Complementarity of Bank and Market Debt in Dynamic Debt Structure[†]

Jessica Shi Li[‡]

Booth School of Business, University of Chicago

First version: September 2025, This version: November 2025

For latest version click here

Abstract

I develop a dynamic model of firm debt structure with renegotiable bank debt and arm's-length market debt. The firm and its bank lender periodically renegotiate bank debt through bilateral bargaining, maximizing the joint value between the firm's equity holders and the bank lender by extracting from the non-bargaining market debt investors. The firm also strategically issues or repurchases market debt to position its equity holders for potential bank debt renegotiation, effectively "front-running" the bank lender and limiting the rents it captures during renegotiation. This strategic interaction creates an element of complementarity between bank and market debt, as the firm orchestrates an architecture of mutually-extractive claims. A key result is that the firm positions for bank debt renegotiation by trading market debt in the opposite direction: it issues market debt if renegotiation would reduce bank debt, and repurchases market debt when renegotiation would increase bank debt. Empirically, firms facing potential bank-debt-increasing renegotiations are three times as likely to repurchase market debt. Another novel prediction is the *dynamic complementarity* of bank and market debt: a higher balance of bank debt leads to greater future market debt issuance. The dynamic complementarity is robust to bargaining power allocation, but assigning more bargaining power to the firm weakens the dynamic complementarity and reduces firm value. The model rationalizes key contractual features observed in practice – such as debt seniority and covenants – and highlights a coordination inefficiency arising from the firm's incentive to strategically pit the bank lender and the market debt investors against each other.

Keywords: Debt Structure, Debt Complementarity, Dynamics, Bargaining

[†]I thank Peter DeMarzo, Doug Diamond, Gene Fama, Maryam Farboodi (discussant), Thomas Geelen (discussant), Lars Hansen, Zhiguo He, Ben Hébert, Ruggero Jappelli (discussant), Anil Kashyap, Arvind Krishnamurthy, Yueran Ma, Stefan Nagel, Marcus Opp, Raghu Rajan, Amir Sufi, Neng Wang, Zijun Wang (discussant), Randy Wright, Anthony Zhang, Hai Zhang (discussant), Jeff Zwiebel, conference participants at the 2025 FMA Annual Meeting, the 2025 NFA Annual Meeting, the 2025 Chicago-Harvard-Wharton Insolvency & Restructuring Conference, the 2025 Vienna Festival of Finance Theory, the 2025 Summer Workshop on Money, Banking, Payments and Finance, the 5th Corporate Policies and Asset Prices Conference, the 2025 Yiran Fan Memorial Conference, the 25th and 26th Macro Finance Society Workshop, and workshop participants at Stanford GSB and Chicago Booth for helpful discussions and comments. All errors are my own.

[‡]5807 S Woodlawn Ave, Chicago, IL 60637. E-mail: jessicali@chicagobooth.edu.

1 Introduction

Firms obtain debt financing through two main sources: bank debt and market debt. A firm's debt choice is not a binary decision. Instead, the vast majority of firms issue both bank debt and market debt. For example, Rauh and Sufi (2010) document that 70% of firm-year observations in their sample have debt structures comprising at least two different types of debt instruments, with bank loans and corporate bonds being the two most commonly used debt types.[1] Despite the prevalence of debt structures that include both bank debt and market debt, a satisfactory theory of debt composition has yet to be fully developed. Existing theories typically focus on the choice between bank debt and market debt rather than how firms combine them in their debt structures. A few models that address the latter either are static in nature or rule out incremental adjustment in debt amounts over time (e.g., Park (2000), Hackbarth, Hennessy and Leland (2007), Crouzet (2018), Halling, Yu and Zechner (2025)), leaving the dynamics crucial to how claimants interact in shaping firms' debt structures underexplored.

In this paper, I develop a dynamic model of firm debt structure with both bank debt and market debt. A key distinction between bank debt and market debt is that the former is relationship-based, whereas the latter is arm's-length. Bank debt is obtained through a bilateral, inherently strategic, financing relationship between the firm and its bank lender. Evolving firm circumstances create incentives for both parties to dynamically renegotiate and re-optimize the contract for their mutual benefit; that is, bank debt is adjusted repeatedly over time through bilateral bargaining in response to changes in the firm's states, leading to changes in the face amount and payments between the bank lender and the firm. This dynamic renegotiability of bank debt is empirically salient: numerous studies document that bank debt – including bilateral bank loans, syndicated loans, and private debt – is frequently renegotiated, both in and outside financial distress, before maturities, and in the absence of covenant violations.[2] By contrast, market debt is held by dispersed market debt investors. Free-riding and coordination problems, as well as legal restrictions, render market debt arm's-length and difficult to renegotiate. Market debt is issued or repurchased at competitive prices.

Given this defining feature of bank debt relative to market debt, how do the two types of debt dynamically interact within a firm's debt structure? The model helps answer this question.

[1] Consistent with their result, I show in Section 5 that among the firm-year observations in my Compustat sample, 55-72% of those with market debt also have bank debt, depending on the definitions of market debt and bank debt.

[2] For example, see Roberts and Sufi (2009), Roberts (2015), and Bidder, Crouzet, Jacobson and Siemer (2023).

The model emphasizes the dynamic interplay among the firm's claimants – specifically, its equity holders, bank lender, and market debt investors. The firm, maximizing its equity holders' value, periodically renegotiates the bank debt with its bank lender. Market debt investors are excluded from these renegotiations, leaving them vulnerable to value extraction by the bargaining coalition of the firm and the bank lender – which can occur, for example, through dilution. During renegotiation, the firm and the bank lender bilaterally bargain to maximize their joint value, trading off the "private" benefit of extracting from the non-bargaining market debt investors against the cost of financial distress. Meanwhile, the firm trades with market debt investors through issuance or repurchase. Market debt trading shapes the outcome of potential bank-firm renegotiation, which is priced in by market debt investors who are forward-looking and fully rational. The firm strategically trades market debt to enhance its equity holders' position in potential bank debt renegotiation while accounting for the renegotiation's impact on the current market debt price, in turn curbing the bank lender's rents during bargaining.

In essence, the firm strategically pits the bank lender and the market debt investors against each other. The firm, acting on behalf of its equity holders, colludes with the bank lender to extract value from the market debt investors through bilateral bargaining that excludes the latter. At the same time, the firm also trades directly with the market debt investors to "front-run" and limit the rents accruing to the bank lender during potential bank-firm renegotiation. Taken together, this strategic interaction creates an element of complementarity between bank debt and market debt: their coexistence reflects a strategic architecture of mutually-extractive claims orchestrated by the firm. As an analogy, market debt serves both as a prey and as a weapon from the firm's perspective: it is preyed upon by the bargaining coalition of the firm and its bank lender, while also being wielded by the firm through strategic trading to blunt the bank lender's rents during renegotiation.

A key result of the model is that the firm positions for potential bank debt renegotiation by trading market debt in the opposite direction: it issues additional market debt if renegotiation would reduce the amount of bank debt, and repurchases existing market debt if renegotiation would increase the bank debt balance. This result may seem surprising at first glance: if the firm intends to increase bank debt to raise leverage, one might expect it to issue market debt at the same time to reach the target leverage more quickly; likewise, if the firm wishes to reduce bank debt to lower leverage, one might expect it to repurchase market debt in tandem. Why is the model result opposite? For example, why would the firm repurchase its market debt if a potential bank debt renegotiation would lead the firm and the bank lender

to increase the bank debt amount? The answer lies in the firm's strategic incentives. Intuitively, if bank debt renegotiation would increase the bank debt level – indicating an unrealized opportunity to extract value from the market debt investors – the firm can limit the bank lender's rents from renegotiation by repurchasing its market debt, thereby reducing the amount of market debt available for extraction.

Particularly transparent is the case of a strong bank lender who holds all the bargaining power. When the bank lender and the firm renegotiate, the firm's equity holders capture no bargaining surplus and are left with their continuation value under the current bank debt contract, which is their outside option and credible threat point in renegotiation. Conditional on renegotiation, the forward-looking market debt price incorporates the expected bargaining outcome, whereas the equity holders' outside option depends on the current states. If renegotiation increases bank debt, the expected dilution of market debt upon renegotiation causes the market debt price to fall below the positive marginal effect of repurchasing market debt on the equity holders' continuation value (i.e., their outside option), prompting the firm to repurchase its market debt. In effect, the cost of repurchasing market debt can be seen as partially borne by the bank lender, allowing the firm to strengthen its equity holders' outside option by opportunistically buying back its market debt at a low price. Conversely, if renegotiation reduces bank debt, the market debt investors benefit from the lower leverage and value their claims more highly. The market debt price can be viewed as effectively subsidized by the bank lender, allowing the firm to issue market debt at a favorable price even if it slightly weakens the equity holders' outside option.

Using granular firm-level data, I test the theory empirically by showing that a firm is about 8% more likely to repurchase market debt, via either open-market repurchases or tender offers, when its existing bank debt is below the implied joint-surplus-maximizing level that would arise from bank-firm renegotiation given the firm's prevailing conditions. The 8 percentage point marginal effect is economically meaningful: given a baseline probability of market debt repurchase just below 5%, a firm facing a potential bank-debt-increasing renegotiation is nearly three times as likely to repurchase its market debt. It is also worth noting that the model provides a natural mechanism for market debt repurchases – which in practice take the form of either debt tender offers or open-market repurchases and are relatively common (around $150 billion in face value per year from 2002 to 2024) – but which existing theories struggle to explain. For instance, the leverage ratchet effect of Admati, DeMarzo, Hellwig and Pfleiderer (2018) and DeMarzo and He (2021) makes the stark prediction that firms would never voluntarily buy back their debt since the benefit due to leverage reduction would accrue to the existing debt holders.

Another novel empirical prediction of the model is what I refer to as the *dynamic complementarity* of bank and market debt: a higher current level of bank debt is followed by an increased intensity of market debt issuance by the firm. This prediction contrasts with the conventional wisdom that bank debt and market debt are substitutes. With a higher current level of bank debt, renegotiation between the bank lender and the firm would result in a greater reduction (or a smaller increase) in the amount of bank debt, prompting the firm to issue more (or repurchase less) of the market debt. The dynamic complementarity between bank debt and market debt is robust to how bargaining power is allocated, although its magnitude declines as the firm's bargaining power increases. Intuitively, this dynamic complementarity arises from the firm's strategic incentive to trade market debt to position for potential bank debt renegotiation, "front-running" the bank lender and limiting the rents it can capture. This incentive is stronger when the firm has less bargaining power, leading to a more pronounced dynamic complementarity between bank and market debt.

I provide suggestive empirical evidence consistent with this novel prediction. Using firm-level data, I show that controlling for firm characteristics, a greater amount of bank debt in the debt structure indeed predicts higher rate of market debt issuance in the following year. The pattern is weaker for firms that renegotiate their bank debt less frequently, highlighting a potential link between the observed dynamic debt complementarity and the bank debt renegotiation mechanism emphasized in the model. Moreover, I show that the degree of dynamic complementarity between bank debt and market debt varies across firm characteristics and time periods. The pattern is more pronounced for smaller, less profitable, and riskier firms as well as during periods of market stress. I consider potential explanations for this heterogeneity across firms and time, and find that it appears to be explained, at least in part, by differences in firm bargaining power.

To my knowledge, this paper is the first to highlight the strategic interactions between a firm's bank debt and market debt in a dynamic setting. The central idea is that the firm bargains bilaterally with its bank lender to extract value from the non-bargaining market debt investors, while simultaneously trading with the market debt investors to discipline the bank lender's rents from renegotiation. The model is deliberately parsimonious to isolate and illuminate this key mechanism. Nonetheless, an important question is what the model's mechanism implies for realistic contractual features observed in practice and for overall efficiency.

An implication of the model is that bank debt is senior to market debt in equilibrium, in the sense that bank debt has higher priority in default and thus greater recovery value. This implication is consistent with one of the most salient empirical features of bank debt (e.g., see Rauh and Sufi (2010), Schwert (2019)). The intuition is as follows. When the bank lender and the firm renegotiate, they choose the bank debt level that maximizes their joint value, trading off the cost of financial distress against any "private" benefit that bank debt may bring to the two bargaining parties. If bank debt is senior, market debt acts as a buffer upon default, enabling the bank lender and the firm's equity holders to extract value by diluting the market debt investors. If bank debt is not senior, future market debt issuances constrain such extraction, eliminating the private benefit and leading to zero bank debt upon renegotiation. Hence, any bank debt that exists in equilibrium must be senior.

The model also sheds light on debt covenants. Debt incurrence imposes cost of financial distress and an efficient covenant package would permit debt reductions while prohibiting any new debt issuance. However, implementing such a package is difficult due to agents' private incentives and the strategic interactions among them. In equilibrium, bank debt covenants place partial restrictions on the firm's ability to issue and repurchase market debt, allowing some flexibility without fully prohibiting or fully permitting these actions. In practice, bank debt agreements often contain leverage tests and debt incurrence covenants that restrict the firm's ability to issue market debt, as well as restricted payment covenants and prepayment restrictions that limit the firm's ability to retire market debt. These covenants are subject to negotiated carve-outs, which determine how restrictive the covenants are. Market debt covenants, by contrast, do not restrict debt reductions, but place some constraints on both bank and market debt issuances, without completely shutting them down. In practice, market debt indentures often contain limitation on indebtedness and negative pledge covenants that constrain the firm's ability to raise additional bank debt and market debt, but rarely limit the firm's ability to retire debt. As a result, covenants cannot entirely resolve the coordination inefficiency that arises from the firm's strategic behavior in pitting the market debt investors and the bank lender against one another.

What does the model imply for efficiency? From the firm-value perspective, it is socially efficient for the firm to rely solely on the renegotiable bank debt. However, the firm has a private incentive to deviate from such a bank-debt-only debt structure, because the firm benefits from having market debt by playing the bank lender and the market debt investors against each other. While bilateral bargaining between the bank lender and the firm would be efficient in isolation, the presence of market debt introduces a

tension: the bank lender and the equity holders share an incentive to extract value from the market debt investors, who are excluded from the bargaining process. The value extraction incentive thus distorts the efficiency of bilateral bargaining between the bank lender and the firm. The firm's strategic trading of its market debt limits the rents accruing to the bank lender during bargaining, but can either mitigate or exacerbate inefficiency. For instance, when the firm issues market debt to position for potential bank debt renegotiation that would reduce the amount of bank debt, the extraction incentive by the bank lender and the firm strengthens during renegotiation, leading to a less efficient reduction in leverage.

Are there ways to improve efficiency? One possibility is optimizing the frequency of bank debt renegotiations. More frequent renegotiations allow the firm to better avoid default via timely bank debt forgiveness when leverage is excessive. However, as default becomes less likely, extracting value by diluting market debt investors appears more attractive to the bank lender and the equity holders during renegotiation, thus creating additional distortion and reducing the total firm value. Finding the right balance in renegotiation frequency – for example, by choosing bank debt maturities or writing covenant tripwires that trigger renegotiation – could enhance efficiency. That being said, it cannot eliminate distortions arising from the strategic interactions among the firm's claimants. More fundamentally, improving efficiency may require state-contingent contractual features that limit inefficient value extraction and better align incentives among the bank lender, the market debt investors, and the equity holders.

Related Literature. This paper contributes to the recent literature on the dynamics of firm debt structure, specifically the composition between bank debt and market debt. Crouzet (2018) and Halling et al. (2025) derive dynamic models where firms fund their investments with both bank debt and market debt. Bank debt differs from market debt in that it can be restructured in financial distress. Crouzet (2018) shows that through endogenous investments, borrowing market debt enables the firm to operate at a larger scale, raising liquidation values and thereby lowering the cost of bank debt. Halling et al. (2025) focus on the demand side, and show that demand for renegotiable bank debt rises in recessions when its flexibility is particularly valuable. My model also distinguishes bank debt from market debt by its renegotiability, but allows renegotiation in both good and bad states: even as earnings improve, bank debt contract entered into previously can be improved upon to generate gains from trade. Moreover, my model departs from the previous models by allowing incremental market debt issuance or repurchase. I abstract from investments and focus instead on the firm's strategic trading in market debt to position for bank debt renegotiation.

The paper thus falls under the broad theoretical literature on debt structures involving multiple forms of debt. For example, the choice between bank debt and market debt has been studied in static settings, with contributions from Diamond (1991), Rajan (1992), Besanko and Kanatas (1993), Chemmanur and Fulghieri (1994), Bolton and Scharfstein (1996), Bolton and Freixas (2000), among others. In continuous-time models, Hackbarth et al. (2007) study the optimal mix between bank debt and market debt under a static trade-off framework, while Morellec, Valta and Zhdanov (2015) embed the trade-off decision into dynamic investment choices. A related line of research examines how seniority is allocated across different debt types, for example, Park (2000), Hackbarth and Mauer (2012), Donaldson, Gromb and Piacentino (2025a).

The paper is fundamentally about the interaction among various claimants of the firm who have differential abilities to bargain. A related paper is Bulow and Shoven (1978). They show in a static model that actions by a bargaining subset of claimants may not maximize the total firm value, an insight echoed in my paper. Unlike their setting and new to the literature, my model is fully dynamic and allows trading between some bargaining claimants (i.e., equity holders) and the non-bargaining claimants (i.e., market debt investors), creating a feedback loop between bank-firm bargaining and market debt trading. A novel insight is that the equity holders strategically pit the bank lender and the market debt investors against one another, by orchestrating an architecture of mutually-extractive claims. Gertner and Scharfstein (1991), Strömberg (2000), and more recently Antill, Wang and Jiang (2025) also demonstrate that conflicts among lenders can lead to efficiency implications for the firm. Further related is Perotti and Spier (1993), who present a static model of labor renegotiation in which the firm temporarily raises its leverage to influence bargaining outcomes with the union.

The paper further connects to the literature on debt dynamics without commitment and on the leverage ratchet effect. In modeling market debt, this paper follows DeMarzo and He (2021) in assuming that the firm cannot commit to a market debt leverage ex-ante and instead adjusts its market debt incrementally through issuance or repurchase. Their paper shows that without commitment, the tax benefits of leverage are fully dissipated. By contrast, my model emphasizes the firm's strategic trading in market debt to position for bank debt renegotiation – a point that is orthogonal to theirs. Unlike the leverage ratchet effect described by Admati et al. (2018) and DeMarzo and He (2021), the firm may optimally decide to repurchase its market debt in my model. Other papers such as Mao and Tserlukevich (2015) and Dangl and Zechner (2021) rely on alternative mechanisms to generate debt repurchases or

partial rollovers. Donaldson, Koont, Piacentino and Vanasco (2025b) argue that credit lines may prevent lenders from making loans by acting as a self-deterring mechanism.

Finally, this paper relates to the empirical literature on debt structures, particularly the composition between bank debt and market debt. Rauh and Sufi (2010) document the prevalence of multi-tiered debt structure consisting of both secured bank debt and subordinated market debt. Colla, Ippolito and Li (2013) show that large rated firms are more likely to diversify across debt types. Becker and Ivashina (2014) and Darmouni and Siani (2025) show debt composition changes upon economic shocks, such as tightening lending standards during crises and policy support to the corporate bond market post-COVID. Furthermore, the paper's modeling choice regarding bank debt renegotiation is motivated by empirical evidence documented by Roberts and Sufi (2009), Roberts (2015), and Bidder et al. (2023).

The rest of the paper is organized as follows. Section 2 describes the setup of the general model. Section 3 provides solution for the case of a strong bank lender who holds all the bargaining power, which allows for several closed-form results thus better illustrating the key mechanism. Section 4 studies how the allocation of bargaining power affects the model predictions as well as the total firm value. Section 5 provides empirical evidence consistent with the model predictions. Section 6 concludes.

2 The Model

I develop a model of dynamic debt structure. In the model, a firm borrows bank debt from a relationship bank lender and market debt from arm's-length market debt investors, while maximizing its equity holders value. The bank debt is dynamically renegotiable: its terms are periodically re-optimized to maximize the joint surplus between the bank lender and the equity holders through bilateral bargaining as the firm's states evolve. By contrast, market debt is non-renegotiable, though the firm may continuously trade with market debt investors by issuing or repurchasing market debt at prevailing competitive prices.

2.1 General Setup

The model is set in continuous time, with $t \in [0, \infty)$. All agents are risk-neutral and discount the future at a constant rate $r > 0$. A firm maximizes the value of its deep-pocketed equity holders. Let Y_t denote the firm's instantaneous earnings before interest and taxes (EBIT) at time t, which evolves

according to the geometric Brownian motion

$$\frac{dY_t}{Y_t} = \mu dt + \sigma dZ_t, \tag{2.1}$$

where the drift μ and the volatility σ are positive constants, and dZ_t is the increment of a standard Brownian motion. I assume that $r > \mu$.

The firm borrows both bank debt from a relationship bank lender and market debt from arm's-length market debt investors. The key distinction between the bank lender and the market debt investors is that the former maintains a strategic relationship with the firm. As the firm's circumstances change, the existing bank debt contract can be improved upon to generate gains that benefit both the bank lender and the firm's equity holders, creating an incentive for them to renegotiate. By contrast, the market debt is held by dispersed institutional investors such as mutual funds, insurance companies, pension funds and so on. The dispersed nature of market debt investors gives rise to free-riding and coordination problems, preventing them from engaging in negotiations with the firm. Hence, the market debt investors are arm's-length and competitive.[3]

For simplicity, I abstract from the benefits of debt, though they can be easily accommodated without affecting the main insights of the model. The firm chooses when to default. Upon default, the firm is assumed to have liquidation value $\alpha Y^\star$, where $Y^\star$ is the firm's EBIT at default and $0 < \alpha < \frac{1}{r-\mu}$. That is, the liquidation value is a fraction of the unlevered firm value at default, implying a deadweight cost of financial distress. If the firm defaults, the equity holders forfeit their equity ownership and receive nothing in recovery. The liquidation value is divided by the bank lender and the market debt investors subject to the absolute priority rule.

A. Market Debt

The firm issues a homogeneous class of market debt. The market debt is modeled as a consol with fixed coupon rate $c > 0$ and no stated maturity. Absent frictions, the firm may continuously adjust its market debt outstanding by issuing additional market debt or repurchasing existing market debt. Let M_t denote the firm's face amount of market debt outstanding at time t. The firm issues market debt at an endogenous rate G_t^m such that the firm's market debt outstanding increases by $G_t^m dt$ over the time

[3] In practice, bank loans, syndicated loans, and private debt can be frequently renegotiated and thus correspond to bank debt in the model. Corporate bonds and notes are typically hard to renegotiate owing to dispersed investors and legal restrictions, and thus correspond to market debt in the model.

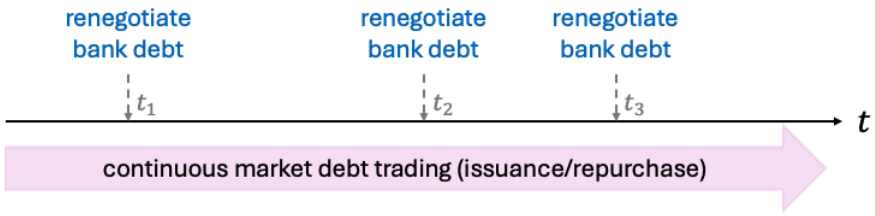


Figure 1: Timing of market debt trading and bank debt renegotiations. The figure illustrates the timing in the model: the firm continuously adjusts its market debt outstanding by issuing additional market debt or repurchasing existing market debt. By contrast, renegotiations of the bank debt occur at Poisson arrival times (e.g., $t_1, t_2, \dots$) with intensity λ.

interval dt. The firm optimally chooses the market debt issuance rate such that the equity holders' value is maximized. Note that G_t^m may be positive or negative, with $G_t^m > 0$ denoting market debt issuance and $G_t^m < 0$ denoting market debt repurchase. The market debt is priced rationally by the market debt investors, and any issuance or repurchase of the market debt takes place at the prevailing competitive price p_t^m.

B. Bank Debt

The bank debt is borrowed from the bank lender through a bilateral financing relationship that is inherently strategic. For tractability, I assume that the bank debt can be renegotiated at Poisson arrival times with an exogenous intensity $\lambda > 0$.[4] Let B_t denote the firm's face amount of bank debt at time t. If renegotiation of the bank debt occurs at time t, the renegotiation leads to an amended face amount $\bar{B}_t$ and a payment $\mathcal{T}_t$ from the bank lender to the equity holders. The payment can be thought of as the net proceeds from issuing (or repurchasing) the bank debt, after accounting for pricing adjustments and amendment fees. Both $\bar{B}_t$ and $\mathcal{T}_t$ are determined through bilateral bargaining à la Nash, between the bank lender and the firm, with the firm's bargaining power given by $\theta \in [0, 1]$. The bank debt is also modeled as a consol with fixed coupon rate c. However, the bank debt's face amount can be periodically adjusted through renegotiations between the bank lender and the firm.

I assume that at $t = 0$, the firm is endowed with a positive initial balance of either market debt or bank debt. This assumption is needed because the model abstracts from the benefits of debt to streamline

[4] As the firm's states evolve, the bank lender and the firm have a shared incentive to renegotiate the bank debt contract whenever there are mutual gains from trade. However, due to bargaining frictions outside the model, renegotiations are only feasible at Poisson times. More generally, the intensity parameter λ is state-dependent (e.g., as a function of the firm's leverage). In the main analysis, I treat λ as a constant, which imparts all the key economics of the model. In Appendix D, I extend the analysis to state-dependent λ and consider endogenization of λ.

the exposition and highlight the key intuition.[5] The assumption ensures that the firm engages in dynamic adjustments in its bank debt and market debt over time, rather than remaining entirely equity-financed. Conceptually, one may ask why the firm holds debt in the first place. A potential justification is that debt once offered benefits that have vanished by the start of the model. For instance, there may have been initial benefits from bank monitoring or from signaling the firm's quality through access to market debt financing.

2.2 Value Functions

The model's equilibrium concept is that of Markov perfect equilibrium. There are three payoff-relevant states, namely the firm's EBIT Y_t, the face amount of market debt M_t, and the face amount of bank debt B_t. Agents' values are functions of these three state variables. Let $V^e(Y_t, M_t, B_t)$ denote the equity holders' value function, then

$$\begin{aligned} V^e(Y_t, M_t, B_t) = \sup_{\tau^\star, \{G_s^m\}} \mathbb{E}_t \Big\{ \int_t^{\min\{\tau^\star, \bar{\tau}\}} e^{-r(s-t)} \big[\underbrace{Y_s - c(B_s + M_s)}_{\substack{\text{net income} \\ \text{after coupon payments}}} + \underbrace{p_s^m G_s^m}_{\substack{\text{market debt} \\ \text{issuance}}} \big] ds \\ + e^{-r(\bar{\tau}-t)} \mathbf{1}_{\{\bar{\tau} < \tau^\star\}} \big[\underbrace{V^e(Y_{\bar{\tau}}, M_{\bar{\tau}}, \bar{B}_{\bar{\tau}}) + \mathcal{T}_{\bar{\tau}}}_{\substack{\text{total payoff from} \\ \text{bank debt renegotiation}}} \big] \Big\}, \end{aligned} \tag{2.2}$$

where $\tau^\star$ is the endogenous default time chosen by the firm, and $\bar{\tau}$ is the (stochastic) arrival time of the next bank debt renegotiation.

Given the current states Y_t, M_t, and B_t, the firm chooses the default time $\tau^\star$ and the path of market debt issuance rate $\{G_s^m\}$ to maximize the equity holders' value. Prior to default or bank debt renegotiation, the equity holders receive the firm's net income after coupon payments as well as the proceeds from continuous market debt issuance.[6] If the bank debt renegotiation occurs before default, the equity holders receive a payment $\mathcal{T}_{\bar{\tau}}$ from the bank lender upon renegotiation, and their continuation value becomes $V^e(Y_{\bar{\tau}}, M_{\bar{\tau}}, \bar{B}_{\bar{\tau}})$ reflecting the amended face amount of bank debt post-renegotiation. If the firm declares default, the equity holders lose their ownership stake and receive zero value in return.

[5] In Appendix C, I incorporate debt benefits into the model and show that the current model with its simplifying assumptions preserves all the key economics.

[6] In the rest of the paper, I often refer to debt issuance and repurchase collectively as *debt issuance*, since debt repurchase can be thought of as negative debt issuance.

Similarly, let $V^b(Y_t, M_t, B_t)$ denote the bank lender's value function, then

$$V^b(Y_t, M_t, B_t) = \mathbb{E}_t\Big\{ \int_t^{\min\{\tau^\star, \bar{\tau}\}} e^{-r(s-t)} c B_s ds + e^{-r(\tau^\star - t)} \mathbf{1}_{\{\tau^\star \leq \bar{\tau}\}} V^{b,\star}(Y_{\tau^\star}, M_{\tau^\star}, B_{\tau^\star}) \\ + e^{-r(\bar{\tau}-t)} \mathbf{1}_{\{\bar{\tau} < \tau^\star\}} \big[V^b(Y_{\bar{\tau}}, M_{\bar{\tau}}, \bar{B}_{\bar{\tau}}) - \mathcal{T}_{\bar{\tau}} \big] \Big\}, \tag{2.3}$$

where $V^{b,\star}$ denotes the bank lender's recovery value upon default, which is a function of the state variables at the time of default. Prior to default or bank debt renegotiation, the bank lender receives coupon payments on the outstanding bank debt. If the bank debt renegotiation occurs before default, the bank lender makes a payment $\mathcal{T}_{\bar{\tau}}$ to the equity holders upon renegotiation, and their continuation value becomes $V^b(Y_{\bar{\tau}}, M_{\bar{\tau}}, \bar{B}_{\bar{\tau}})$. Upon default, the firm's liquidation value is distributed between the bank lender and the market debt investors, and the bank lender receives $V^{b,\star}(Y_{\tau^\star}, M_{\tau^\star}, B_{\tau^\star})$ in recovery.

The firm continuously issues market debt at the prevailing market debt price. In equilibrium, the market debt price p_t^m is also a function of the state variables. Let $p^m(Y_t, M_t, B_t)$ denote the equilibrium price of the market debt. Let $p^{m,\star}$ be the market debt's recovery upon default. Then

$$p^m(Y_t, M_t, B_t) = \mathbb{E}_t\Big\{ \int_t^{\min\{\tau^\star, \bar{\tau}\}} e^{-r(s-t)} c ds + e^{-r(\tau^\star - t)} \mathbf{1}_{\{\tau^\star \leq \bar{\tau}\}} p^{m,\star}(Y_{\tau^\star}, M_{\tau^\star}, B_{\tau^\star}) \\ + e^{-r(\bar{\tau}-t)} \mathbf{1}_{\{\bar{\tau} < \tau^\star\}} p^m(Y_{\bar{\tau}}, M_{\bar{\tau}}, \bar{B}_{\bar{\tau}}) \Big\}. \tag{2.4}$$

2.3 Bank Debt Renegotiation

When the bank lender and the firm renegotiate the bank debt, they bilaterally bargain over the face amount of the bank debt and the payment from the bank lender to the firm's equity holders, according to the Nash bargaining rule. Suppose that $\{B_t', \mathcal{T}_t'\}$ are the proposed terms in the bargaining problem. If the firm and the bank lender agree on the terms, the equity holders' value and the bank lender's value become $V^e(Y_t, M_t, B_t')$ and $V^b(Y_t, M_t, B_t')$ respectively, and the equity holders receive the payment $\mathcal{T}_t'$ from the bank lender. By contrast, if no agreement is reached, the amount of bank debt stays unchanged under the status quo. In this case, the values to the equity holders and the bank lender remain at $V^e(Y_t, M_t, B_t)$ and $V^b(Y_t, M_t, B_t)$, which serve as their respective outside options during renegotiation.

If $\{\bar{B}_t, \mathcal{T}_t\}$ are the outcomes of the bilateral bargaining problem between the bank lender and the firm with the firm having bargaining power θ, then

$$\begin{aligned}\{\bar{B}_t, \mathcal{T}_t\} = \underset{B'_t, \mathcal{T}'_t}{\arg\max} \ &\big[V^e(Y_t, M_t, B'_t) + \mathcal{T}'_t - V^e(Y_t, M_t, B_t)\big]^{\theta} \\ &\big[V^b(Y_t, M_t, B'_t) - \mathcal{T}'_t - V^b(Y_t, M_t, B_t)\big]^{1-\theta},\end{aligned} \tag{2.5}$$

subject to $V^e(Y_t, M_t, B'_t) + \mathcal{T}'_t - V^e(Y_t, M_t, B_t) \geq 0$ and $V^b(Y_t, M_t, B'_t) - \mathcal{T}'_t - V^b(Y_t, M_t, B_t) \geq 0$. The bracketed terms on the right-hand side are the trade surpluses captured by the equity holders and the bank lender from the bank debt renegotiation.

The solution to the bargaining problem (2.5) must satisfy two conditions. First, the transaction terms $\{\bar{B}_t, \mathcal{T}_t\}$ are bilaterally Pareto optimal. Otherwise, one party can always propose an offer that benefits themselves without making the other party worse off. Second, the transaction terms $\{\bar{B}_t, \mathcal{T}_t\}$ are such that the equity holders receive θ fraction of the joint surplus, while the bank lender receives $1-\theta$ fraction of the joint surplus. The solution to the bargaining problem is summarized in the following proposition, with derivations contained in Appendix A.1.

Lemma 1. *Let $\{\bar{B}_t, \mathcal{T}_t\}$ denote the outcomes of the bilateral bargaining problem between the bank lender and the firm, where $\bar{B}_t$ is the bank debt amount post-renegotiation and $\mathcal{T}_t$ is the payment from the bank lender to the equity holders. Then*

$$\bar{B}_t = \underset{B'_t}{\arg\max} \ \big[V^e(Y_t, M_t, B'_t) + V^b(Y_t, M_t, B'_t)\big], \tag{2.6}$$

and

$$\mathcal{T}_t = \theta\big[V^b(Y_t, M_t, \bar{B}_t) - V^b(Y_t, M_t, B_t)\big] - (1-\theta)\big[V^e(Y_t, M_t, \bar{B}_t) - V^e(Y_t, M_t, B_t)\big]. \tag{2.7}$$

Let $V(Y_t, M_t, B_t)$ denote the sum of the equity holders' value and the bank lender's value, that is $V(Y_t, M_t, B_t) \equiv V^e(Y_t, M_t, B_t) + V^b(Y_t, M_t, B_t)$. The condition (2.6) says that given the firm's EBIT Y_t and market debt outstanding M_t, the renegotiated bank debt amount $\bar{B}_t$ must maximize the joint value of the equity holders and the bank lender. $\bar{B}_t$ is thus a function of Y_t and M_t. Using the condition (2.7),

one can rewrite the equity holders' payoff from the bank debt renegotiation as

$$V^e(Y_t, M_t, \bar{B}_t) + \mathcal{T}_t = \underbrace{V^e(Y_t, M_t, B_t)}_{\text{outside option}} + \theta \underbrace{\left[V(Y_t, M_t, \bar{B}_t) - V(Y_t, M_t, B_t)\right]}_{\text{joint bargaining surplus}}. \tag{2.8}$$

In the bargaining game, the equity holders' outside option, which serves as a credible threat point, equals their continuation value under the current bank debt contract, $V^e(Y_t, M_t, B_t)$. This is because if the firm and the bank lender fail to reach an agreement, the status quo prevails leaving the bank lender and the firm stuck with the existing bank debt contract. Thus, any renegotiated agreement must provide the equity holders with at least their current continuation value in order for them to accept the new terms. Analogously, the bank lender's outside option equals its continuation value under the current bank debt contract, $V^b(Y_t, M_t, B_t)$. The joint surplus from bargaining between the equity holders and the bank lender is then the total value generated under the renegotiated agreement in excess of the sum of the two parties' outside options.

The equity holders' total payoff from the renegotiation consists of their outside option $V^e(Y_t, M_t, B_t)$ plus a fraction θ of the joint bargaining surplus they capture, where θ is their bargaining power. It is worth noting that, in the dynamic model, the equity holders' outside option and the joint bargaining surplus are both functions of the prevailing state variables. When the firm has weaker bargaining power, its equity holders capture a smaller share of the joint surplus, making their outside option more important as a credible threat during bargaining. If the bank lender has all the bargaining power (i.e., $\theta = 0$), the equity holders' payoff from the bank debt renegotiation is simply their outside option.

2.4 Market Debt Issuance

The equilibrium market debt price $p^m(Y_t, M_t, B_t)$ is pinned down by the equity holders' first-order condition with respect to market debt issuance, given by

$$p^m(Y_t, M_t, B_t) = -V^e_M(Y_t, M_t, B_t). \tag{2.9}$$

That is, the firm issues market debt until the marginal devaluation of the equity holders' continuation value due to market debt issuance fully offsets the price at which the market debt can be issued. Using

the pricing condition (2.9), one can obtain the equity holders' optimal market debt issuance rate G_t^m. The result is given in the proposition below, with derivations in Appendix A.2.

Proposition 2. *The firm's optimal rate of market debt issuance is*

$$G_t^m = \lambda \frac{p^m(Y_t, M_t, \bar{B}_t) + V_M^e(Y_t, M_t, B_t) + \theta\big[V_M(Y_t, M_t, \bar{B}_t) - V_M(Y_t, M_t, B_t)\big]}{-p_M^m(Y_t, M_t, B_t)}. \tag{2.10}$$

Conditional on renegotiation of the bank debt, the firm's market debt issuance balances the issuance price, which is the additional proceeds from issuing a marginal unit of market debt and depends on the renegotiation outcome, against the market debt issuance's impact on the equity holders' payoff from renegotiation. In other words, market debt serves not only as a source of financing for the firm, but also as a strategic tool that allows the firm to position its equity holders for more favorable outcomes in potential renegotiations with the bank lender. Specifically, the firm takes into account how its market debt issuance influences the bargaining outcome for its equity holders, while anticipating the effect of that bargaining outcome on the market debt price at which they can trade.

The strategic consideration of the trade-off between issuance price and renegotiation payoff becomes less relevant when the bank debt is renegotiated less frequently, that is when λ is lower. In the limiting case with $\lambda = 0$, the bank debt is never renegotiated and the firm neither issues nor repurchases any market debt. Intuitively, without the prospect of bank debt renegotiation, the firm's equity holders derive no strategic value from market debt trading: issuing or repurchasing market debt provides no net gain. This is because issuing market debt lowers the market debt price, and the marginal proceeds from issuance are more than offset by the decline in the equity holders' value due to the increase in leverage. Conversely, repurchasing market debt raises the market debt price, causing the marginal cost of repurchase to exceed the benefit of leverage reduction on the equity holders' value.[7]

2.5 Discrete-Time Example

To build intuition, I present a heuristic discrete-time example. For expositional simplicity, I suppose that the firm's EBIT remains constant over the time interval dt. The timing is as follows. At time t, the

[7] If debt confers benefits, the firm's optimal rate of market debt issuance, as given by (2.10), includes an additional term capturing the incentive to issue market debt until these benefits are fully dissipated, as in DeMarzo and He (2021). See Appendix C for details.

firm has market debt outstanding M and bank debt outstanding B. It issues an incremental amount of market debt Δ, which is determined endogenously. At time $t + dt$, the bank lender and the firm renegotiate the bank debt contract with certainty, resulting in an amended face amount $\bar{B}$ and a payment $\mathcal{T}$ from the bank lender to the equity holders.

The bank debt renegotiation at $t + dt$ determines $\bar{B}$ and $\mathcal{T}$ via Nash bargaining, where the firm has bargaining power θ. As shown in Appendix A.3, the renegotiated bank debt amount $\bar{B}$ maximizes the combined value of the equity holders and the bank lender, and depends on the amount of market debt $M + \Delta$ going into the renegotiation. The payment $\mathcal{T}$, in turn, determines how the bargaining surplus is split between the equity holders and the bank lender. Upon renegotiation, the equity holders receive their outside option plus their share of the bargaining surplus. Thus, the firm's problem is

$$\begin{aligned}\max_{\Delta} \; & \underbrace{V^e(M+\Delta, B) + \theta\big[V(M+\Delta, \bar{B}(M+\Delta)) - V(M+\Delta, B)\big]}_{\text{equity holders' payoff from bank debt renegotiation}} \\ & + \underbrace{\Delta p^m(M+\Delta, \bar{B}(M+\Delta))}_{\text{market debt issuance proceeds}}.\end{aligned} \tag{2.11}$$

At time t, when choosing how much market debt to issue, the firm maximizes its equity holders' total expected value, including their anticipated payoff from the upcoming bank debt renegotiation as well as the proceeds from market debt issuance. The first-order condition implies that the firm's optimal market debt issuance Δ at time t satisfies

$$\begin{aligned}0 = \; & \underbrace{V^e_M(M+\Delta, B)}_{\text{effect on outside option}} + \underbrace{\theta\big[V_M(M+\Delta, \bar{B}(M+\Delta)) - V_M(M+\Delta, B)\big]}_{\text{effect on bargaining surplus}} \\ & + \underbrace{p^m(M+\Delta, \bar{B}(M+\Delta)) + \Delta\big[p^m_M(M+\Delta, \bar{B}(M+\Delta)) + p^m_B(M+\Delta, \bar{B}(M+\Delta))\bar{B}'(M+\Delta)\big]}_{\text{marginal proceeds from market debt issuance}}.\end{aligned} \tag{2.12}$$

In equilibrium, the marginal proceeds from market debt issuance must be offset by the issuance's impact on the equity holders' payoff from the upcoming bank debt renegotiation, consisting of their outside option and their share of the bargaining surplus as captured by the first two terms on the right-hand side of (2.12). If the bank lender has all the bargaining power (i.e., $\theta = 0$), the equity holders receive no bargaining surplus and their renegotiation payoff is simply their outside option, which equals the equity holders' value if the amount of bank debt remains unchanged, that is $V^e(M + \Delta, B)$. Issuing

additional market debt reduces the firm's distance to default, which lowers the equity holders' current value thereby reducing their outside option in the upcoming bank debt renegotiation. Since the outside option serves as the equity holders' credible threat point in bargaining, a lower outside option implies a weaker bargaining position, which represents a strategic cost of market debt issuance.

When the equity holders possess some bargaining power ($\theta > 0$), market debt issuance affects both their outside option and their share of the joint bargaining surplus. Issuing market debt affects the amount of bargaining surplus by influencing the combined post-renegotiation value of the equity holders and the bank lender, both directly by adjusting the amount of market debt and indirectly by impacting the joint-surplus-maximizing bank debt level that arises from the renegotiation. At the same time, issuing market debt also decreases the outside options of both parties, which tends to increase the size of the surplus to be split. In the limiting case with $\theta = 1$, the bank lender gets its outside option and the equity holders capture the remaining value. In this case, the equity holders' outside option does not matter and they benefit from actions that raise the combined value or reduce the bank lender's outside option.

Proposition 3. *The firm's optimal market debt issuance Δ at time t is given by*

$$\Delta = \frac{p^m(M+\Delta,\bar{B}) + V_M^e(M+\Delta,B) + \theta\big[V_M(M+\Delta,\bar{B}) - V_M(M+\Delta,B)\big]}{-p_M^m(M+\Delta,\bar{B})\left[1 + \frac{p_B^m(M+\Delta,\bar{B})}{p_M^m(M+\Delta,\bar{B})}\bar{B}'(M+\Delta)\right]}. \tag{2.13}$$

The equation (2.13) mirrors the market debt issuance policy (2.10) in the continuous-time model.[8] The numerator reflects the net marginal value from issuing market debt, and equals the sum of the forward-looking market debt price (which incorporates the impact due to the upcoming bank debt renegotiation) and the marginal effect of market debt issuance on the equity holders' renegotiation payoff (including both their outside option and their share of the bargaining surplus). The denominator is the sensitivity of market debt price to market debt issuance, incorporating the direct price impact from dilution and the indirect price impact through its effect on the renegotiated bank debt amount.[9]

[8] The denominator of (2.10) captures the total effect of market debt issuance, as the equilibrium market debt price function $p^m(Y_t, M_t, B_t)$ incorporates the anticipated outcomes of future bank debt renegotiations.

[9] Note that (2.13) resembles the markup rule in a standard monopoly problem, as the firm may be viewed as a monopolistic supplier of its risky market debt. However, an important difference is that renegotiation of the bank debt introduces a strategic effect both on the net benefit of market debt issuance and on the sensitivity of market debt price to its issuance. Hence, the problem in this paper is conceptually different.

2.6 Discussion

The model is developed to study debt structure dynamics, with a focus on the dynamic interactions between bank debt and market debt. Nonetheless, the framework is sufficiently general and can be applied to a variety of other settings. In a nutshell, the model is about the strategic interplay among three parties: two bargaining parties and a silent non-bargaining party. Of the two bargaining parties, one is a "control" bargaining party who can trade directly with the non-bargaining party, while the other is a non-control bargaining party who cannot. In the context of this paper, the equity holders act as the control bargaining party, the bank lender as the non-control bargaining party, and the market debt investors as the non-bargaining party.

The central idea of the model is that the two bargaining parties negotiate bilaterally to maximize their joint value, exploiting the silent non-bargaining party who is excluded from the bargaining process and vulnerable to extraction by the bargaining parties. However, the non-bargaining party is fully rational and prices in the expected outcome of the bargaining game between the two bargaining parties, which affects its trade with the control bargaining party. Trade between the non-bargaining party and the control bargaining party, in turn, shapes the bargaining outcome by altering the bargaining parties' outside options and joint value. The control bargaining party thus trades with the non-bargaining party strategically to position itself for bargaining with the non-control bargaining party, while anticipating how this trade will influence the bargaining outcome, which in turn feeds back into its current trade price with the non-bargaining party. This feedback mechanism is illustrated in Figure 2.

Hence, the control bargaining party strategically pits the non-control bargaining party and the non-bargaining party against each other. On the one hand, the control bargaining party colludes with the non-control bargaining party to extract value from the non-bargaining party through bilateral bargaining that excludes the latter. On the other hand, the control bargaining party trades directly with the non-bargaining party to limit the rents accruing to the non-control bargaining party that arise from bilateral bargaining between the two bargaining parties. As an analogy, the non-bargaining party is both a prey and a weapon: it is preyed upon by the colluding bargaining parties, while being wielded by the control bargaining party to blunt the non-control bargaining party's surplus in negotiation which arises from exploiting the non-bargaining party in the first place.

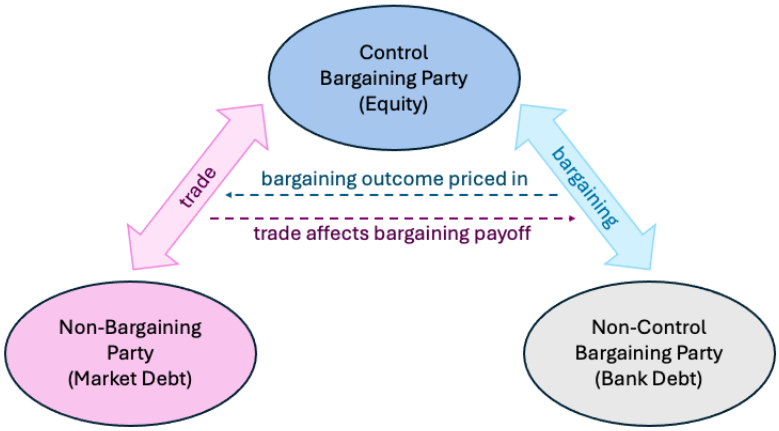


Figure 2: Feedback between trade and bargaining. The stylized diagram illustrates the feedback mechanism between trade and bargaining. The control bargaining party (e.g., equity holders) and the non-control bargaining party (e.g., bank lender) engage in bilateral bargaining to maximize their joint value, at the expense of the non-bargaining party (e.g., market debt investors). The outcome of the bargaining game between the two bargaining parties is priced in by the non-bargaining party, thereby affecting trade between the non-bargaining party and the control bargaining party, which in turn influences the bargaining outcome by altering the bargaining parties' outside options and joint value.

3 Solution with a Strong Bank

In this section, I specialize the model to the case with a strong bank lender who holds all the bargaining power, that is $\theta = 0$. The bank lender can be thought to make a take-it-or-leave-it offer to the firm when they renegotiate the bank debt. This corner case admits several analytical results while preserving the key strategic forces at play in the general setting, thereby clarifying the economic intuition.

When the bank lender has all the bargaining power, it captures the entire joint surplus from renegotiation, leaving the equity holders with only their outside option – namely their continuation value under the status quo. That is, bank debt renegotiations do not affect the equity holders' value. Moreover, the equity holders do not gain from market debt issuance in equilibrium, since the proceeds from issuing an additional unit of market debt are offset by the adverse valuation impact on the equity holders due to the increased leverage. Consequently, the equity holders' value can be solved as if the firm neither issues market debt nor renegotiates its bank debt.

With levels of bank debt and market debt fixed over time, the equity holders' value is then characterized by an ordinary differential equation (ODE), subject to the no-bubble condition as $Y_t \to \infty$ as well as the value matching and smooth pasting conditions at the endogenous default boundary $Y^\star(M_t, B_t)$ – the EBIT threshold at which the equity holders optimally default. The following proposition provides the

closed-form expression for the equity holders' value when $\theta = 0$. Detailed derivations are provided in Appendix A.4.

Proposition 4. *In the case with $\theta = 0$, the equity holders' value is given by*

$$\begin{aligned} V^e(Y_t, M_t, B_t) =& \frac{1}{r-\mu} Y_t - \frac{c}{r}(B_t + M_t) \\ &+ \frac{c}{r(1+\gamma)} \Big[\frac{\gamma}{1+\gamma} \frac{(r-\mu)c}{r} \Big]^{\gamma} (B_t + M_t)^{1+\gamma} Y_t^{-\gamma}, \end{aligned} \tag{3.1}$$

where

$$\gamma \equiv \frac{\mu - \frac{1}{2}\sigma^2 + \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2\sigma^2 r}}{\sigma^2} > 0, \tag{3.2}$$

and the default boundary is

$$Y^{\star}(M_t, B_t) = \frac{\gamma}{1+\gamma} \frac{(r-\mu)c}{r} (B_t + M_t). \tag{3.3}$$

In the case with a strong bank lender, the equity holders do not gain any value from either market debt issuance or bank debt renegotiation. Hence, as is obvious from (3.1), the equity holders' value depends only on the total debt outstanding $B_t + M_t$ and not on the composition of debt. This result is special to the case of $\theta = 0$. However, despite the equity holders' indifference toward debt composition, the firm's market debt issuance decision does depend on it. This is because the firm's prevailing level of market debt influences the joint-surplus-maximizing amount of bank debt upon renegotiation, which in turn is priced in and affects the firm's current incentives to issue market debt.

3.1 Market Debt Issuance and Repurchase

The equilibrium market debt price follows from the first-order condition (2.9), obtained by differentiating (3.1). The market debt issuance rate is then derived using (2.10) of Proposition 2. To allow for a more meaningful interpretation of issuance behavior across leverage levels, I scale the market debt issuance rate by the total debt outstanding $B_t + M_t$, and define $g^m(Y_t, M_t, B_t) \equiv G^m(Y_t, M_t, B_t)/(B_t + M_t)$, which is henceforth referred to as the *market debt issuance intensity*. The following proposition provides the

closed-form expression for the firm's market debt issuance intensity when the bank lender has all the bargaining power (i.e., $\theta = 0$). The derivations are relegated to Appendix A.5.

Proposition 5. *In the case with $\theta = 0$, the equilibrium market debt issuance intensity is*

$$g^m(Y_t, M_t, B_t) = \frac{\lambda}{\gamma}\Big[1 - \Big(\frac{\bar{B}(Y_t, M_t) + M_t}{B_t + M_t}\Big)^{\gamma}\Big]. \tag{3.4}$$

From (3.4), the firm's market debt issuance intensity may be positive or negative, depending on whether bank debt renegotiation – if it occurs – leads to an increase or a decrease in the amount of bank debt. This yields a key result of the model. The firm positions for potential bank debt renegotiation by trading market debt in the opposite direction: it issues additional market debt if renegotiation would reduce the amount of bank debt, and repurchases existing market debt if renegotiation would increase the amount of bank debt. The following theorem formalizes this result.

Theorem 6. *In the case with $\theta = 0$ and given Y_t and M_t,*

$$g^m(Y_t, M_t, B_t) \begin{cases} = 0 & \text{if } B_t = \bar{B}(Y_t, M_t) \\ > 0 & \text{if } B_t > \bar{B}(Y_t, M_t) \\ < 0 & \text{if } B_t < \bar{B}(Y_t, M_t) \end{cases}. \tag{3.5}$$

If $B_t = \bar{B}(Y_t, M_t)$, the firm neither issues nor repurchases any market debt. This is because in equilibrium, $p^m(Y_t, M_t, \bar{B}(Y_t, M_t)) = p^m(Y_t, M_t, B_t) = -V_M^e(Y_t, M_t, B_t)$. That is, the price at which the firm may trade its market debt exactly offsets the impact of any potential trade on the equity holders' current continuation value – which is also their outside option during renegotiation – leaving no surplus from market debt trading to the equity holders. Intuitively, when the bank debt is already at the joint-surplus-maximizing level, bank debt renegotiation would result in no change to the firm's debt structure. It is as if renegotiation does not occur, eliminating the firm's strategic incentive to position for it. Since the firm does not issue or repurchase market debt in the absence of bank debt renegotiation ($\lambda = 0$), the market debt issuance intensity is also zero in this case.

If $B_t > \bar{B}(Y_t, M_t)$, renegotiation would reduce the amount of bank debt, thereby lowering the firm's leverage. Conditional on such renegotiation, the market debt investors benefit from the leverage reduction and in turn value their claims more highly in anticipation. Accordingly, the market debt price, which is

forward-looking and reflects the expected leverage reduction from bank debt renegotiation, exceeds the issuance's adverse impact on the equity holders' current continuation value (i.e., their outside option). That is, $p^m(Y_t, M_t, \bar{B}_t) > -V_M^e(Y_t, M_t, B_t)$, so the firm issues additional market debt in order to capture a positive surplus ahead of the leverage-reducing bank debt renegotiation. In effect, the market debt price can be viewed as if subsidized by the bank lender, allowing the firm to issue market debt at a more favorable price even if it slightly weakens the equity holders' outside option.

If $B_t < \bar{B}(Y_t, M_t)$, renegotiation would lead to an increase in the amount of bank debt, thus diluting the market debt. The market debt investors price in the expected dilution and value the market debt less. As a result, the firm can repurchase the market debt at a lower price relative to the repurchase's positive impact on the equity holders' current continuation value (i.e., their outside option). That is, $p^m(Y_t, M_t, \bar{B}_t) < -V_M^e(Y_t, M_t, B_t)$, and the firm gains surplus by repurchasing its market debt ahead of the leverage-increasing bank debt renegotiation. In effect, the cost of repurchasing market debt can be seen as partially borne by the bank lender. Hence, the anticipated dilution of market debt upon bank debt renegotiation creates an opportunity for the firm to strengthen its equity holders' outside option by repurchasing market debt while it is cheap, improving their bargaining position ahead of renegotiation.

Note that, in contrast to the leverage ratchet effect of Admati et al. (2018) and DeMarzo and He (2021), the firm in this model may voluntarily repurchase its market debt, motivated by strategic positioning for potential bank debt renegotiation. As I will show later, the firm and the bank lender optimally choose a higher level of bank debt upon renegotiation when the firm is closer to default. Hence, when the firm is in distress, bank debt renegotiation is more likely to increase the amount of bank debt and results in dilution of the market debt investors. Knowing that the firm would shift toward more bank debt financing when it renegotiates with the bank lender, the firm opportunistically repurchases market debt at a discount to improve its equity holders' bargaining position going into such renegotiation.

3.2 Dynamic Complementarity of Bank and Market Debt

Given the firm's EBIT Y_t and market debt outstanding M_t, Theorem 6 suggests that the firm issues market debt when its current bank debt balance is sufficiently high, that is $B_t > \bar{B}(Y_t, M_t)$, and repurchases market debt when its bank debt balance is low, that is $B_t < \bar{B}(Y_t, M_t)$. There thus appears to be a positive correlation between the firm's prevailing bank debt level and the intensity of its market

debt issuance. This brings up the question of the dynamic relation between bank debt and market debt. More precisely, how does an increase in the current amount of bank debt affect the firm's market debt issuance intensity? The answer to this question is provided in the following theorem.

Theorem 7. *In the case with* $\theta = 0$, $\partial g^m(Y_t, M_t, B_t)/\partial B_t > 0$.

The theorem shows that the firm's market debt issuance intensity increases with the prevailing level of bank debt, while holding the firm's EBIT and market debt outstanding constant. If $B_t > \bar{B}(Y_t, M_t)$, a higher current bank debt level B_t means that renegotiation between the bank lender and the firm would result in a larger reduction in bank debt, further boosting the market debt price and the surplus from market debt issuance in anticipation of the substantial leverage decrease from potential bank debt renegotiation. Consequently, the firm is incentivized to issue more market debt. Analogously, if $B_t < \bar{B}(Y_t, M_t)$, a higher B_t implies that the bank debt level would increase less upon renegotiation, resulting in less dilution of the market debt and reduced surplus from repurchasing it, which in turn prompts the firm to repurchase less of its market debt.

Consequently, all else equal, a higher current level of bank debt is followed by a greater intensity of market debt issuance by the firm. I describe this pattern as the *dynamic complementarity* between bank debt and market debt. In Figure 3, I illustrate the dynamic complementarity of bank and market debt by plotting the market debt issuance intensity as a function of the bank debt level B_t, where the bank debt is assumed to be senior.[10] The parameters take the values: $\mu = 0.02$, $\sigma = 0.25$, $r = c = 0.05$, $\alpha = 2.33$, and $\lambda = 0.2$ or 1.0. I normalize the face amount of market debt to one, and plot the market debt issuance intensity for various levels of EBIT. The parameter values are illustrative; the rationale for these parameter choices is discussed in Section 3.3.

The figure shows that when the bank debt is periodically renegotiated, the market debt issuance intensity increases with the bank debt level. That is, the bank debt and the market debt are dynamically complementary and the firm issues market debt more aggressively when it has more bank debt in its debt structure. The solid lines are flatter than the dashed lines, indicating that the degree of dynamic complementarity between bank debt and market debt depends on the renegotiation frequency as captured by the Poisson intensity parameter λ. When $\lambda = 0$, the bank debt is never renegotiated and the strategic incentive to trade market debt to position for potential bank debt renegotiation is absent. In this case, the

[10] Discussion of how to solve for the bank lender's value and the joint-surplus-maximizing bank debt level $\bar{B}(Y_t, M_t)$ upon renegotiation can be found in Section 3.3, where I also provide justification for the seniority assumption of the bank debt.

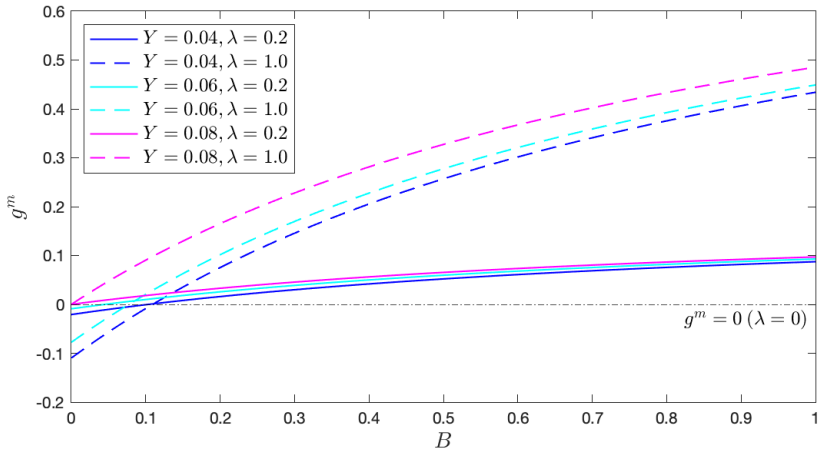


Figure 3: Market debt issuance intensity $g^m(Y_t, M_t, B_t)$ and dynamic complementarity of bank and market debt. The figure plots the market debt issuance intensity as a function of the face amount of bank debt B_t, holding the firm's EBIT Y_t and market debt outstanding M_t constant. Specifically, I plot the relationship between the market debt issuance intensity and the bank debt outstanding for $Y_t = 0.04$, 0.06, and 0.08 while normalizing the amount of market debt outstanding to one. The parameters take the values: $\mu = 0.02$, $\sigma = 0.25$, $r = c = 0.05$, $\alpha = 2.33$, and $\lambda = 0.2$ (solid lines) or 1 (dashed lines).

firm neither issues nor repurchases its market debt.[11] Furthermore, the figure highlights the possibility of market debt repurchase, for instance when bank debt level is very low or when earnings are low and the firm is near default.

3.3 Joint-Surplus-Maximizing Bank Debt and Seniority

So far, I have not explicitly specified the relative seniority of bank debt and market debt, as the analytical results presented in the previous sections do not rely on any particular assumption about debt seniority. Instead, I have treated the firm's EBIT Y_t, market debt outstanding M_t, and bank debt outstanding B_t as the relevant state variables, and derived the firm's decisions as functions of these variables. The joint-surplus-maximizing level of bank debt $\bar{B}(Y_t, M_t)$, which results from bank debt renegotiation, has been left as a general function of the state variables Y_t and M_t.

To characterize the debt structure dynamics, it is necessary to solve for the bank lender's value and the bank debt level that maximizes the joint bargaining surplus between the bank lender and the firm's equity holders in renegotiation. The solution exploits the firm's scale invariance in the setting, which helps reduce the dimensionality of the state space. For instance, the bank lender's value is homogeneous

[11] In Section 3.4, I discuss comparative statics with respect to λ.

of degree one, that is

$$V^b(Y_t, M_t, B_t) = V^b\Big(\frac{Y_t}{M_t}, 1, \frac{B_t}{M_t}\Big) M_t \equiv v^b(y_t, b_t) M_t, \tag{3.6}$$

where $y_t \equiv Y_t/M_t$ is the firm's EBIT-to-market-debt ratio and $b_t \equiv B_t/M_t$ is the *bank debt ratio* which reflects the firm's debt structure.[12] $v^b(y_t, b_t)$ is the bank lender's value scaled by the market debt outstanding, and is referred to as the *bank debt value*. The partial differential equation (PDE) for the bank debt value is given in Appendix A.6. The joint-surplus-maximizing bank debt ratio is

$$\bar{b}(y_t) \equiv \frac{\bar{B}(Y_t, M_t)}{M_t} = \arg\max_{b_t'} \; v(y_t, b_t'), \tag{3.7}$$

where $v(y_t, b_t)$ is the joint value of the equity holders and the bank lender scaled by the amount of market debt outstanding.

In practice, bank debt is typically senior to market debt. With this in mind, I first numerically solve for the bank debt value and the post-renegotiation bank debt ratio assuming that the bank debt is senior. I then show that $\bar{b}(y_t) = 0$ if the bank debt is pari passu with or junior to the market debt. Consequently, any bank debt that arises in equilibrium must be senior. Thus, an implication of the model is that bank debt is optimally senior.

A. Numerical Solution with Senior Bank Debt

The firm chooses to default when its EBIT-to-market-debt ratio $y_t < y^\star(b_t)$, where the default boundary $y^\star(b_t)$ depends on the bank debt ratio b_t. To facilitate numerical solution, it is more convenient to express the value functions in terms of the firm's distance to default $\tilde{y}_t \equiv y_t - y^\star(b_t)$ along with the bank debt ratio b_t. The firm defaults when its distance to default reaches zero from above, regardless of its debt structure.

The boundary value of the bank debt depends on its seniority. If the bank debt is senior to the market debt, the bank lender's recovery value upon default is $\min\{\alpha Y^\star, B_{\tau^\star}\}$. When the liquidation value is enough to cover the principal amount of the bank debt at the time of default, that is $\alpha Y^\star \geq B_{\tau^\star}$, the bank lender receives the face amount of the bank debt and the bank debt is defeased. Otherwise, the

[12] It can be shown that the market debt amount $M_t > 0$. Heuristically, when M_t gets very close to zero, the joint-surplus-maximizing bank debt amount that would arise in bank-firm renegotiation goes to zero, leading to expected bank debt reduction upon renegotiation and prompting the firm to issue market debt away from zero.

bank lender gets the entire liquidation value under the absolute priority rule, leaving the market debt investors with nothing in recovery.

I present a numerical solution using the following baseline parameter values: $\mu = 0.02$, $\sigma = 0.25$, $r = c = 0.05$, $\lambda = 0.2$, and $\alpha = 2.33$. The Poisson intensity $\lambda = 0.2$ implies that the bank debt is renegotiated, on average, every five years – roughly the maturity of a typical term bank loan. $\alpha = 2.33$ implies a liquidation value that is 10% of the unlevered firm value at default, indicating a relatively large cost of default. The remaining parameters are taken from the literature (e.g., DeMarzo and He (2021), Greenwood et al. (2023)).[13]

Panel A of Figure 4 plots the bank debt value as a function of the firm's distance to default $\tilde{y}_t \equiv y_t - y^\star(b_t)$ and its bank debt ratio b_t within the no-default region. Panels B and C fix one variable and plot the bank debt value against the other. Panel B shows that given the bank debt ratio b_t, the bank debt value increases with the distance to default $\tilde{y}_t$, as the firm is further from financial distress and more likely to continue servicing its debt. Panel C shows that given the distance to default $\tilde{y}_t$, the bank debt value increases with the bank debt ratio b_t, since the bank lender has a larger claim on the firm's value.

Given the firm's EBIT Y_t and market debt outstanding M_t, an increase in the bank debt ratio b_t (i.e., more bank debt) also shortens the firm's distance to default. Thus, the bank debt value is generally non-monotonic in b_t. Panel A of Figure 5 illustrates this by showing that the bank debt value is hump-shaped in the bank debt ratio. If there is no bank debt, the bank lender has no claim on the firm's value and the bank debt value is zero. Increasing the bank debt ratio raises the bank lender's claim and hence the bank debt value. However, once bank debt becomes excessive, further adding bank debt makes default imminent, thus driving the bank debt value down toward its boundary value.

On the other hand, more bank debt reduces the equity holders' value. In Panel B of Figure 5, I plot the (normalized) joint equity and bank debt value against the bank debt ratio b_t for given levels of EBIT-to-market-debt ratio y_t.[14] When the bank debt is senior, it turns out that the joint equity and bank debt value is also hump-shaped in b_t. Since renegotiation determines a bank debt ratio that maximizes this joint value, the optimal post-renegotiation bank debt ratio $\bar{b}(y_t)$ is interior, implying a

[13] The parameter values are illustrative and not intended for a quantitative exercise.

[14] I normalize the joint equity and bank debt value by applying the formula $v^{norm} = \frac{v - v^{min}}{v^{max} - v^{min}}$ where v^{min} and v^{max} are the minimum and the maximum values of v in the given support.

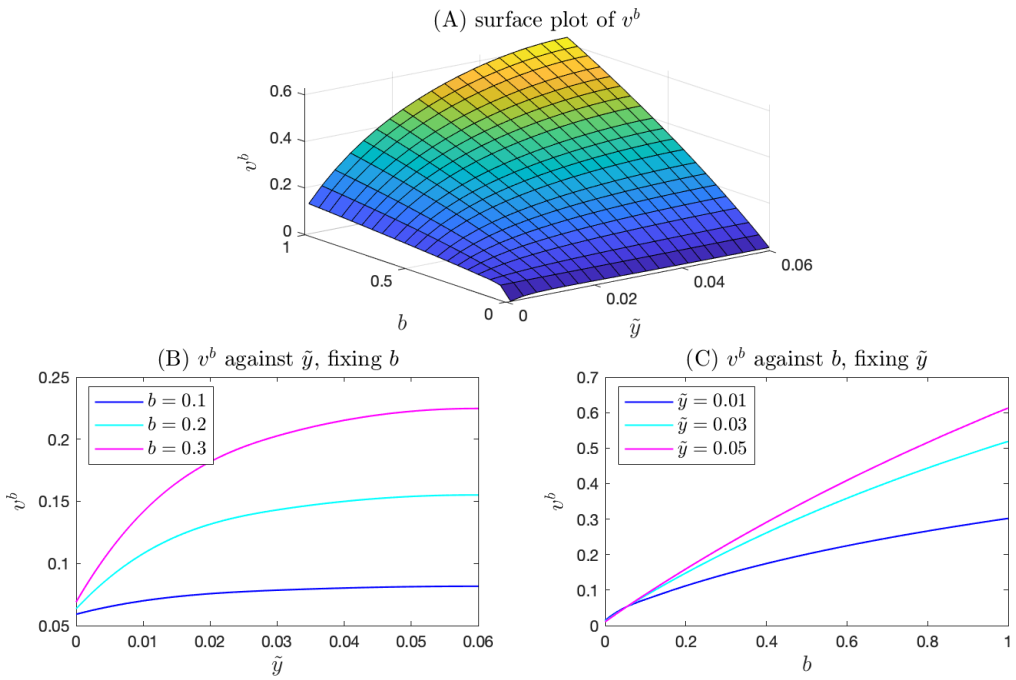


Figure 4: Bank debt value $v^b(\tilde{y}_t, b_t)$. The figure plots the bank debt value v^b as a function of the firm's distance to default $\tilde{y}_t \equiv y_t - y^\star(b_t)$ and the bank debt ratio b_t within the no-default region. Panel A is a two-dimensional surface plot. Panel B plots v^b against $\tilde{y}_t$ for fixed levels of b_t, while Panel C plots v^b against b_t for fixed levels of $\tilde{y}_t$. The parameter values are $\mu = 0.02$, $\sigma = 0.25$, $r = c = 0.05$, $\lambda = 0.2$, and $\alpha = 2.33$.

positive balance of bank debt upon renegotiation. However, as I will show shortly, the joint equity and bank debt value decreases with b_t if the bank debt is pari passu with or junior to the market debt.

When the bank lender and the firm renegotiate the bank debt, they trade off the cost of bank debt against any benefit of bank debt. The cost arises from potential financial distress: additional bank debt shortens the firm's distance to default, increasing the cost of financial distress and reducing the joint value of the equity holders and the bank lender. When the bank debt is senior to the market debt, the market debt acts as a buffer upon default, absorbing losses before the bank debt is impaired. In this case, issuing bank debt allows the bank lender and the equity holders to extract value by diluting the market debt investors. Thus, even though debt provides no intrinsic benefit, bilateral bargaining between the senior bank lender and the firm gives rise to a "private" benefit through the dilution of the market debt investors. By contrast, if the bank debt is not senior, future market debt issuances constrain the ability of the bank lender and the firm's equity holders to extract value from the market debt investors, eliminating any private benefit of bank debt.[15]

[15] Alternatively, seniority of bank debt could also be determined through bilateral bargaining – it is in the shared interest of the bank lender and the firm to give seniority to the bank debt in order to maximize their joint value.

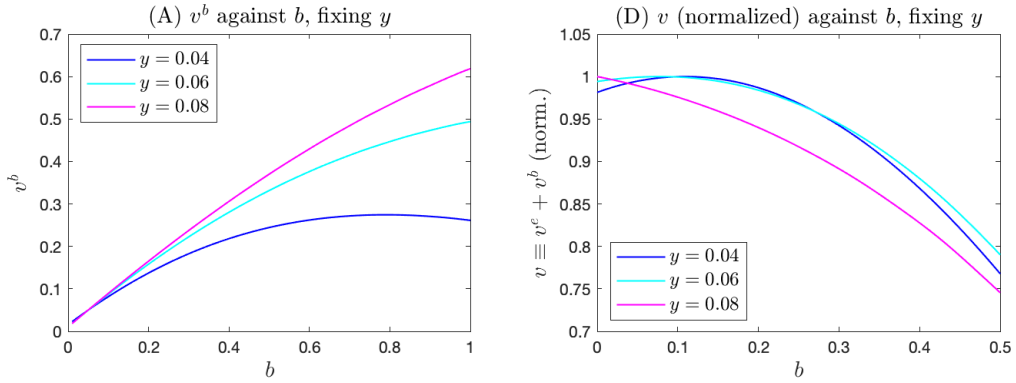


Figure 5: Bank debt value and joint equity and bank debt value as functions of bank debt ratio b_t. The figure plots the bank debt value v^b (Panel A) and the (normalized) joint equity and bank debt value v (Panel B) as functions of the firm's bank debt ratio b_t for given levels of EBIT-to-market-debt ratio y_t. The parameter values are $\mu = 0.02$, $\sigma = 0.25$, $r = c = 0.05$, $\lambda = 0.2$, and $\alpha = 2.33$.

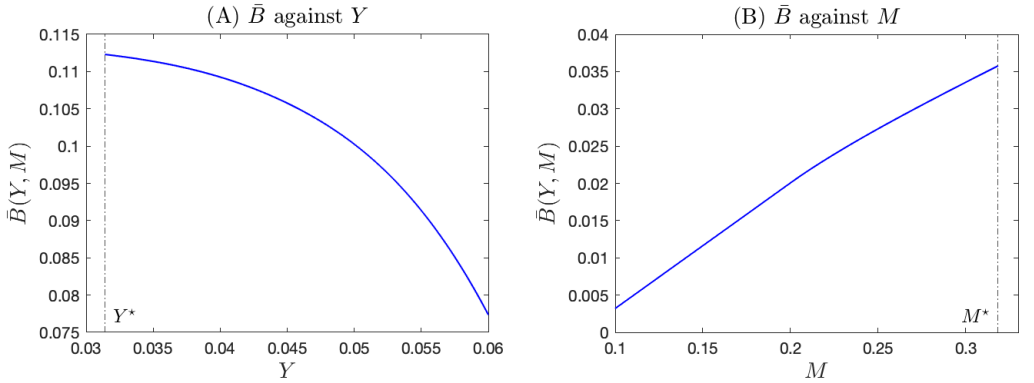


Figure 6: Joint-surplus-maximizing senior bank debt level $\bar{B}(Y_t, M_t)$ from renegotiation. Panel A of the figure plots the joint-surplus-maximizing bank debt level $\bar{B}(Y_t, M_t)$, which would result from bank debt renegotiation, as a function of the firm's EBIT Y_t when the bank debt is senior, while holding the firm's market debt fixed. Panel B plots $\bar{B}(Y_t, M_t)$ as a function of the market debt amount M_t when the bank debt is senior, while holding the EBIT fixed. The parameter values are $\mu = 0.02$, $\sigma = 0.25$, $r = c = 0.05$, $\lambda = 0.2$, and $\alpha = 2.33$. In Panel A, the firm's market debt amount is held constant at $M_t = 1$. In Panel B, the firm's EBIT is held fixed at $Y_t = 0.01$. $Y^\star$ and $M^\star$ denote the respective default boundaries in the two panels.

B. Joint-Surplus-Maximizing Bank Debt Ratio

The joint-surplus-maximizing bank debt level $\bar{B}(Y_t, M_t)$ depends only on the firm's EBIT Y_t and its market debt outstanding M_t. In Figure 6, I plot $\bar{B}(Y_t, M_t)$ as a function of the firm's EBIT Y_t (Panel A) and as a function of the market debt amount M_t (Panel B) respectively, assuming that the bank debt is senior. The parameter values are the same as before. In Panel A, the firm's market debt outstanding is fixed at $M_t = 1$, while in Panel B, the firm's EBIT is held constant at $Y_t = 0.01$.

Panel A illustrates a negative relationship between the joint-surplus-maximizing bank debt level and the firm's EBIT. Holding debt amounts fixed, when the firm's earnings are high and default risk is low, it is in the joint interest of the bank lender and the equity holders to keep the bank debt level low. Panel B

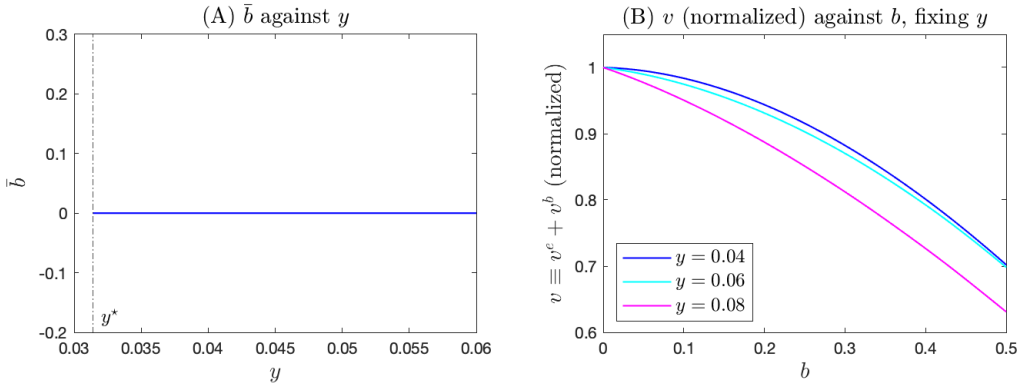


Figure 7: Surplus-maximizing non-senior bank debt ratio $\bar{b}(y_t)$ from renegotiation. Panel A of the figure plots the joint-surplus-maximizing bank debt ratio $\bar{b}(y_t)$ as a function of the firm's EBIT-to-market-debt ratio y_t, when the bank debt is non-senior. Panel B plots the normalized joint equity and bank debt value against the bank debt ratio b_t for various y_t. The parameter values are $\mu = 0.02$, $\sigma = 0.25$, $r = c = 0.05$, $\lambda = 0.2$, and $\alpha = 2.33$.

shows that the joint-surplus-maximizing bank debt level increases with the amount of market debt, that is $\partial \bar{B}(Y_t, M_t)/\partial M_t > 0$. This result will be key for understanding the strategic interplay between bank and market debt, as discussed in more detail in Section 3.5. Intuitively, an increase in M_t means that there is more market debt to dilute, enhancing the ability of the bank lender and the equity holders to extract value from the market debt investors through bilateral bargaining. This raises the private benefit of bank debt to the bargaining parties, which in turn increases the joint-surplus-maximizing bank debt level $\bar{B}(Y_t, M_t)$ upon renegotiation.

C. No Pari Passu or Junior Bank Debt

I now consider the case in which the bank debt is not senior to the market debt. The firm's equity holders are incentivized to dilute the existing lenders by issuing new market debt just before default. If the bank debt is pari passu with or junior to the market debt, the equity holders are able to fully dilute the existing lenders, and the bank lender receives no liquidation value upon default. In Panel A of Figure 7, I plot the joint-surplus-maximizing bank debt ratio $\bar{b}(y_t)$ when the bank debt is not senior. I also plot the (normalized) joint equity and bank debt value against b_t in Panel B. The figure shows that if the bank debt is not senior, the joint equity and bank debt value monotonically decreases with the bank debt ratio b_t. Hence, the optimal bank debt ratio $\bar{b}(y_t)$, which maximizes the joint equity and bank debt value, is zero. In other words, without seniority over market debt, renegotiation would drive the bank debt balance to zero. This implies that bank debt can be sustained in equilibrium only if it is senior to market debt.

3.4 Renegotiation Frequency

The frequency of bank debt renegotiations, captured by the Poisson intensity parameter λ, influences the firm's market debt trading behavior. To build intuition, I consider two limiting cases: one case with $\lambda \to 0$ (that is, renegotiation never takes place) and another case with $\lambda \to \infty$ (that is, renegotiation occurs continuously).

When $\lambda \to 0$, (3.4) implies that the firm's market debt issuance intensity is $g_t^m = 0$. That is, the firm neither issues nor repurchases market debt in the limit where the bank debt is never renegotiated. Without bank debt renegotiation, the firm has no incentive to trade market debt to position its equity holders for potential bank debt renegotiation. Hence, the setting collapses to that of DeMarzo and He (2021) without debt tax shield. Since market debt carries no intrinsic benefit, the firm does not issue any additional market debt. Likewise, the firm also does not wish to repurchase market debt, since repurchase would increase the market debt price such that the equity holders do not gain from it.

It is also useful to examine the limit with $\lambda \to \infty$. In this case, both the bank debt and the market debt may be adjusted continuously. With continuous renegotiation, bank debt adjustments become infinitesimal, that is $\bar{B}(Y_t, M_t) - B_t \to 0$. It may be tempting to conclude that this leaves the firm with no incentive to trade in its market debt. However, the following proposition shows that this is not the case: the firm's strategic trading of market debt to position for bank debt renegotiation remains very much active even as bank debt renegotiations occur continuously.

Proposition 8. *Let $\varepsilon \equiv 1/\lambda$. Then, as $\lambda \to \infty$ (equivalently, $\varepsilon \to 0$), the firm's market debt issuance intensity $g^m(Y_t, M_t, B_t) = O(1)$ even though $\bar{B}(Y_t, M_t) - B_t = O(\varepsilon)$. Furthermore, $sign\big(g^m(Y_t, M_t, B_t)\big) = -sign\big(\bar{B}(Y_t, M_t) - B_t\big)$.*

The proof of the proposition can be found in Appendix A.7. The proposition shows that when the bank debt is renegotiated continuously, the incremental adjustment in bank debt due to renegotiation can have a first-order effect on the firm's market debt issuance policy, even though the adjustment itself is of higher order in $1/\lambda$. As in the finite-λ case, the firm responds to bank debt renegotiation by trading market debt in the opposite direction: it issues market debt if renegotiation reduces bank debt, and repurchases market debt if renegotiation increases bank debt.

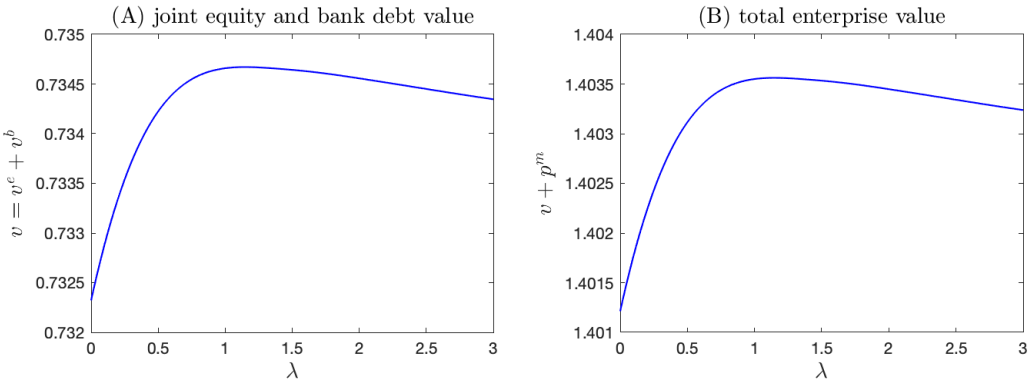


Figure 8: Comparative statics with respect to Poisson intensity of renegotiation λ. Panel A of the figure plots the joint equity and bank debt value $v \equiv v^e + v^b$ against the Poisson intensity parameter λ governing the frequency of bank debt renegotiation. Panel B plots the total enterprise value $v + p^m$ against λ. The parameter values are $\mu = 0.02$, $\sigma = 0.25$, $r = c = 0.05$, $\alpha = 2.33$ and $\theta = 0$. I further normalize the amount of market debt to one, and evaluate the comparative statics for bank debt ratio of $b_t = 0.1$ and distance to default $\tilde{y}_t = 0.03$.

How does the frequency of bank debt renegotiation affect the joint value of the bank lender and the equity holders? Panel A of Figure 8 answers this question by plotting the comparative statics of the joint equity and bank debt value $v \equiv v^e + v^b$ with respect to the Poisson intensity parameter λ governing the renegotiation frequency of the bank debt. The joint value v is hump-shaped in λ. As renegotiations become more frequent, the bank lender and the equity holders can more often capture the bargaining surplus generated by re-optimizing the trade-off between the cost of financial distress and the private benefit of diluting market debt. This tends to increase the joint equity and bank debt value on the extensive margin. However, as shown above, more frequent renegotiations reduce the size of bank debt adjust and thus the surplus captured in each round of renegotiation, lowering the joint value on the intensive margin. Consequently, the joint value of the bank lender and the equity holders is generally non-monotonic in the renegotiation frequency.

When the bank lender has all the bargaining power, the equity holders capture no bargaining surplus and their value is unaffected by the frequency of bank debt renegotiation. Hence, the equilibrium market debt price is also independent of the renegotiation frequency. It is therefore unsurprising that the total firm value is hump-shaped in the renegotiation frequency, as illustrated by Panel B of Figure 8. Intuitively, more frequent renegotiations allow the firm to avoid default more effectively by working with the bank lender to forgive debt when the firm becomes over-leveraged. However, as default becomes less likely, diluting market debt investors appears more attractive to the bank lender and the equity holders when they renegotiate, creating additional distortion.

3.5 Strategic Interplay of Bank Debt and Market Debt

Renegotiation between the bank lender and the firm generates bilateral bargaining surplus for the two bargaining parties by re-optimizing the amount of bank debt to maximize the joint value of the bank lender and the firm's equity holders. This surplus reflects a trade-off between the cost of bank debt from potential financial distress and the private benefit created through dilution of the market debt investors. Put differently, the presence of market debt – whose claim-holders are excluded from the bargaining process – provides a source of value extractable through bilateral bargaining over the bank debt, thereby giving rise to the joint surplus from renegotiation.

Denote by $\mathcal{S}(Y_t, M_t, B_t) \equiv V(Y_t, M_t, \bar{B}(Y_t, M_t)) - V(Y_t, M_t, B_t)$ the joint bargaining surplus between the bank lender and the firm's equity holders. The following proposition shows that fixing the firm's EBIT Y_t and market debt outstanding M_t, the joint bargaining surplus is decreasing in the distance between the joint-surplus-maximizing bank debt level $\bar{B}(Y_t, M_t)$ and the current level of bank debt B_t. The proof of the proposition can be found in Appendix A.8.

Proposition 9. *In the case with $\theta = 0$ and given Y_t and M_t,*

- *if $B_t = \bar{B}(Y_t, M_t)$, the joint bargaining surplus $\mathcal{S}(Y_t, M_t, B_t) = 0$;*
- *if $B_t > \bar{B}(Y_t, M_t)$, the joint bargaining surplus $\mathcal{S}(Y_t, M_t, B_t) > 0$ and decreases with $\bar{B}(Y_t, M_t)$;*
- *if $B_t < \bar{B}(Y_t, M_t)$, the joint bargaining surplus $\mathcal{S}(Y_t, M_t, B_t) > 0$ and increases with $\bar{B}(Y_t, M_t)$.*

When the bank lender is strong and holds all the bargaining power (i.e., $\theta = 0$), the entire joint surplus from bargaining accrues to the bank lender. In other words, although bilateral bargaining between the bank lender and the firm generates surplus through value extraction from the market debt investors, the surplus is fully captured by the bank lender in the form of rents. However, because $\partial \bar{B}(Y_t, M_t)/\partial M_t > 0$ (see Figure 6), the results of Theorem 6 and Proposition 9 together imply that by strategically trading market debt to position for potential bank debt renegotiation, the firm can discipline the rents accruing to the bank lender.

If the current bank debt exceeds the joint-surplus-maximizing level from renegotiation, that is $B_t > \bar{B}(Y_t, M_t)$, the firm issues additional market debt according to Theorem 6. The increase in market debt raises the joint-surplus-maximizing level of the bank debt. As a result, renegotiation of the bank debt leads to a smaller reduction in the amount of bank debt, diminishing the joint bargaining surplus

and thus the rents captured by the bank lender. Similarly, if $B_t < \bar{B}(Y_t, M_t)$, the firm repurchases its market debt, reducing the amount of market debt and the joint-surplus-maximizing level of the bank debt. Consequently, renegotiation of the bank debt results in a smaller increase in the bank debt level, again reducing the joint bargaining surplus and hence the rents going to the bank lender.

Taken together, bank debt and market debt are also "complementary" in a conceptual sense. The firm, acting on behalf of its equity holders, colludes with the bank lender in bilateral bargaining to extract value from the non-bargaining market debt investors, while at the same time trading with the market debt investors to "front-run" and limit the rents captured by the bank lender. Hence, the firm strategically pits the bank lender and the market debt investors against one another. The coexistence of bank debt and market debt reflects a strategic architecture of mutually-extractive claims orchestrated by the firm's equity holders.

A. Efficiency Implications

Since debt provides no intrinsic benefit but imposes the cost of financial distress, the efficient capital structure is an all-equity one. However, when a firm can only access market debt financing, it never has any incentive to repurchase its outstanding market debt, as the equity holders derive no gain from doing so. As a result, a firm relying solely on market debt ends up with too much leverage relative to the first-best. Introducing dynamically-renegotiable bank debt into the debt structure can potentially enhance the firm value by permitting debt reduction through bank debt forgiveness when leverage is excessive, thereby lowering the cost of financial distress. In fact, if the firm borrows exclusively bank debt, the first-best is achieved through bank-firm renegotiations. This is because bank debt renegotiations maximize the joint value of the equity holders and the bank lender, which equals the total firm value when there is only bank debt. Each renegotiation then results in full retirement of any bank debt outstanding, and the firm consistently carries no debt.

Although from the firm-value perspective it is socially efficient for the firm to rely solely on bank debt, the firm has a private incentive to deviate from such a bank-debt-only debt structure, because the firm benefits from having market debt by strategically pitting the bank lender and the market debt investors against each other. While bilateral bargaining between the bank lender and the firm would be efficient in isolation, the presence of market debt introduces a tension because the bank lender and the equity holders share an incentive to extract value by diluting the market debt investors, who are excluded from

the bilateral bargaining process. The value extraction from the market debt investors by the bargaining coalition of the bank lender and the equity holders is not merely a transfer. From a firm-value standpoint, the dilution incentive by the bargaining coalition leads to inefficiently high leverage, as it increases the cost of financial distress. In other words, the presence of market debt distorts the efficiency of bilateral bargaining between the bank lender and the firm.

That said, the firm does not wish to fully extinguish its market debt despite the inefficiency it generates. Instead, the firm trades market debt strategically to discipline the rents accruing to the bank lender from bilateral bargaining.[16] How does the firm's market debt trading affect the efficiency of the bargaining outcome? It turns out that it can be either efficiency-enhancing or efficiency-destroying. If $B_t > \bar{B}(Y_t, M_t)$, the firm issues additional market debt, which increases the private benefit for the bank lender and the equity holders from diluting the market debt investors, thereby further reducing the efficiency of bank debt renegotiation. If $B_t < \bar{B}(Y_t, M_t)$, the firm repurchases its market debt, which lowers the private benefit from market debt dilution and thus improves the efficiency of bank debt renegotiation.

B. Debt Covenants

The model is intentionally stark, abstracting from covenant restrictions both on the firm's ability to issue or repurchase market debt and on the ability of the bank lender and the firm to engage in bilateral bank debt renegotiation. Nevertheless, the analysis above helps shed light on the role of debt covenants and their existence in practice.

I begin by considering what the firm-value-maximizing set of covenants would look like if they could be designed ex-ante and perfectly enforced. Since incurrence of debt imposes the cost of financial distress, in the absence of any intrinsic benefit of debt, the efficient capital structure is all-equity and the firm-value-maximizing covenants should permit debt repurchases while prohibiting any new debt issuance. In principle, such covenants could be implemented through reciprocal restrictions, such as a bank debt covenant prohibiting market debt issuance and a market debt covenant prohibiting bank debt issuance. In practice, however, the implementation of such covenants is complicated by the fact that the private incentives of and interactions among contracting parties may prevent such efficiency-enhancing restrictions from emerging in equilibrium.

[16] The bank lender also does not wish the firm to fully extinguish the market debt, as the presence of market debt allows the creation of private benefits through dilution of the market debt investors.

As discussed earlier, the firm trades market debt strategically to strengthen its equity holders' bargaining position in potential bank debt renegotiation, thereby limiting the rents accruing to the bank lender during bargaining. Suppose that the bank lender and the firm could write covenants restricting market debt issuance or repurchase by the firm, would they choose to do so? The bank lender has an incentive to impose stricter such covenants, as they constrain the firm's ability to "front-run" the bank lender. However, stricter covenants would make the firm's equity holders worse off. A conflict of interest therefore arises between the bank lender and the firm over how restrictive these covenants should be. Hence, if the bank lender and the firm were to negotiate the covenant package bilaterally, the jointly optimal covenants would likely feature an intermediate level of strictness – neither completely prohibiting nor fully permitting market debt issuance and repurchase. In practice, credit agreements governing bank debt often contain leverage tests and debt incurrence covenants that restrict the firm's ability to issue market debt, as well as restricted payment covenants and prepayment restrictions that limit the firm's ability to retire market debt. These covenants are subject to negotiated carve-outs, which determine how restrictive the covenants are.

Now suppose that the market debt investors and the firm could write covenants restricting adjustments to bank debt during bank-firm renegotiation. When the bank lender holds all the bargaining power against the firm (i.e., $\theta = 0$), the equity holders capture no bargaining surplus from renegotiation and any resulting change in the bank debt level. By contrast, the market debt investors benefit if renegotiation reduces the bank debt balance, and are harmed if renegotiation increases the bank debt level. Consequently, it is in the shared interest of the equity holders and the market debt investors to include covenants that restrict the firm's ability to increase the bank debt level, while allowing the firm to reduce its bank debt. If the firm has some bargaining power, however, it is not in the equity holders' interest to fully restrict the firm's ability to increase its bank debt level during renegotiation, as part of the surplus from renegotiation accrues to the equity holders. How about covenants restricting issuances of additional market debt and repurchases of existing market debt? It is in the joint interest of the firm and the existing market debt investors to allow repurchases, but the two parties have opposing incentives when it comes to issuances. In practice, indentures governing market debt often contain limitation on indebtedness and negative pledge covenants that constrain the firm's ability to raise additional bank debt and market debt, but rarely limit the firm's ability to retire debt.

In sum, although the most efficient covenant package would restrict issuances of both bank debt and market debt while permitting debt reductions, implementing such a package is difficult due to agents' private incentives and the strategic interactions among them. In equilibrium, bank debt covenants place partial restrictions on the firm's ability to issue and repurchase market debt, allowing some flexibility without fully prohibiting or fully permitting these actions. Market debt covenants, by contrast, do not restrict debt reductions, but place some constraints on both bank and market debt issuances, without completely shutting them down. As a result, covenants cannot entirely resolve the coordination inefficiency that arises from the firm's strategic behavior in pitting the market debt investors and the bank lender against one another.

4 Effects of Firm Bargaining Power

I now turn to the general case where the firm has positive bargaining power, that is $\theta > 0$. In particular, I am interested in knowing how the firm's bargaining power affects the dynamic relationship between the firm's bank debt amount and its market debt issuance. As discussed in Section 3.5, the firm strategically trades (i.e., issues or repurchases) its market debt to position its equity holders for potential bank debt renegotiation, effectively "front-running" the bank lender to limit the rents it captures from bargaining. The dynamic complementarity of bank and market arises from this strategic incentive, which is stronger when the firm captures less surplus when renegotiating with the bank lender. Hence, one would expect that the degree of dynamic complementarity between bank debt and market debt decreases with the firm's bargaining power θ.

For illustration, I plot $g^m(Y_t, M_t, B_t) \equiv G^m(Y_t, M_t, B_t)/(B_t + M_t)$, the firm's market debt issuance intensity as defined earlier. The market debt issuance intensity is homogeneous of degree zero, and can thus be written as a function of the firm's distance to default $\tilde{y}_t$ and its bank debt ratio b_t. In Figure 9, I plot $g^m(\tilde{y}_t, b_t)$ against the bank debt ratio b_t at various levels of distance to default $\tilde{y}_t$, for both $\theta = 0.3$ (solid lines) and $\theta = 0$ (dashed lines). The general case can only be solved numerically, with the corresponding PDEs provided in Appendix A.9. The parameter values are as follows: $\mu = 0.02$, $\sigma = 0.25$, $r = c = 0.05$, $\lambda = 0.2$, and $\alpha = 2.33$.

For each given distance to default, the slope of the solid line is also positive. The positive slope indicates that holding the amount of market debt constant, the firm's market debt issuance intensity

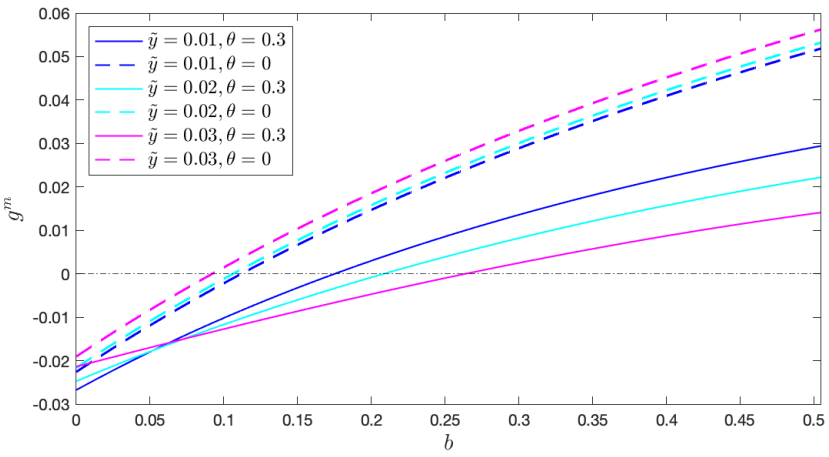


Figure 9: Effect of firm bargaining power θ on market debt issuance intensity $g^m(\tilde{y}_t, b_t)$. The figure plots the market debt issuance intensity $g^m(\tilde{y}_t, b_t)$ as a function of the bank debt ratio b_t at various levels of distance to default $\tilde{y}_t$ (including $\tilde{y}_t = 0.01, 0.02, 0.03$), for $\theta = 0.3$ and $\theta = 0$ respectively. The parameter values are $\mu = 0.02$, $\sigma = 0.25$, $r = c = 0.05$, $\lambda = 0.2$, and $\alpha = 2.33$.

increases with the level of bank debt, thus confirming the dynamic complementarity between bank debt and market debt for $\theta > 0$. However, the slope of the solid line is flatter than that of the dashed line. That is, fixing the amount of market debt, an increase in the firm's bank debt amount leads to a smaller increase in its market debt issuance intensity when the firm has positive bargaining power than when it has none. In other words, assigning bargaining power to the firm weakens the dynamic complementarity between bank debt and market debt.

How does the allocation of bargaining power between the bank lender and the firm affect the total firm value? In Figure 10, I plot the total enterprise value against the firm's bank debt ratio for various levels of distance to default. The figure shows that assigning bargaining power to the firm reduces the total enterprise value. Intuitively, when the firm has greater bargaining power, it can extract more surplus on behalf of its equity holders during bank debt renegotiation. This makes the equity holders less inclined to default, since doing so would cut off the equity holders' access to the bargaining surplus generated through periodic renegotiations with the bank lender. As default becomes more remote, diluting the market debt investors appears increasingly attractive to the bank lender and the equity holders when they renegotiate. Hence, renegotiations of the bank debt give rise to higher leverage, moving the firm further away from the efficient outcome.

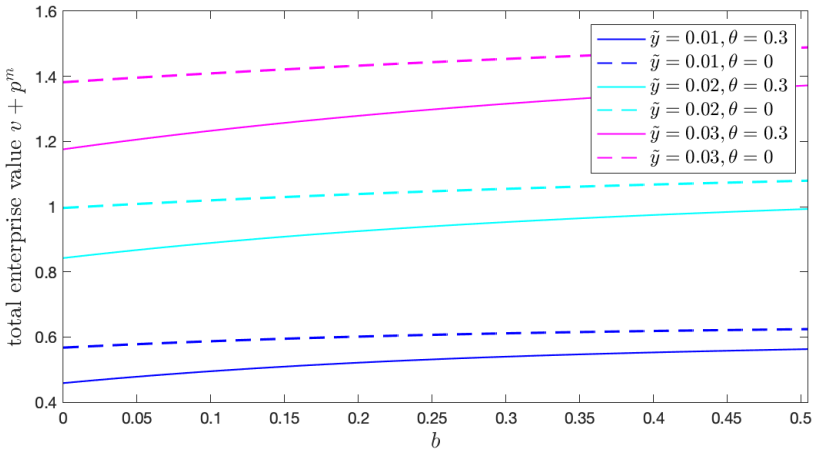


Figure 10: Total enterprise value $v + p^m$. This figure plots the total enterprise value $v + p^m$, defined as the firm value scaled by the amount of market debt outstanding, against the bank debt ratio b_t at various levels of distance to default $\tilde{y}_t$ (including $\tilde{y}_t = 0.01, 0.02, 0.03$), for $\theta = 0.3$ and $\theta = 0$ respectively. The parameter values are $\mu = 0.02$, $\sigma = 0.25$, $r = c = 0.05$, $\lambda = 0.2$, and $\alpha = 2.33$.

5 Empirical Evidence

A key result of the model is that firms position for potential bank debt renegotiations by trading market debt in the opposite direction. This result not only implies that firms may repurchase their own market debt – a behavior observed empirically but difficult to reconcile with existing theories – but also suggests that market debt repurchases and anticipated bank debt increases from potential bank-firm renegotiations naturally go together. The framework developed in this paper provides a unified explanation for why firms repurchase their market debt and how market debt repurchases are inherently linked to expected bank debt issuances. This prediction, which follows directly from Theorem 6, can be summarized as follows.

Prediction 1 (Market Debt Repurchases)**.** *A firm is more likely to repurchase its market debt when its current bank debt level is below the joint-surplus-maximizing level that would arise from bank-firm renegotiation, given the firm's current conditions.*

Another implication of the model is the dynamic complementarity between bank debt and market debt: holding the firm's EBIT and market debt outstanding fixed, the firm's subsequent market debt issuance intensity increases with its current bank debt balance. Because market debt issuance intensity is scale-invariant, this implication can be stated in terms of ratios. That is, holding the market debt leverage (market-debt-to-EBIT ratio) constant, a greater share of bank debt in the debt structure is

associated with a higher intensity of subsequent market debt issuance. Furthermore, the model predicts that the degree of dynamic complementarity between bank and market debt is weaker when the firm has more bargaining power relative to the bank lender.

Prediction 2 (Dynamic Complementarity of Bank and Market Debt)**.** *Controlling for market debt leverage, firms with a larger share of bank debt subsequently issue market debt at a greater rate. Moreover, the positive relation between the share of bank debt and subsequent market debt issuance is weaker for firms with greater bargaining power relative to their bank lenders.*

In this section, I take the above predictions to data and provide suggestive empirical evidence consistent with these predictions.

5.1 Firm-Level Data and Sample Construction

For my main empirical analysis, I rely on firm-level data from multiple data sources. I obtain annual firm characteristics and accounting data from Compustat for the period between 2002 and 2024. I restrict the sample to non-financial firms by excluding all firms whose NAICS codes starting with 52, which correspond to the finance and insurance sector. I end up with a sample containing 144,016 firm-year observations involving 15,383 unique firms. I then supplement the Compustat data with entity-level credit ratings obtained from the S&P Credit Ratings dataset, which provides a complete history of rating changes by Standard & Poor's (S&P). In particular, I focus on the local currency LT credit rating assigned by S&P. To facilitate the analysis, I give a numeric value to each notch of the rating, with 1, 2, 3, 4, ..., denoting AAA, AA+, AA, AA−, ..., respectively.

I obtain debt structure data from the Capital Structure dataset of Capital IQ, and then merge the data into the Compustat dataset. The Capital Structure dataset categorizes firms' total debt into seven mutually exclusive debt types: commercial paper, drawn credit lines, term loans, senior bonds and notes, subordinated bonds and notes, capital leases, and other debt. The dataset allows me to compute firm-year level observations of outstanding bank debt and market debt respectively. In the baseline, I follow Becker and Ivashina (2014) and define bank debt as term loans, while market debt includes senior bonds and notes as well as subordinated bonds and notes. For robustness, I also account for short-term credit by

Table 1: Summary Statistics of Key Financial Variables

	All Observations ($N = 34,424$)			Rated Only ($N = 17,506$)		
	Mean	Median	Std. Dev	Mean	Median	Std. Dev
Book Assets	12,000	2,362	29,414	22,317	6,731	43,065
Total Sales	7,147	1,255	17,755	13,175	3,720	26,400
Market Value	9,556	1,531	23,905	17,577	4,626	34,998
Book Leverage	0.421	0.379	0.258	0.423	0.386	0.223
Debt-to-EBITDA	3.434	2.989	9.723	4.204	3.414	6.059
Tangibility	0.337	0.275	0.259	0.361	0.316	0.249
Turnover	0.786	0.624	0.649	0.736	0.602	0.554
Net Profit Margin	-0.252	0.032	2.168	0.036	0.051	0.205
ROA	0.072	0.098	0.164	0.116	0.109	0.069
Market-to-Book	1.352	1.071	1.026	1.232	1.054	0.689
Dividend Payout	0.290	0.000	1.139	0.385	0.138	1.344
Bank Debt Share	0.399	0.347	0.309	0.305	0.238	0.266
Entity Rating				11.05	11.00	3.627

Note: The table presents summary statistics of key financial variables for all firm-year observations in the estimation sample, and those observations with available S&P credit ratings. Market value equals the product of the closing share price and the total number of shares outstanding. Book leverage is total debt divided by total book assets. Debt-to-EBITDA is the ratio of total debt to operating income before depreciation and amortization. Tangibility is net property, plant, and equipment divided by total assets. Turnover is total sales divided by total assets. Net profit margin is net income divided by total sales. ROA is operating income before depreciation and amortization divided by total assets. Market-to-book is the sum of the market value of equity, total debt, and preferred stock, divided by total assets. Dividend payout is the fraction of net income distributed as dividends. Bank debt share is the amount of bank debt divided by the sum of bank debt and market debt outstanding. All continuous financial variables are winsorized at the 1st and 99th percentiles. The estimation sample is constructed using data from Compustat, S&P Credit Ratings, and Capital IQ for the period between 2002 and 2024.

expanding the definition of bank debt to include drawn credit lines and the definition of market debt to include commercial paper.[17]

After removing any observation with missing debt structure information from the sample, I get 126,227 firm-year observations involving 14,592 unique firms. Following the literature, I further eliminate any firm-year observation with less than $10 million in assets. The ending sample contains 108,046 firm-year observations and 12,608 unique firms. Among the firm-year observations, 55% of those with senior or subordinated bonds and notes also have term loans, while 72% of those with bonds and notes or commercial paper also have term loans or drawn credit lines at the same time. Since I am interested in firms' debt structures and the dynamic interactions between bank and market debt, I focus on firms that have access to both types of debt financing. Accordingly, I restrict the sample to observations with non-zero amounts of both bank debt and market debt outstanding.

Table 1 reports summary statistics of key financial variables for the estimation sample (columns 2-4), and for the subsample of firm-year observations with available S&P credit ratings (columns 5-7). Market value refers to the market value of equity, and is calculated as the product of the closing share price and

[17]The robustness results are presented in Appendix G.1.

the total number of shares outstanding. Book leverage is the sum of current and long-term debt divided by total book assets. Debt-to-EBITDA is another measure of firm leverage, and is the ratio of total debt to operating income before depreciation and amortization. Tangibility is measured as net property, plant, and equipment scaled by total assets. Turnover is calculated as total sales divided by total assets. Net profit margin is net income divided by total sales. Return on assets (ROA) is defined as operating income before depreciation and amortization divided by total assets. Market-to-book is computed as the sum of the market value of equity, total debt, and preferred stock, divided by total assets. Dividend payout is the fraction of net income distributed to shareholders as dividends. Bank debt share is the amount of bank debt divided by the sum of bank and market debt, and captures a firm's debt structure. All continuous financial variables are winsorized at the 1st and 99th percentiles.

Among the firm-year observations with available S&P credit ratings, the mean entity rating is 11 (or BB+). Firms in the rated-only sample tend to be larger (e.g., higher book assets, total sales, and market value) and more profitable (e.g., higher net profit margin and ROA), have higher dividend payout ratio, slightly greater leverage and increased asset tangibility. The mean bank debt share is 0.305 for the rated-only sample, lower than the 0.399 in the full sample. That is, firms with available S&P credit ratings tend to have less bank debt in their debt structures on average.

I also collect bond repurchase data from the LSEG Mergent FISD dataset, which provides information on corporate bond redemptions including calls, open-market repurchases, tender offers, exchanges and other actions. To align with the model, I focus on tender offers (action type "T") and open-market repurchases (action type "IRP"), as these correspond to bond repurchases conducted at prevailing market prices. Between 2002 and 2024, there were 8,163 bond tender offers and 6,908 open-market repurchases, averaging $106 billion and $39 billion in face value per year, respectively. For context, the U.S. corporate bond outstanding amounted to roughly $4-11 trillion over the same period. Hence, corporate bond repurchases are relatively common and account for a non-trivial fraction of the total market.

I then merge the resulting repurchase data into the previously obtained Compustat-Capital IQ Capital Structure dataset. Table 2 reports summary statistics of key financial variables for bond repurchases in the full estimation sample (columns 2-4) and the subsample with available S&P credit ratings (columns 5-7). Comparison of Tables 1 and 2 shows that firms engaging in bond repurchases exhibit characteristics similar to those in the rest of the sample. Firms repurchasing their own bonds tend to have slightly higher leverage as evidenced by higher book leverage and debt-to-EBITDA, slightly lower profitability as

Table 2: Summary Statistics of Key Financial Variables - Bond Repurchases

	All Observations ($N = 1,428$)			Rated Only ($N = 1,144$)		
	Mean	Median	Std. Dev	Mean	Median	Std. Dev
Book Assets	14,771	4,485	34,736	16,713	4,869	38,143
Total Sales	8,257	2,649	19,088	9,203	3,028	20,752
Market Value	10,484	2,260	27,378	11,863	2,529	29,972
Book Leverage	0.508	0.483	0.239	0.502	0.485	0.227
Debt-to-EBITDA	4.843	4.296	9.195	5.094	4.312	9.294
Tangibility	0.337	0.282	0.246	0.343	0.296	0.244
Turnover	0.742	0.591	0.591	0.753	0.590	0.605
Net Profit Margin	-0.051	0.021	0.546	-0.025	0.022	0.379
ROA	0.101	0.104	0.106	0.104	0.104	0.094
Market-to-Book	1.189	1.046	0.618	1.156	1.031	0.539
Dividend Payout	0.263	0.000	1.335	0.270	0.000	1.351
Bank Debt Share	0.300	0.252	0.247	0.288	0.246	0.236
Entity Rating				12.55	13.00	3.019

Note: The table presents summary statistics of key financial variables associated with corporate bond repurchases (including both bond tender offers and open-market repurchases) for all firm-year observations in the estimation sample, and those observations with available S&P credit ratings. Market value equals the product of the closing share price and the total number of shares outstanding. Book leverage is total debt divided by total book assets. Debt-to-EBITDA is the ratio of total debt to operating income before depreciation and amortization. Tangibility is net property, plant, and equipment divided by total assets. Turnover is total sales divided by total assets. Net profit margin is net income divided by total sales. ROA is operating income before depreciation and amortization divided by total assets. Market-to-book is the sum of the market value of equity, total debt, and preferred stock, divided by total assets. Dividend payout is the fraction of net income distributed as dividends. Bank debt share is the amount of bank debt divided by the sum of bank debt and market debt outstanding. All continuous financial variables are winsorized at the 1st and 99th percentiles. The estimation sample is constructed using data from Compustat, S&P Credit Ratings, and Capital IQ for the period between 2002 and 2024. Data on bond repurchases are obtained from the LSEG Mergent FISD dataset.

indicated by the lower net profit margin and market-to-book, as well as lower credit ratings. Furthermore, firms repurchasing their own bonds tend to have lower bank debt share, suggesting that bond repurchases are more likely when a firm's current bank debt level is relatively low.

5.2 Market Debt Repurchases

The data above provide empirical evidence that market debt repurchases – whether through tender offers or open-market repurchases – are relatively common, a pattern that existing theories struggle to explain, but for which this model provides a natural mechanism. The model predicts that a firm is more likely to repurchase its market debt if its current bank debt level is below the joint-surplus-maximizing level implied by bank-firm renegotiation. Because this condition is more likely to hold when a firm carries less bank debt, it helps explain the observation from Table 2 that firms repurchasing their bonds tend to have lower bank debt shares.

It is worth noting that the model not only rationalizes market debt repurchases but also highlights that a firm's decision to repurchase market debt hinges on the expected outcome of potential bank debt renegotiation, should it occur. Accordingly, market debt repurchases are often accompanied by bank debt increases, a stylized pattern that I document empirically using the following regression specification:

$$Bank\ Debt\ Increase_{i,t} = \beta Market\ Debt\ Repurchase_{i,t} + \gamma X_{i,t} + \alpha_i + \alpha_t + \epsilon_{i,t}, \tag{5.1}$$

where i indexes firm and t indexes year. $Bank\ Debt\ Increase_{i,t}$ is an indicator variable equal to one if the bank debt balance for firm i increases in year t, and zero otherwise. $Market\ Debt\ Repurchase_{i,t}$ is an indicator variable equal to one if firm i repurchases its market debt (either via tender offer or open-market repurchase) in year t, and zero otherwise. α_i is firm fixed effect which controls for time-invariant characteristics within a firm. α_t is year fixed effect which controls for time-specific shocks common to all firms. Moreover, $X_{i,t}$ is a set of controls at the firm-year level. Standard errors are double-clustered at the firm and year levels.

The coefficient of interest is β. A positive β implies that, all else equal, firms that repurchase their market debt are more likely to increase their bank debt in the same period. I start by estimating the regression specification (5.1) with only firm and year fixed effects (column 1 of Table 3). I then augment the regression with time-varying firm-level controls (column 2), including variables capturing firms' size, leverage, and profitability such as the log of book assets, tangibility, ROA, book leverage, market-debt-to-EBIT ratio, and debt-to-EBITDA – as well as proxies for investment opportunities such as dividend payout, market-to-book ratio, stock return, and sales growth. Bank debt is not always adjusted in a given firm-year. In columns 3 and 4, I focus on firm-year observations with active bank debt adjustments. In columns 5 and 6, I re-estimate the specification for the subsample of rated firms. Across all specifications, the coefficient on $Market\ Debt\ Repurchase_{i,t}$ is positive and statistically significant, suggesting that firms that repurchase their market debt tend to increase their bank debt balance when renegotiating and amending their bank debt during the year.

The correlational pattern documented in Table 3 provides suggestive evidence consistent with the model's mechanism. A more direct empirical test of the model prediction regarding market debt repurchases, however, would require knowing a firm's expectation of whether bank debt renegotiation – if it were to occur – would result in a joint-surplus-maximizing bank debt level that is higher or lower than its current bank debt amount, given the firm's prevailing conditions. For example, a firm may repurchase its

Table 3: Correlation of Market Debt Repurchase and Bank Debt Increase

	Bank Debt Increase					
	(1)	(2)	(3)	(4)	(5)	(6)
Market Debt Repurchase	0.028*	0.041**	0.041**	0.051**	0.047**	0.065***
	(0.015)	(0.017)	(0.017)	(0.019)	(0.018)	(0.019)
Log(Assets)		0.071***		0.096***	0.031*	0.0670***
		(0.012)		(0.016)	(0.015)	(0.019)
Tangibility		-0.003		0.090	-0.048	0.032
		(0.058)		(0.062)	(0.094)	(0.104)
ROA		-0.388***		-0.365***	-1.227***	-1.095***
		(0.053)		(0.065)	(0.166)	(0.182)
Book Leverage		0.124***		0.221***	0.163**	0.223***
		(0.034)		(0.036)	(0.061)	(0.062)
Market-Debt-to-EBIT		-0.000		-0.000	-0.000**	-0.000*
		(0.000)		(0.000)	(0.000)	(0.000)
Debt-to-EBITDA		0.001*		0.001*	0.002*	0.002*
		(0.000)		(0.000)	(0.001)	(0.001)
Dividend Payout		0.009*		0.008*	0.010**	0.010*
		(0.005)		(0.005)	(0.004)	(0.005)
Market-to-Book		0.011*		-0.001	0.047***	0.049***
		(0.006)		(0.007)	(0.012)	(0.015)
Stock Return		-0.001		-0.002	-0.005	-0.002
		(0.003)		(0.003)	(0.004)	(0.004)
Sales Growth		0.093***		0.081***	0.217***	0.148***
		(0.012)		(0.014)	(0.036)	(0.038)
Cond. on Bank Debt Change	No	No	Yes	Yes	No	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Sample	All	All	All	All	Rated	Rated
R^2	0.219	0.194	0.199	0.216	0.175	0.195
Observations	33,043	23,725	23,806	19,080	12,969	10,632

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Note: The table presents results of the regression specification (5.1) using data from Compustat, S&P Credit Ratings, Capital IQ, and LSEG Mergent FISD for the period 2002-2024. The outcome variable is an indicator variable equal to one if a firm's bank debt balance increased in a given year, and zero otherwise. The coefficients of interest are the coefficients on market debt repurchase, an indicator variable equal to one if the firm repurchased its market debt (either via tender offer or open-market repurchase) in that year, and zero otherwise. Columns 1-4 use all firm-year observations from the estimation sample. Columns 5-6 restrict to observations with available S&P credit ratings. Columns 1, 2 and 5 are unconditional on bank debt adjustments while columns 3, 4 and 6 are conditional on bank debt adjustments. Except for columns 1 and 3, all regressions include both firm and year fixed effects, and firm-year level controls including log of book assets, tangibility, ROA, book leverage, market-debt-to-EBIT, debt-to-EBITDA, dividend payout, market-to-book, stock return, and sales growth. Standard errors are double-clustered at the firm and year levels.

market debt if it anticipates that a bank-debt-increasing renegotiation could occur with some probability, even if no such adjustment ultimately takes place. Firms' expectations for joint-surplus-maximizing bank debt levels are not directly observable, which gives rise to the main empirical challenge. Fortunately, the theory provides guidance that can help address this issue.

A. Predicting Joint-Surplus-Maximizing Bank Debt

The model shows that the joint-surplus-maximizing bank debt level, $\bar{B}(Y_t, M_t)$, which would arise from bank-firm renegotiation, is a function of the firm's current EBIT and its market debt outstanding. While firms' expectations of renegotiation outcomes are unobservable, the data provide realized bank debt levels resulting from actual bank debt adjustments, along with the corresponding earnings and market debt amounts for the same period. This allows me to proxy for the unobserved joint-surplus-maximizing bank debt level anticipated by the firm. Specifically, I regress these realized bank debt levels – scaled by the total book assets to obtain a more stable and comparable measure across firms and time – on contemporaneous market debt outstanding and earnings, also scaled by the total book assets. The fitted values, for each firm-year, offer a data-driven approximation of the bank debt level that the model identifies as joint-surplus-maximizing given the firm's prevailing conditions, thereby enabling an empirical test of the model prediction. More formally, I estimate the following regression:

$$\begin{aligned} \textit{Scaled Bank Debt}|\textit{Adjustment}_{i,t} =& \beta_1 \textit{Scaled Market Debt}_{i,t} + \beta_2 \textit{Scaled EBIT}_{i,t} \\ &+ \alpha_i + \alpha_t + \epsilon_{i,t}, \end{aligned} \tag{5.2}$$

where i indexes firm and t indexes year. $\textit{Scaled Bank Debt}|\textit{Adjustment}_{i,t}$ is the observed bank debt balance for firm i in year t, scaled by the firm's total book assets in that year, conditional on the firm having adjusted its bank debt balance during the period. $\textit{Scaled Market Debt}_{i,t}$ is the market debt balance for firm i in year t scaled by the firm's total book assets in that year, and $\textit{Scaled EBIT}_{i,t}$ is the firm's earnings before interest and taxes as reported by Compustat scaled by its total book assets. α_i and α_t are the firm and year fixed effects. Standard errors are double-clustered at the firm and year levels.

I estimate the specification (5.2) for both the full sample and the rated-only subsample, with results reported in columns 1 and 3 of Table 4. A firm's assets-scaled bank debt conditional on the firm having adjusted its bank debt balance, as captured by the dependent variable $\textit{Scaled Bank Debt}|\textit{Adjustment}_{i,t}$, serves as a proxy for the joint-surplus-maximizing bank debt level (scaled by the total book assets)

Table 4: Predictive Regressions for Joint-Surplus-Maximizing Bank Debt Level

	Conditional on Bank Debt Adjustment					
	Bank Debt Scaled by Assets				Bank-Debt-to-EBIT	
	(1)	(2)	(3)	(4)	(5)	(6)
Market Debt Scaled by Assets	0.250***	0.255***	0.131***	0.131***		
	(0.037)	(0.036)	(0.024)	(0.024)		
EBIT Scaled by Assets	-0.592***	0.943	-2.005**	1.877		
	(0.206)	(1.426)	(0.798)	(2.670)		
Market-Debt-to-EBIT					0.722***	0.491***
					(0.055)	(0.080)
Log(Assets)		-0.104*		-0.091	-4.240	0.827
		(0.058)		(0.104)	(3.755)	(4.556)
Tangibility		0.037		-0.595	20.89	9.942
		(0.424)		(0.765)	(37.38)	(47.17)
ROA		-1.229		-4.207	10.12	-2.474
		(1.387)		(2.658)	(18.69)	(55.08)
Book Leverage		0.914**		0.475	-1.331	-15.46
		(0.356)		(0.355)	(22.48)	(19.55)
Debt-to-EBITDA		0.001		0.003	-0.279	-0.585
		(0.001)		(0.002)	(0.165)	(0.442)
Dividend Payout		0.021		0.014	2.914	5.378*
		(0.025)		(0.014)	(2.059)	(2.708)
Market-to-Book		-0.028		0.041	1.719	6.485
		(0.030)		(0.116)	(3.242)	(8.104)
Stock Return		0.014		0.003	-3.134	-0.261
		(0.009)		(0.007)	(4.084)	(0.820)
Sales Growth		-0.022		-0.054	-5.117	-2.816
		(0.021)		(0.089)	(7.533)	(8.887)
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Sample	All	All	Rated	Rated	All	Rated
R^2	0.887	0.889	0.886	0.883	0.841	0.695
Observations	23,801	19,080	13,087	10,632	19,080	10,632

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Note: The table presents results of the regression specification (5.2) using data from Compustat, S&P Credit Ratings, and Capital IQ for the period 2002-2024. In columns 1-4, the outcome variable is the observed bank debt balance at the firm-year level, scaled by the total book assets, conditional on the firm having adjusted its bank debt balance during the period. The key explanatory variables are the market debt balance and the firm's EBIT, also scaled by the total book assets. Columns 1 and 3 estimate the regression specification for the full sample and the rated-only subsample respectively, while columns 2 and 4 augment the regression with firm-year level controls including log of book assets, tangibility, ROA, book leverage, debt-to-EBITDA, dividend payout, market-to-book, stock return, and sales growth. In columns 5 and 6, the outcome variable is the observed bank debt balance divided by the firm's EBIT, conditional on the firm having adjusted its bank debt balance during the period. The right-hand side includes the market-debt-to-EBIT ration along with the aforementioned controls. All regressions include both firm and year fixed effects, and standard errors are double-clustered at the firm and year levels.

that would arise from bank-firm renegotiation given the firm's current conditions. In both columns, the dependent variable is positively and statistically significantly associated with the firm's assets-scaled market debt while negatively and statistically significantly associated with its assets-scaled EBIT, in line with the model's implication that the joint-surplus-maximizing bank debt level $\bar{B}(Y_t, M_t)$ increases with the market debt outstanding M_t, but tends to decrease with the firm's EBIT Y_t (see Figure 6).

It is indeed reassuring to find empirical evidence of a positive relation between the market debt amount and the proxy for the joint-surplus-maximizing bank debt amount, as this relationship underpins the mechanism through which firms repurchase their market debt to position for potential bank debt renegotiation that would increase the amount of bank debt. As explained earlier, the firm's incentive to repurchase market debt allows it to "front-run" and limit the rents that would otherwise accrue to the bank lender. This occurs because the surplus from bank-firm renegotiation arises from adjustment in the amount of bank debt. With $\partial \bar{B}(Y_t, M_t)/\partial M_t > 0$, repurchasing market debt reduces the joint-surplus-maximizing bank debt level and hence the size of bank debt adjustment during renegotiation, thereby decreasing the surplus that is ultimately captured by the bank lender.

In columns 2 and 4 of Table 4, I augment the regression with firm-year level controls for firm size, leverage, profitability, as well as investment opportunities. The additional controls add little explanatory power: across both specifications, the R^2 remains unchanged at a high level of around 90%. The coefficient estimates on $Scaled\, Market\, Debt_{i,t}$ are also unchanged. By contrast, adding these controls affects the coefficient estimates on $Scaled\, EBIT_{i,t}$, since this variable is highly correlated with the controls which absorb the portion of its variation that they explain. For robustness, I also estimate the specification using financial ratios, with the bank-debt-to-EBIT ratio – conditional on the firm having adjusted its bank debt balance – as the dependent variable, and regressing it on the market-debt-to-EBIT ratio along with the same set of controls. The regressions yield qualitatively similar results.

Using the coefficient estimates from the regression specification (5.2), I can then calculate the predicted values of the renegotiation-implied joint-surplus-maximizing bank debt level for each firm-year observation in the sample, based on the firm's market debt balance, EBIT, and total book assets in the period. These predicted values serve as proxies for the expected joint-surplus-maximizing bank debt levels that would arise from bank-firm renegotiation, and are assumed to enter the firms' decision-making with regard to market debt issuances and repurchases.

B. Joint-Surplus-Maximizing Bank Debt and Market Debt Repurchases

The model predicts that a firm is more likely to repurchase its market debt when its current bank debt level is below the joint-surplus-maximizing level implied by bank-firm renegotiation, given the firm's current conditions. To test this prediction, I first construct – for each firm-year observation – a variable capturing the firm's anticipated bank debt issuance as implied by bank-firm renegotiation, should it occur, based on the firm's current conditions. I leverage the predicted values of the renegotiation-implied joint-surplus-maximizing bank debt levels obtained from the procedure described above. The expected renegotiation-implied bank debt issuance for firm i in year t, which I denote by $\mathbb{E}[Bank\ Debt\ Issuance]_{i,t}$, is computed by subtracting the firm's starting balance of bank debt from the predicted joint-surplus-maximizing value. This variable corresponds to $\bar{B}(Y_t, M_t) - B_t$ in the model.

To test the model prediction, I examine whether a firm is more likely to repurchase its market debt when the expected renegotiation bank debt adjustment points toward an increase, that is $\mathbb{E}[Bank\ Debt\ Issuance]_{i,t} > 0$. To this end, I estimate the following probit model to assess the likelihood of market debt repurchase, where the dependent variable is $Market\ Debt\ Repurchase_{i,t}$ which equals one if firm i repurchases its market debt (either through tender offer or open-market repurchase) in year t, and zero otherwise:

$$\Pr(Market\ Debt\ Repurchase_{i,t} = 1) = \Phi\left(\alpha + \beta Expected\ Bank\ Debt\ Increase_{i,t} + \gamma X_{i,t}\right), \qquad (5.3)$$

where i indexes firm and t indexes year. $Expected\ Bank\ Debt\ Increase_{i,t}$ is an indicator variable equal to one if $\mathbb{E}[Bank\ Debt\ Issuance]_{i,t} > 0$ and zero otherwise. $X_{i,t}$ include the aforementioned time-varying firm-level controls along with year fixed effects. Standard errors are double-clustered at the firm and year indicators. The results are presented in Table 5.

Column 1 of Table 5 reports the estimated coefficients from the probit model using the full sample, while columns 2-6 present results for various subsamples defined by firm characteristics. Across all specifications, the coefficient on $Expected\ Bank\ Debt\ Increase_{i,t}$ is positive and statistically significant, indicating that firms anticipating a bank debt increase in the event of bank debt renegotiation are more likely to repurchase their market debt. The result is thus consistent with the model prediction. In the full-sample specification, the estimated coefficient of 0.872 corresponds to an average marginal effect of 0.079, suggesting that an expected bank debt increase conditional on renegotiation raises the likelihood

Table 5: Probit Model Estimating Likelihood of Market Debt Repurchase

	Market Debt Repurchase					
	(1)	(2)	(3)	(4)	(5)	(6)
Expected Bank Debt Increase	0.872***	1.058***	1.147***	1.186***	0.651***	0.653***
	(0.115)	(0.196)	(0.248)	(0.245)	(0.145)	(0.183)
Log(Assets)	0.133***	0.0242*	0.0281	0.0214	0.0955***	0.0851***
	(0.008)	(0.013)	(0.019)	(0.017)	(0.012)	(0.017)
Tangibility	-0.256***	-0.168**	-0.189**	-0.242***	-0.345***	-0.385***
	(0.068)	(0.081)	(0.086)	(0.091)	(0.110)	(0.121)
ROA	0.467**	0.060	-0.354	-0.319	1.824**	-0.662
	(0.189)	(0.350)	(0.410)	(0.411)	(0.797)	(0.764)
Book Leverage	1.266***	1.205***	1.496***	1.479***	0.680**	1.979***
	(0.069)	(0.094)	(0.110)	(0.105)	(0.280)	(0.237)
Market-Debt-to-EBIT	-0.001***	-0.002***	-0.001**	-0.001*	-0.001	-0.000**
	(0.000)	(0.001)	(0.000)	(0.001)	(0.000)	(0.000)
Debt-to-EBITDA	0.001	0.005	0.001	-0.000	0.129***	-0.0575***
	(0.002)	(0.003)	(0.003)	(0.004)	(0.043)	(0.020)
Dividend Payout	-0.044**	-0.041**	-0.034*	-0.041	-0.023	-0.001
	(0.020)	(0.018)	(0.019)	(0.026)	(0.026)	(0.016)
Market-to-Book	-0.148***	-0.182***	-0.129***	-0.183***	-0.120***	-0.071
	(0.026)	(0.041)	(0.042)	(0.041)	(0.041)	(0.053)
Stock Return	0.014	0.011	0.058**	0.045*	0.003	0.186**
	(0.009)	(0.009)	(0.028)	(0.025)	(0.019)	(0.082)
Sales Growth	-0.262***	-0.188**	-0.145	-0.116	-0.351**	-0.206
	(0.060)	(0.095)	(0.096)	(0.109)	(0.148)	(0.181)
Constant	-3.986***	-3.066***	-3.337***	-3.235***	-3.866***	-3.666***
	(0.168)	(0.263)	(0.351)	(0.328)	(0.257)	(0.340)
Marginal Effect	0.079***	0.134***	0.125***	0.131***	0.060***	0.048**
Baseline Probability	0.048***	0.072***	0.059***	0.061***	0.049***	0.035***
Sample	All	Rated	Large Size	High Sales	High ROA	Div Payer
R^2	0.099	0.079	0.082	0.088	0.101	0.079
Observations	19,546	10,582	10,487	11,054	8,759	7,748

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Note: The table presents results from estimating the probit model (5.3) using data from Compustat, S&P Credit Ratings, Capital IQ, and LSEG Mergent FISD for the period 2002-2024. The dependent variable is $Market\ Debt\ Repurchase_{i,t}$ which equals one if firm i repurchases its market debt (either through tender offer or open-market repurchase) in year t, and zero otherwise. The key right-hand-side variable is $Expected\ Bank\ Debt\ Increase_{i,t}$, an indicator variable equal to one if $\mathbb{E}[Bank\ Debt\ Issuance]_{i,t} > 0$ and zero otherwise, where $\mathbb{E}[Bank\ Debt\ Issuance]_{i,t}$ is computed by subtracted firm i's bank debt level in year t from the predicted joint-surplus-maximizing value obtained by regressing the realized assets-scaled bank debt levels conditional on bank debt adjustments on the firm's prevailing assets-scaled market debt amount and assets-scaled EBIT. All specifications include firm-year level controls including log of book assets, tangibility, ROA, book leverage, market-debt-to-EBIT, debt-to-EBITDA, dividend payout, market-to-book, stock return, and sales growth, as well as year indicators (which are omitted to simplify exposition). Column 1 estimates the probit model for the full sample, while columns 2-6 estimate the probit model for the subsamples of rated firms, firms with book assets above the sample median, firms with total sales above the sample median, firms with ROA above the sample median, and dividend-paying firms respectively. Standard errors are double-clustered at the firm and year levels. The marginal effects of $Expected\ Bank\ Debt\ Increase_{i,t}$ and the baseline probabilities are also reported.

of a market debt repurchase by 7.9%. By contrast, the baseline probability of market debt repurchase is merely 4.8%. That is, firms facing potential bank-debt-increasing renegotiations are nearly three times as likely to repurchase their market debt. The effect remains economically and statistically significant across all subsamples, including for rated firms, larger firms (e.g., firms with book assets above the sample median and firms with total sales above the sample median), more profitable firms (e.g., firms with ROA above the same median), as well as dividend-paying firms.

5.3 Dynamic Complementarity of Bank and Market Debt

To investigate the dynamic complementarity between bank debt and market debt, I begin by plotting the relationship between bank debt share and the following year's market debt net issuance rate of firms. Bank debt share equals a firm's bank debt outstanding divided by the total amount of bank debt and market debt. Market debt net issuance rate is the net increase in market debt scaled by the beginning balance of bank debt and market debt combined, and corresponds to $g^m(Y_t, M_t, B_t)$ in the model. I first sort the observations within each year into 20 equal-sized bins based on the distribution of bank debt share within the year. Each bin thus groups observations with similar values of bank debt share within the given year. Then for each bin-year, I compute the mean bank debt share and the mean market debt net issuance rate in the following year. I then average these bin-year mean values across all years. Finally, I plot the resulting binned means into a two-dimensional binscatter graph, with the bank debt share on the horizontal axis and the subsequent year's market debt net issuance rate on the vertical axis.

The plot is shown in Panel A of Figure 11. It shows a positive relationship between the market debt net issuance rate in the following year and the prevailing bank debt share. Hence, a greater amount of bank debt in the debt structure predicts higher subsequent market debt issuance, suggesting a dynamic complementarity between bank and market debt. Although the plot offers suggestive evidence that bank debt and market debt are dynamically complementary, a couple of main concerns need to be addressed.

First, the observed positive relationship between bank debt share and subsequent market debt net issuance may simply reflect firms rebalancing toward an optimal debt composition. That is, firms with higher-than-optimal bank debt shares accelerate market debt issuance to bring their debt structures back toward the optimal mix. However, the same logic implies that such firms should also reduce their bank debt issuance, leading to a negative relationship between bank debt share and subsequent net issuance of

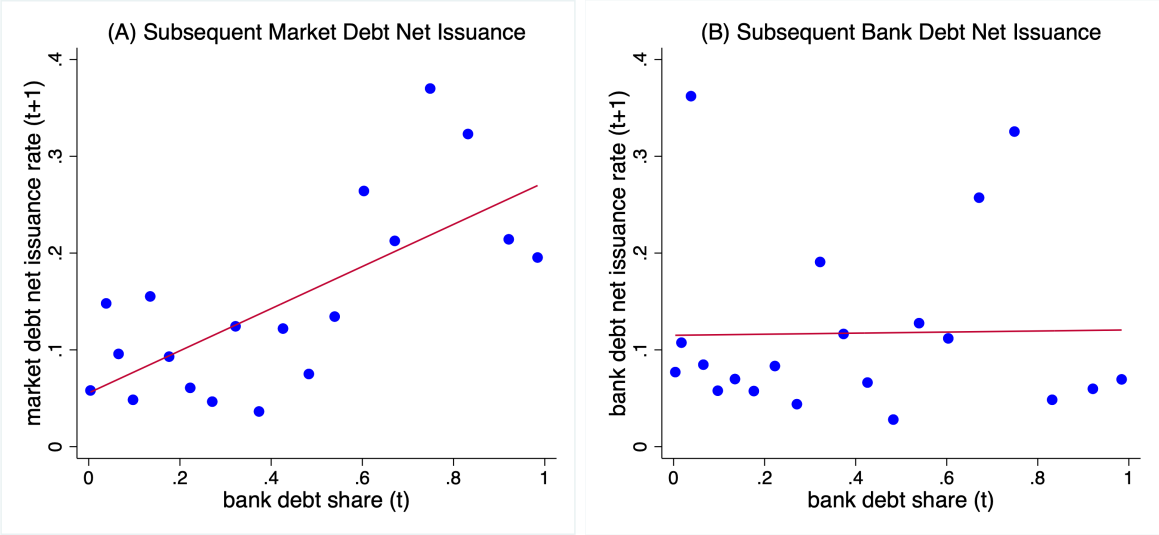


Figure 11: Binscatter Plots of Bank Debt Share and Subsequent Debt Issuances. Panel A of the figure plots market debt net issuance rate (defined as the net increase in market debt outstanding during the following year scaled by the total amount of bank and market debt) against the bank debt share (defined as the amount of bank debt divided by the sum of bank and market debt) in a binscatter graph with 20 equal-sized bins. Panel B of the figure plots bank debt net issuance rate (defined as the net increase in bank debt outstanding during the following year scaled by the total amount of bank and market debt) against the bank debt share in a binscatter graph with 20 equal-sized bins. The estimation sample is constructed using data from Compustat, S&P Credit Ratings, and Capital IQ for the period between 2002 and 2024.

bank debt. Panel B of Figure 11 shows that this is not the case, suggesting that the observed positive relationship between bank debt share and subsequent market debt net issuance rate cannot be simply explained by mean-reversion to a target debt structure.

Another concern is that the bank debt reported by Capital IQ may include bridge loans, which are intended to be replaced by bond issuances in the near term. To finance investment projects or acquisitions, firms often initially turn to bank lenders for debt commitments in the form of bridge loans. Bridge loans are short-term facilities that provide interim financing until long-term securities, such as bonds, can be issued to permanently fund the transactions. However, if the observed increase in market debt issuance following a rise in bank debt share is primarily driven by firms replacing bridge loans with bonds, one would expect to see a corresponding decrease in bank debt net issuance. This is inconsistent with the observed pattern shown in Panel B of Figure 11.

Lastly, the positive relationship between bank debt share and subsequent market debt net issuance rate could be a mechanical artifact of secular trends in debt composition over the sample period. For instance, if bank debt share were in steady decline, earlier years would naturally exhibit higher bank debt shares alongside faster market debt issuance in subsequent periods. The within-year binning and averaging procedure employed in constructing the binscatter addresses this concern. To further mitigate

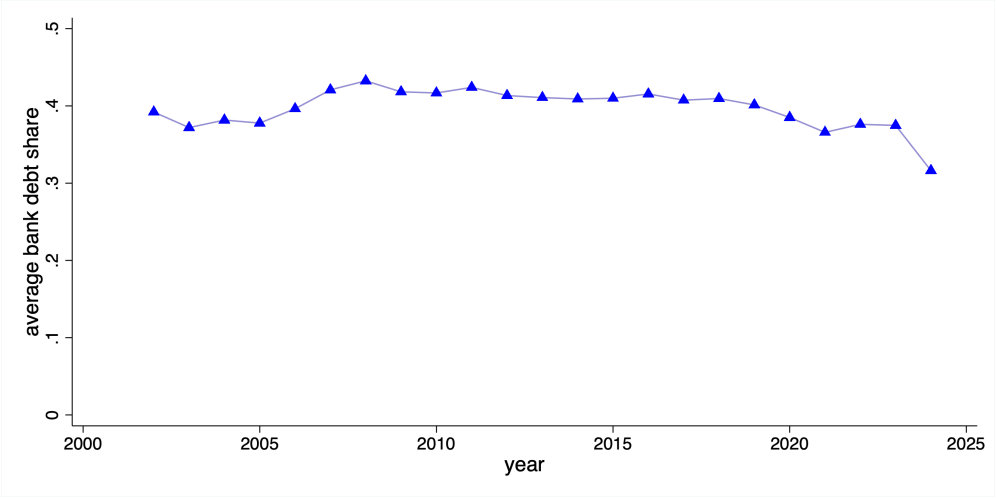


Figure 12: Average Bank Debt Share Over Time. The figure plots the time series of firm-averaged bank debt shares by year, using data from Compustat and Capital IQ for the period between 2002 and 2024. Average bank debt share is computed by taking the average of bank debt shares across all firms within a year, where bank debt share is defined as the amount of bank debt divided by the sum of bank and market debt.

this concern, I plot the time series of firm-averaged bank debt shares by year in Figure 12. Bank debt shares during the sample period appear relatively stable, fluctuating around an average of about 0.4.

A. Panel Regressions

Several confounding factors may distort the interpretation of the positive relationship between bank debt share and subsequent market debt net issuance rate as shown in Figure 11. A major concern is that firm characteristics may affect both the debt structure and market debt issuance behavior. For instance, firms with more tangible assets to pledge as collateral may enjoy better access to both bank debt financing and market debt financing. As a result, these firms may have higher proportions of bank debt in their debt structures and simultaneously issue more market debt, mechanically generating a positive correlation between the two. To address such omitted variable bias, I estimate the following regression specification

$$Market\ Debt\ Net\ Issuance\ Rate_{i,t+1} = \beta Bank\ Debt\ Share_{i,t} + \gamma X_{i,t} + \alpha_i + \alpha_t + \epsilon_{i,t}, \tag{5.4}$$

where i indexes firm and t indexes year. $Market\ Debt\ Net\ Issuance\ Rate_{i,t+1}$ is the net increase in firm i's market debt outstanding from year t to $t+1$ divided by the firm's total amount of bank and market debt in year t. $Bank\ Debt\ Share_{i,t}$ equals firm i's bank debt outstanding in year t scaled by its total

amount of bank and market debt in year t. α_i and α_t are the firm and year fixed effects respectively. $X_{i,t}$ is a set of controls at the firm-year level. Standard errors are double-clustered at the firm and year levels.

The coefficient of interest is β. A positive β means that an increase in the prevailing bank debt share is associated with a higher rate of market debt net issuance in the following year, after controlling for various confounding factors. I start by including the log of book assets, tangibility, ROA, book leverage, market-debt-to-EBIT ratio, and debt-to-EBITDA in the time-varying firm-level controls (X_{it}). The regression results are shown in Table 6. Column 1 is unconditional on market debt net issuance or repurchase by a firm in a given year, while column 3 is conditional on non-zero market debt changes. In both columns, the coefficient estimates on bank debt share are positive and highly statistically significant.

Another potential issue concerns the timing of bank debt and market debt issuances. Firms with better investment opportunities may issue both bank debt and market debt to finance these opportunities. If bank debt is easier or quicker to obtain than market debt (e.g., due to time required for bond roadshows), firms may initially rely more on bank debt before subsequently ramping up market debt issuance. Accordingly, I include several proxies for investment opportunities as additional control variables. These include market-to-book ratio, stock return, and sales growth. I also include dividend payout as a control variable, since distribution of dividend may also affect a firm's demand for credit. The results are shown in column 2 (unconditional on market debt net issuance or repurchase) and in column 4 (conditional on non-zero market debt issuance or repurchase).

Inclusion of the additional control variables reduces the magnitude of the coefficient estimates on bank debt share. However, the estimates remain positive and highly statistically significant. A 1% increase in the prevailing bank debt share is associated with 79-82 basis points higher net issuance rate of market debt in the subsequent year. Finally, I re-estimate the specification with the full set of control variables for the rated-only subsample. The coefficient estimates on bank debt share are positive and statistically significant, though their magnitudes are about half of those in the full estimation sample. Taken together, the regression results in Table 6 are consistent with the pattern documented in Figure 11. In sum, higher bank debt share is associated with greater subsequent net issuance of market debt, implying that bank debt and market debt are dynamically complementary.[18]

[18] In Appendix G.1, I provide additional robustness tests. In the first set of robustness tests, I apply alternative definitions for bank debt and market debt. In the second set of robustness tests, I use a matched-sample regression specification by including industry-year-size-leverage fixed effects as the main controls. Finally, I document complementarity between bank and market debt using time-series aggregate balance sheet data obtained from the Flow of Funds.

Table 6: Panel Regressions: Dynamic Complementarity of Bank and Market Debt

	Market Debt Net Issuance Rate					
	(1)	(2)	(3)	(4)	(5)	(6)
Bank Debt Share	0.918***	0.792***	0.991***	0.820***	0.392**	0.412**
	(0.225)	(0.164)	(0.244)	(0.175)	(0.144)	(0.153)
Log(Assets)	-0.253***	-0.210***	-0.247***	-0.218***	-0.299	-0.339
	(0.047)	(0.068)	(0.051)	(0.072)	(0.174)	(0.198)
Tangibility	0.505	0.284	0.365	0.0411	0.256	0.211
	(0.379)	(0.503)	(0.337)	(0.389)	(0.258)	(0.290)
ROA	-0.123	-0.550**	-0.216	-0.627**	-0.377	-0.373
	(0.267)	(0.232)	(0.309)	(0.268)	(0.642)	(0.777)
Book Leverage	-1.607***	-1.615***	-1.651***	-1.703***	-1.208**	-1.322**
	(0.286)	(0.408)	(0.285)	(0.393)	(0.508)	(0.580)
Market-Debt-to-EBIT	-0.001	0.000	-0.001	0.000	0.000	0.000
	(0.001)	(0.000)	(0.001)	(0.000)	(0.000)	(0.000)
Debt-to-EBITDA	0.002	0.000	0.002	0.000	0.000	0.000
	(0.002)	(0.001)	(0.003)	(0.001)	(0.001)	(0.001)
Dividend Payout		-0.007		0.005	0.000	0.001
		(0.008)		(0.009)	(0.003)	(0.003)
Market-to-Book		0.205***		0.226***	0.233**	0.251*
		(0.048)		(0.054)	(0.112)	(0.122)
Stock Return		-0.011		-0.009	-0.005	-0.005
		(0.009)		(0.010)	(0.007)	(0.007)
Sales Growth		-0.022		-0.037	0.118	0.130
		(0.064)		(0.072)	(0.104)	(0.121)
Sample	All	All	All	All	Rated	Rated
Conditional on Issuance	No	No	Yes	Yes	No	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.150	0.141	0.203	0.204	0.097	0.103
Observations	28,293	21,762	25,008	19,201	12,036	10,619

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Note: The table presents results of the regression specification (5.4) using data from Compustat, S&P Credit Ratings, and Capital IQ for the period 2002-2024. The outcome variable is market debt net issuance rate defined as the net increase in market debt outstanding during the following year scaled by the total amount of bank and market debt. The coefficients of interest are the coefficients on bank debt share defined as the amount of bank debt divided by the sum of bank and market debt. Columns 1-4 use all firm-year observations from the estimation sample. Columns 5-6 restrict to observations with available S&P credit ratings. Columns 1, 2 and 5 are unconditional on non-zero market debt changes while columns 3, 4 and 6 are conditional on non-zero market debt changes. In columns 1 and 3, the firm-year level controls include the log of book assets, tangibility, ROA, book leverage, market-debt-to-EBIT ratio, and debt-to-EBITDA. Columns 2 and 4-6 also include dividend payout, market-to-book, stock return, and sales growth as additional control variables. Standard errors are double-clustered at the firm and year levels.

So far, the results provide suggestive empirical evidence consistent with the model prediction that bank debt and market debt are dynamically complementary in the sense that a larger share of bank debt in the debt structure predicts higher subsequent intensity of market debt issuance. In the model, the dynamic complementarity of bank and market debt arises due to the firm's strategic incentive to issue or repurchase market debt to position for bank debt renegotiations. If the observed dynamic debt complementarity indeed works through this channel, then firms that renegotiate their bank debt more frequently should exhibit stronger dynamic complementarity between their bank debt and market debt, since the strategic incentive is more pronounced for these firms.

I obtain facility-level information on bank loans from the LSEG DealScan, formerly the Loan Pricing Corporation (LPC) DealScan dataset. Borrowers in the data experience frequent changes to their loan facilities. For the period between 2002 and 2024, the median time between loan facility changes is approximately one year, while the median facility maturity is about four years. Changes to loan facilities could be due to new issuances or amendments of existing facilities, both of which result from negotiations between firms and their bank lenders. I re-estimate (5.4) for firms whose bank loan facilities are renegotiated infrequently, defined as having time intervals between facility changes above the sample median. The coefficient estimates are reported in columns (4)-(6) in Table 7, while columns (1)-(3) reproduce the pooled estimates carried over from Table 6. Firms whose bank loan facilities are renegotiated infrequently exhibit weaker dynamic complementarity between their bank debt and market debt, suggesting that bank debt renegotiation underlies the observed pattern of dynamic debt complementarity.

B. Heterogeneity Across Firms and Time

How does the degree of dynamic complementarity between bank and market debt vary depending on firm characteristics and time periods? To answer this question, I introduce various dummy variables capturing firm characteristics and market conditions. For example $Large\ Size_{it}$ is a firm-year level indicator variable equal to one if the amount of book assets is above the sample median and zero otherwise. $High\ Sales_{it}$ and $High\ ROA_{it}$ are analogously defined from total sales and ROA respectively. $Div\ Payer_{it}$ is an indicator variable equal to one if firm i pays dividend in year t. $IG\ Rating_{it}$ is an indicator variable equal to one if the entity credit rating assigned by S&P is BBB- or above. $Crisis_t$ indicates crisis periods (including the GFC and COVID-19), and equals one for years 2008, 2009 and

Table 7: Dynamic Ccomplementarity: Differences Across Renegotiation Frequencies

	Market Debt Net Issuance Rate					
	All Renegotiation Frequencies			Low Renegotiation Frequencies		
	(1)	(2)	(3)	(4)	(5)	(6)
Bank Debt Share	0.792^{***}	0.820^{***}	0.412^{**}	0.602^{***}	0.487^{***}	0.220
	(0.164)	(0.175)	(0.153)	(0.166)	(0.0992)	(0.192)
Sample	All	All	Rated	All	All	Rated
Conditional on Issuance	No	Yes	Yes	No	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.141	0.204	0.103	0.162	0.175	0.169
Observations	21,762	19,201	10,619	8,055	6,928	4,106

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Note: The table presents results of the regression specification (5.4) using data from Compustat, S&P Credit Ratings, and Capital IQ for the period 2002-2024. The outcome variable is market debt net issuance rate defined as the net increase in market debt outstanding during the following year scaled by the total amount of bank and market debt. The coefficients of interest are the coefficients on bank debt share defined as the amount of bank debt divided by the sum of bank and market debt. Columns 1-4 use all firm-year observations from the estimation sample. Columns 5-6 restrict to observations with available S&P credit ratings. Columns 1, 2 ad 5 are unconditional on non-zero market debt changes while columns 3, 4 and 6 are conditional on non-zero market debt changes. In columns 1 and 3, the firm-year level controls include the log of book assets, tangibility, ROA, book leverage, market-debt-to-EBIT ratio, and debt-to-EBITDA. Columns 2 and 4-6 also include dividend payout, market-to-book, stock return, and sales growth as additional control variables. Standard errors are double-clustered at the firm and year levels.

2020. I then interact these variables with $Bank\ Debt\ Share_{it}$ and re-estimate the regression (5.4) with the full set of controls, unconditional on market debt net issuance or repurchase.

The coefficient estimates on the interaction terms are presented in Table 8. The degree of complementarity between bank and market debt tends to be lower for larger firms (e.g., firms with more assets and higher sales), for more profitable firms (e.g., firms with higher ROA), and for firms with investment-grade (IG) credit ratings. Moreover, dividend-paying firms exhibit weaker complementarity between bank and market debt. On the other hand, the degree of complementarity between bank and market debt tends to be amplified during crisis periods such as the GFC and COVID-19.

Several potential explanations may account for this heterogeneity across firms and time periods, depending on the underlying mechanisms that drive the complementarity between bank and market debt. One explanation, as provided by the model, is that the degree of complementarity between bank and market debt is affected by the amount of bargaining power the firm has over its bank lender. The results in Table 8 seem to be consistent with this explanation. Larger and more profitable firms, as well as dividend-paying firms (which are typically larger and more mature) tend to possess greater bargaining power. Firms with little default risk and firms operating under favorable economic conditions are also

Table 8: Complementarity of Bank and Market Debt: Heterogeneity Across Firms and Time

	Market Debt Net Issuance Rate					
	(1)	(2)	(3)	(4)	(5)	(6)
Bank Debt Share×Large Size	-0.668***					
	(0.220)					
Bank Debt Share×High Sales		-0.931***				
		(0.298)				
Bank Debt Share×High ROA			-0.426**			
			(0.204)			
Bank Debt Share×Div Payer				-0.402**		
				(0.161)		
Bank Debt Share×IG Rating					-0.339*	
					(0.172)	
Bank Debt Share×Crisis						0.619**
						(0.274)
FirmFE	Yes	Yes	Yes	Yes	Yes	Yes
YearFE	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.141	0.141	0.141	0.141	0.141	0.141
Observations	21,762	21,762	21,762	21,762	21,762	21,762

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Note: The table regresses market debt net issuance rate in the following year (defined as the net increase in market debt scaled by the total amount of bank debt and market debt) on the interaction terms between the bank debt share variable and various dummy variables capturing firm characteristics and time periods, along with firm-year level controls and firm and year fixed effects. Controls include the log of book assets, tangibility, ROA, book leverage, market-debt-to-EBIT ratio, debt-to-EBITDA, dividend payout, market-to-book, stock return, and sales growth. $Large\ Size_{it}$ is a firm-year level indicator variable equal to one if the amount of book assets is above the sample median. $High\ Sales_{it}$ and $High\ ROA_{it}$ are analogously defined from total sales and ROA respectively. $Div\ Payer_{it}$ is an indicator variable equal to one if firm i pays dividend in year t. $IG\ Rating_{it}$ is an indicator variable equal to one if the entity credit rating assigned by S&P is BBB- or above. $Crisis_t$ indicates crisis periods (including the GFC and COVID-19), and equals one for years 2008, 2009 and 2020. Data are from Compustat, S&P Credit Ratings, and Capital IQ for the period 2002-2024. Standard errors are double-clustered at the firm and year levels.

likely to enjoy stronger bargaining positions. The coefficient estimates in the table thus suggest that the complementarity between bank and market debt decreases as firm bargaining power increases.

Another potential explanation is that the bank lender's monitoring capability or the contractual flexibility of bank debt generates positive spillover effects on market debt, thereby enhancing the firm's ability to raise market debt financing. To assess the validity of this explanation, I re-estimate the specification (5.4) using only observations in the bottom quintiles of book leverage and market leverage (defined as total debt divided by the sum of the market value of equity, total debt, and preferred stock). These observations have very low financial leverage, with a subsample mean book leverage ratio of 0.08 and a subsample mean market leverage ratio of 0.05. I also run the same specification for observations

Table 9: Complementarity of Bank and Market Debt for Low-Risk Firms

	Market Debt Net Issuance Rate					
	Low Book Lev		Low Mkt Lev		High Ratings	
	(1)	(2)	(3)	(4)	(5)	(6)
Bank Debt Share	1.616***	1.452***	1.463**	1.295**	0.302***	0.275**
	(0.520)	(0.463)	(0.559)	(0.506)	(0.101)	(0.099)
Conditional on Issuance	No	Yes	No	Yes	No	Yes
FirmFE	Yes	Yes	Yes	Yes	Yes	Yes
YearFE	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.374	0.384	0.360	0.370	0.065	0.066
Observations	4,430	3,928	4,116	3,577	1,998	1,929

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Note: The table presents coefficient estimates on bank debt share by estimating the regression specification (5.4), using data from Compustat, S&P Credit Ratings, and Capital IQ for the period 2002-2024. The outcome variable is market debt net issuance rate defined as the net increase in market debt outstanding during the following year scaled by the total amount of bank and market debt. Bank debt share is the amount of bank debt divided by the sum of bank and market debt. Columns 1-2 use observations with book leverage in the bottom quintile. Columns 3-4 use observations with market leverage in the bottom quintile. Columns 5-6 use observations with S&P entity credit ratings of A− and above. All regressions include firm and year fixed effects, and the full set of firm-year level control variables including the log of book assets, tangibility, ROA, book leverage, market-debt-to-EBIT ratio, debt-to-EBITDA, dividend payout, market-to-book, stock return, and sales growth. Standard errors are double-clustered at the firm and year levels.

with entity credit ratings of A− or above, which account for 16% of the rated-only sample and 5% of the total estimation sample. In Table 9, I present the coefficient estimates on the bank debt share variable.

Firms with low leverage and high credit ratings face minimal default risk. As a result, the bank lender's ability to monitor and the contractual flexibility of bank debt (such as the ability to renegotiate terms during distress) offer limited additional value. Hence, if the observed pattern of complementarity between bank and market debt were driven by these features, the pattern should be absent for firms with very low leverage and high credit ratings. However, all coefficient estimates in Table 9 are positive and statistically significant. This suggests that the pattern of complementarity between bank and market debt cannot be primarily attributed to bank monitoring or contractual flexibility, as the pattern is evident even among low-risk firms for which bank monitoring and contractual flexibility are of limited importance.

6 Conclusion

This paper develops a dynamic model of firm debt structure that simultaneously incorporates both renegotiable bank debt and competitive market debt. A key insight from the model is that the firm strategically pits the bank lender and the market debt investors against each other. The firm, acting on behalf of its equity holders, engages in dynamic bilateral bargaining with the bank lender to extract value

from the non-bargaining market debt investors. At the same time, the firm also trades with the market debt investors to limit the rents accruing to the bank lender during bilateral bank-firm bargaining. The model generates predictions consistent with empirical evidence and rationalizes key contractual features observed in practice, such as debt seniority and covenants.

Although it is socially efficient for the firm to rely solely on renegotiable bank debt, the firm has a private incentive to deviate, as it benefits from having market debt by playing the bank lender and the market debt investors against each other. The presence of market debt introduces an inefficiency: the bank lender and the equity holders share an incentive to extract value from the non-bargaining market debt investors. The firm's strategic trading of market debt limits the rents to the bank lender but can either mitigate or exacerbate inefficiency. One way to improve efficiency is optimizing the frequency of bank debt renegotiation – finding the right balance in renegotiation frequency by choosing bank debt maturities or writing covenant tripwires that trigger renegotiation. However, this cannot eliminate distortions arising from the strategic interactions among the firm's claimants. Ultimately, improving efficiency may require state-contingent contractual features that limit inefficient value extraction and better align incentives among the firm's claimants – a direction that future research could explore.

Finally, while the model is developed to study debt structure dynamics, the underlying framework is sufficiently general and applicable to other contexts. The model, in a nutshell, centers on the interplay among three parties: two bargaining parties and a silent non-bargaining party. Of the two bargaining parties, one is a "control" bargaining party who can trade directly with the non-bargaining party, while the other is a non-control party who cannot. The central idea is that the non-bargaining party prices in the expected outcome of the bargaining game between the two bargaining parties, shaping its trade with the control bargaining party, which in turn affects the bargaining outcome by influencing the bargaining parties' outside options and joint value. The model may extend to other economic contexts, such as labor contract renegotiation, sovereign debt restructuring, or corporate actions and governance.

References

Admati, Anat R., Peter M. DeMarzo, Martin F. Hellwig, and Paul Pfleiderer, "The Leverage Ratchet Effect," *Journal of Finance*, 2018, *73* (1), 145–198.

Antill, Samuel, Neng Wang, and Zhaoli Jiang, "Creditor-on-Creditor Violence and Secured Debt Dynamics," Working Paper 2025.

Becker, Bo and Victoria Ivashina, "Cyclicality of Credit Supply: Firm Level Evidence," *Journal of Monetary Economics*, 2014, *62*, 76–93.

Besanko, David and George Kanatas, "Credit Market Equilibrium with Bank Monitoring and Moral Hazard," *Review of Financial Studies*, 1993, *6* (1), 213–232.

Bidder, Rhys, Nicolas Crouzet, Maggie M. Jacobson, and Michael Siemer, "Debt Flexibility," Working Paper 2023.

Bolton, Patrick and David S. Scharfstein, "Optimal Debt Structure and the Number of Creditors," *Journal of Political Economy*, 1996, *104* (1), 1–25.

_ **and Xavier Freixas**, "Equity, Bonds, and Bank Debt: Capital Structure and Financial Market Equilibrium under Asymmetric Information," *Journal of Political Economy*, 2000, *108* (2), 324–351.

Bulow, Jeremy I. and John B. Shoven, "The Bankruptcy Decision," *Bell Journal of Economics*, 1978, *9* (2), 437–456.

Chemmanur, Thomas J. and Paolo Fulghieri, "Reputation, Renegotiation, and the Choice between Bank Loans and Publicly Traded Debt," *Review of Financial Studies*, 1994, *7* (3), 475–506.

Colla, Paolo, Filippo Ippolito, and Kai Li, "Debt Specialization," *Journal of Finance*, 2013, *68* (5), 2117–2141.

Crouzet, Nicolas, "Aggregate Implications of Corporate Debt Choices," *Review of Economic Studies*, 2018, *85* (3), 1635–1682.

Dangl, Thomas and Josef Zechner, "Debt Maturity and the Dynamics of Leverage," *Review of Financial Studies*, 2021, *34* (12), 5796–5840.

Darmouni, Olivier and Kerry Y. Siani, "Bond Market Stimulus: Firm-Level Evidence," *Journal of Monetary Economics*, 2025, *151*, 103728.

DeMarzo, Peter M. and Zhiguo He, "Leverage Dynamics without Commitment," *Journal of Finance*, 2021, *76* (3), 1195–1250.

Diamond, Douglas W., "Monitoring and Reputation: The Choice between Bank Loans and Directly Placed Debt," *Journal of Political Economy*, 1991, *99* (4), 689–721.

Donaldson, Jason Roderick, Denis Gromb, and Giorgia Piacentino, "Conflicting Priorities: A Theory of Covenants and Collateral," *Review of Financial Studies*, 2025, *80* (3), 1331–1873.

_ , Naz Koont, Giorgia Piacentino, and Victoria Vanasco, "A New Theory of Credit Lines (with Evidence)," Working Paper 2025.

Gertner, Robert and David Scharfstein, "A Theory of Workouts and the Effects of Reorganization Law," *Journal of Finance*, 1991, *46* (4), 1189–1222.

Greenwood, Robin M., Samuel G. Hanson, and Lawrence J. Jin, "Reflexivity in Credit Markets," *Journal of Finance*, 2023, *forthcoming.*

Hackbarth, Dirk and David C. Mauer, "Optimal Priority Structure, Capital Structure, and Investment," *Review of Financial Studies*, 2012, *25* (3), 747–796.

_ , Christopher A. Hennessy, and Hayne E. Leland, "Can the Trade-off Theory Explain Debt Structure?," *Review of Financial Studies*, 2007, *20* (5), 1389–1428.

Halling, Michael, Jin Yu, and Josef Zechner, "The Dynamics of Corporate Debt Structure," Working Paper 2025.

Mao, Lei and Yuri Tserlukevich, "Repurchasing Debt," *Management Science*, 2015, *61* (7), 1648–1662.

Morellec, Erwan, Philip Valta, and Alexei Zhdanov, "Financing Investment: The Choice Between Bonds and Bank Loans," *Management Science*, 2015, *61* (11), 2580–2602.

Park, Cheol, "Monitoring and Structure of Debt Contracts," *Journal of Finance*, 2000, *55* (5), 2157–2195.

Perotti, Enrico C. and Kathryn E. Spier, "Capital Structure as a Bargaining Tool: The Role of Leverage in Contract Renegotiation," *American Economic Review*, 1993, *83* (5), 1131–1141.

Rajan, Raghuram G., "Insiders and Outsiders: The Choice between Informed and Arm's-Length Debt," *Journal of Finance*, 1992, *47* (4), 1367–1400.

Rauh, Joshua D. and Amir Sufi, "Capital Structure and Debt Structure," *Review of Financial Studies*, 2010, *23* (12), 4242–4280.

Roberts, Michael R., "The role of dynamic renegotiation and asymmetric information in financial contracting," *Journal of Financial Economics*, 2015, *116* (1), 61–81.

_ **and Amir Sufi**, "Renegotiation of financial contracts: Evidence from private credit agreements," *Journal of Financial Economics*, 2009, *93* (2), 159–184.

Schwert, Michael, "Does Borrowing from Banks Cost More than Borrowing from the Market?," *Journal of Finance*, 2019, *75* (2), 905–947.

Strömberg, Per, "Conflicts of Interest and Market Illiquidity in Bankruptcy Auctions: Theory and Tests," *Journal of Finance*, 2000, *55* (6), 2641–2692.

Appendix

A Derivations and Proofs

A.1 Proof of Lemma 1

Suppose that the firm and the bank lender agree on the proposed terms $\{\bar{B}_t, \mathcal{T}_t\}$ when renegotiating the bank debt. The equity holders receive a payment $\mathcal{T}_t$ from the bank lender, and their value becomes $V^e(Y_t, M_t, \bar{B}_t)$. If the firm and the bank lender fail to agree on the proposed terms, the equity holders' value remains at $V^e(Y_t, M_t, B_t)$. Hence, the equity holders' surplus from bargaining is given by

$$V^e(Y_t, M_t, \bar{B}_t) + \mathcal{T}_t - V^e(Y_t, M_t, B_t). \tag{A.1}$$

Similarly, the bank lender's surplus from bargaining is

$$V^b(Y_t, M_t, \bar{B}_t) - \mathcal{T}_t - V^b(Y_t, M_t, B_t). \tag{A.2}$$

The bank debt amount post-renegotiation and the payment from the bank lender to the equity holders are both determined by bilateral bargaining à la Nash, between the bank lender and the firm, where the firm has bargaining power θ. Let $\{\bar{B}_t, \mathcal{T}_t\}$ denote the outcomes of the bargaining problem. Then

$$\begin{aligned}\{\bar{B}_t, \mathcal{T}_t\} = \underset{B'_t, \mathcal{T}'_t}{\arg\max} \; & \left[V^e(Y_t, M_t, B'_t) + \mathcal{T}'_t - V^e(Y_t, M_t, B_t)\right]^{\theta} \\ & \left[V^b(Y_t, M_t, B'_t) - \mathcal{T}'_t - V^b(Y_t, M_t, B_t)\right]^{1-\theta},\end{aligned} \tag{A.3}$$

subject to $V^e(Y_t, M_t, B'_t) + \mathcal{T}'_t - V^e(Y_t, M_t, B_t) \geq 0$ and $V^b(Y_t, M_t, B'_t) - \mathcal{T}'_t - V^b(Y_t, M_t, B_t) \geq 0$. The solution to the bargaining problem is characterized by

$$\bar{B}_t = \underset{B'_t}{\arg\max} \; \left[V^e(Y_t, M_t, B'_t) + V^b(Y_t, M_t, B'_t)\right], \tag{A.4}$$

and given the post-renegotiation bank debt level $\bar{B}_t$,

$$\begin{aligned}(1-\theta)\big[V^e(Y_t,M_t,\bar{B}_t)+\mathcal{T}_t-V^e(Y_t,M_t,B_t)\big]\\ =\theta\big[V^b(Y_t,M_t,\bar{B}_t)-\mathcal{T}_t-V^b(Y_t,M_t,B_t)\big].\end{aligned} \tag{A.5}$$

Rearranging (A.5) yields

$$\mathcal{T}_t=\theta\big[V^b(Y_t,M_t,\bar{B}_t)-V^b(Y_t,M_t,B_t)\big]-(1-\theta)\big[V^e(Y_t,M_t,\bar{B}_t)-V^e(Y_t,M_t,B_t)\big], \tag{A.6}$$

where $\bar{B}_t$ is given by (A.4). Furthermore, substituting (A.4) and (A.6) into the equity holders' surplus (A.1) and the bank lender's surplus (A.2), we have

$$\begin{aligned}&V^e(Y_t,M_t,\bar{B}_t)+\mathcal{T}_t-V^e(Y_t,M_t,B_t)\\ &=\theta\big[V^e(Y_t,M_t,\bar{B}_t)+V^b(Y_t,M_t,\bar{B}_t)-V^e(Y_t,M_t,B_t)-V^b(Y_t,M_t,B_t)\big],\end{aligned} \tag{A.7}$$

and

$$\begin{aligned}&V^b(Y_t,M_t,\bar{B}_t)-\mathcal{T}_t-V^b(Y_t,M_t,B_t)\\ &=(1-\theta)\big[V^e(Y_t,M_t,\bar{B}_t)+V^b(Y_t,M_t,\bar{B}_t)-V^e(Y_t,M_t,B_t)-V^b(Y_t,M_t,B_t)\big].\end{aligned} \tag{A.8}$$

In words, the firm's equity holders capture a fraction θ of the joint surplus and the bank lender captures a fraction $1-\theta$ of the joint surplus. Let $V(Y_t,M_t,B_t)\equiv V^e(Y_t,M_t,B_t)+V^b(Y_t,M_t,B_t)$, it is immediate that

$$\begin{aligned}V^e(Y_t,M_t,\bar{B}_t)+\mathcal{T}_t&=V^e(Y_t,M_t,B_t)+\theta\big[V(Y_t,M_t,\bar{B}_t)-V(Y_t,M_t,B_t)\big],\\ V^b(Y_t,M_t,\bar{B}_t)-\mathcal{T}_t&=V^b(Y_t,M_t,B_t)+(1-\theta)\big[V(Y_t,M_t,\bar{B}_t)-V(Y_t,M_t,B_t)\big].\end{aligned} \tag{A.9}$$

I now verify that the solution $\{\bar{B}_t,\mathcal{T}_t\}$ satisfy $V^e(Y_t,M_t,\bar{B}_t)+\mathcal{T}_t-V^e(Y_t,M_t,B_t)\geq 0$ and $V^b(Y_t,M_t,\bar{B}_t)-\mathcal{T}_t-V^b(Y_t,M_t,B_t)\geq 0$. First, note that (A.4) implies that $V^e(Y_t,M_t,\bar{B}_t)+V^b(Y_t,M_t,\bar{B}_t)\geq V^e(Y_t,M_t,B_t)+V^b(Y_t,M_t,B_t)$. Substituting the inequality into (A.7) and (A.8), it is immediate that the equity holders' surplus and the bank lender's surplus are both non-negative.

A.2 Proof of Proposition 2

The equity holders' value function (2.2) can be expressed recursively through the Hamilton-Jacobi-Bellman (HJB) equation

$$\begin{aligned}(r+\lambda)V^e(Y_t, M_t, B_t) \\ = \max_{G_t^m} \Big\{ & Y_t - c(B_t + M_t) + p_t^m G_t^m + G_t^m V_M^e(Y_t, M_t, B_t) \\ & + \mu Y_t V_Y^e(Y_t, M_t, B_t) + \frac{1}{2}\sigma^2 Y_t^2 V_{YY}^e(Y_t, M_t, B_t) \\ & + \lambda\big[V^e(Y_t, M_t, \bar{B}_t) + \mathcal{T}_t\big]\Big\}.\end{aligned} \tag{A.10}$$

Taking the first-order condition of (A.10) with respect to G_t^m yields

$$p_t^m \equiv p^m(Y_t, M_t, B_t) = -V_M^e(Y_t, M_t, B_t). \tag{A.11}$$

Substituting (A.11) along with (A.9) into the HJB equation (A.10), one can rewrite the HJB equation as

$$\begin{aligned}(r+\lambda)V^e(Y_t, M_t, B_t) = & Y_t - c(B_t + M_t) \\ & + \mu Y_t V_Y^e(Y_t, M_t, B_t) + \frac{1}{2}\sigma^2 Y_t^2 V_{YY}^e(Y_t, M_t, B_t) \\ & + \lambda\Big\{V^e(Y_t, M_t, B_t) + \theta\big[V(Y_t, M_t, \bar{B}_t) - V(Y_t, M_t, B_t)\big]\Big\}.\end{aligned} \tag{A.12}$$

Differentiating both sides of (A.12) with respect to M_t yields

$$\begin{aligned}(r+\lambda)V_M^e(Y_t, M_t, B_t) \\ = & -c + \mu Y_t V_{MY}^e(Y_t, M_t, B_t) + \frac{1}{2}\sigma^2 Y_t^2 V_{MYY}^e(Y_t, M_t, B_t) \\ & + \lambda\Big\{V_M^e(Y_t, M_t, B_t) + \theta\Big[\frac{\partial V(Y_t, M_t, \bar{B}_t)}{\partial M_t} - V_M(Y_t, M_t, B_t)\Big]\Big\}.\end{aligned} \tag{A.13}$$

Note that

$$\frac{\partial V(Y_t, M_t, \bar{B}_t)}{\partial M_t} = V_M(Y_t, M_t, \bar{B}_t) + V_B(Y_t, M_t, \bar{B}_t)\frac{\partial \bar{B}_t}{\partial M_t} = V_M(Y_t, M_t, \bar{B}_t), \tag{A.14}$$

where the second equality follows the envelope theorem. Then substituting (A.14) into (A.13) yields

$$\begin{aligned}(r+\lambda)V_M^e(Y_t,M_t,B_t)&\\ &= -c+\mu Y_t V_{MY}^e(Y_t,M_t,B_t)+\frac{1}{2}\sigma^2 Y_t^2 V_{MYY}^e(Y_t,M_t,B_t)\\ &\quad+\lambda\Big\{V_M^e(Y_t,M_t,B_t)+\theta\big[V_M(Y_t,M_t,\bar{B}_t)-V_M(Y_t,M_t,B_t)\big]\Big\}.\end{aligned} \tag{A.15}$$

Moreover, from (A.11),

$$p_Y^m(Y_t,M_t,B_t) = -V_{MY}^e(Y_t,M_t,B_t), \tag{A.16}$$

$$p_{YY}^m(Y_t,M_t,B_t) = -V_{MYY}^e(Y_t,M_t,B_t). \tag{A.17}$$

Hence, substituting (A.11), (A.16)-(A.17) into (A.15) yields

$$\begin{aligned}-(r+\lambda)p^m(Y_t,M_t,B_t)&\\ &= -c-\mu Y_t p_Y^m(Y_t,M_t,B_t)-\frac{1}{2}\sigma^2 Y_t^2 p_{YY}^m(Y_t,M_t,B_t)\\ &\quad+\lambda\Big\{V_M^e(Y_t,M_t,B_t)+\theta\big[V_M(Y_t,M_t,\bar{B}_t)-V_M(Y_t,M_t,B_t)\big]\Big\}.\end{aligned} \tag{A.18}$$

Moreover from (2.4), the equilibrium market debt price $p^m(Y_t,M_t,B_t)$ satisfies the following HJB equation

$$\begin{aligned}(r+\lambda)p^m(Y_t,M_t,B_t) =& c+G_t^m p_M^m(Y_t,M_t,B_t)+\mu Y_t p_Y^m(Y_t,M_t,B_t)\\ &+\frac{1}{2}\sigma^2 Y_t^2 p_{YY}^m(Y_t,M_t,B_t)+\lambda p^m(Y_t,M_t,\bar{B}_t).\end{aligned} \tag{A.19}$$

Adding up (A.18) and (A.19) yields

$$\begin{aligned}0 &= G_t^m p_M^m(Y_t,M_t,B_t)\\ &+\lambda\Big\{p^m(Y_t,M_t,\bar{B}_t)+V_M^e(Y_t,M_t,B_t)+\theta\big[V_M(Y_t,M_t,\bar{B}_t)-V_M(Y_t,M_t,B_t)\big]\Big\}.\end{aligned} \tag{A.20}$$

Rearranging yields

$$G_t^m = \lambda\frac{p^m(Y_t,M_t,\bar{B}_t)+V_M^e(Y_t,M_t,B_t)+\theta\big[V_M(Y_t,M_t,\bar{B}_t)-V_M(Y_t,M_t,B_t)\big]}{-p_M^m(Y_t,M_t,B_t)}. \tag{A.21}$$

A.3 Discrete-Time Example

Let $\{\bar{B}, \mathcal{T}\}$ be the outcome to the bargaining problem between the bank lender and the firm when renegotiating the bank debt, then as in Appendix A.1,

$$\begin{aligned}\{\bar{B}, \mathcal{T}\} = \underset{B', \mathcal{T}'}{\arg\max} \big[V^e(M+\Delta, B') + \mathcal{T}' - V^e(M+\Delta, B)\big]^{\theta} \\ \big[V^b(M+\Delta, B') - \mathcal{T}' - V^b(M+\Delta, B)\big]^{1-\theta},\end{aligned} \tag{A.22}$$

subject to $V^e(M+\Delta, B') + \mathcal{T}' - V^e(M+\Delta, B) \geq 0$ and $V^b(M+\Delta, B') - \mathcal{T}' - V^b(M+\Delta, B) \geq 0$. Note that at the time of renegotiation, the face amount of market debt is $M+\Delta$ since the firm issues Δ amount of market debt at time t. Then the solution to the Nash bargaining problem is characterized by

$$\bar{B} = \underset{B'}{\arg\max}\ V^e(M+\Delta, B') + V^b(M+\Delta, B'), \tag{A.23}$$

$$\mathcal{T} = \theta\big[V^b(M+\Delta, \bar{B}) - V^b(M+\Delta, B)\big] - (1-\theta)\big[V^e(M+\Delta, \bar{B}) - V^e(M+\Delta, B)\big]. \tag{A.24}$$

Hence, the equity holders' renegotiation payoff is

$$V^e(M+\Delta, \bar{B}) + \mathcal{T} = V^e(M+\Delta, B) + \theta\big[V(M+\Delta, \bar{B}) - V(M+\Delta, B)\big]. \tag{A.25}$$

The condition (A.23) implies that $\bar{B}$ depends on $M+\Delta$ and thus can be expressed as $\bar{B} \equiv \bar{B}(M+\Delta)$. The firm's problem is

$$\max_{\Delta}\ V^e(M+\Delta, \bar{B}) + \mathcal{T} + \Delta p^m(M+\Delta, \bar{B}), \tag{A.26}$$

which can be rewritten as

$$\begin{aligned}\max_{\Delta}\ & V^e(M+\Delta, B) + \theta\big[V(M+\Delta, \bar{B}(M+\Delta)) - V(M+\Delta, B)\big] \\ & + \Delta p^m(M+\Delta, \bar{B}(M+\Delta)).\end{aligned} \tag{A.27}$$

Taking the first-order condition with respect to Δ and applying the envelope theorem,

$$\begin{aligned} &V_M^e(M+\Delta,B)+\theta\big[V_M(M+\Delta,\bar{B})-V_M(M+\Delta,B)\big]+p^m(M+\Delta,\bar{B}) \\ &+\Delta p_M^m(M+\Delta,\bar{B})+\Delta p_B^m(M+\Delta,\bar{B})\bar{B}'(M+\Delta)=0. \end{aligned} \tag{A.28}$$

Rearranging the first-order condition yields

$$\Delta=\frac{p^m(M+\Delta,\bar{B})+V_M^e(M+\Delta,B)+\theta\big[V_M(M+\Delta,\bar{B})-V_M(M+\Delta,B)\big]}{-p_M^m(M+\Delta,\bar{B})\left[1+\frac{p_B^m(M+\Delta,\bar{B})}{p_M^m(M+\Delta,\bar{B})}\bar{B}'(M+\Delta)\right]}. \tag{A.29}$$

A.4 Proof of Proposition 4

In the case with a strong bank lender, that is $\theta=0$, the equity holders' HJB equation (A.12) becomes

$$rV^e(Y_t,M_t,B_t)=Y_t-c(B_t+M_t)+\mu Y_tV_Y^e(Y_t,M_t,B_t)+\frac{1}{2}\sigma^2Y_t^2V_{YY}^e(Y_t,M_t,B_t). \tag{A.30}$$

That is, the equity holders' value can be solved as if the firm neither issues market debt nor renegotiates its bank debt.

Conditional on the face amount of market debt $M_t=M$ and the face amount of bank debt $B_t=B$, let $V^{e,0}(Y_t;M,B)$ denote the equity holders' no-issuance value, that is the equity holders' value when there is never any debt adjustment. Then $V^{e,0}(Y_t;M,B)$ must satisfy the following HJB equation

$$rV^{e,0}(Y_t;M,B)=Y_t-c(B+M)+\mu Y_tV^{e,0\prime}(Y_t;M,B)+\frac{1}{2}\sigma^2Y_t^2V^{e,0\prime\prime}(Y_t;M,B). \tag{A.31}$$

If the firm never defaults, then the cash flows can be evaluated as growing perpetuities. Let $\tilde{V}^{e,0}(Y_t;M,B)$ denote the equity holders' no-issuance value when there is never default, then

$$\tilde{V}^{e,0}(Y_t;M,B)=\frac{1}{r-\mu}Y_t-\frac{c(B+M)}{r}. \tag{A.32}$$

The first term on the right-hand side is the no-default present value of earnings, and the second term on the right-hand side is the no-default present value of coupon payments.

The equity holders' no-issuance value $V^{e,0}(Y_t; M, B)$ must thus equal the no-default value as given by (A.32) plus the value of a default option. That is,

$$V^{e,0}(Y_t; M, B) = \tilde{V}^{e,0}(Y_t; M, B) + \omega(Y_t; M, B)\big[0 - \tilde{V}^{e,0}(Y^\star(M, B); M, B)\big], \tag{A.33}$$

where $Y^\star(M, B)$ is the default boundary at which the equity holders choose to default and $\omega(Y_t; M, B)$ is the discount factor applied to the default option and follows the homogeneous version of the ODE (A.31). That is,

$$r\omega(Y_t; M, B) = \mu Y_t \omega'(Y_t; M, B) + \frac{1}{2}\sigma^2 Y_t^2 \omega''(Y_t; M, B). \tag{A.34}$$

The general solution to (A.34) is

$$\omega(Y_t; M, B) = K_\gamma Y_t^{-\gamma} + K_\eta Y_t^{\eta}, \tag{A.35}$$

where

$$-\gamma \equiv -\frac{\mu - \frac{1}{2}\sigma^2 + \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2\sigma^2 r}}{\sigma^2}, \tag{A.36}$$

$$\eta \equiv -\frac{\mu - \frac{1}{2}\sigma^2 - \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2\sigma^2 r}}{\sigma^2}. \tag{A.37}$$

It is easy to verify that $-\gamma < 0$ and $\eta > 1$. The constants K_γ and K_η as well as the default boundary $Y^\star(M, B)$ are determined by the boundary conditions. First, the no-bubble condition as $Y_t \to \infty$ says that $\omega(\infty; B, M) = 0$, implying that $K_\eta = 0$. The value matching condition at the default boundary requires that $\omega(Y^\star(M, B); M, B) = 1$, thus $K_\gamma = Y^\star(M, B)^\gamma$ and

$$\omega(Y_t; M, B) = \left[\frac{Y_t}{Y^\star(M, B)}\right]^{-\gamma}. \tag{A.38}$$

From (A.33), the equity holders' no-issuance value can thus be rewritten as

$$V^{e,0}(Y_t;M,B) = \frac{1}{r-\mu}Y_t - \frac{c(B+M)}{r} \underbrace{- \left[\frac{1}{r-\mu}Y^\star(M,B) - \frac{c(B+M)}{r}\right]\left[\frac{Y_t}{Y^\star(M,B)}\right]^{-\gamma}}_{\text{default option value}}. \tag{A.39}$$

The smooth pasting condition at the default boundary requires that $V^{e,0\prime}(Y_t;M,B)=0$. From (A.39), the default boundary is thus given by

$$Y^\star(M,B) = \frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}(B+M). \tag{A.40}$$

By inspecting (A.39), the default option as captured by the last term on the right-hand side is indeed maximized at the default boundary $Y^\star(M,B)$. Substituting the default boundary into (A.39) yields

$$V^{e,0}(Y_t;M,B) = \frac{1}{r-\mu}Y_t - \frac{c}{r}(B+M) + \frac{c}{r(1+\gamma)}\left[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\right]^\gamma (B+M)^{1+\gamma}Y_t^{-\gamma}. \tag{A.41}$$

Notice that the default option value as captured by the last term on the right-hand side is decreasing in Y_t. The equity holders choose to default when Y_t reaches $Y^\star(M,B)$ from the above. In equilibrium, the equity holders' value equals the equity holders' no-issuance value. That is, $V^e(Y_t,M_t,B_t) = V^{e,0}(Y_t;M_t,B_t)$. From (A.41),

$$V^{e}(Y_t,M_t,B_t) = \frac{1}{r-\mu}Y_t - \frac{c}{r}(B_t+M_t) + \frac{c}{r(1+\gamma)}\left[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\right]^\gamma (B_t+M_t)^{1+\gamma}Y_t^{-\gamma}. \tag{A.42}$$

Differentiating (A.42) with respect to M_t and B_t respectively, we get

$$\begin{aligned} V_M^e(Y_t, M_t, B_t) &= V_B^e(Y_t, M_t, B_t) \\ &= -\frac{c}{r} + \frac{c}{r}\Big[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\Big]^{\gamma}(B_t + M_t)^{\gamma} Y_t^{-\gamma} \\ &= -\frac{c}{r}\left\{1 - \Big[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\Big]^{\gamma}\Big(\frac{B_t + M_t}{Y_t}\Big)^{\gamma}\right\}. \end{aligned} \tag{A.43}$$

The equity holders choose to default when the firm's EBIT hits the default boundary (A.40) from above. Thus when the firm is not in default, $Y_t > Y^{\star}(M_t, B_t)$. That is

$$Y_t > \frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}(B_t + M_t). \tag{A.44}$$

From (A.43) and (A.44), it is immediate that

$$V_M^e(Y_t, M_t, B_t) = V_B^e(Y_t, M_t, B_t) < 0. \tag{A.45}$$

Moreover, differentiating (A.43) with respect to B_t yields

$$V_{MB}^e(Y_t, M_t, B_t) = \frac{\gamma c}{r}\Big[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\Big]^{\gamma}(B_t + M_t)^{\gamma-1} Y_t^{-\gamma} > 0. \tag{A.46}$$

A.5 Proof of Proposition 5

The equilibrium market debt price satisfies the first-order condition as given by(2.9), that is $p^m(Y_t, M_t, B_t) = -V_M^e(Y_t, M_t, B_t)$. Differentiating (A.42) with respect to M_t yields

$$V_M^e(Y_t, M_t, B_t) = -\frac{c}{r}\left\{1 - \Big[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\Big]^{\gamma}\Big(\frac{B_t + M_t}{Y_t}\Big)^{\gamma}\right\}. \tag{A.47}$$

Thus, the equilibrium market debt price is

$$p^m(Y_t, M_t, B_t) = \frac{c}{r}\left\{1 - \Big[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\Big]^{\gamma}\Big(\frac{B_t + M_t}{Y_t}\Big)^{\gamma}\right\}. \tag{A.48}$$

Differentiating (A.48) with respect to M_t yields

$$p_M^m(Y_t, M_t, B_t) = -\frac{\gamma c}{r}\Big[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\Big]^{\gamma}\frac{(B_t+M_t)^{\gamma-1}}{Y_t^{\gamma}}. \tag{A.49}$$

Note that

$$\begin{aligned} &p^m(Y_t, M_t, \bar{B}_t) + V_M^e(Y_t, M_t, B_t) \\ &= \frac{c}{r}\Big[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\Big]^{\gamma}\Big[\Big(\frac{B_t+M_t}{Y_t}\Big)^{\gamma} - \Big(\frac{\bar{B}_t+M_t}{Y_t}\Big)^{\gamma}\Big]. \end{aligned} \tag{A.50}$$

The firm's optimal market debt issuance rate G_t^m is obtained from (2.10). When $\theta = 0$, it simplifies to

$$G_t^m = \lambda\frac{p^m(Y_t, M_t, \bar{B}_t) + V_M^e(Y_t, M_t, B_t)}{-p_M^m(Y_t, M_t, B_t)}. \tag{A.51}$$

Using (A.49) and (A.50), we get

$$G_t^m \equiv G^m(Y_t, M_t, B_t) = \frac{\lambda}{\gamma}\Big[1 - \Big(\frac{\bar{B}_t+M_t}{B_t+M_t}\Big)^{\gamma}\Big](B_t+M_t). \tag{A.52}$$

Let $g_t^m \equiv G_t^m/(B_t+M_t)$ be market debt issuance intensity. From (A.52),

$$g_t^m = \frac{\lambda}{\gamma}\Big[1 - \Big(\frac{\bar{B}_t+M_t}{B_t+M_t}\Big)^{\gamma}\Big]. \tag{A.53}$$

Since $\gamma > 0$, it is immediate that the first term on the right-hand side is decreasing in B_t. To see that the second term is increasing in B_t, note that

$$\bar{B}_t = \arg\max_{B_t'}\ \big[V^e(Y_t, M_t, B_t') + V^b(Y_t, M_t, B_t')\big]. \tag{A.54}$$

It is immediate that $\bar{B}_t$ depends only on Y_t and M_t. We can thus write $\bar{B}_t = \bar{B}(Y_t, M_t)$. Then (A.53) can be rewritten as

$$g_t^m = \frac{\lambda}{\gamma}\Big[1 - \Big(\frac{\bar{B}(Y_t, M_t)+M_t}{B_t+M_t}\Big)^{\gamma}\Big]. \tag{A.55}$$

A.6 Bank Debt Value When $\theta = 0$

The bank lender's value function (2.3) can be expressed through the HJB equation

$$\begin{aligned}(r+\lambda)V^b(Y_t,M_t,B_t) =& cB_t + G_t^m V_M^b(Y_t,M_t,B_t)\\ &+ \mu Y_t V_Y^b(Y_t,M_t,B_t) + \frac{1}{2}\sigma^2 Y_t^2 V_{YY}^b(Y_t,M_t,B_t)\\ &+ \lambda\big[V^b(Y_t,M_t,\bar{B}_t) - \mathcal{T}_t\big].\end{aligned} \tag{A.56}$$

Since

$$V^b(Y_t,M_t,\bar{B}_t) - \mathcal{T}_t = V^b(Y_t,M_t,B_t) + (1-\theta)\big[V(Y_t,M_t,\bar{B}_t) - V(Y_t,M_t,B_t)\big], \tag{A.57}$$

then the bank lender's HJB equation can be written as

$$\begin{aligned}(r+\lambda)V^b(Y_t,M_t,B_t) =& cB_t + G_t^m V_M^b(Y_t,M_t,B_t)\\ &+ \mu Y_t V_Y^b(Y_t,M_t,B_t) + \frac{1}{2}\sigma^2 Y_t^2 V_{YY}^b(Y_t,M_t,B_t)\\ &+ \lambda\Big\{V^b(Y_t,M_t,B_t) + (1-\theta)\big[V(Y_t,M_t,\bar{B}_t) - V(Y_t,M_t,B_t)\big]\Big\}.\end{aligned} \tag{A.58}$$

When $\theta = 0$, the bank lender's value $V^b(Y_t,M_t,B_t)$ must satisfy the following HJB equation

$$\begin{aligned}(r+\lambda)V^b(Y_t,M_t,B_t) =& cB_t + G_t^m V_M^b(Y_t,M_t,B_t)\\ &+ \mu Y_t V_Y^b(Y_t,M_t,B_t) + \frac{1}{2}\sigma^2 Y_t^2 V_{YY}^b(Y_t,M_t,B_t)\\ &+ \lambda\big[V(Y_t,M_t,\bar{B}_t) - V^e(Y_t,M_t,B_t)\big].\end{aligned} \tag{A.59}$$

The firm is scale invariant in the setting. Thus, the bank lender's value $V^b(Y_t,M_t,B_t)$ is homogeneous of degree one. That is,

$$V^b(Y_t,M_t,B_t) = V^b\Big(\frac{Y_t}{M_t},1,\frac{B_t}{M_t}\Big)M_t \equiv v^b(y_t,b_t)M_t. \tag{A.60}$$

The joint value of the equity holders and the bank lender $V(Y_t, M_t, B_t) \equiv V^e(Y_t, M_t, B_t) + V^b(Y_t, M_t, B_t)$ is also homogeneous of degree one. That is,

$$V(Y_t, M_t, B_t) = V\Big(\frac{Y_t}{M_t}, 1, \frac{B_t}{M_t}\Big) M_t \equiv v(y_t, b_t) M_t, \tag{A.61}$$

where $v(y_t, b_t) \equiv v^e(y_t, b_t) + v^b(y_t, b_t)$ with $v^e(y_t, b_t) \equiv V^e(Y_t, M_t, B_t)/M_t$. From (2.6) of Lemma 1,

$$\bar{B}_t = \arg\max_{B'_t} \; V(Y_t, M_t, B'_t). \tag{A.62}$$

Given the amount of market debt outstanding M_t, (A.62) is equivalent to

$$\bar{b}_t \equiv \frac{\bar{B}_t}{M_t} = \arg\max_{b'_t} \; v(y_t, b'_t). \tag{A.63}$$

Here, $\bar{b}_t$ is the renegotiated face amount of the bank debt scaled by the amount of market debt outstanding, that is the post-renegotiation bank debt ratio. From (A.63), it is easy to see that $\bar{b}_t$ is a function of the firm's EBIT-to-market-debt ratio y_t, that is $\bar{b}_t \equiv \bar{b}(y_t)$. Note that

$$V(Y_t, M_t, \bar{B}_t) = v(y_t, \bar{b}_t) M_t, \tag{A.64}$$

where $\bar{b}_t$ is given by (A.63). Thus, from (A.63) and (A.64),

$$V(Y_t, M_t, \bar{B}_t) = \max_{b'_t} \; v(y_t, b'_t) M_t. \tag{A.65}$$

From (A.60),

$$V^b_M(Y_t, M_t, B_t) = v^b(y_t, b_t) - y_t v^b_y(y_t, b_t) - b_t v^b_b(y_t, b_t), \tag{A.66}$$

$$V^b_Y(Y_t, M_t, B_t) = v^b_y(y_t, b_t), \tag{A.67}$$

$$V^b_{YY}(Y_t, M_t, B_t) = \frac{1}{M_t} v^b_{yy}(y_t, b_t). \tag{A.68}$$

Substituting (A.60), (A.61), and (A.65)-(A.68) into the bank lender's HJB equation (A.59), we obtain the following HJB equation for the case with $\theta = 0$ as

$$\begin{aligned}(r+\lambda)v^b(y_t,b_t)M_t =& cB_t + G_t^m\big[v^b(y_t,b_t) - y_t v_y^b(y_t,b_t) - b_t v_b^b(y_t,b_t)\big] \\ &+ \mu Y_t v_y^b(y_t,b_t) + \frac{1}{2}\sigma^2\frac{Y_t^2}{M_t}v_{yy}^b(y_t,b_t) \\ &+ \lambda\big[\max_{b_t'} v(y_t,b_t') - v^e(y_t,b_t)\big]M_t.\end{aligned} \tag{A.69}$$

Note that

$$\frac{G_t^m}{M_t} = \frac{G_t^m}{B_t+M_t}\frac{B_t+M_t}{M_t} = g_t^m(1+b_t). \tag{A.70}$$

Then dividing both sides of (A.69) by M_t and rearranging, we get

$$\begin{aligned}\big[r+\lambda-g_t^m(1+b_t)\big]v^b(y_t,b_t) =& cb_t - g_t^m b_t(1+b_t)v_b^b(y_t,b_t) + \big[\mu - g_t^m(1+b_t)\big]y_t v_y^b(y_t,b_t) \\ &+ \frac{1}{2}\sigma^2 y_t^2 v_{yy}^b(y_t,b_t) + \lambda\big[\max_{b_t'} v(y_t,b_t') - v^e(y_t,b_t)\big],\end{aligned} \tag{A.71}$$

where $v_b^b(y_t,b_t)$ denotes the first-order partial derivative with respect to b_t, while $v_y^b(y_t,b_t)$ and $v_{yy}^b(y_t,b_t)$ are the first-order and the second-order partial derivatives with respect to y_t.

The market debt issuance intensity g_t^m is given by (3.4), which can be rewritten as

$$g_t^m \equiv g^m(y_t,b_t) = \frac{\lambda}{\gamma}\Big\{1 - \Big(\frac{1+\bar{b}_t}{1+b_t}\Big)^\gamma\Big\}, \tag{A.72}$$

where $\bar{b}_t$ satisfies (A.63). Moreover, $v(y_t,b_t) \equiv v^e(y_t,b_t) + v^b(y_t,b_t)$. From (3.1), we have

$$\begin{aligned}v^e(y_t,b_t) \equiv& \frac{V^e(Y_t,M_t,B_t)}{M_t} \\ =& \frac{1}{r-\mu}y_t - \frac{c}{r}(1+b_t) + \frac{c}{r(1+\gamma)}(1+b_t)\Big[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\Big]^\gamma\Big(\frac{y_t}{1+b_t}\Big)^{-\gamma}.\end{aligned} \tag{A.73}$$

Note that the two state variables are $y_t \equiv Y_t/M_t$ and $b_t \equiv B_t/M_t$. One can transform the state space such that the model is solved in terms of $\tilde{y}_t \equiv [Y_t - Y^\star(M_t,B_t)]/M_t = y_t - y^\star(b_t)$ and b_t. In the transformed state space, the default boundary is $\tilde{y}^\star = 0$. That is, the equity holders choose to default

when the distance to default approaches zero from above. From (A.72), we can rewrite g_t^m as

$$g^m(\tilde{y}_t, b_t) = \frac{\lambda}{\gamma}\Big\{1 - \Big(\frac{1+\bar{b}_t}{1+b_t}\Big)^{\gamma}\Big\}, \tag{A.74}$$

where $\bar{b}_t$ is solution to (A.63), and $y^\star(b_t)$ can be obtained from (3.3), that is

$$y^\star(b_t) = \frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}(1+b_t). \tag{A.75}$$

Similarly, from (A.73), the equity value can be written as

$$\begin{aligned} v^e(\tilde{y}_t, b_t) =& \frac{1}{r-\mu}[\tilde{y}_t + y^\star(b_t)] - \frac{c}{r}(1+b_t) \\ &+ \frac{c}{r(1+\gamma)}(1+b_t)\Big[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\Big]^{\gamma}\Big[\frac{\tilde{y}_t + y^\star(b_t)}{1+b_t}\Big]^{-\gamma}. \end{aligned} \tag{A.76}$$

Note that

$$\frac{\partial v^b}{\partial b_t} = v_b^b(\tilde{y}_t, b_t) - v_{\tilde{y}}^b(\tilde{y}_t, b_t)y^{\star\prime}(b_t), \tag{A.77}$$

$$\frac{\partial v^b}{\partial y_t} = v_{\tilde{y}}^b(\tilde{y}_t, b_t), \tag{A.78}$$

$$\frac{\partial^2 v^b}{\partial y_t^2} = v_{\tilde{y}\tilde{y}}^b(\tilde{y}_t, b_t). \tag{A.79}$$

Substituting (A.77)-(A.79) into (A.71) yields

$$\begin{aligned} \big[r + \lambda - g_t^m(1+b_t)\big]v^b(\tilde{y}_t, b_t) =& cb_t - g_t^m b_t(1+b_t)\big[v_b^b(\tilde{y}_t, b_t) - v_{\tilde{y}}^b(\tilde{y}_t, b_t)y^{\star\prime}(b_t)\big] \\ &+ \big[\mu - g_t^m(1+b_t)\big]\big[\tilde{y}_t + y^\star(b_t)\big]v_{\tilde{y}}^b(\tilde{y}_t, b_t) + \frac{1}{2}\sigma^2\big[\tilde{y}_t + y^\star(b_t)\big]^2 v_{\tilde{y}\tilde{y}}^b(\tilde{y}_t, b_t) \\ &+ \lambda\big[\bar{v}(\tilde{y}_t, b_t) - v^e(\tilde{y}_t, b_t)\big], \end{aligned} \tag{A.80}$$

where

$$\bar{v}(\tilde{y}_t, b_t) = \max_{b_t'} v\big(\tilde{y}_t + y^\star(b_t) - y^\star(b_t'), b_t'\big). \tag{A.81}$$

Rearranging (A.80) yields

$$\begin{aligned}\big[r+\lambda-g_t^m(1+b_t)\big]v^b(\tilde{y}_t,b_t) &= cb_t - g_t^m b_t(1+b_t)v_b^b(\tilde{y}_t,b_t)\\ &+\big\{\big[\mu-g_t^m(1+b_t)\big]\big[\tilde{y}_t+y^\star(b_t)\big]+g_t^m b_t(1+b_t)y^{\star\prime}(b_t)\big\}v_{\tilde{y}}^b(\tilde{y}_t,b_t)\\ &+\frac{1}{2}\sigma^2\big[\tilde{y}_t+y^\star(b_t)\big]^2 v_{\tilde{y}\tilde{y}}^b(\tilde{y}_t,b_t)+\lambda\big[\bar{v}(\tilde{y}_t,b_t)-v^e(\tilde{y}_t,b_t)\big].\end{aligned} \tag{A.82}$$

The boundary value of the bank debt depends on its seniority. If the bank debt is senior to the market debt, the bank lender's recovery value upon default is $\min\{\alpha Y^\star, B_{\tau^\star}\}$. Let $v^{b,\star}$ denote the bank debt value at default, then

$$v^{b,\star}=\frac{\min\{\alpha Y^\star,B_{\tau^\star}\}}{M_{\tau^\star}}=\min\{\alpha y^\star(b_{\tau^\star}),b_{\tau^\star}\}. \tag{A.83}$$

If the bank debt is pari passu with the market debt or junior to the market debt, dilution ahead of default pushes the boundary value of the bank debt to zero.

A.7 Proof of Proposition 8

Define

$$\mathcal{K}(B_t',B_t,M_t)\equiv\frac{1}{\gamma}\Big[1-\Big(\frac{B_t'+M_t}{B_t+M_t}\Big)^\gamma\Big](B_t+M_t). \tag{A.84}$$

Thus from (3.4), the market debt issuance rate is thus

$$G_t^m=\lambda\mathcal{K}(\bar{B}(Y_t,M_t),B_t,M_t). \tag{A.85}$$

Define $\varepsilon\equiv 1/\lambda$, so $\lim_{\lambda\to\infty}\varepsilon=0$. I then use the singular perturbation method. Expanding $\bar{B}_t$ in powers of ε,

$$\bar{B}_t=B_t+\varepsilon\bar{B}^{(1)}+O(\varepsilon^2), \tag{A.86}$$

where $\bar{B}^{(0)}$ is the leading-order post-renegotiation bank debt, and $\bar{B}^{(1)}$ is the first-order correction. From the bank lender's PDE (A.59), it is immediate that $\lim_{\lambda\to\infty}\bar{B}_t \to B_t$. Then

$$\mathcal{K}(B'_t, B_t, M_t) = \frac{1}{\gamma}\left[1 - \left(1 + \gamma\frac{B'_t - B_t}{B_t + M_t}\right)\right](B_t + M_t) = B_t - B'_t = \varepsilon(-\bar{B}^{(1)}) + O(\varepsilon^2). \tag{A.87}$$

Substituting into (A.85) yields

$$G^m_t = -\bar{B}^{(1)} + O(\varepsilon). \tag{A.88}$$

The bank lender's PDE (A.59) can be rewritten as

$$\begin{aligned}(r+\lambda)V^b(Y_t, M_t, B_t) =& cB_t + \lambda\mathcal{K}(\bar{B}_t, B_t, M_t)\partial_M V^b(Y_t, M_t, B_t) + \mathcal{L}_Y V^b(Y_t, M_t, B_t)\\ &+ \lambda\big[V(Y_t, M_t, \bar{B}_t) - V^e(Y_t, M_t, B_t)\big],\end{aligned} \tag{A.89}$$

where the operator $\mathcal{L}_Y = \mu Y_t\partial_Y + \frac{1}{2}\sigma^2 Y_t^2\partial_{YY}$. Dividing both sides by λ and using $\varepsilon \equiv 1/\lambda$, the PDE becomes

$$\begin{aligned}(r\varepsilon+1)V^b(Y_t, M_t, B_t) =& \varepsilon cB_t + \mathcal{K}(\bar{B}_t, B_t, M_t)\partial_M V^b(Y_t, M_t, B_t) + \varepsilon\mathcal{L}_Y V^b(Y_t, M_t, B_t)\\ &+ \big[V(Y_t, M_t, \bar{B}_t) - V^e(Y_t, M_t, B_t)\big].\end{aligned} \tag{A.90}$$

Expanding $V^b(Y_t, M_t, B_t)$ in powers of ε,

$$V^b(Y_t, M_t, B_t) = V^{b,(0)}(Y_t, M_t, B_t) + \varepsilon V^{b,(1)}(Y_t, M_t, B_t) + O(\varepsilon^2). \tag{A.91}$$

Then

$$V(Y_t, M_t, \bar{B}_t) = V(Y_t, M_t, B_t) + \varepsilon\bar{B}^{(1)}\partial_B V(Y_t, M_t, B_t) + O(\varepsilon^2). \tag{A.92}$$

Using (A.86), (A.87), (A.91) and (A.92), one can collect $O(\varepsilon)$ terms,

$$\begin{aligned}rV^{b,(0)}(Y_t, M_t, B_t) + V^{b,(1)}(Y_t, M_t, B_t) =& cB_t - \bar{B}^{(1)}\partial_M V^{b,(0)}(Y_t, M_t, B_t) + \mathcal{L}_Y V^{b,(0)}(Y_t, M_t, B_t)\\ &+ \bar{B}^{(1)}\partial_B V(Y_t, M_t, B_t).\end{aligned} \tag{A.93}$$

Rearranging yields

$$\bar{B}^{(1)} = \frac{cB_t - V^{b,(1)}(Y_t, M_t, B_t) - rV^{b,(0)}(Y_t, M_t, B_t) + \mathcal{L}_Y V^{b,(0)}(Y_t, M_t, B_t)}{\partial_M V^{b,(0)}(Y_t, M_t, B_t) - \partial_B V(Y_t, M_t, B_t)}. \tag{A.94}$$

A.8 Proof of Proposition 9

Given Y_t and M_t, the joint value between the bank lender and the equity holders $V(Y_t, M_t, B_t)$ is maximized at $\bar{B}(Y_t, M_t)$. Thus,

$$V_B(Y_t, M_t, B_t) \begin{cases} > 0 & \text{if } B_t < \bar{B}(Y_t, M_t) \\ < 0 & \text{if } B_t > \bar{B}(Y_t, M_t) \end{cases}. \tag{A.95}$$

It is immediate that $V(Y_t, M_t, \bar{B}(Y_t, M_t)) = V(Y_t, M_t, B_t)$ when $B_t = \bar{B}(Y_t, M_t)$. Let $\mathcal{S}(Y_t, M_t, B_t) \equiv V(Y_t, M_t, \bar{B}(Y_t, M_t)) - V(Y_t, M_t, B_t)$ denote the joint bargaining surplus between the bank lender and the firm's equity holders, it is thus obvious that $\mathcal{S}(Y_t, M_t, B_t) = 0$ if $B_t = \bar{B}(Y_t, M_t)$.

When $B_t > \bar{B}(Y_t, M_t)$, then (A.95) implies that the joint value $V(Y_t, M_t, B_t)$ is decreasing in B_t for given Y_t and M_t, and thus $V(Y_t, M_t, B_t) < V(Y_t, M_t, \bar{B}(Y_t, M_t))$. This implies that $\mathcal{S}(Y_t, M_t, B_t) > 0$. Moreover, increasing $\bar{B}(Y_t, M_t)$ decreases $V(Y_t, M_t, \bar{B}(Y_t, M_t))$, leading to lower joint surplus $\mathcal{S}(Y_t, M_t, B_t)$. Similarly, when $B_t < \bar{B}(Y_t, M_t)$, then (A.95) implies that the joint value $V(Y_t, M_t, B_t)$ is increasing in B_t for given Y_t and M_t, and thus $V(Y_t, M_t, B_t) < V(Y_t, M_t, \bar{B}(Y_t, M_t))$ so the joint surplus $\mathcal{S}(Y_t, M_t, B_t) > 0$. Moreover, increasing $\bar{B}(Y_t, M_t)$ increases $V(Y_t, M_t, \bar{B}(Y_t, M_t))$, resulting in higher joint surplus $\mathcal{S}(Y_t, M_t, B_t)$.

A.9 General Case with $\theta > 0$

When $\theta > 0$, the equity holders' value $V^e(Y_t, M_t, B_t)$ satisfies the HJB equation

$$\begin{aligned}(r+\lambda)V^e(Y_t, M_t, B_t) = \max_{G_t^m} \Big\{ & Y_t - c(B_t + M_t) + p_t^m G_t^m + G_t^m V_M^e(Y_t, M_t, B_t) \\ & + \mu Y_t V_Y^e(Y_t, M_t, B_t) + \frac{1}{2}\sigma^2 Y_t^2 V_{YY}^e(Y_t, M_t, B_t) \\ & + \lambda\big[V^e(Y_t, M_t, \bar{B}_t) + \mathcal{T}_t\big]\Big\}.\end{aligned} \tag{A.96}$$

Substituting the conditions (2.7) and (2.9) into the HJB equation yields

$$\begin{aligned}(r+\theta\lambda)V^e(Y_t,M_t,B_t) =& Y_t - c(B_t+M_t)\\ &+\mu Y_t V^e_Y(Y_t,M_t,B_t)+\frac{1}{2}\sigma^2 Y_t^2 V^e_{YY}(Y_t,M_t,B_t)\\ &+\theta\lambda\big[V(Y_t,M_t,\bar{B}_t)-V^b(Y_t,M_t,B_t)\big].\end{aligned} \tag{A.97}$$

The equity holders' value is homogeneous of degree one, that is

$$V^e(Y_t,M_t,B_t)=V^e\Big(\frac{Y_t}{M_t},1,\frac{B_t}{M_t}\Big)M_t\equiv v^e(y_t,b_t)M_t. \tag{A.98}$$

Similarly,

$$V(Y_t,M_t,B_t)=V\Big(\frac{Y_t}{M_t},1,\frac{B_t}{M_t}\Big)M_t\equiv v(y_t,b_t)M_t. \tag{A.99}$$

Then (A.97) can be rewritten as

$$\begin{aligned}(r+\theta\lambda)v^e(y_t,b_t) =& y_t - c(1+b_t)+\mu y_t v^e_y(y_t,b_t)+\frac{1}{2}\sigma^2 y_t^2 v^e_{yy}(y_t,b_t)\\ &+\theta\lambda\big[\max_{b'_t} v(y_t,b'_t)-v^b(y_t,b_t)\big].\end{aligned} \tag{A.100}$$

Let $y^\star(b_t)$ denote the default boundary, that is the EBIT-to-market-debt ratio at which the equity holders choose to default. It is a function of the bank debt ratio b_t. The value matching condition and the smooth pasting condition require that

$$v^e(y^\star(b_t),b_t)=0, \tag{A.101}$$

$$v^e_y(y^\star(b_t),b_t)=0. \tag{A.102}$$

To facilitate numerical solution, I apply the following transformation to the variable y_t by letting $\tilde{y}_t\equiv y_t-y^\star(b_t)$. Then for all b_t, the equity holders choose to default when $\tilde{y}_t$ is zero. Then,

$$\begin{aligned}(r+\theta\lambda)v^e(\tilde{y}_t,b_t) =& \tilde{y}_t+y^\star(b_t)-c(1+b_t)+\theta\lambda\big[\bar{v}(\tilde{y}_t,b_t)-v^b(\tilde{y}_t,b_t)\big]\\ &+\mu\big[\tilde{y}_t+y^\star(b_t)\big]v^e_{\tilde{y}}(\tilde{y}_t,b_t)+\frac{1}{2}\sigma^2\big[\tilde{y}_t+y^\star(b_t)\big]^2 v^e_{\tilde{y}\tilde{y}}(\tilde{y}_t,b_t),\end{aligned} \tag{A.103}$$

where

$$\bar{v}(\tilde{y}_t, b_t) \equiv \max_{b'_t} \; v(y_t, b'_t). \tag{A.104}$$

The bank lender's value $V^b(Y_t, M_t, B_t)$ satisfies the HJB equation

$$\begin{aligned}\big[r + (1-\theta)\lambda\big] V^b(Y_t, M_t, B_t) =& cB_t + G^m_t V^b_M(Y_t, M_t, B_t) \\ &+ \mu Y_t V^b_Y(Y_t, M_t, B_t) + \frac{1}{2}\sigma^2 Y_t^2 V^b_{YY}(Y_t, M_t, B_t) \\ &+ (1-\theta)\lambda\big[V(Y_t, M_t, \bar{B}_t) - V^e(Y_t, M_t, B_t)\big].\end{aligned} \tag{A.105}$$

Since $V^b(Y_t, M_t, B_t)$ is homogeneous of degree one, the HJB equation can thus be rewritten as

$$\begin{aligned}\big[r + (1-\theta)\lambda - g^m_t\big] v^b(y_t, b_t) =& cb_t - g^m_t b_t v^b_b(y_t, b_t) \\ &+ (\mu - g^m_t) y_t v^b_y(y_t, b_t) + \frac{1}{2}\sigma^2 y_t^2 v^b_{yy}(y_t, b_t) \\ &+ (1-\theta)\lambda\big[\max_{b'_t} \; v(y_t, b'_t) - v^e(y_t, b_t)\big].\end{aligned} \tag{A.106}$$

Again, one can solve the differential equation in terms of $\tilde{y}_t \equiv y_t - y^\star(b_t)$ and b_t, and thus

$$\begin{aligned}\big[r + (1-\theta)\lambda - g^m_t\big] v^b(\tilde{y}_t, b_t) =& cb_t - g^m_t b_t v^b_b(\tilde{y}_t, b_t) + (1-\theta)\lambda\big[\bar{v}(\tilde{y}_t, b_t) - v^e(\tilde{y}_t, b_t)\big] \\ &+ \big\{(\mu - g^m_t)[\tilde{y}_t + y^\star(b_t)] + g^m_t b_t y^{\star\prime}(b_t)\big\} v^b_{\tilde{y}}(\tilde{y}_t, b_t) \\ &+ \frac{1}{2}\sigma^2\big[\tilde{y}_t + y^\star(b_t)\big]^2 v^b_{\tilde{y}\tilde{y}}(\tilde{y}_t, b_t).\end{aligned} \tag{A.107}$$

B Comparative Statics

In this appendix, I analyze how changes in various parameters affect the degree of dynamic complementarity between bank debt and market debt, as well as the total enterprise value. The degree of dynamic complementarity is defined as the derivative of the firm's market debt issuance intensity with respect to the amount of bank debt outstanding, $\partial g^m(Y_t, M_t, B_t)/\partial B_t$. The total enterprise value is defined as the firm value scaled by the amount of market debt outstanding, or the sum of the joint equity and bank debt value and the market debt price. The baseline parameter values are the same as in the main analysis: $\mu = 0.02$, $\sigma = 0.25$, $r = c = 0.05$, $\lambda = 0.2$, $\alpha = 2.33$, and $\theta = 0$. I further normalize the amount of market debt to one, and evaluate the comparative statics for bank debt ratio of $b_t = 0.1$ and distance to default $\tilde{y}_t = 0.03$.

B.1 Liquidation Value Multiple α

In Figure B.1, I plot the effects of the liquidation value multiple α on the degree of dynamic complementarity between bank and market debt, as well as the total enterprise value. α captures the cost of default, and a lower α means that there is a higher deadweight cost of default. The figure shows that both the degree of dynamic complementarity between bank and market debt and the total enterprise value are increasing in α. In words, when the firm has a lower cost of default, its total firm value is higher and the dynamic complementarity between bank debt and market debt is also stronger.

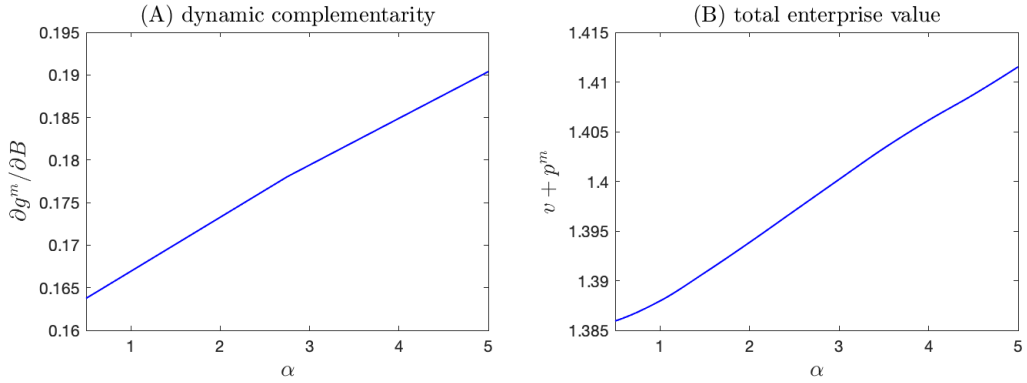


Figure B.1: Comparative statics with respect to liquidation value multiple α. Panel A of the figure plots the degree of dynamic complementarity between bank debt and market debt ($\partial g^m(Y_t, M_t, B_t)/\partial B_t$) against the liquidation value multiple α. Panel B plots the total enterprise value $v + p^m$ against α. The parameter values are $\mu = 0.02$, $\sigma = 0.25$, $r = c = 0.05$, $\lambda = 0.2$ and $\theta = 0$. I further normalize the amount of market debt to one, and evaluate the comparative statics for bank debt ratio of $b_t = 0.1$ and distance to default $\tilde{y}_t = 0.03$.

B.2 Discount Rate r

In Figure B.2, I plot the effects of the agents' discount rate r on the degree of dynamic complementarity between bank and market debt, as well as the total enterprise value. The figure shows that the degree of dynamic complementarity between bank debt and market debt is increasing in the discount rate r, while the total enterprise value is decreasing in r. In words, when the agents are less patient and discount time at a higher rate, the dynamic complementarity of bank and market debt is stronger while the total firm value is lower.

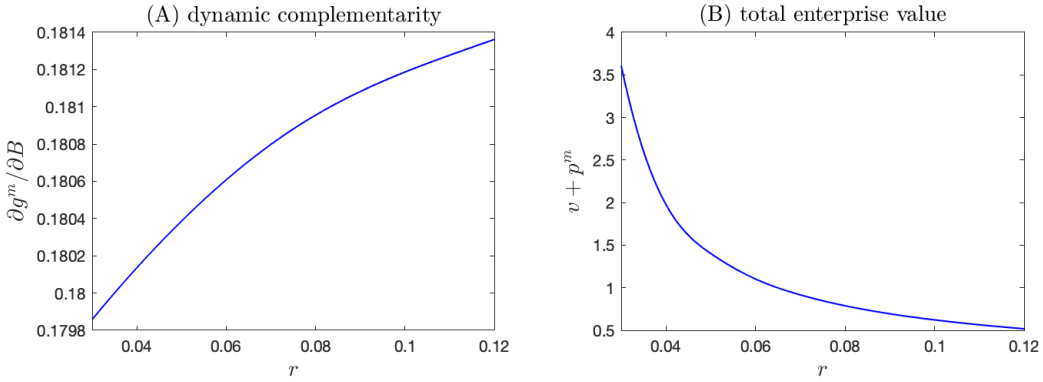


Figure B.2: Comparative statics with respect to discount rate r. Panel A of the figure plots the degree of dynamic complementarity between bank debt and market debt ($\partial g^m(Y_t, M_t, B_t)/\partial B_t$) against the agents' discount rate r. Panel B plots the total enterprise value $v + p^m$ against r. The parameter values are $\mu = 0.02$, $\sigma = 0.25$, $c = 0.05$, $\lambda = 0.2$, $\alpha = 2.33$ and $\theta = 0$. I further normalize the amount of market debt to one, and evaluate the comparative statics for bank debt ratio of $b_t = 0.1$ and distance to default $\tilde{y}_t = 0.03$.

B.3 Earnings Growth μ

In Figure B.3, I plot the effects of the firm's earnings growth, captured by the drift term μ of its earnings process, on the degree of dynamic complementarity between bank debt and market debt, as well as the total enterprise value. The figure shows that the degree of dynamic complementarity between bank debt and market debt is decreasing in the firm's earnings growth μ, while the total enterprise value is increasing in μ. That is, when the firm has higher growth prospect, the dynamic complementarity of bank debt and market debt is weaker while the total firm value is higher.

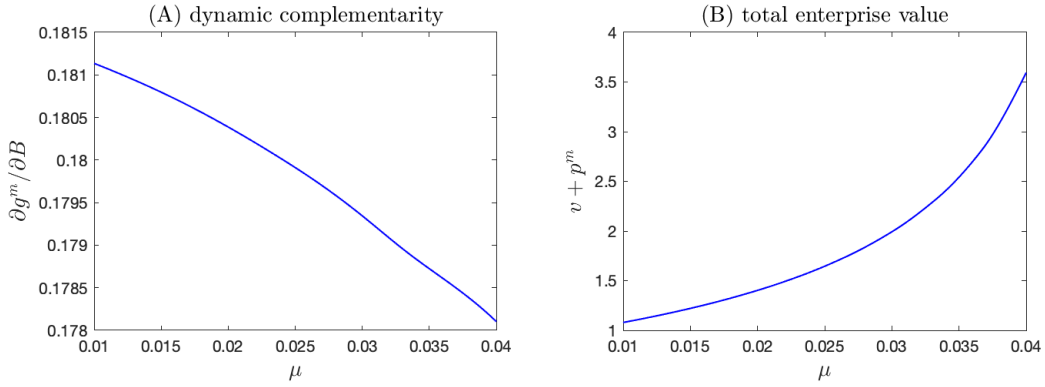


Figure B.3: Comparative statics with respect to earnings growth μ. Panel A of the figure plots the degree of dynamic complementarity between bank debt and market debt $(\partial g^m(Y_t, M_t, B_t)/\partial B_t)$ against the earnings growth μ. Panel B plots the total enterprise value $v + p^m$ against μ. The parameter values are $\sigma = 0.25$, $r = c = 0.05$, $\lambda = 0.2$, $\alpha = 2.33$ and $\theta = 0$. I further normalize the amount of market debt to one, and evaluate the comparative statics for bank debt ratio of $b_t = 0.1$ and distance to default $\tilde{y}_t = 0.03$.

B.4 Tax Rate π

Finally, I consider the effects of the corporate tax rate π, which gives rise to tax benefit of debt. In Figure B.4, I plot the effects of the tax rate π on the degree of dynamic complementarity between bank debt and market debt, as well as the total enterprise value. Both are decreasing in π. That is, when the tax rate is higher, the degree of dynamic complementarity between bank and market debt is weaker, and the total firm value is lower, as higher tax rate amplifies the commitment problem associated with market debt issuance.

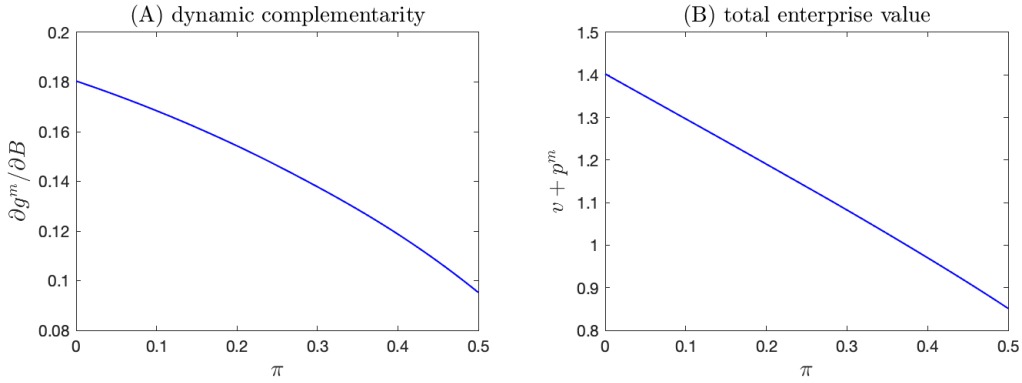


Figure B.4: Comparative statics with respect to corporate tax rate π. Panel A of the figure plots the degree of dynamic complementarity between bank debt and market debt $(\partial g^m(Y_t, M_t, B_t)/\partial B_t)$ against the tax rate π. Panel B plots the total enterprise value $v + p^m$ against π. The parameter values are $\mu = 0.02$, $\sigma = 0.25$, $r = c(1 - \pi) = 0.05$, $\lambda = 0.2$, $\alpha = 2.33$ and $\theta = 0$. I further normalize the amount of market debt to one, and evaluate the comparative statics for bank debt ratio of $b_t = 0.1$ and distance to default $\tilde{y}_t = 0.05$.

C Model with Benefits of Debt

The setting of the main model is rather stark, as it shuts down any benefit of debt. As discussed in Section 2.1, this assumption creates a conceptual inconsistency by leaving unexplained why the firm issues debt in the first place. In this Appendix, I introduce benefits of debt into the model. In particular, I assume that the firm pays corporate taxes on its earnings after interest at a constant rate $\pi \in (0, 1)$. The benefits of debt thus arise from the tax deductibility of interest expenses. More broadly, other forms of debt benefits can be accommodated so long as there is a valuation wedge between debt investors and the equity holders. I will henceforth include the debt tax shield in the remainder of the appendices.

C.1 Market Debt Issuance

The equilibrium market debt price $p^m(Y_t, M_t, B_t)$ is pinned down by the equity holders' first-order condition with respect to market debt issuance, given by (2.9). Using this pricing condition, one can obtain the equity holders' optimal market debt issuance rate G_t^m. Following derivations in Section C.3,

$$\begin{aligned} G_t^m =& \frac{\pi c}{-p_M^m(Y_t, M_t, B_t)} \\ &+ \lambda \frac{p^m(Y_t, M_t, \bar{B}_t) + V_M^e(Y_t, M_t, B_t) + \theta\big[V_M(Y_t, M_t, \bar{B}_t) - V_M(Y_t, M_t, B_t)\big]}{-p_M^m(Y_t, M_t, B_t)}. \end{aligned} \tag{C.1}$$

Compared to (2.10), the difference lies in the first term on the right-hand side of (C.1). This term corresponds to the market debt issuance rate if the bank lender and the firm never renegotiate the bank debt once the bank debt is in place, that is $\lambda = 0$. This is the same expression that appears in DeMarzo and He (2021). Absent bank debt renegotiations, the firm issues market debt at a rate such that the tax benefit of market debt issuance is fully dissipated by the negative valuation effect on the market debt investors due to an increase in leverage.

C.2 Solution with a Strong Bank

When the bank lender has all the bargaining power (i.e., $\theta = 0$), it captures the entire joint surplus from renegotiation, leaving the equity holders with only their outside option - namely their continuation

value. Moreover, the equity holders do not gain from market debt issuance in equilibrium, as any potential tax benefit is fully dissipated by the adverse valuation impact of increased leverage. Consequently, the equity holders' value can be solved as if the firm neither issues market debt nor renegotiates its bank debt.

With levels of bank debt and market debt fixed over time, the only state variable is the firm's EBIT Y_t. The equity holders' value is then characterized by an ODE, subject to the no-bubble condition as $Y_t \to \infty$ as well as the value matching and smooth-pasting conditions at the endogenous default boundary $Y^\star(M_t, B_t)$. As shown in Section C.3, in the case with $\theta = 0$, the equity holders' value is given by

$$\begin{aligned} V^e(Y_t, M_t, B_t) =& \frac{1-\pi}{r-\mu} Y_t - \frac{(1-\pi)c}{r}(B_t + M_t) \\ &+ \frac{1}{1+\gamma}\frac{(1-\pi)c}{r}\Big[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\Big]^\gamma (B_t + M_t)^{1+\gamma} Y_t^{-\gamma}, \end{aligned} \tag{C.2}$$

where γ is a positive constant given in Section C.3 and the default boundary is

$$Y^\star(M_t, B_t) = \frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}(B_t + M_t). \tag{C.3}$$

A. Market Debt Issuance Intensity

As in the main model, I define $g^m(Y_t, M_t, B_t) \equiv G^m(Y_t, M_t, B_t)/(B_t + M_t)$, which is the market debt issuance intensity. Then

$$g^m(Y_t, M_t, B_t) = \frac{\pi c}{\gamma}\frac{r}{(1-\pi)c}\Big[\frac{1+\gamma}{\gamma}\frac{r}{(r-\mu)c}\frac{Y_t}{B_t + M_t}\Big]^\gamma + \frac{\lambda}{\gamma}\Big[1 - \Big(\frac{\bar{B}(Y_t, M_t) + M_t}{B_t + M_t}\Big)^\gamma\Big]. \tag{C.4}$$

The second component of the market debt issuance intensity is the same as the market debt issuance intensity in the model. It reflects the part of the market debt issuance policy driven by the firm's strategic incentive to position for future bank debt renegotiation.

The first component of (C.4) is positive. However, just as in the main model, the second component may be positive or negative. As a result, if the second component is large enough in magnitude, the overall market debt issuance intensity is negative. In other words, just as in the main model, the firm may both issue and repurchase its market debt when benefits of debt are incorporated into the model. Similarly, the first component of (C.4) is strictly decreasing in the amount of bank debt outstanding

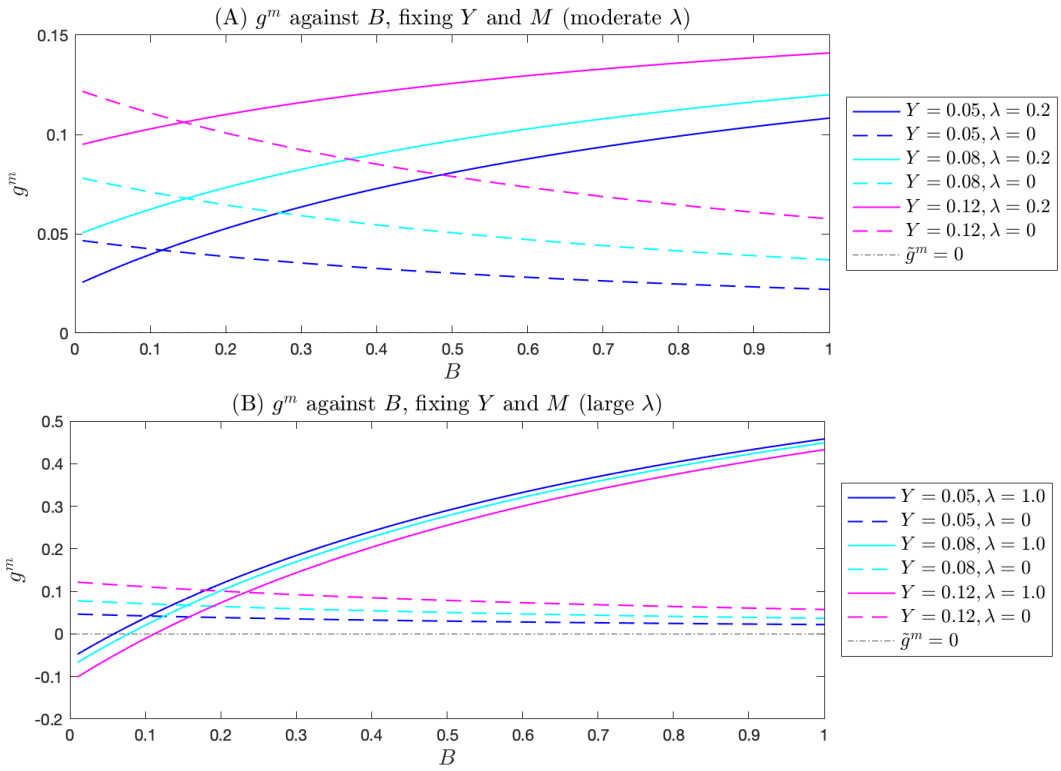


Figure C.1: Market debt issuance intensity $g^m(Y_t, M_t, B_t)$. The figure plots the market debt issuance intensity as a function of the face amount of bank debt B_t, holding the firm's EBIT Y_t and market debt outstanding M_t constant. Specifically, I plot the relationship between the market debt issuance intensity and the bank debt outstanding for $Y_t = 0.05$, 0.08, and 0.12 while normalizing the amount of market debt outstanding to one. Panel A plots this relationship for $\lambda = 0.2$, while Panel B plots this relationship for $\lambda = 1$. The other parameters take the values: $\mu = 0.02$, $\sigma = 0.25$, $\pi = 0.3$, $r = (1 - \pi)c = 0.05$, and $\alpha = 2.33$.

B_t, while the second component is increasing in B_t. Hence, if the second component is large enough in magnitude, the overall market debt issuance intensity is increasing in the amount of bank debt, suggesting a dynamic complementarity between bank debt and market debt.

In Figure C.1, I plot the market debt issuance intensity as a function of the bank debt level B_t using the following parameter values: $\mu = 0.02$, $\sigma = 0.25$, $\pi = 0.3$, $r = c(1 - \pi) = 0.05$, $\alpha = 2.33$, and $\lambda = 0.2$ (Panel A) and 1 (Panel B). I normalize the face amount of market debt to one, and plot the market debt issuance intensity for various levels of EBIT. For comparison, the dashed lines represent $\lambda = 0$ which is the case without bank debt renegotiation such that the second (strategic) component of the market debt issuance intensity drops out.

The dashed lines in Figure C.1 all slope downward, implying that more bank debt is associated with lower market debt issuance intensity absent strategic bank-firm renegotiations of the bank debt.

With periodic bank debt renegotiations, the market debt issuance intensity may increase or decrease with the amount of bank debt, depending on λ. That being said, the figure shows that when the bank debt is renegotiated relatively frequently, the market debt issuance intensity increases with the bank debt level. That is, the bank debt and the market debt are dynamically complementary and the firm issues market debt more aggressively when it has more bank debt in its debt structure. Furthermore, the solid lines in Panel A are flatter than those in Panel B, indicating that the dynamic complementarity between bank debt and market debt is more pronounced as λ increases. Intuitively, more frequent bank debt renegotiations increase the firm's strategic incentive to use market debt issuance to position for renegotiations.

Moreover, Panel B of Figure C.1 shows that the market debt issuance intensity can be negative. That is, in contrast to the leverage ratchet effect, the firm in this model may voluntarily repurchase market debt if the strategic motive is sufficiently strong.

B. Seniority of Bank Debt

As in the main model, I rely on the scale invariance of the setting and reduce the dimensionality of the state space. I then solve numerically for the bank debt value $v^b(y_t, b_t) \equiv V^b(Y_t, M_t, B_t)/M_t$ where $y_t \equiv Y_t/M_t$ is the firm's EBIT-to-market-debt ratio and $b_t \equiv B_t/M_t$ is the bank debt ratio. The firm chooses to default when its EBIT-to-market-debt ratio $y_t < y^\star(b_t)$, where the default boundary $y^\star(b_t)$ depends on the bank debt ratio b_t. To facilitate numerical solution, I express the value functions in the terms of the firm's distance to default $\tilde{y}_t \equiv y_t - y^\star(b_t)$ along with the bank debt ratio b_t. The firm defaults when its distance its distance to default reaches zero from above, regardless of its debt structure.

I first assume that bank debt is senior, and present a numerical solution using the following parameter values: $\mu = 0.02$, $\sigma = 0.25$, $\pi = 0.3$, $r = (1-\pi)c = 0.05$, $\lambda = 0.2$, and $\alpha = 2.33$. Panel A of Figure C.2 plots the bank debt value as a function of the firm's distance to default $\tilde{y}_t \equiv y_t - y^\star(b_t)$ and its bank debt ratio b_t within the no-default region. Panels B and C fix one variable and plot the bank debt value against the other.

In Figure C.3, I plot the surplus-maximizing bank debt ratio against the firm's EBIT-to-market debt ratio y_t (Panel A) and the corresponding distance to default (Panel B). When the firm is far from default, it is in the joint interest of the bank lender and the equity holders to keep the bank debt ratio low and rely primarily on market debt financing. Since dilution works through default, the bank lender and the

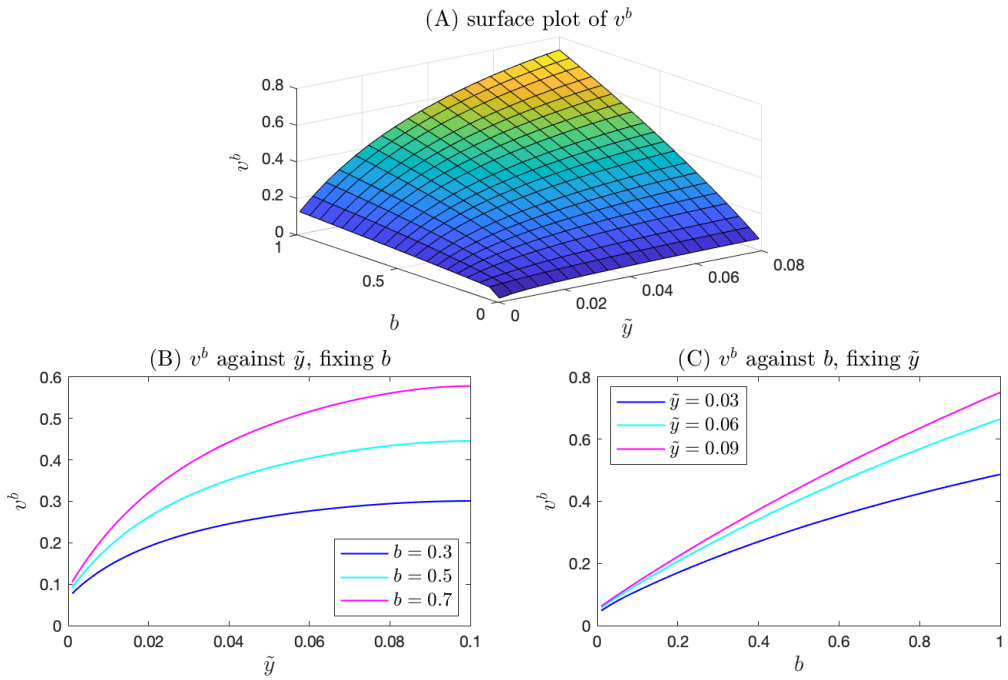


Figure C.2: Bank debt value $v^b(\tilde{y}_t, b_t)$. The figure plots the bank debt value v^b as a function of the firm's distance to default $\tilde{y}_t \equiv y_t - y^\star(b_t)$ and the bank debt ratio b_t within the no-default region. Panel A is a two-dimensional surface plot. Panel B plots v^b against $\tilde{y}_t$ for fixed levels of b_t, while Panel C plots v^b against b_t for fixed levels of $\tilde{y}_t$. The parameter values are $\mu = 0.02$, $\sigma = 0.25$, $\pi = 0.3$, $r = (1-\pi)c = 0.05$, $\lambda = 0.2$, and $\alpha = 2.33$.

equity holders can extract little value from diluting the market debt investors, when both types of debt are relatively safe. As the firm gets closer to default, the incentive to dilute the market debt investors becomes stronger and the surplus-maximizing bank debt ratio increases. However, as default becomes imminent, the optimal level of bank debt emerging from renegotiation decreases again, arising from the lack of commitment on the market debt side.

I next consider the case in which the bank debt is not senior to the market debt. Without commitment, the firm's equity holders are incentivized to dilute the existing lenders by issuing new market debt just before default. If the bank debt is pari passu with or junior to the market debt, the equity holders are able to fully dilute the existing lenders, and the bank lender receives no liquidation value upon default.

In Panel A of Figure C.4, I plot the surplus-maximizing bank debt ratio $\bar{b}(y_t)$ when the bank debt is not senior. I also plot the (normalized) joint equity and bank debt value against b_t in Panel B. The figure shows that if the bank debt is not senior to the market debt, the joint equity and bank debt value is decreasing in the bank debt ratio and the firm never issues any bank debt. The optimal bank debt ratio

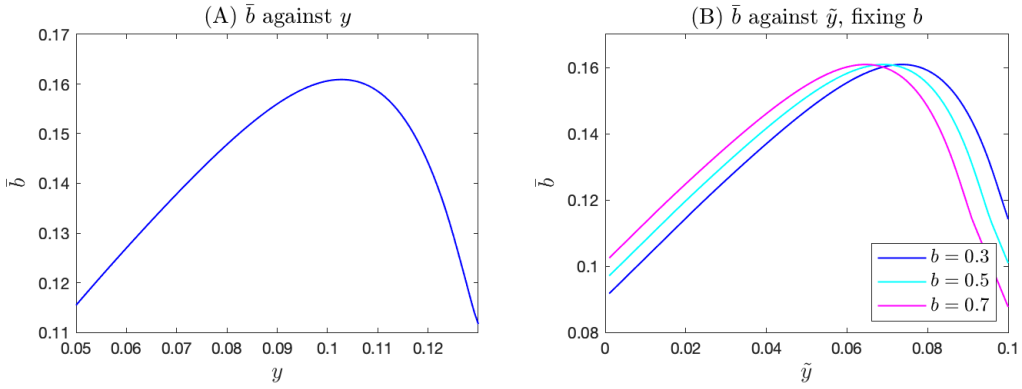


Figure C.3: Surplus-maximizing senior bank debt ratio $\bar{b}(y_t)$ from renegotiation. Panel A of the figure plots the surplus-maximizing bank debt ratio $\bar{b}(y_t)$, which results from bank debt renegotiation, as a function of the firm's EBIT-to-market-debt ratio y_t when the bank debt is senior. Panel B plots the surplus-maximizing bank debt ratio post-renegotiation as a function of the firm's distance to default $\tilde{y}_t \equiv y_t - y^\star(b_t)$ for given levels of bank debt ratio b_t. The parameter values are $\mu = 0.02$, $\sigma = 0.25$, $\pi = 0.3$, $r = (1-\pi)c = 0.05$, $\lambda = 0.2$, and $\alpha = 2.33$.

$\bar{b}(y_t)$, that maximizes the joint equity and bank debt value, is zero. Consequently, any bank debt that exists in equilibrium must be senior.

C.3 Proofs

The equity holders' value function can be expressed recursively through the HJB equation

$$\begin{aligned}(r+\lambda)V^e(Y_t, M_t, B_t) \\ = \max_{G_t^m} \Big\{ &(1-\pi)\big[Y_t - c(B_t + M_t)\big] + p_t^m G_t^m + G_t^m V_M^e(Y_t, M_t, B_t) \\ &+ \mu Y_t V_Y^e(Y_t, M_t, B_t) + \frac{1}{2}\sigma^2 Y_t^2 V_{YY}^e(Y_t, M_t, B_t) \\ &+ \lambda\big[V^e(Y_t, M_t, \bar{B}_t) + \mathcal{T}_t\big]\Big\}.\end{aligned} \tag{C.5}$$

Taking the first-order condition of (C.5) with respect to G_t^m yields

$$p_t^m \equiv p^m(Y_t, M_t, B_t) = -V_M^e(Y_t, M_t, B_t). \tag{C.6}$$

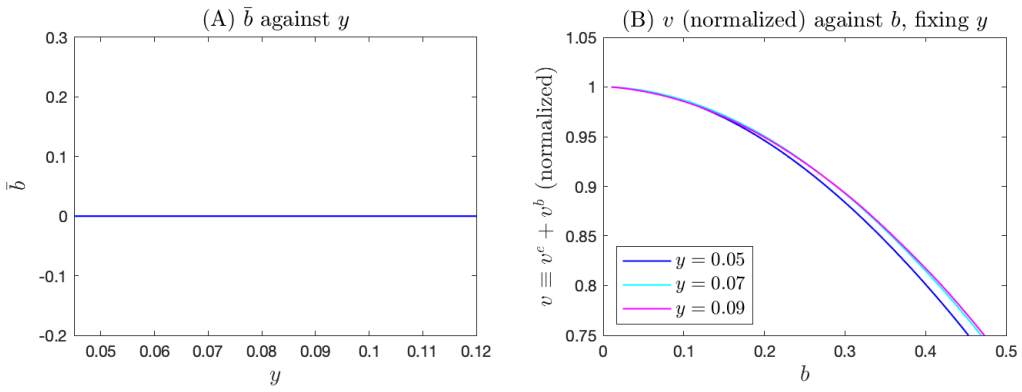


Figure C.4: Surplus-maximizing non-senior bank debt ratio $\bar{b}(y_t)$ from renegotiation. Panel A of the figure plots the surplus-maximizing bank debt ratio $\bar{b}(y_t)$ as a function of the firm's EBIT-to-market-debt ratio y_t, when the bank debt is non-senior. Panel B plots the normalized joint equity and bank debt value against the bank debt ratio b_t for various y_t. The parameter values are $\mu = 0.02$, $\sigma = 0.25$, $\pi = 0.3$, $r = (1-\pi)c = 0.05$, $\lambda = 0.2$, and $\alpha = 2.33$.

Substituting the first-order condition into the HJB equation (C.5), one can rewrite the HJB equation as

$$\begin{aligned}(r+\lambda)V^e(Y_t, M_t, B_t) =& (1-\pi)\big[Y_t - c(B_t + M_t)\big] \\ &+ \mu Y_t V^e_Y(Y_t, M_t, B_t) + \frac{1}{2}\sigma^2 Y_t^2 V^e_{YY}(Y_t, M_t, B_t) \\ &+ \lambda\Big\{V^e(Y_t, M_t, B_t) + \theta\big[V(Y_t, M_t, \bar{B}_t) - V(Y_t, M_t, B_t)\big]\Big\}.\end{aligned} \tag{C.7}$$

A. Market Debt Issuance

Differentiating both sides of (C.7) with respect to M_t and applying the envelope theorem,

$$\begin{aligned}&(r+\lambda)V^e_M(Y_t, M_t, B_t) \\ &\quad= -(1-\pi)c + \mu Y_t V^e_{MY}(Y_t, M_t, B_t) + \frac{1}{2}\sigma^2 Y_t^2 V^e_{MYY}(Y_t, M_t, B_t) \\ &\qquad+ \lambda\Big\{V^e_M(Y_t, M_t, B_t) + \theta\big[V_M(Y_t, M_t, \bar{B}_t) - V_M(Y_t, M_t, B_t)\big]\Big\}.\end{aligned} \tag{C.8}$$

Moreover, from (C.6),

$$p^m_Y(Y_t, M_t, B_t) = -V^e_{MY}(Y_t, M_t, B_t), \tag{C.9}$$

$$p^m_{YY}(Y_t, M_t, B_t) = -V^e_{MYY}(Y_t, M_t, B_t). \tag{C.10}$$

Hence, substituting (C.6), (C.9)-(C.10) into (C.8) yields

$$\begin{aligned}
&-(r+\lambda)p^m(Y_t,M_t,B_t)\\
&\quad=-(1-\pi)c-\mu Y_t p^m_Y(Y_t,M_t,B_t)-\frac{1}{2}\sigma^2 Y_t^2 p^m_{YY}(Y_t,M_t,B_t)\\
&\quad\quad+\lambda\Big\{V^e_M(Y_t,M_t,B_t)+\theta\big[V_M(Y_t,M_t,\bar{B}_t)-V_M(Y_t,M_t,B_t)\big]\Big\}.
\end{aligned}\tag{C.11}$$

Moreover, the equilibrium market debt price $p^m(Y_t,M_t,B_t)$ satisfies the following HJB equation

$$\begin{aligned}
(r+\lambda)p^m(Y_t,M_t,B_t)=&c+G^m_t p^m_M(Y_t,M_t,B_t)+\mu Y_t p^m_Y(Y_t,M_t,B_t)\\
&+\frac{1}{2}\sigma^2 Y_t^2 p^m_{YY}(Y_t,M_t,B_t)+\lambda p^m(Y_t,M_t,\bar{B}_t).
\end{aligned}\tag{C.12}$$

Adding up (C.11) and (C.12) yields

$$\begin{aligned}
0=&\pi c+G^m_t p^m_M(Y_t,M_t,B_t)\\
&+\lambda\Big\{p^m(Y_t,M_t,\bar{B}_t)+V^e_M(Y_t,M_t,B_t)+\theta\big[V_M(Y_t,M_t,\bar{B}_t)-V_M(Y_t,M_t,B_t)\big]\Big\}.
\end{aligned}\tag{C.13}$$

Rearranging yields

$$\begin{aligned}
G^m_t=&\frac{\pi c}{-p^m_M(Y_t,M_t,B_t)}\\
&+\lambda\frac{p^m(Y_t,M_t,\bar{B}_t)+V^e_M(Y_t,M_t,B_t)+\theta\big[V_M(Y_t,M_t,\bar{B}_t)-V_M(Y_t,M_t,B_t)\big]}{-p^m_M(Y_t,M_t,B_t)}.
\end{aligned}\tag{C.14}$$

B. Solution with a Strong Bank

When $\theta=0$, the equity holders' HJB equation (C.7) becomes

$$\begin{aligned}
&rV^e(Y_t,M_t,B_t)\\
&\quad=(1-\pi)\big[Y_t-c(B_t+M_t)\big]+\mu Y_t V^e_Y(Y_t,M_t,B_t)+\frac{1}{2}\sigma^2 Y_t^2 V^e_{YY}(Y_t,M_t,B_t).
\end{aligned}\tag{C.15}$$

That is, the equity holders' value can be solved as if the firm neither issues market debt nor renegotiates its bank debt. Conditional on the face amount of market debt $M_t=M$ and the face amount of bank debt $B_t=B$, let $V^{e,0}(Y_t;M,B)$ denote the equity holders' no-issuance value, that is the equity holders' value

when there is never any debt adjustment. Then $V^{e,0}(Y_t; M, B)$ must satisfy the following HJB equation

$$\begin{aligned} &rV^{e,0}(Y_t; M, B) \\ &\quad = (1-\pi)\big[Y_t - c(B+M)\big] + \mu Y_t V^{e,0\prime}(Y_t; M, B) + \frac{1}{2}\sigma^2 Y_t^2 V^{e,0\prime\prime}(Y_t; M, B). \end{aligned} \tag{C.16}$$

If the firm never defaults, then the cash flows can be evaluated as growing perpetuities. Let $\tilde{V}^{e,0}(Y_t; M, B)$ denote the equity holders' no-issuance value when there is never default, then

$$\tilde{V}^{e,0}(Y_t; M, B) = \frac{1-\pi}{r-\mu} Y_t - \frac{(1-\pi)c(B+M)}{r}. \tag{C.17}$$

The first term on the right-hand side is the no-default present value of after-tax earnings, and the second term on the right-hand side is the no-default present value of after-tax coupon payments.

The equity holders' no-issuance value $V^{e,0}(Y_t; M, B)$ must thus equal the no-default value as given by (C.17) plus the value of a default option. That is,

$$V^{e,0}(Y_t; M, B) = \tilde{V}^{e,0}(Y_t; M, B) + \omega(Y_t; M, B)\big[0 - \tilde{V}^{e,0}(Y^\star(M, B); M, B)\big], \tag{C.18}$$

where $Y^\star(M, B)$ is the default boundary at which the equity holders choose to default and $\omega(Y_t; M, B)$ is the discount factor applied to the default option and follows the homogeneous version of the ODE (C.16). That is,

$$r\omega(Y_t; M, B) = \mu Y_t \omega'(Y_t; M, B) + \frac{1}{2}\sigma^2 Y_t^2 \omega''(Y_t; M, B). \tag{C.19}$$

The general solution to (C.19) is

$$\omega(Y_t; M, B) = K_\gamma Y_t^{-\gamma} + K_\eta Y_t^{\eta}, \tag{C.20}$$

where

$$-\gamma \equiv -\frac{\mu - \frac{1}{2}\sigma^2 + \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2\sigma^2 r}}{\sigma^2}, \tag{C.21}$$

$$\eta \equiv -\frac{\mu - \frac{1}{2}\sigma^2 - \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2\sigma^2 r}}{\sigma^2}. \tag{C.22}$$

It is easy to verify that $-\gamma < 0$ and $\eta > 1$. The constants K_γ and K_η as well as the default boundary $Y^\star(M,B)$ are determined by the boundary conditions. First, the no-bubble condition as $Y_t \to \infty$ says that $\omega(\infty; B, M) = 0$, implying that $K_\eta = 0$. The value matching condition at the default boundary requires that $\omega(Y^\star(M,B); M, B) = 1$, thus $K_\gamma = Y^\star(M,B)^\gamma$ and

$$\omega(Y_t; M, B) = \left[\frac{Y_t}{Y^\star(M,B)}\right]^{-\gamma}. \tag{C.23}$$

From (C.18), the equity holders' no-issuance value can thus be rewritten as

$$\begin{aligned} V^{e,0}(Y_t; M, B) =& \frac{1-\pi}{r-\mu} Y_t - \frac{(1-\pi)c(B+M)}{r} \\ & \underbrace{- \left[\frac{1-\pi}{r-\mu} Y^\star(M,B) - \frac{(1-\pi)c(B+M)}{r}\right] \left[\frac{Y_t}{Y^\star(M,B)}\right]^{-\gamma}}_{\text{default option value}}. \end{aligned} \tag{C.24}$$

The smooth pasting condition at the default boundary requires that $V^{e,0\prime}(Y_t; M, B) = 0$. From (C.24), the default boundary is thus given by

$$Y^\star(M,B) = \frac{\gamma}{1+\gamma} \frac{(r-\mu)c}{r} (B+M). \tag{C.25}$$

By inspecting (C.24), the default option as captured by the last term on the right-hand side is indeed maximized at the default boundary $Y^\star(M,B)$. Substituting the default boundary into (C.24) yields

$$\begin{aligned} V^{e,0}(Y_t; M, B) =& \frac{1-\pi}{r-\mu} Y_t - \frac{(1-\pi)c}{r}(B+M) \\ & + \frac{1}{1+\gamma} \frac{(1-\pi)c}{r} \left[\frac{\gamma}{1+\gamma} \frac{(r-\mu)c}{r}\right]^\gamma (B+M)^{1+\gamma} Y_t^{-\gamma}. \end{aligned} \tag{C.26}$$

Notice that the default option value as captured by the last term on the right-hand side is decreasing in Y_t. The equity holders choose to default when Y_t reaches $Y^\star(M,B)$ from the above. In equilibrium, the equity holders' value equals the equity holders' no-issuance value. That is, $V^e(Y_t, M_t, B_t) = V^{e,0}(Y_t; M_t, B_t)$.

From (C.26),

$$V^e(Y_t, M_t, B_t) = \frac{1-\pi}{r-\mu} Y_t - \frac{(1-\pi)c}{r}(B_t + M_t) + \frac{1}{1+\gamma}\frac{(1-\pi)c}{r}\Big[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\Big]^{\gamma}(B_t + M_t)^{1+\gamma} Y_t^{-\gamma}. \tag{C.27}$$

The equilibrium market debt price satisfies the first-order condition as given by $p^m(Y_t, M_t, B_t) = -V^e_M(Y_t, M_t, B_t)$. Differentiating (C.27) with respect to M_t yields

$$V^e_M(Y_t, M_t, B_t) = -\frac{(1-\pi)c}{r}\left\{1 - \Big[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\Big]^{\gamma}\Big(\frac{B_t + M_t}{Y_t}\Big)^{\gamma}\right\}. \tag{C.28}$$

Thus, the equilibrium market debt price is

$$p^m(Y_t, M_t, B_t) = \frac{(1-\pi)c}{r}\left\{1 - \Big[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\Big]^{\gamma}\Big(\frac{B_t + M_t}{Y_t}\Big)^{\gamma}\right\}. \tag{C.29}$$

Differentiating (C.29) with respect to M_t yields

$$p^m_M(Y_t, M_t, B_t) = -\gamma\frac{(1-\pi)c}{r}\Big[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\Big]^{\gamma}\frac{(B_t + M_t)^{\gamma-1}}{Y_t^{\gamma}}. \tag{C.30}$$

Note that

$$p^m(Y_t, M_t, \bar{B}_t) + V^e_M(Y_t, M_t, B_t) = \frac{(1-\pi)c}{r}\Big[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\Big]^{\gamma}\Big[\Big(\frac{B_t + M_t}{Y_t}\Big)^{\gamma} - \Big(\frac{\bar{B}_t + M_t}{Y_t}\Big)^{\gamma}\Big]. \tag{C.31}$$

When $\theta = 0$, the firm's optimal market debt issuance rate G^m_t becomes

$$G^m_t = \frac{\pi c}{-p^m_M(Y_t, M_t, B_t)} + \lambda\frac{p^m(Y_t, M_t, \bar{B}_t) + V^e_M(Y_t, M_t, B_t)}{-p^m_M(Y_t, M_t, B_t)}. \tag{C.32}$$

Using (C.30) and (C.31), we get

$$\begin{aligned} G_t^m \equiv & G^m(Y_t, M_t, B_t) \\ = & \frac{\pi c}{\gamma} \frac{r}{(1-\pi)c} \Big[\frac{1+\gamma}{\gamma} \frac{r}{(r-\mu)c} \frac{Y_t}{B_t + M_t} \Big]^\gamma (B_t + M_t) \\ & + \frac{\lambda}{\gamma} \Big[1 - \Big(\frac{\bar{B}_t + M_t}{B_t + M_t} \Big)^\gamma \Big] (B_t + M_t). \end{aligned} \tag{C.33}$$

Let $g_t^m \equiv G_t^m/(B_t + M_t)$ be market debt issuance intensity. From (C.33),

$$g_t^m = \frac{\pi c}{\gamma} \frac{r}{(1-\pi)c} \Big[\frac{1+\gamma}{\gamma} \frac{r}{(r-\mu)c} \frac{Y_t}{B_t + M_t} \Big]^\gamma + \frac{\lambda}{\gamma} \Big[1 - \Big(\frac{\bar{B}_t + M_t}{B_t + M_t} \Big)^\gamma \Big]. \tag{C.34}$$

Since $\gamma > 0$, it is immediate that the first term on the right-hand side is decreasing in B_t. To see that the second term is increasing in B_t, note that

$$\bar{B}_t = \arg\max_{B_t'} \ \big[V^e(Y_t, M_t, B_t') + V^b(Y_t, M_t, B_t') \big]. \tag{C.35}$$

It is immediate that $\bar{B}_t$ depends only on Y_t and M_t. We can thus write $\bar{B}_t = \bar{B}(Y_t, M_t)$. Then (C.34) can be rewritten as

$$g_t^m = \frac{\pi c}{\gamma} \frac{r}{(1-\pi)c} \Big[\frac{1+\gamma}{\gamma} \frac{r}{(r-\mu)c} \frac{Y_t}{B_t + M_t} \Big]^\gamma + \frac{\lambda}{\gamma} \Big[1 - \Big(\frac{\bar{B}(Y_t, M_t) + M_t}{B_t + M_t} \Big)^\gamma \Big]. \tag{C.36}$$

D State-Dependent λ

In the main analysis, I have treated the Poisson intensity of bank debt renegotiation λ as a constant. More generally, the intensity parameter is state-dependent. That is, the bank lender and the firm can renegotiate the bank debt at Poisson arrival times with intensity $\lambda(Y_t, M_t, B_t)$. As shown in Section D.3, the optimal market debt issuance rate is written as

$$\begin{aligned} G_t^m =& \frac{\pi c}{-p_M^m(Y_t, M_t, B_t)} + \lambda(Y_t, M_t, B_t) \frac{p^m(Y_t, M_t, \bar{B}_t) + V_M^e(Y_t, M_t, B_t)}{-p_M^m(Y_t, M_t, B_t)} \\ &+ \lambda(Y_t, M_t, B_t) \frac{\theta\big[V_M(Y_t, M_t, \bar{B}_t) - V_M(Y_t, M_t, B_t)\big]}{-p_M^m(Y_t, M_t, B_t)} \\ &+ \lambda_M(Y_t, M_t, B_t) \frac{\theta\big[V(Y_t, M_t, \bar{B}_t) - V(Y_t, M_t, B_t)\big]}{-p_M^m(Y_t, M_t, B_t)}. \end{aligned} \tag{D.1}$$

Compared to (2.10) in the main analysis, (D.13) contains an additional term on the right-hand side. This term arises because the Poisson intensity of bank debt renegotiation depends on the level of market debt. As a result, the firm's market debt issuance policy must also take into account its impact on the renegotiation intensity and thus the likelihood of realizing the bargaining surplus in a given time interval. For example, if $\lambda_M(Y_t, M_t, B_t) > 0$, issuing more market debt increases the the frequency of bank debt renegotiation, thereby raising the equity holders' expected payoff from renegotiation by allowing them to capture the bargaining surplus more often. This creates a strategic benefit of market debt issuance, leading the firm to issue more market debt than in the main analysis.

D.1 Strong Bank Lender ($\theta = 0$)

In the case with a strong bank lender who holds all the bargaining power, that is $\theta = 0$, the firm's optimal market debt issuance rate (D.13) simplifies to

$$G_t^m = \frac{\pi c}{-p_M^m(Y_t, M_t, B_t)} + \lambda(Y_t, M_t, B_t) \frac{p^m(Y_t, M_t, \bar{B}_t) + V_M^e(Y_t, M_t, B_t)}{-p_M^m(Y_t, M_t, B_t)}. \tag{D.2}$$

Moreover, the equity holders' value satisfies the HJB equation

$$\begin{aligned} &rV^e(Y_t, M_t, B_t) \\ &= (1-\pi)\big[Y_t - c(B_t + M_t)\big] + \mu Y_t V^e_Y(Y_t, M_t, B_t) + \frac{1}{2}\sigma^2 Y_t^2 V^e_{YY}(Y_t, M_t, B_t). \end{aligned} \tag{D.3}$$

Notice that this is the same HJB equation as (A.30). Just as before, the equity holders' value can be solved as if the firm neither issues market debt nor renegotiates its bank debt. This is because when the bank lender has all the bargaining power, it captures the entire joint surplus from the bank debt renegotiation, and the equity holders receive only their outside option (or current continuation value). As a result, the renegotiation does not affect the equity holders' value.

As shown in Section D.3, the firm's optimal market debt issuance intensity is

$$\begin{aligned} &g^m(Y_t, M_t, B_t) \equiv \frac{G^m(Y_t, M_t, B_t)}{B_t + M_t} \\ &= \frac{\pi c}{\gamma}\frac{r}{(1-\pi)c}\Big[\frac{1+\gamma}{\gamma}\frac{r}{(r-\mu)c}\frac{Y_t}{B_t + M_t}\Big]^\gamma + \frac{\lambda(Y_t, M_t, B_t)}{\gamma}\Big[1 - \Big(\frac{\bar{B}(Y_t, M_t) + M_t}{B_t + M_t}\Big)^\gamma\Big], \end{aligned} \tag{D.4}$$

which is the same as in the main analysis when $\theta = 0$, except that the Poisson intensity of bank debt renegotiation is now state-dependent. Intuitively, when the firm cannot capture any bargaining surplus, increasing the frequency of bank debt renegotiation by raising the Poisson intensity does not generate additional payoff from renegotiation to the firm's equity holders. In this case, the firm's market debt issuance policy does not account for its impact on the renegotiation intensity.

D.2 Endogenized λ

I now endogenize the Poisson intensity of bank debt renegotiation λ. Specifically, I assume that the bank lender can choose a renegotiation intensity from the interval $\lambda \in [0, \bar{\lambda}]$, subject to a proportional renegotiation cost $\kappa\lambda B_t$ with $\kappa > 0$. For simplicity, I focus on the case with a strong bank lender, that is $\theta = 0$.

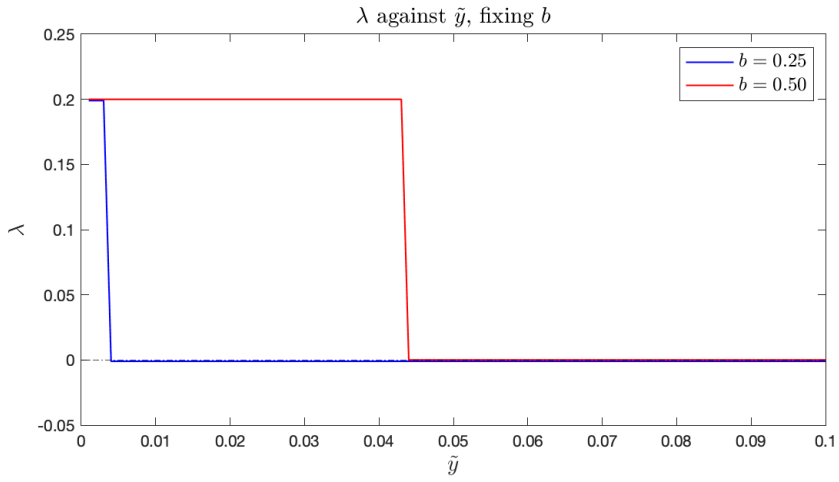


Figure D.1: Endogenous renegotiation intensity. The figure plots the optimal Poisson intensity of bank debt renegotiation λ as a function of the firm's distance to default $\tilde{y}_t$ for fixed levels of bank debt ratio b_t. The parameter values are $\mu = 0.02$, $\sigma = 0.25$, $\pi = 0.3$, $r = c(1-\pi) = 0.05$, $\lambda = 0.2$, and $\alpha = 2.33$.

The bank lender's value $V^b(Y_t, M_t, B_t)$ satisfies the HJB equation

$$\begin{aligned} rV^b(Y_t, M_t, B_t) = \max_{\lambda \in [0,\bar{\lambda}]} \Big\{ & cB_t - \kappa\lambda B_t + G_t^m V_M^b(Y_t, M_t, B_t) \\ & + \mu Y_t V_Y^b(Y_t, M_t, B_t) + \frac{1}{2}\sigma^2 Y_t^2 V_{YY}^b(Y_t, M_t, B_t) \\ & + \lambda\big[V(Y_t, M_t, \bar{B}_t) - V(Y_t, M_t, B_t)\big]\Big\}. \end{aligned} \tag{D.5}$$

As shown in Section D.3, the optimal renegotiation intensity is

$$\lambda(Y_t, M_t, B_t) = \bar{\lambda}\mathbb{I}\{V(Y_t, M_t, \bar{B}_t) - V(Y_t, M_t, B_t) \geq \kappa B_t\}. \tag{D.6}$$

As in the main analysis, the scale invariance in the setting reduces the state space. One can solve the model in terms of the firm's distance to default $\tilde{y}_t \equiv y_t - y^\star(b_t)$ and the bank debt ratio b_t. In Figure D.1, I plot the optimal renegotiation intensity λ as a function of the firm's distance to default $\tilde{y}_t$ for given levels of bank debt ratio b_t. The figure shows that bank debt renegotiations occur more frequently when the firm is closer to default, since renegotiations generate greater gains when the firm is closer to default.

D.3 Proofs

Following the derivations before, the equity holders' HJB equation can be written as

$$\begin{aligned}
&\big[r+\lambda(Y_t,M_t,B_t)\big]V^e(Y_t,M_t,B_t)\\
&\quad=\max_{G_t^m}\Big\{(1-\pi)\big[Y_t-c(B_t+M_t)\big]+p_t^mG_t^m+G_t^mV_M^e(Y_t,M_t,B_t)\\
&\qquad+\mu Y_tV_Y^e(Y_t,M_t,B_t)+\frac{1}{2}\sigma^2Y_t^2V_{YY}^e(Y_t,M_t,B_t)\\
&\qquad+\lambda(Y_t,M_t,B_t)\big[V^e(Y_t,M_t,\bar{B}_t)+\mathcal{T}_t\big]\Big\}.
\end{aligned}\tag{D.7}$$

Taking the first-order condition with respect to G_t^m yields

$$p_t^m\equiv p^m(Y_t,M_t,B_t)=-V_M^e(Y_t,M_t,B_t).\tag{D.8}$$

Substituting (D.8) into the HJB equation (D.7) yields

$$\begin{aligned}
&\big[r+\lambda(Y_t,M_t,B_t)\big]V^e(Y_t,M_t,B_t)\\
&\quad=(1-\pi)\big[Y_t-c(B_t+M_t)\big]+\mu Y_tV_Y^e(Y_t,M_t,B_t)+\frac{1}{2}\sigma^2Y_t^2V_{YY}^e(Y_t,M_t,B_t)\\
&\qquad+\lambda(Y_t,M_t,B_t)\Big\{V^e(Y_t,M_t,B_t)+\theta\big[V(Y_t,M_t,\bar{B}_t)-V(Y_t,M_t,B_t)\big]\Big\}.
\end{aligned}\tag{D.9}$$

Differentiating both sides of (D.9) with respect to M_t and using the envelope theorem, we get

$$\begin{aligned}
&\lambda_M(Y_t,M_t,B_t)V^e(Y_t,M_t,B_t)+\big[r+\lambda(Y_t,M_t,B_t)\big]V_M^e(Y_t,M_t,B_t)\\
&\quad=-(1-\pi)c+\mu Y_tV_{MY}^e(Y_t,M_t,B_t)+\frac{1}{2}\sigma^2Y_t^2V_{MYY}^e(Y_t,M_t,B_t)\\
&\qquad+\lambda_M(Y_t,M_t,B_t)\Big\{V^e(Y_t,M_t,B_t)+\theta\big[V(Y_t,M_t,\bar{B}_t)-V(Y_t,M_t,B_t)\big]\Big\}\\
&\qquad+\lambda(Y_t,M_t,B_t)\Big\{V_M^e(Y_t,M_t,B_t)+\theta\big[V_M(Y_t,M_t,\bar{B}_t)-V_M(Y_t,M_t,B_t)\big]\Big\}.
\end{aligned}\tag{D.10}$$

Applying (D.8) to (D.10) yields

$$\begin{aligned}
&-\big[r+\lambda(Y_t,M_t,B_t)\big]p^m(Y_t,M_t,B_t)\\
&=-(1-\pi)c-\mu Y_t p^m_Y(Y_t,M_t,B_t)-\frac{1}{2}\sigma^2 Y_t^2 p^m_{YY}(Y_t,M_t,B_t)\\
&\quad+\lambda(Y_t,M_t,B_t)\Big\{V^e_M(Y_t,M_t,B_t)+\theta\big[V_M(Y_t,M_t,\bar{B}_t)-V_M(Y_t,M_t,B_t)\big]\Big\}\\
&\quad+\theta\lambda_M(Y_t,M_t,B_t)\big[V(Y_t,M_t,\bar{B}_t)-V(Y_t,M_t,B_t)\big].
\end{aligned}\tag{D.11}$$

Moreover, the equilibrium market debt price $p^m(Y_t,M_t,B_t)$ satisfies the following HJB equation

$$\begin{aligned}
&\big[r+\lambda(Y_t,M_t,B_t)\big]p^m(Y_t,M_t,B_t)\\
&\quad=c+G^m_t p^m_M(Y_t,M_t,B_t)+\mu Y_t p^m_Y(Y_t,M_t,B_t)+\frac{1}{2}\sigma^2 Y_t^2 p^m_{YY}(Y_t,M_t,B_t)\\
&\qquad+\lambda(Y_t,M_t,B_t)p^m(Y_t,M_t,\bar{B}_t).
\end{aligned}\tag{D.12}$$

Adding up (D.11) and (D.12) and rearranging, we get

$$\begin{aligned}
G^m_t=&\frac{\pi c}{-p^m_M(Y_t,M_t,B_t)}+\lambda(Y_t,M_t,B_t)\frac{p^m(Y_t,M_t,\bar{B}_t)+V^e_M(Y_t,M_t,B_t)}{-p^m_M(Y_t,M_t,B_t)}\\
&+\lambda(Y_t,M_t,B_t)\frac{\theta\big[V_M(Y_t,M_t,\bar{B}_t)-V_M(Y_t,M_t,B_t)\big]}{-p^m_M(Y_t,M_t,B_t)}\\
&+\lambda_M(Y_t,M_t,B_t)\frac{\theta\big[V(Y_t,M_t,\bar{B}_t)-V(Y_t,M_t,B_t)\big]}{-p^m_M(Y_t,M_t,B_t)}.
\end{aligned}\tag{D.13}$$

A. State-Dependent λ with Strong Bank Lender

When $\theta=0$, the equity holders' HJB equation (D.9) becomes

$$\begin{aligned}
&rV^e(Y_t,M_t,B_t)\\
&=(1-\pi)\big[Y_t-c(B_t+M_t)\big]+\mu Y_t V^e_Y(Y_t,M_t,B_t)+\frac{1}{2}\sigma^2 Y_t^2 V^e_{YY}(Y_t,M_t,B_t).
\end{aligned}\tag{D.14}$$

This is the same HJB equation as (A.30). Following the derivations in Appendix A.4, the equity holders' value is given by

$$\begin{aligned}
V^e(Y_t,M_t,B_t)=&\frac{1-\pi}{r-\mu}Y_t-\frac{(1-\pi)c}{r}(B_t+M_t)\\
&+\frac{1}{1+\gamma}\frac{(1-\pi)c}{r}\Big[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\Big]^{\gamma}(B_t+M_t)^{1+\gamma}Y_t^{-\gamma},
\end{aligned}\tag{D.15}$$

with

$$\gamma = \frac{\mu - \frac{1}{2}\sigma^2 + \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2\sigma^2 r}}{\sigma^2} > 0, \tag{D.16}$$

and

$$Y^\star(M_t, B_t) = \frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}(B_t + M_t). \tag{D.17}$$

Since $p^m(Y_t, M_t, B_t) = -V_M^e(Y_t, M_t, B_t)$, then the equilibrium market debt price is given by

$$p^m(Y_t, M_t, B_t) = \frac{(1-\pi)c}{r}\left\{1 - \Big[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\Big]^\gamma \Big(\frac{B_t + M_t}{Y_t}\Big)^\gamma\right\}. \tag{D.18}$$

Furthermore, substituting $\theta = 0$ into (D.13) yields the firm's optimal market debt issuance rate when the bank lender has all the bargaining power and the Poisson intensity of bank debt renegotiation is state-dependent. That is,

$$\begin{aligned} G_t^m &\equiv G^m(Y_t, M_t, B_t) \\ &= \frac{\pi c}{-p_M^m(Y_t, M_t, B_t)} + \lambda(Y_t, M_t, B_t)\frac{p^m(Y_t, M_t, \bar{B}_t) + V_M^e(Y_t, M_t, B_t)}{-p_M^m(Y_t, M_t, B_t)}. \end{aligned} \tag{D.19}$$

From (D.15) and (D.18), the market debt issuance rate can be written as

$$\begin{aligned} G^m(Y_t, M_t, B_t) =& \frac{\pi c}{\gamma}\frac{r}{(1-\pi)c}\Big[\frac{1+\gamma}{\gamma}\frac{r}{(r-\mu)c}\frac{Y_t}{B_t + M_t}\Big]^\gamma (B_t + M_t) \\ &+ \frac{\lambda(Y_t, M_t, B_t)}{\gamma}\Big[1 - \Big(\frac{\bar{B}_t + M_t}{B_t + M_t}\Big)^\gamma\Big](B_t + M_t). \end{aligned} \tag{D.20}$$

Thus, the market debt issuance intensity $g^m(Y_t, M_t, B_t) \equiv G^m(Y_t, M_t, B_t)/(B_t + M_t)$ is given by

$$\begin{aligned} &g^m(Y_t, M_t, B_t) \\ &= \frac{\pi c}{\gamma}\frac{r}{(1-\pi)c}\Big[\frac{1+\gamma}{\gamma}\frac{r}{(r-\mu)c}\frac{Y_t}{B_t + M_t}\Big]^\gamma + \frac{\lambda(Y_t, M_t, B_t)}{\gamma}\Big[1 - \Big(\frac{\bar{B}_t + M_t}{B_t + M_t}\Big)^\gamma\Big]. \end{aligned} \tag{D.21}$$

B. Endogenized λ

If $\theta = 0$, then following the preceding derivations, the equity holders' value is

$$\begin{aligned} V^e(Y_t, M_t, B_t) =& \frac{1-\pi}{r-\mu} Y_t - \frac{(1-\pi)c}{r}(B_t + M_t) \\ &+ \frac{1}{1+\gamma}\frac{(1-\pi)c}{r}\Big[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\Big]^{\gamma}(B_t + M_t)^{1+\gamma} Y_t^{-\gamma}, \end{aligned} \tag{D.22}$$

and the firm's market debt issuance rate is given by

$$\begin{aligned} G^m(Y_t, M_t, B_t) =& \frac{\pi c}{\gamma}\frac{r}{(1-\pi)c}\Big[\frac{1+\gamma}{\gamma}\frac{r}{(r-\mu)c}\frac{Y_t}{B_t + M_t}\Big]^{\gamma}(B_t + M_t) \\ &+ \frac{\lambda(Y_t, M_t, B_t)}{\gamma}\Big[1 - \Big(\frac{\bar{B}_t + M_t}{B_t + M_t}\Big)^{\gamma}\Big](B_t + M_t). \end{aligned} \tag{D.23}$$

The Poisson intensity parameter $\lambda(Y_t, M_t, B_t)$ is endogenously determined.

On the other hand, the bank lender's value $V^b(Y_t, M_t, B_t)$ satisfies the following HJB equation

$$\begin{aligned} rV^b(Y_t, M_t, B_t) = \max_{\lambda \in [0, \bar{\lambda}]} \Big\{ & cB_t - \kappa\lambda B_t + G_t^m V_M^b(Y_t, M_t, B_t) \\ & + \mu Y_t V_Y^b(Y_t, M_t, B_t) + \frac{1}{2}\sigma^2 Y_t^2 V_{YY}^b(Y_t, M_t, B_t) \\ & + \lambda\big[V(Y_t, M_t, \bar{B}_t) - V(Y_t, M_t, B_t)\big]\Big\}. \end{aligned} \tag{D.24}$$

Taking the first-order condition with respect to λ and applying the envelope theorem, the endogenous renegotiation intensity is derived as

$$\lambda(Y_t, M_t, B_t) = \begin{cases} \bar{\lambda} & \text{if } V(Y_t, M_t, \bar{B}_t) - V(Y_t, M_t, B_t) \geq \kappa B_t \\ 0 & \text{if } V(Y_t, M_t, \bar{B}_t) - V(Y_t, M_t, B_t) < \kappa B_t \end{cases}. \tag{D.25}$$

The firm is scale invariant in the setting. Thus, the bank lender's value, the equity holders' value, and the joint value of the equity holders and the bank lender are all homogeneous of degree one. That is,

$$V^b(Y_t, M_t, B_t) = V^b\Big(\frac{Y_t}{M_t}, 1, \frac{B_t}{M_t}\Big) M_t \equiv v^b(y_t, b_t) M_t, \tag{D.26}$$

$$V(Y_t, M_t, B_t) = V\Big(\frac{Y_t}{M_t}, 1, \frac{B_t}{M_t}\Big) M_t \equiv v(y_t, b_t) M_t, \tag{D.27}$$

where $y_t \equiv Y_t/M_t$ and $b_t \equiv B_t/M_t$. Then from (D.25),

$$\lambda(Y_t, M_t, B_t) \equiv \lambda(y_t, b_t) = \bar{\lambda}\mathbb{I}\{v(y_t, \bar{b}_t) - v(y_t, b_t) \geq \kappa b_t\}. \tag{D.28}$$

Substituting into the bank lender's HJB equation (D.24) yields

$$\begin{aligned}
&\big[r + \lambda(y_t, b_t) - g_t^m(1 + b_t)\big]v^b(y_t, b_t) \\
&\quad = cb_t - g_t^m b_t(1 + b_t)v_b^b(y_t, b_t) + \big[\mu - g_t^m(1 + b_t)\big]y_t v_y^n(y_t, b_t) \\
&\qquad + \frac{1}{2}\sigma^2 y_t^2 v_{yy}^b(y_t, b_t) + \lambda(y_t, b_t)\big[\max_{b_t'} v(y_t, b_t') - v^e(y_t, b_t)\big].
\end{aligned} \tag{D.29}$$

Following the derivations before, we can write the optimal market debt issuance intensity as

$$g_t^m \equiv g^m(y_t, b_t) = \frac{\pi c}{\gamma}\frac{r}{(1-\pi)c}\Big[\frac{1+\gamma}{\gamma}\frac{r}{(r-\mu)c}\frac{y_t}{1+b_t}\Big]^{\gamma} + \frac{\lambda(y_t, b_t)}{\gamma}\Big[1 - \Big(\frac{1+\bar{b}_t}{1+b_t}\Big)^{\gamma}\Big], \tag{D.30}$$

where

$$\bar{b}_t = \arg\max_{b_t'} v(y_t, b_t'). \tag{D.31}$$

The equity value is given by

$$\begin{aligned}
v^e(y_t, b_t) =& \frac{1-\pi}{r-\mu}y_t - \frac{(1-\pi)c}{r}(1 + b_t) \\
&+ \frac{1}{1+\gamma}\frac{(1-\pi)c}{r}(1 + b_t)\Big[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\Big]^{\gamma}\Big(\frac{y_t}{1+b_t}\Big)^{-\gamma}.
\end{aligned} \tag{D.32}$$

As in Appendix A.6, it is more transparent to transform the state space such that the model is solved in terms of the distance to default $\tilde{y}_t \equiv y_t - y^\star(b_t)$ and the bank debt ratio b_t. From (D.28), the endogenous renegotiation intensity can be rewritten as a function of $\tilde{y}_t$ and b_t, that is

$$\lambda(\tilde{y}_t, b_t) = \bar{\lambda}\mathbb{I}\{\bar{v}(\tilde{y}_t, b_t) - v(\tilde{y}_t, b_t) \geq \kappa b_t\}, \tag{D.33}$$

where

$$\bar{v}(\tilde{y}_t, b_t) = \max_{b_t'} v(\tilde{y}_t + y^\star(b_t) - y^\star(b_t'), b_t'). \tag{D.34}$$

Moreover, (D.30) and (D.32) become

$$g^m(\tilde{y}_t, b_t) = \frac{\pi c}{\gamma} \frac{r}{(1-\pi)c} \Big[\frac{1+\gamma}{\gamma} \frac{r}{(r-\mu)c} \frac{\tilde{y}_t + y^\star(b_t)}{1+b_t} \Big]^\gamma + \frac{\lambda(\tilde{y}_t, b_t)}{\gamma} \Big[1 - \Big(\frac{1+\bar{b}_t}{1+b_t} \Big)^\gamma \Big], \tag{D.35}$$

and

$$\begin{aligned} v^e(\tilde{y}_t, b_t) =& \frac{1-\pi}{r-\mu} \big[\tilde{y}_t + y^\star(b_t) \big] - \frac{(1-\pi)c}{r}(1+b_t) \\ &+ \frac{1}{1+\gamma} \frac{(1-\pi)c}{r}(1+b_t) \Big[\frac{\gamma}{1+\gamma} \frac{(r-\mu)c}{r} \Big]^\gamma \Big[\frac{\tilde{y}_t + y^\star(b_t)}{1+b_t} \Big]^{-\gamma}. \end{aligned} \tag{D.36}$$

The HJB equation for the bank debt value (D.29) is then

$$\begin{aligned} &\big[r + \lambda(\tilde{y}_t, b_t) - g_t^m(1+b_t) \big] v^b(\tilde{y}_t, b_t) \\ &\quad = c b_t - g_t^m b_t (1+b_t) v_b^b(\tilde{y}_t, b_t) \\ &\qquad + \big\{ \big[\mu - g_t^m(1+b_t) \big] \big[\tilde{y}_t + y^\star(b_t) \big] + g_t^m b_t (1+b_t) y^{\star\prime}(b_t) \big\} v_{\tilde{y}}^b(\tilde{y}_t, b_t) \\ &\qquad + \frac{1}{2} \sigma^2 \big[\tilde{y}_t + y^\star(b_t) \big]^2 v_{\tilde{y}\tilde{y}}^b(\tilde{y}_t, b_t) + \lambda(\tilde{y}_t, b_t) \big[\bar{v}(\tilde{y}_t, b_t) - v^e(\tilde{y}_t, b_t) \big]. \end{aligned} \tag{D.37}$$

E Bank Lending Shock

In this appendix, I consider the effect of a bank lending shock, modeled as a shock to the bank lender's discount rate. The economy is in one of two states, $\xi \in \{n, s\}$ with n denoting the normal state and s denoting the shock state. In the normal state, the bank lender discounts time at rate $r^n > 0$. In the shock state, the bank lender has a higher discount rate $r^s > r^n$. The economy transitions between states through a Markov switching process that is independent of the Poisson process governing bank-firm renegotiations. The economy transitions from the shock state to the normal state with intensity ζ^s, and from the normal state to the shock state with intensity ζ^n which I normalize to zero. That is, the normal state is absorbing, and my analysis examines the impact of a bank lending shock whose persistence is characterized by $1/\zeta^s$. For simplicity, I continue to focus on the case with a strong bank lender (i.e., $\theta = 0$).

Let $V^{e,\xi}(Y_t, M_t, B_t)$ denote the equity holders' value when the economy is in state ξ and the firm has EBIT Y_t, market debt outstanding M_t, and bank debt outstanding B_t. Let $p^{m,\xi}(Y_t, M_t, B_t)$ be the corresponding equilibrium market debt price. As shown in Section E.3 and analogous to before, the market debt issuance rate is a function of the state variables and is given by

$$G^{m,\xi}(Y_t, M_t, B_t) = \frac{\pi c}{-p_M^{m,\xi}(Y_t, M_t, B_t)} + \lambda \frac{p^{m,\xi}(Y_t, M_t, \bar{B}_t^{\xi}) + V_M^{e,\xi}(Y_t, M_t, B_t)}{-p_M^{m,\xi}(Y_t, M_t, B_t)}. \tag{E.1}$$

E.1 Permanent Shock

As a benchmark, I first consider a bank lending shock that is permanent. That is, $\zeta^s = 0$. For example, post-GFC banking regulations have tightened banks' balance sheet constraints, making it more costly for banks to lend to firms. I continue to use $g^{m,\xi}(Y_t, M_t, B_t) \equiv G^{m,\xi}(Y_t, M_t, B_t)/(B_t + M_t)$, the market debt issuance intensity, to capture the firm's market debt issuance behavior. As shown in Section E.3, when the bank lending shock is permanent, the firm's market debt issuance intensity in state $\xi \in \{n, s\}$ is given by

$$g^{m,\xi}(Y_t, M_t, B_t) = \frac{\pi c}{\gamma} \frac{r}{(1-\pi)c} \Big[\frac{1+\gamma}{\gamma} \frac{r}{(r-\mu)c} \frac{Y_t}{B_t + M_t} \Big]^{\gamma} + \frac{\lambda}{\gamma} \Big[1 - \Big(\frac{\bar{B}_t^{\xi} + M_t}{B_t + M_t} \Big)^{\gamma} \Big]. \tag{E.2}$$

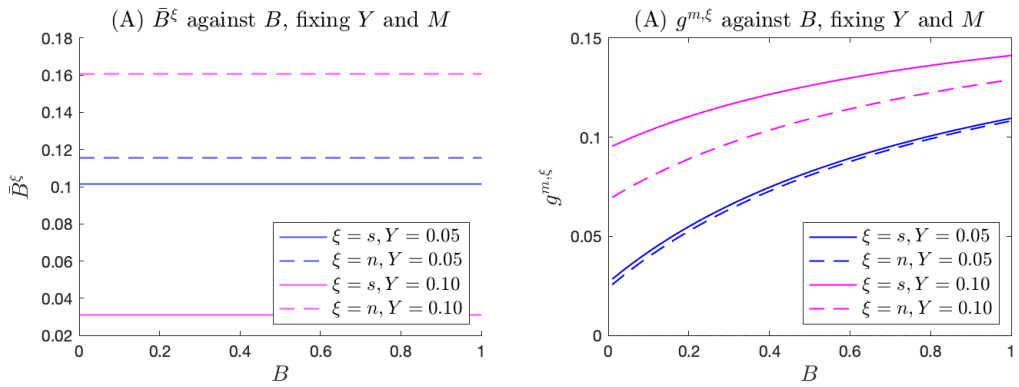


Figure E.1: Effect of permanent bank lending shock on post-renegotiation bank debt level and market debt issuance intensity. Panel A of the figure plots the surplus-maximizing bank debt level post-renegotiation in the normal state and in the shock state, $\bar{B}_t^n$ and $\bar{B}_t^s$, against the prevailing bank debt amount B_t for given levels of EBIT Y_t while normalizing the amount of market debt M_t to one. Panel B plots the corresponding market debt issuance intensity $g^{m,\xi}(Y_t, M_t, B_t)$ in both states of the economy $\xi \in \{n, s\}$. The parameter values are $\mu = 0.02$, $\sigma = 0.25$, $\pi = 0.3$, $r^n = r = c(1-\pi) = 0.05$, $r^s = 0.07$, $\lambda = 0.2$, and $\alpha = 2.33$.

Thus, the market debt issuance intensity differs in the two states of the economy, because the bank lending shock affects the surplus-maximizing level of bank debt upon renegotiation. In Figure E.1, I plot the surplus-maximizing bank debt amount post-renegotiation (Panel A) as well as the corresponding market debt issuance intensity (Panel B) in both the shock state and the normal state, for given levels of EBIT while normalizing the face amount of market debt to one. The parameters take the baseline values: $\mu = 0.02$, $\sigma = 0.25$, $\pi = 0.3$, $r^n = r = c(1-\pi) = 0.05$, $r^s = 0.07$, $\lambda = 0.2$, and $\alpha = 2.33$.

The figure shows that given the prevailing states Y_t, M_t and B_t, the surplus-maximizing bank debt level post-renegotiation is lower when the economy is in the shock state than when it is in the normal state. Intuitively, the shock to the bank lender's discount rate reduces the bank lender's willingness to lend, prompting the bank lender and the firm to agree on a smaller balance of bank debt upon renegotiation. Thus, the bank lending shock can be seen as a shock to the bank credit supply. From (E.2), it is immediate that $g^{m,s}(Y_t, M_t, B_t) > g^{m,n}(Y_t, M_t, B_t)$. This is because less bank debt means less dilution of the market debt, which increases the market debt investors' willingness to lend to the firm, incentivizing the firm to issue more market debt.

In Figure E.2, I plot the market debt issuance intensity in both states of the economy against the bank debt ratio b_t, for given levels of distance of default $\tilde{y}_t$. Again, a permanent bank lending shock results in higher market debt issuance intensity by the firm. The increase in the market debt issuance intensity is more pronounced when the firm is farther from default. The figure thus appears consistent

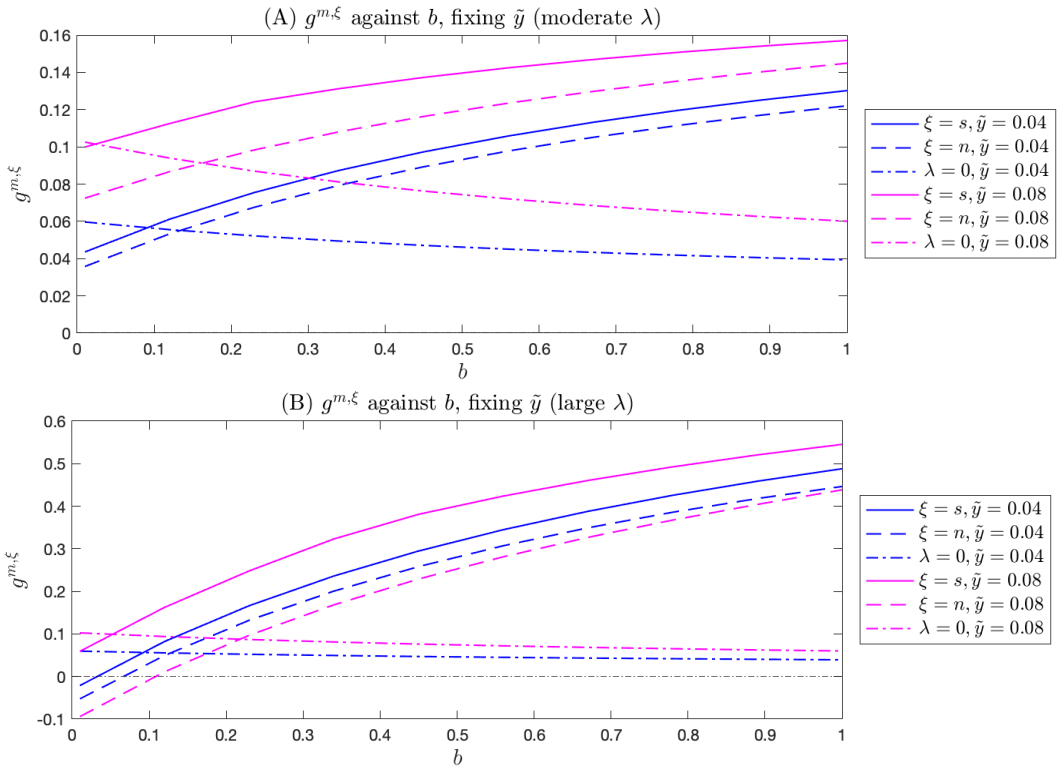


Figure E.2: State-dependent market debt issuance intensity for given distance to default. The figure plots the market debt issuance intensity against the bank debt ratio b_t for given levels of distance to default $\tilde{y}_t$, when the economy is in the normal state and when it is in the shock state. Panel A plots this relationship for $\lambda = 0.2$, while Panel B plots this relationship for $\lambda = 1$. The other parameters take the values: $\mu = 0.02$, $\sigma = 0.25$, $\pi = 0.3$, $r^n = r = (1 - \pi)c = 0.05$, $r^s = 0.07$, and $\alpha = 2.33$.

with the empirical observation that firms increase their market debt issuance following a contraction in bank credit supply (e.g., see Becker and Ivashina (2014)).

Importantly, the dynamic complementarity between bank debt and market debt, as captured by the second term on the right-hand side of (E.2), is still present in both the normal state and the shock state. In both states of the economy, a larger balance of bank debt in the debt structure is associated with higher market debt issuance intensity, even though the firm issues more market debt after the bank lender experiences the discount rate shock.

E.2 Transitory Shock

A bank lending shock is often transitory. I now examine how the persistence of the shock affects the firm's market debt issuance behavior. In Figure E.3, I plot the firm's market debt issuance intensity in

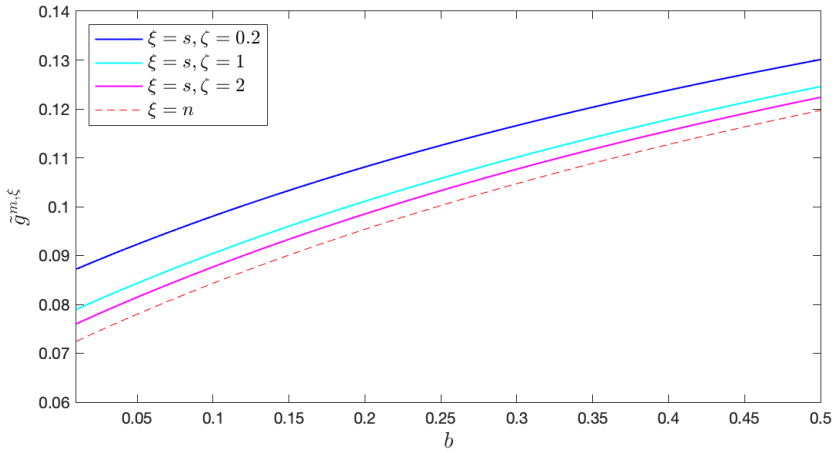


Figure E.3: Effect of shock persistence on market debt issuance intensity. This figure plots the market debt issuance intensity against the bank debt ratio b_t when the distance to default $\tilde{y}_t = 0.08$ for different levels of shock persistence. The parameters values are $\mu = 0.02$, $\sigma = 0.25$, $\pi = 0.3$, $r^n = r = (1-\pi)c = 0.05$, $r^s = 0.07$, $\lambda = 0.2$, and $\alpha = 2.33$.

both the shock state and the normal state, for various levels of shock persistence, ranging from highly persistent ($\zeta^s = 0.2$) to transitory ($\zeta^s = 2$). The pattern that the firm increases market debt issuance upon shock remains robust. However, the extent to which the firm ramps up its market debt issuance depends on the shock's persistence. As shown in the figure, the firm increases its market debt issuance more when the shock is more persistent.

E.3 Proofs

The equity holders' value $V^{e,\xi}(Y_t, M_t, B_t)$ when the economy is in state $\xi \in \{n, s\}$ and the firm has EBIT Y_t, market debt outstanding M_t, and bank debt outstanding B_t, satisfies the following HJB equation

$$\begin{aligned}(r+\lambda)V^{e,\xi}(Y_t, M_t, B_t) = \max_{G_t^{m,\xi}} \Big\{ & (1-\pi)\big[Y_t - c(B_t + M_t)\big] \\ & + p_t^{m,\xi} G_t^{m,\xi} + G_t^{m,\xi} V_M^{e,\xi}(Y_t, M_t, B_t) \\ & + \mu Y_t V_Y^{e,\xi}(Y_t, M_t, B_t) + \frac{1}{2}\sigma^2 Y_t^2 V_{YY}^{e,\xi}(Y_t, M_t, B_t) \\ & + \lambda\big[V^{e,\xi}(Y_t, M_t, \bar{B}_t^{\xi}) + \mathcal{T}_t^{\xi}\big] \\ & + \zeta^{\xi}\big[V^{e,-\xi}(Y_t, M_t, B_t) - V^{e,\xi}(Y_t, M_t, B_t)\big]\Big\},\end{aligned} \tag{E.3}$$

where $-\xi \neq \xi \in \{n, s\}$. $p_t^{m,\xi}$ is the competitive market debt price in state ξ. $\bar{B}_t^{\xi}$ and $\mathcal{T}_t^{\xi}$ are the renegotiated bank debt amount and the payment from the bank lender to the equity holders respectively, which are determined by bargaining. The first-order condition with respect to the market debt issuance rate yields

$$p_t^{m,\xi} = -V_M^{e,\xi}(Y_t, M_t, B_t). \tag{E.4}$$

Hence, the equity holders' HJB equation can be rewritten as

$$\begin{aligned}(r+\lambda)V^{e,\xi}(Y_t, M_t, B_t) = (1-\pi)\big[Y_t - c(B_t + M_t)\big] \\ + \mu Y_t V_Y^{e,\xi}(Y_t, M_t, B_t) + \frac{1}{2}\sigma^2 Y_t^2 V_{YY}^{e,\xi}(Y_t, M_t, B_t) \\ + \lambda\big[V^{e,\xi}(Y_t, M_t, \bar{B}_t^{\xi}) + \mathcal{T}_t^{\xi}\big] + \zeta^{\xi}\big[V^{e,-\xi}(Y_t, M_t, B_t) - V^{e,\xi}(Y_t, M_t, B_t)\big].\end{aligned} \tag{E.5}$$

Similarly, the bank lender's value $V^{b,\xi}(Y_t, M_t, B_t)$ when the economy is in state ξ satisfies

$$\begin{aligned}(r^{\xi}+\lambda)V^{b,\xi}(Y_t, M_t, B_t) =& cB_t + G_t^{m,\xi}V_M^{b,\xi}(Y_t, M_t, B_t) \\ &+ \mu Y_t V_Y^{b,\xi}(Y_t, M_t, B_t) + \frac{1}{2}\sigma^2 Y_t^2 V_{YY}^{b,\xi}(Y_t, M_t, B_t) \\ &+ \lambda\big[V^{b,\xi}(Y_t, M_t, \bar{B}_t^{\xi}) - \mathcal{T}_t^{\xi}\big] \\ &+ \zeta^{\xi}\big[V^{b,-\xi}(Y_t, M_t, B_t) - V^{b,\xi}(Y_t, M_t, B_t)\big].\end{aligned} \tag{E.6}$$

Upon renegotiation of the bank debt, the terms $\{\bar{B}_t^{\xi}, \mathcal{T}_t^{\xi}\}$ are determined by bargaining between the bank lender and the firm. Thus

$$\begin{aligned}\{\bar{B}_t^{\xi}, \mathcal{T}_t^{\xi}\} = \underset{B_t', \mathcal{T}_t'}{\arg\max}\ &\big[V^{e,\xi}(Y_t, M_t, B_t') + \mathcal{T}_t' - V^{e,\xi}(Y_t, M_t, B_t)\big]^{\theta} \\ &\big[V^{b,\xi}(Y_t, M_t, B_t' - \mathcal{T}_t' - V^{b,\xi}(Y_t, M_t, B_t)\big]^{1-\theta},\end{aligned} \tag{E.7}$$

subject to $V^{e,\xi}(Y_t, M_t, B_t') + \mathcal{T}_t' - V^{e,\xi}(Y_t, M_t, B_t) \geq 0$ and $V^{b,\xi}(Y_t, M_t, B_t') - \mathcal{T}_t' - V^{b,\xi}(Y_t, M_t, B_t) \geq 0$. Following derivations in Appendix A.1, when $\theta = 0$ we have

$$\bar{B}_t^{\xi} = \underset{B_t'}{\arg\max}\ \big[V^{e,\xi}(Y_t, M_t, B_t') + V^{b,\xi}(Y_t, M_t, B_t')\big], \tag{E.8}$$

and

$$\mathcal{T}_t^{\xi} = V^{e,\xi}(Y_t, M_t, B_t) - V^{e,\xi}(Y_t, M_t, \bar{B}_t^{\xi}). \tag{E.9}$$

Substituting (E.9) into (E.5) yields

$$\begin{aligned}(r+\lambda)V^{e,\xi}(Y_t, M_t, B_t) =& (1-\pi)\big[Y_t - c(B_t + M_t)\big] \\ &+ \mu Y_t V_Y^{e,\xi}(Y_t, M_t, B_t) + \frac{1}{2}\sigma^2 Y_t^2 V_{YY}^{e,\xi}(Y_t, M_t, B_t) \\ &+ \lambda V^{e,\xi}(Y_t, M_t, B_t) + \zeta^{\xi}\big[V^{e,-\xi}(Y_t, M_t, B_t) - V^{e,\xi}(Y_t, M_t, B_t)\big].\end{aligned} \tag{E.10}$$

Differentiating (E.10) with respect to M_t and using the first-order condition (E.4),

$$\begin{aligned}-(r+\lambda)p^{m,\xi}(Y_t, M_t, B_t) =& -(1-\pi)c - \mu Y_t p_Y^{m,\xi}(Y_t, M_t, B_t) - \frac{1}{2}\sigma^2 Y_t^2 p_{YY}^{m,\xi}(Y_t, M_t, B_t) \\ &+ \lambda V_M^{e,\xi}(Y_t, M_t, B_t) + \zeta^{\xi}\big[-p^{m,-\xi}(Y_t, M_t, B_t) + p^{m,\xi}(Y_t, M_t, B_t)\big].\end{aligned} \tag{E.11}$$

Moreover, let $p^{m,\xi}(Y_t, M_t, B_t)$ denote the equilibrium market debt price in state ξ, then

$$\begin{aligned}(r+\lambda)p^{m,\xi}(Y_t, M_t, B_t) =& c + G_t^{m,\xi} p_M^{m,\xi}(Y_t, M_t, B_t) \\ &+ \mu Y_t p_Y^{m,\xi}(Y_t, M_t, B_t) + \frac{1}{2}\sigma^2 Y_t^2 p_{YY}^{m,\xi}(Y_t, M_t, B_t) \\ &+ \lambda p^{m,\xi}(Y_t, M_t, \bar{B}_t^{\xi}) \\ &+ \zeta^{\xi}\big[p^{m,-\xi}(Y_t, M_t, B_t) - p^{m,\xi}(Y_t, M_t, B_t)\big].\end{aligned} \tag{E.12}$$

Adding up (E.11) and (E.12) and rearranging, we get

$$G_t^{m,\xi} = \frac{\pi c}{-p_M^{m,\xi}(Y_t, M_t, B_t)} + \lambda \frac{p^{m,\xi}(Y_t, M_t, \bar{B}_t^{\xi}) + V_M^{e,\xi}(Y_t, M_t, B_t)}{-p_M^{m,\xi}(Y_t, M_t, B_t)}. \tag{E.13}$$

A. Permanent Shock

I now consider a permanent bank lending shock, that is $\zeta^s = 0$. The equity holders' value can be solved as if the firm neither issues market debt nor adjusts its bank debt. Let $V^{e,\xi,0}(Y_t; M, B)$ denote the equity holders' no-issuance value given the market debt outstanding $M_t = M$ and the bank debt

outstanding $B_t = B$. Then

$$\begin{aligned} rV^{e,\xi,0}(Y_t; M, B) =& (1-\pi)\big[Y_t - c(B+M)\big] \\ &+ \mu Y_t V^{e,\xi,0\prime}(Y_t; M, B) + \frac{1}{2}\sigma^2 Y_t^2 V^{e,\xi,0\prime\prime}(Y_t; M, B), \end{aligned} \tag{E.14}$$

for $\xi \in \{n, s\}$. Following the derivations in Appendix A.4,

$$\begin{aligned} V^{e,\xi,0}(Y_t; M, B) =& \frac{1-\pi}{r-\mu} Y_t - \frac{(1-\pi)c}{r}(B+M) \\ &+ \frac{1}{1+\gamma}\frac{(1-\pi)c}{r}\Big[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\Big]^{\gamma}(B+M)^{1+\gamma} Y_t^{-\gamma}. \end{aligned} \tag{E.15}$$

Hence,

$$\begin{aligned} V^{e,\xi}(Y_t, M_t, B_t) =& \frac{1-\pi}{r-\mu} Y_t - \frac{(1-\pi)c}{r}(B_t+M_t) \\ &+ \frac{1}{1+\gamma}\frac{(1-\pi)c}{r}\Big[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\Big]^{\gamma}(B_t+M_t)^{1+\gamma} Y_t^{-\gamma}. \end{aligned} \tag{E.16}$$

Using the first-order condition (E.4), the equilibrium market debt price in state ξ is then

$$p^{m,\xi}(Y_t, M_t, B_t) = \frac{(1-\pi)c}{r}\left\{1 - \Big[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\Big]^{\gamma}\Big(\frac{B_t+M_t}{Y_t}\Big)^{\gamma}\right\}. \tag{E.17}$$

The market debt issuance intensity in state ξ is given by

$$g_t^{m,\xi} = \frac{\pi c}{\gamma}\frac{r}{(1-\pi)c}\Big[\frac{1+\gamma}{\gamma}\frac{r}{(r-\mu)c}\frac{Y_t}{B_t+M_t}\Big]^{\gamma} + \frac{\lambda}{\gamma}\Big[1 - \Big(\frac{\bar{B}_t^{\xi}+M_t}{B_t+M_t}\Big)^{\gamma}\Big]. \tag{E.18}$$

B. Transitory Shock

I now turn to the case with a transitory bank lending shock. Let $V^{e,\xi,0}(Y_t; M, B)$ denote the equity holders' no-issuance value given the market debt outstanding $M_t = M$ and the bank debt outstanding $B_t = B$. It must then satisfy the following HJB equation

$$\begin{aligned} rV^{e,\xi,0}(Y_t; M, B) =& (1-\pi)\big[Y_t - c(B+M)\big] \\ &+ \mu Y_t V^{e,\xi,0\prime}(Y_t; M, B) + \frac{1}{2}\sigma^2 Y_t^2 V^{e,\xi,0\prime\prime}(Y_t; M, B) \\ &+ \zeta^{\xi}\big[V^{e,-\xi,0}(Y_t; M, B) - V^{e,\xi,0}(Y_t; M, B)\big]. \end{aligned} \tag{E.19}$$

The equity holders' no-issuance value is homogeneous of degree one. Hence

$$V^{e,\xi,0}(Y_t; M, B) = v^{e,\xi,0}(y_t; b)M, \tag{E.20}$$

where $y_t \equiv Y_t/M_t$, $b \equiv B/M$, and $v^{e,\xi,0}(y_t; b)$ is the equity holders' no-issuance value scaled by the amount of market debt outstanding. I refer to $v^{e,\xi,0}(y_t; b)$ as the no-issuance equity value. The HJB equation (E.19) can be rewritten as

$$\begin{aligned} rv^{e,\xi,0}(y_t; b) =& (1-\pi)\big[y_t - c(1+b)\big] + \mu y_t v^{e,\xi,0\prime}(y_t; b) + \frac{1}{2}\sigma^2 y_t^2 v^{e,\xi,0\prime\prime}(y_t; b) \\ &+ \zeta^{\xi}\big[v^{e,-\xi,0}(y_t; b) - v^{e,\xi,0}(y_t; b)\big]. \end{aligned} \tag{E.21}$$

That is, $v^{e,n,0}(y_t; b)$ and $v^{e,s,0}(y_t; b)$ respectively satisfy

$$rv^{e,n,0}(y_t; b) = (1-\pi)\big[y_t - c(1+b)\big] + \mu y_t v^{e,n,0\prime}(y_t; b) + \frac{1}{2}\sigma^2 y_t^2 v^{e,n,0\prime\prime}(y_t; b), \tag{E.22}$$

and

$$\begin{aligned} rv^{e,s,0}(y_t; b) =& (1-\pi)\big[y_t - c(1+b)\big] + \mu y_t v^{e,s,0\prime}(y_t; b) + \frac{1}{2}\sigma^2 y_t^2 v^{e,s,0\prime\prime}(y_t; b) \\ &+ \zeta^{s}\big[v^{e,n,0}(y_t; b) - v^{e,s,0}(y_t; b)\big]. \end{aligned} \tag{E.23}$$

When the economy is in the normal state, at the default boundary $y^{\star,n}(b)$ the no-issuance equity value satisfies the value matching condition $v^{e,n,0}(y^{\star,n}(b); b) = 0$ and the smooth pasting condition $v^{e,n,0\prime}(y^{\star,n}(b); b) = 0$. Following the derivations in Appendix A.4, the default boundary is given by

$$y^{\star,n}(b) = \frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}(1+b), \tag{E.24}$$

where

$$-\gamma \equiv -\frac{\mu - \frac{1}{2}\sigma^2 + \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2\sigma^2 r}}{\sigma^2}. \tag{E.25}$$

The no-issuance equity value in the normal state is then

$$\begin{aligned} v^{e,n,0}(y_t;b) =& \frac{1-\pi}{r-\mu}y_t - \frac{(1-\pi)c}{r}(1+b) \\ &+ \frac{1}{1+\gamma}\frac{(1-\pi)c}{r}\Big[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\Big]^{\gamma}(1+b)^{1+\gamma}y_t^{-\gamma}. \end{aligned} \tag{E.26}$$

In equilibrium, the equity holders' value equals the equity holders' no-issuance value, that is $V^{e,n}(Y_t, M_t, B_t) = V^{e,n,0}(Y_t; M_t, B_t)$. Let $v^{e,\xi}(y_t, b_t) \equiv V^{e,\xi}(Y_t, M_t, B_t)/M_t$ be the equity value in state ξ, where $b_t \equiv B_t/M_t$. Then $v^{e,n}(y_t, b_t) = v^{e,n,0}(y_t; b_t)$. Thus,

$$\begin{aligned} v^{e,n}(y_t,b_t) =& \frac{1-\pi}{r-\mu}y_t - \frac{(1-\pi)c}{r}(1+b_t) \\ &+ \frac{1}{1+\gamma}\frac{(1-\pi)c}{r}\Big[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\Big]^{\gamma}(1+b_t)^{1+\gamma}y_t^{-\gamma}. \end{aligned} \tag{E.27}$$

Let

$$f(y_t;b) \equiv v^{e,n,0}(y_t;b) - v^{e,s,0}(y_t;b). \tag{E.28}$$

Subtracting (E.23) from (E.22) yields

$$(r+\zeta^s)f(y_t;b) = \mu y_t f'(y_t;b) + \frac{1}{2}\sigma^2 y_t^2 f''(y_t;b). \tag{E.29}$$

The general solution to (E.29) is

$$f(y_t;b) = H_\phi y_t^{-\phi} + H_\psi y_t^{\psi}. \tag{E.30}$$

where

$$-\phi \equiv -\frac{\mu - \frac{1}{2}\sigma^2 + \sqrt{(\mu-\frac{1}{2}\sigma^2)^2 + 2\sigma^2(r+\zeta^s)}}{\sigma^2} < 0, \tag{E.31}$$

$$\psi \equiv -\frac{\mu - \frac{1}{2}\sigma^2 - \sqrt{(\mu-\frac{1}{2}\sigma^2)^2 + 2\sigma^2(r+\zeta^s)}}{\sigma^2} > 1. \tag{E.32}$$

Hence, from (E.26), (E.28) and (E.30)

$$\begin{aligned} v^{e,s,0}(y_t;b) =& - H_\phi y_t^{-\phi} - H_\psi y_t^{\psi} + \frac{1-\pi}{r-\mu} y_t - \frac{(1-\pi)c}{r}(1+b) \\ &+ \frac{1}{1+\gamma}\frac{(1-\pi)c}{r}\Big[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\Big]^{\gamma}(1+b)^{1+\gamma} y_t^{-\gamma}. \end{aligned} \tag{E.33}$$

The no-bubble condition at $y_t \to \infty$ implies that $H_\psi = 0$. At the default boundary $y^{\star,s}(b)$, the no-issuance equity value satisfies the value matching condition $v^{e,s,0}(y^{\star,s}(b);b) = 0$ and the smooth pasting condition $v^{e,s,0\prime}(y^{\star,s}(b);b) = 0$. That is

$$\begin{aligned} 0 =& - H_\phi y^{\star,s}(b)^{-\phi} + \frac{1-\pi}{r-\mu} y^{\star,s}(b) - \frac{(1-\pi)c}{r}(1+b) \\ &+ \frac{1}{1+\gamma}\frac{(1-\pi)c}{r}\Big[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\Big]^{\gamma}(1+b)^{1+\gamma} y^{\star,s}(b)^{-\gamma}, \end{aligned} \tag{E.34}$$

and

$$\begin{aligned} 0 =& \phi H_\phi y^{\star,s}(b)^{-\phi-1} + \frac{1-\pi}{r-\mu} \\ &- \frac{\gamma}{1+\gamma}\frac{(1-\pi)c}{r}\Big[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\Big]^{\gamma}(1+b)^{1+\gamma} y^{\star,s}(b)^{-\gamma-1}. \end{aligned} \tag{E.35}$$

The value matching condition and the smooth pasting condition together pin down H_ϕ and $y^{\star,s}(b)$. Then

$$\begin{aligned} v^{e,s}(y_t,b_t) =& \frac{1-\pi}{r-\mu} y_t - \frac{(1-\pi)c}{r}(1+b_t) \\ &+ \frac{1}{1+\gamma}\frac{(1-\pi)c}{r}\Big[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\Big]^{\gamma}(1+b_t)^{1+\gamma} y_t^{-\gamma} - H_\phi y_t^{-\phi}. \end{aligned} \tag{E.36}$$

From (E.4),

$$\begin{aligned} p_t^{m,n} &= y_t v_y^{e,n}(y_t,b_t) + b_t v_b^{e,n}(y_t,b_t) - v^{e,n}(y_t,b_t) \\ &= \frac{(1-\pi)c}{r}\left\{1 - \Big[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\Big]^{\gamma}\Big(\frac{1+b_t}{y_t}\Big)^{\gamma}\right\}. \end{aligned} \tag{E.37}$$

Similarly,

$$\begin{aligned} p_t^{m,s} &= y_t v_y^{e,s}(y_t,b_t) + b_t v_b^{e,s}(y_t,b_t) - v^{e,s}(y_t,b_t) \\ &= \frac{(1-\pi)c}{r}\left\{1 - \Big[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\Big]^{\gamma}\Big(\frac{1+b_t}{y_t}\Big)^{\gamma}\right\} + \left[(1+\phi)H_\phi - b_t\frac{dH_\phi}{db_t}\right] y_t^{-\phi}. \end{aligned} \tag{E.38}$$

Substituting (E.9) into (E.6) yields

$$\begin{aligned} r^{\xi}V^{b,\xi}(Y_t,M_t,B_t) =& cB_t + G_t^{m,\xi}V_M^{b,\xi}(Y_t,M_t,B_t) \\ &+ \mu Y_t V_Y^{b,\xi}(Y_t,M_t,B_t) + \frac{1}{2}\sigma^2 Y_t^2 V_{YY}^{b,\xi}(Y_t,M_t,B_t) \\ &+ \lambda\big[V^{\xi}(Y_t,M_t,\bar{B}_t^{\xi}) - V^{\xi}(Y_t,M_t,B_t)\big] \\ &+ \zeta^{\xi}\big[V^{b,-\xi}(Y_t,M_t,B_t) - V^{b,\xi}(Y_t,M_t,B_t)\big], \end{aligned} \tag{E.39}$$

where $V^{\xi}(Y_t,M_t,B_t) \equiv V^{e,\xi}(Y_t,M_t,B_t) + V^{b,\xi}(Y_t,M_t,B_t)$, and the market debt issuance rate $G_t^{m,\xi}$ is given by (E.13).

Given the scale invariance in the setting, the bank lender's value $V^{b,\xi}(Y_t,M_t,B_t)$ is homogeneous of degree one. That is

$$V^{b,\xi}(Y_t,M_t,B_t) = V^{b,\xi}\Big(\frac{Y_t}{M_t},1,\frac{B_t}{M_t}\Big)M_t \equiv v^{b,\xi}(y_t,b_t)M_t. \tag{E.40}$$

Similarly,

$$V^{\xi}(Y_t,M_t,B_t) = V^{\xi}\Big(\frac{Y_t}{M_t},1,\frac{B_t}{M_t}\Big)M_t \equiv v^{\xi}(y_t,b_t)M_t, \tag{E.41}$$

where $v^{\xi}(y_t,b_t) \equiv v^{e,\xi}(y_t,b_t) + v^{b,\xi}(y_t,b_t)$ with $v^{e,\xi}(y_t,b_t) \equiv V^{e,\xi}(Y_t,M_t,B_t)/M_t$. From (E.8),

$$\bar{b}_t^{\xi} \equiv \frac{\bar{B}_t^{\xi}}{M_t} = \arg\max_{b_t'}\ v^{\xi}(y_t,b_t'). \tag{E.42}$$

Hence, one can rewrite (E.6) as

$$\begin{aligned} &\big[r^{\xi} + \zeta^{\xi} - g_t^{m,\xi}(1+b_t)\big]v^{b,\xi}(y_t,b_t) = cb_t - g_t^{m,\xi}b_t(1+b_t)v_b^{b,\xi}(y_t,b_t) + \zeta^{\xi}v^{b,-\xi}(y_t,b_t) \\ &+ \big[\mu - g_t^{m,\xi}(1+b_t)\big]y_t v_y^{b,\xi}(y_t,b_t) + \frac{1}{2}\sigma^2 y_t^2 v_{yy}^{b,\xi}(y_t,b_t) + \lambda\big[\max_{b_t'}\ v^{\xi}(y_t,b_t') - v^{\xi}(y_t,b_t)\big]. \end{aligned} \tag{E.43}$$

F Comparison of Bank and Market Debt

In the main analysis, the bank debt is renegotiated when renegotiation opportunities arise at Poisson arrival times, and cannot be traded between these Poisson arrivals. By contrast, the market debt is traded continuously at the prevailing competitive price. To facilitate direct comparison between bank debt and market debt, I wish to make renegotiability the only distinguishing feature between the two types of debt. For this purpose, I examine two alternative modeling approaches. First, I consider the setting in which the market debt is traded infrequently. More precisely, the market debt is traded at competitive price, but only at times dictated by a Poisson process with intensity λ, independent of the Poisson process for bank debt renegotiation. Second, I consider a setting in which the bank debt is renegotiated at Poisson arrival times with intensity λ, yet remains tradable at fair competitive price between renegotiation times.

F.1 Infrequent Market Debt Trading

Instead of continuously issuing market debt at the prevailing market debt price, the firm issues market debt at Poisson arrival times with intensity λ. Let $V^e(Y_t, M_t, B_t)$ denote the equity holders' value. It must then satisfy the HJB equation

$$\begin{aligned}(r+2\lambda)V^e(Y_t,M_t,B_t) =&(1-\pi)\big[Y_t - c(B_t+M_t)\big] \\ &+\mu Y_t V^e_Y(Y_t,M_t,B_t) + \frac{1}{2}\sigma^2 Y_t^2 V^e_{YY}(Y_t,M_t,B_t) \\ &+\lambda \max_{M_t'}\ \big[V^e(Y_t,M_t',B_t) + p_t^m(M_t'-M_t)\big] \\ &+\lambda\big[V^e(Y_t,M_t,\bar{B}_t)+\mathcal{T}_t\big].\end{aligned} \tag{F.1}$$

The HJB equation (F.1) is the same as the HJB equation (A.10), except for the term related to the infrequent market debt trading opportunities.

Notice that in (F.1), when market debt trading opportunity arises at time t, the firm issues $M_t' - M_t$ amount of market debt at the market debt price $p_t^m = p^m(Y_t, M_t', B_t)$, which is forward-looking and takes into account the amount of market debt post-issuance. As shown in Section F.3, the following first-order

condition must be satisfied in the action region

$$V_M^e(Y_t, M_t', B_t) + p^m(Y_t, M_t', B_t) + p_M^m(Y_t, M_t', B_t)(M_t' - M_t) = 0. \tag{F.2}$$

Let $p^m(Y_t, M_t, B_t)$ denote the equilibrium price of the market debt, which must satisfy the HJB equation

$$\begin{aligned}(r + 2\lambda)p^m(Y_t, M_t, B_t) =& c + \mu Y_t p_Y^m(Y_t, M_t, B_t) + \frac{1}{2}\sigma^2 Y_t^2 p_{YY}^m(Y_t, M_t, B_t) \\ &+ \lambda p^m(Y_t, M_t', B_t) + \lambda p^m(Y_t, M_t, \bar{B}_t).\end{aligned} \tag{F.3}$$

Define

$$\mathcal{H}(Y_t, M_t, B_t) \equiv V_M^e(Y_t, M_t, B_t) + p^m(Y_t, M_t, B_t), \tag{F.4}$$

which is the marginal gain from issuing one unit of market debt. Suppose that there is no protection against market debt dilution just before default. Then without commitment, the firm's equity holders are incentivized to dilute the existing market debt investors by issuing new market debt just prior to default. Hence, the market debt has zero recovery value upon default. As shown in Section F.3, the marginal gain from market debt issuance satisfies

$$\mathcal{H}(Y_t, M_t, B_t) = \mathbb{E}_t\Big\{ \int_t^{\min\{\tau^\star, \bar{\tau}\}} e^{-r(s-t)}\pi c ds + e^{-r(\bar{\tau}-t)}\mathbf{1}_{\{\bar{\tau}<\tau^\star\}}\mathcal{B}(Y_{\bar{\tau}}, M_{\bar{\tau}}, B_{\bar{\tau}})\Big\}, \tag{F.5}$$

where

$$\begin{aligned}\mathcal{B}(Y_t, M_t, B_t) \equiv& p^m(Y_t, M_t, \bar{B}(Y_t, M_t)) + V_M^e(Y_t, M_t, B_t) \\ &+ \theta\big[V_M(Y_t, M_t, \bar{B}(Y_t, M_t)) - V_M(Y_t, M_t, B_t)\big].\end{aligned} \tag{F.6}$$

From the first-order condition (F.2), the firm issues positive amount of market debt when the marginal benefit of issuance is positive (i.e., $\mathcal{H}(Y_t, M_t, B_t) > 0$). Note that the marginal benefit of market debt issuance includes two components: (1) the debt tax shield, and (2) a strategic component analogous to the numerator of the second term in (2.10). In other words, the strategic consideration emphasized in the main analysis goes through for the setting with infrequent market debt setting. The firm's market debt

issuance helps position its equity holders for bank debt renegotiation, while anticipating its effect on the issuance price of market debt.

F.2 Continuous Trading of Bank Debt Between Renegotiations

I now consider the setting in which the bank debt can be traded continuously at fair competitive price between renegotiations. More specifically, between bank-firm renegotiations of the bank debt, the firm can issue or repurchase the bank debt to competitive, arm's-length bank debt investors. It is thus as if there are two classes of bank debt, one with renegotiation rights and one without. Let B_t^c denote the face amount of "competitive" bank debt outstanding, then the total amount of bank debt is the sum of the amount held by the bank lender and the amount held by the arm's-length bank debt investors, that is $B_t + B_t^c$.

The equity holders' value function thus depends on the firm's EBIT, the amount of market debt, the amount of "competitive" bank debt held by the arm's-length bank debt investors, and the amount of bank debt held by the bank lender. Let $V^e(Y_t, M_t, B_t^c, B_t)$ be the equity holders' value, which is given by

$$\begin{aligned} &V^e(Y_t, M_t, B_t^c, B_t) \\ &= \sup_{\tau^\star, \{G_s^m, G_s^b\}} \mathbb{E}_t\Big\{ \int_t^{\min\{\tau^\star, \bar{\tau}\}} e^{-r(s-t)} \big[(1-\pi)\big(Y_s - c(B_s + B_s^c + M_s)\big) + p_s^m G_s^m + p_s^b G_s^b\big] ds \\ &\qquad + e^{-r(\bar{\tau}-t)} \mathbf{1}_{\{\bar{\tau} < \tau^\star\}} \big[V^e(Y_{\bar{\tau}}, M_{\bar{\tau}}, B_{\bar{\tau}}^c, \bar{B}_{\bar{\tau}}) + \mathcal{T}_{\bar{\tau}}\big] \Big\}. \end{aligned} \tag{F.7}$$

The equilibrium market debt price $p^m(Y_t, M_t, B_t^c, B_t)$ is

$$\begin{aligned} p^m(Y_t, M_t, B_t^c, B_t) = \mathbb{E}_t\Big\{ &\int_t^{\min\{\tau^\star, \bar{\tau}\}} e^{-r(s-t)} c ds + e^{-r(\tau^\star - t)} \mathbf{1}_{\{\tau^\star \le \bar{\tau}\}} p^{m,\star}(Y_{\tau^\star}, M_{\tau^\star}, B_{\tau^\star}^c, B_{\tau^\star}) \\ &+ e^{-r(\bar{\tau}-t)} \mathbf{1}_{\{\bar{\tau} < \tau^\star\}} p^m(Y_{\bar{\tau}}, M_{\bar{\tau}}, B_{\bar{\tau}}^c, \bar{B}_{\bar{\tau}}) \Big\}, \end{aligned} \tag{F.8}$$

and the competitive bank debt price $p^b(Y_t, M_t, B_t^c, B_t)$ is similarly given.

Following the derivations in Section F.3, the optimal market debt issuance rate is

$$G_t^m = \frac{(p_{B^c}^m - p_{B^c}^b)\pi c}{p_M^m p_{B^c}^b - p_M^b p_{B^c}^m} - \frac{p_{B^c}^b}{p_M^m p_{B^c}^b - p_M^b p_{B^c}^m} K^m + \frac{p_{B^c}^m}{p_M^m p_{B^c}^b - p_M^b p_{B^c}^m} K^b, \tag{F.9}$$

$$G_t^b = \frac{(p_M^m - p_M^b)\pi c}{p_M^b p_{B^c}^m - p_M^m p_{B^c}^b} - \frac{p_M^b}{p_M^b p_{B^c}^m - p_M^m p_{B^c}^b} K^m + \frac{p_M^m}{p_M^b p_{B^c}^m - p_M^m p_{B^c}^b} K^b, \tag{F.10}$$

where

$$\begin{aligned} K^m \equiv & \lambda\big[p^m(Y_t, M_t, B_t^c, \bar{B}_t) + V_M^e(Y_t, M_t, B_t^c, B_t)\big] \\ & + \theta\lambda\big[V_M(Y_t, M_t, B_t^c, \bar{B}_t) - V_M(Y_t, M_t, B_t^c, B_t)\big], \end{aligned} \tag{F.11}$$

and

$$\begin{aligned} K^b \equiv & \lambda\big[p^b(Y_t, M_t, B_t^c, \bar{B}_t) + V_{B^c}^e(Y_t, M_t, B_t^c, B_t)\big] \\ & + \theta\lambda\big[V_{B^c}(Y_t, M_t, B_t^c, \bar{B}_t) - V_{B^c}(Y_t, M_t, B_t^c, B_t)\big]. \end{aligned} \tag{F.12}$$

The setting can be viewed as one in which there are two classes of market debt, one of which shares the same cash flow rights and seniority as the bank debt. The firm's strategic incentive to use market debt issuance in order to position its equity holders for bank debt renegotiation remains unchanged, expect that it can now adjust both the market debt amount and the level of "competitive" bank debt held by arm's-length bank debt investors, to pursue this objective.

In the special case with $\theta = 0$, the equity holders are indifferent toward the debt composition and the total issuance of market debt and "competitive" bank debt satisfies

$$\begin{aligned} G_t^m + G_t^b = & \frac{\pi c}{\gamma} \frac{r}{(1-\pi)c} \Big[\frac{1+\gamma}{\gamma} \frac{r}{(r-\mu)c} \frac{Y_t}{B_t + B_t^c + M_t}\Big]^{\gamma} (B_t + B_t^c + M_t) \\ & + \frac{\lambda}{\gamma}\Big[1 - \Big(\frac{\bar{B}_t + B_t^c + M_t}{B_t + B_t^c + M_t}\Big)^{\gamma}\Big](B_t + B_t^c + M_t). \end{aligned} \tag{F.13}$$

F.3 Proofs

I now provide the proofs and derivations to the above analysis.

A. Infrequent Market Debt Trading

The equity holders' value satisfies the HJB equation (F.1). The equity holders' payoff from bank debt renegotiation is

$$V^e(Y_t, M_t, \bar{B}_t) + \mathcal{T}_t = V^e(Y_t, M_t, B_t) + \theta\big[V(Y_t, M_t, \bar{B}) - V(Y_t, M_t, B_t)\big]. \tag{F.14}$$

Hence, the equity holders' HJB equation (F.1) can be rewritten as

$$\begin{aligned}(r+2\lambda)V^e(Y_t, M_t, B_t) =& (1-\pi)\big[Y_t - c(B_t + M_t)\big] \\ &+ \mu Y_t V^e_Y(Y_t, M_t, B_t) + \frac{1}{2}\sigma^2 Y_t^2 V^e_{YY}(Y_t, M_t, B_t) \\ &+ \lambda \max_{M'_t}\ \big[V^e(Y_t, M'_t, B_t) + p^m_t(M'_t - M_t)\big] \\ &+ \lambda\Big\{V^e(Y_t, M_t, B_t) + \theta\big[V(Y_t, M_t, \bar{B}_t) - V(Y_t, M_t, B_t)\big]\Big\}.\end{aligned} \tag{F.15}$$

In the HJB equation, notice that when market debt trading opportunity arises at time t, the firm issues $M'_t - M_t$ amount of market debt at the forward-looking market debt price $p^m_t = p^m(Y_t, M'_t, B_t)$ which depends on the amount of market debt post-issuance. Taking the first-order condition in the action region yields

$$V^e_M(Y_t, M'_t, B_t) + p^m(Y_t, M'_t, B_t) + p^m_M(Y_t, M'_t, B_t)(M'_t - M_t) = 0. \tag{F.16}$$

The equilibrium market debt price $p^m(Y_t, M_t, B_t)$ satisfies the HJB equation (F.3).

Differentiating both sides of (F.15) with respect to M_t and using the envelope theorem, we get

$$\begin{aligned}(r+2\lambda)V^e_M(Y_t, M_t, B_t) =& -(1-\pi)c + \mu Y_t V^e_{MY}(Y_t, M_t, B_t) + \frac{1}{2}\sigma^2 Y_t^2 V^e_{MYY}(Y_t, M_t, B_t) \\ &- \lambda p^m(Y_t, M'_t, B_t) + \lambda\Big\{V^e_M(Y_t, M_t, B_t) + \theta\big[V_M(Y_t, M_t, \bar{B}_t) - V_M(Y_t, M_t, B_t)\big]\Big\}.\end{aligned} \tag{F.17}$$

Let

$$\mathcal{H}(Y_t, M_t, B_t) \equiv V^e_M(Y_t, M_t, B_t) + p^m(Y_t, M_t, B_t). \tag{F.18}$$

Then adding up (F.17) and (F.3) yields

$$\begin{aligned}(r+2\lambda)\mathcal{H}(Y_t,M_t,B_t) &= \pi c + \mu Y_t \mathcal{H}_Y(Y_t,M_t,B_t) + \frac{1}{2}\sigma^2 Y_t^2 \mathcal{H}_{YY}(Y_t,M_t,B_t) \\ &+ \lambda\Big\{p^m(Y_t,M_t,\bar{B}_t) + V_M^e(Y_t,M_t,B_t) + \theta\big[V_M(Y_t,M_t,\bar{B}_t) - V_M(Y_t,M_t,B_t)\big]\Big\}.\end{aligned} \tag{F.19}$$

In the regime where there is no protection against market debt dilution just prior to bankruptcy, the recovery value of market debt upon default is zero. Hence, $\mathcal{H}(Y_t,M_t,B_t)$ can be written as

$$\mathcal{H}(Y_t,M_t,B_t) = \mathbb{E}_t\Big\{\int_t^{\min\{\tau^\star,\bar{\tau}\}} e^{-r(s-t)}\pi c ds + e^{-r(\bar{\tau}-t)}\mathbf{1}_{\{\bar{\tau}<\tau^\star\}}\mathcal{B}(Y_{\bar{\tau}},M_{\bar{\tau}},B_{\bar{\tau}})\Big\}, \tag{F.20}$$

where

$$\begin{aligned}\mathcal{B}(Y_t,M_t,B_t) \equiv & p^m(Y_t,M_t,\bar{B}(Y_t,M_t)) + V_M^e(Y_t,M_t,B_t) \\ & + \theta\big[V_M(Y_t,M_t,\bar{B}(Y_t,M_t)) - V_M(Y_t,M_t,B_t)\big].\end{aligned} \tag{F.21}$$

B. Continuous Trading of Bank Debt Between Renegotiations

When the bank debt is continuously traded at fair competitive prices subject to periodic renegotiations, the equity holders' value function (F.7) can be expressed recursively through the HJB equation as follows

$$\begin{aligned}&(r+\lambda)V^e(Y_t,M_t,B_t^c,B_t) \\ &\quad = \max_{G_t^m,G_t^b}\Big\{(1-\pi)\big[Y_t - c(B_t+B_t^c+M_t)\big] + p_t^m G_t^m + p_t^b G_t^b \\ &\qquad + G_t^m V_M^e(Y_t,M_t,B_t^c,B_t) + G_t^b V_{B^c}^e(Y_t,M_t,B_t^c,B_t) \\ &\qquad + \mu Y_t V_Y^e(Y_t,M_t,B_t^c,B_t) + \frac{1}{2}\sigma^2 Y_t^2 V_{YY}^e(Y_t,M_t,B_t^c,B_t) \\ &\qquad + \lambda\big[V^e(Y_t,M_t,B_t^c,\bar{B}_t) + \mathcal{T}_t\big]\Big\}.\end{aligned} \tag{F.22}$$

Taking the first-order conditions with respect to G_t^m and G_t^b, we get

$$p_t^m \equiv p^m(Y_t,M_t,B_t^c,B_t) = -V_M^e(Y_t,M_t,B_t^c,B_t), \tag{F.23}$$

$$p_t^b \equiv p^b(Y_t,M_t,B_t^c,B_t) = -V_{B^c}^e(Y_t,M_t,B_t^c,B_t). \tag{F.24}$$

Substituting (F.23) and (F.24) into (F.22) and using the bargaining solutions (2.6) and (2.7), one can rewrite the equity holders' HJB equation as

$$\begin{aligned}(r+\lambda)V^e(Y_t,M_t,B_t^c,B_t) &= (1-\pi)\big[Y_t - c(B_t+B_t^c+M_t)\big] \\ &\quad + \mu Y_t V_Y^e(Y_t,M_t,B_t^c,B_t) + \frac{1}{2}\sigma^2 Y_t^2 V_{YY}^e(Y_t,M_t,B_t^c,B_t) \\ &\quad + \lambda\Big\{V^e(Y_t,M_t,B_t^c,B_t) + \theta\big[V(Y_t,M_t,B_t^c,\bar{B}_t) - V(Y_t,M_t,B_t^c,B_t)\big]\Big\}.\end{aligned} \tag{F.25}$$

Differentiating both sides of (F.25) with respect to M_t yields

$$\begin{aligned}&(r+\lambda)V_M^e(Y_t,M_t,B_t^c,B_t) \\ &\quad = -(1-\pi)c + \mu Y_t V_{MY}^e(Y_t,M_t,B_t^c,B_t) + \frac{1}{2}\sigma^2 Y_t^2 V_{MYY}^e(Y_t,M_t,B_t^c,B_t) \\ &\quad\quad + \lambda\Big\{V_M^e(Y_t,M_t,B_t^c,B_t) + \theta\big[V_M(Y_t,M_t,B_t^c,\bar{B}_t) - V_M(Y_t,M_t,B_t^c,B_t)\big]\Big\}.\end{aligned} \tag{F.26}$$

Similarly, differentiating both sides of (F.25) with respect to B_t^c yields

$$\begin{aligned}&(r+\lambda)V_{B^c}^e(Y_t,M_t,B_t^c,B_t) \\ &\quad = -(1-\pi)c + \mu Y_t V_{B^cY}^e(Y_t,M_t,B_t^c,B_t) + \frac{1}{2}\sigma^2 Y_t^2 V_{B^cYY}^e(Y_t,M_t,B_t^c,B_t) \\ &\quad\quad + \lambda\Big\{V_{B^c}^e(Y_t,M_t,B_t^c,B_t) + \theta\big[V_{B^c}(Y_t,M_t,B_t^c,\bar{B}_t) - V_{B^c}(Y_t,M_t,B_t^c,B_t)\big]\Big\}.\end{aligned} \tag{F.27}$$

The equilibrium market debt price $p^m(Y_t,M_t,B_t)$ satisfies the HJB equation

$$\begin{aligned}(r+\lambda)p^m(Y_t,M_t,B_t^c,B_t) =& c + G_t^m p_M^m(Y_t,M_t,B_t^c,B_t) + G_t^b p_{B^c}^m(Y_t,M_t,B_t^c,B_t) \\ &+ \mu Y_t p_Y^m(Y_t,M_t,B_t^c,B_t) + \frac{1}{2}\sigma^2 Y_t^2 p_{YY}^m(Y_t,M_t,B_t^c,B_t) \\ &+ \lambda p^m(Y_t,M_t,B_t^c,\bar{B}_t).\end{aligned} \tag{F.28}$$

Analogously, the competitive bank debt price $p^b(Y_t,M_t,B_t^c,B_t)$ satisfies the HJB equation

$$\begin{aligned}(r+\lambda)p^b(Y_t,M_t,B_t^c,B_t) =& c + G_t^m p_M^b(Y_t,M_t,B_t^c,B_t) + G_t^b p_{B^c}^b(Y_t,M_t,B_t^c,B_t) \\ &+ \mu Y_t p_Y^b(Y_t,M_t,B_t^c,B_t) + \frac{1}{2}\sigma^2 Y_t^2 p_{YY}^b(Y_t,M_t,B_t^c,B_t) \\ &+ \lambda p^b(Y_t,M_t,B_t^c,\bar{B}_t).\end{aligned} \tag{F.29}$$

Adding up (F.26) and (F.28) and using the first-order condition (F.23), we have

$$\begin{aligned} 0 =& \pi c + G_t^m p_M^m(Y_t, M_t, B_t^c, B_t) + G_t^b p_{B^c}^m(Y_t, M_t, B_t^c, B_t) \\ &+ \lambda\big[p^m(Y_t, M_t, B_t^c, \bar{B}_t) + V_M^e(Y_t, M_t, B_t^c, B_t)\big] \\ &+ \theta\lambda\big[V_M(Y_t, M_t, B_t^c, \bar{B}_t) - V_M(Y_t, M_t, B_t^c, B_t)\big]. \end{aligned} \tag{F.30}$$

Similarly, adding up (F.27) and (F.29) and using (F.24),

$$\begin{aligned} 0 =& \pi c + G_t^m p_M^b(Y_t, M_t, B_t^c, B_t) + G_t^b p_{B^c}^b(Y_t, M_t, B_t^c, B_t) \\ &+ \lambda\big[p^b(Y_t, M_t, B_t^c, \bar{B}_t) + V_{B^c}^e(Y_t, M_t, B_t^c, B_t)\big] \\ &+ \theta\lambda\big[V_{B^c}(Y_t, M_t, B_t^c, \bar{B}_t) - V_{B^c}(Y_t, M_t, B_t^c, B_t)\big]. \end{aligned} \tag{F.31}$$

Solving for G_t^m and G_t^b yields

$$G_t^m = \frac{(p_{B^c}^m - p_{B^c}^b)\pi c}{p_M^m p_{B^c}^b - p_M^b p_{B^c}^m} - \frac{p_{B^c}^b}{p_M^m p_{B^c}^b - p_M^b p_{B^c}^m} K^m + \frac{p_{B^c}^m}{p_M^m p_{B^c}^b - p_M^b p_{B^c}^m} K^b, \tag{F.32}$$

$$G_t^b = \frac{(p_M^m - p_M^b)\pi c}{p_M^b p_{B^c}^m - p_M^m p_{B^c}^b} - \frac{p_M^b}{p_M^b p_{B^c}^m - p_M^m p_{B^c}^b} K^m + \frac{p_M^m}{p_M^b p_{B^c}^m - p_M^m p_{B^c}^b} K^b, \tag{F.33}$$

where

$$\begin{aligned} K^m \equiv& \lambda\big[p^m(Y_t, M_t, B_t^c, \bar{B}_t) + V_M^e(Y_t, M_t, B_t^c, B_t)\big] \\ &+ \theta\lambda\big[V_M(Y_t, M_t, B_t^c, \bar{B}_t) - V_M(Y_t, M_t, B_t^c, B_t)\big], \end{aligned} \tag{F.34}$$

and

$$\begin{aligned} K^b \equiv& \lambda\big[p^b(Y_t, M_t, B_t^c, \bar{B}_t) + V_{B^c}^e(Y_t, M_t, B_t^c, B_t)\big] \\ &+ \theta\lambda\big[V_{B^c}(Y_t, M_t, B_t^c, \bar{B}_t) - V_{B^c}(Y_t, M_t, B_t^c, B_t)\big]. \end{aligned} \tag{F.35}$$

In the case with a strong bank lender (i.e., $\theta = 0$), the HJB equation for the equity holders' value (F.25) becomes

$$\begin{aligned} rV^e(Y_t, M_t, B_t^c, B_t) =& (1-\pi)\big[Y_t - c(B_t + B_t^c + M_t)\big] \\ &+ \mu Y_t V_Y^e(Y_t, M_t, B_t^c, B_t) + \frac{1}{2}\sigma^2 Y_t^2 V_{YY}^e(Y_t, M_t, B_t^c, B_t). \end{aligned} \tag{F.36}$$

Again, the equity holders' value can be solved as if there is no adjustment in debt amounts. Following the derivations in Appendix A.4,

$$\begin{aligned} V^e(Y_t, M_t, B_t^c, B_t) =& \frac{1-\pi}{r-\mu} Y_t - \frac{(1-\pi)c}{r}(B_t + B_t^c + M_t) \\ &+ \frac{1}{1+\gamma}\frac{(1-\pi)c}{r}\Big[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\Big]^{\gamma}(B_t + B_t^c + M_t)^{1+\gamma} Y_t^{-\gamma}. \end{aligned} \tag{F.37}$$

Hence,

$$\begin{aligned} & p^m(Y_t, M_t, B_t^c, B_t) = p^b(Y_t, M_t, B_t^c, B_t) \\ &= \frac{(1-\pi)c}{r}\left\{1 - \Big[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\Big]^{\gamma}\Big(\frac{B_t + M_t}{Y_t}\Big)^{\gamma}\right\}. \end{aligned} \tag{F.38}$$

Thus, (F.30) and (F.31) are identical, and can be simplified to

$$\begin{aligned} G_t^m + G_t^b &= \frac{\pi c}{-p_M^m(Y_t, M_t, B_t^c, B_t)} + \lambda \frac{p^m(Y_t, M_t, B_t^c, \bar{B}_t) + V_M^e(Y_t, M_t, B_t^c, B_t)}{-p_M^m(Y_t, M_t, B_t^c, B_t)} \\ &= \frac{\pi c}{\gamma}\frac{r}{(1-\pi)c}\Big[\frac{1+\gamma}{\gamma}\frac{r}{(r-\mu)c}\frac{Y_t}{B_t + B_t^c + M_t}\Big]^{\gamma}(B_t + B_t^c + M_t) \\ &\quad + \frac{\lambda}{\gamma}\Big[1 - \Big(\frac{\bar{B}_t + B_t^c + M_t}{B_t + B_t^c + M_t}\Big)^{\gamma}\Big](B_t + B_t^c + M_t). \end{aligned} \tag{F.39}$$

G Additional Empirical Results

G.1 Further Robustness Tests

In this appendix, I present a number of robustness tests to support the empirical results in Section 5. In the first set of robustness tests, I use alternative definitions for bank debt and market debt. In the second set of robustness tests, I resort to a matched-sample regression specification by including industry-year-size-leverage fixed effects as the main controls. Finally, I show the pattern of complementarity between bank and market debt using time-series aggregate balance sheet data from the Flow of Funds.

A. Alternative Definitions

In the baseline, I define bank debt as term loans, and define market debt as the sum of senior bonds and notes and subordinated bonds and notes. For robustness, I also account for short-term credit by expanding the definition of bank debt to include drawn credit lines and the definition of market debt to include commercial paper. Credit lines and commercial paper are both short-term financing instruments that differ in nature from either term loans or bonds and notes. However, excluding credit lines and commercial paper may impact my results if firms shift between credit lines and term loans or between commercial paper and bonds and notes. I re-estimate the specification (5.4) using the alternative definitions of bank and market debt, and report the results in Table G.1.

The coefficient of interest is the coefficient on the bank debt share variable. In all columns of Table G.1, the coefficient estimates on bank debt share are all positive and highly statistically significant. The magnitudes are also similar to the baseline results reported in Table 6. The results thus show that the baseline findings are robust to alternative definitions of bank debt and market debt.

B. Matched-Sample Analysis

A potential concern is that firms' debt issuance decisions are affected by time-varying credit market conditions. For example, the observed increase in subsequent market debt net issuance following a higher bank debt share could be due to a decline in the relative credit spreads of bonds compared to bank loans. This could arise from increased investor demand for corporate bonds. Moreover, the magnitude of this credit spread differential between bonds and bank loans may vary across firms with different

Table G.1: Panel Regressions: Robustness Using Alternative Definitions

	Market Debt Net Issuance Rate					
	(1)	(2)	(3)	(4)	(5)	(6)
Bank Debt Share	0.857***	0.599***	0.931***	0.607***	0.374***	0.405***
	(0.214)	(0.100)	(0.235)	(0.101)	(0.090)	(0.102)
Log(Assets)	-0.199***	-0.155***	-0.202***	-0.163***	-0.252**	-0.285**
	(0.045)	(0.040)	(0.051)	(0.041)	(0.112)	(0.127)
Tangibility	0.268	0.025	0.203	-0.125	0.027	-0.002
	(0.280)	(0.361)	(0.249)	(0.280)	(0.325)	(0.358)
ROA	-0.040	-0.320*	-0.095	-0.351*	-0.167	-0.205
	(0.181)	(0.159)	(0.209)	(0.185)	(0.355)	(0.428)
Book Leverage	-1.367***	-1.285***	-1.445***	-1.368***	-1.064***	-1.184***
	(0.216)	(0.258)	(0.216)	(0.241)	(0.323)	(0.370)
Debt-to-EBITDA	-0.000	0.000	0.000	0.000	-0.000	-0.000
	(0.001)	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)
Dividend Payout		-0.002		0.007	0.003	0.005*
		(0.005)		(0.006)	(0.002)	(0.002)
Market-to-Book		0.169***		0.190***	0.191***	0.212***
		(0.035)		(0.041)	(0.063)	(0.074)
Stock Return		-0.005		-0.004	-0.005	-0.007
		(0.007)		(0.008)	(0.007)	(0.007)
Sales Growth		-0.032		-0.048	0.044	0.056
		(0.057)		(0.065)	(0.078)	(0.091)
Sample	All	All	All	All	Rated	Rated
Conditional on Issuance	No	No	Yes	Yes	No	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.212	0.140	0.251	0.206	0.099	0.104
Observations	38,502	29,840	34,028	26,289	16,239	14,321

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Note: The table presents results of the regression specification (5.4) using data from Compustat, S&P Credit Ratings, and Capital IQ for the period 2002-2024. The outcome variable is market debt net issuance rate defined as the net increase in market debt outstanding during the following year scaled by the total amount of bank and market debt. The coefficients of interest are the coefficients on bank debt share defined as the amount of bank debt divided by the sum of bank and market debt. Columns 1-4 use all firm-year observations from the estimation sample. Columns 5-6 restrict to observations with available S&P credit ratings. Columns 1, 2 ad 5 are unconditional on non-zero market debt changes while columns 3, 4 and 6 are conditional on non-zero market debt changes. In columns 1 and 3, the firm-year level controls include the log of book assets, tangibility, ROA, book leverage, and debt-to-EBITDA. Columns 2 and 4-6 also include dividend payout, market-to-book, stock return, and sales growth as additional control variables. Standard errors are double-clustered at the firm and year levels.

characteristics and across industries, further complicating identification. To address this concern, I tighten the regression specification by including the interacted industry-year-size-leverage fixed effects.

Table G.2: Panel Regressions: Robustness Using Alternative Definitions

	Market Debt Net Issuance Rate			
	(1)	(2)	(3)	(4)
Bank Debt Share	0.154**	0.159**	0.135**	0.133*
	(0.063)	(0.065)	(0.061)	(0.065)
Debt Definition	Baseline	Baseline	Robustness	Robustness
Conditional on Issuance	No	Yes	No	Yes
Industry-Year-Size-Leverage FEs	Yes	Yes	Yes	Yes
R^2	0.211	0.214	0.221	0.224
Observations	29,353	25,876	40,109	35,358

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Note: The table presents results from regressions of market debt net issuance rate (defined as the net increase in market debt outstanding during the following year scaled by the total amount of bank and market debt) on bank debt share (defined as the amount of bank debt divided by the sum of bank and market debt), while controlling for the interacted industry-year-size-leverage fixed effects. Specifically size corresponds to quintiles of firm size as measured by total assets, and leverage corresponds to quintiles of book leverage. Columns 1-2 follow baseline definitions of bank debt (term loans) and market debt (senior and subordinated bonds and notes), while columns 3-4 follow definitions of bank debt (term loans and drawn credit lines) and market debt (senior and subordinated bonds and notes and commercial paper) in the robustness tests. Columns 1 and 3 are unconditional on non-zero market debt changes while columns 2 and 4 are conditional on non-zero market debt changes. Data are from Compustat and Capital IQ for the period 2002-2024. Standard errors are double-clustered at the firm and year levels.

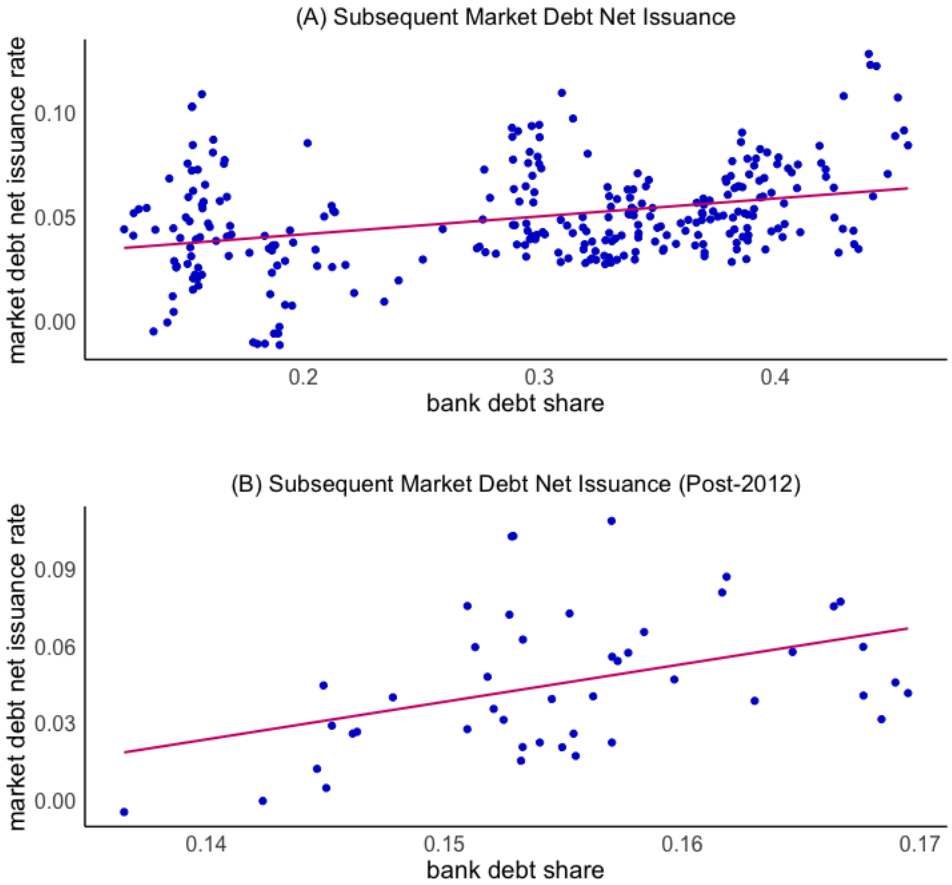
In particular, industry fixed effects are constructed using the first two digits of the NAICS code, capturing broad sectoral classifications. Size fixed effects are defined by quintiles of firm size as measured by book assets. Leverage fixed effects are defined by quintiles of book leverage computed as the total debt outstanding divided by book assets. The coefficient estimates on bank debt share are reported in Table G.2. In columns 1 and 2, I follow the baseline definitions of bank debt and market debt. That is, bank debt is defined as bank loans and market debt includes senior and subordinated bonds and notes. In columns 3 and 4, I use the alternative definitions used in the robustness tests above. That is, bank debt is comprised of bank loans and drawn credit lines, while market debt includes commercial paper as well as senior and subordinated bonds and notes. Columns 1 and 3 are unconditional on market debt net issuance or repurchase, while columns 2 and 4 are conditional on non-zero market debt changes.

In all columns of Table G.2, the coefficient estimates on bank debt share are lower in magnitude compared to those reported in the baseline regressions. However, the coefficient estimates remain positive and statistically significant at conventional levels. The results suggest that the baseline findings are also robust to alternative specifications with tighter controls. These robustness tests thus provide further evidence supporting the complementarity between bank debt and market debt.

C. Aggregate-Level Evidence

I now document the complementarity of bank and market debt using aggregate time-series data. The aggregate data are obtained from the Flow of Funds, which contains quarterly balance sheet information of the U.S. non-financial corporate sector and is accessible via the FRED database of the St. Louis Fed. In Panel A of Figure G.1, I plot the relationship between bank debt share and net issuance rate of market debt in the following year, over the sample period from Q1 1953 to Q4 2024. The bank debt share is the ratio of depository institution loans to corporate bonds (ex. eREITs). The market debt net issuance rate is the net increase in corporate bonds scaled by the combined total of depository institution loans and corporate bonds. The figure shows a positive relationship between the subsequent market debt net issuance rate and the prevailing bank debt share. In other words, firms tend to issue more market debt following periods in which their debt structures contain a greater share of bank debt.

Figure G.1: Bank-to-Market-Debt Ratio and Subsequent Debt Issuances



Note: The figure plots the relationship between bank debt share and market debt net issuance rate in the following year, for the full sample period (Panel A) and for the post-2012 subsample excluding the COVID-19 crisis period (Panel B). The full sample contains quarterly data from Q1 1965 to Q3 2024. The post-2012 subsample contains quarterly data from Q1 2012 to Q3 2024, while the COVID shock period is defined as the first two quarters of 2020. The bank debt share is defined as the ratio of depository institution loans to corporate bonds (ex. eREITs) obtained from the Flow of Funds. The market debt net issuance rate is the net increase in corporate bonds (ex. eREITs) scaled by the sum of depository institution loans and corporate bonds.

The positive relationship between bank debt share and the subsequent market debt net issuance rate may be a mechanical artifact of the secular decline in the bank-to-market-debt ratio over the sample period. To address this concern, I focus on the period between Q1 2012 and Q4 2024 when the bank debt share remained relatively stable. I exclude the first two quarters of 2020 (COVID-19 shock period) from the sample to remove any impact due to unconventional Federal Reserve policies targeting bank lending and the corporate bond market. The positive relationship between bank debt share and market debt net issuance rate remains robust in this subsample period, as seen from Panel B of Figure G.1.