^p)\Big(\omega_t - mrt_{t-1}^* + m_{t-1}^{p*}\Big) \\ + \frac{\beta\mu^p}{1-\beta\mu^p}\Big(\zeta^p\pi_{t-1}^{w*} + (1-\zeta^p)\pi_{t-1}^{p*}\Big). \end{aligned} \tag{E.3}$$

Using (C.3), it gives

$$\pi_t^p = \kappa_c^p\left(\omega_t - \omega_t^e + m_t^p\right) + \kappa_y^p x_t + \zeta^p\beta(1-\mu^p)\left(\omega_t - mrt_{t-1}^* + m_{t-1}^{p*}\right) + \frac{\beta(1-\mu^p)}{1-\beta\mu^p}\left(\zeta^p\pi_{t-1}^{w*} + (1-\zeta^p)\pi_{t-1}^{p*}\right). \tag{E.4}$$

Under the assumption that expectations of y, a, m^p—and so mrt—are constant to their steady-state values, this gives equation (35). Similarly for wage-setting

$$\pi_t^w = \kappa_c^w\left(\omega_t^e - \omega_t + m_t^w\right) + \kappa_y^w x_t + \zeta^w\beta(1-\mu^w)\left(mrs_{t-1}^* - \omega_t + m_{t-1}^{w*}\right) + \frac{\beta(1-\mu^w)}{1-\beta\mu^w}\left(\zeta^w\pi_{t-1}^{p*} + (1-\zeta^w)\pi_{t-1}^{w*}\right), \tag{E.5}$$

which reduces to (36) under the assumption that c, n, e, m^w—and so mrs—are constant to their steady-state values

F Optimal Policy Under Adaptive Expectations

The central bank seeks to maximize (14) subject to (35), (36), (13), (33) and (34). Let $\eta_t^1, \eta_t^2, \eta_t^3, \eta_t^4, \eta_t^5$ be the Lagrange multipliers on the five constraints. The first-order conditions of the central bank's program wrt. x_t, π_t^p, π_t^w; ω_t, π_t^{p*} and π_t^{w*} are

$$\lambda_x x_t + \kappa_y^p\eta_t^1 + \kappa_y^w\eta_t^2 = 0, \tag{F.1}$$

$$\lambda_p\pi_t^p - \eta_t^1 + \gamma\eta_t^4 + \eta_t^3 = 0, \tag{F.2}$$

$$\lambda_w\pi_t^w - \eta_t^2 + \gamma\eta_t^5 - \eta_t^3 = 0, \tag{F.3}$$

$$(\kappa_c^p + \beta(1-\mu^p)\zeta^p)\eta_t^1 - (\kappa_c^w + \beta(1-\mu^w)\zeta^w)\eta_t^2 + \eta_t^3 - \beta\mathbb{E}_t\eta_{t+1}^3 = 0, \tag{F.4}$$

$$\frac{\beta^2(1-\mu^p)}{1-\beta\mu^p}(1-\zeta^p)\mathbb{E}_t\eta_{t+1}^1 + \frac{\beta^2(1-\mu^w)}{1-\beta\mu^w}\zeta^w\mathbb{E}_t\eta_{t+1}^2 - \eta_t^4 + \beta(1-\gamma)\mathbb{E}_t\eta_{t+1}^4 = 0, \tag{F.5}$$

$$\frac{\beta^2(1-\mu^p)}{1-\beta\mu^p}\zeta^p\mathbb{E}_t\eta_{t+1}^1 + \frac{\beta^2(1-\mu^w)}{1-\beta\mu^w}(1-\zeta^w)\mathbb{E}_t\eta_{t+1}^2 - \eta_t^5 + \beta(1-\gamma)\mathbb{E}_t\eta_{t+1}^5 = 0. \tag{F.6}$$

The optimal policy that abstracts from the costs of wage dispersion is characterized in the same way, except that $\lambda_w = 0$ in equation (F.3).

G Optimal Policy After Overly Accommodative Policy under Rational Expectations

Figure G.1 plots the equivalent of Figure 6 under rational expectations. A past policy mistake—e.g. too accommodative policy in the past—has little effect on optimal policy today. It still consists in approximating the output gap.

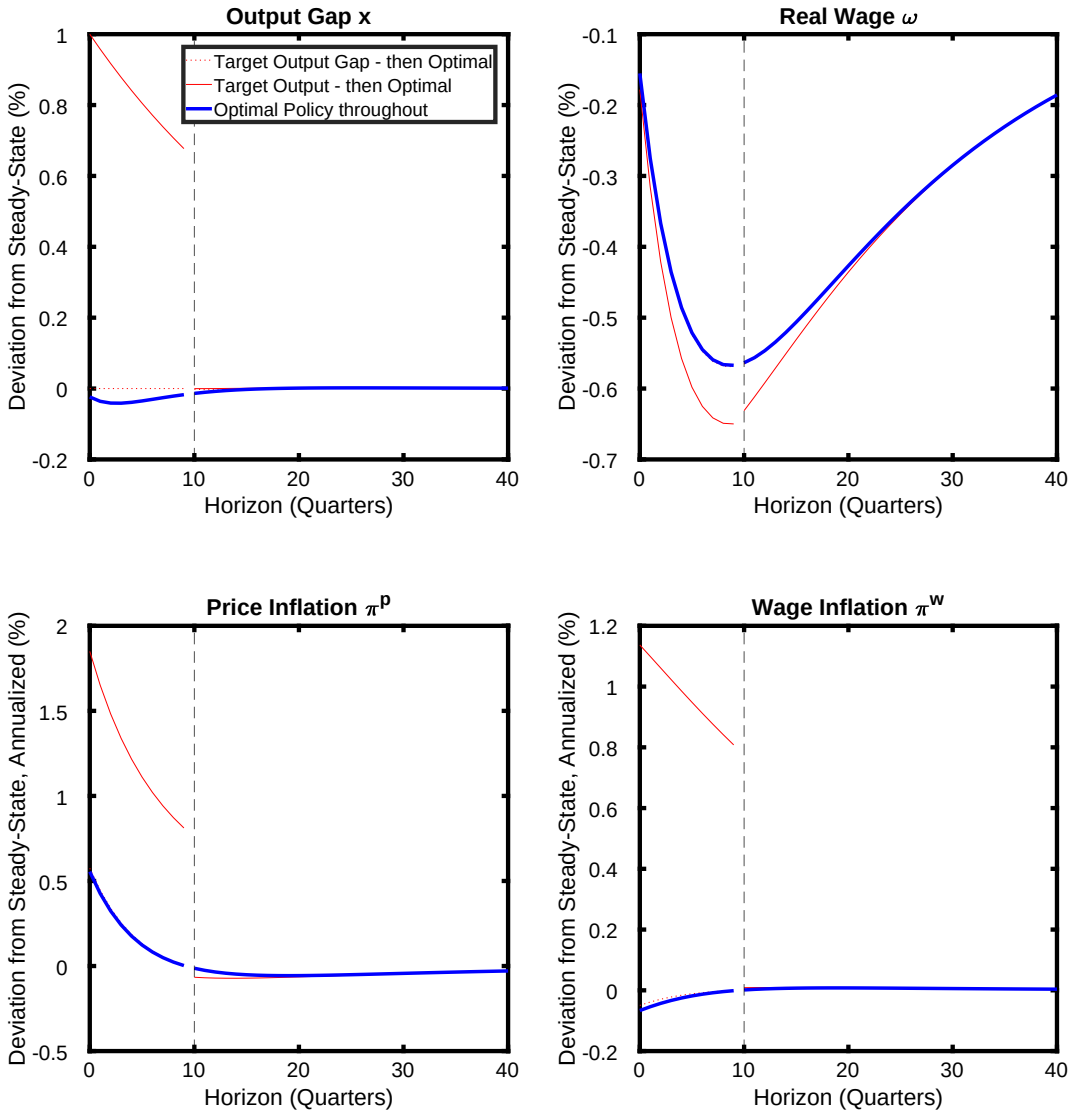


Figure G.1: Optimal Policy After Overly Accommodative Policy, Rational Expectations

Note: The figure plots the response of the output gap, the real wage, price inflation and wage inflation to a persistent negative productivity shock with a half-life of 16 quarters, under three scenarios. First, monetary policy keeps the output gap constant to zero from quarters 0 to 10, then switches to fully optimal policy. Second, monetary policy keeps output constant to its steady-state value from quarters 0 to 10, then switches to optimal policy. Third, monetary policy follows the optimal policy throughout. The calibration is given in the "Default Calibration" column of Table 1.

H Adaptive Expectations Variant

First, derive the price and wage Phillips curves. Start with the price Phillips curve. Subtracting p_t (which is perfectly observed by firms) from the expression of the resetting price (3) gives

$$p_t^* - p_t = (1-\beta\mu^p)\zeta^p(\omega_t - mrt_t + m_t^p) + (1-\beta\mu^p)\sum_{k=1}^{\infty}(\beta\mu^p)^k\tilde{E}_t\left(\zeta^p\left(w_{t+k} - p_t - mrt_{t+k} + m_{t+k}^p\right) + \frac{\zeta^p(1-\alpha)\theta^p}{\alpha}(p_{t+k} - p_t)\right). \tag{H.1}$$

Since the firm forms expectations directly on the real wage (40), we can write

$$\tilde{E}_t(w_{t+k} - p_t) = \tilde{E}_t(\omega_{t+k}) + \tilde{E}_t\left(\sum_{i=1}^{k}\pi_{t+i}^p\right). \tag{H.2}$$

Equation (H.1) then rewrites

$$\begin{aligned} p_t^* - p_t = {} & (1-\beta\mu^p)\zeta^p(\omega_t - mrt_t + m_t^p) \\ & + (1-\beta\mu^p)\zeta^p\sum_{k=1}^{\infty}(\beta\mu^p)^k\tilde{E}_t\left(\omega_{t+k} - mrt_{t+k} + m_{t+k}^p\right) \\ & + \sum_{k=1}^{\infty}(\beta\mu^p)^k\tilde{E}_t\pi_{t+k}^p. \end{aligned} \tag{H.3}$$

Assuming expectations at all horizons are the same,

$$\begin{aligned} p_t^* - p_t = {} & (1-\beta\mu^p)\zeta^p(\omega_t - mrt_t + m_t^p) \\ & + \zeta^p(\beta\mu^p)\left(\omega_{t-1}^* - mrt_{t-1}^* + m_{t-1}^{p*}\right) \\ & + \frac{\beta\mu^p}{1-\beta\mu^p}\pi_{t-1}^{p*}. \end{aligned} \tag{H.4}$$

Using (C.3) and under the assumption that expectations of y, a, m^p are constant to their

steady-state values, it gives

$$\pi_t^p = \kappa_c^p\left(\omega_t - \omega_t^e + m_t^p\right) + \kappa_y^p x_t + \zeta^p\beta(1-\mu^p)\omega_{t-1}^* + \frac{\beta(1-\mu^p)}{1-\beta\mu^p}\left(\pi_{t-1}^{p*}\right). \tag{H.5}$$

Following the same steps the wage Phillips curve is

$$\pi_t^w = \kappa_c^w\left(\omega_t^e - \omega_t + m_t^w\right) + \kappa_y^w x_t - \zeta^w\beta(1-\mu^w)\omega_{t-1}^* + \frac{\beta(1-\mu^w)}{1-\beta\mu^w}\left(\pi_{t-1}^{w*}\right). \tag{H.6}$$

Consider now optimal policy. The central bank seeks to maximize (14) subject to (H.5), (H.6), (13), (33), (34) and (39). Let $\eta_t^1, \eta_t^2, \eta_t^3, \eta_t^4, \eta_t^5, \eta_t^6$ be the Lagrange multipliers on the six constraints. The first-order conditions of the central bank's program wrt. x_t, π_t^p, π_t^w; ω_t, π_t^{p*}, π_t^{w*} and ω_t^* are

$$\lambda_x x_t + \kappa_y^p\eta_t^1 + \kappa_y^w\eta_t^2 = 0, \tag{H.7}$$

$$\lambda_p\pi_t^p - \eta_t^1 + \gamma\eta_t^4 + \eta_t^3 = 0, \tag{H.8}$$

$$\lambda_w\pi_t^w - \eta_t^2 + \gamma\eta_t^5 - \eta_t^3 = 0, \tag{H.9}$$

$$\kappa_c^p\eta_t^1 - \kappa_c^w\eta_t^2 + \eta_t^3 - \beta\mathbb{E}_t\eta_{t+1}^3 + \gamma\eta_t^6 = 0, \tag{H.10}$$

$$\frac{\beta^2(1-\mu^p)}{1-\beta\mu^p}\mathbb{E}_t\eta_{t+1}^1 - \eta_t^4 + \beta(1-\gamma)\mathbb{E}_t\eta_{t+1}^4 = 0, \tag{H.11}$$

$$\frac{\beta^2(1-\mu^w)}{1-\beta\mu^w}\mathbb{E}_t\eta_{t+1}^2 - \eta_t^5 + \beta(1-\gamma)\mathbb{E}_t\eta_{t+1}^5 = 0, \tag{H.12}$$

$$\beta^2(1-\mu^p)\zeta^p\eta_{t+1}^1 - \beta^2(1-\mu^w)\zeta^w\eta_{t+1}^2 - \eta_t^6 + \beta(1-\gamma)\mathbb{E}_t\eta_{t+1}^6 = 0. \tag{H.13}$$

The optimal policy that abstracts from the costs of wage dispersion is characterized in the same way, except that $\lambda_w = 0$ in equation (H.9).

I Real Wage Rigidity under Different Calibrations of Wage Changes

Figures I.1 and I.2 replicate Figure 8, but under different calibrations of the wage-change parameter μ^w: more rigid real wages with a higher probability of not changing real wages

($\mu_w = 0.9$) and more flexible real wages with a lower probability of not changing real wages ($\mu_w = 0.5$)

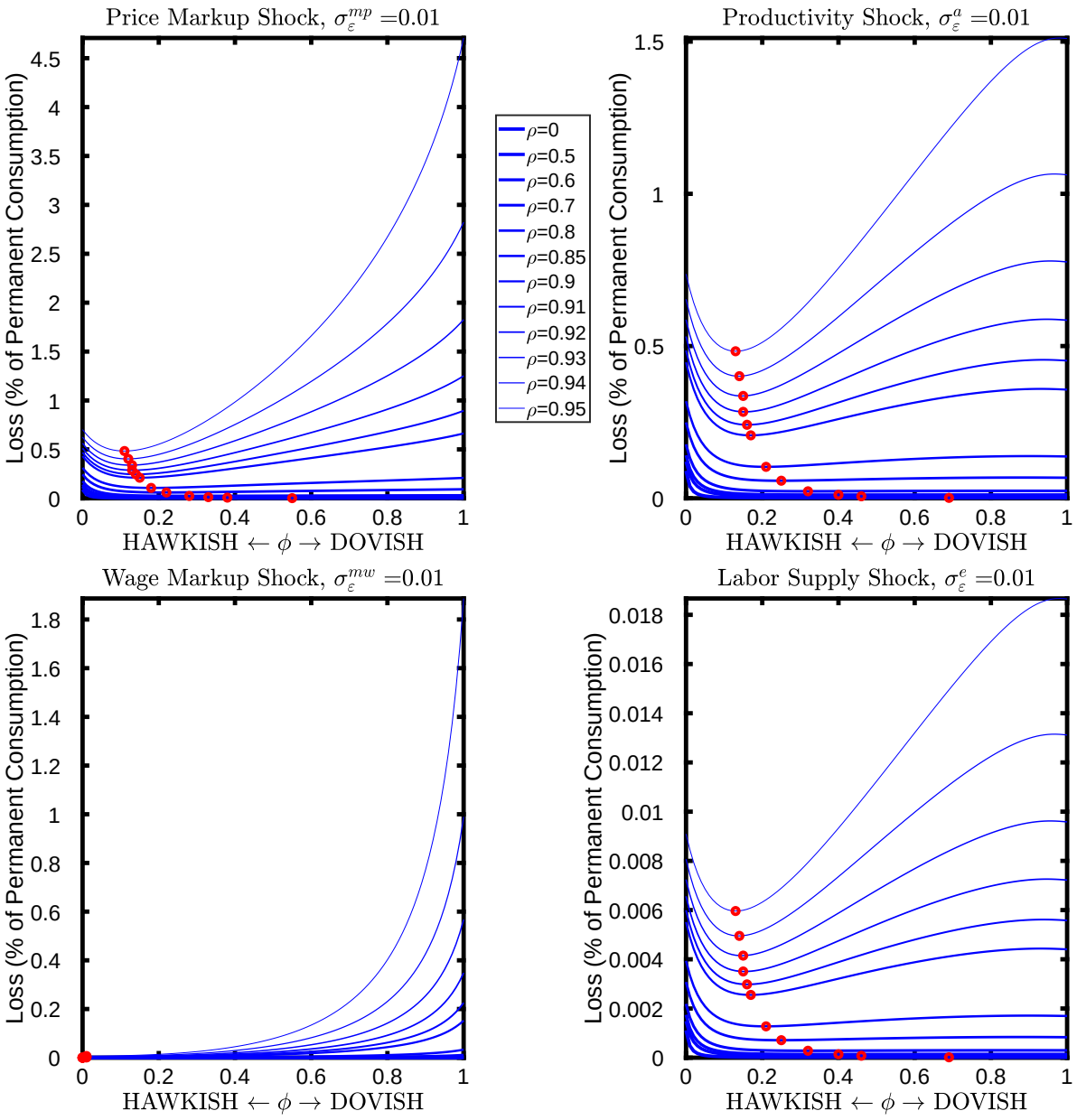


Figure I.1: Loss Function under Real Wage Rigidity and Rational Expectations, $\mu_w = 0.9$

Note: For each type of supply shock, the figure plots the loss function of the central bank (14) as a function of the weights ϕ on the output gap in the objective (20) of the central bank, under the assumption that the economy is hit by this one type of shock only. It does so for different levels of persistence of the shock, captured by the half-life of the AR(1) process. On each curve, the red dot corresponds to the minimum of the curve, that is the optimal policy within the class of policies (20). The calibration is given in the "Default Calibration" column of Table 1.

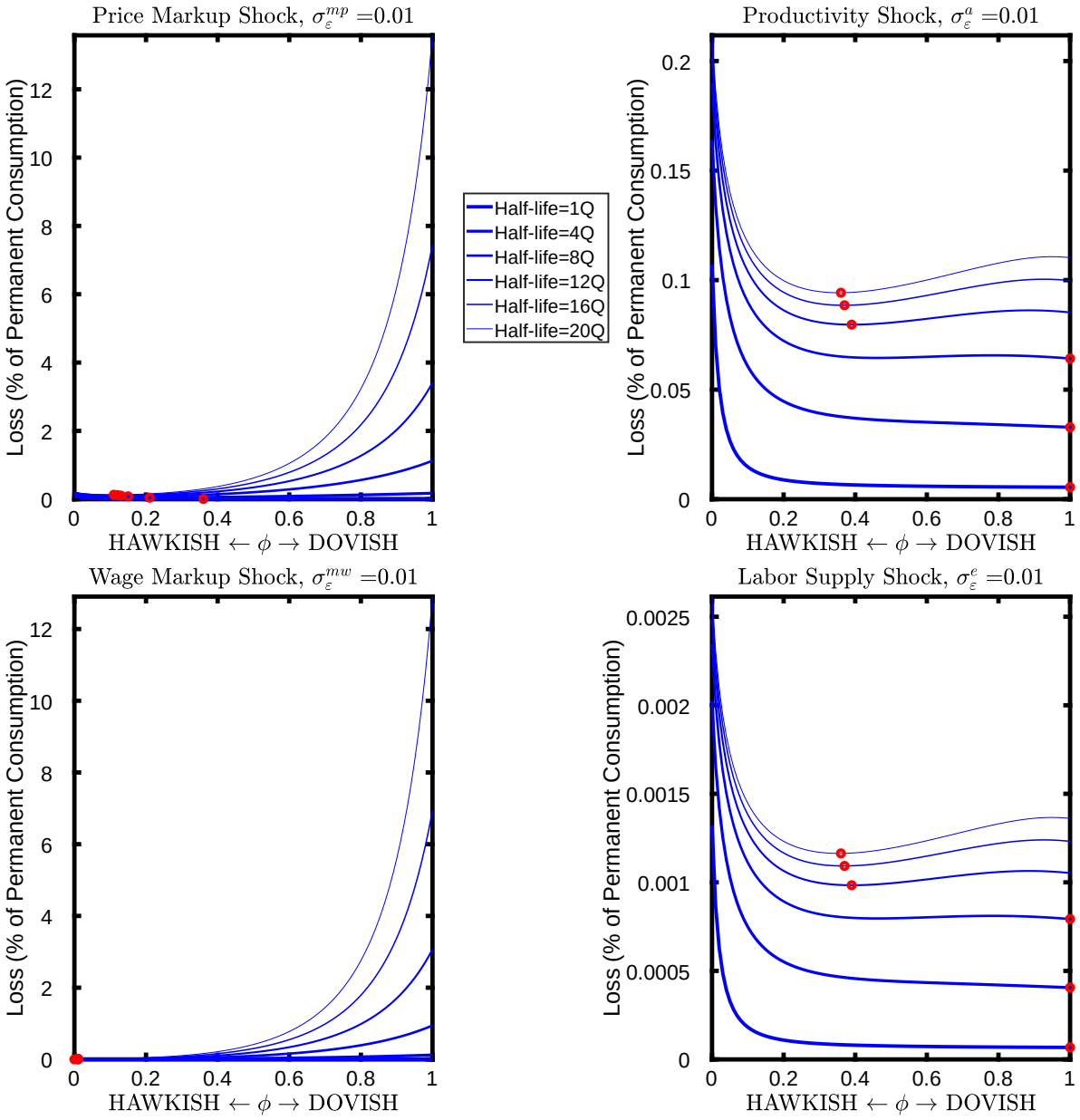


Figure I.2: Loss Function under Real Wage Rigidity and Rational Expectations, $\mu_w = 0.5$

Note: For each type of supply shock, the figure plots the loss function of the central bank (44) as a function of the weights ϕ on the output gap in the objective (20) of the central bank, under the assumption that the economy is hit by this one type of shock only. It does so for different levels of persistence of the shock, captured by the half-life of the AR(1) process. On each curve, the red dot corresponds to the minimum of the curve, that is the optimal policy within the class of policies (20). The calibration is given in the "Default Calibration" column of Table 1.

J Optimal Policy Under Rational Expectations and Real Wage Rigidity

The central bank seeks to maximize (44) subject to (21), (25) and (26). Let $\eta_t^1, \eta_t^2, \eta_t^3$ be the Lagrange multipliers on the three constraints. The first-order conditions of the central bank's program wrt. π^p, x, Δ and ω are

$$\lambda_p \pi_t^p - \eta_t^1 + \eta_{t-1}^1 = 0, \tag{J.1}$$

$$\lambda_x x_t + \kappa_y^p \eta_t^1 + \kappa_y^w \eta_t^2 = 0, \tag{J.2}$$

$$\lambda_w \Delta_t - \eta_t^2 + \eta_{t-1}^2 - \eta_t^3 = 0, \tag{J.3}$$

$$\kappa_c^p \eta_t^1 - \kappa_c^w \eta_t^2 + \eta_t^3 - \beta \mathbb{E}_t \eta_{t+1}^3 = 0. \tag{J.4}$$

K Derivation of the Wage Phillips Curves under Real Wage Rigidity and Adaptive Expectations

The wage-setting equation (41) can be written

$$\omega_t^* - \omega_t = (1 - \beta\mu^w) \sum_{k=0}^{\infty} (\beta\mu^w)^k \tilde{\mathbb{E}}_t \left(\zeta_w \left(mrs_{t+k} + m_{t+k}^w - \omega_t \right) + (1 - \zeta^w) \left(\omega_{t+k} - \omega_t \right) \right) \tag{K.1}$$

Expectations of real wage growth can be written

$$\tilde{E}_t(\omega_{t+k} - \omega_t) = \tilde{E}_t \left(\sum_{i=1}^{k} \pi_{t+i}^w - \pi_{t+i}^p \right), \tag{K.2}$$

Using this, and isolating the time-t term, equation (K.1) can be written

$$\begin{aligned} \omega_t^* - \omega_t = (1 - \beta\mu^w)\zeta^w \left(mrs_t + m_t^w - \omega_t \right) + (1 - \beta\mu^w)\zeta^w \sum_{k=1}^{\infty} (\beta\mu^w)^k \tilde{\mathbb{E}}_t \left(mrs_{t+k} + m_{t+k}^w - \omega_t \right) \\ + (1 - \zeta^w) \sum_{k=1}^{\infty} (\beta\mu^w)^k \tilde{\mathbb{E}}_t \left(\pi_{t+k}^w - \pi_{t+k}^p \right) \end{aligned} \tag{K.3}$$

Using (42) and the fact that expectations at all horizons are the same, it can be rewritten as

$$\omega_t - \omega_{t-1} = \kappa_c^w(\omega_t^e - \omega_t + m_t^w) + \kappa_y^w x_t + \beta(1-\mu^w)\zeta^w(mrs_{t-1}^* - \omega_t + m_{t-1}^{w^*}) + \frac{\beta(1-\mu^w)}{1-\beta\mu^w}(1-\zeta^w)(\pi_{t-1}^{w*} - \pi_{t-1}^{p*}). \tag{K.4}$$

Using the assumption that unions expect future c, n (hence the mrs) and future shocks to be at their steady-state values, it gives equation (47) in the text.

L Derivation of the Wage Phillips Curves under Real Wage Rigidity and Adaptive Expectations Variant

Start again from equation (K.1). Isolating the time-t term, it can be rewritten

$$\omega_t^* - \omega_t = (1-\beta\mu^w)\zeta^w\left(mrs_t + m_t^w - \omega_t\right) + (1-\beta\mu^w)\zeta^w\sum_{k=1}^{\infty}(\beta\mu^w)^k\tilde{\mathbb{E}}_t\left(mrs_{t+k} + m_{t+k}^w - \omega_t\right) + (1-\zeta^w)\sum_{k=1}^{\infty}(\beta\mu^w)^k\tilde{\mathbb{E}}_t\left(\omega_{t+k} - \omega_t\right). \tag{L.1}$$

Using (42) and the fact that expectations at all horizons are the same, it can be rewritten as

$$\omega_t - \omega_{t-1} = \kappa_c^w(\omega_t^e - \omega_t + m_t^w) + \kappa_y^w x_t + \beta(1-\mu^w)\zeta^w(mrs_{t-1}^* - \omega_t + m_{t-1}^{w^*}) + \beta(1-\mu^w)(1-\zeta^w)(\omega_{t-1}^* - \omega_t). \tag{L.2}$$

Using the assumption that unions expect future c, n (hence the mrs) and future shocks to be at their steady-state values, it gives

$$\omega_t - \omega_{t-1} = \kappa_c^w(\omega_t^e - \omega_t + m_t^w) + \kappa_y^w x_t - \beta(1-\mu^w)\zeta^w\omega_{t-1}^* + \beta(1-\mu^w)(\omega_{t-1}^* - \omega_t). \tag{L.3}$$

Together with the price Phillips curve under the adaptive expectations variant (H.5), it forms the economy under real wage rigidity and the adaptive expectations variant.

Figure L.1 represents the ranking of the different policies (20) as in Figure 1 but under the joint assumptions of real wage rigidity and the adaptive expectations variant. Results are very similar to the case of real wage rigidity and the main specification of adaptive expectations on Figure 9.

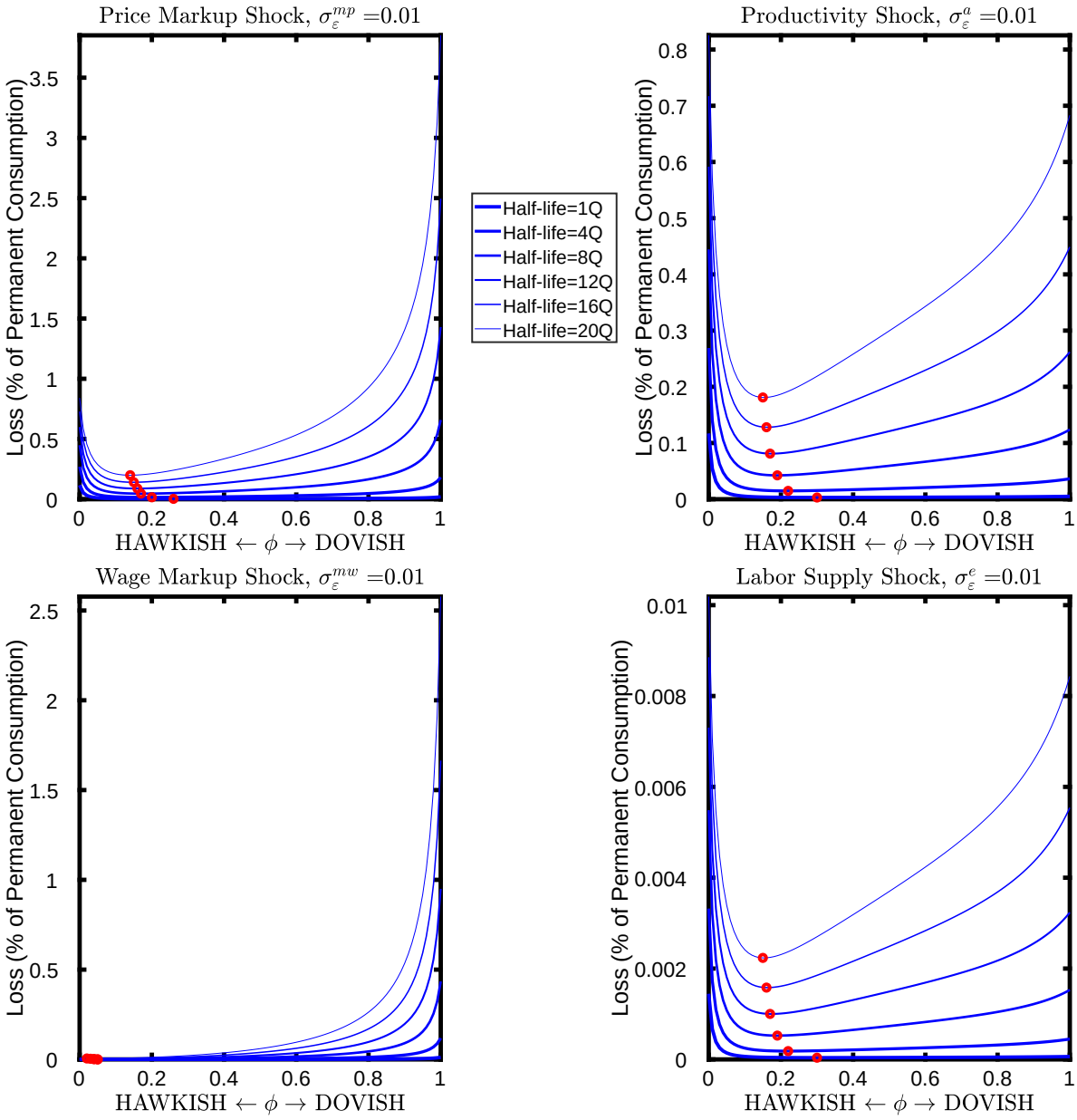


Figure L.1: Loss Function under Real Wage Rigidity and Adaptive Expectations Variant

Note: For each type of supply shock, the figure plots the loss function of the central bank (14) as a function of the weights ϕ on the output gap in the objective (20) of the central bank, under the assumption that the economy is hit by this one type of shock only. It does so for different levels of persistence of the shock, captured by the half-life of the AR(1) process. On each curve, the red dot corresponds to the minimum of the curve, that is the optimal policy within the class of policies (20). The calibration is given in the "Default Calibration" column of Table 1.

M Optimal Policy Under Adaptive Expectations and Real Wage Rigidity

The central bank seeks to maximize (44) subject to (35), (47), (26), (33) and (34). Let $\eta_t^1, \eta_t^2, \eta_t^3, \eta_t^4, \eta_t^5$ be the Lagrange multipliers on the five constraints. The first-order conditions of the central bank's program wrt. x_t, π_t^p, Δ_t; ω_t, π_t^{p*} and π_t^{w*} are

$$\lambda_x x_t + \kappa_y^p \eta_t^1 + \kappa_y^w \eta_t^2 = 0, \quad \text{(M.1)}$$

$$\lambda_p \pi_t^p - \eta_t^1 + \gamma \eta_t^4 + \gamma \eta_t^5 = 0, \quad \text{(M.2)}$$

$$\lambda_w \Delta_t - \eta_t^2 + \gamma \eta_t^5 - \eta_t^3 = 0, \quad \text{(M.3)}$$

$$(\kappa_c^p + \beta(1-\mu^p)\zeta^p)\eta_t^1 - (\kappa_c^w + \beta(1-\mu^w)\zeta^w)\eta_t^2 + \eta_t^3 - \beta \mathbb{E}_t \eta_{t+1}^3 = 0, \quad \text{(M.4)}$$

$$\frac{\beta^2(1-\mu^p)}{1-\beta\mu^p}(1-\zeta^p)\mathbb{E}_t\eta_{t+1}^1 - \frac{\beta^2(1-\mu^w)}{1-\beta\mu^w}(1-\zeta^w)\mathbb{E}_t\eta_{t+1}^2 - \eta_t^4 + \beta(1-\gamma)\mathbb{E}_t\eta_{t+1}^4 = 0, \quad \text{(M.5)}$$

$$\frac{\beta^2(1-\mu^p)}{1-\beta\mu^p}\zeta^p\mathbb{E}_t\eta_{t+1}^1 + \frac{\beta^2(1-\mu^w)}{1-\beta\mu^w}(1-\zeta^w)\mathbb{E}_t\eta_{t+1}^2 - \eta_t^5 + \beta(1-\gamma)\mathbb{E}_t\eta_{t+1}^5 = 0. \quad \text{(M.6)}$$

N Optimal Policy Under Adaptive Expectations Variant and Real Wage Rigidity

The central bank seeks to maximize (44) subject to (H.5), (L.3), (26), (33) and (39). Let $\eta_t^1, \eta_t^2, \eta_t^3, \eta_t^4, \eta_t^6$ be the Lagrange multipliers on the five constraints. The first-order conditions of the central bank's program wrt. x_t, π_t^p, Δ_t; ω_t, π_t^{p*} and ω_t^* are

$$\lambda_x x_t + \kappa_y^p \eta_t^1 + \kappa_y^w \eta_t^2 = 0, \quad \text{(N.1)}$$

$$\lambda_p \pi_t^p - \eta_t^1 + \gamma \eta_t^4 = 0, \quad \text{(N.2)}$$

$$\lambda_w \Delta_t - \eta_t^2 - \eta_t^3 = 0, \quad \text{(N.3)}$$

$$\kappa_c^p \eta_t^1 - (\kappa_c^w + \beta(1-\mu^w))\eta_t^2 + \eta_t^3 - \beta \mathbb{E}_t \eta_{t+1}^3 + \gamma \eta_t^6 = 0, \quad \text{(N.4)}$$

$$\frac{\beta^2(1-\mu^p)}{1-\beta\mu^p}\mathbb{E}_t\eta_{t+1}^1 - \eta_t^4 + \beta(1-\gamma)\mathbb{E}_t\eta_{t+1}^4 = 0, \quad \text{(N.5)}$$

$$(\beta^2(1-\mu^p)\zeta^p)\mathbb{E}_t\eta_{t+1}^1 - (\beta^2(1-\mu^w)(1+\zeta^w))\mathbb{E}_t\eta_{t+1}^2 - \eta_t^6 + \beta(1-\gamma)\mathbb{E}_t\eta_{t+1}^6 = 0. \quad \text{(N.6)}$$